Time Series Models 2023 - Assignment

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Part 1. (A) Replicating the plots

Figure 2.1

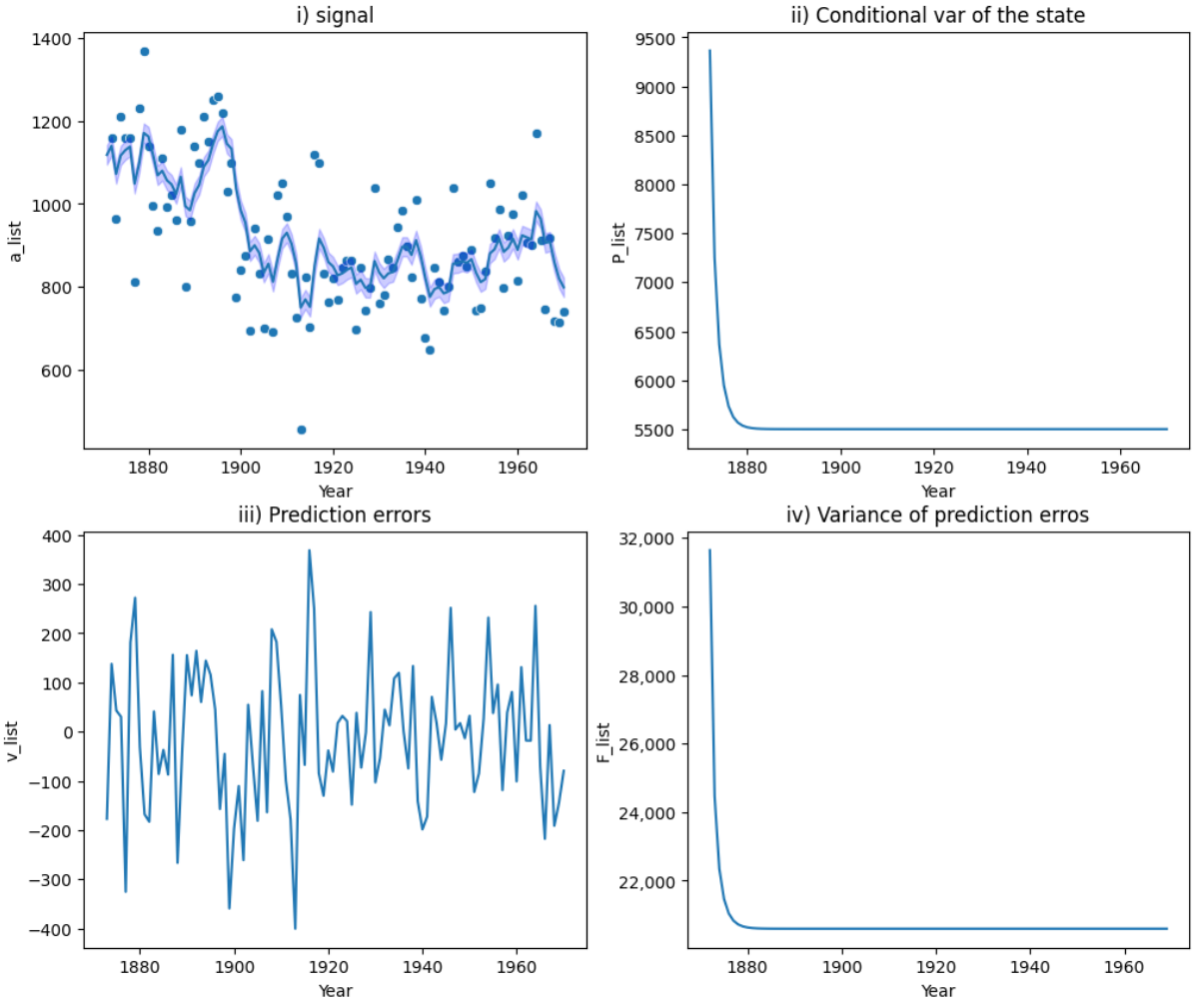


Figure 2.2

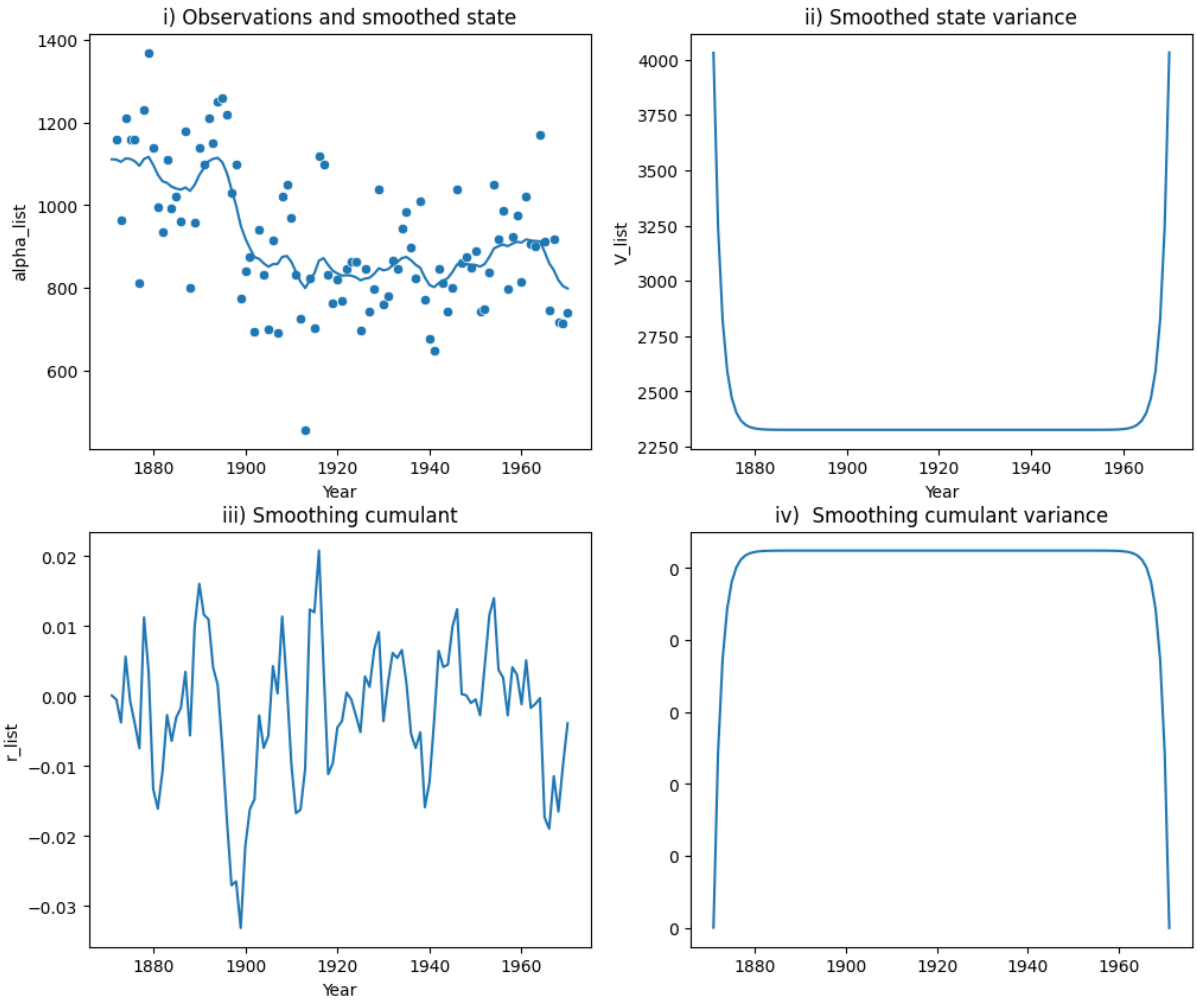

i) Observations and smoothed state
alpha_list
Year
ii) Smoothed state variance
V_list
Year
iii) Smoothing cumulant
r_list
Year
iv) Smoothing cumulant variance

Figure 2.3

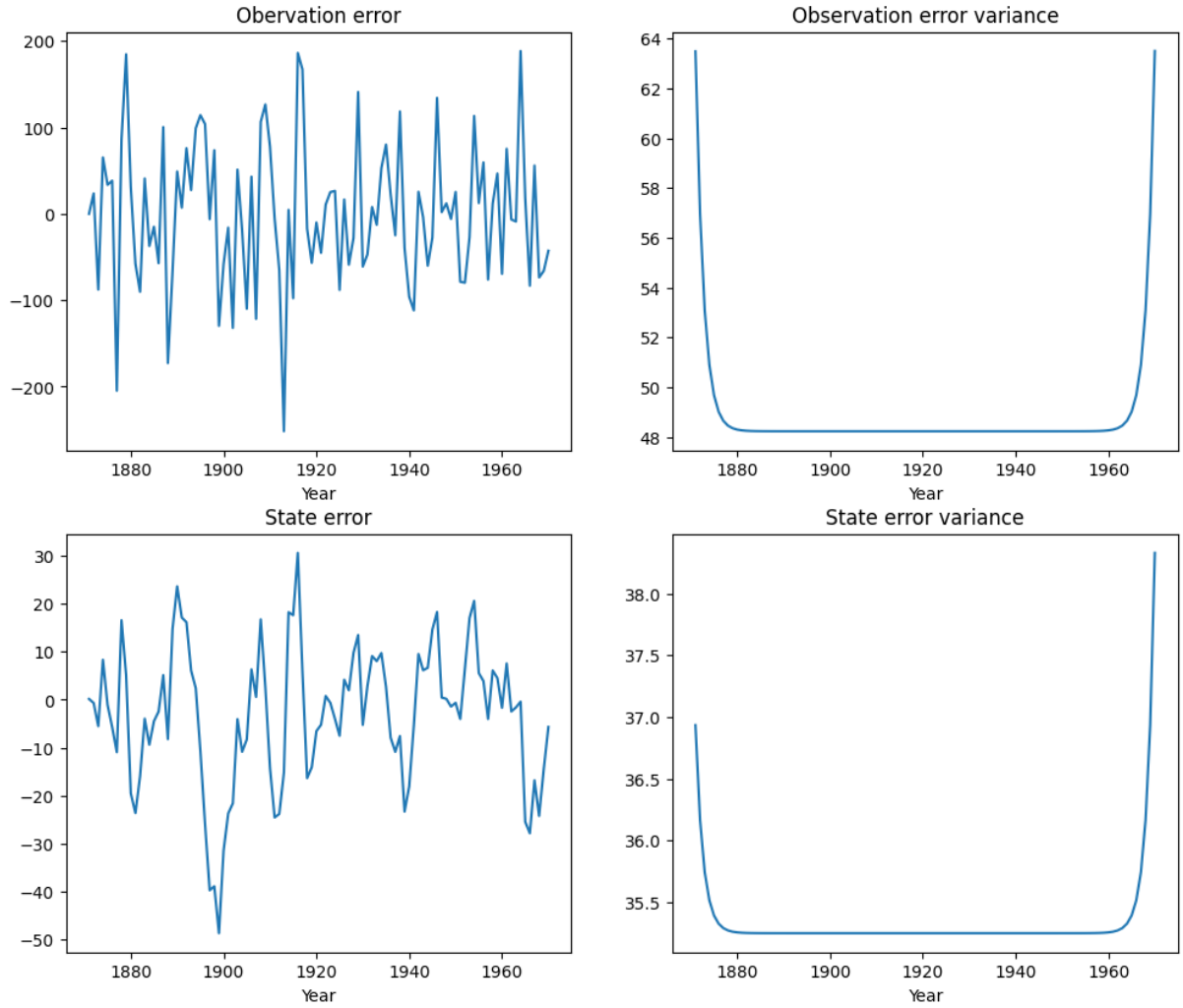

Obervation error
Observation error variance
State error
State error variance
Year
Year
Year
Year

Figure 2.4

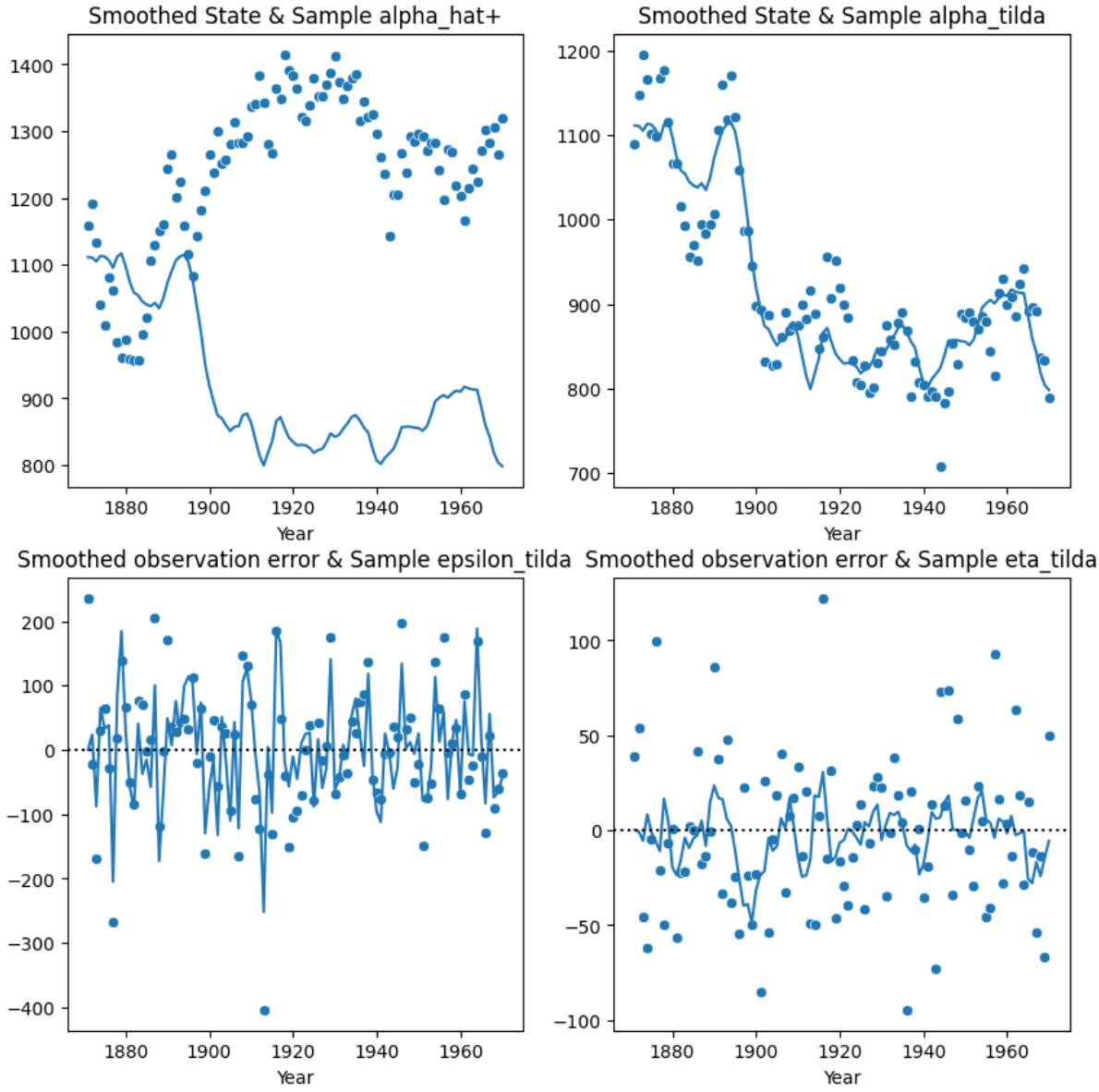

Smoothed State & Sample alpha_hat+
Smoothed State & Sample alpha_tilda
Smoothed observation error & Sample epsilon_tilda
Smoothed observation error & Sample eta_tilda
Year
Year
Year
Year

Figure 2.5

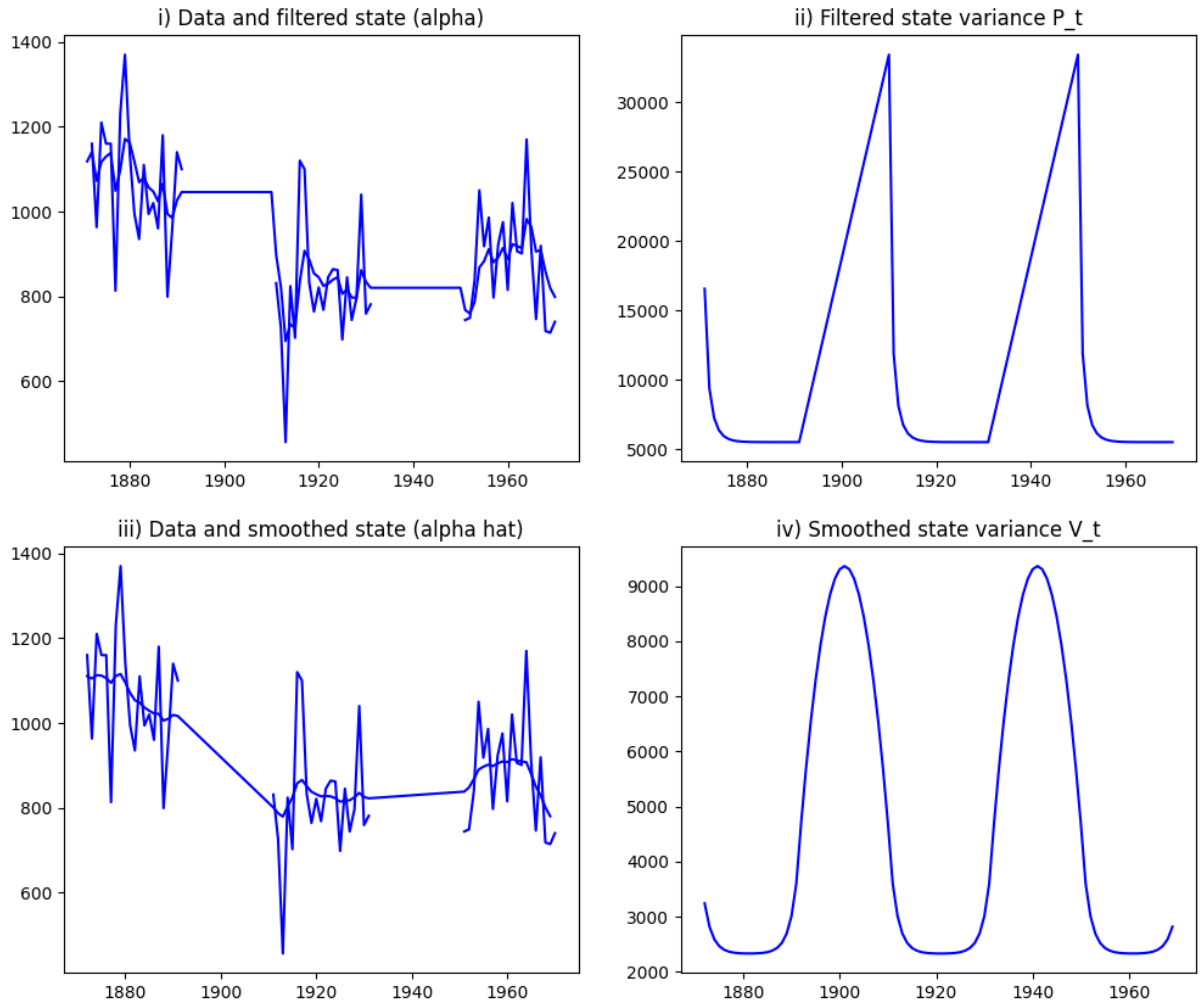

i) Data and filtered state (alpha)
ii) Filtered state variance P_t
iii) Data and smoothed state (alpha hat)
iv) Smoothed state variance V_t
1400
1200
1000
800
600
30000
25000
20000
15000
10000
5000
9000
8000
7000
6000
5000
4000
3000
2000
1880
1900
1920
1940
1960

Figure 2.6

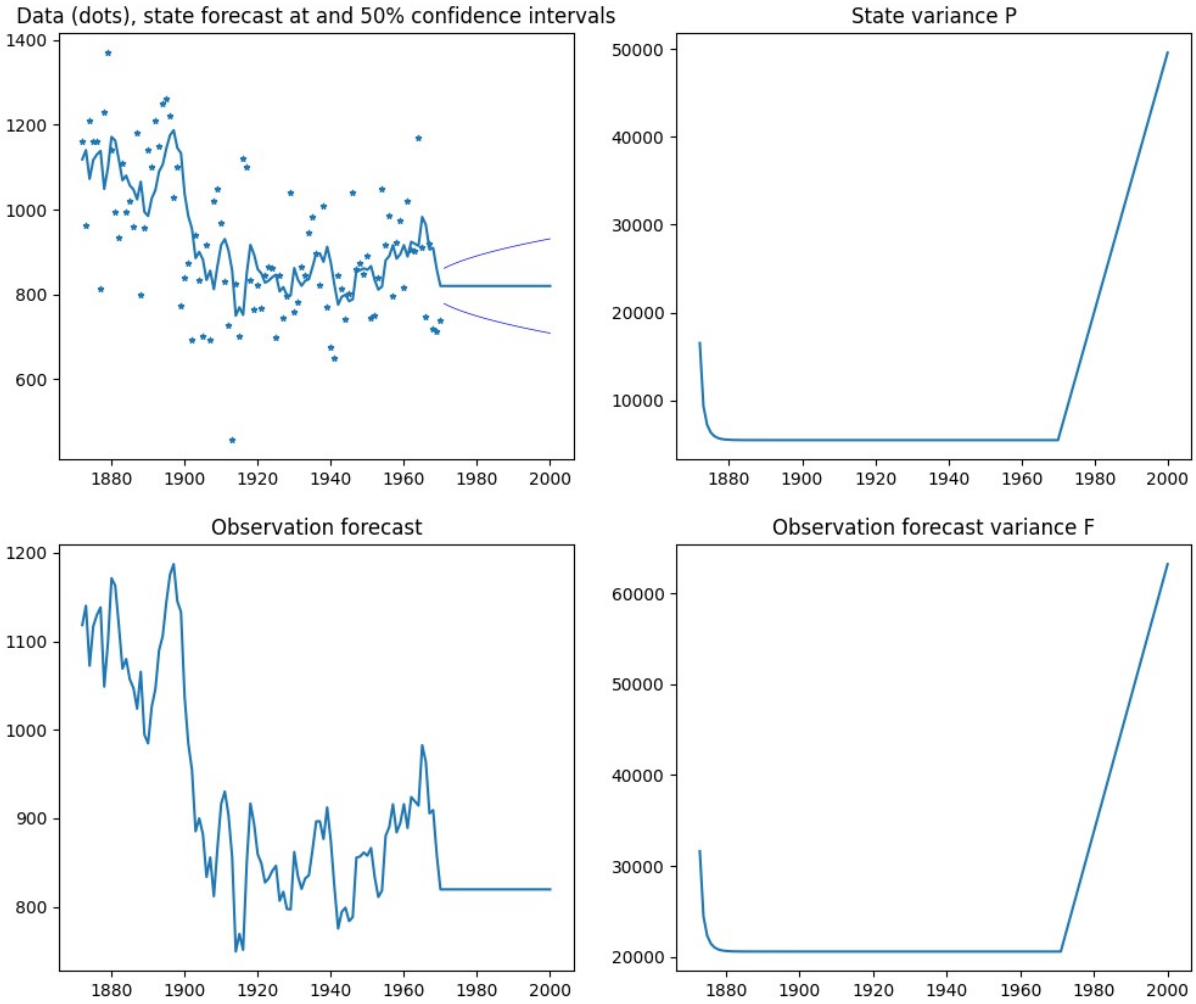

Data (dots), state forecast at and 50% confidence intervals
State variance P
Observation forecast
Observation forecast variance F
1400
1200
1000
800
600
50000
40000
30000
20000
10000
1100
900
60000
1880
1900
1920
1940
1960
1980
2000

Figure 2.7

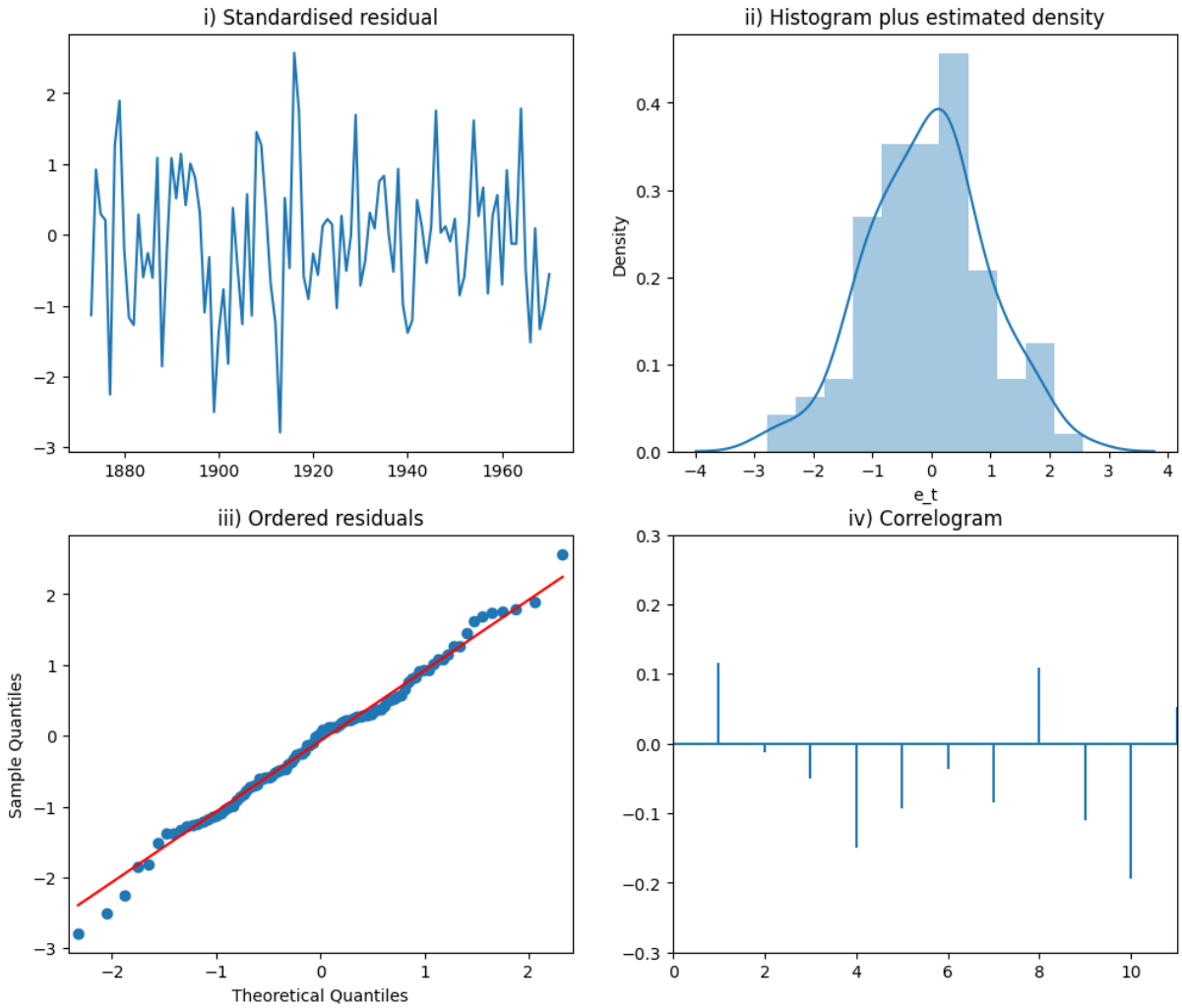
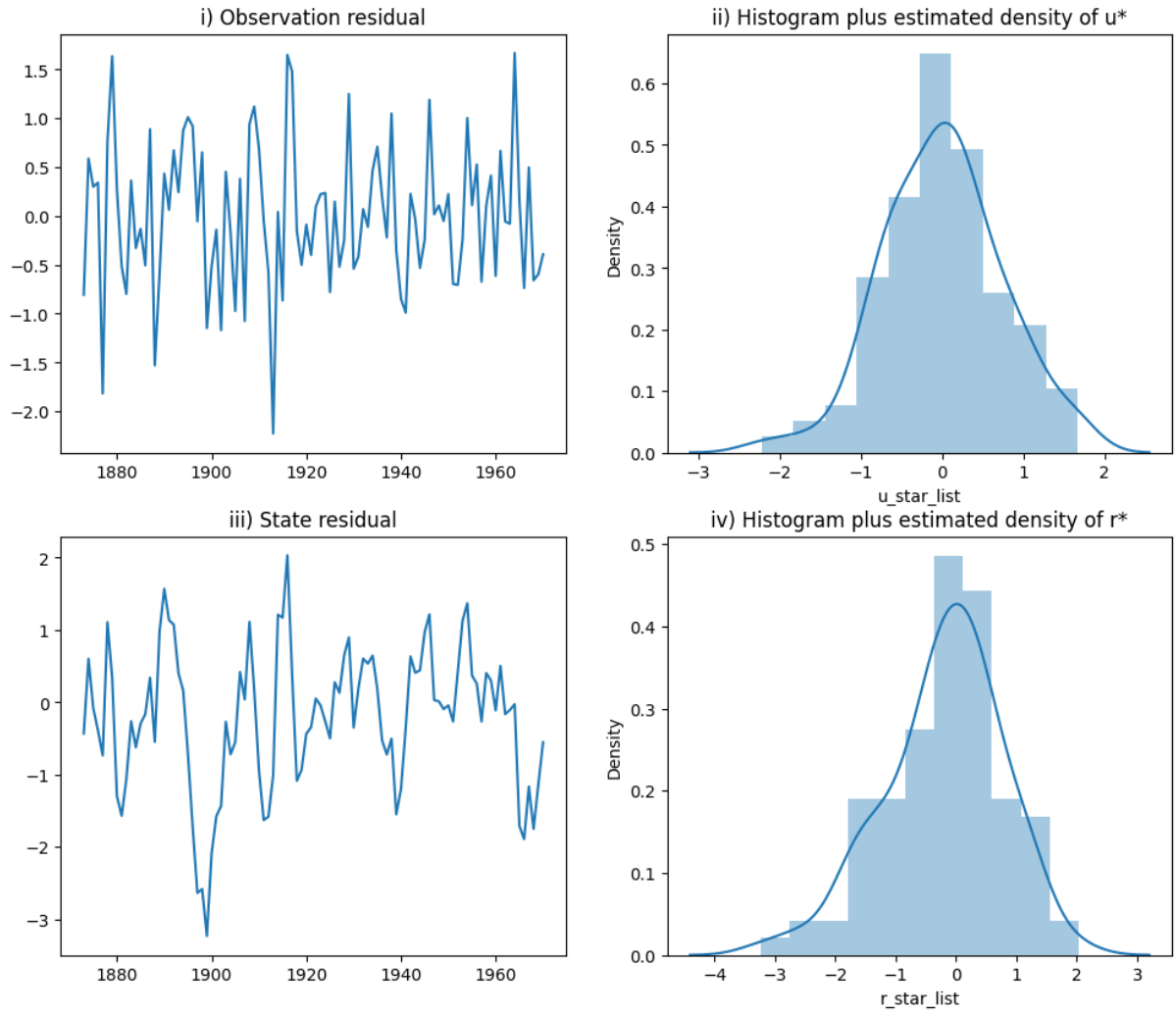

i) Standardised residual
ii) Histogram plus estimated density
Density
e_t
iii) Ordered residuals
Sample Quantiles
Theoretical Quantiles
iv) Correlogram

Figure 2.8



Part 1. (B) Maximum likelihood estimates for σ_ε^2 and σ_n^2.

After estimating the parameters σ_ε^2 and σ_n^2 for the Nile data and checking that both are close to the book results. These are the results we have obtained:

$\sigma_\varepsilon^2 = 15253.927$
$\sigma_n^2 = 1482.757$

```
[157]  print(results)
       ✓ 0.1s

...    final_simplex: (array([[15253.92696055,  1482.75717874],
              [15253.92695675,  1482.75718882],
              [15253.92686799,  1482.75721401]]), array([635.69421112, 635.69421112, 635.69421112]))
                  fun: 635.6942111233221
              message: 'Optimization terminated successfully.'
                 nfev: 112
                  nit: 54
               status: 0
              success: True
                    x: array([15253.92696055,  1482.75717874])
```