Chapter 4. DESIGN OF BEAMS

Beam Behaviour and Modes of Failures

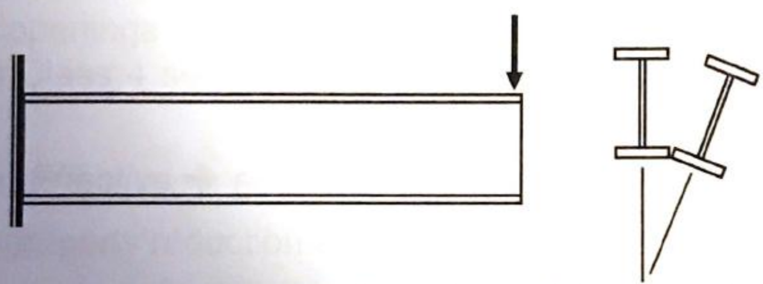
- **Flexure yielding** (Braced, not Class 4)

  Providing the beam is Class 1 (Compact) section and is laterally braced in the lateral plan, then the failure will take place by excessive deformation in the plane of the applied loading.

It is a basic mode of failure if all others are prevented.

- **Lateral torsional buckling** (not fully braced)

  Sudden failure occurs by a combination of lateral deflection and twist, the load at which this being dependent upon the proportions of the beam, the way the loading is applied, and support conditions provided.

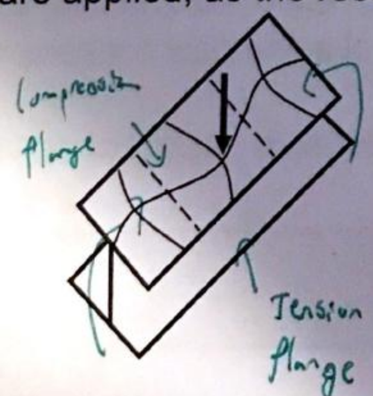


← If load applied at bottom, no lateral torsional buckling. Derivation assumes applied @ shear centre.

It is a common failure mode for unbraced beam and can be prevented by the provision of suitable lateral bracing.

- **Local buckling** (Class 4)

  Failure occurs by buckling of a flange on compression or of the web due to shear or combined shear and bending, or, where concentrated loads are applied, as the result of vertical compression.



It is unlikely for hot-rolled sections for which the proportions have been selected to minimise the local buckling. However, web stiffening is sometimes required under the point of loads and at reaction points.

↑ Only compression flange experience local buckling!

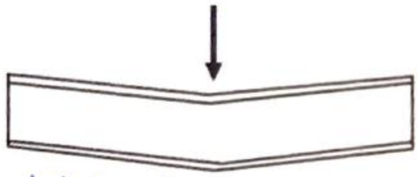
Also, ...

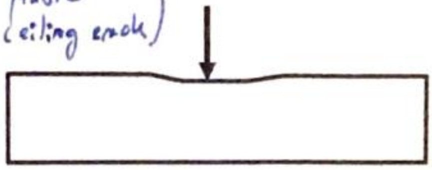
- Shear failure
- Bearing failure (web crippling)
- Excessive deflections

- **Other local failures**

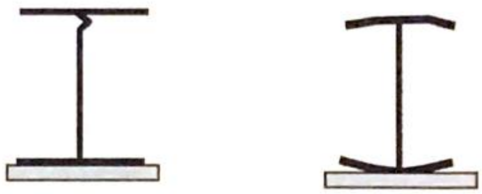
--- Several possibilities including:

1. Shear yield of web; Max shear location
2. Local crushing of web (web crippling);
3. Excessive curling of thin flange; serviceability (eg. plaster ceiling crack)
4. Local failure around web opening.





Those local failures likely occur only for short spans and or deep beams as well as at locations of concentrated forces. The failures can be prevented by suitable web stiffening and or local reinforcement.



- **Proportion** (effect of openings and holes) 14.1

Bending resistance for beams and girders consisting of rolled shapes (with or without cover plates), hollow structural sections or fabricated I-shapes and box-shapes shall be determined using the properties of the gross section or the effective section in accordance with Cl. 14.1. The effect of openings other than holes for fasteners and effect of fastener holes on Class 4 sections shall be considered in accordance with Cl. 14.3.3.

Gross vs. *Effective* ➔ elastic/plastic section modulus: $\underbrace{\mathbf{S}, \mathbf{Z}}_{\text{gross}}$, $\underbrace{\mathbf{S_e} \text{ and } \mathbf{Z_e}}_{\text{effective}}$

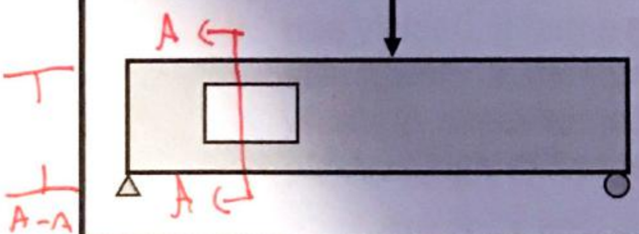
No section property reduction with presence of bolt holes (**S**, **Z**) if either 14.1.2

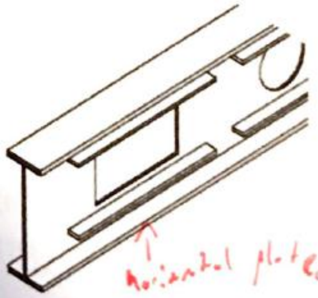
a) holes are in the compression zone only and the holes are standard size holes and filled with correctly sized bolts; or New 2019

b) $F_y \le 350$ MPa, and the reduction in flange area does not exceed 15% of the gross flange area and the total area reduction in the cross-section does not exceed 15%.

For sections with oversize, slotted, unfilled holes in compression zones or sections with holes in tension zones, calculate $\mathbf{S_e}$ and $\mathbf{Z_e}$ per Cl. 14.1.3 New 2019

Large opening: Local reinforcement --- Stiffeners

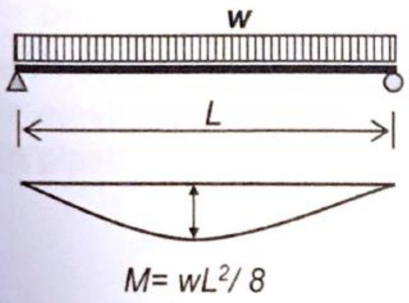




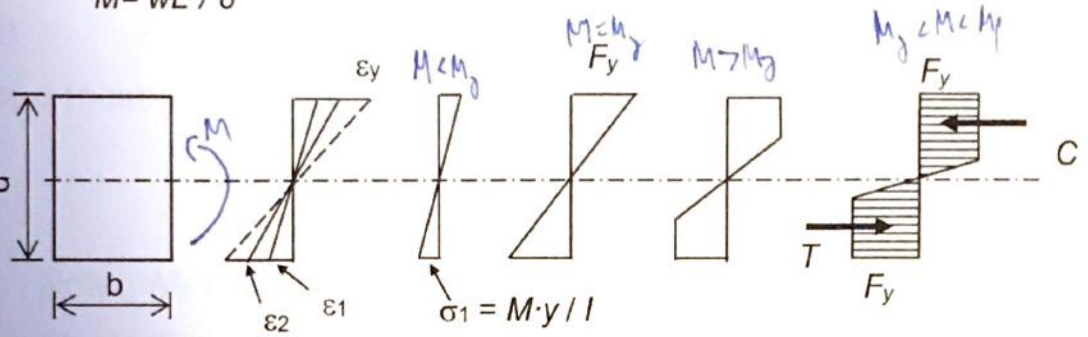
horizontal plate, not vertical for shear resistance moment (to be confirmed)

Laterally Supported Beams

A fully lateral supported (braced) beam shown below is subjected to uniformly distributed load. The maximum moment occurs at the mid-span. Thus, failure would occur at mid-span.



The figure below shows the strain and stress variation on cross-section (assumed to be rectangular) at the mid-span with increasing load. ∴Increasing the mid-span moment.



Stage 1: $\varepsilon_1 < \varepsilon_y$ and $\sigma_1 < F_y$ ∴ Elastic stress
where, ε_y and F_y are the yield strain and yield stress.

$\sigma_1 = E\varepsilon_1$; $F_y = E\varepsilon_y$; $\sigma_1 = M \times y / I$

Stage 2: $\varepsilon_2 \leq \varepsilon_y$ and the section is just yielding at the outer edges.
∴ stress at outer edge $F_y = M_y\, y_{max} / I$

$$M_y = F_y\,(I / y_{max}) = SF_y$$

where, S – elastic section modulus (I / y_{max}); I – moment of inertia of the section; y_{max} – the largest distance to outer edges from Neutral axis.

For a rectangular section, $I = bd^3/12$ and $y_{max} = d/2$; therefore, $S = bd^2/6$

Stage 3: As the load increases, the moment increases and the yield zone spreads across the cross-section. Once the whole section has yielded, it forms a "**Plastic Hinge**". At this time, the section still resists a moment M_p. However, the section could also undergo unrestrained rotation. The maximum rotation of the plastic hinge depends on the ductility of the material.

When a section achieves full plastic moment M_p, in the absence of axial force, shear force, etc.

Total compressive force C = Total tensile force T

$$F_y A_c = F_y A_T \qquad \therefore A_c = A_T$$

$\therefore$ Plastic neutral axis divides the total cross-section area into two parts with equal areas.
In elastic limits, the neutral axis is the centroid-axis.
In plastic limits, the neutral axis is the equal-area-axis.

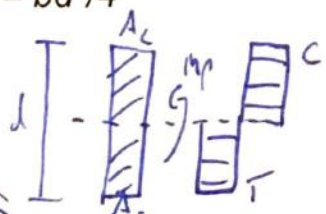
$$M_p = C\,y_c + T\,y_T = A_c\,F_y\,y_c + A_T\,F_y\,y_T = (A_c\,y_c + A_T\,y_T)\,F_y$$

Let $\quad Z = (A_c\,y_c + A_T\,y_T)$; then $\quad M_p = Z\,F_y$

For a rectangular section, Z = plastic section modulus

$$Z = (bd/2) \times (d/4) + (bd/2) \times (d/4) = bd^2/4$$

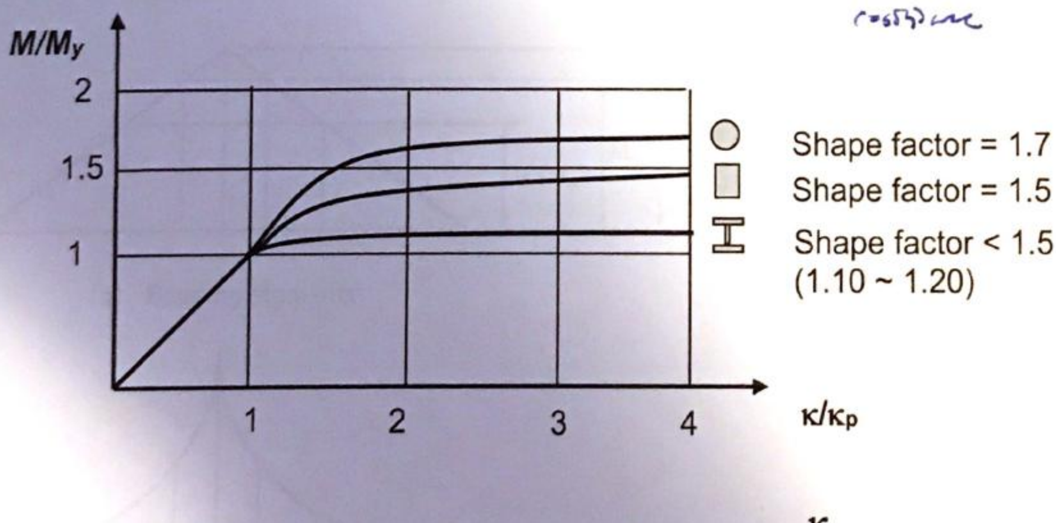
$$\therefore \quad M_p = \frac{bd^2}{4} F_y$$



Note: $M_y = SF_y = (bd^2/6)\,F_y$; $\therefore \quad M_p > M_y$

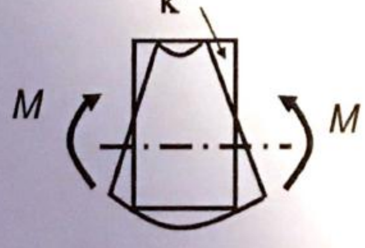
Shape Factor = $M_p / M_y = Z / S = 1.5$

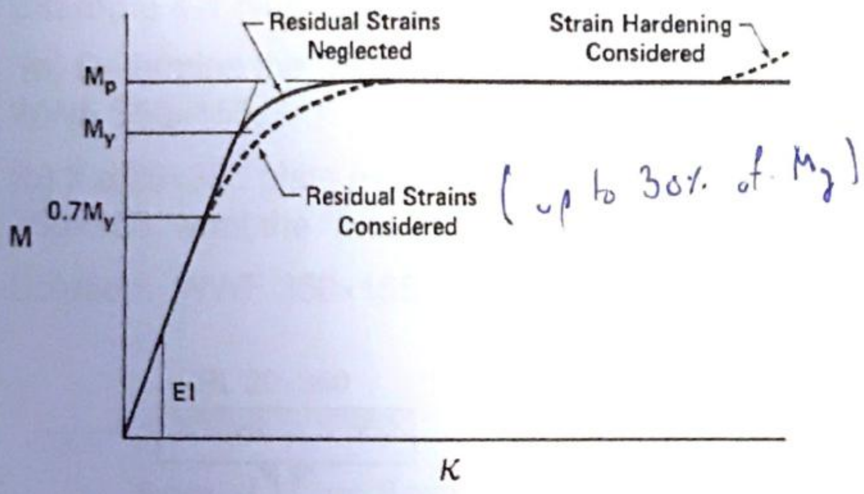
Additional moment carrying capacity (post-yielding strength) depends on "**shape factor**" (depends on the cross-section shape). increase in moment capacity



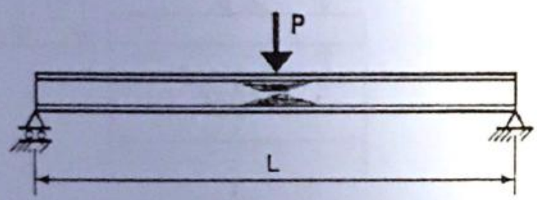
Where, $\kappa = -d^2v/dx^2$ is the curvature and v is the deflection.

In elastic range, $\kappa = M/EI$

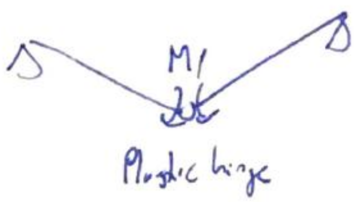


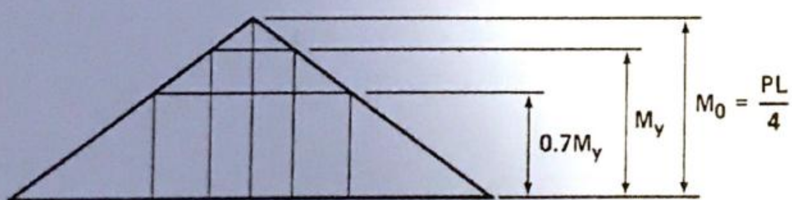


Moment vs. Curvature Relationship

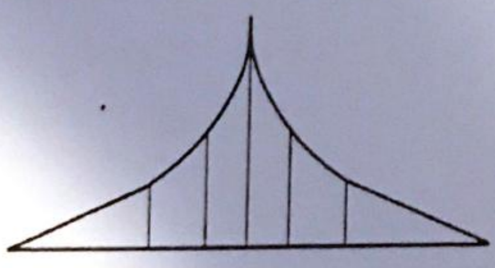


(a)





(b) Bending Moments



(c) Curvatures

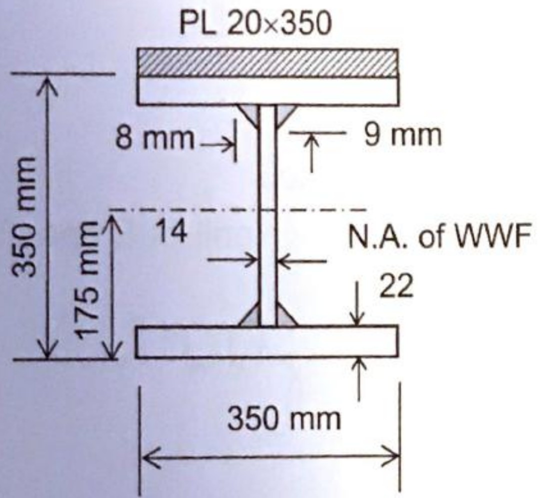
Bending Moment & Curvature Distributions

Example 4-1 (see CivE 310 Example 4-4 for the solution):

(a) Determine the "Plastic Section Modulus" of weld wide flange beam WWF 350×155.

(b) If a 20×350 plate has been welded to the top flange of WWF 350×155, what the "Plastic Section Modulus" of the built-up section?

Solution: WWF 350×155; $A = 19800\ \text{mm}^2$; $Z = 2870\times10^3\ \text{mm}^3$



Elastic section modulus

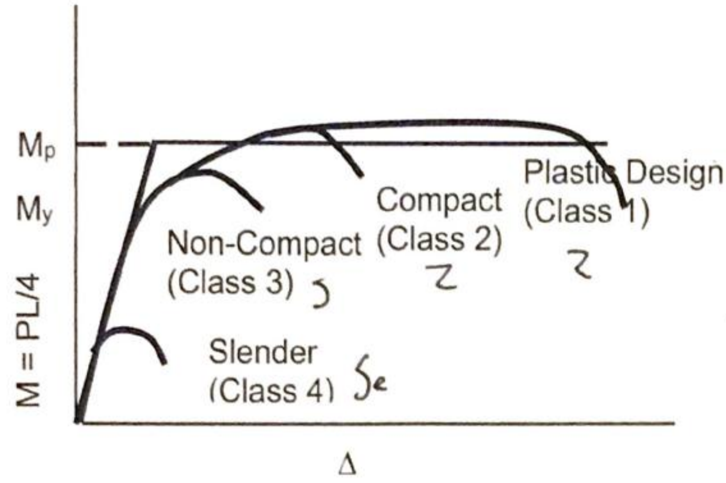
Elastic neutral axis

Plastic section modulus

Plastic neutral axis

Single sym section

Double sym section

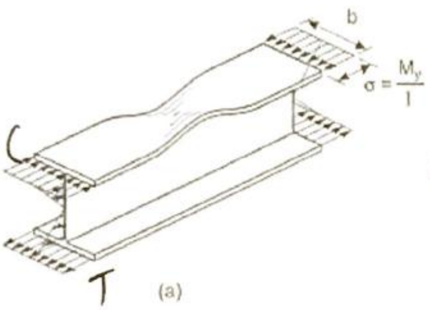


Z = plastic
S = elastic
Se = effective elastic section modulus

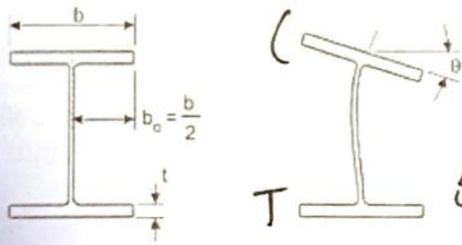
Load Deflection Relations of Different

Flange Buckling:

Local buckling



Local buckling of compression flange is of primary concern



Destabilizing effect

Stabilizing effect

Twisting moment at buckling: $T_w = \sigma(bt)(b^2t/12)(d\theta/dx)$

Torsional resistance of the flange: $T_R = JG(d\theta/dx) = (1/3)bt^3G(d\theta/dx)$

By Equilibrium, $T_w = T_R$

$$\frac{b}{2t} = \sqrt{G/\sigma}$$

$G = 21\times 10^3$ MPa; $\sigma = F_y$ (plastic) Class 1 Flange $b_o/t \le 145/(F_y)^{1/2}$

Based on tests (inelastic) Class 2 Flange $b_o/t \le 170/\sqrt{F_y}$

<u>$G = 77\times 10^3$ MPa</u>; $\sigma = F_y$ (**elastic**) Class 3 Flange $b_o/t \le 277/(F_y)^{1/2}$

↑ use this value for design $200/\sqrt{F_y}$

stand and educated

Table 2: Maximum width-to-thickness ratios of elements in flexural compression

Description of elements	Section classification limits		
	Class 1	Class 2	Class 3
Element supported along one edge and under flexural compression, such as Flanges of I-sections or T- sections under bending about the major axis Plates projecting from element in compression elements Outstanding legs of pairs of angles in continuous contact with an axis of symmetry in the plane of loading	$\frac{b_{el}}{t} \le \frac{145}{\sqrt{F_y}}$	$\frac{b_{el}}{t} \le \frac{170}{\sqrt{F_y}}$	$\frac{b_{el}}{t} \le \frac{200}{\sqrt{F_y}}$
Element supported along one edge under compressive stress due to flexural bending but with a part in traction, such as Stems of T-sections Flange of I-section under flexure around the minor axis	$\frac{b_{el}}{t} \le \frac{145}{\sqrt{F_y}}$	$\frac{b_{el}}{t} \le \frac{170}{\sqrt{F_y}}$	$\frac{b_{el}}{t} \le \frac{340}{\sqrt{F_y}}$
Element supported along two edges mainly under compressive stress due to flexural bending such as Flanges of rectangular hollow sections	$\frac{b_{el}}{t} \le \frac{420}{\sqrt{F_y}}$	$\frac{b_{el}}{t} \le \frac{525}{\sqrt{F_y}}$	$\frac{b_{el}}{t} \le \frac{670}{\sqrt{F_y}}$
Element supported along two edges mainly under compressive stress due to flexural bending such as Flanges of box sections Web of I-section under bending about the minor axis Flange cover plates and diaphragm plates between lines of fasteners or welds	$\frac{b_{el}}{t} \le \frac{525}{\sqrt{F_y}}$	$\frac{b_{el}}{t} \le \frac{525}{\sqrt{F_y}}$	$\frac{b_{el}}{t} \le \frac{670}{\sqrt{F_y}}$
Multi-sided hollow sections (b': Cl. 11.3.4) New to S16-19		$\frac{b'}{t} = \frac{500}{\sqrt{F_y}}$	$\frac{b'}{t} = \frac{560}{\sqrt{F_y}}$
Circular hollow sections	$\frac{D}{t} \le \frac{13\,000}{F_y}$	$\frac{D}{t} \le \frac{18\,000}{F_y}$	$\frac{D}{t} \le \frac{66\,000}{F_y}$
Webs of singly symmetric I-sections subjected to bending about the major axis	$\frac{2d_c}{w} = \frac{1100}{\sqrt{F_y}}$	$\frac{2d_c}{w} = \frac{1100}{\sqrt{F_y}}$	$\frac{2d_c}{w} = \frac{1100}{\sqrt{F_y}}$

Benjamin Klassen

Table 2 (Concluded)

Description of elements	**Section classification limits**	
Element supported along two edges and subject to compression under combined flexure about the major axis and an axial force such as Webs of I-sections $C_y = AF_y$	$\frac{h}{w} \le \frac{1100}{\sqrt{F_y}}\left(1 - 0.39\frac{C_f}{\phi C_y}\right)$ Class 1 $\frac{h}{w} \le \frac{1700}{\sqrt{F_y}}\left(1 - 0.61\frac{C_f}{\phi C_y}\right)$ Class 2 $\frac{h}{w} \le \frac{1900}{\sqrt{F_y}}\left(1 - 0.65\frac{C_f}{\phi C_y}\right)$ Class 3	Bending only, set $C_f/\phi C_y = 0$ Compression only, set $C_f/\phi C_y = 1.0$
Webs of I-sections subjected to compression due to combined member axial compression and bending about the minor axis: a) For $C_r > 0.4\phi C_y$	$\frac{b_{el}}{t} \le \frac{525}{\sqrt{F_y}}$ Class 1 $\frac{b_{el}}{t} \le \frac{525}{\sqrt{F_y}}$ Class 2 $\frac{h}{w} \le \frac{1900}{\sqrt{F_y}}\left(1 - 0.65\frac{C_f}{\phi C_y}\right)$ Class 3	
b) For $C_r \le 0.4\phi C_y$	$\frac{h}{w} = \frac{1100}{\sqrt{F_y}}\left(1 - 1.31\frac{C_f}{\phi C_y}\right)$ Class 1 $\frac{h}{w} = \frac{1100}{\sqrt{F_y}}\left(1 - 1.73\frac{C_f}{\phi C_y}\right)$ Class 2 $\frac{h}{w} \le \frac{1900}{\sqrt{F_y}}\left(1 - 0.65\frac{C_f}{\phi C_y}\right)$ Class 3	
Webs of I-sections subjected to compression due to combined member axial compression and bending about the principal axes, with $\frac{M_{fy}}{S_y} > \frac{0.9M_{fx}}{S_x}$ Note: if $\frac{M_{fy}}{S_y} \le \frac{0.9M_{fx}}{S_x}$, the limits for elements supported along two edges and subjected to combined axial compression and bending about the major axis shall apply.	$\frac{b_{el}}{t} \le \frac{525}{\sqrt{F_y}}$ Class 1 $\frac{b_{el}}{t} \le \frac{525}{\sqrt{F_y}}$ Class 2 $\frac{h}{w} \le \frac{1900}{\sqrt{F_y}}\left(1 - 0.65\frac{C_f}{\phi C_y}\right)$ Class 3	bi-axial flexure + axial compression

Bending --- Laterally Supported Members 13.5

Factored Moment Resistance width-to-thickness ratios of I-shape section

Class-1: $M_r = \phi M_p = \phi Z F_y$; (Sym. axis in loading plane, M redistribution) 11.1.2

Class-2: $M_r = \phi M_p = \phi Z F_y$; (Sym. axis in loading plane*, no M redistribution) *11.1.3

Class-3: $M_r = \phi M_y = \phi S F_y$

Class-4:

(i) If both width-to-thickness ratios for flange and web exceed the limits of Class-3, then both flange and web of the section are likely to be controlled by local buckling. Calculate the factored moment resistance according to CSA-S136 (F_y' based on F_y and F_u of the steel material) ;

(ii) If only the width-to-thickness ratio of web exceeds the limit of Class-3, see Clause 14.3.4 (Chapter 5 of CivE 413 Course Notes); and Plate Girder

(iii) If only the width-to-thickness ratio of flange exceeds the limit of Class-3, compute the factored moment resistance as follows:

The moment resistance may be calculated as

$$M_r = \phi S_e F_y$$

S_e = effective section modulus based on an effective flange width

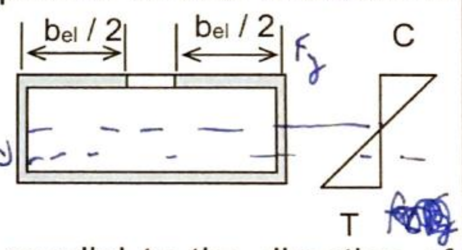
$S = \frac{I}{y}$

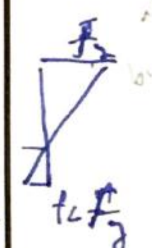
- For flanges supported along two edges parallel to their direction of stress,

$$b_{el} = \frac{670t}{\sqrt{F_y}}$$

Location of elastic NA

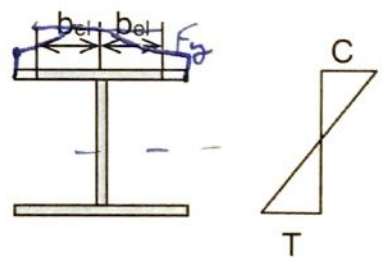
$y_{eff} = ?$

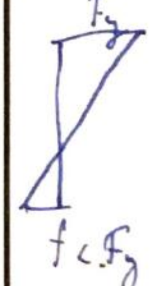




- For flanges supported along one edge parallel to the direction of stress,

$$b_{el} = \frac{200t}{\sqrt{F_y}} \leq 60t$$





~~Alternatively, calculate M_r using an effective yield stress determined from the width-to-thickness ratio meeting the Class 3 limit.~~ S16-14. No longer applicable in S16-19

iv) Circular HSS Class 4 members: $M_r = \phi S F_{ye}$; where $F_{ye} = 66000\ t/D$

BEAM LOAD TABLES
W Shapes

ASTM A992, A572 grade 50
$F_y = 345$ MPa

Total Uniformly Distributed Factored Loads for Laterally Supported Beams (kN)

Designation	W460								Approx. Deflect. (mm)
Mass (kg/m)	106	97	89	82	74	68	60	52	
Span in Millimetres: 2 000						1 560	1 360	1 270	2
2 500	2 200	1 980	1 810	1 700	1 530	1 480	1 270	1 080	3
3 000	1 980	1 810	1 660	1 520	1 370	1 230	1 060	903	5
3 500	1 700	1 550	1 430	1 300	1 170	1 060	908	774	7
4 000	1 480	1 350	1 250	1 140	1 020	925	795	677	9
4 500	1 320	1 200	1 110	1 010	911	822	707	602	11
5 000	1 190	1 080	999	909	820	740	636	542	14
5 500	1 080	985	908	826	745	673	578	492	16
6 000	989	903	832	758	683	617	530	451	20
6 500	913	833	768	699	631	569	489	417	23
7 000	848	774	713	649	586	529	454	387	27
7 500	792	722	666	606	546	493	424	361	31
8 000	742	677	624	568	512	463	397	338	35
8 500	698	637	587	535	482	435	374	319	39
9 000	660	602	555	505	455	411	353	301	44
9 500	625	570	526	478	431	390	335	285	49
10 000	594	542	499	455	410	370	318	271	54
10 500	565	516	476	433	390	352	303	258	60
11 000	540	492	454	413	373	336	289	246	66
11 500	516	471	434	395	356	322	276	235	72
12 000	495	451	416	379	342	308	265	226	78
12 500	475	433	399	364	328	296	254	217	85
13 000	457	417	384	350	315	285	245	208	92
13 500	440	401	370	337	304	274	236	201	99
14 000	424	387	357	325	293	264	227	193	107
PROPERTIES AND DESIGN DATA									
$V_{r\,max}$ (kN)	1 210	1 090	996	933	843	856	746	680	
$V_{r\,min}$ (kN)	1 100	990	906	848	766	778	678	637	
R (kN)	595	519	464	420	368	381	317	282	
G (kN)	32.6	29.5	27.2	25.6	23.3	23.5	20.7	19.7	
B_r' (kN)	593	486	412	366	303	310	239	216	
d (mm)	469	466	463	460	457	459	455	450	
b (mm)	194	193	192	191	190	154	153	152	
t (mm)	20.6	19.0	17.7	16.0	14.5	15.4	13.3	10.8	
w (mm)	12.6	11.4	10.5	9.9	9.0	9.1	8.0	7.6	
IMPERIAL SIZE AND WEIGHT									
Weight (lb/ft)	71	65	60	55	50	46	40	35	
Nominal Depth (in.)	18								

Sections highlighted in yellow are commonly used sizes and are generally readily available.
For information on $V_{r\,max}$ and $V_{r\,min}$, see *Factored Resistance of Beams*.

Example 4-2 (see CivE 310 Example 4-5 for the solution)**:** A W460×60 laterally supported beam of G40.21-M 350W steel spans 7500 mm and has pinned connections at its both ends. Calculate the maximum factored uniformly distributed load that the beam can resist and verify the result with the value listed in BEAM LOAD TABLE of HSC.

Solution: W460×60: F_y =345MPa; Z_x=1280×10^{3}mm^3; S_x=1120×10^3 mm^3

1. Section classification
2. Moment resistance
3. Factored uniformly distributed load
4. HSC Beam load table

Example 4-3 (see CivE 310 Example 4-6 for the solution)**:** A 10000 mm long beam is laterally supported by the joists at every 2000 mm. The maximum factored bending moment of the beam is obtained through structural analysis and is 320 kN-m. The beam is to be selected from W-section with G40.21-M 350W steel and the depth of the beam should not be greater than 480 mm.

Solution: M_f = 320 kN-m

1. Assume section classification (class 1 or 2 for example)
2. Let $M_r = M_f \rightarrow Z_x = M_f / \phi F_y$
3. HSC W-shape section property → select a W-section w/ $d \leq 480$ mm (W410×54)
4. Section Classification
5. Beam Load Table - check unbraced length : L_u
6. Compute $M_r \geq M_f$

Example 4-4: Compute the bending moment capacity of a 5.0 m long fully laterally braced beam with W150×22 section (G40.21-M 350W steel), using

(a) effective section modulus method, and

(b) ~~effective yield stress method~~. S16-14

Solution: *W150x22*, take F_y = 350 MPa.

d = 152 mm, b = 152 mm, t = 6.6 mm, w = 5.8 mm

$I_x = 12.1\times10^6$ mm^4, $I_y = 3.87\times10^6$ mm^4, A = 2,860 mm^2

$S_x = 160\times10^3$ mm^3, $S_y = 50.9\times10^3$ mm^3

$Z_x = 177\times10^3$ mm^3, $Z_y = 77.6\times10^3$ mm^3

Section classification:

Flange: $(b/2)/t = (152/2)/6.6 = 11.5 = 215/\sqrt{F_y} > 200/\sqrt{F_y}$

→ **Flange is Class 4**

Web: $h/w = (152 - 2\times6.6)/5.8 = 23.9 = 448/\sqrt{F_y} < 1100/\sqrt{F_y}$

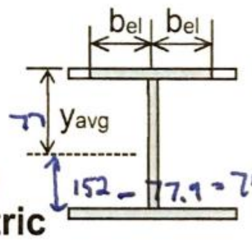
→ Web is Class 1 ∴ The section is Class 4 section

a) Effective section modulus method (laterally supported → *Cl.* 13.5)

Since the flange is Class 4 but the web is not, $\boldsymbol{M_r = \phi S_e F_y}$ Cl. 13.5(c)(iii)

$b_{el} = 200t/\sqrt{F_y} = 200(6.6)/\sqrt{350} = 70.6$ mm $= b_{eff}/2$

→ $\boldsymbol{b_{eff}}$ **=** 141.1mm



Note: part in tension not subjected to local buckling

Therefore, a total width of (152 – 141.1) = 10.9 mm was removed from the compression flange (5.5 mm from each side). → **Effective section is no longer doubly symmetric**

$\boldsymbol{A_e}$ **=** $(141.1)(6.6) + 5.8(152-2\cdot6.6) + 152\cdot6.6 = 2739.5\text{mm}^2$

Locate neutral axis, taking the top surface of top-flange as the reference line:

$y_{avg} = \sum A_i y_i / A_e = [(141.1)(6.6)(6.6/2) + (5.8)(152-2\times6.6)(6.6+138.8/2) + (152)(6.6)(152-6.6/2)] / 2739.5 = 77.9$ mm

Calculate the effective moment of inertia and elastic section modulus:

I_{xe} = 1/12*(141.1)(6.6)³ + (141.1×6.6)(77.9-3.3)² + 1/12*(5.8)(138.8)³ + (5.8×138.8)(77.9-76)² + 1/12*(152)(6.6)³ + (152×6.6)(148.7-77.9)²

$= 11.51 \times 10^6\ mm^4$

Top Flange
Web
Bottom Flange

$S_{xe} = I_{xe} / 77.9 = 147.7 \times 10^3\ mm^3$

→ Note: Must divide by the larger distance to get the smaller value of S_{xe}

→ Note: $S_{xe}/S_x = 0.92$ → 8% *reduction due to local buckling!*

$$M_r = \phi S_e F_y =$$

b) ~~Effective yield stress method:~~

~~Class 3 limit:~~ $\cancel{b_{el}/t \le 200/(F_y)^{1/2}}$

$\cancel{200/\sqrt{F_{ye}} = 11.5 \rightarrow F_{ye} = (200/11.5)^2 = 302.5\ MPa}$

$\cancel{M_r = \phi S_x F_{ye} = 0.9(160\times10^3)(302.5) \div 10^6 = 43.6\ kNm}$

If local buckling is mistakenly neglected,

$$M_r = \phi S_x F_y = 0.9(160\times10^3)(350) \div 10^6 = 50.4\ kNm$$

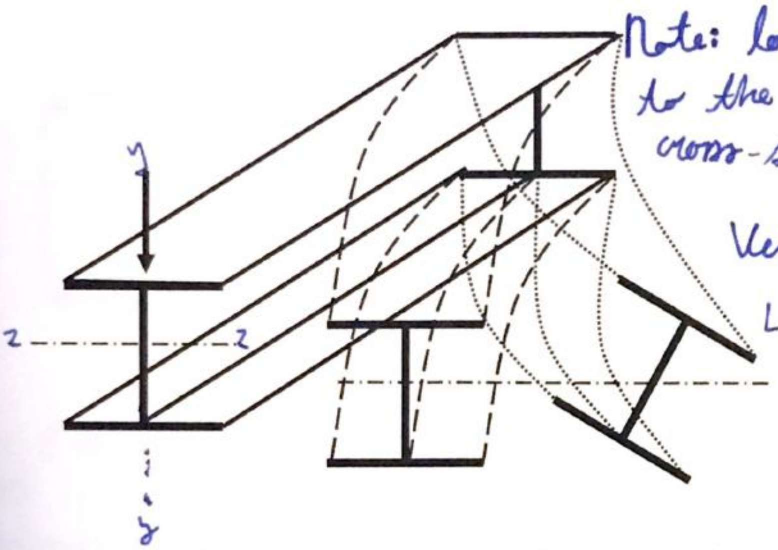
Class 4 Members: gross cross-sectional properties (A, I) → Structural analysis → Response (C_f, M_f)
Effective cross-sectional properties (A_e, I_e) → strength evaluation → member resistance (C_r, M_r)

- Single angles with continuous later-torsional restraint along the length, M_r shall be calculated based on *principal axial bending* (biaxial bending, if applicable). If M_f only applies about one geometric (x, or y) axis, M_r shall be calculated based on bending that axis as stipulated in Cl. 13.5 d).
- For biaxial bending, the member shall meet the following requirement (Cl. 13.5 e):

$$\frac{M_{fx}}{M_{rx}} + \frac{M_{fy}}{M_{ry}} \le 1.0$$

Bending---Laterally Unsupported Members 13.6

For beams whose compression flange is not sufficiently prevented from moving laterally or twisting as load increases, the beam will first deflect in plane, then twist and finally buckle laterally. From a conceptual point of view, the beam can be considered as having a constant tendency to fall over on its weak axis. This failure mode is called lateral-torsional buckling and the bending moment capacity corresponding to this limit state is M_u.

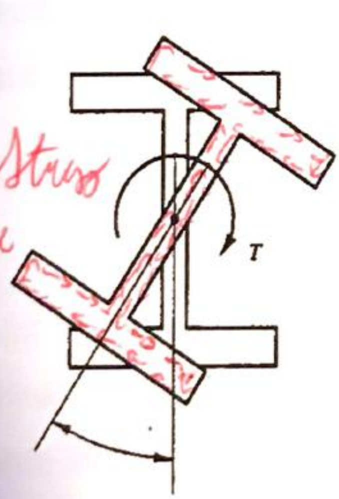


Note: load is applied perpendicular to the strong axis of member cross-section.

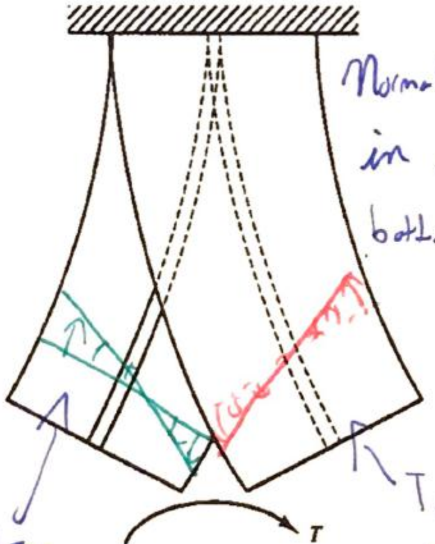
Vertical deflection: z-z axis

Lateral deflection: y-y twisting

Resistance of a bending member to lateral-torsional buckling between points of lateral or torsional support is a function of two torsional characteristics of specific cross sections, St. Venant torsion and warping torsion.



Shear Stress in the cross section



Normal stress distribution in top and bottom flanges

Bottom Flange

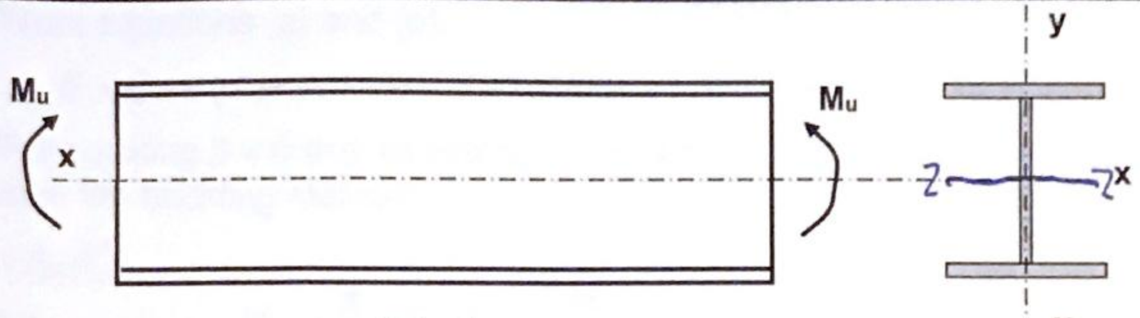
Top Flange

St. Venant Torsion

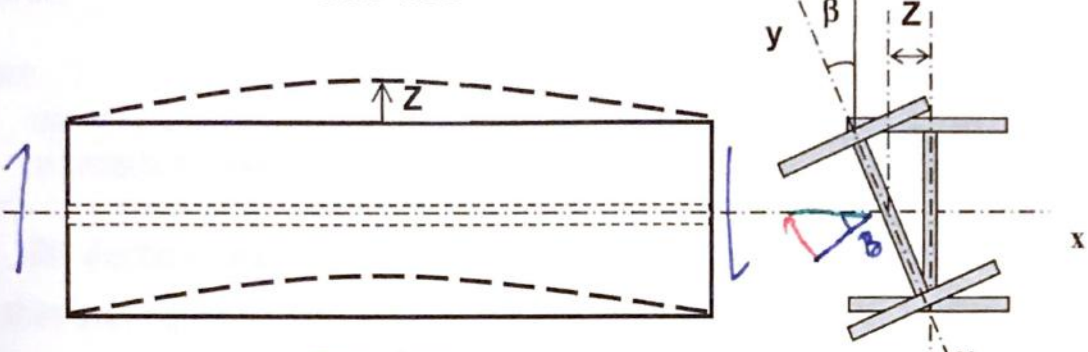
Warping Torsion

Lateral-Torsional buckling in weak axis not going to happen. ∴ Don't check if loaded in weak axis! } Torsion likely if loaded in weak axis

Z - strong axis
Y - weak axis
X - beam longitudinal axis



Side-view



Top-view

where, Z is the out-of-plan deflection and β is the twist rotation of the section

(strong axis x-x bending deformation is not shown and it has been discussed previously under Laterally Supported Beams).

Considering the equilibrium of the beam in deformed state, for bending about weak axis (y-y axis), M_u - ultimate elastic buckling moment

Ext. Moment: $M_{yy\text{-}ext} = M_u \times \sin\beta \approx M_u \times \beta$ - weak axis

Int. Moment: $M_{yy\text{-}int} = -E \times I_y \times (d^2Z/dx^2)$ - weak axis

$$\mathbf{M_{yy\text{-}ext}} = M_{yy\text{-}int}$$

Z: lateral displacement

then,

$$E \times I_y \times (d^2Z / dx^2) = M_u \times \beta \qquad \text{(a)}$$

Similarly, for Torsion,

Ext. Torsional Moment: $M_{T\text{-}ext} = -M_u \times \sin(dZ/dx) \approx -M_u \times (dZ/dx)$

Int. Torsional Moment: $M_{T\text{-}int} = G \times J \times (d\beta/dx) - E \times C_w \times (d^3\beta/dx^3)$

$$M_{T\text{-}ext} = M_{T\text{-}int}$$

then,

$$G \times J \times (d\beta/dx) - E \times C_w \times (d^3\beta/dx^3) + M_u \times (dZ/dx) = 0 \qquad \text{(b)}$$

$\frac{dT_y(b)}{dx} = GJ\frac{d^3\beta}{dx^3} - EC_w\frac{d^4\beta}{dx^4} + M_u\frac{d^2Z}{dx^2} = 0$ Benjamin K

 $\because\ d^2Z/dx^2 = -M_u\beta/EI_y$

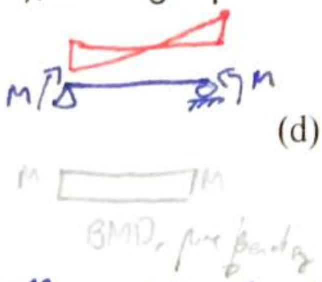
From equations (a) and (b),

$$E \times C_w \times (d^4\beta/dx^4) - G \times J \times (d^2\beta/dx^2) - M_u^2 \times \beta / (E \times I_y) = 0 \qquad \text{(c)}$$

For buckling $\beta \neq 0$ and assuming $\beta = \beta_o \sin(\pi x / L)$, solving equation (c), then the buckling moment,

Elastic Behaviour

$$M_u = \frac{\pi}{L}\sqrt{EI_yGJ + \left(\frac{\pi E}{L}\right)^2 I_yC_w} \qquad \text{(d)}$$ 13.6.1



where,

L --- laterally unsupported length of the beam ← May not be length of the beam

I_y --- moment of inertia respect to weak axis of the cross-section of the beam

J --- St. Venant torsional Constant.

For thin-walled open sections,

$$J = \sum_{i=1}^{n}\left(\frac{b_it_i^3}{3}\right)$$

C_w – warping torsional constant. C_w = 0.0 for hollow structural sections; and

for I (W or WWF) sections, $C_w = I_y\left(\frac{d-t}{2}\right)^2$

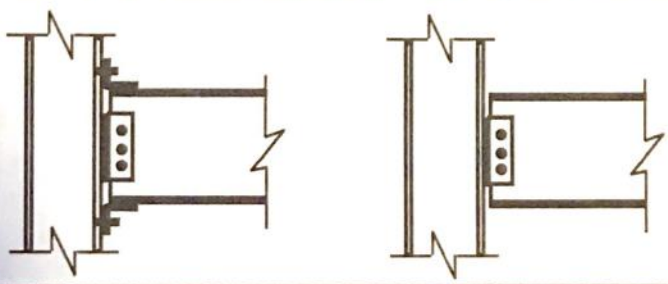
The lateral-torsional buckling is influenced by

- the material and cross-section properties,
- the laterally unsupported length of the beam,
- the end conditions (rotational restraints),
- the applied loads and bending moment distribution along the beam.

End Conditions:

Fixed-end: both twisting and warping are restrainted

Pinned-end: twisting is restrained, but warping is permitted.



BENJAMIN KLASSEN

Benjamin K

Bending - Laterally Unsupported Members

13.6

Where continuous lateral support is not provided to the compression flange of a member subjected to uniaxial strong axis bending, the factored moment resistance, M_r, of **a segment between effective brace points** shall be determined as follows:

(a) For doubly symmetric Class 1 and 2 sections, except closed square and circular sections — no weak axis

(i) when $M_u > 0.67M_p$ Inelastic resistance

residual stress @ 2-3 ends

$$M_r = 1.15\phi M_p\left(1-\frac{0.28M_p}{M_u}\right) \le \phi M_p$$

← empirical formulation

(ii) when $M_u \le 0.67M_p$

$$M_r = \phi M_u$$

where the critical elastic moment of the **unbraced segment** is given by

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_yGJ + \left(\frac{\pi E}{L}\right)^2 I_yC_w}$$

← EI_y = lateral bending; EC_w = warping stiffness

lateral bending; torsion stiffness

in which, L = length of unbraced segment of beam; ω_2 = influence of moment gradient factor

$$\omega_2 = \frac{4M_{max}}{\sqrt{M_{max}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2}} \le 2.5$$

S16-09

M_{max} = max. factored bending moment magnitude in unbraced segment;

M_a =factored bending moment at one-quarter point of unbraced segment;

M_b =factored bending moment at midpoint of unbraced segment; and

M_c=factored bending moment at three-quarter point of unbraced segment

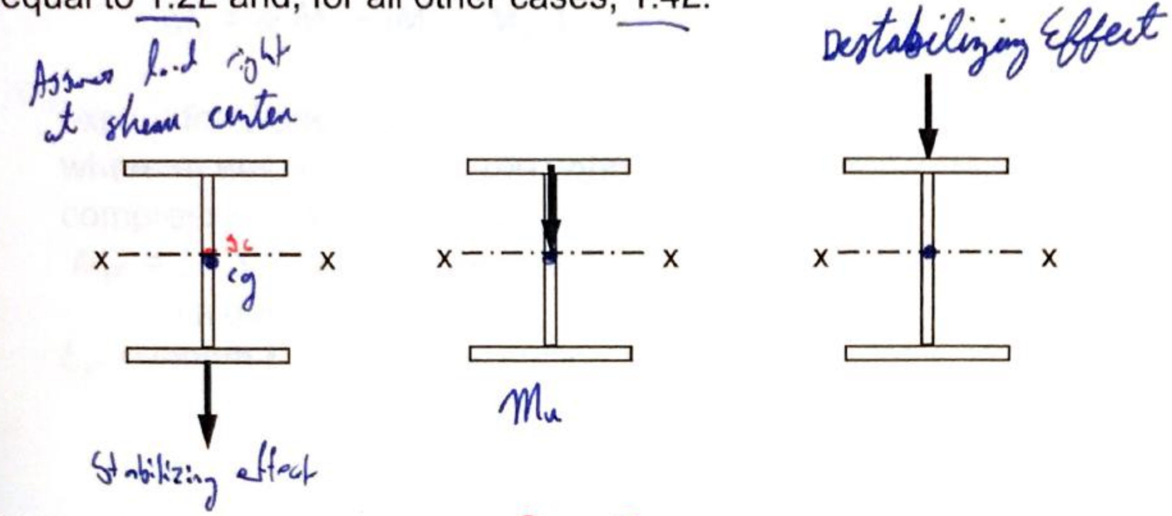
If the bending moment distribution within in the unbraced segment is effectively linear, ω_2, may be taken as — not parabolic

ω_2 = $1.75 + 1.05\kappa + 0.3\kappa^2 \le 2.5$

κ = the ratio of the smaller factored moment to the larger factored moment at opposite ends of the unbraced length, positive for double curvature and negative for single curvature.

$\kappa = \pm\left|\frac{M_1}{M_2}\right|$

For unbraced beam segments **loaded above the shear centre** between brace points, where the method of **load delivery to the member** provides **neither lateral nor rotational restraint to the member**, the associated **destabilizing effect** shall be taken into account **using a rational method**. For loads applied at the level of the top flange, in lieu of a more accurate analysis, M_u may be determined using $\omega_2 = 1.0$ and using an effective length, for pinned-ended beams, equal to $1.2L$ and, for all other cases, $1.4L$.



(b) For doubly symmetric Class 3 and 4 sections, except closed square and circular sections, and for **channels**:

Class 3 : $M_y = S_x F_y$
Class 4 : $M_y = S_{xe} F_y$

(i) when $M_u > 0.67\ M_y$ Inelastic resistance

$$M_r = 1.15\phi M_y\left(1-\frac{0.28M_y}{M_u}\right) \le \phi M_y$$

but not greater than ϕM_y for Class 3 sections and the value given in Clause 13.5 (c)(iii) for Class 4 sections; and

(ii) when $M_u \le 0.67 M_y$ Elastic resistance

For dbl sym Class 3/4 sections M_u is evaluated based on the gross x-sectional properties, I_y, J, C_w

$$M_r = \phi M_u$$

where M_u and ω_2 are as defined in Clause 13.6 (a)(ii)

(c) For closed square and circular sections, M_r shall be determined in accordance with Clause 13.5. Treat as a laterally-braced beam

(d) For cantilever beams, a rational method of analysis taking into account the lateral and torsional restraint conditions at the supports and tips of the cantilever, as well as the load conditions and the flexibility of the backspan, should be used. For fully fixed end I-shapes, M_r may be determined by Cl. 13.6.1 h).

(e) For singly symmetric (monosymmetric) Class 1, 2, or 3 I-sections and T-sections, lateral-torsional buckling strength shall be checked separately for each flange that experiences compression under factored loads at any point along its unbraced length, as follows (except that these sections shall not yield under service loads):

(i) when $M_u > M_{yr}$ inelastic resistance

$$M_r = \phi\left[M_p - \left(M_p - M_{yr}\right)\left(\frac{L - L_u}{L_{yr} - L_u}\right)\right] \le \phi M_p$$

except for Class 3 sections, as well as Class 1 and 2 T-sections where at any point within the unbraced segment the stem tip is in compression, where M_p is replaced with M_y

M_{yr} = $0.7 S_x F_y$ with $\boldsymbol{S_x}$ taken as the **smaller** of $\boldsymbol{M_{yr}}$ the two potential values

L_{yr} = length L obtained by setting $M_u = M_{yr}$

$$L_u = 1.1 r_t \sqrt{\frac{E}{F_y}} == \frac{490 r_t}{\sqrt{F_y}}; \quad \text{and} \quad r_t = \frac{b_c}{\sqrt{12(1 + d_c w / 3 b_c t_c)}}$$

d_c = depth of the web in compression;

b_c = width of compression flange; and

t_c = thickness of compression flange.

(ii) when $M_u \le M_{yr}$

$$M_r = \phi M_u$$

where the critical elastic moment of the unbraced segment is given by

$$M_u = \frac{\omega_3 \pi^2 E I_y}{2L^2}\left[\beta_x + \sqrt{\beta_x^2 + 4\left(\frac{GJL^2}{\pi^2 E I_y} + \frac{C_w}{I_y}\right)}\right]$$

and where in lieu of more accurate values of the section properties β_x and C_w may be evaluated as

$$\beta_x = 0.9(d - t)\left(\frac{2 I_{yc}}{I_y} - 1\right)\left(1 - \left(\frac{I_y}{I_x}\right)^2\right)$$

$$C_w = \frac{I_{yc} I_{yt}}{I_y}(d - t)^2$$

where

β_x = asymmetry parameter for singly symmetric beams

I_{yc} = moment of inertia of the compression flange about the y-axis

I_{yt} = moment of inertia of the tension flange about the y-axis and when singly symmetric beams are in single curvature

$\omega_3 = \omega_2$ for beams with two flanges; $\omega_3 = 1.0$ for T-sections; in all other cases

$\omega_3 = \omega_2\,(0.5 + 2\,(I_{yc}/I_y)^2)$ but ≤ 1.0 for T-sections

For unbraced beam segments **loaded above the section mid-height and between brace points**, where the method of **load delivery to the member provides neither lateral nor rotational restraint** to the member, the associated **destabilizing effect** shall be taken into account using a rational method.

For other singly symmetric shapes, a rational method of analysis shall be used.

(f) For biaxial bending, the member shall meet the following criterion:

$$\frac{M_{fx}}{M_{rx}} + \frac{M_{fy}}{M_{ry}} \leq 1.0$$

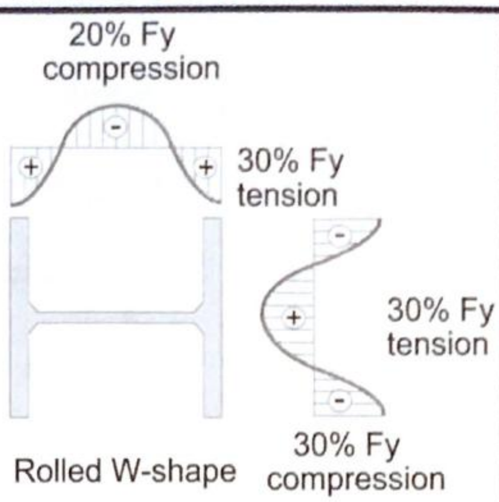
(g) Single angles without continuous lateral-torsional restraint along the length, M_r shall be evaluated based on principal axis bending as stipulated in Cl. 13.6.1 g):

(h) For cantilevers made of doubly symmetric I-shape sections are fully restraint against twisting and warping at one end and subjected to transverse loads only, M_r may be evaluated based on Cl. 13.6.1 h).

no worries

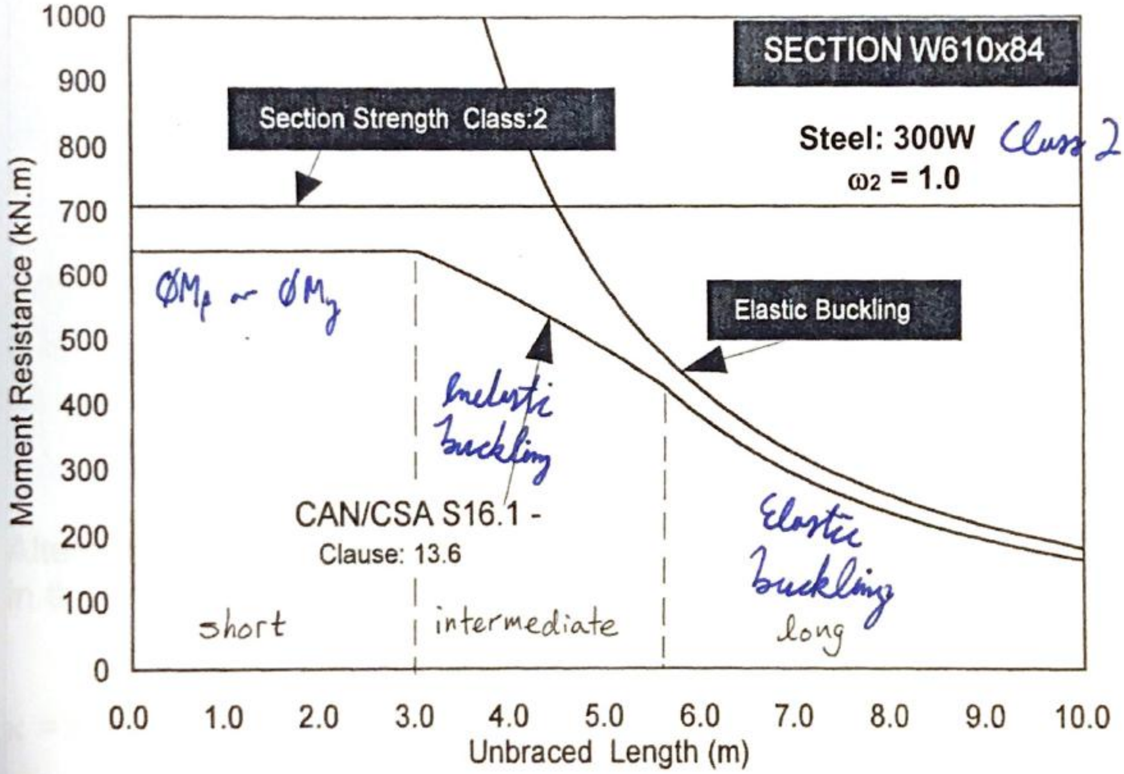
The Effect of Residual Stress:

Because of relatively large compressive residual stress (up to 30% of F_y) at flange tips, yielding will occur when the applied moment reaches approximately two-thirds of the capacity, M_p or M_y. Therefore, Equation (d) is valid until M_u reaches two-thirds (0.67) M_p for doubly symmetric Class 1 and 2 sections or two-thirds (0.67) M_y for doubly symmetric Class 3 and 4 sections.



Instability of Laterally Unbraced Beams

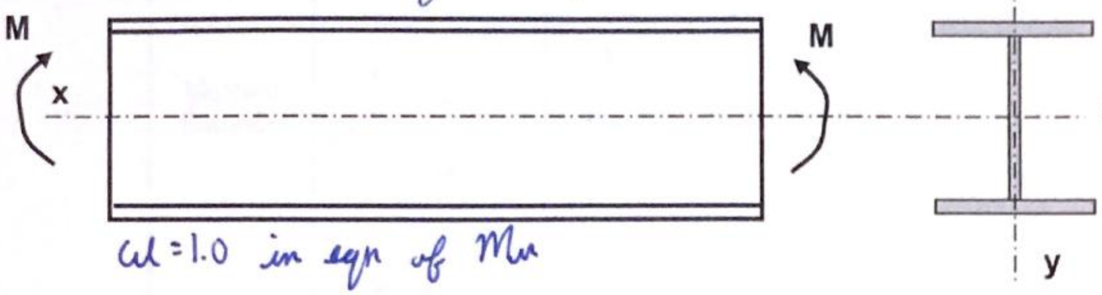
$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$$



Bending Moment Coefficient ω_2:

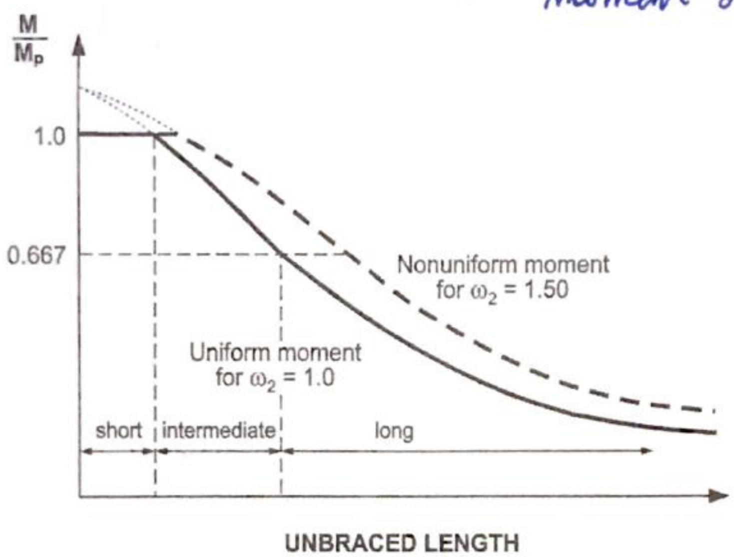
The coefficient ω_2 is introduced to account for that a varying moment is less critical than a uniform moment.

Pure bending [Uniform Moment]



$\omega = 1.0$ in eqn of M_u

$\omega_2 > 1.0$ for non-uniform moment distribution

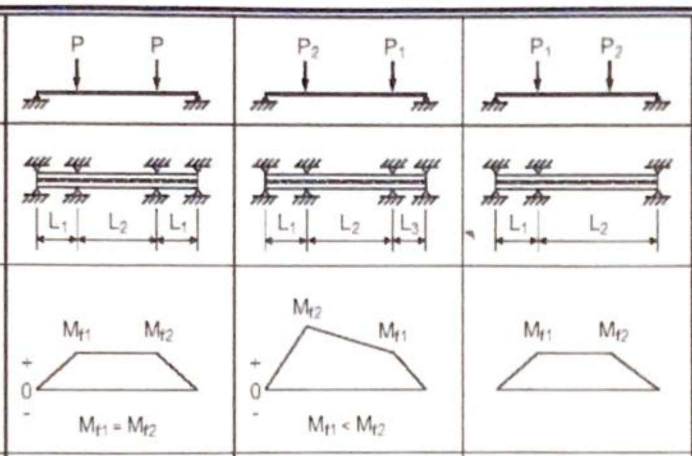


$$\omega_2 = \frac{4M_{max}}{\sqrt{M_{max}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2}} \le 2.5$$ Approximation

Alternative method: Applicable if the bending moment distribution within in the unbraced segment is effectively linear,

$$\omega_2 = 1.75 + 1.05\kappa + 0.3\kappa^2 \le 2.5$$ Approximation

$\kappa = \pm M_{f\text{-small}} / M_{f\text{-large}}$ ("+" for double curvature and "-" for single curvature)



	Case 1	Case 2	Case 3
Loading			
Lateral Restraints (Plan view)			
Moment Diagram	$M_{f1} = M_{f2}$	$M_{f1} < M_{f2}$	
ω_2	1.75 for L_1 1.0 for L_2	1.75 for L_1 $\kappa = \frac{-M_{f1}}{M_{f2}}$ for L_2 1.75 for L_3	1.75 for L_1 1.0 for L_2

$$\omega_2 = \frac{4M_{\max}}{\sqrt{M_{\max}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2}} \leq 2.5$$

Segment L_1 of case 1:

$M_{max} = M_{f1}$; $M_a = 0.25M_{f1}$; $M_b = 0.5M_{f1}$; **M_c =** $0.75 M_{f1}$

$$\omega_2 = \frac{4M_{f1}}{\sqrt{M_{f1}^2 + 4\times0.0625M_{f1}^2 + 7\times0.25M_{f1}^2 + 4\times0.5625M_{f1}^2}} = 1.746$$

Segment L_2 of case 3 (Assume: L_1= 1/3 of L_2; P_1=P_2): No lateral bracing of location of P_2

Assume P_2 is located at the ¾ span of L_2 from the interior support.

a) $M_{max} = M_{f2}$; $M_a = M_{f2}$; $M_b = M_{f2}$; $M_c = M_{f2}$; → **ω_2 =** 1.0

b) Assume P_2 and is located at the mid-span of L_2.

$M_{max} = M_{f2}$; $M_a = M_{f2}$; $M_b = M_{f2}$; $M_c = 0.5M_{f2}$; → ω_2 = 1.11

c) Assume P_2 and is located at the 1/4 span of L_2 from the interior support.

M_{max} = M_{f2}; $M_a = M_{f2}$; $M_b = \frac{2}{3}M_{f2}$; $M_c = \frac{1}{3}M_{f2}$; $\omega_2 = 1.37$

Alternative Method

L_1 of Case 1: $M_{small} = 0$, $M_{large} = M_{f1} \rightarrow \kappa = 0$, $\omega_2 = 1.75$

L_2 of Case 3: $M_{small} = 0$, $M_{large} = M_{f2} \rightarrow \kappa = 0$, $\omega_2 = 1.75$

For L_2 of case 3, use $\omega_2 = 1.0$

∵ A portion of unbraced segment subject to uniform bending

BEAM SELECTION TABLE
W Shapes

ASTM A992, A572 Grade 50

F_y = 345 MPa

, $\omega_2 = 1.0$

Critical unbraced length

Moment resistance of the braced beam

Designation	V_r	I_x	b	L_u	M_r	Factored moment resistance M_r' (kN·m)					
						Unbraced length (mm)					
	kN	10^6 mm^4	mm	mm	$\leq L_u$	2 000	2 500	3 000	3 500	4 000	5 000
W610x82	1 170	560	178	2 110	**683**	—	644	587	522	448	304
W460x97	1 090	445	193	2 650	677	—	—	652	614	574	488
W310x129	854	308	308	5 080	671	—	—	—	—	—	—
W410x100	850	398	260	3 730	661	—	—	—	—	648	596
W530x85	1 130	485	166	2 110	652	—	616	564	507	446	316
W530x82	1 030	477	209	2 660	640	—	—	616	576	531	433
W360x110	841	331	256	3 940	640	—	—	—	—	637	596
W460x89	996	409	192	2 620	624	—	—	598	562	523	439
W310x118	766	275	307	4 920	605	—	—	—	—	—	603
W360x101	768	301	255	3 860	584	—	—	—	—	578	538
W460x82	933	370	191	2 560	568	—	—	540	505	466	384
W530x74	1 050	411	166	2 040	**562**	—	523	474	420	357	247
W310x107	695	248	306	4 800	546	—	—	—	—	—	541
W410x85	931	315	181	2 530	534	—	—	507	475	443	375
W360x91	687	267	254	3 760	522	—	—	—	—	513	475
W460x74	843	332	190	2 530	512	—	—	484	450	414	332
W530x66	927	351	165	1 980	**484**	483	444	398	347	284	195
†W530x72	947	401	207	2 750	475	—	—	462	434	402	331
W410x74	821	275	180	2 470	469	—	467	440	410	379	312
W460x68	856	297	154	2 010	463	—	429	390	348	301	213
†W310x97	625	222	305	4 970	447	—	—	—	—	—	446
W360x79	682	226	205	3 010	444	—	—	—	425	404	361
W310x86	578	198	254	3 900	441	—	—	—	—	438	409
W250x101	644	164	257	4 470	435	—	—	—	—	—	424
W410x67	739	245	179	2 420	422	—	418	392	364	333	264
W460x60	746	255	153	1 970	**397**	396	364	329	289	242	169
W360x72	617	201	204	2 940	397	—	—	395	377	357	315
W310x79	552	177	254	3 810	397	—	—	—	—	392	364
W250x89	570	143	256	4 260	382	—	—	—	—	—	367
W410x60	642	216	178	2 390	369	—	365	341	314	286	218
W310x74	597	164	205	3 100	366	—	—	—	354	339	307
W200x100	680	113	210	4 460	357	—	—	—	—	—	349
W360x64	548	178	203	2 870	354	—	—	350	332	313	273
W460x52	680	212	152	1 890	**338**	333	303	269	231	185	128
W250x80	493	126	255	4 130	338	—	—	—	—	—	321
W410x54	619	186	177	2 310	326	—	318	295	269	241	176
W310x67	533	144	204	3 020	326	—	—	—	312	297	266
W360x57	580	160	172	2 360	314	—	309	289	267	244	192
W250x73	446	113	254	4 010	306	—	—	—	—	—	287
W200x86	591	94.7	209	4 110	305	—	—	—	—	—	292
W310x60	466	128	203	2 960	290	—	—	289	275	261	231
W250x67	469	104	204	3 260	280	—	—	—	275	265	244
W360x51	524	141	171	2 320	**277**	—	271	252	232	210	159
W410x46	578	156	140	1 790	**274**	265	239	210	177	142	99.9
W310x52	494	118	167	2 380	260	—	256	240	223	206	167
W200x71	452	76.6	206	3 730	249	—	—	—	—	246	232

Sections highlighted in yellow are commonly used sizes and are generally readily available.

† Class 3

ASTM A992, A572 Grade 50
F_y = 345 MPa

BEAM SELECTION TABLE
W Shapes

Nominal mass	Factored moment resistance M_r' (kN·m)									Imperial designation
	Unbraced length (mm)									
kg/m	6 000	7 000	8 000	9 000	10 000	11 000	12 000	14 000	16 000	
82	225	177	145	123	106	93.4	83.4	68.9	58.7	**W24x55**
97	389	314	264	227	200	178	161	135	117	W18x65
129	643	612	581	551	520	490	460	390	336	W12x87
100	539	479	411	348	302	266	238	197	168	W16x67
85	240	193	162	139	122	108	97.8	81.9	70.6	W21x57
82	320	249	203	170	147	129	115	94.0	79.8	W21x55
110	553	510	467	423	372	332	300	252	217	W14x74
89	343	276	231	198	174	155	140	117	101	W18x60
118	574	543	513	482	452	422	388	324	279	W12x79
101	497	454	411	363	318	283	255	214	184	W14x68
82	292	234	195	166	146	129	116	97.3	83.6	W18x55
74	186	148	123	105	91.7	81.4	73.2	61.0	52.4	**W21x50**
107	512	483	453	423	392	362	325	271	233	W12x72
85	297	242	205	177	157	140	127	107	92.8	W16x57
91	434	392	350	299	261	232	209	174	150	W14x61
74	249	198	164	140	122	108	96.8	80.6	69.1	W18x50
66	145	115	94.9	80.6	70.0	61.9	55.5	46.0	39.4	**W21x44**
72	246	191	154	129	110	96.3	85.4	69.6	58.8	W21x48
74	239	194	163	140	123	110	99.7	83.8	72.3	W16x50
68	164	133	112	96.7	85.2	76.1	68.9	57.9	50.1	W18x46
97	423	400	375	351	326	302	272	226	193	W12x65
79	317	267	225	194	171	153	139	117	101	W14x53
86	379	347	316	282	248	221	199	167	144	W12x58
101	403	382	362	342	322	302	279	236	205	W10x68
67	201	161	135	116	102	90.4	81.6	68.3	58.9	W16x45
60	129	104	86.6	74.3	65.2	58.1	52.4	43.9	37.8	**W18x40**
72	272	222	186	160	141	126	114	95.6	82.5	W14x48
79	334	304	273	237	207	184	166	139	119	W12x53
89	346	326	306	285	265	242	220	186	161	W10x60
60	165	131	109	93.1	81.3	72.1	64.9	54.1	46.4	W16x40
74	274	240	204	177	156	140	127	107	93.0	W12x50
100	335	321	307	294	280	266	253	222	194	W8x67
64	228	183	153	131	115	102	92.2	77.2	66.5	W14x43
52	96.2	76.7	63.6	54.3	47.4	42.0	37.8	31.5	27.0	**W18x35**
80	302	282	262	242	221	197	179	151	130	W10x54
54	132	104	86.0	73.1	63.5	56.2	50.4	41.8	35.8	W16x36
67	234	198	167	144	127	114	103	86.8	75.1	W12x45
57	147	119	99.7	85.9	75.6	67.5	61.0	51.2	44.2	W14x38
73	268	248	228	209	185	165	149	126	108	W10x49
86	279	265	252	238	225	212	197	168	147	W8x58
60	199	163	137	118	104	92.5	83.6	70.2	60.5	W12x40
67	223	202	180	157	139	125	114	96.5	83.8	W10x45
51	121	96.9	80.9	69.4	60.8	54.1	48.8	40.8	35.2	**W14x34**
46	76.4	61.7	51.8	44.6	39.2	35.0	31.6	26.5	22.9	**W16x31**
52	130	106	89.4	77.4	68.4	61.3	55.5	46.8	40.5	W12x35
71	219	205	192	179	166	150	137	116	101	W8x48

Note: For unbraced beam segments loaded above the shear centre, see CSA S16-14 Clause 13.6.

Example 4-5: For a simply supported beam with span length 7500 mm subjected to a uniformly distributed factored *w*, calculate the moment resistance assuming the beam is laterally unsupported.

Solution: W410×60, F_y =345 MPa; Class 1 section

$$M_p = 410.6kN\cdot m,\ E = 2\times10^5MPa,\ G = 0.77\times10^5MPa$$

$$I_y = 12\times10^6mm^4,\ J = 327\times10^3mm^4,\ C_w = 468\times10^9mm^6$$

$$EI_yGJ = 2\times10^5\times12\times10^6\times0.77\times10^5\times327\times10^3 = 604.3\times10^{20}$$

$$\left(\frac{\pi E}{L}\right)^2 I_yC_w = \left(\frac{\pi\times2\times10^5}{7500}\right)^2\times12\times10^6\times468\times10^9 = 394.2\times10^{20}$$

$$\therefore\sqrt{EI_yGJ+(\pi E/L)^2I_yC_w} = \sqrt{(604.3+394.2)\times10^{20}} = 31.6\times10^{10}$$

ω_2 based on UDL: $M_{max} = \frac{1}{8}wl^2; M_a = M_c = \frac{3}{32}wl^2;\ M_b = \frac{1}{8}wl^2$

$$\therefore\omega_2 = 4M_{max}/\sqrt{M_{max}^2+4M_a^2+7M_b^2+4M_c^2} = 1.13$$

$$\therefore M_u = \frac{\omega_2\pi}{L}\sqrt{EI_yGJ+\left(\frac{\pi E}{L}\right)^2I_yC_w} = \frac{1.13\pi}{7500}\times31.6\times10^{10}$$

$$= 149.6\times10^6N\cdot mm = 149.6kN\cdot m$$ Elastic L-T buckling

$$0.67M_p = 275.1kN\cdot m > M_u = 149.6kN\cdot m$$

$$\therefore M_r = \phi M_u = 0.9\times149.6 = 134.6kN\cdot m$$

HSC Beam Selection Table: W410×60, $\boldsymbol{\omega_2 = 1.0}$

$L = 7000mm;\ M_r = 131kN\cdot m$

$L = 8000mm;\ M_r = 109kN\cdot m$

For $L = 7500mm;\ M_r = 109+(131-109)/2 = 120kN$ $(\omega_2 = 1.0)$

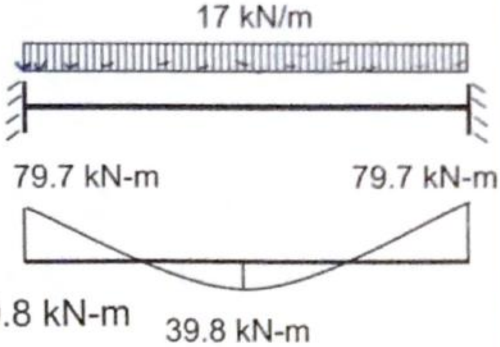
$\because$ $M_u < 0.67M_p$; $\therefore$ Elastic lateral-torsional buckling controls.

$\omega_2 = 1.13$ for UDL: $\therefore M_r = 1.13\times120 = 135.6kN\cdot m$

$$\text{Let } M_r = M_f = \frac{1}{8}\omega l^2,\qquad w_f = \frac{8M_r}{l^2} = \frac{8\times134.6}{7.5^2} = 19.1kN/m$$

If the beam is laterally braced M_r = 369 *kN*-m and $\boldsymbol{w_f}$ **=** 52.6 kN/m

Example 4-6: Redo Example 4-5 by assuming the both ends of the beam are fixed and the beam is subjected to 17 kN/m uniformly distributed factored load.



Solution:

W410×60; F_y =345 MPa; Class 1 section

M_p = 410.6kN-m; L = 7500 mm

End moment: $-wL^2/12$ = -79.7 kN-m

Mid moment: $-wL^2/12+wL^2/8= wL^2/24$ = 39.8 kN-m

$\therefore\ M_{max}$ = 79.7 kN-m; M_b = 39.8 kN-m;

$\boldsymbol{M_a = M_c =}$ $-\frac{wL^2}{12} + \frac{3wL^2}{32} = \frac{wL^2}{96} = 9.96$ kN-m

$$\omega_2 = 4M_{max}/\sqrt{{M_{max}}^2 + 4{M_a}^2 + 7{M_b}^2 + 4{M_c}^2}$$

$$= 4 \times 79.7/\sqrt{79.7^2 + 4 \times 9.96^2 + 7 \times 39.8^2 + 4 \times 9.96^2} = 2.36 \leq 2.5$$

From Example 4-5: $\sqrt{EI_yGJ + (\pi E/L)^2 I_y C_w} = 31.6 \times 10^{10}$

$$\therefore M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_yGJ + (\pi E/L)^2 I_y C_w} = 2.36 \times \pi \times 31.6 \times 10^{10}/7500$$

$$= 312.4 \times 10^6 N \cdot mm = 312.4 kN \cdot m > 0.67M_p = 275.1 kN \cdot m$$

$$\therefore M_r = 1.15\phi M_p(1 - 0.28M_p/M_u)$$ ∴ Inelastic L-T buckling

$$= 1.15 \times 0.9 \times 410.6 \times (1 - 0.28 \times 410.6/312.4)$$

$$= \mathbf{268.6\ kN \cdot m} < \phi M_p = 410.9 \times 0.9 = 369.5kN$$

Let $M_f = M_r = wl^2/12$; fixed ends

$$\therefore w = 12M_r/L^2 = 12 \times 268.6/7.5^2 = \mathbf{57.3\ kN \cdot m}$$

Comparing with Example 4-5: simply supported beam with UDL

If $\omega_2 = 1.0(1.13)$, then, $M_r = 120(134.6) kN \cdot m$; $w = 17.1(19.1) kN/m$

The increase of flexural strength of beam with fixed-ends are primarily due to the effect of end restraints. Moment resistance increases almost 2 times and factored load increases more than 3 times.

Beam Selection Table: L=7500mm; $\omega_2 = 1.0, M_r = 120 kN \cdot m$; for $\omega_2 = 2.36$

$\omega_2 M_r$=2.36×120=283.2 kN-m >268.6 kN-m, ∴ **Can't** use beam selection table

Because inelastic buckling.

Example 4-7: Redo Example 4-4 with assuming the beam is laterally unsupported and $\omega_2 = 1.0$.

Flange: class 4
Web: class 1

Solution: From Example 4-4 W150×22, F_y = 350 MPa; Class 4 section

Gross section properties: $\rightarrow M_u$

$I_y = 3.87 \times 10^6 mm^4,\ J = 41.5 \times 10^3 mm^4,\ C_w = 20.4 \times 10^9 mm^6$

$EI_yGJ = 2 \times 10^5 \times 3.87 \times 10^6 \times 0.77 \times 10^5 \times 41.5 \times 10^3 = 24.7 \times 10^{20}$

$$\left(\frac{\pi E}{L}\right)^2 I_y C_w = \left(\frac{\pi \times 2 \times 10^5}{5000}\right)^2 \times 3.87 \times 10^6 \times 20.4 \times 10^9 = 12.5 \times 10^{20}$$

$$\therefore \sqrt{EI_yGJ + (\pi E/L)^2 I_y C_w} = \sqrt{(24.7 + 12.5) \times 10^{20}} = 6.1 \times 10^{10}$$

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_yGJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} = \frac{1.0\pi}{5000} \times 6.1 \times 10^{10}$$

$$= 38.3 \times 10^6 N \cdot mm = 38.3 kN \cdot m$$

From Example 4-4, Class 4 section, $\boldsymbol{M_y}$= $S_eF_y = 52.9$ kNm ; $S_e = 151.2E3\ mm^3$

$M_u > 0.67 \times M_y = 0.67 \times 52.9 = 35.4\ kN \cdot m$

$M_r = 1.15\phi M_y(1 - 0.28 M_y/M_u)$ ← Inelastic LT buckling

$= 1.15 \times 0.9 \times 52.9 \times (1 - 0.28 \times 52.9/38.3)$

$= 33.9 kN \cdot m < \phi M_y = 0.9 \times 52.9 = 47.6 kN \cdot m$

If the beam length is 7000mm, then

$EI_yGJ = 2 \times 10^5 \times 3.87 \times 10^6 \times 0.77 \times 10^5 \times 41.5 \times 10^3 = 24.7 \times 10^{20}$

$$\left(\frac{\pi E}{L}\right)^2 I_y C_w = \left(\frac{\pi \times 2 \times 10^5}{7000}\right)^2 \times 3.87 \times 10^6 \times 20.4 \times 10^9 = 6.4 \times 10^{20}$$

$$\therefore \sqrt{EI_yGJ + (\pi E/L)^2 I_y C_w} = \sqrt{(24.7 + 6.4) \times 10^{20}} = 5.6 \times 10^{10}$$

$$M_u = \frac{\omega_2 \pi}{L}\sqrt{EI_yGJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} = \frac{1.0\pi}{7000} \times 5.6 \times 10^{10}$$

$$= 25 \times 10^6 N \cdot mm = 25 kN \cdot m$$

$M_u < 0.67 \times M_y = 35.4 kN \cdot m$; $M_r = \phi M_u = 0.9 \times 25 = 22.5 kN \cdot m$

Shear Resistance of Steel Beams

Shear failure may occur, primarily, at the cross-section experiencing the largest shear. However, unless the beam is very short, it should be chosen from bending moment and checking for shear.

The shear failure modes are **Yield of the web** and **Shear buckling of the web.**

The shear stress as computed by simple bending theory at location of the beam web is given by

$$f_v = \frac{VQ}{It}$$

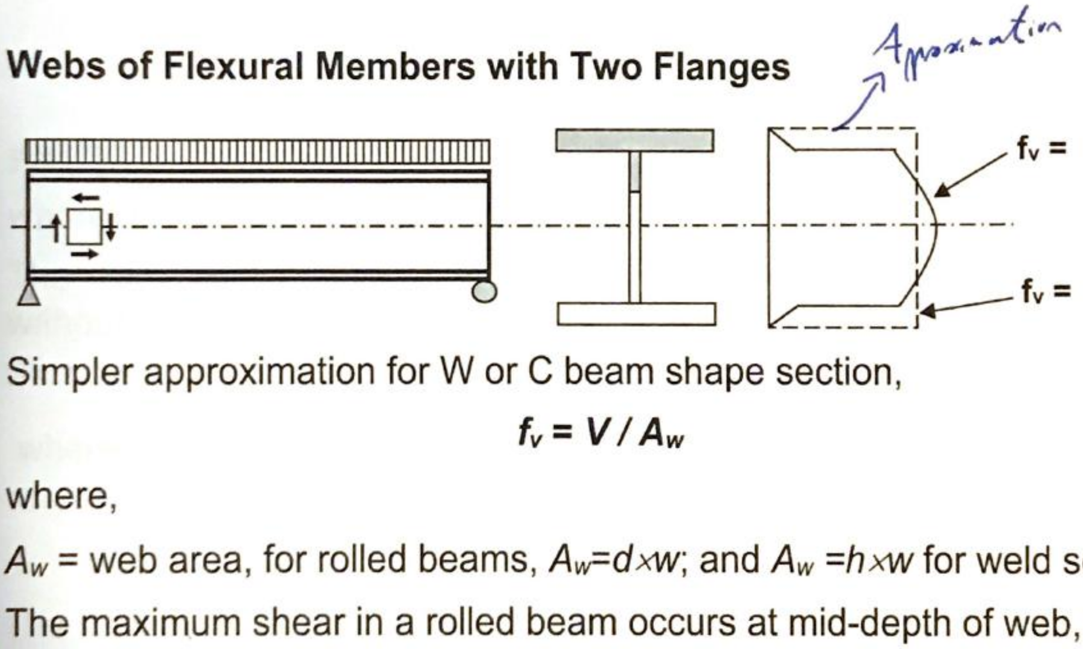
where,

V = total resultant shear force on cross-section

t = web thickness where the shear stress is computed

Q = static moment, taken about the neutral axis, of that portion of the beam area beyond the point at which the shear stress is to be calculated.

Webs of Flexural Members with Two Flanges 13.4.1



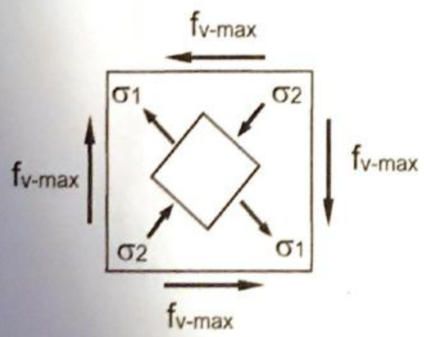
Simpler approximation for W or C beam shape section,

$$f_v = V / A_w$$

where,

A_w = web area, for rolled beams, $A_w = d \times w$; and $A_w = h \times w$ for weld section.

The maximum shear in a rolled beam occurs at mid-depth of web, in general



$f_{v\text{-}max}$ = $[\sigma_1 - \sigma_2]/2$

since the normal stress (longitudinal stress) is zero at this location (neutral axis)

$$\therefore f_{v\text{-}max} = \sigma_1 = -\sigma_2$$

Benjamin Klassen

by the distortion energy theory (von Mises value), the plastic flow will start when

$$\sigma_1\sigma_2 + \sigma_2^2 = F_y^2$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = F_y^2$$

$$3\sigma_1^2 = F_y^2$$

$$f_{v-max} = \sigma_1 = \frac{F_y}{\sqrt{3}} = 0.58F_y$$

(shear yield stress)

In S16, the shear yield stress has been increased as

$$F_s = 0.66\,F_y$$

by accounting for the beneficial effects of strain hardening.

The shear resistance of a flexural member depends on;
---- Web dimensions, yield strength
---- Stiffener condition / spacing (a/h ratio)

If the web is relatively slender, the beam will buckle under the shear forces before it is completely yielding. The S16 imposed the restriction below to ensure that shear buckling will not occur before the web is completely

$$\frac{h}{w} \le 439\sqrt{\frac{k_v}{F_y}} \approx \frac{1014}{\sqrt{F_y}}, \quad k_v = 5.34$$

pin-pin support to web

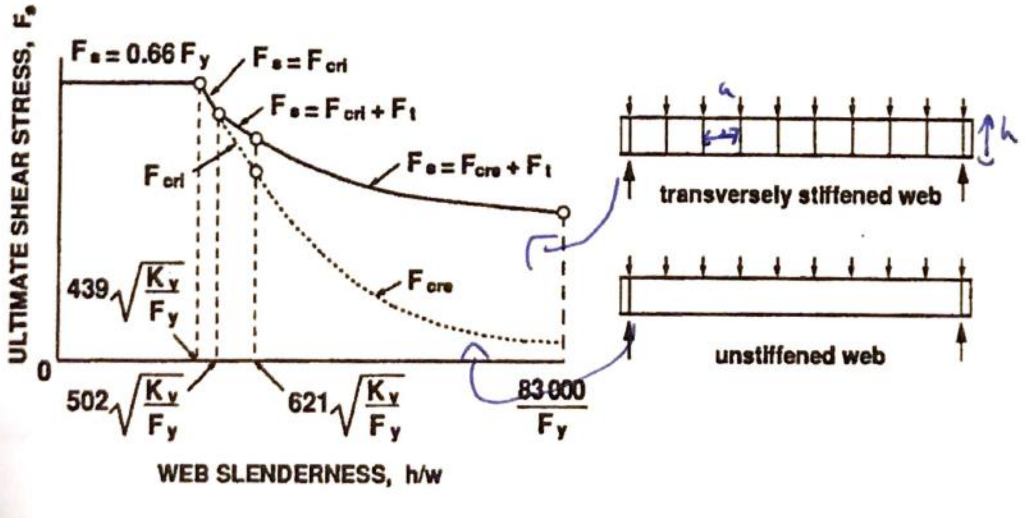
yielded.

where, for an un-stiffened beam, $a/h \to \infty$, $k_v = 5.34$ ($F_t = 0$) Shear buckling coefficient

Therefore, the factored shear resistance V_r, for standard rolled beam without using stiffener is

$$V_r = \phi A_w F_s$$ 13.4.1

where, $F_s = 0.66F_y$



Elastic Analysis (for shear) 13.4.1.1

The factored shear resistance, V_r, developed by the web of a flexural member shall be taken as

$$V_r = \phi A_w F_s$$

A_w = shear area ($d \times w$ for rolled shapes and $h \times w$ for girders, $2h \times t$ for rectangular HSS); and F_s is as *follows:*

(a) for unstiffened webs:

(a) *when* $\frac{h}{w} \le \frac{1014}{\sqrt{F_y}}$; $F_s = 0.66F_y$ (Shear Yielding)

(b) *when* $\frac{1014}{\sqrt{F_y}} < \frac{h}{w} \le \frac{1435}{\sqrt{F_y}}$; $F_s = \frac{670\sqrt{F_y}}{h/w}$ (Inelastic Shear Buckling)

(c) *when* $\frac{h}{w} > \frac{1435}{\sqrt{F_y}}$; $F_s = \frac{961200}{(h/w)^2}$ (Elastic Shear Buckling)

Place stiffener if does not satisfy

Plastic Analysis 13.4.2

When structures design based on the plastic analysis as defined in Clause 8.3.2, the factored shear resistance, V_r, developed by the web of a flexural member shall be taken as

$$V_r = 0.8\phi A_w F_s$$

where F_s is determined in accordance with Clause 13.4.1.1.

Webs of Flexural Members Not Having Two Flanges 13.4.3

The factored shear resistance for cross-section not having two flanges (solid rectangles, round, tees) are to be determined by rational analysis. The factored shear stress at any location must not greater $0.66\phi F_y$ and this limit may be further reduced where shear buckling is a consideration.

Combined shear and moment 14.6

Beams with webs which have $F_s > 0.6F_y$, shall have their shear resistance, V_r determined by Cl.13.4.1.1, multiplied a reduction factor of

$$[2.20 - 1.6\, M_f/M_r] \le 1.0$$

where, M_r = valued determined by Cl. 13.5 or 13.6.1, as applicable

The shear resistance need not be reduced below $0.6\phi A_w F_y$