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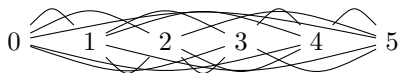
October 21, 2024

Problem 1

Let $A = \{0, 1, 2, 3, 4, 5\}$. Write out the relation R that expresses $>$ on A . Then illustrate it with a diagram.

Include all the ordered pairs where the first coordinate is strictly greater than the second coordinate:

$$R = \{(1, 0), (2, 1), (2, 0), (3, 2), (3, 1), (3, 0), \dots, (5, 1), (5, 0)\}$$



Problem 2

Let $A = \{1, 2, 3, 4, 5, 6\}$. Write out the relation R that expresses $|$ (divides) on A . Then illustrate it with a diagram.

First note that 1 divides everything, so $(1, x)$ will be in R for all $x \in A$. 2 divides itself, 4, and 6, 3 divides itself and 6, and the rest only divide themselves:

$$R = \{(1, 1), (1, 2), \dots, (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)\}$$

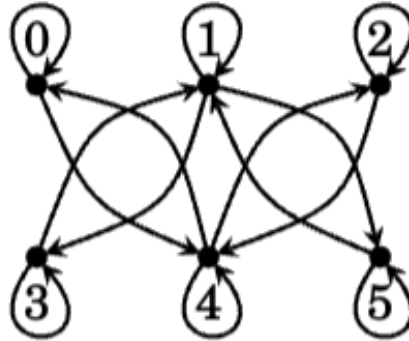
Problem 3

Let $A = \{0, 1, 2, 3, 4, 5\}$. Write out the relation R that expresses \geq on A . Then illustrate it with a diagram.

$$R = \{(0, 0), (1, 1), (1, 0), (2, 2), (2, 1), (2, 0), \dots, (5, 1), (5, 0)\}$$

Problem 4

Here is a diagram for a relation R on a set A . Write the sets A and R .

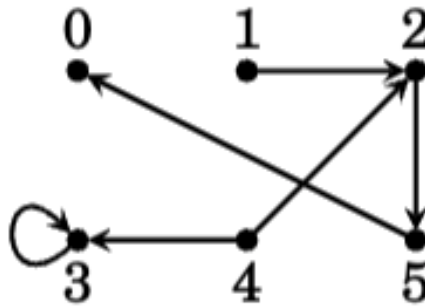


The set A are the letters 0 through 5. We see that relation is reflexive and symmetric but not transitive. It appears to be similar to the 'same parity' relation but missing some elements.

$$R = \{(0, 0), (1, 1), \dots, (5, 5), (0, 4), (4, 0), (1, 3), (3, 1), (2, 4), (4, 2), (1, 5), (5, 1)\}$$

Problem 5

Here is a diagram for a relation R on a set A . Write the sets A and R .



The set A are the nodes on the graph:

$$A = \{0, 1, 2, 3, 4, 5\}$$

The set R is the ordered pairs for the arrows:

$$R = \{(1, 2), (2, 5), (3, 3), (4, 2), (4, 3), (5, 0)\}$$

Problem 6

Congruence modulo 5 is a relation on the set \mathbb{Z} . In this relation xRy means $x \equiv y \pmod{5}$. Write out the set R in set-builder notation.

Elements of R are ordered pairs (x, y) , since $R \subseteq \mathbb{Z} \times \mathbb{Z}$. We also need to specify that a pair (x, y) belongs to R only if x and y are congruent mod 5:

$$R = \{(x, y) \in \mathbb{Z}^2 \mid x \equiv y \pmod{5}\}$$

Problem 7

Write the relation $<$ on the set \mathbb{Z} as a subset R of $\mathbb{Z} \times \mathbb{Z}$. This is an infinite set, so you will have to use set builder notation.

$$R = \{(x, y) \in \mathbb{Z}^2 \mid x < y\}$$

Problem 8

Let $A = \{1, 2, 3, 4, 5, 6\}$. Observe that $\emptyset \subseteq A \times A$, so $R = \emptyset$ is a relation on A . Draw a diagram for this relation.

The diagram would be a graph with nodes 1 through 6, no edges (since there are no relationships in the set).

Problem 9

Let $A = \{1, 2, 3, 4, 5, 6\}$. How many different relations are there on the set A ?

Any subset of $A \times A$ qualifies as a relation, so the answer will be $|\mathcal{P}(A \times A)|$. Since $|A| = 6$ the product $|A \times A| = 36$, and $|\mathcal{P}(A \times A)| = 2^{36}$.

Problem 10

Consider the subset $R = (\mathbb{R} \times \mathbb{R}) - \{(x, x) : x \in \mathbb{R}\} \subseteq \mathbb{R} \times \mathbb{R}$. What familiar relation on \mathbb{R} is this? Explain.

The relation R starts with all ordered pairs in \mathbb{R}^2 but then removes the pairs with the same value on both coordinates. So every real number has this relationship to every other real number but not itself. That's the \neq relation.