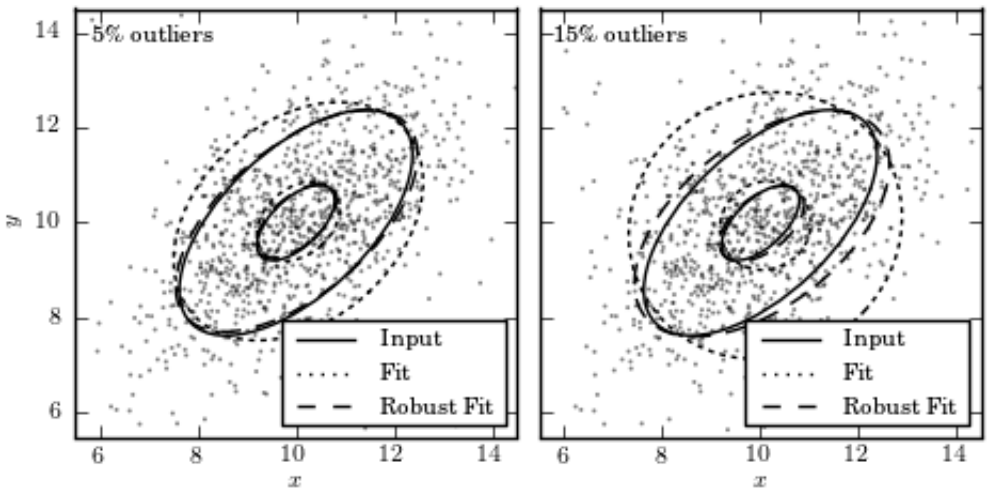
# Bivariate Gaussian: Robust Parameter Estimation

Figure 3.23.

An example of computing the components of a bivariate Gaussian using a sample with 1000 data values (points), with two levels of contamination. The core of the distribution is a bivariate Gaussian with $(\mu_x, \mu_y, \sigma_1, \sigma_2, \alpha) = (10, 10, 2, 1, 45^\odot)$ The "contaminating" subsample contributes 5% (left) and 15% (right) of points centered on the same $(\mu_x, \mu_y)$, and with $\sigma_1 = \sigma_2 = 5$. Ellipses show the 1- and 3-sigma contours. The solid lines correspond to the input distribution. The thin dotted lines show the nonrobust estimate, and the dashed lines show the robust estimate of the best-fit distribution parameters (see Section 3.5.3 for details).




**Code output:**

**Python source code:**

Comments by Ben Rose

```
# Author: Jake VanderPlas
# License: BSD
#   The figure produced by this code is published in the textbo
#   "Statistics, Data Mining, and Machine Learning in Astronomy
#   For more information, see http://astroML.github.com
#   To report a bug or issue, use the following forum:
#    https://groups.google.com/forum/#!forum/astroml-general
import numpy as np
```

```
from scipy import stats
from matplotlib import pyplot as plt
from matplotlib.patches import Ellipse
from astroML.stats import fit_bivariate_normal
from astroML.stats.random import bivariate_normal

# percent sign needs to be escaped if usetex is activated
import matplotlib
if matplotlib.rcParams.get('text.usetex'):
    pct = r'\%'
else:
    pct = r'%'

#------------------------------------------------------------
# This function adjusts matplotlib settings for a uniform feel
# Note that with usetex=True, fonts are rendered with LaTeX.  T
# result in an error if LaTeX is not installed on your system.
# you can set usetex to False.
from astroML.plotting import setup_text_plots
setup_text_plots(fontsize=8, usetex=True)

N = 1000

sigma1 = 2.0
sigma2 = 1.0
mu = [10, 10]
alpha_deg = 45.0
alpha = alpha_deg * np.pi / 180

#------------------------------------------------------------
# Draw N points from a multivariate normal distribution
#
#   we use the bivariate_normal function from astroML.  A more
#   general function for this is numpy.random.multivariate_norm
#   which requires the user to specify the full covariance matr
#   bivariate_normal() generates this covariance matrix for the
#   given inputs

np.random.seed(0)
X = bivariate_normal(mu, sigma1, sigma2, alpha, N)

#------------------------------------------------------------
# Create the figure showing the fits
fig = plt.figure(figsize=(5, 2.5))
fig.subplots_adjust(left=0.1, right=0.95, wspace=0.05,
                    bottom=0.15, top=0.95)


# We'll create two figures, with two levels of contamination
for i, f in enumerate([0.05, 0.15]):
    ax = fig.add_subplot(1, 2, i + 1)

    # add outliers distributed using a bivariate normal.
    X[:int(f * N)] = bivariate_normal((10, 10), 2, 4,
                                      45 * np.pi / 180., int(f
    x, y = X.T

    # compute the non-robust statistics
    (mu_nr, sigma1_nr,
     sigma2_nr, alpha_nr) = fit_bivariate_normal(x, y, robust=F
```

Setting up distribution variables

Generate signal from distribution

Cool trick, to get the value of the list item and its location. He sets up the two subplots with the location in the list.

This is for the first (percent contaminated)*(total number), add contaminated distribution

Got cut off, read online

The blue part is where the statistics/analysis is. Its one line for either the robust or non-robust method.

I personaly hate elipses in matplotlib

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```
    # compute the robust statistics
    (mu_r, sigma1_r,
     sigma2_r, alpha_r) = fit_bivariate_normal(x, y, robust=Tru

    # scatter the points
    ax.scatter(x, y, s=2, lw=0, c='k', alpha=0.5)

    # Draw elipses showing the fits
    for Nsig in [1, 3]:
        # True fit
        E = Ellipse((10, 10), sigma1 * Nsig, sigma2 * Nsig, alp
                    ec='k', fc='none')
        ax.add_patch(E)

        # Non-robust fit
        E = Ellipse(mu_nr, sigma1_nr * Nsig, sigma2_nr * Nsig,
                    (alpha_nr * 180. / np.pi),
                    ec='k', fc='none', linestyle='dotted')
        ax.add_patch(E)

        # Robust fit
        E = Ellipse(mu_r, sigma1_r * Nsig, sigma2_r * Nsig,
                    (alpha_r * 180. / np.pi),
                    ec='k', fc='none', linestyle='dashed')
        ax.add_patch(E)

    ax.text(0.04, 0.96, '%i%s outliers' % (f * 100, pct),
            ha='left', va='top', transform=ax.transAxes)

    ax.set_xlim(5.5, 14.5)
    ax.set_ylim(5.5, 14.5)
    ax.set_xlabel('$x$')

    # This is a bit of a hack:
    # We'll draw some lines off the picture to make our legend
    ax.plot([0], [0], '-k', label='Input')
    ax.plot([0], [0], ':k', label='Fit')
    ax.plot([0], [0], '--k', label='Robust Fit')
    ax.legend(loc='lower right')

    if i == 0:
        ax.set_ylabel('$y$')
    else:
        ax.yaxis.set_major_formatter(plt.NullFormatter())

plt.show()
```

Cool trick to get y-axis label only on the far left

**[download source: fig_robust_pca.py]**