Structural Equation Modeling

M&S Lecture 2, 2017

Measurement Invariance
and
Path Analysis

Ellen Hamaker
Methodology and Statistics
Faculty of Social and Behavioural Sciences

Outline

- Multiple group factor analysis
- Path analysis

Multiple group factor analysis

Thus far we analyzed the **covariance structure**.

We can also analyze the **mean structure**.

Means may be of interest when:

a) we have **multiple groups**

b) we have **multiple occasions** (longitudinal data)

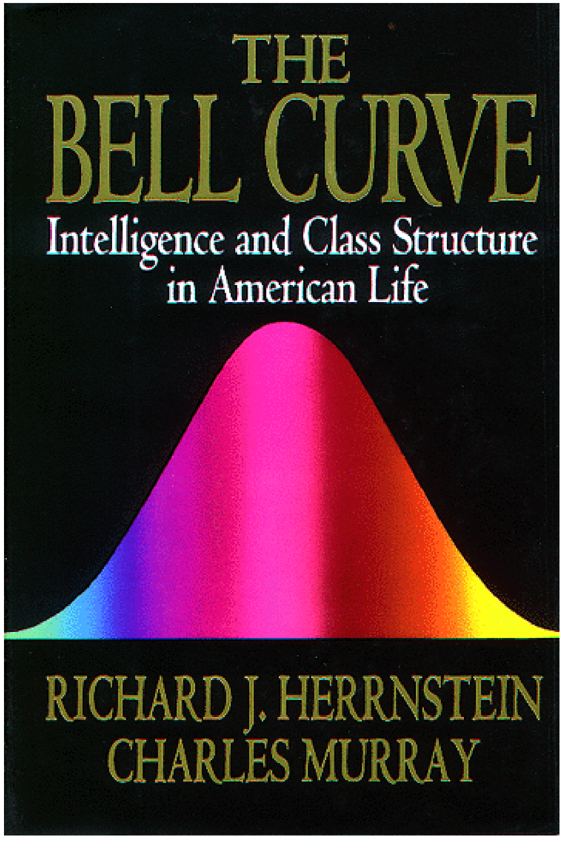
REMEMBER: a latent variable does not have an intrinsic **scale**. We assign it a scale by **scaling** the factor (either through fixing a factor loading, or through fixing its variance).

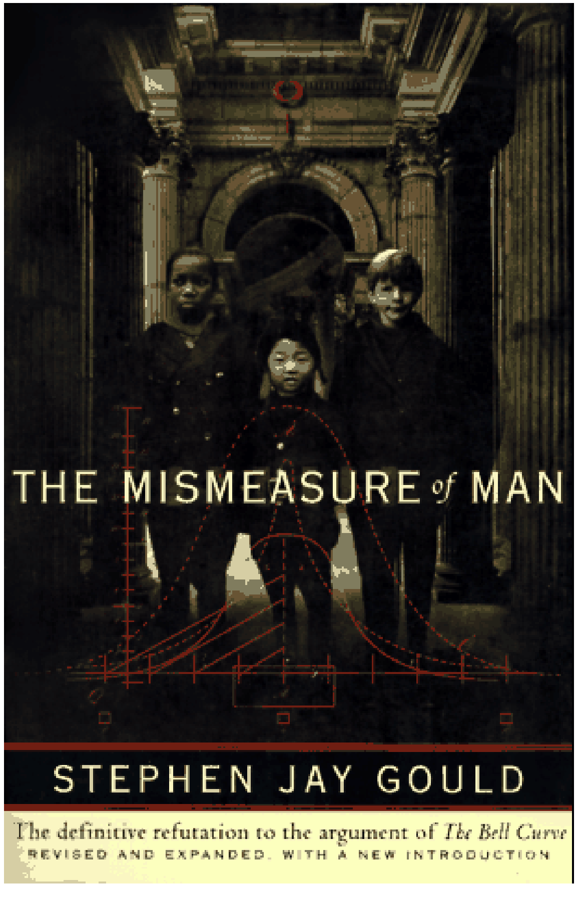
The **latent mean** is also unknown (unidentified).

However, we can estimate the **difference in latent means**, either between groups, or between occasions.

E.g., intelligence research

An important topic in **multiple group CFA** is which of the groups scores higher on the latent variable: This is very important in for instance **intelligence research**.



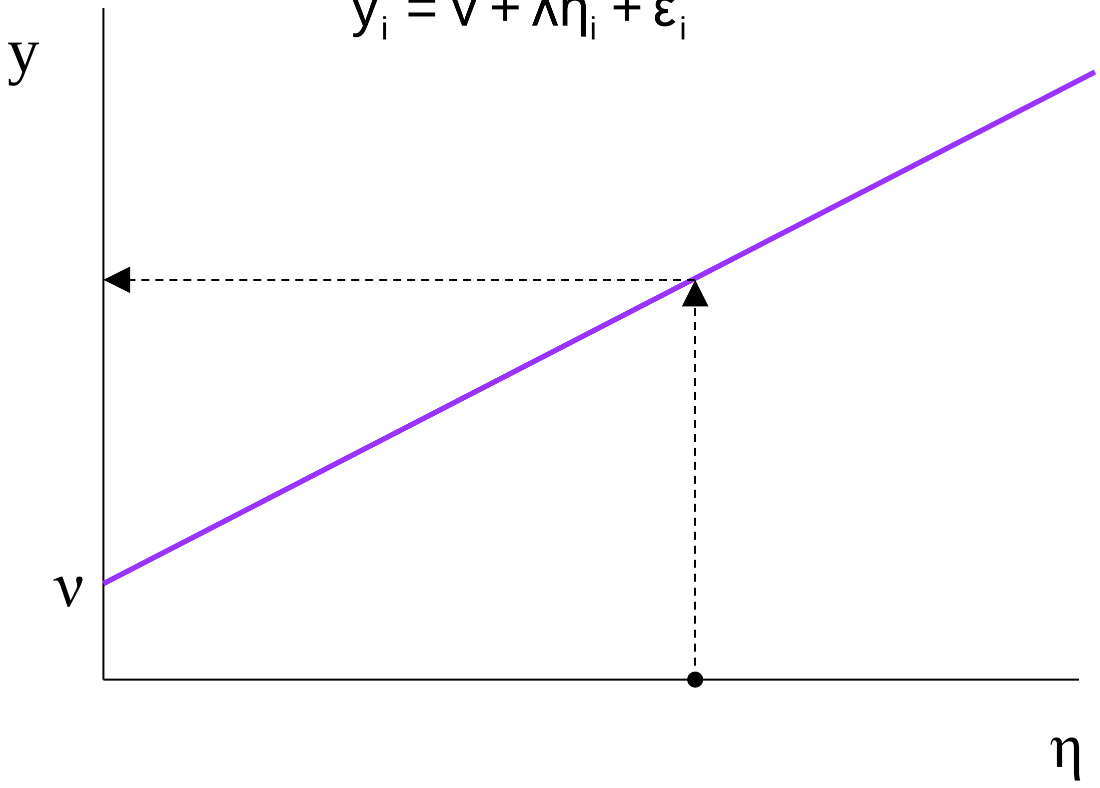


Related to this topic is the question whether a test allows us to make **fair comparisons between individuals from different groups**.

Is a test fair?

In a factor model, the observed variable y is regressed on the latent variable η. Hence, based on some one's standing on the underlying dimension, (s)he has a particular expected score on the observed variable.

$$y_i = \nu + \lambda\eta_i + \varepsilon_i$$

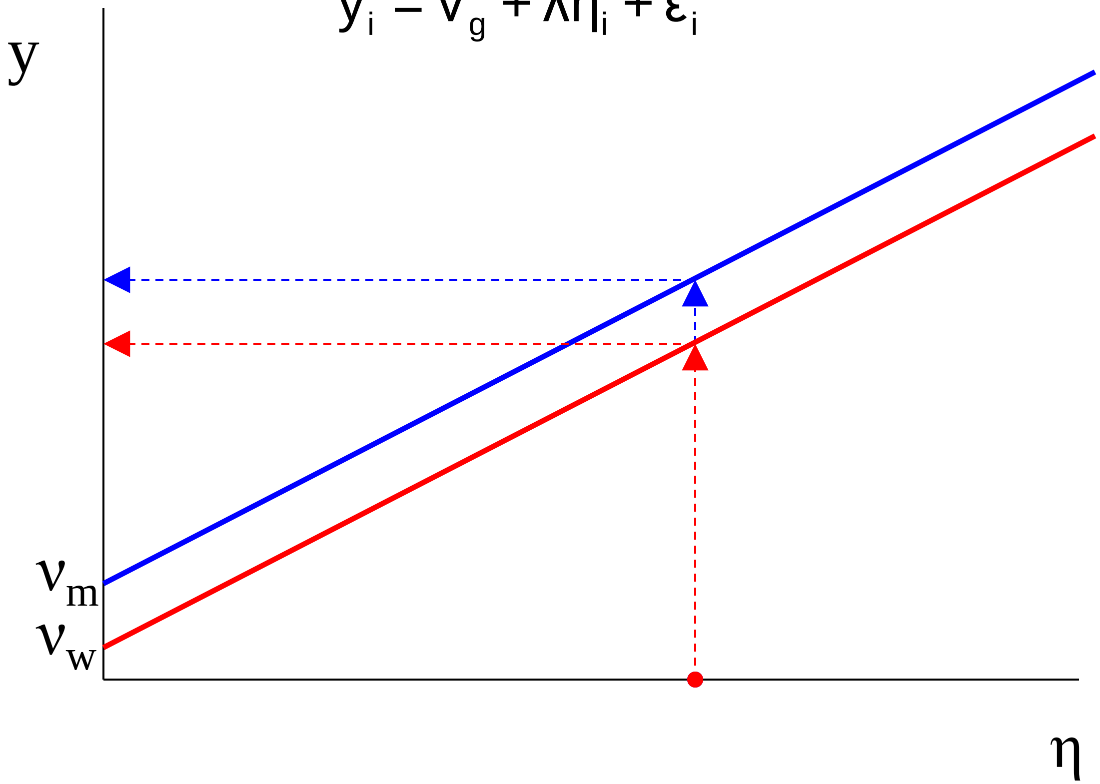


Biased test

Biased implies that two individuals (from **different groups**) with the **same latent score** have **different (expected) observed scores**.

This can be due to a different **intercept**…

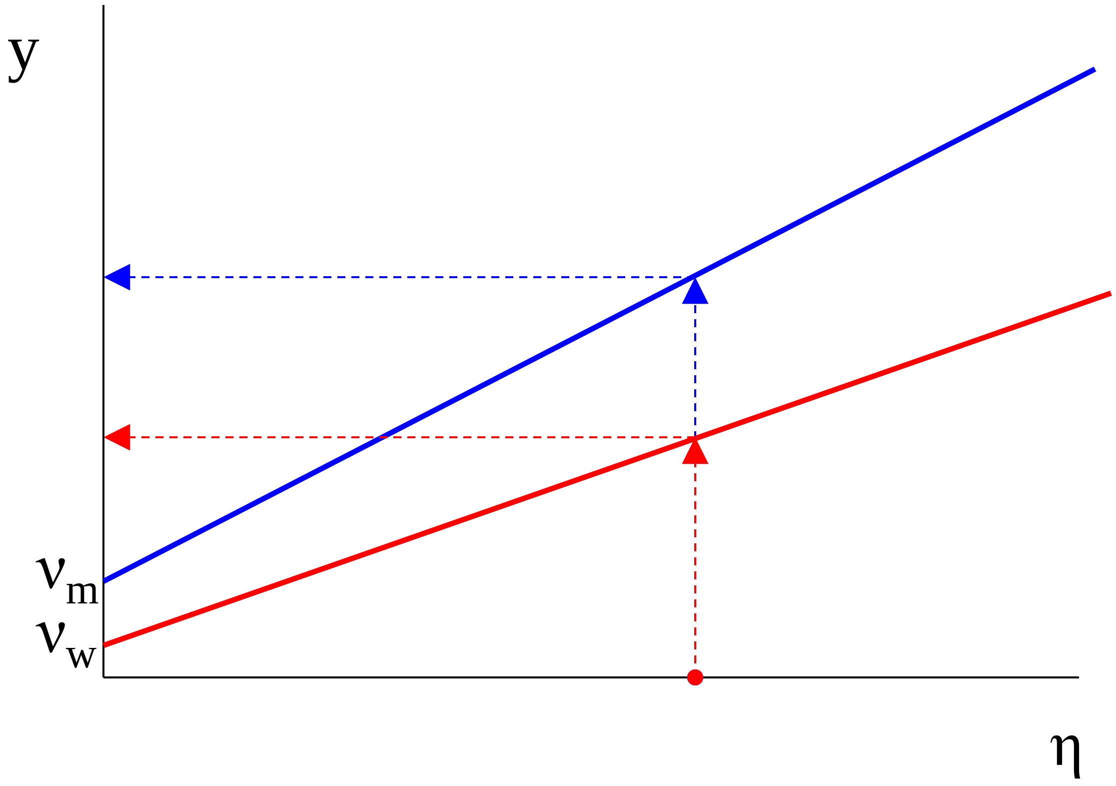
$$y_i = \nu_g + \lambda\eta_i + \varepsilon_i$$



Biased test

… or to a different **slope** (= factor loading):

$$y_i = v_g + \lambda_g \eta_i + \varepsilon_i$$



Covariance and mean structures & likelihood

Measurement equations:

Group 1: $y_i = v_1 + \Lambda_1 \eta_i + \varepsilon_i$

Group 2: $y_i = v_2 + \Lambda_2 \eta_i + \varepsilon_i$

Covariance structures:

Group 1: $\Sigma_1 = \Lambda_1 \Psi_1 \Lambda_1^T + \Theta_1$

Group 2: $\Sigma_2 = \Lambda_2 \Psi_2 \Lambda_2^T + \Theta_2$

Mean structures:

Group 1: $\mu_1 = v_1 + \Lambda_1 \alpha_1$

Group 2: $\mu_2 = v_2 + \Lambda_2 \alpha_2$

Log likelihood: $\log L = \sum_{g=1}^{G} \log L_g$

where:

$$\log L_g = c - \frac{N_g}{2} \log |\boldsymbol{\Sigma}_g| - \frac{N_g}{2} \mathrm{tr}\left(\mathbf{S}_g \boldsymbol{\Sigma}_g^{-1}\right) - \frac{N_g}{2} (m_g - \mu_g)^T \Sigma_g^{-1} (m_g - \mu_g)$$

Measurement invariance

To make sure that the test is **unbiased** (=fair), we have to test whether:

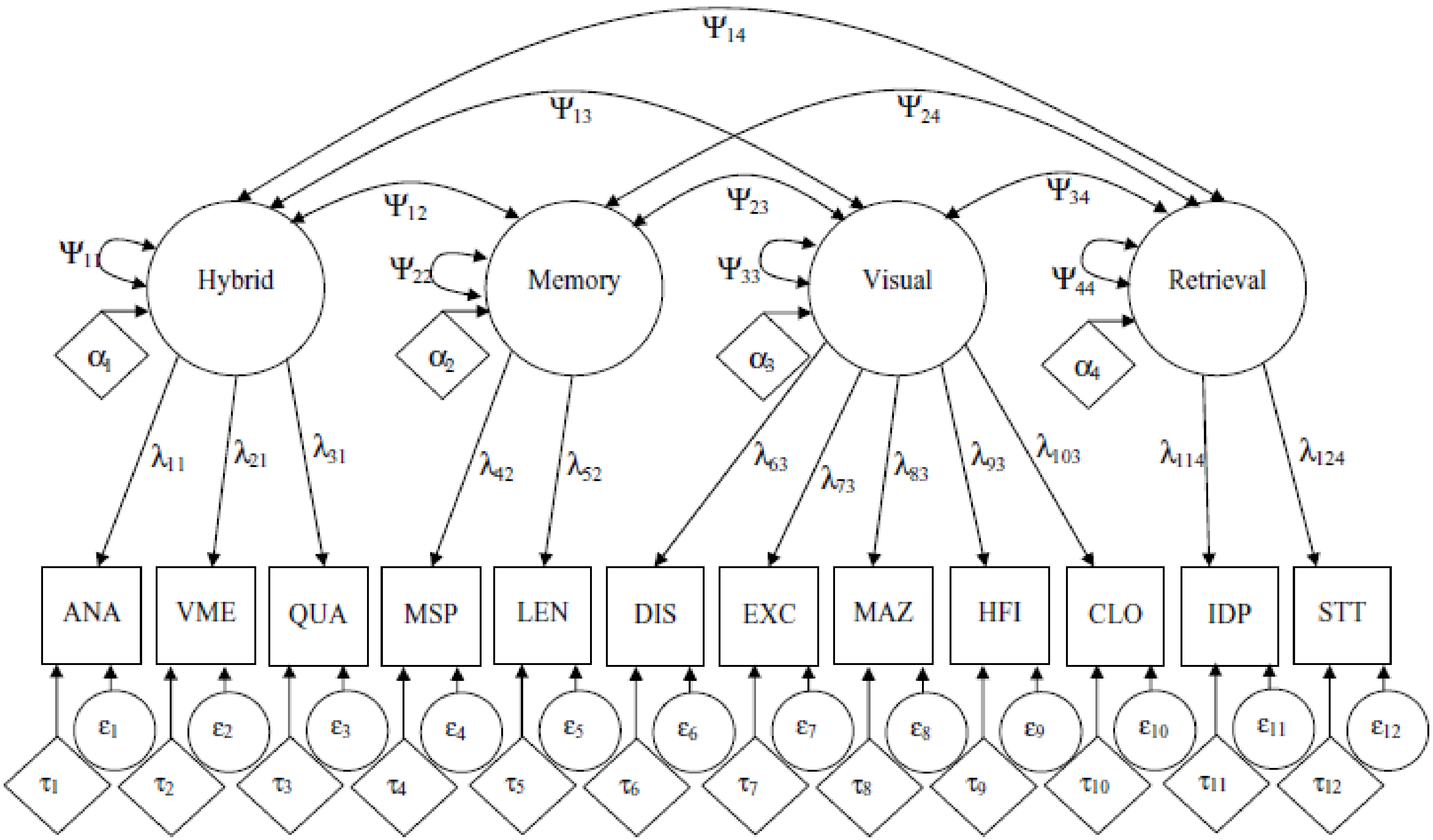
1) **factor loadings** are **identical** across the groups → **weak factorial invariance**: $\Lambda_1 = \Lambda_2$
2) the **intercepts** are **identical** across the groups → **strong factorial invariance**: $v_1 = v_2$

Strong factorial invariance implies that:

- Two people from different groups with the **same latent scores**, have the **same expected observed scores**
- **Group differences in observed means** are the result of **differences in latent means**
- The test allows for a fair comparison between groups, and between people from different groups.

Example: Wicherts & Dolan, 2010

A **4-factor model** for subtests in the RAKIT (an intelligence test):



Sample statistics, parameters, and df

Per group there are 12*13/2 = 78 unique elements in the covariance matrix, and 12 means, so there are 90 sample statistics.

Per group we estimate:

- 12 means
- 12 residual variances
- 4 factor variances
- 6 factor covariances
- 8 factor loadings

So 42 parameters in total.

Hence there are 90-42 = 48 df per group.

Majority group

Specifying this model in Mplus for the **majority group only**:

```
Majority.inp

TITLE:   Majority

DATA:    TYPE IS MEANS COVARIANCE;
         FILE IS majority.dat;
         NOBSERVATIONS ARE 196;


VARIABLE:    NAMES = CLO EXC MSP VME MAZ ANA QUA DIS LEN HFI IDP STT;
             USEVARIABLES ARE ANA VME QUA MSP LEN DIS EXC MAZ HFI CLO IDP STT;

MODEL:       Hybrid BY ANA VME QUA;
             Memory BY MSP LEN;
             Visual BY DIS EXC MAZ HFI CLO;
             Retrieval BY IDP STT;

OUTPUT: SAMPSTAT TECH1 TECH4 MOD(4) STANDARDIZED;
```

NOTE: The statement USEVARIABLES can be used to make a selection of the variables from the original file, or to change the order of the variables.

Oops?

If we run this factor model, Mplus gives the following **warning**:

```
THE MODEL ESTIMATION TERMINATED NORMALLY

WARNING:  THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
DEFINITE.  THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE FOR
LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN TWO LATENT
VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO LATENT VARIABLES.
CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
PROBLEM INVOLVING VARIABLE VISUAL.
```

In the **TECH4 output**, we get the correlations matrix of the latent variables:

ESTIMATED CORRELATION MATRIX FOR THE LATENT VARIABLES

	HYBRID	MEMORY	VISUAL	RETRIEVA
HYBRID	1.000			
MEMORY	0.876	1.000		
VISUAL	0.966	0.677	1.000	
RETRIEVA	0.437	0.418	0.426	1.000

Correlation of almost 1

Model fit majority

```
Chi-Square Test of Model Fit

          Value                                  81.579
          Degrees of Freedom                         48
          P-Value                                0.0018

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                                0.060
          90 Percent C.I.                         0.036  0.082
          Probability RMSEA <= .05                0.224

CFI/TLI

          CFI                                     0.908
          TLI                                     0.874

Chi-Square Test of Model Fit for the Baseline Model

          Value                                 431.961
          Degrees of Freedom                         66
          P-Value                                0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                                   0.051
```

Model fit minority

```
Chi-Square Test of Model Fit

          Value                              71.904
          Degrees of Freedom                     48
          P-Value                            0.0143

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                            0.062
          90 Percent C.I.                     0.028  0.090
          Probability RMSEA <= .05            0.246

CFI/TLI

          CFI                                 0.921
          TLI                                 0.892

Chi-Square Test of Model Fit for the Baseline Model

          Value                             370.034
          Degrees of Freedom                     66
          P-Value                            0.0000

SRMR (Standardized Root Mean Square Residual)

          Value                               0.057
```

Measurement invariance

Models to run & compare:

1. Free model: ***Configural invariance***
2. Constrain **factor loadings** (Λ) across groups: ***Weak factorial invariance***
3. Constrain **intercepts** (ν) across groups (while freeing latent means): ***Strong factorial invariance***

(In addition the residual variances can be contrained across the groups. This is called ***strict factorial invariance***: $f(y|\eta,g) = f(y|\eta)$. Note that there can still be differences between the groups (in α and Ψ).)

Testing for weak factorial invariance

There is some discussion in the literature about whether this procedure is correct: Some claim that if (accidently) you use a biased indicator for scaling, this procedure does not work…

However:

- If there are **no biased indicators** (e.g., strong factorial invariance holds), it does work
- If there are biased indicators, it does not matter whether you use **a biased or an unbiased** indicator for scaling: These models are **statistically indentical** (i.e., they have the same fit, and thus lead to same chi-square difference tests)

Alternative 1: Just imposes the constraints, and look at the Modification Indices when the model does not fit (top-down procedure; cf. Byrne).

Alternative 2: Test each factor loading separately, using different factor loadings for scaling (not doable when there are many indicators…)

Model 1: Overruling the Mplus defaults

MajMin1.inp

```
TITLE:   Majority and minority

DATA:    NGROUPS = 2;
         TYPE IS MEANS COVARIANCE;
         FILE IS majmin.dat;
         NOBSERVATIONS ARE 196 131;


VARIABLE:   NAMES = CLO EXC MSP VME MAZ ANA QUA DIS LEN HFI IDP STT;
            USEVARIABLES ARE ANA VME QUA MSP LEN DIS EXC MAZ HFI CLO IDP STT;

MODEL:      Hybrid BY ANA VME QUA;
            Memory BY MSP LEN;
            Visual BY DIS EXC MAZ HFI CLO;
            Retrieval BY IDP STT;

MODEL G2:   Hybrid BY ANA VME QUA;
            Hybrid BY ANA@1;
            Memory BY MSP LEN;
            Memory BY MSP@1;
            Visual BY DIS EXC MAZ HFI CLO;
            Visual BY DIS@1;
            Retrieval BY IDP STT;
            Retrieval BY IDP@1;
            [Hybrid-Retrieval@0];
            [ANA - STT];

OUTPUT: SAMPSTAT TECH1 TECH4 MOD(4) STANDARDIZED;
```

This is needed to **overrule the defaults**: Mplus automatically imposes the constraints for **strong factorial invariance** (i.e., equal factor loadings and intercepts). Note we also need to **scale the factors** in the second group!

Model 1: Configural Invariance

Write down the expressions for the covariance matrices and mean vectors of both groups.

Model 1: Model fit of both groups

```
Chi-Square Test of Model Fit

          Value                                  153.483
          Degrees of Freedom                          96
          P-Value                                 0.0002

Chi-Square Contributions From Each Group

          G1                                      81.579
          G2                                      71.904

RMSEA (Root Mean Square Error Of Approximation)

          Estimate                                 0.061
          90 Percent C.I.                          0.042  0.078
          Probability RMSEA <= .05                 0.163

CFI/TLI

          CFI                                      0.914
          TLI                                      0.882
…

SRMR (Standardized Root Mean Square Residual)

          Value                                    0.053
```

NOTE: these values are the same as from the separate analyses (so there are absolutely no constraints across the two groups); they sum to the chi-square of this model

BTW: Another very useful way of checking your model and seeing where the constraints and defaults are, is by looking at the **TECH1 output!**

Model 2: Equal factor loadings

In each group, 8 factor loadings were estimated. Hence, constraining these across groups gives us 8 df, such that model 2 should have 96 + 8 = 104 df.

MajMin2.inp

```
TITLE:   Majority and minority

DATA:    NGROUPS = 2;
         TYPE IS MEANS COVARIANCE;
         FILE IS majmin.dat;
         NOBSERVATIONS ARE 196 131;


VARIABLE:    NAMES = CLO EXC MSP VME MAZ ANA QUA DIS LEN HFI IDP STT;
             USEVARIABLES ARE ANA VME QUA MSP LEN DIS EXC MAZ HFI CLO IDP STT;

MODEL:       Hybrid BY ANA VME QUA;
             Memory BY MSP LEN;
             Visual BY DIS EXC MAZ HFI CLO;
             Retrieval BY IDP STT;

MODEL G2:    [Hybrid-Retrieval@0];
             [ANA - STT];


OUTPUT: SAMPSTAT TECH1 TECH4 MOD(4) STANDARDIZED;
```

Now we only overrule the defaults for the means: Fix latent means to zero (@0), and free the intercepts

Model 2: Weak Factorial Invariance

Write down the expressions for the covariance matrices and mean vectors of both groups.

Model 2: Equal factor loadings

```
Chi-Square Test of Model Fit

          Value                                 158.124
          Degrees of Freedom                        104
          P-Value                                0.0005
```

Chi-square difference between this model and the previous model is:

158 – 153 = 5, with 8 df, which is **not significant**.

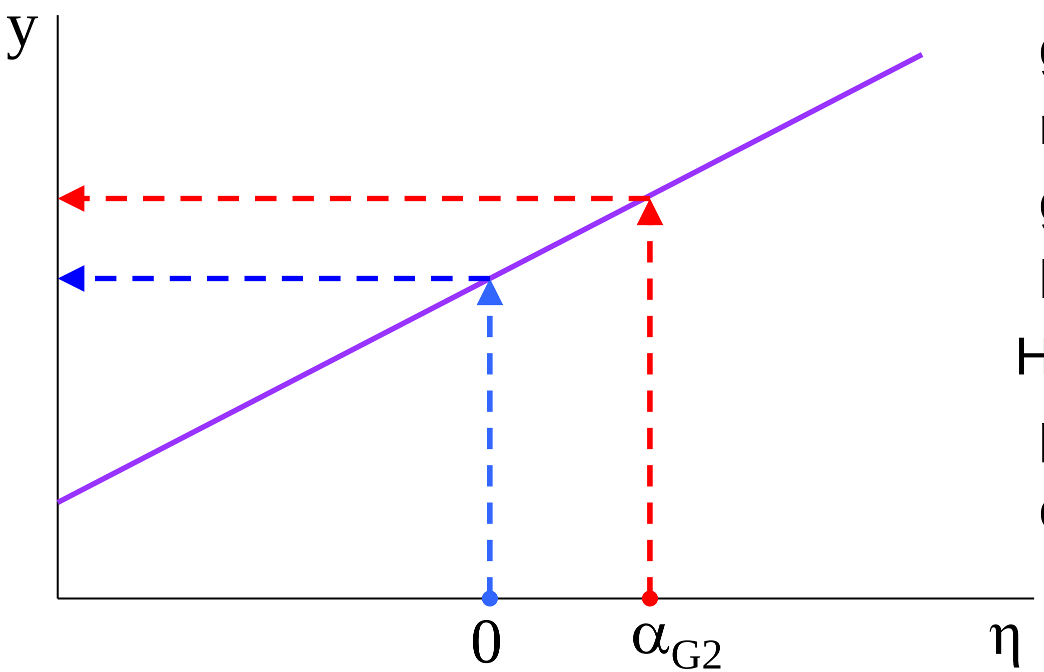
Hence, we do not have to reject the **null model** (Model 2 with equal factor loadings) in favor of the **alternative model** (Model 1 without constraints).

Put differently: We can **assume there are equal factor loadings**.

Model 3: Equal intercepts and factor loadings

In each group, 12 intercepts were estimated: if we would constrain these, we would be saying that the means across the two groups are equal.

However, what we want to say is that the **differences in observed means** are completely explained by **mean differences on the latent variables**.



Therefore, we **constrain the intercepts across the groups**, and **free the latent means in one of the groups**: these represent the **latent mean differences**.

Hence we have: 12 – 4 = 8 parameters less in Model 3 compared to Model 2.

Model 3: Strong Factorial Invariance

Write down the expressions for the covariance matrices and mean vectors of both groups.

Nesting of model 2 and model 3

At first it may be confusing that in model 3 we **constrain the intercepts** AND **free latent means** in the second group: How does this lead to **nested models**?

It helps to write down the **modeled mean differences** between the two groups in each model.

In **model 2** we have: $\mu_2 - \mu_1 = \nu_2 - \nu_1$

This requires **12 additional parameters** (in addition to the 12 mean parameters in group 1).

In **model 3** we have: $\mu_2 - \mu_1 = \nu + \Lambda\alpha_2 - \nu = \Lambda\alpha_2$

This requires only **4 additional parameters** (in addition to the 12 mean parameters in group 1).

Model 3: Equal intercepts and factor loadings

This is the **default model that Mplus will fit** when doing a multiple group analysis: hence, the specification is easy:

MajMin3.inp

```
TITLE:   Majority and minority

DATA:    NGROUPS = 2;
         TYPE IS MEANS COVARIANCE;
         FILE IS majmin.dat;
         NOBSERVATIONS ARE 196 131;


VARIABLE:    NAMES = CLO EXC MSP VME MAZ ANA QUA DIS LEN HFI IDP STT;
             USEVARIABLES ARE ANA VME QUA MSP LEN DIS EXC MAZ HFI CLO IDP STT;

MODEL:       Hybrid BY ANA VME QUA;
             Memory BY MSP LEN;
             Visual BY DIS EXC MAZ HFI CLO;
             Retrieval BY IDP STT;

OUTPUT: SAMPSTAT TECH1 TECH4 MOD(4) STANDARDIZED;
```

Model 3: Equal intercepts and factor loadings

```
Chi-Square Test of Model Fit

          Value                               230.660
          Degrees of Freedom                      112
          P-Value                              0.0000
```

Chi-square difference between this model and the previous model is:

231 – 158 = 73, with 8 df, which is **significant**.

Hence, we **cannot** impose the constraint of equal intercepts.

There is no strong factorial invariance!

To detect **sources of bias**, we look at the **modification indices**: they indicate how much the model would improve if we free this parameter.

Model 3: Check MIs for intercepts

```
MODINDICES (ALL).

Minimum M.I. value for printing the modification index     20.000

                                 M.I.      E.P.C.  Std E.P.C.   StdYX
  E.P.C.
Group G1


BY Statements

VISUAL    BY VME                21.475    -0.980     -2.917       -0.522

Means/Intercepts/Thresholds

[ VME      ]                    42.546     0.471      0.471        0.084
[ QUA      ]                    24.607    -0.430     -0.430       -0.089

Group G2


Means/Intercepts/Thresholds

[ VME      ]                    42.551    -9.159     -9.159       -1.773
[ QUA      ]                    24.609     5.078      5.078        1.006
```

Model 3 adjusted for bias: Check MIs again

Including: `MODEL G2: [VME];`

Model fit:

`Value`	`185.151`
`Degrees of Freedom`	`111`
`P-Value`	`0.0000`

Compared to model 2: 185 – 158 = 27 with 7 df, p=.0003

Modification indices now indicate largest change for:

[MSP]	17.769	-0.488	-0.488	-0.102
[LEN]	17.770	0.138	0.138	0.027

Note it is **no longer** for the intercept of QUA!

That is because **MIs change**, if you change the model.

Model 3 further adjusted for bias

Including: `MODEL G2: [LEN];`

Model fit:

```
Value                          166.477
Degrees of Freedom                 110
P-Value                         0.0004
```

Compared to model 2: 166 – 158 = 8 with 6 df, p=.238

Now we have a model which has:

- equal factor loadings
- equal intercepts except for VME and LEN (to account for bias)

This allows us to **compare the latent means**.

Model 3 further adjusted for bias

Remember: we fixed the latent means in group 1 (majority) to zero, and estimated the latent means in group 2 (minority), which thus represent the **difference in latent means**.

```
Means
    HYBRID            -4.184      0.472     -8.866      0.000
    MEMORY            -0.626      0.635     -0.986      0.324
    VISUAL            -4.155      0.464     -8.962      0.000
    RETRIEVAL         -4.414      0.556     -7.945      0.000
```

Conclusion: Minority group scores on average significantly lower on Hybrid, Visual and Retrieval.

In addition, we should check the **intercepts that were not constrained**, to determine the direction of bias.

Nesting

It is not easy to see that Model 3 is **nested under** Model 2:

a) we are **not only constraining** parameters (intercepts) to be equal

b) **but also freeing** parameters (latent means)

This is called the **Reference-Group method**.

There are two alternative methods (which are **statistically equivalent**!):

the **Marker-variable method** and the **Effects-coding method**.

Marker-variable method

Model 1: Configural invariance

- fix **one of the intercepts** per factor to **zero** in each group (for mean structure); estimate all other intercepts and latent means freely in each group

- fix **one of the factor loadings** per factor to **one** in each group (for covariance structure); estimate all other factor loadings freely in each group

Model 2: Weak factorial invariance

- constrain factor loadings across the groups

Model 3: Strong factorial invariance

- constrain intercepts across the groups

Advantage: nesting is more obvious

Disadvantage: requires to overrule many defaults in Mplus

Effects-coding method

Model 1: Configural invariance

- intercepts are constrained to sum to zero per group; estimate latent means freely per group
- average factor loading is 1

Model 2: Weak factorial invariance

- constrain factor loadings across the groups

Model 3: Strong factorial invariance

- constrain intercepts across the groups

Advantage: latent and observed variables have a comparable scale

Disadantage: makes no sense when the observed variables have very different scales, and it requires overruling defaults in Mplus

Outline

- Multiple group factor analysis
- **Path analysis**

And now for something completely different

Thus far, we have focused on **factor analysis**; another form of SEM is **path analysis.**

Path analysis is closely related to **regression analysis**: observed variables are used to predict other observed variable.

In path analysis, we may have:

- **more than one outcome variable**
- a variable that is both regressed on predictors, and a predictor of other variables (i.e., **be a mediator**).

It is also possible to do **path analysis with latent variables**.

Effect of corporal punishment

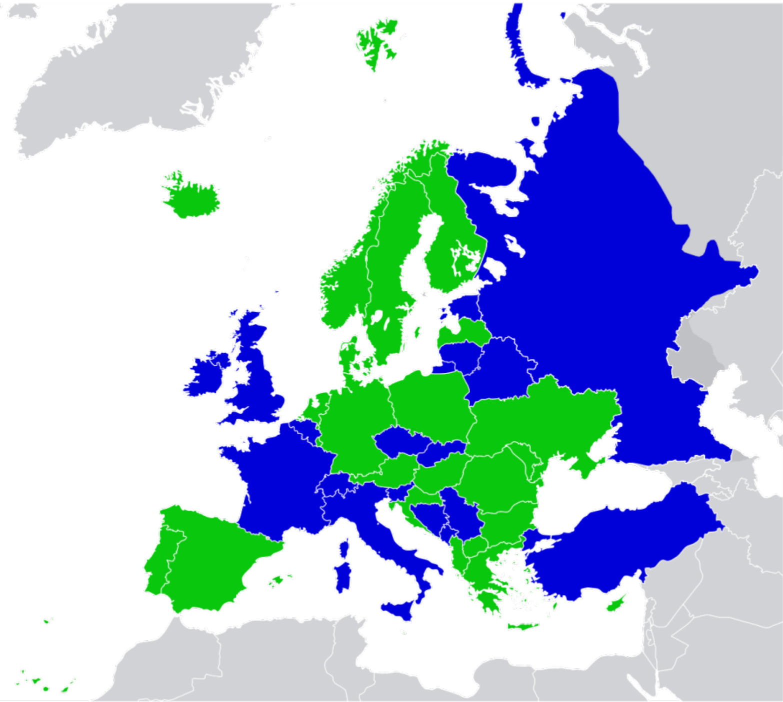
Corporal punishment is: the deliberate infliction of pain as retribution for an offence, or for the purpose of disciplining or reforming a wrongdoer or to change an undesirable attitude or behavior.

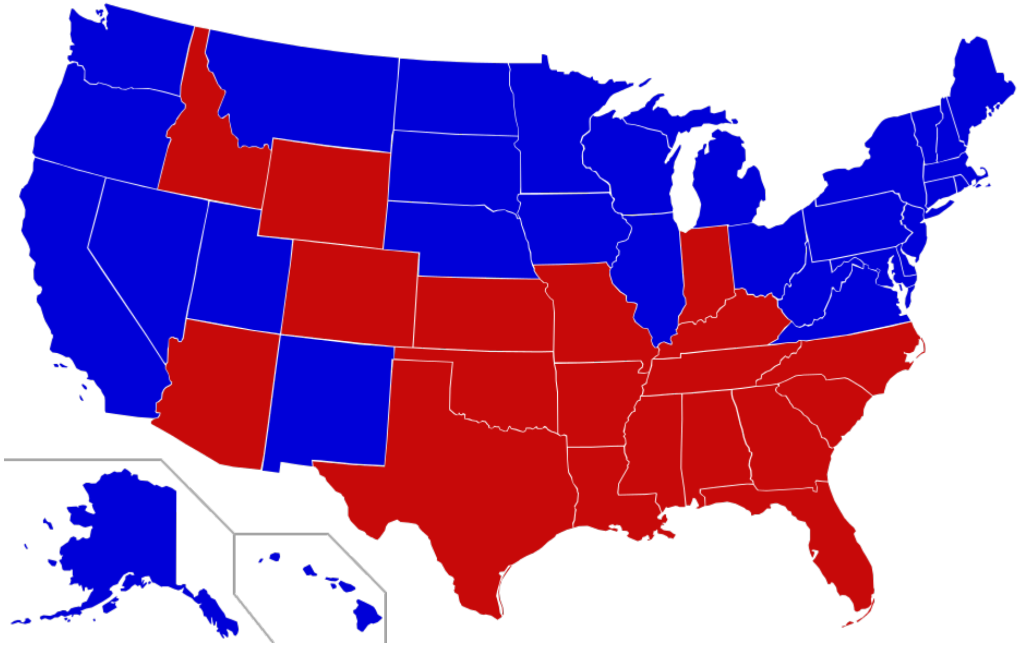
We are interested in how **corporal punishment** influences children's **psychological maladjustment**.

Sample: 175 children between the ages of 8 and 18.

Rohner, R. P., Bourque, S. L., & Elordi, C. A. (1996). Children's perceptions of corporal punishment, caretaker acceptance, and psychological adjustment in a poor, biracial southern community. *Journal of Marriage and the Family, 58*, 842-852.

Laws on corporal punishment





- Corporal punishment prohibited in **schools** and the **home**
- Corporal punishment prohibited in **schools** only
- Corporal punishment **not** prohibited

Research question: mediation

Specifically, is the effect of perceived harshness and perceived justness of physical punishment on children's psychological maladjustment **mediated** by perceived caretaker's rejection?

Variables:

Perceived harshness:	0 = never punished physically in any way
	16 = punished more than 12 times a week, very hard
Perceived justness:	2 = very unfair and almost never deserved
	8 = very fair and almost always deserved
Perceived rejection:	my mother does not really love me
	my mother ignores me as long as I do not bother her
	my mother goes out of her way to hurt my feelings
Psych. maladjustment:	higher score implies more problems

Mediation model: path diagram

 # sample statistics:

4*5/2 = 10

 # free parameters:

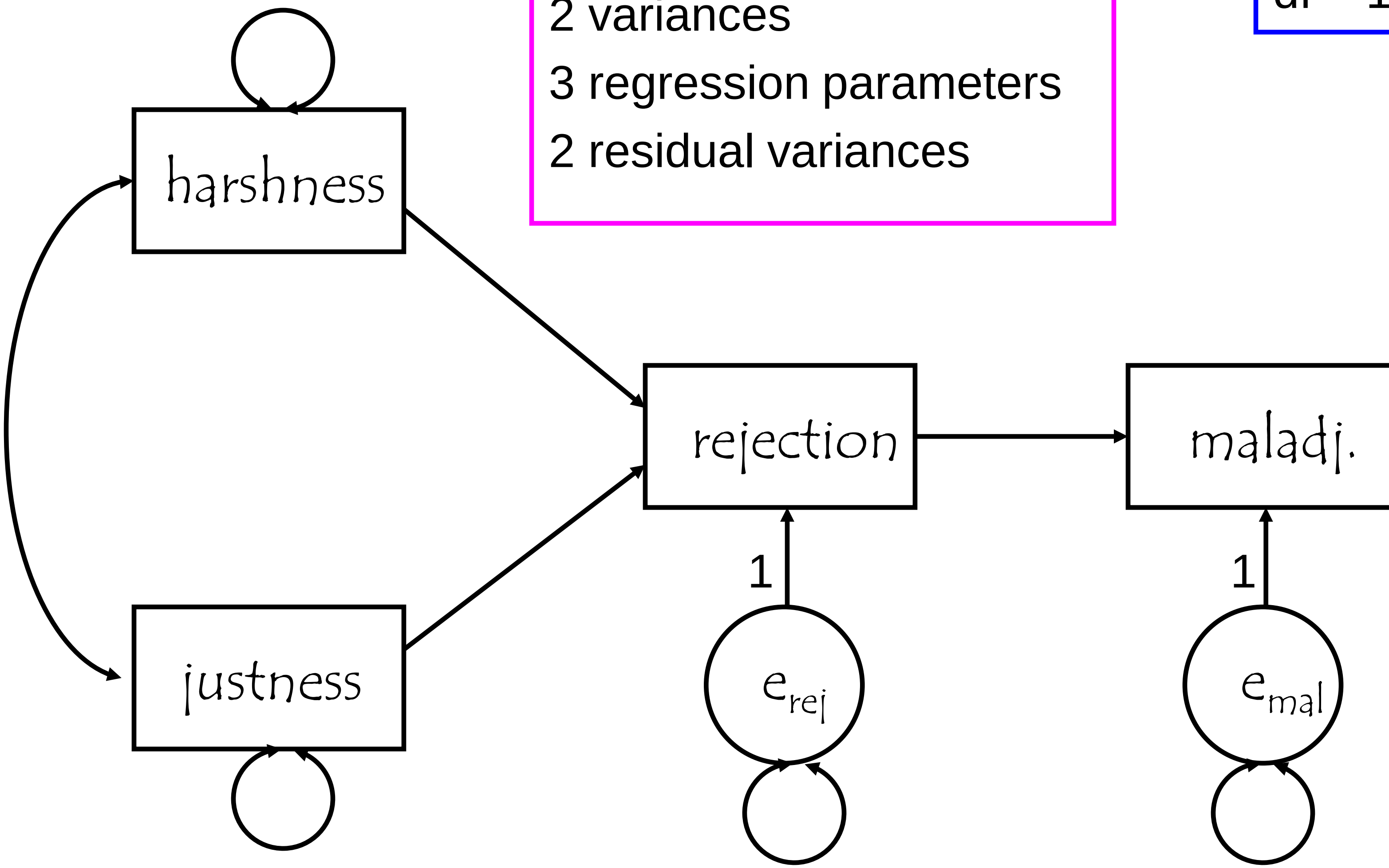
1 covariance

2 variances

3 regression parameters

2 residual variances

df = 10-8 = 2



DIY

How can you get the **measurement equation** to be:

$$\mathbf{y}_i = \nu + \Lambda \eta_i + \varepsilon_i$$
$$= \eta_i$$

DIY

Write down the **structural equation** for this model:

$$\eta_i = \alpha + B\eta_i + \zeta_i$$

DIY

The current expression for η_i is **recursive**. Rewrite it, such that η_i only appears on the left-hand side.

$$\eta_i = \alpha + B\eta_i + \zeta_i$$

Full structural equation model:

Measurement equation: $\mathbf{y}_i = \nu + \Lambda\eta_i + \varepsilon_i$

Structural equation: $\eta_i = \alpha + B\eta_i + \zeta_i$

Mean structure: $\mu = \nu + \Lambda(I - B)^{-1}\alpha$

Covariance structure: $\Sigma = \Lambda(I - B)^{-1}\Psi(I - B)^{-1T}\Lambda^T + \Theta$

Model specification

CorPun.inp

```
TITLE:       Path model for mediation

DATA:        FILE IS CorPun.dat;
             TYPE IS COVA;
             NOBSERVATIONS ARE 175;

VARIABLE:  NAMES ARE harsh just reject malad;

MODEL:      harsh WITH just;
            ! NOTE: If you do not specify this covariance,
            ! it is still being estimated, but not reported
            reject ON harsh just;
            malad ON reject;

MODEL INDIRECT:
            malad IND reject just;
            malad IND reject harsh;

OUTPUT:     Tech1 Standardized MODINDICES (0);
```

Model fit

```
Chi-Square Test of Model Fit
          Value                                 1.546
          Degrees of Freedom                        2
          P-Value                              0.4616

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                              0.000
          90 Percent C.I.                       0.000  0.139
          Probability RMSEA <= .05              0.598

CFI/TLI
          CFI                                   1.000
          TLI                                   1.012

SRMR (Standardized Root Mean Square Residual)
          Value                                 0.023
```

Parameter estimates

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
REJECT ON				
HARSH	2.607	0.584	4.462	0.000
JUST	-4.395	0.763	-5.758	0.000
MALAD ON				
REJECT	0.387	0.051	7.658	0.000
HARSH WITH				
JUST	-0.259	0.262	-0.987	0.324
Variances				
HARSH	4.514	0.483	9.354	0.000
JUST	2.645	0.283	9.354	0.000
Residual Variances				
REJECT	268.220	28.674	9.354	0.000
MALAD	158.956	16.993	9.354	0.000

NOTE: this parameter is only **reported** because we explicitly asked Mplus for it

Standardized results

Mplus gives 3 forms of standardized results when requested:

- **STDYX Standardization**: uses the variances of the continuous latent variables as well as the variances of the background (=exogenous) and outcome (endogenous) variables
- **STDY Standardization**: uses the variances of the continuous latent variables and the variances of the outcome (endogenous) variables; this should be used if one has a binary (=dichotomous) predictor
- **STD Standardization**: uses the variances of the continuous latent variables for standardization

Here we use STDYX; we could also specifically ask for this with:

```
OUTPUT: STDYX
```

Indirect (=mediated) effects

There is **no direct path** from harshness to maladjustment, but there is an **indirect path** (mediated through rejection). Idem for justness.

```
TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

                                                         Two-Tailed
                    Estimate       S.E.  Est./S.E.        P-Value

Effects from JUST to MALAD
  Sum of indirect     -1.700      0.369     -4.602          0.000
  Specific indirect
    MALAD
    REJECT
    JUST              -1.700      0.369     -4.602          0.000

Effects from HARSH to MALAD
  Sum of indirect      1.009      0.262      3.855          0.000
  Specific indirect
    MALAD
    REJECT
    HARSH              1.009      0.262      3.855          0.000
```

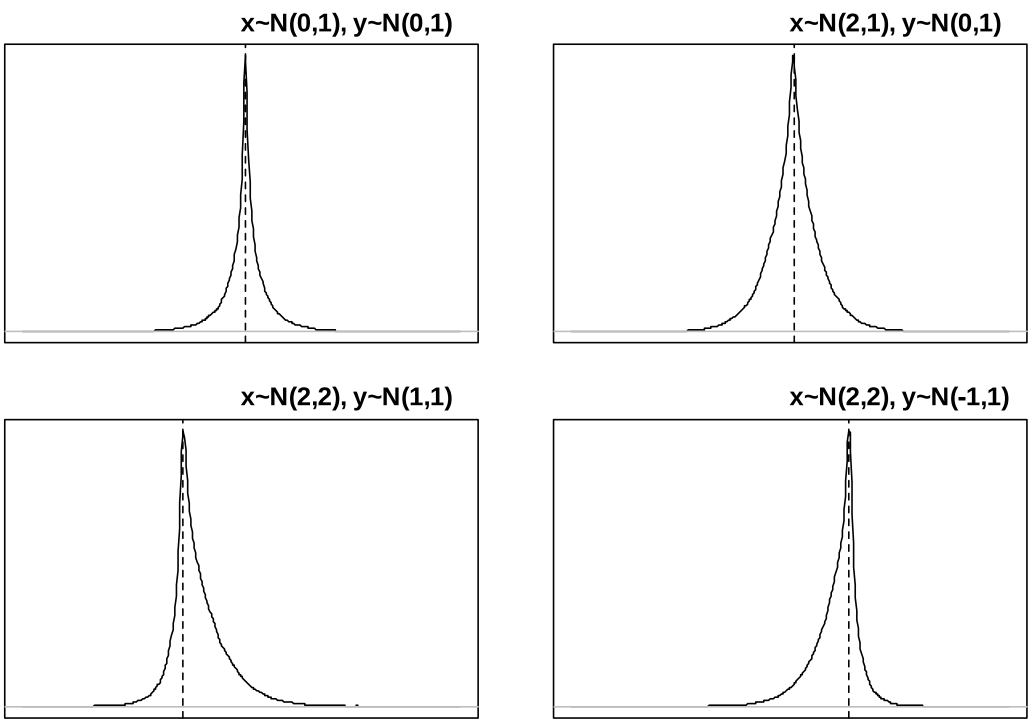
Standardized indirect effects

```
STANDARDIZED TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND
DIRECT EFFECTS

STDYX Standardization

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value
Effects from JUST to MALAD
  Sum of indirect     -0.190      0.039     -4.881      0.000
  Specific indirect
    MALAD
    REJECT
    JUST              -0.190      0.039     -4.881      0.000


Effects from HARSH to MALAD
  Sum of indirect      0.147      0.037      4.012      0.000
  Specific indirect
    MALAD
    REJECT
    HARSH              0.147      0.037      4.012      0.000
```

Cautionary note on testing indirect effects

All the tests for parameters and functions of parameters (e.g., indirect effects, R-square) are **Wald tests** and are thus based on the **assumption of normality.**

A mediation effect is a **product** of two (or more) parameters. Each parameter has a normal sampling distribution. Hence, the indirect effect is the product of two (or more) normally distributed variables.



This is not normal!

As a result, the p-value you obtain with the Wald test is **incorrect** (may be too small or too large).

Solution: Bootstrapping

Bootstrapping is based on **resampling** from your data: For each bootstrap sample the model is estimated, including the **indirect effect**.

If we repeat this procedure many times (e.g., 1000 times), we obtain **1000 estimates** of the indirect effect, and this is used to determine the **95% confidence interval**.

Use:

```
ANALYSIS: BOOTSTRAP = 1000;
OUTPUT:    Cinterval
```

See for details: Mplus User's Guide, p. 548-549 and Example 3.16.

(Note: bootstrapping requires the raw data.)

Multiple mediators

If you have **multiple mediators**, you may be interested in:

- **total** mediated effect
- **specific** mediated effect (through a particular mediator)

Mplus gives both.

When you have **multiple paths that include the same mediator**, you may be interested in all these paths simultaneously, or separately:

- **simultaneously**: y VIA mediator x
- **separately**: y IND mediator x

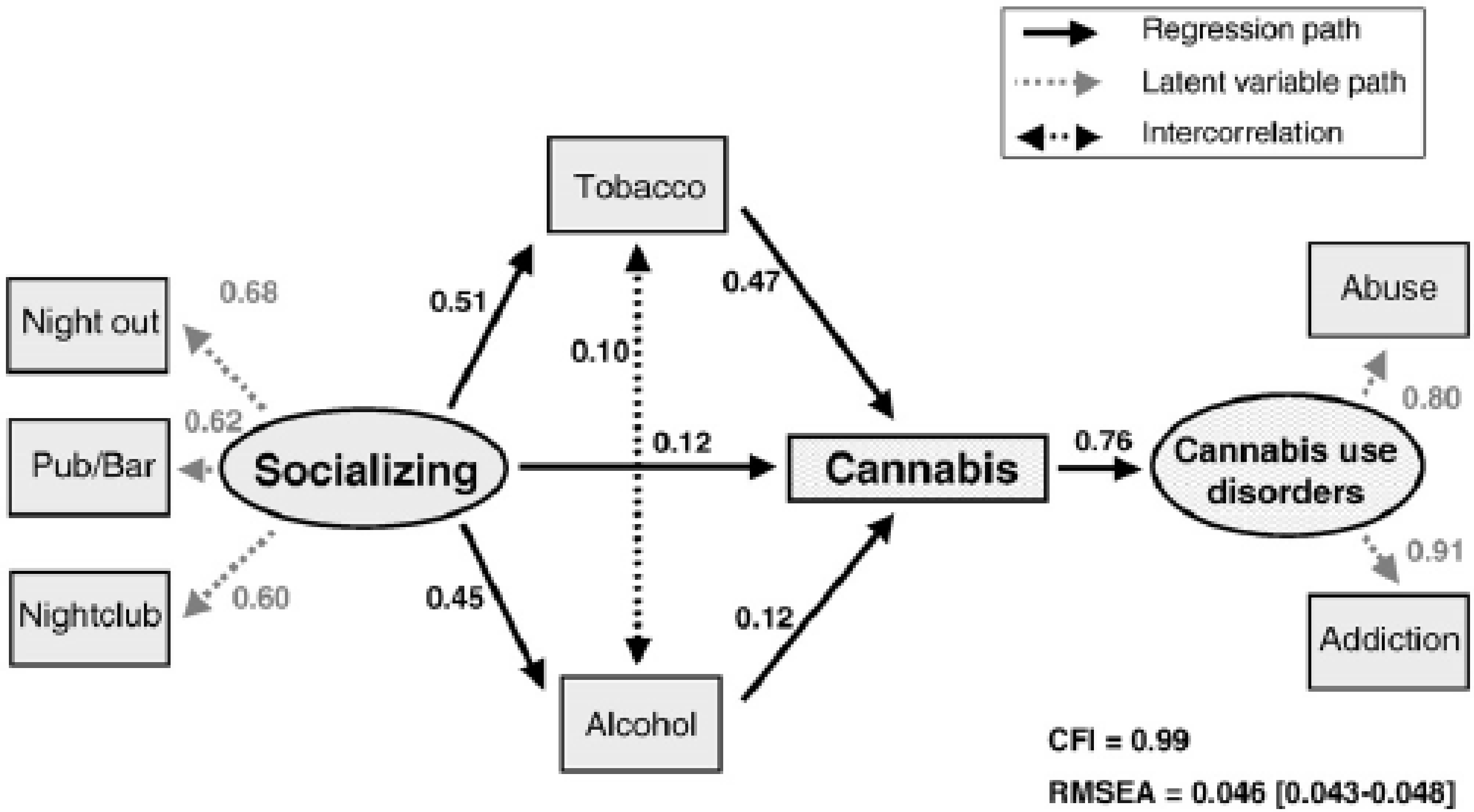
Take home message

- Crucial issue in multiple group CFA is **measurement invariance**
- Mplus imposes constraints for **strong factorial invariance**; note that this does not imply these constraints hold
- **You** have to test each constraint separately, using nested models and chi-square difference tests
- In multiple group analysis, the **mean structure** is of interest (in addition to the **covariance structure**)
- If a test/questionnaire is biased, you can **adjust** your model for this, and still **compare the groups in a fair & meaningful way**
- But when **too many items/subtests** are **biased**, it is hard/impossible to adjust for this.

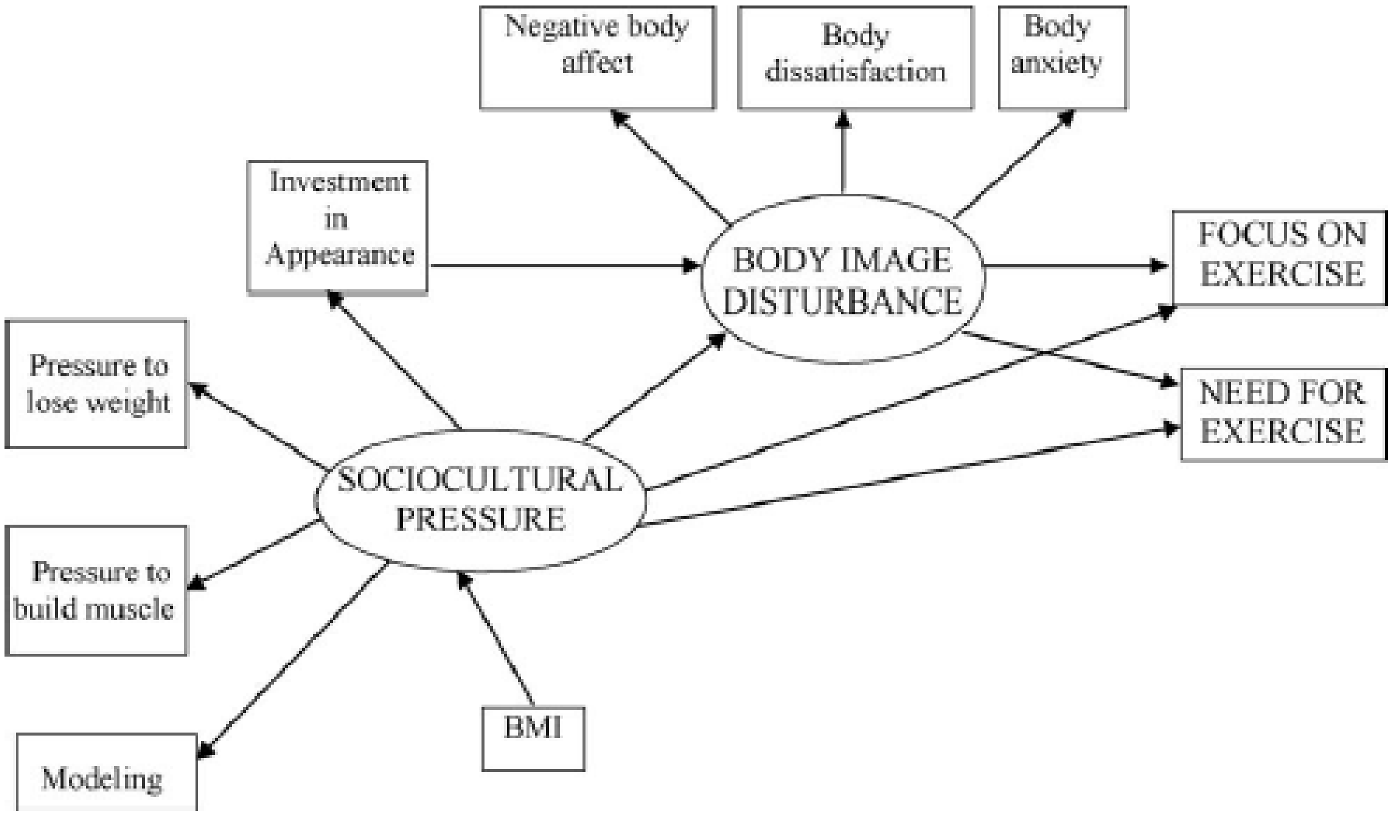
Take home message

- **Path analysis** is another form of SEM
- It requires the use of the **structural equation**, which allows variables to be **regressed on each other** (using the **matrix B**)
- The measurement equation is only used to **transfer** the observed y variables to the structural equation (i.e., there is no “real” measurement model, since $\eta = y$)
- You may consider **different routes of mediation** in you model
- To test whether **indirect (i.e., mediated) effects** are significant, using **confidence intervals based on bootstrapping** should be preferred over the (standard) Wald test
- **Factor analysis (i.e., latent variables)** and **path analysis (i.e., structural relationships)** can also be combined in SEM

Cannabis use disorders



Body image and exercise



Emotional intelligence and life satisfaction