

Das House Kapital

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Abstract: The housing wealth-to-income ratio has been increasing in most developed economies since the 1950s. In this paper we provide a novel theory to explain this long-term pattern. We show analytically that house prices grow in the long term if i) the housing sector is more land intensive than the non-housing sector, or ii) technological progress in the construction sector is weaker than in the non-housing sector. The model is calibrated separately to the US, the UK, France, and Germany. Quantitatively, both channels are equally relevant for rising house prices. We then study the dynamics in the housing wealth-to-income ratio by computing transitions. The model replicates 80, 67, and 76 percent of the observed increase in the housing wealth-to-income ratio of the UK, France, and Germany, respectively. We even slightly over-predict the increase in the US housing wealth-to-income ratio by 35 percentage points. The key for replicating the data is the explicit differentiation between residential land – a non-reproducible factor – and residential structure – a reproducible factor.

Key words: Housing Wealth; Non-Residential Wealth; Wealth-to-Income Ratios; House Prices; Land Prices.

JEL classification: E10, E20, O40.

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1. Introduction

Housing wealth is the largest private wealth component. Figure I displays its size, scaled by aggregate income, and its evolution over time for the USA, UK, Germany, and France since 1950. It has been growing considerably faster than income. In 1950 the aggregate housing stock was on average worth one year of aggregate income, while at the end of our sample (2015 for the US, 2012 for the other countries) the housing stock was on average worth almost three years of aggregate income. Housing wealth also grew faster than non-housing wealth, driving at least half of the increase in overall wealth. For instance, housing wealth grew by a factor of 1.42 and non-housing wealth by a factor of 1.38 in the US between 1950 and 2015, while these numbers read 2.4 and 1.9 for the UK between 1950 and 2012.¹

Why has housing wealth been increasing relative to income in the US, the UK, France, and Germany since 1950? To shed light on this question, we proceed in three steps.

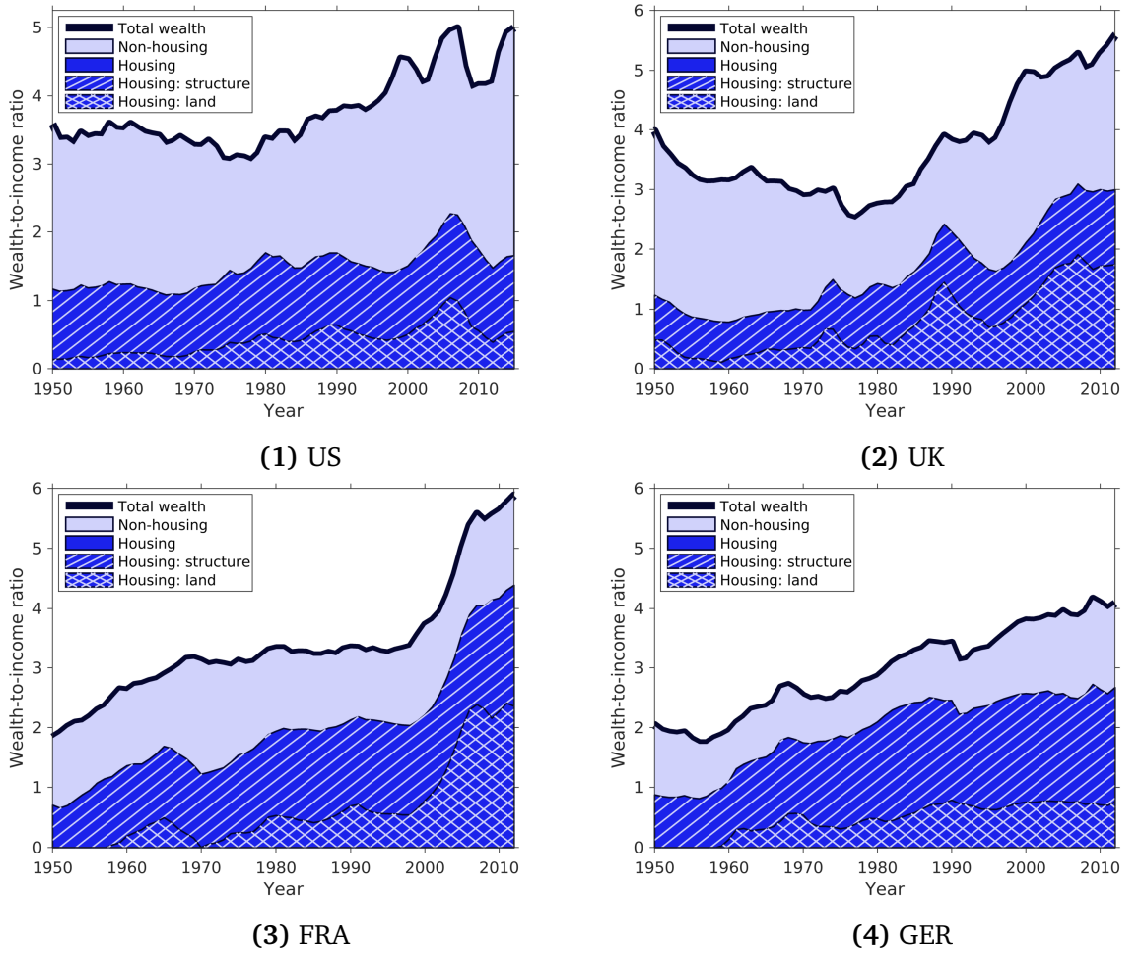
First, we document five stylized facts on housing-related macro variables. We follow [Davis and Heathcote \(2007\)](#) and "[...] conceptualize a house as a bundle comprising a reproducible tangible structure and a non-reproducible plot of land."² It can already be seen in figure I that most of the increase in housing wealth is driven by rising residential land wealth while residential structure wealth has been relatively stable. This explicit distinction between land and structure is therefore key for studying the rise in housing wealth. We document how prices and quantities of residential land and residential structures evolved since 1950 for the US, UK, France, and Germany. The list of stylized facts, presented in section 2, quantifies not only the empirical phenomenon to be explained, but it also documents the evolution of other housing-related macro variables – such as the rent and the house prices – that a plausible theory should also replicate.

Second, we set up a new macroeconomic model with housing that focuses on the supply side and is designed to think long-term. The employed Ramsey growth model comprises two sectors, a numeraire sector and a housing sector. While the numeraire sector is kept standard, the innovative part of our model is the housing sector. It consists of three different types of firms. Real estate development firms purchase land and transform it into residential land by employing labor. Construction firms manufacture structures by employing materials and labor. Property management firms then combine the residential land and the residential structures to produce houses that are rented out to households. The overall supply of land is fixed. The allocation of land across the numeraire sector and the housing sector is endogenous. There is exogenous population growth and differential exogenous technological change entering the non-housing sector and the construction sector.

Third, we analyze the dynamics of housing wealth analytically in steady state and quantitatively along the transition. The tractability of the model allows us to obtain closed-form

¹These numbers and other stylized facts are summarized in section 2.

²Of course, in reality the two housing-related assets (structures and land) are not traded separately. The asset that is being traded is a house. The decomposition into two assets is, however, instructive for understanding the drivers behind rising housing wealth.



Notes: The housing wealth-to-income ratio is the sum of the aggregate value of residential structures and the aggregate value of residential land, both relative to income.

Figure I: Wealth-to-income ratio: the relevance of housing

steady state solutions. We derive conditions under which house prices grow in the long term. The phenomenon of a rising wealth-to-income ratio, however, cannot be explained in steady state.³ We therefore bring our model to the data by studying transitional dynamics.

Why is the increase in the housing wealth-to-income ratio relevant for the macroeconomy? First, rising house prices and rents have raised concerns about the affordability of housing (Albouy *et al.*, 2016; Quigley and Raphael, 2004). The increase in the aggregate housing wealth-to-income ratio points to a potential affordability issue (at the macro level) in that the representative household must devote a larger share of income to purchase the existing housing stock. Second, Gennaioli *et al.* (2014) argue that the long-run increase of the share of financial sector income in GDP is driven by a rising total wealth-to-income ratio that in turn is the result of rising housing wealth. Similarly, based on historical data for 17 advanced economies, Jordà *et al.* (2016) document a pronounced increase in the credit-to-GDP ratio in the 20th century and argue that this is primarily driven by the increase in

³To be precise, a time varying wealth-to-income ratio can either be explained as a sequence of steady states (Borri and Reichlin, 2018; Piketty and Zucman, 2015) or as a transition to a given steady state (Gennaioli *et al.*, 2014).

mortgages and housing wealth.⁴ Lastly, the long-term increase in the total wealth-to-income ratio contributes to a change in the functional income distribution to the advantage of capital income recipients (Piketty and Zucman, 2014). The housing sector seems especially important in this context, as stressed by Rognlie (2015) who argues that the increase in the aggregate capital income share in the group of the G7 economies over the period 1950–2010 is driven exclusively by the housing sector. Since the yield component of the rate of return on housing has been fairly stable (Jordà *et al.*, 2019), the increase in the housing component of the capital income share has to be driven by an increase in the housing wealth-to-income ratio.⁵ Understanding why the housing wealth-to-income ratio has been increasing is therefore key for these three debates.

We use the proposed model to explain why the housing wealth-to-income ratio has been increasing. In our first set of results we show under which conditions house prices increase in steady state. In essence, our model is a two-sectoral growth model with a housing and a non-housing sector. The house price – the price of the stock of housing – grows in steady state at the same rate as the rent – the price of the service flow derived from housing. Both sectors employ labor and experience exogenous technological change. We identify two mechanisms that trigger rising rents and house prices in a growing economy. First, the more land-intensive the housing sector is in comparison to the non-housing sector, the more likely it is that house prices grow at a positive rate in the long term. So it is not the mere existence of a fixed production factor that matters for rising housing wealth, but the relative intensity with which it enters in the two sectors. Second, we allow technological progress to differ between the housing sector – to be precise, the construction sector – and the non-housing sector. We show that the stronger technological growth is in the non-housing sector relative to technological growth in the construction sector, the higher is the long-term growth rate of house prices. The calibrated model shows that both channels – a higher land-intensity in the housing sector and stronger technological progress in the non-housing sector than in the construction sector – are empirically relevant.

Although housing wealth can increase in the steady state, the wealth-to-income ratio and the housing wealth-to-income ratio remain constant in steady state. To explain a rising housing wealth-to-income ratio we therefore study transitional dynamics implications, which constitute the second set of results. We calibrate the model by matching a set of moments along the transition. This is done separately for each of the four economies under study. However, we abstain from matching the increase in housing wealth over time and let the model speak on the implied increase in housing wealth. The calibrated model explains 80, 67, and 76 percent of the increase in the housing wealth-to-income ratio in the UK, France, and Germany over the time period 1950–2012, 1960–2012, and 1962–2012, respectively. In case of the US we even slightly over-predict the increase in the housing

⁴The mortgage-to-GDP ratio is given by $\text{leverage ratio} \times \text{housing wealth-to-income ratio}$. A rising housing wealth-to-income ratio raises – abstracting from log-run changes in the leverage ratio – the mortgage-to-GDP ratio.

⁵The housing capital income share is given by $\text{housing yield} \times \text{housing wealth-to-income ratio}$.

wealth-to-income ratio between 1950 and 2015 by 35 percentage points. This might be due to the fact that US housing wealth has not yet recovered from its previous bust at the end of our sample in 2015. We then run several counterfactual experiments to shed light on the underlying mechanisms. An important role in the story of the increasing housing wealth-to-income ratio is played by a pronounced construction boom.⁶ We show that the construction boom together with an inelastic supply of residential land (a non-reproducible factor in the long run) results in strongly rising housing wealth through the accumulation of residential structures and rising residential land prices. By showing that residential investment as a share of GDP has been – both in the data and in our model simulation – on a downward path since the 1950s for the US, UK, France, and Germany we further corroborate this result. Lastly, we also check that the model-implied transition is also largely in line with the other stylized facts, which are not targeted in the calibration strategy, such as the prices and quantities of houses, structures, and land.

Related literature

In studying why housing wealth increases over the long term our paper adds to the recent literature on housing and macroeconomics. One branch of this literature is primarily concerned with the measurement of housing wealth. [Davis and Heathcote \(2007\)](#) have developed a measurement system to estimate price and quantity indexes for residential land. They show that, on average over 1975 and 2006, land accounts for 36 percent of the value of the aggregate housing stock in the US. Over the same period, the real price of residential land nearly quadrupled, while the real price of structures increased cumulatively by only 33 percent. [Piketty and Zucman \(2014\)](#) have documented the evolution of the aggregate wealth-to-income ratios over 1700 until 2010 for eight developed economies. Their main finding is that the aggregate wealth-to-income ratio has risen gradually between 1970 and 2010, returning to the high values observed in Europe in the eighteenth and nineteenth centuries (600–700 percent). This can be explained, they argue, by a long-run asset price recovery (itself driven by changes in capital taxation policies since the world wars) and by the slowdown in productivity and population growth. Their data shows that housing wealth has gained importance while agricultural land has become less important over time. [Rognlie \(2015\)](#) argues that the housing sector has driven the surge in the net capital income share since 1948. He discusses two alternative explanations: i) the accumulation view according to which capital has been accumulated strongly while its rate of return declined only slightly and ii) the scarcity view according to which residential land became scarcer over time and paid a higher rate of return. Data and theory, he argues, support the scarcity view: the net capital share is rising in part because some forms of wealth are becoming relatively more scarce, not more abundant. [Knoll et al. \(2017\)](#) provide annual house prices for 14 advanced economies since 1870. They show that real house prices stayed constant from

⁶The period 1940 to 1975 is known as the era of mass-suburbanization that was accompanied by the creation of new residential areas at the boundaries of US cities ([Nechyba and Walsh, 2004](#)).

the nineteenth to the mid-twentieth century, but rose strongly and with substantial cross-country variation in the second half of the twentieth century. They point out that land prices, not replacement costs, are key to understanding the trajectory of house prices. We add to this literature by highlighting that most of the increase in housing wealth is driven by the increase in residential land wealth and in particular by offering a theory that demonstrates the importance of the interaction between the construction boom and an inelastic supply of residential land in explaining the observed increase in house prices and housing wealth.

Since the US-housing bust and the subsequent Great Recession many studies contributed to a better understanding of the relationship between housing and the macroeconomy in the short term. One of the earlier papers in this field is [Davis and Heathcote \(2005\)](#) who set up a neoclassical multisector stochastic growth model to investigate the cyclical dynamics of residential investment and other business cycle facts. The housing supply side of their model has formed the basis for many subsequent models. For instance, [Favilukis et al. \(2017\)](#) employing a quantitative heterogeneous agent model that the relaxation of financing constraints and its subsequent reversal was the driving force behind the recent US-housing boom and bust. More recently, [Kaplan et al. \(2019\)](#) come to a different conclusion. Accordingly, it's not the relaxation of financing constraints that drives house prices in the short term but rather changes in beliefs about the likelihood of future housing demand shifts. Whether credit conditions were responsible for the boom and bust in house prices is an ongoing debate. [Greenwald and Guren \(2019\)](#) seem to provide a reconciliation by showing that the segmentation of housing markets matters for the transmission of credit supply shocks. In this paper we do not study short-term fluctuations in housing wealth but focus on the long term. Moreover, the housing supply side of our model differs crucially from these existing models for two reasons. First, existing macro models with housing assume that each period one unit of land becomes available and is incorporated into the housing stock by the construction sector. When explaining the evolution of housing wealth we decompose it into price and quantity components of residential land and structures. Assuming that the quantity of residential land is exogenous would restrict the analysis of the long term evolution of residential land with strong implications for the residential land price. Second, existing models assume that land is not productive in the numeraire sector. This assumption places a strong restriction on differential land intensities of the numeraire and the housing sector. Our analysis shows that differential land intensities can play an important role in the determination of long run rent and house price growth.

We share our focus on housing in the long term with a smaller branch of the literature on housing and macroeconomics. One of the first economists to be concerned about the long-term macroeconomic consequences of the fixed factor land was [Ricardo \(1817\)](#). He argues that economic growth benefits landlords as the owners of the fixed factor disproportionately.⁷ While Ricardo was mainly concerned with agricultural land and the production of corn to feed a growing population, societies in modern times are confronted with the

⁷See also [Piketty \(2014\)](#).

need for residential investments to meet the increasing demand for housing services under the constraint of a fixed supply of land. [Nichols \(1970\)](#) includes a productive factor which is in fixed supply (land) into a Solow model. He shows that the land price increases at the same growth rate as output in steady state. [Hansen and Prescott \(2002\)](#) employ a one-good, two-sector OLG model where land is in fixed supply. The transition from constant to growing living standards, they argue, is inevitable given positive rates of total factor productivity growth and involves no change in the structure of the economy. In particular, the transition from stagnant to growing living standards occurs when profit-maximizing firms, in response to technological progress, begin employing a less land-intensive production process that, although available throughout history, was not previously profitable to operate. [Herkenhoff et al. \(2018\)](#) construct a general equilibrium spatial growth model of the US to analyze how state-level land-use restrictions have impacted on regional and aggregate economic activity between 1950 and 2014. Tightened land use restrictions, they argue, have increasingly limited the availability of land for housing and commercial use, which in turn have raised land prices, slowed interstate migration, reduced factor reallocation, and depressed output and productivity relative to historical trends. [Borri and Reichlin \(2018\)](#) employ a two-sector OLG model with housing services and bequests to show that a rising labor efficiency in the general economy relative to the construction sector pushes the house price, the housing wealth-to-income ratio, and wealth inequality (measured by bequests) upwards. Due to an inelastic housing demand, an increase in the relative labor efficiency triggers not only a strong house price appreciation but also a reallocation of resources to the housing sector. [Miles and Sefton \(2020\)](#) employ a Ramsey growth model with two sectors (numeraire and housing) and a spatial dimension. The amount of residential land is endogenous and depends on the state of transport technology. They derive a condition under which in a growing economy the speed at which transport technology improves relative to the growth in aggregate incomes is such as to generate flat house prices. The model can explain the constancy of the real house price before 1950 and the surge thereafter. While we share the focus on housing over the long term with this literature, our model differs as it explicitly differentiates between i) the stock of residential land and residential structures, ii) differential technological progress and iii) differences in land intensities between the housing and non-housing sectors. The latter allows us to assess the relevance of each of the two mechanisms in driving house prices in the long term. Furthermore, our main focus lies in explaining the rising housing wealth-to-income ratio.

2. Stylized facts

Following [Davis and Heathcote \(2007\)](#) we decompose housing wealth into a sum of the value of structures and the value of residential land,

$$\frac{\text{housing wealth}}{\text{income}} = \frac{\text{value of structures}}{\text{income}} + \frac{\text{value of land}}{\text{income}}. \quad (1)$$

Fact #	Variable	US 1950– 2015	UK 1950– 2012	FR 1960– 2012	DE 1962– 2012
1	wealth-to-income ratio	1.39	1.4	2.2	1.88
	housing wealth-to-income ratio	1.42	2.4	3.2	1.93
	non-housing wealth-to-income ratio	1.38	0.9	1.2	1.79
2	house price	1.9	4.0	6.0	1.6
	residential land price	8.4	9.6	32.2	2.3
	residential structure price	1.2	2.1	1.1	1.4
3	house quantity	4.4	3.3	2.2	4.2
	residential land quantity	2.8	2.1	1.6	3.5
	residential structure quantity	5.5	4.3	6.3	4.3
4	land share in housing wealth	2.8	1.5	3.8	1.2
5	housing rent	1.7	3.1	2.8	1.6

Notes: The sample for FR starts in 1960 because residential land values are negative in some earlier years and the sample for DE starts in 1962 because house and residential land prices are not available for previous periods.

Table I: Stylized facts, growth factors

Moreover, each wealth component can be decomposed into a price and a quantity component according to

$$\frac{P^{\text{housing}} \times Q^{\text{housing}}}{\text{income}} = \frac{P^{\text{structure}} \times Q^{\text{structure}}}{\text{income}} + \frac{P^{\text{land}} \times Q^{\text{land}}}{\text{income}}.$$

where P^{housing} denotes a house price index, Q^{housing} a quantity index for the housing stock, and the notation for structure and land is similar. To understand how housing wealth has been evolving, we look at empirical data that distinguishes between the value of structures and the value of land and between the respective price and quantity components. We also report further housing-related macroeconomic variables such as the land share and the rent.

We study the following four developed economies: USA (1950–2015), UK (1950–2012), France (1960–2012), and Germany (1960–2012). The underlying data, described in the appendix, is not new.⁸ However, we view the data from a specific angle by decomposing the increase in housing wealth into prices and quantities of residential land and structures. This list is helpful to both better understand the long-term evolution of housing wealth and to discipline a theory that aims at providing an explanation. The precise numbers behind all facts are summarized in table I and we elaborate on each of the five stylized facts in the following.

Fact 1: Wealth. The wealth-to-income ratio has been rising, as shown in figure I. This is well known from the work by [Piketty and Zucman \(2014\)](#). Decomposing the wealth-to-income ratio into housing and non-housing wealth-to-income ratios shows that the hous-

⁸We are grateful to Moritz Schularick, Luis Bauluz, and Filip Novokmet for sharing their data on housing wealth.

ing wealth-to-income ratio has been rising much stronger than the non-housing wealth-to-income ratio (see also figure I).⁹ This holds for all four economies and the difference is especially pronounced in the UK and France, as can be seen in table I.

Fact 2: Prices. Real house prices, real construction costs, and real land prices have all been increasing. This has been documented for the US by Davis and Heathcote (2007) and for a group of 14 advanced economies by Knoll *et al.* (2017). Note that the residential land price is typically not observed directly and is therefore usually determined as a residual. Real house prices almost doubled during the post-WWII period in the US and Germany while they even tripled and quintupled in the UK and France as shown in table I. Comparing the increase between prices for residential structures and residential lands reveals a striking pattern. Residential land prices exhibited very strong growth, while the prices of residential structures rose only modestly and house prices lie in between the two. We return to this difference after studying the next fact.

Fact 3: Quantities. By making use of the value indices and price indices for overall housing, residential land, and residential structures we are able to compute quantity indices. The quantity index of residential structures has been steadily increasing, while the quantity index of residential land increased only modestly and the housing index grew at a factor that lies in between the two. Taken together, facts 2 and 3 point to a mirror-like pattern. While the quantity of structures increased strongly, the price of structures increased only moderately. For residential land the reverse holds true, the price increased much stronger than the quantity. This observation points to an important role of the economy's supply side when it comes to understanding the stylized facts. Residential structure is reproducible, implying a higher price elasticity of supply, while residential land is non-reproducible in the long term, implying a lower price elasticity of supply. This asymmetry, it turns out, is the supply-sided characteristic behind this mirror-like pattern. This interpretation of the data guides our modeling choice and we will come back to it frequently.

Fact 4: Land. To understand why the housing wealth-to-income ratio has been increasing (fact 1) it is helpful to decompose it into a residential structure and residential land component as defined in (1). A variable that compactly summarizes the contribution of these two factors is the share of land in total housing wealth.¹⁰ As can be inferred from figure I this share increased considerably, ranging from a factor of 1.2 for Germany to a value of 3.8 for France. This shows that the increase in the housing wealth-to-income ratio is mainly driven by the increase in residential land wealth relative to income, while structure wealth remained relatively stable relative to income.

Fact 5: Rents. The house price – the price of the stock of housing – is closely related to the housing rent – the price of the service flow derived from the stock of housing. Similar to the house price, the real housing rent has also been increasing with a growth factor ranging

⁹The observation that housing wealth has been increasing stronger than non-housing wealth has also been stressed by Rognlie (2015) and Bonnet *et al.* (2019).

¹⁰Divide both sides of (1) by the housing wealth-to-income ratio to obtain $1 = \frac{\text{value of structures}}{\text{housing wealth}} + \frac{\text{value of land}}{\text{housing wealth}}$, where the second fraction on the RHS is the share of land in total housing wealth.

between 1.6 for Germany and 3.1 for the UK. Fact 5 has been stressed by [Knoll \(2017\)](#) and [Jordà et al. \(2019\)](#). A plausible theory that replicates the increase in the housing wealth-to-income ratio – fact 1 – shall also be in line with facts 2 to 5. We will come back to all facts below.

3. The model

Consider a perfectly competitive closed economy. Time is continuous and indexed by $t \in \mathbb{R}$. Infinitely-lived households earn labor and capital income, save, and consume two goods: housing and a non-housing good. We set the non-housing good as the numeraire. The two consumption goods are produced by two different sectors, a numeraire sector and a housing sector. The numeraire sector combines physical capital, labor, and land to produce an output good. The output of this sector can be consumed, invested into physical capital, or used as an input in the construction sector. The innovative part of our model concerns the housing sector. This sector encompasses three types of activities or subsectors. First, property management firms purchase residential land and residential structures to produce houses, which generate revenues from being rented to households. Second, construction firms manufacture structures by employing materials and labor. Third, real estate development firms develop residential land by purchasing non-developed land and employing labor. The overall supply of land is fixed and the allocation of land across the numeraire sector and the housing sector is endogenous. The economy grows in the long run due to exogenous technological change.

3.1. Households

The economy is inhabited by mass one of households. A household consists of $L_t > 0$ homogeneous members. Total population is hence equal to L_t . We allow L_t to grow over time. Households consume two goods, housing services, S_t , and a non-housing good, C_t . We denote all prices in units of the latter and use the terms *numeraire* and *non-housing good* interchangeably. The representative household derives utility from consumption streams $\{C_t, S_t\}_{t=0}^{\infty}$ according to

$$U = \int_0^{\infty} e^{-\rho t} L_t u\left(\frac{C_t}{L_t}, \frac{S_t}{L_t}\right) dt, \quad (2)$$

where $\rho > 0$ is the time preference rate and $u(C_t/L_t, S_t/L_t)$ the period utility function with per-capita consumption of the numeraire and per-capita housing services as arguments. Household utility at t is the product of household size, L_t , and per-capita utility, $u(C_t/L_t, S_t/L_t)$. The period utility function has the following form¹¹

$$u\left(\frac{C}{L}, \frac{S}{L}\right) = \frac{\left[\left(\frac{C}{L}\right)^{1-\theta} \left(\frac{S}{L}\right)^{\theta}\right]^{1-\sigma} - 1}{1-\sigma}, \quad (3)$$

¹¹We suppress the time index t whenever no confusion arises.

such that $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution and $\theta \in (0, 1)$ the housing expenditure share.

Each household member supplies one unit of working time inelastically to the labor market. Labor supply per household and aggregate labor supply are then equal to L . A household's total wealth is given by W and it generates a flow return of rW . The budget constraint reads

$$\dot{W} + C + qS = rW + wL + \Pi^D, \quad (4)$$

where q is the price of housing services and Π^D are profits from real estate development firms that are specified below.¹² Saving, \dot{W} , and total consumption expenditures have to equal total income which consists of wealth income, earnings, and profits.

3.2. Numeraire production

Mass one of firms produce the numeraire, Y , with inputs capital, K , labor, L^Y , and land, N , according to

$$Y = K^\alpha (B^Y L^Y)^\beta N^{1-\alpha-\beta}, \quad (5)$$

where $\alpha, \beta \in (0, 1)$ and $\alpha + \beta < 1$. The variable B^Y captures exogenous labor-augmenting technological change and it grows at the constant rate $g^Y \in \mathbb{R}$. Throughout the paper we will primarily employ Cobb-Douglas production functions. If we used more general CES functions, a steady state with differential technological growth would not exist. The representative firm solves the following problem

$$\max_{K, L^Y, N} \Pi^Y = K^\alpha (B^Y L^Y)^\beta N^{1-\alpha-\beta} - (r + \delta^K)K - wL^Y - R^N N, \quad (6)$$

where $\delta^K \geq 0$ is the capital depreciation rate and R^N the rental rate of non-residential land.

3.3. Housing

A house is a bundle of two conceptually different stocks: underlying land, D , and the residential structure, X , erected upon it. This distinction follows the empirical contribution by Davis and Heathcote (2007), who provide price and quantity indices for housing, land, and structures for the US and show that this distinction is key for understanding aggregate housing market dynamics. This distinction is also necessary for mapping model outcomes to the stylized facts presented in section 2. The main difference between D and X is that the former is not reproducible in the long-run while the latter is. Increasing demand for housing over time then implies that X can be accumulated while D cannot and that prices for X will increase much less than prices for D . This mirror-like pattern is key for explaining the stylized facts and we come back to it repeatedly.

The housing sector consists of three different subsectors.¹³ First, *property management*

¹²We model all households as renters, but the results do not change if we assume owner-occupied housing.

¹³We explain the different economic activities involved in the supply side of the housing sector as undertaken

firms demand residential structures and residential land to build houses which are then rented out to households. Second, *construction* firms demand labor and materials to produce new structures that are then sold to *property management* firms. Lastly, *real estate development* firms demand labor and non-developed land to produce residential land and sell it to *property management* firms. We now go through each of these activities in turn.

3.3.1. Property management

Property management firms combine residential structures, X , and residential land, D , to produce houses, H , according to

$$H = X^\gamma D^{1-\gamma}.$$

The stock of houses H generates a proportional flow of housing services equal to H .¹⁴ Housing services are sold to households at the rental rate q . Property management firms do not only manage an existing stock of housing, but they also invest in additional houses. They commission construction firms to produce I^X new units of structure at the price P^X and real estate development firms to produce I^D new units of residential land at the price P^D . The representative property management firm solves the following dynamic problem

$$\begin{aligned} \max_{\{I_t^D, I_t^X\}_{t=0}^\infty} & \int_0^\infty e^{-\tilde{r}_t} C F_t dt \\ \text{s.t. } & \dot{X}_t = I_t^X - \delta^X X_t \\ & \dot{D}_t = I_t^D, \end{aligned} \tag{7}$$

where

$$C F_t \equiv q_t X_t^\gamma D_t^{1-\gamma} - P_t^X I_t^X - P_t^D I_t^D \quad \text{and} \quad \tilde{r}_t \equiv \int_0^t r_\tau d\tau$$

are the firms' cash flow and the discount factor. The cash flow consists of revenues from renting out the existing housing stock, qH , minus investment expenditures. The law of motions of structures and land reflect that structures depreciate at the rate $\delta^X \in [0, 1]$ while land does not. This seems to be a plausible assumption because maintaining a constant stock of housing necessitates replacement investment into structures, but it is not necessary to produce additional land to keep the housing stock constant.

The property management firm's assets can i) either be expressed as the value of residential structures plus the value of residential land, $P^D D + P^X X$, or ii) as the value of houses, $P^H H$, where P^H is the house price. This equivalence reflects that houses are bundles of structure and land. Since the two have to be equal, $P^D D + P^X X = P^H H$, the resulting house price

by three different types of firms, but other institutional arrangements are equivalent. For example, one can equivalently assume that one firm sector engages in all three activities simultaneously.

¹⁴This is a common assumption in the literature. See, for instance, Favilukis *et al.* (2017); Greenwald and Guren (2019); Kaplan *et al.* (2019); Piazzesi and Schneider (2016).

reads

$$P^H = \frac{D}{H}P^D + \frac{X}{H}P^X. \quad (8)$$

The house prices is a weighted sum of the prices of residential land and residential structures. The weights are the land and structure intensity in housing, respectively.

Property management firms are owned by households. The aggregate valuation of property management firms is given by the value of its assets, $P^H H$. Without loss of generality we set the quantity of shares equal to H such that the share price equals P^H . This implies that the price of a house and the value of a share are both equal to P^H and households effectively trade houses, which they ultimately own. Owning one share entitles the owner to a yield of $\frac{CF + P^H \dot{H}}{H}$. The yield is expressed per unit of housing. It comprises of the operative cash flow, CF , plus the cash flow from issuing new shares, $P^H \dot{H}$.

3.3.2. Construction

The representative construction firm employs materials, M , and labor, L^X , to produce new structures according to

$$I^X = M^\eta (B^X L^X)^{1-\eta},$$

where $\eta \in [0, 1)$. The variable B^X captures exogenous technological progress in the construction sector and grows at the constant rate $g^X \in \mathbb{R}$. The representative firm chooses labor and materials to maximize profits according to

$$\max_{M, L^X} \Pi^X = P^X M^\eta (B^X L^X)^{1-\eta} - M - wL^X \quad \text{subject to} \quad M \geq 0, L^X \geq 0, \quad (9)$$

where P^X is the price of residential structures. Materials are intermediate goods produced with the same technology with which the numeraire good is produced. Therefore their price is equal to one.

3.3.3. Land

The economy's land endowment is denoted by Z . It is allocated between the numeraire sector and the housing sector such that $Z = N_t + D_t$ holds in equilibrium. In order to make land suitable for housing it has to be developed by real estate development firms. These firms purchase (or sell) \dot{N} units of non-developed land in order to increase (or decrease) the stock of developed land by \dot{D} units. We use the terms *residential land* and *developed land* interchangeably. Total land Z is constant such that the total amount of land that can be used economically is fixed. For studying housing in the long run this seems more plausible than the opposite – ever growing land supply.¹⁵ If land formerly used for producing the numeraire can be rededicated to housing without any cost, then N and D are jump variables that adjust

¹⁵This has been acknowledged by previous researchers who analyze land and long run economic growth, starting with [Ricardo \(1817\)](#) and [Nichols \(1970\)](#). The popular statement "buy land, they are not making it anymore", usually ascribed to Mark Twain, illustrates this point.

immediately. We do not impose this assumption and allow that rededicating land from one sector to the other might be costly. Real estate development firms take these cost into account when deciding whether to reallocate land between the sectors. The representative real estate development firm chooses investment into residential land $I^D \geq 0$ to maximize profits according to

$$\max_{I^D} \Pi^D = P^D I^D - \underbrace{[P^N I^D + wL^D(I^D)]}_{\text{cost function}}, \quad (10)$$

where P^D and P^N are prices of developed land and non-developed land, respectively, and $wL^D(I^D)$ are labor cost associated with the reallocation of land. We assume that this cost function takes the form

$$L^D = L^D(I^D) = \frac{\xi}{2} (I^D)^2, \quad (11)$$

where $\xi \geq 0$ captures the importance of adjustment cost. These cost arise from activities like pulling down an existing building, leveling the surface off, and installing utilities like sewage, water, electricity, or roads. We derive the cost function in (10) from a decreasing returns to scale production function with perfect complements in the appendix. Whether ξ is zero or strictly positive determines whether adjustment cost exist or not, as we explain now in greater detail.

If $\xi = 0$, the reallocation of land is costless. Since the overall land endowment is fixed in equilibrium it has to hold that land prices P^D and P^N are always equal.¹⁶ The land allocation would be a jump variable as N and D adjust immediately. For example, during a transition D might increase by 20 percent in an instant of time. Although the model allows for this immediate rededication of land we show in the calibration section that this is empirically implausible.

If $\xi > 0$, the reallocation of land is costly and firms will choose an interior optimum, which exists and is unique because the cost function is convex. In this case I^D is smaller than with $\xi = 0$, the reallocation of land is stretched out over several periods, and N and D are state variables that change slowly over time. The FOC for (10) reads

$$I^D = \frac{P^D - P^N}{\xi w}. \quad (12)$$

In equilibrium the adjustment cost are $wL^D = \frac{(P^D - P^N)^2}{2\xi w}$ and profits can be expressed as $\Pi^D = P^D I^D - wL^D$. The price difference between residential and non-residential land, $P^D - P^N$, determines whether firms develop residential land and how much they develop. First, if residential land is more valuable than non-residential land, firms engage in real estate development and $I^D > 0$. Profits, which are strictly positive, are redistributed to households who own real estate development firms. Second, if land prices are equal, firms do not reallocate land

¹⁶If P^D was larger than P^N , firms would transfer as much land as possible from the numeraire to the housing sector, demanding more and more N and supplying more and more D . The price for land in the numeraire sector would increase and the price for residential land would decrease until they are equal. Hence, P^D cannot be larger than P^N . The same reasoning applies for $P^D < P^N$.

and profits as well as adjustment cost are zero. Lastly, real estate development firms do not only develop land for real estate, but conduct also the reverse activity when the price of non-residential land is higher than the residential land price. In this case I^D is negative and land is reallocated from the housing to the numeraire sector. Profits and adjustment cost are also positive in this case.¹⁷

The real estate development sector becomes inactive in the long run. Since total land endowment is fixed there will be no land reallocation and therefore land prices will equalize such that $L^D = I^D = \Pi^D = 0$. Hence, the real estate development sector matters only for transitional dynamics.

3.4. Assets

The assets in this economy are i) capital, K , ii) non-residential land, N , and iii) houses, H . All assets are ultimately owned by households as can be seen in the decomposition of total household wealth

$$W_t = \underbrace{P_t^H H_t}_{\text{housing wealth}} + \underbrace{K_t + P_t^N N_t}_{\text{non-housing wealth}}. \quad (13)$$

Total wealth consists of housing wealth and non-housing wealth where the latter is the sum of capital and non-developed land. Because markets are complete and the economy is deterministic, all assets yield the same rate of return, r , in equilibrium¹⁸

$$r = \frac{\dot{P}^N + R^N}{P^N} = \frac{\dot{P}^H + \frac{CF + P^H \dot{H}}{H}}{P^H}. \quad (14)$$

Households are indifferent with regards to the allocation of total wealth, W , across the different assets and care only about total wealth W and the rate of return r . It is therefore sufficient to consider only total wealth, W , and r instead of all assets in the household problem.

3.5. Equilibrium

We formally define the competitive equilibrium next.

Definition 1 (Competitive equilibrium). *A competitive equilibrium of the model are sequences $\{C_t, S_t, W_t, q_t, w_t, r_t, \Pi_t^D, Y_t, K_t, L_t^Y, N_t, R_t^N, H_t, X_t, D_t, P_t^X, CF_t, I_t^D, L_t^D, P_t^D, P_t^N, P_t^H, I_t^X, M_t, L_t^X\}_{t=0}^\infty$ for given initial capital, residential land, and residential structures $\{K_0, D_0, X_0\}$ and a population sequence $\{L_t > 0\}_{t=0}^\infty$ such that*

- i) households maximize (2) given (3) subject to (4) and a NPG condition;

¹⁷Our specification of the adjustment cost function $L^D(\cdot)$ has the property that $L^D > 0$ for $I^D \geq 0$ and therefore an interior solution exists also for negative investment in residential land.

¹⁸If the rate of return on an asset deviated from r , households would demand either zero or an infinite amount of the asset and markets would not clear.

- ii) firms in the construction sector and the numeraire sector, real estate developers, and property management firms maximize profits as given by (6), (7), (9) and (10), taking (11) and prices as given;
 - iii) rental market clears: $H_t = S_t$;
 - iv) labor market clears: $L_t^X + L_t^Y + L_t^D = L_t$;
 - v) land market clears: $D_t + N_t = Z$;
 - vi) asset markets clear (13);
 - vii) the house price is given by (8);
 - viii) there are no arbitrage opportunities between assets, as described by (14);
- The market for the numeraire good clears by Walras' law.

3.6. Housing wealth-to-income ratio

In order to express the housing wealth-to-income ratio we need first to define the economy's *income* consistently with the empirical convention of using the net national product (Piketty and Zucman, 2014; Rognlie, 2015). Aggregate income is denoted by NNP and given by

$$NNP = \underbrace{\overbrace{Y}^{\text{numeraire}} + \overbrace{qS}^{\text{housing services}} + \overbrace{P^X I^X - M}^{\text{construction}} + \overbrace{(P^D - P^N) I^D}^{\text{real estate development}} - \overbrace{(\delta^K K + \delta^X P^X X)}^{\text{depreciation}}}_{\text{GDP}}.$$

Since the economy comprises four production sectors, GDP equals the sum of the value-added of these sectors. In order to avoid double accounting, materials M have to be subtracted from output of the construction sector because they already are accounted for by Y . Value added from the real estate development sector is always positive because I^D has the same sign as the land price difference $P^D - P^N$ such that $I^D \times (P^D - P^N) > 0$. Rededicating land from the Y -sector to the S -sector enables real estate development firms to create value added. This equals the value of newly developed land, $P^D I^D$, reduced by the costs for the purchase of non-developed land, $P^N I^D$. Subtracting depreciation from GDP yields net domestic product. Net domestic product is equal to the national product (NNP) because the model economy is closed. The housing wealth-to-income ratio is then – in line with the empirical measurement of section 2 – given by

$$\frac{P^H H}{NNP} = \frac{P^D D}{NNP} + \frac{P^X X}{NNP}. \quad (15)$$

The total wealth-to-income ratio and the non-housing wealth-to-income ratio are defined similarly. Equation (15) shows that the housing wealth-to-income ratio can be decomposed into the residential land wealth-to-income ratio and the residential structure wealth-to-income ratio. This is identical to equation (1) which we used to decompose the empirical housing wealth-to-income ratio. The model hence allows to directly map total wealth, prices, and quantities of houses, residential land, and residential structures into the data.

3.7. Relation to existing theories

We now briefly explain how the model differs from existing macroeconomic models with a housing sector. The model distinguishes two endogenous stocks, namely residential land and residential structures. Both stocks are essential input factors in the production of housing services and both stocks represent the two components of housing wealth. Existing macro models with housing assume that each period one unit of land becomes available and is incorporated into the housing stock by the construction sector.¹⁹ We depart from this model structure because it would be ill-suited to investigate the evolution of housing wealth and its components over the long term. Recall that we decompose housing wealth into four components: the price and quantity components of residential land and the price and quantity components of structures. Assuming that the quantity of residential land is exogenous would restrict the analysis of the long-term evolution of residential land with strong implications for the residential land price.

Moreover, most existing macro models with housing assume that the numeraire sector does not employ land. In the context of our research question, this would be a further restrictive assumption. Below we show that long-term growth in rents and house prices is driven by differential technological change and differential land intensities. Assuming land is not productive in the numeraire sector implies a strong and restrictive assumption with regard to relative land intensities. This assumption would impose a potentially important constraint on the numerical analysis.²⁰

4. Steady state

Can rising housing wealth-to-income ratios and the other stylized facts be explained in steady state of the model economy? In order to answer this question, we study the economy's steady state analytically in section 4.1 and provide the intuition by employing simple demand and supply diagrams in the subsequent section 4.2.

4.1. Analytical results

We define a steady state as a competitive equilibrium according to definition 1 where all variables grow at constant and possibly different rates. From now on we assume that population, described by the sequence $\{L_t > 0\}_{t=0}^{\infty}$, converges to a finite upper bound as time approaches infinity.

¹⁹See, for instance, [Davis and Heathcote \(2005\)](#); [Favilukis et al. \(2017\)](#); [Greenwald and Guren \(2019\)](#); [Kaplan et al. \(2019\)](#).

²⁰Finally, the existing macro model with housing would be ill-suited for a long-term analysis of housing wealth. Even in an economy without long run economic growth the stock of accumulated residential land would tend to infinity as time approaches infinity. In a growing economy the accumulated amount of land would grow to infinity at an even higher rate. The reason is that existing macro models with housing assume that replacement investment requires land.

Variables	Growth rate
$r, \frac{P^H_H}{NNP}, \frac{W}{NNP}, \frac{P^D_D}{P^H_H}, L^D, L^X, L^Y, N, D$	0
$Y, K, M, w, R^N, P^N, P^D, C, NNP, W$	$\frac{\beta}{1-\alpha} g^Y$
X, I^X	$\eta \frac{\beta}{1-\alpha} g^Y + (1-\eta) g^X$
P^X	$(1-\eta) \left(\frac{\beta}{1-\alpha} g^Y - g^X \right)$
S, H	$\gamma \eta \frac{\beta}{1-\alpha} g^Y + \gamma(1-\eta) g^X$
q, P^H	$(1-\gamma\eta) \frac{\beta}{1-\alpha} g^Y - \gamma\eta g^X$

Table II: Steady state growth rates

Proposition 1 (Steady state). *A unique steady state equilibrium exists and variables grow at the rates shown in table II.*

All proofs are relegated to the appendix where we also derive a closed-form solution of the stationary steady state in detrended variables. The source of long run growth in this economy is exogenous technological progress in the numeraire and in the construction sector as described by the growth rates g^Y and g^X . Growth rates of model variables are therefore functions of g^Y , g^X , and other parameters, as shown in table II. We first study the growth rate of house prices and rents before returning to other variables further below. Why are house prices and rents growing in the long-run? The answer is provided by

Proposition 2 (Differential land intensities and differential technological progress). *The steady state growth rate of house prices, P^H , and rents, q , can be expressed as a function of land intensities and technological growth rates as follows*

$$g_{P^H} = g_q = \left[1 - \eta + \eta(\psi^H - \psi^Y) \right] \frac{1 - \psi^Y}{1 - \eta\psi^Y} g^Y - (1 - \eta) \frac{1 - \psi^H}{1 - \eta\psi^Y} g^X,$$

where $\psi^Y \equiv \frac{1-\alpha-\beta}{1-\alpha} \in [0, 1)$ and $\psi^H \equiv \psi^Y \eta \gamma + 1 - \gamma \in [0, 1]$ are the land intensities in the production of the numeraire good and housing, respectively.

- i) If there is no differential technological progress, $g^Y = g^X$, then the growth rate of house prices and rents simplifies to $(\psi^H - \psi^Y) g^Y$. Assuming $g^Y > 0$, the growth rate is positive (negative) iff the land intensity is larger (smaller) in the housing sector than in the numeraire sector, $\psi^H > \psi^Y$ ($\psi^H < \psi^Y$).
- ii) If land intensities are equal, $\psi^Y = \psi^H$, then the growth rate of house prices and rents simplifies to $g_{P^H} = g_q = \frac{(1-\eta)(1-\psi^Y)}{1-\eta\psi^Y} (g^Y - g^X)$. The growth rate is positive (negative) iff technological progress is stronger (weaker) in the numeraire sector than in the construction sector, $g^Y > g^X$ ($g^Y < g^X$).

In a growing economy, the two drivers for a long-term increase in house prices and rents are differential land intensities and differential technological progress. The land intensities of the numeraire and housing sector are denoted by ψ^Y and ψ^H , respectively, and

are derived in the appendix. The land intensity is measured by an elasticity of a reduced-form production function that states by how much – in proportional terms – output of Y (or H) increases if the quantity of employed land is increased. Hence, if $\psi^H > \psi^Y$ for example, then an increase in the quantity of land employed in both sectors increases output of the housing sector by more than it increases output of the numeraire sector. Each of the two mechanisms – differential technological progress and differential land intensities – is sufficient to drive house prices in isolation, but they also reinforce each other.

The economic intuition behind proposition 2 is best understood through the lens of the Rybczynski (1955) theorem.²¹ Consider first part i). Assume that the land intensity of the housing sector is larger than the land intensity of the numeraire sector, $\psi^H > \psi^Y$, but technological change is the same in both sectors, $g^Y = g^X > 0$. Symmetric technological change means that effective labor increases in both sectors – holding prices constant and abstracting from labor reallocations – by the same proportion. The assumption $\psi^H > \psi^Y$ together with constant returns to scale implies a higher labor intensity in numeraire production.²² Hence, the direct effect of technological change is that output in the numeraire sector expands by more than in the housing sector. To restore equilibrium in the goods market, the relative price of the numeraire good must fall. That is, the price of housing services in units of the numeraire good, q , increases.²³

Consider next part ii) of proposition 2. Assume symmetric land intensities, $\psi^H = \psi^Y$, but differential technological change, $g^Y > g^X > 0$.²⁴ The direct effect of differential technological change to the advantage of the numeraire sector – or equivalently disadvantage of the housing sector – is a relatively stronger output expansion of the numeraire sector resulting from the stronger increase in efficient units of labor. Again, the relative price of the numeraire good must fall to restore equilibrium in the goods market. The price of housing services in units of the numeraire good, q , increases. Because the house price equals the present discounted value (PDV) of rental yields and noting that the interest rate is constant in equilibrium, the house price increases as well.

What about the quantitative importance of the two channels – differential land intensities and differential technological growth – for rising house prices? To assess their relative importance we conduct a back-of-the-envelope calculation. Assume the US has been growing along its steady state path over the period 1950–2015. Recall from proposition 2 that

²¹The Rybczynski (1955) theorem focuses on production effects in response to changes in factor endowments under differing factor intensities (Mas-Colell *et al.*, 1995). This is not exactly the case we consider here. The two-step reasoning is nonetheless helpful in the sense that one can first study relative output effects for given prices and then determine the price changes required to restore equilibrium.

²²Notice that the reduced-form production functions exhibit constant returns to scale in labor and land.

²³A slightly different way to describe the result is as follows. Consider the reduced-form production functions in steady state, (23) and (24), and notice that i) technological change is labor augmenting (not land augmenting), ii) the land intensity (measured by partial production elasticities) of the housing sector is greater, and iii) the production functions exhibit CRS. Hence, it is straightforward to see that technological change enlarges output of the numeraire sector by more. Stated differently, the production cost of the numeraire good declines faster. That is, the supply side triggers an ongoing surge in the relative price q .

²⁴This mechanism is related to the Findlay-Grubert theorem, which considers output and price effects of differential technological change.

the steady state growth rate of house prices depends on five parameters: g^Y , g^X , ψ^Y , ψ^H , and η . We take $\eta = 0.556$ and $\psi^Y = 0.15$ from the main calibration (section 5), and conduct a little robustness check reported below. Moreover, we set g^Y , g^X , and ψ^H such that we match the average annual growth rates of 2.8, 1, and 0.3 percent for NNP, the house price, and the price of residential structures (cf. to table A.1). This gives 0.03, 0.02, and 0.34 for g^Y , g^X and ψ^H , respectively. Hence, the difference in sector specific technological change is $g^Y - g^X = 0.01$ and the difference in land intensities reads $\psi^H - \psi^Y = 0.19$. In a first step we shut down the differences in land intensities by setting ψ^H equal to the value of $\psi^Y = 0.15$. The growth rate of house prices then reduces to 0.52 instead of previously 1.00 percent. In a second step we additionally shut down the differences in technological progress by setting g^X to g^Y . The growth rate of house prices then declines to 0. Taken together, differences in land intensities capture 52 percent of the long term increase in house prices while differences in sector specific technological change capture the remaining 48 percent.²⁵ We obtain similar results for the UK, France, and Germany. For instance, in the UK differences in land intensities explain 41 percent of the long-run growth in house prices. This simple exercise shows that both differences in land intensities and differential technological progress are about equally important for rising house prices in the long run.

Proposition 2 is related to existing theoretical explanations for rising house prices in the long run. [Borri and Reichlin \(2018\)](#) study the effect of differential technological progress on house prices, while [Miles and Sefton \(2020\)](#) study how overall economic growth and a fixed long-run supply of land affect house prices. Both assume that land does not enter the production function of the non-housing good. This corresponds, through the lens of proposition 2, to setting the land intensity of the non-housing sector to zero, $\psi^Y = 0$. If we had set ψ^Y equal to zero, then it would be easier to generate rising house prices and differential technological progress would play a smaller role. However, we generalize and allow ψ^Y also to take on strictly positive values. Only then it is possible to analytically show how differences in land intensities affect the long-run growth rate of house prices, as done in proposition 2. It is also not plausible that output of the non-housing good is not affected by changes in the input of land, which is supported by our calibration in section 5 where ψ^Y is endogenously calibrated to 0.15. Setting ψ^Y to zero would therefore underestimate the relevance of differential technological progress. Most importantly, however, by generalizing and isolating the two mechanisms – differential land intensities and differential technological progress – we could assess their relative importance.

Although we focus on the post-WWII period, our theory can also speak to earlier periods. [Knoll et al. \(2017\)](#) show that house prices and residential structure prices were largely constant before the 1950s and started increasing only afterwards. If we study the steady state of our economy and impose both these prices do not grow, then we obtain the restric-

²⁵We further run two robustness checks. First, setting ψ^Y to the value of $\psi^H = 0.34$ yields very similar results. Second, if we set η to 0 (0.99), then differential land intensities explain a third (more than two third) of the overall growth rate of house prices while setting ψ^Y to 0 (0.3) implies that differential land intensities explain 70 (30) percent of the rise in house prices.

tion that i) $g^X = (1 - \psi^Y) g^Y$ and ii) $\psi^H = \eta \psi^Y$. The first restriction implies that before the 1950s technological progress in the construction sector was much stronger than in the second half of the twentieth century where $g^X < (1 - \psi^Y) g^Y$ has to hold. This seems plausible as construction costs were likely held down by technological advances such as the invention of the steel frame at the beginning of the 20th century. The second restriction implies that land was more relevant in the numeraire sector than in the housing sector before 1950, that is, $\psi^H < \psi^Y$. Although this is beyond the scope of this model, changing land intensities may reflect structural change in the sense of the Kuznets hypothesis (Kongsamut *et al.*, 2001). After 1950 an increasing contribution to aggregate economic growth came from the service sector, which is less land-intensive than the manufacturing and the agricultural sectors. All these sectors are subsumed in the numeraire sector of our model and therefore structural change can – in a very reduced form – be thought of as a decrease in ψ^Y . Our theory therefore points to an explanation of the constancy in house prices before 1950 and the take-off thereafter that rests on i) strong technological growth in the construction sector and ii) a high land-intensity in the numeraire sector.

Can the model replicate the other stylized facts, besides rising house prices and rents, in steady state? We provide the answer now in

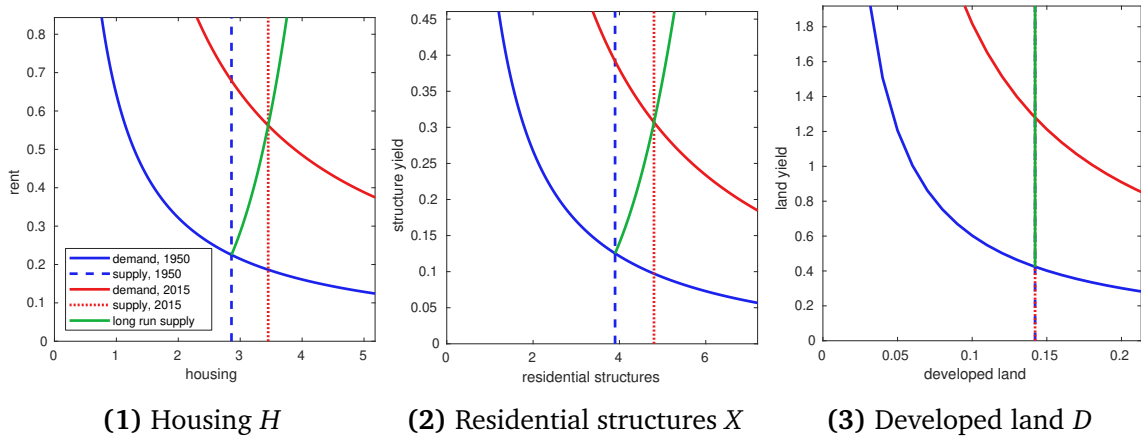
Corollary 2.1 (Stylized facts in steady state). *In the economy's steady state equilibrium*

- #1) *the wealth-to-income ratio, the housing wealth-to-income ratio, and the non-housing wealth-to-income ratio are constant,*
- #2) *prices of residential land, houses, and residential structures grow at strictly positive rates with residential land prices growing at a higher rate than house prices, which in turn grow at a high rate than residential structure prices iff $g^Y > 0$ and $-\frac{\eta}{1-\eta}(1 - \psi^Y) g^Y < g^X < (1 - \psi^Y) g^Y$,*
- #3) *quantities of residential structures grow at a strictly positive rate which is larger than the growth rate of residential land iff $g^X > -\frac{\eta}{1-\eta}(1 - \psi^Y) g^Y$,*
- #4) *the land share, given by $\frac{p^D D}{p^H H}$, is constant, and*
- #5) *rents grow at a strictly positive rate if and only if $g^X < \frac{1-\psi^Y}{1-\psi^H} \frac{1-\eta+\eta(\psi^H-\psi^Y)}{1-\eta} g^Y$ or if $g^X < (1 - \psi^Y) g^Y$.*

Through the lens of our model most of the stylized facts can be explained as steady state phenomena. However, rising housing wealth-to-income ratios as well as a rising land share cannot be explained in steady state. This is not an implication specific to our model, but a very common implication of macroeconomic models.²⁶ In section 6 we will therefore study transitions and show what the main drivers for rising housing wealth-to-income ratios are.

The conditions for the other stylized facts being satisfied in steady state depend crucially on differential land intensities and differential technological progress. First, it is necessary that technological progress in the construction sector is lagging behind technological

²⁶In a one-sectoral economy it intuitively results from the assumption of a constant returns to scale production function which implies that the capital-output ratio is constant.



Notes: Parameter values are from our baseline calibration in section 5. The economy is in steady state and L_t is set to its long-term value. Shift in demand and supply curves result only from long-run growth, but not from transitional dynamics.

Figure II: Demand-supply representation

progress in the rest of the economy. Otherwise the price of residential structures would be declining in steady state. The observation of – although modestly – rising construction cost (see section 2) therefore indicates that technological progress in the construction sector was lagging behind technological progress in the rest of the economy. Second, the larger the land intensity in the housing sector and the smaller the land intensity in the numeraire sector, the more likely it is that the restrictions in corollary 2.1 are satisfied. Hence, relatively weak technological progress in the construction sector and a relatively high land intensity in the housing sector can together explain many of the stylized facts already as steady state phenomena.

4.2. Intuition for mirror-like behavior of prices and quantities

In corollary 2.1 we show under which conditions residential land prices grow at a higher rate than residential structure prices and the quantity of residential structures grows at a higher rate than residential land in steady state. We now explain the intuition behind this mirror-like behavior.

In figure II we plot demand and supply curves for three markets, i) the housing market, ii) the underlying market for residential structures, and iii) the underlying market for developed land.²⁷ For now, we assume that the economy evolves along its steady state and focus on the years 1950 and 2015.

The housing market is depicted in panel (1) of figure II. The (short-run) supply of housing is fixed because housing cannot change instantly. Households demand housing services according to a downward sloping demand schedule. It captures only the direct effect of changes in rents, q , on short run (within-period) housing demand. Since the economy is growing over time, household demand more housing services in 2015 and the demand curve shifts outwards. If supply did not change, rents would increase strongly. The supply

²⁷We derive the analytical expressions behind the demand and supply curves in appendix A.5.

curve can only shift outwards if the stock of residential structures or developed land or a combination of both increases over time. This leads us directly to the market for structures and land. The market for structures is depicted in panel (2) of figure II. The supply schedule is inelastic in the short run and the demand schedule is downward sloping. An increase in housing demand shifts the demand curve for structures outwards. At the same time construction firms anticipate the economy's future evolution, as reflected in the price P^X , and increase the stock of residential structures over time. The faster technological progress in the construction sector, the stronger is this increase in structures. The result is an outward shift in the supply curve of residential structures and a weaker increase in the structures' yield component. In the market for developed land the demand curves behave similarly, but supply cannot increase because land is fixed in the long run. The rental yield for land increases much more than the rental yield for structures, while the quantity of structures increases much more than the quantity of residential land. This mirror-like image is hence the result of rising housing demand that meets relatively inelastic long-run supply of structures and fully inelastic long-run supply of land. The induced changes on the X and D market translate back into the housing market as follows. Houses are bundles of structure and land. Structures increase, but land remains constant and therefore the quantity of housing increases between 1950 and 2015, but less than the quantity of structures. As a result rents increase between 1950 and 2015.²⁸

5. Calibration

We calibrate the model to all four economies – US, UK, France, and Germany – separately on an annual frequency for the post-WWII era. Due to better data availability and quality, we focus on the US in the main text. The calibration of the other economies is documented in appendix B. Although the model economy converges to a steady state, we do not impose that any of the four economies was in steady state since WWII. That is, the model is calibrated out of steady state.

5.1. Calibrated outside the model

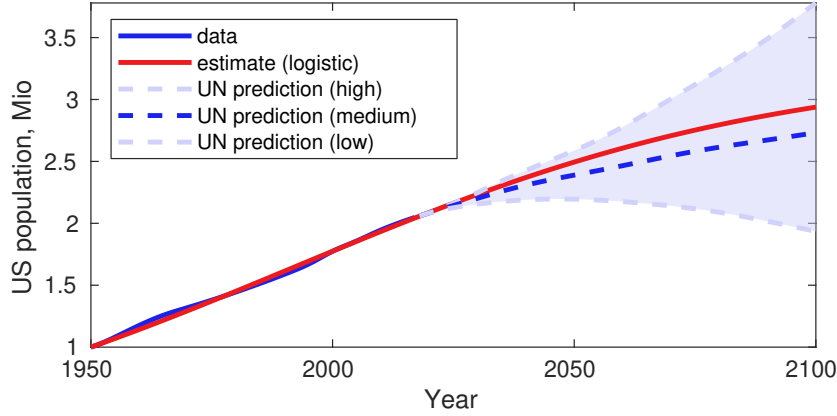
The set of parameters $\{\theta, \sigma, \delta^K, \delta^X, \beta, \{L_t\}_{t=0}^\infty\}$ is calibrated without solving the model numerically. Table III summarizes the calibration. The preference parameter θ equals the aggregate share of housing expenditures in total consumption expenditures. According to data from the National Income and Product Accounts (NIPA) this value is on average 18 percent for the post-WWII US.²⁹ The intertemporal elasticity of substitution (IES) in composite consumption, $C^{1-\theta} S^\theta$, is $1/\sigma$. In a meta-study Havránek (2015) shows that mean-estimates based on micro-studies and asset holders are in the range of 0.3 to 0.4. Best *et al.* (2019)

²⁸The long-run supply curve of housing would be horizontal and rents would not increase at all i) if technological progress in the construction sector were sufficiently strong, ii) if the land intensity in the housing sector were sufficiently low, or iii) if residential land would grow sufficiently fast over time. See proposition 2.

²⁹See also Piazzesi and Schneider (2016, section 2.1).

Parameter	Value	Explanation/Target
θ	0.18	housing expenditure share
σ	10/3	intertemporal elasticity of substitution
δ^K	$\ln(1 + 0.056)$	capital depreciation rate
δ^X	$\ln(1 + 0.015)$	structure depreciation rate
β	0.613	labor income share in Y sector
$\{L_t\}_{t=0}^{\infty}$	logistic differential equation (see figure III)	population dynamics

Table III: Calibrated outside of the model



Notes: Population data is from [UN \(2019\)](#).

Figure III: US population dynamics

estimate an aggregate IES of 0.1 using quasi-experimental variation in UK interest rates. We therefore set $\sigma = 10/3$ to match the lower bound of 0.3 from [Havránek \(2015\)](#). The depreciation rates of capital and structures are set to annual values of 5.6 percent ([Davis and Heathcote, 2005](#)) and 1.5 percent ([Hornstein, 2009](#), p. 13), respectively.³⁰

The parameter β equals the labor income share of the non-housing sector:

$$\frac{wL^Y}{GDP - [wL^X + 2wL^D + qS]} = \frac{wL^Y}{Y} = \beta.$$

We use data from the US Bureau of Economic Analysis (BEA) which is available from 1997 to 2018.³¹ The non-housing labor share is relatively stable over time, declining only by 1.2 percentage points since 2008. The average is 61.3 percent and we hence set $\beta = 0.613$.

Lastly, the exogenous population sequence $\{L_t\}_{t=0}^{\infty}$ is calibrated as follows. We assume that the population will not grow forever and approaches a constant as $t \rightarrow \infty$. Initial population is normalized to $L_0 = 1$, which has no effect on the moments we study. Future

³⁰Our value for δ^X is also very close to the one chosen by [Davis and Heathcote \(2005\)](#), 1.57, and lies in the range documented by [Tuzel \(2010\)](#), 1.5 to 3.

³¹We use the components of value added by industry from <https://apps.bea.gov/iTable/iTable.cfm?ReqID=51&step=1>, accessed on March 8, 2020.

	Moment	Data	Model	relative difference in %
1	$\frac{W}{NNP}$, in 1950	3.6	3.6	0.000
2	$\frac{NNP_{2015}}{NNP_{1950}}$	6.0	6.0	0.000
3	$\frac{P^H_H}{NNP}$, in 1950	1.2	1.2	0.000
4	$\frac{P^D_D}{P^H_H} \times 100$, in 1950	11.8	11.8	0.000
5	$\frac{L^X}{L} \times 100$, long-run	2.5	2.5	0.000
6	$\frac{q_{2015}}{q_{1950}}$	1.7	1.7	0.000
7	$\frac{RESI}{GDP} \times 100$, in 1950	5.6	5.6	0.000
8	$\frac{R^N_N}{NNP} \times 100$, in 1950	10.0	10.0	0.000
9	Half-life of D , in years	22.6	22.6	0.000
10	$\frac{X_{2015}/D_{2015}}{X_{1950}/D_{1950}}$	1.9	1.9	0.000

Table IV: Targeted data and model moments

population dynamics are projected with a simple logistic differential equation

$$\dot{L}_t = bL_t \frac{a - L_t}{a} \Leftrightarrow L_t = \frac{a}{1 + \left(\frac{a}{L_0} - 1\right)e^{-bt}}, \quad (16)$$

where $b \geq 0$ governs the speed of convergence and $a > 0$ is the finite limit of L_t when $t \rightarrow \infty$. The two coefficients are determined by minimizing the unweighted sum of squared residuals between the US population levels from 1950 to 2017 and the values predicted by the logistic equation where $t = 0$ corresponds to 1950. We obtain $a_L = 3.2$ and $b = 0.02$. The result is depicted in figure III. Our simple prediction is very close to the medium scenario prediction by the UN (2019) and lies between its high and low scenario prediction.

5.2. Calibrated jointly inside the model

The remaining 7 parameters $\alpha, \gamma, \eta, \rho, \xi, g^Y$, and g^X as well as the 3 initial states K_0, D_0 and X_0 are calibrated by solving the transition many times for different parameter values until a set of 10 empirical moments is matched. Although all parameters jointly determine the theoretical moments, we explain in the following each parameter in combination with the moment that it most strongly affects.

The time preference rate ρ is chosen to match the level of the total wealth-to-income ratio of 3.6 (see section 2) at the beginning of the transition, in 1950. Similarly, we set the initial capital stock, K_0 , such that we match the level of the housing wealth-to-income ratio of 1.2 in 1950. The technology growth rate of the non-housing sector, g^Y , is set to match the increase in US national income between 1950 and 2015. Taken together, these three moments imply that we match the levels of the total wealth-to-income ratio and the housing

wealth-to-income ratio, as well as the growth in their denominator. The dynamics in total wealth and housing wealth, however, are not targeted by our calibration. By matching the growth in national income we rule out that the model replicates increases in wealth-to-income ratios by implausible dynamics in income.

The parameter γ is the output elasticity of structures in the production of houses. The larger γ , the more relevant are structures, and the less relevant is land in the production of houses. This parameter strongly affects the share of residential land wealth in total housing wealth (land share), i.e. the smaller γ the larger the land share. We therefore calibrate γ by matching the land share's level of 11.8 percent in 1950. Similarly, the parameter η is the output elasticity of materials in the construction sector. The smaller η , the smaller is the demand for materials, and the larger is the demand for labor in the construction sector. We hence set η such that we match the long-run employment share in the residential construction sector of 2.5 percent.³² We target the long-run value of the employment share in residential construction because data is only available for more recent periods, but not for 1950.

We know from proposition 1 that the technological growth rate of the construction sector, g^X , strongly affects the long-run growth rate of rents. We therefore target the growth factor of rents between 1950 and 2015 of 1.7 (see section 2). The initial stock of residential structures, X_0 , is chosen to match the share of residential investment in GDP of 5.6 percent in 1950 as taken from Federal Reserve Economic Data (FRED). In the model residential investment is defined as follows

$$RESI \equiv P^X I^X + P^D \dot{D} + wL^D, \quad (17)$$

where we included investment in residential land because the empirical data does not differentiate between investment into land and structures. The output elasticity of capital in the numeraire sector, α , is set to match the net income share of non-developed land, $R^D D / NNP$ of 10 percent in 1950.³³ This is related to α because the income share of non-developed land in output of the numeraire sector is given by $1 - \alpha - \beta$. The parameter ξ governs the speed of convergence in developed land, D . We set it such that we match the half-life in the transition of residential land of 22.6 years.³⁴ Lastly, the initial stock of residential land, D_0 , is chosen such that we match the growth factor of residential structures relative to residential land of 1.9 between 1950 and 2015, as given by section 2. The targeted moments are shown in table IV and the resulting parameter values in table V.

³²The employment share in residential construction is obtained by multiplying the employment share in the overall construction sector with the output share of residential construction in total construction. The data is from the National Association of Home Builders at <https://www.nahbclassic.org/generic.aspx?sectionID=734&genericContentID=261286> and <http://admin.nahb.org/generic.aspx?sectionID=734&genericContentID=178434&channelID=311> and was accessed on Dec 16 2019. It is very stable since 2010 at a value of 2.5 percent.

³³The data is taken from Rognlie (2015, figure 11).

³⁴The series for residential land, D , is from section 2. We assume that D follows a logistic differential equation and estimate the respective coefficients by minimizing the sum of squared residuals between data and model.

#	Parameter	Explanation	Value
1	ρ	time preference rate	0.040
2	g^Y	technical growth in numeraire sector	0.020
3	K_0/\tilde{K}	initial capital stock	0.227
4	γ	structures' elasticity in H	0.906
5	η	materials' elasticity in I^X	0.556
6	g^X	technical growth in construction sector	-0.014
7	X_0/\tilde{X}	initial stock of residential structures	0.191
8	α	capital elasticity in Y	0.275
9	ξ	intensity of convex adjustment cost in land development	759.05
10	D_0/\tilde{D}	initial stock of residential land	0.403

Notes: Initial states, K_0, D_0, X_0 , are expressed relative to their respective final steady state values.

Table V: Endogenously calibrated parameters

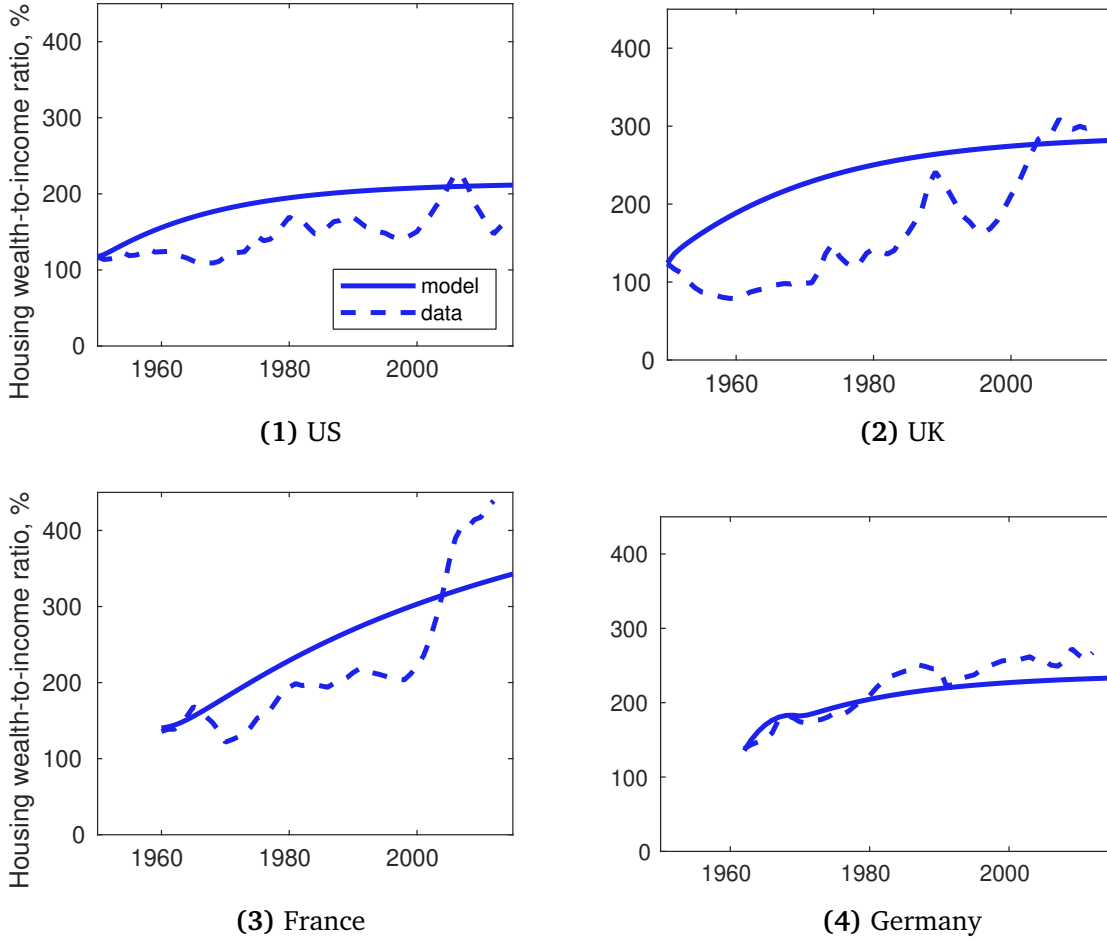
6. Transitional dynamics

6.1. Housing wealth-to-income ratio

The main result of the quantitative part is shown in figure IV where we plot the housing wealth-to-income ratio since 1950 for the US, UK, France, and Germany. The level of the housing wealth-to-income ratio at the start of the transition is matched by construction because we target it in the calibration. The evolution of the housing wealth-to-income ratio over time, however, is not targeted. The calibrated model replicates a rising housing wealth-to-income ratio for all four economies. For the UK, France, and Germany the calibrated model explains 80, 67, and 76 percent of the increase in the housing wealth-to-income ratio between the initial year (1950, 1960, 1962) and 2012, respectively. In the case of the US it over-predicts the increase in the housing wealth-to-income ratio slightly by 35 percentage points. This might be due to the fact that in 2015 the US had not yet fully recovered from its recent housing bust.

Why is the housing wealth-to-income ratio increasing over time? This is not trivial as the housing wealth-to-income ratio is stable in steady state and changes only along the transition. Technically speaking, transitional dynamics result from deviations of initial capital, developed land, and residential structures – K_0 , D_0 , and X_0 – from their respective steady state values.³⁵ To better understand the relative importance of the components associated with the different state variables, we conduct the following experiment. For each initial state – K_0 , D_0 , and X_0 – we compute a counterfactual transition where the respective initial state starts in its steady state value. This way we are shutting down the contribution that results from transitional dynamics triggered by the difference of a respective initial state

³⁵Even if K_0 , D_0 , and X_0 take on their steady state values, the economy will still undergo a transition. The reason is that we feed in exogenous population dynamics.



Notes: Empirical data for the US cover the period 1950–2015, while it is 1950–2012 for the UK, 1960–2012 for France, and 1962–2012 for Germany.

Figure IV: Housing wealth-to-income ratios, 1950–2015

variable from its steady state value. We then compare how the growth factors of the housing wealth-to-income ratio and other variables change. The results are shown in table VI. Column (1) contains the growth factors from the factual transition that is implied by our calibration and columns (2) to (4) the growth factors from the counterfactual transition. For example, comparing the entries for developed land prices, P^D , in columns (1) and (3) shows that if residential land were to start at its steady state value in 1950 instead of 60 percent below it, land prices would increase by a counterfactual factor of 8.1 instead of 4.7.

We now turn to the quantitative importance of the three different initial states – K_0 , D_0 , and X_0 – for the increase in the housing wealth-to-income ratio. The first row of table VI contains all necessary information. The strongest contribution results from the construction sector. At the beginning of the transition, in 1950, the US entered a prolonged construction boom. This is captured by X_0 being 81 percent below its steady state level. A strong increase in residential structures along the transition leads to an increase in the housing wealth-to-income ratio. If we assumed that X_0 starts in its steady state, then the housing wealth-to-income ratio would actually decrease by 20 percent (entry 0.8 for the counterfactual)

	(1) factual	(2)	(3) counterfactual	(4)
		investment $K_0 = \tilde{K}$	land development $D_0 = \tilde{D}$	construction $X_0 = \tilde{X}$
$\frac{P^H H}{NNP}$	1.8 (0.00)	1.7 (0.03)	1.7 (0.08)	0.8 (1.01)
$P^H H$	9.7 (0.00)	6.5 (3.19)	9.3 (0.41)	4.9 (4.80)
NNP	5.9 (0.00)	4.1 (1.87)	5.9 (-0.01)	6.9 (-0.97)
$\frac{P^D D}{NNP}$	1.8 (0.00)	1.1 (0.70)	1.3 (0.49)	1.3 (0.48)
$P^D D$	4.7 (0.00)	2.0 (2.75)	8.1 (-3.41)	4.0 (0.72)
D	2.3 (0.00)	2.3 (-0.02)	1.0 (1.31)	2.3 (-0.03)
$\frac{P^X X}{NNP}$	1.8 (0.00)	1.9 (-0.11)	1.8 (0.00)	0.7 (1.04)
$P^X X$	2.4 (0.00)	1.7 (0.70)	2.4 (-0.01)	5.9 (-3.47)
X	4.3 (0.00)	4.4 (-0.10)	4.3 (0.01)	0.9 (3.46)

Notes: Values are cumulative growth factors of a variable between period $t = 0$ and $t = 65$. Differences to respective growth factors of factual transition are in brackets. Initial states K_0, D_0 , and X_0 start 77, 60, and 81 percent below their respective steady state values.

Table VI: Growth factors of endogenous variables under alternative initial states (US)

instead of increasing by 80 percent (entry 1.8 for the factual) between 1950 and 2015. Hence, the differential effect between the factual and the counterfactual is described by $1.81 - 0.8 = 1.01$. The housing wealth-to-income ratio grows by 101 percentage points more in the factual compared to the counterfactual. In comparison, the other two endogenous states have almost negligible net effects on the housing wealth-to-income ratio.³⁶

Since the construction boom is an important driver for the rising housing wealth-to-income ratio we now discuss it in greater detail. The effect of changes in X_0 on the growth factors of the relevant variables is also shown in column (4) of table VI. First, we can study the numerator and denominator of the housing wealth-to-income ratio. The growth factor of the numerator, $P^H H$, increases by 480 percentage points when X_0 is 81 percent below its steady state value instead of starting directly in its steady state value. The growth factor of the denominator, NNP , decreases by 97 percentage points, also contributing to an increase in the housing wealth-to-income ratio although by much less than the increase in housing wealth. Second, we can further decompose the numerator, housing wealth, into the sum of $P^D D$ and $P^X X$. Most of the effect is driven by rising structure wealth, $P^X X$, which increases by 104 percentage points more relative to NNP , instead of residential land wealth, which increases only by 48 percentage points more. Third, structure wealth in turn is the product of the price and quantity of structures, P^X and X . Most of the increase in structure wealth is due to a 346 percentage point stronger increase in residential structures. This direct effect is quite mechanic. The closer X_0 is to its steady state, the weaker is its subsequent growth. De-

³⁶The results are similar for the UK, France, and Germany. For the UK, for example, initial capital, developed land, and structures start 70, 30, and 80 percent below their respective steady state and contribute with 33, 8, and 117 percentage points to the increase in the housing wealth-to-income ratio.

veloped land, however, even increases slightly. The reason is that with higher X_0 , residential land is relatively scarce and increases therefore at a slightly higher pace. Lastly, differences in initial structures X_0 affect the housing wealth-to-income ratio also through prices. As with a higher X_0 structures are initially more abundant, their price, P_0^X , is smaller. Since the price P^X converges to the same steady state its growth rate therefore increases as can be seen in table VI. The opposite effect operates through P^D . Since D accumulates faster with a higher X_0 , its price increases less because D is less valuable at future dates. To summarize, the construction boom contributes to a rising housing wealth-to-income ratio mainly through its direct quantity effect. This shows up by a pronounced increase in residential structure.

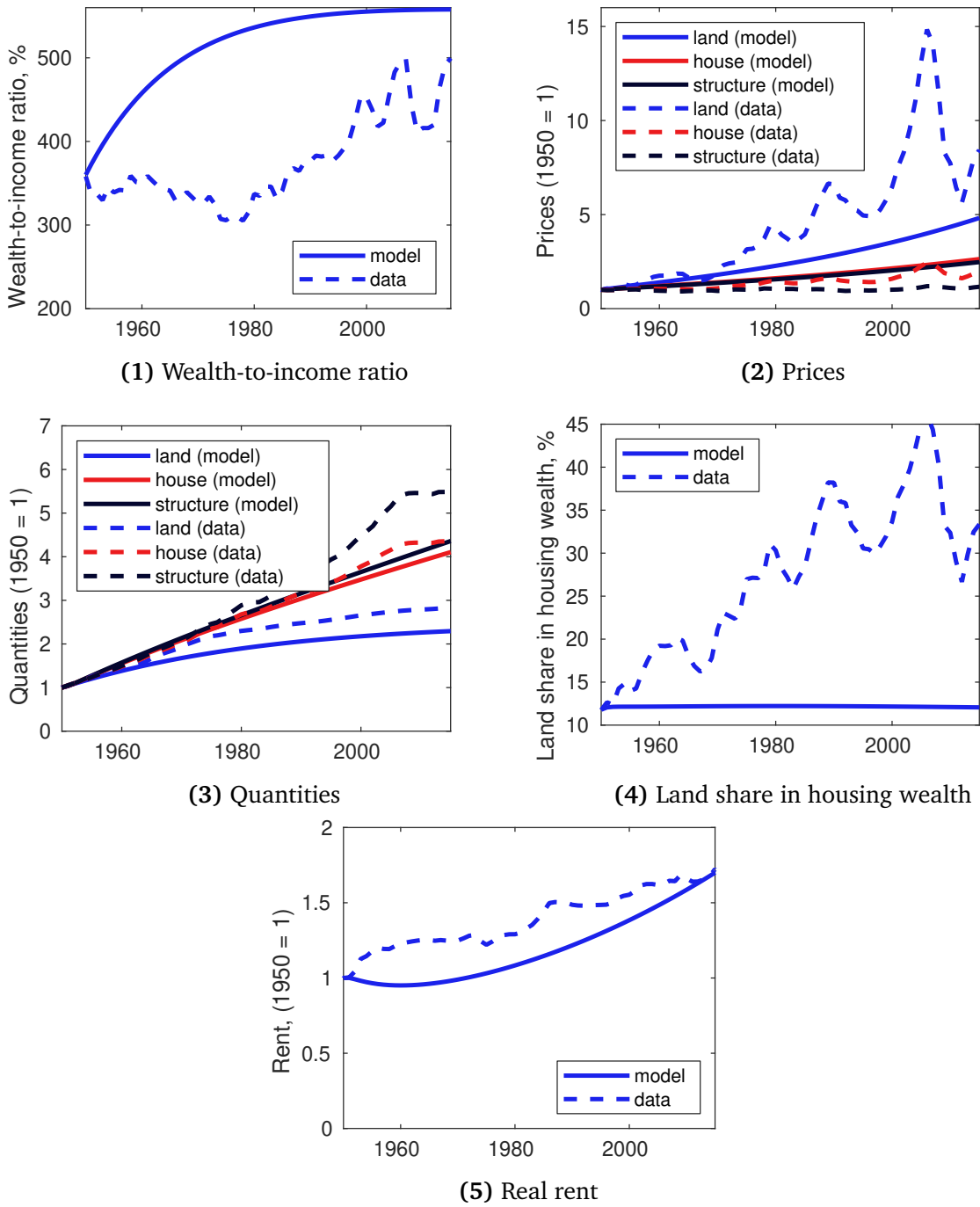
The construction boom, however, implies that residential land wealth grows less than structure wealth, contradicting stylized fact 4. This is where the dynamics through K_0 and D_0 enter the picture. Although their effect on the rising housing wealth-to-income ratio is small, they contribute in generating a stronger increase in residential land wealth (70 and 49 percentage points) than in residential structure wealth (11 and 0 percentage points). The dynamics in K_0 , in particular, result in a stronger increase in residential land prices by 275 percentage points. This works mainly through interest rates. If K_0 is lower, then the interest rate starts at a higher level before converging to the same steady state value. A higher level of the interest rate implies stronger discounting of future yields and hence, since the price is the PDV of future yields, a smaller initial price. As prices also converge to the same steady state values this in turn results in stronger price growth.

The calibrated model replicates the observed increase in housing wealth-to-income ratios quite well. We have shown that the main driver is a construction boom. In the next section we study how our calibration relates to the other stylized facts before discussing further data that corroborate the construction boom.

6.2. Remaining stylized facts

After having discussed how the calibrated model replicates the rising housing wealth-to-income ratio in the previous section, we now turn to the remaining non-targeted moments among the stylized facts described in section 2, focusing on the US. These are plotted in figure V. We match a couple of features by construction, namely the level of the total wealth-to-income ratio in 1950 (figure V1), the level of the land share in housing wealth in 1950 (figure V4), and the growth factor of the rent between 1950 and 2015 (figure V5). Except for rents, the dynamics in all variables of figure V are non-targeted. We discuss each of these stylized fact now in turn.

First, the increase in the wealth-to-income ratio is matched quite accurately, over-predicting the increase between 1950 and 2015 by merely 5 percentage points. According to the model the wealth-to-income ratio increases by 55 percent. This is mostly driven by the housing wealth-to-income ratio, which increases by 80 percent (figure IV), and to a lesser extent by the non-housing wealth-to-income ratio, which increases by 40 percent

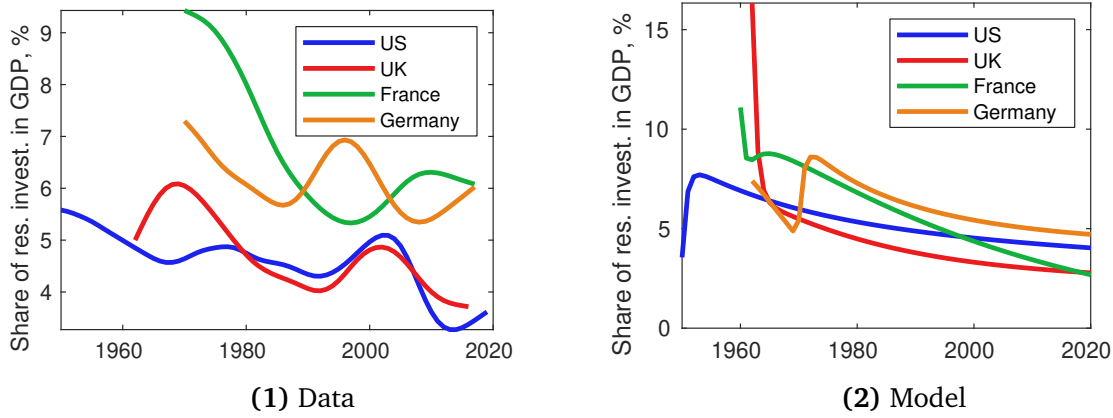


Notes: Empirical data for the US cover the period 1950–2015. The housing wealth-to-income ratio is plotted in figure IV.

Figure V: Stylized facts, US

(not shown). Our theory hence plausibly explains that the increase in the wealth-to-income ratio is mainly driven by the increase in housing wealth as opposed to non-housing wealth, as stated by stylized fact 1.

Second, the model replicates the mirror-like behavior of prices and quantities, which is described in stylized facts 2 and 3. In figure V2 we show that residential land prices grow much stronger than house prices, which in turn grow at a slightly higher rate than construction cost. The opposite holds true for the respective quantities. The quantity of



Notes: Empirical data for the US cover the period 1950–2019, while it is 1960–2016 for the UK, and 1970–2017 for France and Germany. Since residential investment is very volatile in the short run we have HP-filtered the empirical series with a smoothness parameter of 100. In the model residential investment is computed following (17).

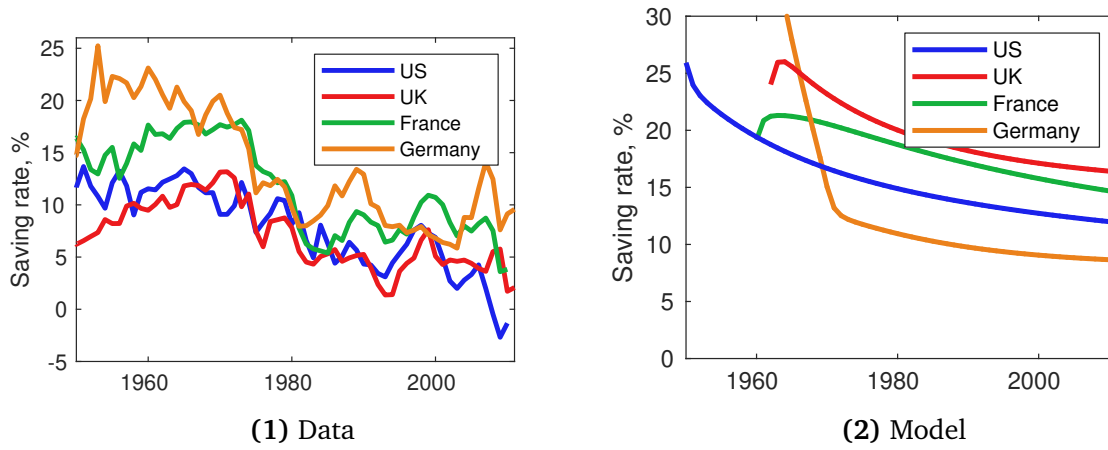
Figure VI: Residential investment

structures grows at a higher rate than the quantity of housing, which grows at a higher rate than the quantity of residential land. We match this pattern qualitatively, but quantitatively the model-generated spread of growth rates between land and structure quantities and prices is too small. We return to this in section 7.2, where we show that the model fit can be improved with a constant elasticity of substitution (CES) function in the production of housing services.

Third, the model endogenously generates a 2 percent increase in the share of residential land wealth in housing wealth, as shown in figure V4. Quantitatively, however, the model performs bad along this dimension as the increase is too small. This is related to the insufficient build up in the stock of residential structures, mentioned before, and will also be addressed in section 7.2. Lastly, the dynamics in real rents is matched by construction as the growth in rent is a targeted moment in the calibration. Note that the model predicts that rents grow at a positive rate of 1.4 percent in the long run, as can already be seen in figure V5.

6.3. Other relevant data moments

Residential investment. In his much cited paper Leamer (2007) argues that *housing is the business cycle*, meaning that residential investment leads the business cycles and explains a considerable share of the fluctuations in GDP. Kohlscheen *et al.* (forthcoming, figure 1) show that residential investment declined since 1970 for every economy out of a set of 15 advanced economies and is becoming less relevant for the business cycle. Although we do not study business cycles, our results are still insightful for the declining relevance of residential investment for the business cycle. In figure VI we plot empirical and model-implied residential investment as a share in GDP for all four economies. Figure VII plots the HP-filtered trend component of residential investment for the US, UK, France, and Germany.



Notes: Data on saving rates is taken from (Piketty and Zucman, 2014, online appendix).

Figure VII: Saving rates

Residential investment as a share of GDP has declined since WWII in all four economies. For the US, for instance, residential investment amounted to 5.6 percent of GDP in 1950 and was almost cut in half in 2019 when it arrived at 3.8 percent. The other economies experienced similar patterns. The model-implied transitions shown in figure VI2 paint the same picture of declining residential investment as a share of GDP. This corroborates our finding that a pronounced construction boom contributed strongly to the increase in housing-wealth-to income ratios. Our paper therefore provides a fundamental long-term theory for better understanding why *housing was the business cycle*.

Saving rates. The aggregate saving rate is crucial in the process of wealth accumulation and hence crucial for the dynamics of the (housing) wealth-to-income ratio (Piketty and Zucman, 2014). We therefore check how the saving rate evolves in the model-based experiment laid out above and how this relates to the data. The empirical data for the considered economies is plotted in figure VII1. Saving rates have been on a persistent downward trend, recently dipping even into negative for the US. This empirical observation has been studied in the previous literature. For example, Parker (1999) discusses several potential explanations and concludes that neither is fully satisfactory. Given that saving rates have been on a downward path empirically, it is reassuring that our model-generated series – shown in figure VII2 – replicate this feature of the data. The behavior of saving rates along the transition depends on income and substitution effects of changes in interest rates and is already ambiguous in a one-sectoral Ramsey growth model (Barro and Sala-i-Martin, 2004, ch. 2.6.4). The interest rate is declining along the transition in our model, implying that the substitution effect dominates the income effect, even with an IES of 0.3.

7. Discussion

Three further channels have been discussed in the literature that may be important when it comes to understanding the surge in house prices and housing wealth: i) increasing

home ownership rates (housing demand), ii) weak elasticity of substitution between residential land and structure in housing production (housing supply), and iii) declining real interest rates (asset prices). We discuss each of these factors in turn and how they relate to the theory outlined above.

7.1. Home ownership revolution

The home ownership rate has risen over the second half of the 20th century in many countries, including the US, UK, France, and Germany (Kohl, 2017). This process is often referred to as *home ownership revolution*. A widely held view is that financial liberalization, by relaxing credit constraints, has primarily triggered this evolution (Ortalo-Magne and Rady, 2006).³⁷ Provided that the surge in the home ownership rate, the story goes, pushes the overall demand for housing upwards, this process has contributed to the increase in house prices.³⁸ However, the descriptive empirical picture on the evolution of the aggregate housing expenditure share is ambiguous. Piazzesi and Schneider (2016) argue that the housing expenditure share is fairly stable over time at 19 percent in the postwar US as implied by data from the US NIPA. Albouy *et al.* (2016) study also alternative data sources and argue that the housing expenditure share may have increased in the US.

Whether or not the home ownership revolution has significantly contributed to the surge in house prices appears an open research question. Let us assume that this channel is important. How can this channel be captured by our analysis? Notice that we model all households as renters, but the results do not change if we assume that all households are home owners. We could employ a short-cut to capture an additional demand effect due to the home ownership revolution by assuming that the expenditure share, which equals θ in our model, increases exogenously over time. This would further add to the growing demand for housing and amplify house price growth and the surge in housing wealth. Our supply-side setting, however, is still necessary to generate a plausible increase in house prices and housing wealth such that land prices increase much more than construction cost. Hence, even when studying this potential channel it is necessary to consider the supply-side features proposed in our model.

7.2. CES technology in housing services

A channel that originates from the supply side concerns the role of land in housing services production. Miles and Sefton (2020) argue that the elasticity of substitution (EoS) between land and structure is less than one. As the economy grows and the demand for

³⁷Jordà *et al.* (2016) stress that banks and households have been heavily leveraging up through mortgages in the second half of the 20th century. Mortgage credit on the balance sheets of banks has been the driving force behind the increasing financialization of advanced economies.

³⁸However, Kaplan *et al.* (2019) argue that the increase in overall housing demand, triggered by credit relaxation, is small. In their model, as in the data, very few households are constrained with respect to the size of the rented house: rather than buying excessively small houses, they prefer to rent a house of the desired size. The credit relaxation induces these renters to become owners, which increases home ownership without pushing up prices.

stylized fact	variable	data (US)	baseline	alternative (1) $g^X = -\frac{\eta}{1-\eta} \frac{1-\alpha}{\beta} g^Y$	alternative (2) $EoS = 0.25$
1	$\frac{W}{NNP}$	1.39	1.55	1.83	2.16
	$\frac{p^D_{D+P^X X}}{NNP}$	1.42	1.80	2.20	2.88
2	p^H	1.93	2.63	3.98	5.42
	p^D	8.45	4.82	6.40	10.75
	p^X	1.16	2.47	3.82	3.37
3	D	2.80	2.29	1.85	1.76
	X	5.49	4.36	3.51	4.40
4	$\frac{p^D D}{p^D D + p^X X}$	2.84	1.02	0.89	1.08
5	q	1.70	1.70	2.23	3.15

Notes: All values are growth factors of the respective variable between 1950 and 2015. Bold numbers highlight targeted moments. Alternative (1) is re-calibrated assuming that $g^X = -\frac{\eta}{1-\eta} \frac{1-\alpha}{\beta} g^Y$ and removing the growth rate of rents from the set of targeted moments. Alternative (2) applies the same parameters from (2) except that the elasticity of substitution between D and X in the production function of H is set to 0.25.

Table VII: Stylized facts with CES technology in housing production

housing surges, the long run price elasticity of housing supply is relatively low as the fixed factor (land) cannot be easily substituted for by the accumulable factor (structure). The consequence is a stronger growth in the rent and the house price compared to the Cobb Douglas case. We have implicitly assumed an elasticity of substitution of one in the production function of housing, $H(X, D)$. We now assume that the production technology for housing services is given the general CES function

$$H(X, D) = \left[\gamma X^{\frac{\chi-1}{\chi}} + (1-\gamma) D^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}},$$

where $\chi > 0$ is the elasticity of substitution between land and structures.

The effect of a weaker EoS between land and structures on the stylized facts is shown in table VII. We study two alternative transitions. If $\chi \neq 1$ a steady state equilibrium only exists if both X and D grow at the same rates and it hence has to hold that $g^X = -\frac{\eta}{1-\eta} \frac{1-\alpha}{\beta} g^Y$. This knife-edge restriction implies differential technological progress that is heavily biased towards the non-housing good. In alternative (1) we therefore first re-calibrate the model with $\chi = 1$ under this knife-edge condition, setting the same set of parameters endogenously as in the baseline calibration except g^X and matching the same set of moments except the growth rate of rents. In alternative (2) we then study how changing χ from 1 to a low value of 0.25 – keeping the parameters from alternative (1) – affects the stylized facts. This two-step approach is necessary because otherwise it would be impossible to tell whether differences occur due to a lower χ or g^X .

The effect of reducing χ can be seen by comparing alternative (1) and (2) in table VII. With a lower elasticity of substitution the spread between growth rates of prices and quantities of residential land and structures increases considerably. The intuition is straightforward.

ward. The weaker the elasticity of substitution, the stronger has X to increase in order to generate additional housing when the economy grows. The relatively stronger growth in residential land prices then also results in much stronger growth of the land share in housing wealth (it actually declines in alternative 1). To summarize, weaker substitutability of structure and land results in stronger growth difference in prices and quantities of land and structure, stronger increase of the land share in housing wealth. It does, however, require to impose a knife-edge condition on differential technological growth to restore the existence of a steady state.

7.3. Declining real interest rate

[Rachel and Summers \(2019\)](#) show that the real interest rate has been trending downwards since the early 1980s.³⁹ Viewing houses as assets suggests that the house price is given by the PDV of future rental yields. A decline in the real interest rate over time then implies that the house price increases as time goes by. This mechanical discounting effect operates as long as the discount factor declines. [Miles and Monro \(2019\)](#) argue that the rise in house prices (relative to incomes) between 1985 and 2018 can be more than accounted for by the substantial decline in the real risk-free interest rate observed over that period.⁴⁰ The interest rate is an endogenous object in our model. This makes it non-trivial to study how its changes affect prices as both variables evolve endogenously. The endogenous dynamics of the interest rate are in line with the empirical observation of declining interest rates as interest rates decline monotonously along the transition. The reason for declining interest rates is best understood by noting that capital starts below its steady state and accumulates over time, reducing the marginal product of capital and hence the interest rate. Our explanation for rising house prices is therefore in line with the empirical trend in the interest rate, providing also an explanation for declining interest rate.

8. Conclusion

Housing wealth has been increasing relative to income in the US, the UK, France, and Germany since 1950. In this paper we provide a theory to explain this increase. We document five stylized facts on housing-related macro variables. The key is the differentiation of housing wealth into the value of residential structure and the value of residential land. Most of the increase in housing wealth is driven by a rising value of residential land. Further decomposing the values of land and structure into price and quantity components reveals a mirror-like pattern. The price of the non-reproducible factor, land, increased strongly, while its quantity remained relatively stable. The opposite holds true for structures – its quantity

³⁹They focus on average yields on long-maturity inflation-protected government securities in the G7, excluding Italy. For alternative measures on the long term real interest rate and long term data series see also [Jordà et al. \(2019\)](#).

⁴⁰They focus on the UK, where the surge in house prices has been especially pronounced. Their main results, they argue, apply to other G7 economies as well.

increased strongly and its price only modestly – which are reproducible.

Motivated by these stylized facts, we propose a novel theory for studying housing wealth in the long term. The employed Ramsey growth model comprises two sectors, a numeraire sector and a housing sector. While the numeraire sector is standard, the innovative part of our model is the housing sector. It consists of three different types of firms. First, property management firms combine residential land and residential structures to produce houses that are rented out to households. Second, real estate development firms purchase land and transform it into residential land by employing labor. Third, construction firms manufacture structures by employing materials and labor. The overall supply of land is fixed. The allocation of land across the numeraire sector and the housing sector is endogenous. There is exogenous population growth and exogenous technological change.

In steady state we show that two forces are driving house prices in the long run. First, the land intensity in the housing sector has to be larger than the land intensity in the non-housing sector – the empirically plausible case. Only then does (equal) technological progress lead to rising house prices. Second, we also allow that technological progress differs between the two sectors of our economy. House prices grow in the long term if technological progress in the construction sector is lagging behind technological progress of the remaining economy – also empirically plausible. Our model therefore focuses on two fundamental forces behind the long-term growth in housing wealth.

Although housing wealth can increase in the steady state of our model, the housing wealth-to-income ratio is constant in steady state. In order to tackle the phenomenon of a rising housing wealth-to-income ratio we therefore study transitions with the calibrated model. The calibrated model explains 80, 67, and 76 percent of the increase in the housing wealth-to-income ratio in the UK, France, and Germany since the 1950s and even slightly over-predicts the increase for the US. We show that the main driver of the increase in the housing wealth-to-income ratio is a pronounced construction boom. This is corroborated by the observation that residential investment as a share of GDP has been – both in the data and in our model simulation – on a downward path since the 1950s for the US, UK, France, and Germany. Lastly, we also show that the model-implied transition is in line with the other stylized facts. If housing will remain relatively land intensive and technological progress in the construction will not catch up with technological progress in the rest of the economy, rising housing wealth, house prices, and rents will also prevail in the future.

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Appendix

A. Model

A.1. First order conditions to various problems

Household problem Households choose streams of $\{C_t, S_t\}_{t=0}^{\infty}$ to maximize (2) subject to (3), (4), and a NPG condition. The associated present-value Hamiltonian reads

$$\mathcal{H} = e^{-\rho t} L_t \frac{\left[\left(\frac{C_t}{L_t} \right)^{1-\theta} \left(\frac{S_t}{L_t} \right)^{\theta} \right]^{1-\sigma} - 1}{1-\sigma} + \lambda [r_t W_t + w_t L_t - C_t - q_t S_t + \Pi_t^N]$$

and the (rearranged) FOC are

$$\begin{aligned} C &= \frac{1-\theta}{\theta} q S \\ \frac{\dot{C}}{C} &= \frac{r-\rho}{\sigma} + \theta \frac{\sigma-1}{\sigma} \frac{\dot{q}}{q} + \frac{\dot{L}}{L}. \end{aligned} \tag{18}$$

The first is a standard *intratemporal* optimality condition that states that the marginal rate of substitution of C and S equals the relative price, q . The second is a modified Keynes-Ramsey rule. The growth rate of rents enters the right hand side and affect the time path of the consumption good C if the intertemporal elasticity of substitution is not 1.

Numeraire firm problem The first order conditions to problem (6) read

$$r + \delta^K = Y_K, \quad w = B^Y Y_{B^Y L^Y}, \quad \text{and} \quad R^N = Y_N, \tag{19}$$

where we use the notation Y_N to denote the derivative of the production function for Y with respect to N . All production factors are rewarded with their marginal products.

Property management firm problem The Hamiltonian associated with problem (7) reads

$$\mathcal{H} = e^{-\tilde{r}} [q X^\gamma D^{1-\gamma} - P^X I^X - P^D I^D] + \mu^X (I^X - \delta^X X) + \mu^D I^D. \tag{20}$$

The FOC are

$$\frac{\partial \mathcal{H}}{\partial I^D} = 0, \quad \frac{\partial \mathcal{H}}{\partial I^X} = 0, \quad \frac{\partial \mathcal{H}}{\partial D} = -\dot{\mu}^D, \quad \text{and} \quad \frac{\partial \mathcal{H}}{\partial X} = -\dot{\mu}^X.$$

Taking the derivatives of (20) modifies the FOC to

$$\mu^D = e^{-\tilde{r}} P^D, \quad \mu^X = e^{-\tilde{r}} P^X, \quad e^{-\tilde{r}} q H_D = -\dot{\mu}^D, \quad \text{and} \quad e^{-\tilde{r}} q H_X - \mu^X \delta^X = -\dot{\mu}^X.$$

Eliminating the co-state variables by taking the derivative with respect to time of the first two equations, obtaining $\dot{\tilde{r}} = r$ by applying the Leibniz rule, inserting the results into the

second two equations and rearranging yields

$$r = \frac{\dot{P}^D + qH_D}{P^D} = \frac{\dot{P}^X + qH_X - \delta^X P^X}{P^X}.$$

These are two no-arbitrage equations. The yield component from investing one additional unit in residential land (structures) are marginal operating profits qH_D (qH_X minus depreciation $\delta^X P^X$). These no-arbitrage equations imply that the property management firm is indifferent with respect to investment into land and structures whenever prices q, P^X, P^D adjust such that these two equations hold. This results from the fact that property management firms discount profits at the economy's interest rate r .

Construction firm problem In (9) we have to explicitly take care of the non-negativity restrictions on the input factors. The resulting unconstrained first order conditions read

$$M = \eta P^X I^X \quad \text{and} \quad L^X = (1 - \eta) \frac{P^X I^X}{w}. \quad (21)$$

Inserting the two equations into the profit equation yields

$$\begin{aligned} \Pi^X &= P^X X^\eta (B^X L^X)^{1-\eta} - M - w L^X \\ &= P^X (\eta P^X I^X)^\eta \left(B^X (1 - \eta) \frac{P^X I^X}{w} \right)^{1-\eta} - \eta P^X I^X - w (1 - \eta) \frac{P^X I^X}{w} \\ &= \left[P^X \eta^\eta (1 - \eta)^{1-\eta} \left(\frac{B^X}{w} \right)^{1-\eta} - 1 \right] P^X I^X. \end{aligned}$$

If the expression in square brackets – which is exogenous to the construction firms – is positive, then I^X , L^X , M , and profits are positive. If, however, the expression in square brackets is negative, then the unconstrained solution would imply negative values for I^X , L^X , and M (and positive profits), violating the non-negativity restriction. In that case the optimal solution is $I^X = L^X = M = 0$. To summarize,

$$\begin{aligned} I^X &= \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{(1-\eta)}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ M^\eta (B^X L^X)^{1-\eta} & \text{else} \end{cases} \\ L^X &= \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{(1-\eta)}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ (1 - \eta) \frac{P^X}{w} I^X & \text{else} \end{cases} \\ M &= \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{(1-\eta)}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ \eta P^X I^X & \text{else} \end{cases}. \end{aligned} \quad (22)$$

A.2. Full dynamic system

Original system

In this section we explicitly formulate the economy as consisting of a differential al-

gebraic system of equations (DAEs). First, we use the asset market clearing condition to substitute W with K in the budget constraint and obtain the law of motion of capital

$$\dot{K} = rK + R^D D + (R^X + \delta^X P^X)X + R^N (Z - D) + wL - C - qS - P^X I^X.$$

Second, with the help of the intratemporal optimality condition, the production function of S , and the laws of motion of X and D we reformulate the Keynes Ramsey by substituting $\frac{\dot{q}}{q}$ by the implied derivative of the rearranged intratemporal FOC:

$$\frac{\dot{C}}{C} = \frac{1}{(1-\theta)\sigma + \theta} \left\{ r - \rho + \sigma \frac{\dot{L}}{L} - \theta(\sigma - 1) \left[\gamma \left(\frac{I^X}{X} - \delta^X \right) + (1-\gamma) \frac{P^D - P^N}{\xi w D} \right] \right\}.$$

Collecting equations the economy can now be described by the following differential algebraic system of equations (DAEs)

$$\begin{aligned} \dot{K} &= rK + R^D D + (R^X + \delta^X P^X)X + R^N (Z - D) + wL - C - qS - P^X I^X \\ \dot{D} &= \frac{P^D - P^N}{\xi w} \\ \dot{X} &= I^X - \delta^X X \\ \frac{\dot{C}}{C} &= \frac{1}{(1-\theta)\sigma + \theta} \left\{ r - \rho + \sigma \frac{\dot{L}}{L} - \theta(\sigma - 1) \left[\gamma \left(\frac{I^X}{X} - \delta^X \right) + (1-\gamma) \frac{P^D - P^N}{\xi w D} \right] \right\} \\ \dot{P}^D &= rP^D - (1-\gamma) \frac{qS}{D} \\ \dot{P}^N &= rP^N - R^N \\ \dot{P}^X &= rP^X - \gamma \frac{qS}{X} + \delta^X P^X \\ C &= \frac{1-\theta}{\theta} qS \\ I^X &= \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{(1-\eta)}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ M^\eta (B^X L^X)^{1-\eta} & \text{else} \end{cases} \\ L^X &= \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{(1-\eta)}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ (1-\eta) \frac{P^X}{w} I^X & \text{else} \end{cases} \\ M &= \begin{cases} 0 & \text{if } \left[\eta^\eta \left[B^X \frac{(1-\eta)}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ \eta P^X I^X & \text{else} \end{cases} \\ S &= X^\gamma N^{1-\gamma} \\ Y &= (K)^\alpha (B^Y (L - L^X - L^N))^\beta (B^Y (Z - N))^{1-\alpha-\beta} \\ r &= \alpha \frac{Y}{K} - \delta^K \\ w &= \beta \frac{Y}{L - L^X - L^N} \end{aligned}$$

$$R^N = (1 - \alpha - \beta) \frac{Y}{Z - D}$$

$$L^D = \frac{(P^D - P^N)^2}{2\xi w^2}.$$

That makes 19 equations in 19 variables, with 3 state variables, 4 jump variables, and 12 auxiliary variables.

Normalized system We divide each variable by its growth factor. For example, variable Y grows at the rate $\frac{\beta}{1-\alpha}g^Y$ and we normalize it by dividing it by the growth factor $\exp[\frac{\beta}{1-\alpha}g^Y t]$. The thus normalized DAEs reads⁴¹

$$\begin{aligned} \dot{K} &= (r - g_Y)K + R^D D + (R^X + \delta^X P^X)X + R^N (Z - D) + wL - C - qS - P^X I^X \\ \dot{D} &= \frac{P^D - P^N}{\xi w} \\ \dot{X} &= I^X - [\delta^X + g_X]X \\ \frac{\dot{C}}{C} &= \frac{1}{(1-\theta)\sigma + \theta} \left\{ r - \rho + \sigma \frac{\dot{L}}{L} - \theta(\sigma - 1) \left[\gamma \left(\frac{I^X}{X} - \delta^X \right) + (1-\gamma) \frac{P^D - P^N}{\xi w D} \right] \right\} - g_Y \\ \dot{P}^D &= (r - g_Y)P^D - (1-\gamma) \frac{qS}{D} \\ \dot{P}^N &= (r - g_Y)P^N - R^N \\ \dot{P}^X &= (r - g_{P^X})P^X - \gamma \frac{qS}{X} + \delta^X P^X \\ C &= \frac{1-\theta}{\theta} qS \\ I^X &= \begin{cases} 0 & \text{if } \left[\eta^\eta \left[\frac{(1-\eta)}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ M^\eta (B^X L^X)^{1-\eta} & \text{else} \end{cases} \\ L^X &= \begin{cases} 0 & \text{if } \left[\eta^\eta \left[\frac{(1-\eta)}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ (1-\eta) \frac{P^X}{w} I^X & \text{else} \end{cases} \\ M &= \begin{cases} 0 & \text{if } \left[\eta^\eta \left[\frac{(1-\eta)}{w} \right]^{1-\eta} P^X - 1 \right] P^X < 0 \\ \eta P^X I^X & \text{else} \end{cases} \\ S &= X^\gamma N^{1-\gamma} \\ Y &= (K)^\alpha ((L - L^X - L^N))^\beta (Z - N)^{1-\alpha-\beta} \\ r &= \alpha \frac{Y}{K} - \delta^K \\ w &= \beta \frac{Y}{L - L^X - L^N} \\ R^N &= (1 - \alpha - \beta) \frac{Y}{Z - D} \end{aligned}$$

⁴¹We do not change the notation and keep in mind that variables now correspond to normalized units of a respective variable.

$$L^D = \frac{(P^D - P^N)^2}{2\xi w^2}.$$

with $g_Y \equiv \frac{\beta}{1-\alpha} g^Y$, $g_X \equiv \eta \frac{\beta}{1-\alpha} g^Y + (1-\eta)g^X$, $g_{PX} \equiv (1-\eta)(\frac{\beta}{1-\alpha} g^Y - g^X)$, and $B_0^Y = B_0^X = 1$.

A.3. Steady state

Closed-form solution We focus on a steady state with $\delta^X > 0$ and $g^X > -\frac{1-\eta}{\eta}(1-\psi^Y)g^Y$. This implies that even in a steady without growth, $g^Y = g^X = 0$ there will be replacement investment for X and that X does not tend to zero as t tends to infinity.⁴² We denote a normalized variable in steady state by a " \sim " above the variable. The DAEs simplifies to a algebraic system of equations:⁴³

$$\begin{aligned} 0 &= (r - g_Y)\tilde{K} + \tilde{R}^D D + (\tilde{R}^X + \delta^X \tilde{P}^X)\tilde{X} + \tilde{R}^N (Z - D) + \tilde{w}L - \tilde{C} - \tilde{q}\tilde{S} - \tilde{P}^X \tilde{I}^X \\ \tilde{P}^D &= \tilde{P}^N \\ \tilde{I}^X &= (\delta^X + g_X)\tilde{X} \\ r &= \rho + [(1-\theta)\sigma + \theta]g_Y + \theta(\sigma-1)\gamma g_X \\ \tilde{P}^D &= (1-\gamma)\frac{\tilde{q}\tilde{S}}{(r - g_Y)D} \\ \tilde{P}^N &= \frac{\tilde{R}^N}{r - g_Y} \\ \tilde{P}^X &= \gamma \frac{\tilde{q}\tilde{S}}{(r - g_{PX} + \delta^X)\tilde{X}} \\ \tilde{C} &= \frac{1-\theta}{\theta}\tilde{q}\tilde{S} \\ \tilde{I}^X &= \tilde{M}^\eta (L^X)^{1-\eta} \\ L^X &= (1-\eta)\frac{\tilde{P}^X}{\tilde{w}}\tilde{I}^X \\ \tilde{M} &= \eta \tilde{P}^X \tilde{I}^X \\ \tilde{S} &= \tilde{X}^\gamma D^{1-\gamma} \\ \tilde{Y} &= (\tilde{K})^\alpha (L - L^X - L^N)^\beta (Z - N)^{1-\alpha-\beta} \\ r &= \alpha \frac{\tilde{Y}}{\tilde{K}} - \delta^K \\ \tilde{w} &= \beta \frac{\tilde{Y}}{L - L^X - L^N} \\ \tilde{R}^N &= (1-\alpha-\beta)\frac{\tilde{Y}}{Z - D} \\ L^D &= 0. \end{aligned}$$

⁴²The other cases can also be studied, but since they seem less relevant to us we eschew the derivation of the steady state under $\delta^X = 0$ and/or $g^X \leq -\frac{1-\eta}{\eta}(1-\psi^Y)g^Y$.

⁴³We do not change the notation of variables and keep in mind that variables within this section are steady state levels of normalized versions of a respective variable.

The fourth equation results from the Keynes Ramsey rule and is already the solution for the steady state interest rate. Solving this equation system yields the closed-form solution⁴⁴

$$\begin{aligned}
r &= \rho + [(1 - \theta)\sigma + \theta]g_Y + \theta(\sigma - 1)\gamma g_X \\
L^X &= \frac{r + (1 - \alpha)\delta^K - \alpha g_Y}{\left[\eta + (1 - \theta) \frac{(r + \delta^K - g_{PX})}{\gamma\theta(g_X + \delta^K)} \right] \frac{\beta}{1 - \eta} (r + \delta^K) + r + (1 - \alpha)\delta^K - \alpha g_Y} L \\
D &= \frac{\frac{(1 - \gamma)\beta}{\gamma(1 - \eta)} \frac{r + \delta^K - g_{PX}}{(g_X + \delta^K)}}{\frac{(1 - \gamma)\beta}{\gamma(1 - \eta)} \frac{r + \delta^K - g_{PX}}{(g_X + \delta^K)} + (1 - \alpha - \beta) \frac{r - g_{PN}}{r - g_Y} \left(\frac{L}{L^X} - 1 \right)} Z \\
\tilde{P}^X &= \eta^{-\eta} \left\{ \frac{\beta}{1 - \eta} \alpha^{\frac{\alpha}{1 - \alpha}} [L - L^X]^{-\frac{1 - \alpha - \beta}{1 - \alpha}} (Z - D)^{\frac{1 - \alpha - \beta}{1 - \alpha}} (r + \delta^K)^{-\frac{\alpha}{1 - \alpha}} \right\}^{1 - \eta} \\
\tilde{C} &= (1 - \theta) \frac{\eta^{\frac{\eta}{1 - \eta}} (r + \delta^K - g_{PX})}{\gamma\theta(g_X + \delta^K)} (\tilde{P}^X)^{\frac{1}{1 - \eta}} L^X \\
\tilde{K} &= \alpha^{\frac{1}{1 - \alpha}} [L - L^X]^{\frac{\beta}{1 - \alpha}} [Z - D]^{\frac{1 - \alpha - \beta}{1 - \alpha}} (r + \delta^K)^{-\frac{1}{1 - \alpha}} \\
\tilde{X} &= \frac{[\eta \tilde{P}^X]^{\frac{\eta}{1 - \eta}} L^X}{(g_X + \delta^K)} \\
\tilde{P}^N &= (1 - \gamma) \frac{\theta}{1 - \theta} \frac{\tilde{C}}{(r - g_Y) \tilde{D}} \\
\tilde{M} &= \eta \tilde{P}^X (g_X + \delta^K) \tilde{X} \\
\tilde{w} &= \beta \tilde{K}^\alpha [L - L^X]^{\beta - 1} [Z - D]^{1 - \alpha - \beta} \\
\tilde{R}^N &= (1 - \alpha - \beta) \tilde{K}^\alpha [L - L^X]^\beta [Z - D]^{-\alpha - \beta} \\
\tilde{Y} &= \tilde{K}^\alpha [L - L^X]^\beta [Z - D]^{1 - \alpha - \beta} \\
\tilde{S} &= \tilde{X}^\gamma D^{1 - \gamma} \\
\tilde{q} &= \frac{\theta}{1 - \theta} \frac{\tilde{C}}{\tilde{S}} \\
L^D &= 0 \\
\tilde{P}^D &= \tilde{P}^N \\
\tilde{I}^X &= (g_X + \delta^K) \tilde{X}
\end{aligned}$$

In the computation we always verify that L^X and D are within their respective bounds, but one could further derive explicit parameter restrictions.

A.4. Proofs

Proof of proposition 1.

⁴⁴In order to avoid very large expressions we formulate the solution recursively. That is, the closed-form solution of L^X is given by inserting the closed-form solution of r in the first line. Similarly, the solution for D is obtained by inserting L^X and r from the above lines. This way all solutions can be easily expressed as functions of parameters only.

Growth rates for land and labor are equal to zero in steady state because aggregate supply of both is constant in the long run. Therefore L^D, L^X, L^N, L, D, N , and Z are all constant.

The Keynes-Ramsey rule (18) implies that for consumption to grow at constant rates the interest rate r has to be constant. We now turn to the production function of the numeraire (5). Since it has constant returns to scale we apply Euler's theorem and use the FOC given by (19) to obtain

$$Y = (r + \delta^K)K + wL^Y + R^N N.$$

Since r, L^Y and N are constant in steady state, it follows that Y, K, w , and R^N all grow at the same rates. This rate can be determined by taking the derivative of the production function of the numeraire (5) with respect to time

$$\begin{aligned}\dot{Y} &= Y_K \dot{K} + Y_{B^Y L^Y} g^Y B^Y L^Y \\ \Rightarrow \frac{\dot{Y}}{Y} &= \frac{Y_K K}{Y} \frac{\dot{K}}{K} + g^Y \frac{Y_{B^Y L^Y} B^Y L^Y}{Y} \\ \frac{\dot{Y}}{Y} &= \alpha \frac{\dot{K}}{K} + \beta g^Y \\ \Rightarrow \frac{\dot{Y}}{Y} &= \frac{\beta}{1-\alpha} g^Y,\end{aligned}$$

where the penultimate equation makes use of the fact that Y and K grow at the same rates. This implies that Y, K, w , and R^N all grow at the exogenous rate $\frac{\beta}{1-\alpha} g^Y$.

The FOC of the real estate development firm problem (12) and the fact that residential land is constant in steady state imply that prices P^D and P^N grow at the same rates and that these are equal to the growth rate of wages, $\frac{\beta}{1-\alpha} g^Y$. The household budget constraint (4) implies that W, C , and qS also grow at the rate $\frac{\beta}{1-\alpha} g^Y$.

The asset market clearing equation, $W = P^D D + P^N N + P^X X + K$, together with the established growth rates of K, W, P^D , and P^N imply that the product $P^X X$ grows at the rate $\frac{\beta}{1-\alpha} g^Y$. The law of motion of structures implies that for X to grow at a constant rate, I^X and X have to grow at the same rates. Applying Euler's theorem to the production function for structures and making use of (22) yields

$$I^X = I_M^X M + I_{B^X L^X}^X B^X L^X = \frac{1}{p^X} M + \frac{w}{p^X} L^X.$$

Both summands have to grow at the same constant rate for I^X to grow at a constant rate. It follows that $\frac{i^X}{I^X} = \frac{\dot{M}}{M} - \frac{\dot{p}^X}{p^X} = \frac{\dot{w}}{w} - \frac{\dot{p}^X}{p^X}$ (L^X is constant). This in turn implies that M grows at the rate $\frac{\beta}{1-\alpha} g^Y$. We now take the derivative of the production function for structures with respect to time

$$\begin{aligned}i^X &= I_M^X \dot{M} + I_{B^X L^X}^X g^X B^X L^X \\ &= I_M^X M \frac{\dot{M}}{M} + I_{B^X L^X}^X B^X L^X g^X\end{aligned}$$

$$\begin{aligned}\frac{\dot{I}^X}{I^X} &= \frac{I_M^X M}{I^X} g^Y + \frac{I_{B^X L^X}^X B^X L^X}{I^X} g^X \\ \frac{\dot{I}^X}{I^X} &= \eta \frac{\beta}{1-\alpha} g^Y + (1-\eta) g^X,\end{aligned}$$

This also gives the growth rate of X , which grows at the same rate as I^X . Since $P^X I^X$ grows at the rate $\frac{\beta}{1-\alpha} g^Y$ it follows that P^X grows at the rate $(1-\eta) \left(\frac{\beta}{1-\alpha} g^Y - g^X \right)$.

Lastly, we need to determine the growth rates of q and S . The derivative of the production function for houses, $H(\cdot)$, with respect to time reads

$$\begin{aligned}\dot{S} &= H_X \dot{X} + H_D \dot{D} = H_X \dot{X} \\ \Rightarrow \frac{\dot{S}}{S} &= \gamma \frac{\dot{X}}{X} = \gamma \eta \frac{\beta}{1-\alpha} g^Y + \gamma (1-\eta) g^X.\end{aligned}$$

From the requirement that qS grows at the rate $\frac{\beta}{1-\alpha} g^Y$ we obtain the growth rate for rents as $(1-\gamma\eta) \frac{\beta}{1-\alpha} g^Y - \gamma\eta g^X$.

To obtain the growth rate of house prices take the derivative of (8) with respect to time

$$\begin{aligned}\dot{P}^H &= \frac{\dot{D}H - D\dot{H}}{H^2} P^D + \frac{D}{H} \dot{P}^D + \frac{\dot{X}H - X\dot{H}}{H^2} P^X + \frac{X}{H} \dot{P}^X \\ &= -\frac{D}{H} \frac{\dot{H}}{H} P^D + \frac{D}{H} \dot{P}^D + \frac{\dot{X}}{X} \frac{X}{H} P^X - \frac{X}{H} \frac{\dot{H}}{H} P^X + \frac{X}{H} \dot{P}^X \\ &= \frac{P^D D}{H} \frac{\dot{P}^D}{P^D} + \frac{P^X X}{H} \frac{\dot{P}^X}{P^X} + \frac{P^X X}{H} \frac{\dot{X}}{X} - P^H \frac{\dot{H}}{H} \\ &= \frac{P^D D}{H} \frac{\dot{P}^D}{P^D} + \frac{P^X X}{H} \left[\frac{\dot{P}^X}{P^X} + \frac{\dot{X}}{X} \right] - P^H \frac{\dot{H}}{H} \\ \Rightarrow \frac{\dot{P}^H}{P^H} &= (1-\gamma\eta) \frac{\beta}{1-\alpha} g^Y - \gamma(1-\eta) g^X.\end{aligned}$$

House prices grow at the same rate as rents. Lastly, all wealth-to-income ratios are constant because aggregate income, NNP , grows at the same rate as the value of a given wealth category. For example, housing wealth $P^H H$ grows at the rate g^Y and NNP grows at the rate g^Y . Similarly, the share of residential land wealth in housing wealth, $P^D D / (P^H H)$, is also constant because numerator and denominator both grow at the rate g^Y . \square

Proof of proposition 2.

We first derive growth rates as functions of long run land intensities. Since K and Y grow at the same rates in steady state, we can express K as a constant share in Y , i.e. $K = \kappa^K Y$. Inserting this into the production function for Y and solving for Y yields then a reduced-form production function

$$Y = (\kappa^K)^{\frac{\alpha}{1-\alpha}} (B^Y L^Y)^{\frac{\beta}{1-\alpha}} N^{\frac{1-\alpha-\beta}{1-\alpha}} \quad (23)$$

The reduced-form production functions captures that, for example, higher land input leads to higher output, which in turn leads to higher capital, amplifying the effect on output. This

Variables	Growth rate
$r, \frac{p^H H}{NNP}, \frac{W}{NNP}, \frac{p^D D}{p^H H}, L^D, L^X, L^Y, N, D$	0
$Y, K, M, w, R^N, P^N, P^D, C, NNP, W$	$(1 - \psi^Y) g^Y$
X, I^X	$\eta(1 - \psi^Y) g^Y + (1 - \eta) g^X$
P^X	$(1 - \eta)[(1 - \psi^Y) g^Y - g^X]$
S, H	$\frac{1 - \psi^H}{1 - \eta \psi^Y} [\eta(1 - \psi^Y) g^Y + (1 - \eta) g^X]$
q, P^H	$[1 - \eta + \eta(\psi^H - \psi^Y)] \frac{1 - \psi^Y}{1 - \eta \psi^Y} g^Y - (1 - \eta) \frac{1 - \psi^H}{1 - \eta \psi^Y} g^X$

Table A.1: Steady state growth rates as functions of land intensities

can be seen from the output elasticity of land, $\psi^Y \equiv \frac{1 - \alpha - \beta}{1 - \alpha}$, which is larger than the output elasticity of land in the original production function, $1 - \alpha - \beta$. We call the parameter ψ^Y the long run land intensity in the Y sector. It measures how an exogenous increase in the supply of land N translates into a long run increase in output of Y .

The land intensity in the housing sector is a bit more complex. The reason is that land enters the Y sector, and that materials M from the Y sector enter the production of structures X , which then in turn enter the production of houses H , where land also enters directly. Since they grow at the same rates in steady states we define $\kappa^M \equiv M/Y$ and $\kappa^X = X/I^X$ and rewrite the production functions for I^X

$$\begin{aligned}
I^X &= (\kappa^M)^\eta [Y]^\eta (B^X L^X)^{1-\eta} \\
&= (\kappa^M)^\eta (\kappa^K)^{\frac{\eta\alpha}{1-\alpha}} (B^Y L^Y)^{\frac{\eta\beta}{1-\alpha}} N^{\frac{(1-\alpha-\beta)\eta}{1-\alpha}} (B^X L^X)^{1-\eta}
\end{aligned}$$

This reduced-form production function for I^X shows that more land and labor in the numeraire sector also raises output in the construction sector. The reduced-form production function for houses then reads

$$\begin{aligned}
H &= (\kappa^X I^X)^\gamma D^{1-\gamma} \\
&= (\kappa^X)^\gamma (\kappa^M)^{\eta\gamma} (\kappa^K)^{\frac{\eta\gamma\alpha}{1-\alpha}} (B^Y L^Y)^{\frac{\eta\gamma\beta}{1-\alpha}} N^{\frac{(1-\alpha-\beta)\eta\gamma}{1-\alpha}} (B^X L^X)^{(1-\eta)\gamma} D^{1-\gamma}
\end{aligned} \tag{24}$$

In this reduced-form function the true role of land for the production of housing can be assessed. It not only enters via D directly, but indirectly through Y and I^X via N . We define the land intensity in the housing sector as $\psi^H \equiv \frac{1 - \alpha - \beta}{1 - \alpha} \eta\gamma + 1 - \gamma = \psi^Y \eta\gamma + 1 - \gamma$. It depends on the land intensity in the Y sector. This channel is shut down if $\eta = 0$, because then materials are irrelevant for producing structures.

Making use of the definitions of ψ^Y and ψ^H the growth rates can be reformulated. We substitute γ and β with ψ^Y and ψ^H . The resulting growth rates are shown in table A.1.

□

Proof of corollary 2.1.

- #1) Wealth-to-income ratios are constant because assets grow at the same rates as income, g^Y , when measured in units of the numeraire. To be precise, it holds for the total wealth to income rate, W/NNP , the housing wealth-to-income ratio, $P^H H/NNP$, and the non-housing wealth-to-income ratio, $(W - P^H H)/NNP$, that the numerator grows at the same rate as the denominator.
- #2) The residential land price grows at a strictly positive rate iff $g^Y > 0$. The residential structure price grows at a strictly positive rate iff $(1 - \psi^Y) g^Y > g^X$. The house price grows at a strictly positive rate iff $g^X < \frac{1 - \psi^Y}{1 - \psi^H} \frac{1 - \eta + \eta(\psi^H - \psi^Y)}{1 - \eta} g^Y$. This restriction holds if $g^X < g^Y$ and $\psi^H \geq \psi^Y$ (sufficient, not necessary condition). P^D grows at a larger rate than P^H iff $g^X > -\frac{\eta}{1 - \eta} (1 - \psi^Y) g^Y$. Similarly, P^H grows at a larger rate than P^X iff $g^X > -\frac{\eta}{1 - \eta} (1 - \psi^Y) g^Y$.
- #3) Residential structures grow at a strictly positive rate that is larger than the growth rate of residential land iff $g^X > -\frac{\eta}{1 - \eta} (1 - \psi^Y) g^Y$.
- #4) The land share in total housing wealth is $P^D D / (P^H H)$. It is constant because numerator and denominator grow both at the rate g^Y .
- #5) Rents grow at a strictly positive rate iff $g^X < \frac{1 - \psi^Y}{1 - \psi^H} \frac{1 - \eta + \eta(\psi^H - \psi^Y)}{1 - \eta} g^Y$.

□

A.5. Demand-supply representation

Housing The supply of housing is constant in t because H is a state variable. The inverse demand can be expressed as⁴⁵

$$q_t = \frac{\theta \mu_t (W_t + \tilde{w}_t)}{H_t}, \quad (25)$$

with

$$\mu_t \equiv \left(\int_t^\infty \left[\left(\frac{q_\tau}{q_t} \right)^\theta \exp \left[-\tilde{r}(\tau, t) - \frac{\rho}{\sigma - 1} (\tau - t) \right] \right]^{\frac{\sigma - 1}{\sigma}} d\tau \right)^{-1},$$

$$\tilde{r}(\tau, t) \equiv \int_t^\tau r_v dv, \quad \text{and} \quad \tilde{w}_t \equiv \int_t^\infty w_\tau L_\tau e^{-\tilde{r}(\tau, t)} d\tau.$$

The expression $(W_t + \tilde{w}_t)$ is total wealth, consisting of physical and human wealth. Physical wealth is W_t and human wealth \tilde{w} is the net-present value of the stream of future earnings. The propensity to consume out of total wealth is μ_t . Total period consumption, $\mu_t (W_t + \tilde{w}_t)$, is allocated between housing and non housing where the share going to housing is θ . The value of housing consumption is therefore given by $\theta \mu_t (W_t + \tilde{w}_t)$. When studying the demand for housing we abstract from the feedback of q through the propensity to consume. If $\sigma = 1$, then $\mu = \rho$ and this channel vanishes anyway.

⁴⁵For the derivation of the consumption function see Grossmann et al. (2019).

In steady state we can directly relate q to house prices. Making use of the no-arbitrage condition for houses yields

$$P^H = \frac{CF}{H\left(r - \frac{\beta}{1-\alpha}g^Y\right)} = \frac{qH - (g_X + \delta^X)P^X X}{H\left(r - \frac{\beta}{1-\alpha}g^Y\right)} = \frac{q}{r - \frac{\beta}{1-\alpha}g^Y} - \frac{g_X + \delta^X}{r - \frac{\beta}{1-\alpha}g^Y} P^X \left(\frac{X}{D}\right)^{1-\gamma}.$$

Structures and land Let $R^D \equiv (1-\gamma)qH/D$ and $R^X \equiv \gamma qH/X - \delta^X P^X$ denote the yield components of developed land D and residential structures X , respectively. Replacing qH by (25) in the definition of R^D and R^X yields

$$R^D = (1-\gamma) \frac{\theta\mu(W + \tilde{w})}{D} \quad \text{and} \quad R^X = \gamma \frac{\theta\mu(W + \tilde{w})}{X} - \delta^X P^X.$$

Due to the CD-function for H both R^D and R^X react to changes in total wealth, $W + \tilde{w}$, in the same proportions (abstracting from $\delta^X P^X$ for the moment). These two equations are inverse demand functions for N and D . Both shift outwards when the demand for housing, qH , increases.

How do changes in yields components finally translate into prices of the stocks X and D ? By solving the no-arbitrage conditions for X and D both P^X and P^D can be expressed

$$P_t^D = \int_t^\infty e^{-\tilde{r}(\tau,t)} R_\tau^D d\tau$$

$$P_t^X = \int_t^\infty e^{-\tilde{r}(\tau,t)} R_\tau^X d\tau.$$

Assume that r is constant (or changes only very little). Then changes in some future R_τ , $\tau \geq t$, translate into changes in P_t . So when we explain why R increases then this directly implies that P_t increases (this is all partial equil. analysis, hence we can ignore r -dynamics).

B. Calibration of UK, France, and Germany

The parameters α , β , δ^K , δ^X , η , σ , and ξ are set to the same values as for the US. We set the housing expenditure share, θ , to 17, 22, and 27 percent for the UK, France, and Germany, respectively.⁴⁶ The exogenous population series are all taken from UN (2019). For each country we fit the same logistic differential equation (16). The obtained values for coefficient a are 1.9, 1.8, and 1.2 for the UK, France, and Germany, respectively. Similarly, for b we get 0.01, 0.02, and 0.04 for the UK, France, and Germany, respectively. The remaining set of parameters $\{\rho, \gamma, g^Y, g^X, K_0, D_0, X_0\}$ is calibrated by matching a corresponding subset of the moments we target in section 5.2. Our sample periods for the UK, France, and Germany are 1950–2012, 1960–2012, and 1962–2012, respectively. The moments we match and the resulting parameter values are shown in table B.2 to B.4.

⁴⁶The data is from each country's national accounts as documented in our companion paper Grossmann *et al.* (2019).

#	Parameter	Explanation	Value
1	ρ	time preference rate	0.011
2	g^Y	technical growth in numeraire sector	0.026
3	K_0/K	initial capital stock (share of final)	0.337
4	γ	structures' elasticity in H	0.743
5	g^X	technical growth in construction sector	-0.035
6	X_0/X	initial stock of residential structures	0.212
7	D_0/D	initial stock of residential land	0.740

(1) Targets

	Moment	Data	Model	rel difference in %
1	$\frac{W}{NNP}$, 1950	4.0	4.0	0.000
2	$\frac{NNP_{2012}}{NNP_{1950}}$	5.5	5.5	0.000
3	$\frac{P^H H}{NNP}$, in 1950	1.2	1.2	0.000
4	$\frac{P^D D}{P^H H} \times 100$, 1950	39.2	39.2	0.000
5	$\frac{q_{2012}}{q_{1950}}$	3.1	3.1	0.000
6	$\frac{RESI}{GDP} \times 100$, 1962	5.0	5.0	0.000
7	$\frac{X_{2012}/D_{2012}}{X_{1950}/D_{1950}}$	2.1	2.1	0.000

(2) Endogenous parameters

Notes: Initial states, K_0, D_0, X_0 , are expressed relative to their respective final steady state values (normalized).

Table B.2: Calibration UK

#	Parameter	Explanation	Value
1	ρ	time preference rate	0.037
2	g^Y	technical growth in numeraire sector	0.015
3	K_0/K	initial capital stock	0.147
4	γ	structures' elasticity in H	0.920
5	g^X	technical growth in construction sector	-0.049
6	X_0/X	initial stock of residential structures	0.141
7	D_0/D	initial stock of residential land	6.562

(1) Targets

	Moment	Data	Model	rel difference in %
1	$\frac{W}{NNP}$, 1960	2.6	2.6	0.027
2	$\frac{NNP_{2012}}{NNP_{1960}}$	4.1	4.1	0.643
3	$\frac{P^H H}{NNP}$, in 1960	1.4	1.4	0.171
4	$\frac{P^D D}{P^H H} \times 100$, 1960	14.1	14.1	0.046
5	$\frac{q_{2012}}{q_{1960}}$	2.8	2.8	-0.653
6	$\frac{RESI}{GDP} \times 100$, 1970	9.4	9.4	0.016
7	$\frac{X_{2012}/D_{2012}}{X_{1960}/D_{1960}}$	4.0	4.0	-0.398

(2) Endogenous parameters

Notes: Initial states, K_0, D_0, X_0 , are expressed relative to their respective final steady state values (normalized).

Table B.3: Calibration France

#	Parameter	Explanation	Value
1	ρ	time preference rate	0.053
2	g^Y	technical growth in numeraire sector	0.021
3	K_0/K	initial capital stock	0.287
4	γ	structures' elasticity in H	0.671
5	g^X	technical growth in construction sector	0.013
6	X_0/X	initial stock of residential structures	0.972
7	D_0/D	initial stock of residential land	0.127

(1) Targets

	Moment	Data	Model	rel difference in %
1	$\frac{W}{NNP}$, 1962	2.2	2.6	20.097
2	$\frac{NNP_{2012}}{NNP_{1962}}$	3.4	3.5	3.785
3	$\frac{P^H H}{NNP}$, in 1962	1.4	1.4	-3.094
4	$\frac{P^D D}{P^H H} \times 100$, 1962	22.9	22.9	0.042
5	$\frac{q_{2012}}{q_{1962}}$	1.6	1.2	-21.882
6	$\frac{RESI}{GDP} \times 100$, 1970	7.3	7.2	-0.84
7	$\frac{X_{2012}/D_{2012}}{X_{1962}/D_{1962}}$	1.2	0.4	-66.507

(2) Endogenous parameters

Notes: Initial states, K_0, D_0, X_0 , are expressed relative to their respective final steady state values (normalized).

Table B.4: Calibration Germany