5 Contrapositive Proof

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Problem 1

If integers a and b are not both zero, then $\gcd(a, b) = \gcd(a - b, b)$.

Solution:

Show inequality both ways. Let $d = \gcd(a, b)$, so $a = dx$ and $b = dy$ for some x, y, and let $e = \gcd(a - b, b)$. We want to show that d divides both $a - b$ and b, so that $d \leq e$. We already have d dividing b since $d = \gcd(a, b)$. We can write $a - b$ as $dx - dy = d(x - y)$, which means $d \mid d(x - y) \implies d \mid a - b$. Since d divides both $a - b$ and b it is a common divisor, and cannot be greater than $e = \gcd(a - b, b)$. Therefore $d \leq e$.
Now to show $e \leq d$, note that e divides b and e divides $a - b$. Therefore e divides $(a - b) + b = a$. Since e divides both a and b it is a common divisor but cannot be greater than the $\gcd(a, b) = d$. Therefore $e \leq d$.

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1 Introduction

This is the introduction section.

2 Main Content

This is the main content section.

2.1 Subsection

This is a subsection.

2.1.1 Subsubsection

This is a subsubsection.

3 Conclusion

This is the conclusion section.