A communicative framework for early word learning

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Abstract

Children do not learn language from passive observation of the world, but from interaction with caregivers who want to communicate with them. These communicative exchanges are structured at multiple levels in ways that support support language learning. We argue this pedagogically supportive structure can result from pressure to communicate successfully with a linguistically immature partner. We first characterize one kind of pedagogically supportive structure in a corpus analysis: caregivers provide more information-rich referential communication, using both gesture and speech to refer to a single object, when that object is rare and when their child is young. In an iterated reference game, we experimentally show that this behavior can arise from pressure to communicate successfully with a less knowledgeable partner. Then, we show that speaker behavior in our experiment can be explained by a rational planning model, without any explicit teaching goal. Finally, in a series of simulations, we explore the language learning consequences of having a communicatively-motivated caregiver. We show that under many parameterizations, simple learning mechanisms interacting with a communicatively-motivated partner outperform more powerful learning mechanisms. In sum, this perspective offers first steps toward a unifying, formal account of both the child's learning and the parent's production: Both are driven by a pressure to communicate successfully.

Keywords: communication; child-directed speech; language learning; computational modeling

Word count: X

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One of the most striking aspects of children's language learning is just how quickly they master the complex system of their natural language (Bloom, 2000). In just a few short years, children go from complete ignorance to conversational fluency in a way that is the envy of second-language learners attempting the same feat later in life (Newport, 1990). What accounts for this remarkable transition?

Distributional learning presents a unifying account of early language learning: Infants come to language acquisition with a powerful ability to learn the latent structure of language from the statistical properties of speech in their ambient environment (Saffran, 2003). Distributional learning mechanisms can be seen in accounts across language including phonemic discrimination (Maye, Werker, & Gerken, 2002), word segmentation (Saffran, 2003), learning the meanings of both nouns (L. B. Smith & Yu, 2008) and verbs (Scott & Fischer, 2012), learning the meanings of words at multiple semantic levels (Xu & Tenenbaum, 2007), and perhaps even the grammatical categories to which a word belongs (Mintz, 2003). A number of experiments clearly demonstrate both the early availability of distributional learning mechanisms, and their potential utility across these diverse language phenomena (DeCasper & Fifer, 1980; DeCasper & Spence, 1986; Estes, Evans, Alibali, & Saffran, 2007; Gomez & Gerken, 1999; Maye, Werker, & Gerken, 2002; Saffran, Aslin, & Newport, 1996; L. B. Smith & Yu, 2008; Xu & Tenenbaum, 2007).

However, there is reason to be suspicious about just how precocious statistical learning abilities are in early development. Although these abilities are available early, they are highly constrained by limits on other developing cognitive capacities. For example, infants' ability to track the co-occurrence information connecting words to their referents is constrained significantly by their developing memory and attention systems (L. B. Smith & Yu, 2013; Vlach & Johnson, 2013). Computational models of these processes show that the rate of acquisition is highly sensitive to variation in environmental statistics (e.g., Vogt, 2012). Models of cross-situational learning have demonstrated that the Zipfian distribution of word frequencies and word meanings yields a learning problem that cross-situational learning alone cannot explain over a reasonable time frame (Vogt, 2012). Further, a great deal of empirical work demonstrates that cross-situational learning even in adults drops off rapidly when participants are asked to track more referents, and also when the number of intervening trials is increased–features likely typical of the naturalistic learning environment (e.g., Yurovsky & Frank, 2015). Thus, precocious unsupervised statistical learning appears to fall short of a complete explanation for rapid early language learning.

Even relatively constrained statistical learning could be rescued, however, if caregivers structured their language in a way that simplified the learning problem and promoted learning. Indeed, infant-directed speech does have distinct structural features compared with typical adult-directed speech, some of which have demonstrated learning benefts across a number of language phenomena. For example, in phoneme learning, infant-directed speech provides examples that seem to facilitate the acquisition of phonemic categories (Eaves Jr, Feldman, Griffiths, & Shafto, 2016). In word segmentation tasks, infant-directed speech facilitates infant learning more than matched adult-directed speech (Thiessen, Hill, & Saffran, 2005). In word learning scenarios, caregivers produce more speech during episodes of joint attention with young infants, which uniquely predicts later vocabulary (Tomasello & Farrar, 1986). Child-directed speech even seems to support learning at multiple levels in parallel–e.g., simultaneous speech segmentation and word learning (Yurovsky, 2012). For each of these language problems faced by the developing learner, caregiver speech exhibits structure that seems uniquely beneficial for learning.

Under distributional learning accounts, the existence of this kind of structure is a theory-external feature of the world that does not have an independently motivated explanation. Such accounts view the generative process of structure in the language environment as a problem separate from language learning. However, across a number of language phenomena, the language environment is not merely supportive, but seems calibrated to children's changing learning mechanisms (Yurovsky, 2018). For example, across development, caregivers engage in more multimodal naming of novel objects than familiar objects, and rely on this synchrony most with young children (Gogate, Bahrick, & Watson, 2000). The prevalence of synchrony in child-directed speech parallels infant learning mechanisms: young infants appear to rely more on synchrony as a cue for word learning than older infants, and language input mirrors this developmental shift (Gogate, Bahrick, & Watson, 2000). Beyond age-related changes, caregiver speech may also support learning through more local calibration to a child's knowledge. Caregivers have been shown to provide more language about referents that are unknown to their child, and adapt their language in-the-moment to the knowledge their child displays during a referential communication game (Leung, Tunkel, & Yurovsky, in press). The calibration of parents' production to the child's learning and knowledge suggests a co-evolution such that these processes should not be considered in isolation.

What then gives rise to the structure in early language input that mirrors children's learning mechanisms? Because of widespread agreement that parental speech is not usually motivated by explicit pedagogical goals (Newport, Gleitman, & Gleitman, 1977), the calibration of speech to learning mechanisms seems a happy accident; parental speech just happens to be calibrated to children's learning needs. Indeed, if parental speech was pedagogically-motivated, we would have a formal framework for deriving predictions and expectations (e.g., Shafto, Goodman, & Griffiths, 2014). Models of optimal teaching have been successfully generalized to phenomena as broad as phoneme discrimination (Eaves Jr, Feldman, Griffiths, & Shafto, 2016) to active learning (Yang, Vong, Yu, & Shafto, 2019). These models take the goal to be teaching some concept to a learner and attempting to optimize that learner's outcomes. However, because the parent's goal is not to teach, this framework gives an incomplete account of parents' behavior, which has features that are not pedagogical even in these same domains (McMurray, Kovack-Lesh, Goodwin, & McEchron, 2013; Tomasello & Todd, 1983).

Instead, the recent outpouring of work exploring optimal communication (the Rational Speech Act model, see Frank & Goodman, 2012) provides a different framework for understanding parent production. Under optimal communication accounts, speakers and listeners engage in recursive reasoning to produce and interpret speech cues by making inferences over one another's intentions (Frank & Goodman, 2012). These accounts have made room for advances in our understanding of a range of language phenomena previously uncaptured by formal modeling, most notably a range of pragmatic inferences (e.g., Frank & Goodman, 2012; Goodman & Frank, 2016). In this work, we consider the communicative structure that emerges from an optimal communication system across a series of interactions where one partner has immature linguistic knowledge. This perspective offers the first steps toward a unifying account of both the child's learning and the parent's production: Both are driven by a pressure to communicate successfully (Brown, 1977).

Early, influential functionalist accounts of language learning focused on the importance of communicative goals (e.g., Brown, 1977). Our goal in this work is to formalize the intuitions in these accounts in a computational model, and to test this model against experimental data. We take as the caregiver's goal the desire to communicate with the child, not about language itself, but instead about the world in front of them. To succeed, the caregiver must produce the kinds of communicative signals that the child can understand and respond contingently, potentially leading caregivers to tune the complexity of their speech as a byproduct of this in-the-moment pressure to communicate successfully (Yurovsky, 2018).

To examine this hypothesis, we focus on ostensive labeling (i.e. using both gesture and speech in the same referential expression) as a case-study phenomenon of information-rich structure in the language learning environment. We first analyze naturalistic parent communicative behavior in a longitudinal corpus of parent-child interaction in the home (Goldin-Meadow et al., 2014). We investigate the extent to which parents tune their ostensive labeling across their child's development to align to their child's developing linguistic knowledge (Yurovsky, Doyle, & Frank, 2016).

We then experimentally induce this form of structured language input in a simple model system: an iterated reference game in which two players earn points for communicating successfully with each other. Modeled after our corpus data, participants are asked to make choices about which communicative strategy to use (akin to modality choice). In an experiment on Mechanical Turk using this model system, we show that pedagogically-supportive input can arise from a pressure to communicate. We then show that participants' behavior in our game conforms to a model of communication as rational planning: People seek to maximize their communicative success while minimizing their communicative cost over expected future interactions. Finally, we demonstrate potential benefits for the learner through a series of simulations to show that communicative pressure on parents' speech facilitates learning. Under a variety of parameter settings, simple learners interacting with communicative partners outperform more complex statistical learners.

Corpus Analysis

We first investigate parent referential communication in a longitudinal corpus of parent-child interaction. We analyze the production of multi-modal cues (i.e. using both gesture and speech) to refer to the same object in the same instance. While many aspects of child-directed speech support learning, multi-modal (e.g., speaking while pointing or looking) are particularly powerful sources of data for young children (e.g., Baldwin, 2000; Gogate, Bahrick, & Watson, 2000). We take the ostensive labeling produced by multi-modal cues to be a case-study phenomenon of pedagogically supportive language input. While our account should hold for other language phenomena, by focusing on one phenomenon we attempt to specify the dynamics involved in the production of such input.

In this analysis of naturalistic communication, we examine the prevalence of multi-modal cues in children's language environment at different ages. We find that this pedagogically-supportive form of input shows a key halmark of adaptive tuning: caregivers using this information-rich cue more for young children and infrequent objects. Thus, parents production of multi-modal reference is tuned to children's developing linguistic knowledge (Yurovsky, Doyle, & Frank, 2016).

Methods

We used data from the Language Development Project–a large-scale, longitudinal corpus of naturalistic parent child-interaction in the home (Goldin-Meadow et al., 2014). The Language Development Project corpus contains transcription of all speech and communicative gestures produced by children and their caregivers over the course of the 90-minute home recordings. We coded each of these communicative instances to identify each time a concrete noun was referenced using speech, gesture, or both in the same referential expression (so called ostenstive labeling). In these analyses, we focus only caregiver's productions of ostenstive labeling in the form of a multi-modal reference.

Participants. The Language Development Project aimed to recruit a sample of families who are representative of the Chicago community in socio-economic and racial diversity (Goldin-Meadow et al., 2014). These data are drawn from a subsample of 10 families from the larger corpus. Our subsample contains data taken in the home every 4-months from when the child was 14-months-old until they were 34-months-old, resulting in 6 timepoints (missing one family at the 30-month timepoint). Recordings were 90 minute sessions, and participants were given no instructions.

Of the ten target children, five were girls, three were Black and two were Mixed-Race. Families spanned a broad range of incomes, with two families earning $15,000 to $34,999 and 1 family earning greater than $100,000. The median family income was $50,000 to $74,999.

Procedure. From the extant transcription and gesture coding, we specifically coded all concrete noun referents produced in either the spoken or gestural modality (or both).

Spoken reference was coded only when a specific noun form was used (e.g., “ball”), to exclude pronouns and anaphoric usages (e.g., “it”). Gesture reference was coded only for deictic gestures (e.g., pointing to or holding an object) to minimize ambiguity in determining the intended referent. In order to fairly compare rates of communication across modalities, we need to examine concepts that can be referred to in either gesture or speech (or both) with similar ease. Because abstract entities are difficult to gesture about using deictic gestures, we coded only on references to concrete nouns.

Reliability. To establish the reliability of the referent coding, 25% of the transcripts were double-coded. Inter-rater reliability was sufficiently high (Cohen’s $\kappa = 0.76$). Disagreements in coding decisions were discussed and resolved by hand.

To ensure that our each referent could potentially be referred to in gesture or speech, we focused on concrete nouns. We further wanted to ensure that the referents were physically present in the scene (and thus accessible to deictic gestures). Using the transcripts, a human rater judged whether the referent was likely to be present, primarily relying on discourse context (e.g., a referent was coded as present if the deictic gesture is used or used at another timepoint for the reference, or if the utterance included demonstratives such as “This is an X”). A full description of the coding criteria can be found in the Supporting Materials.

To ensure our transcript-based coding of referent presence was sufficiently accurate, a subset of the transcripts (5%) were directly compared to corresponding video data observation. Reliability across the video data and the transcript coding was sufficiently high ($\kappa = 0.72$). Based on transcript coding of all the referential communication about concrete nouns, 90% of the references were judged to be about referents that were likely present. All references are included in our dataset for further analysis.

Results

These corpus data were analyzed using a mixed effects regression to predict parents' use of multi-modal reference for a given referent. The model included fixed effects of age in months, frequency of the referent, and the interaction between the two. The model included a random intercept and random slope of frequency by subject and a random intercept for each unique referent. Frequency and age were both log-scaled and then centered both because age and frequency tend to have log-linear effects and to help with model convergence. The model showed that parents use ostensive labeling less with older children (β = -0.78, t = -7.88, p < .001) and marginally less for more frequent referents (β = -0.08, t = -1.81, p = .071). In addition, the interaction between the two was significant, indicating that for parents ostensively label more for younger children when referents are infrequent (β = 0.18, t = 3.26, p = .001). Thus, in these data, we see early evidence that parents are providing richer, structured input about rarer things in the world for their younger children (Figure \ref{fig:corpus-plot).

Discussion

Caregivers are not indiscriminate in their use of multi-modal reference; in these data, they provided more of this support when their child was younger and when discussing less familiar objects. These longitudinal corpus findings are consistent with an account of parental alignment: parents are sensitive to their child's linguistic knowledge and adjust their communication accordingly (Yurovsky, Doyle, & Frank, 2016). Ostensive labeling is perhaps the most explicit form of pedagogical support, so we chose to focus on it for our case study. We argue that these data could be explained by a simple, potentially-selfish pressure: to communicate successfully. The influence of communicative pressure is difficult to draw in naturalistic data, so we developed a paradigm to try to experimentally induce richly-structured, aligned input from a pressure to communicate in the moment.

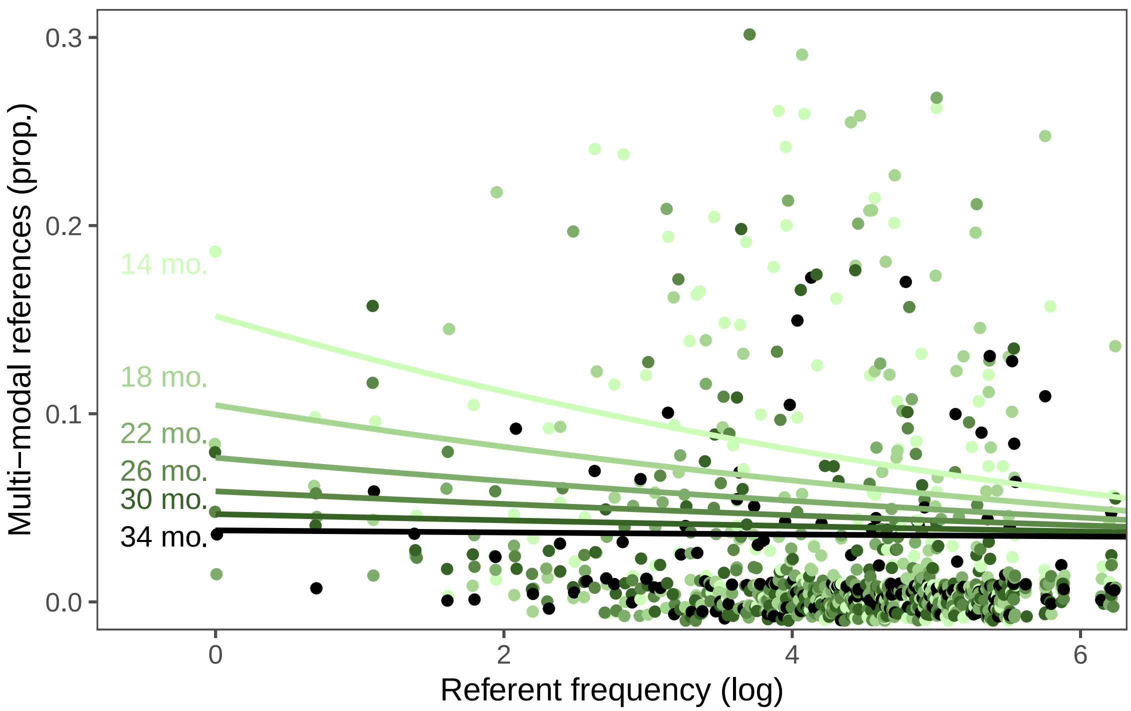

Figure 1. Parents' rate of ostensive labelling via multi-modal reference. Parents used ostensive labeling more for younger children and infrequent referents. Lines show model

Experimental Framework

To study the emergence of pedagogically supportive input from communicative pressure, we developed a simple reference game in which participants would be motivated to communicate successfully. After giving people varying amounts of training on novel names for 9 novel objects, we asked them to play a communicative game in which they were given one of the objects as their referential goal, and they were rewarded if their partner successfully selected this referent from among the set of competitors (Figure 2).

Participants could choose to refer either using the novel labels they had been exposed to, or they could use a deictic gesture to indicate the referent to their partner. The gesture was unambiguous, and thus would always succeed. However, in order for language to be effective, the participant and their partner would have to know the correct novel label for the referent.

Across conditions, we manipulated the relative costs of these two communicative methods (gesture and speech), as we did not have a direct way of assessing these costs in our naturalistic data, and they likely vary across communicative contexts. In all cases, we assumed that gesture was more costly than speech. Though this need not be the case for all gestures and contexts, our framework compares simple lexical labeling and unambiguous deictic gestures, which likely are more costly and slower to produce (see Yurovsky, Meyers, Burke, & Goldin-Meadow, 2018). We set the relative costs by explicitly implementing strategy utility, assigning point values to each communicative method.

If people are motivated to communicate successfully, their choice of referential modality should reflect the tradeoff between the cost of producing the communicative signal with the likelihood that the communication would succeed. We thus predicted that peoples' choice of referential modality would reflect this tradeoff: People should be more likely to use language if they have had more exposures to the novel object's correct label, and they should be more likely to use language as gesture becomes relatively more costly.

Critically, participants were told that they will play this game repeatedly with their partner. In these repeated interactions, participants are then able to learn about an interlocutor and potentially influence their learning. Thus, there is a third type of message: using both gesture and speech within a single trial to effectively teach the listener an object-label mapping. This strategy necessitates making inferences about the listener's knowledge state, so we induced knowledge asymmetries between speaker and listener. To do so, we manipulated how much training they thought their partner had received. Our communicative game was designed to reward in-the-moment communication, and thus teaching required the speaker pay a high cost upfront. However, rational communicators may understand that if one is accounting for future trials, paying the cost upfront to teach the listener allows a speaker to use a less costly message strategy on subsequent trials (namely, speech). Manipulating the listener knowledge and the utility of communicative strategies, we aimed to experimentally determine the circumstances under which richly-structured input emerges, without an explicit pedagogical goal.

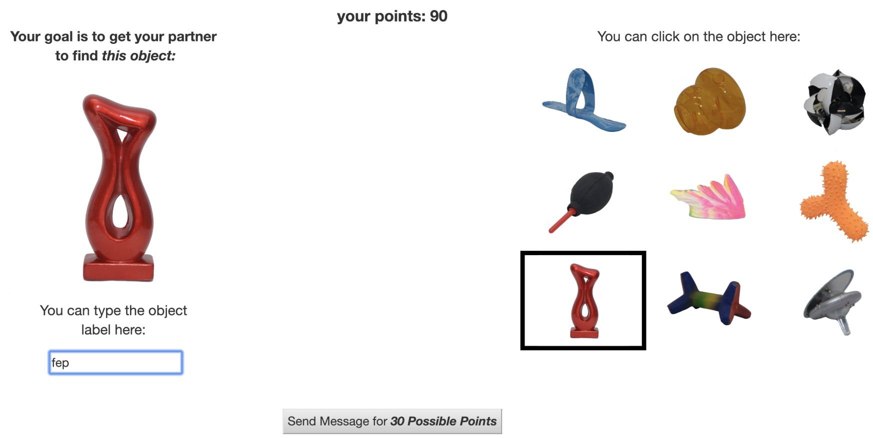

Figure 2. Screenshot showing the participant view during gameplay.

Method

In this experiment, participants were recruited to play our reference game via Amazon Mechanical Turk, an online platform that allows workers to complete surveys and short tasks for payment. In this study, all participants were placed in the role of speaker and listener responses were programmed.

Participants. 480 participants were recruited though Amazon Mechanical Turk and received $1 for their participation. Data from 51 participants were excluded from subsequent analysis for failing the critical manipulation check and a further 28 for producing pseudo-English labels (e.g., "pricklyyone"). The analyses reported here exclude the data from those participants, but all analyses were also conducted without excluding any participants and all patterns hold ($ps < 0.05$).

Design and Procedure. Participants were told they would be introduced to novel object-label pairs and then asked to play a communication game with a partner wherein they would have to refer to a particular target object. Participants were exposed to nine novel objects, each with a randomly assigned pseudo-word label. We manipulated the exposure rate within-subjects: during training participants saw three of the nine object-label mappings four times, two times, or just one time, yielding a total of 21 training trials. Participants were then given a simple recall task to establish their knowledge of the novel lexicon (pretest).

During gameplay, speakers saw the target object in addition to an array of all six objects. Speakers had the option of either directly selecting the target object from the array (deictic gesture)–a higher cost cue but without ambiguity–or typing a label for the object (speech)–a lower cost cue but contingent on the listener's knowledge. After sending the message, speakers are shown which object the listener selected.

We also manipulated participants' expectations about their partner's knowledge to explore the role of knowledge asymmetries. Prior to beginning the game, participants were told how much exposure their partner had to the lexicon. Across 3 between subjects conditions, participants were told that their partner had either no experience with the lexicon, had the same experience as the speaker, or had twice the experience of the speaker. As a manipulation check, participants were then asked to report their partner's level of exposure, and were corrected if they answer incorrectly. Participants were then told that they would be asked to discuss each object three times during the game.

Listeners were programmed with starting knowledge states initialized according to the partner knowledge condition. Listeners with no exposure began the game with knowledge of 0 object-label pairs. Listeners with the same exposure of the speaker began with knowledge of five object-label pairs (3 high frequency, 1 mid frequency, 1 low frequency), based average retention rates found previously. Lastly, the listener with twice as much exposure as the speaker began with knowledge of all nine object-label pairs.

To simulate knowledgeable listener behavior when the speaker typed an object label, the listener was programmed to consult their own knowledge. Messages were evaluate by taking the Levenshtein distance (LD) between the typed label and each possible label in the listener's vocabulary. Listeners then selected the candidate with the smallest edit distance (e.g., if a speaker entered the message "tomi," the programmed listener would select the referent corresponding to "toma," provided toma was found in its vocabulary). If the speaker message had an LD greater than two with each of the words in the listener's vocabulary, the listener selected an unknown object. If the speaker clicked on object (gesture message), the listener was programmed to simply make the same selection.

Speakers could win up to 100 points per trial if the listener correctly selected the target referent based on their message. If the listener failed to identify the target object, the speaker received no points. We manipulated the relative utility of the speech cue between-subjects across two conditions: low relative cost ('Low Relative Cost') and higher relative cost ('Higher Relative Cost'). In the 'Low Relative Cost' condition, speakers received 30 points for gesturing and 100 points for labeling, and thus speech had very little cost relative to gesture and participants should be highly incentivized to speak. In the 'Higher Relative Cost' condition speakers received 50 points for gesturing and 80 points for labeling, and thus gesturing is still costly relative to speech but much less so and participants should be less incentivized to speak.

Participants were told about a third type of possible message using both gesture and speech within a single trial to effectively teach the listener an object-label mapping. This action directly mirrors the multi-modal reference behavior from our corpus data– it presents the listener with an information-rich, potentially pedagogical learning moment. In order to produce this teaching behavior, speakers had to pay the cost of producing both cues (i.e. both gesture and speech). Note that, in all utility conditions, teaching yielded participants 30 points (compared with the much more beneficial strategy of speaking which yielded 100 points or 80 points across our two utility manipulations). Listeners were programmed to integrate new taught words into their knowledge of the lexicon, and check those taught labels on subsequent trials when evaluating speaker messages.

Crossing our 2 between-subjects manipulations yielded 6 conditions (2 utility manipulations: 'Low Relative Cost' and 'Higher Relative Cost'; and 3 levels of partner's exposure: None, Same, Double), with 80 participants in each condition. We expected to find results that mirrored our corpus findings such that rates of teaching would be higher when there was an asymmetry in knowledge where the speaker knew more (None manipulation) compared with when there was equal knowledge (Same manipulation) or when the listener was more familiar with the language (Double manipulation). We expected that participants would also be sensitive to our utility manipulation, such that rates of labeling and teaching would be higher in the 'Low Relative Cost' conditions than the other conditions.

Results

In each trial, participants are able to choose one of 3 communicative strategies: gesture, speech, or teaching. We primarily expect flexible trade-off between the use of each strategy given their relative utilities, participant's knowledge of the lexicon, and the listener's knowledge of the lexicon. To test our predictions about each communicative behavior (gesture, speech, and teaching), we conducted separate logistic mixed effects models for each behavior, reported below. It should be noted that these three behaviors are mutually exhaustive. First, we establish how well participants learned our novel lexicon during training.

Learning. As an initial check of our exposure manipulation, we first conducted a logistic regression predicting accuracy at test from a fixed effect of exposure rate and random intercepts and slopes of exposure rate by participant as well as random intercepts by item. We found a reliable effect of exposure rate, indicating that participants were better able to learn items that appeared more frequently in training ($\beta = 1.08$, $p < .001$, see Figure 3). On average, participants knew at least 6 of the 9 words in the lexicon ($M(sd) = 6.28$ (2.26)). An analysis of variance confirmed that learning did not differ systematically across participants by partner's exposure, utility manipulation, or their interaction ($ps > 0.05$).

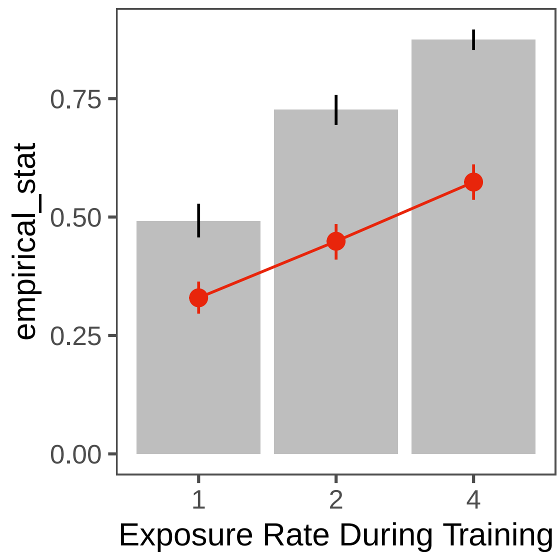

Figure 3. Participants' performance on the baseline recall task for the lexicon, as function of amount of exposure during training (grey bars). The red line shows the propotion of trials in the game in which participants used the learned labels.

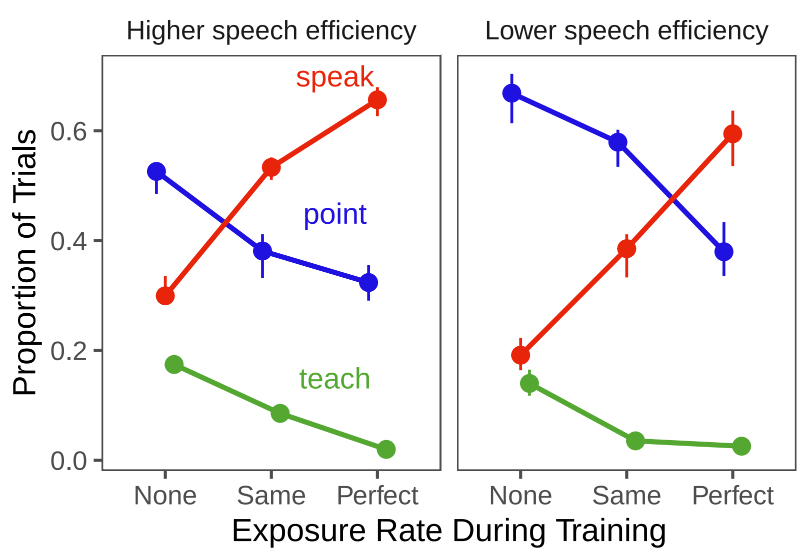

Figure 4. Speaker communicative method choice as a function of exposure and the utility manipulation.

Gesture. When should we expect participants to rely on gesture? Gesturing has the highest utility for words you failed to learn during training, words you think your partner is unlikely to know (i.e., for lower partner knowledge conditions), and when utility scheme is relatively biased toward gesturing (i.e., the 'Higher Relative Cost' condition). To test these predictions, we ran a mixed effects logistic regression to predict whether speakers chose to gesture during a given trial as a function of the target object's exposure rate during training, object instance in the game (first, second, or third), utility manipulation, and partner manipulation. Random effects terms for subject and object were included in the model.

Consistent with our predictions, exposure rate during training was a significant negative predictor of gesturing during the game (see Figure 4), such that participants were less likely to rely on gesture for well trained (and thus well learned) objects (β = -0.50, $p <$ .001). Additionally, participants were significantly more likely to gesture in the Higher Relative Cost condition where gesture is relatively less costly, compared with the Low Relative Cost condition ($\beta = 1.20$, $p < .001$) (see Figure 4). We also found a significant negative effect of partner's knowledge, such that participants used gesture more for partners with less knowledge of the lexicon (β = -0.81, $p < .001$).

Note that these effects cannot be explained by solely speaker knowledge; all patterns above hold when looking *only* at words known by the speaker at pretest (*ps < 0.01*). Further, these patterns directly mirror previous corpus analyses demonstrating adult's use of gesture in naturalistic parental communicative behaviors, and parents likely have lexical knowledge of even even the least frequent referent (see Yurovsky, Meyers, Burke, & Goldin-Meadow, 2018).

Speech. When should we expect participants to use speech? Speech has the highest utility for words you learned during training, words you think your partner is likely to know (i.e., for higher partner knowledge conditions), when utility scheme is relatively biased toward speech (i.e., the 'Low Relative Cost' condition). To test these predictions, we ran a mixed effects logistic regression to predict whether speakers chose to speak during a given trial as a function of the target object's exposure rate during training, object instance in the game (first, second, or third), utility manipulation, and partner manipulation. Random effects terms for subjects and object were included in the model.

Consistent with our predictions, speech seemed to largely tradeoff with gesture. Exposure rate during training was a signficant positive predictor of speaking during the game, such that participants were more likely to utilize speech for well trained (and thus well learned) objects ($\beta = 0.35$, $p < .001$). Additionally, participants were signfinatly less likely to speak in the High Relative Cost condition where speech is relatively more costly, compared with the Low Relative Cost condition (β = -0.87, p .001). We also found a significant positive effect of partner's knowledge, such that particpants used speech more for partners with more knowledge of the lexicon ($\beta = 1.95$, $p < .001$). Unlike for gesture, there is a signifanct effect of object instance in the game (i.e., whether this is the first, second, or third trial with this target object) on the rate of speaking, such that later trials are more likely to elicit speech ($\beta = 0.72$, $p < .001$). This effect of order likely stems from a trade-off with the effects we see in teaching (described below); after a speaker teaches a word on the first or second trial, the utility of speech is much higher on subesequent trials.

Emergence of Teaching. Thus far, we have focused on relatively straightforward scenarios to demonstrate that a pressure to communicate successfully in the moment can lead speakers to trade-off between gesture and speech sensibly. Next, we turn to the emergence of teaching behavior.

When should we expect participants to teach? Teaching has the highest utility for words you learned during training, words you think your partner is unlikely to know (i.e., for lower partner knowledge conditions), when utility scheme is relatively biased toward speech (i.e., the 'Low Relative Cost' condition). To test these predictions, we ran a mixed effects logistic regression to predict whether speakers chose to teach during a given trial as a function of the target object's exposure rate during training, object instance in the game (first, second, or third), utility manipulation, and partner manipulation. Random effects terms for subjects and object were included in the model.

Consistent with our predictions, rates of teaching were higher for better trained words, less knowledgeable partners, and when speech had the highest utility. Exposure rate during training was a signficant positive predictor of teaching during the game, such that participants were more likely to teach for well trained (and thus well learned) objects (β = 0.14, p .001). While costly in the moment, teaching can be a beneifical strategy in our reference game because it subsequently allows for lower cost strategy (i.e. speaking), thus when speaking has a lower cost, participants should be more incentivized to teach. Indeed, participants were significantly less likely to teach in the High Relative Cost condition where speech is relatively more costly, compared with the Low Relative Cost condition (β = -0.96, p .001). We also found a significant negative effect of partner's knowledge, such that participants taught more with partners that had less knowledge of the lexicon (β = -2.23, p < .001). There was also a significant effect of object instance in the game (i.e., whether this is the first, second, or third trial with this target object) on the rate of teaching. The planned utility of teaching comes from using another, cheaper strategy (speech) on later trials, thus the expected utility of teaching should decrease when there are fewer subsequent trials for that object, predicting that teaching rates should drop dramatically across trials for a given object. Participants were significantly less likely to teach on the later appearances of the target object (β = -1.09, p < .001).

Discussion

As predicted, the data from our paradigm corroborate our findings from the corpus analysis, demonstrating that pedagogically supportive behavior emerges despite the initial cost when there is an asymmetry in knowledge and when speech is less costly than other modes of communication. While this paradigm has stripped away much of the interactive environment of the naturalistic corpus data, it provides important proof of concept that the structured and tuned language input we see in those data could arise from a pressure to communicate. The paradigm's clear, quantitative predictions also allow us to build a formal model to predict our empirical results.

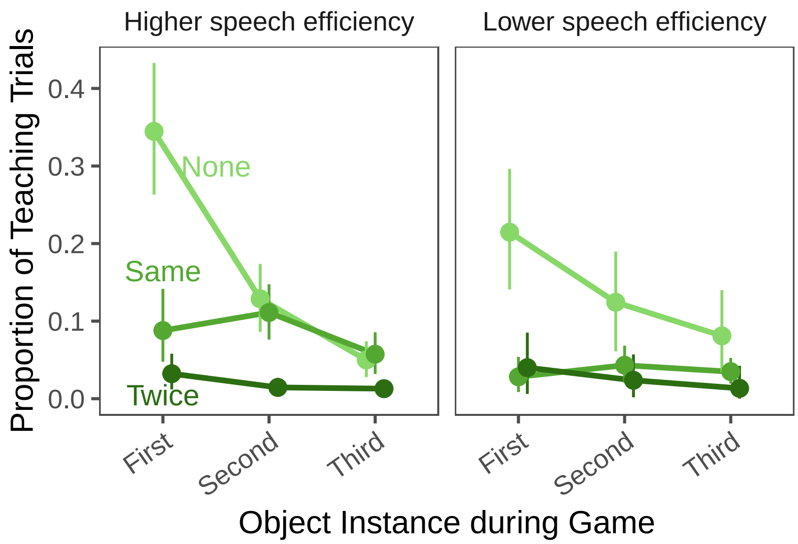

Figure 5. Rates of teaching across the 6 conditions, plotted by how many times an object had been the target object.

The results from this experiment are qualitatively consistent with a model in which participants make their communicative choices to maximize their expected utility from the reference game. We next formalize this model to determine if these results are predicted quantitatively as well.

Model: Communication as planning

In order to model when people should speak, point, or teach, we begin from the problem of what goal people are trying to solve (Marr, 1982). Following a long history of work in philosophy of language, we take the goal of communication to be causing an action in the world by transmitting some piece of information to one's conversational partner (Austin, 1975; e.g., Wittgenstein, 1953). If people are near-optimal communicators, they should choose communicative signals that maximize the probability of being understood while minimizing the cost of producing the signal (Clark, 1996; Grice, 1975). In the special case of reference, solving this problem amounts to producing the least costly signal that correctly specifies one's intended target referent in such a way that one's conversational partner can select it from the set of alternative referents.

Recently, Frank and Goodman (2012) developed the Rational Speech Act framework– a formal instantiation of these ideas. In this model, speakers choose from a set of potential referential expressions in accordance to a utility function that maximizes the probability that a listener will correctly infer their intended meaning while minimizing the number of words produced. This framework has found successful application in a variety of linguistic applications such as scalar implicature, conventional pact formation, and production and interpretation of hyperbole (Goodman & Frank, 2016; see also related work from Franke, 2013). These models leverage recursive reasoning–speakers reasoning about listeners who are reasoning about speakers–in order to capture cases in which the literal meaning and the intended meaning of sentences diverge.

To date, this framework has been applied primarily in cases where both communicative partners share the same linguistic repertoire, and thus communicators know their probability of communicating successfully having chosen a particular signal. This is a reasonable assumption for pairs of adults in contexts with shared common ground. But what if partners do not share the same linguistic repertoire, and in fact do not know the places where their knowledge diverges? In this case, communicators must solve two problems jointly: (1) Figure out what their communicative partner knows, and (2) produce the best communicative signal they can given their estimates of their partner's knowledge. If communicative partners interact repeatedly, these problems become deeply intertwined: Communicators can learn about each-other's knowledge by observing whether their attempts to communicate succeed. For instance, if a communicator produces a word that they believe identifies their intended referent, but their partner fails to select that referent, the communicator can infer that their partner must not share their understanding of that word. They might then choose not to use language to refer to this object in the future, but choose to point to it instead.

Critically, communicators can also change each-other's knowledge. When a communicator both points to an object and produces a linguistic label, they are in effect teaching their partner the word that they use to refer to this object. While this this behavior is costly in the moment, and no more referentially effective than pointing alone, it can lead to more efficient communication in the future–instead of pointing to this referent forever more, communicators can now use the linguistic label they both know they share. This behavior naturally emerges from a conception of communication as planning: Communicators' goal is to choose a communicative signal today that will lead to efficient communication not just in the present moment, but in future communications as well. If they are likely to need to refer to this object frequently, it is worth it to be inefficient in this one exchange in order to be more efficient future. In this way, pedagogically supportive behavior can emerge naturally from a model with no separate pedagogical goal. In the following section, we present a formal instantiation of this intuitive description of communication as planning and show that it accounts for the behavior we observed in our experiments.

Alternatively, pedagogically-supportive input could emerge from an explicit pedagogical goal. Shafto, Goodman, and Griffiths (2014) have developed an framework of rational pedagogy built on the same recursive reasoning principles as in the Rational Speech Act Framework: Teachers aim to teach a concept by choosing a set of examples that would maximize learning for students who reason about the teachers choices as attempting to maximize their learning. Rafferty, Brunskill, Griffiths, and Shafto (2016) et al. expanded this framework to sequential teaching, in which teachers use students in order to infer what they have learned and choose the subsequent example. In this case, teaching can be seen as a kind of planning where teachers should choose a series of examples that will maximize students learning but can change plans if an example they thought would be too hard turns out too easy–or vice-versa. In the case of our reference game, this model is indistinguishable from a communicator who seeks to maximize communicative success but is indifferent to communicative cost. A cost-indifferent model makes poor predictions about parents' behavior in our corpus, and also adults' behavior in our experiments, but we return to it in the subsequent section to consider how differences in parents' goals and differences in children's learning contribute to changes in the rate of language acquisition.

Formal Model

We take as inspiration the idea that communication is a kind of action–e.g., talking is a speech act (Austin, 1975). Consequently, we can understand the choice of *which communicative act* a speaker should take as a question of which act would maximize their utility: achieving successful communication while minimizing their cost (Frank & Goodman, 2012). In this game, speakers can take three actions: talking, pointing, or teaching. The Utilities (U) are given directly by the rules of this game. Because communication is a repeated game, people should take actions that maximize their Expected Utility (EU) not just for the current round, but for all future communicative acts with the same conversational partner. We can think of communication, then as a case of recursive planning. However, people do not have perfect knowledge of each-other's vocabularies (v). Instead, they only have uncertain beliefs (b) about these vocabularies that combine their expectations about what kinds of words people with as much linguistic experience as their partner are likely to know with their observations of their partner's behavior in past communicative interactions. This makes communication a kind of planning under uncertainty well modeled as a Partially Observable Markov Decision Process (POMDP, Kaelbling, Littman, & Cassandra, 1998).

Optimal planning in a Partially Observable Markov Decision Process involves a cycle of three phases: (1) Plan, (2) Act, and (3) Update beliefs. We describe those in turn and finally define how people form initial beliefs about their partner's language now.

Plan. When people plan, they compute the expected utility of each possible action (a) by combining the expected utility of that action now with the Discounted Expected Utility they will get in all future actions. The amount of discounting (γ) reflects how much people care about success now compared to success in the future. Because utilities depend on the communicative partner's vocabulary, people should integrate over all possible vocabularies in proportion to the probability that their belief assigns to that vocabulary ($\mathbb{E}_{v \sim b}$).

$$EU\left[a|\, b\right] = \mathbb{E}_{v \sim b}\left(U(a|v) + \gamma\, \mathbb{E}_{v',o',a'}\ \left(EU\left[a'|b'\right]\right)\right)$$

Act. Next, people take an action as a function of its expected utility. Following other models in the Rational Speech Act framework, we use the Luce Choice Axiom, in which each choice is taken in probability proportional to its exponentiated utility (Frank & Goodman, 2012; Luce, 1959). This choice rule has a single parameter α that controls the noise in this choice–as α approaches 0, choice is random and as α approaches infinity, choice is optimal.

$$P\left(a|b\right) \propto \alpha\, e^{EU[a|b]}$$

Update beliefs. After taking an action, people observe (o) their partner's choice–sometimes they correctly select the intended object, and sometimes they do not. People then update their beliefs about the partner's vocabulary based on this observation. For simplicity, we assume that people think their partner should always select the correct target if they point to it, or if they teach, and similarly should always select the correct target if they produce its label and the label is in their partner's vocabulary. Otherwise, they assume that their partner will select the wrong object. People could of course have more complex inferential rules, e.g., assuming that if their partner does know a word they will choose among the set of objects whose labels they do not know (mutual exclusivity, Markman & Wachtel, 1988). Empirically, however, our simple model appears to accord well with people's behavior.

$$b'(v') \propto P\left(o|v', a\right) \sum_{v \in V} P\left(v'|v, a\right) b\left(v\right)$$

The critical feature of a repeated communication game is that people can change their partner's vocabulary. In teaching, people pay the cost of both talking and pointing together, but can leverage their partner's new knowledge on future trials. Note here that teaching has an upfront cost and the only benefit to be gained comes from using less costly communication modes later. There is no pedagogical goal–the model treats speakers as selfish agents aiming to maximize their own utilities by communicating successfully. We assume for simplicity that teaching is always successful in this very short game, that communicative partners do not forget words once they have learned them, and that no learning happens by inference from mutual exclusivity.

$$P\left(v'|v,a\right) = \begin{cases} 1 & \text{if } v_w \in v\&v' \ \mid \ a = \text{point+talk} \\ 0 & otherwise \end{cases}$$

Initial Beliefs. The final detail is to specify how people estimate their partner's learning rate (p) and initial vocabulary (v). We propose that people begin by estimating their own learning rate by reasoning about the words they learned at the start of the task: Their learning rate (p) is the rate that maximizes the probability of them having learned their initial vocabularies from the trials they observed. People can then expect their partner to have a similar p (per the "like me" hypothesis, Meltzoff, 2005). Having an estimate of their partner's p, they can estimate their vocabulary by simulating their learning from the amount of prior exposure to language their partner had before the game. In our experiments, we explicitly manipulated this expectation by telling participants how much exposure their partner had relative to their own exposure.

Method

We implemented the planning model using the WebPPL– a programming language designed for specifying probabilistic models (Goodman & Stuhlmüller, 2014). To derive predictions from the model, we exposed it to the same trial-by-trial stimuli as the participants in our experiment, and used the probabilistic equations defined above to determine the likelihood of choosing each behavior (e.g., “speak,” “point,” or “teach”) on every trial. Separate predictions were made for each trial for each participant on the basis of all of the information available to each participant at that point in time (e.g., how many words they had learned, their partner’s observed behavior previously, etc).

The model’s behavior is contingent on two parameters–discounting (γ), and it’s rationality (α). In order to determine the values of these parameters that best characterize human participants, we used Bayesian inference to estimate the posterior means of both. Using posterior mean estimates rather than the maximum likelihood estimates naturally penalizes models for their ability to predict patterns of data that were not observed, applying a kind of Bayesian Occam’s razor (MacKay, 1992). Because of we found substantial variability in the best parameter estimates across individual participants, we estimated parameters hierarchically, with group-level hyper-parameters forming the priors for individual participants’ parameters. This hierarchical estimation process achieves the same partial pooling as as subject-level random effects in mixed-effects models, giving estimates of the group-level parameters (Gelman & Hill, 2006). Details of the estimation procedure can be found in the Supplemental Materials.

Model Results

In line with previous work on rational speech act models, and decision making, we expected rationality (α) to be around 1 or 2 (Frank & Goodman, 2012, 2014). We estimated the posterior mean rationality (α) to be 1.33 with 95% credible intervals of [1.24, 1.41]. We did not have strong expectations for the value of the discounting parameter (γ), but estimated it to be 0.42 [0.39, 0.44], suggesting that on average participants weighed the next occurrence of a referent as slightly less than half as important as the current occurrence.

To derive predictions from the model, we ran 100 simulations of the model’s choices participant by participant and trial by trial using our posterior estimates of the hyper-parameters α and γ. Because we did not use our participant-level parameter estimates, this underestimates the correlations between model predictions and empirical data (as it ignores variability across participants). Instead, it reflects the model's best predictions about a the results of a replication of our experiment, where individual participants' parameters will not be known apriori. Figure 6a shows the predictions from the model in analogous format to the empirical data in Figure 4. The model correctly captures the qualitative trends in participants' behavior: It speaks more and points less in the Higher speech efficiency condition. Figure 6b shows the model's predicted teaching behavior in detail in an analogous format to the empirical data in Figure 5. The model again captures the qualitative trends apparent in participants' behavior. The model teaches less knowledgeable partners, especially those who it believes have no language knowledge at all. The model teaches more when speech is relatively more efficient, and thus the future utility of teach a partner is higher. And finally the model teaches most on the first occurrence of each object, and becomes less likely to teach on future occurrences when (1) partners should be more likely to know object labels, and (2) the expected future rewards of teaching are smaller.

To estimate the quantitative fit between model predictions and empirical data, we compute the Pearson correlation between the model's probability of using each action and participants' probability of using that same action as a function of appearance, condition, and partner's exposure. Across experimental manipulations, the model's predictions were highly correlated with participant behavior (r = 0.89 [0.82, 0.94], t(52) = 14.31, $p < .001$; Figure 7).

Discussion

In both qualitative and quantitative analyses, participants' behavior in our communication task was well explained by a model of communication as rational planning under uncertainty. The key intuition formalized by this model is that the value of a

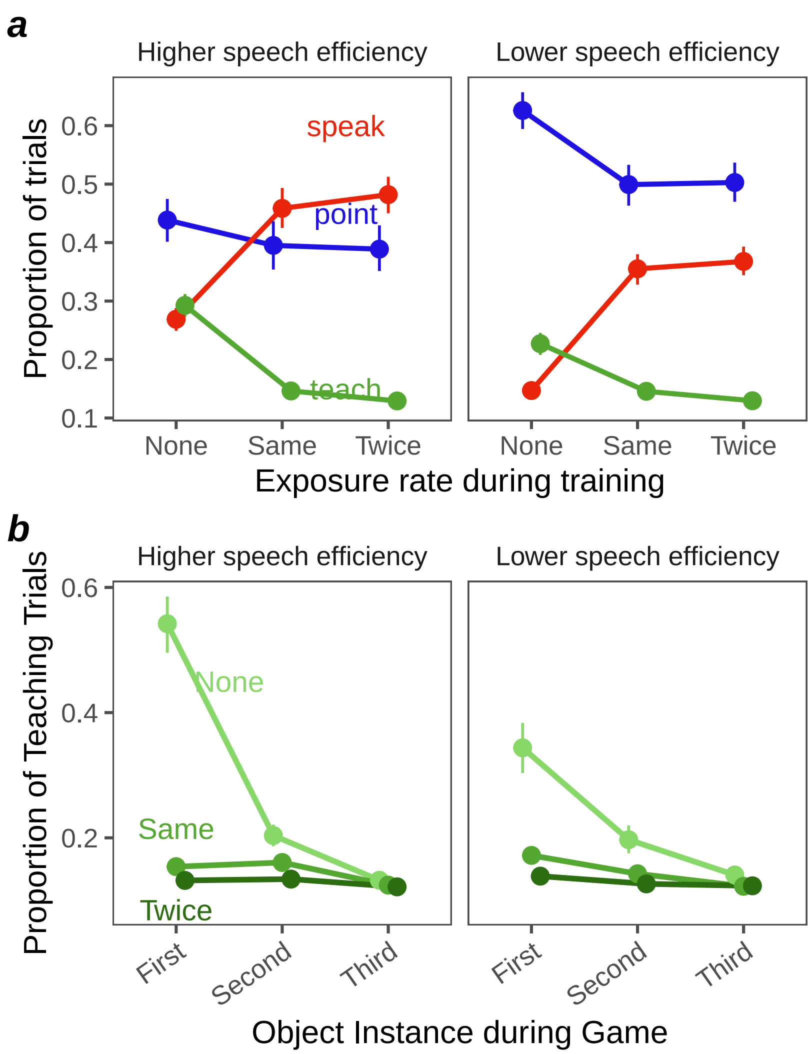

Figure 6. (a) Model prediction choice of communicative method choice as a function of exposure and the utility manipulation. (b) Model predicted probability of teaching by Partner's language knowledge and exposure rate.

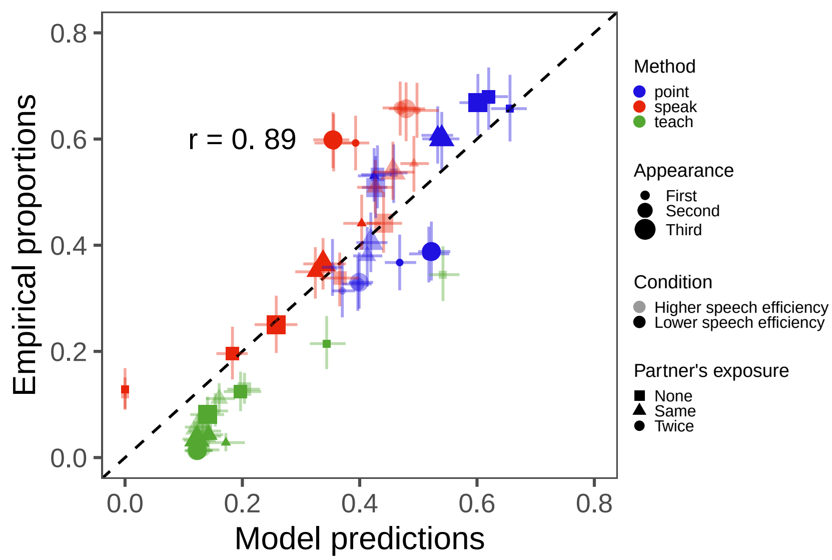

Figure 7. Fit between model predictions and empirical data.

communicative acts derives from (1) the immediate effect on resolving the current communicative need, and (2) the potential benefit of the act for communicative with this conversational partner in the future. Crucially, this model is able to predict a putatively altruistic behavior–teaching by ostenstive labeling–without any altruistic goals at all. Because ostensive labeling can increase the efficiency of future communication, it can be beneficial even under a purely self-interested utility function. What's more, the model correctly predicts the circumstances under which participants will engage in teaching behavior: early interactions with linguistically na:{i}ve communicative partners in circumstances where language is a relatively efficient communicative modality.

Importantly, this model does not rule out the possibility that participants in our experiment–and more broadly people in the real world–may teach because of other more altruistic mechanisms or pressure. The model simply shows that appealing to such mechanisms is not necessary to explain the ostensive labeling observed in parents' conversations with their children, and by extension other behaviors that may at first blush appear to be pedagogically motivated. By the same logic, the model predicts that there should be other pedagogically supportive behaviors in the interactions between parents and their children, and likely in the interactions between any two communicative partners who have some expectation that they will communicate again in the future. This framework thus provides a potential explanation for the occurrence of these behaviors and a framework for understanding their impact on language learning.

Of course, not all potentially pedagogically-supportive behaviors will yield an immediate or future communicative benefit. For instance, correcting children's syntactic errors could be helpful for their language development, but unless it resolves a communicative ambiguity, it will have impact on communicative success. Our framework predicts that these behaviors should be rare, and indeed such behaviors appear to be generally absent in children's input (Marcus, 1993). We return this issue at greater length in the General Discussion. Before turning to that, however, we first consider the consequences of this model of communication for children's language. In the next section, we use simulation methods to ask how much impact parents' communicative motivation may have on their children's learning, and how this impact changes as a function of the complexity of the world and the efficacy of children's learning mechanisms.

Consequences for Learning

In the model and experiments above, we asked whether the pressure to communicate successfully with a linguistically-naive partner would lead to pedagogically supportive input. These results confirmed its sufficiency: As long as linguistic communication is less costly than deictic gesture, speakers should be motivated to teach in order to reduce future communicative costs. Further, the strength of this motivation is modulated by predictable factors (speaker's linguistic knowledge, listener's linguistic knowledge, relative cost of speech and gesture, learning rate, etc.), and the strength of this modulation is well predicted by a rational model of planning under uncertainty about a listener's vocabulary.

In this final section, we take up the consequences of communicatively-motivated linguistic input for a child learning language. To do this, we adapt a framework used by Blythe, Smith, and Smith (2010) to estimate the learning times for an idealized child learning language under a variety of models of both the child and their parent. We derive estimates by simulating exposure to successive communicative events, and measuring the probability that successful learning happens after each event. The question of how different models of the parent impact the learner can then be formalized as a question of how much more quickly learning happens in the context of one parent model than another.

We consider three parents that have three possible goals:

1. *Communication* - The parent's goal in each interaction with their child is to maximize their communicative success while minimizing their communicative cost. This the model described in the Model section above.

2. *Teaching* - The parents' goal in each interaction is to maximize their child's learning (by teaching on every trial). This goal is equivalent to a model in which the goal is to maximize communicative success without minimizing communicative cost.

3. *Talking* - The parents' goal on each interaction is to refer to their intended referent so that an knowledgeable listener would understand them, without accounting for the child's language knowledge. This goal is equivalent to minimizing communicative cost without maximizing communicative success.

Under all of these models, we consider the child's goal to be to learn the correct word-referent mappings that explain the parent's communications. If a communicative event is unambiguous–i.e. the parent is teaching–the child is limited only by their ability to encode this correct mapping. If the event is instead ambiguous, the child needs to both encode potential word-object mappings, and to track their statistical consistency. That is, the child needs to solve the cross-situational learning problem (Yu & Smith, 2007). Across models, we vary both the fidelity of the child's encoding ability, and their capacity for cross-situational learning.

One important point to note is that we are modeling the learning of a single word rather than the entirety of a multi-word lexicon (as in Blythe, Smith, & Smith, 2010). Although learning times for each word could be independent, an important feature of many models of word learning is that they are not (Frank, Goodman, & Tenenbaum, 2009; Yu, 2008; Yurovsky, Fricker, Yu, & Smith, 2014; although c.f. McMurray, 2007). Indeed, positive synergies across words are predicted by the majority of models and the impact of these synergies can be quite large under some assumptions about the frequency with which different words are encountered (Reisenauer, Smith, & Blythe, 2013). We assume independence primarily for pragmatic reasons here–it makes the simulations significantly more tractable (although it is also what our experimental participants appear to assume about learners). Nonetheless, it is an important issue for future consideration. Of course, synergies that support learning under a cross-situational scheme must also support learning from communicators and teachers (Frank, Goodman, & Tenenbaum, 2009; Markman & Wachtel, 1988; Yurovsky, Yu, & Smith, 2013). Thus, the ordering across conditions should remain unchanged. However, the magnitude of the difference across teacher conditions could potentially increase or decrease.

Method

In each fo the sections below, we describe the join models of parents' communication and child's learning that predict learning times under each of the three models of parents' goals.

Teaching. Because the teaching model is indifferent to communicative cost, it engages in ostensive labeling (pointing + speaking) on each communicative event. Consequently, learning on each trial occurs with a probability that depends entirely on the learner's learning rate ($P_k = p$). Because we assume that the learner does not forget, the probability that a learner has failed to successfully learn after n trials is equal to the probability that they have failed to learn on each of n successive independent trials (The probability of zero successes on n trials of a Binomial random variable with parameter p). The probability of learning after n trials is thus:

$$P_k(n) = 1 - (1 - p)^n$$

The expected probability of learning after n trials was thus defined analytically and required no simulation. For comparison to the other models, we computed P_k for values of p that ranged from .1 to 1 in increments of .1.

Communication. To test learner under the communication model, we implemented the same model described in the paper above. However, because our interest was in understanding the relationship between parameter values and learning outcomes rather than inferring the parameters that best describe people's behavior, we made a few simplifying assumptions to allow many runs of the model to complete in a more practical amount of time. First, in the full model above, speakers begin by inferring their own learning parameters (p_s) from their observations of their own learning, and subsequently use their maximum likelihood estimate as a stand-in for their listener's learning parameter (p_l). Because this estimate will converge to the true value in expectation, we omit these steps and simply stipulate that the speaker correctly estimates the listener's learning parameter.

Second, unless the speaker knows a priori how many times they will need to refer to a particular referent, the planning process is an infinite recursion. However, each future step in the plan is less impactful than the previous step (because of exponential discounting). This infinite process is in practice well approximated by a relatively small number of recursive steps. In our explorations we found that predictions made from models which planned over 3 future events were indistinguishable from models that planned over four or more, so we simulated 3 steps of recursion[1]. Finally, to increase the speed of the simulations we re-implemented them in the R programming language. All other aspects of the model were identical.

In our simulations, we varied the child's learning rate (p) from .1 to 1 in steps of .1 as in the Teaching simulation, the parents' future-weighting (λ) from .1 to 1 in steps of .1, the parents' rationality (α) from .5 to 3 in steps of .5, and considered three values each of the cost of speaking ($S = (0, 10, 20)$) and pointing ($P = (50, 60, 70)$). The utility of communicating successfully was always 100.

Talking. The literature on cross-situational learning is rich with a variety of models that could broadly be considered to be "hypothesis testers." In an eliminative hypothesis testing model, the learner begins with all possible mappings between words and objects and prunes potential mappings when they are inconsistent with the data according to some principle. A maximal version of this model relies on the principle that every time a word is heard its referent must be present, and thus prunes any word-object mappings that do not appear on the current trial. This model converges when only one hypothesis remains and is probably the fastest learner when the assumption it relies on is correct (K. Smith, Smith, & Blythe, 2011).

A positive hypothesis tester begins with no hypotheses, and on each trial stores one ore more hypotheses that are consistent with the data, or alternatively strengthens one or more hypotheses that it has already stored that are consistent with the new data. A number of such models have appeared in the literature, with different assumptions about (1) how many hypotheses a learner can store, (2) existing hypotheses are strengthened, (3) how existing hypotheses are pruned, and (4) when the model converges (Siskind, 1996; K. Smith, Smith,

[1] It is an interesting empirical question to determine how the level of depth to which that people plan in this and similar games (see e.g. bounded rationality in Simon, 1991; resource-rationality in Griffiths, Lieder, & Goodman, 2015). This future work is outside the scope of the current project.

& Blythe, 2011; Stevens, Gleitman, Trueswell, & Yang, 2017; Trueswell, Medina, Hafri, & Gleitman, 2013; Yu & Smith, 2012). Finally, Bayesian models have been proposed that leverage some of the strengths of both of these different kinds of model, both increasing their confidence in hypotheses consistent with the data on a given learning event and decreasing their confidence in hypotheses inconsistent with the event (Frank, Goodman, & Tenenbaum, 2009).

Because of its more natural alignment with the learning models we use in the Teaching and Communication simulations, we implemented a positive hypothesis testing model[2]. In this model, learners begin with no hypotheses and add new ones to their store as they encounter data. Upon first encountering a word and a set of objects, the model encodes up to h hypothesized word-object pairs each with probability p. On subsequent trials, the model checks whether any of the existing hypotheses are consistent with the current data, and prunes any that are not. If no current hypotheses are consistent, it adds up to h new hypotheses each with probability p. The model has converged when it has pruned all but the one correct hypothesis for the meaning of a word. This model is most similar to the Propose but Verify model proposed in Trueswell, Medina, Hafri, and Gleitman (2013), with the exception that it allows for multiple hypotheses. Because of the data generating process, storing prior disconfirmed hypotheses (as in Stevens, Gleitman, Trueswell, & Yang, 2017), or incrementing hypotheses consistent with some but not all of the data (as in Yu & Smith, 2012) has no impact on learner and so we do not implement it here. We note also that, as described in Yu and Smith (2012), hypothesis testing models can mimic the behavior of associative learning models given the right parameter settings (Townsend, 1990).

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[2] Our choice to focus on hypothesis testing rather than other learning frameworks is purely a pragmatic choice–the learning parameter p in this models maps cleanly onto the learning parameter in our other models. We encourage other researchers to adapt the code we have provided to estimate the long-term learning for other models.

In contrast to the Teaching and Communication simulations, the behavior of the Talking model depends on which particular non-target objects are present on each naming event. We thus began each simulation by generating a corpus of 100 naming events. On each event, we sampled the correct target as well as (C-1) competitors from a total set of M objects. We then simulated learning over this set of events as described above, and recorded the first trial on which the learner converged (having only the single correct hypothesized mapping between the target word and target object). We repeated this process 1000 times for each simulated combination of $M = (8, 16, 32, 64, 128)$ total objects, $C = (1, 2, 4, 8)$ objects per trial, $h = (1, 2, 3, 4)$ concurrent hypotheses, as the child's learning rate p varied from .1 to 1 in increments of .1.

Results

In order to understand how learning rates vary with model parameters, we first discuss the dependence of each of the three tested models on it's parameters, and then discuss relationships between the models. For clarity of exposition, we analyze the number of events required for 75% of simulated learners to acquire the target word, and plot a representative subset of parameter values.

In addition the results reported here, we have made the full set of simulated results available in an interactive web application at dyurovsky.shinyapps.io/ref-sims. We encourage readers to fully explore the relationships among the models beyond the summary we provide.

Teaching. Because the Teaching model behaves identically on each trial regardless of the learner, the rate of learning under this model depends entirely on the learner's learning rate p. If the learning rate was high (e.g. .8), more than 75% of learners acquired the word after a single learning instance. If the learning rate was medium, closer to the range we estimated for adult learners (.6), more than 75% of learners acquired the word after only 2 instances. Finally, if the learning rate was very low (.2), the same threshold was reached after 7 instances. Thus, the model is predictably sensitive to learning rate, but even very slow learners are expected to acquire words after a small number of communicative events.

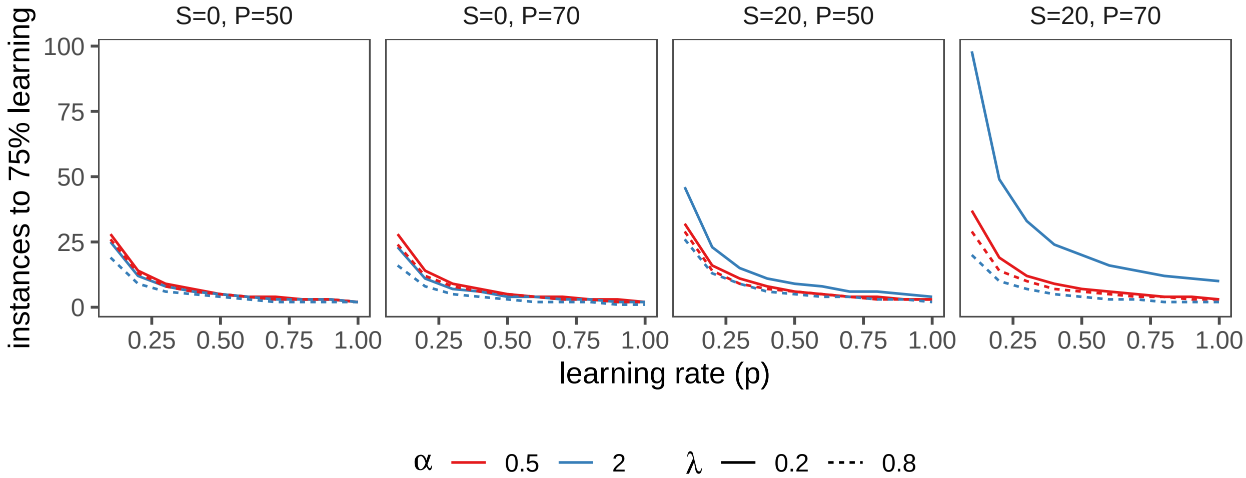

Figure 8. Number of exposures required for 75% of children to learn a word under the Communication model as parameters vary. Color shows rationality (α), Linetype shows future weighting (λ), facets indicate the the cost of speaking (S) and pointing (P). The middle two facets corresponds to Higher Speech Efficiency and Lower Speech efficiency conditions of the experiment.

Communication. The Communication model's behavior depends on parameters of both the child learner and the parent communicator. In general, parameters of both participants had predictable effects on learning: Children learned faster when they had higher learning rates, when parents were more rational, and when parents gave greater weight to the future. Further, the effects of parents' parameters were more pronounced at the lowest learning rates. However, as the cost of speaking increased relative to pointing, the effects of parents' parameters changed. In particular, highly rational parents who heavily discounted the future lead to significantly slower learning. At these parameter settings, the parent becomes very likely to point on any given trial in order to maximize the local utility at the expense of discounted future utility gained from teaching. In addition, as the cost of both modalities increases, the utility of communicating successfully (here defined as 100 points) becomes less motivating. Thus, parents become less discriminating among their communicative choices. Figure 8 shows the number of trials required for 75% of learners to acquire a word as a function of parameters in the Communication model.

Talking. Finally, when parents spoke on each trial and children had to learn from cross-situational statistics, learning was controlled by the the child's learning rate, the number of hypotheses the child could entertain, the number of objects per event, and to a small extent the total vocabulary size. In general, children learned faster when they had a higher learning rate, and could entertain more hypotheses. Learning was also predictably slower when there were more objects on each event and thus ambiguity was higher. Finally, as the total vocabulary size increased, the rate of learning increased slightly, as it does with human cross-situational learners (Yu & Smith, 2007). This counter-intuitive outcome occurs because the rate of spurious co-occurrences, in which the target word consistently co-occurs with an object that is not its referent, decreases as the set of potential foils expands. The the effect of context size (C) and number of hypotheses can be seen along with the learning rates of the other two models in Figure 9.

Comparing the Models

Because the real-world parameters appropriate for each model are difficult to determine, we consider the relationship between the models over the range of their possible parameters. Figure 9 shows the time for 75% of learning to acquire a word in each of the three models. Across all possible child learning rates (p), the Teaching model lead to the fastest learning as expected. We can treat this model as a lower bound how quickly learning could possibly happen.

For the Communication model, we considered the range of all possible rates of learning that could unfold as the parameters of both child and parent varied. The range was substantial. If parents weigh the future near equally to the present, and are highly rational, the child's resultant rate of learning is nearly identical to the rate of learning under the Teaching model: Children required 1.07 times as many learning instances under the Communication model as the Teaching model when averaging over all child learning rates.

In contrast, if the parent weighs the future much less than the present, and is relatively irrational about maximizing utility, the rate of learning can be quite slow–in the worst case requiring children to have 1.07 as many learning instances as under the Teaching model. Despite this bad worst case scenario, if parents' parameters are close to the ones we estimated in our experiment, Communication would require only 1.75 as many instances as Teaching if speech is high efficiency relative to pointing, and 3.12 as many instances if speech is lower efficiency.

For the Talking model, we also observed a wide range of learning times as a function of both the ambiguity of the learning environment and the number of simultaneous hypotheses that the child can maintain. When the environment was unambiguous–only 2 objects were present at a time–and the child could encode both, learning under Talking took only 2.03 times as many instances as Teaching. In contrast, if ambiguity was high, and learners could only track a single hypothesis, learning was significantly slower under Talking than Teaching, (requiring 10.05 times as many instances).

Comparing Communication and Talking to each-other, we find that that Talking can lead to faster learning under some parameter settings. In particular, if events are low in ambiguity, or children can maintain a very large number of hypotheses about the meaning of a word relative the number of objects in each event, children can learn rapidly even if parents are just Talking. This learning can be faster than simpler child models learnign from highly myopic or relatively irrational parents Communicating, especially if speech is high-cost. At medium levels of ambiguity, Communication and Talking are similar and their ordering depends on other parameters. At high levels of ambiguity Communication is the clear winner.

Together, these results suggest that if the set of possible candidate referents is small, even simple cross-situational learners can cope just fine even if their parent is just Talking; they learn roughly two to three times more slowly than if their parent was Teaching them.

However, if the set of possible referents is four, or, eight, or even more on average, cross-situational learners need to have very high bandwidth or their rates of learning will be an order of magnitude slower than if their parent were Teaching them. In these cases, even the simplest learner–who can encode a single hypothesis about the meaning of a word and gets no information from co-occurrence statistics–can learn quite rapidly if they are learning from a parent that Communicates with them.

Discussion

Most of the language that children hear from their parents is unlikely to be designed to teach them language. However, the language that parents direct to them *is* designed to communicate successfully. Here we consider the learning consequences of these differences in design. How different are the learning consequences of language designed for teaching, language designed for communication, and ambient language not designed for the child at all?

If input is not designed for teaching, the rate of learning depends entirely on what the learner brings to the table. In line with prior analyses of cross-situational learning, we find that learning can be quite rapid if environments are low in ambiguity or the learner has very high bandwidth for storing candidate hypotheses Yu & Smith (2012). However, the child's environment is neither guaranteed to be unambiguous nor are young children likely to have high bandwidth for statistical information Woodard, Gleitman, & Trueswell (2016). In fact, when the set of candidate referents is small, it is quite likely to be small in part because parents have designed the context to support communication (Tomasello & Farrar, 1986).

However, the rate of learning from communication is almost as fast as learning from teaching under many possible parameter settings we explored. On average, across all possible parameter values, learning from communication is only 2.5 times slower than learning from teaching. Further, in this model, the learner gets no information co-occurrence statistics at all. Combining learning from communication with low-bandwidth

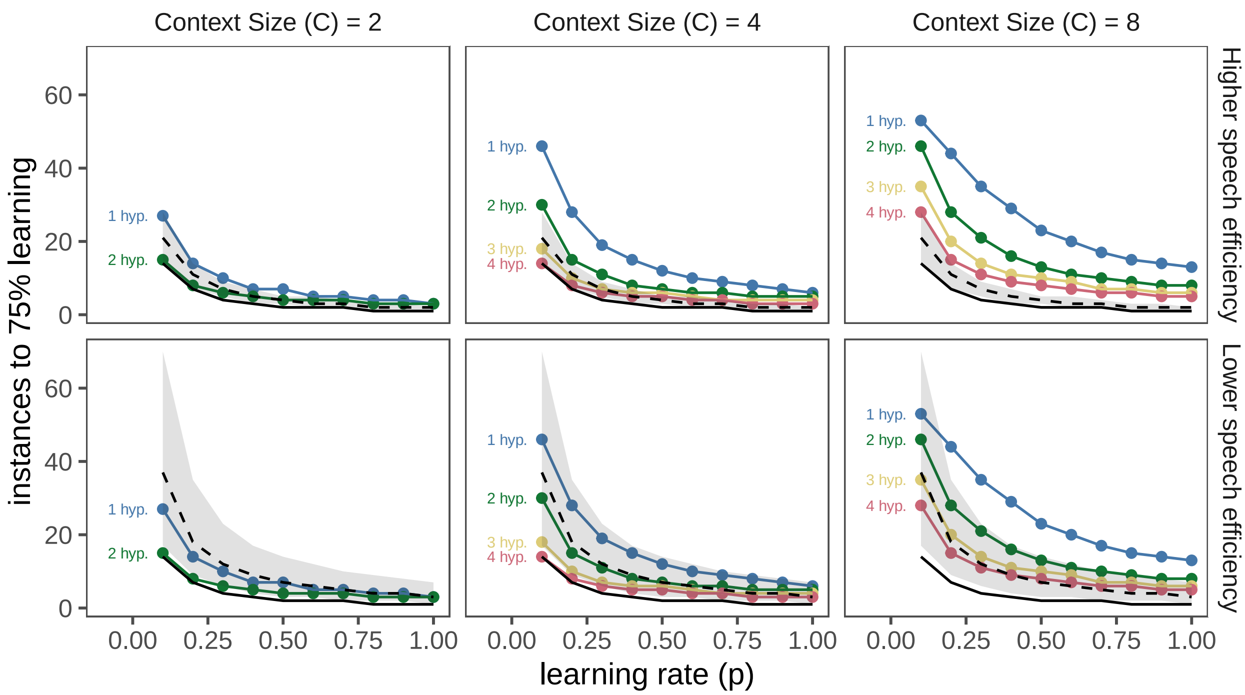

Figure 9. Comparing the number of exposures required for 75% of children to learn a word under all three models as parameters vary. Columns show variation in context size (C), a parameter of the Talking model. Rows show the two variations in the costs of Speech and Pointing for the Communication model used in our experiments. In each facet, the solid black line shows learning under the Teaching model, the light gray region shows an envelope of learning times corresponding to all variations in Communication model parameters, and the black dotted line shows learning time under the Communication model with parameters equal to the empirical estimates from experiments. Colored lines show learning times under the Teaching model with varying numbers of hypotheses. Because there was little effect of total number of objects (M) in the Talking model, all panels show results for 128 objects. Because only Communication model parameters vary across rows, the results of the Talking model are identical in each row.

cross-situational learning could bring the expected rate of learning down to very close to learning from teaching (MacDonald, Yurovsky, & Frank, 2017). We thus might make significanft progress on understanding how children learn language so quickly not just by studying children, but also by understanding how parents design the language they produce in order to support successful communication (Leung, Tunkel, & Yurovsky, in press).

General Discussion

Across naturalistic corpus data, experimental data, and model predictions and simulation, we see evidence that pressure to communicate successfully with a linguistically immature partner could fundamentally structure parent production. In our experiment, we showed that people tune their communicative choices to varying cost and reward structures, and also critically to their partner's linguistic knowledge– providing richer cues when partners are unlikely to know the language and many more rounds remain. These data are consistent with the patterns shown in our corpus analysis of parent referential communication and demonstrate that such pedagogically supportive input could arise from a motivation to maximize communicative success while minimizing communicative cost—no additional motivation to teach is necessary. In simulation, we demonstrate that simple learners whose caregivers want to communicate with them out-learn more powerful statistical learners whose caregivers do not have a communicative goal.

Accounts of language learning often aim to explain its striking speed in light of the sheer complexity of the language learning problem itself. Many such accounts argue that simple (associative) learning mechanisms alone seem insufficient to explain the rapid growth of language skills and appeal instead to additional explanatory factors, such as the so-called language acquisition device, working memory limitations, word learning biases, etc. (e.g., Chomsky, 1965; Goldowsky & Newport, 1993; Markman, 1990). While some have argued for the simplifying role of language distributions (e.g., McMurray, 2007), these accounts largely focus on learner-internal explanations. For example, Elman (1993) simulates language learning under two possible explanations to intractability of the language learning problem: one environmental, and one internal. He first demonstrates that learning is significantly improved if the language input data is given incrementally, rather than all-at-once. He then demonstrates that similar benefits can arise from learning under limited working memory, consistent with the “less-is-more” proposal (Elman, 1993; Goldowsky & Newport, 1993). Elman dismisses the first account arguing that ordered input is implausible, while shifts in cognitive maturation are well-documented in the learner; however, our account’s emphasis on changing calibration to such learning mechanisms suggests the role of ordered or incremental input from the environment may be crucial.

This account is consonant with work in other areas of development, such as recent demonstrations that the infant’s visual learning environment has surprising consistency and incrementality, which could be a powerful tool for visual learning. Notably, research using head mounted cameras has found that infant’s visual perspective privileges certain scenes and that these scenes change across development. In early infancy, the child’s egocentric visual environment is dominated by faces, but shifts across infancy to become more hand and hand-object oriented in later infancy (Fausey, Jayaraman, & Smith, 2016). This observed shift in environmental statistics mirrors learning problems solved by infants at those ages, namely face recognition and object-related goal attribution respectively (Fausey, Jayaraman, & Smith, 2016). These changing environmental statistics have clear implications for learning and demonstrate that the environment itself is a key element to be captured by formal efforts to evaluate statistical learning (L. B. Smith, Jayaraman, Clerkin, & Yu, 2018). Frameworks of visual learning must incorporate both the relevant learning abilities and this motivated, contingent structure in the environment.

By analogy, the work we have presented here aims to draw a similar argument for the language environment, which is also demonstrably beneficial for learning and changes across development. In the case of language, the contingencies between learner and environment are even clearer than visual learning. Functional pressures to communicate and be understood make successful caregiver speech highly dependent on the learner. Any structure in the language environment that is continually suited to changing learning mechanisms must come in large part from caregivers themselves. Thus, a comprehensive account of language learning that can successfully grapple with the infant curriculum must explain parent production as well as learning itself. In this work, we have taken first steps toward providing such an account.

Explaining parental modification is a necessary condition for building a complete theory of language learning, but modification is certainly not a sufficient condition for language learning. No matter how calibrated the language input, non-human primates are unable to acquire language. Indeed, parental modification need not even be a necessary condition for language learning. Young children are able to learn novel words from (unmodified) overheard speech between adults Shneidman & Goldin-Meadow (2012). Our argument is that the rate and ultimate attainment of language learners will vary substantially as a function of parental modification, and that describing the cause of this variability is a necessary feature of models of language learning.

Generalizability and Limitations. Our account aims to think about parent production and child learning in the same system, putting these processes into explicit dialogue. While we have focused on ostensive labeling as a case-study phenomenon, our account should reasonably extend to the changing structure found in other aspects of child-directed speech. Some such phenomena will be easily accounted for; aspects of language that shape communicative efficiency should shift in predictable patterns across development.

While these language phenomena can be captured by our proposed framework, incorporating them will likely require altering aspects of our account and decisions about which alterations are most appropriate. For example, the exaggerated pitch contours seen in infant-directed speech could be explained by our account if we expand the definition of communicative success to include the goal of maintaining attention. Alternatively, one could likely accomplish the same goal by altering the cost structure to penalize loss of engagement. Thus, while this account should generalize to other modifications found in child-directed speech, such generalizations will likely require alterations to the extant structure of the framework.

Of course, not all aspects of language should be calibrated to the child's language development. Our account also provides an initial framework for explaining aspects of communication that would not be modified in child-directed speech: aspects of communication that minimally effect communicative efficiency. In other words, communication goals and learning goals are not always aligned. For example, young children sometimes overregularize past and plural forms, producing incorrect forms such as "runned" or "foots" (rather than the irregular verb "ran" or irregular plural "feet," Marcus et al., 1992). Mastering the proper tense endings (i.e. the learning goal) might be aided by feedback from parent; however, adults rarely provide explicit corrective feedback for these errors (Marcus, 1993). This is perhaps because incorrect grammatical forms nonetheless successfully communicate their intended meaning, and thus do not prevent the successful completion of the communicative goal of language (Chouinard & Clark, 2003). The degree of alignment between communication and learning goals should predict the extent to which a linguistic phenomenon is modified in child-directed speech. Fully establishing the degree to which modification is expected for a given language phenomena will likely require working through a number of limitations in the generalizability of the framework as it stands.

Some aspects of parent production are likely entirely unrepresented in our framework, such as aspects of production driven by speaker-side constraints. Furthermore, our account is formulated primarily around concrete noun learning and future work must address its viability in other language learning problems. We chose to focus on ostensive labeling as a case-study phenomenon because it is an undeniably information-rich cue for young language learners, however ostensive labeling varies substantially across socio-economic, linguistic, and cultural groups (Hoff, 2003). This is to be expected to the extent that parent-child interaction is driven by different goals (or goals given different weights) across these populations–variability in goals could give rise to variability in the degree of modification. Nonetheless, the generalizability of our account across populations remains unknown. Indeed, child-directed speech itself varies cross-linguistically, both in its features (Fernald et al., 1989) and quantity (e.g., Shneidman & Goldin-Meadow, 2012). There is some evidence that child-idrected speech predicts learning even in cultures where it is qualitatively different and less prevalent than in American samples (Shneidman & Goldin-Meadow, 2012). Future work is needed to establish the generalizability of our account beyond the western samples studied here.

We see this account as building on established, crucial statistical learning skills–distributional information writ large and (unmodified) language data from overheard speech are undoubtedly helpful for some learning problems (e.g., phoneme learning). There is likely large variability in the extent to which statistical learning skills drive the learning for a given learning problem. The current framework is limited by its inability to account for such differences across learning problems, which could derive from domain or cultural differences. Understanding generalizability of this sort and the limits of statistical learning will likely require a full account spanning both parent production and child learning. A full account that explains variability in modification across aspects of language will rely on a fully specified model of optimal communication. Such a model will allow us to determine both which structures are predictably unmodified, and which structures must be modified for other reasons. Nonetheless, this work is an important first step in validating the hypothesis that language input that is structured to support language learning could arise from a single unifying goal: The desire to communicate effectively.

Conclusion

Building on early functional account of language learning (e.g., Brown, 1977), our account emphasizes the importance of communicative success in shaping language input and language learning. We have developed an initial formal framework for jointly considering parent productions and child language learning within the same system. We showed that such an account helps to explain parents' naturalistic communicative behavior and participant behavior in an iterated reference game. Formalized model predictions explain these behaviors without an explicit teaching goal, and show the power of communicative partners in supporting learning in simulations. In sum, this work demonstrates that the pressure to communicate successfully may help create a learning environment that fosters language learning.

Acknowledgement

The authors are grateful to XX and YY for their thoughtful feedback on this manuscript. The authors are grateful to Madeline Meyers for her work coding referential communication in the corpus data. This research was supported by a James S McDonnell Foundation Scholars Award to DY.

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