Classical Mechanics

Notes on
the Taylor
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Newton's Laws of Motion

1.1 classical Mechanics

Galileo (1564-1642) + Newton (1642-1727)

These two figures laid the foundation of the mechanics we still use today. We attribute much to Newton and his three laws, but Galileo's work on kinematics and an early concept of inertia were essential for shaping Newton's framework.

Later, other physicists developed formulations of "classical" mechanics that are mathematical equivalent to Newton's laws, but often more powerful or convenient for certain types of problems.

Lagrange (1736-1813) + Hamilton (1805-1865)

Lagrangian and Hamiltonian mechanics provide different ways of describing the same physical laws. These formulations are especially useful when dealing with constrained systems, generalized coordinates, and more complex physical systems.

what is meant here is this:

Newton's laws work best when:

- Objects move in straight lines or simple curves
- Forces are easy to identify
- motion is naturally described with x, y, z coordinates

Constrained Systems:

A constrained system is one where motion is restricted by geometry or physical connections.

In Newton's framework, you often have to introduce extra forces (like tension, normal force, constraint force) just to enforce geometry.

Examples:

- A bead constrained to slide along a string
- A block constrained to move along an inclined plane
- A pendulum constrained to move in a circular arc.
- Two masses connected by a rigid rod.

Newton forces us to include constraint forces that do no work but still clutter the equations.

In Lagrangian Mechanics:

- You directly write equations of θ, s, etc.,
- the equations of motion naturally come out in those coordinates.

So we get to kind of "generalize" the coordinates, more on this later.

"More complex" physical systems

meaning systems that are:

- Multi-body
- highly coupled
- Non-cartesian
- Or where force analysis is messy.

Example include:

- Double Pendulum
- Atwood machine with pulleys that have mass
- Rolling Objects with translation + rotation
- Systems with springs
- Mechanical linkages
- Systems with many degrees of freedom

Newton would have us have you calculate

$\Sigma \vec{F} = m\vec{a}$ for each object

then add constraint equations

then solve a big coupled system

In Lagrangian mechanics you write one scalar function:

$$L = T - V$$

then apply one systemic rule
(Euler-Lagrange equation)

The coupling and constraints are handled cleanly!