Due: August 12, 2024

1. Show that $\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$ is not singular.

2. Is the matrix $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ singular or nonsingular? If it is nonsingular, find the inverse.

3. For the following Matrices, find the inverses if possible:

   a)
   $$\begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$$

   b)
   $$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

   c)
   $$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 3 & 9 & 3 & 6 \end{bmatrix}$$

4. Suppose that $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

   (a) Find A^{-1}

   (b) Find $(A^T)^{-1}$; How do $(A^T)^{-1}$ and A^{-1}compare?

5. For the following matrices evaluate their determinants

   a)
   $$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

   b)
   $$\begin{bmatrix} 0 & 3 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

   c)
   $$\begin{bmatrix} 4 & 2 & 0 \\ 0 & -2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

d)

$$\begin{bmatrix} 4 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

6. Verify that $det(AB) = det(A) \cdot det(B)$ for the following:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -2 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

7. If A and B are nxn matrices with $|A| = 2$ and $|B| = -3$, calculate $|A^{-1}B^T|$.

8. Let

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & 4 \\ 5 & 2 & -3 \end{bmatrix}$$

Compute all cofactors.

9. For the following Matrices, compute determinants by cofactor expansion:

a)

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 2 \\ 3 & 2 & 0 \end{bmatrix}$$

10. Let

$$A = \begin{bmatrix} 6 & 2 & 8 \\ -3 & 4 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

a) Find $adjA$

b) Compute $det(A)$

c) Verify that $A(adjA) = (adjA) \cdot A = detA \cdot I_3$

11. Compute the inverses of the following matrices if they exist by cofactor expansion:

a)

$$\begin{bmatrix} 1 & 2 & -3 \\ -4 & -5 & 2 \\ -1 & 1 & -7 \end{bmatrix}$$

b)

$$\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

12. Find all values of λ for which:

$$det\left(\begin{bmatrix} \lambda - 1 & -4 \\ 0 & \lambda - 4 \end{bmatrix}\right) = 0$$