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NOTE: I'VE CALLED A COMPUTATION A MULTIPLICATION EVENT. I IGNORE ADDITION AND CREATION OF NEW DATASTRUCTURES (OVERHEAD)

Let $\vec{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\vec{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$. Let $\phi(\vec{x})$ map to

$$\phi(\vec{x}) : (x_1, x_2) \rightarrow (1, x_1^2, \sqrt{2}x_1x_2, x_2^2, \sqrt{2}x_1, \sqrt{2}x_2)$$

Let K be a kernal function, $K(\vec{x}, \vec{z}) = (1 + \langle \vec{x}, \vec{z} \rangle)^2$ where $\langle \cdot, \cdot \rangle$ is the dot product. Show $K(\vec{x}, \vec{z}) = \langle \phi(\vec{x}), \phi(\vec{z}) \rangle$.

FIRST SHOW WHAT $\langle \phi(\vec{x}), \phi(\vec{z}) \rangle$ IS. COMPUTING $\phi(\vec{x})$, $\phi(\vec{z})$ IS 6 COMPUTATIONS EACH THEN COMPUTE $\langle \cdot, \cdot \rangle = X^T Z$ WHERE $X = \phi(\vec{x})$, $Z = \phi(\vec{z})$

$$[1, x_1^2, \sqrt{2}x_1x_2, x_2^2, \sqrt{2}x_1, \sqrt{2}x_2] \begin{bmatrix} 1 \\ z_1^2 \\ \sqrt{2}z_1z_2 \\ z_2^2 \\ \sqrt{2}z_1 \\ \sqrt{2}z_2 \end{bmatrix} = 1^2 + x_1^2z_1^2 + 2x_1x_2z_1z_2 + x_2^2z_2^2 + 2x_1z_1 + 2x_2z_2$$

THIS IS ANOTHER 6 COMPUTATIONS

TOTAL 18.

SECOND SHOW WHAT $K(\vec{x}, \vec{z})$ IS. FIRST COMPUTE $X^T Z$

$$[x_1 \;\; x_2] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = x_1z_1 + x_2z_2$$

THIS IS 2 CALCULATIONS

$$K(\vec{x}, \vec{z}) = (1 + x_1z_1 + x_2z_2)^2 = 1^2 + x_1z_1 + x_2z_2 + x_1x_2z_1z_2 + x_2z_2 + x_1x_2z_1z_2 + x_2^2z_2^2$$

$$= 1^2 + x_1^2z_1^2 + 2x_1x_2z_1z_2 + x_2^2z_2^2 + 2x_1z_1 + 2x_2z_2$$

9 COMPUTATIONS. (MULTIPLICATIONS) TOTAL: 11.

MY ANALYSIS THERE WAS LOOKING AT MULTIPLICATIONS IF I KEPT THE VARIABLES SEPERATE.

IN ALL ACTUALLY METHOD 1 $(\langle \phi(\vec{x}), \phi(\vec{z}) \rangle)$ TAKES 18 MULTIPLICATION/TRANSFORMATIONS WHERE AS IN METHOD 2 THERE IS THE INITIAL DOT PRODUCT (2) AND THEN THE SQUARING. (SINCE $x_1z_1 + x_2z_2$ IS JUST A CONSTANT)

SO METHOD ONE IS $3d$, where d IS THE NUMBER OF DIMENSIONS THE VECTOR IS BEING TRANSFORMED TO. SO METHOD ONE STILL HAS 18 CALCULATIONS

METHOD TWO IS $c+1$ WHERE c IS THE ORIGINAL DIMENSIONALITY (IN THIS CASE 2). SO METHOD TWO HAS 3 CALCULATIONS. SAVINGS OF 15 CALCULATIONS.

IF WE SCALE UP TOO 1000 DIMENSIONS, METHOD ONE TAKES $3(1000) = 3000$

METHOD TWO STILL TAKES 3.

SAVINGS ARE 2997 CALCULATIONS, WHICH IS POTENTIALLY LARGE.