

## 1 Framework

Output:

$$Y_t = S_t + I_t + U_t \quad (1)$$

Investment:

$$I_t = \frac{\frac{Y_{t-1} - Y_{t-2}}{v}}{\left(\frac{Y_{t-1} - Y_{t-2}}{v}\right)^4 + q} \quad (2)$$

Consumption:

$$C_t = (1 - s)(Y_{t-1}) + s(Y_{t-2}) \quad (3)$$

Predicted Consumption:

$$U_t = C_{t-1} + \eta_{t-1}(C_{t-1} - C_{t-2}) \quad (4)$$

Adaptive Prediction Parameter:

$$\eta_{t-1} = \frac{C_{t-1} - C_{t-2}}{C_{t-3} - C_{t-4}} \quad (5)$$

Produced for Inventory:

$$S_t = kU_t - Q_{t-1} \quad (6)$$

Inventory Level:

$$Q_t = Q_{t-1} + S_t + (U_t - C_t) \quad (7)$$

## 2 Solving for Growth

Defining Growth:

$$Z_{t-1} = Y_t - Y_{t-1} \quad (8)$$

Consumption in Terms of Growth:

$$C_t - C_{t-1} = (1 - s)Z_{t-2} + sZ_{t-3} \quad (9)$$

Investment in Terms of Growth:

$$I_t - I_{t-1} = \frac{\frac{Z_{t-2}}{v}}{\left(\frac{Z_{t-2}}{v}\right)^4 + q} - \frac{\frac{Z_{t-3}}{v}}{\left(\frac{Z_{t-3}}{v}\right)^4 + q} \quad (10)$$

Adaptive Parameter in Terms of Growth:

$$\eta_{t-1} = \frac{(1 - s)Z_{t-3} + sZ_{t-4}}{(1 - s)Z_{t-4} + sZ_{t-5}} \quad (11)$$

Predicted Consumption in Terms of Growth:

$$U_t - U_{t-1} = [(1 - s)Z_{t-3} + sZ_{t-4}] + \left[ \frac{[(1 - s)Z_{t-3} + sZ_{t-4}]^2}{(1 - s)Z_{t-4} + sZ_{t-5}} \right] - \left[ \frac{[(1 - s)Z_{t-4} + sZ_{t-5}]^2}{(1 - s)Z_{t-5} + sZ_{t-6}} \right] \quad (12)$$

### 2.1 Solving for Inventory Production in terms of Growth

Inventory Level found by substituting  $S_t$

$$\begin{aligned} Q_t &= kU_t - Q_{t-1} + Q_{t-1} + U_t - C_t \\ Q_t &= U_t(k + 1) - C_t \end{aligned} \quad (13)$$

Change in Inventory in Terms of Growth:

$$Q_t - Q_{t-1} = (k + 1)(U_t - U_{t-1}) - (C_t - C_{t-1})$$

$$Q_t - Q_{t-1} = (k+1) \left[ [(1-s)Z_{t-3} + sZ_{t-4}] + \left[ \frac{[(1-s)Z_{t-3} + sZ_{t-4}]^2}{(1-s)Z_{t-4} + sZ_{t-5}} \right] - \left[ \frac{[(1-s)Z_{t-4} + sZ_{t-5}]^2}{(1-s)Z_{t-5} + sZ_{t-6}} \right] \right] - (1-s)Z_{t-2} + sZ_{t-3} \quad (14)$$

Change in Inventory Production in Terms of Growth:

$$\begin{aligned} S_t - S_{t-1} &= k(U_t - U_{t-1}) - (Q_{t-1} - Q_{t-2}) \\ S_t - S_{t-1} &= k \left[ [(1-s)Z_{t-3} + sZ_{t-4}] + \left[ \frac{[(1-s)Z_{t-3} + sZ_{t-4}]^2}{(1-s)Z_{t-4} + sZ_{t-5}} \right] - \left[ \frac{[(1-s)Z_{t-4} + sZ_{t-5}]^2}{(1-s)Z_{t-5} + sZ_{t-6}} \right] \right] - \\ &\quad [(k+1) \left[ [(1-s)Z_{t-4} + sZ_{t-5}] + \left[ \frac{[(1-s)Z_{t-4} + sZ_{t-5}]^2}{(1-s)Z_{t-5} + sZ_{t-6}} \right] - \left[ \frac{[(1-s)Z_{t-5} + sZ_{t-6}]^2}{(1-s)Z_{t-6} + sZ_{t-7}} \right] \right] - (1-s)Z_{t-3} + sZ_{t-4}] \end{aligned} \quad (15)$$

## 2.2 Growth as a 6th Order Difference Equation

$$\begin{aligned} Z_{t-1} &= \frac{\frac{Z_{t-2}}{v}}{\left(\frac{Z_{t-2}}{v}\right)^4 + q} - \frac{\frac{Z_{t-3}}{v}}{\left(\frac{Z_{t-3}}{v}\right)^4 + q} + \\ &\quad [(1-s)Z_{t-3} + sZ_{t-4}] + \left[ \frac{[(1-s)Z_{t-3} + sZ_{t-4}]^2}{(1-s)Z_{t-4} + sZ_{t-5}} \right] - \left[ \frac{[(1-s)Z_{t-4} + sZ_{t-5}]^2}{(1-s)Z_{t-5} + sZ_{t-6}} \right] + \\ &\quad k \left[ [(1-s)Z_{t-3} + sZ_{t-4}] + \left[ \frac{[(1-s)Z_{t-3} + sZ_{t-4}]^2}{(1-s)Z_{t-4} + sZ_{t-5}} \right] - \left[ \frac{[(1-s)Z_{t-4} + sZ_{t-5}]^2}{(1-s)Z_{t-5} + sZ_{t-6}} \right] \right] - \\ &\quad [(k+1) \left[ [(1-s)Z_{t-4} + sZ_{t-5}] + \left[ \frac{[(1-s)Z_{t-4} + sZ_{t-5}]^2}{(1-s)Z_{t-5} + sZ_{t-6}} \right] - \left[ \frac{[(1-s)Z_{t-5} + sZ_{t-6}]^2}{(1-s)Z_{t-6} + sZ_{t-7}} \right] \right] - (1-s)Z_{t-3} - sZ_{t-4}] \end{aligned} \quad (16)$$

$$\begin{aligned} Z_t &= \frac{\frac{Z_{t-1}}{v}}{\left(\frac{Z_{t-1}}{v}\right)^4 + q} - \frac{\frac{Z_{t-2}}{v}}{\left(\frac{Z_{t-2}}{v}\right)^4 + q} + \\ &\quad [(1-s)Z_{t-2} + sZ_{t-3}] + \left[ \frac{[(1-s)Z_{t-2} + sZ_{t-3}]^2}{(1-s)Z_{t-3} + sZ_{t-4}} \right] - \left[ \frac{[(1-s)Z_{t-3} + sZ_{t-4}]^2}{(1-s)Z_{t-4} + sZ_{t-5}} \right] + \\ &\quad k \left[ [(1-s)Z_{t-2} + sZ_{t-3}] + \left[ \frac{[(1-s)Z_{t-2} + sZ_{t-3}]^2}{(1-s)Z_{t-3} + sZ_{t-4}} \right] - \left[ \frac{[(1-s)Z_{t-3} + sZ_{t-4}]^2}{(1-s)Z_{t-4} + sZ_{t-5}} \right] \right] - \\ &\quad [(k+1) \left[ [(1-s)Z_{t-3} + sZ_{t-4}] + \left[ \frac{[(1-s)Z_{t-3} + sZ_{t-4}]^2}{(1-s)Z_{t-4} + sZ_{t-5}} \right] - \left[ \frac{[(1-s)Z_{t-4} + sZ_{t-5}]^2}{(1-s)Z_{t-5} + sZ_{t-6}} \right] \right] - (1-s)Z_{t-2} - sZ_{t-3}] \end{aligned} \quad (17)$$