WORKING PAPERS SES

Das House Kapital

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Abstract: The housing wealth-to-income ratio has been increasing in most developed economies since the 1950s. We provide a novel theory to explain this long-term pattern. We show analytically that house prices grow in the steady state if i) the housing sector is more land-intensive than the non-housing sector, or ii) technological progress in the construction sector is weaker than in the non-housing sector. Despite growing house prices and housing wealth, the housing wealth-*to-income* ratio is constant in steady state. We hence study the dynamics in the housing wealth-to-income ratio by computing transitions. The model is calibrated separately to the US, UK, France, and Germany. On average, we replicate 89 percent of the observed increase in the housing wealth-to-income ratio. The key for replicating the data is the differentiation between residential land as a non-reproducible factor and residential structure as a reproducible factor. The transition process from the calibrated model points to two driving forces of an increasing housing wealth-to-income ratio: i) A long-lasting construction boom that brought about a pronounced build-up in the stock of structures and ii) an increase in the demand for residential land that resulted in surging residential land prices.

Key words: Housing Wealth; Economic Growth; Wealth-to-Income Ratio; House Price; Land Price.
JEL classification: E10, E20, O40.

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1. Introduction

Housing wealth is the largest private wealth component. Figure 1 displays its size, scaled by aggregate income, and evolution over time for the USA, UK, Germany, and France since 1950. It has been growing considerably faster than income. In 1950 the aggregate housing stock was on average worth one year of aggregate income, while at the end of our sample, the housing stock was on average worth almost three years of aggregate income. Housing wealth also grew faster than non-housing wealth, driving at least half of the overall wealth increase. For instance, housing wealth grew by a factor of 1.42 and non-housing wealth by a factor of 1.38 in the US between 1950 and 2015, while these numbers read 2.4 and 1.9 for the UK between 1950 and 2012.[1]

Why has housing wealth been increasing relative to income in the US, the UK, France, and Germany since 1950? To shed light on this question, we proceed in three steps.

First, we document five stylized facts on housing-related macro variables for the US, UK, France, and Germany since 1950. We follow Davis and Heathcote (2007) and "[...] conceptualize a house as a bundle comprising a reproducible tangible structure and a non-reproducible plot of land."[2] It can already be seen from figure 1 that most of the increase in housing wealth is driven by rising residential land wealth, whereas residential structure wealth has been relatively stable. This explicit distinction between land and structure is therefore key for studying the rise in housing wealth. We document a mirror-like pattern in the evolution of prices and quantities of residential land and residential structures: The real price of residential land has been growing at a much higher rate than the real price of residential structures, while the quantity of residential land has been growing by less than the quantity of residential structures. The complete list of stylized facts, presented in section 2, quantifies not only the empirical phenomenon to be explained. It also documents the evolution of other housing-related macro variables — such as the growth differences in real prices of residential structures and residential land — that a plausible theory should replicate.

Second, we set up a new macroeconomic model with housing that focuses on the supply side and is designed to think long-term. The employed Ramsey growth model comprises two sectors, a numeraire sector and a housing sector. While the numeraire

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[1]These numbers and other stylized facts are summarized in section 2.

[2]Of course, the two housing-related assets (structures and land) are not traded separately in reality. The asset that individuals trade is most often a house. The decomposition into two assets is, however, instructive for understanding the drivers behind rising housing wealth.

(i) US   **(ii)** UK

(iii) FRA   **(iv)** GER
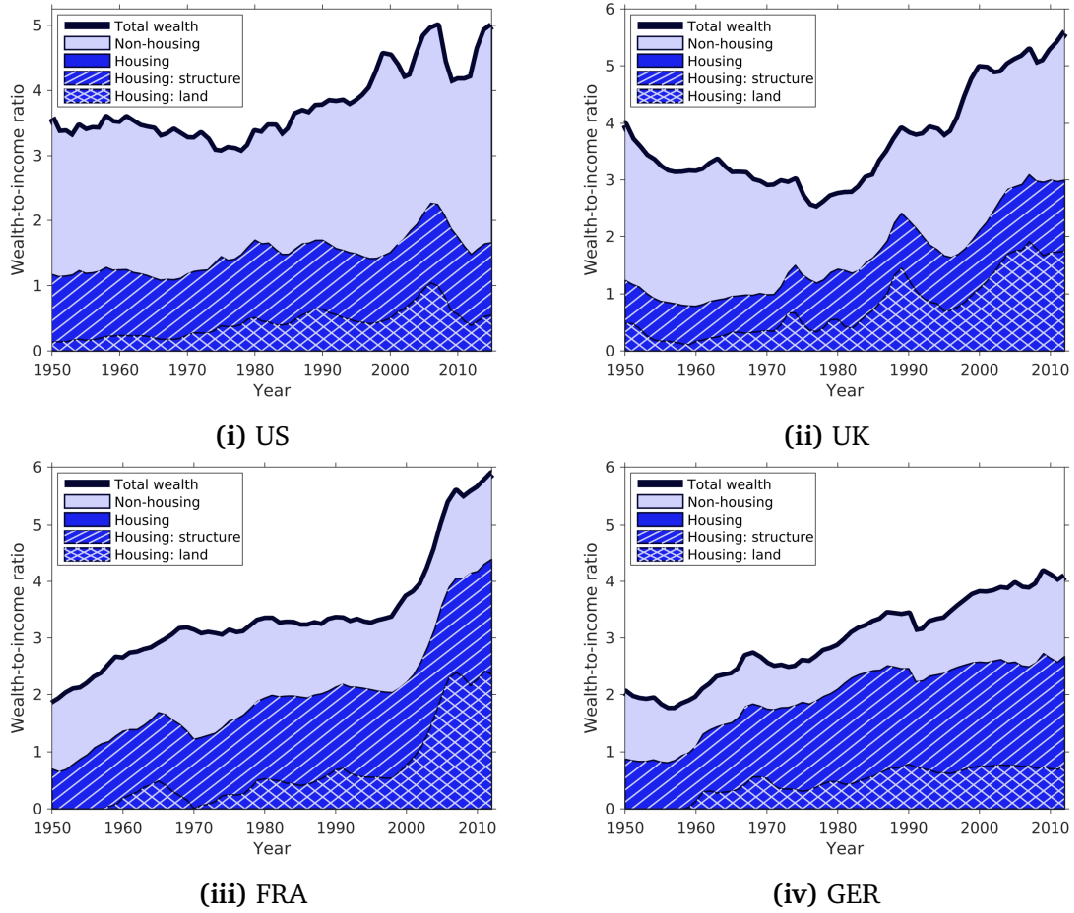
Notes: The housing wealth-to-income ratio is the sum of the aggregate value of residential structures and the aggregate value of residential land, both relative to income. The data source is described in online appendix A.

Figure 1: Wealth-to-income ratio: the relevance of housing

sector is kept standard, the innovative part of our model is the housing sector. It consists of three different types of firms. Land development firms purchase land and transform it into residential land by employing labor. Construction firms manufacture structures by employing materials and labor. Property management firms combine residential land and residential structures to produce houses that they rent out to households. The overall supply of land is fixed. The allocation of land across the numeraire sector and the housing sector is endogenous. The population and the production technology grow exogenously. We allow the growth rates of technological change to differ between the non-housing sector and the construction sector.

Third, we analyze housing wealth dynamics analytically in steady state and quanti-

tatively along the transition. Housing *wealth* is the product of house *price* and house *quantity*. A theory of rising housing wealth should explain a rising house price. Before studying a growing housing wealth-to-income ratio, we hence analyze the determinants of the observed long-term increase in the house price. However, the phenomenon of a rising wealth-to-income ratio cannot be explained in steady state.[3] We bring our model to the data by studying transitional dynamics for the US, UK, France, and Germany.

Why is the increase in the housing wealth-to-income ratio relevant for the macroeconomy? First, rising house prices and rents have raised concerns about the affordability of housing (Quigley and Raphael, 2004; Albouy, Ehrlich, and Liu, 2016). Because the representative household must devote a higher multiple of income to purchase the existing housing stock, the increase in the aggregate housing wealth-to-income ratio points to a potential affordability issue. Second, Gennaioli, Shleifer, and Vishny (2014) argue that the long-run increase in the share of financial sector income in GDP is driven by a rising total wealth-to-income ratio, which results from rising housing wealth, as shown in figure 1. Similarly, based on historical data for 17 advanced economies, Jordà, Schularick, and Taylor (2016) document a pronounced increase in the credit-to-GDP ratio in the 20th century. They argue that rising mortgages and housing wealth are mainly responsible for this increase.[4] Lastly, the long-term increase in the total wealth-to-income ratio contributes to a change in the functional income distribution to the advantage of capital income recipients (Piketty and Zucman, 2014). The housing sector seems especially important in this context, as stressed by Rognlie (2015). He argues that the increase in the aggregate capital income share in the G7 economies over 1950–2010 is driven exclusively by the housing sector. Because the yield component of the rate of return on housing has been reasonably stable (Jordà, Knoll, Kuvshinov, Schularick, and Taylor, 2019), the increase in the housing component of the capital income share has to be driven by an increase in the housing wealth-to-income ratio.[5] Understanding why the housing wealth-to-income ratio has been increasing is, therefore, crucial for these three debates.

We use the proposed model to explain why the housing wealth-to-income ratio has

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[3]To be precise, a time-varying wealth-to-income ratio can either be explained as a sequence of steady states (Borri and Reichlin, 2018; Piketty and Zucman, 2015) or a transition towards a given steady state (Gennaioli, Shleifer, and Vishny, 2014).

[4]The mortgage-to-GDP ratio is given by *leverage ratio × housing wealth-to-income ratio*. A rising housing wealth-to-income ratio raises — for a given leverage ratio — the mortgage-to-GDP ratio.

[5]The housing capital income share is given by *housing yield × housing wealth-to-income ratio*.

been increasing. The first set of results is analytical and applies to the steady state, while the second set of results is quantitative and applies to transitions. Before turning to housing *wealth* in our second set of results, we show under which conditions the house *price* increases in steady state in our first set of results.

In essence, our model is a two-sectoral growth model with a housing and a non-housing sector. The house price — the price of the stock of housing — grows in steady state at the same rate as the rent — the price of the service flow derived from housing. Both sectors employ labor and experience exogenous technological change. We identify two mechanisms that trigger rising rents and house prices in a growing economy. First, the more land-intensive the housing sector is compared to the non-housing sector, the more likely it is that house prices grow at a positive rate in the long term. So it is not the mere existence of a fixed production factor that matters for rising housing wealth but the relative intensity with which it enters the two sectors. Second, we allow technological progress to differ between the housing sector — to be precise, the construction sector — and the non-housing sector. We show that the stronger technological growth is in the non-housing sector relative to technological growth in the construction sector, the higher is the long-term growth rate of house prices. The calibrated model indicates that both channels — a higher land-intensity in the housing sector and stronger technological progress in the non-housing sector than in the construction sector — are empirically relevant.

Although the house price and housing wealth can increase in steady state, the wealth-to-*income* ratio and the housing wealth-to-*income* ratio remain constant in steady state. To explain a rising housing wealth-to-*income* ratio, we, therefore, study transitional dynamics in our second set of results. We calibrate the model by matching a set of moments along the transition. This calibration is done separately for each of the four economies under study. We abstain from matching the increase in housing wealth over time and let the model speak to the implied increase in housing wealth. The calibrated model explains 80, 67, and 76 percent of the increase in the housing wealth-to-income ratio in the UK, France, and Germany over 1950–2012, 1960–2012, and 1962–2012, respectively. In the US, we even slightly over-predict the increase in the housing wealth-to-income ratio by 35 percentage points over 1950–2015. This over-prediction might be because US housing wealth has not yet recovered from its previous bust at the end of our sample in 2015.

We use the calibrated model to run several counterfactual experiments to shed light

on the underlying mechanisms behind a rising housing wealth-to-income ratio. A substantial accumulation of residential buildings at the beginning of the period 1950–2015 — the post-war construction boom — plays a vital role .[6] This pronounced expansion in housing stock is endogenous in our model as the calibration produces a low initial level of residential structures relative to the steady state. Through the accumulation of structures, the construction boom leads to strongly rising housing wealth. However, if taken alone, the construction boom would imply that residential structure wealth grows more strongly than residential land wealth, being at odds with the data shown in figure 1. This is where the accumulation of capital becomes relevant. The calibration also produces a low initial level of capital relative to the steady state. The accumulation of capital increases aggregate income and hence the demand for housing.[7] Producing additional housing necessitates new structures and land. Residential structures are a reproducible production factor, but residential land is not. It can only increase moderately during the transition and is fixed in the long run. This explicit differentiation then implies that rising demand for housing results in pronounced growth in residential land prices and weak growth in residential structure prices.[8] It also implies very weak growth in the quantity of residential land and pronounced growth in residential structures.

We corroborate both mechanisms by comparing model outcomes with two additional observations. First, residential investment as a share of GDP was exceptionally high initially and declined — both in the data and in our model simulation — over 1950–2015 in the US, UK, France, and Germany. This decline in residential investment supports our finding that the post-war construction boom was relevant for rising housing wealth. Second, aggregate saving rates have been declining in all four economies since the 1950s, supporting our finding that capital accumulation also played an important role. Our model replicates both features of the data quite well along the transition. Lastly, we also check that the model-implied transition is in line with the other stylized facts that we do not target in our calibration strategy.

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[6]The period 1940 to 1975 is known as the era of mass-suburbanization, during which new residential areas evolved at the boundaries of US cities (Nechyba and Walsh, 2004).

[7]Pronounced capital accumulation is also associated with a declining interest rate. A declining interest rate amplifies the surge in real asset prices, including the house price and the residential land price.

[8]This prediction is consistent with the finding that 80 percent of the house price growth can be attributed to surges in the residential land price, whereas 20 percent can be attributed to an increase in construction costs (Knoll, Schularick, and Steger, 2017).

Related literature

In studying why housing wealth increases over the long term, our paper adds to the recent literature on housing and macroeconomics. One branch of this literature is primarily concerned with the measurement of housing wealth. Davis and Heathcote (2007) have developed a measurement system to estimate price and quantity indexes for residential land. They show that, on average from 1975 to 2006, land accounts for 36 percent of housing wealth in the US. Over the same period, the real price of residential land nearly quadrupled, while the real price of structures increased by only 33 percent. Piketty and Zucman (2014) have documented the evolution of the aggregate wealth-to-income ratios from 1700 to 2010 for eight developed economies. The main finding is that the aggregate wealth-to-income ratio has risen gradually between 1970 and 2010, returning to the high values observed in Europe in the eighteenth and nineteenth centuries (600–700 percent). Their data also implies that housing wealth has gained importance while agricultural land has become less important over time. Rognlie (2015) argues that the housing sector has driven the surge in the net capital income share since 1948. He discusses two alternative explanations: i) the accumulation view according to which capital has been accumulated strongly while its rate of return declined only slightly and ii) the scarcity view according to which residential land became scarcer over time and paid a higher rate of return. Data and theory, he argues, support the scarcity view: the net capital share is rising because some forms of wealth are becoming relatively more scarce, not more abundant. Knoll, Schularick, and Steger (2017) provide annual house prices for 14 advanced economies since 1870. They show that real house prices stayed constant from the nineteenth to the mid-twentieth century, but rose strongly and with substantial cross-country variation in the second half of the twentieth century. They point out that land prices, not construction costs, are key to understanding the trajectory of house prices. We add to this literature by highlighting that most of the increase in housing wealth is driven by the increase in residential land wealth and by offering a theory that demonstrates the importance of the interaction between the construction boom and an inelastic supply of residential land for explaining the observed increase in house prices and housing wealth.

Since the US-housing bust and the subsequent Great Recession many studies contributed to a better understanding of the relationship between housing and the macroeconomy over the short term. One of the earlier papers in this field is Davis and Heathcote (2005) who set up a neoclassical multisector stochastic growth model to investi-

gate the cyclical dynamics of residential investment and other business cycle facts. The housing supply side of their model has formed the basis for many subsequent models. For instance, Favilukis, Ludvigson, and Nieuwerburgh (2017), employing a quantitative heterogeneous agent model, find that the relaxation of financing constraints and its subsequent reversal was the driving force behind the recent US-housing boom and bust. Kaplan, Mitman, and Violante (2019) come to a different conclusion. Accordingly, it is not the relaxation of financing constraints that drives house prices in the short term but rather changes in beliefs about the likelihood of future housing demand shifts. Whether credit conditions were responsible for the boom and bust in house prices is an ongoing debate. Greenwald and Guren (2019) provide a reconciliation by showing that the segmentation of housing markets matters for the transmission of credit supply shocks. We do not study short-term fluctuations in housing wealth but focus on the long term. Moreover, the housing supply side of our model differs from these existing models for two reasons. First, existing macro models with housing assume that each period one unit of land becomes available and is incorporated into the housing stock by the construction sector. When explaining the evolution of housing wealth we decompose it into price and quantity components of residential land and structures. Assuming that the quantity of residential land is exogenous would restrict the analysis of the long term evolution of residential land with strong implications for the residential land price. Second, existing models assume that land is not productive in the numeraire sector. This assumption places a strong restriction on differential land intensities of the numeraire and the housing sector. Our analysis shows that differential land intensities play an important role in the determination of long-run rent and house price growth.

We share our focus on housing over the long term with a smaller branch of the literature on housing and macroeconomics. One of the first economists to be concerned about the long-term consequences of the fixed factor land was Ricardo (1817). He argues that economic growth benefits landlords as the owners of the fixed factor disproportionately.[9] While Ricardo was mainly concerned with agricultural land and the production of corn to feed a growing population, societies in modern times are confronted with the need for residential investments to meet the increasing demand for housing services under the constraint of a fixed supply of land. Hansen and Prescott (2002) employ a one-good, two-sector OLG model where land is in fixed supply. The transition from constant to growing living standards, they argue, is inevitable given positive rates

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[9]See also Piketty (2014).

of total factor productivity growth. In particular, the transition from stagnant to growing living standards occurs when profit-maximizing firms, in response to technological progress, begin employing a less land-intensive production process that, although available throughout history, was not previously profitable to operate. Herkenhoff, Ohanian, and Prescott (2018) construct a general equilibrium spatial growth model of the US to analyze how state-level land-use restrictions have impacted on regional and aggregate economic activity between 1950 and 2014. Tightened land use restrictions, they argue, have increasingly limited the availability of land for housing and commercial use, which in turn have raised land prices, slowed interstate migration, reduced factor reallocation, and depressed output and productivity growth relative to historical trends. Borri and Reichlin (2018) employ a two-sector OLG model with housing services and bequests to show that a rising labor efficiency in the general economy relative to the construction sector pushes the house price, the housing wealth-to-income ratio, and wealth inequality upwards. Due to an inelastic housing demand, an increase in the relative labor efficiency triggers not only a strong house price appreciation but also a reallocation of resources to the housing sector. Miles and Sefton (2020) employ a spatial Ramsey growth model with two sectors. Residential land is endogenous and depends on the state of transport technology. They derive a condition under which the speed at which transport technology improves relative to the growth in aggregate incomes is such as to generate flat house prices in a growing economy. The model can explain the constancy of the real house price before 1950 and the surge thereafter. While we share the focus on housing over the long term with this literature, our model differs as it explicitly differentiates between i) the stock of residential land and residential structures, ii) differential technological progress and iii) differences in land intensities between the housing and non-housing sectors. The latter allows us to asses the relevance of each of the two mechanisms in driving house prices in the long term. Furthermore, our main focus lies in explaining the rising housing wealth-to-income ratio.

2. Stylized facts

Following Davis and Heathcote (2007) we decompose housing wealth into the sum of the value of structures and the value of residential land,

$$\frac{\text{housing wealth}}{\text{income}} = \frac{\text{value of structures}}{\text{income}} + \frac{\text{value of land}}{\text{income}}. \tag{1}$$

Moreover, each wealth component can be decomposed into a price and a quantity component according to

$$\frac{P^{\text{housing}} \times Q^{\text{housing}}}{\text{income}} = \frac{P^{\text{structure}} \times Q^{\text{structure}}}{\text{income}} + \frac{P^{\text{land}} \times Q^{\text{land}}}{\text{income}}.$$

where P^{housing} denotes a house price index, Q^{housing} a quantity index for the housing stock, and the notation for structure and land is similar. To understand how housing wealth has been evolving, we look at empirical data that distinguishes between the value of structures and the value of land and between the respective price and quantity components. We also report further housing-related macroeconomic variables such as the land share and the rent.

We study the following four developed economies: USA (1950–2015), UK (1950–2012), France (1960–2012), and Germany (1960–2012). The underlying data, described in online appendix A, is not entirely new.[10] However, we view the data from a specific angle by decomposing the increase in housing wealth into prices and quantities of residential land and structures. This list is helpful to both better understand the long-term evolution of housing wealth and to discipline a theory that aims at providing an explanation. The precise numbers behind all facts are summarized in table 1 and we elaborate on each of the five stylized facts in the following.

Fact 1: Wealth. The wealth-to-income ratio has been rising, as shown in figure 1. This is well known from the work by Piketty and Zucman (2014). Decomposing the wealth-to-income ratio into housing and non-housing wealth-to-income ratios shows that the housing wealth-to-income ratio has been rising much more than the non-housing wealth-to-income ratio (see also figure 1).[11] This holds for all four economies and the difference is especially pronounced in the UK and France, as can be seen in table 1.

Fact 2: Prices. Real house prices, real construction costs, and real land prices have all been increasing. This has been documented for the US by Davis and Heathcote (2007) and for a group of 14 advanced economies by Knoll, Schularick, and Steger (2017). Real house prices almost doubled during the post-WWII period in the US and Germany while they even tripled and quintupled in the UK and France as shown in

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[10]We are grateful to Moritz Schularick, Luis Bauluz, and Filip Novokmet for sharing their data on housing wealth.

[11]The observation that housing wealth has been increasing more strongly than non-housing wealth has also been stressed by Rognlie (2015) and Bonnet, Chapelle, Trannoy, and Wasmer (2019).

Fact #	Growth factor of	US 1950–2015	UK 1950–2012	FR 1960–2012	DE 1962–2012
1	wealth-to-income ratio	1.39	1.4	2.2	1.88
	housing wealth-to-income ratio	1.42	2.4	3.2	1.93
	non-housing wealth-to-income ratio	1.38	0.9	1.2	1.79
2	house price	1.9	4.0	6.0	1.6
	residential land price	8.4	9.6	32.2	2.3
	residential structure price	1.2	2.1	1.1	1.4
3	house quantity	4.4	3.3	2.2	4.2
	residential land quantity	2.8	2.1	1.6	3.5
	residential structure quantity	5.5	4.3	6.3	4.3
4	land share in housing wealth	2.8	1.5	3.8	1.2
5	housing rent	1.7	3.1	2.8	1.6

Notes: The sample for FR starts in 1960 because residential land values are negative in some earlier years and the sample for DE starts in 1962 because house and residential land prices are not available for previous periods. The data source is described in online appendix A.

Table 1: Stylized facts, growth factors

table 1. Comparing the increase between prices for residential structures and residential lands reveals a striking pattern. Residential land prices exhibited very strong growth, while the prices of residential structures rose only modestly and house prices lie in between the two. We return to this difference after studying the next fact.

Fact 3: Quantities. By making use of the value indices and price indices for overall housing, residential land, and residential structures we are able to compute quantity indices. The quantity index of residential structures has been steadily increasing, while the quantity index of residential land increased only modestly and the housing index grew at a factor that lies in between the two. Taken together, facts 2 and 3 point to a mirror-like pattern. While the quantity of structures increased strongly, the price of structures increased only moderately. For residential land the reverse holds true, the price increased much stronger than the quantity. This observation points to an important role of the economy's supply side when it comes to understanding the stylized facts. Residential structure is reproducible, implying a higher price elasticity of supply, while residential land is non-reproducible in the long term, implying a lower price elasticity

of supply. Our modeling choice is guided by this observed mirror-like pattern and we will come back to it frequently.

Fact 4: Land. To understand why the housing wealth-to-income ratio has been increasing (fact 1) it is helpful to decompose it into a residential structure and residential land component as defined in (1). A variable that compactly summarizes the contribution of these two factors is the share of land wealth in total housing wealth.[12] As can be inferred from figure 1, this share increased considerably, ranging from a growth factor of 1.2 for Germany to 3.8 for France. This shows that the increase in the housing wealth-to-income ratio is mainly driven by the increase in residential land wealth relative to income, while structure wealth remained relatively stable relative to income.

Fact 5: Rents. The house price – the price of the stock of housing – is closely related to the housing rent – the price of the service flow derived from the stock of housing. Similar to the house price, the real housing rent has also been increasing with a growth factor ranging between 1.6 for Germany and 3.1 for the UK. A plausible theory that replicates the increase in the housing wealth-to-income ratio – fact 1 – shall also be in line with facts 2 to 5. We will come back to all facts below.

3. The model

Consider a perfectly competitive closed economy. Time is continuous and indexed by $t \in \mathbb{R}$. Infinitely-lived households earn labor and capital income, save, and consume two goods: housing and a non-housing good. The two consumption goods are produced by two different sectors, a numeraire sector and a housing sector. The numeraire sector combines physical capital, labor, and land to produce an output good. The output of this sector can be consumed, invested into physical capital, or used as an input in the construction sector. The innovative part of our model concerns the housing sector. This sector comprises three types of firms. First, property management firms purchase residential land and residential structures to produce houses, which generate revenues from being rented to households. Second, construction firms manufacture structures by employing materials and labor. Third, land development firms develop residential land by purchasing non-developed land and employing labor. The overall supply of land is fixed and the intersectoral land allocation is endogenous. The economy grows in the

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[12]Divide both sides of (1) by the housing wealth-to-income ratio to obtain $1 = \frac{\text{value of structures}}{\text{housing wealth}} + \frac{\text{value of land}}{\text{housing wealth}}$, where the second fraction on the RHS is the share of land wealth in total housing wealth.

long run due to exogenous technological change.

3.1. Households

The economy is inhabited by mass one of households. A household consists of $L_t > 0$ homogeneous members. Total population is hence equal to L_t. We allow L_t to grow over time. Households consume two goods, housing services, S_t, and a non-housing good, C_t. We denote all prices in units of the latter and use the terms *numeraire* and *non-housing good* interchangeably. The representative household derives utility from consumption streams $\{C_t, S_t\}_{t=0}^{\infty}$ according to

$$U = \int_0^{\infty} e^{-\rho t} L_t u\left(\frac{C_t}{L_t}, \frac{S_t}{L_t}\right) \mathrm{d}t, \tag{2}$$

where $\rho > 0$ is the time preference rate and $u\left(C_t/L_t, S_t/L_t\right)$ the period utility function with per-capita consumption of the numeraire and per-capita housing services as arguments. Household utility at t is the product of household size, L_t, and per-capita utility, $u\left(C_t/L_t, S_t/L_t\right)$. The period utility function has the following form[13]

$$u\left(\frac{C}{L}, \frac{S}{L}\right) = \frac{\left[\left(\frac{C}{L}\right)^{1-\theta} \left(\frac{S}{L}\right)^{\theta}\right]^{1-\sigma} - 1}{1 - \sigma}, \tag{3}$$

such that $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution and $\theta \in (0, 1)$ the housing expenditure share.

Each household member supplies one unit of working time inelastically to the labor market. Labor supply per household and aggregate labor supply are then equal to L. A household's total wealth is given by W and it generates a flow return of rW. The budget constraint reads

$$\dot{W} + C + qS = rW + wL + \Pi^D, \tag{4}$$

where q is the price of housing services, w is the wage, and Π^D are profits of firms that develop land, as specified below.[14] Saving, $\dot{W}$, and total consumption expenditures have to equal total income which consists of wealth income, earnings, and profits.

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[13]We suppress the time index t whenever no confusion arises.

[14]We model all households as renters, but the results do not change if we assume owner-occupied housing.

3.2. Numeraire production

Mass one of firms produce the numeraire, Y, with inputs capital, K, labor, L^Y, and land, N, according to

$$Y = K^\alpha \left(B^Y L^Y\right)^\beta N^{1-\alpha-\beta}, \tag{5}$$

where $\alpha, \beta \in (0,1)$ and $\alpha + \beta < 1$. The variable B^Y captures exogenous labor-augmenting technological change and it grows at the constant rate $g^Y \in \mathbb{R}$. Throughout the paper we will primarily employ Cobb-Douglas production functions. If we used more general CES functions, a steady state with differential technological growth would not exist. The representative firm solves the following problem

$$\max_{K, L^Y, N} \Pi^Y = K^\alpha \left(B^Y L^Y\right)^\beta N^{1-\alpha-\beta} - (r + \delta^K)K - wL^Y - R^N N, \tag{6}$$

where $\delta^K \geq 0$ is the capital depreciation rate and R^N the rental rate of non-residential land.

3.3. Housing

A house is a bundle of two conceptually different stocks: underlying land, D, and the residential structure, X, erected upon it. This distinction follows the empirical contribution by Davis and Heathcote (2007), who provide price and quantity indices for housing, land, and structures for the US and show that this distinction is key for understanding aggregate housing market dynamics. This distinction is also necessary for mapping model outcomes to the stylized facts presented in section 2. The main difference between D and X is that the former is not reproducible in the long run while the latter is. Increasing demand for housing over time then implies that X can be accumulated while D cannot and that prices for X will increase much less than prices for D. This mirror-like pattern is key for explaining the stylized facts and we come back to it repeatedly.

The housing sector consists of three different subsectors.[15] First, *property management* firms demand residential structures and residential land to build houses which are then rented out to households. Second, *construction* firms demand labor and materials

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[15]We explain the different economic activities involved in the supply side of the housing sector as undertaken by three different types of firms, but other institutional arrangements are equivalent. For example, one can equivalently assume that one firm sector engages in all three activities simultaneously.

to produce new structures that are then sold to *property management* firms. Lastly, *land development* firms demand labor and non-developed land to produce residential land and sell it to *property management* firms. We now go through each of these activities in turn.

3.3.1. Property management

Property management firms combine residential structures, X, and residential land, D, to produce houses, H, according to

$$H = X^\gamma D^{1-\gamma}. \tag{7}$$

The stock of houses H generates a proportional flow of housing services equal to H.[16] Housing services are sold to households at the rental rate q. Property management firms do not only manage an existing stock of housing, but they also invest in additional houses. They commission construction firms to produce I^X new units of structure at the price P^X and land developers to produce I^D new units of residential land at the price P^D. The representative property management firm solves the following dynamic problem

$$\max_{\{I_t^D, I_t^X\}_{t=0}^\infty} \int_0^\infty e^{-\hat{r}_t} CF_t \, \mathrm{d}t \tag{8}$$
$$\text{s.t. } \dot{X}_t = I_t^X - \delta^X X_t$$
$$\dot{D}_t = I_t^D,$$

where

$$CF_t \equiv q_t X_t^\gamma D_t^{1-\gamma} - P_t^X I_t^X - P_t^D I_t^D \quad \text{and} \quad \hat{r}_t \equiv \int_0^t r_\tau \mathrm{d}\tau \tag{9}$$

are the firms' cash flow and the discount factor. The cash flow consists of revenues from renting out the existing housing stock, qH, minus investment expenditures. The law of motions of structures and land reflect that structures depreciate at the rate $\delta^X \in [0,1]$ while land does not. This seems to be a plausible assumption because maintaining a constant stock of housing necessitates replacement investment into structures, but it is not necessary to produce additional land to keep the housing stock constant.

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[16]This is a common assumption in the literature. See, for instance, Piazzesi and Schneider (2016); Favilukis, Ludvigson, and Nieuwerburgh (2017); Greenwald and Guren (2019); Kaplan, Mitman, and Violante (2019).

The property management firm's assets can i) either be expressed as the value of residential structures plus the value of residential land, $P^D D + P^X X$, or ii) as the value of houses, $P^H H$, where P^H is the house price. This equivalence reflects that houses are bundles of structure and land. Since the two have to be equal, $P^D D + P^X X = P^H H$, the resulting house price reads

$$P^H = \frac{D}{H} P^D + \frac{X}{H} P^X. \tag{10}$$

The house prices is a weighted sum of the prices of residential land and residential structures. The weights are the land and structure intensity in housing, respectively.

Property management firms are owned by households. The aggregate valuation of property management firms is given by the value of its assets, $P^H H$. Without loss of generality we set the quantity of shares equal to H such that the share price equals P^H. This implies that the price of a house and the value of a share are both equal to P^H and households effectively trade houses, which they ultimately own. Owning one share entitles the owner to a yield of R^H. In online appendix B.1 we show that this yield is given by $R^H = \frac{CF + P^H \dot{H}}{H}$ in equilibrium. The yield is expressed per unit of housing. It comprises the operative cash flow, CF, plus the cash flow from issuing new shares, $P^H \dot{H}$.

3.3.2. Construction

The representative construction firm employs materials, M, and labor, L^X, to produce new structures according to

$$I^X = M^\eta \left(B^X L^X \right)^{1-\eta},$$

where $\eta \in [0, 1)$. The variable B^X captures exogenous technological progress in the construction sector and grows at the constant rate $g^X \in \mathbb{R}$. The representative firm chooses labor and materials to maximize profits according to

$$\max_{M, L^X} \Pi^X = P^X M^\eta \left(B^X L^X \right)^{1-\eta} - M - w L^X \quad \text{subject to} \quad M \geq 0, L^X \geq 0, \tag{11}$$

where P^X is the price of residential structures. Materials are intermediate goods produced with the same technology with which the numeraire good is produced. Therefore their price is equal to one.

3.3.3. Land development

The economy's land endowment is denoted by Z. It is allocated between the numeraire sector and the housing sector such that $Z = N_t + D_t$ holds in equilibrium. In order to make land suitable for housing it has to be developed by land development firms. These firms purchase (or sell) $\dot{N}$ units of non-developed land in order to increase (or decrease) the stock of developed land by $\dot{D}$ units. We use the terms *residential land* and *developed land* interchangeably. Total land Z is constant such that the total amount of land that can be used economically is fixed. For studying housing in the long run this seems more plausible than the opposite – ever growing land supply.[17] If land formerly used for producing the numeraire can be rededicated to housing without any cost, then N and D are jump variables that may adjust immediately to any reallocation incentive. We do not impose this assumption and allow that rededicating land from one sector to the other might be costly. The representative land development firm chooses investment into residential land $I^D \gtreqless 0$ to maximize profits according to

$$\max_{I^D} \Pi^D = P^D I^D - \underbrace{\left[P^N I^D + w \mathscr{L}^D(I^D) \right]}_{\text{cost function}}, \tag{12}$$

where P^D and P^N are prices of developed land and non-developed land, respectively, and $L^D = \mathscr{L}^D(I^D)$ is the labor input associated with the reallocation of land. We assume that this cost function takes the form

$$\mathscr{L}^D(I^D) = \frac{\xi}{2} \left(I^D \right)^2, \tag{13}$$

where $\xi \geq 0$ captures the importance of adjustment cost. These cost arise from activities like pulling down an existing building, leveling the surface off, and installing utilities like sewage, water, electricity, or roads. We derive the convex cost function in (12) from a decreasing returns to scale production function with perfect complements in the online appendix. Whether ξ is zero or strictly positive determines whether adjustment cost exist or not, as we explain now in greater detail.

If $\xi = 0$, the reallocation of land is costless. The land allocation would be a jump

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[17]This has been acknowledged by previous researchers who analyze land and long-run economic growth, starting with Ricardo (1817) and Nichols (1970). The popular statement "buy land, they are not making it anymore", usually ascribed to Mark Twain, illustrates this point.

variable as N and D adjust immediately until $P^D = P^N$ is restored at each instant.[18] Although the model allows for this immediate rededication of land, we show in the calibration section that this is empirically implausible.

If $\xi > 0$, the reallocation of land is costly and firms will choose an interior optimum. In this case, the reallocation of land is stretched out over time and N and D are state variables. The FOC for (12) reads

$$I^D = \frac{P^D - P^N}{\xi w}. \tag{14}$$

In equilibrium the adjustment cost are $wL^D = \frac{(P^D - P^N)^2}{2\xi w}$ and profits can be expressed as $\Pi^D = wL^D$. The price difference between residential and non-residential land, $P^D - P^N$, determines whether firms develop residential land and how much they develop. First, if residential land is more valuable than non-residential land, firms engage in land development and $I^D > 0$. Profits, which are strictly positive, are redistributed to households who own land development firms. Second, if land prices are equal, firms do not reallocate land and profits as well as adjustment cost are zero. Lastly, land development firms do not only develop residential land, but conduct also the reverse activity when the price of non-residential land is higher than the residential land price. In this case I^D is negative and land is reallocated from the housing to the numeraire sector. Profits and adjustment cost are also positive in this case.[19]

The land development sector becomes inactive in the long run. Since total land endowment is fixed there will be no land reallocation and therefore land prices will equalize such that $L^D = I^D = \Pi^D = 0$. Hence, the land development sector matters only for transitional dynamics.

3.4. Assets

The assets in this economy are i) capital, K, ii) non-residential land, N, and iii) houses, H. All assets are ultimately owned by households as can be seen in the decom-

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[18]If P^D were larger than P^N, firms would transfer as much land as possible from the numeraire to the housing sector, demanding more and more N and supplying more and more D. The price of land in the numeraire sector would increase and the price of residential land would decrease until they are equal. Hence, P^D cannot be larger than P^N. The same reasoning applies for $P^D < P^N$.

[19]Our specification of the adjustment cost function $\mathcal{L}^D(\cdot)$ has the property that $L^D > 0$ for $I^D \gtrless 0$ and therefore an interior solution exists also for negative investment in residential land.

position of total household wealth

$$W_t = \underbrace{P_t^H H_t}_{\text{housing wealth}} + \underbrace{K_t + P_t^N N_t}_{\text{non-housing wealth}}. \tag{15}$$

Total wealth consists of housing wealth and non-housing wealth where the latter is the sum of capital and non-residential land. Because markets are complete and the economy is deterministic, all assets yield the same rate of return, r, in equilibrium[20]

$$r = \frac{\dot{P}^N + R^N}{P^N} = \frac{\dot{P}^H + R^H}{P^H}. \tag{16}$$

Households are indifferent with regard to the allocation of total wealth, W, across the different assets and care only about total wealth W and the rate of return r. It is therefore sufficient to consider only total wealth, W, and r instead of all assets in the household problem.

3.5. Equilibrium

We define the competitive equilibrium next.

Definition 1 (Competitive equilibrium). *A competitive equilibrium of the model are sequences $\{C_t, S_t, W_t, q_t, w_t, r_t, \Pi_t^D, Y_t, K_t, L_t^Y, N_t, R_t^N, H_t, R_t^H, X_t, D_t, P_t^X, CF_t, I_t^D, L_t^D, P_t^D, P_t^N, P_t^H, I_t^X, M_t, L_t^X\}_{t=0}^{\infty}$ for given initial capital, residential land, and residential structures $\{K_0, D_0, X_0\}$ and a population sequence $\{L_t > 0\}_{t=0}^{\infty}$ such that*

  i) households maximize (2) given (3) subject to (4) and a no-Ponzi game condition;
  ii) firms in the construction sector and the numeraire sector, land developers, and property management firms maximize profits as given by (6), (8), (11) and (12), taking (13) and prices as given;
  iii) rental market clears: $H_t = S_t$;
  iv) labor market clears: $L_t^X + L_t^Y + L_t^D = L_t$;
  v) land market clears: $D_t + N_t = Z$;
  vi) asset markets clear (15);
  vii) the house price is given by (10);
  viii) there are no arbitrage opportunities between assets, as described by (16);

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[20]If the rate of return on an asset deviated from r, households would demand either zero or an infinite amount of the asset and markets would not clear.

The market for the numeraire good clears by Walras' law.

3.6. Housing wealth-to-income ratio

In order to express the housing wealth-to-*income* ratio we need first to define the economy's *income* consistently with the empirical convention of using the net national product (Piketty and Zucman, 2014; Rognlie, 2015). It is denoted by NNP and given by

$$NNP = \underbrace{\overbrace{Y}^{\text{numeraire}} + \overbrace{qS}^{\text{housing services}} + \overbrace{P^X I^X - M}^{\text{construction}} + \overbrace{\left(P^D - P^N\right)I^D}^{\text{land development}}}_{\text{GDP}} - \overbrace{\left(\delta^K K + \delta^X P^X X\right)}^{\text{depreciation}}.$$

Since the economy comprises four production sectors, GDP equals the sum of the value-added of these sectors. In order to avoid double accounting, materials M have to be subtracted from output of the construction sector because they already are accounted for by Y. Value added from the land development sector is always positive because I^D has the same sign as the land price difference $P^D - P^N$ such that $I^D \times \left(P^D - P^N\right) > 0$. Rededicating land from the numeraire good sector to the housing sector enables land developers to create value added. This equals the value of newly developed land, $P^D I^D$, reduced by the costs for the purchase of non-developed land, $P^N I^D$. Subtracting depreciation from GDP yields net domestic product. Net domestic product is equal to NNP because the model economy is closed. Using (10), the housing wealth-to-income ratio is – in line with the empirical measurement of section 2 – given by

$$\frac{P^H H}{NNP} = \frac{P^D D}{NNP} + \frac{P^X X}{NNP}. \tag{17}$$

The total wealth-to-income ratio and the non-housing wealth-to-income ratio are defined similarly. Equation (17) shows that the housing wealth-to-income ratio can be decomposed into the residential land wealth-to-income ratio and the residential structure wealth-to-income ratio. The model hence allows to directly map total wealth, prices, and quantities of houses, residential land, and residential structures into the data.

3.7. Relation to existing theories

How does the model differ from existing macroeconomic models with a housing sector? First, the model distinguishes two endogenous stocks, namely residential land and residential structures. Both stocks are essential input factors in the production of housing services and both stocks represent the two components of housing wealth. Existing macro models with housing assume that each period one unit of land becomes available and is incorporated into the housing stock by the construction sector.[21] We depart from this model structure because it would be ill-suited to investigate the evolution of housing wealth and its components over the long term. Recall that we decompose housing wealth into four components: the price and quantity components of residential land and the price and quantity components of structures. Assuming that the quantity of residential land is exogenous would restrict the analysis of the long-term evolution of residential land with strong implications for the residential land price.

Second, most existing macro models with housing assume that the numeraire sector does not employ land. In the context of our research question, this would be a further restrictive assumption. Below we show that long-term growth in rents and house prices is driven by differential technological change and differential long-run land intensities. Assuming land is not productive in the numeraire sector implies a strong and restrictive assumption with regard to relative long-run land intensities. This assumption would impose a potentially important constraint on the numerical analysis.

Third, the existing macro model with housing would be ill-suited for a long-term analysis of housing wealth. Even in an economy without long-run economic growth the stock of accumulated residential land would tend to infinity as time approaches infinity. In a growing economy the accumulated amount of land would grow to infinity at an even higher rate. The reason is that existing macro models with housing assume that replacement investment requires land.

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[21] See, for instance, Davis and Heathcote (2005); Favilukis, Ludvigson, and Nieuwerburgh (2017); Greenwald and Guren (2019); Kaplan, Mitman, and Violante (2019). A notable exception is Herkenhoff, Ohanian, and Prescott (2018), who assume a fixed supply of land but abstract from differential technological growth, a separate construction sector, and adjustment costs in land reallocation.

Variables	Growth rate
$r, \frac{P^H H}{NNP}, \frac{W}{NNP}, \frac{P^D D}{P^H H}, L^D, L^X, L^Y, N, D$	0
$Y, K, M, w, R^N, P^N, P^D, C, NNP, W$	$\frac{\beta}{1-\alpha} g^Y$
X, I^X	$\eta \frac{\beta}{1-\alpha} g^Y + (1-\eta) g^X$
P^X	$(1-\eta)\left(\frac{\beta}{1-\alpha} g^Y - g^X\right)$
S, H	$\gamma \eta \frac{\beta}{1-\alpha} g^Y + \gamma(1-\eta) g^X$
q, P^H	$(1-\gamma\eta)\frac{\beta}{1-\alpha} g^Y - \gamma(1-\eta) g^X$

Table 2: Steady state growth rates

4. Steady state

Can rising housing wealth-to-income ratios and the other stylized facts be explained as steady state phenomena? To answer this question, we study the economy's steady state analytically in section 4.1 and provide the intuition by employing demand and supply diagrams in section 4.2.

4.1. Analytical results

We define a steady state as a competitive equilibrium according to definition 1 where all variables grow at constant and possibly different rates. From now on we assume that population, described by the sequence $\{L_t > 0\}_{t=0}^{\infty}$, converges to a finite upper bound as time approaches infinity.

Proposition 1 (Steady state). *There exists a unique steady state equilibrium and variables grow at the rates shown in table 2.*

All proofs are relegated to the online appendix where we also derive a closed-form solution of the stationary steady state in detrended variables. The source of long-run growth in this economy is exogenous technological progress in the numeraire and in the construction sector as described by the growth rates g^Y and g^X. Growth rates of model variables are therefore functions of g^Y, g^X, and other parameters, as shown in table 2. We first study the growth rate of house prices and rents before we turn to other variables further below. Why are house prices and rents growing in the long-run? The answer is provided by

Proposition 2 (Differential land elasticities and differential technological progress).
The steady state growth rate of house prices, P^H, and rents, q, can be expressed as a function of long-run land elasticities and technological growth rates as follows

$$\mathfrak{g}_{P^H} = \mathfrak{g}_q = \left[1 - \eta + \eta\left(\psi^H - \psi^Y\right)\right]\frac{1 - \psi^Y}{1 - \eta\psi^Y}g^Y - (1 - \eta)\frac{1 - \psi^H}{1 - \eta\psi^Y}g^X,$$

where $\psi^Y \equiv \frac{1-\alpha-\beta}{1-\alpha} \in [0,1)$ and $\psi^H \equiv \psi^Y \eta\gamma + 1 - \gamma \in [0,1]$ are the long-run land elasticities in the production of the numeraire good and housing, respectively.

    i) If there is no differential technological progress, $g^Y = g^X$, then the growth rate of house prices and rents simplifies to $\left(\psi^H - \psi^Y\right)g^Y$. Assuming $g^Y > 0$, the growth rate is positive (negative) iff the long-run land elasticity is larger (smaller) in the housing sector than in the numeraire sector, $\psi^H > \psi^Y$ ($\psi^H < \psi^Y$).

    ii) If long-run land elasticities are equal, $\psi^Y = \psi^H$, then the growth rate of house prices and rents simplifies to $\mathfrak{g}_{P^H} = \mathfrak{g}_q = \frac{(1-\eta)(1-\psi^Y)}{1-\eta\psi^Y}\left(g^Y - g^X\right)$. The growth rate is positive (negative) iff technological progress is stronger (weaker) in the numeraire sector than in the construction sector, $g^Y > g^X$ ($g^Y < g^X$).

    In a growing economy, the two drivers for a long-term increase in house prices and rents are differential long-run land elasticities and differential technological progress. The long-run land elasticities of the numeraire and housing sector are denoted by ψ^Y and ψ^H, respectively, and are derived in the online appendix. The long-run land elasticity of the numeraire sector, ψ^Y, shows by how much output of the numeraire, Y, increases in response to an increase in the overall land endowment, Z. Similarly, the long-run land elasticity of the housing sector, ψ^Y, shows by how much output of the housing sector, H, increases in response to an increase in the overall land endowment, Z. Each of the two mechanisms – differential technological progress and differential long-run land elasticities – is sufficient to drive house prices in isolation, but they also reinforce each other. With a minor abuse of terminology we often call the sector with the higher long-run elasticity of land as being more land intensive.

    The intuition behind part i) of proposition 2 is as follows. Assume that the long-run land elasticity of the housing sector is larger than the long-run land elasticity of the numeraire sector, $\psi^H > \psi^Y$, but technological change is the same in both sectors, $g^Y = g^X > 0$. Symmetric technological change means that effective labor increases in both sectors – holding prices constant and abstracting from labor reallocations – by the same proportion. The assumption $\psi^H > \psi^Y$ together with constant returns to scale

implies a higher importance of labor in numeraire production. Hence, the direct effect of technological change is that output in the numeraire sector expands by more than in the housing sector. To restore equilibrium in the goods market, the relative price of the numeraire good must fall. That is, the price of housing services in units of the numeraire good, q, increases.

In part ii) of proposition 2 we assume equal long-run land elasticities, $\psi^H = \psi^Y$, but differential technological change, $g^Y > g^X > 0$.[22] The direct effect of differential technological change to the advantage of the numeraire sector – or equivalently disadvantage of the housing sector – is a relatively stronger output expansion of the numeraire sector resulting from the stronger increase in efficient units of labor. Again, the relative price of the numeraire good must fall to restore equilibrium in the goods market. The price of housing services in units of the numeraire good, q, increases. Because the house price equals the present discounted value (PDV) of rental yields and noting that the interest rate is constant in equilibrium, the house price increases as well.

What about the quantitative importance of the two channels – differential long-run land intensities and differential technological growth – for rising house prices? To assess their relative importance we conduct a back-of-the-envelope calculation. Assume the US has been growing along its steady state path over the period 1950–2015. Recall from proposition 2 that the steady state growth rate of house prices depends on five parameters: g^Y, g^X, ψ^Y, ψ^H, and η. We take $\eta = 0.556$ and $\psi^Y = 0.15$ from the main calibration (online appendix C), and conduct some robustness checks that we report below. Moreover, we set g^Y, g^X, and ψ^H such that we match the average annual growth rates of 2.8, 1, and 0.3 percent for NNP, the house price, and the price of residential structures (cf. to table B.1), respectively. This gives 0.03, 0.02, and 0.34 for g^Y, g^X and ψ^H, respectively. Hence, the difference in sector specific technological change is $g^Y - g^X = 0.01$ and the difference in long-run land elasticities reads $\psi^H - \psi^Y = 0.19$. In a first step we shut down the differences in long-run land intensities by setting elasticity ψ^H equal to the value of $\psi^Y = 0.15$. The growth rate of house prices then reduces to 0.52 instead of previously 1.00 percent. In a second step we additionally shut down the differences in technological progress by setting g^X to g^Y. The growth rate of house prices then declines to 0. Taken together, differences in long-run land intensities capture 52 percent of the long term increase in house prices while differences in sector

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[22]This mechanism is related to the Findlay-Grubert theorem, which considers output and price effects of differential technological change.

specific technological change capture the remaining 48 percent.[23] We obtain similar results for the UK, France, and Germany. For instance, in the UK differences in long-run land elasticities explain 41 percent of the long-run growth in house prices. This simple exercise shows that both differences in long-run land elasticities and differential technological progress are about equally important for rising house prices in the long run.

Although we focus on the post-WWII period, our theory can also speak to earlier periods. Knoll, Schularick, and Steger (2017) show that house prices and residential structure prices were largely constant before the 1950s and started increasing only afterwards. If we study the steady state of our economy and impose both these prices do not grow, then we obtain the restriction that i) $g^X = \left(1 - \psi^Y\right) g^Y$ and ii) $\psi^H = \eta \psi^Y$. The first restriction implies that before the 1950s technological progress in the construction sector was much stronger than in the second half of the twentieth century where $g^X < \left(1 - \psi^Y\right) g^Y$ has to hold. This seems plausible as construction costs were likely held down by technological advances such as the invention of the steel frame at the beginning of the 20th century. The second restriction implies that land was more relevant in the numeraire sector than in the housing sector before 1950, that is, $\psi^H < \psi^Y$. Although this is beyond the scope of our model, changing long-run land intensities may reflect structural change in the sense of the Kuznets hypothesis (Kongsamut, Rebelo, and Xie, 2001). After 1950 an increasing contribution to aggregate economic growth came from the service sector, which is less land-intensive than the manufacturing and the agricultural sectors. All these sectors are subsumed in the numeraire sector of our model and therefore structural change can – in a reduced form – be thought of as a decrease in ψ^Y. Our theory therefore points to an explanation of the constancy in house prices before 1950 and the take-off thereafter that rests on i) strong technological growth in the construction sector and ii) a high land-intensity in the numeraire sector.

Can the model replicate the other stylized facts, besides rising house prices and rents, in steady state? We provide the answer now in

**Corollary 2.1** (Stylized facts in steady state). *In the economy's steady state equilibrium*

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[23]We further run two robustness checks. First, setting ψ^Y to the value of $\psi^H = 0.34$ yields very similar results. Second, if we set η to 0 (0.99), then differential long-run land intensities explain a third (more than two third) of the overall growth rate of house prices while setting ψ^Y to 0 (0.3) implies that differential long-run land intensities explain 70 (30) percent of the rise in house prices.

#1) the wealth-to-income ratio, the housing wealth-to-income ratio, and the non-housing wealth-to-income ratio are constant,

#2) the price of residential land, the house price, and the price of residential structures grow at strictly positive rates, and the price of residential land grows at a higher rate than the house price, which in turn grows at a higher rate than the price of residential structures iff $g^Y > 0$ and $-\frac{\eta}{1-\eta}\left(1-\psi^Y\right)g^Y < g^X < \left(1-\psi^Y\right)g^Y$,

#3) quantities of residential structures grow at a strictly positive rate which is larger than the growth rate of residential land iff $g^X > -\frac{\eta}{1-\eta}\left(1-\psi^Y\right)g^Y$,

#4) the share of land wealth in housing wealth, given by $\frac{P^D D}{P^H H}$, is constant, and

#5) rents grow at a strictly positive rate if and only if $g^X < \frac{1-\psi^Y}{1-\psi^H}\frac{1-\eta+\eta\left(\psi^H-\psi^Y\right)}{1-\eta}g^Y$ or if $g^X < \left(1-\psi^Y\right)g^Y$.

Through the lens of our model most of the stylized facts can be explained as steady state phenomena. However, rising housing wealth-to-income ratios as well as a rising share of land wealth in housing wealth cannot be explained in steady state. This is not an implication specific to our model, but a very common implication of macroeconomic models.[24] In section 5 we will therefore study transitions and show what the main drivers for rising housing wealth-to-income ratios are.

The conditions for the other stylized facts being satisfied in steady state depend crucially on differential long-run land intensities and differential technological progress. First, it is necessary that technological progress in the construction sector is lagging behind technological progress in the rest of the economy. Otherwise the price of residential structures would be declining in steady state. The observation of – although modestly – rising construction cost (see section 2) therefore indicates that technological progress in the construction sector was lagging behind technological progress in the rest of the economy. Second, the larger the long-run land elasticity in the housing sector and the smaller the long-run land elasticity in the numeraire sector, the more likely it is that the restrictions in corollary 2.1 are satisfied. Hence, relatively weak technological progress in the construction sector and a relatively high long-run land elasticity in the housing sector can jointly explain many of the stylized facts already as steady state phenomena.

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[24]In a one-sectoral economy it intuitively results from the assumption of a constant returns to scale production function which implies that the capital-output ratio is constant.

(i) Housing H **(ii)** Residential structures X **(iii)** Developed land D
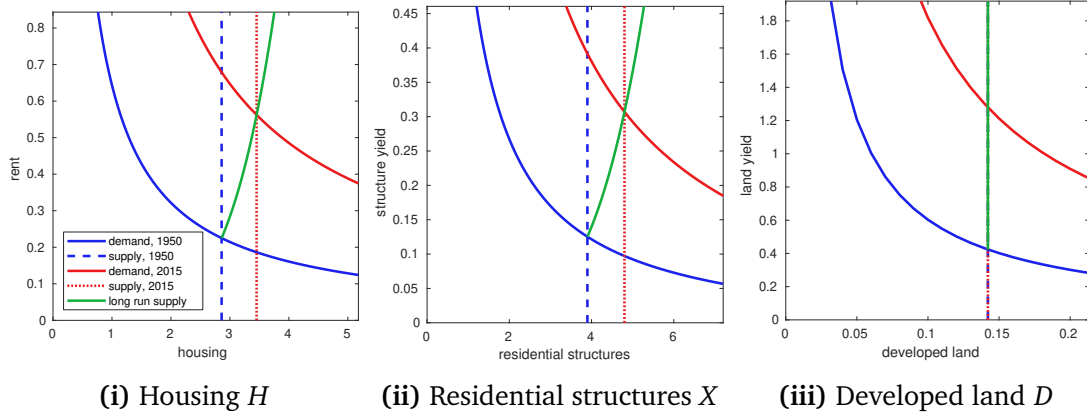
Notes: Parameter values are from our baseline calibration in online appendix C. The economy is in steady state and L_t is set to its long-term value. Shift in demand and supply curves result only from long-run growth, but not from transitional dynamics.

Figure 2: Demand-supply representation

4.2. Intuition for mirror-like behavior of prices and quantities

In corollary 2.1 we show under which conditions residential land prices grow at a higher rate than residential structure prices and the quantity of residential structures grows at a higher rate than residential land in steady state. We now explain the intuition behind this mirror-like behavior.

In figure 2 we plot demand and supply curves for three markets, i) the housing market, ii) the underlying market for residential structures, and iii) the underlying market for developed land.[25] For now, we assume that the economy evolves along its steady state and focus on the years 1950 and 2015.

The housing market is depicted in panel (1) of figure 2. The short-run supply of housing is fixed because housing cannot change instantly. Households demand housing services according to a downward sloping demand schedule. It captures only the direct effect of changes in rents, q, on short-run housing demand. Since the economy is growing over time, household demand more housing services in 2015 and the demand curve shifts outwards. If supply did not change, rents would increase strongly. The supply curve can only shift outwards if the stock of residential structures or developed land or a combination of both increases over time. This leads us directly to the market for structures and land. The market for structures is depicted in panel (2) of figure 2.

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[25]We derive the analytical expressions behind the demand and supply curves in online appendix B.5.

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The supply schedule is inelastic in the short run and the demand schedule is downward sloping. An increase in housing demand shifts the demand curve for structures outwards. At the same time construction firms anticipate the economy's future evolution, as reflected in the price P^X, and increase the stock of residential structures over time. The faster technological progress in the construction sector, the stronger is this increase in structures. The result is an outward shift in the supply curve of residential structures and a weaker increase in the structures' yield component. In the market for developed land the demand curves behave similarly, but supply cannot increase because land is fixed in the long run. The rental yield for land increases much more than the rental yield for structures, while the quantity of structures increases much more than the quantity of residential land. This mirror image is hence the result of rising housing demand that meets relatively inelastic long-run supply of structures and fully inelastic long-run supply of land. The induced changes on the X and D market translate back into the housing market as follows. Houses are bundles of structure and land. Structures increase, but land remains constant and therefore the quantity of housing increases between 1950 and 2015, but less than the quantity of structures. As a result rents increase between 1950 and 2015.[26]

5. Transitional dynamics

5.1. Calibration approach

We calibrate the model to all four economies – US, UK, France, and Germany – separately on an annual frequency for the post-WWII era. Although the model economy converges to a steady state, we do not impose that any of the four economies were in steady state since WWII. That is, the model is calibrated out of steady state. The transition results from two sources, i) deviations of initial state variables K_0, X_0, and D_0 from their respective steady states and ii) exogenous transitory population growth entering through L_t. We calibrate a subset of parameters – including the exogenous time path for L_t – exogenously. The remaining 7 parameters and 3 initial states K_0, X_0, and D_0 are calibrated endogenously by matching 10 moments from the data. We do not

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[26]The long-run supply curve of housing would be horizontal and rents would not increase at all i) if technological progress in the construction sector were sufficiently strong, ii) if the long-run land elasticity in the housing sector were sufficiently low, or iii) if residential land would grow sufficiently fast over time. See proposition 2.
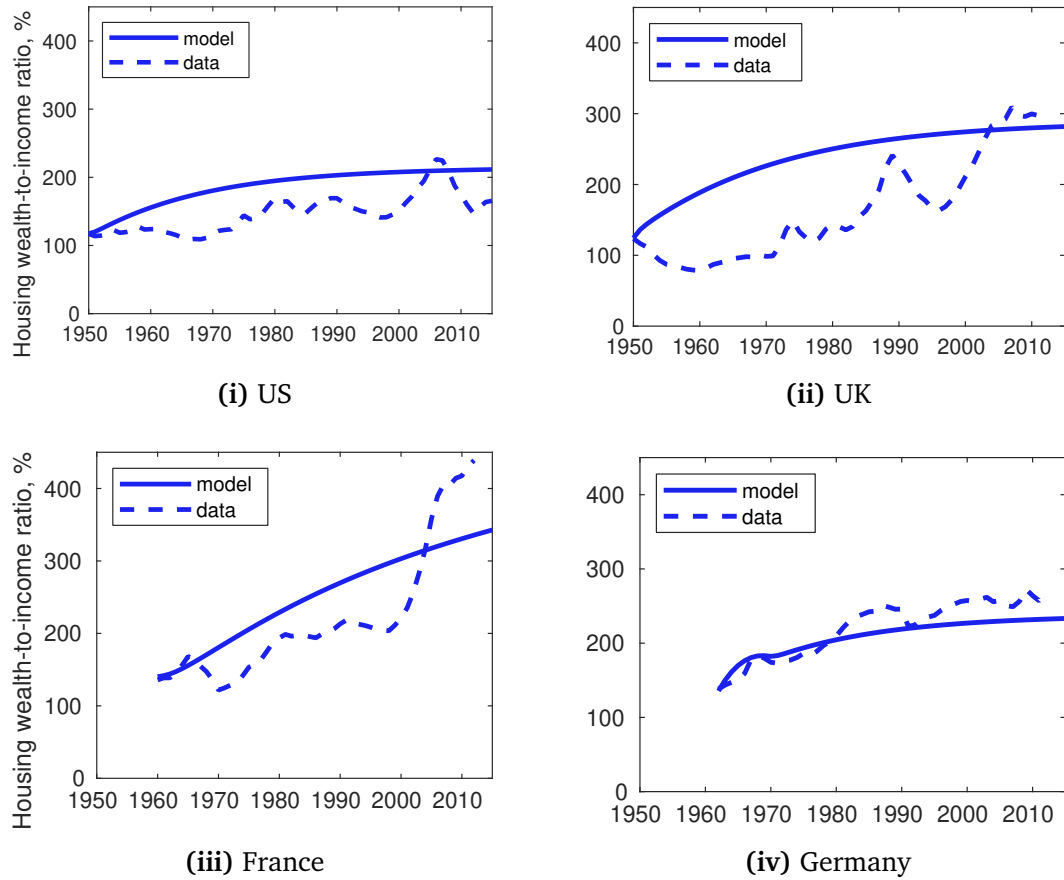
Figure 3: Housing wealth-to-income ratios, 1950–2015

target the growth factor of the housing wealth-to-income ratio or any other variables from the stylized facts of section 2, except for the growth factor of rents. Details can be found in online appendix C.

5.2. Housing wealth-to-income ratio

The main result of the quantitative part is shown in figure 3 where we plot the housing wealth-to-income ratio since 1950 for the US, UK, France, and Germany. The level of the housing wealth-to-income ratio at the start of the transition is matched by construction because we target it in the calibration. The evolution of the housing wealth-to-income ratio over time, however, is not targeted. The calibrated model replicates a

rising housing wealth-to-income ratio for all four economies. For the UK, France, and Germany the calibrated model explains 80, 67, and 76 percent of the increase in the housing wealth-to-income ratio between the initial year (1950, 1960, 1962) and 2012, respectively. In the case of the US it over-predicts the increase in the housing wealth-to-income ratio slightly by 35 percentage points. This might be due to the fact that in 2015 the US had not yet fully recovered from its recent housing bust.

Why is the housing wealth-to-income ratio increasing over time? This is not trivial as the housing wealth-to-income ratio is stable in steady state and changes only along the transition. Technically speaking, transitional dynamics result from deviations of initial capital, developed land, and residential structures – K_0, D_0, and X_0 – from their respective steady state values and from exogenous population dynamics. To better understand the relative importance of the components associated with the different state variables, we conduct the following experiment. For each initial state – K_0, D_0, and X_0 – we compute an alternative transition where the respective initial state starts in its steady state value. This way we are shutting down the contribution that results from transitional dynamics triggered by the difference of a respective initial state variable from its steady state value. We then compare how the growth factors of the housing wealth-to-income ratio and other variables change. In the following we focus on the US. The results are shown in table 3. Column (1) contains the growth factors from the baseline transition that is implied by our calibration and columns (2) to (4) the growth factors from the alternative transitions. For example, comparing the entries for developed land prices, P^D, in columns (1) and (3) shows that if residential land were to start at its steady state value in 1950 instead of 60 percent below it, land prices would increase by a factor of 8.1 instead of 4.7.

Before discussing the results further we now explain why population growth cannot explain the increase in the housing wealth-to-income ratio. In the transition we feed in exogenous population growth. If we instead assume that there is no population growth and the population level in 1950 is at its long-run value, then the housing wealth-to-income ratio increases even more. Hence, population growth rather reduces the housing wealth-to-income ratio than increasing it. The reason is that population growth affects both the numerator and the denominator. The positive effect on the numerator works through the rising demand for housing. The positive effect through the denominator, however, simply results from labor being an important input factor. A one percent increase of labor in the numeraire sector raises output by $\beta = 0.61$ percent

and a one percent increase of labor in the construction sector raises construction output by $1 - \eta = 0.45$ percent. In all our simulations the effect on the denominator dominates the effect on the numerator such that a rising population reduces the housing wealth-to-income ratio. We therefore focus on the dynamics triggered by deviations of the endogenous state variables from their steady states.

We now turn to the quantitative importance of the three different initial states – K_0, D_0, and X_0 – for the increase in the housing wealth-to-income ratio. The first row of table 3 contains all necessary information. The strongest contribution results from the construction sector. At the beginning of the transition, in 1950, the US experienced a prolonged and strong build-up in the stock of residential buildings, also known as post-war construction boom. This is captured by X_0 being 81 percent below its steady state level. A strong increase in residential structures along the transition leads to an increase in the housing wealth-to-income ratio. If we assumed that X_0 starts in its steady state, then the housing wealth-to-income ratio would actually decrease by 20 percent (entry 0.8 for the alternative) instead of increasing by 80 percent (entry 1.8 for the baseline) between 1950 and 2015. Hence, the differential effect between the baseline transition and the alternative transition is described by $1.81 - 0.8 = 1.01$. The housing wealth-to-income ratio grows by 101 percentage points more in the baseline transition compared to the alternative transition. In comparison, the other two endogenous states have almost negligible net effects on the housing wealth-to-income ratio.[27]

We now discuss the construction boom in greater detail. The effect of changes in X_0 on the growth factors of the relevant variables is shown in column (4) of table 3. First, we can study the numerator and denominator of the housing wealth-to-income ratio. The growth factor of the numerator, $P^H H$, increases by 480 percentage points when X_0 is 81 percent below its steady state value instead of starting directly in its steady state value. The growth factor of the denominator, NNP, decreases by 97 percentage points, also contributing to an increase in the housing wealth-to-income ratio. Second, we can further decompose the numerator, housing wealth, into the sum of $P^D D$ and $P^X X$. Most of the effect is driven by rising structure wealth, $P^X X$, which increases by 104 percentage points more relative to NNP, instead of residential land wealth, which increases only by 48 percentage points more. Third, structure wealth in turn

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[27]The results are similar for the UK, France, and Germany. For the UK, for example, initial capital, developed land, and structures start 70, 30, and 80 percent below their respective steady state and contribute with 33, 8, and 117 percentage points to the increase in the housing wealth-to-income ratio, respectively.

	(1) baseline	(2) investment $K_0 = \widetilde{K}$	(3) land development $D_0 = \widetilde{D}$	(4) construction $X_0 = \widetilde{X}$
$\frac{P^H H}{NNP}$	1.8 (0.00)	1.7 (0.03)	1.7 (0.08)	0.8 (1.01)
$P^H H$	9.7 (0.00)	6.5 (3.19)	9.3 (0.41)	4.9 (4.80)
NNP	5.9 (0.00)	4.1 (1.87)	5.9 (-0.01)	6.9 (-0.97)
$\frac{P^D D}{NNP}$	1.8 (0.00)	1.1 (0.70)	1.3 (0.49)	1.3 (0.48)
P^D	4.7 (0.00)	2.0 (2.75)	8.1 (-3.41)	4.0 (0.72)
D	2.3 (0.00)	2.3 (-0.02)	1.0 (1.31)	2.3 (-0.03)
$\frac{P^X X}{NNP}$	1.8 (0.00)	1.9 (-0.11)	1.8 (0.00)	0.7 (1.04)
P^X	2.4 (0.00)	1.7 (0.70)	2.4 (-0.01)	5.9 (-3.47)
X	4.3 (0.00)	4.4 (-0.10)	4.3 (0.01)	0.9 (3.46)

Notes: Column (1) is the baseline simulation resulting from our calibration in online appendix C. Columns (2) to (4) are alternative simulations with one initial state variable set equal to its steady state value. Values are cumulative growth factors of a variable between period $t = 0$ and $t = 65$. Differences to respective growth factors from the baseline simulation are in brackets. Initial states K_0, D_0, and X_0 start 77, 60, and 81 percent below their respective steady state values.

Table 3: Growth factors of endogenous variables under alternative initial states (US)

is the product of the price and quantity of structures, P^X and X. Most of the increase in structure wealth is due to a 346 percentage point stronger increase in residential structures. This direct effect is quite mechanic. The closer X_0 is to its steady state, the weaker is its subsequent growth. Developed land, however, even increases slightly. The reason is that with higher X_0, residential land is relatively scarce and increases therefore at a slightly higher pace. Lastly, differences in initial structures X_0 affect the housing wealth-to-income ratio also through prices. As with a higher X_0 structures are initially more abundant, their price, P_0^X, is smaller. Since the price P^X converges to the same steady state its growth rate therefore increases as can be seen in table 3. The opposite effect operates through P^D. Since D accumulates faster with a higher X_0, its price increases less because D is less valuable at future dates. To summarize, the construction boom contributes to a rising housing wealth-to-income ratio mainly through its direct quantity effect. This shows up by a pronounced increase in residential structure.

The construction boom, however, implies that residential land wealth grows less than

structure wealth, contradicting stylized fact 4. This is where the dynamics through K_0 and D_0 enter the picture. Although their effect on the rising housing wealth-to-income ratio is small, they contribute in generating a stronger increase in residential land wealth (70 and 49 percentage points) than in residential structure wealth (11 and 0 percentage points). The dynamics in K_0, in particular, result in a stronger increase in residential land prices by 275 percentage points. This works through interest rates and rental yields. First, if K_0 is lower, then the interest rate starts at a higher level before converging to the same steady state value. A higher level of the interest rate implies stronger discounting of future yields and therefore a smaller initial price. As prices also converge to the same steady state values this in turn results in stronger price growth. Second, starting with a lower K_0 implies stronger overall income growth. This results in stronger growth in the demand for both structures and land. Following the logic of section 4.2 the demand curves for residential land and residential structures shift outwards over time. The supply curves also move outwards over time. The supply curve of residential structures, however, shifts much more outwards than the supply curve of residential land because land is non-reproducible while structures are. The result is the same mirror image as in section 4.2: the quantity of structures increases much more than the quantity of residential land while the rental yield of residential structures increases much less than the rental yield of residential land. The different growth in rental yields for structures and land translates into different growth rates of the prices of structure and land because prices are the PDV of future yields. The key for this result is, once again, the explicit differentiation of residential structures – a reproducible factor – and residential land – a non-reproducible factor.

The calibrated model replicates the observed increase in the housing wealth-to-income ratio and the increase in residential land wealth quite well. We have shown that the construction boom plays an important role. In the next section we study how the calibrated model matches the remaining stylized facts before discussing further data that corroborate our findings.

5.3. Remaining stylized facts

We now turn to the remaining non-targeted moments among the stylized facts described in section 2, focusing on the US. These are plotted in figure 4. We match a couple of features by construction, namely the level of the total wealth-to-income ratio in 1950 (figure 4i), the level of the land share in housing wealth in 1950 (figure 4iv),

(i) Wealth-to-income ratio

(ii) Prices

(iii) Quantities

(iv) Land share in housing wealth

(v) Real rent
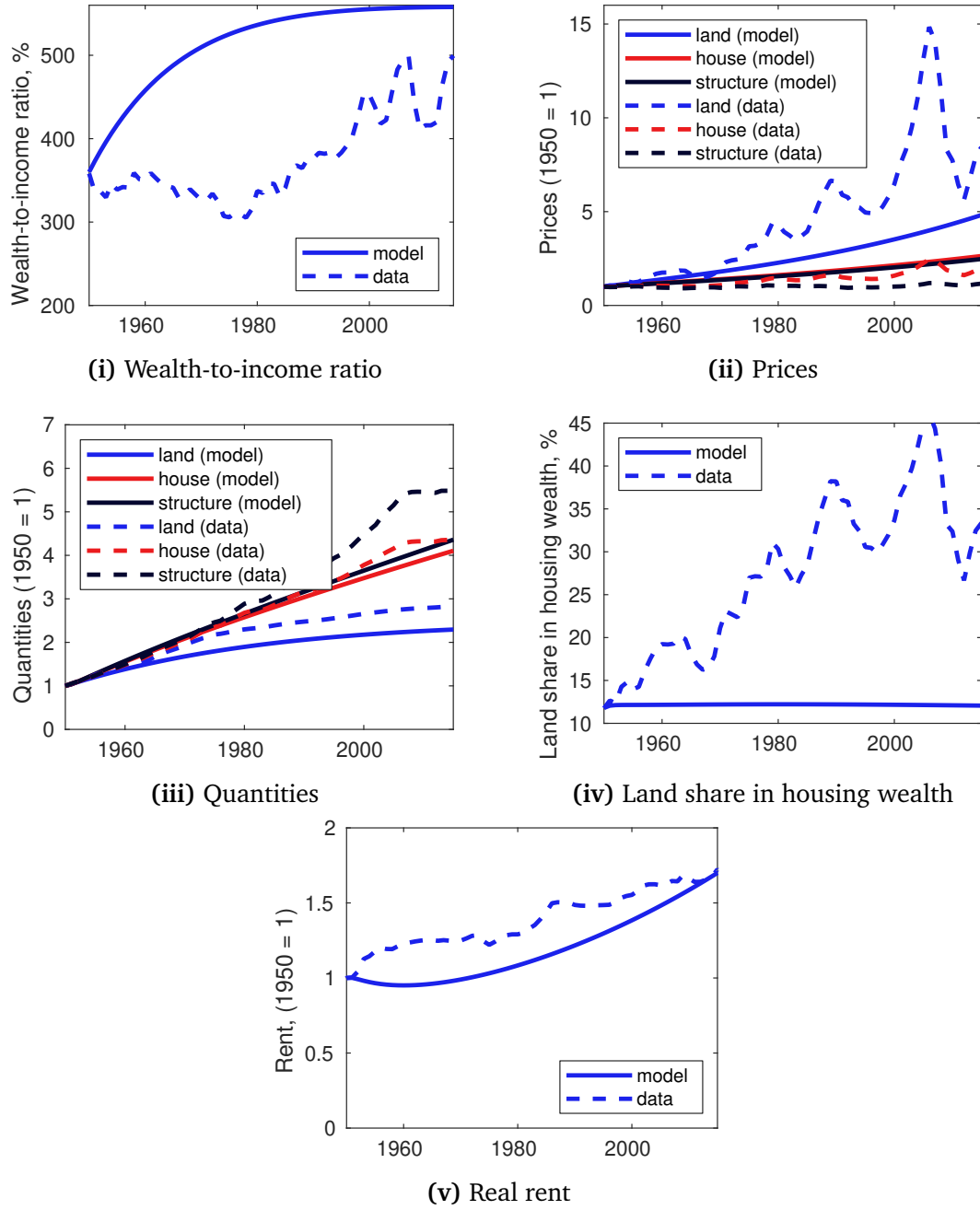
Notes: Empirical data for the US cover the period 1950–2015. The housing wealth-to-income ratio is plotted in figure 3.

Figure 4: Stylized facts, US

and the growth factor of the rent between 1950 and 2015 (figure 4v). Except for rents, the dynamics in all variables of figure 4 are non-targeted. We discuss each of these

stylized fact now in turn.

First, the increase in the wealth-to-income ratio is matched quite accurately, over-predicting the increase between 1950 and 2015 by merely 5 percentage points. According to figure 4i the total wealth-to-income ratio increases by 55 percent. This is mostly driven by the housing wealth-to-income ratio, which increases by 80 percent (figure 3), and to a lesser extent by the non-housing wealth-to-income ratio, which increases by 40 percent (not shown). Our theory hence plausibly explains that the increase in the wealth-to-income ratio is mainly driven by the increase in housing wealth as opposed to non-housing wealth, as stated by stylized fact 1.

Second, the model replicates the mirror-like behavior of prices and quantities, which is described in stylized facts 2 and 3. In figure 4ii we show that residential land prices grow much stronger than house prices, which in turn grow at a slightly higher rate than construction cost. The opposite holds true for the respective quantities. We match this pattern qualitatively, but quantitatively the model-generated spread of growth rates between land and structure quantities and prices is too small. We return to this in section 6.2, where we show that the model fit can be improved with a constant elasticity of substitution (CES) function in the production of housing services.

Third, the model endogenously generates a 2 percent increase in the share of residential land wealth in housing wealth, as shown in figure 4iv. Quantitatively, however, the model performs quite poorly along this dimension as the increase is too small. This is related to the insufficient build-up in the stock of residential structures, mentioned before, and will also be addressed in section 6.2. Lastly, the dynamics in rents is matched by construction as the growth in rent is a targeted moment in the calibration. Note that the model predicts that rents grow at a positive rate of 1.4 percent in the long run, as can already be seen in figure 4v.

5.4. Other relevant data moments

Residential investment. In his much cited paper Leamer (2007) argues that *housing is the business cycle*, meaning that residential investment leads the business cycles and explains a considerable share of the fluctuations in GDP. Kohlscheen, Mehrotra, and Mihaljek (forthcoming, figure 1) show that residential investment declined since 1970 for every economy out of a set of 15 advanced economies and is becoming less relevant for the business cycle. Although we do not study business cycles, our results are still insightful for the declining relevance of residential investment for the business cycle.

(i) Data   **(ii)** Model
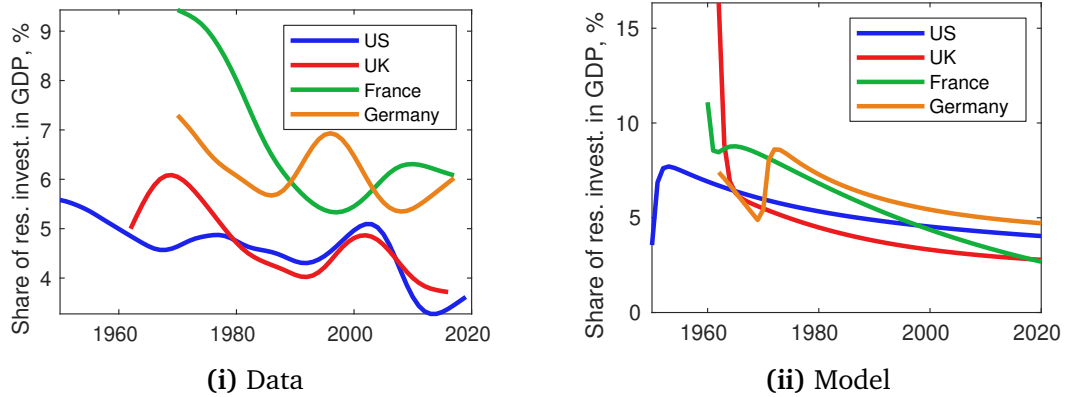
Notes: Empirical data for the US cover the period 1950–2019, while it is 1960–2016 for the UK, and 1970–2017 for France and Germany. Since residential investment is very volatile in the short run we have HP-filtered the empirical series with a smoothness parameter of 100. In the model residential investment is computed following (28).

Figure 5: Residential investment

In figure 5 we plot empirical and model-implied residential investment as a share in GDP for all four economies. Figure 5i plots the HP-filtered trend component of residential investment for the US, UK, France, and Germany. Residential investment as a share of GDP has declined since WWII in all four economies. For the US, for instance, residential investment amounted to 5.6 percent of GDP in 1950 and was almost cut in half in 2019 when it arrived at 3.8 percent. The other economies experienced similar patterns. The model-implied transitions shown in figure 5ii paint the same picture of declining residential investment as a share of GDP. This corroborates our finding that a pronounced construction boom contributed strongly to the increase in housing-wealth-to income ratios. Our paper therefore provides a fundamental, long-term explanation for better understanding why it might hold true that *housing was the business cycle*.

Saving rates. The aggregate saving rate is crucial in the process of wealth accumulation and hence crucial for the dynamics of the (housing) wealth-to-income ratio (Piketty and Zucman, 2014). We therefore check how the saving rate evolves in the model-based experiment laid out above and how this relates to the data. The empirical data for the considered economies is plotted in figure 6i. Saving rates have been on a persistent downward trend, recently dipping even into negative for the US. This empirical observation has been studied in the previous literature. For example, Parker (1999) discusses several potential explanations and concludes that neither is fully satisfactory.

(i) Data

(ii) Model
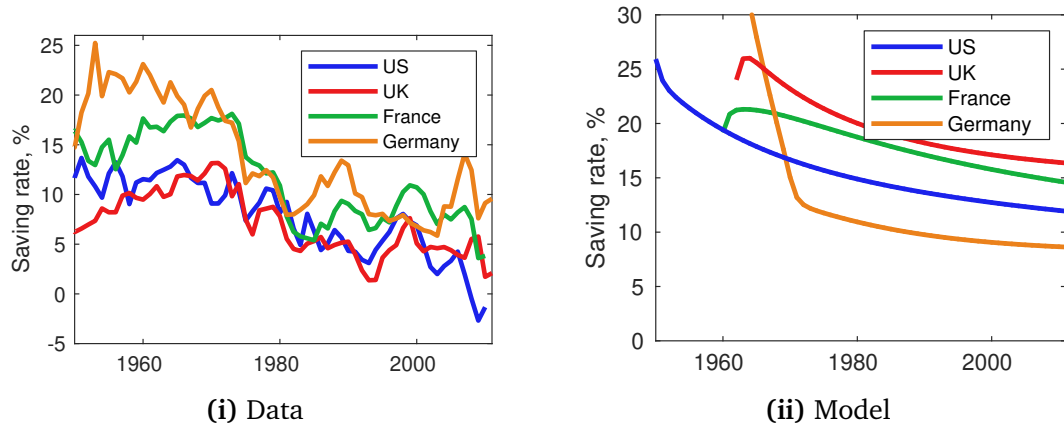
Notes: Data on saving rates is taken from Piketty and Zucman (2014, online appendix).

Figure 6: Saving rates

Given that saving rates have been on a downward path empirically, it is reassuring that our model-generated series – shown in figure 6ii – replicate this feature of the data. The behavior of saving rates along the transition depends on income and substitution effects of changes in interest rates and is already ambiguous in a one-sectoral Ramsey growth model (Barro and Sala-i-Martin, 2004, ch. 2.6.4). The interest rate is declining along the transition in our model, implying that the substitution effect dominates the income effect, even with an intertemporal elasticity of 0.3.

6. Discussion

Three further channels have been discussed in the literature that may be important when it comes to understanding the surge in house prices and housing wealth: i) increasing home ownership rates (housing demand), ii) weak elasticity of substitution between residential land and structure in housing production (housing supply), and iii) declining real interest rates (asset prices). We now discuss how each of these factors relates to the theory outlined above.

6.1. Home ownership revolution

The home ownership rate has risen over the second half of the 20th century in many countries, including the US, UK, France, and Germany (Kohl, 2017). This process is often referred to as *home ownership revolution*. A widely held view is that finan-

cial liberalization, by relaxing credit constraints, has primarily triggered this evolution (Ortalo-Magne and Rady, 2006).[28] Provided that the surge in the home ownership rate, the story goes, pushes the overall demand for housing upwards, this process has contributed to the increase in house prices.[29] However, the descriptive empirical picture on the evolution of the aggregate housing expenditure share is ambiguous. Piazzesi and Schneider (2016) argue that the housing expenditure share is fairly stable over time at 19 percent in the postwar US as implied by data form the US NIPA. Albouy, Ehrlich, and Liu (2016) study also alternative data sources and argue that the housing expenditure share may have increased in the US.

Whether or not the home ownership revolution has significantly contributed to the surge in house prices appears an open research question. Let us assume that this channel is important. How can this channel be captured by our analysis? Notice that we model all households as renters, but the results do not change if we assume that all households are home owners. We could employ a short-cut to capture an additional demand effect due to the home ownership revolution by assuming that the expenditure share, which equals θ in our model, increases exogenously over time. This would further add to the growing demand for housing and amplify house price growth and the surge in housing wealth. Our supply-side setting, however, is still necessary to generate a plausible increase in house prices and housing wealth such that land prices increase much more than construction cost. Hence, even when studying this potential channel it is necessary to consider the supply-side features proposed in our model.

6.2. CES technology in housing services

A channel that originates from the supply side concerns the role of land in housing services production. Miles and Sefton (2020) argue that the elasticity of substitution (EoS) between land and structure is less than one. As the economy grows and the demand for housing surges, the long-run price elasticity of housing supply is relatively low as the fixed factor (land) cannot be easily substituted for by the accumulable fac-

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[28]Jordà, Schularick, and Taylor (2016) stress that banks and households have been heavily leveraging up through mortgages in the second half of the 20th century. Mortgage credit on the balance sheets of banks has been the driving force behind the increasing financialization of advanced economies.

[29]However, Kaplan, Mitman, and Violante (2019) argue that the increase in overall housing demand, triggered by credit relaxation, is small. In their model, as in the data, very few households are constrained with respect to the size of the rented house: rather than buying excessively small houses, they prefer to rent a house of the desired size. The credit relaxation induces these renters to become owners, which increases home ownership without pushing up prices.

tor (structure). The consequence is a stronger growth in the rent and the house price compared to the Cobb Douglas case. We have implicitly assumed an elasticity of substitution of one in the production function of housing, $H(X,D)$. We now assume that the production technology for housing services is given the general CES function

$$H(X,D) = \left[\gamma X^{\frac{\chi-1}{\chi}} + (1-\gamma)D^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}},$$

where $\chi > 0$ is the elasticity of substitution between land and structures.

We study two alternative transitions. If $\chi \neq 1$, a steady state equilibrium only exists if both X and D grow at the same rates and it hence has to hold that $g^X = -\frac{\eta}{1-\eta}\frac{1-\alpha}{\beta}g^Y$. This knife-edge restriction implies differential technological progress that is heavily biased towards the non-housing good. In alternative 1 we therefore first re-calibrate the model with $\chi = 1$ under this knife-edge condition, setting the same set of parameters endogenously as in the baseline calibration except g^X and matching the same set of moments except the growth rate of rents. In alternative 2 we then study how changing χ from 1 to a low value of 0.25 – keeping the parameters from alternative 1 – affects the stylized facts. This two-step approach is necessary because otherwise it would be impossible to tell whether differences occur due to a lower χ or g^X.

A lower elasticity of substitution increases the difference in growth rates of prices and quantities of residential land and structures, improving the mode fit with regards to stylized fact 2 and 3. The intuition is straightforward. The weaker the elasticity of substitution, the stronger has X to increase in order to generate additional housing when the economy grows. The relatively stronger growth in residential land prices then also results in much stronger growth of the land share in housing wealth (it actually declines in alternative 1). The details are shown in table D.6 in the online appendix. To summarize, weaker substitutability of structure and land results in a stronger growth difference in prices and quantities of land and structure, as well as a stronger increase of the land share in housing wealth. It does, however, require to impose a knife-edge condition on differential technological growth to restore the existence of a steady state.

6.3. Declining real interest rate

Rachel and Summers (2019) show that the real interest rate has been trending downwards since the early 1980s.[30] Viewing houses as assets suggests that the house price

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[30]They focus on average yields on long-maturity inflation-protected government securities in the G7,

is given by the present discounted value of future rental yields. A decline in the real interest rate over time then implies that the house price increases as time goes by. This mechanical discounting effect operates as long as the discount factor declines. Miles and Monro (2019) argue that the rise in house prices (relative to incomes) between 1985 and 2018 can be more than accounted for by the substantial decline in the real risk-free interest rate observed over that period.[31] The interest rate is an endogenous object in our model. This makes it non-trivial to study how its changes affect prices as both variables evolve endogenously. The endogenous dynamics of the interest rate are in line with the empirical observation of declining interest rates as interest rates decline monotonously along the transition. The reason for declining interest rates is best understood by noting that capital starts below its steady state and accumulates over time, reducing the marginal product of capital and hence the interest rate. Our explanation for rising house prices is therefore in line with the empirical trend in the interest rate, providing also an explanation for the declining interest rate.

7. Conclusion

We provide a theory to explain the observed increase in housing wealth relative to income in the US, the UK, France, and Germany since 1950. The starting point is a set of five stylized facts on housing-related macro variables. The differentiation of housing wealth into the value of residential structure and the value of residential land is key for a better understanding. Further decomposing the values of land and structure into price and quantity components reveals a mirror-like pattern. The price of the non-reproducible factor – land – increased strongly, while its quantity remained relatively stable. The opposite holds true for the reproducible factor – structures. Its quantity increased strongly and its price only modestly.

Motivated by these observations, we propose a novel theory for studying housing wealth over the long term. The employed Ramsey growth model comprises two sectors, a numeraire sector and a housing sector. Both sectors experience differences in technological progress and operate under different land intensities in equilibrium. The overall supply of land is fixed and the sectoral land allocation is endogenous. Economic

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excluding Italy. For alternative measures on the long term real interest rate and long term data series see also Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019).

[31]They focus on the UK, where the surge in house prices has been especially pronounced. Their main results, they argue, apply to other G7 economies as well.

growth is driven by exogenous population growth and exogenous technological change.

Two forces are driving house prices in the long run. First, the land intensity in the housing sector has to be larger than in the non-housing sector – the empirically plausible case. Under this side condition, (symmetric) technological progress leads to rising house prices. Second, house prices grow in the long term if technological progress in the construction sector is lagging behind technological progress in the rest of the economy, which is also empirically plausible. Our model therefore focuses on two fundamental forces behind the long-term growth in house prices and housing wealth.

Although housing wealth can increase in the steady state, the housing wealth-to-*income* ratio is constant in steady state. To explain a rising housing wealth-to-*income* ratio we therefore study transitions. The calibrated model explains 80, 67, and 76 percent of the increase in the housing wealth-to-income ratio in the UK, France, and Germany since the 1950s, respectively. It even slightly over-predicts the increase for the US. We show that the strong accumulation of residential buildings at the beginning of the period 1950 to 2015 – the post-war construction boom – has played an important role. The expansion in the stock of residential structure has directly contributed to the surge in housing wealth. What is more, a strong accumulation of capital has triggered income and the demand for housing to grow. When the rising housing demand meets an inelastic supply of residential land and an elastic supply of residential structures, land prices grow faster than structure prices. This explanation is consistent with two additional observations. First, residential investment as a share of GDP was exceptionally high at the beginning and has been declining over the period 1950 to 2015 in the US, UK, France, and Germany. Second, aggregate saving rates have been declining in all four economies since the 1950s.

References

ALBOUY, D., G. EHRLICH, AND Y. LIU (2016): "Housing Demand, Cost-of-Living Inequality, and the Affordability Crisis," Working Paper 22816, National Bureau of Economic Research.

BARRO, R. J., AND X. SALA-I-MARTIN (2004): *Economic growth*. MIT Press, Cambridge, 2 edn.

BONNET, O., G. CHAPELLE, A. TRANNOY, AND E. WASMER (2019): "Secular Trends in

Wealth and Heterogeneous Capital: Land is Back...and Should be Taxed," Discussion Paper 92, Sciences Po, LIEPP Working Paper.

BORRI, N., AND P. REICHLIN (2018): "The housing cost disease," *Journal of Economic Dynamics and Control*, 87, 106–123.

DAVIS, M. A., AND J. HEATHCOTE (2005): "Housing and the Business Cycle," *International Economic Review*, 46(3), 751–784.

——— (2007): "The price and quantity of residential land in the United States," *Journal of Monetary Economics*, 54(8), 2595–2620.

FAVILUKIS, J., S. C. LUDVIGSON, AND S. V. NIEUWERBURGH (2017): "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk Sharing in General Equilibrium," *Journal of Political Economy*, 125(1), 140–223.

GENNAIOLI, N., A. SHLEIFER, AND R. VISHNY (2014): "Finance and the Preservation of Wealth," *The Quarterly Journal of Economics*, 129(3), 1221–1254.

GREENWALD, D. L., AND A. GUREN (2019): "Do Credit Conditions Move House Prices?," Discussion paper, working paper.

HANSEN, G. D., AND E. C. PRESCOTT (2002): "Malthus to Solow," *American Economic Review*, 92(4), 1205–1217.

HERKENHOFF, K. F., L. E. OHANIAN, AND E. C. PRESCOTT (2018): "Tarnishing the golden and empire states: Land-use restrictions and the U.S. economic slowdown," *Journal of Monetary Economics*, 93, 89–109.

JORDÀ, Ò., K. KNOLL, D. KUVSHINOV, M. SCHULARICK, AND A. M. TAYLOR (2019): "The Rate of Return on Everything, 1870–2015," *The Quarterly Journal of Economics*.

JORDÀ, Ò., M. SCHULARICK, AND A. M. TAYLOR (2016): "The great mortgaging: housing finance, crises and business cycles," *Economic Policy*, 31(85), 107–152.

KAPLAN, G., K. MITMAN, AND G. L. VIOLANTE (2019): "The housing boom and bust: Model meets evidence," *Journal of Political Economy, forthcoming*.

KNOLL, K., M. SCHULARICK, AND T. STEGER (2017): "No Price Like Home: Global House Prices, 1870–2012," *American Economic Review*, 107(2), 331–353.

KOHL, S. (2017): *Homeownership, Renting and Society: Historical and Comparative Perspectives*. Routledge.

KOHLSCHEEN, E., A. N. MEHROTRA, AND D. MIHALJEK (forthcoming): "Residential investment and economic activity: evidence from the past five decades," *International Journal of Central Banking*.

KONGSAMUT, P., S. REBELO, AND D. XIE (2001): "Beyond Balanced Growth," *The Review of Economic Studies*, 68(4), 869–882.

LEAMER, E. (2007): "Housing IS the Business Cycle," Discussion paper, NBER.

MILES, D., AND J. SEFTON (2020): "House prices and growth with fixed land supply," *The Economic Journal*.

MILES, D. K., AND V. MONRO (2019): "UK House Prices and Three Decades of Decline in the Risk-Free Real Interest Rate," *BAnk of England Working Paper No. 837*.

NECHYBA, T. J., AND R. P. WALSH (2004): "Urban Sprawl," *Journal of Economic Perspectives*, 18(4), 177–200.

NICHOLS, D. A. (1970): "Land and Economic Growth," *The American Economic Review*, 60(3), 332–340.

ORTALO-MAGNE, F., AND S. RADY (2006): "Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints," *Review of Economic Studies*, 73(2), 459–485.

PARKER, J. A. (1999): "Spendthrift in America? On Two Decades of Decline in the U.S. Saving Rate," *NBER Macroeconomics Annual*, 14, 317–370.

PIAZZESI, M., AND M. SCHNEIDER (2016): "Housing and Macroeconomics," in *Handbook of Macroeconomics*, pp. 1547–1640. Elsevier.

PIKETTY, T. (2014): *Capital in the Twenty-First Century*. The Belknap Press of Harvard University Press.

PIKETTY, T., AND G. ZUCMAN (2014): "Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010," *The Quarterly Journal of Economics*, 129(3), 1255–1310.

——— (2015): "Wealth and Inheritance in the Long Run," in *Handbook of Income Distribution*, pp. 1303–1368. Elsevier.

QUIGLEY, J. M., AND S. RAPHAEL (2004): "Is Housing Unaffordable? Why Isn't It More Affordable?," *Journal of Economic Perspectives*, 18(1), 191–214.

RACHEL, L., AND L. SUMMERS (2019): "On Secular Stagnation in the Industrialized World," *Brookings Papers on Economic Activity*.

RICARDO, D. (1817): *On the Principles of Political Economy and Taxation*. London.

ROGNLIE, M. (2015): "Deciphering the fall and rise in the net capital share: accumulation or scarcity?," *Brookings papers on economic activity*, 2015(1), 1–69.

Online Appendix

A. Data

Total wealth – to be precise, nominal net private wealth – for France, Germany, the UK, and the US is an updated series from Piketty and Zucman (2014) that is available at www.wid.world. US nominal housing wealth and US nominal residential structure wealth is from the updated online appendix of Davis and Heathcote (2007), available at www.aei.org/historical-land-price-indicators/. We use the quarterly data from 1975 onwards, because it has a higher quality, and the annual data for earlier periods. Data on nominal housing wealth and nominal residential structure wealth for the other three countries was provided by Moritz Schularick, Luis Bauluz, and Filip Novokmet. Non-housing wealth is obtained residually by subtracting housing wealth from total wealth. Similarly, residential land wealth is also obtained residually by subtracting residential structure wealth from housing wealth.[32] We always consider gross housing wealth as opposed to net housing wealth where mortgage debt is subtracted from gross housing wealth. Nominal net national income (NNP) is also from www.wid.world for all four economies. Wealth-to-income ratios are obtained by dividing nominal wealth series by NNP. The share of residential land wealth in housing wealth is obtained by dividing residential land wealth by housing wealth.

Data on aggregate nominal house prices, residential land prices, and residential structure prices is also based Davis and Heathcote (2007) and obtained at www.aei.org/historical-land-price-indicators/ for the US. We obtain real prices by dividing nominal prices by the CPI which is the series 'CPIAUCNS' as given by https://fred.stlouisfed.org/. House prices, residential land prices, and residential structure prices for the other three economies are from Knoll, Schularick, and Steger (2017), as provided in the online appendix at the journal's webpage and the respecitve CPI's are from http://www.macrohistory.net/.

We compute the quantity indices for housing, residential land, and residential structures by dividing the respective value index by the respective price index, i.e. quantity index $= \frac{\text{value index}}{\text{price index}}$.

Lastly, the nominal rent as well as the interest rate for all four economies are from

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[32]Direct measures of residential land wealth at the aggregate level are very sparse and non-existent for earlier periods. See Albouy, Ehrlich, and Shin (2018) for a recent paper on this issue. The authors provide the first direct measures of residential land wealth for the US starting in 2005.

B. Model

B.1. First order conditions to various problems

Household problem. Households choose streams of $\{C_t, S_t\}_{t=0}^{\infty}$ to maximize (2) subject to (3), (4), and a no Ponzi game (NPG) condition. The associated present-value Hamiltonian reads

$$\mathcal{H} = e^{-\rho t} L_t \frac{\left[\left(\frac{C_t}{L_t}\right)^{1-\theta}\left(\frac{S_t}{L_t}\right)^{\theta}\right]^{1-\sigma} - 1}{1 - \sigma} + \lambda\left[r_t W_t + w_t L_t - C_t - q_t S_t + \Pi_t^N\right]$$

and the (rearranged) FOC are

$$C = \frac{1-\theta}{\theta} q S \tag{18}$$

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\sigma} + \theta \frac{\sigma - 1}{\sigma}\frac{\dot{q}}{q} + \frac{\dot{L}}{L}.$$

The first is a standard *intratemporal* optimality condition that states that the marginal rate of substitution of C and S equals the relative price, q. The second is a modified Keynes-Ramsey rule. The growth rate of rents enters the right hand side and affect the time path of the consumption good C if the intertemporal elasticity of substitution is not 1.

Numeraire firm problem. The first order conditions to problem (6) read

$$r + \delta^K = \alpha \frac{Y}{K}, \quad w = \beta \frac{Y}{L^Y}, \quad \text{and} \quad R^N = (1 - \alpha - \beta)\frac{Y}{N}. \tag{19}$$

All production factors are rewarded with their marginal products.

Property management firm problem. The Hamiltonian associated with problem (8) reads

$$\mathcal{H} = e^{-\hat{r}}\left[q X^{\gamma} D^{1-\gamma} - P^X I^X - P^D I^D\right] + \mu^X\left(I^X - \delta^X X\right) + \mu^D I^D. \tag{20}$$

The FOC are

$$\frac{\partial \mathscr{H}}{\partial I^D} = 0, \quad \frac{\partial \mathscr{H}}{\partial I^X} = 0, \quad \frac{\partial \mathscr{H}}{\partial D} = -\dot{\mu}^D, \quad \text{and} \quad \frac{\partial \mathscr{H}}{\partial X} = -\dot{\mu}^X.$$

Taking the derivatives of (20) modifies the FOC to

$$\mu^D = e^{-\hat{r}} P^D, \quad \mu^X = e^{-\hat{r}} P^X, \quad e^{-\hat{r}} q(1-\gamma) \left(\frac{X}{D}\right)^\gamma = -\dot{\mu}^D, \quad \text{and}$$

$$e^{-\hat{r}} q\gamma \left(\frac{D}{X}\right)^{1-\gamma} - \mu^X \delta^X = -\dot{\mu}^X.$$

Eliminating the co-state variables by taking the derivative with respect to time of the first two equations, obtaining $\dot{\hat{r}} = r$ by applying the Leibniz rule, inserting the results into the two latter equations and rearranging yields

$$r = \frac{\dot{P}^D + qH_D}{P^D} = \frac{\dot{P}^X + qH_X - \delta^X P^X}{P^X}. \tag{21}$$

These are two no-arbitrage equations. The yield component from investing one additional unit in residential land (structures) are marginal operating profits qH_D (qH_X minus depreciation $\delta^X P^X$). These no-arbitrage equations imply that the property management firm is indifferent with respect to investment into land and structures whenever prices q, P^X, P^D adjust such that these two equations hold. This results from the fact that property management firms discount profits at the economy's interest rate r.

Derivation of housing yield (R^H). Differentiating (10) with respect to time and rearranging yields

$$P^D \dot{D} + P^X \dot{X} = \dot{P}^H H + P^H \dot{H} - \dot{P}^D D - \dot{P}^X X. \tag{22}$$

From (8) we know that $I^D = \dot{D}$ and $I^X = \dot{X} + \delta^X X$. This allows us to express the cash flow from (9) as

$$CF = qH - P^X(\dot{X} + \delta^X X) - P^D \dot{D}$$

$$\Leftrightarrow P^D \dot{D} + P^X \dot{X} = qH - P^X \delta^X X - CF.$$

We now use the previous equation to replace $P^D \dot{D} + P^X \dot{X}$ in (22)

$$P^D \dot{D} + P^X \dot{X} = qH - P^X \delta^X X - CF - \dot{P}^D D - \dot{P}^X X.$$

Lastly, we use the FOC (21) to replace $\dot{P}^D$ and $\dot{P}^X$ in the previous equation, make use of Euler's Theorem and the fact that the production function for housing has constant returns to scale, and solve it for r to obtain

$$r = \frac{\dot{P}^H + \frac{CF + P^H \dot{H}}{H}}{P^H}.$$

Construction firm problem. In (11) we have to explicitly take care of the non-negativity restrictions on the input factors. The resulting unconstrained first order conditions read

$$M = \eta P^X I^X \quad \text{and} \quad L^X = (1-\eta)\frac{P^X I^X}{w}.$$

Inserting the two equations into the profit equation yields

$$
\begin{aligned}
\Pi^X &= P^X X^\eta \left(B^X L^X\right)^{1-\eta} - M - w L^X \\
&= P^X \left(\eta P^X I^X\right)^\eta \left(B^X (1-\eta)\frac{P^X I^X}{w}\right)^{1-\eta} - \eta P^X I^X - w(1-\eta)\frac{P^X I^X}{w} \\
&= \left[P^X \eta^\eta (1-\eta)^{1-\eta} \left(\frac{B^X}{w}\right)^{1-\eta} - 1\right] P^X I^X.
\end{aligned}
$$

If the expression in square brackets – which is exogenous to the construction firms – is positive, then I^X, L^X, M, and profits are positiv. If, however, the expression in square brackets is negative, then the unconstrained solution would imply negative values for I^X, L^X, and M (and positive profits), violating the non-negativity restriction. In that case the optimal solution is $I^X = L^X = M = 0$. To summarize,

$$
I^X = \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{(1-\eta)}{w}\right]^{1-\eta} P^X - 1\right] P^X < 0 \\ M^\eta \left(B^X L^X\right)^{1-\eta} & \text{else} \end{cases}
$$

$$
L^X = \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{(1-\eta)}{w}\right]^{1-\eta} P^X - 1\right] P^X < 0 \\ (1-\eta)\frac{P^X}{w} I^X & \text{else} \end{cases}
$$

$$
M = \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{(1-\eta)}{w}\right]^{1-\eta} P^X - 1\right] P^X < 0 \\ \eta P^X I^X & \text{else} \end{cases}.
$$

B.2. Full dynamic system

Original system. In this section we explicitly formulate the economy as consisting of a differential algebraic system of equations (DAEs). First, we use the asset market clearing condition (15) together with (10) to substitute W with K in the budget constraint and obtain the law of motion of capital

$$
\dot{K} = rK + R^D D + (R^X + \delta^X P^X)X + R^N (Z - D) + wL - C - qS - P^X I^X,
$$

where $R^D \equiv (1-\gamma)qH/D$ and $R^X \equiv \gamma qH/X - \delta^X P^X$ denote the yield components of developed land D and residential structures X, respectively.

Second, with the help of the intratemporal optimality condition, the production function of S, and the laws of motion of X and D we reformulate the Keynes Ramsey by substituting $\frac{\dot{q}}{q}$ by the implied derivative of the rearranged intratemporal FOC:

$$
\frac{\dot{C}}{C} = \frac{1}{(1-\theta)\sigma + \theta} \left\{ r - \rho + \sigma \frac{\dot{L}}{L} - \theta(\sigma - 1)\left[\gamma\left(\frac{I^X}{X} - \delta^X\right) + (1-\gamma)\frac{P^D - P^N}{\xi w D}\right]\right\}.
$$

Collecting equations the economy can now be described by the following differential algebraic system of equations (DAEs)

$$
\begin{aligned}
\dot{K} &= rK + R^D D + (R^X + \delta^X P^X)X + R^N (Z - D) + wL - C - qS - P^X I^X \\
\dot{D} &= \frac{P^D - P^N}{\xi w} \\
\dot{X} &= I^X - \delta^X X \\
\frac{\dot{C}}{C} &= \frac{1}{(1-\theta)\sigma + \theta} \left\{ r - \rho + \sigma \frac{\dot{L}}{L} - \theta(\sigma - 1)\left[\gamma\left(\frac{I^X}{X} - \delta^X\right) + (1-\gamma)\frac{P^D - P^N}{\xi w D}\right]\right\} \\
\dot{P}^D &= rP^D - (1-\gamma)\frac{qS}{D} \\
\dot{P}^N &= rP^N - R^N \\
\dot{P}^X &= rP^X - \gamma\frac{qS}{X} + \delta^X P^X
\end{aligned}
$$

$$C = \frac{1-\theta}{\theta} q S$$

$$I^X = \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{1-\eta}{w}\right]^{1-\eta} P^X - 1\right] P^X < 0 \\ M^\eta \left(B^X L^X\right)^{1-\eta} & \text{else} \end{cases}$$

$$L^X = \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{1-\eta}{w}\right]^{1-\eta} P^X - 1\right] P^X < 0 \\ (1-\eta)\frac{P^X}{w} I^X & \text{else} \end{cases}$$

$$M = \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{1-\eta}{w}\right]^{1-\eta} P^X - 1\right] P^X < 0 \\ \eta P^X I^X & \text{else} \end{cases}$$

$$S = X^\gamma N^{1-\gamma}$$

$$Y = K^\alpha \left(B^Y(L - L^X - L^D)\right)^\beta \left(B^Y(Z - N)\right)^{1-\alpha-\beta}$$

$$r = \alpha \frac{Y}{K} - \delta^K$$

$$w = \beta \frac{Y}{L - L^X - L^D}$$

$$R^N = (1 - \alpha - \beta)\frac{Y}{Z - D}$$

$$L^D = \frac{\left(P^D - P^N\right)^2}{2\xi w^2}.$$

That makes 19 equations in 19 variables, with 3 state variables, 4 jump variables, and 12 auxiliary variables.

Normalized system. We divide each variable by its growth factor. For example, variable Y grows at the rate $\frac{\beta}{1-\alpha}g^Y$ and we normalize it by dividing it by the growth factor $\exp\left[\frac{\beta}{1-\alpha}g^Y t\right]$. The thus normalized DAEs reads

$$\dot{K} = (r - \mathfrak{g}_Y)K + R^D D + (R^X + \delta^X P^X)X + R^N (Z - D) + wL - C - qS - P^X I^X$$

$$\dot{D} = \frac{P^D - P^N}{\xi w}$$

$$\dot{X} = I^X - \left[\delta^X + \mathfrak{g}_X\right]X$$

$$\frac{\dot{C}}{C} = \frac{1}{(1-\theta)\sigma + \theta}\left\{r - \rho + \sigma\frac{\dot{L}}{L} - \theta(\sigma - 1)\left[\gamma\left(\frac{I^X}{X} - \delta^X\right) + (1-\gamma)\frac{P^D - P^N}{\xi w D}\right]\right\} - \mathfrak{g}_Y$$

$$\dot{P}^D = (r - \mathfrak{g}_Y)P^D - (1-\gamma)\frac{qS}{D}$$

$$\dot{P}^N = (r - \mathfrak{g}_Y) P^N - R^N$$

$$\dot{P}^X = (r - \mathfrak{g}_{P^X}) P^X - \gamma \frac{qS}{X} + \delta^X P^X$$

$$C = \frac{1-\theta}{\theta} qS$$

$$I^X = \begin{cases} 0 & \text{if } \left[\eta^\eta \left[\frac{1-\eta}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ M^\eta \left(B^X L^X \right)^{1-\eta} & \text{else} \end{cases}$$

$$L^X = \begin{cases} 0 & \text{if } \left[\eta^\eta \left[\frac{1-\eta}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ (1-\eta) \frac{P^X}{w} I^X & \text{else} \end{cases}$$

$$M = \begin{cases} 0 & \text{if } \left[\eta^\eta \left[\frac{1-\eta}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ \eta P^X I^X & \text{else} \end{cases}$$

$$S = X^\gamma N^{1-\gamma}$$

$$Y = K^\alpha \left(L - L^X - L^D \right)^\beta (Z - N)^{1-\alpha-\beta}$$

$$r = \alpha \frac{Y}{K} - \delta^K$$

$$w = \beta \frac{Y}{L - L^X - L^D}$$

$$R^N = (1-\alpha-\beta) \frac{Y}{Z - D}$$

$$L^D = \frac{\left(P^D - P^N \right)^2}{2\xi w^2}.$$

with $\mathfrak{g}_Y \equiv \frac{\beta}{1-\alpha} g^Y$, $\mathfrak{g}_X \equiv \eta \frac{\beta}{1-\alpha} g^Y + (1-\eta) g^X$, $\mathfrak{g}_{P^X} \equiv (1-\eta) \left(\frac{\beta}{1-\alpha} g^Y - g^X \right)$, and $B_0^Y = B_0^X = 1$.

B.3. Steady state

We focus on a steady state with $\delta^X > 0$ and $g^X > -\frac{\eta}{1-\eta} \left(1 - \psi^Y \right) g^Y$. This implies that there will always be replacement investment for X and that X does not tend to zero as t tends to infinity.[33] We denote the steady state value of a normalized variable by a

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[33] The other cases can also be studied, but since they seem less relevant to us we eschew the derivation of the steady state under $\delta^X = 0$ and/or $g^X \leq -\frac{1-\eta}{\eta} \left(1 - \psi^Y \right) g^Y$.

"~" above the variable. The DAEs simplifies to an algebraic system of equations:

$$0 = (r - \mathfrak{g}_Y)\widetilde{K} + \widetilde{R}^D D + (\widetilde{R}^X + \delta^X \widetilde{P}^X)\widetilde{X} + \widetilde{R}^N (Z - D) + \widetilde{w}L - \widetilde{C} - \widetilde{q}\widetilde{S} - \widetilde{P}^X \widetilde{I}^X$$

$$\widetilde{P}^D = \widetilde{P}^N$$

$$\widetilde{I}^X = \left(\delta^X + \mathfrak{g}_X\right)\widetilde{X}$$

$$r = \rho + [(1-\theta)\sigma + \theta]\mathfrak{g}_Y + \theta(\sigma - 1)\gamma\mathfrak{g}_X$$

$$\widetilde{P}^D = (1-\gamma)\frac{\widetilde{q}\widetilde{S}}{(r - \mathfrak{g}_Y)D}$$

$$\widetilde{P}^N = \frac{\widetilde{R}^N}{r - \mathfrak{g}_Y}$$

$$\widetilde{P}^X = \gamma\frac{\widetilde{q}\widetilde{S}}{(r - \mathfrak{g}_{PX} + \delta^X)\widetilde{X}}$$

$$\widetilde{C} = \frac{1-\theta}{\theta}\widetilde{q}\widetilde{S}$$

$$\widetilde{I}^X = \widetilde{M}^\eta \left(L^X\right)^{1-\eta}$$

$$L^X = (1-\eta)\frac{\widetilde{P}^X}{\widetilde{w}}\widetilde{I}^X$$

$$\widetilde{M} = \eta\widetilde{P}^X\widetilde{I}^X$$

$$\widetilde{S} = \widetilde{X}^\gamma D^{1-\gamma}$$

$$\widetilde{Y} = \widetilde{K}^\alpha \left(L - L^X - L^D\right)^\beta (Z - N)^{1-\alpha-\beta}$$

$$r = \alpha\frac{\widetilde{Y}}{\widetilde{K}} - \delta^K$$

$$\widetilde{w} = \beta\frac{\widetilde{Y}}{L - L^X - L^D}$$

$$\widetilde{R}^N = (1-\alpha-\beta)\frac{\widetilde{Y}}{Z - D}$$

$$L^D = 0.$$

The fourth equation results from the Keynes Ramsey rule and is already the solution for the steady state interest rate. Solving this equation system yields the closed-form solution[34]

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[34] In order to avoid very large expressions we formulate the solution recursively. That is, the closed-form solution of L^X is given by inserting the closed-form solution of r in the first line. Similarly, the solution for D is obtained by inserting L^X and r from the above lines. This way all solutions can be easily expressed as functions of parameters only.

$$r = \rho + [(1-\theta)\sigma + \theta]\,\mathfrak{g}_Y + \theta(\sigma - 1)\gamma\mathfrak{g}_X$$

$$L^X = \frac{r + (1-\alpha)\delta^K - \alpha\mathfrak{g}_Y}{\left[\eta + (1-\theta)\frac{r+\delta^X-\mathfrak{g}_{pX}}{\gamma\theta(\mathfrak{g}_X+\delta^X)}\right]\frac{\beta}{1-\eta}(r+\delta^K) + r + (1-\alpha)\delta^K - \alpha\mathfrak{g}_Y}\,L \tag{23}$$

$$D = \frac{\frac{(1-\gamma)\beta}{\gamma(1-\eta)}\frac{r+\delta^X-\mathfrak{g}_{pX}}{\mathfrak{g}_X+\delta^X}}{\frac{(1-\gamma)\beta}{\gamma(1-\eta)}\frac{r+\delta^X-\mathfrak{g}_{pX}}{\mathfrak{g}_X+\delta^X} + (1-\alpha-\beta)\left(\frac{L}{L^X}-1\right)}\,Z \tag{24}$$

$$\widetilde{P}^X = \eta^{-\eta}\left\{\frac{\beta}{1-\eta}\alpha^{\frac{\alpha}{1-\alpha}}\left[L-L^X\right]^{-\frac{1-\alpha-\beta}{1-\alpha}}(Z-D)^{\frac{1-\alpha-\beta}{1-\alpha}}(r+\delta^K)^{-\frac{\alpha}{1-\alpha}}\right\}^{1-\eta}$$

$$\widetilde{C} = (1-\theta)\frac{\eta^{\frac{\eta}{1-\eta}}\left(r+\delta^X-\mathfrak{g}_{pX}\right)}{\gamma\theta(\mathfrak{g}_X+\delta^X)}\left(\widetilde{P}^X\right)^{\frac{1}{1-\eta}}L^X$$

$$\widetilde{K} = \alpha^{\frac{1}{1-\alpha}}\left[L-L^X\right]^{\frac{\beta}{1-\alpha}}\left[Z-D\right]^{\frac{1-\alpha-\beta}{1-\alpha}}(r+\delta^K)^{-\frac{1}{1-\alpha}}$$

$$\widetilde{X} = \frac{\left[\eta\widetilde{P}^X\right]^{\frac{\eta}{1-\eta}}L^X}{(\mathfrak{g}_X+\delta^X)}$$

$$\widetilde{P}^N = (1-\gamma)\frac{\theta}{1-\theta}\frac{\widetilde{C}}{(r-\mathfrak{g}_Y)\widetilde{D}}$$

$$\widetilde{M} = \eta\widetilde{P}^X\left(\mathfrak{g}_X+\delta^X\right)\widetilde{X}$$

$$\widetilde{w} = \beta\widetilde{K}^\alpha\left[L-L^X\right]^{\beta-1}\left[Z-D\right]^{1-\alpha-\beta}$$

$$\widetilde{R}^N = (1-\alpha-\beta)\widetilde{K}^\alpha\left[L-L^X\right]^{\beta}\left[Z-D\right]^{-\alpha-\beta}$$

$$\widetilde{Y} = \widetilde{K}^\alpha\left[L-L^X\right]^{\beta}\left[Z-D\right]^{1-\alpha-\beta}$$

$$\widetilde{S} = \widetilde{X}^\gamma D^{1-\gamma}$$

$$\widetilde{q} = \frac{\theta}{1-\theta}\frac{\widetilde{C}}{\widetilde{S}}$$

$$L^D = 0$$

$$\widetilde{P}^D = \widetilde{P}^N$$

$$\widetilde{I}^X = \left(\mathfrak{g}_X+\delta^X\right)\widetilde{X}$$

Clarification on interior allocations. If we allowed any parameter combination, then this solution could result in an allocation of labor and land outside of their respective bounds, i.e. $L^X/L \notin (0,1)$ or $D/Z \notin (0,1)$. This would not be an equilibrium according to definition 1. Since in equilibrium the NPG condition has to hold, how-

ever, these false solutions are ruled out and land and labor are allocated within their respective bounds, i.e. $L^X/L \in (0,1)$ and $D/Z \in (0,1)$.

Proposition B.1 (Interior steady state allocation)**.** *For any parameter combination that satisfies the NPG condition and $g^X > -\frac{\eta}{1-\eta}\frac{\beta}{1-\alpha}g^Y$ the unique steady state described in proposition 1 represents an interior allocation.*

Notice that the restriction $g^X > -\frac{\eta}{1-\eta}\frac{\beta}{1-\alpha}g^Y$ is necessary to obtain a steady state where the construction sector is active, $I^X > 0$.

Proof of proposition B.1. For the NPG condition to hold the interest rate has to be larger than the growth rate of wealth, $r > \mathfrak{g}_W = \mathfrak{g}_Y$. This is equivalent to assuming that the exogenous parameters satisfy

$$\rho + (\sigma - 1)\left(\frac{\beta[1-\theta(1-\gamma\eta)]}{1-\alpha}g^Y + \theta\gamma(1-\eta)g^X\right) > 0.$$

The condition $r > \mathfrak{g}^Y$ then implies that the numerator of L^X/L is strictly positive, i.e.

$$r + (1-\alpha)\delta^K > \alpha\mathfrak{g}_Y.$$

This in turn implies that the term

$$\left[\eta + (1-\theta)\frac{r + \delta^X - \mathfrak{g}_{PX}}{\gamma\theta(\mathfrak{g}_X + \delta^X)}\right]\frac{\beta}{1-\eta}(r+\delta^K) \tag{25}$$

in (23) has to be strictly positive for $L^X/L \in (0,1)$. We now show that this is indeed the case.[35] With the definition of the growth rate of the price of residential structures we can write

$$r + \delta^X - \mathfrak{g}_{px} = r + \delta^X - \mathfrak{g}_Y + \mathfrak{g}_X = \underbrace{r - \mathfrak{g}_Y}_{>0} + \underbrace{\delta^X + \mathfrak{g}_X}_{>0} > 0.$$

where $r - \mathfrak{g}_Y > 0$ is implied by the NPG condition and $\delta^X + \mathfrak{g}_X$ by the restriction $g^X > -\frac{\eta}{1-\eta}\frac{\beta}{1-\alpha}g^Y$. Taken all together this implies that (25) is strictly positive and hence $L^X/L \in (0,1)$.

We now have only to show that also $D/Z \in (0,1)$ when the NPG condition holds.

---

[35]The expression $(r + \delta^K)$ equals the marginal product of capital, which is always positive due to the Inada conditions.

The numerator of (24),

$$\frac{(1-\gamma)\beta}{\gamma(1-\eta)} \frac{r+\delta^X - \mathfrak{g}_{PX}}{\mathfrak{g}_X + \delta^X} > 0$$

is strictly positive because we have already shown that both $r + \delta^X - \mathfrak{g}_{PX}$ and $\mathfrak{g}_X + \delta^X$ are strictly positive. The denominator is then also strictly positive and larger than the numerator because $(1 - \alpha - \beta)\left(\frac{L}{L^X} - 1\right) > 1$, which is implied by $L^X/L \in (0,1)$. This completes the proof. $\qquad\square$

B.4. Proofs

Proof of proposition 1.

Growth rates for land and labor are equal to zero in steady state because aggregate supply of both is constant in the long run. Therefore L^D, L^X, L^D, L, D, N, and Z are all constant.

The Keynes-Ramsey rule (18) implies that for consumption to grow at constant rates the interest rate r has to be constant. We now turn to the production function of the numeraire (5). Since it has constant returns to scale we apply Euler's theorem and use the FOC given by (19) to obtain

$$Y = (r + \delta^K)K + wL^Y + R^N N.$$

Since r, L^Y and N are constant in steady state, it follows that $Y, K, w,$ and R^N all grow at the same rates. This rate can be determined by taking the derivative of the production function of the numeraire (5) with respect to time

$$\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + \beta g^Y \quad \Rightarrow \quad \frac{\dot{Y}}{Y} = \frac{\beta}{1-\alpha} g^Y,$$

where the latter equation makes us of the fact that Y and K grow at the same rates. This implies that $Y, K, w,$ and R^N all grow at the exogenous rate $\frac{\beta}{1-\alpha} g^Y$.

The FOC of the land development firm problem (14) and the fact that residential land is constant in steady state imply that prices P^D and P^N grow at the same rates and that these are equal to the growth rate of wages, $\frac{\beta}{1-\alpha} g^Y$. The household budget constraint (4) implies that $W, C,$ and qS also grow at the rate $\frac{\beta}{1-\alpha} g^Y$.

The asset market clearing equation, $W = P^D D + P^N N + P^X X + K$, together with the established growth rates of $K, W, P^D,$ and P^N imply that the product $P^X X$ grows at the

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rate $\frac{\beta}{1-\alpha}g^Y$. The law of motion of structures implies that for X to grow at a constant rate, I^X and X have to grow at the same rates. Making use of the fact that equilibrium profits $\Pi^X = 0$ in (11) yields

$$I^X = \frac{1}{P^X}M + \frac{w}{P^X}L^X.$$

Both summands have to grow at the same constant rate for I^X to grow at a constant rate. It follows that $\frac{\dot{I}^X}{I^X} = \frac{\dot{M}}{M} - \frac{\dot{P}^X}{P^X} = \frac{\dot{w}}{w} - \frac{\dot{P}^X}{P^X}$ (L^X is constant). This in turn implies that M grows at the rate $\frac{\beta}{1-\alpha}g^Y$. We now take the derivative of the production function for structures with respect to time

$$\frac{\dot{I}^X}{I^X} = \eta\frac{\beta}{1-\alpha}g^Y + (1-\eta)g^X.$$

This also gives the growth rate of X, which grows at the same rate as I^X. Since $P^X I^X$ grows at the rate $\frac{\beta}{1-\alpha}g^Y$ it follows that P^X grows at the rate $(1-\eta)\left(\frac{\beta}{1-\alpha}g^Y - g^X\right)$.

Lastly, we need to determine the growth rates of q and S. The derivative of the production function for houses, $H(\cdot)$, with respect to time reads

$$\dot{S} = H_X\dot{X} + H_D\dot{D} = H_X\dot{X}$$
$$\Rightarrow \frac{\dot{S}}{S} = \gamma\frac{\dot{X}}{X} = \gamma\eta\frac{\beta}{1-\alpha}g^Y + \gamma(1-\eta)g^X.$$

From the requirement that qS grows at the rate $\frac{\beta}{1-\alpha}g^Y$ we obtain the growth rate for rents as $(1-\gamma\eta)\frac{\beta}{1-\alpha}g^Y - \gamma\eta g^X$.

To obtain the growth rate of house prices take the derivative of (10) with respect to time and rearrange

$$\dot{P}^H = \frac{\dot{D}H - D\dot{H}}{H^2}P^D + \frac{D}{H}\dot{P}^D + \frac{\dot{X}H - X\dot{H}}{H^2}P^X + \frac{X}{H}\dot{P}^X$$
$$\Rightarrow \frac{\dot{P}^H}{P^H} = (1-\gamma\eta)\frac{\beta}{1-\alpha}g^Y - \gamma(1-\eta)g^X.$$

House prices grow at the same rate as rents. Lastly, all wealth-to-income ratios are constant because aggregate income, NNP, grows at the same rate as the value of a given wealth category. For example, housing wealth $P^H H$ grows at the rate g^Y and NNP grows at the rate g^Y. Similarly, the share of residential land wealth in housing wealth, $P^D D/(P^H H)$, is also constant because numerator and denominator both grow

at the rate g^Y. □

Proof of proposition 2.

We first derive growth rates as functions of long-run land elasticities. Since K and Y grow at the same rates in steady state, we can express K as a constant share in Y, i.e. $K = \kappa^K Y$. Inserting this into the production function for Y and solving for Y yields then a reduced-form production function

$$Y = \left(\kappa^K\right)^{\frac{\alpha}{1-\alpha}} \left(B^Y L^Y\right)^{\frac{\beta}{1-\alpha}} N^{\frac{1-\alpha-\beta}{1-\alpha}}$$

The reduced-form production functions captures that, for example, higher land input leads to higher output, wich in turn leads to higher capital, amplifying the effect on output. This can be seen from the output elasticity of land, $\psi^Y \equiv \frac{1-\alpha-\beta}{1-\alpha}$, which is larger than the output elasticity of land in the original production function, $1 - \alpha - \beta$. We call the parameter ψ^Y the long-run land elasticity in the Y sector. It measures how an exogenous increase in the supply of land N translates into a long-run increase in output of Y.

The long-run land elasticity in the housing sector is a bit more complex. The reason is that land enters the Y sector, and that materials M from the Y sector enter the production of structures X, wich then in turn enter the production of houses H, where land also enters directly. Since they grow at the same rates in steady states we define $\kappa^M \equiv M/Y$ and $\kappa^X = X/I^X$ and rewrite the production functions for I^X

$$
\begin{aligned}
I^X &= \left(\kappa^M\right)^\eta [Y]^\eta \left(B^X L^X\right)^{1-\eta} \\
&= \left(\kappa^M\right)^\eta \left(\kappa^K\right)^{\frac{\eta\alpha}{1-\alpha}} \left(B^Y L^Y\right)^{\frac{\eta\beta}{1-\alpha}} N^{\frac{(1-\alpha-\beta)\eta}{1-\alpha}} \left(B^X L^X\right)^{1-\eta}
\end{aligned}
$$

This reduced-form production function for I^X shows that more land and labor in the numeraire sector also raises output in the construction sector. The reduced-form production function for houses then reads

$$
\begin{aligned}
H &= \left(\kappa^X I^X\right)^\gamma D^{1-\gamma} \\
&= \left(\kappa^X\right)^\gamma \left(\kappa^M\right)^{\eta\gamma} \left(\kappa^K\right)^{\frac{\eta\gamma\alpha}{1-\alpha}} \left(B^Y L^Y\right)^{\frac{\eta\gamma\beta}{1-\alpha}} N^{\frac{(1-\alpha-\beta)\eta\gamma}{1-\alpha}} \left(B^X L^X\right)^{(1-\eta)\gamma} D^{1-\gamma}
\end{aligned}
$$

In this reduced-form function the true role of land for the production of housing can

Variables	Growth rate
$r, \frac{P^H H}{NNP}, \frac{W}{NNP}, \frac{P^D D}{P^H H}, L^D, L^X, L^Y, N, D$	0
$Y, K, M, w, R^N, P^N, P^D, C, NNP, W$	$\left(1 - \psi^Y\right) g^Y$
X, I^X	$\eta \left(1 - \psi^Y\right) g^Y + (1 - \eta) g^X$
P^X	$(1 - \eta)\left[\left(1 - \psi^Y\right) g^Y - g^X\right]$
S, H	$\frac{1 - \psi^H}{1 - \eta \psi^Y}\left[\eta\left(1 - \psi^Y\right) g^Y + (1 - \eta) g^X\right]$
q, P^H	$\left[1 - \eta + \eta\left(\psi^H - \psi^Y\right)\right]\frac{1 - \psi^Y}{1 - \eta \psi^Y} g^Y - (1 - \eta)\frac{1 - \psi^H}{1 - \eta \psi^Y} g^X$

Table B.1: Steady state growth rates as functions of long-run land elasticities

be assessed. It not only enters via D directly, but indirectly through Y and I^X via N.[36] We define the long-run land elasticity in the housing sector as $\psi^H \equiv \frac{1 - \alpha - \beta}{1 - \alpha} \eta \gamma + 1 - \gamma = \psi^Y \eta \gamma + 1 - \gamma$. Note that both N and D are proportional to Z in steady state. Thus, ψ^H is the long-run equilibrium elasticity of the housing stock with respect to land endowment. It also depends on the long-run land elasticity in the Y sector. This channel is shut down if $\eta = 0$, because then materials are irrelevant for producing structures.

Making use of the definitions of ψ^Y and ψ^H the growth rates can be reformulated. We substitute γ and β with ψ^Y and ψ^H. The resulting growth rates are shown in table B.1.

$\square$

Proof of corollary 2.1.

#1) Wealth-to-income ratios are constant because assets grow at the same rates as income, g^Y, when measured in units of the numeraire. To be precise, it holds for the total wealth to income rate, W/NNP, the housing wealth-to-income ratio, $P^H H/NNP$, and the non-housing wealth-to-income ratio, $(W - P^H H)/NNP$, that the numerator grows at the same rate as the denominator.

#2) The residential land price grows at a strictly positive rate iff $g^Y > 0$. The residential structure price grows at a strictly positive rate iff $\left(1 - \psi^Y\right) g^Y > g^X$. The house price grows at a strictly positive rate iff $g^X < \frac{1 - \psi^Y}{1 - \psi^H} \frac{1 - \eta + \eta\left(\psi^H - \psi^Y\right)}{1 - \eta} g^Y$. This restriction holds if $g^X < g^Y$ and $\psi^H \geq \psi^Y$ (sufficient, not necessary condition). P^D grows at a larger rate than P^H iff $g^X > -\frac{\eta}{1-\eta}\left(1 - \psi^Y\right) g^Y$. Similarly, P^H grows at a larger rate than P^X iff $g^X > -\frac{\eta}{1-\eta}\left(1 - \psi^Y\right) g^Y$.

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[36] Using the steady state values derived in online appendix B.3, one can verify that κ^M, κ^X, and κ^K are independent of land endowment Z.

#3) Residential structures grow at a strictly positive rate that is larger than the growth rate of residential land iff $g^X > -\frac{\eta}{1-\eta}\left(1-\psi^Y\right)g^Y$.

#4) The land share in total housing wealth is $P^D D/\left(P^H H\right)$. It is constant because numerator and denominator grow both at the rate g^Y.

#5) Rents grow at a strictly positive rate iff $g^X < \frac{1-\psi^Y}{1-\psi^H}\frac{1-\eta+\eta\left(\psi^H-\psi^Y\right)}{1-\eta}g^Y$.

$\square$

B.5. Demand-supply representation

Housing. The supply of housing is constant in t because H is a state variable. The inverse demand can be expressed as[37]

$$q_t = \frac{\theta\mu_t\left(W_t+\hat{w}_t\right)}{H_t},\tag{26}$$

with

$$\mu_t \equiv \left(\int_t^\infty\left[\left(\frac{q_\tau}{q_t}\right)^\theta\exp\left[-\hat{r}(\tau,t)-\frac{\rho}{\sigma-1}(\tau-t)\right]\right]^{\frac{\sigma-1}{\sigma}}\mathrm{d}\tau\right)^{-1},$$

$$\hat{r}(\tau,t)\equiv\int_t^\tau r_v\mathrm{d}v,\quad\text{and}\quad\hat{w}_t\equiv\int_t^\infty w_\tau L_\tau e^{-\hat{r}(\tau,t)}\mathrm{d}\tau.$$

The expression $\left(W_t+\hat{w}_t\right)$ is total wealth, consisting of physical and human wealth. Physical wealth is W_t and human wealth $\hat{w}$ is the net-present value of the stream of future earnings. The propensity to consume out of total wealth is μ_t. Total period consumption, $\mu_t\left(W_t+\hat{w}_t\right)$, is allocated between housing and non housing where the share going to housing is θ. The value of housing consumption is therefore given by $\theta\mu_t\left(W_t+\hat{w}_t\right)$. When studying the demand for housing we abstract from the feedback of q through the propensity to consume. If $\sigma=1$, then $\mu=\rho$ and this channel vanishes anyway.

In steady state we can directly relate q to house prices. Making use of the no-arbitrage condition for houses yields

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[37]For the derivation of the consumption function see Grossmann et al. (2019).

$$P^H = \frac{CF}{\left(r - \frac{\beta}{1-\alpha}g^Y\right)H} = \frac{qH - \left(\mathfrak{g}_X + \delta^X\right)P^XX}{\left(r - \frac{\beta}{1-\alpha}g^Y\right)H} \overset{eq.(7)}{=} \frac{q}{r - \frac{\beta}{1-\alpha}g^Y} - \frac{\mathfrak{g}_X + \delta^X}{r - \frac{\beta}{1-\alpha}g^Y}P^X\left(\frac{X}{D}\right)^{1-\gamma}.$$

Structures and land. Replacing qH by (26) in the definition of R^D and R^X yields

$$R^D = (1-\gamma)\frac{\theta\mu(W+\hat{w})}{D} \quad \text{and} \quad R^X = \gamma\frac{\theta\mu(W+\hat{w})}{X} - \delta^XP^X.$$

Because housing production uses a Cobb-Douglas technology, according to (7), both R^D and R^X react to changes in total wealth, $W + \hat{w}$, in the same proportions (abstracting from δ^XP^X for the moment). These two equations are inverse demand functions for N and D. Both shift outwards when the demand for housing, qH, increases.

How do changes in yields components finally translate into prices of the stocks X and D? By solving the no-arbitrage conditions for X and D both P^X and P^D can be expressed

$$P^D_t = \int_t^\infty e^{-\hat{r}(\tau,t)}R^D_\tau d\tau$$
$$P^X_t = \int_t^\infty e^{-\hat{r}(\tau,t)}R^X_\tau d\tau.$$

Assume that the real interest rate, r, is time-invariant or changes only little over time, consistent with a partial equilibrium analysis. Then changes in some future yield component R_τ translate into changes in current stock prices. Thus, analyzing changes in yield components is qualitatively analogous to analyzing changes in stock prices.

C. Calibration

C.1. US

We calibrate the model to all four economies – US, UK, France, and Germany – separately on an annual frequency for the post-WWII era. Due to better data availability and quality, we focus on the US first, before turning to the other three economies in online appendix C.2.

Calibrated outside the model

Parameter	Value	Explanation/Target
θ	0.18	housing expenditure share
σ	10/3	intertemporal elasticity of substitution
δ^K	$\ln(1 + 0.056)$	capital depreciation rate
δ^X	$\ln(1 + 0.015)$	structure depreciation rate
β	0.613	labor income share in Y sector
$\{L_t\}_{t=0}^{\infty}$	logistic differential equation (see figure 7)	population dynamics

Table C.2: Calibrated outside the model

The set of parameters $\{\theta, \sigma, \delta^K, \delta^X, \beta, \{L_t\}_{t=0}^{\infty}\}$ is calibrated without solving the model numerically. Table C.2 summarizes the calibration. The preference parameter θ equals the aggregate share of housing expenditures in total consumption expenditures. According to data from the National Income and Product Accounts (NIPA) this value is on average 18 percent for the post-WWII US.[38] The intertemporal elasticity of substitution (IES) in composite consumption, $C^{1-\theta}S^{\theta}$, is $1/\sigma$. In a meta-study Havránek (2015) shows that mean-estimates based on micro-studies and asset holders are in the range of 0.3 to 0.4. Best, Cloyne, Ilzetzki, and Kleven (2019) estimate an aggregate IES of 0.1 using quasi-experimental variation in UK interest rates. We therefore set $\sigma = 10/3$ to match the lower bound of 0.3 from Havránek (2015). The depreciation rates of capital and structures are set to annual values of 5.6 percent (Davis and Heathcote, 2005) and 1.5 percent (Hornstein, 2009, p. 13), respectively.[39]

The parameter β equals the labor income share of the non-housing sector, $\left(wL^Y\right)/Y$. We use data from the US Bureau of Economic Analysis (BEA) which is available from 1997 to 2018.[40] The non-housing labor share is relatively stable over time, declining only by 1.2 percentage points since 2008. The average is 61.3 percent and we hence set $\beta = 0.613$.

Lastly, the exogenous population sequence $\{L_t\}_{t=0}^{\infty}$ is calibrated as follows. We assume that the population will not grow forever and approaches a constant as $t \rightarrow \infty$.

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[38]See also Piazzesi and Schneider (2016, section 2.1).

[39]Our value for δ^X is also very close to the one chosen by Davis and Heathcote (2005), 1.57, and lies in the range documented by Tuzel (2010), 1.5 to 3.

[40]We use the *components of value added by industry* from https://apps.bea.gov/iTable/iTable.cfm?ReqID=51&step=1, accessed on March 8, 2020.
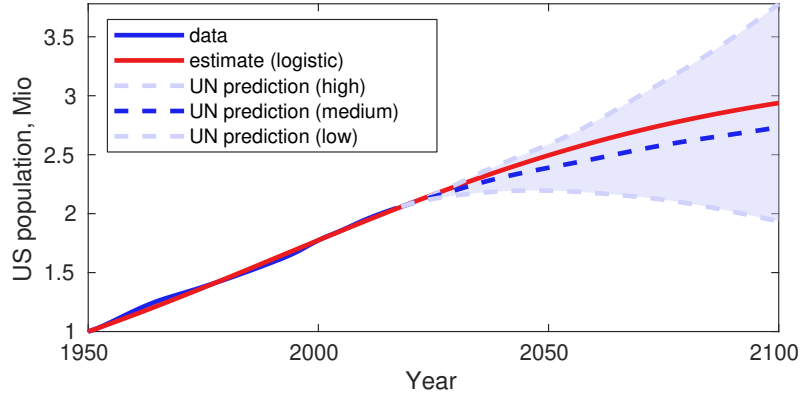
Figure 7: US population dynamics

Initial population is normalized to $L_0 = 1$, which has no effect on the moments we study. Future population dynamics are projected with a simple logistic differential equation

$$\dot{L}_t = b L_t \frac{a - L_t}{a} \Leftrightarrow L_t = \frac{a}{1 + \left(\frac{a}{L_0} - 1\right) e^{-bt}}, \tag{27}$$

where $b \geq 0$ governs the speed of convergence and $a > 0$ is the finite limit of L_t when $t \to \infty$. The two coefficients are determined by minimizing the unweighted sum of squared residuals between the US population levels from 1950 to 2017 and the values predicted by the logistic equation where $t = 0$ corresponds to 1950. We obtain $a = 3.2$ and $b = 0.02$. The result is depicted in figure 7. Our simple prediction is very close to the medium scenario prediction by the UN (2019) and lies between its high and low scenario prediction.

Calibrated jointly inside the model

The remaining 7 parameters $\alpha, \gamma, \eta, \rho, \xi, g^Y$, and g^X as well as the 3 initial states K_0, D_0 and X_0 are calibrated by solving the transition many times for different parameter values until a set of 10 empirical moments is matched. Although all parameters jointly determine the theoretical moments, we explain in the following each parameter in combination with the moment that it most strongly affects.

The time preference rate ρ is chosen to match the level of the total wealth-to-income

	Moment	Data	Model
1	$\frac{W}{NNP}$, in 1950	3.6	3.6
2	$\frac{NNP_{2015}}{NNP_{1950}}$	6.0	6.0
3	$\frac{P^H H}{NNP}$, in 1950	1.2	1.2
4	$\frac{P^D D}{P^H H} \times 100$, in 1950	11.8	11.8
5	$\frac{L^X}{L} \times 100$, long run	2.5	2.5
6	$\frac{q_{2015}}{q_{1950}}$	1.7	1.7
7	$\frac{I^H}{GDP} \times 100$, in 1950	5.6	5.6
8	$\frac{R^N N}{NNP} \times 100$, in 1950	10.0	10.0
9	Half-life of D, in years	22.6	22.6
10	$\frac{X_{2015}/D_{2015}}{X_{1950}/D_{1950}}$	1.9	1.9

Table C.3: Targeted data and model moments

ratio of 3.6 (see section 2) at the beginning of the transition, in 1950. Similarly, we set the initial capital stock, K_0, such that we match the level of the housing wealth-to-income ratio of 1.2 in 1950. The technology growth rate of the non-housing sector, g^Y, is set to match the increase in US national income between 1950 and 2015. Taken together, these three moments imply that we match the levels of the total wealth-to-income ratio and the housing wealth-to-income ratio, as well as the growth in their denominator. The dynamics in total wealth and housing wealth, however, are not targeted by our calibration. By matching the growth in national income we rule out that the model replicates increases in wealth-to-income ratios by implausible dynamics in income.

The parameter γ is the output elasticity of structures in the production of houses. The larger γ, the more relevant are structures, and the less relevant is land in the production of houses. This parameter strongly affects the share of residential land wealth in total housing wealth (land share), i.e. the smaller γ the larger the land share. We therefore calibrate γ by matching the land share's level of 11.8 percent in 1950. Similarly, the parameter η is the output elasticity of materials in the construction sector. The smaller η, the smaller is the demand for materials, and the larger is the demand for labor in the

#	Parameter	Explanation	Value
1	ρ	time preference rate	0.040
2	g^Y	labor efficiency growth in numeraire sector	0.020
3	K_0/K^*	initial capital stock	0.227
4	γ	structures' elasticity in H	0.906
5	η	materials' elasticity in I^X	0.556
6	g^X	labor efficiency growth in construction sector	-0.014
7	X_0/X^*	initial stock of residential structures	0.191
8	α	capital elasticity in Y	0.275
9	ξ	intensity of convex adjustment cost in land development	759.05
10	D_0/D^*	initial stock of residential land	0.403

Notes: Initial states, K_0, D_0, X_0, are expressed relative to their respective final steady state values, denoted by superscript *.

Table C.4: Endogenously calibrated parameters

construction sector. We hence set η such that we match the long-run employment share in the residential construction sector of 2.5 percent.[41] We target the long-run value of the employment share in residential construction because data is only available for more recent periods, but not for 1950.

We know from proposition 1 that the technological growth rate of the construction sector, g^X, strongly affects the long-run growth rate of rents. We therefore target the growth factor of rents between 1950 and 2015 of 1.7 (see section 2). The initial stock of residential structures, X_0, is chosen to match the share of residential investment in GDP of 5.6 percent in 1950 as taken from Federal Reserve Economic Data (FRED). In the model residential investment is defined as follows

$$I^H \equiv P^X I^X + P^D \dot{D} + wL^D, \tag{28}$$

---

[41]The employment share in residential construction is obtained by multiplying the employment share in the overall construction sector with the output share of residential construction in total construction. The data is from the National Association of Home Builders at https://www.nahbclassic.org/generic.aspx?sectionID=734&genericContentID=261286 and http://admin.nahb.org/generic.aspx?sectionID=734&genericContentID=178434&channelID=311 and was accessed on Dec 16 2019. It is very stable since 2010 at a value of 2.5 percent.

where we included investment in residential land because the empirical data does not differentiate between investment into land and structures. The output elasticity of capital in the numeraire sector, α, is set to match the net income share of non-developed land, $R^D D / NNP$ of 10 percent in 1950.[42] This is related to α because the income share of non-developed land in output of the numeraire sector is given by $1 - \alpha - \beta$. The parameter ξ governs the speed of convergence in developed land, D. We set it such that we match the half-life in the transition of residential land of 22.6 years.[43] Lastly, the initial stock of residential land, D_0, is chosen such that we match the growth factor of residential structures relative to residential land of 1.9 between 1950 and 2015, as implied by table 1. The targeted moments are shown in table C.3 and the resulting parameter values in table C.4.[44]

C.2. Calibration of UK, France, and Germany

The parameters α, β, δ^K, δ^X, η, σ, and ξ are set to the same values as for the US. We set the housing expenditure share, θ, to 17, 22, and 27 percent for the UK, France, and Germany, respectively.[45] The exogenous population series are all taken from UN (2019). For each country we fit the same logistic differential equation (27). The obtained values for coefficient a are 1.9, 1.8, and 1.2 for the UK, France, and Germany, respectively. Similarly, for b we get 0.01, 0.02, and 0.04 for the UK, France, and Germany, respectively. The remaining set of parameters $\{\rho, \gamma, g^Y, g^X, K_0, D_0, X_0\}$ is calibrated by matching a corresponding subset of the moments we target for the US. Our sample periods for the UK, France, and Germany are 1950–2012, 1960–2012, and

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[42]The data is taken from **?**, figure 11rognlie2015.

[43]The series for residential land, D, is from the same data source as table 1, see also online appendix A. We assume that D follows a logistic differential equation and estimate the respective coefficients by minimizing the sum of squared residuals between data and model.

[44]We obtain a negative value for g^X. First, there is indeed empirical evidence on negative productivity growth in the construction sector. For instance, Davis and Heathcote (2005) estimate a value of -0.27 percent and Hornstein and Krusell (1996) document negative labor productivity growth and negative TFP growth in the construction sector for most periods after 1950. Second, since the numeraire good enters the construction sector as an intermediate input, labor productivity growth in the numeraire sector adds to productivity growth in the construction sector. Long-run and reduced-form labor productivity growth in the construction sector is indeed given by the growth rate of construction output, I^X, as documented in table 2. This growth rate can be positive even with a negative g^X. In our calibration this is the case. The long-run growth rate of labor productivity in the construction sector, in a reduced-form sense, equals to 0.3 percent.

[45]The data is from each country's national accounts as documented in our companion paper Grossmann, Larin, Löfflad, and Steger (2019). There we assume non-homothetic preferences and study distributional effects.

#	Parameter	Explanation	Value		
			UK	FR	DE
1	ρ	time preference rate	0.011	0.037	0.053
2	g^Y	technical growth in numeraire sector	0.026	0.015	0.021
3	K_0/K	initial capital stock (share of final)	0.337	0.147	0.287
4	γ	structures' elasticity in H	0.743	0.920	0.671
5	g^X	technical growth in construction sector	-0.035	-0.049	0.013
6	X_0/X	initial stock of residential structures	0.212	0.141	0.972
7	D_0/D	initial stock of residential land	0.740	6.562	0.127

(a) Targets

	Moment	UK		FR		DE	
		Data	Model	Data	Model	Data	Model
1	$\frac{W}{NNP}$, 1950	4.0	4.0	2.6	2.6	2.2	2.6
2	$\frac{NNP_{2012}}{NNP_{1950}}$	5.5	5.5	4.1	4.1	3.4	3.5
3	$\frac{P^H H}{NNP}$, in 1950	1.2	1.2	1.4	1.4	1.4	1.4
4	$\frac{P^D D}{P^H H} \times 100$, 1950	39.2	39.2	14.1	14.1	22.9	22.9
5	$\frac{q_{2012}}{q_{1950}}$	3.1	3.1	2.8	2.8	1.6	1.2
6	$\frac{RESI}{GDP} \times 100$, 1962	5.0	5.0	9.4	9.4	7.3	7.2
7	$\frac{X_{2012}/D_{2012}}{X_{1950}/D_{1950}}$	2.1	2.1	4.0	4.0	1.2	0.4

(b) Endogenous parameters

Notes: Initial states, K_0, D_0, X_0, are expressed relative to their respective final steady state values (normalized).

Table C.5: Calibration UK, France, and Germany

1962–2012, respectively. The moments we match and the resulting parameter values are shown in table C.5.

D. CES production function

stylized fact	variable	data (US)	baseline	alternative 1 $g^X = -\frac{\eta}{1-\eta}\frac{1-\alpha}{\beta}g^Y$	alternative 2 $EoS = 0.25$
1	$\frac{W}{NNP}$	1.39	1.55	1.83	2.16
	$\frac{P^D D + P^X X}{NNP}$	1.42	1.80	2.20	2.88
2	P^H	1.93	2.63	3.98	5.42
	P^D	8.45	4.82	6.40	10.75
	P^X	1.16	2.47	3.82	3.37
3	D	2.80	2.29	1.85	1.76
	X	5.49	4.36	3.51	4.40
4	$\frac{P^D D}{P^D D + P^X X}$	2.84	1.02	0.89	1.08
5	q	1.70	**1.70**	2.23	3.15

Notes: All values are growth factors of the respective variable between 1950 and 2015. Bold numbers highlight targeted moments. Alternative 1 is re-calibrated assuming that $g^X = -\frac{\eta}{1-\eta}\frac{1-\alpha}{\beta}g^Y$ and removing the growth rate of rents from the set of targeted moments. Alternative 2 applies the same parameters from alternative 1 except that the elasticity of substitution between D and X in the production function of H is set to 0.25.

Table D.6: Stylized facts under CES technology in housing production

References

ALBOUY, D., G. EHRLICH, AND M. SHIN (2018): "Metropolitan Land Values," *The Review of Economics and Statistics*, 100(3), 454–466.

BEST, M. C., J. S. CLOYNE, E. ILZETZKI, AND H. J. KLEVEN (2019): "Estimating the Elasticity of Intertemporal Substitution Using Mortgage Notches," *The Review of Economic Studies*.

DAVIS, M. A., AND J. HEATHCOTE (2005): "Housing and the Business Cycle," *International Economic Review*, 46(3), 751–784.

——— (2007): "The price and quantity of residential land in the United States," *Journal of Monetary Economics*, 54(8), 2595–2620.

GROSSMANN, V., B. LARIN, H.-T. LÖFFLAD, AND T. STEGER (2019): "Distributional Consequences of Surging Housing Costs under Schwabe's Law," *CESifo working paper*.

HAVRÁNEK, T. (2015): "Measuring Intertemporal Substitution: the Importance of Method Choices and Selective Reporting," *Journal of the European Economic Association*, 13(6), 1180–1204.

HORNSTEIN, A. (2009): "Problems for a fundamental theory of house prices," *Economic Quarterly*, 95(Win), 1–24.

HORNSTEIN, A., AND P. KRUSELL (1996): "Can Technology Improvements Cause Productivity Slowdowns?," *NBER Macroeconomics Annual*, 11, 209–259.

JORDÀ, Ò., K. KNOLL, D. KUVSHINOV, M. SCHULARICK, AND A. M. TAYLOR (2019): "The Rate of Return on Everything, 1870–2015," *The Quarterly Journal of Economics*.

KNOLL, K., M. SCHULARICK, AND T. STEGER (2017): "No Price Like Home: Global House Prices, 1870–2012," *American Economic Review*, 107(2), 331–353.

PIAZZESI, M., AND M. SCHNEIDER (2016): "Housing and Macroeconomics," in *Handbook of Macroeconomics*, pp. 1547–1640. Elsevier.

PIKETTY, T., AND G. ZUCMAN (2014): "Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010," *The Quarterly Journal of Economics*, 129(3), 1255–1310.

TUZEL, S. (2010): "Corporate Real Estate Holdings and the Cross-Section of Stock Returns," *Review of Financial Studies*, 23(6), 2268–2302.

UN (2019): "World Population Prospects 2019, Online Edition. Rev. 1," Discussion paper, United Nations, Department of Economic and Social Affairs, Population Division.

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Abstract

The housing wealth-to-income ratio has been increasing in most developed economies since the 1950s. We provide a novel theory to explain this long-term pattern. We show analytically that house prices grow in the steady state if i) the housing sector is more land-intensive than the non-housing sector, or ii) technological progress in the construction sector is weaker than in the non-housing sector. Despite growing house prices and housing wealth, the housing wealth-to-income ratio is constant in steady state. We hence study the dynamics in the housing wealth-to-income ratio by computing transitions. The model is calibrated separately to the US, UK, France, and Germany. On average, we replicate 89 percent of the observed increase in the housing wealth-to-income ratio. The key for replicating the data is the differentiation between residential land as a non-reproducible factor and residential structure as a reproducible factor. The transition process from the calibrated model points to two driving forces of an increasing housing wealth-to-income ratio: i) A long-lasting construction boom that brought about a pronounced build-up in the stock of structures and ii) an increase in the demand for residential land that resulted in surging residential land prices.

Keywords

Housing Wealth; Economic Growth; Wealth-to-Income Ratio; House Price; Land Price.

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