

Parallels Between Chemical Oscillation and Discrete Time Economic Growth Models

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Introduction
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The Framework
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Quantitative Analysis
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Next Steps
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References

Overview

Introduction

The Framework

Quantitative Analysis

Motivation

Can we apply techniques used in studying chemical oscillations and physical dynamical systems to analyze economic models?

Key Differences

Physical laws make up the foundations of chemical systems

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Physical laws make up the foundations of chemical systems
Macroscale physical systems operate in continuous time

Natural Discrete Time Systems

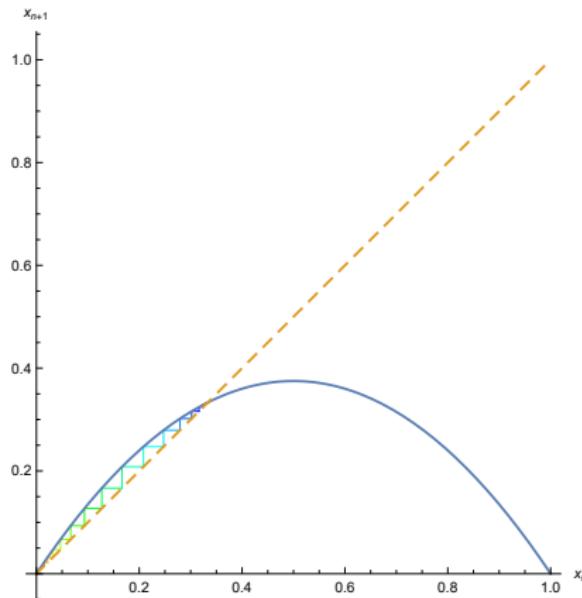
Ecological systems are often modelled using discrete time equations

Natural Discrete Time Systems

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One classically used system is the logistic map¹

The continuous time analogue is used to model autocatalytic reactions

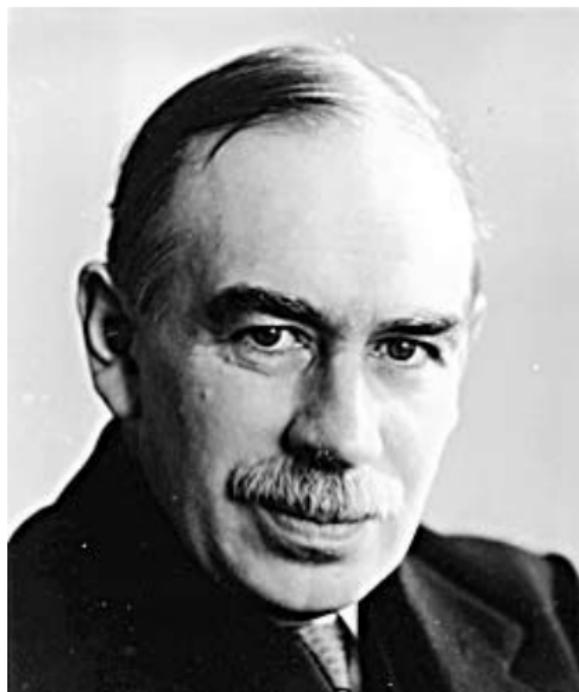


Background

John Maynard Keynes

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John Maynard Keynes
Paul Samuelson and Hicks
attempted to formalize a
business cycle model²



Investment

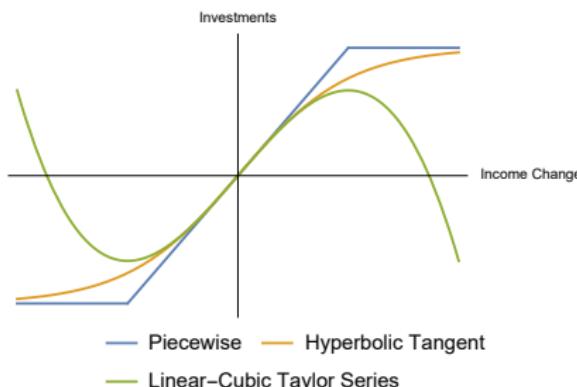
Capital stock is in a given proportion of production³

Investment

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Hicks and Goodwin changed the linear relationship to a hyperbolic tangent curve

This curve can be approximated with a linear-cubic Taylor series expansion



Investment

$$I_t = v(Y_{t-1} - Y_{t-2}) - v(Y_{t-1} - Y_{t-2})^3$$

Consumption

Suppose consumption is a two-period process

All production goes towards consumption and investments

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$$\begin{aligned}C_t &= (1 - s)Y_{t-1} + sY_{t-2} \\Y_t &= C_t + I_t\end{aligned}$$

Production

$$Y_t - Y_{t-1} = (v - s)(Y_{t-1} - Y_{t-2}) - v(Y_{t-1} - Y_{t-2})^3$$

Insert the functions
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$$Z_{t-1} \equiv Y_t - Y_{t-1}$$

$$Z_t = (v - s)Z_{t-1} - vZ_{t-1}^3$$

A Little Economic Trickery

v can be any arbitrary value
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$$\mu \equiv (v - s)$$

This allows the function to
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$$Z_t = \mu Z_{t-1} - (\mu + 1)Z_{t-1}^3$$

Fixed Point Stability

Fixed Points

$$Z = 0$$

Stability

$$|\mu| < 1$$

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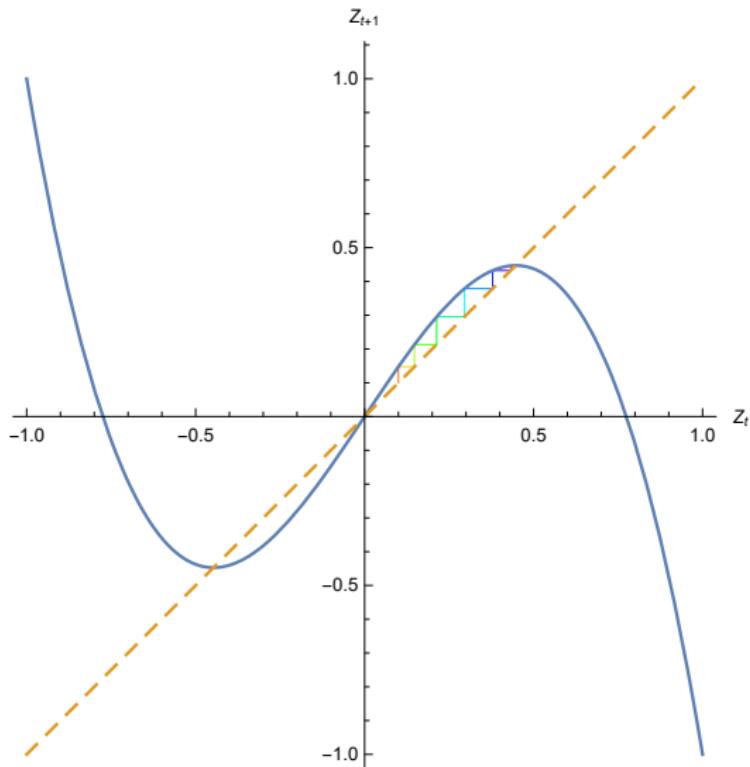
$$Z = \pm \sqrt{\frac{\mu - 1}{\mu + 1}}$$

Stability

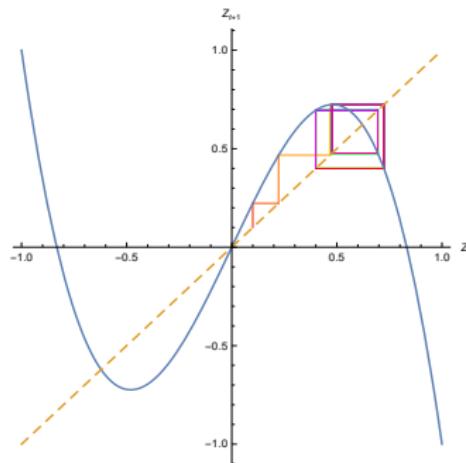
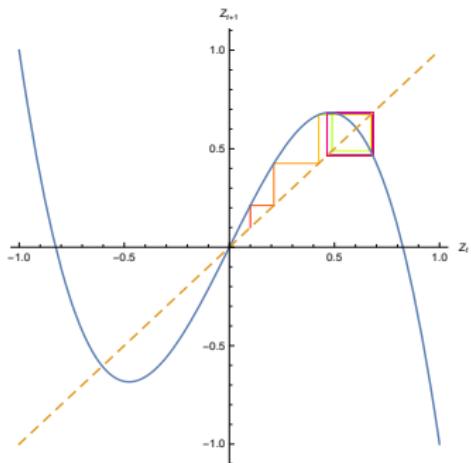
$$|\mu| < 1$$

$$1 < \mu < 2$$

Fixed Point Stability



Cyclic Behavior



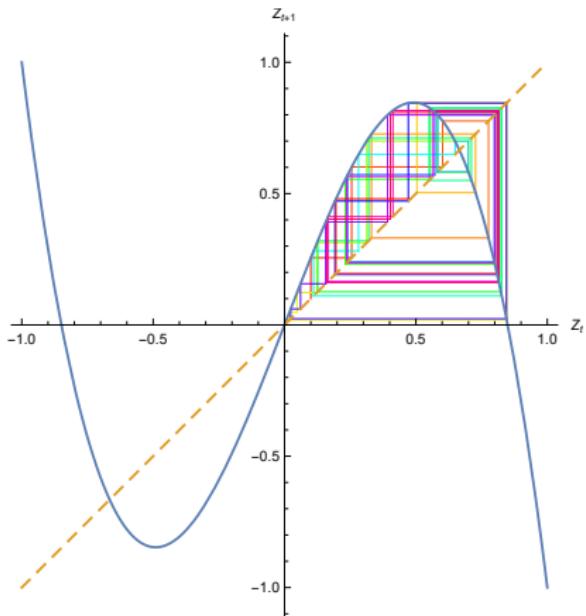
Confined Chaos

Occurs when parameter reaches Feigenbaum point

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$$\mu \approx 2.302$$



Spillover Chaos

Occurs once extremum of
the cubic marginally exceed
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$$f'(Z) = \mu - 3(\mu + 1)Z^2 = 0$$

$$Z = \pm \sqrt{\frac{\mu}{\mu + 1}}$$

$$\mu > \frac{3\sqrt{3}}{2} \approx 2.5981$$

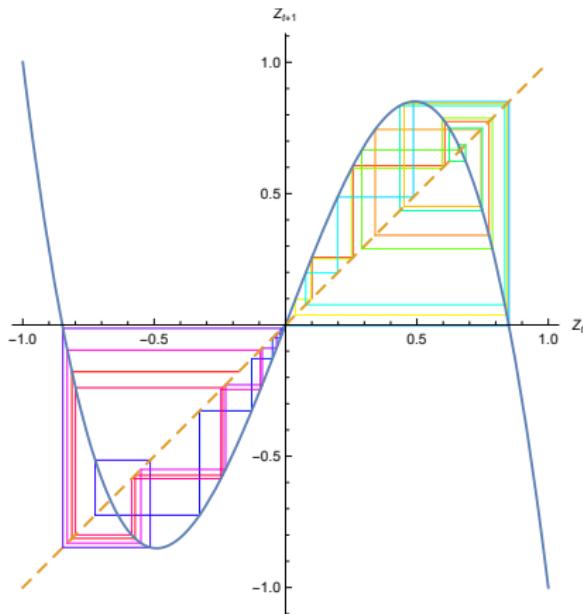
Spillover Chaos

Occurs once extremum of the cubic marginally exceed where the cubic has a zero

$$f'(Z) = \mu - 3(\mu + 1)Z^2 = 0$$

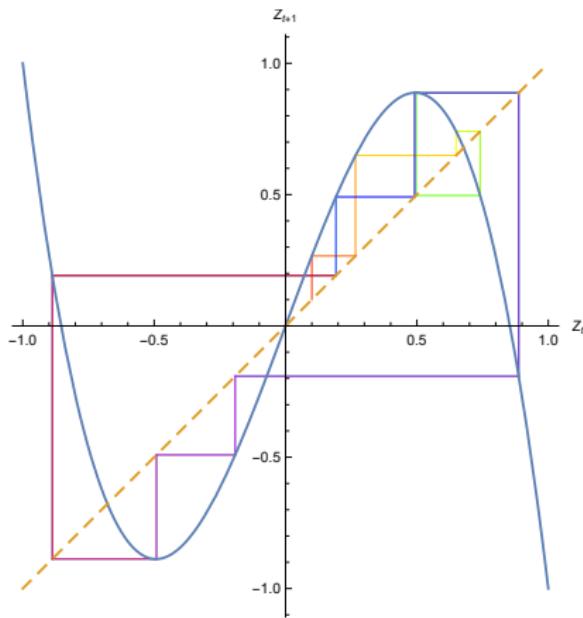
$$Z = \pm \sqrt{\frac{\mu}{\mu + 1}}$$

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True Cyclic Economy

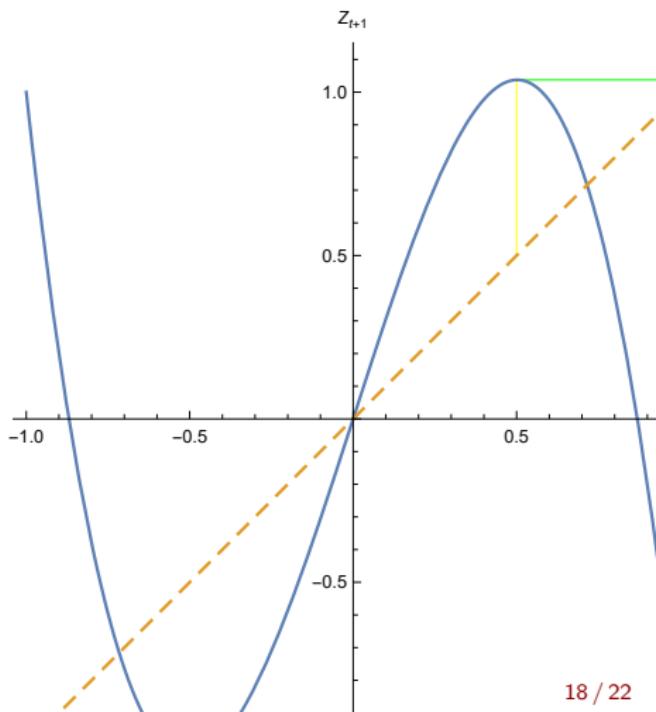
There are windows of order in chaotic regions



Exploding Solutions

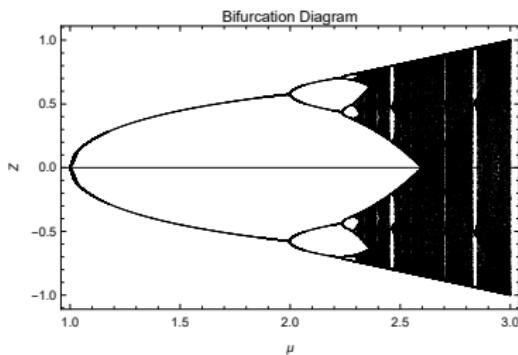
Occur once the cubic
escapes the box

$$\mu > 3$$

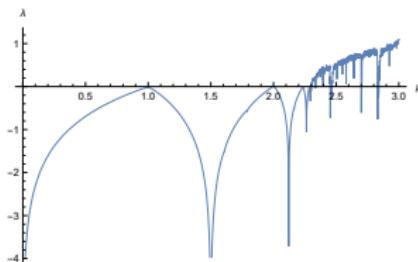


Bifurcations and the Lyapunov Exponent

A way to visualize
progression of stability



$$L(Z_0) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^{i=n} \ln|f'(Z_{i-1})|$$



Next Steps

Samuelson-Hicks is a very simplified model

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Kaldor and Solow-Swann

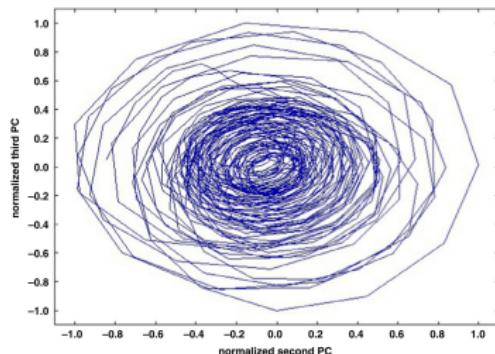


Fig. 4. Strange attractor obtained with RRChaos [17].

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