CSE 101 – Nov 15, 2019 (Week 7)

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DFS

$n = |V|$

$m = |E|$

Item	Cost
Runtime	$\Theta(n)$
Main loop (5-7) except Visit()	$\Theta(n)$
Cost of loop (4-6) apart from cost of Visit()	total length of adj list $= \begin{cases} m - \text{dir} \\ 2m - \text{undir} \end{cases} = \Theta(m)$
Number calls to Visit()	$\Theta(n)$

Total cost $\Theta(n + m)$

Parenthesis string

From example of DFS

Time	String
1	(
2	(
3	(
4	)
...	...

Parenthesis theorem

Let $x, y\ in V(G)$ and suppose $d[x] < d[y]$. Then exactly one of the following holds:

1. $d[x] < f[x] < d[y] < f[y]$

   (( ))

2. $d[x] < d[y] < f[y] < f[x]$

   (( ))

Remark

- (2) is equivalent to saying
  - y is discovered when x is gray
  - y is a descendent of x in some DFS tree
- (1) is equivalent to saying
  - y is **not** a descendent of x
  - cousins in same tree
  - different trees

White path theorem

Let $x, y \in V(G)$ and run DFS(G). Then y is a descendent of $x \iff$ at the the time $d[x]$, G contains an $x \breve{} y$ path consisting entirely of white vertices.

Edge classification

- Tree edges: belong to G_p
- Back edges: join descendent to ancestor
- Forward edges: join ancestor to descendent
- Cross edges
  - join cousin to cousin
  - tree to tree