Assignment 4

ADVANCED ECONOMETRICS

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Notes and instructions:

1. This assignment is mandatory.
2. The assignment is to be made in groups of 4 students. You can create your own group in Canvas > People > Case 3+4 (groups), and then self-enroll or join an incomplete team. Please be professional and welcoming to new team members.
3. Only one of you needs to hand in all files.
4. The deadline for delivery of this assignment is on Tuesday, October 18, at 23:59h

   There will be no tolerance period for late deliveries. Deliveries after the assigned deadline imply that you have a final grade of zero for the assignment (AG4 = 0).
5. To get the full score for this assignment, the following three things must be done:

   (a) upload your final report as a PDF file in Canvas Assignments. Name the file **A4report_2601842_2511351_2661510_2639486.pdf**, where the numbers are replaced by the VU student numbers of the 4 group members.To write you report according to academic standards follow the relevant tips that we have in the questions and also check the example report file under the name 'example_report.pdf' on Canvas > Assignments > Assignment 4: Instructions.

   (b) upload a zip file of your runnable R or Python code in Canvas Assignments. Name the file **A4code_2601842_2511351_2661510_2639486_language.zip**, where the numbers are replaced by the VU student numbers of the 4 group members (or 3 if your group consists of 3 people) and language is either R or Python.

   The code file(s) should be clear, well commented, and directly runnable, so that it reads the datafile and obtains the results of all questions and prints them. Your initial comments in the file should hold your names and student numbers.

(c) upload a pdf of your entire code in Canvas Assignments. Name the file **A4code_2601842_2511351_2661510_2639486_language.pdf**, where the numbers are replaced by the VU student numbers of the 4 group members (or 3 if your group consists of 3 people) and language is either R or Python. The file should be well readable, with proper indentations and should not contain pictures/photos/screenshots of code snippets.

6. As a standard anti-fraud measure, we will at random select a number of you to explain your code and answers. Any one of you must be able to explain any part of the code. Failure to explain your answers will result in a deduction of credits for this assignment.

7. For the support for the assignments, carefully read the announcement we put out at the start of the course and consult the discussion boards related to the assignments.

We wish you success!!

VU VRIJE UNIVERSITEIT AMSTERDAM
School of Business and Economics

1 Background: marketing campaign optimization

A web-based company has contacted you to optimize their marketing campaign. Their web marketing efforts are concentrated on two channels: Google's Adwords directed at Google searches, and advertisement on Youtube.

You have requested the company to implement a small-scale AB-testing procedure for one of their products. As a result, you were given 3 months of hourly data generated by AB-testing marketing expenditures for the selected product. The data set can be found in the file `marketing_data.csv` and it includes de-trended and de-seasonalized online sales (s_t), expenditures on advertisement directed at google search (g_t), and expenditures on advertisement directed at youtube (y_t), all measured in euros. The sample of data shows that the company spent about 26.000 euros over the 3 months of AB-testing on this product, and achieved a total sales volume of 162.000 euros for that product over the same period of time.

In the field of marketing, the adstock created by an advertisement campaign refers to the prolonged effect that an advertisement campaign has on consumers. Adstock is created when consumers are exposed to advertisement, and it fades slowly over time since consumers will remember the advertisement for some time.

The adstock generated by expenditure on Google ads can be filtered using an equation of the type,

$$\underline{\text{gads}_t} = \beta_1 \underline{\text{gads}_{t-1}} + \alpha_1 g_t \quad ,$$

where $\underline{\text{gads}_t}$ denotes the adstock generated by the expenditure on Google search ads. Positive values for α_1 and β_1 will allow the television advertisement to have an enduring effect and generate valuable adstock over future periods. The speed at which the adstock fades is naturally dictated by the parameter β_1.

Similarly, the adstock generated by marketing on Youtube ($\underline{\text{yads}}$) can be filtered using an updating equation of the type,

$$\underline{\text{yads}_t} = \beta_2 \underline{\text{yads}_{t-1}} + \alpha_2 y_t \quad .$$

Finally, the filtered adstock can be used to produce a prediction $\hat{s}_t$ for the product's sales s_t, hour-by-hour, according to the nonlinear equation,

$$\hat{s}_t = \mu + \phi_1 \underline{\text{gads}_t}^{\delta_1} + \phi_2 \underline{\text{yads}_t}^{\delta_2} \quad ,$$

where μ is an intercept parameter, and $\delta_1 > 0$ and $\delta_2 > 0$ are parameters that define the impact that the ad stock of tv and web-based adds have on sales. Note that this equation allows for diminishing returns to scale on marketing expenditures for each individual channel (when $\delta_1 < 1$ and $\delta_2 < 1$).

1. Plot the available data on sales, google search expenditures and youtube expenditures.

2. Estimate the parameters of the following nonlinear dynamic time-varying parameter model

$$\mathrm{s}_t = \mu + \phi_1 \underline{\mathbf{gads}}_t^{\delta_1} + \phi_2 \underline{\mathbf{yads}}_t^{\delta_2} + \varepsilon_t \ ,$$

$$\underline{\mathbf{gads}}_t = \beta_1 \underline{\mathbf{gads}}_{t-1} + \alpha_1 g_t \ ,$$
$$\underline{\mathbf{yads}}_t = \beta_2 \underline{\mathbf{yads}}_{t-1} + \alpha_2 y_t \ .$$

   You can estimate the parameter vector $\boldsymbol{\theta} = (\mu, \phi_1, \phi_2, \delta_1, \delta_2, \alpha_1, \alpha_2, \beta_1, \beta_2)$ by minimizing a least squares function for the prediction of sales,

$$\hat{\boldsymbol{\theta}}_T \ := \ \arg\min_{\boldsymbol{\theta} \in \Theta} \frac{1}{T} \sum_{t=2}^{T} \left(\, s_t - \hat{s}_t(\boldsymbol{\theta}) \, \right)^2.$$

   where $\hat{s}_t = \mu + \phi_1 \underline{\mathbf{gads}}_t^{\delta_1} + \phi_2 \underline{\mathbf{yads}}_t^{\delta_2}$.

   Use $\boldsymbol{\theta} = (\mu, \phi_1, \phi_2, \delta_1, \delta_2, \alpha_1, \alpha_2, \beta_1, \beta_2) = (1, 1, 1, 0.5, 0.5, 5, 5, 0.9, 0.9)$ as an initial value for your optimization algorithm. Report the parameter estimates obtained by your estimator and the value of the minimised objective function.

   - Tip: During the estimation procedure, you may want to restrict the parameters appropriately. In particular, the parameters μ, ϕ_1, ϕ_2, α_1 and α_2 should be positive, and the parameters δ_1, δ_2, β_1 and β_2 should lie between 0 and 1. You can use a constrained optimizer or, alternatively, use an unconstrained optimizer but transform the parameters appropriately. For example, can use an exponential transformation to ensure positivity of the parameters, and you can use a logistic transformation for parameters in $[0, 1]$.

   - Tip: estimation of complex nonlinear dynamic models is fraught with challenges. Note that, in this model, the parameters ϕ_1, ϕ_2, α_1 and α_2 are nearly unidentified. In particular, different combinations of ϕ_i and α_i can deliver very similar results. For this reason, you should not be surprised if small differences in estimation algorithm, parameter restrictions, of initial values, produce vastly different point estimates.

3. Plot the filtered adstock generated by marketing on Google search and Youtube.

   - Tip: If you believe that the parameter estimation did not work properly. Then consider using the following parameter values to answer this and all remaining questions

     $$(\mu, \phi_1, \phi_2, \delta_1, \delta_2, \alpha_1, \alpha_2, \beta_1, \beta_2) = (0.617, 2.88, 19.51, 0.27, 0.60, 0.203, 0.065, 0.949, 0.95)$$

4. Use the fitted model to produce an impulse response function for both the adstock and sales volume generated by spending 100 euros with marketing on Youtube on a given hour. **Note:** The IRF should have an origin that sets adstock to zero $\underline{\text{yads}} = \underline{\text{gads}} = 0$ and sales fixed at the estimated intercept $s = \hat{\mu}_T$. Furthermore, the impulse should be obtained by setting Youtube adds to 100 euros, $y_t = 100$, at time $t = s$.

5. Explain carefully the business interpretation of the IRF produced in the previous question.

6. Use the IRF to calculate the expected accumulated additional sales produced by the impulse of 100 euro expenditure on Youtube on a given hour.

7. Use a similar IRF to calculate the expected accumulated additional sales produced by the impulse of 300 euro expenditure on Youtube on a given hour. Do you obtain three times as much sales when spending 300 euros rather than 100 euros? Provide an explanation for your findings.

8. Is the AB-testing procedure important? Without AB-testing, would you be concerned about simultaneity issues and resulting endogeneity of regressors? How would that affect the interpretation of the results of your model? Explain your reasoning.

Dynamic Pricing

A retailer of electronic products has called upon you to help design a data-driven procedure for optimizing the prices for their products. The activity of this company consists essentially of buying consumer electronics (e.g. cell phones, laptops, etc.) from large manufacturers (Apple, Samsung, etc.) and selling these products directly to consumers both online and on their shops.[1]

As a proof of concept, you will start working on one of their products. You have at your disposal a set of daily de-seazonalised data on sales s_t, selling price p_t, acquisition cost c_t, and marketing expenditures m_t.

9. Plot the available data on sales, prices, acquisition costs and marketing expenditures.

10. Run a simple regression of sales s_t on prices p_t,

$$s_t = \alpha + \beta p_t + \varepsilon_t \ . \tag{1}$$

Report the parameter estimates.

11. Comment on the usefulness of this regression as a predictive model for sales. Suppose your estimator is consistent, to which limit will it converge? What are the properties of that limit?

12. Comment on the value of the same regression as a structural model for analyzing the causal impact that a change in price p_t has on expected sales s_t. Explain your reasoning carefully.

Concerned by the possibility that price p_t is an endogenous regressor in (1) due to simultaneity, you decide to follow an instrumental variable approach.

13. Run a regression of prices p_t on costs c_t,

$$p_t = \delta + \gamma c_t + u_t \ ,$$

and use predicted price $\hat{p}_t$ to find the structural-causal relation between sales and prices,

$$s_t = \alpha + \beta \hat{p}_t + \varepsilon_t \ .$$

Report the parameter estimates for both the instrumental regression and the sales regression. What is the causal impact of an increase in price p_t on expected sales s_t?

[1]The recorded price p_t includes small variations due to promotions and discounts and price differences across stores. The recorded costs include variations due to bundling, transport and exchange rate adjustments. Recorded sales include items sold on different stores and platforms, at different prices and discounts.

14. Test for endogeneity of p_t in the original regression (1) using a Hausman-Durbin-Wu test. Do not use a built-in function or result in a summary table for the Hausman statistic, but code up the test yourself. If this calculation is missing in your code, zero points will be awarded for this question.

    - Tip: For the Hausman-Durbin-Wu test you may use the following test statistic $H^T H = T(\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})^T(\tilde{\boldsymbol{\Sigma}} - \hat{\boldsymbol{\Sigma}})^{-1}(\tilde{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}})$, where $\tilde{\boldsymbol{\theta}}, \hat{\boldsymbol{\theta}}$ are both consistent for $\boldsymbol{\theta}$ under H_0, but $\hat{\boldsymbol{\theta}}$ is an efficient estimator. Under the alternative $\tilde{\boldsymbol{\theta}}$ is consistent, while $\hat{\boldsymbol{\theta}}$ is not. Furthermore, $\sqrt{T}(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} N(\mathbf{0}, \tilde{\boldsymbol{\Sigma}})$ and $\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} N(\mathbf{0}, \hat{\boldsymbol{\Sigma}})$ as $T \to \infty$. Under the null hypothesis the test statistic follows asymptotically the χ^2 distribution with k degrees of freedom, where k is the rank of $(\tilde{\boldsymbol{\Sigma}} - \hat{\boldsymbol{\Sigma}})$.
    - Tip: For the OLS and 2SLS estimator in a linear regression model with data matrix $\boldsymbol{X}$ and instrument matrix $\boldsymbol{Z}$, we have asymptotic distributions
      $\sqrt{T}(\hat{\boldsymbol{\beta}}_{OLS} - \boldsymbol{\beta}) \xrightarrow{d} N\left(\mathbf{0}, \sigma^2(\boldsymbol{X}'\boldsymbol{X})^{-1}\right)$,
      $\sqrt{T}(\hat{\boldsymbol{\beta}}_{2SLS} - \boldsymbol{\beta}) \xrightarrow{d} N\left(\mathbf{0}, \sigma^2(\boldsymbol{Z}'\boldsymbol{X})^{-1}(\boldsymbol{Z}'\boldsymbol{Z})(\boldsymbol{X}'\boldsymbol{Z})^{-1}\right)$ as $T \to \infty$
      When estimating these variance matrices, carefully consider how to evaluate $\hat{\sigma}^2$ for each of the estimators. You may want to consult additional sources for this.

15. Consider improving the sales model and the estimate of the causal relation between sales and prices by including marketing expenditures on the sales regression,

    $$s_t = \alpha + \beta \hat{p}_t + \psi m_t + \varepsilon_t \ .$$

    Report the parameter estimates you obtain.

16. Use this model to calculate the causal impact on expected profit π_t of a unit increase in price p_t. Note that, for a given cost c_t, the expected profit is given by

    $$\mathbb{E}(\pi_t | c_t) = \mathbb{E}(s_t) \cdot (p_t - c_t)$$

    Additionally, use the fact that $\mathbb{E}(s_t) = \alpha + \beta p_t + \psi m_t$ to obtain,

    $$\mathbb{E}(\pi_t | c_t) = (\alpha + \beta p_t + \psi m_t) \cdot (p_t - c_t).$$

    Based on your finding, would you recommend an increase or decrease in price?

    Note: Use the last observed values in the sample to answer your question.