Distributional Consequences of Surging Housing Costs under Schwabe's Law*

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Abstract

Housing costs have been increasing in most industrialized economies since WW2. The burden of rising housing cost is distributed unequally across income groups because the poor spend a larger fraction of their total consumption expenditures on housing than the rich. This negative relation of housing expenditure shares and income is called Schwabe's law. We study how rising housing cost affect the dynamics of wealth inequality and welfare under the explicit consideration of Schwabe's law. We set up a frictionless two-sectoral macroeconomic model with a housing sector. It is shown that in partial equilibrium (i) rising housing cost may reduce wealth inequality, (ii) this is unaffected by Schwabe's law, and (iii) capturing Schwabe's law amplifies welfare effects. We then study in general equilibrium how the abolishment of zoning regulations affects wealth inequality and welfare through housing costs and other prices. Although the effect on wealth inequality is small, the consequence for welfare is pronounced and asymmetric across the wealth distribution. Wealth poor households benefit from the abolishment of zoning regulations while wealth rich households are worse off. These results are amplified by Schwabe's law.

Key words: Macroeconomics and Housing; Long-Term Growth; Schwabe's Law; Wealth Inequality; Welfare.

JEL classification: E10, E20, O40.

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1 Introduction

Since WW2 real housing rents and real house prices have been rising, on average, in most industrialized economies (Knoll 2017; Knoll, Schularick, and Steger 2017). Moreover, the aggregate share of housing expenditures in total consumption expenditures (referred to as housing expenditure share) is the largest single expenditure category and is fairly stable over time at 19 percent in the postwar US (U.S. Bureau of Labor Statistics 2016; Piazzesi and Schneider 2016). At a given point in time, however, the housing expenditure share varies inversely with income. For instance, US households in the first income quintile devoted about 25 percent of their total expenditure to housing in 2015, whereas this number was only 18 percent for the fifth income quintile.[1] This empirical pattern has already been documented more than 150 years ago under the name *Schwabe's law* (Singer 1937; Stigler 1954). Indeed, Stigler (1954, p. 100) characterized it as the second fundamental law of consumer behavior.[2]

> *Hermann Schwabe, the director of the Berlin statistical bureau, proposed a second "law" in 1868. He had salary and rent data for 4,281 public employees receiving less than 1,000 thaler a year, and income and rent data for 9,741 families with incomes in excess of 1,000 thaler. For each group he found the percentage of income (or salary) spent on rent declined as income rose, and proposed the law: 'The poorer any one is, the greater the amount relative to his income that he must spend for housing.' The law seemed to contemporaries less obviously true than Engel's, and a considerable literature arose about it. Ernst Hasse found that it held for Leipzig in 1875, and E. Laspeyres confirmed it for Hamburg. Engel also accepted Schwabe's law.*[3]

Surging housing costs under asymmetric spending patterns for housing across income groups have initiated a debate on the implications for wealth inequality and welfare (Summers 2014; Albouy, Ehrlich, and Liu 2016; Albouy and Ehrlich 2018; Dustmann,

[1] Employing microdata for the US, Albouy, Ehrlich, and Liu (2016) estimate an income elasticity of housing expenditures smaller than unity. This microevidence supports Schwabe's law. Moreover, the cross-sectional variation of housing expenditure shares is even more pronounced in other advanced economies, such as France, Germany, and the UK (Section 5.1).

[2] The first law of consumer behavior is the well-known Engel's law (Stigler 1954).

[3] According to Singer (1937, p. 145) Schwabe's law states that *the proportion of rent in income or expenditure is a continuously diminishing fraction of income.* Schwabe's law can hence be defined in terms of the share of housing expenditures in income or in total expenditures. We follow the latter definition and refer to Schwabe's law as the negative relation between the share of housing expenditures in total expenditures and income.

Fitzenberger, and Zimmermann 2018). In this paper we investigate the dynamics of wealth inequality and the determinants of welfare in a growing economy that experiences surging housing costs.[4] Specifically, our analysis addresses *two research questions*. (1) How do the dynamics in real rents interact with (i) wealth inequality and (ii) welfare in a growing economy? (2) How do these interactions depend on Schwabe's law?[5] We first examine the isolated effects of exogenously increasing rents in partial equilibrium. This step is helpful for our general equilibrium analysis which features endogenous rent growth. As a natural candidate for an exogenous event that triggers changes in the time path of rents, we consider the abolishment of zoning regulations. In fact, zoning regulations are widely recognized as an important amplifier of surging rents in a growing economy (Glaeser, Gyourko, and Saks 2005; Saiz 2010; Albouy and Ehrlich 2018).

We employ a frictionless dynamic general equilibrium model with a housing sector. Abstracting from financial frictions enables us to derive analytical insights into the dynamics of wealth inequality and the determinants of welfare. Our analysis captures the impact of future expected rent growth on the saving decisions of forward-looking households. The supply side of the model, which is introduced to endogenize rents, follows the long-term macro and housing model of Grossmann and Steger (2017). It distinguishes between the extensive margin (the number of houses) and the intensive margin (the size of the average house) of the housing stock. This model structure lends itself to investigating the consequences of removing those policies that regulate the use of land for residential purposes and, therefore, primarily constrain the extensive margin of the housing stock. Households are heterogeneous with respect to initial wealth and labor income (Chatterjee 1994; Caselli and Ventura 2000). The demand side features non-homothetic preferences so as to replicate the inverse variation of housing expenditure shares across income groups (Schwabe's law). Specifically, we assume that households have status concerns with respect to housing, which is in line with empirical evidence (Leguizamon and Ross 2012; Bellet 2017).

[4]Housing costs are either user cost of housing (in the case of homeowners) or rents (in the case of renters). The major part of the paper is framed in terms of renter households. Nonetheless, our analysis applies equally to homeowners as well as renters (Appendix A.1).

[5]Heterogeneity of housing expenditures as percentage of total consumption expenditures across income groups, which turns out to be directly relevant for welfare, is closely related to Schwabe's law, which focuses on housing expenditures relative to income (Section 2).

The analysis proceeds in two steps. In the first, we investigate the dynamics of wealth inequality and the determinants of welfare in partial equilibrium. It is shown that stronger rent growth produces *less* wealth inequality in partial equilibrium, provided that the utility function is sufficiently concave. The reason is that the differences in the saving rates across wealth groups (a force contributing to diverging wealth holdings in the population) shrinks in response to stronger rent growth. This counterintuitive result appears to be robust across a large set of models, as it depends merely on the widely accepted assumptions of forward-looking and optimizing households. The analysis also indicates, somewhat surprisingly at first glance, that Schwabe's law is not important with regard to the dynamics of wealth inequality. This insight is in striking contrast to the welfare implications. Stronger status concerns regarding housing induce greater heterogeneity in housing expenditure shares. This amplifies the welfare differences by enlarging the heterogeneity in household-specific price indices. That is, Schwabe's law is important with regard to welfare inequality. The underlying mechanism appears empirically plausible. For instance, Albouy, Ehrlich, and Liu (2016) show that real income inequality in the US increased 25 percent more since 1970, when deflated with household-specific price indices.

In a second step, we analyze a growing economy in general equilibrium. It is shown that the wealth distribution is stationary in steady state, despite continuously rising housing costs. However, any policy that induces transitional dynamics triggers a permanent change in the wealth distribution. We show, by calibrating the model to the US economy, that removing residential zoning regulations leads to a temporarily slower rent growth, relative to the baseline scenario, which is associated with a reduction in the top 10 percent wealth share by 0.7 percentage points over time. That is, in contrast to the partial equilibrium result, rent growth and wealth inequality are *positively* associated, i.e. slower rent growth induced by abolishing zoning regulations goes hand in hand with a reduction in wealth inequality.[6] Average welfare increases by about 0.5 percent. However, the household-specific welfare effects are asymmetric, so that the poor benefit more

[6]The reason, in a nutshell, is that the induced supply response in the housing sector suppresses future wage growth, in addition to lowering rent growth, such that households increase their saving rates to smooth consumption over time. This effect is especially pronounced for the wealth poor (Section 5.2).

than the rich. The richest wealth decile is even worse off, while welfare of the poorest wealth decile increases by 1.3 percent. The important lessons to be drawn from this policy experiment are twofold. First, despite a potentially negative effect of surging rents on wealth inequality, a policy measure that slows down rent growth may nevertheless lower welfare inequality through its additional general equilibrium effects. Second, under Schwabe's law, surging rents are unambiguously and positively associated with welfare inequality and harmful for the poor but not necessarily for the rich.

There are three strands of related literature, the first of which addresses the importance of the housing sector in macro models. Many models are designed to discuss business cycle phenomena, such as Davis and Heathcote (2005), Iacoviello (2005), Iacoviello and Neri (2010), Kiyotaki, Michaelides, and Nikolov (2011), Favilukis, Ludvigson, and Van Nieuwerburgh (2017), and Kydland, Rupert, and Šustek (2016).[7] More recently, a literature has emerged that focuses on the long term, such as Grossmann and Steger (2017), Miles and Sefton (2017) and Borri and Reichlin (2018). Our research questions necessarily require a long-term perspective. The second strand of literature analyzes one-sector economies under household heterogeneity with the representative household property (Chatterjee 1994; Krusell and Rios-Rull 1999; Caselli and Ventura 2000; Alvarez-Pelaez and Díaz 2005; Garcia-Penalosa and Turnovsky 2006). We add to this literature by analyzing a two-sectoral model, allowing for a continuous relative price change, under household heterogeneity with non-homothetic preferences and the representative household property. A third strand examines savings behavior and wealth inequality, allowing for idiosyncratic shocks under incomplete markets. These contributions typically focus on alternative mechanisms that shape the wealth distribution in steady state.[8] We explore mechanisms that shape the dynamics of wealth inequality and welfare differences, apart from borrowing constraints, and provide analytical insights that apply equally to transitional dynamics and the steady state. Our analysis rests on fundamental market forces that would prevail equally in an economy with borrowing constraints, which we ignore for the sake of analytical results.

[7] Piazzesi and Schneider (2016) provide an excellent survey.

[8] Some exceptions analyze the wealth distribution over time by employing numerical techniques, such as Gabaix et al. (2016), Kaas et al. (2017), Kaymak and Poschke (2016), Hubmer, Krusell, and Smith Jr (2016), and Wälde (2016). De Nardi and Fella (2017) provide an excellent survey.

The structure of this paper is as follows. Section 2 describes the household side and provides some partial equilibrium results. Section 3 discusses analytical insights into the dynamics of wealth inequality and the determinants of household-specific welfare. Section 4 introduces the production side and characterizes the steady state. Section 5 investigates the consequences of removing zoning regulations numerically in general equilibrium.

2 Households

2.1 Setup

Consider a deterministic economy inhabited by mass one of infinitely lived households. There are $J \in \mathbb{N}$ different groups of households indexed by $j \in \{1, 2, \ldots, J\}$. Each group of households j is of equal size and therefore consists of measure $1/J$ identical households. In what follows, when referring to a *household of group* j we use the short formulation *household* j instead, keeping in mind that a group consists of a continuum of households.

We consider two sources of heterogeneity: i) differences in initial wealth and ii) differences in permanent labor productivity. Time is continuous and indexed by $t \geq 0$. Let $W_j(t) \in \mathbb{R}$ denote wealth holdings of household j at point in time t. Initial wealth is then denoted by $W_j(0)$. Household j's labor productivity is time-invariant, denoted by $L_j \in \mathbb{R}^+$, and supplied inelastically to the labor market. To simplify notation, we omit the time indices when this is not confusing and define average variables by omitting the group index j. Since total population is normalized to one, aggregates are equal to averages of a variable. For instance, aggregate wealth is defined as $W = \sum_j^J \frac{1}{J} W_j$ and equal to average wealth. Similarly, aggregate as well as average labor supply is given by $L = \sum_j^J \frac{1}{J} L_j$.

Let $C_j \in \mathbb{R}^+$ and $S_j \in \mathbb{R}^+$ denote consumption of the numeraire good and consumption of housing services by household j, respectively. Life-time utility of household j is described by

$$U_j(0) = \int_0^\infty u\left(C_j(t), S_j(t), S(t)\right) \mathrm{e}^{-\rho t} \mathrm{d}t \tag{1}$$

$$\text{with} \qquad u\left(C_j, S_j, S\right) = \frac{[(C_j)^{1-\theta}(S_j - \phi S)^{\theta}]^{1-\sigma} - 1}{1-\sigma}, \tag{2}$$

where $\sigma > 0$, $\theta \in (0,1)$, and $\rho > 0$ are common utility parameters. Households compare their level of housing services to a share $\phi \in [0,1)$ of the average level of housing services, S, which is exogenous to the household. With $\phi > 0$ utility function (2) is non-homothetic and captures status preferences for housing. Status preferences for housing represent an old topic that has already been discussed by Marx (1847).[9] There is also ample evidence for status preferences for housing.[10] For instance, by employing US microdata, Bellet (2017) shows that suburban homeowners who experienced a relative downscaling of their homes due to the building of larger units in their suburb record lower satisfaction with their home. Alternatively, preferences in (2) can also be interpreted as Stone-Geary preferences with the minimum, possibly time-varying, housing consumption level given by ϕS. Most of the results presented below do not depend on the chosen interpretation.[11] The major motivation for utility specification (2) is that it enables us to replicate Schwabe's law under $\phi > 0$, as shown below.

Each household j chooses consumption paths $\{C_j(t), S_j(t)\}_{t=0}^{\infty}$ that maximize U_j subject to the standard No-Ponzi-game condition and the intertemporal budget constraint[12]

$$\dot{W}_j = rW_j + wL_j - C_j - pS_j, \tag{3}$$

where r, w, and p denote the interest rate, the wage rate per unit of labor, and the housing rent, respectively. All prices are measured in units of a final output good. The budget

[9] According to Marx (1847) *A house may be large or small; as long as the neighboring houses are likewise small, it satisfies all social requirement for a residence. But let there arise next to the little house a palace, and the little house shrinks to a hut. The little house now makes it clear that its inmate has no social position at all to maintain, or but a very insignificant one; and however high it may shoot up in the course of civilization, if the neighboring palace rises in equal or even in greater measure, the occupant of the relatively little house will always find himself more uncomfortable, more dissatisfied, more cramped within his four walls.*

[10] See Bellet (2017), Frank (2005), Turnbull, Dombrow, and Sirmans (2006), and Leguizamon and Ross (2012).

[11] In Online-Appendix C we discuss alternative utility specifications. First, we show that the results are robust to assuming status concerns (or minimum consumption requirements) for both goods, provided they are stronger for housing. Second, we also demonstrate that alternative formulations (CES utility and multiplicative reference level) are inconsistent with empirical observations.

[12] A dot above a variable denotes the partial derivative with respect to time.

constraint states that changes in wealth equal the excess of income over consumption expenditures. Since a household has mass zero, it takes all prices, r, w, and p, as well as the average consumption of housing services, S, as given. We model all households as renters, but the results do not change if we assume owner-occupied housing, as shown in Appendix A.1.

2.2 Representative Household

For given initial wealth, $W_j(0)$, labor endowment, L_j, and price series $\{p(t), r(t), w(t)\}_{t=0}^{\infty}$, the solution of the household problem is fully characterized for all $t \geq 0$ by

$$C_j(t) + p(t)S_j(t) = \mu(t)\left[W_j(t) + \widetilde{w}(t)L_j\right] \tag{4}$$

$$C_j(t) = p(t)\frac{1-\theta}{\theta}\left[S_j(t) - \phi S(t)\right] \tag{5}$$

$$\dot{W}_j(t) = \left[r(t) - \mu(t)\right]W_j(t) + \left[w(t) - \mu(t)\widetilde{w}(t)\right]L_j \tag{6}$$

with

$$\mu(t) \equiv \left(\int_t^{\infty}\left[\left(\frac{p(\tau)}{p(t)}\right)^{\theta}\exp\left[-\widetilde{r}(\tau,t) - \frac{\rho}{\sigma-1}(\tau-t)\right]\right]^{\frac{\sigma-1}{\sigma}}\mathrm{d}\tau\right)^{-1}, \tag{7}$$

$$\widetilde{r}(\tau,t) \equiv \int_t^{\tau} r(v)\mathrm{d}v, \quad \text{and} \quad \widetilde{w}(t) \equiv \int_t^{\infty} w(\tau)e^{-\widetilde{r}(\tau,t)}\mathrm{d}\tau. \tag{8}$$

All derivations and proofs are relegated to the appendix. The variable $\widetilde{w}$ is the present value of the time path of future wages at t and $e^{-\widetilde{r}(\tau,t)}$ is a present-value factor that converts one unit of income at time τ to an equivalent unit of income at time t. Equation (4) states that total consumption expenditures, $C_j + pS_j$, are a fraction μ of *total* wealth, $W_j + \widetilde{w}L_j$, which is the sum of wealth, W_j, and the present value of future labor income, $\widetilde{w}L_j$. Variable μ is hence the propensity to consume out of total wealth. Equation (5) determines the optimal allocation of total consumption expenditures, $C_j + pS_j$, over the numeraire, C_j, and housing, S_j. It is already visible that relative consumption, pS_j/C_j, is not constant across households when $\phi > 0$, implying that expenditure shares vary with household income. Equation (6) is the law of motion of wealth, W_j. This linear differential equation in W_j with time-varying coefficients can easily be solved to yield an explicit solution for W_j as a function of time.

In order to rule out negative utility, we assume that for all j initial wealth, $W_j(0)$, is strictly greater than the threshold $\underline{W}_j(0) \equiv \frac{\phi}{1-(1-\theta)\phi}[W(0) + \widetilde{w}(0)L] - \widetilde{w}(0)L_j$.[13] With $\phi = 0$ this condition implies that total wealth, $W_j(0) + \widetilde{w}(0)L_j$, has to be strictly positive. With $\phi > 0$ the minimum wealth level is larger because households have also to afford $\phi S > 0$ units of housing services.

We define a representative household as a household who owns the economy's endowments and whose individual consumption and asset demand is equal to aggregate consumption and asset demand that result from a set of different households (Mas-Colell, Whinston, and Green 1995; Caselli and Ventura 2000). Since all first order conditions are linear in household-specific variables, aggregation is possible and we obtain

Proposition 1 (Representative household)**.** *An economy populated by a set of households whose preferences are described by (1) together with (2) and who face an intertemporal budget constraint given by (3) admits a representative household.*

As a consequence, the distributions of labor endowment, L_j, and wealth, W_j, play no role for the evolution of aggregate variables. The proposition holds for all values of $\phi \in [0, 1)$ such that a representative household exists also in the case of non-homothetic preferences ($\phi > 0$). It can already be seen from equations (4) and (5) that households face the same linear wealth expansion paths.[14] This also implies that preferences given by (1) and (2) admit an indirect utility function of the Gorman form, which can be obtained with a monotonic transformation of the utility function. Proposition 1 adds to the theoretical literature on dynamic macro models with a representative household and heterogeneity as it shows that a representative household may exist not only in a one-sectoral economy (Chatterjee 1994; Krusell and Rios-Rull 1999; Caselli and Ventura 2000; Alvarez-Pelaez and Díaz 2005; Garcia-Penalosa and Turnovsky 2006), but also in a two-sectoral economy under non-homothetic preferences.

[13] This threshold results from $C_j^{1-\theta}(S_j - \phi S)^\theta = \mu(W_j + \widetilde{w}L_j)/\mathcal{P}_j > 0$, where $\mathcal{P}_j$ is the ideal price index, derived below.

[14] A necessary and sufficient condition for expressing aggregate demand as a function of aggregate wealth and prices is that all households' wealth expansion paths are parallel, straight lines (see, for instance, Mas-Colell, Whinston, and Green 1995, ch. 4).

The assumption of market completeness, which is necessary for a representative household to exist, surely has costs and benefits. On the one hand, it implies that we abstract from feedback effects of changes in the wealth and income distributions on aggregate variables. On the other hand, this setup enables us to gain analytical insights into the dynamics of wealth inequality and the determinants of welfare. In fact, abstracting from the feedback effects of distributional changes on aggregate variables, including prices, is precisely the reason why we are able to derive analytical insights. In this paper we therefore investigate fundamental mechanisms that operate even in the absence of incomplete markets.

2.3 Schwabe's law

We replicate the observed variation of the housing expenditure share in the cross-section – Schwabe's law – and the constancy of the aggregate housing expenditure share over time by employing utility specification (2). Let $e_j \equiv pS_j/(C_j + pS_j)$ denote the housing expenditure share and $\mathcal{W}_j \equiv W_j + \widetilde{w}L_j$ total wealth of household j. From the household's first order conditions (4) and (5) one obtains the optimally chosen housing expenditure share e_j as described by

Proposition 2 (Schwabe's law). *Household j's expenditure share for housing services is time invariant and given by*

$$e_j = \theta \left[1 + \frac{(1-\theta)\phi}{1-(1-\theta)\phi} \left(\frac{\mathcal{W}_j(0)}{\mathcal{W}(0)} \right)^{-1} \right]. \tag{9}$$

If and only if $\phi > 0$, the housing expenditure share, e_j, is declining in relative total wealth of household j at the initial period, $\mathcal{W}_j(0)/\mathcal{W}(0)$, and Schwabe's law can be replicated. The variance in housing expenditure shares is increasing in ϕ.

If $\phi = 0$, the utility function is homothetic, status preferences for housing do not exist and each household optimally chooses the same housing expenditure share $e_j = \theta$. This is clearly at odds with Schwabe's law. Only for strictly positive values of ϕ the housing expenditure share varies across households. This is due to differences in total wealth at the initial period, $\mathcal{W}_j(0)$. Households with larger initial wealth or larger labor income have a lower housing expenditure share. The aggregate housing expenditure share is a function of parameters only and is given by $e \equiv pS/(C + pS) = \frac{\theta}{1-(1-\theta)\phi}$. With status preferences for housing we can hence explain both i) the negative relation of income and housing expenditure shares in the cross section – Schwabe's law – as well as ii) the constant aggregate housing expenditure share over time.

3 Wealth and Welfare in Partial Equilibrium

We are now ready to discuss the dynamics of wealth inequality and the determinants of welfare analytically in partial equilibrium where prices are taken as exogenous. The mechanisms discussed below still hold in general equilibrium when prices are fully endogenous.

3.1 Surging Rents and Wealth Inequality

3.1.1 Wealth Divergence and Wealth Convergence

Let a variable with the superscript R denote the ratio of this variable to the average of the respective variable. For instance, relative wealth is given by $W_j^R(t) \equiv W_j(t)/W(t)$. Following Caselli and Ventura (2000) we define *individual wealth convergence* as a decrease in $|W_j^R(t) - 1|$ over a small time interval $t \in [t, t + \mathrm{d}t]$. This implies increasing relative wealth for a wealth-poor household ($W_j^R < 1$) and declining relative wealth for a wealth-rich household ($W_j^R > 1$). For divergence the opposite holds true.

To understand the different forces that trigger individual wealth divergence or convergence, it is helpful to study saving rates first. The saving rate is defined as the share of income spend on saving, $s_j \equiv \dot{W}_j/y_j$, where $y_j \equiv rW_j + wL_j$ is income. Making use of the budget constraint (3) and equation (4) yields

$$s_j = 1 - \mu \frac{W_j + \widetilde{w}L_j}{rW_j + wL_j} = 1 - \mu \frac{\mathcal{W}_j}{y_j}. \tag{10}$$

The saving rate of household j is one minus the consumption rate and the consumption rate is total consumption expenditures, $\mu\mathcal{W}_j$, over income, y_j. If $r\widetilde{w} > (<) w$, the saving rate is increasing (decreasing) in individual wealth W_j, as can be seen by taking the derivative of (10) with respect to household wealth W_j. The condition $r\widetilde{w} > w$ is satisfied in any steady state with positive wage growth.[15] The result that saving rates are larger for wealth-rich individuals than for wealth-poor individuals is in line with empirical observations as documented by, for instance, Dynan, Skinner, and Zeldes (2004).[16]

Wealth divergence or convergence depends on whether $|W_j^R - 1|$ increases or decreases over time. We therefore study the derivative of relative wealth with respect to time, given by

$$\dot{W}_j^R = \left(\frac{\dot{W}_j}{W_j} - \frac{\dot{W}}{W}\right) W_j^R = \left(s_j \frac{y_j}{W_j} - s\frac{y}{W}\right) W_j^R. \tag{11}$$

The first equation states that the change of household j's relative wealth over time is given by the difference in growth rates of household j's wealth and average wealth, multiplied by the level of household j's wealth. The second equality is obtained by making use of the definition of the saving rate, which implies that the growth rate of wealth equals the saving rate, s_j, multiplied by the income to wealth ratio, y_j/W_j. What matters for individual wealth convergence or divergence is the sign of $\dot{W}_j^R$, which is determined by the term in brackets in (11), as will be studied in the following.

Equation (11) shows that there are two opposing forces at work. On the one hand, wealth-rich households choose a higher saving rate than wealth-poor households, provided that $r\widetilde{w} > w$. If wealth-rich households save sufficiently more than the average, their relative wealth is increasing, while the opposite applies to wealth-poor households. This is a divergence mechanism. In the limiting case where all households have the same income to wealth ratio (which is equivalent to assuming that $W_j^R = L_j^R$), the sign of $\dot{W}_j^R$ depends only on the difference in saving rates, $(s_j - s)$. On the other hand, the income to wealth ratio, $y_j/W_j = r + w\frac{L_j}{W_j}$, is decreasing in W_j. This variation contributes to

[15] Assuming $w(\tau) = w(t)e^{(\tau-t)g}$ with g being the growth rate of wages, we have $\widetilde{w}(t) = \frac{w(t)}{r-g}$. Plugging this into $r\widetilde{w}(t) > w(t)$ results in $g > 0$.

[16] The relation between saving rates and wealth can be best understood by comparing a pure capitalist, defined by $W_j > 0$ and $L_j = 0$, with a pure worker, defined by $W_j = 0$ and $L_j > 0$. The total wealth to income ratio, $\mathcal{W}_j/y_j$, is equal to $1/r$ for the pure capitalist and $\widetilde{w}/w$ for the pure worker. The condition $\widetilde{w}/w > 1/r$ (or $r\widetilde{w} > w$) is thus equivalent to assuming that the total wealth to income ratio of a pure worker exceeds that of a pure capitalist. In this case the total wealth to income ratio is decreasing in W_j. Hence, households characterized by a low W_j choose a high level of consumption relative to income, $\frac{\mu \mathcal{W}_j}{y_j}$, implying that the saving rate, s_j, is low, and vice versa.

$\dot{W}_j^R$ being negative for wealth-rich households and positive for wealth-poor households. This is a convergence mechanism. In the limiting case of equal saving rates, the sign of $\dot{W}_j^R$ is determined by the difference $(L_j/W_j - L/W)$, which is negative for values of W_j sufficiently larger than W.

Under what conditions is the convergence mechanism dominating the divergence mechanism and vice versa? In order to study both effects we substitute s_j and s in equation (11) by (10) to get

$$\dot{W}_j^R = \frac{(\mu\widetilde{w} - w)L}{W}\left(W_j^R - L_j^R\right). \tag{12}$$

Whether individual wealth W_j converges or diverges towards the average W depends on i) the sign of $(\mu\widetilde{w} - w)$, which is the same for all households, ii) the sign of $W_j^R - 1$, which is positive for wealth-rich and negative for wealth-poor households, and iii) the sign of $\left(W_j^R - L_j^R\right)$, which is also household-specific. For instance, consider a wealth-rich household, $W_j^R - 1 > 0$, and assume that $(\mu\widetilde{w} - w) > 0$. If $\left(W_j^R - L_j^R\right) > 0$, the household's wealth diverges because $\dot{W}_j^R$ is positive. We provide a full characterization in Table C.1 in the appendix.

So far we have only analyzed wealth convergence or divergence for a specific *household*, but not for the whole economy. We define *global* wealth convergence (divergence) over a small time interval $t \in [t, t + \mathrm{d}t]$ as a situation where individual wealth converges (diverges) for all j.[17] Depending on the joint distribution of wealth, W_j, and earnings, wL_j, it is possible that some household's wealth converges while it diverges for others, and vice versa, as can be seen in Table C.1 in the appendix. In order to derive predictions about the dynamics of the entire wealth distribution, we therefore postulate

Assumption 1 (Joint distribution of wealth and earnings). *For given initial aggregate*

[17] We use the terms *global wealth convergence (divergence)* and *decreasing (increasing) wealth inequality* interchangeably.

wealth, $W(0)$, and aggregate labor endowment, L, it holds for all j that

$$L_j^R \begin{cases} < W_j^R(0) & \textit{if} \quad W_j^R(0) > 1 \qquad \textit{(wealth-rich)} \\ = W_j^R(0) & \textit{if} \quad W_j^R(0) = 1 \qquad \textit{(representative household)} \\ > W_j^R(0) & \textit{if} \quad W_j^R(0) < 1 \qquad \textit{(wealth-poor)} \end{cases} .$$

Note that relative labor endowment, $L_j^R = L_j/L$, equals relative earnings, wL_j/wL. According to Assumption 1, wealth-rich households' relative earnings are smaller than their relative wealth while it is the opposite for wealth-poor households. We hence abstract from wealth-rich households that have very high relative earnings and from wealth-poor households with very low relative earnings. This assumption does not seem too restrictive, given that wealth is empirically more unequally distributed than earnings (Kuhn and Ríos-Rull 2016). Assumption 1 further implies that if $W_j^R(0) > (<) L_j^R$, it follows that $W_j^R(t) > (<) L_j^R$ for all $t \geq 0$. This results from equation (12) according to which relative wealth, $W_j^R(t)$, can at most converge to relative earnings, L_j^R, but not beyond.

With assumption (1) at hand we are now able to derive insights on the dynamics of the entire wealth distribution.

Proposition 3 (Global wealth convergence and divergence)**.** *Global wealth dynamics are characterized by*

$$\frac{\mathrm{d}}{\mathrm{d}t}|W_j^R(t) - 1| \begin{cases} > 0 & \textit{for all } j \textit{ if} \quad \mu(t)\widetilde{w}(t) > w(t) \qquad \textit{(divergence)} \\ = 0 & \textit{for all } j \textit{ if} \quad \mu(t)\widetilde{w}(t) = w(t) \qquad \textit{(stationarity)} \\ < 0 & \textit{for all } j \textit{ if} \quad \mu(t)\widetilde{w}(t) < w(t) \qquad \textit{(convergence)} \end{cases} . \tag{13}$$

First, the wealth distribution is only stationary if $\mu(t)\widetilde{w}(t) = w(t)$ holds. We will show that this condition holds in steady state. It means that consumption out of human wealth equals contemporaneous earnings. As a result, the growth rate of wealth is the same across households and the divergence mechanism (wealth-rich households' saving rates are higher) compensates the convergence mechanism (wealth-rich households' higher level of wealth reduces their growth rate of wealth). Second, the relation of $\mu(t)\widetilde{w}(t)$ and $w(t)$ depends only on parameters and prices $p(v), w(v), r(v)$ for all $v \in [t, \infty]$. For example, the larger future wage growth, the larger is $\widetilde{w}(t)$, the more likely it is that $\mu(t)\widetilde{w}(t) > w(t)$, and the more likely it is that the wealth distribution diverges.

3.1.2 The Rent Channel

The dynamics of the wealth distribution are affected by the time path of real rents as follows:

Proposition 4 (Rent channel)**.** *An increase in real rent growth between current period t and some future period $\tau > t$, measured by an increase in $\frac{p(\tau)}{p(t)}$, contributes to less (more) wealth inequality in the subsequent period $t + \mathrm{d}t$ if $\sigma > 1$ ($\sigma < 1$). The opposite applies for a decrease in $\frac{p(\tau)}{p(t)}$.*

Let us focus on the empirically relevant case of a sufficiently concave utility function ($\sigma > 1$).[18] Stronger rent growth, measured by an increase in $p(\tau)/p(t)$, induces less wealth inequality. The economic intuition can be explained in two steps. First, all households choose a higher saving rate in order to provide for the future rent burden because they wish to smooth housing consumption over time. Saving rates increase because the propensity to consume, μ, is decreasing in $p(\tau)/p(t)$, a direct implication of equations (7) and (10). Second, this increase in saving rates is asymmetric across households. It is stronger for wealth-poor than for wealth-rich households. To see this, notice that the saving rate, $s_j = 1 - \mu\mathcal{W}_j/y_j$, increases as μ is being reduced. Recall that the total-wealth-to-income ratio, $\mathcal{W}_j/y_j$, is decreasing in the wealth level W_j, provided that $r\widetilde{w} > w$. Thus, the wealth-poor exhibit a comparably high total-wealth-to-income ratio, $\mathcal{W}/y_j$, implying that a reduction in the propensity to consume in response to an increase in the growth factor of rents implies a comparably strong increase in the saving rate. As a result, the differences in the saving rates across wealth groups are being reduced, the divergence mechanism is weakened, and the wealth distribution converges.

An increase in the growth factor of rents, $p(\tau)/p(t)$, may not only be driven by a higher future rent, $p(\tau)$, but also by a lower current rent, $p(t)$. The rent channel in

[18] The calibration is explained in Section 5.1.

Proposition 4 therefore equally applies to a situation where the current rent changes. For example, an increase in the current rent induces all households to save less, but wealth-poor households will reduce their saving rates more than wealth-rich households, resulting in rising wealth inequality.

The negative effect of rising rents on wealth inequality depends on the assumptions of forward-looking, optimizing households together with an empirically plausible intertemporal elasticity of substitution ($\frac{1}{\sigma} < 1$). It can therefore be expected to be robust across a large set of different models.[19] Note further that Schwabe's law does not matter in this context as the rent channel is unaffected by ϕ.[20] This is in stark contrast to welfare effects to which we turn next.

3.2 Schwabe's Law and Welfare

How does the status-induced heterogeneity of housing expenditure shares affect the distribution of household-specific welfare in a growing economy? To discuss this question, we compare a baseline economy B with price series $\{p^B(\tau), w^B(\tau), r^B(\tau)\}_{\tau=t}^{\infty}$ and distributions of wealth at t and labor endowments $\{W_j^B(t), L_j^B\}_{j=1}^J$ to an alternative economy A with $\{p^A(\tau), w^A(\tau), r^A(\tau)\}_{\tau=t}^{\infty}$ and $\{W_j^A(t), L_j^A\}_{j=1}^J$. The household-specific welfare gain that is associated with moving from B to A, measured by consumption-equivalent variations, is defined by

$$\int_t^{\infty} \frac{\left[(1+\psi_j)\,\mathcal{C}_j^B(\tau)\right]^{1-\sigma}-1}{1-\sigma} \mathrm{e}^{-\rho(\tau-t)}\mathrm{d}\tau = \int_t^{\infty} \frac{\left[\mathcal{C}_j^A(\tau)\right]^{1-\sigma}-1}{1-\sigma} \mathrm{e}^{-\rho(\tau-t)}\mathrm{d}\tau, \qquad (14)$$

where $\mathcal{C}_j^B \equiv (C_j^B)^{1-\theta}(S_j^B - \phi S^B)^{\theta}$ and $\mathcal{C}_j^A = (C_j^A)^{1-\theta}(S_j^A - \phi S^A)^{\theta}$ denote the consumption index in the baseline economy B and the alternative economy A, respectively. If $\psi_j(t)$ is positive, household j prefers to live in economy A as opposed to economy B, and vice

[19] As a caveat, if the poor cannot finance going consumption expenditures by running into debt, their propensity to consume is lower compared to the unconstrained case. Stronger rent growth may then not allow these households to lower their propensity to consume and increase their saving rate.

[20] This is consistent with a two-stage logic. First, households maximize life time utility with respect to $\mathcal{C}_j$ (intertemporal problem). Second, households maximize instantaneous utility w.r.t. C_j and S_j (intratemporal problem). Within the setup at hand the decisions at both stages are separable. Moreover, Appendix A.1 shows that the mechanisms discussed above do not depend on whether households are modeled as renters or homeowners.

versa.

Household specific price indices will turn out to be crucial for differences in welfare across households. We therefore first derive ideal price indices for this economy. Household j's ideal price index is defined by $\mathcal{P}_j \mathcal{C}_j \equiv C_j + pS_j$ and can be expressed as

Proposition 5 (Ideal price indices)**.** *The ideal price index of household j in period t is given by*

$$\mathcal{P}_j(t) = \frac{p(t)^\theta}{\theta^\theta (1-\theta)^{-\theta}} \frac{1}{1-e_j}. \tag{15}$$

The aggregate and representative household's ideal price index, $\mathcal{P}$, is obtained for $e_j = e$. If $\phi = 0$, housing expenditure shares e_j are equal to θ for all j such that the price index is also equal across households and given by $\mathcal{P}_j = \frac{p^\theta}{\theta^\theta (1-\theta)^{1-\theta}}$. If $\phi > 0$, wealth-poor households face i) a larger price index and ii) a stronger positive effect of rising rents on their price index than wealth-rich households, as can be seen by taking derivatives of (15). This distributional effect works through the heterogeneity in housing expenditure shares which are, under $\phi > 0$, larger for wealth-poor households than for wealth-rich households.[21]

We can now put the pieces together and show how the heterogeneity in housing expenditure shares affect household-specific welfare.

Proposition 6 (Welfare)**.** *The additional welfare household j enjoys from living in economy A instead of B, at any point in time t, is given by*

$$\psi_j(t) = \left(\frac{\mu^A(t)}{\mu^B(t)}\right)^{\frac{\sigma}{\sigma-1}} \frac{\mathcal{W}_j^A(t)}{\mathcal{W}_j^B(t)} \left(\frac{\mathcal{P}_j(t)^A}{\mathcal{P}_j(t)^B}\right)^{-1} - 1. \tag{16}$$

The first two terms are well known from one-sectoral models (e.g. Caselli and Ventura 2000), except that the relative price, p, now also affects the propensity to consume, μ. The third term, $\left(\mathcal{P}_j^A(t)/\mathcal{P}_j^B(t)\right)^{-1}$, adds a new channel. It enters equation (16) due to the two-sectoral structure and it is heterogeneous across j due to non-homothetic preferences, as can be seen as follows. Under *homothetic* preferences, $\phi = 0$, price indices affect welfare

[21] Albouy, Ehrlich, and Liu (2016) construct an ideal cost-of-living index that varies with income and prices. They show that, based on US microdata, real income inequality, measured by the 90 percentile / 10 percentile ratio, rose by 10 percentage points more when income is deflated by their individual cost-of-living index.

in our two-sectoral economy as opposed to a one-sectoral economy, but the effect is the same across households because the price index is the same across households. Price indices affect welfare differently across households only in a two-sectoral economy with *non-homothetic* preferences, $\phi > 0$, because then price indices do not only differ between economies A and B, but also between households. Capturing Schwabe's law with non-homothetic preferences therefore adds additional heterogeneity in household welfare.

In order to study how Schwabe's law affects welfare comparisons, we consider relative welfare that is given by (omitting the time index)[22]

$$\psi_j^R \equiv \frac{1+\psi_j}{1+\psi} = \frac{\dfrac{\mathcal{W}_j^A}{\mathcal{W}^A} - \dfrac{\phi\theta}{1-(1-\theta)\phi}}{\dfrac{\mathcal{W}_j^B}{\mathcal{W}^B} - \dfrac{\phi\theta}{1-(1-\theta)\phi}}. \tag{17}$$

If household j's relative total wealth, $\frac{\mathcal{W}_j}{\mathcal{W}}$, is larger in the alternative economy A than in the baseline economy B, then its additional welfare, ψ_j, is also larger than the average, ψ. By taking the derivative of (17) with respect to ϕ we obtain

Corollary 6.1 (Amplification of welfare differences)**.** *Stronger status concerns with respect to housing amplify, at any t, the welfare differences, measured by $\psi_j(t)$, i.e.*

$$\frac{\partial \psi_j^R}{\partial \phi} \begin{cases} > 0 & \text{if } \psi_j^R > 1 \\ = 0 & \text{if } \psi_j^R = 1 \\ < 0 & \text{if } \psi_j^R < 1 \end{cases} \cdot \tag{18}$$

The intuition is straightforward. Compare the case without status preferences, $\phi = 0$, to the case with status preferences, $\phi > 0$. For households that benefit more than the average from the the alternative scenario under $\phi = 0$ it holds that $\psi_j^R > 1$. If $\phi > 0$, then these households benefit even more. The opposite holds for households that benefit less than the average from the alternative scenario under $\phi = 0$ because $\psi_j^R < 1$. This general result shows that capturing Schwabe's law through status preferences for housing amplifies welfare differences, independent of the specific scenario under study. It

[22] The subsequent equation results from (9), (15), and (16).

is further highly suggestive that this amplification mechanism will also prevail in more complex model economies where additional features are introduced. Corollary 6.1 applies generally to any two economies A and B. In Section 5 we will study amplification of welfare under a specific policy experiment.

4 General Equilibrium

So far, prices $\{r, w, p\}$ have been taken as given. To endogenize prices, we close the model by introducing the production sector.

4.1 Firms

The aim to study the distributional effect of changes in zoning regulations in general equilibrium guides our modeling choice. Zoning regulations are intended to separate different types of land use (Gyourko and Molloy 2015). We stress that zoning regulations constrain the availability of land for residential purposes and, hence, constrain primarily the number of houses, but not so much the size of a house. We therefore employ the two-sectoral macro model with a housing sector from Grossmann and Steger (2017). This model is designed to think long term and distinguishes the housing stock along an extensive margin – the number of houses – and an intensive margin – the size of the average house. This distinction makes the model especially suited to study the distributional effects of changes in the time path of rents triggered by changes in zoning regulations.

4.1.1 Numeraire Good Sector

This sector consists of mass one of identical firms that act under perfect competition. Each firm produces a final good, Y, chosen as numeraire, according to the Cobb-Douglas production function

$$Y = K^{\alpha}(B^Y L^Y)^{\beta}(B^Y Z^Y)^{1-\alpha-\beta}, \tag{19}$$

where K, L^Y and Z^Y denote inputs of physical capital, labor and land, respectively. The productivity parameter, $B^Y > 0$, grows exponentially at a constant rate $g^Y \geq$

0. Parameters $\alpha \in (0,1)$ and $\beta \in (0,1)$ are output elasticities of capital and labor, respectively, and satisfy $\alpha + \beta < 1$. Firms maximize profits $Y - (r + \delta^K)K - wL^Y - R^Z Z^Y$ implying that factor prices equal their respective marginal products

$$r = \alpha \frac{Y}{K} - \delta^K, \qquad w = \beta \frac{Y}{L^Y}, \quad \text{and} \quad R^Z = (1 - \alpha - \beta) \frac{Y}{Z^Y}. \tag{20}$$

Capital depreciates at rate $\delta^K \geq 0$ such that gross physical capital investment reads $I^K \equiv \dot{K} + \delta^K K$.

4.1.2 Housing Sector

A house is a bundle of two components: structure and land. Both are modeled as stocks. The former is produced by construction firms by employing construction materials and labor. The latter is produced by real estate development firms who transform non-residential land into residential land by using labor as an input. Finally, in each period housing services firms use residential structures and land to generate a consumption flow that is consumed by households.

Construction Mass one of construction firms produce structures that are sold to households by combining labor, L^X, and construction materials, M. The production of one unit of materials requires one unit of the numeraire good. Structures are produced according to

$$I^X = M^\eta (B^X L^X)^{1-\eta}, \tag{21}$$

where $\eta \in (0,1)$ is the output elasticity of materials. Labor augmenting technology, $B^X > 0$, grows exponentially at the constant rate $g^X \geq 0$. Newly produced structures, I^X, is sold to households at the price P^X. Each construction firm maximizes profits $P^X I^X - M - wL^X$ under perfect competition such that factor prices equal their marginal products

$$1 = \eta P^X \frac{I^X}{M}, \quad \text{and} \quad w = (1 - \eta) P^X \frac{I^X}{L^X}. \tag{22}$$

The aggregate stock of structures, X, evolves according to $\dot{X} = I^X - \delta^X X$, where $\delta^X \geq 0$ is the depreciation rate of residential structures.

Real estate development The real estate development sector consists of mass one of firms that operate under perfect competition. The existing, aggregate stock of residential land is denoted by N, it does not depreciate, and additions to this stock are $\dot{N}$. A real estate development firm purchases $Z^N \in \mathbb{R}$ units of non-residential land and employs $L^N \in \mathbb{R}^+$ units of labor to produce residential land according to

$$\dot{N} = \begin{cases} \min\left\{Z^N, \sqrt{\frac{2}{\xi} L^N}\right\} & \text{if } Z^N \geq 0 \\ \max\left\{Z^N, -\sqrt{\frac{2}{\xi} L^N}\right\} & \text{if } Z^N < 0 \end{cases}, \tag{23}$$

with $\xi > 0$. We model labor and land as perfect complements. The individual real estate developer purchases a piece of non-residential land and conducts infrastructure investments on it to transform it into residential land. Production function (23) is symmetric in the sense that also negative units of $\dot{N}$ can be produced with negative inputs of Z^N and symmetrically positive units of labor. The production function is also concave in labor, resulting in a cost function that is convex in $\dot{N}$. This implies that residential land is a state and not a jump variable and hence only slowly adjusting over time.

Newly developed residential land, $\dot{N}$, is sold to households at the price P^N. Each residential land development firm chooses labor, L^N, and land, Z^N, in order to maximize profits $\Pi^N \equiv P^N \dot{N} - P^Z Z^N - wL^N$, where P^Z is the price of non-residential land. First order conditions are given by

$$\dot{N} = Z^N = \frac{P^N - P^Z}{\xi w} \quad \text{and} \quad L^N = \frac{\left(P^N - P^Z\right)^2}{2\xi w^2}. \tag{24}$$

Firms will produce new residential (non-residential) land if the price of residential land, P^N, is larger (smaller) than the price of non-residential land, P^Z. Residential land development firms are, however, only allowed to develop additional residential land when the total stock of residential land is below a fraction $\kappa \in (0, 1]$ of the economy's total amount of land Z, that is $N \leq \kappa Z$. This constraint captures zoning regulations. The parameter κ is an exogenous policy parameter. If $\kappa = 1$, then the zoning regulation is equal to the physical constraint, $N \leq Z$. For sufficiently small values of κ the constraint might be binding such that $N = \kappa Z$ results in equilibrium.[23]

If $\dot{N} \neq 0$, then profits, Π^N, are strictly positive. This is due to the decreasing returns to scale production function (23). We assume that these profits are taxed away lump-sum by the government and used for wasteful government expenditures such that after-tax profits are equal to zero.[24]

Housing services Finally, mass one of firms combine residential structures, X, and residential land, N, to produce housing services, S, according to

$$S = X^{\gamma} N^{1-\gamma}, \tag{25}$$

where $\gamma \in (0, 1)$ is the output elasticity of S with respect to structures. These housing services are consumption flows that are sold to households at the price p, the rental rate of housing. Each firm maximizes profits $pS - R^N N - (R^X + \delta^X P^X)X$, where R^N and R^X are rental rates of residential land and structures, respectively, and P^X denotes the price of structures. Competition is perfect and factor prices equal their respective value marginal products according to

$$R^N = (1-\gamma)p\frac{S}{N} \quad \text{and} \quad R^X = \gamma p \frac{S}{X} - \delta^X P^X. \tag{26}$$

4.2 Assets

Households hold four different assets: physical capital, K, non-residential land, Z^Y, residential structures, X, and residential land, N. Aggregate wealth accordingly comprises

$$W \equiv \underbrace{K + P^Z Z^Y}_{\text{non-housing wealth}} + \underbrace{P^N N + P^X X}_{\text{housing wealth}}. \tag{27}$$

[23] We abstract from potential benefits associated with zoning regulations, such as the efficient provision of local public goods (Duranton and Puga 2015; Gyourko and Molloy 2015).

[24] The main reason for this assumption is that otherwise either i) we would have to introduce stocks of N-firms that are traded, implying an additional differential equation, or ii) we would have to take a stance on how these profits are distributed across households, which in turn would add an additional distributional channel in our analysis. The assumption of wasteful government expenditures allows us to focus on the existing distributional effects without adding another channel. Note further that these profits are zero in a steady state with $\dot{N} = 0$.

Households are indifferent with regard to the allocation of their wealth, W_j, across these different assets given that the following no-arbitrage conditions hold[25]

$$r = \frac{\dot{P}^Z + R^Z}{P^Z} = \frac{\dot{P}^N + R^N}{P^N} = \frac{\dot{P}^X + R^X}{P^X}. \tag{28}$$

At the household level it is therefore sufficient to study total wealth without considering the allocation of wealth across the four asset classes.

4.3 Equilibrium Definition

Definition 1 (Equilibrium)**.** *A competitive equilibrium is a path of aggregate quantities* $\{Y(t),\ K(t),\ X(t),\ N(t),\ M(t),\ L^Y(t),\ L^X(t),\ L^N(t),\ Z^N(t),\ Z^Y(t),\ C(t),\ S(t),\ W(t),\ I^K(t),\ I^X(t)\}_{t=0}^{\infty}$*, group-specific quantities* $\{\{C_j(t),\ S_j(t),\ W_j(t)\}_{j=1}^{J}\}_{t=0}^{\infty}$*, and prices* $\{p(t),\ P^Z(t),\ P^N(t),\ P^X(t),\ w(t),\ r(t),\ R^Z(t),\ R^X(t),\ R^N(t)\}_{t=0}^{\infty}$ *with given aggregate initial states* $K(0)$*,* $N(0)$*,* $X(0)$*, and distributions of labor and initial wealth* $\{L_j,\ W_j(0)\}_{j=1}^{J}$ *such that*

(i) individuals maximize lifetime utility (1) subject to (2) and (3);

(ii) the representative firms in the sectors producing the numeraire good, Y*, structures,* X*, residential land,* N*, and housing services,* S*, maximize their respective profits such that their first order conditions given by (20), (22), (24), and (26) hold;*

(iii) the labor market clears: $L^Y(t) + L^X(t) + L^N(t) = \sum_{j=1}^{J} \frac{1}{J} L_j$*;*

(iv) asset markets clear: $W(t) = \sum_{j=1}^{J} \frac{1}{J} W_j(t)$ *and wealth is the sum of all aggregate assets according to (27);*

(v) perfect arbitrage holds across all asset classes as given by (28);

(vi) consumption markets clear: $S(t) = \sum_{j=1}^{J} \frac{1}{J} S_j(t)$ *and* $C(t) = \sum_{j=1}^{J} \frac{1}{J} C_j(t)$*;*

(vii) the land use regulation holds: $N(t) \leq \kappa Z$*;*

(viii) the market for the numeraire good clears: $Y(t) = C(t) + I^K(t) + M(t)$*.*[26]

[25]If one of the arbitrage conditions would be violated, the demand for the respective asset would not be finite – a violation of equilibrium conditions.

[26] The goods market clearing condition is redundant, according to Walras' law. To exclude conceptual or calculation errors, we analytically and numerically checked that the equilibrium derived from conditions (i)-(vii) fulfills condition (viii).

4.4 Steady State

All variables' steady state growth rates are linear transformations of the growth rates of productivity parameters, g^X and g^Y, as shown in Appendix A.2. Here we focus on the steady state growth rate of the rent which is denoted by $\hat{p}$

$$\hat{p} = (1 - \gamma\eta)\, g^Y - \gamma\,(1 - \eta)\, g^X. \tag{29}$$

Note that GDP and the wage rate grow at the rate g^Y in steady state. The rent is the relative price of housing services, S, that are produced with residential structures, X, and land, N. On one hand, higher income growth (increase in g^Y) raises the demand for housing, but since the long run supply of the number of houses, N, is fixed, this results in a higher rent. On the other hand, higher productivity growth in the construction sector (increase in g^X) makes the production of structures, X, relatively cheaper, resulting in a lower rent. These two opposing forces hence operate through the extensive and intensive margins of housing, N and X, respectively.

How does the wealth distribution behave in a steady state? The answer is given by the following proposition.

Proposition 7 (Stationary wealth distribution)**.** *In a steady state, the wealth distribution is stationary in the sense that for every households j the relative wealth position W_j^R does not change over time.*

The condition for a stationary wealth distribution is $\mu\widetilde{w} = w$ according to (12). The proof of Proposition 7 shows that this condition is indeed satisfied in any steady state. Consequently, a change in wealth inequality over time requires transitional dynamics. The policy experiment analyzed next triggers such transitional dynamics.

5 Numerical Analysis

We investigate the consequences of abolishing residential zoning regulations on wealth inequality and welfare. In the baseline scenario, we consider a steady state with binding zoning regulation. In this scenario the rent grows at constant growth rate, as given by

(29), and the wealth distribution is stationary. In the policy-reform scenario, we consider transitional dynamics in response to the counterfactual abolishment of zoning regulations, implying that the rent grows temporarily at a lower pace than in the baseline scenario.[27]

5.1 Calibration

We calibrate the model economy's steady state to the postwar US economy at an annual frequency. This implies a stationary wealth distribution, which is roughly in line with recent data on the wealth distribution (WID 2017; Kuhn, Schularick, and Steins 2018).[28]

Household sector We calibrate the joint distribution of initial wealth and labor productivity by matching wealth deciles and average earnings of the age group 33-55 from the US Survey of Consumer Finances (SCF) in 2013.[29] Similar to Kuhn and Ríos-Rull (2016) and Krusell and Rios-Rull (1999), we focus on this age-group to calibrate a dynastic model by abstracting from life-cycle effects. We consider $J = 10$ wealth groups, in ascending order, that correspond to the observed wealth deciles and the average earnings within each decile.[30] Moreover, Havránek (2015) shows that the majority of studies find an intertemporal elasticity of substitution (IES) below 0.8. We set $\sigma = 2$, implying an IES of 0.5. It is further assumed that every household holds the same portfolio composition as the representative household.

The preference parameters ϕ and θ are set to match two key moments of the housing expenditure share distribution in the US in 2015, as displayed in Table 1: (i) An aggregate housing expenditure share of 19 percent and (ii) a difference between the expenditure shares of the first and fifth income quintiles of 7 percentage points.[31] This results in $\phi = 0.104$ and $\theta = 0.174$.

[27] The numerical solution procedure is described in detail in Online-Appendix B.

[28] The set of parameters is summarized in Table C.2 in the appendix.

[29] See www.federalreserve.gov/econres/scfindex.htm. We are grateful to Moritz Kuhn for providing the data.

[30] This implies also that each group is of the same size, $n_j = 1/J = 0.1$.

[31] The aggregate expenditure share in the UK amounts to 17 percent, while the expenditure shares in income quintiles 1-5 read as {29, 20, 17, 14, 14} (Office for National Statistics, 2015). The corresponding data for France read 22 percent (averaged over 2011 to 2015) and {26, 24, 24, 23, 18} (Accardo, Billot, and Buron 2017). For Germany (2013) we have 27 percent and {37, 33, 29, 27, 24} (Statistisches Bundesamt 2015).

Table 1: Housing expenditure shares by income quintiles in percent

	aggregate	income quintile				
		1st	2nd	3rd	4th	5th
US data	19	25	21	20	19	18
Model: baseline calibration	19	25	22	20	19	18

Notes. (a) Housing expenditure share is defined as the ratio of expenditures on housing services (including imputed rent) and total consumption expenditures. (b) First row "U.S. data" shows the empirical values for the US in 2015. Data source: www.bls.gov/cex/data.htm (accessed June 19, 2017). (c) Second row "Model: baseline calibrated" shows the model based expenditure shares such that $e = 0.19$ and the difference between first and fifth income quintile according to U.S. data (7 percentage points) is matched.

To study sensitivity, alternative values for ϕ and θ are considered. When changing ϕ, we adjust θ such that the aggregate housing expenditure share, e, remains at 19 percent. For a given ϕ this implies that θ is obtained from

$$\theta = \frac{e(1-\phi)}{1-\phi e}. \tag{30}$$

We additionally consider $\phi = 0$ (no status preferences), implying that housing expenditure shares are homogeneous. This leads to $\theta = 0.19$.

In a two-sectoral model the steady state growth rate of consumption, as implied by the Euler equation, also depends on the growth rate of the relative price of the two consumption goods, p. Hence, the time preference rate, ρ, has to be calibrated jointly with θ and γ. We match the average rate of return on wealth for the postwar US of 5.77 percent (Jordà et al. 2018, Table 12). This yields $\rho = 0.019$.

Although we do not calibrate the model to match the saving rate, the calibrated model is compatible with empirical observations. Our calibrated model implies that the saving rate of the representative household (equal to the aggregate saving rate), $\overline{sav} \equiv \dot{\bar{W}}/\bar{y}$ with income $\bar{y} \equiv r\bar{W} + w\bar{l}$, equals 11.8 percent. This value is in line with the US aggregate saving rate of 9 percent on average from 1950 to 2010 (Piketty and Zucman 2014, Table A86). The saving rates of the 1st to 5th income quintiles are 0.9, 1.8, 4.7, 8.5, and 17.1 percent. These values are in the range of the estimated saving rates by Dynan, Skinner, and Zeldes (2004).

Numeraire sector The total amount of land that can be used economically, Z, is normalized to one. The annual depreciation rate of capital, δ^K, is set to 5.6 percent (Davis and Heathcote 2005). The concavity parameters of the production function for the numeraire good, α and β, are set to match the sector's expenditure shares for labor, β, and land, $1-\alpha-\beta$. Grossmann and Steger (2017) compute $\beta = 0.69$ and $1-\alpha-\beta = 0.03$, implying $\alpha = 0.28$. GDP grows at the rate g^Y in the model economy and therefore g^Y is set equal to the average annual growth rate of real US GDP per capita of 2.0 percent between 1950 and 2017.[32]

Housing sector The annual depreciation rate of structures, δ^X, is set to 1.5 percent (Hornstein 2009, p. 13). The labor expenditure share in the construction sector, $1 - \eta$, amounts to 62 percent on average in the postwar US economy, implying $\eta = 0.38$ (Grossmann and Steger 2017). We choose γ to match the share of residential land value in total housing value $(1 - \gamma)$. Using time series of the aggregate residential land value and the total value of housing from Davis and Heathcote (2007) reveals that the share of residential land in total housing value has been increasing from 10 percent in 1950 to around 30 percent in 1975. Since then it has been fluctuating between 25 and 40 percent. We target an average value of one third, implying a value of γ equal to 0.78.

We choose g^X such that we match (given γ, η, and g^Y) an annual average growth rate of rents of 1 percent. According to Knoll (2017), the annual average growth rate of real rents in the postwar (1953-2017) US economy was about 0.8 percent. Albouy, Ehrlich, and Liu (2016) argue that official data on rent growth is biased downwards due to incomplete accounting for quality improvements. Assuming that rents grow by one percent annually (given γ, η, and g^Y) and using (29) implies $g^X = 0.009$.[33]

The parameter ξ captures the importance of adjustment costs associated with land reallocations between the housing and the numeraire sector. This parameter is difficult to calibrate, as it does not affect the steady state and has an impact only along the transition. We calibrate ξ such that the (average) speed of convergence of residential

[32] The data is obtained from FRED (https://fred.stlouisfed.org/), series *A93RX0Q048SBEA_P* (accessed on November 23, 2018).

[33] The low value for g^X in comparison to g^Y is supported by evidence of low, sometimes even negative, productivity growth in the construction sector (Davis and Heathcote 2005).

land, N, computed in Section 5.2, is identical to the speed of convergence implied by the empirical data for the period 1945 to 1975. This yields $\xi = 765$.[34]

We calibrate κ to match the observed allocation of land in the residential sector. According to geographic land-use data provided by Falcone (2015), 30.2 percent of the total US surface is used economically and 16.9 percent of this land is used as residential land. Hence, we set $\kappa = 0.169$.[35]. Consistent with this observation, average annual growth of residential land was about 3-5 percent during 1945 to 1975 and is almost zero since then (Davis and Heathcote 2007).

5.2 Abolishing Zoning Regulations

Residential zoning regulations are widely considered as an important amplifier of surging housing costs in a growing economy (Glaeser, Gyourko, and Saks 2005; Saiz 2010). For instance, Albouy and Ehrlich (2018) find that, based on data for 230 metropolitan areas in the US from 2005 to 2010, observed land-use restrictions substantially increase housing costs. Moreover, Gyourko and Molloy (2015) argue that zoning regulations were effectively introduced in the US during the 1970s. This is consistent with the data provided by Davis and Heathcote (2007) showing that residential land grew by an average annual growth rate of 5 percent during time period 1945-1975 and grew merely by an average annual growth rate of 0.7 percent during time period 1976-2016.

To address the two research questions set up in the introduction, we compare two scenarios. In the baseline scenario (zoning), the economy is in a steady state, conditional on the binding zoning regulation $N = \kappa Z$ with $\kappa = 0.169$. In the policy-reform scenario (no zoning), residential zoning regulations are abolished completely. That is, we set $\kappa = 1$ such that the zoning constraint $N \leq \kappa Z$ is not binding anymore. The policy-reform

[34]We assume that the long-run dynamics in N came to a halt by the introduction of zoning regulations in the 1970s (Gyourko and Molloy 2015). We then combine the steady state from the model with the observed data between 1945 to 1974 (Davis and Heathcote 2007) to determine the average speed of convergence. This shows that after 30 years about 31 percent of the gap between the initial N and the steady state has been closed. Hence, we set ξ such that residential land, N, has closed 31 percent of the gap between start value and steady state after 30 years of the transition in the experiment of Section 5.2.

[35] Without zoning regulations the model would imply a steady state share, N/Z, equal to 62 percent. Land use regulations started to play a major role in the residential sector in the 1970s (Gyourko and Molloy 2015)

scenario exhibits transitional dynamics, starting from levels of state variables according to the baseline scenario. The analysis captures all general equilibrium effects. That is, all prices $\{w, r, p\}$ are fully endogenous and change in response to an exogenous policy trigger.[36]

Comovement of rents and wealth inequality Figure 1 (a) displays the time path of rents in the baseline scenario (zoning) and in the policy-reform scenario (no zoning). It can be seen that rents grow temporarily at a slower pace in the policy-reform scenario (solid curve) compared to the baseline scenario (dashed curve). This is intuitive as the economy extends the supply of housing along the extensive margin in response to the abolishment of zoning.[37] Figure 1 (b) shows that wealth inequality (measured by the top 10 percent wealth share) declines by about 0.7 percentage points (from 73.7 percent to 73 percent) over time. That is, rent growth and wealth inequality are positively associated in this general equilibrium experiment.

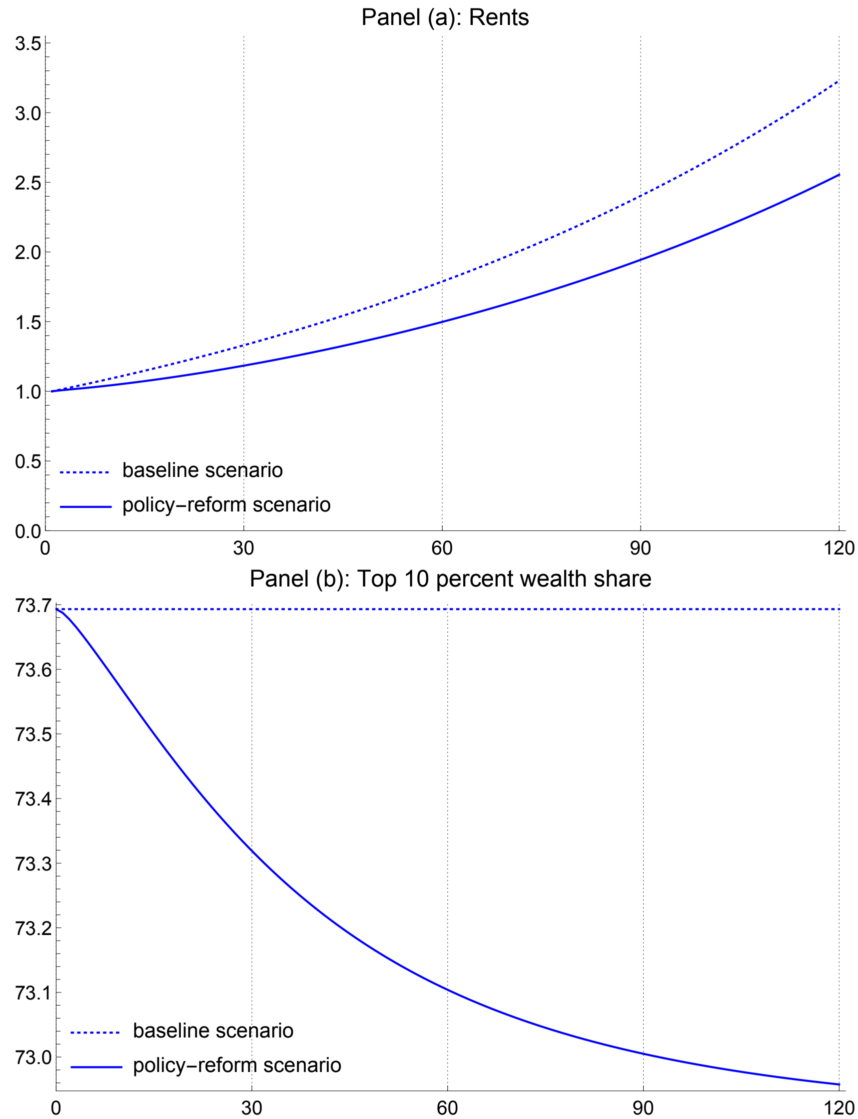
The observation that slower rent growth goes hand in hand with declining wealth inequality is in contrast to the rent channel in partial equilibrium (cf. Section 3.1.2). This seemingly contradiction can be readily explained. The policy experiment under study gives rise to a set of price changes, in addition to the change in rents. The most important one, when it comes to understanding the dynamics of wealth inequality, is the decline of future wages. The deregulation under study triggers a supply response in that the production of aggregate housing services, $S = X^{\gamma} N^{1-\gamma}$, is being expanded. The resulting slower temporary rent growth implies that the competitive wage rate (in terms of final output) of construction workers, $w = P^X \frac{\partial I^X}{\partial L^X}$, declines relative to the baseline scenario. The reason is that the price of new residential buildings, measured by P^X, declines as well.[38] This process is reinforced by two additional mechanisms. First, there is a transformation of commercial land, Z^Y, into residential land, N, in the process of real

[36] Favilukis, Mabille, and Van Nieuwerburgh (2018) consider a similar policy experiment in a model that is calibrated to New York. They model zoning regulations to exert an effect on labor productivity in the construction sector. In our model, relaxing zoning regulation is captured by an increase in κ, which constrains the amount of land allocated to the housing sector.

[37] Notice, however, that this is a temporary effect. The steady state growth rate of rents, given by (29), is unaffected.

[38] In equilibrium, $P^X(t) = \int_t^{\infty} p(\tau) \gamma \frac{S(\tau)}{X(\tau)} e^{\int_t^{\tau} -(r(v)+\delta^X) \mathrm{d}v} \mathrm{d}\tau$ according to (26) and (28).

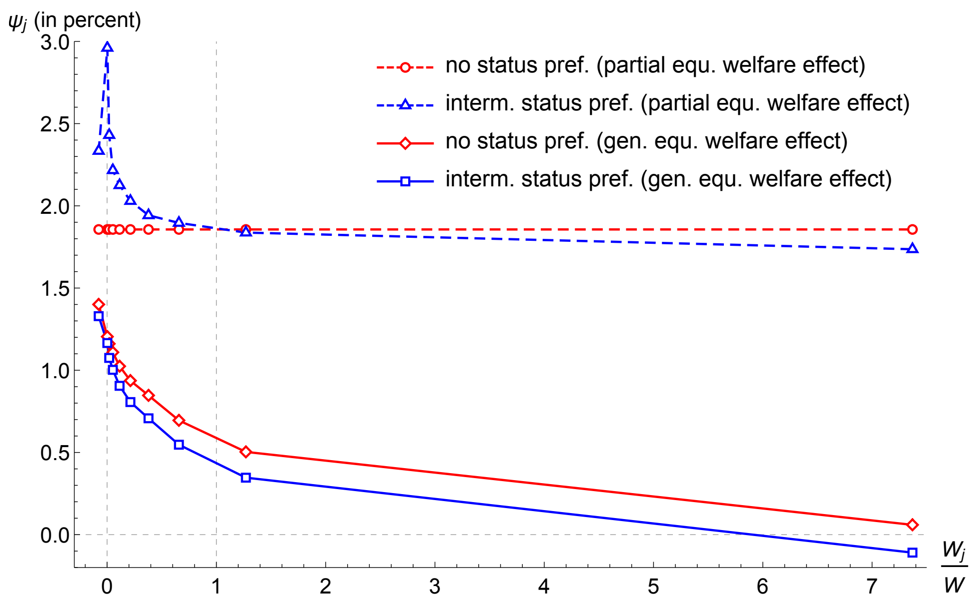
Figure 1: Rent and wealth dynamics in response to abolishment of zoning regulation.



Notes. Panel (a): Evolution of the housing rent in the baseline scenario (zoning) and the policy-reform scenario (no zoning). Panel (b): Evolution of the top 10 percent wealth share in the baseline scenario (zoning) and the policy-reform scenario (no zoning). Calibration as described in Section 5.1.

estate development. Second, investments are temporarily channeled from the numeraire sector to real estate development. As both physical capital and land are complementary to labor in the numeraire sector, the competitive wage of workers in the numeraire sector, $w = \frac{\partial Y}{\partial L^Y}$, declines as well. Lower future wages exert a convergence force as poorer households increase their saving rate by relatively more in order to smooth consumption over time. Hence, the divergence mechanism described in Section 3.1.1 (higher saving rates for the rich) is weakened. This effect dominates the reinforcement of the divergence

Figure 2: Welfare effects of abolishing zoning regulations.



Notes. Measure ψ_j is defined in (14). The baseline scenario is indexed by superscript B, the policy-reform scenario is indexed by superscript A. The calibration is described in Section 5.1.

mechanism due to lower rent growth.

Notably, according to Figure 1 (b), status concerns play only a minor role for the effect of abolishing zoning regulations on wealth inequality. This is in stark contrast to welfare effects to which we turn next.

Welfare Figure 2 displays the welfare gain, expressed in terms of consumption-equivalent variations, as a function of the relative initial wealth position, $W_j^R(t) = W_j(t)/W(t)$, from abolishing zoning regulations. We distinguish, first, between the partial equilibrium effects and the general equilibrium effects and, second, between the case of no status preferences ($\phi = 0$) and the presence of status preferences ($\phi = 0.104$).

Consider first the general equilibrium welfare effects of the abolishment of zoning regulations under status preferences (see the bottom curve marked by squares). Welfare of the representative household (possessing average wealth, $W_j/W = 1$) increases by almost 0.5 percent in general equilibrium under status concerns.[39] However, the welfare effects are asymmetric. The poor benefit by more than the rich. The reason is twofold. The policy reform triggers slower rent growth. The resulting favorable price index effect is especially pronounced for the poor, due to their higher housing expenditure share.[40] The policy reform also reduces house prices. The decline in house prices exerts a negative wealth effect and this effect is stronger for the wealth-rich.[41] Taken together, the price index effect and the wealth effect are both responsible for the asymmetry in welfare gains between wealth-poor and wealth-rich households. The overall welfare gain is even negative for the richest decile. That is, the wealth-rich loose due to lower house prices despite lower rents, or more generally speaking, despite lower housing costs. That the rich oppose the elimination of zoning regulations is well known and often refereed to as NIMBYism (see, for instance, Gyourko and Molloy 2015).

The isolated price index effect is shown by the curve marked by triangles (partial equilibrium). There we evaluate the welfare gain by accounting for a slower rent growth in the policy-reform scenario relative to the baseline scenario, as shown in Figure 1 (a), but keep everything else according the baseline scenario. We find that slower rent growth in response to abolishing zoning regulations, everything else the same, produces an average welfare gain of about 1.8 percent, i.e. a much higher welfare gain than in general equilibrium.[42]

The picture changes if we set $\phi = 0$ (no status preferences). The price index effect is now symmetric across wealth groups (horizontal line marked by circles). This partial equi-

[39] This welfare gain is consistent with the observation (not shown) that the income of the representative household in terms of ideal consumption, $\frac{y}{\mathcal{P}}$, increases by moving from the baseline scenario to the policy-reform scenario.

[40] Recall, from Proposition 5, that $\frac{\partial \mathcal{P}(p,e_j)}{\partial e_j \partial p} > 0$.

[41] The policy reform triggers a drop in the house price, which reduces non-human wealth (W_j). It also triggers a fall in wages, which reduces human wealth ($L_j \widetilde{w}$). The effect on non-human wealth is stronger. The calibration implies that the percentage of an agent's overall wealth in the form of (non-human) wealth increases with (non-human) wealth. Taken together, the policy reform under study lowers overall wealth ($W_j + L_j \widetilde{w}$) and this effect is stronger for the wealth-rich.

[42] The wage grows at a slower pace and the interest rate is slightly higher along the transition in the policy-reform scenario. Both effects suppress welfare. Notice also that the welfare gain is falling from the second wealth decile onward, but is higher for the second decile compared to the first decile. This non-monotonicity of the curve marked by triangles in Figure 2 is an implication of calibrating the joint distribution of wealth and labor endowment. In the second wealth decile labor income is lower compared to the first decile and falling rent is therefore particularly welfare-enhancing.

librium exercise hence shows that Schwabe's law implies that lower rent growth benefits poor households more than rich households. Moreover, the general equilibrium welfare gain from the abolishment of zoning regulations is stronger without status preferences (as indicated by the curve marked by diamonds). In the case with status preferences, there is overconsumption of housing services (relative to the first best solution), due to the negative externality associated with housing. From this perspective, a zoning constraint is a good thing, as it address an inefficiency as a second best instrument (Schünemann and Trimborn 2017).[43] Hence, removing the zoning regulation produces a smaller welfare gain under status preferences, as can be recognized by comparing to two lower curves in Figure 2.

6 Conclusion

This paper aims at better understanding the welfare implications of surging housing costs as well as the long-term comovement between housing costs and wealth inequality in a dynamic general equilibrium model. The demand side features heterogeneity in housing expenditure shares, consistent with Schwabe's law. We have shown that such heterogeneity is important for welfare effects of changes in the path of housing costs, but less so for wealth inequality. The supply side features a distinction between the extensive margin of the housing stock (the number of houses) and the intensive margin (the size of the average house). This model structure lends itself to investigating the consequences of removing those policies that regulate the use of land for residential purposes and, therefore, primarily constrain the extensive margin of the housing stock.

We have clarified, in a first step, the partial equilibrium analytics of surging housing costs on wealth inequality and welfare. In the empirical relevant case of an intertemporal elasticity of substitution below unity, forward-looking behavior implies that rising housing costs lead to a decline in wealth inequality. This, at the first glance surprising effect, reflects that the poor increase their saving rates more than the rich to cope with surging housing costs in the future. We have also shown that larger heterogeneity in

[43]The second-best optimal zoning constraint, assuming $\phi = 0.104$, amounts to $\kappa = 0.49$.

housing expenditure shares is associated with larger heterogeneity of welfare. The general equilibrium analysis, the second step, has investigated numerically the comovement between housing costs and wealth inequality and welfare, as triggered by the abolishment of zoning regulations. This policy reform, despite suppressing rent growth, lowers wealth inequality. The reason, in a nutshell, is that the policy reform suppresses future wage growth, in addition to lowering rent growth, such that households increase their saving rates to smooth consumption over time. This effect is especially pronounced for the wealth-poor. Abolishing zoning regulations also lowers the inequality of welfare. The calibrated model implies that most households gain from the policy reform. Only the richest households are worse off.

Appendix

A Additional analytical results

A.1 Renters vs. Homeowners

Two thirds of US households live in owner-occupied housing while only one third are renters.[44] Our model can equivalently be interpreted as an economy of homeowners and all results still hold true, independent on whether we interpret households as renters or homeowners.

We proof this equivalence by first setting up a homeowner problem and then showing that this is equivalent to the renter problem from the main text. Consider a homeowner household j. Let N_j and P^H denote the homeowner's amount of housing stock owned and the house price, respectively. A unit of housing stocks translates into $h(t)$ units of housing services such that the homeowner's housing services consumption is $S_j(t) = N_j(t)h(t)$. We will define $h(t)$ in a moment, but for now its just an exogenous variable to the household. The homeowner household solves the following problem

$$\max_{\{C_j(t),N_j(t)\}_{t=0}^{\infty}} \int_0^{\infty} u\big(C_j(t), N_j(t)h(t), \overbrace{N(t)h(t)}^{=S(t)}\big)\mathrm{e}^{-\rho t}\mathrm{d}t \tag{31}$$

$$\text{s.t.} \quad \dot{W}_j(t) = r(t)W_j(t) + w(t)L_j - C_j(t) - p^{UC}(t)N_j(t), \tag{32}$$

where $p^{UC}(t) \equiv rP^H + P^X(\delta^X x + \dot{x}) - \dot{P}^H$ is the user cost of housing and $x \equiv X/N$ is the average amount of structures per unit of residential land. The user cost of housing consists of the sum of foregone interest payments, rP^H, and expenditures for maintenance and expansion, $P^X(\delta^X x + \dot{x})$, minus appreciation gains, $\dot{P}^H$.

We now show that this problem is equivalent to the renter problem from the main text. First, substitute N_j with S_j/h in the homeowner's problem. Life time utility (31) is then equivalent to that in the main text, (1) and the thus transformed budget constraint (32) equals the budget constraint of the main text, (3), if $p^{UC}/h = p$ holds. The final

[44] US Bureau of the Census, Homeownership Rate for the United States, retrieved from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/RHORUSQ156N), series RHORUSQ156N (accessed December 21, 2018).

step of the proof is hence to show that p^{UC} equals ph. The house price is given by $P^H \equiv P^N + P^X x$ and the variable h is defined as the average amount of housing services produced per unit of residential land, $h \equiv S/N$.[45] Hence, it remains only to show that p^{UC} equals pS/N:

$$\begin{aligned}
p^{UC} &= rP^H + P^X(\delta^X x + \dot{x}) - \dot{P}^H \\
&= rP^N + rP^X x + P^X(\delta^X x + \dot{x}) - \dot{P}^N - \dot{P}^X x - P^X \dot{x} \\
&= rP^N - \dot{P}^N + \left[rP^X - \dot{P}^X \right] x + P^X \delta^X x \\
&= R^N + R^X + P^X \delta^X x \\
&= (1-\gamma)p\frac{S}{N} + \gamma p \frac{S}{X} x - \delta^X P^X x + P^X \delta^X x \\
&= (1-\gamma)p\frac{S}{N} + \gamma p \frac{S}{X}\frac{X}{N} = p\frac{S}{N},
\end{aligned}$$

where we made use of the definition of P^H, the no-arbitrage condition (28), and the first order conditions of S-firms, (26). This completes the proof.

Since the renter and owner problems are equivalent, all results apply to homeowners as well as renters. The difference is merely in the interpretation. Instead of the *rent* it is the *user cost of housing* that affects the distribution of wealth and welfare. As p^{UC} equals ph this is almost the same.

Under financial frictions additional mechanisms start playing a role. For instance, the rent and the user cost per unit of housing services may diverge implying that renters pay a higher price for housing services. Similarly, if houses pay a rate of return that differs from the rate of return paid by other assets, the portfolio structure plays a role for the wealth effects of surging house prices (Kuhn, Schularick, and Steins 2018).

A.2 Steady State

Proposition A.1 (Steady state). *Assume that productivity parameters B^X and B^Y grow at constant exponential rates $g^X \geq 0$ and $g^Y \geq 0$, respectively. The unique steady state growth rates are:*

[45] The reason for the definition of h becomes apparent when we aggregate housing stocks, $\frac{1}{J}\sum_{j=1}^{J} N_j h = Nh = X^\gamma N^{1-\gamma} = S$.

(i) Variables $K, C, M, R^Z, R^N, P^N, P^Z, w,$ and W grow at the rate g^Y

(ii) Variable X grows at the rate $\eta g^Y + (1-\eta)\, g^X$

(iii) Variable p grows at the rate $(1-\gamma\eta)\, g^Y - \gamma\,(1-\eta)\, g^X$

(iv) Variables P^X, and R^X grow at the rate $(1-\eta)\left(g^Y - g^X\right)$

(v) Variable S grows at the rate $\gamma\left[\eta g^Y + (1-\eta)\, g^X\right]$

(vi) Variables $N, Z^Y, Z^N, L^Y, L^X, L^N$, and r are constant.

This implies that the steady state growth rate of GDP, $GDP \equiv Y + pS + wL^X$, equals g^Y. The proof is in Appendix B.

B Proofs

We first derive the solution to the household problem because this will be relevant for the following proofs. Any household j maximizes lifetime utility as given by (1) and (2) subject to its budget constraint (3) and a standard No-Ponzi game condition, $\lim_{t\to\infty} W_j(t)\mathrm{e}^{-\int_0^t r(v)\mathrm{d}v} \geq 0$, for a given initial wealth endowment $W_j(0)$ and labor endowment L_j. The associated current-value Hamiltonian reads

$$\mathcal{H}_j = \frac{[(C_j)^{1-\theta}(S_j - \phi S)^{\theta}]^{1-\sigma} - 1}{1-\sigma} + \lambda_j[rW_j + wL_j - C_j - pS_j]. \tag{33}$$

The first-order optimality conditions (FOC) are

$$\frac{\theta}{1-\theta}\frac{C_j}{S_j - \phi S} = p \tag{34}$$

$$(1-\theta)(C_j)^{-\theta-\sigma(1-\theta)}(S_j - \phi S)^{\theta(1-\sigma)} = \lambda_j \tag{35}$$

$$-\frac{\dot{\lambda}_j}{\lambda_j} = r - \rho \tag{36}$$

$$\lim_{t\to\infty} W_j(t)\lambda_j(t)\mathrm{e}^{-\rho t} = 0. \tag{37}$$

Using (34) in (35) we obtain

$$\lambda_j = (1-\theta)^{1+(\sigma-1)\theta}\theta^{(1-\sigma)\theta}C_j^{-\sigma}p^{(\sigma-1)\theta}, \text{ and } -\frac{\dot{\lambda}_j}{\lambda_j} = \sigma\frac{\dot{C}_j}{C_j} + (1-\sigma)\theta\frac{\dot{p}}{p}. \tag{38}$$

Combining (36) and (38) yields the following reformulated FOC

$$\begin{aligned} C_j &= p\frac{1-\theta}{\theta}(S_j - \phi S) \\ \frac{\dot{C}_j}{C_j} &= \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma}\frac{\dot{p}}{p} \equiv g_C \\ \dot{W}_j &= rW_j + wL_j - C_j - pS_j \\ \lim_{t\to\infty} W_j(t)C_j(t)^{-\sigma}p(t)^{(\sigma-1)\theta}\mathrm{e}^{-\rho t} &= 0. \end{aligned} \tag{39}$$

The first equation is an intratemporal optimality condition that determines the allocation of total consumption expenditures across housing services and consumption of the numeraire. The second equation is an Euler equation, the third the intertemporal budget constraint, and the last the transversality condition.

We will now derive the consumption function (total consumption expenditures as a function of time only). This comprises three steps: i) integration of the intertemporal budget constraint, ii) integration of the Euler equation, and iii) combining results from i) and ii).

First, define total consumption expenditures as $\mathcal{E}_j \equiv C_j + pS_j$ and rewrite the intertemporal budget constraint as (with period index τ)

$$\dot{W}_j(\tau) = r(\tau)W_j(\tau) + w(\tau)L_j - \mathcal{E}_j(\tau). \tag{40}$$

Multiplying both sides of (40) by $\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}$ and integrating from t to infinity yields

$$\int_t^\infty \dot{W}_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau = \int_t^\infty r(\tau)W_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau + \int_t^\infty \left[w(\tau)L_j - \mathcal{E}_j(\tau)\right]\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau. \tag{41}$$

Integrating the left hand side of this equation by parts results in

$$\begin{aligned} \int_t^\infty \dot{W}_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau &= \lim_{T\to\infty}\left[W_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\right]_t^T + \int_t^\infty r(\tau)W_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau \\ &= -W_j(t) + \int_t^\infty r(\tau)W_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau, \end{aligned} \tag{42}$$

where we make use of the transversality condition $\lim_{T\to\infty} W_j(\tau)\mathrm{e}^{-\int_t^T r(v)\mathrm{d}v}\mathrm{d}\tau = 0$ in the latter equation. Inserting this expression into the left hand side of (41) and rearranging yields total wealth as

$$\mathcal{W}_j(t) \equiv W_j(t) + \widetilde{w}(t)L_j = \int_t^\infty \mathcal{E}_j(\tau)\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau, \tag{43}$$

where $\widetilde{w}(t)$ is defined in (8).

Second, define composite consumption as $\mathcal{C}_j \equiv C_j^{1-\theta}(S_j - \phi S)^\theta$. Making use of the intratemporal optimality condition (34) to substitute S_j in $\mathcal{C}_j$ yields

$$\mathcal{C}_j = \left(\frac{\theta}{1-\theta}\frac{1}{p}\right)^\theta C_j. \tag{44}$$

Solving for C_j, taking the derivative with respect to time, and then substituting C_j with $\mathcal{C}_j$ in the Euler equation (39) results in

$$\frac{\dot{\mathcal{C}}_j}{\mathcal{C}_j} = \frac{r-\rho}{\sigma} - \frac{\theta}{\sigma}\frac{\dot{p}}{p}. \tag{45}$$

This is the Euler equation expressed in terms of composite consumption $\mathcal{C}_j$. It constitutes an ordinary linear differential equation with a time-varying coefficient that can be solved to yield

$$\mathcal{C}_j(\tau) = \mathcal{C}_j(t)\mathrm{e}^{\frac{1}{\sigma}\int_t^\tau\left[r(v)-\rho-\theta\frac{\dot{p}(v)}{p(v)}\right]\mathrm{d}v}. \tag{46}$$

This expresses composite consumption at any date $\tau \geq t$ as a function of consumption in period t and prices r and p.

Third, define the ideal price index as $\mathcal{P}_j = \mathcal{E}_j/\mathcal{C}_j$, substitute $\mathcal{E}_j$ in equation (43) with $\mathcal{P}_j\mathcal{C}_j$, then also substitute $\mathcal{C}_j$ by (46) and multiply both sides by $\mathcal{P}_j(t)$ to obtain

$$\mathcal{P}_j(t)\mathcal{W}_j(t) = \underbrace{\mathcal{P}_j(t)\mathcal{C}_j(t)}_{=\mathcal{E}_j(t)}\int_t^\infty \mathcal{P}_j(\tau)\mathrm{e}^{\frac{1}{\sigma}\int_t^\tau\left[r(v)-\rho-\theta\frac{\dot{p}(v)}{p(v)}\right]\mathrm{d}v}\mathrm{e}^{-\int_t^\tau r(v)\mathrm{d}v}\mathrm{d}\tau. \tag{47}$$

Solving for consumption expenditures in t yields

$$\mathcal{E}_j(t) = \frac{\mathcal{W}_j(t)}{\int_t^\infty \frac{\mathcal{P}_j(\tau)}{\mathcal{P}_j(t)} \mathrm{e}^{\frac{1}{\sigma}\int_t^\tau \left[(1-\sigma)r(v)-\rho-\theta\frac{\dot{p}(v)}{p(v)}\right]\mathrm{d}v} \mathrm{d}\tau}. \tag{48}$$

We now have to replace the relative ideal price index in this expression. Define relative consumption of housing services as $S_j^R \equiv S_j/S$ and use (34) to express total consumption expenditures as

$$\mathcal{E}_j = \frac{pS}{\theta}[S_j^R - (1-\theta)\phi]. \tag{49}$$

Using (44) and (49) together with (34) in $\mathcal{P}_j = \mathcal{E}_j/\mathcal{C}_j$ yields

$$\mathcal{P}_j = \frac{p^\theta}{\theta^\theta(1-\theta)^{1-\theta}} \frac{S_j^R - (1-\theta)\phi}{S_j^R - \phi}. \tag{50}$$

Note that S_j^R is constant over time: First, the Euler equation (39) shows that C_j grows with the same rate g_C for all j and also for the average, that is C grows also at the rate g_C. Second, the intratemporal optimality condition (34) of the average, $C = p\frac{1-\theta}{\theta}(1-\phi)S$, implies that pS grows also at the rate g_C. Third, calculating growth rates in equation (34) where S_j is substituted with $S_j^R S$ yields

$$\underbrace{\frac{\dot{C}_j}{C_j}}_{=g_C} = \underbrace{\frac{\dot{\overline{pS}}}{pS}}_{=g_C} + \frac{\dot{\overline{(S_j^R - \phi)}}}{S_j^R} \Rightarrow \frac{\dot{\overline{(S_j^R - \phi)}}}{S_j^R} = 0 \Rightarrow \dot{S}_j^R = 0. \tag{51}$$

Now, according to (50) and the fact that S_j^R is time-invariant, we obtain the ratio of ideal price indices as

$$\frac{\mathcal{P}_j(\tau)}{\mathcal{P}_j(t)} = \left(\frac{p(\tau)}{p(t)}\right)^\theta. \tag{52}$$

Finally, replacing this ratio in (43) results in

$$\mathcal{E}_j(t) = C_j + pS_j = \mu(t)\left[W_j(t) + \widetilde{w}(t)L_j\right], \tag{53}$$

where the propensity to consume out of total wealth is

$$\mu(t) = \left\{ \int_t^\infty \left(\frac{p(\tau)}{p(t)} \right)^\theta \mathrm{e}^{-\frac{1}{\sigma} \int_t^\tau \left[(\sigma-1) r(v) + \rho + \theta \frac{\dot{p}(v)}{p(v)} \right] \mathrm{d}v} \mathrm{d}\tau \right\}^{-1}. \tag{54}$$

The propensity to consume, μ, can be simplified by solving $\mathrm{e}^{-\frac{1}{\sigma} \int_t^\tau \theta \frac{\dot{p}(v)}{p(v)} \mathrm{d}v}$, resulting in (7). Equation (53) is the consumption function. It expresses total consumption expenditures as a function of endowments (W_j and L_j) and time only (prices are functions of time). This finishes the derivation of the first equation of the household solution, (4). The other equation, (6), is then obtained by replacing $C_j + pS_j$ in the intertemporal budget constraint (3) by (53).

Proof of Proposition 1. Remember, the aggregate of a variable is defined by the sum over all j, weighted by the equal group sizes $1/J$. For example, aggregate consumption is given by $C = \frac{1}{J} \sum_j C_j$. Aggregating this way the left and right hand sides of equations (4), (5), and (6) yields

$$C(t) + p(t)S(t) = \mu(t) \left[W(t) + \widetilde{w}(t) L \right] \tag{55}$$

$$C(t) = p(t) \frac{1-\theta}{\theta} (1-\phi) S(t) \tag{56}$$

$$\dot{W}(t) = \left[r(t) - \mu(t) \right] W(t) + \left[w(t) - \mu(t) \widetilde{w}(t) \right] L. \tag{57}$$

These FOC are identical to the FOC that result from the problem of a single household who owns the entire endowments ($\frac{1}{J} \sum_j L_j$ and $\frac{1}{J} \sum_j W_j$) and makes the aggregate consumption and saving decisions, taking the reference level of housing consumption, $S(t)$, as given. □

Proof of Proposition 2. The housing expenditure share is defined by $e_j \equiv (pS_j)/(C_j + pS_j)$. Make use of the intratemporal first order condition, (34), to obtain

$$e_j = \frac{\theta S_j^R}{S_j^R - (1-\theta)\phi}. \tag{58}$$

Substituting C_j in (5) with (4) S_j yields

$$S_j = \frac{\theta}{p}\mu\mathcal{W}_j + (1-\theta)\phi S \tag{59}$$

and for the representative agent

$$S = \frac{\theta}{1-(1-\theta)\phi}\frac{\mu}{p}\mathcal{W}. \tag{60}$$

Inserting this equation into (59) yields

$$S_j = \frac{\theta\mu}{p}\left[\mathcal{W}_j + \frac{(1-\theta)\phi}{1-(1-\theta)\phi}\mathcal{W}\right]. \tag{61}$$

Using the last two equations to obtain relative housing services consumption then gives

$$S_j^R = \frac{S_j}{S} = (1-\theta)\phi + [1-(1-\theta)\phi]\frac{\mathcal{W}_j}{\mathcal{W}}. \tag{62}$$

Lastly, insert this last expression for S_j^R into (58) to obtain (9). □

Proof of Proposition 3. Assumption (1) rules out columns 3, 4, and 6 in Table (C.1), i.e. the cases $L_j^R > W_j^R > 1$, $L_j^R < W_j^R < 1$, and $L_j^R \neq W_j^R = 1$. All remaining cases are contained in (13). □

Proof of Proposition 4. Wealth divergence or convergence is determined by equation (12). The real rent only affects the marginal propensity to consume, μ, in this expression. From equation (7) we immediately see that μ is declining (increasing) in the growth factor of rents, $\frac{p(\tau)}{p(t)}$ for $\tau > t$, if $\sigma > 1 (< 1)$. If μ is declining (increasing), then $\dot{W}_j^R$ is also declining (increasing), as can be seen from (12). □

Proof of Proposition 5. We can rearrange (58) to obtain

$$S_j^R = \frac{(1-\theta)\phi}{1-\frac{\theta}{e_j}}. \tag{63}$$

Substituting (63) into (50) confirms (15). □

Proof of Proposition 6. Factor out $1/(1-\sigma)$ in (14), split integrals such that

$-\int_t^\infty e^{-\rho(\tau-t)}\mathrm{d}\tau$ is isolated an can be subtracted from both sides, and rearrange to obtain

$$(1+\psi_j)^{1-\sigma} = \frac{\int_t^\infty \mathcal{C}_j^A(\tau)^{1-\sigma}\mathrm{e}^{-\rho(\tau-t)}\mathrm{d}\tau}{\int_t^\infty \mathcal{C}_j^B(\tau)^{1-\sigma}\mathrm{e}^{-\rho(\tau-t)}\mathrm{d}\tau}. \tag{64}$$

Make use of (46) and (7) to rewrite

$$\int_t^\infty \mathcal{C}_j^k(\tau)^{1-\sigma}\mathrm{e}^{-\rho(\tau-t)}\mathrm{d}\tau = \frac{\left[\mathcal{C}_j^k(t)\right]^{1-\sigma}}{\mu^k(t)}, \tag{65}$$

where $k \in \{A, B\}$ denotes the respective economy. Inserting this into (64) yields

$$(1+\psi_j)^{1-\sigma} = \frac{\frac{\left[\mathcal{C}_j^A(t)\right]^{1-\sigma}}{\mu^A(t)}}{\frac{\left[\mathcal{C}_j^B(t)\right]^{1-\sigma}}{\mu^B(t)}} \Leftrightarrow \psi_j = \left[\frac{\mu^A(t)}{\mu^B(t)}\right]^{\frac{1}{\sigma-1}} \frac{\mathcal{C}_j^A(t)}{\mathcal{C}_j^B(t)} - 1. \tag{66}$$

To obtain (16) replace $\mathcal{C}_j^k(t)$ in this expression with $\mathcal{E}_j^k(t)/\mathcal{P}_j^k(t)$ for both $k \in \{A, B\}$, where total expenditures and the ideal price index are taken from (53) and (15), respectively. □

Proof of Corollary 6.1. We take the derivative of (17) with respect to ϕ . Note first that changes in ϕ do not affect total wealth $\mathcal{W} = W_j + \widetilde{w}L_j$. The derivative is hence given by

$$\frac{\partial \psi_j^R}{\partial \phi} = \frac{\theta}{\left(\frac{\mathcal{W}_j^B}{\mathcal{W}^B} - \phi\theta\right)^2}\left(\frac{\mathcal{W}_j^A}{\mathcal{W}^A} - \frac{\mathcal{W}_j^B}{\mathcal{W}^B}\right). \tag{67}$$

To obtain (18) note that i) if $\frac{\mathcal{W}_j^A}{\mathcal{W}^A} > (<) \frac{\mathcal{W}_j^B}{\mathcal{W}^B}$, then $\psi_j^R > (<) 1$, and ii) if $\frac{\mathcal{W}_j^A}{\mathcal{W}^A} = \frac{\mathcal{W}_j^B}{\mathcal{W}^B}$, then $\psi_j^R = 1$. □

Proof of Proposition A.1. Let a "hat" above a variable denote the steady state growth rate of this variable. According to (39), we obtain the steady state interest rate

$$r = \rho + \theta(1-\sigma)\hat{p} + \sigma\hat{C} \equiv r^* \tag{68}$$

The FOC in the numeraire sector, (20), imply

$$\hat{Y} = \hat{K} = \hat{w} = \hat{R^Z}. \tag{69}$$

Writing the production function (19) in growth rates yields $\hat{Y} = \alpha\hat{K} + \beta(g^Y + \hat{L}^Y) + (1-\alpha-\beta)(g^Y + \hat{Z}^Y)$. Suppose that the long run allocation of both labor and land is time invariant (which will be confirmed to be consistent with the derived steady state), i.e. $\hat{L}^Y = \hat{L}^X = \hat{L}^N = \hat{Z}^Y = \hat{N} = 0$. Using $\hat{Y} = \hat{K}$ this implies

$$\hat{Y} = g^Y. \tag{70}$$

With $\dot{N} = 0$ in steady state we obtain from the FOC of the real estate development firm, (24), that $P^N = P^Z$ and

$$\hat{P}^N = \hat{P}^Z. \tag{71}$$

Because the interest rate is constant in steady state, the asset market no arbitrage conditions (28) imply

$$\hat{P}^Z = \hat{R}^Z, \hat{P}^N = \hat{R}^N, \hat{P}^X = \hat{R}^X. \tag{72}$$

Differentiating the construction firms' FOC, (22), with respect to time yields

$$\hat{P}^X = (\eta - 1)(g^X - \hat{M}) = \hat{w} - \eta\hat{M} + (\eta - 1)g^X. \tag{73}$$

Using (73) and recalling $\hat{w} = g^Y$ we obtain

$$\hat{M} \;\; = \;\; \hat{w} = g^Y, \tag{74}$$

$$\hat{P}^X = \hat{R}^X \;\; = \;\; (\eta - 1)(g^X - g^Y). \tag{75}$$

Next, insert the production function (21) in $\dot{X} = I^X - \delta^X X$ and rearrange

$$\frac{\dot{X}}{X} = \frac{M^\eta (B^X L^X)^{1-\eta}}{X} - \delta^X. \tag{76}$$

Together with (74) the growth rate of X is then

$$\hat{X} = \eta g^Y + (1-\eta)g^X. \tag{77}$$

From the production function of housing services producers, (25), we obtain

$$\hat{S} = \gamma \left[\eta g^Y + (1-\eta) g^X \right]. \tag{78}$$

Recall that one FOC of the housing services producer is $R^N = (1-\gamma)p\frac{S}{N}$. Differentiating with respect to time gives $\hat{p} = \hat{R}^N - \hat{S}$. Using (78) and $\hat{R}^N = g^Y$, which results from $\hat{R}^Z = g^Y = \hat{P}^Z = \hat{P}^N = \hat{R}^N$, we confirm (29):

$$\hat{p} = (1-\gamma\eta)\, g^Y - \gamma\,(1-\eta)\, g^X. \tag{79}$$

Finally, aggregating the intertemporal budget constraint (3) and dividing by W yields

$$\hat{W} = r + \frac{wL}{W} - \frac{C}{W} - \frac{pS}{W}. \tag{80}$$

In order for W to grow at a constant rate, it has to hold that $\hat{W} = \hat{w} = g^Y$ as well as that $\hat{C} = \hat{W} = g^Y$. $\square$

Proof of Corollary 7. From Proposition 3 we know that a necessary condition for a stationary wealth distribution is $\mu\widetilde{w} = w$. It hence remains to be shown that this is indeed the case in steady state.

Notice first that the transversality condition of the household optimization problem requires $r^* - g^Y$. Substituting (79) and $\hat{C} = g^Y$ into (68), we find that

$$r^* - g^Y = \rho - \left[(1-\theta+\theta\gamma\eta) g^Y + \theta\gamma\,(1-\eta)\, g^X \right] (1-\sigma) > 0 \tag{81}$$

always holds if $\sigma \geq 1$. Recall from Proposition A.1 that $\hat{w} = g^Y$. Thus, in steady state, $w(\tau) = w(t)\mathrm{e}^{(\tau-t)g^Y}$ and, consequently, the PDV of wages, $\widetilde{w}(t) \equiv \int_t^\infty w(\tau)\mathrm{e}^{\int_t^\tau -r(v)\mathrm{d}v}\mathrm{d}\tau$, can be written as

$$\widetilde{w}(t) = \int_t^\infty \underbrace{w(t)\mathrm{e}^{g^Y(\tau-t)}}_{=w(\tau)} \mathrm{e}^{-r^*(\tau-t)}\mathrm{d}\tau = \frac{w(t)}{r^* - g^Y} \equiv \widetilde{w}^*(t). \tag{82}$$

Aggregate the Euler equation (39) and rewrite

$$\hat{C} = \frac{r^* - \rho + (\sigma - 1)\theta\hat{p}}{\sigma} = g^Y, \tag{83}$$

where the latter follows from Proposition A.1.

Also note from (83) that $\left(\theta\hat{p} - r^* - \frac{\rho}{\sigma-1}\right)\frac{\sigma-1}{\sigma} = -(r^* - g^Y) < 0$ and consider next the propensity to consume, as given by (7). Using that, in steady state, $p(\tau)/p(t) = \mathrm{e}^{\hat{p}(\tau-t)}$ and $\widetilde{r}(\tau,t) \equiv \int_t^\tau r(v)dv = (\tau - t)r^*$, we can rewrite $\mu(t)$ in steady state as

$$\mu(t) = \left(\int_t^\infty \exp\left[\left(\theta\hat{p} - r^* - \frac{\rho}{\sigma - 1}\right)\frac{\sigma - 1}{\sigma}(\tau - t)\right]\mathrm{d}\tau\right)^{-1} = r^* - g^Y \equiv \mu^*(t). \tag{84}$$

Using (82) and (84), we confirm $\mu^*(t)\widetilde{w}^*(t) = w(t)$. □

C Tables

Table C.1: Individual wealth convergence and divergence

	$W_j^R > 1$		$W_j^R < 1$		$W_j^R = 1 \neq L_j^R$	$W_j^R = L_j^R$
	$W_j^R > L_j^R$	$W_j^R < L_j^R$	$W_j^R > L_j^R$	$W_j^R < L_j^R$		
$\mu\widetilde{w} > w$	diverge	converge	converge	diverge	diverge	stationary
$\mu\widetilde{w} = w$	stationary	stationary	stationary	stationary	stationary	stationary
$\mu\widetilde{w} < w$	converge	diverge	diverge	converge	diverge	stationary

Table C.2: Set of parameters for the calibrated model

Parameter	Value	Explanation/Target
L	1	Normalization
J	10	Match deciles
$\left\{\frac{W_j(0)}{W(0)}\right\}_{j=1}^{J}$	see text	Wealth deciles, US, 2013 SCF
$\left\{\frac{L_j}{L}\right\}_{j=1}^{J}$	see text	average earnings within wealth percentile, US, 2013 SCF
σ	2	$IES = 0.5$ (Havránek 2015)
Z	1	Normalization
δ^K	0.056	Davis and Heathcote (2005)
α	0.28	Land income share in Y sector (Grossmann and Steger 2017)
β	0.69	Labor expenditure share in Y sector (Grossmann and Steger 2017)
g^Y	0.02	Growth rate of GDP per capita (FRED)
δ^X	0.015	Hornstein (2009)
η	0.38	Labor expenditure share in X sector (Grossmann and Steger 2017)
g^X	0.009	Implied
κ	0.169	Share of residential land: 16.9 percent (Falcone 2015)
θ	$\{0.19, 0.17, 0.15\}$	Average housing expenditure share 0.19 (U.S. Bureau of Labor Statistics 2016)
ϕ	$\{0.000, \mathbf{0.104}, 0.260\}$	Difference between bottom and top income quintiles of Table 1
ρ	0.019	Real interest rate: 0.0577 (Jordà et al. 2018)
γ	0.78	Land's share in housing wealth: 1/3
ξ	765	Transition speed in N: 31 percent in 30 years (Davis and Heathcote 2007)

References

Accardo, Jérôme, Sylvain Billot, and Maël-Luc Buron (2017). "Household income, consumption and savings per broad category between 2011 and 2015, French economy". In: *Accounts and Files 2017 Edition, INSEE*.

Albouy, David and Gabriel Ehrlich (2018). "Housing productivity and the social cost of land-use restrictions". In: *Journal of Urban Economics* 107, pp. 101–120.

Albouy, David, Gabriel Ehrlich, and Yingyi Liu (2016). *Housing Demand, Cost-of-Living Inequality, and the Affordability Crisis*. Working Paper 22816. National Bureau of Economic Research.

Alvarez-Pelaez, Maria J and Antonia Díaz (2005). "Minimum consumption and transitional dynamics in wealth distribution". In: *Journal of Monetary Economics* 52.3, pp. 633–667.

Bellet, Clément (2017). "The paradox of the joneses: Superstar houses and mortgage frenzy in suburban america". In: *CEP Discussion Paper* 1462.

Borri, Nicola and Pietro Reichlin (2018). "The housing cost disease". In: *Journal of Economic Dynamics and Control* 87, pp. 106–123.

Caselli, Francesco and Jaume Ventura (2000). "A representative consumer theory of distribution". In: *American Economic Review*, pp. 909–926.

Chatterjee, Satyajit (1994). "Transitional dynamics and the distribution of wealth in a neoclassical growth model". In: *Journal of Public Economics* 54.1, pp. 97–119.

Clark, Andrew E, Paul Frijters, and Michael A Shields (2008). "Relative income, happiness, and utility: An explanation for the Easterlin paradox and other puzzles". In: *Journal of Economic Literature* 46.1, pp. 95–144.

Davis, Morris A and Jonathan Heathcote (2005). "Housing and the business cycle". In: *International Economic Review* 46.3, pp. 751–784.

— (2007). "The price and quantity of residential land in the United States". In: *Journal of Monetary Economics* 54.8, pp. 2595–2620.

De Nardi, Mariacristina and Giulio Fella (2017). "Saving and wealth inequality". In: *Review of Economic Dynamics* 26, pp. 280–300.

Duranton, Gilles and Diego Puga (2015). "Urban Land Use". In: *Handbook of Regional and Urban Economics*. Vol. 5. Elsevier, pp. 467–560.

Dustmann, Christian, Bernd Fitzenberger, and Markus Zimmermann (2018). "Housing expenditures and income inequality". In: *ZEW-Centre for European Economic Research Discussion Paper* 48.

Dynan, Karen E, Jonathan Skinner, and Stephen P Zeldes (2004). "Do the rich save more?" In: *Journal of Political Economy* 112.2, pp. 397–444.

Falcone, James A (2015). *US conterminous wall-to-wall anthropogenic land use trends (NWALT), 1974–2012*. Tech. rep. US Geological Survey.

Favilukis, Jack, Sydney C Ludvigson, and Stijn Van Nieuwerburgh (2017). "The macroeconomic effects of housing wealth, housing finance, and limited risk sharing in general equilibrium". In: *Journal of Political Economy* 125.1, pp. 140–223.

Favilukis, Jack Y, Pierre Mabille, and Stijn Van Nieuwerburgh (2018). "Affordable housing and city welfare". In: *Columbia Business School Research Paper* 18-77.

Frank, Robert H (2005). "Positional externalities cause large and preventable welfare losses". In: *AEA Papers and Proceedings* 95.2, pp. 137–141.

Gabaix, Xavier et al. (2016). "The dynamics of inequality". In: *Econometrica* 84.6, pp. 2071–2111.

Garcia-Penalosa, Cecilia and Stephen J Turnovsky (2006). "Growth and income inequality: a canonical model". In: *Economic Theory* 28.1, pp. 25–49.

Glaeser, Edward L, Joseph Gyourko, and Raven Saks (2005). "Why is Manhattan so expensive? Regulation and the rise in housing prices". In: *The Journal of Law and Economics* 48.2, pp. 331–369.

Grossmann, Volker and Thomas Michael Steger (2017). "Das House-Kapital: a theory of wealth-to-income ratios". In: *IMF Working Paper* No. 17/80.

Gyourko, Joseph and Raven Molloy (2015). "Regulation and housing supply". In: *Handbook of Regional and Urban Economics*. Vol. 5. Elsevier, pp. 1289–1337.

Havránek, Tomáš (2015). "Measuring intertemporal substitution: The importance of method choices and selective reporting". In: *Journal of the European Economic Association* 13.6, pp. 1180–1204.

Hornstein, Andreas (2009). "Problems for a fundamental theory of house prices". In: *FRB Richmond Economic Quarterly* 95.1, pp. 1–24.

Hubmer, Joachim, Per Krusell, and Anthony A Smith Jr (2016). "The historical evolution of the wealth distribution: A quantitative-theoretic investigation". In: *National Bureau of Economic Research Working Paper No. 23011.*

Iacoviello, Matteo (2005). "House prices, borrowing constraints, and monetary policy in the business cycle". In: *American Economic Review* 95.3, pp. 739–764.

Iacoviello, Matteo and Stefano Neri (2010). "Housing market spillovers: evidence from an estimated DSGE model". In: *American Economic Journal: Macroeconomics* 2.2, pp. 125–64.

Jordà, Òscar et al. (2018). "The rate of return on everything, 1870-2015". In: *CEPR Discussion Paper No. 12509.*

Kaas, Leo et al. (2017). "Low homeownership in Germany – a quantitative exploration". In: *Journal of the European Economic Association* forthcoming.

Kaymak, Barış and Markus Poschke (2016). “The evolution of wealth inequality over half a century: The role of taxes, transfers and technology”. In: *Journal of Monetary Economics* 77, pp. 1–25.

Kiyotaki, Nobuhiro, Alexander Michaelides, and Kalin Nikolov (2011). “Winners and losers in housing markets”. In: *Journal of Money, Credit and Banking* 43.2–3, pp. 255–296.

Knoll, Katharina (2017). “Our home in days gone by: housing markets in advanced economies in historical perspective”. PhD thesis. Free University of Berlin.

Knoll, Katharina, Moritz Schularick, and Thomas Steger (2017). “No price like home: global house prices, 1870–2012”. In: *American Economic Review* 107.2, pp. 331–353.

Krusell, Per and Jose-Victor Rios-Rull (1999). “On the size of US government: political economy in the neoclassical growth model”. In: *American Economic Review* 89.5, pp. 1156–1181.

Kuhn, Moritz and JV Ríos-Rull (2016). “2013 Update on the US earnings, income, and wealth distributional facts: A View from Macroeconomics”. In: *Federal Reserve Bank of Minneapolis, Quarterly Review* 37.1.

Kuhn, Moritz, Moritz Schularick, and Ulrike Steins (2018). “Income and wealth inequality in America, 1949-2016”. In: *CEPR Discussion Paper No. 12218*.

Kydland, Finn E, Peter Rupert, and Roman Šustek (2016). “Housing dynamics over the business cycle”. In: *International Economic Review* 57.4, pp. 1149–1177.

Leguizamon, Susane J and Justin M Ross (2012). “Revealed preference for relative status: Evidence from the housing market”. In: *Journal of Housing Economics* 21.1, pp. 55–65.

Marx, Karl (1847). *Wage, labour, and capital.* Ed. by Sankar Srinivasan. Independently Published, 2017.

Mas-Colell, Andreu, Michael Dennis Whinston, and Jerry R Green (1995). *Microeconomic theory.* Vol. 1. Oxford University Press New York.

Miles, David and James A Sefton (2017). “Houses across time and across place”. In: *CEPR Working Paper* DP12103.

Piazzesi, Monika and Martin Schneider (2016). “Housing and macroeconomics”. In: *Handbook of Macroeconomics.* Vol. 2. Elsevier, pp. 1547–1640.

Piketty, Thomas and Gabriel Zucman (2014). “Capital is back: wealth-income ratios in rich countries 1700-2010”. In: *The Quarterly Journal of Economics* 129.3, pp. 1255–1310.

Saiz, Albert (2010). “The geographic determinants of housing supply”. In: *The Quarterly Journal of Economics* 125.3, pp. 1253–1296.

Schünemann, Johannes and Timo Trimborn (2017). "Boosting taxes for boasting about houses: Status concerns in the housing market". In: *ECON WPS – Vienna University of Technology Working Papers in Economic Theory and Policy* 05/2017.

Singer, HW (1937). "Income and Rent: A Study of Family Expenditure". In: *The Review of Economic Studies* 4.2, pp. 145–154.

Statistisches Bundesamt (2015). "Einkommens- und Verbrauchsstichprobe 2013 – Aufwendungen privater Haushalte für den Konsum". In: *Fachserie* 5 (15). Accessed: 2017-09-01.

Stigler, George J (1954). "The early history of empirical studies of consumer behavior". In: *Journal of Political Economy* 62.2, pp. 95–113.

Summers, Lawrence H (2014). "The inequality puzzle". In: *Democracy: A Journal of Ideas* 33.2, pp. 65–73.

Turnbull, Geoffrey K, Jonathan Dombrow, and Clemon F Sirmans (2006). "Big house, little house: relative size and value". In: *Real Estate Economics* 34.3, pp. 439–456.

U.S. Bureau of Labor Statistics (2016). *Consumer Expenditure Survey*. https://www.bls.gov/cex/data.htm. Accessed: 2016-10-06.

Wälde, Klaus (2016). "Pareto-improving redistribution of wealth – the case of the NLSY 1979 cohort". In: *Johannes-Gutenberg University, Mainz, mimeo.*

WID (2017). *World Income & Wealth Data Base*. http://wid.world/data/#countrytimeseries/shweal_p0p1_992_j/US/1930/2015/eu/k/p/yearly/s. Accessed: 2017-20-02.

Online Appendix

A Dynamic System

The model is fully described by seven differential equations plus a set of static equations.[46]

$$
\begin{aligned}
\dot{X} &= M^{\eta}(B^X L^X)^{1-\eta} - \delta^X X \\
\dot{N} &= \begin{cases} 0 & \text{if } P^N > P^Z \text{ and } N = \kappa Z \\ \frac{P^N - P^Z}{\xi w} & \text{else} \end{cases} \\
\dot{W} &= rW + wL - C - pS \\
\frac{\dot{C}}{C} &= \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma}\frac{\dot{p}}{p} \\
\dot{P}^X &= -R^X + rP^X \\
\dot{P}^N &= -R^N + rP^N \\
\dot{P}^Z &= -R^Z + rP^Z \\
K &= W - \left(P^N N + P^X X + P^Z Z^Y\right) \\
p &= \frac{\theta}{(1-\theta)(1-\phi)}\frac{C}{S} \\
Y &= K^{\alpha}\left(B^Y L^Y\right)^{\beta}\left(B^Y Z^Y\right)^{1-\alpha-\beta} \\
r &= \alpha\frac{Y}{K} - \delta^K \\
w &= \beta\frac{Y}{L^Y} \\
R^Z &= (1-\alpha-\beta)\frac{Y}{Z^Y} \\
R^X &= \gamma p\frac{S}{X} - \delta_X \qquad \text{should read: deltaX PX} \\
R^N &= (1-\gamma)p\frac{S}{N} \\
S &= X^{\gamma}N^{1-\gamma} \\
L^X &= \left(\frac{(1-\eta)P^X}{w}\right)^{\frac{1}{\eta}}\frac{M}{(B^X)^{\frac{\eta-1}{\eta}}}
\end{aligned}
$$

[46] This is achieved by collecting equations from the main text, aggregating household-level equations, and rearranging.

$$
\begin{aligned}
M &= \left(\eta P^X\right)^{\frac{1}{1-\eta}} B^X L^X \\
L &= L^Y + L^X + L^N \\
Z &= Z^Y + N \\
L^N &= \frac{\xi}{2}\left(\dot{N}\right)^2
\end{aligned}
$$

where $K(0)$, $N(0)$, $X(0)$ are given.[47]

B Computation of transitional dynamics for all agents

The computation of time paths for all J type of agents takes series for (normalized) prices and aggregate quantities – obtained from the solution of the representative household economy in the first step – as given and derives time paths for each agent $j \in \{1, 2, ..., J\}$ by exploiting the recursive structure of the household problem. It is not necessary to employ numerical techniques like solving non-linear equation systems, interpolation, or numerical integration. Given the minor approximation error in the solution of the representative household economy, the computed time paths for all J type of agents are hence exact to machine precision.

B.1 Discretization

In order to solve the model numerically we have to discretize the differential equation system that describes the economy. For a differential equation $\dot{x}(t) = f(x(t), y(t))$ we discretize according to $x_{t+1} - x_t = f(x(t), y(t))$. The growth-adjusted first-order conditions read

$$
\frac{\tilde{C}_j}{1-\theta} = \frac{\tilde{p}}{\theta}(\tilde{S}_j - \phi\tilde{S}) \tag{85}
$$

$$
\frac{\dot{\tilde{C}}_j}{\tilde{C}_j} = \frac{r-\rho}{\sigma} + \frac{\sigma-1}{\sigma}\theta\left(\frac{\dot{\tilde{p}}}{\tilde{p}} + g^p\right) - g^c \tag{86}
$$

$$
\dot{\tilde{W}} = (r - g^c)\tilde{W}_j + \tilde{w}L_j - \tilde{C}_j - \tilde{p}\tilde{S}_j \tag{87}
$$

$$
0 = \lim_{t\to\infty} e^{-\tilde{\rho}t}\tilde{W}_{jt}\tilde{p}_t^{\theta(\sigma-1)}\left(\tilde{c}_{jt}\right)^{-\sigma} \tag{88}
$$

[47] In total, there are 21 equations and 21 endogenous variables: X, N, W, C, P^X, P^N, P^Z, Y, K, p, r, w, R^Z, R^X, R^N, S, L^X, L^N,M, L^Y, and Z^Y.

$$\tilde{W}_{j0} = \text{given}, \tag{89}$$

where g^c is the exogenous growth rate of consumption (the numeraire) in steady state, g^p is the steady state growth rate of rents, and $\tilde{\rho} \equiv \rho + (\sigma - 1)(g^c - \theta g^p)$. The discretized version hence reads (the ˜ above variables is suppressed)

$$\frac{C_{jt}}{1-\theta} = \frac{p_t(S_{jt} - \phi S_t)}{\theta} \tag{90}$$

$$C_{jt+1} - C_{jt} = \frac{r_t - \rho}{\sigma} C_{jt} - \frac{\theta(1-\sigma)}{\sigma} \left(\frac{p_{t+1} - p_t}{p_t} + g^p \right) C_{jt} - g^c C_{jt} \tag{91}$$

$$W_{jt+1} - W_{jt} = (r_t - g^c) W_{jt} + w_t L_j - C_{jt} - p_t S_{jt} \tag{92}$$

$$0 = \lim_{t \to \infty} e^{-\rho t} W_{jt} p_t^{\theta(\sigma-1)} \left(C_{jt} \right)^{-\sigma} \tag{93}$$

$$\tilde{W}_{j0} = given, \tag{94}$$

where the subscript "jt" now denotes the group j and (discrete) time t. This constitutes a linear, non-homogeneous system of first-order difference equations with time-variant coefficients and two boundary conditions. Rearranging yields

$$C_{jt+1} = \underbrace{\frac{(1-g^c)\sigma + r_t - \rho + \theta(\sigma-1)\left(\frac{p_{t+1}-p_t}{p_t} + g^p \right)}{\sigma}}_{\equiv f_t} C_{jt} \tag{95}$$

$$W_{jt+1} = \underbrace{(1 + r_t - g^c)}_{\equiv g_t} W_{jt} - \frac{1}{1-\theta} C_{jt} + \underbrace{[w_t L_j - \phi p_t S_t]}_{\equiv l_t^j} \tag{96}$$

$$0 = \lim_{t \to \infty} e^{-\rho t} W_{jt} p_t^{\theta(\sigma-1)} \left(C_{jt} \right)^{-\sigma} \tag{97}$$

$$W_{j0} = given. \tag{98}$$

The solution is

$$C_{jt} = c_{j0} \prod_{s=0}^{t-1} f_s \tag{99}$$

$$W_{jt} = W_{j0} \prod_{s=0}^{t-1} g_s - \frac{1}{1-\theta} \sum_{k=0}^{t-1} C_{jk} \prod_{s=k+1}^{t-1} g_s + \sum_{k=0}^{t-1} l_k^j \prod_{s=k+1}^{t-1} g_s. \tag{100}$$

B.2 Initial consumption

One obtains c_{j0} by applying the transversality condition (TVC) to (100) and plugging the solution for C_{jt} – as given by (99) – into the result. Define $\Theta_t^1 \equiv W_{j0} \prod_{s=0}^{t-1} g_s$, $\Theta_t^2 \equiv h \sum_{k=0}^{t-1} C_{jk} \prod_{s=k+1}^{t-1} g_s$ and $\Theta_t^3 \equiv \sum_{k=0}^{t-1} l_k^j \prod_{s=k+1}^{t-1} g_s$ and write (100) as

$$W_{jt} = \Theta_t^1 - \Theta_t^2 + \Theta_t^3. \tag{101}$$

We know that $\lim_{t\to\infty} W_{jt}\Xi_t = 0$, where $\Xi_t \equiv e^{-\rho t} p_t^{\theta(\sigma-1)} (C_{jt})^{-\sigma}$, such that

$$\lim_{t\to\infty} W_t^j \Xi_t = 0 = \lim_{t\to\infty} \Xi_t(\Theta_t^1 + \Theta_t^3) - \lim_{t\to\infty} \Xi_t \Theta_t^2 \tag{102}$$

$$\Leftrightarrow 1 = \frac{\lim_{t\to\infty} \Xi_t(\Theta_t^1 + \Theta_t^3)}{\lim_{t\to\infty} \Xi_t \Theta_t^2} \tag{103}$$

$$= \lim_{t\to\infty} \frac{\Xi_t(\Theta_t^1 + \Theta_t^3)}{\Xi_t \Theta_t^2} \tag{104}$$

$$= \lim_{t\to\infty} \frac{\Theta_t^1 + \Theta_t^3}{\Theta_t^2}. \tag{105}$$

Replacing Θ_t^1, Θ_t^2, and Θ_t^3 by their respective expressions yields

$$1 = \frac{W_{j0} \prod_{s=0}^{\infty} g_s + \sum_{k=0}^{\infty} l_k^j \prod_{s=k+1}^{\infty} g_s}{h \sum_{k=0}^{\infty} C_{jk} \prod_{s=k+1}^{\infty} g_s} = \frac{W_{j0} + \sum_{k=0}^{\infty} l_k^j \prod_{s=0}^{k} (g_s)^{-1}}{h \sum_{k=0}^{\infty} C_{jk} \prod_{s=0}^{k} (g_s)^{-1}}. \tag{106}$$

Inserting the solution for C_{jk} as given by (99) gives

$$C_{j0} = \frac{W_{j0} + \sum_{k=0}^{\infty} l_k^j \prod_{s=0}^{k} (g_s)^{-1}}{\sum_{k=0}^{\infty} \frac{1}{1-\theta} \frac{1}{f_k} \prod_{s=0}^{k} \frac{f_s}{g_s}} \tag{107}$$

B.3 How to deal with infinity

In the computation we have to assume that the dynamic system is in its steady state after a some period T, where the number of transition periods, T, is chosen sufficiently large. Then, sums and products can be modified to $\sum_{s=0}^{\infty} x_t = \sum_{s=0}^{T-1} x_t + \sum_{s=T}^{\infty} x$ and $\prod_{s=0}^{\infty} x_t = (\lim_{t\to\infty} x^t) \prod_{s=1}^{T-1} x_t$, where x denotes the respective steady state of x_t. The steady states for the time-dependent parameters are

$$f = 1 \tag{108}$$

$$g = 1 + r - g_c \tag{109}$$

$$l^j = wl^j - \phi pS. \tag{110}$$

Accordingly, the denominator of (107) becomes

$$\begin{aligned}
\sum_{k=0}^{\infty} \frac{h}{f_k} \prod_{s=0}^{k} \frac{f_s}{g_s} &= \frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^{k} \frac{f_s}{g_s} + \frac{1}{1-\theta} \sum_{k=T}^{\infty} f^{-1} \prod_{s=0}^{T-1} \frac{f_s}{g_s} \prod_{s=T}^{k} \frac{f}{g} && (111)\\
&= \frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^{k} \frac{f_s}{g_s} + \frac{1}{1-\theta} \left(\prod_{s=0}^{T-1} \frac{f_s}{g_s} \right) \sum_{k=T}^{\infty} [1 + r - g_c]^{T-k-1} && (112)\\
&= \frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^{k} \frac{f_s}{g_s} + \frac{1}{(1-\theta)(r-g_c)} \prod_{s=0}^{T-1} \frac{f_s}{g_s}. && (113)
\end{aligned}$$

Similarly, the second term in the numerator of (107) can be written as

$$\sum_{k=1}^{\infty} l_k^j \prod_{s=1}^{k} g_s^{-1} = \sum_{k=0}^{T-1} l_k^j \prod_{s=0}^{k} g_s^{-1} + \frac{l^j}{r-g_c} \prod_{s=0}^{T-1} g_s^{-1}.$$

Putting all together gives

$$C_{j0} = \frac{W_{j0} + \sum_{k=0}^{T-1} l_k^j \prod_{s=0}^{k} g_s^{-1} + \frac{l^j}{r-g_c} \prod_{s=0}^{T-1} g_s^{-1}}{\frac{1}{1-\theta} \sum_{k=0}^{T-1} f_k^{-1} \prod_{s=0}^{k} \frac{f_s}{g_s} + \frac{1}{(1-\theta)(r-g_c)} \prod_{s=0}^{T-1} \frac{f_s}{g_s}}. \tag{114}$$

B.4 Solution algorithm

For each $j \in \{1, 2, ..., J\}$:

(i) Obtain initial consumption C_{j0} with (114).

(ii) Obtain individual consumption levels $\{C_{jt}\}_{t=0}^{T}$ from (99) or by iterating over the discretized Euler equation.

(iii) Obtain individual wealth levels $\{W_{jt}\}_{t=0}^{T}$ by making use of (100) or by iterating over the discretized budget constraint.

(iv) Obtain individual housing consumption $\{S_{jt}\}_{t=0}^{T}$ from the intra-temporal optimality condition.

C Robustness

C.1 Status Preferences for Both Goods

If we replaced instantaneous utility (2) by

$$u\left(C_j, S_j\right) = \frac{\left[(C_j - \phi_c C)^{1-\theta}(S_j - \phi_s S)^{\theta}\right]^{1-\sigma} - 1}{1-\sigma}, \tag{115}$$

with $\phi_c, \phi_s \geq 0$, where $\bar{c}$ is average consumption of the numeraire good, then the housing expenditure share would still read as (9), with $\phi \equiv \frac{\phi_s - \phi_c}{1-\phi_c}$. Since $\phi > 0$ iff $\phi_s > \phi_c$, assuming status concerns with respect to housing services only ($\phi_c = 0$) captures, without loss of generality, that status concerns are higher for housing than for non-housing consumption.

C.2 CES Utility

Consider the following utility specification

$$u\left(C_j, S_j\right) = \frac{\left(\mathcal{C}_j\right)^{1-\sigma} - 1}{1-\sigma} \qquad \text{with} \qquad \mathcal{C}_j = \left[\theta\left(S_j - \phi S\right)^{1-\frac{1}{\kappa}} + (1-\theta) C_j^{1-\frac{1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}},$$

where $\kappa > 0$. The housing expenditure share of agent j (e_j) and the aggregate housing expenditure share (e) are then given by

$$e_j = \frac{\theta^{\kappa} p^{1-\kappa}}{\theta^{\kappa} p^{1-\kappa} + (1-\theta)^{\kappa}\left(1 - \frac{\phi}{S_j^R}\right)}, \qquad \text{and} \qquad e = \frac{\theta^{\kappa} p^{1-\kappa}}{\theta^{\kappa} p^{1-\kappa} + (1-\phi)(1-\theta)^{\kappa}},$$

where $S_j^R \equiv \frac{S_j}{S}$. If rents, p, grow over the long run (Piazzesi and Schneider 2016), the aggregate housing expenditure share, e, is only constant if $\kappa = 1$. Notice that the utility specification in the main text, given by (2), is the limiting case of the above stated CES utility function for $\kappa \to 1$.

C.3 Status Preferences: Multiplicative Reference Level

Status preferences are often also captured as ratios instead of differences (Clark, Frijters, and Shields 2008; Schünemann and Trimborn 2017). A typical formulation looks like this

$$v\left(C_j, S_j\right) = \frac{\left[S_j^{\theta}\left(\frac{S_j}{S}\right)^{\phi}\left(C_j\right)^{1-\theta}\right]^{1-\sigma} - 1}{1-\sigma},$$

where $\theta \in (0,1)$, $\phi \in [0,1)$, and $\sigma > 0$. In this case, the housing expenditure share of agent j is given by

$$e_j = \frac{\theta + \phi}{1 + \phi}.$$

Hence, this preference specification is not compatible with heterogenous housing expenditure shares that vary systematically with income.