CSE 101 – Oct 21, 2019 (Week 4)

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Notes

- Take full period
- Alternate seating, directed seating
- Review problems and selected answers
- Sterling's formula **not** provided

PA2

- Zero matrix is an array of empty lists (not NULL)
- First write transpose before product
- Dot product helper function
- Never store 0s in values
- Product is n^2
- PA2 extended 2 days

Helper function dot()

Input lists A, B

A: $(10, 1), (30, 2), (50, -1), (60, 3)$

B: $(20, 2), (40, -1), (50, 5), (70, 4), (80, 5)$

Sum: $0 + (-5)$

Helps with product()

Change Entry

changeEntry(M, i, j, x)

Case 1: $M_{ij} = 0$, $x = 0$

Do nothing

Case 2: $M_{ij} \neq 0$, $x = 0$

Delete an entry object

Case 2: $M_{ij} = 0$, $x \neq 0$

Insert a new entry

Case 4: $M_{ij} \neq 0$, $x \neq 0$

Override existing entry

Inductive Proof Bullet List

1. Clearly stated base case(s)
2. - Introduce "arbitrary" inductive variable
   - State explicit induction hypothesis (weak vs strong)
   - State explicit induction conclusion
   - State point(s) at which inductive hypothesis is used

Example

Define $T(n)$ by

$T(n) = \begin{cases} 1 & \text{if } 1 \leq n \leq 2 \\ 4T(\lfloor \frac{n}{3} \rfloor) + n & \text{if } n \geq 3 \end{cases}$

Prove: $\forall n \geq 1 : T(n) \leq n^2$ (hence $T(n) = O(n^2)$)

I. We need multiple base cases

$n = 1 : T(1) \leq 1^2$, $1 \leq 1$ $n = 2 : T(2) \leq 2^2$, $2 \leq 4$

IId. $\forall n > 2 : (P(1) \wedge ... \wedge P(n-1)) \rightarrow P(n)$

2 *is largest base case,* 1 *in* $P(1)$ *is smallest*

Let n > 2 be arbitrary. Then assume for any k in range $1 \leq k \leq n-1$ that:

$$T(k) \leq k^2$$

We must show that:

$$T(n) \leq n^2$$

Note: $n > 2 \rightarrow n \geq 3$

$\rightarrow \frac{n}{3} \geq 1$

$\rightarrow 1 \leq \lfloor \frac{n}{3} \rfloor < n$

So by inductive hypothesis: $T(\lfloor \frac{n}{3} \rfloor) \leq \lfloor \frac{n}{3} \rfloor^2$

So:

$T(n) = T(\lfloor \frac{4}{3} \rfloor) + n$

$\leq 4 * \lfloor \frac{4}{3} \rfloor^2 + n$ [by the inductive hypothesis with $k = \lfloor \frac{4}{3} \rfloor$]

$\leq 4 * (\frac{n}{3})^2 + n$

$= \frac{4}{9} n^2 + n$

...algebra...

$\leq n^2$

Result follows by 2nd PMI

Note:

n	$\lfloor \frac{n}{3} \rfloor$
1	–
2	–
3	1
4	1
5	1
6	2
7	2
8	2
9	3
10	3
11	3
12	4
13	4

Graphs

Def: A graph in a ? of set

Set:

$$G = (V, E)$$

where

$V \neq \emptyset$: vertex set

$E \subseteq V^{(2)}$

2-element subsets at V unordered pairs