

Exercise 5

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1 Read Chapter 3.1-3.5 from Haykin's book; summarize or sketch your insights in mind-map or an outline or a summary.

- Rosenblatt proved that given linearly separable data, a perceptron is proven to converge.
- Least mean square algorithm is the backbone of linear adaptive filters
- Adaptive filtering
 - m dimensional input produces scalar output
 - data equally distributed
 - data can be spread over space (snapshot) or over time (uniformly spaced in time)
 - Filtering process produces the output and error signals
 - Adaptive process involves adjustments based on errors
 - Error correction is an optimization problem
- Unconstrained optimization techniques
 - Optimal solution is gradient of cost function equal to 0
- Steepest descent
 - converges slowly
 - size of η produces overdamped response when small, under when large
- Newton method
 - needs to be twice continuously differentiable wrt w to form hessian
 - converges quickly and generally not subject to underdamped behavior of steepest descent
 - Needs to be positive definite matrix, however there is no guarantee of that.

- Gauss Newton method

Only requires jacobian of the error vector as opposed to hessian of cost function
Jacobian product must be non singular

- Least mean squares

Inverse of the learning rate eta is the
weight vector traverses random trajectory in contrast with steepest descent
The stability of the system is determined by choosing an appropriate eta for x
Model independent, therefore robust
Needs approx 10x the dimensionality iterations to converge

2 (3.1)

2.1 (a)

2.2 (b)

3 (3.2)

4 (3.4)

The LMS algorithm converges for

$$0 < \eta < \frac{2}{\lambda_{max}}$$

where λ_{max} is the largest eigen value of the correlation matrix.
Given the correlation matrix

$$R_x = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

The SVD of R_x gives 1.5 as the the largest eigen value This particular LMS problem will converge for

$$0 < \eta < \frac{2}{1.5} = 1.33$$

5 (3.8)

5.1 (a)

5.2 (b)

5.3 (c)