How Stars Work

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8 July 2024

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1 Introduction

We live around a star, the Sun. It is overwhelmingly the most important feature of our environment – too bright to look at! – and drives the dynamics of our climate, our economy, and our daily routine. We understand it in remarkable detail: we can measure its internal sounds and buoyancy waves as it rings like a bell, and using the science of helioseismology we can accurately reconstruct its physical properties almost all the way to the core. We can see its magnetised wind as the Southern Lights, and measure it quantitatively on satellites. We we can even measure neutrinos directly reaching us from the nuclear furnace at its core.

More broadly, stars are to astronomy what atoms are to chemistry: innumerable building blocks of a much larger universe of galaxies and cosmology. Each has remarkably simple physics, really parametrized to a very good approximation only by their mass and composition, and at higher order their rotation period; these physics are so simple and reliable that we can accurately model the spectra of whole galaxies in the distant universe just using laboratory physics and our understanding of stars nearby.

Analytic reasoning is usually good enough to get rough understanding of the important phenomena in stellar physics, usually as scaling relations - but these are usually only accurate to an order of magnitude or so, and *real* models require computer calculations. In this course we will attempt to do a bit of both.

This is a series of notes, aimed at a second-year undergraduate level, about how stars work: exploring their simplicity and complexity, assuming only a first year undergrad level of classical mechanics, quantum mechanics, and thermal physics, and other physics that we will introduce *ad hoc* as we go along. These notes are based in part on the excellent lectures from which I originally learned as an undergraduate at Berkeley, by Eliot Quataert; on Peter Tuthill and Mike Ireland's courses at Sydney; and on the PHYS2082 course developed at UQ by Holger Baumgardt, and by myself.

This book is compiled using Jupyter-Books, which allows us to include Python calculations in the text; I aim to make this available in the Open Astrophysics Library when it is in a mature state. We will follow the style guide of Edward Tufte, using margins for asides and illustrations to avoid interrupting the flow of the text.

We will use *cgs* units for most calculations in this text, except where otherwise noted.

2 Hydrostatic Equilibrium

The fundamental physics of stars is determined by a handful of principles:

- the star is everywhere in pressure balance under its own gravity, or *hydrostatic equilibrium*;
- its cooling by radiation must be met by
  - *energy production* in its interior by nuclear fusion or gravitational contraction,
  - and *energy transport* to its surface by radiation, convection, and conduction;
- how its material responds to pressure (parametrized the *equation of state*) and to light (parametrized by *opacity*); and
- in its interior and a star is typically rotating and magnetic, which we will neglect in most of these notes.

hydrostatic from Greek ὕδωρ, 'water', and στάσις, 'standing'; just like water in a tank.

Let's talk about hydrostatic equilibrium first.

An ordinary star like the Sun, throughout its whole body, is to a very good approximation an *ideal gas*, and is fully ionized except in its outermost layer. This means that the gas pressure satisfies the equation of state

$$p = nk_B T$$

where p is the pressure, n is the number density (particles per volume) of the gas, k_B is Boltzmann's constant (1.38×10^{-16} erg/K), and T is the temperature in kelvin.

In a star like the Sun, the pressure is mostly provided by gas pressure. In hotter stars, *photon* or *radiation pressure* is dominant, but in the Sun this is $\sim 10^{-3} p_{\text{gas}}$.

Even in the Sun, though, the gas is not *quite* a classical ideal gas: quantum mechanics is already relevant. There is an equation we will derive later in these notes for the pressure due to the *degeneracy* of a gas where the quantum wavefunctions of its constituent particles are nearly overlapping:

$$p_{\text{degeneracy}} = \frac{\hbar}{5m_e}(\frac{3}{8\pi})^{2/3}n^{5/3}$$

where $\hbar$ is the quantum of action $h/2\pi$ $(1.0546 \times 10^{-27}\text{erg} \cdot \text{s})$, and m_e the mass of the electron.

It turns out this is about a quarter of the gas pressure at the core of the Sun!

2.1 The Equation of Hydrostatic Equilibrium

Consider a thin shell of radius r (and surface area $A = 4\pi r^2$), thickness dr, and mass density ρ, enclosing a mass M_r.

i.e. M_r is the total mass integrated out up to a radius r.

The mass of this shell is $M_{\text{shell}} = \rho A dr$, and from Newton's law of gravitation the magnitude of the gravitational force of the whole shell inwards is

Newton's Shell Theorem states that the gravitational attraction of a symmetric shell of matter, and therefore by linearity of a ball of matter, can be treated as if the mass were concentrated at a point at the centre.

$$\frac{-GM_r M_{\text{shell}}}{r^2}$$

So now we can calculate the net force on this shell (and therefore acceleration a), and require the forces to be in balance:

$$F_{\text{net}} = M_{\text{shell}}a = P_{\text{below}} \cdot A - P_{\text{above}} \cdot A - \frac{GM_r M_{\text{shell}}}{r^2}$$

Letting $P_{\text{above}} = P_{\text{below}} + dP$,

$$M_{\text{shell}}a = a\rho A dr = -AdP - \frac{GM_r M_{\text{shell}}}{r^2}$$

and therefore rearranging, in equilibrium ($a = 0$) we have

$$\frac{dP}{dr} = -\rho\frac{GM_r}{r^2}$$

Tattoo this equation on the back of your eyelids.