Derivative Rules

$\frac{d}{dx}c = 0$

$\frac{d}{dx}x^n = nx^{n-1}$

$\frac{d}{dx}cf(x) = c\frac{d}{dx}cf(x)$

$\frac{d}{dx}(f\pm g) = \frac{d}{dx}f\pm\frac{d}{dx}g$

$\frac{d}{dx}e^x = e^x$

$\frac{d}{dx}fg = f'g + fg'$

$\frac{d}{dx}\frac{f}{g} = \frac{f'g-fg'}{g^2}$

$\frac{d}{dx}f(g) = f'(g) \bullet g'$ rule version)

$\frac{d}{dx}ln|x| = \frac{1}{x}$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\csc x = -\csc x\cot x$

$\frac{d}{dx}\sec x = \sec x\tan x$

$\frac{d}{dx}a^x = a^x lna$ for some constant a

$\frac{d}{dx}a^{f(x)} = a^{f(x)}ln(a) \bullet f'(x)$ for some constant a (chain

$\frac{d}{dx}\log_b a = \frac{1}{f(x)lnb} \bullet f'(x)$

$\frac{d}{dx}sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}tan^{-1}x = \frac{1}{1+x^2}$

$\frac{d}{dx}cot^{-1}x = -\frac{1}{1+x^2}$

$\frac{d}{dx}sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}csc^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}f(x) = f(x) \bullet \frac{d}{dx}lnf(x)$where f(x) = $\text{variable}^{\text{variable}}$ i.e. $\frac{d}{dx}x^{2x} = x^{2x}\frac{d}{dx}\ln(x^{2x})$

- here you could also use exponent rule: $a^b = e^{b\ln(a)}$...$x^{2x} = e^{2x\ln(x)}$ and then do the derivative of that

Integral Rules (for simplicity, dx and "+ C" not included in any of these)

$\int\sin x = -\cos x$

$\int\cos x = \sin x$

$\int\sec^2 x = \tan x$

$\int\csc^2 x = -\cot x$

$\int\sec x\tan x = \sec x$

$\int\csc x\cot x = -\csc x$

$\int\sin(kx) = -\frac{1}{k}cos(kx)$

$\int\cos(kx) = \frac{1}{k}sin(kx)$

$\int\sec^2(kx) = \frac{1}{k}tan(kx)$

$\int\csc^2(kx) = -\frac{1}{k}cot(kx)$

$\int\sec(kx)\tan(kx) = \frac{1}{k}sec(kx)$

$\int\csc(kx)\cot(kx) = -\frac{1}{k}csc(kx)$

$\int\frac{1}{x} = \ln|x|$

$\int a^{kx} = (\frac{1}{klna})a^{kx}$ $a > 0, a \neq 1$

$\int x^n = \frac{x^{n+1}}{n+1}$

$\int e^x = e^x$

$\int e^{kx} = \frac{1}{k}e^{kx}$

$\int f(x) \pm g(x) = \int f(x) \pm \int g(x)$

$\int\frac{1}{\sqrt{1-x^2}} = \sin^{-1}x$

$\int-\frac{1}{\sqrt{1-x^2}} = \cos^{-1}x$

$\int\frac{1}{1+x^2} = \tan^{-1}x$

$\int\frac{1}{\sqrt{1-k^2x^2}} = \frac{1}{k}\sin^{-1}(kx)$

$\int-\frac{1}{\sqrt{1-k^2x^2}} = \frac{1}{k}\cos^{-1}(kx)$

$\int\frac{1}{1+k^2x^2} = \frac{1}{k}\tan^{-1}(kx)$

$\int\frac{1}{\sqrt{k^2-x^2}} = \sin^{-1}\left(\frac{x}{k}\right)$

$\int-\frac{1}{\sqrt{k^2-x^2}} = \cos^{-1}\left(\frac{x}{k}\right)$

$\int\frac{1}{k^2+x^2} = (\frac{1}{k})\tan^{-1}\left(\frac{x}{k}\right)$

$\int -\frac{1}{1+x^2} = \cot^{-1}x$ $\int -\frac{1}{1+k^2x^2} = \frac{1}{k}\cot^{-1}(kx)$ $\int -\frac{1}{k^2+x^2} = (\frac{1}{k})\cot^{-1}\left(\frac{x}{k}\right)$

$\int \frac{1}{x\sqrt{x^2-1}} = \sec^{-1}x$ $\int \frac{1}{x\sqrt{k^2x^2-1}} = \frac{1}{k}\sec^{-1}(kx),\ kx > 1$ $\int \frac{1}{x\sqrt{x^2-k^2}} = (\frac{1}{k})\sec^{-1}\left(\frac{x}{k}\right)$

$\int -\frac{1}{x\sqrt{x^2-1}} = \csc^{-1}x$ $\int -\frac{1}{x\sqrt{k^2x^2-1}} = \frac{1}{k}\csc^{-1}(kx),\ kx > 1$ $\int -\frac{1}{x\sqrt{x^2-k^2}} = (\frac{1}{k})\csc^{-1}\left(\frac{x}{k}\right)$

Special Trig Integrals

$\int \sec x = \ln|\sec x + \tan x|$

$\int \tan x = -\ln|\cos x|$ or $\ln|\sec x|$

$\int \csc x = \ln|\tan(\frac{x}{2})|$

$\int \cot x = \ln|\sin x|$

L'Hopital's Rule

for $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$ ... $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$...keep going until no longer in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form

Exponential/Log Functions

- $\ln x = \log_e x$ $\ln e = 1$ $\ln x = y$ means $e^y = x$ $e^{\ln x} = x$ $\ln e^x = x$
- $\log_b x = y$ means $b^y = x$

$\log_b(b^x) = x$

$\log_b xy = \log_b x + \log_b y$

$\log_b(x/y) = \log_b x - \log_b y$

$\log_b x^r = r\log_b x$

logarithm to natural log conversion:

$$\frac{\log_a x}{\log_b x} = \frac{\frac{\ln x}{\ln a}}{\frac{\ln x}{\ln b}}$$

Functions and inverses:

- given a function y = f(x), solve for x in terms of y. Then swap the x's and y's. This is $f^{-1}(x)$.
- $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
- The range of $f^{-1}(x)$ is the domain of f(x).

Trigonometry

- π radians = 180º $1^o = \frac{\pi}{180}$ radians or 1 radian = $\frac{180}{\pi}$ degrees
- For a circle, s = rθ where s is arc length, r is radius, θ is angle

$\sin = \frac{opp}{hyp}$ $\csc = \frac{hyp}{opp} = \frac{1}{sin}$

$\cos = \frac{adj}{hyp}$ $\sec = \frac{hyp}{adj} = \frac{1}{cos}$

$\tan = \frac{opp}{adj} = \frac{sin}{cos}$ $\cot = \frac{adj}{opp} = \frac{1}{tan} = \frac{cos}{tan}$

Trig Identities

$\sin^2x + \cos^2x = 1$ $1 - \cos^2x = \sin^2x$ $1 - \sin^2x = \cos^2x$

$\tan^2x + 1 = \sec^2x$ $\sec^2x - 1 = \tan^2x$ $\sec^2x - \tan^2x = 1$

$\cos(2x) = \cos^2x - \sin^2x = 2\cos^2x - 1 = 1 - 2\sin^2x$

$\tan(2x) = \frac{2tanx}{1-tan^2x}$

$\sin^2x = \frac{1-cos(2x)}{2}$ $\cos^2x = \frac{1+cos(2x)}{2}$

$\sin(\frac{x}{2}) = \pm\sqrt{\frac{1-cosx}{2}}$ $\cos(\frac{x}{2}) = \pm\sqrt{\frac{1+cosx}{2}}$

$\tan(\frac{x}{2}) = \pm\sqrt{\frac{1-cosx}{1+cosx}} = \frac{sinx}{1+cosx} = \frac{1-cosx}{sinx}$

Odd: f(-x) = -f(x)   sin, tan, csc, cot   **Even:** f(-x) = f(x)   cos, sec

Cofunction:

$\sin(\pi/2 - x) = \cos x$ $\cos(\pi/2 - x) = \sin x$

$\tan(\pi/2 - x) = \cot x$ $\cot(\pi/2 - x) = \tan x$

$\sec(\pi/2 - x) = \csc x$ $\csc(\pi/2 - x) = \sec x$

Product to Sum:

sinAcosB = ½(sin(A+B) + sin(A-B))   sinAsinB = ½(cos(A-B) - cos(A-B))

cosAsinB = ½(sin(A+B) - sin(A-B))   cosAcosB = ½(cos(A-B) + cos(A+B))

Sum to Product:

$\sin x + \sin y = 2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2})$ $\sin x - \sin y = 2\sin(\frac{x-y}{2})\cos(\frac{x+y}{2})$

$\cos x + \cos y = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$ $\cos x - \cos y = 2\sin(\frac{x+y}{2})\sin(\frac{x-y}{2})$

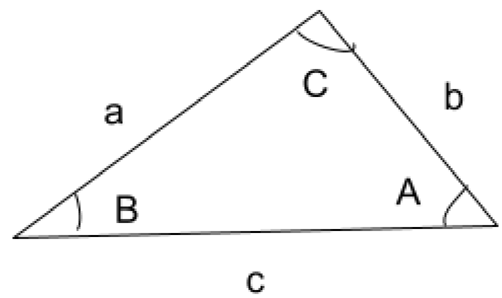
Inner Addition/Subtraction:

$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$

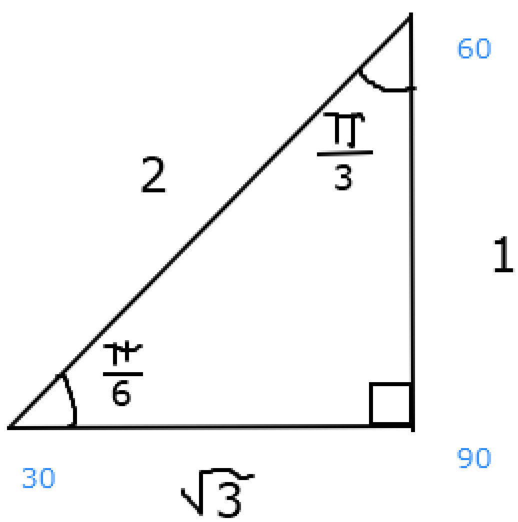
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

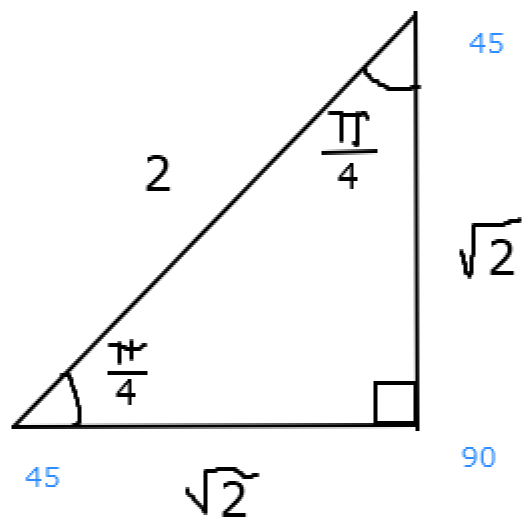
$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$ $\quad$ $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

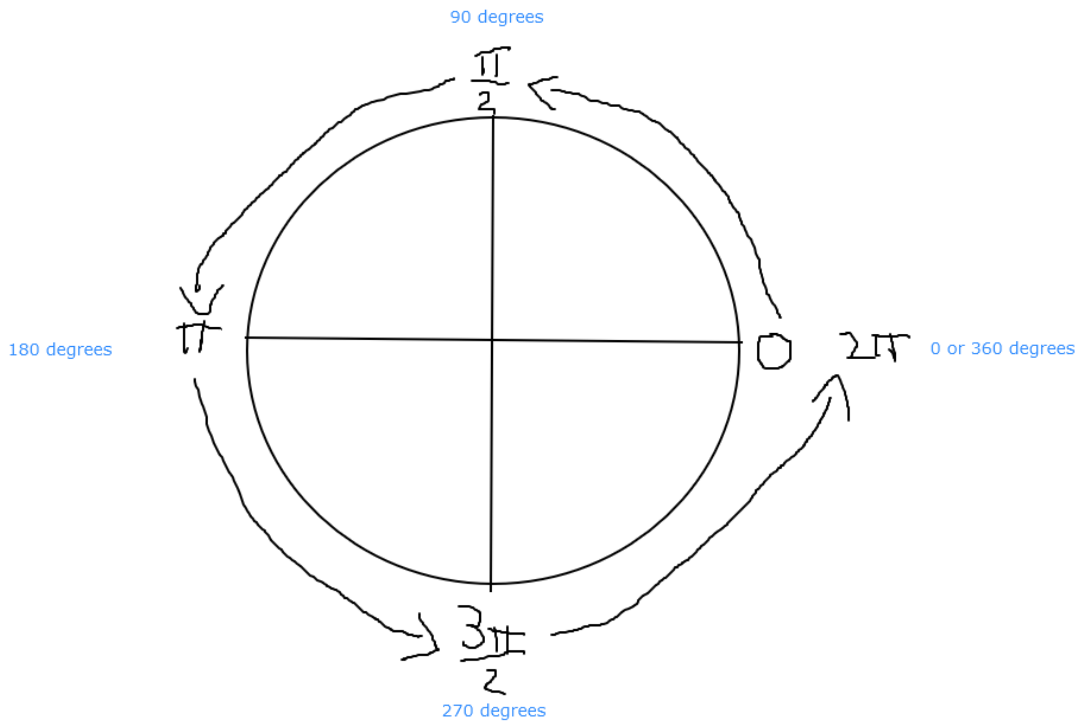
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



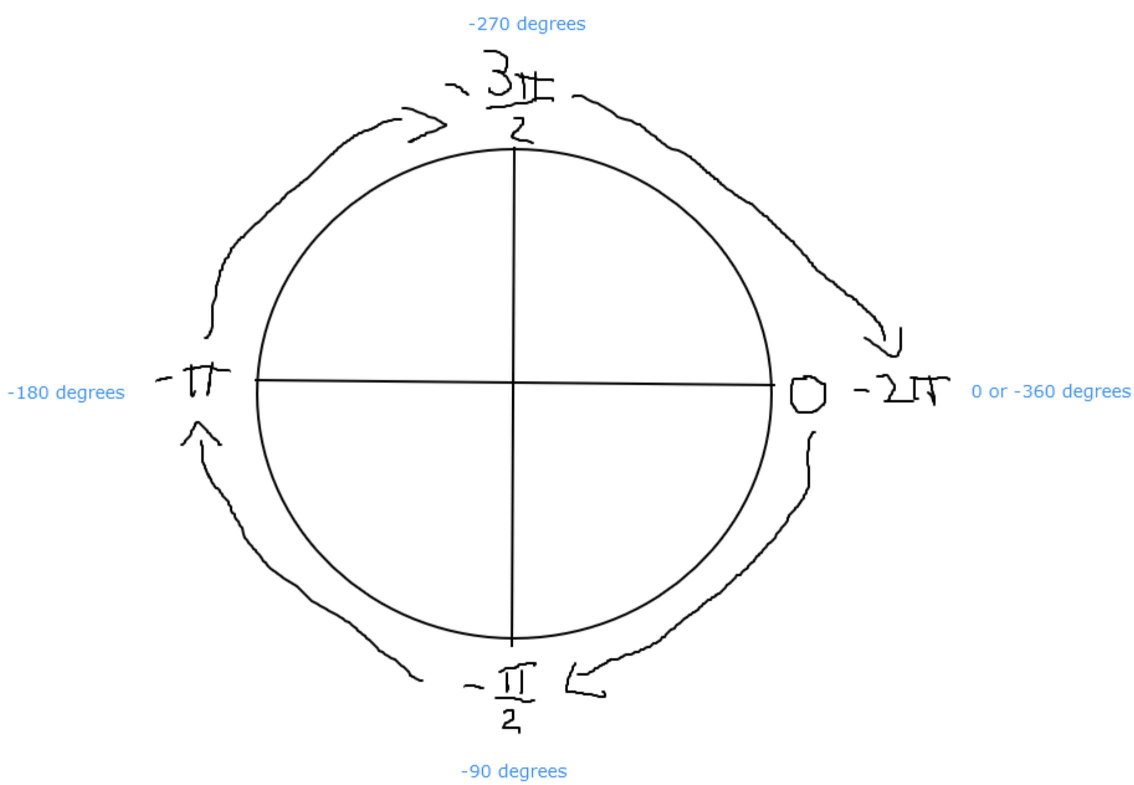
30-60-90 triangle **45-45-90 triangle**







Above: Unit Circle - Counter Clockwise **Below: Unit Circle - Clockwise**



Function Transformations

For some function y = **f(x)**

- y = A**f(**B**(x -** C**))** + D
    - A
        - |A| > 1: function stretches vertically by a factor of A
            - y = 2f(x), stretch vertically by a factor of 2
        - |A| < 1: function compresses vertically by a factor of A
            - y = ½f(x), compress vertically by a factor of 2
        - A < 0: flip across the x-axis
    - B
        - |B| > 1: function compresses horizontally by a factor of B
            - y = f(2x), compress horizontally by a factor of 2
        - |B| < 1: function stretches horizontally by a factor of B
            - y = f(½x), stretch horizontally by a factor of 2
        - B < 0: flip across y-axis
    - C
        - C < 0: function shifts left horizontally by "C"
            - y = f(x + 2), shift left by 2 (the "+" is from the double negative)
        - C > 0: function shifts right horizontally by "C"
            - y = f(x - 2), shift right by 2
    - D
        - D > 0: function shifts up vertically by "D"
            - y = f(x) + 2, shift up by 2
        - D < 0: function shifts down vertically by "D"
            - y = f(x) - 2, shift down by 2

https://www.desmos.com/calculator