one and paired sample tests: wilcoxon signed Rank test and KS one sample goodness of fit test.

1)

Let μ is a average waiting time

$H_0: \mu = 20$ vs $H_1: \mu > 20$ minutes

To calculate the test statistic, we need to construct a following table.

$\mu_0 = 20$

x_i	$x_i - \mu_0$	$d_i = \lvert x_i - \mu_0 \rvert$	Rank
25	5	5	2.5
10	-10	10	7.5
15	-5	5	2.5
20	0	0	Ignore
17	-3	3	1
11	-9	9	6
30	+10	10	7.5
27	7	7	5
36	16	16	10
40	20	20	11
5	-15	15	9
26	6	6	4

sum of +ve rank = $(2.5 + 7.5 + 5 + 10 + 11 + 4) = 40$

Sum of -ve rank = $(7.5 + 2.5 + 1 + 6 + 9) = 26$.

$T = \min(T^+, T^-)$

$= \min(40, 26) = 26$.

$T_{11}(0.05) = 14$.

Conclusion
Hence T^- is greater than T_{tab}.
$26 > 14$. Hence Accept H_0.

2)
Here μ is average no. of seconds an incest survives (119)
$H_0: \mu_0 = 119$ vs $H_1: \mu \neq 119$.

To calculate test statistic, we need to construct following table.

x_i	$x_i - \mu_0$	$d_i = \lvert x_i - \mu_0 \rvert$	Rank
121	2	2	1.5
115	4	4	3.5
79	-40	40	11
52	-67	67	15
102	-17	17	9
106	7	7	6
81	-38	38	10
65	-54	54	13
109	-10	10	7
119	0	0	-
115	-4	4	3.5
121	2	2	1.5
103	-16	16	8
75	-44	44	12
59	-60	60	14
125	6	6	5

Here
Sum of +ve rank = 14
Sum of -ve rank = 106.

Test statistic,
$T = \min(T^-, T^+)$
$= \min(106, 14) = 14$

$n = 15$, sample size '15'

$T_{14}(0.05) = 25$.

Conclusion
Hence
T^- is less than $T_{n,\alpha/2}$
$14 < 25$
We reject H_0.

3) To test verify the claim whether 2nd keyboard is better than 1st keyboard.

Hypothesis:
$H_0: \mu = \mu_0$ vs $H_1: \mu < \mu_0$.

To calculate the test statistic, we need to construct a following table.

x_i	y_i	$x_i - y_i$	$d_i = \lvert x_i - y_i \rvert$	Rank.
40	45	-5	5	9.5
45	50	-5	5	9.5
60	55	+5	5	9.5
50	52	-2	2	2.5
55	52	3	3	5.5
60	60	0	0	-
78	55	23	23	18
80	53	27	27	19
65	70	-5	5	9.5
62	65	-3	3	5.5
70	68	2	2	2.5
71	50	21	21	17
65	58	7	7	14
45	47	-2	2	2.5
48	54	-6	6	13
42	50	-8	8	15.5
48	43	5	5	9.5
68	60	8	8	15.5
50	48	2	2	2.5
55	50	5	5	9.5

Sum of +ve rank = 122.5
Sum of -ve rank = 67.5

Test statistic

$$T = \max(T^+, T^-)$$
$$= \max(122.5, 67.5)$$
$$T = 122.5.$$

$T_{19}(0.05) = \cancel{54}\ 46.$

Conclusion

T^+ is greater than $T_{n,\alpha}$

$122.5 > 46$

Hence we Accept H_0.

4) To test the Hypothesis that drug has no effect on pulse rate

Hypothesis

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$

To calculate the test statistic, we need a construct a following table

(Before) x_i	(After) y_i	$x_i - y_i$	$d_i = \lvert x_i - y_i \rvert$	Rank.
72	74	-2	2	8
70	72	-2	2	8
68	69	-1	1	3.5
67	68	-1	1	3.5
73	72	1	1	3.5
71	72	-1	1	3.5
72	72	0	0	~~3.5~~ -
70	71	-1	1	3.5
69	67	2	2	8
60	73	-13	13	10
68	69	-1	1	3.5

Sum of +ve rank = 43.5
Sum of -ve rank = 11.5

Test Statistic :

$$T = \min(T^+, T^-)$$

$T = \min(42.5, 11.5) = 11.5$

Table value.

$T_{10}(0.05) = 8$.

conclusion:

T is greater than $T_{n,\alpha/2}$

$11.5 > 8$.

we reject H_0.

5) Dataset is claimed to have come from a $U(0,1)$. To verify that

Hypothesis:

H_0: Data comes from $U(0,1)$ or $U(0,1)$ is good fit for data

vs

H_1: Data doesn't come from $U(0,1)$.

if $X \sim U(0,1)$

$$F_0(x) = \int_0^x f(t)\,dt$$

$$= \int_0^x 1 \cdot d(t) = t \Big|_0^x = x$$

Empirical distn function is given.

$$F_n(x) = \begin{cases} 0, & \text{if } x < x_{(1)} \\ i/n, & \text{if } x_{(i)} \le x < x_{(i+1)} \text{ for } i = 1, 2, \dots, n-1 \\ 1, & \text{if } x \ge x_{(n)} \end{cases}$$

To calculate test statistic, we need to construct a following table

x_i	Ascending order $x_{(i)}$	i/n $F_n(x)$	$F_0(x) = x_{(i)}$	$\lvert F_n(x) - F_0(x) \rvert$
0.44	0.1	0.066	0.1	0.033
0.76	0.13	0.133	0.13	0.003
0.13	0.23	0.2	0.23	0.03
0.27	0.27	0.26	0.27	0.01
0.97	0.39	0.3	0.39	0.09
0.45	0.44	0.4	0.44	0.04
0.1	0.45	0.46	0.45	0.01
0.44	0.45	0.533	0.45	0.083
0.39	0.54	0.6	0.5	0.1
0.53	0.53	0.666	0.53	0.136
0.85	0.76	0.733	0.76	0.027
0.45	0.85	0.8	0.85	0.05
0.23	0.94	0.866	0.94	0.074
0.98	0.97	0.933	0.97	0.037
0.5	0.98	1	0.98	0.02

Kolmogorov - Smirnov test statistic

$$D_n = \sup \left[\lvert F_n(x) - F_0(x) \rvert \right]$$

$D_n = 0.136$.

if $D > D_{n,\alpha}$, we reject H_0

$D_{15, 0.05} = 0.388$.

Conclusion

D_n is less than $D_{n,\alpha}$.

0.136 < 0.388.

then, we conclude that we fail to reject H_0.

6) To check if random numbers are come from poisson distribution (mean = 7.6) by using kolmogorov - Smirnov test.

Hypothesis:

H_0: Random numbers follows poisson distn (7.6)

vs

H_1: Random numbers doesn't follows poisson distn with (λ = 7.6).

Calculation:

The distribution function of poisson distn is

$$F_0(x) = \sum_{z=0}^{x} \frac{e^{-\lambda} \lambda^{z}}{z!}$$

Emperical distn function is given by.

$$F_n(x) = Cu(x)/n.$$

Cu(x) → cumulative total of (x).

$$F_n(x) = \begin{cases} 0, & \text{if } z < 0 \\ 1, & \text{if } z \geq \max(x) \\ cu(x)/n, & \text{if otherwise.} \end{cases}$$

x_i	f_i	$Cu(x_i)$	$F_0(x_i)$	$F_n(x_i)$	$\lvert F_n(x_i) - F_0(x_i) \rvert$
0	0	0	0.000	0.001	0.001
1	5	5	0.001	0.004	0.003
2	14	19	0.006	0.019	0.013
3	24	43	0.013	0.055	0.042
4	57	100	0.03	0.125	0.095
5	111	211	0.063	0.231	0.168
6	197	408	0.121	0.365	0.243
7	278	686	0.204	0.510	0.306
8	378	1064	0.316	0.648	0.332
9	418	1432	0.440	0.765	0.325
10	461	1943	0.577	0.854	.776
11	433	2376	0.706	0.915	.209
12	413	2789	0.329	0.954	.125
13	358	3147	0.935	0.976	.041
14	219	3366	1.000	0.989	.011

KS test statistic is

$$D_n = \sup_{x} \lvert F_n(x) - F_0(x) \rvert = 0.332$$

Table value $D_{n,0.05} = 1.36/\sqrt{n}$

$= 1.36/\sqrt{3366} = 0.023.$

Conclusion

D_n is greater than $D_{n,\alpha}$

0.332 > 0.023.

Reject H_0.

7) To test by K.S test datas comes from normal distn at 5% significance level.

Hypothesis:

H_0: Normal distribution is good fit for data
vs
H_1: Normal distribution is not good fit for data.

To calculate test statistic, we need to construct following table.

i	x	$F_n(x) = i/n$	$z_i = \frac{x_i - 3}{1}$	$F_0(x) = p(z \le z_i)$
1	1	$1/10 = 0.1$	$1 - 3/1 = -2$	(normal table) 0.02275
2	1.9	0.2	-1.1	0.1352
3	2.1	0.3	-0.9	0.1841
4	2.7	0.4	-0.3	0.3821
5	2.8	0.5	-0.2	0.4207
6	3.2	0.6	0.2	0.5193
7	3.6	0.7	0.6	0.1258
8	3.9	0.8	0.9	0.8154
9	4.2	0.9	1.2	0.8849
10	5.1	1.0	2.1	0.9821.

Kolmogorov - Smirnov test statistic

$$D_n = \sup_x |F_n(x) - F_0(x)|$$
$$= 0.1159.$$

Table value $D_{n,\alpha} = 0.410.$

conclusion

D_n is less than $D_{n,\alpha}$

$0.1159 < 0.410$

Accept H_0 (or) Fail to reject H_0.

Normal distn is good fit for data.

8) To coin tossed 400 times in sets of four.

Hypothesis:

H_0: Binomial distribution is good fit
vs
H_1: Binomial distribution is not a good fit.

To calculate test statistic, we need to construct a following table.

x_i	f_i	xf	$f_n(x) = p(x=x)$ $4C_x (0.5)^x (0.5)^{4-x}$	$f_0(x)$.	$\lvert f_n(x) - f_0(x) \rvert$
0	23	0	0.0625	0.0625	0.0009
1	95	95	0.25	0.3125	0.0028
2	164	328	0.375	0.6875	0.0028
3	92	276	0.25	0.9375	0.0009
4	26	104	0.0625	1	0.

K.S test statistic is

$$D_n = \sup_x |F_n(x) - F_0(x)| = 0.0028.$$

Table value $D_{4,0.05} = 0.486$

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Conclusion:

D_n is less than $D_{t,\alpha}$.

$0.0028 < 0.486$.

Accept H_0.

we conclude that Binomial distribution is a good fit for data / observation / random variable.

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