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A MARKOV-SWITCHING MODEL OF GNP GROWTH WITH DURATION DEPENDENCE*

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A Markov-switching model of postwar quarterly real GNP growth is used to examine the duration dependence of business cycles. It extends the Hamilton model and the duration-dependent model of Durland and McCurdy, and compares quite favorably to simpler models in out-of-sample forecasting. When an expansion begins, the probability of the expansion ending is 0.2, but it gradually decreases as the expansion ages. When a contraction begins, the probability of the contraction terminating is 0.07, but it increases rapidly as the contraction ages. Output growth slows over the course of an expansion. The hypothesis of the 7–10-year cycle is not supported.

1. INTRODUCTION

Whether the termination probability of an expansion or a contraction is an increasing function of its age or whether business cycles exhibit positive duration dependence is a question that has attracted the attention of a number of authors. The main strand of the literature examines the NBER reference cycle turning point dates. McCulloch (1975), using a simple contingency table test, concludes that business cycles are duration independent. Employing a battery of nonparametric tests, Diebold and Rudebusch (1990) conclude that both duration independence in expansions and duration dependence in contractions are, by and large, consistent with the data. Sichel (1991), estimating a parametric hazard model using the National Bureau of Economic Research (NBER) reference cycle turning points, finds statistically significant duration dependence in postwar contractions and prewar expansions.

A second strand of the literature finds its basis in regime-switching time-series models. In his pioneering work, Hamilton (1989) estimates a two-state Markov

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chain model of output growth, where the two states are interpreted as expansions and contractions. His Markov chain assumes duration independence; as the current phase of the business cycle ages, the probability of moving into the alternative phase remains constant. Durland and McCurdy (1994) introduce duration dependence into the Hamilton model by allowing the transition probabilities to depend upon the age of the current phase of the cycle. They then infer duration dependence in postwar GNP growth rates using the estimated relationship between the transition probabilities and the age of the current phase. They find that as a contraction ages the probability of moving into an expansion increases and that this increase is statistically significant.

As the evidence is far from conclusive, there appears to be room for further investigation. Following Durland and McCurdy (1994), this article empirically investigates duration dependence using a regime-switching model of GNP growth. We use the Hamilton (1989) model extended to contain a general structure for duration dependence. The model allows both the mean growth rates and the transition probabilities to depend upon the age of the current phase of the business cycle. This feature allows us to investigate the duration dependence associated with not only the phase transition but also the amplitude of the cycle. There is now growing consensus that the volatility of output growth declined significantly around the mid-1980s (McConnell and Perez-Quiros, 2000; Kim and Nelson, 1999). Empirical investigations using recent observations must account for the structural break. We followed McConnell and Perez-Quiros (2000) in allowing for switching in volatility in our general model. In doing so, we obtain results that are robust to structural breaks in volatility, and as a bonus, an inference about the fluctuation of output volatility in the sample period.

Instead of using the NBER reference cycle turning points as data, we treat the business cycle as an unobserved stochastic process and infer its duration dependence from GNP growth rate data. Our approach has a number of advantages. First, if the successions of expansions and contractions are driven by a stochastic process, and the NBER identifies turning points with errors, those analyses based upon the NBER reference cycle chronology might not have desirable statistical properties. Our approach, however, yields reliable inferences so long as our regime-switching model captures the driving stochastic process. Second, methodologies that use the NBER reference cycle dates instead of growth rate data are better suited to forecasting turning points as opposed to forecasting growth rates over the course of the business cycle. Our methodology, in contrast, yields a stochastic process that can be used to forecast output growth rates. We present evidence that the general model compares favorably to the simple Hamilton model and the linear autoregressive model in forecasting output growth out-of-sample, when the horizon is long.

We develop an algorithm to estimate the model via maximum likelihood, using growth rate data from postwar quarterly real GNP. The results can be briefly summarized. At the beginning of an expansion, the probability of the expansion ending is close to 0.2, but it gradually decreases as the expansion ages. At the beginning of a contraction, the probability of the contraction ending is less than 0.07, but it increases rapidly as the contraction ages. Also, output growth slows over

the course of an expansion. The first finding is surprising, as it implies duration dependence contrary to the type motivating the literature.² Also, the duration dependence we have estimated implies a clustering of the whole cycle around durations shorter than the 7–10-year type often discussed in the literature (Diebold and Rudebusch, 1990).

The remainder of this article is organized as follows. Section 2 describes the statistical model and develops an algorithm to estimate it via maximum likelihood. Section 3 presents our empirical results. Finally, Section 4 offers some concluding remarks.

2. THE GENERAL MODEL AND ESTIMATION

In this section, we present an univariate regime-switching model based upon Hamilton (1989), Durland and McCurdy (1994), and McConnell and Perez-Quiros (2000). Our model generalizes them by incorporating all their features and introducing mean growth rates that are dependent upon the duration of the current phase.

The section is divided into four subsections. In Section 2.1, we describe the general model. The algorithm used to estimate this model via maximum likelihood is developed in the Appendix. Issues concerning filtering and smoothing are discussed in Section 2.2. The parameterizations needed to implement the estimation are discussed in Section 2.3.

Throughout this article, y is the growth rate of the observed time series, Y is the set containing current value and the past history of y . Also, S and V are the Markov state variables representing fluctuations in business cycle and volatility, respectively, D is the age of the current phase in S , and θ is the set of estimable parameters in the model.

2.1. The General Model. The growth rate of output is comprised of two unobserved components, a noise component denoted by z and a systematic component denoted μ . With the latter of these being a deterministic function of S and D , the model is written as follows:

$$(1) \quad y_t = \mu(S_t, D_t) + z_t$$

The systematic component derives its dynamics from the two-point Markov chain:

$$(2) \quad S_t \in \{0, 1\}$$

² Burns (1968, p. 237) states, “experience strongly suggests that even in the absence of serious external disturbances the course of aggregate activity will in time be reversed by restrictive forces that gradually but insistently come into play as a result of the expansion process itself.” McCulloch (1975) is explicitly motivated by this statement. Diebold and Rudebusch (1990) are motivated partly by the empirical work of Neftci (1982), who assumes that the longer the economy remains in one state, the more likely it is to make a transition to the other. They also allude to the hypothesis often seen in the public press, that “a very long expansion is unstable and is unusually likely to end.” Sichel (1991) begins with the question, “Are periods of expansion or contraction in economic activity more likely to end as they become older?”

where

$$\begin{aligned}\Pr(S_{t+1} = 0/S_t = 0, D_t) &= \pi 0(D_t) \\ \Pr(S_{t+1} = 1/S_t = 0, D_t) &= 1 - \pi 0(D_t) \\ \Pr(S_{t+1} = 1/S_t = 1, D_t) &= \pi 1(D_t) \\ \Pr(S_{t+1} = 0/S_t = 1, D_t) &= 1 - \pi 1(D_t)\end{aligned}$$

The noise component follows a k th order, stationary autoregressive process,

$$(3) \quad \phi(L)z_t = \sigma(V_t)\varepsilon_t$$

where $\varepsilon_t \sim \text{i.i.d. } N(0, 1)$, $\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_k L^k$, and V_t follows a two-point Markov chain

$$(4) \quad V_t \in \{0, 1\}$$

where

$$\begin{aligned}\Pr(V_{t+1} = 0/V_t = 0) &= \pi v 0 \\ \Pr(V_{t+1} = 1/V_t = 0) &= 1 - \pi v 0 \\ \Pr(V_{t+1} = 1/V_t = 1) &= \pi v 1 \\ \Pr(V_{t+1} = 0/V_t = 1) &= 1 - \pi v 1\end{aligned}$$

The random variables ε , S , and V are assumed to be independent of each other.

Equation (2) gives the transition probabilities of the Markov chain representing the business cycle. We also refer to μ as the mean growth rate, as it is the value that the time series takes when the noise component is equal to its unconditional mean. We label the two business cycle states in such a way that $\mu(0, 1) > \mu(1, 1)$. Therefore, $S = 0$ is interpreted as an expansion whereas $S = 1$ is interpreted as a contraction.

Our model allows for two types of duration dependence. The first is the dependence of the level of the systematic component on the age of the current phase. This can be seen in Equation (1), where the age of the current phase is a determinant of the mean growth function. The second is the dependence of the transition probabilities on the age of the current business phase. This is evident in Equation (2), where the probability of exiting the current state depends upon the age of the business cycle phase. Our model also allows for switching of volatility between high-volatility and low-volatility regimes. This can be seen in Equation (3), where the innovation of the noise process is scaled by $\sigma(V)$. When $\sigma(0) \neq \sigma(1)$, the noise component is heteroskedastic. We label the states of volatility such that $\sigma(0) > \sigma(1)$. Therefore, $V = 0$ represents the high-volatility state and $V = 1$ represents the low-volatility state.

When the noise component is homoskedastic, and neither the mean growth rate nor the transition probabilities depend upon the duration variable, the model reduces to the standard Hamilton model. When the noise component is homoskedastic, and the mean growth rate does not depend upon the duration variable, we have the duration-dependent model estimated in Durland and McCurdy (1994). When the noise component is heteroskedastic, and neither the mean growth rate nor the transition probabilities depend on the duration variable, the model reduces to the model of McConnell and Perez-Quiros (2000). The original McConnell and Perez-Quiros model also allows for dependence of mean growth rates on the volatility regime. We ignore the dependence here, not only because McConnell and Perez-Quiros have shown that the dependence lacks statistical significance, but also for the sake of conserving degrees of freedom. Since the models of Hamilton, Durland and McCurdy, and McConnell and Perez-Quiros are all nested within our general model, with a suitable parameterization of the mean growth rates and the transition probabilities, they can in principle be tested.

Although our model is more general than the models of Hamilton, Durland and McCurdy, and McConnell and Perez-Quiros, it is important to note that it is still a descriptive model. It is a parametric summary, albeit a less restrictive one, of the time-series behavior of the data. As a descriptive model, it is incapable of explaining the economic forces behind the behavior of the time series, including duration dependence and switching volatility. The latter task would require modeling the relationship between variables, which is beyond our univariate framework.

We restrict the Markov chain to have a memory of dm periods. More specifically, in Equations (1) and (2), we assume that durations beyond dm have no marginal effect on the mean growth rates and the transition probabilities of the Markov chain. This assumption is motivated by convenience in estimation. It implies that durations exceeding dm are observationally equivalent to, and can be treated the same as, a duration of dm in the evaluation of the likelihood function.

Consider the maximum likelihood estimation of the model.³ Maximum likelihood estimation of a regime-switching model is conceptually straightforward, but the evaluation of the log-likelihood function is nonstandard.⁴ In the Appendix, we devise an algorithm that evaluates the log-likelihood of the sample as a function of the vector θ , consisting of $\phi_1, \phi_2, \dots, \phi_k, \sigma(0), \sigma(1), \pi v_0, \pi v_1$ and the parameters characterizing the dependence of μ, π_0 , and π_1 on S and D . The

³ Our econometric approach is entirely classical. Several important articles in this literature adopt a Bayesian approach, including Kim and Nelson (1999) and Kim (1996). Exploiting the result in Albert and Chib (1993), they use the Gibbs sampling technique to calculate the posterior means of the parameters and marginal likelihood of the model. Their approach is just as tractable. It also handles hypothesis testing easily even when the nuisance parameter is identified only under the alternative, a common problem in this literature when the classical approach is used. Furthermore, it produces a posterior distribution of break points, which is often of central interest. The approach, however, relies on specifying a prior distribution for the parameters, which we plead ignorance to.

⁴ Our evaluation of the likelihood function is exact. Kim (1994) presents a convenient method to approximate the likelihood function. Kim also demonstrated that his approximate maximum likelihood is very close to the exact maximum likelihood in estimating the model in Lam (1990), which is the Hamilton model with a stationary linear process.

algorithm is based on prediction error decomposition using $\{\tilde{S}_i\}_{i=t-k}^t$ and D_{t-k} as state variables, where \tilde{S} is a state variable summarizing S and V , $\tilde{S} = 1$, $\tilde{S} = 2$, $\tilde{S} = 3$, $\tilde{S} = 4$, respectively, when $(S, V) = (0, 0)$, $(S, V) = (1, 0)$, $(S, V) = (0, 1)$ and $(S, V) = (1, 1)$.

2.2. Filtering and Smoothing. One pass through the algorithm not only produces the log-likelihood of the sample, but also the filtered probability distribution of the state variables for each time period in the sample:

$$\Pr[\tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d | Y_t, \theta], \quad t = 1, 2, \dots, T$$

This probability distribution leads to an inference about the unobserved business cycle state, the age of the business cycle phase and the volatility state. By appropriately marginalizing the distribution, we can calculate $\Pr[S_t = s | Y_t, \theta]$, $s = 0, 1$, $\Pr[D_t = d | Y_t, \theta]$, $d = 1, 2, \dots, dm$, and $\Pr[V_t = v | Y_t, \theta]$, $v = 0, 1$, the filtered probability of the business cycle state, the age of the business cycle phase, and the volatility state, respectively, given current information. An alternative inference can be obtained by utilizing future in addition to current observations. By appropriately marginalizing the distribution, we could also calculate $\Pr[S_{t-\ell} = s | Y_t, \theta]$, $s = 0, 1$, $\Pr[D_{t-\ell} = d | Y_t, \theta]$, $d = 1, 2, \dots, dm$, and $\Pr[V_{t-\ell} = v | Y_t, \theta]$, $v = 0, 1$, the smoothed probability of the business cycle state, the age of the business cycle phase, and the volatility state, respectively, ℓ periods ago given current information. Both filtered and smoothed probabilities are evaluated at the maximum likelihood estimates of θ .

2.3. Parameterization. Estimation requires parameterizations for the mean growth rates and the transition probabilities. Following Durland and McCurdy (1994), we assume that the transition probabilities governing business cycles are logistic functions of the duration variable. We assume that transition probabilities governing switching in volatility are also a logistic function, thereby constraining them to lie between zero and one. Furthermore, we assume that the relationship between the mean growth rate and the age of current phase is quadratic. In summary, we have

$$(5) \quad \mu(S_t, D_t) = \alpha_0(S_t) + \alpha_1(S_t)(DD_t - 1) + \alpha_2(S_t)(DD_t - 1)^2, S_t = 0, 1$$

$$(6) \quad \pi_0(D_t) = \frac{\exp[\beta_0(0) + \beta_1(0)(DD_t - 1)]}{1 + \exp[\beta_0(0) + \beta_1(0)(DD_t - 1)]}$$

$$(7) \quad \pi_1(D_t) = \frac{\exp[\beta_0(1) + \beta_1(1)(DD_t - 1)]}{1 + \exp[\beta_0(1) + \beta_1(1)(DD_t - 1)]}, DD_t = \text{Min}(D_t, dm)$$

$$(8) \quad \pi_{v0} = \frac{\exp[\gamma(0)]}{1 + \exp[\gamma(0)]} \quad \text{and} \quad \pi_{v1} = \frac{\exp[\gamma(1)]}{1 + \exp[\gamma(1)]}$$

The dynamics encompassed by the pair of quadratic functions in (5) is quite rich. It includes the growth rate first increasing and then decreasing, and first decreasing and then increasing, over a given phase. It also includes the linear relationship as a special case. In (6) and (7), the coefficients $\beta_1(0)$ and $\beta_1(1)$ determine the relationship between the probability of exiting the current state and the age of the current phase. When $\beta_1(0)$ is negative, the probability of leaving an expansion is positively related to the age of the current expansion. In this case, the expansion is said to exhibit positive duration dependence. Similarly, if $\beta_1(1)$ is negative, the contraction is said to exhibit positive duration dependence. Table 1 summarizes the model and its parameters.

3. EMPIRICAL RESULTS

Using the algorithm described in the Appendix, the model is estimated using maximum likelihood. The log-likelihood function is calculated as a function of the parameters comprising the mean growth rates ($\alpha_0(0)$, $\alpha_1(0)$, $\alpha_2(0)$, $\alpha_0(1)$, $\alpha_1(1)$, $\alpha_2(1)$), the transition probabilities concerning business cycle ($\beta_0(0)$, $\beta_1(0)$, $\beta_0(1)$, $\beta_1(1)$), the transition probabilities concerning volatility ($\gamma(0)$, $\gamma(1)$), and the autoregressive process ($(\sigma(0), \sigma(1), \phi_1, \phi_2, \dots, \text{and } \phi_k)$). The maximization of this function with respect to these parameters is carried out using, sequentially, the Nelder–Mead Simplex, the Davidson–Fletcher–Powell, and the Quadratic Hill-Climbing algorithms. The covariance matrix of the estimates is computed numerically from the negative inverse of the Hessian of the log-likelihood function, evaluated at these estimates.⁵ The number of lags in the autoregressive process, k , and the maximum memory of the business cycle Markov chain, dm , are determined prior to estimation. We follow both Hamilton (1989) and Durland and McCurdy (1994) in setting k equal to 4.⁶ We set dm equal to 40, which is reasonable, as one would normally expect an expansion or a contraction to last for no more than 40 quarters.

The model is estimated using 100 times the growth rate of quarterly real GNP from 1952:2 to 2001:2. The GNP series is chain-weighted, seasonally adjusted, expressed in 1996 dollars, and the rates are annualized. It is the revised and updated version of the fixed-weighted data used in Hamilton (1989) and Durland and McCurdy (1994).⁷ The Bureau of Economic Analysis (BEA) changed from its previous fixed-weighting method to a chain-weighting method at the end of 1995.

The estimates are discussed in Section 3.1. In Section 3.2, the estimates of the benchmark Hamilton model are highlighted and compared to the result in Hamilton (1989). The ability of the models to match the postwar NBER reference

⁵ The quasi-maximum likelihood covariance matrix of White (1982) is robust to nonnormality of error. Virtually all our conclusions remain valid when this covariance matrix is used instead.

⁶ Lam (1990) considers an autoregressive process with two and three lags in levels, which is roughly equivalent to k equal to one or two; his results are not sensitive to the choice of k .

⁷ We obtained the GNP series from the Federal Reserve Bank of St. Louis (<http://www.stls.frb.org/fred/data/gnp96>). Real GNP is available beginning in 1947:1. We follow the literature in starting the sample in 1952:2, thereby avoiding many of the observations surrounding the Korean War. None of the article's major conclusions are sensitive to this choice of sample period.

TABLE 1
THE MODEL AND ITS PARAMETERS

A. The Model		
$y_t = \alpha_0(S_t) + \alpha_1(S_t)(DD_t - 1) + \alpha_2(S_t)(DD_t - 1)^2 + z_t$ $(1 - \phi_1 L - \phi_2 L^2 \cdots \phi_k L^k) z_t = \sigma(V_t) \varepsilon_t, \varepsilon_t \sim N.i.i.d.(0, 1)$ $S_t \in \{0, 1\}, V_t \in \{0, 1\},$ $\Pr[V_{t+1} = v / V_t = v] = \frac{\exp[\gamma(v)]}{1 + \exp[\gamma(v)]}, \quad v = 0, 1,$ $\Pr[S_{t+1} = s / S_t = s, D_t] = \frac{\exp[\beta_0(s) + \beta_1(s)(DD_t - 1)]}{1 + \exp[\beta_0(s) + \beta_1(s)(DD_t - 1)]}, \quad s = 0, 1,$		
where $DD_t = \text{Min}(D_t, dm)$, D is the age of the current run of states, and dm is a truncation parameter. V , S , and ε are independent.		
B. The Parameters		
Mean growth rate	$\alpha_0(0)$	Mean growth rate during the first period of an expansion.
	$\alpha_1(0)$	Linear coefficient in the relationship between the mean growth rate and the age of the expansion.
	$\alpha_2(0)$	Quadratic coefficient in the relationship between the mean growth rate and the age of the expansion.
	$\alpha_0(1)$	Mean growth rate during the first period of a contraction.
	$\alpha_1(1)$	Linear coefficient in the relationship between the mean growth rate and the age of the contraction.
	$\alpha_2(1)$	Quadratic coefficient in the relationship between the mean growth rate and the age of the contraction.
Transition probabilities (mean growth)	$\beta_0(0)$	Constant determining the probability an expansion survives after the first period.
	$\beta_1(0)$	Effect of the age of the current expansion on the probability that an expansion survives. Negative coefficient implies positive duration dependence; positive coefficient implies negative duration dependence.
	$\beta_0(1)$	Constant determining the probability that a contraction survives after the first period.
	$\beta_1(1)$	Effect of the age of the current contraction on the probability that a contraction survives. Negative coefficient implies positive duration dependence; positive coefficient implies negative duration dependence.
Transition probabilities (volatility)	$\gamma(0)$	Parameter determining the probability of staying in the high-volatility regime.
	$\gamma(1)$	Parameter determining the probability of staying in the low-volatility regime.
Autoregressive process	ϕ_i	i th autoregressive parameter.
	$\sigma(0)$	Standard deviation of innovations during the high-volatility regime.
	$\sigma(1)$	Standard deviation of innovations during the low-volatility regime.

cycle turning points is discussed in Section 3.3. In Section 3.4, we investigate the out-of-sample performance of the models. In Section 3.5, we consider the implications of the estimates for the periodicity of the business cycle. In Section 3.6, we discuss the implications for the fluctuation of output volatility. Finally, in

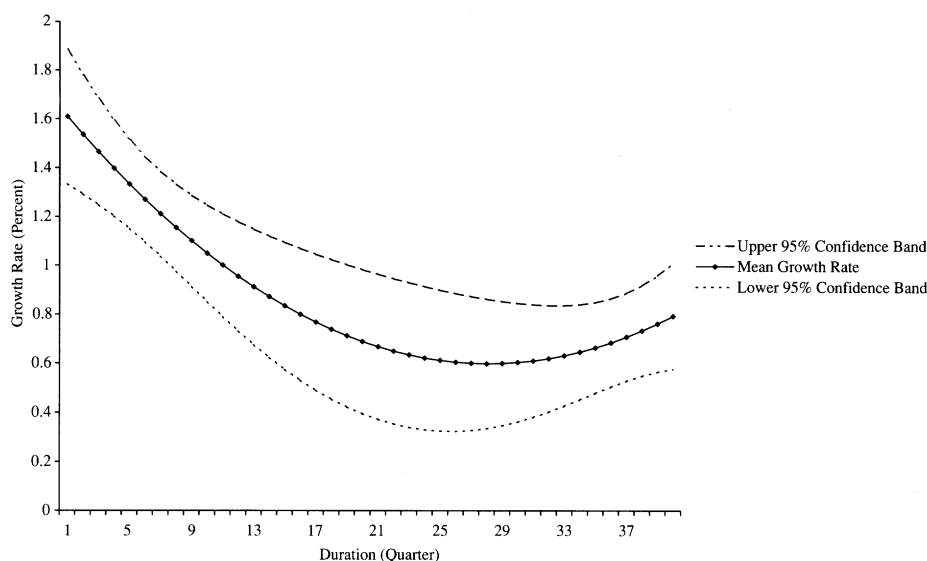


FIGURE 1

MEAN GROWTH RATE AND DURATION OF EXPANSION

Section 3.7, we investigate the sensitivity of our results to two departures from the general model.

3.1. Estimates. The parameter estimates are reported in Table 2. All the parameters, with the exception of ϕ_4 , are precisely estimated. The implications for duration dependence are summarized in a sequence of figures, where we present the mean output growth rates and the transition probabilities over the business cycle using the parameter estimates in Table 2. Figures 1 and 2 plot the mean growth rates and probabilities of remaining in an expansion over a period of 40 quarters. Figures 3 and 4 plot analogous measures for a contraction over a period of five quarters. Also included in the graphs are the 95% confidence bands.⁸

Because $\alpha_1(0)$ is negative and $\alpha_2(0)$ is positive, the mean growth rate first declines as the expansion ages, with this decline gradually leveling off before reaching the maximum memory of the business cycle. As $\alpha_2(0)$ is small, the growth rate of output never regains much ground after reaching its minimum at about seven years. The estimates of $\alpha_1(1)$ and $\alpha_2(1)$, 1.203 and -0.600 , respectively, imply a rich set of dynamics for output growth during a contraction. When a contraction begins, output declines at a rate of 0.2949% per quarter. This decline in output is halted temporarily in the second quarter of a contraction. After the second

⁸ In the case of the transition probabilities, the bands are calculated by simulating the estimated asymptotic covariance matrix of the parameters assuming normality. In the case of the mean growth rates, the bands are calculated by applying the delta method to the estimated asymptotic covariance matrix of the parameters, assuming normality.

TABLE 2
MAXIMUM-LIKELIHOOD ESTIMATES OF THE GENERAL MODEL

Parameter	Estimates
$\alpha_0(0)$	1.6091 (0.1405)
$\alpha_1(0)$	-0.0746 (0.0228)
$\alpha_2(0)$	0.0014 (0.0005)
$\alpha_0(1)$	-0.2949 (0.2345)
$\alpha_1(1)$	1.2031 (0.3217)
$\alpha_2(1)$	-0.6002 (0.1175)
$\beta_0(0)$	1.3946 (0.5006)
$\beta_1(0)$	0.0787 (0.0334)
$\beta_0(1)$	2.5923 (1.1217)
$\beta_1(1)$	-1.8529 (0.7562)
$\sigma(0)$	0.7267 (0.0671)
$\sigma(1)$	0.4580 (0.0628)
$\gamma(0)$	4.6597 (1.2904)
$\gamma(1)$	3.7854 (1.2926)
ϕ_1	0.2844 (0.0874)
ϕ_2	0.2947 (0.1012)
ϕ_3	-0.2064 (0.0925)
ϕ_4	-0.1218 (0.0935)
Log-likelihood	-236.3553
Hypothesis of absence of duration dependence, $\alpha_1(0) = \alpha_1(1) = \alpha_2(0) = \alpha_2(1) = \beta_1(0) = \beta_1(1) = 0$, $Wald = 63.2846, p\text{-value} = 0.0000$	

NOTE: The model is estimated using 100 times the log-difference of quarterly real GNP, 1952:2 to 2001:2. Asymptotic standard errors are in parentheses. *Wald* is computed using the asymptotic covariance matrix of $\alpha_1(0), \alpha_1(1), \alpha_2(0), \alpha_2(1), \beta_1(0)$, and $\beta_1(1)$. See Table 1 for definition of the parameters.

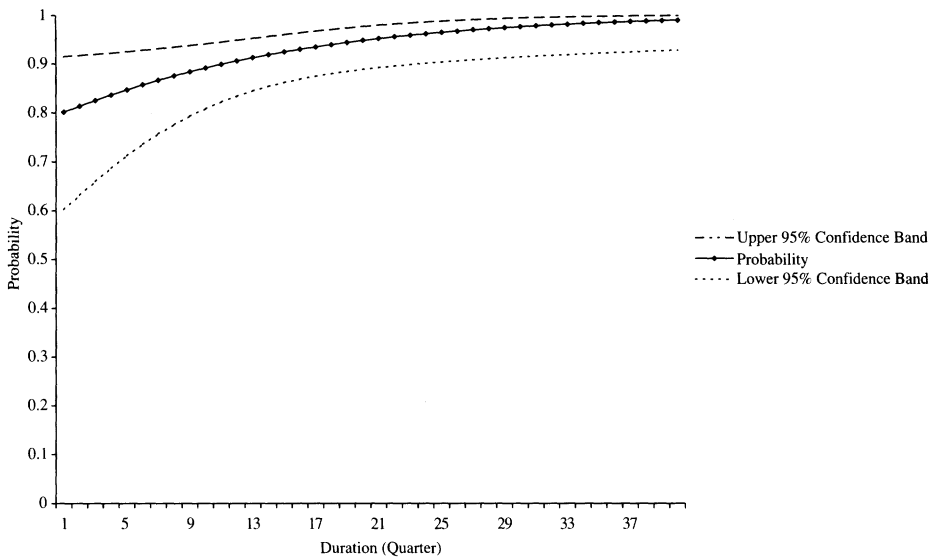


FIGURE 2

PROBABILITY OF REMAINING IN AN EXPANSION AND AGE OF EXPANSION

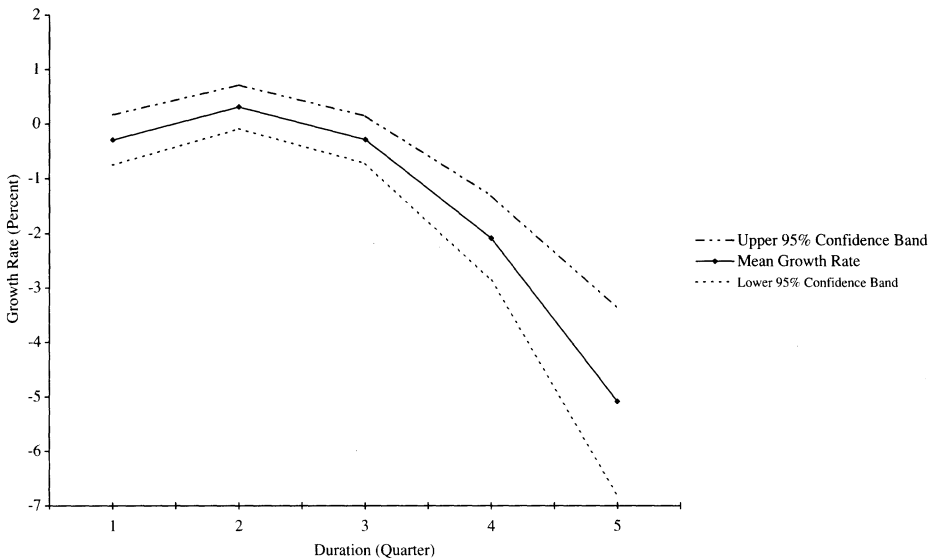


FIGURE 3

MEAN GROWTH RATE AND DURATION OF CONTRACTION

quarter, however, if the contraction continues, output declines again, reaching a rate of 2.088% in four quarters.

It is worth noting that the estimated pattern of mean growth rate over expansion and contraction is broadly consistent with the characterization of the business

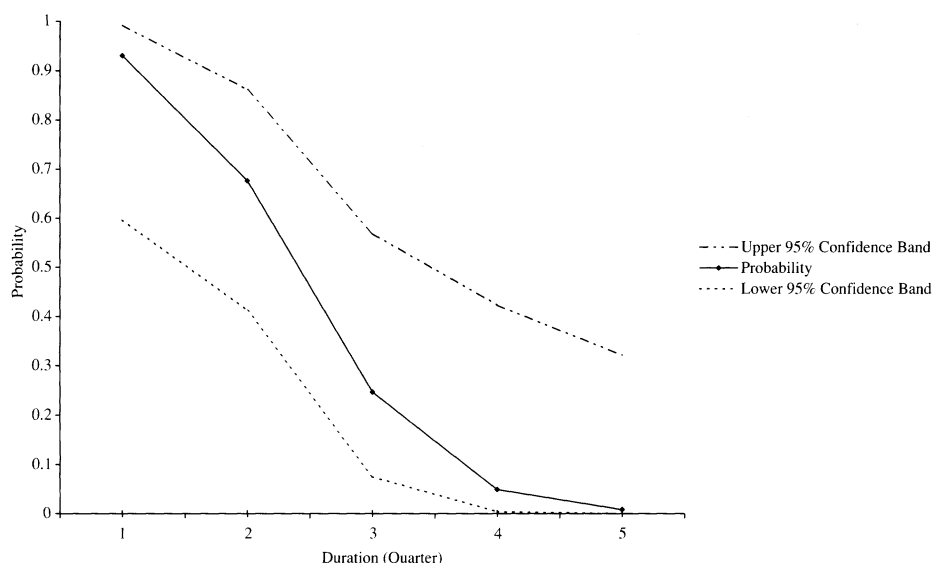


FIGURE 4

PROBABILITY OF REMAINING IN A CONTRACTION AND AGE OF CONTRACTION

cycle given in Burns (1968), Sichel (1994), and Kim and Nelson (1997). Burns argued that expansion tends to be rapid in its early stages and during contractions the rate of decline is usually fastest in the middle stages. Sichel demonstrated that postwar fluctuations in real output had consisted of three sequential phases instead of two—contractions, high growth recoveries, and moderate growth periods following recoveries, due to swings in inventory investment. Kim and Nelson estimated Friedman's Plucking Model using postwar quarterly real GDP and unemployment rate data. Their estimates also imply presence of three distinct phases in business cycle dynamics: a recessionary phase, a high growth recovery phase, and a normal phase.

The duration dependence in the business cycle transition probabilities is statistically significant. The t -statistic for the hypothesis that $\beta_1(0)$ is zero is 2.356, which is significant at 1% in a two-tailed test. Diebold and Rudebusch (1990), Sichel (1991), and Durland and McCurdy (1994) all conclude that duration dependence in postwar expansions is statistically insignificant. Our results contradict their findings.⁹ It is also interesting that the duration dependence we have uncovered

⁹ Three major differences between this article and the previous literature could explain the difference in results. First, Diebold and Rudebusch (1990) and Sichel (1991) use NBER reference cycle dates as data whereas this article uses GNP growth to infer the business cycle. By imposing the maturity criterion, NBER ruled out short expansions. These short expansions would have supported models with large probability of an expansion terminating near the beginning of that expansion. Second, unlike Diebold and Rudebusch (1990), Sichel (1991), and Durland and McCurdy (1994), this article includes the data from the long expansion in the 1990s. The long expansion tilts the evidence toward models with the conditional probability of an expansion terminating decreasing in the age of expansion. Finally, although Durland and McCurdy (1994) also use a similar model and GNP growth to infer the business

is contrary to the type considered in their work, which is motivated by the notion that, as an expansion ages, a contraction is increasingly imminent. Our results demonstrate that a new expansion is fragile and that a reversion to a contraction is most likely to occur when the expansion is in its infancy. Consistent with Durland and McCurdy (1994), we also find evidence for duration dependence in transition probabilities over a contraction. The t -statistic for the hypothesis that $\beta_1(1)$ is zero is -2.450 , which is also significant at 1% in a two-tailed test. The probability of remaining in a contraction tends to decrease as the contraction ages. At the onset of a contraction, this probability is 0.930, but after one quarter it falls to 0.677, becoming negligible after three quarters.¹⁰

To analyze the overall statistical significance of duration dependence, we present four restricted versions of the general model in Table 3. In the second column we present estimates for the case in which the duration dependence in mean growth rates and transition probabilities are absent, the McConnell and Perez-Quiros (2000) model. In the third column we present the case in which the noise component is homoskedastic, but the mean growth rates and transition probabilities are unrestricted. In the fourth column, we present the case in which the noise component is homoskedastic and the mean growth rates are independent of duration, the Durland and McCurdy model. Finally, in the fifth column, we present the benchmark case in which the noise component is homoskedastic and the mean growth rates and transition probabilities are independent of duration, the Hamilton model.

In Table 3, the likelihood ratio statistic for the null hypothesis of absence of duration dependence in mean growth rates and transition probabilities is 11.90. In Table 2, the Wald statistic for the hypothesis is 63.28. Since both statistics are distributed asymptotically as $\chi^2(6)$, the hypothesis can be rejected at a size of about 6% according to the likelihood ratio test and 1% according to the Wald test. The statistical significance of the hypothesis is robust to the assumption about volatility. The likelihood ratio statistic for the same hypothesis, in the case of homoskedasticity, is 31.60. The Wald statistic computed using the covariance matrix of the parameter estimates of the general model under homoskedasticity is 90.25. Both statistics reject the hypothesis at a 1% size.

Table 3 also points to the importance of modeling duration-dependent mean growth rates in testing for duration dependence in transition probabilities. The

cycle, they do not allow the mean growth rate to vary with the age of the business cycle phase. The flexibility in mean growth rate helps detect the tendency of the probability of expansion terminating to decline over the course of an expansion.

¹⁰ Potter (1995) estimated the Self-Exciting Threshold Auto-Regression (SETAR) model to both the pre-WWII (Balke-Gordon) and the post-1945 quarterly real GNP series. For the latter series, Potter obtains a negative and quantitatively large $AR(2)$ coefficient for the contractionary regime. That estimate implies a tendency for output growth to recover two quarters after a negative impulse, and therefore is broadly consistent with positive duration dependence in contraction. We also find positive duration dependence in contraction transition probability. Our estimate indicates, however, that, conditional on remaining in a contraction, output growth tends to further decline instead of increasing.

TABLE 3
MAXIMUM-LIKELIHOOD ESTIMATES OF THE RESTRICTED MODELS

Parameter	Estimates			
	McConnell–Perez- Quiros Model	General Model with Homoskedasticity	Durland–McCurdy Model	Hamilton Model
$\alpha_0(0)$	0.8619 (0.0731)	1.6097 (0.1165)	1.0105 (0.1232)	0.9028 (0.0890)
$\alpha_1(0)$	—	−0.0769 (0.0234)	—	—
$\alpha_2(0)$	—	0.0014 (0.0006)	—	—
$\alpha_0(1)$	−0.0524 (0.5360)	−0.2366 (0.1840)	−0.7566 (0.3714)	−1.2488 (0.4801)
$\alpha_1(1)$	—	1.2317 (0.2487)	—	—
$\alpha_2(1)$	—	−0.6265 (0.0947)	—	—
$\beta_0(0)$	3.7710 (1.1469)	1.1329 (0.4285)	1.6334 (0.7772)	3.3758 (0.6680)
$\beta_1(0)$	—	0.0904 (0.0331)	0.0657 (0.0395)	—
$\beta_0(1)$	0.8380 (1.0821)	2.4958 (0.9581)	0.5606 (1.1168)	−0.9773 (1.3390)
$\beta_1(1)$	—	−1.8818 (0.6727)	−2.6502 (2.9675)	—
$\sigma(0)$	1.0413 (0.0739)	0.6140 (0.0397)	0.7227 (0.0565)	0.7875 (0.0531)
$\sigma(1)$	0.4526 (0.0493)	—	—	—
$\gamma(0)$	5.2144 (1.2116)	—	—	—
$\gamma(1)$	4.8022 (1.3907)	—	—	—
ϕ_1	0.2291 (0.0882)	0.3044 (0.0878)	0.4194 (0.0952)	0.3487 (0.0835)
ϕ_2	0.1318 (0.0795)	0.3866 (0.1393)	0.2221 (0.1013)	0.1266 (0.0962)
ϕ_3	−0.0893 (0.0775)	−0.2467 (0.0909)	−0.2094 (0.1094)	−0.1328 (0.0927)
ϕ_4	−0.0696 (0.0739)	−0.1900 (0.1150)	−0.0938 (0.1004)	−0.1157 (0.0805)
Log-likelihood	−242.3068	−239.0725	−252.3604	−254.8748

The General Model (Table 2) versus the McConnell–Perez-Quiros Model,
 $\alpha_1(0) = \alpha_1(1) = \alpha_2(0) = \alpha_2(1) = \beta_1(0) = \beta_1(1) = 0$, $LR = 11.9030$, $p\text{-value} = 0.0632$
The General Model with Homoskedasticity versus the Hamilton Model,
 $\alpha_1(0) = \alpha_1(1) = \alpha_2(0) = \alpha_2(1) = \beta_1(0) = \beta_1(1) = 0$, $LR = 31.6046$, $p\text{-value} = 0.0001$
The General Model with Homoskedasticity versus the Durland–McCurdy Model,
 $\alpha_1(0) = \alpha_1(1) = \alpha_2(0) = \alpha_2(1) = 0$, $LR = 26.5758$, $p\text{-value} = 0.0001$
The Durland–McCurdy Model versus the Hamilton Model,
 $\beta_1(0) = \beta_1(1) = 0$, $LR = 5.0288$, $p\text{-value} = 0.0810$

NOTE: The model is estimated using 100 times the log-difference of quarterly real GNP, 1952:2 to 2001:2. Standard errors are in parentheses. The log-likelihood of the general model is taken from Table 2. In columns 3, 4, and 5, only one number is reported for $\sigma(0)$ and $\sigma(1)$, because $\sigma(0) = \sigma(1)$.

estimates of $\beta_1(0)$ and $\beta_1(1)$ for the Durland–McCurdy model are not significant at 10% according to a two-tailed t -test. For the general model with homoskedasticity, in contrast, they are significant at 1% according to a two-tailed t -test. The difference can be understood intuitively. In the general model, output is estimated to grow slowly when an expansion ages, and decline sharply when a contraction ages. For a given output growth path, this pattern in the mean growth rates makes rationalizing a long contraction more difficult and a long expansion easier—as a result, a significantly positive $\beta_1(0)$ and a significantly negative $\beta_1(1)$ are obtained.

3.2. The Hamilton Model. The estimates for the Hamilton model in the last column of Table 3 are quite different from the celebrated result in Hamilton (1989). To analyze the differences, we present in Table 4 three sets of estimates for the Hamilton model for the period 1951:2–1984:4, the sample period of Hamilton (1989). In the second column, we present estimates for the Hamilton model using our data series. In the third column, we present the estimates for the Hamilton model using his data series, computed using our algorithm. In the fourth column, we list the estimates presented in Hamilton (1989). Hamilton (1989) parameterizes the mean growth rates as functions of two parameters: the mean growth rate of contractions and the difference in mean growth rate between expansions and contractions. In Table 4, we present results using his parameterization.

In comparing the last columns of Tables 3 and 4, we see a number of differences. First, the probability of remaining in an expansion and the probability of staying in a contraction are 0.97 and 0.27, respectively, compared to 0.90 and 0.76 in Hamilton (1989). In addition, the mean growth rates of expansions and contractions are estimated to be 0.90 and -1.25% , respectively; in Hamilton (1989), they are 1.18 and -0.36% , respectively. Therefore, in comparison to Hamilton (1989), our estimates imply not only that expansions are longer and milder but also that contractions are shorter and more severe. The implied persistence and severity of contraction, unlike in Hamilton (1989), do not resemble those implicit in the business cycle based on the NBER cycle chronology.

The second column of Table 4 shows that the differences between our results and Hamilton (1989) do not stem from the addition of 66 quarters of data. When the Hamilton model is estimated using our data series over his shorter sample period, the probability of a contraction continuing is 0.23, and the mean growth rate of contractions is -1.05 . The differences are, therefore, largely due to the recent data revision by the BEA. This revision is fundamental in nature, unlike the regular ones that reflect newly available information from source data and other definitional and statistical changes. It reflects the change from its previous fixed-weighting to a chain-weighting method in constructing aggregate real output.¹¹ As a result of the revision, both output growth in expansions and output declines

¹¹ In constructing real GNP, BEA previously used the prices of a base period as fixed weights. This is known as the fixed-weighting method. As fast growing industries tend to experience slow growth in prices, the fixed-weighting method overstates the real growth after the base period, and understates the real growth before the base period—a phenomenon known as substitution bias. For example, with the price of computers in 1987 as fixed weight, the real growth of computer output from 1987 to 1992 is overstated, and the real growth of computer output from 1982 to 1987 is understated. Now BEA

TABLE 4
ESTIMATES OF THE HAMILTON MODEL, 1951:2–1984:4

Parameter	Estimates		
	Revised Data	Our Algorithm, Hamilton Data	Hamilton (1989)
$\alpha_0(0) - \alpha_0(1)$	2.022 (0.736)	1.535 (0.266)	1.522 (0.2636)
$\alpha_0(1)$	-1.049 (0.839)	-0.363 (0.269)	-0.3577 (0.2651)
$\frac{\exp[\beta_0(0)]}{1 + \exp[\beta_0(0)]}$	0.950 (0.055)	0.908 (0.035)	0.9049 (0.0374)
$\frac{\exp[\beta_0(1)]}{1 + \exp[\beta_0(1)]}$	0.232 (0.460)	0.756 (0.096)	0.755 (0.09656)
σ	0.916 (0.097)	0.771 (0.065)	0.769 (0.06676)
ϕ_1	0.342 (0.097)	0.019 (0.116)	0.014 (0.120)
ϕ_2	0.082 (0.112)	-0.068 (0.135)	-0.058 (0.137)
ϕ_3	-0.084 (0.111)	-0.253 (0.105)	-0.247 (0.107)
ϕ_4	-0.158 (0.098)	-0.225 (0.111)	-0.213 (0.110)

NOTE: Standard errors are in parentheses. The last column is taken from Table 1 of Hamilton (1989). The first four parameters here correspond to his α_1 , α_0 , p , and q , which are, respectively, the difference in mean growth rate between expansion and contraction, mean growth rate during contraction, the probability of an expansion continuing, and the probability of a contraction continuing. σ is the standard deviation of the innovation in the autoregressive process.

in contractions become larger in magnitude. The larger declines in contractions force our estimate of the mean growth rate in contraction to be large and negative. Because declines of such magnitudes are rare in the postwar data, our estimates of the Hamilton model imply that contractions are short-lived and infrequent.

The third and fourth columns of Table 4 provide a test of the integrity of our algorithm. The comparison reveals that we closely replicated the result of Hamilton (1989). The discrepancies between the parameter estimates, in absolute values, are small. They range from 0.013, in the case of the difference in mean growth rate across states, to 0.001, in the case of the probability of a contraction continuing. The discrepancies are also small according to statistical inference. To gauge the statistical significance of these discrepancies, we computed their t -ratios using our estimated standard errors. The t -ratios average 0.06 across parameters, and their maximum is only 0.11, which occurs in the case of ϕ_4 . Furthermore, the estimates in the third column imply a business cycle chronology identical to Hamilton

uses the prices concurrent with quantities as weights to construct real GNP, period by period. It is called the chain-weighting method, because the real growth from the base period to a future period is obtained by chaining together the period-by-period real growth rates. The change aims at minimizing substitution bias. See the discussion in Landefeld and Parker (1995).

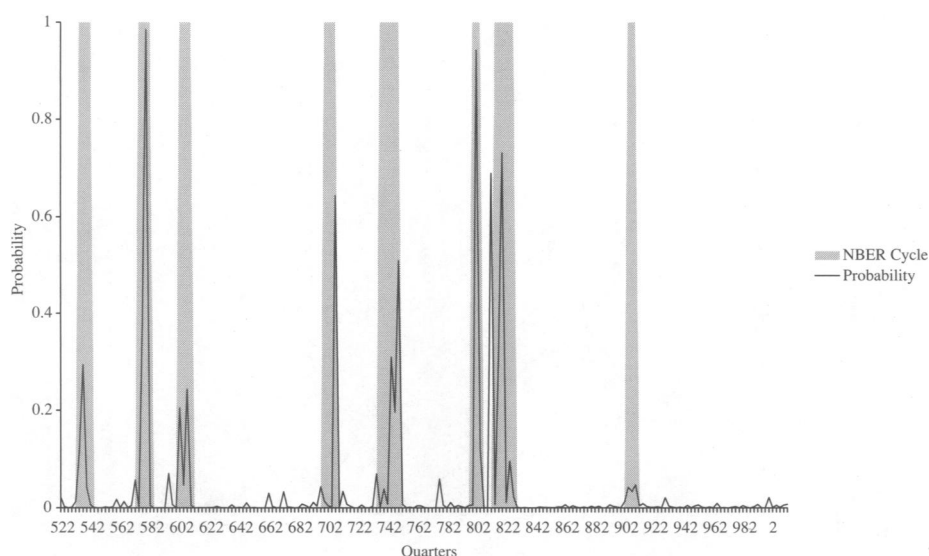


FIGURE 5

PROBABILITY OF BEING IN THE LOW-GROWTH STATE, HAMILTON MODEL

(1989). When we follow Hamilton (1989) in classifying as recessions the quarters with filtered probability of being in low-growth state greater than 0.5, the implied turning points of the business cycle are identical to those described in his Table 2. In contrast, the estimates based on revised data are very different, and therefore do not replicate the business cycle chronology of Hamilton (1989).¹²

3.3. Matching the Postwar NBER Reference Cycle Chronology. Figures 5 through 7 plot the filtered estimates of the probability of being in a contraction, for each point in the sample from 1952:2 to 2001:2. That is, we estimate the probability of being in a contraction. We compute these probabilities using the benchmark Hamilton model, the McConnell and Perez-Quiros model, and the general model. The results from these three models are plotted in Figures 5 through 7, respectively. The recession quarters according to NBER are indicated by the shaded areas in the figures.

A remarkable achievement in Hamilton (1989) is his ability to generate an alternative business cycle chronology that nearly replicates the NBER reference cycle chronology. The parameter estimates for the Hamilton model change substantially with the use of revised chain-weighted data. Figure 5 shows that this match deteriorates quite significantly as a result. If we classify as contractionary those quarters

¹² We also closely replicated the Durland and McCurdy (1994) model. The discrepancies between the parameter estimates, in absolute values, are all less than 0.02, with the exception of the transition probability at the beginning of an expansion. In that case, the discrepancy is 0.21, which is minor qualitatively and statistically: It is 4.62% of the parameter estimate and it implies a *t*-ratio of 0.12.

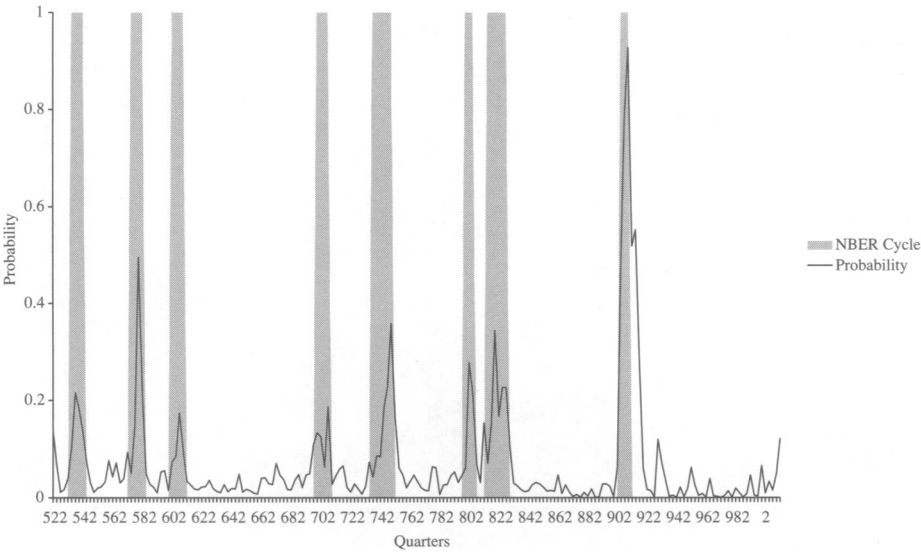


FIGURE 6

PROBABILITY OF BEING IN THE LOW-GROWTH STATE, THE MCCONNEL–PEREZ-QUIROS MODEL

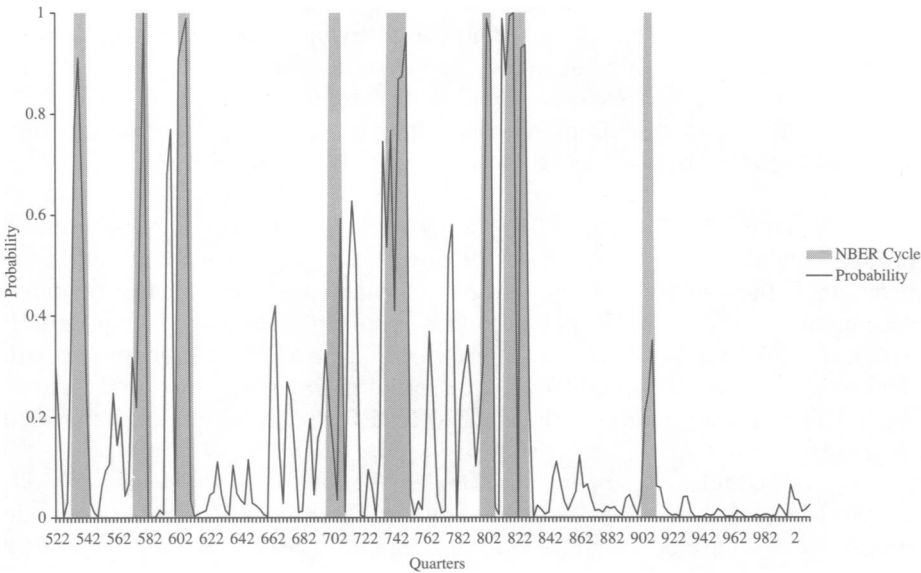


FIGURE 7

PROBABILITY OF BEING IN THE LOW-GROWTH STATE, GENERAL MODEL

with an estimated probability of being in the low-growth state exceeding 0.50, the identified contractionary episodes are generally too brief when compared to the NBER reference cycle chronology. For example, for the NBER-dated contraction occurring between 1969:4 and 1970:4, only 1970:4 is identified as a contraction by

the filter. More importantly, the filter almost completely misses that contraction associated with the 1975 oil shock. Also, the model does not identify the mild NBER-dated recession in the early 1990s as a contraction.

McConnell and Perez-Quiros (2000) emphasize that accounting for structural change in volatility is important for detecting the mild recession in the early 1990s. This is indeed the case in Figure 6. The filtered probability of the low-growth state exceeds 0.9 around 1990. Figure 6 also shows that the filtered probability of the low-growth state moves in the same direction as the NBER reference cycle. However, the level of the filtered probability tends to be quite low. If we classify as contractionary those quarters with an estimated probability of being in the low-growth state exceeding 0.5, there will only be two contractionary episodes, in 1958 and 1990.

The general model matches the NBER reference cycle chronology roughly as well as the McConnell and Perez-Quiros model. As Figure 7 indicates, the filtered probability of the low-growth state is closer to 1, compared to the one in Figure 6, during all NBER recessions except the one in 1990. On the other hand, our result provides a poorer match in two ways to the NBER chronology, compared to the result on the McConnell and Perez-Quiros model in Figure 6. First, the filtered probability of a low-growth state is lower than 0.5 in 1990. Second, the filtered probability of a low-growth state is fairly high and at times exceeding 0.5 in some quarters considered expansionary by NBER. Like the McConnell and Perez-Quiros model, the general model produces a reasonable but imperfect match with the NBER chronology.

The imperfect match with the NBER chronology should not be interpreted as a weakness of the general model relative to the original Hamilton model. When the NBER dating committee decided the business cycle status, it had access to information similar to that contained in the fixed-weighted data used by Hamilton (1989). Had it been provided with information similar to the revised chain-weighted series, the committee likely would have produced a different chronology. A different chronology is likely because, as Figure 5 shows, when the Hamilton (1989) model is applied to the revised chain-weighted data, the resulting chronology is quite different from the NBER chronology. Indeed, the committee could have produced a chronology similar to the one implied by the estimated general model. From that perspective, it is worth noting that the recent empirical works producing close matches with the NBER chronology are largely based on data similar to the fixed-weighted series used by Hamilton. Kim and Yoo (1995) and Foertsch (1997) estimate business cycle status using a multivariate Markov switching model of four monthly coincident indicators. Two of their indicators, personal income less transfer payments in 1987 dollars and the total index of industrial production, are essentially fixed-weighted.¹³

¹³ The four coincident indicators are: (1) employees on nonagricultural payrolls, (2) total personal income less transfer payments in 1987 dollars, (3) total index of industrial production, and (4) total manufacturing and trade sales in 1982 dollars. The personal income series was deflated using the implicit consumption deflator. Constructed before the BEA moved to the chain-weighting scheme, their deflator is fixed-weighted. FRB considered five-year intervals and used the fixed weights (the

3.4. *Out-of-Sample Forecasting Performance.* An advantage of estimating a Markov-switching model based on output data is that it yields a stochastic process that can be used in forecasting output growth, not just turning points. To demonstrate the usefulness of our approach, we therefore compare the performance of the model with duration dependence in mean growth rates and transition probabilities to other prominent models, in forecasting output growth out-of-sample. Specifically, we consider the general model, the Hamilton model, and the linear, one-state model, obtained by estimating the general model with $\alpha_0(0) = \alpha_0(1)$, $\sigma(0) = \sigma(1)$, and $\alpha_1(0), \alpha_1(1), \alpha_2(0), \alpha_2(1), \beta_1(0), \beta_1(1)$ all restricted to be zero.

For the quarters between 1984:1 and 1997:4, we estimate the three models using all the data prior to the quarter, and generate the forecasts of output growth over the next quarter, next year, next two years, next four years, and next six years, using the estimated models. Forecasting errors are obtained by comparing the forecasts to the output growth data over the forecasting horizon. The data reserved for the out-of-sample forecasting include the long expansion in the 1980s, the mild NBER contraction in the early 1990s, and the beginning of another long expansion afterward. It excludes the last three years of the 1990s, a highly volatile period with observations likely to be significantly revised by the BEA. The number of observations on forecasting errors ranges from 56, in the case of forecasting one-quarter growth, to 33, in the case of forecasting six-year growth. Table 5 presents the root-mean-square error of the models, and the significance of their predictive superiority as measured by the Diebold–Mariano (1993) statistics.

For horizons longer than two years, the general model has the smallest mean square error. The statistical significance of the predictive superiority of the general model tends to grow with the length of the horizon. In forecasting six-year growth, its superiority to the linear model and its superiority to the Hamilton model are both statistically significant at 5% according to a one-tailed test. The Hamilton model does not fare well against the linear model, and the predictive superiority of the linear model to the Hamilton model is statistically significant at 5% for horizons longer than one year. Introducing duration dependence into the Hamilton model improves the out-of-sample forecasting performance at horizons longer than two years. The improvement seems to be driven by the implication of the general model that output growth tends to slow down as the expansion ages. Output growth since the mid-1980s, with the exception of the past few years, has been low by historical standards.

3.5. *Periodicity of the Business Cycle.* We now use the estimated transition probabilities to derive the unconditional probability distribution of the length of a whole-cycle. In computing this distribution, we impose the maturity criterion that a contraction must be longer than two quarters to be recognized. The business

unit value-added data at the beginning of the interval) to calculate the production index within each interval. The production index series is the result of chaining together the indexes from different intervals. FRB has recently changed to a pure chain-weighting scheme. See Corrado et al. (1997) for a discussion of the change and its similarity to the recent revision of the national income and product accounts by the BEA.

TABLE 5
OUT-OF-SAMPLE FORECASTING PERFORMANCE OF THE MODELS, 1984:1–1997:4

A: Root Mean-Square Error (RMSE)			
Horizon (k)	Linear Model	Hamilton Model	General Model
1	0.468	0.473	0.476
4	1.362	1.432	1.515
8	2.207	2.273	2.285
12	2.864	2.918	2.788
16	3.470	3.544	3.186
24	4.707	4.788	3.117
B: Diebold–Mariano Test Statistics ($D-M$)			
Null Hypothesis of Equal Accuracy			
Horizon (k)	General–Hamilton	General–Linear	Hamilton–Linear
1	0.069	0.193	1.546
4	0.248	0.493	1.735
8	0.017	0.117	1.672
12	−0.137	−0.081	1.836
16	−0.316	−0.252	2.845
24	−1.717	−1.638	4.305

NOTES:

$$D - M = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{Nf}}}$$

where $\bar{d} = Nf^{-1} \sum_{t=To}^{T-k} (u_{1,t}^2 - u_{2,t}^2)$; To is the first quarter in which forecasts are made; T is the last quarter in the forecasting period; $u_{i,t}$ is the date- t forecast error of k -quarter growth, $\sum_{j=1}^k y_{t+j}$, based on model i , which is estimated using data up to t ; Nf is the number of forecasts, $T - To - k + 1$; $f_d(0)$ is a 4-lag Newey–West (1987) estimate of the spectral density of $u_{1,t}^2 - u_{2,t}^2$ at frequency zero. We set T and To such that they correspond to 1997:4 and 1983:4, respectively. Under the null hypothesis that the two models are equally accurate, $D-M$ is asymptotically standard normal.

$RMSE$ of model i is the square root of $Nf^{-1} \sum_{t=To}^{T-k} u_{i,t}^2$.

cycle measured as such conforms quite well to the NBER reference cycle, which is the focus of the research on the periodicity of the business cycle. The average duration of such expansions, contractions and whole cycles are 20.16 quarters, 3.26 quarters, and 23.42 quarters, respectively, compared to 18.97 quarters, 3.47 quarters, and 22.43 quarters for the NBER expansions, contractions, and peak-to-peak whole cycles in the postwar period.

Figure 8 plots the distribution for peak-to-peak whole cycles and its 95% confidence bands, computed using estimates of the general model in Table 2. In the figure, we also include for comparison the distribution for the estimated benchmark Hamilton model in which all forms of duration dependence are absent.

Figure 8 shows that the length of whole-cycle implied by the general model clusters more around shorter durations than longer durations. If the length of whole cycle is evenly distributed between 4 and 40 quarters, the probability of each quarter will be 2.7%. The lower 95% confidence band lies above 2.7% for

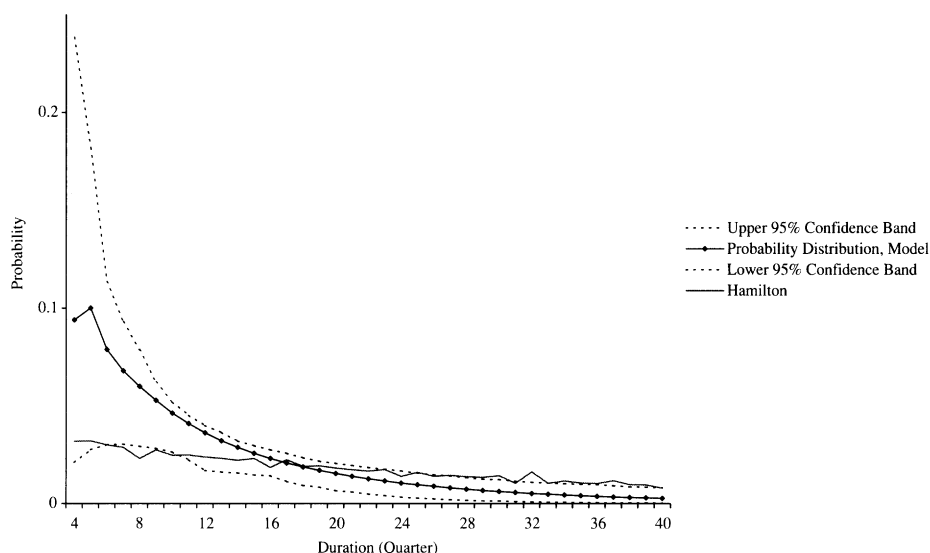


FIGURE 8

PROBABILITY DISTRIBUTION OF THE LENGTH OF WHOLE CYCLE

durations shorter than 14 quarters, and the upper 95% confidence band lies below 2.7% for durations longer than 25 quarters. This result follows mainly from the interplay between the estimated negative duration dependence in expansion and the estimated positive duration dependence in contraction. The tendency of expansions to terminate prematurely, combined with the tendency of contractions to end in a few quarters, causes the whole cycle to cluster around short durations.

Diebold and Rudebusch (1990) reject the hypothesis of no peak-to-peak and trough-to-trough periodicities in the NBER reference cycle turning points. The business cycle process we have estimated exhibits periodicities. These periodicities, however, concentrate around durations shorter than the 7–10-year type that motivated their hypothesis testing. In Figure 8, the probability of a 1–4-year cycle is 68.76%, much higher than the probability of a 7–10-year cycle, which is 6.04%. The difference is statistically significant at 1%, as the t -statistic based on the standard error of the difference, obtained by simulating the estimated covariance matrix of the parameters, is 3.16. In order for business cycles to cluster around 7–10-year durations, positive but small duration dependence must be present in the transition probabilities associated with expansions. We, in contrast, find negative duration dependence in expansions. This negative duration dependence tends to produce cycles that are either very short or very long.

3.6. Changing Volatility. Table 2 indicates the presence of two distinct volatility states. The standard deviation of output growth innovation in the high-volatility regime is almost twice as high as the one in the low-volatility regime. Also, the two volatility regimes are highly persistent. Although the low-volatility regime is slightly less persistent, it still has an expected duration of 44 quarters.

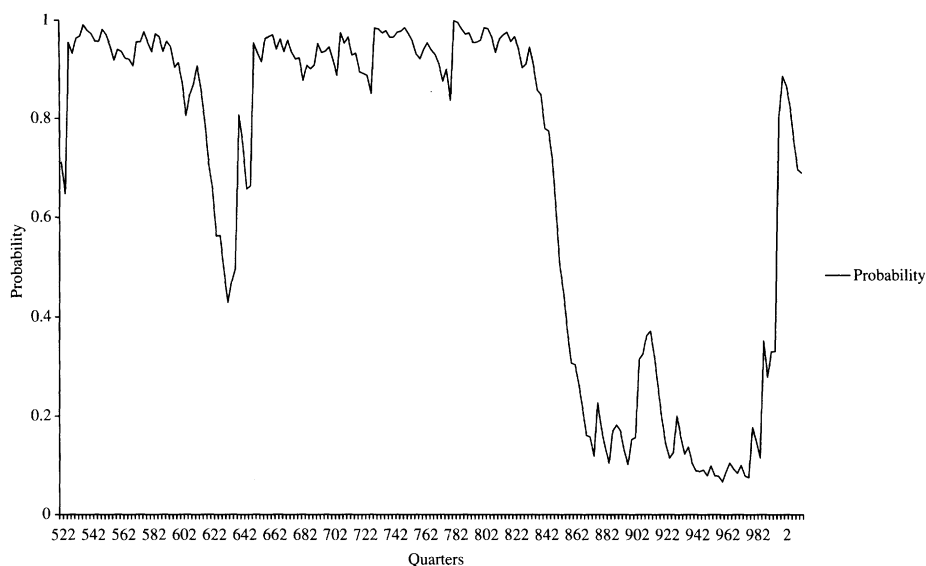


FIGURE 9

PROBABILITY OF BEING IN THE HIGH-VOLATILITY STATE, GENERAL MODEL

Figure 9 plots the filtered probability of the high-volatility regime. As McConnell and Perez-Quiros (2000) and Kim and Nelson (1999) have emphasized, there is evidence of a shift from a high-volatility regime to a low-volatility regime around 1984.¹⁴ There is also a brief period of low-volatility regime around 1963. Interestingly, there is evidence of a reversal to the high-volatility regime after 1999, the period falling outside the sample period of McConnell and Perez-Quiros (2000). Kahn et al. (2001) argued the improvement in inventory policy since the early 1980s, due to advances in information technology, played a primary role in the reduction of output volatility. Since there is no obvious change in inventory policy around 1999, our result suggests that inventory policy is not a complete explanation of reduction in volatility.

3.7. Sensitivity Analysis. We estimated two variants of the model to verify the robustness of the result.¹⁵ First, we considered the maintained assumption that the

¹⁴ We have also estimated the McConnell and Perez-Quiros model using our data series over the sample period of Hamilton, 1951:2–1984:4. The model exceeds the Hamilton model in log-likelihood by only 1.348, while using three more parameters. Also, its parameter estimates are unreasonable. This is consistent with an occurrence of volatility shift around 1984. The model is also rejected in favor of the general model at 5% size according to the likelihood ratio test.

¹⁵ We have also estimated the general model using our data series over the sample period of Hamilton. The pattern of duration dependence estimated using the whole sample mainly survives: The growth rate tends to decline when an expansion ages; the growth rate declines moderately at first and significantly as a contraction ages and the probability of exiting a contraction increases with age. The finding that does not survive is the negative duration dependence during expansion. For this sample period, $\beta_1(0)$ is estimated to be negative but insignificant.

mean growth rate is a quadratic function of the age of the business cycle phase up to 40 quarters. The estimated quadratic function for a contraction implies that output is expected to decline at an unrealistic rate when a contraction lasts longer than five quarters. We therefore reestimate the general model assuming the mean growth rate of contraction is constant beyond the duration of five quarters. The parameter estimates are almost numerically unchanged.

Second, we considered relaxing the tightly specified mean growth function. The quadratic specification is more likely to be restrictive for contractions, as growth rates may vary sharply over short time spans. To address this issue, we reestimated the model, while replacing the quadratic mean growth function with five dummy variables. The five dummies represent the first, second, third, fourth, and other quarters of a contraction. The empirical results are largely unaffected by this change. Output declines at mild rates in the beginning of a contraction, and declines severely when the contraction persists. Duration dependence in transition probability is positive for contractions and negative for expansions. The only substantial change is that the positive duration dependence in contraction transition probability now is insignificant at a 10% size.¹⁶

4. CONCLUDING REMARKS

We have used a regime-switching model of real GNP growth to examine the duration dependence of business cycles. The model is an extended version of Hamilton (1989), Durland and McCurdy (1994), and McConnell and Perez-Quiros (2000) and we estimate using the postwar NIPA quarterly data.

Three empirical findings are worth emphasizing. First, at the beginning of an expansion, the probability of the expansion ending is close to 0.2, but it gradually decreases as the expansion ages. Second, at the beginning of a contraction, the probability that the contraction terminates is less than 0.07, but it increases rapidly as the contraction ages. Third, there is a tendency for output growth to slow down over the course of an expansion. Taken together, the duration dependence that we have estimated implies a clustering of the whole cycle around durations shorter than the 7–10-year type discussed in the literature.

The algorithm presented in this article can be extended to incorporate the effects of the duration of previous phase on the mean growth rates and transition probabilities of the current phase. Such an algorithm can address the issues raised in Diebold (1993), Neftci (1986), and Sichel (1991), such as whether longer expansions tend to be followed by longer recessions and whether longer recessions tend to be followed by longer expansions. Lam (1997) contains one such algorithm and an attempt to estimate the effects in pre-WWII quarterly data.

It would also be interesting to investigate the economic forces behind the estimated duration dependence. A prime candidate is monetary policy. By incorporating monetary variables into the transition probabilities and mean growth rates, we could gauge the contribution of monetary variables to duration dependence. Such a model would be able to address a number of intriguing findings in the

¹⁶ The implied mean growth rates for the first five quarters of a contraction are, respectively, -0.754 , -0.475 , -1.260 , -2.310 , and 0.307 . $\beta_1(1)$ is -0.359 with an asymptotic t -statistic of -1.20 .

literature, such as the finding that negative monetary shocks affect output whereas positive ones do not (Cover, 1992), and the finding that a high Fed funds rate raises both the probability of staying in expansion and the probability of staying in contraction (Filardo, 1994).

APPENDIX

According to prediction error decomposition, the log-likelihood of the sample can be computed by summing the conditional log-likelihoods. Each conditional log-likelihood is the log-likelihood of an observation, conditional upon its past history. If the conditional log-likelihoods depend upon a small set of variables and if these variables are conditionally Markovian, Bayes theorem can be used to evaluate them recursively.

To find the set of state variables, we multiply both sides of equation (1) by the polynomial $\phi(L)$ and use equation (3) to obtain

$$\sigma(V_t)\varepsilon_t = y_t - \sum_{i=1}^k \phi_i y_{t-i} - \mu(S_t, D_t) + \sum_{i=1}^k \phi_i \mu(S_{t-i}, D_{t-i}).$$

This equation shows that the joint probability distribution of $\{S_i\}_{i=t-k}^t$, $\{D_i\}_{i=t-k}^t$ and V_t , is sufficient for evaluating the conditional likelihood of an observation at time t . It follows that the joint distribution of $\{S_i\}_{i=t-k}^t$, $\{D_i\}_{i=t-k}^t$ and $\{V_i\}_{i=t-k}^t$ is sufficient as well. In addition, if the probability distribution of $\{S_i\}_{i=t-k}^t$, $\{D_i\}_{i=t-k}^t$ and $\{V_i\}_{i=t-k}^t$ is known, the probability distribution of $\{S_i\}_{i=t-k+1}^{t+1}$, $\{D_i\}_{i=t-k+1}^{t+1}$ and $\{V_i\}_{i=t-k+1}^{t+1}$, can be derived using the transition probabilities equations (2) and (4). Thus, $\{S_i\}_{i=t-k}^t$, $\{D_i\}_{i=t-k}^t$ and $\{V_i\}_{i=t-k}^t$ are Markovian and can be used as state variables in the recursive evaluation of the likelihood function.

Simplification is possible, however. If we know $\{S_i\}_{i=t-k}^t$, D_{t-k} and $\{V_i\}_{i=t-k}^t$, it is possible to recover $\{S_i\}_{i=t-k}^t$, $\{D_i\}_{i=t-k}^t$ and $\{V_i\}_{i=t-k}^t$. It follows that the set of state variables, $\{S_i\}_{i=t-k}^t$, D_{t-k} and $\{V_i\}_{i=t-k}^t$, is sufficient to evaluate the conditional likelihood of the observation at time t . In addition, using the transition probabilities in (2) and (4), the probability distribution of $\{S_i\}_{i=t-k+1}^{t+1}$, D_{t-k+1} and $\{V_i\}_{i=t-k+1}^{t+1}$ can be computed from the probability distribution of $\{S_i\}_{i=t-k}^t$, D_{t-k} and $\{V_i\}_{i=t-k}^t$. Thus, this smaller set of state variables is Markovian. Therefore, we can evaluate the likelihood function recursively, using $\{S_i\}_{i=t-k}^t$, D_{t-k} and $\{V_i\}_{i=t-k}^t$ as the state variables.

This method is an extension of Hamilton (1989) and Durland and McCurdy (1994) and McConnell and Perez-Quiros (2000). If neither duration dependence nor heteroskedasticity present, $\{S_i\}_{i=t-k}^t$ is sufficient to evaluate the conditional likelihoods. This is the case presented in Hamilton (1989). If there is no heteroskedasticity, and the mean growth rates are not duration dependent, the set of state variables required for evaluating the conditional likelihoods is $\{S_i\}_{i=t-k}^t$ and D_t . This is the case presented in Durland and McCurdy (1994). If there is heteroskedasticity and both transition probabilities and mean growth rates are not duration dependent, the set of state variables required for evaluating the conditional likelihoods is $\{S_i\}_{i=t-k}^t$ and $\{V_i\}_{i=t-k}^t$. This is the case of McConnell and Perez-Quiros (2000).

When there is no heteroskedasticity, and both the transition probabilities and the mean growth rates depend upon the age of the current phase, the set of state variables appropriate for evaluating the conditional likelihood at time t is $\{S_i\}_{i=t-k}^t$ and D_{t-k} . This is not equivalent to the set of state variables employed by Durland and McCurdy (1994). When the mean growth rates depend upon the age of the current phase, to evaluate the conditional likelihood at time t , it is necessary to know $\{D_i\}_{i=t-k}^t$. While it is possible to recover these values from $\{S_i\}_{i=t-k}^t$ and D_{t-k} , it is impossible to do so from $\{S_i\}_{i=t-k}^t$ and D_t .

Below we describe an algorithm for evaluating the likelihood function, using as the state variables $\{S_i\}_{i=t-k}^t$, D_{t-k} and $\{V_i\}_{i=t-k}^t$. Let \tilde{S} be a state variable summarizing S and V , so that

$$\tilde{S} = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \end{cases} \Leftrightarrow \begin{cases} S = 0, V = 0 \\ S = 1, V = 0 \\ S = 0, V = 1 \\ S = 1, V = 1 \end{cases}.$$

The algorithm is summarized in the following five steps:

Step 1: For $s_0 = 1, 2, 3, 4$, $s_1 = 1, 2, 3, 4$, $s_2 = 1, 2, 3, 4, \dots, s_{k-1} = 1, 2, 3, 4$, $s_k = 1, 2, 3, 4$, $d_1 = 1, 2, \dots, dm$, and starting with $\delta_0 = d_1$, calculate

$$\delta_i = \delta_{i-1} \ell(s_{k-i} - s_{k-i-1}) + 1,$$

for $i = 1, 2, \dots, k-1, k$, where ℓ is an indicator function assigning zero to an odd number and one to an even number. Set δ_i equal to dm whenever it exceeds dm . Store this sequence as

$$\tau(s_0, s_1, s_2, \dots, s_k, d_1, i) = \delta_i.$$

Then calculate, for $i = 0, 1, 2, \dots, k-1, k$,

$$\psi(s_0, s_1, s_2, \dots, s_k, d_1, i) = \mu(\ell(s_{k-i}), \delta_i).$$

Step 2: For $s_0 = 1, 2, 3, 4$, $s_1 = 1, 2, 3, 4$, $s_2 = 1, 2, 3, 4, \dots, s_{k-1} = 1, 2, 3, 4$, $s_k = 1, 2, 3, 4$, compute three sets of probabilities:

(i) $d_1 = 1$

$$\begin{aligned} & \Pr[\tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d_1 / Y_{t-1}, \theta] \\ &= \sum_{d_2=1}^{dm} \Pr[\tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, \tilde{S}_{t-k-1} = s, D_{t-k-1} = d_2 / Y_{t-1}, \theta] \\ & \quad \times \Pr[\tilde{S}_t = s_0 / \tilde{S}_{t-1} = s_1, D_{t-1} = \tau(s_1, s_2, \dots, s_k, s, d_2, k)] \\ & \quad + \sum_{d_2=1}^{dm} \Pr[\tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, \tilde{S}_{t-k-1} = s', D_{t-k-1} = d_2 / Y_{t-1}, \theta] \\ & \quad \times \Pr[\tilde{S}_t = s_0 / \tilde{S}_{t-1} = s_1, D_{t-1} = \tau(s_1, s_2, \dots, s_k, s', d_2, k)], \end{aligned}$$

where $(s, s') = (2, 4)$ for $s_k = 1, 3$ and $(s, s') = (1, 3)$ for $s_k = 2, 4$.

(ii) $2 \leq d1 \leq dm - 1$

$$\begin{aligned} & \Pr[\tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d1/Y_{t-1}, \theta] \\ &= \Pr[\tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, \tilde{S}_{t-k-1} = s, D_{t-k-1} = d1 - 1/Y_{t-1}, \theta] \\ & \quad \times \Pr[\tilde{S}_t = s_0/\tilde{S}_{t-1} = s_1, D_{t-1} = \tau(s_1, s_2, \dots, s_k, s, d1 - 1, k)] \\ & \quad + \Pr[\tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, \tilde{S}_{t-k-1} = s', D_{t-k-1} = d1 - 1/Y_{t-1}, \theta] \\ & \quad \times \Pr[\tilde{S}_t = s_0/\tilde{S}_{t-1} = s_1, D_{t-1} = \tau(s_1, s_2, \dots, s_k, s', d1 - 1, k)], \end{aligned}$$

where $(s, s') = (1, 3)$ for $s_k = 1, 3$ and $(s, s') = (2, 4)$ for $s_k = 2, 4$.

(iii) $d1 = dm$

$$\begin{aligned} & \Pr[\tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d1/Y_{t-1}, \theta] \\ &= \sum_{j=0}^1 \Pr[\tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, \tilde{S}_{t-k-1} = s, D_{t-k-1} = dm - j/Y_{t-1}, \theta] \\ & \quad \times \Pr[\tilde{S}_t = s_0/\tilde{S}_{t-1} = s_1, D_{t-1} = \tau(s_1, s_2, \dots, s_k, s, dm - j, k)] \\ & \quad + \sum_{j=0}^1 \Pr[\tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, \tilde{S}_{t-k-1} = s', D_{t-k-1} = dm - j/Y_{t-1}, \theta] \\ & \quad \times \Pr[\tilde{S}_t = s_0/\tilde{S}_{t-1} = s_1, D_{t-1} = \tau(s_1, s_2, \dots, s_k, s', dm - j, k)], \end{aligned}$$

where $(s, s') = (1, 3)$ for $s_k = 1, 3$ and $(s, s') = (2, 4)$ for $s_k = 2, 4$.

In (i), (ii) and (iii), $\Pr[\tilde{S}_t = i/\tilde{S}_{t-1} = j, D_{t-1} = k]$ are computed using the (j, i) element of P_k ,

$$P_k = \begin{bmatrix} \pi o(k)\pi v0 & (1 - \pi o(k))\pi v0 & \pi 0(k)(1 - \pi v0) & (1 - \pi 0(k))(1 - \pi v0) \\ (1 - \pi 1(k))\pi v0 & \pi 1(k)\pi v0 & (1 - \pi 1(k))(1 - \pi v0) & \pi 1(k)(1 - \pi v0) \\ \pi 0(k)(1 - \pi v1) & (1 - \pi 0(k))(1 - \pi v1) & \pi 0(k)\pi v1 & (1 - \pi 0(k))\pi v1 \\ (1 - \pi 1(k))(1 - \pi v1) & \pi 1(k)(1 - \pi v1) & (1 - \pi 1(k))\pi v1 & \pi 1(k)\pi v1 \end{bmatrix}.$$

Step 3: Calculate, for $s_0 = 1, 2, 3, 4, s_1 = 1, 2, 3, 4, s_2 = 1, 2, 3, 4, \dots, s_{k-1} = 1, 2, 3, 4, s_k = 1, 2, 3, 4, d1 = 1, 2, 3, \dots, dm$,

$$\begin{aligned} & \Pr[y_t, \tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d1/Y_{t-1}, \theta] \\ &= \Pr[\tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d1/Y_{t-1}, \theta] \\ & \quad \times \Pr[y_t/\tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d1, Y_{t-1}, \theta], \end{aligned}$$

where

$$\begin{aligned} & \Pr[y_t / \tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d1, Y_{t-1}, \theta] \\ &= \frac{1}{\sqrt{2\pi\omega(s_0)}} \exp \left\{ -\frac{1}{2\omega(s_0)} \left[y_t - \psi(s_0, s_1, \dots, s_k, d1, k) \right. \right. \\ & \quad \left. \left. - \sum_{j=1}^k \phi_j (y_{t-j} - \psi(s_0, s_1, \dots, s_k, d1, k-j)) \right]^2 \right\}, \end{aligned}$$

with $\omega(1) = \omega(2) = \sigma(0)^2$, $\omega(3) = \omega(4) = \sigma(1)^2$.

Step 4: From this, then compute

$$\begin{aligned} \Pr[y_t / Y_{t-1}, \theta] &= \sum_{s_0=1}^4 \sum_{s_1=1}^4 \cdots \sum_{s_k=1}^4 \sum_{d1=1}^{dm} \\ & \Pr[y_t, \tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d1 / Y_{t-1}, \theta]. \end{aligned}$$

Step 5: Finally, calculate, for $s_0 = 1, 2, 3, 4$, $s_1 = 1, 2, 3, 4$, $s_2 = 1, 2, 3, 4, \dots$, $s_{k-1} = 1, 2, 3, 4$, $s_k = 1, 2, 3, 4$, $d1 = 1, 2, \dots, dm$,

$$\begin{aligned} & \Pr[\tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d1 / Y_t, \theta] \\ &= \frac{\Pr[y_t, \tilde{S}_t = s_0, \tilde{S}_{t-1} = s_1, \dots, \tilde{S}_{t-k} = s_k, D_{t-k} = d1 / Y_{t-1}, \theta]}{\Pr[y_t / Y_{t-1}, \theta]}. \end{aligned}$$

Step 1 produces inputs for Step 2 and 3. Step 2 uses as inputs the outputs of Steps 1 and 5 from the previous observation and produces output for Step 3. Step 3 produces output for use in Step 4. In moving through these five steps, the algorithm produces the conditional likelihood of an observation (Step 4) and the filtered probability distribution of the state variables (Step 5). The steps are repeated for the next observation until the end of the sample is reached. Since Step 1 is identical for all observations, after the first observation the algorithm can begin with Step 2.

The log-likelihood function is the sum of log conditional likelihoods of the observations,

$$L(y_T, y_{T-1}, y_{T-2}, \dots, y_2, y_1; \theta) = \sum_{t=1}^T \log(\Pr[y_t / Y_{t-1}, \theta]).$$

To initialize the algorithm, the probability distribution of the states is needed in Step 2. This is provided by the unconditional distribution of the states implied by the transition probabilities, which we compute analytically. The formula for conditional density of y_t in step 3 assumes the conditioning set Y_{t-1} includes k lagged values of y_t , and therefore is valid only when $t \geq k + 1$. For the first k observations ($t \leq k$), we use a modified formula. We compute the auto-covariance function of y_t using the Yule-Walker equations, and then use it to calculate the density of y_t conditional on its lags, assuming they are jointly normal.

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