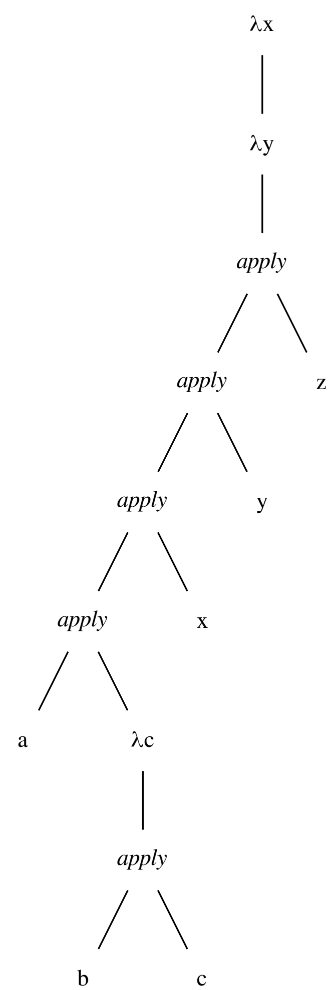
COMPSCI 3MI3

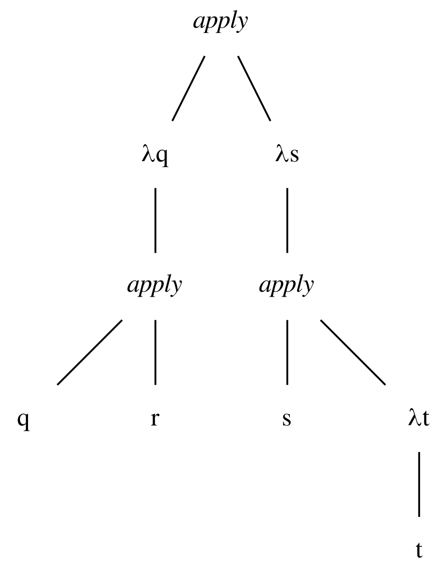
Assignment 4

1 Solution Set

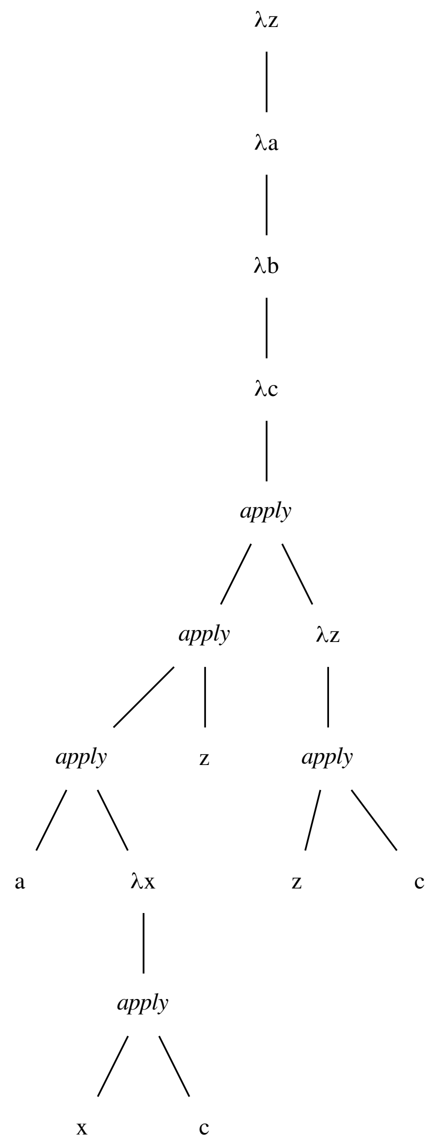
1.1 Q1



(a)



(b)



(c)

1.2 Q2

We define a lambda expression which performs logical or over Church Booleans as follows:

$$\texttt{or} = \lambda b.\lambda c.\, b\, \texttt{tru}\, c$$

We can prove our operation works for all possible inputs.

<u>With input `tru tru`:</u>

$$
\begin{array}{rl}
 & (\lambda b.\lambda c.\ b\ \texttt{tru}\ c)\ \texttt{tru}\ \texttt{tru} \\
\to & (\lambda c.\ \texttt{tru}\ \texttt{tru}\ c)\ \texttt{tru} \\
\to & \texttt{tru}\ \texttt{tru}\ \texttt{tru} \\
\to & (\lambda t.\lambda f.t)\ \texttt{tru}\ \texttt{tru} \\
\to & (\lambda f.\texttt{tru})\ \texttt{tru} \\
\to & \texttt{tru} \\
\not\to &
\end{array}
$$

<u>With input `tru fls`:</u>

$$
\begin{array}{rl}
 & (\lambda b.\lambda c.\ b\ \texttt{tru}\ c)\ \texttt{tru}\ \texttt{fls} \\
\to & (\lambda c.\ \texttt{tru}\ \texttt{tru}\ c)\ \texttt{fls} \\
\to & \texttt{tru}\ \texttt{tru}\ \texttt{fls} \\
\to & (\lambda t.\lambda f.t)\ \texttt{tru}\ \texttt{fls} \\
\to & (\lambda f.\texttt{tru})\ \texttt{fls} \\
\to & \texttt{tru} \\
\not\to &
\end{array}
$$

<u>With input `fls tru`:</u>

$$
\begin{array}{rl}
 & (\lambda b.\lambda c.\ b\ \texttt{tru}\ c)\ \texttt{fls}\ \texttt{tru} \\
\to & (\lambda c.\ \texttt{fls}\ \texttt{tru}\ c)\ \texttt{tru} \\
\to & \texttt{fls}\ \texttt{tru}\ \texttt{tru} \\
\to & (\lambda t.\lambda f.f)\ \texttt{tru}\ \texttt{tru} \\
\to & (\lambda f.f)\ \texttt{tru} \\
\to & \texttt{tru} \\
\not\to &
\end{array}
$$

<u>With input `fls fls`:</u>

$$
\begin{array}{rl}
 & (\lambda b.\lambda c.\ b\ \texttt{tru}\ c)\ \texttt{fls}\ \texttt{fls} \\
\to & (\lambda c.\ \texttt{fls}\ \texttt{tru}\ c)\ \texttt{fls} \\
\to & \texttt{fls}\ \texttt{tru}\ \texttt{fls} \\
\to & (\lambda t.\lambda f.f)\ \texttt{tru}\ \texttt{fls} \\
\to & (\lambda f.f)\ \texttt{fls} \\
\to & \texttt{fls} \\
\not\to &
\end{array}
$$

1.3 Q3

We define a lambda expression which performs exponentiation over Church Numerals, such that m^n is represented `exp m n` where `exp` is defined as

$$\texttt{exp} = \lambda m.\lambda n.\, n\, m$$

We will prove our operation works with inputs c_2, c_2 to evaluate 2^2.

<u>With input $c_2\ c_2$:</u>

$$\begin{array}{rl}
 & (\lambda m.\lambda n.\, n\, m)\, c_2\, c_2 \\
\to\to & (\lambda s.\lambda z.\, s\,(s\,z))(\lambda a.\lambda b.\, a\,(a\,b)) \\
\to & \lambda z.\,(\lambda a.\lambda b.\, a\,(a\,b))\,((\lambda a.\lambda b.\, a\,(a\,b))\,z) \\
\to & \lambda z.\,(\lambda a.\lambda b.\, a\,(a\,b))\,(\lambda c.\, z\,(z\,c)) \\
\to & \lambda z.\lambda b.\,(\lambda c.\, z\,(z\,c))\,((\lambda c.\, z\,(z\,c))\,b) \\
\to & \lambda z.\lambda b.\,(\lambda c.\, z\,(z\,c))\,(z\,(z\,b)) \\
\to & \lambda z.\lambda b.\, z\,(z\,(z\,(z\,b))) \\
\not\to &
\end{array}$$

Our result is equivalent to $\lambda s.\,\lambda z.\, s\,(s\,(s\,(s\,z)))$ which is c_4. Thus we can conclude our exponential expression works for inputs c_2, c_2.