

Distributional Consequences of Surging Housing Rents

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Abstract: The trajectories of real housing rents and house prices have shown an upward sloping trend in most industrialized countries since WW2. The burden of rising housing rents is likely to be distributed unequally because income-poor households devote a larger share of their consumption expenditures on housing than income-rich households. This inverse relation between income and the expenditure share of housing is labeled Schwabe's law. We analyze how the dynamics in housing rents affect household welfare, accounting for Schwabe's law and the endogeneity of housing rents in general equilibrium. Our model features non-homothetic preferences so that we can replicate Schwabe's law. Because a representative household exists, we can discuss the underlying mechanisms analytically. One of our main analytical results is that Schwabe's law considerably amplifies heterogeneous welfare effects that operate through housing rent changes. We then study how zoning deregulation affects household welfare in general equilibrium. Zoning deregulation triggers a temporarily slower growth in housing rents and house prices. The representative household experiences a welfare gain of 0.37 percent. At the micro-level, we show that this welfare effect is highly unequally distributed. The highest welfare gain amounts to 12.52 percent, while the highest welfare loss amounts to 1.63 percent. We study the different welfare channels analytically and quantify their relative strengths. Lower housing rent growth drives most of the overall welfare gain, and Schwabe's law is responsible for most of the variation of the welfare gain across households.

Keywords: Macroeconomics and Housing; Long-Term Growth; Schwabe's Law; Wealth Inequality; Welfare.

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1. Introduction

Housing has become one of the defining economic issues of our times. As an asset, its price has been increasing in most industrialized economies since WW2 (Knoll, Schularick, and Steger, 2017) such that it is now the largest private wealth component (Piketty and Zucman, 2014). As a consumption good, housing expenditures represent the largest single expenditure category, housing rents have been surging (Knoll, 2017), and income-poor households spend a larger fraction of their total consumption expenditures on housing than income-rich households. For instance, US households in the first income quintile devoted 25 percent of their total expenditure to housing in 2015, whereas this number was only 18 percent for the fifth income quintile. This less well-known, inverse variation of the housing expenditure share with income is known as Schwabe's law (Stigler, 1954). Given that housing is the largest asset and the largest consumption category, and income-poor households spent relatively more on housing than income-rich households, surging house prices and housing rents might exert pronounced and heterogeneous welfare effects.

We therefore study how changes in housing rents affect welfare in a growing economy. Our analysis accounts for both Schwabe's law and the endogeneity of housing rents in general equilibrium. Specifically, we address two research questions. (1) How do housing rent dynamics affect the welfare of households? (2) What role does Schwabe's law play for these heterogeneous welfare effects? To answer these questions, we proceed in two steps.

First, before endogenizing housing rents, we focus on the demand side by studying how exogenous changes in housing rents affect households in the economy. This analytical part lays the foundation for understanding the working of the main mechanisms operating in general equilibrium. We set up a dynamic model where infinitely-lived households face a saving and consumption problem with two consumption goods: housing and a non-housing good. Households differ regarding their initial wealth and permanent labor productivity. We abstract from market incompleteness to obtain analytical answers to our research questions. The novel model feature is a non-homothetic utility function. We assume that households compare their housing consumption to housing consumption of others. Empirical evidence by Leguizamon and Ross (2012) and Bellet (2017) supports our assumption of status preferences for housing. Most importantly, status preferences for housing enable us to replicate so-called Schwabe's law. Schwabe's law is similar to the far better known Engel's law. It describes a negative correlation between household income and the share of consumption expenditures devoted to housing. In other words, housing is a necessity good. This correlation was first documented in 1868 by the German statistician Hermann Schwabe (Stigler, 1954). We document it for the present US, UK, France, and Germany. Albouy *et al.* (2016) provide further micro-evidence for the US by estimating an income elasticity of housing expenditures smaller than unity. We apply this demand-side of the model to analytically i) show how status preferences for housing enable reproducing Schwabe's

law, ii) study how changes in housing rents affect household welfare, and iii) analyze which role Schwabe’s law plays for this heterogeneous welfare effect.

Second, housing rents are not exogenous, and the driving force behind housing rent dynamics might also matter for welfare. Therefore, we endogenize housing rents by adding the supply-side to the model economy. To obtain endogenous dynamics in rents, we study residential *zoning deregulation*: removing constraints on land used for residential purposes. This policy reform triggers endogenous housing rent dynamics and thereby affects household welfare in general equilibrium. It is a relevant policy experiment because zoning regulation is a powerful amplifier of surging housing rents in a growing economy (Glaeser *et al.*, 2005; Saiz, 2010). Our employed Ramsey growth model comprises two sectors, a numeraire sector and a housing sector (Grossmann *et al.*, 2020). It distinguishes between an extensive and intensive margin of the housing stock. The extensive margin captures the number of houses, while the intensive margin captures a house’s average size. The overall quantity of land is fixed. Its allocation between housing and non-housing purposes is endogenous. Zoning regulations, however, restrict the total amount of land that can be used for housing. Our model structure lends itself to studying the consequences of residential zoning because this regulation primarily constrains land use for residential purposes and, therefore, constrains the extensive margin of the housing stock (Gyourko *et al.*, 2008; Saiz, 2010; Gyourko *et al.*, 2019). We study the welfare effects of endogenous housing rent dynamics by comparing two scenarios. In the baseline scenario, the economy is in a steady state under binding zoning regulation, and housing rents grow at a constant rate. In the alternative scenario, the zoning regulation is abolished such that the housing supply expands along the transition to a new steady state. Housing rents grow temporarily at a slower pace in response to this policy reform. We study quantitatively how this policy reform affects household welfare in the economy calibrated to the current US, focusing on endogenous housing rent dynamics and Schwabe’s law. We further analytically and quantitatively decompose this welfare effect into different channels.

We obtain the following main results. On the demand-side, we derive analytical solutions to the household problems for given aggregate prices. Based on that solution, we show that a representative household exists in our model economy despite non-homothetic utility (Chatterjee, 1994; Caselli and Ventura, 2000). The representative household’s existence allows us to decompose the welfare effect into different channels analytically because distributions do not feed back on aggregate variables. We then show that households’ optimal consumption choices result in Schwabe’s law under status preferences for housing consumption. The intuition is that a strictly positive housing reference level increases housing’s marginal utility more for income-poor households than for income-rich households. Income-poor households then consume relatively more housing than the non housing good, resulting in a higher housing expenditure share compared to income-rich households. Our

preference structure allows us to replicate not only the variation of housing expenditure shares in the cross-section at a given point in time. It also generates a constant aggregate housing expenditure share in a growing economy. According to the U.S. Bureau of Labor Statistics (2016), the aggregate housing expenditure share has been stable at 19 percent since WW2. This observation might seem puzzling at first sight because a higher income results in a lower housing expenditure share in the cross-section, while growing aggregate income does not lead to a lower housing expenditure share. Our preference specification generates a constant housing expenditure share in a growing economy because the housing reference level is endogenous and growing at the aggregate growth rate of housing consumption.

After showing that our model economy captures Schwabe’s law, we study how changes in housing rents affect welfare and how Schwabe’s law affects this relation. We show analytically that rising housing rents reduce household welfare for all households. More interestingly, this effect is only heterogeneous across households when we assume status preferences to capture Schwabe’s law. We show that Schwabe’s law amplifies existing heterogeneities in welfare gains or losses, independent of the specific experiment. This amplification works through heterogeneous ideal price indices, which react stronger to changes in housing rents for income-poor households than for income-rich households.

With these analytical insights at hand, we turn to our economy’s supply-side and endogenize housing rents. The zoning deregulation triggers a temporarily slower growth in rents and house prices. The representative household experiences a welfare gain of 0.37 percent permanent consumption units. At the micro-level, we see substantial heterogeneity with welfare gains ranging from -1.63 to 12.52 percent. A majority of 93 percent benefits from zoning deregulation. Households at the bottom of the earnings and wealth distribution benefit most because they are not affected by lower house prices, and they benefit more, due to Schwabe’s law, from lower housing rents. The highest welfare losses accrue to wealth-rich households at the bottom of the earnings distribution. These wealthy households care less about lower housing rents but are hurt particularly strongly by lower house prices. We show analytically that the distribution of welfare gains across households is not specific to our calibration but results under many parameter combinations.

We further decompose the welfare effect into different channels, both analytically and numerically. Two channels are of primary importance. According to the rent channel, declining rents increase welfare. The strength of this channel depends on the housing expenditure share relative to the representative household. This result resembles our analytical finding that Schwabe’s law matters for the household-specific welfare gain through the rent channel. According to the asset channel, house prices drop in response to zoning deregulation, thus reducing welfare. Quantitatively, the rent channel drives most of the welfare effect and most of its variation across households.

We conduct the following robustness checks. First, our results do not hinge on the interpretation of our preference specification as status preferences. We show that most results are also obtained with more general Stone-Geary preferences where housing demand is subject to a minimum housing consumption level. Second, we show that households are indifferent between owner-occupied housing or rental housing in the model economy. Our results on the welfare effects of housing rent dynamics therefore equivalently apply to owner-occupiers. Third, we loosen our assumption of portfolio homogeneity when studying zoning deregulation. Allowing for heterogeneous portfolios reveals that the strength of the asset channel depends on the housing wealth portfolio share relative to the representative household. The aggregate welfare effect and the rent channel are unaffected, but the asset channel is now more pronounced for households in the middle of the wealth distribution that hold most of their wealth in the form of housing. This finding mirrors the results of [Kuhn *et al.* \(2020\)](#), who empirically show that declining house prices increase wealth inequality. Although the asset channel becomes quantitatively more relevant for some households, the rent channel remains of first-order importance.

Our analysis clarifies how housing rents affect household welfare and how this effect varies across households when we capture Schwabe’s law. The finding of temporarily slower rent growth, triggered by zoning deregulation, implies that *welfare* inequality declines. Given that the wealth distribution responds to the policy reform under study, we lastly also discuss the effects on *wealth* inequality. Calibrating the model to the realistic case of heterogeneous household portfolios, we find that *wealth* inequality — measured by the top 10 percent wealth share — increases in response to zoning deregulation. This result shows that an increase in *wealth* inequality can go hand in hand with a decrease in *welfare* inequality. We show, both analytically and quantitatively, that the increase in *wealth* inequality is due to declining house prices and lower housing rent growth. The decline in house prices reduces the wealth of the middle-class more than that of the wealthy, increasing wealth inequality. Lower rent growth allows wealth-poor households to better smooth consumption by saving less, reducing saving rates of wealth-poor households more than of wealth-rich households. Overall, both channels operate in the same direction and are of similar magnitude.

Our paper relates to four different strands of literature. The first addresses the importance of the housing sector in macro models. Many models are designed to discuss business cycle phenomena, such as [Davis and Heathcote \(2005\)](#), [Iacoviello \(2005\)](#), [Iacoviello and Neri \(2010\)](#), [Kiyotaki *et al.* \(2011\)](#), [Kydlund *et al.* \(2016\)](#), and [Favilukis *et al.* \(2017\)](#).¹ More recently, a literature has emerged that focuses on the long term, such as [Borri and Reichlin \(2018\)](#), [Miles and Sefton \(2020\)](#), and [Grossmann *et al.* \(2020\)](#). Our research questions require a long-term perspective. The reason is that zoning deregulation triggers an ex-

¹[Piazzesi and Schneider \(2016\)](#) provide an excellent survey.

tended transition to a new steady state with more land allocated to the housing sector and a higher stock of structures. The second strand of literature analyzes one-sector economies under household heterogeneity with the representative household property (Chatterjee, 1994; Krusell and Rios-Rull, 1999; Caselli and Ventura, 2000; Alvarez-Pelaez and Díaz, 2005; Garcia-Penalosa and Turnovsky, 2006). We add to this literature by analyzing a two-sectoral model, allowing for a continuous relative price change under household heterogeneity with non-homothetic preferences and the representative household property. More specifically, our macroeconomic model with a housing sector demonstrates that Schwabe’s law can coexist with a constant aggregate housing expenditure share even under rising real housing rents. A third strand examines the dynamics of wealth inequality. Most contributions in this field have recently employed models allowing for idiosyncratic shocks under incomplete markets (De Nardi and Fella, 2017).² On the empirical side, Kuhn *et al.* (2020) have stressed that middle-class households’ wealth is highly exposed to changes in house prices by analyzing microdata for the US after WW2. Our analysis mirrors their insights as we find that house price changes exert household-specific welfare effects through the asset channel. Finally, there is a strand that focuses on the welfare cost of land-use regulations. Gyourko and Molloy (2015) survey the urban economics literature and conclude that most models and empirical estimates suggest that residential land-use regulations are likely to create a net welfare loss. A small number of macroeconomic papers corroborate this finding, also documenting welfare costs of land use regulations in a general equilibrium perspective (Herkenhoff *et al.*, 2018; Favilukis *et al.*, 2018).³

The structure of this paper is as follows. Section 2 introduces the demand-side of the model. It demonstrates the existence of a representative household, the prevalence of Schwabe’s law, and discusses the determinants of household-specific welfare. Section 3 introduces the production-side of the model, defines general equilibrium, and characterizes the steady state. Section 4 explains the policy experiment and discusses the results on household-specific welfare. Section 5 discusses Stone-Geary preferences, wealth inequality, and the costs of land-use regulation. Section 6 concludes.

2. Demand Side: Welfare and the Rent Channel

This section investigates the direct effect of rent changes on household welfare and how this effect depends on Schwabe’s law. The analysis takes four steps: i) We introduce the household setup, ii) we characterize the demand for housing and non-housing analytically, iii) we show that Schwabe’s law can be replicated, and iv) we discuss how rent changes

²These contributions focus on alternative mechanisms that shape the wealth distribution in steady state. Some exceptions analyze the wealth distribution over time by employing numerical techniques, such as Gabaix *et al.* (2016); Kaas *et al.* (2017); Kaymak and Poschke (2016); Hubmer *et al.* (2016); Wälde (2016).

³As these two papers are closely related to ours, we discuss their results in section 5.

affect household welfare.

2.1. Model: Household Sector

Consider a deterministic economy inhabited by mass one of infinitely lived households. Households are heterogeneous. There are $J \in \mathbb{N}$ different groups of households. Each household group is indexed by $j \in \{1, 2, \dots, J\}$ and consists of measure n_j identical households. Since the total number of households is normalized to unity, it holds that $\sum_{j=1}^J n_j = 1$. In what follows, when referring to a *household of group j* , we use the short formulation *household j* instead, keeping in mind that a group consists of a continuum of households.

We consider two sources of heterogeneity: i) differences in initial wealth and ii) differences in labor productivity. Time is continuous and indexed by $t \geq 0$. Let $W_j(t) \in \mathbb{R}$ denote wealth holdings of household j at point in time t . Initial wealth is then denoted by $W_j(0)$. Household j 's labor endowment (also called labor productivity) is time-invariant, denoted by $L_j \in \mathbb{R}^+$, and supplied inelastically to the labor market. Denote average variables by omitting the group index j . Since the total population is normalized to one, aggregates are equal to a variable's average. For instance, aggregate wealth is defined as $W = \sum_j n_j W_j$ and equal to average wealth. Similarly, aggregate and average labor supply are given by $L = \sum_j n_j L_j$.

Let $C_j \in \mathbb{R}^+$ and $S_j \in \mathbb{R}^+$ denote consumption of the non-housing good and consumption of housing services by household j , respectively. Life-time utility of household j is described by

$$U_j(0) = \int_0^\infty u(C_j(t), S_j(t), S(t)) e^{-\rho t} dt \quad (1)$$

$$\text{with} \quad u(C_j, S_j, S) = \frac{[(C_j)^{1-\theta}(S_j - \phi S)^\theta]^{1-\sigma} - 1}{1 - \sigma}, \quad (2)$$

where $\sigma > 0$, $\theta \in (0, 1)$, and $\rho > 0$ are parameters of the utility function. Households compare their level of housing services to a share $\phi \in [0, 1)$ of the average level of housing services, S , which is exogenous to the household. With $\phi > 0$ utility function (2) is non-homothetic and captures status preferences for housing. Status preferences for housing represent an old topic that has already been discussed by Marx (1891).⁴ There is also ample evidence for status preferences for housing.⁵ For instance, by employing US microdata,

⁴The following statement describes this view (Marx, 1891, pp. 35): *A house may be large or small; as long as the neighboring houses are likewise small, it satisfies all social requirement for a residence. But let there arise next to the little house a palace, and the little house shrinks to a hut. The little house now makes it clear that its inmate has no social position at all to maintain, or but a very insignificant one; and however high it may shoot up in the course of civilization, if the neighboring palace rises in equal or even in greater measure, the occupant of the relatively little house will always find himself more uncomfortable, more dissatisfied, more cramped within his four walls.*

⁵See Bellet (2017); Frank (2005); Turnbull et al. (2006); Leguizamon and Ross (2012).

Bellet (2017) shows that suburban homeowners who experienced a relative downscaling of their homes due to the building of larger units in their suburb record lower satisfaction with their home. Alternatively, preferences in (2) can also be interpreted as a special case of Stone-Geary preferences where the minimum, time-varying housing consumption level is given by ϕS . We show that most results do not change under Stone-Geary preferences in section 5.1.⁶ The primary motivation for utility specification (2) is that it enables us to replicate Schwabe's law under $\phi > 0$ while preserving the representative household property, as shown below.

Each household j chooses consumption paths $\{C_j(t), S_j(t)\}_{t=0}^{\infty}$ that maximize U_j subject to the standard No-Ponzi-game condition and the intertemporal budget constraint⁷

$$\dot{W}_j = rW_j + wL_j - C_j - pS_j, \quad (3)$$

where r, w , and p denote the interest rate, the wage rate per unit of labor, and the housing rent, respectively. Initial wealth, $W_j(0)$, is given. All prices are measured in units of a final output good. The budget constraint states that changes in wealth equal the excess of income over consumption expenditures. Since a household has mass zero, it takes all prices, r, w , and p , as well as the average consumption of housing services, S , as given. We model all households as renters, but the results do not change if we assume owner-occupied housing, as shown in appendix B.

2.2. Analytical Solution

For given initial wealth, $W_j(0)$, labor endowment, L_j , and price series $\{p(t), r(t), w(t)\}_{t=0}^{\infty}$, the solution of the household problem is fully characterized for all $t \geq 0$ by

$$C_j(t) + p(t)S_j(t) = \mu(t)[W_j(t) + \tilde{w}(t)L_j] \quad (4)$$

$$C_j(t) = p(t) \frac{1-\theta}{\theta} [S_j(t) - \phi S(t)] \quad (5)$$

$$\dot{W}_j(t) = [r(t) - \mu(t)]W_j(t) + [w(t) - \mu(t)\tilde{w}(t)]L_j \quad (6)$$

with

$$\mu(t) \equiv \left(\int_t^{\infty} \left[\left(\frac{p(\tau)}{p(t)} \right)^{\theta} \exp \left[-\tilde{r}(\tau, t) - \frac{\rho}{\sigma-1}(\tau - t) \right] \right]^{\frac{\sigma-1}{\sigma}} d\tau \right)^{-1}, \quad (7)$$

⁶In online appendix A we discuss further utility specifications. First, we show that the results are robust to assuming status concerns for both goods as long as the status concern for housing is larger than for the other good. Second, we demonstrate that a general CES utility function and a multiplicative instead of additive reference level are inconsistent with empirical observations.

⁷To simplify notation, we omit the time indices when this is not confusing. Moreover, a dot above a variable denotes the partial derivative with respect to time.

$$\tilde{r}(\tau, t) \equiv \int_t^\tau r(v)dv, \quad \text{and} \quad \tilde{w}(t) \equiv \int_t^\infty w(\tau)e^{-\tilde{r}(\tau, t)}d\tau. \quad (8)$$

All derivations and proofs are relegated to the appendix. The variable \tilde{w} is the present value of the time path of future wages at t and $e^{-\tilde{r}(\tau, t)}$ is a present-value factor that converts one unit of income at time τ to an equivalent unit of income at time t . Equation (4) states that total consumption expenditures, $C_j + pS_j$, are a fraction μ of total wealth, $W_j + \tilde{w}L_j$, which is the sum of wealth, W_j , and the present value of future labor income, $\tilde{w}L_j$. Variable μ is hence the propensity to consume out of total wealth. Equation (5) determines the optimal allocation of total consumption expenditures, $C_j + pS_j$, over the numeraire, C_j , and housing, S_j . It is visible that relative consumption, pS_j/C_j , is not constant across households when $\phi > 0$, implying that expenditure shares vary with household income. Equation (6) is the law of motion of wealth, W_j . This linear differential equation in W_j with time-varying coefficients can be solved to yield an explicit solution for W_j as a function of time.

In order to rule out negative utility, we assume that for all households j initial wealth, $W_j(0)$, is strictly greater than the threshold $\underline{W}_j(0) \equiv \frac{\phi}{1-(1-\theta)\phi}[W(0) + \tilde{w}(0)L] - \tilde{w}(0)L_j$.⁸ With $\phi = 0$ this condition implies that total wealth, $W_j(0) + \tilde{w}(0)L_j$, has to be strictly positive. With $\phi > 0$, the minimum wealth level is larger because households also have to afford $\phi S > 0$ units of housing services.

We define a representative household as a household that owns the economy's endowments and whose individual consumption and asset demand is equal to aggregate consumption and asset demand that results from a set of different households (Mas-Colell *et al.*, 1995; Caselli and Ventura, 2000). Since all first-order conditions are linear in household-specific variables, aggregation is possible, and we obtain

Proposition 1 (Representative household). *An economy populated by a set of households whose preferences are described by (1) together with (2) and who face an intertemporal budget constraint given by (3) admits a representative household.*

Consequently, the distributions of labor endowment, L_j , and wealth, W_j , play no role in the evolution of aggregate variables. The proposition holds for all values of $\phi \in [0, 1]$. Hence, a representative household also exists in the case of non-homothetic preferences ($\phi > 0$). It can already be seen from equations (4) and (5) that households face the same linear wealth expansion paths.⁹ Proposition 1 adds to the theoretical literature on dynamic macro models with a representative household and heterogeneity. It shows that a represen-

⁸This threshold results from $C_j^{1-\theta}(S_j - \phi S)^\theta = \mu(W_j + \tilde{w}L_j)/\mathcal{P}_j > 0$, where \mathcal{P}_j is the ideal price index, derived below.

⁹A necessary and sufficient condition for expressing aggregate demand as a function of aggregate wealth and prices is that all households' wealth expansion paths are parallel, straight lines (see, for instance, Mas-Colell *et al.*, 1995, ch. 4). This co-linearity also implies that preferences given by (1) and (2) admit an indirect utility function of the Gorman form, which can be obtained with a monotonic transformation of the utility function.

	housing expenditure share (in %)					
	aggregate	income quintile				
		1st	2nd	3rd	4th	5th
US (2015)	19	25	21	20	19	18
France (2011–2015)	22	26	24	24	23	18
Germany (2013)	27	37	33	29	27	24
UK (2015)	17	29	20	17	14	14

Notes: The housing expenditure share is the ratio of expenditures on housing services (including imputed rent) to total consumption expenditures. US data is from www.bls.gov/cex/data.htm, accessed June 19, 2017. French data is from [Accardo et al. \(2017\)](#), German data from [Statistisches Bundesamt \(2015\)](#), and UK data from the Office for National Statistics (2015).

Table 1: Schwabe’s Law

tative household may exist not only in a one-sectoral economy ([Chatterjee, 1994](#); [Krusell and Rios-Rull, 1999](#); [Caselli and Ventura, 2000](#); [Alvarez-Pelaez and Díaz, 2005](#); [Garcia-Penalosa and Turnovsky, 2006](#)), but also in a two-sectoral economy under non-homothetic preferences.

We assume that markets are complete. Market completeness is a necessary assumption for a representative household to exist. This assumption has costs and benefits. On the one hand, it implies that we abstract from feedback effects of changes in the wealth and income distributions on aggregate variables. On the other hand, this setup enables us to gain analytical insights into the mechanisms through which changes in rents and house prices affect household welfare. Abstracting from the feedback effects of distributional changes on aggregate variables, including prices, is precisely the reason why we can derive analytical insights. We, therefore, investigate fundamental mechanisms that operate even in the absence of incomplete markets.

2.3. Schwabe’s Law

One of our main insights is that when studying the distribution of welfare effects in housing-related experiments, it is crucial to capture the so-called *Schwabe’s law*. We explain this empirical pattern in the following before showing analytically how our preference structure (2) replicates it.

Schwabe’s law is based on the housing expenditure share. The housing expenditure share is the ratio of expenditures on housing services to total consumption expenditures. In table 1, we show its aggregate value and averages for income quintiles in the US, France, Germany, and the UK. First, housing expenditures are of first-order relevance for household

consumption with aggregate values between 19 and 27 percent. Households spend more on housing than on any other consumption category like food, transport, or entertainment. Second, in each economy, housing expenditure shares decline with income. In the 2015 US, households in the bottom income quintile spend 25 percent of their total consumption expenditures on housing, while only 18 percent is spent by households in the top income quintile. The same applies to other economies, which display an even more pronounced cross-sectional variation. Housing is hence a necessity good. While table 1 depicts only correlations, Albouy *et al.* (2016) provide reduced-form evidence for housing being a necessity good. The variation of housing expenditure shares with income was already documented more than 150 years ago and labeled *Schwabe's law* (Singer, 1937; Stigler, 1954). Stigler (1954, p. 100) characterizes it as the second fundamental law of consumer behavior coming after the well-known Engel's law:

*Hermann Schwabe, the director of the Berlin statistical bureau, proposed a second "law" in 1868. He had salary and rent data for 4,281 public employees receiving less than 1,000 thaler a year, and income and rent data for 9,741 families with incomes in excess of 1,000 thaler. For each group he found the percentage of income (or salary) spent on rent declined as income rose, and proposed the law: "The poorer any one is, the greater the amount relative to his income that he must spend for housing." The law seemed to contemporaries less obviously true than Engel's, and a considerable literature arose about it. Ernst Hasse found that it held for Leipzig in 1875, and E. Laspeyres confirmed it for Hamburg. Engel also accepted Schwabe's law.*¹⁰

Turning from the housing expenditure share's cross-sectional variation to its evolution over time reveals the following pattern. The aggregate housing expenditure share in the US has been mostly constant over time, despite aggregate income growth.¹¹ The fact that the housing expenditure share is decreasing with income in the cross-section but not declining over time presents a puzzle. We now show that status preferences for housing can solve this puzzle.

Let $e_j \equiv pS_j/(C_j + pS_j)$ denote the housing expenditure share and $\mathcal{W}_j \equiv W_j + \tilde{w}L_j$ total wealth of household j . From the household's first order conditions (4) and (5) one obtains the optimally chosen housing expenditure share e_j as described by

¹⁰According to Singer (1937, p. 145) Schwabe's law states that *the proportion of rent in income or expenditure is a continuously diminishing fraction of income*. Hence, one can define Schwabe's law regarding the share of housing expenditures in income or total consumption expenditures. We follow the latter definition and refer to Schwabe's law as the negative relation between the share of housing expenditures in total expenditures and income.

¹¹According to U.S. Bureau of Labor Statistics (2016) the aggregate share of housing expenditures in total consumption expenditures appears relatively stable over time at about 19 percent in the postwar US (Davis and Ortalo-Magné, 2011; Piazzesi and Schneider, 2016). Albouy *et al.* (2016) argue that the share of expenditure devoted to housing in the US has even been rising over time. We could generate a temporarily rising aggregate housing expenditure share by assuming that θ , the preference parameter governing the demand for housing, grows temporarily. A representative household would still exist, and Schwabe's law would still hold in that case.

Proposition 2 (Housing expenditure share). *Household j 's expenditure share for housing services is time invariant and given by*

$$e_j = \theta \left[1 + \frac{(1-\theta)\phi}{1-(1-\theta)\phi} \left(\frac{\mathcal{W}_j(0)}{\mathcal{W}(0)} \right)^{-1} \right]. \quad (9)$$

If and only if $\phi > 0$, the housing expenditure share, e_j , is declining in relative total wealth of household j at the initial period, $\mathcal{W}_j(0)/\mathcal{W}(0)$. The variance in housing expenditure shares is increasing in ϕ .

If $\phi = 0$, utility function (2) is homothetic, status preferences for housing do not exist, and each household chooses the same housing expenditure share $e_j = \theta$. If $\phi > 0$, utility function (2) is non-homothetic, status preferences are operative, and the housing expenditure share varies across household groups. This variation is due to differences in total wealth at the initial period, $\mathcal{W}_j(0)$. Households with higher initial overall wealth have a lower housing expenditure share, replicating Schwabe's law. The aggregate housing expenditure share is a function of parameters only and is given by $e \equiv pS/(C + pS) = \frac{\theta}{1-(1-\theta)\phi}$. The housing expenditure share is constant at the aggregate level, even if the aggregate income grows or fluctuates over time. The reason is that the reference level of housing consumption level, ϕS , which is exogenous to the household, is growing over time at the same rate as the representative household's housing consumption. To sum up, status preferences for housing, captured by utility function (2), can explain the puzzle of a negative relation of housing expenditure shares with income in the cross-section and a stable aggregate housing expenditure share over time.

2.4. Welfare

How does the status-induced heterogeneity in housing expenditure shares affect the household-specific welfare gain in response to an exogenous policy change? How do dynamics in rents affect household-specific welfare? To discuss these questions, we compare a baseline economy B with price series $\{p^B(\tau), w^B(\tau), r^B(\tau)\}_{\tau=t}^{\infty}$ and distributions of wealth at t and labor endowments $\{W_j^B(t), L_j^B\}_{j=1}^J$ to an alternative economy A with $\{p^A(\tau), w^A(\tau), r^A(\tau)\}_{\tau=t}^{\infty}$ and $\{W_j^A(t), L_j^A\}_{j=1}^J$. The household-specific welfare gain that is associated with moving from B to A , measured by consumption-equivalent variations, is defined by

$$\int_t^{\infty} \frac{\left[(1 + \psi_j) \mathcal{C}_j^B(\tau) \right]^{1-\sigma} - 1}{1-\sigma} e^{-\rho(\tau-t)} d\tau = \int_t^{\infty} \frac{\left[\mathcal{C}_j^A(\tau) \right]^{1-\sigma} - 1}{1-\sigma} e^{-\rho(\tau-t)} d\tau, \quad (10)$$

where $\mathcal{C}_j^B \equiv (C_j^B)^{1-\theta} (S_j^B - \phi S^B)^\theta$ and $\mathcal{C}_j^A \equiv (C_j^A)^{1-\theta} (S_j^A - \phi S^A)^\theta$ denote composite consumption in the baseline economy B and the alternative economy A , respectively. If ψ_j is positive,

household j prefers to live in economy A as opposed to economy B , and vice versa.

Household-specific price indices will turn out to be crucial for differences in welfare across households. We therefore first derive ideal price indices for this economy. Household j 's ideal price index is defined by $\mathcal{P}_j \mathcal{C}_j \equiv C_j + pS_j$ and is described by

Proposition 3 (Ideal price indices). *The ideal price index of household j in period t is given by*

$$\mathcal{P}_j(t) = \frac{p(t)^\theta}{\theta^\theta(1-\theta)^{-\theta}} \frac{1}{1-e_j}. \quad (11)$$

The aggregate and representative household's ideal price index, \mathcal{P} , is obtained for $e_j = e$. If $\phi = 0$, housing expenditure shares e_j are equal to θ for all households j such that the price index is also equal across households and given by $\mathcal{P}_j = \frac{p^\theta}{\theta^\theta(1-\theta)^{1-\theta}}$. If $\phi > 0$, wealth-poor households face i) a larger price index and ii) a higher positive effect of rising rents on their price index than wealth-rich households, as can be seen by taking derivatives of (11) with respect to ϕ and noting (9). This distributional effect works through the heterogeneity in housing expenditure shares, which are, under $\phi > 0$, larger for wealth-poor households than for wealth-rich households.¹²

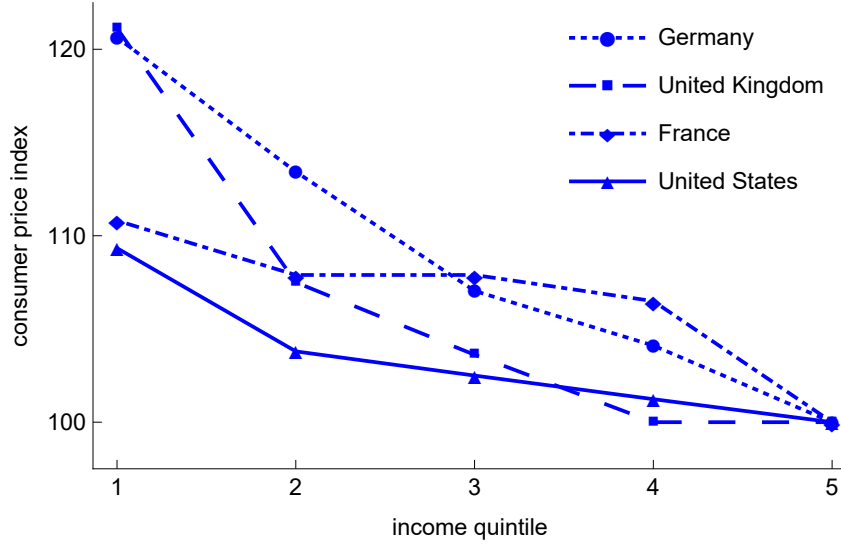
How much variation in price indices results from empirically observed heterogeneity in housing expenditure shares under $\phi > 0$? To illustrate the quantitative importance, we calculate price indices based on equation (11) for five income quintiles. We study an income quintile's price index relative to the price index of the highest income quintile, such that the housing rent cancels out and we do not need to feed in empirical values for the rent. The only empirical measure that we need is the housing expenditure share e_j . Figure 1 displays the results when using the data from table 1. In the US the lowest income group's price index is 9 percent higher than the wealthiest income group's price index. For the UK, this difference amounts to 20 percent, reflecting that the dispersion in housing expenditure shares is much larger in the UK than in the US. These differences are substantial. We show in section 4.3 that a change in the slope of the rent trajectory in response to zoning deregulation exerts very heterogeneous welfare effects, working mainly through differences in household-specific price indices.

We can now put the pieces together and show how the heterogeneity in housing expenditure shares affect household-specific welfare.

Proposition 4 (Welfare). *The additional welfare household j enjoys from living in economy A instead of B , at any point in time t , is given by*

$$\psi_j(t) = \left(\frac{\mu^A(t)}{\mu^B(t)} \right)^{\frac{\sigma}{\sigma-1}} \frac{\mathcal{W}_j^A(t)}{\mathcal{W}_j^B(t)} \left(\frac{\mathcal{P}_j(t)^A}{\mathcal{P}_j(t)^B} \right)^{-1} - 1. \quad (12)$$

¹²Albouy *et al.* (2016) construct an ideal cost-of-living index that varies with income and prices. Based on US microdata, they show that real income inequality, measured by the 90th to 10th percentile ratio, rose by ten percentage points more when income is deflated by an individual cost-of-living index.



Notes: Household-specific price indices are calculated with (11). The empirical data on housing expenditure shares, e_j , is from table 1.

Figure 1: Household-specific price indices

The first two terms are well known from one-sectoral models (e.g. Caselli and Ventura, 2000), except that the relative price, p , now also affects the propensity to consume, μ . The third term, $(\mathcal{P}_j^A(t)/\mathcal{P}_j^B(t))^{-1}$, adds a new channel. It enters equation (12) due to the two-sectoral structure, and it is heterogeneous across j due to non-homothetic preferences, as can be seen as follows. Under *homothetic* preferences, $\phi = 0$, price indices affect welfare in our two-sectoral economy as opposed to a one-sectoral economy. However, the effect is the same across households because the price index is the same across households. Price indices affect welfare differently across households only in a two-sectoral economy with *non-homothetic* preferences, $\phi > 0$, because then price indices do not only differ between economies A and B , but also between households. Capturing Schwabe's law with non-homothetic preferences, therefore, adds additional heterogeneity in household welfare.

Corollary 1 (Amplification of welfare differences). *Stronger status concerns for housing amplify, at any t , the welfare differences, measured by $\psi_j(t)$.*

The intuition is straightforward. The larger ϕ , the more pronounced is the curvature of the utility function (2) with respect to S . Assume a one-period household problem with utility function (2) and a given income \mathcal{W}_j . If income \mathcal{W}_j increases (decreases) by a given amount, utility increases (decreases) more for a large value of ϕ than for a lower value. This intuition translates to the dynamic problem (1). This general result shows that capturing Schwabe's law through status preferences for housing amplifies welfare differences, independent of the specific scenario under study. We expect this amplification mechanism to prevail in more complex model economies where additional features are introduced.

To highlight the amplification effect of status preferences, we study how (temporarily)

higher rent growth affects household welfare.

Proposition 5 (Rent channel). *i) A higher level of current rent $p(t)$ and ii) higher future rents $\tilde{p}(\tau) \equiv p(\tau)/p(t) > 1$ for $\tau > t$ both reduce household welfare. Under $\phi = 0$, both effects are homogeneous across households. Under $\phi > 0$, the poorer a household in terms of total wealth $W_j(t) + \tilde{w}L_j$, the higher is its welfare loss from i) a higher level of current period rents $p(0)$ and ii) from higher future rents, $\tilde{p}(\tau)$.*

We differentiate between i) the level effect and ii) the growth effect of higher rents because the former works primarily through the price index while the latter operates through the marginal propensity to consume. Intuitively, proposition 5 contains two results. First, higher present and future rents reduce household welfare because less consumption can be financed for a given income. Second, and more interestingly, Schwabe’s law amplifies the effect of rents on welfare. This is an application of corollary 1. Poor households spend a larger fraction of their consumption expenditures on housing than rich households. Higher rents consequently affect these households more strongly than richer households. This result shows that Schwabe’s law amplifies the variation in welfare effects of rent dynamics.

We have been silent on the source of changes in rents. For studying welfare, however, this crucially matters. Therefore, we study a specific policy experiment that results in pronounced rent dynamics in section 4. To generate endogenous dynamic in rents, we have to close the model first by introducing the production sector.

3. Supply Side and General Equilibrium

3.1. Sectoral Structure and Production Technologies

We now describe the firm side of the model. Recall that we want to study the welfare consequences of changes in rent dynamics triggered by residential zoning deregulation. One important dimension of this regulation concerns the separation of different types of land use (Gyourko and Molloy, 2015). We stress that zoning regulations constrain the availability of land for residential purposes and, hence, constrain primarily the number of houses, but not so much the size of a house.¹³ We therefore employ the two-sectoral macro model with a housing sector from Grossmann *et al.* (2020). This model distinguishes the housing stock along an extensive margin — the number of houses — and an intensive margin — the size of the average house. This distinction makes the model especially well suited to study the distributional effects of zoning regulations.

¹³For a discussion of the different dimensions of residential land use regulations see Gyourko *et al.* (2008, 2019) who provide the Wharton Regulation Index (WRI) describing local housing supply regulations in the US. Saiz (2010) stresses that metropolitan areas with high values of the WRI have zoning regulations or project approval practices that constrain new residential real estate development.

3.1.1. Non-Housing Good Sector

This sector consists of mass one of identical firms that act under perfect competition. Each firm produces a final good, Y , chosen as numeraire, according to the Cobb-Douglas production function

$$Y = K^\alpha (B^Y L^Y)^\beta (B^Y Z^Y)^{1-\alpha-\beta}, \quad (13)$$

where K , L^Y and Z^Y denote inputs of physical capital, labor and land, respectively. The productivity parameter, $B^Y > 0$, grows exponentially at a constant rate $g^Y \geq 0$. Parameters $\alpha \in (0, 1)$ and $\beta \in (0, 1)$ are output elasticities of capital and labor, respectively, and satisfy $\alpha + \beta < 1$. Firms maximize profits $Y - (r + \delta^K)K - wL^Y - R^Z Z^Y$ implying that factor prices equal their respective marginal products

$$r = \alpha \frac{Y}{K} - \delta^K, \quad w = \beta \frac{Y}{L^Y}, \quad \text{and} \quad R^Z = (1 - \alpha - \beta) \frac{Y}{Z^Y}. \quad (14)$$

Capital depreciates at rate $\delta^K \geq 0$ and gross physical capital investment reads as $I^K \equiv \dot{K} + \delta^K K$.

3.1.2. Housing Sector

A house is a bundle of two components: structure and land. Both are modeled as stocks. The former is produced by construction firms that employ construction materials and labor. The latter is produced by land development firms who transform non-residential land into residential land by using labor as an input. Property management firms use residential structures and land to generate housing services that are sold to households.

Construction Mass one of construction firms produce residential structures by combining labor, L^X , and construction materials, M . The aggregate stock of structures, X , evolves according to $\dot{X} = I^X - \delta^X X$, where $\delta^X \geq 0$ is the depreciation rate. Gross investment in residential structures, I^X , is produced according to

$$I^X = M^\eta (B^X L^X)^{1-\eta}, \quad (15)$$

where $\eta \in (0, 1)$ is the output elasticity of materials. Labor augmenting technology, $B^X > 0$, grows exponentially at the constant rate $g^X \geq 0$. The production of one unit of materials, M , requires one unit of the numeraire good. Newly produced structures, I^X , are sold to households at the price P^X . Each construction firm maximizes profits $P^X I^X - M - wL^X$ under perfect competition such that factor prices equal their marginal products

$$1 = \eta P^X \frac{I^X}{M}, \quad \text{and} \quad w = (1 - \eta) P^X \frac{I^X}{L^X}. \quad (16)$$

Land development The land development sector consists of mass one of firms that operate under perfect competition. The aggregate stock of residential land is denoted by N , it does not depreciate, and additions to this stock are \dot{N} . The representative land development firm chooses investment into residential land $Z^N \geq 0$ to maximize profits, i.e.

$$\max_{Z^N} \Pi^N = P^N Z^N - \underbrace{[P^Z Z^N + w \mathcal{L}^N(Z^N)]}_{\text{cost function}},$$

where P^N and P^Z are prices of developed land and non-developed land, respectively, and $L^N = \mathcal{L}^N(Z^N)$ is the labor input associated with the reallocation of land. We assume that this cost function takes the form

$$\mathcal{L}^N(Z^N) = \frac{\xi}{2} (Z^N)^2,$$

where $\xi > 0$ captures the importance of adjustment cost. These cost arise from activities like pulling down an existing building, leveling the surface off, and installing utilities like sewage, water, electricity, or roads. First-order conditions are given by

$$\dot{N} = Z^N = \frac{P^N - P^Z}{\xi w} \quad \text{and} \quad L^N = \frac{(P^N - P^Z)^2}{2\xi w^2}. \quad (17)$$

Firms produce new residential (non-residential) land if the price of residential land, P^N , is larger (smaller) than the price of non-residential land, P^Z . Newly developed residential land, \dot{N} , is sold to households at the price P^N . Residential land development firms are, however, only allowed to develop additional residential land when the total stock of residential land, N , is below a constant fraction $\kappa \in (0, 1]$ of the economy's total and constant land endowment, Z . That is, $N \leq \kappa Z$ must hold at each point in time. This constraint captures zoning regulations. The parameter κ is an exogenous policy parameter. If $\kappa = 1$, then the zoning regulation is equal to the physical constraint, $N \leq Z$. For sufficiently small values of κ the constraint might be binding such that $N = \kappa Z$ results in equilibrium.¹⁴

If $\dot{N} \neq 0$, then profits, Π^N , are strictly positive. We assume that these profits are taxed away by the government and used for wasteful government expenditures such that after-tax profits are equal to zero.¹⁵

¹⁴Notice that the constraint may be binding along the transition and in steady state. In fact, the baseline scenario considered below represents such a steady state. Relaxing this constraint triggers transitional dynamics towards an unconstrained steady state.

¹⁵The main reason for this assumption is that otherwise either i) we would have to introduce stocks of property management firms that are traded, implying an additional differential equation, or ii) we would have to take a stance on how these profits are distributed across households, which in turn would add an additional distributional channel in our analysis. The assumption of wasteful government expenditures allows us to focus on the existing distributional effects without adding another channel. Note further that these profits are zero in a steady state with $\dot{N} = 0$.

Property management firms Property management firms rent residential structures, X , and residential land, N , from households to produce housing services, S , according to

$$S = X^\gamma N^{1-\gamma}. \quad (18)$$

where $\gamma \in (0, 1)$ is the output elasticity of S with respect to structures. The amount of residential land employed in housing services production, N , can be interpreted as a measure of the extensive margin of housing supply. Housing services are sold to households at the rental rate p . Each firm maximizes profits $pS - R^N N - (R^X + \delta^X P^X)X$, where R^N and R^X are rental rates of residential land and structures, respectively. Competition is perfect and factor prices equal their respective value marginal products according to

$$R^N = (1 - \gamma)p \frac{S}{N} \quad \text{and} \quad R^X = \gamma p \frac{S}{X} - \delta^X P^X. \quad (19)$$

3.1.3. Assets

The assets in this economy are i) physical capital, K , ii) non-residential land, Z^Y , iii) residential structures, X , and iv) residential land, N . Households own these assets and rent them out to firms. Aggregate wealth accordingly comprises

$$W \equiv \underbrace{K + P^Z Z^Y}_{\text{non-housing wealth}} + \underbrace{P^N N + P^X X}_{\text{housing wealth}}. \quad (20)$$

Households are indifferent with regard to the allocation of their wealth, W_j , across different assets given that the following no-arbitrage conditions hold¹⁶

$$r = \frac{\dot{P}^Z + R^Z}{P^Z} = \frac{\dot{P}^N + R^N}{P^N} = \frac{\dot{P}^X + R^X}{P^X}. \quad (21)$$

At the household level it is therefore sufficient to study total wealth without considering the allocation of wealth across the four asset classes.

We now define the stock of housing and the house price. Let $H \equiv X^\gamma N^{1-\gamma}$ be a quantity index that describes the stock of housing in the economy. We choose it such that $H = S$ and one unit of housing stock produces one unit of housing services per instant of time. Housing wealth can be expressed as the value of residential structures plus the value of residential land, $P^N N + P^X X$, or as the value of houses, $P^H H$, where P^H is the house price. This equivalence reflects that houses are bundles of structure and land (Davis and Heathcote, 2007). Since the two have to be equal, $P^N N + P^X X = P^H H$, the resulting house price reads

$$P^H = \frac{N}{H} P^N + \frac{X}{H} P^X.$$

¹⁶If one of the arbitrage conditions would be violated, the demand for the respective asset would not be finite — a violation of equilibrium conditions.

The house prices is a weighted sum of the prices of residential land and residential structures. The weights are the land and structure intensity in housing, respectively.

3.2. General Equilibrium Definition and Steady State

We now define the competitive equilibrium.

Definition 1 (Equilibrium). *A competitive equilibrium is a path of aggregate quantities $\{Y(t), K(t), X(t), N(t), M(t), L^Y(t), L^X(t), L^N(t), Z^N(t), Z^Y(t), C(t), S(t), W(t), I^K(t), I^X(t)\}_{t=0}^{\infty}$, group-specific quantities $\{\{C_j(t), S_j(t), W_j(t)\}_{j=1}^J\}_{t=0}^{\infty}$, and prices $\{p(t), P^Z(t), P^N(t), P^X(t), w(t), r(t), R^Z(t), R^X(t), R^N(t)\}_{t=0}^{\infty}$ with given aggregate initial states $K(0), N(0), X(0)$, and distributions of labor and initial wealth $\{L_j, W_j(0)\}_{j=1}^J$ such that*

- (i) *individuals maximize lifetime utility (1) subject to (2) and (3);*
- (ii) *the representative firms in the sectors producing the numeraire good, Y , structures, X , residential land, N , and housing services, S , maximize their respective profits such that their first order conditions given by (14), (16), (17), and (19) hold;*
- (iii) *the labor market clears: $L^Y(t) + L^X(t) + L^N(t) = \sum_{j=1}^J n_j L_j$;*
- (iv) *asset markets clear: $W(t) = \sum_{j=1}^J n_j W_j(t)$ and wealth is the sum of all aggregate assets according to (20);*
- (v) *perfect arbitrage holds across all asset classes as given by (21);*
- (vi) *the market for housing services clears: $H(t) = S(t) = \sum_{j=1}^J n_j S_j(t)$;*
- (vii) *the land use regulation holds: $N(t) \leq \kappa Z$;*
- (viii) *the market for the numeraire good clears: $Y(t) = C(t) + I^K(t) + M(t)$ with $C(t) = \sum_{j=1}^J n_j C_j(t)$.¹⁷*

Next, we turn to the steady state of the economy. We define a steady state as a competitive equilibrium according to definition 1 where all variables grow at constant and possibly different rates. The steady state growth rates are linear transformations of the growth rates of productivity parameters, g^X and g^Y , as shown in appendix A. Here we focus on the steady state growth rate of the rent which is denoted by \hat{p} and is given by

$$\hat{p} = (1 - \gamma\eta)g^Y - \gamma(1 - \eta)g^X. \quad (22)$$

The rent is the relative price of housing services, S , that are produced with residential structures, X , and land, N . On one hand, higher income growth (increase in g^Y) raises the demand for housing, but since the long run supply of the number of houses, N , is fixed, this results in a higher rent.¹⁸ On the other hand, higher productivity growth in the construction

¹⁷The goods market clearing condition is redundant, according to Walras' law. To exclude conceptual or calculation errors, we analytically and numerically checked that the equilibrium derived from conditions (i)-(vii) fulfills condition (viii).

¹⁸As also shown in appendix A, GDP and the wage rate grow at the rate g^Y in steady state.

sector (increase in g^X) makes the production of structures, X , relatively cheaper, resulting in a lower rent. These two opposing forces hence operate through the extensive and intensive margins of housing, N and X , respectively.

4. Zoning Deregulation

Residential zoning regulations are widely considered as an important amplifier of surging housing rents in a growing economy (Glaeser *et al.*, 2005; Saiz, 2010). For instance, Albouy and Ehrlich (2018) find that, based on data for 230 metropolitan areas in the US from 2005 to 2010, observed land-use restrictions substantially increase housing rents. Moreover, Gyourko and Molloy (2015) argue that zoning regulations were effectively introduced in the US during the 1970s. This is consistent with Davis and Heathcote (2007) who show that residential land grew by an average annual growth rate of 5 percent during the period 1945–1975 but grew merely by 0.7 percent during the period 1976–2016. We now investigate how residential zoning deregulation — interpreted as the removal of constraints on the use of land for residential purposes — changes the slope of the rent trajectory and thereby affects welfare in general equilibrium.

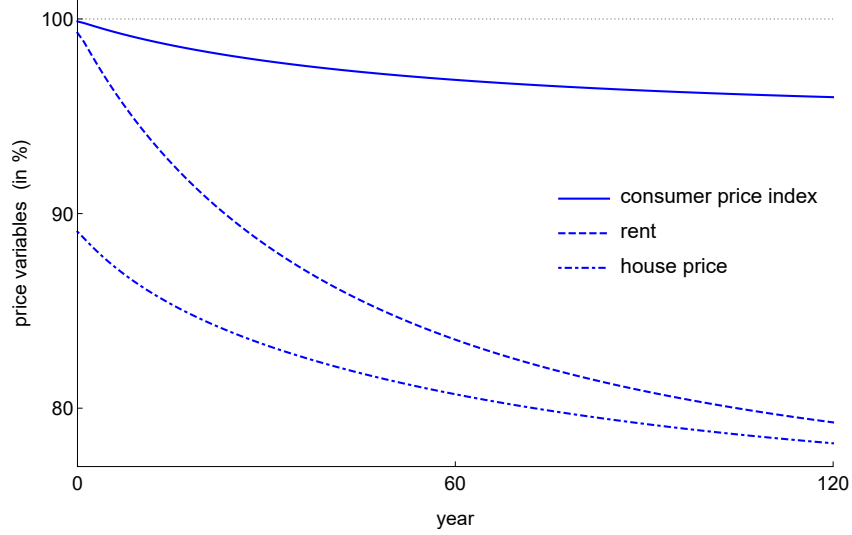
4.1. Calibration and Experiment

We calibrate the model economy in steady state to the current US economy. The calibration of all parameters is described in appendix C. We now briefly explain the calibration of a few central parameters. Schwabe’s law is replicated by setting ϕ and θ such that we match the aggregate housing expenditure share and the differences in housing expenditure shares between the bottom and top income quintiles reported in table 1. The initial joint distribution of wealth and earnings is exogenous in our model. We set it equal to the empirically observed distribution in the US by using data from the 2016 wave of the Survey of Consumer Finances (SCF), taken from Kuhn *et al.*’s (2020) online appendix. Therefore, our economy consists of $J = 10\,180$ different household groups with weights given by the SCF. Lastly, we set the policy parameter governing the zoning restriction, κ , to match the share of residential land in overall economically used land in the US of 16.9 percent. This value follows from geographic land-use data provided by Falcone (2015). Under the resulting $\kappa = 0.169$ the zoning restriction is binding, and N/Z is equal to 0.169. Without zoning regulations, the model would imply a steady state value of $N/Z = 0.54$.

To investigate how residential zoning deregulation affects household welfare, we compare two scenarios. In the baseline scenario (zoning), the economy is in a steady state, conditional on the binding zoning regulation $N = \kappa Z$ with $\kappa = 0.169$. In the alternative scenario (no zoning), residential zoning regulations are entirely removed. That is, we set

$\kappa = 1$.¹⁹ The alternative scenario exhibits transitional dynamics, starting from levels of state variables (K, X, N) according to the baseline scenario. The analysis captures all general equilibrium effects. That is, all prices are fully endogenous and change in response to an exogenous policy event.²⁰

4.2. Aggregate Effects



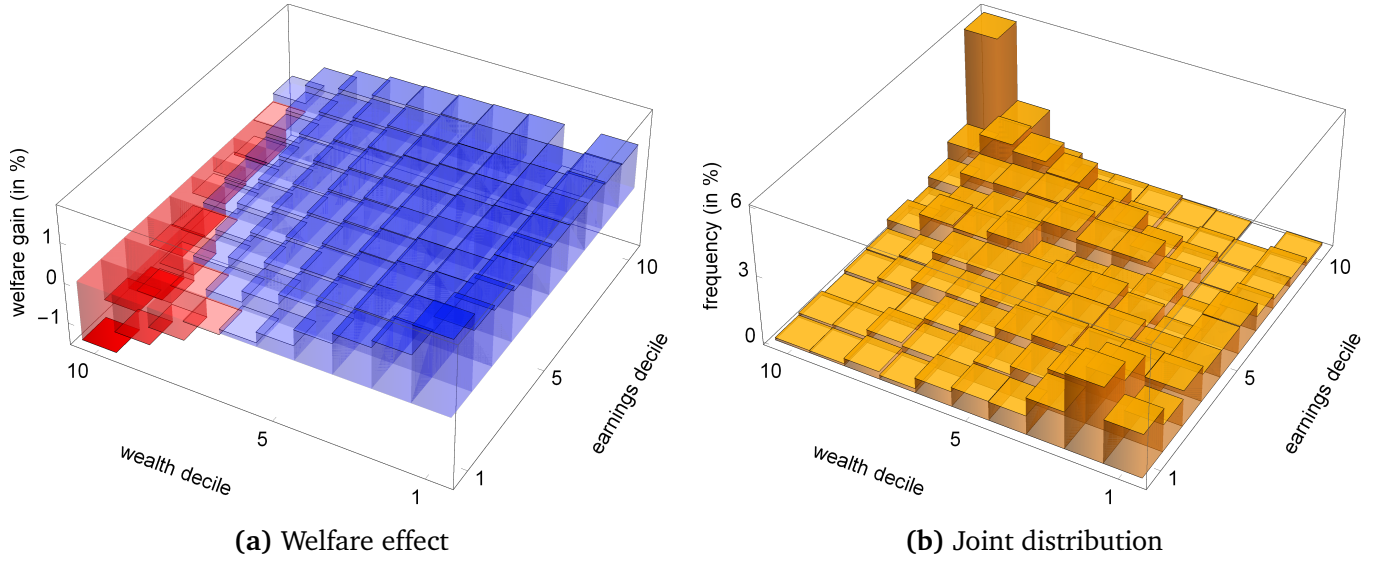
Notes: All variables are detrended and the initial steady state in the baseline scenario (zoning) is normalized to unity.

Figure 2: Evolution of the ideal prices index, \mathcal{P} , the rent, p , and the house price, P^H , in response to residential zoning deregulation.

What are the main effects of the zoning deregulation on the macroeconomy? First and foremost, the removal of the residential zoning regulations triggers an expansion of residential land. More precisely, land is reallocated from the numeraire sector to the housing sector. Land developers stretch this process out over time due to convex land development costs. Property management firms expand the supply of housing services along the transition to the new and unconstrained steady state. This supply response triggers a series of price changes. We focus on those price variables that are especially relevant for household welfare: i) the rent, ii) the price index, and iii) the house price. Figure 2 illustrates the evolution of these prices over time.

¹⁹To be precise notice two points. First, $N \leq \kappa Z$ with $\kappa = 1$ is just a natural resource constraint stating that land in the housing sector, N , cannot exceed the overall land endowment, Z . Second, given concave utility and production functions, any equilibrium is an interior equilibrium such that $N = Z$ does not occur.

²⁰We solve the representative agent economy with Trimborn *et al.*'s (2008) relaxation method. The solution of household-level variables follows then our analytical expressions, (4) – (6) and (25). This two-step approach results from the existence of a representative agent and simplifies the computation considerably (Caselli and Ventura, 2000).



Notes. Panel (a): Welfare effects are measured by consumption equivalent variations according to (12). Blue bars show welfare gains, red bars show welfare losses. The portfolio structure is assumed to be the same for all households. Welfare gains are calculated as averages over individual welfare effects within each of the 100 groups resulting from partitioning the population according to earnings and wealth deciles. Panel (b): Distribution of households over earnings deciles (l_j) and wealth deciles (W_j). Data are taken from the SCF+ data set in 2016.

Figure 3: Welfare effects of zoning deregulation (homogeneous portfolio) and the empirical joint earnings and wealth distribution.

The rent drops on impact by 0.7 percent and declines over the long term by 24 percent.²¹ The aggregate price index drops slightly by 0.1 percent on impact and declines by 4.7 percent over the long term.²² The house price drops by 11 percent on impact and declines by 23.3 percent over the long term.²³ This strong asset price response reflects that the housing wealth was artificially kept scarce in the baseline scenario due to the zoning regulations. Aggregate housing consumption, S , increases over time by 28 percent.²⁴

4.3. Welfare Effects

We now study how zoning deregulation affects household welfare. Therefore, we calculate the consumption-equivalent variation, ψ_j , as given by (12) for all agents in the economy under the dezoning regulation experiment described in section 4.1. It measures the

²¹The immediate drop in the rent results from a reduction in housing demand driven by a negative wealth effect. The short-run supply for housing services is completely price-inelastic as housing supply employs two state variables, namely the stock of residential buildings and residential land. Hence, any immediate price effects must be due to shifts in housing demand.

²²We show the aggregate price index for illustrative purposes. It traces the evolution of the rent in a mitigated manner. Recall that the aggregate housing expenditure share is 19 percent. This value implies that the price index movements amount to about 1/5 of the rent dynamics.

²³The house price declines by the same proportion over the long term as the rent. This decline reflects that i) the house price is the PDV of future housing yields, ii) one house generates one unit of housing services, and iii) given that the interest rate does not change over the long term.

²⁴We plot time paths of all the other variables in figure D.1 of the online appendix.

required percentage change in a household's composite consumption, \mathcal{C}_j , in the baseline scenario for the household to be indifferent between the baseline and the alternative scenario. Under a positive value, the household has to be compensated in the baseline scenario and prefers the alternative scenario. The opposite holds for a negative value. The resulting consumption-equivalent variations are shown in figure 3a. There we plot ψ_j as a function of individual labor productivity L_j and initial wealth endowment $W_j(0)$ as these are the two exogenous sources for heterogeneity in this economy. Because both the earnings and wealth distribution are strongly skewed to the right with fat tails, we do not plot all 10 180 observations but partition the full sample into 100 groups according to wealth and earnings deciles. The reported value for ψ_j in a given wealth and earnings decile in figure 3a is the mean taken over all households within this group.

The representative household, which describes the aggregate economy, experiences a welfare gain of 0.37 percent permanent consumption units. This positive welfare effect reflects that zoning regulations create a distortion and abolishing it typically moves the economy closer to the first-best allocation.²⁵

Moving from the aggregate to the micro-level reveals pronounced heterogeneity in welfare effects. The majority of households are better off in the alternative economy without zoning regulations, but some are worse off and prefer the baseline economy with zoning regulations. Across all households, the welfare gains vary from a maximum loss amounting to -1.63 percent to a maximum gain of 12.52 percent. These are the extreme values across all households in the employed SCF+ dataset. The average welfare gains within the 100 groups, shown in figure 3a, are in the range of -1.48 and 1.84 percent. For most parts of the wealth and earnings distributions, it holds that i) the wealthier a household or ii) the earnings-richer a household, the smaller is the welfare benefit from zoning deregulation. For example, the largest welfare gain of 12.52 percent accrues to poor households who belong to the first earnings and wealth decile, and the largest welfare loss amounting to 1.63 percent falls upon households in the first earnings and the tenth wealth decile. We explain the reason for this variation in the next section.

We also plot the joint distribution of earnings and wealth based on the 2016 SCF+ in figure 3b. This plot is purely descriptive and illustrates the frequency of households over the wealth and earnings dimension. Together with figure 3a, one can see that most households are in the blue area and hence benefit from zoning deregulation. In total, 93 percent of households benefit from zoning deregulation. Very few households are in the first earnings decile and the 10th wealth decile, which is the group with the highest welfare loss. A large group of households is in the 10th earnings and 10th wealth decile. This group is slightly

²⁵Note that there is also an additional distortion in the economy — status preferences for housing. Households do not take the negative externality of housing consumption into account and consume too much housing. Therefore, the economy without any zoning regulation is not first-best, as discussed in greater detail below. For now, it is sufficient to understand that zoning regulations are more distortionary than status preferences.

better off in the alternative scenario.

How does the welfare gain in figure 3a vary in the wealth and earnings dimension? First, to study the variation in the welfare gain along the wealth distribution we form the derivative of (12) with respect to wealth in the baseline scenario. Using the price index (11), the housing expenditure share (9), and employing the simplifying assumption of homogeneous portfolio shares, implying $W_j^A = \frac{W^A}{W^B} W_j^B$, one obtains

$$\frac{\partial \psi_j}{\partial W_j^B} = \left(\frac{\mu^A}{\mu^B} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{p^A}{p^B} \right)^{-\theta} \frac{\omega^2 \tilde{w}^B L}{\left(\omega \mathcal{W}_j^B - \phi \mathcal{W}^B \right)^2} \left(\frac{L_j}{L} - \frac{\phi}{\omega} \right) \left(\frac{W^A}{W^B} - \frac{\tilde{w}^A L}{\tilde{w}^B L} \right), \quad (23)$$

where $\omega \equiv [1 - (1 - \theta)\phi] > 0$ is a constant. The sign of this derivative is given by the product $\left(\frac{L_j}{L} - \frac{\phi}{\omega} \right) \left(\frac{W^A}{W^B} - \frac{\tilde{w}^A L}{\tilde{w}^B L} \right)$. The first term is household-specific, while the second is equal across households.

Assume first that $\phi = 0$. Then, the penultimate term in (23) is positive for all households, and the last term determines the sign of the derivative. If aggregate wealth increases relatively more between baseline and alternative scenario than human wealth, $\frac{W^A}{W^B} > \frac{\tilde{w}^A L}{\tilde{w}^B L}$, then the welfare gain is increasing in household wealth. The opposite applies to $\frac{W^A}{W^B} < \frac{\tilde{w}^A L}{\tilde{w}^B L}$. To summarize, under $\phi = 0$ and $\frac{W^A}{W^B} > \frac{\tilde{w}^A L}{\tilde{w}^B L}$, derivative (23) shows that the wealthier a household, the more it benefits from higher growth in overall wealth than in wages between the scenarios.

Under $\phi > 0$, the sign of the derivative changes along the earnings distribution. Suppose that $\left(\frac{W^A}{W^B} - \frac{\tilde{w}^A L}{\tilde{w}^B L} \right) < 0$. We observe this relationship in our experiment where aggregate wealth and human wealth decline by four and one percent, respectively. For households whose relative earnings, $\frac{L_j}{L} = \frac{w_{L_j}}{w_L}$, are above the threshold $\frac{\phi}{\omega}$, a higher wealth implies a lower welfare gain. That is what we observe in figure 3. For households whose relative earnings, $\frac{L_j}{L} = \frac{w_{L_j}}{w_L}$, are below the threshold $\frac{\phi}{\omega}$, an increase in wealth implies an increase in the welfare gain. This analytical result applies to none of the 100 groups displayed in figure 3 because almost no household's relative earnings are below the threshold. In our calibration, the threshold is at a low value of 0.09. Only 1.5 percent of households have earnings that are lower than 9 percent of average earnings.

Second, to study the variation in the welfare gain along the earnings distribution take the derivative of (12) and rearrange it along the lines of (23) to obtain

$$\frac{\partial \psi_j}{\partial L_j} = - \left(\frac{\mu^A}{\mu^B} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{p^A}{p^B} \right)^{-\theta} \frac{\omega^2 \tilde{w}^B W^B}{\left(\omega \mathcal{W}_j^B - \phi \mathcal{W}^B \right)^2} \left(\frac{W_j^B}{W^B} - \frac{\phi}{\omega} \right) \left(\frac{W^A}{W^B} - \frac{\tilde{w}^A L}{\tilde{w}^B L} \right) \quad (24)$$

This derivative is very similar to (23), and the same reasoning applies. The sign depends on the last two terms in brackets. The last expression in brackets is, as previously explained, negative in our experiment. If ϕ were equal to zero, the right-hand side of (24) would be

positive, and the welfare gain would be increasing in labor productivity. We observe this relationship for most households in figure 3. However, for households in the first two wealth deciles, the relationship is the opposite, the welfare gain is decreasing in labor productivity. This sign-switch results under $\phi > 0$ if $\frac{W_j^B}{W^B} < \frac{\phi}{\omega}$. That is, a household's wealth relative to average wealth is below the threshold $\frac{\phi}{\omega}$. A third of all households' wealth is smaller than $100 \times \frac{\phi}{\omega} = 9$ percent of the average wealth in our calibration. Most households in the first two wealth deciles have zero or negative wealth. Higher labor productivity results in lower welfare gains for households with such low wealth levels, as shown in figure 3.

4.4. Decomposition of Welfare Effects into Different Channels

Zoning deregulation affects household welfare through different channels, namely through changes in the rent, wage, interest rate, aggregate housing consumption, and asset prices. We use an analytical decomposition to study the different channels. The natural logarithm of the welfare gain, defined as $\tilde{\psi}_j \equiv \ln(1 + \psi_j)$ according to (12), can be expressed as²⁶

$$\begin{aligned} \tilde{\psi}_j(0) &= \frac{\sigma}{\sigma - 1} [\ln \mu^A(0) - \ln \mu^B(0)] + \ln [\mathcal{W}_j^A(0) - \phi \tilde{S}^A(0)] - \ln [\mathcal{W}_j^B(0) - \phi \tilde{S}^B(0)] \\ &\quad - \theta (\ln p^A(0) - \ln p^B(0)) \\ &\equiv f \left(\left\{ \{p^k(t), w^k(t), r^k(t), S^k(t)\}_{t=0}^{\infty}, P^{Z,k}(0), P^{H,k}(0) \right\}_{k \in \{A,B\}} \right), \end{aligned} \quad (25)$$

where

$$\phi \tilde{S}^k(0) = \phi p^k(0) \int_0^{\infty} \tilde{p}^k(\tau) S^k(\tau) e^{-\int_0^{\tau} r^k(v) dv} d\tau$$

is the present value of the future reference levels of housing services consumption, $\tilde{p}^k(t) \equiv p^k(t)/p^k(0)$ the growth factor of rents, and $k \in \{A, B\}$ indexes the respective scenario. The main difference compared to (12) is that (25) is a function of the entire time path of aggregate future housing consumption, $\{S(t)\}_{t=0}^{\infty}$, which is exogenous to the household. We use expression (25) in what follows because it allows us to isolate the effect of rent changes — the rent channel — from changes in aggregate housing services consumption — the status channel. The last line of (25) defines the welfare gain as a function of sequences of aggregate prices and quantities.

The decomposition of the welfare gain is then given by its total differential

$$d\tilde{\psi}_j = \overbrace{\frac{\partial f(\bullet)}{\partial p^A(0)} dp^A(0) + \int_0^{\infty} \frac{\partial f(\bullet)}{\partial \tilde{p}^A(t)} d\tilde{p}^A(t) dt}^{\text{rent channel}} + \overbrace{\int_0^{\infty} \frac{\partial f(\bullet)}{\partial w^A(t)} dw^A(t) dt}^{\text{wage channel}} + \overbrace{\int_0^{\infty} \frac{\partial f(\bullet)}{\partial S^A(t)} dS^A(t) dt}^{\text{status channel}}$$

²⁶This expression is a slightly modified version of (12). We derive it in appendix B.4, where it is given by (62). We have replaced $\tilde{S}^A(0)$ by $\phi \tilde{S}(0)$.

$$+ \underbrace{\frac{\partial f(\bullet)}{\partial P^{Z,A}(0)} dP^{Z,A}(0) + \frac{\partial f(\bullet)}{\partial P^{H,A}(0)} dP^{H,A}(0) + \int_0^\infty \frac{\partial f(\bullet)}{\partial r^A(t)} dr^A(t) dt}_{\text{asset channel}}. \quad (26)$$

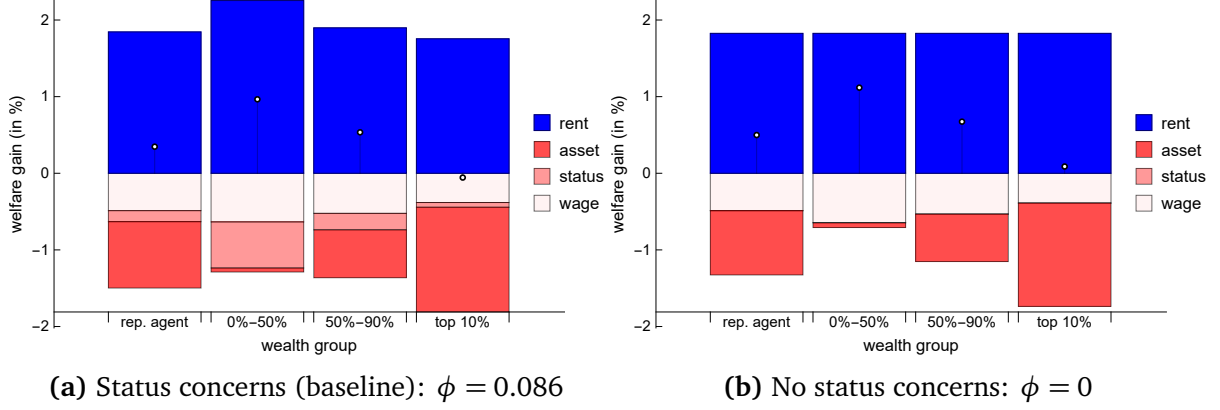
For example, the effect of changes in housing rent in period 0 on welfare is given by $\frac{\partial f(\bullet)}{\partial p^A(0)} dp^A(0)$, where $dp^A(0)$ is the absolute change in real rent between the baseline and the alternative scenario and $\frac{\partial f(\bullet)}{\partial p^A(0)}$ is the partial derivative of the welfare gain with respect to the level of rent in period 0. In the following, we analytically determine the partial derivatives' signs and numerically quantify the decomposition's summands.

Figure 4a shows the decomposition for selected household groups. We put some channels together for better visibility, precisely as indicated in (26), resulting in four different channels. Besides the representative agent, we show the decomposition for the bottom 50 percent of the wealth distribution, the middle class, which are between the 50th and 90th wealth percentile, and the top 10 percent of the wealth distribution. We first focus on the aggregate level and turn to the distribution afterward.

The aggregate welfare effect is positive and equal to a 0.37 percent permanent increase in composite consumption in the baseline scenario. It is, however, very unevenly distributed across agents. On average, households in the bottom 50 percent of the wealth distribution are better off by a 1 percent permanent consumption increase, households in the middle-class benefit by 0.3 percent, and the 10 percent wealthiest households are even slightly worse off. This unequal welfare effect implies that mainly wealthy households benefit from zoning regulations, while wealth-poor households benefit from their abolishment. In the following, we explain the overall welfare effect and its distribution across agents by decomposing it into different channels.

Rent channel The most substantial effect comes from rent changes, as shown by the blue areas in figure 4a. Zoning deregulation reduces rent growth, as can be seen in figure 2. It seems very intuitive that a household benefits from paying lower rent. This effect is not a result of our specific calibration, but it holds under any parameter constellation, as shown in proposition 5. To be precise, both the level of rents in period zero and the growth of rents in future periods decline. We have shown in proposition 5 that this direct effect increases welfare for all households and more for wealth-poor households than for wealthier households. The heterogeneous rent channel results from Schwabe's law. As poor households spent a larger fraction of their consumption expenditures on housing than wealthy households, they benefit relatively more than wealthy households from lower rent growth. To contrast this result, we assume $\phi = 0$ and plot the resulting welfare in figure 4b. The rent channel is then the same across households, and the overall welfare effect is less dispersed. The comparison of figure 4a with figure 4b shows once more that Schwabe's law amplifies the welfare effect of changes in rent dynamics.

Status channel Another strong driver of welfare gains is the status channel. It results



Notes. 0%–50%: poor households with net wealth below the median wealth. 50%–90%: middle class households with net wealth between the median and 90th percentile. Top 10%: 10% richest households in terms of net wealth. Rep. agent: representative agent. Circles: net welfare effect. The decomposition of welfare gains follows (25). In addition to the welfare effect of the representative agent both panels show welfare gains of three wealth groups calculated as averages over the individual welfare effects within each of the three wealth groups.

Figure 4: Decomposition of welfare effects under zoning deregulation.

from an endogenous increase in aggregate housing services, affecting households negatively because they compare their housing services consumption to the average. Aggregate housing consumption in the initial period, $S(0)$, does not change in the experiment because it is a function of $X(0)$ and $N(0)$, both state variables. Therefore, we study only changes in future housing services consumption, $S(t)$ for $t > 0$. We assume that $S(t)$ is increasing over some interval $t \in [t - m, t + m]$, $m > 0$ without reducing $S(t)$ outside of this interval. This assumption holds in the numerical solution. Hence, $\tilde{S}(0)$, the present value of aggregate future housing expenditures, increases. We therefore study how changes in $\tilde{S}(0)$ affect household welfare by taking the derivative

$$\frac{\partial f(\bullet)}{\partial \tilde{S}^A(0)} = -\frac{\phi}{\mathcal{W}_j^A(0) - \phi \tilde{S}^A(0)} \leq 0.$$

If $\phi = 0$, this derivative is zero. Without status preferences, households do not care about changes in aggregate housing consumption. Otherwise, higher future average housing consumption reduces household welfare, in particular for poor households. The reason is that poorer households are closer to the threshold ϕS . Zoning deregulation triggers a housing boom, raising aggregate consumption of housing services. This housing boom negatively affects households, who compare their housing consumption to the average, reducing the total welfare gain and hurting poor households more than wealthy households. In section 5.1 we discuss how the results change if we abstract from endogenous adjustments in ϕS by assuming more general Stone-Geary preferences.

Asset channel The last important channel is the asset channel, particularly the effect of

house price changes on welfare.²⁷ The latter is given by²⁸

$$\frac{\partial f(\bullet)}{\partial P^{H,A}(0)} = \frac{\partial f(\bullet)}{\partial \mathcal{W}_j^A(0)} \frac{\partial \mathcal{W}_j^A(0)}{\partial P^{H,A}(0)} = \frac{N_j(0)}{\mathcal{W}_j^A(0) - \phi \tilde{S}^A(0)} \geq 0, \quad (27)$$

where the house price in the alternative scenario is denoted by $P^{H,A}(t)$ and $N_j(0)$ is the quantity of housing owned by household j in period 0. It is very intuitive that higher house prices increase welfare. The more housing, $N_j(0)$, a household owns, the stronger is the effect of house prices changes on welfare given that ϕ is sufficiently small.²⁹ Our zoning deregulation experiment results in an immediate drop in house prices; see figure 2. This drop reduces welfare, particularly for wealthy households, as displayed in figure 4a.

4.5. Heterogeneous Portfolio Shares

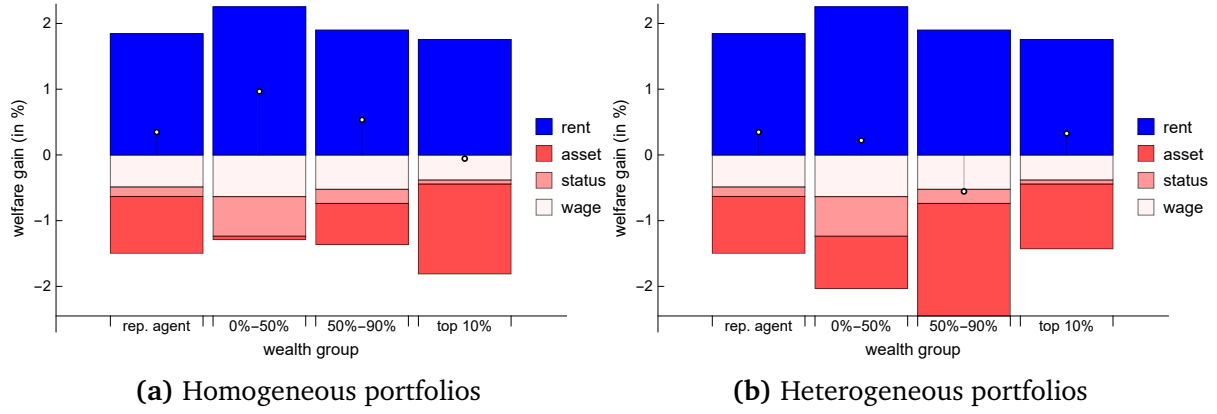
So far, we have abstracted from heterogeneity in household portfolio shares. We now study how zoning deregulation changes household-specific welfare effects when we allow the portfolio composition to differ across households. Recall that the allocation of household wealth across different assets is indeterminate in our economy. We can therefore set the initial distribution of housing and non-housing wealth equal to the distribution observed in the US in 2016 as given by the SCF+ (Kuhn *et al.*, 2020). In figure 5b we show the overall welfare effect and its decomposition into four different channels under portfolio heterogeneity for the representative agent and partition the sample into three subgroups. For a better comparison, the results under homogeneous portfolio shares are reproduced in figure 5a, which is identical to figure 4a.

The differences between figure 5b and figure 5a result from changes in the portfolio composition, which only affects the asset channel. Nothing changes at the aggregate level because we only change the distribution of housing wealth across households in period 0 while the aggregate quantity of housing wealth is unchanged. This can be seen by comparing the asset channel for the representative agent in figure 5a and 5b. In contrast, welfare effects change considerably at the micro-level. The middle-class now experiences a pronounced welfare loss instead of a welfare gain, the welfare gain of the bottom 50 percent of the wealth distribution is smaller than under a homogeneous portfolio, and the top 10's welfare loss turns into a welfare gain. Under homogeneous portfolios, all households hold

²⁷The wage, interest rate, and land price channels can also be studied analytically, but due to their minor quantitative impact, we eschew their analytical discussion.

²⁸Note that house prices only affect welfare in period 0, because afterward, perfect arbitrage holds such that their effect is already fully captured by changes in the interest rate. However, the interest rate does not change much between scenarios.

²⁹The derivative of $\frac{\partial f(\bullet)}{\partial P^{H,A}(0)}$ with respect to $N_j(0)$ is given by $\frac{\mathcal{W}_j^A(0) - \phi \tilde{S}^A(0) - P^{H,A}(0)N_j(0)}{[\mathcal{W}_j^A(0) - \phi \tilde{S}^A(0)]^2}$. It is positive if $\phi < \frac{\mathcal{W}_j^A(0) - P^{H,A}(0)N_j(0)}{\tilde{S}^A(0)}$.



Notes. 0%–50%: poor households with net wealth below the median wealth. 50%–90%: middle class households with net wealth between the median and 90th percentile. Top 10%: 10% richest households in terms of net wealth. Rep. agent: representative agent. Circles: net welfare effect. The decomposition of welfare gains follows (25). In addition to the welfare effect of the representative agent both panels show welfare gains of three wealth groups calculated as averages over the individual welfare effects within each of the three wealth groups.

Figure 5: Decomposition of welfare effects under zoning deregulation: importance of portfolio structure.

49 percent of their wealth in housing, equal to the representative agent’s share of housing in wealth. Remember, house prices decline in this experiment, as shown in figure 2. The drop in house prices hurts wealthier households more than wealth-poor households through a mere scale effect, as shown in section 4.4. Under heterogeneous portfolios, the negative effect of declining house prices on the welfare gain increases in a household’s share of housing in total wealth.³⁰ While the housing portfolio share of the representative agent is still at 49 percent, it is above 100 percent for the bottom 50 percent and the middle class, but only 47 percent for the top 10. The bottom 50’s and middle class’ increased housing exposure explains why the asset channel increases for both groups in figure 5. On the contrary, since the top 10 hold a more diversified portfolio, the decline in house prices harms them less, as can be seen by their smaller asset channel in figure 5. This finding mirrors the results of Kuhn *et al.* (2020) who show that declining house prices increase wealth inequality through the asset channel.

In summary, the zoning deregulation triggers declining rents and house prices, shown in figure 2. Declining rents increase welfare. The relative size of this channel depends on the housing expenditure share relative to the representative household. Declining house prices reduce welfare. The relative size of this channel depends on the share of housing wealth in the portfolio relative to the representative household.

³⁰That amplification of house price changes can be seen by replacing N_j by $\omega_j^N W_j^B / P^{H,B}$ in (27), where ω_j is the housing portfolio share, and taking the derivative of (27) with respect to ω_j .

5. Discussion

We now discuss three additional questions: i) How do more general Stone-Geary preferences affect our main results? ii) How does a changing rent trajectory affect the dynamics of *wealth* inequality? iii) How do the implied costs of land use restrictions compare to the previous literature?

5.1. Stone-Geary Preferences

Consider the utility function

$$u(C_j(t), S_j(t), \mathfrak{S}(t)) = \frac{\left[(C_j(t))^{1-\theta} (S_j(t) - \mathfrak{S}(t))^\theta \right]^{1-\sigma} - 1}{1 - \sigma}, \quad (28)$$

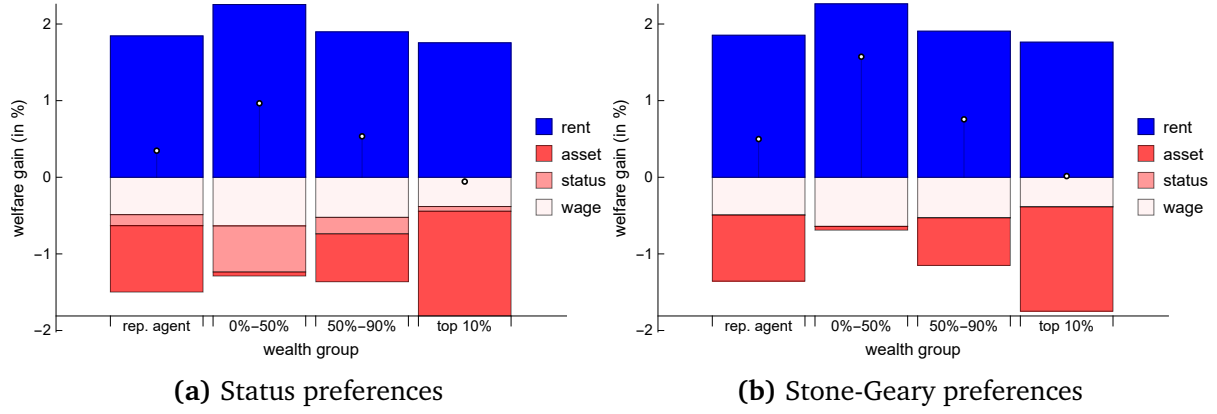
where $\mathfrak{S}(t) \geq 0$ is a minimum housing consumption level that is exogenous to the household. We allow $\mathfrak{S}(t)$ to change over time. Our baseline preference specification (2) is a special case of (28) if $\mathfrak{S}(t) = \phi S(t)$. In this special case the housing subsistence level is endogenous at the aggregate level and proportional to aggregate housing consumption. When studying zoning deregulation under utility function (28) with $\mathfrak{S}(t) = \phi S(t)$, the housing subsistence level would change between the baseline and alternative scenario, as in our main experiment under the status concern interpretation. In contrast, we do now consider the other extreme case by letting $\mathfrak{S}(t)$ unchanged between baseline and alternative scenarios. This experiment is consistent with the Stone-Geary interpretation as the minimum level of housing may be time-varying but should be independent of policies such as zoning deregulation.

Before studying how zoning deregulation affects welfare under more general Stone-Geary preferences, we first show how housing expenditure shares are affected:

Proposition 6 (Stone-Geary preferences). *An economy populated by a set of households whose preferences are described by (1) together with (28) and who face an intertemporal budget constraint given by (3) admits a representative household. In this economy, Schwabe's law holds if $\mathfrak{S} > 0$. The aggregate housing expenditure share is constant over time if and only if $\mathfrak{S} \propto S$, i.e. preferences are of the form (2).*

The proof is in appendix B.4. In the long run there is no difference between more general Stone-Geary preferences and status preferences for housing, because for a steady state to exist, \mathfrak{S} has to grow at the same exogenous rate as aggregate housing consumption S (see assumption 1 in appendix B.4).

We now turn to our experiment of zoning deregulation. To be precise, we assume that $\mathfrak{S}(t)^A = \mathfrak{S}(t)^B = \phi S(t)^B$, for all $t \geq 0$. That is, minimum housing consumption does not change in response to zoning deregulation. As a result, $\mathfrak{S}(t)^A < \phi S(t)^A$, such that the



Notes. 0%–50%: poor households with net wealth below the median wealth. 50%–90%: middle class households with net wealth between the median and 90th percentile. Top 10%: 10% richest households in terms of net wealth. Rep. agent: representative agent. Circles: net welfare effect. The decomposition of welfare gains follows (25). In addition to the welfare effect of the representative agent both panels show welfare gains of three wealth groups calculated as averages over the individual welfare effects within each of the three wealth groups.

Figure 6: Decomposition of welfare effects under zoning deregulation: alternative preference structures.

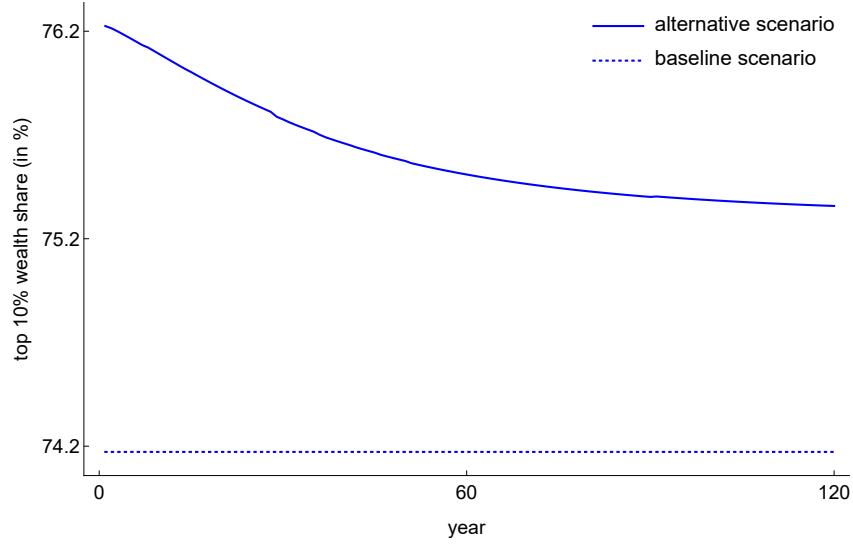
disutility effect of a higher overall housing consumption does not exist. Our main results are even more pronounced under this Stone-Geary interpretation of the utility function, as can be seen by comparing the two panels in figure 6, where figure 6a replicates the baseline experiment under status preferences and figure 6b shows the welfare effects under Stone-Geary preferences. Under Stone Geary preferences the negative externality of status consumption is absent, while the other welfare channels remain the same. Therefore, the representative agent's welfare is larger under Stone-Geary preferences, and also each household's welfare is larger. The bottom 50% of the wealth distribution now benefit relatively more than the top 10 percent of wealthiest households because the status channel hurts poor households more than rich households under status preferences. Hence, Schwabe's law amplifies the differences in household-specific welfare effects more strongly under Stone-Geary preferences.

5.2. Wealth Inequality

The discussion above has focused on *welfare inequality*. We now turn to the effect of rent dynamics on *wealth inequality*. We discuss this question, first, in our specific experiment of zoning deregulation, and, second, analytically.

Figure 7 plots the evolution of the top 10 percent wealth share, assuming empirically plausible portfolio heterogeneity as in section 4.5.³¹ The baseline economy is in a steady

³¹Given the sequences of aggregate prices in general equilibrium, the sequences of net wealth are calculated for every household. This two-stage solution procedure is possible because the distribution does not affect aggregate variables. Sorting households along net wealth in every period, the top 10 percent wealth share



Notes. Evolution of the top 10 percent wealth share in the baseline scenario (zoning) and the alternative scenario (no zoning). Calibration as described in appendix C, except that portfolio shares are heterogeneous as in section 4.5.

Figure 7: Wealth dynamics in response to zoning deregulation.

state such that the wealth distribution does not change.³² The ten percent of wealthiest households own 74.2 percent of total wealth in our calibrated economy according to the data given by the 2016 SCF+. Zoning deregulation increases wealth inequality, as shown by the solid line in figure 7. Initially, the top 10 percent wealth share jumps up by 2 percentage points. During the transition, it declines slightly, and in the long run, it remains at a level that is 1.2 percentage points above the baseline. This is a sizable effect as 1.2 percent of 2016 wealth corresponds to 6 percent of 2016 GDP.³³

We now explain why wealth inequality increases in response to zoning deregulation. House prices and rents drive most of the change in the wealth distribution. We discuss each effect in turn. First, since house prices are a forward-looking variable, they react immediately to zoning deregulation. Because the future supply of housing services increases, future rents decline, and house prices drop in period 0, as shown in figure 2. A lower house price in period 0 directly reduces wealth. Whether this house price decline affects the wealth distribution depends on how housing is distributed across the wealth distribution. Households at the bottom and in the middle of the wealth distribution own most of their assets in the form of housing, while households at the top hold more diversified portfolios than the bottom. Therefore, a drop in the house price reduces the wealth of the bottom 90 percent relatively more than the wealth of the top 10 percent. Wealth inequality increases

is determined.

³²Notice that the wealth distribution is exogenous, as in Caselli and Ventura (2000). However, the change in the wealth distribution due to the zoning deregulation is endogenous.

³³According to data from [wid.world](#), accessed on January 14, 2021, the US net national wealth to net national income ratio is 4.9 in 2016, such that 1.2 percent of total wealth corresponds to $4.9 \times 1.2 = 5.9$ percent of national income.

as a result. This effect mirrors Kuhn *et al.* (2020), who investigate this effect empirically.³⁴

Second, in contrast to house prices, rents do almost not change on impact but decline over time, as shown in figure 2. Therefore, households face a flatter time path of rents. Since households are forward-looking, this affects their saving decision in a non-trivial manner, as explained by the following proposition.

Proposition 7 (Rent effect on wealth distribution). *An increase in real rent growth between the current period 0 and some future period $t > 0$, measured by an increase in $\frac{p(t)}{p(0)}$, contributes to less (more) wealth inequality in the subsequent period dt if $\sigma > 1$ ($\sigma < 1$). The opposite applies to a decrease in $\frac{p(t)}{p(0)}$.*

To obtain proposition 7 we have to assume that wealth is more unequally distributed than earnings and that an individual's distance from the average wealth level is larger than its distance from the average earnings level. This assumption is fairly plausible as it holds for 88 percent of the 2016 SCF+ sample we use. Note, we do not impose that assumption in our quantitative experiment. The details on the assumption underlying proposition 7, its derivation, and further analytical insights on the dynamics of the wealth distribution are in appendix B.5. We focus on the intuition behind proposition 7 now. Consider an intertemporal elasticity of substitution below one ($\sigma > 1$). This value is the empirically relevant case described in our calibration in appendix C. When a household faces a flatter future time path of rents, given by lower $\frac{p(t)}{p(0)}$ for all $t > 0$, it will be able to afford more housing services. Due to the household's strong desire to smooth consumption over time, this results in more present consumption and less saving. Most crucially, poor households' saving rates decline more than rich households' saving rates, resulting in rising wealth inequality.

How do the two effects through house prices and rents work together? In our economy, other prices, like wages, also change in response to zoning deregulation. We quantify how isolated changes in house prices, rents, and wages affect the wealth distribution over time. If only house prices declined and all other prices remained constant, then the top 10 percent wealth share would be 2 percentage points higher after 120 years. If only rents declined over time, the top ten percent wealth share would be 2.2 percentage points higher. Lastly, if only wages changed over time, the top ten percent wealth share would be 2 percentage points lower. These effects do not sum up to the total effect of 1.2 percentage points due to i) non-linearities and ii) other comparatively small general equilibrium feedbacks through land prices and interest rates. Although house price changes immediately affect the wealth distribution, changes in rents affect the wealth distribution by the same magnitude over time. Wealth inequality would decrease in the long run without the rent channel. The

³⁴All of the immediate increase in wealth inequality in figure 7 is due to this mechanism. The interest rate does not change much in response to zoning deregulation, such that asset prices matter almost exclusively in the initial period.

wage channel drives the transitory decline in the top 10 percent wealth share.³⁵ Without it, wealth inequality would increase more.

Finally, we discuss the nexus between changes in wealth and welfare inequality. The policy considered triggers a myriad of price adjustments, which affect the distribution of wealth and welfare. Our analysis indicates that there is no straightforward mapping from changes in wealth inequality to changes in welfare inequality in response to zoning deregulation in general equilibrium. For instance, in section 4.3, we have shown that — under homogeneous portfolios — the wealthiest ten percent are worse off in the alternative scenario, while the bottom 50 percent are considerably better off. This is in line with the observation that the top 10 percent wealth share declines, while the bottom 50 percent wealth share increases. In section 4.5 we have shown that — under heterogeneous portfolios — both the bottom 50 percent and the top 10 percent gain in terms of welfare. At the same time, the top 10 percent wealth share goes up, while the bottom 50 percent wealth share declines. While there seems to be a connection between wealth and welfare for the top 10 percent, there is none for the bottom 50 percent. It is important to notice that welfare is affected by the asset channel in addition to the rent channel. This prevents a straight mapping between wealth and welfare. Hence, changes in the wealth distribution tell us little, if anything, about changes in the welfare distribution. Because we are ultimately interested in the welfare consequences and given the missing direct link between wealth and welfare inequality, we have focused on welfare in the main text.

To summarize, this section provides two main insights: i) changes in the rent exert similarly pronounced effects on the wealth distribution as changes in house prices and ii) wealth and welfare inequality do not need to move in the same direction.³⁶

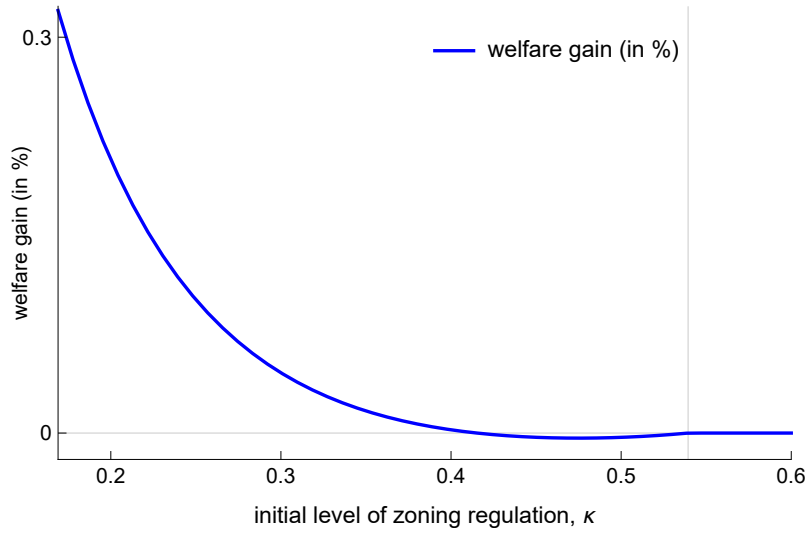
5.3. Land-Use Regulations and Welfare

We now compare our results to the existing literature on the welfare cost of land-use regulations. It has been stressed that land-use regulations can, in principle, increase welfare by correcting market failures.³⁷ It is therefore all but certain that land-use regulations create an aggregate welfare loss. However, [Gyourko and Molloy \(2015\)](#) survey the ur-

³⁵The wage channel is a bit intricate. Zoning deregulation triggers a temporary slower rent growth so that the wage rate of construction workers, $w = P^X \frac{\partial I^X}{\partial L^X}$, declines relative to the baseline scenario. The reason is that the price of new residential buildings, P^X , declines as well. Lower future wages exert a convergence force as poorer households increase their saving rate by relatively more to smooth consumption over time. Notice also that the wage is measured in units of the numeraire good (the non-housing good). The real wage in units of composite consumption does increase because the ideal price index declines stronger than the wage, see appendix D.

³⁶We have also studied how Schwabe's law affects the dynamics of wealth inequality. It plays only a minor role for the dynamics of the wealth distribution. This irrelevance is in stark contrast to the heterogeneity in welfare effects studied in section 4.3.

³⁷Prime examples are the preservation of urban recreational areas and the separation of different types of land use to limit the negative externalities associated with industrial or commercial land use ([Gyourko and Molloy, 2015](#)).



Notes: The horizontal axis start at $\kappa = 0.169$, which is the value for κ in our baseline calibration. For $\kappa \geq 0.54$ the zoning regulation is not binding anymore.

Figure 8: Welfare effect of zoning deregulation for different values of the initial zoning regulation parameter, κ .

ban economics literature and conclude most models and empirical estimates suggest that residential land-use regulations are in fact likely to create a net welfare loss. Two recent papers have addressed the topic by employing general equilibrium models. [Herkenhoff et al. \(2018\)](#) analyze how residential and commercial land-use restrictions in the US have impacted on regional and aggregate economic activity between 1950 and 2014. Lowering the land-use regulation of all states and bringing them 50 percent closer to the level of the state with the weakest land-use regulations in 2014 (Texas) increases aggregate welfare (consumption equivalents) by 10 percent. [Favilukis et al. \(2018\)](#) employ a heterogeneous-agent, incomplete-markets model to evaluate the relaxation of residential land-use restrictions in the city center. This policy generates an aggregate welfare gain equal to a permanent consumption increase of 0.37 percent.

Our analysis shows, incidentally, a similar aggregate welfare gain of 0.37 percent in response to the complete removal of residential zoning regulations starting from the initial level $\kappa = 0.169$. However, this result depends on the initial strength of the regulation. Figure 8 shows the welfare gain in response to complete zoning deregulation conditional on different initial levels, as measured by κ . The welfare gain from zoning deregulation declines if the initial regulation intensity is lower, that is starting with higher initial κ values. For mild initial regulation intensities — relatively high value of κ — the regulation causes a net welfare gain. The reason is that it suppresses the negative status externality and this effect dominates the sectoral misallocation of land. Hence, removing a mild zoning regulation may actually create a welfare loss, as can be seen from figure 8 for $\kappa \in (0.41, 0.54)$.

6. Conclusion

This paper has investigated how a change in the slope of the rent trajectory affects household welfare. The analysis comprises two steps. First, we have set up a model that is consistent with Schwabe's law and allows for a representative household. Second, we have investigated residential zoning deregulation as a relevant policy experiment that changes the slope of the rent trajectory and thereby affects welfare in general equilibrium.

The employed Ramsey growth model distinguishes between the extensive margin and the intensive margin of the housing stock. This feature enables the analysis of zoning deregulation. The demand side features status concerns with respect to housing to replicate the inverse variation of housing expenditure shares across income groups (Schwabe's law). Despite non-homothetic preferences, a representative agent exists. As a result, the distribution does not affect aggregate variables, which enables an analytical discussion of the welfare mechanisms.

The main results can be summarized as follows. The zoning deregulation triggers a temporarily slower growth in rents and house prices. The representative household experiences a welfare gain of 0.37 percent permanent consumption units. At the micro level we see strong heterogeneities. The largest welfare gain of 12.52 percent accrues to poor households who belong to the first earnings and wealth decile. The largest welfare loss of 1.63 percent falls upon households in the first earnings and the tenth wealth decile. We also decompose the welfare effect into different channels. This is done analytically and numerically. Two channels are of primary importance. According to the rent channel declining rents increase welfare. The household-specific size of the rent channel depends on the housing expenditure share relative to the representative household. According to the asset channel declining house prices reduce welfare. Allowing for heterogeneous portfolios reveals that the household-specific size of the asset channel depends on the share of housing wealth in the portfolio relative to the representative household.

Appendix

A. Steady State

Proposition A.1 (Steady state). *The unique steady state growth rates are:*

- (i) Variables $K, C, M, R^Z, R^N, P^N, P^Z, w$, and W grow at the rate g^Y
- (ii) Variable X grows at the rate $\eta g^Y + (1 - \eta) g^X$
- (iii) Variable p grows at the rate $(1 - \gamma\eta) g^Y - \gamma(1 - \eta) g^X$
- (iv) Variables P^X and R^X grow at the rate $(1 - \eta)(g^Y - g^X)$
- (v) Variable S grows at the rate $\gamma[\eta g^Y + (1 - \eta) g^X]$
- (vi) Variables $N, Z^Y, Z^N, L^Y, L^X, L^N$, and r are constant.

This implies that the steady state growth rate of GDP, $GDP \equiv Y + pS + wL^X$, equals g^Y . We derive all growth rates in [B.3](#).

B. Proofs and Derivations

B.1. Derivation of Solution to the Household Problem (Section 2.2)

We first derive the solution to the household problem given by (4) to (6). Any household j maximizes lifetime utility as given by (1) and (2) subject to its budget constraint (3) and a standard No-Ponzi game condition, $\lim_{t \rightarrow \infty} W_j(t) e^{-\int_0^t r(v) dv} \geq 0$, for given initial wealth and labor endowments $W_j(0)$ and L_j . The associated current-value Hamiltonian reads

$$\mathcal{H}_j = \frac{[(C_j)^{1-\theta}(S_j - \phi S)^\theta]^{1-\sigma} - 1}{1 - \sigma} + \lambda_j[rW_j + wL_j - C_j - pS_j].$$

The first-order optimality conditions (FOC) are

$$\frac{\theta}{1 - \theta} \frac{C_j}{S_j - \phi S} = p \quad (29)$$

$$(1 - \theta)(C_j)^{-\theta - \sigma(1 - \theta)}(S_j - \phi S)^{\theta(1 - \sigma)} = \lambda_j \quad (30)$$

$$-\frac{\dot{\lambda}_j}{\lambda_j} = r - \rho \quad (31)$$

$$\lim_{t \rightarrow \infty} W_j(t) \lambda_j(t) e^{-\rho t} = 0.$$

Using (29) in (30) we obtain

$$\lambda_j = (1 - \theta)^{1 + (\sigma - 1)\theta} \theta^{(1 - \sigma)\theta} C_j^{-\sigma} p^{(\sigma - 1)\theta}, \text{ and } -\frac{\dot{\lambda}_j}{\lambda_j} = \sigma \frac{\dot{C}_j}{C_j} + (1 - \sigma)\theta \frac{\dot{p}}{p}. \quad (32)$$

Combining (31) and (32) yields the following reformulated FOC

$$\begin{aligned}
C_j &= p \frac{1-\theta}{\theta} (S_j - \phi S) \\
\frac{\dot{C}_j}{C_j} &= \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma} \frac{\dot{p}}{p} \equiv g_c \\
\dot{W}_j &= rW_j + wL_j - C_j - pS_j \\
\lim_{t \rightarrow \infty} W_j(t) C_j(t)^{-\sigma} p(t)^{(\sigma-1)\theta} e^{-\rho t} &= 0.
\end{aligned} \tag{33}$$

The first equation is an intratemporal optimality condition that determines the allocation of total consumption expenditures across housing services and consumption of the numeraire. The second equation is an Euler equation, the third the intertemporal budget constraint, and the last the transversality condition.

We now derive the consumption function which expresses total consumption expenditures as a function of time only. This comprises three steps: i) integration of the intertemporal budget constraint, ii) integration of the Euler equation, and iii) combining results from i) and ii).

First, define total consumption expenditures as $\mathcal{E}_j \equiv C_j + pS_j$ and rewrite the intertemporal budget constraint as

$$\dot{W}_j(\tau) = r(\tau)W_j(\tau) + w(\tau)L_j - \mathcal{E}_j(\tau). \tag{34}$$

Multiplying both sides of (34) by $e^{-\int_t^\tau r(v)dv}$ and integrating from t to infinity yields

$$\int_t^\infty \dot{W}_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau = \int_t^\infty r(\tau)W_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau + \int_t^\infty [w(\tau)L_j - \mathcal{E}_j(\tau)] e^{-\int_t^\tau r(v)dv} d\tau. \tag{35}$$

Integrating the left hand side of this equation by parts results in

$$\begin{aligned}
\int_t^\infty \dot{W}_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau &= \lim_{T \rightarrow \infty} \left[W_j(\tau) e^{-\int_t^\tau r(v)dv} \right]_t^T + \int_t^\infty r(\tau)W_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau \\
&= -W_j(t) + \int_t^\infty r(\tau)W_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau,
\end{aligned} \tag{36}$$

where we use the transversality condition $\lim_{T \rightarrow \infty} W_j(\tau) e^{-\int_t^T r(v)dv} = 0$ in the last equation. Inserting this expression into the left hand side of (35) and rearranging yields total wealth as

$$\mathcal{W}_j(t) \equiv W_j(t) + \tilde{w}(t)L_j = \int_t^\infty \mathcal{E}_j(\tau) e^{-\int_t^\tau r(v)dv} d\tau, \tag{37}$$

where $\tilde{w}(t)$ is defined in (8).

Second, define composite consumption as $\mathcal{C}_j \equiv C_j^{1-\theta}(S_j - \phi S)^\theta$. Making use of the intratemporal optimality condition (29) to substitute S_j in \mathcal{C}_j yields

$$\mathcal{C}_j = \left(\frac{\theta}{1-\theta} \frac{1}{p} \right)^\theta C_j. \quad (38)$$

Solving for C_j , taking the derivative with respect to time, and then substituting C_j with \mathcal{C}_j in the Euler equation (33) results in

$$\frac{\dot{\mathcal{C}}_j}{\mathcal{C}_j} = \frac{r - \rho}{\sigma} - \frac{\theta}{\sigma} \frac{\dot{p}}{p}.$$

This is the Euler equation expressed in terms of composite consumption \mathcal{C}_j . It is an ordinary linear differential equation with a time-varying coefficient. Its solution is

$$\mathcal{C}_j(\tau) = \mathcal{C}_j(t) e^{\frac{1}{\sigma} \int_t^\tau [r(v) - \rho - \theta \frac{\dot{p}(v)}{p(v)}] dv}. \quad (39)$$

This expresses composite consumption at any date $\tau \geq t$ as a function of consumption in period t and prices r and p .

Third, define the ideal price index as $\mathcal{P}_j = \mathcal{E}_j / \mathcal{C}_j$, substitute \mathcal{E}_j in equation (37) with $\mathcal{P}_j \mathcal{C}_j$, then also substitute \mathcal{C}_j by (39) and multiply both sides by $\mathcal{P}_j(t)$ to obtain

$$\mathcal{P}_j(t) \mathcal{W}_j(t) = \underbrace{\mathcal{P}_j(t) \mathcal{C}_j(t)}_{=\mathcal{E}_j(t)} \int_t^\infty \mathcal{P}_j(\tau) e^{\frac{1}{\sigma} \int_t^\tau [r(v) - \rho - \theta \frac{\dot{p}(v)}{p(v)}] dv} e^{-\int_t^\tau r(v) dv} d\tau.$$

Solving for consumption expenditures in t yields

$$\mathcal{E}_j(t) = \frac{\mathcal{W}_j(t)}{\int_t^\infty \frac{\mathcal{P}_j(\tau)}{\mathcal{P}_j(t)} e^{\frac{1}{\sigma} \int_t^\tau [(1-\sigma)r(v) - \rho - \theta \frac{\dot{p}(v)}{p(v)}] dv} d\tau}.$$

We now have to replace the relative ideal price index in this expression. Define relative consumption of housing services as $S_j^R \equiv S_j / S$ and use (29) to express total consumption expenditures as

$$\mathcal{E}_j = \frac{pS}{\theta} [S_j^R - (1-\theta)\phi]. \quad (40)$$

Using (38) and (40) together with (29) in $\mathcal{P}_j = \mathcal{E}_j / \mathcal{C}_j$ yields

$$\mathcal{P}_j = \frac{p^\theta}{\theta^\theta (1-\theta)^{1-\theta}} \frac{S_j^R - (1-\theta)\phi}{S_j^R - \phi}. \quad (41)$$

Note that S_j^R is constant over time: First, the Euler equation (33) shows that C_j grows with

the same rate g_C for all j and also for the average, that is C grows also at the rate g_C . Second, the intratemporal optimality condition (29) of the average, $C = p^{\frac{1-\theta}{\theta}}(1-\phi)S$, implies that pS grows also at the rate g_C . Third, calculating growth rates in equation (29) where S_j is substituted with $S_j^R S$ yields

$$\underbrace{\frac{\dot{C}_j}{C_j}}_{=g_C} = \underbrace{\frac{\dot{pS}}{pS}}_{=g_C} + \frac{(S_j^R - \phi)}{S_j^R} \Rightarrow \frac{(S_j^R - \phi)}{S_j^R} = 0 \Rightarrow \dot{S}_j^R = 0.$$

Now, according to (41) and the fact that S_j^R is time-invariant, we obtain the ratio of ideal price indices as

$$\frac{\mathcal{P}_j(\tau)}{\mathcal{P}_j(t)} = \left(\frac{p(\tau)}{p(t)} \right)^\theta.$$

Finally, replacing this ratio in (37) results in

$$\mathcal{E}_j(t) = C_j + pS_j = \mu(t) [W_j(t) + \tilde{w}(t)L_j], \quad (42)$$

where the propensity to consume out of total wealth is

$$\mu(t) = \left\{ \int_t^\infty \left(\frac{p(\tau)}{p(t)} \right)^\theta e^{-\frac{1}{\sigma} \int_t^\tau [(\sigma-1)r(v) + \rho + \theta \frac{\dot{p}(v)}{p(v)}] dv} d\tau \right\}^{-1}.$$

The propensity to consume, μ , can be simplified by solving $e^{-\frac{1}{\sigma} \int_t^\tau \theta \frac{\dot{p}(v)}{p(v)} dv}$, resulting in (7). Equation (42) is the consumption function. It expresses total consumption expenditures as a function of endowments (W_j and L_j) and time only (prices are functions of time). This completes the derivation of the first equation of the household solution, (4). The other equation, (6), is then obtained by replacing $C_j + pS_j$ in the intertemporal budget constraint (3) by (42).

B.2. Proofs of Propositions and Corollaries (Section 2)

Proof of proposition 1. The aggregate of a variable is the sum over all j , weighted by the group sizes n_j . For example, aggregate consumption is given by $C = \sum_j n_j C_j$. Aggregating this way the left and right hand sides of equations (4), (5), and (6) yields

$$\begin{aligned} C(t) + p(t)S(t) &= \mu(t) [W(t) + \tilde{w}(t)L] \\ C(t) &= p(t) \frac{1-\theta}{\theta} (1-\phi)S(t) \\ \dot{W}(t) &= [r(t) - \mu(t)]W(t) + [w(t) - \mu(t)\tilde{w}(t)]L. \end{aligned}$$

These FOC are identical to the FOC that result from the problem of a single household who

owns the entire endowments, $\sum_j n_j L_j$ and $\sum_j n_j W_j$, and makes the aggregate consumption and saving decisions, taking the reference level of housing consumption, $S(t)$, as given. \square

Proof of proposition 2. The housing expenditure share is defined by $e_j \equiv (pS_j)/(C_j + pS_j)$. Make use of the intratemporal first order condition, (29), to obtain

$$e_j = \frac{\theta S_j^R}{S_j^R - (1 - \theta)\phi}. \quad (43)$$

Substituting C_j in (5) by (4) and solving for S_j yields

$$S_j = \frac{\theta}{p} \mu \mathcal{W}_j + (1 - \theta)\phi S \quad (44)$$

and for the representative agent

$$S = \frac{\theta}{1 - (1 - \theta)\phi} \frac{\mu}{p} \mathcal{W}.$$

Inserting this equation into (44) yields

$$S_j = \frac{\theta \mu}{p} \left[\mathcal{W}_j + \frac{(1 - \theta)\phi}{1 - (1 - \theta)\phi} \mathcal{W} \right].$$

Using the last two equations to obtain relative housing services consumption then gives

$$S_j^R = \frac{S_j}{S} = (1 - \theta)\phi + [1 - (1 - \theta)\phi] \frac{\mathcal{W}_j}{\mathcal{W}}.$$

Lastly, insert this last expression for S_j^R into (43) to obtain (9). \square

Proof of proposition 3. We can rearrange (43) to obtain

$$S_j^R = \frac{(1 - \theta)\phi}{1 - \frac{\theta}{e_j}}. \quad (45)$$

Substituting (45) into (41) confirms (11). \square

Proof of proposition 4. Factor out $1/(1 - \sigma)$ in (10), split integrals such that $-\int_t^\infty e^{-\rho(\tau-t)} d\tau$ is isolated and can be subtracted from both sides, and rearrange to obtain

$$(1 + \psi_j)^{1-\sigma} = \frac{\int_t^\infty \mathcal{C}_j^A(\tau)^{1-\sigma} e^{-\rho(\tau-t)} d\tau}{\int_t^\infty \mathcal{C}_j^B(\tau)^{1-\sigma} e^{-\rho(\tau-t)} d\tau}. \quad (46)$$

Make use of (39) and (7) to rewrite

$$\int_t^\infty \mathcal{C}_j^k(\tau)^{1-\sigma} e^{-\rho(\tau-t)} d\tau = \frac{[\mathcal{C}_j^k(t)]^{1-\sigma}}{\mu^k(t)},$$

where $k \in \{A, B\}$ denotes the respective economy. Inserting this into (46) yields

$$(1 + \psi_j)^{1-\sigma} = \frac{\left[\frac{\mathcal{C}_j^A(t)}{\mu^A(t)}\right]^{1-\sigma}}{\left[\frac{\mathcal{C}_j^B(t)}{\mu^B(t)}\right]^{1-\sigma}} \Leftrightarrow \psi_j = \left[\frac{\mu^A(t)}{\mu^B(t)}\right]^{\frac{1}{\sigma-1}} \frac{\mathcal{C}_j^A(t)}{\mathcal{C}_j^B(t)} - 1.$$

To obtain (12) replace $\mathcal{C}_j^k(t)$ in this expression with $\mathcal{C}_j^k(t)/\mathcal{P}_j^k(t)$ for both $k \in \{A, B\}$, where total expenditures and the ideal price index are taken from (42) and (11), respectively. \square

Proof of Corollary 1. To study how Schwabe's law affects welfare comparisons, we use the relative welfare given³⁸

$$\psi_j^R \equiv \frac{1 + \psi_j}{1 + \psi} = \frac{\frac{\mathcal{W}_j^A}{\mathcal{W}^A} - \frac{\phi \theta}{1 - (1 - \theta)\phi}}{\frac{\mathcal{W}_j^B}{\mathcal{W}^B} - \frac{\phi \theta}{1 - (1 - \theta)\phi}} \begin{cases} > 1 & \text{if } \frac{\mathcal{W}_j^A}{\mathcal{W}^A} > \frac{\mathcal{W}_j^B}{\mathcal{W}^B} \\ = 1 & \text{if } \frac{\mathcal{W}_j^A}{\mathcal{W}^A} = \frac{\mathcal{W}_j^B}{\mathcal{W}^B} \\ < 1 & \text{if } \frac{\mathcal{W}_j^A}{\mathcal{W}^A} < \frac{\mathcal{W}_j^B}{\mathcal{W}^B} \end{cases}. \quad (47)$$

If household j 's relative total wealth, $\frac{\mathcal{W}_j}{\mathcal{W}}$, is larger in the alternative economy A than in the baseline economy B, then its additional welfare, ψ_j , is also larger than the average, ψ . We take the derivative of (47) with respect to ϕ . Note first that changes in ϕ do not affect total wealth $\mathcal{W}_j = W_j + \tilde{w}L_j$. The derivative is hence given by

$$\frac{\partial \psi_j^R}{\partial \phi} = \frac{\theta}{\left(\frac{\mathcal{W}_j^B}{\mathcal{W}^B} - \phi \theta\right)^2} \left(\frac{\mathcal{W}_j^A}{\mathcal{W}^A} - \frac{\mathcal{W}_j^B}{\mathcal{W}^B}\right) \begin{cases} > 0 & \text{if } \frac{\mathcal{W}_j^A}{\mathcal{W}^A} > \frac{\mathcal{W}_j^B}{\mathcal{W}^B} \\ = 0 & \text{if } \frac{\mathcal{W}_j^A}{\mathcal{W}^A} = \frac{\mathcal{W}_j^B}{\mathcal{W}^B} \\ < 0 & \text{if } \frac{\mathcal{W}_j^A}{\mathcal{W}^A} < \frac{\mathcal{W}_j^B}{\mathcal{W}^B} \end{cases}, \quad (48)$$

If household j 's relative total wealth, $\frac{\mathcal{W}_j}{\mathcal{W}}$, is larger in the alternative economy A than in the baseline economy B, then stronger status preferences imply a stronger reaction of relative welfare, ψ_j^R . Taking together (47) and (48) results in corollary 1. \square

Proof of proposition 5. To calculate the effect of rent changes on the welfare gain, ψ_j , we need to isolate its direct effect from its indirect effect through aggregate housing consumption. That is not the case in expression (12) where we use the fact that changes in the rent, p , also affect the aggregate level of housing services consumption, S . Therefore, we use (25) in the following.

First, the effect of changes in the level of rents on welfare can be seen by taking the derivative of (25) with respect to $p^A(0)$

$$\frac{\partial f(\bullet)}{\partial p^A(0)} = -\frac{1}{p^A(0)} \left[\theta + \frac{\phi \tilde{S}^A(0)}{\mathcal{W}_j^A(0) - \phi \tilde{S}^A(0)} \right] < 0.$$

³⁸Omitting the time index, the subsequent equation results from (9), (11), and (12).

Lower rents in period 0 increase welfare. If $\phi = 0$, the derivative simplifies to $-\theta/p^A(0)$ and changes in rents in period 0 affect all households equally. For $\phi > 0$, the effect is heterogeneous. The wealthier a household (the higher $\mathcal{W}_j^A(0)$), the less it is hurt by higher rents.

Second, the effect of lower future rent growth is given by (25)'s derivative with respect to $\tilde{p}^A(t)$ ³⁹

$$\frac{\partial f(\bullet)}{\partial \tilde{p}^A(t)} = \frac{\sigma}{\sigma - 1} \frac{1}{\mu^A(0)} \frac{\partial \mu^A(0)}{\partial \tilde{p}^A(t)} - \frac{\phi}{\mathcal{W}_j^A(0) - \phi \tilde{S}^A(0)} \frac{\partial \tilde{S}^A(0)}{\partial \tilde{p}^A(t)} < 0. \quad (49)$$

The first term is negative.⁴⁰ Since higher rent growth increases the present value of future minimum housing consumption expenditures, i.e. $\frac{\partial \tilde{S}^A(0)}{\partial \tilde{p}^A(t)} > 0$, the entire derivative (49) is negative. Without status preferences, $\phi = 0$, the second term vanishes and the effect is homogeneous across households. With status preferences, $\phi > 0$, wealthier households are affected less by changes in rents than poor households. Overall, changes in the growth rate of future rents affect welfare similarly as changes in the current level. \square

B.3. Proof of Steady State Proposition (Section 3)

Proof of proposition A.1. Let a "hat" above a variable denote the steady state growth rate of this variable. According to (33), we obtain the steady state interest rate

$$r = \rho + \theta(1 - \sigma)\hat{p} + \sigma\hat{C} \equiv r^* \quad (50)$$

The FOC in the numeraire sector, (14), imply

$$\hat{Y} = \hat{K} = \hat{w} = \hat{R}^Z.$$

Writing the production function (13) in growth rates yields $\hat{Y} = \alpha\hat{K} + \beta(g^Y + \hat{L}^Y) + (1 - \alpha - \beta)(g^Y + \hat{Z}^Y)$. Suppose that the long run allocation of both labor and land is time invariant (which will be confirmed to be consistent with the derived steady state), i.e. $\hat{L}^Y = \hat{L}^X = \hat{L}^N = \hat{Z}^Y = \hat{N} = 0$. Using $\hat{Y} = \hat{K}$ this implies

$$\hat{Y} = g^Y.$$

With $\dot{N} = 0$ in steady state we obtain from the FOC of the real estate development firm,

³⁹We study the derivative of function $f(\bullet)$ with respect to rent growth at one instant of time. Strictly speaking, this should be zero. However, in (26) we consider the derivative over a time interval, implying a non-zero effect. Therefore, (49) represents the derivative with respect to changes in rent growth factors over a non-empty time interval.

⁴⁰If $\sigma > 1$, then $\frac{\partial \mu^A(0)}{\partial \tilde{p}^A(t)} < 0$ and the term is negative. If $\sigma < 1$, then $\frac{\partial \mu^A(0)}{\partial \tilde{p}^A(t)} > 0$ and the term is also negative, because $\frac{\sigma}{\sigma - 1} < 0$.

(17), that $P^N = P^Z$ and

$$\hat{P}^N = \hat{P}^Z.$$

Because the interest rate is constant in steady state, the asset market no arbitrage conditions (21) imply

$$\hat{P}^Z = \hat{R}^Z, \hat{P}^N = \hat{R}^N, \hat{P}^X = \hat{R}^X.$$

Differentiating the construction firms' FOC, (16), with respect to time yields

$$\dot{\hat{P}}^X = (\eta - 1)(g^X - \hat{M}) = \hat{w} - \eta \hat{M} + (\eta - 1)g^X. \quad (51)$$

Using (51) and recalling $\hat{w} = g^Y$ we obtain

$$\hat{M} = \hat{w} = g^Y, \quad (52)$$

$$\dot{\hat{P}}^X = \hat{R}^X = (\eta - 1)(g^X - g^Y). \quad (53)$$

Next, insert the production function (15) in $\dot{X} = I^X - \delta^X X$ and rearrange

$$\frac{\dot{X}}{X} = \frac{M^\eta (B^X L^X)^{1-\eta}}{X} - \delta^X.$$

Together with (52) the growth rate of X is then

$$\hat{X} = \eta g^Y + (1 - \eta)g^X.$$

From the production function of housing services producers, (18), we obtain

$$\hat{S} = \gamma [\eta g^Y + (1 - \eta)g^X]. \quad (54)$$

Recall that one FOC of the housing services producer is $R^N = (1 - \gamma)p \frac{S}{N}$. Differentiating with respect to time gives $\hat{p} = \hat{R}^N - \hat{S}$. Using (54) and $\hat{R}^N = g^Y$, which results from $\hat{R}^Z = g^Y = \hat{P}^Z = \hat{P}^N = \hat{R}^N$, we confirm (22):

$$\hat{p} = (1 - \gamma\eta)g^Y - \gamma(1 - \eta)g^X. \quad (55)$$

Finally, aggregating the intertemporal budget constraint (3) and dividing by W yields

$$\hat{W} = r + \frac{wL}{W} - \frac{C}{W} - \frac{pS}{W}.$$

In order for W to grow at a constant rate, it has to hold that $\hat{W} = \hat{w} = g^Y$ as well as that $\hat{C} = \hat{W} = g^Y$. \square

B.4. Derivation of Results on Stone-Geary Preferences (Section 5.1)

Proof of proposition 6. We now show that a representative household exists under more general Stone-Geary preferences (28). Households maximize (1) subject to the more general utility function (28), the budget constraint (3), and the TVC. The FOC are almost the same, because both \mathfrak{S} and ϕS are exogenous to the household

$$\begin{aligned} C_j &= p \frac{1-\theta}{\theta} (S_j - \mathfrak{S}) \\ \frac{\dot{C}_j}{C_j} &= \frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma} \frac{\dot{p}}{p} \equiv g_c \\ \dot{W}_j &= rW_j + wL_j - C_j - pS_j \\ \lim_{t \rightarrow \infty} W_j(t) C_j(t)^{-\sigma} p(t)^{(\sigma-1)\theta} e^{-\rho t} &= 0 \\ W_j(0) &= \text{given.} \end{aligned} \tag{56}$$

In steady state C_j grows at the constant rate g_c , which is the same across households, according to the second equation. The first equation then demands that \mathfrak{S} grows asymptotically at the same rate as S_j for a steady state to exist. We hence assume

Assumption 1 (Growth rate of minimum housing consumption). *For $t \rightarrow \infty$, the minimum housing consumption level $\mathfrak{S}(t)$ grows at the same rate as aggregate housing consumption $S(t)$, which grows at the rate $\gamma [\eta g^Y + (1-\eta)g^X]$ (see proposition A.1).*

This assumption concerns only the steady state, but it does not assume that \mathfrak{S} grows at the same rate as S along the transition.

Make use of $\mathcal{E}_j \equiv C_j + pS_j$ and the above equations to express (56) as

$$\begin{aligned} \dot{\mathcal{E}}_j &= \left[\frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma} \frac{\dot{p}}{p} \right] \mathcal{E}_j - \left[\frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma} \frac{\dot{p}}{p} \right] p\mathfrak{S} + (\dot{p}\mathfrak{S} + p\dot{\mathfrak{S}}) \\ \dot{W}_j &= rW_j + wL_j - \mathcal{E}_j \end{aligned}$$

Define $\mathcal{G}_j \equiv \mathcal{E}_j - p\mathfrak{S}$ as the consumption expenditures left after paying the minimum housing consumption level \mathfrak{S} and rewrite

$$\begin{aligned} \dot{\mathcal{G}}_j &= \left[\frac{r-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma} \frac{\dot{p}}{p} \right] \mathcal{G}_j \\ \dot{W}_j &= rW_j + wL_j - \mathcal{G}_j - p\mathfrak{S} \end{aligned}$$

This is the solution to a problem with a fixed consumption expenditure $p\mathfrak{S}$. It looks almost as a solution to a homothetic, one-good problem.

Solving both linear differential equations as single equations yields

$$\mathcal{G}_j(t) = \mathcal{G}_j(0)e^{\int_0^t \left[\frac{r(v)-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma} \frac{\dot{p}(v)}{p(v)} \right] dv}$$

$$W_j(t) = W_j(0)e^{\int_0^t r(v)dv} + \int_0^t \left[w(\tau)L_j - \mathcal{G}_j(\tau) - p(\tau)\mathfrak{S}(\tau) \right] e^{\int_\tau^t r(v)dv} d\tau$$

Rearranging the solution for wealth and replacing $\mathcal{G}_j(\tau)$ yields

$$\mathcal{G}_j(0) \int_0^t \left[\left(\frac{p(\tau)}{p(0)} \right)^\theta e^{-\left[\int_0^\tau r(v)dv + \frac{\rho}{\sigma-1}(\tau-0) \right]} \right]^{\frac{\sigma-1}{\sigma}} d\tau = W_j(0) - e^{-\int_0^t r(v)dv} W_j(t)$$

$$+ \int_0^t w(\tau) e^{-\int_0^\tau r(v)dv} d\tau L_j$$

$$- \int_0^t p(\tau) \mathfrak{S}(\tau) e^{-\int_0^\tau r(v)dv} d\tau$$

Take the limit of the previous equation and rearrange to obtain

$$\mathcal{G}_j(0) \int_0^\infty \left[\left(\frac{p(\tau)}{p(0)} \right)^\theta e^{-\left[\int_0^\tau r(v)dv + \frac{\rho}{\sigma-1}(\tau-0) \right]} \right]^{\frac{\sigma-1}{\sigma}} d\tau = W_j(0) - \lim_{t \rightarrow \infty} e^{-\int_0^t r(v)dv} W_j(t)$$

$$+ \tilde{w}(0)L_j - \tilde{\mathfrak{S}}(0) \quad (57)$$

where $\tilde{\mathfrak{S}}(0) \equiv \int_0^\infty p(\tau) \mathfrak{S}(\tau) e^{-\int_0^\tau r(v)dv} d\tau$ is the NPV of future minimum housing consumption. One now has to use the TVC to show that $\lim_{t \rightarrow \infty} e^{-\int_0^t r(v)dv} W_j(t)$ is equal to zero.

Rearrange the TVC by substituting C by \mathcal{G}

$$\lim_{t \rightarrow \infty} W_j(t) (\theta \mathcal{G}_j(t))^{-\sigma} p(t)^{(\sigma-1)\theta} e^{-\rho t} = 0$$

$$\lim_{t \rightarrow \infty} W_j(t) \left(\mathcal{G}_j(0) e^{\int_0^t \left[\frac{r(v)-\rho}{\sigma} + \frac{(\sigma-1)\theta}{\sigma} \frac{\dot{p}(v)}{p(v)} \right] dv} \right)^{-\sigma} p(t)^{(\sigma-1)\theta} e^{-\rho t} = 0$$

$$\lim_{t \rightarrow \infty} W_j(t) e^{\int_0^t \left[-r(v) + \rho - (\sigma-1)\theta \frac{\dot{p}(v)}{p(v)} \right] dv} p(t)^{(\sigma-1)\theta} e^{-\rho t} = 0$$

$$\lim_{t \rightarrow \infty} W_j(t) e^{-\int_0^t r(v)dv} \left(\frac{p(t)}{p(0)} \right)^{-\theta(\sigma-1)} p(t)^{(\sigma-1)\theta} = 0$$

$$\lim_{t \rightarrow \infty} W_j(t) e^{-\int_0^t r(v)dv} = 0.$$

With the last equation we can rewrite current consumption from (57) as

$$\mathcal{G}_j(0) = \mu(0) [W_j(0) + \tilde{w}(0)L_j - \tilde{\mathfrak{S}}(0)].$$

Lastly, use the definition of \mathcal{G} to obtain demand functions for C and S

$$\mathcal{E}_j(0) = \mu(0) [W_j(0) + \tilde{w}(0)L_j - \tilde{\mathfrak{S}}(0)] + p(0)\mathfrak{S}(0)$$

$$\begin{aligned}
C_j(0) &= (1 - \theta)\mu(0)[W_j(0) + \tilde{w}(0)L_j - \tilde{\mathfrak{S}}(0)] \\
S_j(0) &= \theta \frac{1}{p(0)}\mu(0)[W_j(0) + \tilde{w}(0)L_j - \tilde{\mathfrak{S}}(0)] + \mathfrak{S}(0)
\end{aligned} \tag{58}$$

These two Engel curves are linear, but they do not go through the origin if $\mathfrak{S} \neq 0$. As a result, the elasticities of demand for C and S with respect to total wealth are not constant and not equal to one. However, linearity of Engel curves implies that a representative agent exists, independently of the transitory growth rate of \mathfrak{S} .

We now study the housing expenditure share. Making use of (58), it can be expressed as

$$e_j = \frac{pS_j}{\mathcal{E}_j} = \theta + (1 - \theta) \frac{p\mathfrak{S}}{\mu[\mathcal{W}_j - \tilde{\mathfrak{S}}] + p\mathfrak{S}}.$$

The wealthier a household is in terms of total wealth \mathcal{W}_j , the smaller is its housing expenditure share. Stone Geary preferences can hence replicate the variation of the housing expenditure share with income in the cross-section, but can it also replicate a constant aggregate housing expenditure share over time? The aggregate housing expenditure share reads

$$e = \frac{pS}{C + pS} = \frac{\frac{\theta}{1-\theta}C + p\mathfrak{S}}{C + \frac{\theta}{1-\theta}C + p\mathfrak{S}} = \frac{\theta C + p(1 - \theta)\mathfrak{S}}{C + p(1 - \theta)\mathfrak{S}}.$$

The derivative with respect to time reads

$$\dot{e} = \frac{[\theta\dot{C} + (1 - \theta)p\dot{\mathfrak{S}}][C + p(1 - \theta)\mathfrak{S}] - [\theta C + (1 - \theta)p\mathfrak{S}][\dot{C} + (1 - \theta)p\dot{\mathfrak{S}}]}{[C + p(1 - \theta)\mathfrak{S}]^2}$$

For the aggregate housing expenditure share being constant it has to hold that $\dot{e} = 0$ and

$$\begin{aligned}
[\theta\dot{C} + (1 - \theta)p\dot{\mathfrak{S}}][C + p(1 - \theta)\mathfrak{S}] &= [\theta C + (1 - \theta)p\mathfrak{S}][\dot{C} + (1 - \theta)p\dot{\mathfrak{S}}] \\
(1 - \theta)^2 C p \dot{\mathfrak{S}} &= (1 - \theta)^2 p \mathfrak{S} \dot{C} \\
\frac{\dot{C}}{C} &= \frac{p\dot{\mathfrak{S}}}{p\mathfrak{S}} = \frac{\dot{p}\mathfrak{S} + p\dot{\mathfrak{S}}}{p\mathfrak{S}} \\
\Leftrightarrow \frac{\dot{\mathfrak{S}}}{\mathfrak{S}} &= \frac{\dot{C}}{C} - \frac{\dot{p}}{p}.
\end{aligned} \tag{59}$$

Note, according to the intratemporal optimality condition S grows at the rate

$$\begin{aligned}
\dot{C} &= \frac{1 - \theta}{\theta} [\dot{p}S + p\dot{S} - \dot{p}\mathfrak{S} - p\dot{\mathfrak{S}}] \\
\frac{\dot{C}}{C} &= \frac{\dot{p}S + p\dot{S} - \dot{p}\mathfrak{S} - p\dot{\mathfrak{S}}}{pS - p\mathfrak{S}} \\
\frac{\dot{C}}{C} - \frac{\dot{p}}{p} &= \frac{\dot{S} - \dot{\mathfrak{S}}}{S - \mathfrak{S}}
\end{aligned} \tag{60}$$

Equating (60) and (59) yields then

$$\frac{\dot{\mathfrak{S}}}{\mathfrak{S}} = \frac{\dot{S} - \dot{\mathfrak{S}}}{S - \mathfrak{S}} \Rightarrow \frac{\dot{\mathfrak{S}}}{\mathfrak{S}} = \frac{\dot{S}}{S}.$$

The aggregate housing expenditure share is only constant if the minimum housing consumption level grows at the same rate as aggregate housing consumption. Put differently, \mathfrak{S} has to be proportional to S . That is our baseline preference specification with status preferences in (2). Hence, for the aggregate housing expenditure share being constant over time, status preferences have to hold. \square

Welfare How does the expression for welfare change under Stone-Geary preferences? From the main text we have $\psi_j(0)$ given as

$$(1 + \psi_j(0))^{1-\sigma} = \frac{\int_0^\infty \mathcal{C}_j^A(\tau)^{1-\sigma} e^{-\rho(\tau-0)} d\tau}{\int_0^\infty \mathcal{C}_j^B(\tau)^{1-\sigma} e^{-\rho(\tau-0)} d\tau}.$$

Express the KRR in terms of composite consumption

$$\frac{\dot{\mathcal{C}}_j}{\mathcal{C}_j} = \frac{r - \rho}{\sigma} - \frac{\theta}{\sigma} \frac{\dot{p}}{p}$$

and solve it to yield

$$\begin{aligned} \mathcal{C}_j(t) &= \mathcal{C}_j(0) e^{\int_0^t \left[\frac{r(v)-\rho}{\sigma} - \frac{\theta}{\sigma} \frac{\dot{p}(v)}{p(v)} \right] dv} \\ &= \mathcal{C}_j(0) \left(\frac{p(t)}{p(0)} \right)^{-\frac{\theta}{\sigma}} e^{\int_0^t \frac{r(v)}{\sigma} dv - \frac{\rho(t-0)}{\sigma}} \end{aligned}$$

Plugging that into the previous equation yields

$$\begin{aligned} (1 + \psi_j(0))^{1-\sigma} &= \frac{\mathcal{C}_j^A(0)^{1-\sigma} \mu^A(0)^{-1}}{\mathcal{C}_j^B(0)^{1-\sigma} \mu^B(0)^{-1}} \\ 1 + \psi_j(0) &= \frac{\mathcal{C}_j^A(0)}{\mathcal{C}_j^B(0)} \left(\frac{\mu^A(0)}{\mu^B(0)} \right)^{\frac{1}{\sigma-1}} \end{aligned}$$

This is the same expression as in the main text. Make use of $\mathcal{E}_j = \mathcal{P}_j \mathcal{C}_j$ to rewrite

$$\begin{aligned} 1 + \psi_j(0) &= \left(\frac{\mu^A(0)}{\mu^B(0)} \right)^{\frac{1}{\sigma-1}} \frac{\mathcal{E}_j^A(0)}{\mathcal{E}_j^B(0)} \left(\frac{\mathcal{P}_j^A(0)}{\mathcal{P}_j^B(0)} \right)^{-1} \\ &= \left(\frac{\mu^A(0)}{\mu^B(0)} \right)^{\frac{1}{\sigma-1}} \frac{\mu^A(0) [\mathcal{W}_j^A(0) - \tilde{\mathfrak{S}}^A(0)] + p^A(0) \mathfrak{S}^A(0)}{\mu^B(0) [\mathcal{W}_j^B(0) - \tilde{\mathfrak{S}}^B(0)] + p^B(0) \mathfrak{S}^B(0)} \left(\frac{\mathcal{P}_j^A(0)}{\mathcal{P}_j^B(0)} \right)^{-1}. \end{aligned} \quad (61)$$

This expression is a generalization of (12).

The price index can also be more generally expressed as

$$\mathcal{P}_j(0) = \frac{\mathcal{E}_j(0)}{\mathcal{C}_j(0)} = \frac{\mathcal{E}_j(0)}{C_j(0)^{1-\theta} (S_j(0) - \mathfrak{S}(0))^\theta} = \frac{p(0)^\theta}{(1-\theta)^{1-\theta} \theta^\theta} \frac{\mu(0) [\mathcal{W}_j(0) - \tilde{\mathfrak{S}}(0)] + p(0) \mathfrak{S}(0)}{\mu(0) [\mathcal{W}_j(0) - \tilde{\mathfrak{S}}(0)]},$$

where we made use of the solutions for \mathcal{E}_j, C_j, S_j given by (58).

Inserting the price index into (61) and rearranging yields a simpler expression

$$1 + \psi_j(0) = \left(\frac{\mu^A(0)}{\mu^B(0)} \right)^{\frac{\sigma}{\sigma-1}} \frac{\mathcal{W}_j^A(0) - \tilde{\mathfrak{S}}^A(0)}{\mathcal{W}_j^B(0) - \tilde{\mathfrak{S}}^B(0)} \left(\frac{p^A(0)}{p^B(0)} \right)^{-\theta}. \quad (62)$$

B.5. Derivation of Results on Wealth Distribution (Section 5.2)

We have to study the dynamics of the wealth distribution in general before analyzing the effect of rent dynamics on the wealth distribution. Let a variable with the superscript R denote the ratio of this variable to the average of the respective variable. For instance, relative wealth is given by $W_j^R(t) \equiv W_j(t)/W(t)$. Following Caselli and Ventura (2000) we define *individual wealth convergence* as a decrease in $|W_j^R(t) - 1|$ over a small time interval $t \in [t, t+dt]$. This implies increasing relative wealth for a wealth-poor household ($W_j^R < 1$) and declining relative wealth for a wealth-rich household ($W_j^R > 1$). For divergence the opposite holds true.

To understand the different forces that trigger individual wealth divergence or convergence, we have to derive saving rates first. The saving rate is defined as the share of income spend on saving, $s_j \equiv \dot{W}_j/y_j$, where $y_j \equiv rW_j + wL_j$ is income. Making use of the budget constraint (3) and equation (4) yields

$$s_j = 1 - \mu \frac{W_j + \tilde{w}L_j}{rW_j + wL_j} = 1 - \mu \frac{\mathcal{W}_j}{y_j}. \quad (63)$$

The saving rate of household j is one minus the consumption rate and the consumption rate is total consumption expenditures, $\mu \mathcal{W}_j$, over income, y_j . If $r\tilde{w} > (<)w$, the saving rate is increasing (decreasing) in individual wealth W_j , as can be seen by taking the derivative of (63) with respect to household wealth W_j .

Wealth divergence or convergence depends on whether $|W_j^R - 1|$ increases or decreases over time. We therefore study the derivative of relative wealth with respect to time, given by

$$\dot{W}_j^R = \left(\frac{\dot{W}_j}{W_j} - \frac{\dot{W}}{W} \right) W_j^R = \left(s_j \frac{y_j}{W_j} - s \frac{y}{W} \right) W_j^R. \quad (64)$$

The first equation states that the change of household j 's relative wealth over time is given by the difference in growth rates of household j 's wealth and average wealth, multiplied

	$W_j^R > 1$		$W_j^R < 1$		$W_j^R = 1 \neq L_j^R$	$W_j^R = L_j^R$
	$W_j^R > L_j^R$	$W_j^R < L_j^R$	$W_j^R > L_j^R$	$W_j^R < L_j^R$		
$\mu\tilde{w} > w$	diverge	converge	converge	diverge	diverge	stationary
$\mu\tilde{w} = w$	stationary	stationary	stationary	stationary	stationary	stationary
$\mu\tilde{w} < w$	converge	diverge	diverge	converge	diverge	stationary

Table B.1: Individual wealth convergence and divergence

by the level of household j 's wealth. The second equality is obtained by making use of the definition of the saving rate, which implies that the growth rate of wealth equals the saving rate, s_j , multiplied by the income to wealth ratio, y_j/W_j . What matters for individual wealth convergence or divergence is the sign of \dot{W}_j^R , which is determined by the term in brackets in (64), as will be studied in the following.

Equation (64) shows that there are two opposing forces at work. On the one hand, wealth-rich households choose a higher saving rate than wealth-poor households, provided that $r\tilde{w} > w$. If wealth-rich households save sufficiently more than the average, their relative wealth is increasing, while the opposite applies to wealth-poor households. This is a divergence mechanism. In the limiting case where all households have the same income to wealth ratio (which is equivalent to assuming that $W_j^R = L_j^R$), the sign of \dot{W}_j^R depends only on the difference in saving rates, $(s_j - s)$. On the other hand, the income to wealth ratio, $y_j/W_j = r + w \frac{L_j}{W_j}$, is decreasing in W_j . This variation contributes to \dot{W}_j^R being negative for wealth-rich households and positive for wealth-poor households. This is a convergence mechanism. In the limiting case of equal saving rates, the sign of \dot{W}_j^R is determined by the difference $(L_j/W_j - L/W)$, which is negative for values of W_j sufficiently larger than W .

Under what conditions is the convergence mechanism dominating the divergence mechanism and vice versa? In order to study both effects we substitute s_j and s in equation (64) by (63) to get

$$\dot{W}_j^R = \frac{(\mu\tilde{w} - w)L}{W} (W_j^R - L_j^R). \quad (65)$$

Whether individual wealth W_j converges or diverges towards the average W depends on i) the sign of $(\mu\tilde{w} - w)$, which is the same for all households, ii) the sign of $W_j^R - 1$, which is positive for wealth-rich and negative for wealth-poor households, and iii) the sign of $(W_j^R - L_j^R)$, which is also household-specific. For instance, consider a wealth-rich household, $W_j^R - 1 > 0$, and assume that $(\mu\tilde{w} - w) > 0$. If $(W_j^R - L_j^R) > 0$, the household's wealth diverges because \dot{W}_j^R is positive. We provide a full characterization in table B.1.

So far we have only analyzed wealth convergence or divergence for a specific *household*, but not for the whole economy. We define *global* wealth convergence (divergence) over a small time interval $t \in [t, t+dt]$ as a situation where individual wealth converges (diverges)

for all j .⁴¹ Depending on the joint distribution of wealth, W_j , and earnings, wL_j , it is possible that some household's wealth converges while it diverges for others, and vice versa, as can be seen in table B.1. In order to derive predictions about the dynamics of the entire wealth distribution, we therefore postulate

Assumption 2 (Joint distribution of wealth and earnings). *For given initial aggregate wealth, $W(0)$, and aggregate labor endowment, L , it holds for all j that*

$$\frac{W_j^R(0)}{L_j^R} \begin{cases} > 1 & \text{if } W_j^R(0) > 1 & (\text{wealth-rich}) \\ = 1 & \text{if } W_j^R(0) = 1 & (\text{representative household}) \\ < 1 & \text{if } W_j^R(0) < 1 & (\text{wealth-poor}) \end{cases}.$$

Note that relative labor endowment, $L_j^R = L_j/L$, equals relative earnings, wL_j/wL . According to assumption 2, wealth-rich households' relative wealth is larger than their relative earnings while it is the opposite for wealth-poor households. We hence abstract from wealth-rich households that have very high relative earnings and from wealth-poor households with very low relative earnings. This assumption does not seem too restrictive, given that wealth is empirically more unequally distributed than earnings (Kuhn and Ríos-Rull, 2016). In our quantitative application we use the 2016 SCF+, and there 88 percent of all households satisfy assumption 2. Assumption 2 further implies that if $W_j^R(0) > (<)L_j^R$, it follows that $W_j^R(t) > (<)L_j^R$ for all $t \geq 0$. This results from equation (65) according to which relative wealth, $W_j^R(t)$, can at most converge to relative earnings, L_j^R , but not beyond.

With assumption (2) at hand we are now able to derive insights on the dynamics of the entire wealth distribution.

Proposition B.1 (Global wealth convergence and divergence). *Global wealth dynamics are characterized by*

$$\frac{d}{dt}|W_j^R(t) - 1| \begin{cases} > 0 & \text{for all } j \text{ if } \mu(t)\tilde{w}(t) > w(t) & (\text{divergence}) \\ = 0 & \text{for all } j \text{ if } \mu(t)\tilde{w}(t) = w(t) & (\text{stationarity}) \\ < 0 & \text{for all } j \text{ if } \mu(t)\tilde{w}(t) < w(t) & (\text{convergence}) \end{cases}. \quad (66)$$

Proof of proposition B.1. Assumption (2) rules out columns 3, 4, and 6 in table B.1, i.e. the cases $L_j^R > W_j^R > 1$, $L_j^R < W_j^R < 1$, and $L_j^R \neq W_j^R = 1$. All remaining cases are contained in (66). \square

First, the wealth distribution is only stationary if $\mu(t)\tilde{w}(t) = w(t)$ holds. This condition holds in steady state. It means that consumption out of human wealth equals contemporaneous earnings. As a result, the growth rate of wealth is the same across households and

⁴¹We use the terms *global wealth convergence (divergence)* and *decreasing (increasing) wealth inequality* interchangeably.

the divergence mechanism (wealth-rich households' saving rates are higher) compensates the convergence mechanism (wealth-rich households' higher level of wealth reduces their growth rate of wealth). Second, the relation of $\mu(t)\tilde{w}(t)$ and $w(t)$ depends only on parameters and prices $p(v), w(v), r(v)$ for all $v \in [t, \infty]$. For example, the larger future wage growth, the larger is $\tilde{w}(t)$, the more likely it is that $\mu(t)\tilde{w}(t) > w(t)$, and the more likely it is that the wealth distribution diverges.

How does the wealth distribution behave in a steady state? The answer is given by the following proposition.

Proposition B.2 (Stationary wealth distribution). *In a steady state, the wealth distribution is stationary in the sense that for every households j the relative wealth position W_j^R does not change over time.*

Proof of proposition B.2. From the proposition B.1 we know that a necessary condition for a stationary wealth distribution is $\mu\tilde{w} = w$. It hence remains to be shown that this is indeed the case in steady state.

Notice first that the transversality condition of the household optimization problem requires $r^* - g^Y > 0$. Substituting (55) and $\hat{C} = g^Y$ into (50), we find that

$$r^* - g^Y = \rho - [(1 - \theta + \theta\gamma\eta)g^Y + \theta\gamma(1 - \eta)g^X](1 - \sigma) > 0$$

always holds if $\sigma \geq 1$. Recall from proposition A.1 that $\hat{w} = g^Y$. Thus, in steady state, $w(\tau) = w(t)e^{(\tau-t)g^Y}$ and, consequently, the PDV of wages, $\tilde{w}(t) \equiv \int_t^\infty w(\tau)e^{\int_t^\tau -r(v)dv}d\tau$, can be written as

$$\tilde{w}(t) = \int_t^\infty \underbrace{w(t)e^{g^Y(\tau-t)}}_{=w(\tau)} e^{-r^*(\tau-t)} d\tau = \frac{w(t)}{r^* - g^Y} \equiv \tilde{w}^*(t). \quad (67)$$

Aggregate the Euler equation (33) and rewrite

$$\hat{C} = \frac{r^* - \rho + (\sigma - 1)\theta\hat{p}}{\sigma} = g^Y, \quad (68)$$

where the latter follows from proposition A.1.

Also note from (68) that $(\theta\hat{p} - r^* - \frac{\rho}{\sigma-1})\frac{\sigma-1}{\sigma} = -(r^* - g^Y) < 0$ and consider next the propensity to consume, as given by (7). Using that, in steady state, $p(\tau)/p(t) = e^{\hat{p}(\tau-t)}$ and $\tilde{r}(\tau, t) \equiv \int_t^\tau r(v)dv = (\tau - t)r^*$, we can rewrite $\mu(t)$ in steady state as

$$\mu(t) = \left(\int_t^\infty \exp \left[\left(\theta\hat{p} - r^* - \frac{\rho}{\sigma-1} \right) \frac{\sigma-1}{\sigma} (\tau - t) \right] d\tau \right)^{-1} = r^* - g^Y \equiv \mu^*(t). \quad (69)$$

Using (67) and (69), we confirm $\mu^*(t)\tilde{w}^*(t) = w(t)$. □

Consequently, a change in wealth inequality over time requires transitional dynamics. The policy experiment analyzed in the main text triggers such transitional dynamics.

Lastly, proposition 7 describes how the time path of real rents affects the dynamics of the wealth distribution.

Proof of proposition 7. Wealth divergence or convergence is determined by equation (65). The real rent only affects the marginal propensity to consume, μ , in this expression. From equation (7) we immediately see that μ is declining (increasing) in the growth factor of rents, $\frac{p(\tau)}{p(t)}$ for $\tau > t$, if $\sigma > 1 (< 1)$. If μ is declining (increasing), then \dot{W}_j^R is also declining (increasing), as can be seen from (65). \square

C. Calibration

We calibrate the model economy's steady state to the current US economy at an annual frequency. This implies a stationary wealth distribution, which is roughly in line with recent data on the wealth distribution (WID, 2017; Kuhn *et al.*, 2020). The full set of parameters is summarized in table C.2.

C.1. Household Sector

Our model features two exogenous sources for heterogeneity: different initial wealth endowments and different labor productivities. The initial wealth distribution is exogenous and constant in the steady state, as in Caselli and Ventura (2000). Hence, we can set it exogenously. We use the 2016 wave of the harmonized Survey of Consumer Finances (SCF+) from Kuhn *et al.* (2020) to set the joint distribution of earnings and wealth. It comprises 31,240 households, where we count implicates as separate households. We restrict the sample to households with a household head between 35-55 years to abstract from life-cycle effects that we do not capture with our model. This focus on middle-aged households reduces the sample size to 11,305 households. Moreover, to avoid extreme outliers, we drop all households that earn less than 5 percent of the average earnings because most of these households violate the transversality condition. We end up with a final sample of 10,180 households. We further assume that every household holds the same portfolio composition as the representative household.⁴²

The preference parameters ϕ and θ are calibrated endogenously to match two key moments of the housing expenditure share distribution in the US in 2015, as displayed in table 1: An aggregate housing expenditure share of 19 percent and a difference between the expenditure shares of the first and fifth income quintiles of 7 percentage points. The resulting parameter values are $\phi = 0.086$ and $\theta = 0.177$. In order to analyze the relevance of Schwabe's law we also consider $\phi = 0$ as an alternative calibration. When changing ϕ ,

⁴²This assumption is relaxed in section 4.5 where we analyze how the results change under heterogeneous portfolios.

we adjust θ such that the aggregate housing expenditure share, e , remains at 19 percent. For a given ϕ this implies that θ equals

$$\theta = \frac{e(1-\phi)}{1-\phi e}.$$

Under our alternative calibration of $\phi = 0$ we set $\theta = 0.19$.

Havránek (2015) reports that the majority of studies find an intertemporal elasticity of substitution (IES) below 0.8. We set $\sigma = 2$, implying an IES of 0.5.⁴³ The time preference rate, ρ , is calibrated endogenously by matching the average rate of return on wealth for the postwar US of 5.77 percent (**Jordà et al., 2018**, Table 12). In steady state, by re-arranging the Euler equation, the time preference rate can be expressed as

$$\rho = r - \sigma g^Y + (\sigma - 1)\theta \hat{p}.$$

We match the GDP growth rate, g^Y , and the rent growth rate, \hat{p} , with other parameters, as explained below. Targeting an interest rate of 5.44 percent yields then $\rho = 0.0195$.

C.2. Numeraire Sector

We normalize the total amount of land that can be used economically, Z , to one. The annual depreciation rate of capital, δ^K , is set to 5.6 percent (**Davis and Heathcote, 2005**). The concavity parameter β equals in equilibrium the expenditure share for labor in the numeraire sector, i.e., $\beta = wL^Y/Y$. Note further that, in steady state, $GDP^H = pS + wL^X$ is the housing sector's value-added, and total GDP reads $GDP = Y + GDP^H$. Taken together, we can express β as

$$\beta = \frac{wL^Y}{Y} = \frac{\frac{L^Y}{L}}{\frac{Y}{GDP}} \frac{wL}{GDP} = \frac{1 - \frac{L^X}{L}}{1 - \frac{GDP^H}{GDP}} \frac{wL}{GDP}. \quad (70)$$

We calibrate β by measuring three values in the data, i) the share of housing in GDP, $\frac{GDP^H}{GDP}$, ii) the share of labor employed in the residential construction sector, $\frac{L^X}{L}$, and iii) the aggregate labor income share, $\frac{wL}{GDP}$. First, the average value-added of the housing sector and overall construction as percentage of GDP over 1998-2001 (before the housing boom and bust) are, on average, 9.1 and 4.4 percent, respectively (**BEA, 2015b**). In our model, $P^X I^X$ is the value-added of residential construction. The ratio of residential to total investment in structures during 1998-2001 was 61 percent (**BEA, 2015a**), which suggest a value-added of residential construction relative to GDP, $P^X I^X / GDP$, of $0.61 \times 4.4 = 2.7$ percent. Thus, we set GDP^H / GDP to $9.1 + 2.7 = 12$ percent. Second, $\frac{L^X}{L}$ decreased from 4.8 percent in 2004 to 4.1 percent in 2014 (**Henderson, 2015**, Tab. 2.1). Taking an intermediate value of

⁴³Although the assumption $IES < 1$ is widely employed in the macro literature, it should be noted that there is a literature in finance on long-run risk taking which argues that $IES > 1$ (**Bansal and Yaron, 2004**).

Parameter	Value	Explanation/Target
L	1	Normalization
J	10,180	SCF+ sample size
$\left\{\frac{w_j(0)}{w(0)}\right\}_{j=1}^J$	see text	SCF+
$\left\{n_j\right\}_{j=1}^J$	see text	SCF+
$\left\{\frac{L_j}{L}\right\}_{j=1}^J$	see text	SCF+
σ	2	$IES = 0.5$ (Havránek, 2015)
Z	1	Normalization
δ^K	0.056	Davis and Heathcote (2005)
α	0.275	Land income share in Y sector (Grossmann <i>et al.</i> , 2020)
β	0.69	Labor income share in Y sector (BEA)
g^Y	0.02	Growth rate of GDP per capita (FRED)
δ^X	0.015	Hornstein (2009)
η	0.556	Labor expenditure share in X sector (Grossmann <i>et al.</i> , 2020)
g^X	0.004	Implied by matching rent growth of 1%
κ	0.169	Share of residential land: 16.9 percent (Falcone, 2015)
θ	{0.19, 0.177}	Average housing expenditure share 0.19 (U.S. Bureau of Labor Statistics, 2016)
ϕ	{0.000, 0.086 }	Difference between bottom and top income quintiles of Table 1
ρ	0.0195	Real interest rate: 0.0577 (Jordà <i>et al.</i> , 2018)
γ	0.777	Land's share in housing wealth: 1/3
ξ	759.05	Transition speed in N (Grossmann <i>et al.</i> , 2020)

Table C.2: Set of parameters for the calibrated model

4.5 percent and multiplying it by the fraction of residential investment in total investment in structures (61 percent), we arrive at $L^X/L = 0.045 \times 0.61 = 0.027$. Lastly, $\frac{wL}{GDP}$ is 62 percent on average between 1975 and 2012 (Karabarbounis and Neiman, 2013, "CLS KN merged"). Putting all together in (70) yields $\beta = (1 - 0.027)/(1 - 0.12) \times 0.62 = 0.69$. The remaining concavity parameter α is taken from our companion paper (Grossmann *et al.*, 2020). There, we calibrate α by endogenously matching the non-residential land income share. The resulting value is $\alpha = 0.275$

C.3. Housing Sector

The annual depreciation rate of structures, δ^X , is set to 1.5 percent (Hornstein, 2009, p. 13). The construction output elasticity of materials, η , is taken from Grossmann *et al.* (2020), who calibrate it endogenously by matching the labor share in the residential construction sector. We hence set $\eta = 0.556$. We calibrate γ endogenously to match the share

of residential land value in total housing value. According to time series of the aggregate residential land value and the total value of housing from [Davis and Heathcote \(2007\)](#) the share of residential land in total housing value has been increasing from 10 percent in 1950 to around 30 percent in 1975. Since then it has been fluctuating between 25 and 40 percent. We target an average value of one-third, implying a value of $\gamma = 0.777$.

We choose g^X such that we match (given γ , η , and g^Y) an annual average growth rate of rents of 1 percent. According to [Knoll \(2017\)](#), the annual average growth rate of real rents in the postwar (1953-2017) US economy was about 0.8 percent. [Albouy et al. \(2016\)](#) argue that official data on rent growth is biased downwards due to incomplete accounting for quality improvements. Assuming that rents grow by one percent annually (given γ , η , and g^Y) and using (22) implies $g^X = 0.004$.⁴⁴

The parameter ξ captures the importance of adjustment costs associated with land reallocations between the housing and the numeraire sector. This parameter is difficult to calibrate, as it does not affect the steady state and has an impact only along the transition. In [Grossmann et al. \(2020\)](#) we calibrate it endogenously by matching several moments along a transition for the US since 1950. We take the same value of $\xi = 759.05$.

We calibrate κ to match the observed allocation of land in the residential sector. According to geographic land-use data provided by [Falcone \(2015\)](#), 30.2 percent of the total US surface is used economically and 16.9 percent of this land is used as residential land. Hence, we set $\kappa = 0.169$.⁴⁵

⁴⁴The low value for g^X in comparison to g^Y is supported by evidence of low, sometimes even negative, productivity growth in the construction sector ([Davis and Heathcote, 2005](#)).

⁴⁵Without zoning regulations the model would imply a steady state share, N/Z , equal to 58 percent.

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Online Appendix

A. Alternative Utility Specifications

A.1. Status Preferences for Both Goods

If we replaced instantaneous utility (2) by

$$u(C_j, S_j) = \frac{[(C_j - \phi_c C)^{1-\theta} (S_j - \phi_s S)^\theta]^{1-\sigma} - 1}{1-\sigma},$$

with $\phi_c, \phi_s \geq 0$, where \bar{c} is average consumption of the numeraire good, then the housing expenditure share would still read as (9), with $\phi \equiv \frac{\phi_s - \phi_c}{1 - \phi_c}$. Since $\phi > 0$ iff $\phi_s > \phi_c$, assuming status concerns with respect to housing services only ($\phi_c = 0$) captures, without loss of generality, that status concerns are higher for housing than for non-housing consumption.

A.2. CES Utility

Consider the following utility specification

$$u(C_j, S_j) = \frac{(\mathcal{C}_j)^{1-\sigma} - 1}{1-\sigma} \quad \text{with} \quad \mathcal{C}_j = \left[\theta (S_j - \phi S)^{1-\frac{1}{\kappa}} + (1-\theta) C_j^{1-\frac{1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}},$$

where $\kappa > 0$. The housing expenditure share of agent j (e_j) and the aggregate housing expenditure share (e) are then given by

$$e_j = \frac{\theta^\kappa p^{1-\kappa}}{\theta^\kappa p^{1-\kappa} + (1-\theta)^\kappa \left(1 - \frac{\phi}{S_j^R}\right)}, \quad \text{and} \quad e = \frac{\theta^\kappa p^{1-\kappa}}{\theta^\kappa p^{1-\kappa} + (1-\phi)(1-\theta)^\kappa},$$

where $S_j^R \equiv \frac{S_j}{S}$. If rents, p , grow over the long run (Piazzesi and Schneider, 2016), the aggregate housing expenditure share, e , is only constant if $\kappa = 1$. Notice that the utility specification in the main text, given by (2), is the limiting case of the above stated CES utility function for $\kappa \rightarrow 1$.

A.3. Status Preferences: Multiplicative Reference Level

Status preferences are often also captured as ratios instead of differences (Clark *et al.*, 2008; Schünemann and Trimborn, 2017). A typical formulation looks like this

$$v(C_j, S_j) = \frac{\left[S_j^\theta \left(\frac{S_j}{S} \right)^\phi (C_j)^{1-\theta} \right]^{1-\sigma} - 1}{1-\sigma},$$

where $\theta \in (0, 1)$, $\phi \in [0, 1)$, and $\sigma > 0$. In this case, the housing expenditure share of agent j is given by

$$e_j = \frac{\theta + \phi}{1 + \phi}.$$

Hence, this preference specification is not compatible with heterogeneous housing expenditure shares that vary systematically with income.

B. Equivalence: Renters and Homeowners

Two thirds of US households live in owner-occupied housing while only one third are renters.⁴⁶ Our model can equivalently be interpreted as an economy of homeowners and all results still hold true, independent on whether we interpret households as renters or homeowners.

We proof this equivalence by first setting up a homeowner problem and then showing that this is equivalent to the renter problem from the main text. Consider a homeowner household j . Let N_j and P^H denote the homeowner's amount of housing stock owned and the house price, respectively. A unit of housing stocks translates into $h(t)$ units of housing services such that the homeowner's housing services consumption is $S_j(t) = N_j(t)h(t)$. We will define $h(t)$ in a moment, but for now its just an exogenous variable to the household. The homeowner household solves the following problem

$$\max_{\{C_j(t), N_j(t)\}_{t=0}^{\infty}} \int_0^{\infty} u(C_j(t), N_j(t)h(t), \overbrace{N(t)h(t)}^{=S(t)}) e^{-\rho t} dt \quad (71)$$

$$\text{s.t. } \dot{W}_j(t) = r(t)W_j(t) + w(t)L_j - C_j(t) - p^{UC}(t)N_j(t), \quad (72)$$

where $p^{UC}(t) \equiv rP^H + P^X(\delta^X x + \dot{x}) - \dot{P}^H$ is the user cost of housing and $x \equiv X/N$ is the average amount of structures per unit of residential land. The user cost of housing consists of the sum of foregone interest payments, rP^H , and expenditures for maintenance and expansion, $P^X(\delta^X x + \dot{x})$, minus appreciation gains, \dot{P}^H .

We now show that this problem is equivalent to the renter problem from the main text. First, substitute N_j with S_j/h in the homeowner's problem. Life time utility (71) is then equivalent to that in the main text, (1) and the thus transformed budget constraint (72) equals the budget constraint of the main text, (3), if $p^{UC}/h = p$ holds. The final step of the proof is hence to show that p^{UC} equals ph . The house price is given by $P^H \equiv P^N + P^X x$ and the variable h is defined as the average amount of housing services produced per unit of

⁴⁶US Bureau of the Census, Homeownership Rate for the United States, retrieved from FRED, Federal Reserve Bank of St. Louis (<https://fred.stlouisfed.org/series/RHORUSQ156N>), series RHORUSQ156N (accessed December 21, 2018).

residential land, $h \equiv S/N$.⁴⁷ Hence, it remains only to show that p^{UC} equals pS/N :

$$\begin{aligned}
p^{UC} &= rP^H + P^X(\delta^X x + \dot{x}) - \dot{P}^H \\
&= rP^N + rP^X x + P^X(\delta^X x + \dot{x}) - \dot{P}^N - \dot{P}^X x - P^X \dot{x} \\
&= rP^N - \dot{P}^N + [rP^X - \dot{P}^X]x + P^X \delta^X x \\
&= R^N + R^X + P^X \delta^X x \\
&= (1 - \gamma)p \frac{S}{N} + \gamma p \frac{S}{X} x - \delta^X P^X x + P^X \delta^X x \\
&= (1 - \gamma)p \frac{S}{N} + \gamma p \frac{S}{X} \frac{X}{N} = p \frac{S}{N},
\end{aligned}$$

where we made use of the definition of P^H , the no-arbitrage condition (21), and the first order conditions of S -firms, (19). This completes the proof.

Since the renter and owner problems are equivalent, all results apply to homeowners as well as renters. The difference is merely in the interpretation. Instead of the *rent* it is the *user cost of housing* that affects the distribution of wealth and welfare. As p^{UC} equals ph this is almost the same.

Under financial frictions additional mechanisms start playing a role. For instance, the rent and the user cost per unit of housing services may diverge, implying that renters pay a higher price for housing services. Similarly, if houses pay a rate of return that differs from the rate of return paid by other assets, the portfolio structure plays a role for the wealth effects of surging house prices (Kuhn *et al.*, 2020).

C. Dynamic System

The model is fully described by seven differential equations plus a set of static equations.⁴⁸

$$\begin{aligned}
\dot{X} &= M^\eta (B^X L^X)^{1-\eta} - \delta^X X \\
\dot{N} &= \begin{cases} 0 & \text{if } P^N > P^Z \text{ and } N = \kappa Z \\ \frac{P^N - P^Z}{\xi w} & \text{else} \end{cases} \\
\dot{W} &= rW + wL - C - pS \\
\frac{\dot{C}}{C} &= \frac{r - \rho}{\sigma} + \frac{(\sigma - 1)\theta}{\sigma} \frac{\dot{p}}{p} \\
\dot{P}^X &= -R^X + rP^X
\end{aligned}$$

⁴⁷The reason for the definition of h becomes apparent when we aggregate housing stocks, $\sum_{j=1}^J n_j N_j h = Nh = X^\gamma N^{1-\gamma} = S$.

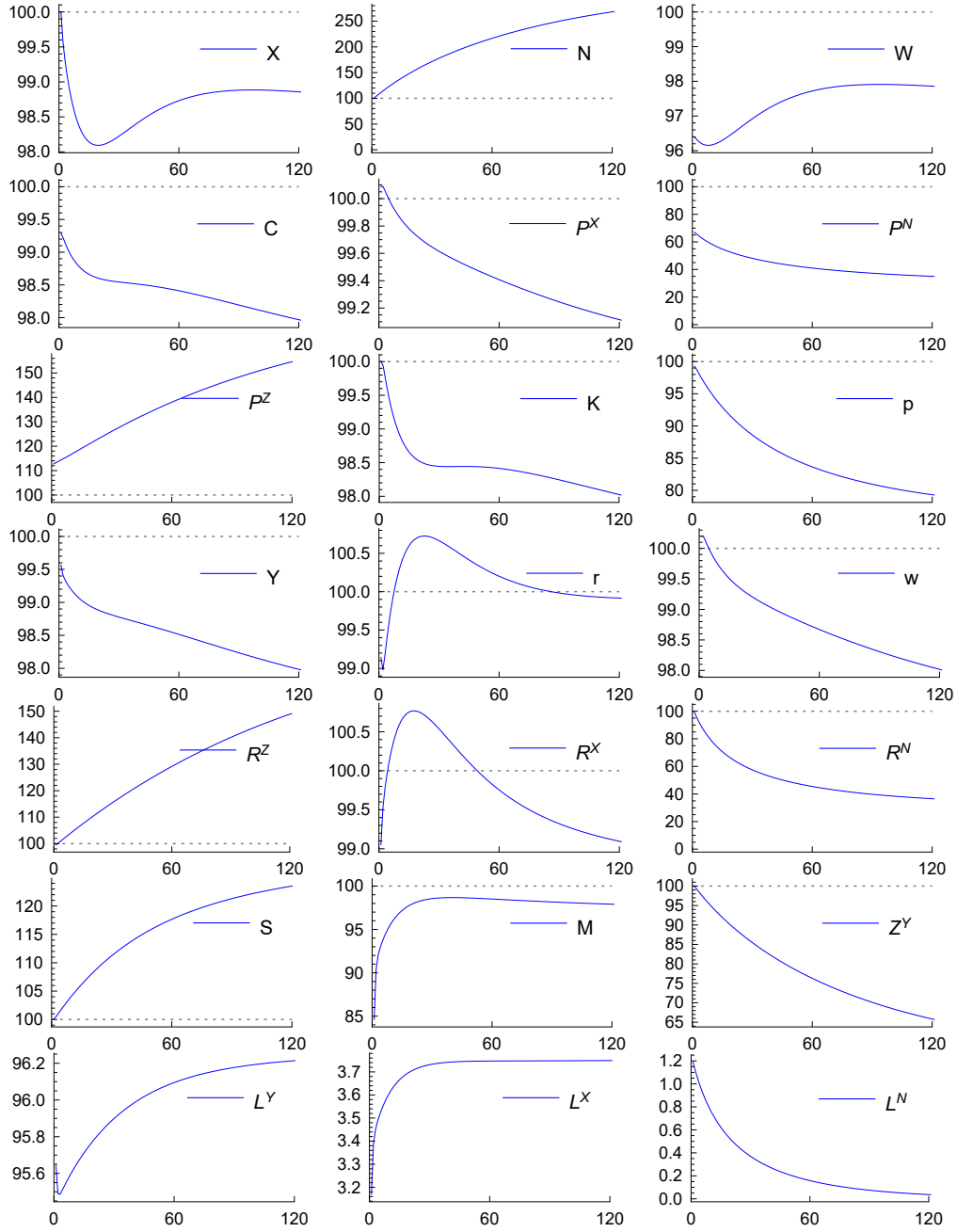
⁴⁸This is achieved by collecting equations from the main text, aggregating household-level equations, and rearranging.

$$\begin{aligned}
\dot{P}^N &= -R^N + rP^N \\
\dot{P}^Z &= -R^Z + rP^Z \\
K &= W - (P^N N + P^X X + P^Z Z^Y) \\
P &= \frac{\theta}{(1-\theta)(1-\phi)} \frac{C}{S} \\
Y &= K^\alpha (B^Y L^Y)^\beta (B^Y Z^Y)^{1-\alpha-\beta} \\
r &= \alpha \frac{Y}{K} - \delta^K \\
w &= \beta \frac{Y}{L^Y} \\
R^Z &= (1-\alpha-\beta) \frac{Y}{Z^Y} \\
R^X &= \gamma p \frac{S}{X} - \delta_X \\
R^N &= (1-\gamma) p \frac{S}{N} \\
S &= X^\gamma N^{1-\gamma} \\
L^X &= \left(\frac{(1-\eta)P^X}{w} \right)^{\frac{1}{\eta}} \frac{M}{(B^X)^{\frac{\eta-1}{\eta}}} \\
M &= (\eta P^X)^{\frac{1}{1-\eta}} B^X L^X \\
L &= L^Y + L^X + L^N \\
Z &= Z^Y + N \\
L^N &= \frac{\xi}{2} (\dot{N})^2
\end{aligned}$$

where $K(0)$, $N(0)$, $X(0)$ are given.⁴⁹

⁴⁹In total, there are 21 equations and 21 endogenous variables: X , N , W , C , P^X , P^N , P^Z , Y , K , p , r , w , R^Z , R^X , R^N , S , L^X , L^N , M , L^Y , and Z^Y .

D. Additional Figures



Notes: Apart from labor allocations (L^N , L^X , and L^Y), all variables are detrended with their respective growth rates and normalized by their steady state value in the baseline scenario (zoning). As in the main text, the horizontal axis is measured in years.

Figure D.1: Evolution of the model economy in response zoning deregulation.