

**Due: August 14, 2023**

1. Show that  $\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$  is not singular.
2. Is the matrix  $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$  singular or nonsingular? If it is nonsingular, find the inverse.
3. For the following Matrices, find the inverses if possible:

a)

$$\begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

c)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 3 & 9 & 3 & 6 \end{bmatrix}$$

4. Suppose that  $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

(a) Find  $A^{-1}$ (b) Find  $(A^T)^{-1}$ ; How do  $(A^T)^{-1}$  and  $A^{-1}$  compare?

5. For the following matrices evaluate their determinants

a)

$$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

b)

$$\begin{bmatrix} 0 & 3 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

c)

$$\begin{bmatrix} 4 & 2 & 0 \\ 0 & -2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

d)

$$\begin{bmatrix} 4 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

6. Verify that  $\det(AB) = \det(A) \cdot \det(B)$  for the following:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -2 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

7. If  $A$  and  $B$  are  $n \times n$  matrices with  $|A| = 2$  and  $|B| = -3$ , calculate  $|A^{-1}B^T|$ .

8. Let

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & 4 \\ 5 & 2 & -3 \end{bmatrix}$$

Compute all cofactors.

9. For the following Matrices, compute determinants by cofactor expansion:

a)

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 5 & 2 \\ 3 & 2 & 0 \end{bmatrix}$$

10. Let

$$A = \begin{bmatrix} 6 & 2 & 8 \\ -3 & 4 & 1 \\ 4 & -4 & 5 \end{bmatrix}$$

a) Find  $\text{adj} A$

b) Compute  $\det(A)$

c) Verify that  $A(\text{adj} A) = (\text{adj} A) \cdot A = \det A \cdot I_3$

11. Compute the inverses of the following matrices if they exist by cofactor expansion:

a)

$$\begin{bmatrix} 1 & 2 & -3 \\ -4 & -5 & 2 \\ -1 & 1 & -7 \end{bmatrix}$$

b)

$$\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

12. Find all values of  $\lambda$  for which:

$$\det \left( \begin{bmatrix} \lambda - 1 & -4 \\ 0 & \lambda - 4 \end{bmatrix} \right) = 0$$