Content

- Added features
- Comparision between linear and non linear H
- Comparision between interpolation methods with/without projective B matrix

Added features

- Cubic H operator and adjoint :
  - H_{nl} : 1) $x \rightarrow x^3$, 2) interpolation on obs points.
  - H : 1) $x \rightarrow 3x^g x^g x^3$, 2) interpolation on obs points.
  - H^* : 1) interpolation on grid points, 2) $x \rightarrow 3x^g x^g x^3$.

- Truth :
  1) $x^t = \mathrm{rand\_normal}(n_h)$.
  2) $x^t = B^{1/2} x^t$.

- Observations from the truth :
  $y = Hx^t + N(0, \sigma^o)$.

- Nearest Neighbor interpolation.

Check the observations from x^t

$$\sigma^o = 0.001$$

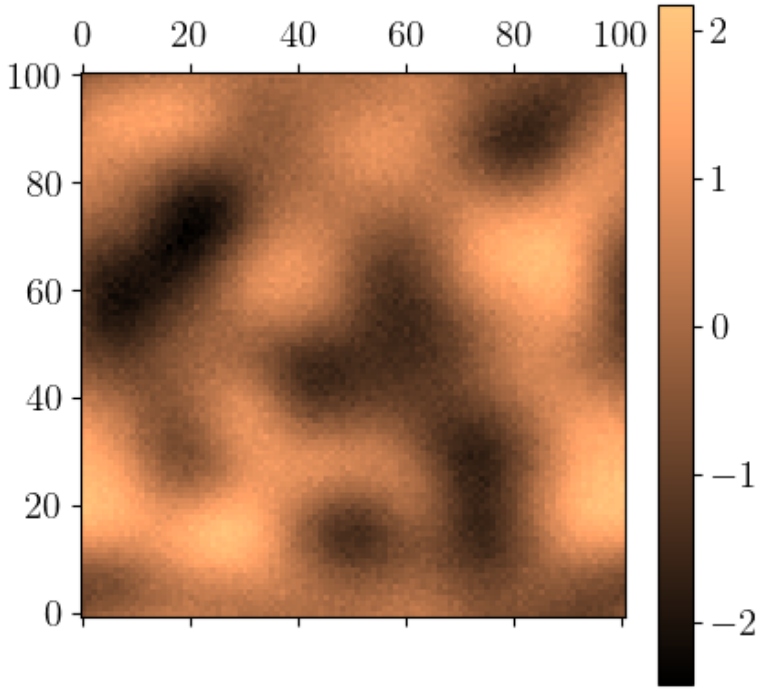


FIGURE – x^t

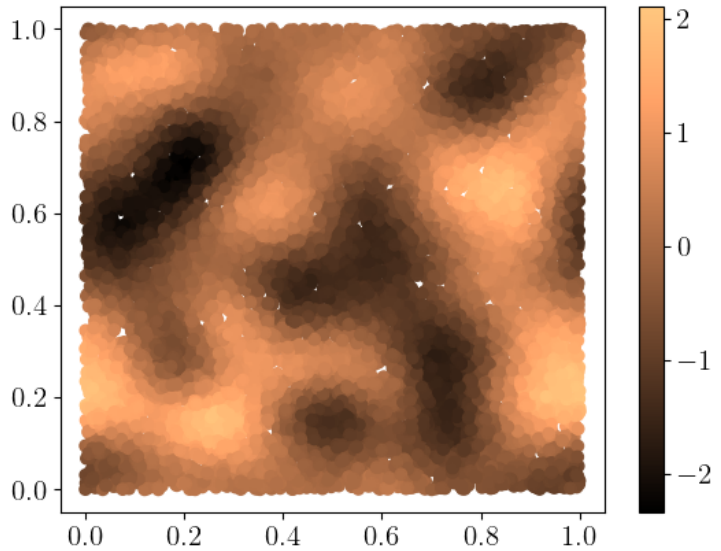


FIGURE – Obs with a linear H

Check the observations from x^t

$$\sigma^o = 0.001$$

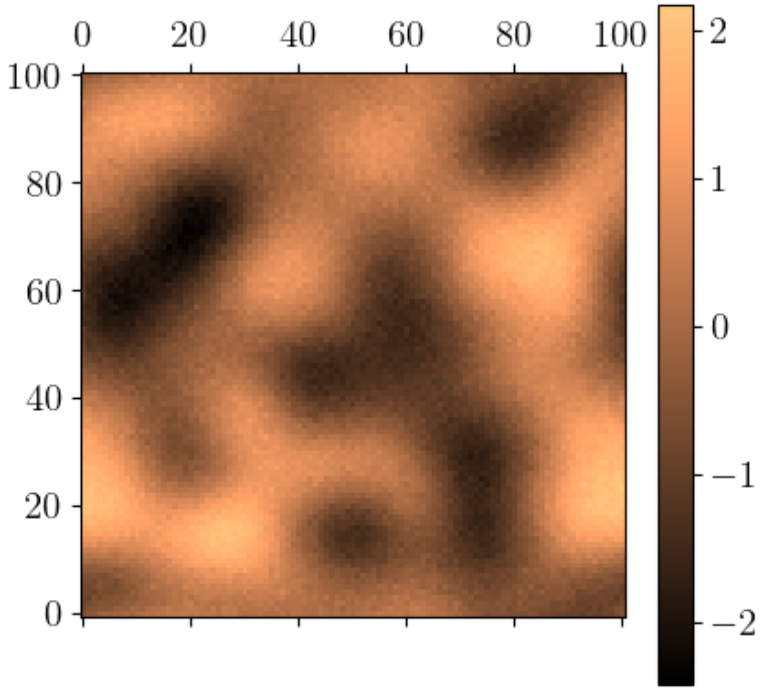


FIGURE – x^t

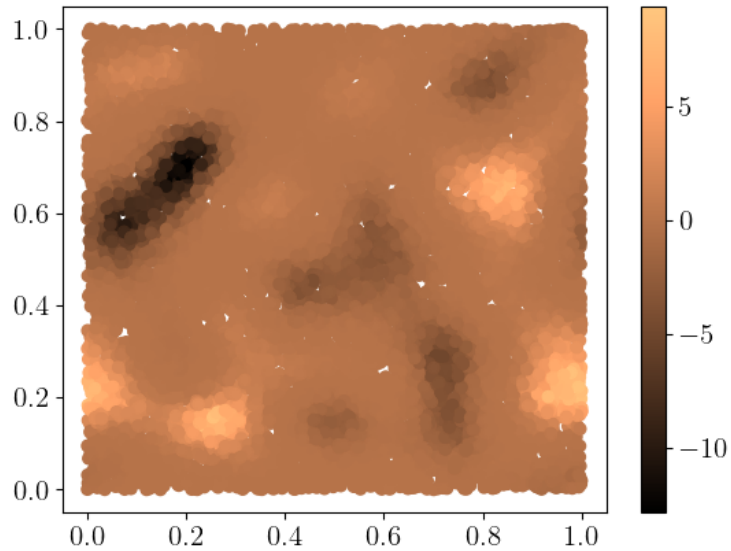
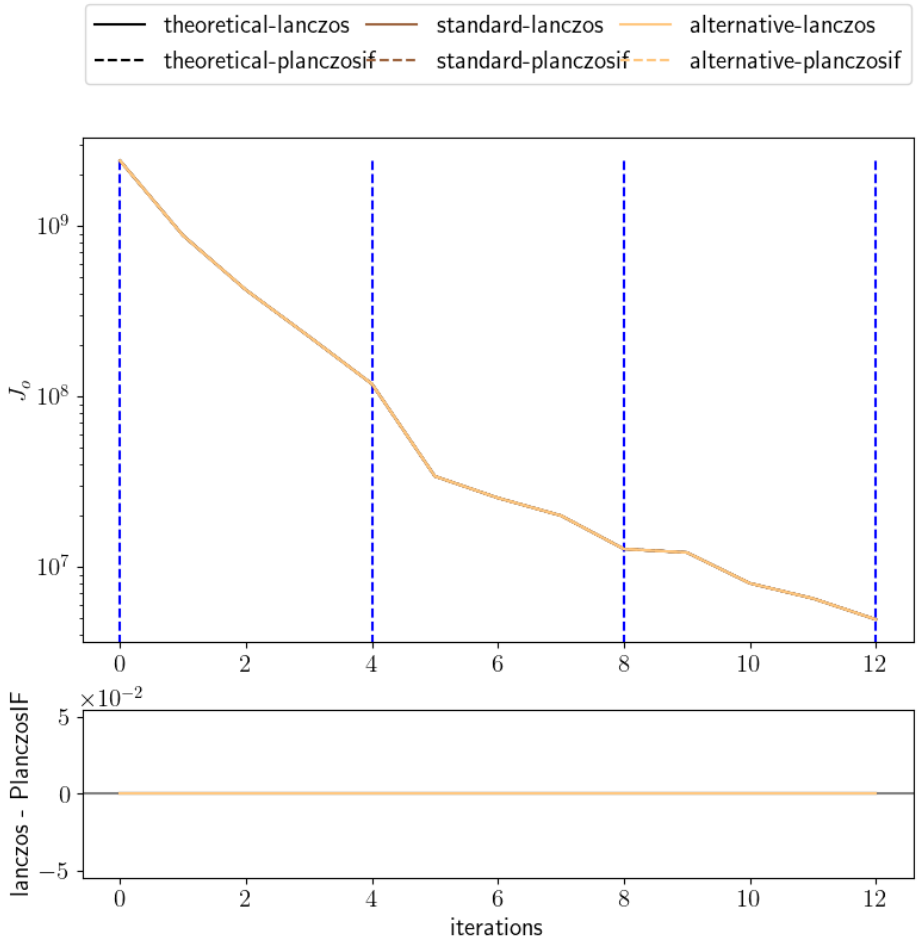


FIGURE – Obs with a cubic H

Comparision between linear and non linear H : J^o

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

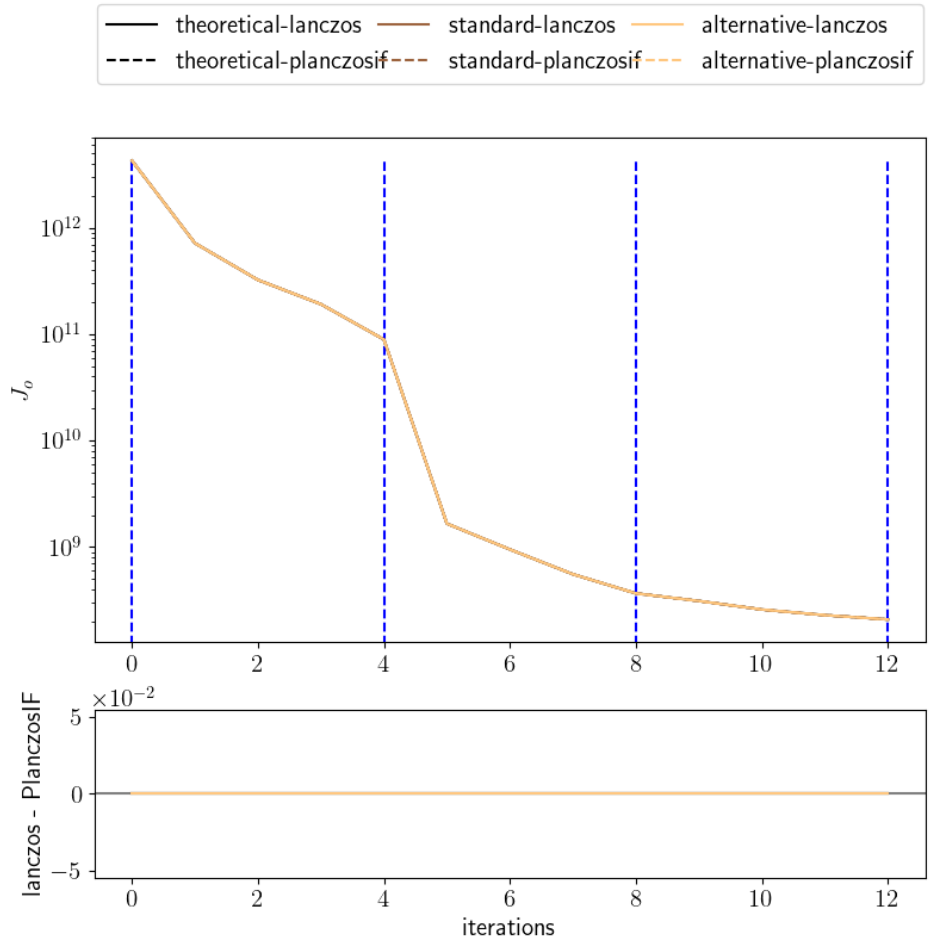
Linear case :



Comparision between linear and non linear H : J^o

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

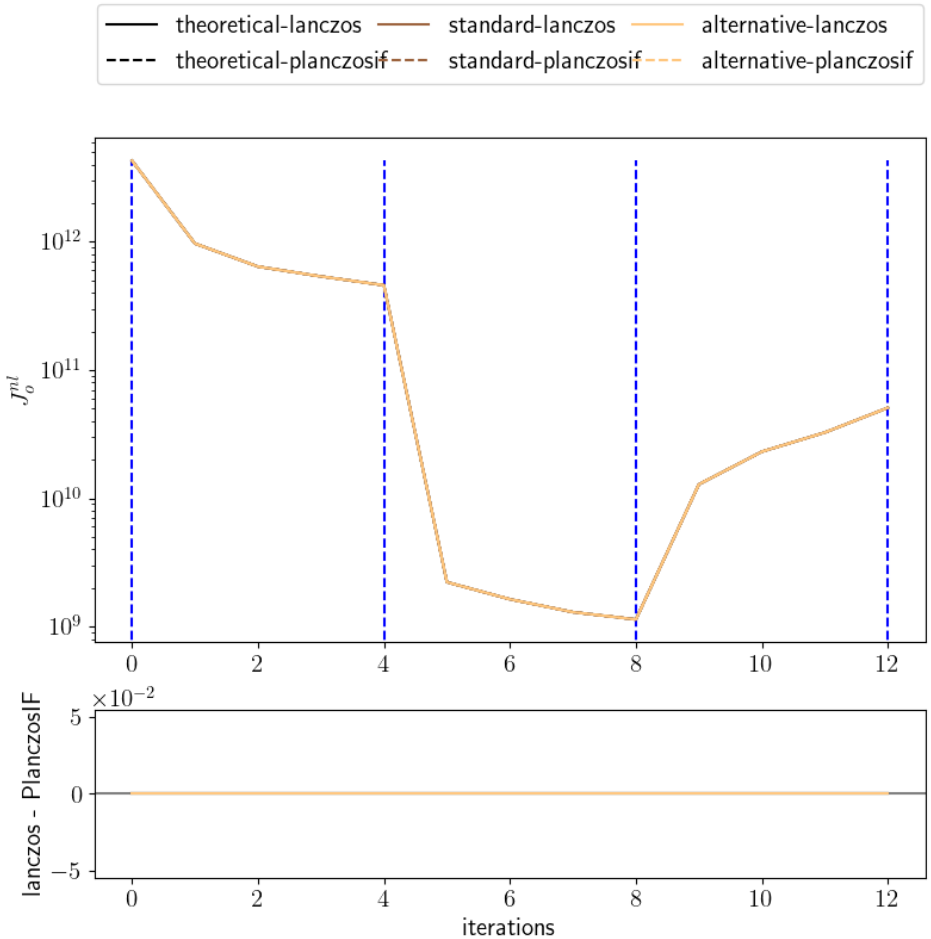
Cubic case :



Comparision between linear and non linear H : J^o_{nl}

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

Cubic case :

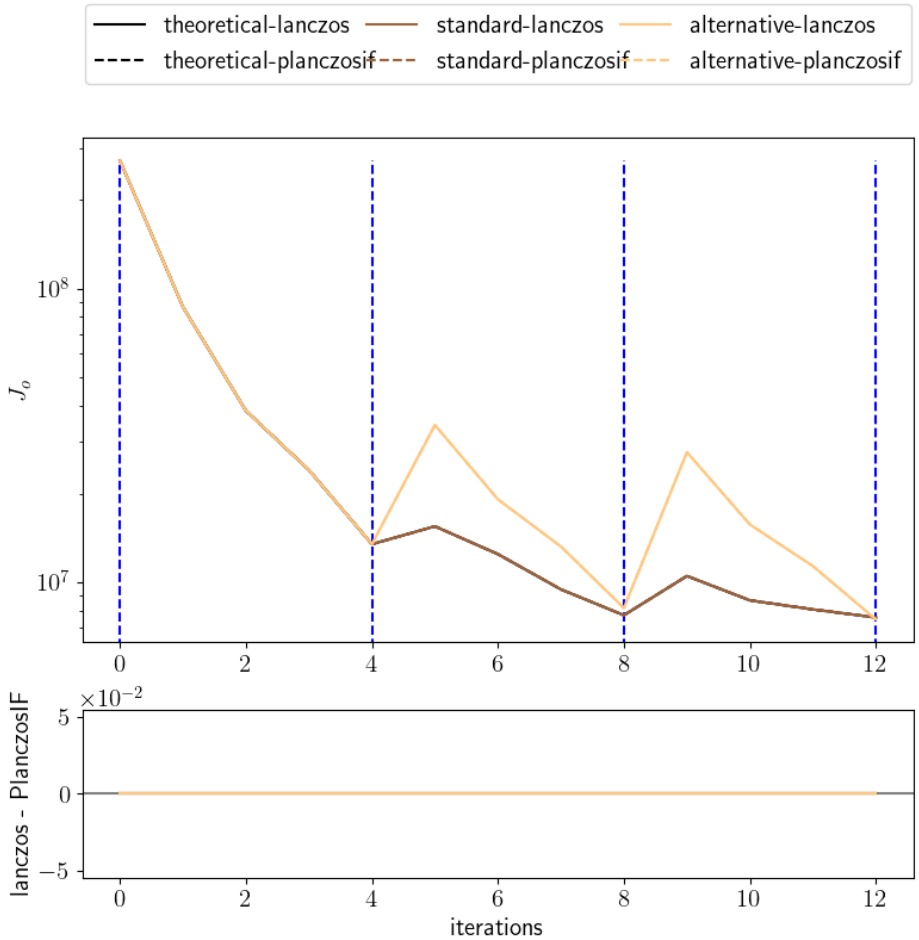


Projective B-matrix

Projective B matrix : nearest interpolation : J^o

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

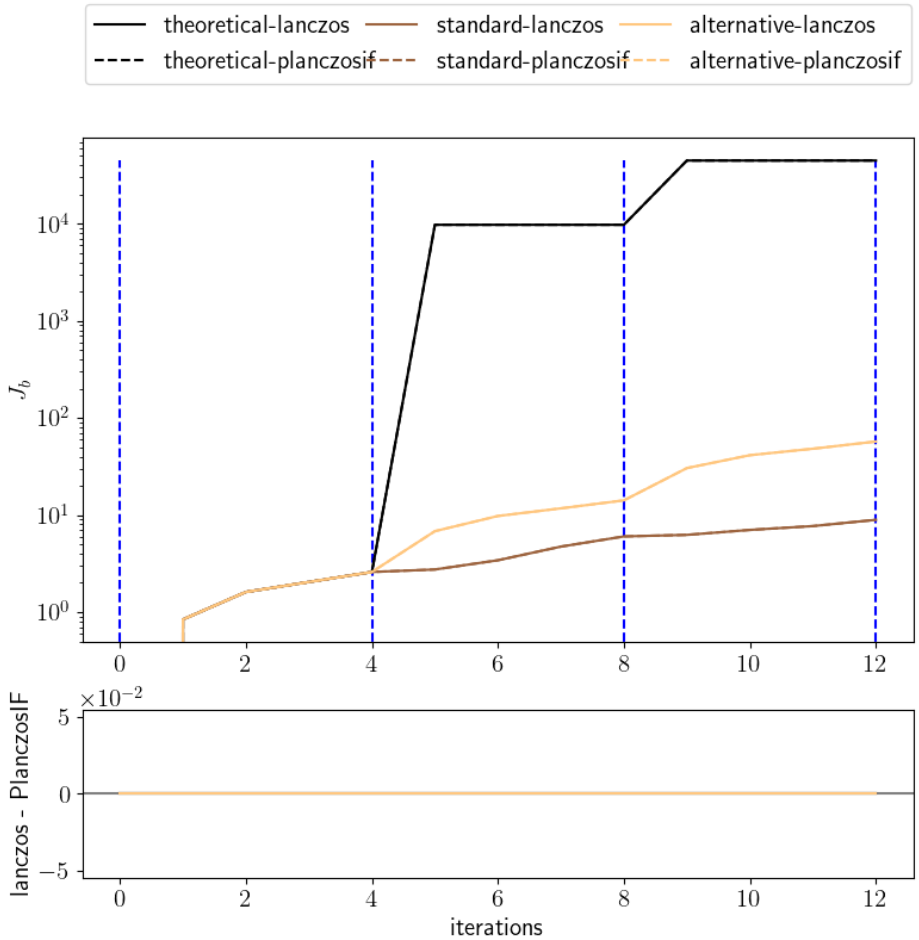
Linear case :



Projective B matrix : nearest interpolation : J^b

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

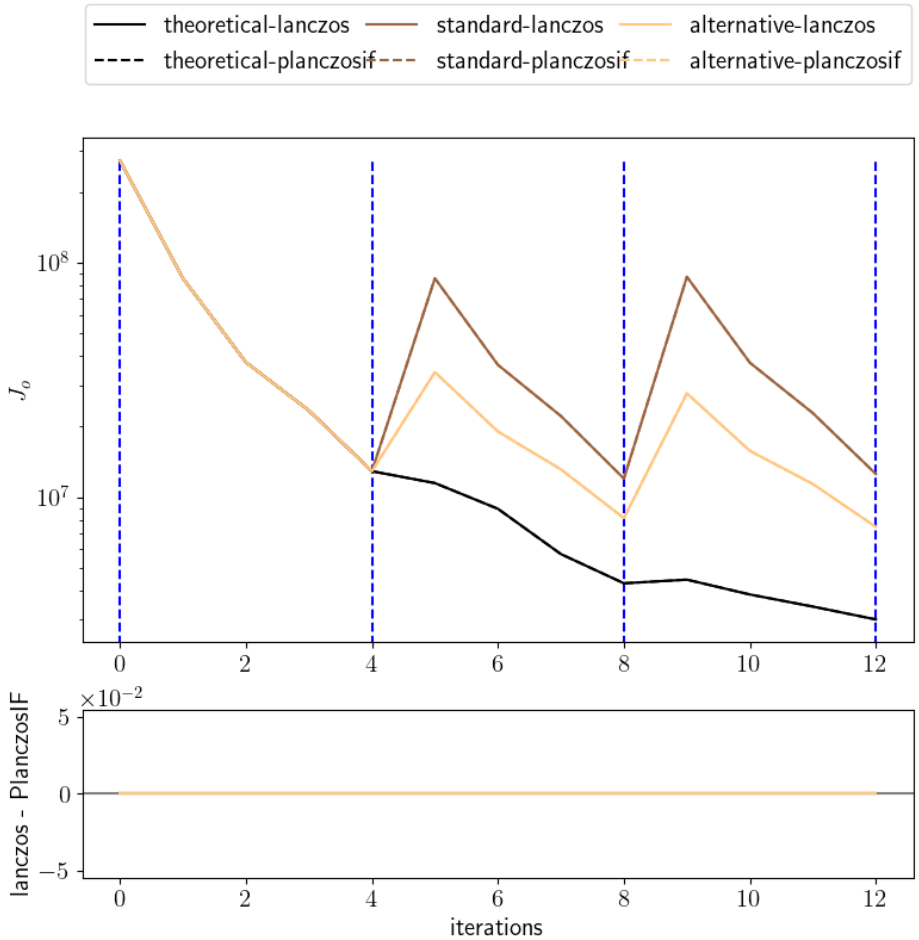
Linear case :



Projective B matrix : bilinear interpolation : J^o

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

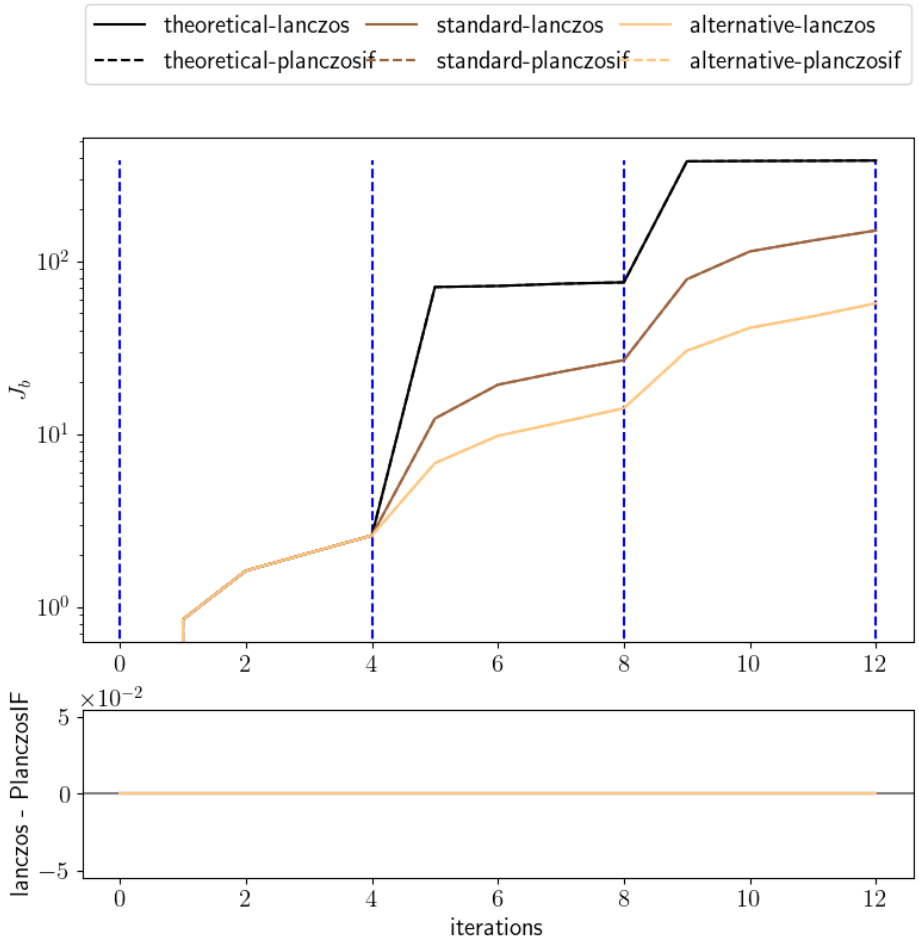
Linear case :



Projective B matrix : bilinear interpolation : J^b

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

Linear case :

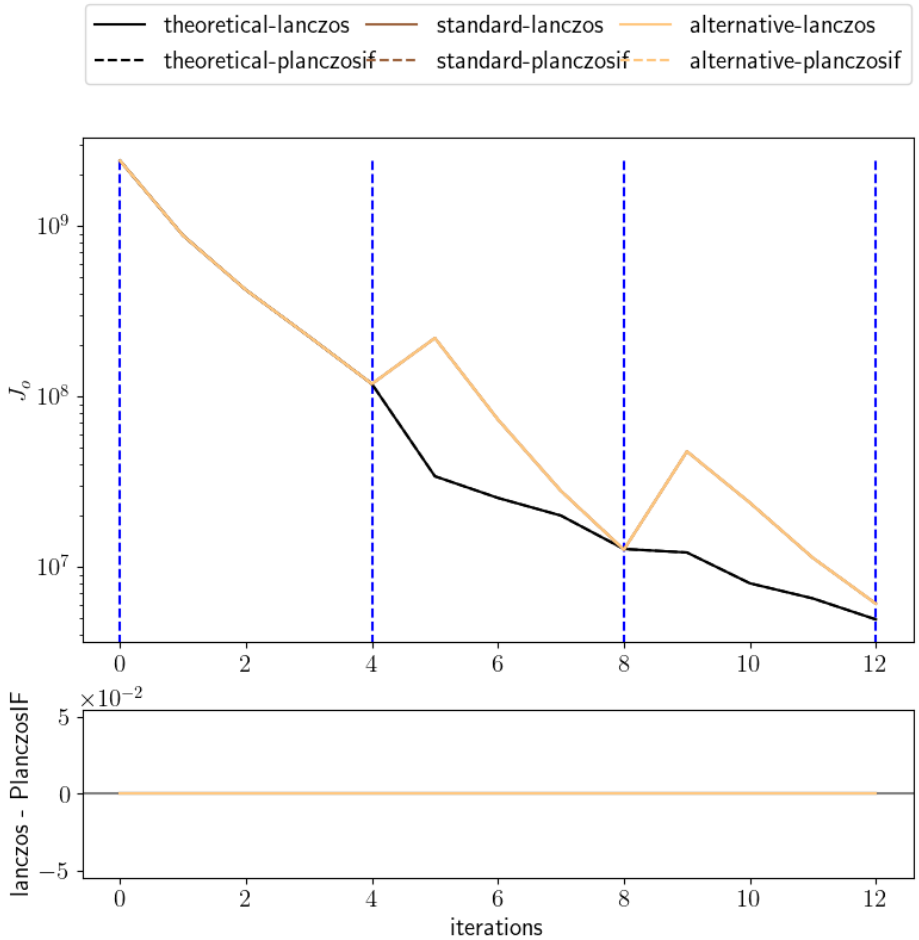


Non projective B-matrix

Non projective B matrix : spectral interpolation : J^o

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

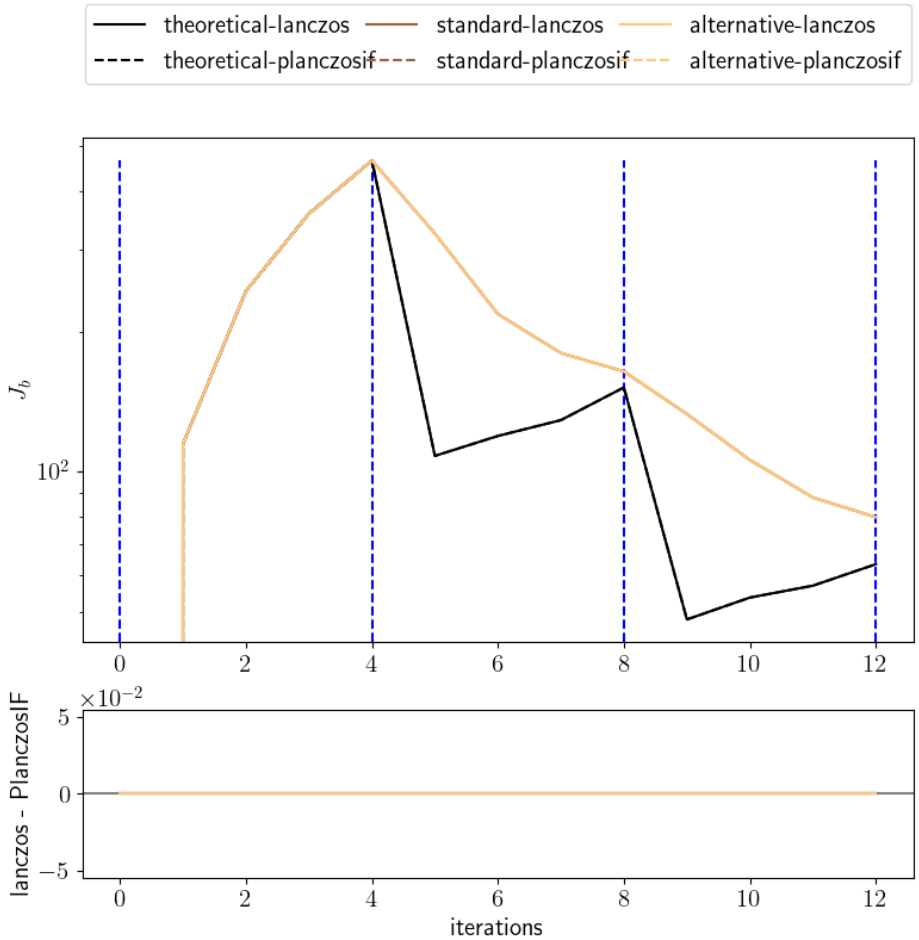
Linear case :



Non projective B matrix : spectral interpolation : J^b

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

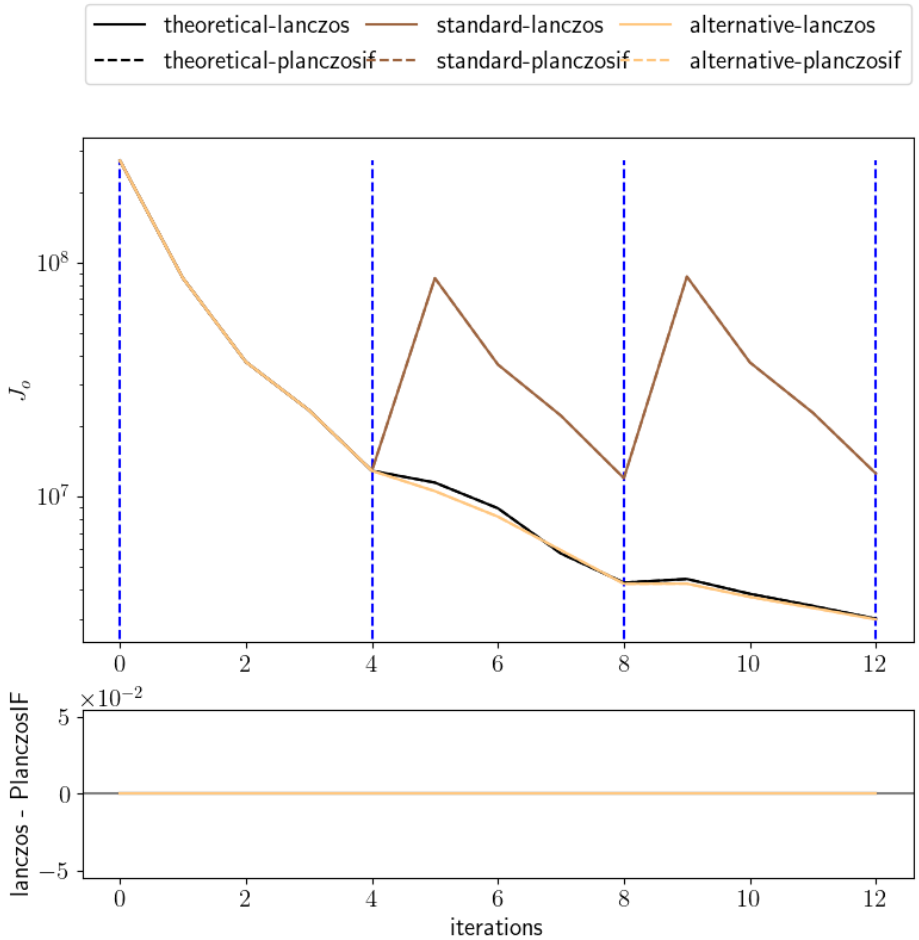
Linear case :



Non projective B matrix : bilinear interpolation : J^o

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

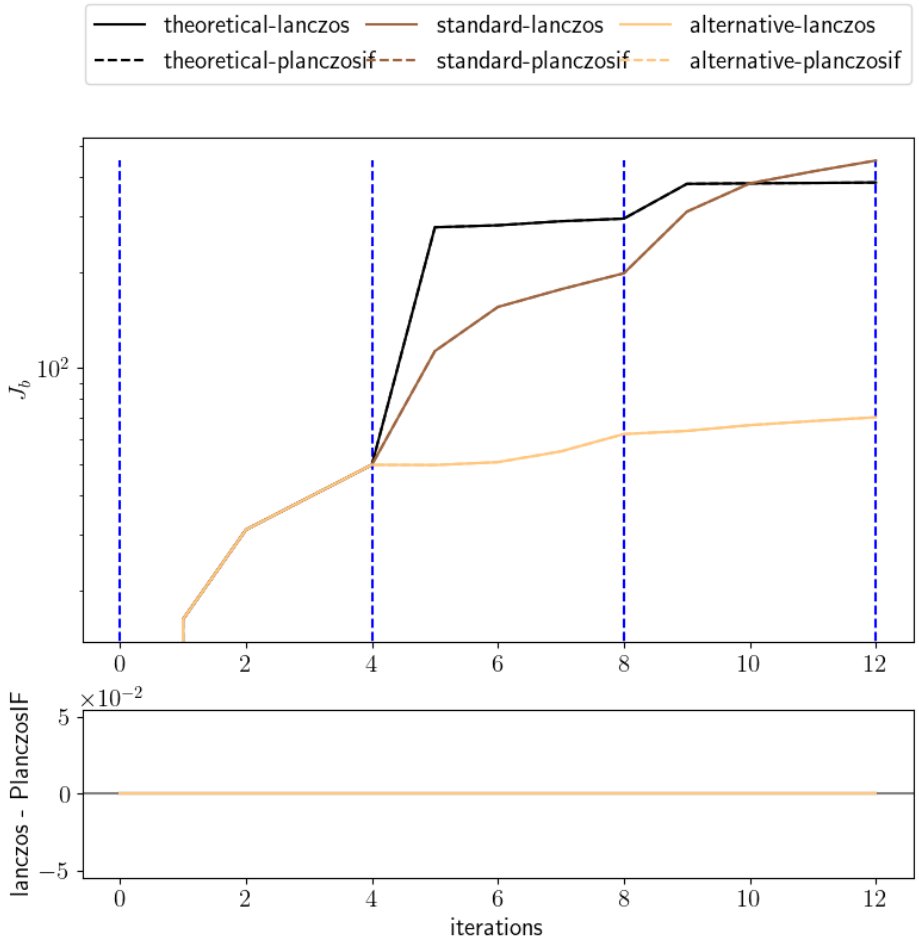
Linear case :



Non projective B matrix : bilinear interpolation : J^b

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

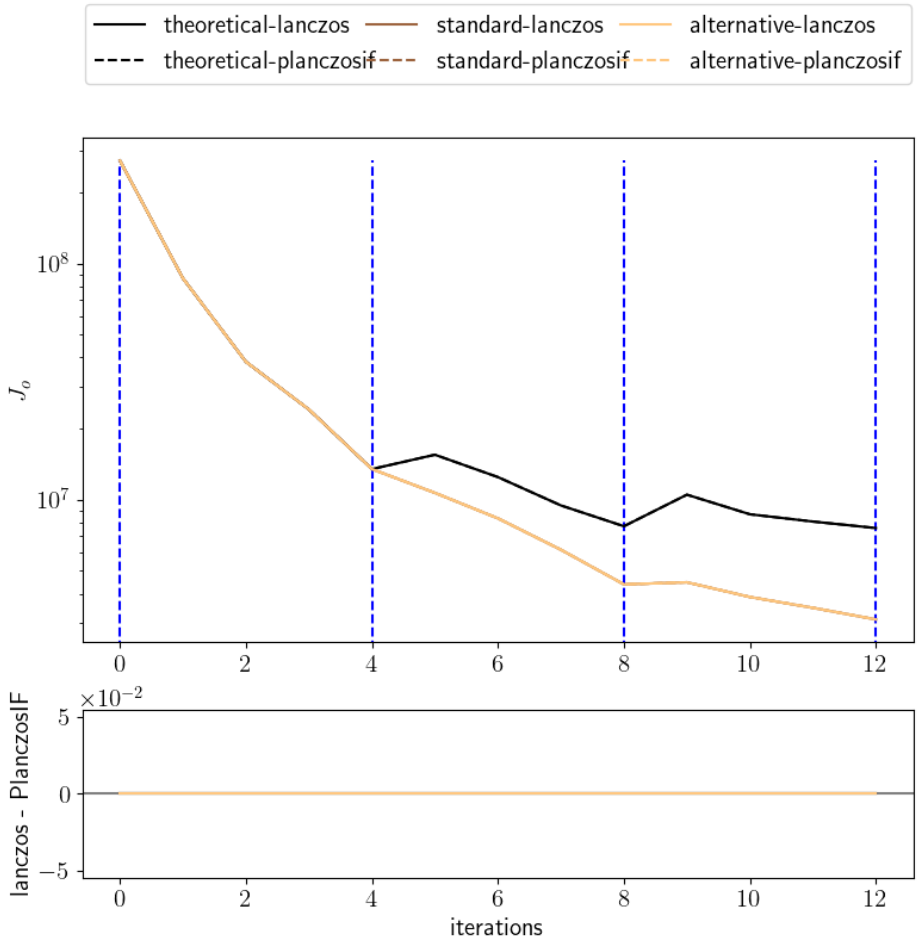
Linear case :



Non projective B matrix : nearest interpolation : J^o

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

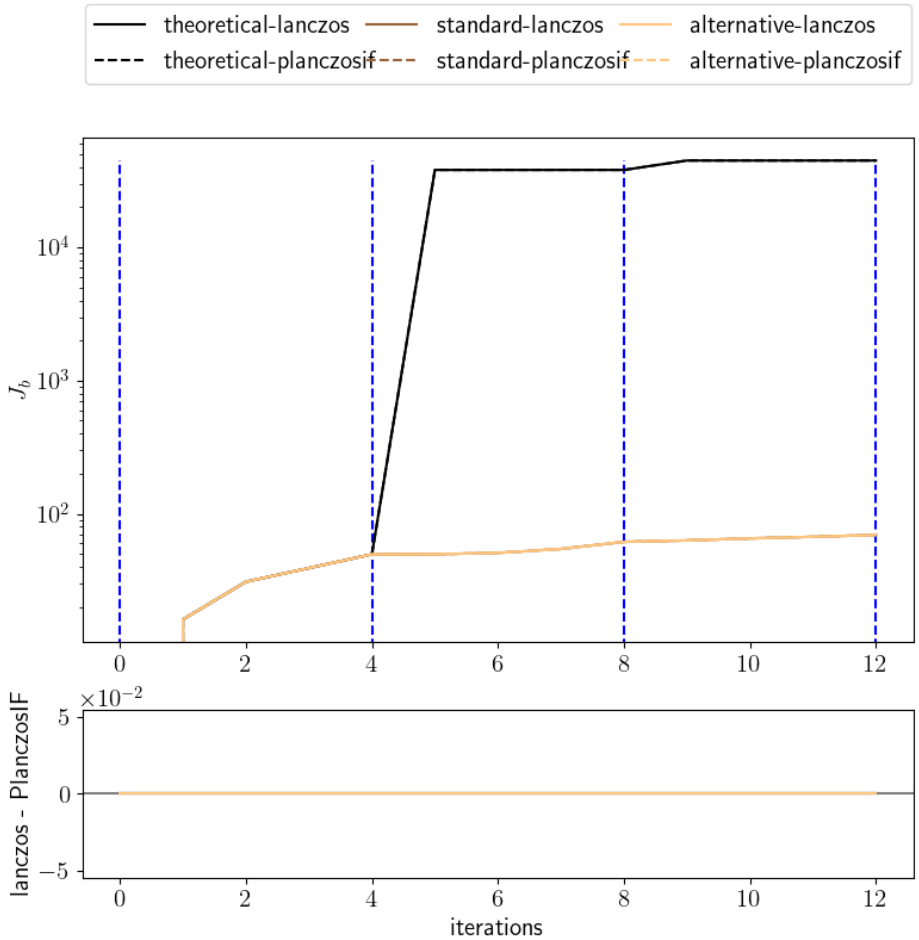
Linear case :



Non projective B matrix : nearest interpolation : J^b

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

Linear case :

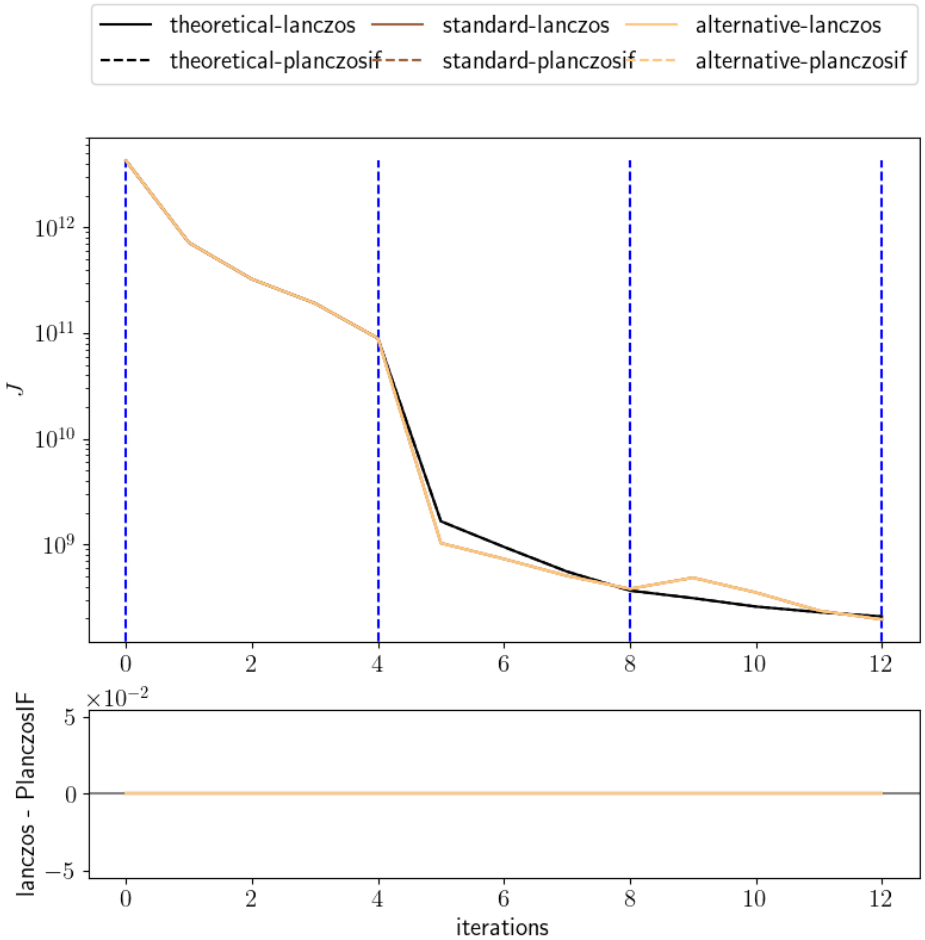


Non projective B-matrix with cubic H

Non projective B matrix : spectral interpolation : J

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

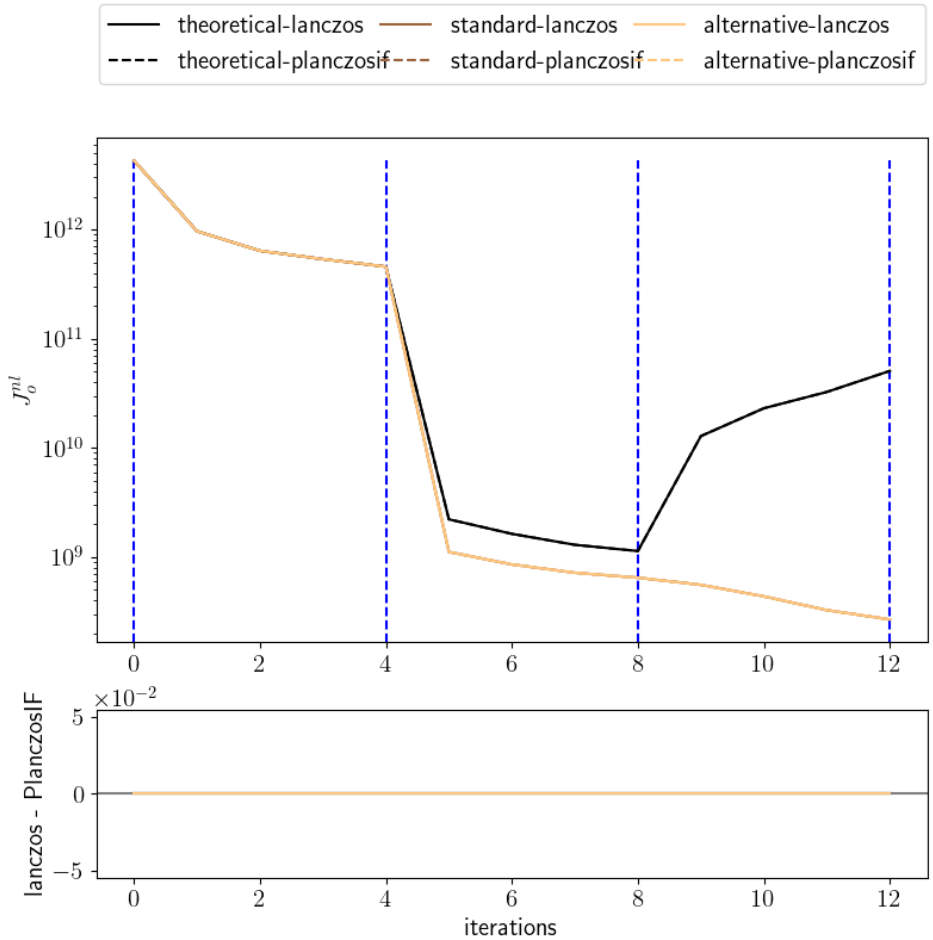
cubic case :



Non projective B matrix : spectral interpolation : J^o_{nl}

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

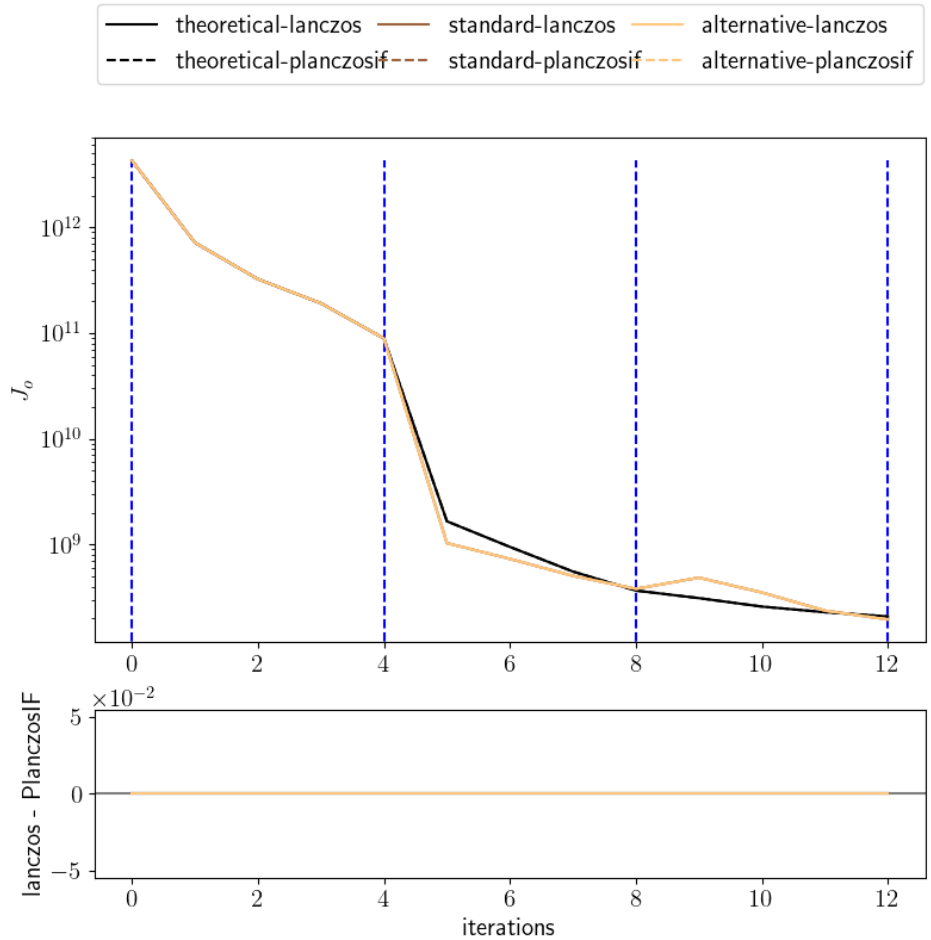
cubic case :



Non projective B matrix : spectral interpolation : J^o

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

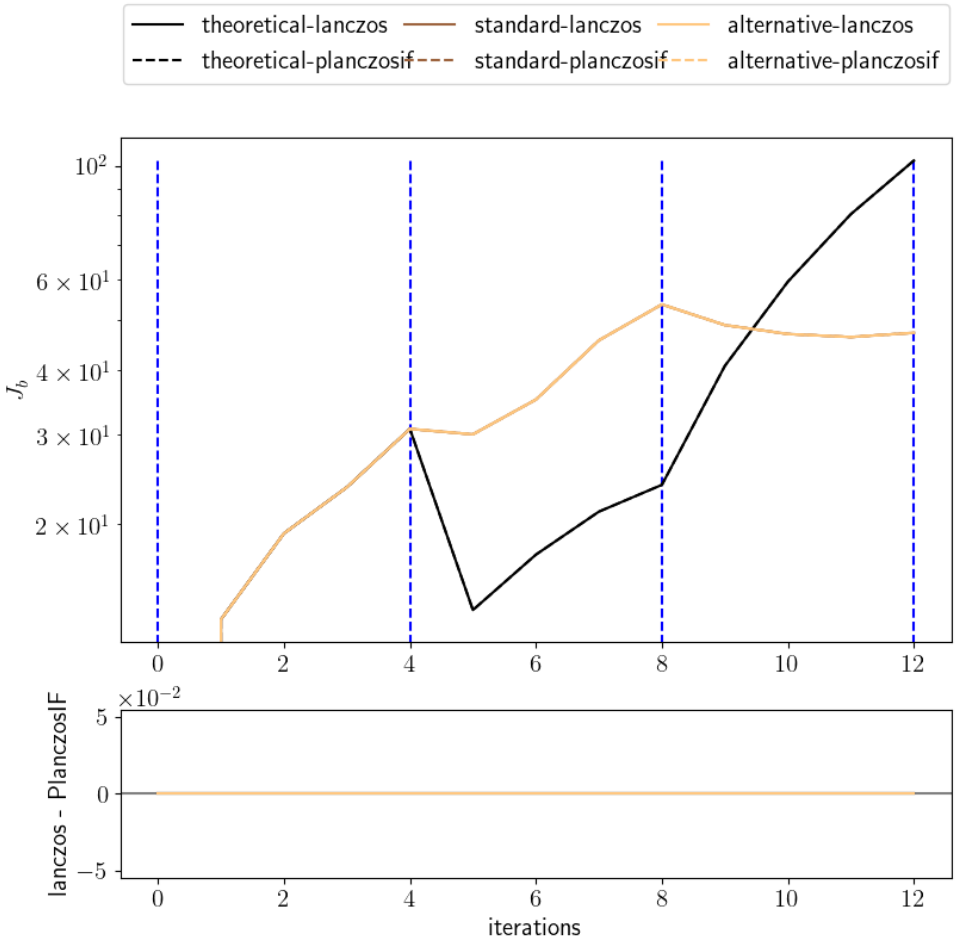
cubic case :



Non projective B matrix : spectral interpolation : J^b

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma^b_{var} = 0; L_b = 0.1; n_{obs} = 300.$

cubic case :



Short conclusion and questions :

- As expected : The only way to get no differences between the methods (th, std, alt) is to have a transitive interpolation AND the projective B matrix condition.
- There is often a change of variation at the first inner loop level (except for the first outer loop).
- Nearest neighbor interpolation seems not transitive (?) (+periodic domain ? + very slow...)
- Similar behaviour with H measuring x^4 (not shown).
- Question : What happens if two observations are interpolated at the same grid point with the bilinear and nearest neighbor interpolations ?
- Question : equation 38 from the notes.