Ch. 3 - Q10

In [1]:
```
import ISLP
import pandas as pd
from sklearn.linear_model import LinearRegression
import statsmodels.api as sm
import matplotlib.pyplot as plt
```

In [2]:
```
# Load in data
Carseats = ISLP.load_data("Carseats")
Carseats.head()
```

Out[2]:

	Sales	CompPrice	Income	Advertising	Population	Price	ShelveLoc	Age	Education	Ur
0	9.50	138	73	11	276	120	Bad	42	17	
1	11.22	111	48	16	260	83	Good	65	10	
2	10.06	113	35	10	269	80	Medium	59	12	
3	7.40	117	100	4	466	97	Medium	55	14	
4	4.15	141	64	3	340	128	Bad	38	13	

In [3]:
```
# Assign design matrix and target vector
X = Carseats[['Price', 'Urban', 'US']].copy()
X[['Urban', 'US']] = X[['Urban', 'US']].apply(lambda col: col.map({'Yes': 1, 'No':
y = Carseats['Sales']
```

Part (a)

In [4]:
```
model = LinearRegression()
model.fit(X, y)

pd.DataFrame({'Variable': ['Intercept'] + list(X.columns),
              'Coefficient': [model.intercept_] + list(model.coef_)})
```

Out[4]:

	Variable	Coefficient
0	Intercept	13.043469
1	Price	-0.054459
2	Urban	-0.021916
3	US	1.200573

Part (b)

Keep in mind that the units of `Sales` are in thousands.

- The coefficient for "Price" means that, on average, increasing the price by $1 decreases sales by 54.46 units, assuming all other factors stay the same.

- The coefficient for "Urban" means that, on average, sales in urban locations are 21.92 units lower than in rural locations, keeping all other factors the same.
- The coefficient for "US" means that, on average, sales in US stores are 1,200.57 units higher than in non-US stores, assuming all other factors remain unchanged.

Part (c)

In [5]:
```
equation = f"Sales = {model.intercept_:.2f}"
for coef, col in zip(model.coef_, X.columns):
    if coef >= 0: equation += f" + {coef:.2f} * {col}"
    else: equation += f" - {-coef:.2f} * {col}"
print(equation)
```

```
Sales = 13.04 - 0.05 * Price - 0.02 * Urban + 1.20 * US
```

Part (d)

In [6]:
```
# Recreating linear model in statsmodels because apparently
#   sci-kit learn doesn't provide p-values :)
X_with_intercept = sm.add_constant(X)
full_model = sm.OLS(y, X_with_intercept).fit()
full_model.pvalues
```

Out[6]:
```
const     3.626602e-62
Price     1.609917e-22
Urban     9.357389e-01
US        4.860245e-06
dtype: float64
```

- We can reject the null hypothesis for the variables, `Price` and `US`.

Part (e)

In [7]:
```
reduced_X = Carseats[['Price', 'US']].copy()
reduced_X['US'] = reduced_X['US'].map({'Yes': 1, 'No': 0})

reduced_X_with_intercept = sm.add_constant(reduced_X)
reduced_model = sm.OLS(y, reduced_X_with_intercept).fit()

print(f"Full Model - R^2:\t{full_model.rsquared}")
print(f"Reduced Model - R^2:\t{reduced_model.rsquared}")
```

```
Full Model - R^2:       0.2392753921840549
Reduced Model - R^2:    0.23926288842678567
```

Part (f)

- The two models are nearly identical. Thus the reduced model is likely the preferred option due to it's simplicity

Part (g)

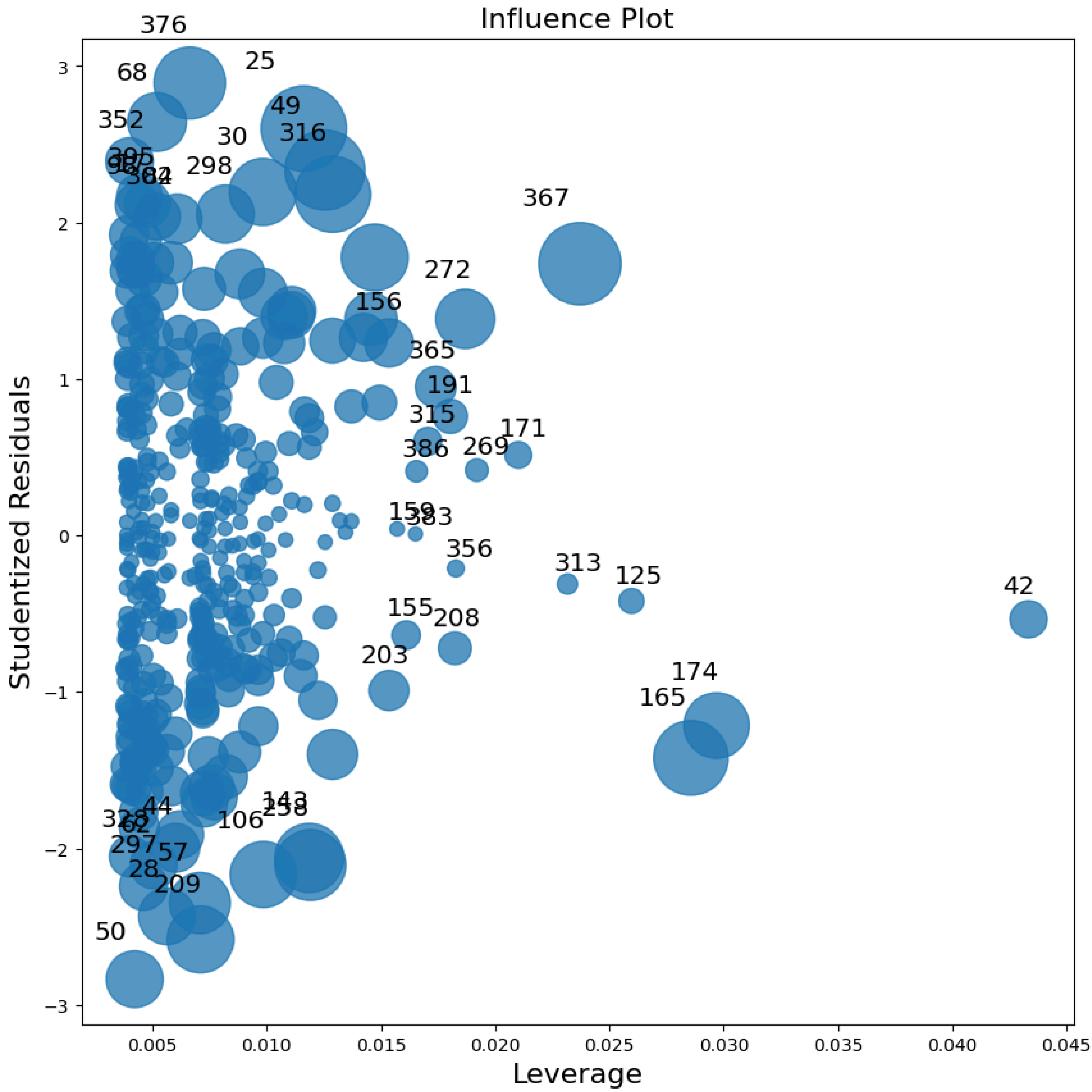
In [8]:
```
confidence_intervals = reduced_model.conf_int(alpha=0.05)  # 95% CI
confidence_intervals.columns = ['Lower Bound', 'Upper Bound']
confidence_intervals
```

Out[8]:

	Lower Bound	Upper Bound
const	11.79032	14.271265
Price	-0.06476	-0.044195
US	0.69152	1.707766

Part (h)

In [9]:
```
fig, ax = plt.subplots(figsize=(10, 10))
sm.graphics.influence_plot(reduced_model, ax=ax)
plt.show()
```



In [10]:
```
n, p = reduced_X_with_intercept.shape
print(f"Threshold: {(p) / n}")
```

```
Threshold: 0.0075
```

Outliers and High Leverage Points

The plot above shows that there are no outliers as all residuals are within ± 3 standard deviations, however some observations come close.

Additionally, many points have high leverage because their leverage values exceed the threshold 0.0075, which is calculated as:

$$\frac{p+1}{n} = \frac{3}{400} = 0.0075$$

where $p = 2$ is the number of predictors and $n = 400$ is the number of observations. However, likewise, these points are not outliers.