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Problem 1

Let X be a topological space; let A be a subset of X. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subset A$. Show that A is open in X.

Proof: direct, using the topology axioms and given information We're given for each $x \in A$ there is an open set U where $x \in U \subset A$. We have arbitrary unions of open sets are open from the topology axioms. So for each x call the containing open set U_x that is itself contained in A, then A is an arbitrary union of open sets, which is open:

$$A = \bigcup_{x \in A} U_x$$

Problem 2

Consider the nine topologies on the set $X = \{a, b, c\}$ indicated in Example 1 of §12. Compare them; that is, for each pair of topologies, determine whether they are comparable, and if so, which is the finer.