Cognitive Uncertainty*

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Abstract

This paper documents the economic relevance of measuring *cognitive uncertainty*: people's subjective uncertainty over their ex-ante utility-maximizing decision. In a series of experiments on choice under risk, the formation of beliefs and forecasts of economic variables, we show that cognitive uncertainty predicts systematic biases in economic decisions and that it can be deployed to provide tests of formal theories. When people are cognitively uncertain – either endogenously or because the problem is designed to be complex – their decisions are heavily attenuated functions of objective probabilities, which gives rise to behavior that is regressive to an intermediate option. This insight ties together a wide range of empirical regularities in behavioral economics that are typically viewed as distinct phenomena or even as reflecting preferences, including the probability weighting function in choice under risk; base rate insensitivity, conservatism and sample size effects in belief updating; and predictable overoptimism and -pessimism in forecasts of economic variables. Our results offer a blueprint for how a simple measurement of cognitive uncertainty generates novel insights about what people find complex and how they respond to it.

Keywords: Cognitive uncertainty, complexity, cognitive noise, beliefs, expectations, choice under risk

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1 Introduction

This paper studies the economic relevance of *cognitive uncertainty*: people's subjective uncertainty over which decision maximizes their expected utility. In the standard economic model, people take decisions that they know may turn out to be ex-post suboptimal, but they never exhibit doubts about their ex-ante optimality. Similarly, in a large majority of behavioral economics models, people may make systematic mistakes, but they are not nervous that they may be committing errors. Yet, casual introspection suggests that people not only struggle with figuring out what to do – instead, they also have some meta-cognitive awareness thereof. For instance, people may have a nagging feeling that they do not really know how much they value a risky asset with a known payoff profile, or they may worry about their ability to rationally process new information. Indeed, a growing body of evidence discussed below – mostly in cognitive psychology and neuroscience, and some in economics – documents that people often express low confidence in their own decisions. Yet, it is not immediately obvious why the insight that people have some meta-cognitive awareness of their own decision errors should be relevant to the interests of economists in formally modeling and predicting behavior.

This paper proposes that measuring cognitive uncertainty is not just a psychological curiosity, but that it can be productively deployed to predict systematic biases in economic behaviors, to help tie together widely-studied empirical regularities in behavioral economics that are typically viewed as distinct phenomena, and to test formal theories. In line with a recent theory literature, the main idea is that when people are cognitively uncertain – for example because a decision problem is very complex – their decisions are severely attenuated functions of objective problem parameters, which gives rise to behavior that is regressive to an intermediate option.

To document this point, we present experiments on decision-making under uncertainty: the ways people reason about probabilities in the valuation of risky lotteries, inference from data, and prediction of future events. As Figure 1 illustrates, these three literatures have established striking similarities about how objective probabilities map into people's decisions. First, the left-hand panel depicts the well-known probability weighting function in choice under risk that goes back to Tversky and Kahneman (1992). It illustrates how experimental subjects implicitly treat objective probabilities in choosing between different monetary gambles. Second, the middle panel illustrates the canonical compressed relationship between participants' posterior beliefs and the Bayesian posterior in experimental belief updating tasks, which shows that people generally overestimate the probability of unlikely events

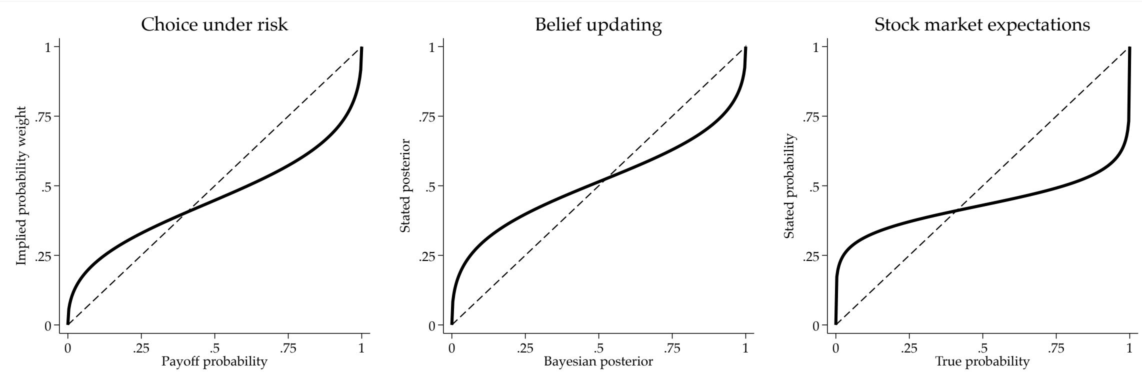

Figure 1: Illustrations of hypothetical decisions as functions of objective probabilities. The left-hand panel illustrates a probability weighting function in choices between monetary gambles. The middle panel illustrates the relationship between stated beliefs and Bayesian posteriors in binary-state belief updating experiments. The right-hand panel illustrates the typical relationship between stated subjective probabilities and objective (historical) probabilities in surveys about stock returns or inflation.

and underestimate the probability of likely ones. Finally, the right-hand panel shows the compressed relationship between respondents' probabilistic estimates and true probabilities that has been documented in a wide range of subjective expectations surveys about, for example, stock market returns, inflation rates or the national income distribution. The characteristic feature of these three functions is that people's decisions implicitly treat different probabilities to some degree alike, which generates a compression effect to an "intermediate" value.

One view in the literature – reflected in the existence of a large number of models of probability weighting and belief updating – is that these phenomena reflect domain-specific heuristics or even preferences. On the other hand, various theoretical contributions discussed below have hypothesized that the patterns summarized in Figure 1 are driven by cognitive noise or other forms of stochasticity that may reflect the inherent complexity of forming beliefs and choosing between monetary gambles. For example, in recent Bayesian cognitive noise models of choice under risk (Khaw et al., 2021; Woodford, 2020), the difficulty of translating objective probabilities into decisions introduces cognitive noise, which induces the decision-maker to partially regress to (or anchor on) an intermediate cognitive default, thus producing probability weighting. In a nutshell, one interpretation of this class of models is that noise generates bias through a mechanism akin to the classical anchoring-and-adjustment heuristic. At the same time, there is little evidence that directly ties together and explains behavior across the three decision domains in Figure 1 as a function of noisy cognition, likely in part due to the difficulty of measuring cognitive noise.

To make progress on these questions, and to document the explanatory power of cognitive uncertainty, we conduct a series of online experiments with a total of more than 3,000 participants. In these experiments and surveys, we elicit entirely standard controlled decisions in each of the three decision domains discussed above. In addition to eliciting payoff-relevant choices and beliefs, we also measure cognitive uncertainty. For example, in lottery valuation tasks, after we elicit a participant's certainty equivalent for a lottery, we ask them how certain they are (in percent) that their true valuation of the lottery actually lies within a one-dollar window around their stated valuation. Similarly, after participants state financially incentivized probabilistic beliefs in canonical belief updating experiments, we ask them how certain they are that the Bayesian posterior is contained in a two percentage point window around their stated belief. These questions elicit people's subjective percent chance that their decision is actually (close to) the ex-ante utility-maximizing one.

This cognitive uncertainty elicitation has five main features. (i) Unlike previous elicitations of decision confidence in cognitive psychology and economics, our measure is quantitative in nature. Indeed, using a stylized model, we formally define our cognitive uncertainty measurement as awareness of cognitive noise. (ii) As documented by our three applications, the elicitation can be tweaked in minor ways to be applicable to a broad set of decision domains with very different experimental paradigms and elicitation protocols. (iii) The question captures people's composite awareness of their own cognitive imperfections, including imperfect perception, preference uncertainty, problems in integrating utils and probabilities, lack of knowledge of Bayes' rule, computational difficulties or memory imperfections. As a result, a productive interpretation of cognitive uncertainty is that it captures people's subjective difficulty or perceived complexity of a problem. (iv) The measure is very simple, quick and costless to elicit, making it easy for researchers to add such a question to their own studies. (v) While we deliberately do not financially incentivize cognitive uncertainty to maintain the simplicity of the elicitation protocol, we provide a direct validation of our measure. In both choice under risk and belief updating, we show that cognitive uncertainty is strongly correlated with the magnitude of decision variability in repetitions of the same decision problem, which is a key choice signature of cognitive noise.

We find large variation in cognitive uncertainty across all of our decision domains. In choice under risk, more than 80% of all certainty equivalents are associated with strictly positive cognitive uncertainty. In our belief updating experiments, less than 10% of all posterior beliefs are associated with full certainty that the stated belief is close to the Bayesian posterior. Participants appear relatively consistent in their degree of cognitive uncertainty, both across decisions within the same domain and across different decision domains, hence suggesting the existence of different cognitive uncertainty (or cognitive noise) types.

Measured cognitive uncertainty strongly predicts observed choices and beliefs in a way that rationalizes the empirical regularities summarized in Figure 1. Across all three decision domains, high cognitive uncertainty decisions are substantially more compressed and less responsive to variation in objective probabilities. For example, in choice under risk, high cognitive uncertainty decisions reflect a substantially shallower slope of the probability weighting function, which implies that cognitive uncertainty is strongly correlated with the well-known fourfold pattern of risk attitudes. Indeed, for decisions with cognitive uncertainty of zero, the median decision exhibits essentially no probability weighting.

In the domains of beliefs and expectations, we likewise see that high cognitive uncertainty beliefs are substantially more compressed towards 50:50. This means that cognitively uncertain people will sometimes appear more optimistic and sometimes more pessimistic than is warranted, purely depending on whether the true probability is high or low. We discuss implications of this for interpreting heterogeneity in economic expectations surveys.

Moving beyond simple notions of overoptimism and -pessimism, we also conduct more structural analyses that shed light on some of the core belief updating biases that the literature has documented. Here, we establish that cognitive uncertainty is strongly predictive of both base rate insensitivity and conservatism. The simple intuition is that beliefs that are compressed towards 50:50 are necessarily conservative and insensitive to the base rate.

Cognitive uncertainty not only predicts well-known compression effects in beliefs and choices but also the deviation from objective performance benchmarks. For example, in belief updating, cognitive uncertainty is significantly correlated with the absolute distance of beliefs to the Bayesian benchmark. These results again suggest that people's awareness of their cognitive imperfections is partly correct.

We are agnostic over whether the strong correlations between cognitive uncertainty and behaviors reflect a causal effect of the true cognitive noise that underlies cognitive uncertainty or whether awareness of potential errors itself drives behaviors. Under either interpretation, our hypothesis is that the link between cognitive uncertainty and decisions partly reflects the complexity of identifying the utility-maximizing decision. To directly investigate this complexity interpretation, we implement different treatments that vary the complexity of the lottery valuation and belief updating tasks. In one set of experiments, we vary the computational complexity of the decision problems by displaying the relevant problem parameters (such as payout probabilities or base rates) as algebraic expressions. In other experiments, we increase problem complexity by turning lotteries or belief updating tasks into compound (multi-stage probabilistic) problems.

We always find that higher complexity leads to higher cognitive uncertainty, which lends credence to our interpretation that cognitive uncertainty partly reflects the subjective complexity of decision problems. Moreover, the compression effects summarized in Figure 1 become substantially more pronounced in the more complex treatments. For instance, contrary to the predictions of (cumulative) prospect theory, the probability weighting function exhibits substantially stronger likelihood insensitivity when the decision problems are more complex. Similarly, in contrast to models of base rate neglect or conservatism that rest on assumptions of fixed parametric biases, the magnitude of base rate insensitivity and conservatism strongly depends on the complexity of the decision problem.

In theoretical models of cognitive noise, compression of decisions towards an intermediate value reflects a cognitive default (or prior). While we remain agnostic about the particular location of such a default, we make some progress on this question by estimating the cognitive default decision that participants endogenously bring to the experimental tasks. To this effect, we estimate a simple model of noisy cognition. In this model, observed decisions reflect a convex combination of a noisy cognitive signal about the utility-maximizing decision and a fixed cognitive default decision, where the relative weight on the cognitive default increases in cognitive uncertainty. Across our experiments, we estimate cognitive default decisions of 40–50%. For instance, in choice under risk, we interpret this result as saying that when people approach lottery valuation tasks, their initial reaction – prior to any deliberation of the specific problem – is that they value a binary lottery at around 40% of the lottery upside. This estimated intermediate cognitive default decision is reminiscent of a large body of evidence on so-called "central tendency" or "compromise" effects in cognitive psychology and economics (e.g. Petzschner et al., 2015; Xiang et al., 2021; Beauchamp et al., 2019). In ancillary experiments, we exogenously manipulate this intermediate cognitive default through so-called partition manipulations. Consistent with a simple model of noisy cognition, we find that such partition manipulations uniformly shift participants' certainty equivalents and posterior beliefs downwards, in particular among those participants with high cognitive uncertainty.

To sum up, this paper documents that cognitive uncertainty helps to predict and tie together various judgment and decision errors that are traditionally viewed as distinct, and that it can be effectively used to test the predictions of models that are difficult to test otherwise. We believe these results provide a rationale for measuring cognitive uncertainty more broadly in experiments and surveys, especially given that it is fast and costless to do.

Our approach directly connects to a recent literature on Bayesian models of cognitive noise, which we build upon in formalizing cognitive uncertainty (Woodford, 2019; Gabaix, 2019; Gabaix and Laibson, 2017; Frydman and Jin, 2021). Most closely related is Khaw et al. (2021), who show theoretically how probability weighting (and other anomalies) can result from cognitive noise.[1] A primary contribution of our work to this literature is to make the magnitude of cognitive noise visible, which makes theoretical models testable.

Related is a diverse set of largely theoretical contributions that have linked probability weighting and over- / underestimation of probabilities to different versions of noise (e.g., Bhatia, 2014; Marchiori et al., 2015; Viscusi, 1985, 1989; Blavatskyy, 2007; Zhang et al., 2020; Erev et al., 1994). Yet, leading recent reviews rarely even mention a potential role of (cognitive) noise for these empirical regularities and instead emphasize models with fixed "probability weighting", "conservatism" or "extreme belief aversion" parameters that are partly even meant to capture preferences (e.g., Wakker, 2010; Fehr-Duda and Epper, 2012; Benjamin, 2019). O'Donoghue and Somerville (2018) note that "the psychology of probability weighting is poorly understood." This view in the literature may reflect that few contributions directly measure cognitive noise or attempt to explain behaviors across different decision domains – both of which we contribute here.

Finally, our work relates to a growing interdisciplinary literature that documents that people often have an awareness of the noisiness of their choices, memories and perceptions, and that they take decisions that are in line with such awareness (e.g., Honig et al., 2020). In psychology and sometimes economics, people's uncertainty about their own decisions is typically measured using Likert scale questions (e.g., Butler and Loomes, 2007; De Martino et al., 2013, 2017; Cubitt et al., 2015; Polania et al., 2019; Xiang et al., 2021; Drerup et al., 2017). Our contribution to this literature is (i) to propose and experimentally validate a more quantitative measure of cognitive uncertainty and (ii) to document that cognitive uncertainty predicts biases across various economic decision tasks.

2 Theoretical Considerations and Hypotheses

Various contributions have hypothesized that the patterns summarized in Figure 1 are driven by cognitive noise or other forms of noise. Most closely aligned with our interpretation and empirical measurement is the Bayesian noisy cognition model of Khaw et al. (2021). They model a decision-maker who exhibits cognitive noise when processing probabilities, which makes him regress towards an intermediate prior, hence producing probability weighting (also see Gabaix, 2019). Earlier related theoretical work modeled probability weighting as resulting from Bayesian updating from imperfect information about objec-

[1] Khaw et al. (2021) and Frydman and Jin (2021) also report experiments on cognitive noise and risk taking, but these do not test predictions related to probability weighting.

tive payout probabilities (Viscusi, 1989; Fennell and Baddeley, 2012), decision or sampling noise (Blavatskyy, 2007; Bhatia, 2014), affective vs. deliberate decision making (Mukherjee, 2010), or random fluctuations in risk preferences (Bhatia and Loomes, 2017). Similarly, multiple contributions have argued that regression of beliefs towards 50:50 may reflect noise or ignorance (Viscusi, 1985; Erev et al., 1994; Marchiori et al., 2015; Moore and Healy, 2008; Fischhoff and Bruine De Bruin, 1999).

Our analysis builds on these contributions. Because each of these models applies to either risky choice or belief updating, we here present a stylized adaptation that illustrates how we think about the commonalities reflected in Figure 1. Our exposition closely builds on the recent Bayesian cognitive noise literature (e.g., Khaw et al., 2021; Woodford, 2020; Frydman and Jin, 2021; Heng et al., 2020), though Bayesian models of objective uncertainty about risks are mathematically similar (e.g., Viscusi, 1985; Fennell and Baddeley, 2012).

Overview. We consider situations in which a decision-maker (DM) with Bernoulli utility function $u(\cdot)$ is tasked with making a decision a that depends on some objective probability p. We denote by $a^*(p) \in \underset{a}{argmax}\, EU(\cdot)$ the DM's true expected-utility maximizing decision. In the spirit of the recent cognitive noise literature, we assume that, through deliberation, the DM only has access to a noisy mental simulation of $a^*(p)$. The noisiness of this mental simulation may depend on the complexity of the decision problem. The DM combines this simulation outcome with a cognitive default decision that we think of as the DM's initial reaction to the problem, prior to any deliberation.

Risky choice. The DM is asked to indicate his certainty equivalent for a lottery that pays \$1 with probability p and nothing otherwise. By standard arguments, normalizing $u(1) = 1$, the expected-utility maximizing decision is given by $a^* = u^{-1}(p)$.

Belief formation. The DM forms beliefs about a binary state of the world, R or B. The DM has prior $b = P(R)$ and receives a binary signal (H or L) with diagnosticity $h = P(H|R) = P(L|B)$. The Bayesian posterior belief is given by $p \equiv P(R|H) = P(B|L) = \frac{bh}{bh+(1-b)(1-h)}$. A widely-used formulation that we also leverage is a so-called Grether (1980) decomposition, which generates a linear relationship between the Bayesian posterior odds, the prior odds, and the likelihood ratio: $ln(p/(1-p)) = ln(b/(1-b)) + ln(h/(1-h))$. We assume the incentive structure is such that it is optimal for the DM to report his true beliefs, such that the utility-maximizing decision is given by $a^* = p$.

Cognitive noise and decisions. We assume that the DM does not have access to the utility-maximizing decision $a^*(p)$.[2] For instance, in risky choice, the DM may not know his true utility function, may find it cognitively hard to integrate payoff probabilities and utils, or have noisy perception. In laboratory belief updating tasks, the DM may not know Bayes' rule or struggle with implementing it computationally. In economic expectations surveys, the DM may not remember his true belief about how the stock market will develop, or he may struggle with processing the financial information available to him.

Whatever the underlying micro-foundations, as we lay out formally in Appendix A, we assume that the DM has access to a cognitive signal S that is (scaled) Binomially distributed with precision N and satisfies $E[S] = a^*(p)$. This cognitive signal could be interpreted as the outcome of a sequential cognitive sampling or deliberation process as in drift-diffusion models. Higher cognitive noise corresponds to a less precise Binomial signal. Relatedly, we can think of the level of cognitive noise – and, hence, the precision of the Binomial signal – as being determined by the complexity of the decision problem. Indeed, we will provide evidence below that higher complexity induces more cognitive noise.

The DM holds a Beta-distributed prior over $a^*(p)$, which has mean d. We refer to d as the "cognitive default decision." For tractability, we assume that the DM's decision is given by the posterior mean over his utility-maximizing decision.[3] Given signal realization $S = s$, a Bayesian DM's decision, a^o, can then be represented as a convex combination of the cognitive signal and the prior mean, see Appendix A:

$$a^o = \lambda(N) \cdot s(a^*(p)) + [1 - \lambda(N)] \cdot d \tag{1}$$

$$E[a^o] = \lambda(N) \cdot a^*(p) + [1 - \lambda(N)] \cdot d \tag{2}$$

Here, the relative weight placed on the cognitive signal, $\lambda(N)$, increases in the signal's precision.

Interpretation of decision rule. Intuitively, eq. (2) is compatible with an anchoring-and-adjustment heuristic (Tversky and Kahneman, 1974), according to which people anchor

[2]Our model features cognitive imprecision at the level of the utility-maximizing decision, rather than of a problem input parameter. We here focus on cognitive noise in decision space to highlight what we believe to be commonalities across decision domains. However, what we model as cognitive noise in the calculation of the utility-maximizing decision could clearly result from cognitive noise in the processing of input parameters. The two formulations are equivalent when the utility-maximizing decision is linear in the problem parameter.

[3]Focusing on the mean is without much loss in the present context because the mean of a Beta(a,b) variable is $a/(a+b)$, the mode is $(a-1)/(a+b-2)$ and the median lies between the two. See footnote 17 in Appendix A for further discussion.

on some initial reaction, d, and then adjust in the direction of the true utility-maximizing decision upon deliberation. Relating this formulation to our applications, the average stated certainty equivalent for a given payout probability is given by a convex combination of the true certainty equivalent and the default. An analogous logic holds for average stated beliefs, which are a convex combination of the rational belief and the default.

The linear equation (2) corresponds to the widely-studied "neo-additive weighting function" that has attracted attention in the literature on choice under risk. To illustrate, if one were to assume linear utility, eq. (2) would simplify to $E[a^o] = \delta + \lambda \cdot p$, where δ does not depend on p but increases in cognitive noise. As discussed in Wakker (2010), this function is appealing due its simplicity and because it can be estimated through simple linear regressions. Our stylized framework motivates this functional form by endogenizing its parameters as a function of cognitive noise. In particular, our framework generates that in this neo-additive regression equation, (i) the intercept increases in cognitive noise and (ii) the slope decreases in cognitive noise. Recall that a higher intercept and a shallower slope are the two characteristic features of the compressed decisions summarized in Figure 1.

Further note that this decision rule implies a "flipping" property: for $a^*(p) < d$, higher cognitive noise is associated with higher decisions, while for $a^*(p) > d$ it is associated with lower decisions. For instance, in lottery valuation tasks, relative to a noiseless DM, a cognitively noisy DM is less risk averse for relatively low payout probabilities yet more risk averse for high payout probabilities.

Finally, observe that eq. (2) implies an attenuated but *linear* mapping between objective probabilities and decisions (when utility is linear). As summarized in Figure 1, this is counterfactual because decisions are often inverted S-shaped functions of objective probabilities. We explain how cognitive noise and cognitive uncertainty shed light on this phenomenon in Section 9 and Appendix D.

Interpretation of cognitive default decision. We interpret the cognitive default as the DM's initial reaction: the decision the DM would take in the absence of any deliberation.[4] We do not provide a theory of what determines the cognitive default decision. For our purposes, all that matters is that the default is sufficiently "intermediate" in nature, on average: for low enough p, $a^*(p) < d$, and for large enough p, $a^*(p) > d$. Indeed, a large literature argues that people's heuristic (or initial) responses to decision problems are intermediate, such

[4]While we here highlight an interpretation of d reflecting a fixed cognitive default, an alternative interpretation of eq. (2) involves random choice. Specifically, the expected action in eq. (2) also follows from a model in which the DM chooses his utility-maximizing decision with probability λ and plays randomly with probability $(1-\lambda)$, where the mean random choice is given by d.

as in research on central tendency effects in judgment and perception (e.g., Hollingworth, 1910; Petzschner et al., 2015; Xiang et al., 2021), compromise effects in choice (Simonson and Tversky, 1992; Beauchamp et al., 2019), and research that interprets 50:50 responses in economic expectations surveys as a manifestation of "I don't know" (Fischhoff and Bruine De Bruin, 1999).[5]

Predictions. Formal statements of predictions and proofs are relegated to Appendix A.

1. *Cognitive noise and compression effects.*
   (a) *In risky choice, cognitive noise is correlated with probability weighting:* $\exists p^*$ *such that, for* $p < p^*$*, certainty equivalents increase in cognitive noise and for* $p > p^*$ *they decrease in cognitive noise.*
   (b) *In stated beliefs, cognitive noise is correlated with overestimation of small and underestimation of large probabilities. Moreover, in Grether decompositions, cognitive noise is correlated with base rate insensitivity and conservatism.*
2. *The distance between the DM's decision and the utility-maximizing decision increases in cognitive noise.*
3. *Cognitive default effects.*
   (a) *An exogenous decrease in the cognitive default decision decreases stated certainty equivalents (shifts the probability weighting function down).*
   (b) *An exogenous decrease in the cognitive default decision decreases stated beliefs.*
   (c) *Both of these effects are stronger among DM with higher cognitive noise.*

Empirical implementation: Cognitive uncertainty. People's actual level of cognitive noise is conventionally unobservable. To render the predictions testable, we make use of the idea that awareness of cognitive noise generates subjectively perceived uncertainty about what the utility-maximizing decision is. This *cognitive uncertainty* is measurable. We define it as

$$p_{CU} \equiv P(|A_{S=s} - a^o| > \kappa). \tag{3}$$

[5]Similarly, in their study of inverse S-shaped probability and frequency estimates across a variety of domains, Zhang and Maloney (2012) report that a 1/N formulation (N being the number of distinct states of the world) appears to capture people's cognitive anchor well.

Here, $A_{S=s}$ denotes the perceived posterior distribution about the maximizing decision, conditional on having received cognitive signal s. Intuitively, cognitive uncertainty captures the likelihood with which the DM thinks his utility-maximizing decision falls outside a window of arbitrary length κ around the decision that he actually chose.

As we show in Appendix A, cognitive uncertainty decreases in the precision of the Binomial cognitive signal. This allows us to use cognitive uncertainty as a proxy for the magnitude of cognitive noise and, hence, λ. Our argument is not that awareness of cognitive noise necessarily causes the economic behavior of interest (though it may), but that it allows for the measurement of a concept that is difficult to quantify otherwise.

3 Experimental Design

3.1 Overview

As summarized in Table 1, we implemented two series of experiments. The main set of experiments reported here was run in early 2022. Earlier experiments, which we also summarize below, were run in 2019. With a slight abuse of language, we refer to these earlier experiments as "conceptual self-replication," because – subject to some important differences highlighted below – they are very similar to the main experiments. We summarize both sets of experiments here but relegate a detailed exposition of the replication experiments to Appendix E. The default manipulation experiments are described in Section 7.2.

3.2 Decision tasks

Choice under risk. To estimate a probability weighting function, treatment *Risk main* elicited certainty equivalents for binary lotteries that paid $\$y \in \{15, 16, \ldots, 25\}$ with probability $p \in \{1, 5, 10, 25, 35, 50, 65, 75, 90, 95, 99\}$ percent, and nothing otherwise. Certainty equivalents were elicited using the BDM technique proposed by Healy (2018). Participants were instructed that for each lottery there is a list of questions that ask whether the participant prefers the lottery or a safe payment, where the safe payment increases as one goes down the list. Following Healy (2018), instead of asking participants to make a decision in every row of the list, we instructed them that they would tell us the safe amount at which they would switch from preferring the lottery to preferring the safe payment, and that we would then fill out the entire choice list based on their decision. Thus, participants simply entered a dollar amount into a text box to indicate their certainty equivalent, where entries

Table 1: Overview of experiments

Experiment	Components	# Particip.	Pool
Risk main	Baseline risky choice tasks (gains) Complex numbers manipulation	500	Prolific
Beliefs main	Baseline belief updating tasks Complex numbers manipulation	500	Prolific
Risk replication	Baseline risky choice tasks (gains and losses) Compound lottery manipulation	700	AMT
Beliefs replication	Baseline belief updating tasks Compound belief manipulation	700	AMT
Risk default manipul.	Manipulation of cognitive default	300	AMT
Beliefs default manipul.	Manipulation of cognitive default	300	AMT

Notes. All experiments additionally included survey questions to elicit economic expectations. All experiments elicited expectations about the one-year return of the S&P 500, and the replication experiments additionally measured expectations about one-year inflation rates and the national income distribution. AMT stands for Amazon Mechanical Turk.

were restricted to be between zero and the lottery upside. Each participant initially stated their valuation of six randomly selected lotteries.

The two main advantages of this design are that (i) it eliminates the need to go through a long choice list that may be mentally tiring for participants and (ii) it is well-known that the choice list procedure has its own effects on behavior (e.g., Beauchamp et al., 2019), and we wanted to ensure that our results on cognitive uncertainty do not just capture such choice list effects.

In treatment *Risk replication*, on the other hand, we instead implemented standard choice lists of the type used by, for example, Tversky and Kahneman (1992); Bruhin et al. (2010); Bernheim and Sprenger (2019). The fact that the results turn out to be very similar suggests that the elicitation technique as such does not generate our results.

We often work with a simple linear transformation of elicited certainty equivalents, *normalized certainty equivalents*, which are given by the certainty equivalent divided by the upside of the lottery (a quantity that is by construction between 0 and 100%).

Belief updating. In designing a structured belief updating task, we follow the recent review by Benjamin (2019) and implement the workhorse paradigm of so-called "balls-and-urns" or "bookbags-and-pokerchips" experiments. In treatment *Beliefs main*, there are two bags, A and B. Both bags contain 100 balls, some of which are red and some of which are blue. The computer randomly selects one of the bags according to a pre-specified base rate. Subjects do not observe which bag was selected. Instead, the computer selects one or more balls from the selected bag at random (with replacement) and shows them to the subject. The subject is then asked to state a probabilistic guess that either bag was selected. We visualized this procedure for subjects using the image in Appendix Figure 11.

The three key parameters of this belief updating problem are: (i) the base rate $b \in \{1, 5, 10, 30, 50, 70, 90, 95, 99\}$ (in percent), which we operationalized as the number of cards out of 100 that had "bag A" as opposed to "bag B" written on them; (ii) the signal diagnosticity $d \in \{65, 75, 90\}$, which is given by the number of red balls in bag A and the number of blue balls in bag B (we only implemented symmetric signal structures such that $P(red|A) = P(blue|B)$); and (iii) the number of randomly drawn balls $M \in \{1, 3, 5\}$. These parameters were randomized across trials but always known to participants.

Each subject initially completed six belief updating tasks. Financial incentives were implemented through the binarized scoring rule (Hossain and Okui, 2013). Here, the probability of receiving a prize of \$10 was given by $\pi = \max\left\{0, 1 - 0.0001 \cdot (g - t)^2\right\}$, where g is the guess (in %) and t the true state (0 or 100).

3.3 Measuring Cognitive Uncertainty

Elicitation. In all decision tasks summarized above, participants' decision is given by a single number. Loosely speaking, we always measure cognitive uncertainty (CU) on the subsequent screen by eliciting the participant's subjective probability that their expected-utility maximizing decision is contained in a window around their actual decision. In choice under risk, we reminded participants of the lottery they were exposed to on the previous screen and then asked:

> *Your decision on the previous screen indicates that you value this lottery as much as receiving \$x with certainty. How certain are you that you actually value this lottery somewhere between getting \$(x-0.50) and \$(x+0.50)?*

Participants answered this question by selecting a radio button between 0% and 100%, in steps of 5%. Appendix F.1 provides screenshots. In line with the discussion in Section 2, we interpret this question as capturing the participant's (posterior) uncertainty about their utility-maximizing decision, after some sampling of cognitive signals has taken place. We refer to (inverted) responses to this question as *cognitive uncertainty* rather than confidence because in economics the latter is commonly used for problems that have an objectively correct solution.

In belief updating, the instructions introduced the concept of an "optimal guess." This guess, we explained, uses the laws of probability to compute a statistically correct statement of the probability that either bag was drawn, based on Bayes' rule. We highlighted that this optimal guess does not rely on information that the subject does not have. After indicating their probabilistic belief, subjects were asked (see Appendix F.2):

> *Your decision on the previous screen indicates that that you believe there is a x% chance that Bag A was selected. How certain are you that the optimal guess is somewhere between (x-1)% and (x+1)%?*

The biggest difference between our main experiments and the ones we conducted earlier is the wording of the CU question. In our earlier experiments, we did not elicit participant's subjective probability that the utility-maximizing decision is within some fixed band around their actual decision, but rather a heuristic confidence interval, see Appendix E for details.[6] We believe the new measure to be superior in that it admits a direct quantitative interpretation and is more intuitive for subjects to use. This being said, the results are qualitatively very similar across both experiments.

Potential origins of cognitive uncertainty. Our measure is deliberately designed to capture participants' overall subjective uncertainty about what their utility-maximizing decision is. This uncertainty could have various potential origins. In choice under risk, people may have imperfect perception, may not know their true preferences, or struggle with integrating utils and probabilities. In belief updating, participants may not know the normatively correct updating rule, or struggle with its computational implementation.

Comparison with alternative measures. Broadly speaking, the literature has proposed two different types of measures for eliciting people's uncertainty about their own decisions. At one extreme, psychologists, neuroscientists and some economists elicit measures of "decision confidence," in which subjects indicate on Likert scales how confident or certain they are in their decision (e.g., De Martino et al., 2013, 2017; Polania et al., 2019; Xiang et al.,

[6]For example, in choice under risk, subjects used a slider to calibrate the statement "I am certain that the lottery is worth between a and b to me." If the participant moved the slider to the very right, a and b corresponded to the previously indicated certainty equivalent. For each of the 20 possible ticks that the slider was moved to the left, a decreased and b increased by 25 cents, in real time.

2021; Butler and Loomes, 2007; Drerup et al., 2017). At the other extreme, economists have used measures of across-trial variability in choices (Khaw et al., 2021) or deliberate randomization (Agranov and Ortoleva, 2017, 2020). Our preferred measure strikes a middle ground between these two approaches. While our approach retains the attractive simplicity of implementing a single question (as in the psychology literature), it is also quantitative in nature. The simplicity of asking one question per decision should be contrasted with the approach of gauging cognitive noise through across-task variability in choices, which requires *many* trials and is usually defined at the level of a study rather than of a single choice problem.

Financial incentives and validation. We deliberately do not financially incentivize our elicitation of CU, for two reasons. First, an additional scoring rule makes the measure itself more complex, which increases the cognitive burden on participants. Indeed, recent work documents that unincentivized measures of beliefs are sometimes superior to incentivized ones because they reduce the strategic incentives to game a potentially complex (and misperceived) scoring rule (Danz et al., 2020). Second, we believe that financially incentivizing the measurement in potentially complicated ways would increase the costs for future researchers to include a CU measure in their experiments and surveys.

We validate our simple-but-unincentivized measure below by documenting correlations with across-trial variability in repetitions of the same decision problem, which is commonly viewed as a key signature of cognitive noise.

3.4 Complexity Manipulations

Our experiments link cognitive noise to decisions in two ways. First, we correlate decisions with cognitive uncertainty (awareness of cognitive noise). Second, we exogenously manipulate cognitive noise by making the decision tasks more complex.

Complex numbers. In our main experiments, *Risk main* and *Beliefs main*, the complexity manipulation is given by representing payout probabilities (in choice under risk) and base rates / signal diagnosticies (in belief updating) as mathematical expressions, such as "Get \$20 with probability $(7 \times 6/2 - 11)$%." These treatments were implemented in a between-subjects design: after each subject had completed six baseline tasks of either risky choice or belief updating, for a second set of six tasks they were randomized into either another set of baseline tasks or a set of the complex numbers tasks.

Compound problems. In our earlier experiments, *Risk replication* and *Beliefs replication*, we instead manipulated complexity by deploying compound problems. We hypothesize that these are more complex for people to think through than the normatively identical reduced problems. The compound problems were randomly interspersed with the respective baseline problems in a within-subjects design. In choice under risk, if a baseline lottery is given by a p% chance of getting $20, then the corresponding compound lottery is to get $20 with probability $p' \sim U\{p-0.05,\ldots,p+0.05\}$. In terms of implementation, we told participants that the probability of receiving the lottery upside was unknown to them and would be randomly determined by drawing from a known interval, such that each integer is equally likely to get drawn. Because expected utility is linear in probabilities, this compound manipulation does not affect the normative benchmark for behavior.

In belief updating, if a baseline updating problem features signal diagnosticity h and base rate $b = 50\%$, then the corresponding compound updating problem features diagnosticity $h' \sim U\{h-0.1,\ldots,h+0.1\}$. It is straightforward to verify that the Bayesian posterior for these two updating problems is identical.

3.5 Main and Replication Experiments

For the sake of completeness, we here again summarize the main differences between treatments *Risk main* and *Beliefs main* on the one hand, and *Risk replication* and *Beliefs replication* on the other hand. (i) The CU measurement differs in wording and quantitative interpretation. (ii) The risky choice tasks were implemented using different procedures: with a BDM mechanism à la Healy (2018) in the main experiments and as a visual multiple price list in the replication experiments. (iii) The complexity manipulations differ. Moreover, these were implemented in a between-subjects format in the main experiments and in a within-subjects format in the replication experiments. (iv) The main experiments feature some repeated, identical problems that allow us to study choice variability. (iv) The replication experiments include a broader set of questions measuring economic forecasts, as discussed in Section 8.

3.6 Logistics and Participant Pool

As summarized in Table 1, our main experiments were conducted on Prolific, while the conceptual self-replication was run on Amazon Mechanical Turk. In both sets of experiments, we took two measures to achieve high data quality. First, our financial incentives are unusually large both by AMT and Prolific standards. Average hourly earnings in our experiments are given by \$12 and \$18, exceeding the target compensation on those platforms by roughly 190% and 250%, respectively. Second, we screened out inattentive prospective subjects through comprehension questions and attention checks. Screenshots of instructions and comprehension check questions can be found in Appendix F.

The timeline of *Risk main* and *Beliefs main* was as follows: (i) main incentivized task (lottery valuation or belief updating); (ii) hypothetical economic forecast question; (iii) incentivized Raven matrices test; (iv) demographic questionnaire. Participants received a completion fee of \$3 in both treatments. In addition, each participant potentially earned a bonus. With probability 30%, a randomly-selected task of part (i) was payoff-relevant and with probability 70% part (iii) was paid out. Average earnings in were \$8.10 *Risk main* and \$4.80 in *Beliefs main*.

4 Cognitive Uncertainty: Variation and Validation

Variation. Figure 2 shows histograms of task-level CU in the baseline tasks of *Risk main* and *Beliefs main*. The magnitude of CU should not be compared across decision domains because the length of the interval with respect to which CU is measured is not comparable across risky choice and belief formation problems. Rather, we show these histograms side-by-side to illustrate (i) that a large majority of decisions reflect strictly positive CU and (ii) the large heterogeneity in CU. We find that 83% of the certainty equivalents in *Risk main* and 93% of beliefs in *Beliefs main* are associated with strictly positive CU.

The histograms capture both across-subject and within-subject-across-task variation. Across the two studies, 51–54% of the variation in the CU data is explained by participant fixed effects, see columns (1) and (4) of Table 2. Given that some of the residual variation likely reflects measurement error, this suggests that across-subject variation is the dominant source of variation in the CU data, and that participants are relatively consistent in their degree of CU within a given domain.

To shed further light on the potential presence of "cognitive uncertainty types," we also investigate cross-domain stability by correlating average CU in choice under risk with CU in stock market expectations, and average CU in lab beliefs with CU in stock market expectations.[7] The Spearman correlations are given by $\rho = 0.19$ for risky choice and $\rho = 0.35$ for belief updating ($p < 0.01$ for both correlations). This further suggests some within-person stability of CU.

[7]We describe the stock market expectations data and CU measure in detail in Section 8.

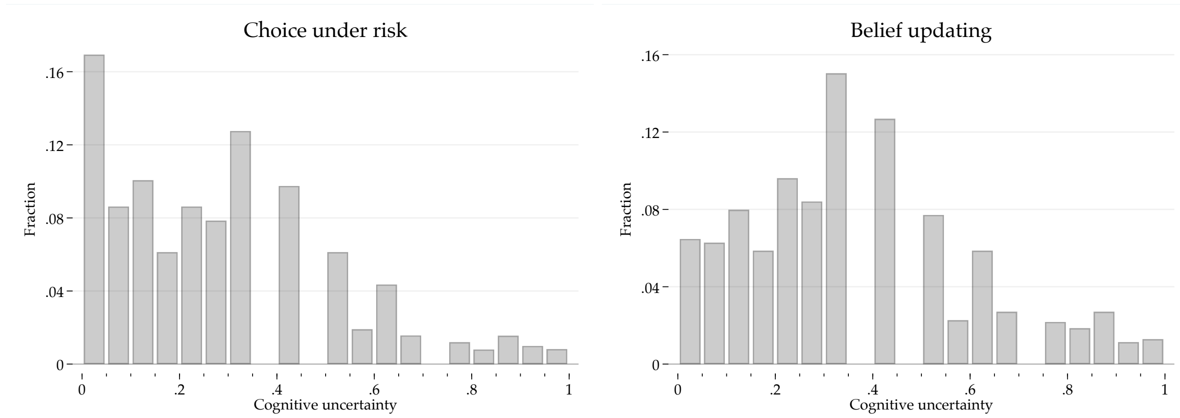

Figure 2: Histograms of cognitive uncertainty in the baseline tasks in *Risk main* (left panel, $N = 4,524$) and *Beliefs main* (right panel, $N = 4,590$).

Table 2 studies demographic correlates of cognitive uncertainty. The most consistent pattern is that women report 6.3 percentage points higher cognitive uncertainty, akin to a large body of evidence on other domains of confidence. We also find that older participants report lower cognitive uncertainty, though the quantitative magnitude of this relationship is small. Meanwhile, response times and proxies for cognitive ability (score on a Raven matrices test and a college degree) are only weakly related to CU.

Cognitive uncertainty and choice variability. Some researchers have used choice variability as an empirical measure of cognitive noise (Khaw et al., 2021). We examine the empirical correspondence between our CU question and variability for two reasons. First, data on choice variability is useful to understand whether people's subjective perception of their own cognitive noise is roughly accurate. Second, a correlation between CU and choice variability may be seen as validation of our quantitative-but-unincentivized question, in the spirit of recent experimental validation studies in the literature (e.g., Falk et al., 2015).

We compute across-trial variability as absolute difference in decisions across two repetitions of the same problem. We find that decisions that are associated with higher average CU across the two trials are more variable, see Appendix Figure 8. The Spearman correlation is $\rho = 0.27$ in choice under risk and $\rho = 0.26$ in belief updating ($p < 0.01$ in both datasets). These results resonate with those from our own work on cognitive uncertainty in intertemporal choice, in which cognitive uncertainty and across-trial variability in responses are likewise significantly correlated (Enke and Graeber, 2021).

Table 2: Correlates of cognitive uncertainty

	Dependent variable: Cognitive uncertainty					
	Choice under risk			Belief updating		
	(1)	(2)	(3)	(4)	(5)	(6)
1 if female		0.063***	0.064***		0.054***	0.052***
		(0.02)	(0.02)		(0.02)	(0.02)
Age		-0.0017**	-0.0017***		-0.0016***	-0.0016***
		(0.00)	(0.00)		(0.00)	(0.00)
Ln [Total time taken for study]			-0.018			0.011
			(0.02)			(0.02)
Raven matrices score (0-4)			-0.022**			-0.0023
			(0.01)			(0.01)
1 if college degree			0.00045			0.020
			(0.02)			(0.02)
Experimental round			0.0014			-0.00072
			(0.00)			(0.00)
Constant	0.55***	0.29***	0.46***	0.40***	0.36***	0.29**
	(0.00)	(0.03)	(0.14)	(0.00)	(0.03)	(0.14)
Subject FE	Yes	No	No	Yes	No	No
Observations	4524	4524	4524	4590	4590	4590
R^2	0.54	0.03	0.04	0.51	0.02	0.02

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

5 Results: Cognitive Uncertainty Predicts Bias

5.1 Visual Illustration of Compression Effects

We begin by analyzing the data in the baseline tasks.[8] Figure 3 summarizes the link between cognitive uncertainty and compression effects in the treatment of probabilities. Both panels are constructed following the same logic, by plotting participants' (normalized) decisions against objective probabilities. Panel A shows normalized certainty equivalents as a function of payout probabilities in *Risk main*. Panel B shows posterior beliefs as a function of Bayesian posteriors in *Beliefs main*. The dots show medians within the samples of above- and below-

[8]Recall that in both *Risk main* and *Beliefs main*, each subject first completed six such baseline tasks, after which half the subjects completed six additional baseline tasks, while the remaining half completed the complex math problems. As a result, the data in this section consist of twelve tasks for some subjects and six tasks for others.

median cognitive uncertainty decisions, respectively.

We see that decisions are always substantially more compressed towards intermediate options in the presence of higher cognitive uncertainty. For instance, in choice under risk, the median decision of low cognitive uncertainty subjects is frequently visually indistinguishable from the benchmark of no probability weighting. This pattern implies the "flipping" property discussed in the theoretical framework: cognitively uncertain decisions are less risk averse for low probabilities but more risk averse for high probabilities. We interpret these patterns as showing that what the literature often refers to as "probability-dependent risk preferences" are, in fact, due to bounded rationality (cognitive noise).

In the belief updating task, the median posteriors of low cognitive uncertainty decisions are likewise relatively close to the rational benchmark. In contrast, cognitively uncertain beliefs reflect pronounced overestimation of small and underestimation of high probabilities. Thus, the phenomenon of "extreme belief aversion" discussed in the review by Benjamin (2019) reflects cognitive noise rather than preferences.

5.2 Regression Evidence

5.2.1 Choice under risk

Table 3 studies the link between CU and likelihood insensitivity (probability weighting) in risky choice more formally, through regression analyses. We estimate the neo-additive weighting function in eq. (2). To this effect, we regress certainty equivalents on the payout probability, cognitive uncertainty and their interaction. The framework in Section 2 predicts that (i) the interaction coefficient is negative (indicating a shallower slope) and (ii) the raw cognitive uncertainty term is positive, indicating a higher intercept.

Columns (1)–(2) of Table 3 document that both of these predictions are indeed borne out in the data. In quantitative terms, an increase in cognitive uncertainty from 0% to 50% is associated with a decrease in the slope of certainty equivalents with respect to payout probabilities by 33.5 percentage points, a very large magnitude.

We likewise find that cognitive uncertainty is strongly related to the regression intercept, as predicted by the model. In other words, the positive cognitive uncertainty raw term does not mean that the probability weighting function of cognitively uncertain subjects has higher elevation on average – it just means that the elevation at $p = 0$ is higher.[9]

Columns (3)–(6) provide further evidence that these patterns imply the characteristic

[9]Recall that in this type of interaction regression, the raw CU term captures the "effect" of cognitive uncertainty at $p = 0$, which corresponds to the regression intercept.

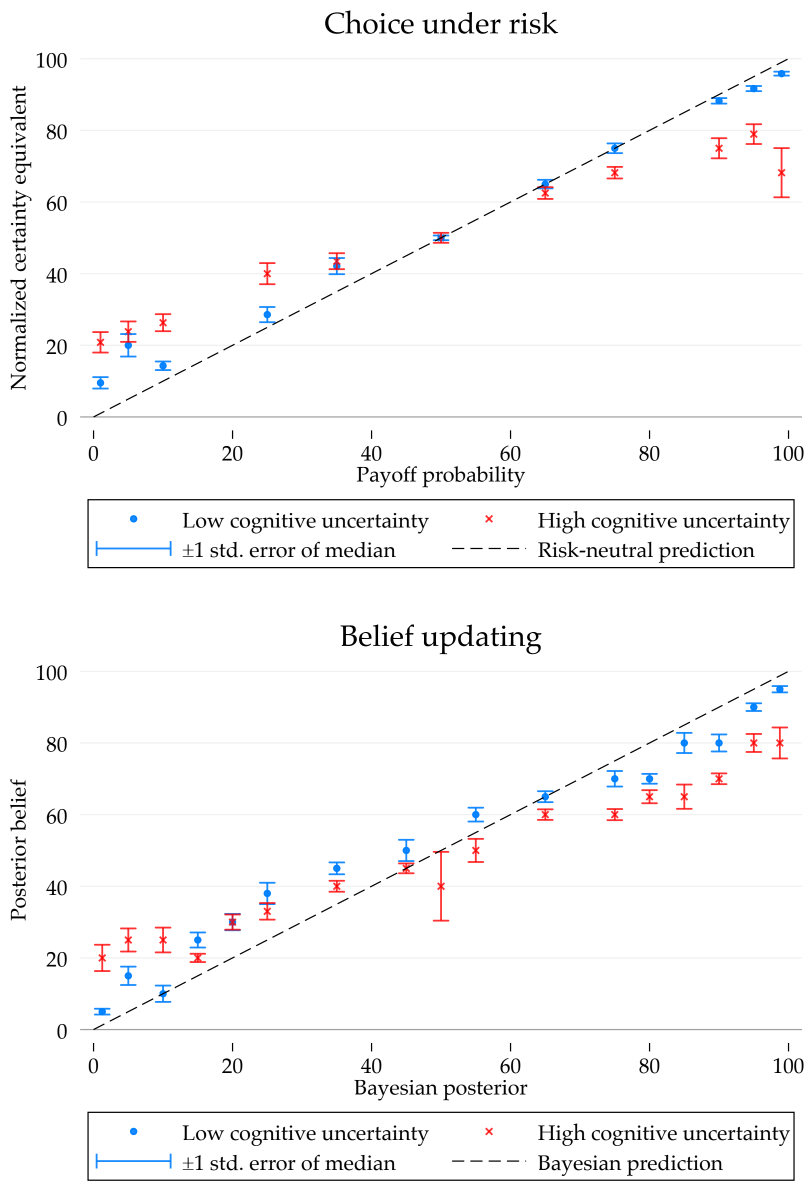

Figure 3: Median decisions as a function of objective probabilities. The top panel shows normalized certainty equivalents as a function of payout probabilities in *Risk main* ($N = 4,524$) and the bottom panel posterior beliefs as a function of Bayesian posteriors in *Beliefs main* ($N = 4,590$). Bayesian posteriors are binned into the midpoint of intervals of length five. The figure displays bins with 30 or more observations. Low CU is defined as below median. Whiskers show standard error bars.

"flipping" pattern that we anticipated in the discussion of the theoretical framework: for small probabilities, cognitively uncertain decisions reflect significantly *more* risk seeking, while for high probabilities they reflect less risk seeking.

Table 3: Cognitive uncertainty and likelihood insensitivity in *Risk main*

	Dependent variable: Normalized certainty equivalent					
	Full sample		$p < 50\%$		$p \geq 50\%$	
	(1)	(2)	(3)	(4)	(5)	(6)
Payout probability	0.73***	0.73***	0.56***	0.55***	0.60***	0.59***
	(0.03)	(0.03)	(0.06)	(0.05)	(0.04)	(0.04)
Payout probability × Cognitive uncertainty	-0.67***	-0.67***				
	(0.08)	(0.08)				
Cognitive Uncertainty	25.1***	22.7***	15.0***	11.0**	-26.3***	-27.0***
	(6.18)	(6.02)	(5.64)	(5.48)	(3.51)	(3.60)
Constant	19.7***	31.5***	22.3***	39.4***	30.7***	38.1***
	(2.35)	(4.38)	(2.35)	(6.09)	(3.23)	(4.66)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	4524	4524	2035	2035	2489	2489
R^2	0.49	0.50	0.10	0.15	0.32	0.32

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Demographic controls include age, gender, college education and performance on a Raven matrices test. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Losses and MPL elicitation technique. Our earlier "replication" experiments allow us to probe the robustness of our results along two dimensions. First, we studied both gain and loss lotteries. Second, the certainty equivalents were elicited using standard visual multiple price lists. The results in these experiments are very similar to those reported above, in the sense that cognitively uncertain decisions are significantly more compressed. This is true for both gains and losses, see Appendix E.

The results in the replication study imply a nuanced pattern about how CU is correlated with risk seeking vs. risk averse behavior. Because CU is associated with "overweighting" of small and "underweighting" of large probabilities for both gains and losses, we have that high CU decisions reflect risk-seeking behavior for low probability gains and high probability losses, but risk-averse behavior for high probability gains and low probability losses. In other words, CU is predictive of the so-called fourfold pattern of risk attitudes.

5.2.2 Belief Updating

Table 4 studies the link between cognitive uncertainty and belief updating in *Beliefs main*. Again, the framework predicts that cognitive uncertainty should be related to (i) lower sen-

Table 4: Cognitive uncertainty and belief updating

	Dependent variable:					
	Posterior belief		Ln [Posterior odds]			
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.71***	0.71***				
	(0.02)	(0.02)				
Bayesian posterior × Cognitive uncertainty	-0.42***	-0.42***				
	(0.07)	(0.07)				
Cognitive Uncertainty	12.3***	12.3***	-0.47***	-0.49***	-0.48***	-0.48***
	(3.76)	(3.74)	(0.16)	(0.16)	(0.16)	(0.16)
Ln [Bayesian odds]			0.55***	0.55***		
			(0.03)	(0.03)		
Ln [Bayesian odds] × Cognitive uncertainty			-0.40***	-0.41***		
			(0.08)	(0.08)		
Log[Prior Odds]					0.69***	0.69***
					(0.04)	(0.04)
Log[Likelihood Ratio]					0.37***	0.37***
					(0.03)	(0.03)
Ln [Prior odds] × Cognitive uncertainty					-0.52***	-0.52***
					(0.11)	(0.11)
Ln [Likelihood ratio] × Cognitive uncertainty					-0.19**	-0.19**
					(0.08)	(0.08)
Constant	19.6***	18.8***	0.24***	0.29**	0.25***	0.28**
	(1.68)	(2.48)	(0.06)	(0.13)	(0.06)	(0.12)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	4590	4590	4590	4590	4590	4590
R^2	0.49	0.49	0.45	0.46	0.48	0.49

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. To avoid a mechanical loss of observations resulting from the log odds definition, the log posterior odds in columns (3)–(6) are computed by replacing stated posterior beliefs of 100% and 0% by 99% and 1%, respectively. The results are virtually identical without this replacement. Demographic controls include age, gender, college education and performance on a Raven matrices test. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

sitivity of beliefs to variation in objective probabilities and (ii) a higher intercept. Columns (1)–(2) directly estimate the neo-additive decision rule (2) that our cognitive noise framework motivates. Here, we link observed beliefs to Bayesian posteriors, cognitive uncertainty and their interaction. Consistent with the visual impression from Figure 3, cognitively uncertain beliefs are substantially less sensitive to variation in Bayesian posteriors, and their intercept is higher. In terms of quantitative magnitude, the regression coefficients imply that moving from cognitive uncertainty of 0% to 50% is associated with a decrease of the slope by 21 percentage points.

Grether regressions: Inelasticity to base rate and likelihood ratio (conservatism). The literature typically highlights not only deviations of stated from Bayesian beliefs, but also the ways in which people implicitly respond to variation in the base rate, the likelihood ratio and the sample size (see Benjamin, 2019, for a review). As discussed in Section 2, we are interested in whether cognitive noise could generate the well-known phenomena of base rate insensitivity, conservatism (likelihood ratio insensitivity) and sample size insensitivity.

To analyze this empirically, we resort to so-called Grether regressions (Grether, 1980). This specification is derived by expressing Bayes' rule in logarithmic form, which implies a linear relationship between the posterior odds, the prior odds, and the likelihood ratio. The canonical finding in the literature is that in these regressions the observed coefficients of the log prior odds and the log likelihood ratio are usually considerably smaller than the Bayesian benchmark of one. As discussed in Section 2 and shown in Appendix A, our stylized cognitive noise model predicts that higher cognitive noise leads to higher insensitivity in these regressions. A simple intuition is that if someone always stated posterior beliefs of 50:50, their sensitivity of beliefs to the base rate and likelihood ratio would be zero.

Columns (3) and (4) of Table 4 estimate a restricted version of a Grether regression, in which we relate the subject's log posterior odds to the log Bayesian odds. This analysis is instructive because it takes place in log odds space (as motivated by the Grether decomposition), but essentially uses the same variables as in columns (1) and (2). Again, we find that cognitive uncertainty is strongly predictive of the degree of insensitivity of log posterior odds with respect to the Bayesian benchmark.

Finally, columns (5) and (6) estimate a standard Grether regression, except that we also account for interactions with cognitive uncertainty. The negative interaction coefficients show that cognitive uncertainty is strongly related to base rate insensitivity and likelihood insensitivity (conservatism). The quantitative magnitudes of the regression coefficients suggest that, for example, base rate sensitivity decreases from 0.69 with CU of 0% to 0.43 with CU of 50%.[10]

These patterns document that (at least a part of) what this literature has identified as base rate neglect, conservatism and extreme belief aversion are in fact not independent psychological phenomena but instead all generated by cognitive noise and resulting compression effects.

[10]The interaction coefficients are larger for the log prior odds than for the log likelihood ratio. We can only speculate about why this is the case. In our experiment, base rates are displayed using sets of cards, while diagnosticities are displayed using urns that are filled with 100 colored balls. We cannot rule out that this difference in the way in which information is presented affects the perceived complexity of these decision parameters and / or their interaction with cognitive noise.

Sample size effects. At the most basic level, our stylized model provides a mapping from Bayesian posteriors to stated posteriors. However, as is well known in the literature, experimental data also reveal systematic variation in stated beliefs *conditional* on Bayesian posteriors. For instance, for a given base rate, the draw of one blue ball gives rise to the same Bayesian posterior as the draw of two blue balls and one red ball, yet experimental participants consistently update more strongly after observing one blue ball (Benjamin, 2019). A common explanation is that subjects update based on sample *proportions* (which are more extreme for smaller samples), while Bayesian updating prescribes updating based on sample *differences*. Our account of cognitive uncertainty also provides an explanation for this pattern. The straightforward reason is that stated cognitive uncertainty significantly increases in the sample size, holding the sample difference and the Bayesian posterior fixed (see Appendix Table 6). That is, subjects appear to find it easier to form beliefs based on one blue ball than based on two blue balls and one red ball. As a result of this systematic variation in cognitive noise, our account correctly predicts that subjects respond more to the sample difference when the sample size is smaller.

Replication. All of the patterns summarized above also hold in our conceptual self-replication, see Appendix E.

5.3 Cognitive Uncertainty and Distance to the Optimal Decision

Thus far, the analyses documented that *average* decisions are more compressed and further away from normative benchmarks when they are associated with higher cognitive uncertainty. In itself, however, this does not imply that cognitively uncertain decisions are located further away from normative benchmarks, on average. To see this, consider a simple example in which the Bayesian posterior is 80%. Then, the average of beliefs of 79% and 77% is located further away from the Bayesian benchmark than the average of beliefs of 60% and 100%, yet the average absolute distance is still smaller in the former case.

Our stylized model predics that cognitive noise produces not only stronger compression of the average, but also that it leads to larger average absolute distances to the normatively optimal decision. We here test this additional prediction. For belief updating, we use the Bayesian posterior as the normative benchmark. For choice under risk, we assume that subjects' objective is to maximize expected value. However, we have verified that very similar results hold when we infer the "true" utility-maximizing decisions by estimating a population-level CRRA parameter.

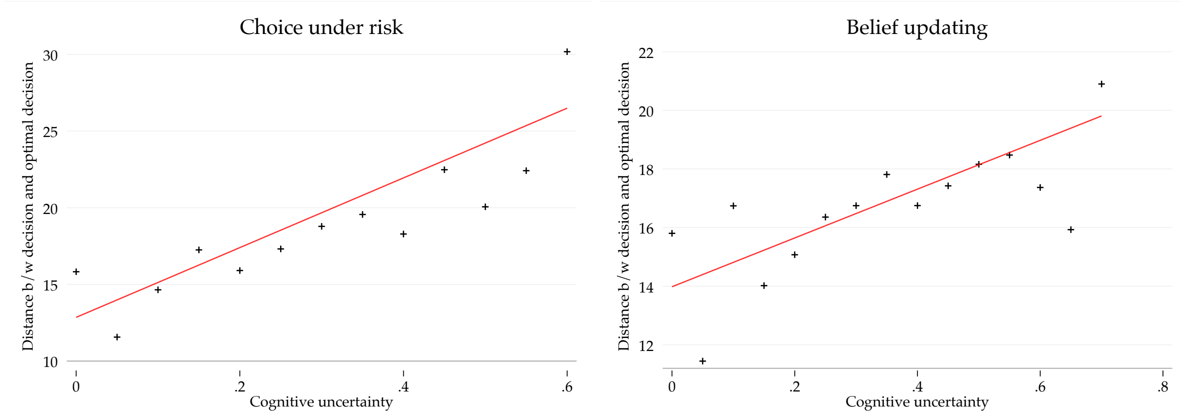

Figure 4: Absolute distance between decisions and normatively optimal decisions as a function of cognitive uncertainty. In panel A ($N = 4,524$), the normative benchmark is assumed to be expected value maximization and in panel B ($N = 4,590$) it is the Bayesian posterior. Cognitive uncertainty is winsorized at the 90th percentile in each dataset for ease of readability.

Figure 4 summarizes the results. Across both domains, cognitive uncertainty and absolute distances to the normative benchmark are significantly correlated (Spearman's $\rho = 0.31$ in risky choice and $\rho = 0.17$ in beliefs, $p < 0.01$ for both comparisons).

6 Complexity, Cognitive Noise and Compression Effects

In the conceptual framework in Section 2, we took the magnitude of cognitive noise (captured by N) as given. More realistically, cognitive noise will be higher if the complexity of a decision problem is high. As outlined in Section 3, our main experiments manipulated problem complexity by expressing probabilities as math problems. The replication experiments instead manipulated complexity through compound problems.

Given that there is no widely accepted theory of what is (not) complex, neither of these two treatments is directly theoretically motivated. However, multiple previous contributions have hypothesized that compound problems or complex numbers can make decision problems harder (e.g. Huck and Weizsäcker, 1999; Gillen et al., 2019). Moreover, an added benefit of our cognitive uncertainty measurement is that it allows us to directly test whether a complexity intervention actually increases cognitive noise. Both experimental manipulations had large effects on cognitive uncertainty. The complex numbers manipulation increased CU by 45% in risky choice and by 48% in belief updating. The compound manipulations lead to an increase in CU by 23% in risky choice and by 33% in belief updating.[11]

[11]Recall that we used a different CU measure in the replication experiments, such that the magnitudes of

Panels A–D of Figure 5 document that this increase in complexity (and resulting cognitive noise) has a large effect on decisions. As we predicted, responses are always substantially more compressed towards an intermediate value than in our baseline experiments. This is true for both the math manipulation and the compound problems.[12] Appendix Tables 8–11 provide corroborating regression evidence that shows that these patterns are statistically highly significant.[13] Overall, we interpret these patterns as evidence that cognitive noise actually causes compression towards an intermediate value, rather than that it only correlates with it.

We also note that all of these results are inconsistent with a large class of models of probability weighting and belief updating biases that rest on the assumption of fixed parametric biases, such as "base rate neglect parameters" or a "probability weighting sensitivity parameter". Instead, our results suggest that the complexity of the decision environment partly determines the level of cognitive noise, which, in turn, drives the magnitude of errors.

7 What's the Cognitive Default?

In this paper, we think of the cognitive default decision as people's initial reaction: the decision they would have taken prior to deliberating about the problem at hand with its specific parameter values. We do not have a general theory of what determines this default / initial reaction. Instead, we attempt to make some progress first by directly estimating the cognitive default, and then by documenting that it can be manipulated.

7.1 Model Estimations

Recall that the average decision in our framework can be expressed as a convex combination of the expected-utility-maximizing decision and the cognitive default, with the relative weight λ being a function of the magnitude of (unobserved) cognitive noise. We proceed by heuristically approximating $\lambda = max\{0; (1 - \gamma p_{CU})\}$, where γ is a nuisance parameter

the CU increase should not be directly compared across experiments.

[12]It is interesting to relate these results to Harbaugh et al. (2010). They identify evidence for probability weighting in one elicitation mechanism but not another one, and interpret this by suggesting that the mechanism that produces probability weighting is "more complex."

[13]In experiment *Risk replication*, we also implemented compound lotteries for loss gambles. The results are very similar, see Appendix E.

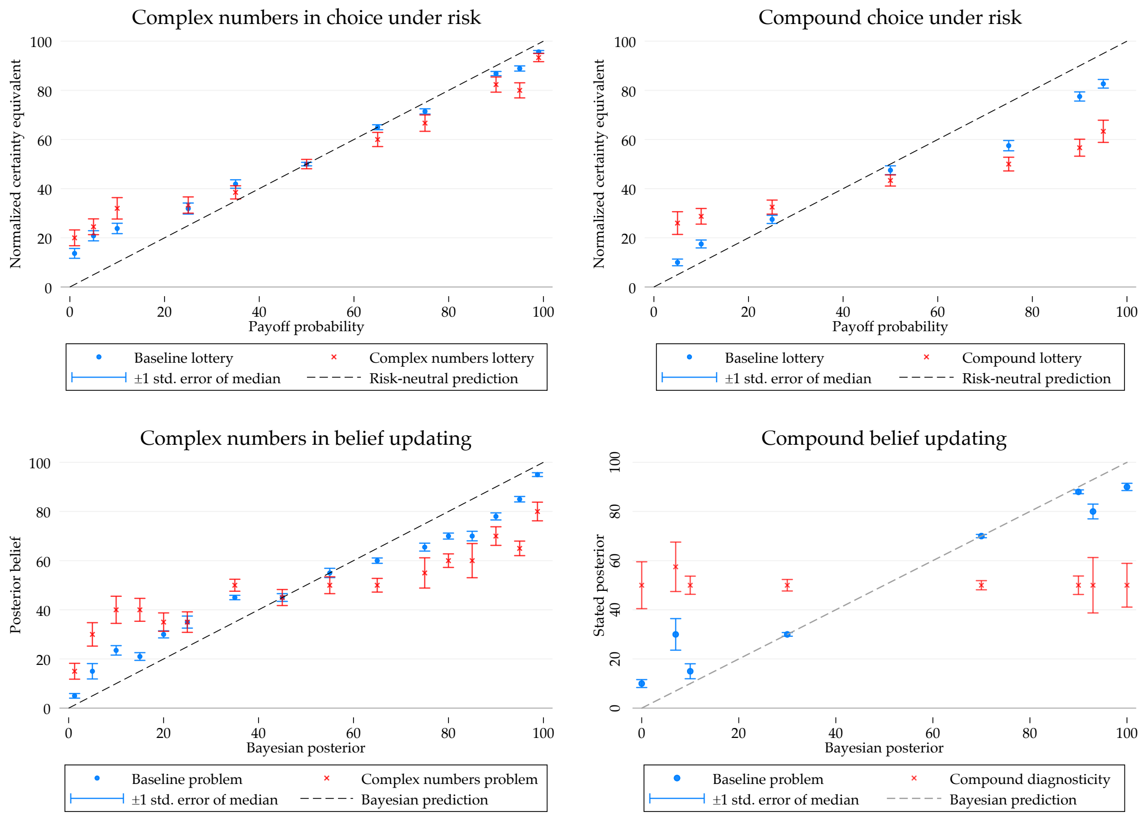

Figure 5: Complexity and decisions. Panel A shows median normalized certainty equivalents separately for baseline and complex numbers lotteries in the *Risk main* experiment ($N = 3,000$). Panel B shows median normalized certainty equivalents separately for baseline and compound lotteries in the *Risk replication* experiment ($N = 1,958$). Panel C shows median posterior beliefs separately for baseline and complex numbers updating problems in the *Beliefs main* experiment ($N = 3,000$). Panel D shows median posterior beliefs separately for baseline and compound updating problems in the *Beliefs replication* experiment ($N = 2,056$). Whiskers show standard error bars. The Beliefs figures show bins with more than 30 observations.

to be estimated. We can then estimate the decision rule in (2) as:

$$a^o = \underbrace{max\{1 - \gamma p_{CU}; 0\}}_{\lambda} a^*(p) + \underbrace{min\{\gamma p_{CU}; 1\}}_{1-\lambda} d + \epsilon, \tag{4}$$

where p_{CU} is observed, γ and d are to be estimated and ϵ is a disturbance term. The utility-maximizing decision a^* is assumed to be the Bayesian posterior in belief updating. For choice under risk, we assume that the utility-maximizing decision reflects CRRA utility, with utility curvature to be estimated.[14] We estimate this equation using standard non-linear least squares techniques. For benchmarking purposes, we also estimate a "restricted

[14]The estimating equation with CRRA utility curvature parameter α is given by $a^o = max\{1 - \gamma p_{CU}; 0\}p^{1/\alpha} + min\{\gamma p_{CU}; 1\}d + \epsilon$.

Table 5: Estimates of cognitive default across experiments

	Risk main		*Beliefs main*		*Risk Replication*		*Beliefs Replication*		*Risk low default*	*Beliefs low default*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Restr.	CU	Restr.	CU	Restr.	CU	Restr.	CU	CU	CU
$\hat{d}$	N/A	0.41	N/A	0.52	N/A	0.40	0.32	0.52	0.32	0.36
AIC	18805	17762	299	-765	7996	7707	211	-935	2793	0.52

Notes. Estimates of different versions of (4). Columns (1) and (3): set $\gamma = 1$ and $p_{CU} = 0$. All estimated standard errors (computed based on clustering at the subject level) are smaller than 0.02. In choice under risk, column the CRRA utility curvature parameter is estimated as $\alpha = 1.54$ in clumn (2), as $\alpha = 1.13$ in column (6) and as $\alpha = 0.63$ in column (9).

model" that excludes cognitive noise, i.e., setting $p_{CU} = 0$.

Table 5 reports the model estimates for both our main experiments and the self-replication. There are three main takeaways. First, we consistently estimate an "intermediate" default decision. The estimated default is very close to 0.5 in the beliefs experiments and somewhat lower at around 0.4 in choice under risk. The estimates of the default decision jive well both with the visual impressions from Figure 3 and with the large body of work on central tendency or compromise effects in psychology and economics that argues that people's heuristic reactions to decision problems are intermediate in nature.

The second main takeaway from the model estimations is that allowing for a role of cognitive noise (columns (2), (4), (6) and (8)) increases model fit substantially relative to the restricted model that does not include cognitive uncertainty (columns (1), (3), (5) and (7)). This can be inferred from the lower values of Akaike's Information Criterion. We explain columns (9) and (10) in the next section.

7.2 Manipulating the Cognitive Default

Our model estimations suggest that in both risky choice and belief updating, the cognitive default reflects a "central tendency effect," akin to much work on the psychology of perception (see, e.g., Petzschner et al., 2015; Xiang et al., 2021). We acknowledge that this default is most likely context-specific. In our relatively abstract experiments with which participants have very limited (if any) experience, a moderate, regression-to-the-center default seems intuitively plausible. On the other hand, when people have experience with a decision problem, a potential cognitive default may be influenced by habit or contextual features that determine "what comes to mind."

As a simple proof of concept, we now aim to exogenously manipulate the cognitive default to test Prediction 4 from Section 2. Given that the decision contexts that we study in this paper feature a binary state space, an intermediate cognitive default decision could reflect a type of ignorance prior. In choice under risk, prior to any deliberation of the problem, the paying and non-paying state may seem roughly equally likely from an ex-ante perspective. Similarly, in belief formation, both states of the world may appear equally likely prior to deliberating on a specific task, which again makes an intermediate default posterior belief plausible. This logic suggests that the cognitive default decision can be manipulated through a partition manipulation that increases the number of states, holding all normatively relevant aspects of the decision problem constant. Indeed, a large body of prior work suggests that decisions are sensitive to partition manipulations (e.g., Fox and Clemen, 2005; Starmer and Sugden, 1993; Fischhoff et al., 1978).

Design. We designed treatment conditions that increase the number of states from two to ten. We further designed these treatments with the objective of holding cognitive uncertainty fixed (which we verify below). In choice under risk, we do so by further replicating treatment *Risk replication*, except that we now frame probabilities in terms of the number of colored balls in a bag. For example, we describe a lottery as:

> Out of 100 balls, 80 are red. If a red ball gets drawn: get $20.
> 20 balls are blue. If a blue ball gets drawn: get $0.

In addition to this treatment, labeled *Risk replication high default*, we also implemented treatment *Risk replication low default*. Here, we implemented the same lotteries as in *Risk high default*, yet we split the zero-payout state into nine payoff-equivalent states with different probability colors. For example, the lottery above would be described as:

> Out of 100 balls, 80 are red. If a red ball gets drawn: get $20.
> 2 balls are blue. If a blue ball gets drawn: get $0.
> 2 balls are black. If a black ball gets drawn: get $0.
> 2 balls are white. If a white ball gets drawn: get $0.
> . . .
> 4 balls are yellow. If a yellow ball gets drawn: get $0.

We designed a similar manipulation for the balls-and-urns updating task. Recall that in treatment *Beliefs replication*, an example updating problem is that the base rates for bags A and B are 70% and 30%, and the signal diagnosticity (number of red balls in bag A and number of blue balls in bag B) is 70%. Now, in treatment *Beliefs low default*, we split the probability mass for bag B up into nine different bags. That is, there are now ten bags, labeled A through J. In the specific example above, the base rate for A would again be 70%, the one for B through I 3% each and the one for J 6%. Bag A would contain 70 red and 30 balls, and all bags B through J 30 red and 70 blue balls. That is, these bags have identical ball compositions. Observe that in both the lottery choice problem and the belief updating task the normatively relevant structure of the problem (which consists of the objective lottery payoff profile and the Bayesian posterior) is held constant.

Importantly, the elicitation of decisions was held exactly constant across treatments. For instance, in both *Beliefs replication* and *Beliefs low default*, subjects only enter their subjective probability that Bag A got selected. The implied probability for the other events was displayed automatically. In *Beliefs replication*, if a subject entered probability p% for Bag A, then our computer interface automatically showed the joint subjective probability for Bags B–J as (1-p)%.

In total, 300 subjects participated in *Risk replication low default* and *Risk replication high default*, which we implemented in a between-subjects design with random assignment to treatments within sessions. 300 subjects participated in treatment *Beliefs replication low default*, which was randomized within the same experimental sessions as treatment *Beliefs replication*. Appendix F shows screenshots of the experimental instructions. Appendix E.3 discusses the pre-registration, including the pre-specified exclusion of extreme outliers.

Interpretation. This experimental manipulation lends itself to two interpretations, both of which we embrace. First, as discussed above, the default decision could be influenced by a type of ignorance prior or 1/N heuristic. A second interpretation is that the manipulation makes the zero-payout state in the risky choice problems more visually salient because it now appears nine times on the decision screen. Similarly, in the belief updating problems, Bag A (the one for which we elicit the subjective posterior probability) becomes less salient because there are nine other bags in the partition manipulation. These two interpretations share the common theme that they emphasize how the partition manipulation changes people's heuristic (or intuitive) response, prior to actually thinking about the specific problem at hand. This is what we intend to capture and manipulate.

Results. Cognitive uncertainty is unaffected by the partition manipulation, in both choice under risk ($p = 0.90$) and belief updating ($p = 0.85$). This lends credence to the assumption that our experimental manipulation affects the cognitive default but not cognitive noise.

As foreshadowed in Section 2, we test two predictions of our cognitive noise framework: (i) that a decrease in the cognitive default decreases decisions and (ii) that this effect is

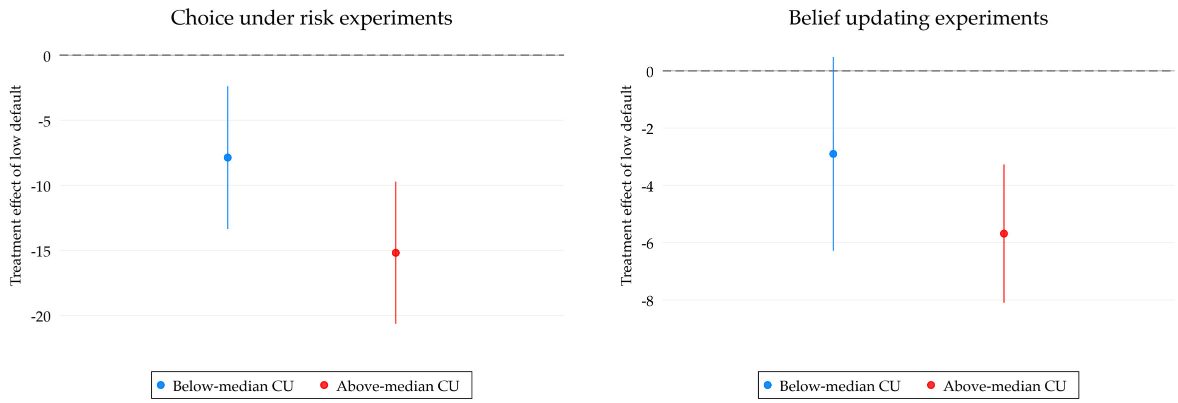

Figure 6: Treatment effect of the partition manipulations, separately for below- and above-median CU decisions. Displayed are the coefficients of regressing normalized certainty equivalents in *Risk default manipulation* experiments (left-hand panel) or stated beliefs in the *Beliefs default manipulation* experiments (right-hand panel) on an indicator that equals one in the *low default* condition, separately for the sample of above- vs. below median CU decisions. Whiskers show 95% confidence intervals. (Risk: $N = 881$; Beliefs: $N = 5,372$)

particularly pronounced among high cognitive uncertainty decisions. Intuitively, this should be the case because if a decision-maker does not have cognitive noise, he does not place any weight on the cognitive default.

Figure 6 summarizes the results through a coefficient plot that shows the treatment effect of the low default conditions, separately for below- and above-median CU decisions. Appendix Table 12 reports the regression estimates. For both risky choice and beliefs, we find that the partition manipulation significantly decreases average decisions. Moreover, this response to the change in the default is more pronounced among cognitively uncertain decisions. Overall, these results highlight that cognitive uncertainty is not associated with a general form of insensitivity. Instead, as predicted by a cognitive noise model, it produces insensitivity to objective probabilities but excessive sensitivity to variation in the normatively irrelevant cognitive default.

To further quantify these patterns, we use the same techniques as in Section 7.1 to estimate the cognitive default decision in the default treatments, see Table 5. The estimated default decision drops from 0.52 to 0.36 in the beliefs experiments, and from 0.40 to 0.32 in the risk experiments.

8 Economic Forecasts and Expectations

Large and growing literatures measure people's beliefs and expectations about economic variables such as the stock market, inflation, returns to education, earnings, and others. A well-known finding in this literature is that people's beliefs and expectations are strongly compressed to 50% (e.g., Fischhoff and Bruine De Bruin, 1999; Drerup et al., 2017; Hvidberg et al., 2020). We here study the link between cognitive noise and such economic expectations, and discuss the benefits of measuring cognitive uncertainty for the extant subjective expectations literature.

A conceptual difference between expectations about real-life quantities and the types of experimental tasks we studied above is that in the latter the experimenter supplies all information that the subject needs to make a well-defined rational decision, while in expectations surveys the experimenter does not have access to the respondent's information set. Still, as we show below, cognitive uncrtainty can be measured in a very similar way. Indeed, intuitively, people may well exhibit cognitive uncertainty about their economic expectations: they may not perfectly remember their current beliefs about the stock market (or the information they received in the past), or they may worry that they have incorrectly processed past information.

Design. In our main study ($N = 1,000$, see Table 1), we elicit probabilistic forecasts of the performance of the S&P 500. Because incentivizing expectations about future events creates various logistical issues such as credibility concerns and the necessity to wait for future variables to have realized, we elicited them without financial incentives. This is line with the vast majority of the literature on survey expectations.

> *The S&P 500 is an American stock market index that includes 500 of the largest companies based in the United States. Jon invested $100 in the S&P 500 today. What is the percent chance that the value of his investment will be less than $y in one year from now?*

In our earlier experiments, we also elicited beliefs about future inflation rates and the national income distribution, see Appendix E.

The elicitation of cognitive uncertainty closely mimics the one for the belief updating task, asking how certain the respondent is that their probabilistic guess is within and one percentage point band around the guess that's optimal *given the information available to the respondent*. Thus, the question does not elicit people's awareness of their lack of information, but instead their perceived ability to appropriately remember and process the information available to them. Appendix Figure F.3 shows screenshots. Appendix Figure 9 shows a histogram of CU in stock market forecasts.

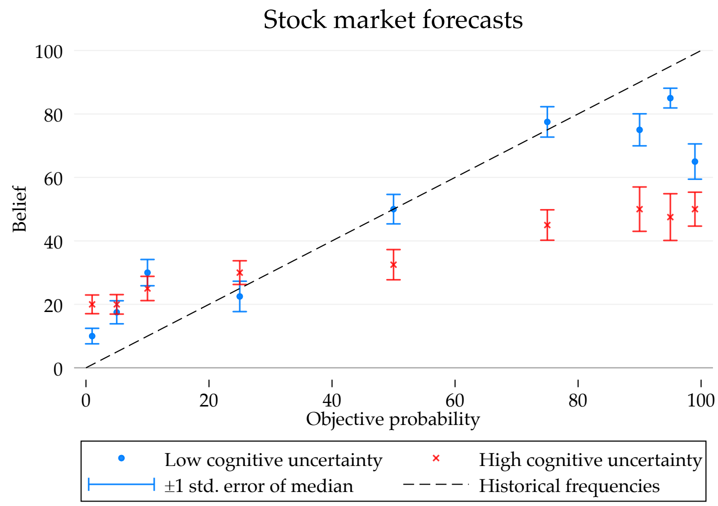

Figure 7: Median stock market forecasts as a function of historical probabilities ($N = 1,000$). Low CU is defined as below median. Whiskers show standard error bars.

Results. Figure 7 summarizes the results. Similarly to the lab belief updating task, we see that cognitive uncertainty is strongly predictive of overestimation of small and underestimation of large probabilities. Appendix Table 7 presents regression analyses that confirm this visual impression.

In our conceptual self-replication, we find almost identical patterns for the same measure of stock market expectations. Moreover, we find very similar patterns of cognitive uncertainty predicting compression towards 50:50 also for inflation expectations and beliefs about the income distribution. See Appendix E for details.

Implications for research linking expectations measures to field behaviors. If stated expectations are systematically distorted due to the types of compression effects that we document in this paper, documented demographic differences in expectations could just reflect heterogeneity in cognitive noise rather than true beliefs. Moreover, when researchers estimate links between expectations and field behaviors, cognitive noise could attenuate these relationships. In line with this conjecture, Drerup et al. (2017) and Giglio et al. (2019) find that the relationship between expectations and investment behavior is considerably more pronounced among people with high confidence in their expectations answers. We conjecture that cognitive uncertainty will be predictive of the strength of the relationship between behaviors and expectations more generally. Thus, at a minimum, measuring cognitive uncertainty in surveys allows researchers to conduct heterogeneity analyses regarding the predictability of field behaviors.

9 Discussion

This paper has argued that measuring cognitive uncertainty in a simple, fast and costless manner allows experimental and survey researchers to predict behavior and biases, to shed light on the decision modes that underlie commonalities in errors across different domains, and to provide tests of formal economic models. Instead of recapitulating the paper's results, we here discuss extensions, limitations and directions for future research.

Extension: S-shaped response functions. While our main empirical analyses focus on the observation that people's beliefs and choices are *compressed* towards some intermediate value, it is well-known in the literature that decisions are often non-linear (inverse S-shaped) in objective probabilities (see Figure 1). As we discuss in detail in Appendix D, our account of cognitive uncertainty also sheds light on this regularity. The reason is that, in our data, measured cognitive uncertainty is hump-shaped in objective probabilities. For example, it appears to be easier for people to value a lottery that has a payout probability close to the boundaries. Similarly, people report lower cognitive uncertainty in belief updating problems that have Bayesian posteriors close to the boundaries. The model estimations in Appendix D show that these non-linearities in how cognitive uncertainty depends on objective probabilities translate into the canonical S-shaped response functions commonly observed in the literature.

Extension: Ambiguity attitudes. While in this paper we focus on how cognitive uncertainty sheds light on the pattern that people treat different objective probabilities to some degree alike, there is also a direct connection to research on ambiguity. The reason is that recent reviews highlight the concept of "ambiguity-insensitivity," which asserts that people are excessively insensitive to changes in the likelihood of ambiguous events (Trautmann and Van De Kuilen, 2015). In the working paper version of this paper, we document that measured cognitive uncertainty also strongly predicts the magnitude of ambiguity insensitivity (Enke and Graeber, 2019). Indeed, we find that cognitively uncertain people often act as though they are ambiguity-seeking when the ambiguous event is very unlikely.

Limitations. An obvious limitation of our approach is that we do not have a general theory of what the cognitive default is. We here work with the idea that the default reflects people's initial reaction to a decision problem: the decision they would have taken prior to deliberating about the problem at hand. Yet, casual introspection suggests that other factors might also shape people's initial reactions. For instance, if a choice option is displayed in red font, it might be visually salient and therefore serve as a cognitive anchor from which people's deliberation process adjusts.

More generally, bounded rationality research in economics can arguably be partitioned into work that focuses on (i) the effects of complexity and (ii) the role of misleading intuitions, as they result from salience, focusing, or memory-based cueing effects (e.g., Kahneman, 2011; Bordalo et al., 2013, 2017; Kőszegi and Szeidl, 2013; Enke et al., 2020). While our paper is part of the former strand, we conjecture that the (unspecified) cognitive default provides a potential link between these two literatures. We speculate that strong intuitions, salient choice options or associations-based recall shape people's initial reaction to a choice problem (the cognitive default), while cognitive uncertainty captures the degree to which people adjust away from these initial reactions. If true, such a perspective would suggest the testable prediction that salience, focusing and memory-based cueing effects are particularly pronounced among people with high cognitive uncertainty.

More closely integrating cognitive noise with attention and memory research is also relevant because prior work has shown that both probability weighting in risky choice and probability estimates are influenced by salience and asymmetric recall (e.g., Stewart et al., 2006; Bordalo et al., 2012, 2021). Similarly, a broad body of work often identifies the opposite of probability weighting when people decide based on experience rather than problem descriptions (Hertwig and Erev, 2009). It is not obvious that our approach of measuring cognitive uncertainty can reconcile these patterns. In our view, a promising avenue for future research is to integrate research on cognitive noise with that on attention and memory, both regarding how salience and recall may shape a potential cognitive default (see above) and how imperfect memory and attention generate cognitive noise in the first place.

A third limitation of our work is that we do not have a theory of which aspects of a decision actually generate cognitive noise and resulting cognitive uncertainty. As we saw above, more complex decisions lead to higher cognitive uncertainty. Yet, a general theory of what makes a task (not) complex is not available. Other aspects that generate cognitive uncertainty may pertain to the decision-maker: the availability of cognitive resources, time spent on the problem, or the amount of experience. Future research could helpfully shed light on this.

Open questions and potential applications. We conjecture that the measurement of cognitive uncertainty could shed light on behavior in multiple other domains of economic decision-making. Most fundamentally, people likely don't just have cognitive uncertainty in choosing between lotteries or in updating their beliefs, but also in other domains. For instance, in Enke and Graeber (2021), we study how cognitive uncertainty helps to shed light on "anomalies" in intertemporal choice. Yet, we speculate that there may be many more applications in which a measurement of cognitive uncertainty could shed light on biases and anomalies that have a "compression" flavor. For example, in the widely-studied newsvendor game that is of relevance to researchers in economics, management and operations research, people generally succumb to a pull-to-the-center bias (Schweitzer and Cachon, 2000). Similarly, laboratory experiments on effort choice often find that the elasticity of labor supply with respect to piece rates is very low; we again speculate that this insensitivity / compression effect could be explained by measuring cognitive uncertainty.

Another open question relates to the link between objective cognitive noise and cognitive uncertainty. In the decision contexts that we study in this paper, people's awareness of their own cognitive noise is at least partly accurate. Yet, in other decision domains, people's meta-cognition may be less well-calibrated, as in Enke et al. (2021). This immediately raises the question of when people's cognitive uncertainty is (not) reflective of actual noise.

Finally, another open question concerns the choice implications of cognitive noise. In this paper, we have highlighted the empirical regularity that cognitive uncertainty is associated with an attenuated relationship between decisions and problem parameters. In other contexts, cognitive uncertainty may predict a form of "caution" (Cerreia-Vioglio et al., 2015) or "complexity aversion," according to which people shy away from choice options regarding which they have high cognitive uncertainty. Future research could helpfully shed light on when compression effects or caution dominate.

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ONLINE APPENDIX

A Theoretical Framework

A.1 Baseline Model

Below we discuss the main behavioral predictions of a Bayesian cognitive noise model as outlined in Section 2. Suppose the DM does not know their ex-ante utility-maximizing action, a^*, but has access to a mental simulation, S, which is an unbiased cognitive signal of a^*,

$$S \sim \frac{1}{N} Bin(N, a^*), \tag{5}$$

such that $0 < S < 1$. The parameter N controls the precision of the mental simulation.

The DM holds a prior about his subjective utility-maximizing action, A. We assume that this prior can be represented by a Beta distribution, $a^* \sim Beta(nd\ ,\ n(1-d))$. Here, d is the prior mean and carries the interpretation of a "cognitive default" action that the DM would take before deliberating about the problem. The parameter n, on the other hand, reflects the DM's confidence in (or precision of) his prior.[15] Given the fact she has a prior, the cognitive signal from the DM's perspective is seen as:

$$S \sim \frac{1}{N} Bin(N, A). \tag{6}$$

The subjective likelihood of choosing the true utility-maximizing action based on a randomly drawn cognitive signal $\{S = s\}$ can then be represented by a binomial distribution:

$$\mathscr{L}(A = a^* | S = s) = P(S = s | a^*, N) = \binom{N}{sN} (a^*)^{sN} (1 - a^*)^{(1-s)N}. \tag{7}$$

A Bayesian DM accounts for the noisiness of his mental simulation by implicitly forming a posterior assessment of the utility-maximizing action. Given a Beta-distributed prior and a Binomial signal, this posterior belief, $A|S = s$, is also Beta-distributed.[16] We assume the

[15] Note that $n = a + b$ is a re-parameterization of the shape parameters a and b of the Beta distribution. n is inversely related to the variance of the prior, $\sigma_A^2 = \frac{d \cdot (1-d)}{1+n}$.

[16] Specifically, $A_{S=s} \sim\ Beta(sN + nd\ ,\ N(1-s) + n(1-d))$.

DM's decision is given by his posterior mean:[17]

$$a^o = E[A|S=s] = \lambda s + (1-\lambda)d \quad \text{with} \quad \lambda = N/(n+N). \tag{8}$$

Crucially, a more precise mental simulation (higher N) has a direct effect on the weighting factor λ which implies a lower weight on the cognitive default action. In the following subsection, we will thus focus on deriving behavioral predictions for changes in λ. In subsection A.3 we characterize cognitive uncertainty in the context of this model.

For the purposes of the belief updating experiments it will be helpful to define the following terms:

$$b := \text{ The prior / base rate} \tag{9}$$
$$h := \text{ The signal diagnosticity} \tag{10}$$
$$d := \text{ The cognitive default decision} \tag{11}$$
$$n := \text{ The number of balls in the sample} \tag{12}$$
$$k := \text{ The number of red balls in the sample} \tag{13}$$

These quantities allows us to derive / define the following quantities:

$$\begin{aligned}
a^* &:= \frac{h^k(1-h)^{n-k}b}{h^k(1-h)^{n-k}b + (1-h)^k h^{n-k}(1-b))} && \text{(Bayesian Posterior)} \\
a &:= \lambda a^* + (1-\lambda)d && \text{The mean observed action} \\
o &:= \frac{b}{1-b} && \text{The prior odds} \\
LR &:= \frac{h^k(1-h)^{n-k}}{(1-h)^k h^{n-k}} = \left(\frac{h}{1-h}\right)^{2k-n} && \text{The likelihood ratio} \\
lo &:= \frac{a}{1-a} && \text{Log[Subjective posterior odds]}
\end{aligned}$$

A.2 Proofs of Predictions in Main Text

We restate the predictions from the main text more formally here and provide proofs.

[17]We focus on the mean for tractability. This is without much loss in the present context because the mean of a Beta(a,b) variable is $a/(a+b)$, the mode is $(a-1)/(a+b-2)$ and the median lies between the two. In our belief formation experiments, it is indeed optimal for the DM to play the posterior mean given the quadratic loss function implied by our scoring rule. In the case of our risky choice experiments, whether the DM optimally plays the mean, median or mode depends on the specific loss function assumed, but as argued above, plausible assumptions such as L1 or L2 loss will lead to almost identical decisions.

Prediction 1 (Cognitive noise and compression effects)**.**

(a) *In risky choice, cognitive noise is correlated with probability weighting. Specifically, the mean error,* $e := \mathbb{E}[a^o - p]$*, when faced with the same situation satisfies:* $\partial e / \partial \lambda < 0$ *for* $p < u(d)$ *and* $\partial e / \partial \lambda > 0$ *for* $p > u(d)$*.*

(b) *i. In stated beliefs, cognitive noise is correlated with overestimation of small and underestimation of large probabilities. Specifically, the mean error,* $e := \mathbb{E}[a^o - p]$*, when faced with the same situation satisfies:* $\partial e / \partial \lambda < 0$ *for* $p < d$ *and* $\partial e / \partial \lambda > 0$ *for* $p > d$*.*

ii. In Grether decompositions, when taking the default position to be in the interior $d \in (0, 1)$*, cognitive noise is correlated with base rate insensitivity and conservatism (likelihood ratio insensitivity).*

Proof.

(a) We consider the expression for e, that is, $\mathbb{E}[a^o - p]$ and compute the derivative:

$$\frac{\partial e}{\partial \lambda} = a^* - d \tag{14}$$

hence, we see that $\frac{\partial e}{\partial \lambda} > 0$ when:

$$a^* > d \tag{15}$$

$$p > u(d) \tag{16}$$

The result immediately follows.

(b) *i.* This follows given the result above and noting that $a^* = p$, that is, the utility maximizing choice is the Bayesian posterior, by *mutatis mutandis*.

ii. In a Grether regression framework, our prediction concerns how the derivative of the log posterior odds with respect to the log prior odds depends on λ. Base rate insensitivity that increases in cognitive noise would mean that:

$$\frac{\partial^2 lo}{\partial \lambda \partial \ln |o|} \geq 0 \tag{17}$$

and we begin by deriving this inequality. Since b is not a function of λ, the desired derivative may be expanded as:

$$\frac{\partial^2 lo}{\partial \lambda \partial \ln|o|} = \frac{\partial^2 lo}{\partial \lambda \partial b}\frac{db}{d\ln|o|} \tag{18}$$

Noting that:

$$b = \frac{e^{\ln|o|}}{1+e^{\ln|o|}} \tag{19}$$

so that we find:

$$\frac{\partial b}{\partial \ln|o|} = \frac{e^{\ln|o|}}{(1+e^{\ln|o|})^2} \tag{20}$$

$$= b(1-b) \geq 0 \tag{21}$$

Accordingly, our claim will be proven if:

$$\frac{\partial^2 lo}{\partial \lambda \partial b} \geq 0 \tag{22}$$

For simplicity, we define the following quantities:

$$e_1 = b^2(1-h)^{2n}h^{4k}(\lambda^2(1-d)^2 + d(1-d)) \tag{23}$$

$$e_2 = 2((1-h)h)^{2k+n}b(1-b)(1-\lambda^2)d(1-d) \tag{24}$$

$$e_3 = (1-h)^{4k}h^{2n}(1-b)^2 d(1-d(1-\lambda^2)) \tag{25}$$

and it may be seen that for $0 \leq k \leq n$ and $b, h, \lambda, d \in (0,1)$ that the quantities above are positive.

We then proceed to directly compute the value of this mixed partial and after combining and canceling out terms find it to be:

$$\frac{\partial^2 lo}{\partial \lambda \partial b} = \frac{((1-h)h)^{2k+n}(e_1+e_2+e_3)}{x^2 y^2} \tag{26}$$

where we have

$$x = (1-h)^{2k}h^n(1+d(1-\lambda)(1-b) - (1-d)(1-\lambda)(1-h)^n h^{2k} b) \tag{27}$$

$$y = (1-h)^n h^{2k}\lambda b + d(\lambda-1)((1-h)^{2k}h^n(b-1) - (1-h)^n h^{2k} b) \tag{28}$$

The denominator is the product of two squares and is accordingly non-negative. Since the terms $(1-h)h, e_1, e_2, e_3 > 0$ given our assumptions, we have accordingly shown that base rate insensitivity decreases in signal precision, λ. In other words, base rate insensitivity increases in cognitive noise, $(1-\lambda)$.

We now consider likelihood ratio insensitivity. If we define the log of the subject's log posterior odds as lo, then likelihood insensitivity that increases in cognitive noise would mean that:

$$\frac{\partial^2 lo}{\partial \lambda \partial \ln |LR|} \geq 0 \tag{29}$$

and we again begin by deriving this inequality. Since h is not a function of λ, the desired derivative may be expanded as:

$$\frac{\partial^2 lo}{\partial \lambda \partial \ln |LR|} = \frac{\partial^2 lo}{\partial \lambda \partial h} \frac{dh}{d \ln |LR|} \tag{30}$$

Now, the sign of $dg/d \ln |LR|$ is the same[18] as that of $d|LR|/dh$ and we see that the latter may computed to be:

$$\frac{d|LR|}{dh} = (2k-n)\left(\frac{h^{2k-n-1}}{(1-h)^{2k-n+1}}\right) \tag{31}$$

hence, its sign depends on that of $2k-n$. Accordingly, our claim will be proven if:

$$\operatorname{sgn}\left(\frac{\partial^2 lo}{\partial \lambda \partial h}\right) = \operatorname{sgn}(2k-n) \tag{32}$$

We may directly compute the value of this mixed partial and after combining and canceling out terms find it to be:

$$\frac{\partial^2 lo}{\partial \lambda \partial h} = \frac{(2k-n)((1-h)h)^{2k+n-1} b(1-b)(e_1+e_2+e_3)}{x^2 y^2} \tag{33}$$

Once again the denominator is the product of two squares and is accordingly non-negative. Since the terms $(1-h)h, e_1, e_2, e_3 > 0$ given our assumptions, we have accordingly proven our claim.

$\square$

[18]Recall, that $\frac{d \ln |LR|}{dh} = \frac{1}{|LR|} \frac{d|LR|}{dh}$.

Prediction 2. *The squared difference between the DM's decisions and his utility-maximizing decision decreases in signal precision on average when the signal is more informative than the prior. Stated formally, we have that the mean squared error*

$$\mathbb{E}[(a^o - a^*)^2] \tag{34}$$

satisfies:

$$\partial \mathbb{E}[(a^o - a^*)^2]/\partial N < 0 \tag{35}$$

when $N > n$.

Proof. We recall that for a given task:

$$a^o = \lambda S + (1-\lambda)d$$

where $NS \sim Bin(N, a^*)$ and $\lambda = N/(n+N)$. Accordingly, we may compute:

$$\mathbb{E}[(a^o - a^*)^2] = \frac{d^2n^2 - 2dn^2a^* + a^*(N + n^2a^* - Na^*)}{(n+N)^2} \tag{36}$$

taking the derivative we find that:

$$\frac{\partial \mathbb{E}[(a^o - a^*)^2]}{N} = \frac{(N-n)a^*(a^*-1) - 2n^2(a^*-d)^2}{(n+N)^3} \tag{37}$$

which is negative since $N > n$ and $a^* < 1$ thereby proving the claim. □

Prediction 3 (Cognitive default effects)**.**

(a) *An exogenous decrease in the cognitive default decision decreases stated certainty equivalents (shifts the probability weighting function down).*

(b) *An exogenous decrease in the cognitive default decision decreases stated beliefs.*

(c) *Both of these effects are stronger among DM with higher cognitive noise.*

Proof.

(a) As in Prediction 1, we recall that the probability weighting error, e is defined by:

$e := \lambda a^* + (1-\lambda)d - p$. Taking the derivative:

$$\frac{\partial e}{\partial d} = (1-\lambda) > 0 \tag{38}$$

yields the result.

(b) This follows given Prediction 4(a).

(c) Here we need only compute the mixed partial, $\partial^2 e/\partial d \partial \lambda$. Computing it results in:

$$\frac{\partial^2 e}{\partial d \partial \lambda} = -1 < 0 \tag{39}$$

yielding the result.

□

A.3 Cognitive Uncertainty and Cognitive Noise

As laid out in Section 2, the DM subjectively perceives his ex-ante utility-maximizing decision as a *distribution* conditional on his noisy signal. This means: while the agent is assumed to choose $a^o = \mathbb{E}[A_{S=s}]$, the underlying perceived posterior distribution of the utility-maximizing decision is Beta-distributed:

$$A_{S=s} \sim \; Beta\left(\underbrace{sN + nd}_{\equiv a}, \underbrace{N(1-s) + n(1-d)}_{\equiv b}\right) \tag{40}$$

where N is the signal precision. Now, let us restate our definition of cognitive uncertainty,

$$p_{CU} := \mathbb{P}(|A_{S=s} - \mathbb{E}[A_{S=s}]| > \kappa), \tag{41}$$

for fixed constant κ. The objective of this subsection is to establish that increases in signal precision decrease cognitive uncertainty. Below, we develop two sets of results about this relationship. First, Corollary 1 provides a limit argument showing that any desired decrease in cognitive uncertainty can be achieved by an increase in signal precision. Second, to shed light on the case with low signal precision, Theorem 1 shows that cognitive uncertainty decreases with signal precision when using the Gaussian approximation of the Beta distribution.

To begin, we prove:

Proposition 1. $\forall \kappa > 0, \forall \varepsilon > 0, \exists N \in \mathbb{N}$ *such that* $p_{CU} < \varepsilon$ *for* $N > N$.

Proof. By Chebyshev's inequality we see that for any positive number, κ:

$$p_{CU} < \frac{\text{Var}(A_{S=s})}{\kappa^2} \tag{42}$$

and, since $A_{S=s} \sim Beta(Ns + nd, N(1-s) + n(1-d))$ its variance is found to be:

$$\text{Var}(A_{S=s}) = \frac{(Ns+nd)(N(1-s)+n(1-d)),}{(n+N)^2(n+N+1)} = O(N^{-1}) \tag{43}$$

Accordingly, we find

$$\lim_{N \to \infty} p_{CU} = 0, \tag{44}$$

which in turn yields the proposition. □

This proposition yields the following corollary:

Corollary 1. *Holding the signal value constant* $\{S = s\}$ *and given a base level of signal precision,* N*, there exists a constant* Δn *such that a desired decrease in cognitive uncertainty may be accomplished by increasing the signal precision by more than* Δn*.*
Formally, given a base signal precision, N*, and a desired decrease in cognitive uncertainty,* $\delta \in (0, p_{CU})$*, there exists a quantity,* $\Delta n \in \mathbb{N}$*, such that:*

$$N\prime > N + \Delta n \rightarrow p_{CU} - p_{CU}\prime > \delta \tag{45}$$

with $N\prime$ *and* $p_{CU}\prime$ *being the new signal precision and cognitive uncertainty respectively.*

Proof. Given a signal precision N and cognitive uncertainty, p_{CU}, we may apply the proposition to $\varepsilon = p_{CU} - \delta$. We then find that $\Delta n = N - N$. The result follows. □

In essence, this corollary formally states the intuition that any desired decrease in cognitive uncertainty may be accomplished through an increase in signal precision.

In general, though, we will employ the standard Gaussian approximation,[19] which provides a good approximation when $\alpha = \beta$ even for smaller values of α, β. Under the Gaussian approximation we may illustrate our claim concerning the decrease of p_{CU} with respect to signal precision as follows:

[19] This is a commonly used approximation that follows from the fact that $X/(X+Y)$ has a Beta distribution if X, Y are Gamma(λ,1) random variables; that the Gamma distribution is asymptotically normal and an application of the Delta method.

Theorem 1. *Holding the signal* $\{S = s\}$ *constant, an increase in signal precision causes a decrease in cognitive uncertainty in the Gaussian approximation:*

$$\frac{\Delta p_{CU}}{\Delta N} < 0 \tag{46}$$

Proof. This trivially follows from the definition of p_{CU}, the fact that κ is constant and the fact that the variance decreases. $\square$

B Additional Figures

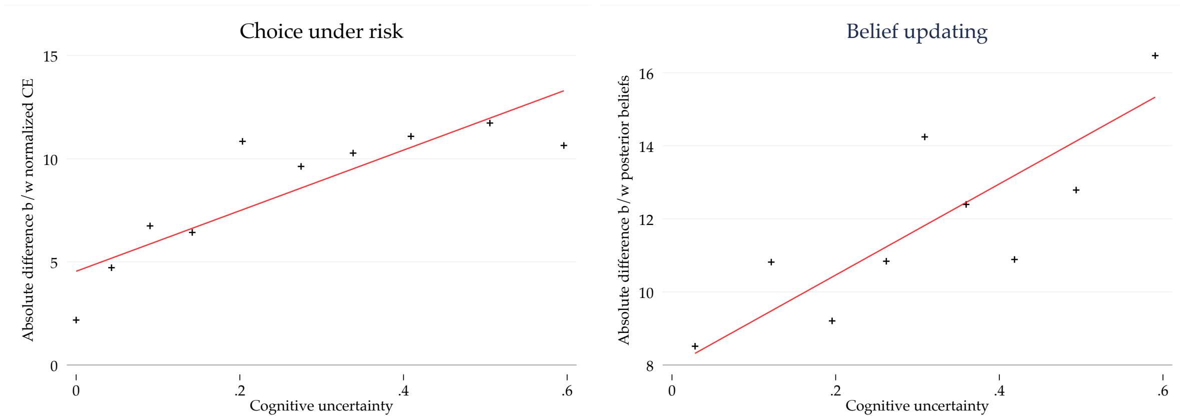


Figure 8: Link between cognitive uncertainty and across-task variability in decisions in *Risk main* (left panel, $N = 1,000$) and *Beliefs main* (right panel, $N = 1,000$). The y-axis captures the average absolute difference between the decisions that a subject took across two implementations of the exact same problem configuration. Cognitive uncertainty is winsorized at the 90th percentile for ease of readability.

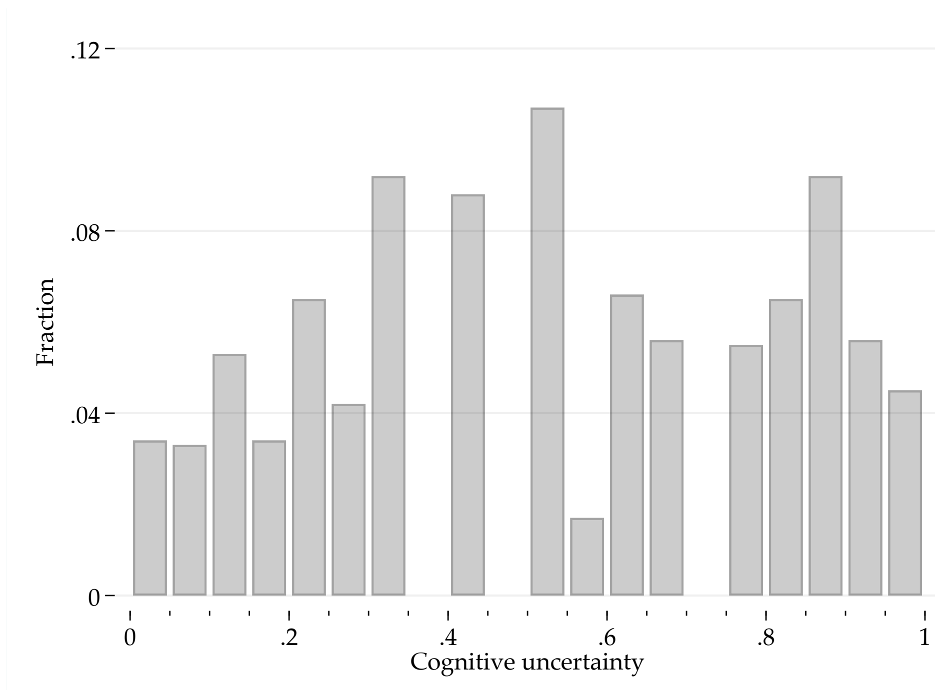


Figure 9: Histogram of cognitive uncertainty in stock market forecasts ($N = 1,000$).

C Additional Tables

Table 6: Cognitive uncertainty in belief updating as a function of sample size

	Dependent variable: Cognitive uncertainty			
	(1)	(2)	(3)	(4)
Sample size (Total number of drawn balls)	0.013***	0.014***	0.014***	0.014***
	(0.00)	(0.00)	(0.00)	(0.00)
Absolute difference between number of red and blue balls	-0.018***	-0.017***		
	(0.01)	(0.01)		
Distance b/w Bayesian posterior and 50			-0.0033***	-0.0033***
			(0.00)	(0.00)
Constant	0.34***	0.35***	0.42***	0.43***
	(0.01)	(0.04)	(0.01)	(0.04)
Demographic controls	No	Yes	No	Yes
Observations	4590	4590	4590	4590
R^2	0.00	0.03	0.04	0.06

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 7: Cognitive uncertainty and likelihood insensitivity in economic forecasts

	Dependent variable:			
	Posterior belief		Ln [Posterior odds]	
	(1)	(2)	(3)	(4)
Historical probability	0.64***	0.63***		
	(0.04)	(0.04)		
Historical probability × Cognitive uncertainty	-0.49***	-0.47***		
	(0.07)	(0.07)		
Cognitive Uncertainty	9.51**	10.3**	-0.76***	-0.67***
	(3.90)	(4.06)	(0.19)	(0.20)
Ln [Historical odds]			0.55***	0.55***
			(0.04)	(0.04)
Ln [Historical odds] × Cognitive uncertainty			-0.47***	-0.47***
			(0.06)	(0.06)
Constant	17.5***	6.18	-0.076	-0.77***
	(2.57)	(4.15)	(0.12)	(0.27)
Demographic controls	No	Yes	No	Yes
Observations	1000	1000	1000	1000
R^2	0.34	0.35	0.30	0.31

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: Complex numbers manipulation in *Risk main*

	Dependent variable: Normalized certainty equivalent			
	(1)	(2)	(3)	(4)
Payout probability	0.62***	0.62***	0.71***	0.70***
	(0.02)	(0.02)	(0.03)	(0.03)
Payout probability × 1 if *Complex numbers*	-0.23***	-0.23***	-0.16***	-0.16***
	(0.04)	(0.04)	(0.04)	(0.04)
1 if *Complex numbers*	6.83**	6.80**	5.26*	5.44*
	(2.80)	(2.75)	(2.85)	(2.80)
Payout probability × Cognitive uncertainty			-0.39***	-0.39***
			(0.06)	(0.06)
Cognitive Uncertainty			7.68*	6.02
			(4.55)	(4.53)
Constant	24.9***	32.4***	23.4***	32.1***
	(1.90)	(4.49)	(2.47)	(4.62)
Demographic controls	No	Yes	No	Yes
Observations	3000	3000	3000	3000
R^2	0.37	0.38	0.40	0.41

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 9: Compound lottery manipulation in *Risk replication*

	Dependent variable: Normalized certainty equivalent			
	(1)	(2)	(3)	(4)
Probability of payout	0.62***	0.62***	0.67***	0.66***
	(0.02)	(0.02)	(0.02)	(0.02)
Payout probability × 1 if compound lottery	-0.30***	-0.29***	-0.27***	-0.27***
	(0.03)	(0.03)	(0.03)	(0.03)
1 if compound	12.3***	12.3***	11.7***	11.6***
	(1.89)	(1.90)	(1.91)	(1.91)
Probability of payout × Cognitive uncertainty			-0.30***	-0.29***
			(0.06)	(0.06)
Cognitive uncertainty			8.07**	7.53*
			(3.95)	(3.96)
Demographic controls	No	Yes	No	Yes
Observations	1918	1918	1918	1918
R^2	0.44	0.45	0.45	0.46

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 10: Complex numbers manipulations in *Beliefs main*

	Dependent variable:					
	Posterior belief		Ln [Posterior odds]			
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.60***	0.71***				
	(0.02)	(0.03)				
Bayesian posterior × 1 if *Complex numbers*	-0.28***	-0.21***				
	(0.04)	(0.04)				
1 if *Complex numbers*	9.71***	8.18***	-0.26***	-0.17**	-0.26***	-0.17**
	(2.26)	(2.35)	(0.08)	(0.08)	(0.07)	(0.08)
Bayesian posterior × Cognitive uncertainty		-0.36***				
		(0.07)				
Cognitive Uncertainty		6.26		-0.63***		-0.62***
		(3.88)		(0.16)		(0.16)
Ln [Bayesian odds]			0.48***	0.55***		
			(0.03)	(0.04)		
Ln [Bayesian odds] × 1 if *Complex numbers*			-0.22***	-0.14***		
			(0.04)	(0.04)		
Ln [Bayesian odds] × Cognitive uncertainty				-0.32***		
				(0.07)		
Log[Prior Odds]					0.62***	0.73***
					(0.04)	(0.05)
Ln [Prior odds] × 1 if *Complex numbers*					-0.27***	-0.15**
					(0.06)	(0.06)
Log[Likelihood Ratio]					0.32***	0.32***
					(0.03)	(0.04)
Ln [Likelihood ratio] × 1 if *Complex numbers*					-0.17***	-0.14***
					(0.04)	(0.04)
Ln [Prior odds] × Cognitive uncertainty						-0.48***
						(0.09)
Ln [Likelihood ratio] × Cognitive uncertainty						-0.084
						(0.09)
Constant	22.4***	18.8***	0.12**	0.28	0.12**	0.27
	(1.64)	(3.59)	(0.05)	(0.19)	(0.05)	(0.19)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	3000	3000	3000	3000	3000	3000
R^2	0.37	0.40	0.33	0.36	0.37	0.40

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 11: Compound diagnosticity manipulation in *Beliefs replication*

	Dependent variable:					
	Posterior belief		Ln [Posterior odds]			
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.72***	0.80***				
	(0.02)	(0.02)				
Bayesian posterior × 1 if *Compound Lottery*	-0.51***	-0.47***				
	(0.03)	(0.03)				
1 if *Compound Lottery*	26.4***	25.4***	0.0051	0.033	0.0058	0.034
	(1.75)	(1.77)	(0.05)	(0.05)	(0.05)	(0.05)
Bayesian posterior × Cognitive uncertainty		-0.28***				
		(0.05)				
Cognitive uncertainty		10.5***		-0.14		-0.14
		(3.05)		(0.09)		(0.09)
Log[Posterior Odds]			0.43***	0.48***		
			(0.02)	(0.02)		
Ln [Bayesian odds] × 1 if *Compound Lottery*			-0.26***	-0.24***		
			(0.02)	(0.02)		
Ln [Bayesian odds] × Cognitive uncertainty				-0.20***		
				(0.04)		
Log [Likelihood ratio]					0.45***	0.50***
					(0.02)	(0.02)
Log [Likelihood ratio] × 1 if *Compound Lottery*					-0.28***	-0.25***
					(0.02)	(0.02)
Log [Likelihood ratio] × Cognitive uncertainty						-0.21***
						(0.04)
Constant	15.0***	16.0***	0.052*	0.25**	0.051*	0.25**
	(0.95)	(2.66)	(0.03)	(0.12)	(0.03)	(0.12)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1947	1947	1890	1890	1890	1890
R^2	0.60	0.61	0.52	0.53	0.53	0.54

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 12: Cognitive default manipulations

	Dependent variable:					
	Normalized CE in Risk repl.			Posterior belief in Beliefs repl.		
	(1)	(2)	(3)	(4)	(5)	(6)
1 if *Low Default*	-11.2***	-8.05***	-7.64***	-4.30***	-0.62	-0.65
	(2.06)	(2.61)	(2.61)	(1.06)	(1.82)	(1.82)
1 if *Low Default* × Cognitive uncertainty		-15.6**	-17.0**		-11.7***	-11.7***
		(7.84)	(7.96)		(3.61)	(3.62)
Cognitive uncertainty		-0.70	0.22		-3.42**	-3.45*
		(6.64)	(6.78)		(1.72)	(1.77)
Demographic controls	No	No	Yes	No	No	Yes
Observations	881	881	881	5372	5372	5372
R^2	0.04	0.06	0.07	0.00	0.01	0.01

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. All regressions exclude extreme outliers as pre-registered and discussed in Appendix Section E.3. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

D On Inverse S-Shapes

The objective of this section is to shed light on the pronounced non-linearities (inverse S-shapes) in the response patterns established in the empirical literatures that we built on. After all, the neo-additive function that we derived in Section 2 posits that decisions are a *linear* function of objective probabilities. While these linear representations are popular due to their simplicity, they have the drawback that they do not capture the canonical inverse S-shaped response patterns summarized in Figure 1.

In particular, recall that the estimating equation for our model estimates in eq. (4) is prima facie linear in p. However, a crucial observation is that, empirically, cognitive uncertainty, p_{CU}, is not constant across problems with varying objective probability, p. The left-hand panels of Figure 10 show the empirical relationship between measured cognitive uncertainty and objective probabilities in our main experiments. In the top left panel, the x-axis shows the objective payout probability of a gamble. In the bottom left panel, the x-axis denotes the Bayesian posterior in belief updating tasks. Across domains, cognitive uncertainty exhibits a pronounced hump shape: our experimental participants tell us that they find it easier to think about lotteries with extreme payout probabilities, or about belief formation tasks that have extreme solutions. Intuitively, such higher cognitive noise at intermediate probabilities may generate the well-known empirical pattern that decisions are less sensitive to variation in objective probabilities over the intermediate probability range.

To investigate whether these non-linearities could generate an inverse S-shaped decision function, we return to our model estimations. Specifically, to reduce the role of attenuating measurement error in the cognitive uncertainty measurement, we re-estimate equation (4) by replacing each participant's stated CU for a given decision problem with the average CU for a given objective probability p. For example, in choice under risk, we compute the average level of cognitive uncertainty for each payout probability, and then estimate the model based on these average levels of CU. This is justified for the purposes of the present exercise because here our focus is precisely on the variation in cognitive uncertainty *across objective probabilities* rather than across subjects.

The right-hand panels of Figure 10 show the fit of these model estimates. We see that the CU model accounts for the non-linearity in decisions and attributes it partly to the hump-shaped relationship between cognitive uncertainty and objective probabilities.

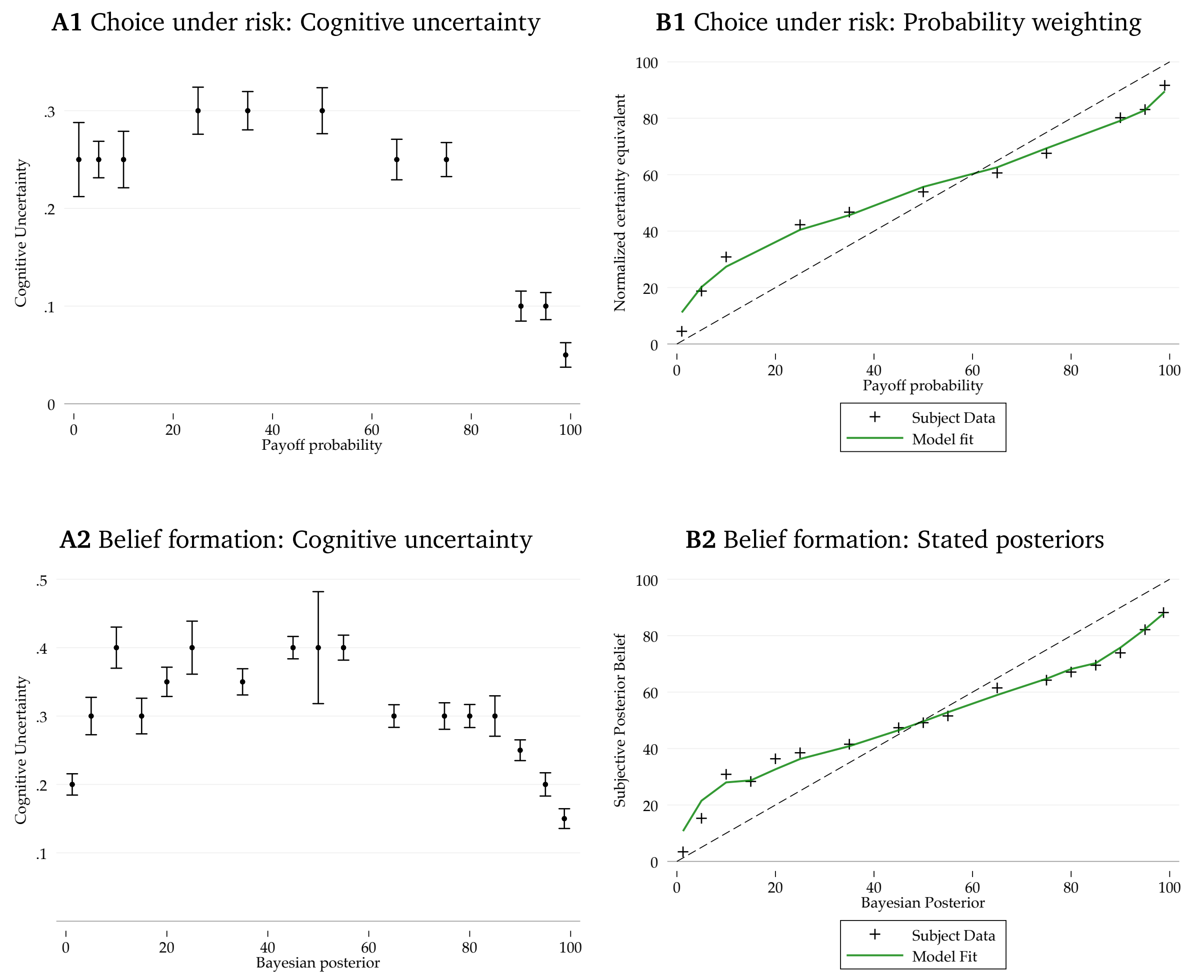


Figure 10: Left panels: median cognitive uncertainty as a function of probabilities. Right panels: fitted values of estimates of eq. (4), where for each decision problem a subject is assigned the across-subject average CU for this problem. Figure B2 displays bins with 30 or more observations. (Risk: $N = 4,524$, Beliefs: $N = 4,590$)

E Details on Replication (Earlier Experiments)

We here briefly summarize the experiments that formed the core of our earlier NBER working paper with the same title. For more details please refer to Enke and Graeber (2019).

E.1 Decision Tasks

Choice under risk. In experiment *Risk replication*, we followed a large set of previous works and implemented multiple price lists that elicit certainty equivalents for lotteries (see, e.g. Tversky and Kahneman, 1992; Bruhin et al., 2010; Bernheim and Sprenger, 2019). Each subject completed a total of six price lists. On the left-hand side of the decision screen, a simple lottery was shown that paid $\$y$ with probability p and nothing otherwise. On the right-hand side, a safe payment $\$z$ was offered that increased by $1 for each row that one proceeds down the list. As in Bruhin et al. (2010) and Bernheim and Sprenger (2019), the end points of the list were given by $z = 0$ and $z = y$.

Throughout, we did not allow for multiple switching points. This facilitates a simpler elicitation of cognitive uncertainty. To enforce unique switching points, we implemented an auto-completion mode: if a subject chose Option A in a given row, the computer implemented Option A also for all rows above this row. Likewise, if a subject chose Option B in a given row, the computer instantaneously ticked Option B in all lower rows. However, participants could always revise their decision and the auto-completion before moving on.

The parameters y and p were drawn uniformly randomly and independently from the sets $y \in \{15, 20, 25\}$ and $p \in \{5, 10, 25, 50, 75, 90, 95\}$. We implemented both gain and loss gambles, where the loss amounts are the mirror images of y. In the case of loss gambles, the lowest safe payment was given by $z = -\$y$ and the highest one by $z = \$0$. In loss choice lists, subjects received a monetary endowment of $\$y$ from which potential losses were deducted. Out of the six choice lists that each subject completed, three concerned loss gambles and three gain gambles. We presented either all loss gambles or all gain gambles first, in randomized order.

Finally, with probability 1/3, a choice list was presented in a compound lottery format, as described in the main text.

Belief updating. The procedures in *Beliefs replication* were essentially the same as in *Beliefs main*, with slight changes in the experimental instructions used.

Survey expectations. As in the main experiments, we elicited expectations about the 12-months return of the S&P 500. In addition, we also measured inflation expectations:

> *[Explanation of inflation rates.] We randomly picked a year X between 1980 and 2018. Imagine that, at the beginning of year X, the set of products that is used to compute the inflation rate cost $100. What do you think is the probability that, at the end of that same year, the same set of products cost less than $y? (In other words, what do you think is the probability that the inflation rate in year X was lower than z%?)*

Finally, we also elicited respondents' beliefs about the national income distribution:

Assume that in 2018, we randomly picked a household in the United States. What do you think is the probability that this household earned less than USD y in 2018, before taxes and deductions?

E.2 Measuring Cognitive Uncertainty

Choice under risk. After stating a switching interval in a price list, a participant was reminded of their valuation (switching interval) for the lottery on the previous price list screen. They were then asked to indicate how certain they are that to them the lottery is worth exactly the same as their previously indicated certainty equivalent. To answer this question, subjects used a slider to calibrate the statement "I am certain that the lottery is worth between a and b to me." If the participant moved the slider to the very right, a and b corresponded to the previously indicated switching interval. For each of the 20 possible ticks that the slider was moved to the left, a decreased and b increased by 25 cents, in real time. In gain lotteries, a was bounded from below by zero and b bounded from above by the lottery's upside. Analogously, for losses, a was bounded from below by the lottery's downside and b from above by zero. The slider was initialized at cognitive uncertainty of zero, but subjects had to click somewhere on the slider in order to be able to proceed.

Belief updating. The instructions introduced the concept of an "optimal guess." This guess, we explained to subjects, uses the laws of probability to compute a statistically correct statement of the probability that either bag was drawn, based on Bayes' rule. We highlighted that this optimal guess does not rely on information that the subject does not have.

After subjects had indicated their probabilistic belief that either bag was drawn, the next decision screen elicited cognitive uncertainty. Here, we asked subjects how certain they are that their own guess equals the optimal guess for this task. Operationally, similarly to the case of choice under risk, subjects navigated a slider to calibrate the statement "I am certain that the optimal guess is between a and b.", where a and b collapsed to the subject's own previously indicated guess in case the slider was moved to the very right. For each of the 30 possible ticks that the slider was moved to the left, a decreased and b increased by one percentage point. a was bounded from below by zero and b bounded from above by 100. Again, the slider was initialized at cognitive uncertainty of zero and we forced subjects to click somewhere on the slider to be able to proceed.

E.3 Logistics and Pre-Registration

Based on a pre-registration, we recruited $N = 700$ completes. We restricted our sample to AMT workers that were registered in the United States, but we did not impose additional participation constraints. After reading the instructions, participants completed three comprehension questions. Participants who answered one or more control questions incorrectly were immediately routed out of the experiment and do not count towards the number of completes. In addition, towards the end of the experiment, a screen contained a simple attention check. Subjects that answered this attention check incorrectly are excluded from the data analysis and replaced by a new complete, as specified in the pre-registration. In total, 62% of all prospective participants were screened out of the experiment in the comprehension checks. Of those subjects that passed, 2% were screened out in the attention check.

In terms of timeline, subjects first completed six of the choice under risk tasks. Then, we elicited their survey expectations about various economic variables, as discussed below. Finally, participants completed a short demographic questionnaire and an eight-item Raven matrices IQ test.

Participants received a completion fee of \$1.70. In addition, each participant earned a bonus. The experiment comprised three financially incentivized parts: the risky choice lists, the survey expectations questions, and the Raven IQ test. For each subject, one of these parts of the experiment was randomly selected for payment. If choice under risk was selected, a randomly selected decision from a randomly selected choice list was paid out.

The experiments reported in this appendix were pre-registered in the AEA RCT registry, see `https://www.socialscienceregistry.org/trials/4493`. As we pre-registered, all regression analyses of the replication data exclude extreme outliers. In choice under risk, these are observations for which (i) the normalized certainty equivalent is strictly larger than 95% while the objective payout probability is at most 10%, or (ii) the normalized certainty equivalent is strictly less than 5% while the objective payout probability is at least 90%. In belief updating, outliers are defined analogously.

E.4 Results for Choice Under Risk

Table 13 provides a regression analysis of the data. As in the main paper, our object of interest is the extent to which a subject's normalized certainty equivalent is (in)sensitive to variations in the probability of the non-zero payout state. Thus, we regress a participant's absolute normalized certainty equivalent on (i) the probability of receiving the non-zero

Table 13: Inelasticity with respect to probability and cognitive uncertainty in *Risk replication*

	Dependent variable: Absolute normalized certainty equivalent					
	Gains		Losses		Pooled	
	(1)	(2)	(3)	(4)	(5)	(6)
Probability of payout	0.68***	0.68***	0.59***	0.59***	0.65***	0.65***
	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)
Probability of payout × Cognitive uncertainty	-0.41***	-0.41***	-0.20**	-0.20**	-0.31***	-0.31***
	(0.09)	(0.09)	(0.09)	(0.09)	(0.07)	(0.07)
Cognitive uncertainty	11.6**	11.3**	14.8***	14.8***	13.5***	13.5***
	(5.19)	(5.18)	(5.26)	(5.17)	(3.84)	(3.84)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1271	1271	1254	1254	2525	2525
R^2	0.54	0.54	0.41	0.41	0.47	0.47

Notes. Demographic controls include age, gender, college education and performance on the Raven matrices test. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. The dependent variable is a subject's absolute normalized certainty equivalent. The sample includes choices from all baseline gambles with strictly interior payout probabilities. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

gain / loss; (ii) cognitive uncertainty; and (iii) an interaction term.

The results show that higher cognitive uncertainty is associated with lower responsiveness to variations in objective probabilities, in both the gains and the loss domain. In terms of quantitative magnitude, the regression coefficients suggest that with cognitive uncertainty of zero, the slope of the neo-additive weighting function is given by 0.65, yet it is only 0.34 for maximum cognitive uncertainty of one. A different way to gauge quantitative magnitudes is to standardize cognitive uncertainty into a z-score. When doing so, the regression results (not reported) suggest that an one standard deviation increase in cognitive uncertainty decreases the slope of the neo-additive weighting function by about 0.11. These are arguably large effect sizes that underscore the quantitative relevance of cognitive uncertainty in generating probability weighting.

E.5 Results For Belief Updating

Columns (1)–(3) of Table 14 provide an econometric analysis, which again corresponds to the neo-additive weighting function. Here, we regress a subject's stated posterior on (i) the Bayesian posterior; (ii) cognitive uncertainty; and (iii) their interaction term. We find that with cognitive uncertainty of zero, the slope of the neo-additive weighting function is given by 0.83 but it is only 0.41 with cognitive uncertainty of one.

Table 14: Belief updating: Regression analyses

	Dependent variable:					
	Posterior belief		Ln [Posterior odds]			
	(1)	(2)	(3)	(4)	(5)	(6)
Bayesian posterior	0.80***	0.80***				
	(0.01)	(0.01)				
Bayesian posterior × Cognitive uncertainty	-0.39***	-0.39***				
	(0.04)	(0.04)				
Cognitive uncertainty	16.6***	16.5***	-0.17**	-0.17**	-0.16**	-0.16**
	(2.32)	(2.32)	(0.07)	(0.07)	(0.07)	(0.07)
Log[Posterior Odds]			0.50***	0.50***	0.58***	0.58***
			(0.01)	(0.01)	(0.03)	(0.03)
Ln [Bayesian odds] × Cognitive uncertainty			-0.24***	-0.24***		
			(0.04)	(0.04)		
Log [Likelihood ratio]					-0.099**	-0.10**
					(0.04)	(0.04)
Log [Prior odds] × Cognitive uncertainty					-0.40***	-0.40***
					(0.07)	(0.07)
Log [Likelihood ratio] × Cognitive uncertainty					-0.20***	-0.19***
					(0.05)	(0.05)
Constant	11.1***	11.2***	0.046	0.036	0.043	0.035
	(0.90)	(1.86)	(0.03)	(0.10)	(0.03)	(0.10)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	3187	3187	3012	3012	3012	3012
R^2	0.73	0.73	0.63	0.63	0.63	0.63

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. To avoid a mechanical loss of observations resulting from the log odds definition, the log posterior odds in columns (3)–(6) are computed by replacing stated posterior beliefs of 100% and 0% by 99% and 1%, respectively. The results are virtually identical without this replacement. Demographic controls include age, gender, college education and performance on a Raven matrices test. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

E.6 Results for Economic Forecasts

Table 15 summarizes the results. Again, we see that the responsiveness of stated expectations with respect to the objective / historical probabilities strongly decreases in measured cognitive uncertainty.

Table 15: Survey expectations: Regression analyses

	Dependent variable: Probability estimate about:					
	Income distr.		Stock market		Inflation rate	
	(1)	(2)	(3)	(4)	(5)	(6)
Objective probability	0.90***	0.90***	0.69***	0.69***	0.76***	0.76***
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Objective probability × Cognitive uncertainty	-0.41***	-0.41***	-0.53***	-0.52***	-0.60***	-0.60***
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Cognitive uncertainty	18.9***	18.9***	24.2***	24.6***	27.5***	27.4***
	(2.37)	(2.37)	(2.27)	(2.31)	(2.86)	(2.86)
Demographic controls	No	Yes	No	Yes	No	Yes
Observations	1980	1980	1892	1892	1848	1848
R^2	0.83	0.83	0.52	0.52	0.54	0.54

Notes. OLS estimates, robust standard errors (in parentheses) are clustered at the subject level. Demographic controls include age, gender, college education and performance on the Raven matrices test. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

F Experimental Instructions and Control Questions

Below we provide screenshots of the instructions, control questions and decision screens of the main experiments. Corresponding information for the self-replication experiments as well as the default manipulation experiments can be found in a working paper version of this paper (Enke and Graeber, 2019).

F.1 Treatment *Risk Main*

Part 1: Instructions (1/3)

Please read these instructions carefully. "If you correctly answer three qualification questions and complete this study you will additionally receive a fixed bonus of $1.50.

In this study, there are various lotteries with different probabilities of winning. An example is:

With probability **5%: Get $ 18**
With probability **95%: Get $ 0**

The lotteries will actually be played out by the computer and determine your earnings in exactly the way we describe on the following pages.

Decision screen 1

You will be asked to choose which of two payment options you prefer: a lottery or a payment that you would receive *with certainty*. For the lottery displayed above, for example, you would be asked the following list of questions:

Question #		Option A		Option B
1	Would you rather have:		or	$0.01 with certainty
2	Would you rather have:		or	$0.02 with certainty
3	Would you rather have:	With probability **5 %** : **Get $18** With probability **95 %**: **Get $0**	or	$0.03 with certainty
...	...		...	...
1,799	Would you rather have:		or	$17.99 with certainty
1,800	Would you rather have:		or	$18.00 with certainty

In each question you pick either Option A (the lottery) or Option B (the certain payment). We will randomly pick one question and pay you according to what you chose on that one question. Each question is equally likely to be chosen for payment.

We assume that you prefer the lottery (Option A) when the certain payment in Option B is very small, but that you switch to preferring Option B when the certain payment is very large. Therefore, to save time, we will not actually ask you 1,800 questions. Instead, we will simply ask you to state the amount at which you would switch from Option A to B in the table. This is the same as asking you:

Which certain payment is worth as much to you as this lottery?

Based on your response to this question, we can then fill out your answers to all 1,800 questions in the list. That is, you will choose Option A for all questions with a lower certain payment than your response, and Option B for all questions with a certain payment that is at least as high as your response.

Example: Suppose you were to tell us that a given lottery is worth as much to you as a certain payment of $12. Then this would mean that you prefer the lottery over any certain payment that is smaller than $12. On the other hand, it would also mean that you prefer any certain payment greater than $12 over the lottery.

Next

Part 1: Instructions (2/3)

Example

With probability **35% : Get $ 18**
With probability **65% : Get $0**

Which certain payment is worth as much to you as this lottery?

Getting $ ______ with certainty is worth as much to me as this lottery.

Next

Part 1 of this study: Instructions (3/3)

Decision screen 2: Your certainty about your decision

When you make your decision, you may feel **uncertain about which certain payment is worth as much to you as the lottery**. On decision screen 2, we will ask you to select a button to indicate **how certain** you are that you value the lottery within +/- $0.50 of the amount you entered on the previous screen.

Example

Suppose that on the first decision screen you indicated that you value a 5% chance of getting $18 as much as receiving $5.50 with certainty. Your second decision screen would look like this:

With probability **5% : Get $18**
With probability **95% : Get $0**

How certain are you that you actually value this lottery somewhere between getting $5.00 and $6.00?

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

Qualification Questions

Important: If you answer any of the below questions incorrectly, the study ends after this page and you will receive your participation payment, but you do not qualify for a bonus.

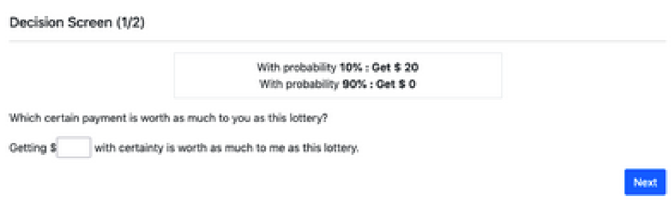
1. Which one of the following statements is correct if the following lottery is played for you?

With probability **60%: Get $ 15**
With probability **40%: Get $ 5**

Please select one of the statements:

- ○ It is possible that I get paid both $15 and $5, i.e., I may receive a total amount of $20 from this lottery.
- ○ I receive EITHER $15 OR $5 from this lottery.
- ○ It is possible that I receive no money from this lottery.

2. Jon is faced with the following lottery:


Decision Screen (1/2)

With probability **10% : Get $ 20**
With probability **90% : Get $ 0**

Which certain payment is worth as much to you as this lottery?

Getting $ [ ] with certainty is worth as much to me as this lottery.

Next

Suppose Jon enters $13. Which one of the following statements is correct? :

- ○ Jon would prefer $12 over the lottery.
- ○ Jon would prefer the lottery over any certain payment.
- ○ Jon would prefer the lottery over $10.

3. Now imagine that Jon is uncertain about how much exactly the lottery is worth to him.
Jon; however, is 60% certain that the lottery is worth between $12.5 and $13.5 to him. Please select the correct button to accurately reflect this level of certainty:

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

Task 1 of 12

Decision Screen (1/2)

With probability **65% : Get $ 22**
With probability **35% : Get $ 0**

Which certain payment is worth as much to you as this lottery?

Getting $ ______ with certainty is worth as much to me as this lottery.

Next

Task 1 of 12

Decision Screen (2/2)

With probability **65% : Get $ 22**
With probability **35% : Get $ 0**

Your decision on the previous screen indicates that you value this lottery as much as getting $1.00 with certainty.

How certain are you that you actually value this lottery somewhere between getting $0.50 and $1.50?

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

F.2 Treatment *Beliefs Main*

Part 1 of this Study: Instructions (1/4)

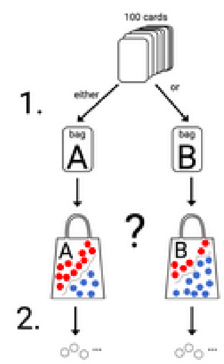
Please read these instructions carefully. There will be bonus qualification questions based on the instructions. If you do not answer all bonus qualification questions correctly, you will not earn an additional bonus for this study.

In this study, you will be asked to complete **12 guessing tasks**.
In each guessing task, there are two bags, "Bag A" and "Bag B". A bag contains exactly 100 balls, each of which is either red or blue. However, the two bags contain **different numbers of red vs. blue balls:** one bag contains more red balls, the other contains more blue balls. One of the bags is selected at random by the computer as described below. You will not observe which bag was selected. Instead, the computer will randomly draw one or several balls from the secretly selected bag, and will show these balls to you. Your task is to **guess which bag was selected** based on the available information. The exact procedure is described below:

Task setup

- There is a deck of cards that consists of 100 cards. Each card in the deck either has "Bag A" or "Bag B" written on it. You will be informed about **how many** of these 100 cards have "Bag A" or "Bag B" written on them.
- There are two bags, "Bag A" and "Bag B". Each bag contains 100 balls. One of the bags contains more red balls, and the other bag contains more blue balls. You will be informed about exactly **how many red and blue balls** each bag contains.

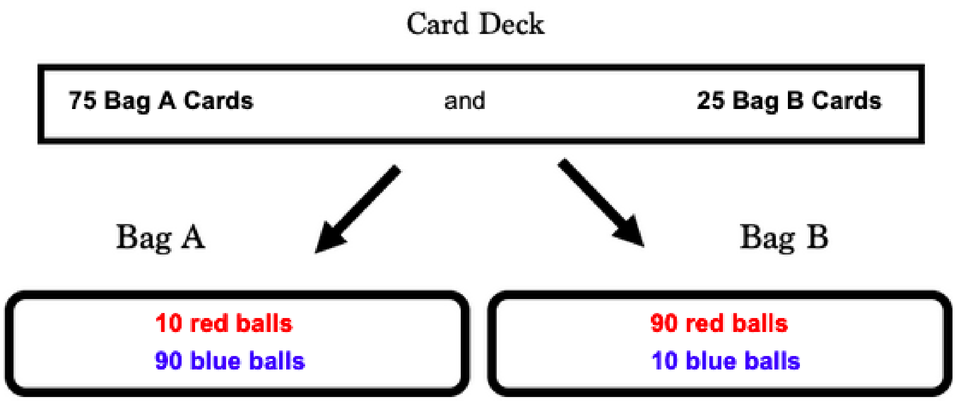
Sequence of events



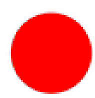
1. The computer **randomly selects one** of the 100 cards, with equal probability.
   If a "Bag A" card was drawn, Bag A is selected.
   If a "Bag B" card was drawn, Bag B is selected.
2. Next, the computer **randomly draws one or more** balls from the secretly selected bag. Each ball is equally likely to get selected. Importantly, if more than one ball is drawn, the computer draws these balls **with replacement**. This means that after a ball has been drawn and taken out of the bag, it gets replaced by a ball of the same color. Thus, the probability of a red or blue ball being drawn does not depend on whether previous draws were red or blue.
   The computer will **inform you about the color** of the randomly drawn balls.

Instructions (2/4)

Here's an example of what a decision screen will look like. At the top of the screen, you see the information you need to solve the guessing task. At the bottom, you enter your guess.



The computer randomly selected a card from the deck and then randomly drew the following ball(s) from the selected bag:



Decision 1: Your guess

Given that these balls were drawn, **how likely** do you think it is that Bag A (as opposed to Bag B) has been selected (in %)?

I believe it is [ ] % likely that Bag A was selected.

Submit your guess

Instructions (3/4)

The optimal guess

Using the laws of probability, the computer computes a **statistically correct statement of the probability that Bag A was selected**, based on all the information available to you. This **optimal guess** does not rely on information that you do not have. It is just the best possible (this means: payoff-maximizing) estimate given the available information. In technical terms, this guess is based on a statistical rule called Bayes' Law.

Decision screen 2: Your certainty about your guess

In any given task, you may actually be **uncertain about whether your probability guess corresponds to the optimal guess**. On decision screen 2, we will ask you to select a button to indicate **how certain** you are that the optimal guess is within ± 1 percentage point of your answer.

Example

Suppose that you stated that Bag A was selected with probability 80%. Your second decision screen would then look like this:

Your decision on the previous screen indicates that you believe there is a 80% chance of Bag A having been selected.

How certain are you that the optimal guess is somewhere between 79.0% and 81.0%?

○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

Instructions (4/4)

Summary: Sequence of events in each task

You will be asked to complete 12 guessing tasks. For each task, there will be **2 decision screens:**

Decision screen 1

You will be asked to enter probabilities that express **how likely** you think it is that Bag A or Bag B has been selected.

The following aspects will vary across the different guessing tasks:

- How many Bag A cards and Bag B cards are contained in the card deck.
- How many red and blue balls are contained in bags A and B.
- The number and color of the drawn balls.

Decision screen 2

You will be asked to indicate **how certain** you are that the guess you provided on decision screen 1 **is close to the optimal guess in this task**.

Your payment for part 1

You can potentially earn $5.00 with your guess. Your probability of winning $5.00 is higher the higher the probability you assign to the bag that actually got selected. If Part 1 determines your payment, the computer will randomly select one of your guesses to be relevant for your payment.

To maximize your earnings, you should therefore simply try to guess as accurately as possible which bag was selected. In case you're interested, the specific formula that determines whether you get the prize is explained here.

Next

Qualification Questions

Important: If you answer any of the below questions incorrectly, the study ends after this page and you will receive your participation payment, but you do not qualify for a bonus.

1. Which statement about the number of cards corresponding to each bag is correct?
   - ◯ The number of "Bag A" cards is the same in all tasks.
   - ◯ The exact number of cards corresponding to each bag may vary across tasks.

2. Suppose there are 70 Bag A cards and 30 Bag B cards in the deck. Further suppose there are more red balls in Bag A than there are red balls in Bag B. Next, one red ball is drawn from the secretly selected bag.
   Which one of the following guesses is closest to the statistically optimal guess?
   - ◯ 25%
   - ◯ 50%
   - ◯ 80%

3. Suppose that in a given task you guess that the probability that Bag A was selected is 70%. Further suppose you are 60% certain that the optimal guess is actually somewhere between 69% and 71% (close to your guess). Please select the appropriate button below to indicate this level of certainty.

◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯	◯
0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

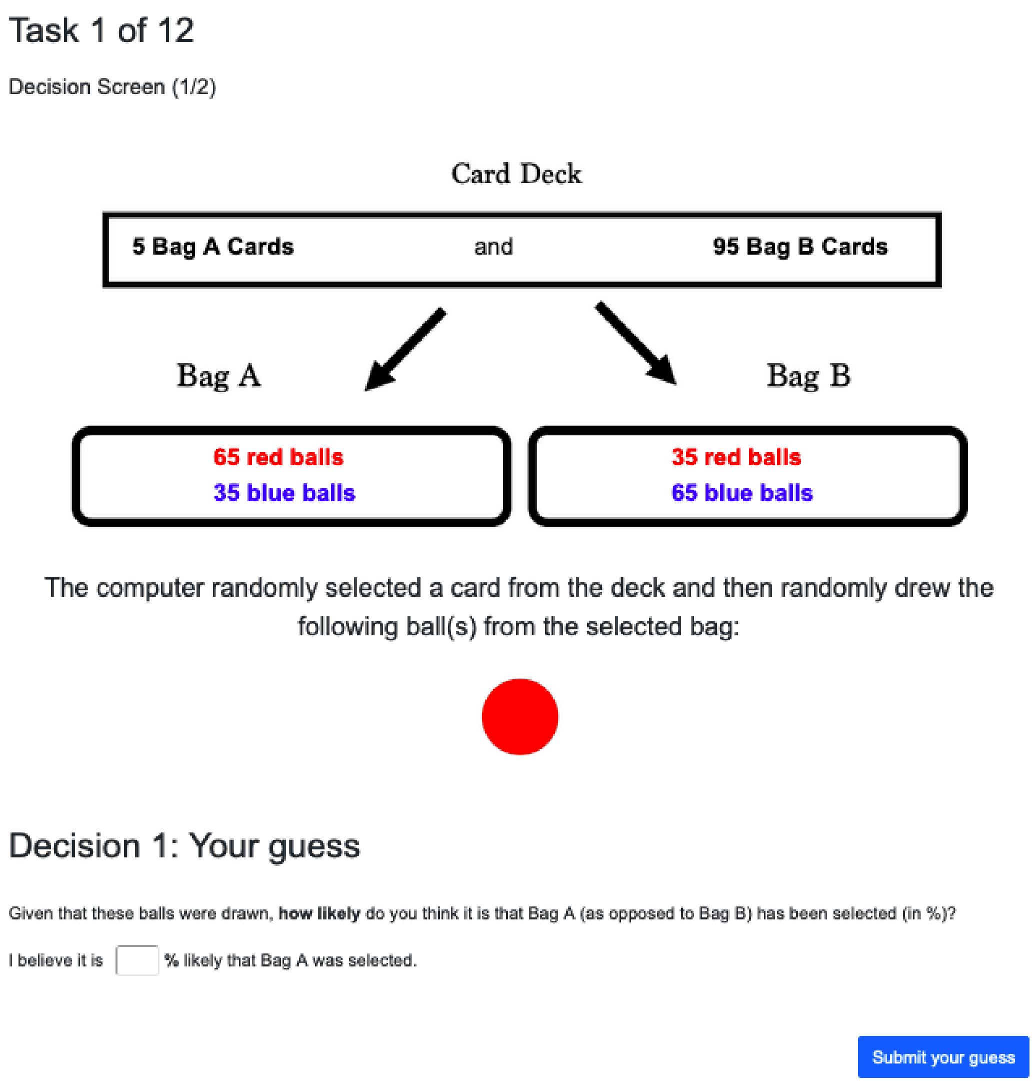


Figure 11: Beliefs elicitation

Task 1 of 12

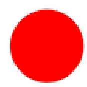
Decision Screen (2/2)

Card Deck

5 Bag A Cards	and	95 Bag B Cards

Bag A	Bag B
65 red balls 35 blue balls	35 red balls 65 blue balls

The computer randomly selected a card from the deck and then randomly drew the following ball(s) from the selected bag:



Decision 2: Your certainty

Your decision on the previous screen indicates that you believe there is a 1.0% chance that Bag A was selected.

How certain are you that the optimal guess is somewhere between 0.0% and 2.0%?

0% 5% 10% 15% 20% 25% 30% 35% 40% 45% 50% 55% 60% 65% 70% 75% 80% 85% 90% 95% 100%

very uncertain **completely certain**

Next

F.3 Stock market expectations

Question about the Stock Market

The S&P 500 is an American stock market index that includes 500 of the largest companies based in the United States.

Jon invested $100 in the S&P 500 today.
What is the **percent chance** that the **value of his investment will be less than $ 123** in one year from now?

In other words, what do you think is the percent chance that Jon will gain less than $23.0 or lose money on his investment over the next year?

Percent chance that Jons investment will be worth **less than $ 123** : %

Next

Your certainty about your estimate

On the previous screen, you indicated that you think there's a 1.0 % chance that a $100 investment into the S&P 500 today will be worth less than $123 in one year from now.

How certain are you that the statistically optimal guess (given the information you have) is somewhere between 0.0% and 2.0%?

0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%

very uncertain **completely certain**

Next

F.4 Complex numbers in choice under risk

Information

In the next few rounds, there will be an additional complication. **In the following lotteries some of the amounts and/or probabilities may be expressed as mathematical expressions.** The lotteries will still be played out by the computer in exactly the way we describe.

Example

With probability **(6 x 9) / 3 - 7% : Get $ 18**
With probability **100 - ((6 x 9)/ 3 - 7)% : Get $0**

Which certain payment is worth as much to you as this lottery?

Getting $[   ] with certainty is worth as much to me as this lottery.

Task 7 of 12

Decision Screen (1/2)

With probability **(6 x 14) / 6 + 61% : Get $ 22**
With probability **100 - ((6 x 14) / 6 + 61)% : Get $ 0**

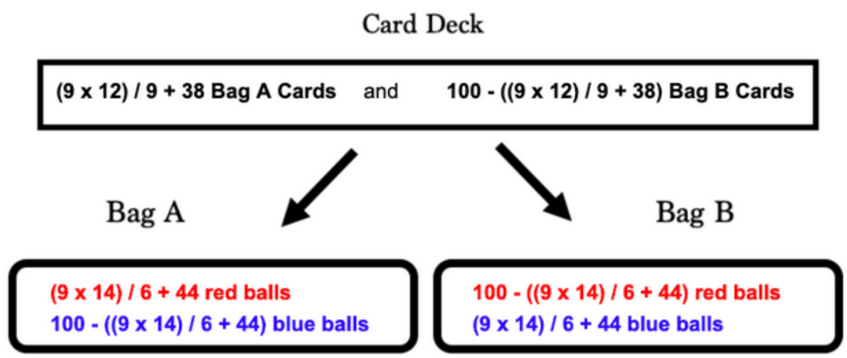
Which certain payment is worth as much to you as this lottery?

Getting $[   ] with certainty is worth as much to me as this lottery.

F.5 Complex numbers in belief updating

Information:

In the following rounds the information (i) on the number of Bag A and Bag B cards in the deck and (ii) on the number of red and blue balls in bags A and B will be expressed as mathematical expressions. The scenarios will still be played out by the computer in exactly the way we described. An example is:



The computer randomly selected a card from the deck and then randomly drew the following ball(s) from the selected bag:

Task 7 of 12

Decision Screen (1/2)

Card Deck

(5 x 10) / 5 + 85 Bag A Cards and 100 - ((5 x 10) / 5 + 85) Bag B Cards

Bag A

(4 x 18) / 2 + 29 red balls
100 - ((4 x 18) / 2 + 29) blue balls

Bag B

100 - ((4 x 18) / 2 + 29) red balls
(4 x 18) / 2 + 29 blue balls

The computer randomly selected a card from the deck and then randomly drew the following ball(s) from the selected bag:

Decision 1: Your guess

Given that these balls were drawn, **how likely** do you think it is that Bag A (as opposed to Bag B) has been selected (in %)?

I believe it is [ ] % likely that Bag A was selected.