Chery

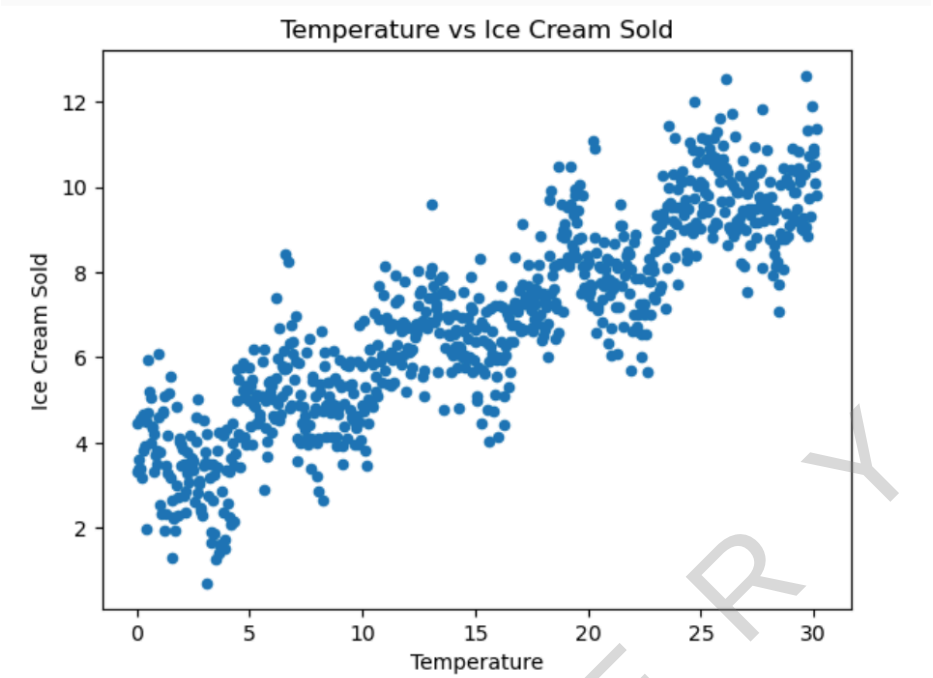
Predictive Regression Model for Ice Cream Sales Based on Temperature Variations


```
df_tr['X'] = df_tr['X'] + 0.000001
df_tr
```

	X	y
0	0.000002	4.456329
1	0.037739	3.301815
2	0.075476	3.595955
3	0.113213	3.401495
4	0.150949	4.571147
...	...	...
795	30.000797	10.928634
796	30.038534	10.106108
797	30.076270	10.521322
798	30.114007	9.814676
799	30.151744	11.364662

	X	y
count	800.000000	800.000000
mean	15.075873	6.785090
std	8.720397	2.352965
min	0.000002	0.694817
25%	7.537937	5.008300
50%	15.075873	6.764510
75%	22.613808	8.689887
max	30.151744	12.621870

The table provides a statistical summary of 800 observations related to temperature (X) and the number of ice creams sold (y). The temperatures range from 0.000001 ℃ to 30.15 ℃ . For the number of ice cream sold, the average is approximately 6.79 units with a standard deviation of 2.35 units.



The graph provides insights into the linear regression model's assumptions. The scatter plot of temperatures versus ice cream sales shows a positive linear relationship, suggesting that as temperature increases, ice cream sales tend to rise.

2) Simple Linear Regression function to fit the training data.

```
reg.fit(df_tr[['X']],df_tr['y'])
```

```
▸ LinearRegression
```

```
reg.coef_
```

```
array([0.23839763])
```

```
reg.intercept_
```

```
3.1910373234577953
```

(a) The coefficient estimates of the linear regression model are $\beta_0 \simeq 3.1910$ and $\beta_1 \simeq 0.2383$

(b) Since the $\beta_1 \neq 0$, this confirms the linear trend that can be seen in the previous scatterplot.

```
# B1 t-statistic
t_stat = reg.coef_[0]/std_B1
t_stat
```

```
53.28879296911331
```

```
pvalue = 2*(1-stats.t.cdf(t_stat,df=n-2))
pvalue
```

```
0.0
```

The t-statistic of β_1 is 53.2887 and its p-value = 0, it can be inferred that there is an association between temperature and the number of ice cream sold.

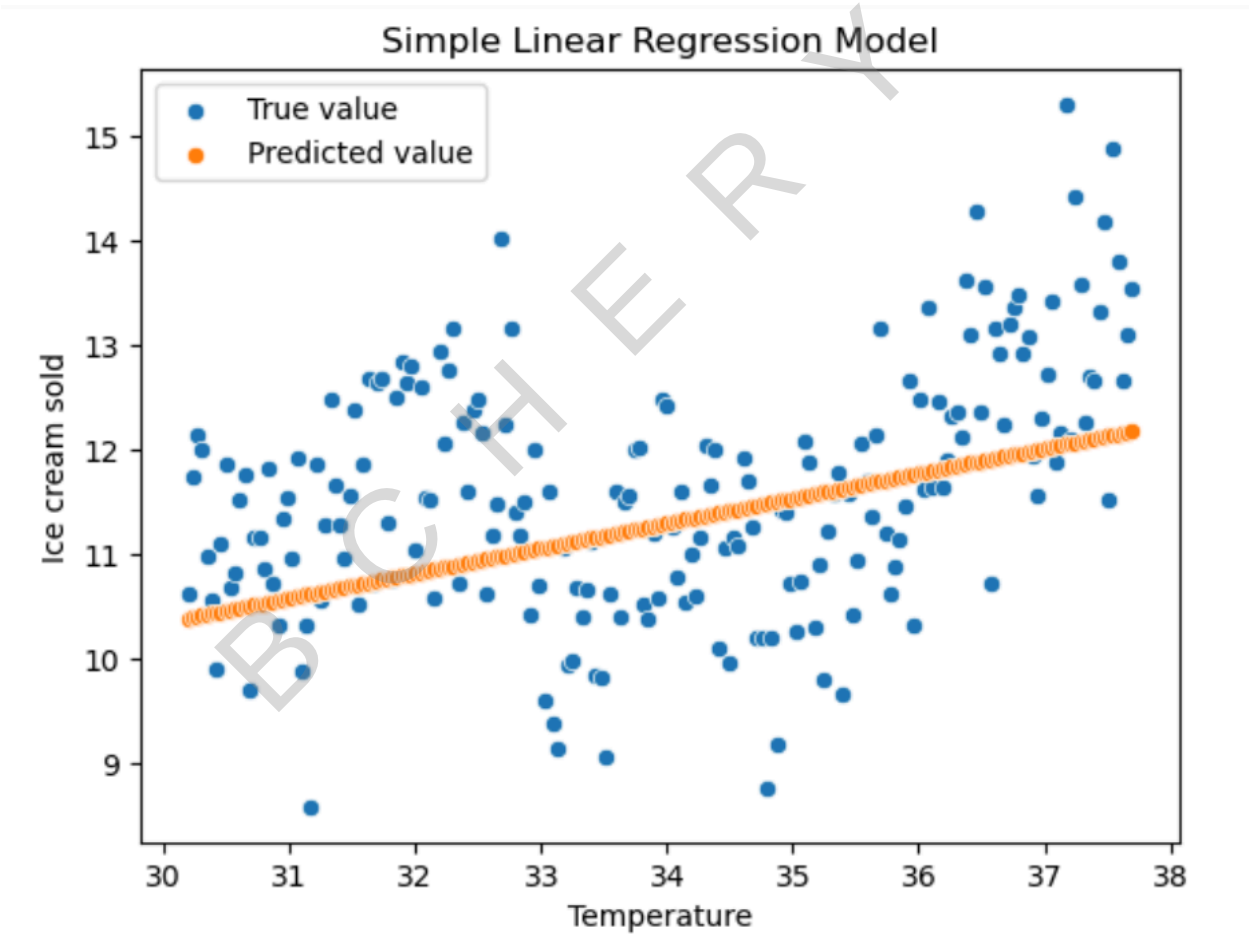
(c) The t-statistic of β_0 is 40.9617 and its p-value is also 0 so it is significant. That is, ice creams are sold even if the temperature is low.

```
# B0 t-statistic
t_stat = reg.intercept_/std_B0
t_stat
```

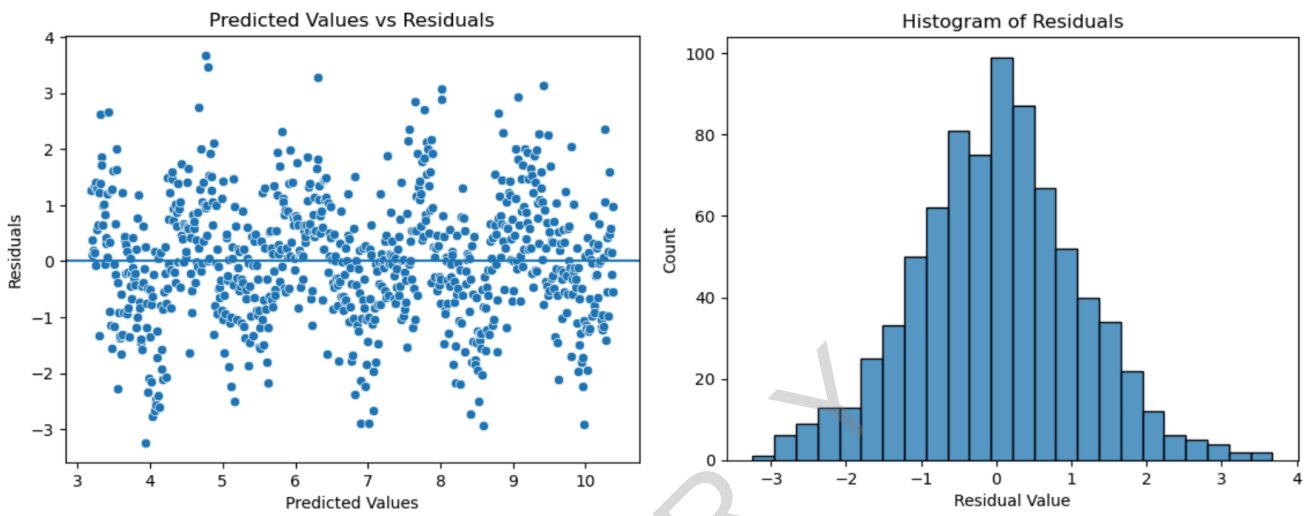
```
40.961717937070304
```

```
pvalue = 2*(1-stats.t.cdf(t_stat,df=n-2))
pvalue
```

```
0.0
```



The simple linear regression model's predictions, represented by the orange line, show a clear linear trend that increases with temperature, but fails to capture the high variability of the actual ice cream sales represented by the blue points. This suggests a more complex model that can capture nonlinear trends might improve the prediction performance.



The residuals plot displays a random scatter around zero, with no clear pattern, indicating that the assumption of constant variance is likely met. The histogram of residuals shows a roughly normal distribution, supporting the normality assumption of the errors.

3) Improvement of the model by adding non-linear features.

To select the best feature set for the model, both Sequential Feature Selector (**SFS**) and Recursive Feature Elimination with Cross-Validation (**RFECV**) were used, with each method evaluated based on *Adjusted* R^2. SFS adds (forward selection) or removes (backward selection) features to form a feature subset in a greedy fashion. At each stage, it chooses the best feature to add or remove based on the cross-validation performance. RFECV, in contrast, recursively reduces the feature set by training the estimator and removing the least important features in each iteration. The SFS process selected features: X, cos(x), sin(3x), cos(2x), while RFECV selected only 3: X, cos(x), sin(3x). In terms of performance, RFECV slightly outperformed SFS on the test set achieving an Adjusted R^2 of 0.4157 compared to SFS 0.4098. Given its performance and fewer selected features, RFECV's feature set may be preferable for better generalization and simplicity.

Linear Regression Model based on SFS variable selection.

```
print(f"Selected features: {features}")
```

```
Selected features: ['X', 'cos(x)', 'sin(3x)', 'cos(2x)']
```

```
print(model.coef_)
```

```
[[ 0.24375646  0.80560674  0.37283376 -0.05764484]]
```

```
adjusted_r2_train = adjusted_r2_scorer(model, X_train[features], y_train)
adjusted_r2_test = adjusted_r2_scorer(model, X_test[features], y_test)

print(f"sfs Adjusted R² on the training set: {adjusted_r2_train}")
print(f"sfs Adjusted R² on the test set: {adjusted_r2_test}")
```

```
sfs Adjusted R² on the training set: 0.8525671804523196
sfs Adjusted R² on the test set: 0.4098169921744048
```

Linear Regression Model based on RFECV variable selection.

```
print(f"Selected features: {feat}")
```

```
Selected features: ['X', 'cos(x)', 'sin(3x)']
```

```
print(model.coef_)
```

```
[[0.24390028 0.80723528 0.37146658]]
```

```
adjusted_r2_train = adjusted_r2_scorer(model, X_train[feat], y_train)
adjusted_r2_test = adjusted_r2_scorer(model, X_test[feat], y_test)

print(f"rfecv Adjusted R² on the training set: {adjusted_r2_train}")
print(f"rfecv Adjusted R² on the test set: {adjusted_r2_test}")
```

```
rfecv Adjusted R² on the training set: 0.8524488858820521
rfecv Adjusted R² on the test set: 0.4157832955818095
```

4) Lasso and Ridge regression

Ridge

```
Ridge(alpha=0.005)
```

```
print(reg.coef_)
```

```
[[ 0.24368559  0.00136115  0.80578882 -0.02308458  0.37265904  0.02261838
   0.00612593 -0.0580463  -0.02850832]]
```

```
adjusted_r2_train = adjusted_r2_scorer(reg, X_train, y_train)
adjusted_r2_test = adjusted_r2_scorer(reg, X_test, y_test)

print(f"Ridge Adjusted R² on the training set: {adjusted_r2_train}")
print(f"Ridge Adjusted R² on the test set: {adjusted_r2_test}")
```

```
Ridge Adjusted R² on the training set: 0.851752973933268
Ridge Adjusted R² on the test set: 0.3980786478053212
```

Lasso

```
Lasso(alpha=0.01)
```

```
print(np.round(reg.coef_, 4))
```

```
[ 0.2435  0.      0.7864 -0.0028  0.3531  0.0019  0.     -0.038  -0.    ]
```

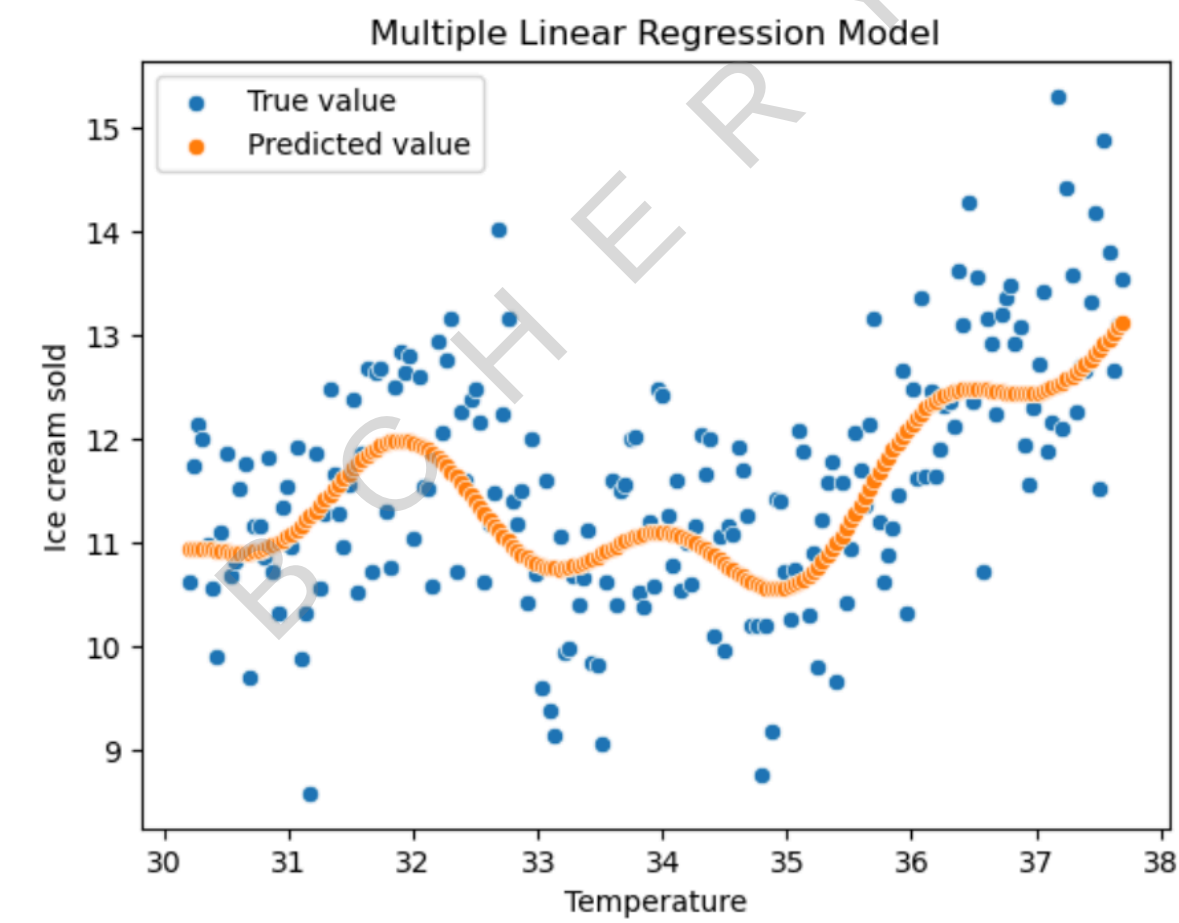
```
adjusted_r2_train = adjusted_r2_scorer(reg, X_train, y_train)
adjusted_r2_test = adjusted_r2_scorer(reg, X_test, y_test)

print(f"sfs Adjusted R² on the training set: {adjusted_r2_train}")
print(f"sfs Adjusted R² on the test set: {adjusted_r2_test}")
```

```
sfs Adjusted R² on the training set: 0.851544576269941
sfs Adjusted R² on the test set: 0.3938389985858648
```

The adjusted R^2 scores for Ridge and Lasso regression models on both the training and test sets appear to be lower than the scores achieved previously using SFS and RFECV.

5) Model prediction vs real value

	X	y	y_pred	residuals
0	30.189480	10.632211	10.951414	0.319203
1	30.227217	11.743522	10.951832	-0.791690
2	30.264954	12.148739	10.949863	-1.198876
3	30.302690	11.999377	10.945968	-1.053409
4	30.340427	10.991670	10.940629	-0.051041
...	...	...	...	...
195	37.548165	14.873723	12.918955	-1.954768
196	37.585902	13.807789	12.970946	-0.836843
197	37.623639	12.669681	13.023379	0.353698
198	37.661376	13.106141	13.075733	-0.030408
199	37.699113	13.544400	13.127476	-0.416924



The model predictions, represented by the orange points, follow the general trend of the data but do not fully capture its variability. Adjusted R^2 can be used to quantify the extent to which the model explains the variability in ice cream sales. Previously, the RFECV model demonstrated an Adjusted R^2 of approximately 42%, indicating that it explains about 42% of the variability in the data.