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Small Project 1

First Deliverable: Faster versus Accurate

Mesh Size will naturally affect any analysis, so it's important to understand how different size and speeds will change the results from each analysis. Altair Inspire uses element configuration and element size for its analysis.

Stress Calculation: Analytical Approach

The model of a notched flat bar that will be used has the following dimensions:

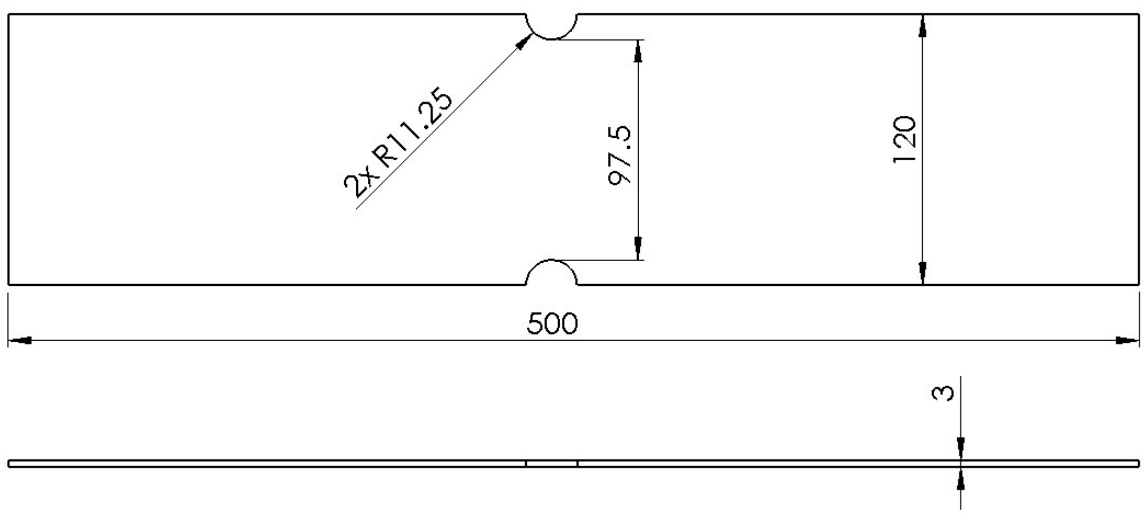


Figure 1: Dimensions of a notched flat bar.

The theoretical value for a geometric stress concentration factor, K_t, for a notched flat bar in axial tension can be found using the following equation:

$$K_t = A\left(\frac{r}{d}\right)^b \tag{1}$$

Where, the values for A and b come from a table of values that depend on $\frac{D}{d}$ from Norton's Machine Design, An Integrated Approach, 5th ed. The values of r, d, and D are defined. In this case, those values are found to be 11.25mm, 97.5mm, and 120mm, respectively, to achieve a stress concentration factor, K_t, of 2.2993 as seen in the Table 1. The nominal tension stress is found using the following equation:

$$\sigma_{\text{nom}} = \frac{P}{A} \tag{2}$$
$$= \frac{P}{(D-2r)\,h} \tag{3}$$

Where, P is axial force of 20,000 N, and h is the thickness of the plate. The maximum tension stress is found through manipulating the following equation and solving for maximum tension stress:

$$K_t = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}} \tag{4}$$
$$\sigma_{\text{max}} = K_t\sigma_{\text{nom}} \tag{5}$$

Table 1: Results from stress calculation

Parameter	**Symbol**	**Value**
Stress concentration factor	K_t	2.2993
Nominal tension stress	σ_{nom}	68.4 MPa
Maximum tension stress at the end of the notch	σ_{max}	157.2 MPa

FEM Analyses of the Plate

The different element sizes below in Table 2 will be used to investigate the accuracy of the results.

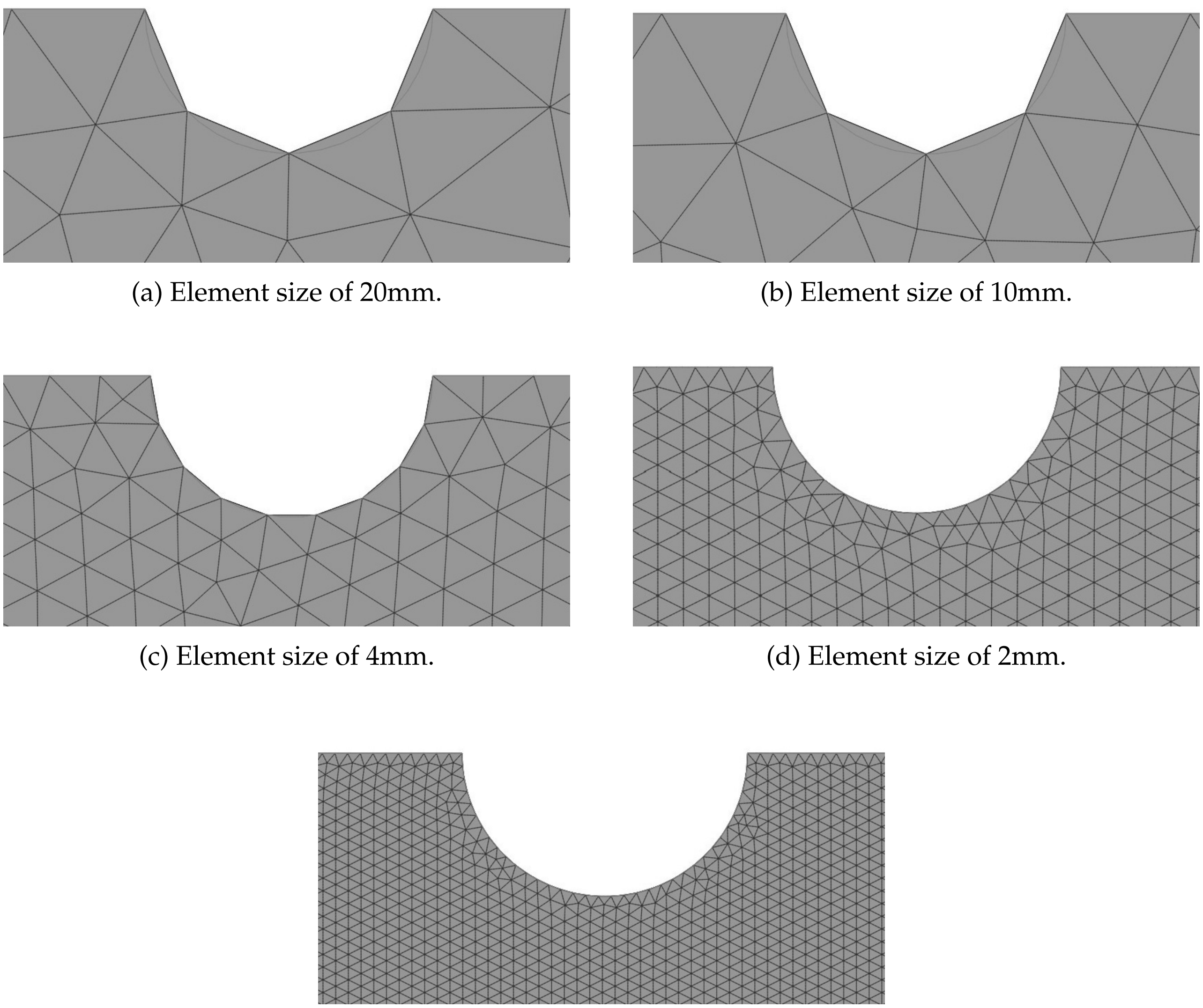
Table 2: Element size

N^o	**Element Size**
1	20mm
2	10mm
3	4mm
4	2mm
5	1 mm

The notched flat bar will be modeled using constraints and forces. The left hand size of the plat will be fixed using a support, and the right hand side will have an axial force in tension.

Figure 2: FEM model with constraints and forces applied.

The element distribution near the notch is presented in the following figures..



(a) Element size of 20mm.

(b) Element size of 10mm.

(c) Element size of 4mm.

(d) Element size of 2mm.

(e) Element size of 1mm.

Figure 3: Magnified images of upper notch of the model under different element sizes.

Results & Conclusions

Faster:

The results from the faster analysis are listed below in Table 3. There is a gradual trend of decreasing error as the element size also decreases. However, the element size of 2 mm experienced a lower relative error of 1.47% than the 1 mm element size at 2.14%. The element size of 4mm experienced 16.69% relative error. This shows that decreasing the element size drastically improves the results depending on the acceptable limit of relative error. Additionally, it shows that there is only a certain point in which decreasing element size is beneficial to obtaining more accurate results. In this case, that element size is 2mm.

Table 3: Results of von Mises Stresses from faster run setting at different element sizes.

Faster							
Element Size		**Maximum von Mises Stress**		**Theoretical Value**		**Relative Error**	
20	mm	94.84	MPa	157.72	MPa	39.87%	%
10		93.36				40.81%	
4		131.40				16.69%	
2		155.40				1.47%	
1		161.10				2.14%	

Accurate:

The results from the accuracy analysis are listed below in Table 4. It's clear that in the "more accurate" study, smaller element size does not improve accuracy to the theoretical value. In fact, the largest element size, 20 mm, has the most comparative value of 7.24% with the smallest element size, 1 mm, at 7.15%. Given these results, if being within 10% relative error is acceptable, then there is no reason to spend more time running a 1 mm element size analysis. That being said, the element size of 4 mm experienced the lowest relative error value at 3.98%.

Table 4: Results of von Mises Stresses from more accurate run setting at different element sizes.

Accurate							
Element Size		**Maximum von Mises Stress**		**Theoretical Value**		**Relative Error**	
20	mm	146.30	MPa	157.72	MPa	7.24%	%
10		171.40				8.67%	
4		164.00				3.98%	
2		168.30				6.71%	
1		169.00				7.15%	

Second Deliverable: Shortened Bar with "Best" Element Size

The best element size used for this analysis was 2mm on the faster setting. The trend for run time versus number of elements in the model was linear as seen in Fig. 4. The shortening of the bar resulted in less material to analyze, therefore the total number of elements also decreased. Naturally, it took less time to analyze fewer elements. The relative error was less consistent. The relative error grew slowly as the bar shortened, which is expected. However, there was an unexpected jump in relative error at 82.5 mm bar length. The slow trend of increasing relative error continued after this jump and is shown in Table 5. It's assumed that there is a certain point in the analysis when the analytical values hit nodes that cause this difference that is seen around 5%. It's also assumed that a similar jump occurs somewhere between 75 mm and 50 mm, or 8% and 25%, respectively.

Table 5: Run time versus element size when length is changed.

Bar Length		**Max. von Mises Stress [MPa]**	**Theoretical Value [MPa]**	**Relative Error**	**Time**		**Element Size**	
50	mm	195.00	157.72	23.64%	9	s	16624	#
75		171.10		8.48%	8		12701	
80		168.10		6.58%	9		13609	
82.5		169.30		7.34%	8		14097	
85		159.90		1.38%	9		14425	
90		155.70		1.28%	9		15056	
100		156.00		1.09%	13		16821	
125		156.80		0.58%	10		20236	
200		156.00		1.09%	11		32070	
300		152.60		3.25%	14		47100	
400		158.50		0.49%	19		62535	
500		156.20		0.96%	20		77682	

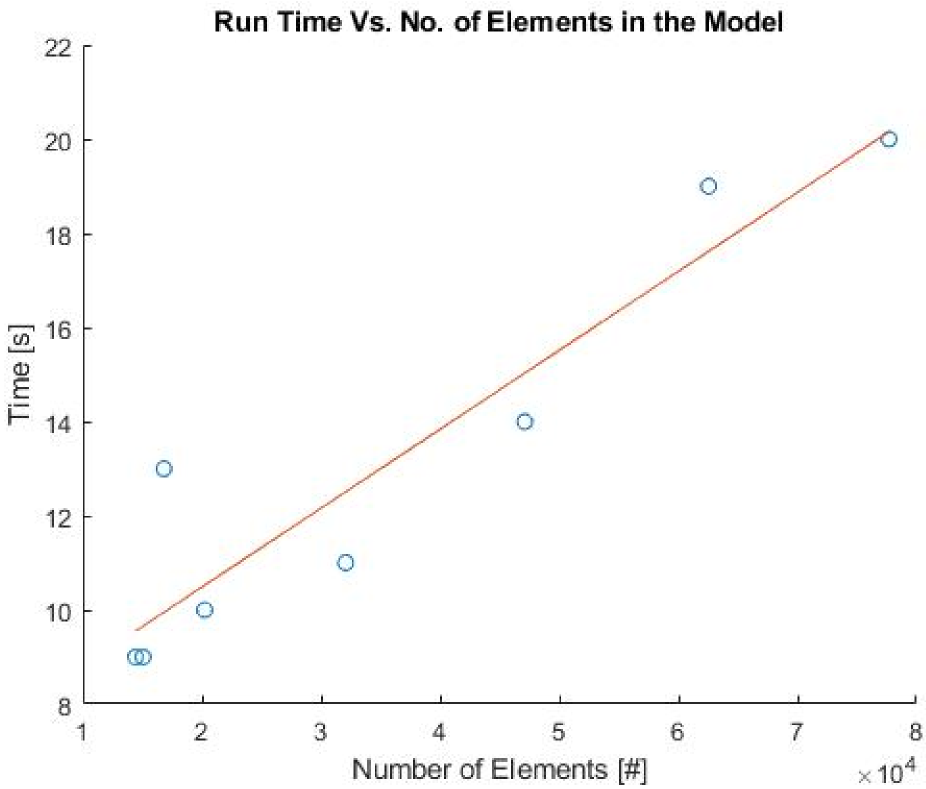


Figure 4: Graph of run time versus element size using the best settings and minimizing total elements through shortening the length of the bar.

Third Deliverable: Mass Reduction

The results from the third deliverable are shown in Fig. 5 and Fig. 7. Topological optimization was used to reduce the mass of the bar from 0.08234 kg to 0.039757 kg. The 51.7% mass reduction still allows for a 1.2 factor of safety in the system. All material that was experiencing low von Mises stresses was removed from the system. For reference, the stresses are displayed clearly in Fig. 6. As seen in Fig. 5, only the lowest point in the notch will experience high stress, thus this is the only location that really needs material. At first, this was thought to be a mistake in the design space. However, after further analysis and comparison was made, it was determined that the topological optimization clearly matches the stress locations.

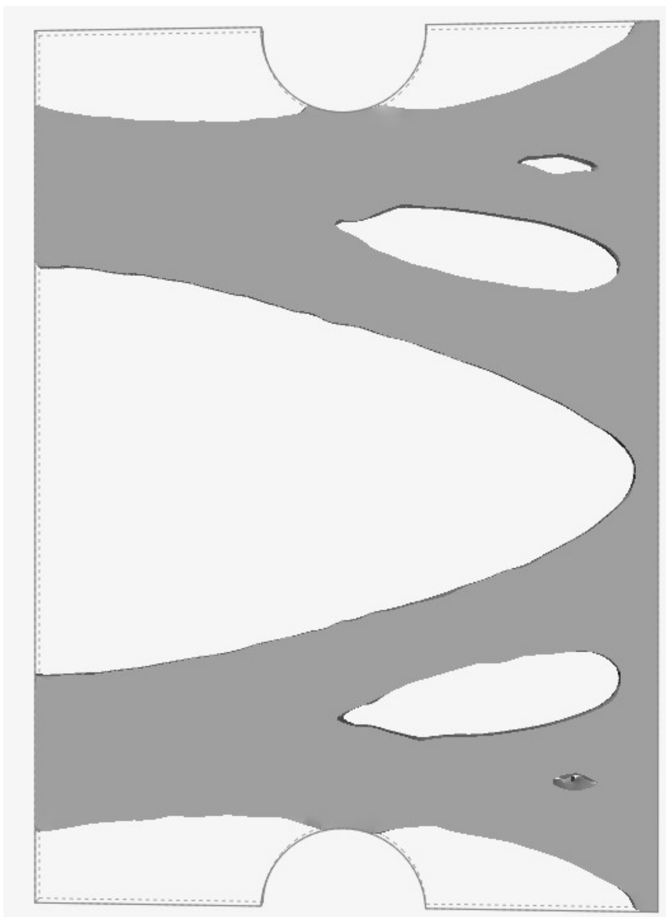


Figure 5: Mass reduction of the bar using topological optimization with outline of the original shape.

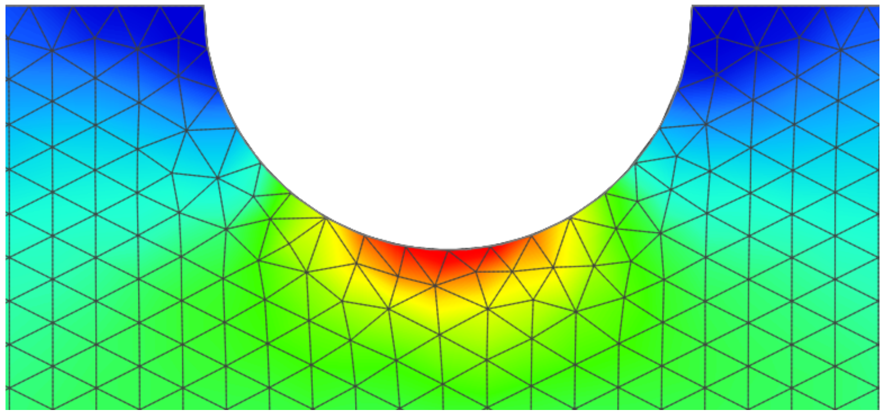


Figure 6: Close up of the upper notch on the bar with a 2mm element size.

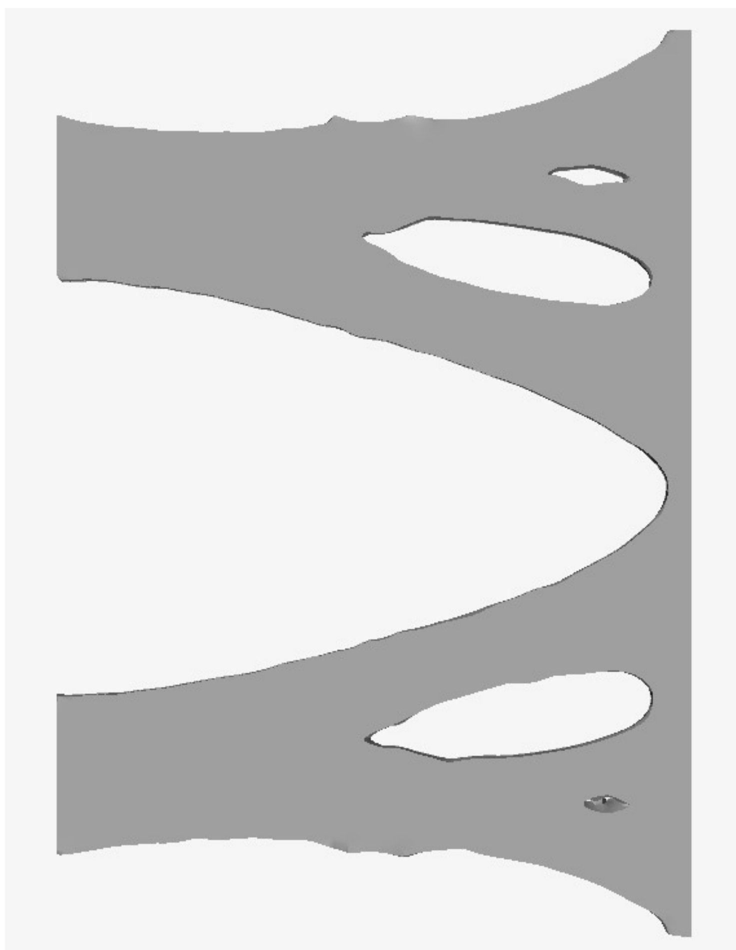


Figure 7: Mass reduction of the bar using topological optimization.