2 Lecture 2: Basic Mathematics II

2.1 Summation Operator

- Consider a set $X = \{x_1, x_2, ..., x_n\}$
  - $\sum_{i=1}^{n} x_i = x_1 + x_2 + ... + x_n$
    - "The sum of x_i, over the range from $i = l$ through $i = n$."
- Let $Y = \{y_1, y_2, ..., y_n\}$
  - $\sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + ... + x_n y_n$
    - Here, we sum the outcome of $x_i y_i$ n times.
- But, what if there is more than one summation operator and subscript?

$$\begin{aligned}\Sigma_{j=1}^{m}\Sigma_{i=1}^{n} x_i y_j &= (y_1 x_1 + y_1 x_2 + ... + y_1 x_n)\\ &+ (y_2 x_1 + y_2 x_2 + ... + y_2 x_n)\\ &+ ... + (y_m x_1 + y_m x_2 + ... + y_m x_n)\end{aligned}$$

  We have m parentheses, with n x's in each parenthesis. We end up summing each combination of x_i and y_j.

- What if there is a constant, c? Drawing upon the *distributive property*:
  - $\sum_{i=1}^{n} c x_i = c x_1 + c x_2 + ... + c x_n = c \sum_{i=1}^{n} x_i$
- The *associative property* can also be applied to summation:
  - $\sum_{i=1}^{n} (x_i + y_i) = (x_1 + y_1) + (x_2 + y_2) + ... + (x_n + y_n) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$

2.2 Product Operator

- $X = \{x_1, x_2, ..., x_n\}$ and $Y = \{y_1, y_2, ..., y_n\}$
- $\prod_{i=1}^{n} x_i = x_1 \cdot x_2 \cdot ... \cdot x_n$
- $\prod_{i=1}^{n} (x_i + y_i) = (x_1 + y_1)(x_2 + y_2)...(x_n + y_n)$
  - We can't split $(x_i + y_i)$ like in summation. Instead, we multiply $(x_i + y_i)$ repeatedly.
- What about a constant?
  - $\prod_{i=1}^{n} c x_i = (c x_1)(c x_2) \cdot ... \cdot (c x_n) = c^n \prod_{i=1}^{n} x_i$

  We can move c to the front, but we have to exponentiate it to c^n.

2.3 Factorials, Permutations, and Combinations

Most of our quantitative coursework is about modeling probabilities, where:

$$\text{probability} = \frac{\#\text{occurences}}{\#\text{possibilities}}$$

For both the numerator and denominator we are dealing with *counting* the number of relevant outcomes. Factorials, permutations, and combinations are foundational concepts when it comes to counting.

- Some useful illustrations/properties:
  - $0! = 1$
  - $x! = x \cdot (x-1) \cdot (x-2) \cdot \ldots \cdot 0!$
  - $2! = 2 \cdot 1$
  - $3! = 3 \cdot 2 \cdot 1$
  - $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot \ldots \cdot 1$
  - If n objects in boxes of size x where *order matters*, then the number of *permutations* is:

$$\frac{n!}{(n-x)!}$$

  - If order does not matter, then the number of *combinations* is:

$$\frac{n!}{(n-x)!x!} = nCx = \binom{n}{x}$$

    - Pronounced: 'n choose x'

2.4 Solving equations, inequalities, and for roots

- Solving an equation example:

$$\begin{aligned} 3x + 4y + 8 &= 0 \\ 4y &= -(3x+8) \\ y &= \frac{-(3x+8)}{4} \\ y &= -\frac{3}{4}x - 2 \end{aligned}$$

We can also write this answer as:

$$\{(x, y) \in \mathbb{R} | y = -\frac{3}{4}x - 2\}$$

- Solving an inequality ($x >$, $x \geqslant y$, $y < x$, $y \leqslant x$) example:

$$\begin{aligned} -4y &> 2x + 12 \\ y &< -\frac{2x}{4} - \frac{12}{4} \\ y &< \frac{x}{2} - 3 \end{aligned}$$

  Note that dividing by a negative flips the sign

- Solving for a quadratic
  - Quadratic formula:

$$\begin{aligned} &ax^2 + bx + c \\ &x \in \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

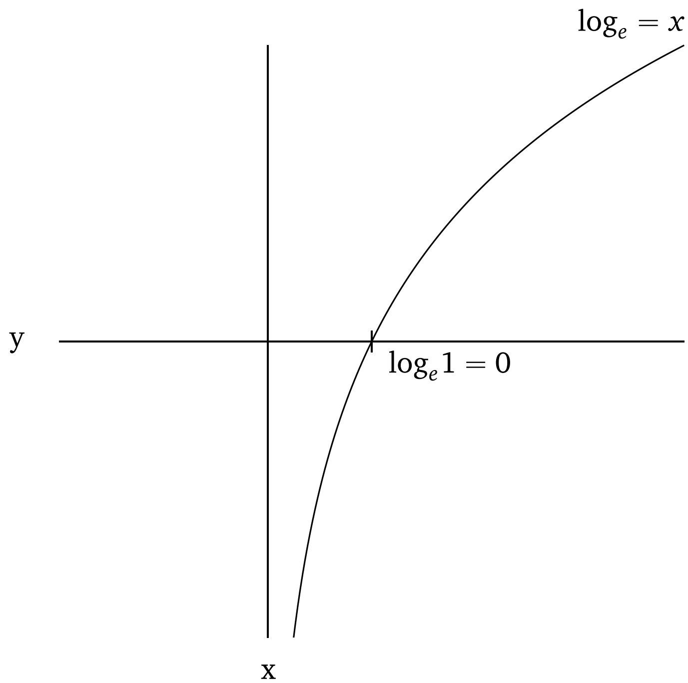
    Example:

$$\begin{aligned} &1.4x^2 + 3.7x + 1.1 = 0 \\ &a = 1.4, b = 3.7, c = 1.1 \\ &x = \frac{-3.7 \pm \sqrt{3.7^2 - 4 \times 1.4 \times 1.1}}{2.8} \\ &x = -0.341 \text{ or } x = -2.301 \end{aligned}$$

2.5 Logarithms

- If $y = a^x$, then we can rewrite as $\log_a y = x$
- Examples:
  - $\log_e e = 1$, because $e^1 = e$
  - Solving for x:
    - If $8 = 2^x$, then $\log_2 8 = x = 3$
- **Rules**
  - $\log(m \cdot n) = \log(m) + \log(n)$
  - $\log(\frac{m}{n}) = \log(m) - \log(n)$
  - $\log(b^a) = a \log b$
  - $\log(b^a) = (\log_b e)(\log_e a)$
  - $\log(b^a) = \frac{1}{\log_a b}$

- **Visual**



- Note: $\log_e(x) \equiv ln(x)$, pronounced the 'natural logarithm'