

Homework 4

Reading: *Book of Proof* Sections 3.3-3.5, 4.1-4.5, Handout 1 and 5.3
(Handout 1 can be found in eCommons under Resources.)

1 point will be awarded if there is a staple keeping all of the pages of your homework together, even if there is only one page.

Problems: Due Wednesday October 29, 2014 9:30am PDT in class.

Use other sheets. There is not enough room for correct answers here. Show work.

1. **(12 points)** Bridge is a card game which uses a standard deck of 52 cards. All 52 cards are dealt to four players, so each player receives a 13-card hand. The order of the cards within a hand is not important.
 - (a) How many 13-card hands have exactly 9 spades?
 - (b) How many 13-card hands have exactly 2 aces, 2 kings and 4 queens?
 - (c) How many 13-card hands have exactly 7 face cards?
(A face card is a king, queen or jack of any suit.)
 - (d) How many 13-card hands have at least 4 spades and exactly 5 hearts?
(You may want to use an on-line Pascal's triangle to find the values of the binomials for this problem.)
 - (e) How many 13-card hands have 4 aces or 4 kings?
 - (f) How many 13-card hands have cards from at most two suits?
2. **(12 points)** In this problem you will calculate the number of UCSC ID numbers that meet certain criteria, yet again. But this time, to make things simpler we will allow the first digit to be 0. So UCSC ID numbers have 7 digits and each digit can be 0 through 9. Be careful! There are still subtleties lurking here.
 - (a) How many UCSC ID numbers have exactly three 4's ?
 - (b) How many UCSC ID numbers have exactly two odd digits?
 - (c) How many UCSC ID numbers have at least 5 even digits?
 - (d) How many UCSC ID numbers have exactly two 5's or exactly three 8's?
 - (e) How many UCSC ID numbers **do not** have 3 consecutive 6's?
 - (f) How many UCSC ID numbers with no repeated digits and have digits in alphabetical order (i.e. 8, 5, 4, 9, 1, 7, 6, 3, 2, 0)?
3. **(4 points)** In each case below give the coefficient of the specified term.
 - (a) What is the coefficient of x^4y^3 in $(x+y)^7$?
 - (b) What is the coefficient of x^7y^4 in $(x+y)^{11}$?
 - (c) What is the coefficient of x^4 in $(x+2)^{10}$?
 - (d) What is the coefficient of x^3 in $(2x+1)^{13}$?
4. **(8 points)** In this problem you will prove the following statement in two ways

$$\forall i \in \mathbb{N}, \forall j \in \mathbb{N}, i \binom{i-1}{j-1} = j \binom{i}{j}$$
 - (a) **(5 points)** using the formula for $\binom{n}{k}$.
 - (b) **(3 points)** using a combinatorial argument

5. **(20 points)** Provide direct proofs for each of the following statements.

(a) If n is an odd integer, then n^3 is an odd integer.

(b) If n is an odd integer, then 4 divides $n^2 - 1$.

(c) For any real number x ,

$$\lceil x \rceil - 1 < x \leq \lceil x \rceil.$$

(This is Proposition 2 in Handout 1 whose proof was left to you.)

(d) For any integers a , b and c , if $a \mid b$ and $a \mid (b + c)$ then $a \mid c$.

(e) For any integer n ,

$$n = \left\lfloor \frac{n}{2} \right\rfloor + \left\lceil \frac{n}{2} \right\rceil.$$

(Hint: cases.)