Chapter 25

Electric Potential

▲ *Processes occurring during thunderstorms cause large differences in electric potential between a thundercloud and the ground. The result of this potential difference is an electrical discharge that we call lightning, such as this display over Tucson, Arizona. (© Keith Kent/Photo Researchers, Inc.)*

The concept of potential energy was introduced in Chapter 8 in connection with such conservative forces as the gravitational force and the elastic force exerted by a spring. By using the law of conservation of energy, we were able to avoid working directly with forces when solving various problems in mechanics. The concept of potential energy is also of great value in the study of electricity. Because the electrostatic force is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy. This idea enables us to define a scalar quantity known as *electric potential.* Because the electric potential at any point in an electric field is a scalar quantity, we can use it to describe electrostatic phenomena more simply than if we were to rely only on the electric field and electric forces. The concept of electric potential is of great practical value in the operation of electric circuits and devices we will study in later chapters.

25.1 Potential Difference and Electric Potential

When a test charge q_0 is placed in an electric field $\mathbf{E}$ created by some source charge distribution, the electric force acting on the test charge is $q_0\mathbf{E}$. The force $q_0\mathbf{E}$ is conservative because the force between charges described by Coulomb's law is conservative. When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative of the work done by the external agent causing the displacement. This is analogous to the situation of lifting an object with mass in a gravitational field—the work done by the external agent is mgh and the work done by the gravitational force is $-mgh$.

When analyzing electric and magnetic fields, it is common practice to use the notation $d\mathbf{s}$ to represent an infinitesimal displacement vector that is oriented tangent to a path through space. This path may be straight or curved, and an integral performed along this path is called either a *path integral* or a *line integral* (the two terms are synonymous).

For an infinitesimal displacement $d\mathbf{s}$ of a charge, the work done by the electric field on the charge is $\mathbf{F} \cdot d\mathbf{s} = q_0\mathbf{E} \cdot d\mathbf{s}$. As this amount of work is done by the field, the potential energy of the charge–field system is changed by an amount $dU = -q_0\mathbf{E} \cdot d\mathbf{s}$. For a finite displacement of the charge from point A to point B, the change in potential energy of the system $\Delta U = U_B - U_A$ is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad (25.1)$$

Change in electric potential energy of a system

The integration is performed along the path that q_0 follows as it moves from A to B. Because the force $q_0\mathbf{E}$ is conservative, **this line integral does not depend on the path taken from A to B.**

For a given position of the test charge in the field, the charge–field system has a potential energy U relative to the configuration of the system that is defined as $U = 0$. Dividing the potential energy by the test charge gives a physical quantity that depends only on the source charge distribution. The potential energy per unit charge U/q_0 is

PITFALL PREVENTION

25.1 Potential and Potential Energy

The *potential is characteristic of the field only,* independent of a charged test particle that may be placed in the field. *Potential energy is characteristic of the charge–field system* due to an interaction between the field and a charged particle placed in the field.

independent of the value of q_0 and has a value at every point in an electric field. This quantity U/q_0 is called the **electric potential** (or simply the **potential**) V. Thus, the electric potential at any point in an electric field is

$$V = \frac{U}{q_0} \tag{25.2}$$

The fact that potential energy is a scalar quantity means that electric potential also is a scalar quantity.

As described by Equation 25.1, if the test charge is moved between two positions A and B in an electric field, the charge–field system experiences a change in potential energy. The **potential difference** $\Delta V = V_B - V_A$ between two points A and B in an electric field is defined as the change in potential energy of the system when a test charge is moved between the points divided by the test charge q_0:

Potential difference between two points

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{25.3}$$

Just as with potential energy, only *differences* in electric potential are meaningful. To avoid having to work with potential differences, however, we often take the value of the electric potential to be zero at some convenient point in an electric field.

Potential difference should not be confused with difference in potential energy. The potential difference between A and B depends only on the source charge distribution (consider points A and B *without* the presence of the test charge), while the difference in potential energy exists only if a test charge is moved between the points. **Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.**

If an external agent moves a test charge from A to B without changing the kinetic energy of the test charge, the agent performs work which changes the potential energy of the system: $W = \Delta U$. The test charge q_0 is used as a mental device to define the electric potential. Imagine an arbitrary charge q located in an electric field. From Equation 25.3, the work done by an external agent in moving a charge q through an electric field at constant velocity is

$$W = q\,\Delta V \tag{25.4}$$

PITFALL PREVENTION

25.2 Voltage

A variety of phrases are used to describe the potential difference between two points, the most common being **voltage,** arising from the unit for potential. A voltage *applied* to a device, such as a television, or *across* a device is the same as the potential difference across the device. If we say that the voltage applied to a lightbulb is 120 volts, we mean that the potential difference between the two electrical contacts on the lightbulb is 120 volts.

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a **volt** (V):

$$1\ \text{V} \equiv 1\ \frac{\text{J}}{\text{C}}$$

That is, 1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

Equation 25.3 shows that potential difference also has units of electric field times distance. From this, it follows that the SI unit of electric field (N/C) can also be expressed in volts per meter:

$$1\ \frac{\text{N}}{\text{C}} = 1\ \frac{\text{V}}{\text{m}}$$

Therefore, **we can interpret the electric field as a measure of the rate of change with position of the electric potential.**

A unit of energy commonly used in atomic and nuclear physics is the **electron volt** (eV), which is defined as **the energy a charge–field system gains or loses when a charge of magnitude *e* (that is, an electron or a proton) is moved through a potential difference of 1 V.** Because $1\ \text{V} = 1\ \text{J/C}$ and because the fundamental charge is 1.60×10^{-19} C, the electron volt is related to the joule as follows:

The electron volt

$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{C}\cdot\text{V} = 1.60 \times 10^{-19}\ \text{J} \tag{25.5}$$

For instance, an electron in the beam of a typical television picture tube may have a speed of 3.0×10^7 m/s. This corresponds to a kinetic energy of 4.1×10^{-16} J, which is equivalent to 2.6×10^3 eV. Such an electron has to be accelerated from rest through a potential difference of 2.6 kV to reach this speed.

Quick Quiz 25.1 In Figure 25.1, two points A and B are located within a region in which there is an electric field. The potential difference $\Delta V = V_B - V_A$ is (a) positive (b) negative (c) zero.

Quick Quiz 25.2 In Figure 25.1, a negative charge is placed at A and then moved to B. The change in potential energy of the charge–field system for this process is (a) positive (b) negative (c) zero.

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25.3 The Electron Volt

The electron volt is a unit of *energy*, NOT of potential. The energy of any system may be expressed in eV, but this unit is most convenient for describing the emission and absorption of visible light from atoms. Energies of nuclear processes are often expressed in MeV.

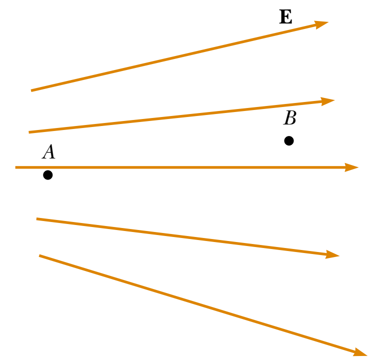


Figure 25.1 (Quick Quiz 25.1) Two points in an electric field.

25.2 Potential Differences in a Uniform Electric Field

Equations 25.1 and 25.3 hold in all electric fields, whether uniform or varying, but they can be simplified for a uniform field. First, consider a uniform electric field directed along the negative y axis, as shown in Figure 25.2a. Let us calculate the potential difference between two points A and B separated by a distance $|\mathbf{s}| = d$, where $\mathbf{s}$ is parallel to the field lines. Equation 25.3 gives

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B (E \cos 0°)\, ds = -\int_A^B E\, ds$$

Because E is constant, we can remove it from the integral sign; this gives

$$\Delta V = -E \int_A^B ds = -Ed \qquad (25.6)$$

Potential difference between two points in a uniform electric field

The negative sign indicates that the electric potential at point B is lower than at point A; that is, $V_B < V_A$. **Electric field lines always point in the direction of decreasing electric potential,** as shown in Figure 25.2a.

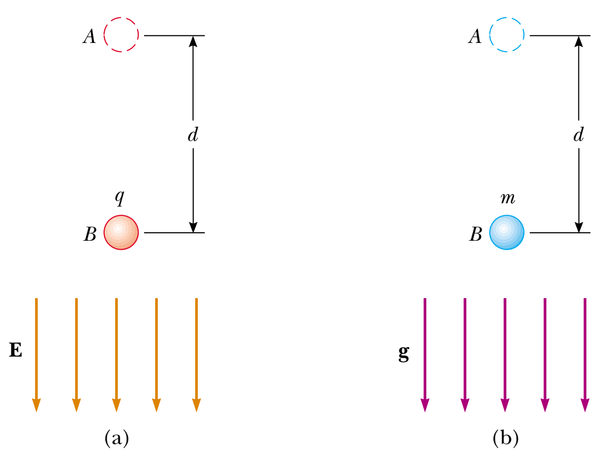


Figure 25.2 (a) When the electric field **E** is directed downward, point B is at a lower electric potential than point A. When a positive test charge moves from point A to point B, the charge–field system loses electric potential energy. (b) When an object of mass m moves downward in the direction of the gravitational field **g**, the object–field system loses gravitational potential energy.

Now suppose that a test charge q_0 moves from A to B. We can calculate the change in the potential energy of the charge–field system from Equations 25.3 and 25.6:

$$\Delta U = q_0 \, \Delta V = -q_0 E d \qquad (25.7)$$

From this result, we see that if q_0 is positive, then ΔU is negative. We conclude that **a system consisting of a positive charge and an electric field loses electric potential energy when the charge moves in the direction of the field.** This means that an electric field does work on a positive charge when the charge moves in the direction of the electric field. (This is analogous to the work done by the gravitational field on a falling object, as shown in Figure 25.2b.) If a positive test charge is released from rest in this electric field, it experiences an electric force $q_0\mathbf{E}$ in the direction of $\mathbf{E}$ (downward in Fig. 25.2a). Therefore, it accelerates downward, gaining kinetic energy. **As the charged particle gains kinetic energy, the charge–field system loses an equal amount of potential energy.** This should not be surprising—it is simply conservation of energy in an isolated system as introduced in Chapter 8.

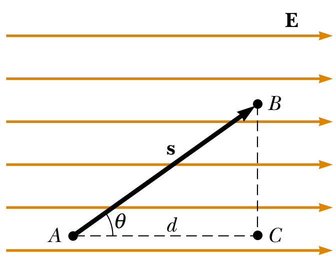


Figure 25.3 A uniform electric field directed along the positive x axis. Point B is at a lower electric potential than point A. Points B and C are at the *same* electric potential.

If q_0 is negative, then ΔU in Equation 25.7 is positive and the situation is reversed: **A system consisting of a negative charge and an electric field gains electric potential energy when the charge moves in the direction of the field.** If a negative charge is released from rest in an electric field, it accelerates in a direction opposite the direction of the field. In order for the negative charge to move in the direction of the field, an external agent must apply a force and do positive work on the charge.

Now consider the more general case of a charged particle that moves between A and B in a uniform electric field such that the vector $\mathbf{s}$ is not parallel to the field lines, as shown in Figure 25.3. In this case, Equation 25.3 gives

$$\Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\mathbf{E}\cdot\int_A^B d\mathbf{s} = -\mathbf{E}\cdot\mathbf{s} \qquad (25.8)$$

where again we are able to remove $\mathbf{E}$ from the integral because it is constant. The change in potential energy of the charge–field system is

Change in potential energy when a charged particle is moved in a uniform electric field

$$\Delta U = q_0 \, \Delta V = -q_0 \, \mathbf{E}\cdot\mathbf{s} \qquad (25.9)$$

Finally, we conclude from Equation 25.8 that all points in a plane perpendicular to a uniform electric field are at the same electric potential. We can see this in Figure 25.3, where the potential difference $V_B - V_A$ is equal to the potential difference $V_C - V_A$. (Prove this to yourself by working out the dot product $\mathbf{E}\cdot\mathbf{s}$ for $\mathbf{s}_{A\rightarrow B}$, where the angle θ between $\mathbf{E}$ and $\mathbf{s}$ is arbitrary as shown in Figure 25.3, and the dot product for $\mathbf{s}_{A\rightarrow C}$, where $\theta = 0$.) Therefore, $V_B = V_C$. **The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.**

The equipotential surfaces of a uniform electric field consist of a family of parallel planes that are all perpendicular to the field. Equipotential surfaces for fields with other symmetries are described in later sections.

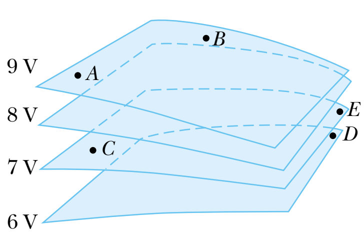


Figure 25.4 (Quick Quiz 25.3) Four equipotential surfaces.

Quick Quiz 25.3 The labeled points in Figure 25.4 are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from A to B; from B to C; from C to D; from D to E.

Quick Quiz 25.4 For the equipotential surfaces in Figure 25.4, what is the *approximate* direction of the electric field? (a) Out of the page (b) Into the page (c) Toward the right edge of the page (d) Toward the left edge of the page (e) Toward the top of the page (f) Toward the bottom of the page.

Example 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery produces a specified potential difference ΔV between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in Figure 25.5. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and if we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.

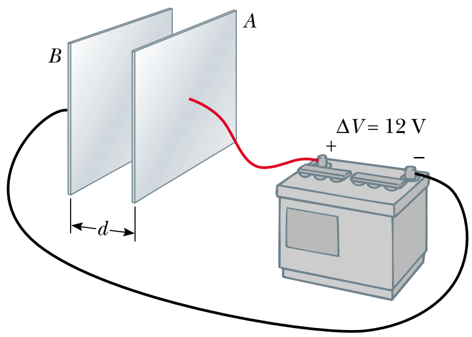


Figure 25.5 (Example 25.1) A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation d.

Solution The electric field is directed from the positive plate (A) to the negative one (B), and the positive plate is at a higher electric potential than the negative plate is. The potential difference between the plates must equal the potential difference between the battery terminals. We can understand this by noting that all points on a conductor in equilibrium are at the same electric potential[1]; no potential difference exists between a terminal and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is, from Equation 25.6,

$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = \boxed{4.0 \times 10^3\text{ V/m}}$$

The configuration of plates in Figure 25.5 is called a *parallel-plate capacitor,* and is examined in greater detail in Chapter 26.

Example 25.2 Motion of a Proton in a Uniform Electric Field

Interactive

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m (Fig. 25.6). The proton undergoes a displacement of 0.50 m in the direction of $\mathbf{E}$.

(A) Find the change in electric potential between points A and B.

Solution Because the positively charged proton moves in the direction of the field, we expect it to move to a position of lower electric potential. From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4\text{ V/m})(0.50\text{ m}) = \boxed{-4.0 \times 10^4\text{ V}}$$

(B) Find the change in potential energy of the proton–field system for this displacement.

Solution Using Equation 25.3,

$$\begin{aligned}\Delta U &= q_0\,\Delta V = e\,\Delta V\\ &= (1.6 \times 10^{-19}\text{ C})(-4.0 \times 10^4\text{ V})\\ &= \boxed{-6.4 \times 10^{-15}\text{ J}}\end{aligned}$$

The negative sign means the potential energy of the system decreases as the proton moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time the system loses electric potential energy.

(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

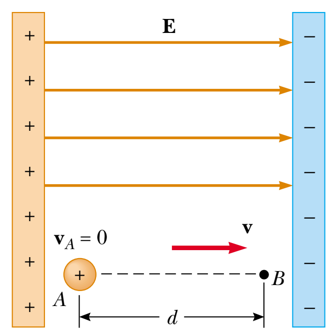


Figure 25.6 (Example 25.2) A proton accelerates from A to B in the direction of the electric field.

Solution The charge–field system is isolated, so the mechanical energy of the system is conserved:

$$\Delta K + \Delta U = 0$$

$$(\tfrac{1}{2}mv^2 - 0) + e\,\Delta V = 0$$

$$\begin{aligned}v &= \sqrt{\frac{-(2e\,\Delta V)}{m}}\\ &= \sqrt{\frac{-2(1.6 \times 10^{-19}\text{ C})(-4.0 \times 10^4\text{ V})}{1.67 \times 10^{-27}\text{ kg}}}\\ &= \boxed{2.8 \times 10^6\text{ m/s}}\end{aligned}$$

What If? **What if the situation is exactly the same as that shown in Figure 25.6, but no proton is present? Could both parts (A) and (B) of this example still be answered?**

[1] The electric field vanishes within a conductor in electrostatic equilibrium; thus, the path integral between any two points in the conductor must be zero. A more complete discussion of this point is given in Section 25.6.

Answer Part (A) of the example would remain exactly the same because the potential difference between points A and B is established by the source charges in the parallel plates. The potential difference does not depend on the presence of the proton, which plays the role of a test charge. Part (B) of the example would be meaningless if the proton is not present. A change in potential energy is related to a change in the charge–field system. In the absence of the proton, the system of the electric field alone does not change.

At the Interactive Worked Example link at **http://www.pse6.com,** *you can predict and observe the speed of the proton as it arrives at the negative plate for random values of the electric field.*

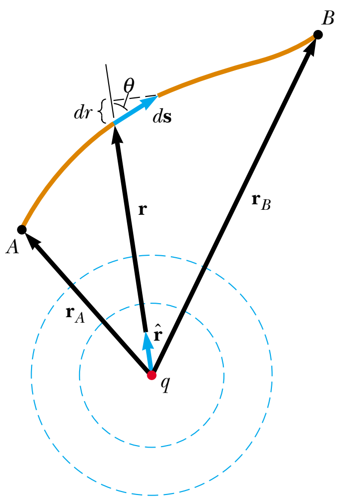


Figure 25.7 The potential difference between points A and B due to a point charge q depends *only* on the initial and final radial coordinates r_A and r_B. The two dashed circles represent intersections of spherical equipotential surfaces with the page.

25.3 Electric Potential and Potential Energy Due to Point Charges

In Section 23.4 we discussed the fact that an isolated positive point charge q produces an electric field that is directed radially outward from the charge. To find the electric potential at a point located a distance r from the charge, we begin with the general expression for potential difference:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

where A and B are the two arbitrary points shown in Figure 25.7. At any point in space, the electric field due to the point charge is $\mathbf{E} = k_e q\hat{\mathbf{r}}/r^2$ (Eq. 23.9), where $\hat{\mathbf{r}}$ is a unit vector directed from the charge toward the point. The quantity $\mathbf{E} \cdot d\mathbf{s}$ can be expressed as

$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2}\, \hat{\mathbf{r}} \cdot d\mathbf{s}$$

Because the magnitude of $\hat{\mathbf{r}}$ is 1, the dot product $\hat{\mathbf{r}} \cdot d\mathbf{s} = ds \cos\theta$, where θ is the angle between $\hat{\mathbf{r}}$ and $d\mathbf{s}$. Furthermore, $ds \cos\theta$ is the projection of $d\mathbf{s}$ onto $\mathbf{r}$; thus, $ds \cos\theta = dr$. That is, any displacement $d\mathbf{s}$ along the path from point A to point B produces a change dr in the magnitude of $\mathbf{r}$, the position vector of the point relative to the charge creating the field. Making these substitutions, we find that $\mathbf{E} \cdot d\mathbf{s} = (k_e q/r^2)\,dr$; hence, the expression for the potential difference becomes

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r}\Bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad (25.10)$$

This equation shows us that the integral of $\mathbf{E} \cdot d\mathbf{s}$ is *independent* of the path between points A and B. Multiplying by a charge q_0 that moves between points A and B, we see that the integral of $q_0\mathbf{E} \cdot d\mathbf{s}$ is also independent of path. This latter integral is the work done by the electric force, which tells us that the electric force is conservative (see Section 8.3). We define a field that is related to a conservative force as a **conservative field.** Thus, Equation 25.10 tells us that the electric field of a fixed point charge is conservative. Furthermore, Equation 25.10 expresses the important result that the potential difference between any two points A and B in a field created by a point charge depends only on the radial coordinates r_A and r_B. It is customary to choose the reference of electric potential for a point charge to be $V = 0$ at $r_A = \infty$. With this reference choice, the electric potential created by a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r} \qquad (25.11)$$

PITFALL PREVENTION

25.4 Similar Equation Warning

Do not confuse Equation 25.11 for the electric potential of a point charge with Equation 23.9 for the electric field of a point charge. Potential is proportional to $1/r$, while the field is proportional to $1/r^2$. The effect of a charge on the space surrounding it can be described in two ways. The charge sets up a vector electric field $\mathbf{E}$, which is related to the force experienced by a test charge placed in the field. It also sets up a scalar potential V, which is related to the potential energy of the two-charge system when a test charge is placed in the field.

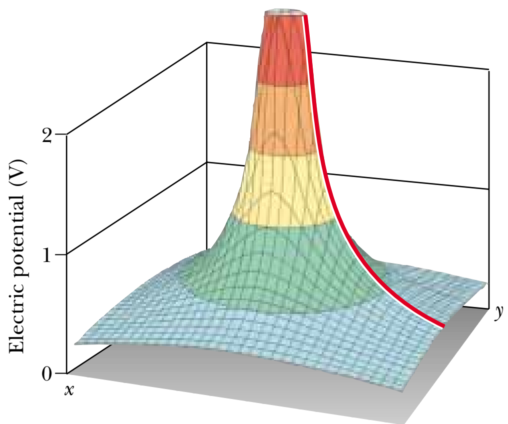


Figure 25.8 The electric potential in the plane around a single positive charge is plotted on the vertical axis. (The electric potential function for a negative charge would look like a hole instead of a hill.) The red line shows the $1/r$ nature of the electric potential, as given by Equation 25.11.

Figure 25.8 shows a plot of the electric potential on the vertical axis for a positive charge located in the xy plane. Consider the following analogy to gravitational potential: imagine trying to roll a marble toward the top of a hill shaped like the surface in Figure 25.8. Pushing the marble up the hill is analogous to pushing one positively charged object toward another positively charged object. Similarly, the electric potential graph of the region surrounding a negative charge is analogous to a "hole" with respect to any approaching positively charged objects. A charged object must be infinitely distant from another charge before the surface in Figure 25.8 is "flat" and has an electric potential of zero.

We obtain the electric potential resulting from two or more point charges by applying the superposition principle. That is, the total electric potential at some point P due to several point charges is the sum of the potentials due to the individual charges. For a group of point charges, we can write the total electric potential at P in the form

Electric potential due to several point charges

$$V = k_e \sum_i \frac{q_i}{r_i} \qquad (25.12)$$

where the potential is again taken to be zero at infinity and r_i is the distance from the point P to the charge q_i. Note that the sum in Equation 25.12 is an algebraic sum of scalars rather than a vector sum (which we use to calculate the electric field of a group of charges). Thus, it is often much easier to evaluate V than to evaluate $\mathbf{E}$. The electric potential around a dipole is illustrated in Figure 25.9. Notice the steep slope of the potential between the charges, representing a region of strong electric field.

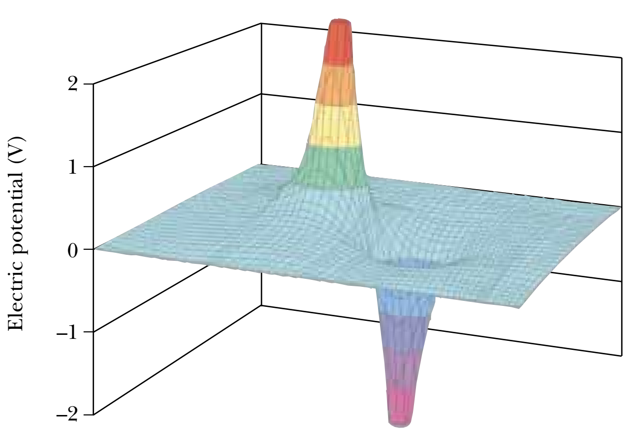
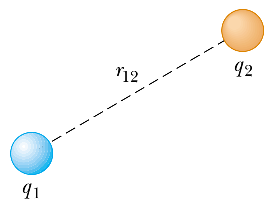
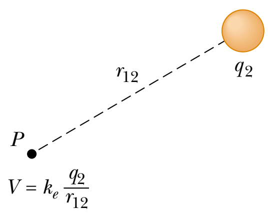


Figure 25.9 The electric potential in the plane containing a dipole.



(a)



(b)

Active Figure 25.10 (a) If two point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by $k_e q_1 q_2/r_{12}$. (b) If charge q_1 is removed, a potential $k_e q_2/r_{12}$ exists at point P due to charge q_2.

At the Active Figures link at http://www.pse6.com, you can move charge q_1 or point P and see the result on the electric potential energy of the system for part (a) and the electric potential due to charge q_2 for part (b).

We now consider the potential energy of a system of two charged particles. If V_2 is the electric potential at a point P due to charge q_2, then the work an external agent must do to bring a second charge q_1 from infinity to P without acceleration is $q_1 V_2$. This work represents a transfer of energy into the system and the energy appears in the system as potential energy U when the particles are separated by a distance r_{12} (Fig. 25.10a). Therefore, we can express the potential energy of the system as[2]

$$U = k_e \frac{q_1 q_2}{r_{12}} \tag{25.13}$$

Note that if the charges are of the same sign, U is positive. This is consistent with the fact that positive work must be done by an external agent on the system to bring the two charges near one another (because charges of the same sign repel). If the charges are of opposite sign, U is negative; this means that negative work is done by an external agent against the attractive force between the charges of opposite sign as they are brought near each other—a force must be applied opposite to the displacement to prevent q_1 from accelerating toward q_2.

In Figure 25.10b, we have removed the charge q_1. At the position that this charge previously occupied, point P, we can use Equations 25.2 and 25.13 to define a potential due to charge q_2 as $V = U/q_1 = k_e q_2/r_{12}$. This expression is consistent with Equation 25.11.

If the system consists of more than two charged particles, we can obtain the total potential energy by calculating U for every pair of charges and summing the terms algebraically. As an example, the total potential energy of the system of three charges shown in Figure 25.11 is

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \tag{25.14}$$

Physically, we can interpret this as follows: imagine that q_1 is fixed at the position shown in Figure 25.11 but that q_2 and q_3 are at infinity. The work an external agent must do to bring q_2 from infinity to its position near q_1 is $k_e q_1 q_2/r_{12}$, which is the first term in Equation 25.14. The last two terms represent the work required to bring q_3 from infinity to its position near q_1 and q_2. (The result is independent of the order in which the charges are transported.)

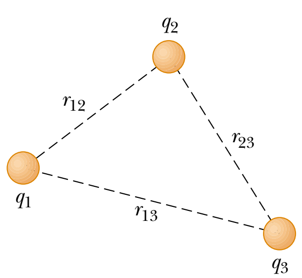


Figure 25.11 Three point charges are fixed at the positions shown. The potential energy of this system of charges is given by Equation 25.14.

PITFALL PREVENTION

25.5 Which Work?

There is a difference between work done *by one member of a system on another member* and work done *on a system by an external agent.* In the present discussion, we are considering the group of charges to be the system and an external agent is doing work on the system to move the charges from an infinite separation to a small separation.

Quick Quiz 25.5 A spherical balloon contains a positively charged object at its center. As the balloon is inflated to a greater volume while the charged object remains at the center, does the electric potential at the surface of the balloon (a) increase, (b) decrease, or (c) remain the same? Does the electric flux through the surface of the balloon (d) increase, (e) decrease, or (f) remain the same?

Quick Quiz 25.6 In Figure 25.10a, take q_1 to be a negative source charge and q_2 to be the test charge. If q_2 is initially positive and is changed to a charge of the same magnitude but negative, the potential at the position of q_2 due to q_1 (a) increases (b) decreases (c) remains the same.

Quick Quiz 25.7 Consider the situation in Quick Quiz 25.6 again. When q_2 is changed from positive to negative, the potential energy of the two-charge system (a) increases (b) decreases (c) remains the same.

[2] The expression for the electric potential energy of a system made up of two point charges, Equation 25.13, is of the *same* form as the equation for the gravitational potential energy of a system made up of two point masses, $-Gm_1m_2/r$ (see Chapter 13). The similarity is not surprising in view of the fact that both expressions are derived from an inverse-square force law.

Example 25.3 The Electric Potential Due to Two Point Charges

Interactive

A charge $q_1 = 2.00\ \mu\text{C}$ is located at the origin, and a charge $q_2 = -6.00\ \mu\text{C}$ is located at (0, 3.00) m, as shown in Figure 25.12a.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.

Solution For two charges, the sum in Equation 25.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{2.00 \times 10^{-6}\ \text{C}}{4.00\ \text{m}} - \frac{6.00 \times 10^{-6}\ \text{C}}{5.00\ \text{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \text{V}}$$

(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu\text{C}$ as the latter charge moves from infinity to point P (Fig. 25.12b).

Solution When the charge q_3 is at infinity, let us define $U_i = 0$ for the system, and when the charge is at P, $U_f = q_3 V_P$; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \text{C})(-6.29 \times 10^3\ \text{V})$$

$$= \boxed{-1.89 \times 10^{-2}\ \text{J}}$$

Therefore, because the potential energy of the system has decreased, positive work would have to be done by an external agent to remove the charge from point P back to infinity.

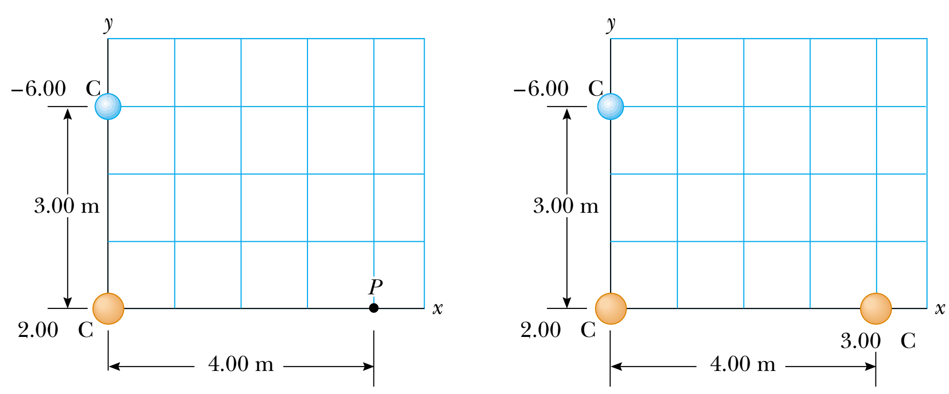
What If? **You are working through this example with a classmate and she says, "Wait a minute! In part (B), we ignored the potential energy associated with the pair of charges q_1 and q_2!" How would you respond?**

Answer Given the statement of the problem, it is not necessary to include this potential energy, because part (B) asks for the *change* in potential energy of the system as q_3 is brought in from infinity. Because the configuration of charges q_1 and q_2 does not change in the process, there is no ΔU associated with these charges. However, if part (B) had asked to find the change in potential energy when *all three* charges start out infinitely far apart and are then brought to the positions in Figure 25.12b, we would need to calculate the change as follows, using Equation 25.14:

$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{(2.00 \times 10^{-6}\ \text{C})(-6.00 \times 10^{-6}\ \text{C})}{3.00\ \text{m}} + \frac{(2.00 \times 10^{-6}\ \text{C})(3.00 \times 10^{-6}\ \text{C})}{4.00\ \text{m}} + \frac{(3.00 \times 10^{-6}\ \text{C})(-6.00 \times 10^{-6}\ \text{C})}{5.00\ \text{m}}\right)$$

$$= -5.48 \times 10^{-2}\ \text{J}$$



(a) (b)

Figure 25.12 (Example 25.3) (a) The electric potential at P due to the two charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) A third charge $q_3 = 3.00\ \mu\text{C}$ is brought from infinity to a position near the other charges.

Explore the value of the electric potential at point P and the electric potential energy of the system in Figure 25.12b at the Interactive Worked Example link at **http://www.pse6.com.**

25.4 Obtaining the Value of the Electric Field from the Electric Potential

The electric field $\mathbf{E}$ and the electric potential V are related as shown in Equation 25.3. We now show how to calculate the value of the electric field if the electric potential is known in a certain region.

From Equation 25.3 we can express the potential difference dV between two points a distance ds apart as

$$dV = -\mathbf{E} \cdot d\mathbf{s} \tag{25.15}$$

If the electric field has only one component E_x, then $\mathbf{E} \cdot d\mathbf{s} = E_x\, dx$. Therefore, Equation 25.15 becomes $dV = -E_x\, dx$, or

$$E_x = -\frac{dV}{dx} \tag{25.16}$$

That is, the x component of the electric field is equal to the negative of the derivative of the electric potential with respect to x. Similar statements can be made about the y and z components. Equation 25.16 is the mathematical statement of the fact that the electric field is a measure of the rate of change with position of the electric potential, as mentioned in Section 25.1.

Experimentally, electric potential and position can be measured easily with a voltmeter (see Section 28.5) and a meter stick. Consequently, an electric field can be determined by measuring the electric potential at several positions in the field and making a graph of the results. According to Equation 25.16, the slope of a graph of V versus x at a given point provides the magnitude of the electric field at that point.

When a test charge undergoes a displacement $d\mathbf{s}$ along an equipotential surface, then $dV = 0$ because the potential is constant along an equipotential surface. From Equation 25.15, we see that $dV = -\mathbf{E} \cdot d\mathbf{s} = 0$; thus, $\mathbf{E}$ must be perpendicular to the displacement along the equipotential surface. This shows that the **equipotential surfaces must always be perpendicular to the electric field lines passing through them.**

As mentioned at the end of Section 25.2, the equipotential surfaces for a uniform electric field consist of a family of planes perpendicular to the field lines. Figure 25.13a shows some representative equipotential surfaces for this situation.

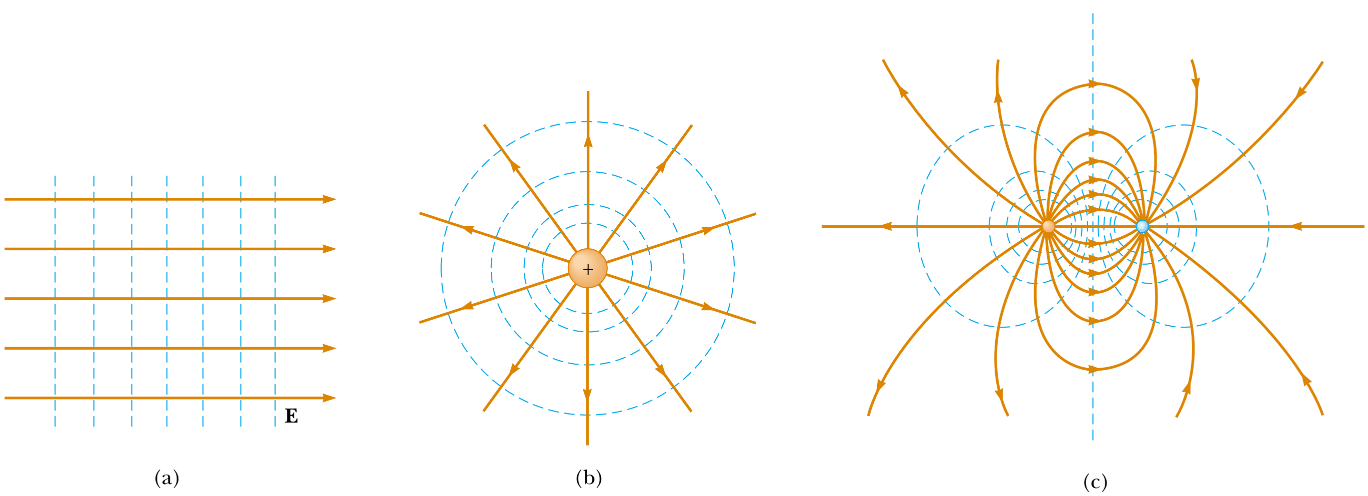


Figure 25.13 Equipotential surfaces (the dashed blue lines are intersections of these surfaces with the page) and electric field lines (red-brown lines) for (a) a uniform electric field produced by an infinite sheet of charge, (b) a point charge, and (c) an electric dipole. In all cases, the equipotential surfaces are *perpendicular* to the electric field lines at every point.

If the charge distribution creating an electric field has spherical symmetry such that the volume charge density depends only on the radial distance r, then the electric field is radial. In this case, $\mathbf{E} \cdot d\mathbf{s} = E_r\, dr$, and we can express dV in the form $dV = -E_r\, dr$. Therefore,

$$E_r = -\frac{dV}{dr} \tag{25.17}$$

For example, the electric potential of a point charge is $V = k_e q/r$. Because V is a function of r only, the potential function has spherical symmetry. Applying Equation 25.17, we find that the electric field due to the point charge is $E_r = k_e q/r^2$, a familiar result. Note that the potential changes only in the radial direction, not in any direction perpendicular to r. Thus, V (like E_r) is a function only of r. Again, this is consistent with the idea that **equipotential surfaces are perpendicular to field lines.** In this case the equipotential surfaces are a family of spheres concentric with the spherically symmetric charge distribution (Fig. 25.13b).

The equipotential surfaces for an electric dipole are sketched in Figure 25.13c.

In general, the electric potential is a function of all three spatial coordinates. If $V(r)$ is given in terms of the Cartesian coordinates, the electric field components E_x, E_y, and E_z can readily be found from $V(x, y, z)$ as the partial derivatives[3]

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z} \tag{25.18}$$

Finding the electric field from the potential

For example, if $V = 3x^2y + y^2 + yz$, then

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(3x^2y + y^2 + yz) = \frac{\partial}{\partial x}(3x^2y) = 3y\frac{d}{dx}(x^2) = 6xy$$

Quick Quiz 25.8 In a certain region of space, the electric potential is zero everywhere along the x axis. From this we can conclude that the x component of the electric field in this region is (a) zero (b) in the $+x$ direction (c) in the $-x$ direction.

Quick Quiz 25.9 In a certain region of space, the electric field is zero. From this we can conclude that the electric potential in this region is (a) zero (b) constant (c) positive (d) negative.

Example 25.4 The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$, as shown in Figure 25.14. The dipole is along the x axis and is centered at the origin.

(A) Calculate the electric potential at point P.

Solution For point P in Figure 25.14,

$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \boxed{\frac{2k_e qa}{x^2 - a^2}}$$

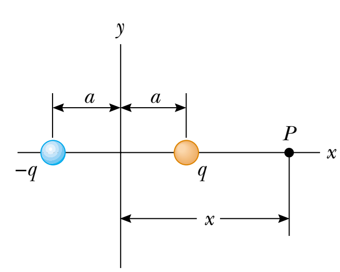


Figure 25.14 (Example 25.4) An electric dipole located on the x axis.

[3] In vector notation, $\mathbf{E}$ is often written in Cartesian coordinate systems as

$$\mathbf{E} = -\nabla V = -\left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)V$$

where ∇ is called the *gradient operator.*

(B) Calculate V and E_x at a point far from the dipole.

Solution If point P is far from the dipole, such that $x \gg a$, then a^2 can be neglected in the term $x^2 - a^2$ and V becomes

$$V \approx \boxed{\frac{2k_e qa}{x^2}} \qquad (x \gg a)$$

Using Equation 25.16 and this result, we can calculate the magnitude of the electric field at a point far from the dipole:

$$E_x = -\frac{dV}{dx} = \boxed{\frac{4k_e qa}{x^3}} \qquad (x \gg a)$$

(C) Calculate V and E_x if point P is located anywhere between the two charges.

Solution Using Equation 25.12,

$$V = k_e \sum \frac{q_i}{r_i} = k_e \left(\frac{q}{a-x} - \frac{q}{a+x} \right) = \boxed{\frac{2k_e qx}{a^2 - x^2}}$$

and using Equation 25.16,

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{2k_e qx}{a^2 - x^2} \right) = \boxed{-2k_e q \left(\frac{a^2 + x^2}{(a^2 - x^2)^2} \right)}$$

We can check these results by considering the situation at the center of the dipole, where $x = 0$, $V = 0$, and $E_x = -2k_e q/a^2$.

What If? **What if point P in Figure 25.14 happens to be located to the left of the negative charge? Would the answer to part (A) be the same?**

Answer The potential should be negative because a point to the left of the dipole is closer to the negative charge than to the positive charge. If we redo the calculation in part (A) with P on the left side of $-q$, we have

$$V = k_e \sum \frac{q_i}{r_i} = k_e \left(\frac{q}{x+a} - \frac{q}{x-a} \right) = \boxed{-\frac{2k_e qa}{x^2 - a^2}}$$

Thus, the potential has the same value but is negative for points on the left of the dipole.

25.5 Electric Potential Due to Continuous Charge Distributions

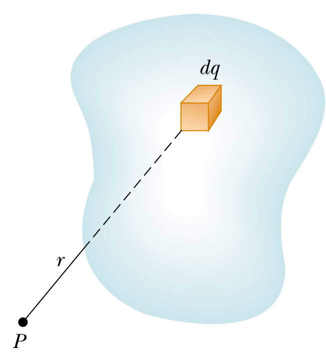


Figure 25.15 The electric potential at the point P due to a continuous charge distribution can be calculated by dividing the charge distribution into elements of charge dq and summing the electric potential contributions over all elements.

We can calculate the electric potential due to a continuous charge distribution in two ways. If the charge distribution is known, we can start with Equation 25.11 for the electric potential of a point charge. We then consider the potential due to a small charge element dq, treating this element as a point charge (Fig. 25.15). The electric potential dV at some point P due to the charge element dq is

$$dV = k_e \frac{dq}{r} \tag{25.19}$$

where r is the distance from the charge element to point P. To obtain the total potential at point P, we integrate Equation 25.19 to include contributions from all elements of the charge distribution. Because each element is, in general, a different distance from point P and because k_e is constant, we can express V as

Electric potential due to a continuous charge distribution

$$V = k_e \int \frac{dq}{r} \tag{25.20}$$

In effect, we have replaced the sum in Equation 25.12 with an integral. Note that this expression for V uses a particular reference: the electric potential is taken to be zero when point P is infinitely far from the charge distribution.

If the electric field is already known from other considerations, such as Gauss's law, we can calculate the electric potential due to a continuous charge distribution using Equation 25.3. If the charge distribution has sufficient symmetry, we first evaluate $\mathbf{E}$ at any point using Gauss's law and then substitute the value obtained into Equation 25.3 to determine the potential difference ΔV between any two points. We then choose the electric potential V to be zero at some convenient point.

PROBLEM-SOLVING HINTS

Calculating Electric Potential

- Remember that electric potential is a scalar quantity, so vector components do not exist. Therefore, when using the superposition principle to evaluate the electric potential at a point due to a system of point charges, simply take the algebraic sum of the potentials due to the various charges. However, you must keep track of signs. The potential is positive for positive charges and negative for negative charges.
- Just as with gravitational potential energy in mechanics, only *changes* in electric potential are significant; hence, the point where you choose the potential to be zero is arbitrary. When dealing with point charges or a charge distribution of finite size, we usually define $V = 0$ to be at a point infinitely far from the charges.
- You can evaluate the electric potential at some point P due to a continuous distribution of charge by dividing the charge distribution into infinitesimal elements of charge dq located at a distance r from P. Then, treat one charge element as a point charge, such that the potential at P due to the element is $dV = k_e dq/r$. Obtain the total potential at P by integrating dV over the entire charge distribution. In performing the integration for most problems, you must express dq and r in terms of a single variable. To simplify the integration, consider the geometry involved in the problem carefully. Study Examples 25.5 through 25.7 below for guidance.
- Another method that you can use to obtain the electric potential due to a finite continuous charge distribution is to start with the definition of potential difference given by Equation 25.3. If you know or can easily obtain $\mathbf{E}$ (from Gauss's law), then you can evaluate the line integral of $\mathbf{E} \cdot d\mathbf{s}$. This method is demonstrated in Example 25.8.

Example 25.5 Electric Potential Due to a Uniformly Charged Ring

(A) Find an expression for the electric potential at a point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.

Solution Figure 25.16, in which the ring is oriented so that its plane is perpendicular to the x axis and its center is at the origin, helps us to conceptualize this problem. Because the ring consists of a continuous distribution of charge rather than a set of discrete charges, we categorize this problem as one in which we need to use the integration technique represented by Equation 25.20. To analyze the problem, we take point P to be at a distance x from the center of the ring, as shown in Figure 25.16. The charge element dq is at a distance $\sqrt{x^2 + a^2}$ from point P. Hence, we can express V as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

Because each element dq is at the same distance from point P, we can bring $\sqrt{x^2 + a^2}$ in front of the integral sign, and V reduces to

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \boxed{\frac{k_e Q}{\sqrt{x^2 + a^2}}} \qquad (25.21)$$

The only variable in this expression for V is x. This is not surprising because our calculation is valid only for points along the x axis, where y and z are both zero.

(B) Find an expression for the magnitude of the electric field at point P.

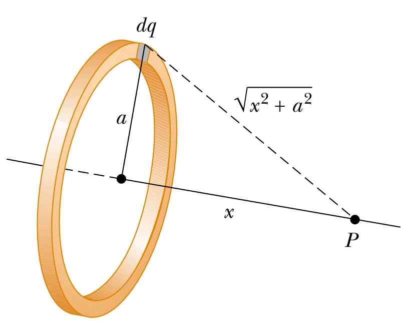


Figure 25.16 (Example 25.5) A uniformly charged ring of radius a lies in a plane perpendicular to the x axis. All elements dq of the ring are the same distance from a point P lying on the x axis.

Solution From symmetry, we see that along the x axis $\mathbf{E}$ can have only an x component. Therefore, we can use Equation 25.16:

$$E_x = -\frac{dV}{dx} = -k_eQ\frac{d}{dx}(x^2 + a^2)^{-1/2}$$
$$= -k_eQ(-\tfrac{1}{2})(x^2 + a^2)^{-3/2}(2x)$$

$$E_x = \boxed{\frac{k_eQx}{(x^2 + a^2)^{3/2}}} \qquad (25.22)$$

To finalize this problem, we see that this result for the electric field agrees with that obtained by direct integration (see Example 23.8). Note that $E_x = 0$ at $x = 0$ (the center of the ring). Could you have guessed this?

Example 25.6 Electric Potential Due to a Uniformly Charged Disk

A uniformly charged disk has radius a and surface charge density σ. Find

(A) the electric potential and

(B) the magnitude of the electric field along the perpendicular central axis of the disk.

Solution (A) Again, we choose the point P to be at a distance x from the center of the disk and take the plane of the disk to be perpendicular to the x axis. We can simplify the problem by dividing the disk into a series of charged rings of infinitesimal width dr. The electric potential due to each ring is given by Equation 25.21. Consider one such ring of radius r and width dr, as indicated in Figure 25.17. The surface area of the ring is $dA = 2\pi r\,dr$. From the definition of surface charge density (see Section 23.5), we know that the charge on the ring is $dq = \sigma\,dA = \sigma 2\pi r\,dr$. Hence, the potential at the point P due to this ring is

$$dV = \frac{k_e dq}{\sqrt{r^2 + x^2}} = \frac{k_e\sigma 2\pi r\,dr}{\sqrt{r^2 + x^2}}$$

where x is a constant and r is a variable. To find the *total* electric potential at P, we sum over all rings making up the disk. That is, we integrate dV from $r = 0$ to $r = a$:

$$V = \pi k_e\sigma\int_0^a \frac{2r\,dr}{\sqrt{r^2 + x^2}} = \pi k_e\sigma\int_0^a (r^2 + x^2)^{-1/2}\,2r\,dr$$

This integral is of the common form $\int u^n\,du$ and has the value $u^{n+1}/(n+1)$, where $n = -\frac{1}{2}$ and $u = r^2 + x^2$. This gives

$$V = \boxed{2\pi k_e\sigma[(x^2 + a^2)^{1/2} - x]} \qquad (25.23)$$

(B) As in Example 25.5, we can find the electric field at any axial point using Equation 25.16:

$$E_x = -\frac{dV}{dx} = \boxed{2\pi k_e\sigma\left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)} \qquad (25.24)$$

The calculation of V and $\mathbf{E}$ for an arbitrary point off the axis is more difficult to perform, and we do not treat this situation in this text.

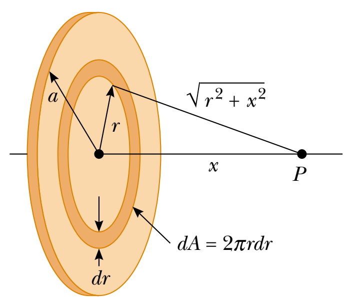


Figure 25.17 (Example 25.6) A uniformly charged disk of radius a lies in a plane perpendicular to the x axis. The calculation of the electric potential at any point P on the x axis is simplified by dividing the disk into many rings of radius r and width dr, with area $2\pi r\,dr$.

Example 25.7 Electric Potential Due to a Finite Line of Charge

A rod of length ℓ located along the x axis has a total charge Q and a uniform linear charge density $\lambda = Q/\ell$. Find the electric potential at a point P located on the y axis a distance a from the origin (Fig. 25.18).

Solution The length element dx has a charge $dq = \lambda\,dx$. Because this element is a distance $r = \sqrt{x^2 + a^2}$ from point P, we can express the potential at point P due to this element as

$$dV = k_e\frac{dq}{r} = k_e\frac{\lambda\,dx}{\sqrt{x^2 + a^2}}$$

To obtain the total potential at P, we integrate this expression over the limits $x = 0$ to $x = \ell$. Noting that k_e and λ are

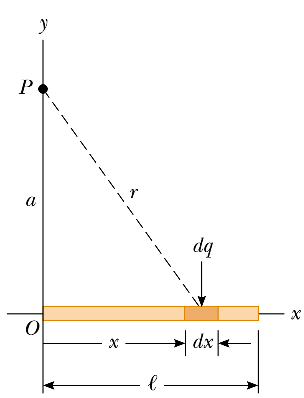


Figure 25.18 (Example 25.7) A uniform line charge of length ℓ located along the x axis. To calculate the electric potential at P, the line charge is divided into segments each of length dx and each carrying a charge $dq = \lambda\,dx$.

constants, we find that

$$V = k_e\lambda \int_0^\ell \frac{dx}{\sqrt{x^2 + a^2}} = k_e \frac{Q}{\ell}\int_0^\ell \frac{dx}{\sqrt{x^2+a^2}}$$

This integral has the following value (see Appendix B):

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right)$$

Evaluating V, we find

$$\boxed{V = \frac{k_e Q}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right)} \qquad (25.25)$$

What If? **What if we were asked to find the electric field at point P? Would this be a simple calculation?**

Answer Calculating the electric field by means of Equation 23.11 would be a little messy. There is no symmetry to appeal to, and the integration over the line of charge would represent a vector addition of electric fields at point P. Using Equation 25.18, we could find E_y by replacing a with y in Equation 25.25 and performing the differentiation with respect to y. Because the charged rod in Figure 25.18 lies entirely to the right of $x = 0$, the electric field at point P would have an x component to the left if the rod is charged positively. We cannot use Equation 25.18 to find the x component of the field, however, because we evaluated the potential due to the rod at a specific value of x ($x = 0$) rather than a general value of x. We would need to find the potential as a function of both x and y to be able to find the x and y components of the electric field using Equation 25.25.

Example 25.8 Electric Potential Due to a Uniformly Charged Sphere

An insulating solid sphere of radius R has a uniform positive volume charge density and total charge Q.

(A) Find the electric potential at a point outside the sphere, that is, for $r > R$. Take the potential to be zero at $r = \infty$.

Solution In Example 24.5, we found that the magnitude of the electric field outside a uniformly charged sphere of radius R is

$$E_r = k_e \frac{Q}{r^2} \qquad (\text{for } r > R)$$

where the field is directed radially outward when Q is positive. This is the same as the field due to a point charge, which we studied in Section 23.4. In this case, to obtain the electric potential at an exterior point, such as B in Figure 25.19, we use Equation 25.10, choosing point A as $r = \infty$:

$$V_B - V_A = k_e Q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_B - 0 = k_e Q\left[\frac{1}{r_B} - 0\right]$$

$$V_B = \boxed{k_e \frac{Q}{r}} \qquad (\text{for } r > R)$$

Because the potential must be continuous at $r = R$, we can use this expression to obtain the potential at the surface of the sphere. That is, the potential at a point such as C shown in Figure 25.19 is

$$V_C = k_e \frac{Q}{R} \qquad (\text{for } r = R)$$

(B) Find the potential at a point inside the sphere, that is, for $r < R$.

Solution In Example 24.5 we found that the electric field inside an insulating uniformly charged sphere is

$$E_r = \frac{k_e Q}{R^3} r \qquad (\text{for } r < R)$$

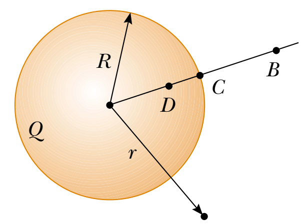


Figure 25.19 (Example 25.8) A uniformly charged insulating sphere of radius R and total charge Q. The electric potentials at points B and C are equivalent to those produced by a point charge Q located at the center of the sphere, but this is not true for point D.

We can use this result and Equation 25.3 to evaluate the potential difference $V_D - V_C$ at some interior point D:

$$V_D - V_C = -\int_R^r E_r\,dr = -\frac{k_e Q}{R^3}\int_R^r r\,dr = \frac{k_e Q}{2R^3}(R^2 - r^2)$$

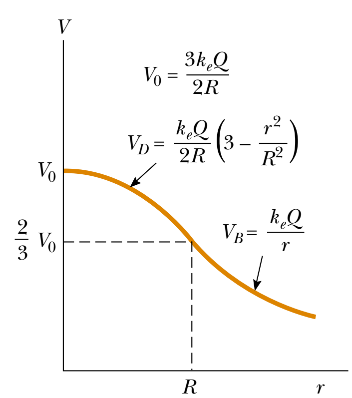


Figure 25.20 (Example 25.8) A plot of electric potential V versus distance r from the center of a uniformly charged insulating sphere of radius R. The curve for V_D inside the sphere is parabolic and joins smoothly with the curve for V_B outside the sphere, which is a hyperbola. The potential has a maximum value V_0 at the center of the sphere. We could make this graph three dimensional (similar to Figures 25.8 and 25.9) by revolving it around the vertical axis.

Substituting $V_C = k_eQ/R$ into this expression and solving for V_D, we obtain

$$V_D = \boxed{\frac{k_eQ}{2R}\left(3 - \frac{r^2}{R^2}\right)} \qquad (\text{for } r < R) \quad (25.26)$$

At $r = R$, this expression gives a result that agrees with that for the potential at the surface, that is, V_C. A plot of V versus r for this charge distribution is given in Figure 25.20.

25.6 Electric Potential Due to a Charged Conductor

In Section 24.4 we found that when a solid conductor in equilibrium carries a net charge, the charge resides on the outer surface of the conductor. Furthermore, we showed that the electric field just outside the conductor is perpendicular to the surface and that the field inside is zero.

We now show that **every point on the surface of a charged conductor in equilibrium is at the same electric potential.** Consider two points A and B on the surface of a charged conductor, as shown in Figure 25.21. Along a surface path connecting these points, $\mathbf{E}$ is always perpendicular to the displacement $d\mathbf{s}$; therefore $\mathbf{E} \cdot d\mathbf{s} = 0$. Using this result and Equation 25.3, we conclude that the potential difference between A and B is necessarily zero:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

This result applies to any two points on the surface. Therefore, V is constant everywhere on the surface of a charged conductor in equilibrium. That is,

> the surface of any charged conductor in electrostatic equilibrium is an equipotential surface. Furthermore, because the electric field is zero inside the conductor, we conclude that the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

Because this is true, no work is required to move a test charge from the interior of a charged conductor to its surface.

Consider a solid metal conducting sphere of radius R and total positive charge Q, as shown in Figure 25.22a. The electric field outside the sphere is k_eQ/r^2 and points radially outward. From Example 25.8, we know that the electric potential at the interior and surface of the sphere must be k_eQ/R relative to infinity. The potential outside the sphere is k_eQ/r. Figure 25.22b is a plot of the electric potential as a function of r, and Figure 25.22c shows how the electric field varies with r.

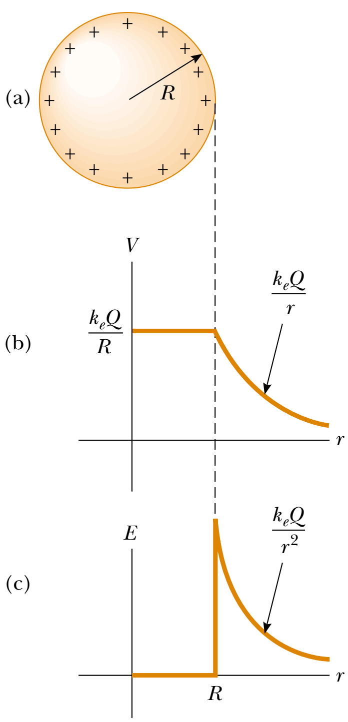


Figure 25.22 (a) The excess charge on a conducting sphere of radius R is uniformly distributed on its surface. (b) Electric potential versus distance r from the center of the charged conducting sphere. (c) Electric field magnitude versus distance r from the center of the charged conducting sphere.

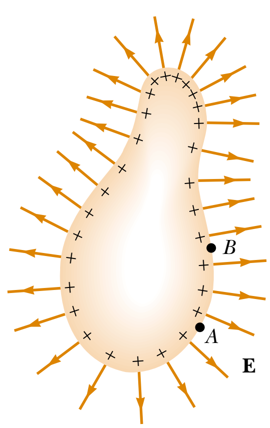


Figure 25.21 An arbitrarily shaped conductor carrying a positive charge. When the conductor is in electrostatic equilibrium, all of the charge resides at the surface, $\mathbf{E} = 0$ inside the conductor, and the direction of $\mathbf{E}$ just outside the conductor is perpendicular to the surface. The electric potential is constant inside the conductor and is equal to the potential at the surface. Note from the spacing of the positive signs that the surface charge density is nonuniform.

When a net charge is placed on a spherical conductor, the surface charge density is uniform, as indicated in Figure 25.22a. However, if the conductor is nonspherical, as in Figure 25.21, the surface charge density is high where the radius of curvature is small (as noted in Section 24.4), and it is low where the radius of curvature is large. Because the electric field just outside the conductor is proportional to the surface charge density, we see that **the electric field is large near convex points having small radii of curvature and reaches very high values at sharp points.** This is demonstrated in Figure 25.23, in which small pieces of thread suspended in oil show the electric field lines. Notice that the density of field lines is highest at the sharp tip of the left-hand conductor and at the highly curved ends of the right-hand conductor. In Example 25.9, the relationship between electric field and radius of curvature is explored mathematically.

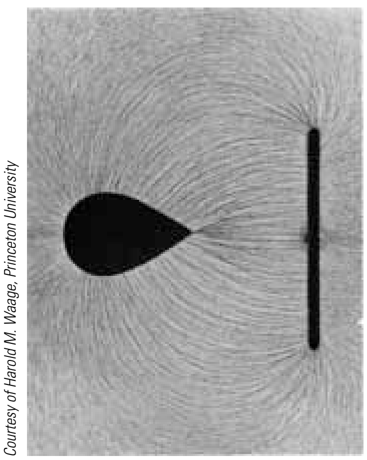
Figure 25.24 shows the electric field lines around two spherical conductors: one carrying a net charge Q, and a larger one carrying zero net charge. In this case, the surface charge density is not uniform on either conductor. The sphere having zero net charge has negative charges induced on its side that faces the charged sphere and positive charges induced on its side opposite the charged sphere. The broken blue curves in the figure represent the cross sections of the equipotential surfaces for this charge configuration. As usual, the field lines are perpendicular to the conducting surfaces at all points, and the equipotential surfaces are perpendicular to the field lines everywhere.

PITFALL PREVENTION

25.6 Potential May Not Be Zero

The electric potential inside the conductor is not necessarily zero in Figure 25.22, even though the electric field is zero. From Equation 25.15, we see that a zero value of the field results in no *change* in the potential from one point to another inside the conductor. Thus, the potential everywhere inside the conductor, including the surface, has the same value, which may or may not be zero, depending on where the zero of potential is defined.

Quick Quiz 25.10 Consider starting at the center of the left-hand sphere (sphere 1, of radius a) in Figure 25.24 and moving to the far right of the diagram, passing through the center of the right-hand sphere (sphere 2, of radius c) along the way. The centers of the spheres are a distance b apart. Draw a graph of the electric potential as a function of position relative to the center of the left-hand sphere.



Courtesy of Harold M. Waage, Princeton University

Figure 25.23 Electric field pattern of a charged conducting plate placed near an oppositely charged pointed conductor. Small pieces of thread suspended in oil align with the electric field lines. The field surrounding the pointed conductor is most intense near the pointed end and at other places where the radius of curvature is small.

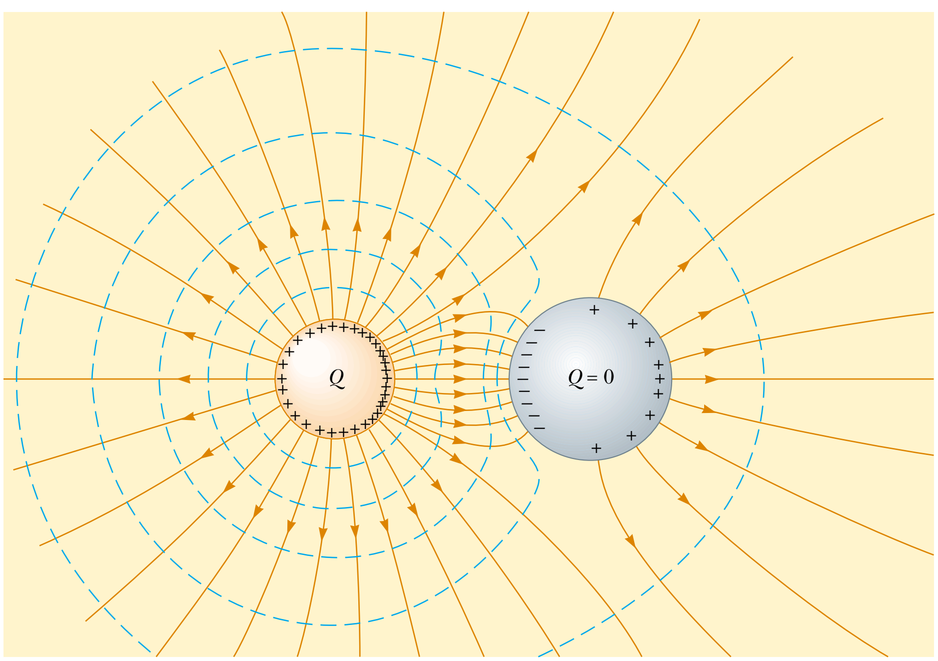


Figure 25.24 The electric field lines (in red-brown) around two spherical conductors. The smaller sphere has a net charge Q, and the larger one has zero net charge. The broken blue curves are intersections of equipotential surfaces with the page.

Example 25.9 Two Connected Charged Spheres

Two spherical conductors of radii r_1 and r_2 are separated by a distance much greater than the radius of either sphere. The spheres are connected by a conducting wire, as shown in Figure 25.25. The charges on the spheres in equilibrium are q_1 and q_2, respectively, and they are uniformly charged. Find the ratio of the magnitudes of the electric fields at the surfaces of the spheres.

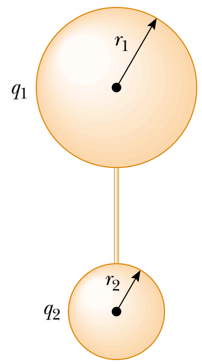


Figure 25.25 (Example 25.9) Two charged spherical conductors connected by a conducting wire. The spheres are at the *same* electric potential V.

Solution Because the spheres are connected by a conducting wire, they must both be at the same electric potential:

$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$$

Therefore, the ratio of charges is

$$(1) \qquad \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

Because the spheres are very far apart and their surfaces uniformly charged, we can express the magnitude of the electric fields at their surfaces as

$$E_1 = k_e \frac{q_1}{r_1^2} \quad \text{and} \quad E_2 = k_e \frac{q_2}{r_2^2}$$

Taking the ratio of these two fields and making use of Equation (1), we find that

$$(2) \qquad \frac{E_1}{E_2} = \boxed{\frac{r_2}{r_1}}$$

Hence, the field is more intense in the vicinity of the smaller sphere even though the electric potentials of both spheres are the same.

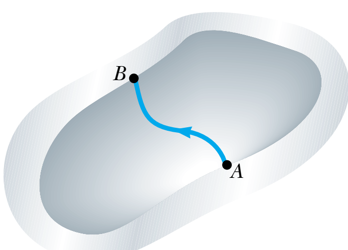


Figure 25.26 A conductor in electrostatic equilibrium containing a cavity. The electric field in the cavity is zero, regardless of the charge on the conductor.

A Cavity Within a Conductor

Now suppose a conductor of arbitrary shape contains a cavity as shown in Figure 25.26. Let us assume that no charges are inside the cavity. **In this case, the electric field inside the cavity must be zero** regardless of the charge distribution on the outside surface of the conductor. Furthermore, the field in the cavity is zero even if an electric field exists outside the conductor.

To prove this point, we use the fact that every point on the conductor is at the same electric potential, and therefore any two points A and B on the surface of the cavity must be at the same potential. Now imagine that a field $\mathbf{E}$ exists in the cavity and evaluate the potential difference $V_B - V_A$ defined by Equation 25.3:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Because $V_B - V_A = 0$, the integral of $\mathbf{E} \cdot d\mathbf{s}$ must be zero for all paths between any two points A and B on the conductor. The only way that this can be true for *all* paths is if $\mathbf{E}$ is zero *everywhere* in the cavity. Thus, we conclude that **a cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity.**

Corona Discharge

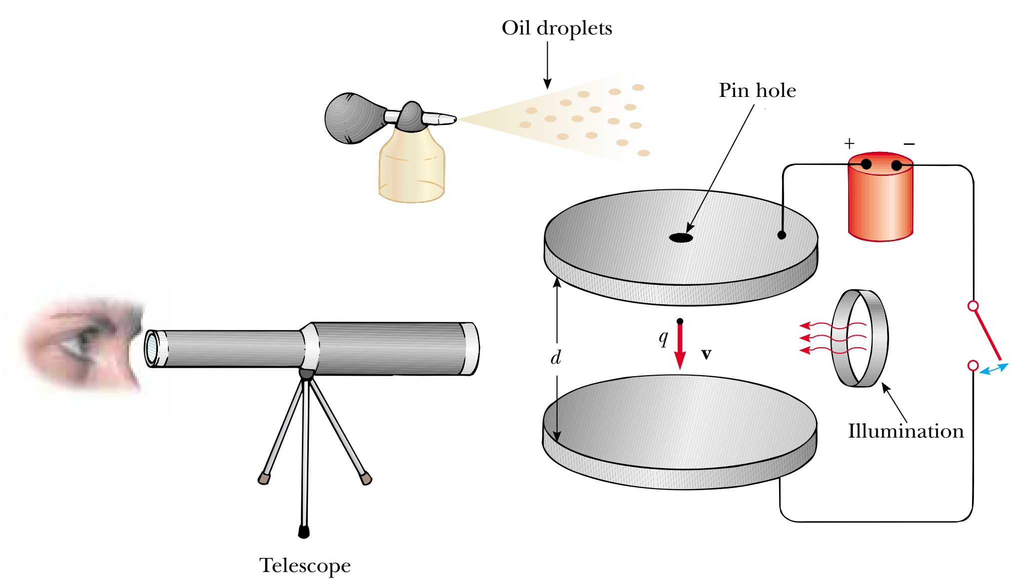
A phenomenon known as **corona discharge** is often observed near a conductor such as a high-voltage power line. When the electric field in the vicinity of the conductor is sufficiently strong, electrons resulting from random ionizations of air molecules near the conductor accelerate away from their parent molecules. These rapidly moving electrons can ionize additional molecules near the conductor, creating more free electrons. The observed glow (or corona discharge) results from the recombination of these free electrons with the ionized air molecules. If a conductor has an irregular shape, the electric field can be very high near sharp points or edges of the conductor; consequently, the ionization process and corona discharge are most likely to occur around such points.

Corona discharge is used in the electrical transmission industry to locate broken or faulty components. For example, a broken insulator on a transmission tower has sharp edges where corona discharge is likely to occur. Similarly, corona discharge will occur at the sharp end of a broken conductor strand. Observation of these discharges is difficult because the visible radiation emitted is weak and most of the radiation is in the ultraviolet. (We will discuss ultraviolet radiation and other portions of the electromagnetic spectrum in Section 34.6.) Even use of traditional ultraviolet cameras is of little help because the radiation from the corona discharge is overwhelmed by ultraviolet radiation from the Sun. Newly developed dual-spectrum devices combine a narrow-band ultraviolet camera with a visible light camera to show a daylight view of the corona discharge in the actual location on the transmission tower or cable. The ultraviolet part of the camera is designed to operate in a wavelength range in which radiation from the Sun is very weak.

25.7 The Millikan Oil-Drop Experiment

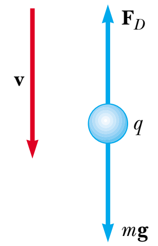
During the period from 1909 to 1913, Robert Millikan performed a brilliant set of experiments in which he measured e, the magnitude of the elementary charge on an electron, and demonstrated the quantized nature of this charge. His apparatus, diagrammed in Figure 25.27, contains two parallel metallic plates. Oil droplets from an atomizer are allowed to pass through a small hole in the upper plate. Millikan used x-rays to ionize the air in the chamber, so that freed electrons would adhere to the oil drops, giving them a negative charge. A horizontally directed light beam is used to illuminate the oil droplets, which are viewed through a telescope whose long axis is perpendicular to the light beam. When the droplets are viewed in this manner, they appear as shining stars against a dark background, and the rate at which individual drops fall can be determined.

Let us assume that a single drop having a mass m and carrying a charge q is being viewed and that its charge is negative. If no electric field is present between the plates,

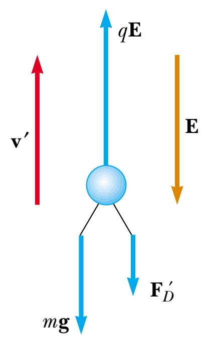


Active Figure 25.27 Schematic drawing of the Millikan oil-drop apparatus.

At the Active Figures link at http://www.pse6.com, you can do a simplified version of the experiment for yourself. You will be able to take data on a number of oil drops and determine the elementary charge from your data.



(a) Field off



(b) Field on

Figure 25.28 The forces acting on a negatively charged oil droplet in the Millikan experiment.

the two forces acting on the charge are the gravitational force $m\mathbf{g}$ acting downward[4] and a viscous drag force $\mathbf{F}_D$ acting upward as indicated in Figure 25.28a. The drag force is proportional to the drop's speed. When the drop reaches its terminal speed v, the two forces balance each other ($mg = F_D$).

Now suppose that a battery connected to the plates sets up an electric field between the plates such that the upper plate is at the higher electric potential. In this case, a third force $q\mathbf{E}$ acts on the charged drop. Because q is negative and $\mathbf{E}$ is directed downward, this electric force is directed upward, as shown in Figure 25.28b. If this force is sufficiently great, the drop moves upward and the drag force $\mathbf{F}'_D$ acts downward. When the upward electric force $q\mathbf{E}$ balances the sum of the gravitational force and the downward drag force $\mathbf{F}'_D$, the drop reaches a new terminal speed v' in the upward direction.

With the field turned on, a drop moves slowly upward, typically at rates of hundredths of a centimeter per second. The rate of fall in the absence of a field is comparable. Hence, one can follow a single droplet for hours, alternately rising and falling, by simply turning the electric field on and off.

After recording measurements on thousands of droplets, Millikan and his co-workers found that all droplets, to within about 1% precision, had a charge equal to some integer multiple of the elementary charge e:

$$q = ne \qquad n = 0, -1, -2, -3, \ldots$$

where $e = 1.60 \times 10^{-19}$ C. Millikan's experiment yields conclusive evidence that charge is quantized. For this work, he was awarded the Nobel Prize in Physics in 1923.

25.8 Applications of Electrostatics

The practical application of electrostatics is represented by such devices as lightning rods and electrostatic precipitators and by such processes as xerography and the painting of automobiles. Scientific devices based on the principles of electrostatics include electrostatic generators, the field-ion microscope, and ion-drive rocket engines.

The Van de Graaff Generator

Experimental results show that when a charged conductor is placed in contact with the inside of a hollow conductor, all of the charge on the charged conductor is transferred to the hollow conductor. In principle, the charge on the hollow conductor and its electric potential can be increased without limit by repetition of the process.

In 1929 Robert J. Van de Graaff (1901–1967) used this principle to design and build an electrostatic generator. This type of generator is used extensively in nuclear physics research. A schematic representation of the generator is given in Figure 25.29. Charge is delivered continuously to a high-potential electrode by means of a moving belt of insulating material. The high-voltage electrode is a hollow metal dome mounted on an insulating column. The belt is charged at point A by means of a corona discharge between comb-like metallic needles and a grounded grid. The needles are maintained at a positive electric potential of typically 10^4 V. The positive charge on the moving belt is transferred to the dome by a second comb of needles at point B. Because the electric field inside the dome is negligible, the positive charge on the belt is easily transferred to the conductor regardless of its potential. In practice, it is possible to increase the electric potential of the dome until electrical discharge occurs through the air. Because the "breakdown" electric field in air is about 3×10^6 V/m, a

[4] There is also a buoyant force on the oil drop due to the surrounding air. This force can be incorporated as a correction in the gravitational force $m\mathbf{g}$ on the drop, so we will not consider it in our analysis.

sphere 1 m in radius can be raised to a maximum potential of 3×10^6 V. The potential can be increased further by increasing the radius of the dome and by placing the entire system in a container filled with high-pressure gas.

Van de Graaff generators can produce potential differences as large as 20 million volts. Protons accelerated through such large potential differences receive enough energy to initiate nuclear reactions between themselves and various target nuclei. Smaller generators are often seen in science classrooms and museums. If a person insulated from the ground touches the sphere of a Van de Graaff generator, his or her body can be brought to a high electric potential. The hair acquires a net positive charge, and each strand is repelled by all the others, as in the opening photograph of Chapter 23.

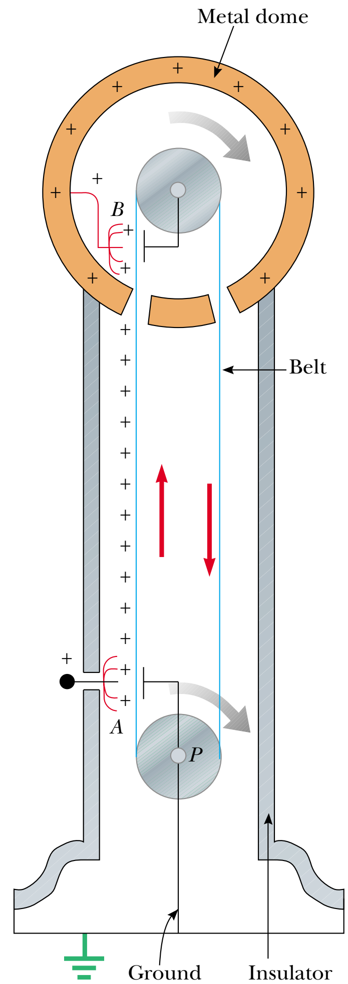


Figure 25.29 Schematic diagram of a Van de Graaff generator. Charge is transferred to the metal dome at the top by means of a moving belt. The charge is deposited on the belt at point *A* and transferred to the hollow conductor at point *B*.

The Electrostatic Precipitator

One important application of electrical discharge in gases is the *electrostatic precipitator.* This device removes particulate matter from combustion gases, thereby reducing air pollution. Precipitators are especially useful in coal-burning power plants and in industrial operations that generate large quantities of smoke. Current systems are able to eliminate more than 99% of the ash from smoke.

Figure 25.30a shows a schematic diagram of an electrostatic precipitator. A high potential difference (typically 40 to 100 kV) is maintained between a wire running down the center of a duct and the walls of the duct, which are grounded. The wire is maintained at a negative electric potential with respect to the walls, so the electric field is directed toward the wire. The values of the field near the wire become high enough to cause a corona discharge around the wire; the air near the wire contains positive ions, electrons, and such negative ions as O_2^-. The air to be cleaned enters the duct and moves near the wire. As the electrons and negative ions created by the discharge are accelerated toward the outer wall by the electric field, the dirt particles in the air become charged by collisions and ion capture. Because most of the charged dirt particles are negative, they too are drawn to the duct walls by the electric field. When the duct is periodically shaken, the particles break loose and are collected at the bottom.

In addition to reducing the level of particulate matter in the atmosphere (compare Figs. 25.30b and c), the electrostatic precipitator recovers valuable materials in the form of metal oxides.

(a) (b) (c)

Figure 25.30 (a) Schematic diagram of an electrostatic precipitator. The high negative electric potential maintained on the central coiled wire creates a corona discharge in the vicinity of the wire. Compare the air pollution when the electrostatic precipitator is (b) operating and (c) turned off.

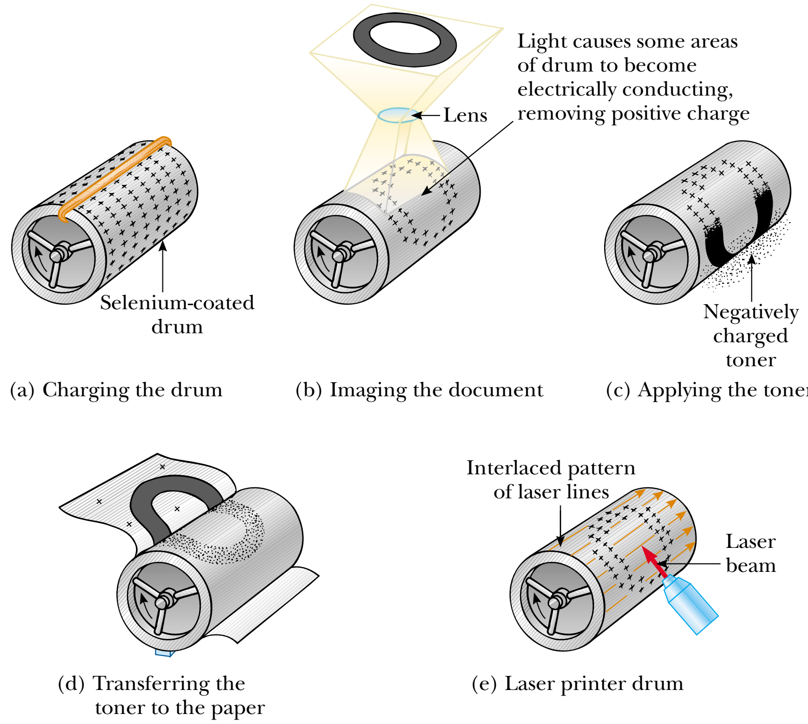


Figure 25.31 The xerographic process: (a) The photoconductive surface of the drum is positively charged. (b) Through the use of a light source and lens, an image is formed on the surface in the form of positive charges. (c) The surface containing the image is covered with a negatively charged powder, which adheres only to the image area. (d) A piece of paper is placed over the surface and given a positive charge. This transfers the image to the paper as the negatively charged powder particles migrate to the paper. The paper is then heat-treated to "fix" the powder. (e) A laser printer operates similarly except the image is produced by turning a laser beam on and off as it sweeps across the selenium-coated drum.

Xerography and Laser Printers

The basic idea of xerography[5] was developed by Chester Carlson, who was granted a patent for the xerographic process in 1940. The unique feature of this process is the use of a photoconductive material to form an image. (A *photoconductor* is a material that is a poor electrical conductor in the dark but becomes a good electrical conductor when exposed to light.)

The xerographic process is illustrated in Figure 25.31a to d. First, the surface of a plate or drum that has been coated with a thin film of photoconductive material (usually selenium or some compound of selenium) is given a positive electrostatic charge in the dark. An image of the page to be copied is then focused by a lens onto the charged surface. The photoconducting surface becomes conducting only in areas where light strikes it. In these areas, the light produces charge carriers in the photoconductor that move the positive charge off the drum. However, positive charges remain on those areas of the photoconductor not exposed to light, leaving a latent image of the object in the form of a positive surface charge distribution.

Next, a negatively charged powder called a *toner* is dusted onto the photoconducting surface. The charged powder adheres only to those areas of the surface that contain the positively charged image. At this point, the image becomes visible. The toner (and hence the image) is then transferred to the surface of a sheet of positively charged paper.

Finally, the toner is "fixed" to the surface of the paper as the toner melts while passing through high-temperature rollers. This results in a permanent copy of the original.

A laser printer (Fig. 25.31e) operates by the same principle, with the exception that a computer-directed laser beam is used to illuminate the photoconductor instead of a lens.

[5] The prefix *xero-* is from the Greek word meaning "dry." Note that liquid ink is not used in xerography.

SUMMARY

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

When a positive test charge q_0 is moved between points A and B in an electric field $\mathbf{E}$, the **change in the potential energy of the charge–field system** is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{25.1}$$

The **electric potential** $V = U/q_0$ is a scalar quantity and has the units of J/C, where 1 J/C ≡ 1 V.

The **potential difference** ΔV between points A and B in an electric field $\mathbf{E}$ is defined as

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{25.3}$$

The potential difference between two points A and B in a uniform electric field $\mathbf{E}$, where $\mathbf{s}$ is a vector that points from A to B and is parallel to $\mathbf{E}$ is

$$\Delta V = -Ed \tag{25.6}$$

where $d = |\mathbf{s}|$.

An **equipotential surface** is one on which all points are at the same electric potential. Equipotential surfaces are perpendicular to electric field lines.

If we define $V = 0$ at $r_A = \infty$, the electric potential due to a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r} \tag{25.11}$$

We can obtain the electric potential associated with a group of point charges by summing the potentials due to the individual charges.

The **potential energy associated with a pair of point charges** separated by a distance r_{12} is

$$U = k_e \frac{q_1 q_2}{r_{12}} \tag{25.13}$$

This energy represents the work done by an external agent when the charges are brought from an infinite separation to the separation r_{12}. We obtain the potential energy of a distribution of point charges by summing terms like Equation 25.13 over all pairs of particles.

If we know the electric potential as a function of coordinates x, y, z, we can obtain the components of the electric field by taking the negative derivative of the electric potential with respect to the coordinates. For example, the x component of the electric field is

$$E_x = -\frac{dV}{dx} \tag{25.16}$$

The **electric potential due to a continuous charge distribution** is

$$V = k_e \int \frac{dq}{r} \tag{25.20}$$

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential. The potential is constant everywhere inside the conductor and equal to its value at the surface.

Table 25.1 lists electric potentials due to several charge distributions.

Table 25.1

Electric Potential Due to Various Charge Distributions

Charge Distribution	Electric Potential	Location
Uniformly charged ring of radius a	$V = k_e \dfrac{Q}{\sqrt{x^2 + a^2}}$	Along perpendicular central axis of ring, distance x from ring center
Uniformly charged disk of radius a	$V = 2\pi k_e \sigma [(x^2 + a^2)^{1/2} - x]$	Along perpendicular central axis of disk, distance x from disk center
Uniformly charged, *insulating* solid sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$	$r \geq R$
	$V = \dfrac{k_e Q}{2R}\left(3 - \dfrac{r^2}{R^2}\right)$	$r < R$
Isolated *conducting* sphere of radius R and total charge Q	$V = k_e \dfrac{Q}{r}$	$r > R$
	$V = k_e \dfrac{Q}{R}$	$r \leq R$

QUESTIONS

1. Distinguish between electric potential and electric potential energy.

2. A negative charge moves in the direction of a uniform electric field. Does the potential energy of the charge–field system increase or decrease? Does the charge move to a position of higher or lower potential?

3. Give a physical explanation of the fact that the potential energy of a pair of charges with the same sign is positive whereas the potential energy of a pair of charges with opposite signs is negative.

4. A uniform electric field is parallel to the x axis. In what direction can a charge be displaced in this field without any external work being done on the charge?

5. Explain why equipotential surfaces are always perpendicular to electric field lines.

6. Describe the equipotential surfaces for (a) an infinite line of charge and (b) a uniformly charged sphere.

7. Explain why, under static conditions, all points in a conductor must be at the same electric potential.

8. The electric field inside a hollow, uniformly charged sphere is zero. Does this imply that the potential is zero inside the sphere? Explain.

9. The potential of a point charge is defined to be zero at an infinite distance. Why can we not define the potential of an infinite line of charge to be zero at $r = \infty$?

10. Two charged conducting spheres of different radii are connected by a conducting wire as shown in Figure 25.25. Which sphere has the greater charge density?

11. What determines the maximum potential to which the dome of a Van de Graaff generator can be raised?

12. Explain the origin of the glow sometimes observed around the cables of a high-voltage power line.

13. Why is it important to avoid sharp edges or points on conductors used in high-voltage equipment?

14. How would you shield an electronic circuit or laboratory from stray electric fields? Why does this work?

15. Two concentric spherical conducting shells of radii $a = 0.400$ m and $b = 0.500$ m are connected by a thin wire as shown in Figure Q25.15. If a total charge $Q = 10.0\ \mu$C is placed on the system, how much charge settles on each sphere?

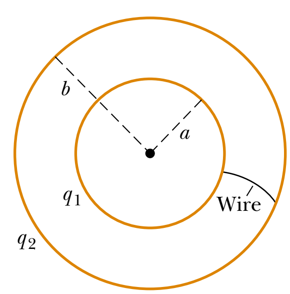


Figure Q25.15

16. Study Figure 23.4 and the accompanying text discussion of charging by induction. You may also compare to Figure

25.24. When the grounding wire is touched to the rightmost point on the sphere in Figure 23.4c, electrons are drained away from the sphere to leave the sphere positively charged. Suppose instead that the grounding wire is touched to the leftmost point on the sphere. Will electrons still drain away, moving closer to the negatively charged rod as they do so? What kind of charge, if any, will remain on the sphere?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 25.1 Potential Difference and Electric Potential

1. How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is -5.00 V? (The potential in each case is measured relative to a common reference point.)

2. An ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of 7.37×10^{-17} J. Calculate the charge on the ion.

3. (a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference.

4. What potential difference is needed to stop an electron having an initial speed of 4.20×10^5 m/s?

Section 25.2 Potential Differences in a Uniform Electric Field

5. A uniform electric field of magnitude 250 V/m is directed in the positive x direction. A $+12.0$-μC charge moves from the origin to the point $(x, y) = (20.0 \text{ cm}, 50.0 \text{ cm})$. (a) What is the change in the potential energy of the charge–field system? (b) Through what potential difference does the charge move?

6. The difference in potential between the accelerating plates in the electron gun of a TV picture tube is about 25 000 V. If the distance between these plates is 1.50 cm, what is the magnitude of the uniform electric field in this region?

7. An electron moving parallel to the x axis has an initial speed of 3.70×10^6 m/s at the origin. Its speed is reduced to 1.40×10^5 m/s at the point $x = 2.00$ cm. Calculate the potential difference between the origin and that point. Which point is at the higher potential?

8. Suppose an electron is released from rest in a uniform electric field whose magnitude is 5.90×10^3 V/m. (a) Through what potential difference will it have passed after moving 1.00 cm? (b) How fast will the electron be moving after it has traveled 1.00 cm?

9. A uniform electric field of magnitude 325 V/m is directed in the negative y direction in Figure P25.9. The coordinates of point A are $(-0.200, -0.300)$ m, and those of point B are $(0.400, 0.500)$ m. Calculate the potential difference $V_B - V_A$, using the blue path.

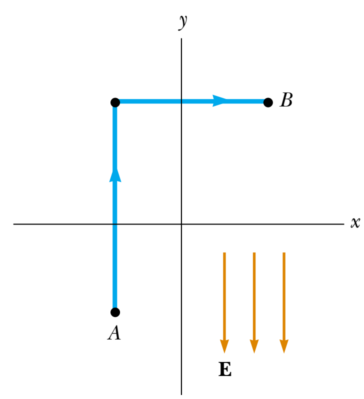


Figure P25.9

10. Starting with the definition of work, prove that at every point on an equipotential surface the surface must be perpendicular to the electric field there.

11. **Review problem.** A block having mass m and charge $+Q$ is connected to a spring having constant k. The block lies on a frictionless horizontal track, and the system is immersed in a uniform electric field of magnitude E, directed as shown in Figure P25.11. If the block is released from rest when the spring is unstretched (at $x = 0$), (a) by what maximum amount does the spring expand? (b) What is the equilibrium position of the block? (c) Show that the block's motion is simple harmonic, and determine its period. (d) **What If?** Repeat part (a) if the coefficient of kinetic friction between block and surface is μ_k.

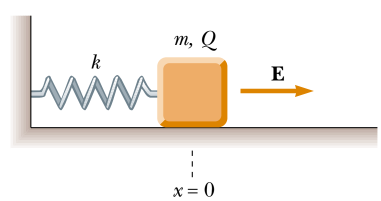


Figure P25.11

12. On planet Tehar, the free-fall acceleration is the same as that on Earth but there is also a strong downward electric field that is uniform close to the planet's surface. A 2.00-kg ball having a charge of 5.00 μC is thrown upward at a speed of 20.1 m/s, and it hits the ground after an interval of 4.10 s. What is the potential difference between the starting point and the top point of the trajectory?

13. An insulating rod having linear charge density λ = 40.0 μC/m and linear mass density μ = 0.100 kg/m is released from rest in a uniform electric field E = 100 V/m directed perpendicular to the rod (Fig. P25.13). (a) Determine the speed of the rod after it has traveled 2.00 m. (b) **What If?** How does your answer to part (a) change if the electric field is not perpendicular to the rod? Explain.

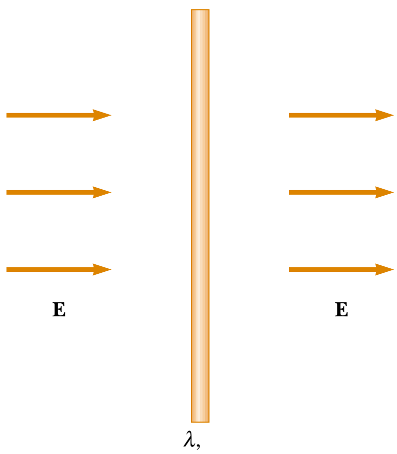


Figure P25.13

14. A particle having charge q = +2.00 μC and mass m = 0.010 0 kg is connected to a string that is L = 1.50 m long and is tied to the pivot point P in Figure P25.14. The particle, string and pivot point all lie on a frictionless horizontal table. The particle is released from rest when the string makes an angle θ = 60.0° with a uniform electric field of magnitude E = 300 V/m. Determine the speed of the particle when the string is parallel to the electric field (point a in Fig. P25.14).

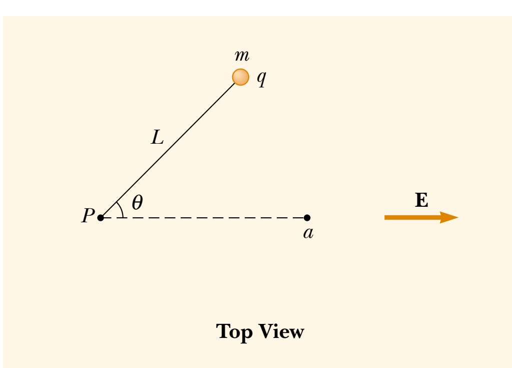


Figure P25.14

Section 25.3 Electric Potential and Potential Energy Due to Point Charges

Note: Unless stated otherwise, assume the reference level of potential is $V = 0$ at $r = \infty$.

15. (a) Find the potential at a distance of 1.00 cm from a proton. (b) What is the potential difference between two points that are 1.00 cm and 2.00 cm from a proton? (c) **What If?** Repeat parts (a) and (b) for an electron.

16. Given two 2.00-μC charges, as shown in Figure P25.16, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted by the two 2.00-μC charges on the test charge q? (b) What is the electric field at the origin due to the two 2.00-μC charges? (c) What is the electric potential at the origin due to the two 2.00-μC charges?

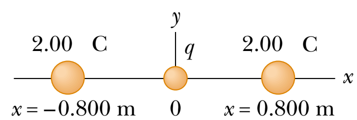


Figure P25.16

17. At a certain distance from a point charge, the magnitude of the electric field is 500 V/m and the electric potential is −3.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

18. A charge $+q$ is at the origin. A charge $-2q$ is at x = 2.00 m on the x axis. For what finite value(s) of x is (a) the electric field zero? (b) the electric potential zero?

19. The three charges in Figure P25.19 are at the vertices of an isosceles triangle. Calculate the electric potential at the midpoint of the base, taking q = 7.00 μC.

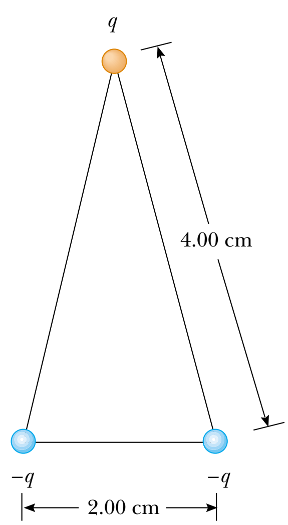


Figure P25.19

20. Two point charges, $Q_1 = +5.00$ nC and $Q_2 = -3.00$ nC, are separated by 35.0 cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

21. *Compare this problem with Problem 57 in Chapter 23.* Four identical point charges ($q = +10.0\ \mu$C) are located on the corners of a rectangle as shown in Figure P23.57. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm. Calculate the change in electric potential energy of the system as the charge at the lower left corner in Figure P23.57 is brought to this position from infinitely far away. Assume that the other three charges in Figure P23.57 remain fixed in position.

22. *Compare this problem with Problem 20 in Chapter 23.* Two point charges each of magnitude 2.00 μC are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric potential on the y axis at $y = 0.500$ m. (b) Calculate the change in electric potential energy of the system as a third charge of $-3.00\ \mu$C is brought from infinitely far away to a position on the y axis at $y = 0.500$ m.

23. Show that the amount of work required to assemble four identical point charges of magnitude Q at the corners of a square of side s is $5.41 k_e Q^2/s$.

24. *Compare this problem with Problem 23 in Chapter 23.* Five equal negative point charges $-q$ are placed symmetrically around a circle of radius R. Calculate the electric potential at the center of the circle.

25. *Compare this problem with Problem 41 in Chapter 23.* Three equal positive charges q are at the corners of an equilateral triangle of side a as shown in Figure P23.41. (a) At what point, if any, in the plane of the charges is the electric potential zero? (b) What is the electric potential at the point P due to the two charges at the base of the triangle?

26. **Review problem.** Two insulating spheres have radii 0.300 cm and 0.500 cm, masses 0.100 kg and 0.700 kg, and uniformly distributed charges of $-2.00\ \mu$C and $3.00\ \mu$C. They are released from rest when their centers are separated by 1.00 m. (a) How fast will each be moving when they collide? (*Suggestion:* consider conservation of energy and of linear momentum.) (b) **What If?** If the spheres were conductors, would the speeds be greater or less than those calculated in part (a)? Explain.

27. **Review problem.** Two insulating spheres have radii r_1 and r_2, masses m_1 and m_2, and uniformly distributed charges $-q_1$ and q_2. They are released from rest when their centers are separated by a distance d. (a) How fast is each moving when they collide? (*Suggestion:* consider conservation of energy and conservation of linear momentum.) (b) **What If?** If the spheres were conductors, would their speeds be greater or less than those calculated in part (a)? Explain.

28. Two particles, with charges of 20.0 nC and -20.0 nC, are placed at the points with coordinates (0, 4.00 cm) and (0, -4.00 cm), as shown in Figure P25.28. A particle with charge 10.0 nC is located at the origin. (a) Find the electric potential energy of the configuration of the three fixed charges. (b) A fourth particle, with a mass of 2.00×10^{-13} kg and a charge of 40.0 nC, is released from rest at the point (3.00 cm, 0). Find its speed after it has moved freely to a very large distance away.

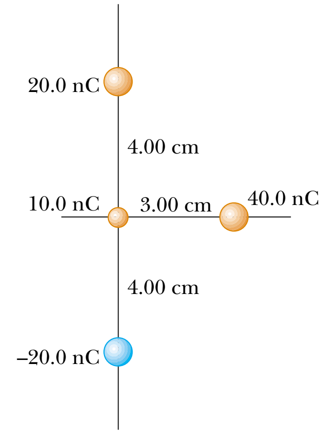


Figure P25.28

29. **Review problem.** A light unstressed spring has length d. Two identical particles, each with charge q, are connected to the opposite ends of the spring. The particles are held stationary a distance d apart and then released at the same time. The system then oscillates on a horizontal frictionless table. The spring has a bit of internal kinetic friction, so the oscillation is damped. The particles eventually stop vibrating when the distance between them is $3d$. Find the increase in internal energy that appears in the spring during the oscillations. Assume that the system of the spring and two charges is isolated.

30. Two point charges of equal magnitude are located along the y axis equal distances above and below the x axis, as shown in Figure P25.30. (a) Plot a graph of the potential at points along the x axis over the interval $-3a < x < 3a$. You should plot the potential in units of $k_e Q/a$. (b) Let the charge located at $-a$ be negative and plot the potential along the y axis over the interval $-4a < y < 4a$.

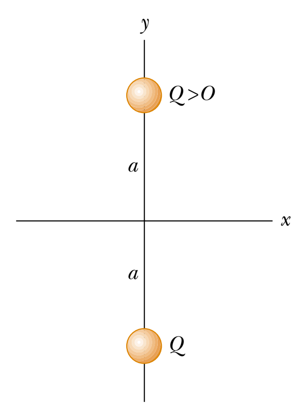


Figure P25.30

31. A small spherical object carries a charge of 8.00 nC. At what distance from the center of the object is the potential equal to 100 V? 50.0 V? 25.0 V? Is the spacing of the equipotentials proportional to the change in potential?

32. In 1911 Ernest Rutherford and his assistants Geiger and Marsden conducted an experiment in which they scattered alpha particles from thin sheets of gold. An alpha particle, having charge $+2e$ and mass 6.64×10^{-27} kg, is a product of certain radioactive decays. The results of the experiment led Rutherford to the idea that most of the mass of an atom is in a very small nucleus, with electrons in orbit around it—his planetary model of the atom. Assume an alpha particle, initially very far from a gold nucleus, is fired with a velocity of 2.00×10^7 m/s directly toward the nucleus (charge $+79e$). How close does the alpha particle get to the nucleus before turning around? Assume the gold nucleus remains stationary.

33. An electron starts from rest 3.00 cm from the center of a uniformly charged insulating sphere of radius 2.00 cm and total charge 1.00 nC. What is the speed of the electron when it reaches the surface of the sphere?

34. Calculate the energy required to assemble the array of charges shown in Figure P25.34, where $a = 0.200$ m, $b = 0.400$ m, and $q = 6.00\ \mu$C.

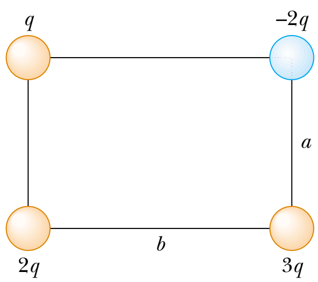


Figure P25.34

35. Four identical particles each have charge q and mass m. They are released from rest at the vertices of a square of side L. How fast is each charge moving when their distance from the center of the square doubles?

36. How much work is required to assemble eight identical point charges, each of magnitude q, at the corners of a cube of side s?

Section 25.4 Obtaining the Value of the Electric Field from the Electric Potential

37. The potential in a region between $x = 0$ and $x = 6.00$ m is $V = a + bx$, where $a = 10.0$ V and $b = -7.00$ V/m. Determine (a) the potential at $x = 0$, 3.00 m, and 6.00 m, and (b) the magnitude and direction of the electric field at $x = 0$, 3.00 m, and 6.00 m.

38. The electric potential inside a charged spherical conductor of radius R is given by $V = k_eQ/R$, and the potential outside is given by $V = k_eQ/r$. Using $E_r = -dV/dr$, derive the electric field (a) inside and (b) outside this charge distribution.

39. Over a certain region of space, the electric potential is $V = 5x - 3x^2y + 2yz^2$. Find the expressions for the x, y, and z components of the electric field over this region. What is the magnitude of the field at the point P that has coordinates $(1, 0, -2)$ m?

40. Figure P25.40 shows several equipotential lines each labeled by its potential in volts. The distance between the lines of the square grid represents 1.00 cm. (a) Is the magnitude of the field larger at A or at B? Why? (b) What is **E** at B? (c) Represent what the field looks like by drawing at least eight field lines.

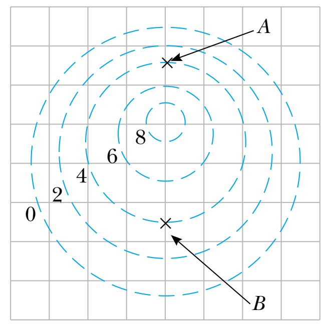


Figure P25.40

41. It is shown in Example 25.7 that the potential at a point P a distance a above one end of a uniformly charged rod of length ℓ lying along the x axis is

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right)$$

Use this result to derive an expression for the y component of the electric field at P. (*Suggestion:* Replace a with y.)

Section 25.5 Electric Potential Due to Continuous Charge Distributions

42. Consider a ring of radius R with the total charge Q spread uniformly over its perimeter. What is the potential difference between the point at the center of the ring and a point on its axis a distance $2R$ from the center?

43. A rod of length L (Fig. P25.43) lies along the x axis with its left end at the origin. It has a nonuniform charge density $\lambda = \alpha x$, where α is a positive constant. (a) What are the units of α? (b) Calculate the electric potential at A.

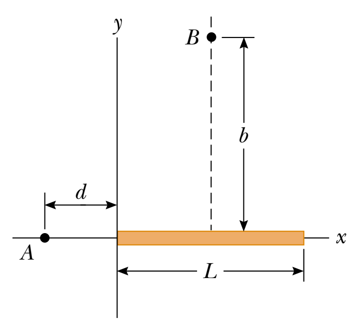


Figure P25.43 Problems 43 and 44.

44. For the arrangement described in the previous problem, calculate the electric potential at point B, which lies on the perpendicular bisector of the rod a distance b above the x axis.

45. *Compare this problem with Problem 33 in Chapter 23.* A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure P23.33. The rod has a total charge of $-7.50\ \mu$C. Find the electric potential at O, the center of the semicircle.

46. Calculate the electric potential at point P on the axis of the annulus shown in Figure P25.46, which has a uniform charge density σ.

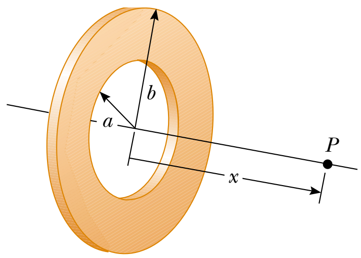


Figure P25.46

47. A wire having a uniform linear charge density λ is bent into the shape shown in Figure P25.47. Find the electric potential at point O.

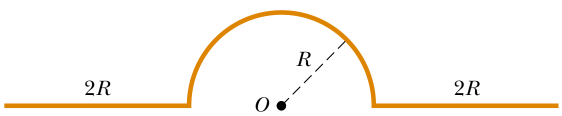


Figure P25.47

Section 25.6 Electric Potential Due to a Charged Conductor

48. How many electrons should be removed from an initially uncharged spherical conductor of radius 0.300 m to produce a potential of 7.50 kV at the surface?

49. A spherical conductor has a radius of 14.0 cm and charge of 26.0 μC. Calculate the electric field and the electric potential (a) $r = 10.0$ cm, (b) $r = 20.0$ cm, and (c) $r = 14.0$ cm from the center.

50. Electric charge can accumulate on an airplane in flight. You may have observed needle-shaped metal extensions on the wing tips and tail of an airplane. Their purpose is to allow charge to leak off before much of it accumulates. The electric field around the needle is much larger than the field around the body of the airplane, and can become large enough to produce dielectric breakdown of the air, discharging the airplane. To model this process, assume that two charged spherical conductors are connected by a long conducting wire, and a charge of 1.20 μC is placed on the combination. One sphere, representing the body of the airplane, has a radius of 6.00 cm, and the other, representing the tip of the needle, has a radius of 2.00 cm. (a) What is the electric potential of each sphere? (b) What is the electric field at the surface of each sphere?

Section 25.8 Applications of Electrostatics

51. Lightning can be studied with a Van de Graaff generator, essentially consisting of a spherical dome on which charge is continuously deposited by a moving belt. Charge can be added until the electric field at the surface of the dome becomes equal to the dielectric strength of air. Any more charge leaks off in sparks, as shown in Figure P25.51. Assume the dome has a diameter of 30.0 cm and is surrounded by dry air with dielectric strength 3.00×10^6 V/m. (a) What is the maximum potential of the dome? (b) What is the maximum charge on the dome?

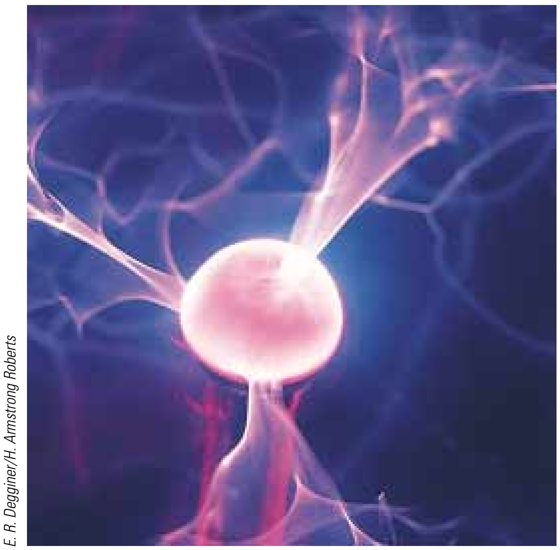


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Figure P25.51 Problems 51 and 52.

52. The spherical dome of a Van de Graaff generator can be raised to a maximum potential of 600 kV; then additional charge leaks off in sparks, by producing dielectric breakdown of the surrounding dry air, as shown in Figure P25.51. Determine (a) the charge on the dome and (b) the radius of the dome.

Additional Problems

53. The liquid-drop model of the atomic nucleus suggests that high-energy oscillations of certain nuclei can split the nucleus into two unequal fragments plus a few neutrons. The fission products acquire kinetic energy from their mutual Coulomb repulsion. Calculate the electric potential energy (in electron volts) of two spherical fragments from a uranium nucleus having the following charges and radii: $38e$ and 5.50×10^{-15} m; $54e$ and 6.20×10^{-15} m. Assume that the charge is distributed uniformly throughout the volume of each spherical fragment and that just before separating they are at rest with their surfaces in contact. The electrons surrounding the nucleus can be ignored.

54. On a dry winter day you scuff your leather-soled shoes across a carpet and get a shock when you extend the tip of one finger toward a metal doorknob. In a dark room you see a spark perhaps 5 mm long. Make order-of-magnitude estimates of (a) your electric potential and (b) the charge on your body before you touch the doorknob. Explain your reasoning.

55. The Bohr model of the hydrogen atom states that the single electron can exist only in certain allowed orbits

around the proton. The radius of each Bohr orbit is $r = n^2(0.0529 \text{ nm})$ where $n = 1, 2, 3, \ldots$. Calculate the electric potential energy of a hydrogen atom when the electron (a) is in the first allowed orbit, with $n = 1$, (b) is in the second allowed orbit, $n = 2$, and (c) has escaped from the atom, with $r = \infty$. Express your answers in electron volts.

56. An electron is released from rest on the axis of a uniform positively charged ring, 0.100 m from the ring's center. If the linear charge density of the ring is $+0.100\ \mu\text{C/m}$ and the radius of the ring is 0.200 m, how fast will the electron be moving when it reaches the center of the ring?

57. As shown in Figure P25.57, two large parallel vertical conducting plates separated by distance d are charged so that their potentials are $+V_0$ and $-V_0$. A small conducting ball of mass m and radius R (where $R \ll d$) is hung midway between the plates. The thread of length L supporting the ball is a conducting wire connected to ground, so the potential of the ball is fixed at $V = 0$. The ball hangs straight down in stable equilibrium when V_0 is sufficiently small. Show that the equilibrium of the ball is unstable if V_0 exceeds the critical value $k_e d^2 mg/(4RL)$. (*Suggestion:* consider the forces on the ball when it is displaced a distance $x \ll L$.)

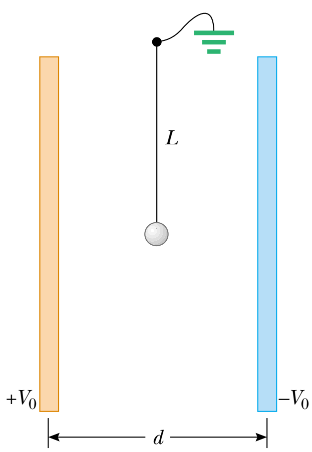

Figure P25.57

58. *Compare this problem with Problem 34 in Chapter 23.* (a) A uniformly charged cylindrical shell has total charge Q, radius R, and height h. Determine the electric potential at a point a distance d from the right end of the cylinder, as shown in Figure P25.58. (*Suggestion:* use the result of Example 25.5 by treating the cylinder as a collection of ring charges.) (b) **What If?** Use the result of Example 25.6 to solve the same problem for a solid cylinder.

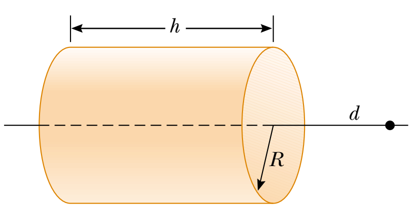

Figure P25.58

$\boxed{59.}$ Calculate the work that must be done to charge a spherical shell of radius R to a total charge Q.

60. Two parallel plates having charges of equal magnitude but opposite sign are separated by 12.0 cm. Each plate has a surface charge density of 36.0 nC/m^2. A proton is released from rest at the positive plate. Determine (a) the potential difference between the plates, (b) the kinetic energy of the proton when it reaches the negative plate, (c) the speed of the proton just before it strikes the negative plate, (d) the acceleration of the proton, and (e) the force on the proton. (f) From the force, find the magnitude of the electric field and show that it is equal to the electric field found from the charge densities on the plates.

61. A Geiger tube is a radiation detector that essentially consists of a closed, hollow metal cylinder (the cathode) of inner radius r_a and a coaxial cylindrical wire (the anode) of radius r_b (Fig. P25.61). The charge per unit length on the anode is λ, while the charge per unit length on the cathode is $-\lambda$. A gas fills the space between the electrodes. When a high-energy elementary particle passes through this space, it can ionize an atom of the gas. The strong electric field makes the resulting ion and electron accelerate in opposite directions. They strike other molecules of the gas to ionize them, producing an avalanche of electrical discharge. The pulse of electric current between the wire and the cylinder is counted by an external circuit. (a) Show that the magnitude of the potential difference between the wire and the cylinder is

$$\Delta V = 2k_e \lambda \ln\left(\frac{r_a}{r_b}\right)$$

(b) Show that the magnitude of the electric field in the space between cathode and anode is given by

$$E = \frac{\Delta V}{\ln(r_a/r_b)}\left(\frac{1}{r}\right)$$

where r is the distance from the axis of the anode to the point where the field is to be calculated.

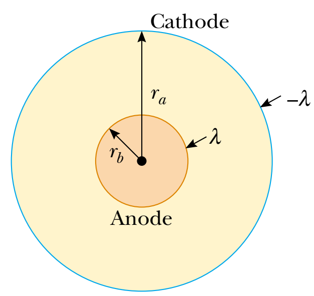

Figure P25.61 Problems 61 and 62.

62. The results of Problem 61 apply also to an electrostatic precipitator (Figures 25.30 and P25.61). An applied voltage $\Delta V = V_a - V_b = 50.0$ kV is to produce an electric field of magnitude 5.50 MV/m at the surface of the central wire. Assume the outer cylindrical wall has uniform radius $r_a = 0.850$ m. (a) What should be the radius r_b of the central wire? You will need to solve a transcendental equation. (b) What is the magnitude of the electric field at the outer wall?

63. From Gauss's law, the electric field set up by a uniform line of charge is

$$\mathbf{E} = \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing radially away from the line and λ is the linear charge density along the line. Derive an expression for the potential difference between $r = r_1$ and $r = r_2$.

64. Four balls, each with mass m, are connected by four nonconducting strings to form a square with side a, as shown in Figure P25.64. The assembly is placed on a horizontal nonconducting frictionless surface. Balls 1 and 2 each have charge q, and balls 3 and 4 are uncharged. Find the maximum speed of balls 1 and 2 after the string connecting them is cut.

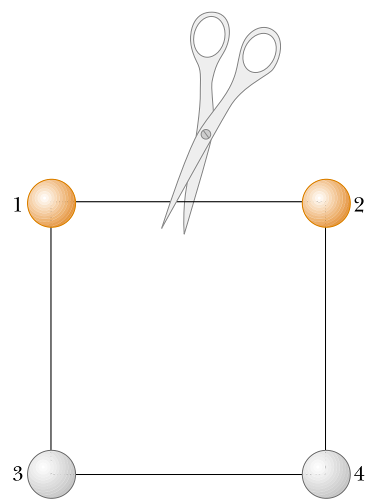

Figure P25.64

65. A point charge q is located at $x = -R$, and a point charge $-2q$ is located at the origin. Prove that the equipotential surface that has zero potential is a sphere centered at $(-4R/3, 0, 0)$ and having a radius $r = 2R/3$.

66. Consider two thin, conducting, spherical shells as shown in Figure P25.66. The inner shell has a radius $r_1 = 15.0$ cm and a charge of 10.0 nC. The outer shell has a radius $r_2 = 30.0$ cm and a charge of -15.0 nC. Find (a) the electric field $\mathbf{E}$ and (b) the electric potential V in regions A, B, and C, with $V = 0$ at $r = \infty$.

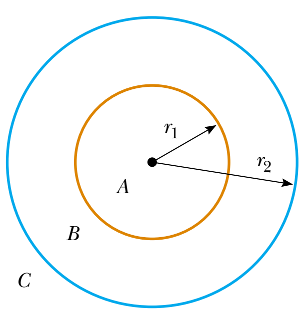

Figure P25.66

67. The x axis is the symmetry axis of a stationary uniformly charged ring of radius R and charge Q (Fig. P25.67). A point charge Q of mass M is located initially at the center of the ring. When it is displaced slightly, the point charge accelerates along the x axis to infinity. Show that the ultimate speed of the point charge is

$$v = \left(\frac{2k_eQ^2}{MR}\right)^{1/2}$$

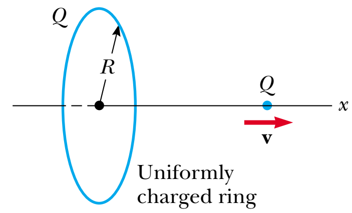

Figure P25.67

68. The thin, uniformly charged rod shown in Figure P25.68 has a linear charge density λ. Find an expression for the electric potential at P.

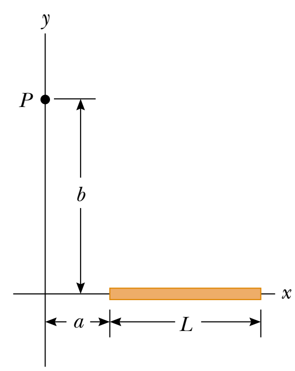

Figure P25.68

69. An electric dipole is located along the y axis as shown in Figure P25.69. The magnitude of its electric dipole moment is defined as $p = 2qa$. (a) At a point P, which is far from the dipole $(r \gg a)$, show that the electric potential is

$$V = \frac{k_e p\cos\theta}{r^2}$$

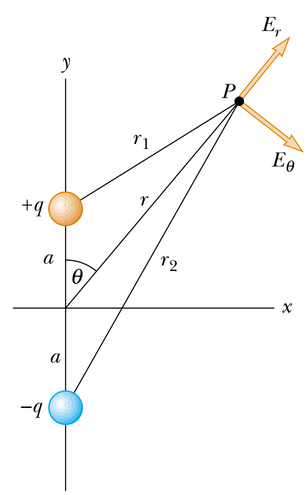

Figure P25.69

(b) Calculate the radial component E_r and the perpendicular component E_θ of the associated electric field. Note that $E_\theta = -(1/r)(\partial V/\partial \theta)$. Do these results seem reasonable for $\theta = 90°$ and $0°$? for $r = 0$? (c) For the dipole arrangement shown, express V in terms of Cartesian coordinates using $r = (x^2 + y^2)^{1/2}$ and

$$\cos\theta = \frac{y}{(x^2 + y^2)^{1/2}}$$

Using these results and again taking $r \gg a$, calculate the field components E_x and E_y.

70. When an uncharged conducting sphere of radius a is placed at the origin of an xyz coordinate system that lies in an initially uniform electric field $\mathbf{E} = E_0\hat{\mathbf{k}}$, the resulting electric potential is $V(x, y, z) = V_0$ for points inside the sphere and

$$V(x, y, z) = V_0 - E_0 z + \frac{E_0 a^3 z}{(x^2 + y^2 + z^2)^{3/2}}$$

for points outside the sphere, where V_0 is the (constant) electric potential on the conductor. Use this equation to determine the x, y, and z components of the resulting electric field.

71. A disk of radius R (Fig. P25.71) has a nonuniform surface charge density $\sigma = Cr$, where C is a constant and r is measured from the center of the disk. Find (by direct integration) the potential at P.

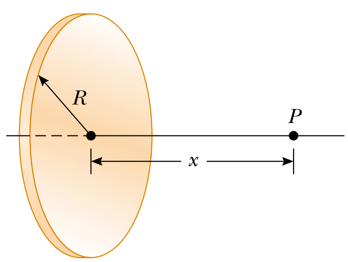

Figure P25.71

72. A solid sphere of radius R has a uniform charge density ρ and total charge Q. Derive an expression for its total electric potential energy. (*Suggestion:* imagine that the sphere is constructed by adding successive layers of concentric shells of charge $dq = (4\pi r^2\, dr)\rho$ and use $dU = V\, dq$.)

73. Charge is uniformly distributed with a density of 100.0 μC/m^3 throughout the volume of a cube 10.00 cm on each edge. (a) Find the electric potential at a distance of 5.000 cm from the center of one face of the cube, measured along a perpendicular to the face. Determine the potential to four significant digits. Use a numerical method that divides the cube into a sufficient number of smaller cubes, treated as point charges. Symmetry considerations will reduce the number of actual calculations. (b) **What If?** If the charge on the cube is redistributed into a uniform sphere of charge with the same center, by how much does the potential change?

Answers to Quick Quizzes

25.1 (b). When moving straight from A to B, $\mathbf{E}$ and $d\mathbf{s}$ both point toward the right. Thus, the dot product $\mathbf{E} \cdot d\mathbf{s}$ in Equation 25.3 is positive and ΔV is negative.

25.2 (a). From Equation 25.3, $\Delta U = q_0\,\Delta V$, so if a negative test charge is moved through a negative potential difference, the potential energy is positive. Work must be done to move the charge in the direction opposite to the electric force on it.

25.3 $B \rightarrow C$, $C \rightarrow D$, $A \rightarrow B$, $D \rightarrow E$. Moving from B to C decreases the electric potential by 2 V, so the electric field performs 2 J of work on each coulomb of positive charge that moves. Moving from C to D decreases the electric potential by 1 V, so 1 J of work is done by the field. It takes no work to move the charge from A to B because the electric potential does not change. Moving from D to E increases the electric potential by 1 V, and thus the field does -1 J of work per unit of positive charge that moves.

25.4 (f). The electric field points in the direction of decreasing electric potential.

25.5 (b) and (f). The electric potential is inversely proportion to the radius (see Eq. 25.11). Because the same number of field lines passes through a closed surface of any shape or size, the electric flux through the surface remains constant.

25.6 (c). The potential is established only by the source charge and is independent of the test charge.

25.7 (a). The potential energy of the two-charge system is initially negative, due to the products of charges of opposite sign in Equation 25.13. When the sign of q_2 is changed, both charges are negative, and the potential energy of the system is positive.

25.8 (a). If the potential is constant (zero in this case), its derivative along this direction is zero.

25.9 (b). If the electric field is zero, there is no change in the electric potential and it must be constant. This constant value *could be* zero but does not *have to be* zero.

25.10 The graph would look like the sketch below. Notice the flat plateaus at each conductor, representing the constant electric potential inside a conductor.