Structural Equation Modeling & Mediation

Introduction to SEM with Lavaan

Kyle M. Lang

Department of Methodology & Statistics
Utrecht University

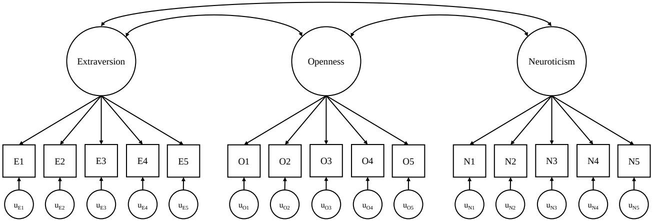
Outline

Full SEM

A full structural equation model (SEM) simply combines path analysis and CFA.

- SEM allows us to model complicated structural relations among latent variables.

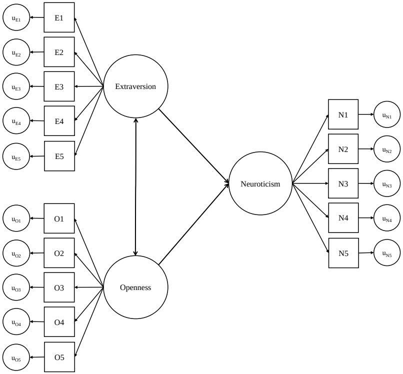
Let's consider a simple, three-factor CFA model.



CFA → SEM

We first evaluate the validity of the measurement model via CFA.

- We then convert the CFA to an SEM by converting some covariances to latent regression paths.



CFA Example

```r
## Load the lavaan package and some data:
library(lavaan)
data(bfi, package = "psych")

## Specify the CFA model:
cfaMod <- '
extra =~ E1 + E2 + E3 + E4 + E5
open  =~ O1 + O2 + O3 + O4 + O5
neuro =~ N1 + N2 + N3 + N4 + N5
'

## Estimate the model:
cfaOut <- cfa(cfaMod, data = bfi, missing = "fiml", std.lv = TRUE)

## Check the fit:
fitMeasures(cfaOut,
            c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr")
            )

   chisq        df    pvalue       cfi       tli     rmsea      srmr
2251.679    87.000     0.000     0.809     0.769     0.094     0.081
```

CFA Example

```
partSummary(cfaOut, 7)

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  extra =~
    E1                0.973    0.032   30.607    0.000
    E2                1.163    0.030   38.171    0.000
    E3               -0.815    0.027  -30.358    0.000
    E4               -0.979    0.028  -35.254    0.000
    E5               -0.714    0.027  -26.638    0.000
  open =~
    O1                0.630    0.025   24.886    0.000
    O2               -0.605    0.036  -16.781    0.000
    O3                0.897    0.029   30.765    0.000
    O4                0.290    0.028   10.402    0.000
    O5               -0.602    0.031  -19.734    0.000
  neuro =~
    N1                1.272    0.027   47.254    0.000
    N2                1.218    0.026   46.491    0.000
    N3                1.157    0.029   40.195    0.000
    N4                0.892    0.030   29.356    0.000
    N5                0.823    0.031   26.163    0.000
```

CFA Example

```
partSummary(cfaOut, 8)

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  extra ~~
    open             -0.444    0.024  -18.472    0.000
    neuro             0.240    0.023   10.551    0.000
  open ~~
    neuro            -0.117    0.025   -4.667    0.000
```

CFA Example

```
partSummary(cfaOut, 9)

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .E1                2.974    0.031   96.223    0.000
   .E2                3.143    0.030  103.424    0.000
   .E3                4.002    0.026  156.117    0.000
   .E4                4.421    0.028  160.350    0.000
   .E5                4.417    0.025  174.595    0.000
   .O1                4.816    0.021  224.964    0.000
   .O2                2.713    0.030   91.745    0.000
   .O3                4.436    0.023  191.555    0.000
   .O4                4.892    0.023  211.519    0.000
   .O5                2.490    0.025   98.932    0.000
   .N1                2.932    0.030   98.589    0.000
   .N2                3.508    0.029  121.459    0.000
   .N3                3.217    0.030  106.147    0.000
   .N4                3.185    0.030  106.894    0.000
   .N5                2.969    0.031   96.663    0.000
    extra             0.000
    open              0.000
    neuro             0.000
```

CFA Example

```
partSummary(cfaOut, 10)

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .E1                1.713    0.054   31.442    0.000
   .E2                1.224    0.049   24.952    0.000
   .E3                1.163    0.038   30.388    0.000
   .E4                1.166    0.041   28.522    0.000
   .E5                1.272    0.039   32.789    0.000
   .O1                0.878    0.031   28.320    0.000
   .O2                2.083    0.062   33.705    0.000
   .O3                0.686    0.043   16.130    0.000
   .O4                1.407    0.039   36.236    0.000
   .O5                1.401    0.044   31.837    0.000
   .N1                0.848    0.037   23.029    0.000
   .N2                0.842    0.035   24.184    0.000
   .N3                1.228    0.043   28.308    0.000
   .N4                1.666    0.051   32.808    0.000
   .N5                1.942    0.056   34.465    0.000
    extra             1.000
    open              1.000
    neuro             1.000
```

SEM Example

```r
## Add structural paths:
semMod <- '
extra =~ E1 + E2 + E3 + E4 + E5
open  =~ O1 + O2 + O3 + O4 + O5
neuro =~ N1 + N2 + N3 + N4 + N5

neuro ~ extra + open
'

## Estimate the model:
semOut <- sem(semMod, data = bfi, missing = "fiml", std.lv = TRUE)

## Check the fit:
fitMeasures(semOut,
            c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr")
            )

   chisq       df   pvalue      cfi      tli    rmsea     srmr
2251.679   87.000    0.000    0.809    0.769    0.094    0.081
```

SEM Example

```
partSummary(semOut, 7)

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  extra =~
    E1                0.973    0.032   30.607    0.000
    E2                1.163    0.030   38.172    0.000
    E3               -0.815    0.027  -30.358    0.000
    E4               -0.979    0.028  -35.254    0.000
    E5               -0.714    0.027  -26.638    0.000
  open =~
    O1                0.630    0.025   24.886    0.000
    O2               -0.605    0.036  -16.781    0.000
    O3                0.897    0.029   30.765    0.000
    O4                0.290    0.028   10.402    0.000
    O5               -0.602    0.031  -19.734    0.000
  neuro =~
    N1                1.235    0.027   45.916    0.000
    N2                1.183    0.026   45.360    0.000
    N3                1.123    0.028   39.976    0.000
    N4                0.866    0.029   29.605    0.000
    N5                0.799    0.031   26.204    0.000
```

SEM Example

```
partSummary(semOut, 8:9)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  neuro ~
    extra             0.241    0.030    8.169    0.000
    open             -0.014    0.031   -0.448    0.654

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  extra ~~
    open             -0.444    0.024  -18.472    0.000
```

SEM Example

```
partSummary(semOut, 10)

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .E1                2.974    0.031   96.223    0.000
   .E2                3.143    0.030  103.424    0.000
   .E3                4.002    0.026  156.117    0.000
   .E4                4.421    0.028  160.350    0.000
   .E5                4.417    0.025  174.595    0.000
   .O1                4.816    0.021  224.964    0.000
   .O2                2.713    0.030   91.745    0.000
   .O3                4.436    0.023  191.555    0.000
   .O4                4.892    0.023  211.520    0.000
   .O5                2.490    0.025   98.932    0.000
   .N1                2.932    0.030   98.589    0.000
   .N2                3.508    0.029  121.459    0.000
   .N3                3.217    0.030  106.146    0.000
   .N4                3.185    0.030  106.894    0.000
   .N5                2.969    0.031   96.663    0.000
    extra             0.000
    open              0.000
   .neuro             0.000
```

SEM Example

```
partSummary(semOut, 11)

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .E1                1.713    0.054   31.442    0.000
   .E2                1.224    0.049   24.952    0.000
   .E3                1.163    0.038   30.388    0.000
   .E4                1.166    0.041   28.522    0.000
   .E5                1.272    0.039   32.789    0.000
   .O1                0.878    0.031   28.320    0.000
   .O2                2.083    0.062   33.705    0.000
   .O3                0.686    0.043   16.130    0.000
   .O4                1.407    0.039   36.236    0.000
   .O5                1.401    0.044   31.837    0.000
   .N1                0.848    0.037   23.029    0.000
   .N2                0.842    0.035   24.184    0.000
   .N3                1.228    0.043   28.308    0.000
   .N4                1.666    0.051   32.808    0.000
   .N5                1.942    0.056   34.465    0.000
    extra             1.000
    open              1.000
   .neuro             1.000
```

Why SEM?

The beauty of SEM is that we get to model the types of complex relations we can specify via path models while leveraging all the strengths of latent variables.

- Multiple-group SEM models moderation by group.
  - The latent variables give us the ability to evaluate measurement invariance across groups.
  - We'll see more of these ideas in the next lecture.
- Path analysis and SEM lend themselves especially well to mediation analysis and conditional process analysis.

MEDIATION

Mediation vs. Moderation

What do we mean by *mediation* and *moderation*?

Mediation and moderation are types of hypotheses, not statistical methods or models.

- Mediation tells us *how* one variable influences another.
- Moderation tells us *when* one variable influences another.

Contextualizing Example

Say we wish to explore the process underlying exercise habits.

Our first task is to operationalize “exercise habits”

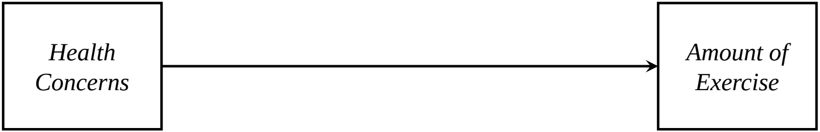
- DV: Hours per week spent in vigorous exercise (*exerciseAmount*).

We may initial ask: what predicts devoting more time to exercise?

- IV: Concerns about negative health outcomes (*healthConcerns*).

Focal Effect Only

The *healthConcerns* → *exerciseAmount* relation is our *focal effect*

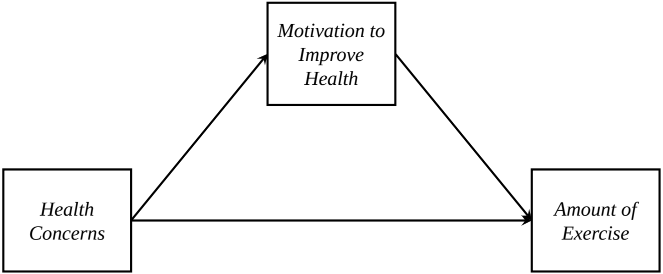


- Mediation, moderation, and conditional process analysis all attempt to describe the focal effect in more detail.
- We always begin by hypothesizing a focal effect.

The Mediation Hypothesis

A mediation analysis will attempt to describe how health concerns affect amount of exercise.

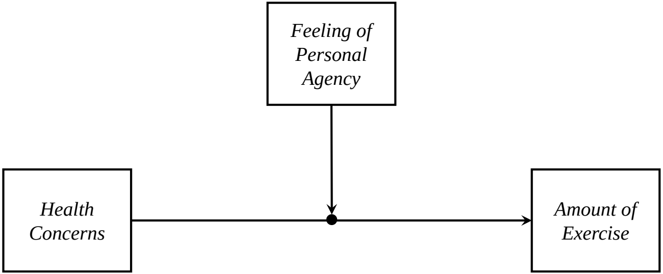
- The *how* is operationalized in terms of intermediary variables.
- Mediator: Motivation to improve health (*motivation*).



Moderation Hypothesis

A moderation hypothesis will attempt to describe when health concerns affect amount of exercise.

- The *when* is operationalized in terms of interactions between the focal predictor and contextualizing variables
- Moderator: Sense of personal agency relating to physical health (*agency*).

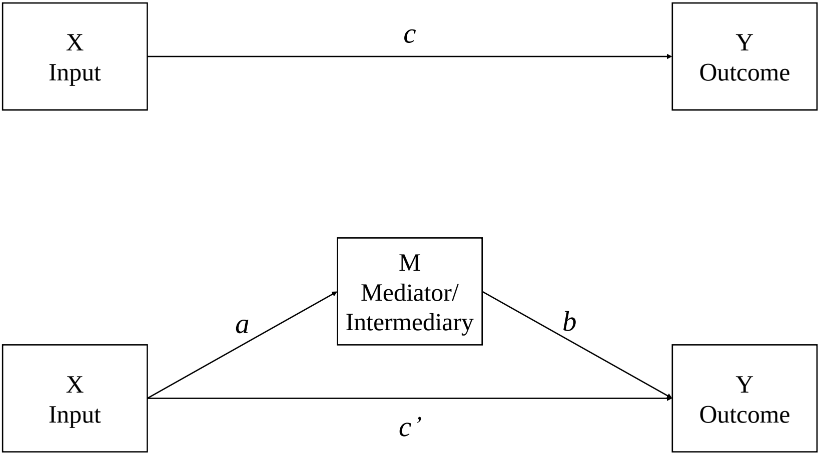


Conditional Process Analysis

Conditional process analysis combines the mediation and moderation hypotheses into models of moderated mediation.

- Given a mediation model describing *how* health concerns affect exercise amount, what other variables may modulate the indirect effect.

Path Diagrams



Necessary Equations

To get all the pieces of the preceding diagram using OLS regression, we'll need to fit three seperate models.

$$Y = i_1 + cX + e_1 \tag{1}$$
$$Y = i_2 + c'X + bM + e_2 \tag{2}$$
$$M = i_3 + aX + e_3 \tag{3}$$

- Equation 1 gives us the total effect (c).
- Equation 2 gives us the direct effect (c') and the partialled effect of the mediator on the outcome (b).
- Equation 3 gives us the effect of the input on the outcome (a).

Two Measures of Indirect Effect

Indirect effects can be quantified in two different ways:

$$IE_{diff} = c - c' \tag{4}$$

$$IE_{prod} = a \cdot b \tag{5}$$

IE_{diff} and IE_{prod} are equivalent in simple mediation.

- Both give us information about the proportion of the total effect that is transmitted through the intermediary variable.
- IE_{prod} provides a more direct representation of the actual pathway we're interested in testing.
- IE_{diff} gets at our desired hypothesis indirectly.

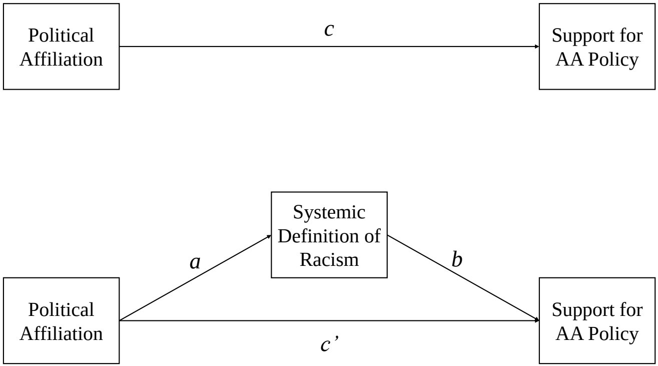
The Causal Steps Approach

Baron and Kenny (1986, p. 1176) describe three/four conditions as being sufficient to demonstrate statistical "mediation."

1. Variations in levels of the independent variable significantly account for variations in the presumed mediator (i.e., Path *a*).
   - Need a significant a path.
2. Variations in the mediator significantly account for variations in the dependent variable (i.e., Path *b*).
   - Need a significant b path.
3. When Paths *a* and *b* are controlled, a previously significant relation between the independent and dependent variables is no longer significant.
   - Need a significant total effect
   - The direct effect must be "less" than the total effect

Example Process Model

Consider the following process.



Causal Steps Example

```r
## Load some data:
dat1 <- readRDS("../data/adamsKlpsScaleScore.rds")

## Check pre-conditions:
mod1 <- lm(policy ~ polAffil, data = dat1)
mod2 <- lm(policy ~ sysRac, data = dat1)
mod3 <- lm(sysRac ~ polAffil, data = dat1)

## Partial out the mediator's effect:
mod4 <- lm(policy ~ sysRac + polAffil, data = dat1)
```

Causal Steps Example

```
summary(mod1)


Call:
lm(formula = policy ~ polAffil, data = dat1)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.7357 -0.8254  0.0643  0.6827  3.2481 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2.71516    0.35648   7.617 3.32e-11 ***
polAffil     0.23675    0.07775   3.045   0.0031 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.134 on 85 degrees of freedom
Multiple R-squared:  0.09836,Adjusted R-squared:  0.08775 
F-statistic: 9.273 on 1 and 85 DF,  p-value: 0.003096
```

Causal Steps Example

```
summary(mod2)


Call:
lm(formula = policy ~ sysRac, data = dat1)

Residuals:
     Min       1Q   Median       3Q      Max
-2.28970 -0.53821  0.08866  0.64015  3.08343

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.1218     0.4883   2.297   0.0241 *
sysRac        0.6649     0.1210   5.494 4.03e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.026 on 85 degrees of freedom
Multiple R-squared:  0.262,Adjusted R-squared:  0.2534
F-statistic: 30.18 on 1 and 85 DF,  p-value: 4.029e-07
```

Causal Steps Example

```
summary(mod3)


Call:
lm(formula = sysRac ~ polAffil, data = dat1)

Residuals:
    Min      1Q  Median      3Q     Max
-2.2187 -0.5449 -0.2115  0.6182  1.9516

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.19726    0.27634  11.570   <2e-16 ***
polAffil     0.17023    0.06027   2.825   0.0059 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8788 on 85 degrees of freedom
Multiple R-squared:  0.08581,Adjusted R-squared:  0.07505
F-statistic: 7.978 on 1 and 85 DF,  p-value: 0.005898
```

Causal Steps Example

```
summary(mod4)


Call:
lm(formula = policy ~ sysRac + polAffil, data = dat1)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.1370 -0.6338 -0.0020  0.6658  3.4674 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.80704    0.51013   1.582   0.1174    
sysRac       0.59680    0.12478   4.783  7.3e-06 ***
polAffil     0.13515    0.07252   1.864   0.0658 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.011 on 84 degrees of freedom
Multiple R-squared:  0.2913,Adjusted R-squared:  0.2745 
F-statistic: 17.27 on 2 and 84 DF,  p-value: 5.228e-07
```

Causal Steps Example

```
## Extract important parameter estimates:
a      <- coef(mod3)["polAffil"]
b      <- coef(mod4)["sysRac"]
c      <- coef(mod1)["polAffil"]
cPrime <- coef(mod4)["polAffil"]

## Compute indirect effects:
ieDiff <- unname(c - cPrime)
ieProd <- unname(a * b)

ieDiff

[1] 0.1015958

ieProd

[1] 0.1015958
```

Sobel's Z

In the previous example, do we have a *significant* indirect effect?

- The direct effect is “substantially” smaller than the total effect, but is the difference statistically significant?
- Sobel (1982) developed an asymptotic standard error for IE_{prod} that we can use to assess this hypothesis.

$$SE_{sobel} = \sqrt{a^2 \cdot SE_b^2 + b^2 \cdot SE_a^2} \tag{6}$$

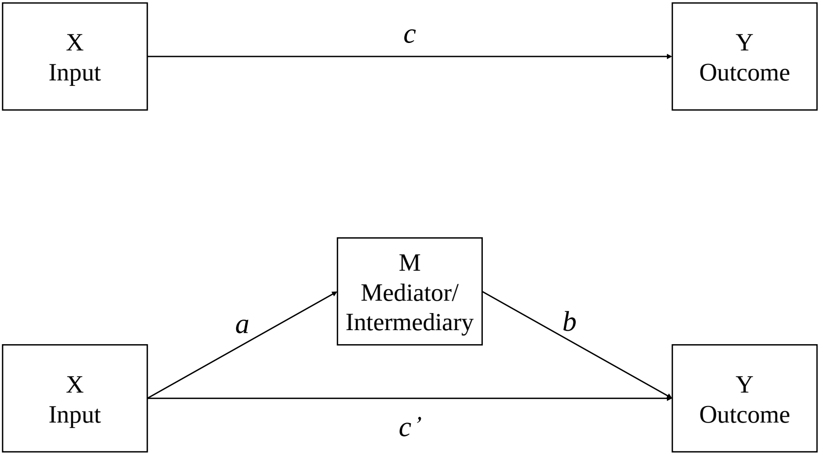
$$Z_{sobel} = \frac{ab}{SE_{sobel}} \tag{7}$$

$$95\%CI_{sobel} = ab \pm 1.96 \cdot SE_{sobel} \tag{8}$$

Sobel Example

```
## SE:
seA <- (mod3 %>% vcov() %>% diag() %>% sqrt())["polAffil"]
seB <- (mod4 %>% vcov() %>% diag() %>% sqrt())["sysRac"]

se <- sqrt(b^2 * seA^2 + a^2 * seB^2) %>% unname()

## z-score:
(z <- ieProd / se)

[1] 2.432107

## p-value:
(p <- 2 * pnorm(z, lower = FALSE))

[1] 0.01501126

## 95% CI:
c(ieProd - 1.96 * se, ieProd + 1.96 * se)

[1] 0.01972121 0.18347034
```

Recall our Basic Path Diagram



Two Measures of Indirect Effect

Recall the two definitions of an indirect effect:

$$IE_{diff} = c - c' \tag{9}$$

$$IE_{prod} = a \cdot b \tag{10}$$

It pays to remember a few key points:

- IE_{diff} and IE_{prod} are equivalent in simple mediation.
- IE_{diff} is only an indirect indication of IE_{prod}.
- A significant indirect effect can exist without a significant total effect.
- If we only care about the indirect effect, then we don't need to worry about the total effect.

Two Measures of Indirect Effect

Recall the two definitions of an indirect effect:

$$IE_{diff} = c - c' \tag{9}$$

$$IE_{prod} = a \cdot b \tag{10}$$

It pays to remember a few key points:

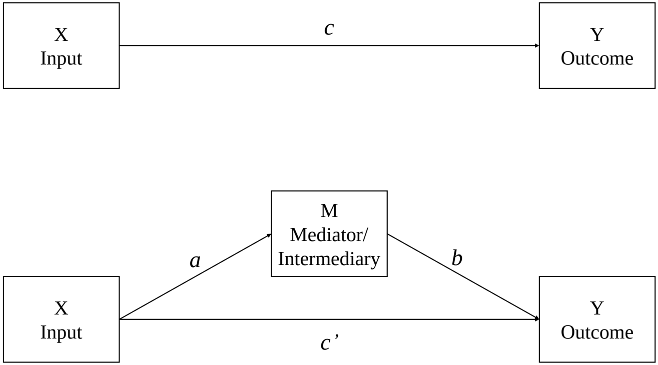
- IE_{diff} and IE_{prod} are equivalent in simple mediation.
- IE_{diff} is only an indirect indication of IE_{prod}.
- A significant indirect effect can exist without a significant total effect.
- If we only care about the indirect effect, then we don't need to worry about the total effect.

These points imply something interesting:

- We don't need to estimate c!

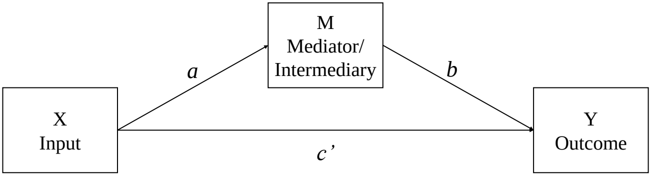
Simplifying our Path Diagram

QUESTION: If we don't care about directly estimating *c*, how can we simplify this diagram?

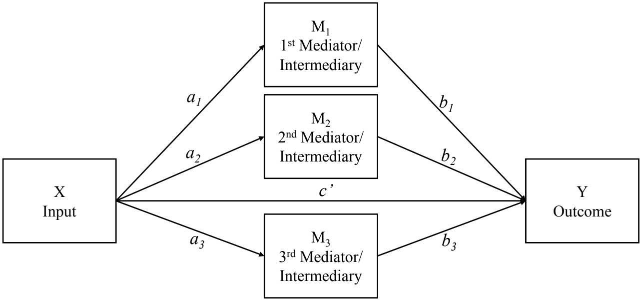


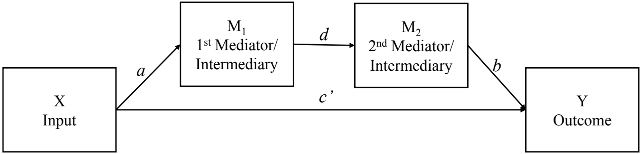
Simplifying our Path Diagram

ANSWER: We don't fit the upper model.



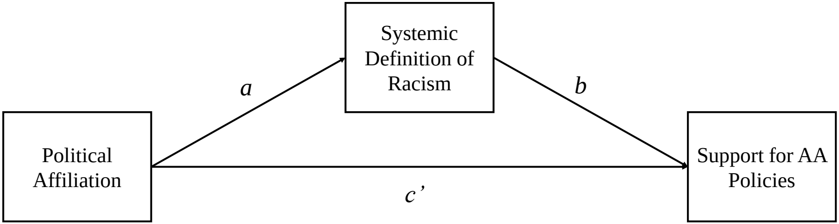
Why Path Analysis?





Example

Let's revisit the above example using path analysis in **lavaan**.



Example

```r
## Load the lavaan package:
library(lavaan)

## Specify the basic path model:
mod1 <- '
policy ~ 1 + sysRac + polAffil
sysRac ~ 1 + polAffil
'

## Estimate the model:
out1 <- sem(mod1, data = dat1)
```

Example

```
## Look at the results:
partSummary(out1, 7:9)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    sysRac            0.597    0.123    4.867    0.000
    polAffil          0.135    0.071    1.897    0.058
  sysRac ~
    polAffil          0.170    0.060    2.858    0.004

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.807    0.501    1.610    0.107
   .sysRac            3.197    0.273   11.705    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.987    0.150    6.595    0.000
   .sysRac            0.755    0.114    6.595    0.000
```

Example

```r
## Include the indirect effect:
mod2 <- '
policy ~ 1 + b*sysRac + polAffil
sysRac ~ 1 + a*polAffil

ab := a*b # Define a parameter for the indirect effect
'

## Estimate the model:
out2 <- sem(mod2, data = dat1)
```

Example

```r
## Look at the results:
partSummary(out2, 7:8)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    sysRac    (b)     0.597    0.123    4.867    0.000
    polAffil          0.135    0.071    1.897    0.058
  sysRac ~
    polAffil  (a)     0.170    0.060    2.858    0.004

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.807    0.501    1.610    0.107
   .sysRac            3.197    0.273   11.705    0.000
```

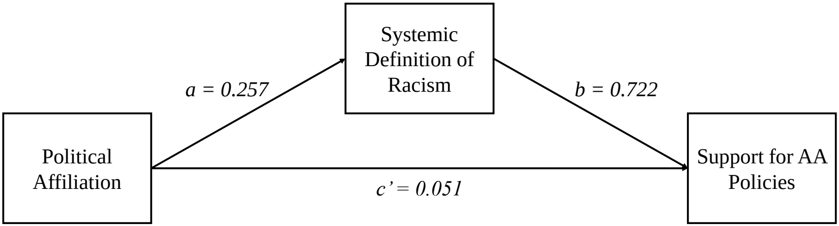
Example

```
partSummary(out2, 9:10)

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.987    0.150    6.595    0.000
   .sysRac            0.755    0.114    6.595    0.000

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab                0.102    0.041    2.464    0.014
```

Example

```
## We can also get CIs:
parameterEstimates(out2, zstat = FALSE, pvalue = FALSE, ci = TRUE)

        lhs op      rhs label   est    se ci.lower ci.upper
1    policy ~1                0.807 0.501   -0.175    1.789
2    policy  ~   sysRac     b 0.597 0.123    0.356    0.837
3    policy  ~ polAffil       0.135 0.071   -0.005    0.275
4    sysRac ~1                3.197 0.273    2.662    3.733
5    sysRac  ~ polAffil     a 0.170 0.060    0.053    0.287
6    policy ~~   policy       0.987 0.150    0.694    1.280
7    sysRac ~~   sysRac       0.755 0.114    0.530    0.979
8  polAffil ~~ polAffil       2.444 0.000    2.444    2.444
9  polAffil ~1                4.310 0.000    4.310    4.310
10       ab :=      a*b    ab 0.102 0.041    0.021    0.182
```

Results


Systemic Definition of Racism
a = 0.257
b = 0.722
Political Affiliation
Support for AA Policies
c' = 0.051

We’re not there yet...

Path analysis allows us to directly model complex (and simple) relations, but the preceding example still suffers from a considerable limitation.

- The significance test for the indirect effect is still conducted with the Sobel Z approach.

Path analysis (or full SEM) doesn’t magically get around distributional problems associated with Sobel’s Z test.

- To get a robust significance test of the indirect effect, we need to use *bootstrapping*.

Bootstrapping

Bootstrapping was introduced by Efron (1979) as a tool for non-parametric inference.

- Traditional inference requires that we assume a parametric sampling distribution for our focal parameter.
- We need to make such an assumption to compute the standard errors we require for inferences.
- If we cannot safely make these assumptions, we can use bootstrapping.

Bootstrapping

Assume our observed data $Data_0$ represent the population and:

1. Sample rows of $Data_0$, with replacement, to create B new samples $\{Data_b\}$.
2. Calculate our focal statistic on each of the B bootstrap samples.
3. Make inferences based on the empirical distribution of the B estimates calculated in Step 2

Bootstrapping

$\text{Data}_0 =$

ID	X	Y
1	$x_{i=1}$	$y_{i=1}$
2	$x_{i=2}$	$y_{i=2}$
3	$x_{i=3}$	$y_{i=3}$
4	$x_{i=4}$	$y_{i=4}$
5	$x_{i=5}$	$y_{i=5}$

$\text{Data}_{b=1} =$

ID	X	Y
3	$x_{i=3}$	$y_{i=3}$
2	$x_{i=2}$	$y_{i=2}$
4	$x_{i=4}$	$y_{i=4}$
2	$x_{i=2}$	$y_{i=2}$
3	$x_{i=3}$	$y_{i=3}$

, $\text{Data}_{b=2} =$

ID	X	Y
1	$x_{i=1}$	$y_{i=1}$
3	$x_{i=3}$	$y_{i=3}$
3	$x_{i=3}$	$y_{i=3}$
4	$x_{i=4}$	$y_{i=4}$
5	$x_{i=5}$	$y_{i=5}$

, $\text{Data}_{b=3} =$

ID	X	Y
3	$x_{i=3}$	$y_{i=3}$
5	$x_{i=5}$	$y_{i=5}$
1	$x_{i=1}$	$y_{i=1}$
4	$x_{i=4}$	$y_{i=4}$
3	$x_{i=3}$	$y_{i=3}$

, $\cdots$ $\text{Data}_{b=B} =$

ID	X	Y
2	$x_{i=2}$	$y_{i=2}$
5	$x_{i=5}$	$y_{i=5}$
1	$x_{i=1}$	$y_{i=1}$
1	$x_{i=1}$	$y_{i=1}$
4	$x_{i=4}$	$y_{i=4}$

Example

Suppose I'm on the lookout for a retirement location. Since I want to relax in my old-age, I'm concerned with ensuring a low probability of dragon attacks, so I have a few salient considerations:

- Shooting for a location with no dragons, whatsoever, is a fools errand (since dragons are, obviously, ubiquitous).
- I merely require a location that has at least two times as many dragon-free days as other kinds.

Example

I've been watching several candidate locales over the course of my (long and illustrious) career, and I'm particularly hopeful about one quiet hamlet in the Patagonian highlands.

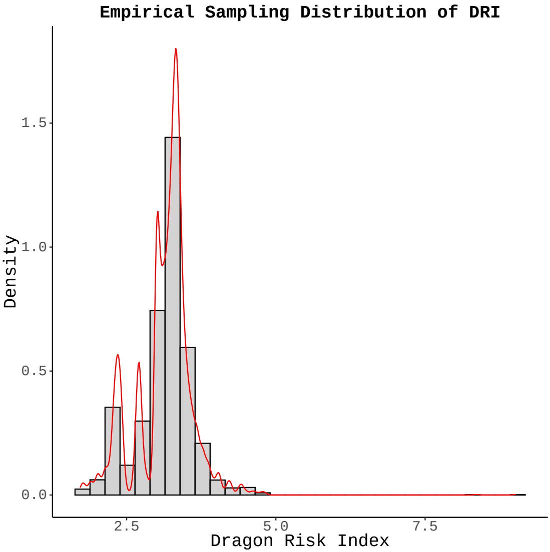
- To ensure that my required degree of dragon-freeness is met, I'll use the *Dragon Risk Index* (DRI):

$$DRI = Median\left(\frac{\text{Dragon-Free Days}}{\text{Dragonned Days}}\right)$$

Example

```r
## Read in the observed data:
rawData <- readRDS("../data/daysData.rds")

## Compute the observed test statistic:
obsDRI <- median(rawData$goodDays / rawData$badDays)
obsDRI

[1] 3.24476

## Draw the bootstrap samples:
set.seed(235711)
nSams <- 5000
bootDRI <- rep(NA, nSams)
for(b in 1:nSams) {
    bootSam    <- rawData[sample(1:nrow(rawData), replace = TRUE), ]
    bootDRI[b] <- median(bootSam$goodDays / bootSam$badDays)
}
```

Example



Empirical Sampling Distribution of DRI
Density
1.5
1.0
0.5
0.0
2.5
5.0
7.5
Dragon Risk Index

Example

To see if I can be confident in the dragon-freeness of my potential home, I'll summarize the preceding distribution with a (one-tailed) percentile confidence interval:

```
bootLB <- sort(bootDRI)[0.05 * nSams]
bootUB <- Inf

## The bootstrapped Percentile CI:
c(bootLB, bootUB)

[1] 2.288555      Inf
```

Bootstrapped Inference for Indirect Effects

We can apply the same procedure to testing the indirect effect.

- The problem with Sobel's Z is exactly the type of issue for which bootstrapping was designed
    - We don't know a reasonable finite-sample sampling distribution for the ab parameter.
- Bootstrapping will allow us to construct an empirical sampling distribution for ab and construct confidence intervals for inference.

Bootstrapped Inference for Indirect Effects

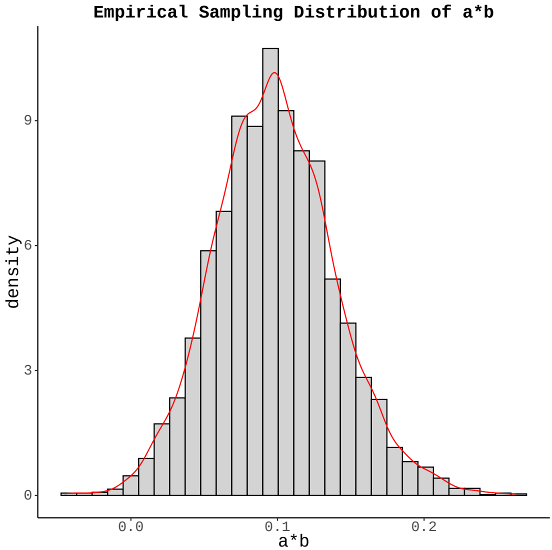
PROCEDURE:

1. Resample our observed data with replacement
2. Fit our hypothesized path model to each bootstrap sample
3. Store the value of ab that we get each time
4. Summarize the empirical distribution of ab to make inferences

Example

```r
abVec <- rep(NA, nSams)
for(i in 1:nSams) {
    ## Resample the data:
    bootSam <- dat1[sample(1:nrow(dat1), replace = TRUE), ]

    ## Fit the path model:
    bootOut <- sem(mod2, data = bootSam)

    ## Store the estimated indirect effect:
    abVec[i] <- coef(bootOut)[c("a", "b")] %>% prod()
}
```

Example



Empirical Sampling Distribution of a*b
density
9
6
3
0
0.0
0.1
0.2
a*b

Example

```r
## Calculate the percentile CI:
lb <- sort(abVec)[0.025 * nSams]
ub <- sort(abVec)[0.975 * nSams]
c(lb, ub)

[1] 0.01983165 0.18521052
```

Example

```r
## Much more parsimoniously:
bootOut2 <- sem(mod2, data = dat1, se = "boot", bootstrap = nSams)

parameterEstimates(bootOut2, zstat = FALSE, pvalue = FALSE)
```

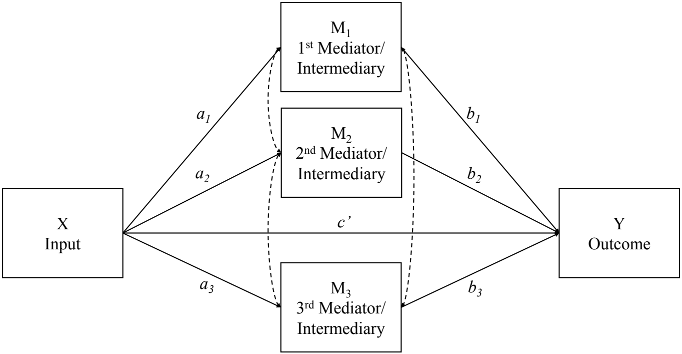
```
        lhs op      rhs label   est    se ci.lower ci.upper
1    policy ~1                0.807 0.554   -0.269    1.912
2    policy  ~   sysRac     b 0.597 0.136    0.314    0.858
3    policy  ~ polAffil       0.135 0.084   -0.030    0.303
4    sysRac ~1                3.197 0.276    2.674    3.782
5    sysRac  ~ polAffil     a 0.170 0.063    0.039    0.292
6    policy ~~   policy       0.987 0.166    0.650    1.291
7    sysRac ~~   sysRac       0.755 0.109    0.530    0.959
8  polAffil ~~ polAffil       2.444 0.000    2.444    2.444
9  polAffil ~1                4.310 0.000    4.310    4.310
10       ab :=      a*b    ab 0.102 0.042    0.024    0.189
```

Simple Mediation is Too Simple

We can justify multiple mediator models by asking: "What mediates the effects in a simple mediation model?"

- Mediation of the direct effect leads to *parallel multiple mediator models*.
- Mediation of the a or b paths produces *serial multiple mediator models*.

Parallel Multiple Mediation



Parallel Multiple Mediation

To get all of the information in the preceding diagram, we need to estimate four equations:

$$
\begin{aligned}
Y &= i_Y + b_1M_1 + b_2M_2 + b_3M_3 + c'X + e_Y \\
M_1 &= i_{M1} + a_1X + e_{M1} \\
M_2 &= i_{M2} + a_2X + e_{M2} \\
M_3 &= i_{M3} + a_3X + e_{M3}
\end{aligned}
$$

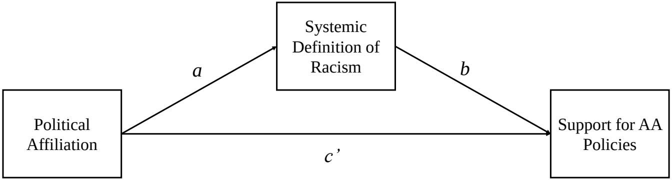
In general, a parallel mediator model with K mediator variables will required $K + 1$ separate equations.

Path modeling can make this task much simpler.

- Also allows us to explicitly estimate the correlations between parallel mediators.

Parallel Multiple Mediation

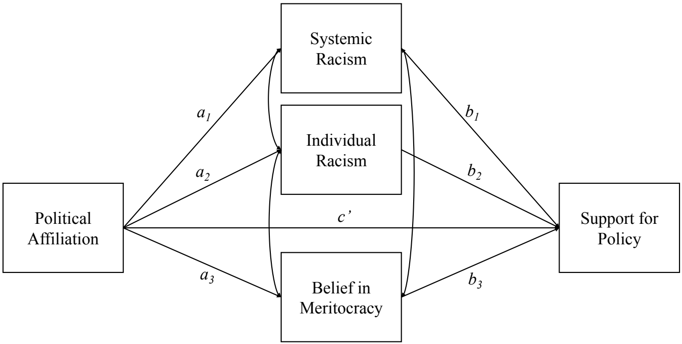
Let's reconsider the last example:



QUESTION: What might be mediating the residual direct effect?

Parallel Multiple Mediation

POTENTIAL ANSWER:



A Quick Note on Inference

In parallel multiple mediation:

- We have K *specific indirect effects*, where K is the number of mediators: $a_1b_1, a_2b_2, \ldots, a_Kb_K$.
- The *Total Indirect Effect* is equal to the sum of all the specific indirect effects: $IE_{tot} = \sum_{k=1}^{K} a_kb_k$.
- The *Total Effect* is equal to the direct effect plus the total indirect effect: $c = c' + IE_{tot}$

Inference for the specific indirect effects is basically the same as it is for the sole indirect effect in simple mediation.

- CAVEAT: Each specific indirect effect must be interpreted as conditional on all other mediators in the model.

Example

```r
## Read in the data
dat1 <- readRDS("../data/adamsKlpsScaleScore.rds")

## Parallel Multiple Mediator Model:
mod1.1 <- '
policy ~ 1 + b1*sysRac + b2*indRac + b3*merit + cp*polAffil
sysRac ~ 1 + a1*polAffil
indRac ~ 1 + a2*polAffil
merit  ~ 1 + a3*polAffil

sysRac ~~ indRac + merit
indRac ~~ merit

ab1 := a1*b1
ab2 := a2*b2
ab3 := a3*b3
totalIE := ab1 + ab2 + ab3
'

## Fit the model:
out1.1 <- sem(mod1.1, data = dat1, se = "boot", bootstrap = 5000)
```

Example

```
## Look at results:
partSummary(out1.1, 7)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    sysRac    (b1)    0.601    0.142    4.243    0.000
    indRac    (b2)    0.143    0.105    1.359    0.174
    merit     (b3)   -0.036    0.153   -0.238    0.812
    polAffil  (cp)    0.125    0.077    1.637    0.102
  sysRac ~
    polAffil  (a1)    0.170    0.065    2.623    0.009
  indRac ~
    polAffil  (a2)   -0.004    0.077   -0.055    0.956
  merit ~
    polAffil  (a3)   -0.266    0.060   -4.429    0.000
```

Example

```
partSummary(out1.1, 9)

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.490    0.886    0.553    0.580
   .sysRac            3.197    0.282   11.325    0.000
   .indRac            3.398    0.319   10.656    0.000
   .merit             4.977    0.259   19.252    0.000
```

Example

```
partSummary(out1.1, c(10, 8))

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.963    0.177    5.454    0.000
   .sysRac            0.755    0.110    6.880    0.000
   .indRac            1.188    0.152    7.816    0.000
   .merit             0.719    0.112    6.444    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  .sysRac ~~
   .indRac           -0.076    0.102   -0.742    0.458
   .merit            -0.217    0.092   -2.343    0.019
  .indRac ~~
   .merit             0.154    0.098    1.577    0.115
```

Example

```
partSummary(out1.1, 11)

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab1               0.102    0.045    2.283    0.022
    ab2              -0.001    0.015   -0.042    0.966
    ab3               0.010    0.042    0.230    0.818
    totalIE           0.111    0.053    2.109    0.035
```

Example

```
parameterEstimates(out1.1, boot.ci.type = "bca.simple") %>%
    select(c("label", "est", "ci.lower", "ci.upper")) %>%
    tail(4)

     label    est ci.lower ci.upper
21     ab1  0.102    0.030    0.213
22     ab2 -0.001   -0.035    0.029
23     ab3  0.010   -0.075    0.090
24 totalIE  0.111    0.015    0.220
```

Comparing Specific Indirect Effects

When we have multiple specific indirect effects in a single model, we can test if they are statistically different from one another.

QUESTION: How might we go about doing such a test (assuming we're using path modeling)?

Comparing Specific Indirect Effects

When we have multiple specific indirect effects in a single model, we can test if they are statistically different from one another.

QUESTION: How might we go about doing such a test (assuming we're using path modeling)?

ANSWER: There are, at least, two reasonable methods:

1. Use nested model $\Delta\chi^2$ tests
2. Define a new parameter to encode the constraint and use bootstrapping

Example

```r
## Test differences in specific indirect effects:
mod1.2 <- '
policy ~ 1 + b1*sysRac + b2*indRac + b3*merit + cp*polAffil
sysRac ~ 1 + a1*polAffil
indRac ~ 1 + a2*polAffil
merit  ~ 1 + a3*polAffil

sysRac ~~ indRac + merit
indRac ~~ merit

ab1 := a1*b1
ab2 := a2*b2
ab3 := a3*b3
totalIE := ab1 + ab2 + ab3

ab1 == ab2 # The first two IEs are constrained to equality
'

out1.2 <- sem(mod1.2, data = dat1)
```

Example

```
## Look at results:
partSummary(out1.2, 7)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    sysRac    (b1)    0.575    0.123    4.662    0.000
    indRac    (b2)    0.192    0.096    2.004    0.045
    merit     (b3)   -0.055    0.131   -0.416    0.678
    polAffil  (cp)    0.125    0.074    1.696    0.090
  sysRac ~
    polAffil  (a1)    0.027    0.025    1.082    0.279
  indRac ~
    polAffil  (a2)    0.082    0.067    1.222    0.222
  merit ~
    polAffil  (a3)   -0.217    0.055   -3.943    0.000
```

Example

```
partSummary(out1.2, 9)

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.497    0.965    0.514    0.607
   .sysRac            3.813    0.146   26.178    0.000
   .indRac            3.025    0.313    9.668    0.000
   .merit             4.766    0.254   18.730    0.000
```

Example

```
partSummary(out1.2, c(10, 8))

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.967    0.147    6.595    0.000
   .sysRac            0.804    0.122    6.595    0.000
   .indRac            1.206    0.183    6.595    0.000
   .merit             0.724    0.110    6.595    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .sysRac ~~
   .indRac           -0.106    0.106   -0.995    0.320
   .merit            -0.234    0.086   -2.731    0.006
 .indRac ~~
   .merit             0.164    0.102    1.615    0.106
```

Example

```
partSummary(out1.2, 11)

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab1               0.016    0.014    1.093    0.274
    ab2               0.016    0.014    1.093    0.274
    ab3               0.012    0.029    0.412    0.680
    totalIE           0.043    0.042    1.038    0.299
```

Example

```
## Conduct a chi-squared difference test:
chiDiff <- fitMeasures(out1.2)["chisq"] - fitMeasures(out1.1)["chisq"]
dfDiff  <- fitMeasures(out1.2)["df"] - fitMeasures(out1.1)["df"]

pchisq(chiDiff, dfDiff, lower = FALSE)

      chisq
0.009440066
```

Example

```
## Same test as above using bootstrapping:
mod1.3 <- '
policy ~ 1 + b1*sysRac + b2*indRac + b3*merit + cp*polAffil
sysRac ~ 1 + a1*polAffil
indRac ~ 1 + a2*polAffil
merit  ~ 1 + a3*polAffil

sysRac ~~ indRac + merit
indRac ~~ merit

ab1 := a1*b1
ab2 := a2*b2
ab3 := a3*b3
totalIE := ab1 + ab2 + ab3

test1 := ab2 - ab1
'

out1.3 <- sem(mod1.3, data = dat1, se = "boot", bootstrap = 5000)
```

Example

```r
## Look at results:
partSummary(out1.3, 7)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    sysRac    (b1)    0.601    0.144    4.161    0.000
    indRac    (b2)    0.143    0.108    1.326    0.185
    merit     (b3)   -0.036    0.152   -0.239    0.811
    polAffil  (cp)    0.125    0.079    1.592    0.111
  sysRac ~
    polAffil  (a1)    0.170    0.063    2.710    0.007
  indRac ~
    polAffil  (a2)   -0.004    0.078   -0.055    0.956
  merit ~
    polAffil  (a3)   -0.266    0.061   -4.390    0.000
```

Example

```
partSummary(out1.3, 9)

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.490    0.894    0.548    0.584
   .sysRac            3.197    0.274   11.684    0.000
   .indRac            3.398    0.325   10.463    0.000
   .merit             4.977    0.260   19.155    0.000
```

Example

```
partSummary(out1.3, c(10, 8))

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.963    0.178    5.424    0.000
   .sysRac            0.755    0.109    6.903    0.000
   .indRac            1.188    0.155    7.680    0.000
   .merit             0.719    0.111    6.479    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .sysRac ~~
   .indRac           -0.076    0.102   -0.742    0.458
   .merit            -0.217    0.090   -2.416    0.016
 .indRac ~~
   .merit             0.154    0.099    1.553    0.120
```

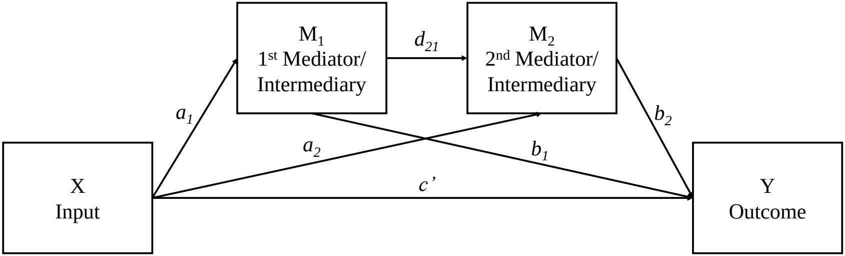
Example

```
partSummary(out1.3, 11)

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab1               0.102    0.044    2.340    0.019
    ab2              -0.001    0.015   -0.042    0.967
    ab3               0.010    0.042    0.231    0.818
    totalIE           0.111    0.050    2.216    0.027
    test1            -0.103    0.048   -2.158    0.031
```

Example

```r
parameterEstimates(out1.3, boot.ci.type = "bca.simple") %>%
    select(c("label", "est", "ci.lower", "ci.upper")) %>%
    tail(5)

     label    est ci.lower ci.upper
21     ab1  0.102    0.033    0.209
22     ab2 -0.001   -0.037    0.027
23     ab3  0.010   -0.079    0.088
24 totalIE  0.111    0.019    0.220
25   test1 -0.103   -0.215   -0.022
```

Serial Multiple Mediation



Serial Multiple Mediation

To get all of the information in the preceding diagram, we need to estimate three equations:

$$
\begin{aligned}
Y &= i_Y + b_1 M_1 + b_2 M_2 + c' X + e_Y \\
M_2 &= i_{M2} + d_{21} M_1 + a_2 X + e_{M2} \\
M_1 &= i_{M1} + a_1 X + e_{M1}
\end{aligned}
$$

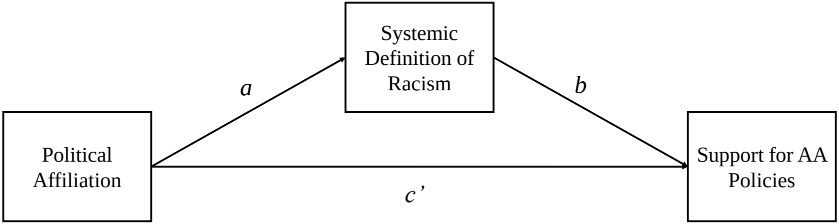
As with parallel mediator models, a serial mediator model with K mediator variables will required $K + 1$ separate equations.

Again, path modeling can make this task much simpler.

- Also allows us to fit more parsimonious, restricted models.

Serial Multiple Mediation

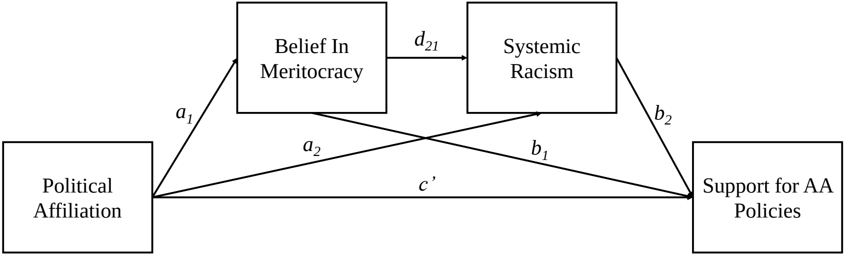
OK, back to our simple mediation example:



QUESTION: What could be mediating the *a* path?

Serial Multiple Mediation

POTENTIAL ANSWER:



A Quick Note on Inference

Parallel multiple mediation operates much like a number of combined simple mediation models.

- Serial multiple mediation is not so straight-forward.

In serial multiple mediation:

- Every possible path from X to Y that passes through, at least, one mediator is a specific indirect effect.
  - With the saturated two-mediator model shown above, we have: $IE_{spec} = \{a_1b_1, a_2b_2, a_1d_{21}b_2\}$
- The *Total Indirect Effect* is, again, equal to the sum of all the specific indirect effects: $IE_{tot} = \sum_{k=1}^{|\{IE_{spec}\}|} IE_{spec,k}$.
- The *Total Effect* is equal to the direct effect plus the total indirect effect: $c = c' + IE_{tot}$

A Quick Note on Inference

Inference for the specific indirect effects is basically the same as it is for the sole indirect effect in simple mediation.

- CAVEAT: Normal-theory, Sobel-Type, standard errors for the specific indirect effects that involve more than two constituent paths can be very complex.
  - This isn't really a problem since you should always use bootstrapping, anyway!

Example

```r
## Serial Multiple Mediator Model:
mod2.1 <- '
policy ~ 1 + b1*merit + b2*sysRac + cp*polAffil
sysRac ~ 1 + d21*merit + a2*polAffil
merit  ~ 1 + a1*polAffil

ab1 := a1*b1
ab2 := a2*b2
fullIE := a1*d21*b2
totalIE := ab1 + ab2 + fullIE
'

out2.1 <- sem(mod2.1, data = dat1, se = "boot", bootstrap = 5000)
```

Example

```
## Check the results:
partSummary(out2.1, 7)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    merit     (b1)   -0.008    0.147   -0.051    0.959
    sysRac    (b2)    0.595    0.145    4.087    0.000
    polAffil  (cp)    0.134    0.076    1.750    0.080
  sysRac ~
    merit    (d21)   -0.301    0.112   -2.700    0.007
    polAffil  (a2)    0.090    0.072    1.248    0.212
  merit ~
    polAffil  (a1)   -0.266    0.061   -4.391    0.000
```

Example

```
partSummary(out2.1, 8:9)

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.851    0.917    0.928    0.353
   .sysRac            4.698    0.651    7.210    0.000
   .merit             4.977    0.260   19.117    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.987    0.165    5.973    0.000
   .sysRac            0.689    0.092    7.513    0.000
   .merit             0.719    0.115    6.261    0.000
```

Example

```
partSummary(out2.1, 10)

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab1               0.002    0.040    0.050    0.960
    ab2               0.053    0.044    1.209    0.226
    fullIE            0.048    0.026    1.803    0.071
    totalIE           0.103    0.049    2.095    0.036
```

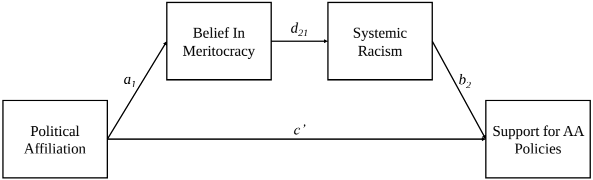
Example

```
parameterEstimates(out2.1, boot.ci.type = "bca.simple") %>%
    select(c("label", "est", "ci.lower", "ci.upper")) %>%
    tail(4)

      label   est ci.lower ci.upper
15      ab1 0.002   -0.079    0.082
16      ab2 0.053   -0.030    0.147
17   fullIE 0.048    0.010    0.116
18  totalIE 0.103    0.011    0.209
```

Restricted Models

In the preceding example, the a_2 and b_1 paths and the specific indirect effects $a_1 b_1$ and $a_2 b_2$ were all non-significant.

- There is a school of thinking that would prescribe constraining the a_2 and b_1 paths to zero as in:



- This model will ascribe a larger effect size to $a_1 d_{21} b_2$ since it must convey all of the indirect influence of X on Y.

Example

```
mod2.2 <- '
policy ~ 1 + cp*polAffil + b2*sysRac
merit  ~ 1 + a1*polAffil
sysRac ~ 1 + d21*merit

fullIE := a1*d21*b2
'

out2.2 <- sem(mod2.2, data = dat1, se = "boot", bootstrap = 5000)
```

Example

```
partSummary(out2.2, 7:8)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    polAffil  (cp)    0.135    0.082    1.650    0.099
    sysRac    (b2)    0.597    0.135    4.435    0.000
  merit ~
    polAffil  (a1)   -0.266    0.061   -4.371    0.000
  sysRac ~
    merit    (d21)   -0.367    0.097   -3.767    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.807    0.563    1.433    0.152
   .merit             4.977    0.260   19.106    0.000
   .sysRac            5.337    0.394   13.539    0.000
```

Example

```
partSummary(out2.2, 9:10)

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.987    0.167    5.895    0.000
   .merit             0.719    0.113    6.333    0.000
   .sysRac            0.705    0.092    7.635    0.000

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    fullIE            0.058    0.025    2.303    0.021
```

Example

```
parameterEstimates(out2.2, boot.ci.type = "bca.simple") %>%
    select(c("label", "est", "ci.lower", "ci.upper")) %>%
    filter(label != "")

   label    est ci.lower ci.upper
1     cp  0.135   -0.034    0.287
2     b2  0.597    0.304    0.846
3     a1 -0.266   -0.390   -0.152
4    d21 -0.367   -0.545   -0.159
5 fullIE  0.058    0.021    0.126
```

Example

As in parallel multiple mediation, we can test for differences in the specific indirect effects of a serial multiple mediator model:

```
mod2.3 <- '
policy ~ 1 + cp*polAffil + b1*merit + b2*sysRac
merit  ~ 1 + a1*polAffil
sysRac ~ 1 + a2*polAffil + d21*merit

ab1 := a1*b1
ab2 := a2*b2
fullIE := a1*d21*b2
totalIE := ab1 + ab2 + fullIE

fullIE == ab1
fullIE == ab2
'

out2.3 <- sem(mod2.3, data = dat1)
```

Example

```
partSummary(out2.3, 7:8)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    polAffil  (cp)    0.108    0.074    1.469    0.142
    merit     (b1)   -0.150    0.046   -3.243    0.001
    sysRac    (b2)    0.521    0.113    4.624    0.000
  merit ~
    polAffil  (a1)   -0.271    0.057   -4.769    0.000
  sysRac ~
    polAffil  (a2)    0.078    0.023    3.364    0.001
    merit    (d21)   -0.287    0.073   -3.925    0.000

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            1.794    0.435    4.124    0.000
   .merit             4.998    0.261   19.122    0.000
   .sysRac            4.695    0.237   19.826    0.000
```

Example

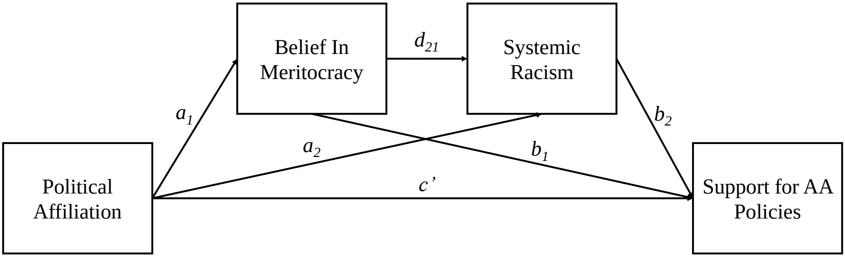
```
partSummary(out2.3, 9:10)

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            1.001    0.152    6.595    0.000
   .merit             0.719    0.109    6.595    0.000
   .sysRac            0.690    0.105    6.595    0.000

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab1               0.041    0.014    2.873    0.004
    ab2               0.041    0.014    2.873    0.004
    fullIE            0.041    0.014    2.873    0.004
    totalIE           0.122    0.042    2.873    0.004
```

Example

```r
## Conduct a chi-squared difference test:
chiDiff <- fitMeasures(out2.3)["chisq"] - fitMeasures(out2.1)["chisq"]
dfDiff  <- fitMeasures(out2.3)["df"] - fitMeasures(out2.1)["df"]

pchisq(chiDiff, dfDiff, lower = FALSE)

   chisq
0.513125
```

Serial Multiple Mediation

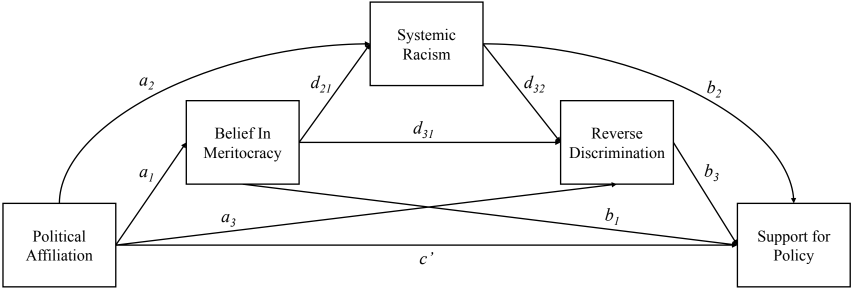
OK. We've supported an interesting hypothesis with the following model, but why stop there?



QUESTION: What might mediated the b_2 path?

Serial Multiple Mediation

Potential Answer:



Serial Multiple Mediation

QUESTION: How many equations do we need to get the information in the preceding diagram?

Serial Multiple Mediation

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$$
\begin{aligned}
Policy &= i_Y + b_1 Merit + b_2 SysRac + b_3 RevDisc + c' PolAff + e_Y \\
RevDisc &= i_{M3} + d_{31} Merit + d_{32} SysRac + a_3 PolAff + e_{M3} \\
SysRac &= i_{M2} + d_{21} Merit + a_2 PolAff + e_{M2} \\
Merit &= i_{M1} + a_1 PolAff + e_{M1}
\end{aligned}
$$

Which produces the following set of specific indirect effects:

- $a_1 b_1$
- $a_2 b_2$
- $a_3 b_3$
- $a_1 d_{31} b_3$
- $a_1 d_{21} b_2$
- $a_2 d_{32} b_3$
- $a_1 d_{21} d_{32} b_3$

Example

```
## Serial Multiple Mediator Model with 3 Mediators:
mod3.1 <- '
policy  ~ 1 + b1*merit + b2*sysRac + b3*revDisc + cp*polAffil
revDisc ~ 1 + d31*merit + d32*sysRac + a3*polAffil
sysRac  ~ 1 + d21*merit + a2*polAffil
merit   ~ 1 + a1*polAffil

ab1 := a1*b1
ab2 := a2*b2
ab3 := a3*b3

partIE1 := a1*d31*b3
partIE2 := a1*d21*b2
partIE3 := a2*d32*b3

fullIE := a1*d21*d32*b3

totalIE := ab1 + ab2 + ab3 + partIE1 + partIE2 + partIE3 + fullIE
'

out3.1 <- sem(mod3.1, data = dat1, se = "boot", bootstrap = 5000)
```

Example

```
partSummary(out3.1, 7)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    merit      (b1)   0.005    0.141    0.035    0.972
    sysRac     (b2)   0.589    0.147    4.003    0.000
    revDisc    (b3)  -0.026    0.082   -0.323    0.747
    polAffil   (cp)   0.130    0.079    1.633    0.103
  revDisc ~
    merit     (d31)   0.473    0.190    2.493    0.013
    sysRac    (d32)  -0.196    0.240   -0.818    0.413
    polAffil   (a3)  -0.149    0.133   -1.122    0.262
  sysRac ~
    merit     (d21)  -0.301    0.112   -2.681    0.007
    polAffil   (a2)   0.090    0.073    1.236    0.217
  merit ~
    polAffil   (a1)  -0.266    0.061   -4.382    0.000
```

Example

```
partSummary(out3.1, 8:9)

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.933    1.005    0.928    0.353
   .revDisc           3.108    1.572    1.977    0.048
   .sysRac            4.698    0.660    7.115    0.000
   .merit             4.977    0.263   18.894    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.985    0.165    5.968    0.000
   .revDisc           2.361    0.307    7.683    0.000
   .sysRac            0.689    0.093    7.387    0.000
   .merit             0.719    0.112    6.434    0.000
```

Example

```
partSummary(out3.1, 10)

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab1              -0.001    0.039   -0.034    0.973
    ab2               0.053    0.044    1.199    0.231
    ab3               0.004    0.017    0.232    0.816
    partIE1           0.003    0.012    0.268    0.788
    partIE2           0.047    0.026    1.796    0.072
    partIE3           0.000    0.003    0.160    0.873
    fullIE            0.000    0.002    0.177    0.859
    totalIE           0.107    0.052    2.055    0.040
```

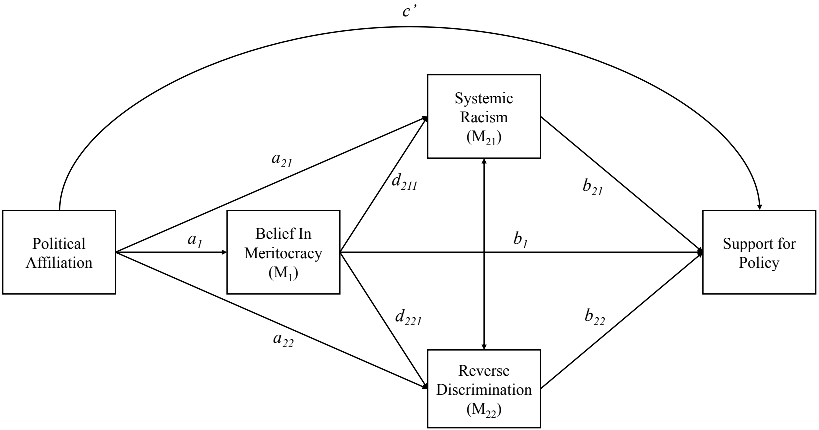
Example

```
parameterEstimates(out3.1, boot.ci.type = "bca.simple") %>%
    select(c("label", "est", "ci.lower", "ci.upper")) %>%
    tail(8)

     label    est ci.lower ci.upper
21     ab1 -0.001   -0.078    0.076
22     ab2  0.053   -0.029    0.146
23     ab3  0.004   -0.018    0.060
24 partIE1  0.003   -0.014    0.039
25 partIE2  0.047    0.012    0.121
26 partIE3  0.000   -0.002    0.016
27  fullIE  0.000   -0.002    0.011
28 totalIE  0.107    0.012    0.219
```

Hybrid Multiple Mediation

We can also combine parallel and serial mediation models:



Example

```
## Hybrid Multiple Mediator Model:
mod4.1 <- '
policy  ~ 1 + b1*merit + b21*sysRac + b22*revDisc + cp*polAffil
sysRac  ~ 1 + d211*merit + a21*polAffil
revDisc ~ 1 + d221*merit + a22*polAffil
merit   ~ 1 + a1*polAffil

sysRac ~~ revDisc

ab1 := a1*b1
ab21 := a21*b21
ab22 := a22*b22

fullIE21 := a1*d211*b21
fullIE22 := a1*d221*b22

totalIE := ab1 + ab21 + ab22 + fullIE21 + fullIE22
'

out4.1 <- sem(mod4.1, data = dat1, se = "boot", bootstrap = 5000)
```

Example

```
partSummary(out4.1, 7)

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    merit     (b1)    0.005    0.143    0.035    0.972
    sysRac   (b21)    0.589    0.149    3.969    0.000
    revDisc  (b22)   -0.026    0.080   -0.329    0.742
    polAffl   (cp)    0.130    0.079    1.634    0.102
  sysRac ~
    merit   (d211)   -0.301    0.110   -2.752    0.006
    polAffl  (a21)    0.090    0.072    1.249    0.212
  revDisc ~
    merit   (d221)    0.532    0.189    2.822    0.005
    polAffl  (a22)   -0.167    0.136   -1.227    0.220
  merit ~
    polAffl   (a1)   -0.266    0.061   -4.358    0.000
```

Example

```
partSummary(out4.1, 8:10)

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .sysRac ~~
   .revDisc          -0.135    0.161   -0.841    0.400

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.933    1.014    0.921    0.357
   .sysRac            4.698    0.646    7.267    0.000
   .revDisc           2.187    1.167    1.873    0.061
   .merit             4.977    0.264   18.866    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .policy            0.985    0.166    5.940    0.000
   .sysRac            0.689    0.092    7.499    0.000
   .revDisc           2.388    0.306    7.793    0.000
   .merit             0.719    0.112    6.421    0.000
```

Example

```
partSummary(out4.1, 11)

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab1              -0.001    0.040   -0.034    0.973
    ab21              0.053    0.044    1.198    0.231
    ab22              0.004    0.018    0.249    0.803
    fullIE21          0.047    0.026    1.828    0.068
    fullIE22          0.004    0.013    0.277    0.782
    totalIE           0.107    0.052    2.067    0.039
```

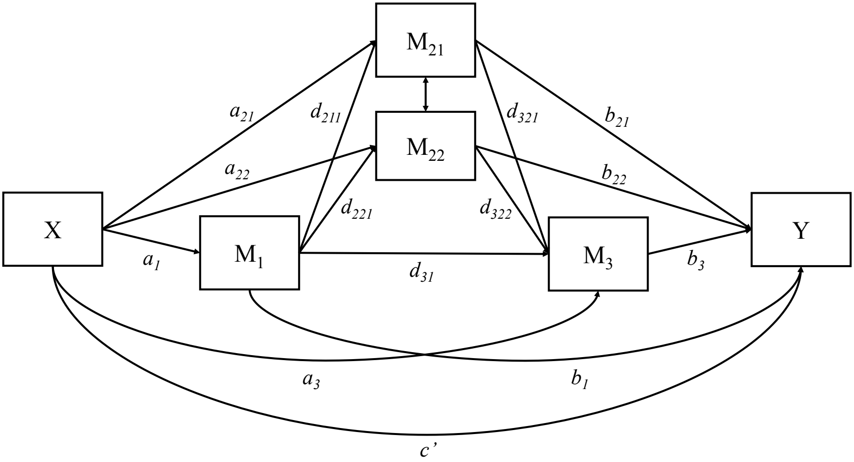
Example

```
parameterEstimates(out4.1, boot.ci.type = "bca.simple") %>%
    select(c("label", "est", "ci.lower", "ci.upper")) %>%
    tail(6)

      label    est ci.lower ci.upper
21      ab1 -0.001   -0.084    0.077
22     ab21  0.053   -0.024    0.157
23     ab22  0.004   -0.018    0.062
24 fullIE21  0.047    0.011    0.117
25 fullIE22  0.004   -0.016    0.042
26  totalIE  0.107    0.013    0.224
```

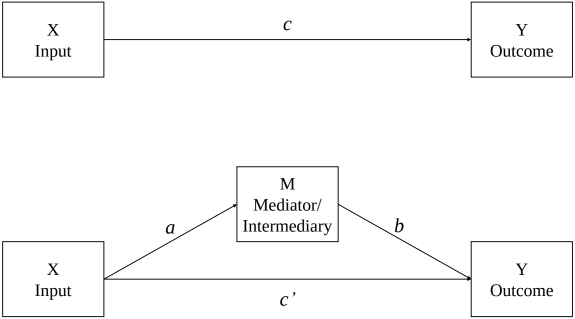
Practice

List all of the specific indirect effects present in this model:



Boring Model

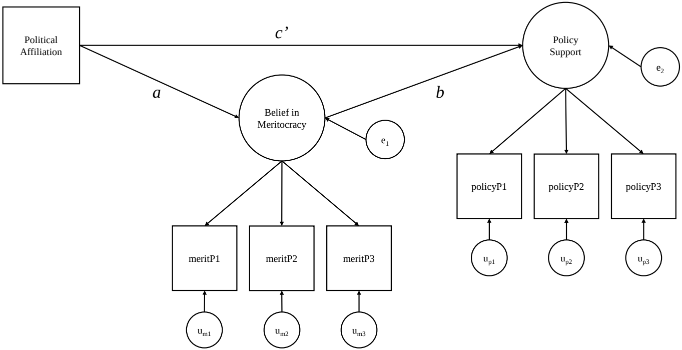
So far, all of our models have looked something like:



But there is no reason that we need to restrict ourselves to mucking around with observed variables.

Better Model

We can (and should) test for indirect effects using full SEMs such as:



Measurement error can be a big problem for mediation analysis, so latent variable modeling is highly recommended.

Example

```r
dat1 <- readRDS("../data/adamsKlpsData.rds") %>% select(-merit, -policy)

## Specify the CFA model:
mod5.1 <- '
merit  =~ meritP1 + meritP2 + meritP3
policy =~ policyP1 + policyP2 + policyP3
'

## Fit the CFA and check model:
out5.1 <- cfa(mod5.1, data = dat1, std.lv = TRUE)

## Check model fit:
fitMeasures(out5.1,
            c("chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr")
            )

 chisq     df  pvalue    cfi    tli  rmsea   srmr
16.869  8.000  0.031  0.922  0.853  0.113  0.065
```

Example

```
partSummary(out5.1, 7)

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  merit =~
    meritP1           0.690    0.134    5.155    0.000
    meritP2           0.968    0.142    6.830    0.000
    meritP3           0.748    0.137    5.458    0.000
  policy =~
    policyP1          0.851    0.186    4.570    0.000
    policyP2          0.996    0.167    5.967    0.000
    policyP3          1.121    0.177    6.339    0.000
```

Example

```
partSummary(out5.1, 8:9)

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  merit ~~
    policy           -0.336    0.131   -2.563    0.010

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .meritP1           0.865    0.165    5.248    0.000
   .meritP2           0.445    0.201    2.211    0.027
   .meritP3           0.833    0.172    4.857    0.000
   .policyP1          1.836    0.324    5.671    0.000
   .policyP2          0.942    0.256    3.683    0.000
   .policyP3          0.857    0.297    2.882    0.004
    merit             1.000
    policy            1.000
```

Example

```r
## Specify the structural model:
mod5.2 <- '
merit  =~ meritP1 + meritP2 + meritP3
policy =~ policyP1 + policyP2 + policyP3

policy ~ b*merit + polAffil
merit  ~ a*polAffil

ab := a*b
'

## Fit the structural model and test the indirect effect:
out5.2 <-
    sem(mod5.2, data = dat1, std.lv = TRUE, se = "boot", bootstrap = 2500)
```

Example

```
partSummary(out5.2, 7:8)

Latent Variables:
                     Estimate  Std.Err  z-value  P(>|z|)
  merit =~
    meritP1             0.545    0.124    4.396    0.000
    meritP2             0.858    0.132    6.506    0.000
    meritP3             0.609    0.116    5.252    0.000
  policy =~
    policyP1            0.799    0.194    4.107    0.000
    policyP2            0.924    3.111    0.297    0.766
    policyP3            1.001    1.708    0.586    0.558

Regressions:
                     Estimate  Std.Err  z-value  P(>|z|)
  policy ~
    merit      (b)     -0.195    0.210   -0.929    0.353
    polAffil            0.169    0.148    1.148    0.251
  merit ~
    polAffil   (a)     -0.411    0.102   -4.034    0.000
```

Example

```
partSummary(out5.2, 9:10)

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .meritP1           0.922    0.182    5.078    0.000
   .meritP2           0.341    0.226    1.507    0.132
   .meritP3           0.869    0.183    4.755    0.000
   .policyP1          1.801    0.337    5.338    0.000
   .policyP2          0.918  166.242    0.006    0.996
   .policyP3          0.922  103.049    0.009    0.993
   .merit             1.000
   .policy            1.000

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    ab                0.080    0.109    0.732    0.464
```

Example

```
parameterEstimates(out5.2, boot.ci.type = "bca.simple") %>%
    select(c("label", "est", "ci.lower", "ci.upper")) %>%
    filter(label != "")

  label    est ci.lower ci.upper
1     b -0.195   -0.593    0.214
2     a -0.411   -0.642   -0.240
3    ab  0.080   -0.091    0.274
```

Interpretation of Indirect Effects

Indirect effects are composed parameters, but they can be interpreted independently of their constituent paths.

- The $X \rightarrow M \rightarrow Y$ indirect effect, ab, is interpreted as:
    - The expected change in Y for a unit change in X that is transmitted indirectly through M.
    - For a unit change in X, Y is expected to change by ab units, indirectly through M.
    - Participants who differ by one unit on X are expect to differ by ab units on Y as a results of the effect of X on M which, in turn, affects Y.

- The interpretation/scaling of the indirect effect is entirely defined by the input, X, and outcome, Y.
    - The scaling of the intermediary variable, M, does not affect the interpretation of the indirect effect.

References

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