Assignment

Multivariate Econometrics

2022

MSc Econometrics and Operations Research

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Group 24

09-12-22

Abstract

The current paper examines the relationship between climate change and economic production in India between 1961 and 2016. It takes rainfall and temperature as climate variables, and agricultural land and agricultural GDP as economic variables. Further, by using a combination of them, we also obtain land productivity and agricultural GDP per capita. Throughout this paper, we use multivariate time series econometrics methods such as the Engle-Granger two-step approach or the Augmented Dickey-Fuller test to base our findings on model estimation, hypothesis testing and cointegration analysis. Specifically, we find evidence of a unit root presence in agricultural GDP and agricultural land, whereas there is not a clear statement for the rainfall and temperature variables.At the cointegration level, we find unclear results; thus policy recommendation is rather hard. Further research is suggested instead. The paper consists of four sections, such that section 1 on a time series analysis based on simulation where wide-sense and unit root non-stationary is discussed. Section 2 carries out a graphical analysis, and section 3 analyzes the order of integration of the chosen variables. Finally, section 4 realizes a deep cointegration analysis.

1 Time series analysis based on simulation

The next section of the work will discuss the behaviour and the necessary conditions for wide-sense stationary and unit root non-stationary processes based on multivariate time-series simulated data. Further, we will close the section by discussing the presence of a unit root process in a particular univariate time-series data by using the Dickey-Fuller test. In subsection 1, we first introduce the assumptions needed for the wide-sense stationary VAR process and then comment on the properties and features, focusing on the averages and variances of the simulated data and the time series processes. Throughout section 2, we follow a similar procedure to section 1, but with unit root non-stationary processes, i.e., we start by setting the proper assumption and simulating the data and then comment on their properties. Finally, in section 3, we employ a univariate time-series non-stationary process to implement the Dickey-Fuller test. Along the section, when needed, we make use of plots to support our analysis and interpretation of this descriptive part.

1.1 Wide-sense stationarity

1.1.1 Simulation and necessary conditions

We are first asked to set our assumption in a 3 x 1 multivariate time-series data by using the following DGP: $x_t = \delta + \Lambda x_{t-1} + \epsilon_t$ [1]. By setting the following assumptions about all components, we can ensure it:

1. ϵ_t follows a N(0,1).
2. Λ is a diagonal matrix with absolute values less than one, such that:

$$\Lambda = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.9 \end{bmatrix} \tag{1}$$

By setting those assumptions, we can confirm that x_t is wide-sense stationarity since the necessary conditions for stationary of VAR processes are satisfied. Those conditions require to have a time-invariant intercept, errors are identically distributed for all t and the stability condition is met. More details bellow:

1. $\delta_t = \delta$.
2. The vectors ϵ_t are identically distributed for all t.
3. The stability condition is satisfy on Λ, i.e., $\Lambda^n \to 0$

For our particular simulation:

1. $\delta_t = \delta$ is set in the statement of the question, and we gave a value of δ, which is time-invariant:

$$\delta_t = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{2}$$

[1] For more details, refer to the assignment questions.

2. Since ϵ_t follows a N(0,1), the vectors ϵ_t are identically distributed for all t.

3. $\Lambda^n \to 0$ is satisfied when the eigenvalues of Λ strictly lie inside the unit circle. This is implied if the roots of $|\Lambda - zI| = 0$ strictly lie inside the unit circle. For described values of Λ, $|\Lambda - zI| = (0.1 - z_1) \cdot (0.5 - z_2) \cdot (0.9 - z_3) = 0$ and $z_1 = 0.1, z_2 = 0.5, z_3 = 0.9$. Since the roots are less than 1 in absolute value, they lie inside the unit circle, and the eigenvalues of Λ strictly lie inside the unit circle. Thus the stability condition is satisfied for Λ.

Since the three conditions above are satisfied, we can conclude that x_t is wide-sense stationary.

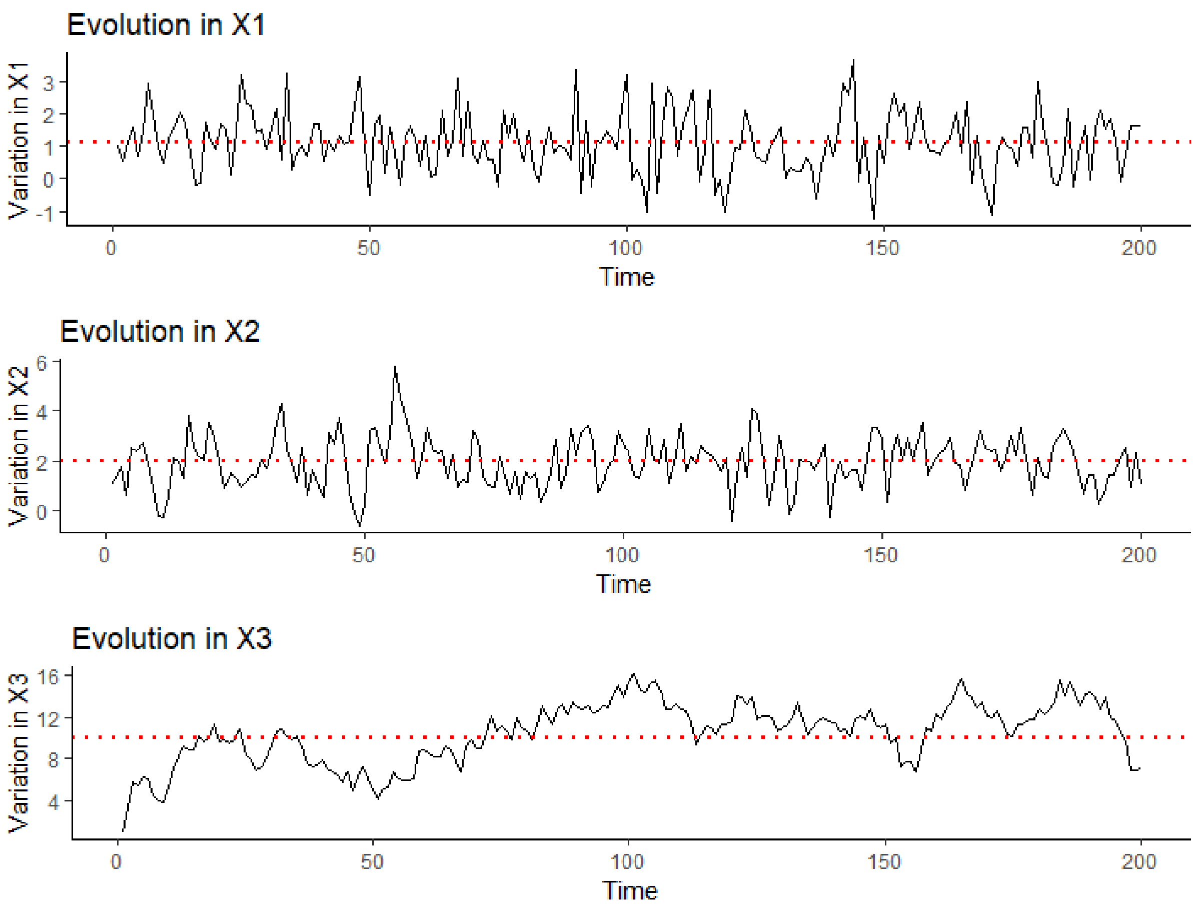

Figure 1: Evolution of the components of the generated vector x_t throughout the whole period T

Notes: The Figure shows the data generated on each of the components of the vector $x_t = (x_{1,t}, x_{2,t}, x_{3,t})$, $x_{1,t}$ *(top panel),* $x_{2,t}$ *(middle panel) and* $x_{3,t}$ *(bottom panel). The x-axis shows a time axes, whereas the y-axis displays the variation in each component. The red dotted line represents the expected value of each of the components subtracted from the geometric series convergence that states:* $\sum_{i=0}^{\inf} r^i = 1/(1-r)$ *if* $|r| < 1$*, where* r *symbolize the coefficient of* x_{t-1}.

1.1.2 Comparation of the averages, the mean, and the variances between the simulations and the processes.

Along this subsection, we are first obtaining and comparing the averages ($\bar{x_t}$) and the mean $[E(x_t)]$. Since Λ is stable (proved before), the matrix can be inverted, and we have that $[E(x_t)] = (I - \Lambda)^{-1} \cdot \delta$.

Following the assumptions imposed before and the given value of $\delta_t = (2)$, we have that:

1. $x_{1,t} = 1 + 0.1x_{1,t-1} + \epsilon_t$, thus $E(x_{1,t}) = 1/(1-0.1) = 1.111$

2. $x_{2,t} = 1 + 0.5x_{2,t-1} + \epsilon_t$, thus $E(x_{2,t}) = 1/(1-0.5) = 2$

3. $x_{3,t} = 1 + 0.9x_{3,t-1} + \epsilon_t$, thus $E(x_{3,t}) = 1/(1-0.9) = 10$

From these three processes, we get the average and the mean as follows:

Table 1: Average for each time series data simulated, the mean of the time series processes and the difference between the two.

	λ_i	$(\bar{x_t})$	$[E(x_t)]$	$(\bar{x_t}) - [E(x_t)]$
X1	0.1	1.124	1.111	0.012
X2	0.5	1.937	2	-0.063
X3	0.9	10.448	10	0.448

Notes: The table above displays by columns λ_i, the average for each time series simulated, the mean of the time series processes and the difference between the last two, respectively.

In table 1, we can see the average of the simulation data and the mean process, as well as the values of λ_i, i.e., the coefficient of x_{t-1}. The lower the values of λ_i, the lower the mean of the process and the average of the simulation and vice versa. We can also see how the bigger λ_i, the more significant the difference between the average of the data simulated and the mean of the process.

Second, we are obtaining and comparing the variance for each 3 time series data simulated and the variance of the 3 time series processes.

The variance of the process can be defined as[2]:

1. $x_{1,t} = 1 + 0.1x_{1,t-1} + \epsilon_t$, thus $var(x_{1,t}) = 1/(1-0.1^2) = 1.010$

2. $x_{2,t} = 1 + 0.5x_{2,t-1} + \epsilon_t$, thus $var(x_{2,t}) = 1/(1-0.5^2) = 1.333$

3. $x_{3,t} = 1 + 0.9x_{3,t-1} + \epsilon_t$, thus $var(x_{3,t}) = 1/(1-0.9^2) = 5.263$

Table 2: Variance for the simulation and the time series processes and the difference between the two.

	λ_i	**Variance simulation**	**Variance time series processes**	**Difference on variances**
X1	0.1	0.893	1.010	-0.117
X2	0.5	1.025	1.333	-0.308
X3	0.9	8.300	5.263	3.036

Notes: The table above displays by columns λ_i, the variance of the simulation, the variance of the time series processes and the difference between the last two, respectively.

[2] Derived from $var(x_{i,t}) = var(\delta) + var(\lambda_i \cdot x_{i,t-1}) + var(\epsilon_{i,t}) + 2 \cdot cov(\delta, \lambda_i \cdot x_{i,t-1}) + 2 \cdot cov(\delta, \lambda_i \cdot \epsilon_{i,t}) + 2 \cdot cov(x_{i,t-1}, \epsilon_{i,t}) \overset{Var/cov(constant)=0; cov(\epsilon_t,)=0}{=} 0 + {\lambda_i}^2 \cdot var(x_{i,t-1}) + 1 + 0 + 0 + 0 \overset{x_t stationary: var(x_t)=var(x_{t-1})}{=} {\lambda_i}^2 \cdot var(x_{i,t}) + 1 \Leftrightarrow var(x_{i,t}) = \frac{1}{1-{\lambda_i}^2}$

Table 2 shows the variance of the simulation data and the process, as well as the values of λ_i, i.e., the coefficient of x_{t-1}. The lower the values of λ_i, the lower the variance of the process and the variance for the simulation and vice versa. We can also see how the bigger λ_i, the more significant the difference between the variance of the data simulated and the variance of the process.

1.2 Unit root non-stationarity

As in the first part of the subsection, we are given the DGP: $x_t = \Lambda x_{t-1} + \epsilon_t$ and we are asked to impose assumptions to ensure unit root non-stationarity processes. Assumptions:

1. ϵ_t follows a N(0,1).

2. Λ is the identity matrix:

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

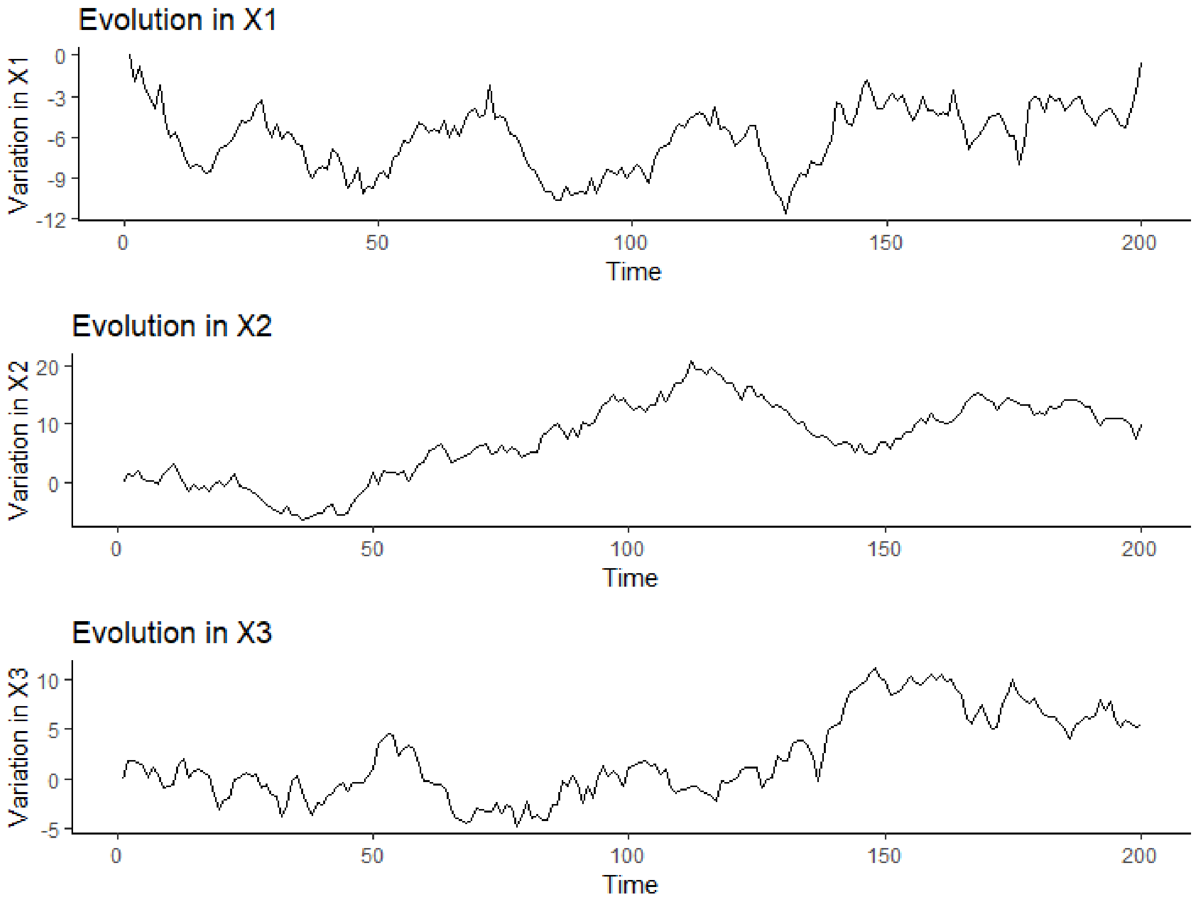

Figure 2: Evolution of the components of the generated vector x_t throughout the whole period T

Notes: The Figure shows the data generated on each of the components of the vector $x_t = (x_{1,t}, x_{2,t}, x_{3,t})$, $x_{1,t}$ *(top* *panel), $x_{2,t}$ (middle panel) and $x_{3,t}$ (bottom panel). The x-axis shows a time axes, whereas the y-axis displays the variation in each component.*

The simulated multivariate time-series can be decomposed in three AR(1) processes with $\lambda = 1$ since there is no intercept ($\delta = 0$) and Λ is equal to the identity matrix, that is to say, each of the component's lags values only affect itself. Thus for $x_{i,t} = \lambda x_{i,t-1} + u_{i,t}$ where $i \in \{1, 2, 3\}$ and $\lambda = 1$. This is by definition a I(1) process, also called unit root process.

Since every shock persists infinitely in the memory of the process, non-stationary processes have a sample mean $\bar{x}_t$ that is diverging and, as a consequence, distinctive non-standard asymptotic properties.

1.3 Univariate unit root non-stationary process and the Dickey-Fuller test

Within this subsection, we first simulate a univariate time-series data: $x_t = x_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, 1)$, and then, we test for a unit root by using the Dickey-Fuller test:

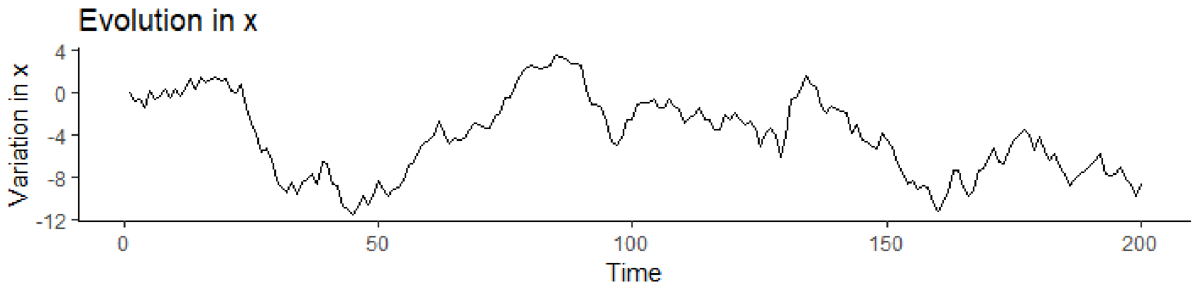

Figure 3: Evolution of x_t throughout the whole period T

Notes: The Figure shows the data generated for x_t. The x-axes show time axes, whereas the y-axes display variations for x_t.

We close the subsection by testing for a unit root in the simulated data by using the Dickey-Fuller test.

We first want to check the presence of a deterministic trend by assuming the following model: $x_t = B_0 + B_1 \cdot T + u_t$, where x_t is the simulated data, T is Time and u_t the residual. Then we set as null hypothesis $H_0 : B_1 = 0$ (determinist trend nonexistent) and alternative hypothesis $H_1 : B_1 \neq 0$ (existence of a deterministic trend).

We ran a regression of the simulation on time, and we got $B_1 = -0.0197$ as the coefficient of the time variable. Such a coefficient close to 0 allows us to reject the idea of having a deterministic trend that changes with time.

Then we want to test for the existence of a stochastic trend on the simulated data with a Dickey-Fuller test. For a model such that $x_t = \lambda x_{t-1} + \epsilon_t$ this test holds as null hypothesis $H_0 : \lambda = 1$ (non-stationary) and as alternative hypothesis $H_1 : |\lambda| < 1$ (stationary).

Note that when testing for the existence of a unit root, the Dickey-Fuller statistic does not have a normal distribution, even in large samples, so we need to compare it with different critical values. Since we discarded the option of a time trend based on the previous regression on time, we consider the critical values where the regression only includes the intercept: -2.57 at 10% significance, -2.86 at 5% significance and -3.43 at 1% significance.

The test on the simulated data displayed the following results:

$$\text{Test statistic} = -2.1081 \text{ and P-value} = 0.5308$$

Since the test statistic is not negatively big enough (and then the p-value is not lower than 0.05), we do not have enough evidence to reject the null hypothesis; thus, we cannot say that the univariate unit root process simulated is stationary [3].

2 Graphical Analysis

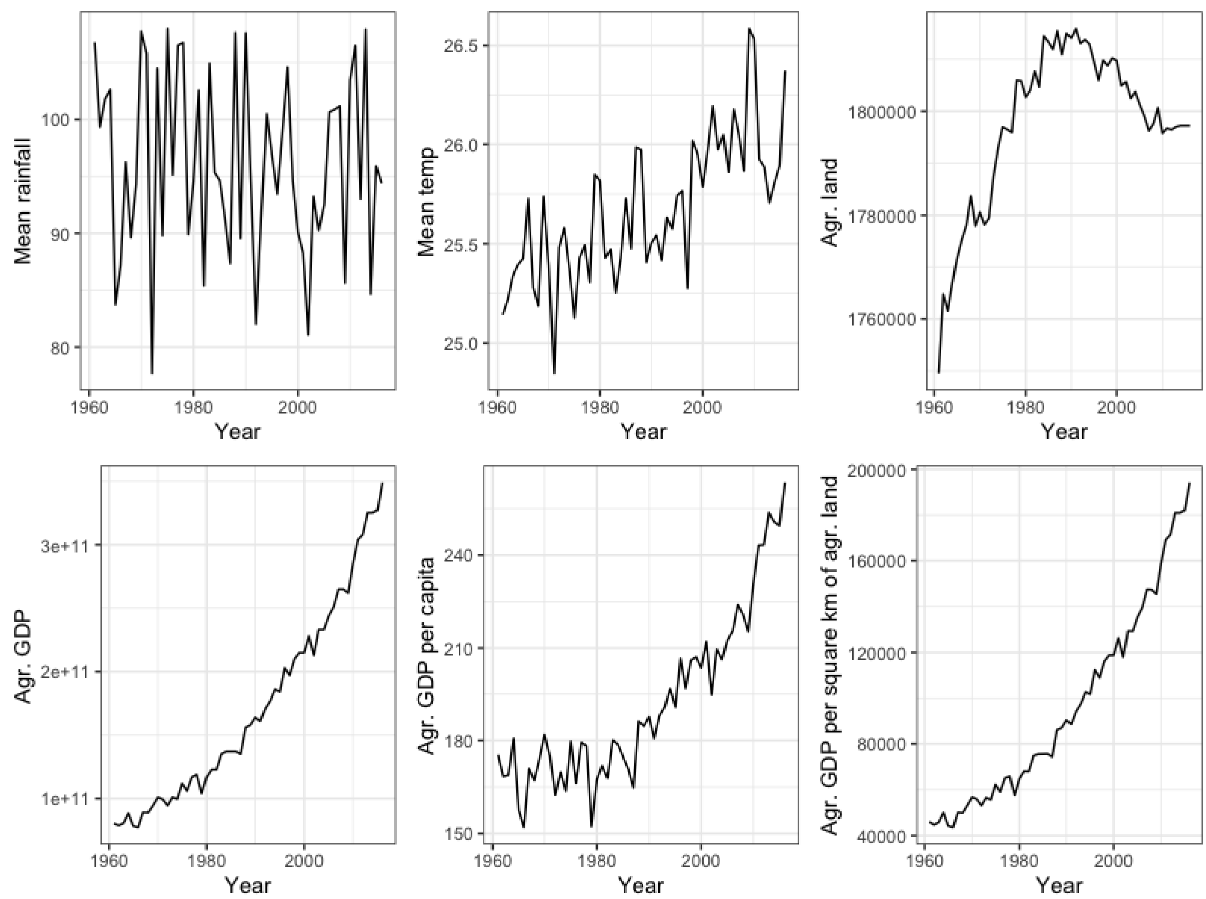

Figure 4: Plotting of raw data for all variables

We have decided to choose two climate variables: Annual average monthly total rainfall, Annual average temperature, as well as two economic variables: Agricultural land and Agricultural GDP. We also add in figure 4 two extra variables that will be used in the analysis, namely, Agricultural GDP per km^2 of agricultural land, as well Agricultural GDP per capita. These variables are based on the four we chose.

[3]Note that by definition we already know that this type of process is non-stationary.

Furhtermore, Figure 8 in the appendix shows the four main variables in levels, log, first and second differences.

These variables were chosen based on theory, and potentially interesting relationships among the variables. Firstly, a warmer atmosphere holds more moisture. More specifically, an increase in 1 degree Celsius increases the air's ability to hold 7 percent more moisture. Evidence shows that global warming not only increases rainfall variability (more erratic monsoon seasons and longer drought periods in India), it also increases overall annual rainfall, although large spatial variations exists (Wang et al., 2015).

Secondly, both increases in temperatures and rainfall have a positive effect on agriculture. Crops are able to grow more due to an increase in CO_2 concentration in the atmosphere, and rainfall is especially important for crops under water stress.

Lastly, at very high temperatures, some crops are not able to grow optimally, which would significantly affect Agricultural GDP. At the same time very high temperatures can also make some crop fields unsustainable since it decreases favorable growing conditions in the soil.

When focusing on Figure 4, agricultural GDP seems to be the only variable to exhibit some sort of deterministic component. More specifically, is may exhibit trend stationarity. The variable trends upwards, so it might be the case that there is a deterministic component in its data generating process. If the trend were to be removed, the process would be integrated of order zero. (stationary). The data slightly curves up, indicating that there might actually be a quadratic trend.

Mean rainfall is the only variable that might be a stationary $I(0)$ series. The other variables are likely to be non-stationary, since they do not exhibit constant mean nor variance throughout the time series.

It's common practice in the literature to take the logarithm for GDP, so we will do the same here for Agricultural GDP. Furthermore, We will define two new variables which are agricultural GDP divided by total agricultural land (in km^2). This would be a measure for the productivity of the land, i.e., the amount of economic value per km^2 in agriculture. The second variable will be Agricultural GDP per capita. We will take the logarithm of both variables.

3 Analysis of order of integration

3.1 Choice of deterministic components

It is important to find deterministic trends (is there a linear trend or intercept), after which the models are tested for the presence of a unit root. First, we looked at the plots in figure 4 to graphically test whether a unit root and/or deterministic trend is present for the four variables. The following statements can be argued:

1. The annual average of monthly total rainfall (mean pre) could be a random walk model if it would be non-stationary. There is no evidence of a drift since the mean seems constant. However, by looking at the graph it seems that the model could be stationary. This will be determined later by conducting unit root tests.

2. The annual average temperature could be non-stationary. It can be argued that a drift is present since there is an increase in the average temperature as the year increases.

3. The agricultural land could have a deterministic trend and an intercept if it is non-stationary. This can be seen in the graph as there is an upwards increase in agricultural land as time continues on. It can be argued that the model is trend stationary and follows a negative quadratic relationship that flattens out eventually.

4. The agricultural GDP could also have a deterministic trend with an intercept if the model is non-stationary. The model represented in figure 4 shows an upwards increase in GDP. Also, the model seems trend stationary as there is an exponential increase.

3.2 Unit root testing

Many tests exist to test for unit roots because none have a much larger statistical power than other tests. The Dickey-Fuller test does not account for serial correlation while the Augmented-Dickey-Fuller does. This test is suited for large and complex models. However, it has considerably high Type I error rates. Next, the Phillips-Perron test corrects for autocorrelation and heteroscedasticity of the errors and is considered an extension of the ADF test. Another example is the Zivot-Andrews test, which allows for structural breaks. Lastly, you also have the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test. This test is based on linear regression but also has high Type I errors. Correcting this results in lower test power. A solution is to combine it with an ADF test (Kotz and et al, 2006; Vogt, 2005).

3.2.1 Specification of (Augmented) Dickey-Fuller tests

The process follows a Dickey-Fuller distribution if a unit-root is present. There are three specifications of the (Augmented) Dickey-Fuller tests ((R-bloggers, 2021)):

1. $\delta y_t = \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \delta y_{t-i+1} + \eta_t$
   (τ) $H_0 : \gamma = 0$,a unit root is present

2. $\delta y_t = \gamma y_{t-1} + \alpha_0 + \sum_{i=1}^{p} \beta_i \delta y_{t-i+1} + \eta_t$
   ($\phi 2$) $H_0 : \gamma = 0$ & $\alpha_0 = 0$, a unit root with no intercept
   (τ) $H_0 : \gamma = 0$, a unit root is present

3. $\delta y_t = \gamma y_{t-1} + \alpha_0 + \alpha_2 t + \sum_{i=1}^{p} \beta_i \delta y_{t-i+1} + \eta_t$
   ($\phi 3$) $H_0 : \gamma = 0$ & $\alpha_0 = 0$ & $\alpha_2 = 0$, unit root with no intercept or trend
   ($\phi 2$) $H_0 : \gamma = 0$ & $\alpha_0 = 0$, a unit root with no intercept
   (τ) $H_0 : \gamma = 0$, a unit root is present

3.2.2 The Dickey-Fuller test

Table 3: Dickey-Fuller Test

	τ	$\phi2$	$\phi3$	**CV 1**	**CV 2**	**CV 3**
average rainfall	-0.39			-1.95		
average temperature	-2.79	4.04		-2.89	4.71	
agricultural land	-1.62	7.32	5.81	-3.45	6.49	4.88
agricultural GDP per capita	-1.82	3.12	3.24	-3.45	6.49	4.88

Notes: The table above displays the results of the Dickey-Fuller test.

H_0: The process is a unit root process.
H_a: The process is stationary.

According to the results, the test statistic of average rainfall is less extreme than the critical value. Thus, we fail to reject the null hypothesis that a unit root is present. Second, the test statistic of average temperature is for both τ and $\phi2$ smaller than the corresponding critical values, meaning that there is not enough evidence to reject the null hypothesis; the average temperature has a unit root. Third, the agricultural land test statistic for τ is smaller than the critical value while $\phi2$ and $\phi3$ are larger than the corresponding critical values. Thus, we reject the null hypothesis of $\phi2$ and $\phi3$, suggesting the process is stationary. Finally, the three test statistics of agricultural GDP per capita are all less than their critical values, meaning that we fail to reject the null hypothesis; the process has a unit root with intercept and trend. These results seem contradictory; the results can be explained due to the poor performance of the DF test when the models are near-unit-root alternatives and when the number of observations is low (n=54).

3.2.3 Evidence of serial correlation of the DF regression

Some ways to check for serial correlation are the Breusch-Godfrey test, a correlogram, and the Ljung-Box test. The Breusch-Godrey test is a Lagrange multiplier test and is very powerful in an asymptotic setting. Since our sample size is not very large (T=56) we will disregard this test and choose better options. The Ljung-Box is based on second moments and has lots of power when a wide range of hypotheses are considered. The correlogram is always good to analyze and works in most settings.

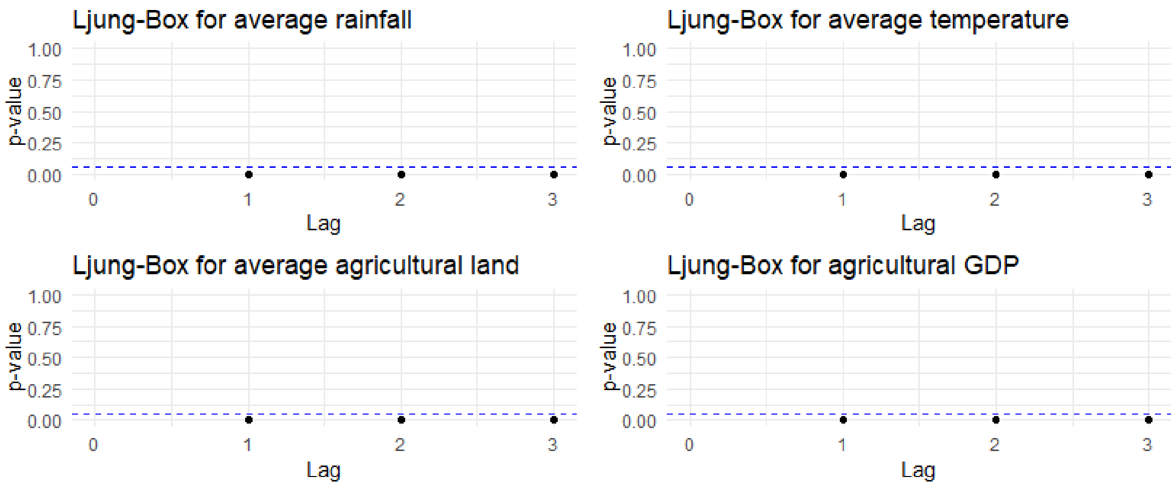

Figure 5: The p-values of the Ljung-Box statistic for the four analyzed variables. The variables are from left to right, starting from the top row: an annual average of monthly total rainfall, annual average temperature, agricultural land, agricultural GDP

H_0: The data is independently distributed.

H_a: The data is not independently distributed; it exhibits serial correlation.

The number of lags in the Ljung-Box test is chosen based on the paper 'Selecting optimal lag order in Ljung-Box test (Hassani and Yeganegi, 2019). Their study suggests that a small value of about 3 should be chosen for a time series that has length ¡ 500. Looking at the plots above, we fail to reject the null hypothesis for all variables. This suggests that none of the variables exhibit serial correlation.

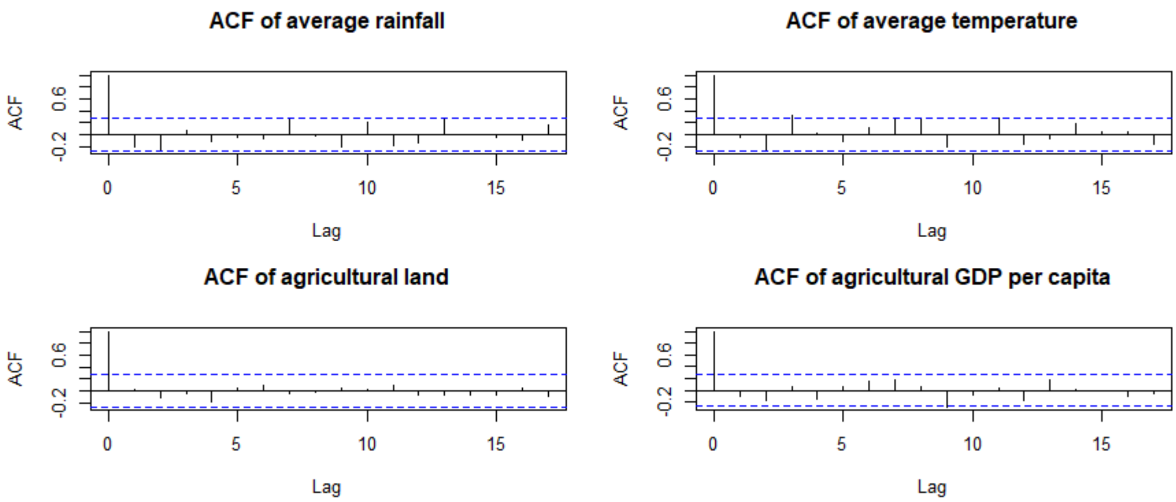

Figure 6: A correlogram representing the ACF of the four variables: annual average of monthly totagl rainfall, annual average temperature, agricultural land, agricultural GDP.

Afterward, a correlogram was plotted for the four variables. The results of the Ljung-Box test are confirmed since none of the variables are serially correlated because the ACF stays within the lower and upper bounds.

3.2.4 The Augmented-Dickey-Fuller test

Table 4: Augmented-Dickey-Fuller Test

	τ	$\phi 2$	$\phi 3$	**CV 1**	**CV 2**	**CV 3**
average rainfall	-0.16			-1.95		
average temperature	-1.48	1.38		-2.89	4.71	
agricultural land	-1.65	5.77	4.38	-3.45	6.49	4.88
agricultural GDP per capita	-1.77	2.60	3.16	-3.45	6.49	4.88

Notes: The table above displays the results of the Augmented-Dickey-Fuller test.

H_0: The process has a unit root.
H_a: The process is stationary.

By looking at the critical values, we can conclude that we fail to reject the null hypothesis for average rainfall, suggesting that the process has a unit root. Furthermore, the test statistics of average temperature are also lower than their critical values. Therefore, we fail to reject the null hypothesis and the process is said to have a unit root. Moreover, we can conclude that agricultural land also has a unit root since we fail to reject the null hypothesis. Finally, agricultural GDP per capita is also smaller than the corresponding critical values, meaning consequently that the process has a unit root. These results do not match with the graphical analysis; a discussion of the performance of the ADF is discussed in section 3.4.

3.2.5 The Philips-Perron test

Table 5: Philips-Perron Test

	test-statistic	**p-value**
average rainfall	-8.53	0.01
average temperature	-5.96	0.01
agricultural land	-2.68	0.301
agricultural GDP per capita	-2.81	0.246

Notes: The table above displays the results of the Philips-Perron unit root test.

H_0: The process is integrated of order 1; it has a unit root.
H_a: The process is stationary.

The Philips-Perron (PP) test is robust to unspecified autocorrelation and heterescedasticity. We fail to reject the null hypothesis for agricultural land and agricultural GDP per capita, suggesting that these processes are non-stationary. Average rainfall and average temperature both have a p-value of 0.01, hereby rejecting the null hypothesis; these processes are considered to be stationary according to the PP test.

3.2.6 The Zivot-Andrews test

Table 6: Zivot-Andrews Test with for structural breaks

	break trend, t-stat	**break intercept, t-stat**	**break both, t-stat**	**CV1**	**CV2**	**CV3**
average rainfall	-4.20	-4.51	-4.66	-4.42	-4.80	-5.08
average temperature	-4.16	-4.90	-4.86	-4.42	-4.80	-5.08
agricultural land	-4.10	-4.16	-4.30	-4.42	-4.80	-5.08
agricultural GDP per capita	-2.33	-2.19	-2.33	-4.42	-4.80	-5.08

Notes: The table above displays the results of the Zivot-Andrews unit root test. 'Break both trend' tests if the process is trend stationary with one time break in the trend and intercept. Similarly, 'break intercept' and 'break trend' test if the process is trend stationary with one time break in intercept or trend, respectively.

H_0: The process has a unit root with drift, which excludes exogenous structural change: $y_t = \mu + y_{t-1} + \epsilon_t$
H_a: The process is trend stationary allowing for one time break in the level, the trend, or both.

In addition to the previous test, we conducted the Zivot-Andrews (ZA) test since it allows for a single structural break (Osabuohien-Irabor, 2020). This means that the time-series can be split into two parts at a certain point (at the intercept or in trend). We can determine whether the process is trend stationary with this break. If we look at the plots of average temperature we could argue that a structural break is present since the time-series has an upward trend, then a more flattened dynamic, and then a downward trend. The ZA test confirms this since the null hypothesis is rejected: the process is trend stationary with a single break in the level. The ZA test is less relevant for the other variables but for completeness, the results are noted in the table.

3.2.7 The Kwiatkowski–Phillips–Schmidt–Shin test

Table 7: KPSS Test

	p-value level stationarity	test-statistic	p-value trend stationarity	test-statistic
average rainfall	0.1	0.11	0.1	0.048
average temperature	0.01	1.39	0.01	0.23
agricultural land	0.01	1.32	0.1	0.047
agricultural GDP per capita	0.01	-2.19	0.1	0.074

Notes: The table above displays the results of the KPSS test for level or trend stationarity.

H_0: The time series is level/trend stationary.

H_A: The time series is not level/trend stationary; it has a unit root.

The KPSS test is often combined with the ADF test since the null hypothesis and the alternative hypothesis are switched around for each test. We fail to reject the null hypothesis for average rainfall, suggesting that the time series is level/trend stationary ((Kocenda, 2017),(et al, 1992)). Average temperature rejects the null hypothesis and is not stationary. It is interesting to note that agricultural land and GDP per capita fail to reject the null hypothesis of trend stationarity but not for level stationarity. This suggests these variables are not level stationary but are trend stationary. This is feasible since we assume that these variables have a deterministic trend.

This is important to know since it is possible for a time series to be non-stationary and have no unit root, yet still be trend-stationary. By combining the trend stationary hypothesis with the unit root hypothesis, it is possible to distinguish between series that are stationary, have a unit root, or it is not possible to identify either option.

3.3 Final results of unit-root analysis

A confidence level of 5% is chosen throughout all the unit root testing. This default cut-off value seems to be a reasonable threshold for T=56 observations.

The ZA is irrelevant for all variables except for average temperature. We will not take this test into account for those variables. Furthermore, the ADF test presumes that all variables are non-stationary. A possible explanation could be that the ADF does not account for seasonal non-stationarity. Also, due to the non-consensus on the methodology to determine lag length p, it can mean that the remaining serial correlation can affect the test size; while a large p can mean a deterioration in test power. A lag-length of 4 was chosen as a standard. However, by using a specific-to-general approach, the test statistic stayed below the critical value until lag 10. Another reason for underperformance could be that the ADF test performs well in asymptotic data and our sample set is finite (T=56) (Kocenda, 2017). This could be why the ADF test performs poorly with our data. Therefore, we will disregard this test.

The order of integration is discussed for each variable.

1. The annual average of monthly total rainfall is not stationary according to the DF test. However, the PP and KPSS tests infer that the process is (level/trend) stationary. A justification could be that the process is trend stationary, explaining why the DF test concluded non-stationarity. However, further research needs to be done to find out the order of integration as the data does not provide enough observations to make a sensible conclusion.

2. The annual average temperature is stationary according to the PP test. The DF test suggests that the process has a unit root. Moreover, the KPSS test states that the process is non-stationary. Besides that, the ZA test deduces that the process is trend stationary with a single time break in level. This is in line with the graphical analysis. Thus, we can probably suppose trend stationarity with a time break in level.

3. The agricultural land is most likely to have a unit root. The PP test concludes that the process has a unit root. The KPSS test specifies that the time-series is trend stationary. Lastly, the DF test confirms that the process has a unit-root. Combining these results, we infer that the model has a unit-root with a deterministic trend since it is trend stationary.

4. The results for agricultural GDP per capita are similar to the results of agricultural land. All (used) tests state that the model has a unit-root, while the KPSS test shows trend stationarity. We can deduce that the model has a unit-root, but possibly has a deterministic trend since it is trend stationary.

4 Cointegration analysis

4.1 Cointegrating relationships in the literature

Among the four variables that have been chosen for this analysis, there may exist some links driven by common stochastic factors. First, it is likely that agricultural GDP is related to average rainfall and temperatures. In particular, as Srinivasa Rao et al. (2015) show, India's crop production strictly relies on the amount of rainfall that the country faces. This cointegrating relationship has already been shown to exist in other countries, such as China (Chandio et al., 2020) for overall agricultural output, for Pakistan (Chandio et al., 2019) in rice production, and for Bangladesh (Chandio et al., 2022) in cereal production. Warsame et al. (2021) show cointegrating relationships among crop production, carbon dioxide levels, temperature and rainfall using data from Somalia. Moreover, rainfall is revealed to improve crop production in the long-run, but hamper it in the short-run, whereas temperature has a negative effect in both the short and long-run.

Ben Zaied and Ben Cheikh (2014) study the cointegrating relationship between cereal and date production, temperature and rainfall, using a panel dataset of Tunisian regions. They find spatial variation of the effects of temperature and rainfall on crop production, mainly due to differences in elevation and climate conditions. Bhardwaj et al. (2022) use a similar approach using panel data, but use maximum and minimum temperature instead. This allows to study the effect of climate extremes, which will be more prominent as the mean temperature rises (IPCC, 2014). The authors examine the impact of temperature and rainfall on wheat and rice yield (per hectare), so, taking into account the available land for each crop. The panel dataset is based on regions in the Punjab state of India. The analysis show minimum temeperature has a positive effect on both crops, while maximum temperature has a negative effect. Rainfall is found to have a negative effect on the production of wheat. This is because 80 percent of all rainfall happen between June and September, during the south-west monsoon season. This increased rainfall is detrimental for yield since it mostly happens during the ripening stage.

Many articles do find evidence of cointegrating relationships between these variables. However, the effect of the variables is often inconsistent, since some studies find positive effects for temperature and rainfall on crop production/agricultural GDP, while others find negative effects.

4.2 Model specifications

Among the different options available given our dataset, and based on the aforementioned literature analysis, we chose to explore four different cointegrating relationships:

1. $\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right) = \alpha + \beta_1 temperature_t + \beta_2 rainfall_t + \varepsilon_t$
2. $\ln\left(\frac{Agr.GDP_t}{population_t}\right) = \alpha + \beta_1 temperature_t + \beta_2 rainfall_t + \epsilon_t$
3. $\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right) = \alpha + \beta temperature_t + \nu_t$
4. $\ln\left(\frac{Agr.GDP_t}{population_t}\right) = \alpha + \beta temperature_t + u_t$.

As can be seen, we opted for analyzing one specification with the natural logarithm of agricultural GDP per square kilometer, one with the natural logarithm of agricultural GDP per capita, once regressed against both $temperature$ and $rainfall$ and once with average temperatures. The reason why we chose to keep temperatures only in the specifications with one independent variable only is that, as shown in Section 3, rainfall appears to be I(0). Hence, the cointegrating relationship is likely non-existing in the latter case. Furthermore, the output variables have been transformed in logarithm, in order to make the series exhibit more linear dynamics – unlike in Figure 4. The plot for Agr. GDP in the second row of Figure 8 in the Appendix shows this clearly.

4.3 Cointegration tests

4.3.1 Engle-Granger two-step approach

The Engle and Granger (EG thereafter) approach is the simplest form of cointegration test. It is based on the idea that if two or more times series are cointegrated, then the residuals from a simple OLS regression should be stationary as well. More precisely, if one knows that there exist a cointegrating relationship such as $1 - \rho$ between two variables x_t and y_t such that $y_t - \rho x_t$ is a stationary series, then the error term (ψ_t) of the following regression

$$y_t = \rho x_t + \psi_t$$

is I(0). Therefore, unit root tests can be applied to the residuals of an OLS regression to analyze the presence of a potential cointegrating relationship between two or more variables. In this case, a normalization to unity of the first element of the cointegreting relationship – i.e. the dependent variable of each specification – is made (Hamilton, 1994).

We used the EG testing approach for the four model specifications above-mentioned. Specifically, after using OLS and having estimated the residuals of the four specifications, an Augmented Dickey Fuller test with no deterministic trend is applied. For additional robustness in the results, two batteries of tests have been run, depending on how the number of lags for the ADF test has been computed. First, a rule of thumb from the literature that sets nr. lags = $T^{1/3}$ was employed. Second, the lag choice was based on the highest fit from a vector-autoregressive model according to the Akaike information criteria. Note that we cannot use the critical values from Dickey and Fuller, since the asymptotic behavior of the test differs when applying it on residuals from a cointegration model. Therefore, MacKinnon (2010) offers critical values when using Augmented Dickey-Fuller for testing cointegration. The critical value is -3.897 for a cointegrating relation with three variables ($T = 3$), and -3.447 for $T = 2$, at the 5 percent significanc level. The two critical values differ because the limiting distributions shifts to the left as more variables are added in the cointegrating relationship.The results indicate that for all the four specifications, the null-hypothesis of no cointegration is not rejected (all test results are shown in Table 8 below). The results are robust to the choice of the lags. Nonetheless, it should be mentioned that having to choose a certain specification for the first-stage regression can lead to different test results, unless the R^2 of the first-stage regressions is equal to 1 (Hamilton, 1994). Furthermore, the fact that the no-cointegration hypothesis is not rejected for all the four reltionships might indicate that some of the variables concerned are stationary or integrated of order higher than 1.

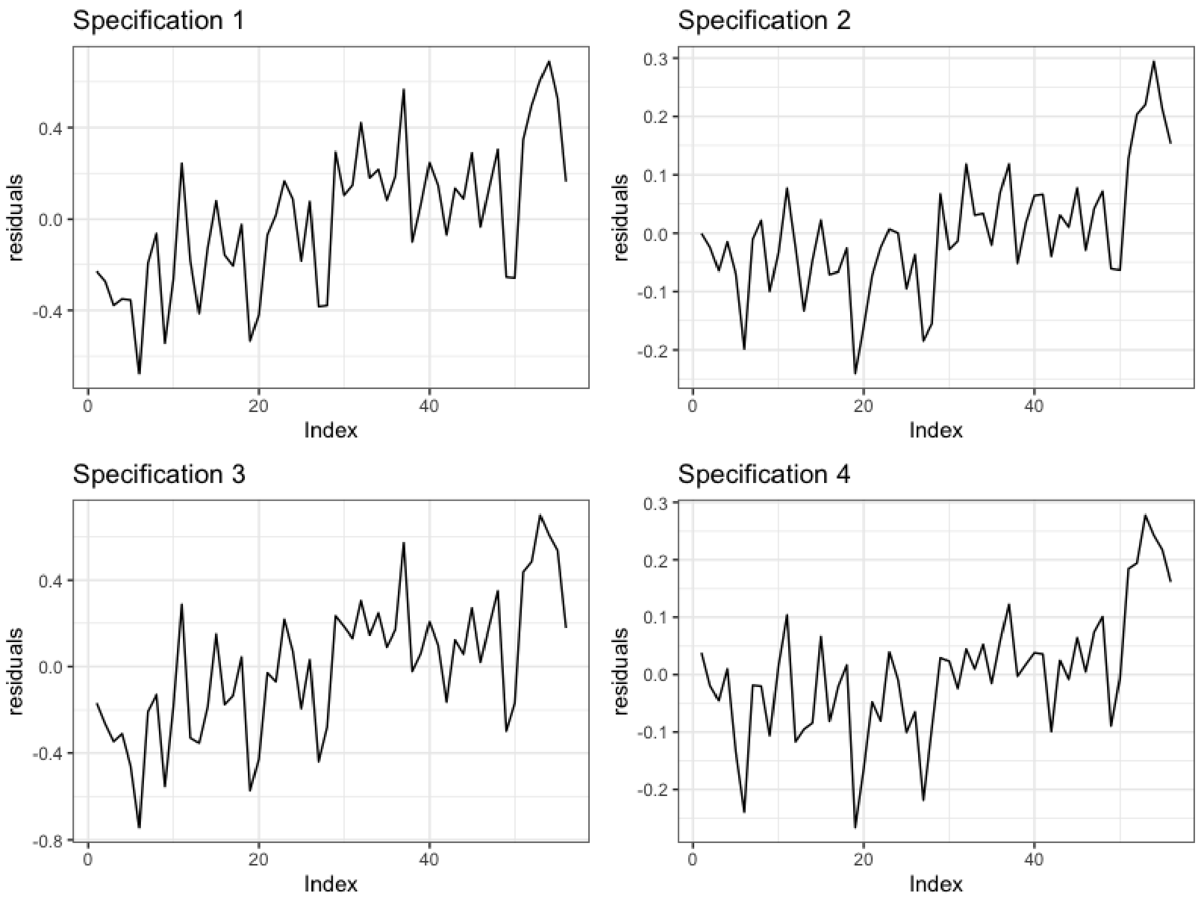

Figure 7: residuals extracted from the simple OLS regression

The figure above shows that the residuals for the first specification may be non-stationary, since the mean of the series increases slightly.

4.3.2 Philips-Ouilaris test

The Philips-Ouilaris test handles autocorrelation in the same way as the Philips-Perron test, so it can be seen as an extension of the latter for testing cointegration. Moreover, unlike for the EG approach, there is no need to specify an OLS model at first instance. This makes the testing procedure more versatile and less ambiguous. The test is corrected by estimating the nuisance parameters. We can extract the critical values of the test with the tables in Phillips and Ouliaris (1990). For the first specification, we reject the no-cointegration hypothesis at the 10 percent. For the second specification, the null-hypothesis cannot be rejected. For the remaining two models, the tests let one reject the no-cointegration hypothesis at 5% (the results can be seen on Table 8).

4.3.3 Johansen's method

The last method employed to test the presence of cointegration relationship is the Johansen approach. Specifically, the trace test has been applied to the specifications. This method is used to test the null hypothesis that the rank of the cointegrating relationship is smaller than the number of time series in the relationship. Likeilihood-ratio test statistics are computed sequentially for different values of the rank r. As soon as the null-hypothesis is not rejected, it is possible to determine the number of cointegrating relationships r (see Table 8 for a summary of the results). Like for the EG testing approach, two methods have been used for computing the appropriate number of lags to use. For all four specifications, the null-hypothesis of no more than one cointegrating vectors has been accepted. Hence, this implies that according to this test there is only one cointegrating relationship for each model. Such results are obtained when using the rule of thumb to compute the lags. When the latter are derived using the AIC, very similar results are gathered, exept for the fourth specification for which the null-hypothesis is not rejected. Hence, for all of them, this indicates the presence of one cointegrating relationship, even if not too solid for some of the models considered.

	EG ADF		Phillips-Ouliaris		Johansen	
Specification	T.stat.	CV	T.stat.	p-value	r	p-value
$\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right) = \alpha + \beta_1 temperature_t + \beta_2 rainfall_t + \varepsilon_t$	-1.7845	-3.897	-24.496	0.068	1	< 0.01
$\ln\left(\frac{Agr.GDP_t}{population_t}\right) = \alpha + \beta_1 temperature_t + \beta_2 rainfall_t + \epsilon_t$	-1.810	-3.897	-22.167	0.101	1	< 0.05
$\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right) = \alpha + \beta temperature_t + \nu_t$	-1.8607	-3.447	-24.612	0.022	1	< 0.1
$\ln\left(\frac{Agr.GDP_t}{population_t}\right) = \alpha + \beta temperature_t + u_t$	-1.4341	-3.447	-23.896	0.025	1	< 0.05

Table 8: Cointegration test results. The number of lags for the EG ADF and Johansen tests used to obtain such results is $T^{1/3} \approx 4$. In the code, it is possible to obtain the test statistics and outputs using the AIC-derived number of lags. The critical values for the EG ADF have been computed based on Table 2 in MacKinnon (2010).

4.3.4 Final results of cointegration tests

Overall, the EG approach does not indicate the presence of a cointegrating relationship for all the specifications considered. However, the Phillip-Ouliaris' and Johansen's tests show a different picture. This heterogeneity in the results could stem from the need to normalize one of the variables for the first stage regression. Concerning the Phillip-Ouliaris test, a cointegration relationship can be identified in all the specifications but the second one. Regarding the Johansen method, a cointegrating relationship is shown to exist for all models, when the number of lags is 4. Table 8 sums up the results just mentioned. The r in the first column shows the maximum number of cointegrating relationships for which the null-hypothesis of no cointegration can be rejected.

In short, given such results, it is hard to find too clear and coherent evidence of cointegration among the four specifications.

4.4 Estimation of the cointegrating vector

By assuming the presence of cointegration in the four models, it is possible to estimate such relationships. This has been done by running the Static OLS, the Error Correction Model, the Dynamic LS and the Fully Modified LS. To begin with, the Static OLS (SOLS thereafter) estimator is discussed. Such model is consistent, if the variables of the system are cointegrated, as it is assumed in this case. Moreover, SOLS does not take into account the dynamic structure of the variables, as the estimation occurs at the same time period for all of them. As shown in the literature, however, such estimation is subject to bias. This occurs because of two reasons. First, the model is subject to simultaneity, due to cross-sectional dependence. Second, there might be autocorrelation in the error term. Both are very likely to be present in this case as well. In fact, a Durbin-Watson test has been run for all the four SOLS regressions and the null-hypothesis of no-autocorrelation has been firmly rejected in all cases. A consequence of this is misleading inference of the estimated coefficients (Stock and Watson, 2019).

Second, the Error Correction Model is employed. This modelling approach hinges on the fact that the variables of the specifications are I(1) and by taking first differences it is possible to obtain a stationary model. For instance, if one looks at the first specification and assumes that there exist a cointegrating relationship as

$$\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right) = \alpha + \beta_1 temperature_t + \beta_2 rainfall_t + \varepsilon_t$$

Then, the first-difference of $Agr.GDP_t/Agr.land_t$, which is stationary, is a function of the deviations from the equilibrium relationship in the previous period (Lütkepohl, 2007), expressed by the lagged cointegrating relationship:

$$\Delta\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right) = \alpha\left[\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right) - \beta_1 rainfall_{t-1} - \beta_2 temperature_{t-1}\right] + \varepsilon_t.$$

To this, the model can also be endowed with lagged first-differences from all variables in the system:

$$\begin{aligned}\Delta\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right) = \alpha\left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right) - \beta_1 rainfall_{t-1} - \beta_2 temperature_{t-1}\right] + \\ \rho_1\Delta rainfall_{t-1} + \\ \rho_2\Delta temperature_{t-1} + \varepsilon_t.\end{aligned}$$

The advantage of this model, compared to SOLS, is that the issue of autocorrelation is eliminated. This is confirmed by the Durbin-Watson test, with which the null-hypothesis of no-correlation in the residuals is accepted for each specification. Moreover, using this model setting for all the specifications, an estimation of the error correction term can be obtained. The intercept term has been added for the first specification only, as for the others it was not statistically distinguishable from zero. From the estimates obtained, it is possible to get the estimated cointegrating relationships:

Spec. 1): the coefficient of the speed of adjustment – i.e. the coefficient for $\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right)$ – is not significant. Hence, according to the ECM there is no cointegration.

$$\begin{aligned}\textbf{Spec. 2):} &\left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) - \hat{\beta}_1 rainfall_{t-1} - \hat{\beta}_2 rainfall_{t-1}\right] \\ &= \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) + \frac{0.001}{0.117} rainfall_{t-1} + \frac{0.021}{0.117} temperature_{t-1}\right] \\ &= \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right) + 0.18 temperature_{t-1}\right]\end{aligned}$$

Spec. 3): the coefficient of the speed of adjustment – i.e. the coefficient for $\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right)$ – is not significant. Hence, according to the ECM there is no cointegration.

$$\begin{aligned}
\textbf{Spec. 4):} &\left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) - \hat{\beta}_1 rainfall_{t-1}\right] \\
&= \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) + \frac{0.028}{0.138} temperature_{t-1}\right] = \ln\left(\frac{Agr.GDP_t}{Agr.population_t}\right) \\
&= \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) + 0.2 temperature_{t-1}\right].
\end{aligned}$$

The above results have been obtained from the coefficients shown in Table 11 in the Appendix. When a coefficient is not statistically significant, it has been equalized to zero and removed from the calculation. What is perhaps interesting to see from these results is that there seems to be some coherence between the two specifications modelling the output per capita measure. Specifically, there appears to be a similar long-run relationship between log GDP per capita and temperature. Alternatively, this could mean that the ECM cannot identify a cointegrating relationship between agricultural GDP per km^2 and the other variables concerned. That said, other models are used to provide a more precise assessment.

The next model in scrutiny is the Dynamic Least Squares estimator (DOLS). Such model is a refinement of the Static OLS, as it aims at solving the issue of autocorrelation in the disturbances, like the ECM. In the DOLS, this is done by including on the right-hand side past, present and future values of the first difference of the independent variables, i.e. I(0) terms. It should be however emphasized that the estimation bias cannot be removed completely if the independent variables are not strongly exogenous. The results of the DOLS can be observed in Table 12. From the estimated coefficients, we obtain the following estimated cointegrating relationships:

$$\begin{aligned}
\textbf{Spec. 1):} &\left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right) + 0.014 rainfall_{t-1} - 0.5 rainfall_{t-1}\right] \\
&= \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right) - 0.5 rainfall_{t-1}\right] \\
\textbf{Spec. 2):} &\left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) + 0.002 rainfall_{t-1} - 0.211 rainfall_{t-1}\right] \\
&= \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) - 0.211 rainfall_{t-1}\right] \\
\textbf{Spec. 3):} &\left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right) - 0.443 rainfall_{t-1}\right] \\
\textbf{Spec. 4):} &\left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) - 0.204 rainfall_{t-1}\right].
\end{aligned}$$

According to the DOLS results, the long-run equilibrium is only composed by the output measure and rainfall, without temperature, since its estimated coefficients are not significant.

Finally, we estimated the Fully Modified Least Squares (FMOLS) for the four specifications. This model seeks to solve both the issues of simultaneity and autocorrelation above-mentioned by applying a correction to the data and the estimator. This type of model yields super-consistent estimators in the limit. Once again, from the estimated coefficients, we obtain the following estimated cointegrating relationships:

$$
\begin{aligned}
\textbf{Spec. 1):} & \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right) + 0.09 rainfall_{t-1} - 0.479 rainfall_{t-1}\right] \\
&= \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right) - 0.479 rainfall_{t-1}\right] \\
\textbf{Spec. 2):} & \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) - 0.002 rainfall_{t-1} - 0.198 rainfall_{t-1}\right] \\
&= \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) - 0.198 rainfall_{t-1}\right] \\
\textbf{Spec. 3):} & \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right) - 0.442 rainfall_{t-1}\right] \\
\textbf{Spec. 4):} & \left[\ln\left(\frac{Agr.GDP_{t-1}}{Agr.population_{t-1}}\right) - 0.205 rainfall_{t-1}\right].
\end{aligned}
$$

Again, insignificant coefficients have been rounded to 0. Overall, it can be seen that the results of the FMOLS are very similar to those of the DOLS. What these relationships show is that there is a quite remarkable difference compared to the ECM estimated cointegrating relationships. In the ECM, only temperatures enter the relationship with GDP, whereas for the DOLS and FMOLS only rainfall does.

4.5 Johansen's System approach

As explained by Davidson (2000), Johansen's analysis consists of estimating a vector error correction model (VECM) of the cointegrated variables. The model specification is as follows:

$$\boldsymbol{\Delta x_t = \delta + \alpha\beta' x_{t-1} + \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \varepsilon_t} \tag{4}$$

Johansen's suggest to use reduced rank regression. This estimation method imposes the rank to be $r < m$ on the $m \times m$ matrix of coefficients of coefficients of $\boldsymbol{x_{t-1}}$. The estimation method for this can be done using concentrated maximum likelihood (CML). In the context of estimating VECM's, the method first maximizes the likelihood with respect to β by taking α as given, then insert the maximizing value for β in the likelihood function and maximize for α. The key advantage of Johansen's analysis is that it allows for the estimation of multiple cointegrating vectors, as long as $r < m$. This contrasts with other OLS-based estimators, which only allow for one cointegrating vector.

The results can be found in the following tables. Table 9 shows the estimated cointegrating vectors for the four specification. Since the results of of the Johansen's test in Section 4.3.3 show that for all specifications there is at least one cointegrating vector, the rank is chosen to be one when running the model. Furthermore, table 10 and 11 show the estimated coefficients of the VEC models for all specifications.

The α's in the VECM can be interpreted as the adjustment to the long run equilibrium. The α's for the first equation in all four specification are negative and at least statistically significant at 5 percent, suggesting that the long-equilibrium does exist and that it is stable. This is because any deviation from

	Spec. 1	Spec. 3		Spec. 2	Spec. 4
$\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right)$	1	1	$\ln\left(\frac{Agr.GDP_{t-1}}{Pop_{t-1}}\right)$	1	1
$temperature_{t-1}$	-1.564	-1.699	$temperature_{t-1}$	-0.452	-0.552
$rainfall_{t-1}$	0.168	-	$rainfall_{t-1}$	0.064	-

Table 9: Estimated cointegrating vector β

	Spec. 1			Spec. 3	
Estimated coefficients	equation 1	equation 2	equation 3	equation 1	equation 2
δ	-0.270(0.105)*	0.908(0.554)	-78.253(17.918)***	-1.780(0.835)*	14.384(3.926)***
α	-0.025(0.008)**	0.065(0.043)	-6.019(1.407)***	-0.056(0.025)*	0.443(0.121)***
$\Delta\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right)$	-0.459(0.184)*	-2.146(0.974)*	40.995(31.489)	-0.442(0.151)**	-2.118(0.713)**
$\Delta temperature_{t-1}$	0.003(0.029)	-0.466(0.154)**	2.376(4.989)	-0.024(0.039)	-0.095(0.183)
$\Delta rainfall_{t-1}$	0.002(0.001)*	-0.005(0.007)	-0.157(0.224)		
$\Delta\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-2}}\right)$	-0.237(0.198)	0.296(1.045)	30.734(33.766)	-0.191(0.157)	-0.674(0.742)
$\Delta temperature_{t-2}$	-0.023(0.026)	-0.397(0.138)**	-2.220(4.467)	-0.037(0.030)	-0.154(0.143)
$\Delta rainfall_{t-2}$	0.001(0.001)	-0.008(0.005)	-0.085(0.170)		

Table 10: Estimated VECM results for specification 1 and 3, using Johansen's ML estimation technique

this equilibrium will be multiplied by α in the next period, indicating a return to the equilibrium. This interpretation is also used by Mukhtar and Rasheed (2010). The fact that the coefficient for the α's are statistically significant suggests that cointegration exists, since the test on α can be considered a test for cointegration. Furthermore, the short-run dynamic can be determined from the coefficients of lagged and differenced variables. None of the variables are statistically significant at 5 percent, for any of the specifications, suggesting that temperature and rainfall have no effect on agricultural GDP per km^2 or agricultural GDP per capita, in the short run.

4.6 Discussion

In the previous sections we described different techniques for estimating the cointegrating vectors. The single equation OLS-based estimators (static and dynamic OLS, FMOLS) have the advantage that they are simple to understand and to implement. Furthermore, since we concluded from the Johansen test that the rank is equal to one, the resulsts from the single equation OLS based estimators and Johansen's system approach shouldn't differ by much. Nevertheless, Caporale and Pittis (2004) show by means of Monte-Carlo analysis that static OLS, dynamic OLS and FMOLS perform poorly under small sample sizes, which is the case for this analysis. Johansen's system approach is often regarded as the standard of estimating cointegrating vectors and VECM's, as explained by Lütkepohl (2007), and thus, we would favor the results from this methodology.

	Spec. 2			Spec. 4	
Estimated coefficients	equation 1	equation 2	equation 3	equation 1	equation 2
δ	0.002(0.007)	0.064(0.041)	-3.319(1.345)*	-1.648(0.584)**	9.126(2.909)**
α	-0.064(0.021)**	0.145(0.112)	-15.363(3.604)***	-0.185(0.065)**	1.014(0.325)**
$\Delta \ln\left(\frac{Agr.GDP_{t-1}}{Pop_{t-1}}\right)$	-0.392(0.185)*	-2.124(0.979)*	44.706(31.425)	-0.355(0.150)*	-2.333(0.751)**
$\Delta temperature_{t-1}$	0.010(0.028)	-0.489(0.152)**	3.648(4.887)	-0.030(0.036)	-0.1911(0.1795)
$\Delta rainfall_{t-1}$	0.002(0.001)*	-0.004(0.006)	-0.162(0.222)		
$\Delta \ln\left(\frac{Agr.GDP_{t-1}}{Pop_{t-2}}\right)$	-0.176(0.198)	0.365(1.045)	30.079(33.547)	-0.155(0.154)	-0.636(0.769)
$\Delta temperature_{t-2}$	-0.020(0.026)	-0.413(0.138)**	-1.402(4.433)	-0.042(0.028)	-0.217(0.143)
$\Delta rainfall_{t-2}$	0.001(0.001)	-0.008(0.005)	-0.080(0.168)		

Table 11: Estimated VECM results for specification 2 and 4, using Johansen's ML estimation technique

We can also see that the results of the cointegrating vector differ substantially between the OLS-based techniques and the VECM model. We are unable to explain why this is the case, but we suspect it might be due to the small sample size.

The results for the VECM can be found in table 10 and 11. The first equation is the one we are interested in, since it has Agricultural GDP per km^2 of land or Agricultural GDP per capita as the dependent variable. Nevertheless, many of the coefficients estimated in the VECM are not statistically significant. This result is odd, since the lag order of the model was decided using information criteria (3 lags in VAR form). One way to improve the model is by restricting some of the coefficents to be equal to zero, to increase estimation precision (Lütkepohl, 2007).

Furthermore, we investigated the possibility that the long-run relationships of the four models changed over the years. One reason why this could have occurred is due to the presence of a breakpoint in the temperatures, as mentioned in Section 3. After looking more in depth, a breakpoint at observation 37 was found. Then, a new VECM has been estimated for the four specifications to check potential major differences. The new estimated ECT is shown in Table 12. As can be seen, comparing these with Table

	Spec. 1	Spec. 3		Spec. 2	Spec. 4
$\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right)$	1	1	$\ln\left(\frac{Agr.GDP_{t-1}}{Pop_{t-1}}\right)$	1	1
$temperature_{t-1}$	-3.661	-2.941	$temperature_{t-1}$	-0.473	-0.562
$rainfall_{t-1}$	0.258	-	$rainfall_{t-1}$	0.047	-

Table 12: Estimated cointegrating vector β

9's results, the coefficients for temperature are more negative when the GDP measure is per km^2 and there is not much difference when GDP is per capita. The estimated speed of adjustment parameters (α) for the equations with GDP as dependent variable are now significant for all the four models and not too different in magnitude from those in Table 11. Once again, the estimates are all negative. In summary, the influence of drastic changes in the average temperatures seems to be rather large for the error correction term, even if it is hard to determine a clear and direct effect of this on output. Nonetheless, tackling global warming with climate policy could further change the relationship, if a solid emission reduction is brought about in the next decades.

In general, the results from the cointegration analysis are mixed, since some techniques suggest a cointegrating relationships while others do not. We expected the evidence for cointegration to be much stronger, due to the small literature review in section 4.1. We suggest this analysis to be expanded, by increasing the number of observations in the time series, or even consider a panel data set, of similar countries.

5 Appendix

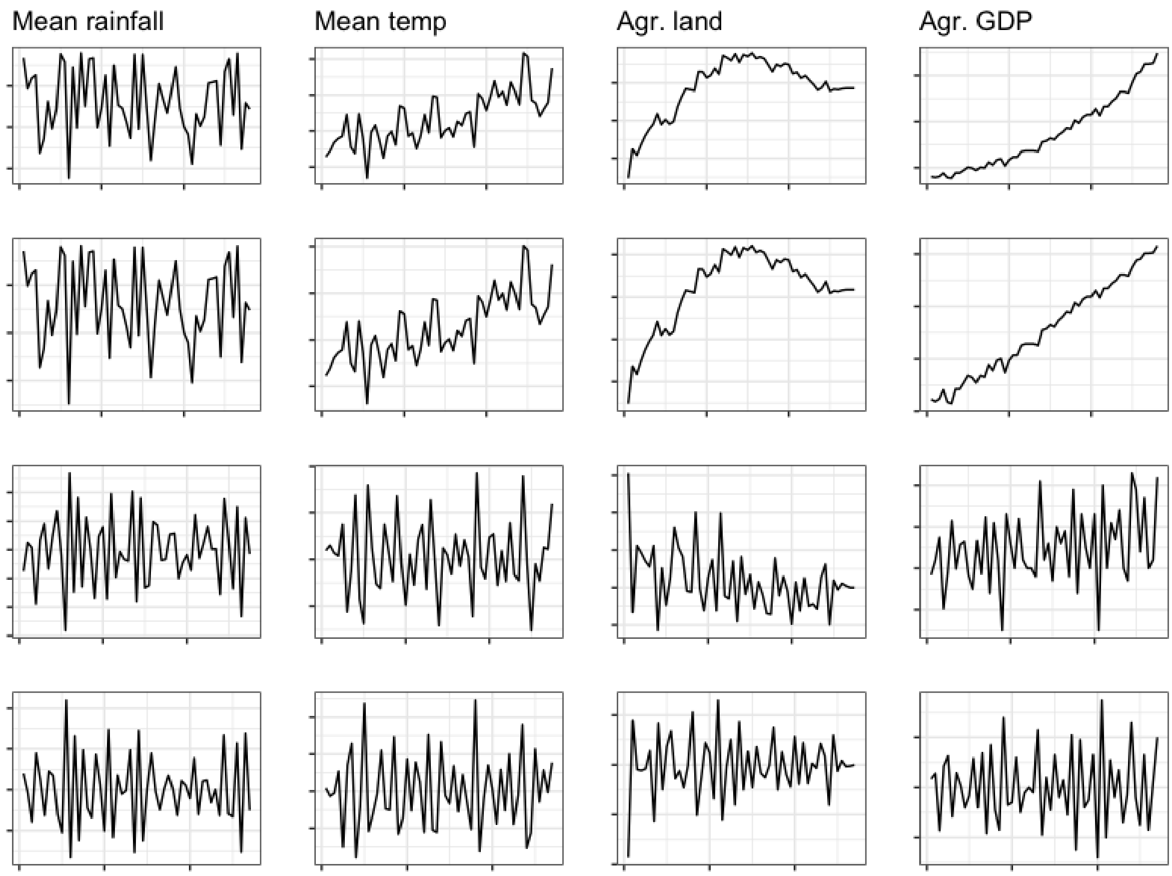

Figure 8: plot of the four variables. First row is the variables in levels, second is in logs, third in first differences, and fourth in second differences.

	Spec. 1 Dep. var.: $\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right)$		Spec. 2 Dep. var.: $\ln\left(\frac{Agr.GDP_t}{population_t}\right)$		Spec. 3 Dep. var.: $\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right)$		Spec. 4 Dep. var.: $\ln\left(\frac{Agr.GDP_t}{population_t}\right)$	
	SOLS	*ECM*	*SOLS*	*ECM*	*SOLS*	*ECM*	*SOLS*	*ECM*
$rainfall_t$	0.008 (0.005)		0.004^{**} (0.002)					
$temperature_t$	0.926^{***} (0.122)		0.191^{***} (0.006)		0.443^{***} (0.002)		0.205^{***} (0.001)	
$\ln\left(\frac{Agr.GDP_{t-1}}{Agr.land_{t-1}}\right)$		0.003 (0.022)				0.017 (0.022)		
$\ln\left(\frac{Agr.GDP_{t-1}}{population_{t-1}}\right)$				0.117^{**} (0.056)				0.138^{*} (0.072)
$rainfall_{t-1}$		0.0002 (0.001)		-0.001 (0.001)				
$temperature_{t-1}$		0.021 (0.033)		-0.021^{*} (0.012)		-0.007 (0.010)		-0.028^{*} (0.015)
$\Delta rainfall_t$		0.003^{***} (0.001)		0.003^{***} (0.001)				
$\Delta temperature_t$		-0.033 (0.026)		-0.012 (0.021)		-0.047^{*} (0.025)		-0.031 (0.026)
Constant	-13.120^{***} (3.284)	-0.564 (0.692)						
Observations	56	55	56	55	56	55	56	55
R^2	0.522	0.488	1.000	0.520	0.999	0.246	1.000	0.153
Adjusted R^2	0.504	0.436	1.000	0.472	0.999	0.203	1.000	0.105
								*p<0.1; **p<0.05; ***p<0.01

Table 13: Static OLS and Error Correction Model estimates for the four specifications. Note that the dependent variable for the ECM is always in first-difference.

	Spec. 1 Dep. var.: $\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right)$		Spec. 2 Dep. var.: $\ln\left(\frac{Agr.GDP_t}{population_t}\right)$		Spec. 3 Dep. var.: $\ln\left(\frac{Agr.GDP_t}{Agr.land_t}\right)$		Spec. 4 Dep. var.: $\ln\left(\frac{Agr.GDP_t}{population_t}\right)$	
	DOLS	*FMOLS*	*DOLS*	*FMOLS*	*DOLS*	*FMOLS*	*DOLS*	*FMOLS*
$rainfall_t$	-0.014 (0.034)	-0.009 (0.0128)	-0.002 (0.008)	0.002 (0.003)				
$temperature_t$	0.495^{***} (0.127)	0.479^{***} (0.048)	0.211^{***} (0.028)	0.198^{***} (0.013)	0.443^{***} (0.006)	0.442^{***} (0.006)	0.204^{***} (0.001)	0.205^{***} (0.001)
								*p<0.1; **p<0.05; ***p<0.01

Table 14: Dynamic OLS and Fully Modified OLS estimates for the four specifications

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