7.7 $\quad \underset{①}{(D-2)y_1 + 3y_2 - 3y_3 = 0}\ , \quad \underset{②}{-4y_1 + (D+5)y_2 - 3y_3 = 0}\ , \quad \underset{③}{-4y_1 + 4y_2 + (D-2)y_3 = 0}$

$$\phi(D) = \begin{vmatrix} D-2 & 3 & -3 \\ -4 & D+5 & -3 \\ -4 & 4 & D-2 \end{vmatrix} = \overset{(D-2)\times}{\left[(D+5)(D-2) - (-3)(4)\right]} - \overset{3\times}{\left[(D-2)(-4) - (-3)(-4)\right]} + \overset{-3\times}{\left[-4(4) - (D+5)(-4)\right]}$$

$$= (D-2)(D^2+3D-10+12) - 3(-4D+8-12) - 3(-16+4D+20)$$

$$\phi(D) = D^3+3D^2-10D+12D-2D^2-6D+20-24+12D-24+36+48-12D-60$$

$$D^3+D^2-4D-4 \ , \quad D=1 \text{ makes zero}$$

$$\phi(D) = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -4 & -4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

D^2-4

$(D-2)(D+2)$

$\therefore \lambda = -1, 2, -2$

$$y_1 = A_1 e^{-2x} + A_2 e^{-x} + A_3 e^{2x}$$

$$y_2 = B_1 e^{-2x} + B_2 e^{-x} + B_3 e^{2x}$$

$$y_3 = C_1 e^{-2x} + C_2 e^{-x} + C_3 e^{2x}$$

, since $\phi(D)$ is 3rd order, 3 constants
also since $[0,0,0]$ RHS, no particular

$\therefore$ Plug into originals

① $(D-2)\left[A_1e^{-2x}+A_2e^{-x}+A_3e^{2x}\right] + 3\left[B_1e^{-2x}+B_2e^{-x}+B_3e^{2x}\right] - 3\left[C_1e^{-2x}+C_2e^{-x}+C_3e^{2x}\right] = 0$

$= \left[-2A_1e^{-2x} - A_2e^{-x} + 2A_3e^{2x}\right] + \left[-2A_1e^{-2x} - 2A_2e^{-x} - 2A_3e^{2x}\right] + \left[3B_1e^{-2x} + 3B_2e^{-x} + 3B_3e^{2x}\right] + \left[-3C_1e^{-2x} - 3C_2e^{-x} - 3C_3e^{2x}\right]$

$= e^{-2x}\left[-2A_1 - 2A_1 + 3B_1 - 3C_1\right] + e^{-x}\left[-A_2 - 2A_2 + 3B_2 - 3C_2\right] + e^{2x}\left[2A_3 - 2A_3 + 3B_3 - 3C_3\right] = 0$

$= e^{-2x}\left[-4A_1 + 3B_1 - 3C_1\right] + e^{-x}\left[-3A_2 + 3B_2 - 3C_2\right] + e^{2x}\left[3B_3 - 3C_3\right]$

$$② \ -4(A_2e^{-x}+A_1e^{-2x}+A_3e^{2x})+(-B_2e^{-x}-2B_1e^{-2x}+2B_3e^{2x})+(5B_2e^{-x}+5B_1e^{-2x}+5B_3e^{2x})$$

$$+(-3C_2e^{-x}-3C_1e^{-2x}-3C_3e^{2x})=0$$

$$=e^{-2x}(-4A_1+3B_1-3C_1)+e^{-x}(-4A_2+4B_2-3C_2)+e^{2x}(-4A_3+7B_3-3C_3)$$

$\therefore$, if ① = ②,

$-3A_2+3B_2-3C_2=-4A_2+4B_2-3C_2$

i) $A_2=B_2$, let $C_2=0$

$-4A_1+3B_1-3C_1=-4A_1+3B_1-3C_1$, let $A_1=0$

ii) $3B_1-3C_1=3B_1-3C_1$, $B_1=C_1$

$4A_3+3C_3=7B_3$

$-\ 3C_3=3B_3$, $C_3=B_3$

$4A_3=4B_3$

$A_3=B_3$

$\therefore y_1=B_2e^{-x}+B_3e^{2x}$, $y_2=B_2e^{-x}+B_1e^{-2x}+B_3e^{2x}$

$y_3=B_1e^{-2x}+B_3e^{2x}$

(7.13)

$$(D+2)x+5y=0 \quad ① , \quad -x+(D-2)y=\sin 2t \quad ②$$

$$\phi(D)=\begin{vmatrix} D+2 & 5 \\ -1 & D-2 \end{vmatrix} = (D+2)(D-2)-(-1)(5)$$
$$= D^2-4+5$$
$$= D^2+1$$

Complementary: $\lambda^2+1=0$

$\lambda=\pm i1$

$$x_c = A_1\cos t + B_2 \sin t$$

Particular

$$x_p, \ \frac{\begin{vmatrix} 0 & 5 \\ \sin 2t & D-2 \end{vmatrix}}{\phi(D)} = 5\frac{1}{D^2+1}\sin 2t = 5\frac{1}{-2^2+1}\sin 2t = \frac{5}{3}\sin 2t$$

$$\therefore x = A_1\cos t + B_2\sin t + \frac{5}{3}\sin 2t$$

$$y = \frac{-1}{5}(D+2)x$$
$$= \frac{-1}{5}(D+2)\left(A\cos t + B\sin t + \frac{5}{3}\sin 2t\right)$$
$$= \frac{-1}{5}\left(-A\sin t + B\cos t + \frac{10}{3}\cos 2t + 2A\cos t + 2B\sin t + \frac{10}{3}\sin 2t\right)$$
$$= \frac{-1}{5}\left[(2B-A)\sin t + (B+2A)\cos t + \frac{10}{3}(\sin 2t + \cos 2t)\right]$$
$$= \frac{1}{5}(A-2B)\sin t - 1/5(2A+B)\cos t - 2/3(\cos 2t + \sin 2t)$$

7.15

$$\overset{①}{(3D+2)x+(D-6)y=5e^{t}}, \quad \overset{②}{(4D+2)x+(D-8)y=5e^{t}+2t-3}$$

Complementary soln

$$\Delta = \begin{vmatrix} 3D+2 & D-6 \\ 4D+2 & D-8 \end{vmatrix} = (3D+2)(D-8)-(D-6)(4D+2)$$

$$= 3D^2-24D+2D-16-4D^2-2D+24D+12$$

$$= -D^2-4 = -1(D^2+4)$$

$$D^2=-4$$

$$\therefore \lambda = \pm i2$$

$$x_c = A_1\cos 2t + A_2 \sin 2t \quad ; \quad y_c = B_1 \cos 2t + B_2 \sin 2t$$

Particular

$$x_p = \frac{\Delta_i}{\Delta} = \frac{\begin{vmatrix} 5e^t & D-6 \\ 5e^t+2t-3 & D-8 \end{vmatrix}}{-1(D^2+4)} = \frac{5e^t-40e^t-5e^t-2+30e^t+12t-18}{-1(D^2+4)}$$

$$= \frac{1}{D^2+4}10e^t + \frac{1}{D^2+4}20 - \frac{12}{D^2+4}t$$

$$= 10\cdot\frac{1}{5}e^t + \frac{\frac{1}{4}}{1+\frac{D^2}{4}}20 - \frac{\frac{12}{4}}{1+\frac{D^2}{4}}t$$

$$= 2e^t + \left(1-\frac{D^2}{4}+\dots\right)2 - 3\left(1-\frac{D^2}{4}\right)t$$

$$= 2e^t + 5 - 3t$$

$$y_p = \frac{\Delta_i}{\Delta} = \frac{-1}{D^2+4}\begin{vmatrix} 3D+2 & 5e^t \\ 4D+2 & 5e^t+2t-3 \end{vmatrix} = \frac{-1}{D^2+4}\left[15e^t+6+10e^t+4t-6-20e^t-10e^t\right]$$

$$= \frac{5e^t-4t}{D^2+4}$$

$$5 \cdot \frac{1}{D^2+4} e^t \quad -4 \cdot \frac{1}{D^2+4} t$$

$$= e^t \quad - \frac{1}{1+\frac{D^2}{4}} t$$

$$= e^t - \left(1 - \frac{D^2}{4} \ldots\right) t$$

$$= e^t - t$$

$\therefore x = A_1 \cos 2t + A_2 \sin 2t + 2e^t + 5 - 3t$, since $\phi(D)$ 2nd degree, 2 constants

$y = B_1 \cos 2t + B_2 \sin 2t + e^t - t$

Plug into ①

$$3\left(-2A_1 \sin 2t + 2A_2 \cos 2t + 2e^t - 3\right) + 2\left(A_1 \cos 2t + A_2 \sin 2t + 2e^t + 5 - 3t\right)$$

$$+\left(-2B_1 \sin 2t + 2B_2 \cos 2t + e^t - 1\right) - 6\left(B_1 \cos 2t + B_2 \sin 2t + e^t - t\right) = 5e^t$$

$$= 2\cos 2t\left(3A_2 + A_1 + B_2 - 3B_1\right) + 2\sin 2t\left(-3A_1 + A_2 - B_1 - 3B_2\right) + 5e^t = 5e^t$$

0 0

i) $A_1 + B_2 = 3(B_1 - A_2)$

ii) $-3(A_1 + B_2) = B_1 - A_2$, $\therefore$ i → ii, $-3(3(B_1 - A_2)) = B_1 - A_2$

$-9B_1 + 9A_2 = B_1 - A_2$

iii) $A_2 = B_1$

iii) → i, $-3(A_1 + B_2) = 0$

$A_1 = -B_2$

$\therefore x = A_1 \cos 2t + A_2 \sin 2t + 2e^t + 5 - 3t$, $y = A_2 \cos 2t - A_1 \sin 2t + e^t - t$