

## Homework 6

**Reading:** *Book of Proof* Sections 7.1-7.4, 8.1-8.3 and 9.1

**1 point will be awarded if there is a staple keeping all of the pages of your homework together, even if there is only one page.**

**Problems:** Due Wednesday November 12, 2014 9:30am PST in class.

Use other sheets. There is not enough room for correct answers here. Show work.

1. **(5 points)** Prove or disprove that for any real number  $x$  if  $\sqrt{x}$  is irrational then  $x$  is irrational.
2. **(5 points)** Prove or disprove that for any real number  $x$  if  $x$  is irrational then  $\sqrt{x}$  is not a rational number.
3. **(5 points)** Prove or disprove that the product of an irrational number and a non-zero rational number is irrational.
4. **(5 points)** In each of the following give a value for  $x$  greater than -1 and less than the modulus (the number after **mod** inside the parentheses).
  - (a)  $x \equiv -75 \pmod{11}$
  - (b)  $x \equiv 895 \pmod{7}$
  - (c)  $x \equiv 2^{126} \pmod{5}$
  - (d)  $x^2 \equiv 9 \pmod{11}$

5. **(5 points)** Prove the following theorem:

**Theorem** For a prime number  $p$  and integer  $i$ ,

$$\text{if } 0 < i < p \text{ then } p \mid \binom{p}{i}.$$

6. **(5 points)** Prove the following theorem:

For an integer  $n$ ,  $n$  is an odd number if and only if  $n^2 - 1$  is a multiple of 4.

7. Consider the two sets  $S_1$  and  $S_2$

$$S_1 = \{k^2 : k \text{ is an odd integer}\} \quad \text{and} \quad S_2 = \{4m + 1 : m \text{ is an integer}\}$$

- (a) **(3 points)** Prove that  $S_1$  is a subset of  $S_2$ .
  - (b) **(2 points)** Prove that  $S_2$  is not a subset of  $S_1$ .
8. **(6 points)** Prove that the two sets  $A$  and  $B$  below are equal.

$$A = \{7m - 5 : m \text{ is an integer}\} \quad \text{and} \quad B = \{14k + b : k \text{ is an integer, and } b = 2 \text{ or } 9\}$$

9. (6 points) Prove that the following are equivalent for any integer  $n$ :

- (S1)  $n$  is odd
- (S2)  $(n + 1)^2$  is even
- (S3)  $(n - 1)/2$  is an integer

**Extra Credit** In class Friday we talked about RSA encryption. For the two problems below, assume that your bank's public key is the pair of integers  $(9, 35143)$ . You will want to use a calculator for these problems.

a. (2 points) Suppose you want to send the message consisting of the integer 6 to your bank. How would the integer 6 be encoded using your bank's public key?

b. (4 points) In doing your discrete math homework you happened to notice that  $35143 = 113 \cdot 311$ . With this knowledge, decode the message consisting of the integer 9443 which was encoded using your bank's public key.