Übungs Blatt 1

1

Trivial.

2

Additive polynomial:

$$P(T_1 + T_2) = P(T_1) + P(T_2)$$

a)

Characteristic:
n summands for $1 + ... + 1 = 0 \Rightarrow char(R) = n$

b)

Trivial.

c)

Trivial.

d)

Trivial.

3

An integrity domain is a nonzero commutative ring in which the product of any two nonzero elements is nonzero.

a)

Injective:

$$\forall a, b \in X, f(a) = f(b) \Rightarrow a = b$$

b)

For finite fields:
injectivity $\Leftrightarrow$ surjectivity $\Leftrightarrow$ bijectivity
Requirements for a ring to be a field:

1. $$1 \neq 0$$

2. $$\forall x \in R \exists x^{-1} : xx^{-1} = x^{-1}x = 1$$

4

a)

A monic polynomial is a polynomial where the leading coefficient is equal to 1.

b)

A is a divisor of B $\Rightarrow B = A \times C$.

Übungs Blatt 2

1

a)

Equivalent matrix: $B = T^{-1}AS$
Equivalence relation:

1. reflexive property $x = x$
2. symmetric property $x = y \Rightarrow y = x$
3. transitive property $a = b \land b = c \Rightarrow a = c$

b)

Invertible Matrix:

$$AA^{-1} = A^{-1}A = I$$

For square matrices:

$$det(A) \neq 0$$

c)

Trivial.

d)

Similar:

$$B = T^{-1}AS, \text{ where } T = S$$

2

$$\dim(V) = \dim(\mathrm{Ker}(V)) + d\mathrm{im}(\mathrm{im}(V))$$

3

a)

Determinant:

- 2×2:

$$\det(A) = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

- 3×3:

$$\det(A) = \det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \det \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \det \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \det \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

- $n \times n$:

$$\det(A) = \sum_{\sigma \in S_n} (\mathrm{sgn}(\sigma) \prod_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{a}_\mathrm{i}, \sigma_\mathrm{i})$$

  Do note this is mostly irrelevant, since you can just perform something equivalent to a 3×3 determinant over and over again for larger matrices.

b)

Inverse matrix for 3×3 matrices:

$$A^{-1} = \frac{1}{\det(A)} \times \mathrm{adj(A)}$$

`https://en.wikipedia.org/wiki/Adjugate_matrix`

4

a)

Trivial.

b)

$F(U) \subset U$ if all bases of $F(U)$ are in U, but $\dim(F(U)) < \dim(U)$.

Übungs Blatt 3

1

$$\dim(V) = \dim(U) + \dim(V/U)$$

2

$$Danke$$

a)

b)

c)

d)

3

A polynomial of an odd-numbered degree will always go through zero at least once.

4

a)

b)

Übungs Blatt 4

Übungs Blatt 5

Übungs Blatt 6

Übungs Blatt 7

Übungs Blatt 8

Übungs Blatt 9

Übungs Blatt 10

1

$$\|v\| = \sqrt{\sum_{i=1}^{n} (v_i)^2}$$

$$\mathrm{orthogonal} \Leftrightarrow < \mathrm{v}, \mathrm{u} >= 0 \Leftrightarrow \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{v_i u_i} = 0$$

2

`https://en.wikipedia.org/wiki/Orthogonal_complement`

Norm: 1

3

a)

"$(V, <, >)$ ist ein euklidischer Raum, wenn V ein endlichdimensionaler $\mathbb{R}$-Vektorraum und $<, >$ ein Skalarprodukt ist."

"Ein Skalarprodukt auf einem endlichdimensionalen $\mathbb{R}$-Vektorraum V ist eine positiv definite symmetrische Bilinearform: $\varphi(u, v) \mapsto < u, v >$."

A matrix M is positive definite if $\forall v > 0 \in \mathbb{N} : v^\top M v > 0$.

A symmetrical bilinear form is a bilinear form so that $B(u, v) = B(v, u)$.

b)

For V vector space, $W \subset V$ a subspace, B the bilinear form of V:

$$W^\perp = \{x \in V : B(x, y) = 0 \text{ for all } y \in W\}.$$

c)

"Eine Orthonormalbasis von V [euklidischer oder unitärer Raum $(V, <, >)$] ist eine Basis $\mathcal{B} = \{b_i\}$ / $b_i \perp b_j,\ i \neq j,\ \|b_i\| = 1$."

d)

4

a)

A symmetrical matrix is a matrix A so that $A = A^\top$.

b)

a) Positive definite matrix.

Übungs Blatt 11

Übungs Blatt 12