TEXAS TECH UNIVERSITY™

Lecture 9: Moderation Wrap-Up

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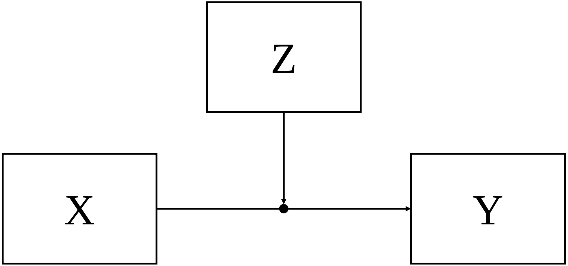
Outline

- Multiple Moderators
- Moderated Moderation
- Categorical Moderation
- Multiple Group Modeling

Starting Point

So far, we've been looking at this type of model:



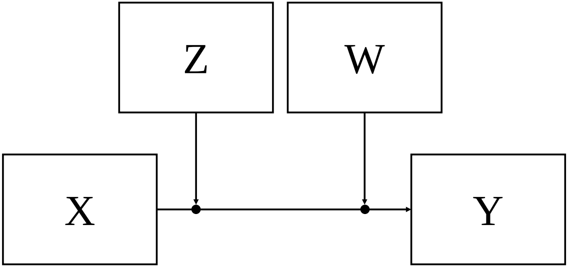
We've had one focal variable and one moderator.

- We've been asking questions about how the focal effect changes as a function of the moderator.
- There's no reason we need to restrict ourselves to a single moderator.

Multiple Moderation

Maybe we suspect that the focal effect changes as a function of two other variables.

- We could fit this type of model:



Now, the focal effect of X on Y changes as a function of both Z and W.

Multiple Moderation

The preceding diagram implies the following formula:

$$Y = \alpha + f(Z, W)X + \beta_2 Z + \beta_3 W + e,$$

Taking $f(Z, W)$ to be the following simple slope:

$$f(Z, W) = \beta_1 + \beta_4 Z + \beta_5 W$$

Produces the following analytic equation:

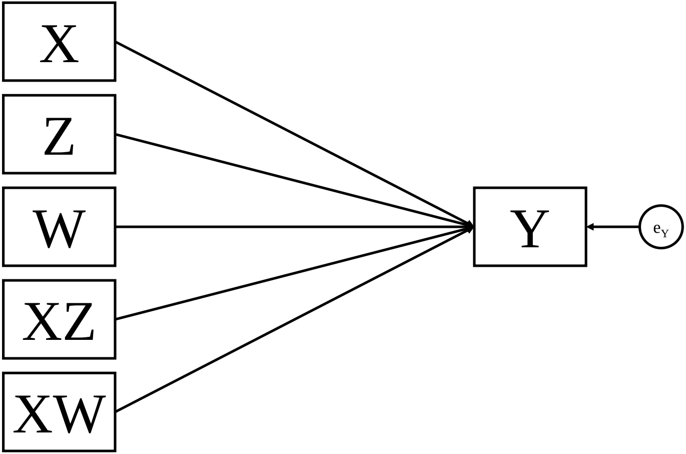
$$Y = \alpha + \beta_1 X + \beta_2 Z + \beta_3 W + \beta_4 XZ + \beta_5 XW + e$$

We can easily fit this model in any regression software

- We can test for significant moderating effects of Z and W by testing for non-zero β_4 and β_5, respectively.

Multiple Moderation

Our analytic diagram is predictably extended:



Example

```
library(psych)
library(rockchalk)
dat1 <- readRDS("../data/bfiData1.rds")
## Additive model:
out1.1 <- lm(agree ~ conc + open + neuro, data = dat1)
summary(out1.1)
```

```
Call:
lm(formula = agree ~ conc + open + neuro, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.78733 -0.41707  0.09673  0.47476  2.12198

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.23373    0.12379  26.123  < 2e-16 ***
conc         0.06890    0.02647   2.603  0.00929 **
open         0.27661    0.02647  10.449  < 2e-16 ***
neuro       -0.10633    0.01205  -8.826  < 2e-16 ***
---
```

Example

```
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1          1

Residual standard error: 0.6966 on 2548 degrees of freedom
Multiple R^2:  0.06473,  Adjusted R^2:  0.06363
F-statistic: 58.79 on 3 and 2548 DF,  p-value: < 2.2e-16
```

Example

```
## Additive two-way interaction model:
out1.2 <- lm(agree ~ open*conc + open*neuro, data = dat1)
summary(out1.2)
```

```
Call:
lm(formula = agree ~ open * conc + open * neuro, data = dat1
    )

Residuals:
     Min       1Q   Median       3Q      Max
-2.78651 -0.41308  0.09699  0.47778  2.18968

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.05739    0.50931   4.040 5.51e-05 ***
open         0.58398    0.13111   4.454 8.78e-06 ***
conc         0.49645    0.14540   3.414 0.000649 ***
neuro       -0.24314    0.08373  -2.904 0.003719 **
open:conc   -0.11115    0.03716  -2.991 0.002803 **
open:neuro   0.03542    0.02124   1.667 0.095589 .
---
```

Example

```
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1         1

Residual standard error: 0.6957 on 2546 degrees of freedom
Multiple R^2:  0.06809,  Adjusted R^2:  0.06626
F-statistic: 37.21 on 5 and 2546 DF,  p-value: < 2.2e-16
```

Example

```
## Center 'conc' on interesting values for SS analysis:
dat1$concLo  ← with(dat1, conc - quantile(conc, 0.25,
    na.rm = TRUE))
dat1$concMid ← with(dat1, conc - quantile(conc, 0.5, na.rm
     = TRUE))
dat1$concHi  ← with(dat1, conc - quantile(conc, 0.75,
    na.rm = TRUE))
```

Example

```r
## Test simple slopes via centering:
out1.2.1 <- lm(agree ~ open*concLo + neuro, data = dat1)
summary(out1.2.1)
```

```
Call:
lm(formula = agree ~ open * concLo + neuro, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.78565 -0.41336  0.09706  0.47676  2.15172

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.34421    0.11416  29.294  < 2e-16 ***
open         0.30929    0.02943  10.508  < 2e-16 ***
concLo       0.39874    0.13311   2.996  0.00277 **
neuro       -0.10499    0.01205  -8.716  < 2e-16 ***
open:concLo -0.08563    0.03387  -2.528  0.01152 *
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1          1
```

Example

```
Residual standard error: 0.6959 on 2547 degrees of freedom
Multiple R^2:  0.06707,  Adjusted R^2:  0.06561
F-statistic: 45.78 on 4 and 2547 DF,  p-value: < 2.2e-16
```

Example

```
out1.2.2 ← lm(agree ∼ open*concMid + neuro, data = dat1)
summary(out1.2.2)
```

```
Call:
lm(formula = agree ∼ open * concMid + neuro, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.78565 -0.41336  0.09706  0.47676  2.15172

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   3.50370    0.10564  33.166  < 2e-16 ***
open          0.27503    0.02645  10.398  < 2e-16 ***
concMid       0.39874    0.13311   2.996  0.00277 **
neuro        -0.10499    0.01205  -8.716  < 2e-16 ***
open:concMid -0.08563    0.03387  -2.528  0.01152 *
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1         1

Residual standard error: 0.6959 on 2547 degrees of freedom
```

Example

```
Multiple R^2:  0.06707,  Adjusted R^2:  0.06561
F-statistic: 45.78 on 4 and 2547 DF,  p-value: < 2.2e-16
```

Example

```
out1.2.3 ← lm(agree ~ open*concHi + neuro, data = dat1)
summary(out1.2.3)
```

```
Call:
lm(formula = agree ~ open * concHi + neuro, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.78565 -0.41336  0.09706  0.47676  2.15172

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.66320    0.12230  29.952  < 2e-16 ***
open         0.24078    0.03000   8.026 1.52e-15 ***
concHi       0.39874    0.13311   2.996  0.00277 **
neuro       -0.10499    0.01205  -8.716  < 2e-16 ***
open:concHi -0.08563    0.03387  -2.528  0.01152 *
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1         1

Residual standard error: 0.6959 on 2547 degrees of freedom
```

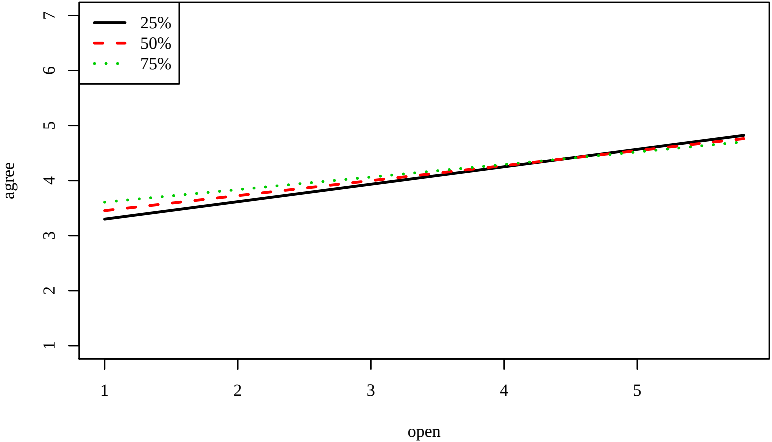
Example

```
Multiple R^2:  0.06707,  Adjusted R^2:  0.06561
F-statistic: 45.78 on 4 and 2547 DF,  p-value: < 2.2e-16
```

Example

```
## Plot the simple slopes:
par(family = "serif", cex = 0.75)
plotOut1.2 <- plotSlopes(model = out1.2,
                         plotx = "open",
                         modx = "conc",
                         plotPoints = FALSE,
                         modxVals =
                             quantile(dat1$conc,
                                      c(0.25, 0.5, 0.75),
                                      na.rm = TRUE)
                         )
```

Example



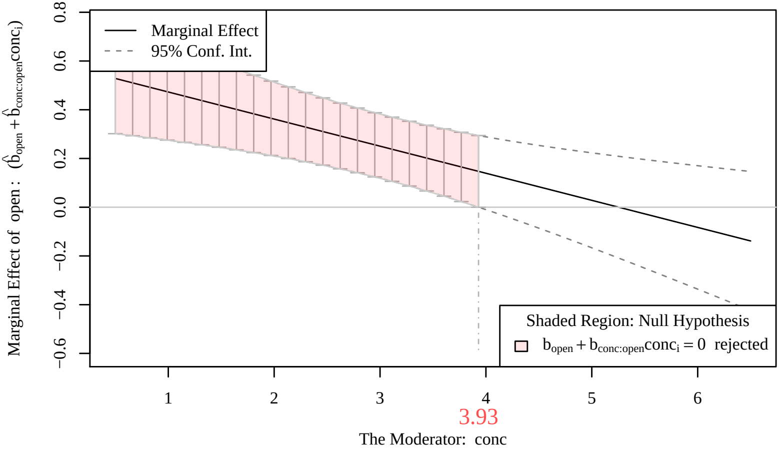
Example

```
par(family = "serif", cex = 0.75)
testOut1.2 ← testSlopes(plotOut1.2)
```

```
Values of conc OUTSIDE this interval:
      lo        hi
3.930435 9.927895
cause the slope of (b1 + b2*conc)open to be statistically
    significant
```

```
plot(testOut1.2)
```

Example



Marginal Effect
95% Conf. Int.
Shaded Region: Null Hypothesis
$b_{open} + b_{conc:open}conc_i = 0$ rejected
Marginal Effect of open : $(\hat{b}_{open} + \hat{b}_{conc:open}conc_i)$
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
1
2
3
4
5
6
3.93
The Moderator: conc

Moderated Moderation

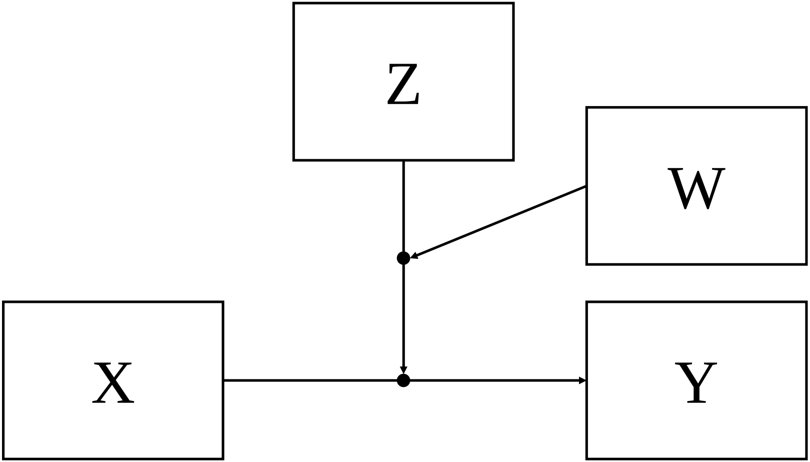
The additive two-way interaction model is more flexible than the simple single-moderator model, but it still imposes constraints.

- The moderating effect of Z (or W) on the $X \rightarrow Y$ relation is assumed to be constant across levels of W (or Z).
- I.e., the moderation is not moderated

We can relax this constraint by modeling moderation of the moderated effect using a three-way interaction.

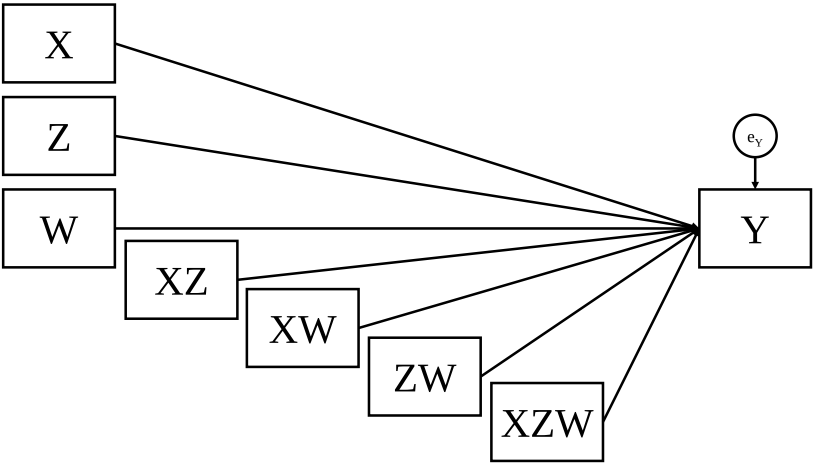
Moderated Moderation

Moderated moderation implies the following conceptual diagram:



Moderated Moderation

The preceding conceptual diagram implies this analytic diagram:



Moderated Moderation

The preceding diagram represents the following equation:

$$Y = \alpha + \beta_1 X + \beta_2 Z + \beta_3 W + \\ \beta_4 XZ + \beta_5 XW + \beta_6 ZW + \beta_7 XZW + e$$

Which can be restructured into:

$$\begin{aligned} Y &= \alpha + (\beta_1 + \beta_4 Z + \beta_5 W + \beta_7 ZW)X + \\ &\quad \beta_2 Z + \beta_3 W + \beta_6 ZW + e \\ &= \alpha + g(Z, W)X + \beta_2 Z + \beta_3 W + \beta_6 ZW + e \end{aligned}$$

With moderated moderation, the simple slope is given by:

$$g(Z, W) = \beta_1 + \beta_4 Z + \beta_5 W + \beta_7 ZW$$

Which has the same structure as a single moderator model.

- Three-way simple slopes represent the moderated effect of Z on the $X \rightarrow Y$ relation at conditional values of W.

Example

```
## Three-way interaction model:
out1.3 ← lm(agree ~ open*conc*neuro, data = dat1)
summary(out1.3)
```

```
Call:
lm(formula = agree ~ open * conc * neuro, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.79789 -0.41779  0.09925  0.47556  2.10928

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)     -0.58747    0.96633  -0.608  0.54328
open             1.27903    0.25747   4.968 7.23e-07 ***
conc             1.20831    0.26559   4.550 5.63e-06 ***
neuro            0.73766    0.32240   2.288  0.02222 *
open:conc       -0.29722    0.06935  -4.286 1.89e-05 ***
open:neuro      -0.21616    0.08091  -2.672  0.00760 **
conc:neuro      -0.25632    0.08244  -3.109  0.00190 **
open:conc:neuro  0.06541    0.02028   3.225  0.00128 **
---
```

Example

```
Signif. codes:  0   ***   0.001   **   0.01   *   0.05
        .   0.1        1

Residual standard error: 0.6945 on 2544 degrees of freedom
Multiple R^2:  0.07189,  Adjusted R^2:  0.06933
F-statistic: 28.15 on 7 and 2544 DF,  p-value: < 2.2e-16
```

Example

```
## Test simple slopes via centering:
dat1$neuroLo ←
    with(dat1,
         neuro - quantile(neuro, 0.05, na.rm = TRUE)
         )
dat1$neuroMid ←
    with(dat1,
         neuro - quantile(neuro, 0.5, na.rm = TRUE)
         )
dat1$neuroHi ←
    with(dat1,
         neuro - quantile(neuro, 0.95, na.rm = TRUE)
         )
```

Example

```
out1.4.1 <- lm(agree ~ open*conc*neuroLo, data = dat1)
summary(out1.4.1)
```

```
Call:
lm(formula = agree ~ open * conc * neuroLo, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.79789 -0.41779  0.09925  0.47556  2.10928

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)        0.29772    0.68387   0.435  0.66335
open               1.01964    0.18561   5.494 4.33e-08 ***
conc               0.90073    0.19180   4.696 2.79e-06 ***
neuroLo            0.73766    0.32240   2.288  0.02222 *
open:conc         -0.21872    0.05099  -4.289 1.86e-05 ***
open:neuroLo      -0.21616    0.08091  -2.672  0.00760 **
conc:neuroLo      -0.25632    0.08244  -3.109  0.00190 **
open:conc:neuroLo  0.06541    0.02028   3.225  0.00128 **
---
```

Example

```
Signif. codes:  0   ***   0.001   **   0.01   *   0.05
        .   0.1        1

Residual standard error: 0.6945 on 2544 degrees of freedom
Multiple R^2:  0.07189,  Adjusted R^2:  0.06933
F-statistic: 28.15 on 7 and 2544 DF,  p-value: < 2.2e-16
```

Example

```
out1.4.2 <- lm(agree ~ open*conc*neuroMid, data = dat1)
summary(out1.4.2)
```

```
Call:
lm(formula = agree ~ open * conc * neuroMid, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.79789 -0.41779  0.09925  0.47556  2.10928

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)         1.62552    0.57429   2.831  0.00468 **
open                0.63055    0.14845   4.248 2.24e-05 ***
conc                0.43936    0.15119   2.906  0.00369 **
neuroMid            0.73766    0.32240   2.288  0.02222 *
open:conc          -0.10098    0.03883  -2.601  0.00936 **
open:neuroMid      -0.21616    0.08091  -2.672  0.00760 **
conc:neuroMid      -0.25632    0.08244  -3.109  0.00190 **
open:conc:neuroMid  0.06541    0.02028   3.225  0.00128 **
---
```

Example

```
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1         1

Residual standard error: 0.6945 on 2544 degrees of freedom
Multiple R^2:  0.07189,  Adjusted R^2:  0.06933
F-statistic: 28.15 on 7 and 2544 DF,  p-value: < 2.2e-16
```

Example

```
out1.4.3 <- lm(agree ~ open*conc*neuroHi, data = dat1)
summary(out1.4.3)
```

```
Call:
lm(formula = agree ~ open * conc * neuroHi, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.79789 -0.41779  0.09925  0.47556  2.10928

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)        3.24838    1.03730   3.132  0.00176 **
open               0.15500    0.25391   0.610  0.54161
conc              -0.12454    0.25621  -0.486  0.62694
neuroHi            0.73766    0.32240   2.288  0.02222 *
open:conc          0.04293    0.06158   0.697  0.48578
open:neuroHi      -0.21616    0.08091  -2.672  0.00760 **
conc:neuroHi      -0.25632    0.08244  -3.109  0.00190 **
open:conc:neuroHi  0.06541    0.02028   3.225  0.00128 **
---
```

Example

```
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1          1

Residual standard error: 0.6945 on 2544 degrees of freedom
Multiple R^2:  0.07189,  Adjusted R^2:  0.06933
F-statistic: 28.15 on 7 and 2544 DF,  p-value: < 2.2e-16
```

Example

```
## Construct product terms to facilitate J-N technique:
dat1$openXneuro <- with(dat1, neuro*open)
dat1$concXneuro <- with(dat1, neuro*conc)
dat1$openXconc <- with(dat1, open*conc)
dat1$openXconcXneuro <- with(dat1, open*conc*neuro)
```

Example

```
out1.5 ← lm(agree ~ open + conc + neuro +
                 openXconc + openXneuro + concXneuro +
                     openXconc*neuro,
             data = dat1)
summary(out1.5)
```

```
Call:
lm(formula = agree ~ open + conc + neuro + openXconc +
    openXneuro +
    concXneuro + openXconc * neuro, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.79789 -0.41779  0.09925  0.47556  2.10928

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)     -0.58747    0.96633  -0.608  0.54328
open             1.27903    0.25747   4.968 7.23e-07 ***
conc             1.20831    0.26559   4.550 5.63e-06 ***
neuro            0.73766    0.32240   2.288  0.02222 *
openXconc       -0.29722    0.06935  -4.286 1.89e-05 ***
```

Example

```
openXneuro      -0.21616    0.08091  -2.672  0.00760 **
concXneuro      -0.25632    0.08244  -3.109  0.00190 **
neuro:openXconc  0.06541    0.02028   3.225  0.00128 **
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1          1

Residual standard error: 0.6945 on 2544 degrees of freedom
Multiple R^2:  0.07189,  Adjusted R^2:  0.06933
F-statistic: 28.15 on 7 and 2544 DF,  p-value: < 2.2e-16
```

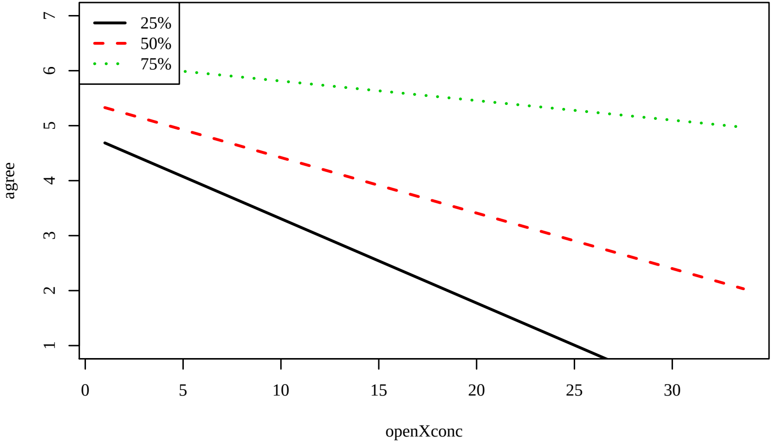
```
sum(coef(out1.3) - coef(out1.5))# Same as above
```

```
[1] 0
```

Example

```
par(family = "serif", cex = 0.75)
plotOut1.5 <- plotSlopes(model = out1.5,
                         plotx = "openXconc",
                         modx = "neuro",
                         plotPoints = FALSE,
                         modxVals =
                         quantile(dat1$neuro,
                                  c(0.25, 0.5, 0.75),
                                  na.rm = TRUE)
                         )
```

Example



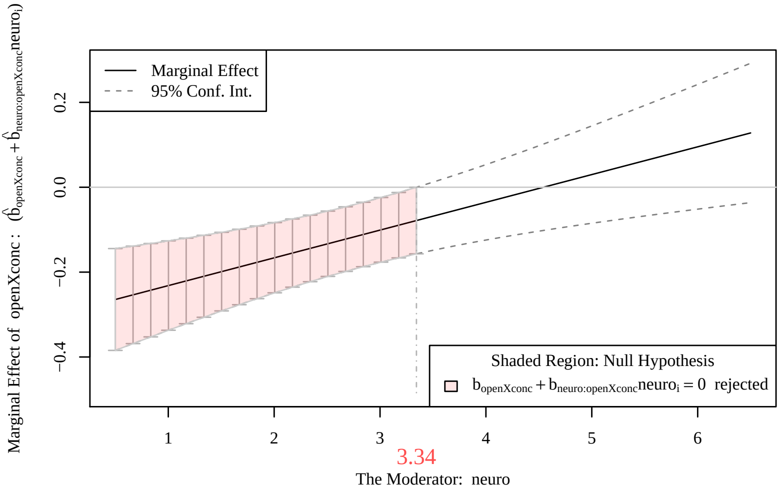
Example

```
par(family = "serif", cex = 0.75)
testOut1.5 ← testSlopes(plotOut1.5)
```

```
Values of neuro OUTSIDE this interval:
      lo         hi
3.343832 7.745048
cause the slope of (b1 + b2*neuro)openXconc to be
    statistically significant
```

```
plot(testOut1.5)
```

Example



Marginal Effect
95% Conf. Int.
Shaded Region: Null Hypothesis
$b_{openXconc} + b_{neuro:openXconc} neuro_i = 0$ rejected
0.2
0.0
−0.2
−0.4
1
2
3
4
5
6
3.34
The Moderator: neuro
Marginal Effect of openXconc : $(\hat{b}_{openXconc} + \hat{b}_{neuro:openXconc} neuro_i)$

Categorical Variable Moderation

When the moderator is a categorical variable, moderation implies between-group differences in the focal effect.

- This simplifies probing considerably
- The simple slopes are given (almost) directly in the output

Recall the simple slope formula:

$$SS = \beta_1 + \beta_3 Z$$

Because Z is a dummy code, this formula reduces to:

$$\begin{aligned} SS &= \beta_1, \text{ or} \\ SS &= \beta_1 + \beta_3 \end{aligned}$$

Example

```
## Marginal focal effect:
out2.1 <- lm(conc ~ neuro, data = dat1)
summary(out2.1)
```

```
Call:
lm(formula = conc ~ neuro, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.55547 -0.33353  0.00824  0.36098  1.85381

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.437327   0.029659  115.90   <2e-16 ***
neuro       0.118144   0.008844   13.36   <2e-16 ***
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1         1

Residual standard error: 0.533 on 2550 degrees of freedom
Multiple R^2:  0.0654, Adjusted R^2:  0.06504
F-statistic: 178.4 on 1 and 2550 DF,  p-value: < 2.2e-16
```

Example

```
## Moderated by highest education attained:
out2.2 <- lm(conc ~ neuro*educ, data = dat1)
summary(out2.2)
```

```
Call:
lm(formula = conc ~ neuro * educ, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.52324 -0.34119  0.01457  0.36247  1.86213

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.72924    0.10864  34.326  < 2e-16 ***
neuro        0.01259    0.03156   0.399 0.689990
educ2       -0.32892    0.11497  -2.861 0.004258 **
educ3       -0.30738    0.12102  -2.540 0.011146 *
neuro:educ2  0.11033    0.03346   3.297 0.000990 ***
neuro:educ3  0.12755    0.03552   3.591 0.000336 ***
---
Signif. codes:  0  ***  0.001  **  0.01  *  0.05
     .  0.1   1
```

Example

```
Residual standard error: 0.5308 on 2546 degrees of freedom
Multiple R^2:  0.0746, Adjusted R^2:  0.07278
F-statistic: 41.05 on 5 and 2546 DF,  p-value: < 2.2e-16
```

```
## Test for omnibus moderation:
anova(out2.1, out2.2)
```

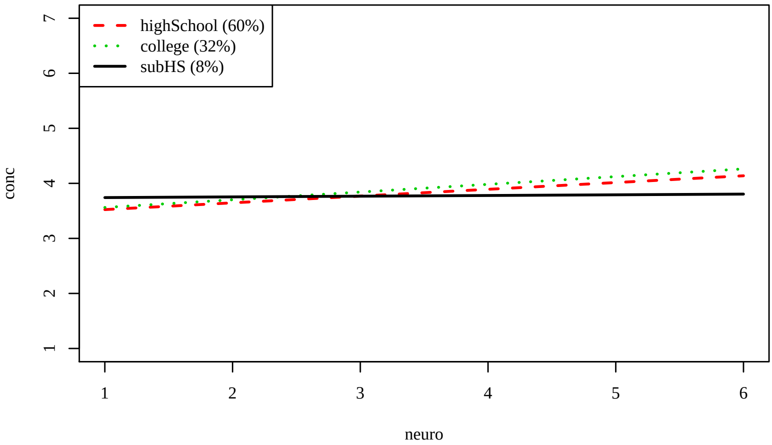
```
Analysis of Variance Table

Model 1: conc ~ neuro
Model 2: conc ~ neuro * educ
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1   2550 724.47
2   2546 717.35  4    7.1285 6.3251 4.617e-05 ***
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
        .    0.1          1
```

Example

```
par(family = "serif", cex = 0.75)
plotSlopes(out2.2,
           plotx = "neuro",
           modx = "educ",
           plotPoints = FALSE)
```

Example



Example

```r
## Compute simple slopes by hand:
ssSubHS <- coef(out2.2)[2]
ssHighSchool <- sum(coef(out2.2)[c(2, 5)])
ssCollege <- sum(coef(out2.2)[c(2, 6)])
## Compute simple slopes using centering:
dat1$educ2 <- relevel(dat1$educ, ref = "highSchool")
dat1$educ3 <- relevel(dat1$educ, ref = "college")
out2.3 <- lm(conc ~ neuro*educ2, data = dat1)
out2.4 <- lm(conc ~ neuro*educ3, data = dat1)
```

Example

```r
## By hand:
ssSubHS
```

```
    neuro
0.0125915
```

```r
## By centering:
as.matrix(coef(out2.2))
```

```
                  [,1]
(Intercept)  3.7292366
neuro        0.0125915
educ2       -0.3289176
educ3       -0.3073786
neuro:educ2  0.1103337
neuro:educ3  0.1275497
```

Example

```r
## By hand:
ssHighSchool
```

```
[1] 0.1229252
```

```r
## By centering:
as.matrix(coef(out2.3))
```

```
                           [,1]
(Intercept)          3.40031894
neuro                0.12292519
educ2subHS           0.32891761
educ2college         0.02153898
neuro:educ2subHS    -0.11033369
neuro:educ2college   0.01721601
```

Example

```
## By hand:
ssCollege
```

```
[1] 0.1401412
```

```
## By centering:
as.matrix(coef(out2.4))
```

```
                              [,1]
(Intercept)             3.42185792
neuro                   0.14014120
educ3subHS              0.30737863
educ3highSchool        -0.02153898
neuro:educ3subHS       -0.12754971
neuro:educ3highSchool  -0.01721601
```

Example

```
summary(out2.2)
```

```
Call:
lm(formula = conc ~ neuro * educ, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.52324 -0.34119  0.01457  0.36247  1.86213

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.72924    0.10864  34.326  < 2e-16 ***
neuro        0.01259    0.03156   0.399 0.689990
educ2       -0.32892    0.11497  -2.861 0.004258 **
educ3       -0.30738    0.12102  -2.540 0.011146 *
neuro:educ2  0.11033    0.03346   3.297 0.000990 ***
neuro:educ3  0.12755    0.03552   3.591 0.000336 ***
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
       .    0.1          1

Residual standard error: 0.5308 on 2546 degrees of freedom
Multiple R^2:  0.0746, Adjusted R^2:  0.07278
F-statistic: 41.05 on 5 and 2546 DF,  p-value: < 2.2e-16
```

Example

```
summary(out2.3)
```

```
Call:
lm(formula = conc ~ neuro * educ2, data = dat1)

Residuals:
     Min        1Q    Median        3Q       Max
-2.52324 -0.34119   0.01457  0.36247  1.86213

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)         3.40032    0.03761  90.401  < 2e-16 ***
neuro               0.12293    0.01111  11.063  < 2e-16 ***
educ2subHS          0.32892    0.11497   2.861  0.00426 **
educ2college        0.02154    0.06525   0.330  0.74134
neuro:educ2subHS   -0.11033    0.03346  -3.297  0.00099 ***
neuro:educ2college  0.01722    0.01972   0.873  0.38277
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05
       .    0.1          1

Residual standard error: 0.5308 on 2546 degrees of freedom
Multiple R^2:  0.0746, Adjusted R^2:  0.07278
F-statistic: 41.05 on 5 and 2546 DF,  p-value: < 2.2e-16
```

Example

```
summary(out2.4)
```

```
Call:
lm(formula = conc ~ neuro * educ3, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.52324 -0.34119  0.01457  0.36247  1.86213

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)           3.42186    0.05331  64.183  < 2e-16 ***
neuro                 0.14014    0.01629   8.601  < 2e-16 ***
educ3subHS            0.30738    0.12102   2.540 0.011146 *
educ3highSchool      -0.02154    0.06525  -0.330 0.741340
neuro:educ3subHS     -0.12755    0.03552  -3.591 0.000336 ***
neuro:educ3highSchool -0.01722   0.01972  -0.873 0.382770
---
Signif. codes:  0  ***  0.001  **  0.01  *  0.05
      .  0.1       1

Residual standard error: 0.5308 on 2546 degrees of freedom
Multiple R^2:  0.0746, Adjusted R^2:  0.07278
F-statistic: 41.05 on 5 and 2546 DF,  p-value: < 2.2e-16
```

Moderation via Multiple Group SEM

When our moderator is a categorical variable, we can use multiple group CFA/SEM to test for moderation.

- Categorical moderators define groups
- Significant moderation with categorical moderators implies between-group differences in the focal effect
- These hypotheses are easily tested with multiple group SEM

WHITEBOARD TIME!

Example

```r
library(lavaan)
library(semTools)
dat2 <- readRDS("../data/bfiData2.rds")
## Multiple group moderation:
mod1 <- "
conc =~ C1 + C2 + C3 + C4 + C5
neuro =~ N1 + N2 + N3 + N4 + N5
"
```

Example

```
fit1 ← measurementInvariance(mod1,
                             data = dat2,
                             group = "educ",
                             std.lv = TRUE)
```

```
Measurement invariance models:

Model 1 : fit.configural
Model 2 : fit.loadings
Model 3 : fit.intercepts
Model 4 : fit.means

Chi Square Difference Test

                Df   AIC   BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit.configural 102 85428 85971 1039.1
fit.loadings   118 85427 85877 1070.0     30.927      16  0.0137462
     *
fit.intercepts 134 85456 85813 1131.4     61.399      16  3.037e-07
     ***
fit.means      138 85468 85801 1150.8     19.324       4  0.0006788
     ***
---
Signif. codes:  0   ***   0.001   **   0.01   *   0.05
       .   0.1        1


Fit measures:

                 cfi rmsea cfi.delta rmsea.delta
fit.configural 0.874 0.104        NA          NA
fit.loadings   0.871 0.097     0.002       0.007
fit.intercepts 0.865 0.094     0.006       0.004
fit.means      0.863 0.093     0.002       0.001
```

Example

```
mod2 <- "
conc =~ C1 + C2 + C3 + C4 + C5
neuro =~ N1 + N2 + N3 + N4 + N5

conc ~ neuro

conc ~~ c(1.0, NA, NA)*conc
neuro ~~ c(1.0, NA, NA)*neuro

conc ~ c(0.0, NA, NA)*1.0
neuro ~ c(0.0, NA, NA)*1.0
"
```

Example

```
fit2 <- lavaan(mod2,
               data = dat2,
               std.lv = FALSE,
               auto.fix.first = FALSE,
               auto.var = TRUE,
               int.ov.free = TRUE,
               group = "educ",
               group.equal = c("loadings", "intercepts")
               )
```

Example

```
summary(fit2)
```

```
lavaan (0.5-20) converged normally after  79 iterations

  Number of observations per group
  highSchool                                          1536
  subHS                                                192
  college                                              824

  Estimator                                             ML
  Minimum Function Test Statistic                 1131.438
  Degrees of freedom                                   134
  P-value (Chi-square)                               0.000

Chi-square for each group:

  highSchool                                       573.289
  subHS                                            108.925
  college                                          449.224

Parameter Estimates:
```

Example

```
  Information                                  Expected
  Standard Errors                              Standard


Group 1 [highSchool]:

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
    C1      (.p1.)    0.573    0.027   21.471    0.000
    C2      (.p2.)    0.678    0.029   23.706    0.000
    C3      (.p3.)    0.634    0.028   22.666    0.000
    C4      (.p4.)   -0.897    0.031  -29.235    0.000
    C5      (.p5.)   -0.947    0.036  -26.307    0.000
  neuro =~
    N1      (.p6.)    1.305    0.033   39.285    0.000
    N2      (.p7.)    1.247    0.032   38.701    0.000
    N3      (.p8.)    1.205    0.034   35.309    0.000
    N4      (.p9.)    0.909    0.034   26.982    0.000
    N5      (.10.)    0.837    0.035   23.998    0.000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
```

Example

```
  conc ~
    neuro              -0.359    0.035  -10.208    0.000

Intercepts:
                     Estimate  Std.Err  Z-value  P(>|z|)
    conc                0.000
    neuro               0.000
    C1        (.26.)    4.571    0.027  170.017    0.000
    C2        (.27.)    4.442    0.029  152.180    0.000
    C3        (.28.)    4.379    0.028  154.381    0.000
    C4        (.29.)    2.434    0.032   75.721    0.000
    C5        (.30.)    3.186    0.037   85.494    0.000
    N1        (.31.)    2.940    0.039   75.566    0.000
    N2        (.32.)    3.509    0.038   93.494    0.000
    N3        (.33.)    3.224    0.038   83.772    0.000
    N4        (.34.)    3.197    0.035   91.203    0.000
    N5        (.35.)    2.975    0.035   84.171    0.000

Variances:
                     Estimate  Std.Err  Z-value  P(>|z|)
    conc                1.000
    neuro               1.000
    C1                  1.109    0.045   24.781    0.000
```

Example

```
    C2                    1.187    0.050   23.872    0.000
    C3                    1.201    0.049   24.414    0.000
    C4                    0.893    0.048   18.572    0.000
    C5                    1.600    0.073   22.053    0.000
    N1                    0.840    0.046   18.316    0.000
    N2                    0.830    0.044   19.003    0.000
    N3                    1.219    0.055   22.298    0.000
    N4                    1.701    0.067   25.564    0.000
    N5                    1.962    0.075   26.138    0.000


Group 2 [subHS]:

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
    C1        (.p1.)    0.573    0.027   21.471    0.000
    C2        (.p2.)    0.678    0.029   23.706    0.000
    C3        (.p3.)    0.634    0.028   22.666    0.000
    C4        (.p4.)   -0.897    0.031  -29.235    0.000
    C5        (.p5.)   -0.947    0.036  -26.307    0.000
  neuro =~
    N1        (.p6.)    1.305    0.033   39.285    0.000
```

Example

```
    N2          (.p7.)    1.247    0.032   38.701    0.000
    N3          (.p8.)    1.205    0.034   35.309    0.000
    N4          (.p9.)    0.909    0.034   26.982    0.000
    N5          (.10.)    0.837    0.035   23.998    0.000

Regressions:
                        Estimate  Std.Err  Z-value  P(>|z|)
  conc ~
    neuro                 -0.252    0.105   -2.396    0.017

Intercepts:
                        Estimate  Std.Err  Z-value  P(>|z|)
    conc                  -0.261    0.095   -2.741    0.006
    neuro                  0.016    0.081    0.202    0.840
    C1          (.26.)    4.571    0.027  170.017    0.000
    C2          (.27.)    4.442    0.029  152.180    0.000
    C3          (.28.)    4.379    0.028  154.381    0.000
    C4          (.29.)    2.434    0.032   75.721    0.000
    C5          (.30.)    3.186    0.037   85.494    0.000
    N1          (.31.)    2.940    0.039   75.566    0.000
    N2          (.32.)    3.509    0.038   93.494    0.000
    N3          (.33.)    3.224    0.038   83.772    0.000
    N4          (.34.)    3.197    0.035   91.203    0.000
```

Example

```
    N5        (.35.)    2.975    0.035    84.171    0.000

Variances:
                       Estimate  Std.Err  Z-value  P(>|z|)
    conc                  1.061    0.170    6.237    0.000
    neuro                 0.905    0.120    7.540    0.000
    C1                    1.263    0.142    8.919    0.000
    C2                    1.270    0.148    8.568    0.000
    C3                    1.437    0.162    8.855    0.000
    C4                    1.192    0.160    7.466    0.000
    C5                    1.748    0.217    8.039    0.000
    N1                    1.014    0.146    6.965    0.000
    N2                    1.236    0.161    7.681    0.000
    N3                    1.310    0.165    7.937    0.000
    N4                    1.507    0.170    8.888    0.000
    N5                    2.042    0.221    9.232    0.000


Group 3 [college]:

Latent Variables:
                       Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
```

Example

```
    C1          (.p1.)      0.573     0.027    21.471     0.000
    C2          (.p2.)      0.678     0.029    23.706     0.000
    C3          (.p3.)      0.634     0.028    22.666     0.000
    C4          (.p4.)     -0.897     0.031   -29.235     0.000
    C5          (.p5.)     -0.947     0.036   -26.307     0.000
  neuro =~
    N1          (.p6.)      1.305     0.033    39.285     0.000
    N2          (.p7.)      1.247     0.032    38.701     0.000
    N3          (.p8.)      1.205     0.034    35.309     0.000
    N4          (.p9.)      0.909     0.034    26.982     0.000
    N5          (.10.)      0.837     0.035    23.998     0.000

Regressions:
                         Estimate   Std.Err   Z-value   P(>|z|)
  conc ~
    neuro                  -0.278     0.052    -5.354     0.000

Intercepts:
                         Estimate   Std.Err   Z-value   P(>|z|)
    conc                   -0.168     0.053    -3.139     0.002
    neuro                  -0.092     0.045    -2.056     0.040
    C1          (.26.)      4.571     0.027   170.017     0.000
    C2          (.27.)      4.442     0.029   152.180     0.000
```

Example

```
    C3          (.28.)    4.379    0.028  154.381    0.000
    C4          (.29.)    2.434    0.032   75.721    0.000
    C5          (.30.)    3.186    0.037   85.494    0.000
    N1          (.31.)    2.940    0.039   75.566    0.000
    N2          (.32.)    3.509    0.038   93.494    0.000
    N3          (.33.)    3.224    0.038   83.772    0.000
    N4          (.34.)    3.197    0.035   91.203    0.000
    N5          (.35.)    2.975    0.035   84.171    0.000

Variances:
                       Estimate  Std.Err  Z-value  P(>|z|)
    conc                  1.139    0.098   11.634    0.000
    neuro                 0.865    0.063   13.807    0.000
    C1                    1.178    0.064   18.364    0.000
    C2                    1.142    0.065   17.467    0.000
    C3                    1.093    0.062   17.713    0.000
    C4                    0.952    0.067   14.255    0.000
    C5                    1.633    0.100   16.405    0.000
    N1                    0.807    0.058   13.850    0.000
    N2                    0.820    0.057   14.498    0.000
    N3                    1.122    0.068   16.394    0.000
    N4                    1.630    0.087   18.807    0.000
    N5                    1.882    0.098   19.207    0.000
```

Example

```
mod3 <- "
conc =~ C1 + C2 + C3 + C4 + C5
neuro =~ N1 + N2 + N3 + N4 + N5

conc ~ c(b1, b1, b1)*neuro

conc ~~ c(1.0, NA, NA)*conc
neuro ~~ c(1.0, NA, NA)*neuro

conc ~ c(0.0, NA, NA)*1.0
neuro ~ c(0.0, NA, NA)*1.0
"
```

Example

```
fit3 <- lavaan(mod3,
               data = dat2,
               std.lv = FALSE,
               auto.fix.first = FALSE,
               auto.var = TRUE,
               int.ov.free = TRUE,
               group = "educ",
               group.equal = c("loadings", "intercepts")
               )
```

Example

```
summary(fit3)
```

```
lavaan (0.5-20) converged normally after  82 iterations

  Number of observations per group
  highSchool                                          1536
  subHS                                                192
  college                                              824

  Estimator                                             ML
  Minimum Function Test Statistic                 1133.785
  Degrees of freedom                                   136
  P-value (Chi-square)                               0.000

Chi-square for each group:

  highSchool                                       574.222
  subHS                                            109.473
  college                                          450.090

Parameter Estimates:
```

Example

```
  Information                                  Expected
  Standard Errors                              Standard


Group 1 [highSchool]:

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
    C1       (.p1.)    0.575    0.027   21.516    0.000
    C2       (.p2.)    0.679    0.029   23.743    0.000
    C3       (.p3.)    0.635    0.028   22.707    0.000
    C4       (.p4.)   -0.898    0.031  -29.262    0.000
    C5       (.p5.)   -0.949    0.036  -26.367    0.000
  neuro =~
    N1       (.p6.)    1.307    0.033   39.323    0.000
    N2       (.p7.)    1.249    0.032   38.734    0.000
    N3       (.p8.)    1.207    0.034   35.334    0.000
    N4       (.p9.)    0.910    0.034   26.987    0.000
    N5       (.10.)    0.839    0.035   24.010    0.000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
```

Example

```
  conc ~
    neuro       (b1)    -0.330     0.029  -11.365     0.000

Intercepts:
                     Estimate   Std.Err  Z-value   P(>|z|)
    conc                0.000
    neuro               0.000
    C1        (.26.)    4.571     0.027  170.418     0.000
    C2        (.27.)    4.442     0.029  152.673     0.000
    C3        (.28.)    4.379     0.028  154.796     0.000
    C4        (.29.)    2.434     0.032   76.041     0.000
    C5        (.30.)    3.187     0.037   85.740     0.000
    N1        (.31.)    2.940     0.039   75.506     0.000
    N2        (.32.)    3.509     0.038   93.434     0.000
    N3        (.33.)    3.224     0.039   83.699     0.000
    N4        (.34.)    3.197     0.035   91.151     0.000
    N5        (.35.)    2.975     0.035   84.126     0.000

Variances:
                     Estimate   Std.Err  Z-value   P(>|z|)
    conc                1.000
    neuro               1.000
    C1                  1.107     0.045   24.765     0.000
```

Example

```
    C2                  1.184     0.050    23.859     0.000
    C3                  1.199     0.049    24.404     0.000
    C4                  0.898     0.048    18.626     0.000
    C5                  1.603     0.073    22.047     0.000
    N1                  0.838     0.046    18.279     0.000
    N2                  0.828     0.044    18.968     0.000
    N3                  1.219     0.055    22.286     0.000
    N4                  1.703     0.067    25.564     0.000
    N5                  1.963     0.075    26.136     0.000


Group 2 [subHS]:

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
    C1        (.p1.)    0.575     0.027    21.516     0.000
    C2        (.p2.)    0.679     0.029    23.743     0.000
    C3        (.p3.)    0.635     0.028    22.707     0.000
    C4        (.p4.)   -0.898     0.031   -29.262     0.000
    C5        (.p5.)   -0.949     0.036   -26.367     0.000
  neuro =~
    N1        (.p6.)    1.307     0.033    39.323     0.000
```

Example

```
    N2          (.p7.)     1.249     0.032    38.734     0.000
    N3          (.p8.)     1.207     0.034    35.334     0.000
    N4          (.p9.)     0.910     0.034    26.987     0.000
    N5          (.10.)     0.839     0.035    24.010     0.000

Regressions:
                       Estimate   Std.Err   Z-value   P(>|z|)
  conc ~
    neuro         (b1)    -0.330     0.029   -11.365     0.000

Intercepts:
                       Estimate   Std.Err   Z-value   P(>|z|)
    conc                  -0.259     0.095    -2.721     0.007
    neuro                  0.016     0.081     0.201     0.841
    C1          (.26.)     4.571     0.027   170.418     0.000
    C2          (.27.)     4.442     0.029   152.673     0.000
    C3          (.28.)     4.379     0.028   154.796     0.000
    C4          (.29.)     2.434     0.032    76.041     0.000
    C5          (.30.)     3.187     0.037    85.740     0.000
    N1          (.31.)     2.940     0.039    75.506     0.000
    N2          (.32.)     3.509     0.038    93.434     0.000
    N3          (.33.)     3.224     0.039    83.699     0.000
    N4          (.34.)     3.197     0.035    91.151     0.000
```

Example

```
    N5        (.35.)     2.975     0.035    84.126     0.000

Variances:
                      Estimate   Std.Err   Z-value   P(>|z|)
    conc                 1.054     0.169     6.216     0.000
    neuro                0.896     0.119     7.551     0.000
    C1                   1.264     0.142     8.917     0.000
    C2                   1.272     0.148     8.569     0.000
    C3                   1.434     0.162     8.851     0.000
    C4                   1.196     0.160     7.477     0.000
    C5                   1.742     0.217     8.026     0.000
    N1                   1.013     0.145     6.978     0.000
    N2                   1.242     0.161     7.705     0.000
    N3                   1.312     0.165     7.951     0.000
    N4                   1.507     0.169     8.893     0.000
    N5                   2.045     0.221     9.236     0.000


Group 3 [college]:

Latent Variables:
                      Estimate   Std.Err   Z-value   P(>|z|)
  conc =~
```

Example

```
    C1          (.p1.)    0.575    0.027   21.516    0.000
    C2          (.p2.)    0.679    0.029   23.743    0.000
    C3          (.p3.)    0.635    0.028   22.707    0.000
    C4          (.p4.)   -0.898    0.031  -29.262    0.000
    C5          (.p5.)   -0.949    0.036  -26.367    0.000
  neuro =~
    N1          (.p6.)    1.307    0.033   39.323    0.000
    N2          (.p7.)    1.249    0.032   38.734    0.000
    N3          (.p8.)    1.207    0.034   35.334    0.000
    N4          (.p9.)    0.910    0.034   26.987    0.000
    N5          (.10.)    0.839    0.035   24.010    0.000

Regressions:
                       Estimate  Std.Err  Z-value  P(>|z|)
  conc ~
    neuro       (b1)     -0.330    0.029  -11.365    0.000

Intercepts:
                       Estimate  Std.Err  Z-value  P(>|z|)
    conc                 -0.173    0.053   -3.239    0.001
    neuro                -0.092    0.045   -2.056    0.040
    C1          (.26.)    4.571    0.027  170.418    0.000
    C2          (.27.)    4.442    0.029  152.673    0.000
```

Example

```
    C3          (.28.)     4.379    0.028  154.796    0.000
    C4          (.29.)     2.434    0.032   76.041    0.000
    C5          (.30.)     3.187    0.037   85.740    0.000
    N1          (.31.)     2.940    0.039   75.506    0.000
    N2          (.32.)     3.509    0.038   93.434    0.000
    N3          (.33.)     3.224    0.039   83.699    0.000
    N4          (.34.)     3.197    0.035   91.151    0.000
    N5          (.35.)     2.975    0.035   84.126    0.000

Variances:
                    Estimate  Std.Err  Z-value  P(>|z|)
    conc               1.132    0.097   11.640    0.000
    neuro              0.860    0.062   13.828    0.000
    C1                 1.179    0.064   18.362    0.000
    C2                 1.148    0.066   17.486    0.000
    C3                 1.096    0.062   17.718    0.000
    C4                 0.951    0.067   14.262    0.000
    C5                 1.621    0.099   16.366    0.000
    N1                 0.810    0.058   13.894    0.000
    N2                 0.824    0.057   14.542    0.000
    N3                 1.122    0.068   16.399    0.000
    N4                 1.627    0.086   18.806    0.000
    N5                 1.880    0.098   19.206    0.000
```

Example

```r
diffVec <- fitMeasures(fit3)[c("chisq", "df")] -
    fitMeasures(fit2)[c("chisq", "df")]
pchisq(diffVec[1], diffVec[2], lower = FALSE)
```

```
    chisq
0.3093433
```

Example

```
mod4 <- "
conc =~ C1 + C2 + C3 + C4 + C5
neuro =~ N1 + N2 + N3 + N4 + N5

conc ~ c(b1, b1, b2)*neuro

conc ~~ c(1.0, NA, NA)*conc
neuro ~~ c(1.0, NA, NA)*neuro

conc ~ c(0.0, NA, NA)*1.0
neuro ~ c(0.0, NA, NA)*1.0
"
```

Example

```
fit4 <- lavaan(mod4,
               data = dat2,
               std.lv = FALSE,
               auto.fix.first = FALSE,
               auto.var = TRUE,
               int.ov.free = TRUE,
               group = "educ",
               group.equal = c("loadings", "intercepts")
               )
```

Example

```r
summary(fit4)
```

```
lavaan (0.5-20) converged normally after  75 iterations

  Number of observations per group
  highSchool                                        1536
  subHS                                              192
  college                                            824

  Estimator                                           ML
  Minimum Function Test Statistic               1132.387
  Degrees of freedom                                 135
  P-value (Chi-square)                             0.000

Chi-square for each group:

  highSchool                                     573.494
  subHS                                          109.779
  college                                        449.114

Parameter Estimates:
```

Example

```
  Information                                Expected
  Standard Errors                            Standard


Group 1 [highSchool]:

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
    C1      (.p1.)    0.574    0.027   21.493    0.000
    C2      (.p2.)    0.679    0.029   23.735    0.000
    C3      (.p3.)    0.635    0.028   22.691    0.000
    C4      (.p4.)   -0.897    0.031  -29.229    0.000
    C5      (.p5.)   -0.947    0.036  -26.305    0.000
  neuro =~
    N1      (.p6.)    1.306    0.033   39.305    0.000
    N2      (.p7.)    1.248    0.032   38.715    0.000
    N3      (.p8.)    1.205    0.034   35.313    0.000
    N4      (.p9.)    0.909    0.034   26.975    0.000
    N5      (.10.)    0.837    0.035   23.992    0.000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
```

Example

```
  conc ~
    neuro       (b1)    -0.349    0.034  -10.380    0.000

Intercepts:
                   Estimate  Std.Err  Z-value  P(>|z|)
    conc              0.000
    neuro             0.000
    C1        (.26.)  4.571    0.027  170.141    0.000
    C2        (.27.)  4.442    0.029  152.312    0.000
    C3        (.28.)  4.379    0.028  154.494    0.000
    C4        (.29.)  2.434    0.032   75.849    0.000
    C5        (.30.)  3.186    0.037   85.603    0.000
    N1        (.31.)  2.940    0.039   75.537    0.000
    N2        (.32.)  3.509    0.038   93.472    0.000
    N3        (.33.)  3.224    0.038   83.753    0.000
    N4        (.34.)  3.197    0.035   91.190    0.000
    N5        (.35.)  2.975    0.035   84.164    0.000

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    conc              1.000
    neuro             1.000
    C1                1.108    0.045   24.771    0.000
```

Example

```
    C2                    1.186    0.050   23.857    0.000
    C3                    1.200    0.049   24.403    0.000
    C4                    0.895    0.048   18.603    0.000
    C5                    1.601    0.073   22.063    0.000
    N1                    0.839    0.046   18.296    0.000
    N2                    0.829    0.044   18.988    0.000
    N3                    1.219    0.055   22.298    0.000
    N4                    1.702    0.067   25.566    0.000
    N5                    1.963    0.075   26.139    0.000


Group 2 [subHS]:

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
    C1      (.p1.)    0.574    0.027   21.493    0.000
    C2      (.p2.)    0.679    0.029   23.735    0.000
    C3      (.p3.)    0.635    0.028   22.691    0.000
    C4      (.p4.)   -0.897    0.031  -29.229    0.000
    C5      (.p5.)   -0.947    0.036  -26.305    0.000
  neuro =~
    N1      (.p6.)    1.306    0.033   39.305    0.000
```

Example

```
    N2          (.p7.)     1.248    0.032   38.715    0.000
    N3          (.p8.)     1.205    0.034   35.313    0.000
    N4          (.p9.)     0.909    0.034   26.975    0.000
    N5          (.10.)     0.837    0.035   23.992    0.000

Regressions:
                         Estimate  Std.Err  Z-value  P(>|z|)
  conc ~
    neuro         (b1)    -0.349    0.034  -10.380    0.000

Intercepts:
                         Estimate  Std.Err  Z-value  P(>|z|)
    conc                  -0.259    0.095   -2.716    0.007
    neuro                  0.016    0.081    0.201    0.841
    C1          (.26.)     4.571    0.027  170.141    0.000
    C2          (.27.)     4.442    0.029  152.312    0.000
    C3          (.28.)     4.379    0.028  154.494    0.000
    C4          (.29.)     2.434    0.032   75.849    0.000
    C5          (.30.)     3.186    0.037   85.603    0.000
    N1          (.31.)     2.940    0.039   75.537    0.000
    N2          (.32.)     3.509    0.038   93.472    0.000
    N3          (.33.)     3.224    0.038   83.753    0.000
    N4          (.34.)     3.197    0.035   91.190    0.000
```

Example

```
    N5        (.35.)    2.975    0.035    84.164    0.000

Variances:
                    Estimate  Std.Err  Z-value  P(>|z|)
    conc               1.056    0.170    6.208    0.000
    neuro              0.896    0.119    7.550    0.000
    C1                 1.264    0.142    8.916    0.000
    C2                 1.272    0.148    8.564    0.000
    C3                 1.433    0.162    8.848    0.000
    C4                 1.197    0.160    7.476    0.000
    C5                 1.742    0.217    8.030    0.000
    N1                 1.012    0.145    6.980    0.000
    N2                 1.243    0.161    7.710    0.000
    N3                 1.314    0.165    7.957    0.000
    N4                 1.507    0.169    8.895    0.000
    N5                 2.046    0.221    9.238    0.000


Group 3 [college]:

Latent Variables:
                    Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
```

Example

```
    C1          (.p1.)    0.574    0.027   21.493    0.000
    C2          (.p2.)    0.679    0.029   23.735    0.000
    C3          (.p3.)    0.635    0.028   22.691    0.000
    C4          (.p4.)   -0.897    0.031  -29.229    0.000
    C5          (.p5.)   -0.947    0.036  -26.305    0.000
  neuro =~
    N1          (.p6.)    1.306    0.033   39.305    0.000
    N2          (.p7.)    1.248    0.032   38.715    0.000
    N3          (.p8.)    1.205    0.034   35.313    0.000
    N4          (.p9.)    0.909    0.034   26.975    0.000
    N5          (.10.)    0.837    0.035   23.992    0.000

Regressions:
                       Estimate  Std.Err  Z-value  P(>|z|)
  conc ~
    neuro         (b2)   -0.277    0.052   -5.348    0.000

Intercepts:
                       Estimate  Std.Err  Z-value  P(>|z|)
    conc                 -0.168    0.053   -3.137    0.002
    neuro                -0.092    0.045   -2.056    0.040
    C1          (.26.)    4.571    0.027  170.141    0.000
    C2          (.27.)    4.442    0.029  152.312    0.000
```

Example

```
    C3          (.28.)     4.379    0.028  154.494    0.000
    C4          (.29.)     2.434    0.032   75.849    0.000
    C5          (.30.)     3.186    0.037   85.603    0.000
    N1          (.31.)     2.940    0.039   75.537    0.000
    N2          (.32.)     3.509    0.038   93.472    0.000
    N3          (.33.)     3.224    0.038   83.753    0.000
    N4          (.34.)     3.197    0.035   91.190    0.000
    N5          (.35.)     2.975    0.035   84.164    0.000

Variances:
                        Estimate  Std.Err  Z-value  P(>|z|)
    conc                   1.138    0.098   11.640    0.000
    neuro                  0.864    0.063   13.809    0.000
    C1                     1.178    0.064   18.359    0.000
    C2                     1.141    0.065   17.460    0.000
    C3                     1.093    0.062   17.706    0.000
    C4                     0.953    0.067   14.269    0.000
    C5                     1.634    0.100   16.409    0.000
    N1                     0.807    0.058   13.845    0.000
    N2                     0.820    0.057   14.497    0.000
    N3                     1.122    0.068   16.397    0.000
    N4                     1.630    0.087   18.808    0.000
    N5                     1.882    0.098   19.208    0.000
```

Example

```r
diffVec ← fitMeasures(fit4)[c("chisq", "df")] -
    fitMeasures(fit2)[c("chisq", "df")]
pchisq(diffVec[1], diffVec[2], lower = FALSE)
```

```
    chisq
0.3299714
```

Probing Multiple Group Moderation

Several advantages to testing moderation with multiple group SEM

- Remove measurement error from the estimates
- Test for factorial invariance
- *All information needed to plot/probe the simple slopes is contained directly in the output from the unrestricted model*

Example

```
summary(fit2)
```

```
lavaan (0.5-20) converged normally after  79 iterations

  Number of observations per group
  highSchool                                        1536
  subHS                                              192
  college                                            824

  Estimator                                           ML
  Minimum Function Test Statistic               1131.438
  Degrees of freedom                                 134
  P-value (Chi-square)                             0.000

Chi-square for each group:

  highSchool                                     573.289
  subHS                                          108.925
  college                                        449.224

Parameter Estimates:
```

Example

```
  Information                                   Expected
  Standard Errors                               Standard


Group 1 [highSchool]:

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
    C1      (.p1.)    0.573    0.027   21.471    0.000
    C2      (.p2.)    0.678    0.029   23.706    0.000
    C3      (.p3.)    0.634    0.028   22.666    0.000
    C4      (.p4.)   -0.897    0.031  -29.235    0.000
    C5      (.p5.)   -0.947    0.036  -26.307    0.000
  neuro =~
    N1      (.p6.)    1.305    0.033   39.285    0.000
    N2      (.p7.)    1.247    0.032   38.701    0.000
    N3      (.p8.)    1.205    0.034   35.309    0.000
    N4      (.p9.)    0.909    0.034   26.982    0.000
    N5      (.10.)    0.837    0.035   23.998    0.000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
```

Example

```
  conc ~
    neuro            -0.359    0.035  -10.208    0.000

Intercepts:
                   Estimate  Std.Err  Z-value  P(>|z|)
    conc              0.000
    neuro             0.000
    C1        (.26.)  4.571    0.027  170.017    0.000
    C2        (.27.)  4.442    0.029  152.180    0.000
    C3        (.28.)  4.379    0.028  154.381    0.000
    C4        (.29.)  2.434    0.032   75.721    0.000
    C5        (.30.)  3.186    0.037   85.494    0.000
    N1        (.31.)  2.940    0.039   75.566    0.000
    N2        (.32.)  3.509    0.038   93.494    0.000
    N3        (.33.)  3.224    0.038   83.772    0.000
    N4        (.34.)  3.197    0.035   91.203    0.000
    N5        (.35.)  2.975    0.035   84.171    0.000

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    conc              1.000
    neuro             1.000
    C1                1.109    0.045   24.781    0.000
```

Example

```
    C2                   1.187    0.050   23.872    0.000
    C3                   1.201    0.049   24.414    0.000
    C4                   0.893    0.048   18.572    0.000
    C5                   1.600    0.073   22.053    0.000
    N1                   0.840    0.046   18.316    0.000
    N2                   0.830    0.044   19.003    0.000
    N3                   1.219    0.055   22.298    0.000
    N4                   1.701    0.067   25.564    0.000
    N5                   1.962    0.075   26.138    0.000


Group 2 [subHS]:

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
    C1        (.p1.)    0.573    0.027   21.471    0.000
    C2        (.p2.)    0.678    0.029   23.706    0.000
    C3        (.p3.)    0.634    0.028   22.666    0.000
    C4        (.p4.)   -0.897    0.031  -29.235    0.000
    C5        (.p5.)   -0.947    0.036  -26.307    0.000
  neuro =~
    N1        (.p6.)    1.305    0.033   39.285    0.000
```

Example

```
    N2          (.p7.)     1.247    0.032   38.701    0.000
    N3          (.p8.)     1.205    0.034   35.309    0.000
    N4          (.p9.)     0.909    0.034   26.982    0.000
    N5          (.10.)     0.837    0.035   23.998    0.000

Regressions:
                         Estimate  Std.Err  Z-value  P(>|z|)
  conc ~
    neuro                  -0.252    0.105   -2.396    0.017

Intercepts:
                         Estimate  Std.Err  Z-value  P(>|z|)
    conc                   -0.261    0.095   -2.741    0.006
    neuro                   0.016    0.081    0.202    0.840
    C1          (.26.)     4.571    0.027  170.017    0.000
    C2          (.27.)     4.442    0.029  152.180    0.000
    C3          (.28.)     4.379    0.028  154.381    0.000
    C4          (.29.)     2.434    0.032   75.721    0.000
    C5          (.30.)     3.186    0.037   85.494    0.000
    N1          (.31.)     2.940    0.039   75.566    0.000
    N2          (.32.)     3.509    0.038   93.494    0.000
    N3          (.33.)     3.224    0.038   83.772    0.000
    N4          (.34.)     3.197    0.035   91.203    0.000
```

Example

```
    N5        (.35.)    2.975    0.035   84.171    0.000

Variances:
                     Estimate  Std.Err  Z-value  P(>|z|)
    conc                1.061    0.170    6.237    0.000
    neuro               0.905    0.120    7.540    0.000
    C1                  1.263    0.142    8.919    0.000
    C2                  1.270    0.148    8.568    0.000
    C3                  1.437    0.162    8.855    0.000
    C4                  1.192    0.160    7.466    0.000
    C5                  1.748    0.217    8.039    0.000
    N1                  1.014    0.146    6.965    0.000
    N2                  1.236    0.161    7.681    0.000
    N3                  1.310    0.165    7.937    0.000
    N4                  1.507    0.170    8.888    0.000
    N5                  2.042    0.221    9.232    0.000


Group 3 [college]:

Latent Variables:
                     Estimate  Std.Err  Z-value  P(>|z|)
  conc =~
```

Example

```
    C1        (.p1.)    0.573    0.027   21.471    0.000
    C2        (.p2.)    0.678    0.029   23.706    0.000
    C3        (.p3.)    0.634    0.028   22.666    0.000
    C4        (.p4.)   -0.897    0.031  -29.235    0.000
    C5        (.p5.)   -0.947    0.036  -26.307    0.000
  neuro =~
    N1        (.p6.)    1.305    0.033   39.285    0.000
    N2        (.p7.)    1.247    0.032   38.701    0.000
    N3        (.p8.)    1.205    0.034   35.309    0.000
    N4        (.p9.)    0.909    0.034   26.982    0.000
    N5        (.10.)    0.837    0.035   23.998    0.000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conc ~
    neuro            -0.278    0.052   -5.354    0.000

Intercepts:
                   Estimate  Std.Err  Z-value  P(>|z|)
    conc             -0.168    0.053   -3.139    0.002
    neuro            -0.092    0.045   -2.056    0.040
    C1        (.26.)    4.571    0.027  170.017    0.000
    C2        (.27.)    4.442    0.029  152.180    0.000
```

Example

```
    C3        (.28.)     4.379    0.028  154.381    0.000
    C4        (.29.)     2.434    0.032   75.721    0.000
    C5        (.30.)     3.186    0.037   85.494    0.000
    N1        (.31.)     2.940    0.039   75.566    0.000
    N2        (.32.)     3.509    0.038   93.494    0.000
    N3        (.33.)     3.224    0.038   83.772    0.000
    N4        (.34.)     3.197    0.035   91.203    0.000
    N5        (.35.)     2.975    0.035   84.171    0.000

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    conc              1.139    0.098   11.634    0.000
    neuro             0.865    0.063   13.807    0.000
    C1                1.178    0.064   18.364    0.000
    C2                1.142    0.065   17.467    0.000
    C3                1.093    0.062   17.713    0.000
    C4                0.952    0.067   14.255    0.000
    C5                1.633    0.100   16.405    0.000
    N1                0.807    0.058   13.850    0.000
    N2                0.820    0.057   14.498    0.000
    N3                1.122    0.068   16.394    0.000
    N4                1.630    0.087   18.807    0.000
    N5                1.882    0.098   19.207    0.000
```

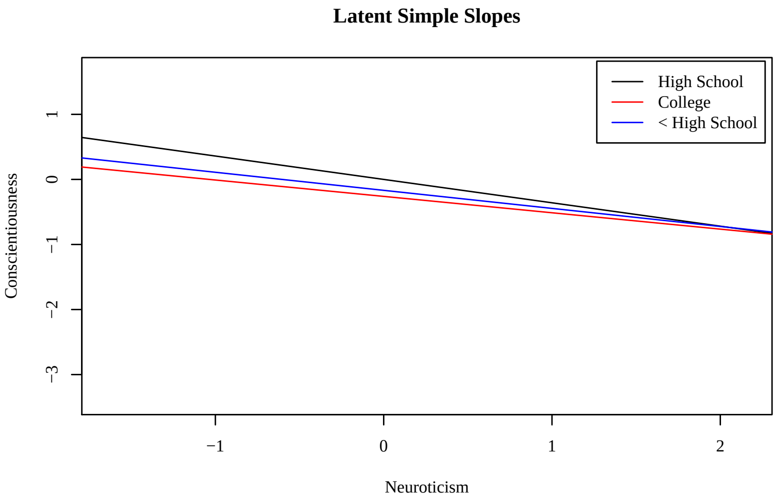
Example

```
## Extract info needed to plot simple slopes:
ints <- c(0,
          coef(fit2)[c("conc~1.g2",
                       "conc~1.g3")]
          )
slopes <- coef(fit2)[c("conc~neuro",
                       "conc~neuro.g2",
                       "conc~neuro.g3")]
fScores <- do.call(rbind, predict(fit2))
```

Example

```
par(family = "serif", cex = 0.75)
plot(y = fScores[ , "conc"],
     x = fScores[ , "neuro"],
     type = "n",
     main = "Latent Simple Slopes",
     xlab = "Neuroticism",
     ylab = "Conscientiousness")
abline(a = ints[1], b = slopes[1])
abline(a = ints[2], b = slopes[2], col = "red")
abline(a = ints[3], b = slopes[3], col = "blue")
legend(x = "topright",
       inset = 0.01,
       legend =
           c("High School",
             "College",
             "< High School"),
       col =
           c("black",
             "red",
             "blue"),
       lty = 1)
```

Example



Latent Simple Slopes
High School
College
< High School
Conscientiousness
1
0
−1
−2
−3
−1
0
1
2
Neuroticism