Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
E		2218354

In the past few decades, global warming has rapidly moved into the limelight, becoming the hotspot for mainstream discussion on environmental issues. As such, greater public awareness of the impacts of global warming has led to the creation of guidelines designed to combat rising temperatures. However, precedent demonstrates that there is not enough being done to meet the necessary thresholds. The most noticeable shortcoming is the Paris Climate Accords, which set goals for limiting carbon emissions to be met by 2030. Presently, halfway to the agreed goal year, only one country is on track to meet the standards set at the agreement. It is clear that countries are having difficulty reducing their emission of greenhouse gases, especially with continued industrial growth. Combined with political pressure on governments to not inhibit economic growth through restrictive emissions control, it does not appear as if things will take a sudden turn for the better. Instead, we must find alternative ways of controlling atmospheric greenhouse gases that do not interfere with the workings of a given nation's economy.

To do so, we shift our goals away from limiting the initial emission of gases instead to the control of gases already present in the atmosphere. Rather than pursuing some panacea in the form of a newly developing technology, we draw inspiration from what nature has been doing for millions of years. Indeed, in the long run, a novel technology may be the cure to the climate crisis, but given the severity of the problem in the present, we need actionable plans that can be immediately put into motion. A promising line of inquiry involves the forests. More specifically, we are interested in their ability to sequester carbon as trees grow, which would allow for the recapture of greenhouse gases.

This process of carbon sequestration is clearly a naturally occuring phenomenon. What we then seek is to determine ways of more efficiently utilizing forests' sequestering ability to increase the rate at which we can remove carbon dioxide from the atmosphere. To do so, we create a discretized, stochastic model for tree growth in the context of various types of forests. We then analyze the total carbon sequestration over time by altering parameters such as harvest frequency, thresholds for harvesting, and tree composition to determine the forest management strategy that will maximize carbon sequestration.

Our varying of the parameters yields several general conclusions, as well as specific quantitative parameters specific to a forest. The model recommends that, in general, forest managers maximize the time between successive harvests. Our model shows that as long as harvests are conducted prior to the plateauing of carbon sequestration, the total carbon sequestered is similar, while the amount of carbon dioxide lost back into the atmosphere is reduced as we increase the time between harvests, As for which and how many trees to cut in a given harvest, the model provides specific numerical thresholds that apply to the makeup and distribution of trees within a given forest. Forest managers can then consider the tools at their disposal to determine what level of action is feasible while trying to adhere to the model's recommendations where possible.

We can now take the results from the model and combine the quantitative analysis with qualitative analysis of features specific to a given forest to create a decision framework for forest management. While the model deals with just the carbon impact of forest management, the decision framework extends it to consider the impact on local biodiversity. These two elements work in concert to enable forest managers to maximize carbon removal from the atmosphere while remaining considerate of local fauna and flora.

We acknowledge that the assumptions made throughout the model may make the cutoffs for the parameters not fully accurate when applied to the actual harvesting of a forest. However, we believe that this trade-off allows for a model that is more flexible when presented with forests of various conditions. While our model does not create a perfect representation of forest conditions, we hope that it will serve as an easy-to-use baseline tool for forest managers seeking a starting point.

Contents

1 Introduction

1.1 Problem Explanation

Year after year, global warming is tossed around by the media and politicians alike as a key environmental crisis that needs to be addressed. However, despite the attention it receives, the issue remains poorly resolved by actual policy. While it is true that efforts directed towards curbing global emissions, such as the Paris Climate Accords, have been taken, the implementation of methods to actually meet the outlined goals has been extremely lacking [1]. Due to this, it appears as if the current focus on simply reducing emissions and transitioning to cleaner energy sources may not be enough, especially considering the rate at which the planet continues to warm.

It is evident, then, that alternative approaches to addressing global warming is necessary. One such approach that holds considerable promise is the management of forests. Specifically, we are interested in their ability to sequester carbon dioxide, and in doing so, remove greenhouse gases that are accelerating the warming of our planet.

Plants, most notably as part of forests, have served as natural carbon sinks for the Earth, holding onto carbon derived from the absorption of carbon dioxide from the atmosphere. In recent years, however, human industrial activity has resulted in exponential growth of atmospheric carbon dioxide concentrations to levels which the forests cannot fully process. To overcome this new problem, we must approach forest management in a new light so that we can most effectively utilize our forests' ability to sequester carbon.

Past efforts have largely been centered around maximizing a forest's bio-productivity, usually through the introduction of varied plant species. Increasing the biodiversity in this artificial way has allowed plants with various niches to coexist within the same environment, increasing the density of plant-life within forests and therefore increase the total amount of carbon a forest is able to sequester [2].

Since this approach has been explored quite thoroughly within existing literature, we will instead look at an alternative means of forest management, focusing on the regular and structured harvesting of trees to maximize continuous forest growth. Hence, the primary goal of our model is to identify optimal parameters that can be used to determine harvesting rates and strategies. We will then use data collected regarding various types of forests to fine tune our parameters to maximize the carbon sequestration capabilities of any individual forest.

1.2 Existing Literature

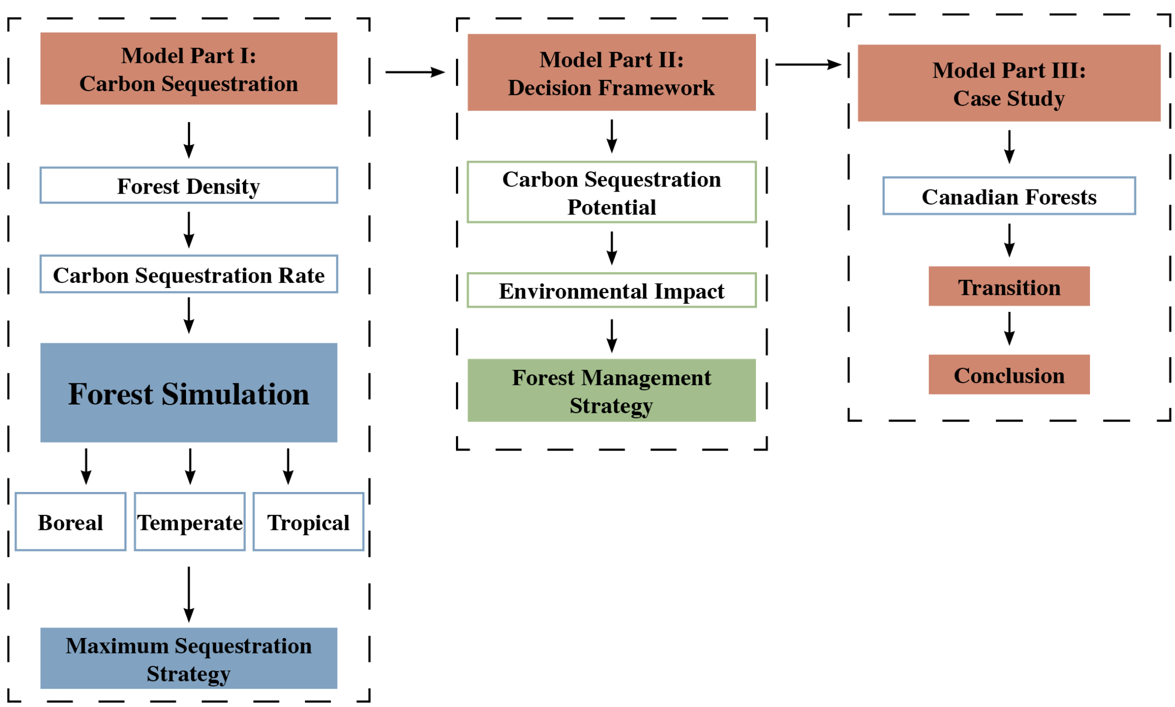
Many proactive organizations in regards to the climate issue have already enacted plans to further incorporate biological carbon sequestration as part of their methods. One such actor is the United States Geological Survey, monitoring and developing ways of increasing carbon sequestration through their LandCarbon program [3]. However, their efforts have been centered around establishing current baselines and creating projections for future carbon stores, rather than taking a more active role in forest management. The reasoning for this may be that increased harvesting poses a threat to the fauna and flora of forest ecosystems. This, combined with uncertainty regarding the realizable benefits of controlled harvesting, has led to minimal action being taken despite the signs pointing to the need for some sort of action.

We seek to add to the existing literature in this paper by developing a model that can pinpoint key measures of carbon sequestration. Forest managers will then be able to combine the output of our model with their specific knowledge of the local forest to determine an optimal forest management plan for the coming decades. Rather than simply acknowledging or confirming the existence of a problem, we hope to create a guideline for active harvesting that will enable forest managers to be proactive in their care for the forests.

1.3 Problem Restatement

The goal of this paper is then to address the following:

1. Create a model that evaluates different forest harvesting strategies to produce the maximum carbon sequestration given characteristics of various forests.
2. Produce a decision framework that unifies the carbon sequestration model with other criterion to determine a holistic assessment of how to manage a given forest.
3. Apply the carbon sequestration model and decision framework to real world forests in the form of case studies to confirm the validity of the model's recommendations.



2 Model I: Carbon Sequestration

2.1 Modeling Framework

Table 1: Notation

Symbol	Definition
m	Harvesting mass threshold
t	Time
α	Tree age
ϕ	Density ratio threshold
Φ	Set of possible density thresholds
θ	Forest type
λ	Maturity ratio
ρ	Tree Density Threshold
c_0	Initial carbon mass
tC	Tons of carbon
ζ	Forest management rate
Z	Set of possible management rates
p	Carbon mass release percentage
E	Endangerment index
V	Total carbon mass of a mature tree
δ	Level of endangerment
$\psi(t;\theta)$	Carbon mass of a tree
$\xi(r,V,\rho)$	Carbon sequestration rate of a tree
$C(m,\phi,t;\theta)$	Total carbon sequestered
$CR(m,\phi,t;\theta)$	Total carbon release to atmosphere

2.2 Overview

Our model seeks to find the set of optimal harvesting parameters which maximizes the total carbon sequestered over time by a forest. More specifically, our problem is the following:

$$\underset{\phi,\zeta\in(\phi,Z)}{\arg\max}\, C(m,\phi,t,\zeta;\theta) \tag{1}$$

$$C(m,\phi,t,\zeta;\theta) = \int_A c_0 + \psi(t;\theta)\big|_{(m,\phi,\zeta)} dA \tag{2}$$

Essentially, we seek the combination of harvesting mass threshold and density ratio threshold that will maximize C for a forest area of size A over a certain time horizon t. We denote the carbon sequestration of a single tree of forest type θ at a time t with harvesting condition thresholds (m,ϕ,ζ) applied to the tree as $\psi(t;\theta)\big|_{(m,\phi,\zeta)}$. Through simulation, we aim to model the differences in total carbon sequestration as a result of the different harvesting strategies.

2.3 Landscape

In our model, we represent a forest landscape as a discretized $n \times n$ grid of a forest biome $\theta \in$ {Boreal, Temperate, Tropical}, with each cell representing one tree in the forest. We can rewrite the total carbon sequestered to account for discretization over the $n \times n$ forest as:

$$C(m, \phi, t, \zeta; \theta) = \sum_{i=0}^{n} \sum_{j=0}^{n} c_{0_{i,j}} + \psi_{i,j}(t; \theta)\Big|_{(m,\phi,\zeta)} \tag{3}$$

where $\psi_{i,j}$ represents the carbon content of the tree at the (i, j)-th index within the model forest. Each type of forest has certain characteristics such as carbon stock and carbon sequestration rate, with which the model constructs a normally distributed forest. These characteristics are further discussed below.

2.3.1 Forest Biomes Details

1. **Boreal**: Boreal forests are dry environments characterized by their high latitude where freezing temperatures persist for the greater half of the year. Boreal forests typically have larger, more woody trees that hold onto more carbon. Forests here usually have low canopy cover, and most of the carbon is stored in the frozen soils
2. **Temperate**: Temperate forests are found in between tropical and boreal regions in terms of latitude. Temperate forests are characterized by tree sizes ranging between those of boreal and tropical forests. The soil in these forests is generally fertile and is a good environment for young plants to thrive. Certain temperate forests have sensitive wildlife such as those in the United States, which are home to 12 species of endangered mammals [4].
3. **Tropical**: Tropical forests are located in the tropics above and below the equator. Tropical forests have high biodiversity and have very complex and vulnerable inter-species relationships. Tropical forests typically have fast nutrient cycles and regrowth of young plants is difficult because soil nutrients are quickly absorbed by nearby established trees.

2.3.2 Landscape Assumptions

1. **Nutrient Distribution:** We assume that the nutrient and resource distribution across different parts of the forest is constant, such that no part of the forest will grow faster than another due to skewed resource allocation.
2. **Tree Biodiversity:** We assume that all the trees in the model forest to share similar characteristics, purely because they inhabit the same biome rather than because they are all identical species. This allows us to reduce the noise of any inter-species interactions that may occur.
3. **Tree Growth:** We assume that trees grow at a rate that is determined by their carbon mass. In reality, various other factors including nutrient availability, sunlight, and yearly temperatures can all affect a forests growth. We do not include such factors in the model because they vary spatially and temporally and can introduce variations in our model that we are not seeking to measure.
4. **Tree Age:** We assume at the beginning of the simulation, the forest is at the middle of its lifespan. The initial forest is fairly established, yet retains a high potential for growth in the coming decades. This allows us to observe the forest as it transitions from early maturity to old age.

Given the above assumptions and applying characteristics specific to the three chosen biomes, we determine the following distributions from which we can draw particular properties of a given tree.

Table 2: Tree Characteristics

	Tropical	Boreal	Temperate
Height (h) [ft]	40	80	60
Radius (r) [ft]	1	1.5	1.25
Max age (α_{max}) [yr]	$N(95, 15)$	$N(150, 25)$	$N(120, 20)$
Current age (α) [yr]	$N(0.4 \cdot \alpha_{max}, 0.2 \cdot \alpha_{max})$	$N(0.4 \cdot \alpha_{max}, 0.2 \cdot \alpha_{max})$	$N(0.4 \cdot \alpha_{max}, 0.2 \cdot \alpha_{max})$

The values in the table above were selected by determining the most common tree types for each forest type, then averaging the measures across the selected types for each of the characteristics. The set of trees used for tropical forests was palm, nut, and rubber trees, for boreal forests it was spruce, maple and pine, and for temperate forests it was oak, fir, and birch [5] [6] [7]. The average measurements for height and radius for each tree type at full maturity were researched then averaged for each forest type. For the max age of a tree, We did the same averaging, but also determined the standard deviation of the three chosen tree types for each forest to create a probabilistic distribution for the variable [8]. We then set the current age of the individual trees based on the max age of trees in the given forest type using a normal distribution to simulate the distribution of tree ages in a forest.

2.3.3 Forest Density

Forest density is a critical factor in the regrowth of trees; that is, a sapling which is surrounded by tall trees blocking out sunlight and competing for nutrients will grow much slower than a tree which has access to the necessary nutrients. As such, we include density into our model by defining it as the ratio between any given tree's carbon content to the average carbon content of the trees immediately surrounding it. We calculate the density value for the (i, j)-th indexed cell in the forest as:

$$\rho_{i,j} = \frac{\psi_{i,j}}{\frac{1}{n}\sum_{(i,j)\in\mathscr{A}} \psi_{i,j}} \tag{4}$$

where n is the number of adjacent cells (i.e. $n = 3$ for a corner cell, $n = 8$ for any cell in the middle of the forest) and $\mathscr{A}$ is the set of cells adjacent to (i, j) and within the forest outer boundaries.

2.3.4 Carbon Sequestration Rate

Carbon sequestration rate is analogous to the growth of a tree, as its carbon content increases with age. We will therefore use growth and carbon sequestration in a synonymous manner for the remainder of the paper.

We estimate the carbon sequestration rate by taking the product of three key factors which are each standardized prior to taking the product. Additionally, we add a coefficient of 0.1 as a factor to scale growth down to a reasonable rate. The coefficient was determined by reasoning that beginning from zero mass, a tree should, across its lifetime, sequester carbon at a rate that would lead to it having the expected carbon mass at full maturity.

The carbon sequestration rate ξ is as follows:

$$\xi_{i,j}(\lambda, V, \rho; \theta) = 0.1 \cdot \lambda \rho_{i,j} V \tag{5}$$

where r, V, ρ are the tree maturity factor, total carbon mass of the tree, and relative tree density, respectively. Note that the type of tree implicitly factors into each of the variables.

We obtain the tree maturity ratio by evaluating λ as follows:

$$\lambda = 6.37x - 14.27x^2 + 8.3x^3\Big|_{x=\alpha/\alpha_{max}}$$

where α is the current age of the tree and α_{max} is the maximum age of a tree with type θ [9] [10] [11].

The above cubic equation was derived by fitting regression lines to data gathered by Keith et al., which characterizes the relationship between a tree's age and its carbon sequestration rate. We use the ratio between a tree's current and maximum age to normalize across different tree types with varying life expectancies.

The value of the next factor, the total carbon mass of a mature tree, is obtained by evaluating:

$$V = \frac{0.5 d \pi r^2 h}{2000}$$

for which the radius r and height h vary for every tree type θ as described in Table 2. The variable d represents the average density of the most common tree type in each forest setting, and was determined from the data included in the sources above.

The final factor is the relative density, ρ, as determined by the methods described in the above section.

Additionally, as a point of reference, the initial carbon mass c_0 of a tree is distributed as follows:

$$c_o \sim N\left(V\frac{\alpha}{\alpha_{max}}, 0.1 \cdot V\frac{\alpha}{\alpha_{max}}\right)$$

This distribution allows for both the tree type and level of maturity to directly influence the mass of a tree in a manner faithful to realistic tree growth.

2.4 Harvesting

2.4.1 Harvesting Threshhold Parameters

In our model, we are primarily interested in investigating the effect that the following parameters have on total carbon sequestration in a forest: the harvesting mass threshold m, density ratio threshold ϕ, as well as the forest management rate ζ. The harvesting scheme implemented in our model is as follows: harvest years occur every ζ years; i.e. if $\zeta = 10$, then we check the forest for trees to harvest every 10 years. During a harvest year, the total mass of each 3×3 group of adjacent cells is measured against the density threshold ϕ; if the total mass is greater than ϕ, then each cell in the group is checked against m. Finally, for any cell in the group whose mass is greater than m, then we cut/harvest that particular cell.

2.4.2 Carbon Losses from Harvesting

There are many factors that determine how much carbon is captured from harvesting a tree. Throughout harvesting processes, carbon losses take forms such as leftover tree decomposition or inefficiently made forest goods. Additionally, carbon intensive harvesting techniques from industrial machinery can also mitigate the benefits of harvesting and storing sequestered carbon.

As such, not all of the carbon that was within the carbon mass of a tree is sequestered when it is harvested. We take this into account by letting a harvested tree lose a certain percentage p of its carbon mass to the atmosphere while retaining the rest. From literature search [12], we decided that $p = 0.55$ is an appropriate percentage; i.e. at harvest, about half of the carbon is lost to the atmosphere.

2.4.3 Harvesting Assumptions

1. **Tree Selection and Acquisition:** We assume that it is possible to measure the mass of any particular tree through observation. Further, we assume that we can locate all such trees above the harvesting thresholds and harvest them within the time frame t.
2. **Removal Process:** In the real world, tree removal takes on many forms and sometimes parts of the tree such as roots are left in the ground. In our model, we assume that any removal of a tree is complete without any remnants in the original location. We assume that any amount of trees can removed as long as they meet the harvest threshold. Factors like financial and technical feasibility does not hinder harvesting of trees.
3. **Percentage of Carbon Captured from Harvest:** We assume that all trees have the same percentage, p, and have same percentage losses to the atmosphere. In reality, trees are harvested in different ways and made into different products that have different rates of carbon loss.

2.5 Simulation

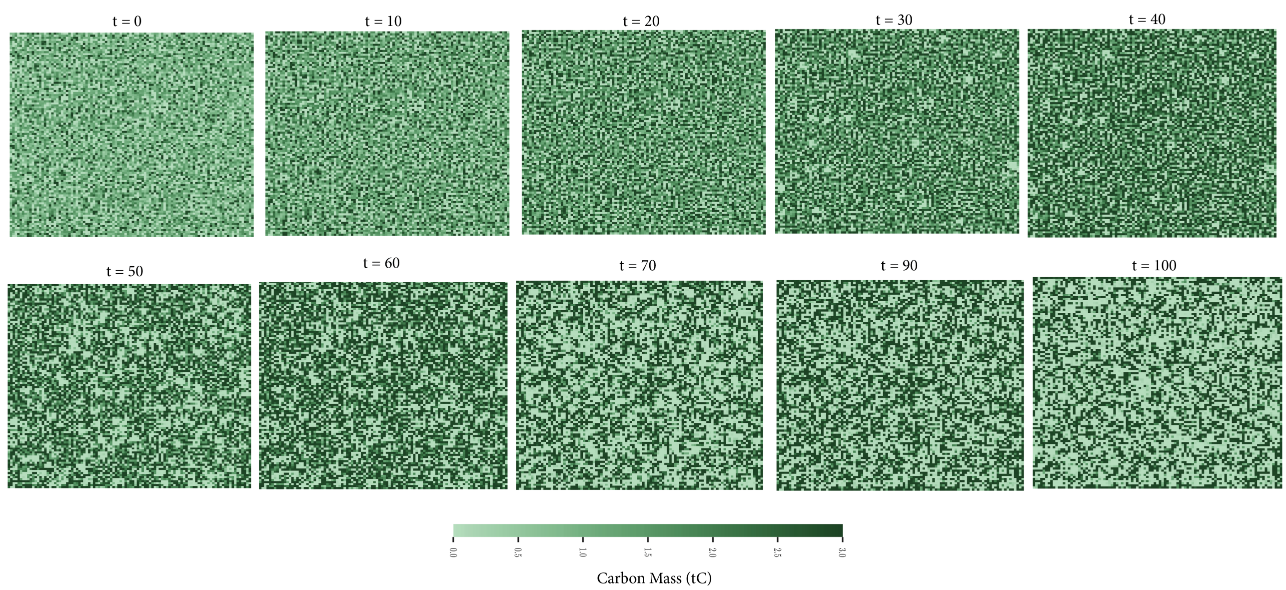
2.5.1 Simulation Assumptions

1. **Asynchronous Growth:** In reality, changes in the biosphere occur simultaneously and as such, there is no notion of one tree growing before another at a certain discrete moment in time. As our model is a discretized model of the forest in both time and land, it is therefore limited to the stochastic processes through which we simulate changes in the forest. However, best mimic nature, we update cells in a random order during each time step as opposed to conventional array iteration from left to right, top to bottom.

We simulate different forest management plans according to the following procedure:

Algorithm 1 Simulation loop

$t \leftarrow 0$
for $t <$ `max_time` **do**
    if t is a harvest year **then**
        for $(i, j) \in$ `land` **do**
            if $\sum_{(i,j)\in\mathscr{A}} \psi_{i,j} > \phi$ **then**
                $\forall(i, j) \in \mathscr{A}$ such that $\psi_{i,j} > m,$ `harvest(i, j)`
            end if
        end for
    else
        for $(i, j) \in$ `land` **do**
            $\psi_{i,j} \leftarrow \psi_{i,j} + \xi_{i,j}$ ▷ Increase the carbon mass tree at `(i, j)` by the relevant ξ
        end for
    end if
end for



(a) Total Carbon Sequestered (C)

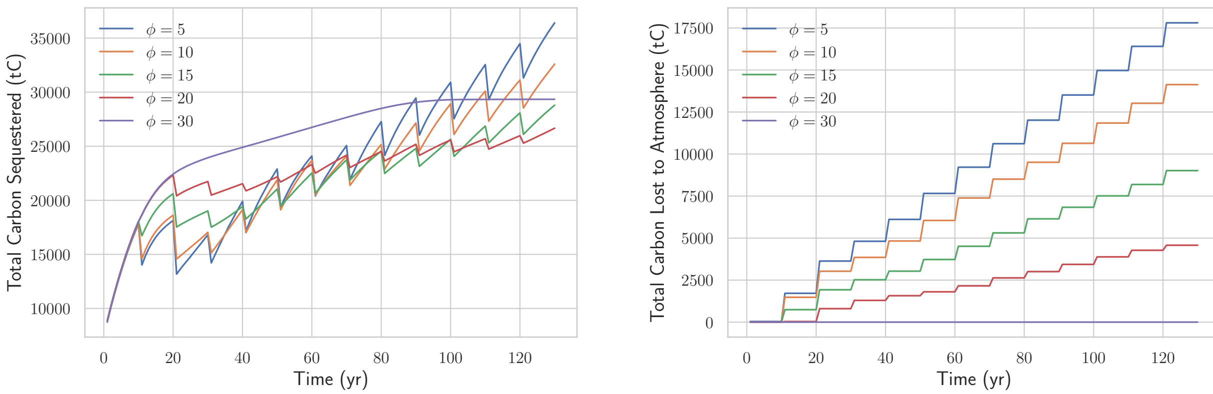
Figure 1: Sample Visual Simulation of the Forest Model

3 Carbon Sequestration Model Results

3.1 Results and Analysis

Through our simulation we investigate how different harvesting density thresholds and harvesting rates affect the total carbon sequestration. We also want to find out whether these findings are generalized across all three forests biomes. To do this, we one of ϕ, ζ, or θ at a time while keeping everything else constant.

3.1.1 Results and analysis on ϕ



(a) Total Carbon Sequestered (C) (b) Total Carbon Released to Atmosphere (CR)

Figure 2: Behavior of Carbon Rates in Tropical Forests Varying ϕ, $p = .55, \zeta = 10$

We refer to total carbon sequestered under density threshold ϕ as $C(\phi)$, and total carbon released into the atmosphere as $CR(\phi)$. In Figure 2a, we notice that $\phi = 30$ represents an upper bound on harvesting density threshold since the forest is not harvested at all. This is because the forest never reaches a density level beyond this threshold. On average, we note that on the shorter time horizon from year 0 to 40, it holds that

$$C(5) < C(10) < C(15) < C(20) < C(30)$$

However, on a larger time scale, from year 100-120, this trend breaks and instead, the following holds:

$$C(20) < C(15) < C(30) \leq C(10) < C(5)$$

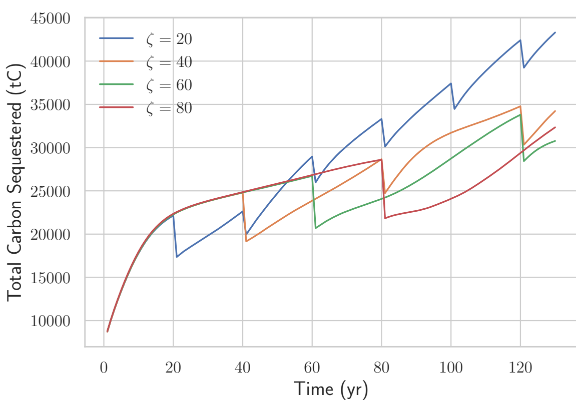
As a result, we note that in the short run, not harvesting yields the maximal carbon sequestration; however, in the long run, a harvesting strategies with smaller ϕ(more aggressive) results in a larger C. Intuitively, this makes sense as during early growing years of the forest, trees are young and still have a lot of potential growth which is analogous to their carbon sequestration potential. However, as time passes, these trees mature and their sequestration abilities plateau. Hence, strategies that harvest old trees with weaker growth will make room for newer trees to grow and sequester more carbon.

Harvesting trees to promote young tree growth does have drawbacks, though. In Figure 2b, we notice steady step like increases in CR as a result of harvesting years. The magnitude of these steps depend on the value of ϕ; in particular, smaller ϕ values correspond to more drastic and sudden changes to atmospheric carbon release, while larger ϕ values correlate with a slower, steadier carbon release. Note that this atmospheric carbon release occurs because tree harvest is imperfect. Oftentimes, 50 percent of carbon mass is lost on the forest floor and even more may be lost through manufacturing forest goods.

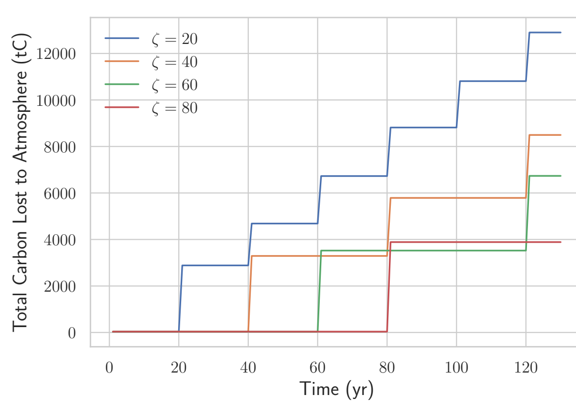
Therefore, the ideal strategy is to harvest as many trees as possible while maintaining an non-harmful level of CR. This non-harmful threshold depends on what climate experts of the world or region deem is acceptable. Since this value is varies across different literature sources, for the purposes of model analysis we use $\phi = 15$ as the most optimal value for maximum carbon sequestration rate and minimum environmental impact for the following sections.

3.1.2 Results and analysis on ζ

In this section, we investigate the effects of the management rate, or harvesting frequency, ζ, on the total carbon sequestration.



(a) Total Carbon Sequestered (C)



(b) Total Carbon Released to Atmosphere (CR)

Figure 3: Behavior of Carbon Rates in Tropical Forests Varying ζ, $p = .55, m = 2.3, \phi = 15$

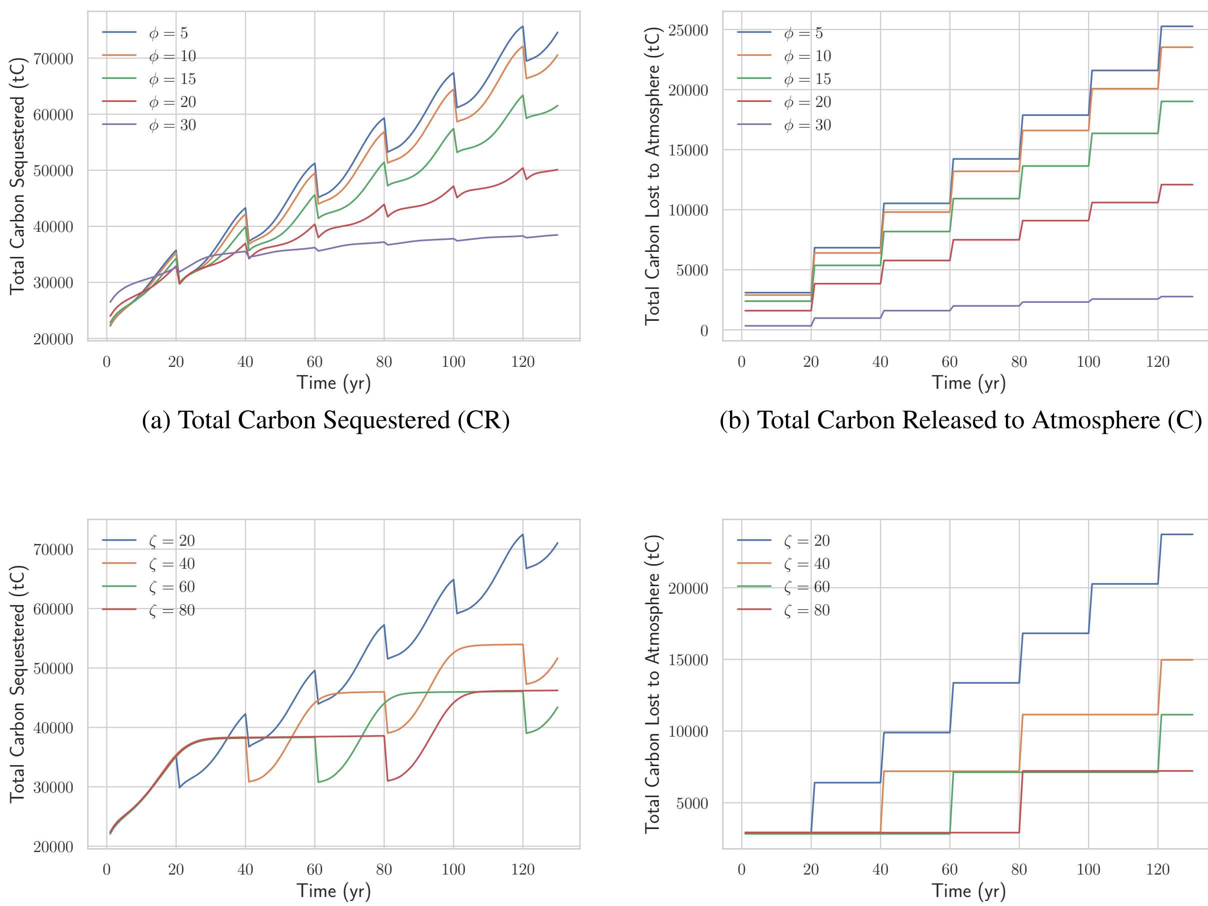
In the first 80 years, we observe that smaller values of ζ are associated with lower carbon sequestration due to the harvesting of trees before they achieve their peak growth and sequestration rate. Larger values of ζ such as 80 perform better in this regard because they take advantage of the trees' peak growth rate during this time. At year t=80 when the first harvest for the $\zeta = 80$ forest happens, $C(80)$ dips below C for the other management frequencies; however, at year 120, the forest recovers and $C(80)$ becomes greater than $C(60)$ but less than $C(40)$ and $C(20)$.

Despite $C(80)$ being less than $C(20)$, we notice a significant difference in Figure 3b, where $CR(80)$ is more than 3 times lower than $CR(20)$ at t=120. This suggests that slower harvesting strategies can match or surpass the sequestration rates of more frequent harvesting strategies while producing less atmospheric carbon. Our model thus demonstrates that less frequent harvesting strategies are a more efficient and environmentally neutral way to maximize carbon sequestration. The same comparison can be made to the $\zeta = 40$ and $\zeta = 60$ harvesting strategies.

Therefore, the ideal management rate for a tropical forest with $p = .55, m = 2.3, \phi = 15$ is to harvest in longer cycles upwards of 60 to 80 years.

3.1.3 Results and analysis on different forest types θ

Using the optimal parameters from 3.1.1 and 3.1.2, we apply the harvesting techniques to boreal and temperate forests.



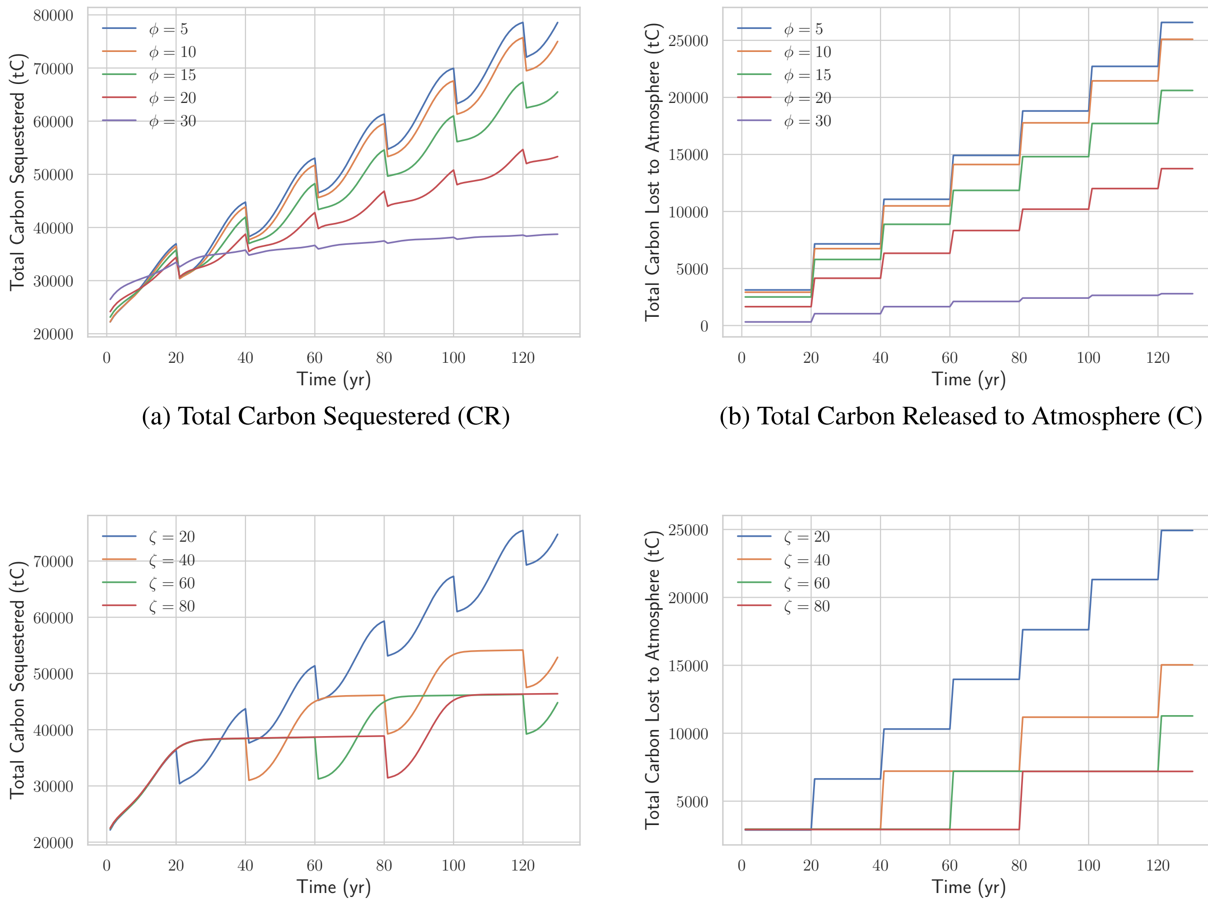
(a) Total Carbon Sequestered (CR)

(b) Total Carbon Released to Atmosphere (C)

(c) Total Carbon Sequestered (CR)

(d) Total Carbon Released to Atmosphere (C)

Figure 4: Behavior of Carbon Rates in Boreal Forests, $p = .55$



(a) Total Carbon Sequestered (CR)

(b) Total Carbon Released to Atmosphere (C)

(c) Total Carbon Sequestered (CR)

(d) Total Carbon Released to Atmosphere (C)

Figure 5: Behavior of Carbon Rates in Temperate Forests, $p = .55$

3.2 Optimal Management Plan

From the simulations above, we find that in general, decreasing ϕ maximizes the total carbon sequestration as a lower ϕ threshold implies more older trees are being cleared out for younger trees to grow. We also find that decreasing the harvesting frequency or increasing ζ is an effective and more environmentally optimal way to maximize a forest's carbon sequestration. After optimizing ϕ and ζ, we applied this strategies to temperate and boreal forests to see if similar trends hold.

When running the simulations on boreal and temperate forests with the same ranges of ϕ and ζ we see a plateau that didn't exist in the Tropical forest. This is because our model predicted that boreal and temperate forests grow back faster and their growth rates are less dependent on surrounding forest density. This can be explained by the different rates of nutrient cycling in the different forests; tropical forests typically have faster nutrients cycles that slow down the growth rate.

This plateau in boreal and temperate forests indicates that the forest has hit max carbon sequestration and is waiting on the next harvest cycle to open up room for more growth. Therefore for boreal and temperate forests, the optimal ζ is smaller than that of a tropical forest; i.e. these forest types should be harvested more frequently. It is important to note that despite ζ being lower for boreal and temperate forests, the relationship between total carbon sequestration and atmospheric release remains.

In conclusion, the optimal management plan is one that is associated with lower harvesting density threshold ϕ, which also achieves the lowest amount of atmospheric release. From the graphs that our model produced, this optimum occurs at about $\zeta = 80, \phi = 15$ for tropical forests. Meanwhile, for boreal and temperate forests, the optimal harvesting rates and density thresholds are $\zeta = 40$ and $\phi = 15$, respectively.

4 Model Part II: Decision Framework

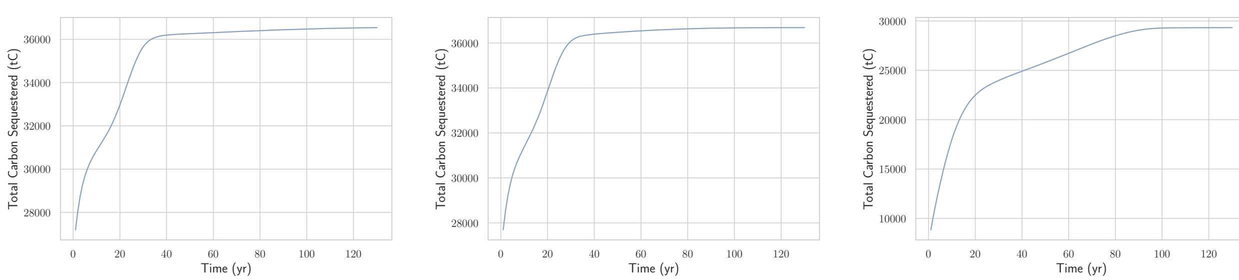
In order to develop a decision model to inform forest managers of the best use of a forest, we use the carbon sequestration model presented above while also considering the environmental effects of harvesting. The resulting decision model will resolve whether a particular forest should be harvested and if so, with what harvesting strategies in order to maximize carbon sequestration over a century.

4.1 Assumptions

1. **Simplification of Environmental Impact:** In reality there are many factors that influence the environmental impact of harvesting, however in this model we use endangerment and biodiversity as the only measures of environmental impact.
2. **Perfect Population Count:** We assume that when evaluating an environment, we are able to perfectly count the endangered species and we also can find the total population in the environment.

4.2 Carbon Sequestration Potential

Carbon sequestration potential is the predicted amount of carbon a forest is expected to sequester in the future. Applying our model to the different θ-type forests, we observe in Figure 6 that without any harvesting, boreal and temperate forests have the greatest carbon sequestration potential.



(a) boreal (b) Temperate (c) Tropical

Figure 6: Total Carbon Sequestered per Forest Type Without Harvesting

4.3 Impact on Wildlife

While the responsible harvesting of trees to enable newer younger tress to grow and sequester carbon is a beneficial action to counteract global warming, it should not be done at the cost of the environment and the life within. The smallest alterations to an environment can result in significant compounding effects because of the tight interdependence of natural ecosystems; as such, harvesting in biologically-unstable forests is extremely dangerous. Specifically, harvesting can result in imbalances within forests which could potentially hinder forest productivity and detract from the long-run potential of the given forest as a carbon sink. This only exacerbates the existing problem with the greenhouse gases due to various harvesting techniques. There are several factors that forest managers can evaluate to gauge the impact of their harvesting, which we discuss next.

4.3.1 Endangered Species:

According to the IUCN, a species is endangered if it meets any one of the following criteria:

a. A 50–70% population decrease over 10 years.
b. A total geographic area less than 5,000 km^2 (or local population area less than 500 km^2).
c. A population size less than 2,500 adult or restricted population of 250 adults.

In order to create a measure of endangered species of a population, we create a model derived from the Claude Shannon Entropy formula and inspired by the Shannon-Winer index.

$$\text{H} = -\sum_{i=0}^{k} x_i \ln(x_i) \tag{6}$$

We note that species with higher level of endangerment take up a smaller portion of the total population in a given environment. Further, more endangered species should have be more heavily considered when determining if a forest is at a higher endangerment level. To represent the level on endangerment of a species, we use the IUCN Red List of endangered species which ranks endangerment on a discrete scale of "least concerned", "near threatened", "vulnerable", "endangered", "critically endangered", and "extinct in the wild". We assign a value $\delta = \{0 \text{ (non-endangered)} \ldots 6 \text{ (extinct in the wild)}\}$ to each respective ranking.

With this in mind, the modified endangerment index is as follows where δ is the level of endangerment of species i , and p_i is the portion of the total population of species i. Where more endangered species with a smaller p_i value contributes more to the endangerment index. Essentially, an ecosystem with many endangered species will result in a larger E index value.

$$\text{E} = -\sum_{i=0}^{k} \delta_i \cdot \sqrt{p_i} \ln(p_i) \tag{7}$$

4.3.2 Biodiversity

In the context of this model, biodiversity refers to the variety of plant and animal species in the environment. Previous research has shown that forests with greater biodiversity show greater productivity through observations like increased tree biomass and soil carbon storage [13].

Forest management can combine these indices with real world data about forest species distributions and population counts to identify forests with valuable biodiversity that need to be preserved. For example, tropical rain forests are known to be the most biodiverse ecosystems on the planet. As such, over-harvesting destroys the habitat of many species, putting survival and resource pressures on the entire environment. Beyond a certain tipping point, the detrimental stresses that over-harvesting puts on a forest environment can result in the endangerment and extinction of forest inhabitants. Such a result counteracts the motivation behind forest harvesting strategies which aim to increase carbon sequestration.

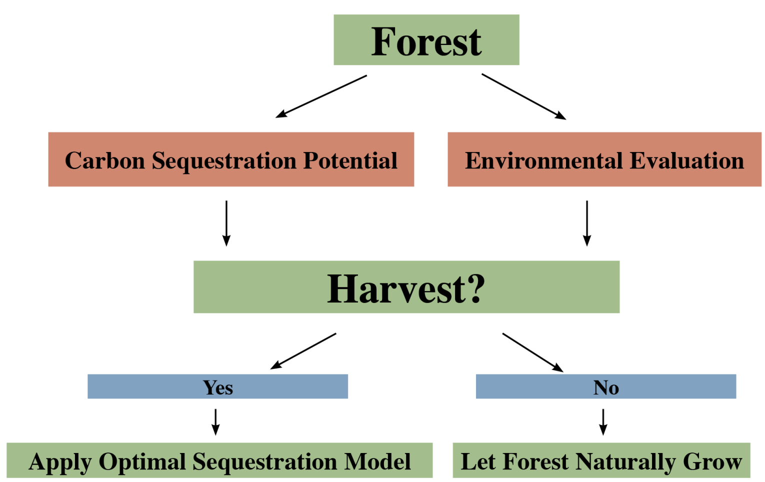
4.4 Overall Decision Framework

Having discussed the carbon and biological valuations of forests, we can now formulate a comprehensive management plan based on these conditions. Our motivation is to find an optimal balance between environmental welfare and carbon sequestration.

Evaluation 1: Should harvesting be included in the management plan?
The first question to ask before harvesting a forest is whether or not the forest has large amounts of biodiversity and if endangered animals live there. Forest managers can use real world data and the Endangerment index, E, to help evaluate these criteria. If a forest consists of many endangered populations or is a haven to biodiversity such as the tropical rain forest, harvesting should not take place because the environmental drawbacks can decrease forest productivity in the long term which can hinder or destroy the existing forests that are large carbon sinks.

If the forest management deems the harvest of the forest to have minimal effects on biodiversity and endangerment and decides harvesting is possible, we can find the optimal way of harvest to maximize sequestration.

Evaluation 2: What is the best harvesting plan to maximize carbon sequestration?
To answer this question, we can apply the optimizations found in Section 3: Carbon Sequestration Model Results.



(a)

Figure 7: Decision Model Framework

5 Case Study: Canadian Forests

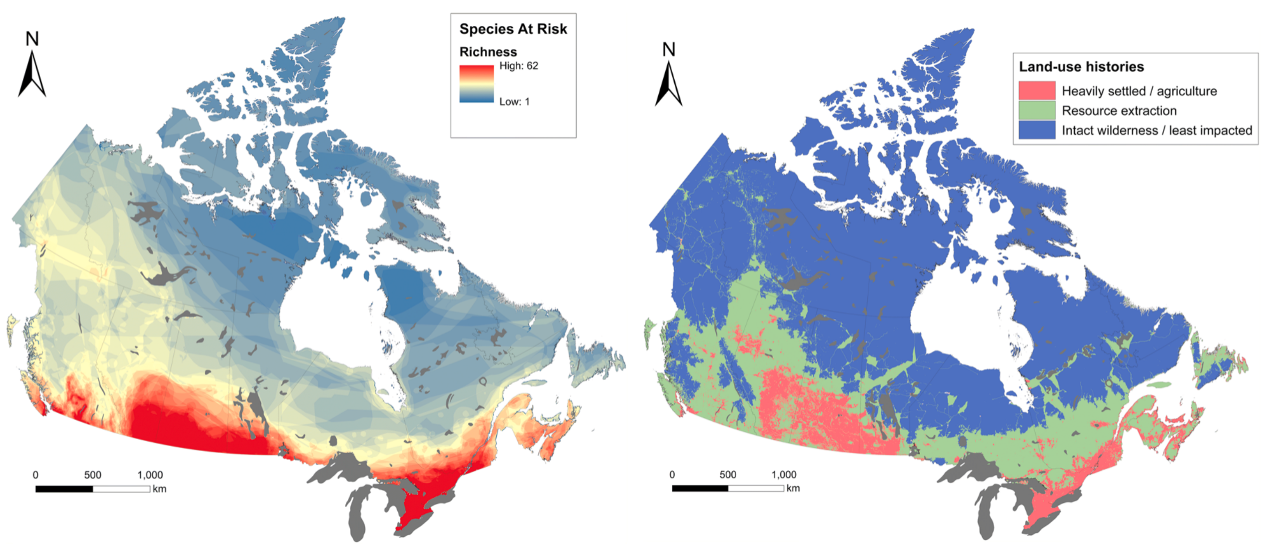
Here, we apply our decision framework to determine the optimal forest management plan for a sample forest, chosen from the Canadian boreal forests.

5.1 Carbon Sequestration Potential:

Over a 100 year time frame, our model projects that a boreal forest patch with 100 trees will sequester approximately 31000 tC without any harvesting practices. Using the determined optimal strategy with harvest density threshold of $\zeta = 15$ and harvest rate of $\phi = 40$, we see that the forest potentially sequesters up to approximately 55 tC at 100 years. This is a significant amount of carbon, enough to offset regional emissions of Northern Canada.

5.2 Environmental Impact

In terms of biodiversity and endangered species, Canadian forests have certain regions where wildlife is more endangered than others. From Figure 8, we see that most of the species endangerment is along the southern border of Canada where human activity is putting species at risk. In these high risk regions, harvesting should not take place because the environmental risks of irreversibly hurting certain ecosystems is to great. Looking towards the northern parts of Canada where species risk is at a minimum, optimal harvesting strategies can be applied with a smaller risk of environmental damage.



(a) (b)

Figure 8: Endangerment [14]

5.3 Transitioning from current practices:

Since 1990, Canada's low annual deforestation rate has declined even further, dropping from 64,000 hectares (ha) per year to about 34,300 ha per year in 2018 [15]. This decrease in deforestation has allowed certain forests to recover and will begin to reach its peak rates of carbon sequestration. With that being said, Canada should not harvest these young forest until the forest density is so large that no new trees are growing. By carefully monitoring recovering forests, Canada can maximize its forest's total carbon sequestration. Likewise, forest management can find overly dense forest whose carbon sequestration has plateaued to trim off trees in approximately 40 year cycle to maximize total carbon sequestration.

6 Strengths and Weaknesses

6.1 Strengths

1. **Terrain Customization and Generation:** The core strength of our model is its ability to model forest growth and carbon sequestration for different forest types. In our investigation, we simulated the growth and harvesting techniques across 3 different forest types (boreal, temperate, and tropical), and using our discretized model, we are able to generate entirely unique forests with different biological and harvesting characteristics. This allows the model to be expanded to further forest types and allows for users to customize harvesting standards and goals. As such, our model is able to adapt, recreate, and simulate forests under different biological and harvesting conditions.
2. **Visual Simulation of Forest Growth:** Due to the careful considerations of real world forest features taken into account for modeling assumptions, our simulation engine is able to visually represent the state of the forest at any given time step while growing and harvesting. This allows us to get a birds-eye view of carbon concentrations across the forest as well as harvesting patterns, creating a visually intuitive representation of carbon sequestration and harvesting.

6.2 Weaknesses

1. **Simplified Tree Growth:** The simplification of tree growth to depend on only tree mass and density was a necessary simplification for the feasibility of the project. However, our decision to do so resulted in the model losing a lot of nuance about forest behavior. With more time and in improved models, we would aim to incorporate measures of soil characteristics such as nutrient content, to better approximate tree growth dynamics akin to a real forest.
2. **Simplified Biomes Growth:** The simplification of biomes to 3 major categories was necessary for the completion of this model. However in reality, there are drastic differences even between forests of the same biome type. For example, tropical regions have dry and wet tropical forests that have very different carbon cycles. With more time, an improved model would incorporate additional research about different growth and sequestration patterns in different forests types. This could include creating distinctions based on key characteristics such as the previously mentioned dry and wet tropical forests.
3. **Excess Wood Production:** Our model suggests that in the long run, cutting down trees at a more significant level compared to current standards can help sequester additional carbon out of the atmosphere. However, this assumes that the additional wood obtained as a result of harvesting can be utilized in a way such that the carbon stored within is not released back in to the atmosphere. Although this could be a limitation in the short-run due to a lack of need for

excess wood, technologies already exist that seek to convert biomass into stable stores of carbon. A prime example of this is the generation of biochar through pyrolysis of the harvested wood. Using such a method would allow us to generate biochar while simultaneously producing a net gain in energy. This has the two-fold benefit of condensing carbon into a form that can buried and remain stable for centuries, while generating energy that can be utilized in the place of fossil fuel combustion [16].

7 Potential Improvements and Future Work

The model we created and the subsequently conducted analysis assumed that it was only possible for forests to exist in a homogeneous form, without differentiation on the bases of different tree species or topographical conditions. However, this may not be the case in reality as forests may not be normally distributed throughout their entirety with regards to age and carbon mass. Rather, they may have inter-species interactions which cause certain sections of forest to differ in the various characteristics.

Breaking away from the landscape assumptions we made would allow for a richer and more nuanced model at the expense of needing to implement different harvesting methods for different parts of the forest. Such a change in a future model would exponentially increase the ways in which we can vary the parameters for harvesting a given forest. While this would allow us to much more accurately mimic the behaviors of a real forest, it would also make it impossible to draw useful conclusions by fixing some variables while varying others.

However, with the aid of machine learning; in particular reinforcement learning, we believe that our model can indeed be adapted to provide nuanced and mixed harvesting strategies for a non-uniform forest model. Provided that we set up the environment and state representation correctly based on data more fine-tuned with what our model requires, we could implement a reinforcement learning agent that is rewarded for implementing harvesting techniques which result in higher total carbon sequestration. We would allow the model to train on multiple samples of forest types in order to explore harvesting strategy compatibility with certain types of forests, then expose it to mixed forests and introduce non-uniformity. The model would then be able to learn how to create an optimal mix of harvesting options throughout the varying characteristics of a single forest, further enabling our model's ability to be applied in a real forest management setting.

8 Conclusion

Forests and ecosystems alike are complex. They are defined by an intertwining of processes and relationships that determine not only their own ecological health, but also that of the entire planet. In this paper, we investigated the utilization of one of these particular processes - carbon sequestration. Despite the number of assumptions and generalizations that were made, we believe that the visualization and simulation methods generated by our model provides insightful representations of carbon sequestration over time. The model conveyed different levels of potential carbon sequestration across various biomes and the our parameter analysis allowed us to gain insight into the advantages and disadvantages of varying harvest frequency and aggressiveness. Using these findings, we were able to create a general decision framework that serves as an accessible starting point for forest managers who are deciding how to best enable their forests in development.

9 Non-Technical Newsletter Article

In the light of growing concerns over the impact that global warming will have on our everyday lives, it may be disconcerting to some that we have proposed for the cutting down of some trees in the forests of northern Canada. Furthermore, it may seem counterintuitive that we are proposing such a plan in the name of combating global warming. However, please rest assured that our proposal is backed by research and experimentation as well as careful consideration of the local ecosystem.

Deforestation as a general plan of action is certainly damaging to the environment and exacerbates the ongoing warming of our planet. This is not what we are recommending. Whereas deforestation involves the removal of a forest in its entirety without consideration for the environmental impacts of such an action, our plan instead centers around managing the density of the forest so that it can continue in its growth and development in a way that would not be possible if no actions were to be taken.

Trees, and more generally, forests, are great ways of counteracting the carbon emissions that have been the result of modern-day energy needs. However, for forests to properly serve this purpose, they must be kept healthy and allowed to continue growing. The issue with very mature forests such as the one at hand is that there is not much room left for growth, as the vast majority of trees comprising the forest have reached full maturity. Without this growth, forests lose their ability to continue extracting greenhouse gases from the atmosphere.

Moreover, there is the issue of the local dry climate to be concerned about. The dryness of the local terrain combined with its rather remote location presents a great risk of forest fires. As you well know, the low annual precipitation has resulted in close calls with forest fires in the past. Should such a fire begin spreading, the forest in its present state presents a considerable threat to both the local population and the global atmosphere. The high carbon mass of the forest as it is right now means that there is more fuel for the fire to consume. This would make potential fires much more difficult to control, and a result, could lead to unnecessary dangers being posed against the local population. Simultaneously, the burning of such a mature and unmanaged forest would lead to the release of an immense amount of greenhouse gases.

To combat both the lack of forest growth and the high risk of forest fires, we propose the controlled and structured cutting of select trees in the forest at a regular interval. The benefits of the plan is two-fold. First, it removes fully mature trees, freeing up nutrients, sunlight, and space for younger trees to grow, removing carbon dioxide from the atmosphere in the process. Second, it reduces the total carbon present as well as the carbon density of the forest, reducing the risk of fire spread and mitigating the threat that a fire would pose to the community.

Even considering the above reasons for why active management of the forest is vital, there may be some who suspect the possibility of detrimental environmental impacts to the local wildlife populations. We have already taken the characteristics of the local wildlife into consideration in our decision to propose this plan. The wildlife in the northern boreal forests of Canada is considered to be under low risk of species endangerment, While we cannot claim that harvesting operations will not impact wildlife in any way, we have chosen this forest due to its stable wildlife populations that will be able to easily recover from the minimally invasive procedures we seek to implement.

Without a doubt, it is an amazing thing that you care deeply for the health of the local ecosystem and wish to protect your forests from harm. We hope to work with the local population in helping protect both the forests and the local community from environmental dangers. Once again, we ask you to reconsider the proposal in this light and look forward to our cooperation in striving to do our parts in the fight against global warming.

10 Appendix

10.1 Source Code

```
#Imports and visual settings omitted for brevity
class Cell:
    def __init__(self, max_age: int):
        self.max_age = max_age
        temp_age = np.random.normal(0.5 * self.max_age, 0.2)
        self.curr_age = 1 if temp_age < 0 else temp_age
        self.carbon_mass, self.cs_rate = 0, 0.025
    def get_cs(self):
        return self.cs_rate
    def grow(self):
        self.curr_age += 1
        if self.carbon_mass + self.cs_rate < 3:
            self.carbon_mass += self.cs_rate
    def get_mass(self) -> float:
        return self.carbon_mass
    def cut(self) -> None:
        self.curr_age = 1
        self.set_mass()
def csr_mat(x):
    return (6.37*x) - (14.27*x**2) + (8.31*x**3)
class TropicalCell(Cell):
    def __init__(self, max_age: int):
        self.max_age = max_age
        temp_age = np.random.normal(0.5 * self.max_age, 0.5 * self.max_age)
        self.curr_age = 1 if temp_age < 0 else temp_age
        self.flat_rate = 0
    def set_cs_rate(self, density: float):
        mat_dist = csr_mat( 0.5-((0.5-(self.curr_age/self.max_age)) / 1.1))
        mat = 0.4 - (0.4 - mat_dist)/3
        mu = (0.5 * np.pi *  40 * 43 / 2000)
        self.cs_rate = self.flat_rate + 0.5 *((mat * 0.5 * density) * np.random.normal(mu, 0.1 * mu))
    def set_mass(self):
        mu = (0.5 * np.pi * 40 * 43 / 2000) * (self.curr_age / self.max_age)
        self.carbon_mass = np.random.normal(mu, 0.1 * mu)
class BorealCell(Cell):
    ...
class TemperateCell(Cell):
    ...
class Forest:
    def __init__(self, land, size: int, max_time: int, maturity: str, plotting: bool, phi: float, m: float, management_rate: int, begin_mgmt: int, ftype):
        self.size, self.maturity, self.plotting, self.max_time, self.land = size, maturity, plotting, max_time, land
        self.ftype, self.management_rate, self.phi, self.begin_mgmt, self.m = ftype, management_rate, phi, begin_mgmt, m
        self.net_sequestration, self.tot_atm_carbon, self.rmvd_carbon, self.tot_rmvd_carbon = [], [], [], []
        self.tot_rmvd_carbon, self.curr_carbon, self.total_carbon, self.curr_atm_release = [], [], [], []
    def simulate(self) -> None:
        p = .55
        q = 1-p
        for t in range(self.max_time):
            if t == 0:
                for (ii, ij) in np.ndindex(self.land.shape):
                    self.land[ii, ij].set_mass()
            if t % 10 == 0:
                if self.plotting == True:
                    self.plot_2d_mass(time=t)
                    plt.title(r'$\mathrm{{Time}}={{{}}}$'.format(str(t)))
            if t % self.management_rate == 0 and t >= self.begin_mgmt:
                curr_rmv = self.deforest()
                self.rmvd_carbon.append(p * curr_rmv)
                self.tot_rmvd_carbon.append(sum(self.rmvd_carbon))
                if self.plotting == True:
                    self.plot_2d_mass(time=t)
                    plt.title(r'$\mathrm{{Harvest Time}}={{{}}}$'.format(str(t)))
            rand_order = np.random.permutation([(x, y) for x in range(self.size) for y in range(self.size)])
            for (ii, ij) in rand_order:
                assert(isinstance(self.land[ii, ij], Cell))
                density = self.land[ii, ij].get_mass() / self.get_surrounding_density(ii, ij)
                self.land[ii, ij].set_cs_rate(density)
                self.land[ii, ij].grow()
            self.curr_carbon.append(self.get_curr_carbon_mass())
        self.total_carbon = [v if i < self.begin_mgmt else v + self.tot_rmvd_carbon[math.floor((i-self.begin_mgmt)//self.management_rate)] \
                             for i, v in enumerate(self.curr_carbon)]
        self.total_atm_carbon = [0 if i < self.begin_mgmt else q * self.tot_rmvd_carbon[math.floor((i-self.begin_mgmt)//self.management_rate)] \
                                 for i, v in enumerate(self.curr_carbon)]
        self.net_sequestration = [x-y for x, y in zip(self.total_carbon, self.total_atm_carbon)]
        if self.plotting:
            fig, axes = plt.subplots(3,1, sharex=True, figsize=(6, 6))
            x = list(range(1, self.max_time + 1))
            sns.set_style(style = "whitegrid")
            sns.lineplot(x=x, y=self.total_carbon, color='#81A1C1')
            sns.lineplot(ax = axes[1], x=x, y=self.total_atm_carbon, color='#81A1C1')
            sns.lineplot(ax = axes[2], x=x, y=self.net_sequestration, color='#81A1C1')
            if USE_TEX:
                plt.xlabel(r'Time (yr)')
                plt.ylabel(r"Total Carbon Sequestered (tC)")
                axes[0].set(ylabel=r"$\mathrm{Total\ Carbon\ Sequestered \ (tC)}$")
```

```python
            axes[1].set(ylabel=r"$\mathrm{Total\ Carbon\ Lost\ to\ Atmosphere\ (tC)}$")
            axes[2].set(ylabel=r"$\mathrm{Net\ Carbon\ Sequestration \ (tC)}$")
        axes[0].yaxis.get_label().set_fontsize(7)
        axes[1].yaxis.get_label().set_fontsize(7)
        axes[2].yaxis.get_label().set_fontsize(7)
        plt.suptitle('Carbon Levels Over Time',fontsize=15)
        fig.align_ylabels()
        plt.tight_layout(pad=2.0)
        if PRODUCTION:
            path = "./better_images/" + self.ftype + "_" + str(self.phi) + "_" + str(self.management_rate)
            plt.savefig(path + ".pdf")
            with open(path+'.txt', 'w') as f:
                f.writelines(["Total Carbon Sequestration: " + str(self.total_carbon[-1]) + "\n", \
                              "Total Carbon Lost to Atmosphere: " + str(self.total_atm_carbon[-1]) + "\n", \
                              "Net Sequestration: " + str(self.total_carbon[-1] - self.total_atm_carbon[-1])])
    else:
        return self.total_carbon, self.total_atm_carbon, self.net_sequestration
def deforest(self) -> int:
    curr_rmvd = 0
    rand_order = np.random.permutation([(x, y) for x in range(2,self.size - 2) for y in range(2,self.size - 2)])
    for (ii, ij) in rand_order:
        surr_mass = self.get_surrounding_density(ii, ij) * 8
        if surr_mass > self.phi:
            for (ix, iy) in [(x, y) for x in range(ii - 1, ii + 2) for y in range(ij - 1, ij + 2)]:
                curr_mass = self.land[ix, iy].get_mass()
                if curr_mass > self.m:
                    rand = np.random.random()
                    if rand > .9:
                        self.land[ix, iy].cut()
                        curr_rmvd += curr_mass
    return curr_rmvd
def get_surrounding_density(self, ii: int, ij: int) -> float:
    #Omitted for brevity: averages carbon mass of surrounding cells
def get_curr_carbon_mass(self) -> int:
    return sum([self.land[ii, ij].get_mass() for (ii, ij) in np.ndindex(self.land.shape)])
def populate_state(self):
    ret = [[0 for _ in range(self.size)] for _ in range(self.size)]
    for i in range(self.size):
        for j in range(self.size):
            ret[i][j] = self.land[i, j].get_mass()
    return ret
def plot_2d_mass(self, time=0) -> None:
    curr_state = self.populate_state()
    ax = sns.heatmap(curr_state, cmap =sns.cubehelix_palette(start=2, rot=0, dark=0.2, light=.8,as_cmap=True),\
                     vmin = 0, vmax = 3, xticklabels=False, yticklabels=False, cbar_kws={'label': '$\mathrm{Carbon\ Mass\ (tC)}$'})
    plt.savefig("./tropical_landscape/" + self.ftype + "_" + str(time) + ".png")
    plt.show()
class Tropical(Forest):
    def __init__(self, size: int,max_time: int, maturity: str, plotting: bool, phi: float, m: float, management_rate: int, begin_mgmt: int,
↪   ftype="tropical"):
        super().__init__(np.array([[TropicalCell(np.random.normal(95, 15)) for _ in range(size)] for _ in range(size)]), \
                         size, max_time, maturity, plotting, phi, m, management_rate, begin_mgmt, ftype="tropical")
class Boreal(Forest):
    ...
class Temperate(Forest):
    ...
phi_VALS = [10]
ov_total_carbon, ov_total_atm_carbon, ov_net_sequestration = [], [], []
for i in phi_VALS:
    tropical = Tropical(100, 130, "test", False, i, 2.3, 20, 0, ftype="Tropical")
    vals = tropical.simulate()
    ov_total_carbon.append(vals[0])
    ov_total_atm_carbon.append(vals[1])
    ov_net_sequestration.append(vals[2])
def plot_total_carbon(pl="tot_carbon"):
    x = list(range(1, 130 + 1))
    sns.set_style(style = "whitegrid")
    fig, ax = plt.subplots(sharex=True)
    y = ov_total_carbon if pl == "tot_carbon" else ov_total_atm_carbon if pl == "tot_atm" \
        else ov_net_sequestration
    for i in range(len(phi_VALS)):
        plt.plot(x, y[i], label=r'$\phi = {}$'.format(str(phi_VALS[i])))
    plt.legend()
    plt.xlabel(r'Time (yr)')
    plt.ylabel(r"Total Carbon Sequestered (tC)" if pl == "tot_carbon" else \
               r'Total Carbon Lost to Atmosphere (tC)' \
               if pl == "tot_atm" else r'Net Carbon Sequestration (tC)')
    plt.savefig("better_images/" + pl + "_overlay_new.pdf")
plot_total_carbon()
plot_total_carbon("tot_atm")
plot_total_carbon("net_seq")
MGM_VALS = [40,50,60,70,80,90]
for i in MGM_VALS:
    tropical = Tropical(100, 130, "test", False, 10, 2.3, i, 0, ftype="Tropical")
    vals = tropical.simulate()
    ov_total_carbon.append(vals[0])
    ov_total_atm_carbon.append(vals[1])
    ov_net_sequestration.append(vals[2])
```

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