COMPSCI 3MI3

Assignment 10

1 Solution Set

1.1 Q1

We assume the syntax and semantics for our extended simply-typed λ-Calculus defined in the slides for topic 11. We propose the following extension to the syntax and semantics for pattern-matching and *match*.

$\langle t \rangle ::= \ldots$

$\langle v \rangle ::= \ldots$

$\langle p \rangle ::= \ldots$
$\quad | \quad \text{nil}[\langle T \rangle]$
$\quad | \quad \text{cons}[\langle T \rangle]\ \langle p \rangle\ \langle p \rangle$

We extend *match* with the following matching rules:

$$match(nil[T], nil[T]) = [x \mapsto x] \tag{M-Nil}$$

$$\frac{match(p_1, v_1) = \sigma_1 \qquad match(p_2, v_2) = \sigma_2}{match(cons[T]\ p_1\ p_2, cons[T]\ v_1\ v_2) = \sigma_1 \circ \sigma_2} \tag{M-Cons}$$

Where T is a type and $\circ$ is function composition.

1.2 Q2

We assume the syntax and semantics for our simply-typed λ-Calculus extended with lists defined in the slides for topic 11. We define the syntax for our new terms as follows:

$\langle t \rangle$ ::= ...
  | index[$\langle T \rangle$] $\langle t \rangle$ $\langle t \rangle$
  | length[$\langle T \rangle$] $\langle t \rangle$
  | concat[$\langle T \rangle$] $\langle t \rangle$ $\langle t \rangle$
  | filter[$\langle T \rangle$] $\langle t \rangle$ $\langle t \rangle$

$\langle v \rangle$ ::= ...

$\langle T \rangle$ ::= ...

We define the semantics of each of our new terms below.

(a) List indexing We define the following evaluation and typing semantics for list indexing:

$$\mathit{index}[S]\ (\mathit{cons}[T]\ v_1\ v_2)\ 0 \to v_1 \qquad \text{(E-IndexZero)}$$

$$\mathit{index}[S]\ (\mathit{cons}[T]\ v_1\ v_2)\ (\mathit{succ}\ nv) \to \mathit{index}[S]\ v_2\ nv \qquad \text{(E-IndexSucc)}$$

$$\frac{t_1 \to t_1'}{\mathit{index}[T]\ t_1\ t_2 \to \mathit{index}[T]\ t_1'\ t_2} \qquad \text{(E-Index1)}$$

$$\frac{t_2 \to t_2'}{\mathit{index}[T]\ v_1\ t_2 \to \mathit{index}[T]\ v_1\ t_2'} \qquad \text{(E-Index2)}$$

$$\frac{\Gamma \vdash t_1 : \mathit{List}\ T \qquad \Gamma \vdash t_2 : \mathit{Nat}}{\Gamma \vdash \mathit{index}[T]\ t_1\ t_2 : T} \qquad \text{(T-Index)}$$

(b) Length
We define the following evaluation and typing semantics for length:

$$\mathit{length}[S](\mathit{nil}[T]) \to 0 \qquad \text{(E-LengthNil)}$$

$$\mathit{length}[S](\mathit{cons}[T]\ v_1\ v_2) \to \mathit{succ}\ (\mathit{length}[S]\ v_2) \qquad \text{(E-LengthCons)}$$

$$\frac{t \to t'}{\mathit{length}[T]\ t \to \mathit{length}[T]\ t'} \qquad \text{(E-Length)}$$

$$\frac{\Gamma \vdash t : \mathit{List}\ T}{\Gamma \vdash \mathit{length}[T]\ t : \mathit{Nat}} \qquad \text{(T-Length)}$$

(c) Concatenation
We define the following evaluation and typing semantics for concatenation:

$$\mathit{concat}[S]\ (\mathit{nil}[T])\ v_2 \to v_2 \qquad \text{(E-ConcatNil)}$$

$$\mathit{concat}[S]\ (\mathit{cons}[T]\ v_1\ v_2)\ v_3 \to \mathit{cons}[T]\ v_1\ (\mathit{concat}[S]\ v_2\ v_3) \qquad \text{(E-ConcatCons)}$$

$$\frac{t_1 \to t_1'}{\mathit{concat}[T]\ t_1\ t_2 \to \mathit{concat}[T]\ t_1'\ t_2} \qquad \text{(E-Concat1)}$$

$$\frac{t_2 \to t_2'}{\mathit{concat}[T]\ v_1\ t_2 \to \mathit{concat}[T]\ v_1\ t_2'} \qquad \text{(E-Concat2)}$$

$$\frac{\Gamma \vdash t_1 : \mathit{List}\ T \qquad \Gamma \vdash t_2 : \mathit{List}\ T}{\Gamma \vdash \mathit{concat}[T]\ t_1\ t_2 : \mathit{List}\ T} \qquad \text{(T-Concat)}$$

(d) Filtering
We define the following evaluation and typing semantics for filtering:

$$\mathit{filter}[S]\ v_1\ (\mathit{nil}[T]) \to \mathit{nil}[T] \qquad \text{(E-FilterNil)}$$

$$\mathit{filter}[S]\ v_1\ (\mathit{cons}[T]\ v_2\ v_3) \to \texttt{if}\ v_1\ v_2\ \texttt{then}\ \mathit{cons}[T]\ v_2\ (\mathit{filter}[S]\ v_1\ v_3)\ \texttt{else}\ \mathit{filter}[S]\ v_1\ v_3 \quad \text{(E-FilterCons)}$$

$$\frac{t_1 \to t_1'}{\mathit{filter}[T]\ t_1\ t_2 \to \mathit{filter}[T]\ t_1'\ t_2} \quad \text{(E-Filter1)}$$

$$\frac{t_2 \to t_2'}{\mathit{filter}[T]\ v_1\ t_2 \to \mathit{filter}[T]\ v_1\ t_2'} \quad \text{(E-Filter2)}$$

$$\frac{\Gamma \vdash t_1 : T \Rightarrow \mathit{Bool} \qquad \Gamma \vdash t_2 : \mathit{List}\ T}{\Gamma \vdash \mathit{filter}[T]\ t_1\ t_2 : \mathit{List}\ T} \quad \text{(T-Filter)}$$