

**Due: August 11, 2023**

1. Consider the following linear system:

$$\begin{aligned}2x + w &= 7 \\3x + 2y + 3z &= -2 \\2x + 3y - 4z &= 3 \\x + 3z &= 5\end{aligned}$$

- a) Find the coefficient matrix.  
b) Write the linear system in matrix form.  
c) Find the augmented matrix.
2. Write the linear system for the augmented matrix:

$$\left[ \begin{array}{ccc|c} 2 & 0 & -4 & 3 \\ 0 & 1 & 2 & 5 \\ 1 & 3 & 4 & -1 \end{array} \right]$$

3. Write the following linear system as a linear combination of the columns of the coefficient matrix:

a)

$$\begin{aligned}x + 2y &= 3 \\2x - y &= 5\end{aligned}$$

b)

$$\begin{aligned}2x - 3y + 5z &= -2 \\x + 4y - z &= 3\end{aligned}$$

4. Find the value of  $r$  so that  $AB^T = 0$ , where

$$A = \begin{bmatrix} r & 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & -1 \end{bmatrix}$$

5. Let

$$A = \begin{bmatrix} 2 & 1 & -2 \\ 3 & 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 1 & -2 \end{bmatrix}$$

If possible, compute:

a)  $(AB)^T$

b)  $B^T A^T$

c)  $A^T B^T$

d)  $BB^T$

e)  $B^T B$

6. Let

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

Find,

a)  $A^2 + 3A$

b)  $2A^3 + 3A^2 + 4A + 5I_2$

7. Determine a scalar  $r$  such that  $Ax = rx$ , where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

8. For the following matrices, determine whether it is in reduced row echelon form, row echelon form, or neither:

a)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

9. Let

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -3 & 1 & 4 \\ 4 & 2 & 2 \\ 5 & -1 & 5 \end{bmatrix}$$

Find the matrices obtained by performing the following elementary row operations on  $A$ .

- a) Interchanging the second and the fourth rows of the original matrix.
- b) Multiplying the third row of the original matrix by 3.
- c) Adding  $(-3)$  times the first row to the fourth row (original matrix).

10. Find the solution to this linear system:

$$\begin{aligned} x + y + z &= 1 \\ x + y - 2z &= 3 \\ 2x + y + z &= 2 \end{aligned}$$

11. Solve the linear system with the following augmented matrix:

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 \end{array} \right]$$