

## CSE 101 – Oct 21, 2019 (Week 4)

Notes provided by Ben Sihota bsihota@ucsc.edu

---

### Notes

- Take full period
- Alternate seating, directed seating
- Review problems and selected answers
- Sterling's formula **not** provided

### PA2

- Zero matrix is an array of empty lists (not NULL)
- First write transpose before product
- Dot product helper function
- Never store 0s in values
- Product is  $n^2$
- PA2 extended 2 days

### Helper function dot()

Input lists A, B

A: (10, 1), (30, 2), (50, -1), (60, 3)

B: (20, 2), (40, -1), (50, 5), (70, 4), (80, 5)

**Sum:**  $0 + (-5)$

Helps with product()

### Change Entry

changeEntry(M, i, j, x)

**Case 1:**  $M_{ij} = 0, x = 0$

Do nothing

**Case 2:**  $M_{ij} \neq 0, x = 0$

Delete an entry object

**Case 3:**  $M_{ij} = 0, x \neq 0$

Insert a new entry

**Case 4:**  $M_{ij} \neq 0, x \neq 0$

Override existing entry

---

### Inductive Proof Bullet List

1. Clearly stated base case(s)
2.
  - Introduce “arbitrary” inductive variable
  - State explicit induction hypothesis (weak vs strong)
  - State explicit induction conclusion
  - State point(s) at which inductive hypothesis is used

### Example

Define  $T(n)$  by

$$T(n) = \begin{cases} 1 & \text{if } 1 \leq n \leq 2 \\ 4T(\lfloor \frac{n}{3} \rfloor) + n & \text{if } n \geq 3 \end{cases}$$

**Prove:**  $\forall n \geq 1 : T(n) \leq n^2$  (hence  $T(n) = O(n^2)$ )

**I.** We need multiple base cases

$$n = 1 : T(1) \leq 1^2, 1 \leq 1 \quad n = 2 : T(2) \leq 2^2, 2 \leq 4$$

**II d.**  $\forall n > 2 : (P(1) \wedge \dots \wedge P(n-1)) \rightarrow P(n)$

*2 is largest base case, 1 in  $P(1)$  is smallest*

Let  $n > 2$  be arbitrary. Then assume for any  $k$  in range  $1 \leq k \leq n-1$  that:

$$T(k) \leq k^2$$

We must show that:

$$T(n) \leq n^2$$

**Note:**  $n > 2 \rightarrow n \geq 3$

$$\rightarrow \frac{n}{3} \geq 1$$

$$\rightarrow 1 \leq \lfloor \frac{n}{3} \rfloor < n$$

So by inductive hypothesis:  $T(\lfloor \frac{n}{3} \rfloor) \leq \lfloor \frac{n}{3} \rfloor^2$

So:

$$T(n) = T(\lfloor \frac{n}{3} \rfloor) + n$$

$$\leq 4 * \lfloor \frac{4}{3} \rfloor^2 + n \text{ [by the inductive hypothesis with } k = \lfloor \frac{4}{3} \rfloor]$$

$$\leq 4 * (\frac{n}{3})^2 + n$$

$$= \frac{4}{9}n^2 + n$$

... algebra...

$$\leq n^2$$

Result follows by 2nd PMI

**Note:**

n	$\lfloor \frac{n}{3} \rfloor$
1	—
2	—
3	1
4	1
5	1
6	2
7	2
8	2
9	3
10	3
11	3
12	4
13	4

## Graphs

**Def:** A graph in a ? of set

Set:

$$G = (V, E)$$

where

$V \neq \emptyset$  : vertex set

$$E \subseteq V^{(2)}$$

2-element subsets at  $V$  unordered pairs