CMPE 16 Homework # 2

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1. *Give the set represented by each of the expressions below where* $A_1 = \{\square, 2, 8, a, g\}$, $A_2 = \{\triangle, -2, 8, a\}$, $A_3 = \{\square, 12, 7, a, g\}$, *and* $A_4 = \{\square, \triangle, 2, 7, a, b, g\}$. *List each element in the set only once (i.e.* $\{1, 2\}$ *instead of* $\{1, 2, 2\}$*).*

   (a) $A_1 \cup A_2$
   Answer : $A_1 \cup A_2 = \{\square, 2, 8, a, g, -2, \triangle\}$

   (b) $A_3 \cap A_4$
   Answer : $A_3 \cap A_4 = \{\square, a, 7, g\}$

   (c) $A_4 - A_1$
   Answer : $A_4 - A_1 = \{\triangle, 7, b\}$

   (d) $A_1 - A_4$
   Answer : $A_1 - A_4 = \{8\}$

   (e) $\bigcup_{i=1}^{4} A_i$
   Answer : $\bigcup_{i=1}^{4} A_i = \{\square, 2, 8, a, g, \triangle, -2, 12, b\}$

   (f) $\bigcap_{i=1}^{4} A_i$
   Answer : $\bigcap_{i=1}^{4} A_i = \{a\}$

2. *For each of the sets below fill in the corresponding regions of a general Venn diagram for 3 sets. (The Venn diagram should have 3 sets in each case.)* **Answers on last page.**

   (a) $A \cup \overline{B} \cup C$

   (b) $C - (A \cap B)$

   (c) $\overline{(B - C) \cup A}$

3. *Write* $\bigcup_{i \in \mathbb{Z}} (i, i+1)$ *as the difference of two well known sets. Here* $(i, i+1)$ *is the open interval of the real line with endpoints* i *and* $i+1$*. (That is,* $(i, i+1) = x \in \mathbb{R} : i < x < i+1$*).*

   Answer : The set $Y = \bigcup_{i \in \mathbb{Z}} (i, i+1)$ can be written as the difference of $\mathbb{R}$ and $\mathbb{N}$; $Y = \mathbb{R} - \mathbb{N}$. This is because Y includes all numbers in $\mathbb{R}$ up-to but not including the actual integer values in $\mathbb{Z}$

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4. Using only the symbols 4, Z, S, P, W, $\emptyset, \subseteq, \in, \cup, \cap$, -, =, {, },),(, and $\neq$, express the following statements

   (a) 4 is pale and shy
   Answer : $4 \in (P \cap S)$

   (b) All worried integers are pale.
   Answer : $W \subseteq P$

(c) Every integer is shy, worried, or pale.
Answer : $\mathbb{Z} \subseteq (P \cup S \cup W)$

(d) There are worried integers that are not shy.
Answer : $(W - S) \neq \emptyset$

5. Let P and Q be the statements
P I eat garlic.
Q I go to the dentist.
Rewrite each of the statements below using P and Q and logical connectives ($\neg, \wedge, \vee, \implies$).

(a) I don't eat Garlic
Answer : $\neg P$

(b) I don't go to the Dentist, but I eat garlic.
Answer : $\neg Q \wedge P$

(c) I eat garlic or I don't go to the dentist.
Answer : $P \vee \neg Q$

(d) Whenever I go to the dentist, I don't eat garlic.
Answer : $Q \implies \neg P$

6. Let P, Q and R be the statements
P I use plastic bags.
Q I use paper bags.
R I help the environment.

(a) $\neg P$
Answer : I do not use plastic bags

(b) $P \wedge Q$
Answer : I use plastic bags and I use paper bags

(c) $Q \implies \neg R$
If I use paper bags, then I am not helping the environment.

(d) $\neg(P \implies R)$
It is not the case that if I use plastic bags I will help the environment.

7. Use a truth table to determine the values of each of the logical expressions below. Both of your truth tables should have at least 3 intermediate columns.

(a) $\neg(P \vee Q) \wedge \neg Q$

(b) $(P \vee Q) \wedge (Q \vee R) \wedge \neg(P \wedge R)$

8. Convert each of the following statements into the form If P then Q without changing their meanings. (Some of these statements might not be True and thats okay.)

(a) For a number to be prime, it must be greater than 1.
Answer : If x is prime then $x > 1$

(b) Whenever you bring an umbrella, it rains.
Answer : If I bring an umbrella, then it will rain.

(c) A number is prime only if it is not even.
Answer : If a number is not even, then it is prime.

(d) Either $n > a$ or $m > a$ if $n + m > 2a$
Answer : If $n + m > 2a$, then either $n > a \vee m > a$

9. As discussed in class, given a finite set S of size n and an ordering $s_1, s_2, ..., s_n$ of the n elements in S, we can represent the subsets of s using bit vectors of length n ($\{0,1\}^n$). For a subset $A \subseteq S$, the corresponding bit vector $b(A) = (b_1, b_2, ..., b_n)$ where $b_i = 1$ if $s_i \in A$ and $b_i = 0$ if $s_i \notin A$. Let S be the elements from the four sets in Problem 1 ordered as $\square, \triangle, -2, 2, 7, 8, 12, a, b, g$.

   (a) Give the bit vector corresponding to $\emptyset$
   Answer : $\{0,0,0,0,0,0,0,0,0,0\}$

   (b) Give the bit vector corresponding to the subset A_1 in Problem 1
   Answer : $\{1,0,0,1,0,1,0,1,0,1\}$

   (c) Give the bit vector corresponding to the subset A_2 in Problem 1
   Answer : $\{0,1,1,0,0,1,0,1,0,0\}$

   (d) Give the bit vector corresponding to the subset A_3 in Problem 1
   Answer : $\{1,0,0,0,1,0,1,1,0,1\}$

   (e) Give the bit vector corresponding to the subset A_4 in Problem 1
   Answer : $\{1,1,0,1,1,0,0,1,1,1\}$

   (f) Give the bit vector corresponding to S
   Answer : $\{1,1,1,1,1,1,1,1,1,1\}$

10. Given the bit vectors $\mathbf{b}(B) = (b_1, b_2, ..., b_n)$ and $\mathbf{b}(D) = (d_1, d_2, ..., d_n)$ representing two subsets B and D. In each case below explain how you would calculate the required bit vector (in general), and then apply your method to obtain the result for $B = A_1$ and $D = A_4$ from Problem 1.

   (a) $\mathbf{b}(B \bigcup D)$
   Answer : In general, if I want to find the union of two bit vectors representing subsets of the same set, I would simply perform a bitwise OR operation. The OR operation would put a 1 at any index where either B or D have 1's, which is the definition of union.
   In the specific case of $U = S$, $B = A_1$, and $D = A_4$ ...

$$\mathbf{b}(A_1 \cup A_4) = \{x_1, x_2, ..., x_n : x_i = (B_i \text{ OR } D_i)\}$$

$$\mathbf{b}(A_1 \cup A_4) = \{1,1,0,1,1,1,0,1,1,1\}$$

   (b) $\mathbf{b}(B \bigcap D)$
   Answer : In general, if I want to find the intersection of two bit vectors representing subsets of the same set, I would simply perform a bitwise AND operation.
   In the specific case of $U = S$, $B = A_1$, and $D = A_4$ ...

$$\mathbf{b}(A_1 \cap A_4) = \{x_1, x_2, ..., x_n : x_i = (B_i \text{ AND } D_i)\}$$

$$\mathbf{b}(A_1 \cap A_4) = \{1,0,0,1,0,0,0,1,0,1\}$$

   (c) $\mathbf{b}(B - D)$
   Answer : To find the difference between the two subsets B and D, I would need to only take members which are in B but also not in D. More formally,

$$\mathbf{b}(B - D) = \{x_1, x_2, ..., x_n : x_i = (B_i \text{ AND } (\neg D_i))\}$$

$$\mathbf{b}(A_1 - A_4) = \{0,0,0,0,0,1,0,0,0,0\}$$

(d) $\mathbf{b}(\overline{B})$
Answer : For sets, taking the compliment of a subset just means taking everything in the Universe that is not in that subset. In other words

$$\mathbf{b}(\overline{B}) = \{x_1, x_2, ..., x_n : x_i = (\neg B_i)\}$$

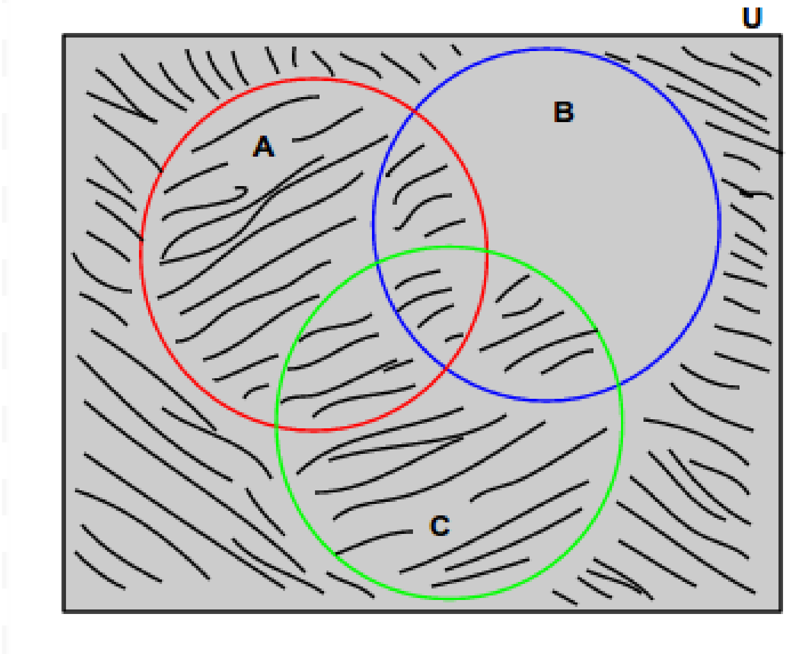

Figure 1: Venn Diagram for Problem 2a

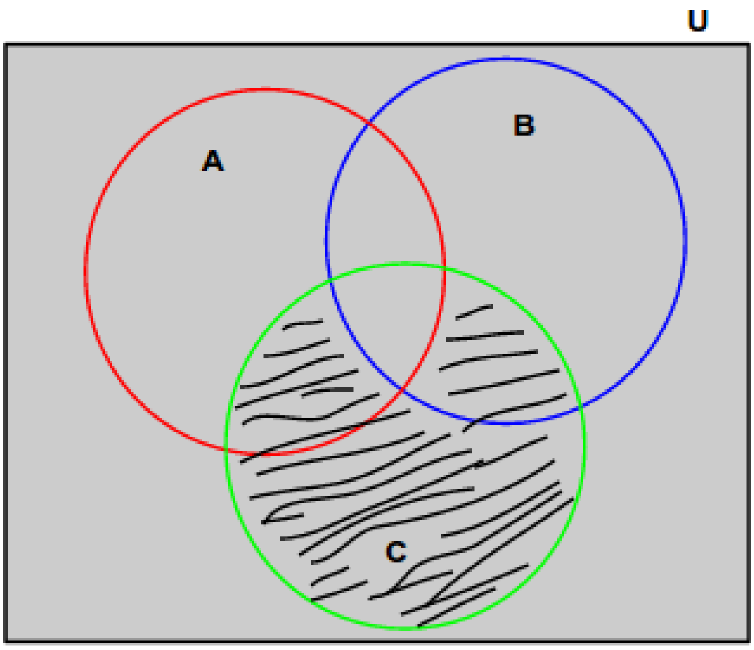

Figure 2: Venn Diagram for Problem 2b

Figure 3: Venn Diagram for Problem 2c