Problem Set 1: Demand Estimation

14.273 – Spring 2025

Due: March 1, 2025

Overview: This problem set is designed to help you understand the nuts and bolts of discrete choice demand models and imperfect competition. We will simulate data with known (true) parameters so that you can compare alternative estimation approaches. Please read the entire problem set before you begin.

If you have any questions, please contact tishara@mit.edu.

Market Simulation

Demand

Consider a market with $m = 1, \ldots, 100$ and products $j = 1, 2, 3$. Consumer i's utility for product j in market m is given by

$$U_{ijm} = X_{jm}\beta - \alpha_i \, p_{jm} + \xi_{jm} + \epsilon_{ijm}, \tag{1}$$

$$\alpha_i = \alpha + \sigma_\alpha \, \nu_{ip}, \tag{2}$$

$$U_{i0m} = \epsilon_{i0m}, \tag{3}$$

where:

- X_{jm} is a vector of observable product characteristics (with the first element equal to 1).
- p_{jm} is the price of product j in market m.
- ξ_{jm} is a product-market-specific demand shock.
- ϵ_{ijm} is an idiosyncratic shock drawn from a Type I Extreme Value distribution.
- ν_{ip} captures consumer-specific taste heterogeneity; assume

  $$\nu_{ip} \sim \text{LN}(0, 1),$$

  where LN denotes the lognormal distribution.
- The outside option is normalized by setting $U_{i0m} = \epsilon_{i0m}$.

Supply

Each product is produced by a single-product firm. Although markets are regional, the firms operate nationally. The marginal cost for product j in market m is given by

$$MC_{jm} = \gamma_0 + \gamma_1 \, W_j + \gamma_2 \, Z_{jm} + \eta_{jm},$$

where:

- W_j is a firm-specific cost shifter (common across markets).
- Z_{jm} is a market-specific cost shifter.
- $\eta_{jm} \sim \text{LN}(0, 1)$.

Data and True Parameters

The researcher observes the following for each market:

- Product characteristics X_{jm},
- Prices p_{jm},
- Market shares s_{jm},
- Cost shifters W_j and Z_{jm}.

The parameters of interest for demand are $\theta = (\beta, \alpha, \sigma_\alpha)$ and for supply are $\gamma = (\gamma_0, \gamma_1, \gamma_2)$. The true parameter values are:

$$\beta = (5, 1, 1), \quad \alpha = 1, \quad \sigma_\alpha = 1, \quad (\gamma_0, \gamma_1, \gamma_2) = (2, 1, 1).$$

Problem 0: Simulate Data

Task: Generate simulated data $\{X_{jm}, p_{jm}, s_{jm}, W_j, Z_{jm}\}$ for 3 products in 100 markets under the following assumptions:

1. **Product Characteristics:**
   - $X_{jm} = (X_{1jm}, X_{2jm}, X_{3jm})$ with $X_{1jm} = 1$ (intercept).
   - $X_{2jm} \sim U[0, 1]$ and $X_{3jm} \sim N(0, 1)$.
2. **Demand Shocks:** $\xi_{jm} \sim N(0, 1)$.
3. **Cost Shifters:** $W_j \sim \text{LN}(0, 1)$ and $Z_{jm} \sim \text{LN}(0, 1)$.
4. **True Parameters:** Use the true values given above.
5. **Market Equilibrium:** Firms behave oligopolistically in each market. That is, the prices set by the firm satisfy the following relationship between the Lerner index and the demand elasticities:

$$\frac{p_{jm} - MC_{jm}}{p_{jm}} = \frac{-1}{\frac{d \ln s_{jm}}{d \ln p_{jm}}}. \tag{4}$$

Instructions:

- Derive the market shares s_{jm} as a function of the mean utilities $\{\delta_{jm}\}$, where
$$\delta_{jm} = X_{jm}\beta - \alpha\, p_{jm} + \xi_{jm}.$$
- Use the share equation derived above, together with Equation 4, to numerically solve for equilibrium prices p_{jm} and shares s_{jm} for each product j and market m. One suggested approach is to implement a fixed-point iteration algorithm.

Problem 1: BLP and Hausman Instruments

1. **Moment Conditions:** Consider the following conditional moment restrictions:
$$E[\xi \mid X] = 0, \quad E[\xi \mid p] = 0, \quad E[\xi \mid W, Z] = 0.$$
   a. Discuss which of these moment conditions are valid and relevant given that you know the true model. Explain your reasoning.
   b. Compute the empirical counterparts of the moments
$$E[\xi_{jm}X_{jm}], \quad E[\xi_{jm}\bar{X}_{jm}], \quad E[\xi_{jm}p_{jm}], \quad \text{and} \quad E[\xi_{jm}\bar{p}_{jm}],$$
   where $\bar{X}_{jm}$ and $\bar{p}_{jm}$ denote the average characteristics and prices of the competing products in the same market.
   c. Discuss the use of BLP instruments (based on competitor characteristics) versus Hausman instruments (based on cost shifters) in this setting. What are the advantages and drawbacks of each instrument set for identifying the demand-side parameters?
2. **BLP Estimation:** Estimate the demand parameters $\theta = (\beta, \alpha, \sigma_\alpha)$ using the BLP approach. In your estimation, use instruments based on:
   - Competitor characteristics (i.e., assuming $E[\xi_{jm} \mid X_{-j,m}] = 0$).
   - Cost shifters (i.e., $E[\xi_{jm} \mid W_j, Z_{jm}] = 0$).

   Do *not* impose any supply-side restrictions in this part.
   a. Write down the moment conditions and construct the corresponding GMM objective function.
   b. Estimate θ and report the point estimates for each parameter.
   c. Derive the expression for the asymptotic standard errors. Provide pseudocode that outlines how you would implement a function to compute these standard errors.
   d. Write the expressions for the own-price and cross-price elasticities of demand. Include pseudocode for a function that calculates these elasticities.
   e. Write the expression for consumer surplus. Provide pseudocode for a function that computes consumer surplus.
3. **Misspecification Exercise:** Suppose you estimated θ under the incorrect assumption that $E[\xi \mid p] = 0$ holds in each market. How would you expect the parameter estimates to differ from the true values and from those obtained using the valid instruments?

Problem 2: Adding the Supply-Side

1. **Marginal Cost Estimation:** Given your estimated θ, compute the markups and hence the marginal costs.

   a. Write down the expressions for marginal costs under each of the following pricing assumptions:

      i. Perfect Competition,

      ii. Perfect Collusion,

      iii. Oligopoly.

   b. Provide pseudocode for estimating marginal costs under each pricing assumption.

   c. Given that the true model is oligopolistic, discuss how the marginal cost estimates under the assumptions of perfect competition and perfect collusion are expected to compare with the true marginal costs.

2. **Joint Estimation of Demand and Supply:** Now suppose you wish to estimate both θ and γ jointly while imposing the equilibrium conditions on both the demand and supply sides. Assume that

$$E[\xi,\, \eta \mid X, W] = 0.$$

   a. Provide pseudocode for an estimation procedure that jointly estimates θ and γ under each pricing assumption from Problem 2(1)(a).

   b. Explain how you would use your joint estimates to test the different pricing assumptions.

Problem 3: Merger Exercise

1. **Merger Analysis:** Suppose firms 1 and 2 plan to merge.

   a. Derive the pricing equation for the merged firm.

   b. Explain the interpretation of each term in the merged firm's pricing equation.

   c. Provide pseudocode for a function that calculates counterfactual prices following the merger, using your estimated parameters.

   d. Based on your demand estimates, discuss qualitatively how you expect the merger to affect markups.