Development of a double-sided ladder for tracking in high energy physics

Ph. D. defense

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Outlines

1. Introduction
2. Higgs boson study
3. Detector development
4. Mechanical deformation
5. Radiation length measurement
6. Conclusion and outlook

Electroweak symmetry breaking

Standard Model:

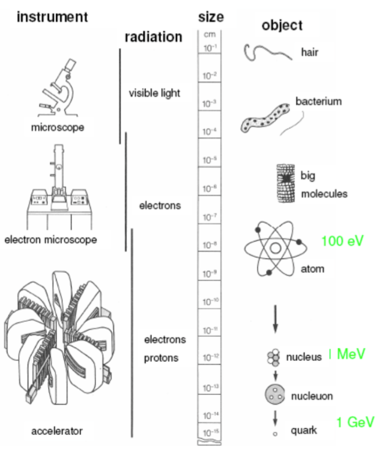
- Well-tested physics theory: predict precisely a wide variety of phenomena
- Electro-Weak Symmetry Breaking (EWSB): explain how particles acquire mass via Higgs mechanism
- Last milestone: discovery of the Higgs Boson

Open questions

Limitations:

- Why are they 3 generations of leptons and quarks?
- Neutrino oscillation (theory does not predict mass of neutrino)
- What are dark matter and dark energy?
- Why electroweak symmetry is breaking?
- Is there only one Higgs boson as defined in SM?

How to study particle physics?

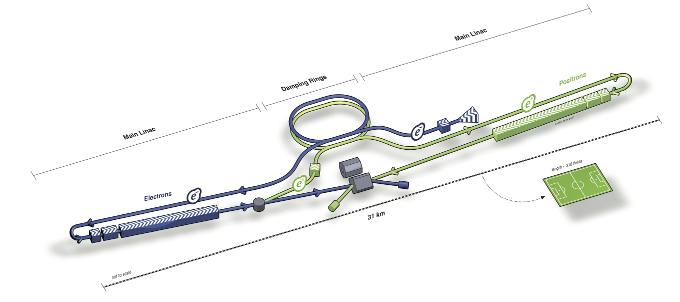


LHC: a discovery machine

- Centre-of-mass energy $\sqrt{s} = 14$ TeV
- Collision with composites particles (protons or Pb)
  - Unknown momentum distribution of partons
  - Unknown polarisation of colliding partons
  - Trigger needed
  - Background made of complex Standard Model reactions

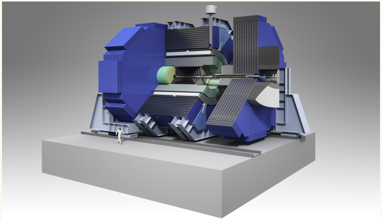
$\Rightarrow$ **Need new experimental program**

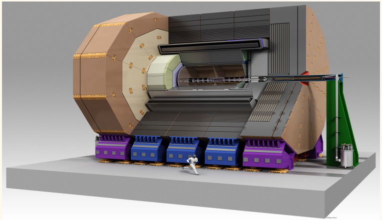
International Linear Collider



- Future e^+e^- linear collider at $\sqrt{s} = 250 - 500$ GeV (upgrade up to $\sqrt{s} = 1$ TeV)
- Polarised beam
- Luminosity $\simeq 2 \times 10^{34}$ cm^{-2}s^{-1}
- Candidate site: Kitakami in nothern Japan
- To study properties of the Higgs boson, top physics...

SiD and ILD





Silicon Detector

- Silicon tracking (radius = 1.2 m)
- B_{field} = 5 T

International Linear Detector

- TPC + silicon envelope (radius = 1.8 m)
- B_{field} = 3.5 T

Both detectors designed for Particle Flow Calorimetry

- High granularity calorimeters (ECAL and HCAL) inside solenoid
- Low mass tracker to reduce interactions and conversions

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2 Higgs boson study
- Motivation
- Study of the Hνν final state
- Reduction of the background

3 Detector development

4 Mechanical deformation

5 Radiation length measurement

6 Conclusion and outlook

Higgs boson study

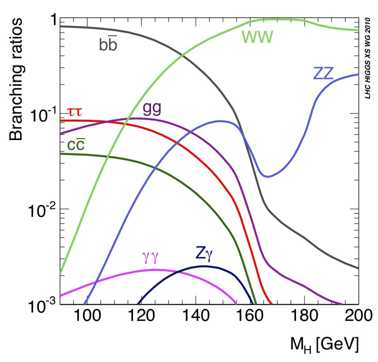
Measurements @ LHC:

- Higgs boson discovered in 2012 (ATLAS and CMS collaborations)
- Mass: 125.7 ± 0.4 GeV
- Spin: 0
- Only a subset of Higgs decays can be observed (with uncertainties of 5 %)

Missing measurements

- Couplings of Higgs boson to c-quarks and gluons
- Higgs self-coupling
- Precision recquired ~ 1 %

Higgs boson study



At LHC

- Higgs boson to quarks difficult to observe
- $H \to b\bar{b}$ observed in special kinematics
- $H \to c\bar{c}$ and $H \to gg$ are challenging to observe

At ILC

- $H \to b\bar{b}, c\bar{c}, WW^*, \tau\tau$ and gg able to be separately identified with high efficiency

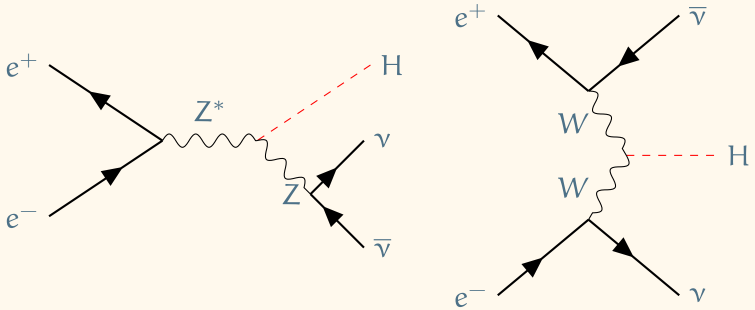
Analysis of simulated data at the ILC @ 350 GeV with different polarisations:

- $e^+_L e^-_R$
- $e^+_R e^-_L$

Study of H$\nu\nu$ final state

- Study final state leading to H$\nu\nu$ channel where the Higgs boson decays into a pair of quarks or gluons
- Focus on Higgs Strahlung and WW fusion:

$m_H \simeq$ 125 GeV and $\sqrt{s} = 350$ GeV $\Rightarrow$ Higgs Strahlung and WW-fusion have comparable cross sections.



Using polarised beam to separate the processes.

Reconstruction of the $H\nu\nu$ channel

Final state signature:

- 2 jets coming from the Higgs boson decay
- Missing energy

Events selection:

1. Reject events with isolated leptons
2. Remove $\gamma\gamma$ overlay interactions
3. Look for jets
4. Find displaced vertices of the jets
5. Tag 2 jets coming from Higgs boson decay

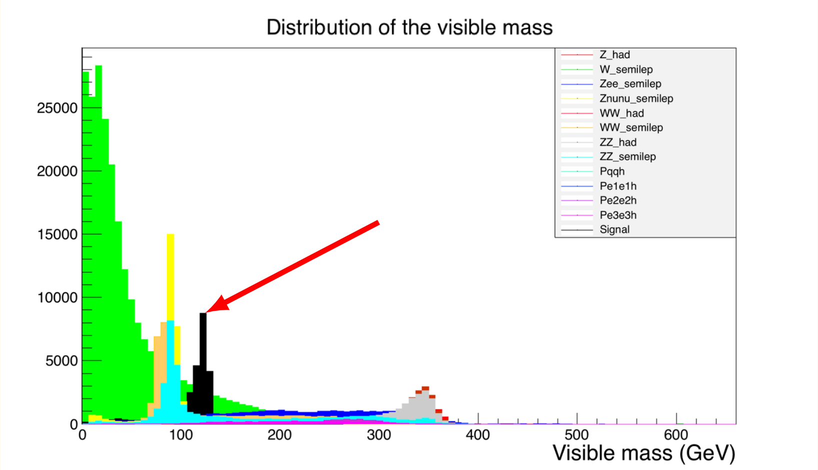
Background processes

Events which give same detector response or same final state

- W-boson pair production
  - Semi-leptonic decay: $e^+e^- \rightarrow W^+W^- \rightarrow \nu_l l^\pm q\bar{q}$
  - Hadronic decay: $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$
- Z-boson pair production
  - $e^+e^- \rightarrow ZZ \rightarrow \nu_l\bar{\nu_l}q\bar{q}$
  - $e^+e^- \rightarrow ZZ \rightarrow l^+l^-q\bar{q}$
  - $e^+e^- \rightarrow ZZ \rightarrow q\bar{q}q\bar{q}$
- Single W-boson production
  - $e^+e^- \rightarrow W^\pm e^\pm \nu_e \rightarrow \nu_e e^\pm q\bar{q}$
- Single Z-boson production
  - $e^+e^- \rightarrow Ze^-e^+ \rightarrow q\bar{q}e^-e^+$
  - $e^+e^- \rightarrow Z\,q\bar{q} \rightarrow q\bar{q}q\bar{q}$
- Higgsstrahlung:
  - $e^+e^- \rightarrow ZH \rightarrow q\bar{q}q\bar{q}$
  - $e^+e^- \rightarrow ZH \rightarrow l^+l^-q\bar{q}$

Distribution of the visible invariant mass with background

$\sqrt{s} = 350$ GeV, luminosity: 250 fb^{-1} and polarisation: e^-_L, e^+_R



Distribution of each processes

Process	Expected events
Z hadronic decay	$1.3 \cdot 10^7$
WW hadronic decay	$2.2 \cdot 10^6$
WW semi-leptonic decay	$3.7 \cdot 10^6$
ZZ hadronic decay	$2.0 \cdot 10^5$
ZZ semi-leptonic decay	$1.9 \cdot 10^5$
W semi-leptonic decay	$5.4 \cdot 10^5$
Zee semi-leptonic decay	$1.0 \cdot 10^5$
$Z\nu\nu$ semi-leptonic decay	$1.2 \cdot 10^5$
Higgs BG	$5.3 \cdot 10^4$
Other Higgs boson decay	$1.0 \cdot 10^4$
Background	$1.9 \cdot 10^7$
Signal	$2.2 \cdot 10^4$

Reducing the background

Find optimized cuts:

- For each cut, try to find the one which reduces the signal the least

$$significance = \frac{signal}{\sqrt{signal + background}}$$

- Apply the cuts from the one which gives best significance to the one gives the worst

Sequential cuts strategy:

cut0 Number of isolated lepton (niso): niso = 0

cut1 Transverse Momentum visible (P_t^{vis}): 35 < P_t^{vis} < 155 GeV

cut2 Visible mass (m_{vis}): 95 < m_{vis} < 140 GeV

cut3 Angle between the momentum axis of both jets ($\cos\alpha$): -1 < $\cos\alpha$ < 0.22

...

Reduction table after applying cuts

Process	Background	Signal	Significance
Cross-section (fb)	$5.69 \cdot 10^4$	$6.82 \cdot 10^2$	
Expected event number	$1.88 \cdot 10^7$	$2.25 \cdot 10^4$	5.2
No isolated leptons	$1.65 \cdot 10^7$	$2.23 \cdot 10^4$	5.5
$35 < P_t^{vis} < 155$ GeV	$9.31 \cdot 10^5$	$1.82 \cdot 10^4$	18.7
$95 < m_{vis} < 140$ GeV	$1.50 \cdot 10^5$	$1.66 \cdot 10^4$	40.6
$-1 < \cos_{,} < 0.22$	$8.76 \cdot 10^4$	$1.57 \cdot 10^4$	48.8
$26 < (N.R.C > 1GeV) < 99$	$2.25 \cdot 10^4$	$1.19 \cdot 10^4$	56.3
$0.11 < DurhamjD2ym < 1$	$1.78 \cdot 10^4$	$1.05 \cdot 10^4$	62.3
$0 < abs(P_z^{vis}) < 113$ GeV	$1.51 \cdot 10^4$	$1.01 \cdot 10^4$	63.5
$156 < E_{miss} < 230$ GeV	$1.37 \cdot 10^4$	$9.85 \cdot 10^3$	64.1

Outlook

- Higher significance is needed to study Higgs decay (TMVA solution)
- Focus on Higgs boson decay mode, especially $H \rightarrow c\bar{c}$

$\Rightarrow$ determine vertex detector geometry

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1 Introduction

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3 Detector development
- ILD vertex detector
- Design
- Test beam

4 Mechanical deformation

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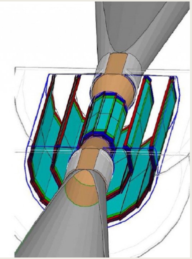
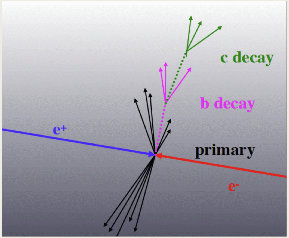
The ILD Vertex Detector

Vertex detector





Impact parameter resolution

- $\sigma_{r\phi} \simeq \sigma_{rz} \simeq a \oplus \frac{b}{p \cdot \sin^{3/2}\theta}$
- Hit resolution: $a \simeq 5\mu m \Rightarrow \sigma_{spatial} < 3\mu m$
- Multiple scattering: $b \simeq 10 - 15\mu m \Rightarrow$ material budget per layer $\simeq 0.15\ \%\ X_0$

Main aims

- Constraint material budget $\Rightarrow < 0.3\ \%\ X_0$
- Study impact of the mechanical structure on sensor performance
- Study the added value of double-sided measurement (mini vectors)

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Double-sided VXD: PLUME

PLUME = **P**ixelated **L**adder with **U**ltra-low **M**aterial **E**mbedding

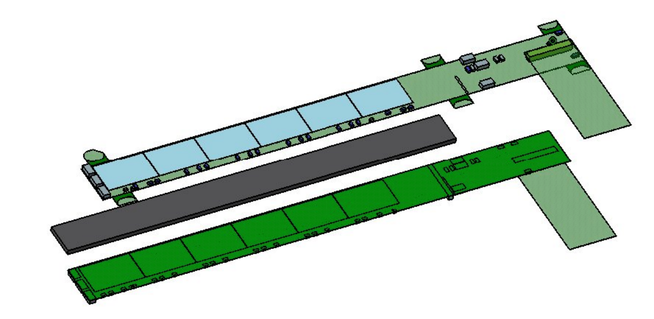
Motivation

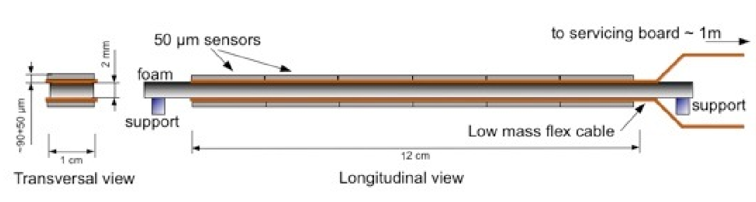
ILD Vertex detector at ILC

Design

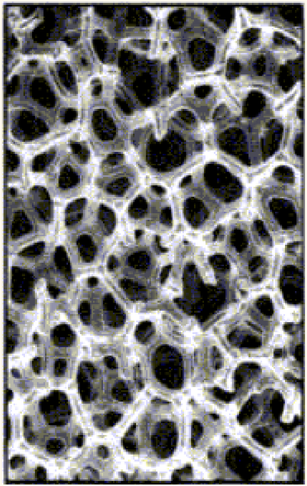
- Double-sided ladder with an active area of 1x12cm^2
- On each side: six MIMOSA-26 CMOS sensors thinned down to $\sim$ 50 μm on a kapton-metal flex cable
- 2 mm of silicon carbide foam as mechanical support and spacer between two modules

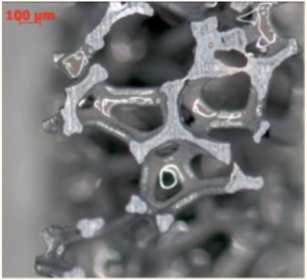
What does it look like?





Silicon-Carbide foam support structure

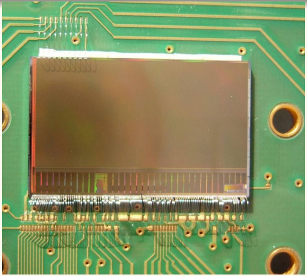


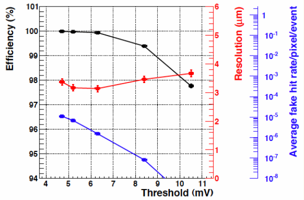


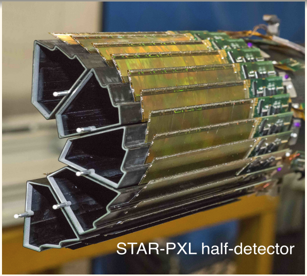
Properties

- Open-cell foam
- Macroscopically uniform
- No tensioning needed
- Density: 4 to 8 % (2-3 % possible)
- Low thermal and electrical conductivity (50 W/m/K)

MIMOSA-26 sensor







Monolithic Active Pixels Sensor (MAPS)

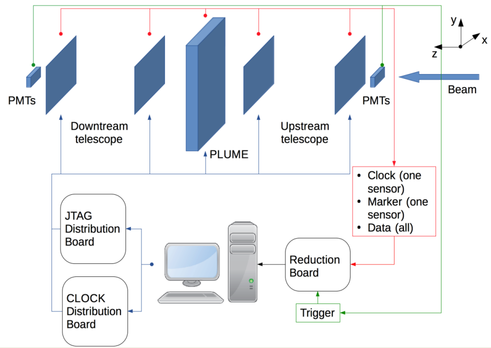
- Pitch: 18.4 μm (square pixels)
- Active area: $10.6 \times 21.2\text{mm}^2$ (576 rows x 1152 columns)
- Integration time: 115.2 $\mu(s)$ (200 ns per line)
- Binary output with Zero suppression
- Well known sensors $\Rightarrow$ used for EUDET telescope
- Extended to MIMOSA-28 exploited in STAR-PXL vertex detector @ RHIC-BNL since 2014

Test beam

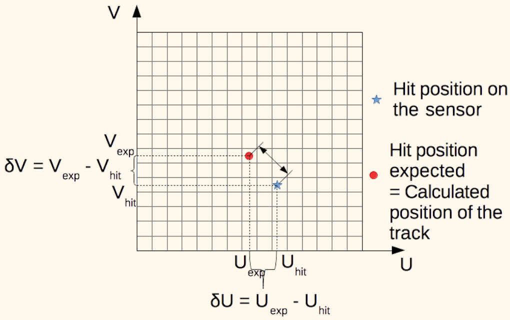
Motivation

Test detector under real conditions to determine its performance

Set-up



Track-hit residual



Device Under Test (DUT) alignment:

- Telescope planes defined particle's track
- Alignment of DUT in local coordinate system:
  - Define a maximal range in which a hit can be associated to a track
  - Find the best tilt and position to minimise the distance between a hit and its associated track.

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1. Introduction
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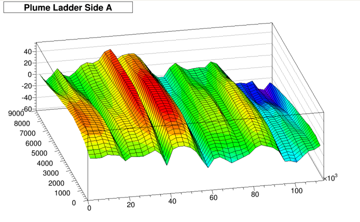
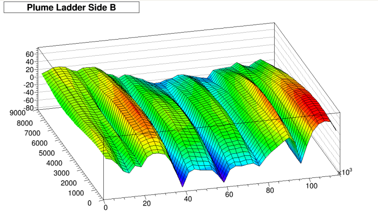
4. Mechanical deformation
   - Surface's survey
   - Origin of deviations and how to take them into account
   - Results on the correction of deviations
5. Radiation length measurement
6. Conclusion and outlook

Metrology of module's surface

Are our ladders completely flat?

Peak-to-peak flatness ~ 100 μm



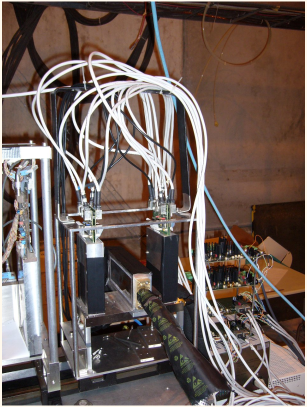


Test performed at Bristol with a dummy ladder

Question

What is the impact of such deformations on ladder's performance?

Test of the first fully functional prototype



Test beam 2011

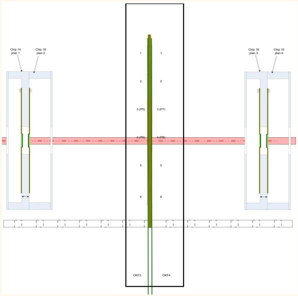
- CERN-SPS with 120 GeV π^-
- Reference plane: 4 MIMOSA-26
- DUT: first double-sided ladder equipped with 12 MIMOSA-26 sensors
- Ladder performance studies with different configurations

Impact of deformations

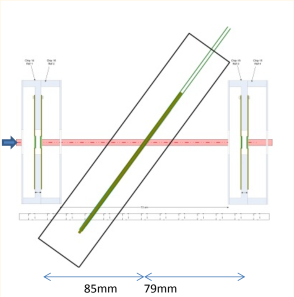
- Already observed and studied by 2 Ph. D. students
- Method to correct manually and locally these deviations

Is it possible to include the deformations observed during the offline analysis?

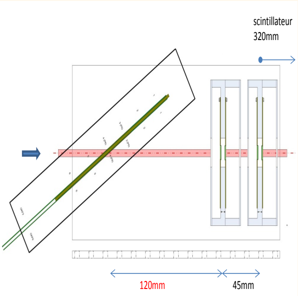
Geometrical configurations studied



Module perpendicular to the beam.



Module tilted (between 28°and 40°).



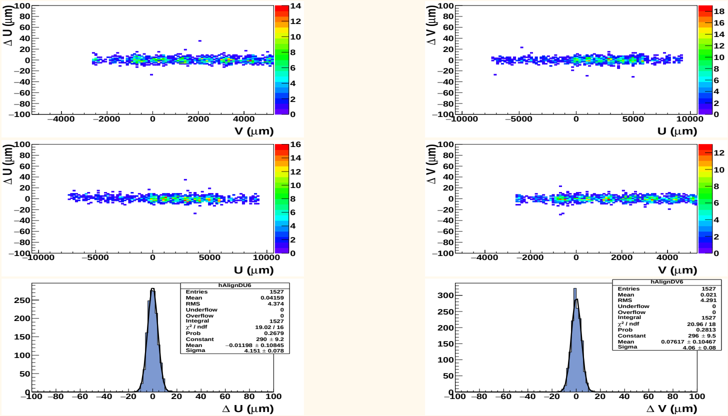
Module tilted ($\simeq 60°$).

⇒ Study track-hit residual and the distribution of this residual as a function of the relative position of the beam on the sensor.

Analysis performed with TAF (TAPI Analysis Framework).

Module perpendicular to the beam

Threshold 6 σ, air flow speed < 5m/s and 1.8M events.

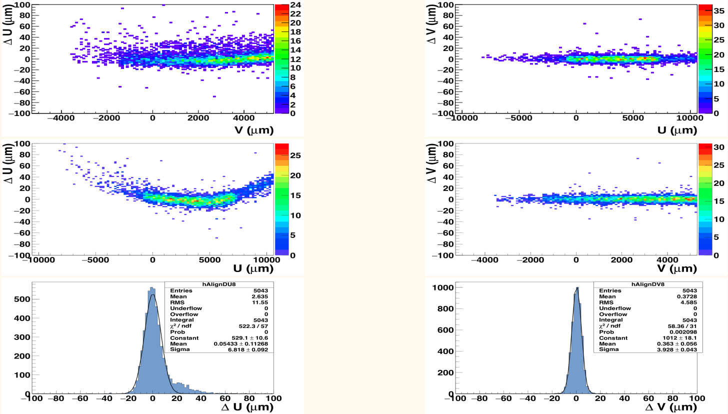


Spatial residual obtained after alignment:

$\sigma_U \simeq 4.2$ μm and $\sigma_V \simeq 4.1$ μm

Module titled in one direction (w.r.t. to the beam axis)

Threshold 6 σ, air flow speed < 5m/s, 720k events and 36°tilt.



Spatial residual obtained after alignment:

$\sigma_U \simeq 6.8$ μm and $\sigma_V \simeq 4.0$ μm

Origin of deviations

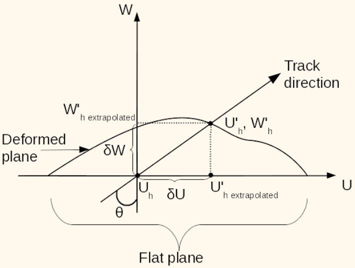
Consequence of the ladder's characteristics

- Use of ultra-thin (50 μm) and precise sensors (spatial resolution less than 4 μm)
- Mechanical constraints induce permanent deformations ($\simeq$ 100 μm) which can not be flattened during the ladder assembly

Origin of the deviations

Artefacts from the modelling of our sensors during the analysis

- Sensors modeled as completely flat planes
- The track extrapolation is sensitive to the exact position of the hit on the plane and the angle of incidence



Deviations of the residual

$$\delta W = \frac{\delta U}{\tan\theta}$$

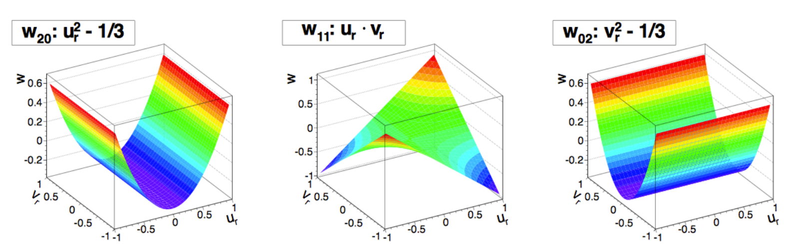
How to describe deviations from the flat plane?

arXiv:1403.2286 [physics.ins-det] CMS paper

- Sensor shape parametrised as a sum of products of modified Legendre polynomials:

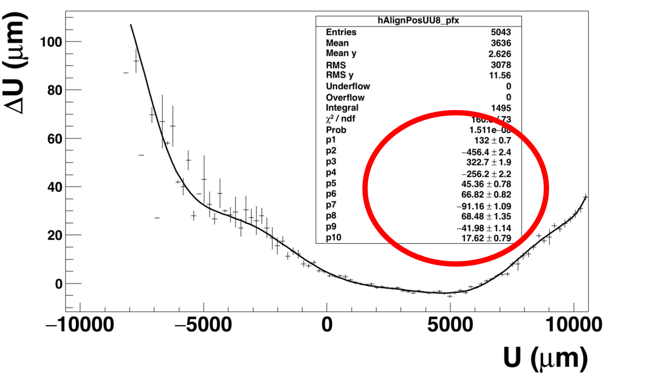
$$
\begin{aligned}
w(u_r, v_r) &= w \\
&+ w_{10} \cdot u_r + w_{01} \cdot v_r \\
&+ w_{20} \cdot (u_r^2 - 1/3) + w_{11} \cdot (u_r \cdot v_r) + w_{02} \cdot (v_r^2 - 1/3)
\end{aligned}
$$

- In our case, we used Legendre polynomials of the 11^{th} order only in the direction of the deformation.



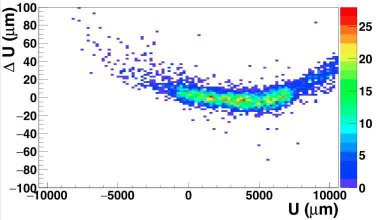
Deformation's parametrisation

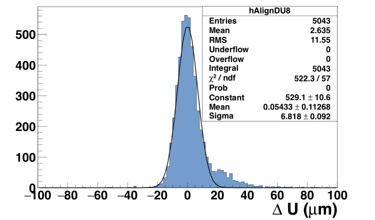
Possibility to parametrise the deformation with Legendre polynomials of the 11^{th} order .



Summary before/after correction

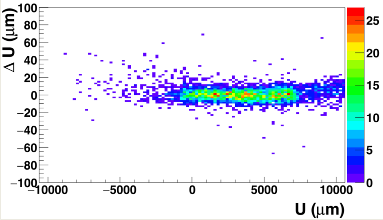
Before correction

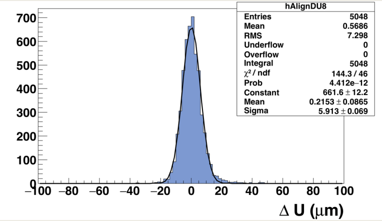




$\sigma_u = 6.8\ \mu m$

After correction





$\sigma_u = 5.9\ \mu m$

Summary of correction for different angles and same planes

Spatial residuals

Side	Tilted angle (°)	σ_u^{Def} (μm)	σ_u^{Cor} (μm)	Improvement
Front	28	9.0 ± 0.1	4.9 ± 0.1	46.6 %
Back	28	5.7 ± 0.1	4.7 ± 0.1	17.5 %
Front	36	14.1 ± 0.1	6.1 ± 0.1	56.0 %
Back	36	6.8 ± 0.1	5.9 ± 0.1	13.2 %
Front	60	41.2 ± 0.15	25.8 ± 0.2	37.4 %
Back	60	23.3 ± 0.13	21.7 ± 0.1	6.8 %

$\sigma_{tel} = 2.2\mu m$ for 36°and $\sigma_{tel} = 18.8\mu m$ for 60°.

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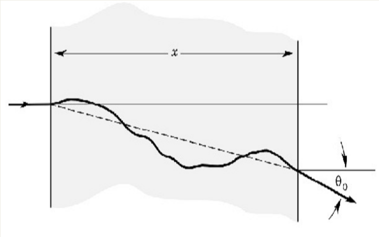
- Motivation
- Test beam @ DESY
- Theoretical estimation
- Results

6 Conclusion and outlook

Multiple scattering

Charged particles traveling through matter:

- Lose energy via inelastic collisions with atomic electrons
- Deflection by many small angles (Coulomb scattering from nuclei)
- Standard deviation of the scattering angle distribution described by Highland formula

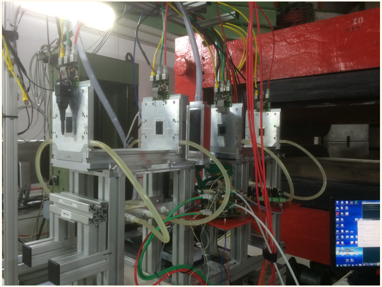


$$\theta_0 = \frac{13.6(MeV)}{p}\left(\frac{x}{X_0}\right)^{0.555}$$

Motivation of measuring the radiation length:

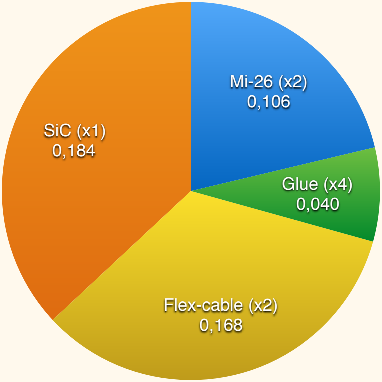
- Tracking system has to detect particle's path and minimise its energy degradation
- Physics analysis: reconstruction of events depends strongly on the energy loss inside the different part of detectors

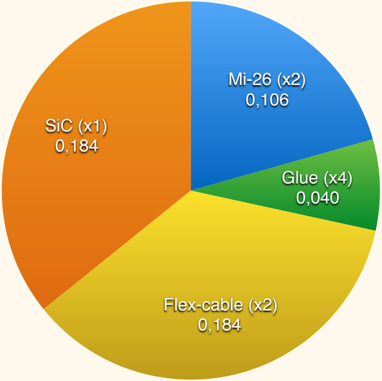
Test beam @ DESY with 5 GeV e^- (April 2016)



- Test Beam 21
- Reference plane: 4 EUDET telescope planes
- Goal: radiation length measurement

Estimation of the radiation length



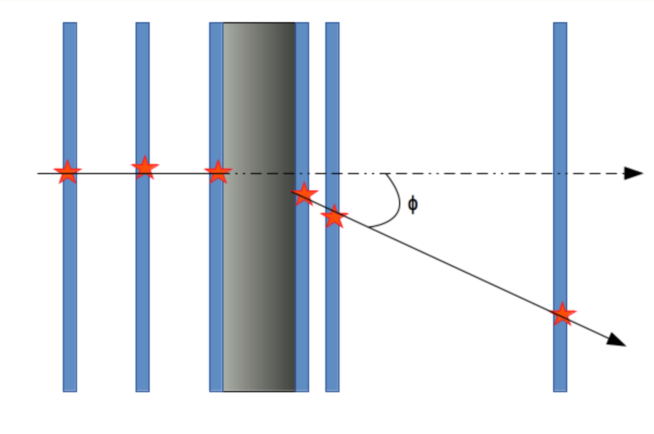


Total material budget

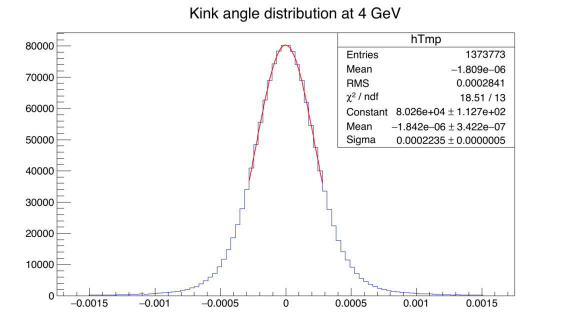
Depends on flex-cable fill factor (25 % or 30 %)

$$\Rightarrow \frac{x}{X_0} \simeq 0.498 \;-\; 0.515\ \%\ X_0$$

Kink angle measurement



Kink angle measurement at 4 GeV



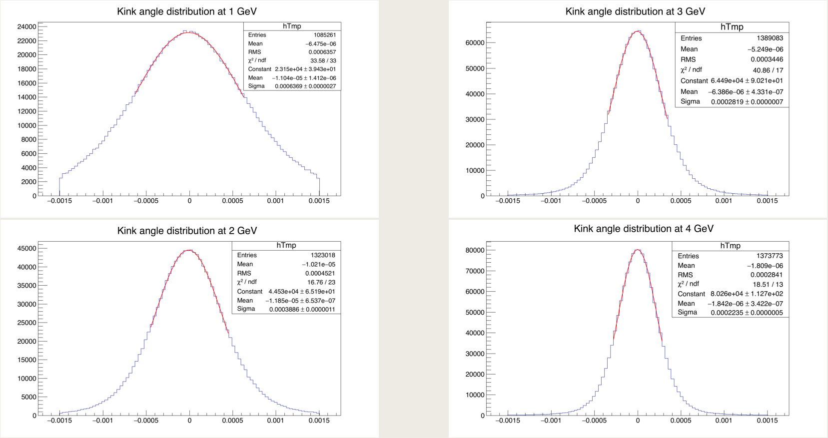
Determination of θ_0

$$\theta_0 = \sqrt{k^2 - F}$$

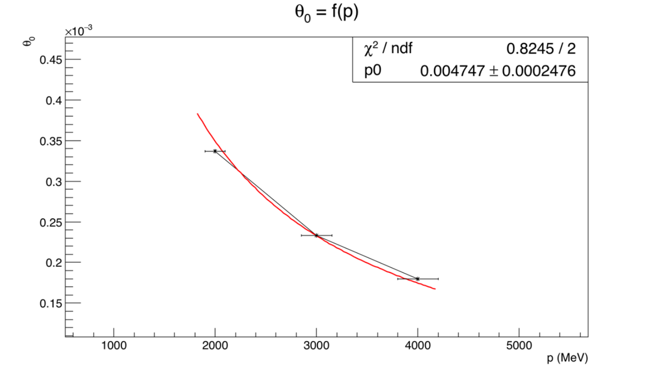
- k = width of the kink angle distribution fit
- F = Offset parameter from the GBL track fitting algorithm

Kink angle measurement between 1 and 4 GeV

Fitted kink angle distributions



Material budget



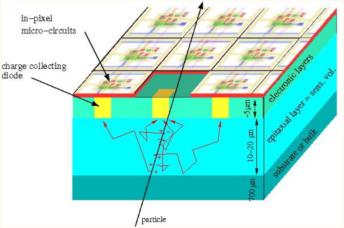
Measurement

$$\text{Highland formula: } \theta_0 = \frac{13.6(MeV)}{p}\left(\frac{x}{X_0}\right)^{0.555}$$

$$\frac{x}{X_0}\Big|_{estimated} \simeq 0.498 - 0.515\ \%\ X_0$$

$$\frac{x}{X_0}\Big|_{measured} \simeq 0.47 \pm 0.02\ \%\ X_0$$

Lower estimation of X_0



Possible explanation

- During analysis, hit position is located in middle of the epitaxial layer
- Missing 6 μm of the electronic layers and 7 μm of epitaxial layer
- For 2 sensors $\Rightarrow$ ~ 0.028 % X_0 not included in the calculation

Conclusion

Context:

- ILC: next collider to study precisely EWSB (and other physics scenarios)
- Performances to achieve imposes R&D detector development

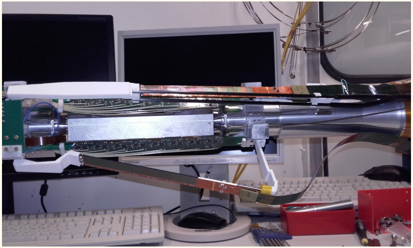
blabla

- Mechanical deformations observed are impacting ladder's performance
- Algorithm based on Legendre polynomials reduces impact of these deformations during the analysis
- Radiation length measurement:

$$\frac{x}{X_0}\Big|_{estimated} \simeq 0.498 \; - \; 0.515 \; \% \; X_0$$
$$\frac{x}{X_0}\Big|_{measured} \simeq 0.47 \pm 0.02 \; \% \; X_0$$

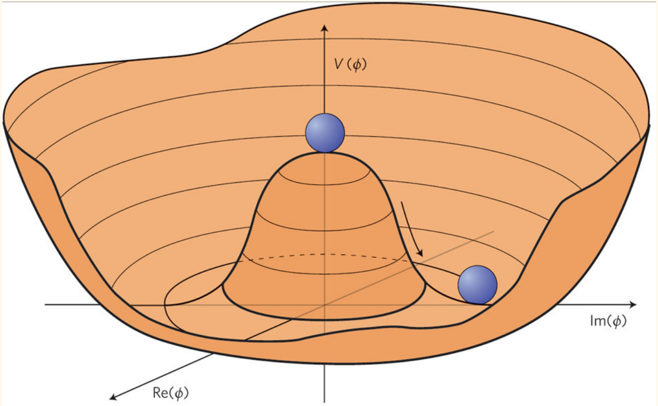
Outlook

- Study ladder performance for low energy (test beam April 2016)
- New prototype with a material budget of 0.35 % X_0 has been built, tested in laboratory but not yet in real conditions
- Double-sided ladders could be enriched with sensors having different characteristics (fast integration time VS good spatial resolution)
- Using the beam structure to apply a power-pulsing method
- Application of PLUME in SuperKEKB for the BEAST project



Thanks for your attention !!!

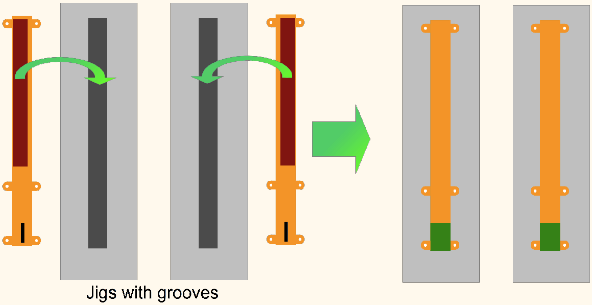
Higgs boson potential



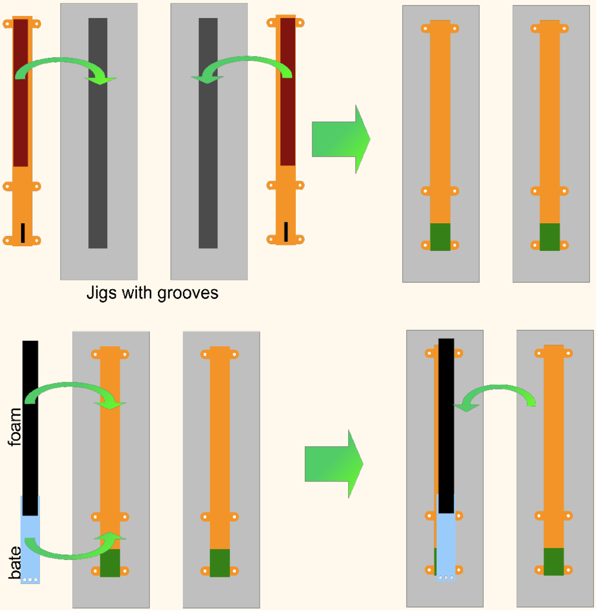
Higgs boson physics at the ILC

- Same measurements as LHC: couplings, mass and spin
- Model independent measurement: no dependence on theory
- Total Higgs width
- $\mathrm{H} \rightarrow c\bar{c}/gg$
- Higgs self couplings

Ladder assembly

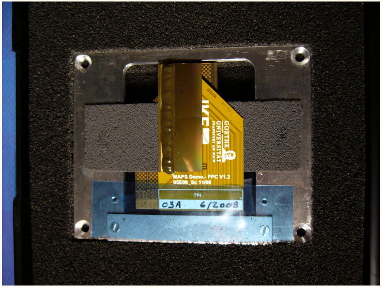


Ladder assembly

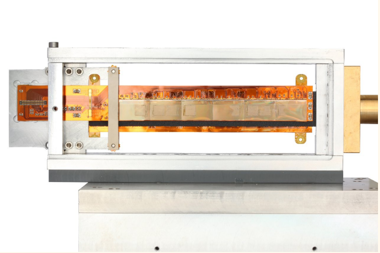


PLUME history

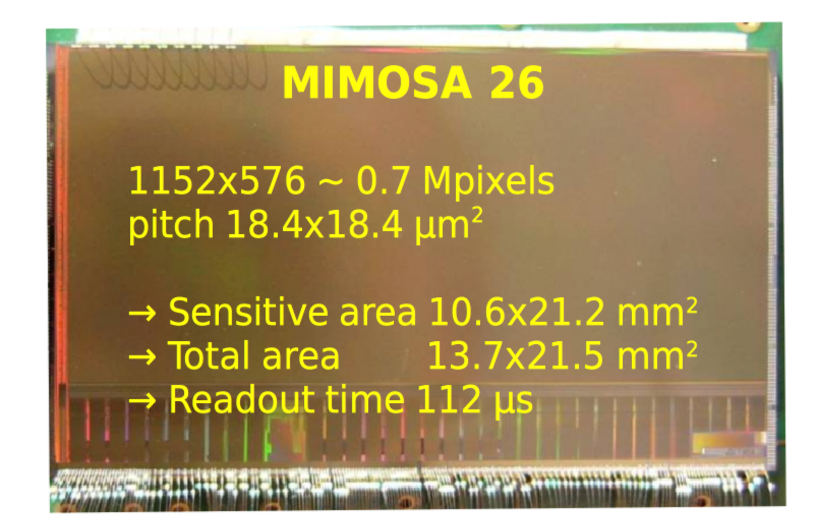
PLUME-V0 (2009)



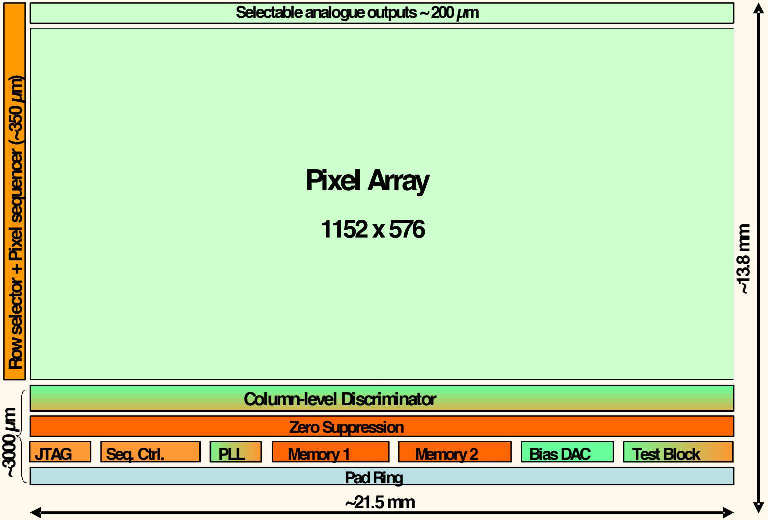
PLUME-V1 (2010)



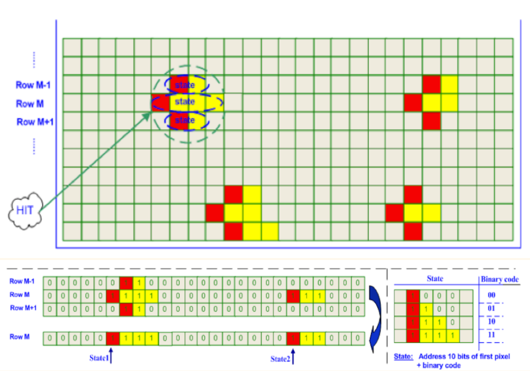
MIMOSA-26 description



MIMOSA-26 architecture



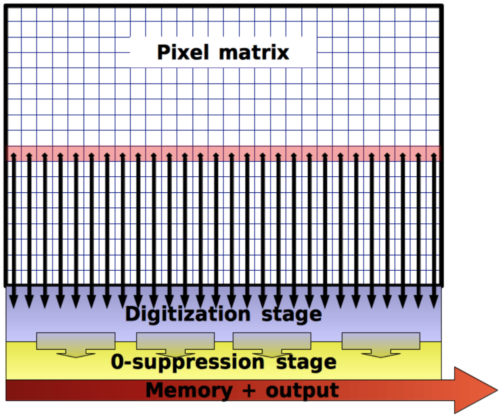
Zero Suppression logic (SUZE)



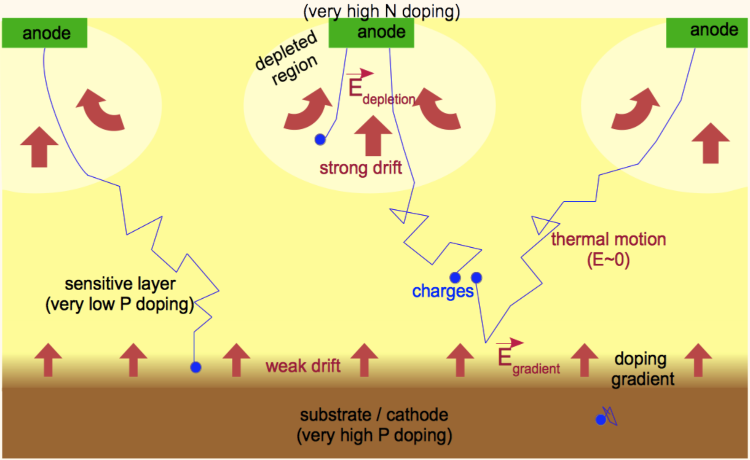
SUZE logic split in 3 blocks:

- **Sparse Data Scan (SDS)** Hit detection per line and data encoding, until 6 states consecutive pixels (1 to 4 pixels) per block of 64 columns;
- **Multiplexing Logic (Mux)** giving up to 9 states;
- **Memory storage** 2 blocks to store the states of the full frame, switching to avoid dead time (during one acquire states of event N, the other one transfer the information of frame N-1).

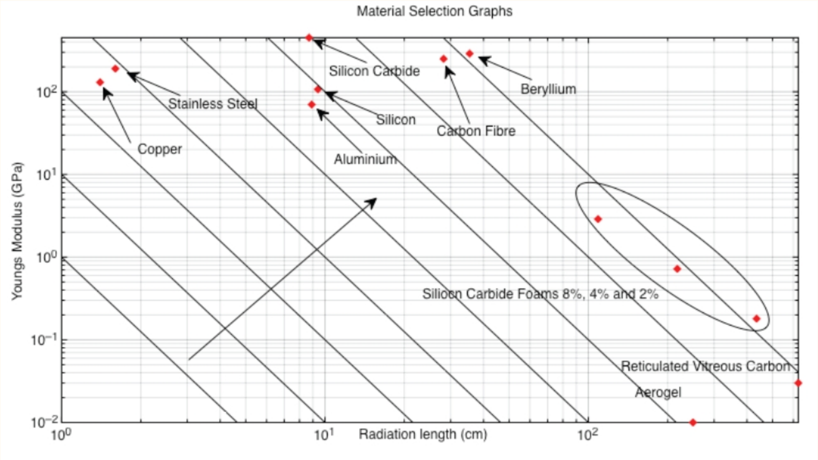
Column parallel readout



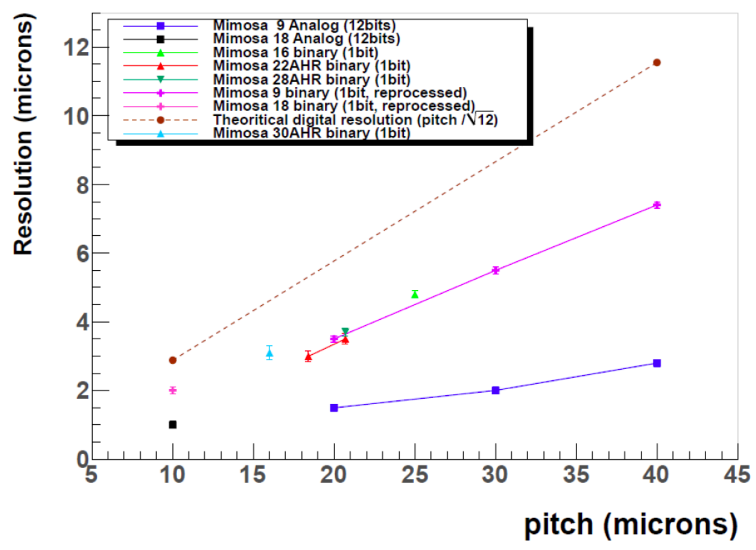
MAPS principle



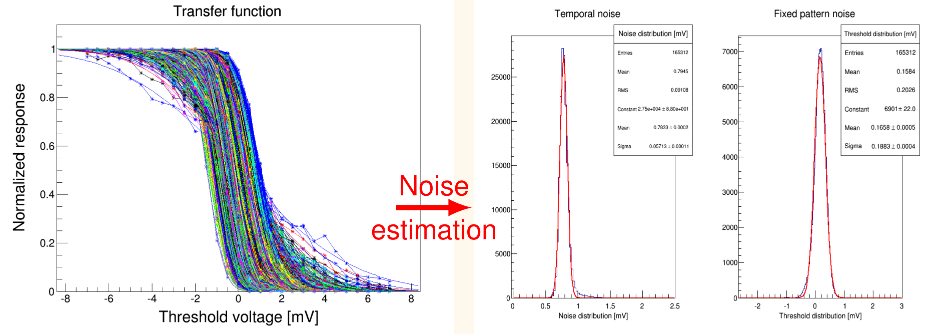
Young Modulus



Spatial resolution for different pitch (IPHC-Strasbourg)



Characterization of one sensor



Threshold scan

Normalised response of pixels in a sub-matrix (288 discriminators) as a function of threshold applied (mV).

Noise performances

- Temporal noise (derivative of the S-curve): 0.79 mV
- Fixed pattern noise (thresholds' dispersion for a mid-point): 0.2 mV
- Offset: 0.16 mV

$\Rightarrow$ **Can now define different thresholds**

Higgs Strahlung kinematics

$$E_H = \frac{s - M_Z^2 + M_H^2}{2\sqrt{s}}$$

$$E_Z = \frac{s - M_H^2 + M_Z^2}{2\sqrt{s}}$$

$$|\overrightarrow{p_H}| = |\overrightarrow{p_Z}| = \frac{\sqrt{[s - (M_H + M_Z)^2] \cdot [s - (M_H - M_Z)^2]}}{2\sqrt{s}}$$

If $M_H = 125$ GeV, $M_Z = 91.2$ GeV and $\sqrt{s} = 350$ GeV, then:

$$E_H \simeq 185.4 \text{ GeV}$$

$$E_Z \simeq 164.6 \text{ GeV}$$

$$|\overrightarrow{p_H}| = |\overrightarrow{p_Z}| \simeq 68.5 \text{ GeV}$$

Detector performances

Vertexing

$$\sigma_{IP} = 5 \oplus \frac{10}{p \sin^{3/2}\theta} (\mu m)$$

Tracking

$$\sigma(1/p) = 2x10^{-5} (GeV^{-1})$$

Jet ernergy

$$\sigma_E / E = 0.3/\sqrt{E(GeV)}$$

Particle Flow Algorithm

- Typical jet:
  - Charged hadrons $\simeq$ 60 %
  - Photons $\simeq$ 30 %
  - Neutral $\simeq$ 10 %
- Standard approch
  - All jet components energy measured in ECAL/HCAL
  - $E_{jet} = E_{ECAL} + E_{HCAL}$
- Particle flow calorimetry
  - Measurement of charged particles in tracker
  - Measurement of photon in ECAL
  - Measurement of hadrons in HCAL
  - $E_{jet} = E_{Track} + E_{\gamma} + E_{n}$

Why a linear collider?

Limitations of e^+e^- colliders

- Synchroton radiation loss $\sim \mathrm{E}^4/r$
- Synchroton cost: $\sim$ quadratically with energy
- Power consumption

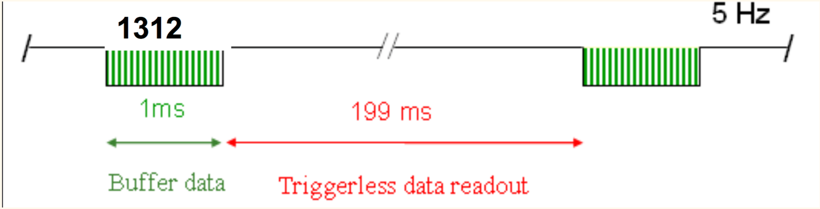
Avantages of linear colliders

- Not limited by synchrotron radiation
- Cost: $\sim$ linear with energy
- Polarisation of both beams
- Detectors close to the IP $\Rightarrow$ optimum for c-tagging

ILC interaction region

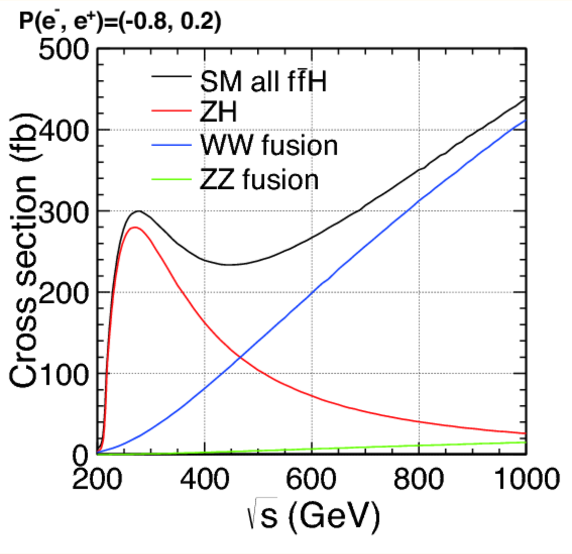
- 1 interaction region for 2 detectors
- Push-pull:
  - Detectors mounted on movable platforms
  - Sharing of beam time
  - Switching time: 24h to 48h
  - Allow cross-checking

ILC environment

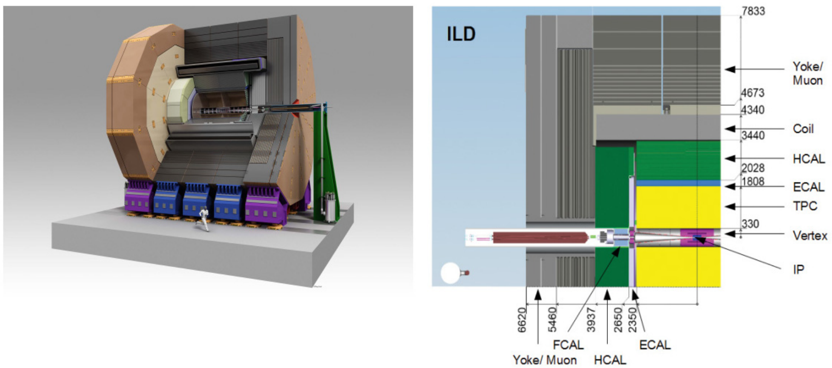


- Bunch spacing of $\sim$ 554 ns
- 1312 bunches in a 1 ms long pulse (train)
- Quiet time: 199 ms
- Occupancy dominated by beam background and noise
- Reading during quiet time possible

Higgs production cross-section



Overview of the ILD



Sequential cuts strategy

cut0: Number of isolated lepton (niso): niso = 0

cut1: Transverse Momentum visible (Ptvis): $35 < P_t^{vis} < 155$ GeV

cut2: Visible mass (mvis): $95 < m_{vis} < 140$ GeV

cut3: Angle between the momentum axis of both jets ($\cos\alpha$): $-1 < \cos\alpha < 0.22$

cut4: Number of reconstructed particle

cut5: D2YM

cut6: Visible longitudial momentum ($abs(P_z^{vis})$)

cut7: E_{miss}

Beam polarisation

Simulated data: 100 % left or right events

$$\sigma_{P_{(e+,e-)}} = \left(\frac{1 - P_{e-}}{2}\right)\left(\frac{1 + P_{e+}}{2}\right)\sigma_{RL} + \left(\frac{1 + P_{e+}}{2}\right)\left(\frac{1 - P_{e-}}{2}\right)\sigma_{LR}$$

$$\sigma_{P_{(e+,e-=\mathbf{0.3},-\mathbf{0.8})}} = 0.585 \cdot \sigma RL + 0.035 \cdot \sigma_{LR}$$