

CMPE 16 Homework 1

John Allard, id:1437547

October 6th, 2014

1. Express each of the sets below by listing all of their elements when the set is finite, and otherwise listing at least 6 elements.

(a) $\{9n - 7 : n \in \mathbb{Z}\}$

Answer : $\{\dots, -7, 2, 11, 20, 29, 38 \dots\}$

(b) $\{x \in \mathbb{Z} : 2x^2 - 7 < 43\}$

Answer : $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

(c) $\{x \in \mathbb{N} : 2x^2 - 7 \leq 43\}$

Answer : $\{1, 2, 3, 4, 5\}$

(d) $\{n \in \mathbb{Z} : 0 < n^2 - 4 < 38\}$

Answer : $\{-7, -6, -5, -4, -3, 3, 4, 5, 6, 7\}$

(e) $\{\sin \frac{n\pi}{2} : n \in \mathbb{Z} \text{ and } n \text{ is odd}\}$

Answer : $\{1, -1\}$

(f) $\{X \subseteq \{1,2,3,4\} : |X| = 2\}$

Answer : $\{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$

(g) $\{1,2,3\} \times \{1,2\}$

Answer : $\{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)\}$

(h) $\{1,2,3\} \times \mathbb{N} \times \emptyset$

Answer : \emptyset

2. For each of the sets below give the size of the set if it is finite, and otherwise state that it is infinite.

(a) $\{1,2,3\}$

Answer : 3

(b) \emptyset

Answer : 0

(c) $\{\emptyset\}$

Answer : 1

(d) $\{\{\emptyset\}, \emptyset\}$

Answer : 2

(e) (Save me the trouble of having to rewrite the purposely tedious nested brackets)

Answer : 3

(f) $\{\mathbb{N}, \emptyset, \mathbb{Z}\}$

Answer : 3

(g) $\{1,2,3,4,5\} \times \{7,8,9\} \times \{10,11,12,13\}$

Answer : 60

(h) The power set of $\{a,b,c,d,e,f\}$

Answer : $2^6 = 64$

3. For the problem let $A = \{3n + 4 \mid n \in \mathbb{N}\}$, $B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ and $C = \{n^2 - 3 \mid n \in \mathbb{Z}\}$. State whether each of the statements below is True or False and justify your answer.

(a) $B \subseteq A$

Answer : False. $0 \in B$ but $0 \notin A$ Therefore $B \not\subseteq A$

(b) $C \subseteq \mathbb{N}$

Answer : False. $-3 \in C$ but $a > 0 \forall a \in \mathbb{N}$. Therefore $C \not\subseteq \mathbb{N}$

(c) $A \subseteq \mathbb{Z}$

Answer : True, $a \in \mathbb{N} \forall a \in A, \mathbb{N} \subseteq \mathbb{Z} \implies a \in \mathbb{Z} \forall a \in A$, therefore $A \subseteq \mathbb{Z}$

(d) $A \subsetneq \mathbb{Z}$

Answer : True. $a \in \mathbb{Z} \forall a \in A$, also $\exists x < 0 \in \mathbb{Z}$ while $x < 0 \notin A$ therefore $A \subsetneq \mathbb{Z}$

4. Give the power set of each of the sets below.

(a) $\{1, 2, 3\}$

Answer : $\{\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$

(b) $\{\emptyset\}$

Answer : $\{\emptyset\}$

(c) $\{1, \mathbb{R}\}$

Answer : $\{\emptyset, \{1\}, \{\mathbb{R}\}, \{1, \mathbb{R}\}\}$

(d) $\{\emptyset, \{\emptyset\}\}$

Answer : $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$ ¹

¹This one was confusing, the emptyset would appear twice so one occurrence should be removed because you can't have redundant items in a single set, I think ... ?