

Let us first find the MSE of $\hat{\mu}_{MSE}$ by simulation:

```
n = 10;  
a = 1;  
B = 3;  
m = 10^4;  
mu = (a + B)/2;  
sum = 0;  
for i = 1:m  
    X = a + (B-a) * rand(n,1);  
    sum = sum + (((min(X) + max(X))/2) - mu)^2;  
end  
MSE_sim = sum/m;  
disp("MSE of MLE of mu"); disp(MSE_sim);
```

```
MSE of MLE of mu  
0.0152
```

Let us now find the MSE of the plug-in estimate $\hat{\mu}_n = \bar{X}_n$ of μ analytically:

We know $MSE[\hat{\mu}_n] = bias[\hat{\mu}_n]^2 + se[\hat{\mu}_n]^2$:

$$\rightarrow bias[\hat{\mu}_n] = \mathbb{E}[\hat{\mu}_n] - \mu = \mu - \mu = 0$$

$$\rightarrow se[\hat{\mu}_n] = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{n}} \cdot \sqrt{\frac{(\beta - \alpha)^2}{12}} = \frac{1}{\sqrt{10}} \cdot \sqrt{\frac{(3 - 1)^2}{12}} = \frac{2}{\sqrt{120}} \quad (\text{ignore sampling fraction})$$

$$\rightarrow MSE[\hat{\mu}_n] = 0^2 + \left(\frac{2}{\sqrt{120}}\right)^2 = \frac{1}{30} \approx 0.0333$$

Therefore, we see that the MSE of $\hat{\mu}_n$ is about 2 times greater than the simulated MSE of $\hat{\mu}_{MSE}$.