

**KU Leuven**  
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**THE EFFECT OF MISSING DATA ON THE ESTIMATION  
BIAS, VARIANCE, AND STATISTICAL POWER IN  
MULTILEVEL AUTOREGRESSIVE(1) MODELS**

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degree of Master of Science in  
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# Contents

<b>Introduction</b>	<b>2</b>
Emotional inertia and psychological well-being . . . . .	2
Problems with the estimation of inertia . . . . .	3
Evidence from simulation studies . . . . .	3
Compliance and missing data in ESM studies . . . . .	5
<b>Methods</b>	<b>6</b>
Multilevel AR(1) model . . . . .	6
Assumptions of the MLAR(1) model . . . . .	6
Estimation procedures for the MLAR(1) model . . . . .	7
Multilevel AR(1) model and statistical power . . . . .	7
Simulation study . . . . .	7
Simulation procedure . . . . .	7
Reproducibility and code/data availability . . . . .	11
<b>Results</b>	<b>12</b>
Simulation A . . . . .	12
Outcome: Estimation bias (MSE) . . . . .	12
Outcome: Standard error . . . . .	13
Outcome: Statistical power . . . . .	16
Simulation B . . . . .	18
<b>Discussion</b>	<b>19</b>
Limitations . . . . .	20
<b>References</b>	<b>21</b>

# Introduction

IDK INTRO SENTENCE. In the recent years, the focus in diverse subfields of psychology has been shifting towards complexity, dynamics and within-person perspective in psychology (Hamaker, 2012). Among other things, this shift has been facilitated by the growing availability of smartphones and wearables, which allow researchers to use the Experience Sampling Method (ESM) to collect intensive longitudinal data with a high level of ecological validity (Myin-Germeys et al., 2018). Intensive longitudinal data consist of repeated (self-report) measurements nested within individual participants (Larson & Csikszentmihalyi, 2014). The use of intensive longitudinal data considerably broadens the extent of research questions psychological researchers can investigate and statistical analyses they can conduct. Importantly, the multilevel structure of intensive longitudinal data allows researchers to investigate both within-person dynamic processes, and the individual differences therein between persons (CITE). One of the new research avenues that emerged with the growing popularity of intensive longitudinal data is the study of affect dynamics (for an overview, see Houben et al. (2015)). The dynamic measures make use of the structure of intensive longitudinal data and take the fluctuating nature of affect/emotions into account.

Apart from dynamic measures such as emotional variance or instability, inertia (operationalised as autoregression in psychological studies using ESM) emerged as a dynamic measure that can contribute to the knowledge about emotional regulation (Kuppens & Verduyn, 2017). The autoregressive (AR) parameter quantifies the degree to which the value of a process at time  $t$  is influenced by the lagged variable (i. e., the process value at a previous timepoint; usually,  $t-1$ ). In other words, the AR parameter allows us to approximate to what degree the intensity of an emotion/affect is carried onto the intensity of the emotional experience at the subsequent timepoint (for the same emotion), and to what degree the process is influenced by other factors. The part of the process value that is not explained by the lagged variable is considered to be caused by *innovation*: the ensemble of (both within- and between-person factors) that have influenced the process at a given timepoint (Ariens et al., 2020). Inertia is usually modeled using a multilevel autoregressive (MLAR) model (Koval et al., 2021). Usually, the model has two levels: the within-person Level 1, in which the intra-person autoregressive process is modeled, and the between-person Level 2, in which the individual differences in the autoregressive parameter and intercept are modelled. A more detailed description of the model is described in the Methods section.

## Emotional inertia and psychological well-being

Although some degree of inertia is to be expected in human emotional experiences, a high level of emotional inertia (i. e., a high temporal persistence of emotional states) has been linked to psychological maladjustment (Kuppens et al., 2010). A negative emotion process with a high inertia can get caught in a self-reinforced feedback loop (also called *critical slowing down*, Leemput 2014). This causes the process to be partially resistant to both external influences and inner processes, including emotional regulation (Koval et al., 2015).

The evidence about the association between emotional inertia and the well-being/psychopathology spectrum has grown steadily over the last two decades. A recent meta-analysis indicated an association between emotional inertia (of both positive and negative emotions) and psychological well-being/psychopathology (Houben et al., 2015). More specifically, higher emotional inertia has been linked to lower well-being and higher occurrence of depressive symptoms (Brose et al., 2015), bipolar disorder (Mneimne et al., 2018), and lower response of depression and anxiety symptoms to cognitive-behavioral therapy (Bosley et al., 2019).

However, a recent study demonstrated that when the affect intensity (i.e., the mean of the affective process) is taken into account in the regression model, the dynamic measures such as inertia, instability and variability no longer meaningfully contribute to the prediction of well-being and psychopathology (Dejonckheere, 2019). Similarly, recent empirical studies did not show a significant association between inertia and depression/anxiety symptoms (Bosley et al., 2019) and borderline personality disorder (Houben & Kuppens, 2020). One reason for this is the statistical overlap between the emotion intensity (i. e., the process mean), inertia, and the other dynamic measures (Bos et al., 2019; Koval et al., 2013).

### **Problems with the estimation of inertia**

In addition to the statistical overlap between dynamic measures, another reason for the limited contribution of inertia to the prediction of psychopathology/well-being is the fact that the current ways of estimating emotional inertia from ESM data do not provide sufficient precision in the estimation of inertia. Wenzel & Brose (2022) show that the neglect of estimating reliability of inertia leads to an attenuation of its relation to depressive symptoms. When reliability is taken into account via multilevel dynamic structural equation modeling and inertia is modeled as latent, the inertia of negative affect contributes to the prediction of depressive symptoms even when the emotion mean is included in the model (Wenzel & Brose, 2022). Furthermore, measurement decisions (such as the choice of items comprising the positive/negative affect composite measures), which are often made ad hoc in ESM research (Cloos et al., 2022), can lead to very different estimates of inertia.

More generally, low estimation precision will lead to the attenuation of the meaningful associations between inertia and other psychological constructs. Several simulation studies investigated the effect of varying parameters (such as number of participants and number of observations per participant) and assumption violations on measures related to the estimation precision of the MLAR(1) model. In the following sub-chapter, the evidence from simulation studies will be summarised.

### **Evidence from simulation studies**

Jongerling et al. (2015) investigated the effect of modeling innovation variance as fixed (identical for all participants) instead of random. They found that modeling innovation as fixed when it differs across participants leads to a considerable bias in the estimation of the fixed AR effect. There is an upward bias

(overestimation) present when the correlation between the individual AR effects and individual innovation variances is positive, and vice versa. Additionally, Jongerling et al. point out that using the person-means to center the lagged predictor variable leads to a downward bias in the estimation of the fixed AR effect. The effect of person-mean centering the predictor on the estimation performance of the MLAR model was further studied by Hamaker & Grasman (Hamaker & Grasman, 2015). Their simulation study confirmed that person-mean centering leads to an underestimation of the fixed autoregressive effect, especially when the number of timepoints per participant ( $T.obs$ ) is low. Still, they recommend using person-mean centering when one is interested in the effect of a between-person predictor on inertia.

In their simulation study comparing the maximum likelihood and Bayesian approaches to estimating the MLAR model, Krone et al. (2016) show that the two estimation procedures have a very similar performance. Furthermore, a higher  $T.obs$  leads to more precise estimates, while the effect of  $N$  on the estimation performance is small. They also show that a higher variance of the random AR effects lead to a lesser estimation precision, and that the estimation bias gets smaller when the real fixed AR effect increases. Liu (2017) assessed how violating the normality of the random AR effect distribution influences the estimation performance of the MLAR model. The different distributions of the random AR effects were found to only have a small effect on the estimation performance.

TODO: Gistelinck.

While the simulation studies mentioned above provide an extensive body of evidence about the statistical properties of the MLAR model under different conditions, several questions remain unanswered. One of them is the effect of missing observations on estimation performance. The presence of missing values in intensive longitudinal dataset decreases the number of observations per participant (or, more specifically, the number of observation-pairs that can be used for the estimation of the model). As such, it can be expected that lower compliance (i. e., lower proportion of ESM beeps that the given participant answered) will worsen the estimation bias. Additionally, different patterns of missingness might have different consequences on the estimation performance. Ji et al. (2018) show that the presence of data missing completely at random (MCAR), missing at random (MAR) and not missing at random (MNAR) leads to a considerable bias in point estimated of cross-lagged and autoregressive parameters in vector autoregressive models when list-wise deletion is used. However, no similar evidence is available about the MLAR(1) model.

As such, the goal of the present thesis is to investigate whether compliance (or the proportion of missing data) and the different patterns of missingness have an effect on estimation bias, variability, and statistical power of the multilevel AR(1) model. Additionally, the number of participants ( $N$ ), number of timepoints per participant ( $T.obs$ ), the simulated fixed AR effect ( $\beta_{10}$ ) and the variance of the random slopes/AR effects ( $\sigma_{\nu 1}$ ) will be manipulated. More details about the simulation study are provided in the Methods section. In the remainder of the Introduction, I will present the evidence about compliance in ESM studies.

## Compliance and missing data in ESM studies

Due to the nature of intensive longitudinal data collection in psychology, the presence of missing data in the datasets is more of a rule than an exception. In their meta-analysis of 477 published ESM studies (with total  $N$  of 677,536), Wrzus and Neubauer (2022) found the average compliance to be 79.19% ( $SD = 13.64\%$ ). Importantly, both within-person, momentary factors (e. g., a participant might be less likely to answer an ESM beep during social situations, compared to when they are alone) and between-person factors could influence compliance.

On the study design level, compliance is usually higher in studies that provide financial incentives to participants Wrzus & Neubauer (2022), while increasing questionnaire length is associated with lower compliance (Eisele et al., 2020). On the other hand, sampling frequency (i. e., the number of ‘beeps’ per day) was not found to be associated with compliance in both an experimental study (Eisele et al., 2020) and a large-scale meta-analysis (Wrzus & Neubauer, 2022).

The evidence about participant characteristics associated with compliance is more ambiguous. While some studies found men to have lower compliance than women Vachon et al. (2019), the meta-analysis by Wrzus & Neubauer (2022) found the evidence for gender differences in compliance to be limited. Psychotic disorders appear to be associated with lower compliance (Vachon et al., 2019), and participants with depression tend to have higher compliance (Rintala et al., 2019). Additionally, Rintala et al. (2019) report that participants with higher overall positive affect and lower overall negative affect tend to have higher compliance. Furthermore, higher compliance was found to be associated with lower overall activity stress, lower event unpleasantness, feeling less physically tired/unwell, and using less alcohol (Rintala et al., 2020). The evidence regarding the momentary factors of compliance is similarly limited. While Rintala (2019) found some momentary measures (higher positive affect, feeling less disturbed by the ESM beep, and using tobacco) to be associated with higher compliance they conclude that the majority of the momentary factors are not correlated with compliance. Even though the evidence is limited, we can expect both within- and between-person differences in compliance to arise from a complex interaction of study design, sample characteristics and momentary.

To summarise the introductory part: I have introduced the application of the multilevel autoregressive model to operationalise inertia in ESM research, summarised the simulation studies about the statistical properties of the multilevel autoregressive model, and introduced the evidence about compliance in ESM research. In the rest of the thesis, I will describe the design, results and implications of two Monte Carlo simulation studies conducted to address the knowledge gap about the effect of missing data on the estimation performance of the multilevel autoregressive model.

# Methods

## Multilevel AR(1) model

In this subchapter, I will describe the mathematical basis, assumptions and estimation procedures of the first-order multilevel autoregressive (MLAR(1)) model with random intercepts and random autoregressive effects, which is the focus of the simulation part of the thesis. While the notation for the model varies across different papers, the notation used by Lafit et al. (2020) will be adhered to throughout the thesis.

The MLAR model consists of two levels: the within-person Level 1 and the between-person Level 2. At level 1, described by Equation (1) (Lafit et al., 2020), each participant's first-order autoregressive process is modelled: The person-specific autoregressive parameter (inertia)  $\gamma_{1i}$  quantifies to what degree the process value  $esm_{it}$  of participant  $i$  at time  $t$  depends on the lagged process value  $esm_{i,t-1}$ . The person-specific intercept  $\gamma_{0i}$  represents the expected process value  $esm_{it}$  when the lagged variable  $esm_{i,t-1}$  equals 0 (Jongerling et al., 2015). Following the recommendations by (Hamaker & Grasman, 2015), the predictor (lagged variable) was person-mean centered. The innovation  $\epsilon_{it}$  (i.e., residuals, the part of the variance that is not explained by the lagged variable  $esm_{i,t-1}$ ) is assumed to be independent and coming from a normal distribution with mean of 0 and variance  $\sigma_e^2$  (Lafit et al., 2020). The model used in the present thesis assumes that the innovation variance is identical for all participants.

$$esm_{it} = \gamma_{0i} + \gamma_{1i} * esm_{i,t-1} + \epsilon_{it} \quad (1)$$

In the multilevel AR(1) model, the person-specific autoregressive effects  $\gamma_{1i}$  and the person-specific intercepts  $\gamma_{0i}$  are allowed to vary between participants. The Level 2 of the MLAR(1) model describes this between-person variability. The Level 2 is defined in Equation (2)). Each person-specific autoregressive effect  $\gamma_{1i}$  is a sum of a fixed effect  $\beta_{10}$  and a person-specific random effect  $\nu_{1i}$ . The random effects  $\nu_{1i}$  themselves come from a normal distribution with mean of 0 and variance  $\sigma_{\nu_1}^2$  (Lafit et al., 2020). The same holds for the person-specific intercepts  $\gamma_{0i}$ : they are a sum of a fixed effect  $\beta_{00}$  and a random effect  $\nu_{0i}$  that comes from  $N(0, \sigma_{\nu_0}^2)$ .

$$\begin{aligned} \gamma_{0i} &= \beta_{00} + \nu_{0i} \\ \gamma_{1i} &= \beta_{10} + \nu_{1i} \end{aligned} \quad (2)$$

## Assumptions of the MLAR(1) model

In this part, the assumptions of the MLAR(1) model and the way they were taken into account in the present simulation study will be explained.

**Stationarity.** The MLAR(1) model is used to model stable processes in which no temporal trends are present. As such, it assumes weak stationarity: the (person-specific) process mean, innovation variance,

and autoregressive parameter are assumed to not change through the time series (Rovine & Walls, 2006). For this reason, the person-specific autoregressive effects  $\gamma_{1i}$  are assumed to be bounded by -1 and 1, as autoregressive effects larger than 1 (or lower than -1) cause a change in the process mean (Krone et al., 2016).

**Exogeneity.** TODO (see word file)

**Equally spaced measurements.** TODO (see word file again)

## Estimation procedures for the MLAR(1) model

**Initial conditions problem.** TODO (see Word)

## Multilevel AR(1) model and statistical power

## Simulation study

The goal of the present exploratory simulation study is to assess the effects of four different patterns of missing data (data missing completely random, data missing in blocks, and two patterns of data missing dependent on process value) on estimation performance/bias, standard error and statistical power for the estimation of the fixed autoregressive effect in the MLAR(1) model.

## Simulation procedure

The study followed the general principles of a Monte Carlo simulation procedure described by Lane & Hennes (2018).

**Simulation conditions.** Two simulation studies, Simulation A and Simulation B, were carried out to investigate the research questions. In Simulation A, no random autoregressive effects were simulated and estimated (i.e., each subject’s time-series in the simulation had the same simulated autoregressive effect, and only fixed autoregressive effects were estimated). In Simulation B, random autoregressive effects were simulated and estimated (with the random effects variance set to either 0.05 or 0.1). Both random and fixed intercepts were estimated in Simulations A and B.

Simulation A followed a  $4 \times 2 \times 3 \times 4 \times 3$  factorial design (yielding 288 simulation conditions in total), and Simulation B followed a  $4 \times 2 \times 2 \times 4 \times 2 \times 2$  design (256 conditions in total). Each of the conditions was simulated in 1,000 simulation runs. As such, 544,000 datasets were generated (and the same number of models was estimated) in this simulation study. The manipulated variables (as well as the other parameters that remained fixed throughout all simulation conditions) are listed in Table 1 and 2.

**Data generation.** First, for each of the simulation conditions (i.e., combination of the parameters listed below), 1000 synthetic datasets were generated. Each dataset contained observations from  $N$  participants. A temporally dependent time-series of length  $T.obs$  was generated as nested within each simulated participant via a recursive equation. Additionally, for each time-series, a burn-in period with



Table 1: Values of the manipulated parameters used in the two simulation studies

Manipulated parameter	Simulation A	Simulation B
Missingness pattern	MCAR, block, extreme-onesided, extreme-twosided	MCAR, block, extreme-onesided, extreme-twosided
Simulated fixed AR effect	0.3, 0.5, 0.7	0.3, 0.7
Variance of random AR effects	-	0.05, 0.1
Compliance	0.4, 0.6, 0.8, 1	0.4, 0.6, 0.8, 1
Number of participants (N)	20, 50	20, 50
Timepoints per participant (T.obs)	20, 50, 100	50, 100

Table 2: Parameters used for the two simulation studies.

Simulation parameter	Simulation A	Simulation B
Fixed intercept	0	0
Variance of random intercepts	3	3
Innovation variance	3	3
Correlation between random intercepts and random slopes	0	0
Significance threshold	0.05	0.05
Simulation runs per condition	1000	1000

1,000 observation was generated. The within-person error (innovation) vector  $\epsilon_i$  was generated from a  $N(0, \sigma)$  distribution with  $\sigma$  set to 3 in all simulations. The fixed intercept  $\beta_{00}$  was set to 0 across all conditions. The random intercepts  $\nu_{0i}$  for each simulated time-series were sampled from a  $N(0, 3)$  distribution in both studies. In Simulation A, only fixed autoregressive effects  $\beta_{10}$  were simulated and manipulated, while both fixed and random autoregressive effects  $\nu_{1i}$  were included in Simulation B. For an overview of the values of all manipulated simulation parameters, please refer to Table 1.

Each time-series was then generated using Equation (1). The initial value was generated as a sum of the person-specific intercept  $\gamma_{0i}$  and the innovation  $\epsilon_{ij}$ , and the following observations were calculated by multiplying the value of the time-series at  $t-1$  by the person-specific autoregressive effect  $\gamma_{1i}$  and adding the person-specific intercept  $\gamma_{0i}$  and the innovation  $\epsilon_{ij}$ . Subsequently, after removing the burn-in datapoints, the first-order lagged version of the time-series was generated, setting the first lagged value as missing.

The non-manipulated simulation parameters ( $\beta_{00}, \sigma_{\nu 0}, \sigma, \rho_{\nu}$ ) were set following a simulation design from Hamaker & Grasman (2015).

**Introduction of missing values.** Secondly, missing data were introduced to each of the generated datasets according to the missing data pattern and compliance of the given simulation condition. Four different missingness patterns were introduced to the data: a) data missing completely at random (MCAR); b) data missing in blocks of consecutive observations; c) lowest (100%-compliance) observations set as missing, and d) highest and lowest (100%-compliance)/2 observations set as missing.

Each of these missingness patterns correspond to a hypothetical scenario in an ESM study. The MCAR pattern assumes that the participants miss responding to beeps randomly, and each beep has the same probability of being missed, regardless on any other factors (e.g., whether the previous beep was missed, or the intensity of the emotion measured by ESM). When there is a block of missing data present, all missing observations follow each other, there is a block of non-missing values and missing values. The start and the endpoint of the missing block do not depend on the intensity of the emotion. This can correspond to a situation where a participant misses a series of beeps because they are attending a social event. For patterns c) and d), the missingness is dependent on the value of the process itself. Pattern c) represent a situation in which a participant does not respond to an ESM measure of a positive mood because they are not feeling well, while pattern d) represents a situation where a participant misses an ESM beep when they either do not feel well enough, or they feel too good to respond to their phone beeping.

It can be expected that the different missingness patterns will differ in their effects on the simulation outcomes (estimation bias, standard error, power etc.). This is because with identical proportion in missing data, datasets with different missingness patterns will have different proportions of effective observation-pairs (i.e., proportion of timepoints for which both the observation at  $t$  and the observation at  $t-1$  are not missing) used to estimate the autoregressive effect. Figure 1 illustrates the four different missingness patterns on the same ESM time-series.

**Fitting a multilevel autoregressive model.** After missing values were introduced to the data, a MLAR(1) model was fitted to each of the simulated datasets using the *lme* function from the *nlme* R package (Pinheiro et al., 2022) with the value of the time-series at  $t$  as the outcome, the lagged ( $t-1$ ) value of the time-series as the predictor, and the participant number as the grouping variable. We then extracted relevant parameters from the models that converged successfully. Missing values were treated by list-wise deletion. The restricted maximum log-likelihood method was used to estimate the model.

The predictor (lagged) variable was person-mean centered. Although person-mean centering results in an underestimation of the autoregressive effect (Hamaker & Grasman, 2015), it allows for a clearer interpretation of the within-person effects in multilevel models Hamaker & Muthén (2020).

TODO: Describe person-mean centering more in detail

**Simulation outcomes.** Estimation bias (MSE), the standard error of the estimation, and the statistical power to estimate the fixed autoregressive effect  $\beta_{10}$  were the focal outcomes of the study. Additionally, we examined the effect of the manipulated variables on the proportion of models that successfully converged and the bias in the estimation of the person-mean used for centering of the predictor (lagged) variable.

Estimation bias was computed as the difference between the real (simulated) fixed autoregressive effect  $\beta_{10}$  and the estimated fixed autoregressive effect  $\hat{\beta}_{10}$  in each simulation run. As such, the dataset with estimation bias contained 1,000 rows per simulation condition.

Standard error (SE) and statistical power were calculated for each simulation condition (i.e., 1 row per

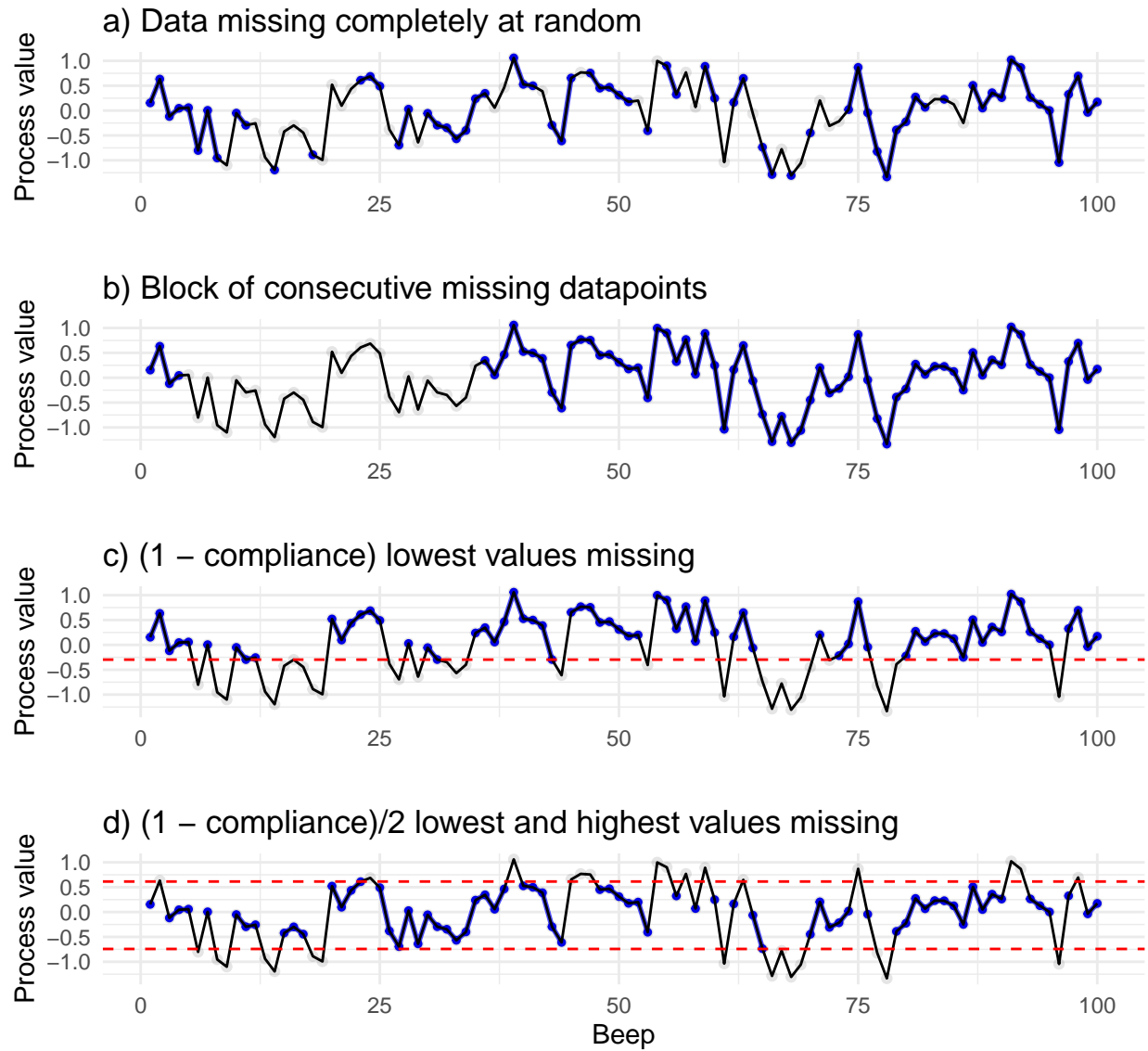


Figure 1: Illustration of the four different missingness pattern used in the simulation study. The blue dots represent observed datapoints, while the light gray dots represent the missing values.

condition). TODO: Describe how SE was calculated. Statistical power was calculated as the proportion of simulation runs (within the given simulation condition) in which the p-value for the estimated fixed autoregressive effect  $\hat{\beta}_{10}$  was below the significance threshold ( $\alpha = 0.05$ ) and the number of simulation runs that converged successfully.

The bias in the estimation of the person-mean of the time-series was computed as the average difference between the real process mean  $\mu_i$  (3) and the observed person-mean  $\hat{\mu}_i$  (computed after the missing data were introduced).

$$\mu_i = \beta_{00} + \nu_{0i}/1 - (\beta_{10} + \nu_{1i}) \quad (3)$$

**Manipulated simulation parameters.** Apart from the four different missingness patterns (described above), the following parameters were manipulated in both Simulation A and Simulation B: The number of participants of each simulated study (N), the number of ESM observations within each participant (T.obs), the compliance rate (i.e., the proportion of timepoints that are not missing for each participant), and the simulated autoregressive effect.

Furthermore, in Simulation B, the variance of the random autoregressive effects was varied. The values of the manipulated variables were set considering realistic research questions in psychological research. The values of the manipulated variables for both studies are reported in Table 1.

## Reproducibility and code/data availability

The simulations were conducted in R version 4.2.1 (R Core Team, 2021). The study was conducted with emphasis on reproducibility of the results (Pawel et al., 2022). As such, we provide all data used for the reported analyses, as well as the full reproducible R code for the simulations, (including the custom functions created for the purposes of the study), and the code used to generate the plots and result tables (available at <https://github.com/benjsimsa/AR-missing-simulations>..) The repository also includes a *sessionInfo* document that lists the versions of the packages used for the study. The present thesis was written using R Markdown (Allaire et al., 2022).

Additionally, the *renv* R package (Ushey, 2022) was used to set up a reproducible R environment and improve reproducibility by creating a project-local package library. For reproducible file referencing, the R package *here* (Müller, 2020) was used. For more information about the custom functions, simulation code, and the structure of the GitHub repository itself, please refer to the file README.md in the repository.

# Results

## Simulation A

The descriptive results for all 288 conditions included in Simulation A are reported in the appendix (TODO).

### Outcome: Estimation bias (MSE)

**ANOVA.** We used a  $4 \times 2 \times 3 \times 4 \times 3$  factorial Type I ANOVA (with estimation bias as an outcome and number of participants, number of timepoints per participant, missingness type, compliance, and the simulated fixed autoregressive effect) to assess which of the manipulated factors had a considerable influence on estimation bias. The results from every simulation run (i. e., 1,000 results per condition = 288,000 rows) were combined into a single dataset for the analysis. Given the very large sample size (which would make any difference significant) and the exploratory character of the analysis,  $p$ -values and significance thresholds were not used make inferences. Instead, we used a threshold of 0.14 for the partial  $\omega^2$ , indicating a large effect size (Field et al., 2012). This cutoff will be used for all ANOVA results throughout the results section. The partial  $\omega^2$  was chosen as the less biased alternative to partial  $\eta^2$  (Okada, 2013). The results and effect sizes are reported in Table 3.

Four main effects above the effect size threshold of 0.14 were found: the main effect of missingness type ( $\omega^2 = 0.73$ ), compliance ( $\omega_p^2 = 0.63$ ), the number of timepoints per participant ( $\omega_p^2 = 0.26$ ), and the simulated fixed slope ( $\omega_p^2 = 0.14$ ). Furthermore, the interaction between the missingness type and compliance ( $\omega_p^2 = 0.54$ ) had an effect size above the cut-off.

The main effects of missingness type and compliance are visualised in Figure 2 and Figure 3 (respectively), while the interaction between missingness type and compliance are depicted in Figure 4.

Figure 2 shows that while the underestimation of the fixed slopes is fairly low (although still considerable) when the observations are missing completely at random or in block, it becomes severe when only the most extreme values (both at one side and at both sides) are missing. SOMETHING ABOUT JANNE’S PAPER! Additionally, the underestimation of the fixed slopes becomes more severe as the compliance gets lower.

The average estimation bias when compliance is 0.8 (which is very close to the average compliance of ESM studies in psychology) is -0.13. As a consequence, many estimates of inertia in psychological research could be seriously downward biased. Furthermore, the estimates are slightly biased even when compliance is 1 (i. e., there are no missing data; average bias: -0.04). This is in line with the findings about estimation biased caused by person-mean centering in multilevel autoregressive models (Hamaker & Grasman, 2015).

Zooming in on the interaction between compliance and missingness type (Figure 4) suggests that the effect of compliance on estimation bias is dramatically more severe for the two conditions in which the most extreme values of the process were set as missing (as compared to the other two conditions, i. e., data

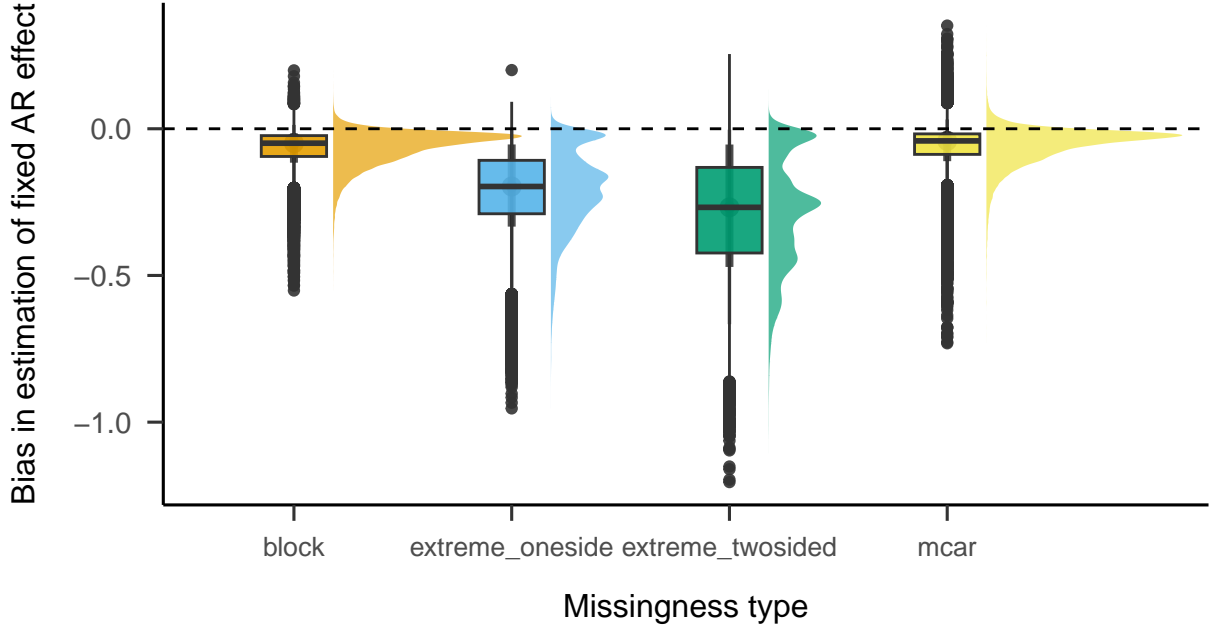


Figure 2: The effect of compliance on the bias in estimation of the fixed slopes.

MCAR and missing in blocks). In the worst-case scenario (low compliance of 0.4; the most extreme values at both sides missing), the average estimation bias was -0.48). Given that the average simulated fixed slope was 0.5, these results imply that even rather large autocorrelations can be estimated as close to 0 in studies with the combination of low compliance and a non-random missingness pattern. At the same time, the results about data MCAR and missing in blocks are encouraging. Even in a low-compliance (0.4) condition, the average estimation bias was -0.08 for the former and -0.09 for the latter.

The average estimation bias for all combinations of missingness type and compliance (averaged over the different values of the number of participants, timepoints per participant and simulated fixed slope) is reported in Table 4.

### Outcome: Standard error

**Descriptive statistics.** The average standard errors for the different combinations of number of participants, timepoints per participant and compliance are reported in Table \ref{tab:tab\_aov\_se}.

**ANOVA.** To examine the effect of the manipulated parameters on the standard error of the estimation of the fixed slopes, we combined the results for each condition (1,000 simulation runs) into a single row. As such, the dataset used for the following analyses had one row per simulation condition (288 rows in total). A  $4 \times 2 \times 3 \times 4 \times 3$  factorial Type I ANOVA was used to analyse the data. The full ANOVA results and effect sizes are reported in Table DS.

The main effects of the number of participants ( $\omega_p^2 = 0.68$ ), number of timepoints per participant ( $\omega_p^2 = 0.68$ ) and compliance ( $\omega_p^2 = 0.66$ ) crossed the cut-off for effect size.

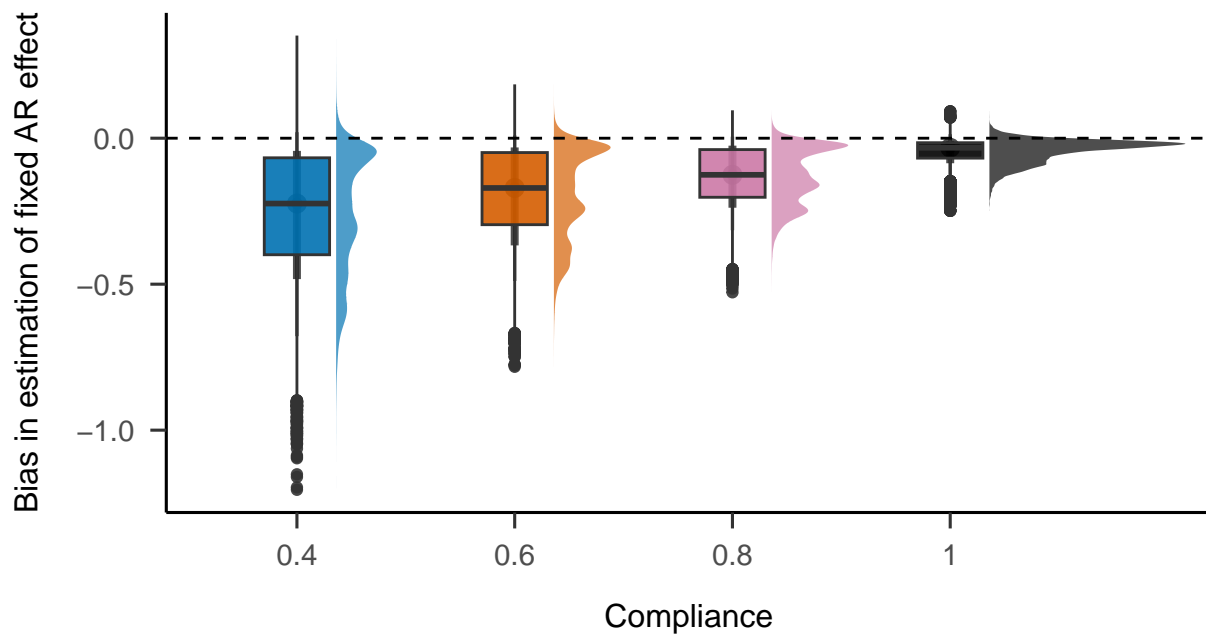


Figure 3: The effect of missingness type on the bias in estimation of the fixed slopes.

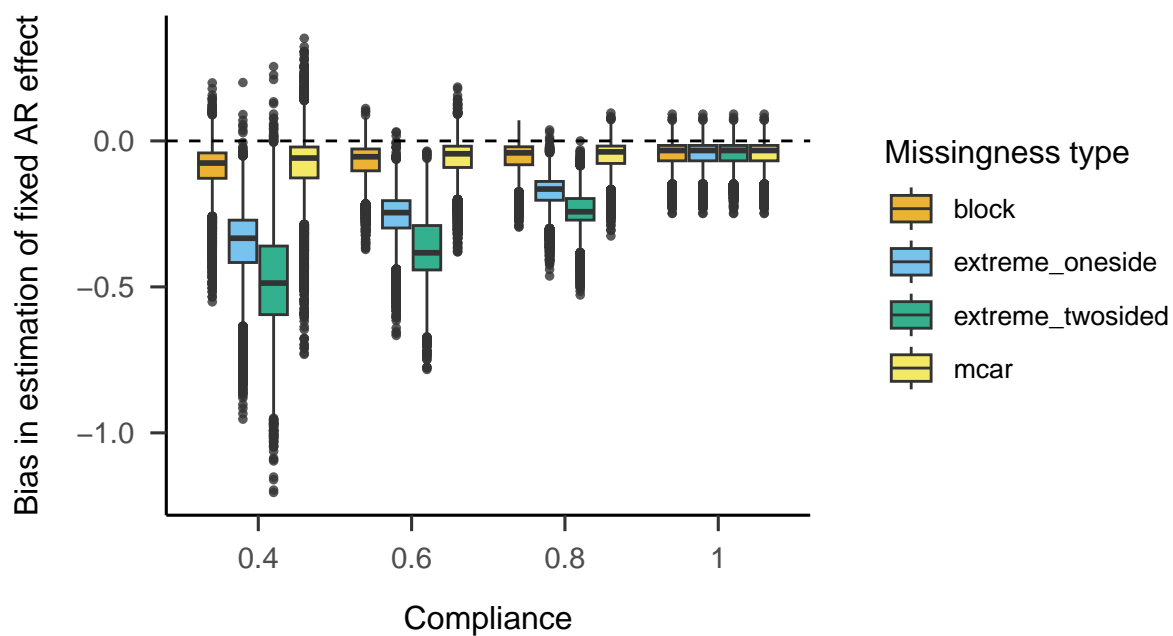


Figure 4: The effect of the interaction between missingness type and compliance on the bias in estimation of the fixed slopes.

Table 3: ANOVA results, simulation A. Outcome: Estimation bias

	Df	Sum Sq	Mean Sq	F value	p-value	Partial omega-squared
N	1	0.11	0.11	32.53	<0.001	0.00
T.obs	1	354.93	354.93	101753.95	<0.001	0.26
miss_type	3	2657.10	885.70	253921.29	<0.001	0.73
compliance	1	1706.99	1706.99	489377.69	<0.001	0.63
B1_sim	1	169.00	169.00	48449.29	<0.001	0.14
N:T.obs	1	0.02	0.02	4.40	0.0360	0.00
N:miss_type	3	0.00	0.00	0.31	0.8216	0.00
T.obs:miss_type	3	14.11	4.70	1348.32	<0.001	0.01
N:compliance	1	0.03	0.03	8.09	0.0044	0.00
T.obs:compliance	1	22.78	22.78	6529.83	<0.001	0.02
miss_type:compliance	3	1157.71	385.90	110634.77	<0.001	0.54
N:B1_sim	1	0.03	0.03	8.41	0.0037	0.00
T.obs:B1_sim	1	1.75	1.75	502.84	<0.001	0.00
miss_type:B1_sim	3	148.38	49.46	14179.70	<0.001	0.13
compliance:B1_sim	1	59.28	59.28	16994.38	<0.001	0.06
Residuals	287974	1004.48	0.00		NA	

Table 4: Simulation A. Average bias in estimation of the fixed slope for each combination of missingness type and compliance.

compliance	Missingness type			
	block	extreme_oneside	extreme_twosided	mcar
0.4	-0.09	-0.36	-0.48	-0.08
0.6	-0.07	-0.26	-0.37	-0.06
0.8	-0.05	-0.17	-0.24	-0.05
1.0	-0.04	-0.04	-0.04	-0.04

Table 5: Simulation A. Average standard error in the estimation of the fixed slope for each combination of number of participants, number of timepoints/participant, and compliance.

N	T.obs	Compliance			
		0.4	0.6	0.8	1
20	20	0.14	0.08	0.06	0.05
	50	0.07	0.05	0.04	0.03
	100	0.05	0.03	0.02	0.02
100	20	0.06	0.04	0.03	0.02
	50	0.03	0.02	0.02	0.01
	100	0.02	0.01	0.01	0.01



Table 6: ANOVA results, simulation A. Outcome: Standard error

	Df	Sum Sq	Mean Sq	F value	p-value	Partial omega-squared
N	1	0.06	0.06	625.92	<0.001	0.68
T.obs	1	0.06	0.06	621.16	<0.001	0.68
miss_type	3	0.00	0.00	14.11	<0.001	0.12
compliance	1	0.05	0.05	556.59	<0.001	0.66
B1_sim	1	0.00	0.00	21.75	<0.001	0.07
N:T.obs	1	0.01	0.01	91.92	<0.001	0.24
N:miss_type	3	0.00	0.00	2.13	0.096	0.01
T.obs:miss_type	3	0.00	0.00	1.48	0.220	0.00
N:compliance	1	0.01	0.01	82.89	<0.001	0.22
T.obs:compliance	1	0.01	0.01	114.06	<0.001	0.28
miss_type:compliance	3	0.00	0.00	13.31	<0.001	0.11
N:B1_sim	1	0.00	0.00	3.14	0.078	0.01
T.obs:B1_sim	1	0.00	0.00	1.37	0.243	0.00
miss_type:B1_sim	3	0.00	0.00	0.20	0.895	0.00
compliance:B1_sim	1	0.00	0.00	1.55	0.214	0.00
Residuals	262	0.03	0.00		NA	

Additionally, the interaction between the number of timepoints per participant and compliance ( $\omega_p^2 = 0.28$ ), number of participants and timepoints per participants ( $\omega_p^2 = 0.07$ ), and between the number of participants and compliance ( $\omega_p^2 = 0.22$ ) was found.

Figure 5 depicts the interaction between the number of timepoints per participant and compliance, while Figure 6 shows the interaction between the number of participants and compliance.

**Bias in the estimation of the fixed slope and bias in the estimation of the person-specific means.** TODO.

### Outcome: Statistical power

**Descriptive statistics.** The statistical power for each combination of the manipulated parameters is reported in Table (BIG TABLE TODO). As an illustration, the effect of compliance, missingness type, the number of participants and the number of timepoints per participant when the simulated fixed slope is 0.3 are visualised in Figure 7. Consistent with the results about estimation bias, statistical power is the lowest in the two conditions with the most extreme datapoints missing. For the conditions with data missing completely at random and data missing in consecutive blocks, power is very high even when the compliance is low for most conditions (except for the two conditions with  $T = 20$ ).

A peculiar pattern is worth pointing out in the plot: in the two conditions with  $T = 20$  and the most extreme data missing at both sides (green dashed line), the statistical power is higher when compliance is 0.4 compared to when compliance is 0.6. This counterintuitive result is likely due to the fact that the underestimation is the most severe when the most extreme values at both sides. As such, some of the estimates of the fixed slope will be negative, and their magnitude will be large enough for them to reach

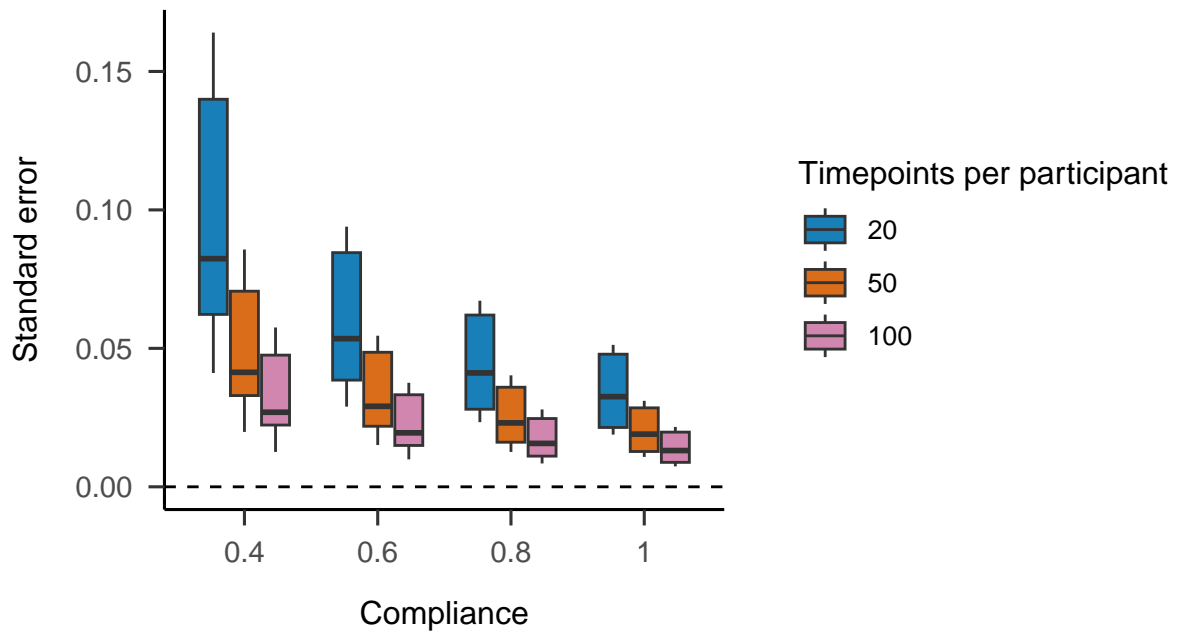


Figure 5: The effect of the interaction between number of timepoints and compliance on standard error of estimation of the fixed slopes. Simulation A.

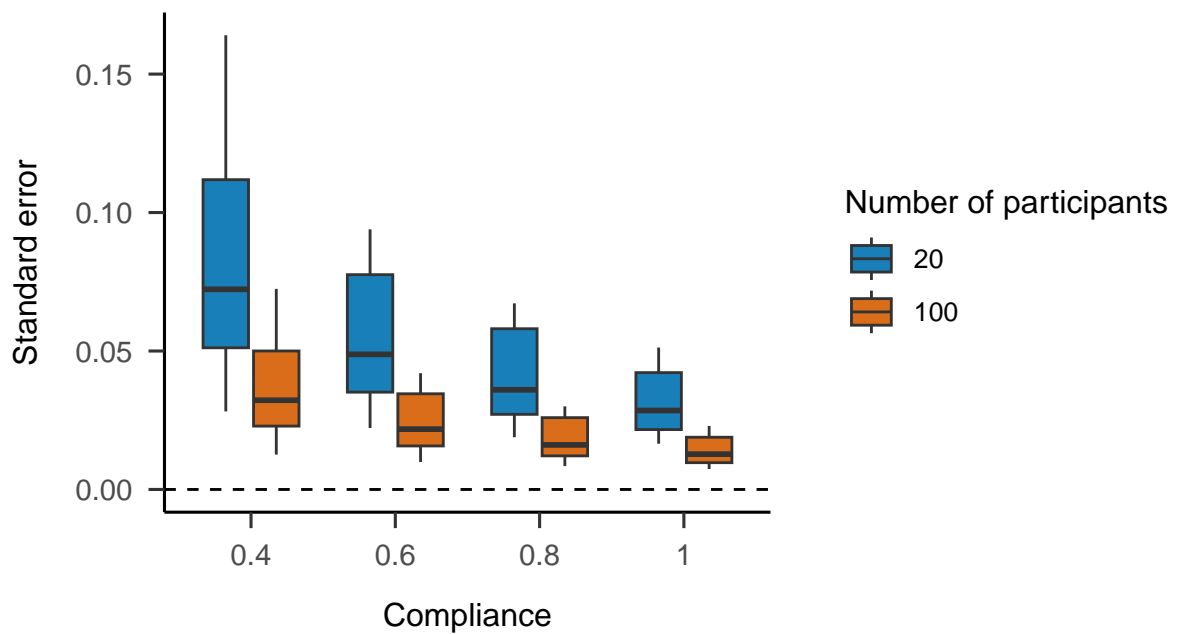


Figure 6: The effect of the interaction between number of participants and compliance on standard error of estimation of the fixed slopes. Simulation A.

statistical significance.

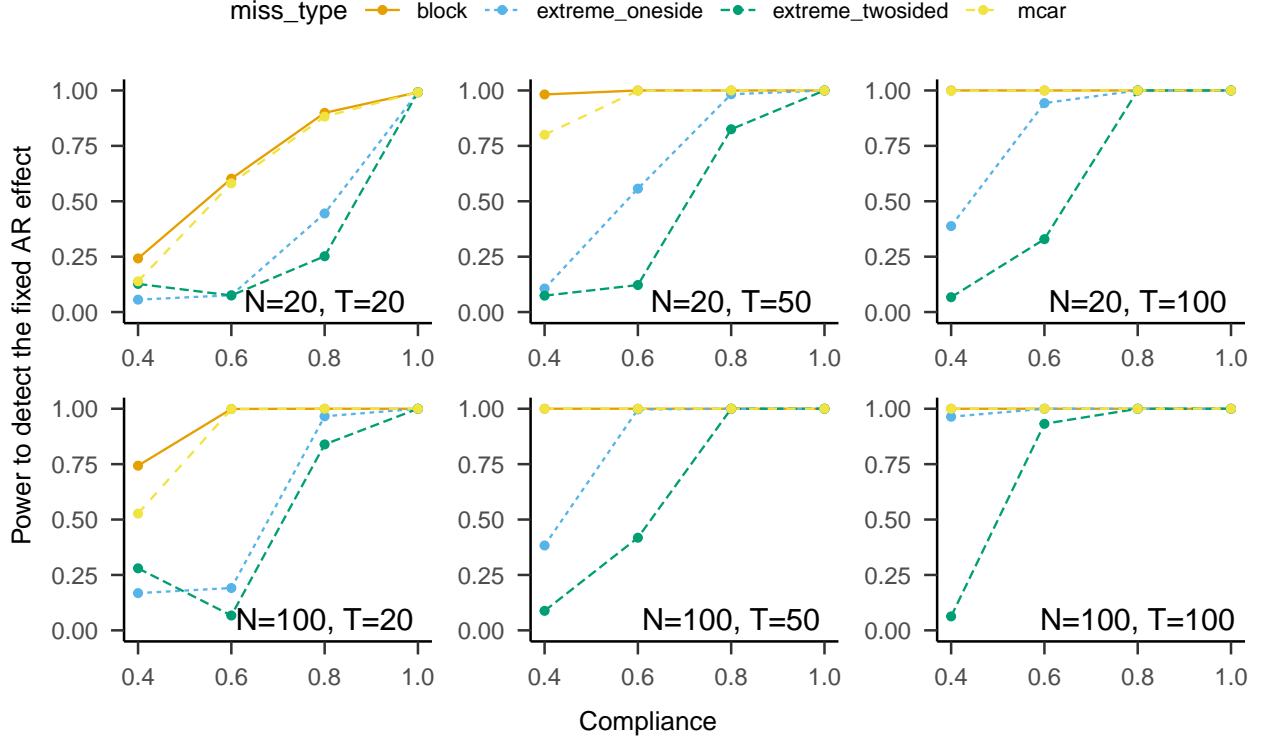


Figure 7: Simulation A. Statistical power to detect the fixed slope for all combinations of compliance, missingness type, number of participants and timepoints per participant when the simulated fixed slope is 0.3.

**ANOVA.** Aa  $4 \times 2 \times 3 \times 4 \times 3$  factorial Type I ANOVA was used to analyse the effect of the manipulated parameters (288 conditions in total) on statistical power. The results are reported in Table 7.

Four main effect above the cut-off for the effect size were found: the effect of compliance ( $\omega_p^2 = 0.43$ ), of missingness type ( $\omega_p^2 = 0.36$ ), simulated fixed slope ( $\omega_p^2 = 0.18$ ), and the effect of the number of timepoints per participant ( $\omega_p^2 = 0.17$ ).

## Simulation B

Table 7: ANOVA results, simulation A. Outcome: Power to detect the fixed slope

	Df	Sum Sq	Mean Sq	F value	p-value	Partial omega-squared
N	1	0.42	0.42	20.32	<0.001	0.06
T.obs	1	1.21	1.21	58.92	<0.001	0.17
miss_type	3	3.36	1.12	54.56	<0.001	0.36
compliance	1	4.47	4.47	217.19	<0.001	0.43
B1_sim	1	1.35	1.35	65.78	<0.001	0.18
N:T.obs	1	0.08	0.08	3.80	0.0524	0.01
N:miss_type	3	0.10	0.03	1.66	0.1753	0.01
T.obs:miss_type	3	0.34	0.11	5.49	0.0011	0.04
N:compliance	1	0.22	0.22	10.93	0.0011	0.03
T.obs:compliance	1	0.79	0.79	38.66	<0.001	0.12
miss_type:compliance	3	3.12	1.04	50.65	<0.001	0.34
N:B1_sim	1	0.09	0.09	4.25	0.0403	0.01
T.obs:B1_sim	1	0.24	0.24	11.50	<0.001	0.04
miss_type:B1_sim	3	0.44	0.15	7.19	<0.001	0.06
compliance:B1_sim	1	0.76	0.76	37.10	<0.001	0.11
Residuals	262	5.39	0.02		NA	

## Discussion

- we did this and that ....
- confirmed the evidence about high T being important (and especially so when there are weird missing data patterns)
- The effect of compliance on all 3 outcomes is quite high, and it largely depends on the missingness pattern.
- observed the opposite of Krone 2016: the estimation bias actually got higher, not smaller, as the real fixed AR effect gets larger
- while the previous research suggested that increasing T is very effective (and this is true), it is very important to keep compliance as high as possible
- consequences of misestimation: could this be one of the reasons why the inertia of NA/PA does not predict psychopathology/well-being (Dejonckheere, 2019)?
- in studies estimating inertia, there might be some individual differences in inertia estimates that are due to different missingness patterns, not due to changes in real inertia (i. e., two participants with the same real inertia but different missingness patterns could have different inertia estimates)
- also, this makes computing power a bit less straightforward (ideally, to get realistic estimates of power, one should have an idea about what the average compliance could be and what missingness patterns might be present in the data - of course, in real life, we can expect there to be a mix of missing data patterns in every dataset).

- of course, not entirely realistic - real life data will likely have a mixture of missing data patterns at both the within- and between-person level.

## Limitations

- focused on normally distributed processes. a normal dist can be assumed for positive affective processes, however, negative affective processes are usually heavily right-skewed in the general population (Haslbeck et al., 2022).
- o We assumed that the analysed time-series is measured without any error; this is not the case in real-world research Schuurman & Hamaker (2019) . Also, this unreliability can lead to further attenuation of the estimated parameters (Wenzel & Brose, 2022).
  - o The person-mean centering was carried out using observed means. However, using latent means might be more appropriate; they describe this in (Gistelinck et al., 2021).
- o Only used very specific combination of parameters, BUT I provide a framework for people to reproduce the study
- Limitation-code: Although I took steps to maximise reproducibility of the simulations (shared all code and results, made a package containing the custom functions, used packages `here()` and `renv()`, and reported the `sessionInfo` for every simulation), I also used quite a lot of packages with many dependencies, which might be detrimental to reproducibility

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