Cognitive Uncertainty*

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Abstract

This paper identifies and sets out to explain a set of striking similarities in boundedly rational behavior across four domains: choice under risk, choice under ambiguity, belief updating, and survey expectations about economic variables. In each of these domains, behavior in experiments and surveys tends to be insensitive to variation in objective probabilities, generating a characteristic response pattern of compression towards 50:50 as in the classical probability weighting function. Building on existing models of cognitive noise, we formally propose that the unifying underlying mechanism is *cognitive uncertainty*: people's subjective uncertainty about what the rational action is, which induces them to shrink probabilities towards a mental default. We introduce an experimental measurement of cognitive uncertainty and show that the responses of individuals with higher cognitive uncertainty exhibit stronger compression of objective probabilities in choice under uncertainty, belief updating, and survey expectations. Our framework makes additional predictions that we test using exogenous manipulations of both cognitive uncertainty and the location of the mental default. The results provide causal evidence for the role of cognitive uncertainty in belief formation and choice, which we quantify through structural estimations.

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1 Introduction

In many contexts of economic interest, behaving optimally is difficult. For instance, a long literature in behavioral economics suggests that decision-making under uncertainty – forming and updating beliefs, and making choices among risky options – is prone to errors. Our paper builds on the premise that people's awareness of their own potential errors induces *cognitive uncertainty*: subjective uncertainty about what the optimal action or solution to a decision problem is. For example, people may think that they do not know how exactly to compute rational beliefs in light of new information, or how exactly to determine their certainty equivalent of a lottery. By formalizing and introducing an experimental measurement of cognitive uncertainty, our paper brings together and seeks to explain a set of striking similarities in previously unconnected anomalies across four economic decision domains: choice under risk, choice under ambiguity, belief updating, and survey expectations about economic variables.

Figure 1 depicts the set of behavioral anomalies that motivate our investigation. All four functions share in common a characteristic inverse S-shape. First, panel A depicts the well-known probability-weighting function in choice under risk that goes back to Kahneman and Tversky's (1979) prospect theory. It illustrates how experimental subjects implicitly treat objective probabilities in choosing between different monetary gambles. Relative to an expected utility maximizer, people behave as if small probabilities were larger than they really are, and high probabilities as if they were smaller than they really are, leading to a compression effect. Second, depicted in panel B is an "ambiguity-weighting function" adapted from Li et al. (2019) that depicts the emerging consensus that, in choices between monetary gambles over gains, people are ambiguity-averse for likely events, yet ambiguity-seeking for unlikely events. This reflects a compression effect that is labeled "a-insensitivity" in the literature (Dimmock et al., 2015; Trautmann and Van De Kuilen, 2015). Third, in panel C, we illustrate a less well-known stylized fact, which is an inverse S-shaped relationship between participants' posterior beliefs and the Bayesian posterior in balls-and-urns belief updating tasks of the type recently reviewed by Benjamin (2019). People learn from information in a way that makes their posterior beliefs too high if the normatively correct belief is below 50%, and too low otherwise (Ambuehl and Li, 2018). Finally, panel D of Figure 1 shows the relationship between objectively correct probabilities and the level of respondents' probabilistic estimates in subjective expectations surveys about, e.g., stock market returns, inflation rates, the shape of the income distribution. Here, again, people's beliefs are compressed towards 50:50 relative to the correct answers (Fischhoff and Bruine De Bruin, 1999).

Why do these four functions, drawn from different decision contexts and experimental paradigms, look so strikingly similar? The objective of this paper is to propose

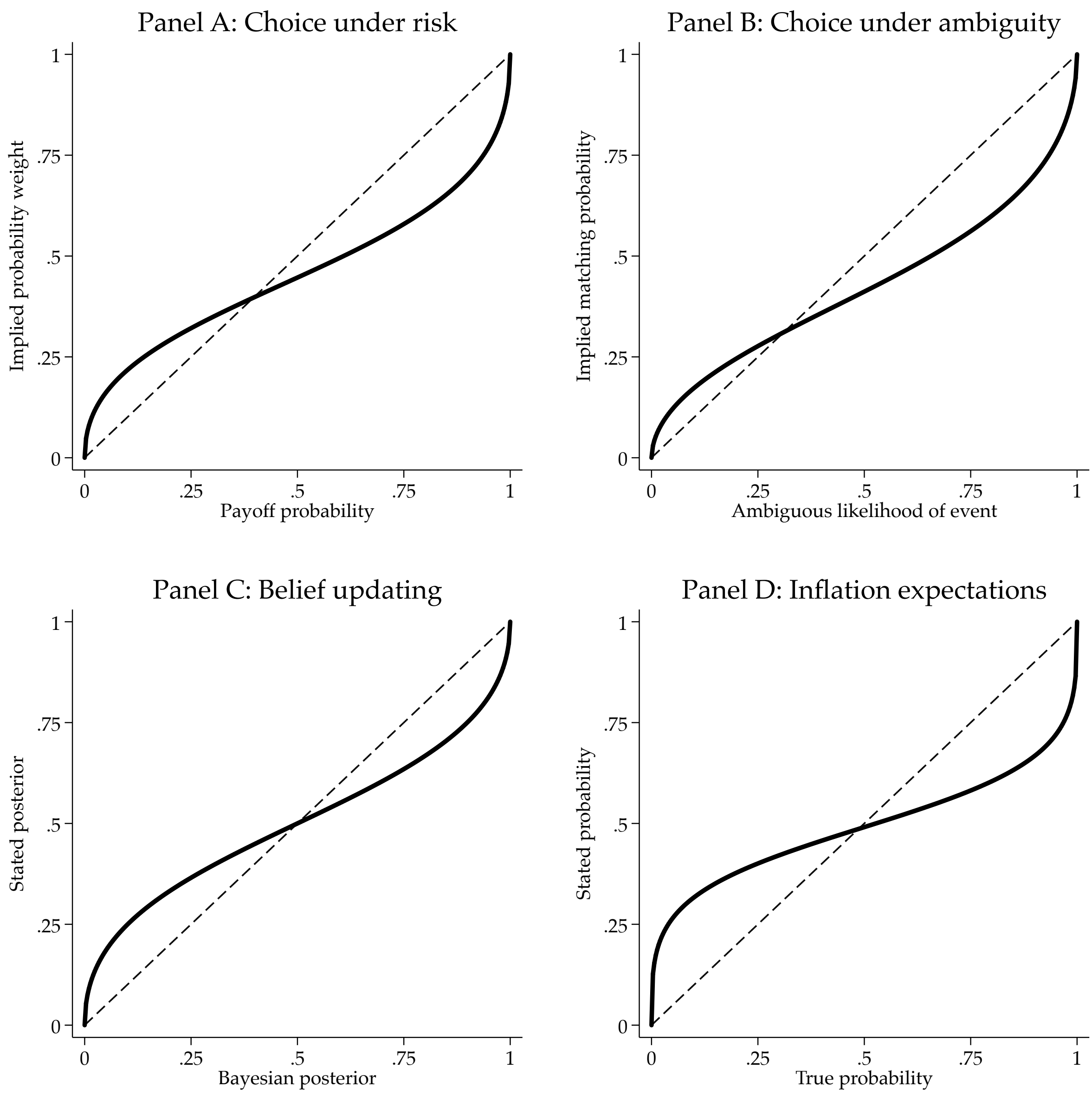


Figure 1: "Weighting functions" in choices and beliefs. Panel A depicts a probability weighting function in choice under risk, estimated from the data described in Section 3. Panel B illustrates an "ambiguity weighting function," where the x-axis represents the likelihood of an ambiguous event and the y-axis the matching probability (adapted from Li et al., 2019). Panel C visualizes the relationship between Bayesian posteriors and stated beliefs in binary-state balls-and-urns belief updating experiments (constructed from the data described in Section 4). Finally, panel D depicts the relationship between objective probabilities and stated subjective probabilities in a survey on inflation expectations, described in Section 5.

cognitive uncertainty as a new lens through which these anomalies can be interpreted and unified. Our experimental analysis is based on a theoretical framework that builds on the mathematical machinery of noisy Bayesian cognition models, in particular Gabaix (2019) and Khaw et al. (2017). We take a broad interpretation of these models as capturing (i) noise that primarily results from high-level reasoning in optimization rather than perceptual imperfections alone and (ii) measurable awareness of the resulting subjective uncertainty about the optimal action.

In the model, people exhibit cognitive noise in processing probabilistic information into an optimal response or action. Similarly to standard Bayesian signal extraction models, this cognitive noise induces people to shrink objective probabilities towards a prior, or mental default. We assume that in unfamiliar environments, this default is influenced by an ignorance prior. Given this setup, we formally define and characterize an empirically measurable notion of cognitive uncertainty as subjective uncertainty about the rational action. Building on insights from the noisy cognition literature, we show that our formal notion of cognitive uncertainty governs and endogenizes the sensitivity parameter in the familiar two-parameter version of the probability weighting function proposed by Gonzalez and Wu (1999). However, our framework clarifies that we expect this two-parameter weighting function to apply not only to choice under uncertainty but also to belief formation. Moreover, endogenizing these weighting function parameters clarifies that the precise shape of these functions will not be constant, even within a given decision domain. Instead, the curvature and elevation should depend on the magnitude of cognitive noise and the location of the mental default.

This theoretical framework makes three types of new predictions: (a) correlationally, individuals with higher cognitive uncertainty exhibit response functions that are more compressed towards 50:50 and hence more insensitive; (b) an exogenous increase in cognitive uncertainty generates more compressed response functions; and (c) an exogenous decrease in the location of the mental default shifts the entire response function downwards. All of these predictions have both a reduced-form and a structural interpretation in terms of Gonzalez and Wu's weighting function.

To test these predictions, we implement a series of pre-registered experiments with a total of $N = 2,000$ participants on Amazon Mechanical Turk (AMT). Like the motivating examples, our experiments cover the domains of choice under uncertainty, balls-and-urns belief updating tasks, and survey expectations about economic variables. Throughout the paper, our analysis rests on a new experimental paradigm to measure cognitive uncertainty, which is readily portable across decision domains.

In choice under risk, each participant completes two decision screens per lottery. On the first screen, we elicit subjects' certainty equivalents for two-outcome gambles such as "Get \$20 with probability 75%; get \$0 with probability 25%" in a standard price list format. On the next screen, participants are asked how certain they are that to them the lottery is worth exactly the same as the switching interval that they stated on the previous screen. To answer this question, participants use a slider to calibrate the statement "I am certain that the lottery is worth betwen x and y to me." If a subject moves the slider to the very right, x and y collapse to their own switching interval in the price list. The further a subject moves the slider to the left, the wider the range of cognitive uncertainty becomes. Thus, our measure of cognitive uncertainty (i) directly reflects subjects' own assessment of uncertainty; (ii) is quantitative in nature; and (iii) can be conceptualized as a heuristic version of a cognitive confidence interval. Our data show that about 50% of the time, subjects exhibit cognitive uncertainty that is strictly wider than the switching interval of \$1.

Through hypotheses (a)–(c) spelled out above, our framework makes nuanced and novel predictions about how cognitive uncertainty or exogenous problem parameters should be related to observed risk-taking, as a function of whether the lottery in question is defined over gains or losses and whether it features a high or low payout probability.

As a test of prediction (a), we show that cognitive uncertainty is strongly correlated with the magnitude of probability weighting, in both the gains and loss domains. Specifically, as would be expected from the perspective of our framework, cognitive uncertainty is positively correlated with risk taking for low probability gains and high probability losses, yet negatively correlated with risk taking for high probability gains and low probability losses. Thus, subjects with higher cognitive uncertainty exhibit a more pronounced fourfold pattern of risk attitudes (Kahneman and Tversky, 1979) and their behavior is less sensitive to variations in objective payout probabilities.

As called for by our model, we quantify these patterns by estimating a two-parameter version of the probability weighting function, where one parameter primarily governs the function's sensitivity to variation in objective probabilities. Here, we find that below-average cognitive uncertainty subjects are 46% more sensitive to variation in payout probabilities than above-average cognitive uncertainty participants.

In a second step of the analysis of choice under risk, we test prediction (b) above by exogenously manipulating the magnitude of cognitive uncertainty using compound lotteries. To take an example, a compound lottery is a lottery that pays a non-zero amount with probability p, where p is determined by a random draw from, e.g., $U[0, 20]$. We hypothesize and empirically verify that compound lotteries induce substantially higher cognitive uncertainty than the corresponding reduced lotteries, which are identical for an expected utility maximizer. Building on Halevy (2007), Gillen et al. (2019) recently documented that attitudes towards compound lotteries and ambiguity attitudes are almost perfectly correlated. We hence view our experimental implementation of compound lotteries as closely related to introducing ambiguity, in line with our opening examples.

Our framework predicts that the increase in cognitive uncertainty that is generated by compound lotteries translates into a more compressed weighting function. This again implies predictions about how compound lotteries should induce higher or lower risk aversion depending on whether one considers gains or losses, and high or low probabilities. In our experiments, we find consistent support for this hypothesis: while subjects act as if they are aversive to compound lotteries under high probability gains and low probability losses, they are more risk seeking under compound lotteries over small probability gains and high probability losses.

In a final step of the analysis of choice under risk, we test prediction (c) above by exogenously manipulating the location of the mental default. To this effect, we leverage our assumption that in unfamiliar environments the default is influenced by an ignorance prior, i.e., 50:50 in two-states lotteries. To manipulate the location of the mental default, we translate these two-states lotteries into ten-states lotteries, without changing the objective payoff profile. We hypothesize that this shifts the mental default downwards, which should move the entire probability weighting function closer towards zero. Our experimental results show that the probability weighting function with ten states is indeed significantly shifted towards zero, although these patterns are more pronounced for gains than for losses.

In a second set of experiments, we conduct conceptually analogous exercises for belief updating. Here, we implement canonical balls-and-urns updating tasks of the type recently reviewed by Benjamin (2019). In these experiments, a computer randomly selects one of two bags according to a known base rate, yet subjects do not observe which bag got selected. The two bags both contain 100 balls, where one bag contains $q > 50$ red and $(100-q)$ blue balls, while the other bag contains q blue and $(100-q)$ red balls. The computer randomly draws one or more balls from the selected bag and shows these balls to the subject, who is then asked to provide a probabilistic assessment of which bag was actually drawn. Across experimental tasks, the base rate, the signal diagnosticity q and the number of random draws from the bags vary, but are always known to subjects.

After subjects state their posterior belief, we again elicit cognitive uncertainty. In a conceptually very similar fashion to the case of choice under risk, we ask subjects to use a slider to calibrate the statement "I am certain that the optimal guess is between x and y." We explain that the optimal Bayesian guess relies on the same information that is available to subjects, and combines this information in a statistically optimal way. Again, if a subject moves the slider to the very right, x and y collapse to the subject's own previously stated belief. The further the slider is moved to the left, the wider the cognitive confidence interval becomes. In addition, we also elicit subjects' willingness-to-pay to replace their own guess by the guess of the optimal machine as a complementary and incentivized measure of cognitive uncertainty.

Our experimental strategy again proceeds in three steps: (a) document a correlation between cognitive uncertainty and the magnitude of compression of posterior beliefs; (b) exogenously manipulate cognitive uncertainty; and (c) exogenously manipulate the location of the mental default. To begin, we find that cognitive uncertainty is correlated with compression of posterior beliefs towards 50:50. Moreover, we document that cognitive uncertainty strongly predicts the magnitude of base rate insensitivity and likelihood ratio insensitivity, two of the key underreaction anomalies highlighted in Benjamin's (2019) meta-analysis.

We again quantify these patterns by structurally estimating a "belief weighting function" that has the same functional form as the probability weighting function. Here, we find that below-average cognitive uncertainty subjects are 57% more sensitive to variation in objective probabilities than above-average cognitive uncertainty participants. Moreover, the estimated sensitivity parameters are very similar to those obtained for choice under risk, even though the choice domain is evidently an entirely different one.

Next, we exogenously manipulate cognitive uncertainty in a parallel fashion to the case of choice under risk. Specifically, we implement compound belief updating tasks, in which the base rate is given by 50:50 and the signal diagnosticity q is itself a random draw, such as $q \sim U[60, 80]$. These compound updating tasks deliver the same Bayesian posterior as a corresponding reduced updating task with $q = 70$. However, we hypothesize that cognitive uncertainty will be higher under compound signal diagnosticities, hence giving rise to more compressed belief distributions. In our experimental data, we find that cognitive uncertainty indeed increases by 33% under compound diagnosticities. Moreover, the distribution of beliefs becomes substantially more compressed towards 50:50, as predicted by our framework.

In a last step of the analysis of belief updating tasks, we exogenously vary the location of the mental default. Here, we once more employ the same methodology as in our risky choice experiments: we increase the number of states (bags) from two to ten, without changing the relevant Bayesian posterior. The results show that this manipulation induces a substantial and statistically significant downward shift of the entire distribution of posterior beliefs towards zero. Thus, in summary, our experimental analysis provides comprehensive support for the roles of cognitive uncertainty and a mental default also in stylized laboratory belief updating tasks.

In the third part of the paper, we leave the realm of structured laboratory decision problems and study the relationship between cognitive uncertainty and survey expectations about the performance of the stock market, inflation rates, and the structure of the national income distribution. For instance, we ask respondents to guess the probability that in a randomly selected year between 1980 and 2018, the inflation rate was less than x%, where x varies across respondents. Similarly, we ask respondents to guess the probability that a randomly selected U.S. household earns less than $x, where x again varies across respondents. We measure cognitive uncertainty after we elicit these beliefs, using the same methodology as before.

As described in the opening examples and Figure 1, across domains average stated beliefs are inverse S-shaped and thus compressed functions of the true probabilities. Crucially, we find that subjects with higher cognitive uncertainty exhibit survey expectations that are more regressive towards 50:50. Again, we confirm and quantify these patterns by structurally estimating a parametric "survey expectations weighting function." Thus, cognitive uncertainty is linked to more compressed response patterns in all of the decision domains that we consider in this paper.

In the last part of the paper, we study whether variation in cognitive uncertainty is largely due to between- or within-subject variation. In a heuristic decomposition exercise, we find that – as a very weak lower bound – at least 50% of the variation in our data is due to between-subjects heterogeneity. This suggests that participants exhibit meaningfully stable "types." Across our different sets of experiments, this subject-level heterogeneity is correlated with observables: women, participants with low cognitive skills, and subjects with faster response times exhibit higher cognitive uncertainty.

In summary, the central contributions of our paper are (i) to propose that cognitive uncertainty is the mechanism behind strikingly similar response functions in belief formation and choice under uncertainty; (ii) to introduce a formal definition and a new experimental measure of cognitive uncertainty; and (iii) to provide both correlational and causal evidence for the role of cognitive uncertainty and a mental default across four previously unconnected domains of economic decision making, each of which has received substantial interest in the literature on its own.

Our paper builds on recent theoretical work on cognitive imprecision and resulting shrinkage processes, see Woodford (2012, 2019), Khaw et al. (2017), Gabaix and Laibson (2017), Gabaix (2019), Frydman and Jin (2019), and Steiner and Stewart (2016).[1] In contrast to some of these theories, we posit that cognitive uncertainty arises in the mental process of optimizing rather than from perceptual distortions of numeric quantities. We also do not require the neural coding of noise to be Bayes-optimal. Abstracting from these interpretive differences, we view our experiments as encouraging support for this emerging body of theoretical work. We show that cognitive noise is not just a low-level subconscious phenomenon but instead measurable and that it applies to, and unifies anomalies across, a much broader range of economic settings than prior literature has theorized.

In the experimental literature, Butler and Loomes (2007) propose a measurement of preference imprecision in choice under risk, which is related in spirit to our measure. Agranov and Ortoleva (2017) show that experimental subjects often deliberately randomize between lottery options. However, these authors do not conceptualize their measures as cognitive uncertainty and resulting shrinking processes, and do not study the types of behavioral anomalies that we focus on.

[1]This line of work in turn builds on an active recent literature on Bayesian cognition in cognitive science (Chater et al., 2008; Gershman and Bhui, 2019; Griffiths et al., 2008; Tenenbaum et al., 2006), including work on subjective confidence in one's decisions (Meyniel et al., 2015).

Our paper also builds on various literatures on the anomalies that we attempt to bring together in this paper, which are too voluminous to review here. For example, in the belief updating literature, our work is indebted to the comprehensive review by Benjamin (2019), in particular his discussion of the reduced-form observations of underreaction and "extreme belief aversion." More broadly, our paper fits into the recent theoretical and experimental literature on bounded rationality that has focused on the mechanisms behind different behavioral anomalies (Bordalo et al., 2012, 2017; Enke, 2017; Enke and Zimmermann, 2019; Enke et al., 2019; Esponda and Vespa, 2016; Graeber, 2019; Martínez-Marquina et al., 2019).

The paper proceeds as follows. Section 2 lays out a theoretical framework of the role of cognitive uncertainty in the types of environments that we study in this paper. Sections 3, 4, and 5 present the experiments on choice under risk, belief updating, and survey expectations. Section 6 studies the correlates of cognitive uncertainty and Section 7 concludes.

2 A Model of Cognitive Uncertainty

2.1 Overview

Our formal framework directly builds on the cognitive imprecision models of Khaw et al. (2017) and Gabaix (2019). Following these contributions, our central assumption is the existence of cognitive noise in decision-making.[2] In contrast to some earlier work, we interpret this noise not as necessarily being perceptual imperfection, but as resulting primarily from higher-level reasoning during optimization.[3] Translating a set of problem inputs (e.g., probabilities) into an optimal response (e.g., a certainty equivalent) is often difficult, which could introduce noise through various psychological mechanisms, including computational errors, retrieval from memory, or even from reading off and implementing one's own preferences. Such cognitive noise creates cognitive uncertainty: subjective uncertainty about what the optimal action is. We focus on a notion of cognitive uncertainty that people are aware and to which they therefore have access through introspection. To illustrate informally, suppose your prior belief that it rains tomorrow is 15%. Next, a weather forecast predicts that it will rain. You know from experience that the weather forecast is correct 80% of the time. What is your posterior belief that it will rain tomorrow? 45%? Really? Not 40%? Or perhaps 52%? To take another example, suppose you were asked to state your certainty equivalent of a 25% chance of

[2]The notion of processing noise is well-grounded in neuroscientific research (e.g., Faisal et al., 2008; Shadlen and Newsome, 1998; Stocker and Simoncelli, 2006).

[3]We hence follow the behavioral science tradition going back to at least Prospect Theory of recognizing that evidence about low-level processes can shed light on high-level decision-making.

getting \$15. Suppose you are small-stakes risk averse. You announce that your certainty equivalent is \$3. But is it really \$3? Or maybe \$2.50 or \$3.20?

Our second key assumption is that people represent quantities (probabilities) in log odds space. An extensive body of research in the cognitive sciences suggests that the brain tends to deal with quantities in log space, which is closely linked to the notion of a Weber's Law that describes perceptual patterns across various domains (Izard and Dehaene, 2008; Zhang and Maloney, 2012). Cognitive noise and log coding are independent elements of the model. The model's key prediction about the effect of cognitive uncertainty on decisions (which is a form of insensitivity) does not depend on the assumption of log coding. However, log coding in combination with cognitive noise generates inverse S-shaped response functions.

In this paper, we are interested in the application of cognitive uncertainty to decision-making under external uncertainty. We focus on noisy processing of probabilities. However, since cognitive noise may come from processing problem inputs other than probabilities, our general framework of cognitive uncertainty is readily applied to other contexts.

2.2 Cognitive Noise and Shrinkage

We focus our presentation on the case with normally distributed data and linear-quadratic utility but provide various generalizations in Appendix A. Assume a decision-maker takes an action a and derives utility $u(a,x)$ that depends on a one-dimensional quantity x:

$$u(a,x) = -\frac{1}{2}(a - Bx)^2. \tag{1}$$

The quantity x may be a problem parameter explicitly presented to the decision-maker, or a value calculated by or retrieved form the agent's memory. By "action," we generically refer to the solution to a decision problem such as a stated posterior belief or a stated certainty equivalent. The rational action is

$$a^r(x) = Bx. \tag{2}$$

We assume that the cognitive process required to identify an optimal action a is subject to cognitive noise. We model this as the agent receiving a signal $s = x + \varepsilon$ instead of having direct access to x.

The agent is aware of the existence of cognitive noise. He *perceives* the noise term to be distributed according to $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. The agent's subjectively perceived cognitive noise need not be equal to the true noise that he is exposed to, $\tilde{\varepsilon} \sim \mathcal{N}(0, \sigma_{\tilde{\varepsilon}}^2)$. This assumption highlights that we do not require that an agent's cognitive uncertainty as defined below reflects the objective level of noise in his internal processing.

The agent holds a prior $x \sim \mathcal{N}(x^d, \sigma_x^2)$, where we refer to x^d as the "mental default." The prior may be influenced by a multitude of factors, such as the objective distribution of x across situations, the agent's retrieved personal experience, or some "anchor" value in the problem description. Here, we do not model how exactly the prior is determined. As we discuss in greater detail below, in all of our applications x will represent a probability and we will specify x^d as influenced by a discrete ignorance prior that assigns equal probability mass to all possible states. This assumption is plausible in our particular set of experiments because these are setups that are unfamiliar to most participants. We do not posit that the prior is always shaped by an ignorance prior.

Agents account for their cognitive noise by forming an implicit update about x. For a Bayesian agent, this creates a standard Gaussian signal extraction problem:

$$\mathbb{P}(x|s) \sim \mathcal{N}(\lambda s + (1-\lambda)x^d, (1-\lambda)\sigma_x^2), \tag{3}$$

with the shrinkage factor

$$\lambda = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\varepsilon^2} \in [0, 1]. \tag{4}$$

A rational agent takes an action by solving: $\max_a \mathbb{E}\left[-\frac{1}{2}(a - Bx)^2 \,|s\right]$. Only expectations matter in the linear first-order condition, leading to a rational action

$$a^r(s) = B(\mathbb{E}[x|s]) = B\lambda s + B(1-\lambda)x^d \tag{5}$$

For a given x, the median action a^e across many agents with individual realizations of cognitive noise is then

$$a^e(x) = \text{Median}\,(a^r(s)|x) = B\lambda x + B(1-\lambda)x^d, \tag{6}$$

which should be compared with equation (2). We see that the agent dampens his response by λ, generating shrinkage towards the default (prior). The key takeaway is that the existence of cognitive noise makes the rational action *insensitive* to variations in the problem parameter x.

2.3 Cognitive Uncertainty

Awareness of cognitive noise generates subjectively perceived uncertainty about what the optimal action is. We label this *cognitive uncertainty*. Our objective is to characterize this uncertainty that predicts the degree of insensitivity *at the level of an individual action*, and derive empirical implications. Note that, in contrast to the concept of cognitive noise, cognitive uncertainty is defined over actions and turns out to be an empirically measurable quantity. Next, we provide a definition within our model.

The agent's subjective uncertainty about his optimal action takes as given his individual draw of s, and reflects how the agent's rational action a^r (equation (2)) subjectively varies due to the agent's own posterior distribution of x (equation (3)), i.e., based on[4]

$$\mathbb{P}(a^r(x)|s) \sim \mathcal{N}(B\lambda s + B(1-\lambda)x^d, B^2(1-\lambda)\sigma_x^2). \tag{7}$$

Definition. *The agent's cognitive uncertainty is given by*

$$\sigma_{CU} = \sigma_{a^r(x)|s} = |B|\sqrt{1-\lambda}\sigma_x = |B|\frac{\sigma_\varepsilon \sigma_x}{\sqrt{\sigma_\varepsilon^2 + \sigma_x^2}}. \tag{8}$$

To take an illustrative example, the agent might compute a posterior belief based on a base rate and a likelihood ratio, but he might be aware that this posterior belief could have turned out to be slightly different had he drawn a different computational error.[5]

In our empirical analysis, we will measure a variant of (8). We reiterate that we do not require cognitive uncertainty to reflect the true amount of cognitive noise. While our model is Bayesian, cognitive uncertainty is based on the agent's subjectively perceived level of cognitive noise. Because we are concerned with empirical applications, all that matters for us is perceived – and hence inherently subjective – cognitive uncertainty.

We make the following observation about the shrinkage factor λ:

$$\lambda = 1 - \frac{\sigma_{CU}^2}{B^2\sigma_x^2} \tag{9}$$

That is, higher cognitive uncertainty generates more shrinking towards the default and makes people more insensitive to variation in x.[6]

[4]Note that the mean of $a^r(x)|s$ is $B(\lambda s + (1-\lambda)x^d)$ as in equation (5).

[5]Cognitive uncertainty cannot be estimated from data on actions alone. An alternative construct estimated from the variation of actions across subjects, or a measure of consistency of actions for a single subject is (a) not available at the level of a single action, (b) will by construction incorporate variation due to the random realizations of cognitive noise and (c) may differ conceptually because actions reflect the true noise, not only the subjectively perceived one. Empirically, measuring $\sigma_{a^r(x)}$ from having a person take the same choice repeatedly is both experimentally tedious and has been associated with additional, confounding factors – such as deliberate randomization – that cannot be attributed to random realizations of cognitive noise (see, e.g., Agranov and Ortoleva, 2017).

[6]While in literal terms our model posits shrinkage of the "input" quantity x, it also permits an equivalent interpretation of shrinkage of the response a. Using $a^r(x) = Bx$ and letting $a^d = Bx^d$ we get

$$a^e(x) = B\lambda x + B(1-\lambda)x^d = \lambda a^r(x) + (1-\lambda)a^d. \tag{10}$$

2.4 Log Coding

Following prior work in both cognitive science and economics (Gabaix, 2019; Zhang and Maloney, 2012), we assume that a probability p is transformed into a quantity q in log odds space by applying

$$q = Q(p) = \ln\frac{p}{1-p}. \tag{11}$$

In our model of probabilistic reasoning, people process probabilities with cognitive noise that occurs in log odds space. This means we now assume that the decision-relevant quantity x from (1) is a probability in log odds space q about which an agent receives a signal $s = q + \varepsilon$. This will result in shrinkage of probabilities in log odds space:

$$q(s) = \lambda s + (1-\lambda)q^d. \tag{12}$$

In the following, we will focus on medians, which have the attractive property that for any strictly monotone function Y, $\text{Median}(Y(x)) = Y(\text{Median}(x))$. Over many draws of s – fixing x, but varying ε – the median posterior q^e about probability p after encoding in log odds space and shrinkage is:

$$q^e(q) := \text{ Median}\left(q(s)|q\right) = \lambda q + (1-\lambda)q^d. \tag{13}$$

From this we can derive the implied median posterior probability p by applying the inverse log odds function $P(q) = Q^{-1}(q) = \frac{1}{1+e^{-q}}$:

$$p^e(p) = P(q^e) = \frac{1}{1+\exp\left(-\lambda\ln\frac{p}{1-p} - (1-\lambda)\ln\frac{p^d}{1-p^d}\right)}. \tag{14}$$

The rational action is, as before, $a^r(s) = B\lambda s + B(1-\lambda)q^d$, so at the median

$$a^e(p) = B \cdot q^e = B \cdot Q(p^e). \tag{15}$$

2.5 Empirical Applications and Predictions: "Weighting" Functions

Equations (14) and (15) deliver the microfoundation for our empirical analysis. Similarly to the models in Khaw et al. (2017), equation (14) can be reformulated as

$$w(p) := p^e(p) = \frac{\delta p^\lambda}{\delta p^\lambda + (1-p)^\lambda}, \tag{16}$$

where $\delta = \exp\left((1-\lambda)\ln\frac{p^d}{1-p^d}\right)$. This reformulation is instructive because it corresponds to the well-known two-parameter specification of a probability weighting function suggested by Gonzalez and Wu (1999). The original motivation by Gonzalez and Wu (1999) is that the log odds transformation allows a convenient characterization of the weighting function in which one parameter, λ, primarily represents the sensitivity of the weighting function to changes in probabilities, while another parameter, δ, controls the function's elevation. Our model motivates this functional form by endogenizing its parameters: λ directly corresponds to our shrinkage factor (and hence implicitly depends on cognitive uncertainty), while δ is a transformation of the constant term that jointly reflects the default and λ.

An important implication of our approach, however, is that we expect this "weighting" function to adequately capture decision making not just in choice under risk. Instead, in this paper, we will consider three types of applications of cognitive uncertainty in decision-making under (external) uncertainty: (i) choice under risk and ambiguity; (ii) laboratory belief updating tasks; and (iii) survey expectations about economic variables.

Our stylized model maps into each of these contexts as follows. First, in the case of choice under risk (or ambiguity), cognitive noise with respect to probabilities arises through the process of computing expected utilities. Cognitive uncertainty then refers to the agent's subjective uncertainty about their own certainty equivalent for a lottery.

Second, in belief updating tasks, cognitive noise arises in the complicated process of combining base rate, diagnosticity and signal into a posterior. Cognitive uncertainty then refers to the agent's subjective uncertainty about what the Bayesian posterior is.

Third, in the case of expectation elicitation in surveys, we posit that cognitive noise arises through the process of retrieving information from memory (Azeredo da Silveira and Woodford, 2019). Then, cognitive uncertainty refers to the agent's subjective uncertainty about the correct value of the state variable of interest.

In all of these applications, we will operate under the assumption that the mental default about probabilities is influenced by an ignorance prior. To be clear, we do not posit that the default is *always* affected by this ignorance prior – we just posit that this is the case in our experimental applications, with which people have no or very little prior experience.

Prediction 1. *Higher cognitive uncertainty is associated with more compressed weighting functions. Thus:*

(a) Cognitive uncertainty is positively correlated with risk seeking for low probability gains and high probability losses, and negatively correlated with risk seeking for high probability gains and low probability losses.

(b) Cognitive uncertainty is positively correlated with stated posterior beliefs or survey expectations if the rational belief is below 50%, and negatively if the rational belief is above 50%.

Prediction 2. *An exogenous increase in cognitive uncertainty induces more compressed weighting functions, with identical implications as in Prediction 1 (a)–(b).*

Prediction 3. *An exogenous decrease in the mental default induces the entire weighting function to move closer towards zero.*

We test these predictions in both reduced-form and structural analyses in which we estimate the "weighting function" from equation (16) across decision domains.

3 Choice Under Risk

3.1 Experimental Design

All experiments reported in this paper were designed with the same objective in mind: replicate standard experimental designs from the literature to elicit choices and beliefs, and supplement these tasks with a measurement of cognitive uncertainty.

3.1.1 Measuring Choice Behavior

To estimate a probability weighting function, we follow a large set of previous works and implement price lists that elicit certainty equivalents for lotteries (see, e.g. Bernheim and Sprenger, 2019; Bruhin et al., 2010; Tversky and Kahneman, 1992). Recent work suggests that these types of price lists are particularly easy for subjects to understand and give rise to more internally consistent and externally predictable choices than alternative measurement tools (Andreoni and Kuhn, 2019).

In treatment *Baseline Risk*, each subject completed a total of six price lists. On the left-hand side (Option A), a simple lottery was shown that paid a with probability p and nothing otherwise. On the right-hand side (Option B), a safe payment s was offered that increased by \$1 for each row that one proceeds down the list. As in Bruhin et al. (2010) and Bernheim and Sprenger (2019), the end points of the list were given by $s = \$0$ and $s = \$a$. Thus, each decision screen required $a + 1$ choices. A subject would typically start out by preferring Option A at the top of the list and then switch to Option B at some point as the safe payment increases.

Throughout, we enforce that subjects behave in internally consistent ways within a given choice list. That is, we do not allow for multiple switching points. This facilitates a simpler elicitation of cognitive uncertainty, as discussed below. To aid subjects' decision-making, we implemented an auto-completion mode: if a subject chose Option A in a given row, the computer implemented Option A also for all rows above this row. Likewise, if a subject chose Option B in a given row, the computer automatically and instantaneously ticked Option B in all lower rows. However, participants could always revise their decision and the auto-completion before moving on. See Figure 12 in Appendix B.1 for a screenshot of a decision screen.

The non-zero payout of the lottery x and the payout probability p were drawn randomly and independently from the sets $a \in \{15, 20, 25\}$ and $p \in \{5, 10, 25, 50, 75, 90, 95\}$. Note that these lotteries are rather simple and well in line with previous work on probability weighting. We implemented both gain and loss gambles, where the loss amounts are just the mirror images of a. In the case of loss gambles, the lowest safe payment was given by $s = -\$x$ and the highest one by $s = \$0$. In loss choice lists, subjects received a monetary endowment of $\$a$ from which potential losses were deducted. Out of the six choice lists that each subject completed, three concerned loss gambles and three gain gambles. We presented either all loss gambles or all gain gambles first, in randomized order.

Finally, with probability 1/3, a lottery choice list in treatment *Baseline Risk* was presented in a compound lottery format. We will describe, motivate and analyze these data in Section 3.3. For now we focus on the baseline (reduced) lotteries discussed above.

3.1.2 Measuring Cognitive Uncertainty

After a participant had completed a choice list, the next screen elicited their cognitive uncertainty with respect to this decision. Conceptually, we are interested in measuring the analog of σ_{CU} in the model (equation (8)). However, many people are not naturally familiar with the concept of a standard deviation. To strike a balance between conceptual clarity and quantitative interpretation on the one hand and participant comprehension on the other hand, we hence elicit an intuitive version of a subjective confidence interval.

Figure 2 provides a screenshot. Here, a participant was reminded of their valuation (switching interval) for the lottery. They were then asked to indicate how certain they are that to them the lottery is worth exactly the same as getting their previously indicated certainty equivalent. To answer this question, subjects used a slider to calibrate the statement "I am certain that the lottery is worth between a and b to me." If the participant moved the slider to the very right, a and b corresponded to the previously indicated switching interval. For each of the 20 possible ticks that the slider was moved to the left, a decreased and b increased by 25 cents, in real time. In gain lotteries, a was bounded from below by zero and b bounded from above by the lottery's upside. Analogously, for losses, a was bounded from below by the lottery's downside and b from above by zero. The slider did not have a default value, meaning that subjects had to click somewhere on the slider in order to proceed.

This measure of cognitive uncertainty is quantitative in nature and can be thought

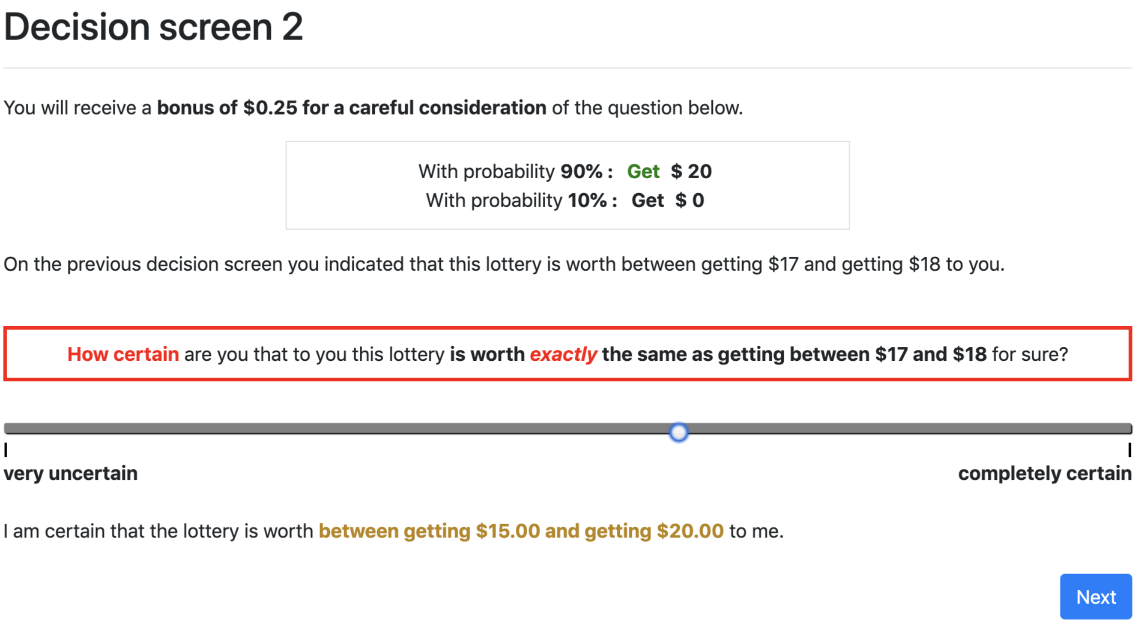


Figure 2: Decision screen to elicit cognitive uncertainty in choice under risk

of as a subjective confidence interval. We note that we deliberately did not financially incentivize the elicitation of cognitive uncertainty. The reason is that we do not know the objective truth (subjects' valuation for a lottery) because we do not know subjects' true preferences.[7] For ease of interpretation, we normalize this measure to be between zero and one. Figure 13 in Appendix B.1 shows a histogram of the distribution of cognitive uncertainty, which shows considerable variation. Average cognitive uncertainty is 0.20, with a median of 0.10 and a standard deviation of 0.20. 57% of our data indicate cognitive uncertainty that is strictly larger than the one-dollar switching interval.[8]

3.1.3 Subject Pool

All experiments reported in this paper were conducted on Amazon Mechanical Turk (AMT). AMT is becoming an increasingly used resource in experimental economics (e.g. DellaVigna and Pope, 2018; Imas et al., 2016), including in work on bounded rationality (Martínez-Marquina et al., 2019). Review papers suggest that experimental results on AMT and in the lab closely correspond to each other (Paolacci and Chandler, 2014). An important advantage of AMT that we also leverage in this study is that the pool of potential subjects is very large, which allows for high-powered analyses and a relatively

[7]Butler and Loomes (2007) elicit a measure of preference imprecision in price lists that is defined as the range of those rows in which subjects indicate that they "are not sure" which option they prefer. The authors link this measure to random choice.

[8]As a basic validity check, in a small sample of 272 price lists, we implemented payout probabilities of $p = 0\%$ or $p = 100\%$, so that there is no external uncertainty. In these tasks, cognitive uncertainty drops considerably to an average of 0.10 and a median of zero.

large number of different treatments and tasks (Robinson et al., 2019).

Still, a potential concern with using an AMT subject pool is that AMT workers may be less motivated or less attentive than traditional lab experimental subjects. This is of particular importance here because we wanted our analyis of the role of cognitive uncertainty to build on the same set of response patterns that one typically identifies in laboratory experiments.

We hence took four measures to achieve high data quality. First, our financial incentives are unusually large by AMT standards. Average realized earnings in the choice under risk experiments are \$6.10 for a median completion time of 20 minutes. This implies average hourly earnings of \$18, compared to a typical hourly wage of about \$5 on AMT. Second, we aggressively screened out inattentive prospective subjects through comprehension questions and attention checks, described in detail below. Third, we pre-registered analyses that remove extreme outliers and speeders. Fourth, as explained above, subjects only completed six choice lists, which is considerably less than in typical choice under risk experiments.

3.1.4 Logistics and Pre-Registration

Based on the pre-registration (see below), we recruited $N = 700$ completes for treatment *Baseline Risk*. After reading the instructions, participants completed a set of three comprehension questions that tested their understanding of the choice lists and the cognitive uncertainty question. Participants who answered one or more control questions incorrectly were immediately routed out of the experiment and do not count towards the number of completes. In addition, towards the end of the experiment, a screen contained a simple attention check. Subjects that answered this attention check incorrectly are excluded from the data analysis and replaced by a new complete, as specified in the pre-registration. In total, 62% of all prospective participants were screened out in the comprehension checks. Of those subjects that passed, 2% were screened out in the attention check towards the end of the experiment. Thus, all of our results should be understood as being conditional on a pretty attentive participant pool – given the link between cognitive uncertainty and response times discussed in Section 6, we imagine that we would have identified even more variation in cognitive uncertainty had we not restricted the sample.

In terms of timeline, subjects first completed six of the choice under risk tasks discussed above. Second, we elicited their survey expectations about various economic variables, as discussed in Section 5. Finally, participants completed a short demographic questionnaire and an eight-item Raven matrices IQ test.

Participants received a completion fee of \$1.70. In addition, each participant potentially earned a bonus. The experiment comprised three financially incentivized parts: the risky choice lists, the survey expectations questions, and the Raven IQ test. For each subject, one of these parts of the experiment was randomly selected for payment. If choice under risk was selected, we randomly selected a choice list and a decision on that list for payment. We recognize that this incentive scheme introduces a compound lottery over subjects' earnings. See Bernheim and Sprenger (2019) for a corresponding discussion and a similar strategy to incentivize subjects.

All experiments reported in this paper were pre-registered in the AEA RCT registry, see `https://www.socialscienceregistry.org/trials/4493`. The pre-registration includes (i) the sample size in each treatment; (ii) data exclusion criteria such as the aforementioned attention checks or the handling of extreme outliers; and (iii) predictions about the relationship between cognitive uncertainty and our outcome measures.

Screenshots of the entire experiment, including instructions and control questions, can be found in Appendix E.

3.2 Cognitive Uncertainty and the Probability Weighting Function

3.2.1 Non-Parametric Analyses

Because of the simple structure of our lotteries with only one non-zero payout state, an instructive non-parametric way to visualize our data is to compute *normalized certainty equivalents* as $NCE = CE/x$, where the certainty equivalent CE is defined as the midpoint of the switching interval[9] and x is the non-zero payout. This measure hence effectively only represents the raw data, normalized by the non-zero payout. An attractive feature of NCE is that it directly corresponds to subjects' implied probability weights if one assumes that utility is linear. Because this will be instructive, these normalized certainty equivalents are negative for lotteries with losses.

For the purposes of the baseline analysis, we exclude extreme outliers as defined in the pre-registration: these are observations for which (i) the normalized certainty equivalent is strictly larger than 95% while the objective payout probability is at most 10%, or (ii) the normalized certainty equivalent is strictly less than 5% while the objective payout probability is at least 90%. For example, this excludes observations that state that the certainty equivalent for a 5% chance of receiving \$20 is strictly larger than \$19. This procedure of excluding outliers affects 3% of all data points. We report robustness checks using all data below.

Because the normalized certainty equivalents reflect implied probability weights, Figure 3 plots these normalized certainty equivalents against objective payout probabilities

[9]We restrict normalized certainty equivalents to be between zero and one.

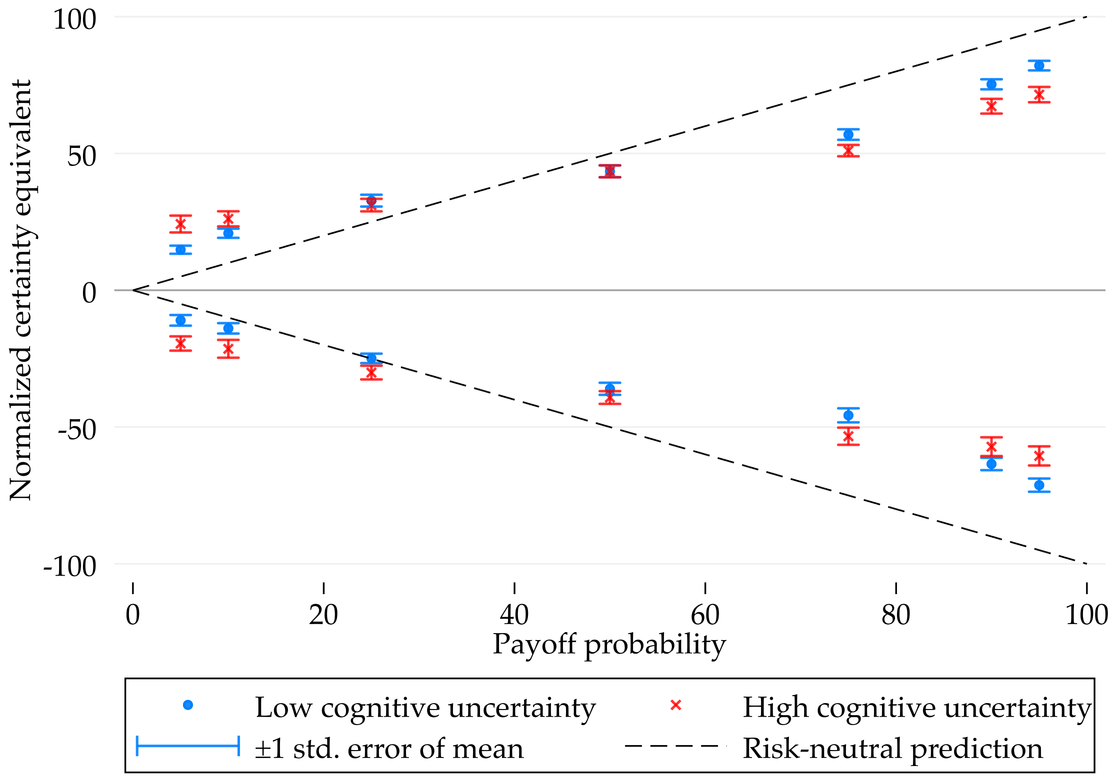


Figure 3: Probability weighting function separately for subjects above / below average cognitive uncertainty. The partition is done separately for each probability × gains / losses bucket. The plot shows averages and corresponding standard error bars. Normalized certainty equivalents (implied probability weights) are computed as certainty equivalent divided by payout probability. The figure is based on 2,525 certainty equivalents of 700 subjects.

to visualize a heuristic version of the probability weighting function in our data. The figure distinguishes between subjects above and below average cognitive uncertainty within a given payoff probability bucket.

Focusing on the upper half of the figure (gain lotteries), first note that we replicate prior findings on the shape of the weighting function: implied probability weights are above the risk-neutral prediction for low probabilities but below the risk-neutral prediction for intermediate and high probabilities. Thus, our subject pool doesn't seem to be unusual in this regard.

More importantly, we find that subjects with higher cognitive uncertainty exhibit more pronounced probability weighting functions: still focusing on the top half, high cognitive uncertainty subjects are slightly more risk seeking for small probability gains and more risk averse for high probability gains. Thus, overall, cognitive uncertainty is associated with more pronounced shrinking and hence a flatter relationship between implied probability weights and objective payout probabilities.[10]

[10]This result resonates with the findings reported in Bruhin et al. (2010), who uncover substantial het-

Note that the heuristic probability weighting function depicted in Figure 2 crosses the 45-degree line to the left of $p = 50\%$. This pattern is well-known in the literature and in line with our hypothesis as long as subjects both (i) shrink towards 50:50 because of cognitive uncertainty and (ii) exhibit genuine preference-based risk or loss aversion, which shifts the function towards zero.

Next, we turn to the bottom panel of Figure 3, which depicts the analogous observed data for losses. By the construction of our figure, the weighting function is now given by the mirror image of the weighting function in the gain domain. Again, we see that the implied probability weights of subjects with higher cognitive uncertainty are more compressed. An attractive feature of visualizing the data as in Figure 3 is that it highlights that the relationship between cognitive uncertainty and risk aversion reverses in predictable ways depending on whether the payouts are positive or negative and whether the payout probability is high or low. For instance, subjects with higher cognitive uncertainty are more risk seeking for small probability gains, but less risk seeking for small probability losses. Similarly, high cognitive uncertainty participants are more risk averse for high probability gains, yet less risk averse for high probability losses. In other words, as predicted, high cognitive uncertainty subjects exhibit a more pronounced fourfold pattern of risk attitudes, by the logic of shrinking towards 50:50.

Table 1 provides a regression analysis of these patterns. Our object of interest in these non-parametric analyses is the extent to which a subject's normalized certainty equivalent is (in)sensitive to variations in the probability of the non-zero payout state. Thus, we regress a participant's absolute normalized certainty equivalent on (i) the probability of receiving the non-zero gain / loss; (ii) cognitive uncertainty; and (iii) a corresponding interaction term. The regression analysis hence immediately corresponds to the setup of Figure 2. To facilitate comparability with the analyses of belief formation below, we implement OLS regressions where the dependent variable is the (normalized) midpoint of the switching interval.

The results show that, naturally, the probability of the non-zero payout state is positively predictive of stated absolute certainty equivalents. However, there is a strong interaction with cognitive uncertainty: higher cognitive uncertainty subjects respond less to variations in objective probabilities, in both the gains and the loss domain. In terms of quantitative magnitude, the regression coefficients suggest the following: if one were to increase the probability of the non-zero payout state from zero to one, then, on average, the increase in valuation for that lottery of subjects with cognitive uncertainty of zero is 20–40 probability points higher than for subjects with cognitive uncertainty of one.

erogeneity in individual-level probability weighting. Our notion of heterogeneity in cognitive uncertainty provides a possible micro-foundation for their results.

Table 1: Insensitivity to probability and cognitive uncertainty

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
Probability of payout	0.68***	0.68***	0.59***	0.59***	0.65***	0.65***
	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)
Probability of payout × Cognitive uncertainty	-0.41***	-0.41***	-0.20**	-0.19**	-0.31***	-0.31***
	(0.09)	(0.09)	(0.09)	(0.09)	(0.07)	(0.07)
Cognitive uncertainty	11.6**	11.4**	14.8***	14.6***	13.5***	13.9***
	(5.19)	(5.27)	(5.26)	(5.25)	(3.84)	(3.87)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1271	1271	1254	1254	2525	2525
R^2	0.54	0.55	0.41	0.42	0.47	0.47

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's absolute normalized certainty equivalent, computed as midpoint of the switching interval divided by the non-zero payout. The sample includes choices from all baseline gambles with strictly interior payout probabilities. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

3.2.2 Parametric Estimations

To supplement these reduced-form analyses, we estimate the two-parameter version of the probability weighting function that we endogenized in Section 2. To this effect, we estimate the parameters $\hat{\lambda}$ and $\hat{\delta}$ by minimizing the sum of squared residuals for the non-linear equation (16).

Table 2 summarizes the estimates in our data, pooled across gains and losses. For now, we focus on the top three rows and return to the bottom two rows in the sections below. Pooled across all subjects, we estimate a sensitivity parameter of $\hat{\lambda} = 0.49$ and an elevation parameter of $\hat{\delta} = 0.81$. These estimates are very close to the estimates of $\hat{\lambda} = 0.44$ and $\hat{\delta} = 0.77$ in the classic by Gonzalez and Wu (1999), which again suggests that our sample and results are very similar to previous findings.

Comparing subjects with above and below average cognitive uncertainty, we estimate that the sensitivity parameter λ is 46% higher for low than for high cognitive uncertainty subjects. Figure 16 in Appendix B.1 plots the resulting estimated probability weighting functions for the three different groups of subjects.

Recall that the two key parameters of our framework (the magnitude of cognitive uncertainty and the location of the mental default) have direct counterparts in equation (16): cognitive uncertainty determines the degree of insensitivity λ, while the elevation of the function δ is largely governed by the location of the mental default. In what follows, we seek to provide causal support for our framework by separately exogenously

Table 2: Estimates of probability weighting function in choice under risk

Treatment / group	Sensitivity $\hat{\lambda}$	Elevation $\hat{\delta}$
Baseline Risk: all subjects	0.49 (0.02)	0.81 (0.03)
Baseline Risk: high CU subjects	0.37 (0.03)	0.77 (0.04)
Baseline Risk: low CU subjects	0.54 (0.02)	0.83 (0.04)
Compound Risk: all subjects	0.39 (0.01)	0.79 (0.02)
Low Default Risk: all subjects	0.44 (0.04)	0.54 (0.03)

Notes. Estimates of equation (16), standard errors (clustered at subject level) reported in parentheses. CU = cognitive uncertainty (split at average). The analysis is implemented on the pooled gains and losses data.

manipulating cognitive uncertainty and the location of the default, and thereby moving around our estimates of λ and δ in predictable ways.

3.3 Exogenous Manipulation of Cognitive Uncertainty

3.3.1 Experimental Design

To exogenously manipulate cognitive uncertainty, we operate with compound lotteries. To take an example, a compound lottery is given by: "We randomly draw an integer between 60 and 80. Call this number n. With probability n%, you receive \$20. With probability 100%-n%, you receive \$0." The analogous reduced lottery has payout probability $p = 70\%$. These two lotteries are identical under expected utility theory because EU is linear in probabilities.

As is well-known in the literature, subjects are typically averse to compound lotteries when the probability of the non-zero payout state is relatively high and when the lottery is defined in the gain domain. Moreover, compound lottery aversion and ambiguity aversion are strongly correlated (Halevy, 2007), to the extent that other researchers have concluded that they are "virtually identical" (Gillen et al., 2019). We hence view compound lotteries as a simple and attractive way to implement ambiguous-ish lotteries, while avoiding the many logistical difficulties that are associated with implementing ambiguity, in particular maintaining credibility vis-a-vis experimental subjects.

Our hypothesis is that compound lotteries induce higher cognitive uncertainty and hence more shrinking towards 50:50. We hence hypothesize that, in compound lotteries, subjects will be more risk averse under high probability gains (this is the case studied in Halevy (2007)) and low probability losses, yet less risk averse under low probability gains and high probability losses.

A causal interpretation of our experiments with respect to cognitive uncertainty requires the assumption that the introduction of compound lotteries affects choices *only* through cognitive uncertainty. While this is a strong assumption, we are not aware of alternative theories that would predict the nuanced pattern of how risk aversion changes as a function of reduced versus compound lotteries, depending on whether the lottery features high or low probabilities and gains or losses.

As noted above, we implemented these compound lotteries as part of treatment *Baseline Risk*, where each lottery had a 1 in 3 chance of being presented in compound form. We collected 1,241 observations on compound lotteries.

3.3.2 Results

Relative to reduced lotteries, compound lotteries increase stated cognitive uncertainty by 32%, on average. Figure 14 in Appendix B.1 shows corresponding histograms. Thus, our experiments produce a strong "first stage."

Figure 4 shows the results. Here, we plot average normalized certainty equivalents separately for the baseline lotteries discussed above and for compound lotteries. We find that the probability weighting function is substantially more compressed under compound than under reduced lotteries, for both gains and losses. That is, subjects are actually compound lottery seeking for low probability gains and high probability losses, consistent with our hypothesis of more pronounced shrinking towards 50:50 once cognitive uncertainty increases.

Table 3 provides a corresponding regression analysis, akin to Table 1 above. We find that subjects' certainty equivalents are considerably less responsive to the objective payout probabilities under compound than under reduced lotteries, for both gains and losses. In terms of quantitative magnitude, subjects respond 23–29 percentage points less under compound lotteries. Moreover, we again find a within-treatment correlation between responsiveness to payout probabilities and cognitive uncertainty.

Table 2 provides a parametric analysis of the compound data by estimating equation (16). We find that the estimated sensitivity parameter decreases from $\hat{\lambda} = 0.49$ in the baseline gambles to $\hat{\delta} = 0.39$ with compound gambles, where both of these parameters are tightly estimated. Figure 16 in Appendix B.1 plots the estimated weighting function.

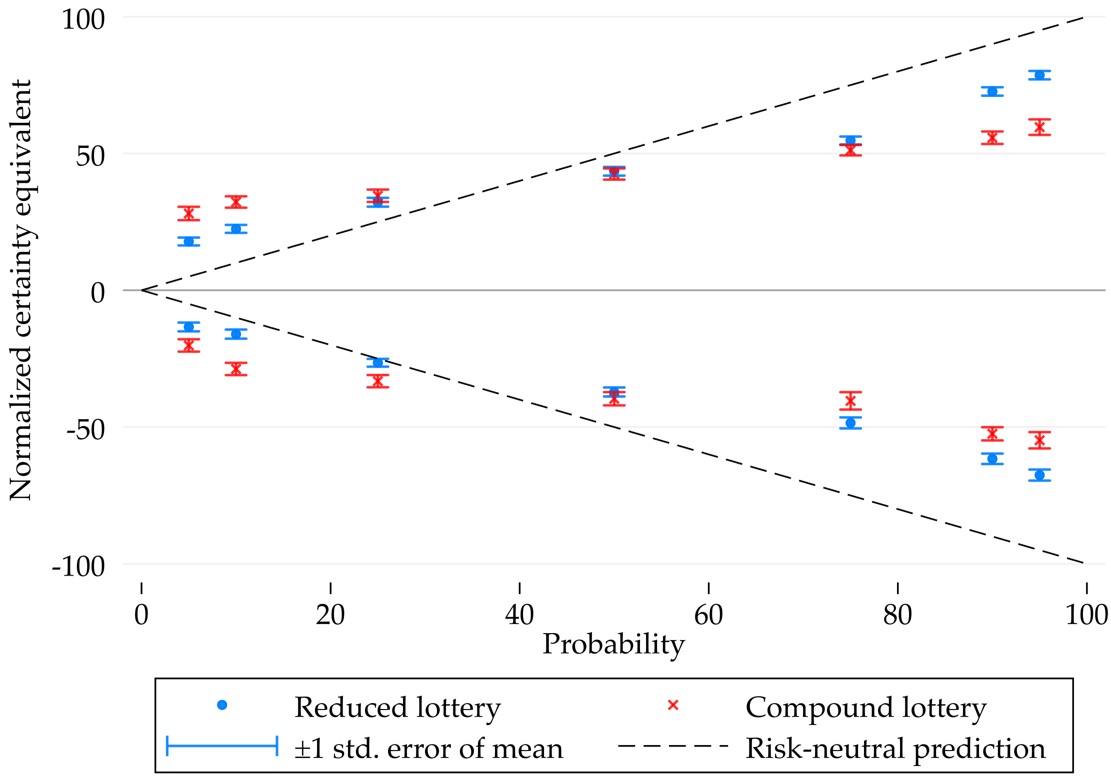


Figure 4: Probability weighting function separately for reduced and compound lotteries. The plot shows averages and corresponding standard error bars. Normalized certainty equivalents (implied probability weights) are computed as certainty equivalent divided by payout probability. The figure is based on 3,766 certainty equivalents of 700 subjects.

3.4 Exogenous Manipulation of the Mental Default

3.4.1 Experimental Design

In a final step of the analysis of choice under risk, we exogenously manipulate the location of the mental default. Recall that we operate under the assumption that the default is influenced by the ignorance prior. With two states of the world, the ignorance prior is 50:50. To vary the default, we implement a partition manipulation and increase the number of states of the world to ten. Under the maintained assumption that the default is given by the ignorance prior, this means that the ignorance prior for each state is now given by 10%. Following the logic of the model in Section 2, we hence predict that the entire probability weighting function shifts towards zero as the number of states increases.

To experimentally implement this manipulation, we replicate treatment *Baseline Risk*, but now frame probabilities in terms of number of colored balls in an urn. For example, we describe a lottery as:

> Out of 100 balls, 80 are red. If a red ball gets drawn: get $20.

Table 3: Choice under risk: Baseline versus compound lotteries

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
Probability of payout	0.62***	0.66***	0.56***	0.60***	0.60***	0.64***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Probability of payout × 1 if compound lottery	-0.30***	-0.27***	-0.25***	-0.23***	-0.27***	-0.25***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)
Probability of payout × Cognitive uncertainty		-0.29***		-0.23***		-0.28***
		(0.06)		(0.07)		(0.05)
1 if compound lottery	12.3***	11.8***	12.3***	11.3***	12.3***	11.8***
	(1.89)	(1.90)	(1.84)	(1.87)	(1.34)	(1.33)
Cognitive uncertainty		7.77*		14.8***		12.0***
		(4.01)		(4.52)		(3.25)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1918	1918	1848	1848	3766	3766
R^2	0.44	0.46	0.35	0.36	0.39	0.41

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's absolute normalized certainty equivalent, computed as midpoint of the switching interval divided by the non-zero payout. The sample includes choices from the baseline and compound gambles, where for comparability the set of baseline gambles is restricted to gambles with payout probabilities of 10%, 25%, 50%, 75%, and 90%, see Figure 4. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

20 balls are blue. If a blue ball gets drawn: get $0.

In addition to this treatment, labeled *High Default Risk*, we also implement treatment *Low Default Risk*. Here, we implement the same lotteries as in *High Default Risk*, yet we split the zero-payout state into nine payoff-equivalent states with different probability colors. For example, the lottery above would be described as:

Out of 100 balls, 80 are red. If a red ball gets drawn: get $20.
2 balls are blue. If a blue ball gets drawn: get $0.
2 balls are black. If a black ball gets drawn: get $0.
2 balls are white. If a white ball gets drawn: get $0.
...
4 balls are yellow. If a yellow ball gets drawn: get $0.

This lottery is identical to the one described above in terms of the objective payout profile. Still, we hypothesize that this manipulation shifts the probability weighting function towards zero.

In total, 300 subjects participated in these two treatments, which we implemented in a between-subjects design with random assignment to treatments within experimental

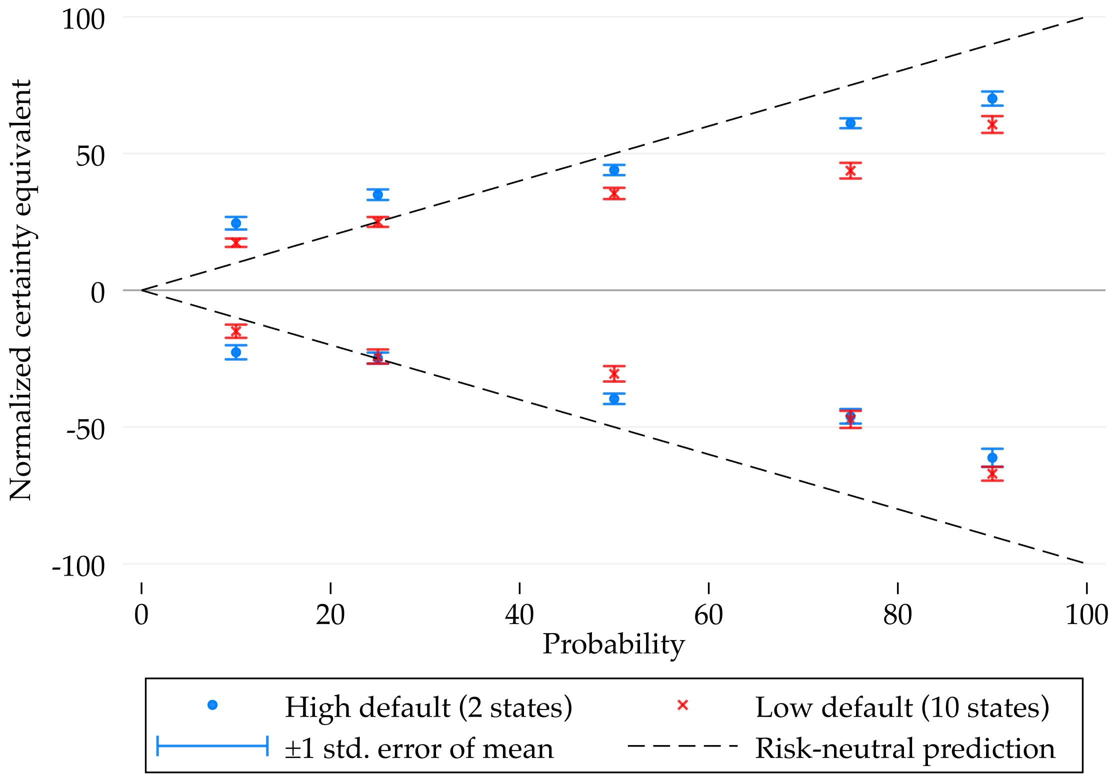


Figure 5: Probability weighting function separately for treatments *High Default Risk* and *Low Default Risk*. The plot shows averages and corresponding standard error bars. Normalized certainty equivalents (implied probability weights) are computed as certainty equivalent divided by payout probability. The figure is based on 1,757 certainty equivalents of 300 subjects.

sessions. All procedures other than the ones described here (and corresponding comprehension questions) were identical to the ones in condition *Baseline Risk*.

3.4.2 Results

First note that cognitive uncertainty does not vary across these two treatments ($p = 0.898$), see the histograms in Figure 15 in Appendix B.1. This lends credence to our implicit assumption that our experimental manipulation only affects the mental default but not cognitive uncertainty.

Figure 5 shows average normalized certainty equivalents, separately for treatments *High Default Risk* and *Low Default Risk*. We find that, in the gain domain, the probability weighting function is significantly shifted downwards towards zero with 10 states (a low default), as hypothesized. In the loss domain, our framework would predict that the weighting function is shifted upwards towards zero. We only find mixed evidence for this prediction: the weighting function appears to move up for low and intermediate probabilities but not for high probabilities.

Table 4: Choice under risk: Treatments *Low Default Risk* and *High Default Risk*

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
0 if *High Default*, 1 if *Low Default*	-10.3***	-9.95***	-2.35	-2.10	-6.33***	-5.97***
	(1.82)	(1.84)	(2.15)	(2.13)	(1.49)	(1.49)
Probability of payout	0.61***	0.61***	0.57***	0.57***	0.59***	0.59***
	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)
Probability of payout × Cognitive uncertainty	-0.47***	-0.47***	-0.24*	-0.25**	-0.34***	-0.35***
	(0.10)	(0.10)	(0.13)	(0.13)	(0.09)	(0.09)
Cognitive uncertainty	15.6***	15.4***	21.9***	22.1***	18.4***	18.6***
	(5.93)	(5.90)	(8.32)	(8.45)	(5.12)	(5.16)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	881	881	876	876	1757	1757
R^2	0.41	0.42	0.30	0.32	0.34	0.35

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's absolute normalized certainty equivalent, computed as midpoint of the switching interval divided by the non-zero payout. The sample includes choices from treatments *Low Default Risk* and *High Default Risk*. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 4 provides a corresponding regression analysis that confirms these visual patterns. Columns (1)–(3) analyze gain lotteries. Here, normalized certainty equivalents (observed risk tolerance) are 10 percentage points lower in the *Low Default Risk* condition. In the case of losses, the regression coefficient of the low default condition is negative – as predicted by our framework – but not statistically significant ($p = 0.15$). A potential (post-hoc) explanation for this null result is that, in all treatments, the choice data in the loss domain appear to be considerably more noisy than in the gain domain. This can be inferrred from the difference in R^2 between columns (1) and (3) in Table 4 and similar patterns in all other tables above. Either way, the treatment effect of the low default is statistically significant in the pooled gains and losses sample.

Table 2 provides a parametric analysis of the *Low Default Risk* data, pooled across gains and losses.[11] We find that the elevation parameter δ decreases from $\hat{\delta} = 0.81$ in the baseline gambles to $\hat{\delta} = 0.54$ with 10 states of the world, where these parameters are again tightly estimated. Figure 16 in Appendix B.1 plots this estimated probability weighting function and compares it to those estimated for the other treatment conditions.

[11] The estimates for *High Default Risk* are almost identical to those in *Baseline* and hence suppressed for brevity.

3.5 Robustness Checks

The pre-registration specified that we will conduct our analyses on three different samples: (i) excluding extreme outliers, as done thus far; (ii) using all data; and (iii) excluding "speeders," defined as subjects in the bottom decile of the response time distribution. Thus, Appendices B.2 and B.3 reproduce the analysis above on the full sample and excluding speeders. The results are always very similar. Two minor exceptions are that (i) the treatment difference between *Low Default Risk* and *High Default Risk* is statistically significant also for losses when we exclude speeders (the p-value was $p = 0.15$ in the baseline analysis above) and (ii) the interaction between cognitive uncertainty and payout probability in treatment *Baseline Risk* is marginally not significant for losses (it is statistically significant in the baseline analysis).

4 Belief Updating

4.1 Experimental Design

Our experimental design strategy for belief updating closely mirrors the one for choice under risk: we (i) supplement an established experimental design from the literature with a measurement of cognitive uncertainty; (ii) document a correlation between cognitive uncertainty and the magnitude of compression in subjects' beliefs; (iii) exogenously manipulate cognitive uncertainty using a compound manipulation; and (iv) vary the location of the mental default by increasing the number of states of the world.

4.1.1 Measuring Belief Updating

In designing a structured belief updating task, we follow the recent review and meta-study by Benjamin (2019) on so-called "balls-and-bags" or "bookbags-and-pokerchips" experiments. In treatment *Baseline Beliefs*, there are two bags, A and B. Both bags contain 100 balls, some of which are red and some of which are blue. The computer randomly selects one of the bags according to a pre-specified base rate. Subjects do not observe which bag was selected. Instead, the computer selects one or more of the balls from the selected bag at random (with replacement) and shows them to the subject. The subject is then asked to state a probabilistic guess that either bag was selected. We visualized this procedure for subjects using the image at the top right of Figure 6.

The three key parameters of this belief updating problem are: (i) the base rate $r \in \{10, 30, 50, 70, 90\}$ (in percent), which we operationalized as the number of cards out of 100 that had "bag A" or "bag B" written on them; (ii) the signal diagnosticity $q \in \{70, 90\}$, which is given by the number of red balls in bag A and the number of blue balls in bag B (we only implemented symmetric signal structures such that $P(red|A) = P(blue|B)$); and (iii) the number of randomly drawn balls N. These parameters were randomized across trials.

Each subject completed six belief updating tasks. In each task, they were asked to state a probabilistic belief (0-100) that bag A got selected. The computer automatically and instantaneously showed the corresponding subjective probability that bag B got selected. The decision screen contained information on the base rate, the signal diagnosticity, the number of drawn balls, and their color, see Figure 23 in Appendix C.1 for a decision screenshot.

Financial incentives were implemented through the binarized scoring rule (Hossain and Okui, 2013). Here, subjects had a chance of winning a prize of \$10. The probability of receiving the prize was given by $\pi = 100 - 0.04 * (b - t)^2$, where b is the guess (in %) and t the truth (0 or 100).

With probability 5 in 6, a belief updating task was implemented using the design discussed above, and with probability 1 in 6 in a compound design. We return to the compound data in Section 4.3 and focus on the baseline problems for now.

4.1.2 Measuring Cognitive Uncertainty

Our main measure of cognitive uncertainty in belief updating is very similar to the one for choice under risk, both conceptually and implementation-wise. The instructions explained the concept of an "optimal guess." This guess, we explained to subjects, uses the laws of probability to compute a statistically correct statement of the probability that either bag was drawn, based on Bayes' rule. We highlighted that this optimal guess does not rely on information that the subject does not have.

After subjects had indicated their probabilistic belief that either bag was drawn, the next decision screen elicited cognitive uncertainty. Here, we asked subjects how certain they are that their own guess equals the optimal guess for this task. Operationally, similarly to the case of choice under risk, subjects navigated a slider to calibrate the statement "I am certain that the optimal guess is between a and b.", where a and b collapsed to the subject's own previously indicated guess in case the slider was moved to the very right. For each of the 30 possible ticks that the slider was moved to the left, a decreased and b increased by one unit. a was bounded from below by zero and b bounded from above by 100. Again, we did not set a default: subjects had to click somewhere on the slider in order to proceed. Figure 6 shows a screenshot of the elicitation screen. For ease of interpretation, we again normalize this measure to be between zero and one.

Just like our measure of cognitive uncertainty in choice under risk, this one is not financially incentivized. However, in the case of belief updating, it is possible to devise

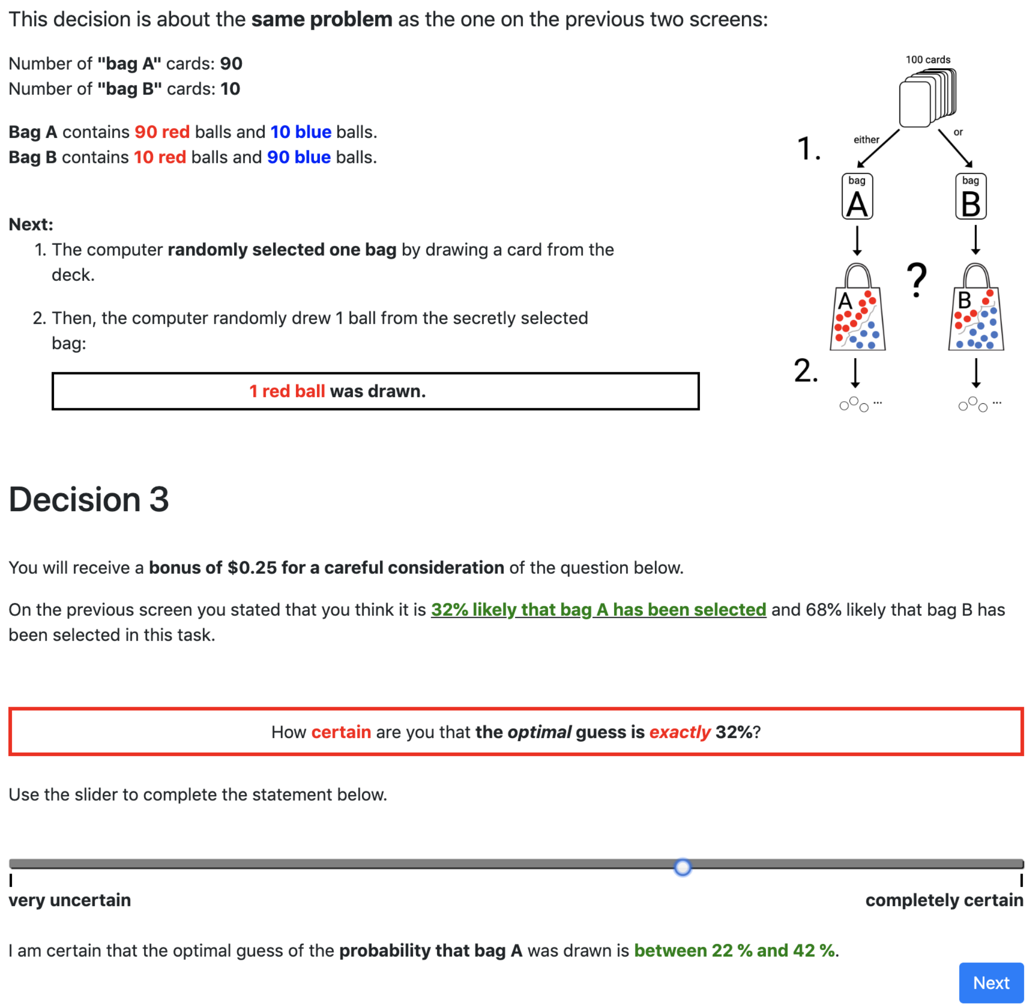


Figure 6: Decision screen to elicit cognitive uncertainty in belief updating

an incentivized measure because here an objectively correct response (the Bayesian posterior) exists. Thus, we additionally elicited a second measure of cognitive uncertainty from each participant: their willingness-to-pay (WTP) for replacing their own guess with the optimal (Bayesian) guess. To this effect, before subjects stated their own guess, they received an endowment of $3 for each task and then indicated how much of this budget they would at most be willing to pay to replace their guess. Subjects' WTP was elicited using a direct Becker-deGroot-Marschak elicitation mechanism. That is, we randomly drew a price $p \sim U[0,3]$ and the guess was replaced iff $p \leq$ WTP. See Figure 24 in Appendix C.1 for a screenshot.

To maximize statistical power, subjects' WTP and the resulting replacement of their own decision was only implemented in randomly selected 10% of all tasks. To avoid concerns about hedging, this uncertainty was resolved before subjects stated their own posterior guess. The timeline of each task was hence as follows: (i) observe game parameters; (ii) indicate WTP; (iii) find out whether own guess or Bayesian guess potentially counts for payment; (iv) state own posterior guess; and (v) indicate cognitive uncertainty range. The analysis below excludes those tasks in which a subject's guess got replaced by the optimal guess (3% of all data), though we have verified that virtually identical results hold if these (non-incentivized) guesses are included.

Figures 25 and 26 in Appendix C.1 show histograms of the cognitive uncertainty measure as well as subjects' WTP. Both measures exhibit considerable variation. Average cognitive uncertainty is 0.31, with a median of 0.33 and a standard deviation of 0.27. 85% of our data indicate a strictly positive cognitive uncertainty. The average WTP is \$0.85 with a median of \$0.50 and a standard deviation of 0.93.[12]

The two measures exhibit a correlation of $\rho = 0.21$. While not incentivized, we view the cognitive uncertainty measure as our primary measure because (i) by its nature, and as exemplified by this paper, it is easily portable across different experimental contexts and decision situations; (ii) it is more fine-grained and exhibits more variation (26% of all WTPs are zero, perhaps due to some loss aversion vis-a-vis giving up safe money); and (iii) it is not confounded by risk aversion. Still, below we verify that all of our results are robust to using the WTP measure.

4.1.3 Logistics and Pre-Registration

Based on a pre-registration, we recruited $N = 700$ completes for treatment *Baseline Beliefs*. After reading the instructions, participants completed a set of four comprehension questions. As in the choice under risk experiments, participants who answered one or more questions incorrectly were immediately routed out of the experiment and do not count towards the number of completes. Similarly, subjects are excluded from the analysis if they failed an attention check, as specified in the pre-registration. In total, 49% of all prospective participants were screened out in the comprehension checks. Of those subjects that passed, 6% were screened out based on the attention check.

In terms of timeline, subjects first completed the belief updating tasks discussed above. Second, as in the choice under risk experiments, we elicited their survey expectations about various economic variables, discussed in Section 5. Finally, participants completed a short demographic questionnaire and an eight-item Raven matrices IQ test. As in the risky choice experiments, one of the three parts of the experiments (belief updating, survey expectations, or Raven test) was randomly selected for payment.

Average earnings are \$4.80 with a median completion time of 23 minutes. The experiments were pre-registered under the same AEA RCT trial as discussed above. Screenshots

[12]As a basic validity check, in a small sample of 161 updating tasks, we implemented a signal diagnosticity of $d = 100$, so that the selected bag is deterministically revealed. In these tasks, cognitive uncertainty essentially drops to zero: the distribution of both the cognitive uncertainty range and subjects' WTP has a median of zero, with means of 0.06 and 0.26.

of the entire experiment, including instructions and control questions, can be found in Appendix E.

4.2 Cognitive Uncertainty and Belief Updating

4.2.1 Non-Parametric Analyses

As in the analysis of choice under risk, we begin by excluding extreme outliers to keep the analysis clean. As specified in the pre-registration, these are defined as subjective probability p_s and Bayesian posteriors p_o such that $p_s < 25 \wedge p_o > 75$ or $p_s > 75 \wedge p_o < 25$. This is the case for 5% of all data. We report robustness checks using the full sample below.

Figure 1 in the Introduction depicts the "belief weighting function" that we estimate in our data: the inverse S-shaped relationship between average stated and Bayesian posteriors. Figure 7 replicates this figure separately for subjects above or below average cognitive uncertainty as defined by our unincentivized cognitive uncertainty range. We see that, over the entire support of Bayesian posteriors, stated posteriors are more compressed towards 50:50 for subjects with higher cognitive uncertainty. Figure 30 in Appendix C.1 replicates this figure based on the financially incentivized WTP measure, with very similar results.

Columns (1)–(3) of Table 5 provide a corresponding econometric analysis. Here, we regress a subject's stated posterior on (i) the Bayesian posterior; (ii) cognitive uncertainty; and (iii) their interaction term. We find that subjects with higher cognitive uncertainty respond considerably less to variation in the objectively correct answer: the quantitative magnitude of the regression coefficients suggests that the slope of the regression line between stated posterior and Bayesian posterior is 0.80 for subjects with measured cognitive uncertainty of zero, yet only 0.40 for subjects who state maximal cognitive uncertainty of one.

4.2.2 Parametric Estimations

Grether regressions. A different way of analyzing our data is through the lens of so-called Grether regressions, see Grether (1992), El-Gamal and Grether (1995), and Benjamin (2019). This specification is derived by expressing Bayes' rule in logarithmic form, which implies a linear relationship between the posterior odds, the prior odds, and the likelihood ratio:

$$ln\left(\frac{b(A|s)}{b(B|s)}\right) = \beta_1 ln\left(\frac{p(A)}{p(B)}\right) + \beta_2 ln\left(\frac{p(s|A)}{p(s|B)}\right), \tag{17}$$

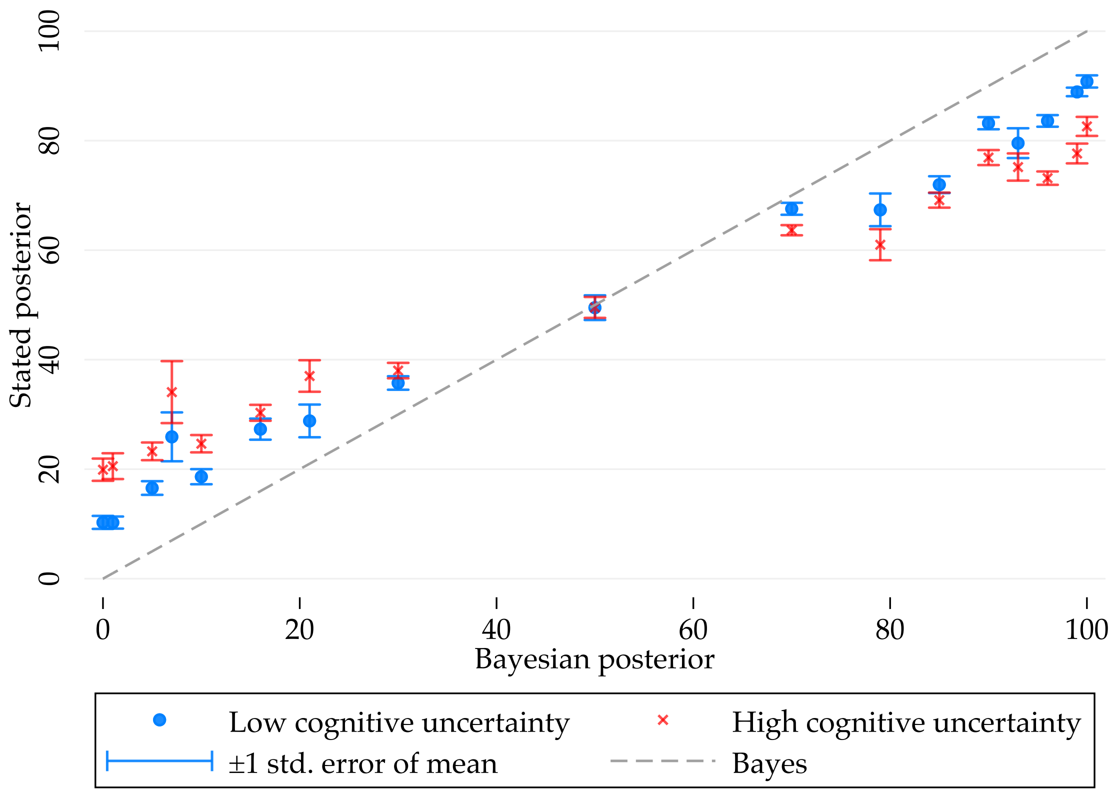


Figure 7: Relationship between average stated and Bayesian posteriors, separately for subjects above / below average cognitive uncertainty. The partition is done separately for each Bayesian posterior. Bayesian posteriors are rounded to the nearest integer. We only show buckets with more than ten observations. The figure is based on 3,187 beliefs of 700 subjects.

where $b(\cdot)$ denotes the stated posterior belief, A and B the two bags (states of the world), s a signal history, the first fraction on the right-hand side the prior odds, and the second term on the right-hand side the likelihood ratio. This formulation is attractive because it allows an assessment of the sensitivity of people's posteriors to variation in both the base rate and the likelihood ratio in a simple linear regression framework. The standard finding in the literature is that $\hat{\beta}_1 < 1$ and $\hat{\beta}_2 < 1$, even though Bayesian updating implies coefficients of one. This evidence hence points to paramount underreaction (insensitivity) to both the prior odds and the likelihood ratio (Benjamin, 2019).

Columns (4)–(7) of Table 5 implement these regressions using our data. As shown in column (4), similarly to past work, we find regression coefficients that are substantially smaller than one. In fact, our estimates are well within the range of results discussed in Benjamin's (2019) meta-study. However, as shown in columns (5)–(7), these insensitivities are significantly more pronounced for subjects with higher cognitive uncertainty: the responsiveness to the likelihood ratio (prior odds) decreases by 36% (65%) for subjects with maximal cognitive uncertainty relative to those with zero cognitive uncertainty.

These patterns suggest that (at least a part of) what this literature has identified as

Table 5: Belief updating: Baseline tasks

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.69***	0.80***	0.80***			
	(0.01)	(0.01)	(0.01)			
Bayesian posterior × Cognitive uncertainty		-0.39***	-0.39***			
		(0.04)	(0.04)			
Cognitive uncertainty		16.6***	16.4***		-0.14**	-0.16**
		(2.32)	(2.32)		(0.07)	(0.07)
Log [Likelihood ratio]				0.41***	0.44***	0.44***
				(0.01)	(0.02)	(0.02)
Log [Prior odds]				0.42***	0.52***	0.52***
				(0.02)	(0.03)	(0.03)
Log [Likelihood ratio] × Cognitive uncertainty					-0.16***	-0.16***
					(0.04)	(0.04)
Log [Prior odds] × Cognitive uncertainty					-0.34***	-0.35***
					(0.07)	(0.07)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	3187	3187	3187	3104	3104	3104
R^2	0.72	0.73	0.74	0.62	0.63	0.63

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

base rate neglect or conservatism are in fact not independent psychological phenomena but instead generated by people shrinking their responses towards 50:50 due to cognitive uncertainty. Table 16 in Appendix C.2 replicates Table 5 using the WTP instead of the cognitive uncertainty measure, with very similar results.

A "Belief Weighting Function". To formally document the close analogy between compressed response patterns in choice under risk and belief formation, we proceed by estimating a "belief weighting function" that has the same functional form as the probability weighting function that we estimated in Section 3.2.2:

$$b = \frac{\delta p^{\lambda}}{\delta p^{\lambda} + (1-p)^{\lambda}} + \epsilon, \tag{18}$$

where b now denotes a subject's posterior belief and p the relevant Bayesian posterior. Again, λ largely governs the sensitivity of the function to variations in objective probabilities and δ its elevation.

Table 6 summarizes the estimation results. We find that $\hat{\lambda}$ is significantly lower for subjects with above-average cognitive uncertainty ($\hat{\lambda} = 0.37$) than for those with below-

Table 6: Estimates of "belief weighting function"

Treatment / group	Sensitivity $\hat{\lambda}$	Elevation $\hat{\delta}$
Baseline Beliefs: all	0.48 (0.01)	1.01 (0.17)
Baseline Beliefs: high CU	0.37 (0.01)	1.01 (0.37)
Baseline Beliefs: low CU	0.58 (0.02)	1.01 (0.03)
Compound Beliefs: all	0.12 (0.01)	1.09 (0.12)
Low Default Beliefs: all	0.43 (0.01)	0.84 (0.03)

Notes. Estimates of equation (18), standard errors (clustered at subject level) reported in parentheses. CU = cognitive uncertainty (split at average).

average cognitive uncertainty ($\hat{\lambda} = 0.58$). Figure 31 in Appendix C.1 plots these estimated functions. Furthermore, the estimate of the insensitivity parameter λ is strikingly similar to that estimated in choice under risk, compare Table 2: in the full sample of subjects, we estimate $\hat{\lambda}_{beliefs} = 0.48$, while the corresponding estimate for choice under risk was $\hat{\lambda}_{risk} = 0.49$.

4.3 Exogenous Manipulation of Cognitive Uncertainty

4.3.1 Experimental Design

To manipulate cognitive uncertainty, we again resort to turning "reduced" problems into compound problems. Consider belief updating problems in which the base rate is given by 50:50 and the signal diagnosticity by $d \equiv P(A|red) = P(B|blue)$. In the compound version of these problems, the base rate is again 50:50, yet the diagnosticity is the outcome of a random integer draw, $d \sim U[d-10, d+10]$. It is straightforward to verify that these two problems give rise to the same mean Bayesian posterior. For instance, if a red ball gets drawn, the posterior in the reduced version equals the signal diagnosticity d because the prior is 50:50. In the compound version, the posterior is equally likely to be $d-10, d-9, \dots, d+10$, hence d in expectation.

As in choice under risk, we hypothesize that subjects exhibit higher cognitive uncertainty in compound than in reduced updating problems. Hence, by the logic of our framework, we expect that participants' beliefs in compound problems will be more compressed towards 50:50 and hence higher for low Bayesian posteriors and lower for high

Bayesian posteriors.[13]

As noted above, we implemented these compound belief updating problems as part of treatment *Baseline Beliefs*, where each belief updating problem had a 1 in 6 chance of being presented in a compound form. We collected 592 observations on compound belief updating problems.

4.3.2 Results

Relative to reduced updating problems, compound signal diagnosticities increase stated cognitive uncertainty by 33% and subjects' WTP for the Bayesian guess by 43%, on average. Figures 27 and 28 in Appendix C.1 show corresponding histograms. Thus, as in choice under risk, the compound manipulation produces a strong "first stage."

Figure 8 shows the results on stated beliefs. Here, we plot average stated posteriors as a function of Bayesian posteriors, separately for baseline and compound updating problems. Because in compound problems the base rate is always 50:50, the figure only includes data from tasks with a 50:50 base rate also for the baseline updating problems. We see that subjects' posteriors are substantially more compressed towards 50:50 in compound updating problems.

Columns (1) and (2) of Table 7 provide a corresponding regression analysis. The regression coefficients suggest that the sensitivity of stated posteriors to the Bayesian posterior is 0.72 in baseline updating problem, yet only 0.21 in compound updating problems. To provide an alternative perspective on the data, we again resort to Grether regressions, see columns (4)–(6). Because in compound updating problems the base rate is fixed at 50:50, the only explanatory variable of interest here is the log likelihood ratio. The results show that subjects always underreact to variations in the likelihood ratio (the regression coefficient is smaller than one in both reduced and compound updating tasks), yet this underreaction is substantially more pronounced under compound lotteries.

Finally, Table 6 again reports the results for estimations of the "belief weighting function" in equation (18). As suggested by the linear regression analyses above, we find that the sensitivity parameter is only $\hat{\lambda} = 0.12$ with compound signal diagnosticities, compared to $\hat{\lambda} = 0.48$ in the baseline updating problems. Figure 31 plots the "belief weighting function" that we estimate from the compound updating problems and compares them to the functions that we estimate in the baseline treatment.

[13] In contemporaneous work, Liang (2019) identifies increased underreaction under compound relative to reduced updating problems. This is in line with our hypothesis, but he does not measure cognitive uncertainty.

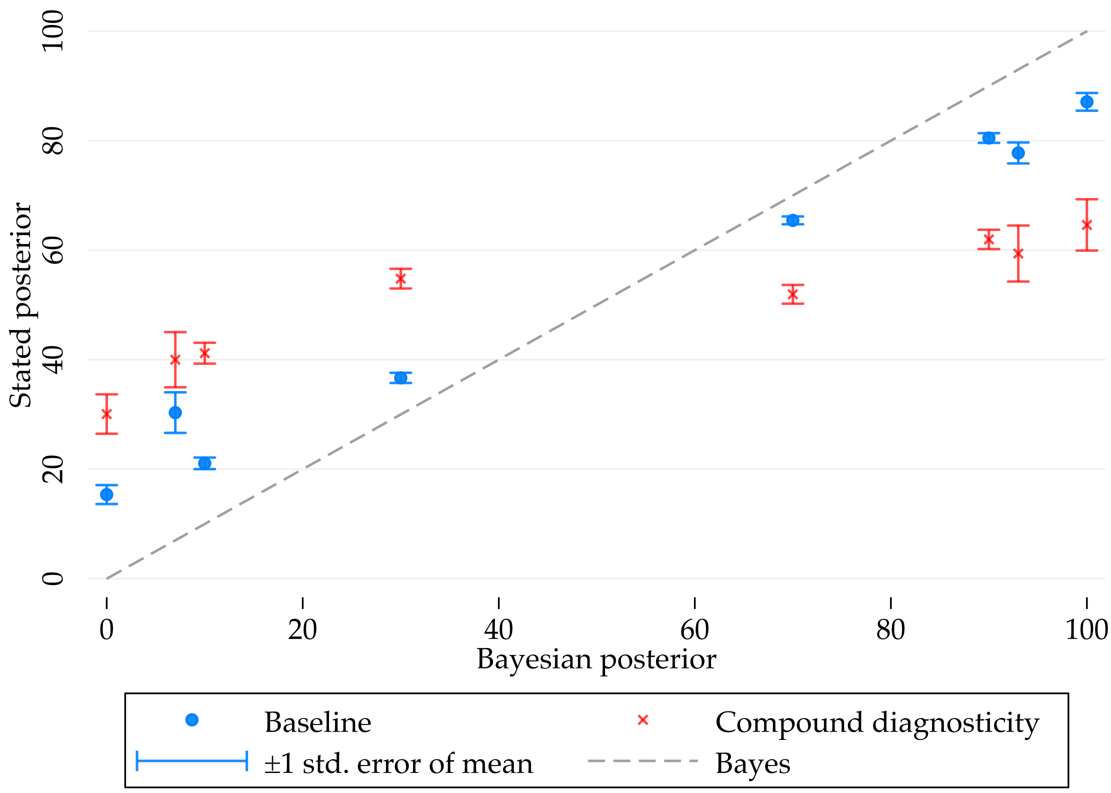


Figure 8: Stated average posteriors as a function of Bayesian posteriors, separately for reduced and compound belief updating problems. The plot shows averages and corresponding standard error bars. To allow for a valid comparison between baseline and compound updating problems, the sample is restricted to updating tasks in which the base rate is 50:50. Bayesian posteriors are rounded to the nearest integer. We only show buckets with more than ten observations. The figure is based on 1,947 beliefs of 691 subjects.

4.4 Exogenous Manipulation of the Mental Default

4.4.1 Experimental Design

In a final step of the analysis of belief updating, we exogenously manipulate the location of the mental default. To manipulate the default, we employ the same type of partition manipulation as in choice under risk and increase the number of states to ten. Under the maintained assumption that the default is given by the ignorance prior, this means that the default for each state is given by 10%. Our framework then predicts that the entire distribution of posterior beliefs is shifted downwards, leading to a drecrease in the parametric estimate of the elevation parameter δ.

Recall that in treatment *Baseline Beliefs*, an example updating problem is that the base rates for bags A and B are 70% and 30%, and the signal diagnosticity (number of red balls in bag A and number of blue balls in bag B) 70%. Now, in treatment *Low Default Beliefs*, we split the probability mass for bag B up into nine different bags. That is, there are now ten bags, labeled A through J. In the specific example above, the base

Table 7: Belief updating: Reduced versus compound signal diagnosticities

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.57***	0.72***	0.72***			
	(0.01)	(0.02)	(0.02)			
Bayesian posterior × 1 if compound problem		-0.51***	-0.51***			
		(0.03)	(0.03)			
1 if compound diagnosticity		26.4***	26.5***		0.0058	0.0091
		(1.75)	(1.74)		(0.05)	(0.05)
Log [Likelihood ratio]				0.37***	0.45***	0.45***
				(0.01)	(0.02)	(0.02)
Log [Likelihood ratio] × 1 if compound problem					-0.28***	-0.27***
					(0.02)	(0.02)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	1947	1947	1947	1890	1890	1890
R^2	0.51	0.60	0.60	0.47	0.53	0.53

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

rate for A would again be 70%, the one for B through I 3% each and the one for J 6%. Again, bag A would contain 70 red and 30 balls, and all bags B through J 30 red and 70 blue balls. That is, these bags have identical ball compositions.

Note that, regardless of what the actual draws of balls are, the Bayesian posterior for bag A having been selected is identical in the baseline version and the version with 10 bags. The reason is that under the state space {A; not A} the base rates and signal diagnosticities are identical. Thus, in treatment *Low Default Beliefs*, we asked participants to indicate their belief that bag A got selected, and the computer automatically showed the corresponding composite probability for one of the other bags having been selected.

300 subjects participated in treatment *Low Default Beliefs*, which was randomized within the same experimental sessions as treatment *Baseline Beliefs*. All procedures other than the ones described above were identical to the ones in *Baseline Beliefs*.

4.4.2 Results

Stated cognitive uncertainty is almost identical across conditions *Baseline Beliefs* and *Low Default Beliefs*, $p = 0.85$. This corroborates our implicit assumption that the experimental manipulation of increasing the number of bags only manipulates the mental default but not cognitive uncertainty.

Figure 9 shows average stated posteriors as a function of Bayesian posteriors, separately for treatments *Baseline Beliefs* and *Low Default Beliefs*. The results show that the

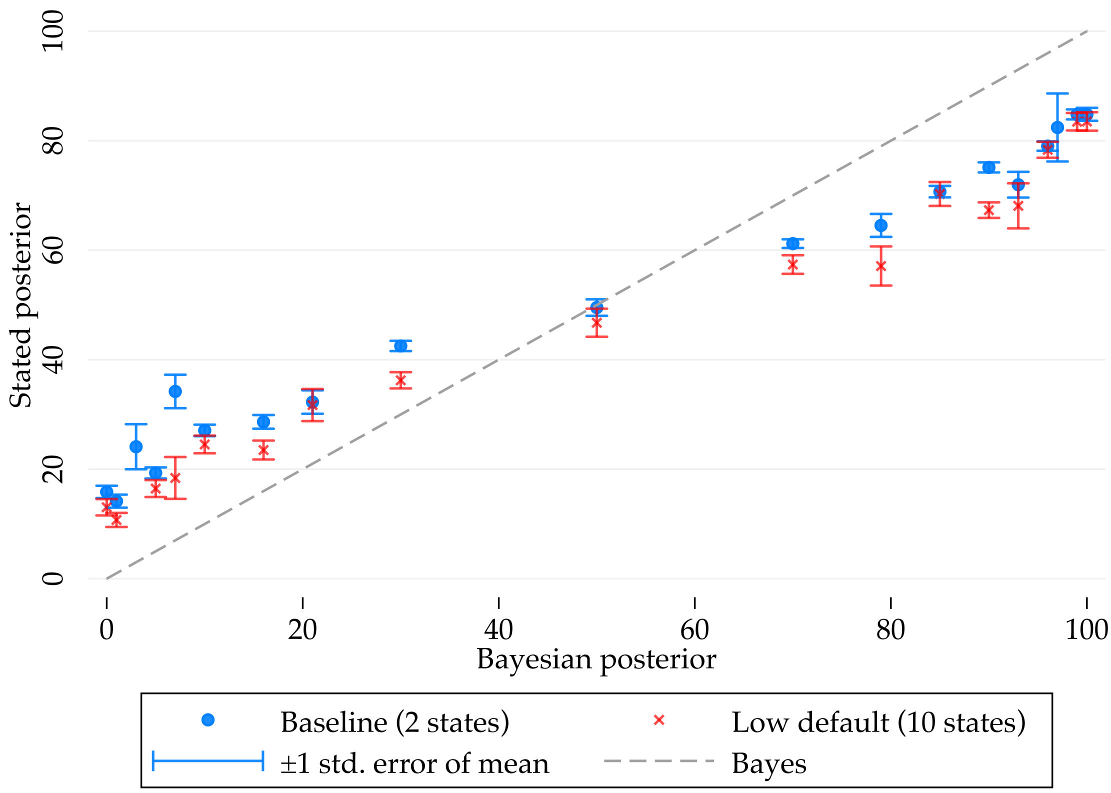


Figure 9: Stated average posteriors as a function of Bayesian posteriors, separately for treatments *Baseline Beliefs* and *Low Default Beliefs*. Bayesian posteriors are rounded to the nearest integer. We only show buckets with more than ten observations. The figure is based on 5,372 beliefs of 1,000 subjects.

entire distribution of subjects' beliefs is shifted down towards zero, consistent with our hypothesis that a larger state space induces shrinking towards a lower mental default. Table 8 provides a corresponding regression analysis that confirms these visual patterns.

Finally, Table 6 again reports the results for estimations of the "belief weighting function" in equation (18). We find that the elevation parameter in treatment *Low Default Beliefs* if only $\hat{\delta} = 0.84$ and hence substantially lower than in *Baseline beliefs* ($\hat{\delta} = 1.01$). Figure 31 visualizes the estimated functions.

4.5 Robustness Checks

As in choice under risk, the pre-registration for belief updating specified that we would conduct our analyses on three different samples: (i) excluding extreme outliers, as done thus far; (ii) using all data; and (iii) excluding "speeders," defined as subjects in the bottom decile of the response time distribution. Appendices C.3 and C.4 provide the analyses for the full same and excluding speeders. The results are always very similar.

Table 8: Belief updating: Low versus high mental default

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
0 if *Baseline*, 1 if *Low Default*	-3.75***	-4.15***	-4.32***	-0.25***	-0.27***	-0.27***
	(0.71)	(0.70)	(0.76)	(0.05)	(0.04)	(0.05)
Bayesian posterior	0.64***	0.75***	0.75***			
	(0.01)	(0.01)	(0.01)			
Bayesian posterior × Cognitive uncertainty		-0.39***	-0.39***			
		(0.03)	(0.03)			
Cognitive uncertainty		14.8***	14.7***		-0.20***	-0.21***
		(1.87)	(1.89)		(0.06)	(0.06)
Log [Likelihood ratio]				0.36***	0.40***	0.40***
				(0.01)	(0.01)	(0.01)
Log [Prior odds]				0.48***	0.58***	0.58***
				(0.02)	(0.03)	(0.03)
Log [Likelihood ratio] × Cognitive uncertainty					-0.16***	-0.16***
					(0.03)	(0.03)
Log [Prior odds] × Cognitive uncertainty					-0.35***	-0.35***
					(0.06)	(0.06)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	5372	5372	5372	5226	5226	5226
R^2	0.63	0.64	0.64	0.57	0.58	0.58

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

5 Survey Expectations

5.1 Experimental Design

Miscalibrated survey expectations about economic variables have been documented in a wide range of contexts, including beliefs about the macroeconomy, the stock market, or personal life experiences (Hurd, 2009; Manski, 2004). A common theme is the presence of 50:50 answers (Fischhoff and Bruine De Bruin, 1999). Drerup et al. (2017) suggest that the low explanatory power of survey expectations for economic behavior might reflect that some people do not even hold meaningful belief distributions, which is reminiscent of cognitive uncertainty. To illustrate the link between cognitive uncertainty and survey expectations, we elicit beliefs about three variables that have attracted attention in the literature: the structure of the national income distribution, inflation rates, and the development of the stock market. Again, we supplement measurements of beliefs about these variables with our measure of cognitive uncertainty.

5.1.1 Measuring Beliefs

To be able to financially incentive participants without going through the logistical hassle of waiting for future stock returns or inflation rates to have realized, we elicited beliefs about contemporaneous or past variables. Each participant was asked three questions that elicited their beliefs about some specific aspect of the income distribution, stock returns, and the inflation rate. The question about the income distribution reads as:

> *Assume that in 2018, we randomly picked a household in the United States. What do you think is the probability that this household earned less than USD y in 2018, before taxes and deductions?*

Beliefs about the performance of the stock market were elicited as:

> *The S&P 500 is an American stock market index that includes 500 of the largest companies based in the United States. We randomly picked a year X between 1980 and 2018. Imagine that someone invested $100 into the S&P 500 at the beginning of year X. What do you think is the probability that, at the end of that same year, the value of the investment was less than $y?*
> *(In other words, what do you think is the probability that the S&P 500 [lost more than z% of its value / gained less than z%, or decreased in value]?*

Finally, beliefs about the inflation rate were measured as:

> *The inflation rate in the United States measures the percentage change in the consumer price index, which reflects the price level of a comprehensive set of consumer goods and services purchased by households. The inflation rate in a given time period captures how much more or less expensive goods and services have become on average. We randomly picked a year X between 1980 and 2018. Imagine that, at the beginning of year X, the set of products that is used to compute the inflation rate cost $100. What do you think is the probability that, at the end of that same year, the same set of products cost less than $y?*
> *(In other words, what do you think is the probability that the inflation rate in year X was lower than z%?)*

The order of topics was randomized across participants. Across participants, y (and hence z) varies randomly such that the true probability ranges fom 0% to 100%.

Subjects' beliefs were financially incentivized using the same binarized scoring rule as discussed in Section 4, except that the prize a subject could win was $2. One of the three questions was randomly selected for payment.

5.1.2 Measuring Cognitive Uncertainty

To measure cognitive uncertainty, we again make use of the same elicitation tool as before. That is, subjects were asked how certain they are that their probabilistic guess is correct. To provide a response, subjects used a slider to calibrate the statement: "I am certain that the actual probability that [...] is between a and b.", where a and b collapsed to the subject's own previously indicated guess if the slider was moved to the very right. For each of the 30 possible ticks that the slider was moved to the left, a decreased and b increased by one probability point. a was bounded from below by zero and b bounded from above by 100. Figure 38 in Appendix D.1 shows a screenshot of the elicitation screen.

Figures 39–41 in Appendix D.1 show histograms of cognitive uncertainty for each question type. Overall, cognitive uncertainty is substantial in these contexts, in particular regarding the stock market and inflation rates.

5.1.3 Logistics and Pre-Registration

The elicitation of survey expectations took place with the same set of subjects that completed the choice under risk or belief updating tasks discussed in Sections 3 and 4. Thus, the total sample size is $N = 2,000$. As explained above, one of the three parts of the experiments (choice under risk or belief updating, survey expectations, or Raven matrices test) was randomly selected for payment. The elicitation of survey expectations and the corresponding hypotheses are all included in the pre-registration discussed above.

5.2 Results

As in Section 4, and as pre-registered, we begin by excluding extreme outliers, defined as $p_s < 25 \wedge p_o > 75$ or $p_s > 75 \wedge p_o < 25$, where p_s is the subjective probability and p_o the objective one. This results in the exclusion of 5% of all data.

Figure 10 shows average beliefs as a function of objective probabilities, separately for subjects with above and below average cognitive uncertainty. Again, we see that these "survey belief weighting functions" exhibit an inverse S-shape, yet this pattern is substantially more pronounced for subjects who indicate higher cognitive uncertainty. Table 23 in Appendix D.2 provides a corresponding non-parametric econometric analysis that confirms the statistical significance of this pattern. Furthermore, Appendices D.3 and D.4 again provide robustness checks in which we reproduce the analysis above (i) on the full sample of observations and (ii) excluding speeders.

Finally, Table 9 provides parametric estimations of the sensitivity parameter λ and the elevation parameter δ, following the same methodology as for belief updating by

Table 9: Estimates of "survey expectations weighting function"

Task / group	Sensitivity $\hat{\lambda}$	Elevation $\hat{\delta}$
Income distribution: high CU	0.51 (0.02)	1.08 (0.04)
Income distribution: low CU	0.75 (0.02)	1.27 (0.04)
Stock market performance: high CU	0.19 (0.01)	0.82 (0.03)
Stock market performance: low CU	0.45 (0.02)	0.84 (0.04)
Inflation rates: high CU	0.22 (0.01)	0.97 (0.03)
Inflation rates: low CU	0.47 (0.02)	0.98 (0.05)

Notes. Estimates of equation (18) for survey expectations, standard errors (clustered at subject level) reported in parentheses. CU = cognitive uncertainty (split at average).

estimating equation (18). The estimates confirm that the sensitivity parameter is always substantially lower for high cognitive uncertainty subjects.

6 Heterogeneity in Cognitive Uncertainty

In the final part of the paper, we shed some more light on the heterogeneity in cognitive uncertainty. From an ex ante perspective, there are two plausible accounts of variation in cognitive uncertainty. First, it is conceivable that individuals exhibit reasonably stable cognitive uncertainty "types," so that a large part of the variation in cognitive uncertainty is between rather than within subjects. Second, it is conceivable that cognitive uncertainty varies dramatically across tasks, with little evidence for consistent types at the individual level.

To decompose cognitive uncertainty into between- and within-subject variation, we separately look at the choice under risk, belief updating, and survey expectations data. Within each dataset, we regress the collection of cognitive uncertainty statements on subject fixed effects.[14] We find that the variance explained is 44% in choice under risk, 53% in belief updating, and 60% in survey expectations. It is worth pointing out that these numbers represent lower bounds for the fraction of the true variation that is due to between-subject variation, as all measurement error gets soaked up by the residual

[14]Treatment fixed effects always explain less than 1% of the variation in the data.

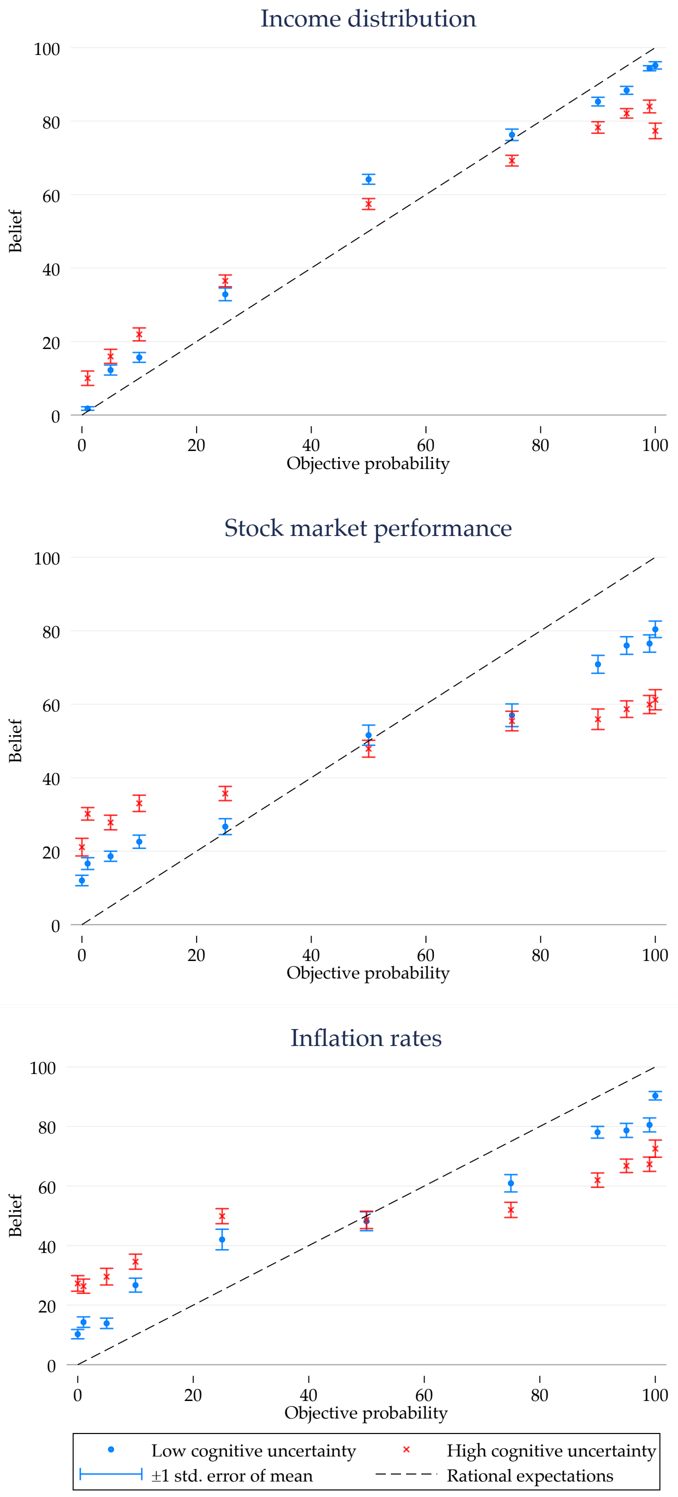


Figure 10: Survey beliefs as a function of objective probabilities, separately for subjects above / below average cognitive uncertainty. The partition is done separately for each probability bucket. In the top panel, the question asks for the probability that a randomly selected U.S. household earns less than \$x ($N = 1,974$). In the middle panel, the question asks for the probability that in a randomly selected year the S&P500 increased by less than x% ($N = 1,887$). In the bottom panel, the question asks for the probability that in a randomly selected year the inflation rate was less than x% ($N = 1,842$).

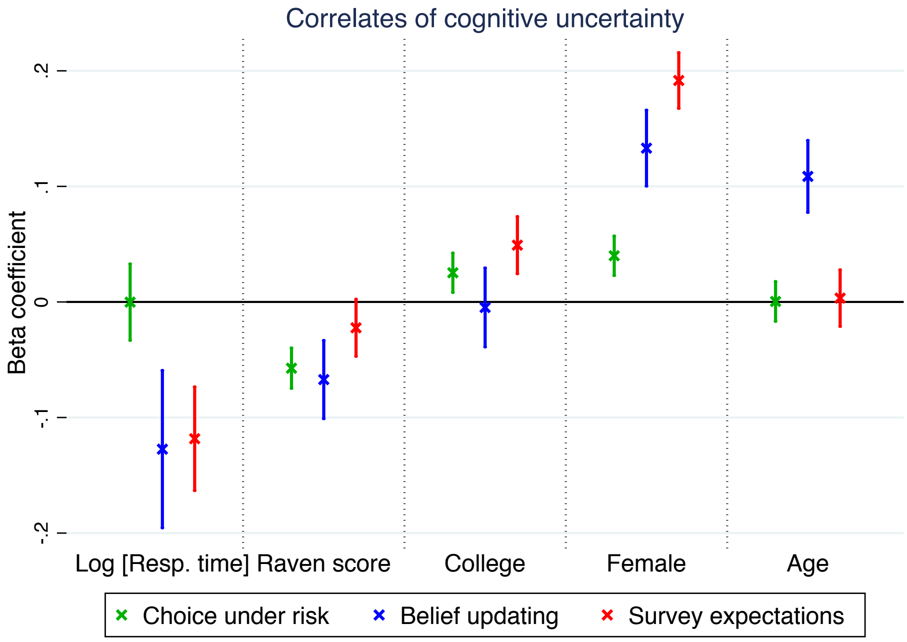


Figure 11: Correlates of average cognitive uncertainty. The figure shows the standardized beta coefficients of regressions of a subject's average cognitive uncertainty on different variables, controlling for treatment fixed effects. The values on the y-axis show the percent change in cognitive uncertainty that is associated with a 1% increase in the explanatory variable of interest. The beta coefficients are estimated conditional on treatment fixed effects. Response times are computed as total completion time within the relevant part of the experiment. $N = 1,000$ observations for choice under risk and belief updating and $N = 2,000$ observations for survey expectations.

and hence by "within-subject variation." We conclude from this exercise that cognitive uncertainty varies systematically across participants.

Next, we investigate correlates of the variation in cognitive uncertainty across individuals. For this purpose, we relate subjects' cognitive uncertainty in each of the three decision domains to a vector of individual characteristics: the score on an eight-item Raven matrices IQ test as a proxy for cognitive skills, educational attaiment, response times as a proxy for cognitive effort, gender, and age. All of these correlational analyses except for the one with response times were pre-registered. Figure 11 summarizes the results, separately for each of the types of experiments reported above. The figure shows the results of different regressions, each of which relates average (subject-level) cognitive uncertainty to a different subject-level variable, controlling for treatment fixed effects. The figure reports standardized beta coefficients, so that the y-axis shows the percent change in cognitive uncertainty that is associated with a 1% increase in the explanatory variable of interest. While the results are mixed overall, the strongest and most consistent correlations reflect that women, people who take less time to complete the experiment, and people with lower IQ test scores report higher cognitive uncertainty.

7 Conclusion

This paper has formally defined and introduced an experimental measurement of *cognitive uncertainty*: people's subjective uncertainty about the rational solution to a decision problem. As we have documented using belief updating and survey expectations data, such cognitive uncertainty does not just reflect preference uncertainty but captures a more general uncertainty about how to behave optimally.

Based on a simple formal framework that draws from existing theories, we have argued that cognitive uncertainty induces people to shrink probabilities towards a simple ignorance prior. This idea both reconciles existing evidence and makes new predictions. To argue our case, the paper has brought together decision tasks on choice under risk, belief updating, and survey expectations, all of which generate a stylized pattern of inverse S-shaped response functions. Our results show that, across all of these perhaps seemingly-unrelated decision domains, experimental participants with higher cognitive uncertainty exhibit more strongly compressed response functions. Moreover, in an attempt to provide causal evidence for our formal framework, we have exogenously manipulated both the magnitude of cognitive uncertainty and the location of the ignorance prior, and have identified predictable changes in subjects' beliefs and behaviors in response to these treatment manipulations.

We believe that the concept of cognitive uncertainty is likely to be important also outside of the specific domains of belief formation and choice under risk that we study in this paper. By providing a simple and portable experimental tool that allows to measure cognitive uncertainty in a quantitative fashion, our paper hence opens up the possibility for future experimental work on the relationship between cognitive uncertainty and economic decision making. A key implication of cognitive uncertainty in combination with an ignorance prior is that it generates insensitivity to objective probabilities, or, more generally, to the parameters of a decision problem. We believe that there are many potential applications of this idea to decision domains outside of risk and belief updating, in particular given that insensitivities of unknown origin pervade the empirical and experimental literatures.

References

Agranov, Marina and Pietro Ortoleva, "Stochastic choice and preferences for randomization," *Journal of Political Economy*, 2017, *125* (1), 40–68.

Ambuehl, Sandro and Shengwu Li, "Belief updating and the demand for information," *Games and Economic Behavior*, 2018, *109*, 21–39.

Andreoni, James and Michael A Kuhn, "Is It Safe to Measure Risk Preferences? A Comparison of Four Techniques," *Working Paper*, 2019.

Benjamin, Daniel J, "Errors in Probabilistic Reasoning and Judgmental Biases," in "Handbook of Behavioral Economics" 2019.

Bernheim, B Douglas and Charles Sprenger, "Direct tests of cumulative prospect theory," *Available at SSRN 3350196*, 2019.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer, "Salience theory of choice under risk," *Quarterly Journal of Economics*, 2012, *127* (3), 1243–1285.

_ , _ , and _ , "Memory, Attention, and Choice," *Working Paper*, 2017.

Bruhin, Adrian, Helga Fehr-Duda, and Thomas Epper, "Risk and rationality: Uncovering heterogeneity in probability distortion," *Econometrica*, 2010, *78* (4), 1375–1412.

Butler, David J and Graham C Loomes, "Imprecision as an account of the preference reversal phenomenon," *American Economic Review*, 2007, *97* (1), 277–297.

Chater, Nick, Mike Oaksford et al., *The probabilistic mind: Prospects for Bayesian cognitive science*, OUP Oxford, 2008.

da Silveira, Rava Azeredo and Michael Woodford, "Noisy Memory and Over-Reaction to News," in "AEA Papers and Proceedings," Vol. 109 2019, pp. 557–61.

DellaVigna, Stefano and Devin Pope, "Predicting experimental results: who knows what?," *Journal of Political Economy*, 2018, *126* (6), 2410–2456.

Dimmock, Stephen G, Roy Kouwenberg, and Peter P Wakker, "Ambiguity attitudes in a large representative sample," *Management Science*, 2015, *62* (5), 1363–1380.

Drerup, Tilman, Benjamin Enke, and Hans-Martin Von Gaudecker, "The precision of subjective data and the explanatory power of economic models," *Journal of Econometrics*, 2017, *200* (2), 378–389.

El-Gamal, Mahmoud A. and David M. Grether, "Are People Bayesian? Uncovering Behavioral Strategies," *Journal of the American statistical Association*, 1995, *90* (432), 1137–1145.

Enke, Benjamin, "What You See Is All There Is," *Working Paper*, 2017.

_ **and Florian Zimmermann**, "Correlation Neglect in Belief Formation," *Review of Economic Studies*, 2019, *86* (1), 313–332.

_ **, Frederik Schwerter, and Florian Zimmermann**, "Associative Memory and Belief Formation," *Working Paper*, 2019.

Esponda, Ignacio and Emanuel Vespa, "Hypothetical Thinking: Revisiting Classic Anomalies in the Laboratory," *Working Paper*, 2016.

Faisal, A Aldo, Luc PJ Selen, and Daniel M Wolpert, "Noise in the nervous system," *Nature reviews neuroscience*, 2008, *9* (4), 292.

Fischhoff, Baruch and Wändi Bruine De Bruin, "Fifty–fifty= 50%?," *Journal of Behavioral Decision Making*, 1999, *12* (2), 149–163.

Frydman, Cary and Lawrence Jin, "Efficient Coding and Risky Choice," *Working Paper*, 2019.

Gabaix, Xavier, "A sparsity-based model of bounded rationality," *The Quarterly Journal of Economics*, 2014, *129* (4), 1661–1710.

_ , "Behavioral inattention," in Douglas Bernheim, Stefano DellaVigna, and David Laibson, eds., *Handbook of Behavioral Economics-Foundations and Applications 2*, Elsevier, 2019, p. 261.

_ **and David Laibson**, "Myopia and discounting," Technical Report, National bureau of economic research 2017.

Gershman, Samuel J and Rahul Bhui, "Rationally inattentive intertemporal choice," *bioRxiv*, 2019, p. 680652.

Gillen, Ben, Erik Snowberg, and Leeat Yariv, "Experimenting with Measurement Error: Techniques with Applications to the Caltech Cohort Study," *Journal of Political Economy*, 2019.

Gonzalez, Richard and George Wu, "On the shape of the probability weighting function," *Cognitive psychology*, 1999, *38* (1), 129–166.

Graeber, Thomas, "Inattentive Inference," *Working Paper*, 2019.

Grether, David M, "Testing Bayes rule and the representativeness heuristic: Some experimental evidence," *Journal of Economic Behavior & Organization*, 1992, *17* (1), 31–57.

Griffiths, TL, C Kemp, and JB Tenenbaum, "Bayesian models of cognition," *Cambridge Handbook of Computational Cognitive Modeling*, 2008.

Halevy, Yoram, "Ellsberg revisited: An experimental study," *Econometrica*, 2007, *75* (2), 503–536.

Hossain, Tanjim and Ryo Okui, "The binarized scoring rule," *Review of Economic Studies*, 2013, *80* (3), 984–1001.

Hurd, Michael D, "Subjective probabilities in household surveys," *Annu. Rev. Econ.*, 2009, *1* (1), 543–562.

Imas, Alex, Michael Kuhn, and Vera Mironova, "Waiting to choose," *Working Paper*, 2016.

Izard, Véronique and Stanislas Dehaene, "Calibrating the mental number line," *Cognition*, 2008, *106* (3), 1221–1247.

Kahneman, Daniel and Amos Tversky, "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 1979, *47* (2), 263–292.

Khaw, Mel Win, Ziang Li, and Michael Woodford, "Risk aversion as a perceptual bias," Technical Report, National Bureau of Economic Research 2017.

Li, Chen, Uyanga Turmunkh, and Peter P Wakker, "Social and Strategic Ambiguity versus Betrayal Aversion," *Working Paper*, 2019.

Liang, Yucheng, "Learning from Unknown Information Sources," *Working Paper*, 2019.

Manski, Charles F, "Measuring expectations," *Econometrica*, 2004, *72* (5), 1329–1376.

Martínez-Marquina, Alejandro, Muriel Niederle, and Emanuel Vespa, "Failures in Contingent Reasoning: The Role of Uncertainty," *American Economic Review*, 2019.

Meyniel, Florent, Mariano Sigman, and Zachary F Mainen, "Confidence as Bayesian probability: From neural origins to behavior," *Neuron*, 2015, *88* (1), 78–92.

Paolacci, Gabriele and Jesse Chandler, "Inside the Turk: Understanding Mechanical Turk as a participant pool," *Current Directions in Psychological Science*, 2014, *23* (3), 184–188.

Robinson, Jonathan, Cheskie Rosenzweig, Aaron J Moss, and Leib LItman, "Tapped Out or Barely Tapped? Recommendations for How to Harness the Vast and Largely Unused Potential of the Mechanical Turk Participant Pool," 2019.

Shadlen, Michael N and William T Newsome, "The variable discharge of cortical neurons: implications for connectivity, computation, and information coding," *Journal of neuroscience*, 1998, *18* (10), 3870–3896.

Steiner, Jakub and Colin Stewart, "Perceiving prospects properly," *American Economic Review*, 2016, *106* (7), 1601–31.

Stocker, Alan A and Eero P Simoncelli, "Noise characteristics and prior expectations in human visual speed perception," *Nature neuroscience*, 2006, *9* (4), 578.

Tenenbaum, Joshua B, Thomas L Griffiths, and Charles Kemp, "Theory-based Bayesian models of inductive learning and reasoning," *Trends in cognitive sciences*, 2006, *10* (7), 309–318.

Trautmann, Stefan T and Gijs Van De Kuilen, "Ambiguity attitudes," *The Wiley Blackwell handbook of judgment and decision making*, 2015, *1*, 89–116.

Tversky, Amos and Daniel Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," *Journal of Risk and uncertainty*, 1992, *5* (4), 297–323.

Woodford, Michael, "Prospect theory as efficient perceptual distortion," *American Economic Review*, 2012, *102* (3), 41–46.

_ , "Modeling Imprecision in Perception, Valuation and Choice," *Working Paper*, 2019.

Zhang, Hang and Laurence T Maloney, "Ubiquitous log odds: a common representation of probability and frequency distortion in perception, action, and cognition," *Frontiers in neuroscience*, 2012, *6*, 1.

ONLINE APPENDIX

A Model Extensions

A.0.1 Estimation of Model Parameters

We can structurally calibrate the model by making additional assumptions. Given (5), the median action across many agents is

$$a^e(x) = \text{Median}(a^r(s)|x) = B\lambda x + B\lambda(1-\lambda)x^d. \tag{19}$$

The response of the median action to variation in x thus allows to identify the slope parameter b:

$$b = a^e_x(x) = \frac{da^e(x)}{dx} = B\lambda. \tag{20}$$

Data about people's actions alone thus provide no separate identification for λ. However, it is the additional measurement of cognitive uncertainty that provides the necessary traction: reformulating our definition from (8) using $B = \frac{b}{\lambda}$, we get

$$\frac{\lambda}{\sqrt{1-\lambda}} = \frac{|b|\sigma_x}{\sigma_{CU}}, \tag{21}$$

from which λ can be estimated given suitable assumptions about σ_x.

A.0.2 Nonlinear Version

We now allow the rational action to be a nonlinear function of x, so the analogue of (2) becomes

$$a^r = A(x). \tag{22}$$

We make the simplifying assumptions that, first, the agent still chooses an action based on the posterior expectation about x, as has been done in the previous literature (Gabaix, 2014, 2019),

$$a(s) = A(\mathbb{E}[x|s]), \tag{23}$$

second, that the function A is strictly monotone, such that it can again be identified from the median action a^e,

$$a^e(x) = \text{Median}(a(s)|x) = A(\lambda x + (1-\lambda)x^d), \tag{24}$$

and third, that $x^d = 0$, which is merely a notational simplification. In our empirical applications we will slightly deviate from this and elicit a different type of interval that is wider than the interquartile range, but we here stick to the notation of cognitive uncertainty as denoting one perceived standard deviation around the action for simplicity.

We define cognitive uncertainty analogously to (8) as the agent's perceived uncertainty about his rational action,

$$\sigma_{CU}(x) = \left| A\left(\lambda x + \frac{1}{2}\sqrt{1-\lambda}\sigma_x \right) - A\left(\lambda x - \frac{1}{2}\sqrt{1-\lambda}\sigma_x \right) \right|. \tag{25}$$

At the median, using $a^e(x) = A(\lambda x)$ yields

$$\sigma_{CU}(x) = \left| a^e\left(x + \frac{1}{2}\frac{\sqrt{1-\lambda}}{\lambda}\sigma_x \right) - a^e\left(x - \frac{1}{2}\frac{\sqrt{1-\lambda}}{\lambda}\sigma_x \right) \right|. \tag{26}$$

A Taylor expansion of (26) gives

$$\sigma_{CU} = \left| a^{e\prime}(x) \right| \frac{\sqrt{1-\lambda}}{\lambda}\sigma_x, \tag{27}$$

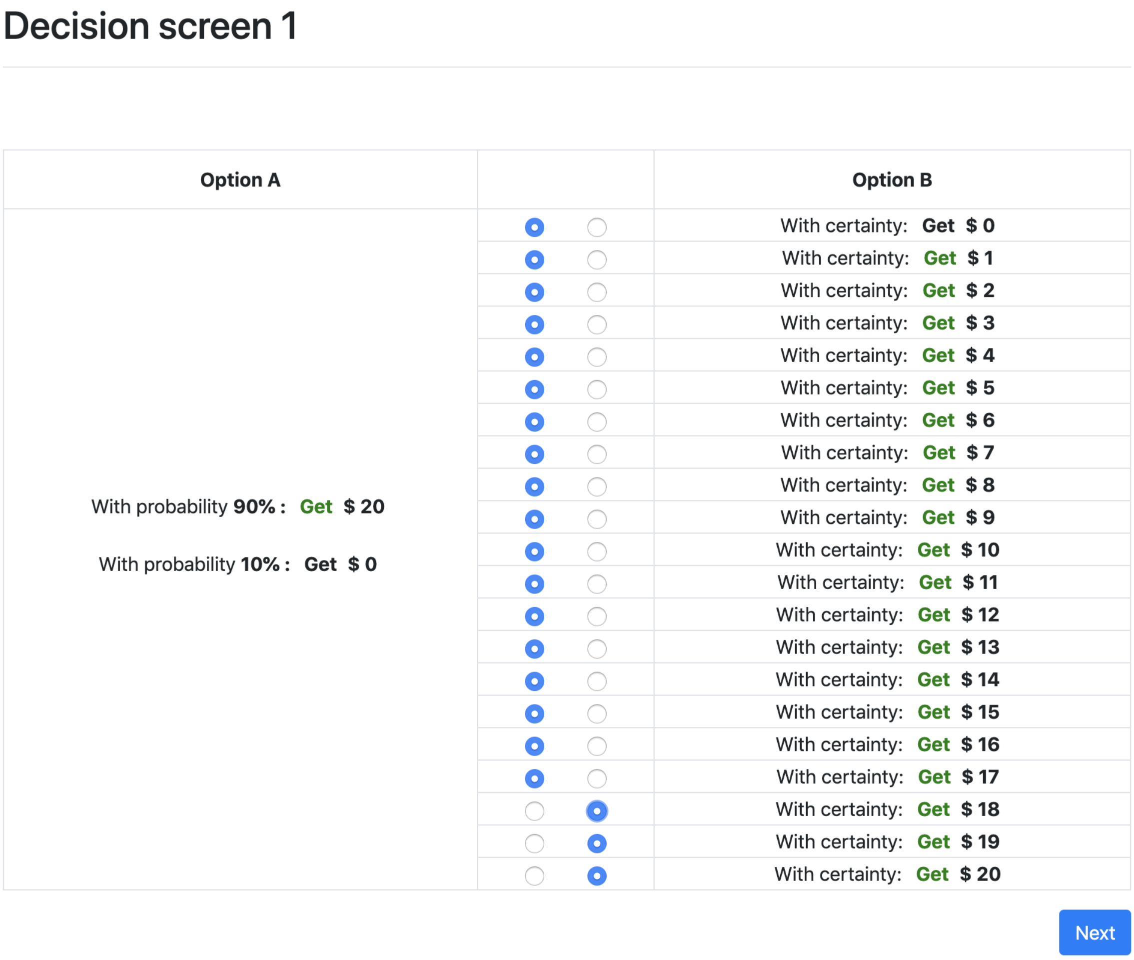
which is the nonlinear equivalent of equation (21):

$$\frac{\lambda}{\sqrt{1-\lambda}} = \frac{|a^{e\prime}(x)|\,\sigma_x}{\sigma_{CU}}. \tag{28}$$

A.1 Multidimensional Input Space

B Additional Details and Analyses for Choice under Risk Experiments

B.1 Additional Figures



Decision screen 1

Option A		Option B
With probability **90%** : Get $ 20 With probability **10%** : Get $ 0	A	With certainty: Get $ 0
	A	With certainty: Get $ 1
	A	With certainty: Get $ 2
	A	With certainty: Get $ 3
	A	With certainty: Get $ 4
	A	With certainty: Get $ 5
	A	With certainty: Get $ 6
	A	With certainty: Get $ 7
	A	With certainty: Get $ 8
	A	With certainty: Get $ 9
	A	With certainty: Get $ 10
	A	With certainty: Get $ 11
	A	With certainty: Get $ 12
	A	With certainty: Get $ 13
	A	With certainty: Get $ 14
	A	With certainty: Get $ 15
	A	With certainty: Get $ 16
	A	With certainty: Get $ 17
	B	With certainty: Get $ 18
	B	With certainty: Get $ 19
	B	With certainty: Get $ 20

Next

Figure 12: Decision screen to elicit certainty equivalents for lotteries

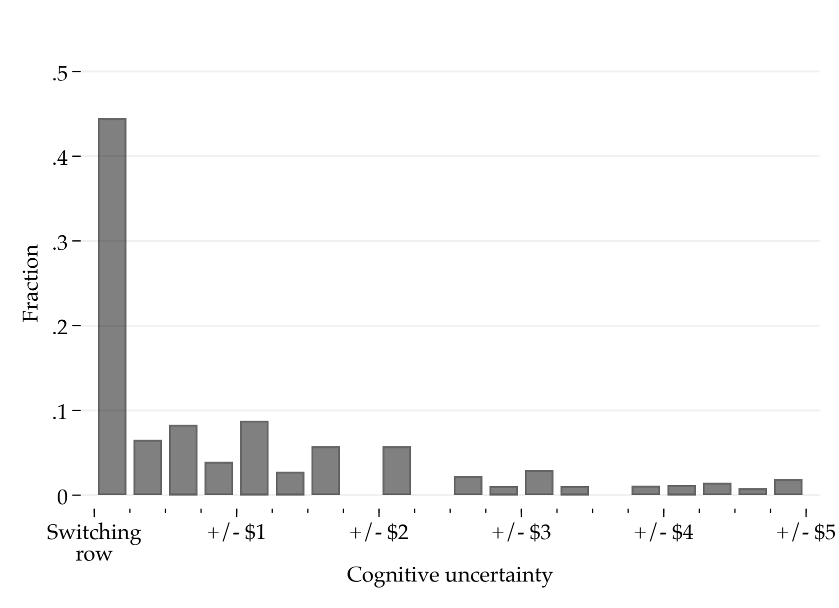


Figure 13: Histogram of cognitive uncertainty in baseline choice under risk tasks

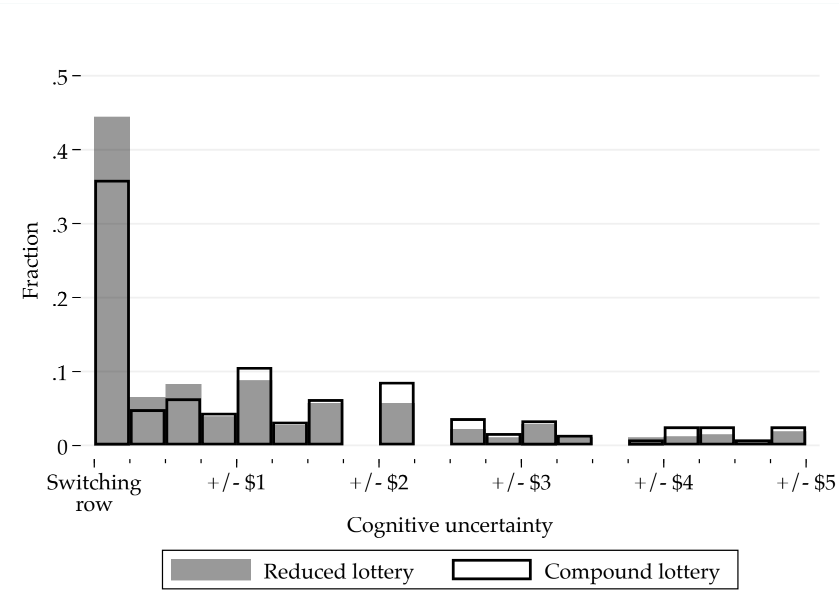


Figure 14: Histograms of cognitive uncertainty in choice under risk tasks, separately for reduced and compound lotteries

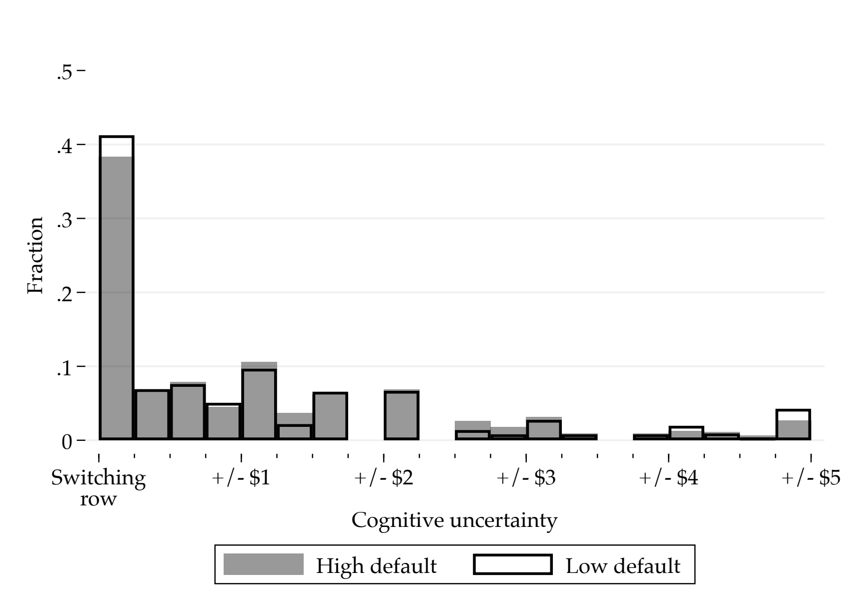


Figure 15: Histograms of cognitive uncertainty in choice under risk tasks, separately for treatments *High Default Risk* and *Low Default Risk*.

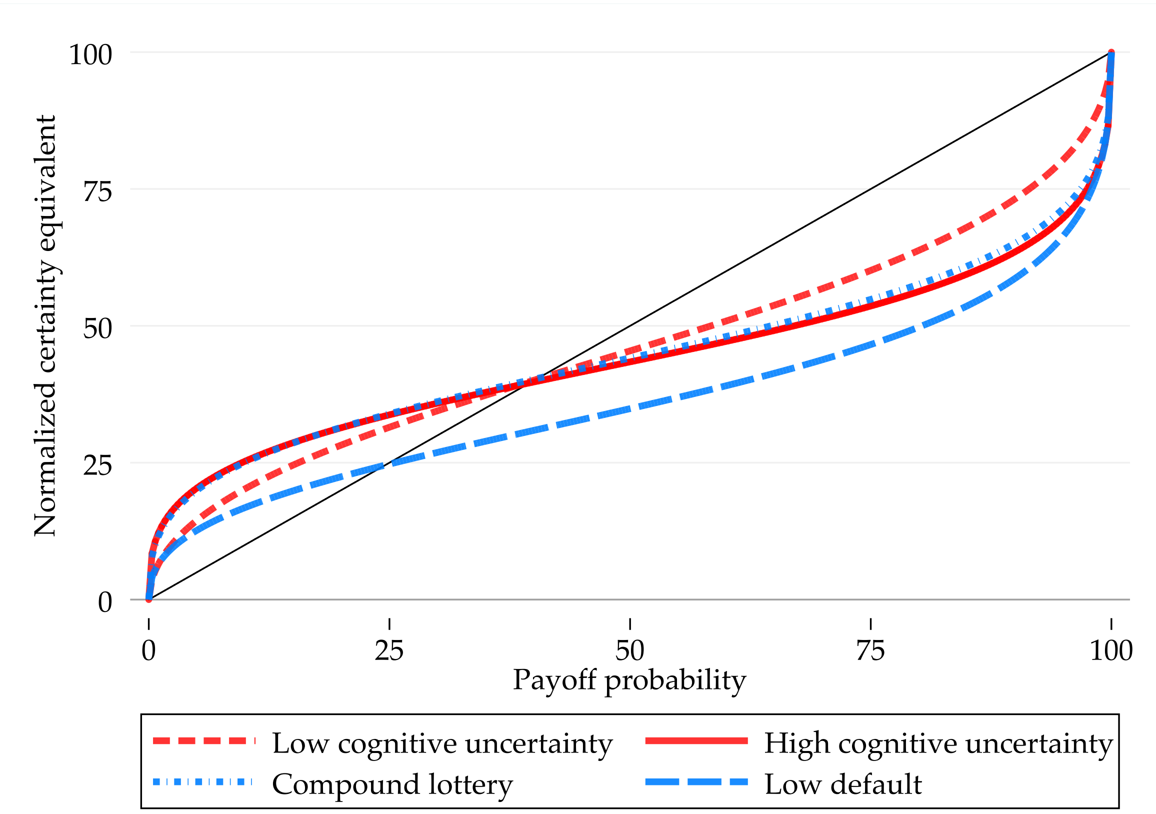


Figure 16: Estimated probability weighting functions across treatments and groups of subjects.

B.2 Results with Full Sample

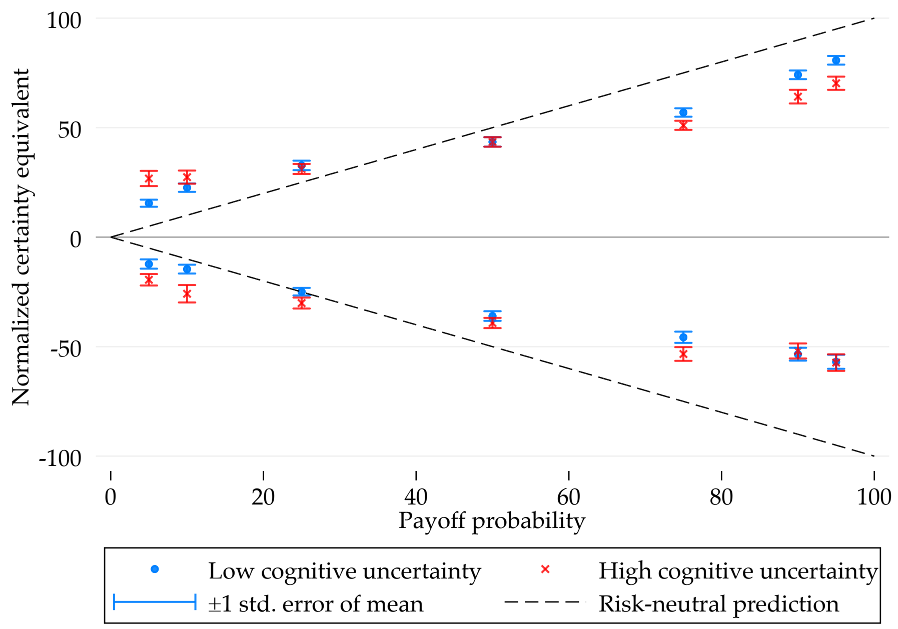


Figure 17: Probability weighting function separately for subjects above / below average cognitive uncertainty (full sample). The partition is done separately for each probability × gains / losses bucket. The plot shows averages and corresponding standard error bars. Normalized certainty equivalents (implied probability weights) are computed as certainty equivalent divided by payout probability. The figure is based on 2,601 certainty equivalents of 700 subjects.

Table 10: Insensitivity to probability and cognitive uncertainty (full sample)

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
Probability of payout	0.67***	0.67***	0.46***	0.46***	0.57***	0.57***
	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)
Probability of payout × Cognitive uncertainty	-0.51***	-0.51***	-0.087	-0.073	-0.29***	-0.28***
	(0.10)	(0.10)	(0.09)	(0.09)	(0.07)	(0.07)
Cognitive uncertainty	16.1***	16.1***	13.7**	13.2**	14.9***	15.4***
	(5.77)	(5.82)	(5.34)	(5.30)	(4.09)	(4.13)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1286	1286	1315	1315	2601	2601
R^2	0.49	0.50	0.27	0.29	0.36	0.36

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's normalized certainty equivalent, computed as midpoint of the switching interval divided by the non-zero payout. The sample includes choices from all baseline gambles with strictly interior probabilities. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

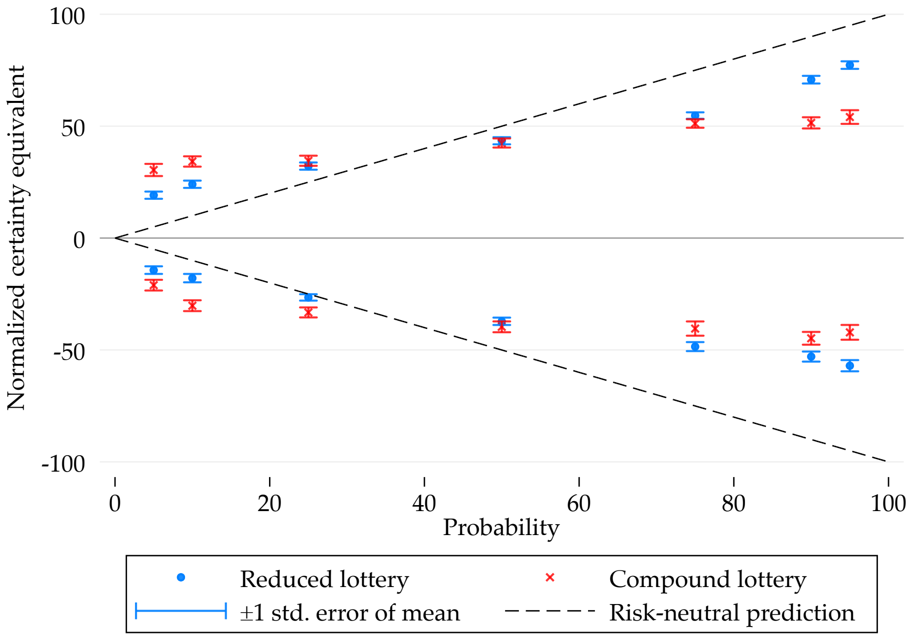


Figure 18: Probability weighting function separately for reduced and compound lotteries (full sample). The plot shows averages and corresponding standard error bars. Normalized certainty equivalents (implied probability weights) are computed as certainty equivalent divided by payout probability. The figure is based on 3,905 certainty equivalents of 700 subjects.

Table 11: Choice under risk: Baseline versus compound lotteries (full sample)

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
Probability of payout	0.59***	0.65***	0.45***	0.47***	0.52***	0.57***
	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)
Probability of payout × 1 if compound lottery	-0.34***	-0.31***	-0.25***	-0.24***	-0.29***	-0.28***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)
Probability of payout × Cognitive uncertainty		-0.36***		-0.12		-0.25***
		(0.07)		(0.07)		(0.05)
1 if compound lottery	13.6***	12.8***	12.3***	11.2***	12.9***	12.2***
	(2.09)	(2.08)	(1.98)	(2.01)	(1.46)	(1.44)
Cognitive uncertainty		10.9**		14.3***		13.2***
		(4.42)		(4.65)		(3.42)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1958	1958	1947	1947	3905	3905
R^2	0.37	0.40	0.21	0.24	0.28	0.29

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's normalized certainty equivalent, computed as midpoint of the switching interval divided by the non-zero payout. The sample includes choices from the baseline and compound gambles, where for comparability the set of baseline gambles is restricted to gambles with payout probabilities of 10%, 25%, 50%, 75%, and 90%, see Figure 4. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

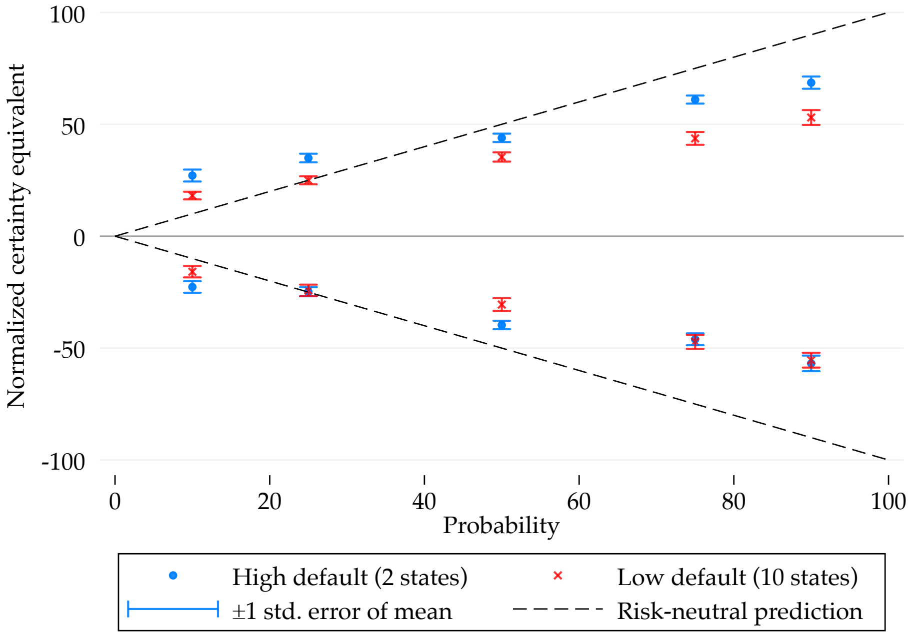


Figure 19: Probability weighting function separately for treatments *High Default Risk* and *Low Default Risk* (full sample). The plot shows averages and corresponding standard error bars. Normalized certainty equivalents (implied probability weights) are computed as certainty equivalent divided by payout probability. The figure is based on 1,800 certainty equivalents of 700 subjects.

Table 12: Choice under risk: Treatments *Low Default* and *High Default* (full sample)

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
0 if *High Default*, 1 if *Low Default*	-11.9***	-11.4***	-3.28	-2.82	-7.61***	-7.08***
	(1.93)	(1.98)	(2.28)	(2.25)	(1.60)	(1.60)
Probability of payout	0.56***	0.56***	0.51***	0.51***	0.54***	0.54***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)
Probability of payout × Cognitive uncertainty	-0.49***	-0.50***	-0.26**	-0.28**	-0.36***	-0.38***
	(0.10)	(0.10)	(0.13)	(0.13)	(0.09)	(0.09)
Cognitive uncertainty	16.7***	16.9***	22.3***	22.5***	19.1***	19.4***
	(6.08)	(6.05)	(8.21)	(8.40)	(5.19)	(5.25)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	900	900	900	900	1800	1800
R^2	0.35	0.36	0.23	0.26	0.27	0.29

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's normalized certainty equivalent, computed as midpoint of the switching interval divided by the non-zero payout. The sample includes choices from treatments *Low Default* and *High Default*. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

B.3 Results excluding Speeders

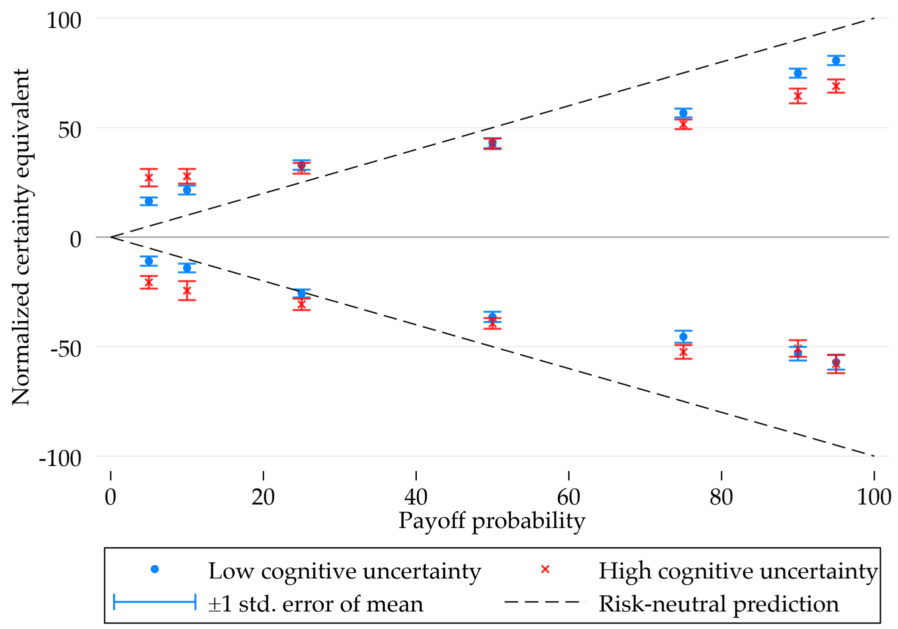


Figure 20: Probability weighting function separately for subjects above / below average cognitive uncertainty (excl. speeders). The partition is done separately for each probability × gains / losses bucket. The plot shows averages and corresponding standard error bars. Normalized certainty equivalents (implied probability weights) are computed as certainty equivalent divided by payout probability. The figure is based on 2,349 certainty equivalents of 630 subjects.

Table 13: Insensitivity to probability and cognitive uncertainty (excl. speeders)

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
Probability of payout	0.68***	0.68***	0.47***	0.47***	0.58***	0.58***
	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)
Probability of payout × Cognitive uncertainty	-0.57***	-0.56***	-0.12	-0.10	-0.33***	-0.32***
	(0.10)	(0.10)	(0.09)	(0.09)	(0.08)	(0.08)
Cognitive uncertainty	19.0***	19.0***	15.1***	14.4**	17.0***	17.3***
	(6.29)	(6.26)	(5.62)	(5.59)	(4.38)	(4.41)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1162	1162	1187	1187	2349	2349
R^2	0.49	0.50	0.27	0.29	0.36	0.36

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's normalized certainty equivalent, computed as midpoint of the switching interval divided by the non-zero payout. The sample includes choices from all baseline gambles with strictly interior probabilities. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

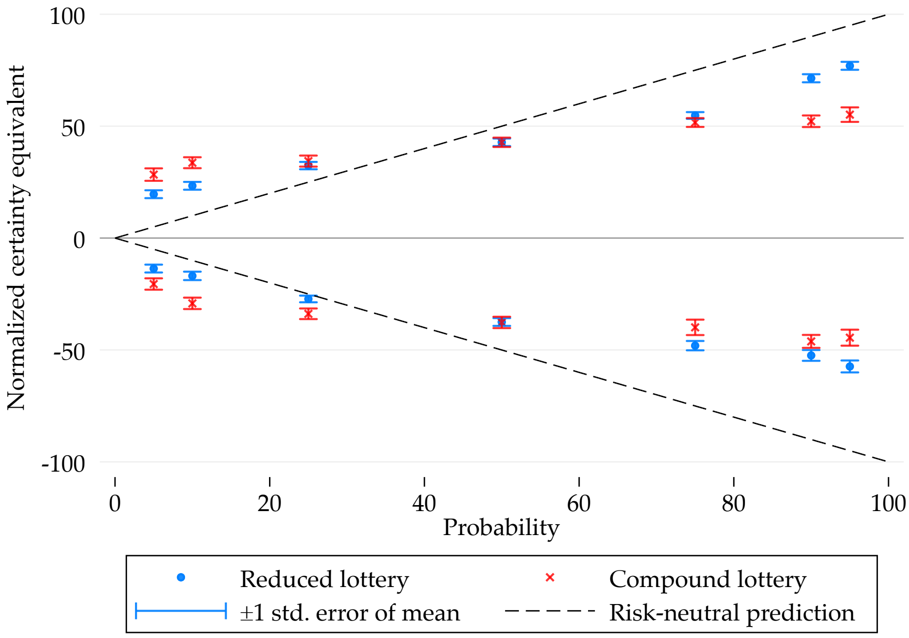


Figure 21: Probability weighting function separately for reduced and compound lotteries (excl. speeders). The plot shows averages and corresponding standard error bars. Normalized certainty equivalents (implied probability weights) are computed as certainty equivalent divided by payout probability. The figure is based on 3,519 certainty equivalents of 700 subjects.

Table 14: Choice under risk: Baseline versus compound lotteries (excl. speeders)

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
Probability of payout	0.59***	0.65***	0.46***	0.48***	0.53***	0.57***
	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)
Probability of payout × 1 if compound lottery	-0.32***	-0.29***	-0.23***	-0.22***	-0.27***	-0.25***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)
Probability of payout × Cognitive uncertainty		-0.36***		-0.15*		-0.26***
		(0.07)		(0.08)		(0.06)
1 if compound lottery	12.5***	11.8***	11.6***	10.1***	12.0***	11.2***
	(2.18)	(2.18)	(2.05)	(2.08)	(1.52)	(1.51)
Cognitive uncertainty		11.6**		15.8***		14.2***
		(4.85)		(4.96)		(3.70)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1766	1766	1753	1753	3519	3519
R^2	0.38	0.40	0.22	0.24	0.29	0.30

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's normalized certainty equivalent, computed as midpoint of the switching interval divided by the non-zero payout. The sample includes choices from the baseline and compound gambles, where for comparability the set of baseline gambles is restricted to gambles with payout probabilities of 10%, 25%, 50%, 75%, and 90%, see Figure 4. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

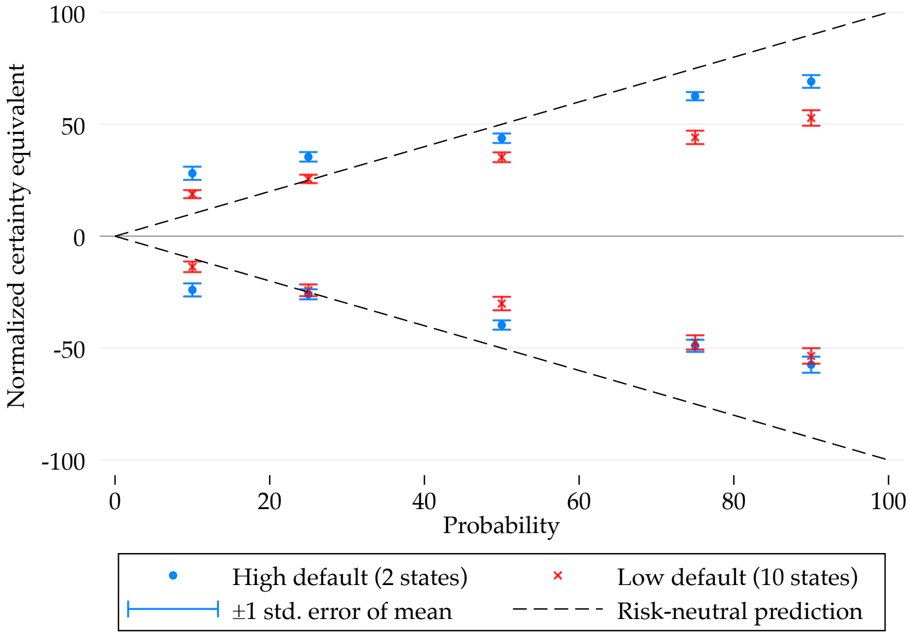


Figure 22: Probability weighting function separately for treatments *High Default Risk* and *Low Default Risk* (excl. speeders). The plot shows averages and corresponding standard error bars. Normalized certainty equivalents (implied probability weights) are computed as certainty equivalent divided by payout probability. The figure is based on 1,620 certainty equivalents of 270 subjects.

Table 15: Choice under risk: Treatments *Low Default* and *High Default* (excl. speeders)

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
0 if *High Default*, 1 if *Low Default*	-12.4***	-12.1***	-5.26**	-4.78**	-8.80***	-8.33***
	(2.05)	(2.10)	(2.39)	(2.38)	(1.69)	(1.71)
Probability of payout	0.55***	0.56***	0.51***	0.52***	0.54***	0.54***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)
Probability of payout × Cognitive uncertainty	-0.49***	-0.49***	-0.28**	-0.29**	-0.37***	-0.38***
	(0.10)	(0.10)	(0.12)	(0.13)	(0.09)	(0.09)
Cognitive uncertainty	15.8**	15.8**	23.7***	23.7***	19.5***	19.8***
	(6.59)	(6.52)	(8.16)	(8.43)	(5.38)	(5.46)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	810	810	810	810	1620	1620
R^2	0.34	0.36	0.24	0.27	0.28	0.29

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's normalized certainty equivalent, computed as midpoint of the switching interval divided by the non-zero payout. The sample includes choices from treatments *Low Default* and *High Default*.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C Additional Details and Analyses for Belief Updating Experiments

C.1 Additional Figures

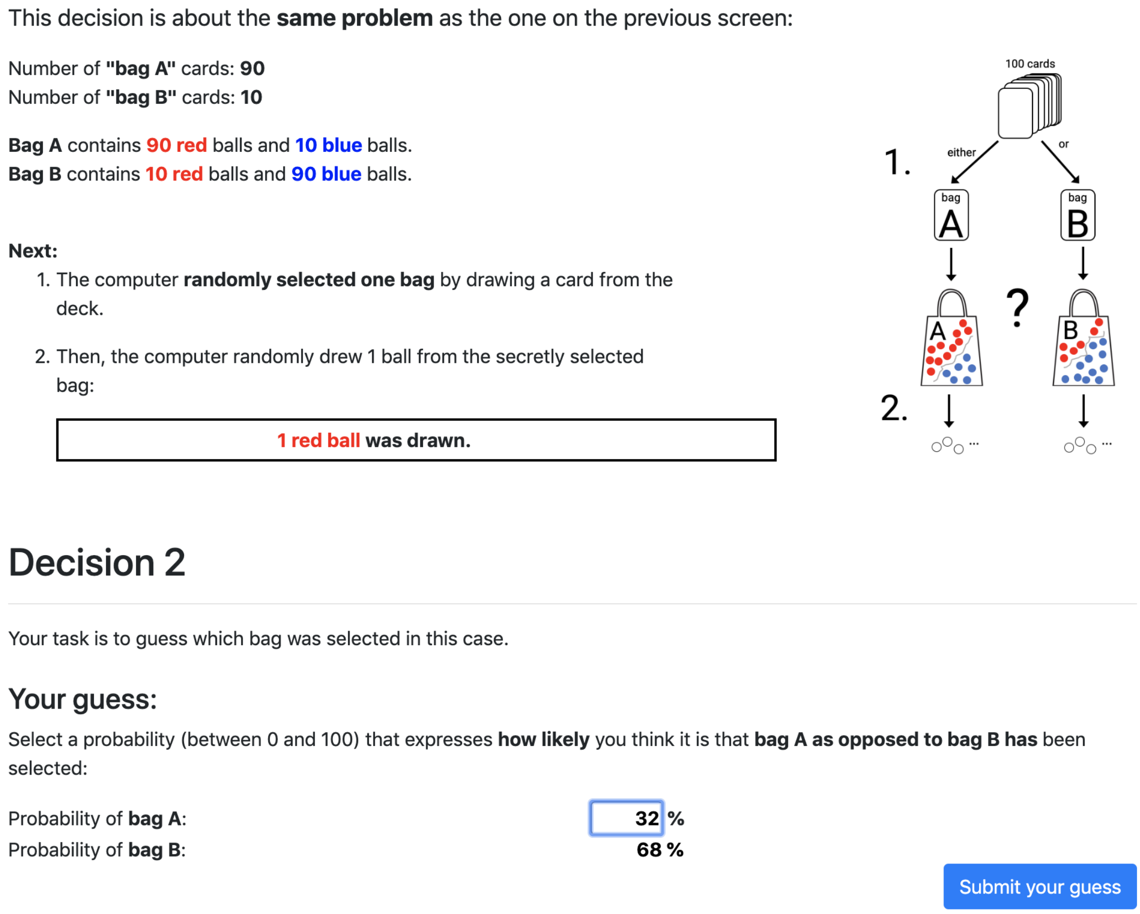


Figure 23: Decision screen to elicit posterior belief in belief updating tasks

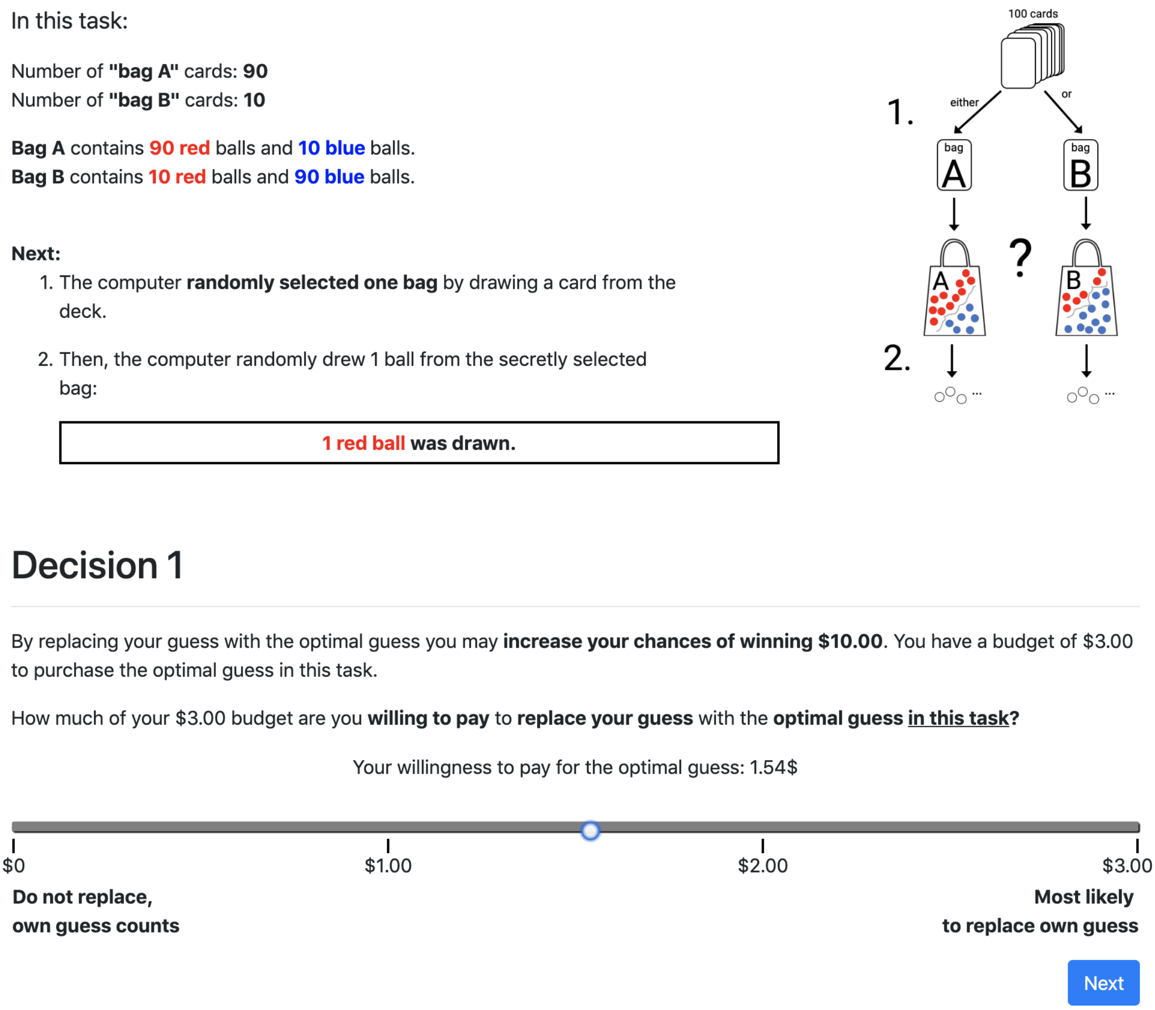


Figure 24: Decision screen to elicit willingness-to-pay for optimal guess in belief updating

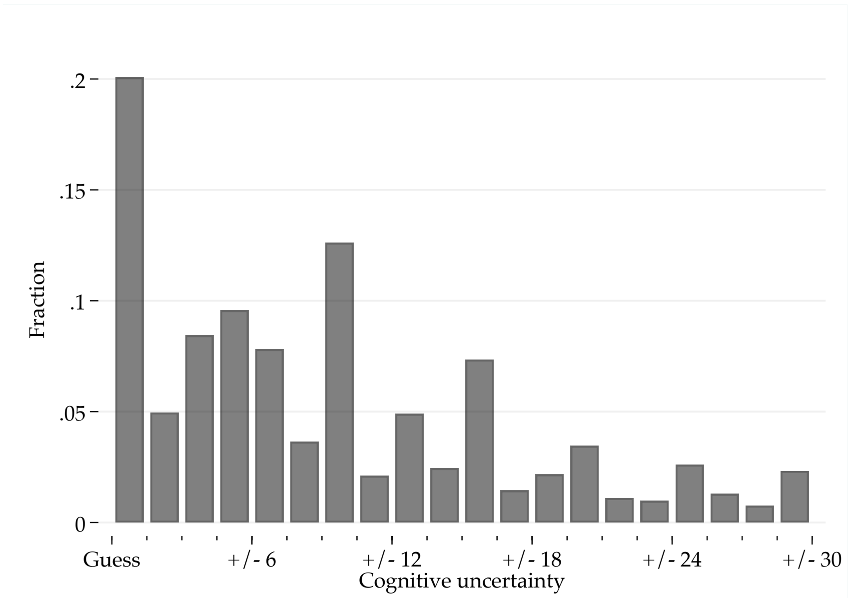


Figure 25: Histogram of cognitive uncertainty in baseline belief updating tasks

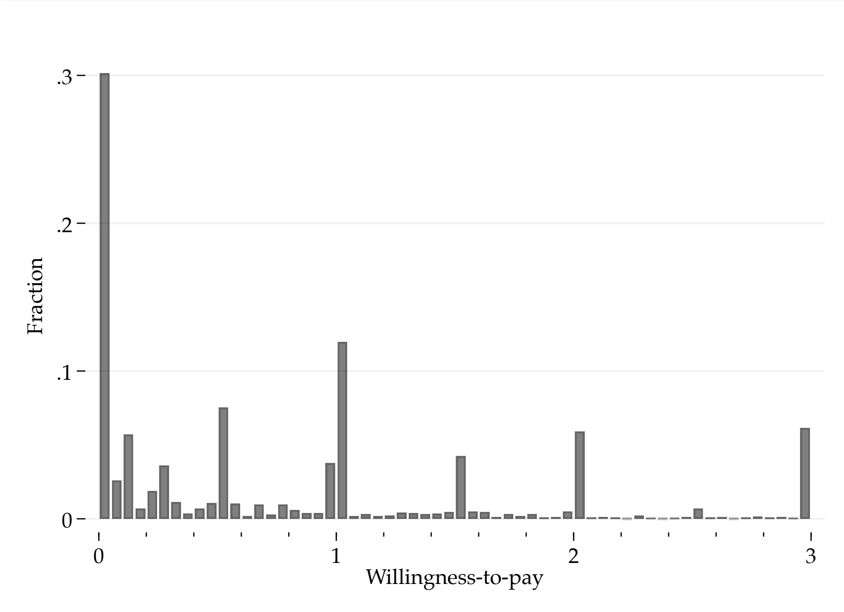


Figure 26: Histogram of willingness-to-pay to replace own guess by Bayesian posterior in baseline belief updating tasks

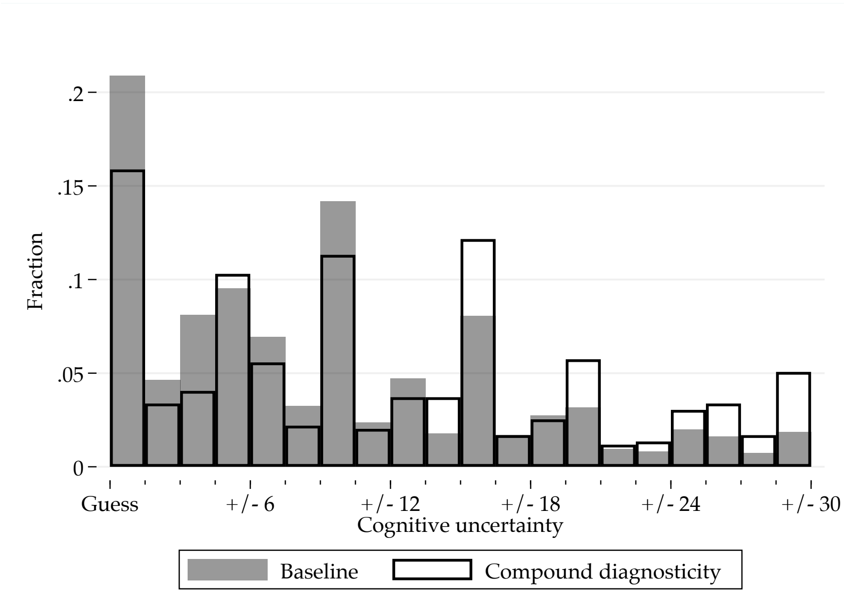


Figure 27: Histograms of cognitive uncertainty in belief updating tasks, separately for baseline and compound diagnosticities

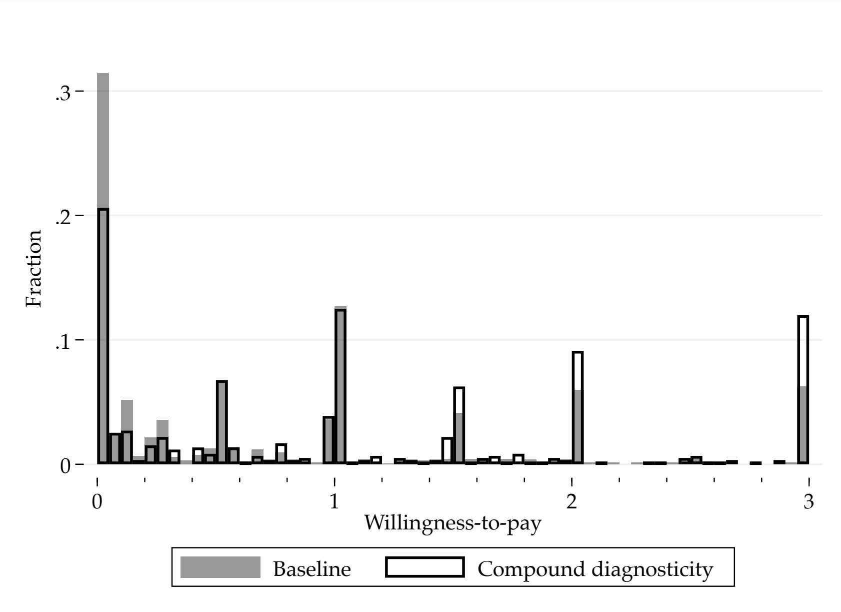


Figure 28: Histograms of willingness-to-pay to replace own guess by Bayesian posterior in belief updating tasks, separately for baseline and compound diagnosticities

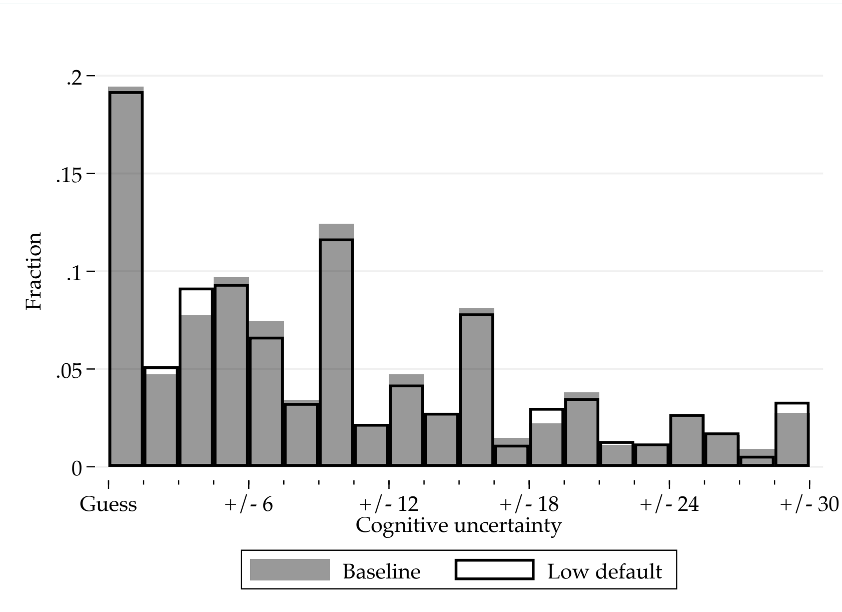


Figure 29: Histograms of cognitive uncertainty in belief updating tasks, separately for treatments *Baseline* and *Low Default Beliefs*.

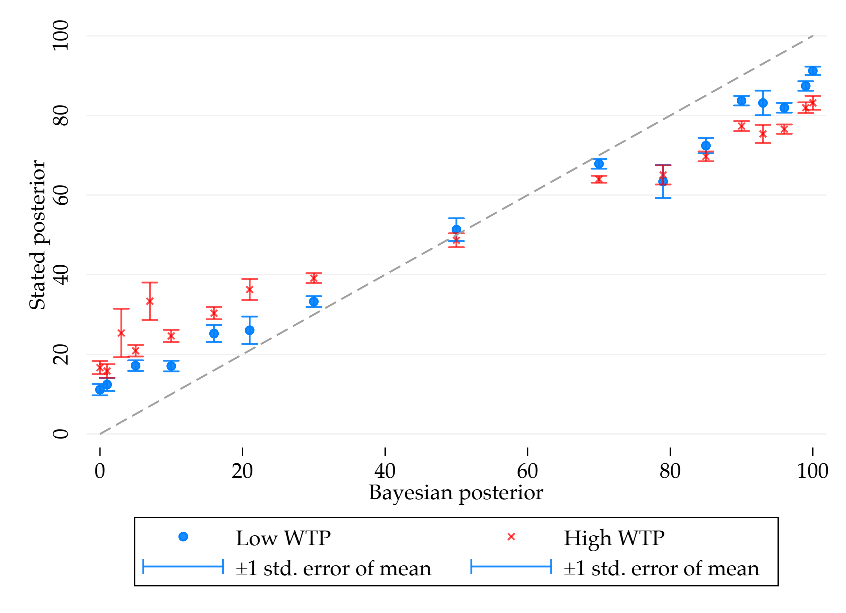


Figure 30: Relationship between stated and Bayesian posteriors, separately for subjects above / below median WTP for the Bayesian guess. The partition is done separately for each Bayesian posterior. The plot shows averages and corresponding standard error bars.

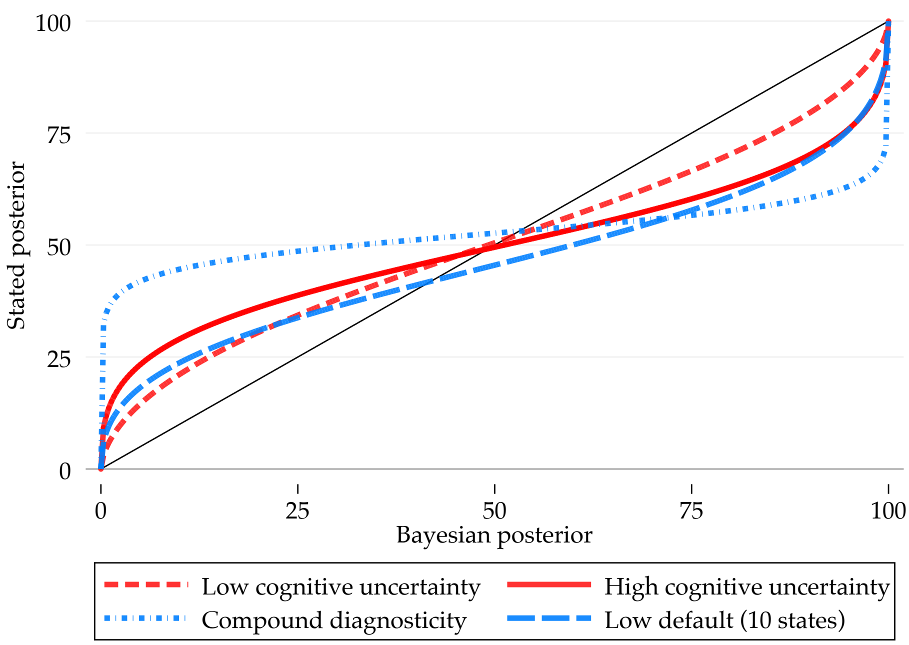


Figure 31: Estimated belief weighting functions across treatments and groups of subjects.

C.2 Additional Tables

Table 16: Belief updating: Baseline tasks: WTP measure

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.69***	0.76***	0.76***			
	(0.01)	(0.01)	(0.01)			
Bayesian posterior × WTP for Bayes		-0.096***	-0.096***			
		(0.01)	(0.01)			
WTP for Bayesian posterior		5.49***	5.47***		0.027	0.024
		(0.76)	(0.76)		(0.02)	(0.02)
Log [Likelihood ratio]				0.41***	0.43***	0.43***
				(0.01)	(0.01)	(0.01)
Log [Prior odds]				0.42***	0.44***	0.44***
				(0.02)	(0.03)	(0.03)
Log [Likelihood ratio] × WTP for Bayes					-0.042***	-0.043***
					(0.01)	(0.01)
Log [Prior odds] × WTP for Bayes					-0.028	-0.027
					(0.02)	(0.02)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	3187	3187	3187	3104	3104	3104
R^2	0.72	0.73	0.73	0.62	0.63	0.63

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C.3 Results with Full Sample

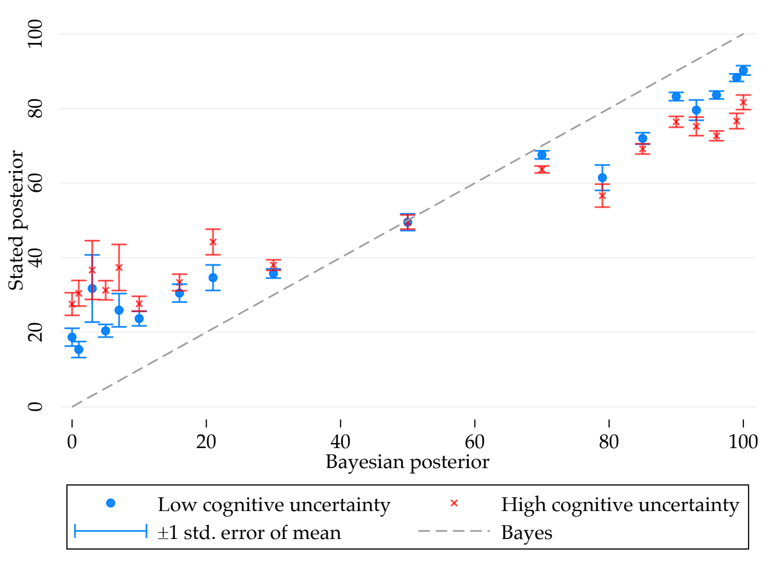


Figure 32: Relationship between average stated and Bayesian posteriors, separately for subjects above / below median cognitive uncertainty (full sample). The partition is done separately for each Bayesian posterior. Bayesian posteriors are rounded to the nearest integer. We only show buckets with more than ten observations. The figure is based on 3,310 beliefs of 700 subjects.

Table 17: Belief updating: Baseline tasks (full sample)

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.62***	0.75***	0.75***			
	(0.01)	(0.02)	(0.02)			
Bayesian posterior × Cognitive uncertainty		-0.46***	-0.45***			
		(0.05)	(0.05)			
Cognitive uncertainty		19.9***	19.9***		-0.14*	-0.15*
		(3.10)	(3.12)		(0.08)	(0.08)
Log [Likelihood ratio]				0.36***	0.41***	0.41***
				(0.01)	(0.02)	(0.02)
Log [Prior odds]				0.36***	0.49***	0.49***
				(0.02)	(0.03)	(0.03)
Log [Likelihood ratio] × Cognitive uncertainty					-0.21***	-0.21***
					(0.05)	(0.05)
Log [Prior odds] × Cognitive uncertainty					-0.45***	-0.45***
					(0.08)	(0.08)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	3310	3310	3310	3222	3222	3222
R^2	0.57	0.59	0.60	0.48	0.50	0.50

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

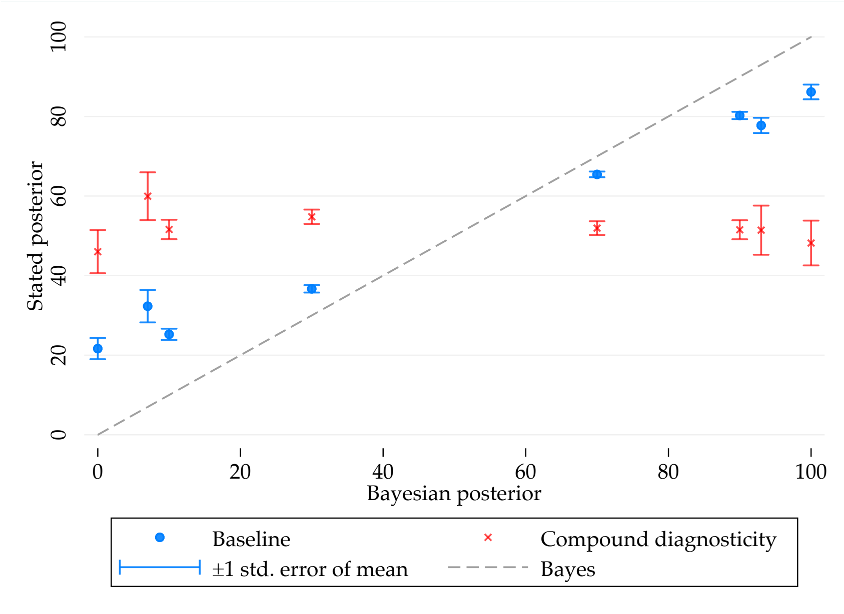


Figure 33: Stated average posteriors as a function of Bayesian posteriors, separately for reduced and compound belief updating problems (full sample). The plot shows averages and corresponding standard error bars. To allow for a valid comparison between baseline and compound updating problems, the sample is restricted to updating tasks in which the base rate is 50:50. Bayesian posteriors are rounded to the nearest integer. We only show buckets with more than ten observations. The figure is based on 2,056 beliefs of 697 subjects.

Table 18: Belief updating: Reduced versus compound signal diagnosticities (full sample)

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.44***	0.67***	0.67***			
	(0.02)	(0.02)	(0.02)			
Bayesian posterior × 1 if compound problem		-0.69***	-0.69***			
		(0.04)	(0.04)			
1 if compound diagnosticity		34.5***	34.7***		-0.046	-0.043
		(2.17)	(2.15)		(0.06)	(0.06)
Log [Likelihood ratio]				0.31***	0.40***	0.40***
				(0.01)	(0.02)	(0.02)
Log [Likelihood ratio] × 1 if compound problem					-0.32***	-0.32***
					(0.03)	(0.03)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	2056	2056	2056	1954	1954	1954
R^2	0.29	0.45	0.46	0.33	0.40	0.41

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

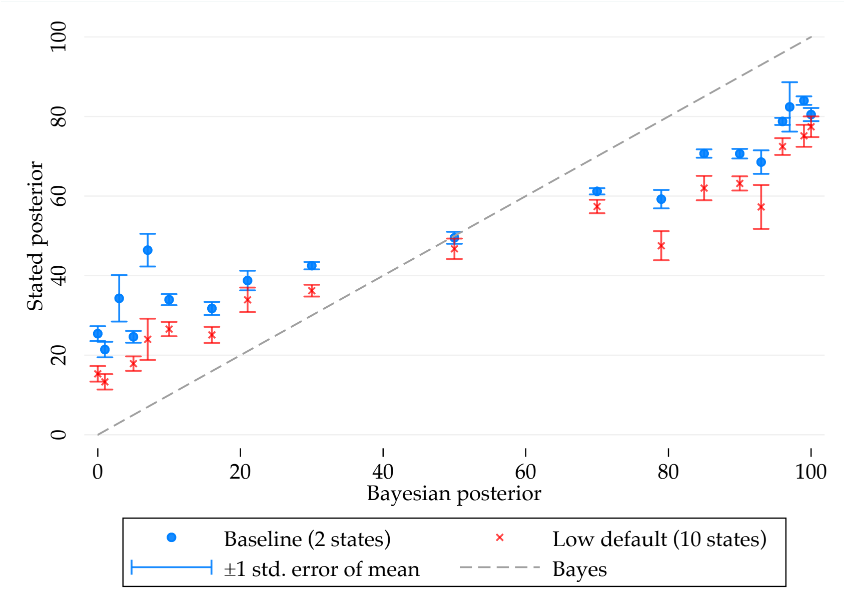


Figure 34: Stated average posteriors as a function of Bayesian posteriors, separately for treatments *Baseline Beliefs* and *Low Default Beliefs* (full sample). Bayesian posteriors are rounded to the nearest integer. We only show buckets with more than ten observations. The figure is based on 5,668 beliefs of 1,000 subjects.

Table 19: Belief updating: Low versus high mental default (full sample)

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
0 if *Baseline*, 1 if *Low Default*	-6.94***	-7.22***	-7.68***	-0.41***	-0.43***	-0.46***
	(0.97)	(0.95)	(1.01)	(0.06)	(0.06)	(0.06)
Bayesian posterior	0.54***	0.65***	0.65***			
	(0.01)	(0.02)	(0.02)			
Bayesian posterior × Cognitive uncertainty		-0.39***	-0.39***			
		(0.04)	(0.04)			
Cognitive uncertainty		13.5***	13.4***		-0.26***	-0.26***
		(2.37)	(2.39)		(0.08)	(0.08)
Log [Likelihood ratio]				0.31***	0.37***	0.37***
				(0.01)	(0.02)	(0.02)
Log [Prior odds]				0.41***	0.54***	0.54***
				(0.02)	(0.03)	(0.03)
Log [Likelihood ratio] × Cognitive uncertainty					-0.21***	-0.21***
					(0.04)	(0.04)
Log [Prior odds] × Cognitive uncertainty					-0.43***	-0.43***
					(0.07)	(0.07)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	5668	5668	5668	5473	5473	5473
R^2	0.44	0.45	0.46	0.42	0.44	0.44

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C.4 Results excluding Speeders

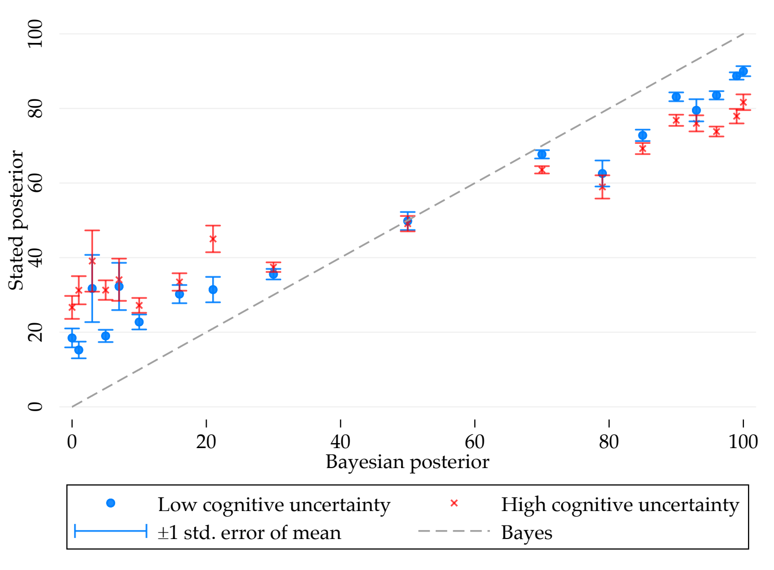


Figure 35: Relationship between average stated and Bayesian posteriors, separately for subjects above / below median cognitive uncertainty (excl. speeders). The partition is done separately for each Bayesian posterior. Bayesian posteriors are rounded to the nearest integer. We only show buckets with more than ten observations. The figure is based on 3,006 beliefs of 635 subjects.

Table 20: Belief updating: Baseline tasks (excl. speeders)

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.63***	0.75***	0.76***			
	(0.01)	(0.02)	(0.02)			
Bayesian posterior × Cognitive uncertainty		-0.46***	-0.46***			
		(0.05)	(0.05)			
Cognitive uncertainty		21.4***	21.6***		-0.085	-0.085
		(3.18)	(3.20)		(0.09)	(0.09)
Log [Likelihood ratio]				0.36***	0.41***	0.41***
				(0.01)	(0.02)	(0.02)
Log [Prior odds]				0.38***	0.51***	0.50***
				(0.02)	(0.04)	(0.04)
Log [Likelihood ratio] × Cognitive uncertainty					-0.22***	-0.22***
					(0.06)	(0.05)
Log [Prior odds] × Cognitive uncertainty					-0.44***	-0.44***
					(0.08)	(0.08)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	3006	3006	3006	2925	2925	2925
R^2	0.59	0.61	0.61	0.49	0.51	0.51

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

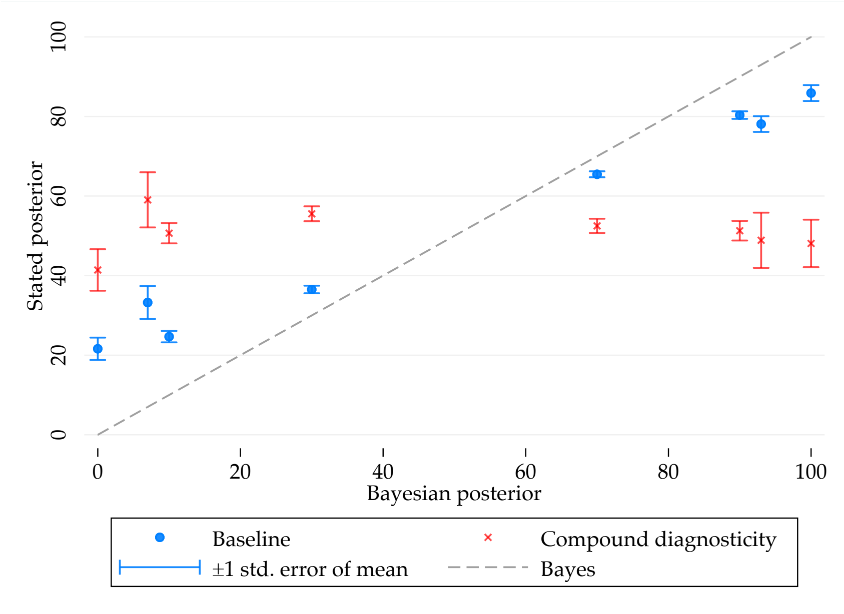


Figure 36: Stated average posteriors as a function of Bayesian posteriors, separately for reduced and compound belief updating problems (excl. speeders). The plot shows averages and corresponding standard error bars. To allow for a valid comparison between baseline and compound updating problems, the sample is restricted to updating tasks in which the base rate is 50:50. Bayesian posteriors are rounded to the nearest integer. We only show buckets with more than ten observations. The figure is based on 1,874 beliefs of 632 subjects.

Table 21: Belief updating: Reduced versus compound signal diagnosticities (excl. speeders)

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.45***	0.68***	0.68***			
	(0.02)	(0.02)	(0.02)			
Bayesian posterior × 1 if compound problem		-0.68***	-0.68***			
		(0.04)	(0.04)			
1 if compound diagnosticity		33.9***	34.1***		-0.071	-0.069
		(2.25)	(2.24)		(0.06)	(0.06)
Log [Likelihood ratio]				0.31***	0.41***	0.41***
				(0.02)	(0.02)	(0.02)
Log [Likelihood ratio] × 1 if compound problem					-0.31***	-0.31***
					(0.03)	(0.03)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	1874	1874	1874	1779	1779	1779
R^2	0.30	0.46	0.46	0.34	0.40	0.41

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

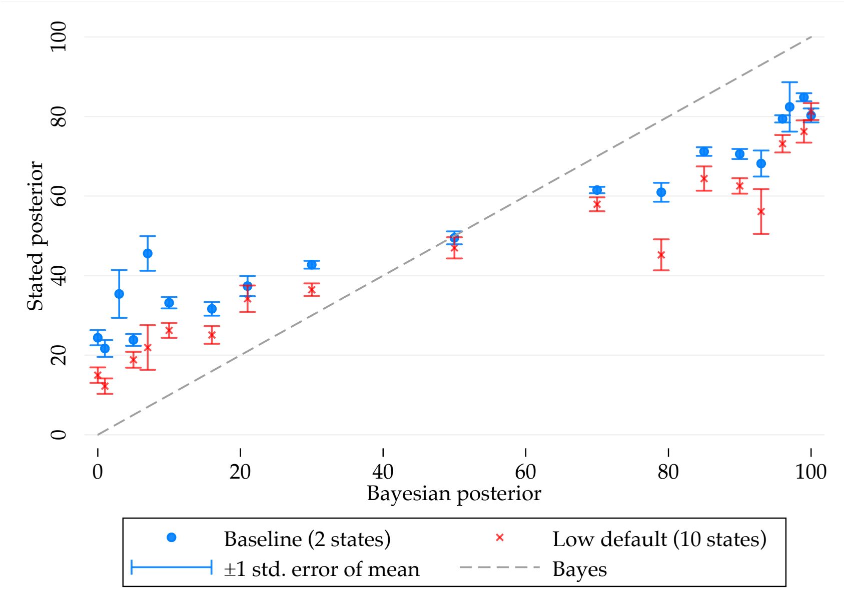


Figure 37: Stated average posteriors as a function of Bayesian posteriors, separately for treatments *Baseline Beliefs* and *Low Default Beliefs* (full sample). Bayesian posteriors are rounded to the nearest integer. We only show buckets with more than ten observations. The figure is based on 5,107 beliefs of 899 subjects.

Table 22: Belief updating: Low versus high mental default (excl. speeders)

	Dependent variable:					
	Posterior belief			Log [Posterior odds]		
	(1)	(2)	(3)	(4)	(5)	(6)
0 if *Baseline*, 1 if *Low Default*	-6.64***	-6.91***	-7.21***	-0.40***	-0.42***	-0.43***
	(0.98)	(0.97)	(1.02)	(0.06)	(0.06)	(0.06)
Bayesian posterior	0.55***	0.66***	0.65***			
	(0.01)	(0.02)	(0.02)			
Bayesian posterior × Cognitive uncertainty		-0.39***	-0.38***			
		(0.04)	(0.04)			
Cognitive uncertainty		14.7***	14.7***		-0.19**	-0.19**
		(2.48)	(2.51)		(0.08)	(0.08)
Log [Likelihood ratio]				0.32***	0.37***	0.37***
				(0.01)	(0.02)	(0.02)
Log [Prior odds]				0.43***	0.55***	0.55***
				(0.02)	(0.03)	(0.03)
Log [Likelihood ratio] × Cognitive uncertainty					-0.20***	-0.20***
					(0.04)	(0.04)
Log [Prior odds] × Cognitive uncertainty					-0.42***	-0.42***
					(0.07)	(0.07)
Session FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	5107	5107	5107	4930	4930	4930
R^2	0.45	0.47	0.47	0.44	0.45	0.46

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

D Additional Details and Analyses for Survey Expectations

D.1 Additional Figures

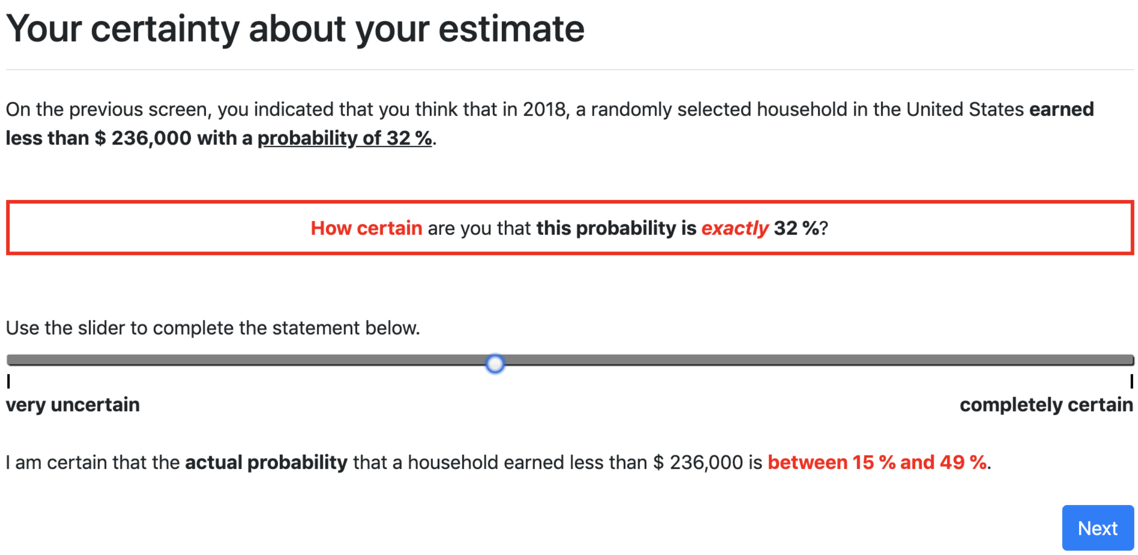


Figure 38: Decision screen to elicit cognitive uncertainty in survey expectations

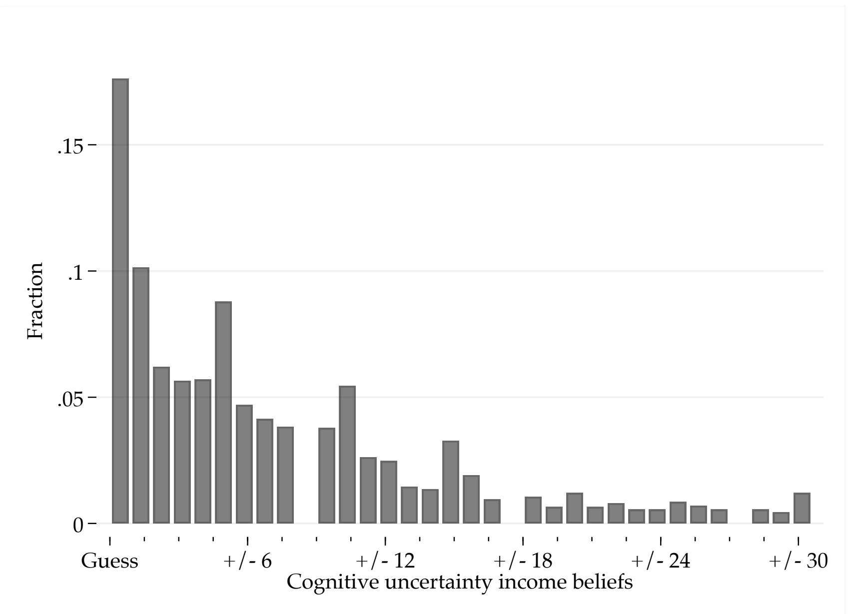


Figure 39: Histogram of cognitive uncertainty in survey expectations about income distribution

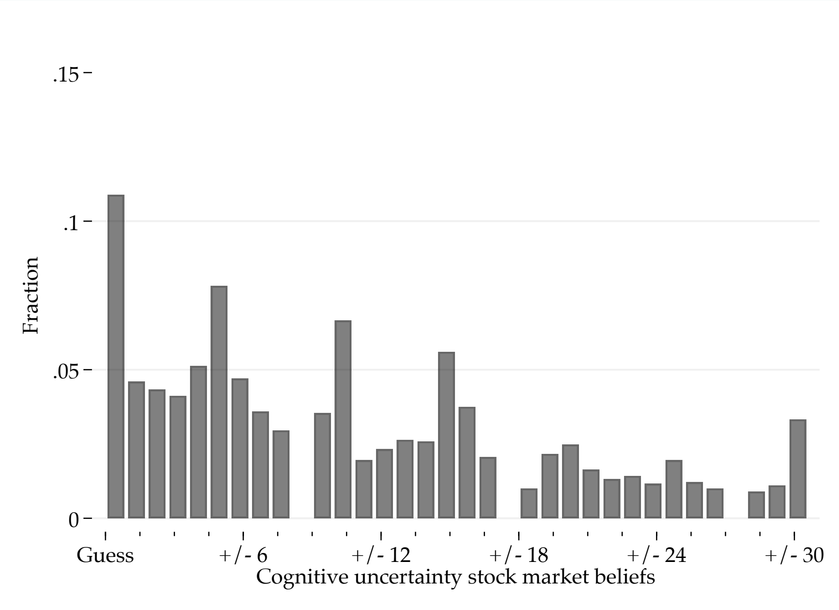


Figure 40: Histogram of cognitive uncertainty in survey expectations about the stock market

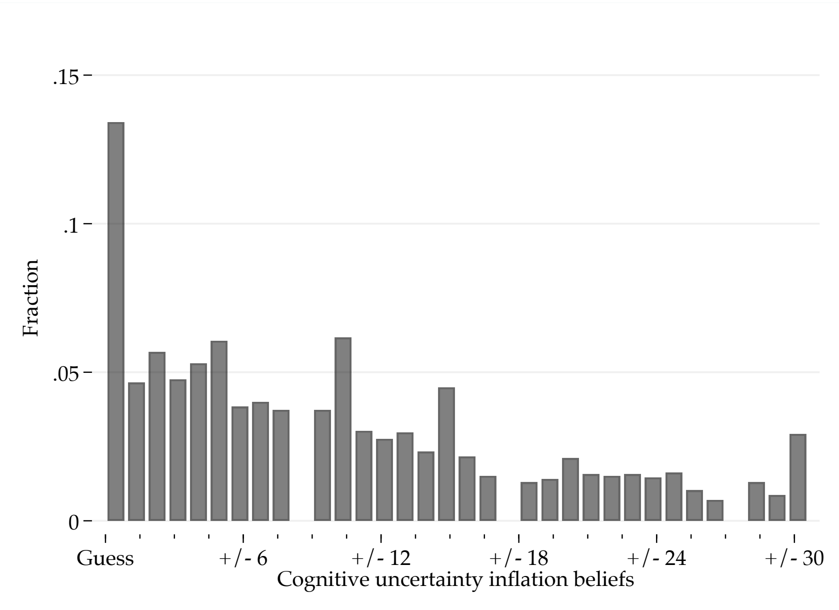


Figure 41: Histogram of cognitive uncertainty in survey expectations about inflation rates

D.2 Additional Tables

Table 23: Survey expectations and cognitive uncertainty

	Dependent variable: Probability estimate about:					
	Income distr.		Stock market		Inflation rate	
	(1)	(2)	(3)	(4)	(5)	(6)
Objective probability	0.90***	0.90***	0.69***	0.69***	0.76***	0.76***
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Objective probability × Cognitive uncertainty	-0.41***	-0.41***	-0.53***	-0.52***	-0.60***	-0.60***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Cognitive uncertainty	18.9***	18.6***	24.2***	24.6***	27.5***	27.0***
	(2.37)	(2.41)	(2.27)	(2.30)	(2.86)	(2.89)
Session FE	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1980	1980	1892	1892	1848	1848
R^2	0.83	0.84	0.52	0.53	0.54	0.54

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. In columns (1)–(2), the question about income distribution asks participants for the probability that a randomly selected U.S. household earns less than $x. In columns (3)–(4), the question about the stock market asks participants for the probability that in a randomly selected year the S&P500 increased by less than x%. In columns (5)–(6), the question about inflation rates asks participants for the probability that in a randomly selected year the inflation rate was less than x%. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

D.3 Results with Full Sample

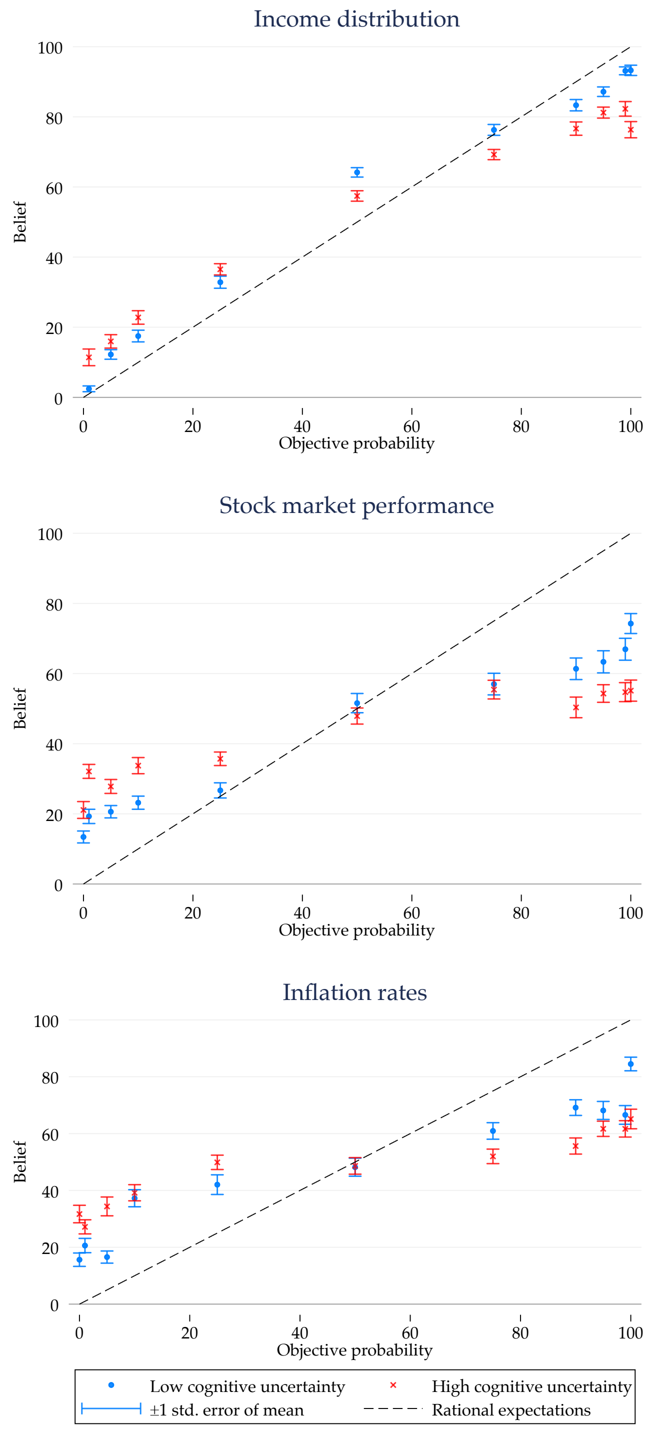


Figure 42: Survey beliefs as a function of objective probabilities, separately for subjects above / below median cognitive uncertainty (full sample). The partition is done separately for each probability bucket. In the top panel, the question asks for the probability that a randomly selected U.S. household earns less than \$x. In the middle panel, the question asks for the probability that in a randomly selected year the S&P500 increased by less than x%. In the bottom panel, the question asks for the probability that in a randomly selected year the inflation rate was less than x%. $N = 2,000$ observations each.

D.4 Results excluding Speeders

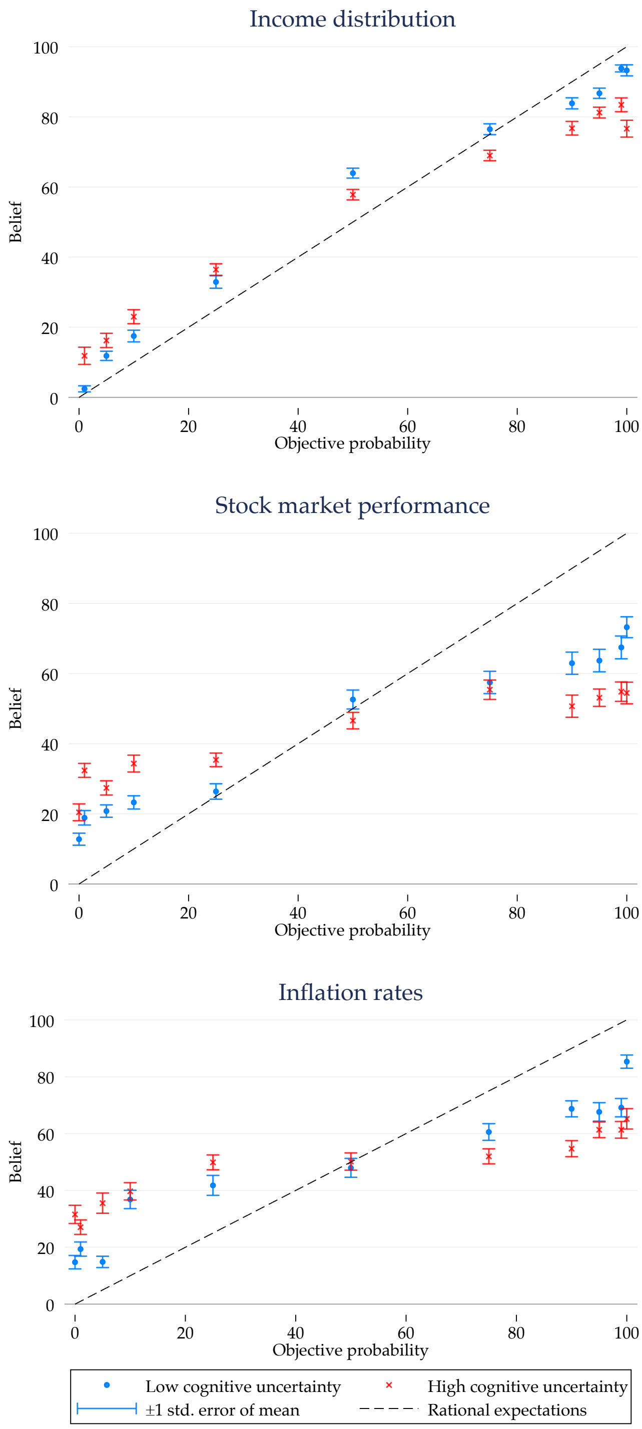


Figure 43: Survey beliefs as a function of objective probabilities, separately for subjects above / below median cognitive uncertainty (excl. speeders). The partition is done separately for each probability bucket. In the top panel, the question asks for the probability that a randomly selected U.S. household earns less than \$x. In the middle panel, the question asks for the probability that in a randomly selected year the S&P500 increased by less than x%. In the bottom panel, the question asks for the probability that in a randomly selected year the inflation rate was less than x%. $N = 1,896$ observations each.

E Experimental Instructions and Decision Screens

E.1 Treatment *Baseline Risk*

E.2 Treatment *Low Default Risk*

E.3 Treatment *Baseline Beliefs*

E.4 Treatment *Low Default Beliefs*

E.5 Survey Expectations

TBD