

- Added features
- Comparision between linear and non linear H
- Comparision between interpolation methods with/without projective B matrix

- Cubic H operator and adjoint :
 - $H_{nl} : 1) x \rightarrow x^3$, 2) interpolation on obs points.
 - $H : 1) x \rightarrow 3x^g x^g x^3$, 2) interpolation on obs points.
 - $H^* : 1) \text{ interpolation on grid points, } 2) x \rightarrow 3x^g x^g x^3$.
- Truth :
 - 1) $x^t = \text{rand_normal}(n_h)$.
 - 2) $x^t = B^{1/2} x^t$.
- Observations from the truth :
$$y = Hx^t + N(0, \sigma^o).$$
- Nearest Neighbor interpolation.

Check the observations from x^t

$$\sigma^o = 0.001$$

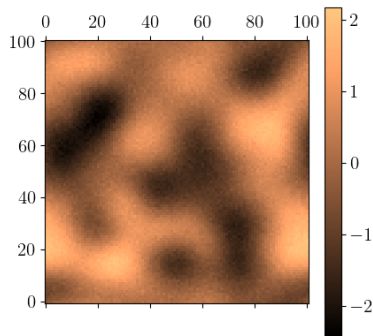


FIGURE – x^t

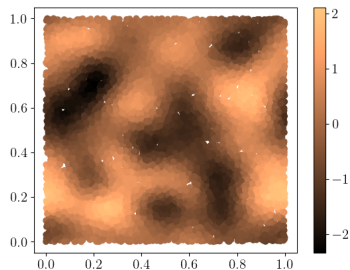


FIGURE – Obs with a linear H

Check the observations from x^t

$$\sigma^o = 0.001$$

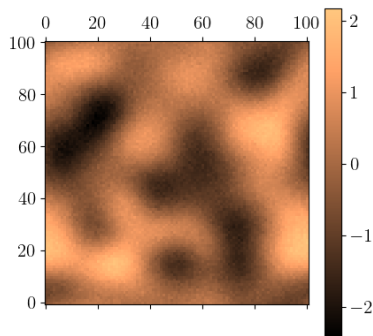


FIGURE – x^t

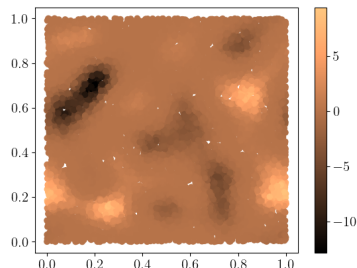
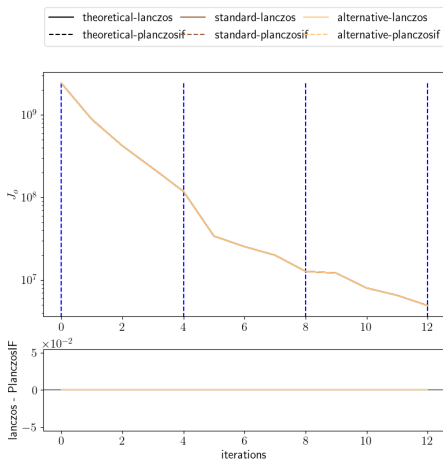


FIGURE – Obs with a cubic H

Comparison between linear and non linear H : J^o

$$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma_{var}^b = 0; L_b = 0.1; n_{obs} = 300.$$

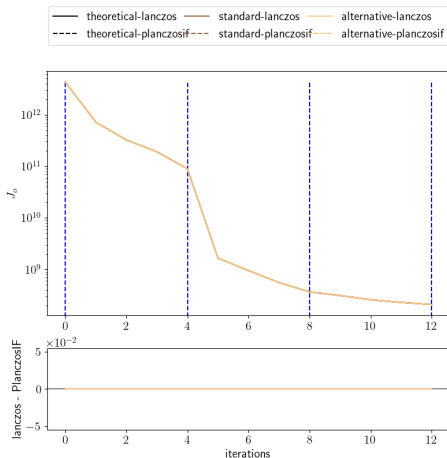
Linear case :



Comparison between linear and non linear H : J^o

$$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma_{var}^b = 0; L_b = 0.1; n_{obs} = 300.$$

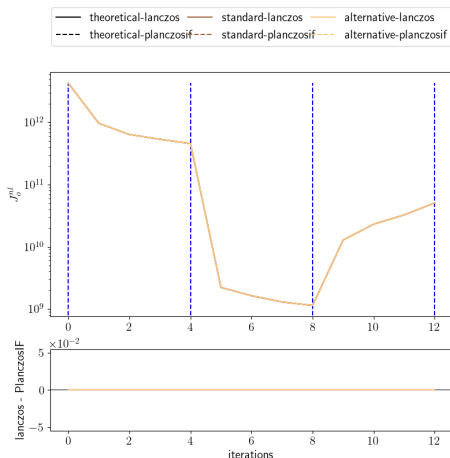
Cubic case :



Comparison between linear and non linear H : J_{nl}^o

$$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma_{var}^b = 0; L_b = 0.1; n_{obs} = 300.$$

Cubic case :

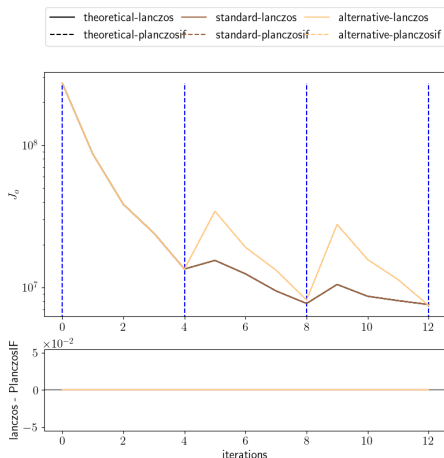


Projective B-matrix

Projective B matrix : nearest interpolation : J^o

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.

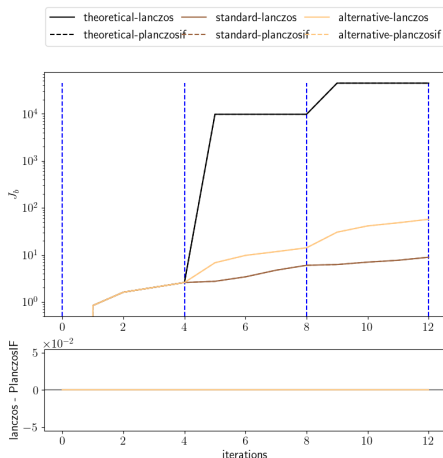
Linear case :



Projective B matrix : nearest interpolation : J^b

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.

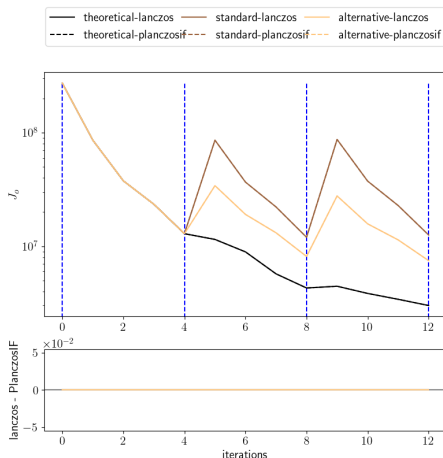
Linear case :



Projective B matrix : bilinear interpolation : J^o

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.

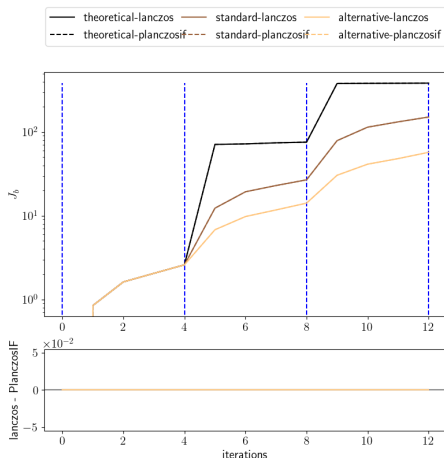
Linear case :



Projective B matrix : bilinear interpolation : J^b

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.

Linear case :

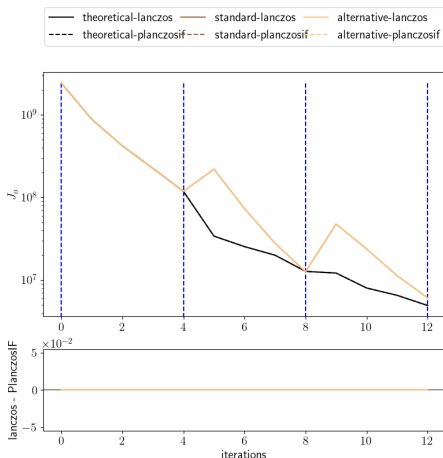


Non projective B-matrix

Non projective B matrix : spectral interpolation : J^o

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.

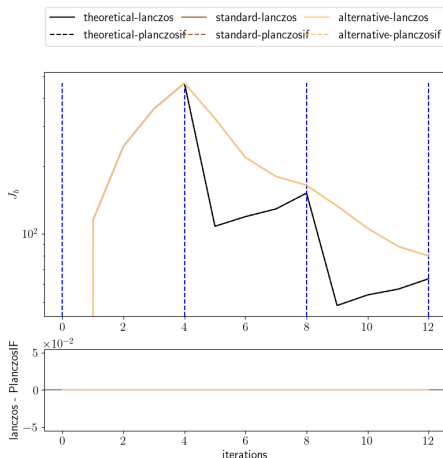
Linear case :



Non projective B matrix : spectral interpolation : J^b

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.

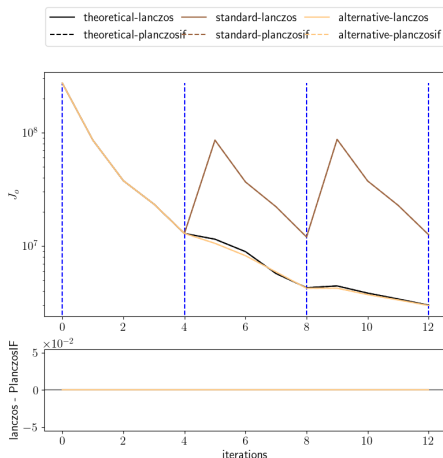
Linear case :



Non projective B matrix : bilinear interpolation : J^o

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.

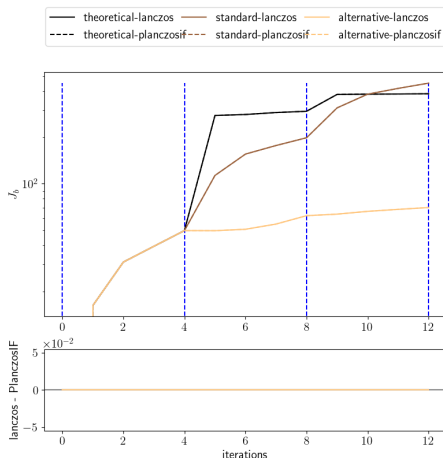
Linear case :



Non projective B matrix : bilinear interpolation : J^b

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.

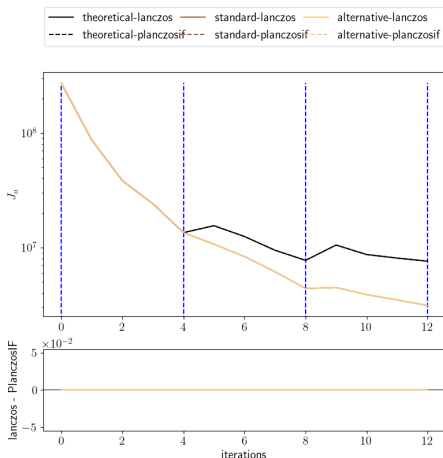
Linear case :



Non projective B matrix : nearest interpolation : J^o

$$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma_{var}^b = 0; L_b = 0.1; n_{obs} = 300.$$

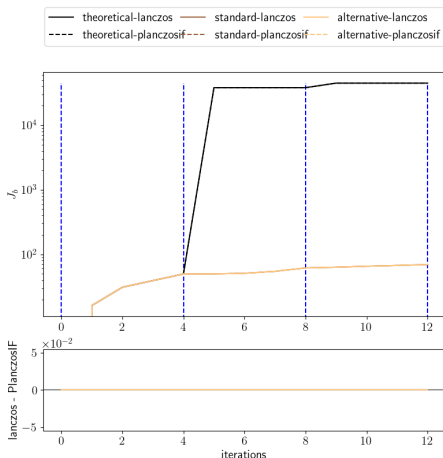
Linear case :



Non projective B matrix : nearest interpolation : J^b

$n_x = n_y = 23, 51, 101; \sigma^o = 0.001; \sigma_{var}^b = 0; L_b = 0.1; n_{obs} = 300.$

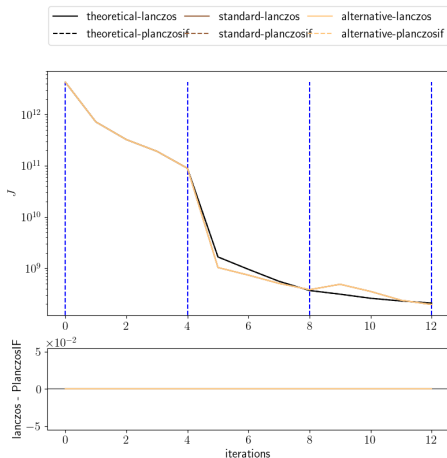
Linear case :



Non projective B-matrix with cubic H

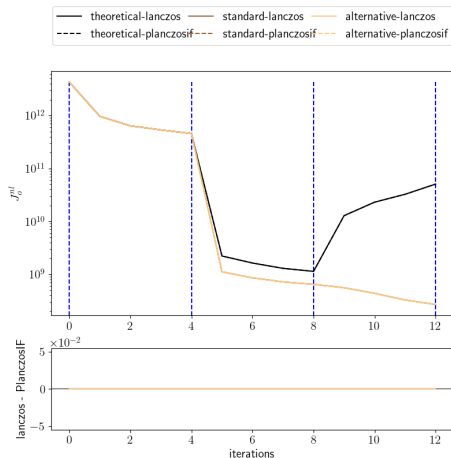
Non projective B matrix : spectral interpolation : J

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.
cubic case :



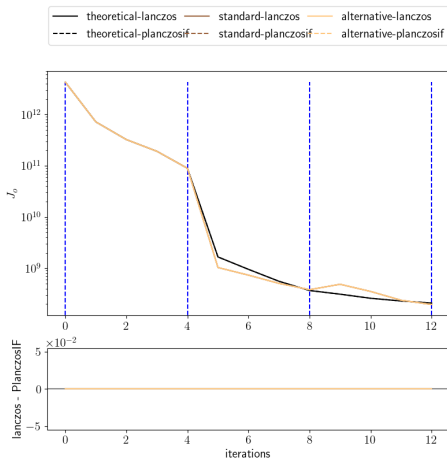
Non projective B matrix : spectral interpolation : J_n^o

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.
cubic case :



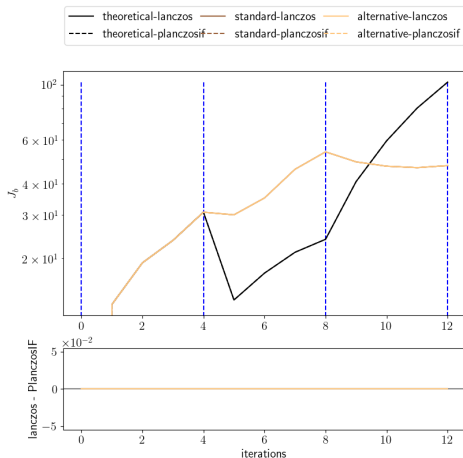
Non projective B matrix : spectral interpolation : J^o

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.
cubic case :



Non projective B matrix : spectral interpolation : J^b

$n_x = n_y = 23, 51, 101$; $\sigma^o = 0.001$; $\sigma_{var}^b = 0$; $L_b = 0.1$; $n_{obs} = 300$.
cubic case :



Short conclusion and questions :

- As expected : The only way to get no differences between the methods (th, std, alt) is to have a transitive interpolation AND the projective B matrix condition.
- There is often a change of variation at the first inner loop level (except for the first outer loop).
- Nearest neighbor interpolation seems not transitive (?) (+periodic domain? + very slow...)
- Similar behaviour with H measuring x^4 (not shown).
- Question : What happens if two observations are interpolated at the same grid point with the bilinear and nearest neighbor interpolations?
- Question : equation 38 from the notes.