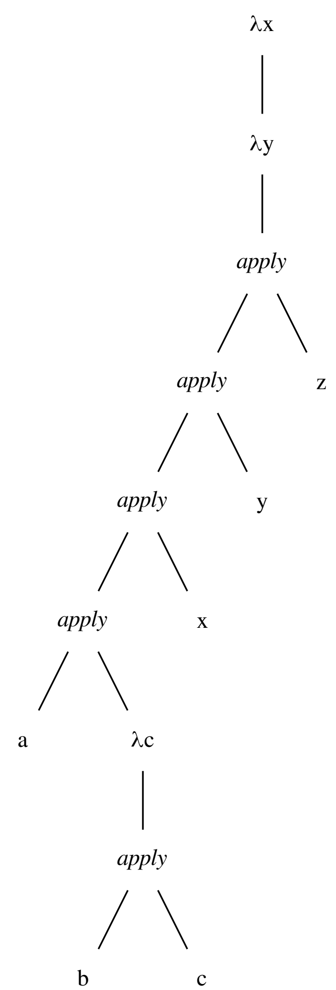
COMPSCI 3MI3

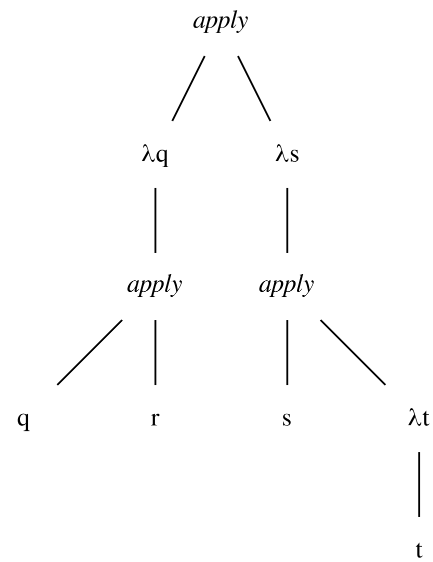
Assignment 4

1 Solution Set

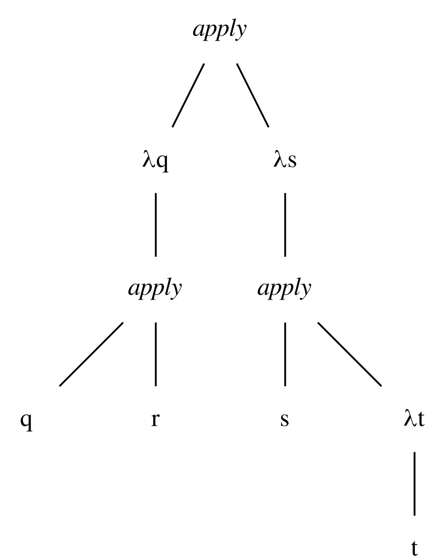
1.1 Q1



(a)


apply
λq
λs
apply
apply
q
r
s
λt
t

(b)


apply
λq
λs
apply
apply
q
r
s
λt
t

(c)

1.2 Q2

We define a lambda expression which performs logical or over Church Booleans as follows:

$$\texttt{or} = \lambda b.\lambda c.\, b\, \texttt{tru}\, c$$

We can prove our operation works for all possible inputs.

<u>With input tru tru:</u>

$$\begin{array}{ll} & (\lambda b.\lambda c.\, b\, \texttt{tru}\, c)\, \texttt{tru}\, \texttt{tru} \\ \to & (\lambda c.\, \texttt{tru}\, \texttt{tru}\, c)\, \texttt{tru} \\ \to & \texttt{tru}\, \texttt{tru}\, \texttt{tru} \\ \to & (\lambda t.\lambda f.t)\, \texttt{tru}\, \texttt{tru} \\ \to & (\lambda f.\texttt{tru})\, \texttt{tru} \\ \to & \texttt{tru} \\ \not\to & \end{array}$$

<u>With input tru fls:</u>

$$\begin{array}{ll} & (\lambda b.\lambda c.\, b\, \texttt{tru}\, c)\, \texttt{tru}\, \texttt{fls} \\ \to & (\lambda c.\, \texttt{tru}\, \texttt{tru}\, c)\, \texttt{fls} \\ \to & \texttt{tru}\, \texttt{tru}\, \texttt{fls} \\ \to & (\lambda t.\lambda f.t)\, \texttt{tru}\, \texttt{fls} \\ \to & (\lambda f.\texttt{tru})\, \texttt{fls} \\ \to & \texttt{tru} \\ \not\to & \end{array}$$

<u>With input fls tru:</u>

$$\begin{array}{ll} & (\lambda b.\lambda c.\, b\, \texttt{tru}\, c)\, \texttt{fls}\, \texttt{tru} \\ \to & (\lambda c.\, \texttt{fls}\, \texttt{tru}\, c)\, \texttt{tru} \\ \to & \texttt{fls}\, \texttt{tru}\, \texttt{tru} \\ \to & (\lambda t.\lambda f.f)\, \texttt{tru}\, \texttt{tru} \\ \to & (\lambda f.f)\, \texttt{tru} \\ \to & \texttt{tru} \\ \not\to & \end{array}$$

<u>With input fls fls:</u>

$$\begin{array}{ll} & (\lambda b.\lambda c.\, b\, \texttt{tru}\, c)\, \texttt{fls}\, \texttt{fls} \\ \to & (\lambda c.\, \texttt{fls}\, \texttt{tru}\, c)\, \texttt{fls} \\ \to & \texttt{fls}\, \texttt{tru}\, \texttt{fls} \\ \to & (\lambda t.\lambda f.f)\, \texttt{tru}\, \texttt{fls} \\ \to & (\lambda f.f)\, \texttt{fls} \\ \to & \texttt{fls} \\ \not\to & \end{array}$$

1.3 Q3

We define a lambda expression which performs exponentiation over Church Numerals, such that m^n is represented `exp m n` where `exp` is defined as

$$\texttt{exp} = \lambda m.\lambda n.\, n\, m$$

We will prove our operation works with inputs c_2, c_2 to evaluate 2^2.

<u>With input $c_2\ c_2$:</u>

$$\begin{array}{rl} & (\lambda m.\lambda n.\, n\, m)\, c_2\, c_2 \\ \to\to & (\lambda s.\lambda z.\, s\, (s\, z))(\lambda a.\lambda b.\, a\, (a\, b)) \\ \to & \lambda z.\, (\lambda a.\lambda b.\, a\, (a\, b))\, ((\lambda a.\lambda b.\, a\, (a\, b))\, z) \\ \to & \lambda z.\, (\lambda a.\lambda b.\, a\, (a\, b))\, (\lambda c.\, z\, (z\, c)) \\ \to & \lambda z.\lambda b.\, (\lambda c.\, z\, (z\, c))\, ((\lambda c.\, z\, (z\, c))\, b) \\ \to & \lambda z.\lambda b.\, (\lambda c.\, z\, (z\, c))\, (z\, (z\, b)) \\ \to & \lambda z.\lambda b.\, z\, (z\, (z\, (z\, b))) \\ \not\to & \end{array}$$

Our result is equivalent to $\lambda s.\ \lambda z.\ s\ (s\ (s\ (s\ z)))$ which is c_4. Thus we can conclude our exponential expression works for inputs c_2, c_2.