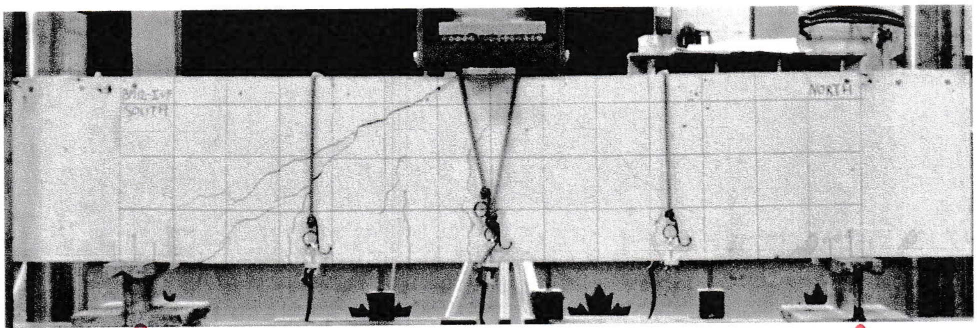
CIVE 414
STRUCTURAL CONCRETE DESIGN

SHEAR

in REINFORCED

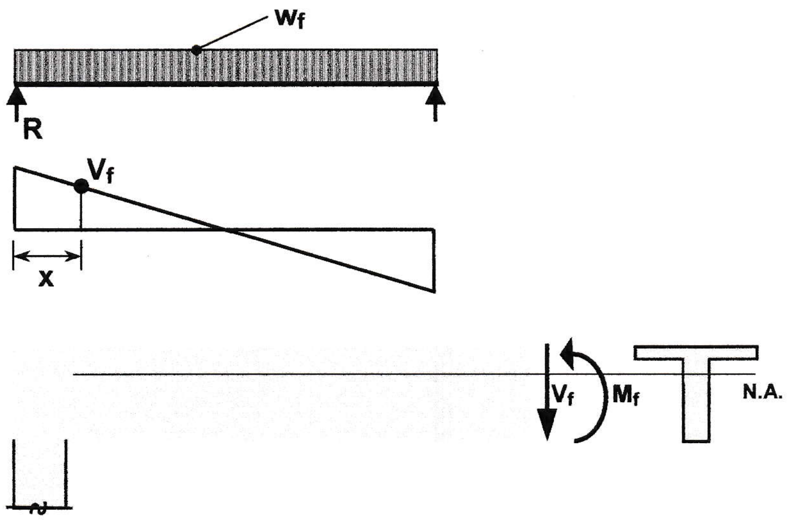
CONCRETE MEMBERS



Majority of failures are result of shear or development length

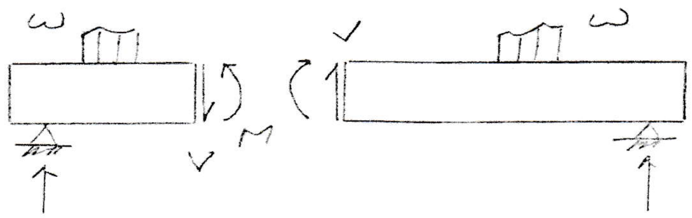
SHEAR IN REINFORCED CONCRETE

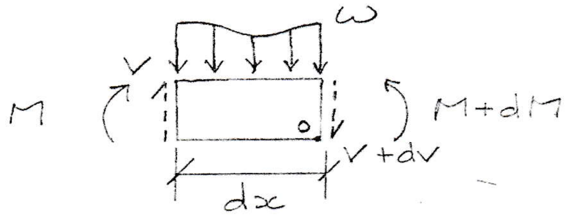
- Brittle failure mode → very undesirable
- Shear in reinforced concrete is complex:
  - non-linear material
  - non-homogeneous material
  - cracking
  - presence of reinforcement



MECHANICS OF SHEAR IN BEAMS

In beams, the external load is resisted by moment and shear, i.e.:





From moment equilibrium of a differential element around point O

$$Vdx = dM$$

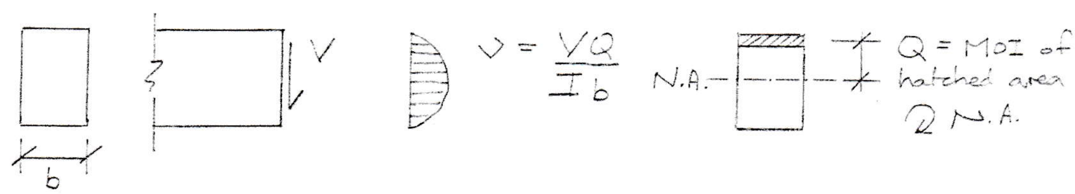
$$V = \frac{dM}{dx}$$

Shear exists where moment is changing

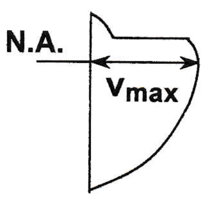
For uncracked beams in shear we have homogenous, linear elastic material:

$$v_c = \frac{V_f Q}{Ib}$$

For Rectangular Section:



For T-Section:



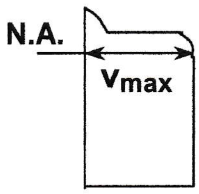
After cracking:

(nominal) $v_c = \dfrac{V_f}{b_w d}$

shear force

shear stress

web width



non-homogeneous concrete member

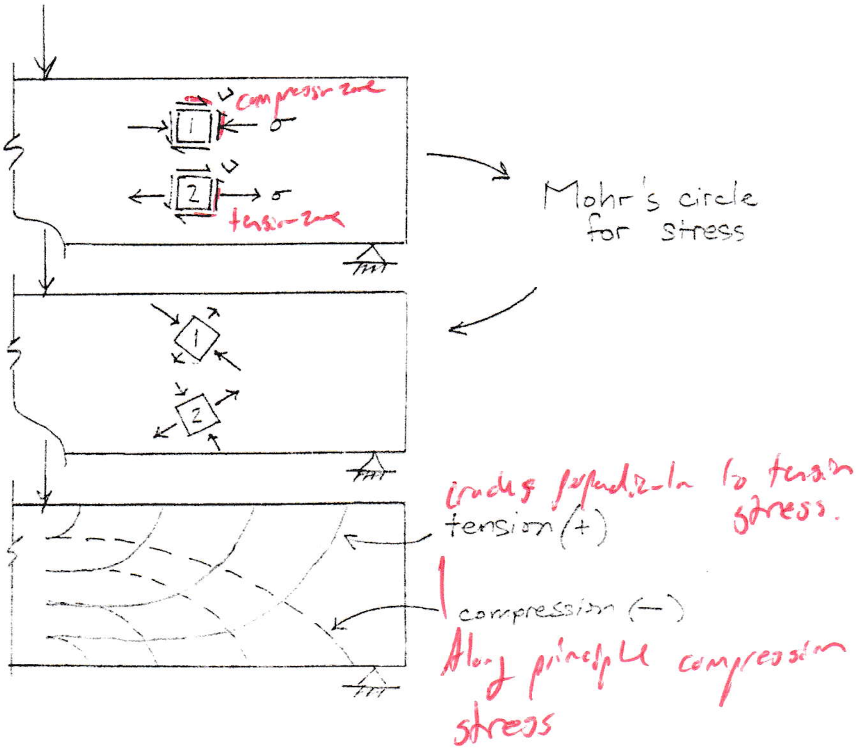
Plain stress for beam

Flexural and shear stresses:

Along principal directions, Shear stress is zero

Principal Stresses:

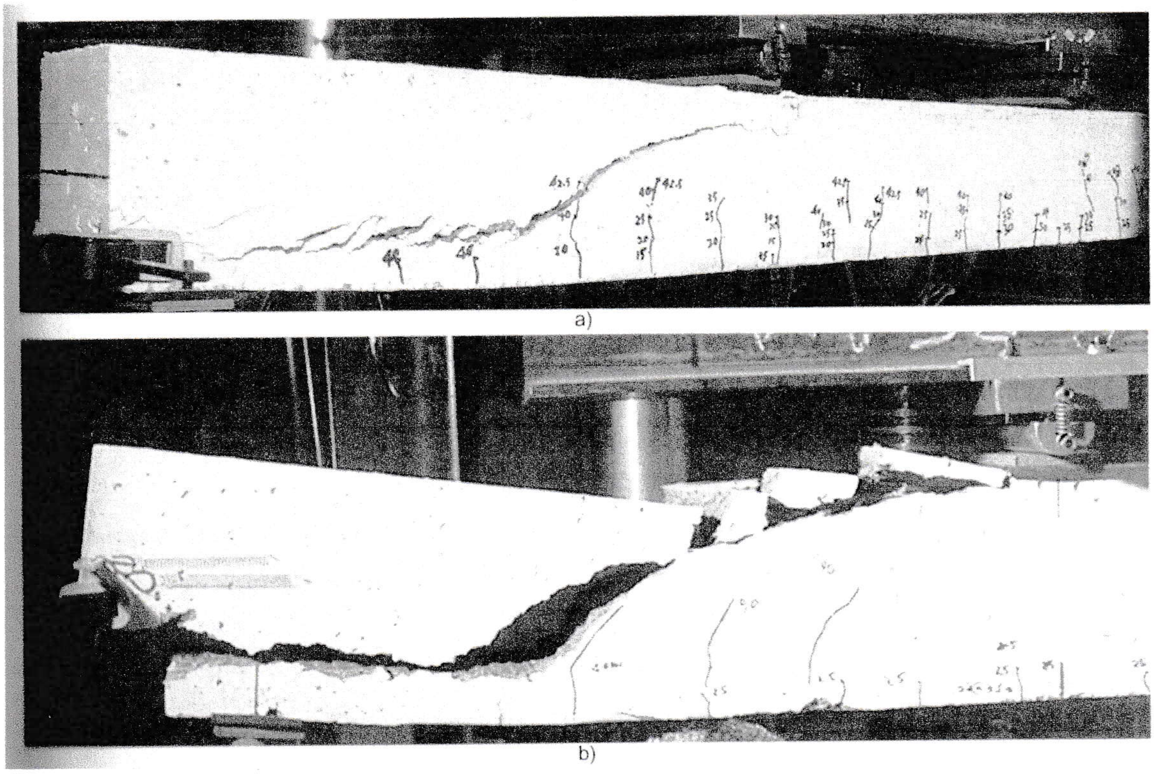
Principal stress trajectories:



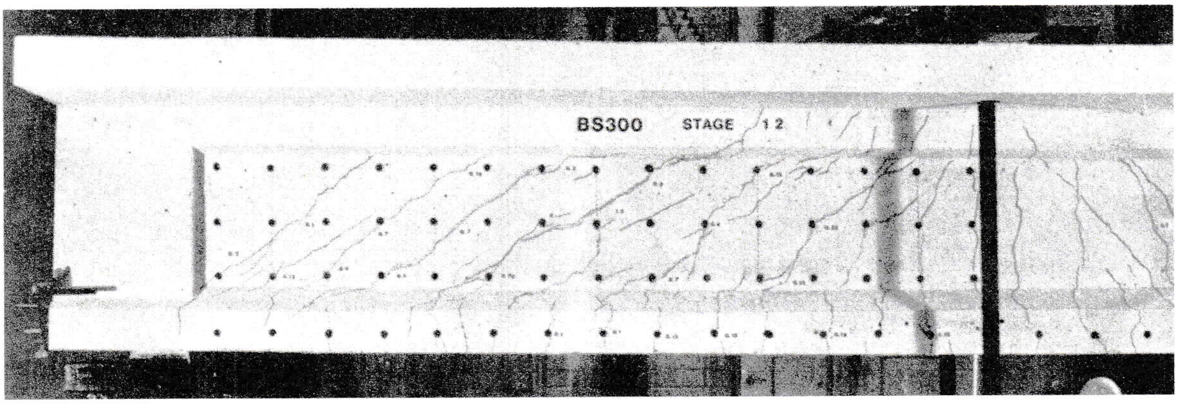
Note: (principal stresses)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



→ beam with no shear reinforcement [Brzev & Pao]



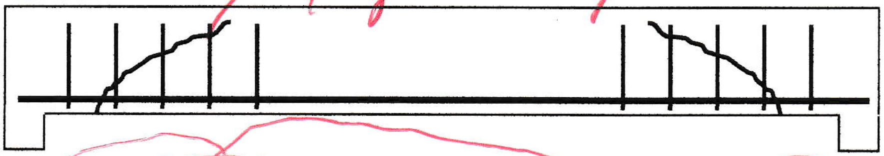
Prestressed! ∴ Parallel cracks

SHEAR REINFORCEMENT

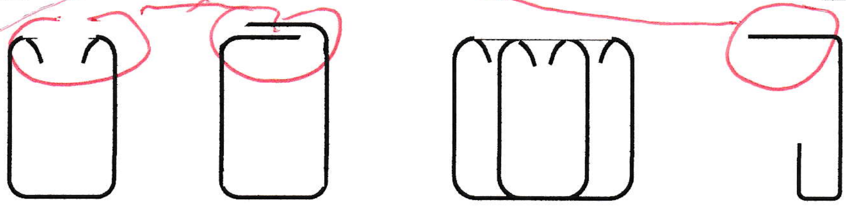
CSA A23.3 ***Clause 11.2.4***

1. Stirrups

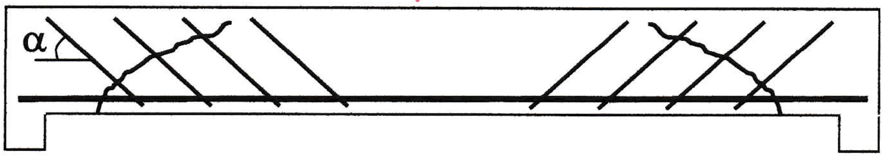
spacing small enough so each crack is covered by at least 1 stirrup



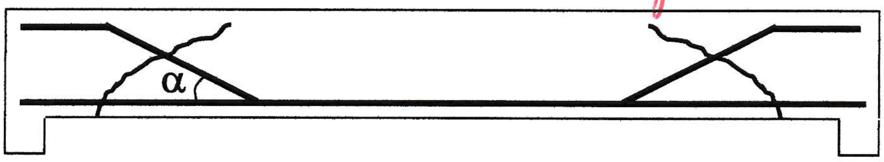
to ensure bond
[illegible]



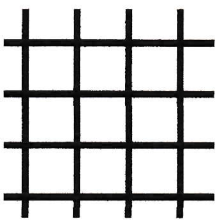
2. Stirrups with $\alpha \geq 45$ deg. - practical, but not constructable



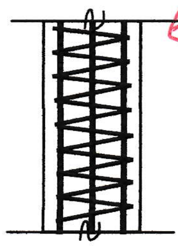
3. Bent Bars with $\alpha \geq 30$ deg. - not used anymore



4. Welded-wire Fabric

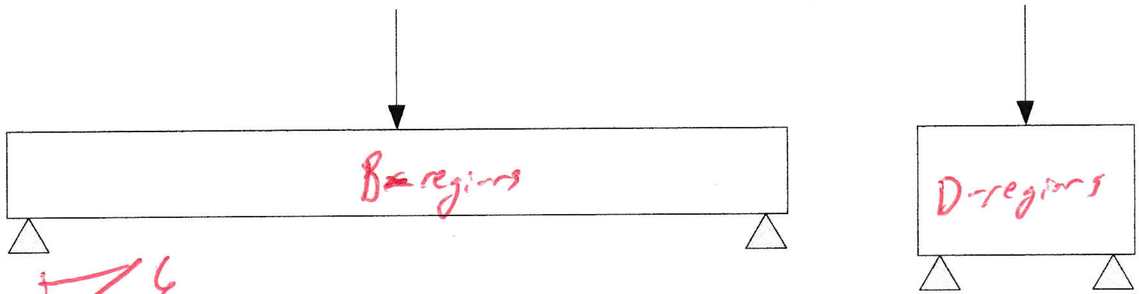


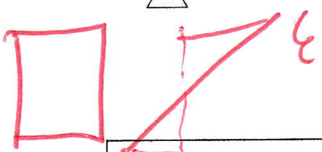
5. Spirals and Ties



← column

In the design of reinforced concrete structures, so-called ‘B’ and ‘D’ regions are distinguished *[CSA A23.3 §11.1.1-2]*:





for slender, Bernoulli beams

Plane sections remain plane, Linear axial strain distribution	Nonlinear axial strain distribution through the thickness
Slender beams, Bernoulli beams **B Regions**	Deep Beams **D Regions**
“sectional analysis” Based on truss model	Strut and Tie Analysis

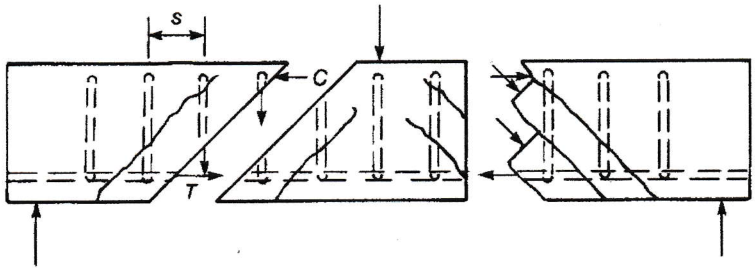
Shear reinforcement in beams   The purpose of shear reinforcement is to restrain diagonal cracks and ensure that full flexural capacity can be developed.

– Note: Shear reinforcement does not prevent diagonal shear cracks from occurring altogether.

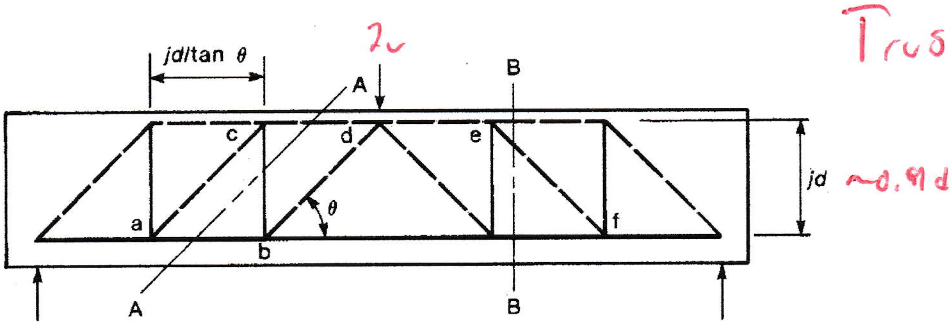
Analysis for Shear

Truss Models

Given a reinforced concrete beam with shear cracks:



(a) Internal forces in a cracked beam.



(b) Pin-jointed truss.

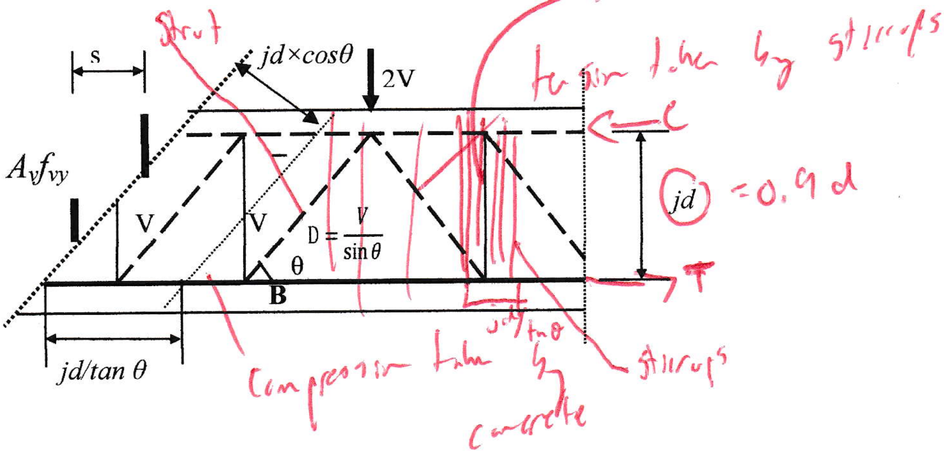
Truss Model

MacGregor, J.G., Bartlett, M. "Reinforced Concrete Mechanics and Design", Prentice Hall, 2000

VARIABLE ANGLE TRUSS MODEL

Assumes:

- tensile strength of concrete is ignored
  - θ° crack angle



Equilibrium in vertical direction (vertical tie)

$$\frac{\left(\frac{jd}{\tan\theta}\right)}{s} \times A_v f_{vy} = V$$

$\frac{\left(\frac{jd}{\tan\theta}\right)}{s}$ *is number of stirrups within jd/tan θ*

used to calculate required cross-sectional area of a stirrup (knowing the spacing):

$$A_v = \frac{V \times s}{f_{vy} \times (jd/\tan\theta)}$$

or spacing if stirrups (knowing the cross-sectional area):

$$s = \frac{A_v \times f_{vy} \times (jd/\tan\theta)}{V}$$

Or the stirrup contribution:

$$V = \frac{A_v \times f_{vy} \times (jd/\tan\theta)}{s}$$

Equilibrium along the inclined strut

Used to dimension the overall dimensions of the beam to avoid concrete crushing

D (force in strut)

$$f_2 = \frac{V}{b \times jd \times \sin\theta \cos\theta} \leq compressive\ strength\ of\ concrete$$

width of beam

; jd cotθ = width strut

As θ ↓, compression ↑, less stirrups req'd

Note: The truss models predict that a beam with no stirrups will have no shear capacity:

concrete contribution

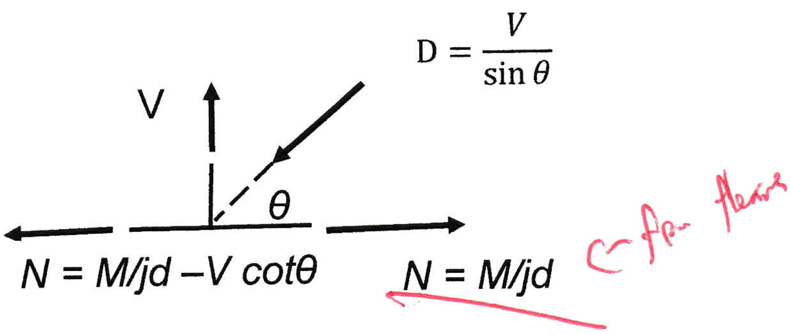
steel stirrup contribution to shear capacity

→ conservative!

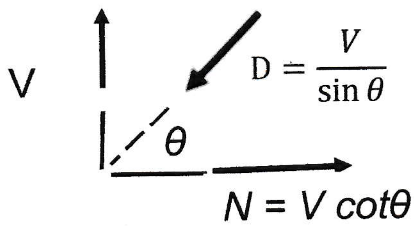
→ basis for traditional "$V_c + V_s$" approach

Effect of Shear on tensile forces in flexural reinforcement

Equilibrium of Joint B



Equilibrium at support of simply supported beam:



45° Truss Model by Ritter and Mörsch

***[Ritter 1899, Mörsch 1902]*: angle θ = 45 ^{0}**

sin 45° = cos 45°= $1/\sqrt{2}$

$$\frac{jd}{s} \times A_v f_{vy} = V$$

$$A_v = \frac{V \times s}{f_{vy} \times jd}$$

$$s = \frac{A_v \times f_{vy} \times jd}{V}$$

Traditional "V_c + V_s" Approach

This approach was the basis for the "simplified method" in the old Canadian Concrete Design Code *[CSA A23.3 1994]*.

It can be summarized as follows:

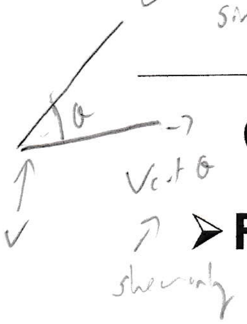
- $V_r \geq V_f$
- $V_r = V_c + V_s$
- $V_c = 0.2 \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \cdot b_w \cdot d$

  (for beams with > min. shear reinf. or d < 300 mm)

  $= \left(\dfrac{260}{1000 + d}\right) \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \cdot b_w \cdot d$ but $\geq 0.1 \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \cdot b_w \cdot d$

  (all other cases)
- $V_s = \dfrac{\phi_s \cdot A_v \cdot f_y \cdot d}{s}$

$D = \frac{V}{\sin\theta}$ More economical to use low angle, $jd \sim 0.9d$



CSA A23.3 -2014 Design for Shear

- **Required Shear Resistance** *Cl 11.3.1*

Is 30° unconservative?

$$V_r \geq V_f$$

Method 1: Simplified Design Method – Clause 11.3.6.3

(up to 60 MPa f'_c)

Factored Shear Resistance

CSA A23.3-14 *Clause 11.3.3*

concrete steel

$$\mathbf{V_r = V_c + V_s}$$

V_c = shear resistance provided by concrete, For many years, $V_c = 0.2\lambda\phi_c\sqrt{f'_c}\,b_w d$, See fig. 11

V_s = shear resistance provided by steel

(provide stirrups if necessary!)

Since $\sqrt{f'_c}$ related to tensile strength

Maximum Factored Shear Resistance (Clause 11.3.3)

- Upper limit to prevent web crushing failure

  CSA A23.3-14 *Clause 11.3.3*

(0.25f'c when diagonal crushing happens)

$$\mathbf{V_{r,max} = 0.25\phi_c f'_c b_w d_v}$$

$d_v = jd = 0.9d$, $D = \frac{V}{\sin\theta}$

where,

ϕ_c = 0.65

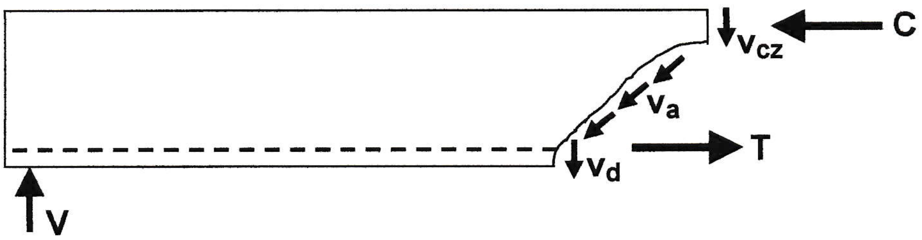
d_v = effective shear depth

= 0.9d or 0.72h, whichever is greater

b_w = web width

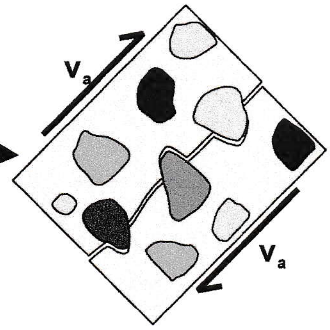
Concrete Resistance in Shear, V_c

CSA A23.3-14 *Clause 11.3.4*



Three contributions:

- v_{cz} shear in compression zone
- v_a aggregate interlock
- v_d dowel action (longitudinal bar)
  - → v_a is the largest contribution to concrete shear strength



$$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$$

← In Eurocode depends on flexural, longitudinal reinforcement

Where,

ϕ_c = 0.65

λ = factor to account for low-density concrete
= 1 for normal density concrete

β = factor accounting for shear resistance of cracked concrete, determined in *Clause 11.3.6*

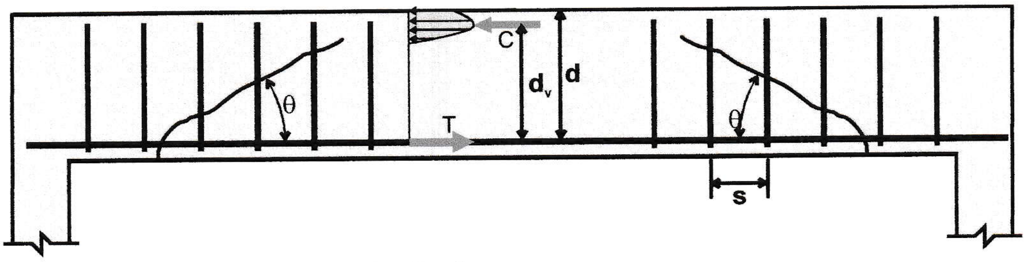
Note that $\sqrt{f'_c} \le 8\,\text{MPa}$ when computing V_c.

- For $f'_c > 64$ MPa, use $\sqrt{f'_c} = 8\,\text{MPa}$
- Accounts for reduced aggregate interlock in high strength concrete

If HPC, cracks go through aggregate. Not necessarily work like here w/ aggregate interlock

STEEL RESISTANCE IN SHEAR, V_s

CSA A23.3-14 *Clause 11.3.5*



Force in one stirrup: $\phi_s A_v f_y$

number of stirrups crossing a crack: $n = \dfrac{d_v \cot\theta}{s}$

$$V_s = \frac{\phi_s A_v f_y d_v \cot\theta}{s}$$ (from truss model)

Where,

A_v = area of shear reinforcement within distance "s"
= A_b x no. of legs in stirrup

s = stirrup spacing

θ = angle of inclination of compression stresses, determined in *Clause 11.3.6*
≈ angle of inclined cracks due to shear

For design:

$$(V_s)_{req'd} = V_f - V_c = \frac{\phi_s A_v f_y d_v \cot\theta}{s}$$

Req'd # stirrups based on
$V_f - V_c$
Total concrete can carry

Thus: $$s_{req'd} = \frac{\phi_s A_v f_y d_v \cot\theta}{V_f - V_c}$$

DETERMINATION OF β AND θ

CSA A23.3-14 *Clause 11.3.6*

Clause 11.3.6.2 - Special Member Types

β = 0.21
θ = 42 deg.

$\rangle$ $\beta \uparrow$, $\theta \downarrow$ slightly. $\therefore$ not much Δ from 0.2 & 45

For

- slabs with thickness $\leq$ 350 mm
- beams with overall thickness $\leq$ 250 mm
- concrete joist construction (Clause 10.4)
- beams cast integrally with slabs where the depth of the beam below the slab is not greater than one-half of the web width or 350 mm

Clause 11.3.6.3 – Simplified Method

➤ Applicable to cases other than Clause 11.3.6.2 and members not subject to significant axial tension

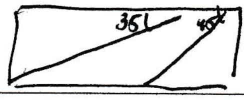
➤ Limitations: $f'_c \leq 60$ MPa
$f_y \leq 400$ MPa

θ = 35 deg.

β = 0.18 for sections containing at least the minimum transverse reinforcement (Clause 11.2.8.2)

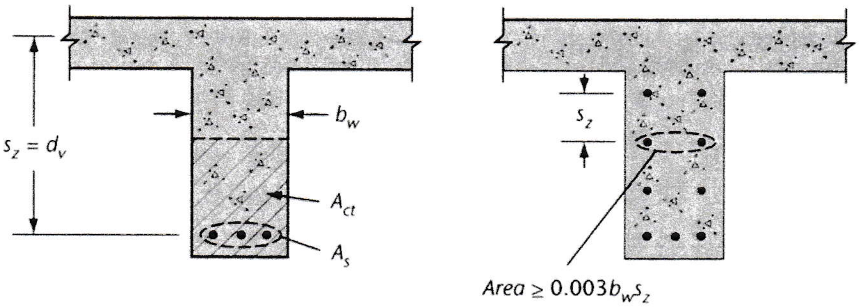
$$\beta = \frac{230}{1000 + d_v}$$ for sections containing no transverse reinforcement and having maximum aggregate size $\geq$ 20 mm

$$\beta = \frac{230}{1000 + s_{ze}}$$ for sections containing no transverse reinforcement and all aggregate sizes



s_{ze} = equivalent crack spacing parameter

$$= \frac{35\,s_z}{15 + a_g} \geq 0.85\,s_z$$



Additional requirements:

deep member

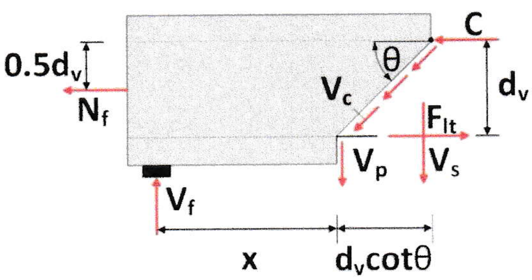
1) Must provide strirrups if: $V_f \geq V_c$ or $h > 750$ mm
2) $A_{v,min}$ $= 0.06 \cdot \dfrac{\sqrt{f'_c} \cdot b_w \cdot s}{f_y}$ (so strength maintained after cracking)
3) s_{max} = 600 mm or $0.7 \cdot d_v$

   $\frac{1}{2} V_{r,max}$

   = 300 mm or $0.35 \cdot d_v$ if $V_f > 0.125\lambda\phi_c f_c' b_w d_v$
4) $V_{r,max}$ $= 0.25 \cdot \phi_c \cdot f_c' b_w d_v$
5) Extend tension reinforcement $d_v \cot\theta$ past location it is required for flexure alone (*cl 11.3.9.1)* or

5a) provide tension reinforcement such as: (*cl 11.3.9.2*)

$$F_{lt} = \underbrace{\frac{M_f}{d_v}}_{\text{flexure}} + \overbrace{0.5N_f}^{\text{axial loads}} + \underbrace{\left(V_f - 0.5V_s - V_p\right)\cot\theta}_{\text{prestressing shear}}$$



$$F_{lt} \times d_v = V_f(x + d_v \cot\theta) + N_f 0.5 d_v - V_s 0.5 d_v \cot\theta - V_p d_v \cot\theta$$

$$F_{lt} = \frac{M_f}{d_v} + V_f \cot\theta + N_f 0.5 - V_s 0.5 \cot\theta - V_p \cot\theta$$

6) At exterior direct bearing supports, the longitudinal reinforcement on the flexural tension side of the member shall be capable of resisting a tensile force of $(V_f - 0.5 \cdot V_s)\ cot\ \theta$

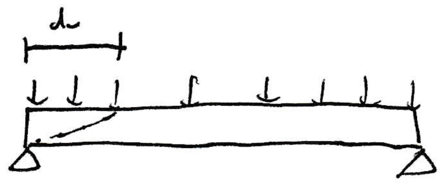
7) Sections near supports (*clause 11.3.2*)

➢ **Sections located less than a distance d_v from the face of the support may be designed for the same shear, V_f ,as that computed at a distance d_v, provided that**

a) there action force in the direction of applied shear introduces compression into the member;

b) no concentrated load that causes a shear force greater than is applied within the distance d_v from the face of the support; and

c) loads applied within distance dv from the face of the support do not increase the absolute magnitude of the shear at the face by more than 20%.

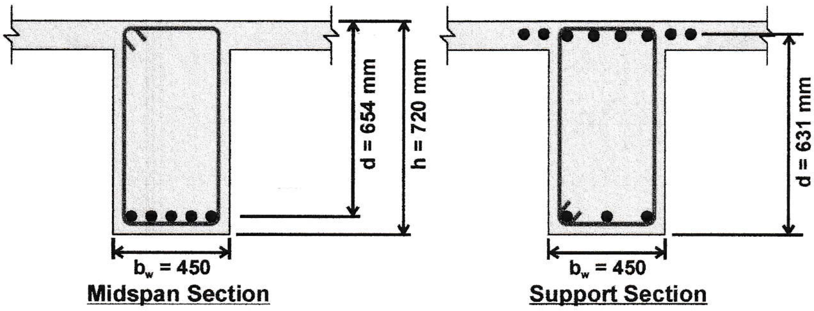


Example 1: *The factored shear force envelope for a continuous interior beam is shown below. Design the shear reinforcement for the beam.*

f'_c = 25 MPa

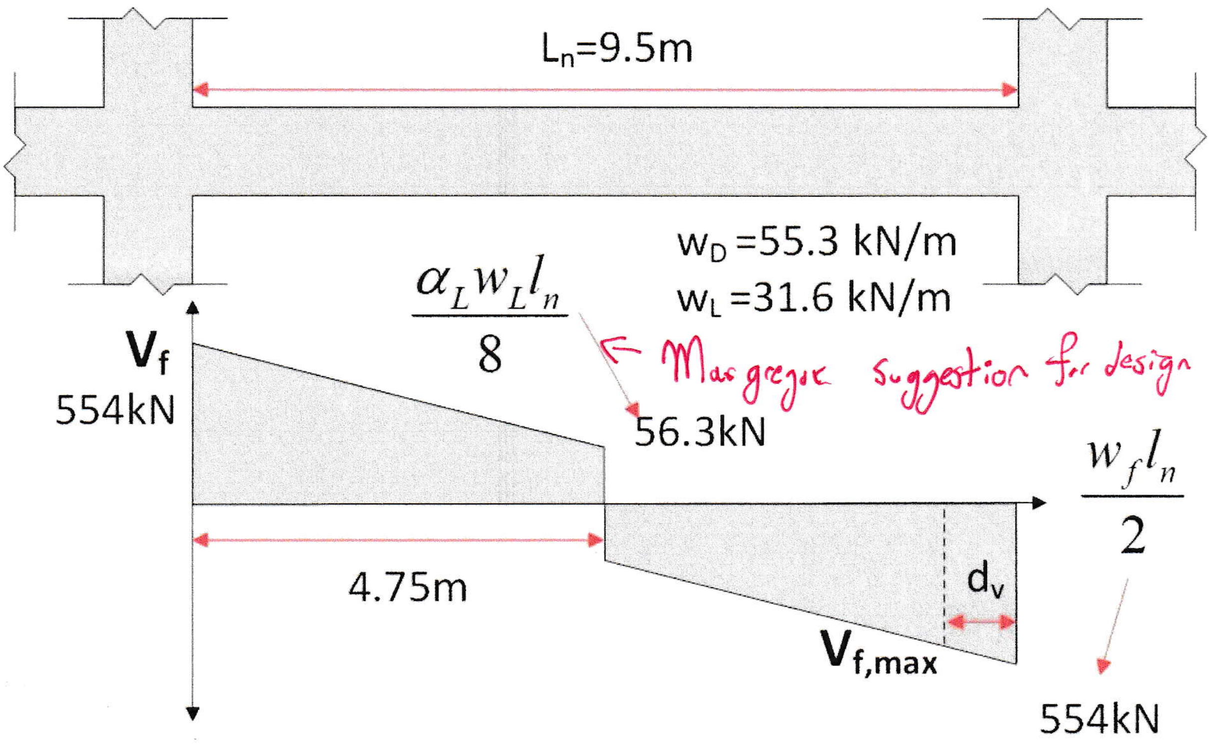
f_y = 400 MPa

Max. C.A. size = 20 mm



(Tension reinforcement on top)

$$V_f(x) = 554 - \left(\frac{554 - 56.3}{4750}\right)x = 554 - 0.104x$$



$\boldsymbol{\alpha_L = 1.5, \quad \alpha_D = 1.25}$

$\boldsymbol{w_f = 116.6\ \mathrm{kN/m}}$

$w_D = 55.3$ kN/m
$w_L = 31.6$

Benjamin Klassen

Determine d_v:

$$d_v \geq \begin{Bmatrix} 0.9d = 0.9(631mm) = 568mm \rightarrow governs \\ 0.72h = 0.72(720mm) = 518mm \end{Bmatrix}$$

Since the maximum shear occurs at the support, use d=631mm when determining d_v for the critical section (at support). Although we could consider increasing d_v based on d=654mm in the positive moment region, it is conservative to use the ~~smaller~~ bigger value throughout. However, use bigger anyway!! l.l

1) Determine if section is adequate

Check $V_{f,max} \leq V_{r,max}$

$V_{f,max}$ → calculate at critical section (at d_v from support, CL 11.3.2) 568

$V_{r,max}$ → Clause 11.3.3

$V_{f,max}$ occurs at $d_v = 568mm$

$$V_f(568) = 554 - 0.104(568mm) = 494kN$$

the horizontal tie (stirrup), the diagonal crush (concrete failure)

$$V_{r,max} = 0.25\phi_c f_c' b_w d_v$$

$$= 0.25(0.65)(25MPa)(450mm)(568mm) \div 10^3 = 1038kN > V_{f,max}$$ ∴ max

∴ section is adequate

*If $V_{f,max} > V_{r,max}$ →must increase b_w, d_v or f_c' to make section adequate

2) θ and β, Clause 11.3.6

Simplified method, $\theta = 35°$

With stirrups, $\beta = 0.18$

Without stirrups, $\beta = \frac{230}{1000+d_v} = \frac{230}{1000+568} = 0.14$

∴, concrete contribution larger w/ stirrups, as concrete-resistance is maintained after cracks

3) Compute $V_c \rightarrow 2\ values$

$V_c = \phi_c \lambda \beta \sqrt{f_c'} b_w d_v$ $\quad\quad$ $\boldsymbol{\lambda} = 1$, normal density concrete

With stirrups: $V_c = (0.65)(\mathbf{0.18})(\sqrt{25})(450)(568) \div 10^3 = 150kN$

Without stirrups: $V_c = (0.65)(\mathbf{0.14})(\sqrt{25})(450)(568) \div 10^3 = 122kN$

4) Design Stirrups Near Support ($V_{f,max} = 494kN$)

Check whether stirrups are needed

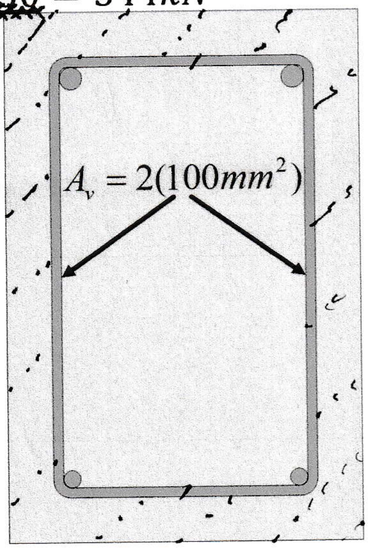
$V_f = V_{f,max} = 494kN > V_c = 122kN\ (without\ stirrups)$

$\rightarrow stirrups\ are\ required$

Vc w/ stirrups

Thus $V_{s,req'd} = V_{f,max} - V_c\ [with\ stirrups] = 494 - 150 = 344kN$

Use: 10M stirrup, double-legged ← 2 legs



$A_v = 2(100mm^2) = 200mm^2$

Area of single leg of 10M stirrup $= 100mm^2$

$$V_s = \frac{\phi_s A_v f_y d_v cot\theta}{s}$$

$$s_{req'd} = \frac{\phi_s A_v f_y d_v cot\theta}{V_f - V_c}$$

$$= \frac{(0.85)(200mm^2)(400MPa)(568mm)\cot(35°)}{344 \times 10^3 N} = 160mm$$

$\therefore use\ s = 150mm$

spacing

$$Check: A_{v,min} = \frac{0.06\sqrt{f_c'} b_w s}{f_y} = \frac{0.06\sqrt{25}(450)(150)}{400}\quad s = spacing$$

$= 51mm^2 < A_v = 200mm^2 \rightarrow OK$

Benjamin Klasser

Check max spacing (Clause 11.3.8)

Two limits based on V_f at section of interest

check if $V_f > or \underline{\leq \frac{1}{2} V_{r,max}}$

$$\frac{1}{2}(V_{r,max}) = \frac{1}{2}(1038kN) = 519kN$$

Since $V_f = V_{f,max} = 494kN < 519kN$ 400

Then $s_{max} \leq \begin{Bmatrix} 0.7d_v = 0.7(568) = 398mm \rightarrow governs \\ 600mm \end{Bmatrix}$

$\because s = 150mm < s_{max} = 398mm \rightarrow OK$

For $s = 150mm$: $V_r = V_c + V_s = 150kN + \frac{\phi_s A_v f_y d_v cot\theta}{s}$ [s=150mm]

$= 150kN + 368kN = 518kN$

Use s=150mm near support, $V_r = 518kN$

5) Design Stirrups for Other Regions of Beam

Since $V_f < V_{f,max}$ along length of beam, $V_{s,req'd}$ will decrease as we move away from support

- s increases
- Fewer stirrups are required

Balance computational effort, construction considerations and material/labour savings

Consider:

A) Regions where no stirrups are required or where minimum stirrups are required $\rightarrow V_{r,min}$
If $h \leq 750mm$, look for regions where $V_f \leq V_c$ [$no\ stirrups\ \beta = 0.14$] 0.14 (**applies to this example since h=720mm**)
If $h > 750mm$, look for regions where s_{max} governs
$V_f \leq V_r = V_c[with\ stirrups, \beta = 0.18] + V_s[based\ on\ s_{max}]$
s_{max} based on Clause 11.3.8 or 11.2.8.2

B) One or two other regions

$$V_{f,max} > V_f > V_{r,min}\ [V_{r,min}\ from\ Region\ A]$$

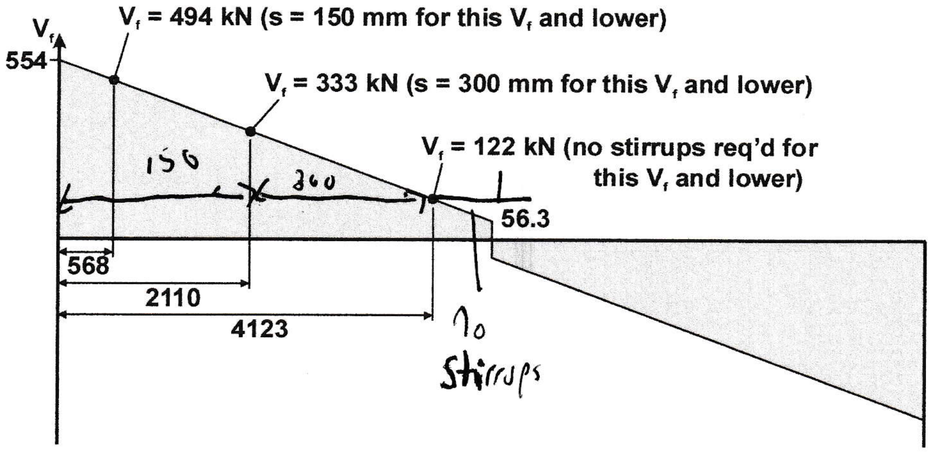
→ choose an intermediate spacing and solve for corresponding

V_r = 333 kN for s=300mm (not nat)

A) Region where no stirrups are required since $h = 720mm < 750mm$

$$V_{r,min} = V_c = 122kN\ (no\ stirrups)$$

Where $V_f \leq 122kN \rightarrow no\ stirrups\ required$



B) Intermediate Region (spacing)

Consider:

1) maximum spacing allowed in regions of low shear (Clause 11.3.8)

For $V_f < 519kN\ [from\ step\ 4] \rightarrow s_{max} = 398mm$

398mm is largest spacing we can use for this beam

2) For $V_f = V_{f,max} = 494kN \rightarrow use\ s = 150mm$

150mm is smallest spacing we can use in this problem

→choose spacing between these values

→choose s=300mm → **now calculate V_r for s=300mm**

$$V_r = V_c + V_s = 150kN + \frac{\phi_s A_v f_y d_v \cot\theta}{s}$$

$$= 150kN + \frac{(0.85)(200)(400)(568)\cot(35°)}{300mm} \div 10^3$$

$V_r = 333kN$ for s=300mm

For $127kN \le V_f \le 333kN$ use $s = 300mm \rightarrow V_r = 333kN$

6) Determine Stirrup Layout and V_r Diagram

* Determine the location, x, where $V_f = V_r$ for the various design spacings and for $V_f = V_{r,min} = V_{c,nostirrups}$ (see next page)

Start stirrups at 50mm from face of column (must not exceed $s/2$)

Region 1: s=150mm

X=50mm to at least x=2110mm

Use 14 spaces @ s=150mm, x = 50+14(150) = 2150mm

Region 2: s=300mm

X=2150mm to at least x=4123mm

Use 7 spaces @ s=300mm, x = 2150+7(300)=4250mm

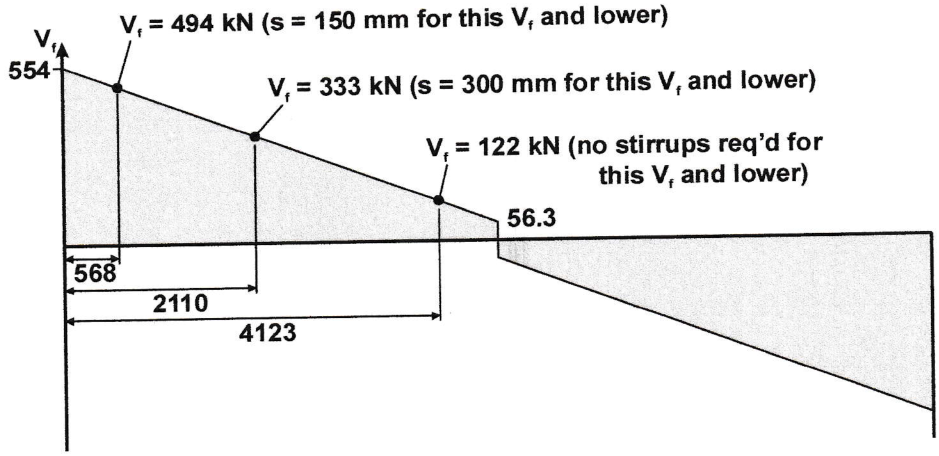
Region 3: no stirrups

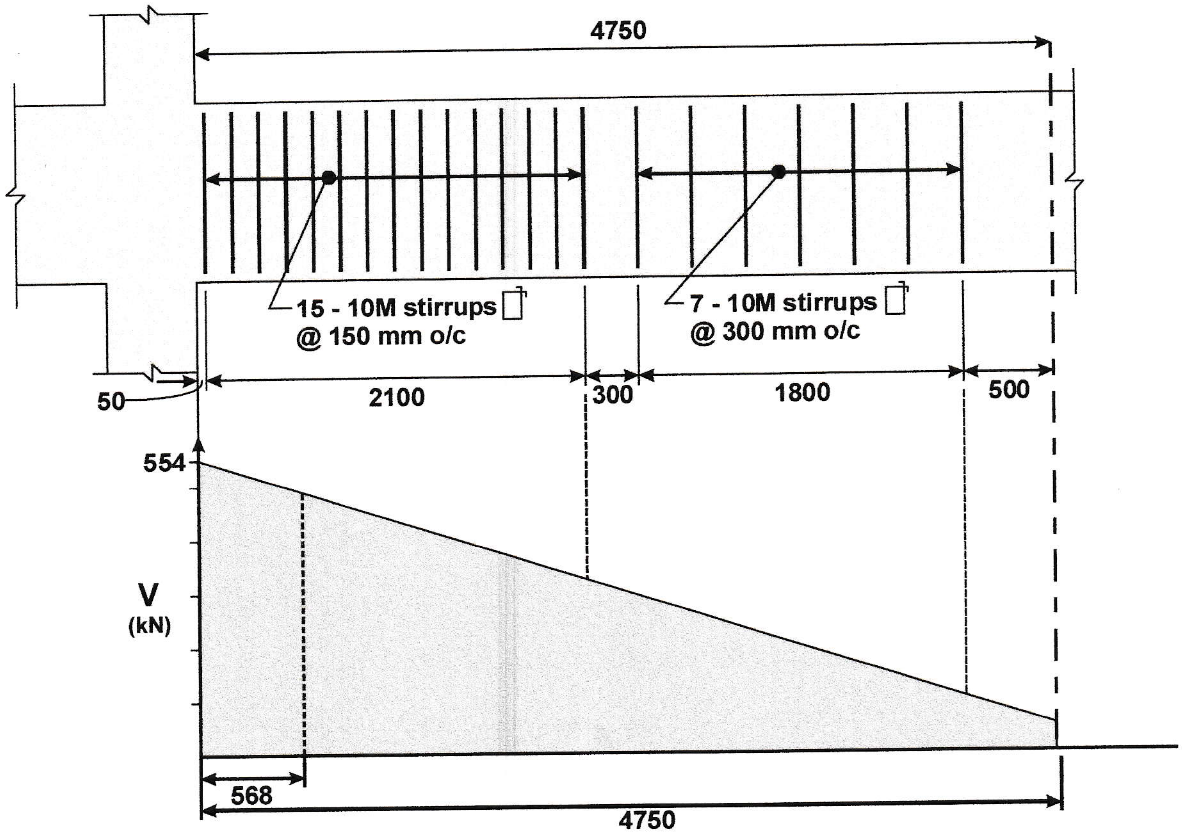
X=4250mm to beam centerline (CL)

Draw V_r Diagram, Plot V_r for 3 regions

Clause 11.3.7 – where spacing, s, changes, V_s is assumed to vary linearly over a length of “h” centered on the location of the spacing change

Develop stirrup layout and shear resistance envelope:





CSA A23.3-2014 General Method: clause 11. 3.6.4

- Use for:
  - $f'_c > 60$ MPa
  - Members subject to significant tension
  - Prestressed concrete elements
  - Situations where designer wants a more rigorous approach → non-typical members/structures

- The **general method is essentially the same as the simplified method** (in fact it is the basis for the simplified method) in all respects except in the calculation of β and θ.

- **The general method can be summarized as follows:**

  - $$\beta = \left(\frac{0.40}{1+1500\cdot\varepsilon_x}\right)\cdot\left[\left(\frac{1300}{1000+s_{ze}}\right)\right] = 1$$ (for beams w/ shear reinforcement)

"strain effect" → less aggregate interlock as cracks get wider

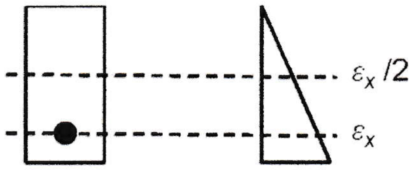
"size effect" → increase in beam size w.r.t. aggregate size

$s_{ze} = 300$ mm if $A_v \geq A_{v,min}$ for beams w/ shear reinforcement

otherwise, s_{ze} can be determined using the same formula given for the simplified method → for beams w/o shear reinforcement

Same location

- $\varepsilon_x = \dfrac{(M_f/d_v + V_f + 0.5 \cdot N_f)}{2 \cdot E_s \cdot A_s}$ = estimate of long. strain at mid-height
- $\theta = 29 + 7000 \cdot \varepsilon_x$



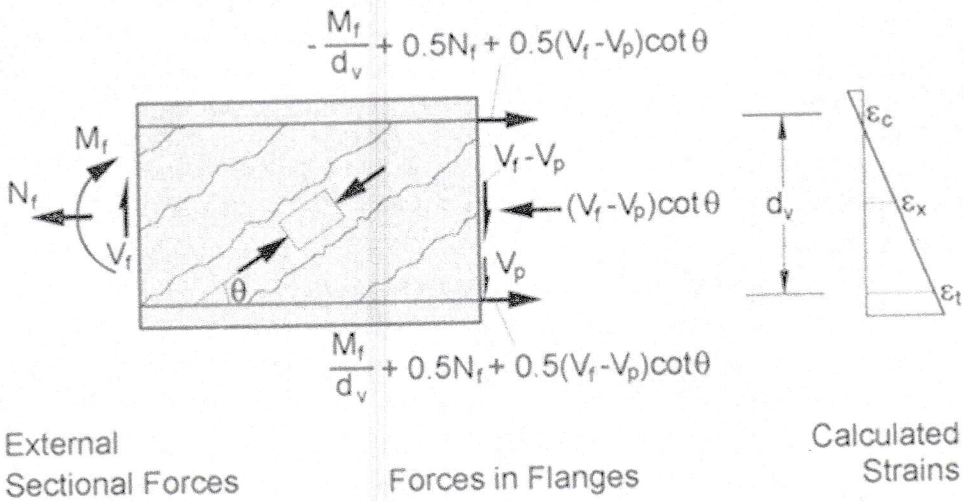
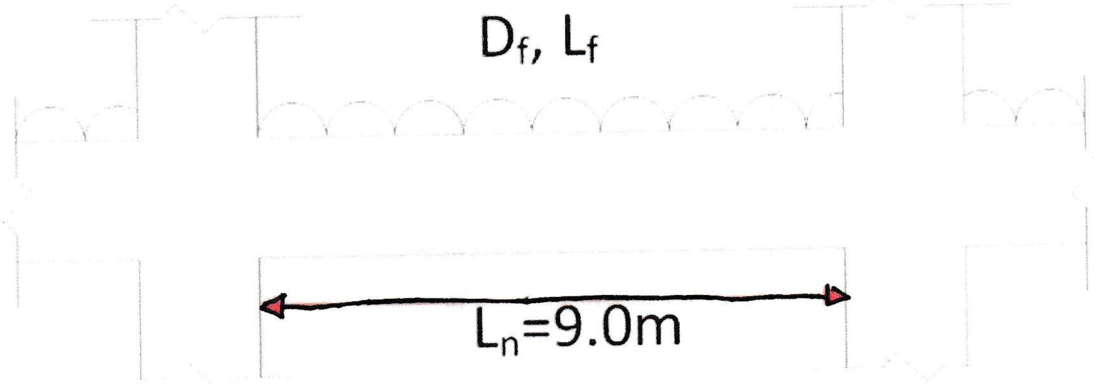


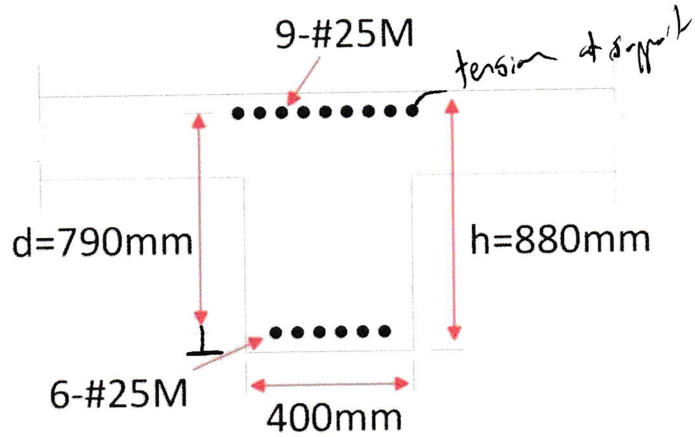
Fig. N11.3.6.4(a)
More accurate procedure for determining ε_x.

for shear

EXAMPLE 2 Design the beam below using General Method- – from Macgregor and Bartlett (example 6.5)

factored , $\alpha_L = 1.5$
$\alpha_D = 1.25$





"May as well use smaller one for shear"

$$D_f = 60\frac{kN}{m},$$

$$L_f = 75\frac{kN}{m}, \quad \text{->} \quad w_f = 135\frac{kN}{m}$$

$$f'_c = 25MPa, \qquad f_y = 300MPa,$$

Benjamin [illegible]

Solution

1) Calculate the factored Shear force diagram and Bending Moment Diagram

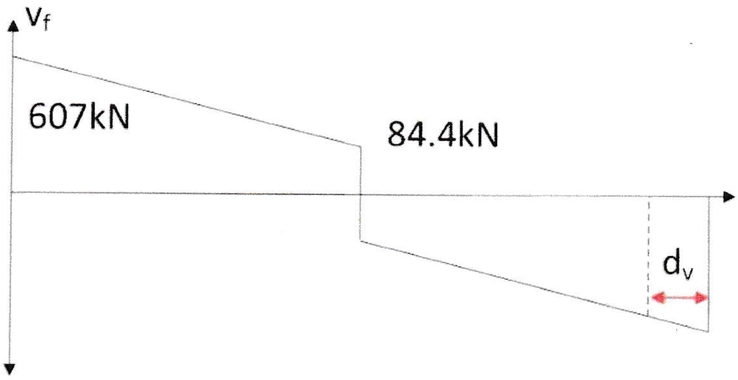
- From fig N9.3.3 (approximate Frame analysis, CSA A 23.3-04

At the support

$$V_f = \frac{w_f \times l_n}{2} = \frac{135 \times 9}{2} = 607kN$$

To consider pattern loading loading, MacGregor and Bartlett suggest the following at midspan:

$$V_f = \frac{L_f \times l_n}{8} = \frac{75 \times 9}{8} = 84.4kN$$

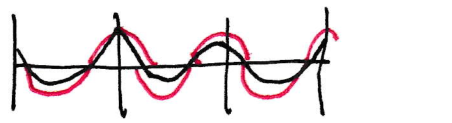


$$d_v = MAX(0.9d, 0.72h) = MAX(711,634) = 711mm$$

$$V_f{}' = 84.4 + \frac{607 - 84.4}{4500}(4500 - 711) = 525\ kN$$ → @ d_v from support

At the support

$$M_f = \frac{-w_f \cdot l_n^2}{11} = -994\ kN \cdot m$$



From MacGregor and Bartlett Figure A1:

$$x = \frac{711}{9000} = 0.079 \rightarrow M_f' = 0.6 \cdot M_f = -596kN \cdot m$$

Note: The major difference between the simplified method and the general method is that the shear resistance depends on the applied moment.

Necessary for exam

Is the section large enough? Avoiding Crushing of diagonal struts

$$V_{r\,max} = 0.25 \cdot \phi_c \cdot f_c' \cdot b_w \cdot d_v$$

$$V_{r\,max} = 0.25 \times 0.65 \times 25 \times 400 \times 711 = 1155\ kN > 525kN$$

$V_{r\,max} > V_f'$

2) Are stirrups required?

$$V_c = \lambda \cdot \phi_c \cdot \beta \cdot \sqrt{f_c'} \cdot b_w \cdot d_v$$

$$\beta = \left(\frac{0.40}{1 + 1500 \cdot \varepsilon_x}\right)\left(\frac{1300}{1000 + s_{ze}}\right)$$

ε_x large, larger cracks, smaller β

$$\varepsilon_x = \frac{\left(\frac{M_f}{d_v} + V_f\right)}{2 \cdot E_s \cdot A_s} = 0.000758$$

$$s_{ze} = \frac{35 \cdot s_z}{15 + a_g} = \quad s_z = d_v\ ,with\ no\ stirrups, and\ 20mm\ aggregate$$

$$\beta = 0.142$$

$$V_c = 1 \cdot 0.65 \cdot 0.142 \cdot \sqrt{25} \cdot 400 \cdot 711 = 131\ kN < 525kN$$

Stirrups required

$V_c < V_f'$
∴ Need stirrups

3) Determine the max stirrup spacing

$$0.125 \cdot \lambda \cdot \phi_c \cdot f_c' \cdot b_w \cdot d_v = 577\ kN \rightarrow \tfrac{1}{2} V_{r,max}$$

concrete density
$\lambda = 1$ for normal density

$$525 < 577$$

therefore

$$s \leq MIN(600, 0.7 \cdot d_v) = (600, 498)$$

Based on A_v min

$$A_{v\,min} = \frac{0.06 \cdot \sqrt{f_c'} \cdot b_w \cdot s}{f_y}$$

$$s \leq \frac{f_y \cdot A_v}{0.06 \cdot \sqrt{f_c'} \cdot b_w}$$

Assuming 10M stirrups

↳ Always go with 10M stirrups

$$s \leq \frac{300 \cdot 200}{0.06 \cdot \sqrt{25} \cdot 400} \leq 500mm$$

Maximum spacing is 500 mm

4) Determine s to resist V_f'

$$S_{req'd} = \frac{\phi_s \cdot A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{V_f' - V_c}$$

$$V_c = \lambda \cdot \phi_c \cdot \beta \cdot \sqrt{f_c'} \cdot b_w \cdot d_v$$

from P.25

If $A_v > A_{v\,min}$ then $s_{ze} = 300$

$$\beta = \left(\frac{0.40}{1 + 1500 \cdot \varepsilon_x}\right)\left(\frac{1300}{1000 + 300}\right)$$

$$\varepsilon_x = 0.000758$$ → doesn't change from step 2.

from P.26

$$\beta = 0.187$$

$$V_c = 173.1\ kN$$

$$\theta = 29 + 7000 \cdot 0.000758 = 34.3°$$

$S_{req} < S_{max}$ from step 3

$$S_{req'd} = \frac{0.85 \cdot 200 \cdot 300 \cdot 711 \cdot \cot(34.3)}{525 - 173.1} = 151mm$$

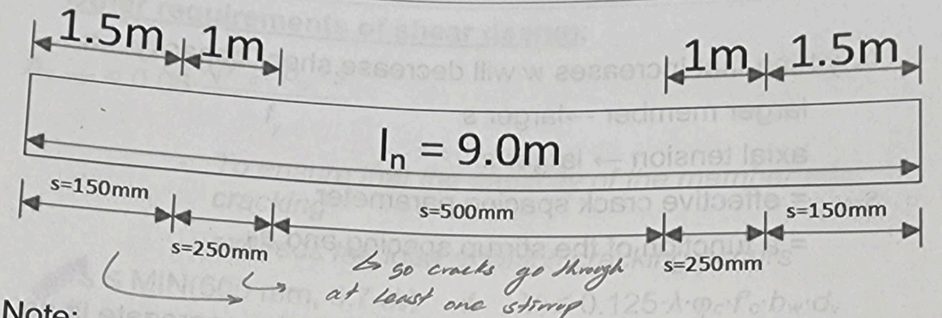
$\frac{1}{\tan(34.3)}$

5) Repeat steps 1-4 @ several other locations

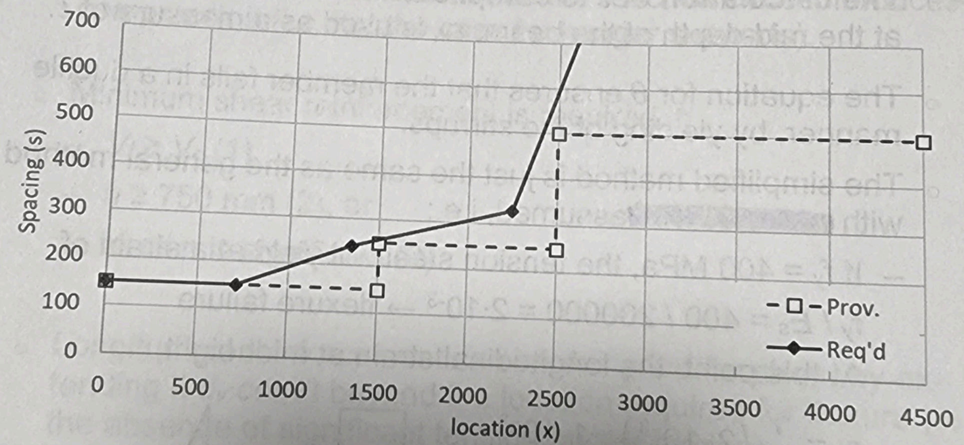
X	711	1350	2250	3600	mm
V_f'	525	451	346	189	kN
M_f'	-596	-298	342	615	$kN \cdot m$
A_s	4500	4500	3000	3000	mm^2
$S_{req'd}$	151	242	330	1747	mm

One possible stirrup layout

↳ max spacing is 500mm so at 3600mm distance, 1747 → 500.

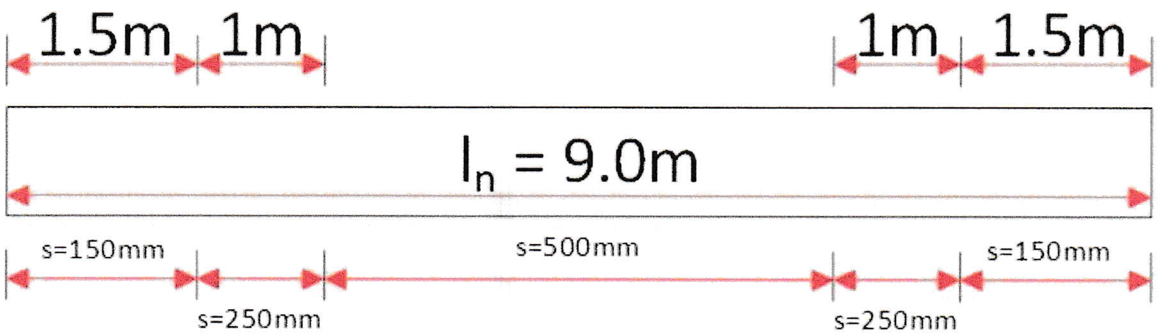


Note:
- Further check of spacing at 2500 should be done to ensure it is 500 mm
- no stirrups required when V_f drops below V_c (if h < 750mm), however some engineers would provide minimum stirrups regardless.



Final Comments and Review:

-
- Aggregate interlock depends on the concrete strength, aggregate size, and crack width.
- $w = \varepsilon \cdot s$, where ε = average strain perpendicular to crack, and s = crack spacing



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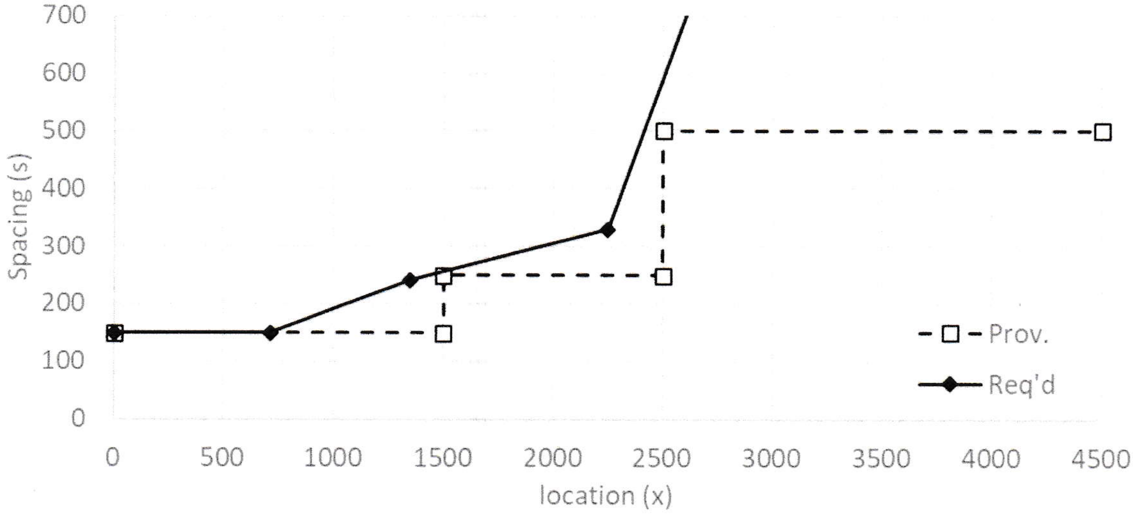


Final Comments and Review:

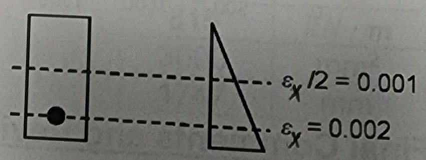
-
- Aggregate interlock depends on the concrete strength, aggregate size, and crack width.
- $w = \varepsilon \cdot s$, where ε = average strain perpendicular to crack, and s = crack spacing

- Anything that increases w will decrease shear capacity, i.e.:
  larger member → larger s
  axial tension → larger ε
- s_{ze} = effective crack spacing parameter
  = a function of the stirrup spacing and a_g

- For high strength concrete ($f'_c \geq 60$ MPa), the aggregate is assumed to fracture (i.e. the crack runs through the aggregate), and therefore the roughness parameter, a_g, is reduced:
  $a_g = 0$ for $f'_c \geq 70$ MPa

➤ **The calculation of ε is complicated**. The longitudinal strain at the mid-depth of the beam, ε_x, is used as a measure of ε.

- The equation for θ ensures that the member fails in a ductile manner, by yielding of the stirrups.
- The simplified method is just the same as the general method with $\varepsilon_x = 0.85 \cdot 10^{-3}$ assumed, i.e.:
  - If $f_y = 400$ MPa, the tension steel will yield at a strain of:
    $f_y / E_s = 400 / 200000 = 2 \cdot 10^{-3}$ → flexure failure
  - At this point, the longitudinal strain at midheight,
    $\varepsilon_x = \frac{1}{2} \cdot \left(2 \cdot 10^{-3}\right) = 1 \cdot 10^{-3}$



Simplified method

- The assumed $\varepsilon_x = 0.85 \cdot 10^{-3}$ limit ensures that shear failure does not occur prior to flexural failure.
- $\varepsilon_x = 0.85 \cdot 10^{-3}$ → $\theta = 35°$,
  → $\beta = 0.18$ (with stirrups)
  $= \dfrac{230}{1000 + d_v \text{(or } s_{ze})}$ (without stirrups)

W5L2

- **Other requirements of shear design:**

$$A_{v,min} = 0.06 \cdot \frac{\sqrt{f'_c} \cdot b_w \cdot s}{f_y}$$

- *To ensure that the capacity of the member after cracking exceeds the load at which cracking occurs*

- $s \le \mathrm{MIN}(600\text{ mm}, 0.7 \cdot d_v)$ if $V_f \le 0.125 \cdot \lambda \cdot \varphi_c \cdot f'_c \cdot b_w \cdot d_v$
- $s \le \mathrm{MIN}(300\text{ mm}, 0.35 \cdot d_v)$ if $V_f > 0.125 \cdot \lambda \cdot \varphi_c \cdot f'_c \cdot b_w \cdot d_v$

Not at the support

- The cross-section near a support may be designed for V_f at a distance, d_v, from support, provided there are no concentrated forces within the distance d_v and the reaction force introduces compression (i.e. not the case for hanger supports).

- Minimum shear reinforcement is required if:
  - $V_f \ge V_c$ (1)
  - $h \ge 750$ mm (2), or
  - torsion is significant (3).

- Longitudinal reinforcement can be designed for shear by extending it $d_v \cdot cot(\theta)$ beyond the location required for flexure (in the absence of significant tension and/or torsion).
- Or provide tension reinforcement such as: *Clause 11.3.9.2*
- $F_{lt} = \frac{M_f}{d_v} + 0.5N_f + \left(V_f - 0.5V_s - V_p\right)\cot\theta$

- At exterior direct bearing supports, design the longitudinal reinforcement for a tension force equal to:

$$\left(V_f - 0.5 \cdot V_s\right) \cdot \mathrm{COT}(\theta) + 0.5 \cdot N_f$$

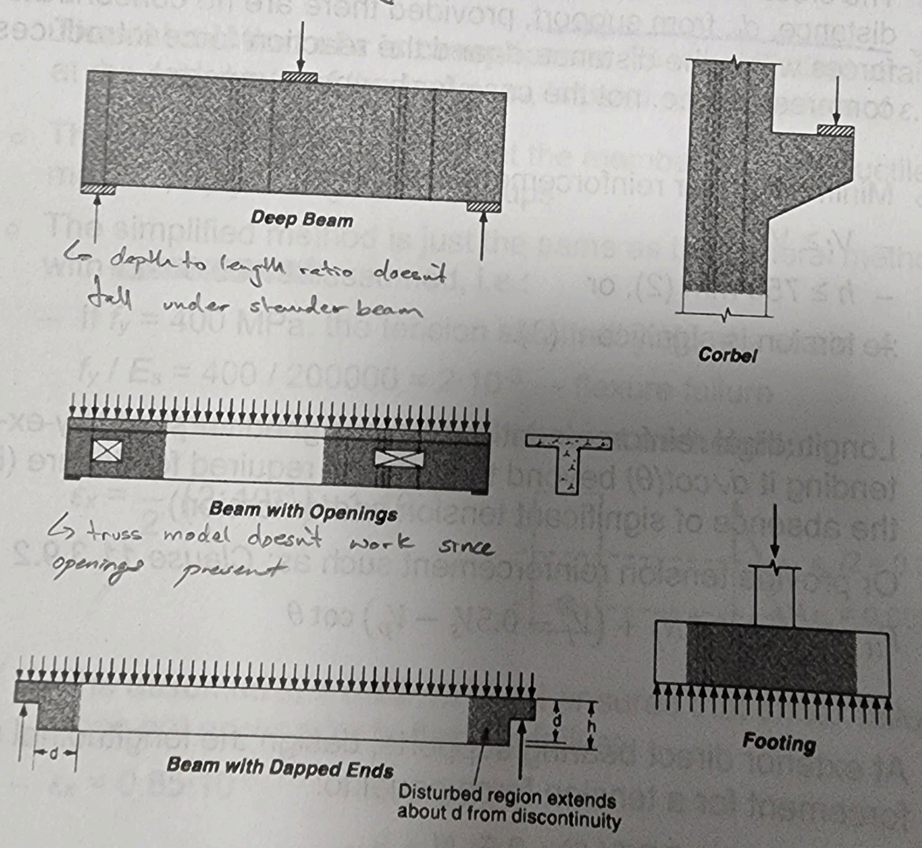
62522

SHEAR DESIGN IN D- REGIONS USING STRUT-AND-TIE METHOD

- Simple truss models and the modified compression field theory were developed for B-regions, which are characterized by a linear strain distribution (i.e. "plane sections remain plane").
- In D-regions, these approaches are not directly applicable.
- In D-regions, a significant portion of the shear is carried by in-plane forces (i.e. "arch action").
- examples of D-regions:

Disturbed-Regions

for beams that are not slender



↳ depth to length ratio doesn't fall under slender beam

↳ truss model doesn't work since openings present

Fig. N11.1.2(a)
Examples of disturbed regi... [CSA A23.3 2004]

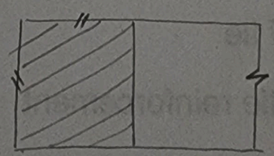
- D-regions are characterized by abrupt changes in geometry and/or concentrated loads.
- Historically, D-regions designed using rules of thumb. These led to inconsistent performance and failures in some cases.
- Work to replace rules of thumb with a rational design procedure has resulted in the strut-and-tie modelling approach.
- Landmark article: [Schlaich et al. 1987] first formalized the idea of modelling D-regions using strut-and-tie models.

– [ACI Special Publication SP-208]:

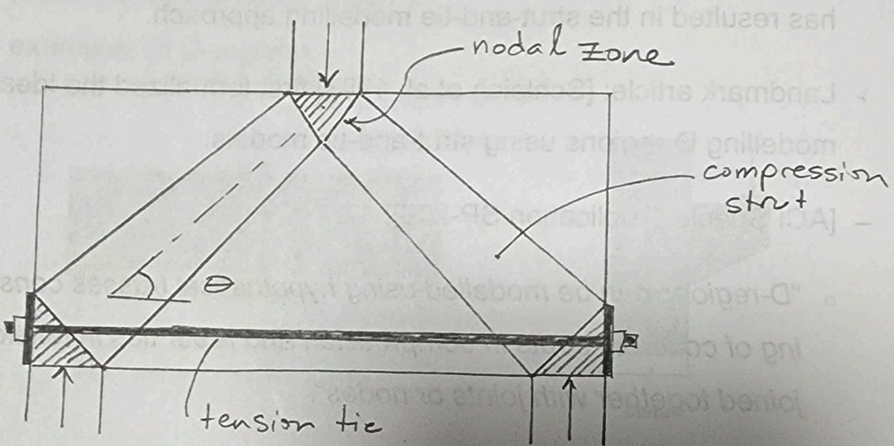
  - *"D-regions can be modelled using hypothetical trusses consisting of concrete struts in compression and rebar ties in tension, joined together with joints or nodes".*

– definition of D-regions:

  - based on St. Venant's principal: *"Stresses caused by a self-equilibrating stress system tend to die out over a distance equal to the extent of the self-equilibrating system."*



- Prior to cracking, elastic (i.e. finite element) analysis can be a useful tool for predicting the behaviour of D-regions.
- After cracking, D-region can be modeled with strut-and-tie model consisting of: 1) concrete compression struts, 2) steel tension ties, and 3) joints or nodal zones.

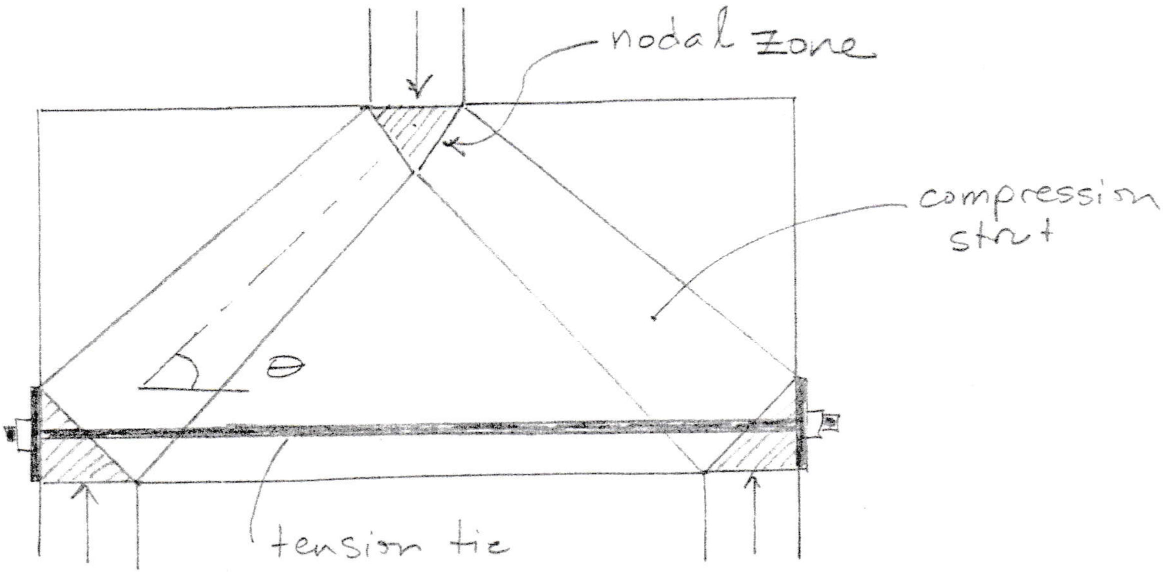


i.e. deep beam:

- D-region can fail by:
  1. compression failure of strut
  2. yielding of tie
  3. pullout of tie reinforcement
  4. crushing of concrete in nodal region

➢ Prior to cracking, elastic (i.e. finite element) analysis can be a useful tool for predicting the behaviour of D-regions.

➢ After cracking, D-region can be modeled with strut-and-tie model consisting of: 1) concrete compression struts, 2) steel tension ties, and 3) joints or nodal zones.



i.e. deep beam:

➢ D-region can fail by:

1. compression failure of strut
2. yielding of tie
3. pullout of tie reinforcement
4. crushing of concrete in nodal region

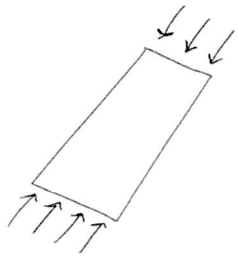
- Essentially, a strut-and-tie model is a truss-like representation of the D-region. The main purpose of the model is to provide a consistent and rational approach for: 1) verifying the four failure modes listed above, and 2) determining where to place the reinforcing steel and how much to provide.
- Note: Strut-and-tie models can also be used in B-regions!
- Why does strut-and-tie modelling work?
    - Lower bound plasticity theory:

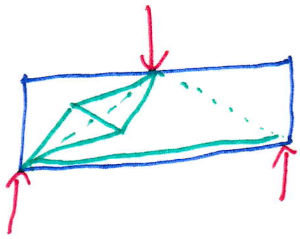
      "A load system based on a statically allowable stress field (i.e. a stress field satisfying equilibrium and static boundary conditions) that does not violate the yield condition provides a lower bound of the ultimate load."

    - Strut-and-tie models provide a lower bound solution and thus a safe estimate of the true D-region capacity, provided that: 1) the structure is sufficiently ductile to accommodate any redistribution of forces needed to carry the load, and 2) the model is in equilibrium with the applied loads and the strut-and-tie forces satisfy the rules of statics.

Model Components:

Compression strut:





1) Compression strut:

$$f_c = \phi_c f_{cu}$$

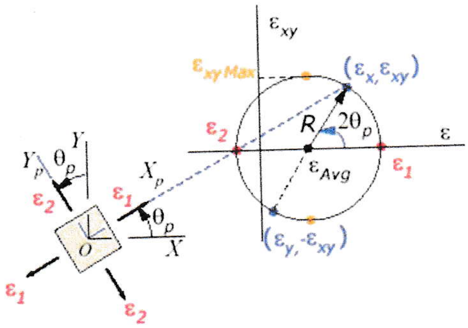
f_c = factored compressive strength of strut

f_{cu} = effective compressive strength of strut

$f_{cu} = \alpha \times f'_c$ from cylinder testing

α = efficiency factor

↳ accounts for tensile strain ⊥ strut direction



$$f_{cu} = \frac{f'_c}{0.8+170\varepsilon_1} \leq 0.85 f'_c$$

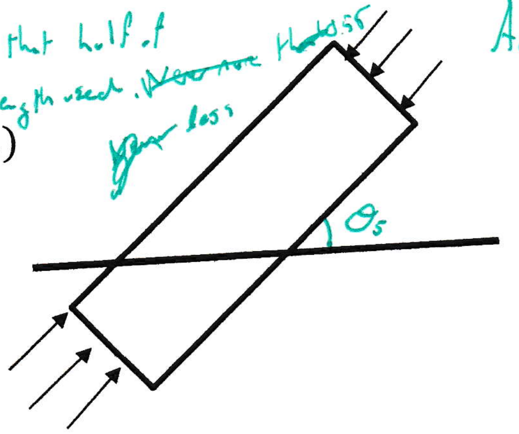
$\alpha = \frac{1}{0.8+170\varepsilon_1}$, if no code, use $\alpha = 0.5$ so that half of strength used. ~~Never more than~~ 0.55 your loss

According to Schlaich, never less than 0.55.

$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cdot \cot^2(\theta_s)$

ε_1 is tensile strain

(compatibility condition)



$\varepsilon_s = \frac{f_s}{E_s}$ or

$\varepsilon_s = \frac{f_y}{E_s}$ $\qquad \varepsilon_s = 0.002$

Assuming $\varepsilon_s = 0.002$ →

θ	60°	45°	30°
f_{cu} / f'_c	0.73	0.55	0.31

very conservative

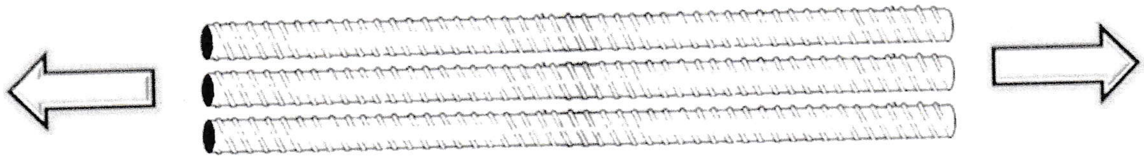
($\theta \geq 40°$ recommended, min 25°)

$f_{cu} = f'_c / (1.14 + 0.68 \cot^2(\theta))$ but $\leq 0.85 \cdot f'_c$ if $\varepsilon_s = 0.002$

cl. 11.4.2.3

Important: don't have full strength of compression strut

2. Tension tie:



$T_f \leq T_r$ **where:** $T_r = \phi_s \cdot A_s \cdot f_y$

Note: Anchorage of tension ties is also important.

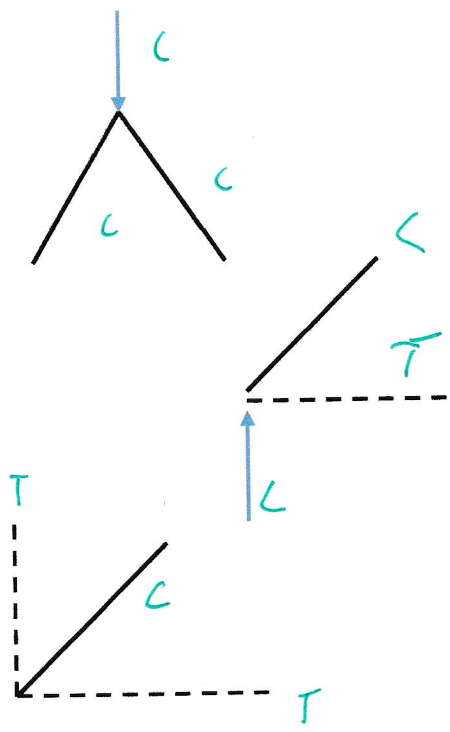
2) **Nodal Zones:**

- Although not required, it is sometime useful to consider the nodal zones as "hydrostatic elements" (i.e. assume the same stress level on each face of the node).
- must be in equilibrium
- **types of nodes:**

  1) CCC

  2) CCT

  3) CTT or TTT



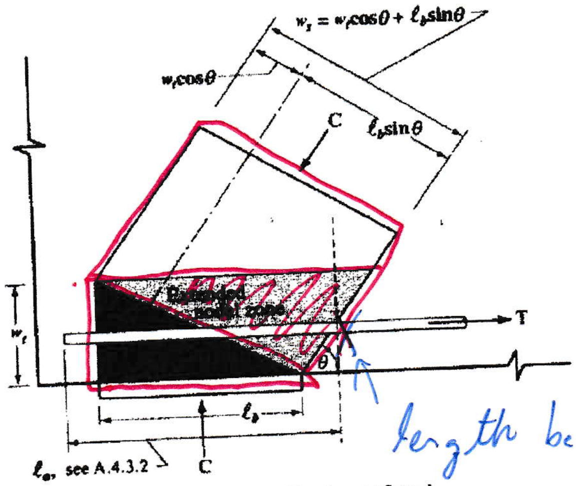
– stress limits for nodes:

CCC nodes → $0.85 \cdot \phi_c \cdot f'_c$

CCT nodes → $0.75 \cdot \phi_c \cdot f'_c$

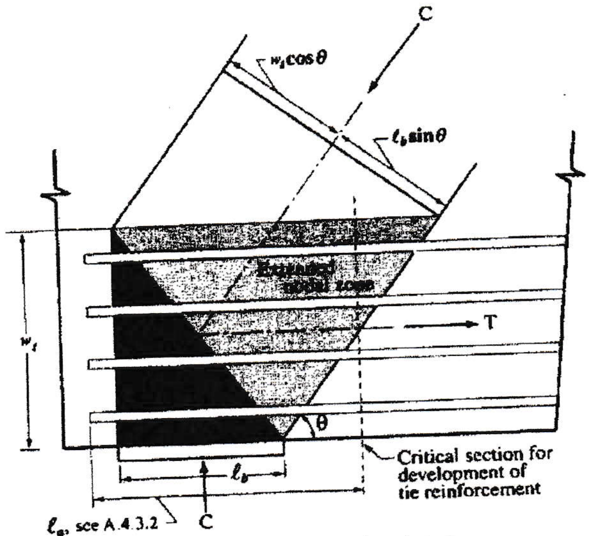
CTT or TTT nodes → $0.65 \cdot \phi_c \cdot f'_c$

– anchorage of tie reinforcement at nodes:



(a) One layer of steel

length before crossing is development length



(b) Distributed steel

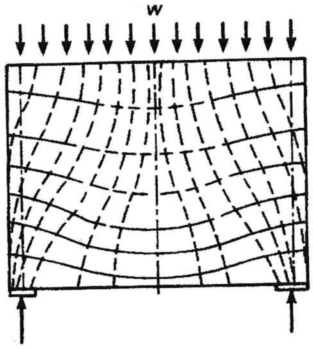
Solution Method:

1) Isolate the D-region.
2) Compute boundary stresses (these are obtained from structural analysis and often from B-region design).
3) Subdivide the boundary and compute the resultant forces.
4) Draw a truss to transmit the forces between the boundaries.
5) Check the stresses in the truss elements.
6) Draw model to scale!!!

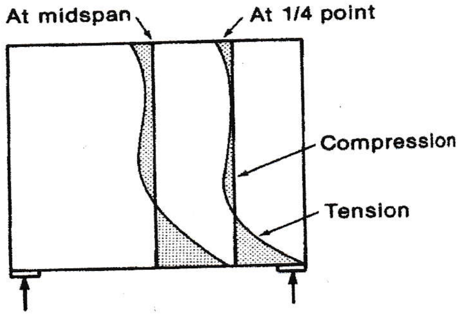
Guidelines:

- Elastic analysis is useful for predicting 1st crack location and for guiding the engineer as to where to place the reinforcement.
- Accuracy of strut-and-tie model depends on closeness of approximation to reality. If excessive deformation is required to reach the plastic state, the D-region may fail prematurely.
- Compressive struts should follow comp. stress trajectories ±15°.
- Model with the fewest and shortest ties will usually be the "best".
- if Statically indeterminate- assume steel yields
- Anchorage of the bars very important

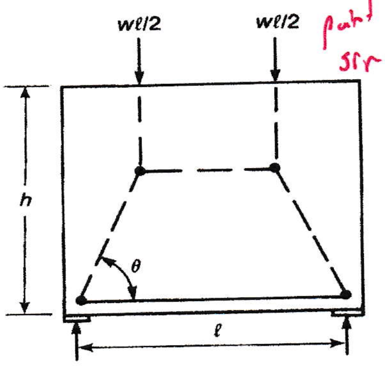
--- C
— T



(a) Stress trajectories.

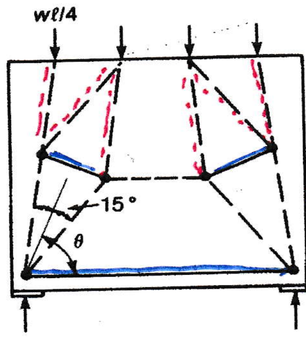


(b) Distribution of theoretical horizontal elastic stresses.



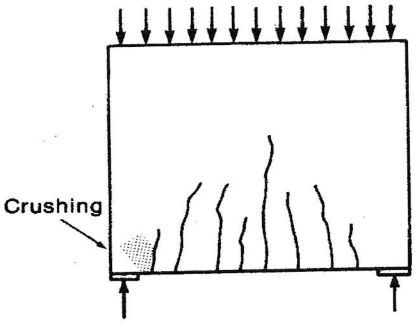
point loads
space truss

(c) Truss model.
$\theta = 68°$ if $\ell/h \leq 1$
$= 54°$ if $\ell/h = 2$



make this horizontal

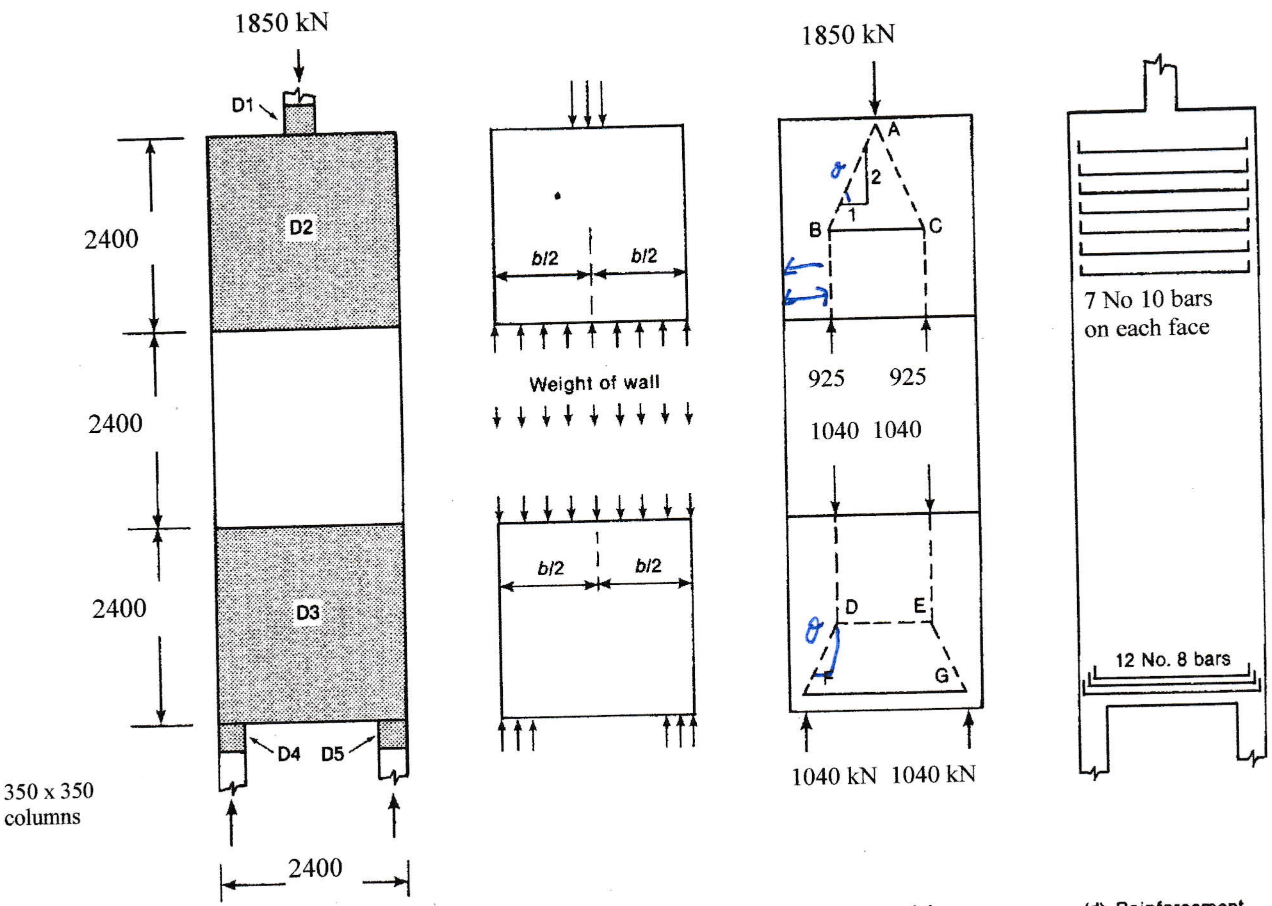
(d) Refined truss model.



(e) Crack pattern.

Example 3 : Wall :, Design D2 Region

Wall: 350mm thick
Columns 350 x 350 mm
$f_c^{'} = 35MPa$



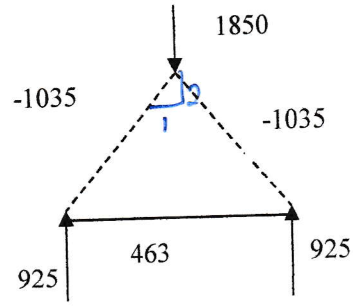
(a) Wall and D-regions. (b) Stresses on D-regions. (c) Truss model. (d) Reinforcement.

MacGregor, J.G., Bartlett, M. "Reinforced Concrete Mechanics and Design", Prentice Hall,2000

1. **Assume 2:1** slope for the struts (if geometry of the wall allows it)
   could also assume less, e.g. 1:1

Benjamin Klassen

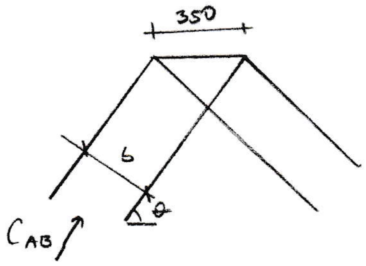
2. Calculate Member forces (kN)



3. Check strength of the struts and the ties

STRUT AB

/ column width



$$\Theta = 63.4^o$$

$$b = \frac{350mm}{2 \times \sin 63.4^o} = 196mm$$

force in strut

$$f_{cAB} = \frac{C_{AB}}{b \times t} = \frac{1035}{196 \times 350} = 15.1\,\text{MPa}$$

cross-sectional area in strut

Using CSA limiting stresses

$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cdot cot^2(\theta_s)$

assume $\varepsilon_s = \varepsilon_y = 0.002$

$\theta_s = 63.4^0$ $\quad \varepsilon_1 = 0.003$

$$f_c = \frac{\phi_c f_c'}{0.8 + 170\epsilon_1} = \frac{0.65 \times 35 \times 1}{0.8 + 170 \times 0.003} = 17.3\ MPa > 15.1\ MPa$$

$\leftarrow \lambda$

$f_c > f_{cAB}$ OK

Tension in bar is assumed to be a constant force

TIE B-C

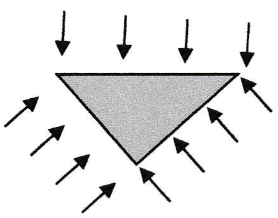
$$\boxed{T_s = 463kN}$$

$$A_s = \frac{T_s}{\phi_s f_y} = \frac{463 \times 10^{-3}}{0.85 \times 400\text{MPa}} = 1362\, mm^2$$

Provide 14 No. 10 bars → A_s = 1400 mm^2

- The centroid of the bars must be where we assumed the tie in the ST model.
- Bars must be properly anchored (development length, hooks, or mechanical ancorage)
- Bars should be uniformly distributed: *0.3 x d* from each side of the tie.

Node A: C-C-C- type



$$f_1 = \frac{1850}{350 \times 350} = 15.1\,\text{MPa - hydrostatic node}$$

$$f_2 = f_3 = 15.1MPa$$ ccc

$f_c = 0.85\, \phi_c\, f_c' = 0.85 \times 0.65 \times 35 = 19.0\ MPa$

$f_c > f_1$ OK

In the vast majority of cases, the strut forces govern , not nodal forces.

Crack Control Reinforcement

0.2% reinforcement must be provided in each orthogonal direction, near each face of the wall. Maximum spacing 300 mm *cl. 11.4.5*