CSE 101 – Nov 11, 2019 (Week 7) (Veterans Day Webcasts)

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CMPS 101 Summer 2019 Lecture 14

Digraph Analog of Handshake lemma

$$\sum_{x \in V} id(x) = \sum_{x \in V} od(x) = |E|$$

where id is in degrees and od is out degrees

Similar definitions of directed

- walk
- trail
- path
- trivial (walk, trail, path)
- cycle
- subgraph

Two kinds of connectedness

Weak connectivity: Underlying multi-graph is connected

Strong connectivity:

Definition We say y is **reachable** from $x \iff G$ contains a directed $x \breve{} y$ path (walk, trail)

Definition A digraph G is called **strongly connected** iff for all $x, y \in V(G)$, y is reachable from x (and x from y)

More generally, $S \subseteq V(G)$ is called **strongly connected** iff $\forall x, y\ inS : y$ is reachable from x (and x from y)

Example *box graph*

Definition A subset $S \subseteq V(G)$ is called a **strongly connected component** (SCC) iff

1. S is strongly connected and
2. S is maximal with respect to **(1)**

CMPS 101 Summer 2019 Lecture 15

Definition

Given $x, y \in V(G)$

$$S(x,y) = \begin{cases} \text{min length of an } x \breve{} y \text{ path if } y \text{ is reachable from } x \\ \infty \text{ otherwise} \end{cases}$$

SSSP

Given a source vertex $S \in V(G)$

1. For each $x \in V(G)$, determine $S(s,x)$
2. For each $x \in V(G)$ with $S(s,x) < \infty$ determine a shortest $s \breve{} x$ path in G

Breadth First Search (BFS)

Solves SSSP.

Require: $V(G) = \{x_1, x_2, ..., x_n\}$

Each vertex x has attributes

- $color(x)$
  - **White**: Undiscovered
  - **Gray**: Discovered, but unfinished
  - **Black**: Finished
- $d(x)$: Distance estimate from source s. When complete $d(x) = d(s,x)$
- $p(x)$: Predecessor of x along a shortest $s \breve{} x$ path encoded shortest $s \breve{} x$ paths

Called BFS because it discovers all vertices at distance k before it discovers any vertices at distance $k+1$

Example

Source $s = 3$

Starting table

adj	color	d	p
1: 2 3	w	∞	n
2: 1 4 5 6	w	∞	n
3: 1 4	g	0	n
4: 2 3 5	w	∞	n
5: 2 4 6	w	∞	n
6: 2 5	w	∞	n

Definition

Predecessor-Subgraph

$V_p = \{x \in V(G) | P[x] \neq \text{ nil } \cup \{s\}\}$

$E_p = \{(P[x], x) | P[x] \neq \text{ nil }\}$

PrintPath(G, s, x)

Precondition: BFS(G, s) was run

1. if $x == s$
    2. print(s)
2. else if $P[x] ==$ nil
    4. print("x not reachable from s")
3. else
    6. PrintPath$(G, s, P[x])$
    7. print(x)