# 1 Base Model

## Set-Up

$$Y_t = I_t + S_t + U_t$$

National income $Y$ is equal to the sum of consumption goods produced for sale $U$, inventory stock $S$, and investment goods $I$. This model fixes investment good production to such that $I_t = \bar{I}$.

Inventory stock is adjusted by producers every period in order to achieve a desired level of inventory $\hat{Q}_t$. $S_t$ inventory is purchased to raise inventory stock to this desired level:

$$S_t = \hat{Q}_t - Q_{t-1}$$

Let the desired level of inventory be positively related to expected sale of consumption goods:

$$\hat{Q}_t = kU_t$$

Realized inventory does not always match desired level of inventory due to unexpected changes in inventory. These changes occur when actual consumption does not match expected consumption, i.e. $U_t > C_t$. This gives the relationship:

$$Q_{t-1} = \hat{Q}_{t-1} - (C_{t-1} - U_{t-1})$$

As taxes are not a factor of this economy, consumption is a fixed proportion of income.

$$C_t = bY_t$$

$0 < b < 1$ is the aggregate marginal propensity to consume.

Producers must thus predict consumption behavior. This model uses 2 different expectation formation rules that they can switch between: $U_t^E$ and $U_t^R$. The first strategy involved the firms attempting to extrapolate future consumption:

$$U_t^E = C_{t-1} + c(C_{t-1} - \bar{C})$$

such that $\bar{C}$ is the equilibrium level of consumption and $0 \leq c$ is a measure of speed of expected deviation from equilibrium.

The other strategy has producers who believe in a return to equilibrium:

$$U_t^R = C_{t-1} + f(\bar{C} - C_{t-1})$$

such that $0 \leq f \leq 1$. $f = 1$ denotes a state where it is believed that the economy will immediately return to equilibrium.

Aggregate expected consumption is a weighted average of the two strategies:

$$U_t = w_t U_t^E + (1 - w_t) U_t^R$$

The distribution is determined by the weight in the time period:

$$w_t = \frac{1}{1 + d(\bar{C} - C_{t-1})^2}$$

Intuitively, the farther from equilibrium the current state, the more producers will believe that production will shift back towards equilibrium. Likewise, at equilibrium, firms will choose to extrapolate. $d$ indicates the popularity of extrapolation.

## Model

Combining equations gives the function for income:

$$Y_t = U_t + kU_t - (1+k)U_{t-1} + C_{t-1} + \bar{I}$$

This gives the fixed point:

$$\bar{Y} = \frac{1}{1-b}\bar{I}$$

which is analogous to the Keynesian multiplier solution. By parameter restrictions, the model lacks stability iff

$$1 - b(1+c)(1+k) > 0$$

## List of Equations

$$Y_t = \bar{I} + S_t + U_t$$
$$S_t = \hat{Q}_t - Q_{t-1}$$
$$\hat{Q}_t = kU_t$$
$$Q_{t-1} = \hat{Q}_{t-1} - (C_{t-1} - U_{t-1})$$
$$U_t^E = C_{t-1} + c(C_{t-1} - \bar{C})$$
$$U_t^R = C_{t-1} + f(\bar{C} - C_{t-1})$$
$$U_t = w_t U_t^E + (1-w_t)U_t^R$$
$$w_t = \frac{1}{1 + d(\bar{C} - C_{t-1})^2}$$
$$Y_t = U_t + kU_t - (1+k)U_{t-1} + C_{t-1} + \bar{I}$$

## Possible Expansions

This model is similar in terms of economic scope to the multiplier-accelerator model in that it only takes into account consumption and investment. Government expenditures and taxes can be incorporated into the model in order to incorporate fiscal policy. The investment goods condition can also be relaxed and allowed to vary, perhaps via a system similar to the Solow growth model. This would allow for a naive incorporation of monetary policy via control of interest rates; however, this may introduce theoretical complications due to the debate on the effect of nominal interest rates on real interest rates.

The key aspect of the model given by producer expectations can also be expanded to allow for other possible game strategies or to incorporate learning into the model.

# 2 Expanded with Cobb-Douglas Production

Based on the Metzlerian model previously explained. This alternative model attempts to provide endogenous change to investment in addition to the pre-existing endogenous inventory change by way of a Cobb-Douglas production taking capital as input similar to a Solow model. Labor is allowed to grow exogenously for the model to experience growth in output.

## Mathematical Setup

$$
\begin{aligned}
Y_t &= I_t + U_t + S_t \\
Y_t &= K_t^{\alpha} L_t^{1-\alpha} \\
L_t &= (1+n)L_{t-1} \\
K_t &= I_{t-1} + (1-\delta)K_{t-1} \\
C_t &= cY_t \\
S_t &= \hat{Q}_t - Q_{t-1} \\
\hat{Q}_t &= kU_t \\
Q_{t-1} &= \hat{Q}_{t-1} - (C_{t-1} - U_{t-1}) \\
U_t^E &= C_{t-1} + c(C_{t-1} - \bar{C}) \\
U_t^R &= C_{t-1} + f(\bar{C} - C_{t-1}) \\
U_t &= w_t U_t^E + (1-w_t)U_t^R \\
w_t &= \frac{1}{1 + d(\bar{C} - C_{t-1})^2}
\end{aligned}
$$