Chapter 1

Assignment Sheet 1

1.1

Trivial.

1.2

Additive polynomial:

$$P(T_1 + T_2) = P(T_1) + P(T_2)$$

1.2.1

Characteristic:
n summands for $1 + ... + 1 = 0 \Rightarrow char(R) = n$

1.2.2

Trivial.

1.2.3

Trivial.

1.2.4

Trivial.

1.3

An integrity domain is a nonzero commutative ring in which the product of any two nonzero elements is nonzero.

1.3.1

Injective:

$$\forall a, b \in X, f(a) = f(b) \Rightarrow a = b$$

1.3.2

For finite fields:
injectivity $\Leftrightarrow$ surjectivity $\Leftrightarrow$ bijectivity
Requirements for a ring to be a field:

1. $$1 \neq 0$$

2. $$\forall x \in R \exists x^{-1} : xx^{-1} = x^{-1}x = 1$$

1.4

1.4.1

A monic polynomial is a polynomial where the leading coefficient is equal to 1.

1.4.2

A is a divisor of B $\Rightarrow B = A \times C$.

Chapter 2

Assignment Sheet 2

2.1

2.1.1

Equivalent matrix: $B = T^{-1}AS$

Equivalence relation:

1. reflexive property $x = x$
2. symmetric property $x = y \Rightarrow y = x$
3. transitive property $a = b \wedge b = c \Rightarrow a = c$

2.1.2

Invertible Matrix:

$$AA^{-1} = A^{-1}A = I$$

For square matrices:

$$det(A) \neq 0$$

2.1.3

Trivial.

2.1.4

Similar:

$$B = T^{-1}AS, \text{ where } T = S$$

2.2

Trivial.

2.3

2.3.1

Determinant:

- 2×2:

$$det(A) = det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

- 3×3:

$$det(A) = det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = adet \begin{bmatrix} e & f \\ h & i \end{bmatrix} - bdet \begin{bmatrix} d & f \\ g & i \end{bmatrix} + cdet \begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

- $n \times n$:

$$det(A) = \sum_{\sigma \in S_n} (sgn(\sigma) \prod_{i=1}^{n} a_i, \sigma_i)$$

  Do note this is mostly irrelevant, since you can just perform something equivalent to a 3×3 determinant over and over again for larger matrices.

2.3.2

Inverse matrix for 3×3 matrices:

$$A^{-1} = \frac{1}{det(A)} \times A^T$$

2.4

2.4.1

Mathematical induction.

2.4.2

$F(U) \subset U$ if all bases of F(U) are in U, but $dim(F(U)) < dim(U)$.