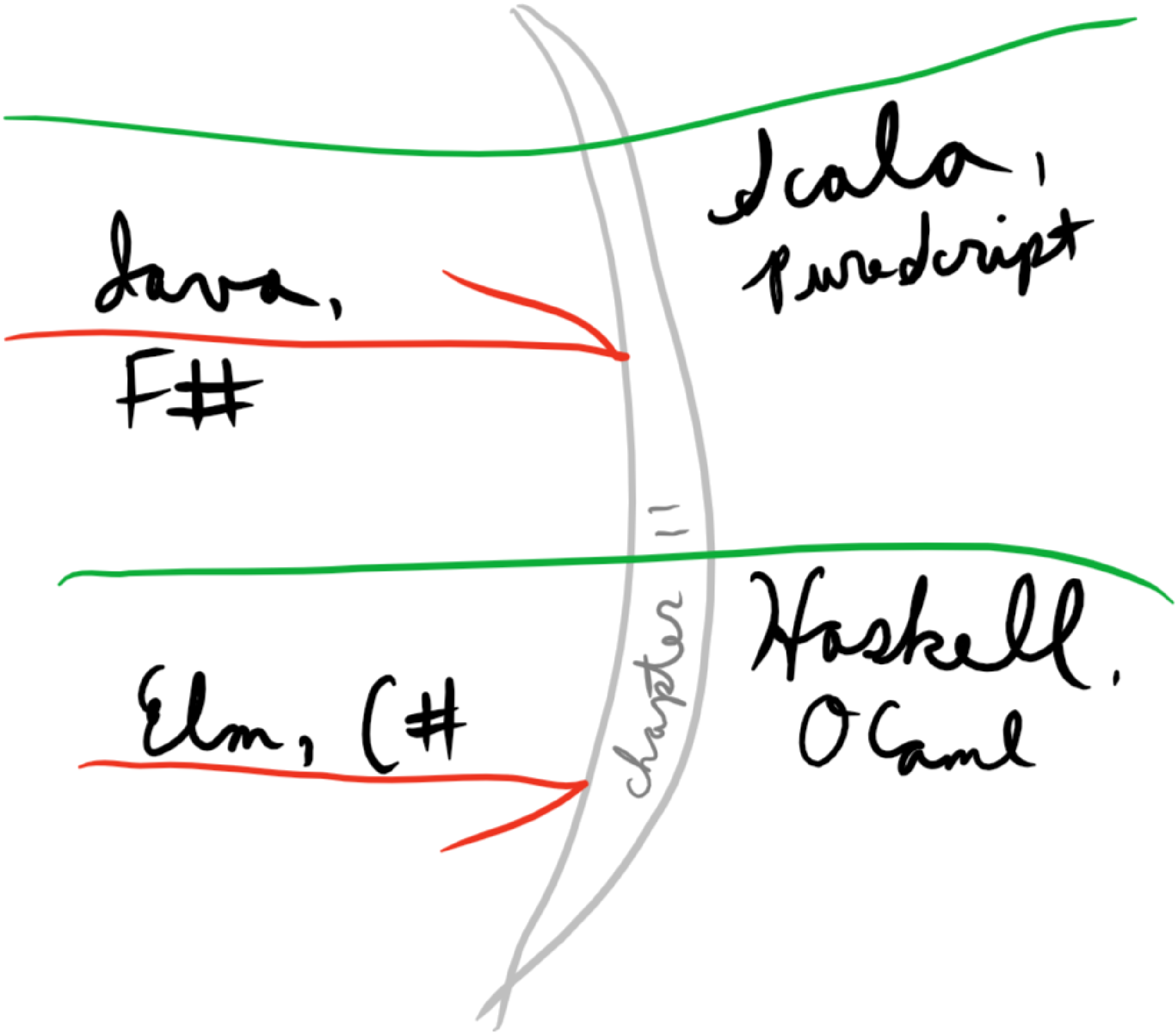
Higher-kinded types: the difference between giving up, and moving forward

As its opening sentence reminds the reader—a point often missed by many reviewers—the book *Functional Programming in Scala* is not a book about Scala. This [wise] choice occasionally manifests in peculiar ways.

For example, you can go quite far into the book implementing its exercises in languages with simpler type systems. Chapters 1–8 and 10 port quite readily to Java [8] and C#. So *Functional Programming in Scala* can be a very fine resource for learning some typed functional programming, even if such languages are all you have to work with. Within these chapters, you can remain blissfully unaware of the limitations imposed on you by these languages' type systems.

However, there is a point of inflection in the book at chapter 11. You can pass through with a language such as OCaml, Scala, Haskell, PureScript, or one of a few others. However, users of Java, C#, F#, Elm, and many others may proceed no further, and must turn back here.



Here is where abstracting over type constructors, or "higher-kinded types", comes into play. At this point in the book, you can give up, or proceed with a sufficiently powerful language. Let's see how this happens.

Functional combinators

The bread and butter of everyday functional programming, the "patterns" if you like, is the implementation of standard functional combinators for your datatypes, and more importantly the comfortable, confident use of these combinators in your program.

For example, confidence with `bind`, also known as `>>=` or `flatMap`, is very important. The best way to acquire this comfort is to reimplement it a bunch of times, so *Functional Programming in Scala* has you do just that.

```
def flatMap[B](f: A => List[B]): List[B] // in List[A]
def flatMap[B](f: A => Option[B]): Option[B] // in Option[A]
def flatMap[B](f: A => Either[E, B]): Either[E, B] // in Either[E, A]
def flatMap[B](f: A => State[S, B]): State[S, B] // in State[S, A]
```

All `flatMap`s are the same

The similarity between these functions' types is the most obvious surfacing of their 'sameness'. (Unless you wish to count their names, which I do not.) That sameness is congruent: when you write functions using `flatMap`, in any of the varieties above, these functions inherit a sort of sameness from the underlying `flatMap` combinator.

For example, supposing we have `map` and `flatMap` for a type, we can 'tuple' the values within.

```
def tuple[A, B](as: List[A], bs: List[B]): List[(A, B)] =
  as.flatMap{a =>
    bs.map((a, _))}

def tuple[A, B](as: Option[A], bs: Option[B]): Option[(A, B)] =
  as.flatMap{a =>
    bs.map((a, _))}

def tuple[E, A, B](as: Either[E, A], bs: Either[E, B]): Either[E, (A, B)] =
  as.flatMap{a =>
    bs.map((a, _))}

def tuple[S, A, B](as: State[S, A], bs: State[S, B]): State[S, (A, B)] =
  as.flatMap{a =>
    bs.map((a, _))}
```

Functional Programming in Scala contains several such functions, such as `sequence`. These are each implemented for several types, each time with potentially the same code, if you remember to look back and try copying and pasting a previous solution.

To parameterize, or not to parameterize

In programming, when we encounter such great sameness—not merely similar code, but *identical* code—we would like the opportunity to *parameterize*: extract the parts that are different to arguments, and recycle the common code for all situations.

In `tuple`'s case, what is different are

1. the `flatMap` and `map` implementations, and
2. the type constructor: `List`, `Option`, `State[S, ...]`, what have you.

We have a way to pass in implementations; that's just higher-order functions, or 'functions as arguments'. For the type constructor, we need 'type-level functions as arguments'.

```
def tuplef[F[_], A, B](fa: F[A], fb: F[B]): F[(A, B)] = ???
```

We've handled 'type constructor as argument', and will add the `flatMap` and `map` implementations in a moment. First, let's learn how to read this.

Reading a higher-kinded type

Confronted with a type like this, it's helpful to sit back and muse on the nature of a function for a moment.

Functions are given meaning by substitution of their arguments.

```
def double(x: Int) = x + x
```

`double` remains "an abstraction" until we *substitute for x*; in other words, pass an argument.

```
double(2)     double(5)
2 + 2         5 + 5
4             10
```

But this isn't enough to tell us *what* `double` *is*; all we see from these tests is that `double` sometimes returns 4, sometimes 10, sometimes maybe other things. We must imagine what `double` does in common *for all possible arguments*.

Likewise, we give meaning to type-parameterized definitions like `tuplef` by substitution. The parameter declaration `F[_]` means that `F` may not be a simple type, like `Int` or `String`, but instead a one-argument type constructor, like `List` or `Option`. Performing these substitutions for `tuplef`, we get

```scala
// original, as above
def tuplef[F[_], A, B](fa: F[A], fb: F[B]): F[(A, B)]

// F = List
def tupleList[A, B](fa: List[A], fb: List[B]): List[(A, B)]

// F = Option
def tupleOpt[A, B](fa: Option[A], fb: Option[B]): Option[(A, B)]
```

More complicated and powerful cases are available with other kinds of type constructors, such as by partially applying. That's how we can fit `State`, `Either`, and other such types with two or more parameters into the `F` parameter.

```scala
// F = Either[E, ...]
def tupleEither[E, A, B](fa: Either[E, A], fb: Either[E, B])
    : Either[E, (A, B)]

// F = State[S, ...]
def tupleState[S, A, B](fa: State[S, A], fb: State[S, B])
    : State[S, (A, B)]
```

Just as with `double`, though this isn't the whole story of `tuplef`, its true meaning arises from the common way in which it treats *all possible* `F` arguments. That is where higher kinds start to get interesting.

Implementing functions with higher-kinded type

The type of `tuplef` expresses precisely our intent—the idea of "multiplying" two `F`s, tupling the values within—but cannot be implemented as written. That's because we don't have functions that operate on `F`-constructed values, like `fa: F[A]` and `fb: F[B]`. As with any value of an ordinary type parameter, these are opaque.

In Scala, there are a few ways to pass in the necessary functions. One option is to implement a `trait` or `abstract class` that itself uses a higher-kinded type parameter or abstract type constructor. Here are a couple possibilities.

```scala
trait Bindable[F[_], +A] {
  def map[B](f: A => B): F[B]
  def flatMap[B](f: A => F[B]): F[B]
```

```
}

trait BindableTM[+A] {
  type F[X]
  def map[B](f: A => B): F[B]
  def flatMap[B](f: A => F[B]): F[B]
}
```

Note that we must use higher-kinded trait type signatures to support our higher-kinded method types; otherwise, we can't write the return types for `map` and `flatMap`.

```
trait BindableBad[F] {
  def map[B](f: A => B): F ???
          // where is the B supposed to go?
```

Now we make every type we'd like to support either inherit from or implicitly convert to `Bindable`, such as `List[+A] extends Bindable[List, A]`, and write `tuplef` as follows.

```
def tupleBindable[F[_], A, B](fa: Bindable[F, A], fb: Bindable[F, B])
    : F[(A, B)] =
  fa.flatMap{a =>
    fb.map((a, _))}
```

Escaping two bad choices

There are two major problems with `Bindable`'s representation of `map` and `flatMap`, ensuring its wild unpopularity in the Scala functional community, though it still appears in some places, such as in Ermine.

1. The choices of inheritance and implicit conversion are both bad in different ways. Implicit conversion propagates very poorly—it doesn't compose, after all, and fails as soon as we do something innocent like put the value-to-be-converted into a tuple. Inheritance leaves its own mess: modifying a type to add new, nonessential operations, and the weird way that `F` is declared in the method type parameters above.

2. The knowledge required to work out the new type signature above is excessively magical. There are rules about when implicit conversion happens, how much duplication of the reference to `Bindable` is required to have the `F` parameter infer correctly, and even how many calls to `Bindable` methods are performed. For example, we'd have to declare the `F` parameter as `F[X] <: Bindable[F, X]` if we did one more trailing `map` call. But then we wouldn't support implicit conversion cases anymore, so we'd have to do something

else, too.

As a result of all this magic, generic functions over higher kinds with OO-style operations tend to be ugly; note how much `tuplef` looked like the `List`-specific type, and how little `tupleBindable` looks like either of them.

But we still really, really want to be able to write this kind of generic function. Luckily, we have a Wadler-made alternative.

Typeclasses constrain higher-kinded types elegantly

To constrain `F` to types with the `flatMap` and `map` we need, we use typeclasses instead. For `tuplef`, that means we leave `F` abstract, and leave the types of `fa` and `fb` as well as the return type unchanged, but add an implicit argument, the "typeclass instance", which is a first-class representation of the `map` and `flatMap` operations.

```
trait Bind[F[_]] {
  // note the new ↓ fa argument
  def map[A, B](fa: F[A])(f: A => B): F[B]
  def flatMap[A, B](fa: F[A])(f: A => F[B]): F[B]
}
```

Then we define instances for the types we'd like to have this on: `Bind[List]`, `Bind[Option]`, and so on, as seen in chapter 11 of *Functional Programming in Scala*.

Now we just add the argument to `tuplef`.

```
def tupleTC[F[_], A, B](fa: F[A], fb: F[B])
           (implicit F: Bind[F]): F[(A, B)] =
  F.flatMap(fa){a =>
    F.map(fb)((a, _))}
```

We typically mirror the typeclass operations back to methods with an implicit conversion—unlike with `Bindable`, this has no effect on exposed APIs, so is benign. Then, we can remove the `implicit F` argument, replacing it by writing `F[_]: Bind` in the type argument list, and write the method body as it has been written before, with `flatMap` and `map` methods.

There's another major reason to prefer typeclasses, but let's get back to *Functional Programming in Scala*.

Getting stuck

I've just described many of the practical mechanics of writing useful functions that abstract over type constructors, but *all this is moot if you cannot abstract over type constructors*. The fact that Java provides no such capability is not an indicator that they have sufficient abstractions to replace this missing feature: it is simply an abstraction that they do not provide you.

Oh, you would like to factor this common code? Sorry, you are stuck. You will have to switch languages if you wish to proceed.

Don't get stuck on the second order

`map` functions are obvious candidates for essential parts of a usable library for functional programming. This is the first-order abstraction—it eliminates the concrete loops, recursive functions, or `State` lambda specifications, you would need to write otherwise.

When we note a commonality in patterns and define an abstraction over that commonality, we move "one order up". When we stopped simply defining functions, and started taking functions as arguments, we moved from the first order to the second order.

It is not enough for a modern general-purpose functional library in Scala to simply have a bunch of `map` functions. It must also provide the second-order feature: the ability to *abstract over* `map` functions, as well as many, many other functions numerous type constructors have in common. Let's not give up; let's move forward.

This article was tested with Scala 2.11.7 and fpinscala 5b0115a answers, with the addition of the method variants of `List#map` *and* `List#flatMap`*.*

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by Stephen Compall on Aug 21, 2016

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Radek Micek • 2 years ago

> Here is where abstracting over type constructors, or “higher-kinded types”, comes into play. At this point in the book, you can give up, or proceed with a sufficiently powerful language.

I don't have the book but you can certainly proceed with a language without higher-kinded types - eg. in Kotlin you simply write

```
data class App<F, A>(val x: Any)

abstract class Monad<M> {
    abstract fun <A> pure(a: A): App<M, A>
    abstract fun <A, B> bind(ma: App<M, A>, f: (A) -> App<M, B>): App<M, B>
}

fun <M, A> join(m: Monad<M>, mma: App<M, App<M, A>>): App<M, A> =
        m.bind(mma) { it }
```

and as you can see no higher-kinded type (in join) is involved.

Then you can implement a concrete monad:

```
class Seq
```

see more

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Christophe Calvès → Radek Micek • 2 years ago

Sure you can encode it. But as any encoding, this is painful to use. Furthermore, you seem to lose kind-checking (`App<Int, Boolean>` seems to be legal).

1 • Reply • Share ›

Def → Radek Micek • 2 years ago

That's known as defunctionalization. You can read mode about the technique in https://www.cl.cam.ac.uk/~j... .

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Yawar Amin → Radek Micek • 2 years ago

Neat trick. I've seen F#-ers do it before. Can you make it safer by making the `App` constructor private? Perhaps to module level?

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