Content

- Non linearity induced by H operator :
  $(Hx = (1 - \alpha)x + \alpha x^3)$.
- Number of outer loops vs. inner loops.
- Non linearity induced by changing the resolution at outer loop level.
- Varying the projective B matrix option.
- Varying the interpolation method.

Non linearity induced by H operator :

Full resolution, varying α parameter with the same relinearization scheme : $no = 4, ni = 6$,
spectral interpolation and projective B matrix, $\sigma^o = 0.01$

Full resolution ; linear H ; J vs J^{nl}

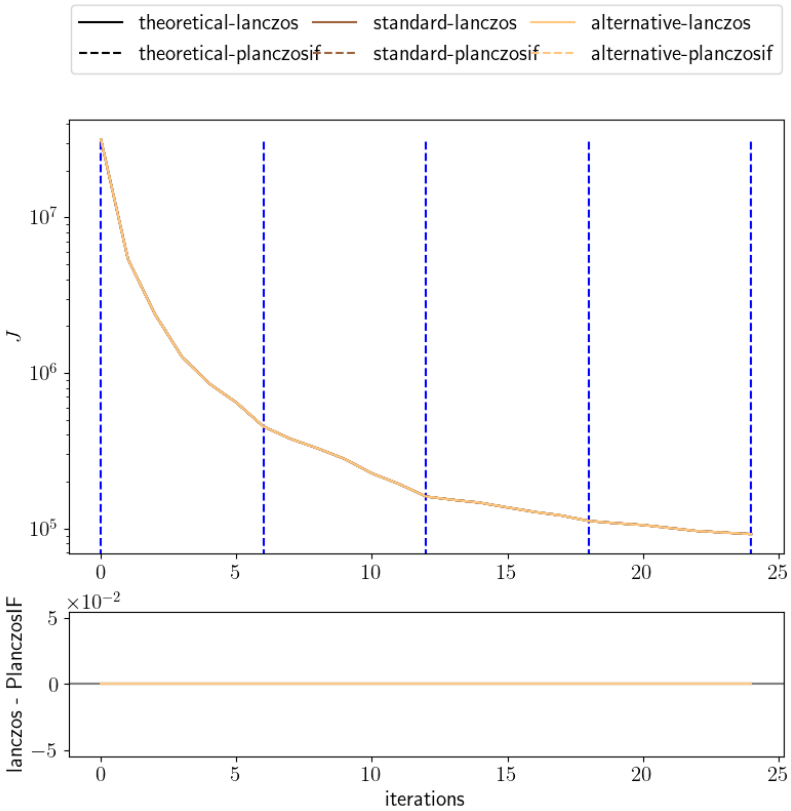

FIGURE – $\alpha = 0$

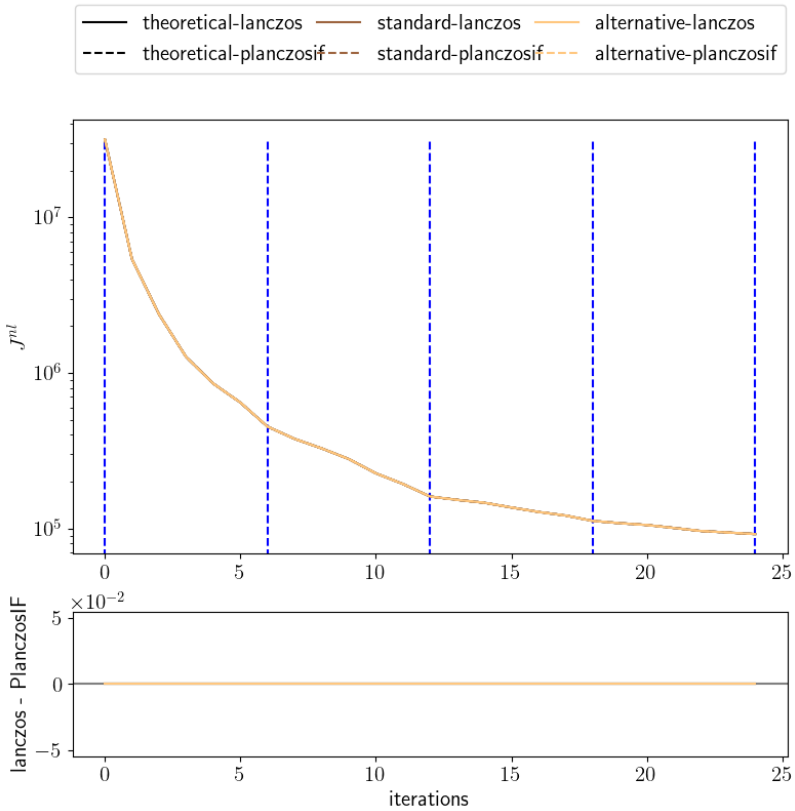

FIGURE – $\alpha = 0$

Full resolution ; non linear H ; J vs J^{nl}

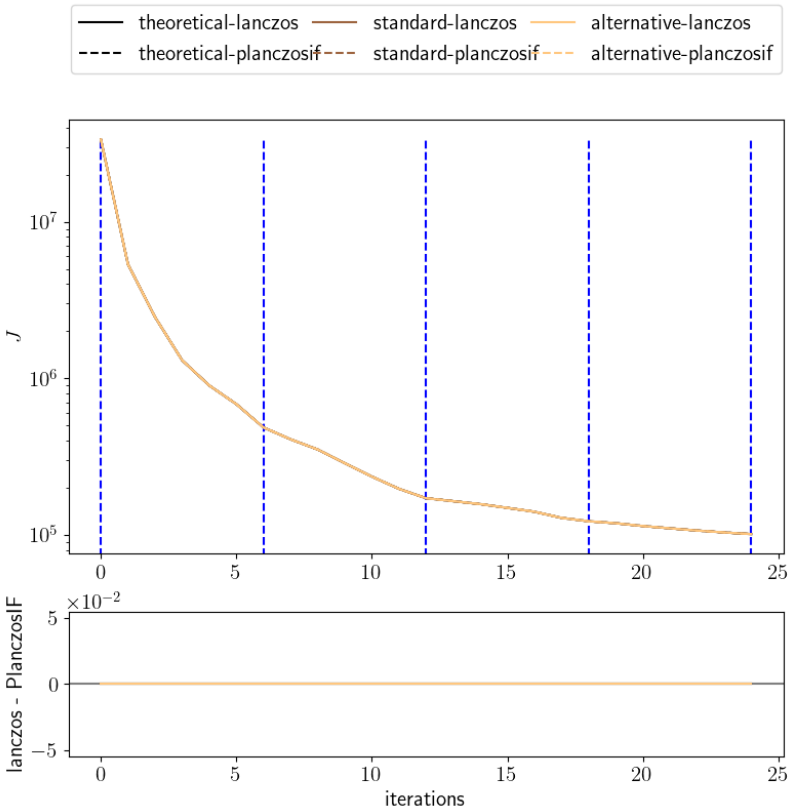

FIGURE – $\alpha = 0.01$

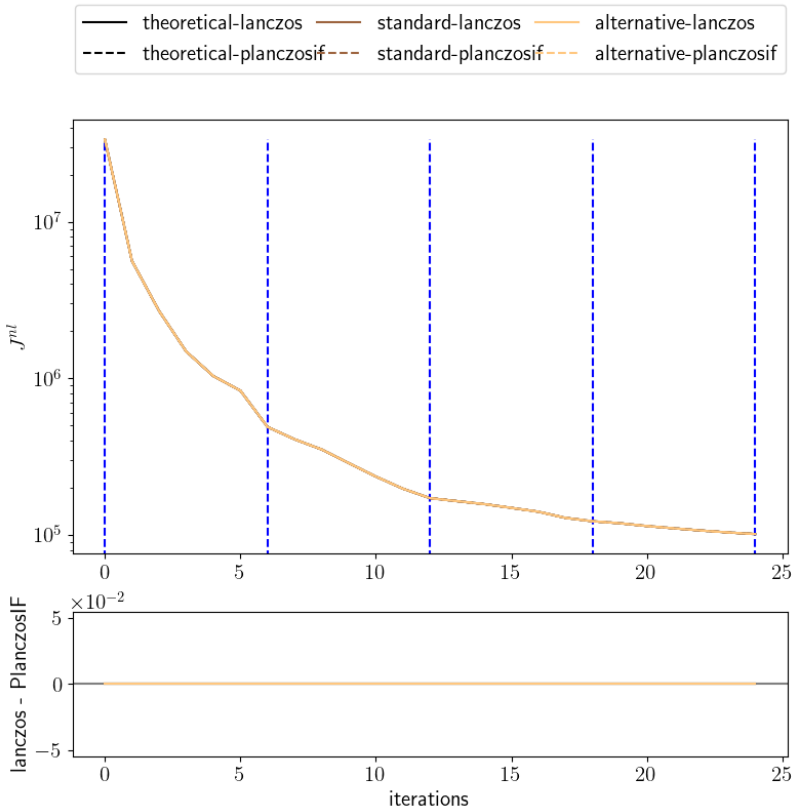

FIGURE – $\alpha = 0.01$

Full resolution ; non linear H ; J vs J^{nl}

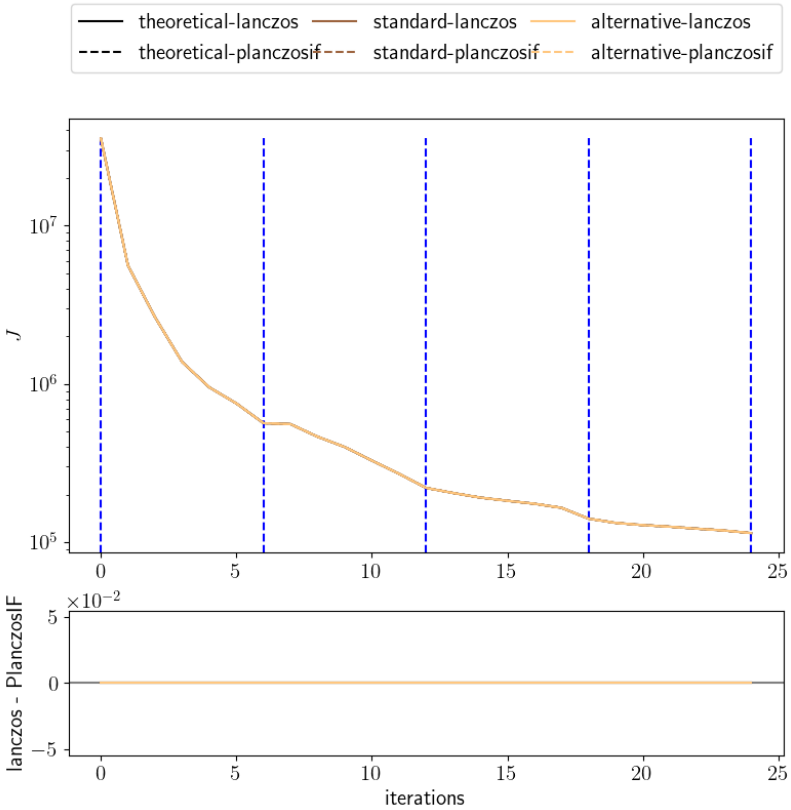

FIGURE – $\alpha = 0.02$

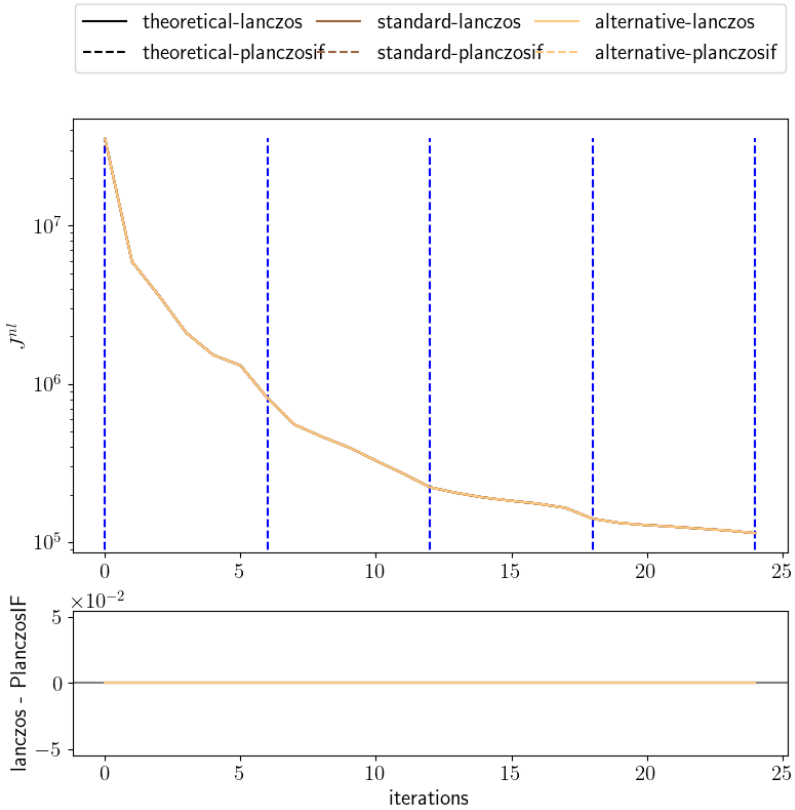

FIGURE – $\alpha = 0.02$

Full resolution ; non linear H ; J vs J^{nl}

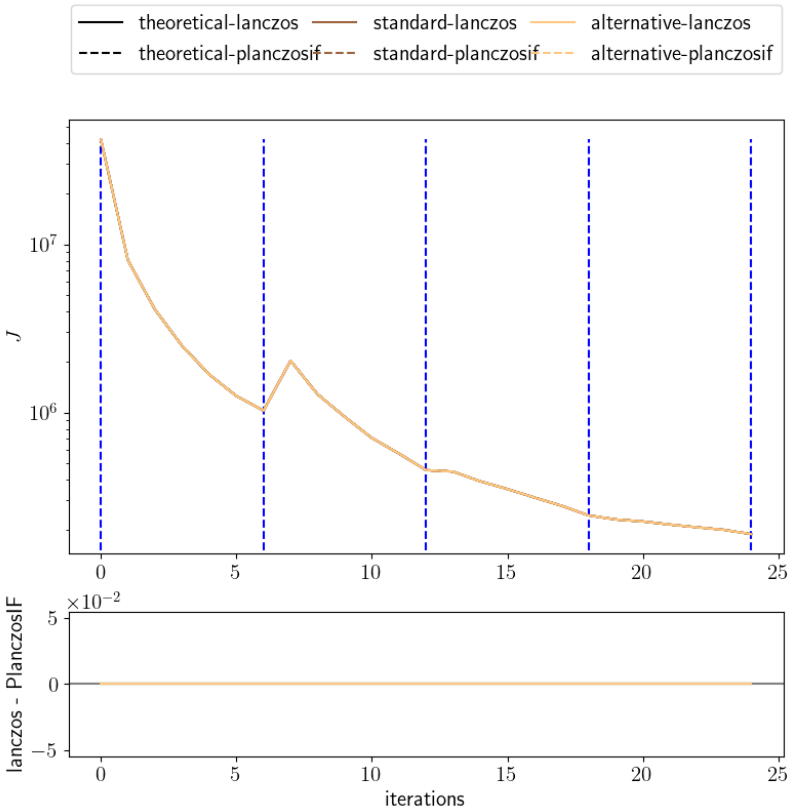

FIGURE – $\alpha = 0.05$

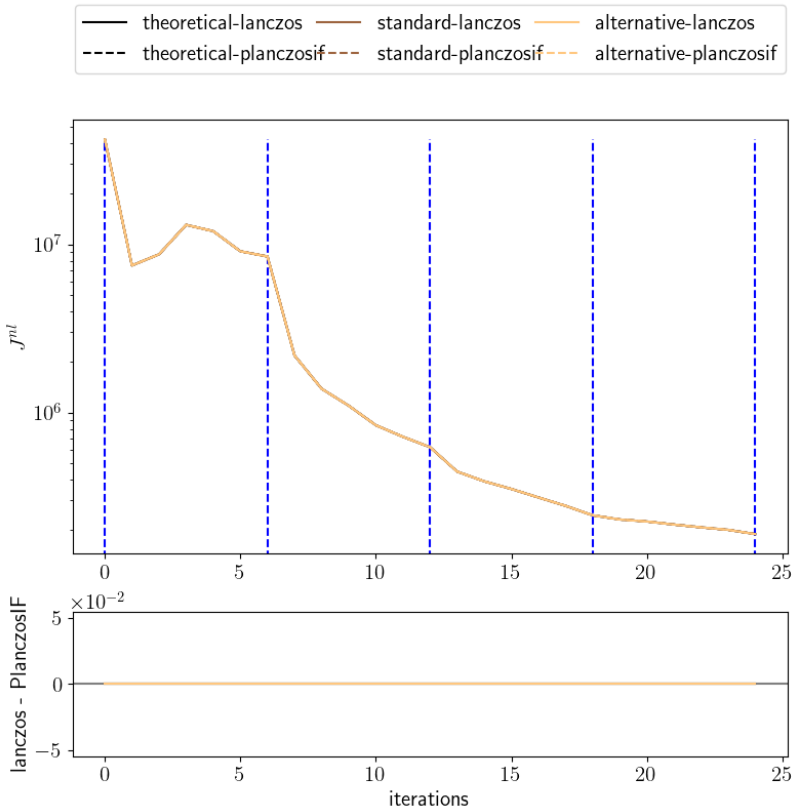

FIGURE – $\alpha = 0.05$

Full resolution ; non linear H ; J vs J^{nl}

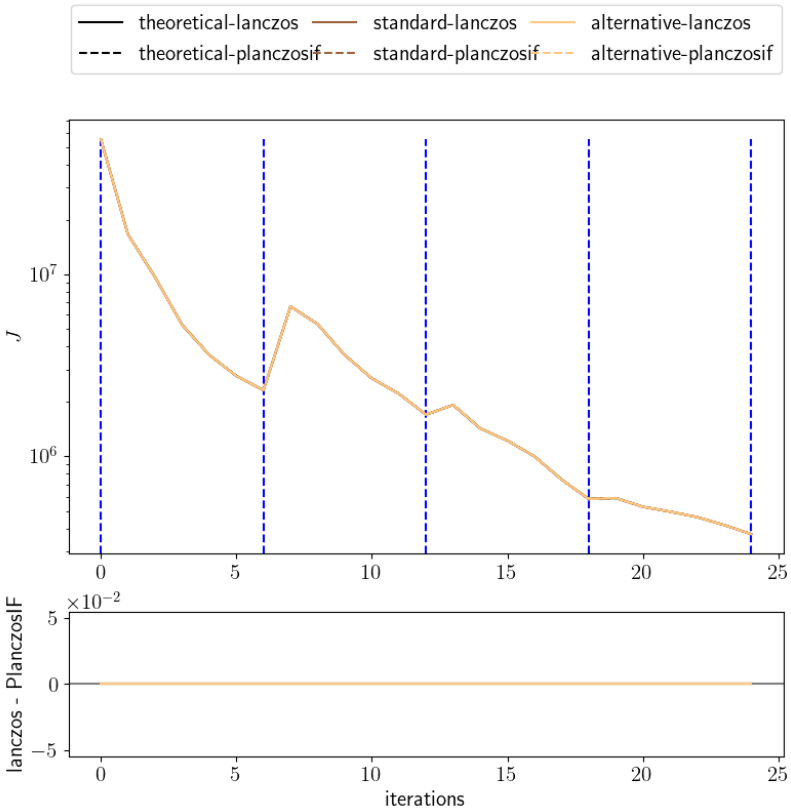

FIGURE – $\alpha = 0.1$

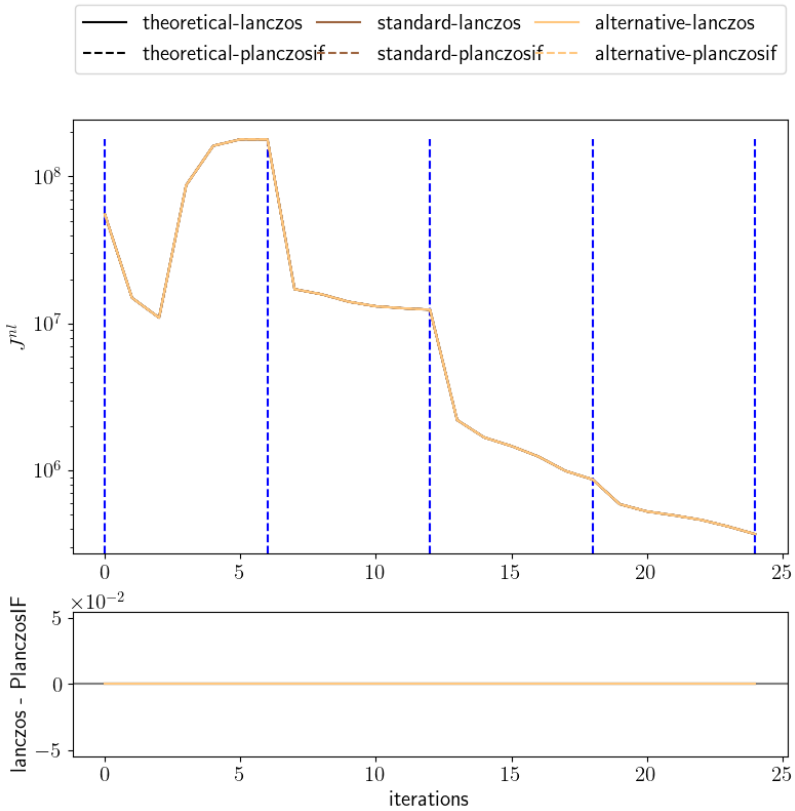

FIGURE – $\alpha = 0.1$

Full resolution ; non linear H ; J vs J^{nl}

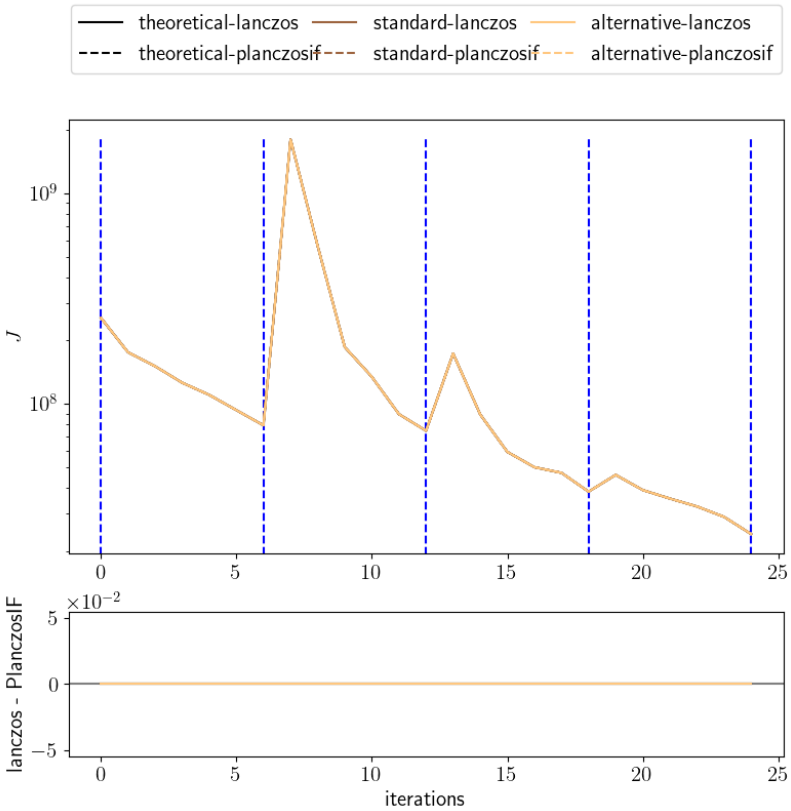

FIGURE – $\alpha = 0.5$

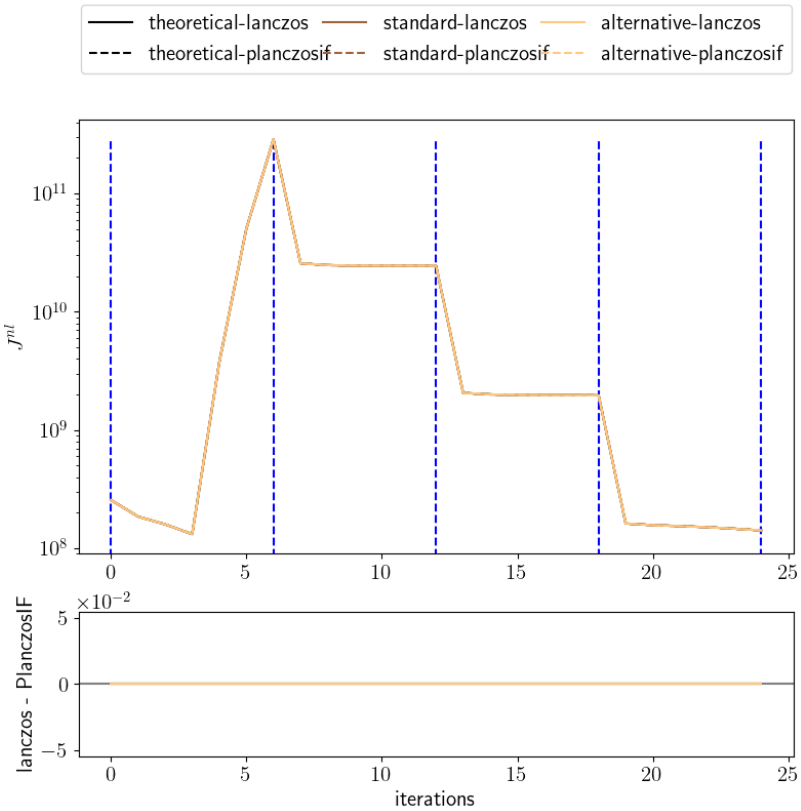

FIGURE – $\alpha = 0.5$

Full resolution ; non linear H ; J vs J^{nl}

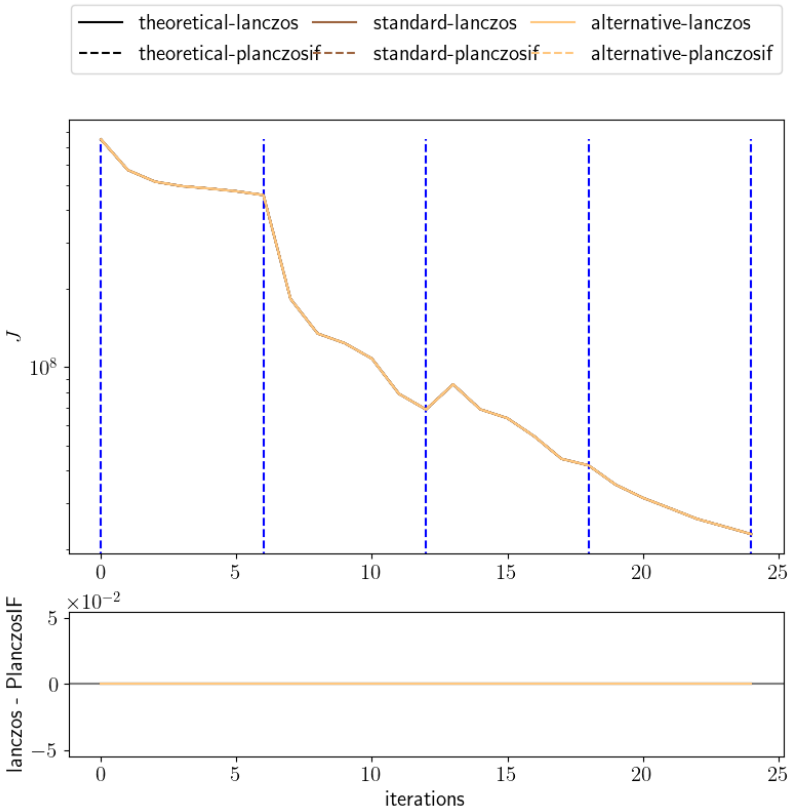

FIGURE – $\alpha = 1$

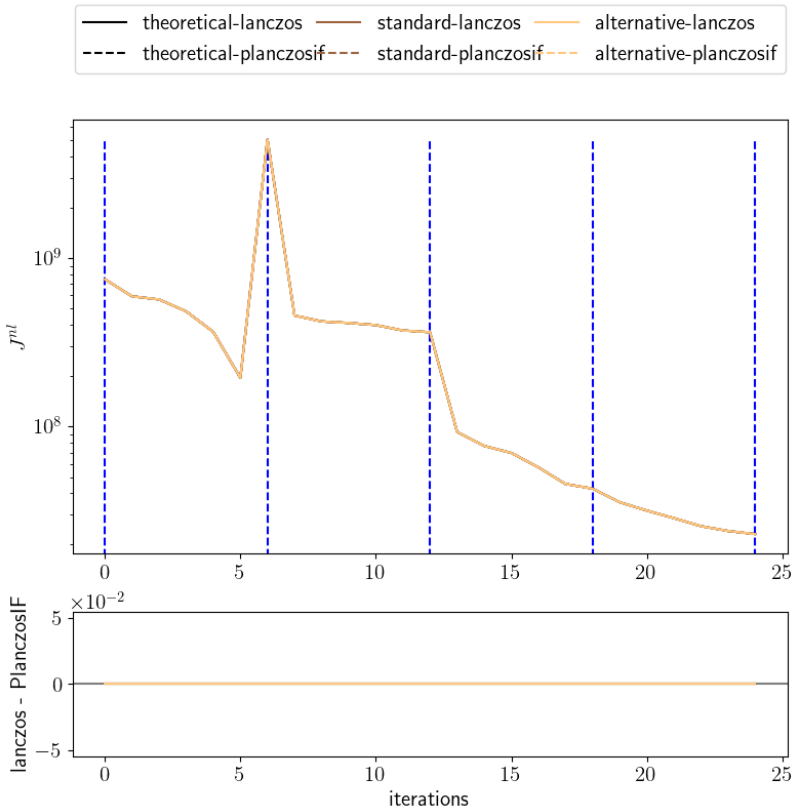

FIGURE – $\alpha = 1$

Conclusion on the non linearity induced by H

- Very sensitive to α even for small values.
- The case with $\alpha = 1$ seems better than the case with $\alpha = 0.5$...
  $\longrightarrow$ What could be the reason for it ?
- It seems that there are too much inner loops before relinearization but the iteration at which the "jump" occurs seems NOT correlated to the value of α.

$\longrightarrow$ Need to study the number of inner iterations vs. outer iterations.

Full resolution, varying the number of inner and outer loops with a non linear H and the same total number of iterations ($n_o \times n_i = 24$) (spectral interpolation and projective B matrix, $\sigma^o = 0.01$)

Full resolution ; non linear H ($\alpha = 0.05$) : J^{nl}

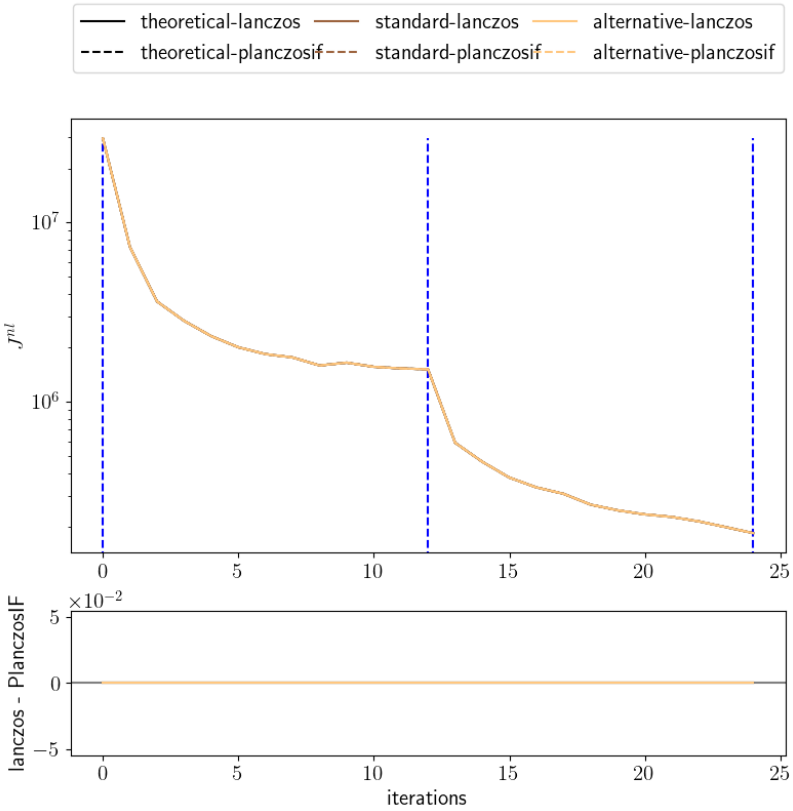

FIGURE – $n_o = 2, n_i = 12$

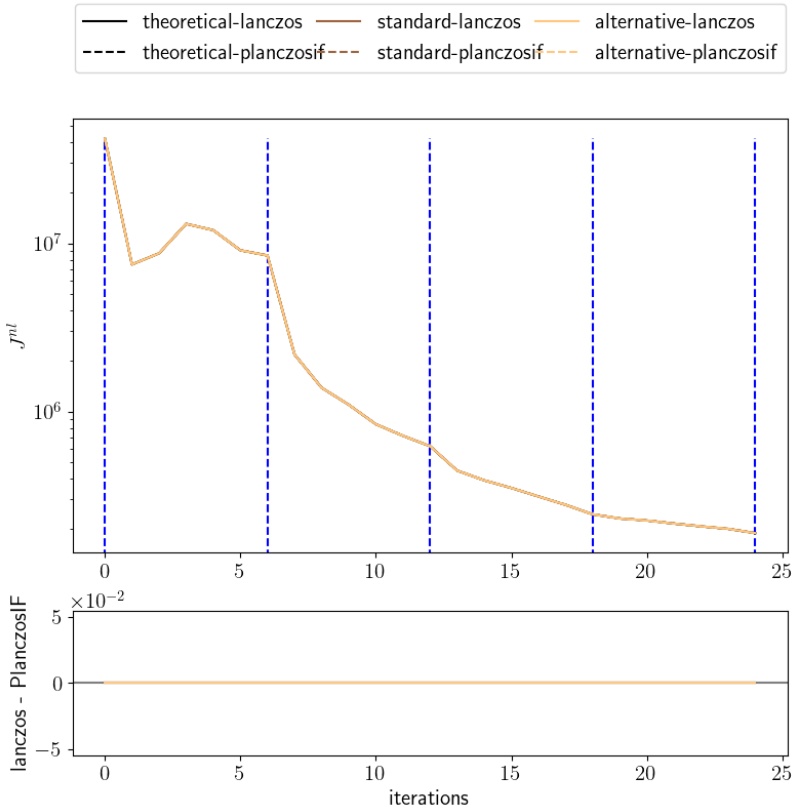

FIGURE – $n_o = 4, n_i = 6$

Full resolution ; non linear H ($\alpha = 0.05$) : J^{nl}

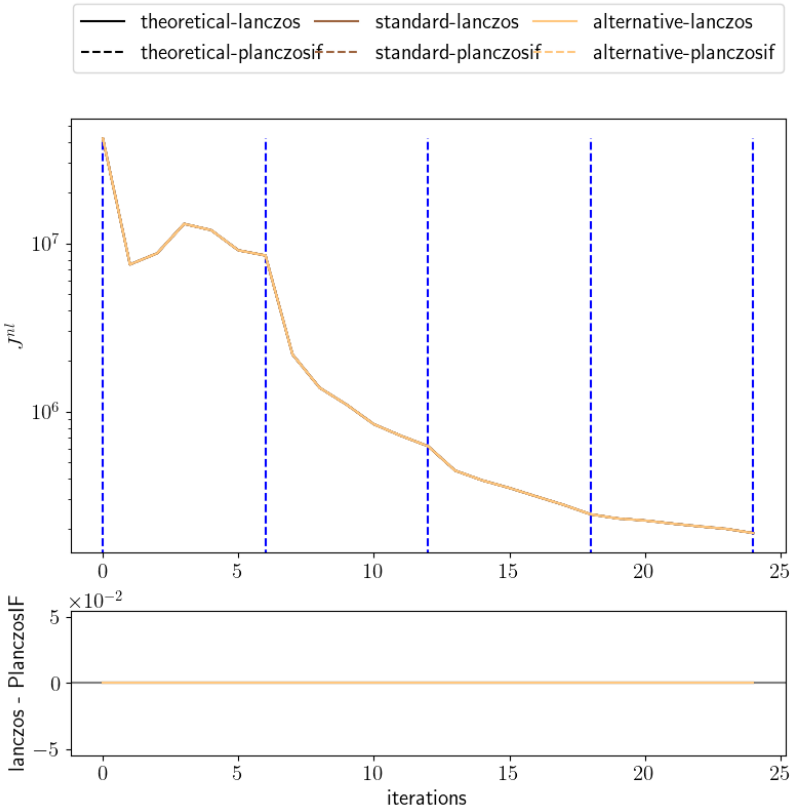

FIGURE – $n_o = 4, n_i = 6$

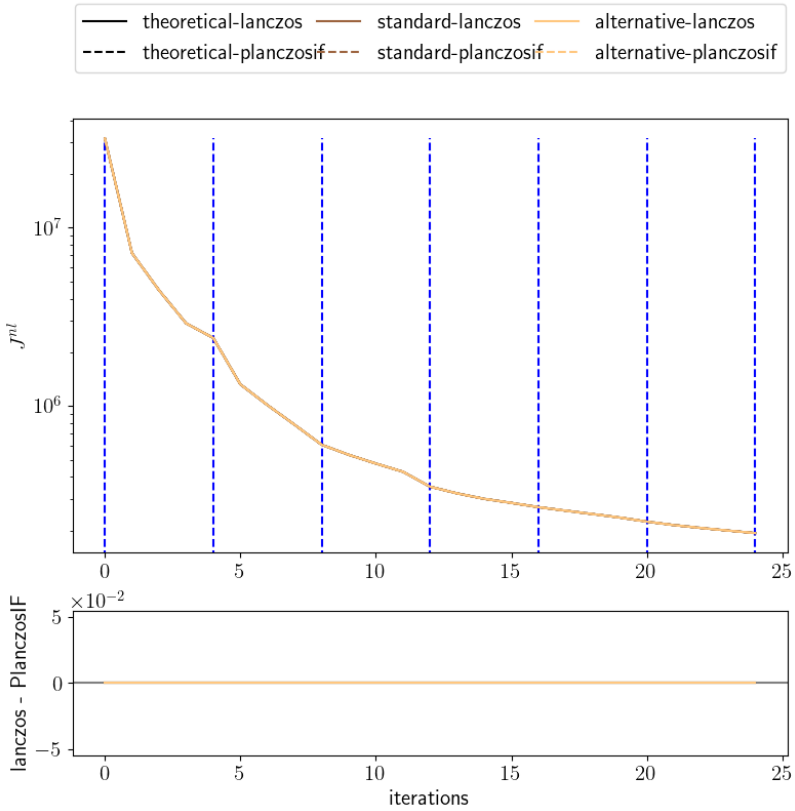

FIGURE – $n_o = 6, n_i = 4$

Full resolution ; non linear H ($\alpha = 0.05$) : J

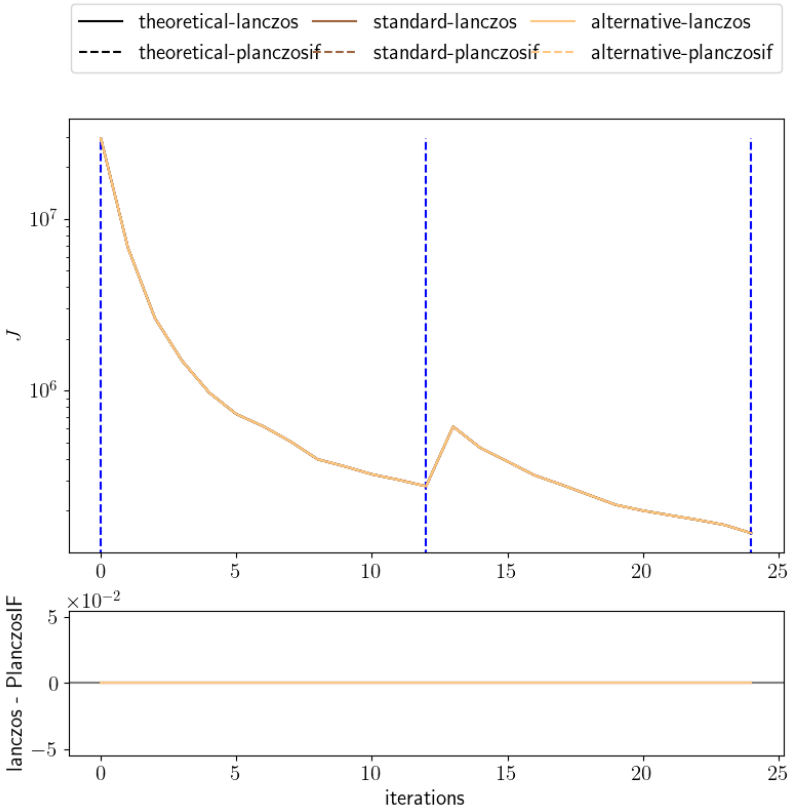

FIGURE – $n_o = 2, n_i = 12$

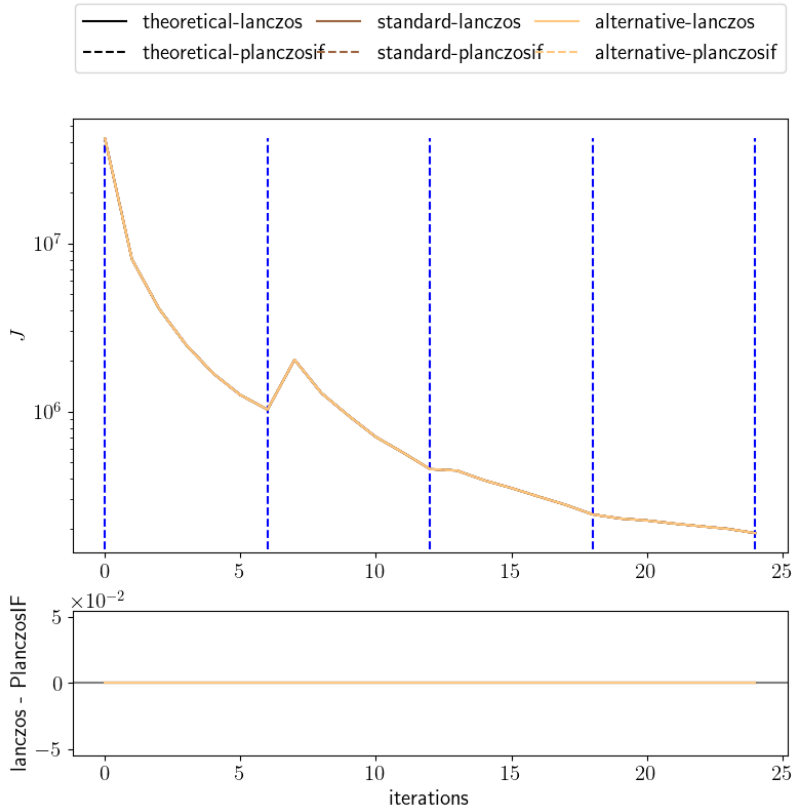

FIGURE – $n_o = 4, n_i = 6$

Full resolution ; non linear H ($\alpha = 0.05$) : J

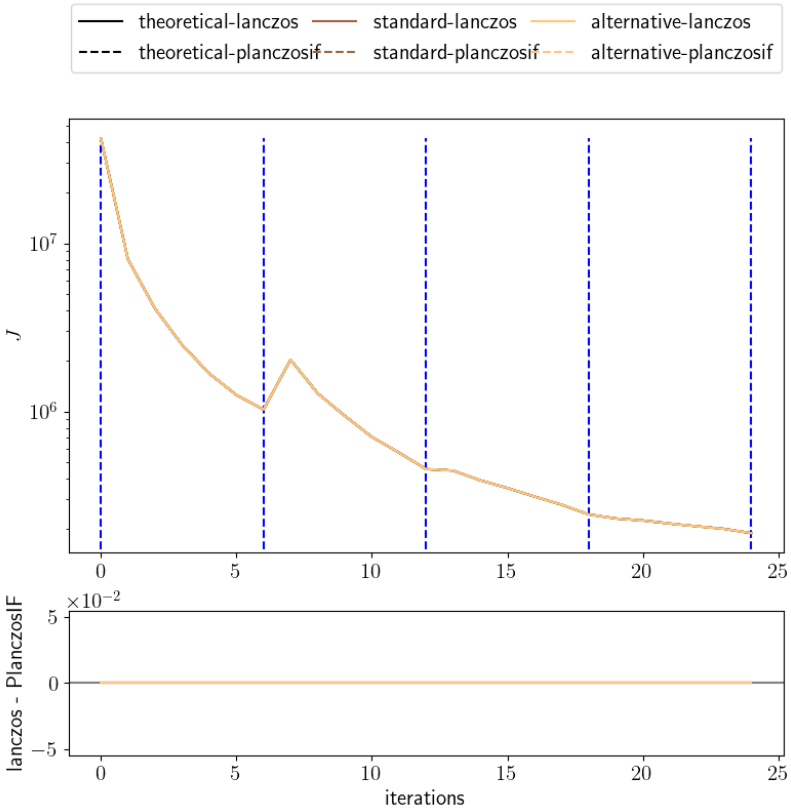

FIGURE – $n_o = 4, n_i = 6$

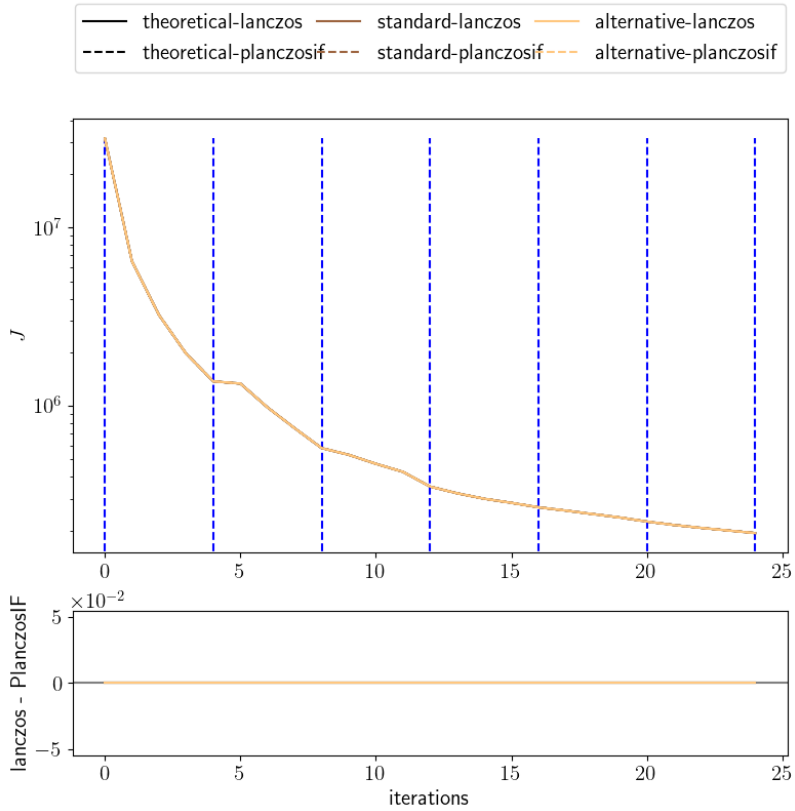

FIGURE – $n_o = 6, n_i = 4$

Full resolution ; non linear H ($\alpha = 0.1$) : J^{nl}

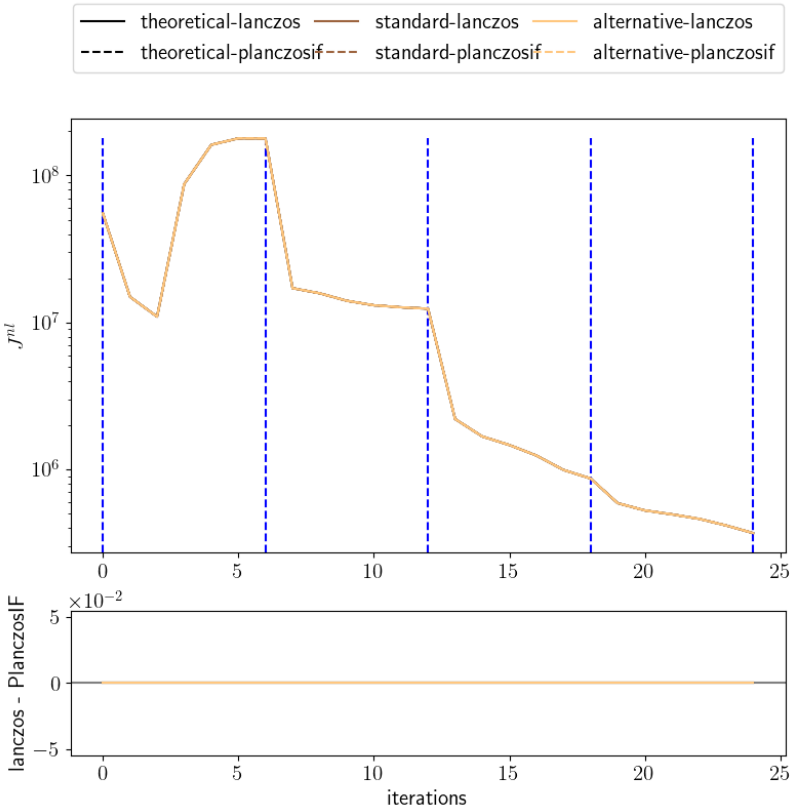

FIGURE – $n_o = 4, n_i = 6$

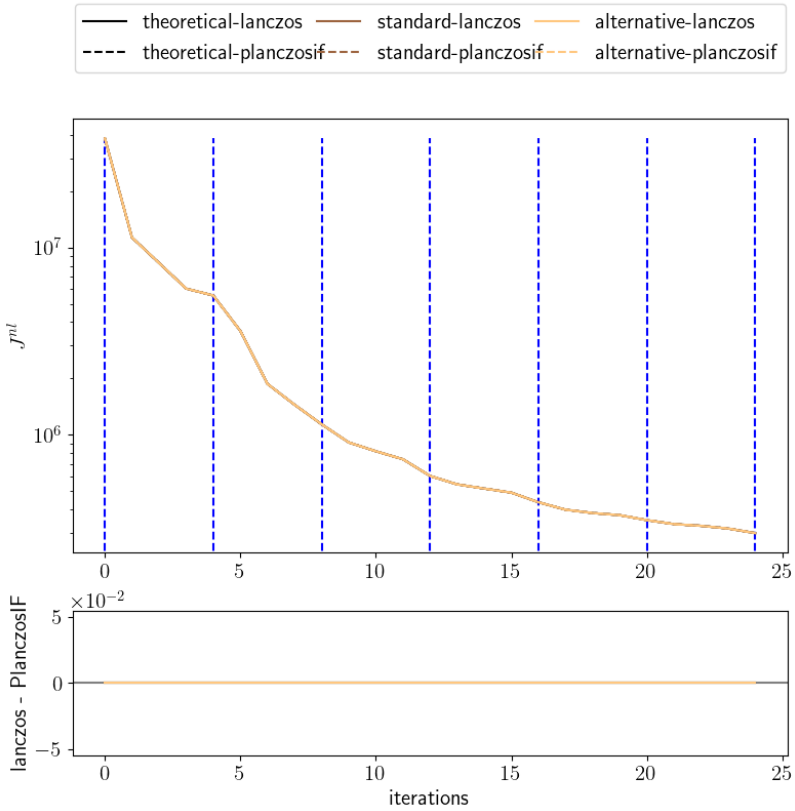

FIGURE – $n_o = 6, n_i = 4$

Full resolution ; non linear H ($\alpha = 0.1$) : J

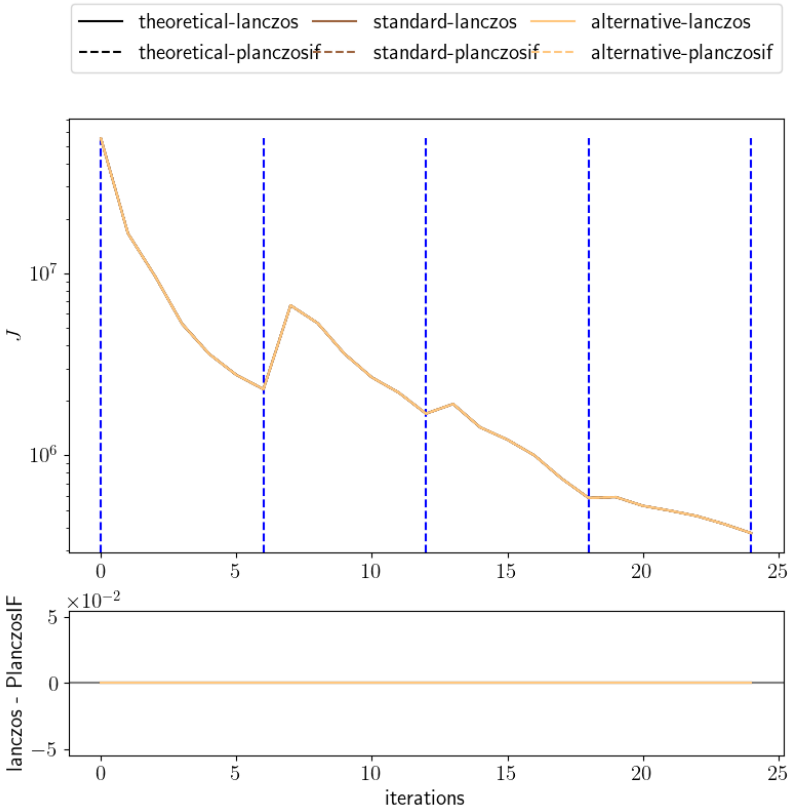

FIGURE – $n_o = 4, n_i = 6$

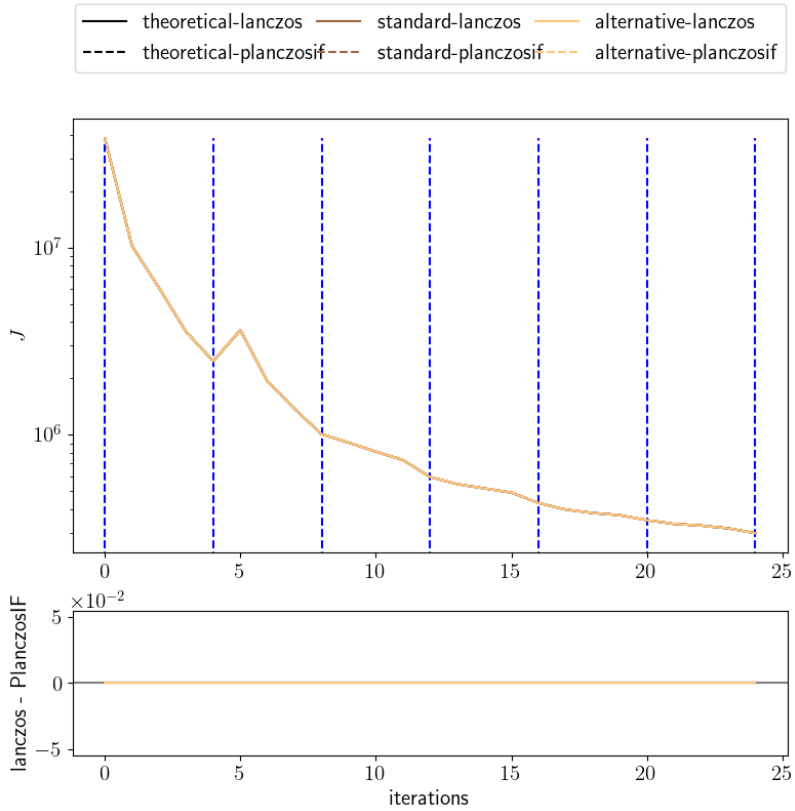

FIGURE – $n_o = 6, n_i = 4$

Full resolution ; non linear H ($\alpha = 1$) : J^{nl}

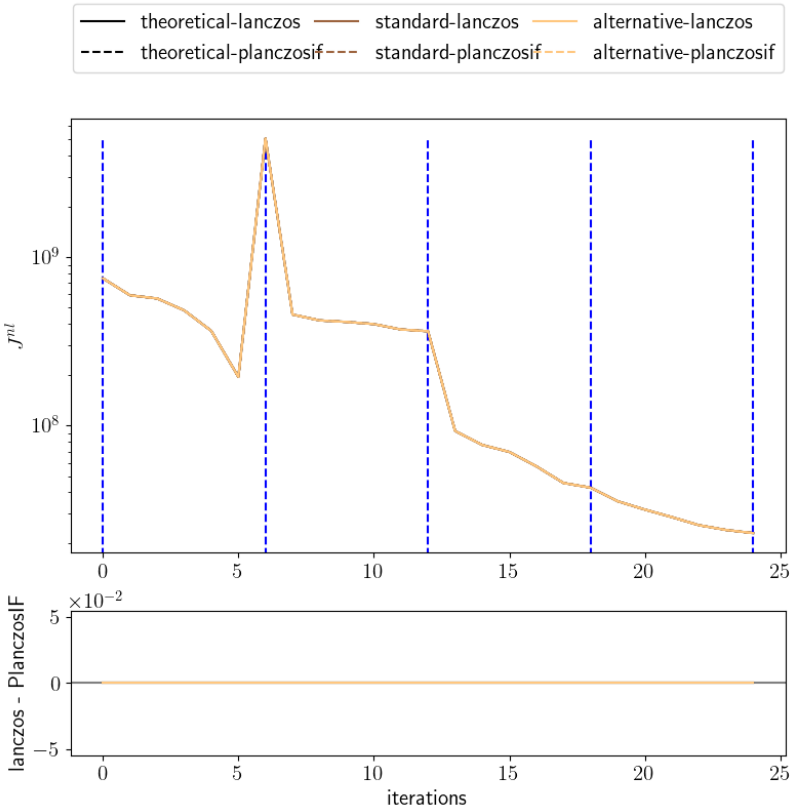

FIGURE – $n_o = 4, n_i = 6$

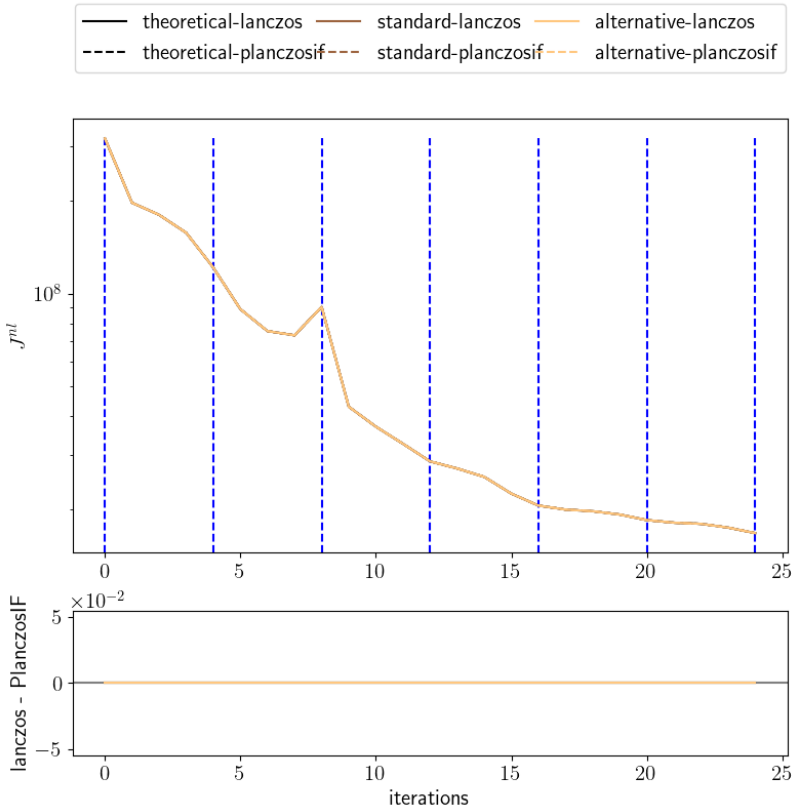

FIGURE – $n_o = 6, n_i = 4$

Full resolution ; non linear H ($\alpha = 1$) : J

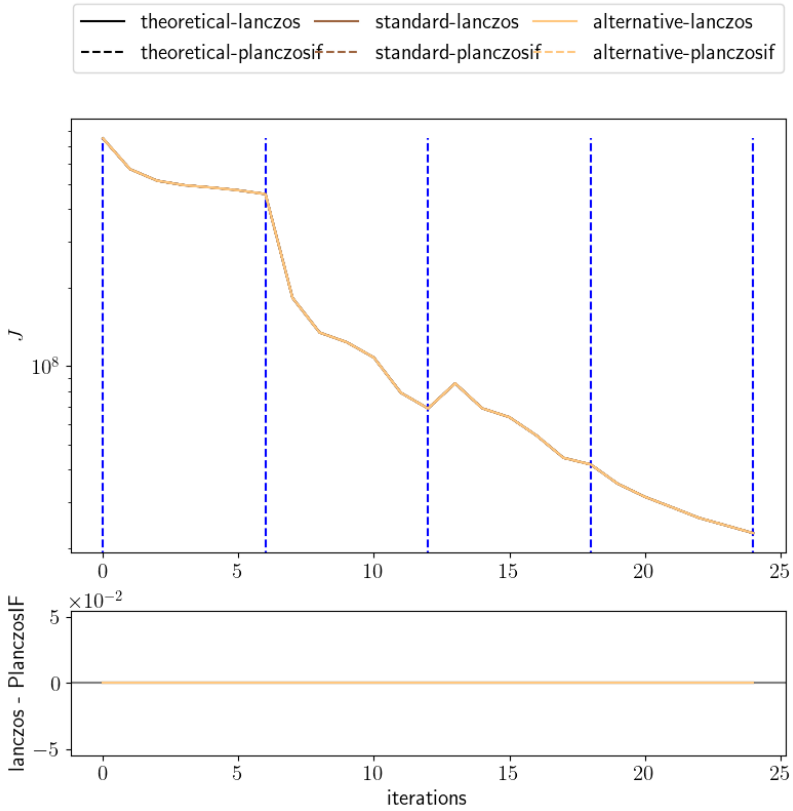

FIGURE – $n_o = 4, n_i = 6$

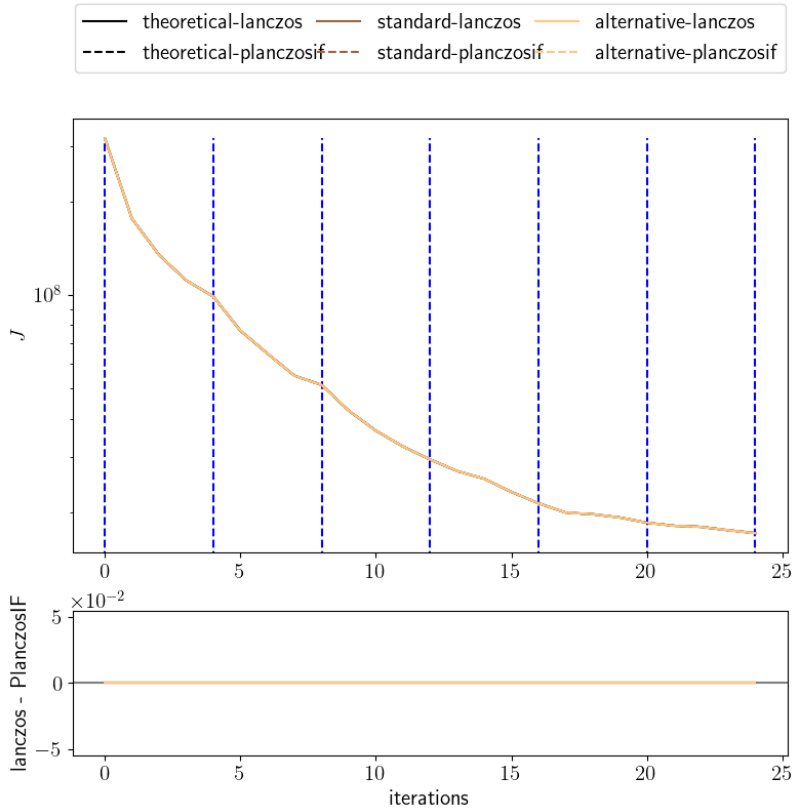

FIGURE – $n_o = 6, n_i = 4$

Conclusion on the number of inner and outer loops

- As expected, the assimilation scheme with the more outer loops is equal or better than the others.
- There is often a "jump" in the linear cost functions just after relinearization, BUT it seems not necessarily correlated to the behaviour of the non linear cost function (or it is not trivial).
- Problem : The first inner iterations in the first case with 12 inner iterations seems better than the case with 6 inner iterations whereas the case with 4 inner iterations seems better than the case with 6 inner iterations :
  $\longrightarrow$ The problem is too much dependant on the initial background and observation states that are randomly generated (?) : there is a difference of $10^7 - 10^8$ in the cost function at the beggining in these cases !

Non linearity induced by the change of resolution between the outer loops :

(Linear H operator, spectral interpolation and projective B matrix, $\sigma^o = 0.01$, $no = 2, ni = 12$)

Linear H, spectral interpolation and projective B matrix

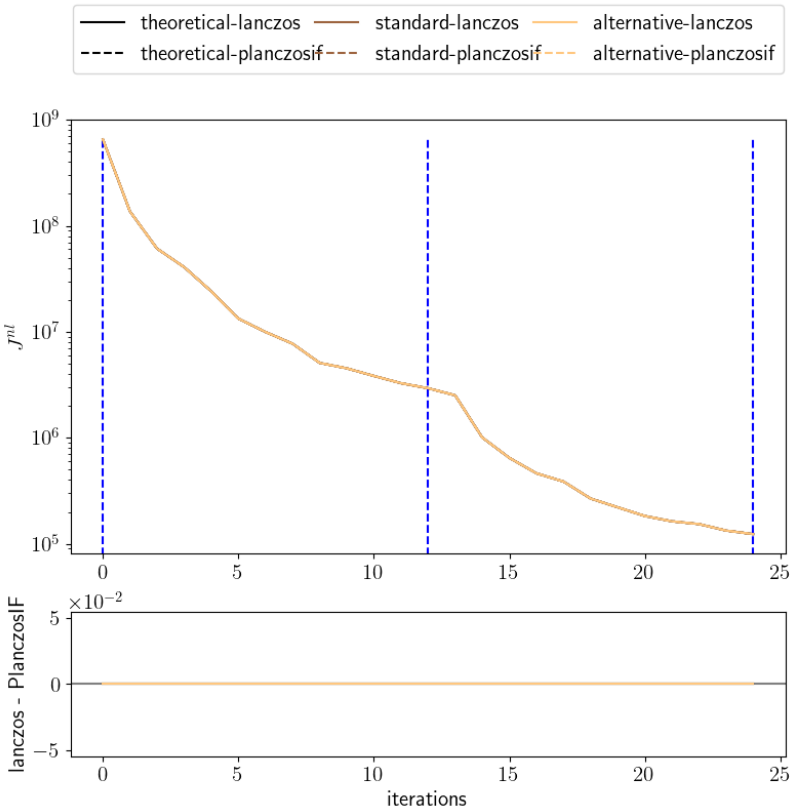

FIGURE – resolutions : 11 > 101

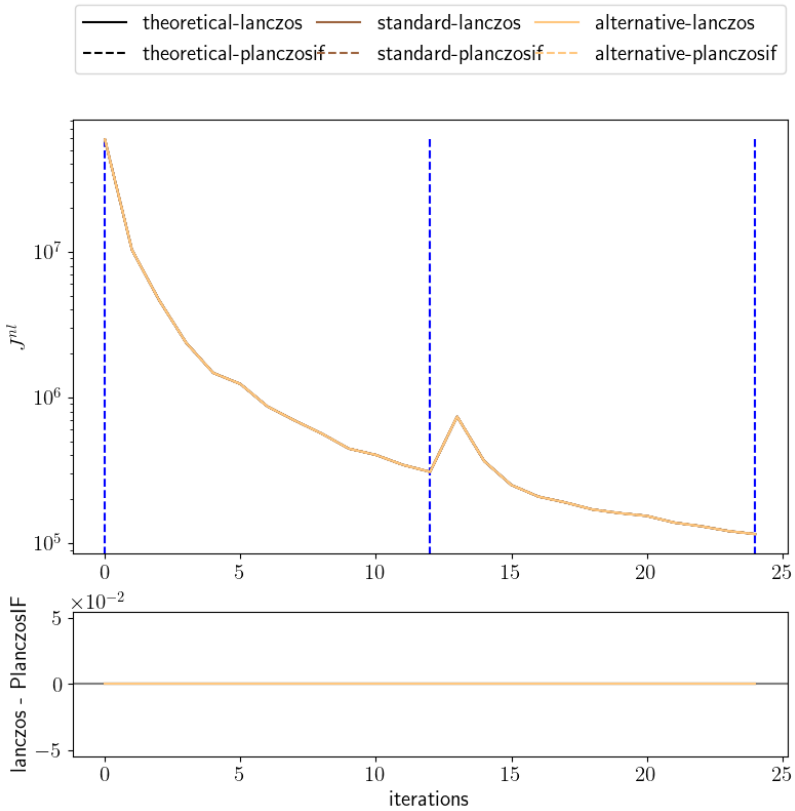

FIGURE – resolutions : 51 > 101

(The jump is also present in the J^o and the residue)

Linear H, spectral interpolation and projective B matrix

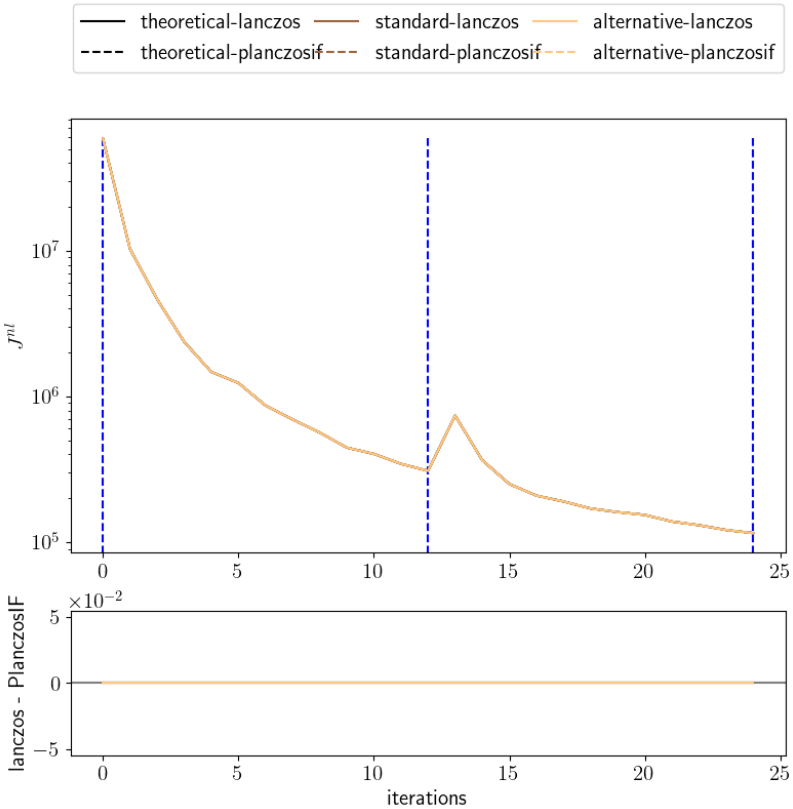

FIGURE – resolutions : 51 $>$ 101

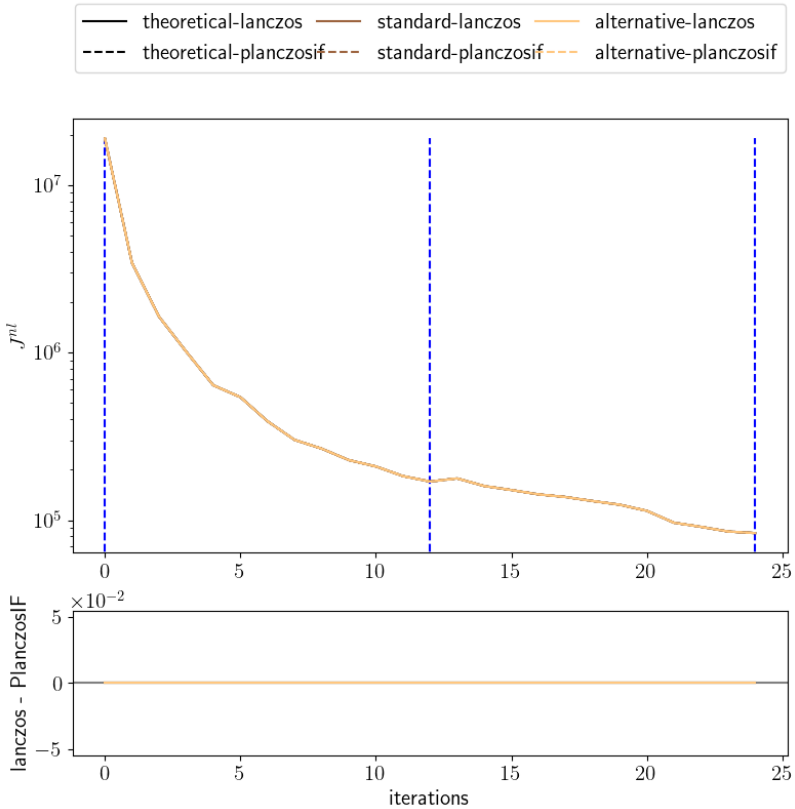

FIGURE – resolutions : 91 $>$ 101

Linear H, spectral interpolation and projective B matrix

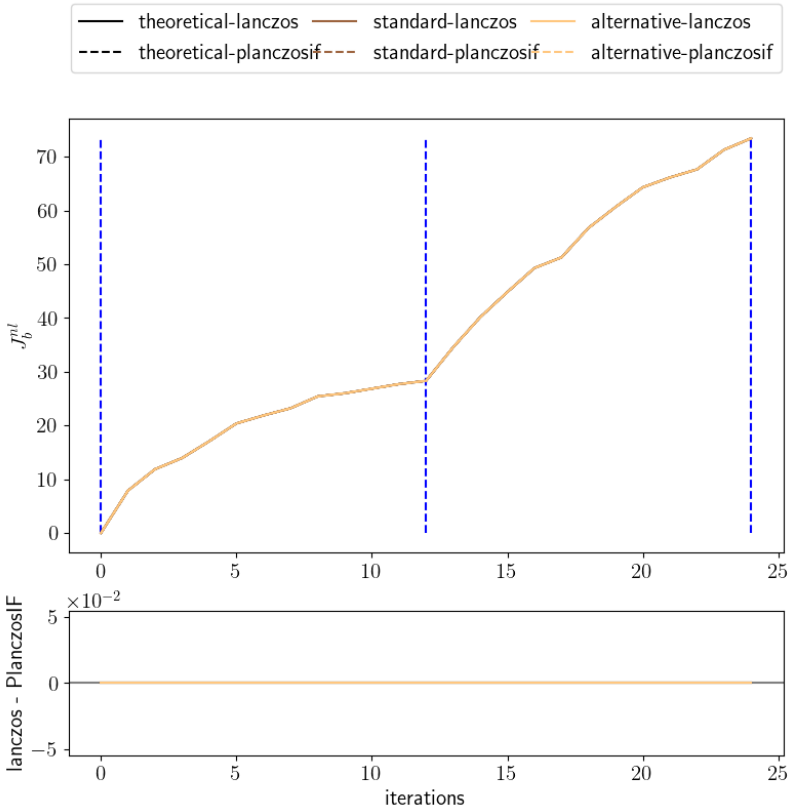

FIGURE – resolutions : 11 > 101

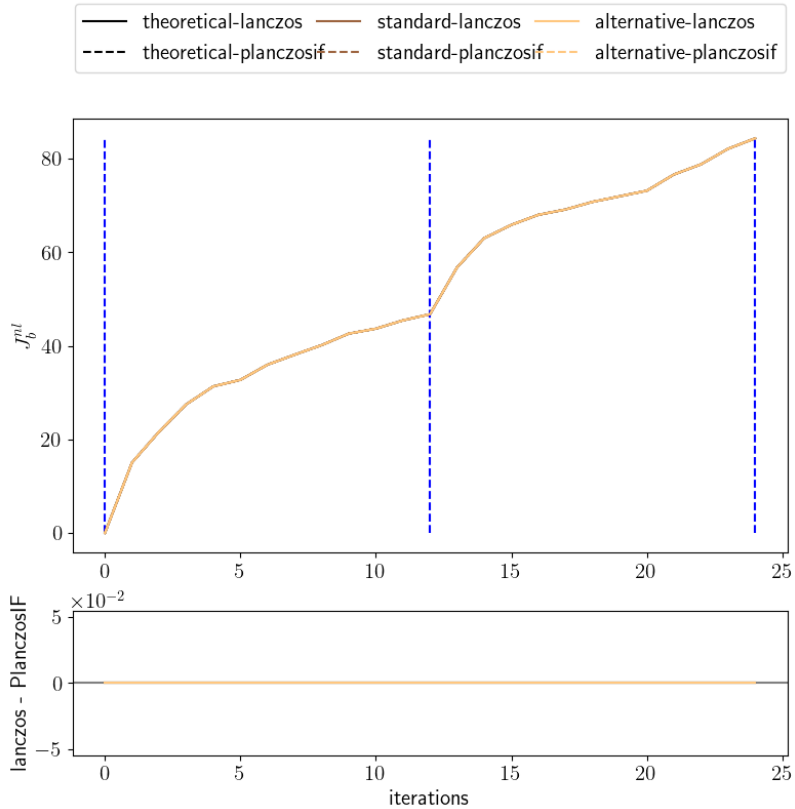

FIGURE – resolutions : 51 > 101

Linear H, spectral interpolation and projective B matrix

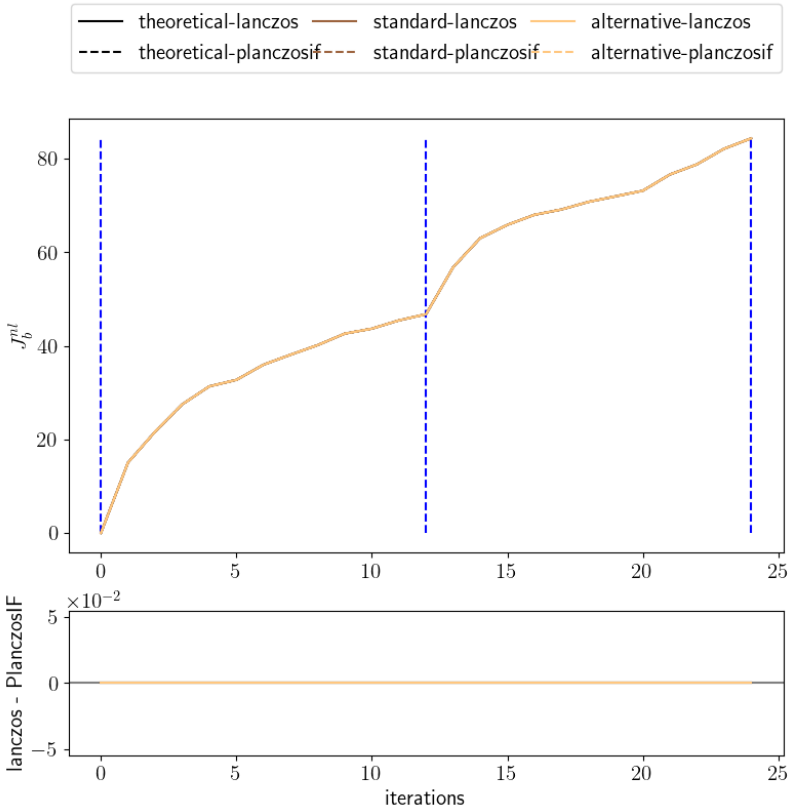

FIGURE – resolutions : 51 $>$ 101

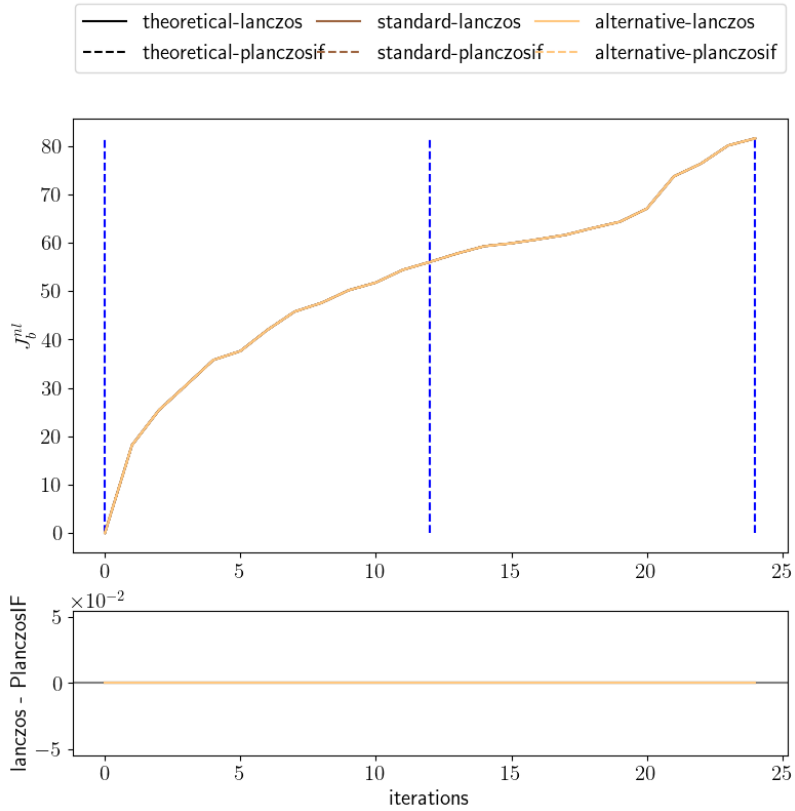

FIGURE – resolutions : 91 $>$ 101

Varying the projective B matrix option :

(Linear H operator, spectral interpolation, $\sigma^o = 0.01$, $no = 2, ni = 12$)

Linear H, spectral interpolation and NON projective B matrix

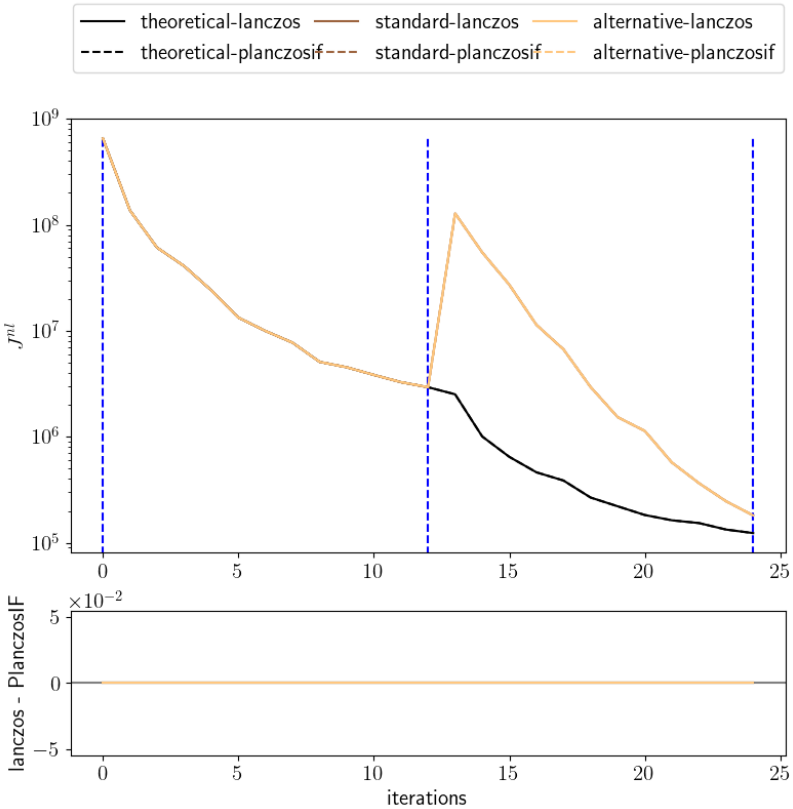

FIGURE – resolutions : 11 > 101

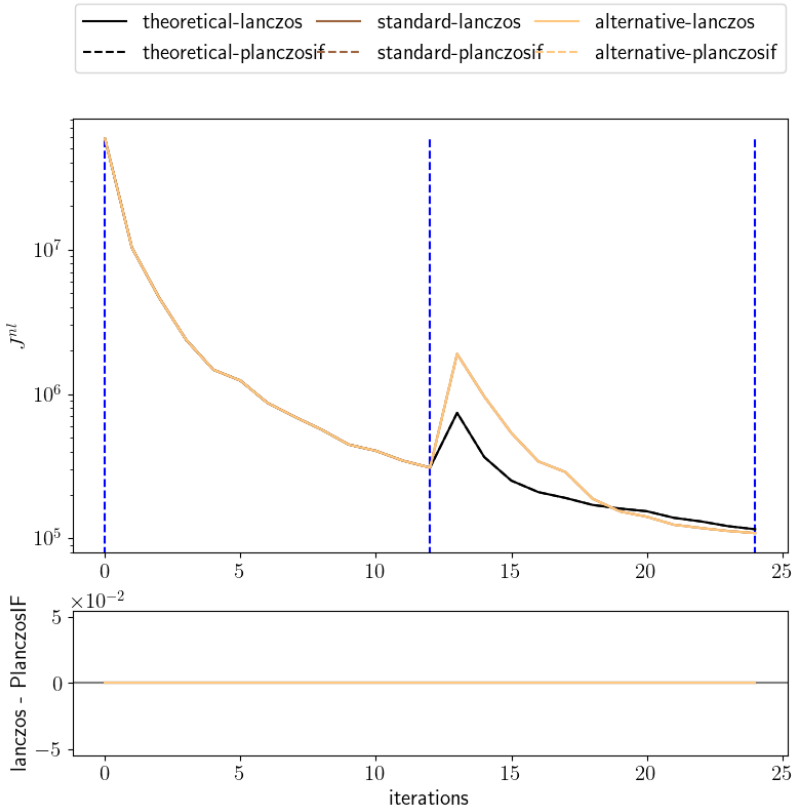

FIGURE – resolutions : 51 > 101

Linear H, spectral interpolation and NON projective B matrix

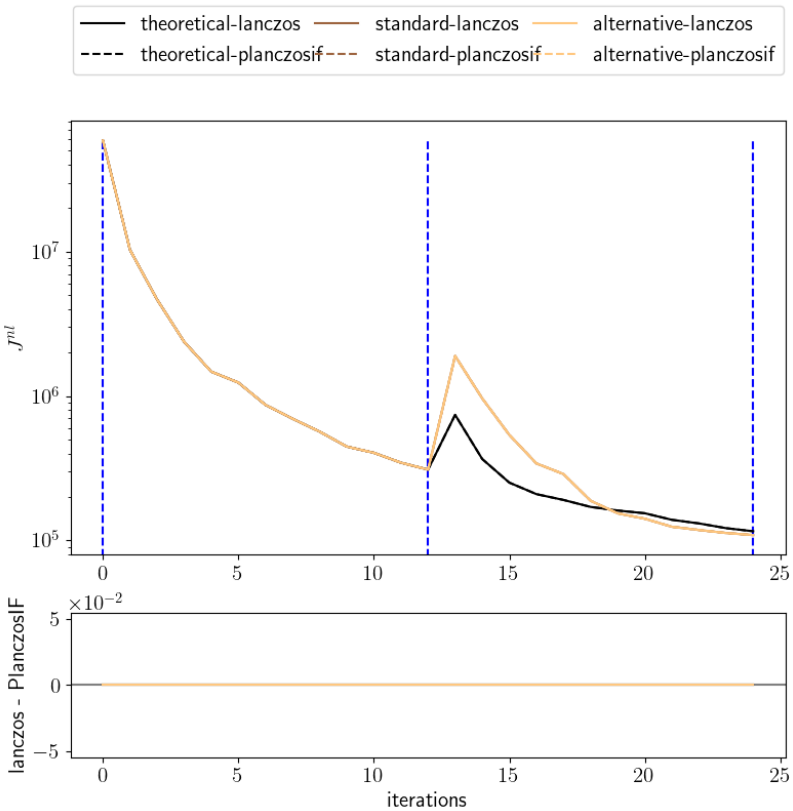

FIGURE – resolutions : 51 > 101

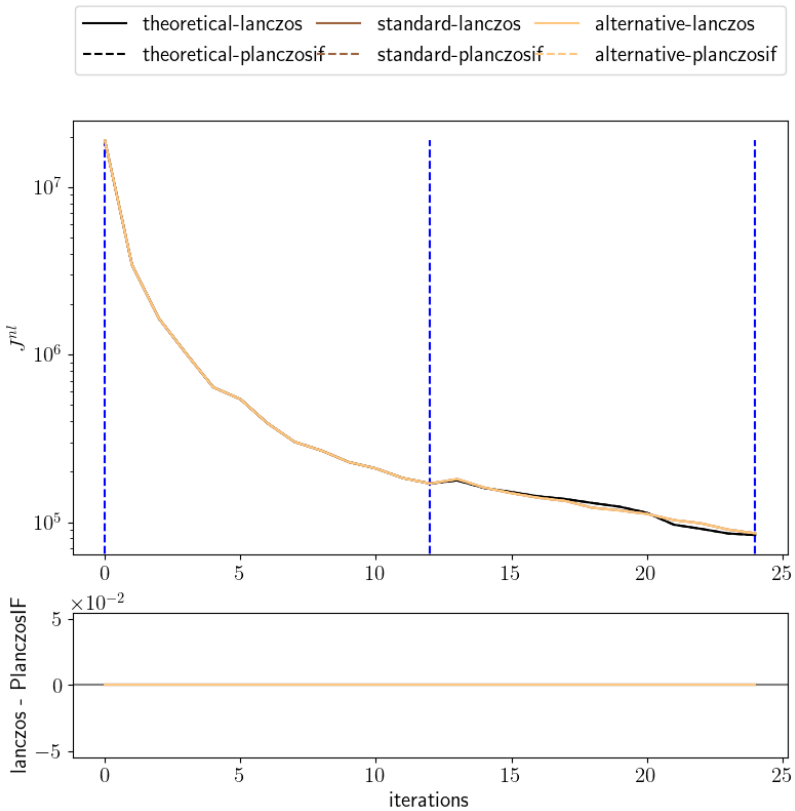

FIGURE – resolutions : 91 > 101

Linear H, spectral interpolation and NON projective B matrix

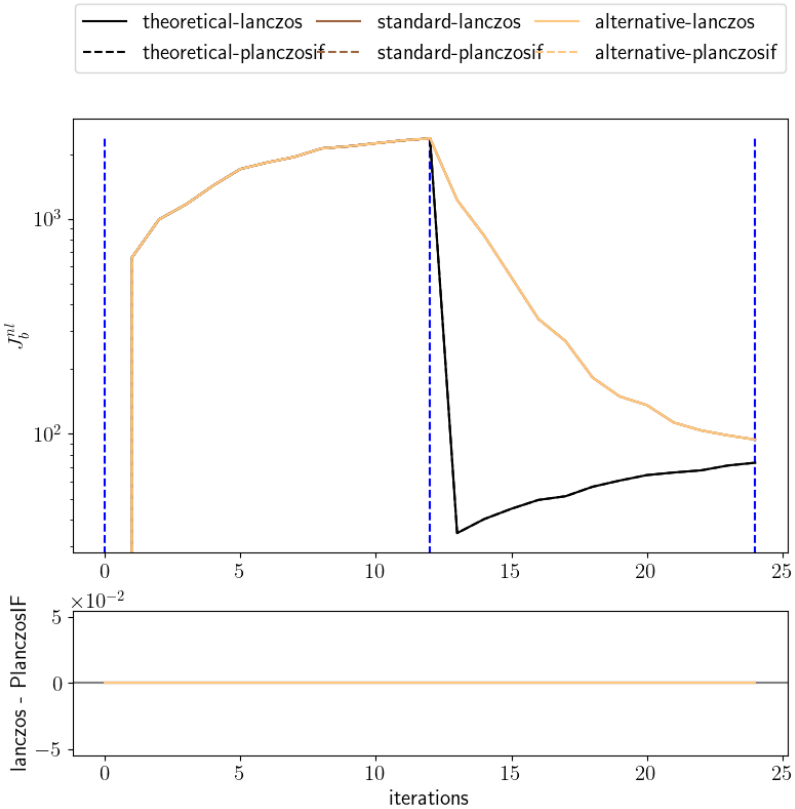

FIGURE – resolutions : 11 > 101

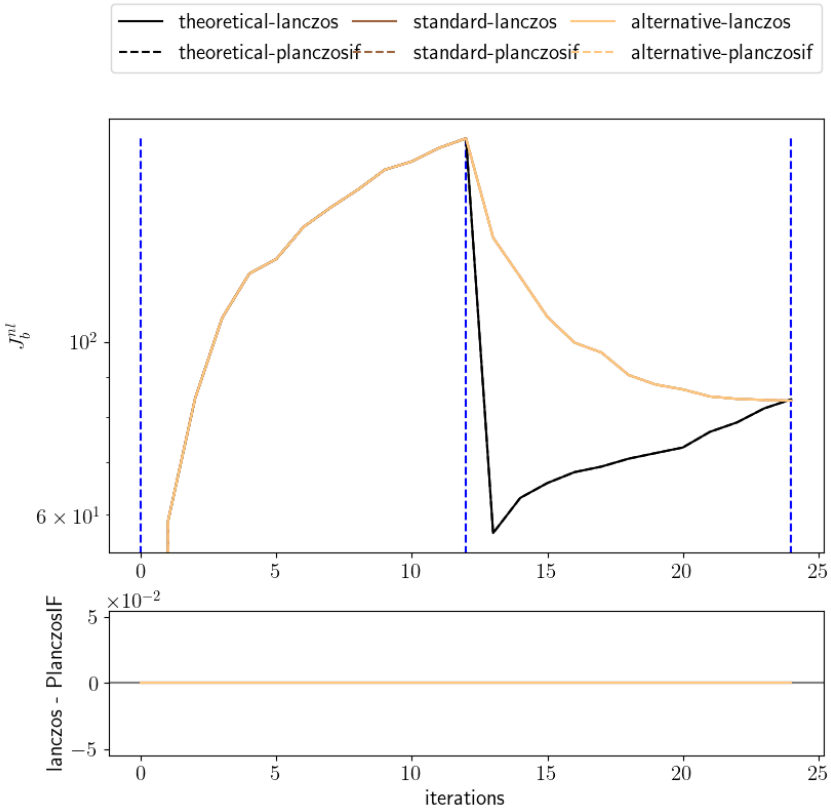

FIGURE – resolutions : 51 > 101

Linear H, spectral interpolation and NON projective B matrix

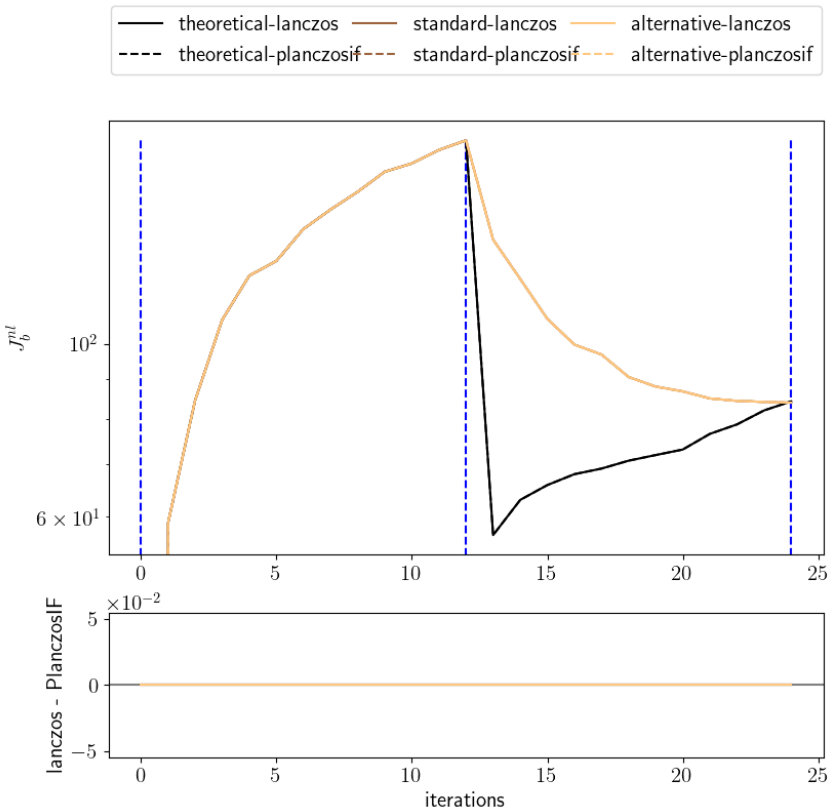

FIGURE – resolutions : 51 > 101

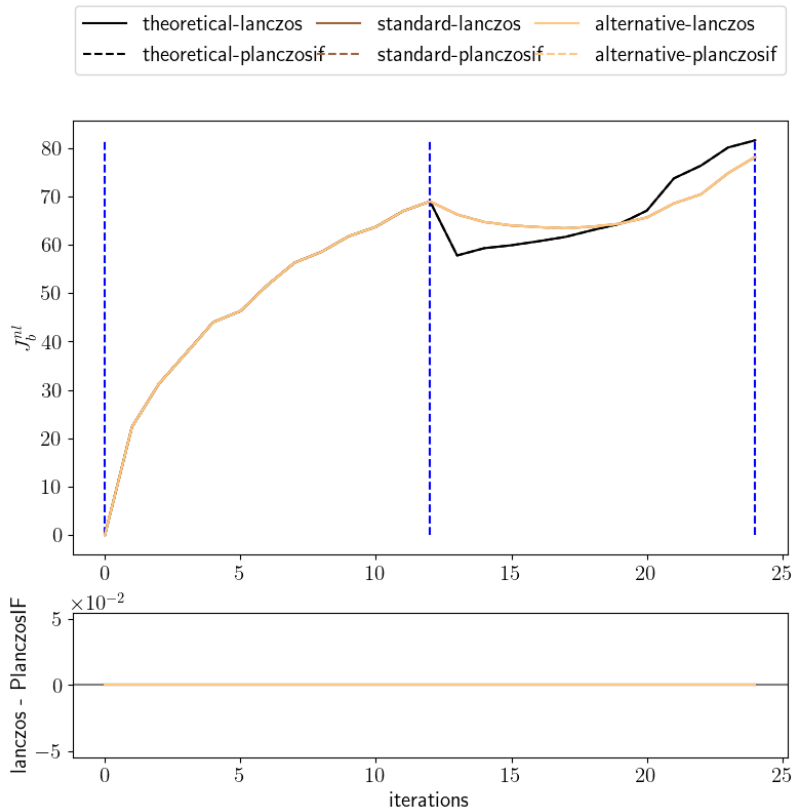

FIGURE – resolutions : 91 > 101

Short conclusions on the projective B matrix (1)

- There might be a jump in the cost function at the first inner iteration of a new outer loop which is also present in the residue and in the J^o and seems due to the change of resolution..
- There is no differences between the methods if the B matrix is projective and if the interpolation is transitive (spectral).
- There are differences if the B matrix is not projective, and it occurs after the first outer loop.
- The higher the change of resolution, the higher the difference between the methods.

Short conclusions on the projective B matrix (2)

- In these cases, the alternative and standard methods give the same results and differ from the theoretical one when the B matrix is not projective.
- The theoretical method seems to give the same results for the J^o with or without a projective B matrix while the J^b is different (and shows a decrease after the first outer loop).

Checking the effects of σ^b_{var}

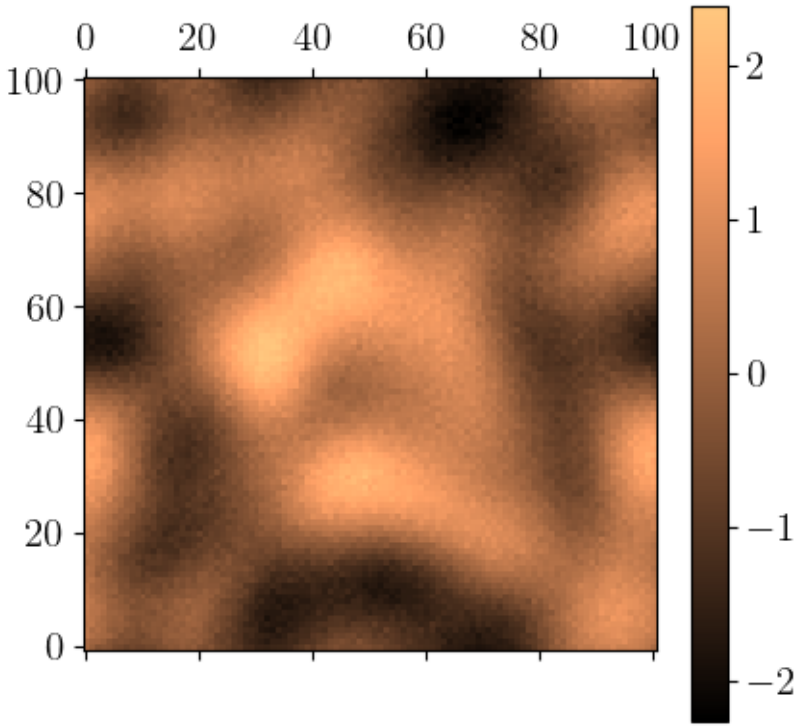

FIGURE – $\sigma^b_{var} = 0.1$

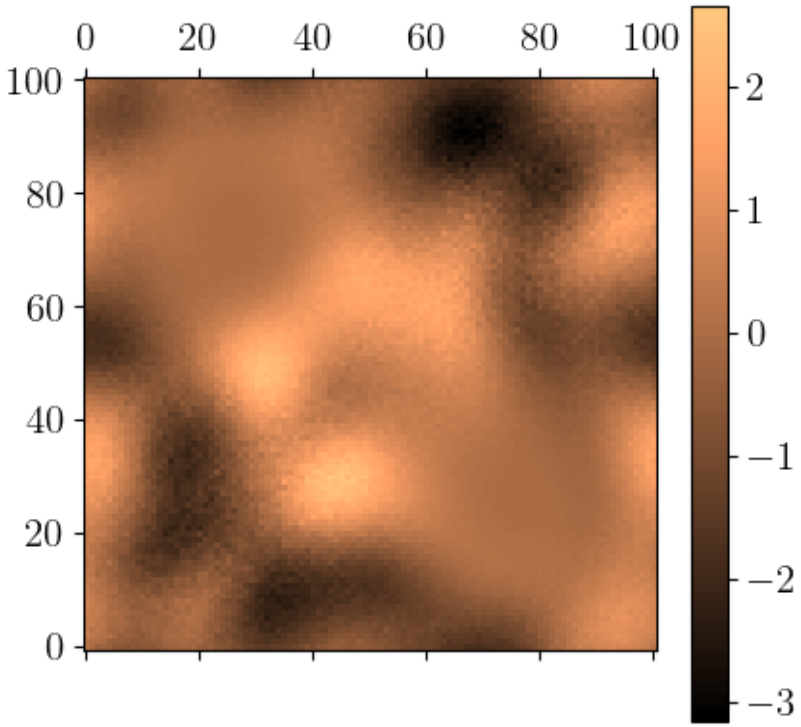

FIGURE – $\sigma^b_{var} = 1$