Exponential/Logistic models Exercises

L3 - Exercises

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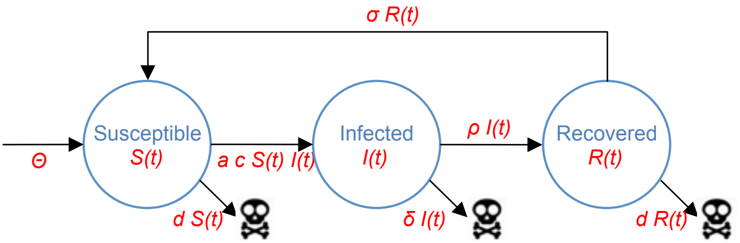
Exercise 1.4 Calculate a logistic model for the last 100 years with r of 2.8 %, initial population of 2.55 billions and carrying capacity of 11.5 billions. Plot the result! (L3, p. 15-17)

Exercise 1.5 (EXTRA) Calculate a simple predator-prey model (L3, p. 21-23) and plot the temporal dynamics. For the parameter values r=0.5, a=1.0, c=0.01, ϵ=0.02, and δ=0.1. For an initial population of the prey of 500 and predators of 30. For a time range of 150 years. Plot phase plane (Predator vs. Prey) for different initial conditions, describe the plot you obtain .

Exercise 2 - SIR Model

Exercise 2.1 By using the parameter values and the initial conditions given in the lecture, code and graphically represent the SIR model. Get familiar with the dynamics of the model by varying the parameter values one by one and annotating the model behaviour.



$$\frac{dS}{dt} = \theta - d\,S(t) - a\,c\,S(t)\,I(t) + \sigma\,R(t)$$

$$\frac{dR}{dt} = \rho\,I(t) - \sigma\,R(t) - d\,R(t)$$

$$\frac{dI}{dt} = a\,c\,S(t)\,I(t) - \delta\,I(t) - \rho\,I(t)$$

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Symbol	Description	Value (per day)
c	Contact rate per susceptible individual	1.0
a	Probability that the disease is transmitted upon contact	0.025
ρ	Recovery rate per infected individual	0.03
d	Death rate of healthy individuals (i.e. susceptible and recovered individuals)	0.1
σ	Rate of recovered individuals that become susceptible of being infected again	0.3
θ	Immigration rate of individuals entering the population of those that are susceptible of being infected	0.15
δ	Death rate of infected individuals	0.25

As for initial conditions, use: 85 susceptible, 10 infected, and 5 recovered.

Exercise 3 - HIV Model

Exercise 3.1 Code the Phillips' HIV model in python and reproduce the main results obtained by Phillips in his 1996 article, which were shown in the lecture (L4, p. 20). Try some sensitivity analysis (i.e. change parameters and observe how the results may change).

$$\frac{dR(t)}{dt} = \Gamma\,\tau - \mu\,R(t) - \beta\,V(t)\,R(t)\,,$$

$$\frac{dL(t)}{dt} = p\,\beta\,V(t)\,R(t) - \mu\,L(t) - \alpha\,L(t)\,,$$

$$\frac{dE(t)}{dt} = (1 - p)\,\beta\,V(t)\,R(t) + \alpha\,L(t) - \delta\,E(t)\,,$$

$$\frac{dV(t)}{dt} = \pi\,E(t) - \sigma\,V(t)\,.$$

Exercise 3 - HIV Model

Symbol	Description	Value (per day)	Comments
Γ	Rate of CD4 cell production	1.36	Equivalent to 3.4×10^{8} cells per day in whole body
μ	HIV-independent death rate of susceptible CD4 cells	1.36×10^{-3}	Average life span of uninfected CD4 cells: 2 years
τ	Fraction of cells becoming susceptible of HIV attack	0.2	
β	Rate of infection of cells per virion	0.00027	Cannot be estimated directly
p	Proportion of cells becoming latently infected upon infection	0.1	
α	Activation rate of latently infected cells	3.6×10^{-2}	
σ	removal rate of cell-free virus	2.0	Average life span: 1/2 day
δ	Removal (death) rate of actively infected cells	0.33	Average life span: 3 days
π	Reate of production of virions by an actively infected cell	100.0	

- The total number of CD4 cells is: $1000 \times (1 - \tau) + R(t) + L(t) + E(t)$
- Initial conditions are: $R(t=0) = 200$, $L(t=0) = 0$, $E(t=0) = 0$, $V(t=0) = 4.0x10^{-7}$