Spot Kinematic Simulator

State Update Equation:

$$s_{t+1} = As_t + Ba_t$$

State Vector:

$$s_t = \begin{pmatrix} x^W(t) \\ y^W(t) \\ \theta^W(t) \\ v_x^W(t) \\ v_y^W(t) \\ v_\theta^W(t) \\ \\ x_{ee}^W(t) \\ y_{ee}^W(t) \\ z_{ee}^W(t) \\ v_{x,ee}^W(t) \\ v_{y,ee}^W(t) \\ v_{z,ee}^W(t) \end{pmatrix}$$

Action Vector:

$$a_t = \begin{pmatrix} v_{x,\mathrm{cmd}}^B(t) \\ v_{y,\mathrm{cmd}}^B(t) \\ v_{\theta,\mathrm{cmd}}^B(t) \\ \\ v_{x,ee,\mathrm{cmd}}^B(t) \\ v_{y,ee,\mathrm{cmd}}^B(t) \\ v_{z,ee,\mathrm{cmd}}^B(t) \end{pmatrix}$$

Matrix A:

$$A = \begin{pmatrix}
1 & 0 & 0 & \frac{\Delta t}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & \frac{\Delta t}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & \frac{\Delta t}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{\Delta t}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{\Delta t}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{\Delta t}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Matrix B:

$$B = \begin{pmatrix} \frac{\Delta t}{2}\cos(\theta^W(t)) & -\frac{\Delta t}{2}\sin(\theta^W(t)) & 0 & 0 & 0 & 0 \\ \frac{\Delta t}{2}\sin(\theta^W(t)) & \frac{\Delta t}{2}\cos(\theta^W(t)) & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\Delta t}{2} & 0 & 0 & 0 \\ \cos(\theta^W(t)) & -\sin(\theta^W(t)) & 0 & 0 & 0 & 0 \\ \sin(\theta^W(t)) & \cos(\theta^W(t)) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\Delta t}{2}\cos(\theta^W(t)) & -\frac{\Delta t}{2}\sin(\theta^W(t)) & 0 \\ 0 & 0 & 0 & \frac{\Delta t}{2}\sin(\theta^W(t)) & \frac{\Delta t}{2}\cos(\theta^W(t)) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\Delta t}{2} \\ 0 & 0 & 0 & \cos(\theta^W(t)) & -\sin(\theta^W(t)) & 0 \\ 0 & 0 & 0 & \sin(\theta^W(t)) & \cos(\theta^W(t)) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Detailed Equations:

Base Position Updates:

$$x^W(t+1) = x^W(t) + \frac{v_x^W(t) + v_{x,\text{cmd}}^B(t)\cos(\theta^W(t)) - v_{y,\text{cmd}}^B(t)\sin(\theta^W(t))}{2}\Delta t$$

$$y^W(t+1) = y^W(t) + \frac{v_y^W(t) + v_{x,\text{cmd}}^B(t)\sin(\theta^W(t)) + v_{y,\text{cmd}}^B(t)\cos(\theta^W(t))}{2}\Delta t$$

Base Orientation Update:

$$\theta^W(t+1) = \theta^W(t) + \left(\frac{v_\theta^W(t) + v_{\theta,\text{cmd}}^B(t)}{2}\right)\Delta t$$

Base Velocity Updates:

$$v_x^W(t+1) = v_{x,\text{cmd}}^B(t)\cos(\theta^W(t)) - v_{y,\text{cmd}}^B(t)\sin(\theta^W(t))$$

$$v_y^W(t+1) = v_{x,\text{cmd}}^B(t)\sin(\theta^W(t)) + v_{y,\text{cmd}}^B(t)\cos(\theta^W(t))$$

$$v_\theta^W(t+1) = v_{\theta,\text{cmd}}^B(t)$$

End Effector Position Updates:

$$x_{ee}^W(t+1) = x_{ee}^W(t) + \left(\frac{v_{x,ee}^W(t) + v_{x,ee,\text{cmd}}^B(t)\cos(\theta^W(t)) - v_{y,ee,\text{cmd}}^B(t)\sin(\theta^W(t))}{2}\right)\Delta t$$

$$y_{ee}^W(t+1) = y_{ee}^W(t) + \left(\frac{v_{y,ee}^W(t) + v_{x,ee,\text{cmd}}^B(t)\sin(\theta^W(t)) + v_{y,ee,\text{cmd}}^B(t)\cos(\theta^W(t))}{2}\right)\Delta t$$

$$z_{ee}^W(t+1) = z_{ee}^W(t) + \left(\frac{v_{z,ee}^W(t) + v_{z,ee,\text{cmd}}^B(t)}{2}\right)\Delta t$$

End Effector Velocity Updates:

$$v_{x,ee}^W(t+1) = v_{x,ee,\text{cmd}}^B(t)\cos(\theta^W(t)) - v_{y,ee,\text{cmd}}^B(t)\sin(\theta^W(t))$$

$$v_{y,ee}^W(t+1) = v_{x,ee,\text{cmd}}^B(t)\sin(\theta^W(t)) + v_{y,ee,\text{cmd}}^B(t)\cos(\theta^W(t))$$

$$v_{z,ee}^W(t+1) = v_{z,ee,\text{cmd}}^B(t)$$