

CMPE 16 Homework # 2

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1. Give the set represented by each of the expressions below where $A_1 = \{\square, 2, 8, a, g\}$, $A_2 = \{\triangle, -2, 8, a\}$, $A_3 = \{\square, 12, 7, a, g\}$, and $A_4 = \{\square, \triangle, 2, 7, a, b, g\}$. List each element in the set only once (i.e. $\{1, 2\}$ instead of $\{1, 2, 2\}$).

(a) $A_1 \cup A_2$

Answer : $A_1 \cup A_2 = \{\square, 2, 8, a, g, -2, \triangle\}$

(b) $A_3 \cap A_4$

Answer : $A_3 \cap A_4 = \{\square, a, 7, g\}$

(c) $A_4 - A_1$

Answer : $A_4 - A_1 = \{\triangle, 7, b\}$

(d) $A_1 - A_4$

Answer : $A_1 - A_4 = \{8\}$

(e) $\bigcup_{i=1}^4 A_i$

Answer : $\bigcup_{i=1}^4 A_i = \{\square, 2, 8, a, g, \triangle, -2, 12, b\}$

(f) $\bigcap_{i=1}^4 A_i$

Answer : $\bigcap_{i=1}^4 A_i = \{a\}$

2. For each of the sets below fill in the corresponding regions of a general Venn diagram for 3 sets. (The Venn diagram should have 3 sets in each case.) **Answers on last page.**

(a) $A \cup \overline{B} \cup C$

(b) $C - (A \cap B)$

(c) $\overline{(B - C) \cup A}$

3. Write $\bigcup_{i \in \mathbb{Z}} (i, i+1)$ as the difference of two well known sets. Here $(i, i+1)$ is the open interval of the real line with endpoints i and $i+1$. (That is, $(i, i+1) = \{x \in \mathbb{R} : i < x < i+1\}$).

Answer : The set $Y = \bigcup_{i \in \mathbb{Z}} (i, i+1)$ can be written as the difference of \mathbb{R} and \mathbb{N} ; $Y = \mathbb{R} - \mathbb{N}$. This is because Y includes all numbers in \mathbb{R} up-to but not including the actual integer values in \mathbb{Z}

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4. Using only the symbols $\forall, \exists, \subseteq, \in, \cup, \cap, -, =, \{, \}, \neg, \neq$, express the following statements

(a) 4 is pale and shy

Answer : $4 \in (P \cap S)$

(b) All worried integers are pale.

Answer : $W \subseteq P$

- (c) Every integer is shy, worried, or pale.
Answer : $\mathbb{Z} \subseteq (P \cup S \cup W)$
- (d) There are worried integers that are not shy.
Answer : $(W - S) \neq \emptyset$
5. Let P and Q be the statements
P I eat garlic.
Q I go to the dentist.
Rewrite each of the statements below using P and Q and logical connectives ($\neg, \wedge, \vee, \implies$).
- (a) I don't eat Garlic
Answer : $\neg P$
- (b) I don't go to the Dentist, but I eat garlic.
Answer : $\neg Q \wedge P$
- (c) I eat garlic or I don't go to the dentist.
Answer : $P \vee \neg Q$
- (d) Whenever I go to the dentist, I don't eat garlic.
Answer : $Q \implies \neg P$
6. Let P, Q and R be the statements
P I use plastic bags.
Q I use paper bags.
R I help the environment.
- (a) $\neg P$
Answer : I do not use plastic bags
- (b) $P \wedge Q$
Answer : I use plastic bags and I use paper bags
- (c) $Q \implies \neg R$
If I use paper bags, then I am not helping the environment.
- (d) $\neg(P \implies R)$
It is not the case that if I use plastic bags I will help the environment.
7. Use a truth table to determine the values of each of the logical expressions below. Both of your truth tables should have at least 3 intermediate columns.
- (a) $\neg(P \vee Q) \wedge \neg Q$
- (b) $(P \vee Q) \wedge (Q \vee R) \wedge \neg(P \wedge R)$
8. Convert each of the following statements into the form If P then Q without changing their meanings. (Some of these statements might not be True and that's okay.)
- (a) For a number to be prime, it must be greater than 1.
Answer : If x is prime then $x > 1$
- (b) Whenever you bring an umbrella, it rains.
Answer : If I bring an umbrella, then it will rain.
- (c) A number is prime only if it is not even.
Answer : If a number is not even, then it is prime.
- (d) Either $n > a$ or $m > a$ if $n + m > 2a$
Answer : If $n + m > 2a$, then either $n > a \vee m > a$

9. As discussed in class, given a finite set S of size n and an ordering s_1, s_2, \dots, s_n of the n elements in S , we can represent the subsets of s using bit vectors of length n ($\{0, 1\}^n$). For a subset $A \subseteq S$, the corresponding bit vector $b(A) = (b_1, b_2, \dots, b_n)$ where $b_i = 1$ if $s_i \in A$ and $b_i = 0$ if $s_i \notin A$. Let S be the elements from the four sets in Problem 1 ordered as $\square, \triangle, -2, 2, 7, 8, 12, a, b, g$.

- (a) Give the bit vector corresponding to \emptyset

Answer : $\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$

- (b) Give the bit vector corresponding to the subset A_1 in Problem 1

Answer : $\{1, 0, 0, 1, 0, 1, 0, 1, 0, 1\}$

- (c) Give the bit vector corresponding to the subset A_2 in Problem 1

Answer : $\{0, 1, 1, 0, 0, 1, 0, 1, 0, 0\}$

- (d) Give the bit vector corresponding to the subset A_3 in Problem 1

Answer : $\{1, 0, 0, 0, 1, 0, 1, 1, 0, 1\}$

- (e) Give the bit vector corresponding to the subset A_4 in Problem 1

Answer : $\{1, 1, 0, 1, 1, 0, 0, 1, 1, 1\}$

- (f) Give the bit vector corresponding to S

Answer : $\{1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$

10. Given the bit vectors $\mathbf{b}(B) = (b_1, b_2, \dots, b_n)$ and $\mathbf{b}(D) = (d_1, d_2, \dots, d_n)$ representing two subsets B and D . In each case below explain how you would calculate the required bit vector (in general), and then apply your method to obtain the result for $B = A_1$ and $D = A_4$ from Problem 1.

- (a) $\mathbf{b}(B \cup D)$

Answer : In general, if I want to find the union of two bit vectors representing subsets of the same set, I would simply perform a bitwise OR operation. The OR operation would put a 1 at any index where either B or D have 1's, which is the definition of union.

In the specific case of $U = S$, $B = A_1$, and $D = A_4$...

$$\mathbf{b}(A_1 \cup A_4) = \{x_1, x_2, \dots, x_n : x_i = (B_i \text{ OR } D_i)\}$$

$$\mathbf{b}(A_1 \cup A_4) = \{1, 1, 0, 1, 1, 1, 0, 1, 1, 1\}$$

- (b) $\mathbf{b}(B \cap D)$

Answer : In general, if I want to find the intersection of two bit vectors representing subsets of the same set, I would simply perform a bitwise AND operation.

In the specific case of $U = S$, $B = A_1$, and $D = A_4$...

$$\mathbf{b}(A_1 \cap A_4) = \{x_1, x_2, \dots, x_n : x_i = (B_i \text{ AND } D_i)\}$$

$$\mathbf{b}(A_1 \cap A_4) = \{1, 0, 0, 1, 0, 0, 0, 1, 0, 1\}$$

- (c) $\mathbf{b}(B - D)$

Answer : To find the difference between the two subsets B and D , I would need to only take members which are in B but also not in D . More formally,

$$\mathbf{b}(B - D) = \{x_1, x_2, \dots, x_n : x_i = (B_i \text{ AND } (\neg D_i))\}$$

$$\mathbf{b}(A_1 - A_4) = \{0, 0, 0, 0, 0, 1, 0, 0, 0, 0\}$$

(d) $\mathbf{b}(\overline{B})$

Answer : For sets, taking the compliment of a subset just means taking everything in the Universe that is not in that subset. In other words

$$\mathbf{b}(\overline{B}) = \{x_1, x_2, \dots, x_n : x_i = (\neg B_i)\}$$

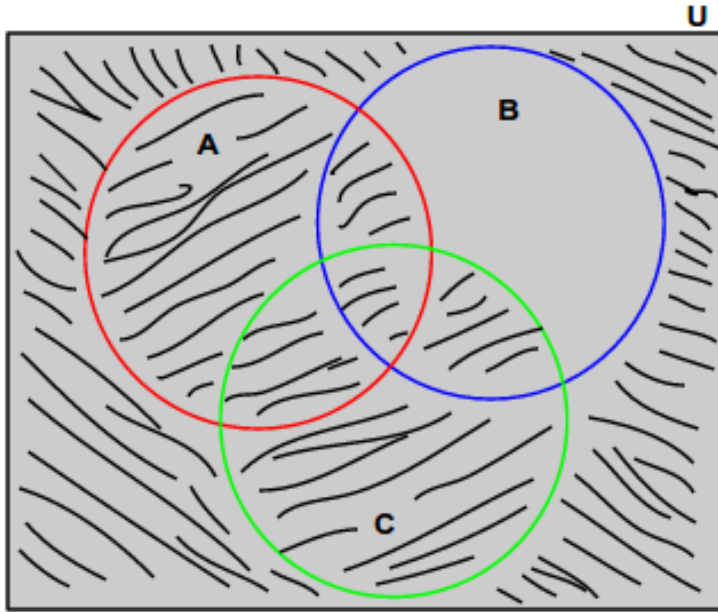


Figure 1: Venn Diagram for Problem 2a

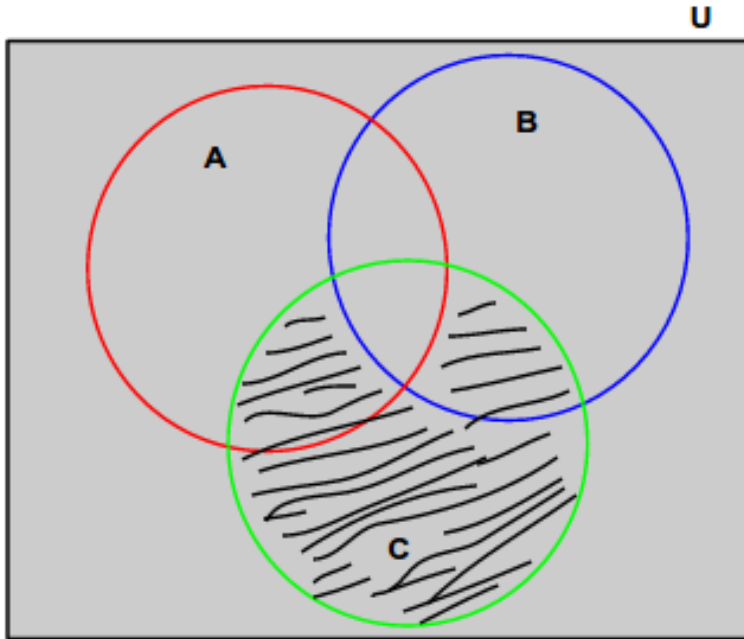


Figure 2: Venn Diagram for Problem 2b

Figure 3: Venn Diagram for Problem 2c