

cluster weights

$$r_{ik} = \frac{\mathcal{N}(x_i | \mu_k, \Sigma_k) \cdot \pi_k}{\sum_{j=1}^k \mathcal{N}(x_i | \mu_j, \Sigma_j) \cdot \pi_j}$$

$$\log(r_{ik}) = \frac{\log \left[\pi_k (2\pi)^{-\frac{r}{2}} \det(\Sigma_k)^{-\frac{1}{2}} e^{-\frac{1}{2}(x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k)} \right]}{\log \left[\sum_{j=1}^k \pi_j (2\pi)^{-\frac{r}{2}} \det(\Sigma_j)^{-\frac{1}{2}} e^{-\frac{1}{2}(x_i - \mu_j)^T \Sigma_j^{-1} (x_i - \mu_j)} \right]}$$

$$= \log(\pi_k) - \frac{r}{2} \log(2\pi) - \frac{1}{2} \log |\Sigma_k| - \frac{1}{2} (x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k)$$

$$- \log \left[\sum_{j=1}^k e^{-\frac{1}{2}(x_i - \mu_j)^T \Sigma_j^{-1} (x_i - \mu_j)} - \frac{1}{2} \log |\Sigma_j| - \frac{r}{2} \log(2\pi) + \log[\pi_j] \right]$$

$$= \log(\pi_k) - \frac{1}{2} \left[r \log(2\pi) + \log |\Sigma_k| + M_k \right] - \log \sum_{j=1}^k \exp \left[\log(\pi_j) - \frac{1}{2} \left[r \log(2\pi) + \log |\Sigma_j| + M_j \right] \right]$$

$$\text{where } M_k = (x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k)$$

GMM Update

$$\pi_k := \frac{1}{N} \sum_i r_{ik}$$

$$\mu_k := \frac{\sum_i r_{ik} x_i}{\sum_i r_{ik}}$$

$$\Sigma_k = \frac{\sum_i r_{ik} (x_i - \mu_k)(x_i - \mu_k)^T}{\sum_i r_{ik}}$$