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Pre-Session Exam 2018

Problem 1

1.1 $\frac{x^{n+2}}{x^{n-2}} = x^{n+2-n+2} = \underline{x^4}$

1.2 $x^{-1} \cdot 8 = 2$

$\frac{8}{x} = 2$

$2x = 8$

$\underline{x = 4}$

1.3 $a = 5 \quad b = 10$

$(5^{10})^0 = 5^0 = \underline{1}$

1.4 $\frac{\sqrt{4x}}{\sqrt{x}} = \sqrt{\frac{4\cancel{x}}{\cancel{x}}} = \sqrt{\frac{4}{1}} = \sqrt{4} = \underline{2}$

1.5 $x^2 + (x+1)^2 = (x+2)^2$

$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$

$x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0$

$\underline{x = 3}, \quad \underline{x = -1}$

1.6 $2^x > 1024$

$x > \log_2 1024$

$\underline{x > 10}$

Problem 2

2.1 C 7

$\overset{+100}{\curvearrowright}$ 0 32 $\overset{+180}{\curvearrowleft}$

100 212

$32 + 1{,}8x = x$

$32 = x - 1{,}8x \Rightarrow -0{,}8x = 32$

$y = x = \underline{-40}$

2.2 $f(x) = 5x + 4$

$y = ?$

$f(3) = 5 \cdot 3 + 4 = 19$

$y = \underline{19}$

2.3 $x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x_1 = 1$

$\underline{x_2 = 3}$

2.4 $10 \cdot 1{,}02^{90} = \underline{59{,}43}$

2.5 $e^{\ln 5} = \underline{5}$

Problem 4

4.1 $A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$

4.2 $B = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 2 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$

$B \cdot A = \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$

4.3 $A = \begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$

4.4 Det = ?

$A = \begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix} \Rightarrow \text{Det } A = 8 - 18 = \underline{-10}$

Problem 5

5.1 $\Omega = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31,$
$32, 33, 34, 35, 36, \cancel{41}, 42, 43, 44, 45, 46, 51,$
$52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$

Sample Size = 36

5.2 Uses drugs = 1% = 0.01
Drug free = 99% = 0.99
True positive = 99% = 0.99
False negative = 1% = 0.01
True negative = 99.5% = 0.995
False positive = 0.5% = 0.005

Positive result = $(T.P)(U.D) + (F.P)(D.F) = 0.99 + 0.01 + 0.005 \cdot 0.99 =$
$= 0.01485$

Positive res. = 0.01485 = 1.485%

5.3 $P(U.D. \mid P.R.) = \dfrac{U.D. \cdot T.P.}{P.R.} = \dfrac{0.98 \cdot 0.01}{0.01485} = 0.66 = 66\%$

Problem 3

3.1 $\sum_{i=1}^{\infty} \dfrac{12}{6^i} = 3$

$\sum_{i=1}^{\infty} \dfrac{a}{1-r} \Rightarrow a = \dfrac{12}{6} = 2 \qquad r = \dfrac{12}{36} = \dfrac{1}{3} \qquad |r| < 1$

$= \dfrac{2}{1-\frac{1}{3}} = \dfrac{2}{\frac{2}{3}} = \dfrac{2 \cdot 3}{2} = 3$

3.2 $\lim_{x \to \infty} \dfrac{6^{1-x}}{x} = \dfrac{1}{1} = 1$

3.3 $f(x) = x^5 - 8$ slope at $x-3$

$f'(x) = 5x^4$

$x = -3$

$f'(-3) = 5(-3)^4 = 5 \cdot 81 = 405$

3.4 $\dfrac{d}{dx} \dfrac{x^3 + 2x - 1}{x-2} = \dfrac{(3x^2+2)(x-2) - (x^3+2x-1)}{(x-2)^2} = \dfrac{2x^3 - 6x^2 - 3}{(x-2)^2}$

3.5 $\dfrac{d^2}{dx^2} 4x^4 + 4x^2 \Rightarrow \dfrac{d}{dx} 16x^3 + 8x \Rightarrow 48x^2 + 8$

3.6 $\dfrac{\log(x)}{e^x} \dfrac{d}{dx} = (e^{-x} \log(x)) \dfrac{d}{dx}$

$\dfrac{d}{dx}(u \cdot v) = v\dfrac{du}{dx} + u\dfrac{dv}{dx} \qquad u = e^{-x} \quad v = \log x$

$\log(x) \cdot \left(\dfrac{d}{dx}(e^{-x})\right) + e^{-x}\left(\dfrac{d}{dx} \cdot \log(x)\right) =$
$= -e^{-x}\log(x) + \dfrac{e^{-x}}{x} = e^{-x}\left(\dfrac{1}{x} - \log(x)\right)$

3.7 $3x^2-5x+2$

$$(3x-2)(x-1)=0$$

$$x=\frac{2}{3} \quad x=1$$

$$f'(x)=6x-5$$

$$f''(x)=6$$

$$6x-5=0$$

$$\underline{x=\frac{5}{6}}$$

x	$-\infty$	$x<\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}<x<\frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6}<x<1$	1	∞
$f(x)$	∞ +	+	0	−	−	−	0	+
$f'(x)$	−	−	−	−	0	+	+	+
SHAPE	↘	↘	↘	↘	min	↗	↗	↗
$f''(x)$	+	+	+	+	+	+	+	+
SHAPE	$\cup$	$\cup$	$\cup$	$\cup$	$\cup$	$\cup$	$\cup$	$\cup$

3.8 $f(x,y)=x^2+y^3$

$$x=2 \quad y=3$$

$$f(2,3)=2^2+3^3=4+27=\underline{31}$$

3.9 $f(x,y)=\ln(x-y)$

$$(x,y)\in\mathbb{R}^2: x>y$$

$\ln(-R)$ - doesn't exist

3.10 Partial derivative

$$\frac{\partial}{\partial x} \quad x^5+xy^3$$

$$\frac{\partial}{\partial x}(x^5+xy^3)=5x^4+y^3$$

$$\frac{\partial}{\partial y}(x^5+xy)=3xy^2$$

3.11 $f(x,y)=x^2y^2+10$

$$\frac{\partial}{\partial x}=2xy^2 \qquad 2xy^2=0$$
$$x=0$$

$$\frac{\partial}{\partial y}=2x^2y \qquad 2x^2y=0$$
$$y=0$$

$f(x,0)=10$ - local min.

$f(0,y)=10$ - local min.

Point $(0,0)$ - global min.

3.12 Lagrange max x^2y^2

$x^2y^2-\lambda(x+y-10)$ s.t. $x+y=10$

$$\frac{\partial}{\partial x}=2\cancel{x}y^2-\lambda=0$$

$$\frac{\partial}{\partial y}=2\cancel{x}^2y-\lambda=0$$

$$\frac{\partial}{\partial \lambda}=-x-y+10=x+y-10=0$$

$$x+y=10$$

$$2xy^2-\lambda=0$$
$$2x^2y-\lambda=0$$
$$x+y=10$$
$$x=10-y$$

$$2xy^2=2x^2y \quad /:2$$
$$xy^2=x^2y$$
$$y^2=\cancel{x^2}y$$
$$\cancel{x}$$
$$y^2=xy$$
$$x=y$$

$$x=10-x$$
$$2x=10$$
$$\underline{x=5}$$
$$\underline{y=5}$$