CMPE 16 Homework #5

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1. Prove the following theorum

$$\text{For } x \in \mathbb{Z},\ \text{if } x^3 - 1 \text{ is even, then } x \text{ is odd}$$

Answer :

- This will be a proof by contrapositive.
- Let $\mathbb{E}$ and $\mathbb{O}$ represent the even and odd integers for this proof.
- $x^2 - 1 \in \mathbb{E} \implies x \in \mathbb{O}$ is logically equivilent to $x \in \mathbb{E} \implies x^3 - 1 \in \mathbb{O}$.
- I will prove the later to will prove the former.

Given some general $x \in \mathbb{E}$	$\exists a \in \mathbb{Z} : x = 2a$
$x^3 = (2a)^3 = 8a^3$	Cubing and substituting
$8a^3 - 1 = 2(4a^3) - 1$	Factor out a two
$\forall a \in \mathbb{Z} :\ 2(4a^3) - 1 \in \mathbb{O}$	Shows this will always be odd

Thus if x is even, $x^3 - 1$ will always be odd, which is logically equivalent to the the fact that if $x^3 - 1$ is even, then x must be odd.

2. Prove the following theorum

$$\text{If } x^2 \text{ is a prime number, then } x \text{ is not an integer}$$

Answer :

(a) This will be proof by contrapositive.
(b) $x^2 \in \mathbb{P} \implies x \notin \mathbb{Z}$ is the same as $x \in \mathbb{Z} \implies x^2 \notin \mathbb{P}$
(c) I will directly prove the later which will prove the former.
(d) If $x \in \mathbb{Z}, then x^2 \in \mathbb{Z}$
(e) $x^2 = x * x$ which is the product of two integers that are not 1 and x^2.
(f) This means that $x^2 \notin \mathbb{P}$

3. EXTRA CREDIT - Prove the following theorum

If a, b, and c are integers that satisfy $a^2 + b^2 + c^2$, then either a or b is even.

Two cases, first if c is odd, second if c is even.

(a) c is odd. If c is odd, then $c^2 = (2d + 1)^2 = 4d^2 + 4d + 1$
(b) Thus $a^2 + b^2 = 4d^2 + 4d + 1$