Lecture 5: A Bit Advanced Mediation

Kyle M. Lang

Institute for Measurement, Methodology, Analysis & Policy
Texas Tech University
Lubbock, TX

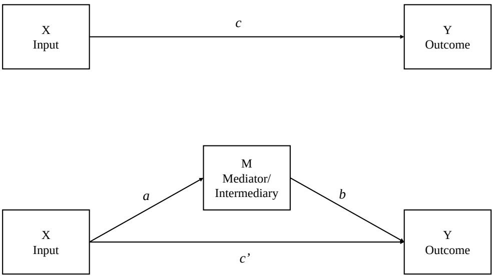
2016 Stats Camp

Outline

- Show how to test for indirect effects in latent variable models
- Discuss the interpretation of indirect effects
- Discuss effect size measures for indirect effects

Boring Model

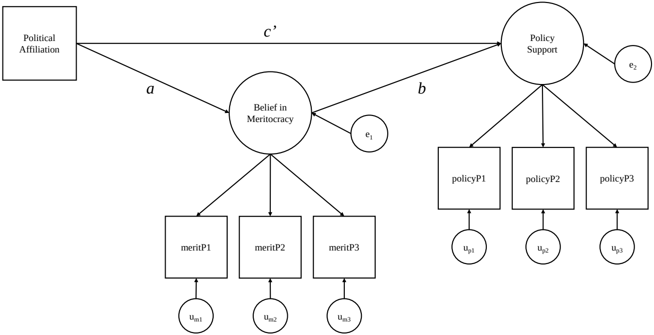
So far, all of our models have been similar to:



But there is no reason that we need to restrict ourselves to mucking about with observed variables.

Better Model

We can (and should) test for indirect effects using *latent variable models* such as:



Measurement error can be a big problem for mediation analysis, so latent variable modeling is highly recommended.

Example

```r
library(lavaan)
dataDir <- "../data/"
dat1 <- readRDS(paste0(dataDir, "adamsKlpsData.rds"))
## Specify the CFA model:
mod1.1 <- "
merit =~ meritP1 + meritP2 + meritP3
policy =~ policyP1 + policyP2 + policyP3
"
## Fit the CFA and check model:
out1.1 <- cfa(mod1.1, data = dat1, std.lv = TRUE)
## Check model fit:
round(fitMeasures(out1.1)[c("chisq", "df", "pvalue", "cfi",
                            "tli", "rmsea", "srmr")], 4)
```

```
  chisq       df   pvalue      cfi      tli    rmsea     srmr
16.8695   8.0000   0.0315   0.9215   0.8529   0.1129   0.0653
```

```r
summary(out1.1)
```

Example

```
lavaan (0.5-20) converged normally after  22 iterations

  Number of observations                            87

  Estimator                                         ML
  Minimum Function Test Statistic               16.869
  Degrees of freedom                                 8
  P-value (Chi-square)                           0.031

Parameter Estimates:

  Information                                 Expected
  Standard Errors                             Standard

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  merit =~
    meritP1           0.690    0.134    5.155    0.000
    meritP2           0.968    0.142    6.830    0.000
    meritP3           0.748    0.137    5.458    0.000
  policy =~
    policyP1          0.851    0.186    4.570    0.000
```

Example

```
    policyP2          0.996    0.167    5.967    0.000
    policyP3          1.121    0.177    6.339    0.000

Covariances:
                   Estimate  Std.Err  Z-value  P(>|z|)
  merit ~~
    policy           -0.336    0.131   -2.563    0.010

Variances:
                   Estimate  Std.Err  Z-value  P(>|z|)
    meritP1           0.865    0.165    5.248    0.000
    meritP2           0.445    0.201    2.211    0.027
    meritP3           0.833    0.172    4.857    0.000
    policyP1          1.836    0.324    5.671    0.000
    policyP2          0.942    0.256    3.683    0.000
    policyP3          0.857    0.297    2.882    0.004
    merit             1.000
    policy            1.000
```

Example

```
## Specify the structural model:
mod1.2 <- "
merit =~ meritP1 + meritP2 + meritP3
policy =~ policyP1 + policyP2 + policyP3

policy ~ b*merit + polAffil
merit ~ a*polAffil

ab := a*b
"
## Fit the structural model and test the indirect effect:
out1.2 <- sem(mod1.2, data = dat1, std.lv = TRUE,
              se = "boot", boot = 2500)
summary(out1.2)
```

Example

```
lavaan (0.5-20) converged normally after  24 iterations

  Number of observations                              87

  Estimator                                           ML
  Minimum Function Test Statistic                 20.665
  Degrees of freedom                                  12
  P-value (Chi-square)                             0.056

Parameter Estimates:

  Information                                   Observed
  Standard Errors                              Bootstrap
  Number of requested bootstrap draws               2500
  Number of successful bootstrap draws              2475

Latent Variables:
                   Estimate  Std.Err  Z-value  P(>|z|)
  merit =~
    meritP1           0.545    0.125    4.374    0.000
    meritP2           0.858    0.150    5.737    0.000
    meritP3           0.609    0.116    5.251    0.000
```

Example

```
  policy =~
    policyP1            0.799     0.193     4.141     0.000
    policyP2            0.924     2.899     0.319     0.750
    policyP3            1.001     2.037     0.491     0.623

Regressions:
                     Estimate   Std.Err   Z-value   P(>|z|)
  policy ~
    merit      (b)     -0.195     0.202    -0.962     0.336
    polAffil            0.169     0.139     1.217     0.224
  merit ~
    polAffil   (a)     -0.411     0.100    -4.097     0.000

Variances:
                     Estimate   Std.Err   Z-value   P(>|z|)
    meritP1             0.922     0.181     5.109     0.000
    meritP2             0.341     0.460     0.740     0.459
    meritP3             0.869     0.185     4.700     0.000
    policyP1            1.801     0.337     5.352     0.000
    policyP2            0.918   173.253     0.005     0.996
    policyP3            0.922   140.529     0.007     0.995
    merit               1.000
    policy              1.000
```

Example

```
Defined Parameters:
                   Estimate  Std.Err  Z-value  P(>|z|)
    ab                0.080    0.090    0.890    0.373
```

```
parameterEstimates(out1.2, boot = "bca.simple")[-c(1 : 3)]
```

```
   label    est      se      z pvalue ci.lower ci.upper
1         0.545   0.125  4.374  0.000    0.337    0.831
2         0.858   0.150  5.737  0.000    0.630    1.119
3         0.609   0.116  5.251  0.000    0.383    0.833
4         0.799   0.193  4.141  0.000    0.422    1.146
5         0.924   2.899  0.319  0.750    0.662    1.424
6         1.001   2.037  0.491  0.623    0.575    1.783
7       b -0.195   0.202 -0.962  0.336   -0.610    0.173
8         0.169   0.139  1.217  0.224   -0.089    0.455
9       a -0.411   0.100 -4.097  0.000   -0.642   -0.235
10        0.922   0.181  5.109  0.000    0.600    1.304
11        0.341   0.460  0.740  0.459   -0.100    0.679
12        0.869   0.185  4.700  0.000    0.562    1.308
13        1.801   0.337  5.352  0.000    1.226    2.583
14        0.918 173.253  0.005  0.996    0.342    1.507
```

Example

15		0.922	140.529	0.007	0.995	-0.038	1.614
16		1.000	0.000	NA	NA	1.000	1.000
17		1.000	0.000	NA	NA	1.000	1.000
18		2.444	0.000	NA	NA	2.444	2.444
19	ab	0.080	0.090	0.890	0.373	-0.063	0.275

Interpretation of Indirect Effects

Although indirect effects are composed parameters, they have direct interpretations, independent of the interpretations of their constituent paths:

- The $X \rightarrow M \rightarrow Y$ indirect effect ab is interpreted as:
  - The expected change in Y for a unit change in X that is transmitted indirectly through M, or...
  - For a unit change in X, Y is expected to change by ab units, indirectly through M, or...
  - Participants who differ by one unit on X are expect to differ by ab units on Y as a results of the effect of X on M which, in turn, affects Y.
- The interpretation/scaling of the indirect effect is entirely defined by the input X and outcome Y
  - The scaling of the intermediary variable M does not affect the interpretation of the indirect effect.

Partially Standardized Indirect Effect

$$ab_{ps} = \frac{ab}{SD_Y}$$
$$c'_{ps} = \frac{c'}{SD_Y}$$
$$c_{ps} = \frac{c}{SD_Y} = ab_{ps} + c'_{ps}$$

- Simple
- Removes binding to the scale of Y
- Still scale-bound by X
- Not clear what constitutes a “large” effect

Completely Standardized Indirect Effect

$$ab_{cs} = \frac{SD_X\, ab}{SD_Y}$$

$$c'_{cs} = \frac{SD_X\, c'}{SD_Y}$$

$$c_{cs} = \frac{SD_X\, c}{SD_Y} = ab_{cs} + c'_{cs}$$

- Simple
- Removes all scale binding
- Not clear what constitutes a “large” effect

Ratio of the Indirect Effect to the Total Effect

$$P_M = \frac{ab}{c} = \frac{ab}{c' + ab}$$

- Very simple
- Not bounded by 0 and 1
- Explodes toward $\pm\infty$ as $c \to 0$
- Very unstable
  - High between-sample variability
  - Requires $N \geq 500$

Ratio of the Indirect Effect to the Direct Effect

$$R_M = \frac{ab}{c'} = \frac{P_M}{1 - P_M}$$

- Very simple
- Not bounded by 0 and 1
- Explodes toward $\pm\infty$ as $c' \to 0$
- Very unstable
  - High between-sample variability
  - Requires $N \geq 2000$

Proportion of Variance in Y Explained by the

Developed by Fairchild, MacKinnon, Taborga, and Taylor (2009).

- Given a non-zero total effect, represents the proportion of variance in Y accounted for by the indirect effect.

$$R^2_{med} = r^2_{MY} - \left(R^2_{Y.MX} - r^2_{XY}\right)$$

- Mostly sensible interpretation
- Predicated on the assumption that $\beta_{YX} \neq 0$
- $|ab| > |c| \Rightarrow R^2_{med} < 0$
  - Not a strict proportion

Kappa Squared

Developed by Preacher and Kelley (2011).

- Gives the proportion of the *maximum possible* indirect effect represented by ab.

$$\kappa^2 = \frac{ab}{\max(ab)}$$

- Bounded by 0 and 1
- Values closer to 1.0 indicate a bigger effect
- A bit of a pain to calculate.

Computing $\max(ab)$

$$a \in \left\{ \frac{\sigma_{YM}\sigma_{YX} \pm \sqrt{\sigma_M^2\sigma_Y^2 - \sigma_{YM}^2}\sqrt{\sigma_X^2\sigma_Y^2 - \sigma_{YX}^2}}{\sigma_X^2\sigma_Y^2} \right\} = [a_{low}, a_{high}],$$

$$b \in \left\{ \pm \frac{\sqrt{\sigma_X^2\sigma_Y^2 - \sigma_{YX}^2}}{\sqrt{\sigma_X^2\sigma_M^2 - \sigma_{MX}^2}} \right\} = [b_{low}, b_{high}],$$

$$\max(a) = \begin{cases} a_{high}, & \text{if} \quad \hat{a} > 0 \\ a_{low}, & \text{if} \quad \hat{a} < 0 \end{cases}, \quad \max(b) = \begin{cases} b_{high}, & \text{if} \quad \hat{b} > 0 \\ b_{low}, & \text{if} \quad \hat{b} < 0 \end{cases},$$

$$\max(ab) = \max(a)\max(b)$$

Example

```r
## Specify the model:
mod2 <- "
policy ~ b*sysRac + cp*polAffil
sysRac ~ a*polAffil

ab := a*b
"
## Estimate the model:
out2 <- sem(mod2, data = dat1)
##
## Extract/compute the necessary quantities:
ab <- prod(coef(out2)[c("a", "b")])
ab
```

```
[1] 0.1015958
```

Example

```
cPrime ← coef(out2)["cp"]
##
sdY ← sd(dat1$policy)
sdX ← sd(dat1$polAffil)
##
r2MY ← with(dat1, cor(policy, sysRac))^2
r2XY ← with(dat1, cor(policy, polAffil))^2
R2Y.MX ← inspect(out2, "r2")["policy"]
```

Example

```
## Partially Standardized:
abPS <- ab / sdY
abPS
```

```
[1] 0.08559454
```

```
cPrimePS <- cPrime / sdY
cPrimePS
```

```
       cp
0.1138675
```

```
cPS <- abPS + cPrimePS
cPS
```

```
      cp
0.199462
```

Example

```
## Completely Standardized:
abCS ← (sdX * ab) / sdY
abCS
```

```
[1] 0.1345859
```

```
cPrimeCS ← (sdX * cPrime) / sdY
cPrimeCS
```

```
       cp
0.1790413
```

```
cCS ← abCS + cPrimeCS
cCS
```

```
       cp
0.3136272
```

Example

```
## Proportions:
pm ← ab / (cPrime + ab)
pm
```

```
       cp
0.429127
```

```
rm ← ab / cPrime
rm
```

```
        cp
0.7517031
```

```
## R2:
R2med ← r2MY - (R2Y.MX - r2XY)
R2med
```

```
    policy
0.06905689
```

Compute κ^2

```r
## Subset the data:
tmpData <- dat1[ , c("polAffil", "sysRac", "policy")]
colnames(tmpData) <- c("x", "m", "y")
##
## Extract pertinent variance/covariance elements:
cov1 <- cov(tmpData)
sYM <- cov1["x", "m"]
sYX <- cov1["y", "x"]
sMX <- cov1["m", "x"]
s2X <- cov1["x", "x"]
s2M <- cov1["m", "m"]
s2Y <- cov1["y", "y"]
```

Compute κ^2

```
## Possible range of a:
aMarg <- sqrt(s2M * s2Y - sYM^2) * sqrt(s2X * s2Y - sYX^2)
aInt <- c(
    (sYM * sYX - aMarg) / (s2X * s2Y),
    (sYM * sYX + aMarg) / (s2X * s2Y)
)
aInt
```

```
[1] -0.4378558  0.5793099
```

```
##
## Possible range of b:
bMarg <- sqrt(s2X * s2Y - sYX^2) / sqrt(s2X * s2M - sMX^2)
bInt <- c(-1 * bMarg, bMarg)
bInt
```

```
[1] -1.289996  1.289996
```

Compute κ^2

```r
##
## max(a):
aMax <- ifelse(coef(out2)["a"] < 0,
               aInt[1],
               aInt[2])
aMax
```

```
        a
0.5793099
```

```r
##
## max(b)
bMax <- ifelse(coef(out2)["b"] < 0,
               bInt[1],
               bInt[2])
bMax
```

```
       b
1.289996
```

Compute κ^2

```
##
## max(ab)
abMax <- aMax * bMax
abMax
```

```
        a
0.7473075
```

```
##
## Kappa Squared:
k2 <- ab / abMax
k2
```

```
        a
0.1359491
```

Practice

Suppose:

1. Σ is given by:

	x	m	y
x	1.5		
m	0.3	1.4	
y	0.6	0.45	1.55

2. The estimated paths are:
   - $a = 0.2$
   - $b = 0.246$
   - $ab = 0.049$

Compute κ^2 for the estimated ab.

References

Fairchild, A. J., MacKinnon, D. P., Taborga, M. P., & Taylor, A. B. (2009). R-squared effect-size measures for mediation analysis. *Behavior Research Methods*, *41*(2), 486–498.

Preacher, K. J., & Kelley, K. (2011). Effect size measures for mediation models: Quantitative strategies for communicating indirect effects. *Psychological Methods*, *16*(2), 93.