CMPE 16 - HomeWork 1

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1. *Express each of the sets below by listing all of their elements when the set is finite, and otherwise listing at least 6 elements.*

   (a) $\{9n - 7 : n \in \mathbb{Z}\}$
   Answer : $\{\ldots -7, 2, 11, 20, 29, 38 \ldots\}$

   (b) $\{x \in \mathbb{Z} : 2\text{x}^2 - 7 < 43\}$
   Answer : $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

   (c) $\{x \in \mathbb{Z} : 2\text{x}^2 - 7 \leq 43\}$
   Answer : $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

   (d) $\{n \in \mathbb{Z} : 0 < n^2 - 4 < 38\}$
   Answer : $\{3, 4, 5, 6, 7\}$

   (e) $\{\sin \frac{n\pi}{2} : n \in \mathbb{Z} \text{ and } n \text{ is } odd\}$
   Answer : $\{1, -1\}$

   (f) $\{X \subseteq \{1,2,3,4\} : |X| = 2\}$
   Answer : $\{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$

   (g) $\{1,2,3\} \times \{1,2\}$
   Answer : $\{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)\}$

   (h) $\{1,2,3\} \times \mathbb{N} \times \emptyset$
   Answer : $\emptyset$

2. *For each of the sets below give the size of the set if it is finite, and otherwise state that it is infinite.*

   (a) $\{1,2,3\}$
   Answer : 3

   (b) $\emptyset$
   Answer : 0

   (c) $\{\emptyset\}$
   Answer : 1

   (d) $\{\{\emptyset\}, \emptyset\}$
   Answer : 2

   (e) (Save me the trouble of having to rewrite the purposfully tedious nested brackets)
   Answer : 3

   (f) $\{\mathbb{N}, \emptyset, \mathbb{Z}\}$
   Answer : 1

   (g) $\{1,2,3,4,5\} \times \{7,8,9\} \times \{10,11,12,13\}$
   Answer : 60

   (h) The power set of $\{a,b,c,d,e,f\}$
   Answer : $2^6 = 128$

3. For the problem let $A = \{3\text{n} + 4 \mid n \in \mathbb{N}\}$, $B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ and $C = \{n^2 - 3 \mid n \in \mathbb{Z}\}$. State whether each of the statements below is True or False and justify your answer.

   (a) $B \subseteq A$
   Answer : False. $0 \in B$ but $0 \notin A$ Therefor $B \nsubseteq A$

   (b) $C \subseteq N$
   Answer : False. $\{-3, -1\} \in C$ but $a > 0\ \forall a \in \mathbb{N}$. Therefor C contains members that $\mathbb{N}$ does not and thus $C \nsubseteq \mathbb{N}$

   (c) $A \subseteq \mathbb{Z}$
   Answer : True, $a \in \mathbb{Z}\ \forall\ a \in A$, therefor $A \subseteq \mathbb{Z}$

   (d) $A \subsetneq \mathbb{Z}$
   Answer : True. Here is a short proof-like justification.
   $a \in \mathbb{Z}\ \forall\ a \in A$, but also $-5000 \in \mathbb{Z}$ while $-5000 \notin A$ therefor $A \subsetneq \mathbb{Z}$

4. Give the power set of each of the sets below.

   (a) $\{1, 2, 3\}$
   Answer : $\{\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$

   (b) $\{\emptyset\}$
   Answer : $\{\emptyset\}$

   (c) $\{1, \mathbb{R}\}$
   Answer : $\{\emptyset, \{1\}, \{\mathbb{R}\}, \{1, \mathbb{R}\}\}$

   (d) $\{\emptyset, \{\emptyset\}\}$
   Answer : $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$ (This one was confusing, the emptyset would appear twice so one should be removed because you can't have redundant items in a single set, I think ... ?)