Parallels Between Chemical Oscillation and Discrete Time Economic Growth Models

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October 5, 2018

Overview

Motivation

Can we apply techniques used in studying chemical oscillations and physical dynamical systems to analyze economic models?

Key Differences

Physical laws make up the foundations of chemical systems

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Macroscale physical systems operate in continuous time

Natural Discrete Time Systems

Ecological systems are often modelled using discrete time equations

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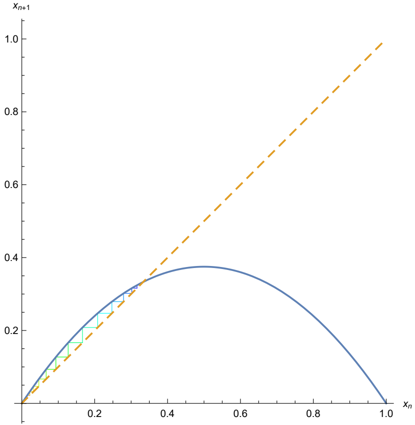
One classically used system is the logistic map[3]

The continuous time analogue is used to model autocatalytic reactions



Background

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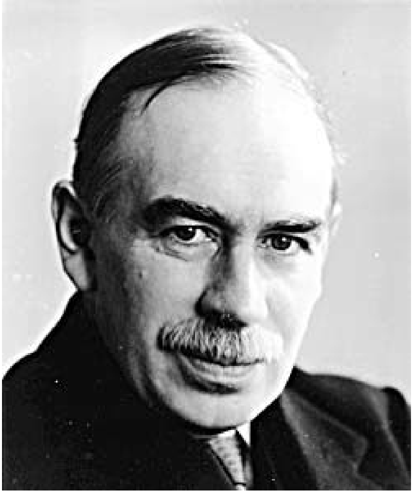
Paul Samuelson and Hicks attempted to formalize a business cycle model[1]



Investment

Capital stock is in a given proportion of production[2]

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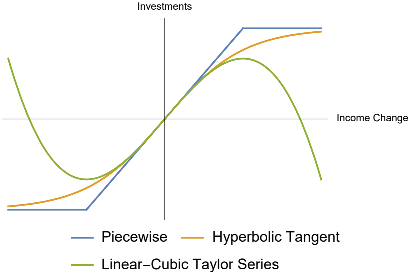
Hicks and Goodwin changed the linear relationship to a hyperbolic tangent curve

This curve can be approximated with a linear-cubic Taylor series expansion



Investment

$$I_t = v(Y_{t-1} - Y_{t-2}) - v(Y_{t-1} - Y_{t-2})^3$$

Consumption

Suppose consumption is a two-period process

All production goes towards consumption and investments

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$$C_t = (1 - s)Y_{t-1} + sY_{t-2}$$
$$Y_t = C_t + I_t$$

Production

Insert the functions for investment and consumption into production

$$Y_t - Y_{t-1} = (v - s)(Y_{t-1} - Y_{t-2}) \\ -v(Y_{t-1} - Y_{t-2})^3$$

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$$Z_{t-1} \equiv Y_t - Y_{t-1}$$

$$Z_t = (v - s)Z_{t-1} - vZ_{t-1}^3$$

A Little Economic Trickery

v can be any arbitrary value by rescaling how we measure income

This allows the function to be contained in a box of interval $[-1, 1]$

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$$Z_t = \mu Z_{t-1} - (\mu + 1) Z_{t-1}^3$$

Fixed Point Stability

Fixed Points

$$Z = 0$$

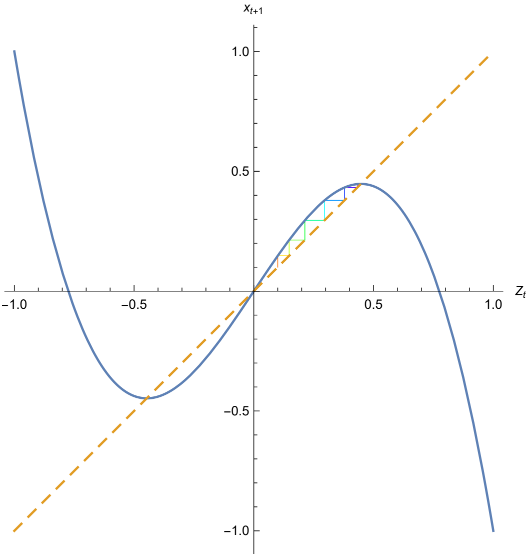
Stability

$$|\mu| < 1$$

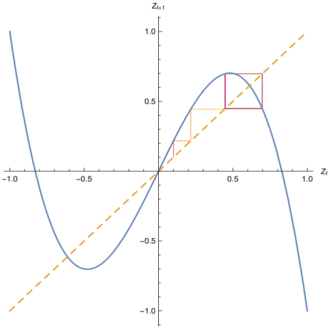
Fixed Point Stability

Fixed Points	**Stability**
$Z = 0$	$\lvert\mu\rvert < 1$
$Z = \pm\sqrt{\frac{\mu - 1}{\mu + 1}}$	$1 < \mu < 2$

Fixed Point Stability



Cyclic Behavior



Confined Chaos

Occurs when parameter reaches Feigenbaum point

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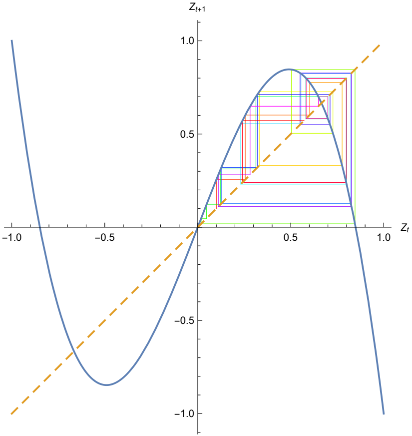
$\mu \approx 2.302$



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Occurs once extremum of
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$$Z = \pm\sqrt{\frac{\mu}{\mu + 1}}$$

$$\mu > \frac{3\sqrt{3}}{2} \approx 2.5981$$

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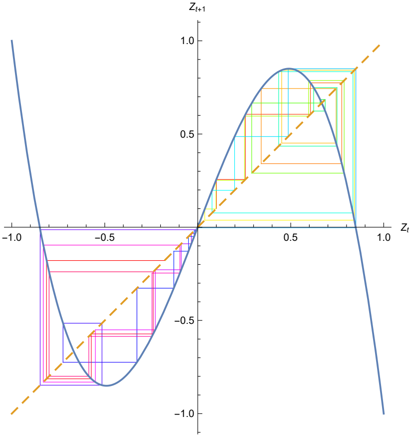
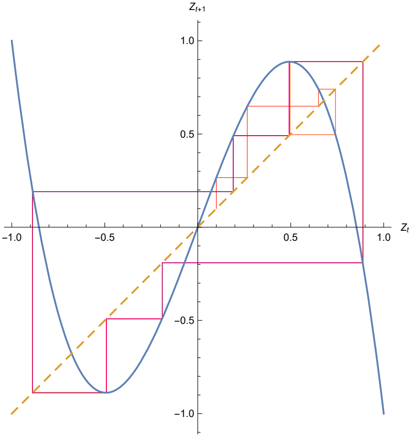


True Cyclic Economy

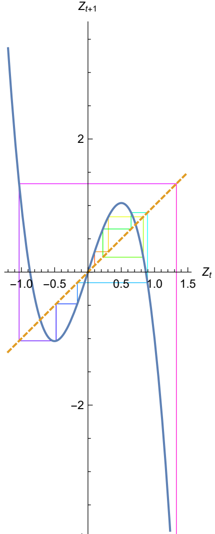
There are windows of order in chaotic regions



Exploding Solutions

Occur once the cubic escapes the box

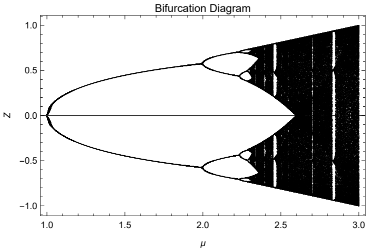
$$\mu > 3$$

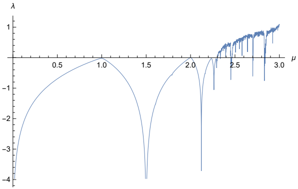


Bifurcations and the Lyapunov Exponent

A way to visualize progression of stability

$$L(Z_0) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{i=n} \ln|f'(Z_{i-1}|)$$





Next Steps

Samuelson-Hicks is a very simplified model

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Kaldor and Solow-Swann

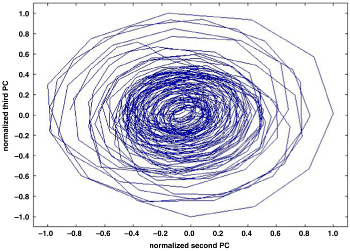

Fig. 4. Strange attractor obtained with RRChaos [17].

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