11.1 Relations

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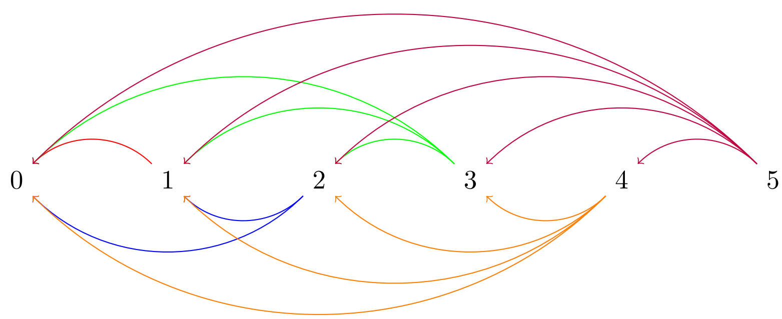
Problem 1

Let $A = \{0, 1, 2, 3, 4, 5\}$. Write out the relation R that expresses $>$ on A. Then illustrate it with a diagram.

Include all the ordered pairs where the first coordinate is strictly greater than the second coordinate:

$$R = \{(1, 0), (2, 1), (2, 0), (3, 2), (3, 1), (3, 0), \ldots, (5, 1), (5, 0)\}$$



Problem 2

Let $A = \{1, 2, 3, 4, 5, 6\}$. Write out the relation R that expresses $|$ (divides) on A. Then illustrate it with a diagram.

First note that 1 divides everything, so $(1, x)$ will be in R for all $x \in A$. 2 divides itself, 4, and 6, 3 divides itself and 6, and the rest only divide themselves:

$$R = \{(1, 1), (1, 2), \ldots, (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)\}$$

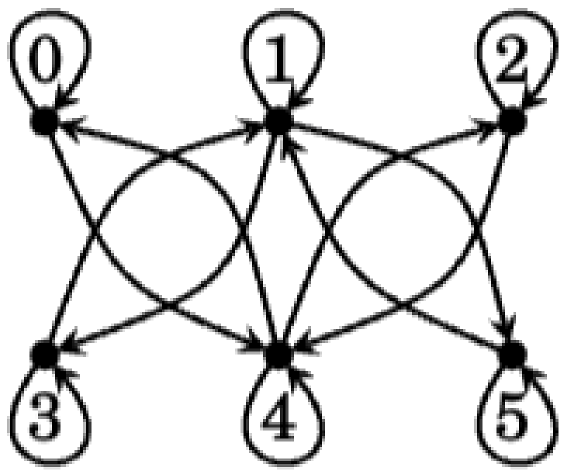
Problem 3

Let $A = \{0, 1, 2, 3, 4, 5\}$. Write out the relation R that expresses $\geq$ on A. Then illustrate it with a diagram.

$$R = \{(0, 0), (1, 1), (1, 0), (2, 2), (2, 1), (2, 0), \ldots, (5, 1), (5, 0)\}$$

Problem 4

Here is a diagram for a relation R on a set A. Write the sets A and R.

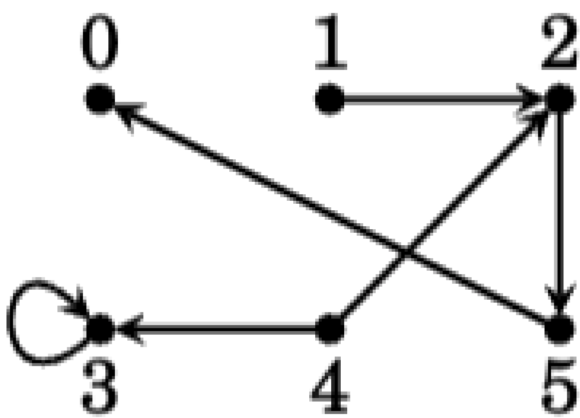


The set A are the letters 0 through 5. We see that relation is reflexive and symmetric but not transitive. It appears to be similar to the 'same parity' relation but missing some elements.

$$R = \{(0,0),(1,1),\ldots,(5,5),(0,4),(4,0),(1,3),(3,1),(2,4),(4,2),(1,5),(5,1)\}$$

Problem 5

Here is a diagram for a relation R on a set A. Write the sets A and R.



The set A are the nodes on the graph:

$$A = \{0,1,2,3,4,5\}$$

The set R is the ordered pairs for the arrows:

$$R = \{(1,2),(2,5),(3,3),(4,2),(4,3),(5,0)\}$$

Problem 6

Congruence modulo 5 is a relation on the set $\mathbb{Z}$. In this relation xRy means $x \equiv y \pmod 5$. Write out the set R in set-builder notation.

Elements of R are ordered pairs (x, y), since $R \subseteq \mathbb{Z} \times \mathbb{Z}$. We also need to specify that a pair (x, y) belongs to R only if x and y are congruent mod 5:

$$R = \{(x, y) \in \mathbb{Z}^2 \mid x \equiv y \pmod 5\}$$

Problem 7

Write the relation ¡ on the set $\mathbb{Z}$ as a subset R of $\mathbb{Z} \times Z$. This is an infinite set, so you will have to use set builder notation.

$$R = \{(x, y) \in \mathbb{Z}^2 \mid x < y\}$$

Problem 8

Let $A = \{1, 2, 3, 4, 5, 6\}$. Observe that $\varnothing \subseteq A \times A$, so $R = \varnothing$ is a relation on A. Draw a diagram for this relation.

The diagram would be a graph with nodes 1 through 6, no edges (since there are no relationships in the set).

Problem 9

Let $A = \{1, 2, 3, 4, 5, 6\}$. How many different relations are there on the set A?

Any subset of $A \times A$ qualifies as a relation, so the answer will be $|\mathcal{P}(A \times A)|$. Since $|A| = 6$ the product $|A \times A| = 36$, and $|\mathcal{P}(A \times A)| = 2^{36}$.

Problem 10

Consider the subset $R = (\mathbb{R} \times \mathbb{R}) - \{(x, x) : x \in \mathbb{R}\} \subseteq \mathbb{R} \times \mathbb{R}$. What familiar relation on $\mathbb{R}$ is this? Explain.

The relation R starts with all ordered pairs in $\mathbb{R}^2$ but then removes the pairs with the same value on both coordinates. So every real number has this relationship to every other real number but not itself. That's the $\neq$ relation.

Problem 11

Given a finite set A, how many different relations are there on A?

As we saw in a previous problem, it's the number of subsets the product $A \times A$ has: $|\mathcal{P}(A \times A)|$.