Monte Carlo Simulation of View Factors for Differential and Finite Areas

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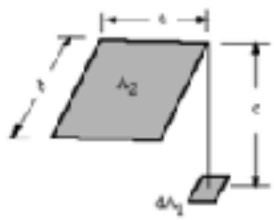
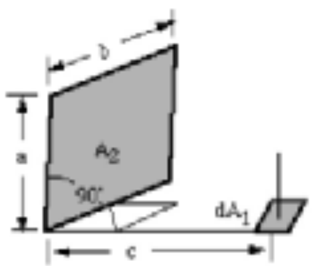
Problem: Determine view factors for the given configurations below, where A_2 is located at a point (x, y, z) and dA_1 is a diffuse, grey emitter.





Monte Carlo Solution:

From the Stefan-Boltzmann Law, the probability density function for emitting a bundle of wavelength λ at an angle (θ, ϕ) is

$$p(\lambda, T, \theta, \phi) = \frac{I_{\lambda,b}\cos\theta}{\sigma T^4}$$

Note that the emissivity ϵ cancels because dA_1 is diffuse and gray. If we consider the PDF to have the following form,

$$p(\lambda, T, \theta, \phi) = p_{\lambda T}\, p_\theta\, p_\phi$$

We can separate our function and integrate to obtain the cumulative distribution functions

$$R_\theta = \sin^2(\theta)$$
$$R_\phi = \frac{\phi}{2\pi}$$

λT can be disregarded, since it has not impact on the exchange factor. Solving for the angles (θ, ϕ), the final position of the bundle can be found by converting from spherical coordinates.

For case 1:

$$z_2 = r\cos(\theta) = z \Rightarrow r = z\sec(\theta)$$
$$x_2 = r\sin(\theta)\cos(\phi) = z\tan(\theta)\cos(\phi)$$

$$y_2 = r\,\sin(\theta)\sin(\phi) = z\,\tan(\theta)\sin(\phi)$$

If $x \leq x_2 \leq x + D_x$ and $y \leq y_2 \leq y + D_y$, the bundle hits.

For case 2:

$$x_2 = r\,\sin(\theta)\cos(\phi) = x \Rightarrow r = x\,\csc(\theta)\sec(\phi)$$
$$y_2 = x\,\tan(\phi)$$
$$z_2 = x\,\cot(\theta)\sec(\phi)$$

Like the first case, the bundle must hit at $y \leq y_2 \leq y + D_y$ and $z \leq z_2 \leq z + D_z$, but there is the additional condition that $z \geq 0$ due to the arrangement of the areas.

Solution by superposition

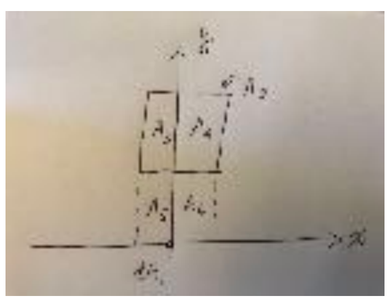
For the first position, if A_2 has a corner at the origin, the view factor is

$$F_{12} = \frac{1}{2\pi}\left(\frac{A}{\sqrt{1+A^2}}\tan^{-1}\left(\frac{B}{\sqrt{1+A^2}}\right) + \frac{B}{\sqrt{1+B^2}}\tan^{-1}\left(\frac{A}{\sqrt{1+B^2}}\right)\right)$$

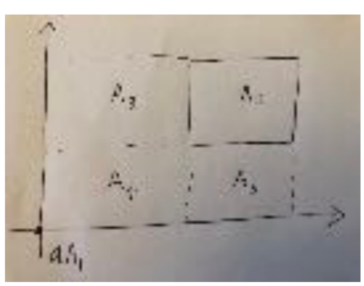
Where

$$A = \frac{D_x}{D_z}$$
$$B = \frac{D_y}{D_z}$$

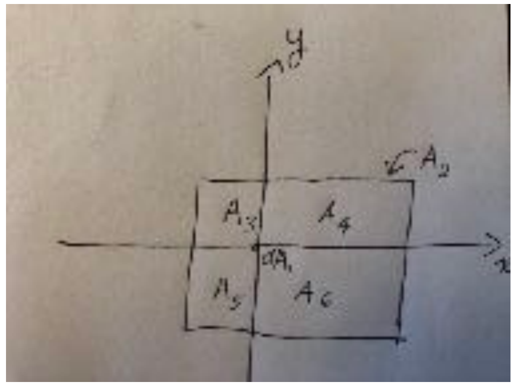
If A_2 is not centered at the origin, we can calculate the solution by superposition, as shown in the examples below.



$$F_{12} = F_{1-(3,5)} + F_{1,(4,6)} - F_{15} - F_{16}$$



$$F_{12} = F_{1-(2,3,4,5)} - F_{1-(3,4)} - F_{1-(4,5)} + F_{14}$$

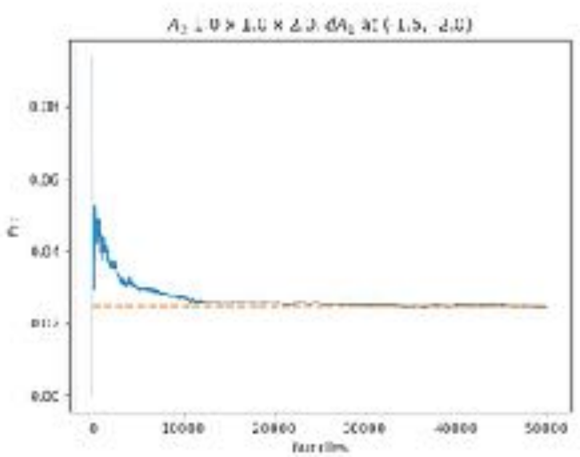


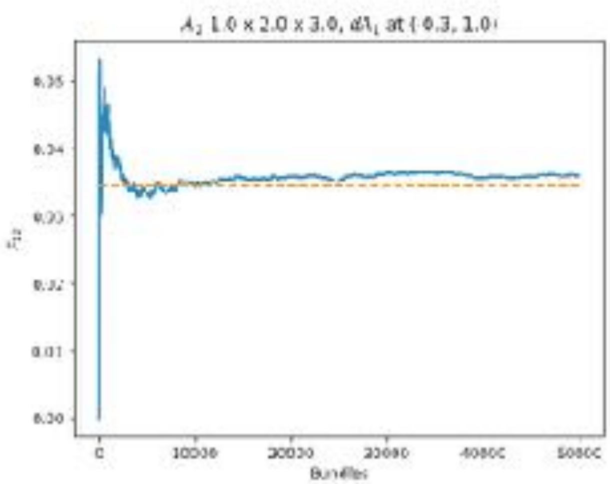
$$F_{12} = F_{13} + F_{14} + F_{15} + F_{16}$$

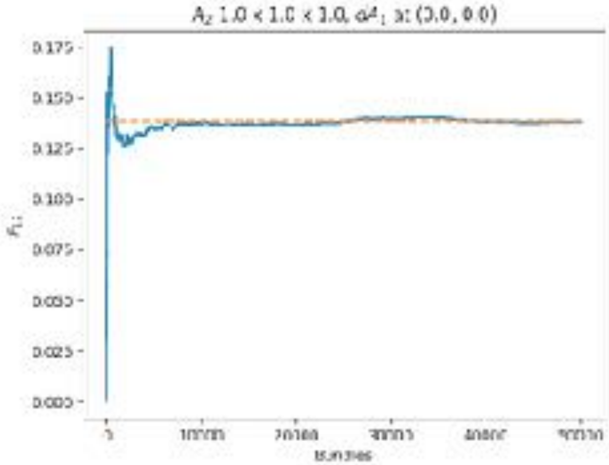
If solving for configuration one, only the area above $z \geq 0$ must been considered when setting up the areas used to calculate the view factor.

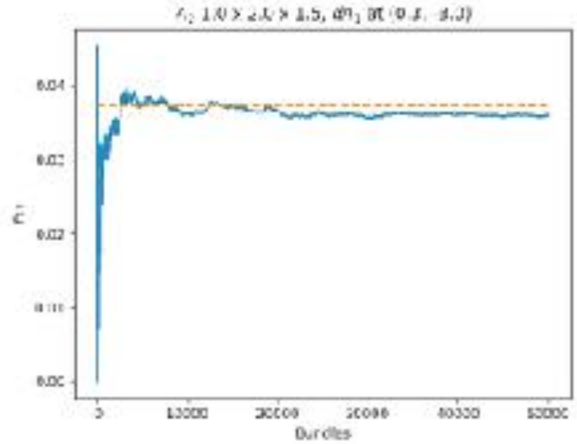
Solutions for Case 1:

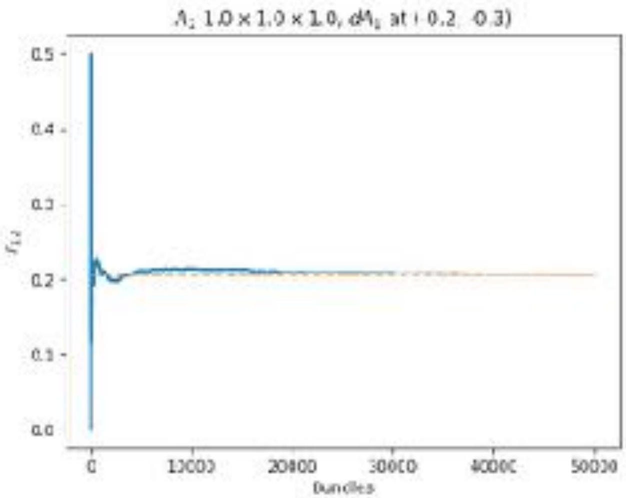
The following plots are results of simulations based on the solutions above. On each plot, the title lists the dimensions and location of A_2 in space. The orange line represents the exact solution which the Monte Carlo solution, given as a blue line, must converge to as the number of bundles increases.

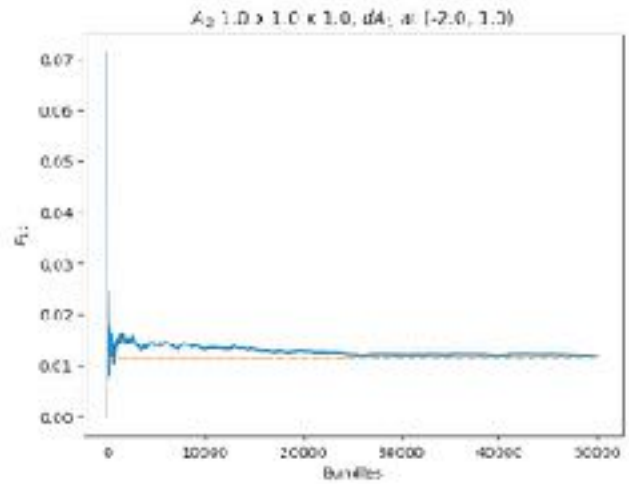






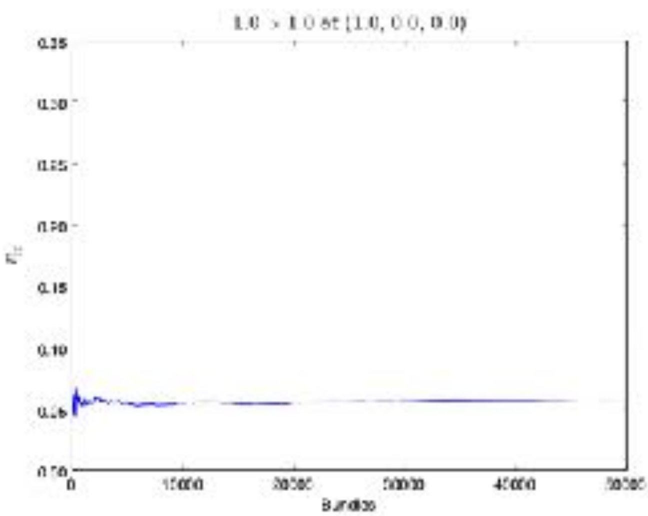


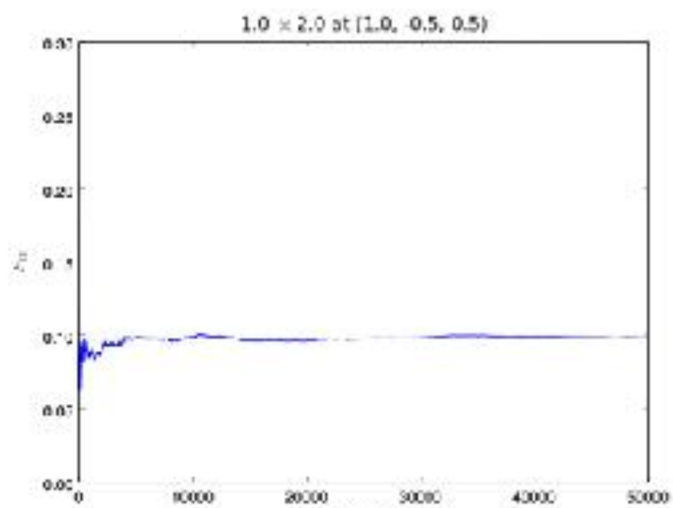


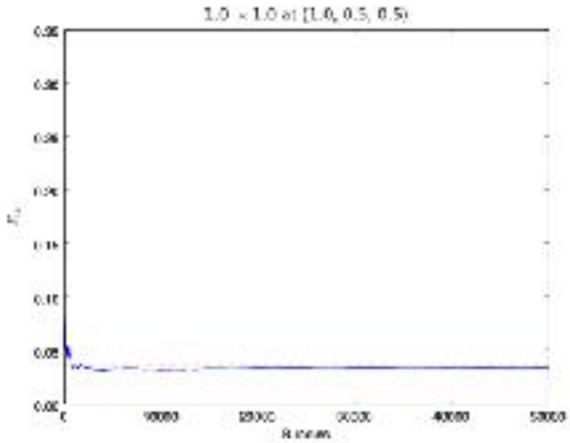


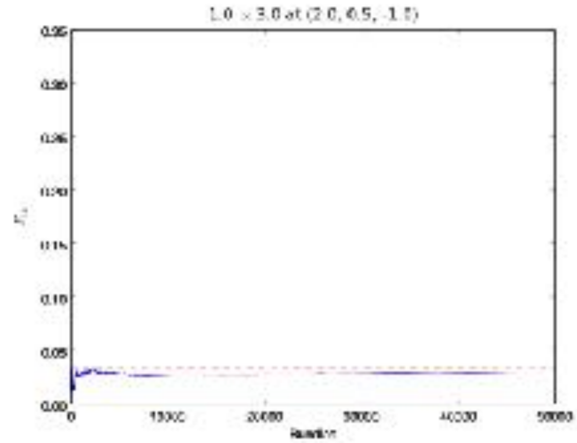
Solutions for Case 2:

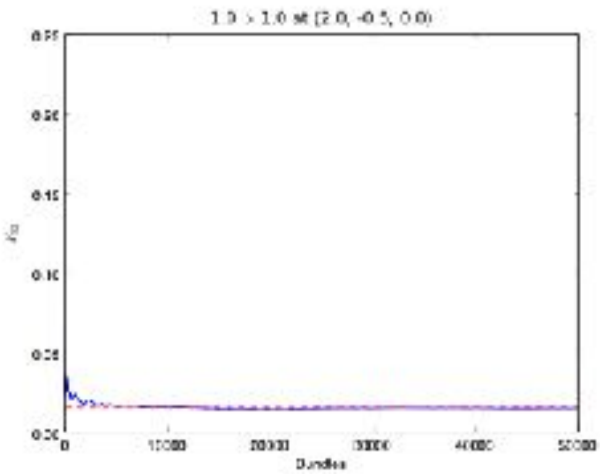
The plots show the results for various configurations as explained above.

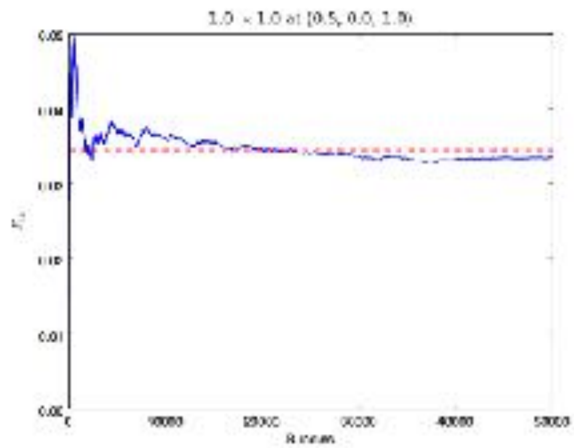












Conclusions:

The view factor calculations seem to converge rapidly for most configurations. The view factors for the second configuration are much lower, which makes sense considering Lambert's Cosine Law states that the radiation is less intense at higher polar angles.