Classical and Modern Geometry Lecture Notes

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March 12, 2021

Contents

1 Origami Geometry

1.1 30° - 60° - 90° Right Triangle by Paper Folding

First, we need a rectangular paper is needed to proceed.
Next, follow the steps outlined below

1. Fold the paper lengthwise into 2 equal parts
2. Fold the upper right (upper left) corner such that it intersects with the line that separates the paper into 2 equal parts.

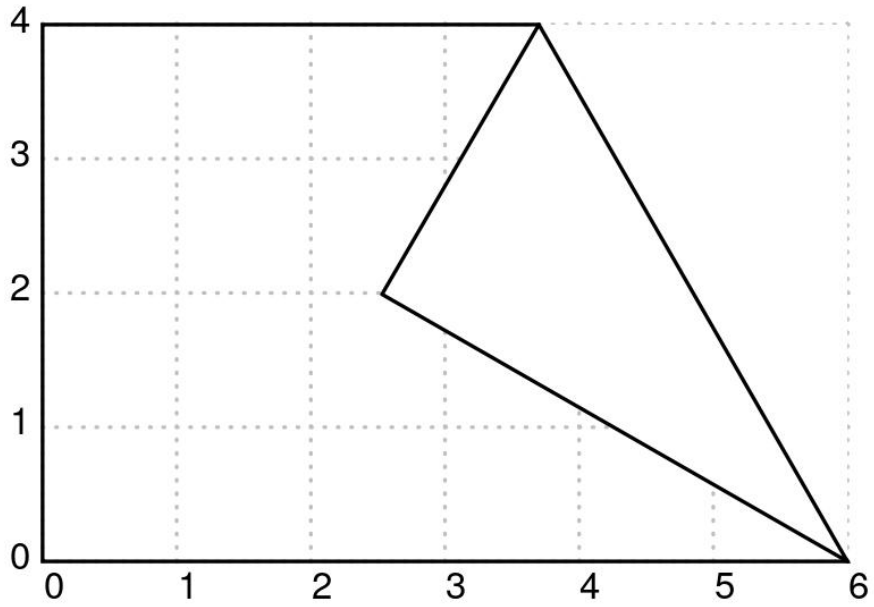

Figure 1: Resulting Fold

```
\documentclass[pstricks,border=12pt]{standalone}
\usepackage{pst-eucl,multido,fp}

\FPset\Width{6.00}
\FPset\Height{4.00}


\psset
{
    PointName=none,
    PointSymbol=none,
}

\def\Picture#1#2{%
\def\X{#1}\def\Y{#2}
\begin{pspicture}[showgrid=tru](\Width,\Height)
        \pstGeonode(0,0){BottomLeft}(\Width,\Height){TopRight}(\X,\Height){Top}(\Width,\Y){Right}
        \pstOrtSym{Top}{Right}{TopRight}
        \pspolygon[dimen=inner](BottomLeft)(BottomLeft|Top)(Top)(Right)(TopRight|BottomLeft)

        \pspolygon[fillstyle=solid,linestyle=none](Top)(TopRight)(Right)
        \pspolygon[fillstyle=solid,linejoin=1](Top)(TopRight')(Right)
\end{pspicture}%
}
\begin{document}
    \multido{\nx=\Width+-0.1}{24}{\Picture{\nx}{0}}
    \FPeval\Start{round(Width+14*(-0.25):2)}%
\end{document}
```

Figure 2: Code to Generate the Origami Animation

Proof. Consider the diagram below. Notice that the hypotenuse of ΔABC is equal to the height of our paper say x (in this specific case, that is equal to 4). It is also clear, that the shorter leg is equal to $\frac{x}{2}$.
Now, borrowing from plane geometry, it follows that ΔABC is a 30-60-90 triangle.
That fact will imply the following:

1. $\angle CAB = 30°$
2. $\angle DAC = 60°$ since $\angle DAB$ is a right triangle
3. $\angle DAE = \angle EAD = \angle DEA = \angle AEC = 30°$ since $\overline{\text{AE}}$ is a bisector of $\angle DAC$ and $\angle DEC$
4. $\angle ADE = 60°$ since ΔADE is an isoceles triangle.

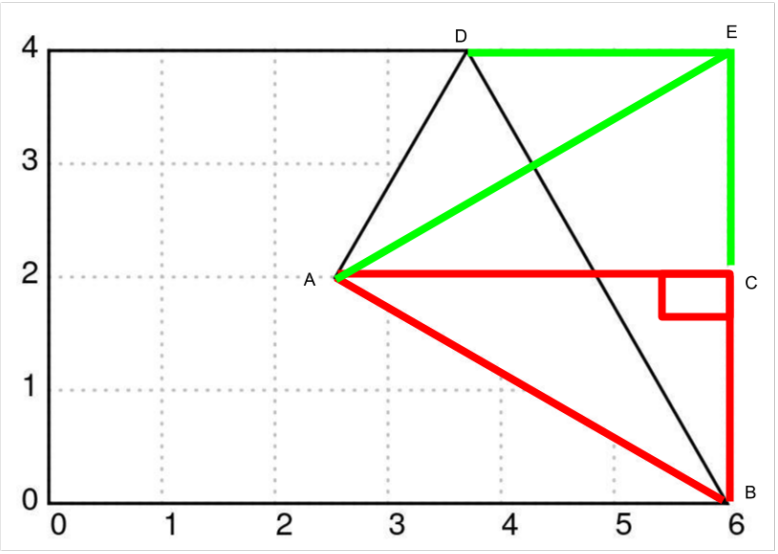

Figure 3: Code to Generate the Origami Animation

$\therefore \Delta DAB$ is a 30° - 60° - 90° triangle since by contruction $\angle DAB$ is a right angle and we have shown that $\angle ADE = 60°$ The sum of the angles must be 180° thereby forcing $\angle DBA = 30°$.

□