Derivative Rules

$\frac{d}{dx}c = 0$ $\frac{d}{dx}$sinx = cosx $\frac{d}{dx}sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}x^n = nx^{n-1}$ $\frac{d}{dx}$cosx = -sinx $\frac{d}{dx}cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}cf(x) = c\frac{d}{dx}cf(x)$ $\frac{d}{dx}$tanx = sec^2x $\frac{d}{dx}tan^{-1}x = \frac{1}{1+x^2}$

$\frac{d}{dx}(f\pm g) = \frac{d}{dx}f\pm\frac{d}{dx}g$ $\frac{d}{dx}$cotx = $-csc^2x$ $\frac{d}{dx}cot^{-1}x = -\frac{1}{1+x^2}$

$\frac{d}{dx}e^x = e^x$ $\frac{d}{dx}$cscx = -cscxcotx $\frac{d}{dx}sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}fg = f'g + fg'$ $\frac{d}{dx}$secx = secxtanx $\frac{d}{dx}csc^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}\frac{f}{g} = \frac{f'g-fg'}{g^2}$ $\frac{d}{dx}a^x = a^x lna$ for some constant a

$\frac{d}{dx}f(g) = f'(g) \bullet g'$ $\frac{d}{dx}a^{f(x)} = a^{f(x)}ln(a) \bullet f'(x)$ for some constant a (chain rule version)

$\frac{d}{dx}ln|x| = \frac{1}{x}$ $\frac{d}{dx}\log_b a = \frac{1}{f(x)lnb} \bullet f'(x)$

$\frac{d}{dx}f(x) = f(x) \bullet \frac{d}{dx}lnf(x)$ where f(x) = variable^{variable} i.e. $\frac{d}{dx}x^{2x} = x^{2x}\frac{d}{dx}\ln(x^{2x})$

- here you could also use exponent rule: $a^b = e^{b\ln(a)}$...$x^{2x} = e^{2x\ln(x)}$ and then do the derivative of that

Integral Rules (for simplicity, dx and "+ C" not included in any of these)

$\int$sinx = -cosx $\int$sin(kx) = $-\frac{1}{k}cos(kx)$ $\int\frac{1}{x}$ = ln|x|

$\int$cosx = sinx $\int$cos(kx) = $\frac{1}{k}sin(kx)$ $\int a^{kx} = (\frac{1}{klna})a^{kx}$ a > 0, a ≠ 1

$\int sec^2x$ = tanx $\int sec^2(kx) = \frac{1}{k}tan(kx)$ $\int x^n = \frac{x^{n+1}}{n+1}$

$\int csc^2x$ = -cotx $\int csc^2(kx) = -\frac{1}{k}cot(kx)$ $\int e^x = e^x$

$\int$secxtanx = secx $\int$sec(kx)tan(kx) = $\frac{1}{k}sec(kx)$ $\int e^{kx} = \frac{1}{k}e^{kx}$

$\int$cscxcotx = -cscx $\int$csc(kx)cot(kx) = $-\frac{1}{k}csc(kx)$ $\int f(x) \pm g(x) = \int f(x) \pm \int g(x)$

$\int\frac{1}{\sqrt{1-x^2}} = \sin^{-1}x$ $\int\frac{1}{\sqrt{1-k^2x^2}} = \frac{1}{k}\sin^{-1}(kx)$ $\int\frac{1}{\sqrt{k^2-x^2}} = \sin^{-1}\left(\frac{x}{k}\right)$

$\int-\frac{1}{\sqrt{1-x^2}} = \cos^{-1}x$ $\int-\frac{1}{\sqrt{1-k^2x^2}} = \frac{1}{k}\cos^{-1}(kx)$ $\int-\frac{1}{\sqrt{k^2-x^2}} = \cos^{-1}\left(\frac{x}{k}\right)$

$\int\frac{1}{1+x^2} = \tan^{-1}x$ $\int\frac{1}{1+k^2x^2} = \frac{1}{k}\tan^{-1}(kx)$ $\int\frac{1}{k^2+x^2} = (\frac{1}{k})\tan^{-1}\left(\frac{x}{k}\right)$

$\int -\frac{1}{1+x^2} = \cot^{-1}x$ $\int -\frac{1}{1+k^2x^2} = \frac{1}{k}\cot^{-1}(kx)$ $\int -\frac{1}{k^2+x^2} = (\frac{1}{k})\cot^{-1}\left(\frac{x}{k}\right)$

$\int \frac{1}{x\sqrt{x^2-1}} = \sec^{-1}x$ $\int \frac{1}{x\sqrt{k^2x^2-1}} = \frac{1}{k}\sec^{-1}(kx)$, $kx > 1$ $\int \frac{1}{x\sqrt{x^2-k^2}} = (\frac{1}{k})\sec^{-1}\left(\frac{x}{k}\right)$

$\int -\frac{1}{x\sqrt{x^2-1}} = \csc^{-1}x$ $\int -\frac{1}{x\sqrt{k^2x^2-1}} = \frac{1}{k}\csc^{-1}(kx)$, $kx > 1$ $\int -\frac{1}{x\sqrt{x^2-k^2}} = (\frac{1}{k})\csc^{-1}\left(\frac{x}{k}\right)$

Special Trig Integrals

$\int \sec x = \ln|\sec x + \tan x|$

$\int \tan x = -\ln|\cos x|$ or $\ln|\sec x|$

$\int \csc x = \ln|\tan(\frac{x}{2})|$

$\int \cot x = \ln|\sin x|$

L'Hopital's Rule

for $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$ ... $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$...keep going until no longer in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form

Exponential/Log Functions

- $\ln x = \log_e x$ $\ln e = 1$ $\ln x = y$ means $e^y = x$ $e^{\ln x} = x$ $\ln e^x = x$
- $\log_b x = y$ means $b^y = x$

$\log_b(b^x) = x$

$\log_b xy = \log_b x + \log_b y$

$\log_b(x/y) = \log_b x - \log_b y$

$\log_b x^r = r\log_b x$

logarithm to natural log conversion:

$$\frac{\log_a x}{\log_b x} = \frac{\frac{\ln x}{\ln a}}{\frac{\ln x}{\ln b}}$$

Functions and inverses:

- given a function y = f(x), solve for x in terms of y. Then swap the x's and y's. This is $f^{-1}(x)$.
- $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
- The range of $f^{-1}(x)$ is the domain of f(x).

Trigonometry

- π radians = 180°   1° = $\frac{\pi}{180}$ radians or 1 radian = $\frac{180}{\pi}$ degrees
- For a circle, s = rθ where s is arc length, r is radius, θ is angle

$\sin = \frac{opp}{hyp}$ $\csc = \frac{hyp}{opp} = \frac{1}{sin}$

$\cos = \frac{adj}{hyp}$ $\sec = \frac{hyp}{adj} = \frac{1}{cos}$

$\tan = \frac{opp}{adj} = \frac{sin}{cos}$ $\cot = \frac{adj}{opp} = \frac{1}{tan} = \frac{cos}{tan}$

Trig Identities

$\sin^2 x + \cos^2 x = 1$ $1 - \cos^2 x = \sin^2 x$ $1 - \sin^2 x = \cos^2 x$

$\tan^2 x + 1 = \sec^2 x$ $\sec^2 x - 1 = \tan^2 x$ $\sec^2 x - \tan^2 x = 1$

$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan(2x) = \frac{2tanx}{1-tan^2 x}$

$\sin^2 x = \frac{1-cos(2x)}{2}$ $\cos^2 x = \frac{1+cos(2x)}{2}$

$\sin(\frac{x}{2}) = \pm\sqrt{\frac{1-cosx}{2}}$ $\cos(\frac{x}{2}) = \pm\sqrt{\frac{1+cosx}{2}}$

$\tan(\frac{x}{2}) = \pm\sqrt{\frac{1-cosx}{1+cosx}} = \frac{sinx}{1+cosx} = \frac{1-cosx}{sinx}$

Odd: f(-x) = f(x)   sin, tan, csc, cot   **Even:** f(-x) = -f(x)   cos, sec

Cofunction:

$\sin(\pi/2 - x) = \cos x$ $\quad\quad$ $\cos(\pi/2 - x) = \sin x$
$\tan(\pi/2 - x) = \cot x$ $\quad\quad$ $\cot(\pi/2 - x) = \tan x$
$\sec(\pi/2 - x) = \csc x$ $\quad\quad$ $\csc(\pi/2 - x) = \sec x$

Product to Sum:

$\sin A\cos B = ½(\sin(A+B) + \sin(A-B))$ $\quad\quad$ $\sin A\sin B = ½(\cos(A-B) - \cos(A-B))$
$\cos A\sin B = ½(\sin(A+B) - \sin(A-B))$ $\quad\quad$ $\cos A\cos B = ½(\cos(A-B) + \cos(A+B))$

Sum to Product:

$\sin x + \sin y = 2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2})$ $\quad\quad$ $\sin x - \sin y = 2\sin(\frac{x-y}{2})\cos(\frac{x+y}{2})$

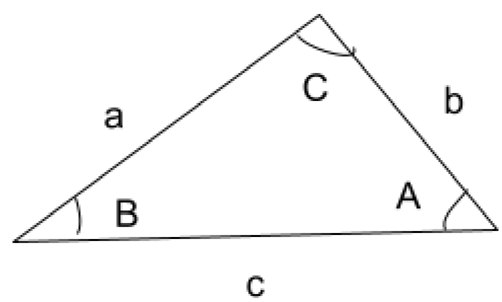
$\cos x + \cos y = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$ $\quad\quad$ $\cos x - \cos y = 2\sin(\frac{x+y}{2})\sin(\frac{x-y}{2})$

Inner Addition/Subtraction:

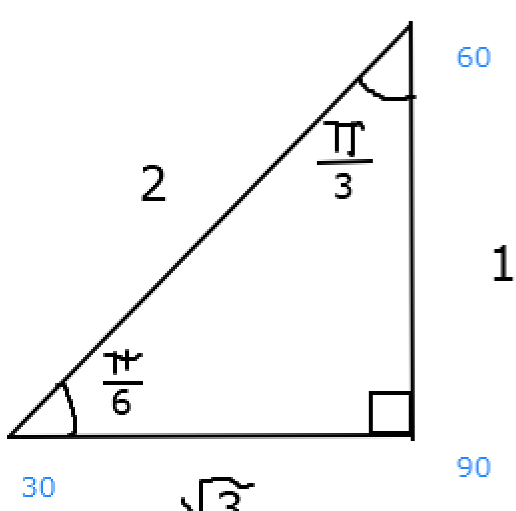
$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$ $\quad\quad$ $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



30-60-90 triangle $\quad\quad$ **45-45-90 triangle**



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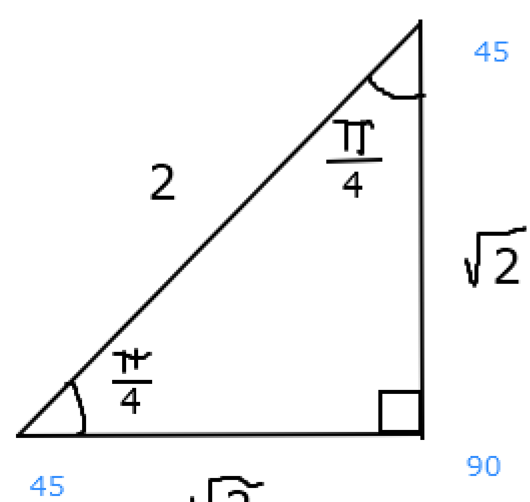
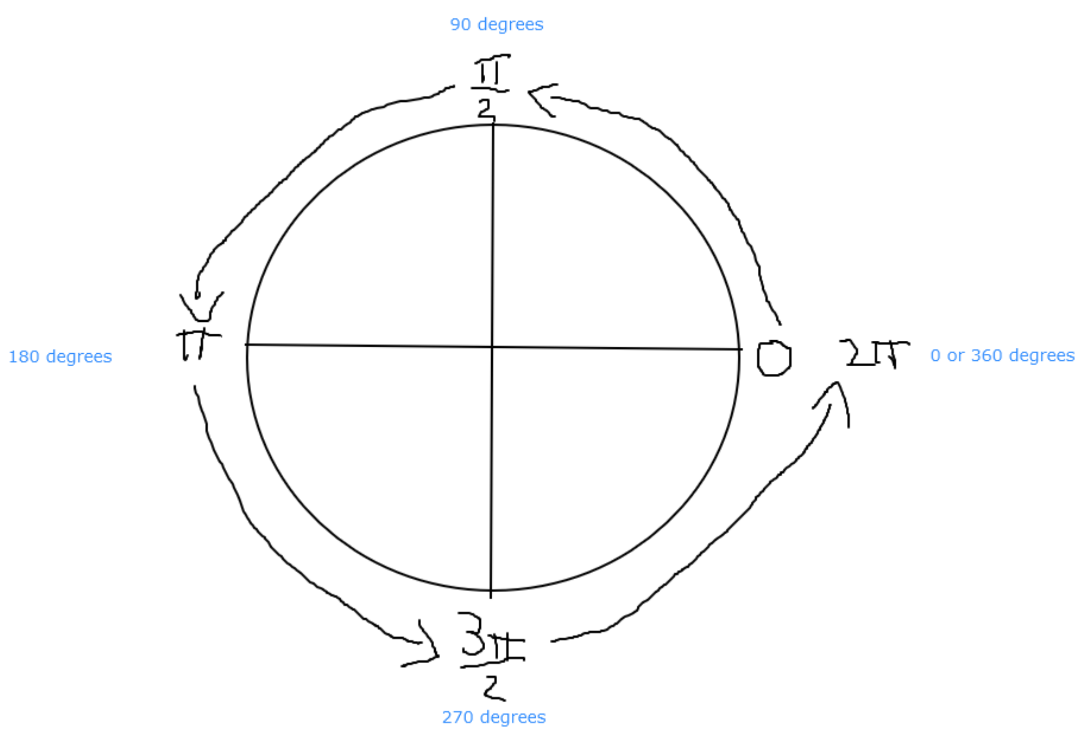
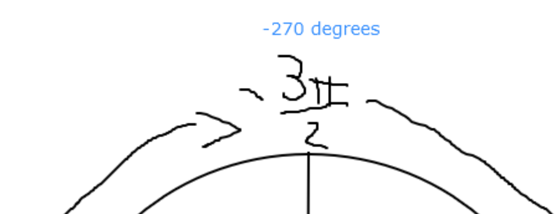




Above: Unit Circle - Counter Clockwise **Below: Unit Circle - Clockwise**



Function Transformations

For some function y = **f(x)**

- $y = A\mathbf{f}(B(\mathbf{x} - C)) + D$
  - A
    - |A| > 1: function stretches vertically by a factor of A
      - y = 2f(x), stretch vertically by a factor of 2
    - |A| < 1: function compresses vertically by a factor of A
      - y = ½f(x), compress vertically by a factor of 2
    - A < 0: flip across the y-axis
  - B
    - |B| > 1: function compresses horizontally by a factor of B
      - y = f(2x), compress horizontally by a factor of 2
    - |B| < 1: function stretches horizontally by a factor of B
      - y = f(½x), stretch horizontally by a factor of 2
    - B < 0: flip across x-axis
  - C
    - C < 0: function shifts left horizontally by "C"
      - y = f(x + 2), shift left by 2 (the "+" is from the double negative)
    - C > 0: function shifts right horizontally by "C"
      - y = f(x - 2), shift right by 2
  - D
    - D > 0: function shifts up vertically by "D"
      - y = f(x) + 2, shift up by 2
    - D < 0: function shifts down vertically by "D"
      - y = f(x) - 2, shift down by 2

https://www.desmos.com/calculator