Thermal Radiation I
Chapter 5
Radiative Exchange between Surfaces
(Network Analysis)

Radiative Exchange Between Diffuse, Gray Surface in an Enclosure - The Network Analogy

The basic assumptions

1. All surfaces are opaque
2. The surface temperature is uniform
3. The surface properties are uniform
4. The surface is diffuse and gray ($\varepsilon = \alpha = 1 - \rho$)
5. The incident and reflected energy flux is uniform over each individual surface

Comments:

Assumptions 2 and 3 are not too severe as you can subdivide a give surface into small surfaces with uniform temperature and properties

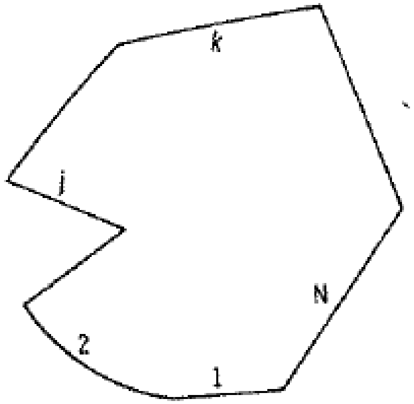
Radiative Exchange Between Diffuse, Gray Surface in an Enclosure - The Network Analogy

<u>The net radiation method</u>

The general problem is

1. To find the required energy supplied to a surface when its temperature is known, or

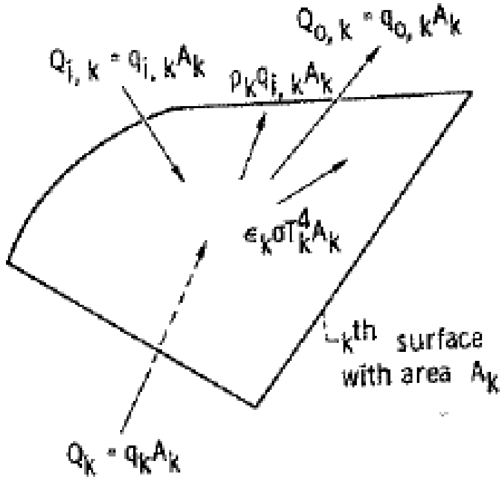
2. To find the temperature that a surface will achieve when a known heat input is imposed



Radiative Exchange Between Diffuse, Gray Surface in an Enclosure - The Network Analogy

The net radiation method

The concept of radiosity, irradiation and net radiative heat flux



$q_{o,k}(W/m^2, J_k)$ = net outgoing energy flux (reflection and emission) from surface A_k, (Radiosity at A_k)

$q_{i,k}(W/m^2, G_k)$ = net incoming energy flux (from all surfaces, as well as the medium) incident upon surface A_k, (Irradiation at A_k)

$q_k(W/m^2) = q_{o,k} - q_{i,k}$, net radiative heat flux leaving from surface A_k

Radiative Exchange Between Diffuse, Gray Surface in an Enclosure - The Network Analogy

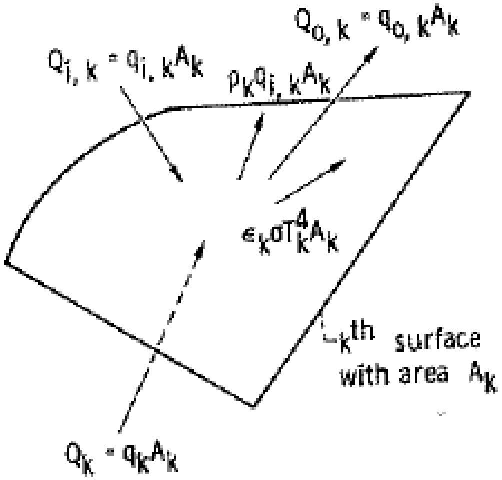
The net radiation method

Net radiative exchange at a surface

$$Q_i = A_i\left(J_i - G_i\right)$$

$$J_i = E_i + \rho_i G_i = \varepsilon_i E_{bi} + \left(1 - \varepsilon_i\right) G_i$$

$$Q_i = \frac{E_{bi} - J_i}{\left(1 - \varepsilon_i\right) / \left(A_i \varepsilon_i\right)}$$



Radiative Exchange Between Diffuse, Gray Surface in an Enclosure - The Network Analogy

The net radiation method

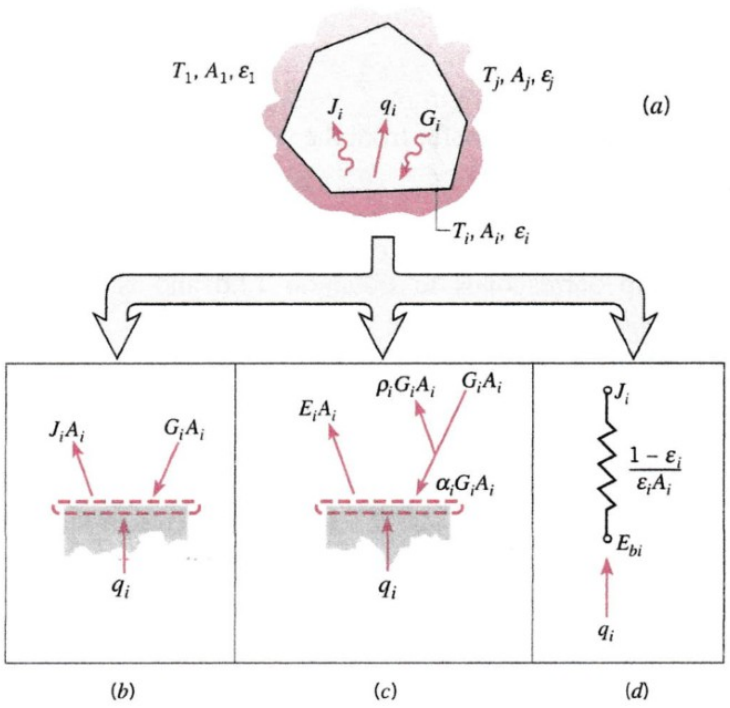
The network analogy for radiative heat transfer at a surface based on its local values of radiosity J_i, irradiation G_i, and heat flux q_i

J_i = External Potential at Surface A_i

E_{bi} = Internal Potential at Surface A_i

$\frac{1-\varepsilon_i}{A_i\varepsilon_i}$ = Internal Resistance at Surface A_i

Q_i = Current Flow out of Surface A_i

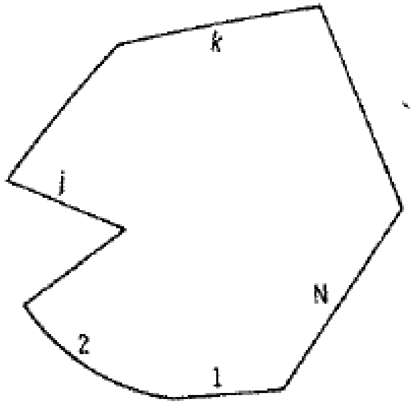


Radiative Exchange Between Diffuse, Gray Surface in an Enclosure - The Network Analogy

The net radiation method

Radiative Exchange Between Surfaces:

a. Relations between irradiation at surface i, G_i, and the radiosities of othter surfaces, J_k, $k = 1, \ldots, N$



$$A_i G_i = A_1 J_1 F_{1-i} + A_2 J_2 F_{2-i} + \cdots + A_i J_i F_{i-i} + \cdots + A_N J_N F_{N-i}$$

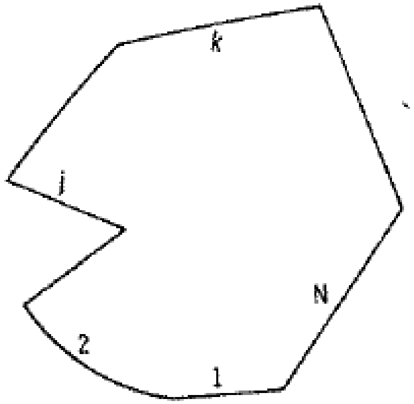
$$= \sum_{k=1}^{N} A_k J_k F_{k-i}$$

$$= A_i \sum_{k=1}^{N} J_k F_{i-k} \quad \text{(Reciprocity)}$$

Radiative Exchange Between Diffuse, Gray Surface in an Enclosure - The Network Analogy

<u>The net radiation method</u>

Radiative Exchange Between Surfaces:



b. Expressions of net radiative heat flux

$$
\begin{aligned}
Q_i &= A_i (J_i - G_i) \\
&= A_i \left(J_i - \sum_{k=1}^{N} J_k F_{i-k} \right) \\
&= A_i \left[\sum_{k=1}^{N} (J_i - J_k) F_{i-k} \right] \qquad \left(\sum_{k=1}^{N} F_{i-k} = 1 \right) \\
&= \sum_{k=1}^{N} \frac{(J_i - J_k)}{1/(A_i F_{i-k})} \\
&= \frac{E_{bi} - J_i}{(1-\varepsilon_i)/(A_i \varepsilon_i)}
\end{aligned}
$$

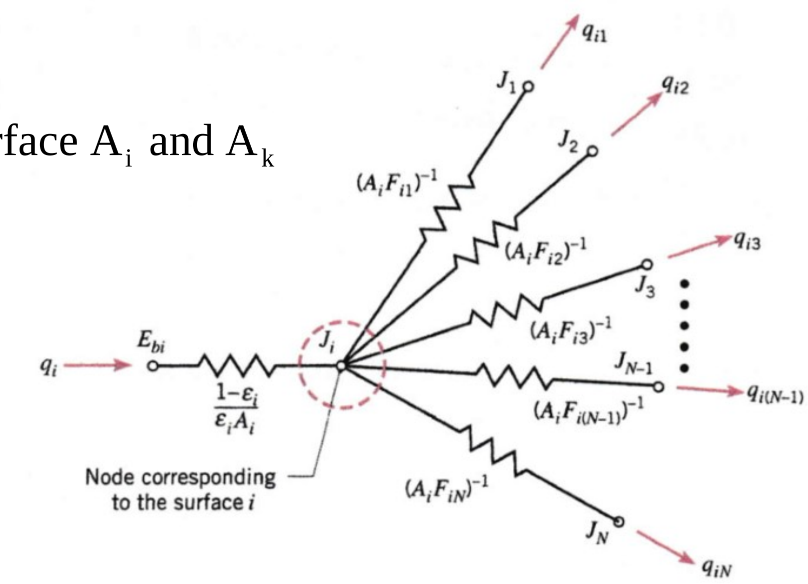
Radiative Exchange Between Diffuse, Gray Surface in an Enclosure - The Network Analogy

The net radiation method

Radiative Exchange Between Surfaces:

c. The network analogy

$$\frac{1}{A_i F_{i-k}} = \frac{1}{A_k F_{k-i}} = R_{i-k}$$ = Resistance Between Surface A_i and A_k



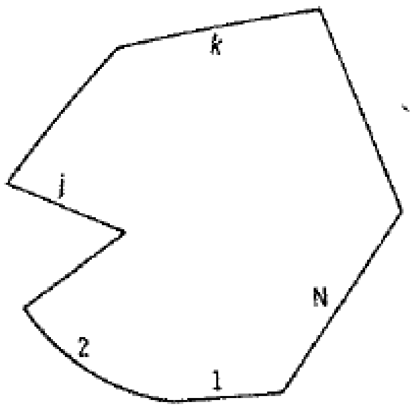
Radiative Exchange Between Diffuse, Gray Surface in an Enclosure - The Network Analogy

<u>The net radiation method</u>

Summary of Equations for the net radiation method

$$Q_i = \sum_{k=1}^{N} \frac{(J_i - J_k)}{1/(A_i F_{i-k})} \qquad i = 1, 2, \cdots, N$$

$$Q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/(A_i \varepsilon_i)} \qquad i = 1, 2, \cdots, N$$



2N equations with 2N unknowns (J_i and q_i(or E_{bi}))
Note that either q_i or E_{bi} must be specified on each surface

Examples of Network Analysis, 1
The two-surface enclosure

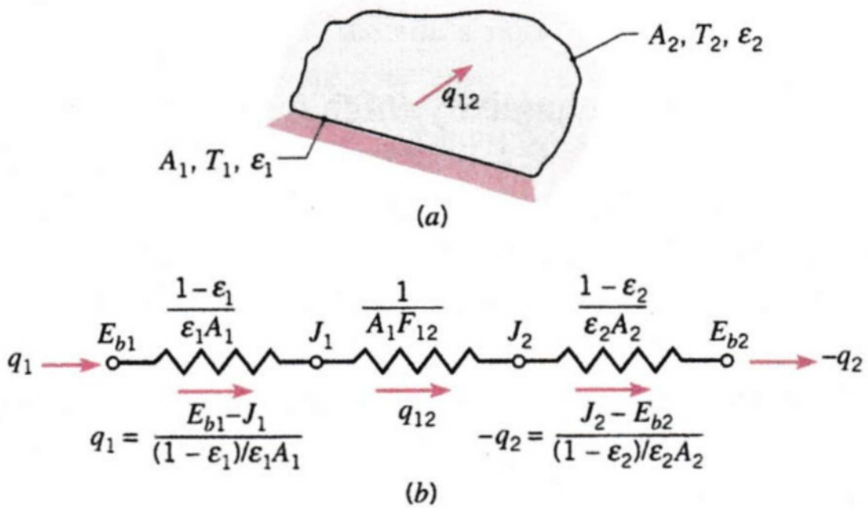
Solution is

$$Q = Q_1 = -Q_2 = \frac{E_{b1} - E_{b2}}{R_{tot}}$$

$$R_{tot} = \frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1-\varepsilon_2}{A_2\varepsilon_2} + \frac{1}{A_1F_{1-2}}$$

$$J_1 = E_{b1} - Q_1\frac{1-\varepsilon_1}{A_1\varepsilon_1}$$

$$J_2 = E_{b2} - Q_2\frac{1-\varepsilon_2}{A_2\varepsilon_2}$$

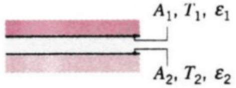


Examples of Network Analysis, 1
The two-surface enclosure

Example of application of the two-surface enclosure solution

TABLE 13.3 Special Diffuse, Gray, Two-Surface Enclosures

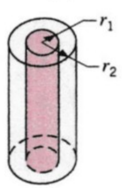
Large (Infinite) Parallel Planes



$$A_1 = A_2 = A \qquad F_{12} = 1$$

$$q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} \qquad (13.24)$$

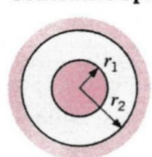
Long (Infinite) Concentric Cylinders



$$\frac{A_1}{A_2} = \frac{r_1}{r_2} \qquad F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)} \qquad (13.25)$$

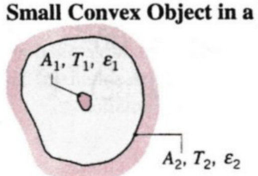
Concentric Spheres



$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} \qquad F_{12} = 1$$

$$q_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2} \qquad (13.26)$$

Small Convex Object in a Large Cavity



$$\frac{A_1}{A_2} \approx 0 \qquad F_{12} = 1$$

$$q_{12} = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4) \qquad (13.27)$$

Examples of Network Analysis, 1
The two-surface enclosure

A two-surface enclosure with one flat surface A_1 at $T_1 = 0$ and $\varepsilon_1 = 1.0$ (an opening to a "cold" environment)

$$Q_{out} = -Q_1 = \frac{E_{b2}}{R_{tot}}$$

$$R_{tot} = \frac{1-\varepsilon_2}{A_2\varepsilon_2} + \frac{1}{A_1F_{12}}$$

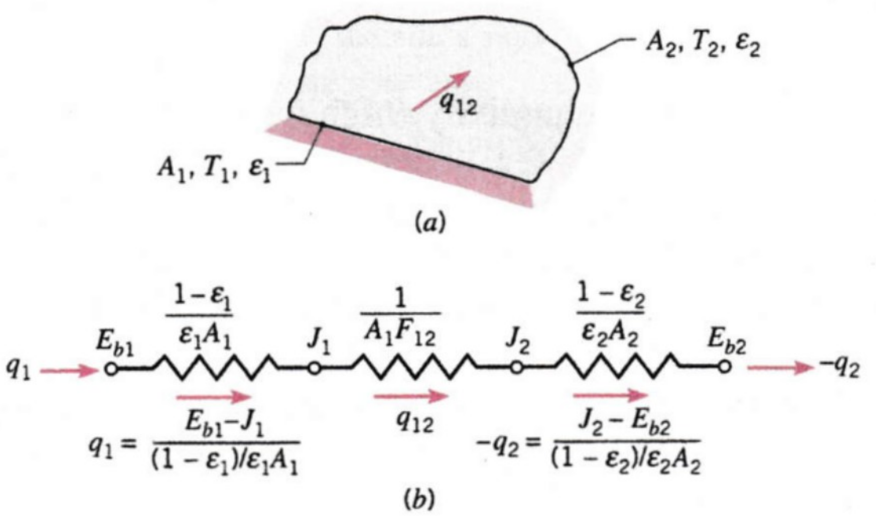
but $F_{12} = 1$

$$q_{out} = \frac{Q_{out}}{A_1} = \frac{E_{b2}}{A_1R_{tot}}$$

$$A_1R_{tot} = A_1\frac{1-\varepsilon_2}{A_2\varepsilon_2} + 1$$

$$q_{out} = \frac{E_{b2}}{A_1\frac{1-\varepsilon_2}{A_2\varepsilon_2} + 1}$$

$$q_{out} \to E_{b2} \text{ as } \frac{A_1}{A_2} \to 0 \text{ (small opening)}$$

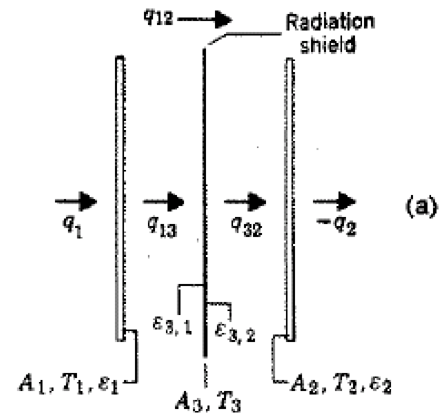


Examples of Network Analysis, 2
The Radiation Shield

Example of resistances in series

$$Q_1 = -Q_2 = \frac{A_1\left(E_{b1} - E_{b2}\right)}{\left(1-\varepsilon_1\right)/\varepsilon_1 + \left(1-\varepsilon_2\right)/\varepsilon_2 + \left(1-\varepsilon_{3,1}\right)/\varepsilon_{3,1} + \left(1-\varepsilon_{3,2}\right)/\varepsilon_{3,2} + 2}$$

$$= \frac{A_1\left(E_{b1} - E_{b2}\right)}{1/\varepsilon_1 + 1/\varepsilon_2 + \left(1-\varepsilon_{3,1}\right)/\varepsilon_{3,1} + \left(1-\varepsilon_{3,2}\right)/\varepsilon_{3,2}}$$

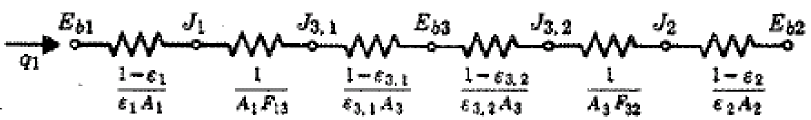


when $\quad \varepsilon_1 = \varepsilon_2 = \varepsilon_{3,1} = \varepsilon_{3,2} = \varepsilon$

$$Q_1 = -Q_2 = \frac{1}{2}\left(Q_1\right)_0$$

with

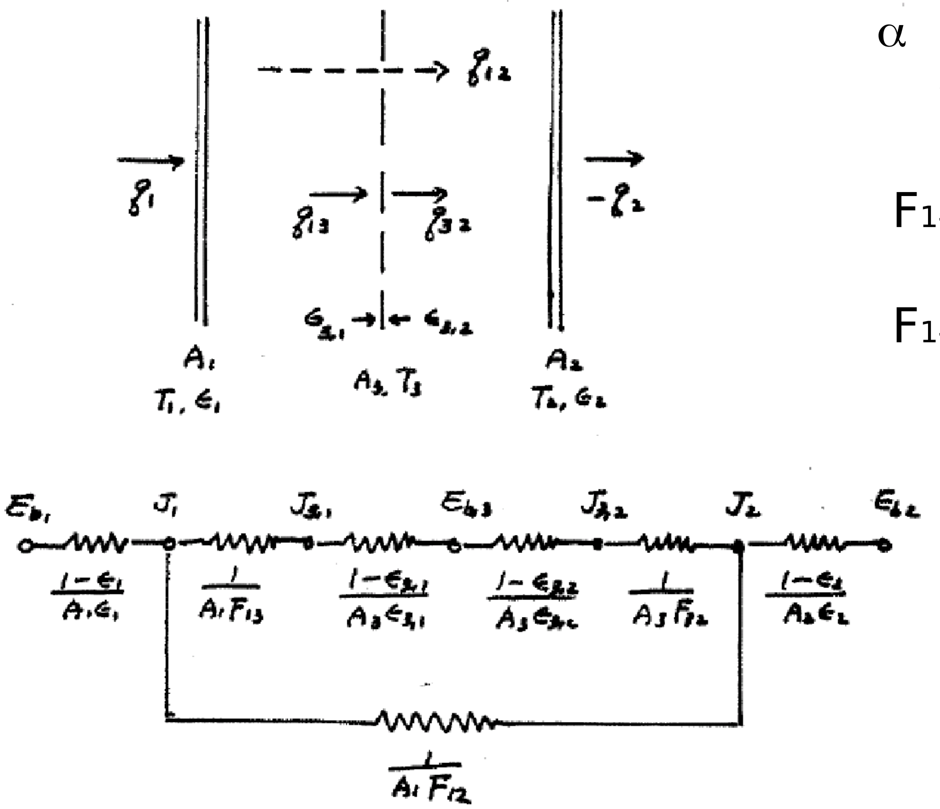
$$\left(Q_1\right)_0 = \frac{A_1\left(E_{b1} - E_{b2}\right)}{2/\varepsilon - 1} = \text{heat transfer without shield}$$



For N shields $\qquad \left(Q_1\right)_N = \frac{1}{N+1}\left(Q_1\right)_0$

Examples of Network Analysis, 3
The Perforated Radiation Shield

Example of resistances in series and in parallel



α = Fraction of Opening of the Shield

$F_{1-2} = \alpha$

$F_{1-3} = 1 - \alpha$

Examples of Network Analysis, 4
A Re-radiating Surface

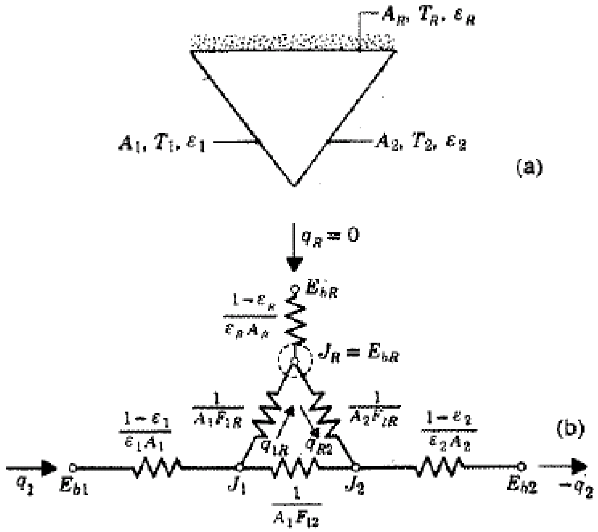
Example of a re-radiating (adiabatic) surface

Energy Balance (Kirchoff's Law in Network Analysis)

$$\frac{J_1 - J_R}{1/(A_1 F_{1-R})} + \frac{J_2 - J_R}{1/(A_2 F_{2-R})} = 0$$

$$\frac{J_R - J_1}{1/(A_R F_{R-1})} + \frac{J_2 - J_1}{1/(A_2 F_{2-1})} + \frac{E_{b1} - J_1}{(1-\varepsilon_1)/(A_1\varepsilon_1)} = 0$$

$$\frac{J_R - J_2}{1/(A_R F_{R-2})} + \frac{J_1 - J_2}{1/(A_1 F_{1-2})} + \frac{E_{b2} - J_2}{(1-\varepsilon_2)/(A_2\varepsilon_2)} = 0$$



would yield solutions for J_1, J_2 and J_R

$E_{bR} = J_R$ note that this is independent of ε_R

$$Q_1 = \frac{E_{b1} - J_1}{(1-\varepsilon_1)/(A_1\varepsilon_1)} \quad \text{and} \quad Q_2 = \frac{E_{b2} - J_2}{(1-\varepsilon_2)/(A_2\varepsilon_2)} = -Q_1$$

Examples of Network Analysis, 4
A Re-radiating Surface

Example of a re-radiating (adiabatic) surface

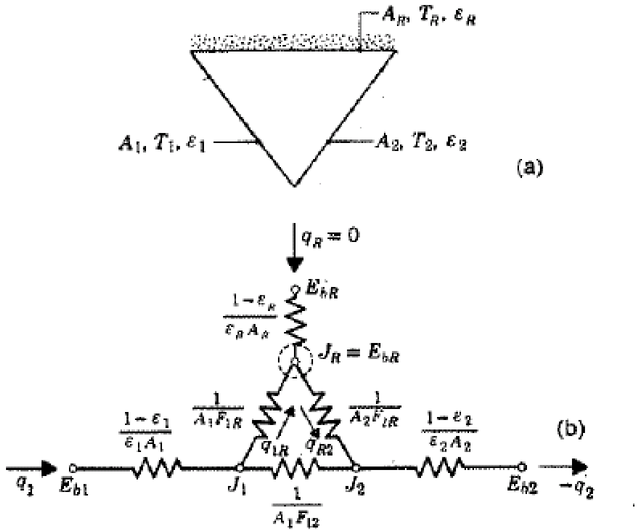
Heat Transfer Can be Obtained Based on Network Analogy

$$Q_1 = -Q_2 = \frac{E_{b1} - E_{b2}}{R_{total}}$$

with

$$R_{total} = \frac{1-\varepsilon_1}{A_1\varepsilon_1} + R_{net,1-2} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}$$

$$\frac{1}{R_{net,1-2}} = \frac{1}{1/(A_1F_{1-2})} + \frac{1}{1/(A_1F_{1-R}) + 1/(A_RF_{R-2})}$$



Examples of Network Analysis, 4
A Re-radiating Surface

Example of a re-radiating (adiabatic) surface

Example 5.15 (text)

$$F_{13} = \frac{1}{2}\left[X - \sqrt{X^2 - 4\left(\frac{R_3}{R_1}\right)^2}\right]$$

$$R_1 = \frac{r_1}{h}, R_3 = \frac{r_3}{h}, X = 1 + \frac{1+R_3^2}{R_1^2}$$

$$R_1 = \frac{7.5}{10} = 0.75, R_3 = \frac{5}{10} = 0.5, X = 1 + \frac{1+0.5^2}{0.75^2} = 3.22$$

$$F_{13} = \frac{1}{2}\left[3.22 - \sqrt{3.22^2 - 4\left(\frac{0.5}{0.75}\right)^2}\right] = 0.1444$$

$$E_{b1} = E_{b3} + Q_1 R_{tot}$$

$$T_1 = T_3\left(1 + \frac{Q_1 R_{tot}}{E_{b3}}\right)^{\frac{1}{4}}$$

$$= 550\left(1 + \frac{3000 \times \pi(0.075)^2 \times 192}{5.67 \times 10^{-8}(550)^4}\right)^{\frac{1}{4}}$$

$$= 721.5K$$

$$\frac{1}{R_{net,13}} = A_1 F_{13} + \frac{1}{\frac{1}{A_1 F_{1R}} + \frac{1}{A_3 F_{3R}}}$$

$$= A_1\left[F_{13} + \frac{1}{\frac{1}{1-F_{13}} + \frac{A_1}{A_3\left(1 - \frac{A_1}{A_3}F_{13}\right)}}\right]$$

$$= 64.7 cm^2$$

$$R_{tot} = \frac{1-\varepsilon_1}{A_1\varepsilon_1} + R_{net,13}$$

$$= \frac{1-0.6}{\pi(7.5)^2 \times 0.6} + \frac{1}{64.7}$$

$$= 0.01921 cm^2$$

Examples of Network Analysis, 4
A Re-radiating Surface

Example of a re-radiating (adiabatic) surface

Example 5.15 (text)

$$J_1 = E_{b1} - Q_1\left(\frac{1-\varepsilon_1}{A_1\varepsilon_1}\right)$$

$$= 5.67\times10^{-8}(721.5)^4 - 3000\left(\frac{1-0.6}{0.6}\right)$$

$$= 13364\frac{W}{m^2}$$

$$J_1 = E_{b3}$$

$$= 5.67\times10^{-8}(550)^4$$

$$= 5188\frac{W}{m^2}$$

$$J_R = \frac{J_1A_1F_{1R} + J_3A_3F_{3R}}{A_1F_{1R} + A_3F_{3R}}$$

$$= \frac{J_1A_1(1-F_{13}) + J_3A_3\left(1-\frac{A_1}{A_3}F_{13}\right)}{A_1(1-F_{13}) + A_3\left(1-\frac{A_1}{A_3}F_{13}\right)}$$

$$= \frac{J_1A_1(1-F_{13}) + J_3A_3\left(1-\frac{A_1}{A_3}F_{13}\right)}{A_1 + A_3 - 2A_1F_{13}}$$

$$= \frac{13364(7.5)^2(1-0.1444) + 5188(5)^2\left(1-\frac{(7.5)^2}{25}(0.1444)\right)}{(7.5)^2 + 25 - 2(7.5)^2(0.1444)}$$

$$= 11241\frac{W}{m^2}$$

$$T_2 = \left(\frac{J_2}{\sigma}\right)^{\frac{1}{4}} = \left(\frac{11241}{5.67\times10^{-8}}\right)^{\frac{1}{4}} = 667.3K$$

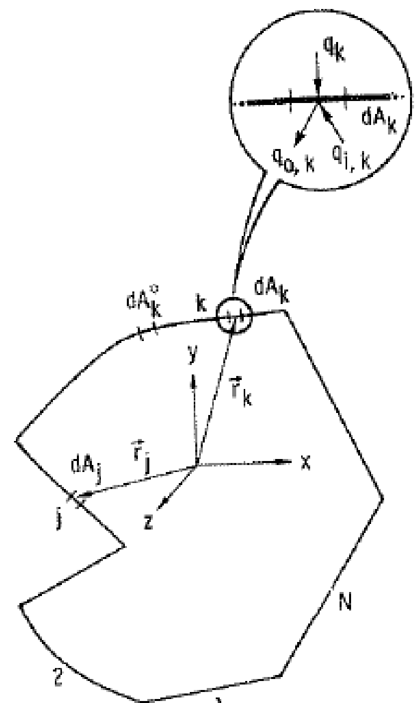
Radiative Exchange Between Infinitesimal Diffuse, Gray Surface – The Generalized Net-Radiation Method

Energy Balance on Infinitesimal Area Elements dA_k

Local energy balance at surface A_k

$$q_k(\vec{r}_k) = q_{o,k}(\vec{r}_k) - q_{i,k}(\vec{r}_k)$$

$$q_k(\vec{r}_k) = \varepsilon_k(\vec{r}_k)\, E_b(\vec{r}_k) - \left[1 - \varepsilon_k(\vec{r}_k)\right] q_{i,k}(\vec{r}_k)$$



Energy exchange between different surfaces

$$dA_k q_{i,k}(\vec{r}_k) = \int_{A_1} q_{o,1}(\vec{r}_1)\, dF_{d1-dk}(\vec{r}_1, \vec{r}_k)\, dA_1 + \cdots$$

$$+ \int_{A_k} q_{o,k}(\vec{r}_k')\, dF_{dk'-dk}(\vec{r}_k', \vec{r}_k)\, dA_k' + \cdots$$

$$+ \int_{A_N} q_{o,N}(\vec{r}_N)\, dF_{dN-dk}(\vec{r}_N, \vec{r}_k)\, dA_N$$

$$q_{i,k}(\vec{r}_k) = \sum_{j=1}^{N} \int_{A_j} q_{o,j}(\vec{r}_j)\, dF_{dk-dj}(\vec{r}_j, \vec{r}_k) = \sum_{j=1}^{N} \int_{A_j} q_{o,j}(\vec{r}_j)\, K(\vec{r}_j, \vec{r}_k)\, dA_j$$

$$K(\vec{r}_j, \vec{r}_k) = \frac{dF_{dk-dj}(\vec{r}_j, \vec{r}_k)}{dA_j} = \text{kernel of the integral equation}$$

Radiative Exchange Between Infinitesimal Diffuse, Gray Surface – The Generalized Net-Radiation Method

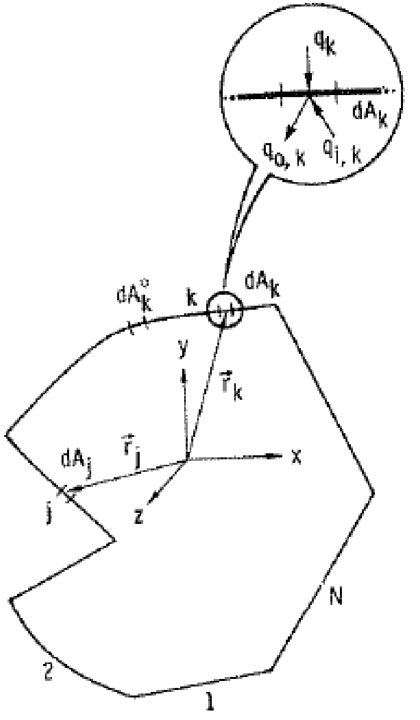
Energy Balance on Infinitesimal Area Elements dA_k

Summary of Governing Equations

$$q_k(\vec{r}_k) = \frac{E_b(\vec{r}_k) - q_{o,k}(\vec{r}_k)}{\left[1 - \varepsilon_k(\vec{r}_k)\right] / \varepsilon_k(\vec{r}_k)}$$

$$q_k(\vec{r}_k) = q_{o,k}(\vec{r}_k) - \sum_{j=1}^{N} \int_{A_j} q_{o,j}(\vec{r}_j) K_{dk-dj}(\vec{r}_j, \vec{r}_k)\, dA_j$$

$$k = 1, 2, \cdots; N$$

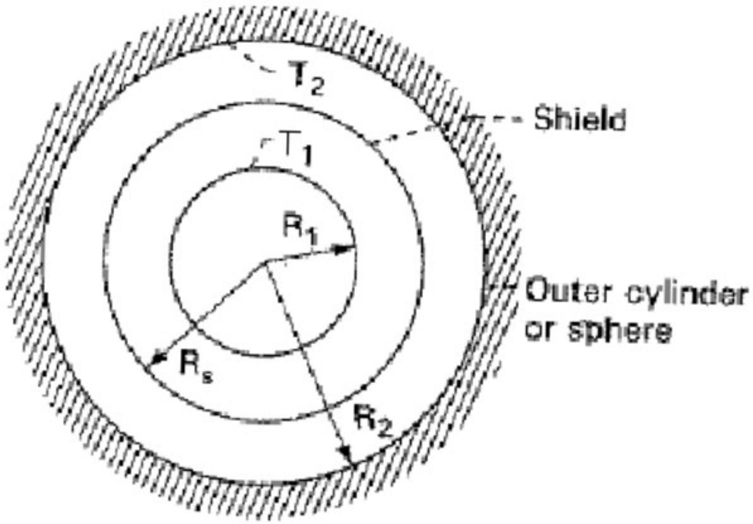


2N (integral) equations,
2N unknown functions $q_{o,k}(r_k)$, $q_k(r_k)$ or $T_k(r_k)$

$$k = 1, N$$

Homework 1

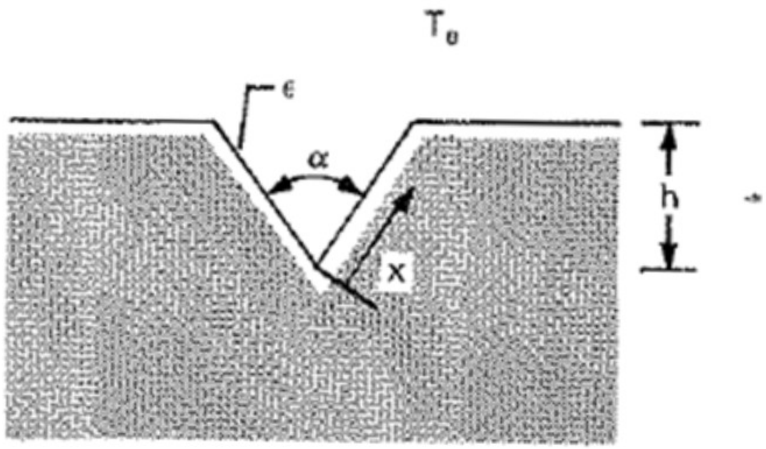
7-16 (a) What is the effect of a single thin radiation shield on the transfer of energy between two concentric cylinders? Assume the cylinder and shield surfaces are diffuse-gray with emissivities independent of temperature. Both sides of the shield have emissivity ϵ_s, and the inner and outer cylinders have respective emissivities ϵ_1 and ϵ_2.



(b) What is the effect of a single thin radiation shield on the transfer of energy between two concentric spheres? Assume the sphere and shield surfaces are diffuse-gray with emissivities independent of temperature. Both sides of the shield have emissivity ϵ_s, and the inner and outer spheres have respective emissivities ϵ_1 and ϵ_2.

Homework 2

7-38 A long groove is cut into a metal surface as shown in cross section below. The groove surface is diffuse-gray and has emissivity ϵ. The temperature profile along the groove sides, as measured from the apex, is found to be $T(x)$. The environment is at temperature T_e.



(a) Derive the equations for the heat flux distribution $q(x)$ along the groove surface.

(b) Examine the kernel of the integral equation found in part (a), and show whether it is symmetrical and/or separable.