Appendix: Time Series Analysis

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Cohen's paper [1] discusses analyzing the data with time series using FILM (FMRIBs Improved Linear Model). While we are not familiar with the FILM method, we did try modeling individual voxels in the framework of an autoregressive integrated moving average (ARIMA) process. We focused only on a single voxel from the first subject, but the method could easily be extended to additional or aggregate voxels. Let $\{Y_t\}$ be a single volume's value at time t and assume that the dth difference $W_t = \nabla^d Y_t$ is weakly stationary, defined to be when W_t has a constant mean function and autocovariance dependent only on lag k and not time t. Then we can try to model W_t as a linear combination of p autoregressive terms (or the number of most recent values to include) and q moving average terms (the number of lags to include for the white noise error terms):

$$W_t = \phi_1 W_{t-1} + \phi_2 W_{t-2} + ... + \phi_p W_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - ... - \theta_q e_{t-q}.$$

White noise is defined as a sequence of independent, identically distributed random variables. In order to fit an ARIMA process, the three orders p, d, and q must be first be specified, and the the associated coefficients estimated. We used a combination of visual inspection and quantitative methods to specify the ARIMA orders, and then used the maximum likelihood method to estimate parameters.

Having specified the order for d, we turned to the problem of specifying p and q. We used a combination of visually inspecting the autocorrelation and partial autocorrelation plots of the first difference, and looking at the Akaike information criteria (AIC) and Bayesian information criteria (BIC) computed from a grid of possible models. The latter method suggested specifying $p = 1$ and $q = 1$ (based on either the AIC or the BIC), which was also supported by the visual inspections.

We estimated the parameters for an ARIMA(1,1,1) model using the exact maximum likelihood estimator via Kalman filter. The residuals appear to be normally distributed, and its autocorrelation and partial autocorrelation plots also do not raise any red flags. Furthermore, when visually comparing the fitted time series to the true observed data, the ARIMA process seems to approximate the observed data much better than any of the linear regression models. However, since specifying the correct ARIMA process orders and estimating the associated parameters must be done separately by hand for each individual voxel of interest, we decided to eliminate this direction of analysis from our main pipeline.

Had we decided to continue pursuing time series analysis, we may have tried to forecast future observations based on previous ones. As an example, we modeled an ARIMA(1,1,1) process based on the first half of the observations for a single voxel. This process was then used to forecast the second half of the observations. A comparison between the true observations and the forecasted predictions is shown in [Figure 1]. While the forecasted observations look reasonable for approximating the true values, more quantitative and robust metrics for assessing performance need to be implemented.

One such procedure for assessing performance would be to design a permutation test. A null voxel time course could be simulated from by performing a Fourier transform on the observed time course, permuting the phases, and then transforming back to the original space. That way, the simulated time course has the same autocovariance as the observed time course, but random signal (as under the null case). We can then fit the same ARIMA model to the permuted process and examine how much, if at all, the ARIMA process fitted to the observed data makes improvements over the null case. Generating confidence intervals for the

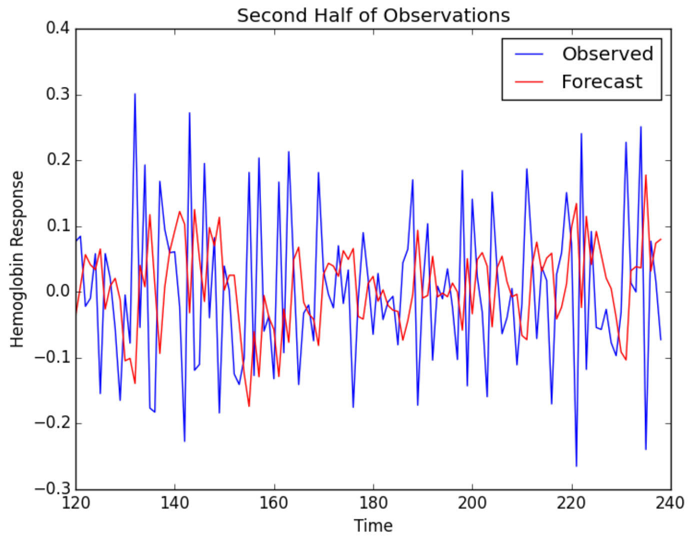

Figure 1: Forecasting the second half of observations based on the first half.

parameter estimates and forecasting future observations may also be of interest. Other considerations for modeling voxels as time series include exploring efficient and reasonable techniques for comparing multiple voxels both within and across subjects.

References

[1] J. R. Cohen, *The development and generality of self-control*, ProQuest, (2009), p. 164.