

Development of a double-sided ladder for tracking in high energy physics

Ph. D. defense

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- 1 Introduction
- 2 Higgs boson study
- 3 Detector development
- 4 Mechanical deformation
- 5 Radiation length measurement
- 6 Conclusion and outlook

Electroweak symmetry breaking

Standard Model:

- Well-tested physics theory: predict precisely a wide variety of phenomena
- Electro-Weak Symmetry Breaking (EWSB): explain how particles acquire mass via Higgs mechanism
- Last milestone: discovery of the Higgs Boson

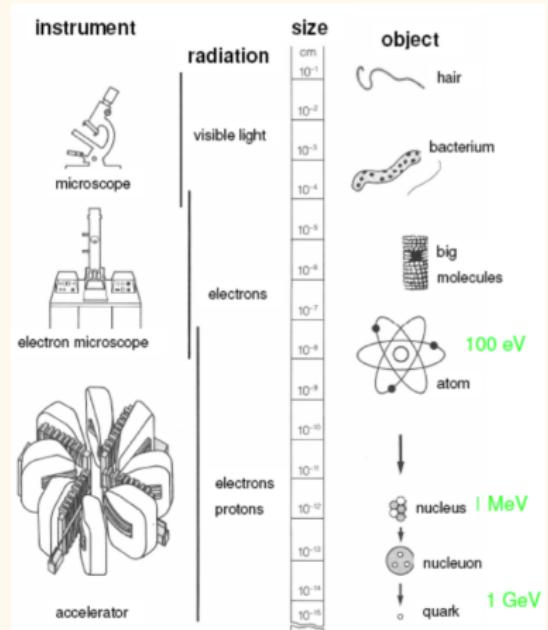
Electroweak symmetry breaking

Open questions

Limitations:

- Why are there 3 generations of leptons and quarks?
- Neutrino oscillation (theory does not predict mass of neutrino)
- What are dark matter and dark energy?
- Why electroweak symmetry is breaking?
- Is there only one Higgs boson as defined in SM?

How to study particle physics?

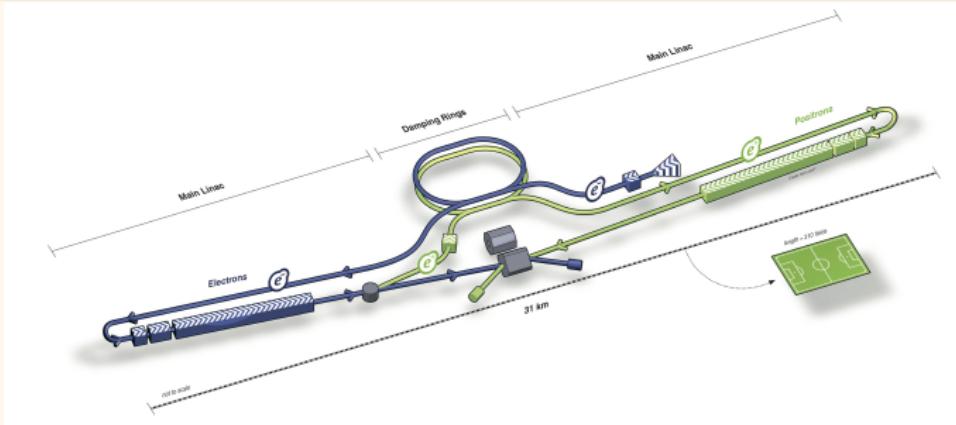


LHC:

- Centre-of-mass energy $\sqrt{s} = 14 \text{ TeV}$
- Collision with composites particles (protons or Pb)
 - Unknown momentum distribution of partons
 - Unknown polarisation of colliding partons
 - Trigger needed
- Discovery machine

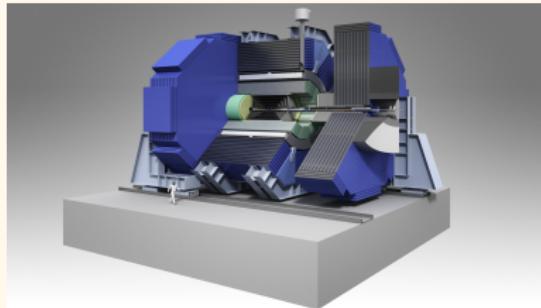
⇒ Need new experimental program

International Linear Collider



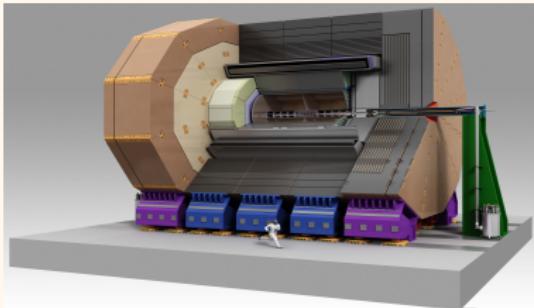
- Future e^+e^- linear collider at $\sqrt{s} = 250 - 500$ GeV (upgrade up to $\sqrt{s} = 1$ TeV)
- Polarised beam
- Luminosity $\simeq 2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$
- Candidate site: Kitakami in northern Japan
- To study properties of the Higgs boson, top physics...

SiD and ILD



Silicon Detector

- Silicon tracking
(radius = 1.2 m)
- $B_{field} = 5\text{ T}$



International Linear Detector

- TPC + silicon envelope
(radius = 1.8 m)
- $B_{field} = 3.5\text{ T}$

Both detectors designed for Particle Flow Calorimetry

- High granularity calorimeters (ECAL and HCAL) inside solenoid
- Low mass tracker to reduce interactions and conversions

Outlines

- 1 Introduction
- 2 Higgs boson study
 - Motivations
- 3 Detector development
- 4 Mechanical deformation
- 5 Radiation length measurement
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Higgs boson study

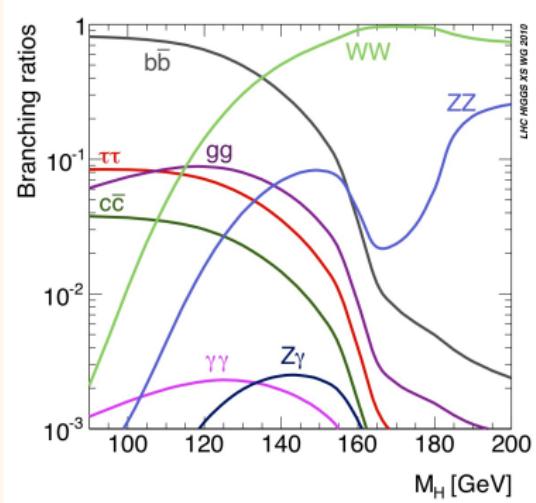
Electroweak Symmetry Breaking

- 2012: Discovery of the Higgs boson (CMS and ATLAS)
- Particle's mass: electroweak symmetry breaking via Higgs-Brout-Englert mechanism

Measurements @ LHC:

- Mass: 125.7 ± 0.4 GeV
- Spin: 0
- Couplings of Higgs boson to fermions and bosons, **except c-quarks, gluons and Higgs self-coupling**

Higgs boson study at the ILC

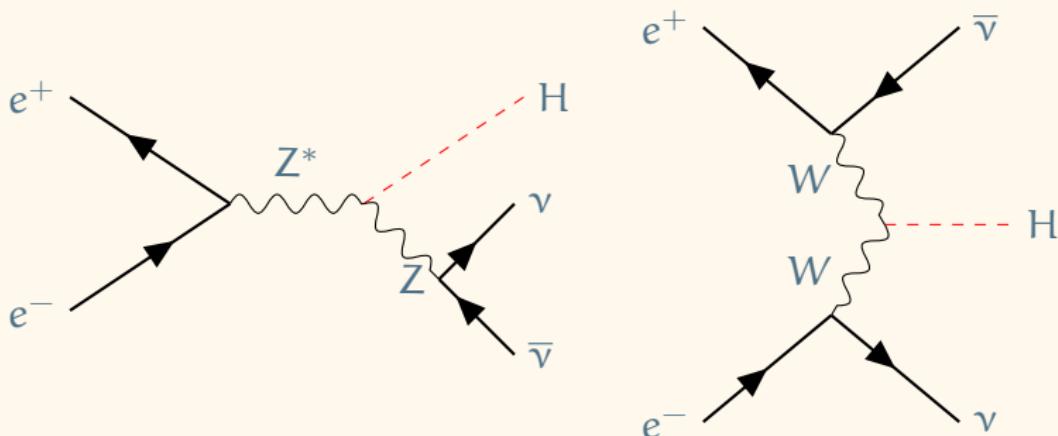


- Analysis of simulated data at the ILC @ 350 GeV with different polarisations:

- $e_L^+ e_R^-$
- $e_R^+ e_L^-$

How to perform the analysis?

- Study final state leading to $H\nu\nu$ channel where the Higgs boson decays into a pair of quarks or gluons
- Focus on Higgs Strahlung and WW fusion:
 $m_H \simeq 125$ GeV and $\sqrt{s} = 350$ GeV \Rightarrow Higgs Strahlung and WW-fusion have comparable cross sections.



Using polarized beam to separate the processes.

Reconstruction of the $H\gamma\gamma$ channel

Final state signature:

- 2 jets coming from the Higgs boson decay
- Missing energy

Events selection:

- ➊ Reject events with isolated leptons
- ➋ Remove $\gamma\gamma$ overlay interactions
- ➌ Look for jets
- ➍ Find displaced vertices of the jets
- ➎ Tag 2 jets coming from Higgs boson decay

Background processes

Events which give same detector response or same final state

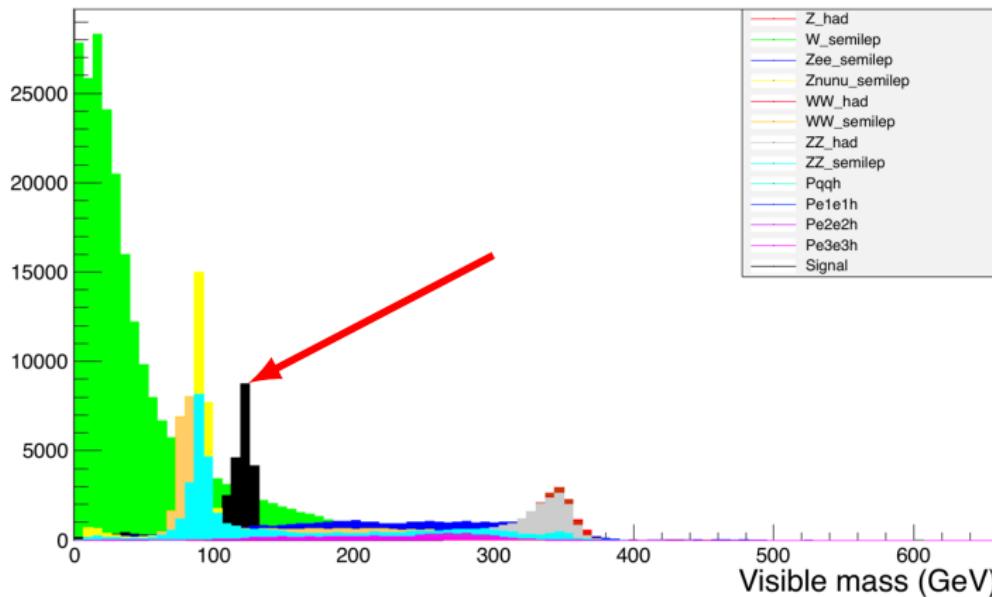
- W-boson pair production
 - Semi-leptonic decay: $e^+e^- \rightarrow W^+W^- \rightarrow \nu_l l^\pm q\bar{q}$
 - Hadronic decay: $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$
- Z-boson pair production
 - $e^+e^- \rightarrow ZZ \rightarrow \nu_l \bar{\nu}_l q\bar{q}$
 - $e^+e^- \rightarrow ZZ \rightarrow l^+l^- q\bar{q}$
 - $e^+e^- \rightarrow ZZ \rightarrow q\bar{q}q\bar{q}$
- Single W-boson production
 - $e^+e^- \rightarrow W^\pm e^\pm \nu_e \rightarrow \nu_e e^\pm q\bar{q}$
- Single Z-boson production
 - $e^+e^- \rightarrow Ze^-e^+ \rightarrow q\bar{q}e^-e^+$
 - $e^+e^- \rightarrow Z q\bar{q} \rightarrow q\bar{q}q\bar{q}$
- Higgsstrahlung:
 - $e^+e^- \rightarrow ZH \rightarrow q\bar{q}q\bar{q}$
 - $e^+e^- \rightarrow ZH \rightarrow l^+l^- q\bar{q}$



Distribution of the visible invariant mass with background

$\sqrt{s} = 350 \text{ GeV}$, luminosity: 250 fb^{-1} and polarisation: e_L^-, e_R^+

Distribution of the visible mass



Distribution of each processes

Process	Expected events
Z hadronic decay	$1.3 \cdot 10^7$
WW hadronic decay	$2.2 \cdot 10^6$
WW semi-leptonic decay	$3.7 \cdot 10^6$
ZZ hadronic decay	$2.0 \cdot 10^5$
ZZ semi-leptonic decay	$1.9 \cdot 10^5$
W semi-leptonic decay	$5.4 \cdot 10^5$
Zee semi-leptonic decay	$1.0 \cdot 10^5$
Zvv semi-leptonic decay	$1.2 \cdot 10^5$
Higgs BG	$5.3 \cdot 10^4$
Other Higgs boson decay	$1.0 \cdot 10^4$
Background	$1.9 \cdot 10^7$
Signal	$2.2 \cdot 10^4$

Reducing the background

Find optimized cuts:

- For each cut, try to find the one which reduces the signal the least

$$\text{significance} = \frac{\text{signal}}{\sqrt{\text{signal} + \text{background}}}$$

- Apply the cuts from the one which gives best significance to the one gives the worst

Sequential cuts strategy:

cut0 Number of isolated lepton (niso): niso = 0

cut1 Transverse Momentum visible (P_t^{vis}): $35 < P_t^{\text{vis}} < 155 \text{ GeV}$

cut2 Visible mass (m_{vis}): $95 < m_{\text{vis}} < 140 \text{ GeV}$

cut3 Angle between the momentum axis of both jets ($\cos \alpha$): $-1 < \cos \alpha < 0.22$



Reduction table after applying cuts

Process	Background	Signal	Significance
Cross-section (fb)	$5.69 \cdot 10^4$	$6.82 \cdot 10^2$	
Expected event number	$1.88 \cdot 10^7$	$2.25 \cdot 10^4$	5.2
No isolated leptons	$1.65 \cdot 10^7$	$2.23 \cdot 10^4$	5.5
$35 < P_t^{\text{vis}} < 155 \text{ GeV}$	$9.31 \cdot 10^5$	$1.82 \cdot 10^4$	18.7
$95 < m_{\text{vis}} < 140 \text{ GeV}$	$1.50 \cdot 10^5$	$1.66 \cdot 10^4$	40.6
$-1 < \cos \theta < 0.22$	$8.76 \cdot 10^4$	$1.57 \cdot 10^4$	48.8
$26 < (\text{N.R.C} > 1\text{GeV}) < 99$	$2.25 \cdot 10^4$	$1.19 \cdot 10^4$	56.3
$0.11 < \text{DurhamjD2ym} < 1$	$1.78 \cdot 10^4$	$1.05 \cdot 10^4$	62.3
$0 < \text{abs}(P_z^{\text{vis}}) < 113 \text{ GeV}$	$1.51 \cdot 10^4$	$1.01 \cdot 10^4$	63.5
$156 < E_{\text{miss}} < 230 \text{ GeV}$	$1.37 \cdot 10^4$	$9.85 \cdot 10^3$	64.1

Outlook

- Other method of selection could improve the significance (TMVA)
- Focus on Higgs boson decay mode, especially $H \rightarrow c\bar{c}$

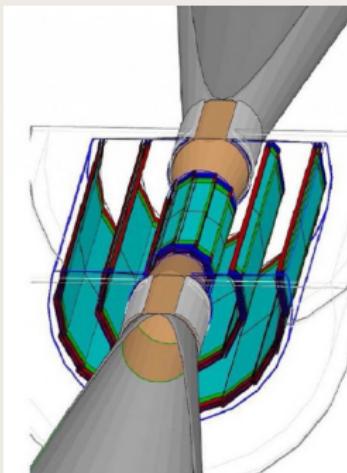
⇒ determine vertex detector geometry

Outlines

- 1 Introduction
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- 3 Detector development
 - ILD vertex detector
 - Design
 - Test beam
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The ILD Vertex Detector

Geometry option: 3 double-sided layers



Impact parameter resolution

- $\sigma_{r\phi} \simeq \sigma_{rz} \simeq a \oplus \frac{b}{p \cdot \sin^{3/2}\theta}$
- Hit resolution: $a \simeq 5\mu\text{m} \Rightarrow \sigma_{\text{spatial}} < 3\mu\text{m}$
- Multiple scattering: $b \sim 10 - 15\mu\text{m} \Rightarrow$ material budget per

Main aims

- Constraint material budget $\Rightarrow < 0.3 \% X_0$
- Study impact of the mechanical structure on sensor performance
- Study the advantage of having two measurement points (mini vectors)

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- **Constraint material budget** $\Rightarrow < 0.3 \% X_0$
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Double-sided VXD: PLUME



PLUME = Pixelated Ladder with Ultra-low Material Embedding



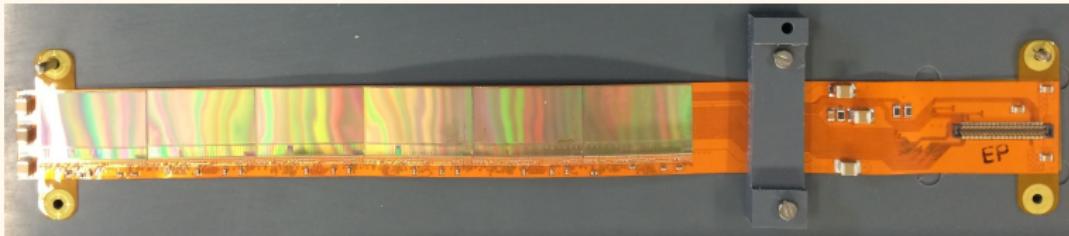
Motivation

ILD Vertex detector at ILC

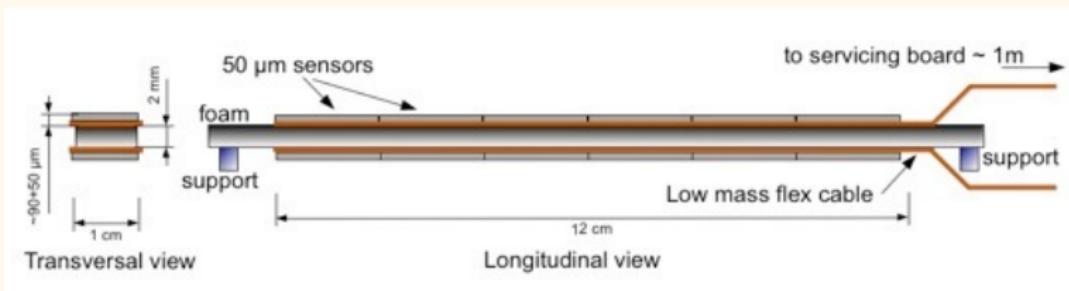
Design

- Double-sided ladder with an active area of $1 \times 12\text{cm}^2$
- On each side: six MIMOSA-26 CMOS sensors thinned down to $\sim 50\text{ }\mu\text{m}$ on a kapton-metal flex cable
- 2 mm of silicon carbide foam as mechanical support and spacer between two modules

What does it look like?

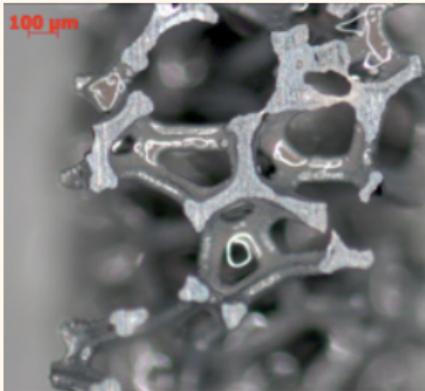
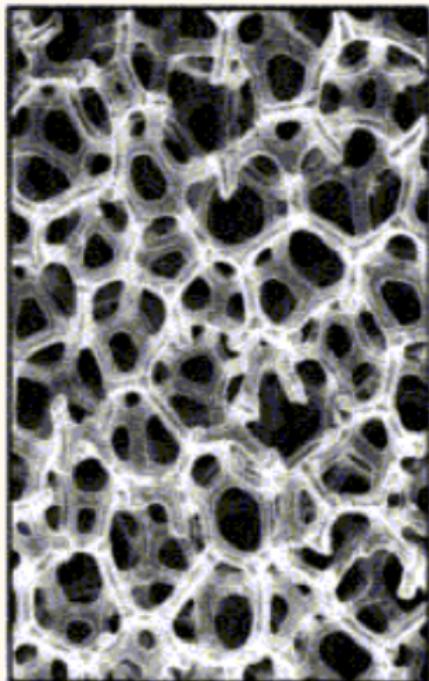


Picture of one module with copper traces.



Scheme of one PLUME ladder.

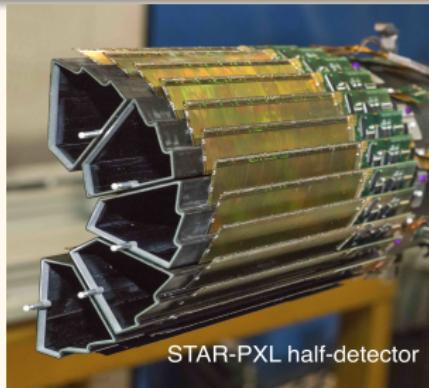
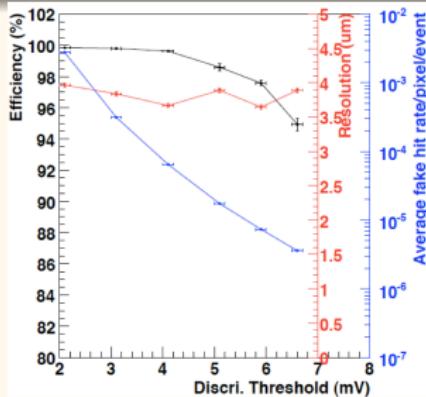
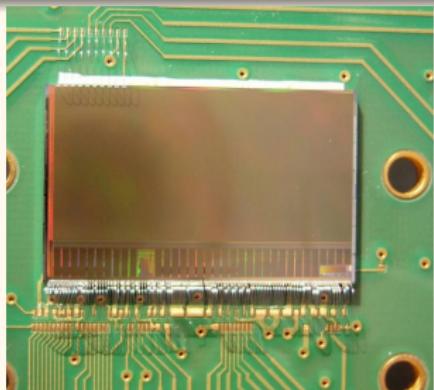
Silicon-Carbide foam support structure



Properties

- Open-cell foam
- Macroscopically uniform
- No tensioning needed
- Density: 4 to 8 %
(2-3 % possible)
- Low thermal and electrical conductivity
(50 W/m/K)

MIMOSA-26 sensor



STAR-PXL half-detector

Monolithic Active Pixels Sensor (MAPS)

- Pitch: 18.4 μm (square pixels)
- Active area: $10.6 \times 21.2\text{mm}^2$ (576 rows x 1152 columns)
- Integration time: 115.2 μs (200 ns per line)
- Binary output with Zero suppression
- Well known sensors \Rightarrow used for EUDET telescope
- Extended to MIMOSA-28 exploited in STAR-PXL vertex detector @ RHIC-BNL since 2014

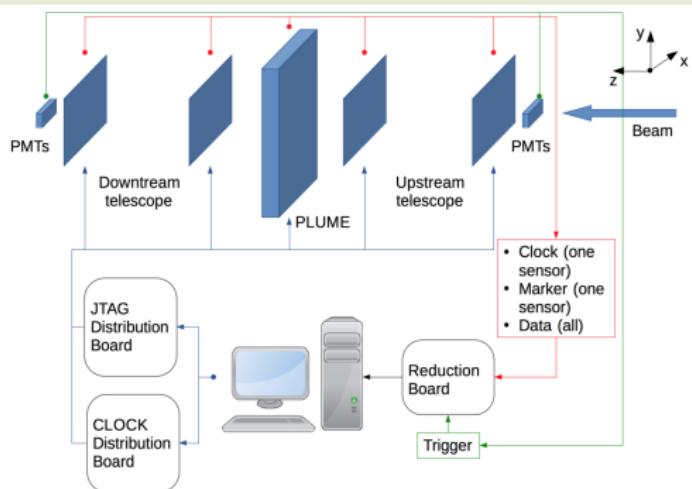


Test beam

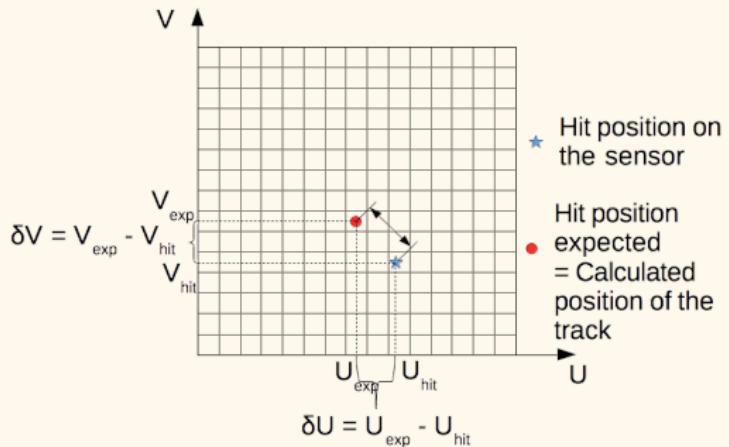
Motivation

Test detector under real conditions to determine its performance

Set-up



Track-hit residual



Device Under Test (DUT) alignment:

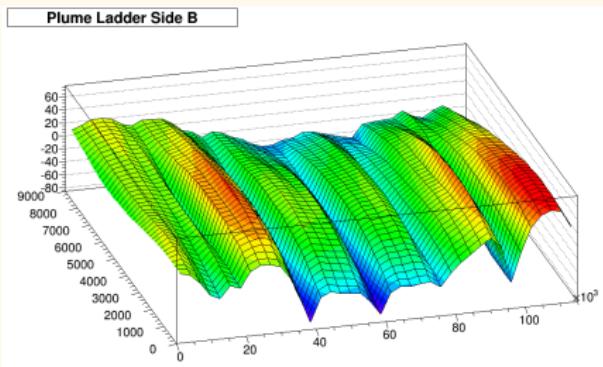
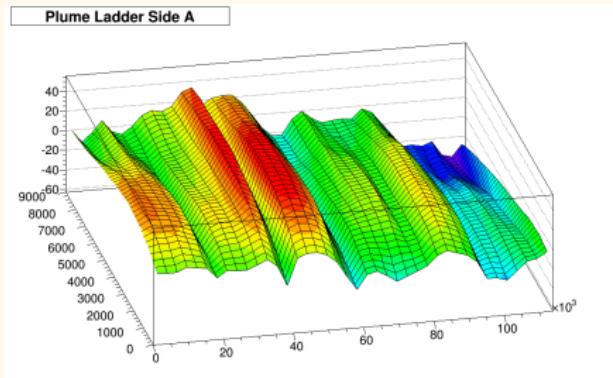
- Telescope planes defined particle's track
- Alignment of DUT in local coordinate system:
 - Define a maximal range in which a hit can be associated to a track
 - Find the best tilt and position to minimise the distance between a hit and its associated track.

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 - Test Beam @ SPS
 - Origin of deviations and how to take them into account
 - Results on the correction of deviations
- 5 Radiation length measurement
- 6 Conclusion and outlook

Metrology of module's surface

Performed at Bristol with a dummy ladder



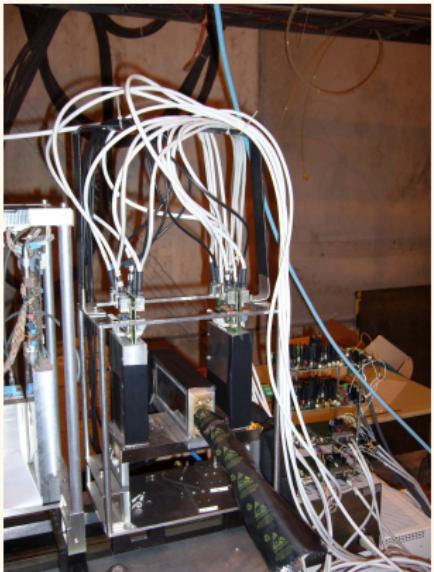
Does it have an impact on ladder's performance?

Test beam @ SPS with 120 GeV π^- in November 2011

- Beam test on line H6a @ SPS
- Reference plane: 4 Mimosa 26
- Validation of the first double-sided ladder equipped with 12 Mi26 sensors

Ladder performance studies:

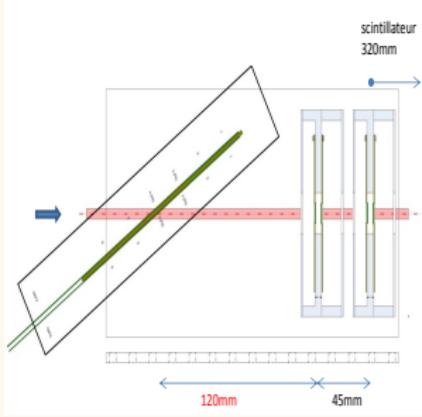
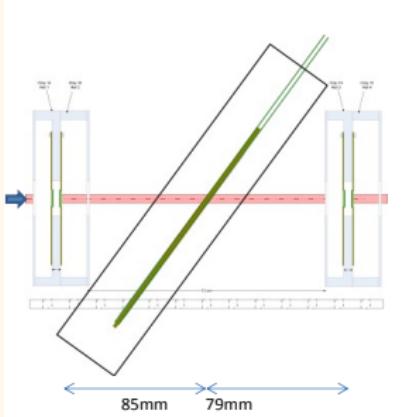
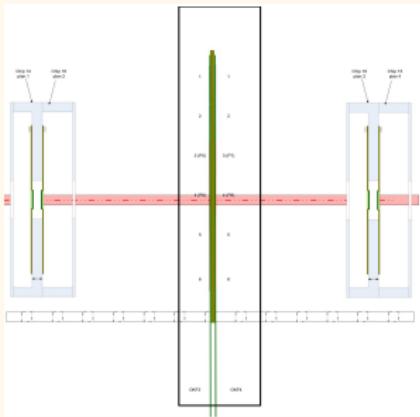
- In threshold (5 and 6 mV)
- In position (sensor 1-2, 3-4 and 5-6)
- Without and with angle (between 30° and 40°)
- With two different air flow speeds (\simeq 3 to 6 ms^{-1})



Jérôme Baudot, Gilles Claus, Loïc Cousin, Mathieu Goffe, Rohrry Gold, Joel Goldstein, Ingrid Gregor, Robert Maria.

Test beam @ SPS with 120 GeV π^- in November 2011

Three configurations studied:



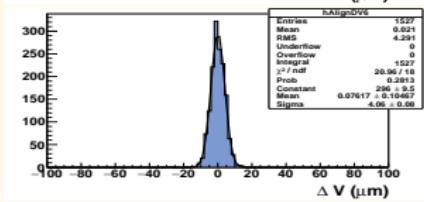
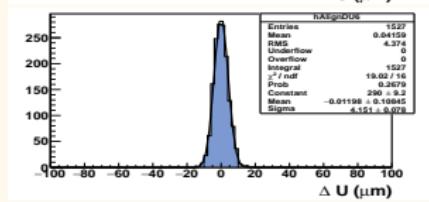
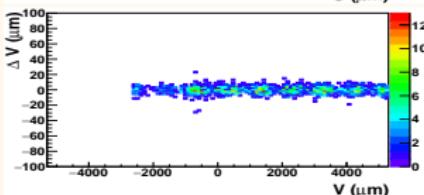
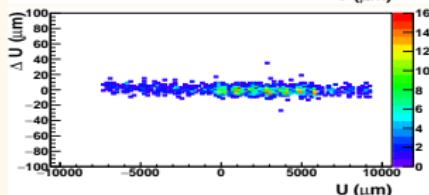
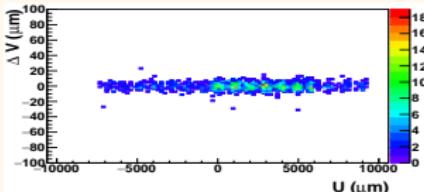
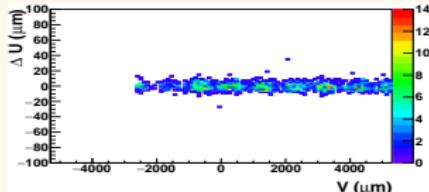
Module perpendicular to the beam.

⇒ Study track-hit residual and the distribution of this residual as a function of the relative position of the beam on the sensor.

Analysis performed with TAF (TAPI Analysis Framework).

Module perpendicular to the beam

Threshold 6σ , air flow speed $< 5\text{m/s}$ and 1.8M events.

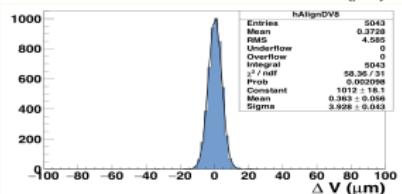
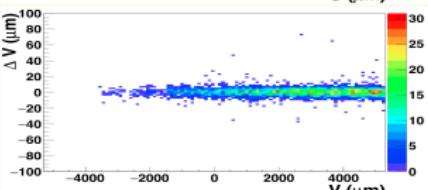
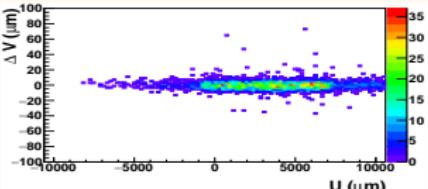
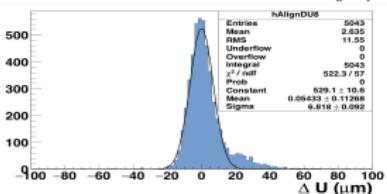
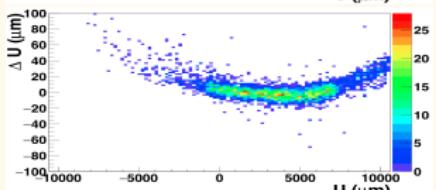
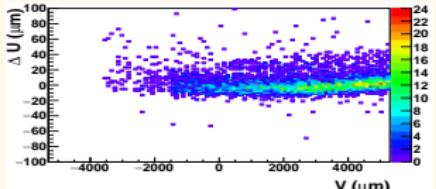


Spatial residual obtained after alignment:

$$\sigma_U \simeq 4.2 \text{ } \mu\text{m} \text{ and } \sigma_V \simeq 4 \text{ } \mu\text{m}$$

Module titled in one direction (w.r.t. to the beam axis)

Threshold 6σ , air flow speed $< 5\text{m/s}$, 720k events and 36° tilt.



Spatial residual obtained after alignment:

$$\sigma_U \simeq 6.1 \text{ } \mu\text{m} \text{ and } \sigma_V \simeq 3.8 \text{ } \mu\text{m}$$

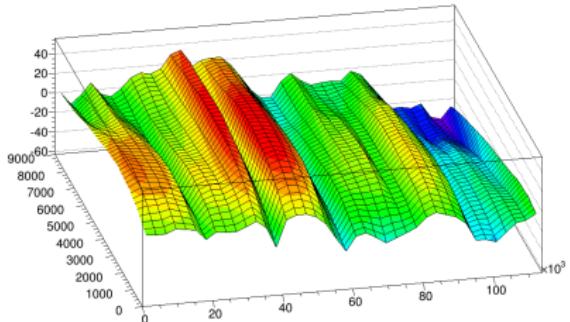
Origin of deviations

Consequence of the ladder's characteristics

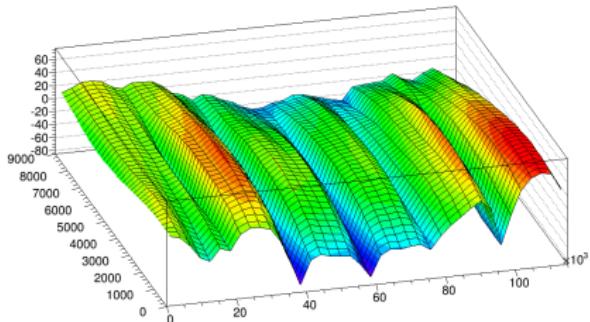
- Use of ultra-thin ($50 \mu\text{m}$) and precise sensors (spatial resolution less than $4 \mu\text{m}$)
- Mechanical constraints induce permanent deformations ($\simeq 10 \mu\text{m}$) which can not be flattened during the ladder assembly

Metrology of the module's surface (performed at Bristol)

Plume Ladder Side A



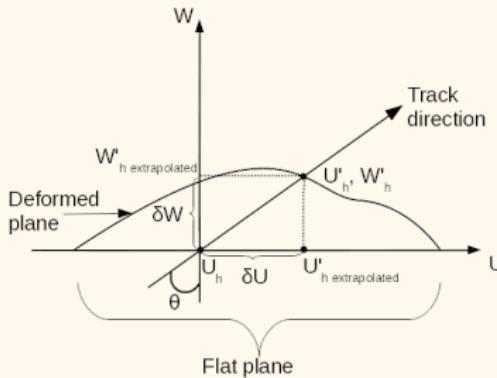
Plume Ladder Side B



Origin of the deviations

Artefacts from the modelling of our sensors during the analysis

- Sensors modeled as completely flat planes
- The track extrapolation is sensitive to the exact position of the hit on the plane and the angle of incidence



Deviations of the residual

$$\delta W = \frac{\delta U}{\tan \theta}$$

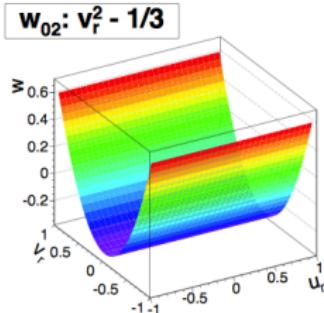
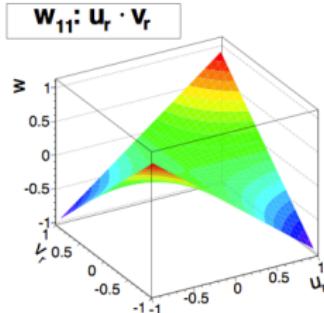
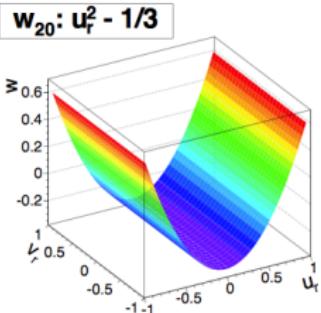
How to describe deviations from the flat plane?

arXiv:1403.2286 [physics.ins-det] CMS paper

- Sensor shape parametrised as a sum of products of modified Legendre polynomials:

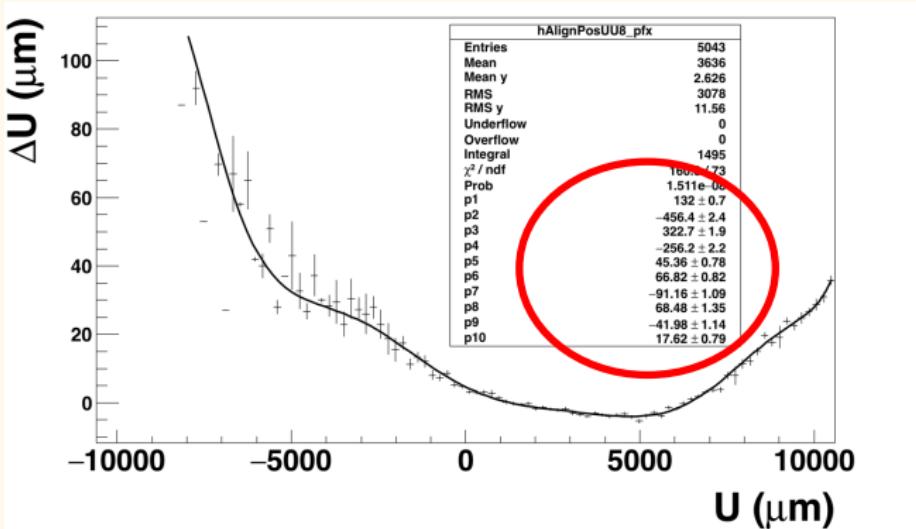
$$\begin{aligned} w(u_r, v_r) &= w \\ &+ w_{10} \cdot u_r + w_{01} \cdot v_r \\ &+ w_{20} \cdot (u_r^2 - 1/3) + w_{11} \cdot (u_r \cdot v_r) + w_{02} \cdot (v_r^2 - 1/3) \end{aligned}$$

- In our case, we used Legendre polynomials of the 11th order only in the direction of the deformation.

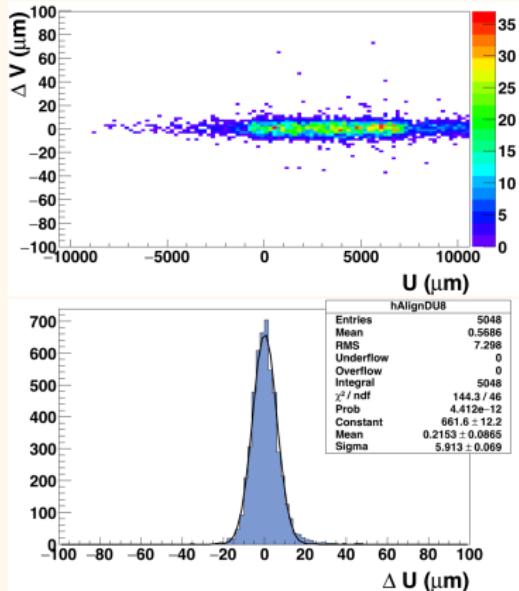
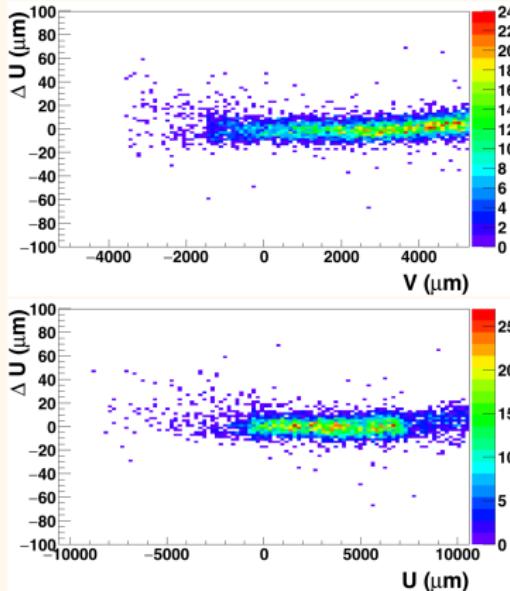


Deformation's parametrisation

Possibility to parametrise the deformation with Legendre polynomials of the 11th order .



Correction of the deviations between real hits and extrapolated ones

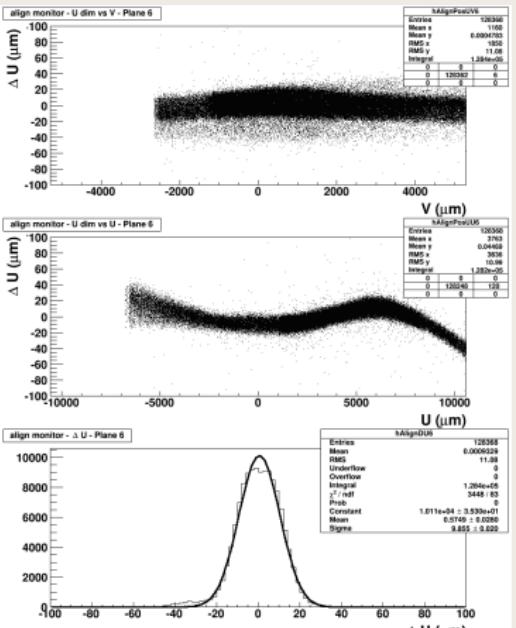


Spatial residual obtained after correction:
 $\sigma_U \simeq 5.9 \mu\text{m}$ instead of $\sigma_U \simeq 6.1 \mu\text{m}$

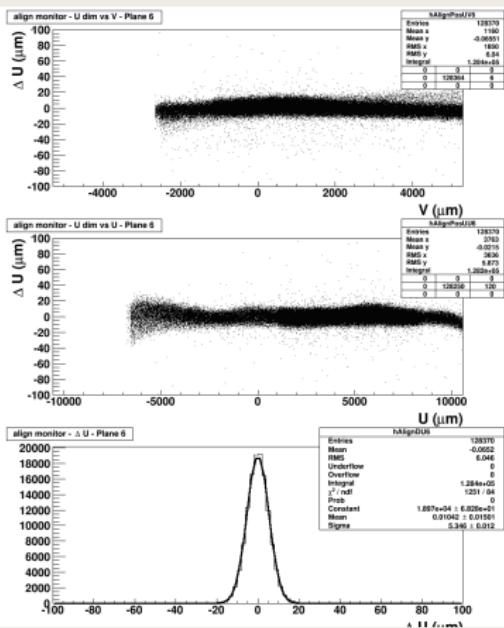
Module tilted with an angle of 28°

Threshold: 5 σ , air flow speed: 6 m/s, 720k events.

Before correction



After correction



Summary of correction for different angles and same planes

Spatial residuals

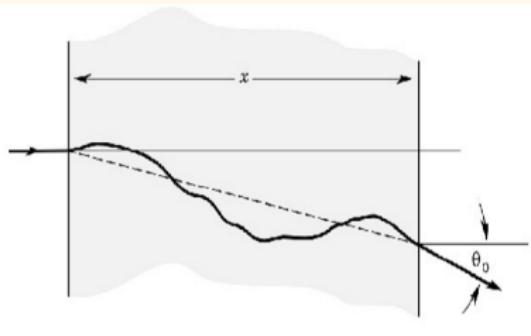
Side	Tilted angle (°)	$\sigma_u^{\text{Def}} (\mu\text{m})$	$\sigma_u^{\text{Cor}} (\mu\text{m})$	Improvement
Front	28	9.0 ± 0.1	4.9 ± 0.1	46.6 %
Back	28	5.7 ± 0.1	4.7 ± 0.1	17.5 %
Front	36	14.1 ± 0.1	6.1 ± 0.1	56.0 %
Back	36	6.8 ± 0.1	5.9 ± 0.1	13.2 %
Front	60	41.2 ± 0.15	25.8 ± 0.2	37.4 %
Back	60	23.3 ± 0.13	21.7 ± 0.1	6.8 %

$\sigma_{\text{tel}} = 2.2 \mu\text{m}$ for 36° and $\sigma_{\text{tel}} = 18.8 \mu\text{m}$ for 60° .

Outlines

- 1 Introduction
- 2 Higgs boson study
- 3 Detector development
- 4 Mechanical deformation
- 5 Radiation length measurement
 - Motivation
 - Test beam @ DESY
 - Theoretical estimation
 - Results

Multiple scattering



Charged particles traveling through matter:

- Lose energy via inelastic collisions with atomic electrons
- Deflection by many small angles (Coulomb scattering from nuclei)

$$\theta_0 = \frac{13.6(\text{MeV})}{p} \left(\frac{x}{X_0} \right)^{0.555}$$

Motivation of measuring the radiation length:

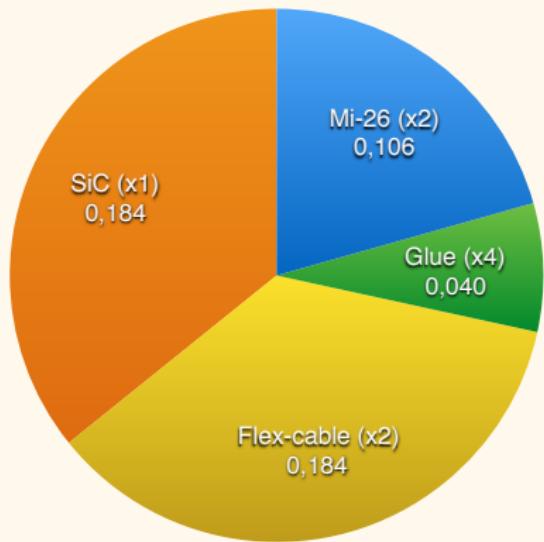
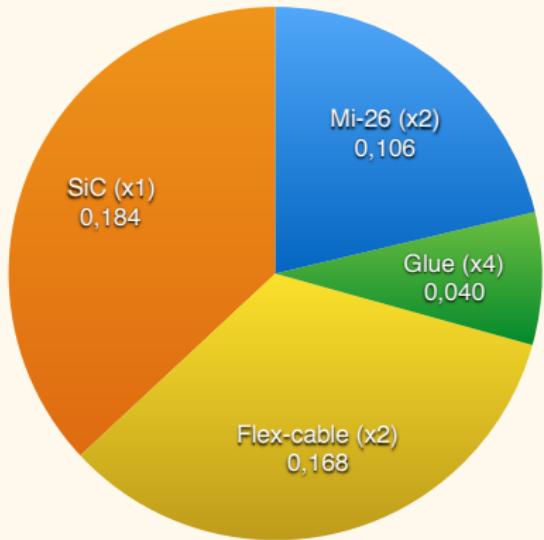
- Tracking system has to detect particle's path and minimise its energy degradation
- Physics analysis: reconstruction of events depends strongly on the energy loss inside the different part of detectors

Test beam @ DESY with 5 GeV e^- (April 2016)



- Test Beam 21
- Reference plane: 4 EUDET telescope planes
- Goal: radiation length measurement

Estimation of the radiation length



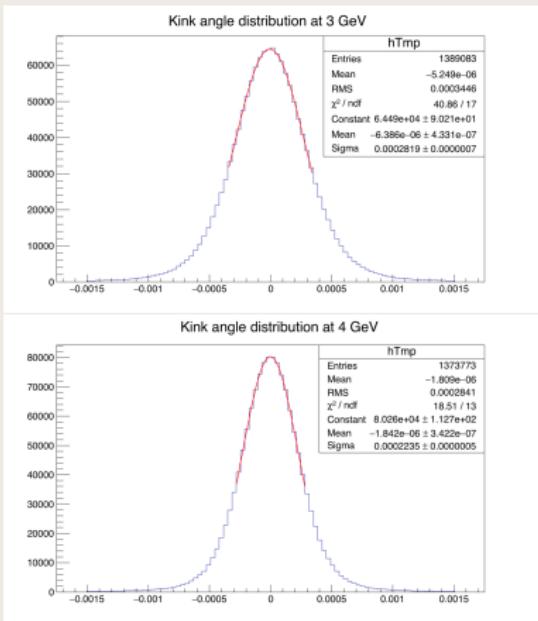
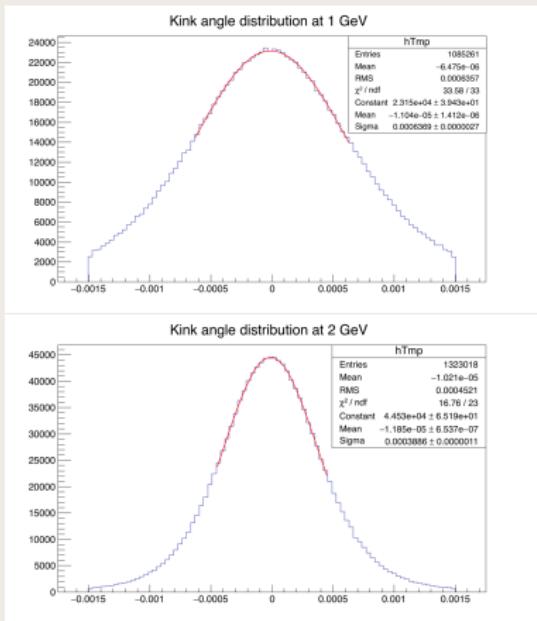
Total material budget

Depends on flex-cable fill factor (25 % or 30 %)

$$\Rightarrow \frac{x}{X_0} \simeq 0.498 - 0.515 \% X_0$$

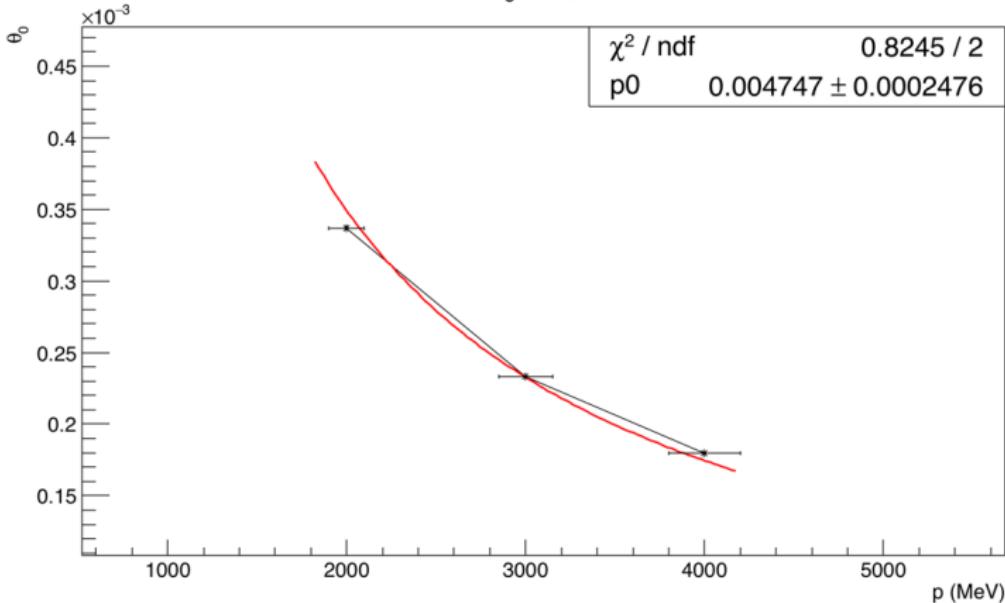
Kink angle measurement between 1 and 4 GeV

Fitted kink angle distributions



Material budget

$$\theta_0 = f(p)$$



$$\frac{x}{X_0} \mid_{\text{estimated}} \simeq 0.498 - 0.515 \% X_0$$
$$\frac{x}{X_0} \mid_{\text{measured}} \simeq 0.47 \pm 0.02 \% X_0$$

Conclusion

Context:

- ILC: next collider to study precisely EWSB (and other physics scenarios)
- Performances to achieve imposes R&D detector development

blabla

- Mechanical deformations observed are impacting ladder's performance
- Radiation length measurement:

$$\frac{x}{X_0} \mid_{\text{estimated}} \simeq 0.498 - 0.515 \% X_0$$

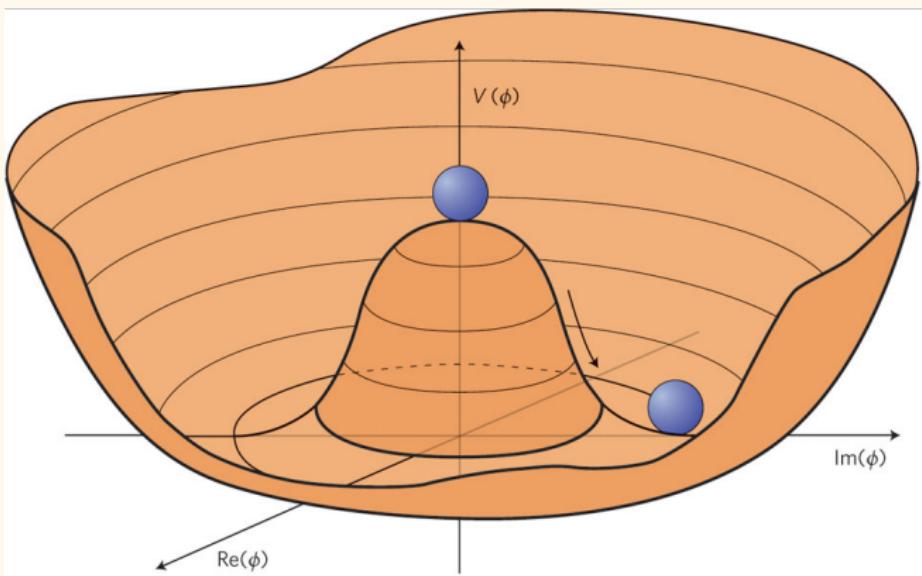
$$\frac{x}{X_0} \mid_{\text{measured}} \simeq 0.47 \pm 0.02 \% X_0$$

Outlook

- Study ladder performance for low energy (test beam April 2016)
- New prototype with a material budget of 0.35 % X_0 has been built, tested in laboratory but not yet in real conditions
- Double-sided ladders could be enriched with sensors having different characteristics (fast integration time VS good spatial resolution)
- Using the beam structure to apply a power-pulsing method
- Application of PLUME in SuperKEKB (BEAST)

Thanks for your attention !!!

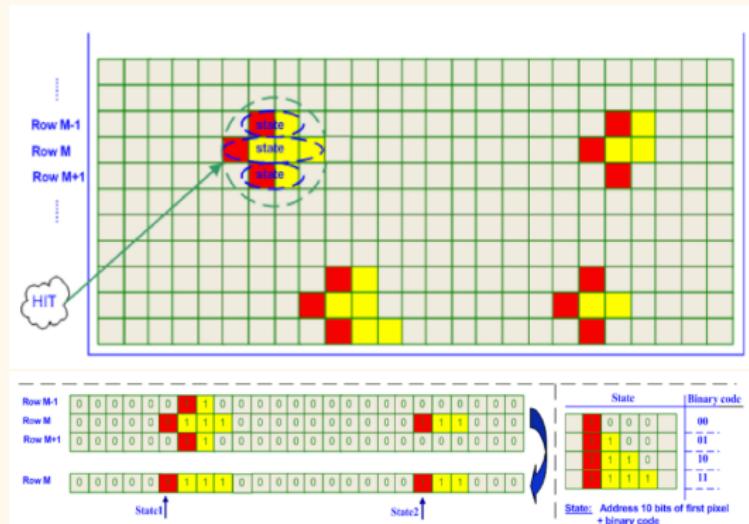
Higgs boson potential



Higgs boson physics at the ILC

- Same measurements as LHC: couplings, mass and spin
- Model independent measurement: no dependence on theory
- Total Higgs width
- $H \rightarrow c\bar{c}/gg$
- Higgs self couplings

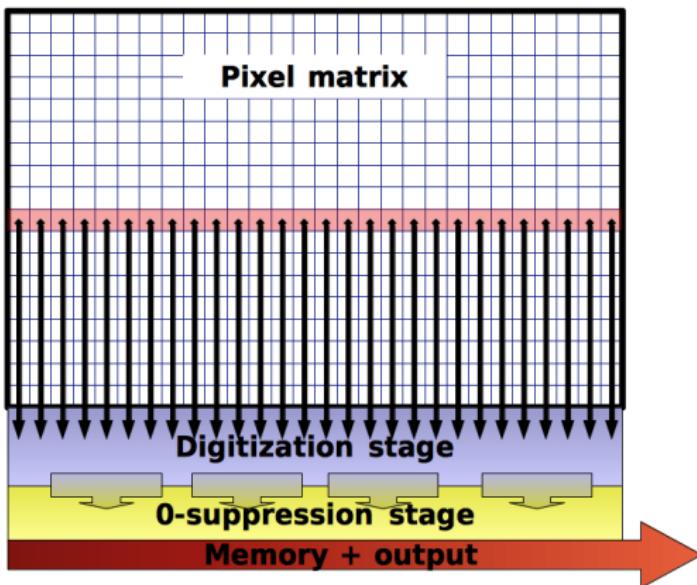
Zero Suppression logic (SUZE)



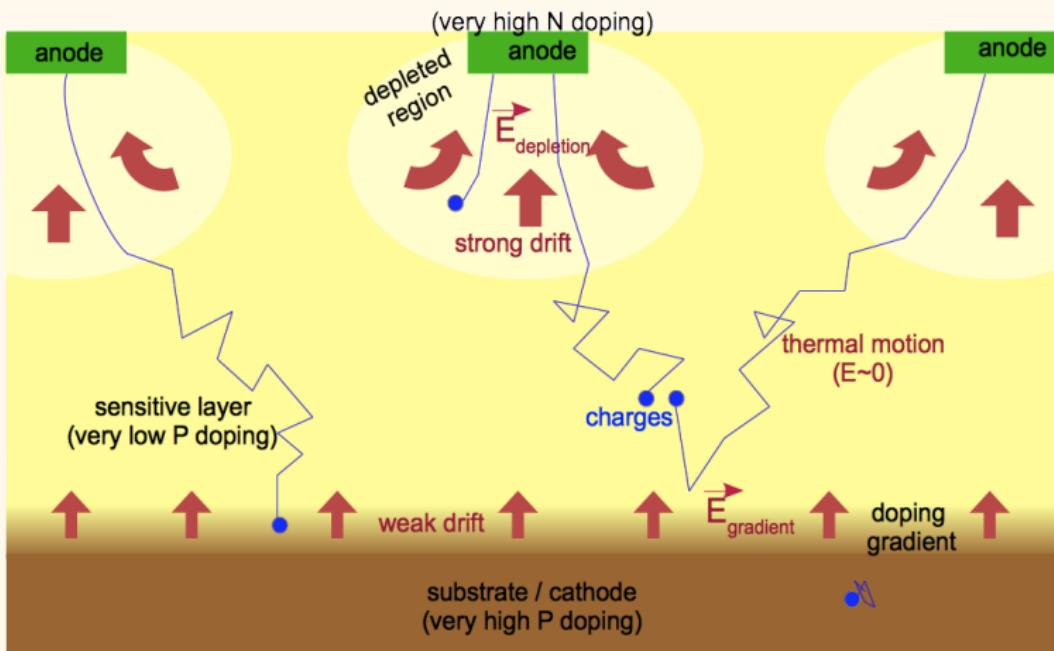
SUZE logic split in 3 blocks:

- **Sparse Data Scan (SDS)** Hit detection per line and data encoding, until 6 states consecutive pixels (1 to 4 pixels) per block of 64 columns;
- **Multiplexing Logic (Mux)** giving up to 9 states;
- **Memory storage** 2 blocks to store the states of the full frame, switching to avoid dead time (during one acquire states of event N, the other one transfer the information of frame N-1).

Column parallel readout

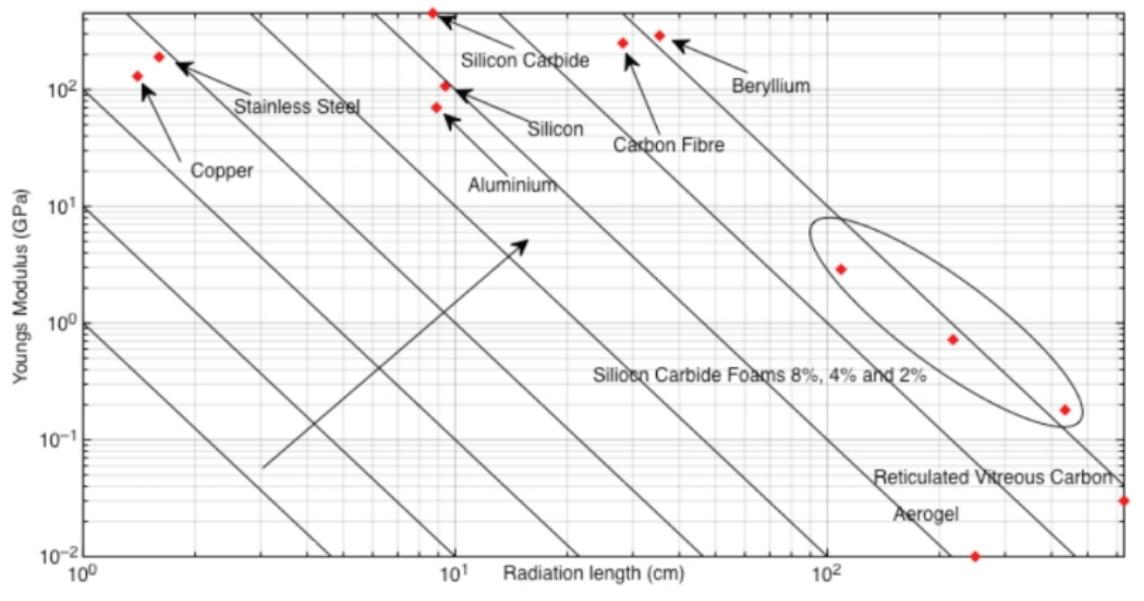


MAPS principle

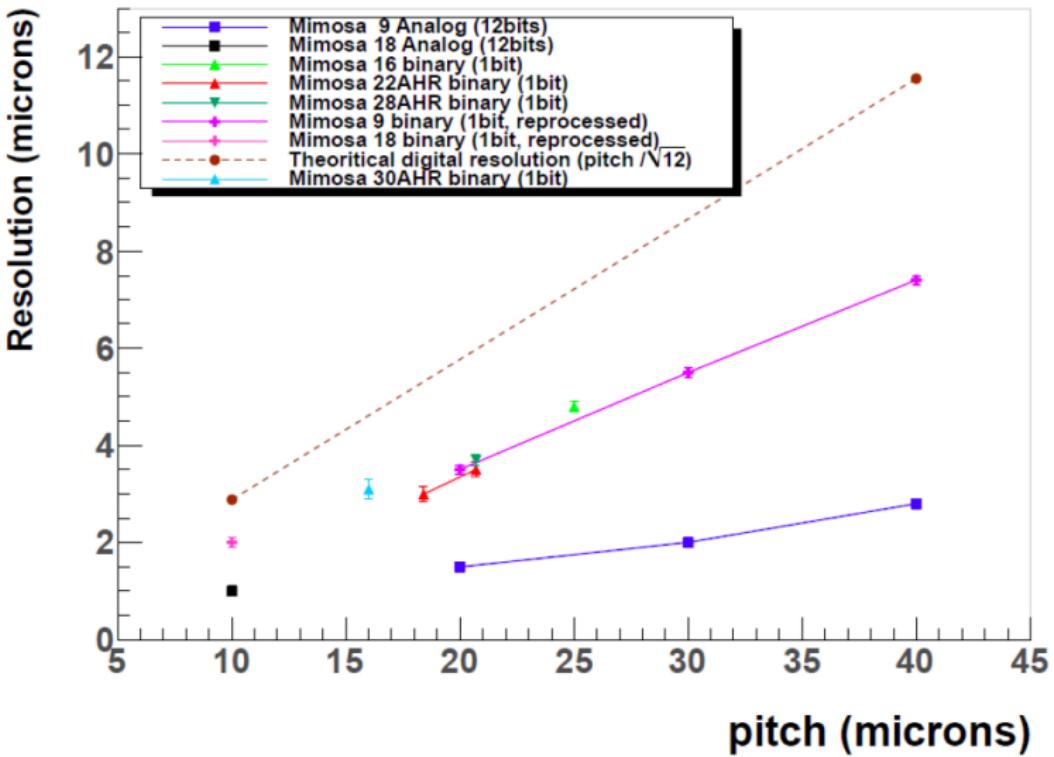


Young Modulus

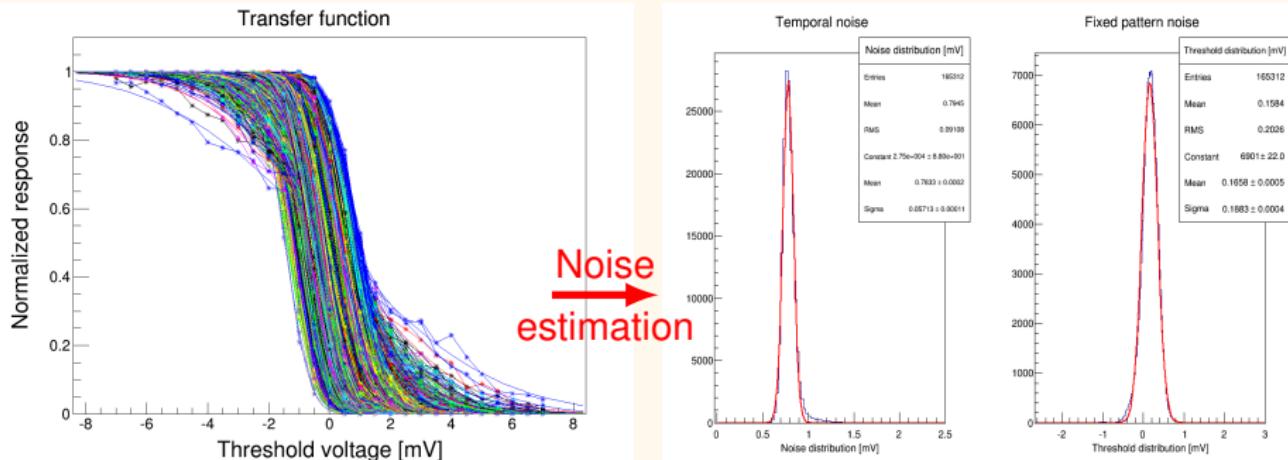
Material Selection Graphs



Spatial resolution for different pitch (IPHC-Strasbourg)



Characterization of one sensor



Threshold scan

Normalised response of pixels in a sub-matrix (288 discriminators) as a function of threshold applied (mV).

Noise performances

- Temporal noise (derivative of the S-curve): 0.79 mV
- Fixed pattern noise (thresholds' dispersion for a mid-point): 0.2 mV
- Offset: 0.16 mV

⇒ Can now define different thresholds

Higgs Strahlung kinematics

$$E_H = \frac{s - M_Z^2 + M_H^2}{2\sqrt{s}}$$

$$E_Z = \frac{s - M_H^2 + M_Z^2}{2\sqrt{s}}$$

$$|\vec{p}_H| = |\vec{p}_Z| = \frac{\sqrt{[s - (M_H + M_Z)^2] \cdot [s - (M_H - M_Z)^2]}}{2\sqrt{s}}$$

If $M_H = 125$ GeV, $M_Z = 91.2$ GeV and $\sqrt{s} = 350$ GeV, then:

$$E_H \simeq 185.4 \text{ GeV}$$

$$E_Z \simeq 164.6 \text{ GeV}$$

$$|\vec{p}_H| = |\vec{p}_Z| \simeq 68.5 \text{ GeV}$$

Detector performances

Vertexing

$$\sigma_{\text{IP}} = 5 \oplus \frac{10}{p \sin^{3/2} \theta} (\mu\text{m})$$

Tracking

$$\sigma(1/p) = 2 \times 10^{-5} (\text{GeV}^{-1})$$

Jet energy

$$\sigma_E/E = 0.3/\sqrt{E(\text{GeV})}$$

Particle Flow Algorithm

- Typical jet:
 - Charged hadrons \simeq 60 %
 - Photons \simeq 30 %
 - Neutral \simeq 10 %
- Standard approach
 - All jet components energy measured in ECAL/HCAL
 - $E_{jet} = E_{ECAL} + E_{HCAL}$
- Particle flow calorimetry
 - Measurement of charged particles in tracker
 - Measurement of photon in ECAL
 - Measurement of hadrons in HCAL
 - $E_{jet} = E_{Track} + E_{\gamma} + E_n$

Why a linear collider?

Limitations of e^+e^- colliders

- Synchrotron radiation loss $\sim E^4/r$
- Synchrotron cost: \sim quadratically with energy
- Power consumption

Advantages of linear colliders

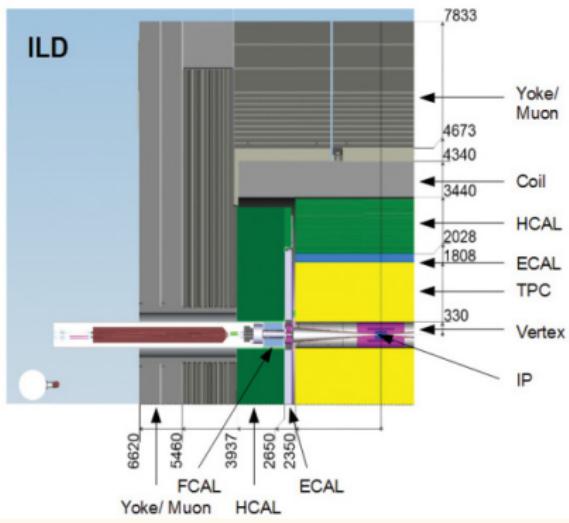
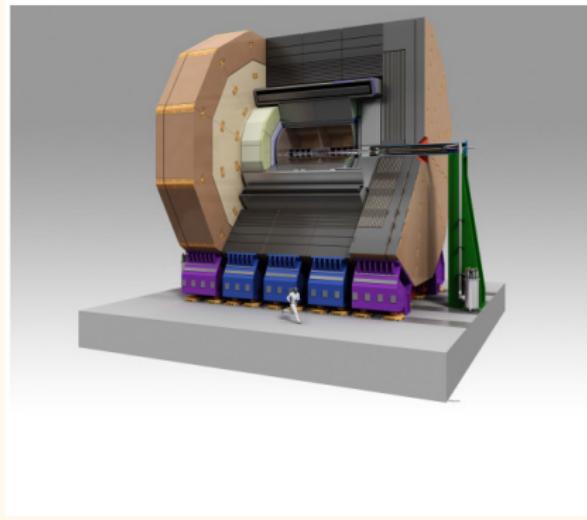
- Not limited by synchrotron radiation
- Cost: \sim linear with energy
- Polarisation of both beams
- Detectors close to the IP \Rightarrow optimum for c-tagging

ILC interaction region

- 1 interaction region for 2 detectors
- Push-pull:
 - Detectors mounted on movable platforms
 - Sharing of beam time
 - Switching time: 24h to 48h
 - Allow cross-checking

- Bunch spacing of ~ 554 ns
- 1312 bunches in a 1 ms long pulse (train)
- Quiet time: 199 ms
- Occupancy dominated by beam background and noise
- Reading during quiet time possible

Overview of the ILD



Sequential cuts strategy

cut0: Number of isolated lepton (niso): $n_{\text{iso}} = 0$

cut1: Transverse Momentum visible (P_t^{vis}): $35 < P_t^{\text{vis}} < 155 \text{ GeV}$

cut2: Visible mass (m_{vis}): $95 < m_{\text{vis}} < 140 \text{ GeV}$

cut3: Angle between the momentum axis of both jets ($\cos \alpha$): $-1 < \cos \alpha < 0.22$

cut4: Number of reconstructed particle

cut5: D2YM

cut6: Visible longitudinal momentum ($\text{abs}(P_z^{\text{vis}})$)

cut7: E_{miss}

Beam polarisation

Simulated data: 100 % left or right events

$$\sigma_{P(e^+, e^-)} = \left(\frac{1 - P_{e^-}}{2} \right) \left(\frac{1 + P_{e^+}}{2} \right) \sigma_{RL} + \left(\frac{1 + P_{e^+}}{2} \right) \left(\frac{1 - P_{e^-}}{2} \right) \sigma_{LR}$$

$$\sigma_{P(e^+, e^- = 0.3, -0.8)} = 0.585 \cdot \sigma_{RL} + 0.035 \cdot \sigma_{LR}$$