Phase instability of the Stokes V parameter

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Abstract

Our numerical experiments in computation of the polarization evolution of the low frequency (80 to 300 MHz) radiation in the coronal magnetoactive plasmas using the [Kravtsov *et al.* (2007)] method elicited a phenomenon of extreme variability of the Stokes V parameter, which is regarded as the measure of circular polarization. The simulated solar images in the Stokes V component of brighness temperature obtained through the ray tracing demonstrate "carpet" structure. The power of adjacent pixels (corresponding to adjacent rays) takes almost arbitrary levels, sometimes from opposite ends of the dynamic range. This behavior is normal for the Q and U Stokes parameters since they quickly oscillate along the ray representing the Faraday rotation in magnetized plasmas. It was expected that the circular polarization parameter V would change monotonically with small amplitude oscillations picked from Q and U along the ray. The V dynamics, however, is not that simple. The numerical study allows to make several statements (or conclusions?).

The method for computing the Stokes vector evolution developed by [Kravtsov *et al.* (2007)] is restricted to smoothly inhomogeneous, weakly dissipative, and weakly anisotropic media. Here we shall show that all these conditions are satisfied in our numerical experiments. Consider three standard dimensionless plasma parameters together with L, the characteristic scale of medium inhomogeneity:

$$u = \left(\frac{\omega_c}{\omega}\right)^2 = \left(\frac{e\,\mathbf{B}}{mc\omega}\right)^2, \quad v = \left(\frac{\omega_p}{\omega}\right)^2 = \frac{4\pi e^2 N}{m\omega^2}, \quad \text{and} \quad w = \frac{\nu_{\text{eff}}}{\omega}, \tag{1}$$

where ω is the electromagnetic wave frequency, ω_c is the cyclotron frequency, ω_p is the Langmuir plasma frequency, ν_{eff} is the effective electron-ion collision frequency, m is the electron mass in grams, e is the electron charge in statcoulombs, and N is the electron number density in cm^{-3}.

The condition of smooth inhomogeneity means that the geometrical optics approximation is applicable, of, formally,

$$\frac{\lambda}{2\pi L} \ll 1, \tag{2}$$

where λ is the wavelength in a vacuum. The weak dissipation condition implies

$$w \ll 1. \tag{3}$$

The weak anisotropy condition requires

$$v \ll 1 \;\text{ or }\; u \ll 1. \tag{4}$$

Either u or v may be comparable with unity: only one of them must be small.

In order to abstract away from any irrelevant detail we assumed idealized conditions. All of the following plasma parameters are constant and uniform.

- Magnetic field $\mathbf{B} = 0.1$ G.
- Collision frequency $\nu_{\text{eff}} = 30$ rad s^{-1}.
- Electron number density $N = 3.8 \times 10^5$ cm^{-3} (at $\sim 2R_\odot$ altitude).

Then for the wave frequency band $f = [80..300]$ MHz, the dimensionless plasma parameters (1) variation will be:

Denote as θ the angle between the magnetic field vector B and the wave propagation direction.

The magnetic fields in a plasma are responsible for its birefringence (i.e. anisotropy of refraction). The magnetic fields and collisions in a plasma are responsible for its dichroism (i.e. anisotropy of absorption) The Q and U Stokes parameters oscillate with the frequency approximately proportional to $\sin\theta$. For their oscillation the plasma must be magnetized, but no collisions required. The oscillation of Q and U is a manifestation of the Faraday rotation: the (Q,U) vector draws circles. The V Stokes parameter generally has two components: monotonic and harmonic. The harmonic component of V is picked from the Q and U oscillations in magnetized plasmas. The monotonic component of V is only present in collisional plasmas. When a ray turns from fieldwise to counterfieldwise the change of θ sign causes the phase reversal of the Q and U oscillation near the point where $\theta = 90°$. The phase of U, calculated as its integral along the ray path, making small-amplitude oscillations around zero while θ is far from 90°, undergoes an abrupt jump in the vicinity of $\theta = 90°$. The sign of phase of the U jump near $\theta = 90°$ depends on the phase at which U arrived at $\theta = 90°$. The Stokes V is approximately proportional to the phase of U. The Stokes V and the phase of U are unpredictable at large distances.

The sine and magnitude of V in a long ray are unpredictable.

References

[Kravtsov *et al.* (2007)] Kravtsov, Yu. A., Bieg, B., and Bliokh, K. Yu., Stokes-vector evolution in a weakly anisotropic inhomogeneous medium, *J. O Opt. Soc. Am*, Vol. **24**, No. 10, pp. 3388-3396, 2007