Parental loss from drugs and firearms

Methodological points

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2023-06-08

Contents

Matrix Kinship model

Kin: parents

- Cause-specific (drugs, firearms, other) mortality (Caswell, Margolis, and Verdery 2023)
  - Track parents' deaths by age and cause
- Two-sex: male fertility = shifted female fertility (Schoumaker 2019)
- Time-variant rates (2000-2020)

Model run independently on each ethnic group

Notation

- $\boldsymbol{F_t}$ is the $(\omega \cdot \omega)$ fertility matrix in year t. The first row contains the fertility rates at the different ages of reproduction.
- $\tilde{\boldsymbol{U_t}}$ is a $((2\omega + 2\omega\alpha) \cdot (2\omega + 2\omega\alpha))$ block-matrix containing the survival probabilities (and cause-specific probabilities of dying), in year t.
- $\boldsymbol{n_t}$ is the vector of the population counts by age in year t.
- $\boldsymbol{\pi_t}$ is the $(2\omega \cdot 1)$ vector of age distribution of parents of offspring in year t.

- ω is the highest age + 1, here 86 yo.

Visualization convention

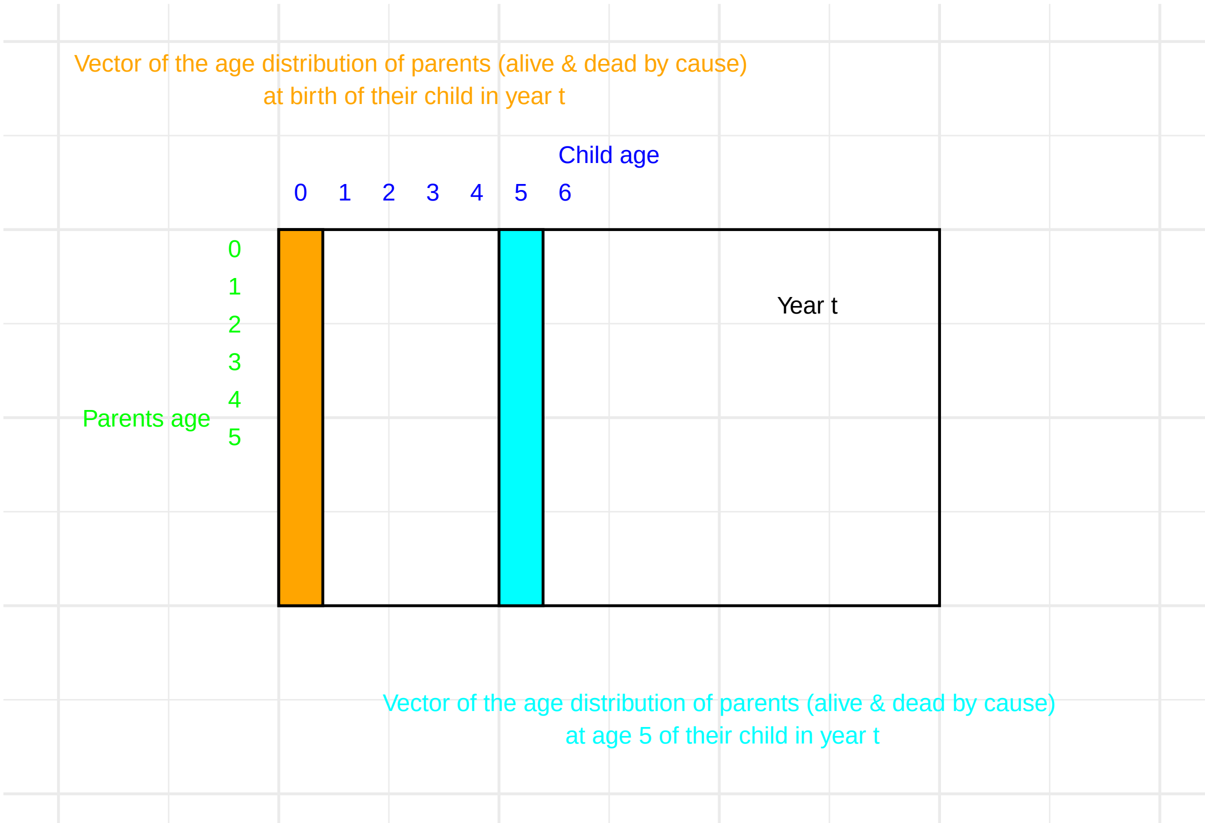

Figure 1: 3D arrays to visualize the model

The model assumes that we randomly select a child in the population, hence this child could be any age in year t. This explains the different vectors of age distribution of parents over the ages of their child in year t. An additional assumption is that the two parents are alive at the time of birth of their child.

The aim of the model is to project the vector of age distribution of parents over time and age of their child.

Starting the dynamic

Suppose our analysis focuses on children aged <5 yo (<18 yo in reality but it simplifies the visualization).

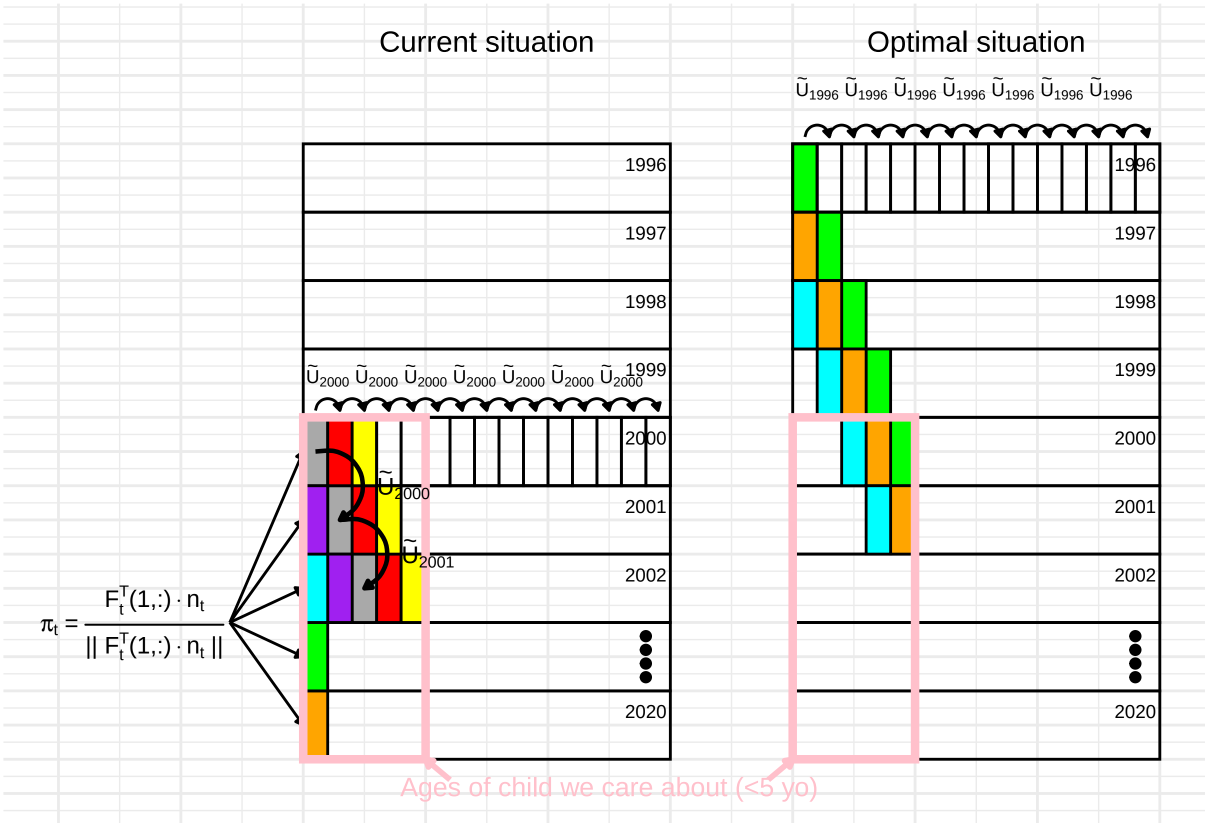

Figure 2: Parents population projection

The issue with the current case (Figure 2) is that in the year 2000, the age distribution of parents for a child aged <5 yo might not correctly represent the reality. This comes from assuming a stable population in that year to start the projection (no data before 2000). This means that in order to obtain the vector of the age distribution of parents at age x of their child in the year 2000, we assume

$$\boldsymbol{n_{x,t=2000}} = \boldsymbol{\tilde{U}}^{x}_{t=2000} \cdot \boldsymbol{\pi_{t=2000}}$$

where $\boldsymbol{\tilde{U}_{t=2000}}$ is raised to the power equal to the age of the child, and $\boldsymbol{\pi_t}$ is the age distribution of parents of offspring (see formula in Figure 2).

The optimal situation would be to start the dynamics in the year 1996 (see Figure 2) if we were to focus on children aged <5 yo (but in the year 1983 in reality as we focus on children aged <18 yo). This requires additional data that we currently don't have:

- Female fertility rates by age, and ethnic group for the years 1983-1999
  - Found for Black, White, and Hispanic and require to perform some temporal intrapolation (CDC: brth1980_2000.xlsx)
- Population counts by age, sex, and ethnic group for the years 1983-1999
  - Only found for White and require to perform some temporal intrapolation (CDC: pop1980_2016.xlsx)

- All-causes deaths (to compute mortality rates) by age, sex and ethnic group for the years 1983-1999. This could be relaxed by obtaining estimated life expectancy at birth and converting it into survival probabilities using the Coale-Demeny Model West Life Tables (Verdery and Margolis 2017)
  - Before the year 2000, I will simplify the model to only look at parents surviving, not accounting for parents' death. This nevertheless allows to obtain more realistic age distributions of parents of children aged <18 yo in the year 2000.

Number of children losing a parent

Following a similar method as in Alburez-Gutierrez, Kolk, and Zagheni (2021), we want to compute the number of children aged 1 yo, <5 yo, and <18 yo losing a parent (CLP_x) over the years and ethnic groups. This requires additional data:

- Found real number of births 2000-2015 for Black & White non Hispanic and Hispanic (CDC: t5_birth_2015_1989.pdf)
- Found real number of births 2016-2020 for all ethnic groups (CDC: t1_birth_2021_2016.pdf)

The computation is visualized for children aged <5 yo belonging to the cohort born in the year 2000 (Figure 3). The calculation follows the steps below,

1. Get the number of births (B) in year 2000
2. Compute the fraction of cohort surviving (l_x) from period mortality rates (blue squares in Figure 3), both sexes combined (a randomly selected child could be any sex)
3. Get the number of parental deaths by cause at age x of a child ($d_D(x)$) from the matrix kinship model
4. Compute $CLP_x = \sum_{x=0}^{5}(l_x * B) * d_D(x)$

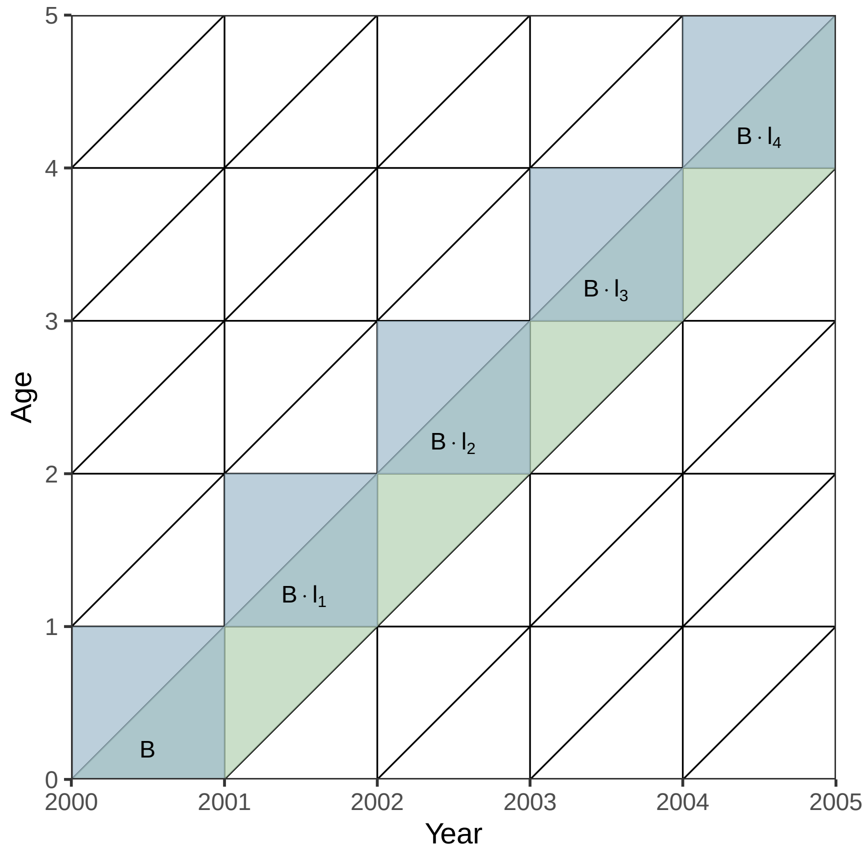

Figure 3: Cohort rates approximated by period rates

Uncertainty in estimates

We can simulate an i-th series of death counts for every combination of sex, ethnic group, year, and cause of death (other, drugs, firearms) at all ages using the Chiang method (Chiang 1984) as follows:

$$_nD_{x,i} \sim Binomial(\frac{_nD_{x,i}}{_nq_x}, {_nq_x})$$

We repeat this procedure N_{sim} times for each combination and obtain N_{sim} associated survival probabilities and cause-specific probabilities of dying that we can use as inputs in the matrix kinship model (use the Poisson distribution to do the same for the fertility rates?)

This is similar to the UN WPP projectionss using Bayesian prosterior draws as inputs for the cohort-component model.

Note that this allows to account for the stochasticity stemming from the mortality rates but not from projecting a small population of kin (parents).

Kitagawa-Keyfitz decomposition

Contribution of m_x, F_x, and the population structure in the difference in outputs between Black and White (Kitagawa 1955; Caswell 2010).

Male fertility

Focusing on parents means that fertility enters the model only through the age distribution of parents at the birth of their child (π_t).

Currently, the kin dynamics consists of a full two-sex model ($\boldsymbol{F_t^f}$ and $\boldsymbol{F_t^m}$). Male fertility rates = shifted female fertility rates. The shift is equal to the difference in the mean age at childbearing between the two sexes. The shifts are modeled with a linear model over the years (data from the UN Demographic Yearbook).

- Sensitivity analysis: use female fertility rates to compute both female and male age distribution at the birth of their child.
- A call with Bruno Schoumaker is possible to substantially refine the computation of male fertility rates (using IPUMS data).

References

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