Application of Mathematical Methods in the Study of Chemical Dynamical Systems to an Inventory Multiplier-Accelerator Business Cycle Model

Benjamin Bui

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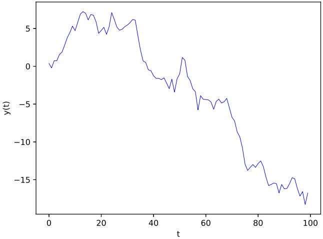
Motivation

- The study of stochastic processes was deeply influential in explaining macroscale themodynamic properties
- This quickly spread into the fields of economics and finance



Motivation

Fractals and chaotic processes are a more recent addition to the chemist's toolkit[1,2]

Could chaos play a role in economics as well?

Overview

Wegener et al.: Inventory cycle model with a Lundberg lag

Tönu Puu: Multiplier-acclerator growth model with a Robertson lag

Benjamin Bui: Inventory Cycles with endogenous investment

Background

Lloyd Metzler developed an inventory cycle model in 1941 in order to explore the effect of a Lundberg lag[3]

Wegener, Westerhoff, and Zaklan expanded on this model in 2009[4]

Consumption and Investment

Investment is held as endogenous

Consumption is a lagged proportion of income

$$I_t = \bar{I} \tag{1}$$

$$C_t = bY_t \tag{2}$$

Inventory

- Desired inventory is a proportion of expected income
- Stock output is made to match desired inventory
- Actual inventory depends on the accuracy of the expectation

$$\hat{Q}_t = kU_t$$
$$S_t = \hat{Q}_t - Q_{t-1}$$
$$Q_t = \hat{Q}_t - (C_t - U_t)$$

Predicting Consumption

- "Extrapolaters" believe that consumption will diverge from the steady-state
- "Regressers" believe that consumption will return to the steady-state
- Firms will choose from the types depending on the consumption level

$$U_t^E = C_{t-1} + c(C_{t-1} - \bar{C})$$
$$U_t^R = C_{t-1} + f(\bar{C} - C_{t-1})$$
$$U_t = w_t U_t^E + (1 - w_t) U_t^R$$
$$w_t = \frac{1}{1 + d(\bar{C} - C_{t-1})^2}$$

Aggregate Output

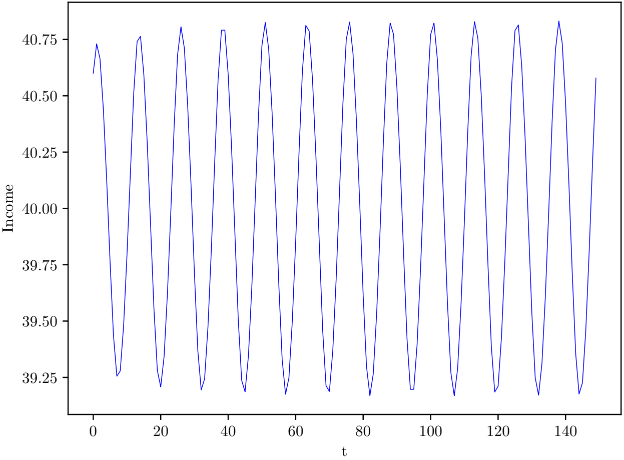
Output is the sum of these production factors

Can solve for a single fixed point

$$Y_t = I_t + U_t + S_t$$

$$\bar{Y} = \frac{1}{1-b}\bar{I}$$

A Possible Trajectory



Framework

Consumption has a Robertson lag

Investment is cubic relative to income change

There is no Lundberg lag

$$C_t = (1-s)Y_{t-1} + sY_{t-2}$$

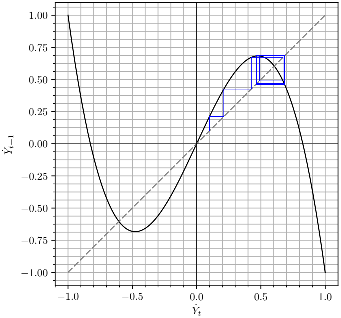
$$I_t = v(Y_{t-1} - Y_{t-2}) - v(Y_{t-1} - Y_{t-2})^3$$

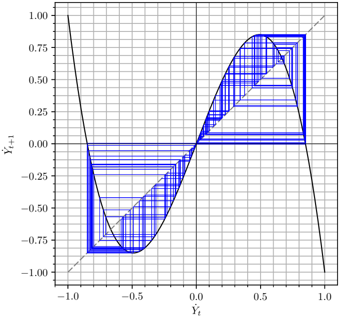
The Growth Model

Change in income is normalized to be within the range (-1, 1)[5]

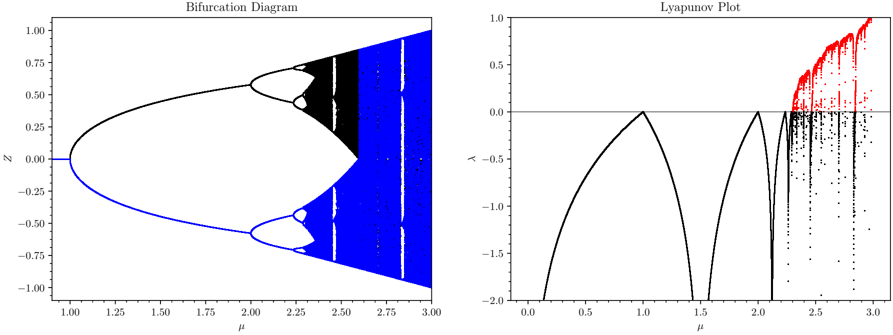
$$\mu \equiv v - s$$
$$\dot{Y}_t = \mu \dot{Y}_{t-1} - (\mu + 1)\dot{Y}_{t-1}^3$$

Qualitatively Different Trajectories Possible





Growth Dynamics



Goal

To incorporate a Roberson and Lundberg lag in the same model

To provide a reasonable basis for the boundedly rational behavior of firms

Consumption and Investment

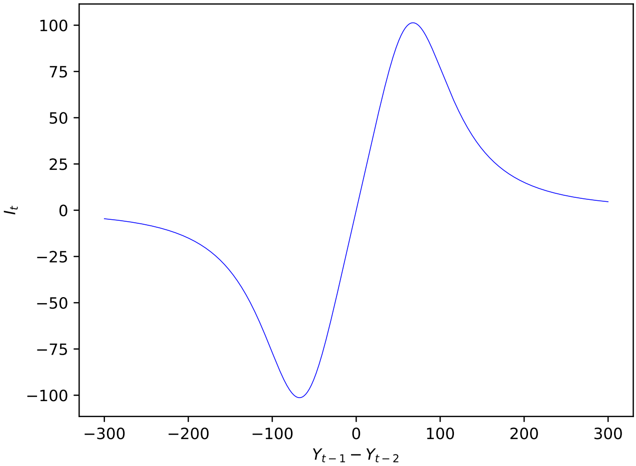
Consumption is identical to the multiplier-accelerator model

The investment function is qualitatively similar but asymptotically approaches 0

$$C_t = (1-s)Y_{t-1} + sY_{t-2}$$

$$I_t = \frac{\frac{Y_{t-1}-Y_{t-2}}{v}}{\frac{Y_{t-1}-Y_{t-2}}{v} + q}$$

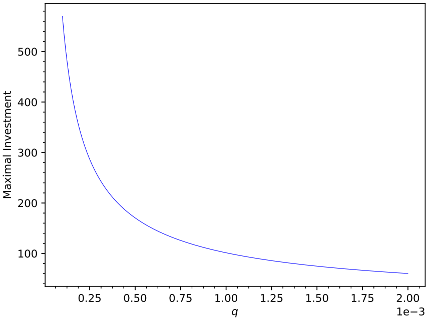
Investment Curve



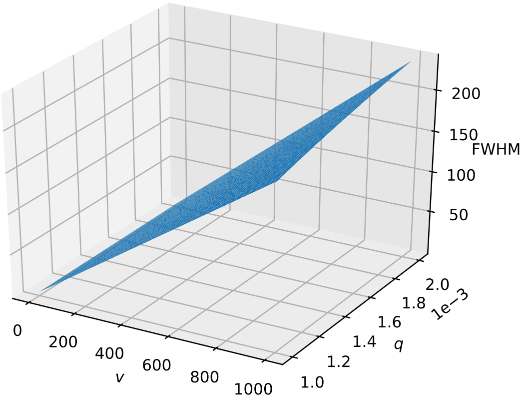
Maximal/Minimal Investment

$$Y_{t-1} - Y_{t-2} = \pm\frac{q^{1/4}v}{3^{1/4}}$$

$$I_t = \frac{3^{3/4}}{3q^{3/4}}$$



Investment FWHM



Inventory

Predicted consumption is an average of lagged consumption

Inventory production is otherwise identical to Wegener's model

$$U_t = \frac{C_{t-1} + C_{t-2} + C_{t-2}}{3}$$

$$S_t = kU_t - Q_{t-1}$$

$$Q_t = Q_{t-1} + S_t + (U_t - C_t)$$

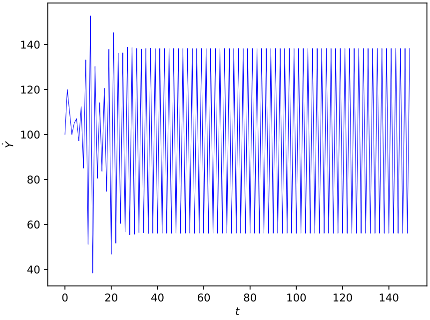
Output Growth

Endogenous investment allows for long-run growth or decay

$$Y_t = I_t + S_t + U_t$$
$$\dot{Y}_t = Y_t - Y_{t-1}$$

Output Growth

$$\dot{Y}_t = \frac{\frac{\dot{Y}_{t-1}}{v}}{\left(\frac{\dot{Y}_{t-1}}{v}\right)^4 + q} - \frac{\frac{\dot{Y}_{t-2}}{v}}{\left(\frac{\dot{Y}_{t-2}}{v}\right)^4 + q} + \frac{k+1}{3}\left[(1-s)(\dot{Y}_{t-2} - Y_{t-5}) + s(\dot{Y}_{t-3} - Y_{t-6})\right] + (1-s)\dot{Y}_{t-2} + s\dot{Y}_{t-3}$$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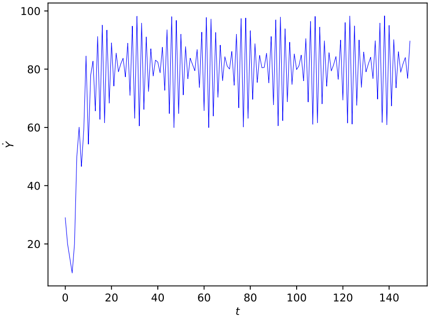
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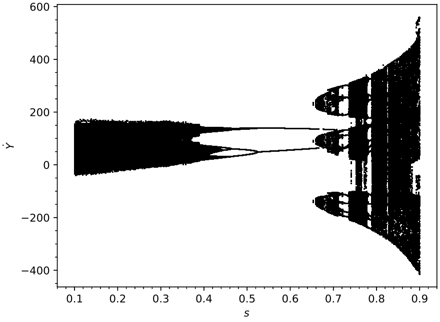
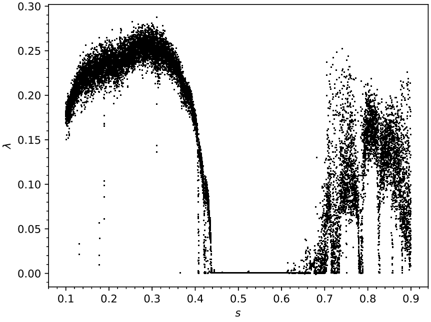
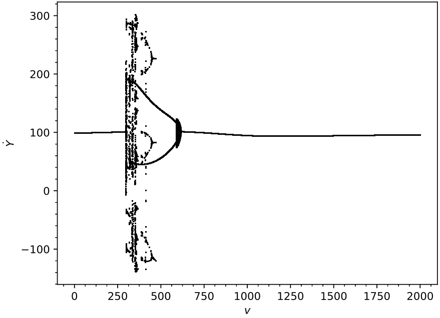
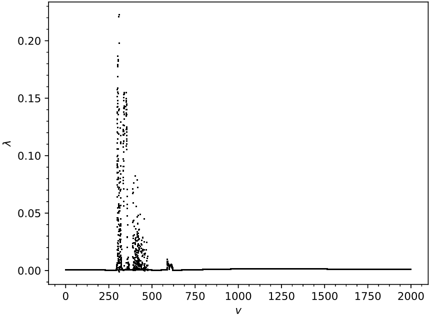
Propensity to Consume



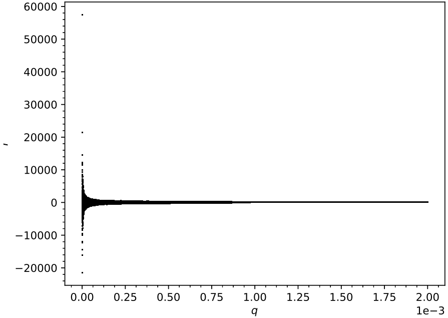


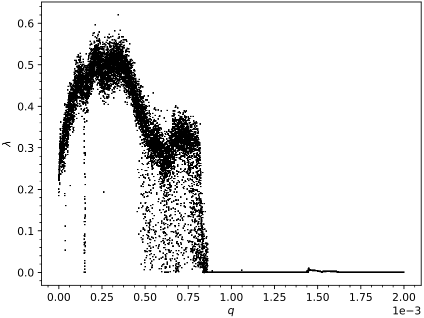
Investment (FWHM)



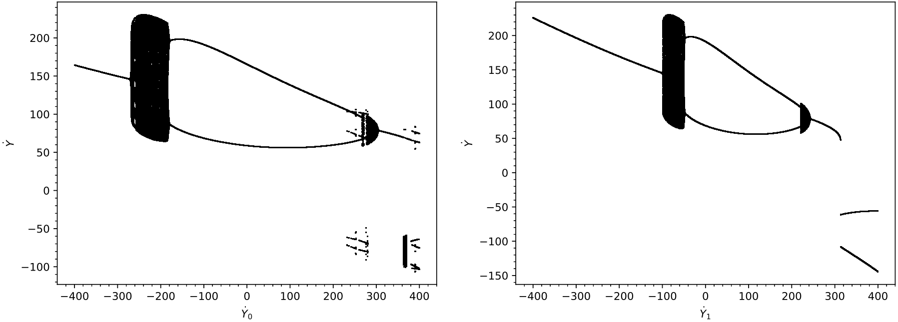


Investment (Maximum/Minimum)





Sensitivity to Initial Conditions



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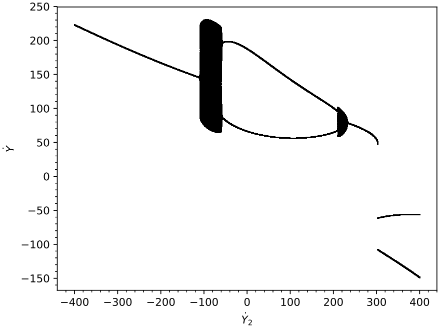

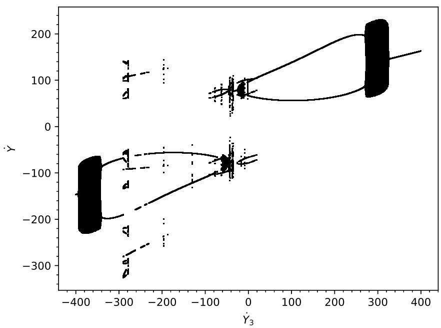

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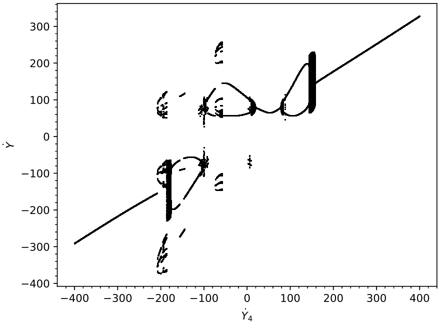

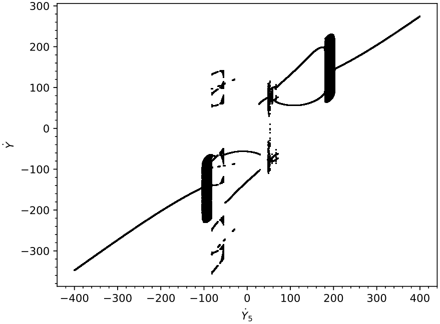

Next steps

- Further investigate the fractal structure of the strange attractors
- Open the economy to foreign trade and improve the investment mechanism
- Improve consumption prediction mechanism

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