Amat I

Metodos de integracion:

Substitucion

1) $\int 3x^2(x^3+5)^7\,dx$ $\qquad u = x^3+5$

$du = 3x^2\,dx$

$$\int \underbrace{3x^2\,(u)^7\,dx}_{=du} \Rightarrow \int (u)^7\,du$$

remplazo $\qquad$ regla potencia

$$\Rightarrow \frac{u^8}{8} + C \Rightarrow \boxed{\frac{(x^3+5)^8}{8} + C}$$

$$\int (2x-3)^{-7}\,dx = \int u^{-7}\,dx$$

$u = 2x-3 \quad ; \quad du = 2\,dx \Rightarrow \boxed{\frac{du}{2} = dx}$

$$\Rightarrow \int \frac{1}{u^7}\,dx = \int \frac{1}{2u^7}\,du = \frac{1}{2}\int \frac{1}{u^7}\,du$$

$$\frac{1}{2}\left(-\frac{u^{-6}}{6}\right) + C = \frac{1}{2}\left(-\frac{(2x-3)^{-6}}{6}\right) + C$$

$$\frac{1}{2}\left(-\frac{1}{6(2x-3)^6}\right) = \boxed{\frac{-1}{12(2x-3)^6}}$$

$$\int (11x-7)^{-3}\,dx$$

$u = 11x-7$

$du = 11\,dx$

$\frac{du}{11} = dx$

$$\int \frac{1}{u^3}\,dx = \int \frac{1}{11u^3}\,du$$

Sereo constante

$$\frac{1}{11}\int \frac{1}{u^3}\,du \Rightarrow \frac{1}{11}\left(-\frac{1}{2u^2}\right)+C$$

$$\frac{-1}{22\,u^2}+C \xrightarrow{\text{remplazo}} \boxed{\frac{-1}{22(11x-7)^2}+C}$$

Es importante chequear si se puede hacer algo con el diferencial de x para luego reemplazarlo

$$\int \frac{x^2+1}{x^3+3x}\,dx$$

$u = x^3+3x$

$du = (3x^2+3)\,dx$

$du = 3(x^2+1)\,dx$

$\frac{du}{3} = (x^2+1)\,dx$

$$\int \frac{1}{x^3+3x}\cdot(x^2+1)\,dx = \int \frac{1}{u}\,\frac{du}{3}$$

$$\int \frac{du}{3u} = \frac{1}{3}\int \frac{1}{u}\,du \Rightarrow \frac{1}{3}\ln|u|+c$$

$$\frac{\ln|u|}{3}+C \xrightarrow{\text{remp.}} \boxed{\frac{\ln(x^3+3x)}{3}+C}$$

$$\int (2x+5)(x^2+5x)^7\,dx$$

$u = x^2+5x$

$du = (2x+5)\,dx$

$$\int u^7\,du = \frac{u^8}{8}+C = \frac{(x^2+5x)^8}{8}+C$$

$$\int (3-x)^{10}\,dx \qquad u = 3-x$$
$$du = -dx \Rightarrow -du = dx$$

$$\int u^{10}\,dx = \int u^{10}(-1)\,du = -\int u^{10}\,du$$

$$-\frac{u^{11}}{11} + c = \boxed{-\frac{(3-x)^{11}}{11} + c}$$

$$\int e^{5x+2}\,dx \qquad u = 5x+2$$
$$du = 5dx$$
$$\frac{du}{5} = dx$$

$$\| $$

$$\int e^{u}\,dx = \int \frac{e^{u}\,du}{5} = \frac{1}{5}\int e^{u}\,du = \frac{1}{5}e^{u} + C$$

$$\frac{1}{5}e^{5x+2} + C = \boxed{\frac{e^{5x+2}}{5} + C}$$

$$\int \frac{e^{x}}{1+e^{2x}}\,dx \qquad du \qquad u = e^{x}$$
$$du = e^{x}\,dx$$

$$\int \frac{1}{1+(e^{x})^{2}} \cdot e^{x}\,dx \Rightarrow \int \frac{1}{1+u^{2}}\,du$$

substitucion

$$\tan^{-1}u + C \Rightarrow \tan^{-1}(e^{x}) + c$$

$$\int 4x\cos(2-3x)\,dx$$

integration by parts

$\underline{\text{I}}$ late

alg trig

$u = 4x \qquad dv = \cos(2-3x)\,dx$

$\int 4x\cos(2-3x)\,dx$

$du = 4dx$

$v = -\frac{1}{3}(\operatorname{sen}(2-3x))$

$dv = -\frac{1}{\cancel{3}}((\cos(2-3x))\cdot\cancel{-3}$

$dv = \cos(2-3x)\,dx$

$\int u\,dv = uv - \int v\,du$

$= (4x)\left(-\frac{1}{3}(\operatorname{sen}(2-3x))\right) - \int\left(\frac{-\operatorname{sen}(2-3x)}{3}\right) 4\,dx$

$= -\frac{4}{3}x\operatorname{sen}(2-3x) - \int -\frac{4}{3}\operatorname{sen}(2-3x)\,dx$

$= -\frac{4}{3}x\operatorname{sen}(2-3x) + \frac{4}{3}\int \operatorname{sen}(2-3x)\,dx$

$= -\frac{4}{3}x\operatorname{sen}(2-3x) + \frac{4}{3}\left(-\frac{1}{3}\right) - \cos(2-3x) + C$

$= -\frac{4}{3}x\operatorname{sen}(2-3x) + \frac{4}{9}\cos(2-3x) + C$

tengo que fijarme que para integrar una funcion que usa la regla de la cadena tengo que pensarlo como estaba antes de aplicar la regla de la cadena, osea fijarme el numero que anu le la derivada de lo de adentro de la cadena

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$\int (3t+t^2)\operatorname{sen}(2t)\,dt$

$u = 3t + t^2$

$du = (3+2t)\,dx$

$u\,dv = uv - \int v\,du$

$dv = \operatorname{sen}(2t)\,dt$

$v = \frac{1}{2}(-\cos(2t))$

$= (3t+t^2)\left(\frac{-\cos(2t)}{2}\right) - \int \frac{-\cos(2t)}{2}(3+2t)\,dx$

$$= \frac{3t - t^2 \cdot \cos(2t)}{2} + \frac{1}{2}\int \cos(2t)(3+2t)\,dx$$

tengo que integrar por partes de nuevo en la segunda parte de la ecuacion

$u = 3+2t \qquad\qquad dv = \cos(2t)\,dx$

$du = 2\,dx \qquad\qquad v = \frac{1}{2}(\operatorname{sen}(2t))$

$$u\,dv = uv - \int v\,du$$

$$= (3+2t)\frac{1}{2}\operatorname{sen}(2t) - \int \frac{1}{\cancel{2}}\operatorname{sen}(2t)\cancel{2}\,dx$$

$$= \frac{(3+2t)\operatorname{sen}(2t)}{2} - \int \operatorname{sen}(2t)\,dx$$

$$= \frac{(3+2t)\operatorname{sen}(2t)}{2} - \frac{1}{2}(-\cos(2t)) + C$$

$$= \frac{(3+2t)\operatorname{sen}(2t)}{2} + \frac{1}{2}\cos(2t) + C$$

reemplazo con lo que tenia originlamente

$$\frac{-3t - t^2\cos(2t)}{2} + \frac{1}{2}\left(\frac{(3+2t)\operatorname{sen}(2t)}{2} + \frac{\cos(2t)}{2}\right) + C$$

$$= \frac{-(3t + t^2)\cos(2t)}{2} + \frac{(3+2t)\operatorname{sen}(2t)}{4} + \frac{\cos(2t)}{4} + C$$

$$\int t^7 \operatorname{Sen}(2t^4)\,dt \qquad u = t^4$$

$$du = 4t^3\,dt$$

$$\int t^4 \cdot t^3 \operatorname{Sen}(2t^4)\,dt \qquad dv = t^3 \operatorname{Sen}(2t^4)\,dt$$

$$v = \frac{1}{8} \cdot \int -\cos(2t^4)\,dt$$

$$udv = uv - \int v\,du \qquad v = -\frac{1}{8}\cos(2t^4)$$

$$= t^4\left(-\frac{1}{8}\cos(2t^4)\right) - \int -\frac{1}{8}\cos(2t^4) \cdot 4t^3\,dt$$

$$= -\frac{1}{8}\left(t^4\cos(2t^4) + \frac{1}{\cancel{8}}\int \cos(2t^4)\,\cancel{4}t^3\,dt\right.$$

$$= -\frac{1}{8}\left(t^4\cos(2t^4) + \frac{1}{2}\int \cos(2t^4)\,t^3\,dt\right.$$

substitucion u

$$u = 2t^4$$

$$du = 8t^3\,dt$$

$$\frac{du}{8} = t^3\,dt \longrightarrow \int \cos(u)\,\underbrace{t^3\,dt}_{\frac{du}{8}}$$

$$\int \frac{\cos(u)}{8}\,du = \frac{1}{8}\int \cos(u)\,du$$

$$= \frac{1}{8} \cdot \operatorname{Sen}(u) + C = \frac{\operatorname{Sen}(2t^4)}{8} + C$$

finalmente queda

$$-\frac{1}{8}\left(t^4\cos(2t^4) + \frac{1}{\cancel{2}}\left(\frac{\operatorname{Sen}(2t^4)}{\cancel{8}}\right)\right) + C \quad 16$$

$$\int x^2 \operatorname{sen}(x)\,dx$$

$u = x^2$
$du = 2x\,dx$
$dv = \operatorname{sen} x\,dx$
$v = -\cos x$

$uv - uv - \int v\,du$

$= -x^2 \cos x - \int -\cos x \cdot 2x\,dx$

Integro de nuevo

$u = 2x$ $\qquad dv = -\cos x\,dx$
$du = 2\,dx$ $\qquad v = -\operatorname{sen} x$

$2x(-\operatorname{sen} x) - \int -\operatorname{sen} x \cdot 2\,dx$

$2x(-\operatorname{sen} x) - 2\int -\operatorname{sen} x \cdot dx$

$2x(-\operatorname{sen} x) - 2\cos x + C$

$= -2(\ x \operatorname{sen} x + \cos x) + C$

ahora si reemplazo

$-x^2 \cos x - (-2(x \operatorname{sen} x + \cos x)) + C$

$-x^2 \cos x + 2(x \operatorname{sen} x + \cos x) + C$

$$\int x^2 \operatorname{sen}(3x^3 + 2)\,dx$$

Sustitution
$u = 3x^3 + 2$
$du = 9x^2\,dx$
$\frac{du}{9} = x^2\,dx$

$\int x^2 \operatorname{sen}(u)\,dx$

$\int \operatorname{sen}(u) \cdot \frac{du}{9}$

$$\frac{1}{9}\int \underbrace{\operatorname{sen}(u)\,du}_{\text{reemplazo 'u'}} \Longrightarrow \frac{1}{9}(-\cos(u))+C$$

$$\Longrightarrow \frac{1}{9}\left(-\cos(3x^3+2)\right)+C = \frac{-\cos(3x^3+2)}{9}+C$$

$$\int x\,e^{x^2-3}\,dx =$$

$u = x^2-3$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\int x e^u\,dx$$

$$\int e^u \frac{du}{2} = \frac{e^u}{2}+C = \frac{e^{x^2-3}}{2}+C$$

$$\int x^2 \cos\frac{x}{2}\,dx$$

$u = x^2$ $\quad dv = \cos\frac{x}{2}dx$

$du = 2x\,dx$ $\quad v = 2\operatorname{sen}\frac{x}{2}$

$dv = 2\cos\frac{x}{2}\cdot\frac{dx}{2}$

$$\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$$

$dv = \cancel{2}\cos\left(\frac{x}{2}\right)\cdot\frac{1}{\cancel{2}}$

$f(x)=x$ $\quad g(x)=2$

$f'(x)=1$ $\quad g'(x)=0$

$$\frac{1\cdot 2-\cancel{x\cdot 0}}{4} = \frac{\cancel{2}}{\cancel{4}}\ \frac{1}{2}$$

$$u\,dv = uv - \int v\,du \Longrightarrow = x^2\cdot 2\operatorname{sen}\frac{x}{2} - \int 2\operatorname{sen}\frac{x}{2}\,2x\,dx$$

$$= x^2\cdot 2\operatorname{sen}\frac{x}{2} - \int 4\operatorname{sen}\frac{x}{2}\cdot x\,dx$$

$$= x^2\cdot 2\operatorname{sen}\frac{x}{2} - 4\int \operatorname{sen}\frac{x}{2}\cdot x\,dx$$

integro de nuevo

$u = x \qquad dv = \operatorname{sen}\frac{x}{2}\,dx \qquad dv = -\cancel{2}\left(-\operatorname{sen}\frac{x}{2}\right)\frac{1}{\cancel{2}}$

$du = 1dx \qquad v = 2\left(-\cos\frac{x}{2}\right) \qquad = \operatorname{sen}\frac{x}{2}$

$v = -2\cos\frac{x}{2}$

$= x\cdot\left(-2\cos\frac{x}{2}\right) - \int\left(-2\cos\frac{x}{2}\right)dx$

$= -2x\cos\frac{x}{2} + 2\int\cos\frac{x}{2}\,dx$

$= -2x\cos\frac{x}{2} + 2\cdot\operatorname{sen}\left(\frac{x}{2}\right)\cdot 2 + C$

$= -2x\cos\frac{x}{2} + 4\operatorname{sen}\left(\frac{x}{2}\right) + C \Rightarrow$ ahora reemplazo en la integral inicial

$2x^2\operatorname{sen}\frac{x}{2} - 4\left(-2x\cos\frac{x}{2} + 4\operatorname{sen}\frac{x}{2}\right) + C$

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integracino por fracciones parciales

$\int \frac{x^2+1}{x^2-5x+6}\,dx \qquad (x-3)(x-2)$

$\int \frac{x^2+1}{(x-3)(x-2)}\,dx \Longrightarrow \left[\frac{A}{(x-3)} + \frac{B}{(x-2)}\right](x-3)(x-2)$

$A(x-2) + B(x-3) = x^2+1$

Si $x = 2$

$-B = 5 \Rightarrow B = -5$

Si $x = 3$

$A = 9+1 \Longrightarrow A = 10$

$\int\left(\frac{10}{(x-3)} + \frac{(-5)}{(x-2)}\right)dx$

$10\int\frac{1}{x-3}\,dx - 5\int\frac{1}{x-2}\,dx$

$10\ln|x-3| - 5\ln|x-2| + C$

$\int\frac{x}{(x-1)(x-2)^2}\,dx = \left(\frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}\right)(x-1)(x-2)^2$

$$A(x-2)^2 + B(x-1)(x-2) + C(x-1) = x$$

Cuando $x=2$ | Cuando $x=1$
$\boxed{C=2}$ | $A(-1)^2 = 1$
| $\boxed{A=1}$

Sabiendo C y A
reemplazo para saber B
Cuando $x=3$

$$A(1)^2 + B(2)(1) + C(2) = 3$$

$$A + 2B + 2C = 3$$

$$1 + 2B + 4 = 3$$

$$2B = -2$$

$$\boxed{B=-1}$$

entonces la integral queda

$$\int \frac{1}{(x-1)}dx + \int \frac{-1}{(x-2)}dx + \int \frac{2}{(x-2)^2}dx$$

tengo que hacer sustit.

$$\int \frac{1}{(x-1)}dx - \int \frac{1}{(x-2)}dx + 2\int \frac{1}{(x-2)^2}dx$$

$$\int \frac{1}{(x-2)^2}dx \qquad u = x-2 \qquad du = dx$$

$$\int \frac{1}{u^2}dx = \int u^{-2}dx = -\frac{u^{-1}}{1} + C \rightarrow -\frac{1}{u} + C$$

$-\frac{1}{x-2} + c$ reemplazo entonces queda:

$$\ln|x-1| - \ln|x-2| - \frac{2}{x-2} + C$$