Due: August 12, 2021

1. Consider the following linear system:

$$\begin{aligned} 2x + w &= 7 \\ 3x + 2y + 3z &= -2 \\ 2x + 3y - 4z &= 3 \\ x + 3z &= 5 \end{aligned}$$

   a) Find the coefficient matrix.

   b) Write the linear system in matrix form.

   c) Find the augmented matrix.

2. Write the linear system for the augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & 0 & -4 & 3 \\ 0 & 1 & 2 & 5 \\ 1 & 3 & 4 & -1 \end{array}\right]$$

3. Write the following linear system as a linear combination of the columns of the coefficient matrix:

   a)

$$\begin{aligned} x + 2y &= 3 \\ 2x - y &= 5 \end{aligned}$$

   b)

$$\begin{aligned} 2x - 3y + 5z &= -2 \\ x + 4y - z &= 3 \end{aligned}$$

4. Find the value of r so that $AB^T = 0$, where

$$A = \begin{bmatrix} r & 1 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & -1 \end{bmatrix}$$

5. Let

$$A = \begin{bmatrix} 2 & 1 & -2 \\ 3 & 2 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ 1 & -2 \end{bmatrix}$$

If possible, compute:

a) $(AB)^T$

b) $B^T A^T$

c) $A^T B^T$

d) BB^T

e) $B^T B$

6. Let

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

Find,

a) $A^2 + 3A$

b) $2A^3 + 3A^2 + 4A + 5I_2$

7. Determine a scalar r such that $Ax = rx$, where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

8. For the following matrices, determine whether it is in reduced row echelon form, row echelon form, or neither:

a)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

9. Let

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -3 & 1 & 4 \\ 4 & 2 & 2 \\ 5 & -1 & 5 \end{bmatrix}$$

Find the matrices obtained by performing the following elementary row operations on A.

a) Interchanging the second and the fourth rows of the original matrix.

b) Multiplying the third row of the original matrix by 3.

c) Adding (-3) times the first row to the fourth row (original matrix).

10. Find the solution to this linear system:

$$\begin{aligned} x + y + z &= 1 \\ x + y - 2z &= 3 \\ 2x + y + z &= 2 \end{aligned}$$

11. Solve the linear system with the following augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 \end{array}\right]$$