Assignment 3

Advanced Econometrics

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Notes and instructions:

1. This assignment is mandatory.
2. The assignment is to be made in groups of 4 students. You can create your own group in Canvas > People > Case 3+4 (groups), and then self-enroll or join an incomplete team. Please be professional and welcoming to new team members.
3. Only one of you needs to hand in all files.
4. The deadline for delivery of this assignment is on Wednesday, October 5, at 23:59h

   There will be no tolerance period for late deliveries. Deliveries after the assigned deadline imply that you have a final grade of zero for the assignment (AG3 = 0).
5. To get the full score for this assignment, the following three things must be done:

   (a) upload your final report as a PDF file in Canvas Assignments. Name the file **A3report_2601842_2511351_2661510_2639486.pdf**, where the numbers are replaced by the VU student numbers of the 4 group members.To write you report according to academic standards follow the relevant tips that we have in the questions and also check the example report file under the name 'example_report.pdf' on Canvas > Assignments > Assignment 3: Instructions.

   (b) upload a zip file of your runnable R or Python code in Canvas Assignments. Name the file **A3code_2601842_2511351_2661510_2639486_language.zip**, where the numbers are replaced by the VU student numbers of the 4 group members (or 3 if your group consists of 3 people) and language is either R or Python.

   The code file(s) should be clear, well commented, and directly runnable, so that it reads the datafile and obtains the results of all questions and prints them. Your initial comments in the file should hold your names and student numbers.

(c) upload a pdf of your entire code in Canvas Assignments. Name the file **A3code_2601842_2511351_2661510_2639486_language.pdf**, where the numbers are replaced by the VU student numbers of the 4 group members (or 3 if your group consists of 3 people) and language is either R or Python. The file should be well readable, with proper indentations and should not contain pictures/photos/screenshots of code snippets.

6. As a standard anti-fraud measure, we will at random select a number of you to explain your code and answers. Any one of you must be able to explain any part of the code. Failure to explain your answers will result in a deduction of credits for this assignment.

7. For the support for the assignments, carefully read the announcement we put out at the start of the course and consult the discussion boards related to the assignments.

We wish you success!!

VU VRIJE UNIVERSITEIT AMSTERDAM
School of Business and Economics

1 Background: modeling financial returns

In economics and finance, it is well known that the conditional volatility of financial returns evolves slowly over time. Financial returns are thus characterized by the presence of 'clusters of volatility'. Recent evidence suggests that robust volatility filters might be needed for correctly capturing the time-varying conditional volatility in stock returns. Furthermore, there are also reasons to believe that large negative returns produce more volatility than large positive returns. This characteristic, first noted by Black (1976), is known as the leverage effect. Some analysts claim that the leverage effect plays a very important role in financial markets. As an econometrician, you surely want to test these claims! Do robust filters really perform better? Does the data support the existence of a 'leverage effect'?

2 Getting the data

You will have to download the data yourself, such that you also experience some of the data licenses VU has in store for you to do high-quality research. For stock price research, Yahoo Finance is not an academically trusted source. Sources that are deemed trustworthy are for instance Datastream, Factset, Compustat-Capital IQ, Center of Security Price Research (CRSP), and others. For this assignment, we take the CRSP data. Be sure to have the UBVU off campus add-in installed if you work off campus: see this link. Go to the link, fill out your VU email address on the left, and do the 'no-robot' test. Then wait for an email (in spam?) from WRDS with your one-day pass for the databases. As data licenses are very expensive, all these safeguards are installed by the data provider.

Clicking the one-day pass link in the email you receive, you enter the WRDS shell. Many high-quality databases are in there, not all of which we have a license for. Some of them may be interesting for your thesis too.

- Find CRSP and click on it.
- Then find 'Stock/Security files' and click it.
- Then find 'Daily stock files'.
- You now land on a download request form for the data.
- Download the stocks JNJ (Johnson & Johnson), MRK (Merck), PFE (Pfizer), and KO (Coca Cola).

- There are many data types that can be downloaded. You should download the TICKER and the holding period returns (HPR). These include dividend payments, and adjust for stock splits and other complications. They are the returns you as an investor would be interested in.
- Download the data from Monday 1 Jan, 2001 to Friday 31 Dec 2021.
- To simplify a number of issues in this assignment, transform the data by first subtracting the sample mean from each time-series and then multiplying the result by 100 such that 3.5 means 3.5 per cent above average holding period return. That will make your code numerically much more stable than working with small numbers like 0.035 meaning 3.5 per cent. Subtracting the mean, by the way, is sometimes done in empirical work for the models below as well, but not always. In our case, it simplifies a number of the computations, which is good enough for this case.

At the very end of this assignment description, you find relevant output to help you on your way.

3 Univariate volatility modeling

Consider the following Robust-GARCH-with-Leverage-Effect model for time-varying volatilities

$$x_t = \sigma_t \varepsilon_t \quad \text{for every} \quad t \in \mathbb{Z} \quad \text{where} \quad \{\varepsilon_t\}_{t\in\mathbb{Z}} \sim \text{TID}(\lambda),$$

$$\sigma_t^2 = \omega + \frac{\alpha x_{t-1}^2 + \delta x_{t-1}^2 \cdot 1_{\{x_{t-1}<0\}}}{1 + \frac{x_{t-1}^2}{\lambda \sigma_{t-1}^2}} + \beta \sigma_{t-1}^2,$$

where 1_A is an indicator function with the value 1 if event A holds, and zero otherwise. Here, TID denotes the standard Student's t distribution with pdf

$$p_\varepsilon(\varepsilon_t) = \frac{\Gamma\left(\frac{\lambda+1}{2}\right)}{\Gamma\left(\frac{\lambda}{2}\right)\sqrt{\lambda\pi}} \left[1 + \frac{\varepsilon_t^2}{\lambda}\right]^{-(\lambda+1)/2}.$$

Note that the conditional density $p_x(x_t \mid \mathscr{F}_{t-1})$ of x_t that you need for the construction of the log-likelihood function via the prediction error decomposition is not given by $p_\varepsilon(x_t/\sigma_t)$, where $\mathscr{F}_{t-1} = \{x_{t-1}, x_{t-2}, \ldots, x_1\}$ is the information set containing the past observations; see also the tips following Question 3.

Question 1. Analyze, plot, compare and interpret the news-impact-curves (see Chapter 3, slide 34) of this model for different values of $\lambda = 2, 5, 10, 50$, and different values of $\delta = 0, 1, 0.2, 0.4$. What happens as $\lambda \to \infty$? Do you find a familiar model? What if you let both $\lambda \to \infty$ and $\delta \to 0$? Make sure your figures for this question have max 4 panels each (one for each value of δ) and satisfy the standards for good scientific reporting (see the tip below Question 2). To plot your curves, use $\sigma_{t-1}^2 = 1$, $\omega = 0$, $\alpha = 0.05$, and $\beta = 0.9$.

- Tip: In general, the news-impact-curve shows how the shocks affect the conditional volatility. Therefore, the news-impact curve shows the values of σ_{t+1}^2 for different values of x_t, holding all other elements in the transition equation for σ_t^2 constant.

Question 2. For this question, use all the data in the sample. Make a table with the descriptive statistics (number of observations, means, medians, standard deviations, skewness, kurtosis, minimum, maximum) of your four stocks and briefly discuss it. Also make a figure with four panels, each panel holding the returns for one stock. Also briefly discuss this plot.

- Tip: make sure to obey good academic reporting standards. A good example is given by the figures in the pdf version of this paper, and for tables in Table 3 of the same paper. If you have trouble accessing the paper, follow these steps from the University Library to get automatic access with your VU account (a feature that will prove useful at many occasions during your program). Sometimes, the explanatory notes may seem excessive to you, like in Table 3, but that is not true: you should do everything to make the figures and tables stand-alone, i.e., understandable without reading anything else in the paper. (Note, however, that the explanatory note does not give an interpretation of the data in the figure or table.) Very important: if you make a time series plot, put dates on the axis, and NOT the indices 1,...,1000 of the observations.

Question 3. For this question, **only use the first 2,500 observations**. Using the stocks of the pharmaceutical companies, i.e. JNJ, MRK and PFE, using the robust GARCH without leverage effect ($\delta = 0$) model as well as the Robust GARCH-with-Leverage-Effect model ($\delta \neq 0$) above, filter the time-varying conditional volatility present in these stocks and report the (maximum likelihood) estimated parameters of your models and (below each estimated parameter) the standard error of the parameters calculated as the square roots of the diagonals of $-H^{-1}$, with $H = \frac{\partial^2 \sum_{t=1}^{T} \ell(x_t, \hat{\sigma}(\theta, \hat{\sigma}_1), \theta)}{\partial\theta\partial\theta'}$. Note there are 3 stocks, and 2 models ($\delta = 0$, $\delta \neq 0$) for each stock, giving a total of 6 sets of estimates and standard errors. Report the results in your pdf with all results in one table. Make your own (beautiful latex or word) table that satisfies good academic reporting standards (see previous two assignments), e.g.,

Table example

Here I explain everything that is in the table without interpreting the table of course. This text can actually be pretty long to ensure that the table is really stand alone.

	Stock 1		Stock 2		Stock 3	
ω	1.11	1.12	1.11	1.12	1.11	1.12
	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)
α	1.11	1.12	1.11	1.12	1.11	1.12
	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)
β	1.11	1.12	1.11	1.12	1.11	1.12
	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)
δ		1.12		1.12		1.12
		(0.01)		(0.03)		(0.03)
λ	5.11	5.12	5.11	5.12	5.11	5.12
	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.03)
Log-lik	100	100	100	100	100	100
AIC	100	100	100	100	100	100
BIC	100	100	100	100	100	100

Use a sensible number of digits as in academic journals.

Also at the bottom of each column in the table (as corresponding to a (stock, model) pair, report the total log likelihood, AIC and BIC attained by each model and discuss which model is best for each stock? Justify your answer.

- Tip: to help you on your way, we have provided for you our estimates of the model using stock four (KO) at the end of these instructions.
- Tip: please recall that if σ_t is given and $x_t = \sigma_t \varepsilon_t$ and $\epsilon_t \sim f(\varepsilon_t, \boldsymbol{\theta})$. Then, the classical theorem for the pdf of a transformation of a random variable tells us that, $x_t/\sigma_t \sim f(x_t/\sigma_t, \boldsymbol{\theta})(1/\sigma_t)$. This means that a log density takes the form $\log f(x_t/\sigma_t, \boldsymbol{\theta}) - \frac{1}{2}\log\sigma_t^2$.
- Tip: in order to avoid programming errors, you may be tempted to use built-in pdfs. For example in R, `dnorm(x)` gives the density of a Standard Normal at point `x`, and `dt(x,lambda)` gives the density of a Student's-t with `lambda` degrees of freedom, at point `x`. However, this may be a very bad idea in some cases for numerical stability, as we actually need the log density rather than the density. You can use the built-in functions however to numerically check your outcomes.
- Tip: for numerical stability, you do not want to use the log of a gamma function in your log-likelihood expressions. The gamma function rises more than exponentially fast, and the log partly reverts that. This is a recipe for numerical problems. It is better to use the numerically stable log-gamma function, which is built in in all major languages such as R

(lgamma), Python (math.lgamma), and Matlab (gammaln) and even Excel (gammaln). Just try taking the log of gamma(10000), or computing loggamma(10000).

- Tip: for the robust ($\delta \neq 0$) model, it is possible that certain parameter values will generate negative filtered volatilities. There are multiple solutions to this. In our experiments, we found the solution with a penalized estimator to work best. After computing all the values of σ_t^2, check whether the lowest value is smaller than 0 or whether λ is smaller than $c = 0.0001$. If either of these two is, do not return the likelihood value, but return $-100 \cdot T - 10000 \cdot T \cdot \left[(\lambda - c)^2 \cdot 1_{(\lambda<c)} + \sum_t \left((\sigma_t^2)^2 \cdot 1_{(\sigma_t^2<0)}\right)\right]$ instead of the log-likelihood. Otherwise, return the log-likelihood. Note that you want to maximize the likelihood, such that the optimizer wants to shy away from these regions now. If you use a minimizer rather than a maximizer, make sure to multiply everything by -1.

- Tip: selecting initial parameter values for the estimation of nonlinear dynamic models is a tricky business! You may want to start the ML parameter optimization with initial parameter values for the Robust GARCH model without leverage effect as

$$(\omega, \alpha, \beta, \lambda) \approx (s_T^2/50, 0.02, 0.96, 5),$$

  and with leverage effect as

$$(\omega, \alpha, \beta, \delta, \lambda) \approx (s_T^2/50, 0.02, 0.96, 0, 5),$$

  such that you allow for fat tailed innovations ($\lambda = 5$) at the start and no leverage, and where s_T^2 the sample variance of the returns for that stock based on the $T = 2,500$ observations in the estimation sample. Such values are typically inspired by earlier work in the academic literature on similar models and similar datasets.

- Tip: there is an issue on how to choose the initial value σ_1^2. In this assignment, we want you to take the sample variance of the first 50 observations for $\sigma_1^2 = s_{50}^2$. Other choices are possible, but we want you to adopt this specific one for the current case. Note we want you to use $\sigma_1^2 = \frac{1}{50}\sum_{t=1}^{50}(x_t - \bar{x}_{50})^2)$ with $\bar{x}_{50} = \frac{1}{50}\sum_{t=1}^{50} x_t$. Note that some packages for the internal var( ) command divide by $T - 1 = 49$ rather than $T = 50$. Check this and correct if necessary in order not to loose any credits, e.g., by checking whether the variance of $\{1, -1\}$ is computed as 1 or 2 (should be 1).

- Tip: note that you have to estimate two models for each stock. This is very different from only estimating the general model with $\delta \neq 0$ and then putting $\delta = 0$ if there is no leverage. In fact, if you impose no-leverage, all the other parameters like α and λ etc. have to be re-estimated under this constraint $\delta = 0$. See our outcomes for the example stock KO.

Question 4. Use the maximum likelihood parameter estimates obtained in the previous question to plot and compare the estimated news-impact-curves and the filtered volatilities obtained from the two alternative models for each of the 3 stocks over the entire sample (so both for the 2,500 in-sample observations, as well as for the remaining out-of-sample observations). Put a vertical line at the *date* where the in-sample observations stop. Summarize all this in a 3×2 panel plot: each row corresponding to a stock (put it in the title of the panel), left-hand plot the two news impact curves, right-hand plot the time series of the filtered volatilities for the two models. Obey good academic reporting standards for the figure. Briefly discuss all plots. As in Question 1, you may use $\sigma^2_{t-1} = 1$.

Question 5. In this question you compute VaR estimates based on simulation. Use your estimates based on the first 2,500 observations as obtained before. With those estimates, run the filter to compute the volatility up to and including April 1, 2020 (so just after covid really kicks in). On April 1, 2020, you are interested to compute the 1, 5, and 20 day-ahead VaR for the compound returns over these horizons at the levels 1%, 5%, and 10%. Report the results (for each stock you have two models, and for each (stock, model) pair you have 3 horizons and 3 confidence levels, so $3 \times 2 \times 3 \times 3 = 54$ numbers in total.

- Tip: though you are forecasting the VaR, you should not do this in a parametric way. In particular, do not use the approach on slide 14 of Chapter 9, for instance. Instead, base your whole result on pure simulation. Forecasting in non-linear time-series models using simulations was dealt with in slide 28 of Chapter 9. As an example, using $\hat\sigma^2_{t_0}(\hat\theta, \hat\sigma^2_1)$ at t_0 equal to April 1, 2020, compute $\hat\sigma^2_{t_0+1}(\hat\theta, \hat\sigma^2_1)$ using the robust GARCH equation. Then draw $\varepsilon^{(s)}_{t_0+1}$ from $T(\hat\lambda)$ and construct $x^{(s)}_{t_0+1} = \hat\sigma_{t_0+1}(\hat\theta, \hat\sigma^2_1) \cdot \varepsilon^{(s)}_{t_0+1}$, where the (s) stands for a simulated HPR (return). Next, compute $\hat\sigma^{2,(s)}_{t_0+2}(\hat\theta, \hat\sigma^2_1)$ using $\hat\sigma^2_{t_0+1}(\hat\theta, \hat\sigma^2_1)$ and $x^{(s)}_{t_0+1}$ and the robust GARCH equation. Then draw $\varepsilon^{(s)}_{t_0+2}$ and construct $x^{(s)}_{t_0+2} = \hat\sigma^{(s)}_{t_0+2}(\hat\theta, \hat\sigma^2_1) \cdot \varepsilon^{(s)}_{t_0+2}$, and compute $\hat\sigma^{2,(s)}_{t_0+3}(\hat\theta, \hat\sigma^2_1)$ using $\hat\sigma^{2,(s)}_{t_0+2}(\hat\theta, \hat\sigma^2_1)$ and $x^{(s)}_{t_0+2}$ and the robust GARCH equation; etcetera, until you reach $t_0 + 20$. You can do this for $s = 1, \ldots, S$ simulations and empirically compute the relevant quantiles of your simulated multi-period compound returns to compute the VaR.

- Tip: note that the question is on compound returns. As you have obtained holding period returns x_t from WRDS that you demeaned and scaled by 100, your compound returns at for instance horizons 1 and 5 are x_{t_0+1}, there t_0 is April 1, 2020, and $100 \cdot [(1 + x_{t_0+1}/100)(1 + x_{t_0+2}/100) \cdots (1 + x_{t_0+5}/100) - 1]$. We abstract from further demeaning complications in this assignment.

- Tip: you may want to structure the table as follows: organize the rows in 3 blocks of 2 lines (separated by an empty line), one for each stock. Each block's first line is for the model without, and the second line for the model with leverage. First column holds the name of the stock, second column has 'δ free' or '$\delta = 0$'. Then 9 columns in 3 blocks of 3: 1%, 5% and 10 %VaRs, and that for 1, 5, and 20 day ahead. Make sure the column headings are clear, and that the explanatory note states how you did the simulations and how many simulations (thus making the table stand-alone). In this way you can provide the reader in a digestible way with all the results at ones, rather than having many small tables.

- Tip: to help you on your way, we provide the results for KO. Beware, the results are based on simulations, so you cannot expect to get the same results. You should however be able to get reasonable close. See the output at the end of the assignment. The differences between your outcomes and ours can be substantial for the longer horizons (20) due to compounding slight differences in estimates (even up to plus or minus 2 sometimes). Do not be be super-alarmed then (but do tripple check your code). In the end, be satisfied if your outcomes land in the rough same ballpark as ours.

- Tip: use (at least) 10,000 simulations in the end (or even more). However, until your code works well, use only 10 simulations or so to prevent having to wait for ages each time. Also note that by simulating for $h = 20$, you have also directly obtained the simulations for $h = 1$ and $h = 5$ if you cleverly store intermediate results.

Question 6. For this question, again use your estimates of the model parameters as obtained earlier on the 2,500 observations only. Use observations 2,501 until the end of the sample to perform a backtesting procedure on which model performs best. The backtest uses the 1-day-ahead VaR predictions (so not the longer horizons) at 1%, 5%, and 10%. To compute the one-day-ahead VaRs, you can use the model and the Student's t distribution, in particular, its quantiles (rather than the simulation approach of Question 4). Describe in detail your backtesting approach before presenting the results.

For your three stocks and the model without leverage, provide the out-of-sample hit rate, below it in parentheses its usual standard error, below that the mis-specification robust standard error (see tips). Below that, report the same entries for the model with leverage.

Discuss your results briefly.

- Tip: unlike for Question 4, in this question we use the parametric VaR for the one-period-ahead backtest. Given your estimate $\hat{\theta}$ based on the 2,500 observations, you can compute all values of $\hat{\sigma}_t^2(\hat{\theta}, \hat{\sigma}_1^2)$ also over the forecasting period. The γ VaR for $\gamma = 1\%$ for instance is then given by $\hat{\sigma}_{t+1} \cdot T^{-1}(\gamma, \hat{\lambda})$, where $T^{-1}(\cdot, \lambda)$ denotes the inverse cdf of the Student's t distribution with λ degrees of freedom.

- Tip: A VaR violation is counted if the realized return at time $t+1$ exceeds your VaR forecast for time $t+1$. Of course, if your VaR model is good, this can only happen in 1% of the cases for the 99%-level VaR, or 5% for the 95%-level VaR.

- Tip: mis-specification robust standard errors are needed, because VaR violations are often autocorrelated. A standard way to do that is to use Newey-West standard errors, rather than the usual ones. See for a short intro Wikipedia, or consult the original paper in Econometrica. Here, we want you to program it yourself using

$$s_{NW}^2 = \frac{1}{H} \sum_{h=1}^{H} y_{t_0+h}^2 + \frac{2}{H} \sum_{\ell=1}^{L} \sum_{h=\ell+1}^{H} w_\ell y_{t_0+h} y_{t_0+h-\ell},$$

  where H is the number of observations in the forecasting period (so 2,501 until the end), $L = \text{int}(H^{1/5})$ (note 1/5 not 1/2), t_0 is 2,500, and $w_\ell = 1 - \ell/(L+1)$. Finally y_t in the above equation is the binary (0/1) VaR violation indicator. Note that the standard error of the hit rate is $s_{NW}/\sqrt{H}$ and not s_{NW}.

  IMPORTANT: we are aware that there are many libraries out there that compute robust standard errors for you. We want you, however, to code them up yourself to better understand what the method does: accounting for the autocorrelation in the series. You can use the libraries to check yourself if needed.

- Tip: a good idea is to have the table structured as follows: create 3 panels of rows, one panel for each stock. Each panel contains 2 blocks of 3 lines, separated by an empty line. Each block corresponds to either the model without leverage ($\delta = 0$) or with leverage ($\delta \neq 0$). Each block's first line holds the hit ratios, followed by the standard error and robust standard error in the lines below.

  The table should have 5 column. The first column holds the name of the stock and the second column has 'δ free' or '$\delta = 0$'. Then 3 columns for the 3 VaR confidence levels. Make sure the column headings are clear, and that the explanatory note states how you computed the entries (thus making the table stand-alone). In this way you can provide the reader in a digestible way with all the results at ones, rather than having many small tables.

- Tip: to help you on your way, we provide the results for "KO". See the output at the end of the assignment description.

- Tip: in reality, people typically update the parameter estimates after a number of forecast observations have elapsed, e.g., a month of observations. We <u>do not</u> want you to do that in this assignment! But later in your thesis or real work, of course, you should.

```
OUTPUT FOR "KO". NOT IN NICE TABLE FORMAT (AS YOU ARE SUPPOSED TO MAKE)!!
FOR ONE THING, THIS OUTPUT HAS WAY TOO MANY DIGITS FOR A DECENT REPORT TABLE.

Descriptives

      nobs         means       medians          sdvs
5284.00000000    0.00000000    0.01314822    1.22477769
     skewness      kurtosis      minimums      maximums
  -0.07095353   10.68374444  -10.09220178   13.84829822

[1] "KO estimation results on 2500 obs"
converged
[1] "KO result without leverage"
           omega      alpha       beta    lambda
[1,] 0.005665683 0.09922153 0.91055318 5.5415955
[2,] 0.002739142 0.01817242 0.01605118 0.5696382
[1] "Loglik: -3774.146"

converged
[1] "KO result with leverage"
           omega      alpha      beta   lambda      delta
[1,] 0.007442577 0.02670845 0.9217446 6.056328 0.11266453
[2,] 0.002598288 0.01481172 0.0138582 0.667616 0.02191671
```

```
[1] "Loglik: -3758.596"

[1] "*** FORECASTING compound HPR VaR of KO on Apr 1, 2020"

No Leverage
level         h=1       h=5      h=20
  0.01 -12.639028 -24.87684 -43.11051
  0.05  -7.655525 -16.80573 -30.59565
  0.1   -5.637303 -12.95664 -24.27458

Leverage
level         h=1       h=5      h=20
  0.01 -11.698841 -25.33607 -43.69626
  0.05  -7.448495 -17.04078 -31.15104
  0.1   -5.441814 -12.79001 -25.01897

[1] "*** BACKTESTING 1-step VaR of KO"
metric              VAR(0.01)   VAR(0.05)     VAR(0.1)
  no_lev_hit_rate 0.013663535 0.047822374  0.097779675
  no_lev_se       0.002399349 0.004410358  0.006138749
  no_lev_NW_se    0.002576106 0.005284970  0.008013233
  lev_hit_rate    0.014944492 0.050384287  0.097779675
  lev_se          0.002507669 0.004520857  0.006138749
  lev_NW_se       0.002720672 0.005367124  0.007940093
```