CHAPTER 4

Radiation Processes and Properties

Objectives

To learn the basic radiation processes and terminology associated with radiative properties of materials so that one can determine

a. the radiative emission from a surface as a function of direction, wavelength and temperature
b. Blackbody and the Planck Function
c. the absorption and transmission of radiation by a surface
d. the energy balance at a surface including the effect of radiation

Major Topics

1. Effect of surface orientation on radiative exchange (Ex. 12.1, Hmwk. Text 12.9)
2. Concept of spectral irradiation and total irradiation, spectral emission and total emission, characteristics of the Planck Function (Ex. 12.2, 12.4, 12.6, Hmwk, Text 12.20, 12.34, 12.54)
3. Concepts of absorptivity and reflectivity (Ex. 12.8, 12.9, greenhouse effect example, Hmwk, Text 12.51)

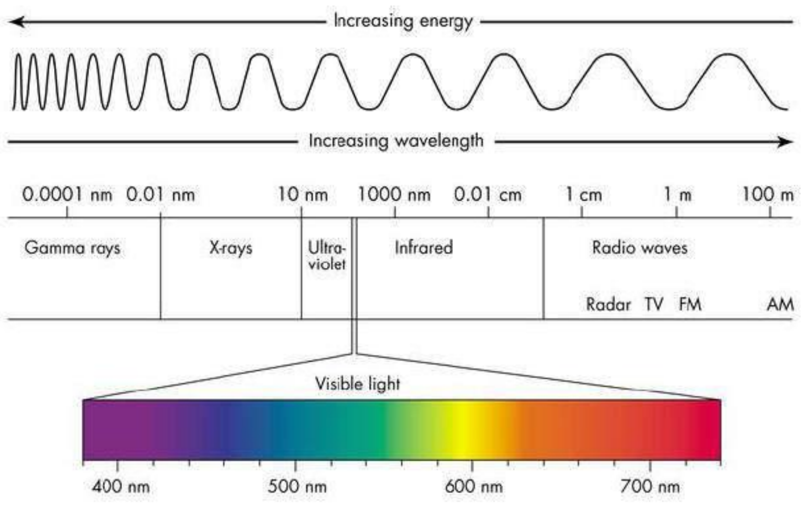
Fundamental Concepts

1. Radiation is a form of energy, it is the electromagnetic radiation emitted by a body as a result of its temperature. (All materials with temperature T > 0°K emits radiation)

2. Materials emit radiation in all direction, away from the surface

3. Radiation can be treated as "photon" particles traveling at the constant speed of light (c = 3 x 10^{10} cm/sec.) In a vacuum, radiation travels in a straight line and is un-attenuated.

Fundamental Concepts

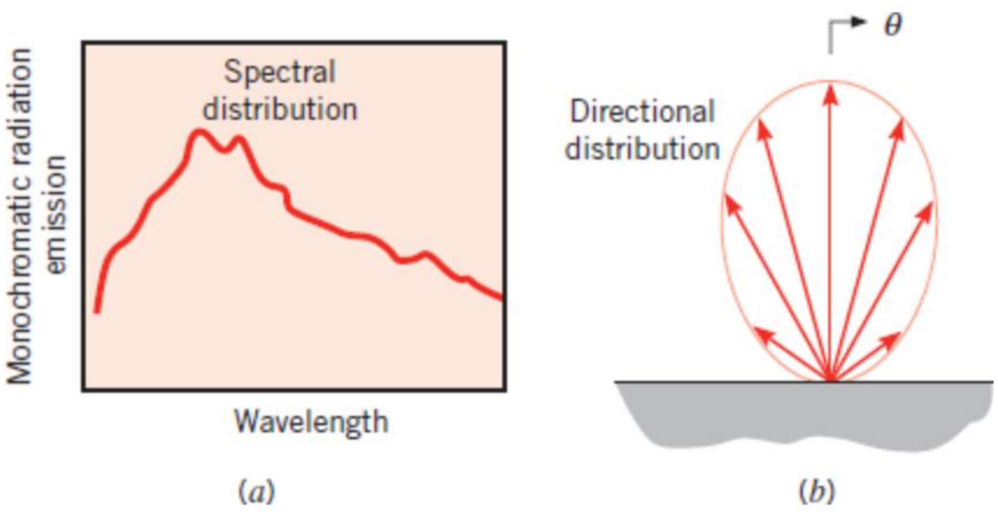


- Radiation is characterized by its <u>wavelength</u> λ, and <u>frequency</u> ν, which are related by $c = \lambda\nu$.

- Energy per photon is $e = h\nu/2\pi$,
  - higher energy per photon (x-ray, gamma ray), higher damage and thus higher danger to health
  - Low energy radiation (radio, microwave, cell phone), generally expected to have no harmful effect
  - In heat transfer, thermal radiation is generally in the infrared

Fundamental Concepts

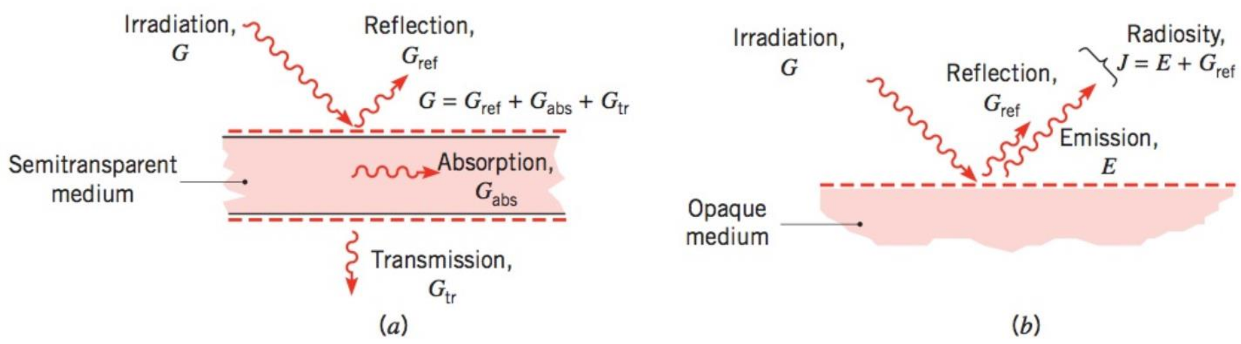
- Radiation emitted by a surface depends on both wavelength and direction



Surface Radiative Heat Fluxes

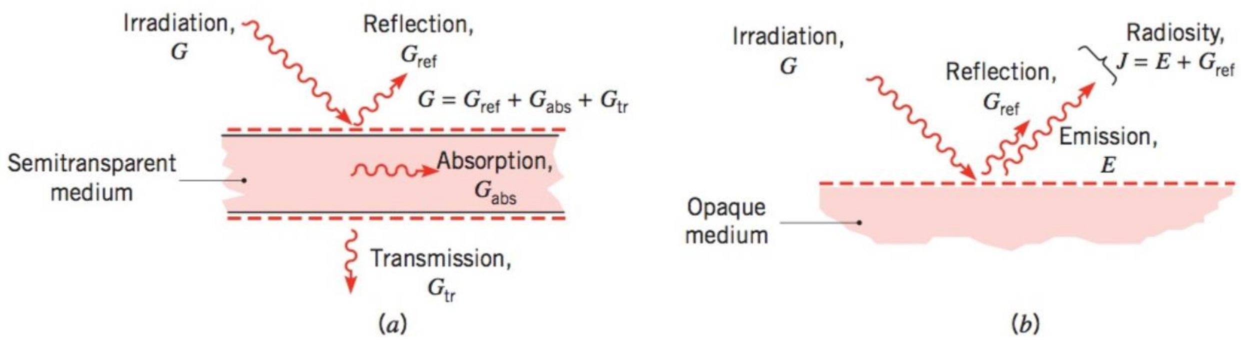
TABLE 12.1 Radiative fluxes (over all wavelengths and in all directions)

Flux (W/m²)	Description	Comment
Emissive power, E	Rate at which radiation is emitted from a surface per unit area	$E = \varepsilon \sigma T_s^4$
Irradiation, G	Rate at which radiation is incident upon a surface per unit area	Irradiation can be reflected, absorbed, or transmitted
Radiosity, J	Rate at which radiation leaves a surface per unit area	For an opaque surface $J = E + \rho G$
Net radiative flux, $q''_{rad} = J - G$	Net rate of radiation leaving a surface per unit area	For an opaque surface $q''_{rad} = \varepsilon \sigma T_s^4 - \alpha G$



Surface Radiative Heat Fluxes

Surface properties



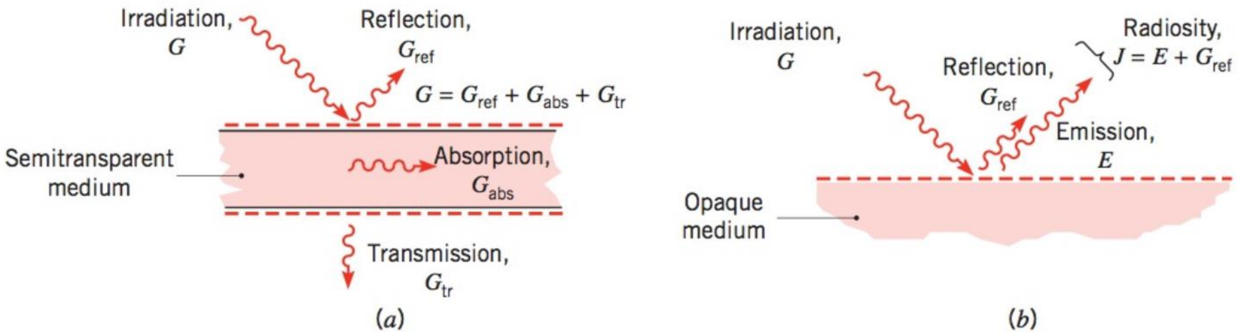
reflectivity, ρ = fraction of the irradiation which is reflected
absorptivity, α = fraction of the irradiation which is absorbed
Transmissivity, τ = fraction of the irradiation which is transmitted

$$G_{ref} = \rho G \qquad G_{abs} = \alpha G \qquad G_{tr} = \tau G$$

$\rho + \alpha + \tau = 1$ Energy conservation

Surface Radiative Heat Fluxes

Surface properties



Energy balance at a surface

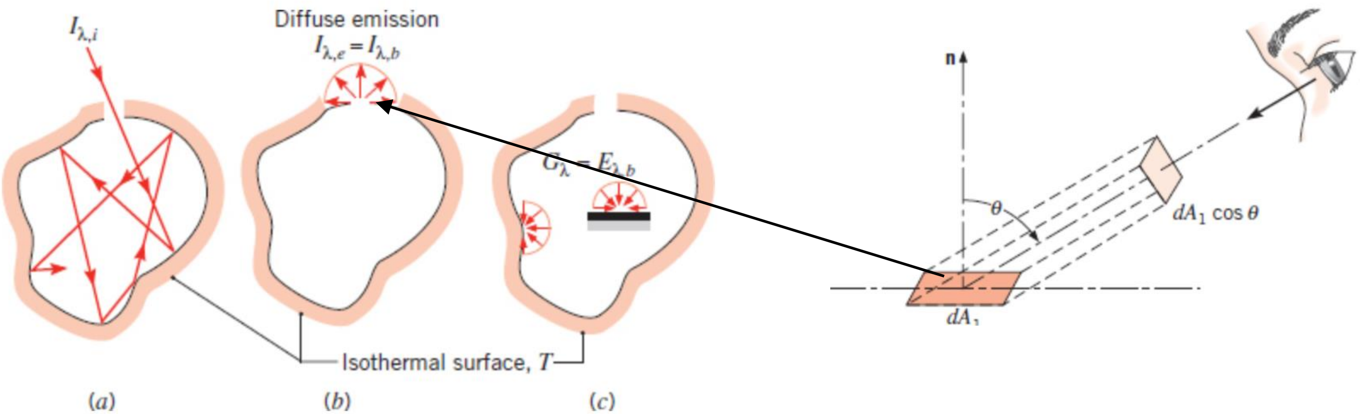
$$J = E + G_{ref} = \varepsilon\sigma T^4 + \varrho G$$

$$q"_{rad} = J - G = \varepsilon\sigma T^4 + \varrho G - G = \varepsilon\sigma T^4 - (\alpha + \tau)G$$

net radiative heat flux from a surface

Cavity Radiation
(Concept of projected area)

Consider radiation emitted from a homogeneous isothermal cavity (not a function of the content of the cavity)



If dA_1 is taken to be a small opening of such a cavity, then from quantum mechanic and statistical mechanic consideration, one can show

$$I_{\lambda,b} = n_\lambda \frac{hc}{\lambda}$$

n_λ = number of photon with leaving the opening

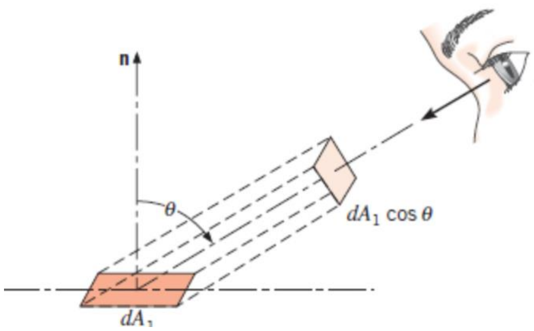
$$dq = energy\ measured\ by\ detector$$
$$\propto I_{\lambda b} dA_1 cos\theta = I_{\lambda b} dA_{1,p}$$

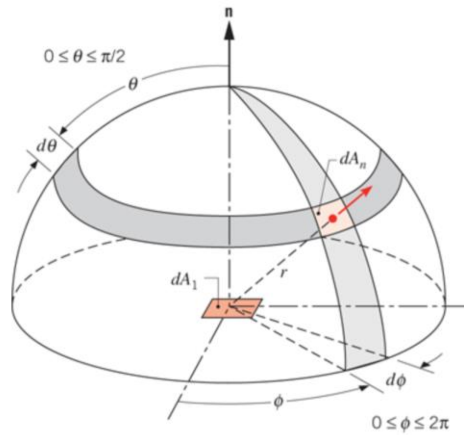
<u>Energy measured be detector is proportional to the projected area</u>

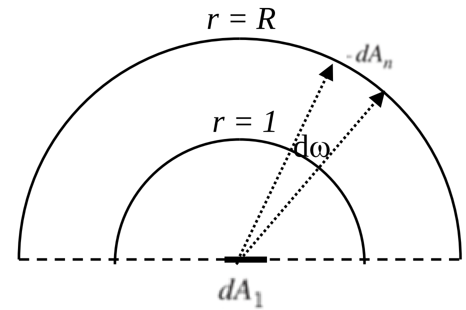
$$\boldsymbol{dA_{1,p} = dA_1 \cos\theta}$$ **Area seen by the observer (detector)**

Cavity Radiation
Concept of Solid Angle

Concept of Solid Angle







- Since all photos travel in a straight line (in all direction) coming out of the surface, the same number of photon which is intercepted by the detector dA_n (at a distance R from the source) must also go through the area $d\omega$ at a unit distance from the source

- The energy intercepted by the detector dA_n is thus given by

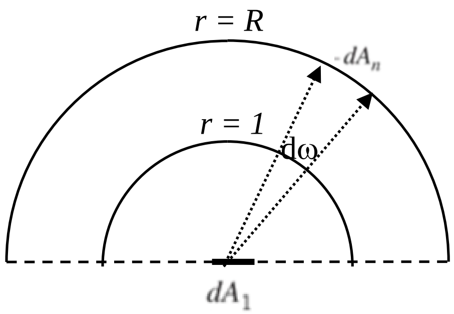
$$dq = energy\ measured\ by\ detector = I_{\lambda b} dA_1 cos\theta d\omega$$

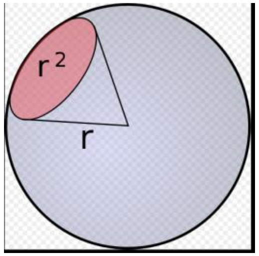
- The energy intercepted by the detector dA_n is a function of $d\omega$

- $d\omega$ is call the **solid angle**

Concept of Solid Angle

Concept of Solid Angle





- $d\omega$ is call the **solid angle**

- From geometry, $\frac{dA_n}{d\omega} = \frac{R^2}{1}$

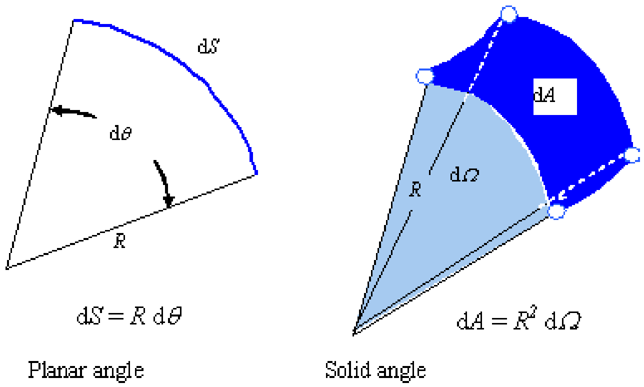
- For any finite area dA on a sphere with radius r, the solid angle corresponded to that area is given by

$$d\omega = \frac{dA}{r^2}$$

- The surface area of a sphere with r = 1 is given by 4π

- The total solid angle subtended for a sphere of any radius is thus $\omega = 4\pi$

Concept of "Solid Angle"



Analogy between a planar angle and a solid angle

Planar angle (dS/R)

$\frac{dS}{2\pi R}$=fraction of the circumference occupied by dS

2π = total angle for the full circle

$d\theta = \frac{dS}{2\pi R} \times 2\pi$

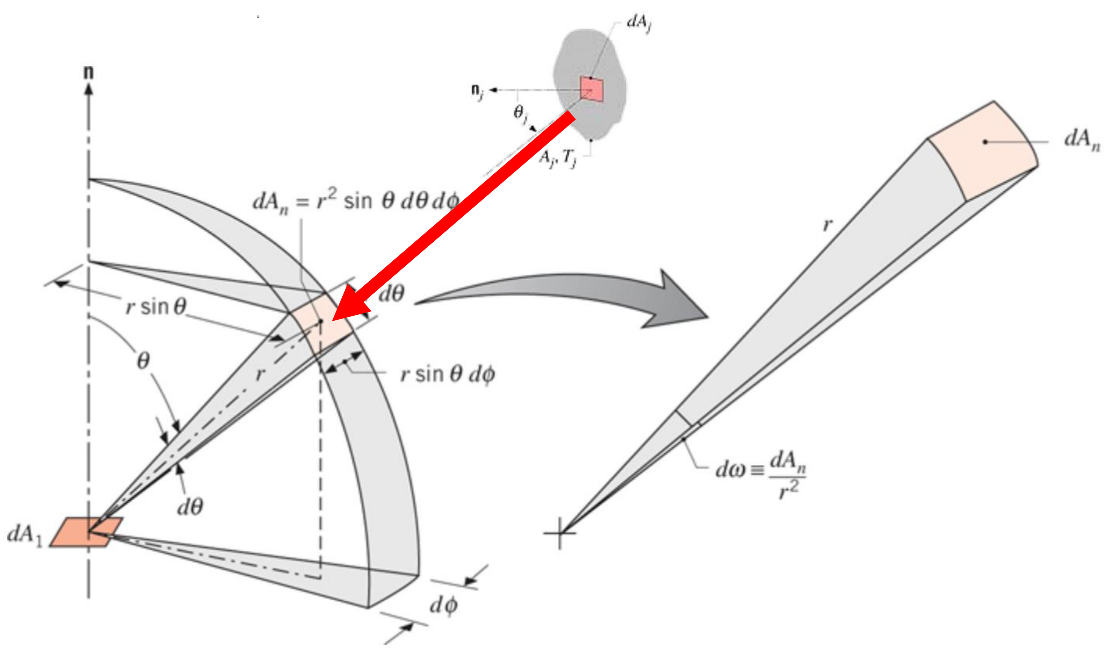
Solid angle (dA/R^2)

$\frac{dA}{4\pi R^2}$=fraction of the total spherical surface occupied by dA

4π = total solid angle for the full sphere

$d\omega = \frac{dA}{4\pi R^2} \times 4\pi$

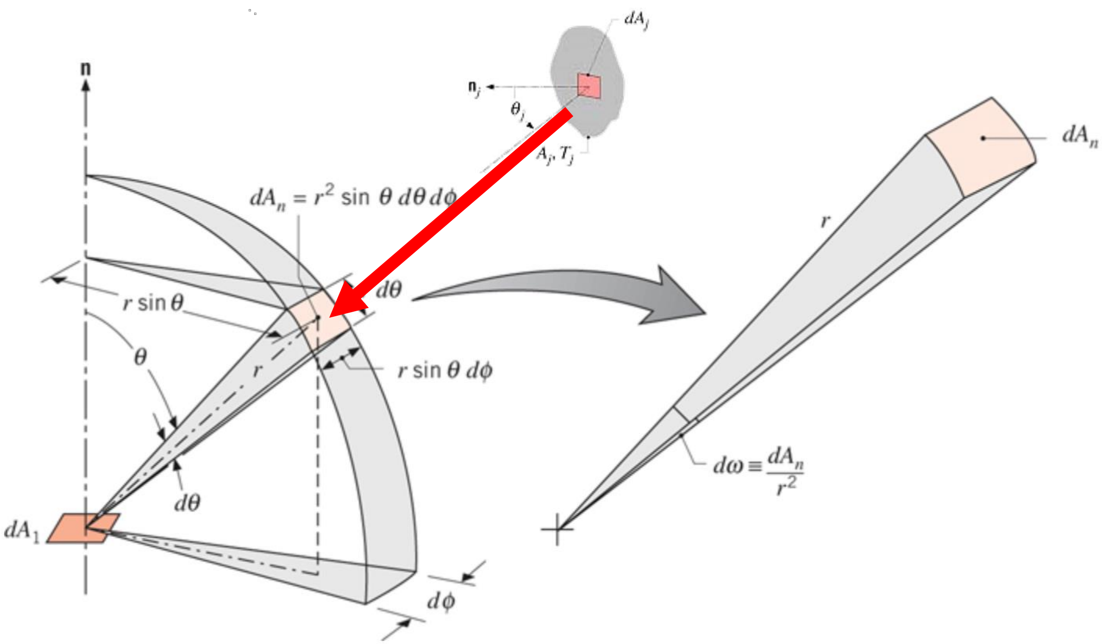
Concept of "Projected Receiving Area"



$dA_j cos\theta_j$ = dA_n = projected receiving area = projected area at the surface of a hemisphere of radius r with its center at dA_i

$$dq = I_{\lambda b} dA_1 cos\theta d\omega = I_{\lambda b} dA_1 cos\theta \frac{dA_n}{r^2} = I_{\lambda b} \frac{dA_1 cos\theta dA_j cos\theta_j}{r^2}$$

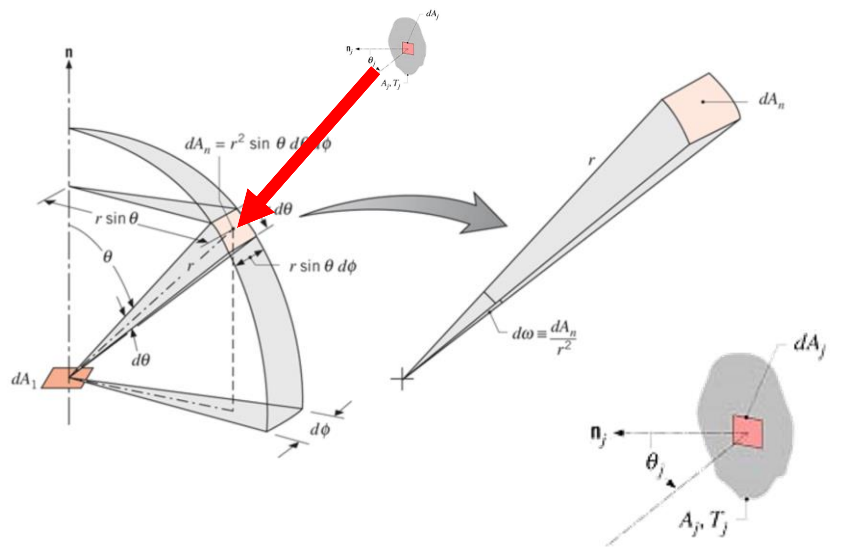
Evaluation of Solid Angle



$$d\omega = \frac{dA_n}{r^2} = \frac{dA_j cos\theta_j}{r^2} \qquad d\omega = \frac{dA_n}{r^2} = \frac{r^2 sin\theta d\theta d\phi}{r^2} = sin\theta d\theta d\phi$$

$$\boldsymbol{dq = I_{\lambda b} dA_1 cos\theta d\omega = I_{\lambda b} dA_1 cos\theta sin\theta d\theta d\phi}$$

Summary of Concepts
Radiative Exchange



Projected area of emitter $dA_1 cos\theta$

Projected area of detector $dA_j cos\theta_j = dA_n$

Solid angle subtended by detector

$$d\omega = \frac{dA_n}{r^2} = \frac{dA_j cos\theta_j}{r^2}$$

Solid angle subtended by detector (in terms of the angular coordinate relative to the sour

$$d\omega = \frac{dA_n}{r^2} = \frac{r^2 sin\theta d\theta d\phi}{r^2} = sin\theta d\theta d\phi$$

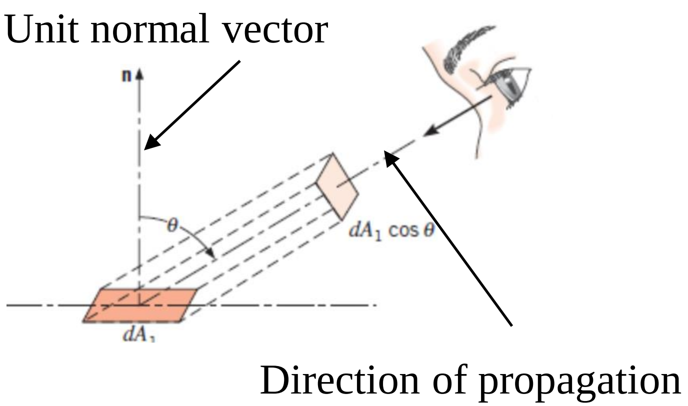
Radiative energy transfer from the emitter to the detector

$$dq = I_1 dA_1 cos\theta \frac{dA_n}{r^2} = I_1 dA_1 cos\theta d\omega = I_1 dA_1 cos\theta sin\theta d\theta d\phi$$

How to find projected emitter area ?

Projected area of emitter $dA_1 cos\theta$

θ = angle between a unit normal to area dA_1 (a vector pointing in the direction perpendicular to the surface) and the direction of the radiative energy leaving the surface

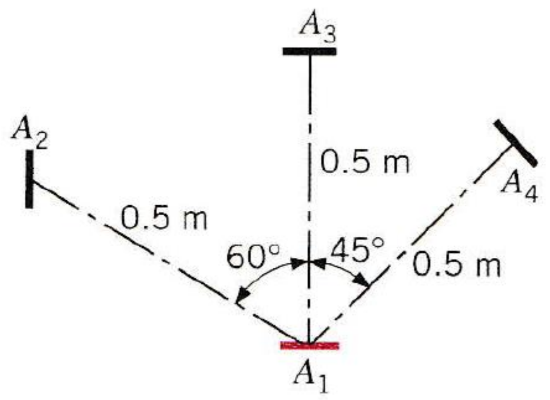


Example:

For energy radiated from A_1 to A_2 $\theta = 60°$

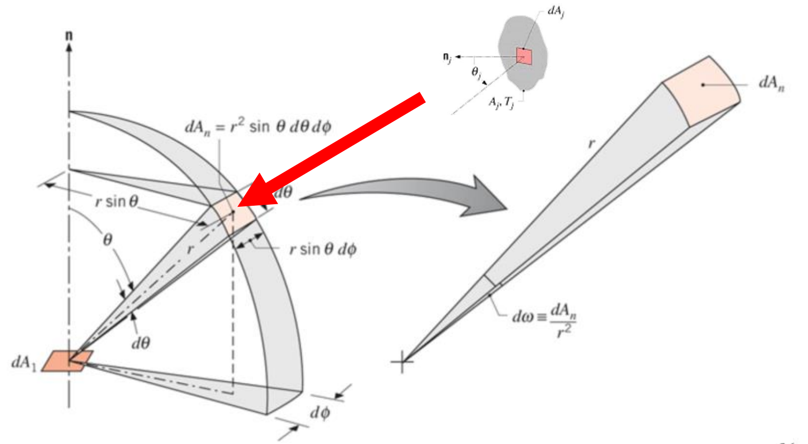
For energy radiated from A_1 to A_3 $\theta = 0°$

For energy radiated from A_1 to A_4 $\theta = 45°$



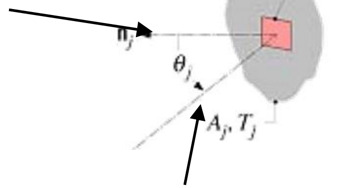
How to find projected detector area ?

<u>Projected area of detector</u> $dA_j cos\theta_j = dA_n$



θ_j = angle between a unit normal to area dA_j (a vector pointing in the direction perpendicular to the surface) and the direction of the radiative energy coming to the surface

Unit normal vector



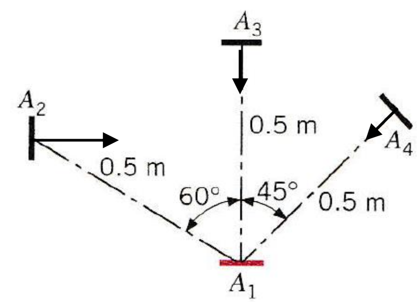
Direction of propagation

<u>Example:</u>

For energy radiated from A_1 to A_2 $\theta_2 = 30°$

For energy radiated from A_1 to A_3 $\theta_3 = 0°$

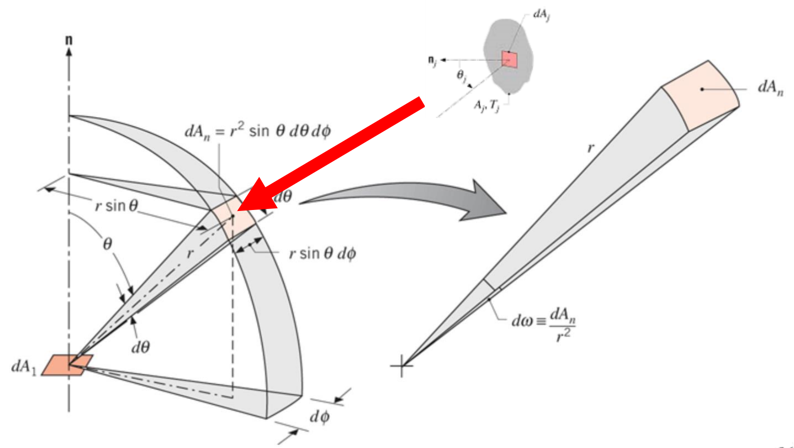
For energy radiated from A_1 to A_4 $\theta_4 = 0°$

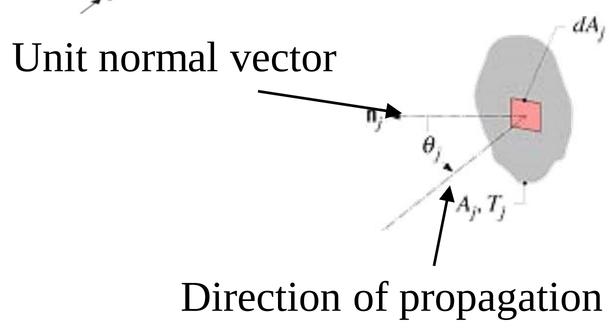


How to find solid angle of the detecting area?

Solid angle subtended by detector

$$d\omega = \frac{dA_n}{r^2} = \frac{dA_j cos\theta_j}{r^2}$$



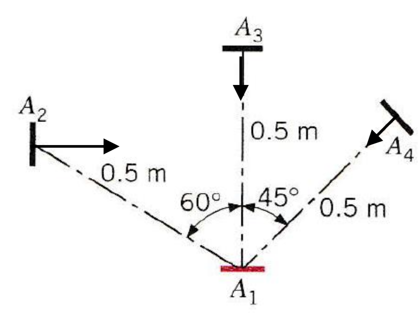


Example:

For energy radiated from A_1 to A_2 $\quad \omega_2 = \frac{A_2 cos(30°)}{(0.5m)^2}$

For energy radiated from A_1 to A_3 $\quad \omega_3 = \frac{A_3}{(0.5m)^2}$

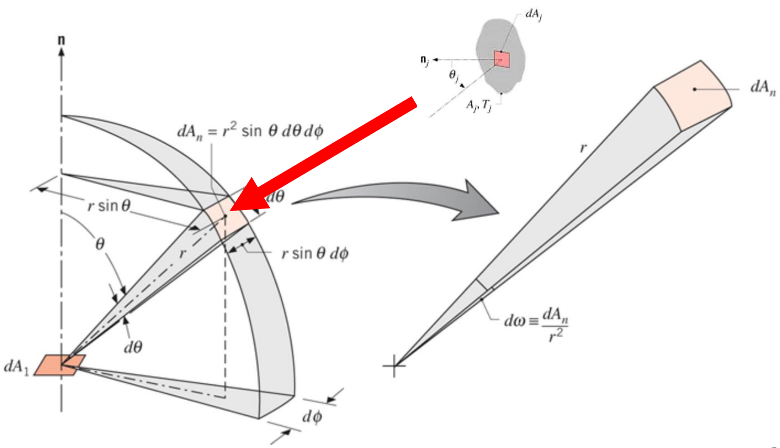
For energy radiated from A_1 to A_4 $\quad \omega_4 = \frac{A_4}{(0.5m)^2}$

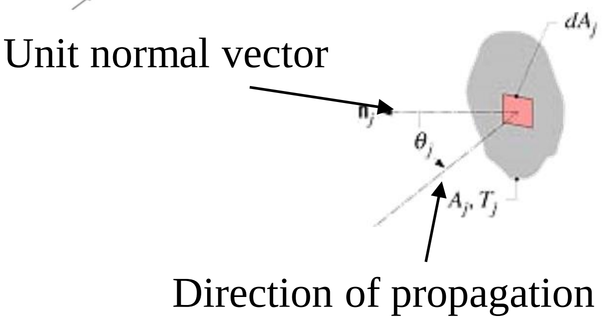


How to find radiative heat transfer ?

Radiative Energy Transfer

$$dq = I_1 dA_1 cos\theta \frac{dA_n}{r^2} = I_1 dA_1 cos\theta d\omega$$



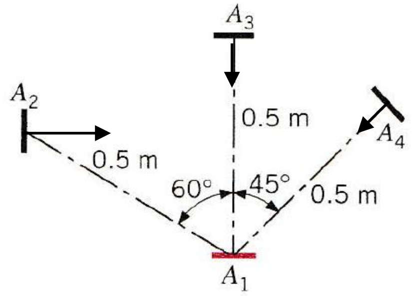


Example:

From A_1 to A_2 $\quad dq_{1-2} = I_1 \frac{A_1 cos(60°) A_2 cos(30°)}{(0.5m)^2}$

From A_1 to A_3 $\quad dq_{1-3} = I_1 \frac{A_1 A_3}{(0.5m)^2}$

From A_1 to A_4 $\quad dq_{1-4} = I_1 \frac{A_1 cos(45°) A_4}{(0.5m)^2}$

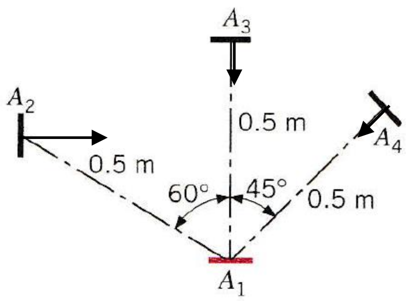


I_1 is either specified or determined from surface properties of A_1

When do we use expression based on angular coordinate from the source ?

$$d\omega = \frac{dA_n}{r^2} = \frac{r^2 sin\theta d\theta d\phi}{r^2} = sin\theta d\theta d\phi$$

$$dq = I_1 dA_1 cos\theta \frac{dA_n}{r^2} = I_1 dA_1 cos\theta d\omega = I_1 dA_1 cos\theta sin\theta d\theta d\phi$$



Answer:

Only if either the emitting area or the detecting area is finite (not infinitesimally small)

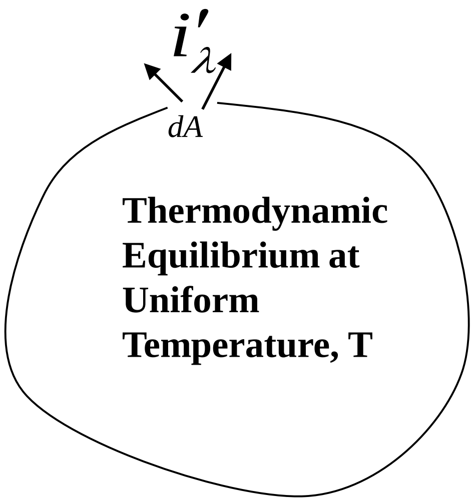
Radiative energy transfer and solid angle determined by numerical integration

<u>Example:</u>

- For the evaluation of $q_{1\text{-}3}$, $\sin\theta = 0$ only applied to the infinitesimal surface in A_3 which is directly above an infinitesimal area directly underneath in A1
- $\sin\theta \neq 0$ for other combination of differential emitting area and detecting area
- Use the expression for angular coordinate only if the evaluation is done by integration (e.g. one of the area is large, like in lab4)

Blackbody Intensity

From quantum mechanic, it can be derived theoretically (and also verified experimentally) that if dA is infinitesimal, the radiative intensity leaving the cavity through dA (cavity radiation) is independent of direction and is given by



Planck's Law (Blackbody Radiation Intensity)

$$I_\lambda(\lambda,\theta,\phi) = I_{\lambda,b}(\lambda,T) = \frac{C_1}{\pi\lambda^5\left(e^{C_2/\lambda T}-1\right)}$$

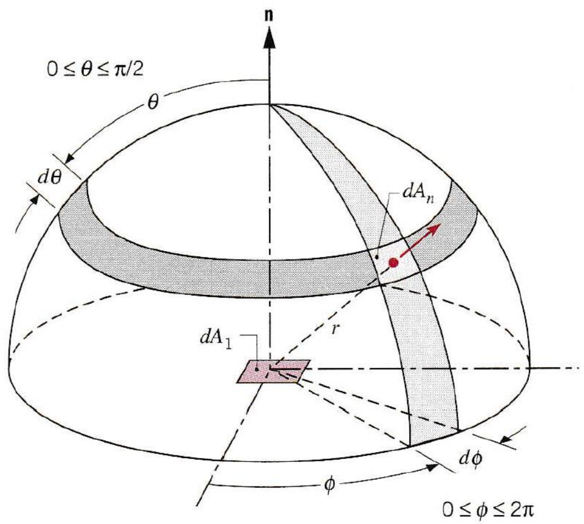
with

$$C_1 = 2\pi h c_0^2, \quad C_2 = \frac{hc_0}{k_B}$$

Intensity has a unit of of energy per unit time, per unit projected source area and per unit solid angle of the receiving area

The heat transfer is $dq = I_{\lambda b} dA_1 cos\theta d\omega = I_{\lambda b} dA_1 cos\theta \dfrac{dA_n}{r^2} = I_{\lambda b} \dfrac{dA_1 cos\theta dA_j cos\theta_j}{r^2}$

Blackbody Emissive Power

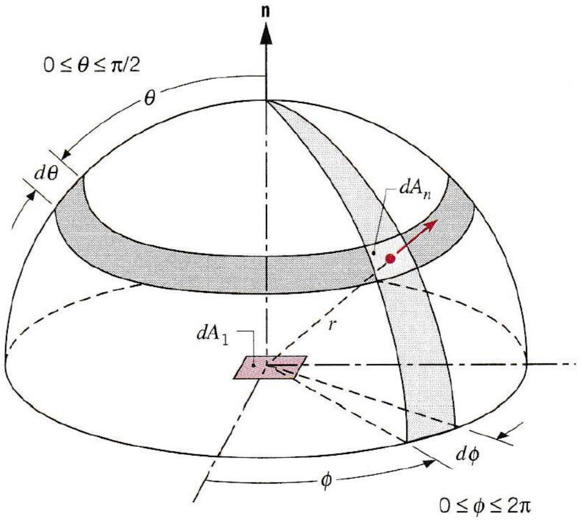


The blackbody emissive power, $E_{\lambda,b}$ (W/m^2),is the total energy flux at a wavelength λ leaving per unit source area dA_s given by

$$E_{\lambda,b}(\lambda,T) = q''_{\lambda}(\lambda,T) = \int_0^{2\pi}\int_0^{\pi} I_{\lambda,b}(\lambda,T) cos\theta sin\theta d\theta d\phi$$

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)}$$

Blackbody Total Emissive Power (Stefan-Boltzmann Law)



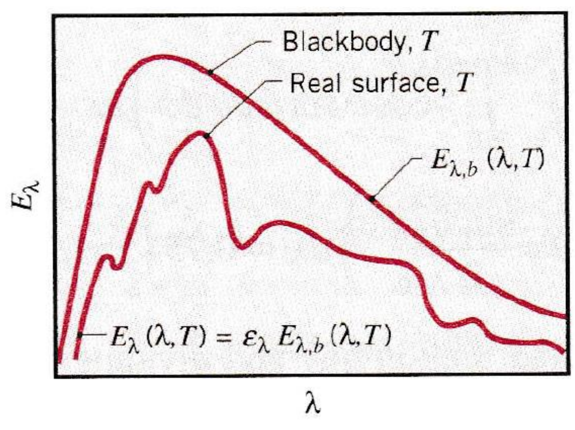
The blackbody total emissive power, E_b (W/m^2),is the total energy flux integrated over all wave length leaving per unit source area dA_s given by

$$E_b(T) = \int_0^\infty E_{\lambda,b}(\lambda, T)\, d\lambda$$

$$= \int_0^\infty \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)} d\lambda = \sigma T^4$$

Emission from Real Surfaces

- Blackbody is a “perfect” emitter
- All real surfaces emitted less than a blackbody at the same temperature
- Emissivity is defined to be the ratio of the energy emitted by a surface to the blackbody emission at the same temperature



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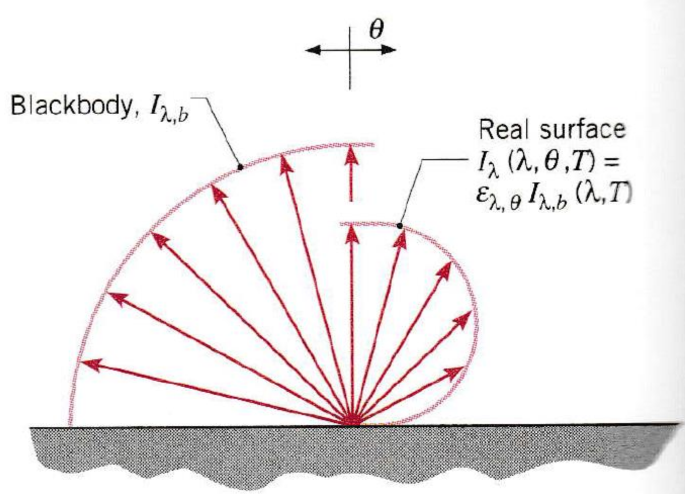
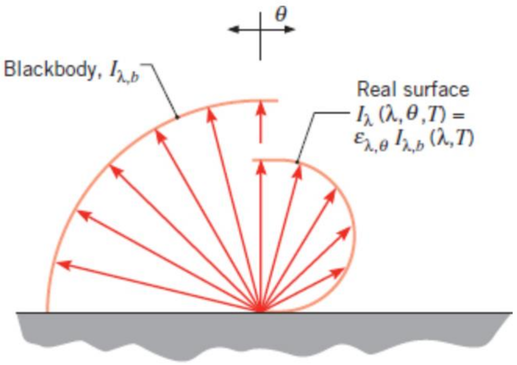


(b)

Emission from Real Surfaces

- **Definition of surface emissivity**

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\varphi,T) \equiv \frac{I_{\lambda,e}(\lambda,\theta,\varphi,T)}{I_{\lambda,b}(\lambda,T)}$$



Concept of a diffuse surface

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\varphi,T) = \varepsilon_\lambda(\lambda,T)$$

Not a function of direction

$$I_{\lambda,e}(\lambda,\theta,\varphi,T) = I_{\lambda,e}(\lambda,T) = \varepsilon_\lambda(\lambda,T) I_{\lambda,b}(\lambda,T)$$

$$E_{\lambda,e}(\lambda,T) = \pi I_{\lambda,e}(\lambda,T) = \varepsilon_\lambda(\lambda,T) E_{\lambda,b}(\lambda,T)$$

Spectral and total Emissivity

$$\varepsilon_\lambda = \frac{\text{Energy emitted at wavelength } \lambda \text{ into all direction}}{\text{Corresponding energy emitted by blackbody at the same temperature}}$$

$$\varepsilon_\lambda(\lambda, T) \equiv \frac{E_\lambda(\lambda, T)}{E_{\lambda,b}(\lambda, T)}$$

$$\varepsilon = \frac{\text{Total energy emitted by the surface (over all wavelength and direction)}}{\text{Total energy emitted by blackbody at the same temperature}}$$

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{E(T)}{\sigma T^4}$$

$$\varepsilon(T) = \frac{1}{\sigma T^4} \int_0^\infty \varepsilon_\lambda(\lambda, T) E_{\lambda,b}(\lambda, T)\, d\lambda$$

Relation between Emissive Power and Intensity from a Diffuse Surface

For a black surface

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)}$$

$$E_b = \pi I_b = \sigma T^4$$

For a diffuse non-black surface $\varepsilon_\lambda < 1$

$$I_\lambda = \frac{E_\lambda}{\pi} = \frac{\varepsilon_\lambda(\lambda,T)E_{\lambda,b}}{\pi}$$

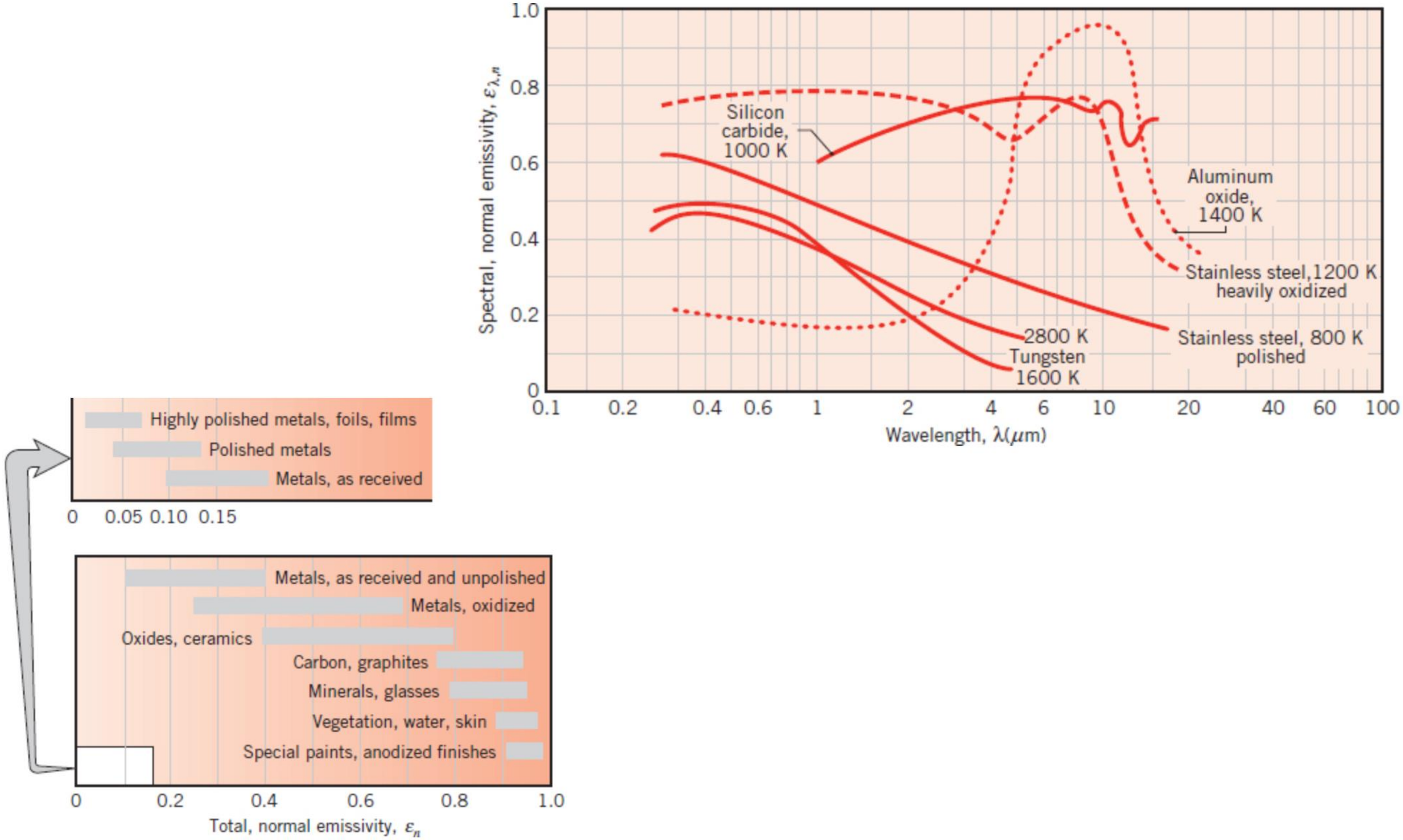
$$I = \frac{E}{\pi} = \frac{\varepsilon(T)\sigma T^4}{\pi}$$

Relation between Emissive Power and Intensity from a Diffuse Surface

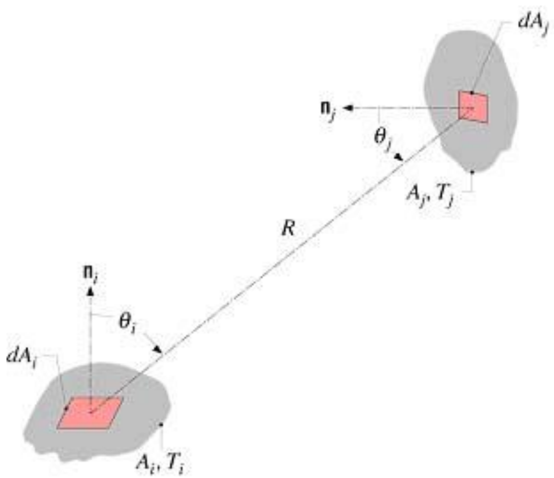
	black surface	diffuse non-black surface
Intensity	$I_{\lambda,b}(\lambda,T)=\dfrac{C_1}{\pi\lambda^5\left(e^{C_2/\lambda T}-1\right)}$	$I_{\lambda,e}(\lambda,T)=\varepsilon_\lambda(\lambda,T)I_{\lambda,b}(\lambda,T)$
Emissive power	$E_{\lambda,b}(\lambda,T)=\pi I_{\lambda,b}(\lambda,T)$	$E_{\lambda,e}(\lambda,T)=\pi I_{\lambda,e}(\lambda,T)$ $E_{\lambda,e}(\lambda,T)=\varepsilon_\lambda(\lambda,T)E_{\lambda,b}(\lambda,T)$
Total Intensity	$I_b=\dfrac{E_b}{\pi}=\dfrac{\sigma T^4}{\pi}$	$I=\dfrac{E}{\pi}=\dfrac{\varepsilon(T)\sigma T^4}{\pi}$
Total Emissive power	$E_b=\sigma T^4$	$E=\varepsilon(T)\sigma T^4$

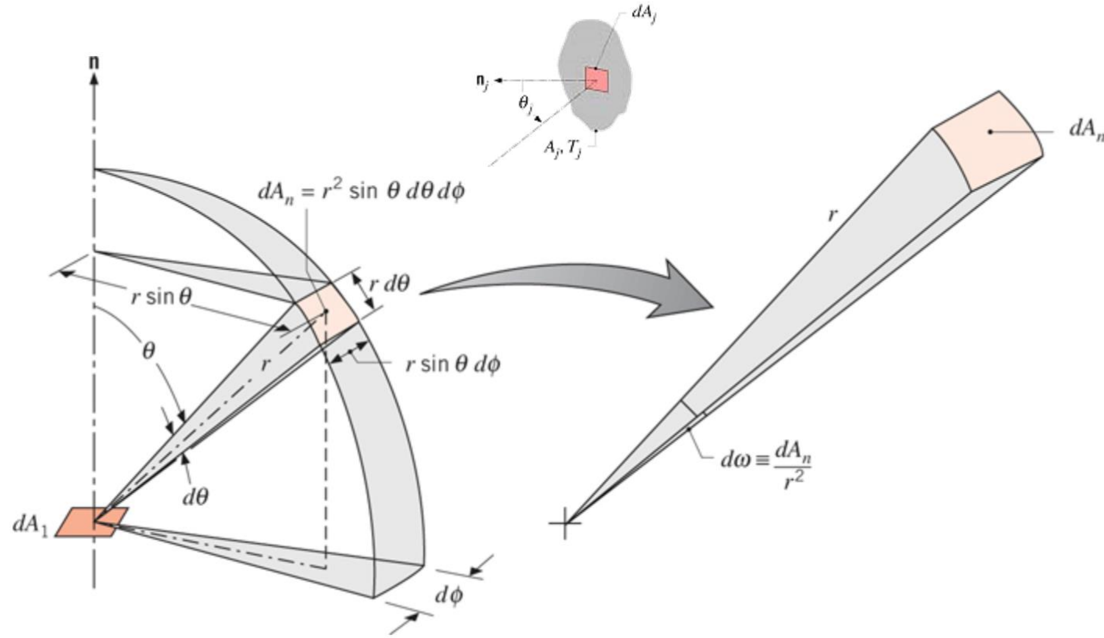
$$\varepsilon(T)=\frac{1}{\sigma T^4}\int_0^\infty \varepsilon_\lambda(\lambda,T)E_{\lambda,b}(\lambda,T)d\lambda$$

Emissivity of Real Materials



The radiative exchange between two differential diffuse surfaces





For radiation emitted from dA_i, the e
the projected area ($dA_i cos\theta_i$) and the solid angle $d\omega_j$, subtended by dA_j

$$dq_{i-j} = I_i dA_i cos\theta_i d\omega_j$$

$$dq_{i-j} = \frac{I_i dA_i cos\theta_i dA_j cos\theta_i}{R^2} \qquad d\omega_j = \frac{dA_j cos\theta_j}{R^2}$$

$$dq_{i-j} = I_i dA_i cos\theta_i sin\theta_i d\theta_i d\phi_i \qquad d\omega_j = sin\theta_i d\theta_i d\phi_i$$

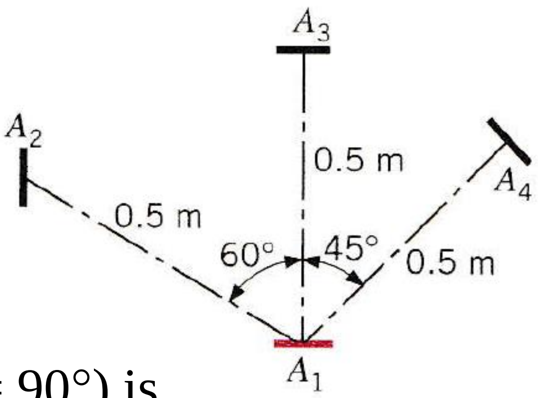
Ex. 12.1

Radiation Heat Transfer from one small area to three other small areas at different locations and orientations

Given:

$$A_1 = A_2 = A_3 = A_4 = 10^{-3} m^3 \quad \text{(small areas)}$$

$$r_2 = r_3 = r_4 = 0.5\,m$$



A_1 is a diffuse emitter

Emitted intensity in the normal direction (θ = 90°) is

$$I_n = I(90^0) = 7000 \frac{\mathrm{W}}{\mathrm{m}^2 sr}$$

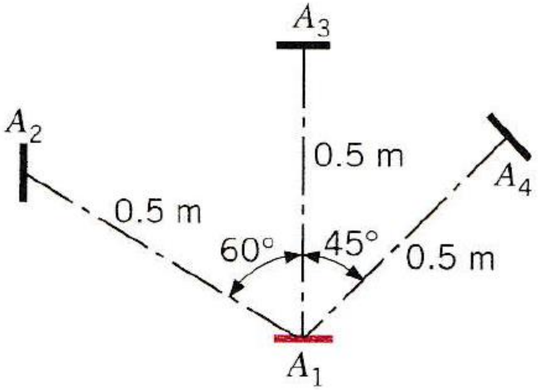
Find:

1. Intensity of emission in each of the three directions.
2. Solid angles subtended by the three surfaces.
3. Rate at which radiation is intercepted by the three surfaces.

Ex. 12.1

Radiation Heat Transfer from one small area to three other small areas at different locations and orientations

Solution:



1. Diffuse emitter and

$$I_n = I\left(90^0\right) = 7000\,\frac{\text{W}}{\text{m}^2 sr}$$

$$I = I_n = 7000\,\frac{\text{W}}{\text{m}^2 sr}$$

Ex. 12.1

Radiation Heat Transfer from one small area to three other small areas at different locations and orientations

Solution:

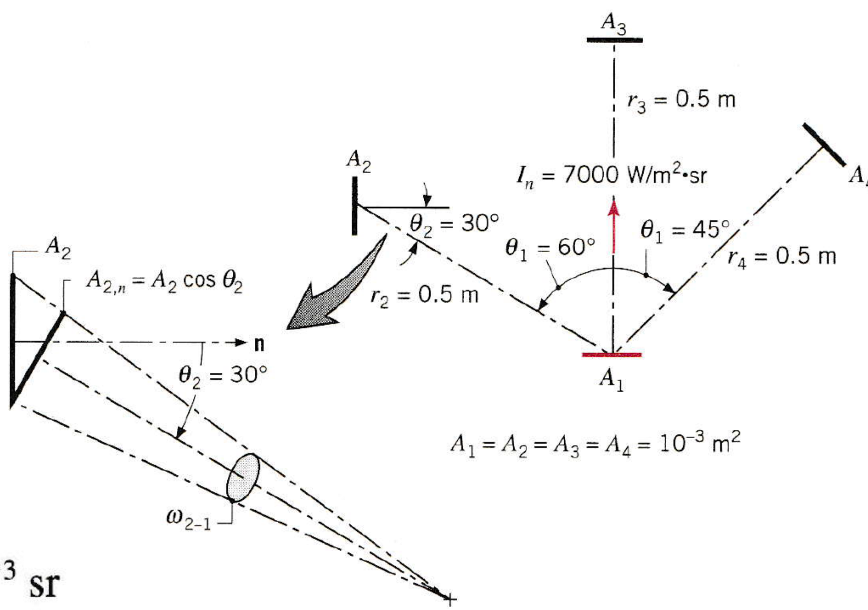
2. Treating all areas as differential areas

$$d\omega \equiv \frac{dA_n}{r^2}$$

$$dA_{4,n} = dA_4,\, dA_{3,n} = dA_3,\, dA_{2,n} = dA_2\cos\theta_2$$

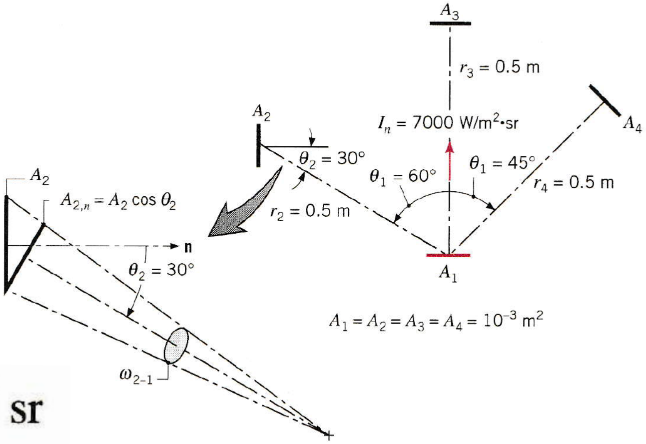
$$\omega_{3-1} = \omega_{4-1} = \frac{A_3}{r^2} = \frac{10^{-3}\ \text{m}^2}{(0.5\ \text{m})^2} = 4.00 \times 10^{-3}\ \text{sr}$$

$$\omega_{2-1} = \frac{A_2 \cos\theta_2}{r^2} = \frac{10^{-3}\ \text{m}^2 \times \cos 30^\circ}{(0.5\ \text{m})^2} = 3.46 \times 10^{-3}\ \text{sr}$$



Ex. 12.1

Radiation Heat Transfer from one small area to three other small areas at different locations and orientations



Solution:

3. $q_{1-j} = I \times A_1 \cos\theta_1 \times \omega_{j-1}$

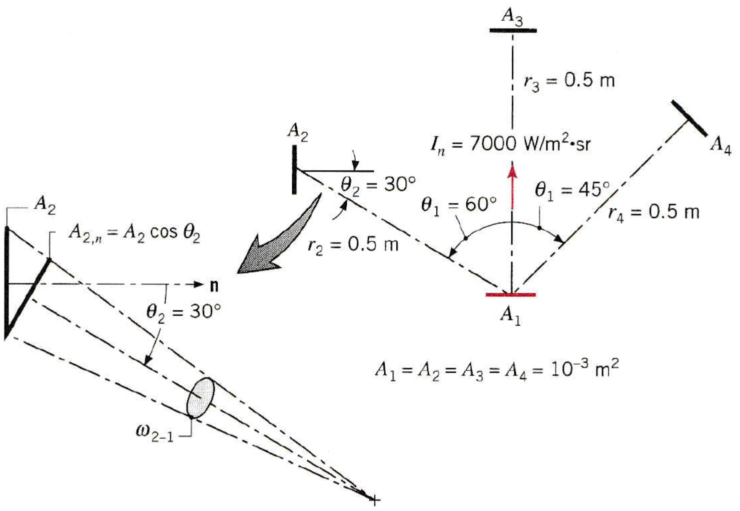
$$q_{1-2} = 7000\ \mathrm{W/m^2 \cdot sr}\ (10^{-3}\ \mathrm{m^2} \times \cos 60°)\ 3.46 \times 10^{-3}\ \mathrm{sr} = 12.1 \times 10^{-3}\ \mathrm{W}$$

$$q_{1-3} = 7000\ \mathrm{W/m^2 \cdot sr}\ (10^{-3}\ \mathrm{m^2} \times \cos 0°)\ 4.00 \times 10^{-3}\ \mathrm{sr} = 28.0 \times 10^{-3}\ \mathrm{W}$$

$$q_{1-4} = 7000\ \mathrm{W/m^2 \cdot sr}\ (10^{-3}\ \mathrm{m^2} \times \cos 45°)\ 4.00 \times 10^{-3}\ \mathrm{sr} = 19.8 \times 10^{-3}\ \mathrm{W}$$

Ex. 12.1

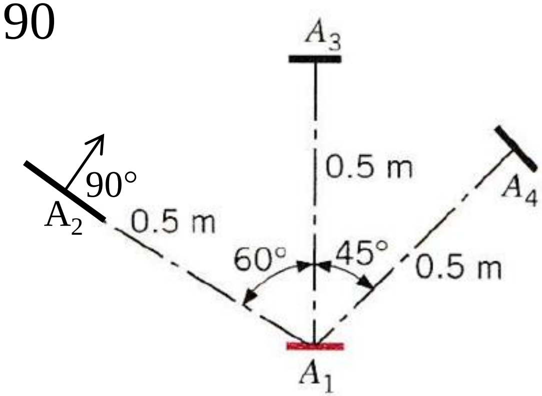
Radiation Heat Transfer from one small area to three other small areas at different locations and orientations



Comments:

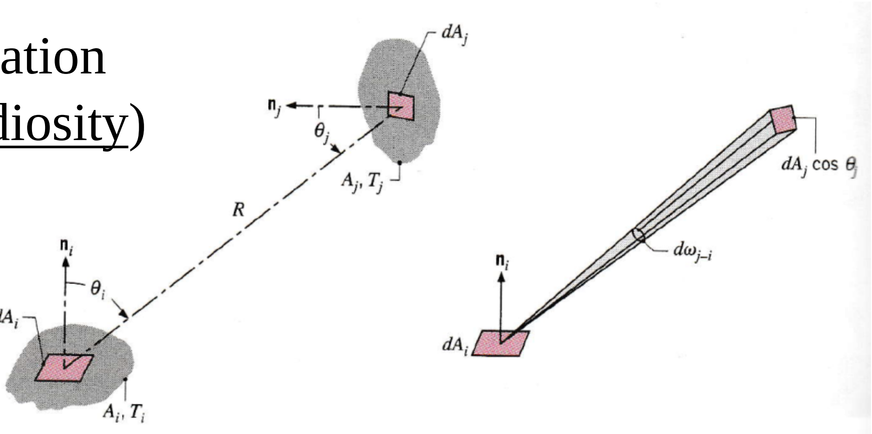
Eventhough dA_1 is a diffuse emitter, the energy received by the different areas varies with the area location and its orientation.

For example: $\omega_{2-1} = 0$ and $q_{1-2} = 0$ with $\theta_2 = 90$



The View Factor

View Factor F_{ij} = Fraction of the radiation leaving <u>diffuse</u> surface i (with <u>uniform radiosity</u>) that is intercepted by surface j



$dq_{i\to j}$ = rate at which radiation leaving surface dA and is intercepted by dAj

$$dq_{i\to j} = I_{e+r,i}\cos\theta_i\, dA_i\, d\omega_{j-i} = I_{e+r,i}\frac{\cos\theta_i\cos\theta_j}{R^2}dA_i\,dA_j = J_i\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_i\,dA_j$$

$$q_{i\to j} = J_i\int_{A_i}\int_{A_j}\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_i\,dA_j$$ when J_i = constant at A_i

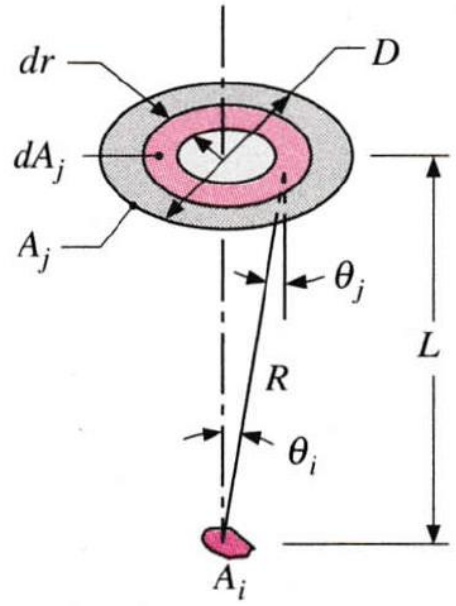
$$F_{ij} = \frac{q_{i\to j}}{A_i J_i} = \frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_i\,dA_j$$ and $$F_{ji} = \frac{1}{A_j}\int_{A_i}\int_{A_j}\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_i\,dA_j$$

Example 13.1

Evaluation of View Factor by direct integration

Known: Orientation of small surface relative to large circular disk.

Find: View factor of small surface with respect to disk, F_{ij}.



$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i \, dA_j$$

Since A_i is small

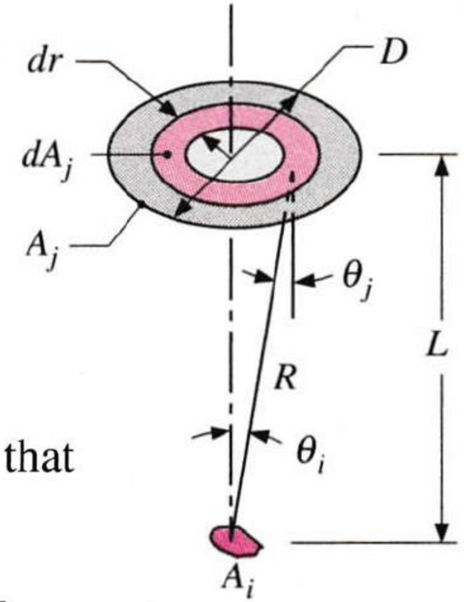
θ_i, θ_j, and R are approximately independent of position on A_i,

so

$$F_{ij} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j$$

Example 13.1

Evaluation of View Factor by direct integration



Known: Orientation of small surface relative to large circular disk.

Find: View factor of small surface with respect to disk, F_{ij}.

with $\theta_i = \theta_j \equiv \theta$,

$$F_{ij} = \int_{A_j} \frac{\cos^2\theta}{\pi R^2}\, dA_j$$

With $R^2 = r^2 + L^2$, $\cos\theta = (L/R)$, and $dA_j = 2\pi r\, dr$, it follows that

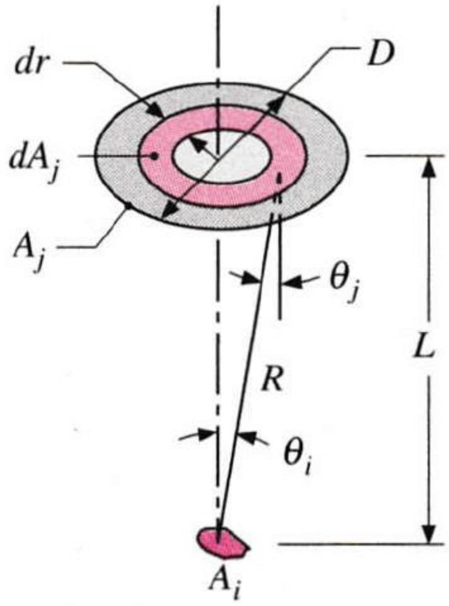
$$F_{ij} = 2L^2 \int_0^{D/2} \frac{r\, dr}{(r^2 + L^2)^2} = \frac{D^2}{D^2 + 4L^2}$$

Example 13.1a (Lab 4)

Evaluation of View Factor by direct integration

Known: Orientation of small surface relative to large circular disk.

Find: View factor of small surface with respect to disk, F_{ij}.



In Ex. 13.1, we calculated

$$F_{ij} = \frac{q_{i \to j}}{A_i J_i} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i \, dA_j$$

In Lab #4, we need

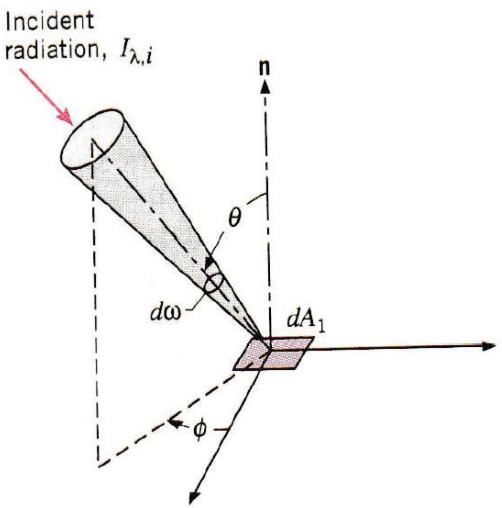
$$F_{ji} = \frac{q_{j \to i}}{A_j J_j} = \frac{1}{A_j} \int_{A_j} \int_{A_i} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j = \frac{A_i F_{ij}}{A_j}$$

So the flux measured by the radiometer (A_i) is given by

$$q''_{rad} = \frac{q_{j \to i}}{A_i} = J_j F_{ij} = J_j \frac{D^2}{D^2 + 4L^2}$$

Irradiation

Within a solid angle dω, the incident radiation energy per unit area normal to the direction of propagation, is called the incident intensity $I_{\lambda,i}(\lambda,\theta,\phi)$



The rate of energy received by the surface dA_1 from a direction (θ, ϕ) is given by

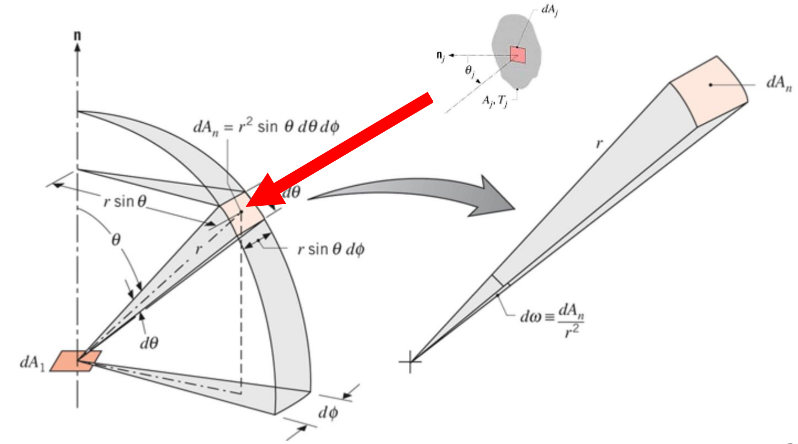
$$dq_{\lambda,i} = I_{\lambda,i}(\lambda,\theta,\phi)dA_1 cos\theta d\omega = I_{\lambda,i}(\lambda,\theta,\phi)dA_1 cos\theta sin\theta d\theta d\phi$$

The spectral energy flux received by the surface dA_1 from a direction (θ, ϕ) is given by

$$dq"_{\lambda,i} = \frac{dq_{\lambda,i}}{dA_1} = I_{\lambda,i}(\lambda,\theta,\phi)cos\theta sin\theta d\theta d\phi$$

Irradiation
(From small emitting area)

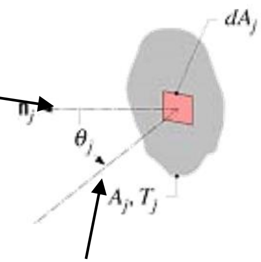
Within a solid angle dω, the incident radiation energy per unit area normal to the direction of propagation, is called the incident intensity



The rate of energy received by the surface dA_1 from a direction (θ, ϕ) is given by

$$dq_{\lambda,i} = I_{\lambda,i}(\lambda,\theta,\phi)dA_1 cos\theta d\omega = I_{\lambda,i}(\lambda,\theta,\phi)dA_1 cos\theta \frac{dA_n}{r^2}$$

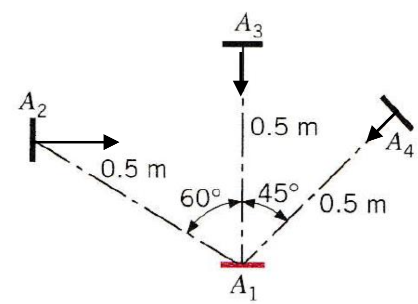
Unit normal vector



Direction of propagation

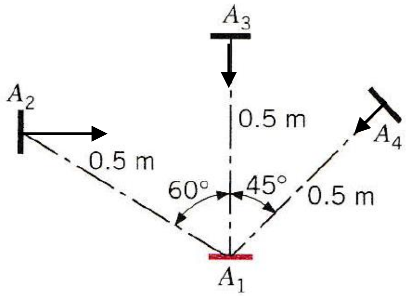
The spectral energy flux received by the surface dA_1 from a direction (θ, ϕ) is given by

$$dq''_{\lambda,i} = \frac{dq_{\lambda,i}}{dA_1} = I_{\lambda,i}(\lambda,\theta,\phi)cos\theta \frac{dA_n}{r^2}$$



Irradiation (From small emitting area), Example

What is the irradiation (heat flux) onto surface A_1, from radiation emitted from surfaces A_2, A_3, A_4 (assume all surfaces are diffuse)



The spectral energy flux received by the surface dA_1 from a direction (θ, ϕ) is given by

$$dq''_{\lambda,i} = \frac{dq_{\lambda,i}}{dA_1} = I_{\lambda,i}(\lambda, \theta, \phi) cos\theta \frac{dA_n}{r^2}$$

From surface A_2

$$dq''_{\lambda,2} = \frac{dq_{\lambda,2}}{dA_1} = I_{\lambda,2}(\lambda) cos\theta \frac{dA_{n,2}}{r^2} = I_{\lambda,2}(\lambda)\cos(60°)\frac{A_2 cos(30°)}{(0.5m)^2}$$

From surface A_3

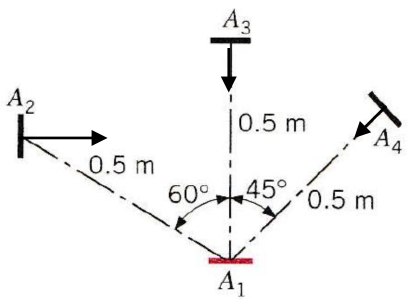
$$dq''_{\lambda,3} = \frac{dq_{\lambda,3}}{dA_1} = I_{\lambda,3}(\lambda) cos\theta \frac{dA_{n,3}}{r^2} = I_{\lambda,3}(\lambda)\frac{A_3}{(0.5m)^2}$$

From surface A_4

$$dq''_{\lambda,4} = \frac{dq_{\lambda,4}}{dA_1} = I_{\lambda,4}(\lambda) cos\theta \frac{dA_{n,4}}{r^2} = I_{\lambda,4}(\lambda)\cos(45°)\frac{A_4}{(0.5m)^2}$$

Irradiation
(From small emitting area), Example

What is the irradiation (heat flux) onto surface A_1, from radiation emitted from surfaces A_2, A_3, A_4 (assume all surfaces are diffuse)



The spectral irradiation is the rate of radiation energy per unit area incident from all direction onto a surface dA, is given by

G_λ = sum of irradiation from all different outside surfaces

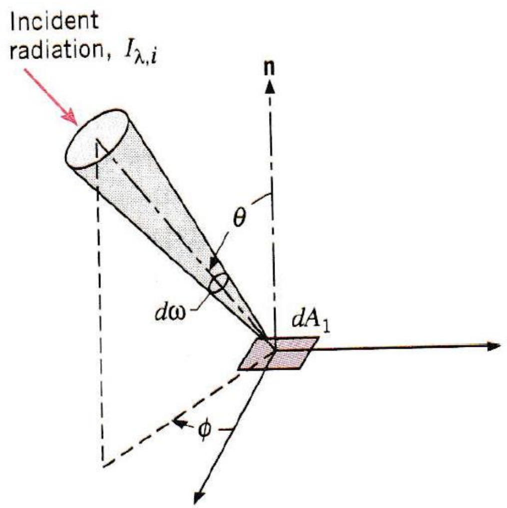
$$G_\lambda = dq"_{\lambda,2} + dq"_{\lambda,3} + dq"_{\lambda,4}$$

$$G_\lambda = I_{\lambda,2}(\lambda)\cos(60°)\frac{A_2 cos(30°)}{(0.5m)^2} + I_{\lambda,3}(\lambda)\frac{A_3}{(0.5m)^2} + I_{\lambda,4}(\lambda)\cos(45°)\frac{A_4}{(0.5m)^2}$$

For numerical evaluation, need to specify intensity from the different surfaces or temperature and emissivity of the different surfaces

Irradiation

The spectral irradiation is the rate of radiation energy per unit area incident from all direction onto a surface dA, is given by



$$G_\lambda = \int_0^{2\pi}\int_0^{\pi} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

The total irradiation is the rate of radiation energy per unit area incident from all direction, over all wavelength, onto a surface dA, is given by

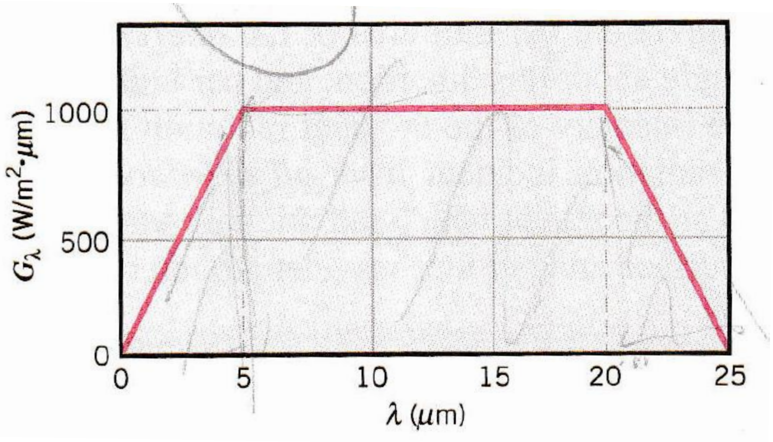
$$G = \int_0^{\infty} G_\lambda d\lambda = \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi\, d\lambda$$

Ex. 12.2

Total Irradiation onto a Surface

Given: Spectral Irradiation onto a surface

Find: Total Irradiation



$$G = \int_0^\infty G_\lambda \, d\lambda$$

$$G = \int_0^{5\,\mu\text{m}} G_\lambda \, d\lambda + \int_{5\,\mu\text{m}}^{20\,\mu\text{m}} G_\lambda \, d\lambda + \int_{20\,\mu\text{m}}^{25\,\mu\text{m}} G_\lambda \, d\lambda + \int_{25\,\mu\text{m}}^{\infty} G_\lambda \, d\lambda$$

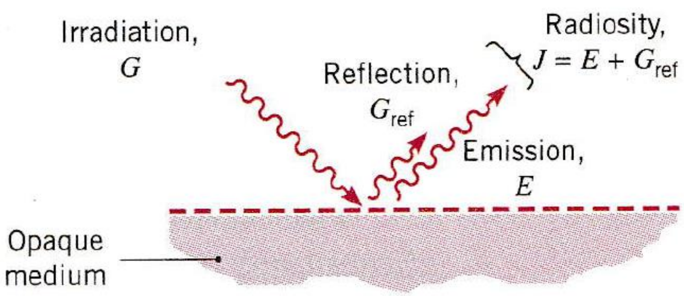
$$G = \tfrac{1}{2}(1000\ \text{W/m}^2 \cdot \mu\text{m})(5 - 0)\ \mu\text{m} + (1000\ \text{W/m}^2 \cdot \mu\text{m})(20 - 5)\ \mu\text{m} + \tfrac{1}{2}(1000\ \text{W/m}^2 \cdot \mu\text{m})(25 - 20)\ \mu\text{m} + 0$$

$$= (2500 + 15{,}000 + 2500)\ \text{W/m}^2$$

$$G = 20{,}000\ \text{W/m}^2$$

Radiosity (from an opaque surface)

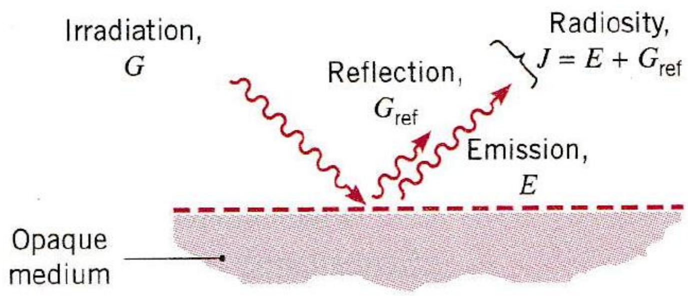
Radiosity (J) = all radiant energy leaving a surface (emission +reflection)



$$J = E + G_{ref}$$

$$J = E + G_{ref} = \varepsilon\sigma T^4 + \varrho G$$

Net Radiative Heat Flux for an opaque surface



$$q''_{rad} = J - G = \varepsilon\sigma T^4 + \varrho G - G = \varepsilon\sigma T^4 - \alpha G$$

net radiative heat flux from an opaque surface

The theoretical significance of the Planck Function

$$I_{\lambda,b}(\lambda,T)=\frac{C_1}{\pi\lambda^5\left(e^{C_2/\lambda T}-1\right)}$$

From Statistical Thermodynamics and Quantum Mechanical consideration, one can prove theoretically that

- For all real surfaces (at thermodynamic equilibrium) radiating at a temperature T,

$$I_\lambda(\lambda,\theta,\phi)\le I_{\lambda,b}(\lambda,T)$$

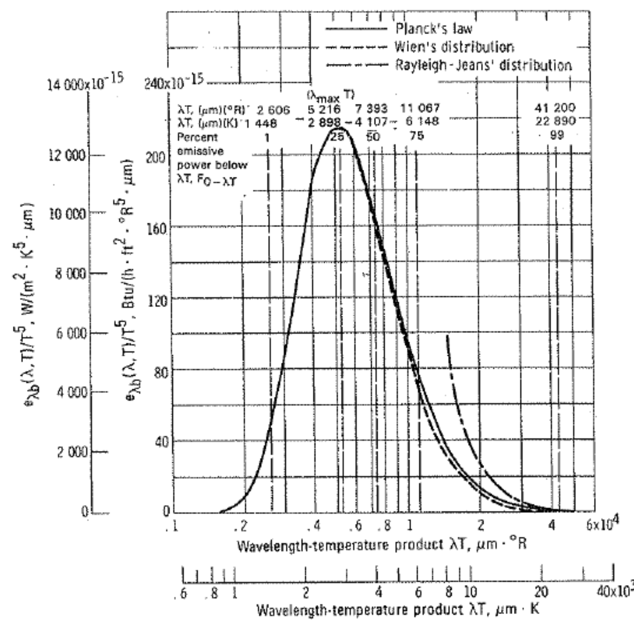
- A surface which radiates with the Planck function at a particular wavelength is called a “black surface” at that wavelength

Important Mathematical Properties of the Planck Function (1)

Universal Blackbody Function

$$\frac{E_{\lambda,b}(\lambda,T)}{T^5} = \frac{C_1}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)} = f(\lambda T)$$

A function of λT only for all T



Important Mathematical Properties of the Planck Function (2)

<u>Wien's Displacement Law</u>

$$\frac{E_{\lambda,b}(\lambda,T)}{T^5} = \frac{C_1}{(\lambda T)^5\left(e^{C_2/\lambda T}-1\right)} = f(\lambda T)$$ **A function of λT only for all T**

The maximum of *f(λT)* occurs at a fixed value of $\lambda_{max}T$, given by

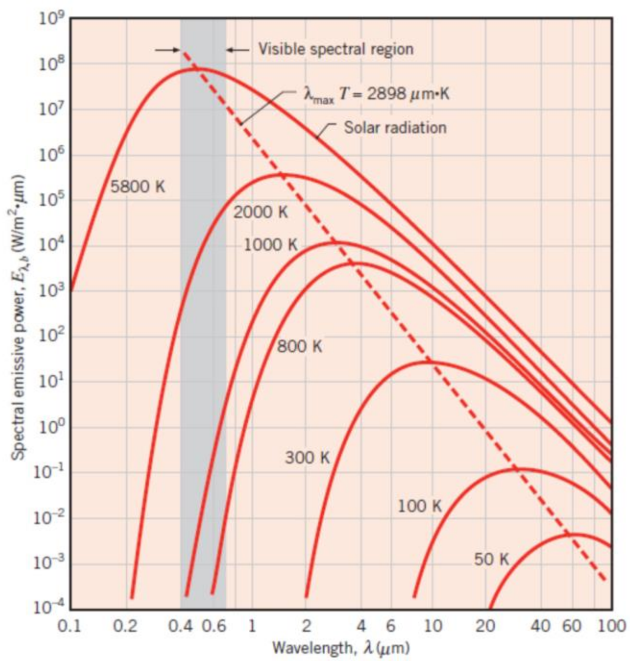
$$\lambda_{\max}T = C_3 = 2898\mu\text{m-K (Third Radiation Constant)}$$

The maximum of the Planck Function occurs at λ_{max}, given by

$$E_{\lambda,b}(\lambda_{\max},T) = \frac{C_1}{\lambda_{\max}^5\left(e^{C_2/\lambda_{\max}T}-1\right)} = T_{\max}^5\, f(\lambda_{\max}T = 2898\mu m\cdot K)$$

Important Mathematical Properties of the Planck Function (3)

Wien’s Displacement Law (“color of radiation”)



T (K)	λ_{max} (μm)	
294	9.85	Infrared
1000	2.897	
2000	1.449	Near Infrared
3000	0.966	
4000	0.724	(red)
5000	0.579	visible
6000	0.483	(violet)

For $T_2 > T_1$

$$\lambda_{\max}\left(T_2\right) < \lambda_{\max}\left(T_1\right)$$

$$E_{\lambda b}\left(\lambda_{\max,2}, T_2\right) > E_{\lambda b}\left(\lambda_{\max,1}, T_1\right)$$

Important Mathematical Properties of the Planck Function (4)

Integrated Blackbody Function $F_{0-\lambda T}$

$$\int_0^{\lambda} E_{\lambda,b}(\lambda',T)\,d\lambda' = T^5\int_0^{\lambda}\frac{E_{\lambda,b}(\lambda',T)}{T^5}\,d\lambda' = \sigma T^4\int_0^{\lambda}\frac{E_{\lambda,b}(\lambda',T)}{\sigma T^5}\,d(\lambda' T) = \sigma T^4 F_{0-\lambda T}$$

with

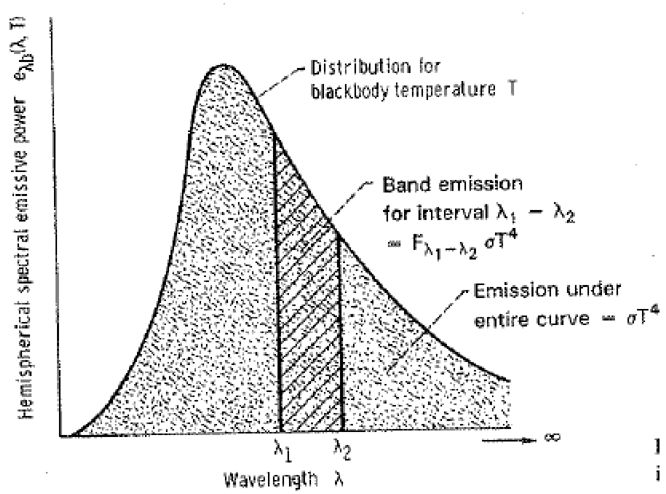
$$F_{0-\lambda T} = \int_0^{\lambda}\frac{E_{\lambda,b}(\lambda',T)}{\sigma T^5}\,d(\lambda' T)$$ is a function of λT only

Important Mathematical Properties of the Planck Function (6)

<u>Properties of the Integrated Blackbody Function</u> $F_{0\text{-}\lambda T}$

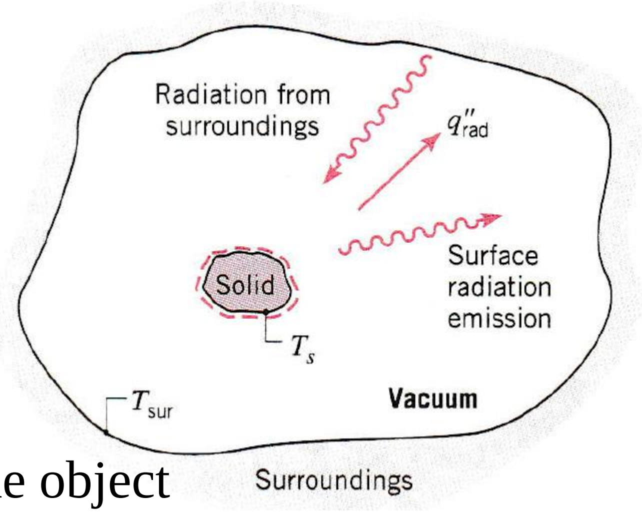
$$\int_{\lambda_1}^{\lambda_2} E_{\lambda,b}(\lambda',T)\,d\lambda' = \int_{0}^{\lambda_2} E_{\lambda,b}(\lambda',T)\,d\lambda' - \int_{0}^{\lambda_1} E_{\lambda,b}(\lambda',T)\,d\lambda' = \sigma T^4\left[F_{0-\lambda_2 T} - F_{0-\lambda_1 T}\right]$$

$$F_{0-\infty} = \frac{1}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} E_{\lambda,b}(\lambda',T)\,d\lambda' = 1.0$$



Ex. 12-3 (read)

Radiative heat transfer between a small solid blackbody and a surrounding blackbody enclosure



Given: Surface temperature of the small body T_s and the surrounding T_{surr}

Find: Net radiative heat flux at the surface of the object

Since the surface is black

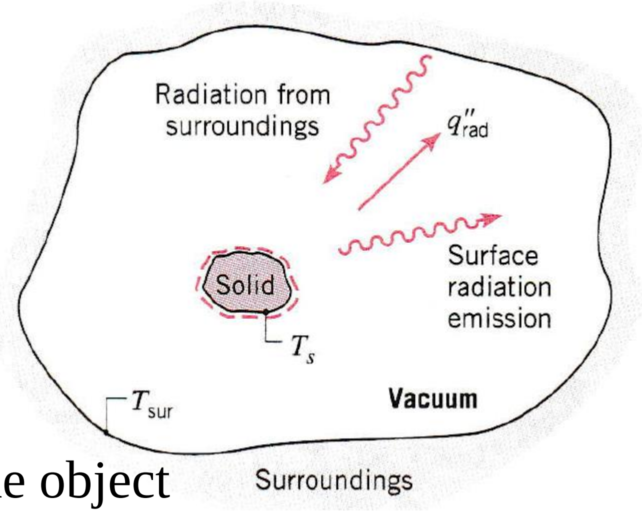
$$q''_{rad} = \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda - \int_0^\infty \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda, \theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda$$

and

$$I_{\lambda,e}(\lambda, \theta, \phi) = I_{\lambda,b}(\lambda, T_s) \qquad I_{\lambda,i}(\lambda, \theta, \phi) = I_{\lambda,b}(\lambda, T_{sur})$$

Ex. 12-3

Radiative heat transfer between a small solid blackbody and a surrounding blackbody enclosure



Given: Surface temperature of the small body T_s and the surrounding T_∞

Find: Net radiative heat flux at the surface of the object

$$q''_{rad} = \int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\phi \times \int_0^\infty I_{\lambda,b}(\lambda, T_{sur}) \, d\lambda$$

$$-\int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta \, d\phi \times \int_0^\infty I_{\lambda,b}(\lambda, T_s) \, d\lambda$$

$$= \pi\left[\int_0^\infty I_{\lambda,b}(\lambda, T_{sur}) \, d\lambda - \int_0^\infty I_{\lambda,b}(\lambda, T_s) \, d\lambda\right]$$

$$q''_{rad} = \sigma(T_s^4 - T_{sur}^4)$$

Comment: Blackbody is generally a reasonable approximation for the surrounding incoming radiation

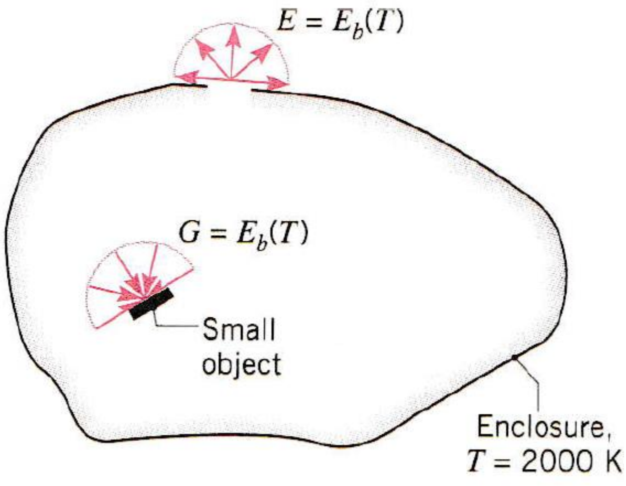
Ex. 12-4

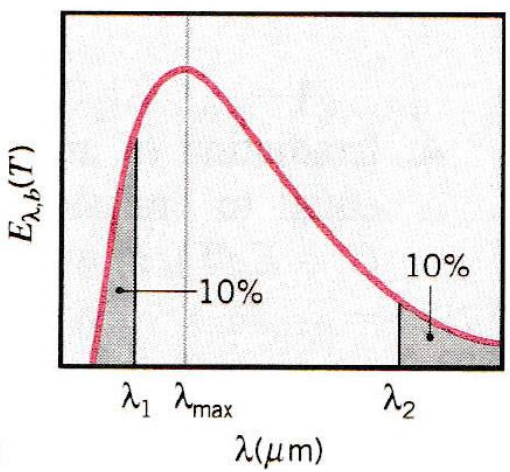
Emission from a large isothermal enclosure

Given: Large isothermal enclosure at uniform temperature T = 2000 K

Find:

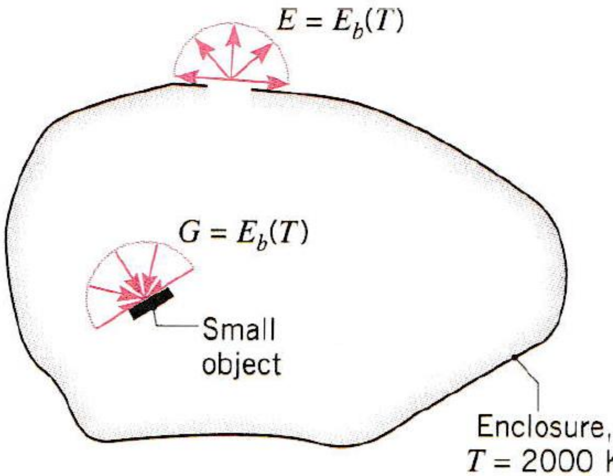
1. Emissive power of a small aperture on the enclosure.
2. Wavelengths below which and above which 10% of the radiation is concentrated.
3. Spectral emissive power and wavelength associated with maximum emission.
4. Irradiation on a small object inside the enclosure.





Ex. 12-4

Emission from a large isothermal enclosure



Solutions:

1. Emissive power of a small aperture on the enclosure.

Emission from a small aperture of an enclosure is like that of a blackbody, so

$$E = E_b(T) = \sigma T^4 = 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}(2000\ \mathrm{K})^4$$

$$E = 9.07 \times 10^5\ \mathrm{W/m^2}$$

Ex. 12-4

Emission from a large isothermal enclosure

Solutions:

2. Wavelengths below which and above which 10% of the radiation is concentrated.

To fine λ_1, look for value of λT such that $F_{0-\lambda T} = 0.1$

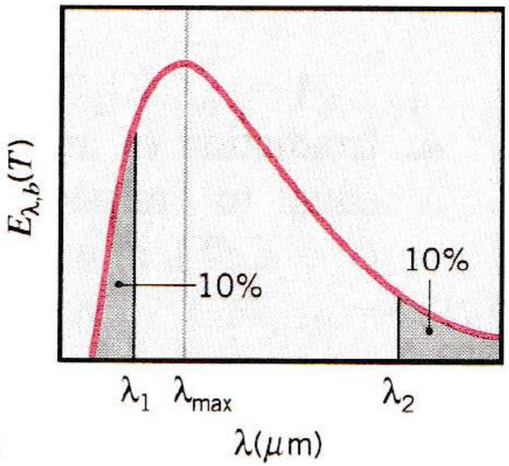


TABLE 12.2 Blackbody Radiation Functions

λT ($\mu m \cdot K$)	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155

$$F_{0-\lambda T} = 0.1 \quad \text{occurs at } \lambda T = 2195\ \mu m \cdot K$$

$$\lambda_1 = \frac{2195 \mu m \cdot K}{2000 K} = 1.1 \mu m$$

Ex. 12-4

Emission from a large isothermal enclosure

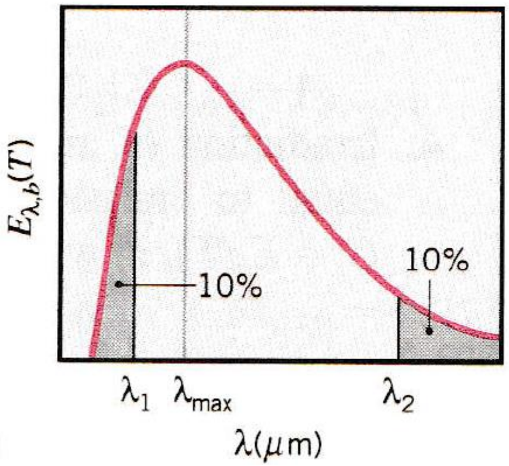
Solutions:

2. Wavelengths below which and above which 10% of the radiation is concentrated.

To fine λ_2, look for value of λT such that $F_{0-\lambda T} = 0.9$

TABLE 12.2 Blackbody Radiation Functions

λT ($\mu m \cdot K$)	$F_{(0\rightarrow\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
9,000	0.890029	0.901463×10^{-5}	0.124801
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442



$$F_{0-\lambda T} = 0.9 \quad \text{occurs at } \lambda T = 9382\ \mu m \cdot K$$

$$\lambda_1 = \frac{9382 \mu m \cdot K}{2000K} = 4.69 \mu m$$

Ex. 12-4

Emission from a large isothermal enclosure

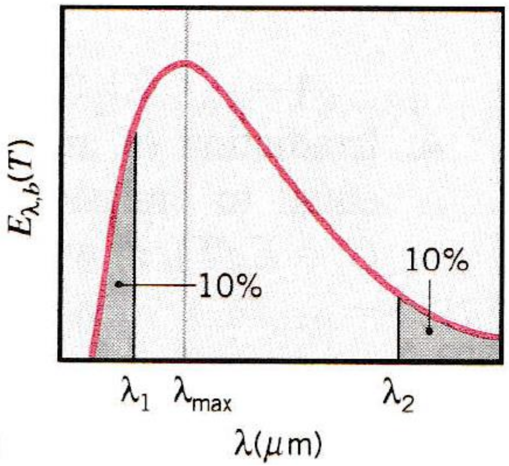
Solutions:

3. Spectral emissive power and wavelength associated with maximum emission.

By Wein's displacement law, $\lambda_{max}T = 2898\ \mu m \cdot K$

$$\lambda_{max} = \frac{2898 \mu m \cdot K}{2000K} = 1.45 \mu m$$

and at $\lambda_{max}T = 2898\ \mu m \cdot K$ $\quad \dfrac{I_{\lambda b}}{\sigma T^5} = 0.722 \times 10^{-4}$



$$I_{\lambda,b}(1.45\ \mu m, 2000\ K) = 0.722 \times 10^{-4}\ (\mu m \cdot K \cdot sr)^{-1} \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (2000\ K)^5$$

$$I_{\lambda,b}(1.45\ \mu m, 2000\ K) = 1.31 \times 10^5\ W/m^2 \cdot sr \cdot \mu m$$

$$E_{\lambda,b} = \pi I_{\lambda,b} = 4.12 \times 10^5\ W/m^2 \cdot \mu m$$

Ex. 12-4

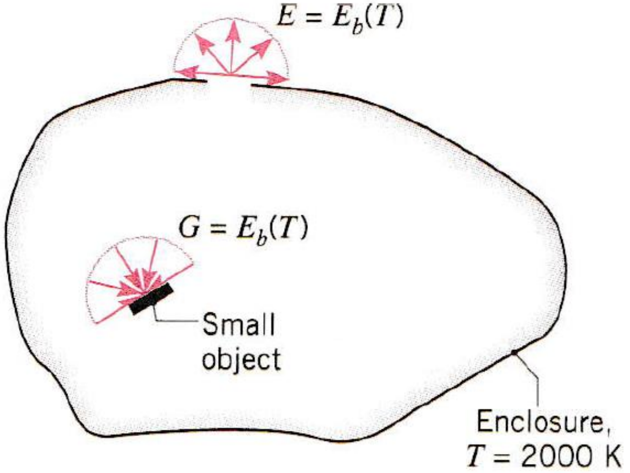
Emission from a large isothermal enclosure

Solutions:

4. Irradiation on a small object inside the enclosure.

Since the interior of the isothermal enclosure will "see" the same radiation as the cavity, the irradiation is the blackbody emission, so

$$G = E_b\left(T\right) = \sigma T^4 = 9.07 \times 10^5 \,\mathrm{W/m^3}$$

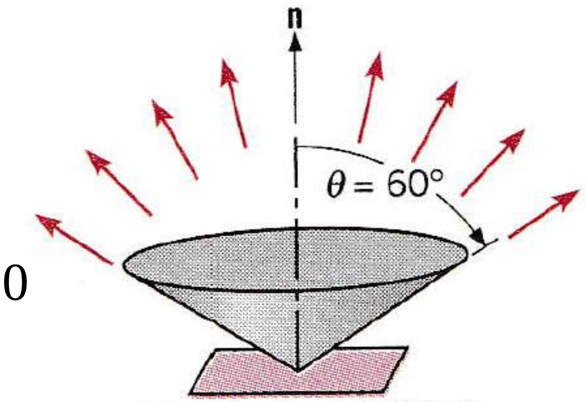


Ex. 12.5 (read)

Energy received from a blackbody over a finite angular range and wavelength range

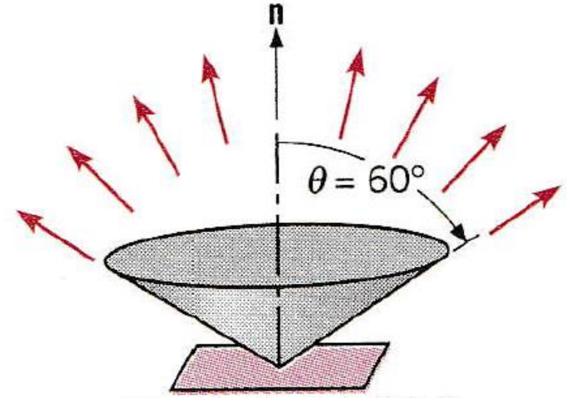
Given: A blackbody at 1500 K

Find: Rate of emission per unit area between θ = 0 and 60 and over the wavelength region between λ = 2 and 4 μm



Ex. 12.5

Energy received from a blackbody over a finite angular range and wavelength range



Given: A blackbody at 1500 K

Solution:

$$\Delta E = \int_2^4 \int_0^{2\pi} \int_0^{\pi/3} I_{\lambda,b} \cos\theta \sin\theta \, d\theta \, d\phi \, d\lambda$$

since a blackbody emits diffusely,

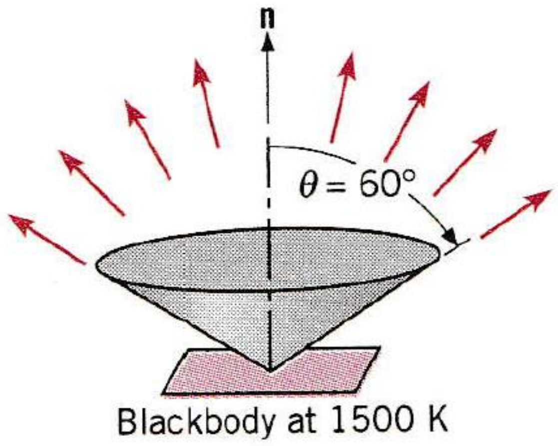
$$\Delta E = \int_2^4 I_{\lambda,b} \left(\int_0^{2\pi} \int_0^{\pi/3} \cos\theta \sin\theta \, d\theta \, d\phi \right) d\lambda$$

$$\Delta E = \int_2^4 I_{\lambda,b} \left(2\pi \frac{\sin^2\theta}{2} \Big|_0^{\pi/3} \right) d\lambda = 0.75 \int_2^4 \pi I_{\lambda,b} \, d\lambda$$

$$\Delta E = 0.75 E_b \int_2^4 \frac{E_{\lambda,b}}{E_b} d\lambda = 0.75 E_b \left[F_{(0\rightarrow 4)} - F_{(0\rightarrow 2)} \right]$$

Ex. 12.5

Energy received from a blackbody over a finite angular range and wavelength range



Given: A blackbody at 1500 K

Solution:

$$\Delta E = 0.75 E_b \int_2^4 \frac{E_{\lambda,b}}{E_b} d\lambda = 0.75 E_b \left[F_{(0\to4)} - F_{(0\to2)}\right]$$

from Table 12.2

$$\lambda_1 T = 2\ \mu\text{m} \times 1500\ \text{K} = 3000\ \mu\text{m}\cdot\text{K}: \qquad F_{(0\to2)} = 0.273$$

$$\lambda_2 T = 4\ \mu\text{m} \times 1500\ \text{K} = 6000\ \mu\text{m}\cdot\text{K}: \qquad F_{(0\to4)} = 0.738$$

$$\Delta E = 0.75(0.738 - 0.273)E_b = 0.75(0.465)E_b$$

$$\Delta E = 0.75(0.465)5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ (1500\ \text{K})^4 = 10^5\ \text{W/m}^2$$

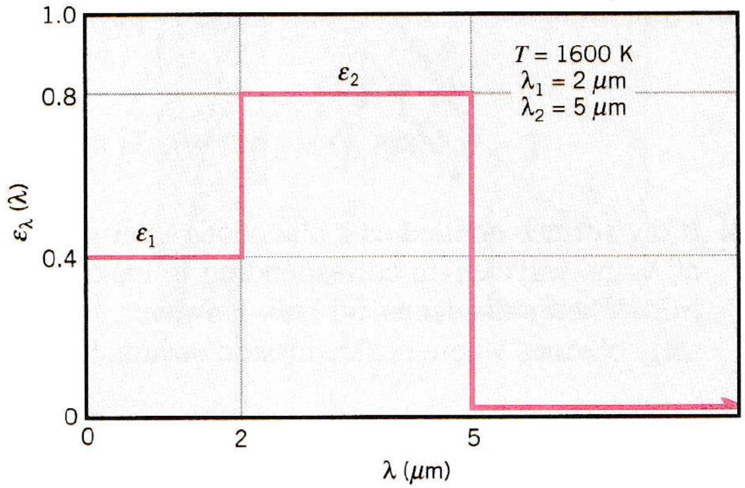
Ex.12.6

Determine total, hemispherical emissivity and total emissive power for a diffuse surface

Given: Spectral, hemispherical emissivity of a diffuse surface at 1600 K

Find:

1. Total, hemispherical emissivity.
2. Total emissive power.
3. Wavelength at which spectral emissive power will be a maximum.



Ex.12.6

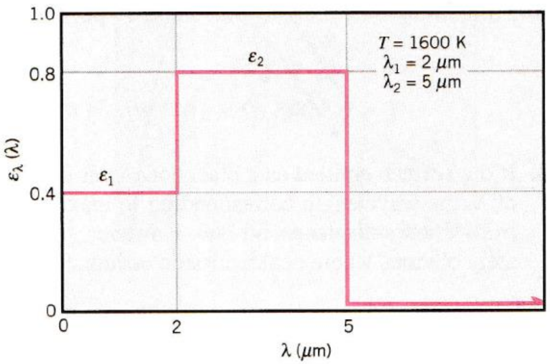
Determine total, hemispherical emissivity and total emissive power for a diffuse surface

Given: Spectral, hemispherical emissivity of a diffuse surface at 1600 K

Solution: **1.** Total, hemispherical emissivity.

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}\,d\lambda}{E_b} = \frac{\varepsilon_1 \int_0^2 E_{\lambda,b}\,d\lambda}{E_b} + \frac{\varepsilon_2 \int_2^5 E_{\lambda,b}\,d\lambda}{E_b}$$

$$\varepsilon = \varepsilon_1 F_{(0\rightarrow 2\,\mu\text{m})} + \varepsilon_2[F_{(0\rightarrow 5\,\mu\text{m})} - F_{(0\rightarrow 2\,\mu\text{m})}]$$



From Table 12.2 we obtain

$$\lambda_1 T = 2\,\mu\text{m} \times 1600\text{ K} = 3200\,\mu\text{m}\cdot\text{K}: \qquad F_{(0\rightarrow 2\,\mu\text{m})} = 0.318$$

$$\lambda_2 T = 5\,\mu\text{m} \times 1600\text{ K} = 8000\,\mu\text{m}\cdot\text{K}: \qquad F_{(0\rightarrow 5\,\mu\text{m})} = 0.856$$

$$\varepsilon = 0.4 \times 0.318 + 0.8[0.856 - 0.318] = 0.558$$

Ex.12.6

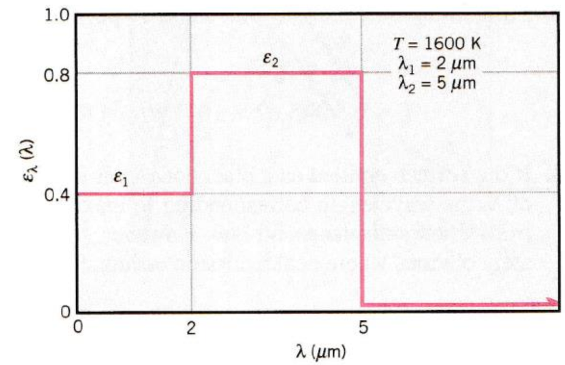
Determine total, hemispherical emissivity and total emissive power for a diffuse surface

Given: Spectral, hemispherical emissivity of a diffuse surface at 1600 K

Solution: **2.** Total emissive power.

$$E = \varepsilon E_b = \varepsilon \sigma T^4$$

$$E = 0.558(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})(1600\ \mathrm{K})^4 = 207\ \mathrm{kW/m^2}$$



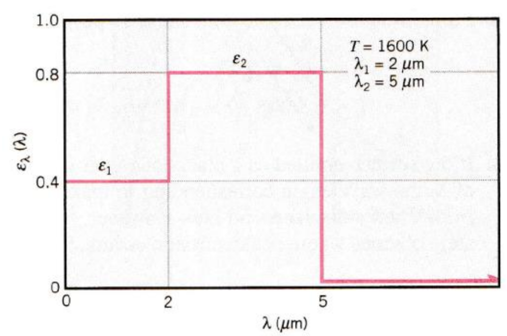
Ex.12.6

Determine total, hemispherical emissivity and total emissive power for a diffuse surface

Given: Spectral, hemispherical emissivity of a diffuse surface at 1600 K

Solution: **3.** Wavelength at which spectral emissive power will be a maximum.

Since ε_λ varies with λ, the wavelength for maximum emission might not be the one given by the Wien's displacement law, if the emissivity at that wavelength is small.



Need to find maximum wavelength with a two-step procedure

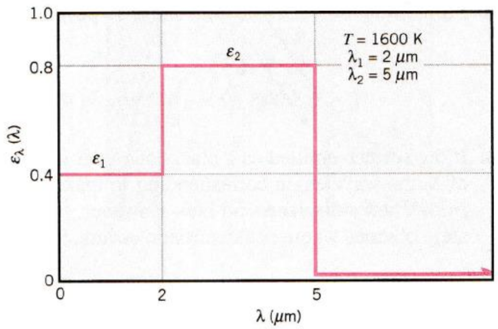
Ex.12.6

Determine total, hemispherical emissivity and total emissive power for a diffuse surface

Given: Spectral, hemispherical emissivity of a diffuse surface at 1600 K

Solution: **3.** Wavelength at which spectral emissive power will be a maximum.

First, find the maximum wavelength given by the Wien's displacement law and its corresponding emissivity and emissive power



$$\lambda_{\max} = \frac{2898\ \mu\text{m}\cdot\text{K}}{1600\ \text{K}} = 1.81\ \mu\text{m}$$

$$E_\lambda(\lambda_{\max}, T) = \varepsilon_\lambda(\lambda_{\max})E_{\lambda,b}(\lambda_{\max}, T)$$

$$E_\lambda(\lambda_{\max}, T) = \pi\varepsilon_\lambda(\lambda_{\max})I_{\lambda,b}(\lambda_{\max}, T)$$

$$= \pi\varepsilon_\lambda(\lambda_{\max})\frac{I_{\lambda,b}(\lambda_{\max}, T)}{\sigma T^5} \times \sigma T^5$$

$$E_\lambda(1.81\ \mu\text{m}, 1600\ \text{K}) =$$

$$\pi \times 0.4 \times 0.722 \times 10^{-4}\ (\mu\text{m}\cdot\text{K}\cdot\text{sr})^{-1} \times 5.67$$

$$\times\ 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 \times (1600\ \text{K})^5 = 54\ \text{kW/m}^2\cdot\mu\text{m}$$

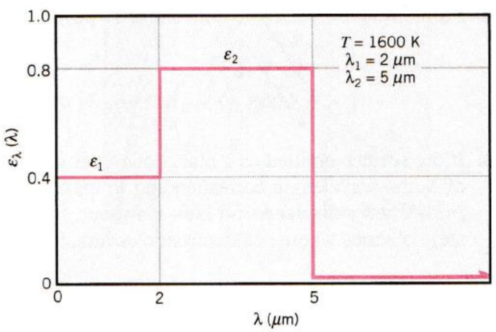
However, this might not the maximum emissive power since the emissivity is higher at other region

Ex.12.6

Determine total, hemispherical emissivity and total emissive power for a diffuse surface

Given: Spectral, hemispherical emissivity of a diffuse surface at 1600 K

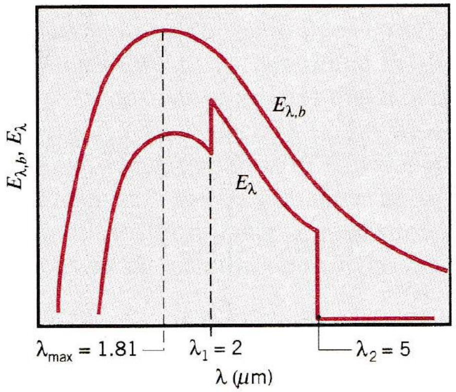
Solution: **3.** Wavelength at which spectral emissive power will be a maximum.



The likely candidate for maximum emission is at 2 μm, since the emissivity is at the highest value 0.8 and the blackbody emissive power is decreasing with wavelength after 2 μm

For λ = 2μm, λT = 3200 μm-K

$$[I_{\lambda,b}(\lambda_1, T)/\sigma T^5] = 0.706 \times 10^{-4}\ (\mu\mathrm{m}\cdot\mathrm{K}\cdot\mathrm{sr})^{-1}.$$

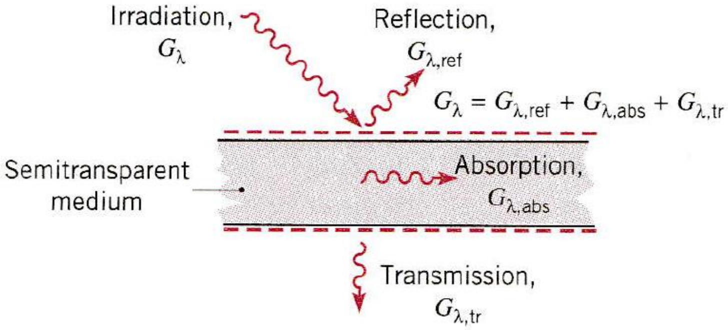


$$E_\lambda(2\ \mu\mathrm{m}, 1600\ \mathrm{K}) = \pi \times 0.80 \times 0.706 \times 10^{-4}\ (\mu\mathrm{m}\cdot\mathrm{K}\cdot\mathrm{sr})^{-1} \times 5.67 \times 10^{-8}\ \mathrm{W/m^2\cdot K^4}\ (1600\ \mathrm{K})^5$$

$$E_\lambda(2\ \mu\mathrm{m}, 1600\ \mathrm{K}) = 105.5\ \mathrm{kW/m^2\cdot\mu m} > E_\lambda(1.81\ \mu\mathrm{m}, 1600\ \mathrm{K})$$

Absorptivity

When the irradiation arrives at a surface, it can be absorbed, reflected and transmitted



$$G_\lambda = G_{\lambda,\mathrm{ref}} + G_{\lambda,\mathrm{abs}} + G_{\lambda,\mathrm{tr}}$$

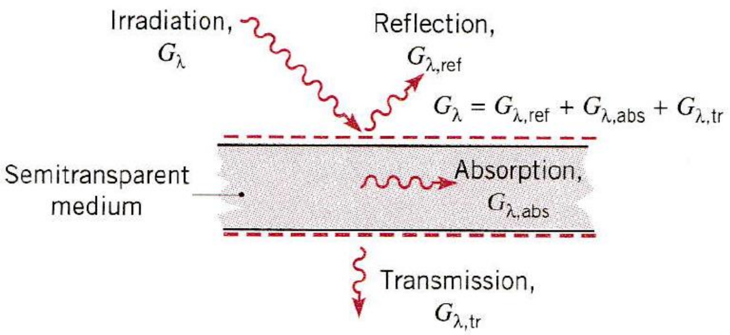
<u>Absorptivity</u>

$$\alpha_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{abs}}(\lambda)}{G_\lambda(\lambda)}$$

Spectral, hemispherical absorptivity

Absorptivity

When the irradiation arrives at a surface, it can be absorbed, reflected and transmitted



$$G_\lambda = G_{\lambda,\mathrm{ref}} + G_{\lambda,\mathrm{abs}} + G_{\lambda,\mathrm{tr}}$$

<u>Absorptivity</u>

$$\alpha \equiv \frac{G_{\mathrm{abs}}}{G} \qquad \alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda}$$

Total, hemispherical absorptivity

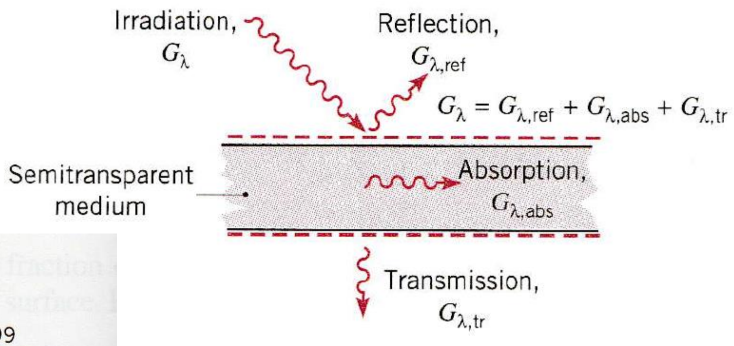
<u>**α is a function of the irradiation, so it is a function of the source temperature**</u>.

Treating the sun as a blackbody at T = 5800 K, the solar absorptivity is given by

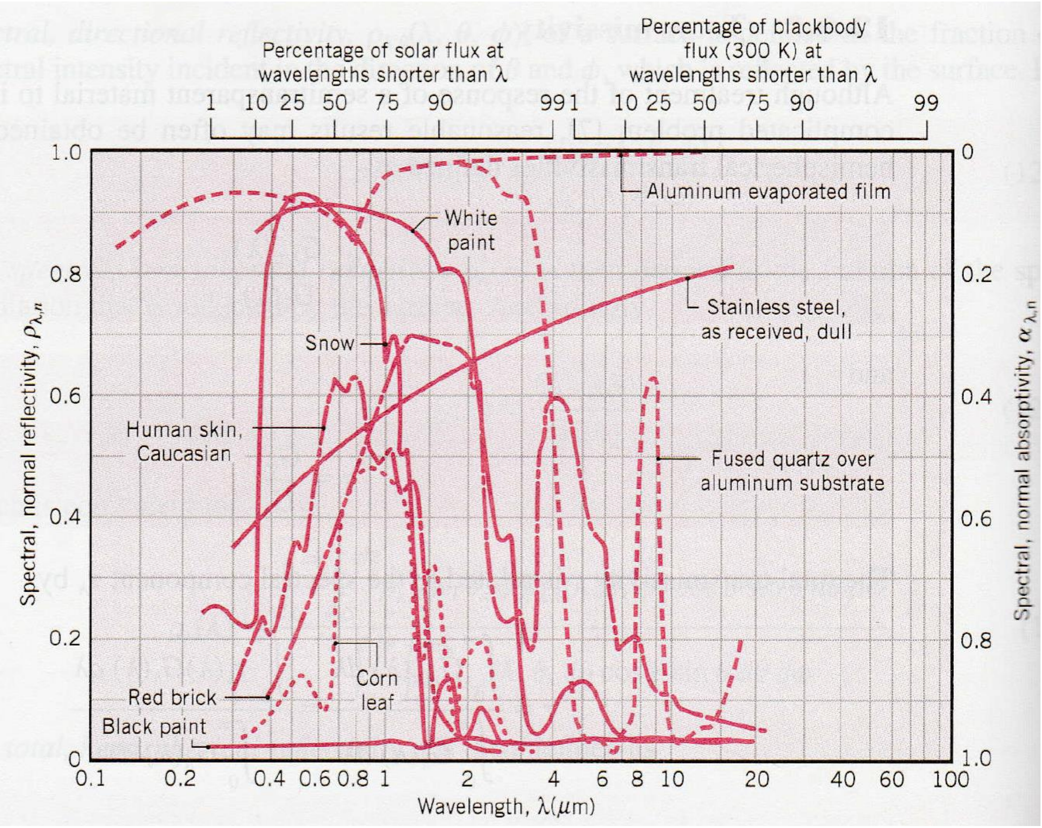
$$\alpha_S \approx \frac{\int_0^\infty \alpha_\lambda(\lambda) E_{\lambda,b}(\lambda, 5800\ \mathrm{K})\, d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda, 5800\ \mathrm{K})\, d\lambda}$$

Absorptivity

When the irradiation arrives at a surface, it can be absorbed, reflected and transmitted

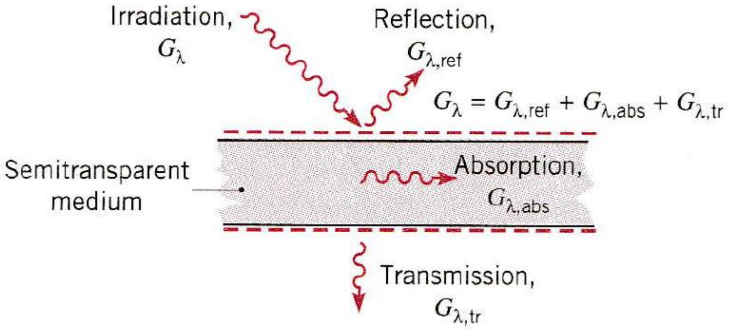


$$G_\lambda = G_{\lambda,\mathrm{ref}} + G_{\lambda,\mathrm{abs}} + G_{\lambda,\mathrm{tr}}$$



Reflectivity

When the irradiation arrives at a surface, it can be absorbed, reflected and transmitted



$$G_\lambda = G_{\lambda,\mathrm{ref}} + G_{\lambda,\mathrm{abs}} + G_{\lambda,\mathrm{tr}}$$

<u>Reflectivity</u>

$$\rho_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{ref}}(\lambda)}{G_\lambda(\lambda)}$$

Spectral, hemispherical reflectivity

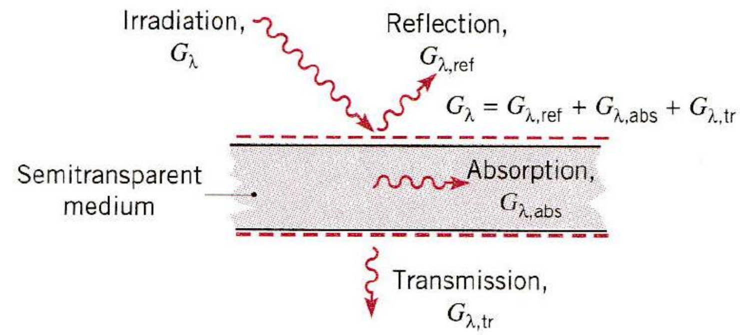
$$\rho \equiv \frac{G_{\mathrm{ref}}}{G}$$

Total, hemispherical reflectivity

$$\rho = \frac{\int_0^\infty \rho_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda}$$

Reflectivity

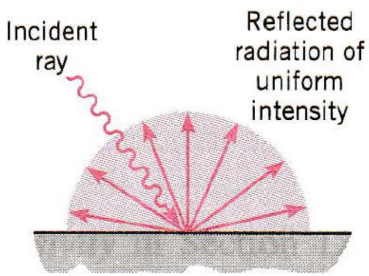
When the irradiation arrives at a surface, it can be absorbed, reflected and transmitted

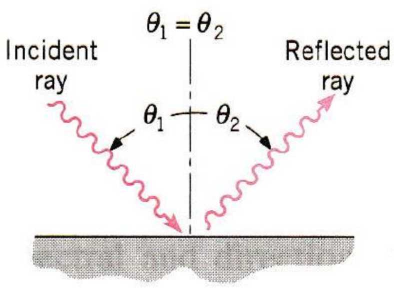


$$G_\lambda = G_{\lambda,\text{ref}} + G_{\lambda,\text{abs}} + G_{\lambda,\text{tr}}$$

<u>Reflectivity</u>

<u>Comments of diffuse reflection and specular reflection</u>

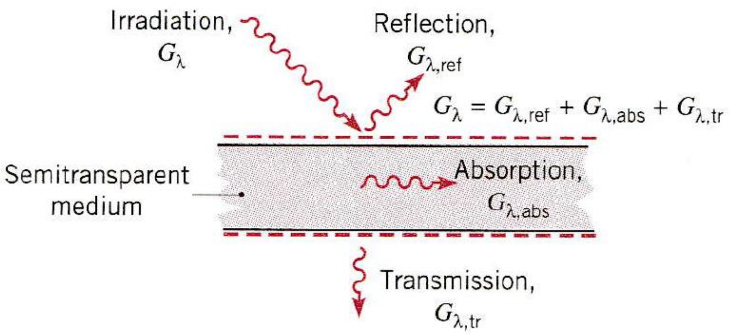




- diffuse reflection means that the reflected radiation intensity is uniform in all direction
- diffuse reflection is reasonable for most heat transfer engineering application

Transmissivity

When the irradiation arrives at a surface, it can be absorbed, reflected and transmitted



$$G_\lambda = G_{\lambda,\mathrm{ref}} + G_{\lambda,\mathrm{abs}} + G_{\lambda,\mathrm{tr}}$$

<u>Transmissivity</u>

$$\tau_\lambda = \frac{G_{\lambda,\mathrm{tr}}(\lambda)}{G_\lambda(\lambda)}$$

Spectral, hemispherical reflectivity

$$\tau = \frac{G_{\mathrm{tr}}}{G}$$

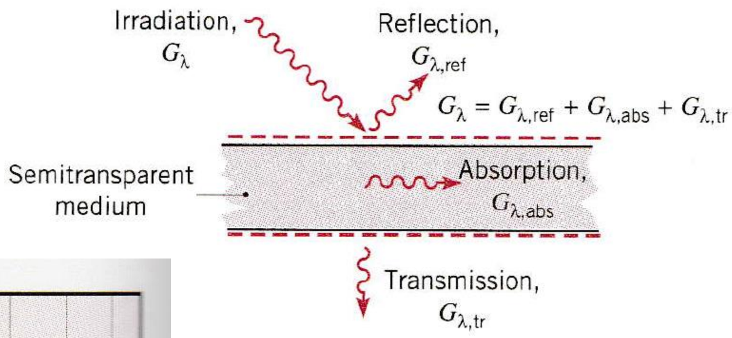
Total, hemispherical reflectivity

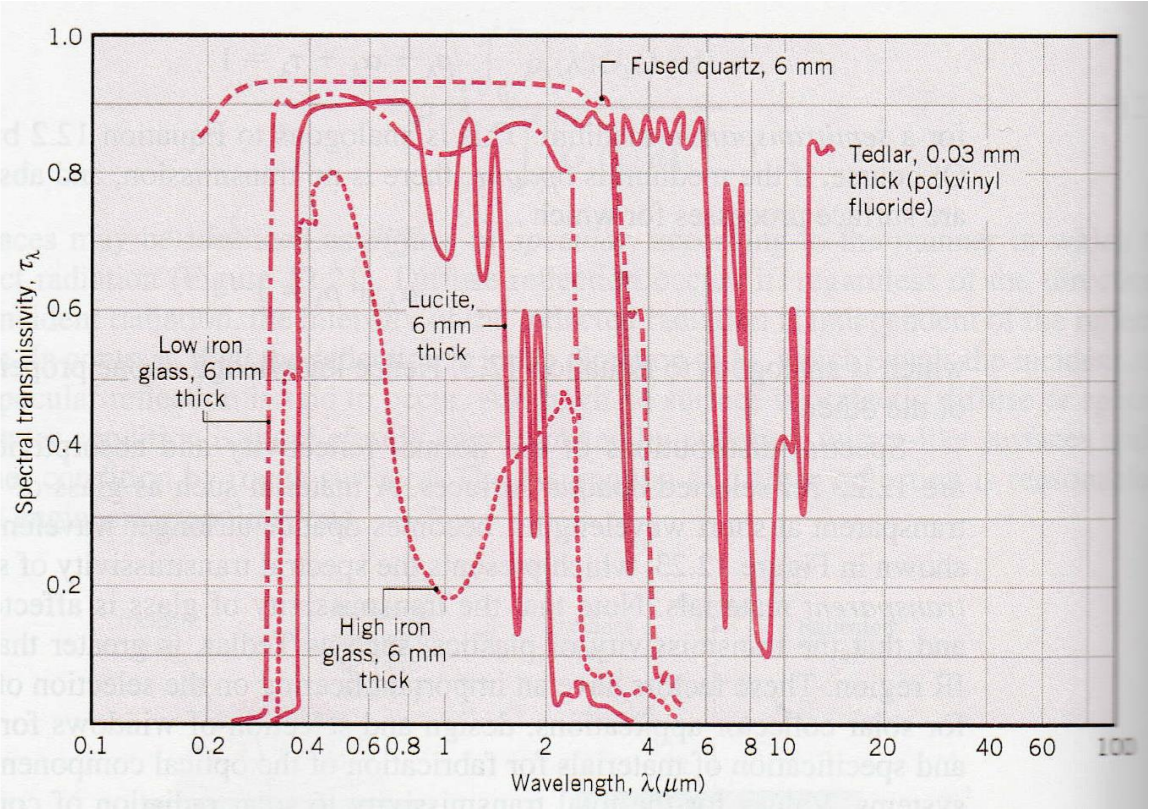
$$\tau = \frac{\int_0^\infty G_{\lambda,\mathrm{tr}}(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda} = \frac{\int_0^\infty \tau_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda}$$

Transmissivity

When the irradiation arrives at a surface, it can be absorbed, reflected and transmitted

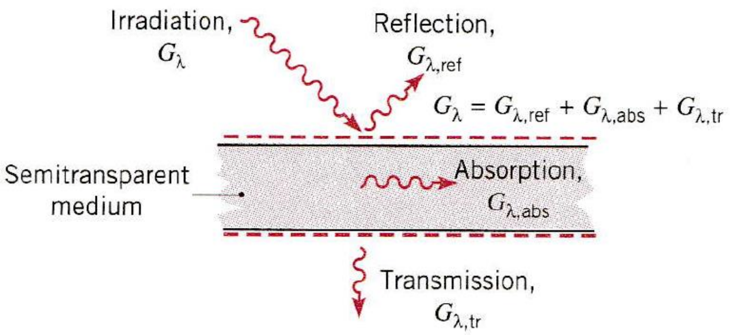
$$G_\lambda = G_{\lambda,\mathrm{ref}} + G_{\lambda,\mathrm{abs}} + G_{\lambda,\mathrm{tr}}$$





Radiative Energy Balance at a Surface

When the irradiation arrives at a surface, it can be absorbed, reflected and transmitted



$$G_\lambda = G_{\lambda,\mathrm{ref}} + G_{\lambda,\mathrm{abs}} + G_{\lambda,\mathrm{tr}}$$

<u>For semi-transparent surface ($\tau_\lambda \neq 0$)</u>

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1$$

<u>For opaque surface ($\tau_\lambda = 0$)</u>

$$\alpha_\lambda + \rho_\lambda = 1$$

Kirchoff's Law

From thermodynamic consideration

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T)=\alpha_{\lambda,\theta}(\lambda,\theta,\phi,T)$$ no restriction

$$\varepsilon_\lambda(\lambda,T)=\alpha_\lambda(\lambda,T)$$ Surface is diffuse

$$\varepsilon(T)=\alpha(T)$$

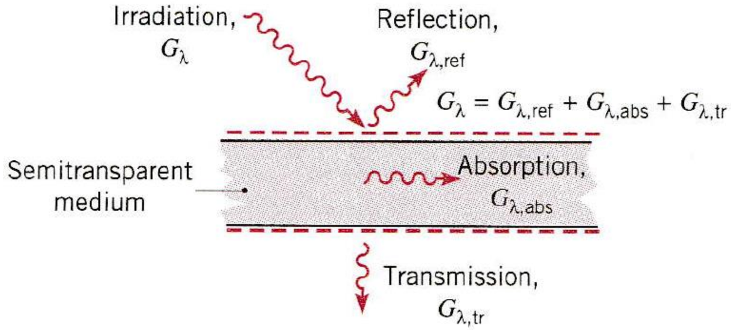
Irradiation is diffuse and proportional to $E_{\lambda,b}(T)$
or
Surface is diffuse, gray

A diffuse-gray surface is a surface with emissivity which is independent of direction (diffuse) and wavelength (gray)

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T)=\varepsilon(T)=\alpha(T)$$

Radiative Energy Balance at a Diffuse Surface

When the irradiation arrives at a surface, it can be absorbed, reflected and transmitted



$$G_\lambda = G_{\lambda,\text{ref}} + G_{\lambda,\text{abs}} + G_{\lambda,\text{tr}}$$

<u>For semi-transparent surface ($\tau_\lambda \neq 0$)</u>

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1 \quad \Rightarrow \quad \rho_\lambda + \varepsilon_\lambda + \tau_\lambda = 1$$

<u>For opaque surface ($\tau_\lambda = 0$)</u>

$$\alpha_\lambda + \rho_\lambda = 1 \quad \Rightarrow \quad \rho_\lambda + \varepsilon_\lambda = 1$$

Ex. 12.8

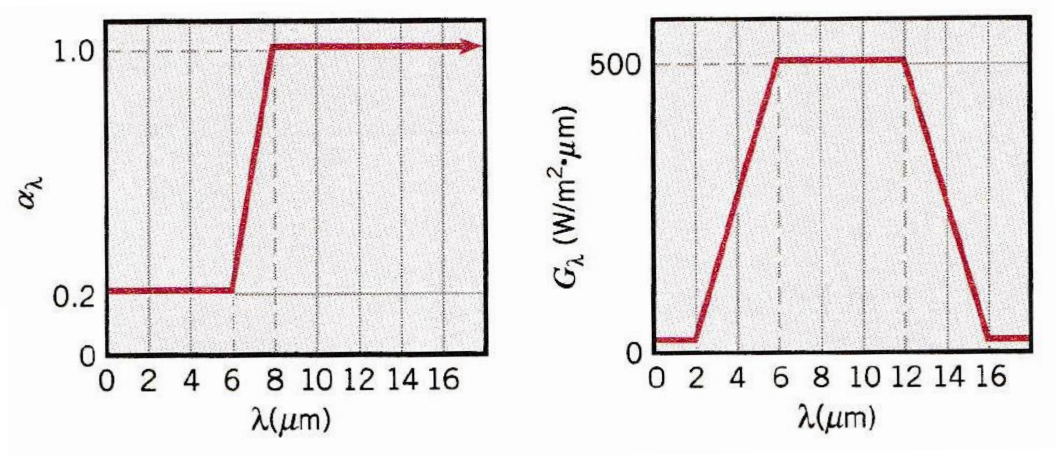
Determine reflectivity from absorptivity for an opaque surface and the impact of absorption on temperature change

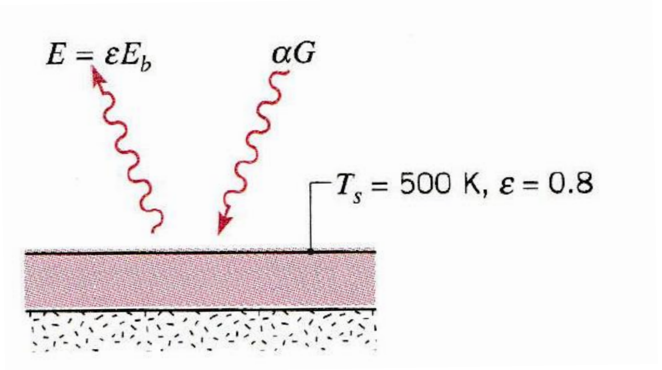
Given: Spectral hemispherical absorptivity and irradiation of an opaque surface with

$T_s = 500$ K
$\varepsilon = 0.8$

Find:

1. Spectral distribution of reflectivity.
2. Total, hemispherical absorptivity.
3. Nature of surface temperature change.





Ex. 12.8

Determine reflectivity from absorptivity for an opaque surface and the impact of absorption on temperature change

Given: Spectral hemispherical absorptivity and irradiation of an opaque surface with

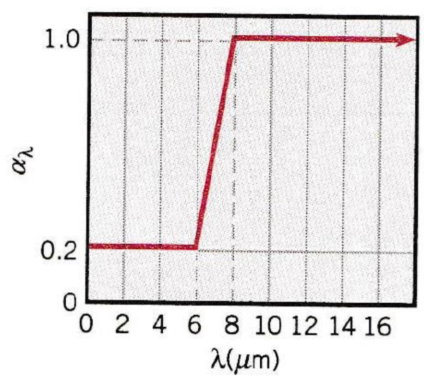
T_s = 500 K

ε = 0.8

Solution: **1.** Spectral distribution of reflectivity.

$$\rho_\lambda = 1 - \alpha_\lambda$$

1 -



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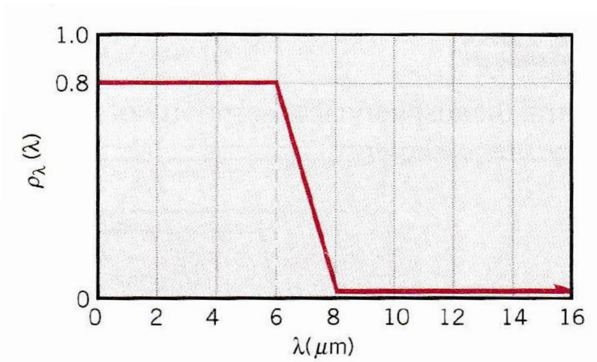

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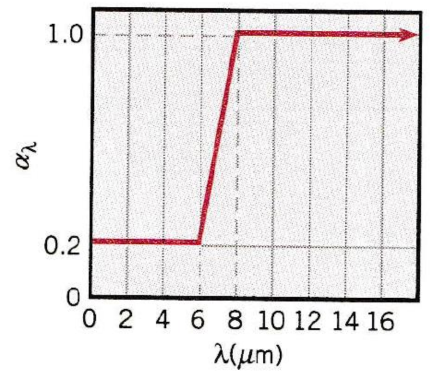
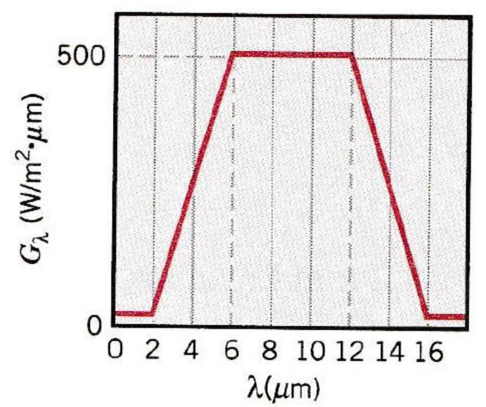


Solution: **2.** Total, hemispherical absorptivity.

$$\alpha = \frac{G_{\text{abs}}}{G} = \frac{\int_0^\infty \alpha_\lambda G_\lambda \, d\lambda}{\int_0^\infty G_\lambda \, d\lambda}$$

$$\alpha = \frac{0.2 \int_2^6 G_\lambda \, d\lambda + 500 \int_6^8 \alpha_\lambda \, d\lambda + 1.0 \int_8^{16} G_\lambda \, d\lambda}{\int_2^6 G_\lambda \, d\lambda + \int_6^{12} G_\lambda \, d\lambda + \int_{12}^{16} G_\lambda \, d\lambda}$$



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Determine reflectivity from absorptivity for an opaque surface and the impact of absorption on temperature change

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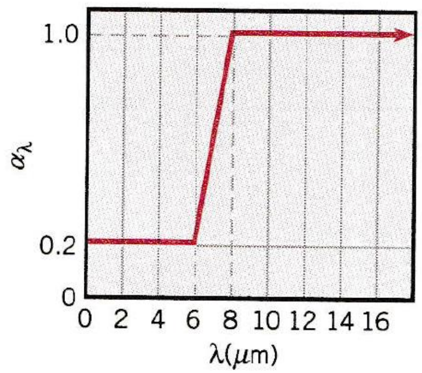
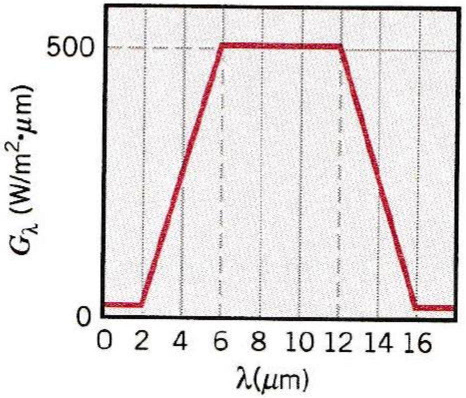


Solution: **2.** Total, hemispherical absorptivity.

$$\alpha = \{0.2(\tfrac{1}{2})500\ \text{W/m}^2\cdot\mu\text{m}\,(6-2)\ \mu\text{m} + 500\ \text{W/m}^2\cdot\mu\text{m}\,[0.2(8-6)\ \mu\text{m} + (1-0.2)(\tfrac{1}{2})(8-6)\ \mu\text{m}] + [1\times 500\ \text{W/m}^2\cdot\mu\text{m}\,(12-8)\ \mu\text{m} + 1(\tfrac{1}{2})500\ \text{W/m}^2\cdot\mu\text{m}\,(16-12)\ \mu\text{m}]\} \div [(\tfrac{1}{2})500\ \text{W/m}^2\cdot\mu\text{m}\,(6-2)\ \mu\text{m} + 500\ \text{W/m}^2\cdot\mu\text{m}\,(12-6)\ \mu\text{m} + (\tfrac{1}{2})500\ \text{W/m}^2\cdot\mu\text{m}\,(16-12)\ \mu\text{m}]$$

$$\alpha = \frac{G_{abs}}{G} = \frac{(200+600+3000)\ \text{W/m}^2}{(1000+3000+1000)\ \text{W/m}^2} = \frac{3800\ \text{W/m}^2}{5000\ \text{W/m}^2} = 0.76$$



Ex. 12.8

Determine reflectivity from absorptivity for an opaque surface and the impact of absorption on temperature change

Given: Spectral hemispherical absorptivity and irradiation of an opaque surface with

$T_s = 500$ K

$\varepsilon = 0.8$

Solution: **3.** Nature of surface temperature change.

Neglecting convection effects, the net heat flux *to* the surface is

$$q''_{net} = \alpha G - E = \alpha G - \varepsilon \sigma T^4$$

$$q''_{net} = 0.76(5000\ \mathrm{W/m^2}) - 0.8 \times 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}(500\ \mathrm{K})^4$$

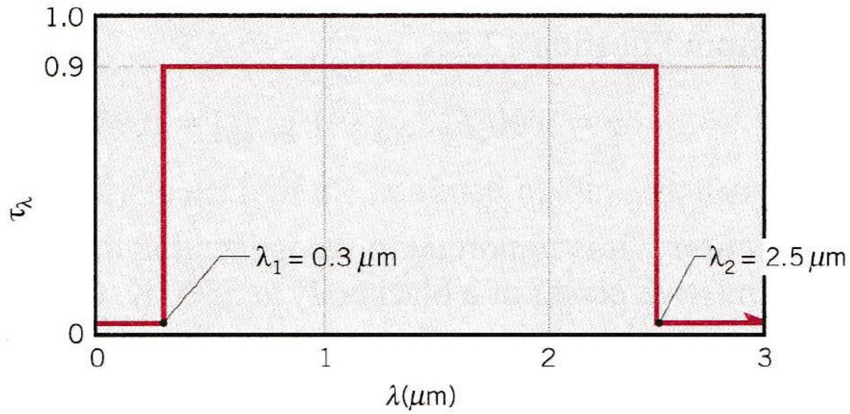
$$q''_{net} = 3800 - 2835 = 965\ \mathrm{W/m^2}$$

Since $q''_{net} > 0$, the surface temperature will *increase* with time.

Ex. 12.9

Determine total transmissivity of cover glass to solar radiation

<u>**Given**</u>: Spectral hemispherical transmissivity of cover glass



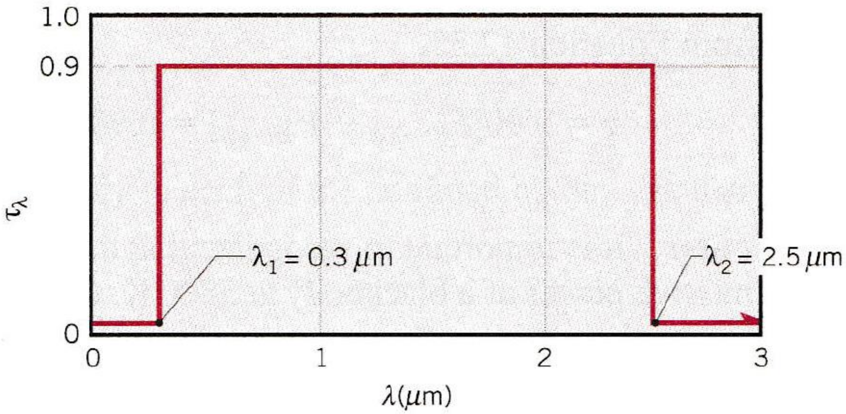
<u>**Find**</u>: Total transmissivity of cover glass to solar radiation.

Ex. 12.9

Determine total transmissivity of cover glass to solar radiation

Solution: Total transmissivity of cover glass to solar radiation.

Assume:
The sun is radiating as a blackbody at 5800 K and a fraction of its energy (due to the long distance) is reaching the earth's surfac



$$G_\lambda = FE_{\lambda,b}(5800K)$$

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda \, d\lambda}{\int_0^\infty G_\lambda \, d\lambda} \Rightarrow \tau = \frac{\int_0^\infty \tau_\lambda E_{\lambda,b}(5800\text{ K})\, d\lambda}{\int_0^\infty E_{\lambda,b}(5800\text{ K})\, d\lambda} \Rightarrow \tau = 0.90\frac{\int_{0.3}^{2.5} E_{\lambda,b}(5800\text{ K})\, d\lambda}{E_b(5800\text{ K})}$$

$\lambda_1 = 0.3\ \mu\text{m}, T = 5800\text{ K}$: $\quad \lambda_1 T = 1740\ \mu\text{m}\cdot\text{K}, F_{(0\rightarrow\lambda_1)} = 0.0335$

$\lambda_2 = 2.5\ \mu\text{m}, T = 5800\text{ K}$: $\quad \lambda_2 T = 14{,}500\ \mu\text{m}\cdot\text{K}, F_{(0\rightarrow\lambda_2)} = 0.9664$

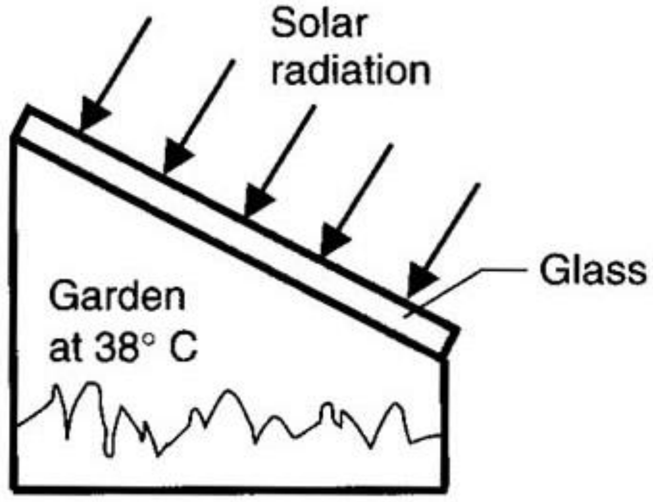
$$\tau = 0.90[F_{(0\rightarrow\lambda_2)} - F_{(0\rightarrow\lambda_1)}] = 0.90(0.9664 - 0.0335) = 0.84$$

Example on the Greenhouse Effect

- **Greenhouse effect**

Transmissivity of silica glass

λ	τ_λ
$\lambda < 0.33\ \mu m$	0
$0.33\ \mu m < \lambda < 2.5\ \mu m$	0.9
$\lambda > 2.5\ \mu m$	0



Answer: 82.1%; 0.001%

What percent of the solar radiation (consider the sun as a blackbody at 5780 K) and the garden radiation (consider as a blackbody at 38 ^oC) will be transmitted through the glass ?

Greenhouse Effect (Cont.)

Incident solar flux:

$$G_\lambda = C E_{\lambda b}(T_s) \qquad G = C \int_0^\infty E_{\lambda b}(T_s)\, d\lambda = C \sigma T^4$$

Transmitted solar energy:

$$G_{\lambda,tr} = \tau_\lambda C E_{\lambda b}(T_s) \qquad G_{tr} = C \int_0^\infty \tau_\lambda E_{\lambda b}(T_s)\, d\lambda$$

$$\text{Total transmissivity} = \tau = \frac{E_{trans}}{E_{inc}} = \frac{1}{\sigma T_s^4} \int_0^\infty \tau_\lambda E_{\lambda b}(T_s)\, d\lambda$$

Greenhouse Effect (Cont.)

Evaluation of Transmissivity :

$$\text{Total transmissivity} = \tau = \frac{E_{trans}}{E_{inc}} = \frac{1}{\sigma T_s^4}\int_0^{\infty} \tau_\lambda E_{\lambda b}(T_s)\,d\lambda$$

$$\tau = \frac{1}{\sigma T_s^4}\int_0^{\infty} \tau_\lambda E_{\lambda b}(T_s)\,d\lambda = \frac{1}{\sigma T_s^4}\int_{0.33}^{2.5} 0.9 E_{\lambda b}(T_s)\,d\lambda$$

$$= \frac{1}{\sigma T_s^4}\int_{0.}^{2.5} 0.9 E_{\lambda b}(T_s)\,d\lambda - \frac{1}{\sigma T_s^4}\int_{0}^{0.33} 0.9 E_{\lambda b}(T_s)\,d\lambda$$

$$= 0.9\left[F_{0-2.5T_s} - F_{0-0.33T_s}\right]$$

Fractional Black Body Emissive Power

$$\tau = 0.9\left[F_{0-2.5T_s} - F_{0-0.33T_s}\right]$$

$$= 0.9\left[F_{0-14675} - F_{0-1937}\right] = 0.9\left[0.96726 - 0.0828\right] = 0.796$$

Greenhouse Effect (Cont.)

For radiation from the graden at 38 C:

$$\tau = \frac{1}{\sigma T_g^4}\int_0^{\infty} \tau_\lambda E_{\lambda b}(T_s)d\lambda = \frac{1}{\sigma T_g^4}\int_{0.33}^{2.5} 0.9E_{\lambda b}(T_s)d\lambda$$

$$\tau = 0.9\left[F_{0-2.5T_g} - F_{0-0.33T_g}\right]$$

$$\tau = 0.9[F_{0-777.5} - F_{0-10.26}]\sim 0$$