CMPE 16 Homework #9

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Decmber 9th, 2014

1. Suppose you roll a six-sided fair die twice.

   (a) What is the probability that either roll is a 1 or 2?
   Experiment : Rolling a fair 6-sided die twice;
   Sample Space : $\{x_1x_2 : \forall x_1x_2 \in \{1,2,3,4,5,6\}\}$
   Event Space : $\{x_1x_2 : \forall x_1x_2 \in \{1,2\}^2\}$
   Answer : $1/3 + 1/3 - 2/36 = 22/36 = 11/18$
   The probability of rolling either a one or two for the first roll is 1/3, same with the second roll, then we subtract the intersection so we don't overcount.

   (b) What is the probability that the sum of the two rolls is at least 10 or at least one of the rolls is an odd number?
   Experiment : Rolling a fair 6-sided die twice;
   Sample Space : $\{x_1x_2 : 1 \leq i \leq 2\ \forall x_i \in \{1,2,3,4,5,6\}\}$
   Event Space : $\{x_1x_2 : (x_1 + x_2 >= 10) \forall x_i, x_j \in \{1,2\}\}$
   Answer :
   Proabability of rolling a sum at least 10 :
   $|\mathcal{E}_1| = \frac{2+8-1}{8} - 1 * 3 = 9 - 3 = 6$, $|\mathcal{S}| = 36, p\{\mathcal{E}_\infty\} = \frac{1}{6}$
   Probability of rolling at least one odd number :
   $|\mathcal{E}_2| = 3 * 3 + 3 * 3 + 3 * 3 = 27.$ $p(\mathcal{E}_2) = 27/36 = 3/4.$
   (Corresponding to the number of $\{(even, odd), (odd, even), (odd, odd)\}$ pairs that can be made to satisfy the event requirements). Combined, we add the probabilities above and subtract their intersection to get :
   $p(\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 - \mathcal{E}_1 \cap \mathcal{E}_2) = 3/4 + 1/6 - 3/36 = 30/36 = 5/6$

   (c) What is the probability that you rolled doubles given that the sum is at least 10?
   Experiment : Rolling a fair 6-sided die twice;
   Sample Space : $\{x_1x_2 : 1 \leq i \leq 2\ \forall x_i \in \{1,2,3,4,5,6\}\}$
   Event Space : $\{x_1x_1 : \forall x_1 \in \{1,2,3,4,5,6\}\}$
   Answer :
   There are 36 items in the sample space, and only 2 ways to roll doubles and a sum greater than 10, so ..
   $p(\mathcal{E}) = 2/36 = 1/18$

2. Suppose the six-sided die you used in the previous problem is not fair. It is biased so that rolling a 6 is three times more likely than any other roll.

   (a) What is the probability that either roll is a 1 or 2?

   (b) What is the probability that either roll is a 1 or 2?
   Experiment : Rolling a fair 6-sided die twice;
   Sample Space : $\{x_1x_2 : \forall x_1x_2 \in \{1,2,3,4,5,6\}\}$
   Event Space : $\{x_1x_2 : \forall x_1 \in \{1,2\} \forall x_2 \in \{1,2\}\}$
   To start, I will split up the probabilities associated with the items in the sample space into those items that have 0 sixes, 1 six, or 2 sixes.
   $p(r_1r_2) : r_1r_2 \in \mathcal{E}_{no6} = (1/8)^2$
   $p(r_1r_2) : r_1r_2 \in \mathcal{E}_{one6} = (1/8) * (3/8)$
   $p(r_1r_2) : r_1r_2 \in \mathcal{E}_{two6} = (3/8)^2$

   Now these are all dijoint sets, so ..
   $p(\mathcal{E}) = (\mathcal{E} \cap \mathcal{E}_{no6}) * (1/8)^2 \cup (\mathcal{E} \cap \mathcal{E}_{one6}) * (3/64) \cup (\mathcal{E} \cap \mathcal{E}_{two6}) * (9/64)$
   $p(\mathcal{E}) = 18/64 + 3/64 + 0/64 = 21/64$

   (c) What is the probability that the sum of the two rolls is at least 10 or at least one of the rolls is an odd number?
   Once again, I will break this up into cases that involve 0 sixes, 1 six, or 2 sixes.

   (d) What is the probability that you rolled doubles given that the sum is at least 10?

3. The following questions are about a game in which you draw randomly from a bag containing 4 red balls, 4 blue balls, 3 green balls and 3 yellow balls. The ball is not replaced after each draw, and each ball in the bag is equally likely to be selected

   (a) If you draw two balls from the bag without replacing them, what is the probability of drawing two red balls?
   Answer : $(4/14) * (3/13) = 12/182 = 6/91$

   (b) Starting over with all 14 balls back in the bag, what is the probability of drawing two balls that are the same color?
   $p(\mathcal{E}) = p(\mathcal{E}_2 r) + p(\mathcal{E}_{2b}) + p(\mathcal{E}_{2g}) + p(\mathcal{E}_{2y})$
   $p(\mathcal{E}) = (4/14 * 3/13) + (4/14 * 3/13) + (3/14 * 2/13) + (3/14 * 3/13)$
   $p(\mathcal{E}) = 24/182 + 12/182 = 36/182 = 18/91$

(c) Starting over with all 14 balls back in the bag, what is the probability of drawing at least two yellow balls if you draw 4 times (without replacement)?

4. A bag contains a fair die and a biased die which is three times more likely to produce a 6. A die is selected at random from the bag and then rolled twice.

   (a) If the two rolls were both 6, what is the probability that the fair die was picked?

   (b) If exactly one of the two rolls resulted in a 6, what is the probability that the fair die was picked?

5. The following questions are about a game in which you draw randomly from a bag containing 2 red balls, 2 blue balls and a single yellow ball. The ball is not replaced after each draw, and each ball in the bag is equally likely to be selected. A red ball is worth \$10 and a blue ball is worth \$5. Yellow balls are worthless. In each case below the random variable X associated with each outcome is the value of the balls drawn.

   (a) What is the expectation for X of drawing one ball?
   Answer : $2/5 * 10 + 2/5 * 5 + 1/5 * 0 = 4 + 2 = \$6$

   (b) Starting over, what is the expectation for X of drawing two balls?
   Answer :

   (c) Suppose you are allowed to keep drawing balls until you draw the yellow ball. (So you will draw at most five times, since the balls are not replaced after each draw.) What is the expectation for X of balls drawn?