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Digital Image Processing
12

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12 Laboratory, Homography

Learning goals

- You can apply the notion of geometric image transformation.
- You can identify the parameters of a homography.
- You can apply a homography to undistort the image of a plane object that has been captured from any perspective.

Important! To represent image coordinates by a tuple (a, b), different image processing tools use different tuple order. When reading out image point coordinates manually in this laboratory, be aware which order your current tool applies.

Exercise 1. Undistortion by Homographic Transformation

Assume that a picture is taken from a plane object and the camera's optical axis is not perpendicular to the object plane. In such a situation the image of the plane object will show a perspective distortion which is described by a homography. See the image `chessboard_perspective.jpg`. According to a homographic mapping, the coordinates $\boldsymbol{b}^d = (x^d, y^d)$ within the distorted image are related to the coordinates $\boldsymbol{b}^u = (x^u, y^u)$ within the undistorted image by

$$\underbrace{\begin{bmatrix} x^d\, z \\ y^d\, z \\ z \end{bmatrix}}_{\check{\boldsymbol{b}}_i^d} = \underbrace{\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}}_{\boldsymbol{H}_u^d} \cdot \underbrace{\begin{bmatrix} x^u \\ y^u \\ 1 \end{bmatrix}}_{\check{\boldsymbol{b}}_i^u} \tag{1}$$

For a specific point correspondence i this transform can be written in the form

$$x^d \cdot (gx^u + hy^u + 1) = ax^u + by^u + c \tag{2}$$

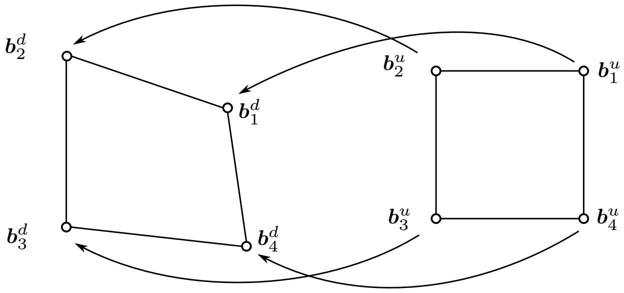
$$y^d \cdot (gx^u + hy^u + 1) = dx^u + ey^u + f. \tag{3}$$

The eight unknowns a to h can be abtained by applying equations (2) and (3) separately on four pairs of image points i.e. for $\boldsymbol{b}_i^d = (x_i^d, y_i^d)$ and $\boldsymbol{b}_i^u = (x_i^u, y_i^u)$ with $i = 1, 2 \ldots 4$. These resulting eight equations can be written as a linear system of equations. The unknown is a vector $\boldsymbol{\theta}$ whose components are the desired parameters a to h.

a) Determine the point coordinates of four points $\boldsymbol{b}_i^d$ by reading them from the perspectively distorted image. Then define image coordinates $\boldsymbol{b}_i^u$ for the four points in the undistorted image.

b) Determine the eight parameters a to h. In Python you can solve a linear system of quations by calling `x = numpy.linalg.solve(Matrix, y)`.

c) Now, the image can be undistorted: For each image coordinate $\boldsymbol{b}^u$ in the undistorted image, the corresonding point $\boldsymbol{b}^d$ can be computed. With this the color of the new image point $\boldsymbol{b}^u$ can just be read out form the distorted image. However, as $\boldsymbol{b}^d$ will not have integer components in general, the color must be found by, e.g., taking the color of the pixel in the nearest neighborhood of $\boldsymbol{b}^d$. A more advanced method would be a *linear* or *cubic* interpolation.



OpenCV offers the following functions to estimate and perform perspective transformations:

- `findHomography()` … estimates the homography for more than 3 point correspondences and even can handle outliers (RANSAC)
- `warpPerspective()` … computes the geometrically transformed image according to a homography