PRACTICAL-03

QUENOUILLE'S AND JACKKNIFE ESTIMATION

QUESTION-01

The pdf of uniform distribution is given by

$$f(x) = \frac{1}{\theta}, \quad 0 \le x \le \theta, \quad \theta > 0$$

The jackknife estimator of θ is

$$J(\hat{\theta}) = Y_n + \frac{(n-1)}{n}(Y_n - Y_{n-1})$$

where, $Y_n = X_{(n)}$ and $Y_{n-1} = X_{(n-1)}$

Here; $Y_n = 0.92$
$Y_{n-1} = 0.91$

$$\therefore J(\hat{\theta}) = 0.92 + \frac{(7-1)}{7}(0.92 - 0.91)$$

$$= 0.9286 \;//$$

QUESTION-02:

Given; $X \sim N(\mu, \sigma^2)$

The Jackknife estimator for σ^2 starting from S_n^2 is given by

$$J(\hat{\sigma}^2) = S_{n-1}^2$$

$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

x_i : 104.1, 85.2, 94.1, 112.7 and 98.8

$$\bar{x} = 98.98$$

$$J(\hat{\sigma}^2) = \frac{(104.1 - 98.98)^2 + (85.2 - 98.98)^2 + (94.1 - 98.98)^2 + (112.7 - 98.98)^2 + (98.8 - 98.98)^2}{5-1}$$

$$= 107.047$$ //

QUESTION - 03

Given, $x \sim N(10, 4)$

Mean = 10, variance = 4

$n = 20$.

The Jackknife estimator of r is

$$J(r) = \frac{2(1+6h+60h^2)r - (1+2h+15h^2)\left(\frac{r_1+r_2}{2}\right)}{1+9h+105h^2}$$

where; $r = \frac{\bar{Y}}{\bar{X}}$, $r_1 = \frac{\bar{Y}_1}{\bar{X}_1}$, $r_2 = \frac{\bar{Y}_2}{\bar{X}_2}$

$\bar{x}_1$ = mean of first 10 terms in the data

$\bar{x}_2$ = mean of second 10 terms in the data

$\bar{y}_1$ = mean of first 10 terms in the data

$\bar{y}_2$ = mean of second 10 terms in the data

$h \rightarrow$ variance.

$\bar{x} = 78.65$

$\bar{y} = 7.35$

$\bar{x}_1 = 72.1$ $\quad \bar{y}_1 = 5.9$ $\quad h = 4.$

$\bar{x}_2 = 85.2$ $\quad \bar{y}_2 = 8.8.$

$$q = \frac{\bar{y}}{\bar{x}} = \frac{7.35}{78.65} = 0.0934 \,//$$

$$q_1 = \frac{\bar{y}_1}{\bar{x}_1} = \frac{5.9}{72.1} = 0.0818 \,//$$

$$q_2 = \frac{\bar{y}_2}{\bar{x}_2} = \frac{8.8}{85.2} = 0.1033 \,//$$

$$J(\hat{\theta}) = \frac{2(1+6\times4+60\times4^2)(0.0934)-(1\times2\times4+15\times4^2)\left(\frac{0.0818+0.1033}{2}\right)}{1+9\times4+105\times4^2}$$

$$= \frac{160.9531}{1717}$$

$$= 0.0937 \,//$$

The Quenouille's Jackknife estimator is

$$\hat{f}_Q = Nr - \frac{N-1}{N} \sum_{j=1}^{N} \hat{f}^j$$

where; $\hat{f}_j = \frac{\bar{y}^j}{\bar{x}^j}$

$$\bar{y}^j = \frac{n\bar{y} - M\bar{y}_j}{n-M} \quad , \quad \bar{x}^j = \frac{n\bar{x} - M\bar{x}_j}{n-M}$$

$N = 5$ of size $M = 4$, $n = 20$.

put $j=1$

$$\bar{y}^1 = \frac{n\bar{y} - M\bar{y}_1}{n-M}$$

$$\bar{y}_1 = \left(\frac{9+7+3+2}{4}\right) = 5.25$$

$$\bar{y}^1 = \frac{20(7.35) - 4(5.25)}{20-4}$$

$$\boxed{\bar{y}^1 = 7.875}$$

put $j=2$

$$\bar{y}^2 = \frac{n\bar{y} - M\bar{y}_2}{n-M} = \frac{20(7.35) - 4\left(\frac{3+8+9+10}{4}\right)}{20-4}$$

$$\boxed{\bar{y}^2 = 7.3125}$$

put $j=3$

$$\bar{y}^3 = \frac{n\bar{y} - M\bar{y}_3}{n-M} = \frac{20(7.35) - 4\left(\frac{6+2+16+4}{4}\right)}{20-4}$$

$$\boxed{\bar{y}^3 = 7.4375}$$

put $j=4$

$$\bar{y}^4 = \frac{n\bar{y} - M\bar{y}_3}{n-M} = \frac{20(7.35) - 4\left(\frac{5+13+4+9}{4}\right)}{20-4}$$

$$\boxed{\bar{y}^4 = 7.25}$$

put $j=5$,

$$\bar{y}^5 = \frac{n\bar{y} - M\bar{y}_j}{n-M} = \frac{20(7.35) - 4\left(\frac{12+8+9+8}{4}\right)}{20-4}$$

$$\boxed{\bar{y}^5 = 6.875}$$

$$\Rightarrow \quad \bar{x}^j = \frac{n\bar{x} - M\bar{x}_j}{n-M}$$

$$\bar{x}^1 = \frac{n\bar{x} - M\bar{x}_1}{n-M} = \frac{20(78.65) - 4\left(\frac{95+79+30+45}{4}\right)}{20-4}$$

$$\boxed{\bar{x}^1 = 82.75}$$

$$\bar{x}^2 = \frac{n\bar{x} - M\bar{x}_2}{n-M} = 20(78.65) - 4\left(\frac{28+142+125+61}{4}\right)$$

$$\boxed{\bar{x}^2 = 74.8125}$$

$$\bar{x}^3 = \frac{n\bar{x} - M\bar{x}_3}{n-M} = \frac{20(78.65) - 4\left(\frac{43+53+148+89}{4}\right)}{20-4}$$

$$\boxed{\bar{x}^3 = 77.5}$$

$$\bar{x}^4 = \frac{n\bar{x} - M\bar{x}_4}{n-M} = \frac{20(78.65) - 4\left(\frac{57+132+47+43}{4}\right)}{20-4}$$

$$\boxed{\bar{x}^4 = 80.875}$$

$$\bar{x}^5 = \frac{n\bar{x} - M\bar{x}_5}{n-M} = \frac{20(78.65) - 4\left(\frac{116+65+103+52}{4}\right)}{20-4}$$

$$\boxed{\bar{x}^5 = 77.3125}$$

$$\hat{\beta}_1 = \frac{\bar{y}^1}{\bar{x}^1} = \frac{7.875}{82.75} = 0.0952$$

$$\hat{\beta}_2 = \frac{\bar{y}^2}{\bar{x}^2} = \frac{7.3125}{74.8125} = 0.0977$$

$$\hat{\beta}_3 = \frac{\bar{y}^3}{\bar{x}^3} = \frac{7.4375}{77.5} = 0.0959$$

$$\hat{\rho}_i^4 = \frac{\bar{y}^4}{\bar{x}^4} = \frac{7.25}{80.875} = 0.0896$$

$$\hat{\rho}^5 = \frac{\bar{y}^5}{\bar{x}^5} = \frac{6.675}{77.3125} = 0.0889 \,//$$

$$\sum \hat{\rho}^j = 0.0952 + 0.0977 + 0.0959 + 0.0896 + 0.0889$$
$$= 0.4673 \,//$$

$$\therefore \hat{\rho}_Q = N.r - \frac{N-1}{N} \times \sum_{j=1}^{N} \hat{\rho}^j$$

$$= 5(0.0934) - \frac{4}{5} \times 0.4673$$

$$= 0.0932 \,//$$

Question 2

The Jackknife estimator for μ

$$J(\hat{\mu}) = n\hat{\mu} - (n-1)\bar{\hat{\mu}}_{(\cdot)}$$

where $\hat{\theta}_{(\cdot)} = \sum_{i=1}^{n} \hat{\theta}_{(i)}$

$$\hat{\theta}_{(i)} = \frac{1}{(n-1)} \left(\sum_{j=1}^{n} x_j - x_i \right)$$

$$\hat{\mu} = 98.98 \quad , \quad \Sigma x_i = 494.9$$

$$\hat{\theta}_{(1)} = \frac{1}{4}(494.9 - 104.1) = 97.7 \,//$$

$$\hat{\theta}_{(2)} = \frac{1}{4}(494.9 - 85.2) = 102.425 \,//$$

$$\hat{\theta}_{(3)} = \frac{1}{4}(494.9 - 94.1) = 100.2 \,//$$

$\hat{\theta}_4 = \frac{1}{4}(494.9 - 112.7) = 95.55$ //

$\hat{\theta}_5 = \frac{1}{4}(494.9 - 98.8) = 99.025$ //

$\Sigma \hat{\theta}_{(i)} = 494.9$

$\hat{\theta}_{(i)} = \frac{494.9}{5} = 98.98$.

$J(\hat{\mu}) = 494.9 - 395.92$

$= 98.98$ //

$\therefore$ $\bar{x}$ is an unbiased estimator for μ

$\therefore$ The Jackknife estimator for μ is $\bar{x}$ (98.98) //.

——— x ——— x ——— x ———