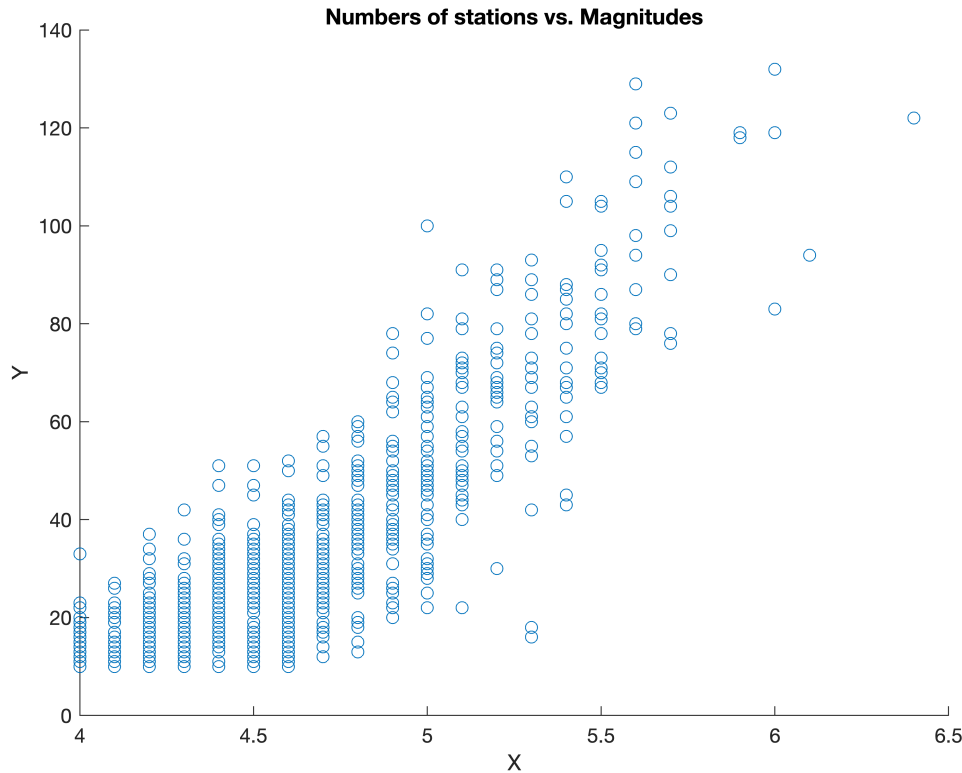


## Part a

```
fiji = importdata("fiji.txt");  
X = fiji(:,5); % magnitudes  
Y = fiji(:,6); % numbers of stations  
scatter(X,Y)  
title("Numbers of stations vs. Magnitudes")  
xlabel("X")  
ylabel("Y")
```



## Part b

Given the representation of  $\theta$ , let us calculate the following plug-in estimate  $\hat{\theta}_n$  :

$$\hat{\theta}_n = \frac{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2 \frac{1}{n} \sum_{i=1}^n (Y_i - \bar{Y})^2}} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2 \sum_{i=1}^n (Y_i - \bar{Y})^2}}$$

Let us solve with  $n1 = \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$ ,  $d1 = \sum_{i=1}^n (X_i - \bar{X})^2$ ,  $d2 = \sum_{i=1}^n (Y_i - \bar{Y})^2$  such that  $\hat{\theta}_n = \frac{n1}{\sqrt{d1 \cdot d2}}$

```
n = 10^3;  
n1 = 0;
```

```

d1 = 0;
d2 = 0;
for i = 1:n
    n1 = n1 + ((X(i) - mean(X)) * (Y(i) - mean(Y)));
    d1 = d1 + (X(i) - mean(X))^2;
    d2 = d2 + (Y(i) - mean(Y))^2;
end
plugin = n1/sqrt(d1*d2);
disp("Plug-in estimate: "); disp(plugin);

```

```

Plug-in estimate:
    0.8512

```

## Part c

```

B = 10^3;
theta_hat = zeros(B, 1);
for i = 1:B
    samp = datasample([X Y], n);
    Xsamp = samp(:,1);
    Ysamp = samp(:,2);
    theta_hat(i) = corr(Xsamp, Ysamp);
end
bs_se = sqrt(var(theta_hat));
disp("Bootstrap estimate of standard error: "); disp(bs_se);

```

```

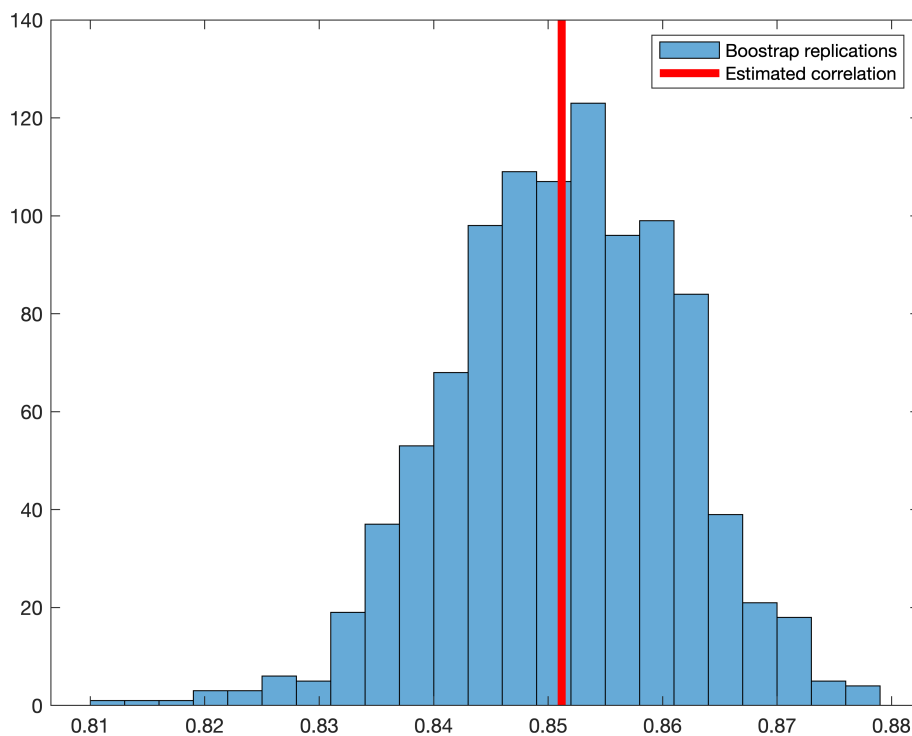
Bootstrap estimate of standard error:
    0.0099

```

```

figure
histogram(theta_hat)
hold on
line([plugin, plugin], ylim, 'LineWidth', 4, 'Color', 'red');
legend('Bootstrap replications', 'Estimated correlation');
hold off;

```



We notice a decently strong bell-shaped normal distribution for the bootstrap replications of correlation estimates. Furthermore, we clearly see the plug-in estimate lines essentially directly through the center of the distribution of values.

## Part d

```
normal_interval = norminv([0.025 0.975], plugin, bs_se);
disp("Normal 95% confidence interval: "); disp(normal_interval);
```

```
Normal 95% confidence interval:
    0.8317    0.8707
```

```
pivotal_interval = bootci(B, @corr, X, Y);
disp("Pivotal 95% confidence interval: "); disp(transpose(pivotal_interval));
```

```
Pivotal 95% confidence interval:
    0.8278    0.8678
```

We notice that the two intervals are very similar.