

# Classical and Modern Geometry Lecture Notes

Benjamin Cabalona Jr.

March 12, 2021

## Contents

<b>1</b>	<b>Origami Geometry</b>	<b>2</b>
1.1	30° - 60° - 90° Right Triangle by Paper Folding . . . . .	2

# 1 Origami Geometry

## 1.1 $30^\circ - 60^\circ - 90^\circ$ Right Triangle by Paper Folding

First, we need a rectangular paper is needed to proceed.

Next, follow the steps outlined below

1. Fold the paper lengthwise into 2 equal parts
2. Fold the upper right (upper left) corner such that it intersects with the line that separates the paper into 2 equal parts.

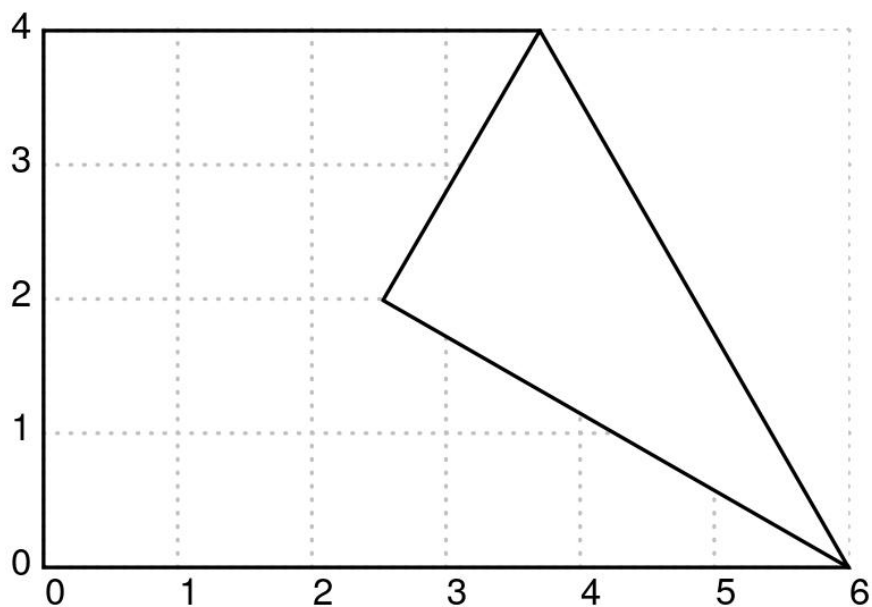


Figure 1: Resulting Fold

```

1 \documentclass[pstricks,border=12pt]{standalone}
2 \usepackage{pst-eucl,multido,fp}
3
4 \FPset\Width{6.00}
5 \FPset\Height{4.00}
6
7
8 \psset
9 {
10   PointName=none,
11   PointSymbol=none,
12 }
13
14 \def\Picture#1#2{%
15 \def\X{#1}\def\Y{#2}
16 \begin{pspicture}[showgrid=tru](\Width,\Height)
17   \pstGeonode(0,0){BottomLeft}(\Width,\Height){TopRight}(\X,\Height){Top}(\Width,\Y){Right}
18   \pstOrtSym{Top}{Right}{TopRight}
19   \pspolygon[dimen=inner](BottomLeft)(BottomLeft|Top)(Top)(Right)(TopRight|BottomLeft)
20
21   \pspolygon[fillstyle=solid,linestyle=none](Top)(TopRight)(Right)
22   \pspolygon[fillstyle=solid,linejoin=1](Top)(TopRight')(Right)
23 \end{pspicture}%
24 }
25 \begin{document}
26   \multido{\nx=\Width+-0.1}{24}{\Picture{\nx}{0}}
27   \FPeval\Start{round(Width+14*(-0.25):2)}%
28 \end{document}

```

Figure 2: Code to Generate the Origami Animation

*Proof.* Consider the diagram below. Notice that the hypotenuse of  $\triangle ABC$  is equal to the height of our paper say  $x$  (in this specific case, that is equal to 4). It is also clear, that the shorter leg is equal to  $\frac{x}{2}$ . Now, borrowing from plane geometry, it follows that  $\triangle ABC$  is a 30-60-90 triangle.

That fact will imply the following:

1.  $\angle CAB = 30^\circ$
2.  $\angle DAC = 60^\circ$  since  $\angle DAB$  is a right triangle
3.  $\angle DAE = \angle EAD = \angle DEA = \angle AEC = 30^\circ$  since  $\overline{AE}$  is a bisector of  $\angle DAC$  and  $\angle DEC$
4.  $\angle ADE = 60^\circ$  since  $\triangle ADE$  is an isosceles triangle.

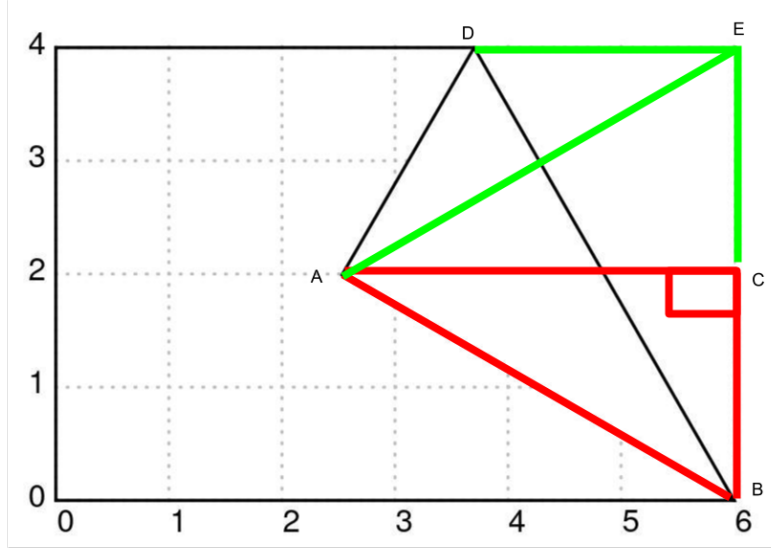


Figure 3: Code to Generate the Origami Animation

$\therefore \triangle DAB$  is a  $30^\circ - 60^\circ - 90^\circ$  triangle since by construction  $\angle DAB$  is a right angle and we have shown that  $\angle ADE = 60^\circ$ . The sum of the angles must be  $180^\circ$  thereby forcing  $\angle DBA = 30^\circ$ .

□