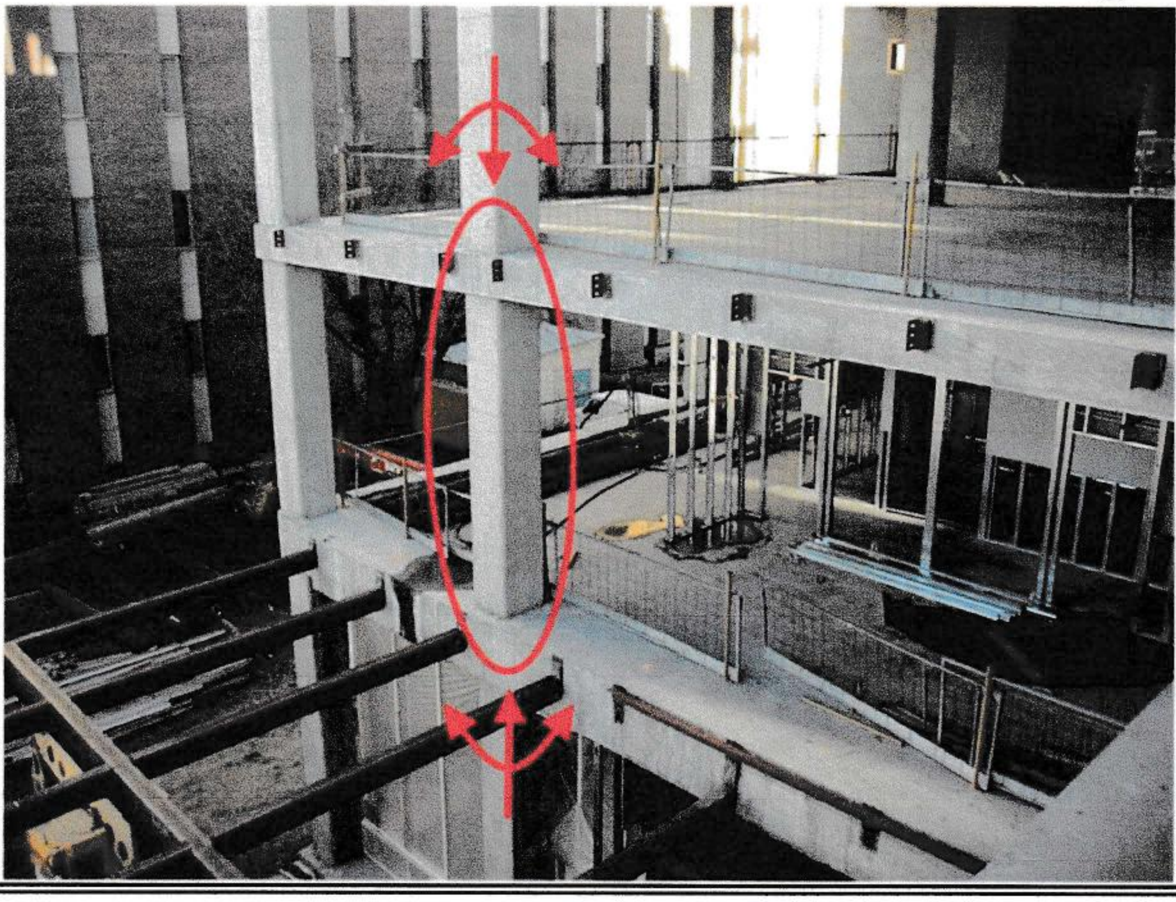
CivE 414
Structural Concrete Design

Topic 6
COLUMNS
Axial Compression and Bending



CONCRETE COLUMNS

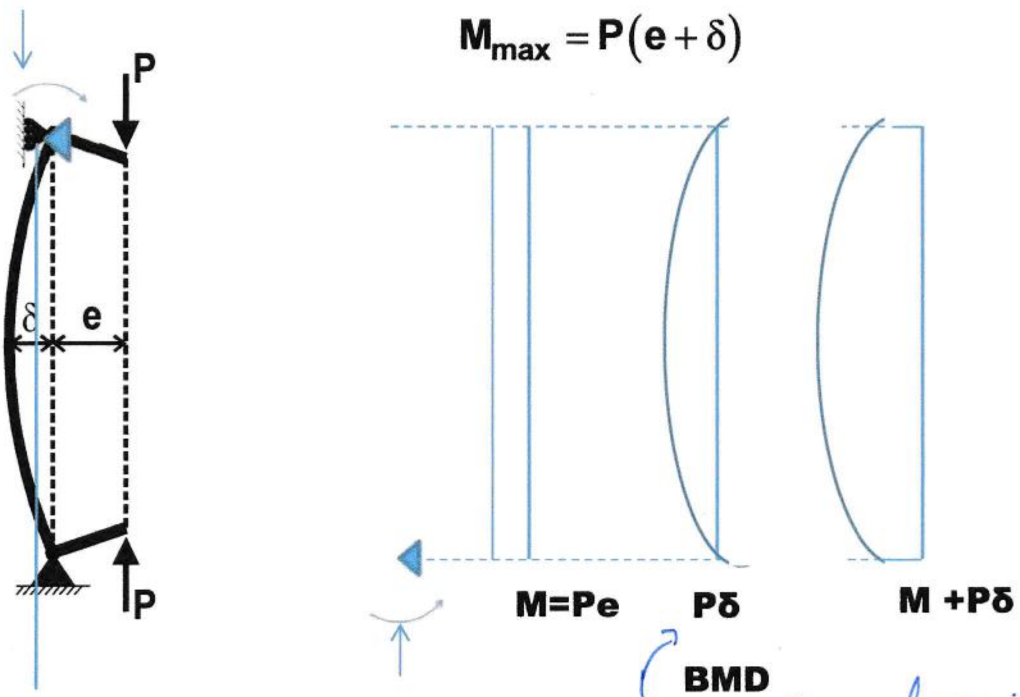
- Primarily compression members
- In general, must design for combined axial load and bending
- Usually vertical, but may be inclined or horizontal in trusses and frames

Because columns are subjected primarily to compression loading, stability effects must be considered. This is done as follows:

- If these effects have little or no impact on the column capacity, we have what we call a "short" column. In this case, stability effects can be safely ignored.
- If stability effects significantly reduce the column capacity, we have a "slender" column. In this case, stability effects must be considered explicitly in the design.

- **Short columns:** no second order effects or buckling
- **Slender columns**: influenced by second order effects (buckling may occur)

$$M_{max} = P(e + \delta)$$

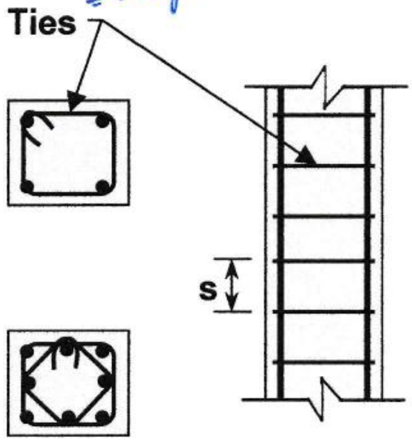


Question: how large is δ

TYPES OF COLUMNS

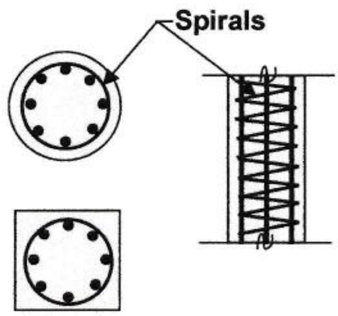
1. Tied Columns

= always a closed loop, unlike U-bars



- Ties are used mainly to prevent **buckling** of the longitudinal bars and consequently, to prevent the concrete cover from spalling off.
- For a large number of bars, other tie arrangements may be required.

2. Spiral Columns

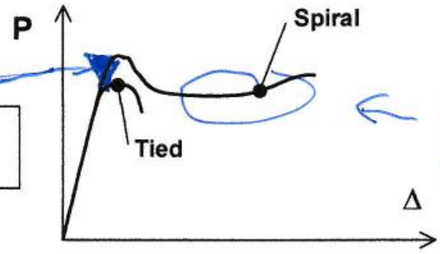


- Spirals are helical ties (continuous) which contain the concrete and prevent local buckling.
- Spirals may be used in square columns.
- Ductility and ultimate strength are increased with the use of spirals.

! good for seismic

confinement increases compressive strength

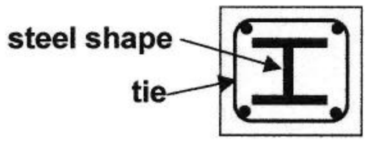
Load vs Deflection for Tied & Spiral Columns



← increases ductility

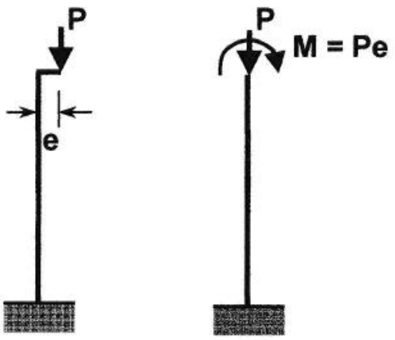
3. Composite Columns

Combination of structural steel shape and reinforced concrete



COLUMNS IN BENDING

- Very rare for a column to be subjected to pure axial load
- Both vertical loads and lateral loads produce moments in frame columns

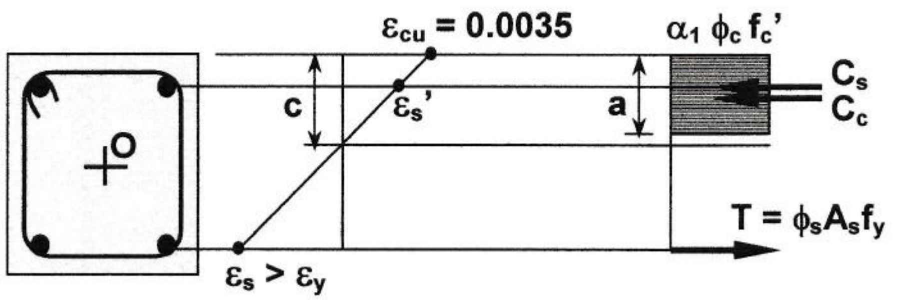


For $e = 0$ → pure axial load ($M = 0$)
$e = \infty$ → pure bending ($P = 0$)

PURE BENDING

$M > 0$, $P = 0$ → $e = \infty$

Short column to stretch



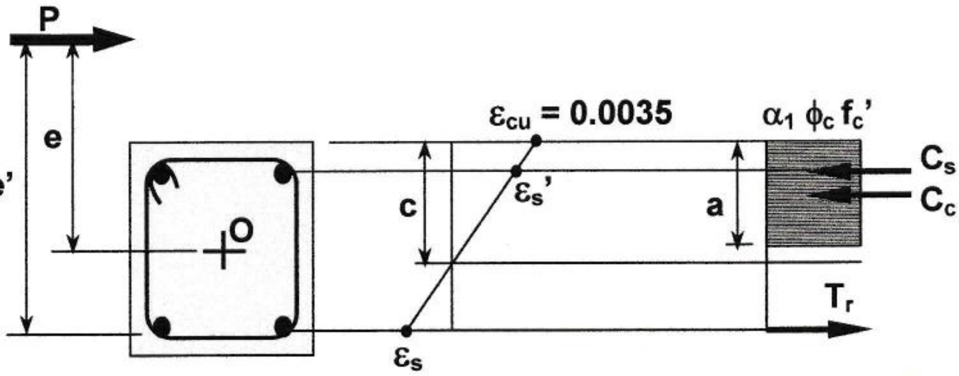
Summation of forces: $\mathbf{C_c + C_s - T = 0}$

Summation of moments: $\mathbf{M_r = C_c(d - a/2) + C_s(d - d')}$

(no axial load, any point can be used for summation of moments)

MOMENT AND AXIAL LOAD

$M > 0, P > 0 \rightarrow 0 < e < \infty$



1. Summation of forces: $\mathbf{P = C_c + C_s - T}$ Axial load

2 Summation of moments:

a) about O *(centroid)*: ← preferred as this corresponds to structural analysis results

M $$\mathbf{Pe = C_c(h/2 - a/2) + C_s(h/2 - d') + T_r(d - h/2)}$$

or

b) about T_r: location of tensile reinforcement
$$\mathbf{Pe' = C_c(d - a/2) + C_s(d - d')}$$

where $\mathbf{e' = e + (d - h/2)}$

Two unknowns: **P and a** (for a given e)
e and a (for a given P)

(a is related to strain in tension reinforcement)

Two equations: $\mathbf{\Sigma F = 0}$

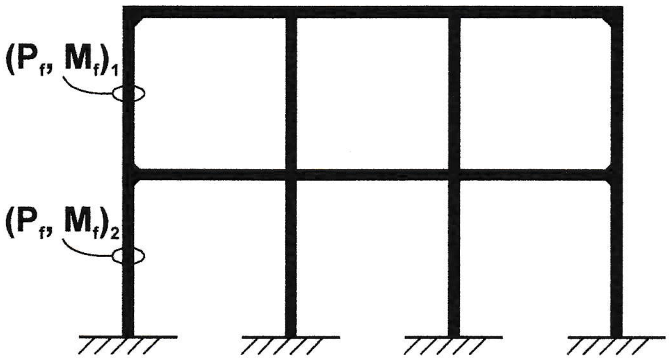
$\mathbf{\Sigma M = 0}$

➢ **of P_r & M_r**

Analysis of Column Capacity

- Frame analysis results give P_f and M_f for each column

- Check adequacy of a given column section and reinforcement



Approach 1 – Analysis for a Given Eccentricity

Given: $e = \dfrac{M_f}{P_f}$ $\qquad e' = e + (d - h/2)$

- Looking for $\mathbf{P_r}$ **&** $\mathbf{M_r}$

Equilibrium: $\qquad P = C_c + C_s - T$

Moment about T: $\qquad Pe' = C_c(d - a/2) + C_s(d - d')$

Unknowns: a, P

Solution: $P_r = P$ $\qquad$ for given "e"

$M_r = P\,e$

Verify: $P_r \geq P_f$ for column in question
$M_r \geq M_f$

APPROACH 2 – ANALYSIS FOR A GIVEN AXIAL LOAD

Given: $P_r = P_f$ find M_r *(and e)*

Equations:

Equilibrium: $$P_f = C_c + C_s - T$$

Moment about T: $$P_f e' = C_c\left(d - a/2\right) + C_s\left(d - d'\right)$$

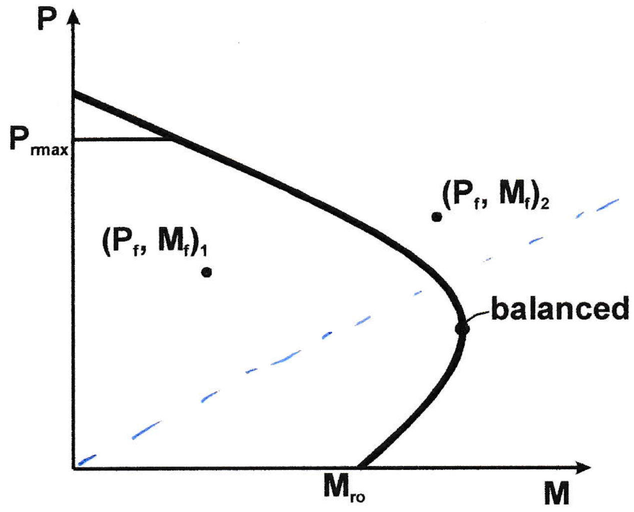
Unknowns: a, e'

Solution: $e = e' - \left(d - h/2\right)$ for given "P_f"
$M_r = P_f e$

Verify: $M_r \geq M_f$ for column in question

APPROACH 3 – GENERATE P-M INTERACTION DIAGRAM

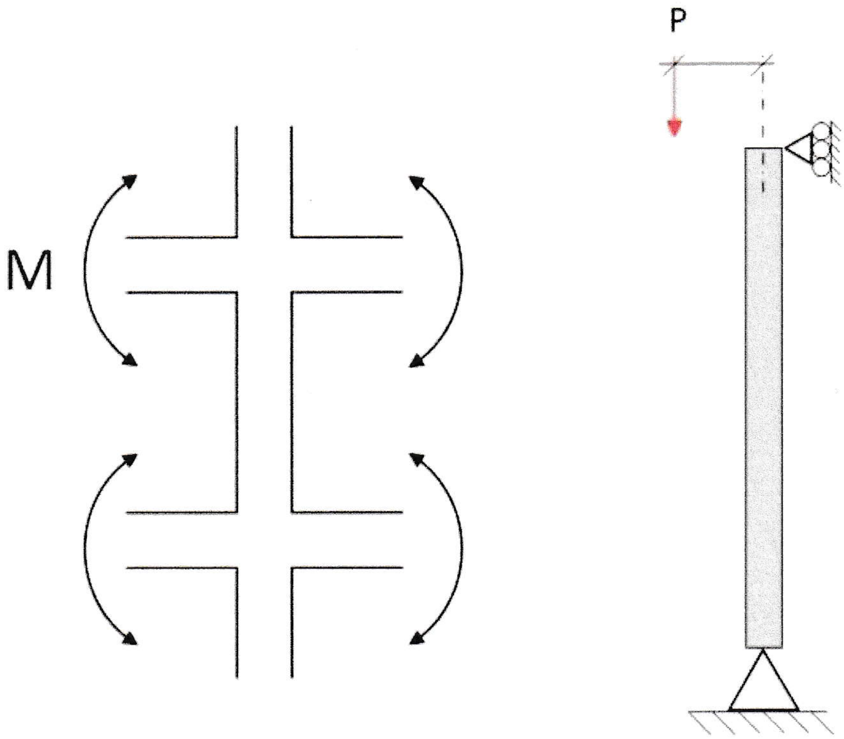
➢ Useful for several combinations of P_f and M_f



P_f, M_f must be within interaction diagram

Interaction Diagrams

- Columns are usually loaded by a compressive axial load and a moment. Combination
- Bending moments can be introduced at column ends via monolithically cast beams or slabs or due to eccentricity of the point of axial load application, with respect to the column centroid.



Under combined axial and bending loads, the strength of a column is generally lower than in the pure axial load case, because the axial and bending stresses add together, reaching the concrete compressive strength at a lower load level. This is accounted for in design through the use of "interaction diagrams".

For an ideal elastic, brittle material, the theoretical interaction diagram can be determined as follows:

- Failure occurs when the strength, $f_{cu} = f_{tu} = f$, is reached.
- Failure of the x-section is thus defined by the locus of points:

$$f = \frac{P}{A} \pm \frac{M \cdot y}{I}$$ *(Stress is a combination of stress due to axial load and moment)*

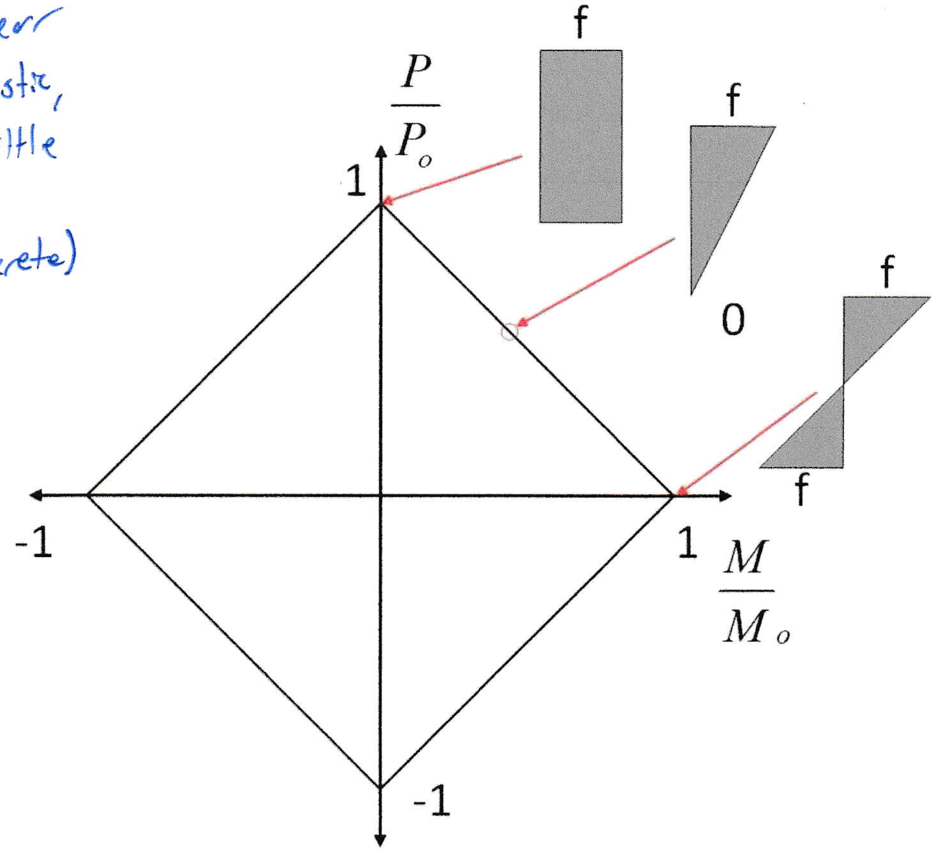
– This expression can be rewritten (normalized) :

$$1 = \frac{P}{A \cdot f} \pm \frac{M \cdot y}{I \cdot f} = \frac{P}{P_0} \pm \frac{M}{M_0}$$

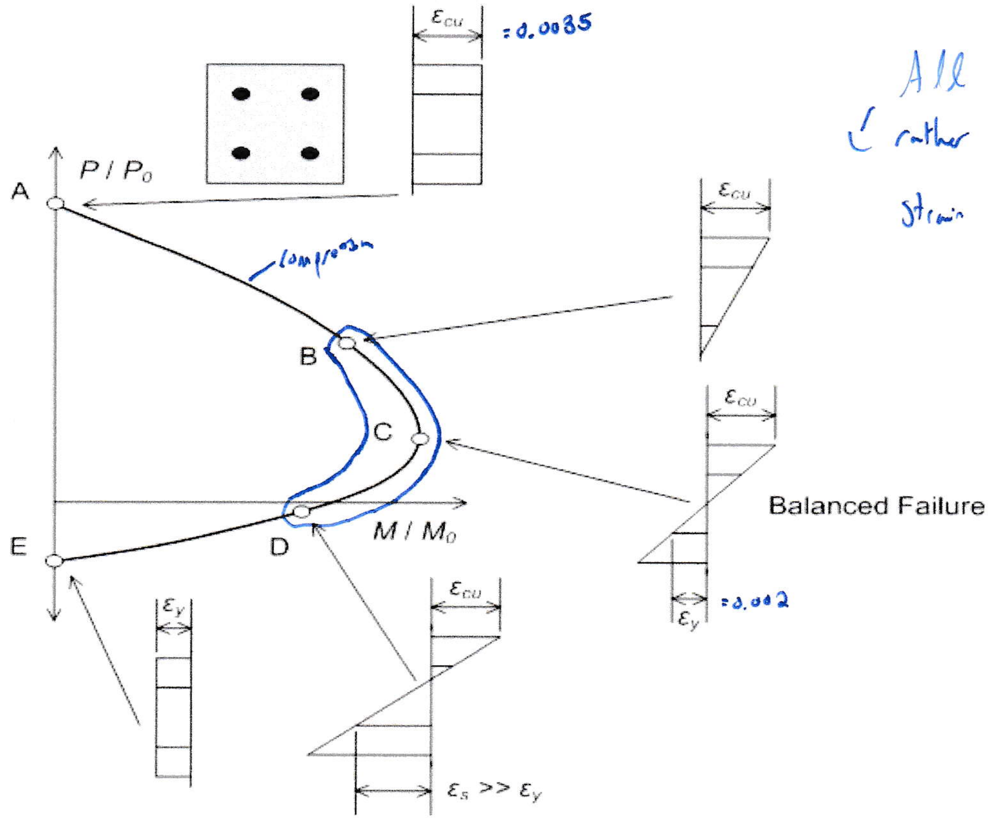
– The resulting "interaction diagram" takes the following form:

linear elastic, brittle

(Not concrete)



For reinforced concrete columns, the interaction diagram can be determined by analyzing the column x-section under various strain distributions, corresponding with different P-M combinations:



All based on strain distribution, rather than stress, as strain easier to predict.

Each point on the interaction diagram represents the maximum axial load that can be applied at a given eccentricity.

Three Types of Failure:

1. Tension failure $\quad \varepsilon_s > \varepsilon_y$
   $e > e_b$
2. Balanced failure $\quad \varepsilon_s = \varepsilon_y$
   $e = e_b$
3. Compression failure $\quad \varepsilon_s < \varepsilon_y$
   $e < e_b$

- To determine points A-E indicated in the preceding diagram (and others), the following assumptions are made:

– ε_{cu} = 0.0035

– $\varepsilon_y = f_y / E_s$ = 400 MPa / 200 000 MPa = 0.002

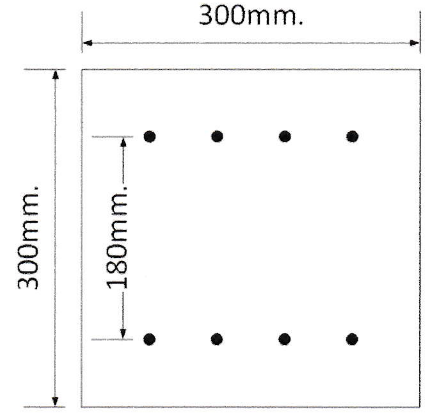
Example 1

- Calculate factored balanced failure point for the column in the Figure

$$f_c' = 40\ MPa$$

$$8 - 20M\ bars$$

$$f_y = 400\ MPa$$



Solution *(compression positive in this example)*

$$C = 240 \cdot \frac{0.0035}{0.002} = 152.7mm$$

$$\varepsilon_{s1} = -0.002 \quad \varepsilon_c = -0.0035 \quad \text{(given)}$$

$$\varepsilon_{s2} = 0.0035\left(\frac{152.7-60}{152.7}\right) = 0.00213$$

$$f_{s1} = -400\ MPa\ (tension)$$

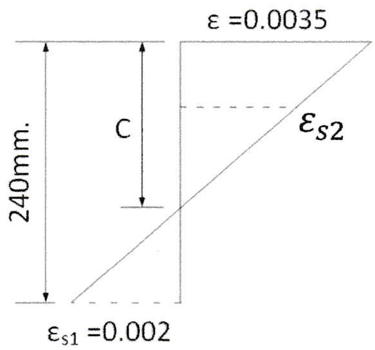
$$f_{s2} = MIN(0.00213 \cdot 200GPA, 400MPa)$$

$$f_{s2} = 400\ MPa$$

$$\beta_1 = 0.97 - 0.0025(40) = 0.87$$

$$a = \beta_1 \cdot c = 0.87 \cdot 152.7 = 132.9mm$$

$$\alpha_1 = 0.85 - 0.0015(40) = 0.79 > 0.67\ \ ok$$



Concrete

$$C_{rc} = \alpha_1 \cdot \phi_c \cdot f_c' \cdot a \cdot b = 0.79 \cdot 0.65 \cdot 40 \cdot 132.9 \cdot 300 = 818.8\ kN$$

Steel

$$F_{rs1} = \phi_s \cdot f_{s1} \cdot A_{s1} = 0.85 \cdot 400 \cdot 1200\ \times 10^{-3} = -408\ kN$$

$$F_{rs} = (\phi_s \cdot f_s - \alpha_1 \cdot \phi_c \cdot f_c')A_{s2} = (0.85 \cdot 400 - 0.79 \cdot 0.65 \cdot 40) \cdot 1200 \times 10^{-3}$$

$\alpha_1 \cdot \phi_c \cdot f_c'$ already accounted for when calculating C_{rc} where area replaced by compression steel was not accounted for.

$$F_{rs2} = 383.4\ kN$$

$$P_r = +818.8 + 383.4 - 408 = 794\ kN$$

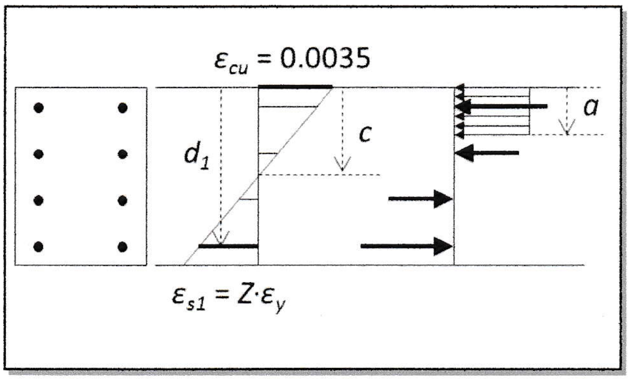
Moment around centroidal axis

$$M_r = C_{rc}\left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n} F_{rsi}\left(\frac{h}{2} - d_i\right)$$

$$M_r = 818.8\left(150 - \frac{132.9}{2}\right) - 408(150 - 240) + 383.4(150 - 60)$$

$$M_r = 139.6\ kN \cdot m$$

Steps for calculating points on the interaction diagram:



1. Choose the strain distribution (through the depth of the column)

2. Determine depth to neutral axis, *c*, by inspection

3. Calculate depth of rectangular stress block, $a = \beta_1 \cdot c$ where $\beta_1 = 0.97 - 0.0025 \cdot f_c' \geq 0.67$

4. Calculate compression force in the concrete, $C_{rc} = \alpha_1 \cdot \phi_c \cdot f_c' \cdot a \cdot b$ where *b* is the column width, where

$\alpha_1 \quad = \quad 0.85 \quad - \quad 0.0015\, f'_c \ \geq \ 0.67$

5. Calculate forces in compression and tension reinforcement:
   a. Tension reinforcement force, $F_r = \phi_s \cdot f_s \cdot A_s$
   b. Compression reinforcement force, $F_r = \left(\phi_s \cdot f_s - \alpha_1 \cdot \phi_c \cdot f_c'\right) \cdot A_{st}$

6. Calculate the axial force and moment, *P* and *M*, by assuming force and moment equilibrium of the section

Short Columns

Under purely compressive loading, the unfactored resistance of a column is a function of: the x-sectional areas and strengths of the concrete and longitudinal reinforcement present, i.e.:

– $P_0 = \underbrace{\alpha_1 \cdot f'_c \cdot (A_g - A_{st})}_{\text{concrete}} + \underbrace{f_y \cdot A_{st}}_{\text{steel}}$

– where:
A_g = gross x-section area of column
A_{st} = area of reinforcing steel
f'_c = specified compressive strength of concrete
f_y = specified yield strength of reinforcing steel

– $\alpha_1 = 0.85 - 0.0015 \cdot f'_c \geq 0.67$ *[CSA A23.3 §10.1.7]*. *(equivalent stress block factor)*

Short Column Analysis and Design

In order to account for unintended eccentricities, the maximum load that a column can carry is limited *[CSA A23.3 §10.10.4]*:

– $P_{r,max} = 0.80 \cdot P_{r0}$ for tied columns (80% theoretical strength

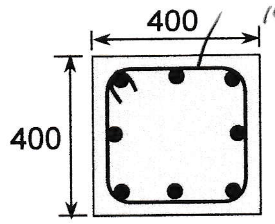
– $P_{r,max} = 0.85 \cdot P_{r0}$ for spiral columns (85%)

Where: $P_{r0} = \alpha_1 \cdot \phi_c \cdot f'_c \cdot (A_g - A_{st}) + \phi_s \cdot f_y \cdot A_{st}$

Example 2: ***Calculate the factored axial resistance of the tied column shown.***

ties so longitudinal reinforcement doesn't buckle

Also provide some confinement



8 – 20M

Assume:

f'_c = 30 MPa

f_y = 400 Mpa

α_1 =0.81 (α_1 =0.85 – 0.0015f'_c) ≥ 0.67

$$A_g = 400 \times 400 = 160{,}000\,\text{mm}^2$$

$$A_s = 8 \times 300\,\text{mm}^2 = 2{,}400\,\text{mm}^2$$

$$P_{ro} = \alpha_1 \phi_c f'_c \left(A_g - A_s\right) + \phi_s A_s f_y$$

$$= (0.81)(0.65)(30\text{MPa})\left(160{,}000 - 2400\,\text{mm}^2\right)$$

$$+ (0.85)(400\text{MPa})\left(2{,}400\,\text{mm}^2\right)$$

$$P_{ro} = 3305\,\text{kN}$$

Tied Column: k = 0.80 *(unintended eccentricities)*

$$P_{r\max} = 0.80(3305\,\text{kN})$$

$$P_{r\max} = 2644\,\text{kN}$$

Interaction diagrams for design

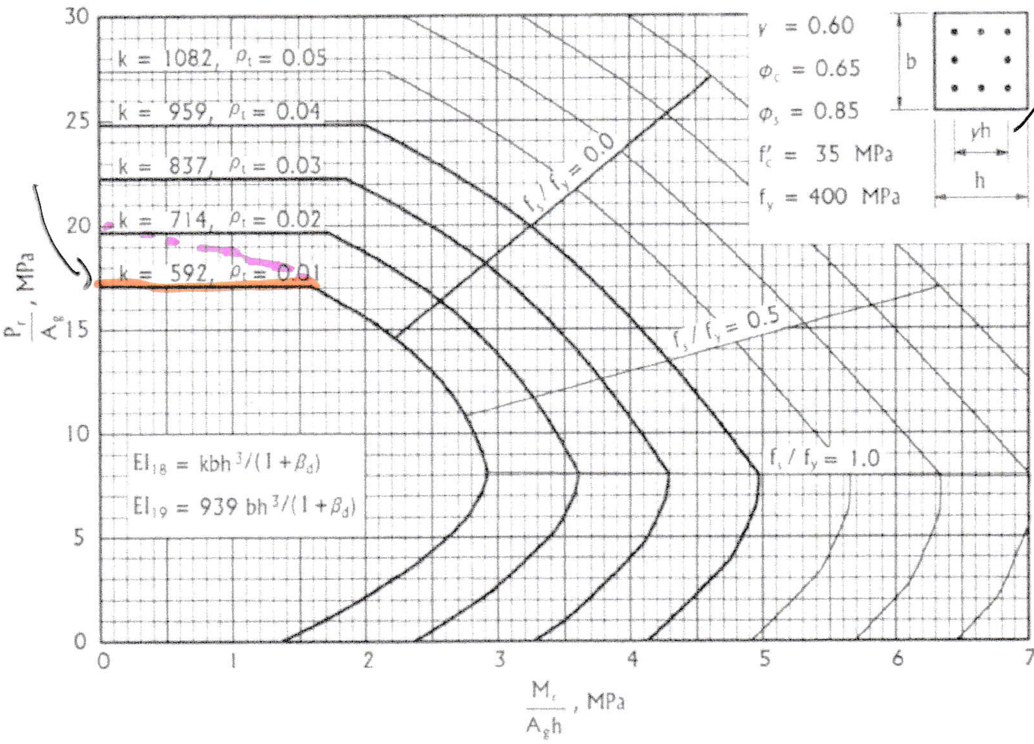
- Can be generated and then compared with the calculated factored axial load and moment, or
- non-dimensional diagrams provided in *[CAC Handbook 2006]* can be used.
- Currently available for download and included in course material

$\left(\frac{P_r}{A_g}, \frac{M_r}{A_g h}\right)$ **for rectangular columns**

$\left(\frac{P_r}{h^2}, \frac{M_r}{h^3}\right)$ **for circular columns**

γ defines distance between reinforcement

Cutoff at 0.8



ANALYSIS FOR GIVEN ECCENTRICITY, e (FIND P_R AND M_R)

- $P_r e = M_r \quad \Rightarrow \quad e = \dfrac{M_r}{P_r}$ and $\dfrac{1}{e} = \dfrac{P_r}{M_r}$
- Slope of a line in the chart $= \dfrac{P_r/A_g}{M_r/A_g h}$

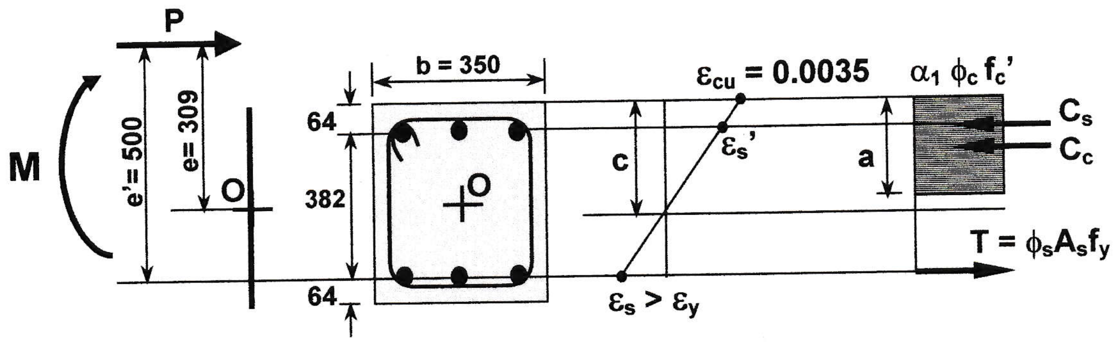
$$= \frac{P_r h}{M_r} = \frac{h}{e}$$

- Draw a straight line from the origin with a slope of h/e, and where it crosses the interaction curve for a given value of ρ_g determines the corresponding values of P_r/A_g and $M_r/A_g h$

ANALYSIS FOR GIVEN FACTORED AXIAL LOAD, P_F

- Set $P_r = P_f$ and calculate P_r/A_g
- Draw horizontal line at calculated value of P_r/A_g, and where it crosses the interaction curve for a given value of ρ_g determines the corresponding value of $M_r/A_g h$.
- Compute $e = M_r/P_r$.

Example 3: ***Calculate P_r and M_r when e = 309 mm for the cross-section shown below using Handbook Interaction Diagram.***
f'_c = 30 MPa, f_y = 400 MPa, A_s = 6 – 25M bars.



Compute section properties, γ, A_g, ρ_g

$$\gamma h = 382mm$$

$$h = 382 + 2(64) = 510mm$$

$$\therefore \gamma = \frac{\gamma h}{h} = \frac{382}{510} = 0.75$$

$$A_g = bh = 350mm(510mm) = 178500mm^2$$

$$\rho_g = \frac{A_s}{A_g} = \frac{6(500mm^2)}{178500mm^2} = 1.68\%$$

$= P_{re}$

Use interaction diagrams to estimate P_r and M_r

To do this need to plot line with slope of $^h/_e$

e = 309mm (given)

$$\therefore \frac{h}{e} = \frac{510}{309} = 1.65$$

Since interaction diagrams for $\gamma = 0.75$ are not provided, we need to interpolate for $\gamma = 0.7$ *and* $\gamma = 0.8$

Use Table 7.11.7 and 7.11.8 provided by CAC

$\gamma = 0.7 \rightarrow Table\ 7.11.7$

$\rho = 1\% \rightarrow$ "x" = 2.8 ∴"y" =mx = 1.65(2.8) = 4.62

$\rho = 2\% \rightarrow$ "x" = 4.24, "y" = 6.996

Use linear interpolation to find values for $\rho = 1.68\%$

$$\frac{x - 2.8}{1.68 - 1} = \frac{4.24 - 2.8}{2 - 1} \rightarrow x = 3.78$$

$$\frac{y - 4.62}{1.68 - 1} = \frac{6.996 - 4.62}{2 - 1} \rightarrow y = 6.235$$

$\gamma = 0.8 \rightarrow Table\ 7.11.8$

$\rho = 1\% \rightarrow$ "x" = 3.05, "y" = 5.0325

$\rho = 2\% \rightarrow$ "x" = 4.6, "y" = 7.59

Use linear interpolation to find values for $\rho = 1.68\%$

$$\frac{x - 3.05}{1.68 - 1} = \frac{4.6 - 3.05}{2 - 1} \rightarrow x = 4.104$$

$$\frac{y - 5.0325}{1.68 - 1} = \frac{7.59 - 5.0325}{2 - 1} \rightarrow y = 6.7716$$

Since $\gamma = 0.75$ is the midpoint of 0.7 and 0.8 take the average of the points found for $\rho = 1.68\%$

$$x_{avg} = \frac{3.7792 + 4.104}{2} = 3.94 = \frac{M_r}{A_g h}$$

$$y_{avg} = \frac{6.235 + 6.7716}{2} = 6.5 = P_r/A_g$$

$$\therefore M_r = 3.94(178500mm^2)(510mm) \div 1000^2 = 358.6kNm$$

$$P_r = 6.5(178500mm^2) \div 1000 = 1160.2kN$$

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Actual values computed using Approach #1

$$M_r = 370kNm, P_r = 1196kN$$

Table 7.11.7 Rectangular Columns with Bars on End Faces Only

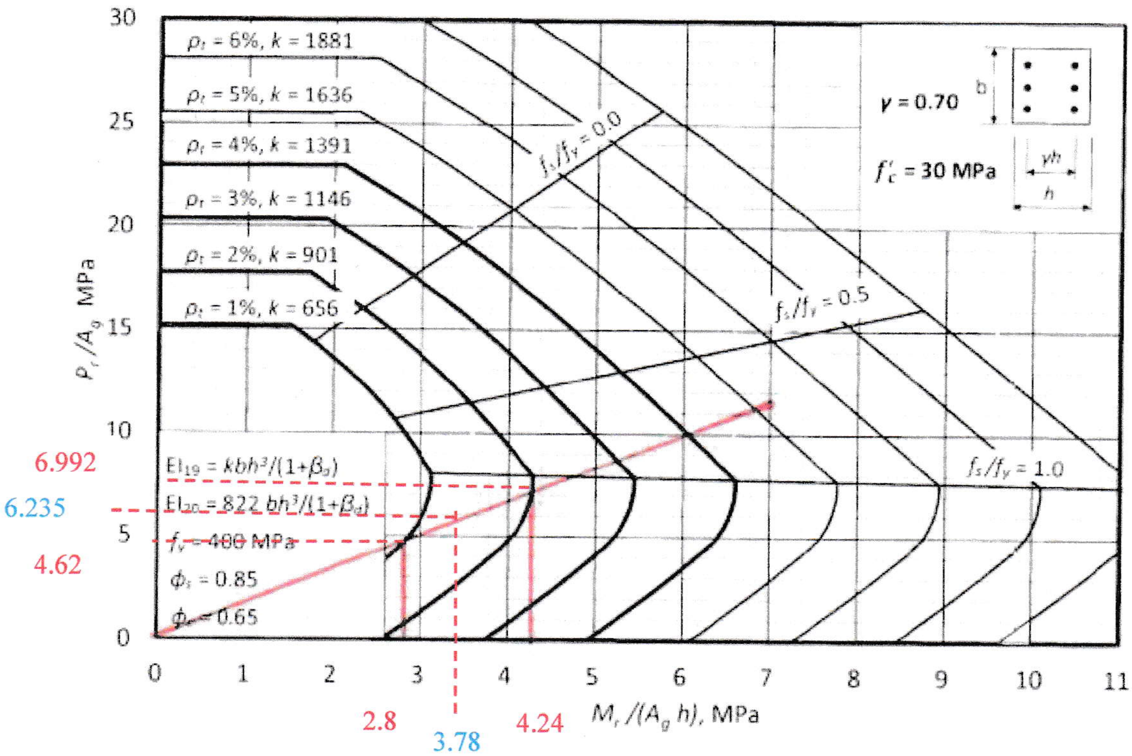

Table 7.11.8 Rectangular Columns with Bars on End Faces Only