CMPE 16 Homework # 2

John Allard, id:1437547

October 12th, 2014

1. *Give the set represented by each of the expressions below where* $A_1 = \{\square, 2, 8, a, g\}$, $A_2 = \{\triangle, -2, 8, a\}$, $A_3 = \{\square, 12, 7, a, g\}$, *and* $A_4 = \{\square, \triangle, 2, 7, a, b, g\}$. *List each element in the set only once (i.e.* $\{1, 2\}$ *instead of* $\{1, 2, 2\}$*).*

   (a) $A_1 \cup A_2$
   Answer : $A_1 \cup A_2 = \{\square, 2, 8, a, g, -2, \triangle\}$

   (b) $A_3 \cap A_4$
   Answer : $A_3 \cap A_4 = \{\square, a, 7, g\}$

   (c) $A_4 - A_1$
   Answer : $A_4 - A_1 = \{\triangle, 7, b\}$

   (d) $A_1 - A_4$
   Answer : $A_1 - A_4 = \{8\}$

   (e) $\bigcup_{i=1}^{4} A_i$
   Answer : $\bigcup_{i=1}^{4} A_i = \{\square, 2, 8, a, g, \triangle, -2, 12, b\}$

   (f) $\bigcap_{i=1}^{4} A_i$
   Answer : $\bigcap_{i=1}^{4} A_i = \{a\}$

2. *For each of the sets below fill in the corresponding regions of a general Venn diagram for 3 sets. (The Venn diagram should have 3 sets in each case.)* **Answers on last page.**

   (a) $A \cup \overline{B} \cup C$

   (b) $C - (A \cap B)$

   (c) $\overline{(B - C) \cup A}$

3. *Write* $\bigcup_{i \in \mathbb{Z}} (i, i+1)$ *as the difference of two well known sets. Here* $(i, i+1)$ *is the open interval of the real line with endpoints* i *and* $i+1$*. (That is,* $(i, i+1) = x \in \mathbb{R} : i < x < i + 1$*).*

   Answer : The set $Y = \bigcup_{i \in \mathbb{Z}} (i, i + 1)$ can be written as the difference of $\mathbb{R}$ and $\mathbb{N}$; $Y = \mathbb{R} - \mathbb{N}$. This is because Y includes all numbers in $\mathbb{R}$ up-to but not including the actual integer values in $\mathbb{Z}$

4. Using only the symbols 4, Z, S, P, W, $\emptyset, \subseteq, \in, \cup, \cap$, -, =, {, },),(, and $\neq$, express the following statements

   (a) 4 is pale and shy
   Answer : $4 \in (P \cap S)$

   (b) All worried integers are pale.
   Answer : $W \subseteq P$

   (c) Every integer is shy, worried, or pale.
   Answer : $\mathbb{Z} \subseteq (P \cup S \cup W)$

(d) There are worried integers that are not shy.
Answer : $(W - S) \neq \emptyset$

5. Let P and Q be the statements
P I eat garlic.
Q I go to the dentist.
Rewrite each of the statements below using P and Q and logical connectives ($\neg, \wedge, \vee, \implies$).

   (a) I don't eat Garlic
   Answer : $\neg P$

   (b) I don't go to the Dentist, but I eat garlic.
   Answer : $\neg Q \wedge P$

   (c) I eat garlic or I don't go to the dentist.
   Answer : $P \vee \neg Q$

   (d) Whenever I go to the dentist, I don't eat garlic.
   Answer : $Q \implies \neg P$

6. Let P, Q and R be the statements
P I use plastic bags.
Q I use paper bags.
R I help the environment.

   (a) $\neg P$

   (b) $P \wedge Q$

   (c) $Q \implies \neg R$

   (d) $\neg(P \implies R)$

7. Use a truth table to determine the values of each of the logical expressions below. Both of your truth tables should have at least 3 intermediate columns.

   (a) $\neg(P \vee Q) \wedge \neg Q$

   (b) $(P \vee Q) \wedge (Q \vee R) \wedge \neg(P \wedge R)$

8. Convert each of the following statements into the form If P then Q without changing their meanings. (Some of these statements might not be True and thats okay.)

   (a)

9. As discussed in class, given a finite set S of size n and an ordering s1, s2, . . . , sn of the n elements in S, we can represent the subsets of S using bit vectors of length n (0, 1 n). For a subset A S, the corresponding bit vector b(A) = (b1, b2, . . . , bn) where bi = 1 if si A and bi = 0 if si 6 A. Let S be the elements from the four sets in Problem 1 ordered as , 4, 2, 2, 7, 8, 12, a, b, g.

   (a)

10. Given the bit vectors b(B) = (b1, b2, . . . , bn) and b(D) = (d1, d2, . . . , dn) representing two subsets B and D. In each case below explain how you would calculate the required bit vector (in general), and then apply your method to obtain the result for B = A1 and D = A4 from Problem 1.

    (a)

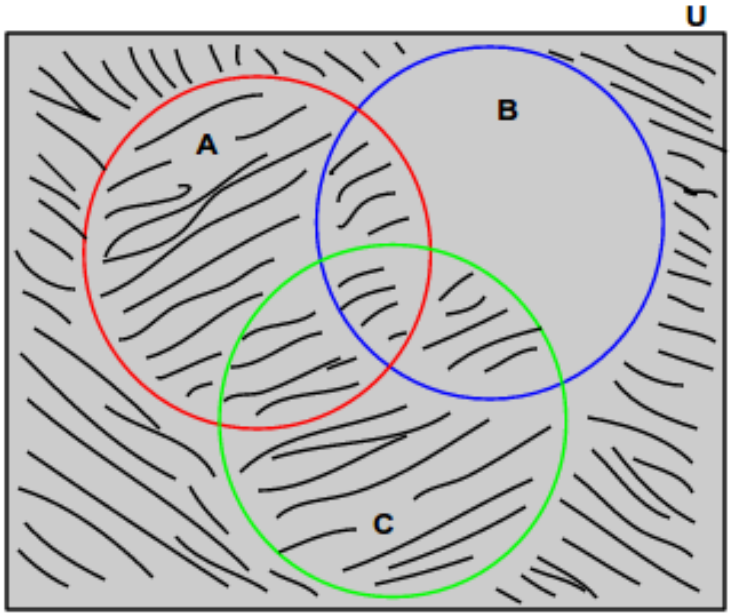

Figure 1: Venn Diagram for Problem 2a

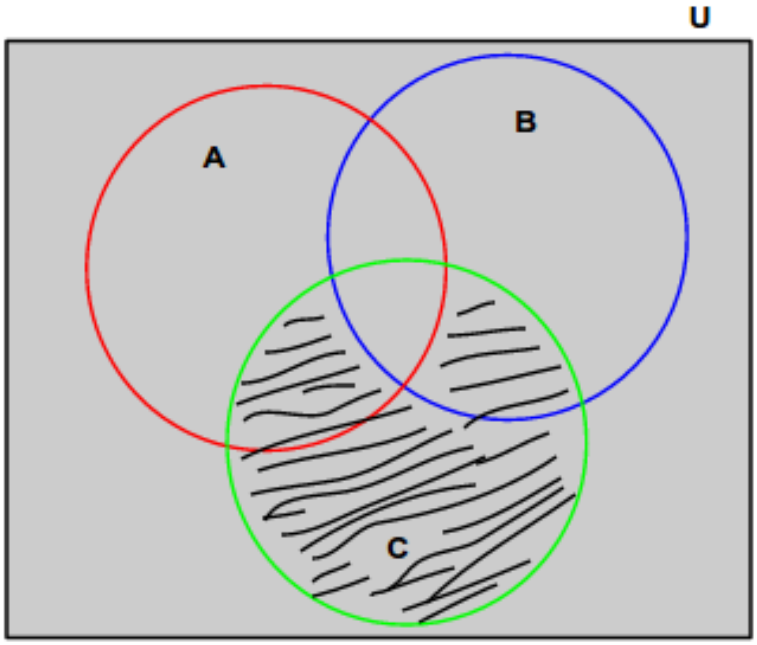

Figure 2: Venn Diagram for Problem 2b

Figure 3: Venn Diagram for Problem 2c