Cognitive Uncertainty in Intertemporal Choice*

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Abstract

This paper experimentally documents the relevance of cognitive uncertainty for understanding and predicting intertemporal choice. First, cognitive uncertainty sheds light on various empirical regularities that are difficult to explain with non-standard preferences alone, including extreme short-run impatience, per-period impatience that decreases in the length of the delay, and subadditive discounting. Second, accounting for bounded rationality in the form of cognitive uncertainty is also quantitatively important and generates large improvements in model fit. Third, measuring and manipulating cognitive uncertainty yields insights for both choice architecture and predicting the context-dependence of (im)patience.

Keywords: Cognitive uncertainty, intertemporal choice, complexity

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1 Introduction

Due to the relevance of intertemporal tradeoffs for a broad set of economic decisions, economists have long been interested in the psychological principles that undergird intertemporal choice behavior. Perhaps most prominently, the behavioral economics research program has successfully shed light on how motivations such as a desire for immediate gratification contribute to the stylized fact that people often appear extremely short-run impatient in present-future tradeoffs. Yet, as highlighted by recent review papers, other commonly-identified – and economically no less important – empirical regularities are less well-understood and not easily explained by accounts of present-focused preferences (Ericson and Laibson, 2019; Cohen et al., 2020).

A key stylized fact is that people's observed discounting behavior tends to be very inelastic with respect to the length of the time delay. This basic principle manifests in three distinct well-known empirical regularities. First, as visualized in Panel A of Figure 1, people often act in very impatient ways in decisions over relatively short horizons, yet appear considerably less impatient over longer horizons, in both lab and field (Thaler, 1981; Loewenstein and Prelec, 1992; Giglio et al., 2015). This implies that people's per-period impatience strongly decreases in the length of the time delay. This is puzzling because the extreme flattening out of observed discounting behavior is not predicted by present bias models (Laibson, 1997), which converge to exponential discounting over long horizons. Second, as visualized in Panel B of Figure 1, an inelasticity of discounting with respect to the time delay is also observed for tradeoffs in which the early consumption opportunity is not today but in the future, again at odds with a pure present bias account (Kable and Glimcher, 2010).[1] Third, experimental studies robustly identify a particular type of transitivity violation called subadditivity, according to which people appear considerably more patient in tradeoffs over one long interval than in choices where that same interval is partitioned into two sub-intervals (Read, 2001). Again, this can be understood as people being insensitive to the time delay.

A dominant approach in the economics literature has been to attempt to explain these stylized facts through non-standard discount functions, such as the generalized hyperbola and its variants (e.g., Mazur, 1987; Loewenstein and Prelec, 1992; Kable and Glimcher, 2010). Either implicitly or explicitly, such accounts usually take the perspective that "anomalous" discounting behavior reflects "anomalous" preferences.

Our point of departure is orthogonal and complementary to the specification of people's (potentially non-standard) preferences. Instead, we note that intertemporal deci-

[1]Decreasing impatience is the dominant finding in the literature (see, e.g. Cohen et al., 2020; Kable and Glimcher, 2010; He et al., 2019). However, it is not universal, neither when the early date is today nor when it is in the future (see, e.g., Andersen et al., 2014; Harrison et al., 2005).

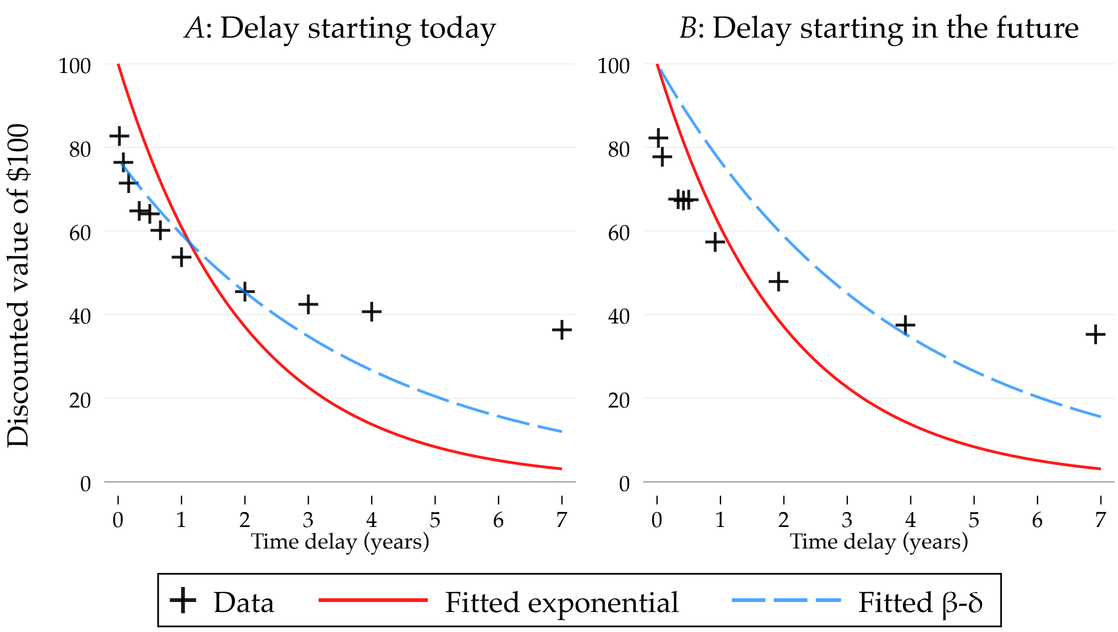


Figure 1: The figure shows the discounted value of $100 to be received with different time delays, partitioned by whether the early payment date is today (Panel A) or in the future (Panel B). The black markers indicate average behavior in our experiments described in Section 3. The red solid line fits an exponential discounting model and the blue dashed line a $\beta - \delta$ model. Both models are estimated on the joint data.

sions may often be complex and, hence, introduce cognitive imprecision or noise. For instance, people may find it hard to determine how exactly to trade off the costs of current exercise against the benefits of future health. We label the decision-maker's resulting subjective uncertainty about his utility-maximizing action "cognitive uncertainty" (Enke and Graeber, 2020). Indeed, both previous work on cognitive uncertainty and a wide range of psychological studies have documented that people frequently report less than full confidence in their decisions, but this insight has not been applied to intertemporal choice yet.[2] This paper proposes that, (i) qualitatively, the measurement of cognitive uncertainty sheds light on the empirical puzzles described above, (ii) quantitatively, it explains a substantial portion of choice variation and creates large improvements in model fit, and (iii) it has predictable implications for choice architecture and the context-dependence of (im)patient behavior.

Several classes of random choice models are broadly consistent with the insight that people's behavior is too inelastic with respect to the time delay: (i) Bayesian cognitive imprecision models, according to which the decision-maker anchors on an intermediate "cognitive default" action and, upon deliberation, adjusts in the direction of his true utility-maximizing action; (ii) random preference models, in which preference parameters fluctuate due to noise; and (iii) random response models, in which the action is perturbed by a stochastic element. For example, behavior will *appear* very impatient over short horizons but very patient over long ones if decisions partly reflect an "intermediate"

[2] See, for example, De Martino et al. (2013, 2017); Polania et al. (2019); Xiang et al. (2021).

cognitive default action in Bayesian cognitive noise models (as in well-established central tendency or compromise effects (Hollingworth, 1910; Simonson and Tversky, 1992)); or if random responses in random response models have an "intermediate" mean.

Our experimental strategy is to link observed discounting behavior to a quantitative measure of cognitive uncertainty. In a first experimental paradigm, experimental participants trade off different time-dated UberEats vouchers that can be used for restaurant delivery and takeout. For example, using a standard multiple price list technique, participants state which voucher value with a validity period of seven days starting from today makes them indifferent to receiving a \$40 voucher with a validity period of seven days starting in six months. In a second, complementary paradigm, we implement analogous decisions, except that these are defined over hypothetical monetary amounts. Third, as a robustness check, we replicate our findings using an alternative, direct elicitation technique, that does not have a response grid or an obvious middle option. We discuss in detail how our study design relates to ongoing discussions about experimental intertemporal choice methodology, including reliability and fungibility of payments.

Following Enke and Graeber (2020), we measure cognitive uncertainty as a person's subjective probability (in percent) that their revealed switching interval in a choice list actually contains their true valuation of the delayed payment. We interpret this question as capturing the participant's (posterior) uncertainty about their utility-maximizing decision, after some cognitive sampling has taken place. This cognitive uncertainty question is simple to understand, quantitative and fast and easy to administer. Below, we discuss which specific psychological mechanisms could give rise to cognitive uncertainty. To provide evidence for the behavioral validity of our measure, we document that cognitive uncertainty is strongly correlated with across-trial variation in responses across repetitions of the same choice problem. This suggests that cognitive uncertainty indeed captures a revealed-preferences signature of cognitive noise, and that participants' cognitive uncertainty is to some extent accurate.

In our data, 75–80% of all decisions are associated with strictly positive cognitive uncertainty. The main insight of our analysis, from which almost all of our results follow, is that the indifference points of cognitively uncertain participants are strongly compressed towards a valuation of roughly 50% of the delayed payment, and exhibit a hyperbolic shape. As a result of this attenuation, we observe a distinctive "flipping" relationship between cognitive uncertainty and behavior: decisions associated with cognitive uncertainty *look like* they reflect relatively high impatience over short horizons, but relatively low impatience over long horizons. Thus, cognitive uncertainty is predictive of decreasing impatience. Importantly, this inelasticity pattern holds both when the intertemporal tradeoff involves the present and when it does not. This is relevant because canonical models of present bias do not generate such a pattern when the present is not involved.

All of these correlations are quantitatively large. For instance, we find that the magnitude of decreasing impatience is five times larger for decisions that are associated with strictly positive cognitive uncertainty.

We find similar results for subadditivity, a canonical transivity violation according to which people are more impatient over two short than over one long interval. This pattern is likewise strongly correlated with cognitive uncertainty. Indeed, in many specifications, we cannot statistically reject the null hypothesis that there is no subadditivity in the set of decisions that are not associated with cognitive uncertainty.

We also pre-registered the prediction and find empirically that cognitive uncertainty is *unrelated* to front-end delay effects, which are widely believed to be a revealed preferences signature of present bias. Intuitively, cognitive uncertainty as such should not be predictive of present-focused behavior because it captures *inelasticity* with respect to the length of a time interval, which however is held constant in documentations of front-end delay effects. This provides another indication that bounded rationality in the form of cognitive noise and present-focused preferences are complementary objects.

Our main objective is to document that the measurement of cognitive uncertainty sheds light on intertemporal choice, rather than to definitively disentangle different classes of random choice models. This is partly because – especially with full flexibility on the functional form of the noise – different models often make very similar predictions. This being said, we tentatively highlight that random preference models are inconsistent with some of our data. On the other hand, both a Bayesian cognitive imprecision model and a random response model (which make indistinguishable predictions about average behavior) are consistent with all of the results summarized above.

To further examine the quantitative importance of cognitive uncertainty, we estimate a model in which observed actions are given by a weighted average of discounted-utility maximization and an estimated "central tendency effect," which could either reflect a cognitive default action or the mean random response. Here, measured cognitive uncertainty determines the relative weights of utility maximization and the central tendency effect. In these estimations, accounting for cognitive uncertainty substantially increases model fit relative to only allowing for present bias. At the same time, we also find that present bias of $\beta < 1$ is needed to rationalize the data, even when cognitive uncertainty is accounted for. In line with the evidence on front-end delay effects, this again suggests that a desire for immediate gratification (preferences) and cognitive noise are orthogonal and complementary objects that both matter.

The reader may wonder why it is important for economists to understand that discounting behavior is to a large degree governed by bounded rationality rather than non-standard discount functions per se. After all, variants of the generalized hyperbola often fit data relatively well, even if they may be getting the underlying reason wrong.

We document the relevance of understanding cognitive microfoundations in two ways.

First, cognitive noise predicts systematic context-dependence of (im)patience: contexts with higher decision complexity, or situations in which people are cognitively busy, should produce more pronounced inelasticities. According to stable discount functions, on the other hand, behavior does not vary as a function of complexity or cognitive states. To test these ideas, we implement additional treatment arms in which we either induce cognitive load or manipulate the complexity of the intertemporal decision tasks by embedding them into a math problem. As predicted, we find that both cognitive load and increased complexity lead to discounting behavior that is much more inelastic, which leads to predictably high or low impatience.

Second, we highlight the relevance of distinguishing between preferences and bounded rationality by studying choice architecture. A main implication of our account is that people are potentially "nervous" about making mistakes and might therefore desire expert advice. In contrast, in pure preferences-based accounts of intertemporal choice, people may behave in impatient ways, but at the time the decision is taken they do not worry that the decision reflects a mistake. We study this distinction through a variation of our incentivized UberEats voucher experiments. Here, after participants have indicated their choices, we surprise them with information about advice from a poll of economists, who recommend that the participant chooses the most patient action. We find that about 1/3 of participants revise their actions in the direction of greater patience, where the probability of revising is twice as high among cognitively uncertain participants.

Linking this paper to the literature, work on intertemporal choice may be separated by whether it speaks to the representation of preferences or the nature of decision processes conditional on preferences. As we discussed in the opening paragraph, a broad class of decision anomalies in intertemporal choice research appear to reflect not just present focus, but also a more general insensitivity of behavior with respect to the time delay. Various pioneering contributions to the literature focused on identifying (or even openly reverse-engineering) reduced-form discount functions that fit this pattern well (Mazur, 1987; Loewenstein and Prelec, 1992; Ebert and Prelec, 2007). Our approach builds on this work, but shows that these phenomena fundamentally reflect bounded rationality rather than non-standard time preferences (discount functions) per se.

A second line of work, often in psychology, studies the role of randomness for intertemporal choice behavior, see Regenwetter et al. (2018) for a recent overview. The main insight of this literature is that allowing for random preferences and / or random responses can often generate hyperbolic discounting (e.g., Lu and Saito, 2018; He et al., 2019). Yet, researchers in this literature typically argue their case through model-fitting exercises rather than direct measurements of cognitive noisiness. This is problematic because, with a multitude of different random preferences and random response models as well as functional form specifications at the researcher's disposal, a large set of different results can potentially be rationalized. More recently, a Bayesian cognitive imprecision literature emerged in economics, which generally predicts phenomena related to inelasticity (Woodford, 2020; Khaw et al., 2021; Gabaix, 2019; Frydman and Jin, 2021; Frydman and Nunnari, 2021; Enke and Graeber, 2020). As discussed in detail below, existing intertemporal choice applications of these models inspire our work (Gabaix and Laibson, 2017; Gershman and Bhui, 2019). Relative to these papers, our contribution is (i) to measure cognitive imprecision, which allows us to provide much sharper and more direct tests of the ideas in this literature; (ii) to show that the measurement of cognitive uncertainty leads to large improvements in model fit; and (iii) to highlight implications for choice architecture and predicting context-dependence.

Our work directly ties into an active recent literature that suggests that what seem like non-standard preferences is sometimes better thought of as reflecting bounded rationality (Nielsen and Rehbeck, 2020; Esponda and Vespa, 2016; Martínez-Marquina et al., 2019; Bordalo et al., 2020) and complexity (Oprea, 2019). This includes documentations of "cognitive effects" in intertemporal choice, such as linkages with time perception (Brocas et al., 2018), waiting periods (Imas et al., 2016), focusing effects (Dertwinkel-Kalt et al., 2021), similarity (Rubinstein, 2003), cognitive ability (Dohmen et al., 2010), and GARP violations (Choi et al., 2021).

Finally, our focus on *cognitive* uncertainty also links to the "implicit risk" literature, which highlights the importance of *objective* uncertainty about whether or when a delayed reward is received (Sozou, 1998; Dasgupta and Maskin, 2005; Halevy, 2008; Chakraborty et al., 2020). As we discuss below, our account is complementary to this literature, and our experiments allow us to ensure that our results on cognitive uncertainty do not spuriously pick up effects of (objective or subjective) payment uncertainty.

The paper proceeds as follows. Section 2 discusses theoretical background. Section 3 presents the experimental design, Sections 4–5 the results and Section 6 the model estimations. Section 7 shows results on choice architecture and context-dependence, and Section 8 concludes.

2 Theoretical Considerations and Hypotheses

Consider a choice context in which a decision-maker (DM) is prompted to specify the units of consumption a at an earlier point in time t_1 that make him indifferent to consuming $c_{t_2} = 1$ at $t_2 > t_1$. Denote by $D(t) = \delta^t$ the DM's discount function, and by $u(\cdot)$ a weakly concave utility function. Later, we will allow for taste-based present bias. A helpful theoretical benchmark is that of a rational DM's utility-maximizing decision, which equates the discounted utilities of both options:

$$D(t_1)u(a) = D(t_2)u(1) \quad \Rightarrow \quad a^* = u^{-1}(\delta^{t_2 - t_1}) \in [0, 1], \tag{1}$$

where we have normalized $u(1) = 1$.

Inelasticity with respect to time and intertemporal choice regularities. As noted in the Introduction, a generic inelasticity to the time delay pervades intertemporal choice research. To intuitively see how inelasticity can produce widely studied regularities, consider the extreme thought experiment in which a decision-maker treats all time delays exactly the same and maximizes discounted utility conditional on treating any objective time delay as $t_2 - t_1 = c$. Evidently, decreasing per-period impatience mechanically follows from such a decision rule. Extreme short-run impatience also follows, provided that the imagined constant delay c is not very short.

Finally, subadditivity suggests that people generally behave as if they are less impatient in a single tradeoff between t_0 and t_2, compared to the combined impatience that is revealed in two tradeoffs between t_0 and t_1, and between t_1 and t_2. In the exponential model, the constant rate of discounting implies a transitivity feature that rules out subadditive discounting. Our hypothetical DM, on the other hand, treats each of the two sub-intervals exactly like the original long interval, and therefore appears more impatient once the long interval is divided up.

Our objective in this section is to summarize the extant random choice literature in a way that highlights that three broad classes of models often generate an inelasticity of decisions with respect to the delay. We only provide an informal discussion here because, as reviewed in the formal treatment of Regenwetter et al. (2018), random choice models exhibit large diversity in precise modeling approaches and functional form assumptions.

Bayesian cognitive imprecision models. Bayesian cognitive imprecision (or cognitive noise) models presume that people do not have direct access to their utility-maximizing action a^* (or a specific problem parameter) but only to a noisy mental simulation thereof. This mental simulation or cognitive signal could equivalently be understood as resulting from a sampling process in which people gather cognitive "evidence" about their optimal action through deliberation, as in drift-diffusion models. DMs are hypothesized to combine this cognitive signal with a prior over their utility-maximizing action, which we call cognitive default (see, e.g., Woodford, 2020; Khaw et al., 2021; Frydman and Jin, 2021). Because the DM's decisions (or perceptions) partly reflect an invariant cognitive default, a first-order implication of this class of models is the presence of inelasticities to variation in problem inputs. In the intertemporal choice domain, cognitive noise could arise for a variety of reasons, which we discuss in Section 3 below.

Appendix A presents a simple intertemporal cognitive imprecision model that is an adaptation of atemporal applications (Fennell and Baddeley, 2012; Heng et al., 2020). In this model, the DM holds a Beta-distributed prior over his discounted-utility maximizing action, where the prior has mean d. Through deliberation, the DM generates a cognitive signal about what his discounted-utility maximizing decision is. This signal S is (scaled) Binomially distributed and satisfies $E[S] = a^*$. Under these assumptions, a Bayesian DM's posterior mean over his optimal action can instructively be represented as

$$a^o = \lambda s(a^*(\delta, \Delta t)) + (1-\lambda)d \qquad \Rightarrow \qquad E[a^o] = \lambda a^*(\delta, \Delta t) + (1-\lambda)d \tag{2}$$

This formulation intuitively captures an anchoring-and-adjustment heuristic (Tversky and Kahneman, 1974), according to which people anchor on some default action (an action they would take in the absence of deliberation) and then adjust based upon the outcome of their deliberation process. Here, the weight λ partly captures the precision of the mental simulation of the utility-maximizing action.[3]

The main implication of eq. (2) is that decisions are insufficiently sensitive to the time delay because decisions partly reflect the delay-invariant cognitive default. As we show in Appendix A, eq. (2) implies the following predictions, which we pre-registered.

Pre-registered predictions. *A DM with $\lambda < 1$ (cognitive imprecision) exhibits:*

1. *More pronounced short-run impatience, both when the time delay starts in the present and when it starts in the future.*
2. *More pronounced decreasing impatience, both when the time delay starts in the present and when it starts in the future.*
3. *More pronounced subadditivity.*
4. *The same degree of front-end delay effects: the pattern that people appear more patient when a constant is added to both the early and the later date.*

Predictions 1 and 2 imply a distinctive "flipping" property of the relationship between impatience and cognitive imprecision: while cognitively imprecise agents are *more* impatient for short delays, the more pronounced decreasing impatience can make them

[3]Another potential microfoundation for regression to a cognitive default d are models of caution (e.g., Cerreia-Vioglio et al., 2015; Chakraborty, 2020). These models are intuitively related in the sense that – due to subjective uncertainty over their utility function – agents regress towards preferring a certain option. This setup could potentially be modified such that, because of uncertainty about their preferences and caution, agents regress to a "simple" or "intuitive" option d, rather than a certain one. However, at this point, such a model of caution has not been formulated yet.

behave *less* impatient for very long delays. Prediction 4 clarifies that a cognitive imprecision framework like the one sketched above does not predict a link between cognitive imprecision and front-end delay effects, which are usually thought of as a canonical signature of present-focused preferences.[4] The reason is that, in eq. (2), cognitive precision λ only affects how people respond to a given time delay, rather than whether it starts in the present or future. This prediction is instructive because it highlights that noise-induced inelasticities and temptation effects are orthogonal to each other.

Finally, we note that not all cognitive imprecision models generate the full set of predictions above. The main intertemporal choice applications of Bayesian noisy cognition in the literature are Gabaix and Laibson (2017) and Gershman and Bhui (2019). Their setup is slightly different from the one above because they assume that all decision-relevant cognitive imprecision stems from the mental simulation of future *utils*. The stylized framework above, on the other hand, takes a broader perspective by assuming that the *utility-maximizing action* is perceived with noise, regardless of what the underlying sources of noise are (noisy mental simulation of future utils may be one of them). This distinction matters for predictions. While Gabaix and Laibson's model generates decreasing impatience, their model makes two predictions that differ from the ones above. First, because their model maintains transitivity, it does not predict subadditivity. Second, their model predicts front-end delay effects and related preference reversals, see Section 2.7 of Gabaix and Laibson (2017).

Random response models. This class of models is broad. One incarnation that relates to the preceding discussion is that the DM probabilistically either plays his utility-maximizing action or chooses at random, $\epsilon \sim F(\cdot) \in [0,1]$, with $E[\epsilon] = d$. Formally, we say that a trembling action a^{tr} is given by

$$a^{tr} = \begin{cases} a^*(\lambda, \Delta t) & \text{with prob. } \lambda \\ \epsilon & \text{otherwise} \end{cases} \qquad \Rightarrow \qquad E[a^{tr}] = \lambda a^*(\lambda, \Delta t) + (1-\lambda)d. \tag{3}$$

This expression for the average action is identical to the one in (2). Thus, the two models make identical predictions about average behavior. Moreover, the models are also difficult to tease apart looking at individual decisions because they both predict that actions will be random (even conditional on potential anchoring on a cognitive default). Thus, depending on the precise assumptions about the distributions of noise, various different individual-level response patterns can be rationalized.[5]

[4] Front-end delay effects refer to the regularity that people generally behave less patiently in a tradeoff between consumption dates t_0 and t_1 than in a tradeoff between $t_0 + z$ and $t_1 + z$, for $z > 0$.

[5] Another type of random response model is that the DM's action is given by $a^{t,2} = a^*(\delta, \Delta t) + \eta$, with $E[\eta] = 0$. Then, because the DM's action is bounded by zero and one, random decision errors may

Random preference models. This class of models assumes that the DM's discount function is stochastic and fluctuates over time (e.g., Regenwetter et al., 2018; Lu and Saito, 2018; He et al., 2019). In its most widespread incarnation, random intertemporal preferences models assume that "true" discounting is exponential, yet the decision-relevant discount factor $\tilde{\delta}$ varies randomly across trials, such that $\tilde{\delta} = \delta + \mu$, with $E[\mu] = 0$. Thus, in the setup sketched above, the DM's random preference action a^r would be given by

$$a^r = a^*(\tilde{\delta}, \Delta t). \tag{4}$$

It is widely understood that variation in δ can produce behavior that implies "decreasing impatience" because the average of multiple exponential functions is not necessarily exponential and can be hyperbolic. This insight was first noted in interpersonal (social welfare) contexts (Weitzman, 2001; Jackson and Yariv, 2014). However, a mathematically identical insight applies when a single DM's discount factor varies across time (Lu and Saito, 2018; He et al., 2019), or when the DM's beliefs about his discount factor varies over time. Thus, as in the models described above, higher noisiness (variance of μ) should be correlated with stronger decreasing impatience.

At the same time, models that only feature random variation in preferences do predict front-end delay effects, see Proposition 1 in Jackson and Yariv (2014). This distinguishes random preference models from the models discussed above. Moreover, in contrast to the models above, natural random preference models do not predict subadditivity.[6]

Summary and empirical implementation. The different classes of random choice models make similar predictions, in particular as far as a link between noise on the one hand and decreasing impatience as well as short-run impatience on the other hand are concerned. Moreover, the models afford varying degrees of flexibility, in particular once they are combined with one another (see Regenwetter et al., 2018). Hence, our objective is not to definitely tease these models apart, but to generically show that cognitive noise is instrumental for understanding intertemporal choice. At the same time, to the degree that the different classes of models *do* make different predictions, corre-

"bounce off the boundary" and push decisions to be intermediate, on average. Indeed, the idea that random decision noise in combination with boundary effects may produce both hyperbolic and subadditive discounting behavior was well-undestood by early contributions to the literature (e.g., Read, 2001). We do not highlight this type of model because, in our data, decisions that are associated with strictly positive cognitive uncertainty are rarely located at or close to the boundaries, see Appendix Figure 13.

[6] To see this, consider a model à la Lu and Saito (2018), in which the DM draws a separate discount factor $\tilde{\delta}+\mu$ for each potential calendar time prior to observing the specific payout dates in an experimental trial. In a subadditivity documentation, there are three time delays, $t_0 \to t_1$, $t_1 \to t_2$ and $t_0 \to t_2$. Let the decision-relevant discount factors for the first two delays be $\tilde{\delta}_{t_0 \to t_1} = \delta + \mu_1$ and $\tilde{\delta}_{t_1 \to t_2} = \delta + \mu_2$. Then, with $\tilde{\delta}_{t_0 \to t_2} = \tilde{\delta}_{t_0 \to t_1} \tilde{\delta}_{t_1 \to t_2}$, there is no subadditivity.

sponding empirical results will allow us to draw some tentative conclusions about the relative explanatory power of the different approaches.

Because the actual form and realizations of cognitive noise are unobservable, we empirically measure a signature of cognitive imprecision that is compatible with all three different classes of models. Following Enke and Graeber (2020), we define *cognitive uncertainty* as people's self-reported degree of certainty that their response to an intertemporal choice problem equals their true utility-maximizing action. In the context of a Bayesian cognitive noise model, we define:

$$p_{CU} \equiv P(\left|A|\{S=s\} - a^o\right| > c). \tag{5}$$

$A\Big|\{S = s\}$ is the perceived posterior distribution about the utility-maximizing action, conditional on the mental simulation s. Intuitively, cognitive uncertainty captures the likelihood with which the DM thinks his optimal action might fall outside a window of length $2c$ around the action that he actually chose.Our empirical study leverages cognitive uncertainty as a (probably imperfect) proxy for cognitive imprecision to examine the relationship between cognitive imprecision and intertemporal choice regularities as discussed above.

3 Experimental Design

3.1 Choice Tasks

Incentivized UberEats Voucher Experiments. In treatment *Voucher Main*, rewards are given by UberEats food delivery vouchers.[7] Participants complete multiple price lists (MPLs) that elicit interval information about indifference points. In each list, the left-hand side Option A is a fixed delayed UberEats voucher with value $y_2 \in \{40, 42, \ldots, 50\}$. The delayed payout date $t = t_2$ varies between one week and one year. The right-hand side Option B is an UberEats voucher whose value increases as one goes down the list, from \$2 to \$$y_2$, in steps of \$2 each. The payment date for Option B, t_1, is always strictly earlier than the one for Option A, though not necessarily today.

Participants had to indicate a choice between Options A and B in each row of the MPL. We implemented a computerized auto-completion mode that enforces a single switching row: whenever a subject chose Option A in a given row, Option A automatically got selected in all rows above. Likewise, whenever a subject chose Option B in a given

[7] The currently most widely used experimental economics paradigm to implement primary rewards in an intertemporal choice context consists of real effort tasks (Augenblick et al., 2015). These are infeasible in our context, however, because our research hypothesis requires a consumption good that can plausibly be implemented with long time delays, while real effort studies focus on horizons of a few weeks at most.

row, Option B automatically got selected in all rows below. Participants could revisit and change their choices at any time, and choices only became locked in when a participant decided to proceed to the next screen. Appendix Figure 10 shows a screenshot.

UberEats is the largest online food ordering and delivery service in the world. The service can be used to order food for takeout or delivery from a wide array of restaurants and is widely available throughout the United States, with an estimated market share of between one fifth to one third (Curry, 2021). Our UberEats vouchers are valid for a period of only seven days. For example, when a choice option is given by "$40 voucher that is valid in 6 months," then this means that the voucher will become valid six months after the participant's study date, and will remain valid for a period of seven days. We implemented a comprehension check to verify that participants understood that the voucher would expire after seven days, rather than be valid indefinitely. Participants' vouchers were directly credited to their personal UberEats accounts within 10 hours of completion of the study, such that subjects did not have to actively claim the voucher. The vouchers were always visible in their accounts, they could just not be used before the validity period. Participants received two automatic reminders 24 hours before a voucher became valid and 24 hours before it expired.

Hypothetical Money-Early-versus-Later Experiments. Treatment *Money Main* has the same structure as the UberEats voucher experiments, except that the rewards are given by hypothetical dollar amounts. While the hypothetical nature of the payouts has obvious disadvantages, it also confers various advantages, in particular in conjunction with our financially incentivized UberEats experiments. First, we could explicitly instruct participants to make their choices assuming that there is no payment risk. We verify participants' understanding of this through a comprehension check. Second, hypothetical payments allow us to use some very long time delays (up to "in 7 years") that would not be credible with real payments or food vouchers. This is an important advantage because, as discussed above, the inelasticity of discounting to the time delay leads us to expect that the relationship between cognitive uncertainty and impatience will flip as a function of the time delay. Finally, money experiments allow us to replicate the setup in which regularities such as diminishing impatience or subadditivity have predominantly been documented in the literature (Cohen et al., 2020).

Choice configurations. First, for choice lists with an early date of today, we implement delayed dates that range from one week to seven years in the hypothetical money experiments, and from one week to one year in the incentivized UberEats study. Second, in both experiments, we implement a broad set of lists that have an early payment date of "in one month," again with large variation in the corresponding later payment dates.

These choice lists allow us to study short-run impatience and decreasing impatience, starting from both the present and the future.

Third, we implement two sets of three choices each that serve to test for subadditivity effects. Subadditivity means that people appear more patient in a single decision than when the respective time delay is broken up into multiple delay periods. Our first subadditivity set consists of the following intertemporal tradeoffs: $(t_1 = 0, t_2 = 12m)$, $(t_1 = 0, t_2 = 6m)$, $(t_1 = 6m, t_2 = 12m)$. The second subadditivity set is constructed analogously, except with payment dates of zero, four and eight months.

Fourth, the subadditivity sets also allow for an analysis of front-end delay effects: the extent to which people are more patient in, e.g., $(t_1 = 6, t_2 = 12)$ than in $(t_1 = 0, t_2 = 6)$. Fifth, for each participant, two randomly selected choice configurations were presented twice in random locations in the sequence of twelve price lists. These are exact repetitions of the same choice problems, including the same earlier and delayed payment dates as well as the same delayed payout amount y_2. These choice problems facilitate an analysis of across-trial choice variability. The order of all choice lists was was randomized at the participant level.

Study components. The hypothetical money study consisted of four parts. In the first, each participant completed a total of twelve MPLs. In the second part, each subject completed six additional intertemporal choice problems that were administered in a direct elicitation format rather than using MPLs. We discuss these data in greater detail in Section 5.4. In the third part of the study, participants completed three choice under risk MPLs that (i) facilitate an analysis of the cross-domain stability of cognitive uncertainty and (ii) allow to disentangle time discounting from the role of utility curvature in our structural analyses (Section 6). In the fourth part, participants completed a Raven matrices IQ test. The structure of the UberEats study was identical, except that we did not implement the direct elicitation choice problems.

3.2 Measuring Cognitive Uncertainty

Elicitation. In both paradigms described above, participants make choices in MPLs that carry interval information about indifference points. In our experiments, the switching intervals have width \$2. Our experimental instructions explain that we use this switching interval to determine how much the participant values the later payment at the earlier date. Immediately after each choice list, we measure cognitive uncertainty (CU) as the participant's subjective probability that their true valuation of the later payment / voucher is actually contained in their stated switching interval. Specifically, after a participant completes a choice list with switching interval given by $[\$a, \$b]$, the subsequent screen reminds them of their previous decision and elicits cognitive uncertainty:

> *Your choices on the previous screen indicate that you value $\$y_2$ in t_2 somewhere between $\$a$ and $\$b$ in t_1. How certain are you that you actually value $\$y_2$ in t_2 somewhere between $\$a$ and $\$b$ in t_1?*

Participants answer this question by selecting a radio button between 0% and 100%, in steps of 5%. Appendix Figure 11 provides a screenshot. This cognitive uncertainty measurement follows the same protocol as proposed in a revised version of Enke and Graeber (2020) for choice under risk, here adapted to an intertemporal choice context. In line with the discussion in Section 2, we interpret this question as capturing the participant's (posterior) uncertainty about their utility-maximizing decision, after some sampling of cognitive signals has taken place.

Potential origins of cognitive uncertainty. Our measure is deliberately designed to capture participants' overall subjective uncertainty about what their preferred action is. This uncertainty could have various potential origins. First, people may not know their true preferences. This preference uncertainty could either be about one's true discount factor, or about the instantaneous utils that one will derive from future consumption, as in Gabaix and Laibson (2017).

Second, even conditional on knowing their preferences, people may cognitively struggle with choosing an action that maximizes discounted utility. For example, people may find it hard to cognitively integrate their discount factor with the time delay that is implied by different choice options, or they may suffer from imperfect time perception (Brocas et al., 2018), or they may make random trembling errors in implementing their preferred choice in experimental interfaces. A hypothetical special case of this class of mechanisms is that there is no true discounting and people attempt to maximize the net present value of payments, but find it cognitively difficult to do so.

Comparison with alternative measures. Broadly speaking, the literature has proposed two different types of measures for eliciting people's uncertainty about their own decisions, though we are not aware of prior applications to intertemporal choice. At one extreme, psychologists, neuroscientists and some economists elicit measures of "decision confidence," in which subjects indicate on Likert scales how "confident" or "certain" they are in their decision (e.g., De Martino et al., 2013, 2017; Polania et al., 2019; Lebreton et al., 2015; Xiang et al., 2021; Butler and Loomes, 2007). At the other extreme, economists have proposed to use measures of across-trial variability (Khaw et al., 2021) or even deliberate randomization (Agranov and Ortoleva, 2017, 2020). Our preferred measure strikes a middle ground between these two approaches. First, while our approach retains the attractive simplicity of implementing a single question (as in the psychology literature), it is also quantitative in nature. The simplicity of asking one question per decision screen should be contrasted with the approach of gauging cognitive imprecision through across-task variability in choices, which requires *many* trials and is usually defined at the level of a study rather than of a single choice problem. Second, as illustrated in Enke and Graeber (2020), our simple measure can also be deployed in contexts other than price lists. This is particularly attractive here because some of our experiments do not rely on choice lists. Despite these arguably attractive features, incentivized data on across-trial choice variability are an additional attractive tool to gauge cognitive imprecision. Below, we therefore correlate our simple-but-unincentivized CU measure with choice variability.

3.3 Design Considerations

Time discounting studies are complicated by a range of methodological considerations (Frederick et al., 2002; Andreoni and Sprenger, 2012; Augenblick et al., 2015; Cohen et al., 2020). We discuss prominent concerns and implications for interpretation below.

External uncertainty / payment credibility. According to the so-called "implicit risk" hypothesis, intertemporal decisions could reflect not only genuine discounting but also external uncertainty (e.g. Benzion et al., 1989; Sozou, 1998; Halevy, 2008). This could be due to a lack of trust in the experimenter, uncertainty about the future purchasing power of money or vouchers, or the subjective probability of forgetting about the existence of the later reward. To address this, we put various measures in place. First, we deliberately implemented the money experiments in hypothetical terms. This allows us to emphasize that subjects should make their decisions by assuming that they know with certainty that they will receive all payments with certainty. We verified understanding of this through a comprehension check question.

Second, in the UberEats experiments, because vouchers appear in the participant's UberEats account within a few hours of the study regardless of the precise validity period, there is no differential payment risk across vouchers with different time delays. Participants could always view vouchers in their account, they could just not be used. We view this as a main advantage of our method relative to traditional monetary payments.

Third, those participants that actually won a voucher were asked to state their subjective probability that they will actually receive and use their voucher. The median (average) response is 95% (84%). Most importantly, we find that subjects' beliefs are uncorrelated with the delay of the voucher's validity period. This suggests that future vouchers were not perceived as more uncertain. All of our results are robust to only including participants in the analysis who indicate 100% certainty.[8]

Cognitive vs. external uncertainty. A related concern is that participants misinterpret the CU question as asking about their subjective probability of actually receiving the later reward. To address this, our money experiments include a comprehension check question that directly asks participants to indicate whether the CU elicitation question asks about (i) the subject's subjective probability of actually receiving the money or (ii) their certainty about own their valuation, given that they know they will receive the money with certainty. In addition, notice that an account of CU capturing perceived payment uncertainty would predict that CU is always negatively correlated with observed patience. However, we will see that, over sufficently long time horizons, CU is actually positively correlated with revealed patience.

Fungibility. A common argument is that intertemporal choice experiments over money do not capture preferences-based discounting because money is fungible. From such a perspective, behavior in experiments reveals participants' attempt to maximize the net present value of payments, given (perceived) real interest rates. An alternative view is that experimental participants narrowly bracket their choices and treat monetary amounts in experiments as proxy for utils (Halevy, 2014; Sprenger, 2015). We acknowledge this discussion, but note that it only affects the precise interpretation of our cognitive uncertainty question. Under the interpretation that our experimental paradigms do not capture true discounting, our CU measure picks up participants' cognitive limitations in computing discounted utility (here: NPVs), conditional on knowing their preferences ($\delta = 1$). On the other hand, if experiments over money also capture real discounting, the CU question potentially captures all of the various psychological mechanisms discussed in the previous subsection. Regardless of whether the participant's objective is to maximize NPV or discounted utility more generally, our hypothesis is that subjective uncertainty about the maximizing action is associated with a compression effect.

Utility curvature. It is well-established that estimates of discount rates from price list choices are confounded unless the curvature of the utility function is taken into account. To address this, we use the "double price list method" that estimates utility curvature from separate risky choices.

[8]Regarding actual consumption of our vouchers, at the time of the writing of this paper, 77% of subjects had used their UberEats credit, which is arguably a high usage rate for a voucher. This percentage fluctuates across delays but does not systematically decrease in the length of the delay. For example, usage rates were 80% if the voucher is valid immediately to 70% after two weeks and 100% after 6 months.

Transaction costs. A main concern with traditional time discounting experiments is that they capture differential transaction costs between present and future. In our hypothetical money experiments, transaction costs are implausible. In the UberEats experiments, there are likewise no transaction costs because participants automatically receive their vouchers credited to their UberEats app.

3.4 Logistics and Participant Pool

The study was conducted on Prolific, an online worker platform. Recent experimental economics work suggests that data quality on Prolific is much higher than on Amazon Mechanical Turk, and comparable to that in a canonical lab subject pool (Gupta et al., 2021). For the hypothetical money experiments, we made use of Prolific's "representative sample" option to collect data from a broad and diverse (though not actually nationally representative) set of participants.[9] We pre-registered a sample size of $N = 600$ participants. However, because of the discreteness of Prolific's representative sample procedure, we eventually ended up sampling $N = 645$ people. Since we view throwing away data as questionable, we keep the full sample, but we have verified that all results hold quantitatively unchanged if we restrict the sample to the first 600 completes.

In the UberEats experiments, the study description that was visible to prospective participants announced that study bonuses would be paid in the form of UberEats vouchers. In addition, we implemented a screening in which participants were again asked whether they possess an UberEats account, and we immediately routed all people out of the experiment if they did not.[10] As we pre-registered, $N = 500$ workers participated in the UberEats study.

Participants in both studies completed a comprehension check of three questions each. Any participant who failed one or more of these questions was immediately routed out of the experiment. This was the case for 16% in the money and 37% in the UberEats experiments. We additionally implemented an attention check at the end of the study, and exclude all participants who failed it (2% in the money and 1% in the UberEats experiments). Our procedures imply that, if anything, we are working with a relatively attentive and cognitively sophisticated participant pool.

[9]In our money experiments, average age is 42 years, 54% are female, and 45% have a college degree. In our UberEats experiments, average age is 28 years, 58% are female and 59% have a college degree.

[10]Because our experiments were conducted from late March through May 2021, we took various measures to ensure that only those prospective participants signed up for the study who were not concerned about ordering food for delivery due to COVID-19. First, the study description clarifies that people should not participate if they are concerned about ordering food for delivery due to COVID-19. Second, we restricted the sample to participants of age 45 and under. Third, we ask prospective participants whether they are worried about ordering delivery food due to COVID-19, and we immediately exclude anyone from the study whose response is affirmative. Finally, by late March 2021 it had become increasingly evident that delivery food is not a main source of COVID-19 transmission.

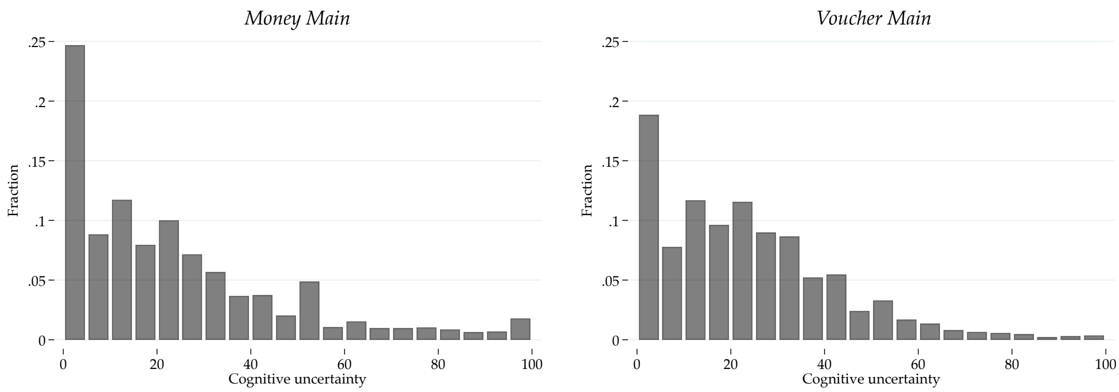


Figure 2: Histogram of cognitive uncertainty in *Money Main* (left panel) and *Voucher Main* (right panel).

In the hypothetical money experiments, participants received \$4.50 as a flat payment for completion of the study. In the UberEats study, participants received a completion fee of \$4.00. In addition, one of the three parts of the experiment (intertemporal choice, risky choice, Raven IQ test) was randomly selected for payout, with associated probabilities of 25:5:70. When the intertemporal or risky decision part was selected, one randomly selected choice from one randomly selected list was implemented and subjects received the UberEats voucher as described above. Appendix G contains screenshots of all experimental instructions and comprehension checks.

3.5 Pre-Registration

Appendix Table 5 provides an overview of all treatments conducted for this paper, including pre-registration details.[11] Our pre-registration includes (i) predictions 1–4 in Section 2, (ii) the prediction that cognitive uncertainty is correlated with across-trial choice variability, and (iii) descriptive analyses of the correlates of cognitive uncertainty to be discussed in Section 4.

4 Variation in Cognitive Uncertainty

4.1 Variation Across Participants and Decision Problems

To help build intuition for the CU measure in an intertemporal choice context, we begin by summarizing descriptive evidence. Figure 2 shows histograms of task-level CU in the MPL decisions in treatments *Money Main* (left panel) and *Vouchers Main* (right panel), such that each participant corresponds to twelve observations. We see that 75% of all

[11]Pre-registrations can be found at `https://aspredicted.org/kg7zs.pdf` (*Moneay Main*) and `https://aspredicted.org/b4pw2.pdf` (*Voucher Main*).

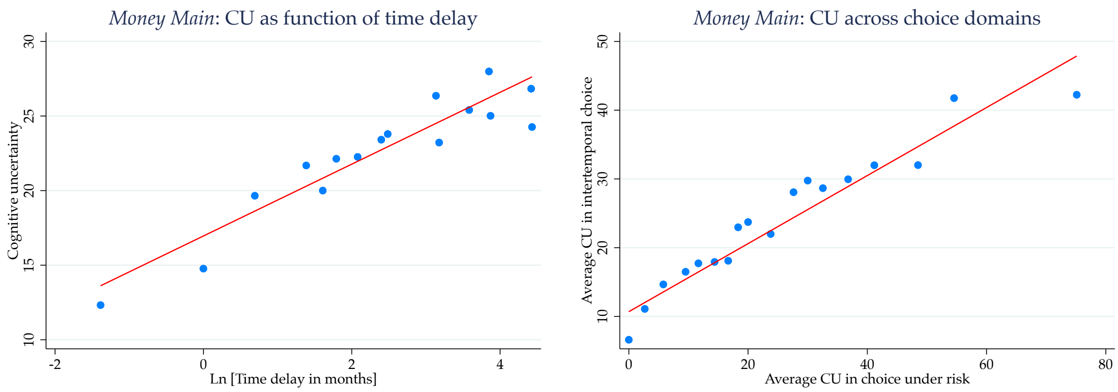


Figure 3: Binscatter plots. The left panel shows the relationship between task-level CU and the time delay in a decision problem (N=7,740 decisions). The right panel shows the correlation between participant-level average CU in intertemporal choice and average CU in choice under risk (N=645 participants).

decisions in *Money Main* and 81% of decisions in *Voucher Main* are associated with strictly positive CU.

The heterogeneity in Figure 2 reflects both across-participant heterogeneity and systematic variation across choice problems. Figure 3 illustrates correlates of CU in treatment *Money Main* using binned scatter plots; the analogous figures for treatment *Voucher Main* look almost identical. The left panel shows that CU strongly increases in the length of the log time delay ($\rho = 0.16$), suggesting that payouts or consumption in two temporally distant periods are more dfficult to compare and evaluate against each other.

In light of this across-task variation, a relevant question is how consistent or stable people are in exhibiting high or low CU. In our data, participant fixed effects explain about 45-54% of the variation in CU. Thus, CU appears to have reasonably high within-domain stability. Looking at across-domain stability, the right panel of Figure 3 documents that a participant's average CU is strongly correlated with the participant's average CU in separate risky choice (lottery) experiments that we implemented in the final part of our study. The raw correlation between average CU in intertemporal choice and average CU in choice under risk is $r = 0.62$ in *Money Main* and $r = 0.50$ in *Voucher Main*, which is arguably very high. This result is consistent with those reported in Enke and Graeber (2020), who document that CU in choice under risk is likewise highly correlated with CU in economic expectations.[12] In combination, these results suggest that cognitive uertainty may be a reasonably stable domain-general trait.

[12]Other correlations (omitted for brevity) between average subject-level CU and demographics are mostly small. The first value refers to the money study and the second one to the voucher study: $r = -0.08$ (0.01) with the score on Raven matrices IQ test, $r = -0.10$ (0.08) with age, $r = 0.06$ (0.06) with a female indicator, $r = -0.03$ (-0.05) with a college degree indicator, and $r = 0.07$ (-0.07) with log study completion time.

4.2 Is Cognitive Uncertainty Reflective of Actual Noise?

As discussed in Section 3.2, some researchers have used the presence of choice variability as empirical measure of cognitive imprecision. We deem it useful to establish an empirical correspondence between our CU question and variability for two reasons. First, data on choice variability is useful to understand whether people's subjective perception of their own cognitive imprecision is roughly accurate. Second, a correlation between CU and choice variability may be seen as a validation of a quantitative-but-unincentivized question with financially incentivized decisions, in the spirit of recent experimental validation studies in the literature (e.g. Falk et al., 2015; Enke et al., forthcoming).

Figure 4 shows the magnitude of across-trial variability as a function of cognitive uncertainty. Variability is computed as absolute difference in normalized switching points across two repetitions of the same choice list. We see that decisions that are associated with higher average CU across the two trials are more variable. In quantitative terms, an increase in average CU from zero to fifty is associated with a threefold increase in variability. In both datasets, the raw correlation is $\rho \approx 0.17$, $p < 0.01$.

5 Cognitive Uncertainty and Intertemporal Choice

5.1 Cognitive Attenuation: Inelasticity of Decisions to Time Delay

We begin by displaying the raw data: how intertemporal decisions vary as a function of the delay. For each choice list, a useful summary statistic is a participant's *normalized indifference point*, $NIP \in [0, 1]$, which is given by the midpoint of the switching interval, divided by the later payment amount. This measure represents which payment at the earlier payment date makes the participant indifferent to receiving \$1 at the later date.

Figure 5 shows average normalized indifference points (in percent). The top panel shows results for treatment *Money Main* and the bottom panel those for *Voucher Main*. To make the results comparable, the x-axes are kept identical even though the maximal time delay in the vouchers study is only twelve months. We show results separately for participants with CU of zero and strictly positive CU.[13] For ease of illustration, we restrict attention to decision problems in which the early payment date is today, $t_1 = 0$. The analogous figure for $t_1 > 0$ looks very similar (Figure 12 in Appendix B).

The figure's main takeaway is that CU is strongly associated with compression of indifference points towards the center (roughly 50%). Notably, in treatment *Money Main*, this CU-associated inelasticity is sufficiently strong that cognitively uncertain partici-

[13] This specific split illustrates how well the 20–25% of decisions associated with zero cognitive uncertainty are explained by exponential discounting. Our regressions analyses, however, leverage the entire variation in cognitive uncertainty.

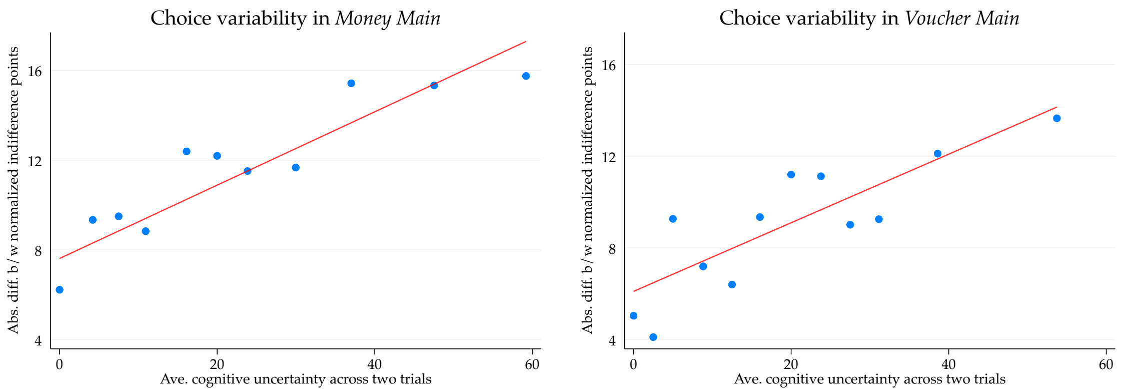
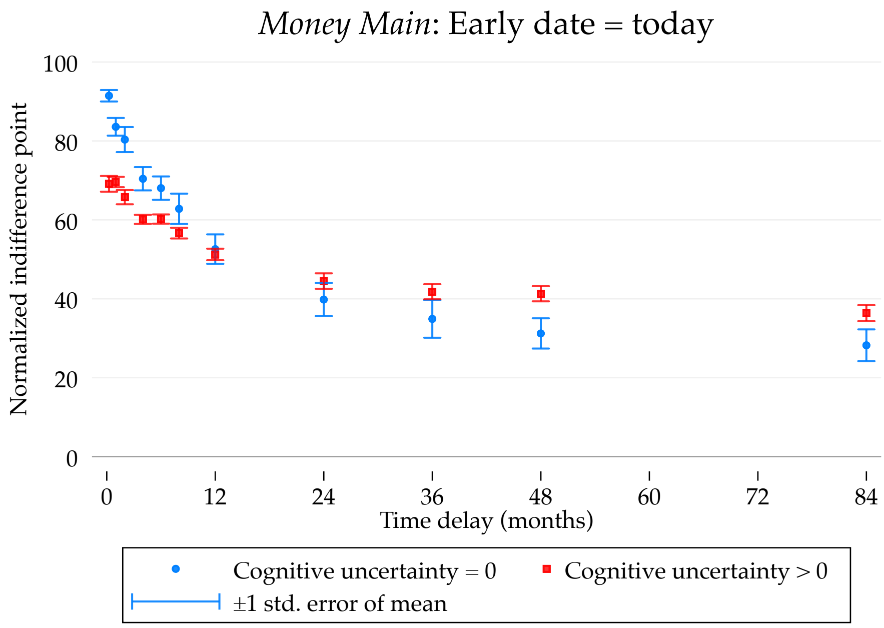


Figure 4: Link between cognitive uncertainty and across-task variability in normalized switch points in an exact repetition of the same decision problem in *Money Main* (left panel, $N = 1,290$) and *Voucher Main* (right panel, $N = 1,000$). The y-axis captures the absolute difference between the normalized indifference points in two exact repetitions of the same choice list. Average cognitive uncertainty is winsorized at 60 (roughly the 95th percentile in both datasets) for ease of visibility.

pants act as if they are *less* patient over relatively short horizons, yet *more* patient over relatively long horizons, with a crossover point at around one year. This indicates that the main behavioral implication of CU in intertemporal choice is indeed insensitivity to time delays, rather than generically higher impatience. A second takeaway from Figure 5 is that behavior is very similar in *Money Main* and *Voucher Main*, including in its link to CU. In particular, cognitively uncertain decisions in *Voucher Main* also reflect lower patience over short time delays, yet as a result of insensitivity, this difference becomes ever smaller as the length of the time delay increases.

Table 1 presents corresponding OLS regression estimates. Here, we relate participant's normalized indifference point to the length of the time delay, interacted with CU. Columns (1)–(4) show the results for *Money Main*, separately for whether the early payment date is today or in the future. Columns (5)–(8) show analogous results for *Voucher Main*. The results confirm the visual impression from Figure 5: (i) CU is associated with a lower sensitivity of indifference values with respect to time delays, as can be inferred from the positive interaction coefficient and (ii) the regression intercept (patience over very short horizons) is negatively correlated with CU, as we can infer from the significant raw CU term. These results are very similar for $t_1 = 0$ and $t_1 > 0$. A final comment regards the coefficient magnitudes. For example, in column (1), the coefficients suggest that increasing CU from zero to fifty (which is the 90th percentile) is associated with a decrease in sensitivity from 8.1 to 2.6 (or 68%), arguably a very large magnitude.[14]

[14]The main reason why the coefficient magnitudes are so different between *Money Main* and *Voucher Main* is the large difference in the average time delay between these two experiments. Once the data in *Money Main* are restricted to delays of at most one year, the coefficients are similar across the two experiments.



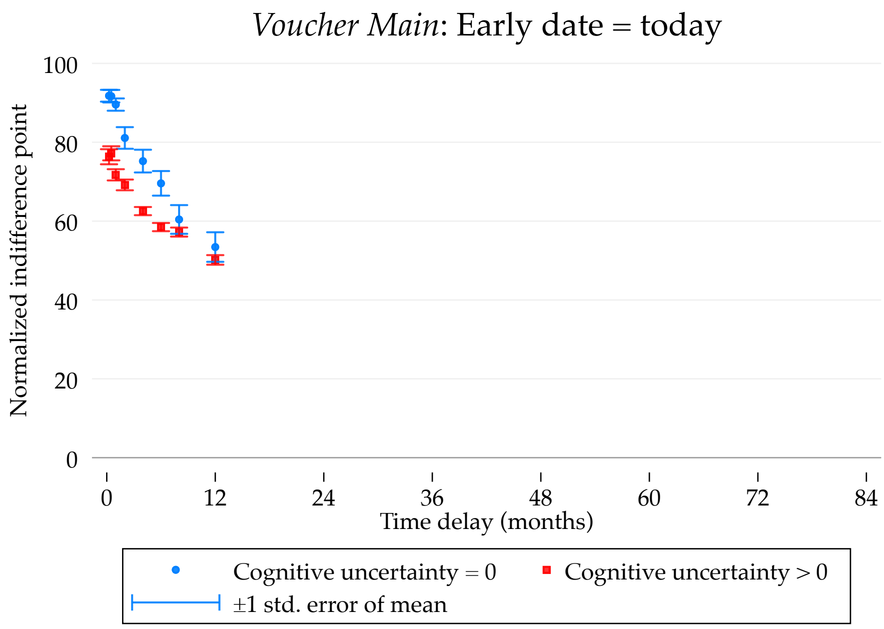


Figure 5: Observed discounting with $t_1 = 0$ in *Money Main* (top panel, $N = 4,948$) and *Voucher Main* (bottom panel, $N = 3,846$). Normalized indifference points are given by the midpoint of the switching interval in a choice list, divided by the larger-later payout amount (in %). The figure shows averages across decisions. Whiskers show standard error bars, computed based on clustering at the subject level.

5.2 Linking Cognitive Uncertainty to Empirical Regularities

5.2.1 Short-Run Impatience

Figure 5 provided strong visual evidence for the hypothesis that, over very short horizons, cognitively uncertain subjects are more impatient than cognitively certain ones, in

Table 1: Cognitive uncertainty and inelasticity with respect to time delays

	Dependent variable: Normalized indifference point							
Treatment:	*Money Main*				*Voucher Main*			
Sample:	$t1 = 0$		$t1 > 0$		$t1 = 0$		$t1 > 0$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Time delay (years)	-8.08***	-8.08***	-7.76***	-7.72***	-39.2***	-38.9***	-39.1***	-39.1***
	(0.39)	(0.39)	(0.39)	(0.39)	(2.16)	(2.14)	(3.88)	(3.86)
Time delay × Cognitive uncertainty	0.11***	0.11***	0.073***	0.071***	0.61***	0.59***	0.58***	0.59***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.08)	(0.08)	(0.14)	(0.14)
Cognitive uncertainty	-0.38***	-0.37***	-0.32***	-0.31***	-0.59***	-0.58***	-0.57***	-0.57***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.06)	(0.06)	(0.07)	(0.07)
Payment amount FE	No	Yes	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes	No	Yes
Observations	4948	4948	2792	2792	3846	3846	2154	2154
R^2	0.17	0.19	0.19	0.21	0.20	0.21	0.13	0.14

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Columns (1)–(4) include data from *Money Main*, where columns (1)–(2) restrict attention to decision problems with $t_1 = 0$ and columns (3)–(4) to problems with $t_1 > 0$. An analogous logic applies to columns (5)–(8) for *Voucher Main*. Demographic controls include age, gender and income bucket. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

both *Money Main* and *Voucher Main*. More formally, in *Money Main*, the raw correlation between normalized indifference points for one-week delays and cognitive uncertainty is $\rho = -0.45$ both when $t_1 = 0$ and when $t_1 > 0$. In *Voucher Main*, the same correlations are given by $\rho = -0.39$ and $\rho = -0.45$. All of these correlations are statistically significant at the 1% level. Appendix Table 7 provides complementary regressions.

5.2.2 Decreasing Impatience

To study decreasing impatience, we follow the literature and define a *required rate of return* for a given normalized indifference point a^o as $RRR_{t_1,t_2}(a^o) \equiv \ln\left(\frac{c_{t_2}}{c_{t_1}}\right) = \ln\left(\frac{1}{a^o}\right)$. The RRR is a metric of impatience that depends on the delay. The literature frequently computes a per-period measure of patience as $RRR/\Delta t$. A transformation of this measure that captures per-period patience in an intuitive and structurally meaningful way is

$$\delta_H(a^o) \equiv e^{-RRR/\Delta t} = (a^o)^{1/\Delta t}. \tag{6}$$

This monotone transformation is attractive because – in a standard exponential discounting model without utility curvature and present bias – it directly corresponds to the exponential annual discount factor that is implied by the indifference point a^o. Thus, decreasing impatience says that $\delta_H(a^o)$ increases in the time delay, while under exponential discounting $\delta_H(a^o)$ is constant in the time delay.

Figure 6 shows the link between CU and decreasing impatience in four different panels: treatments *Money Main* and *Voucher Main*, separately for $t_1 = 0$ and $t_1 > 0$. Again, to make the figures comparable across experiments, we scale the x-axis to accommodate the longer time delays in *Money Main*. For each of the samples, we compute the average implied $\delta_H(a^o)$ across subjects for a given time horizon.[15]

The figures show that average per-period patience strongly increases in the time delay for cognitively uncertain participants. This is true in all four panels. For participants with CU of zero, however, per-period patience increases much more weakly. For example, for decisions in *Money Main* involving a 1-week vs. a 7-year delay starting today, we find that implied per-period patience increases by a factor of 9.4 for choices associated with positive cognitive uncertainty, but by a factor of only 1.8 for decisions with zero cognitive uncertainty.[16] Table 8 in Appendix C confirms these visual impressions through regressions. The strong increase in per-period patience for high-CU decisions cannot be explained by present bias alone even if one asserted that CU and a desire for immediate gratification are correlated. This is because we find very similar patterns for $t_1 = 0$ and $t_1 > 0$, while present bias only predicts diminishing impatience for $t_1 = 0$. Section 6 calibrates the relative importance of CU and present bias in generating observed behavior.

5.2.3 Subadditivity

We turn to the two "subadditivity sets" in our data, each of which consists of three decisions that involve the following points in time: set 1: {0, 6m, 12m}; set 2: {0, 4m, 8m}.[17] Following standard procedures in the literature, we compare the normalized indiffer-

[15] This visualization procedure is not subject to the aggregation insight of Weitzman (2001) and Jackson and Yariv (2014), which is that if the true data-generating process consists of subjects having different exponential discount functions, the average choice cannot necessarily be represented by an exponential function. This is not a problem here because we do not compute an implied δ_H for the average choice, but instead average the implied δ_H. Therefore, if the true process was exponential and participants had heterogeneous but constant discount factors, the average implied δ_H in Figure 6 should be constant in the time horizon. In any case, in our regression analyses, we always work with decision-level (rather than average) implied δ_H, which implies that potential aggregation issues never matter for our statistical tests.

[16] For delays starting in the future in *Money Money*, per-period patience increases by a factor of 6.3 (positive CU) vs. 1.6 (zero CU). In *Voucher Main*, where decisions only involve delays up to six months, the corresponding factors are 3.4 (positive CU) vs. 1.2 (zero CU) when the early date is today and 2.7 (positive CU) vs. 1.1 (zero CU).

[17] Because we randomly selected some choice lists to be presented twice to the same participant, we sometimes have more than one observation for one of the three decisions that constitute a subadditivity set. In those cases, we average the decisions in the two identical choice lists before analyzing the data for subadditivity effects.

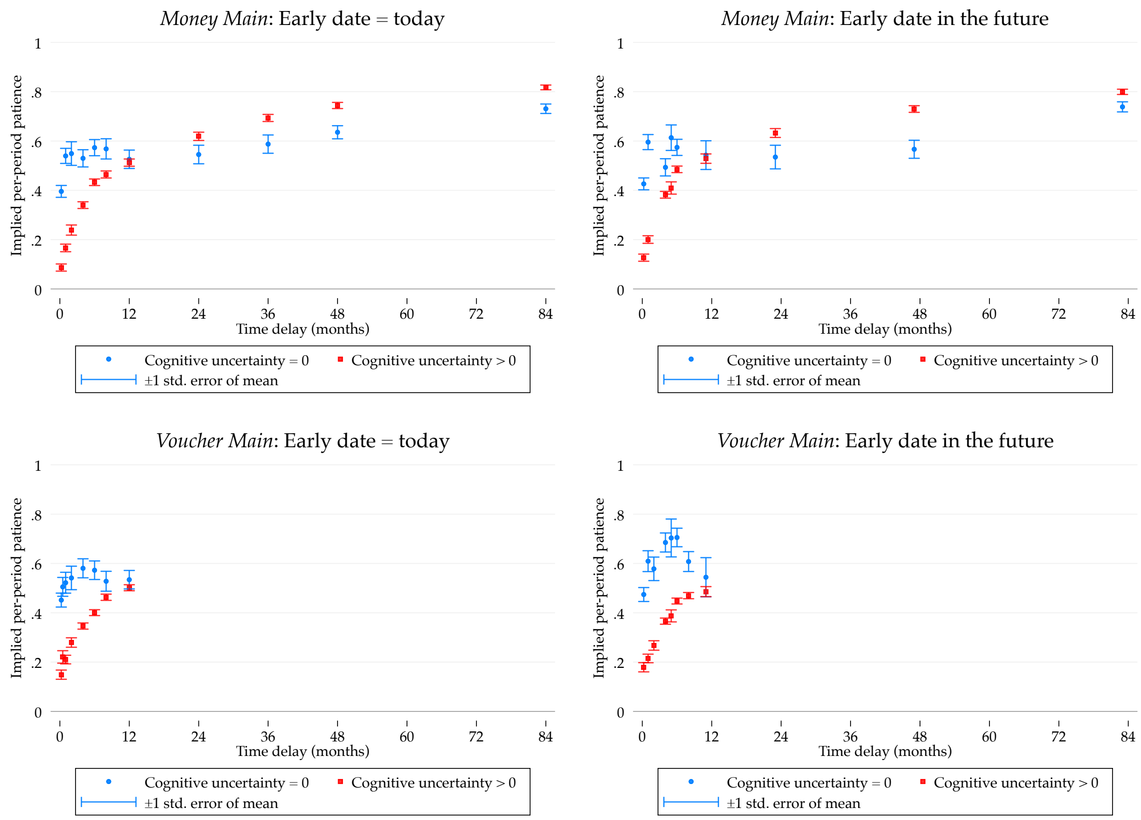


Figure 6: Implied per-period patience in *Money Main* (top panels) and *Voucher Main* (bottom panels), partitioned by whether the early payment date is today or in the future. Per-period patience is computed as $\delta_H(a^o) \equiv e^{-RRR/\Delta t} = (a^o)^{1/\Delta t}$, where a^o is the observed normalized indifference point. The figure shows average δ_H across decisions. Whiskers show standard error bars, computed based on clustering at the subject level.

ence point obtained from the problem involving the one long interval with the product of the two normalized indifference points obtained from the respective two short intervals ("composite normalized indifference point"). Thus, although each subject makes three decisions for a given set, these give rise to two observations. Subadditivity occurs if the former quantity is larger than the latter. Table 2 summarizes the results for both *Money Main* and *Voucher Main*. In both sets of experiments, we see strong evidence for the existence of subadditivity, see columns (1) and (4). In line with our hypothesis, the difference in observed patience between long and short intervals increases significantly in CU, see the interaction term in columns (2)–(3) and (5)–(6).

5.2.4 Front-End Delay Effects

Finally, we study the link between CU and front-end delay effects. These refer to the regularity that people exhibit greater patience in a decision problem in which both payment dates are moved forward by a constant. For example, people frequently appear more patient in tradeoffs between {6m, 12m} than between {0, 6m}. As summarized in Cohen

Table 2: Cognitive uncertainty and subadditivity

	Dependent variable: Composite normalized indifference point					
Treatment:	*Money Main*			*Voucher Main*		
	(1)	(2)	(3)	(4)	(5)	(6)
1 if one long interval	8.53***	3.35**	3.63***	9.50***	1.51	1.56
	(0.62)	(1.32)	(1.32)	(0.60)	(1.61)	(1.60)
1 if one long interval × Cognitive uncertainty		0.25***	0.23***		0.32***	0.32***
		(0.06)	(0.06)		(0.06)	(0.06)
Cognitive uncertainty		-0.44***	-0.42***		-0.42***	-0.42***
		(0.06)	(0.06)		(0.08)	(0.08)
Set FE	Yes	Yes	Yes	Yes	Yes	Yes
Payment amount FE	Yes	Yes	Yes	Yes	Yes	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	1948	1948	1948	2000	2000	2000
R^2	0.02	0.07	0.09	0.05	0.08	0.09

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Each subject makes three decisions for a given set, which give rise to two observations / composite normalized indifference points. The first is given by the normalized indifference point for a decision over the respective long horizon. The second is given by the product of the two normalized indifference points for the decisions over the two respective short horizons. Set fixed effects include fixed effect for each pair of decision problems that exhibit a front-end delay structure. Set 1: {0, 6m}, {6m, 12m} and {0m, 12m}. Set 2: {0, 4m}, {4m, 8m} {0m, 8m}. Demographic controls include age, gender and income. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

et al. (2020), front-end delay effects are often but not always present in choices over monetary amounts. In our context, columns (1) and (4) document that we find highly significant and quantitatively large evidence for the presence of front-end delay effects: participants appear as if they are more patient when a given time delay is moved forward. More importantly for our purposes, we find that the correlation between front-end delay effects and cognitive uncertainty is either small and statistically insignificant (columns (2)–(3)) or even goes in the opposite direction (columns (5)–(6)). This is despite a relatively large sample size of $N = 2,393$ decisions (645 subjects) in *Money Main* and $N = 2,337$ decisions (500 subjects) in *Voucher Main*.

5.3 Taking Stock: Modeling Approaches vs. Evidence

As noted earlier, our primary contribution is to empirically document the relevance of noisy cognition for intertemporal choice, rather than to definitively disentangle between different classes of random choice models that often make similar predictions (and each of which come in different variants). This being said, a comparison of the empirical results with the discussion in Section 2 allows us to draw some tentative conclusions about which types of models appear to explain the patterns better than others.

Table 3: Cognitive uncertainty and front-end delay effects

	Dependent variable: Normalized indifference point					
Treatment:	*Money Main*			*Voucher Main*		
	(1)	(2)	(3)	(4)	(5)	(6)
1 if front end delay	3.07***	2.56*	2.47*	2.74***	4.98***	5.18***
	(0.85)	(1.32)	(1.30)	(0.86)	(1.68)	(1.67)
Front-end delay × Cognitive uncertainty		0.048	0.049		-0.055	-0.064
		(0.05)	(0.05)		(0.05)	(0.05)
Cognitive uncertainty		-0.30***	-0.28***		-0.24***	-0.23***
		(0.05)	(0.05)		(0.06)	(0.06)
Set FE	Yes	Yes	Yes	Yes	Yes	Yes
Payment amount FE	No	No	Yes	No	No	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	2393	2393	2393	2337	2337	2337
R^2	0.00	0.05	0.06	0.01	0.05	0.05

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Set fixed effects include fixed effect for each pair of decision problems that exhibit a front-end delay structure. Set 1: {0, 6m} and {6m, 12m}. Set 2: {0, 4m} and {4m, 8m}. Demographic controls include age, gender and income. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

A crucial role in this regard play the choice patterns regarding subadditivity and front-end delay effects. The main reason is that the cognitive-imprecision-in-action-space framework and the random response model that we sketched in Section 2 predict that cognitive imprecision is correlated with subadditivity but not with front-end delay effects. Random preference models and the cognitive noise model of Gabaix and Laibson (2017), on the other hand, both predict that cognitive imprecision is linked to front-end delay effects but not to subadditivity. Given that we find that cognitive uncertainty is predictive of subadditivity but not of front-end delay effects, we tentatively conclude that random preference models and the approach of Gabaix and Laibson (2017) and Gershman and Bhui (2019) do not explain all aspects of the evidence.

5.4 Robustness

Omitted variables. Given that all analyses up to this point are correlational in nature, a potential concern is the existence of a stable participant characteristic other than cognitive uncertainty that somehow generates the results. While we are not aware of other characteristics that could plausibly lead to higher implied impatience over short horizons, yet lower implied impatience over long horizons, we perform a robustness check by including participant fixed effects in our main regression in Table 1. By definition, these soak up fixed subject characteristics such as overall cognitive ability. As a result, inelasticity with respect to variation in the time delay is identified purely off of within-participant-across-task variation in CU. As we document in Appendix Table 9, the results remain statistically significant conditional on these subject fixed effects.

Direct elicitation experiments. Up to this point, all results were derived from experiments in which intertemporal choice behavior was elicited using choice lists. To document that the logic of CU and inelasticity extends to another elicitation technique, treatment *Money Main* also included a direct elicitation component, see Section 3. Here, subjects were directly asked how much they value a hypothetical payment of \$y in $t = t_2$ in terms of a payment to be received today. To answer this question, subjects directly typed a dollar amount into a text box. After each decision, subjects indicate their cognitive uncertainty by indicating their subjective probability that their true valuation for the later payment actually lies within ±\$1 of their stated valuation.

Appendix D shows that these direct elicitation experiments deliver very similar results as the ones reported thus far. Specifically, we find that (i) CU is significantly correlated with across-trial choice variability; (ii) CU is strongly correlated with short-run impatience over one week; (iii) CU is correlated with decreasing impatience; (iv) CU is correlated with subadditivity; and (iv) CU is again uncorrelated with front-end delay effects. Thus, all of our results from the MPL experiments replicate using the direct elicitation technique.

6 Model Estimations

Following the discussion in Section 5.3, we proceed by estimating specification (2) in Section 2. This serves two purposes. First, to gauge how well such a reduced-form model fits the data, and how much the measurement of cognitive uncertainty contributes to model fit. Second, to shed light on the location of a potential cognitive default action (or the mean random action), including heterogeneity across participants.

In eq. (2), the weight λ depends on the magnitude of cognitive imprecision. We do not observe cognitive imprecision itself but cognitive uncertainty, denoted p_{CU}. We proceed by using the heuristic approximation $\lambda = 1 - \alpha p_{CU}$, where $\alpha \geq 0$ is a nuisance parameter to be estimated. With CRRA utility and larger-later payment $x \equiv c_{t_2} \geq 1$, eq. (2) suggests that the mean observed choice in our experiments is determined as

$$\begin{aligned} \mathbb{E}[a^o] &= \lambda(p_{CU}) \cdot \mathbb{E}[s] \cdot x + (1 - \lambda(p_{CU})) \cdot d \cdot x \\ &= (1 - \alpha \cdot p_{CU}) \cdot (\delta^{\Delta t})^{1/\gamma} \cdot x + (\alpha \cdot p_{CU}) \cdot d \cdot x \end{aligned} \tag{7}$$

This equation, amended by a mean-zero error term, can be estimated using straightforward nonlinear least squares techniques. To assess and compare model fit, we estimate four model variants. First, a baseline exponential discounting model in which we ignore CU (i.e., we set $\alpha = 0$). Second, also for benchmarking purposes, a $\beta - \delta$ model, which also precludes a role for CU. Finally, we estimate both of these variants by including a potential role for CU.[18]

Notice that the estimate of d has two potential interpretations. Under the Bayesian cognitive imprecision interpretation, d is a constant cognitive default action that people anchor on. Under the random response interpretation, d is the mean of the distribution function $F(\cdot)$ from which random responses are drawn.

Aggregate estimates. We begin by estimating the model across subjects, treating the data as if it was generated by one representative agent. Table 4 summarizes the model estimates across the three different types of experiments that we report in this paper. The are five main takeaways. First, in line with prior research, a pure exponential discounting model fits the data very poorly. Second, a beta-delta model fits the data considerably better, but not nearly as well as a model that includes both exponential discounting and CU (see the Akaike Information Criterion values in the last row). Third, a model that includes both a role for taste-based present bias and CU performs best. This – in line with our results on front-end delay effects – again highlights that a desire for immediate gratification and cognitive imprecision are distinct and complementary objects. Fourth, ignoring CU in the estimations considerably inflates the role of present bias β. Fifth, the estimates are strikingly similar across experiments; in particular, the estimated d is always around 50% of the larger-delayed reward.

To put these estimates in perspective, note that our setup in which the earliest reward lies several hours in the future likely underestimates the true extent of present bias. Even though we find clear evidence for $\beta < 1$, recent experimental work suggests that most discounting occurs in the first few hours following a decision (e.g., Augenblick, 2018), something that is not captured in our experimental paradigm.

Figure 7 visualizes the fit of the various estimated models for treatment *Money Main*, separately for decision problems in which the early payment date is today or in the future.[19] The figures are constructed by generating predicted values, based on the parameter estimates in Table 4. We again see that exponential discounting fits the data very poorly. Likewise, almost by construction, the canonical beta-delta model fits poorly when the early payment date is in the future.[20] On the other hand, when the early pay-

[18]The amended estimation equation for $\beta - \delta - CU$ is given by $a^o = (1 - \alpha \cdot p_{CU}) \cdot (\beta \delta^{\Delta t})^{1/\gamma} + (\alpha \cdot p_{CU}) \cdot d$.

[19]Appendix Figure 14 shows analogous results for *Voucher Main*.

[20]The different model fit for the exponential discounting and the beta-delta model for the case of

Table 4: Estimates of model parameters across experiments

	Money Main MPL				*Money Main Direct Elicitation*				*Voucher Main MPL*			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	δ	$\beta-\delta$	$\delta-CU$	$\beta-\delta$ $-CU$	δ	$\beta-\delta$	$\delta-CU$	$\beta-\delta$ $-CU$	δ	$\beta-\delta$	$\delta-CU$	$\beta-\delta$ $-CU$
$\hat{\delta}$	0.96	0.98	0.97	0.98	0.97	0.99	0.98	0.99	0.94	0.95	0.95	0.95
$\hat{\beta}$		0.77		0.86		0.76		0.85		0.89		0.95
$\hat{d}$			0.51	0.49			0.52	0.49			0.57	0.56
AIC	64,148	63,165	61,904	61,701	32,247	31,391	30,993	30,791	46,980	46,652	45,853	45,817

Notes. Estimates of different versions of (7). MPL = multiple price list. AIC = Akaike Information Criterion. Each column corresponds to a separate model estimation. Columns (1), (5), (9): set $\beta = 1$ and $\alpha = 0$. Columns (2), (6), (10): set $\alpha = 0$. Columns (3), (7), (11): set $\beta = 1$. All estimated standard errors (computed based on clustering at the subject level) are smaller than 0.02. In estimations that include CU, we also estimate the nuisance parameter α (not reported). All estimations are conducted by setting a CRRA parameter of $\gamma = 0.94$, which is the population-level risk aversion that was separately estimated on the risky choice data. The exponential parameter δ is the monthly discount factor.

ment date is today, the beta-delta model performs well in fitting behavior over relatively short time delays, but (as is well-known) performs relatively poorly in capturing the strong flattening out of the observed data for long time delays.

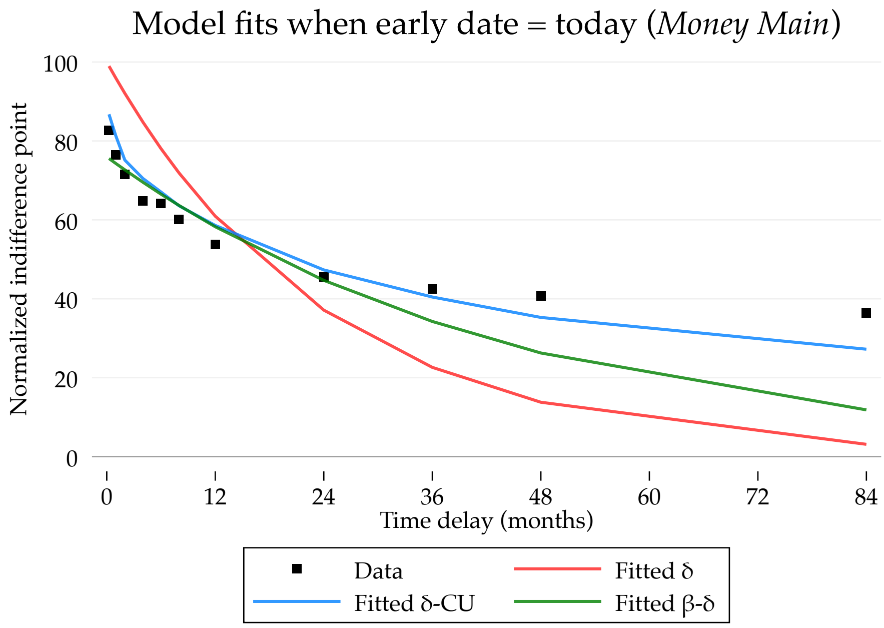
The delta-CU model, on the other hand, captures several key aspects of the data. First, it partly (though imperfectly) accounts for some of the extreme impatience over short horizons. Second, the model accounts much better (though also somewhat imperfectly) for the strong compression effects over long horizons. Third, the delta-CU model matches the data reasonably well both when the early payment date is today and when it is in the future.

Individual-Level Estimates. Estimating any intertemporal choice model at an aggregate level is problematic because participants might have heterogeneous discount factors (Weitzman, 2001; Jackson and Yariv, 2014). Therefore, we proceed by estimating the model at the level of individual subjects.[21] We report the results in Appendix Table 10. To summarize, there is substantial individual-level variation in estimated model parameters. For most parameters, the center of the estimated coefficient distributions is well in line with the parameters in our representative-agent estimation.[22] The empirical distribution of $\hat{d}$ is roughly bell-shaped with a center around $\hat{d} \approx 0.5$, with two pronounced spikes at $\hat{d} = 1$ and $\hat{d} = 0$.

$t_1 > 0$ result from the fact that we estimate both models on *all* data, including those with $t_1 = 0$.

[21] To increase power in these iundividual-level estimations, we restrict attention to treatment *Money Main*, in which each subject completed both 12 MPLs and 6 direct elicitation tasks, for a total of 18 decisions per subject.

[22] An exception is the the present bias parameter β. We find *less* pronounced present bias (larger β) in our individual estimations than the aggregate ones, in line with the theoretical insight that aggregate quasi-hyperbolic discounting can partly result from the aggregation of preferences of individuals with different discount factors.



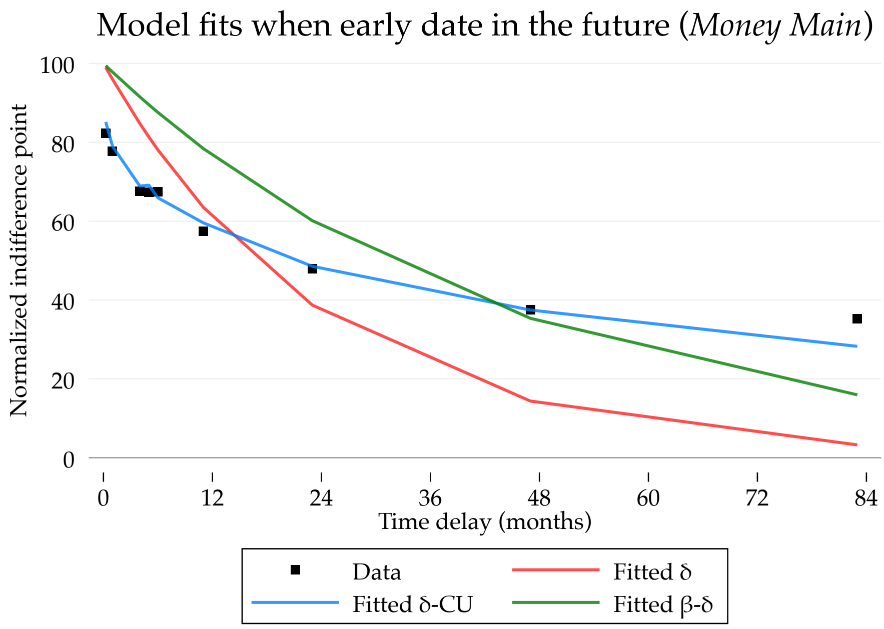


Figure 7: Model fit vs. data in *Money Main*. The model predictions are computed as fitted values of the parameter estimates in Table 4.

Discussion. Our estimations consistently suggest that a potential cognitive default action or mean random response is given by roughly 50% of the larger-later payment. This is true both in experiments with price lists and in direct elicitation experiments. Of course, given the available evidence, we do not intend to take a strong stance on whether this estimate will be context-specific. While we suspect that it will be, it is also interesting to note that the "central" nature of the estimated d jives well with a large body of work in both economics and psychology that suggests that people's heuristic responses to decision problems tend to be "intermediate" in nature. In psychology, this well-known finding has come to be known as "central tendency effect" (Hollingworth, 1910), which appears across a large set of decision domains. Indeed, in joint work with cognitive psychologists, we have documented that central tendency effects in various perceptual domains are strongly linked to cognitive uncertainty (Xiang et al., 2021), which also suggests the presence of an intermediate cognitive default. In economics, a related effect is the so-called compromise effect (see, e.g., Beauchamp et al., 2019, for an example in risky choice), which captures that people tend to indicate indifference values that are in the "center" of a choice set.

7 Why Cognitive Microfoundations Matter

It may not be immediately obvious why it is important for economists to understand that intertemporal choice is to a large degree governed by bounded rationality rather than only preferences, if reduced-form discount functions such as variants of the generalized hyperbola generally perform reasonably well in fitting data (even if for the wrong reasons). While we believe that understanding cognitive microfoundations is scientifically valuable in its own right (especially in light of the voluminous literature that has proposed a multitude of different discount functions), we now additionally show that understanding cognitive mechanisms matters for both prediction and policy.

7.1 Context-Dependence of Inelasticity

A main implication of a preferences-based account is that inelasticity with respect to the delay (and resulting "excessive" patience or impatience) is fixed. Our account, on the other hand, predicts that inelasticity and its implications will be more pronounced in environments that increase cognitive imprecision. This context-dependence and the resulting potential gains in prediction accuracy are a main implication of frameworks such as the one we estimated in the previous section. In the absence of theoretical guidance for what determines cognitive noise, we conjecture that, plausibly, the magnitude of cognitive imprecision will be a function of (i) the complexity of the decision problem and (ii) the availability of cognitive resources for mental simulation of the problem. Crucially, there is arguably reason to believe that, in the wild, these aspects vary systematically, which raises the question of when we should expect to see more or less elastic discounting.

Task complexity. Because there is no widely accepted definition for task complexity, we implement two treatments that plausibly increase the perceived complexity of the intertemporal decision problems. Here, one treatment is aimed at increasing the complexity of the time delay, while the other treatment increases the complexity of mentally simulating payoffs and resulting utils. Specifically, in *Money Complex Dates*, we implemented the same procedures as in *Money Main*, except that all payout dates in the choice lists were represented as a math task. For instance, "In 1 year" could be represented as "In (6*2/3-3) years AND (3*6/2-9) months AND (5*4/2-10) days." Appendix Figure 18 provides a screenshot.

In treatment *Money Complex Amounts*, we again implemented the same procedures as in *Money Main*, except that, for the delayed option A in a choice list, the monetary amount was again represented as a math problem, such as "$(4*8/2)+(8*9/2)-12". Appendix Figure 19 provides a screenshot. Relative to our baseline condition, this treatment leaves the complexity of the display of payment dates constant, but makes determining the consumption implications of a choice option more difficult.

Availability of cognitive resources. To manipulate the cognitive resources that people have at their disposal to mentally simulate their true indifference point, prior literature has worked with cognitive load or time pressure / waiting periods (Deck and Jahedi, 2015; Imas et al., 2016; Ebert, 2001). The main result in this literature is that having fewer cognitive resources available generally leads to lower revealed patience. Yet, in these studies, researchers implemented relative short time delays. Our account predicts that people act as if they are actually *more* patient over long horizons when they have fewer cognitive resources available. In our treatment *Money Load*, participants are tasked with simultaneously (i) completing the intertemporal choice problems over money described in the previous section and (ii) adding up red numbers that appeared at random intervals next to the choice list.[23]

An obvious issue with our complexity and load manipulations is that cognitive effort and resulting response times are endogenous: in principle, it is conceivable that subjects in the more complex conditions take substantially longer to complete the tasks, so that no effect on CU would be visible. To prevent this, we implemented a time limit of 25 seconds per choice list in each of these conditions, including in a replication of treatment *Money Main* that we administered in the same experimental sessions. See Appendix E for example screenshots for all treatments.

We conducted these experiments with a separate sample, in which each participant was randomly assigned to one of the four treatments (*Money Complex Dates*, *Money Complex Amounts*, *Money Load* and *Money Main Replication*). We pre-registered at `https://aspredicted.org/77xp6.pdf` that we would sample 150 subjects per condition,

[23]We also separately implemented a within-subjects design that manipulated the presence of the number counting task within subjects across tasks. The results are very similar, see Appendix F.

but due to the randomness in allocating subjects to treatments we ended up with 161 in *Money Main replication*, 153 in *Money Complex Amounts*, 149 in *Money Complex Dates* and 154 in *Money Load*. See a list of all treatments in Appendix Table 5.

Results. We find that all three treatment variations substantially increase stated CU relative to the replication of our main treatment. The magnitude of the increase is between 5 and 12 percentage points (20% to 50%, respectively), $p < 0.01$ for all comparisons to the baseline replication. Turning to intertemporal choice behavior, Figure 8 summarizes the results for treatment *Money Complex Amounts*. We see that the indifference points in the more complex treatment are much more compressed around 50% compared to the replication of the baseline treatment. As a result, participants in the more complex treatment behave as if they are more impatient over short horizons but less impatient over long ones. Appendix E summarizes the results for treatments *Money Complex Dates* and *Money Load*, which look very similar to those reported for *Money Complex Amounts*. See Appendix Table 17 for statistical tests. In all, we see that acknowledging a role of cognitive imprecision facilitates improved predictions of the context-dependence of (im)patience. A tentative conclusion from this series of experiments is that the specific *source* of cognitive imprecision may not matter much for its behavioral effect: different features of decision contexts that plausibly increase the degree of cognitive noise lead to very similar compression in choices.

7.2 Choice Architecture

As we emphasize throughout the paper, a desire for immediate gratification ("present bias") and bounded rationality in the form of cognitive imprecision are complementary, not substitutes. Yet, they potentially have different policy implications. For example, an account of cognitive uncertainty predicts that short-run impatient choices will often be associated with a sense of "nervousness" that the decision reflects an error. Thus, people may be open to (or even actively seek out) advice about how to behave.

To study the relevance of cognitive uncertainty for choice architecture, we test whether it is indeed true that people with cognitive uncertainty about a given decisions are more likely to follow the advice of an outside expert. It is worth pointing out that this is a strong hypothesis because variation in intertemporal decisions surely partly reflects genuine heterogeneity in preferences (e.g., in δ). Given that outside experts will rarely know the decision-maker's true preferences, following the advice of an expert is a double-edged sword: it may reduce the probability of making mistakes, but increase the probability of doing something that goes against one's preferences.

To assess the relevance of cognitive uncertainty for advice-seeking and choice ar-

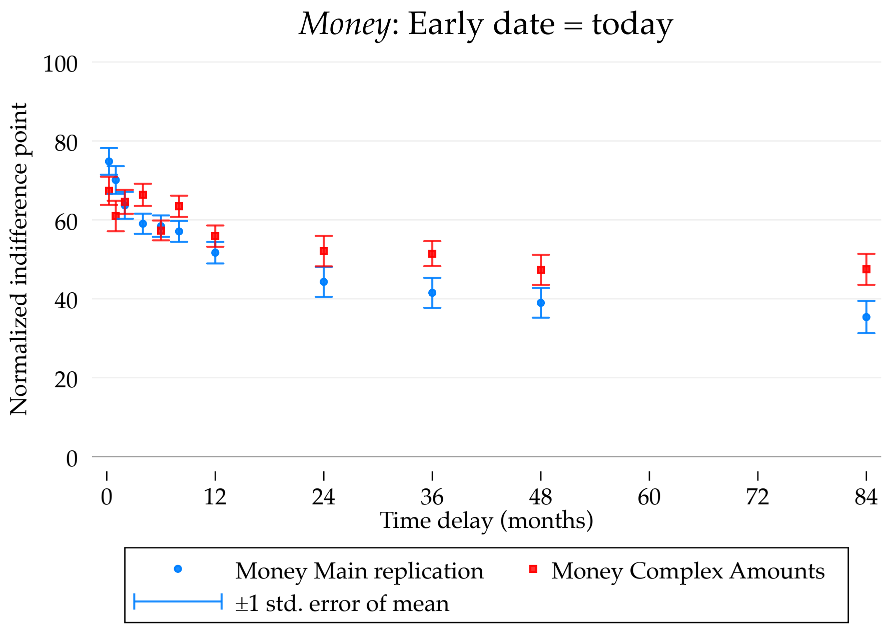


Figure 8: Observed discounting with $t_1 = 0$ in *Money Main replication* ($N = 161$) and *Money Complex Amounts* ($N = 153$). Normalized indifference points are given by the midpoint of the switching interval in a choice list, divided by the larger-later payout amount (in %). The figure shows averages across decisions. Whiskers show standard error bars, computed based on clustering at the subject level.

chitecture, we implement treatment *Voucher Advice*. This treatment follows exactly the same protocol as *Voucher Main*, except that it introduces a piece of advice. In the first choice list, we fixed the early payment date at today and varied the delayed payment date between one week and two months. After the experimental participant had indicated their decisions in this choice list and their cognitive uncertainty, we presented a surprise announcement:[24]

> *We surveyed a few academic economists about which advice they would give to participants in this study regarding which decisions to make. These academic economists recommend that participants choose the delayed Voucher A in all rows of the choice list you just completed. We recognize that decisions like these depend on your own preferences, so we neither encourage nor discourage you to follow this advice. However, should you wish to revise your decision, you can do so in the choice list below. The choices that are indicated right now are those that you made yourself a few seconds ago.*

We pre-registered the sample size and our prediction that cognitive uncertainty is associated with a higher likelihood of following expert advice by revising a previous decision

[24]No deception was involved in the design of the study because we actually polled Harvard-based economists for advice. We suspect that the reason why people are comfortable articulating advice in such situations is that – over timeframes of one week to two months as in our study – even mildly impatient decisions imply absurdly high discount rates.

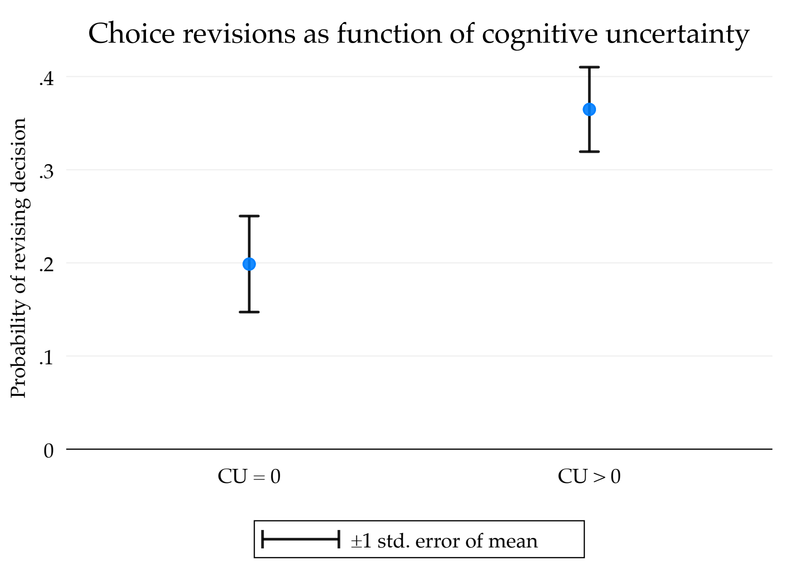


Figure 9: Probability of revising decision towards higher patience, as function of cognitive uncertainty ($N = 153$). The figure is constructed controlling for the normalized indifference point before seeing advice. In other words, the y-axis shows the residual probability of revising the decision after the initial choice is partialed out through an OLS regression.

at `https://aspredicted.org/jk5s5.pdf`.

In our data, 34% of participants revise their decision upon seeing advice, where almost all revisions are in the direction of higher patience. Figure 9 shows the relationship between cognitive uncertainty and choice revisions towards the advice of full patience.[25] We see that participants with strictly positive cognitive uncertainty are 16 percentage points (80%) more likely to revise their choice, $p < 0.01$.

8 Discussion

Contribution. This paper has made three contributions. First, we documented that cognitive uncertainty sheds light on a myriad of empirical regularities, including extreme short-run impatience, decreasing impatience, sub-additivity and choice variability. Second, through model estimations, we showed that accounting for cognitive uncertainty leads to large improvements in model fit, and that bounded rationality and a desire for immediate gratification are complementary factors in intertemporal choice. Third, we documented that understanding cognitive foundations is relevant both for choice architecture and for predicting the context-dependence of (im)patience.

[25]Because subjects with higher cognitive uncertainty on average state lower indifference points in their initial decision, they have more "room" to adjust. We control for this by residualizing the y-axis of Figure 9 from the initial normalized indifference point through a linear regression (the results are even stronger without this adjustment).

Link to cognitive effects in intertemporal choice research. We conjecture that our account of cognitive uncertainty provides a rationale for extant empirical findings about "cognitive" effects in intertemporal choice research. The perhaps most widely-known result on cognition and intertemporal choice is that, if the time horizon over which the intertemporal tradeoff is defined is relatively short, a lower availability of cognitive resources is associated with less patient decisions. At the same time, Ebert (2001) presents evidence that suggests that, over long horizons, a lower availability of resources makes people *more* patient. Our account of the link between inelasticity and cognitive uncertainty reconciles this somewhat puzzling combination of results.

Limitations. Our paper does not purport to explain nearly all intertemporal choice anomalies. One regularity that our study does not address are well-known framing effects, such as the speed-up / delay asymmetry (Loewenstein and Prelec, 1992) or date / delay effects (Read et al., 2005). At the same time, we do conjecture a potential link between such framing effects and our work: if one choice option is presented to people as default that they can "speed up" at a cost, it seems plausible that people use that option as a cognitive default. Based on this idea, we conjecture that speed-up / delay asymmetries are more pronounced when cognitive uncertainty is high. More generally, this conjecture highlights that further research is needed to understand the location of a potential cognitive default action, and how it depends on the context.

Broader agenda and open questions. An important step in taking this agenda a step further is to identify a link between cognitive uncertainty and (intertemporal) decisions also in field settings. Indeed, we do not believe that the presence of cognitive uncertainty is a result of the abstract and unfamiliar decision tasks that experimental economists deploy. Instead, we predict that just like people find it cognitively difficult to identify their indifference points in choice lists or direct elicitation mechanisms, they also find it challenging to think through their intertemporal decisions in more naturalistic settings. After all, while these settings may be more familiar to people, they are also usually considerably more complex.

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ONLINE APPENDIX

A Derivations for Bayesian Cognitive Noise Model

A.1 Model setup

Below we discuss the main behavioral predictions of a Bayesian cognitive imprecision model as outlined in Section 2. Suppose the DM has access to a mental simulation of the optimal action that we conceptualize as a "cognitive signal" S. We assume that S is an unbiased estimate of a^* and follows a scaled binomial distribution,

$$S \sim \frac{1}{n_2} Bin(n_2, a^*), \tag{8}$$

such that $0 < S < 1$. The parameter n_2 controls the precision of the mental simulation. The subjective likelihood of the utility-maximizing action based on a randomly drawn internal representation $\{S = s\}$ can then be represented by a binomial distribution:

$$\mathscr{L}(a^*|S = s) = P(S = s|a^*, n_2) = \binom{n_2}{sn_2}(a^*)^{sn_2}(1 - a^*)^{(1-s)n_2}. \tag{9}$$

The DM holds a prior about his utility-maximizing action, A, which we broadly think of as the mathematical analogue of a decision maker's initial reaction to a choice problem. We assume that this prior can be represented by a Beta distribution, $A \sim Beta(n_1 d\ ,\ n_1(1 - d))$. Here, d is the prior mean and carries the interpretation of a "cognitive default" action that the DM would take before deliberating about the problem. The parameter n_1, on the other hand, reflects the DM's confidence in (or precision of) their prior.[26] Note that the default action represents a *fraction* of the larger-later consumption, rather than an absolute consumption level.

A Bayesian DM accounts for the noisiness of his mental simulation by implicitly forming a posterior assessment of the utility-maximizing action. Given a Beta-distributed prior and a Binomial signal, this posterior belief, $A\big|\{S = s\}$, is also Beta-distributed.[27] The DM's observed action given a mental signal is assumed to be the posterior mean:[28]

$$a^o = E[A\big|\{S = s\}] = \lambda s + (1 - \lambda)d \quad \text{with} \quad \lambda = n_2/(n_1 + n_2). \tag{10}$$

[26]Note that $n_1 = a + b$ is a re-parameterization of the typical shape parameters a and b of the Beta distribution. n_1 is inversely related to the variance of the prior, $\sigma_A^2 = \frac{d\cdot(1-d)}{1+n_1}$.

[27]Specifically, $A_{S=s} \sim\ Beta(sn_2 + n_1 d\ ,\ n_2(1 - s) + n_1(1 - d))$.

[28]Focusing on the posterior mean is without much loss in the present context because the mean of a Beta(a,b) variable is $a/(a + b)$, the mode is $(a - 1)/(a + b - 2)$ and the median lies between the two.

This endogenizes the decision rule we posited in equation (2) of Section 2. Crucially, a more precise mental simulation (higher n_2) has a direct, negative effect on the weighting factor λ, which implies a lower weight on the cognitive default action. In the following subsection, we will thus focus on deriving behavioral predictions for changes in λ. In subsection A.3, we characterize cognitive uncertainty in the context of this model.

A.2 Derivations for behavioral predictions

All theorems and derivations in this subsection will solely concern a given subject's mean observed action, i.e., their average response aggregating across many unbiased signals. Given $\mathbb{E}[s(a^*)] = a^*$, we define:

$$a^e := \mathbb{E}[a^o] = \lambda \cdot a^* + (1-\lambda) \cdot d. \tag{11}$$

where $\lambda \in [0,1]$ is our representation of cognitive precision. We derive the following behavioral predictions under the assumption of a canonical exponential discount function, $D(t) = \delta^t$, in order to disentangle the effect of shrinkage to a default from the predictions generated by present bias. Hence, $a^* = u^{-1}(\delta^{\Delta t})$ with $u(c) = c^a, a > 0$. Since $a > 0$, we may take $a = 1$ in the proofs. As before (Section 5.2.2), we define:

$$RRR := \ln(\frac{1}{a^e}) = -\ln(a^e), \qquad \delta_H := e^{-\frac{RRR}{\Delta t}} \tag{12}$$

as the required rate of return and implied annualized discount factor. We will use the required rate of return per unit of time,

$$r := \frac{RRR}{\Delta t}, \tag{13}$$

as our measure of per-period impatience. We define short horizons as those time horizons where an exponential discounter behaves more patiently than a subject playing the default action:

$$SH := \{\Delta t \mid a^* > d\} \tag{14}$$

Long horizons, LH, are similarly defined by:

$$LH := \{\Delta t \mid a^* < d\} \tag{15}$$

We now turn to the theoretical predictions underlying the pre-registered predictions spelled out in Section 2.

Theorem 1 (Impatience over different time horizons)**.**

(i) Higher cognitive precision leads to less per-period impatience over short horizons.

$$\frac{\partial r}{\partial \lambda}\Big|_{\Delta t \in SH} < 0 \tag{16}$$

(ii) Higher cognitive precision leads to more per-period impatience over long horizons.

$$\frac{\partial r}{\partial \lambda}\Big|_{\Delta t \in LH} > 0 \tag{17}$$

Proof. Note that:

$$\frac{\partial a^e}{\partial \lambda} = a^* - d, \tag{18}$$

by definition. Hence, the sign of eq. (18) depends on whether it is evaluated over a short or long time horizon. We may now differentiate:

$$\frac{\partial r}{\partial \lambda} = \frac{1}{\Delta t}\frac{\partial RRR}{\partial \lambda} \tag{19}$$

$$= -\frac{1}{a^e \Delta t}\frac{\partial a^e}{\partial \lambda} \tag{20}$$

Since we trivially have $\Delta t, a^e > 0$, the sign of $\partial r / \partial \lambda$ is given by eq. (18) and the definitions (14), (15) yielding the result. □

We note the trivial corollary that delivers Prediction 1 in the main text:

Corollary 1.1. *Subjects with perfect cognitive precision, $\lambda = 1$, have less pronounced short run impatience than those with imperfect cognitive precision, whereas the opposite is true concerning long run impatience.*

Given our measure of per-period impatience, we may show that per-period impatience decreases in the time delay (Δt).

Proposition 1 (Decreasing per-period impatience)**.**

(i) For those with complete cognitive precision, $\lambda = 1$, per-period impatience is constant in the time delay. Formally,

$$\frac{\partial r}{\partial \Delta t}\Big|_{\lambda=1} = 0 \tag{21}$$

(ii) For those with imperfect cognitive precision, $\lambda \in (0, 1)$, per-period impatience decreases in the time delay. In other words:

$$\frac{\partial r}{\partial \Delta t}\Big|_{\lambda<1} < 0 \tag{22}$$

Proof. We will consider the two cases: (i) $\lambda = 1$; (ii) $\lambda \in [0,1)$ separately.

In the case $\lambda = 1$ we trivially note that:

$$r = \frac{-\ln|a^e|}{\Delta t} = -\ln|\delta| \tag{23}$$

Hence, we have that:

$$\frac{\partial r}{\partial \Delta t} = 0 \tag{24}$$

For $\lambda \in (0,1)$, note that $\delta < 1$ implies:

$$\frac{\partial RRR}{\partial \Delta t} = -\frac{\lambda \delta^{\Delta t} \ln|\delta|}{a^e} > 0 \tag{25}$$

$$\frac{\partial^2 RRR}{(\partial \Delta t)^2} = -\frac{\lambda(1-\lambda)\delta^{\Delta t} d \ln^2|\delta|}{(a^e)^2} < 0 \tag{26}$$

meaning that the RRR is concave in Δt. The following expression describes how the RRR per unit of time changes in the time delay:

$$\frac{\partial r}{\partial \Delta t} = \frac{\frac{\partial RRR}{\partial \Delta t}\Delta t - RRR}{\Delta t^2} \tag{27}$$

A sufficient condition for (27) to have negative sign is therefore:

$$\Delta t \cdot \frac{\partial RRR}{\partial \Delta t} < RRR \tag{28}$$

We may now define the function:

$$g := RRR - \frac{\partial RRR}{\partial \Delta t}\Delta t \tag{29}$$

and differentiate to find:

$$\frac{\partial g}{\partial \Delta t} = -\frac{\partial^2 RRR}{(\partial \Delta t)^2}\Delta t \geq 0 \tag{30}$$

We note that at $\Delta t = 0$ we have:

$$g(0) = RRR(0) = -\ln|\lambda + (1-\lambda)d| > 0 \tag{31}$$

since $0 < d, \lambda < 1$. Hence, we find that g is positive for all $\Delta t > 0$:

$$g > 0 \;\Big|\Delta t > 0 \tag{32}$$

substituting in the definition of g shows that (28) is satisfied yielding the result. □

The following corollary underlies Prediction 2 in the main text:

Corollary 1.1. *The magnitude of per-period impatience's decrease in the time delay is smaller for those with perfect cognitive precision than for those with imperfect cognitive precision. Locally, this provides:*

$$\left.\frac{\partial^2 r}{\partial \lambda \partial \Delta t}\right|_{\lambda=1} > 0 \tag{33}$$

Proof. Note that the previous proposition provides that $dr/d\Delta t < 0$ for $\lambda < 1$ and is equal to zero for $\lambda = 1$. The result follows. □

It is important to note that the above theorems make no assumptions concerning the start time t_1 or end time t_2; but rather, only depend on the time delay $\Delta t = t_2 - t_1$. This is in line with our Predictions 1 and 2, which cover both delays starting in the present and in the future.

Next, we turn to the phenomenon of subadditivity. Subadditivity arises purely as a result of cognitive noise – as is well-known, beta-delta preferences do not generate subadditivity.

Theorem 2 (Subadditivity)**.** *Those subjects reporting cognitive uncertainty and an interior default will exhibit subadditivity in their choices. In other words, for $\lambda \in [0,1), d \in (0,1)$ we claim:*

$$SA := (RRR_{t_1,t_2} + RRR_{t_2,t_3}) - RRR_{t_1,t_3} > 0 \tag{34}$$

Since *RRR* only depends on the time delay and not the start time, it will be convenient to replace t_1, t_2, t_3 with the variables:

$$\Delta t_1 := t_2 - t_1, \qquad \Delta t_2 := t_3 - t_2$$

Taking a^e as a function of the time delay, our subadditivity condition can be rewritten as:

$$SA > 0 \tag{35}$$

$$\ln\left|\frac{a^e(\Delta t_1 + \Delta t_2)}{a^e(\Delta t_1)a^e(\Delta t_2)}\right| > 0 \tag{36}$$

$$\frac{a^e(\Delta t_1 + \Delta t_2)}{a^e(\Delta t_1)a^e(\Delta t_2)} > 1 \tag{37}$$

$$a^e(\Delta t_1 + \Delta t_2) > a^e(\Delta t_1)a^e(\Delta t_2) \tag{38}$$

Before we proceed to the proof, let us illustrate the effect of cognitive uncertainty on strict subadditivity through consideration of the edge cases $\lambda \in \{0,1\}, d \in \{0,1\}$. With perfect cognitive precision, $\lambda = 1$, the model reduces to standard exponential discounting and using (38) there is no subadditivity:

$$\delta^{\Delta t_1 + \Delta t_2} = \delta^{\Delta t_1} \cdot \delta^{\Delta t_2} \tag{39}$$

In the presence of no cognitive precision, $\lambda = 0$, using (38) there is subadditivity for any interior cognitive default $d \in (0, 1)$:

$$d > d^2 \tag{40}$$

Having discussed the corner cases we now proceed to the proof.

Proof. By (38) the existence of subadditivity is equivalent to:

$$a^e(\Delta t_1 + \Delta t_2) > a^e(\Delta t_1)a^e(\Delta t_2) \tag{41}$$

$$\lambda\delta^{\Delta t_1+\Delta t_2} + (1-\lambda)d > [\lambda\delta^{\Delta t_1} + (1-\lambda)d][\lambda\delta^{\Delta t_2} + (1-\lambda)d] \tag{42}$$

$$> \lambda^2\delta^{\Delta t_1+\Delta t_2} + (1-\lambda)^2 d^2 + \lambda(1-\lambda)(\delta^{\Delta t_1} + \delta^{\Delta t_2}) \tag{43}$$

Gathering our terms to the left hand side we find:

$$(\lambda-\lambda^2)\delta^{\Delta t_1+\Delta t_2} + (1-\lambda)d(1-(1-\lambda)d) - (\lambda-\lambda^2)d(\delta^{\Delta t_1} + \delta^{\Delta t_2}) > 0 \tag{44}$$

Since $\lambda \neq 1$ we may divide both sides by $(1-\lambda)$ to yield:

$$\lambda(\delta^{\Delta t_1+\Delta t_2} - d(\delta^{\Delta t_1} + \delta^{\Delta t_2})) + d(1-(1-\lambda)d) > 0 \tag{45}$$

We may define a function, $g(d)$ by:

$$g(d) := \lambda(\delta^{\Delta t_1+\Delta t_2} - d(\delta^{\Delta t_1} + \delta^{\Delta t_2})) + d(1-(1-\lambda)d) \tag{46}$$

so that subadditivity is equivalent to $g > 0 \,\big|\, d \in (0, 1)$. We now prove this claim. Note that g is quadratic in d with negative second derivative:

$$\frac{\partial^2 g}{\partial d^2} = -2(1-\lambda) < 0 \tag{47}$$

Accordingly, its unique minima on an interval will be found on the boundary points of the interval. In our case the boundary points are $d \in \{0, 1\}$. At $d = 0$ we find:

$$g(0) = \lambda\delta^{\Delta t_1+\Delta t_2} > 0 \tag{48}$$

At $d = 1$ we have:

$$g(1) = \lambda(1 + \delta^{\Delta t_1+\Delta t_2} - \delta^{\Delta t_1} - \delta^{\Delta t_2}) \tag{49}$$

If we view $g(1)$ as function of Δt_1, with, $h(\Delta t_1) := g(1)$, then we may note that:

$$h(0) = 0 \tag{50}$$

$$\frac{dh}{d\Delta t_1} = \lambda(\delta^{\Delta t_2} - 1)\delta^{\Delta t_1} \ln|\delta| \geq 0 \tag{51}$$

since $\delta \in (0,1)$, $\Delta t_1 \geq 0$, $\Delta t_2 \geq 0$. Consequently, we see that $g(1) \geq 0$ and may conclude that $g > 0$ for $d \in (0,1)$. □

The following corollary delivers Prediction 3 in the main text.

Corollary 2.1. *The magnitude of subadditive behavior is greater for those with lower cognitive precision than for those who are certain ($\lambda = 1$).*

Proof. For those that are certain, we have that $SA = 0$, whereas for those that exhibit any uncertainty we have $SA > 0$. □

Theorem 3. *There are no front-end delay effects.*

Proof. As mentioned earlier, *RRR* is a function of the time delay, Δt, not the individual start and end times. This precludes the existence of front-end delay effects. Formally, for any $l > 0$,

$$\Delta FE := RRR_{0,t_2} - RRR_{l,t_2+l} = \ln\left(\frac{\lambda u^{-1}(\delta^{t_2}) + (1-\lambda)d}{\lambda u^{-1}(\delta^{t_2}) + (1-\lambda)d}\right) = 0 \tag{52}$$

□

The corresponding corollary underlying Prediction 4 in the main text is:

Corollary 3.1. *An increase in cognitive precision doesn't affect front-end delay effects.*

A.3 Derivations for Cognitive Uncertainty Measure

As laid out in Section 2, the DM subjectively perceives his optimal action as a *distribution* conditional on his noisy signal. This means: while the agent's loss function induces him to play $a^o = \mathbb{E}[A \big| \{S = s\}]$, the underlying perceived posterior distribution of the optimal action is Beta-distributed:

$$A \Big| \{S = s\} \sim \; Beta\left(\underbrace{sn_2 + n_1 d}_{\equiv a}, \; \underbrace{n_2(1-s) + n_1(1-d)}_{\equiv b}\right) \tag{53}$$

where n_2 is the signal precision. Now, let us restate our definition of cognitive uncertainty,

$$p_{CU} := \mathbb{P}(|A \big| \{S = s\} - \mathbb{E}[A \big| \{S = s\}]| > c), \tag{54}$$

for fixed constant c. The objective of this subsection is to establish that increases in signal precision decrease cognitive uncertainty. Below, we develop two sets of results about this relationship. First, Corollary 2.1 provides a limit argument showing that any desired decrease in cognitive uncertainty can be achieved by an increase in signal precision. Second, to shed light on the case with low signal precision, Theorem 4 shows that cognitive uncertainty decreases with signal precision when using the closest Gaussian approximation of the Beta distribution.

To begin, we prove:

Proposition 2. $\forall \kappa > 0, \forall \varepsilon > 0, \exists N \in \mathbb{N}$ *such that* $p_{CU} < \varepsilon$ *for* $n_2 > N$.

Proof. By Chebyshev's inequality we see that for any positive number, κ:

$$p_{CU} < \frac{\mathrm{Var}(A\big|\{S=s\})}{\kappa^2} \tag{55}$$

and, since $A\big|\{S=s\} \sim Beta(n_2 s + n_1 d, n_2(1-s) + n_1(1-d))$ its variance is found to be:

$$\mathrm{Var}(A\big|\{S=s\}) = \frac{(n_2 s + n_1 d)(n_2(1-s) + n_1(1-d)),}{(n_1+n_2)^2(n_1+n_2+1)} = O(n_2^{-1}) \tag{56}$$

Accordingly, we find:

$$\lim_{n_2 \to \infty} p_{CU} = 0 \tag{57}$$

Which in turn yields the proposition. □

This proposition yields the following corollary:

Corollary 2.1. *Holding the signal value constant* $\{S=s\}$ *and given a base level of signal precision,* n_2*, there exists a constant* Δn *such that a desired decrease in cognitive uncertainty may be accomplished by increasing the signal precision by more than* Δn*.*
Formally, given a base signal precision, n_2*, and a desired decrease in cognitive uncertainty,* $\delta \in (0, p_{CU})$*, there exists a quantity,* $\Delta n \in \mathbb{N}$*, such that:*

$$n_2\prime > n_2 + \Delta n \rightarrow p_{CU} - p_{CU}\prime > \delta \tag{58}$$

with $n_2\prime$ *and* $p_{CU}\prime$ *being the new signal precision and cognitive uncertainty respectively.*

Proof. Given a signal precision n_2 and cognitive uncertainty, p_{CU}, we may apply the proposition to $\varepsilon = p_{CU} - \delta$. We then find that $\Delta n = N - n_2$. The result follows. □

In essence, this corollary formally states the intuition that any desired decrease in cognitive uncertainty may be accomplished through an increase in signal precision.

For a better approximation in cases of low signal precision, it is necessary to develop approximations of the Beta distribution. One such approximation follows from the Central Limit Theorem. We first prove a useful result:

Proposition 3. *Let $B_i \sim Beta(a_i, b_i)$, $n_i = a_i + b_i$ and $\forall i \in \mathbb{N}$, $\frac{a_i}{a_i+b_i} = \mu$, then*

$$B_i \xrightarrow{d} \mathcal{N}\left(\mu, \frac{\mu(1-\mu)}{n_i}\right)$$

as $a_i, b_i \to \infty$.

To prove this proposition we require the following lemma:

Lemma 3.1. *Let $Y_n \sim Gamma(na, 1)$ then $Y_n \xrightarrow{d} \mathcal{N}(na, a)$ as $n \to \infty$.*

Proof. Since the sum of Gamma variables follows a Gamma distribution,[29] we see that Y_n has the same distribution as:

$$\tilde{X} = \sum_{i=1}^{n} X_i$$

where X_i are i.i.d. random variables sampled from $Gamma(a, 1)$.

Now, by the Central Limit Theorem, we have:

$$\sqrt{\frac{n}{a}}\left(\frac{\tilde{X}}{n} - a\right) \xrightarrow{d} \mathcal{N}(0,1) \tag{59}$$

which yields:

$$Y_n \xrightarrow{d} \mathcal{N}(na, na) \tag{60}$$

□

We are now in a position to prove our proposition.

Proof. Let $X_i \sim Gamma(a_i, 1)$ and $Y_i \sim Gamma(b_i, 1)$ be independent random variables. Then we know[30] that:

$$Z_i = g(X_i, Y_i) \coloneqq \frac{X_i}{X_i + Y_i} \sim Beta(a_i, b_i)$$

Note, that if we scale both X_i and Y_i by $(a_i + b_i)^{-1}$ that Z_i remains unchanged. Further-

[29]Let $X_n \sim Gamma(a_n, 1)$ and $Y_n \sim Gamma(b_n, 1)$ be independent Gamma variables. Then $Z_n = X_n + Y_n \sim Gamma(a_n + b_n, 1)$.

[30]This may be verified by consideration of the joint density of X_i, Y_i; making the transformation $V = X_i + Y_i$, with W_i as defined earlier; and finding the marginal density of W_i.

more, by our lemma above, we have:

$$\begin{pmatrix} \frac{X_i}{a_i+b_i} \\ \frac{Y_i}{a_i+b_i} \end{pmatrix} \xrightarrow{d} \mathcal{N}\left(\begin{pmatrix} \frac{a_i}{a_i+b_i} \\ \frac{b_i}{a_i+b_i} \end{pmatrix}, \begin{pmatrix} \frac{a_i}{(a_i+b_i)^2} & 0 \\ 0 & \frac{b_i}{(a_i+b_i)^2} \end{pmatrix}\right) \tag{61}$$

$$\xrightarrow{d} \mathcal{N}\left(\begin{pmatrix} \mu \\ 1-\mu \end{pmatrix}, \begin{pmatrix} \frac{\mu}{n_i} & 0 \\ 0 & \frac{1-\mu}{n_i} \end{pmatrix}\right) \tag{62}$$

We now employ the Delta method. A Taylor expansion of $g(x, y)$ yields that to first order:

$$g(x, y) \approx g(x_0, y_0) + \nabla g(x_0, y + 0) \cdot (x - x_0, y - y_0) \tag{63}$$

$$\approx g(x_0, y_0) + (\frac{y_0}{(x_0 + y_0)^2}, \frac{-x_0}{(x_0 + y_0)^2}) \cdot (x - x_0, y - y_0) \tag{64}$$

Accordingly, we find that:

$$Z_i = g(X_i, Y_i) \tag{65}$$

$$\approx g(\mu, 1-\mu) + ((1-\mu), -\mu) \cdot (X_i - \mu, Y_i - (1-\mu)) \tag{66}$$

$$\xrightarrow{d} \mu + ((1-\mu), -\mu) \cdot \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{\mu}{n} & 0 \\ 0 & \frac{1-\mu}{n} \end{pmatrix}\right) \tag{67}$$

$$\xrightarrow{d} \mathcal{N}\left(\mu, \begin{pmatrix} 1-\mu & -\mu \end{pmatrix} \begin{pmatrix} \frac{\mu}{n} & 0 \\ 0 & \frac{1-\mu}{n} \end{pmatrix} \begin{pmatrix} 1-\mu \\ -\mu \end{pmatrix}\right) \tag{68}$$

$$\xrightarrow{d} \mathcal{N}\left(\mu, \frac{\mu(1-\mu)}{n_i}\right) \tag{69}$$

□

This proposition provides the simplest Gaussian approximation of the Beta distribution; however, this approximation may be improved by taking into account geometric aspects of density function. In this case, we will consider the peak and the points of inflection. For a Gaussian, the mode is found at μ and the points of inflection are found at $\mu \pm \sigma$. For the Beta distribution we have that the mode is given by:

$$m = \frac{a-1}{a+b-2} = \frac{n\mu - 1}{n-2} \tag{70}$$

When $a, b > 2$ the Beta's density function is bell shaped and we may find the points of inflection about the mode. If we define the constant:

$$\kappa := \frac{1}{a+b-2}\sqrt{\frac{(a-1)(b-1)}{a+b-3}} \tag{71}$$

the points of inflection may be written as $m \pm \kappa$. Note that $\kappa/\sigma \to 1$ and $m \to \mu$ as $n \to \infty$. Accordingly, we may employ the superior approximation (especially for lower values of a, b):

$$Beta(a, b) \approx \mathcal{N}(m, \kappa) \tag{72}$$

The goodness of fit for this approximation for our purposes may shown empirically. Using this approximation, we may return to our original goal of demonstrating that cognitive uncertainty decreases in the signal precision and claim:

Proposition 4. *Let $N_1(m_1, \kappa_1^2)$ be the normal approximation to $A_1 \sim Beta(a_1, b_1)$ and $N_2(m_2, \kappa_2^2)$ the normal approximation to $A_2 \sim Beta(a_2, b_2)$ with: $a_1, a_2, b_1, b_2 > 2$, $\frac{a_1}{a_1+b_1} = \frac{a_2}{a_2+b_2}$ and $a_2 > a_1, b_2 > a_1$ then for fixed $c \in (0, min_{i=1,2}\{m_i, 1-m_i\})$:*

$$\mathbb{P}(|N_2 - m_2| < c) > \mathbb{P}(|N_1 - m_1| < c) \tag{73}$$

where m_i is the mode of A_i and κ_i is the distance from the mode to the points of inflection for A_i.

Proof. We note that:

$$\mathbb{P}(|N_i - m_i| < c) = \mathbb{P}(|Z| < c/\kappa_i) \tag{74}$$

and that κ as defined in (71) satisfies[31] $\partial\kappa/\partial a, \partial\kappa/\partial b < 0$ for $a, b > 2$. Accordingly, we have that $\kappa_2 < \kappa_1$. Hence, we see that:

$$\mathbb{P}(|N_1 - m_1| < c) = \mathbb{P}(|Z| < c/\kappa_1) < \mathbb{P}(|Z| < c/\kappa_2) = \mathbb{P}(|N_2 - m_2| < c) \tag{75}$$

□

This provides us with our final result:

Theorem 4. *Holding the signal $\{S = s\}$ constant, cognitive uncertainty decreases with increases the signal precision in the Gaussian approximation eq.* (72).

$$\frac{\Delta p_{CU}}{\Delta n_2} < 0 \tag{76}$$

Proof. Apply the previous proposition with respect to the values of a, b from eq. (53). □

[31] Just reparametrize it under $a = x + 1, b = y + 1$ and square the expression.

B Additional Figures

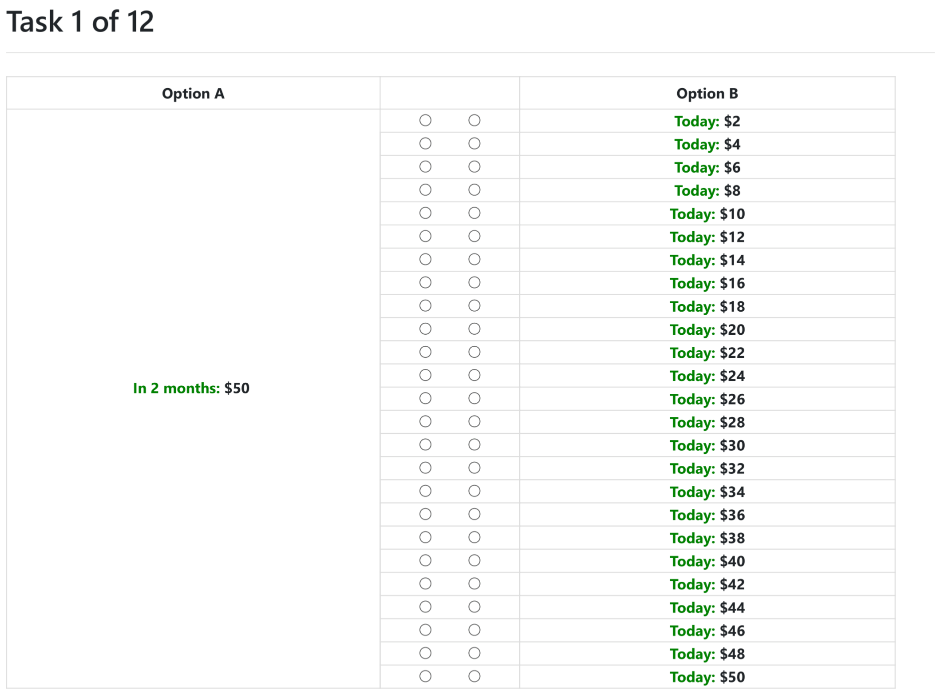


Figure 10: Screenshot of an example decision screen in *Money Main*

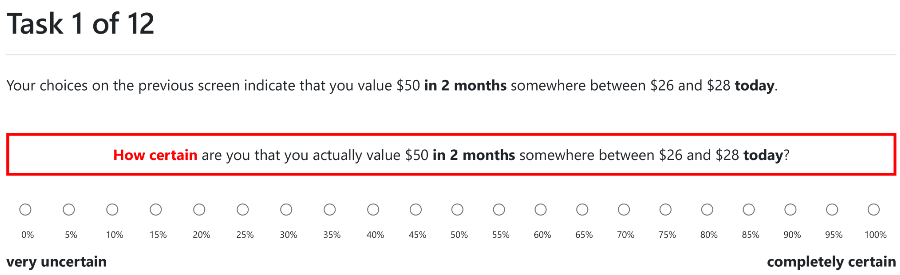


Figure 11: Screenshot of an example cognitive uncertainty elicitation screen in *Money Main*

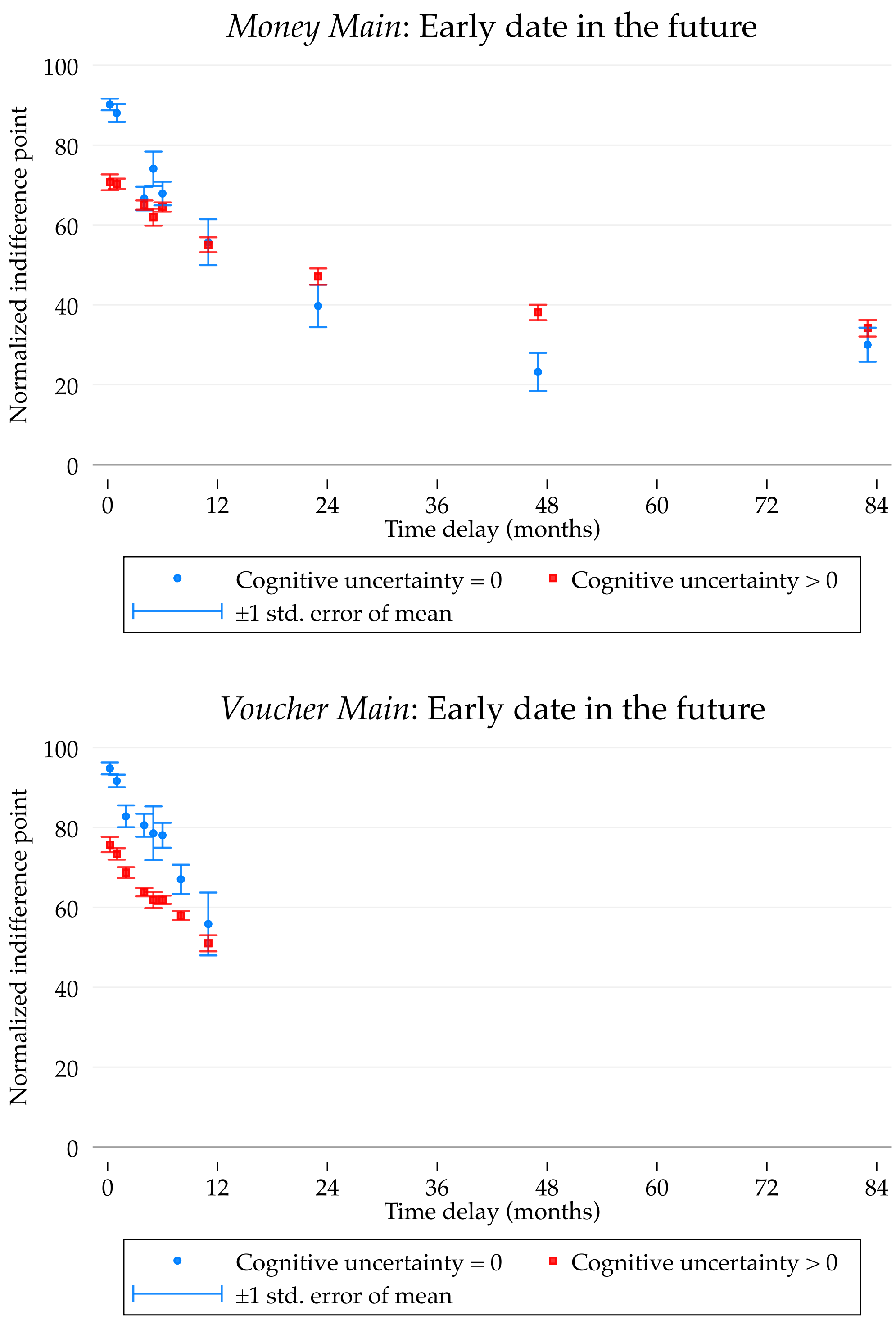
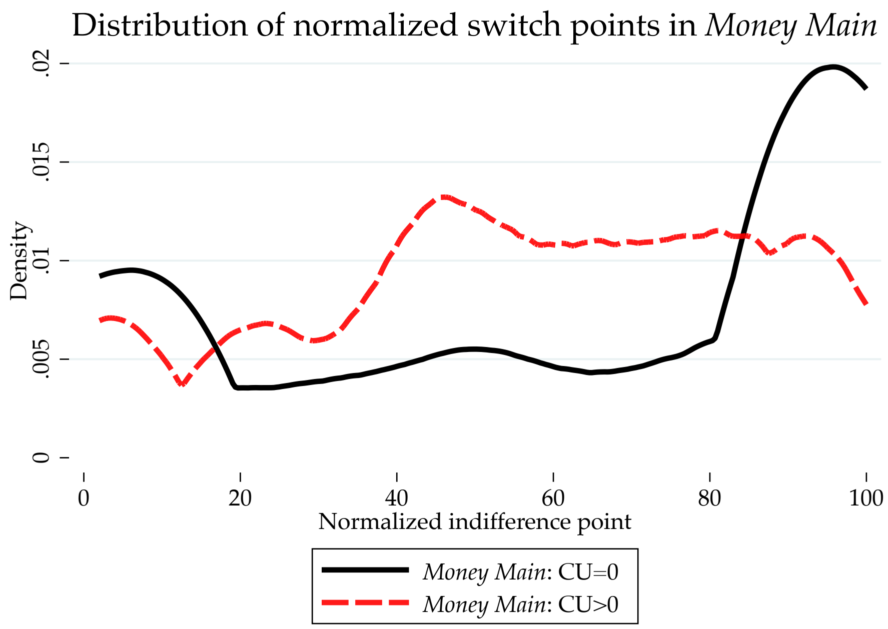


Figure 12: Observed discounting with $t_1 > 0$ in *Money Main* (top panel) and *Voucher Main* (bottom panel). The figure shows averages across decisions. Whiskers show standard error bars, computed based on clustering at the subject level.



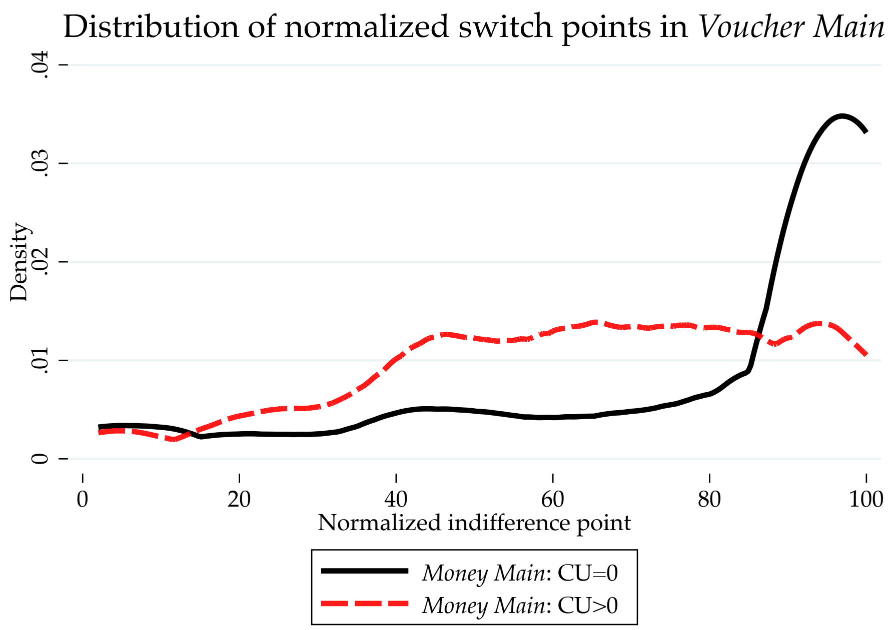
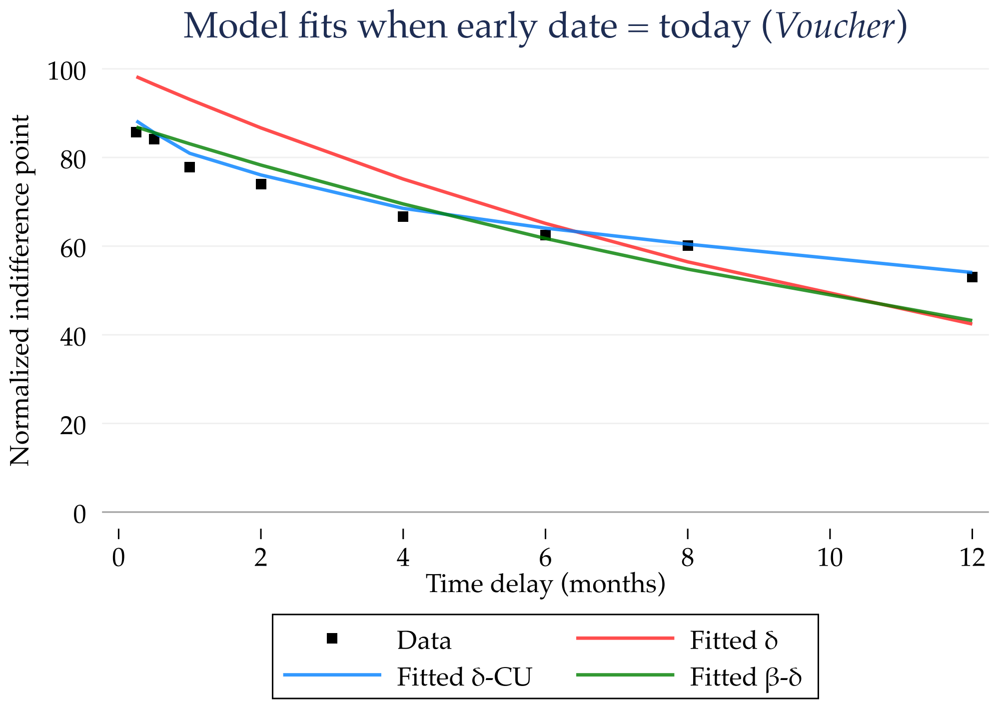


Figure 13: Kernel density plots of the distribution of normalized indifference points in the baseline experiments, separately for decisions that reflect zero or strictly positive cognitive uncertainty. Kernel is Epanechnikov.



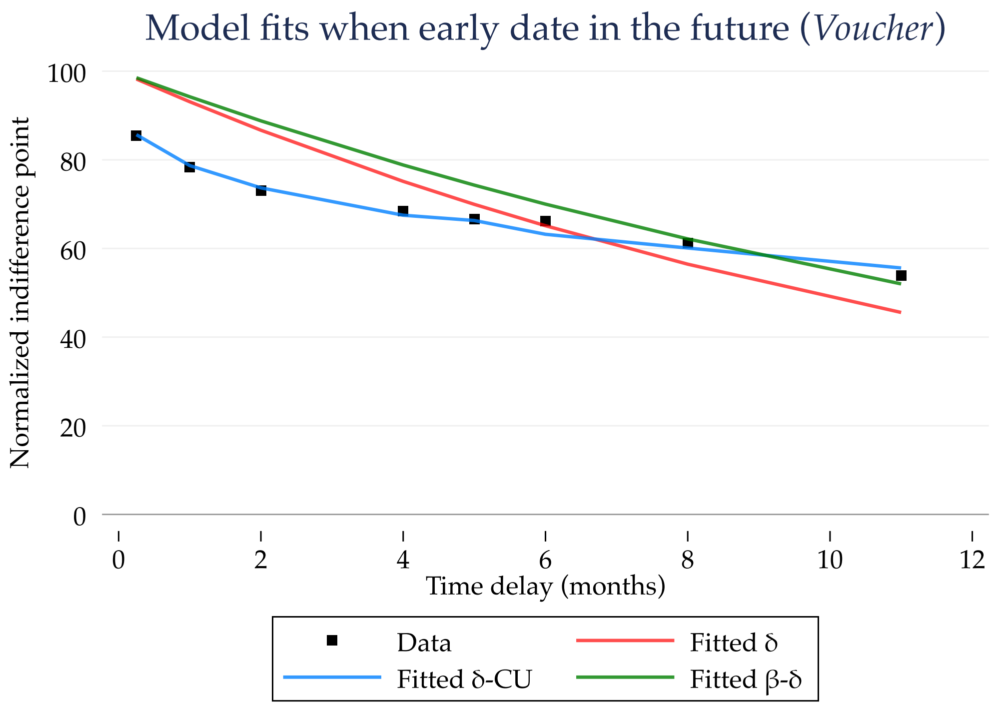


Figure 14: Model fit vs. data in *Voucher Main*. The model predictions are computed as fitted values of the parameter estimates in Table 4.

C Additional Tables

Table 5: List of treatment conditions

Treatment	**Description**	**Sample**	*Pre-registration*	**Covered in**
Money Main	Hypothetical money-early-versus-later experiments	645	`https://aspredicted.org/kg7zs.pdf`	Sections 3–5
Voucher Main	Incentivized UberEats Voucher Experiments	500	`https://aspredicted.org/b4pw2.pdf`	Sections 3–5
Money Complex Dates	Like *Voucher Main*, except with later payment date displayed as math expression	149	`https://aspredicted.org/77xp6.pdf`	Section 7.1
Money Complex Amounts	Like *Voucher Main*, except with payoff amount displayed as math expression	153	`https://aspredicted.org/77xp6.pdf`	Section 7.1
Money Load	Like *Voucher Main*, except with cognitive load manipulation	154	`https://aspredicted.org/77xp6.pdf`	Section 7.1
Money Main Replication	Like *Voucher Main*, used as control group for *Money Complex Dates*, *Money Complex Amounts* and *Money Load* (within-session randomization)	161	`https://aspredicted.org/77xp6.pdf`	Section 7.1
Voucher Advice	Like *Voucher Main*, except introducing piece of advice	153	`https://aspredicted.org/jk5s5.pdf`	Section 7.2
Money Within-Subject Load	Like *Money Main*, except across-round variation in cognitive load	400	`https://aspredicted.org/av2y2.pdf`	Appendix F

Notes. List of all treatments included in this paper.

Table 6: Cognitive uncertainty and across-trial choice variability

	Dependent variable: Abs. diff. b/w normalized indifference points					
Treatment:	*Money Main*			*Voucher Main*		
	(1)	(2)	(3)	(4)	(5)	(6)
Ave. cognitive uncertainty	0.14***	0.14***	0.14***	0.14***	0.13***	0.13***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Time delay FE	No	Yes	Yes	No	Yes	Yes
Demographic controls	No	No	Yes	No	No	Yes
Observations	1290	1290	1290	1000	1000	1000
R^2	0.03	0.04	0.04	0.03	0.04	0.04

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is computed as absolute difference between the normalized indifference points in two repetitions of the exact same choice list. The independent variable is average cognitive uncertainty across the two repetitions of the choice list. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 7: Cognitive uncertainty and impatience over one week

	Dependent variable: Normalized indifference point							
Treatment:	*Money Main*				*Voucher Main*			
Sample:	$t1 = 0$		$t1 > 0$		$t1 = 0$		$t1 > 0$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cognitive uncertainty	-0.66***	-0.65***	-0.58***	-0.55***	-0.66***	-0.65***	-0.61***	-0.64***
	(0.10)	(0.11)	(0.11)	(0.10)	(0.13)	(0.13)	(0.16)	(0.14)
Payment amount FE	No	Yes	No	Yes	No	Yes	No	Yes
Round FE	No	Yes	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes	No	Yes
Observations	350	350	218	218	404	404	152	152
R^2	0.20	0.23	0.20	0.30	0.15	0.18	0.21	0.34

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The sample includes decisions in which the time delay is given by one week. Columns (1)–(2) and (5)–(6) include those trials in which the early payment date is today, and columns (3)–(4) and (7)–(8) those in which the early payment date is in the future. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: Cognitive uncertainty and increasing per-period patience

	Dependent variable: Implied per-period patience δ_H							
Treatment:	*Money Main*				*Voucher Main*			
Sample:	$t1 = 0$		$t1 > 0$		$t1 = 0$		$t1 > 0$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Time delay (years)	0.058***	0.058***	0.049***	0.050***	0.18***	0.18***	0.20***	0.20***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.03)	(0.03)	(0.05)	(0.05)
Time delay × Cognitive uncertainty	0.00076***	0.00074***	0.00059***	0.00056***	0.0059***	0.0056***	0.0062***	0.0063***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Cognitive uncertainty	-0.0028***	-0.0027***	-0.0027***	-0.0026***	-0.0064***	-0.0063***	-0.0070***	-0.0071***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Payment amount FE	No	Yes	No	Yes	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes	No	Yes	No	Yes
Observations	4948	4948	2792	2792	3846	3846	2154	2154
R^2	0.20	0.21	0.16	0.18	0.11	0.13	0.12	0.12

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Columns (1)–(2) and (5)–(6) include those trials in which the early payment date is today, and columns (3)–(4) and (7)–(8) those in which the early payment date is in the future. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 9: Cognitive uncertainty and insensitivity to time delays: Including participant fixed effects

	Dependent variable: Normalized indifference point							
Treatment:	*Money Main*				*Voucher Main*			
Sample:	$t1 = 0$		$t1 > 0$		$t1 = 0$		$t1 > 0$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Time delay (years)	-6.99***	-6.96***	-6.77***	-6.81***	-33.9***	-34.0***	-30.7***	-30.9***
	(0.36)	(0.36)	(0.40)	(0.39)	(1.99)	(1.96)	(3.88)	(3.89)
Time delay × Cognitive uncertainty	0.055***	0.055***	0.044***	0.045***	0.25***	0.25***	0.30*	0.30*
	(0.01)	(0.01)	(0.01)	(0.01)	(0.07)	(0.07)	(0.16)	(0.16)
Cognitive uncertainty	-0.26***	-0.26***	-0.28***	-0.28***	-0.30***	-0.30***	-0.42***	-0.41***
	(0.04)	(0.04)	(0.05)	(0.05)	(0.06)	(0.06)	(0.09)	(0.08)
Payment amount FE	No	Yes	No	Yes	No	Yes	No	Yes
Round FE	No	Yes	No	Yes	No	Yes	No	Yes
Participant FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4948	4948	2792	2792	3846	3846	2154	2154
R^2	0.66	0.66	0.68	0.69	0.73	0.74	0.71	0.71

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Columns (1)–(4) include data from *Money Main*, where columns (1)–(2) restrict attention to decision problems with $t_1 = 0$ and columns (3)–(4) to problems with $t_1 > 0$. An analogous logic applies to columns (5)–(8) for *Voucher Main*. Demographic controls include age, gender and income bucket. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 10: Distribution of participant-level estimates of model parameters

	Money Main (MPL & Direct Elicitation)			
	(1)	(2)	(3)	(4)
	δ	$\beta-\delta$	$\delta-CU$	$\beta-\delta$ $-CU$
	Median (25 / 75 pctl.)	Median (25 / 75 pctl.)	Median (25 / 75 pctl.)	Median (25 / 75 pctl.)
$\hat{\delta}$	0.96 (0.90 / 0.99)	0.97 (0.92 / 0.99)	0.97 (0.91 / 0.99)	0.97 (0.92 / 0.99)
$\hat{\beta}$		0.88 (0.66 / 0.99)		0.96 (0.74 / 1.00)
$\hat{d}$			0.51 (0.27 / 0.73)	0.51 (0.25 / 0.73)

Notes. Distribution of estimates of different versions of (7) estimated at the subject level. MPL = multiple price list. Each column corresponds to a separate model specification. Column (1): set $\beta = 1$ and $p_{CU} = 0$. Column (2): set $p_{CU} = 0$. Column (3): set $\beta = 1$. All estimations accommodate utility curvature: a representative-agent CRRA parameter of $\hat{\gamma} = 0.94$ was separately estimated on the risky choice data and used in the participant-level estimations on the intertemporal choice data. The exponential parameter δ is the monthly discount factor.

Table 11: Correlations between participant-level estimates of model parameters in $\beta{-}\delta{-}CU$ specification

	$\hat{\delta}$	$\hat{\beta}$	$\hat{d}$	Mean stated CU
$\hat{\delta}$	1.000			
$\hat{\beta}$	0.198*** (0.000)	1.000		
$\hat{d}$	-0.169*** (0.000)	-0.015 (0.705)	1.000	
Mean stated CU	-0.100* (0.011)	-0.024 (0.546)	-0.096* (0.015)	1.000

Notes. Pairwise correlations of participant-level estimates of equation (7). p-values shown in brackets. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

D Direct Elicitation Experiments

As part of our *Money Main* experiments, each subject completed six additional intertemporal choice problems that were administered in a direct elicitation format rather than using MPLs. That is, in each of these decisions, subjects were directly asked which monetary amount to be received in $t = t_1$ is worth as much to them as receiving \$$y_2$ in

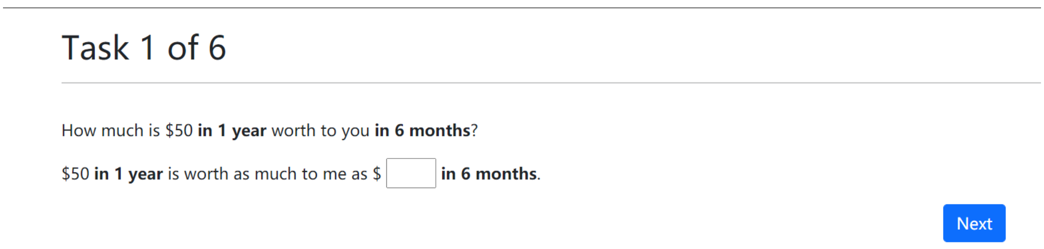


Figure 15: Screenshot of an example decision screen in the direct elicitation part of *Money Main*

$t = t_2$, see Figure 15 for an example screenshot.[32] After participants had indicated their indifference amount, the next screen again elicited cognitive uncertainty, see Figure 16.

We here replicate all of our main analyses using these direct elicitation data. First, Table 12 shows that cognitive uncertainty is again significantly correlated with the magnitude of across-trial inconsistencies (choice variability), as defined by the absolute difference in normalized indifference points across two repetitions of the same question.

Second, Table 13 documents that cognitive uncertainty is again strongly and significantly correlated with impatience over a horizon of one week. Third, columns (1)–(2) of Table 14 documents that cognitive uncertainty is again strongly predictive of a reduced sensitivity of intertemporal choice behavior with respect to variation in the time delay, as we can infer from the significant interaction term. Columns (3)–(4) show the same patterns by documenting that cognitive uncertainty is strongly predictive of decreasing impatience as the time delay increases, as we can again infer from the significant interaction term. Figure 17 visualizes these patterns.

Next, Table 15 documents that subadditivity effects strongly increase in cognitive uncertainty, see columns (2)–(3), (5)–(6) and (8)–(9). Indeed, as we can see from the usually insignificant raw term "1 if long interval", there is no significant evidence for subadditivity among subjects who indicate cognitive uncertainty of zero.

Finally, Table 16 replicates the result that cognitive uncertainty is uncorrelated with front-end delay effects. This again highlights that "not anything goes" but that cognitive uncertainty is only predictive of a specific class of (pre-registered) empirical regularities.

[32] The only difference between the choice problems in the direct elicitation experiments and the MPL is that (to save time) we only elicited direct elicitation problems in which the early payment date was today.

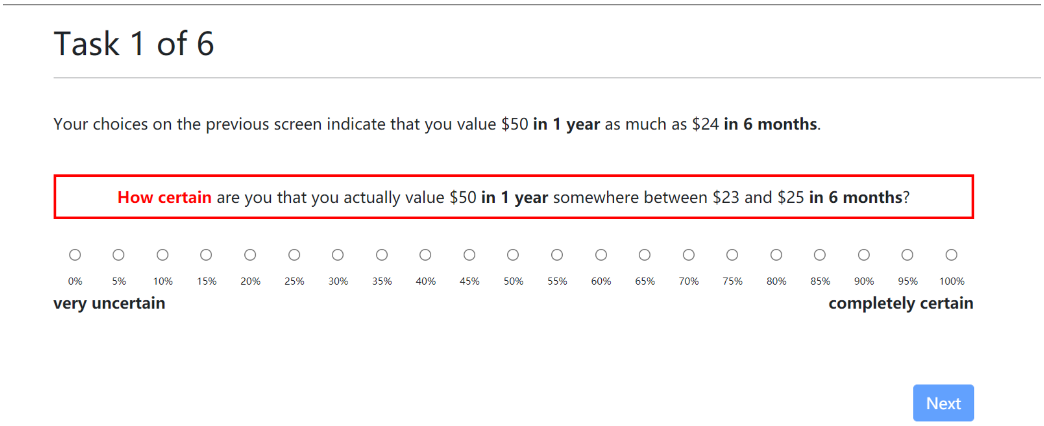


Figure 16: Screenshot of an example cognitive uncertainty elicitation screen in the direct elicitation part of *Money Main*

Table 12: Cognitive uncertainty and across-trial choice variability: Direct elicitation

	Dependent variable:		
	Abs. diff. b/w normalized indiff. points		
	(1)	(2)	(3)
Ave. cognitive uncertainty	0.083**	0.079**	0.079**
	(0.03)	(0.04)	(0.04)
Time delay FE	No	Yes	Yes
Demographic controls	No	No	Yes
Observations	645	645	645
R^2	0.01	0.02	0.02

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is computed as absolute difference between the normalized indifference points in two repetitions of the exact same choice list. The independent variable is average cognitive uncertainty across the two repetitions of the choice task. All observations are from the direct elicitation experiments. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 13: Cognitive uncertainty and impatience over one week: Direct elicitation

	Dependent variable: Normalized indifference point		
	(1)	(2)	(3)
Cognitive uncertainty	-0.59***	-0.59***	-0.57***
	(0.10)	(0.11)	(0.11)
Payment amount FE	No	Yes	Yes
Demographic controls	No	No	Yes
Observations	327	327	327
R^2	0.13	0.17	0.17

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The sample includes decisions in which the time delay is given by one week. All observations are from the direct elicitation experiments. In these experiments, the early payment date is always today. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 14: Cognitive uncertainty and diminishing impatience: Direct elicitation

	Dependent variable: Normalized indifference point		Implied per-period patience δ_H	
	(1)	(2)	(3)	(4)
Time delay (years)	-7.16***	-7.08***	0.043***	0.044***
	(0.49)	(0.48)	(0.00)	(0.00)
Time delay × Cognitive uncertainty	0.091***	0.084***	0.0011***	0.0011***
	(0.02)	(0.01)	(0.00)	(0.00)
Cognitive uncertainty	-0.43***	-0.40***	-0.0050***	-0.0047***
	(0.05)	(0.05)	(0.00)	(0.00)
Payment amount FE	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes
Observations	3870	3870	3870	3870
R^2	0.17	0.19	0.17	0.19

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. All observations are from the direct elicitation experiments. In these experiments, the early payment date is always today. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

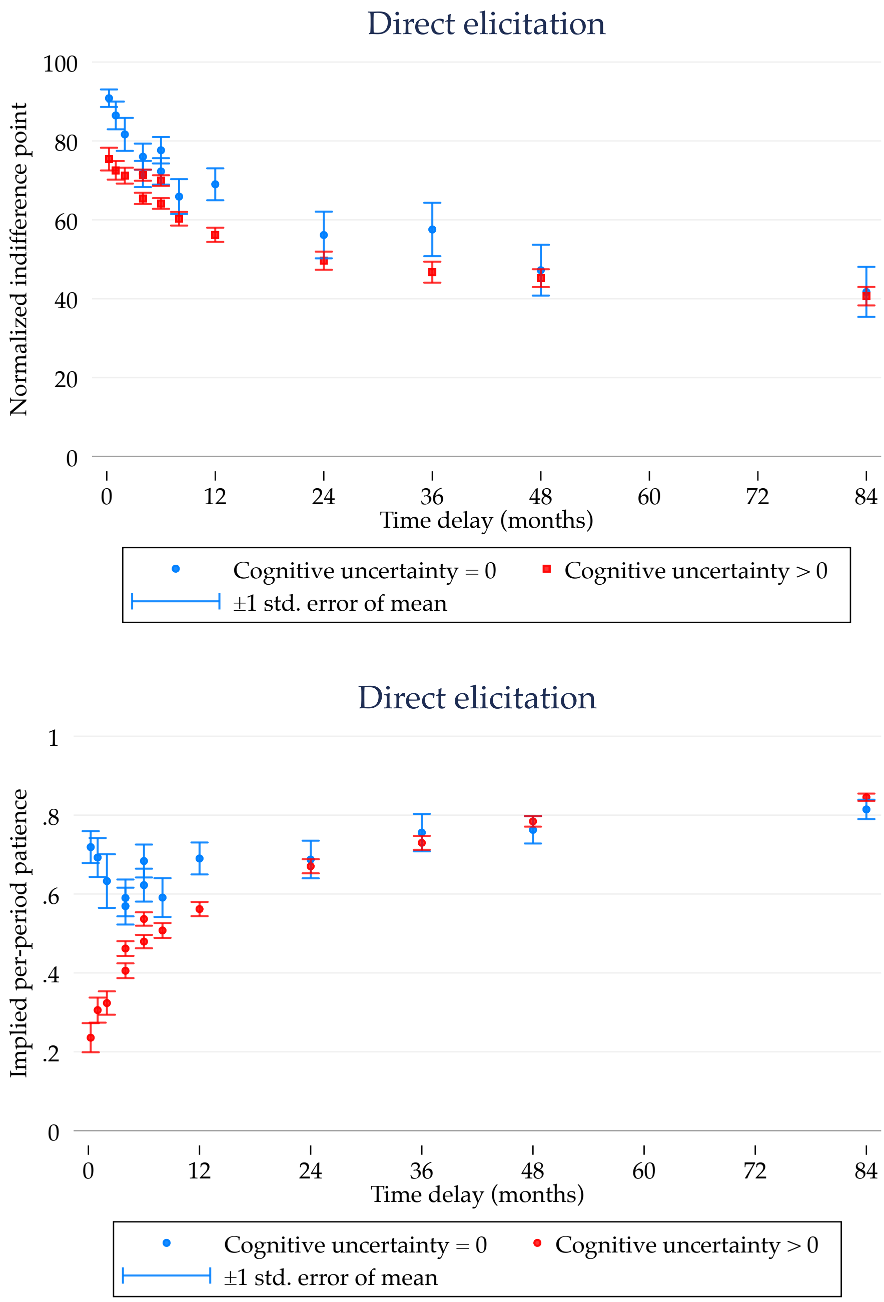


Figure 17: Observed discounting in the direct elicitation experiments (top panel, $N = 4,614$). Normalized indifference points are given by the midpoint of the switching interval in a choice list, divided by the larger-later payout amount (in %). Per-period patience is computed as $\delta_H(a^o) \equiv e^{-RRR/\Delta t} = (a^o)^{1/\Delta t}$, where a^o is the observed normalized indifference point.

Table 15: Cognitive uncertainty and subadditivity: Direct elicitation

	Dependent variable: Composite indifference point								
Sample:	Full			Set 1			Set 2		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1 if one long interval	7.45***	3.81***	3.93***	7.05***	3.72*	3.72*	7.86***	3.66*	3.77*
	(0.70)	(1.40)	(1.41)	(1.05)	(1.94)	(1.94)	(0.93)	(2.02)	(2.04)
1 if one long interval × Cognitive uncertainty		0.20***	0.19***		0.18**	0.18**		0.23***	0.22**
		(0.06)	(0.06)		(0.08)	(0.08)		(0.09)	(0.09)
Cognitive uncertainty		-0.47***	-0.47***		-0.53***	-0.53***		-0.41***	-0.39***
		(0.07)	(0.07)		(0.10)	(0.10)		(0.10)	(0.11)
Set FE	Yes	Yes	Yes	No	No	No	No	No	No
Payment amount FE	No	No	Yes	No	No	Yes	No	No	Yes
Observations	1290	1290	1290	654	654	654	636	636	636
R^2	0.02	0.06	0.07	0.01	0.08	0.10	0.02	0.05	0.07

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. All observations are from the direct elicitation experiments. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 16: Cognitive uncertainty and front-end delay effects: Direct elicitation

	Dependent variable: Normalized indifference point			
	(1)	(2)	(3)	(4)
1 if front end delay	5.54***	5.28***	5.26***	5.27***
	(0.69)	(1.32)	(1.32)	(1.31)
Front-end delay × Cognitive uncertainty		0.060	0.061	0.056
		(0.06)	(0.06)	(0.06)
Cognitive uncertainty		-0.35***	-0.35***	-0.32***
		(0.06)	(0.06)	(0.06)
Set FE	Yes	Yes	Yes	Yes
Payment amount FE	No	No	Yes	Yes
Demographic controls	No	No	No	Yes
Observations	1290	1290	1290	1290
R^2	0.01	0.06	0.07	0.10

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. All observations are from the direct elicitation experiments. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

E Complexity and Load Experiments

E.1 Screenshots of Decision Screens

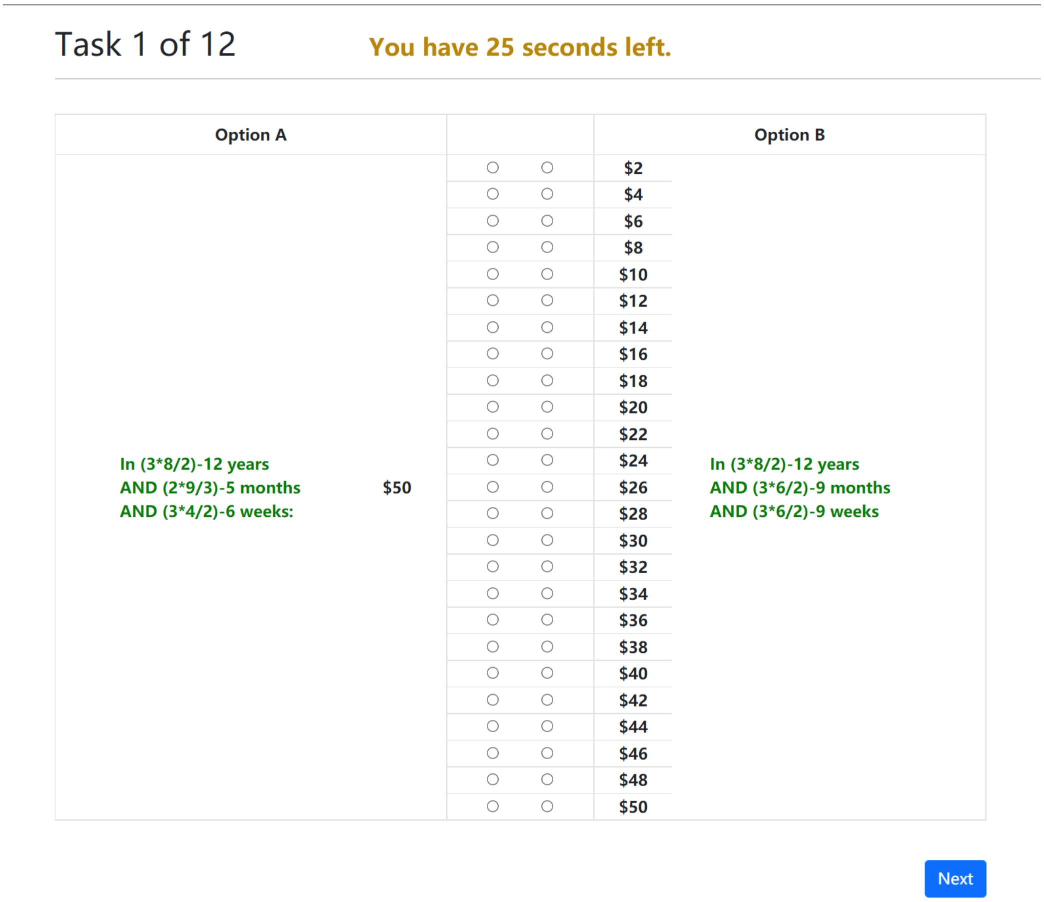
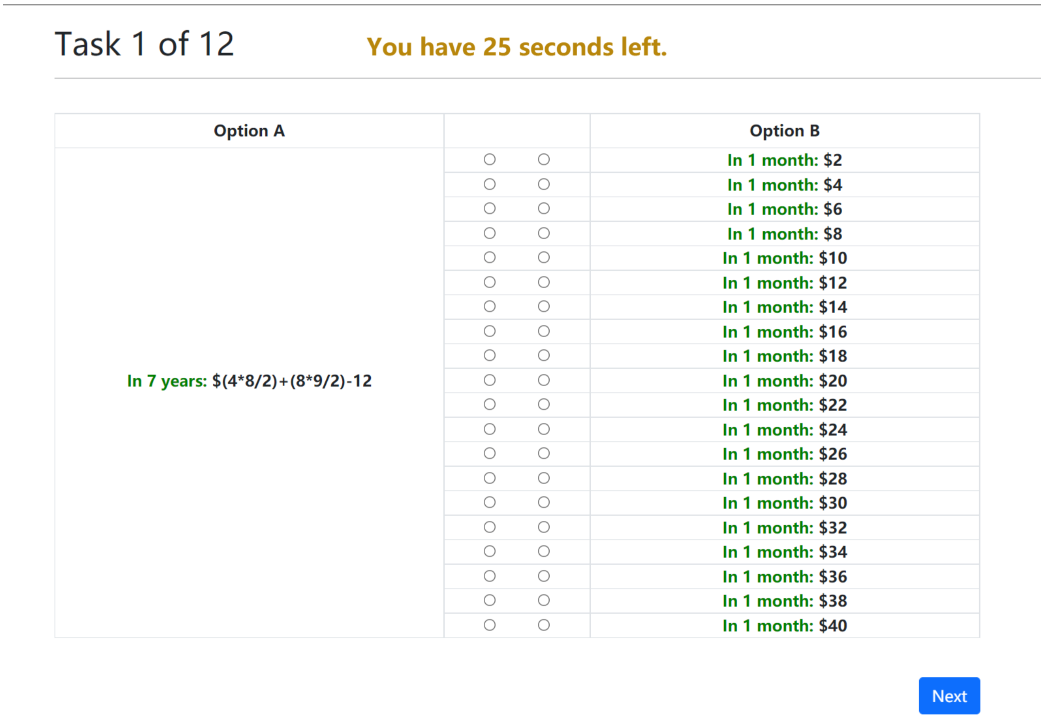


Figure 18: Screenshot of an example decision screen in *Money Complex Dates*

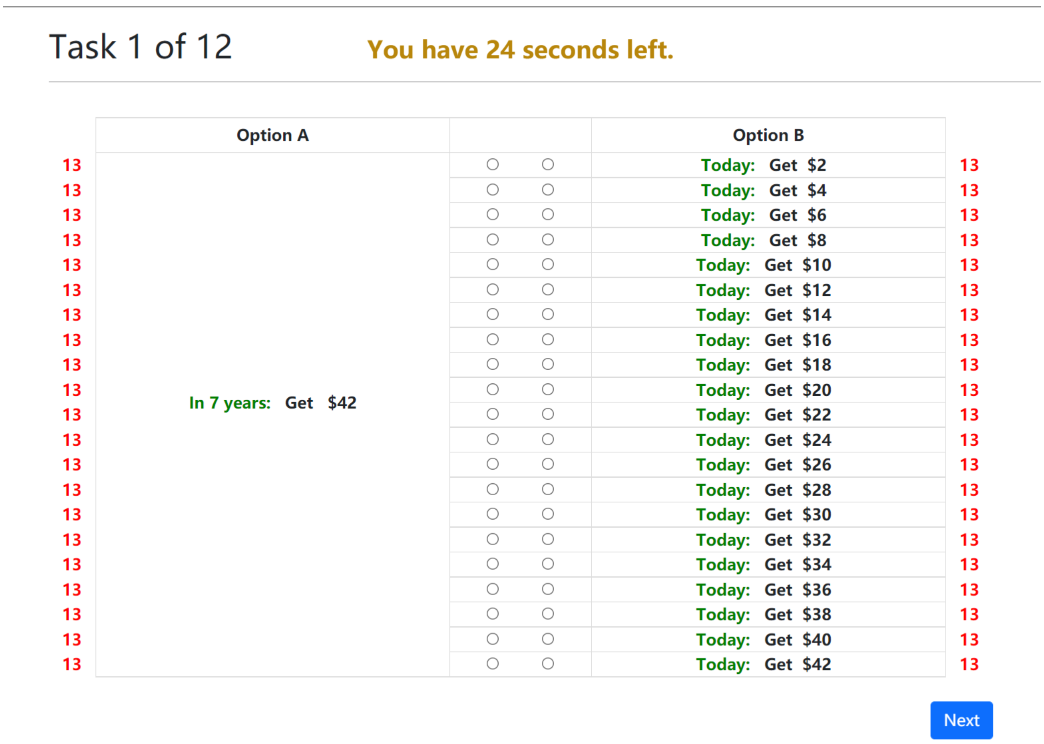


Task 1 of 12 You have 25 seconds left.

Option A		Option B
In 7 years: $(4*8/2)+(8*9/2)-12		In 1 month: $2
		In 1 month: $4
		In 1 month: $6
		In 1 month: $8
		In 1 month: $10
		In 1 month: $12
		In 1 month: $14
		In 1 month: $16
		In 1 month: $18
		In 1 month: $20
		In 1 month: $22
		In 1 month: $24
		In 1 month: $26
		In 1 month: $28
		In 1 month: $30
		In 1 month: $32
		In 1 month: $34
		In 1 month: $36
		In 1 month: $38
		In 1 month: $40

Next

Figure 19: Screenshot of an example decision screen in *Money Complex Amounts*



Task 1 of 12 You have 24 seconds left.

	Option A		Option B	
13	In 7 years: Get $42		Today: Get $2	13
13			Today: Get $4	13
13			Today: Get $6	13
13			Today: Get $8	13
13			Today: Get $10	13
13			Today: Get $12	13
13			Today: Get $14	13
13			Today: Get $16	13
13			Today: Get $18	13
13			Today: Get $20	13
13			Today: Get $22	13
13			Today: Get $24	13
13			Today: Get $26	13
13			Today: Get $28	13
13			Today: Get $30	13
13			Today: Get $32	13
13			Today: Get $34	13
13			Today: Get $36	13
13			Today: Get $38	13
13			Today: Get $40	13
13			Today: Get $42	13

Next

Figure 20: Screenshot of an example decision screen in *Money Load*

E.2 Results

Table 17 summarizes the results for all treatments that either manipulate complexity or cognitive load. For each of the three treatment variations, we compare behavior with treatment *Money Main replication*, which was administered along with the complexity and load treatments. Columns (1)–(3) show the results for choice problems in which the early date is today, while columns (4)–(6) summarize analogous results for $t_1 > 0$. Througout, the positive interaction coefficients between the time delay and the more complex / load treatments indicate that people's decisions are more inelastic to the time delay when the choice problems are more complex or they are placed under cognitive load. Figures 21–25 visualize these patterns.

Table 17: Complexity and load manipulations

	Dependent variable: Normalized indifference point					
Sample:	$t1 = 0$			$t1 > 0$		
	(1)	(2)	(3)	(4)	(5)	(6)
Time delay (years)	-4.97***	-4.94***	-4.89***	-4.84***	-4.84***	-4.85***
	(0.55)	(0.54)	(0.55)	(0.62)	(0.62)	(0.62)
1 if *Complex Dates*	3.17			1.38		
	(3.00)			(2.94)		
Time delay × 1 if *Complex Dates*	2.97***			3.36***		
	(0.79)			(0.88)		
1 if *Complex Amounts*		0.86			-2.63	
		(2.91)			(3.00)	
Time delay × 1 if *Complex Amounts*		2.07***			2.43***	
		(0.75)			(0.84)	
1 if *Load*			-2.34			-2.68
			(3.04)			(3.04)
Time delay × 1 if *Load*			1.64**			2.00**
			(0.78)			(0.82)
Payment amount FE	Yes	Yes	Yes	Yes	Yes	Yes
Demographic controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2381	2405	2428	1339	1363	1352
R^2	0.08	0.08	0.10	0.07	0.06	0.07

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Columns (1)–(6) include data from *Money Main Replication*, *Money Complex Dates*, *Money Complex Amounts* and *Money Load*. Columns (1)–(3) restrict attention to decision problems with $t_1 = 0$ and columns (4)–(6) to problems with $t_1 > 0$. Demographic controls include age, gender and income bucket. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

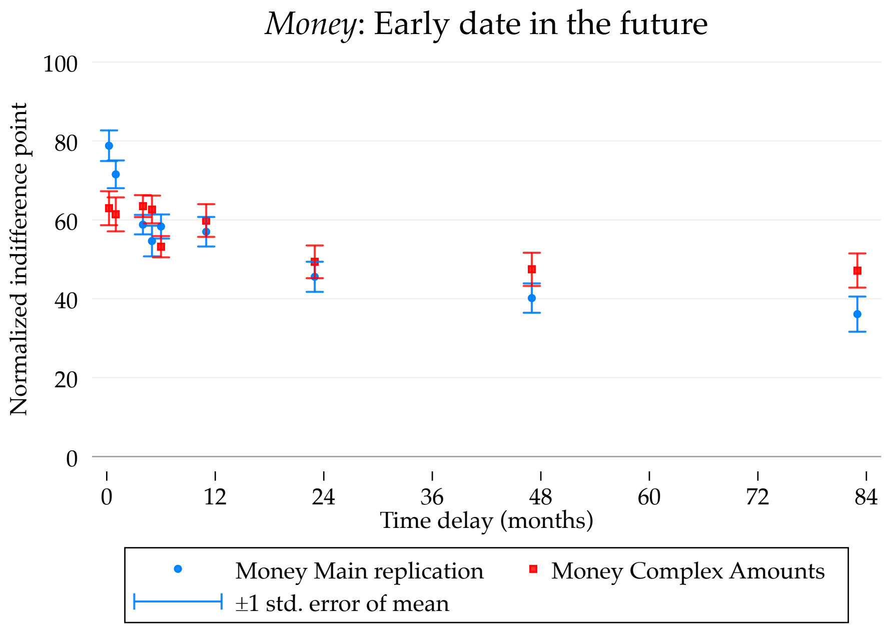


Figure 21: Observed discounting with $t_1 > 0$ in *Money Main replication* ($N = 161$) and *Money Complex Amounts* ($N = 153$). Normalized indifference points are given by the midpoint of the switching interval in a choice list, divided by the larger-later payout amount (in %). The figure shows averages across decisions. Whiskers show standard error bars, computed based on clustering at the subject level.

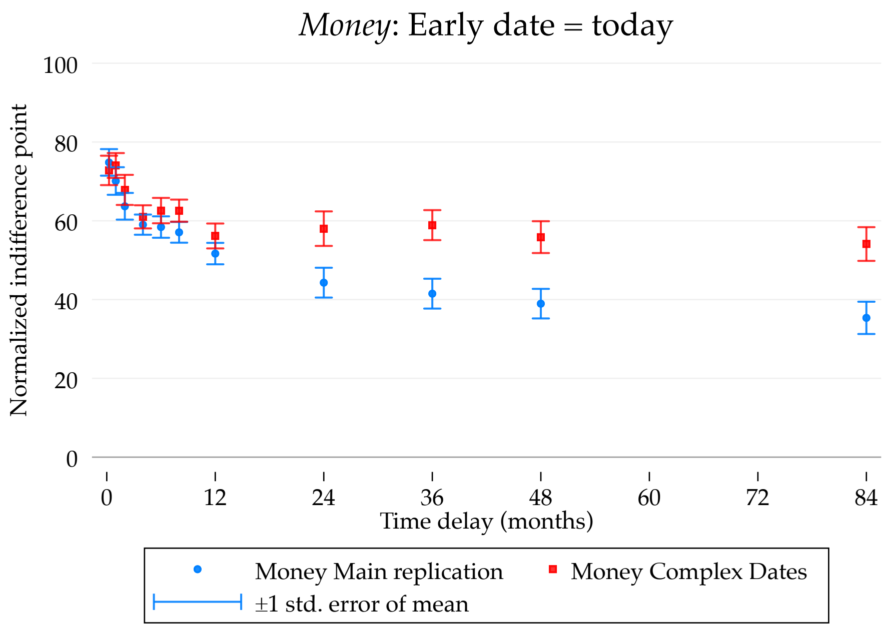


Figure 22: Observed discounting with $t_1 = 0$ in *Money Main replication* ($N = 161$) and *Money Complex Dates* ($N = 149$). Normalized indifference points are given by the midpoint of the switching interval in a choice list, divided by the larger-later payout amount (in %). The figure shows averages across decisions. Whiskers show standard error bars, computed based on clustering at the subject level.

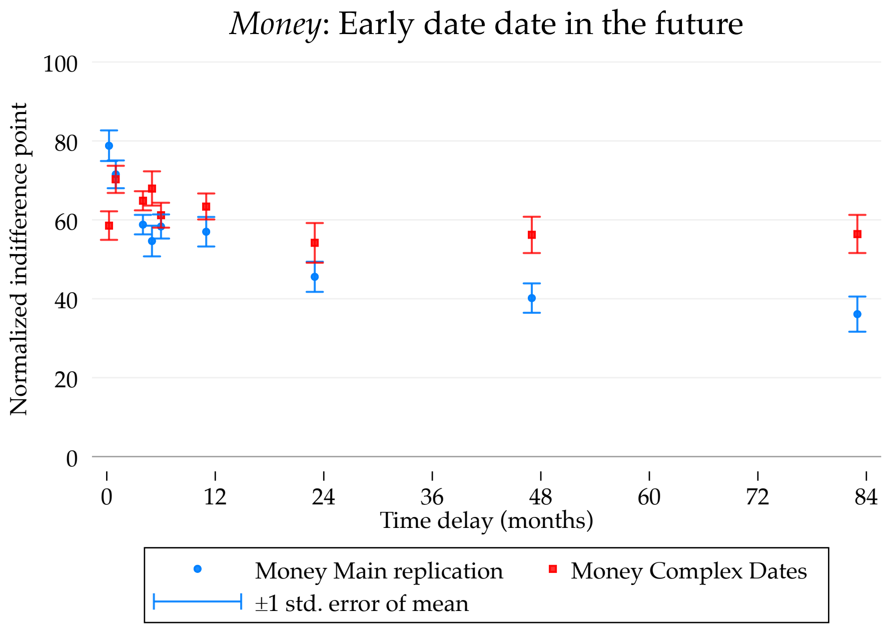


Figure 23: Observed discounting with $t_1 > 0$ in *Money Main replication* ($N = 161$) and *Money Complex Dates* ($N = 149$). Normalized indifference points are given by the midpoint of the switching interval in a choice list, divided by the larger-later payout amount (in %). The figure shows averages across decisions. Whiskers show standard error bars, computed based on clustering at the subject level.

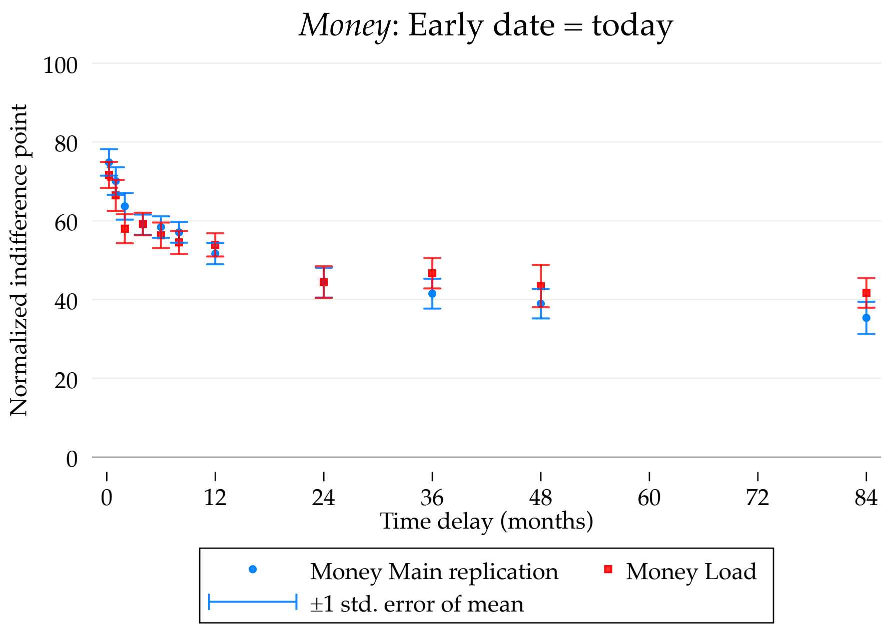


Figure 24: Observed discounting with $t_1 = 0$ in *Money Main replication* ($N = 161$) and *Money Complex Dates* ($N = 154$). Normalized indifference points are given by the midpoint of the switching interval in a choice list, divided by the larger-later payout amount (in %). The figure shows averages across decisions. Whiskers show standard error bars, computed based on clustering at the subject level.

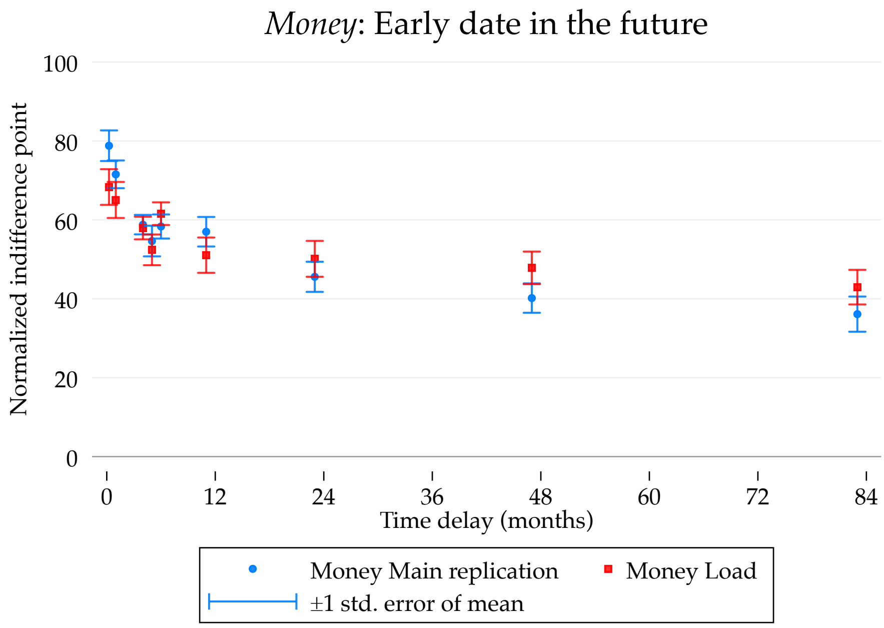


Figure 25: Observed discounting with $t_1 > 0$ in *Money Main replication* ($N = 161$) and *Money Complex Dates* ($N = 154$). Normalized indifference points are given by the midpoint of the switching interval in a choice list, divided by the larger-later payout amount (in %). The figure shows averages across decisions. Whiskers show standard error bars, computed based on clustering at the subject level.

F Results for Within-Subject Cognitive Load Experiment

In addition to the between-subject manipulation of cognitive load presented in Appendix E, we ran a within-subject version of this experiment. As in *Money Main*, subjects completed 12 choice lists involving hypothetical monetary payments. 6 of those rounds were NOLOAD rounds which were exactly identical to *Money Main*. In the other 6, randomly selected LOAD rounds, subjects faced a non-binding time limit of 15 seconds to complete the choice list and were instructed to sum up numbers that were flashed on the choice list screen in random intervals. There was no time limit and not number-counting task on the subsequent cognitive uncertainty elicitation screen. Following their cognitive uncertainty statement, subjects were prompted to enter the sum of numbers that we flashed on the choice list. Similar to treatment *Money Load*, our prediction was that in LOAD rounds subjects have relatively fewer cognitive resources available to fill out the choice list than in NOLOAD rounds. The manipulation should therefore lead to more pronounced insensitivity to time delays in LOAD rounds.

The predictions and sample size for this treatment were pre-registered at `https://aspredicted.org/av2y2.pdf`. We find that relative to NOLOAD rounds, the cognitive load manipulation increases average stated cognitive uncertainty by 9.2 percentage points (41%). Table 18 summarizes the effect of this treatment on intertemporal choices. They are in line with our predictions. In rounds with cognitive load, observed choices display substantially reduced sensitivity to the length of the time delay. These findings confirm our results from treatment *Money Load* at the subject level and suggest that the availability of cognitive resources is a source of cognitive imprecision that leads to more compressed intertemporal choice behavior.

Table 18: Cognitive load and insensitivity: Within-subject evidence

	Dependent variable: Normalized indifference point			
Sample:	$t1 = 0$		$t1 > 0$	
	(1)	(2)	(3)	(4)
Time delay (years)	-5.26***	-5.27***	-6.37***	-6.45***
	(0.43)	(0.43)	(0.50)	(0.50)
Time delay × 1 if cognitive load	2.33***	2.37***	3.81***	3.90***
	(0.63)	(0.63)	(0.70)	(0.71)
1 if cognitive load	-1.14	-1.15	-7.90***	-7.99***
	(1.03)	(1.03)	(1.33)	(1.33)
Payment amount FE	No	Yes	No	Yes
Demographic controls	No	Yes	No	Yes
Subject FE	Yes	Yes	Yes	Yes
Observations	2894	2894	1966	1966
R^2	0.70	0.70	0.68	0.68

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The sample includes decisions from the within-subject cognitive load experiment. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

G Experimental Instructions

G.1 Money Main

Part 1 of this study: Instructions (1/3)

Please read these instructions carefully. There will be comprehension checks. If you fail these checks, you will immediately be excluded from the study and you will not receive the completion payment.

In this part of the study, you will **choose between various hypothetical payments, which pay different amounts of money at different points in time**. An example decision is between the following two hypothetical payments:

In 30 days: $ 40	OR	**Today: $ 12**

For all hypothetical payments in this study, please treat them as if you knew that you would receive them with certainty, even if they are delayed. That is, please assume that there is no risk that you wouldn't actually get paid. Further assume that all payments were made by leaving a check in your mailbox.

Throughout the experiment, there are no right or wrong answers, because how much you like an option depends on your personal taste. There will be two types of decision screens.

Decision screen 1

On decision screen 1, you will be asked to choose which of two payment options you prefer. You will see choice lists such as the one below, where each row is a separate choice. In every list, the left-hand side option (Option A) is a delayed payment that is identical in all rows. The right-hand side option (Option B) is a payment *with an earlier payment date than Option A*. The earlier, right-hand side payment increases as you go down the list. **An effective way to complete these choice lists is to determine in which row you would like to switch from preferring Option A to preferring Option B.**

Based on where you switch from Option A to Option B in this list, we assess which amount at the early payment date (Option B) you value as much as the amount specified at the later payment date (Option A). For example, in the choice list below, you would value $40 in 30 days somewhere between $12 and $14 today, because this is where switching occurs.

Option A		Option B
	◉ ○	**Today:** $2
	◉ ○	**Today:** $4
	◉ ○	**Today:** $6
	◉ ○	**Today:** $8
	◉ ○	**Today:** $10
	◉ ○	**Today:** $12
	○ ◉	**Today:** $14
	○ ◉	**Today:** $16
	○ ◉	**Today:** $18
In 30 days: $40	○ ◉	**Today:** $20
	○ ◉	**Today:** $22
	○ ◉	**Today:** $24
	○ ◉	**Today:** $26
	○ ◉	**Today:** $28
	○ ◉	**Today:** $30
	○ ◉	**Today:** $32
	○ ◉	**Today:** $34
	○ ◉	**Today:** $36
	○ ◉	**Today:** $38
	○ ◉	**Today:** $40

On the next page, you will see an example choice list, and you can practice making your selections.

Click "Next" to proceed to the example page.

Part 1 of this study: Instructions (2/3)

Auto-completion: The table auto-completes your choices so you don't have to click through all of the rows. You do not have to start at the top of the table. If you select Option A in any one row, we assume that you will also prefer Option A in all rows *above* that row. If you select Option B in any one row, we assume that you will also prefer Option B in all rows *below* that row.

Option A			Option B
In 30 days: $40	○	○	Today: $2
	○	○	Today: $4
	○	○	Today: $6
	○	○	Today: $8
	○	○	Today: $10
	○	○	Today: $12
	○	○	Today: $14
	○	○	Today: $16
	○	○	Today: $18
	○	○	Today: $20
	○	○	Today: $22
	○	○	Today: $24
	○	○	Today: $26
	○	○	Today: $28
	○	○	Today: $30
	○	○	Today: $32
	○	○	Today: $34
	○	○	Today: $36
	○	○	Today: $38
	○	○	Today: $40

Part 1 of this study: Instructions (3/3)

Decision screen 2

When you fill out a choice list, you may feel **uncertain about whether you prefer the left or right payment option**. On decision screen 2, we will ask you to select a button to indicate **how certain** you are how much money the larger later payment is worth to you in terms of dollars at the earlier payment date.

In answering this question, we ask you to assume that you would receive both payment options with certainty. We are interested in **your uncertainty about your own preferences regarding these payments**, not in your potential uncertainty about whether you would actually receive the money.

Example

Suppose that on the first decision screen you indicated that you valued $40 in 30 days somewhere between $12 and $14 today. Your second decision screen would look like this.

How certain are you that you actually value $40 **in 30 days** somewhere between $12 and $14 **today**?

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Comprehension questions

The questions below test your understanding of the instructions.

Important: If you fail to answer any one of these questions correctly, you will not be allowed to participate in the study.

1. Which of the following statements is true?

   - ○ In making my decisions, I am asked to assume that I will actually receive all payments as indicated, regardless of whether they take place now or in the future.
   - ○ In making my decisions, I am asked to assume that it is less likely that I will actually receive payments that are meant to take place in the future.
   - ○ In making my decisions, I am asked to assume that it is less likely that I will actually receive payments that are meant to take place now.

2. Suppose you are 80% certain that your decisions actually correspond to how much the different choice options are worth to you. Which button should you click in this case?

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

   very uncertain **completely certain**

3. When we ask you how certain you are about how much different payments are worth to you at different points in time, then which type of uncertainty are we interested in?

   - ○ Uncertainty about whether I would actually receive the payments.
   - ○ Uncertainty about how much I value the payments, assuming that I know I would receive them with certainty.

G.2 Voucher Main

Part 1 of this study: Instructions (1/4)

Please read these instructions carefully. There will be comprehension checks. If you fail these checks, we will have to exclude you from the study and you will not receive the completion payment.

In this part of the study, you will **choose between different UberEats food delivery vouchers. These vouchers will vary along two dimensions**:

- **The vouchers will have different values**
- **The vouchers will be valid at different points in time**

How do the vouchers work?

Each voucher is valid for food delivery during a period of only seven days. A voucher can be used starting **from the indicated date**, and **it remains valid for exactly 7 days after** that date. Specifically, the vouchers work as follows:

- If you win a voucher, you will be informed about the voucher amount and the validity period on the last page of this study. You will then be asked to provide an email address associated with an UberEats account. The voucher will directly be credited to the corresponding UberEats account within the next 10 hours.
- However, the voucher amount **can only be spent during the validity period** of the voucher.
- Vouchers can be used to order from the entire range of restaurants, cafes, and bars that partner with UberEats in your area.
- You do not need to worry about forgetting the validity period: **UberEats will automatically send reminders** about your voucher 24 hours before the validity period starts and 24 hours before it ends.

What decisions will you be asked to make?

An example decision is between the following two vouchers:

Valid in 30 days: $40 Voucher OR **Valid today: $20 Voucher**

The left-hand side voucher carries an amount of $40 and can be spent in the 7-day period starting in 30 days from now. The right-hand side voucher is for an amount of only $20, but can be spent in the 7-day period starting immediately.

Throughout the experiment, there are no right or wrong answers because how much you like a voucher depends on your personal taste.

Part 1 of this study: Instructions (2/4)

Decision screen 1

On decision screen 1, you will be asked to choose which of two vouchers you prefer. You will see choice lists such as the one below, where each row is a separate choice. In every list, the left-hand side option (Voucher A) is a voucher that is identical in all rows. The right-hand side option (Voucher B) is a voucher *with an earlier validity period than Voucher A*. The amount associated with the earlier, right-hand side voucher increases as you go down the list. **An effective way to complete these choice lists is to determine in which row you would like to switch from preferring Voucher A to preferring Voucher B.**

Based on where you switch from Voucher A to Voucher B in this list, we assess which voucher amount in the early validity period (Voucher B) you value as much as the voucher amount specified in the later validity period (Voucher A). For example, in the choice list below, you would value a $40 voucher that is valid in 30 days somewhere between a $12 and a $14 voucher that is valid today, because this is where switching occurs.

Voucher A			Voucher B
	●	○	Valid Today: $2 Voucher
	●	○	Valid Today: $4 Voucher
	●	○	Valid Today: $6 Voucher
	●	○	Valid Today: $8 Voucher
	●	○	Valid Today: $10 Voucher
	●	○	Valid Today: $12 Voucher
	○	●	Valid Today: $14 Voucher
	○	●	Valid Today: $16 Voucher
	○	●	Valid Today: $18 Voucher
Valid In 30 days: $40 Voucher	○	●	Valid Today: $20 Voucher
	○	●	Valid Today: $22 Voucher
	○	●	Valid Today: $24 Voucher
	○	●	Valid Today: $26 Voucher
	○	●	Valid Today: $28 Voucher
	○	●	Valid Today: $30 Voucher
	○	●	Valid Today: $32 Voucher
	○	●	Valid Today: $34 Voucher
	○	●	Valid Today: $36 Voucher
	○	●	Valid Today: $38 Voucher
	○	●	Valid Today: $40 Voucher

If you are selected to receive an additional reward from part 1 of the study, your reward will be determined as follows:
Your choice in a randomly selected row of a randomly selected choice list determines the amount of your personal voucher. Each choice list and each row are equally likely to get selected.

Important:

- Your choices may matter for real money! If you are selected to receive a bonus, one of your choices will actually be implemented, and your decision will determine which type of voucher you receive.
- Since only one of your decisions will be randomly selected to count, you should consider each choice list independently of the others. There is no point in strategizing across decisions.

On the next page, you will see an example choice list, and you can practice making your selections.

Click "Next" to proceed to the example page.

Part 1 of this study: Instructions (3/4)

Auto-completion: The table auto-completes your choices so you don't have to click through all of the rows. You do not have to start at the top of the table. If you select Voucher A in any one row, we assume that you will also prefer Voucher A in all *above* that row. If you select Voucher B in any one row, we assume that you will also prefer Voucher B in all rows *below* that row.

Reminder: both vouchers are valid for 7 days starting on the day indicated for each voucher.

Voucher A		Voucher B
	○ ○	**Valid today: $2 Voucher**
	○ ○	**Valid today: $4 Voucher**
	○ ○	**Valid today: $6 Voucher**
	○ ○	**Valid today: $8 Voucher**
	○ ○	**Valid today: $10 Voucher**
	○ ○	**Valid today: $12 Voucher**
	○ ○	**Valid today: $14 Voucher**
	○ ○	**Valid today: $16 Voucher**
	○ ○	**Valid today: $18 Voucher**
Valid in 30 days: $40 Voucher	○ ○	**Valid today: $20 Voucher**
	○ ○	**Valid today: $22 Voucher**
	○ ○	**Valid today: $24 Voucher**
	○ ○	**Valid today: $26 Voucher**
	○ ○	**Valid today: $28 Voucher**
	○ ○	**Valid today: $30 Voucher**
	○ ○	**Valid today: $32 Voucher**
	○ ○	**Valid today: $34 Voucher**
	○ ○	**Valid today: $36 Voucher**
	○ ○	**Valid today: $38 Voucher**
	○ ○	**Valid today: $40 Voucher**

Part 1 of this study: Instructions (4/4)

Decision screen 2

When you fill out a choice list, you may feel **uncertain about whether you prefer the left or right voucher**. On decision screen 2, we will ask you to select a button to indicate **how certain** you are about how much the larger voucher amount with the later validity period is worth to you in terms of voucher credit that can be spent in the earlier validity period.

Example

Suppose that on the first decision screen you indicated that you value a $40 voucher that is valid in 30 days somewhere between a $12 and a $14 voucher that is valid today. Your second decision screen would look like this.

How certain are you that you actually value a $40 voucher that is valid **in 30 days** somewhere between a $12 and a $14 voucher that is valid **today**?

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Comprehension questions

The questions below test your understanding of the instructions.

Important: If you fail to answer any one of these questions correctly, you will not be allowed to participate in the study, and you will not receive the completion payment.

1. Which of the following statements about the voucher below is true?

Valid in 1 month: $30 Voucher

- ○ This voucher can be used to order food starting from today until no later than 1 month.
- ○ This voucher can be used to order food any time after 1 month. The validity period has no end date.
- ○ This voucher can be used to order food in the 7-day period starting in 1 month.

2. Suppose you are 80% certain that your decisions actually correspond to how much the different voucher options are worth to you.

Which button should you click in this case?

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

3. Which of the following statements is true?

- ○ Even if the validity period starts in the future, my voucher will be credited to my UberEats account shortly after the experiment. I do not have to remember the validity period because UberEats will send me reminders.
- ○ If the validity period of the voucher starts in the future, I should expect to get my voucher credited to my UberEats account only shortly before the validity period starts. I have to memorize the validity period, otherwise I may forget to use the voucher amount. There is also some risk that I will not actually receive the voucher.