CSE 101 – Oct 28, 2019 (Week 5) (Unofficial Lecture)

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Class Notes

- Pat will find older webcasts
- PA2 closing tomorrow at 10am

PA3

(these vertical equations were hard to format with macros :/ ... check scanned handwritten notes on class website)

Example 1

Normalize:

Index: 2 1 0

91	87	64
19	29	45
−−	−−	−−
109	116	108

$\Rightarrow$ (01 10 17 08)

Example 2

(98	17	12)	
(81	29	52)	
−−	−−	−−	subtract
−1	−1		
(17	−12	−40)	
	−13		
	100	100	
		−	
(16	87	60)	

Example 3

Issue where MSB is out of range... just change signs of all the first results $\Rightarrow (-17\ 12\ 40 \Rightarrow 17\ -12\ -40)$

Recurrence Relations

Example

$$T(n) = \begin{cases} 0 & \text{if } n = 1 \\ T(\lfloor \dfrac{n}{2} \rfloor) + 1 & \text{if } n \geq 2 \end{cases}$$

Iteration method

$T(smile) = 1 + T(\lfloor \frac{smile}{2} \rfloor)$

$\therefore T(n) = 1 + T(\lfloor \frac{n}{2} \rfloor)$

$= 1 + 1 + T(\lfloor \frac{\lfloor \frac{n}{2} \rfloor}{2} \rfloor)$

$= 2 + 1 + T(\lfloor \frac{\lfloor \frac{n}{2^2} \rfloor}{2} \rfloor)$

$= 3 + 1 + T(\lfloor \frac{n}{2^3} \rfloor)$

...

$= k + T(\lfloor \frac{n}{2^k} \rfloor)$

Recursion halts when

$$\lfloor \frac{n}{2^k} \rfloor = 1$$

$\iff 1 \leq \frac{n}{2^k} < 2$

$\iff 2^k \leq n < 2 * 2^k$

$\iff 2^k \leq n < 2^{k+1}$

$\iff k \leq \lg(n) < k + 1$

$\therefore k = \lfloor \lg(n) \rfloor$

$\therefore T(n) = \lfloor \lg(n) \rfloor$ (exact solution)

Therefore: $T(n) = \Theta(\log(n))$ (asymptotic solution)

Exercise: Check that $T(n) = \lfloor \lg(n) \rfloor$ is a solution to a recurrence

Exercise Let

$$S(n) = \begin{cases} 0 & \text{if } n = 1 \\ S(\lceil \dfrac{n}{2} \rceil) + 1 & \text{if } n \geq 2 \end{cases}$$

Show that

$S(n) = \lceil \lg(n) \rceil$

- Derive solution using iteration method
- Check solution by substitution

Thus again $S(n) = \Theta(\log(n))$

Example

$T(n) = \begin{cases} c & \text{if } 1 \leq n \leq n_0 \\ T(\lfloor \dfrac{n}{2} \rfloor) + d & \text{if } n \geq n_0 \end{cases}$

$T(n) = d + T(\lfloor \frac{n}{2} \rfloor)$

$= d + d + T(\lfloor \frac{\lfloor \frac{n}{2} \rfloor}{2} \rfloor)$

$= 2d + T(\lfloor \frac{n}{2^2} \rfloor)$

...

$kd + T(\lfloor \frac{n}{2^k} \rfloor)$

Recursion halts at k s.t.

$$1 \leq \lfloor \frac{n}{2^k} \rfloor < n_0$$

We seek largest k s.t.

$$\lfloor \frac{n}{2^k} \rfloor < n_0$$

i.e. largest k s.t.

$$\frac{n}{2^k} < n_0$$

i.e.

$$\frac{n}{n_0} < 2^k$$

i.e.

$$\lg(\frac{n}{n_0}) < k$$

Since largest such k:

$$k - 1 \leq \lg(\frac{n}{n_0}) < k$$

$\therefore k = 1 + \lfloor \lg(\frac{n}{n_0}) \rfloor$

$\therefore k = 1 + \lfloor \lg(n) - \lg(n_0) \rfloor$

Thus

$T(n) = d(1 + \lfloor \lg(n) - \lg(n_0) \rfloor) + c$ (exact solution)

So

$T(n) = \Theta(\log(n))$ (asymptotic solution)