TEXAS TECH UNIVERSITY

Lecture 6: Longitudinal Mediation

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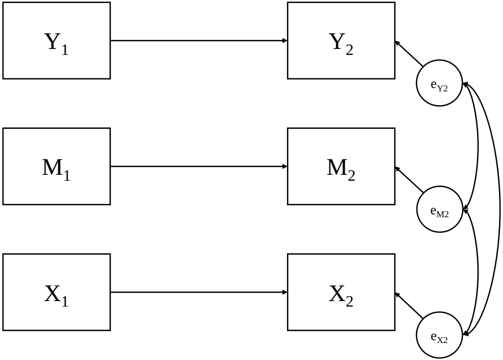
2016 Stats Camp

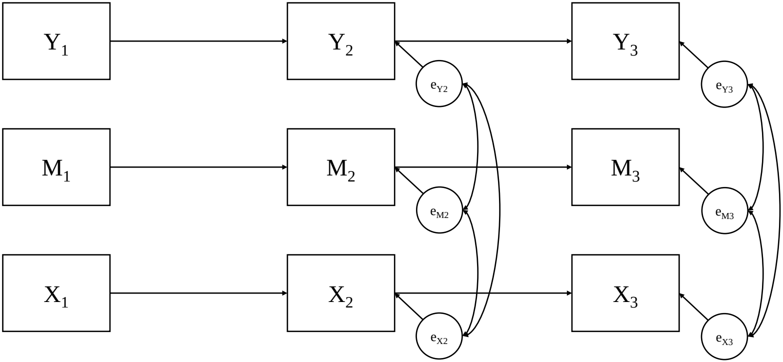
Outline

- Show how to test indirect effects with longitudinal models
- Briefly discuss causal inference
- Discuss how to make *more* causal conclusions about mediation

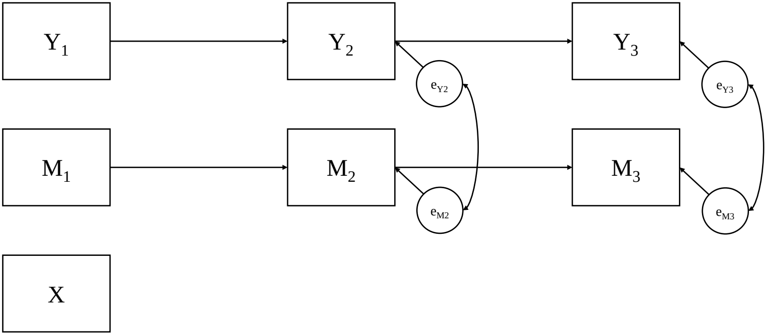
Simplest Longitudinal Model



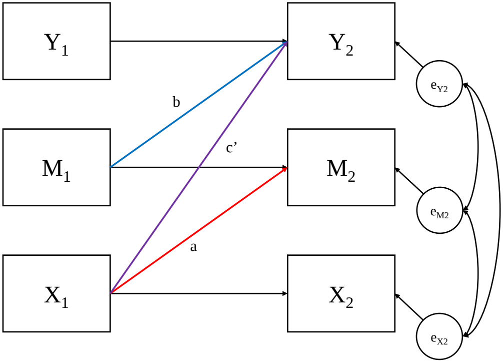
Better Longitudinal Model



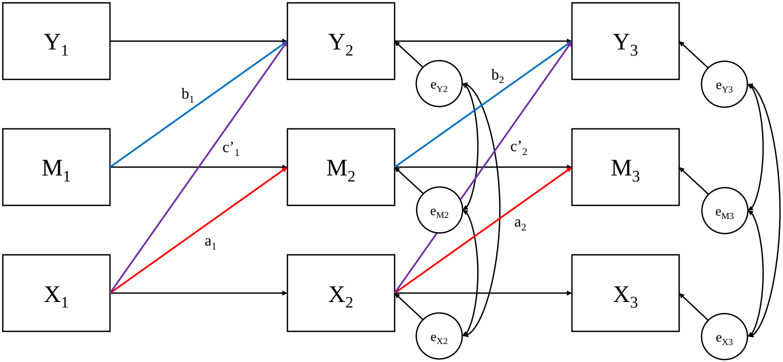
With Experimental Manipulation



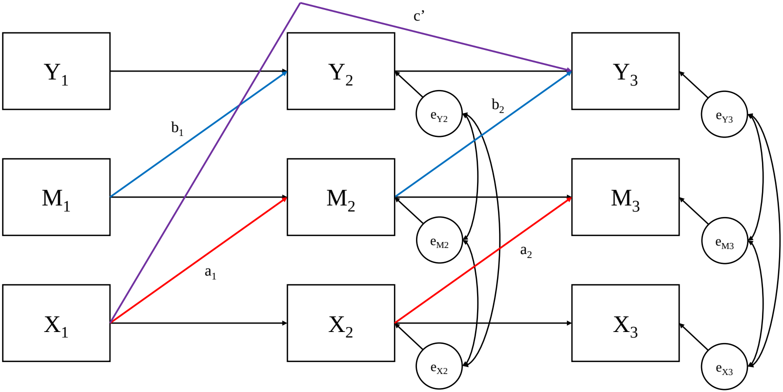
With Mediated Paths



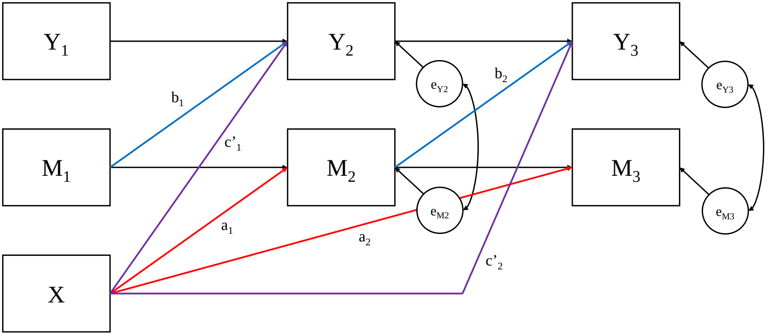
With Mediated Paths



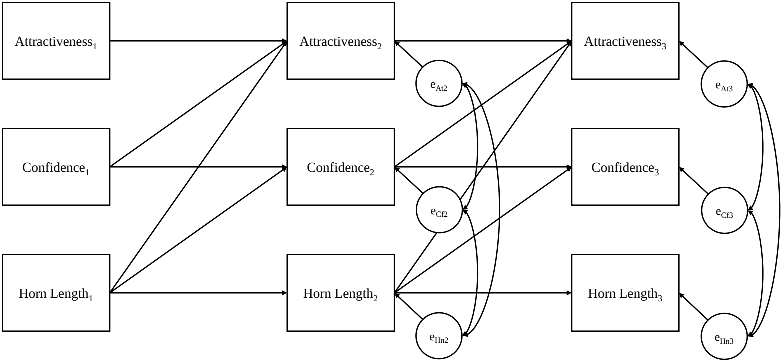
Alternative c' Path



Experimental Model



Let's Try It



Attractiveness$_1$
Confidence$_1$
Horn Length$_1$
Attractiveness$_2$
Confidence$_2$
Horn Length$_2$
Attractiveness$_3$
Confidence$_3$
Horn Length$_3$
e_{At2}
e_{Cf2}
e_{Hn2}
e_{At3}
e_{Cf3}
e_{Hn3}

Example

```
library(lavaan)
dataDir <- "../data/"
dat1 <- readRDS(paste0(dataDir, "unicorn2.rds"))
mod1 <- "
att3 ~ att2 + b2*conf2 + cp2*horn2
att2 ~ att1 + b1*conf1 + cp1*horn1

conf3 ~ conf2 + a2*horn2
conf2 ~ conf1 + a1*horn1

horn3 ~ horn2
horn2 ~ horn1

horn3 ~~ conf3 + att3
conf3 ~~ att3

horn2 ~~ conf2 + att2
conf2 ~~ att2

ab1 := a1*b1
ab2 := a2*b2
"
```

Example

```r
out1 ← sem(mod1, data = dat1, se = "boot", boot = nBoot)
summary(out1)
```

```
lavaan (0.5-20) converged normally after  52 iterations

  Number of observations                           500

  Estimator                                         ML
  Minimum Function Test Statistic              285.028
  Degrees of freedom                                15
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws             2000
  Number of successful bootstrap draws            2000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  att3 ~
```

Example

```
    att2                 0.525    0.037   14.086    0.000
    conf2       (b2)     0.068    0.025    2.756    0.006
    horn2      (cp2)     0.070    0.095    0.731    0.465
  att2 ~
    att1                 0.498    0.043   11.522    0.000
    conf1       (b1)     0.127    0.026    4.795    0.000
    horn1      (cp1)     0.112    0.100    1.110    0.267
  conf3 ~
    conf2                0.636    0.040   16.048    0.000
    horn2       (a2)     0.780    0.173    4.506    0.000
  conf2 ~
    conf1                0.657    0.033   20.188    0.000
    horn1       (a1)     0.293    0.155    1.897    0.058
  horn3 ~
    horn2                0.835    0.029   29.228    0.000
  horn2 ~
    horn1                0.708    0.024   29.601    0.000

Covariances:
                      Estimate  Std.Err  Z-value  P(>|z|)
  conf3 ~~
    horn3                1.039    0.159    6.524    0.000
  att3 ~~
```

Example

```
    horn3          0.332    0.092    3.613    0.000
    conf3          3.548    0.459    7.724    0.000
  conf2 ~~
    horn2          0.812    0.121    6.740    0.000
  att2 ~~
    horn2          0.264    0.088    3.007    0.003
    conf2          2.028    0.410    4.946    0.000

Variances:
                Estimate  Std.Err  Z-value  P(>|z|)
    att3           5.988    0.383   15.642    0.000
    att2           6.017    0.354   17.013    0.000
    conf3         15.800    0.932   16.948    0.000
    conf2         12.455    0.765   16.279    0.000
    horn3          0.694    0.039   17.783    0.000
    horn2          0.559    0.035   16.099    0.000

Defined Parameters:
                Estimate  Std.Err  Z-value  P(>|z|)
    ab1            0.037    0.021    1.785    0.074
    ab2            0.053    0.021    2.504    0.012
```

Example

```
parameterEstimates(out1, boot = "bca.simple")[ , -c(1 : 3)]
```

```
   label   est    se      z pvalue ci.lower ci.upper
1        0.525 0.037 14.086  0.000    0.452    0.596
2     b2 0.068 0.025  2.756  0.006    0.015    0.112
3    cp2 0.070 0.095  0.731  0.465   -0.118    0.265
4        0.498 0.043 11.522  0.000    0.417    0.584
5     b1 0.127 0.026  4.795  0.000    0.074    0.180
6    cp1 0.112 0.100  1.110  0.267   -0.091    0.296
7        0.636 0.040 16.048  0.000    0.555    0.711
8     a2 0.780 0.173  4.506  0.000    0.443    1.133
9        0.657 0.033 20.188  0.000    0.596    0.725
10    a1 0.293 0.155  1.897  0.058    0.000    0.606
11       0.835 0.029 29.228  0.000    0.780    0.893
12       0.708 0.024 29.601  0.000    0.662    0.756
13       1.039 0.159  6.524  0.000    0.734    1.371
14       0.332 0.092  3.613  0.000    0.161    0.516
15       3.548 0.459  7.724  0.000    2.684    4.445
16       0.812 0.121  6.740  0.000    0.593    1.068
17       0.264 0.088  3.007  0.003    0.101    0.448
18       2.028 0.410  4.946  0.000    1.278    2.903
19       5.988 0.383 15.642  0.000    5.267    6.790
```

Example

20		6.017	0.354	17.013	0.000	5.412	6.807
21		15.800	0.932	16.948	0.000	14.186	17.777
22		12.455	0.765	16.279	0.000	11.095	14.124
23		0.694	0.039	17.783	0.000	0.625	0.782
24		0.559	0.035	16.099	0.000	0.495	0.632
25		8.748	0.000	NA	NA	8.748	8.748
26		8.475	0.000	NA	NA	8.475	8.475
27		1.470	0.000	NA	NA	1.470	1.470
28		28.025	0.000	NA	NA	28.025	28.025
29		3.939	0.000	NA	NA	3.939	3.939
30		1.809	0.000	NA	NA	1.809	1.809
31	ab1	0.037	0.021	1.785	0.074	0.003	0.085
32	ab2	0.053	0.021	2.504	0.012	0.019	0.103

Example

```
mod2 <- "
att3 ~ att2 + b2*conf2 + cp*horn1
att2 ~ att1 + b1*conf1

conf3 ~ conf2 + a2*horn2
conf2 ~ conf1 + a1*horn1

horn3 ~ horn2
horn2 ~ horn1

horn3 ~~ conf3 + att3
conf3 ~~ att3

horn2 ~~ conf2 + att2
conf2 ~~ att2

ab := a1*b2
"
out2 <- sem(mod2, data = dat1, se = "boot", boot = nBoot)
summary(out2)
```

Example

```
lavaan (0.5-20) converged normally after  50 iterations

  Number of observations                           500

  Estimator                                         ML
  Minimum Function Test Statistic              286.206
  Degrees of freedom                                16
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws             2000
  Number of successful bootstrap draws            2000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  att3 ~
    att2              0.531    0.036   14.556    0.000
    conf2     (b2)    0.082    0.025    3.285    0.001
    horn1     (cp)   -0.065    0.082   -0.797    0.425
```

Example

```
  att2 ~
    att1                0.503    0.043   11.722    0.000
    conf1      (b1)     0.141    0.024    5.914    0.000
  conf3 ~
    conf2               0.645    0.040   16.303    0.000
    horn2      (a2)     0.708    0.172    4.112    0.000
  conf2 ~
    conf1               0.661    0.033   20.177    0.000
    horn1      (a1)     0.258    0.152    1.698    0.090
  horn3 ~
    horn2               0.830    0.029   29.025    0.000
  horn2 ~
    horn1               0.705    0.024   28.882    0.000

Covariances:
                     Estimate  Std.Err  Z-value  P(>|z|)
  conf3 ~~
    horn3               1.035    0.161    6.419    0.000
  att3 ~~
    horn3               0.343    0.092    3.711    0.000
    conf3               3.562    0.465    7.663    0.000
  conf2 ~~
    horn2               0.810    0.121    6.705    0.000
```

Example

```
  att2 ~~
    horn2          0.253    0.089    2.837    0.005
    conf2          2.013    0.408    4.932    0.000

Variances:
                Estimate  Std.Err  Z-value  P(>|z|)
    att3           5.996    0.370   16.190    0.000
    att2           6.021    0.367   16.395    0.000
    conf3         15.793    0.946   16.689    0.000
    conf2         12.448    0.743   16.749    0.000
    horn3          0.694    0.041   16.918    0.000
    horn2          0.559    0.035   15.892    0.000

Defined Parameters:
                Estimate  Std.Err  Z-value  P(>|z|)
    ab             0.021    0.014    1.486    0.137
```

```
parameterEstimates(out2, boot = "bca.simple")[ , -c(1 : 3)]
```

Example

	label	est	se	z	pvalue	ci.lower	ci.upper
1		0.531	0.036	14.556	0.000	0.460	0.605
2	b2	0.082	0.025	3.285	0.001	0.034	0.132
3	cp	-0.065	0.082	-0.797	0.425	-0.237	0.091
4		0.503	0.043	11.722	0.000	0.417	0.584
5	b1	0.141	0.024	5.914	0.000	0.093	0.187
6		0.645	0.040	16.303	0.000	0.569	0.724
7	a2	0.708	0.172	4.112	0.000	0.360	1.026
8		0.661	0.033	20.177	0.000	0.593	0.723
9	a1	0.258	0.152	1.698	0.090	-0.039	0.559
10		0.830	0.029	29.025	0.000	0.774	0.888
11		0.705	0.024	28.882	0.000	0.659	0.754
12		1.035	0.161	6.419	0.000	0.716	1.367
13		0.343	0.092	3.711	0.000	0.173	0.534
14		3.562	0.465	7.663	0.000	2.688	4.485
15		0.810	0.121	6.705	0.000	0.584	1.046
16		0.253	0.089	2.837	0.005	0.083	0.436
17		2.013	0.408	4.932	0.000	1.259	2.920
18		5.996	0.370	16.190	0.000	5.319	6.823
19		6.021	0.367	16.395	0.000	5.357	6.847
20		15.793	0.946	16.689	0.000	14.112	17.819
21		12.448	0.743	16.749	0.000	11.134	14.003

Example

```
22          0.694 0.041 16.918  0.000     0.621     0.782
23          0.559 0.035 15.892  0.000     0.493     0.629
24          1.809 0.000     NA     NA     1.809     1.809
25          1.470 0.000     NA     NA     1.470     1.470
26          3.939 0.000     NA     NA     3.939     3.939
27          8.748 0.000     NA     NA     8.748     8.748
28          8.475 0.000     NA     NA     8.475     8.475
29         28.025 0.000     NA     NA    28.025    28.025
30      ab  0.021 0.014  1.486  0.137     0.000     0.059
```

Assumptions

We need a few assumptions to fully generalize our findings:

- Stability: Do mean levels follow a stable trend?
- Stationarity: Are lagged associations equal for equivalent lags?
- Equilibrium: Are cross-sectional variances and covariances equal at all waves?

Another definition of stationarity is used in time-series analysis:

- Weak Stationarity:
  1. Finite variance
  2. No trend in mean levels
  3. Lag-K auto-covariances are equal
- Strict Stationarity: The distribution of the process is the same at all time points.

Test Assumptions

```
mod3 <- "
att3 ~ att2 + b2*conf2 + cp2*horn2
att2 ~ att1 + b1*conf1 + cp1*horn1

conf3 ~ conf2 + a2*horn2
conf2 ~ conf1 + a1*horn1

horn3 ~ horn2
horn2 ~ horn1

horn3 ~~ conf3 + att3
conf3 ~~ att3

horn2 ~~ conf2 + att2
conf2 ~~ att2

a1 == a2
b1 == b2
cp1 == cp2
"
out3 <- sem(mod3, data = dat1)
summary(out3)
```

Test Assumptions

```
lavaan (0.5-20) converged normally after  46 iterations

  Number of observations                           500

  Estimator                                         ML
  Minimum Function Test Statistic              294.220
  Degrees of freedom                                18
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  att3 ~
    att2              0.497    0.035   14.234    0.000
    conf2     (b2)    0.098    0.019    5.200    0.000
    horn2    (cp2)    0.083    0.072    1.157    0.247
  att2 ~
    att1              0.530    0.040   13.345    0.000
```

Test Assumptions

```
    conf1       (b1)    0.098     0.019     5.200     0.000
    horn1      (cp1)    0.083     0.072     1.157     0.247
  conf3 ~
    conf2               0.684     0.035    19.602     0.000
    horn2       (a2)    0.493     0.107     4.596     0.000
  conf2 ~
    conf1               0.623     0.032    19.546     0.000
    horn1       (a1)    0.493     0.107     4.596     0.000
  horn3 ~
    horn2               0.826     0.030    27.609     0.000
  horn2 ~
    horn1               0.714     0.024    29.181     0.000

Covariances:
                     Estimate  Std.Err  Z-value  P(>|z|)
  conf3 ~~
    horn3               1.016     0.155     6.556     0.000
  att3 ~~
    horn3               0.322     0.093     3.483     0.000
    conf3               3.574     0.465     7.691     0.000
  conf2 ~~
    horn2               0.836     0.124     6.721     0.000
  att2 ~~
```

Test Assumptions

```
    horn2              0.273    0.083    3.289    0.001
    conf2              2.027    0.400    5.067    0.000

Variances:
                    Estimate  Std.Err  Z-value  P(>|z|)
    att3               6.019    0.381   15.811    0.000
    att2               6.041    0.382   15.811    0.000
    conf3             15.814    1.000   15.811    0.000
    conf2             12.570    0.795   15.811    0.000
    horn3              0.695    0.044   15.811    0.000
    horn2              0.560    0.035   15.811    0.000

Constraints:
                                                |Slack|
    a1 - (a2)                                     0.000
    b1 - (b2)                                     0.000
    cp1 - (cp2)                                   0.000
```

Test Assumptions

```
chiDiff <- fitMeasures(out3)["chisq"] -
    fitMeasures(out1)["chisq"]
dfDiff <- fitMeasures(out3)["df"] -
    fitMeasures(out1)["df"]
pchisq(chiDiff, dfDiff, lower = FALSE)
```

```
     chisq
0.02684148
```

Weirdness of Causation

Causal modeling/inference is underpinned by an odd contradiction:

- Correlation does not exist in the real world
- An observed correlation is merely an artifact of inadequate measurement.
- If X and Y covary, then there are three possible *underlying* reasons:
    1. X causes Y
    2. Y causes X
    3. An unmeasured third variable causes both X and Y

Weirdness of Causation

- Probability theory and statistical analysis can only encode correlational relations
  - Probability theory can only describe the (in)dependence of X and Y in terms of their joint distribution.
  - Statistics can only test for a (non)linear association between X and Y.
- Causal information must be externally imposed by the researcher in the form of assumptions and theory.

Sufficient Conditions to Infer Causation

Three conditions are generally sufficient to suggest that X causes Y:

1. X and Y must covary
2. X must temporally precede Y
3. All alternative explanations for the covariance of X and Y must be eliminated

Sufficient Conditions to Infer Causation

Three conditions are generally sufficient to suggest that X causes Y:

1. X and Y must covary
2. X must temporally precede Y
3. All alternative explanations for the covariance of X and Y must be eliminated

Condition 1 is easy to confirm in any data analytic context.

Condition 2 can be easily assessed with longitudinal data.

Condition 3 is impossible to satisfy statistically.

- Even the most rigorous experimental designs must make assumptions to satisfy Condition 3.
- Tenability decreases without random assignment.

What do Longitudinal Data Give Us?

QUESTION: Do we get to claim causation if we find mediation with a panel model?

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ANSWER: No!

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QUESTION: Do we get to claim causation if we find mediation with a panel model?

ANSWER: No!

But we can make stronger claims than we can with cross-sectional data.

- Panel models can satisfy Conditions 1 and 2.
- Panel models can help with Condition 3
  - Due to the autoregressive paths, each case acts as its own control
  - Autoregression only controls for time-invariant traits
- We can further improve the strengths of our inferences by including appropriate covariates
  - Same idea as matching non-experimental cases for causal inference

Example

```
mod4 <- "
att3 ~ att2 + b2*conf2 + cp*horn1
att2 ~ att1 + b1*conf1

conf3 ~ conf2 + a2*horn2
conf2 ~ conf1 + a1*horn1

att2 + att3 ~ income
conf2 + conf3 ~ income
horn2 + horn3 ~ income

horn3 ~ horn2
horn2 ~ horn1

horn3 ~~ conf3 + att3
conf3 ~~ att3

horn2 ~~ conf2 + att2
conf2 ~~ att2

ab := a1*b2
"
```

Example

```
out4 <- sem(mod4, data = dat1, se = "boot", boot = nBoot)
summary(out4)
```

```
lavaan (0.5-20) converged normally after  63 iterations

  Number of observations                           500

  Estimator                                         ML
  Minimum Function Test Statistic              219.789
  Degrees of freedom                                16
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Information                                 Observed
  Standard Errors                            Bootstrap
  Number of requested bootstrap draws             2000
  Number of successful bootstrap draws            2000

Regressions:
                   Estimate  Std.Err  Z-value  P(>|z|)
  att3 ~
```

Example

```
    att2                0.513    0.039   13.099    0.000
    conf2     (b2)      0.022    0.027    0.826    0.409
    horn1     (cp)     -0.119    0.084   -1.418    0.156
  att2 ~
    att1                0.477    0.043   10.980    0.000
    conf1     (b1)      0.084    0.028    3.048    0.002
  conf3 ~
    conf2               0.488    0.041   11.803    0.000
    horn2     (a2)      0.492    0.160    3.076    0.002
  conf2 ~
    conf1               0.543    0.037   14.594    0.000
    horn1     (a1)      0.175    0.146    1.204    0.228
  att2 ~
    income              0.052    0.013    4.108    0.000
  att3 ~
    income              0.057    0.012    4.853    0.000
  conf2 ~
    income              0.110    0.016    6.871    0.000
  conf3 ~
    income              0.148    0.018    8.206    0.000
  horn2 ~
    income              0.016    0.003    5.698    0.000
  horn3 ~
```

Example

```
    income            0.013    0.004    3.555    0.000
    horn2             0.780    0.033   23.921    0.000
  horn2 ~
    horn1             0.654    0.026   25.525    0.000

Covariances:
                   Estimate  Std.Err  Z-value  P(>|z|)
  conf3 ~~
    horn3             0.915    0.152    6.020    0.000
  att3 ~~
    horn3             0.322    0.092    3.489    0.000
    conf3             2.971    0.412    7.215    0.000
  conf2 ~~
    horn2             0.678    0.112    6.026    0.000
  att2 ~~
    horn2             0.203    0.083    2.428    0.015
    conf2             1.601    0.382    4.194    0.000

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    att3              5.771    0.351   16.431    0.000
    att2              5.821    0.362   16.099    0.000
    conf3            14.066    0.828   16.988    0.000
```

Example

```
    conf2              11.512    0.729   15.792    0.000
    horn2               0.529    0.033   16.004    0.000
    horn3               0.677    0.040   16.788    0.000

Defined Parameters:
                    Estimate  Std.Err  Z-value  P(>|z|)
    ab                 0.004    0.007    0.559    0.576
```

```
parameterEstimates(out4, boot = "bca.simple")[ , -c(1 : 3)]
```

```
   label    est    se      z pvalue ci.lower ci.upper
1          0.513 0.039 13.099  0.000    0.438    0.589
2     b2   0.022 0.027  0.826  0.409   -0.030    0.076
3     cp  -0.119 0.084 -1.418  0.156   -0.275    0.051
4          0.477 0.043 10.980  0.000    0.396    0.565
5     b1   0.084 0.028  3.048  0.002    0.029    0.139
6          0.488 0.041 11.803  0.000    0.410    0.569
7     a2   0.492 0.160  3.076  0.002    0.186    0.818
8          0.543 0.037 14.594  0.000    0.464    0.614
9     a1   0.175 0.146  1.204  0.228   -0.099    0.468
10         0.052 0.013  4.108  0.000    0.028    0.077
11         0.057 0.012  4.853  0.000    0.033    0.079
```

Example

12	0.110	0.016	6.871	0.000	0.079	0.141
13	0.148	0.018	8.206	0.000	0.114	0.183
14	0.016	0.003	5.698	0.000	0.011	0.022
15	0.013	0.004	3.555	0.000	0.005	0.019
16	0.780	0.033	23.921	0.000	0.714	0.844
17	0.654	0.026	25.525	0.000	0.601	0.703
18	0.915	0.152	6.020	0.000	0.633	1.225
19	0.322	0.092	3.489	0.000	0.143	0.508
20	2.971	0.412	7.215	0.000	2.207	3.834
21	0.678	0.112	6.026	0.000	0.466	0.903
22	0.203	0.083	2.428	0.015	0.040	0.369
23	1.601	0.382	4.194	0.000	0.870	2.387
24	5.771	0.351	16.431	0.000	5.130	6.509
25	5.821	0.362	16.099	0.000	5.171	6.590
26	14.066	0.828	16.988	0.000	12.623	15.934
27	11.512	0.729	15.792	0.000	10.207	13.077
28	0.529	0.033	16.004	0.000	0.472	0.602
29	0.677	0.040	16.788	0.000	0.606	0.765
30	1.809	0.000	NA	NA	1.809	1.809
31	1.470	0.000	NA	NA	1.470	1.470
32	3.939	0.000	NA	NA	3.939	3.939
33	5.753	0.000	NA	NA	5.753	5.753
34	8.748	0.000	NA	NA	8.748	8.748

Example

35		8.475	0.000	NA	NA	8.475	8.475
36		16.763	0.000	NA	NA	16.763	16.763
37		28.025	0.000	NA	NA	28.025	28.025
38		39.156	0.000	NA	NA	39.156	39.156
39		136.353	0.000	NA	NA	136.353	136.353
40	ab	0.004	0.007	0.559	0.576	-0.003	0.031