

Derivative Rules

$$\frac{d}{dx}c = 0$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$$

$$\frac{d}{dx}(f \pm g) = \frac{d}{dx}f \pm \frac{d}{dx}g$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}fg = f'g + fg'$$

$$\frac{d}{dx}\frac{f}{g} = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}f(g) = f'(g) \cdot g'$$

rule version)

$$\frac{d}{dx}\ln|x| = \frac{1}{x}$$

$$\frac{d}{dx}f(x) = f(x) \cdot \frac{d}{dx}\ln f(x) \text{ where } f(x) = \text{variable}^{\text{variable}} \text{ i.e. } \frac{d}{dx}x^{2x} = x^{2x}\frac{d}{dx}\ln(x^{2x})$$

- here you could also use exponent rule: $a^b = e^{b\ln(a)} \dots x^{2x} = e^{2x\ln(x)}$ and then do the derivative of that

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}a^x = a^x \ln a \text{ for some constant } a$$

$$\frac{d}{dx}a^{f(x)} = a^{f(x)} \ln(a) \cdot f'(x) \text{ for some constant } a \text{ (chain rule version)}$$

$$\frac{d}{dx}\log_b a = \frac{1}{f(x)\ln b} \cdot f'(x)$$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\csc^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$$

Integral Rules (for simplicity, dx and “+ C” not included in any of these)

$$\int \sin x = -\cos x$$

$$\int \sin(kx) = -\frac{1}{k}\cos(kx)$$

$$\int \frac{1}{x} = \ln|x|$$

$$\int \cos x = \sin x$$

$$\int \cos(kx) = \frac{1}{k}\sin(kx)$$

$$\int a^{kx} = \left(\frac{1}{k\ln a}\right)a^{kx} \quad a > 0, a \neq 1$$

$$\int \sec^2 x = \tan x$$

$$\int \sec^2(kx) = \frac{1}{k}\tan(kx)$$

$$\int x^n = \frac{x^{n+1}}{n+1}$$

$$\int \csc^2 x = -\cot x$$

$$\int \csc^2(kx) = -\frac{1}{k}\cot(kx)$$

$$\int e^x = e^x$$

$$\int \sec x \tan x = \sec x$$

$$\int \sec(kx)\tan(kx) = \frac{1}{k}\sec(kx)$$

$$\int e^{kx} = \frac{1}{k}e^{kx}$$

$$\int \csc x \cot x = -\csc x$$

$$\int \csc(kx)\cot(kx) = -\frac{1}{k}\csc(kx)$$

$$\int f(x) \pm g(x) = \int f(x) \pm \int g(x)$$

$$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1}x$$

$$\int \frac{1}{\sqrt{1-k^2x^2}} = \frac{1}{k}\sin^{-1}(kx)$$

$$\int \frac{1}{\sqrt{k^2-x^2}} = \sin^{-1}\left(\frac{x}{k}\right)$$

$$\int -\frac{1}{\sqrt{1-x^2}} = \cos^{-1}x$$

$$\int -\frac{1}{\sqrt{1-k^2x^2}} = \frac{1}{k}\cos^{-1}(kx)$$

$$\int -\frac{1}{\sqrt{k^2-x^2}} = \cos^{-1}\left(\frac{x}{k}\right)$$

$$\int \frac{1}{1+x^2} = \tan^{-1}x$$

$$\int \frac{1}{1+k^2x^2} = \frac{1}{k}\tan^{-1}(kx)$$

$$\int \frac{1}{k^2+x^2} = \left(\frac{1}{k}\right)\tan^{-1}\left(\frac{x}{k}\right)$$

$$\begin{aligned}
\int -\frac{1}{1+x^2} &= \cot^{-1}x & \int -\frac{1}{1+k^2x^2} &= \frac{1}{k}\cot^{-1}(kx) & \int -\frac{1}{k^2+x^2} &= \left(\frac{1}{k}\right)\cot^{-1}\left(\frac{x}{k}\right) \\
\int \frac{1}{x\sqrt{x^2-1}} &= \sec^{-1}x & \int \frac{1}{x\sqrt{k^2x^2-1}} &= \frac{1}{k}\sec^{-1}(kx), kx > 1 & \int \frac{1}{x\sqrt{x^2-k^2}} &= \left(\frac{1}{k}\right)\sec^{-1}\left(\frac{x}{k}\right) \\
\int -\frac{1}{x\sqrt{x^2-1}} &= \csc^{-1}x & \int -\frac{1}{x\sqrt{k^2x^2-1}} &= \frac{1}{k}\csc^{-1}(kx), kx > 1 & \int -\frac{1}{x\sqrt{x^2-k^2}} &= \left(\frac{1}{k}\right)\csc^{-1}\left(\frac{x}{k}\right)
\end{aligned}$$

Special Trig Integrals

$$\int \sec x = \ln|\sec x + \tan x|$$

$$\int \tan x = -\ln|\cos x| \text{ or } \ln|\sec x|$$

$$\int \csc x = \ln\left|\tan\left(\frac{x}{2}\right)\right|$$

$$\int \cot x = \ln|\sin x|$$

L'Hopital's Rule

for $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$... $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$... keep going until no longer in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form

Exponential/Log Functions

$$\begin{aligned}
- \ln x &= \log_e x & \ln e &= 1 & \ln x = y &\text{ means } e^y = x & e^{\ln x} &= x & \ln e^x &= x \\
- \log_b x &= y &\text{ means } b^y &= x
\end{aligned}$$

$$\log_b(b^x) = x$$

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b(x/y) = \log_b x - \log_b y$$

$$\log_b x^r = r \log_b x$$

logarithm to natural log conversion:

$$\frac{\log_a x}{\log_b x} = \frac{\frac{\ln x}{\ln a}}{\frac{\ln x}{\ln b}}$$

Functions and inverses:

- given a function $y = f(x)$, solve for x in terms of y . Then swap the x 's and y 's. This is $f^{-1}(x)$.
- $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
- The range of $f^{-1}(x)$ is the domain of $f(x)$.

Trigonometry

- π radians = 180° $1^\circ = \frac{\pi}{180}$ radians or 1 radian = $\frac{180}{\pi}$ degrees
- For a circle, $s = r\theta$ where s is arc length, r is radius, θ is angle

$$\begin{aligned}\sin &= \frac{\text{opp}}{\text{hyp}} & \csc &= \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin} \\ \cos &= \frac{\text{adj}}{\text{hyp}} & \sec &= \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos} \\ \tan &= \frac{\text{opp}}{\text{adj}} = \frac{\sin}{\cos} & \cot &= \frac{\text{adj}}{\text{opp}} = \frac{1}{\tan} = \frac{\cos}{\sin}\end{aligned}$$

Trig Identities

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 & 1 - \cos^2 x &= \sin^2 x & 1 - \sin^2 x &= \cos^2 x \\ \tan^2 x + 1 &= \sec^2 x & \sec^2 x - 1 &= \tan^2 x & \sec^2 x - \tan^2 x &= 1 \\ \cos(2x) &= \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x \\ \tan(2x) &= \frac{2\tan x}{1 - \tan^2 x}\end{aligned}$$

$$\begin{aligned}\sin^2 x &= \frac{1 - \cos(2x)}{2} & \cos^2 x &= \frac{1 + \cos(2x)}{2} \\ \sin\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 - \cos x}{2}} & \cos\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 + \cos x}{2}} \\ \tan\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}\end{aligned}$$

Odd: $f(-x) = -f(x)$ \sin, \tan, \csc, \cot **Even:** $f(-x) = f(x)$ \cos, \sec

Cofunction:

$$\begin{aligned}\sin(\pi/2 - x) &= \cos x & \cos(\pi/2 - x) &= \sin x \\ \tan(\pi/2 - x) &= \cot x & \cot(\pi/2 - x) &= \tan x \\ \sec(\pi/2 - x) &= \csc x & \csc(\pi/2 - x) &= \sec x\end{aligned}$$

Product to Sum:

$$\begin{aligned}\sin A \cos B &= \frac{1}{2}(\sin(A+B) + \sin(A-B)) & \sin A \sin B &= \frac{1}{2}(\cos(A-B) - \cos(A+B)) \\ \cos A \sin B &= \frac{1}{2}(\sin(A+B) - \sin(A-B)) & \cos A \cos B &= \frac{1}{2}(\cos(A-B) + \cos(A+B))\end{aligned}$$

Sum to Product:

$$\begin{aligned}\sin x + \sin y &= 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) & \sin x - \sin y &= 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right) \\ \cos x + \cos y &= 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) & \cos x - \cos y &= 2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\end{aligned}$$

Inner Addition/Subtraction:

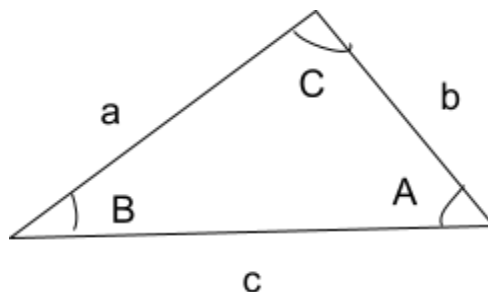
$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

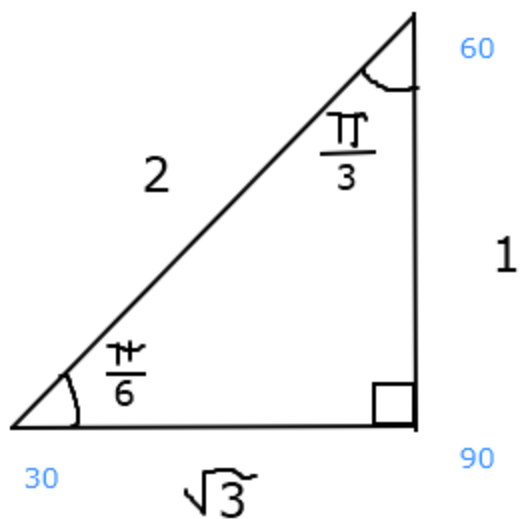
$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

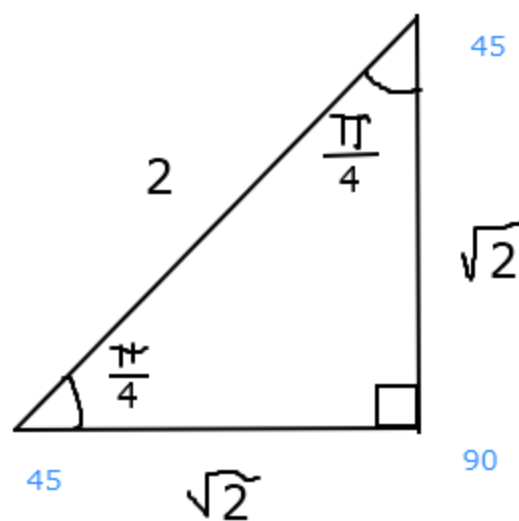
$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

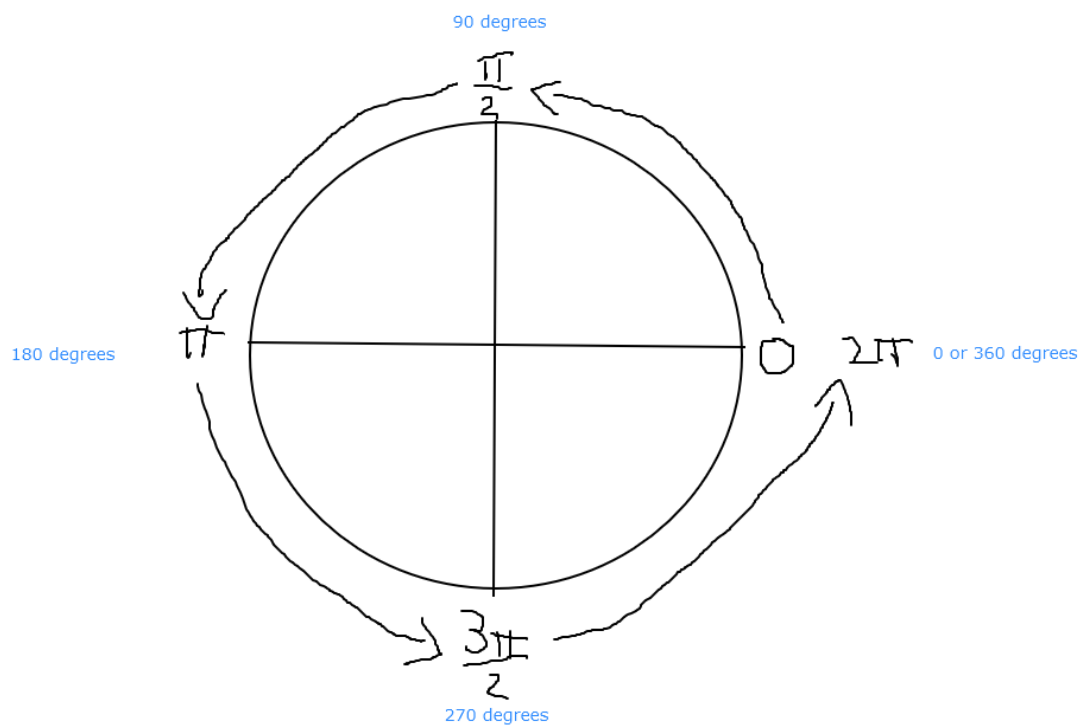


30-60-90 triangle



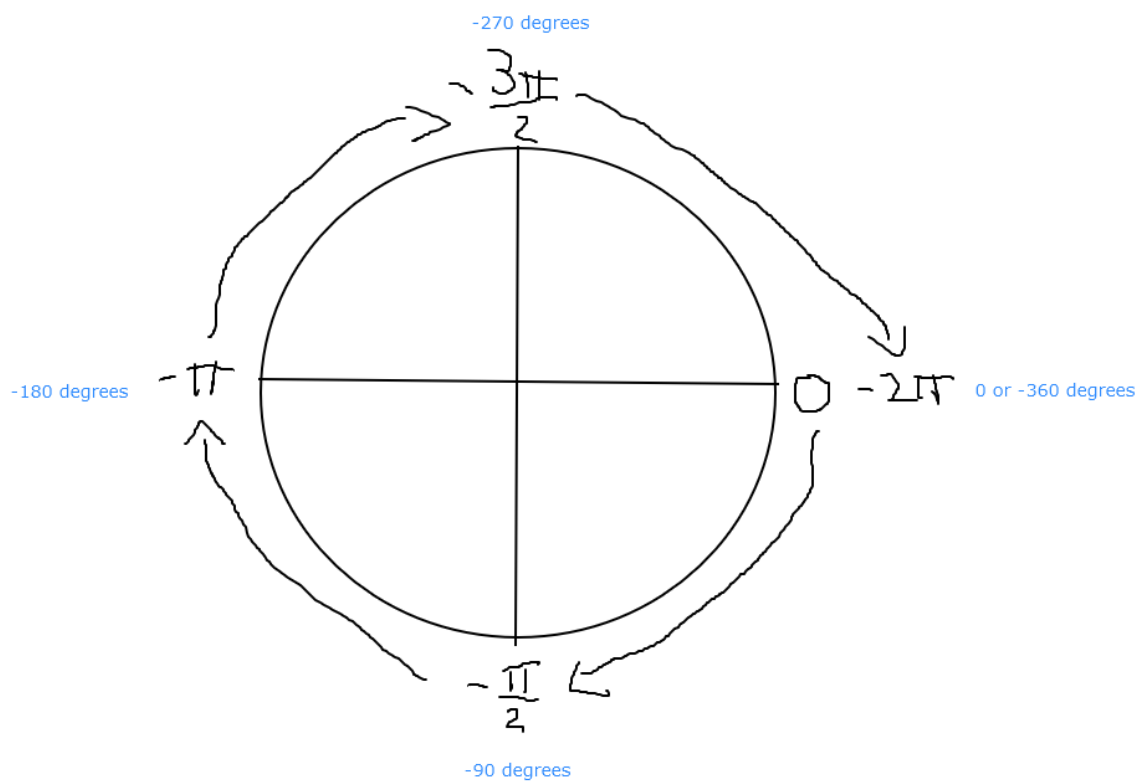
45-45-90 triangle





Above: Unit Circle - Counter Clockwise

Below: Unit Circle - Clockwise



Function Transformations

For some function $y = f(x)$

- $y = A f(B(x - C)) + D$
 - A
 - $|A| > 1$: function stretches vertically by a factor of A
 - $y = 2f(x)$, stretch vertically by a factor of 2
 - $|A| < 1$: function compresses vertically by a factor of A
 - $y = \frac{1}{2}f(x)$, compress vertically by a factor of 2
 - $A < 0$: flip across the x-axis
 - B
 - $|B| > 1$: function compresses horizontally by a factor of B
 - $y = f(2x)$, compress horizontally by a factor of 2
 - $|B| < 1$: function stretches horizontally by a factor of B
 - $y = f(\frac{1}{2}x)$, stretch horizontally by a factor of 2
 - $B < 0$: flip across y-axis
 - C
 - $C < 0$: function shifts left horizontally by “C”
 - $y = f(x + 2)$, shift left by 2 (the “+” is from the double negative)
 - $C > 0$: function shifts right horizontally by “C”
 - $y = f(x - 2)$, shift right by 2
 - D
 - $D > 0$: function shifts up vertically by “D”
 - $y = f(x) + 2$, shift up by 2
 - $D < 0$: function shifts down vertically by “D”
 - $y = f(x) - 2$, shift down by 2

<https://www.desmos.com/calculator>