Homework 3

Reading: *Book of Proof* Sections 2.4-2.12, 3.1-3.2

1 point will be awarded if there is a staple keeping all of the pages of your homework together, even if there is only one page.

Problems: Due Wednesday October 22, 2014 9:30am PDT in class.
Use other sheets. There is not enough room for correct answers here. Show work.

1. (**5 points**) You have six friends, Ann, Bob, Doris, Fay, Joe and Matt. One of them always tells the truth and the other five always lie. They each make a statement as indicated below.

Ann says	"Fay tells the truth."
Bob says	"Ann tells the truth."
Doris says	"Matt or Bob tells the truth."
Fay says	"Doris tells the truth."
Joe says	"Fay lies."
Matt says	"Joe and I lie."

   Determine who is the honest friend by completing the table below. The first section of the table has been filled in with the six possibilities for the veracity (truthfulness) of your six friends, In each row, there is only one honest friend (H) and the other 5 friends are liars (L).

   (a) Fill in the middle section, with the truth value for each of the statements based on who the liars are in that row.

   (b) Fill in the last section on the right, with (Y)es or (N)o, to indicate whether friend X would make statement S_X. Friend X makes statement S_X if either friend X is honest (H) and S_X is True, or if friend X is a liar and S_X is False.

   (c) Determine who the honest friend is from the contents of the last section.

Veracity of 6 friends						Truth of statement S_X						Would X say S_X?					
A	B	D	F	J	M	S_A	S_B	S_D	S_F	S_J	S_M	S_A	S_B	S_D	S_F	S_J	S_M
H	L	L	L	L	L												
L	H	L	L	L	L												
L	L	H	L	L	L												
L	L	L	H	L	L												
L	L	L	L	H	L												
L	L	L	L	L	H												

2. (**5 points**) You have four friends, Meg, Pat, Zoe and Tim. Two of them always tell the truth and the other two always lie. They each make a statement as indicated below.

Meg says	"I tell the truth, but Tim does not."
Pat says	"Tim and I are different when it comes to telling the truth."
Tim says	"Pat or Zoe lie. "
Zoe says	"I tell the truth, but Tim does not."

   Determine which two friends tell the truth using the same technique as in the previous problem.

   If you enjoy logic puzzles you'll like the Zebra Puzzle:
   `http://en.wikipedia.org/wiki/Zebra_Puzzle#Text_of_the_Life_International_puzzle`.
   This is NOT a homework problem. Do not attempt unless you have completed all other classwork.

3. **(8 points)** Detemine the truth value of each of the statements below. Justify your answer. The domain for x in all cases is the real numbers. You may use the fact that for all real numbers $x^2 \geq 0$. Start by stating clearly whether it is **True** or **False**.

   (a) $\forall x \in \mathbb{R},\ (3x \leq 2^x)$.
   (b) $\exists x \in \mathbb{R},\ (3x \leq 2^x)$.
   (c) $\forall x \in \mathbb{R},\ (x \leq x^2)$.
   (d) $\forall x \in \mathbb{R},\ (x < (x+1)^2 - x)$.

4. **(8 points)** Express the **negation** ($\sim$) of each of the statements below so that all negation symbols immediately precede only the P's or Q's.

   a.) $\exists x\, \forall y\, \exists z\, [P(x, y, z)]$
   b.) $\forall x\, \exists y\, [P(x, y)] \vee \forall x\, \forall y\, [Q(x, y)]$
   c.) $\exists x\, \forall y\, [P(x, y) \Leftrightarrow P(y, x)]$
   d.) $\forall y\, \exists x\, \forall z\, [P(x, y, z) \Rightarrow Q(z, y)]$

5. **(8 points)** Detemine the truth value of each of the statements below. Justify your answer. The domain for x is the real numbers ($\mathbb{R}$) and the domain for y is the **non-negative** real numbers ($\mathbb{R}^+ \cup \{0\}$).

   (a) $\forall x \in \mathbb{R},\ \forall y \in \mathbb{R}^+ \cup \{0\},\ (4x = y + 4)$.
   (b) $\exists x \in \mathbb{R},\ \exists y \in \mathbb{R}^+ \cup \{0\},\ (4x = y + 4)$.
   (c) $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R}^+ \cup \{0\},\ (4x = y + 4)$.
   (d) $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R}^+ \cup \{0\},\ (4x = y + 4)$.
   (e) $\exists y \in \mathbb{R}^+ \cup \{0\},\ \forall x \in \mathbb{R},\ (4x = y + 4)$.
   (f) $\forall y \in \mathbb{R}^+ \cup \{0\},\ \exists x \in \mathbb{R},\ (4x = y + 4)$.
   (g) $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R}^+ \cup \{0\},\ (4x = xy)$.
   (h) $\exists y \in \mathbb{R}^+ \cup \{0\},\ \forall x \in \mathbb{R},\ (4x = xy)$.

6. **(7 points)** In this problem you will calculate the number of UCSC that meet certain criteria. UCSC ID numbers all have 7 digits from 0 to 9 and the first digit is not 0. Be careful! There are subtleties lurking here.

   (a) How many UCSC ID numbers start with a 1 or 2?
   (b) How many UCSC ID numbers have only odd digits?
   (c) How many UCSC ID numbers have only even digits?
   (d) How many UCSC ID numbers start and end with the same digit?
   (e) How many UCSC ID numbers start and end with the different digits?
   (f) How many UCSC ID numbers have no repeated digits?
   (g) How many UCSC ID numbers are palindromes?
   (A palindrome is a string that is the same when all of its characters are reversed (e.g., "1012101" or "noon" or "rotator").

7. **(4 points)** Using only paper and pencil calculate the following values. All of the answers are integers. You must show your calculations to receive credit.

   (a) $\dfrac{100!}{96! \cdot 4!} =$
   (b) $\dfrac{400!}{397! \cdot 3!} =$
   (c) $\dfrac{2^{20}!}{2^{19}!} =$
   (d) $\dfrac{19!}{12! \cdot 7!} =$