

P51

- 1) a) $\pi_{\text{person.name}} (\sigma_{\text{company.name} = \text{"First Bank Corporation"}} (\text{works}))$
 b) $\pi_{\text{person.name}, \text{city}} (\sigma_{\text{company.name} = \text{"First Bank Corporation"}} (\text{employee} \bowtie \text{works}))$
 c) $\pi_{\text{person.name}, \text{street}, \text{city}} (\sigma_{\text{salary} > 10,000} (\sigma_{\text{company.name} = \text{"First Bank Corporation"}} (\text{employee} \bowtie \text{works})))$
 d) $e-x-v \leftarrow \text{employee} \bowtie \text{works}$
 $\pi_{\text{person.name}} (\sigma_{e-x-v.\text{city} = \text{company.city}} (\sigma_{e-x-v.\text{company.name} = \text{company.company.name}} (e-x-v \times \text{company})))$
 e) $\pi_{\text{test.city}} (\sigma_{\text{company.city} = \text{test.city}} (\sigma_{\text{company.company.name} = \text{"Small Bank Corporation"}} (\text{company} \times \rho_{\text{test}}(\text{company}))))$

- 2) $\pi_{\text{customer.name}, \text{customer.city}, \text{loan.number}, \text{amount}} (\text{borrower} \bowtie \text{loan} \bowtie \text{customer})$
 a) Jackson is not included in this result b/c the natural join w/ customer will eliminate the tuple for Jackson (since Jackson is not in customer relation)
 b) The database could be modified such that a new tuple is added for Jackson in customer relation:
- | customer.name | customer.street | customer.city |
|---------------|-----------------|---------------|
| Jackson | Libertaria | Pasadena |
- c) $\pi_{\text{customer.name}, \text{customer.city}, \text{loan.number}, \text{amount}} ((\text{borrower} \bowtie \text{loan}) \bowtie \text{customer})$

- 3) a) $\text{works} \leftarrow \pi_{\text{person.name}, \text{company.name}, (\text{salary} * 1.1)} (\sigma_{\text{company.name} = \text{"First Bank Corporation"}} (\text{works}))$
 b) $\text{managers} \leftarrow \pi_{\text{manager.name as person.name}} (\text{works})$
 $\text{works} \leftarrow \pi_{\text{person.name}, \text{company.name}, (\text{salary} * 1.1)} (\sigma_{\text{salary} * 1.1 \geq 100,000 \vee \text{loan} * 1.1 \geq 100,000} (\text{works} \bowtie \text{managers})) \cup \pi_{\text{person.name}, \text{company.name}, (\text{salary} * 1.03)} (\sigma_{\text{salary} * 1.1 > 100,000} (\text{works} \bowtie \text{managers})) \cup (\text{works} - (\text{works} \bowtie \text{managers}))$
 c) $\text{works} \leftarrow \text{works} - \sigma_{\text{company.name} = \text{"Small Bank Corporation"}} (\text{works})$

- 4) a) $\pi_{\text{account.number}} (\sigma_{\text{account.number} \in \text{count}(\text{customer.name}) \text{ as } u} (\text{depositor}))$
 b) $d \leftarrow \text{depositor}$
 $\pi_{d.\text{account.number}} (\sigma_{d.\text{customer.name} \neq t1.\text{customer.name} \vee d.\text{customer.name} \neq t2.\text{customer.name} \vee t1.\text{customer.name} \neq t2.\text{customer.name} \wedge d.\text{account.number} = t1.\text{account.number} \wedge d.\text{account.number} = t2.\text{account.number}} (d \times \rho_{t1}(d) \times \rho_{t2}(d)))$

- 5) a) $\text{company_info} \leftarrow \text{company.name} \bowtie_{\text{count}(\text{person.name}) \text{ as } \text{employees}} (\text{works})$
 $\pi_{\text{company.name}(\text{company_info})} - \pi_{\text{company_info}, \text{company.name}} (\sigma_{\text{company_info.employees} < \text{test.employees}} (\text{company_info} \times \rho_{\text{test}}(\text{company_info})))$
 b) $\text{payroll_info} \leftarrow \text{company.name} \bowtie_{\text{sum}(\text{salary}) \text{ as } \text{payroll}} (\text{works})$
 $\pi_{\text{company.name}(\text{payroll_info})} - \pi_{\text{payroll_info}, \text{company.name}} (\sigma_{\text{payroll_info.payroll} > \text{test.payroll}} (\text{payroll_info} \times \rho_{\text{test}}(\text{payroll_info})))$

① $\text{company_salaries} \leftarrow \text{company_name} \text{ } G_{\text{average(salaries) as avg_salary (works)}}$

$\text{FBL} \leftarrow \sigma_{\text{company_name} = \text{"First Bank Corporation"}} (\text{company_salaries})$

$\text{other} \leftarrow \text{company_salaries} - \text{FBL}$

$\pi_{\text{other_company_name}} (\sigma_{\text{FBL.avg_salary} < \text{other.avg_salary}} (\text{FBL} \times \text{other}))$

monkey_likes.name	monkey_food.food
Bobo	apple
Bobo	bananas



name
Bobo
Jojo...

name	food
Bobo	apples

monkey_likes

$$\textcircled{6} r \div s = \pi_{R-S}(r) - \pi_{R-S}((\pi_{R-S}(r) \times s) - \pi_{R-S,S}(r)) \Rightarrow \text{monkey_likes} \div \text{monkey_foods} = \pi_{\text{name}}(\text{monkey_likes}) - \pi_{\text{name}}((\pi_{\text{name}}(\text{monkey_likes}) \times \text{monkey_foods}) - \pi_{\text{name, food}}(\text{monkey_likes}))$$

Gunter would appear in $\text{monkey_likes} \div \text{monkey_foods}$ b/c Gunter satisfies the 2 conditions given by the definition of relational division:

① t is in $\pi_{R-S}(r) \Rightarrow$ Gunter is in $\pi_{\text{name}}(\text{monkey_likes})$ ✓

② For every food item in monkey_foods , there is a tuple $t_{\text{monkey_likes}}$ in monkey_likes satisfying:

① $t_{\text{monkey_likes}}[\text{food}] = t_{\text{monkey_foods}}[\text{food}] \Rightarrow$ each food item in monkey_foods appears in a tuple w/ Gunter in monkey_likes ✓

② $t_{\text{monkey_likes}}[\text{name}] = t \Rightarrow \text{Gunter} = \text{Gunter}$ ✓

$$\textcircled{7} r \div_E s = (r \div s) - \pi_{R-S}(\pi_{R-S,S}(r) - \pi_{R-S}(r) \times s)$$

- This definition would work in this case (and all other cases) because we know that $r \div s$ gives us $\{(Jojo), (Gunter)\}$, but we want to get rid of Gunter since there is a tuple with Gunter in monkey_likes that does not match the contents of monkey_foods (tofu not in monkey_foods). So, the right side of this definition $(\pi_{R-S}(\pi_{R-S,S}(r) - \pi_{R-S}(r) \times s))$ will end up projecting the name Gunter because it basically gives us the tuples in monkey_likes that are leftover after taking the set difference with all of the possibilities of name, food combinations from monkey_foods . Specifically, $\pi_{R-S,S}(r) - \pi_{R-S}(r) \times s$, in regards to Gunter, can be imagined as so:

Gunter	apples
Gunter	oranges
Gunter	bananas
Gunter	tofu

—

Gunter	apples
Gunter	oranges
Gunter	bananas

which will include Gunter in the result since $(\text{Gunter}, \text{tofu})$ will not be eliminated so $\pi_{R-S}()$ will project, in this case, names. Therefore, Gunter will be included on the right side of the set difference in the definition of $r \div_E s$, meaning that the final, accurate result of $\text{monkey_likes} \div_E \text{monkey_foods}$ will be $\{(Jojo)\}$, as desired.
Note that this logic applies in all cases.

⑧ $\pi_{A-S}(\sigma_{\text{count} = c(R-S \text{ } G_{\text{count}(S)} \text{ as count}(rows) \times G_{\text{count}(S)} \text{ as } c(S))})$

⑨ $\sigma_B(\pi_A G_F(r))$ vs. $\pi_A G_F(\sigma_B(r))$

These expressions would be equivalent since $\sigma_B()$ only uses predicates from A , so it will always select the same tuples whether or not $\pi_A G_F()$ is applied before or after $\sigma_B()$ (because it has to be grouped by A). If σ could use predicates not from A , then these expressions would not be equivalent. Again, the same tuples will always be selected in this case due to given definitions in this case. This goes both ways such that the aggregation is not affected by the selection.

\Rightarrow equivalent

⑩ $\pi_A(r-s)$ vs. $\pi_A(r) - \pi_A(s)$ let π_A be π_a

a	b
dog	red
dog	white
fish	blue

r

a	b
dog	red
cat	white
turtle	blue

s

\Rightarrow

a	b
dog	white
fish	blue

$r-s \Rightarrow$

$\Rightarrow \pi_a \left(\begin{array}{|c|c|} \hline a & b \\ \hline \text{dog} & \text{white} \\ \text{fish} & \text{blue} \\ \hline \end{array} \right) \Rightarrow$

a
dog
fish

$\pi_A(r) - \pi_A(s) \Rightarrow$

a
dog
fish

-

b
dog
cat
turtle

\Rightarrow

a
fish

We have clearly provided a counterexample where $\pi_A(r-s) \neq \pi_A(r) - \pi_A(s)$

\Rightarrow NOT equivalent

(11) $(r \bowtie s) \bowtie t$ $r \bowtie (s \bowtie t)$ $r(a, b1) \quad s(a, b2) \quad t(a, b3)$

a	b1
0	v
1	w

r

a	b2
2	x
3	y

s

a	b3
0	v
1	w

t

$r \bowtie s$:

a	b1	b2
0	v	null
1	w	null



$(r \bowtie s) \bowtie t$:

a	b1	b2	b3
0	v	null	v
1	w	null	w

$s \bowtie t$:

a	b2	b3
2	x	null
3	y	null



$r \bowtie (s \bowtie t)$:

a	b1	b2	b3
0	v	null	null
1	w	null	null

\Rightarrow Thus we have shown that $(r \bowtie s) \bowtie t \neq r \bowtie (s \bowtie t)$ when r and t have at least 1 equivalent value under attribute a (while s has zero common values in attribute a) \Rightarrow NOT equivalent

(12) $\sigma_\theta(r \bowtie s)$ vs. $\sigma_\theta(r) \bowtie s$

We can determine that these expressions are equivalent since we are only able to select using predicates using attributes from r. With this, the same tuples will be selected appropriately whether or not the left outer join was applied first. If θ could be a predicate using attributes from s, then these expressions would not be equivalent. Again, since it is a left outer join and we can only select from attributes from r (the "left" side), we can see how this holds true.

\Rightarrow equivalent

(13) $\sigma_{b_1=x}(r \bowtie s)$ vs. $r \bowtie \sigma_{b_2=x}(s)$ $\theta \rightarrow b_2=x$

a	b1
0	w
1	x

r

a	b2
1	x
2	y

s

$r \bowtie s$:

a	b1	b2
0	w	null
1	x	x

$\sigma_{b_1=x}(r \bowtie s)$:

a	b1	b2
1	x	x

$\sigma_{b_1=x}(s)$:

a	b2
1	x

$r \bowtie \sigma_{b_2=x}(s)$:

a	b1	b2
0	w	null
1	x	x

By counter-example, we can clearly see that $\sigma_{b_1=x}(r \bowtie s) \neq r \bowtie \sigma_{b_2=x}(s) \Rightarrow$ NOT equivalence