Lecture 10: Introduction to Conditional Process Analysis

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TEXAS TECH UNIVERSITY

Outline

- Conceptual introduction to conditional process analysis
- Define conditional direct and indirect effects
- Examples of some basic conditional process models
- Run through some basic examples of conditional process models

Starting Point

So far, we've been discussing mediation and moderation as independent hypotheses.

- With mediation, we're interested in describing the chain of events by which X influences Y.
  - We want to model the process by which X affects Y.
  - We're asking questions about *how* X impacts Y.
- With moderation, we're interested in discovering how the relation between X and Y changes as a function of some moderating variable (or set of moderating variables).
  - We want to know at what levels of the moderator is the $X \rightarrow Y$ relation statistically significant.
  - We're asking questions about *when* X affects Y.

Conditional Process Analysis

We can combine the *how*-type questions answer by mediation models and the *when*-type questions answered by moderation analysis via *conditional process analysis*.

- With conditional process analysis, we're interested in assessing how an indirect effect (i.e., a process) changes as a function of some set of moderating variables (i.e., is conditional on those moderators).
- We want to estimate *conditional indirect (direct) effects*.
- This type of model is often called *moderated mediation*.

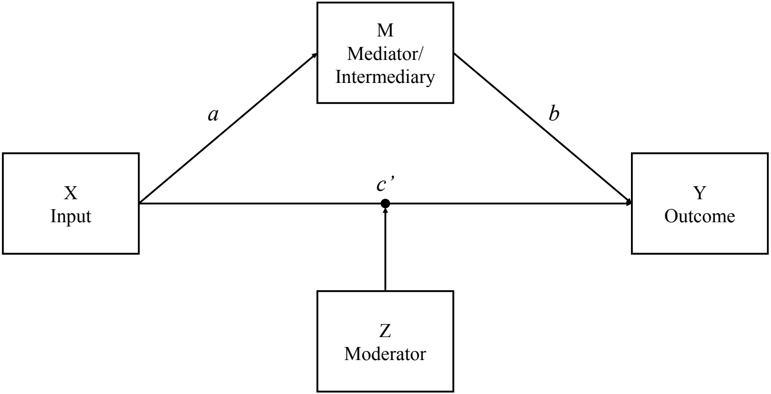
Conditional Process Analysis

We can also ask questions about how the effect of an interaction term is transmitted through a mediator to the focal outcome.

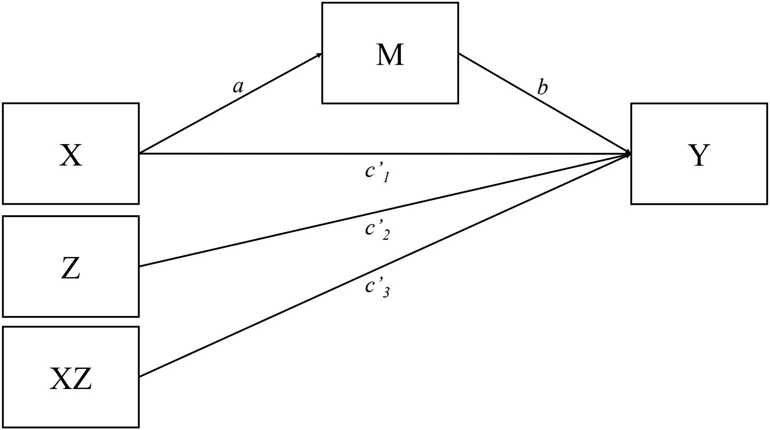
- This type of model is called *mediated moderation*.
- It turns out that mediated moderation is mathematically equivalent to moderated mediation.
- Mediated moderation is, nearly always, impossible to interpret (more on that later).

Simplest Example

The simplest example of a conditional process model includes only the direct effect as conditional:



The preceding conceptual diagram corresponds to the following analytic diagram:



Simplest Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'_1 X + c'_2 Z + c'_3 XZ + e_Y \tag{1}$$

$$M = i_2 + aX + e_M \tag{2}$$

Simplest Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'_1X + c'_2Z + c'_3XZ + e_Y \tag{1}$$
$$M = i_2 + aX + e_M \tag{2}$$

In this simple case, the indirect effect is not conditional, so it is defined as before:

$$IE = ab$$

Simplest Example

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$$Y = i_1 + bM + c'_1 X + c'_2 Z + c'_3 XZ + e_Y \tag{1}$$

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$$IE = ab$$

The direct effect, on the other hand, must be interpreted as conditional on Z.

Simplest Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'_1 X + c'_2 Z + c'_3 XZ + e_Y \tag{1}$$

$$M = i_2 + aX + e_M \tag{2}$$

In this simple case, the indirect effect is not conditional, so it is defined as before:

$$IE = ab$$

The direct effect, on the other hand, must be interpreted as conditional on Z.

We can rearrange Equation 1 to get:

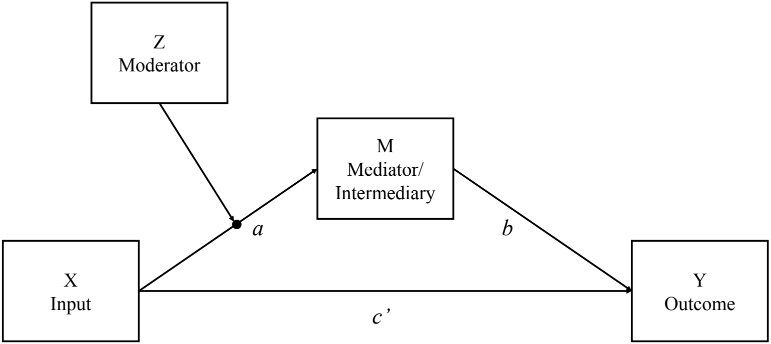
$$Y = i_1 + bM + c'_2 Z + \left(c'_1 + c'_3 Z\right) X + e_Y$$

The conditional direct effect is the simple slope linking X to Y:

$$DE = c'_1 + c'_3 Z$$

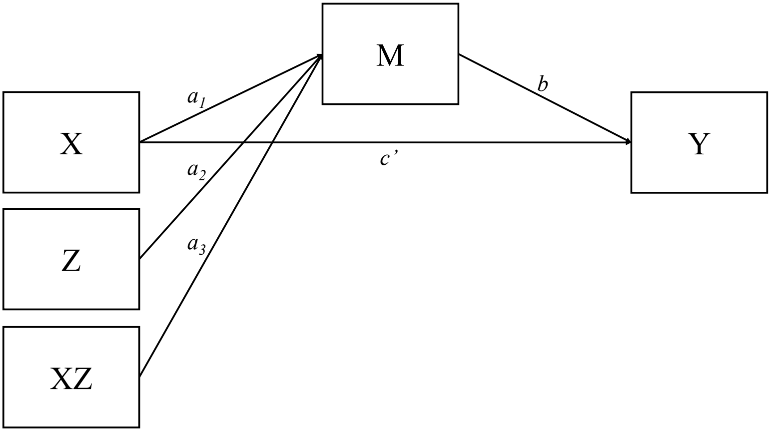
Another Example

A somewhat more interesting example of a conditional process model includes a conditional indirect effect induced by moderation of the a path:



Another Example

The preceding conceptual diagram corresponds to the following analytic diagram:



Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'X + e_Y \tag{3}$$

$$M = i_2 + a_1X + a_2Z + a_3XZ + e_M \tag{4}$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'X + e_Y \tag{3}$$

$$M = i_2 + a_1X + a_2Z + a_3XZ + e_M \tag{4}$$

In this case, the direct effect is now unconditional, so it is defined as in simple mediation analysis:

$$DE = c'$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'X + e_Y \tag{3}$$

$$M = i_2 + a_1X + a_2Z + a_3XZ + e_M \tag{4}$$

In this case, the direct effect is now unconditional, so it is defined as in simple mediation analysis:

$$DE = c'$$

Now, the indirect effect must be interpreted as conditional on Z due to the a path being moderated by Z.

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'X + e_Y \tag{3}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{4}$$

In this case, the direct effect is now unconditional, so it is defined as in simple mediation analysis:

$$DE = c'$$

Now, the indirect effect must be interpreted as conditional on Z due to the a path being moderated by Z.

We can rearrange Equation 4 to get:

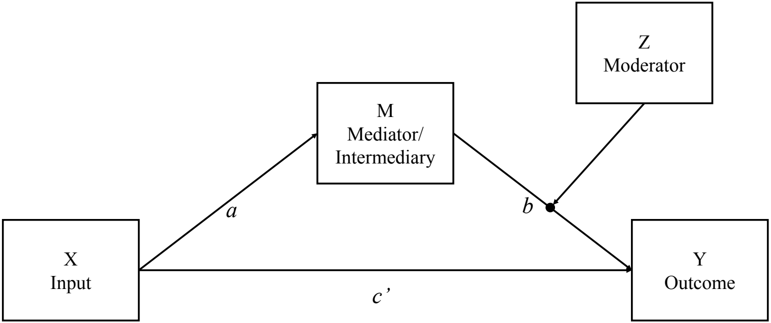
$$M = i_2 + a_2 Z + (a_1 + a_3 Z) X + e_M$$

The conditional indirect effect is now defined by the following product:

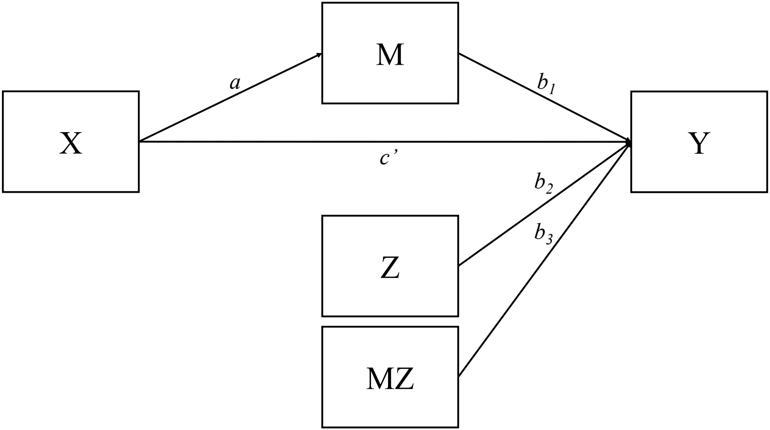
$$IE = (a_1 + a_3 Z) b$$

Another Example

The conditional indirect effect can also be induced by moderation of the b path:



The preceding conceptual diagram corresponds to the following analytic diagram:



Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'X + b_1 M + b_2 Z + b_3 MZ + e_Y \tag{5}$$

$$M = i_2 + aX + e_M \tag{6}$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'X + b_1 M + b_2 Z + b_3 MZ + e_Y \quad (5)$$

$$M = i_2 + aX + e_M \quad (6)$$

As above, the direct effect is unconditional:

$$DE = c'$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'X + b_1M + b_2Z + b_3MZ + e_Y \tag{5}$$

$$M = i_2 + aX + e_M \tag{6}$$

As above, the direct effect is unconditional:

$$DE = c'$$

Again, the indirect effect is conditional on Z due to the b path being moderated by Z.

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'X + b_1 M + b_2 Z + b_3 MZ + e_Y \tag{5}$$

$$M = i_2 + aX + e_M \tag{6}$$

As above, the direct effect is unconditional:

$$DE = c'$$

Again, the indirect effect is conditional on Z due to the b path being moderated by Z.

We can rearrange Equation 5 to get:

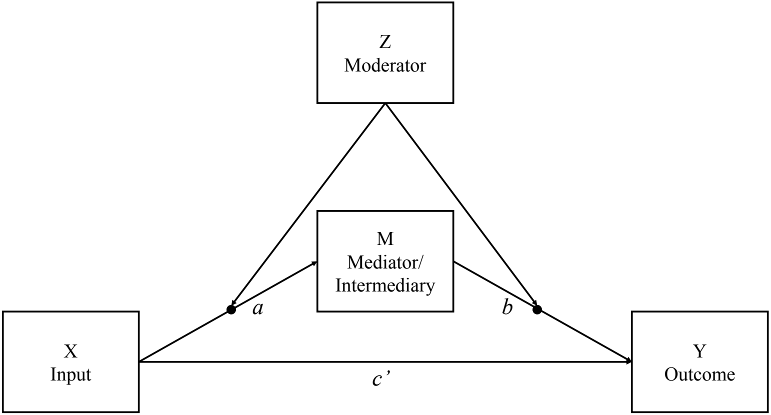
$$Y = i_1 + b_2 Z + (b_1 + b_3 Z) M + e_Y$$

So, the conditional indirect effect is defined by the following product:

$$IE = a(b_1 + b_3 Z)$$

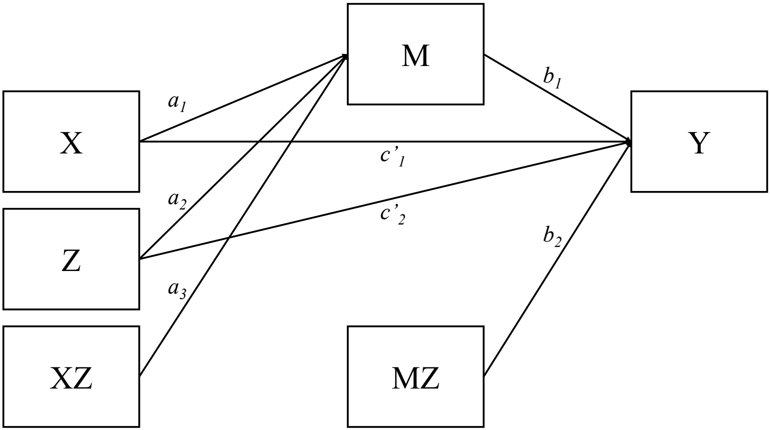
Another Example

Maybe, we have a conditional indirect effect because Z moderates both the a and b paths:



Another Example

The preceding conceptual diagram corresponds to the following analytic diagram:



Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'_1 X + c'_2 Z + b_1 M + b_2 MZ + e_Y \tag{7}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{8}$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'_1 X + c'_2 Z + b_1 M + b_2 MZ + e_Y \tag{7}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{8}$$

The direct effect is still unconditional:

$$DE = c'_1$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c_1'X + c_2'Z + b_1M + b_2MZ + e_Y \tag{7}$$

$$M = i_2 + a_1X + a_2Z + a_3XZ + e_M \tag{8}$$

The direct effect is still unconditional:

$$DE = c_1'$$

The indirect effect is conditional on Z due to the a and b paths being moderated by Z.

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'_1 X + c'_2 Z + b_1 M + b_2 MZ + e_Y \tag{7}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{8}$$

The direct effect is still unconditional:

$$DE = c'_1$$

The indirect effect is conditional on Z due to the a and b paths being moderated by Z.

We can rearrange Equations 7 and 8 to get:

$$Y = i_1 + c'_2 Z + (b_1 + b_2 Z) M + e_Y$$

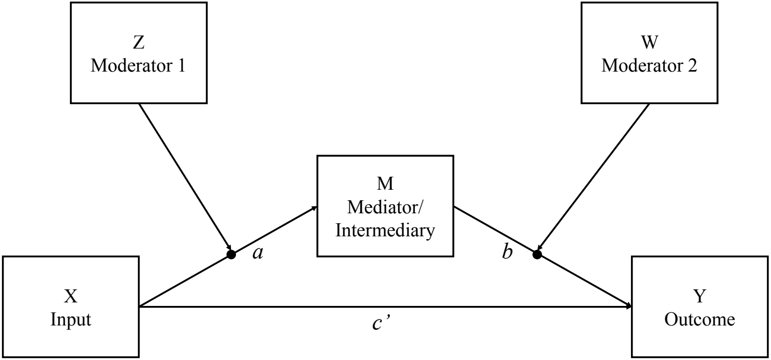
$$M = i_2 + a_2 Z + (a_1 + a_3 Z) X + e_M$$

So, the conditional indirect effect is defined by the following product:

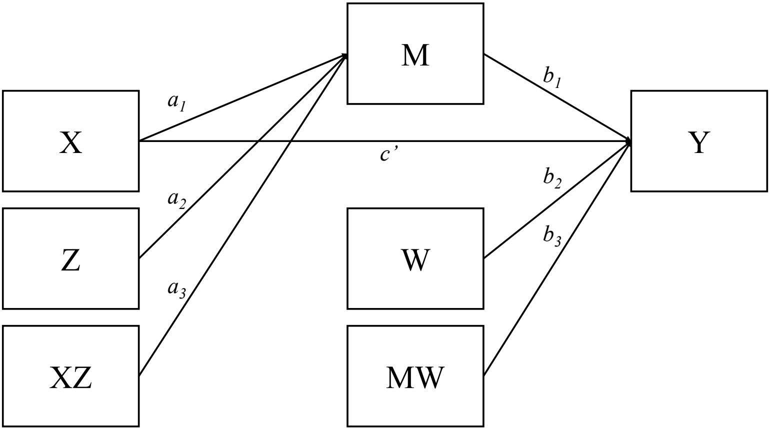
$$IE = (a_1 + a_3 Z)(b_1 + b_2 Z)$$

Another Example

A conditional indirect effect can arise when the a and b paths are moderated by separate variables:



The preceding conceptual diagram corresponds to the following analytic diagram:



Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'X + b_1 M + b_2 W + b_3 MW + e_Y \tag{9}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{10}$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'X + b_1 M + b_2 W + b_3 MW + e_Y \tag{9}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{10}$$

The direct effect is still unconditional:

$$DE = c'$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'X + b_1M + b_2W + b_3MW + e_Y \tag{9}$$

$$M = i_2 + a_1X + a_2Z + a_3XZ + e_M \tag{10}$$

The direct effect is still unconditional:

$$DE = c'$$

The indirect effect is now conditional on both Z and W since these variables moderate the a and b paths, respectively.

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + c'X + b_1 M + b_2 W + b_3 MW + e_Y \tag{9}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{10}$$

The direct effect is still unconditional:

$$DE = c'$$

The indirect effect is now conditional on both Z and W since these variables moderate the a and b paths, respectively.

We can rearrange Equations 9 and 10 to get:

$$Y = i_1 + b_2 W + (b_1 + b_3 W) M + e_Y$$

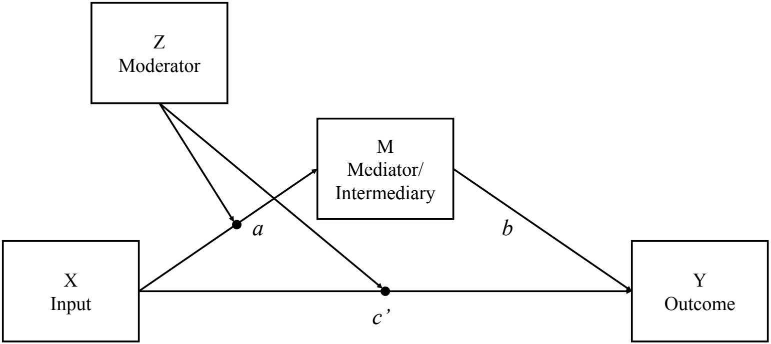
$$M = i_2 + a_2 Z + (a_1 + a_3 Z) X + e_M$$

So, the conditional indirect effect is defined by the following product:

$$IE = (a_1 + a_3 Z)(b_1 + b_3 W)$$

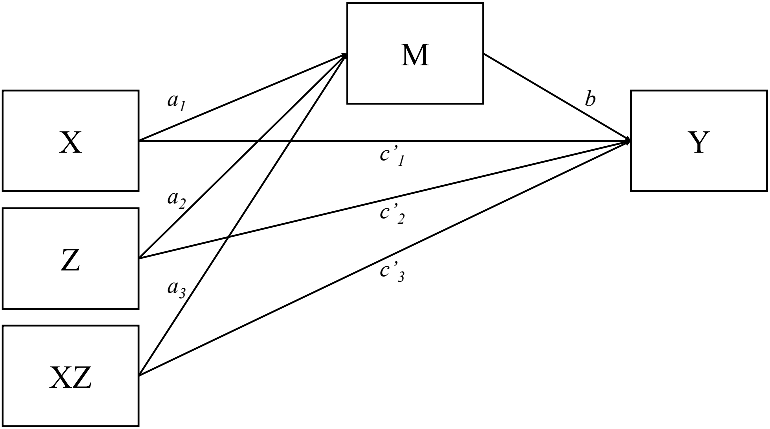
Another Example

We could have conditional indirect and direct effects due to moderation by a single variable:



Another Example

The preceding conceptual diagram corresponds to the following analytic diagram:



Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'_1 X + c'_2 Z + c'_3 XZ + e_Y \tag{11}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{12}$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'_1 X + c'_2 Z + c'_3 XZ + e_Y \tag{11}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{12}$$

Both the direct and indirect effects are now conditional on Z since it moderates the a and c' paths.

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + bM + c'_1X + c'_2Z + c'_3XZ + e_Y \tag{11}$$
$$M = i_2 + a_1X + a_2Z + a_3XZ + e_M \tag{12}$$

Both the direct and indirect effects are now conditional on Z since it moderates the a and c' paths.

We can rearrange Equations 11 and 12 to get:

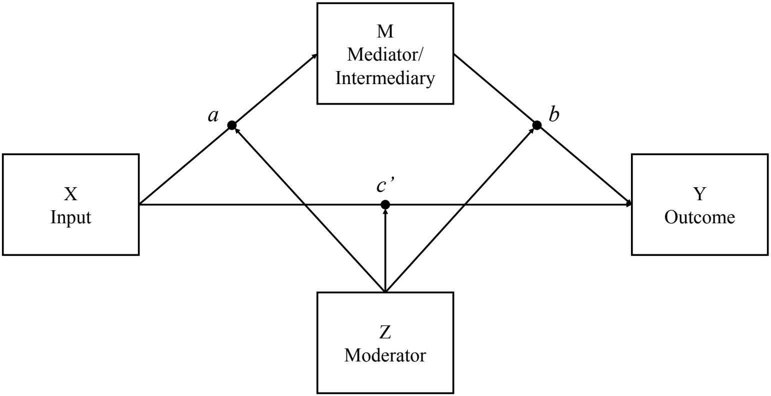
$$Y = i_1 + bM + c'_2Z + \left(c'_1 + c'_3Z\right)X + e_Y$$
$$M = i_2 + a_2Z + \left(a_1 + a_3Z\right)X + e_M$$

So, the conditional direct and indirect effects are defined by the following:

$$DE = c'_1 + c'_3Z$$
$$IE = \left(a_1 + a_3Z\right)b$$

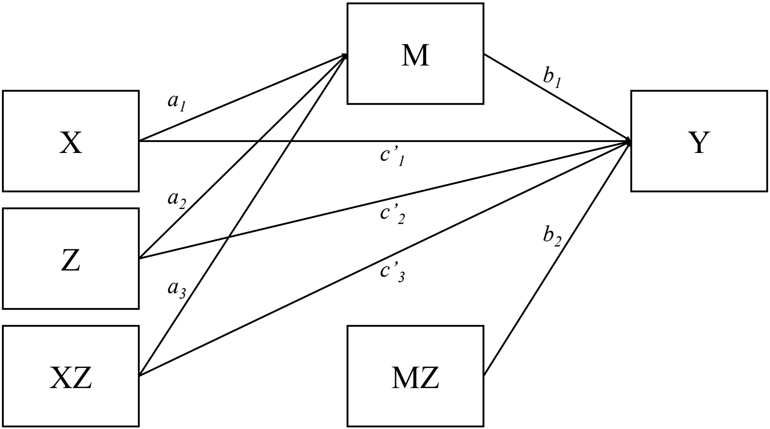
Another Example

We could have one moderator of the a, b, and c' paths inducing conditional indirect and direct effects:



Another Example

The preceding conceptual diagram corresponds to the following analytic diagram:



Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + b_1 M + c'_1 X + c'_2 Z + b_2 MZ + c'_3 XZ + e_Y \tag{13}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{14}$$

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + b_1 M + c_1' X + c_2' Z + b_2 MZ + c_3' XZ + e_Y \tag{13}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{14}$$

Both the direct and indirect effects are again conditional on Z since it moderates the a, b, and c' paths.

Another Example

This analytic diagram implies the following equations:

$$Y = i_1 + b_1 M + c'_1 X + c'_2 Z + b_2 MZ + c'_3 XZ + e_Y \tag{13}$$

$$M = i_2 + a_1 X + a_2 Z + a_3 XZ + e_M \tag{14}$$

Both the direct and indirect effects are again conditional on Z since it moderates the a, b, and c' paths.

We can rearrange Equations 13 and 14 to get:

$$Y = i_1 + c'_2 Z + (b_1 + b_2 Z) M + \left(c'_1 + c'_3 Z\right) X + e_Y$$

$$M = i_2 + a_2 Z + (a_1 + a_3 Z) X + e_M$$

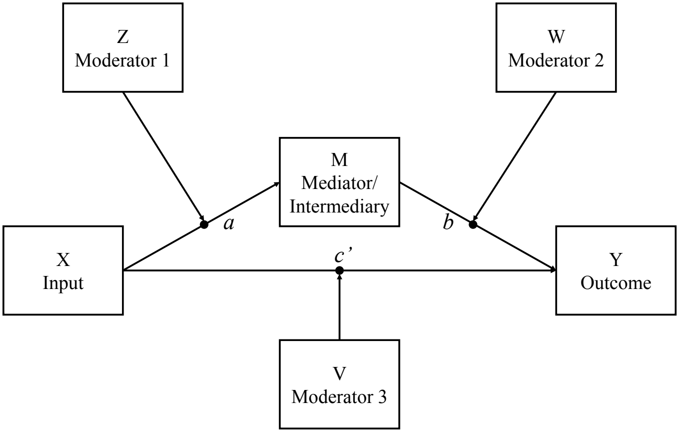
So, the conditional direct and indirect effects are defined by the following:

$$DE = c'_1 + c'_3 Z$$

$$IE = (a_1 + a_3 Z)(b_1 + b_2 Z)$$

Another Example

Or maybe, the a, b, and c' paths are each moderated by a separate variable to induce the conditional indirect and direct effects:



Another Example

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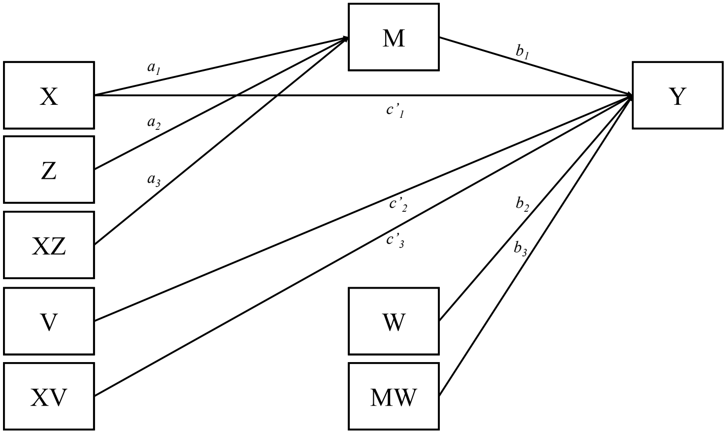


Another Example

This analytic diagram implies the following equations:

$$
\begin{aligned}
Y &= i_1 + b_1 M + b_2 W + c'_1 X + c'_2 V + c'_3 XV + b_3 MW + e_Y \\
M &= i_2 + a_1 X + a_2 Z + a_3 XZ + e_M
\end{aligned}
$$

Another Example

This analytic diagram implies the following equations:

$$
\begin{aligned}
Y &= i_1 + b_1 M + b_2 W + c_1' X + c_2' V + c_3' XV + b_3 MW + e_Y \\
M &= i_2 + a_1 X + a_2 Z + a_3 XZ + e_M
\end{aligned}
$$

The direct effect is now conditional on V, while the indirect effect is conditional on Z and W.

Another Example

This analytic diagram implies the following equations:

$$
\begin{aligned}
Y &= i_1 + b_1 M + b_2 W + c'_1 X + c'_2 V + c'_3 XV + b_3 MW + e_Y \\
M &= i_2 + a_1 X + a_2 Z + a_3 XZ + e_M
\end{aligned}
$$

The direct effect is now conditional on V, while the indirect effect is conditional on Z and W.

We can rearrange the preceding equations to get:

$$
\begin{aligned}
Y &= i_1 + b_2 W + c'_2 V + (b_1 + b_2 W)\, M + \left(c'_1 + c'_3 V\right) X + e_Y \\
M &= i_2 + a_2 Z + (a_1 + a_3 Z)\, X + e_M
\end{aligned}
$$

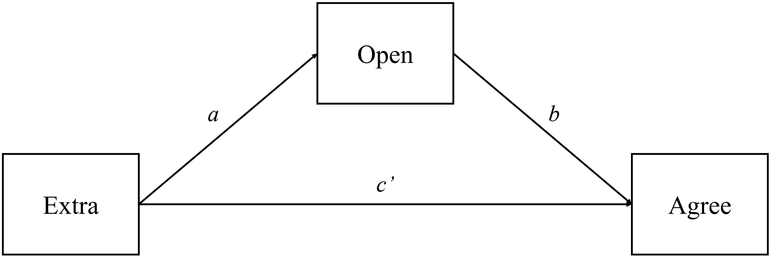
So, the conditional direct and indirect effects are defined by the following:

$$
\begin{aligned}
DE &= c'_1 + c'_3 V \\
IE &= (a_1 + a_3 Z)(b_1 + b_3 W)
\end{aligned}
$$

Example

Okay, let's do an example analysis.

We'll start by fitting the following simple indirect effects model to the `bfi` data from the **psych** package:



Example

```r
library(lavaan)
dat1 <- readRDS("../data/lecture10Data.rds")
nBoot <- 5000
## Simple indirect effects model:
mod1 <- "
agree ~ b*open + cp*extra
open ~ a*extra

ab := a*b
"
out1 <- sem(mod1, data = dat1, se = "boot", boot = nBoot)
summary(out1)
```

```
lavaan (0.5-19) converged normally after  16 iterations

  Number of observations                          2800

  Estimator                                         ML
  Minimum Function Test Statistic                0.000
  Degrees of freedom                                 0
  Minimum Function Value               0.0000000000000
```

Example

```
Parameter Estimates:

  Information                                  Observed
  Standard Errors                             Bootstrap
  Number of requested bootstrap draws              5000
  Number of successful bootstrap draws             5000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  agree ~
    open      (b)     0.166    0.028    5.955    0.000
    extra    (cp)     0.358    0.026   13.610    0.000
  open ~
    extra     (a)     0.268    0.023   11.698    0.000

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    agree             0.487    0.015   33.403    0.000
    open              0.294    0.010   29.466    0.000

Defined Parameters:
                   Estimate  Std.Err  Z-value  P(>|z|)
    ab                0.045    0.009    5.026    0.000
```

Example

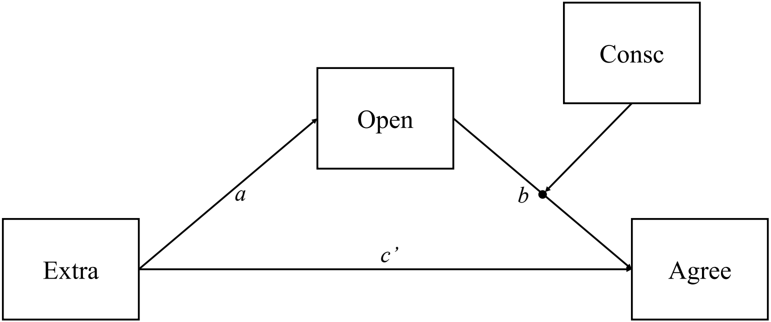
```
tab1 <-
    parameterEstimates(out1, boot.ci.type = "bca.simple")
tab1[grep("ab", tab1$label),
     c("label", "est", "ci.lower", "ci.upper")]
```

```
  label   est ci.lower ci.upper
7    ab 0.045    0.029    0.064
```

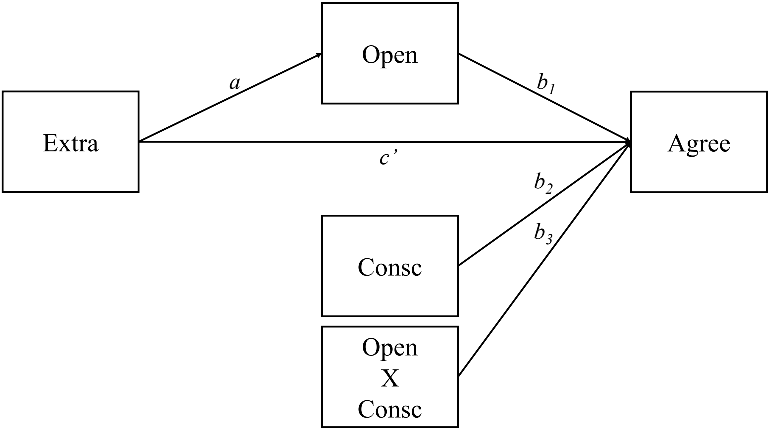
Example

Maybe we suspect that *conscientiousness* moderates the effect of *openness* on *agreeableness* (i.e., the b path).

We can assess this conditional process via the following model:



The preceding conceptual model translates into the following analytic model:



Example

```r
## Construct the product term:
dat1$openXconsc <- dat1$open*dat1$consc
## Find interesting quantiles of 'consc':
quantile(dat1$consc, c(0.25, 0.50, 0.75))
```

```
    25%     50%     75%
-0.4045 -0.0045  0.3955
```

```r
## Conditional process model with b path moderated:
mod2 <- "
agree ~ b1*open + cp*extra + b2*consc + b3*openXconsc
open ~ a*extra

abLo  := a*(b1 + b3*(-0.4045))
abMid := a*(b1 + b3*(-0.0045))
abHi  := a*(b1 + b3*0.3955)
"
```

Example

```r
out2 <- sem(mod2, data = dat1, se = "boot", boot = nBoot)
summary(out2)
```

```
lavaan (0.5-19) converged normally after  17 iterations

  Number of observations                          2800

  Estimator                                         ML
  Minimum Function Test Statistic              161.865
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws             5000
  Number of successful bootstrap draws            5000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  agree ~
```

Example

```
    open      (b1)    0.167    0.027    6.079    0.000
    extra     (cp)    0.361    0.027   13.512    0.000
    consc     (b2)   -0.028    0.026   -1.071    0.284
    openXcnsc (b3)   -0.079    0.039   -2.027    0.043
  open ~
    extra      (a)    0.268    0.023   11.614    0.000

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    agree             0.485    0.015   32.593    0.000
    open              0.294    0.010   29.733    0.000

Defined Parameters:
                   Estimate  Std.Err  Z-value  P(>|z|)
    abLo              0.053    0.010    5.242    0.000
    abMid             0.045    0.008    5.321    0.000
    abHi              0.036    0.009    4.166    0.000
```

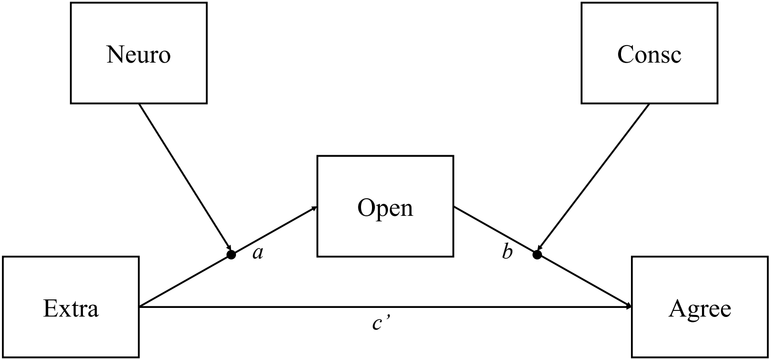
Example

```r
tab2 <-
    parameterEstimates(out2, boot.ci.type = "bca.simple")
tab2[grep("ab", tab2$label),
     c("label", "est", "ci.lower", "ci.upper")]
```

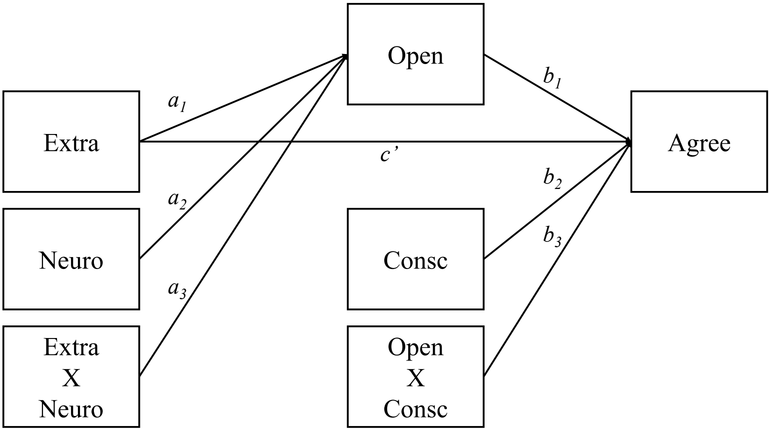
```
   label   est ci.lower ci.upper
14  abLo 0.053    0.035    0.076
15 abMid 0.045    0.030    0.063
16  abHi 0.036    0.021    0.054
```

Suppose we also think that *neuroticism* moderates the effect of *extroversion* on *openness* (i.e., the a path).

We can assess this conditional process via the following model:



The preceding conceptual model translates into the following analytic model:



Example

```r
## Construct another product term:
dat1$extraXneuro <- dat1$extra*dat1$neuro
## Find interesting quantiles of 'neuro':
quantile(dat1$neuro, c(0.25, 0.50, 0.75))
```

```
       25%        50%        75%
-0.9622679 -0.1622679  0.8377321
```

```r
## Conditional process model with a and b paths moderated:
mod3 <- "
agree ~ b1*open + cp*extra + b2*consc + b3*openXconsc
open ~ a1*extra + a2*neuro + a3*extraXneuro

abLoLo  := (a1 + a3*(-0.962268)) * (b1 + b3*(-0.4045))
abLoMid := (a1 + a3*(-0.962268)) * (b1 + b3*(-0.0045))
abLoHi  := (a1 + a3*(-0.962268)) * (b1 + b3*0.3955)

abMidLo  := (a1 + a3*(-0.162268)) * (b1 + b3*(-0.4045))
abMidMid := (a1 + a3*(-0.162268)) * (b1 + b3*(-0.0045))
abMidHi  := (a1 + a3*(-0.162268)) * (b1 + b3*0.3955)

abHiLo  := (a1 + a3*0.837732) * (b1 + b3*(-0.4045))
abHiMid := (a1 + a3*0.837732) * (b1 + b3*(-0.0045))
abHiHi  := (a1 + a3*0.837732) * (b1 + b3*0.3955)
"
```

Example

```
out3 ← sem(mod3, data = dat1, se = "boot", boot = nBoot)
summary(out3)
```

```
lavaan (0.5-19) converged normally after  18 iterations

  Number of observations                          2800

  Estimator                                         ML
  Minimum Function Test Statistic              214.855
  Degrees of freedom                                 4
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws             5000
  Number of successful bootstrap draws            5000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  agree ~
```

Example

```
    open       (b1)    0.167    0.028    6.014    0.000
    extra      (cp)    0.361    0.026   13.880    0.000
    consc      (b2)   -0.028    0.027   -1.044    0.297
    openXcnsc  (b3)   -0.079    0.039   -2.055    0.040
  open ~
    extra      (a1)    0.261    0.023   11.345    0.000
    neuro      (a2)    0.072    0.009    8.007    0.000
    extraXner  (a3)    0.005    0.022    0.216    0.829

Variances:
                    Estimate  Std.Err  Z-value  P(>|z|)
    agree              0.485    0.015   33.192    0.000
    open               0.287    0.010   30.118    0.000

Defined Parameters:
                    Estimate  Std.Err  Z-value  P(>|z|)
    abLoLo             0.051    0.012    4.197    0.000
    abLoMid            0.043    0.010    4.483    0.000
    abLoHi             0.035    0.009    4.001    0.000
    abMidLo            0.052    0.010    4.957    0.000
    abMidMid           0.043    0.008    5.115    0.000
    abMidHi            0.035    0.008    4.157    0.000
    abHiLo             0.053    0.010    5.441    0.000
```

Example

```
abHiMid          0.044    0.009    5.198    0.000
abHiHi           0.036    0.009    3.903    0.000
```

Example

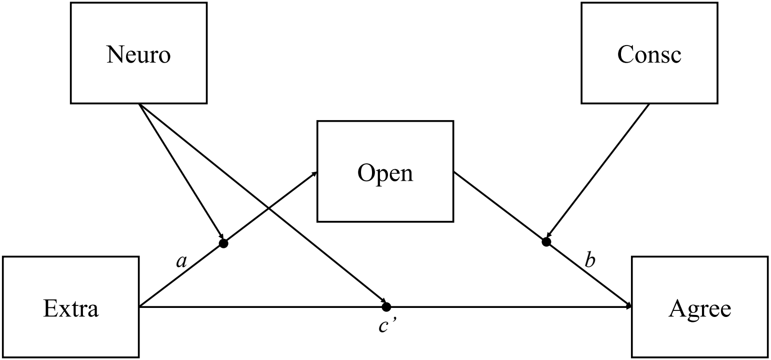
```r
tab3 <-
    parameterEstimates(out3, boot.ci.type = "bca.simple")
tab3[grep("ab", tab3$label),
     c("label", "est", "ci.lower", "ci.upper")]
```

```
      label   est ci.lower ci.upper
25   abLoLo 0.051    0.030    0.078
26  abLoMid 0.043    0.026    0.064
27   abLoHi 0.035    0.019    0.054
28  abMidLo 0.052    0.033    0.074
29 abMidMid 0.043    0.029    0.061
30  abMidHi 0.035    0.019    0.053
31   abHiLo 0.053    0.035    0.073
32  abHiMid 0.044    0.029    0.063
33   abHiHi 0.036    0.019    0.055
```

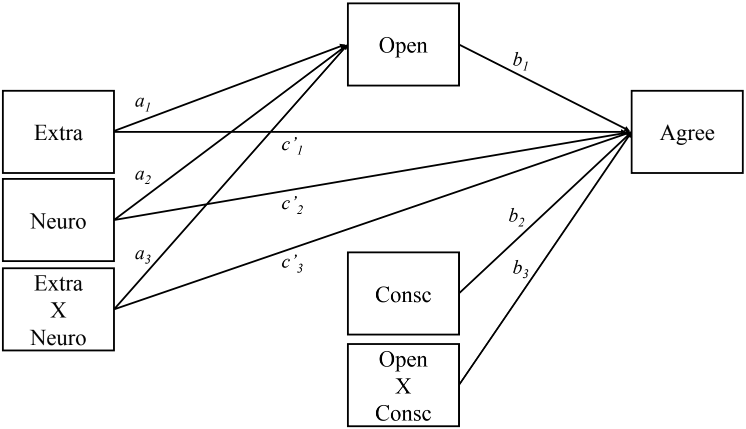
Example

Finally, maybe we think that *neuroticism* also moderates the direct effect of *extroversion* on *agreeableness* (i.e., the c' path).

We can assess this conditional process via the following model:



The preceding conceptual model translates into the following analytic model:



Example

```
## Conditional process model with a, b, c paths moderated:
mod4 ← "
agree ~ b1*open + b2*consc + cp1*extra + cp2*neuro +
        cp3*extraXneuro + b3*openXconsc
open ~ a1*extra + a2*neuro + a3*extraXneuro

cpLo  := cp1 + cp3*(-0.962268)
cpMid := cp1 + cp3*(-0.162268)
cpHi  := cp1 + cp3*0.837732

abLoLo  := (a1 + a3*(-0.962268)) * (b1 + b3*(-0.4045))
abLoMid := (a1 + a3*(-0.962268)) * (b1 + b3*(-0.0045))
abLoHi  := (a1 + a3*(-0.962268)) * (b1 + b3*0.3955)

abMidLo  := (a1 + a3*(-0.162268)) * (b1 + b3*(-0.4045))
abMidMid := (a1 + a3*(-0.162268)) * (b1 + b3*(-0.0045))
abMidHi  := (a1 + a3*(-0.162268)) * (b1 + b3*0.3955)

abHiLo  := (a1 + a3*0.837732) * (b1 + b3*(-0.4045))
abHiMid := (a1 + a3*0.837732) * (b1 + b3*(-0.0045))
abHiHi  := (a1 + a3*0.837732) * (b1 + b3*0.3955)
"
```

Example

```
out4 <- sem(mod4, data = dat1, se = "boot", boot = nBoot)
summary(out4)
```

```
lavaan (0.5-19) converged normally after  20 iterations

  Number of observations                          2800

  Estimator                                         ML
  Minimum Function Test Statistic              128.914
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws             5000
  Number of successful bootstrap draws            5000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  agree ~
```

Example

```
    open      (b1)    0.191    0.027    7.072    0.000
    consc     (b2)    0.021    0.027    0.764    0.445
    extra    (cp1)    0.349    0.026   13.301    0.000
    neuro    (cp2)   -0.102    0.012   -8.356    0.000
    extraXnr (cp3)    0.049    0.022    2.194    0.028
    opnXcnsc  (b3)   -0.074    0.044   -1.669    0.095
  open ~
    extra     (a1)    0.261    0.023   11.468    0.000
    neuro     (a2)    0.072    0.009    7.962    0.000
    extraXnr  (a3)    0.005    0.021    0.218    0.827

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    agree             0.471    0.015   32.438    0.000
    open              0.287    0.010   29.854    0.000

Defined Parameters:
                   Estimate  Std.Err  Z-value  P(>|z|)
    cpLo              0.302    0.035    8.579    0.000
    cpMid             0.341    0.027   12.740    0.000
    cpHi              0.390    0.031   12.585    0.000
    abLoLo            0.057    0.013    4.275    0.000
    abLoMid           0.049    0.010    4.828    0.000
```

Example

```
abLoHi       0.041   0.009   4.603   0.000
abMidLo      0.057   0.011   5.156   0.000
abMidMid     0.050   0.009   5.703   0.000
abMidHi      0.042   0.009   4.877   0.000
abHiLo       0.058   0.010   5.865   0.000
abHiMid      0.051   0.009   5.934   0.000
abHiHi       0.043   0.009   4.531   0.000
```

Example

```
tab4 ←
    parameterEstimates(out4, boot.ci.type = "bca.simple")
tab4[grep("Lo|Mid|Hi", tab4$label),
     c("label", "est", "ci.lower", "ci.upper")]
```

```
      label   est ci.lower ci.upper
27     cpLo 0.302    0.235    0.372
28    cpMid 0.341    0.291    0.394
29     cpHi 0.390    0.331    0.451
30   abLoLo 0.057    0.034    0.087
31  abLoMid 0.049    0.032    0.073
32   abLoHi 0.041    0.026    0.061
33  abMidLo 0.057    0.037    0.082
34 abMidMid 0.050    0.034    0.070
35  abMidHi 0.042    0.026    0.060
36   abHiLo 0.058    0.041    0.081
37  abHiMid 0.051    0.036    0.070
38   abHiHi 0.043    0.026    0.063
```

Example

The a path was not significantly moderated in any of the previous models.

- Maybe we should try culling that path to see if we can get a more parsimonious description of the process.

```
## Conditional process model with b, c paths moderated:
mod5 <- "
agree ~ b1*open + b2*consc + cp1*extra + cp2*neuro +
        cp3*extraXneuro + b3*openXconsc
open ~ a*extra

cpLo  := cp1 + cp3*(-0.962268)
cpMid := cp1 + cp3*(-0.162268)
cpHi  := cp1 + cp3*0.837732

abLo := a * (b1 + b3*(-0.4045))
abMid := a * (b1 + b3*(-0.0045))
abHi := a * (b1 + b3*0.3955)
"
```

Example

```
out5 <- sem(mod5, data = dat1, se = "boot", boot = nBoot)
summary(out5)
```

```
lavaan (0.5-19) converged normally after  18 iterations

  Number of observations                          2800

  Estimator                                         ML
  Minimum Function Test Statistic              201.443
  Degrees of freedom                                 4
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws             5000
  Number of successful bootstrap draws            5000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  agree ~
```

```
    open      (b1)    0.191    0.028    6.951    0.000
    consc     (b2)    0.021    0.027    0.778    0.436
    extra    (cp1)    0.349    0.027   12.833    0.000
    neuro    (cp2)   -0.102    0.012   -8.493    0.000
    extraXnr (cp3)    0.049    0.022    2.175    0.030
    opnXcnsc  (b3)   -0.074    0.044   -1.695    0.090
  open ~
    extra      (a)    0.268    0.023   11.488    0.000

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    agree             0.471    0.014   33.121    0.000
    open              0.294    0.010   30.146    0.000

Defined Parameters:
                   Estimate  Std.Err  Z-value  P(>|z|)
    cpLo              0.302    0.036    8.333    0.000
    cpMid             0.341    0.028   12.286    0.000
    cpHi              0.390    0.032   12.305    0.000
    abLo              0.059    0.011    5.293    0.000
    abMid             0.051    0.009    5.743    0.000
    abHi              0.043    0.009    4.805    0.000
```

Example

```r
tab5 <-
    parameterEstimates(out5, boot.ci.type = "bca.simple")
tab5[grep("Lo|Mid|Hi", tab5$label),
     c("label", "est", "ci.lower", "ci.upper")]
```

```
   label   est ci.lower ci.upper
25  cpLo 0.302    0.233    0.375
26 cpMid 0.341    0.288    0.396
27  cpHi 0.390    0.325    0.450
28  abLo 0.059    0.038    0.082
29 abMid 0.051    0.035    0.070
30  abHi 0.043    0.027    0.062
```