CSE 101 – Oct 7, 2019 (Week 2)

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PA 1 Questions

Recall $f(n) = \Omega(g(n)) \iff \exists c, n_0$ s.t. $\forall n \geq n_0$

$$0 \geq c * g(n) \geq f(n)$$

holds

Diagram

If $f(n)$, $g(n)$ are asymptotically positive, then we have a picture

$$0 < c \geq \frac{f(n)}{g(n)}$$

Diagram

Fact: $an^2 + bn + c = \Omega(dn + e)$

Note: Both sides must be positive

Theorem

$f(n) = O(g(n)) \iff g(n) = \Omega(f(n))$

Proof $(\Rightarrow)$

Assume $f(n) = O(g(n))$. Then there exist positive c_1, n_1 such that for all $n \geq n_1$:

(1)

$$0 \leq f(n) \leq c_1 g(n)$$

is true. We must show that $g(n) = \Omega(f(n))$, i.e. that there exist positive c_2, n_2 such that for all $n \geq n_2$:

(2)

$$0 \leq c_2 f(n) \leq g(n)$$

is true. Let $c_2 = \frac{1}{c_1}$ and $n_2 = n_1$. Then, if $n \geq n_2$, we have $n \geq n_1$ and therefore (1)is true. Hence

(3)

$$0 \leq \frac{1}{c_1} f(n) \leq g(n)$$

Since $c_2 = \frac{1}{c_1}$, (3) gives us that (2) is true.

$$\therefore f(n) = O(g(n)) \Rightarrow g(n) = \Omega(f(n))$$

Exercise: Do $\Leftarrow$

Analogy

$f(n) = O(g(n))$ is analogous to $x \leq y$

$f(n) = \Omega(g(n))$ is analogous to $x \geq y$

$f(n) = \Theta(g(n))$ is analogous to $x = y$

$f(n) = o(g(n))$ is analogous to $x < y$

$f(n) = \omega(g(n))$ is analogous to $x > y$ ### Definition

$\Theta(g(n)) = O(g(n)) \bigcap \Omega(g(n))$

Equivalently:

$f(n) = \Theta(g(n)) \iff$ there exist positive c_1, c_2, n_0 such that for all $n \geq n_0$

$$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$$

We say: $g(n)$ is a tight asymptotic bound on $f(n)$

If $f(n), g(n)$ are asymptotically positive, then

$$0 \leq c_1 \leq \frac{f(n)}{g(n)} \leq c_2$$

Exercise: Prove $f(n) = \Theta(g(n)) \iff g(n) = \Theta(f(n))$

Exercise: Let $c > 0$ and $g(n)$ be a function. Prove that

$$c * g(n) = O(g(n))$$

$$c * g(n) = \Omega(g(n))$$

$$c * g(n) = \Theta(g(n))$$

Example

$\sqrt{n+10} = \Theta(\sqrt{n})$

Proof

We must find positive c_1, c_2, n_0 such that for all $n \geq n_0$

$$0 \leq c_1\sqrt{n} \leq \sqrt{n+10} \leq c_2\sqrt{n}$$