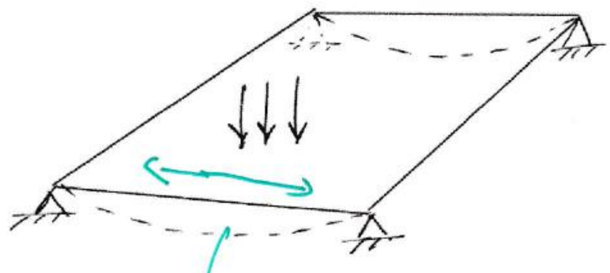
TWO-WAY CONCRETE SLABS

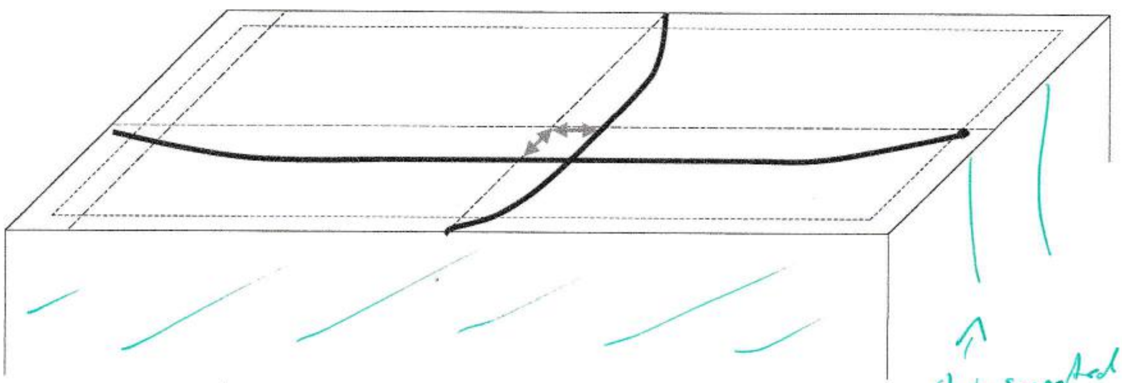
- One-way slabs deform cylindrically, transmit load in one direction.



force transmissible movement

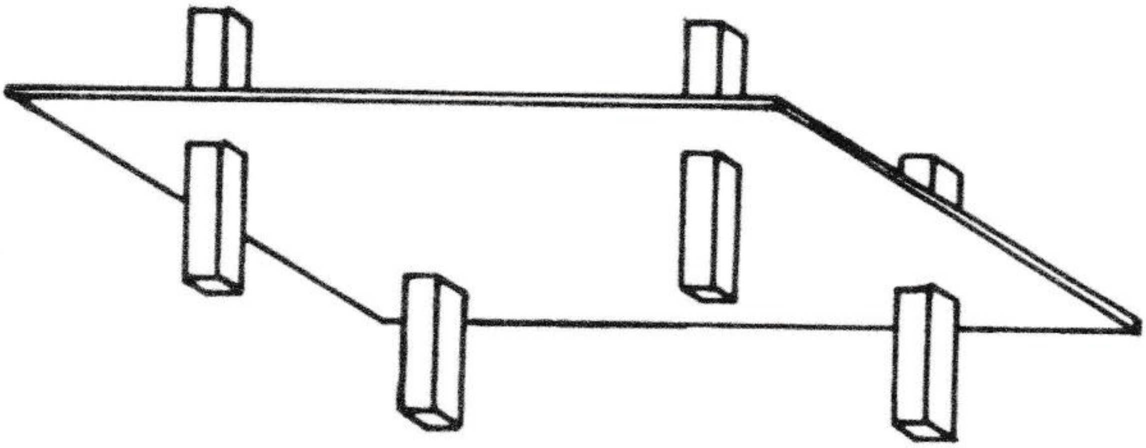
- Two-way slabs exhibit curvature, transmit load in two directions.

↳ If designed as one-way slab by accident, it will crack in other direction and then act as one-way slab.



Slab supported on column

Slab supported on walls (stiff support)
• or beam if it is stiff enough



[CSA A23.3 2004]

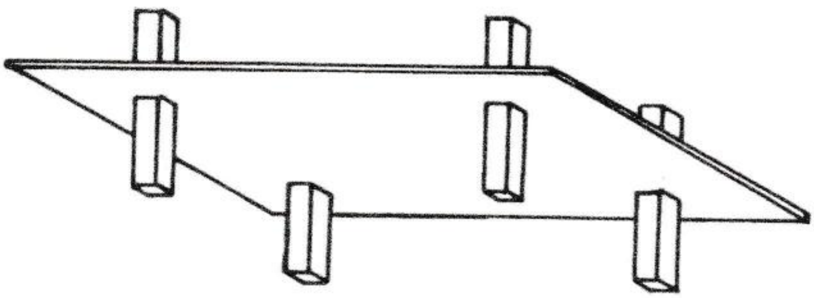
- Two-way slabs are unique to concrete constructi
- They are an efficient and versatile structural form.
  - They can be adapted to virtually any floor plan.

Two-way slabs come in a variety of forms.

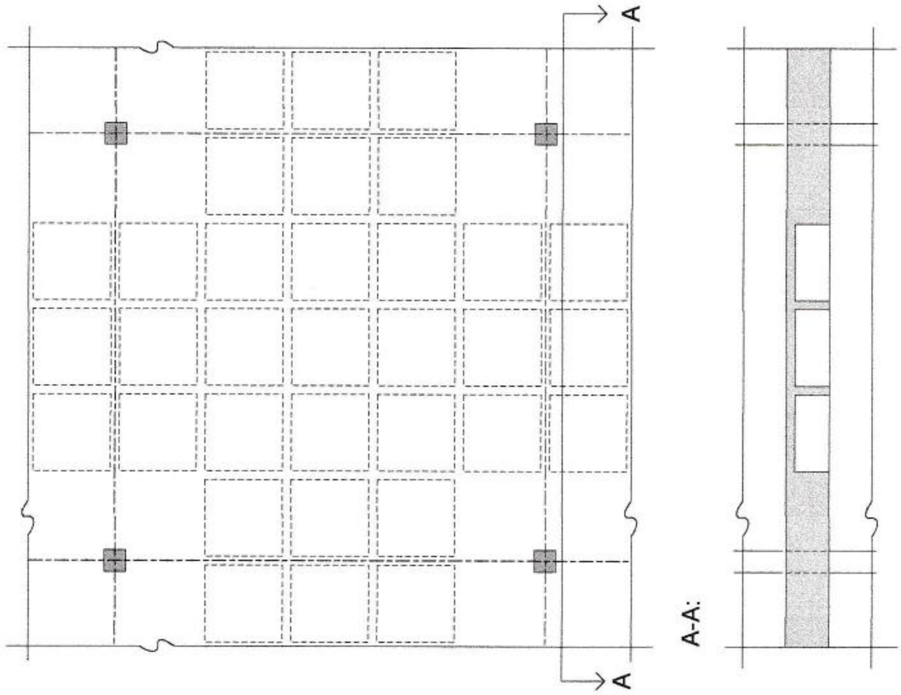
The optimal form depends on (among other things) the column spacing and loading:

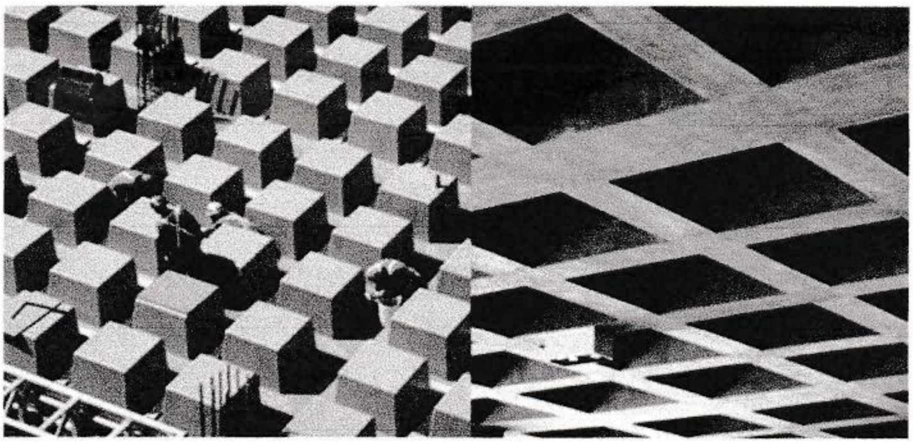
- **<u>Flat plate</u>:** 4.5 → 6 m span (not economical for larger spans)

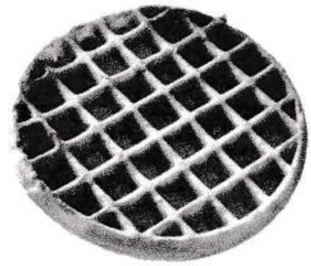
[CAC Handbook 2006]



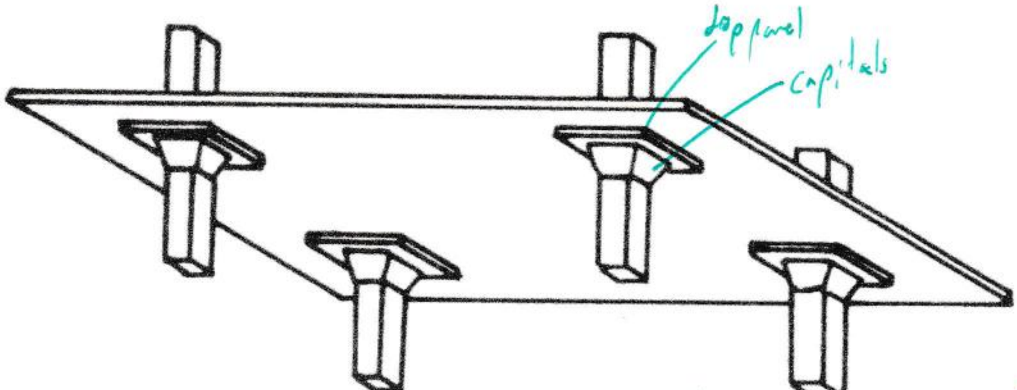
- **<u>Waffle slab</u>:** 7.5 → 12 m span (the slab thickness is retained at the column and effectively reduced at the midspan)





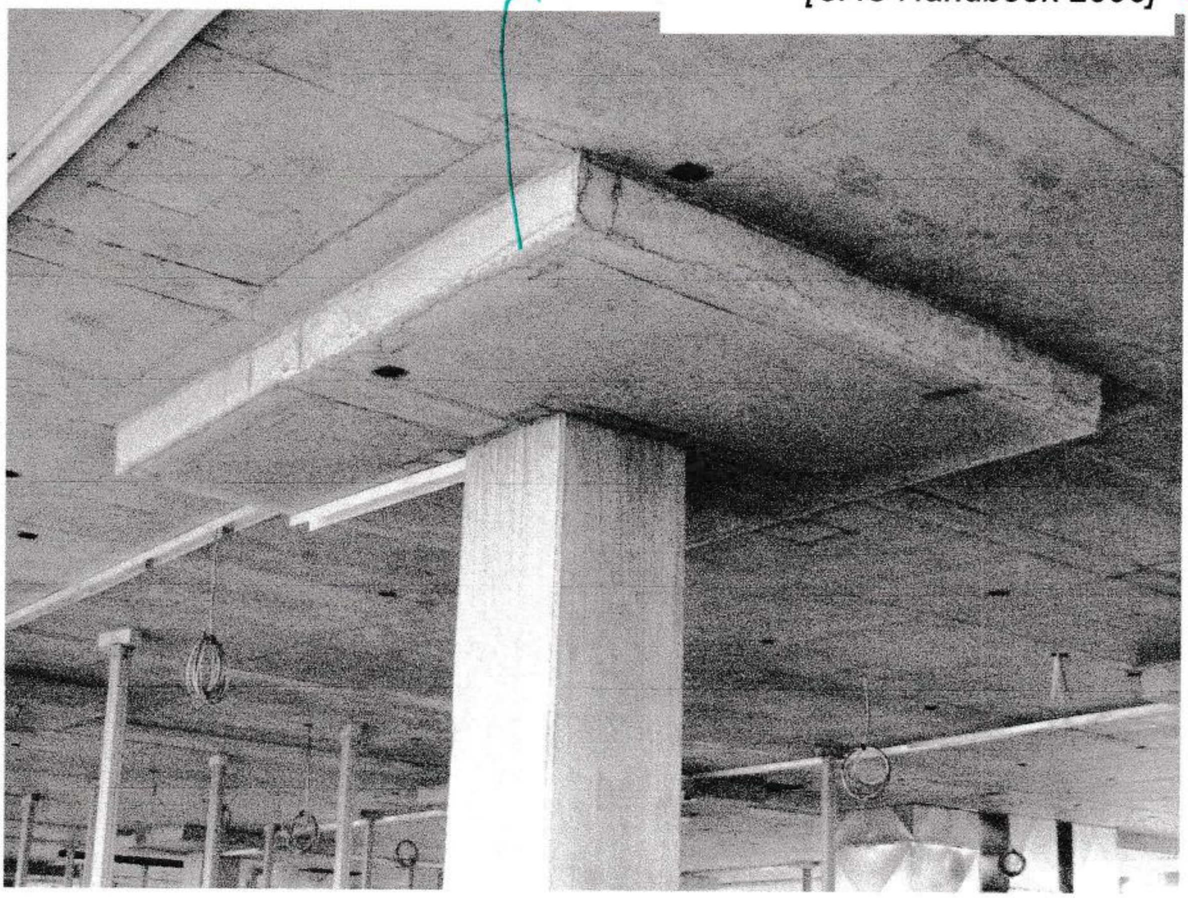


- **Flat slab with drop panels**: 6 → 9 m span (larger thickness near columns, panels extend ~ 1/6 of span on either side)

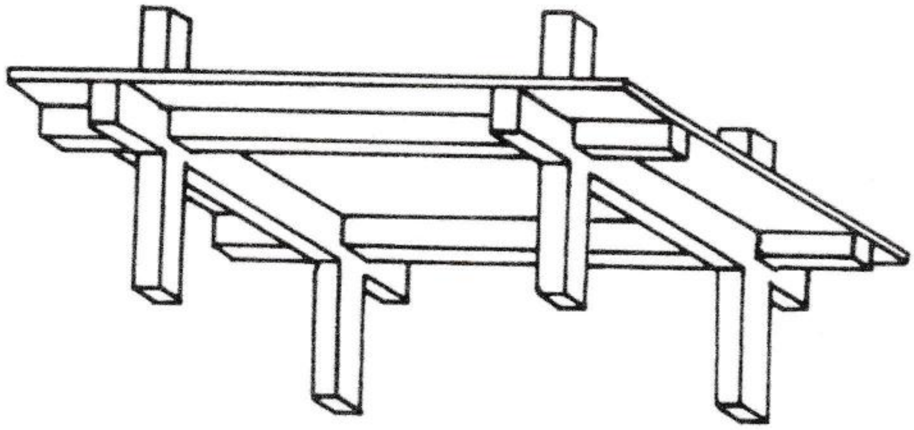


prevent punching shear. Alt: shear reinforcement can be used. However, do decrease deflections

[CAC Handbook 2006]



> **Two-way slabs with beams**.



Two-way slabs need to be designed (primarily) for:

- shear strength
- flexural strength
- serviceability: deflections

Two way concrete slabs can be divided into two categories:

1. Two way slabs supported on stiff beam or walls
2. Two way slabs supported on columns

TWO-WAY SLABS SUPPORTED ON WALLS

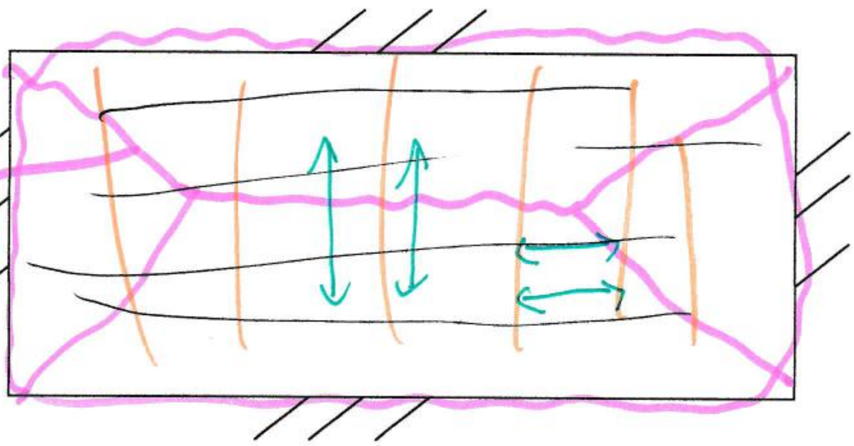
- Wall supported two-way slab loaded to flexural failure (3 stages):
  1) Elastic behaviour:
     - classic or numerical elastic plate analysis applies
  2) First cracking at maximum moment location:
     - elastic analysis is still adequate
     - most slabs are partly cracked under service loads
  3) Yielding and formation of a "failure mechanism":

Yield line method

• rather than studying equilibrium, based on collapse pattern

— = reinforcement

— = different direction



Partially carried in each direction.

$m_p = 1\,kN/m^2$

- Two-way slabs are highly redundant structural elements.
- Flexural failure is ductile, preceded by moment redistribution.
- Shear failure is brittle!
- Before cracking, the slab acts like an elastic plate.
- After cracking, but before yielding of the reinforcement, the slab stiffness is no longer constant and no longer isotropic. The stiffness depends on the amount of cracking. The crack pattern and extent of cracking generally differ in the two directions.
- Yielding starts in one or more region(s) of high moment and spreads throughout the slab as the moments are redistributed.

DESIGN OF SLABS SUPPORTED ON STIFF WALLS

Design Tables Based on Elastic Analysis

[CSA A23.3 Annex B] allows the design of slabs supported along all four sides through the use of moment coefficients.

These coefficients were determined through the semi-elastic analysis of homogeneous isotropic slabs.

Rectangular, uniformly loaded slabs supported on all four sides by stiff walls can be designed using these moment coefficients:

Note:

- The calculated moments are for the "middle strip".
- Moment in the "column strip" = (⅔) · middle strip moment.

$M_{a,neg} = C_{a,neg} \cdot w_f \cdot \ell_a^2$

$M_{b,neg} = C_{b,neg} \cdot w_f \cdot \ell_b^2$

$M_{al,pos} = C_{al} \cdot w_{lf} \cdot \ell_a^2$

$M_{bl,pos} = C_{bl} \cdot w_{lf} \cdot \ell_b^2$

$M_{ad,pos} = C_{ad} \cdot w_{df} \cdot \ell_a^2$

$M_{bd,pos} = C_{bd} \cdot w_{df} \cdot \ell_b^2$

l_a = short side clear span
l_b = long side clear span

C_{ad} = moment coefficient for positive dead load moment in short span

C_{al} = moment coefficient for positive live load moment in short span

$C_{a,neg}$ = moment coefficient for negative moment in short span

C_{bd} = moment coefficient for positive dead load moment in long span

C_{bl} = moment coefficient for positive live load moment in long span
$C_{b,neg}$ = moment coefficient for negative moment in long span

$M_{ad,\ pos}$ = positive dead load moment in short span

$M_{al,pos}$ = positive live load moment in short span

$M_{a,neg}$ = negative moment in short span

$M_{bd,pos}$ = positive dead load moment in long span

$M_{bl,pos}$ = positive live load moment in long span

$M_{b,neg}$ = negativemomentinlongspan

w_{df} = factored dead load per unit area

w_f = factored load per unit area

w_{lf} = factored live load per unit area

Coefficients can be found in Tables in the Annex B

- They are valid for slabs supported on four walls or "stiff" beams.

- A beam is considered to be stiff if:

  $b_w \cdot h_b^3 / (\ell_n \cdot h_s^3) \geq 2.0$ where:

  b_w = beam width

  h_s = slab thickness

  h_b = depth

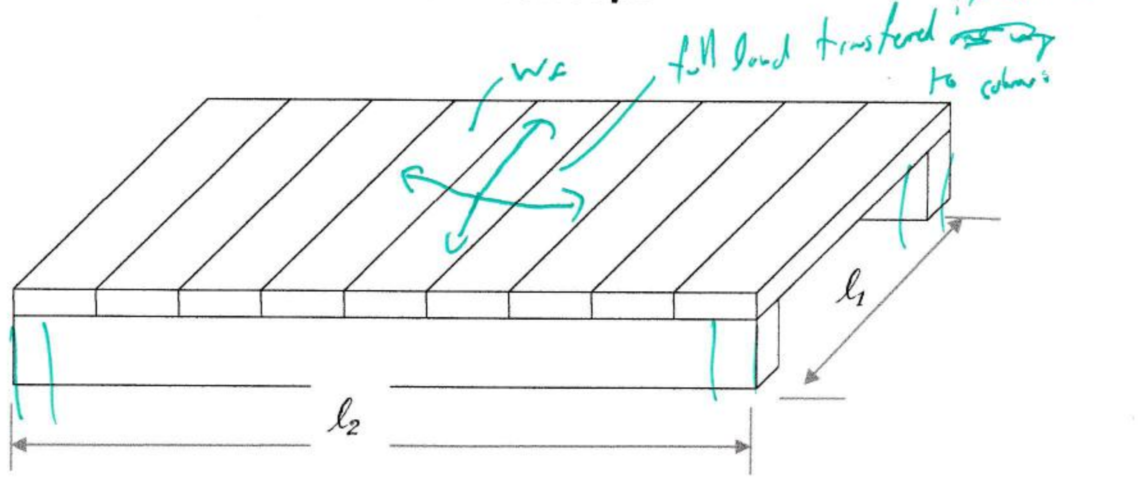
  ℓ_n = clear span

– Design shorter support beam for uniform load = $w_f \cdot \ell_a / 3$.

– Design longer beam for uniform load = $w_f \cdot \dfrac{\ell_a}{3} \cdot \left(\dfrac{3 - (\ell_a / \ell_b)^2}{2} \right)$.

TWO-WAY SLABS SUPPORTED ON COLUMNS

Total Statical Moment - Mechanical Concept



- The moment per metre of width in the planks (section A-A):

  $m = w \cdot \ell_1^2 / 8$

- The total moment in all of the planks:

  $$M_0 = \ell_2 \cdot \left(w \cdot \frac{\ell_1^2}{8} \right) = w \cdot \ell_2 \cdot \ell_1^2 / 8$$

- The planks impose a line load (= $w \cdot \ell_1 / 2$) on each beam.
- The moment in one beam:

  $$m = \left(w \cdot \frac{\ell_1}{2} \right) \cdot \ell_2^2 / 8$$

- The total moment in both beams:

  $M_0 = w \cdot \ell_1 \cdot \ell_2^2 / 8$

In the design of two-way concrete slabs supported on columns, it is similarly assumed that the total moment must be transferred in both directions to ensure static equilibrium of the slab.

Design Steps and Guidelines

- Concrete slabs should be designed as two-way slabs when the ratio: $\dfrac{\text{long span}}{\text{short span}} \leq 2.0$

- When this ratio is greater than 2.0 the slab can be designed as one-way slab spanning in the short direction.

Design steps:

1) Choose the layout and type.
2) Select the slab thickness (based on deflection considerations and preliminary shear check).
3) Choose the design method (for flexural design).
4) Compute the positive and negative moments.
5) Distribute the moments across the slab width (the procedure for this step is independent of the chosen flexural design method).
6) Assign moments to the beams.
7) Design the slab reinforcement.
8) Check shear.
9) Design beams (if present).

Guidelines for selecting slab thickness:

- $h \geq 120$ mm.
- 10 mm increments normally used (best height may depend on availability of "high chairs" used to support top reinforcement).
  - Do a deflection calculation <u>or</u> use $h_s > h_{min}$, where:
  - for flat plate slabs, *clause 13.2.3*

$$h_s \geq \frac{\ell_n \cdot \left(0.6 + f_y / 1000\right)}{30}, \qquad \text{where:}$$

ℓ_n = longer <u>clear</u> span.

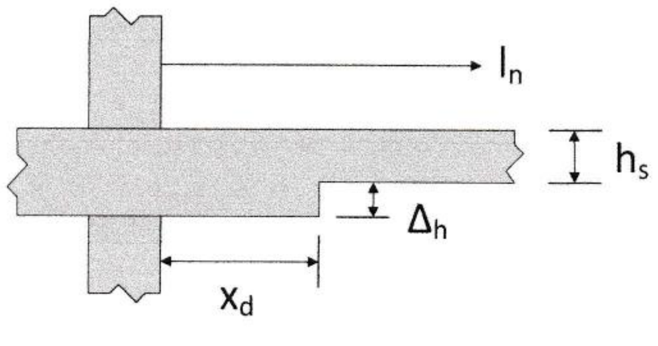
  - for slabs with drop panels, *clause 13.2.4*

$$h_s \geq \frac{\ell_n \cdot \left(0.6 + f_y / 1000\right)}{30} - \frac{2 \cdot x_d}{\ell_n} \cdot \Delta_h, \qquad \text{where:}$$

  - for slabs with beams between all supports, *clause 13.2.5*

$$h_s \geq \frac{\ell_n \cdot \left(0.6 + f_y / 1000\right)}{30 + 4 \cdot \beta \cdot \alpha_m}, \text{ where:}$$

β	=	aspect ratio (long side / short side length)
α_m	=	mean stiffness ratio
	=	ratio of beam to slab stiffness (average along 4 panel edges)
	≤	2.0



Preliminary shear check: - Not req'd but wise to do.

- Slab moments are not yet known at this stage.
- Design for v_f at the column:

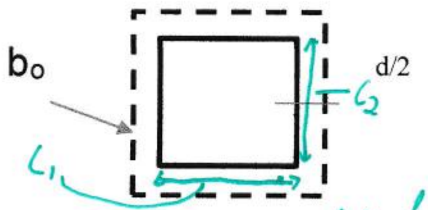
  = $1.2 \cdot v_f$ for interior columns

  $1.6 \cdot v_f$ for edge columns

  $2.0 \cdot v_f$ for corner columns

Shear resistance (same for preliminary and final shear check):
clause 13.3.4

- $v_f \leq v_r$
- $v_f = \dfrac{V_f}{b_0 \cdot d}$ - shear stress acting on critical perimeter
- Tributary area for calculation of v_f is bound by lines of zero shear. Normally, these pass through centre of panel.



d = effective depth of flexural reinforcement

$b_0 = 2 \times (c_1 + d) + 2 \times (c_2 + d)$

- $v_r = v_c + v_s$ (note: v_s is normally zero)

concrete contribution

reinforcement contribution

$$v_c = \text{MIN}\begin{cases} \left(1+\dfrac{2}{\beta_c}\right) \cdot 0.19 \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \\ \left(\dfrac{\alpha_s \cdot d}{b_0} + 0.19\right) \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \\ 0.38 \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \end{cases}$$

← probably govern

PRELIM CHECK TO determine IF shear reinforcement req'd

> Note: if $d > 300$ mm, multiply v_c by $\left(\dfrac{1300}{1000+d}\right)$

- where: β_c = ratio of long-to-short column side dimension

  $\alpha_s = 4$ ····· interior column

  $= 3$ ····· edge column

  $= 2$ ····· corner column

shear reinforcement adds ductility; good for seismic. However has constructability issue

- If $v_c < v_f$, shear capacity can be increased by:
  1. thickening the entire slab panel,
  2. using drop panels,
  3. adding a column capital to increase b_o,
  4. increasing column size to increase b_o, or
  5. adding shear reinforcement.

     ↳ + ductility

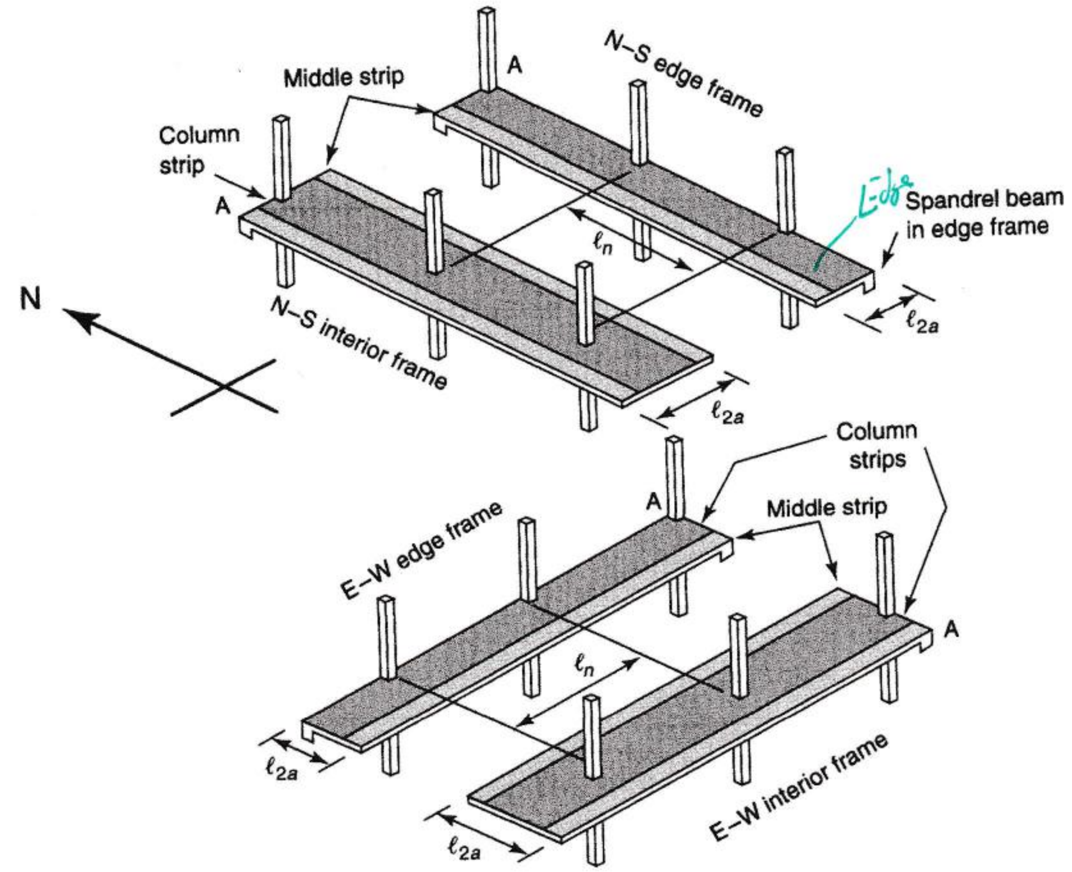
Methods for flexural analysis and design: – Find M at different locations

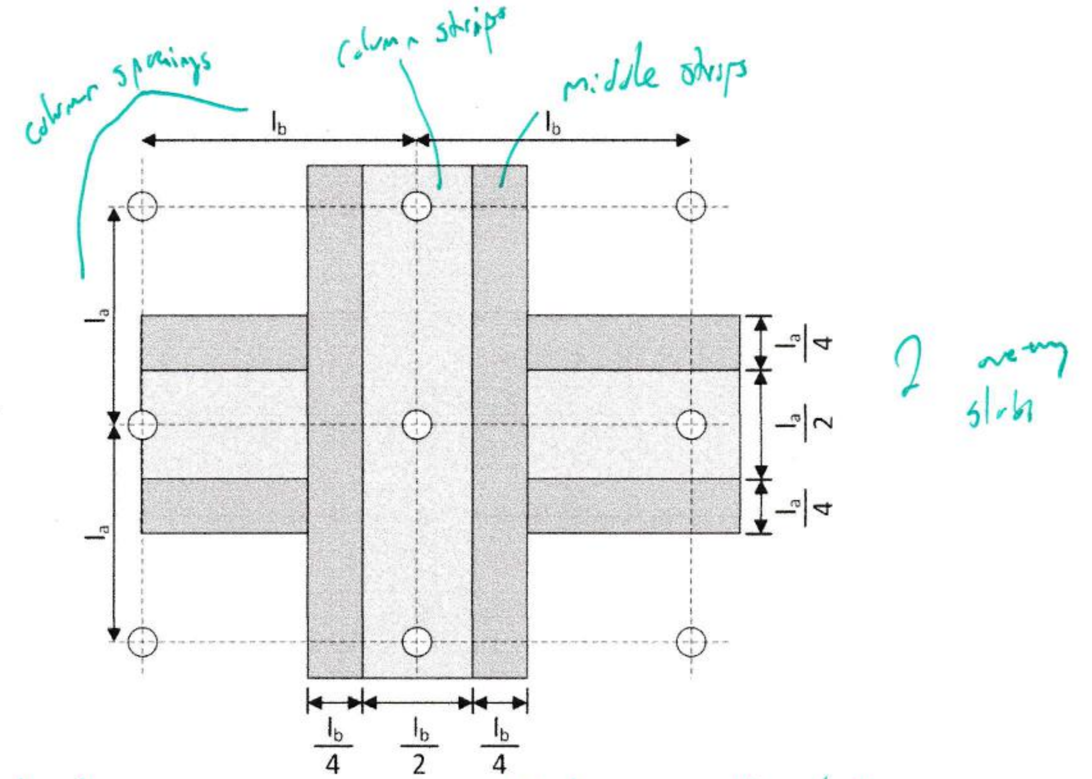
1) Design tables based on elastic analysis
2) Theorems of plasticity (i.e. yield line theory)
3) **Direct design method**
4) Elastic frame method

} Similar

5) Classical or numerical (finite element) elastic plate theory

Turns slab system into equivalent frames





Spandrel beam requires more attention due to torsion

Direct Design Method *clause 13.9*

Based on distribution of Statical Moment M_0

Moments in slabs on isolated columns:

- The stiffest portion of the slab is along the column lines
  → maximum positive and negative moments along these lines:
- For design, the slab is divided into "middle" and "column" strips.
- Flexural reinforcement provided for average positive and negative moments in middle and column strips.

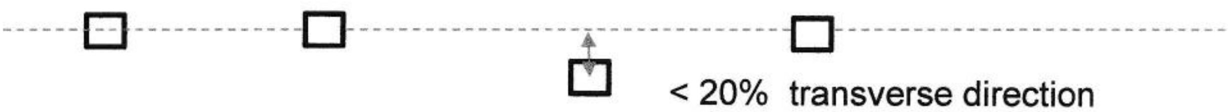
Limitations for elastic frame and direct methods for flexural design:

Both methods can be used for "regular two-way slabs", where:

a) The ratio of the longer to shorter slab span (measured from centreline to centreline of supports) ≤ 2.0

b) For slabs with beams between supports, the relative stiffness of these beams, $0.2 \le \alpha_1 \cdot \ell_2^2 / (\alpha_2 \cdot \ell_1^2) \le 5.0$

c) Columns not offset by more than 20% of span (in offset direction).



d) The reinforcement is placed in an orthogonal grid.

Additional limitations for direct method:

e) A minimum of three (3) continuous spans in each direction.

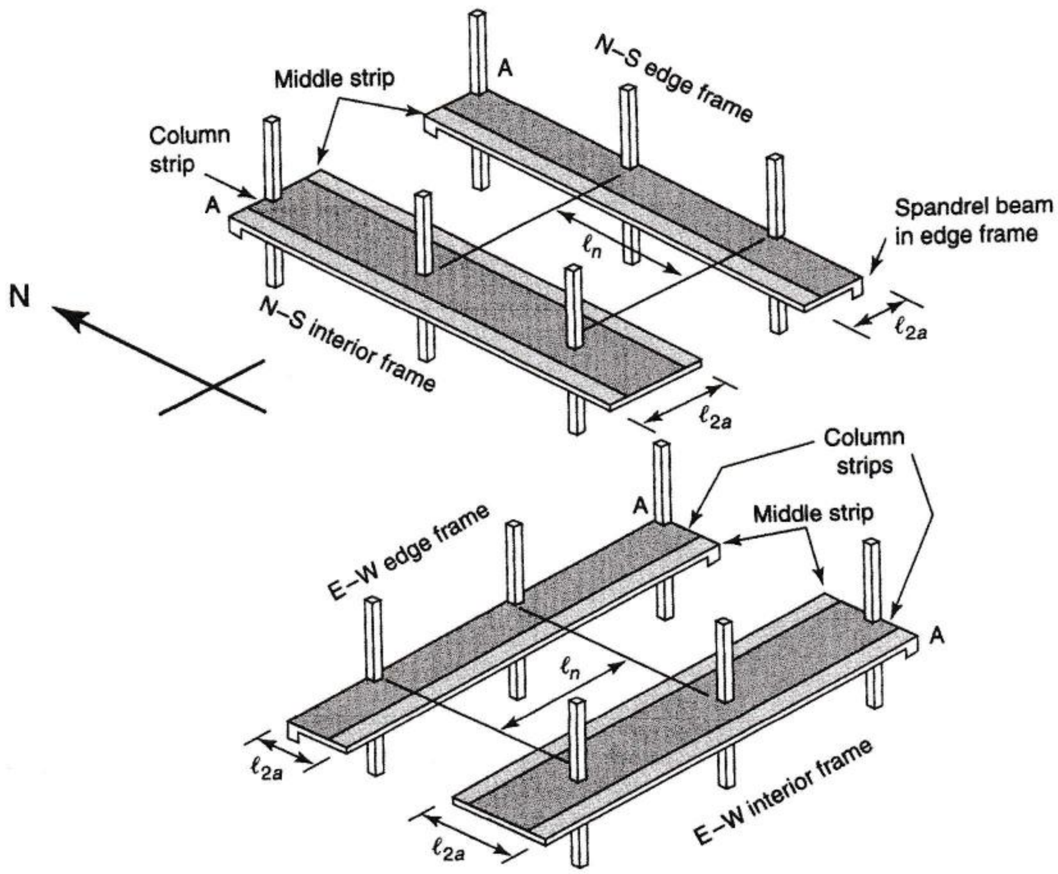
f) Successive spans do not differ by more than (⅓) x longer span.

g) $w_{lf} \leq 2.0 \cdot w_{df}$, where: $w_{lf} = 1.5 \cdot L$ and $w_{df} = 1.25 \cdot D$.

(because of pattern loading)

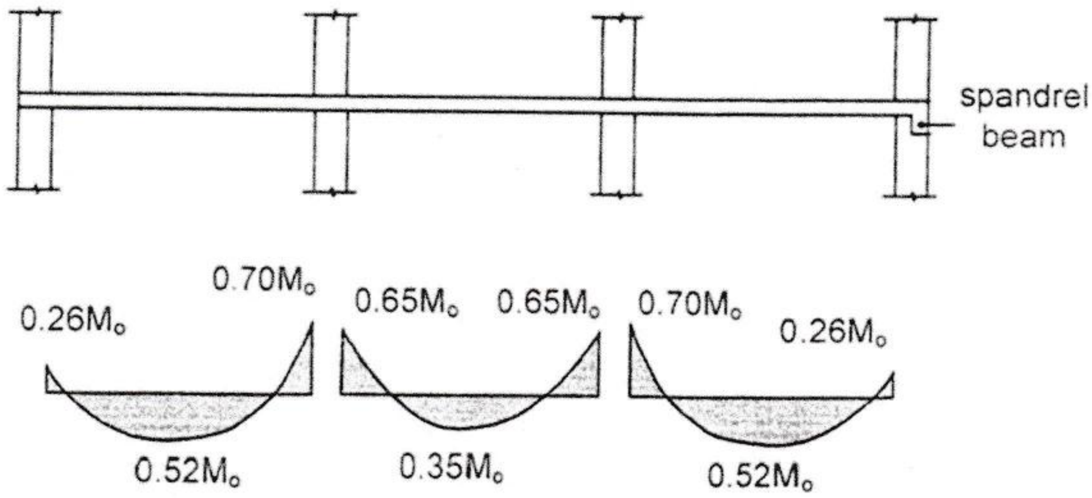
Direct Design of Flat Plate Slabs and Slabs with Drop Panels

- -define equivalent frames

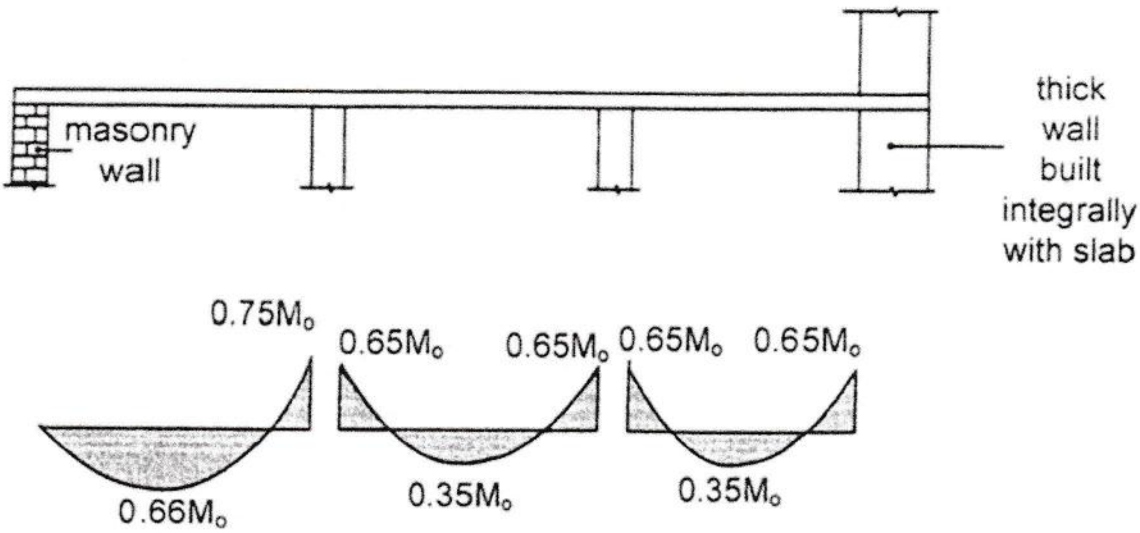


- assign the percentage of statical moments to supports and spans

$M_o = \frac{wL^2}{8}$



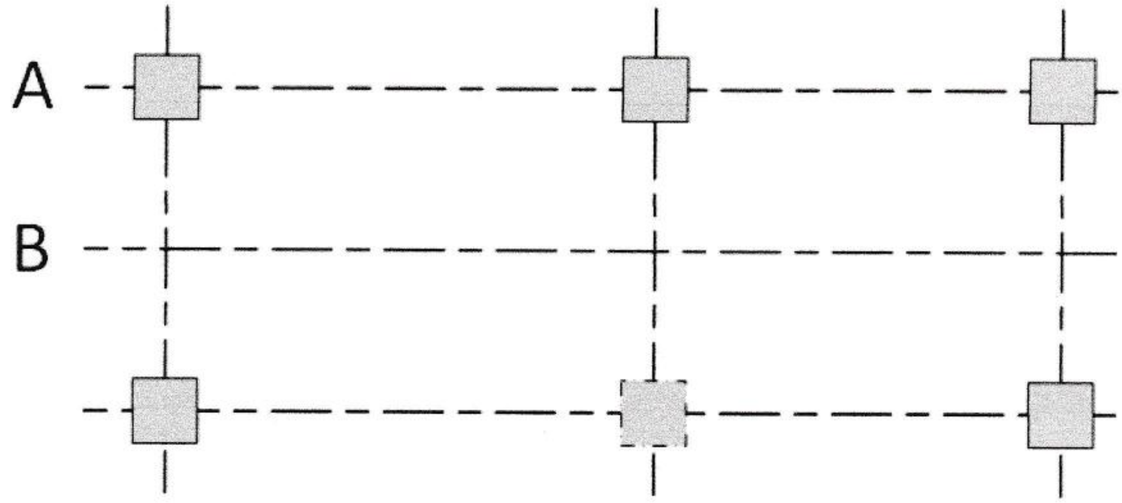
(a) Slab without beams between interior supports

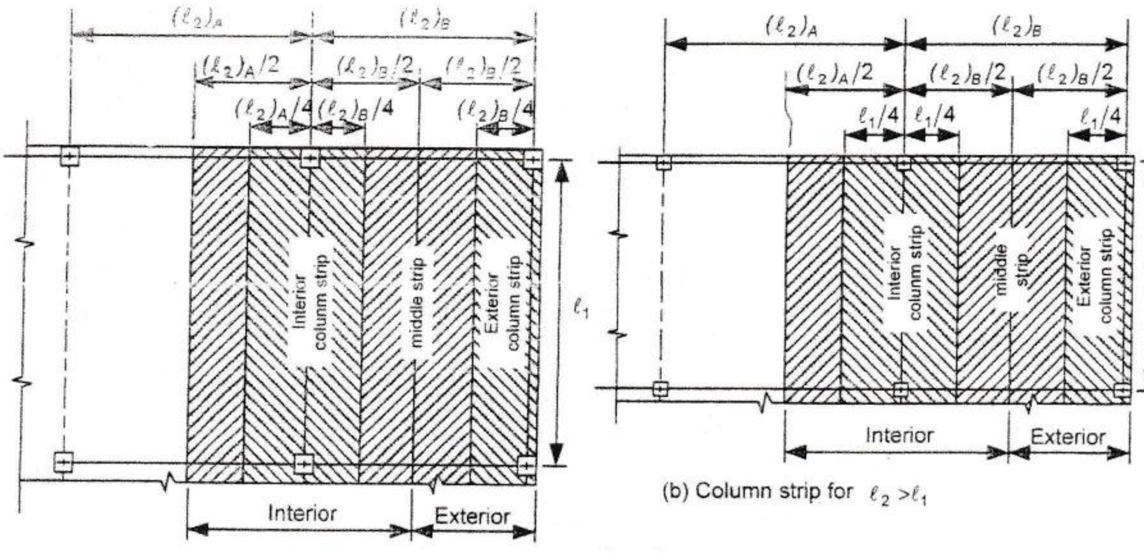


(a) Slab supported on walls

➢ Distribute the moments to column strips and middle strips

The stiffest portion of the slab is along the column lines. The largest moments are there.





(a) Column strip for $\ell_2 \leq \ell_1$

(b) Column strip for $\ell_2 > \ell_1$

Fig. N13.1.2(c) Definition of Design Strip

[CSA A23.3 2014]

➢ The total factored **statical moment** is calculated as follows:

kN/m²

$$M_O = \frac{w_f \cdot \ell_{2,avg} \cdot \ell_n^{\,2}}{8}$$, where:

ℓ_n = clear span and $\ell_{2,avg} = \text{AVERAGE}\left(\ell_{2,A}, \ell_{2,B}\right)$

- The sum of the absolute values of 1) the positive moment and 2) the average negative moment must be ≥ M_O
- Negative and positive moments obtained from §13.9.3.
- Negative and positive moments can be modified by up to 15%, provided that the total static moment is not less than required.
- At supports, if the negative moments on each side are not equal, design for the greater of the two or redistribute up to 15% of the moment (on one side or both) to make them equal.
- Distribution of moment between column and middle strips: §13.11.2.2- 3

plastic, ?range

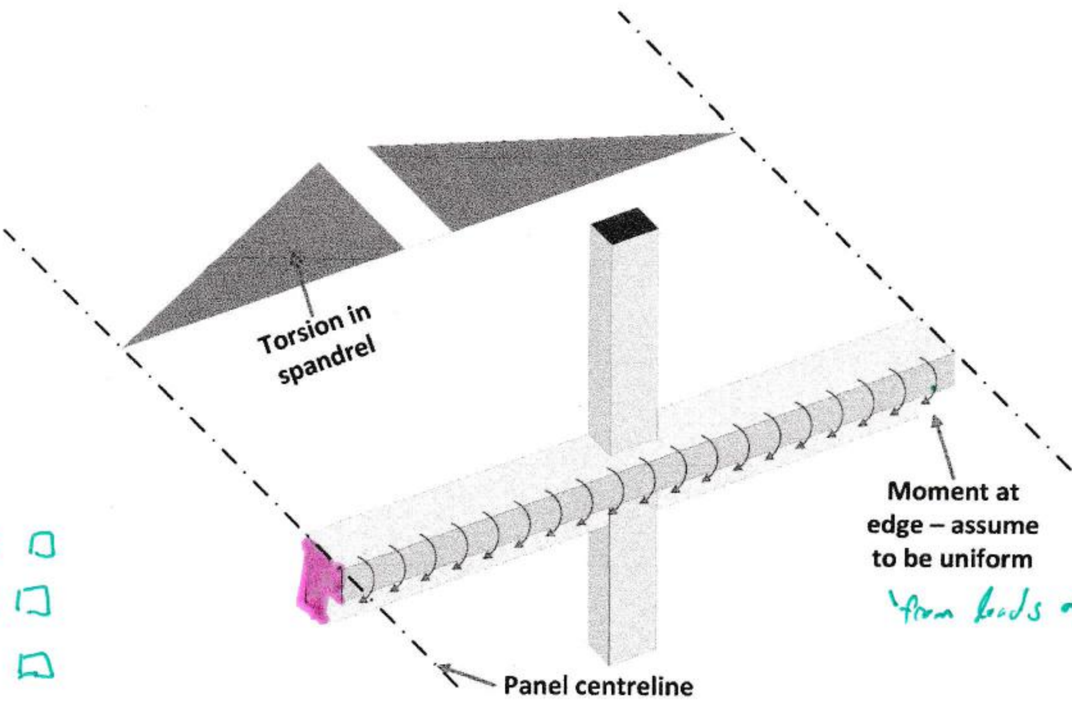
Moment	Column Strip
Negative, interior column	..... 70 (75)-90%*
Negative, exterior column	..... 100%**
Positive, all spans	..... 55-65% (some of this is "integrity" reinforcement)

* 33% with band of width, b_b §13.11.2.7

** 100% within band of width, b_b §13.10.3

b_b= column width + 2 x 1.5 slab depth (on each side of the column)

TORSION in the spandrel beam



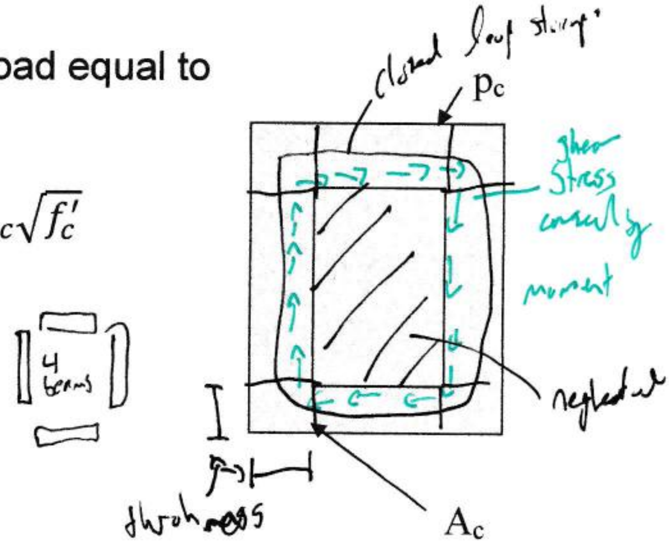
from loads on slab

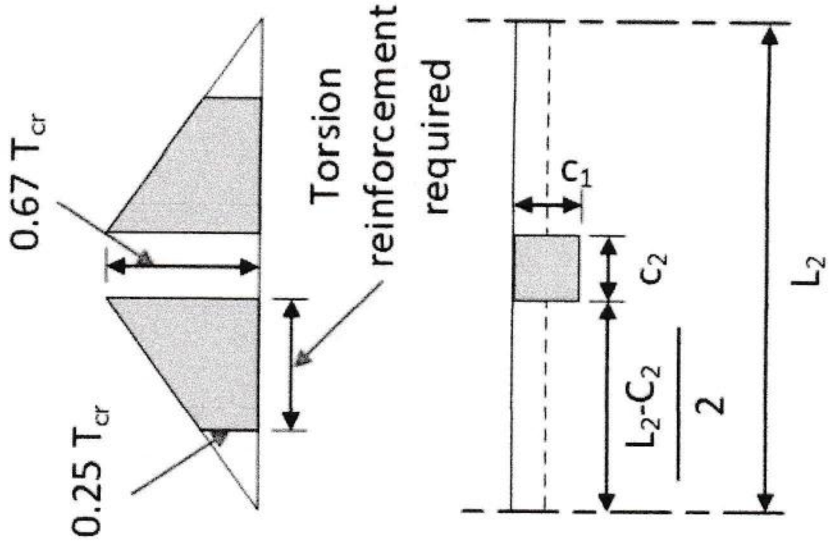
Fig N13.11.2.6a
Calculation of Torsion at Exterior Edge

ADDITIONAL NOTES FOR TORSION IN SPANDREL BEAM

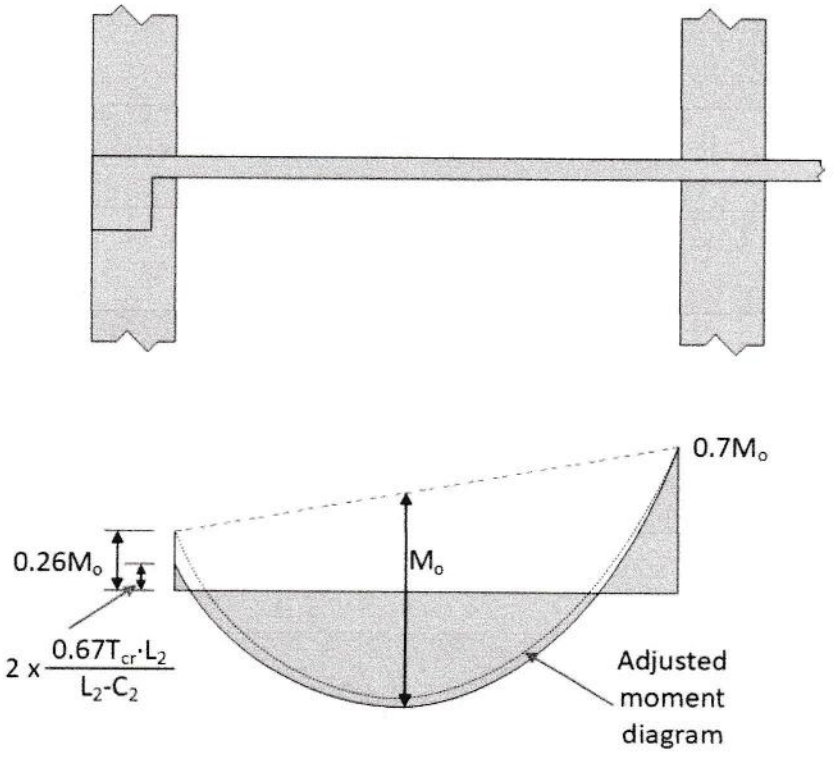
- Spandrel beams are subject to torsion because of the action of the attached slab.
- Spandrel beams are designed for the load equal to
  - $0.67\ T_{cr}$

$$T_{cr} = (A_c^2/p_c)0.38\lambda\phi_c\sqrt{f_c'}$$





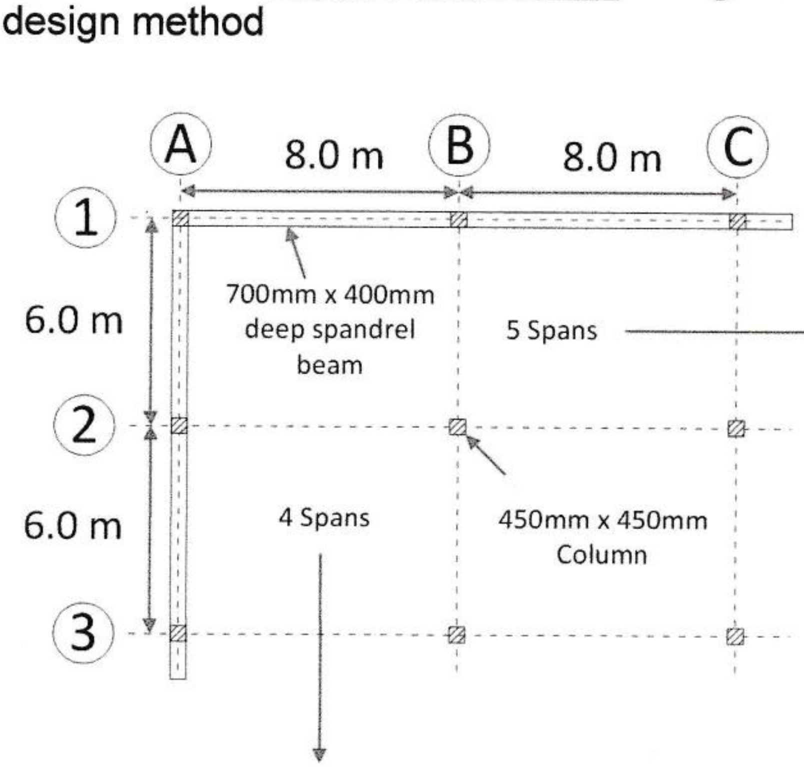
(b) Torsion diagram if calculated torsion exceeds 0.67 T_{cr} (see clause 11.2.9.2)



(c) adjusted moment diagram

Fig. N13.11.2.6(b)
Redistribution of Moment Caused by Reduced Torsional Stiffness of Spandral Beam After Cracking

EXAMPLE 1 **2 way slab design:** Design for flexure using direct design method



$f'_c = 30\ MPa$
$f_y = 400\ MPa$
$LL = 4.8\ kPa$
$SD = 1.0\ kPa$

*

Before starting we must confirm that the slab meets the requirements for the direct method:

a) $\frac{8.0}{6.0} \leq 2.0$ OK

b) $N.A.$ no beams

c) No column offset OK

d) Use orthogonal reinforcement grid. Slab is a "regular 2-way slab" OK

e) 3 spans in each direction?

OK

f) Successive spans are equal?

OK

g) $w_{lf} \leq 2.0 \cdot w_{df}$? Check after slab thickness is determined

1) Choose slab layout and type: given

2) Select slab thickness: we will consider different options for the slab

For flat slab:

$$h_s \geq \frac{l_n}{30} \ where\ l_n = 8.0 - 0.45 = 7.55m$$

$$h_s \geq \frac{7550}{30} = 252mm$$

With drop panels:

$$h_s \geq \frac{l_n}{30} - 2 \cdot x_d \cdot \frac{\Delta h}{l_n}$$

$$x_d \ngtr \frac{l_n}{4}, \Delta h \quad \ngtr h_s$$

$$\rightarrow try\ 2500\ mm\ by\ 2500mm\ by\ 200mm\ \ drop\ panel$$

$$x_d = \frac{2500}{2} - \frac{450}{2} = 1025mm$$

$$h_s \geq \frac{7550}{30} - 2 \cdot 1025 \cdot \frac{200}{7550} \rightarrow h_s \geq 197mm$$

With beams between all supports

$$h_s \geq \frac{l_n}{30 + 4 \cdot \beta \cdot \alpha_m}$$

$$beam\ design\ not\ known\ yet. For\ very\ stiff\ beam\ (\alpha_m = 2.0)$$

$$\beta = \frac{7550}{5550} = 1.36$$

$$h_s = \frac{7550}{30 + 4 \cdot 1.36 \cdot 2} = 185mm$$

*therefore, assume slab with drop panels selected

Calculate w_f for selected slab thickness:

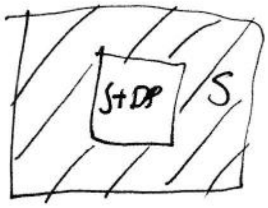
slab + drop panel

$$\underline{Average}\ slab\ thickness = \overset{slab}{0.2} \cdot \frac{41.75}{48} + 0.4 \cdot \frac{6.25}{48} = 226mm$$

drop panel + slab

$$DL = 1.0 + 0.226 \cdot 24 = 6.43\ kPa$$ area of slab

$$LL = 4.8\ kPa\ \left(w_{lf} < 2 \cdot w_{df} \rightarrow OK\right)$$

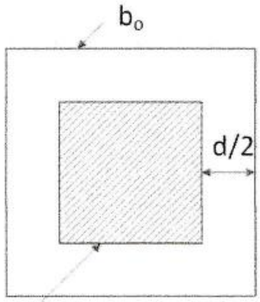


$$w_f = 1.25 \cdot 6.43 + 1.5 \cdot 4.8 = 15.23\ kPa$$

Perform preliminary shear check for an interior column

Clause 13.3.3 → critical section for 2-way shear = perimeter, b_o at distance d/2 from face of column or drop panel.

If don't want to use shear reinforcement



Column or drop panel

length of critical parameter

LOCATION	d	bo
@ COLUMN	$= 400 - 25 - 20 = 355mm$	$= 4(450 + 355) = 3220mm$
@AT DROP PANEL	$= 200 - 25 - 20 = 155mm$	$= 4(2500 + 155) = 10620mm$

Applied shear @ Column:

column + critical perimeter

$$V_f = (48m^2 - (0.45m + 0.355m)^2) \cdot 15.23kPa$$

$$= 721.2\ kN$$

$$v_{f(for\ check)} = \frac{1.2 \cdot V_f}{b_0\ d} = \frac{1.2 \cdot 721.2}{(3220mm \cdot 355mm)} = 0.76MPa$$

d

Applied shear @ Drop Panel:

$$V_f = (48 - (2.5 + 0.155)^2) \cdot 15.23 = 623.7kN$$

$$v_{f(for\ check)} = \frac{1.2 \cdot V_f}{b_0\ d} = \frac{1.2 \cdot 623.7}{(10620 \cdot 155)} = 0.45MPa$$

d

Shear strength (concrete only)

$$v_c = MIN \begin{Bmatrix} \left(1 + \frac{2}{\beta_c}\right) \cdot 0.19 \\ \left(\frac{\alpha_s \cdot d}{b_o} + 0.19\right) \\ 0.38 \end{Bmatrix} \lambda \phi_c \sqrt{f_c'}$$

Shear strength @ Column: → Reduce for size effect if d > 300mm

- Fracture mechanics
- Higher probability of errors

$$v_c = MIN \begin{Bmatrix} \left(1 + \frac{2}{2}\right) \cdot 0.19 = 0.57 \\ \left(\frac{4 \cdot 355}{3220} + 0.19 = 0.63\right) \\ 0.38 \end{Bmatrix} \cdot 1.0 \cdot 0.65 \cdot \sqrt{30} = 1.35MPa$$

$$\therefore, d > 300mm$$

$$v = v_c \left(\frac{1300}{1000 + d}\right) = 1.35 \left(\frac{1300}{1000 + 355}\right) = 1.30MPa > 0.76MPa$$

If no shear reinforcement, brittle failure possible if there is an issue. In seismic zones, always have shear reinforcement

Shear strength @ drop panel:

$$v_c = MIN\begin{Bmatrix} \left(1+\frac{2}{2}\right)\cdot 0.19 = 0.57 \\ \left(\frac{4\cdot 155}{10620}+0.19 = 0.25\right) \\ 0.38 \end{Bmatrix}\cdot 1.0\cdot 0.65\cdot\sqrt{30}$$

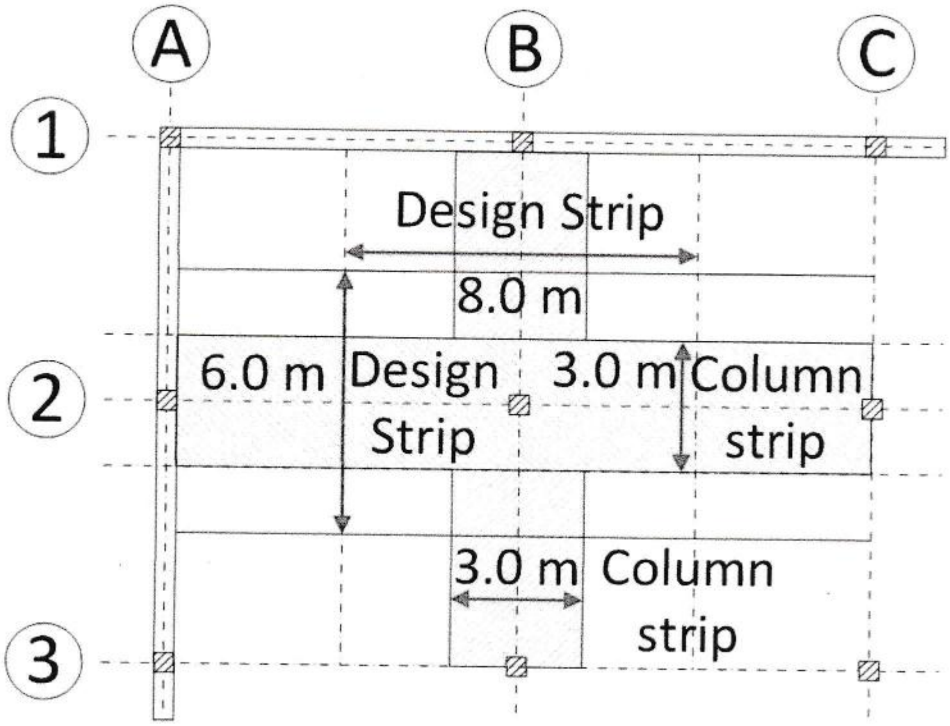
$$= 0.88MPa > 0.45MPa$$

3) Choose flexural design method:

-given

4) Compute positive and negative moments in design strips

-consider N-S and E-W interior strips



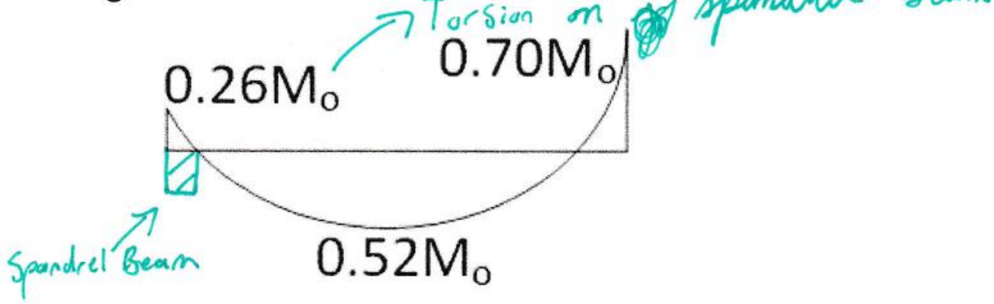
North – South:

$$M_0 = w_f \cdot l_{2,avg} \cdot \frac{l_n^2}{8} = 15.23 \cdot 8 \cdot \frac{5.55^2}{8} = 469.2kN \cdot m$$

East – West:

$$M_0 = 15.23 \cdot 6.0 \cdot \frac{7.55^2}{8} = 651.2\ kN \cdot m$$

@ edge span design for: Table 13.1



Also, 1) design spandrel beam for resulting torsional moment or 2) design spandrel beam for $0.67 \cdot T_{cr}$ and modify slab design moments accordingly:

700 by 400 spandrel beam:

$$T_{cr} = \left(\frac{A_c^2}{P_c}\right) 0.38 \cdot \lambda \cdot \phi_c \cdot \sqrt{f_c'} = \left(\frac{(700 \cdot 400)^2}{2(700 + 400)}\right) 0.38 \cdot 0.65 \cdot \sqrt{30}$$
$$= 48.2\ kN \cdot m$$

Moment at support (north – South) *(two beams framing into the support)*

$$M = 0.67 \cdot 48.2 \cdot 2\left(\frac{l}{l - c}\right) = 0.67 \cdot 48.2 \cdot 2\left(\frac{8}{7.55}\right) = 68.45kN \cdot m$$

Moment at support (East – West)

$$M = 69.84\ kN \cdot m$$

North – South Strip

Span:	Exterior			Interior	
Moment Coefficient	$\mathbf{0.26}$	0.52	0.70	0.65	0.35
$\boldsymbol{M-(kN\cdot m)}$	$\mathbf{122}$ 68.45	244 $\mathbf{469.2}-\left(\mathbf{328.4}+\frac{\mathbf{68.45}}{\mathbf{2}}\right)$ **=270.7**	328.4	305	164.2

East - West Strip

Span:	Exterior			Interior	
Moment Coefficient	$\mathbf{0.26}$	0.52	0.70	0.65	0.35
$\boldsymbol{M-(kN\cdot m)}$	$\mathbf{169.3}$ 69.84	338.6	455.8 $\mathbf{388.3}$	423.3	227.9

BOTTOM REINFORCING DESIGN

N-S		Span	1-2		2-3	
		M_{f_Total}	270.7		164.2	
N-S Bottom Reinforcing		Strip	Column	Middle	Column	Middle
		$\%M_f$	65	35	55	45
		M_f	176.0	94.7	90.3	73.9
		Width	3000	5000	3000	5000
		t	200	200	200	200
		d	145	145	145	145
	Reinf.	10	0	0	0	0
		15	6	10	6	10
		20	10	0	3	0
		A_s	4200	2000	2100	2000
		a	30.32	8.66	15.16	8.66
		M_r	185.4	95.7	98.1	95.7
		A_s / A_g	*0.0070*	*0.0020*	*0.0035*	*0.0020*

f'_c	30
f_y	400
ϕ_s	0.85
ϕ_c	0.65
α_1	0.81
β_1	0.90

E-W		Span	A-B		B-C	
		M_{f_Total}	388.3		227.9	
E-W Bottom Reinforcing		Strip	Column	Middle	Column	Middle
		$\%M_f$	55	45	55	45
		M_f	213.6	174.7	125.3	102.6
		Width	3000	3000	3000	3000
		t	200	200	200	200
		d	165	165	165	165
	Reinf.	10	0	0	0	0
		15	10	10	10	10
		20	8	5	3	0
		A_s	4400	3500	2900	2000
		a	31.77	25.27	20.94	14.44
		M_r	223.1	181.3	152.4	107.3
		A_s / A_g	*0.0073*	*0.0058*	*0.0048*	*0.0033*

TOP MAT DESIGN

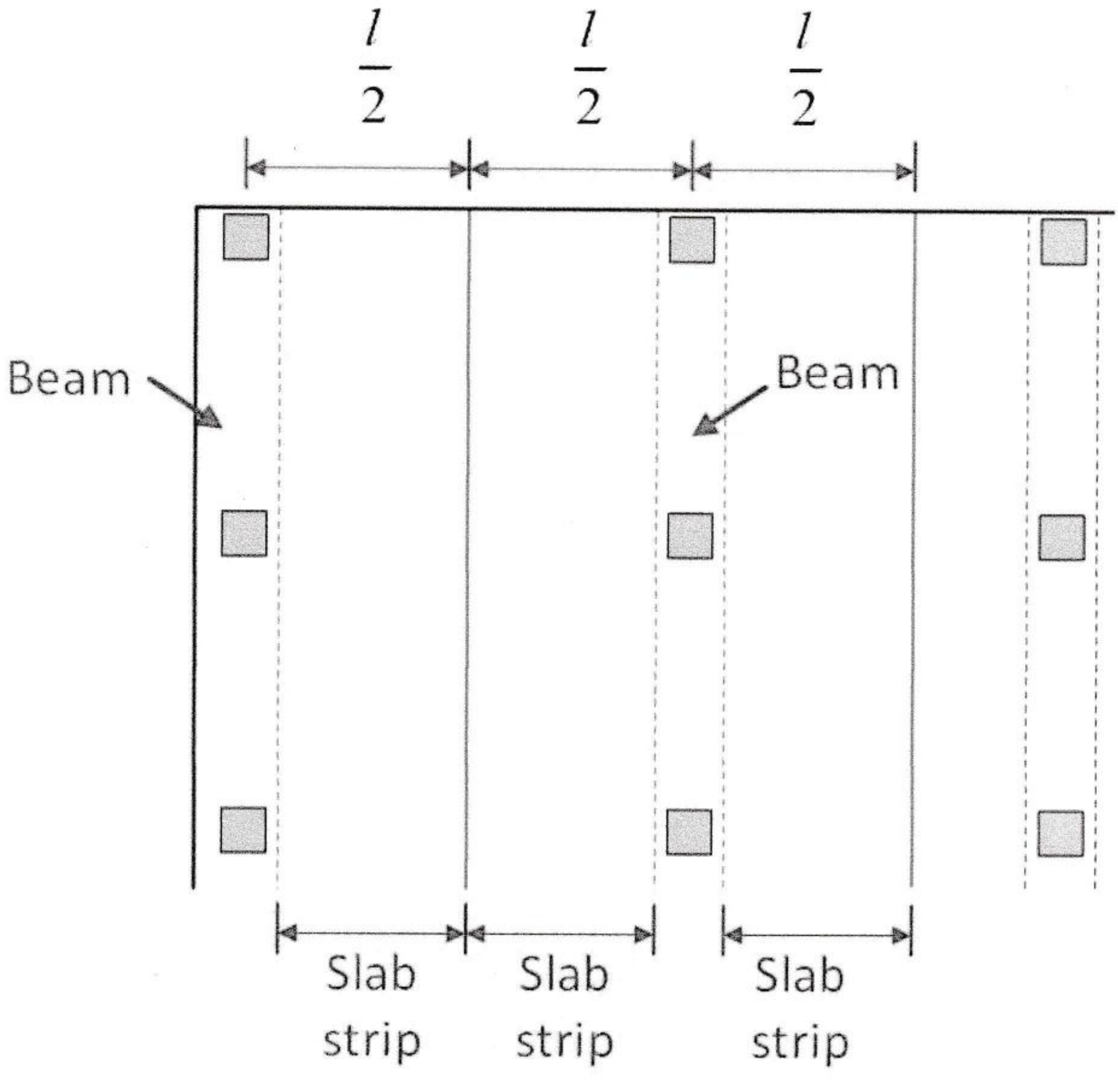
N-S						
Column	1		2		3	
$M_{f\ Total}$	68.5		328.4		305.0	
N-S Top Mat Reinforcing						
Strip	Column	Middle	Column	Middle	Column	Middle
$\%M_f$	100	0	90	10	90	10
M_f	68.5	0.0	295.6	32.8	274.5	30.5
Width	2500	5000	2500	5000	2500	5000
t	400	200	400	200	400	200
d	345	145	345	145	345	145
Reinf. 10	0	0	0	0	0	0
Reinf. 15	6	10	6	10	6	10
Reinf. 20	0	0	5	0	5	0
A_s	1200	2000	2700	2000	2700	2000
a	10.40	8.66	23.39	8.66	23.39	8.66
M_r	138.6	95.7	306.0	95.7	306.0	95.7
A_s / A_g	0.0020	0.0020	0.0045	0.0020	0.0045	0.0020

f'_c	30
f_y	400
α_1	0.81
β_1	0.90
ϕ_s	0.85
ϕ_c	0.65

E-W						
Column	A		B		C	
$M_{f\ Total}$	69.8		455.8		423.3	
E-W Top Mat Reinforcing						
Strip	Column	Middle	Column	Middle	Column	Middle
$\%M_f$	100	0	90	10	90	10
M_f	69.8	0.0	410.2	45.6	381.0	42.3
Width	2500	3000	2500	3000	2500	3000
t	400	200	400	200	400	200
d	365	165	365	165	365	165
Reinf. 10	0	0	0	0	0	0
Reinf. 15	6	6	6	6	6	6
Reinf. 20	0	0	8	0	7	0
A_s	1200	1200	3600	1200	3300	1200
a	10.40	8.66	31.19	8.66	28.59	8.66
M_r	146.8	65.6	427.7	65.6	393.5	65.6
A_s / A_g	0.0020	0.0020	0.0060	0.0020	0.0055	0.0020

Design for moment:

– Similar to design for slabs without beams, except “middle” and “column” strips replaced with “slab strip” and “beam”.



- Calculation of M_0 and division of M_0 into positive and negative moments same as for slabs without beams.
- For positive and negative interior moments, the beam is assumed to resist a fraction of the moment:

$$= \left(\frac{\alpha_1}{0.3+\alpha_1}\right)\cdot\left(1-\frac{\ell_2}{3\cdot\ell_1}\right) \quad \textit{[CSA A23.3 §13.12.2.1]}$$

- Beams proportioned for 100% of the exterior negative moment.
- Also, beams take 100% of loading applied directly to them.
- The slab carries the remainder of the moment, with flexural reinforcement uniformly distributed over width.

Example 2: Design edge strip with spandrel beam

-calculate α:

$$a = 400, h = 200, \frac{a}{h} = 2.0$$
$$b = 700, h = 200, \frac{b}{h} = 3.5 \quad \rightarrow f = 1.12 \text{ (from tables)}$$

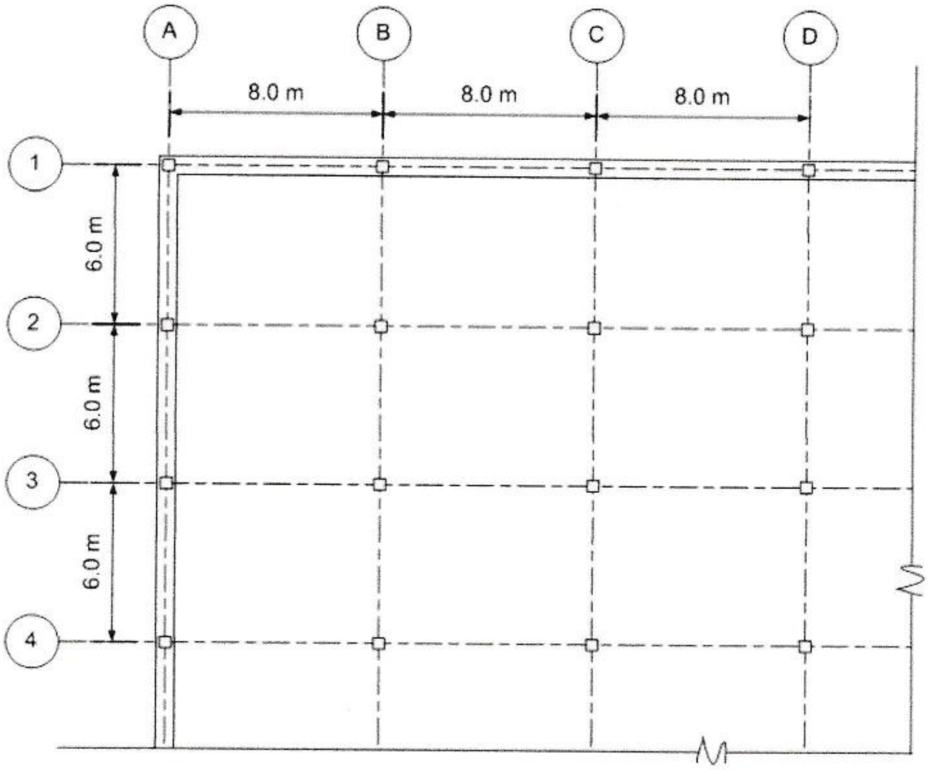
$$\alpha = \frac{E_{c,b} \cdot b}{E_{c,s} \cdot l}\left(\frac{a}{h}\right)^3 \cdot f = \frac{700}{4000 + 350}\left(\frac{400}{200}\right)^3 \cdot 1.12$$

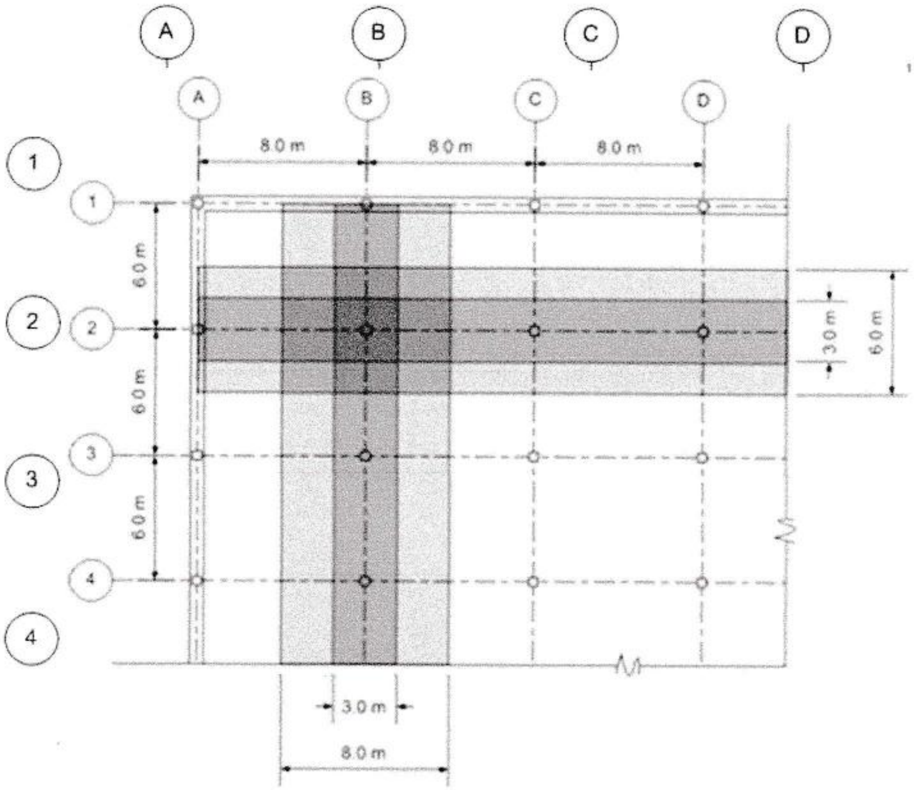
$$\alpha = 1.44\ (N - S)$$
$$\alpha = 1.87\ (E - W)$$

-proportion beams for 100% of negative exterior moment

$$\left(\frac{\alpha}{0.3 + \alpha_1}\right)\left(1 - \frac{l_2}{3 \cdot l_1}\right) x\ 100\%\ of\ neg\ and\ pos\ moments$$
$$\left(\frac{1.44}{0.3 + 1.44}\right)\left(1 - \frac{8}{3 \cdot 6}\right) \cdot 100\%$$
$$= 46\%\ (N - S)$$
$$62\%\ (E - W)$$

-design slab strip for remainder of moment

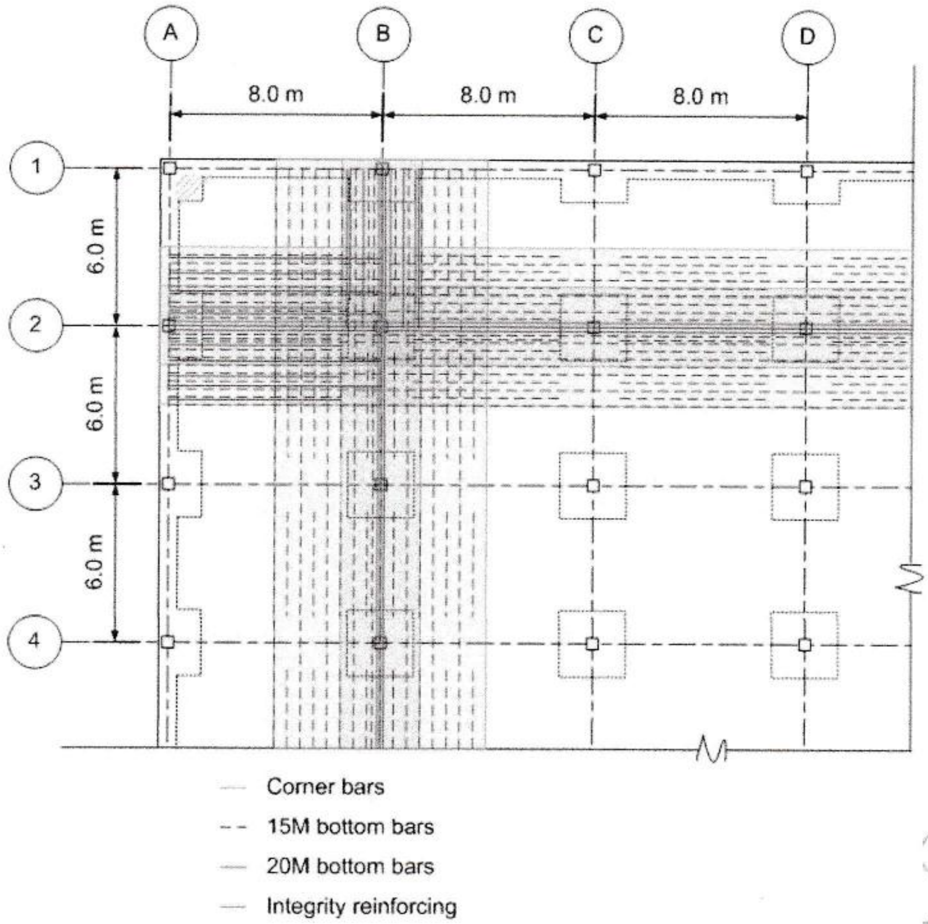




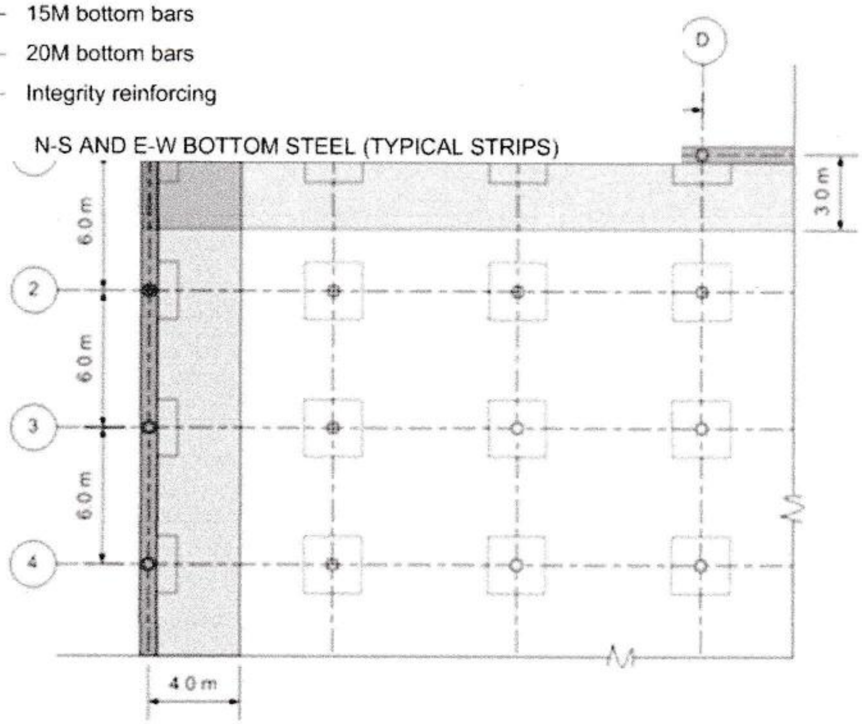
10

— Integrity reinforcing

N-S AND E-W BOTTOM STEEL (TYPICAL STRIPS)



N-S AND E-W BOTTOM STEEL (TYPICAL STRIPS)



N-S AND E-W EDGE DESIGN (BEAM + SLAB) STRIPS

13

REINFORCEMENT OF SLABS: special considerations

- Miniumum slab reinforcement *[CSA A23.3 §7.8.1]*:

  $A_{s,min} = 0.002 \cdot A_g$ (in each direction).

- Maximum slab reinforcement spacing, *s*, *[CSA A23.3 §13.10.4]*:

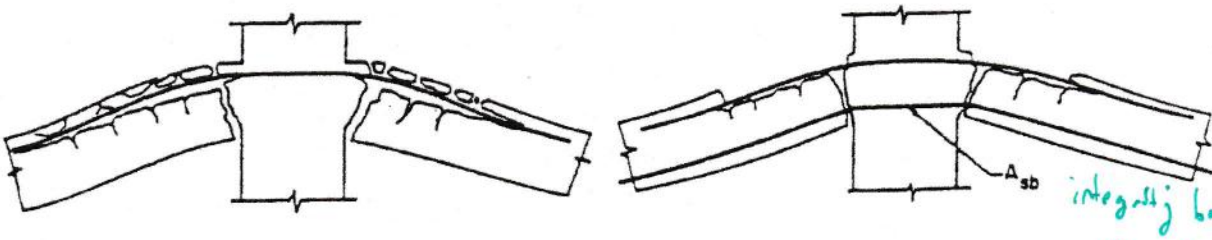
  $1.5 \cdot h_s$ but ≤ 250 mm (negative moment reinf. within b_b)

  $3.0 \cdot h_s$ but ≤ 500 mm (everywhere else)

- Distribution of reinforcement:
  - The area of reinforcement must be at least equal to that required by flexure at critical sections.
  - Special provisions also exist for the detailing of flexural reinforcement in exterior space.
  - Corner reinforcement is required for slabs supported on stiff beams (and walls) $\alpha > 1.0$
  - Locations of bar cut-off points should be determined based on moment envelopes, development length requirements.
  - Tables may be used for bend point locations and minimum extensions for the reinforcement of slabs without beams.
  - For frames subjected to lateral loads, further analysis is required to determine cut-off points.

Reinforcement for Structural Integrity

➢ Two-way (punching) shear failure may cause tearing out of the top reinforcement over the support.



(a) Tearing out of top reinforcement after shear failure

(b) Bottom reinforcement for hanging up slab

integrity bars not for flexure, and don't increase punching shear capacity. Only to avoid progressive collapse post-punching shear failure.

[CSA A23.3 2014]

➢ To avoid progressive collapse, a certain amount of the bottom reinforcement is required to go through the column in each span direction, the minimum area of this reinforcement:

$$\sum A_{sb} = \frac{2 \cdot V_{se}}{f_y} \qquad [CSA\ A23.3\ §13.10.6]$$

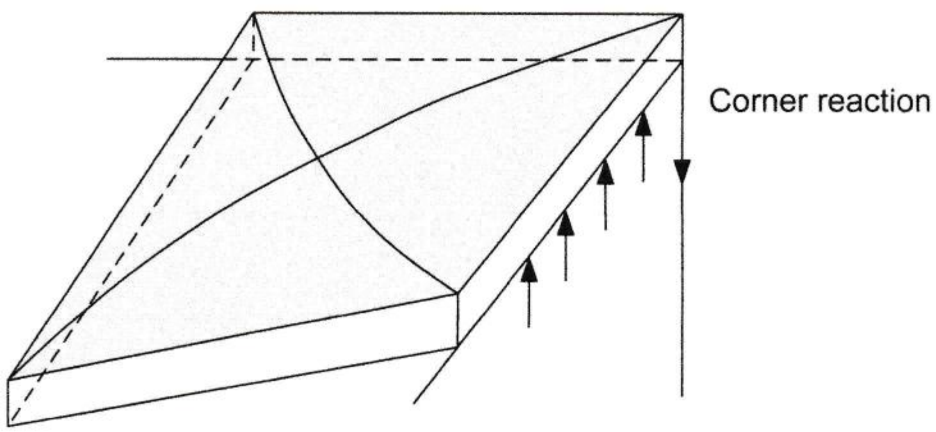
where: V_{se} = shear transmitted to column or column capital due to specified loads but not less than shear corresponding to 2 · self weight of the slab

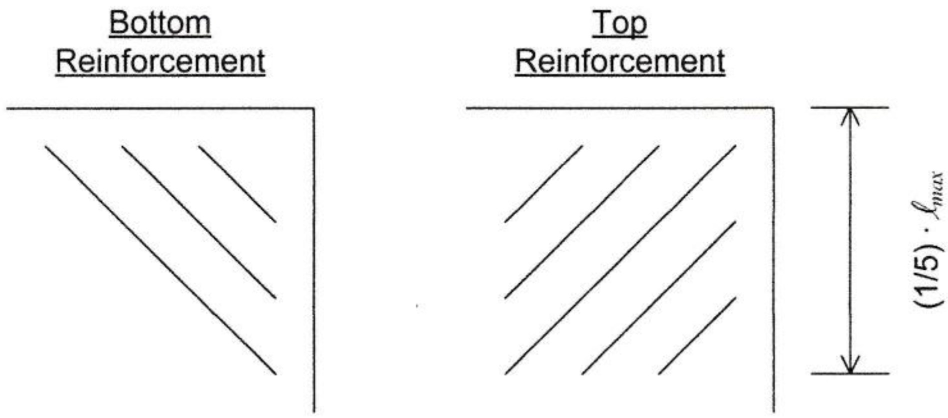
$\sum A_{sb}$ = area of bottom reinforcement connecting the slab to the column or column capital on all faces of its periphery

Corner Reinforcement

– Required for slabs supported on stiff beams (and walls) if α > 1.0.

- Top reinforcement is required to resist moments about an axis perpendicular to a diagonal drawn through the corner.
- Bottom reinforcement is needed to resist moments about an axis parallel to the diagonal.
- Both the top and bottom reinforcement should be such that it can resist a moment equal to the maximum positive moment per metre width in the slab.
- This corner reinforcement should be provided along a distance = (1/5) · longer span.

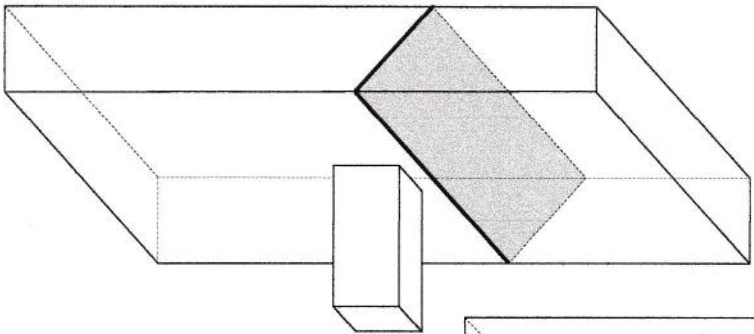


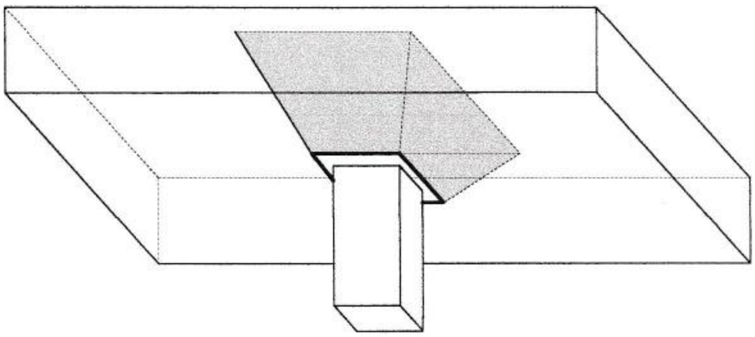


SHEAR AND MOMENT TRANSFER IN CONCRETE SLABS

Shear in reinforced concrete slabs:

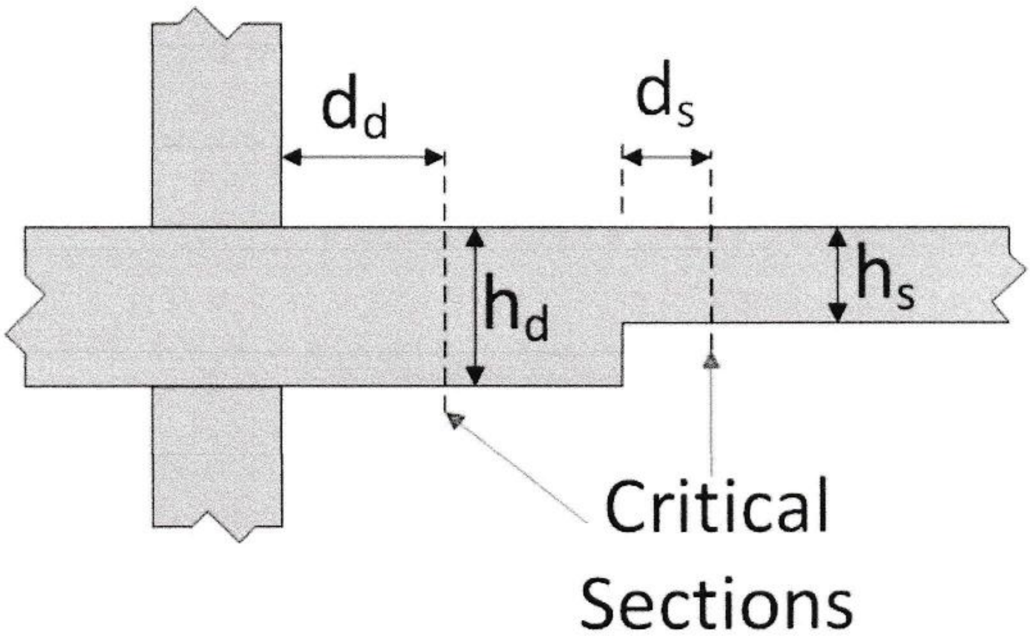
- Shear failure results from inclined cracks caused by flexural and shear stresses.
- <u>One-way shear</u>: shear crack extending across entire width of a slab, similar to beam shear.
- <u>Two-way shear</u> (or "punching shear"): pyramid shaped crack surface around column perimeter.
- Two-way shear usually governs (but we still need to check both).
- Shear failure is normally sudden and brittle, and therefore highly undesirable.





One-Way Shear

- Design such that: $v_f \leq v_c$
- $v_c = \phi_c \cdot \lambda \cdot \beta \cdot \sqrt{f'_c} \cdot b_w \cdot d_v$ *[CSA A23.3 §11.3.4]*
- β = 0.21 for slabs with $h \leq$ 350 mm
- <u>or</u> determine β using simplified or general shear design methods.
- The critical section for one-way shear is located a distance, *d*, from the face of the column or drop panel:



- Calculate shear for the design strip.

- Assume that the one-way shear force is distributed between the column and middle strips in proportion to the design negative moments in each strip *[CSA A23.3 §13.3.6.1]*.

- For corner columns *[CSA A23.3 §13.3.6.2]*:

– If corner columns pass this check, then they don't need to be checked for two-way shear.

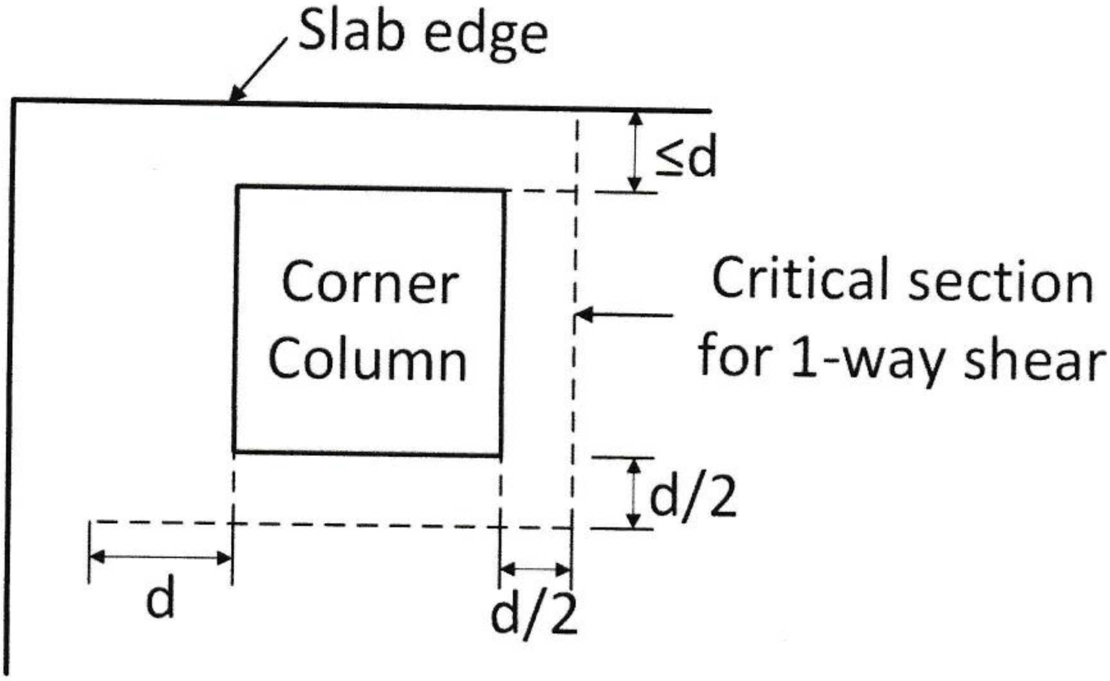


Fig. 13.3.6.2 – Critical Section for One-Way Shear at Corner Columns

Two-Way Shear

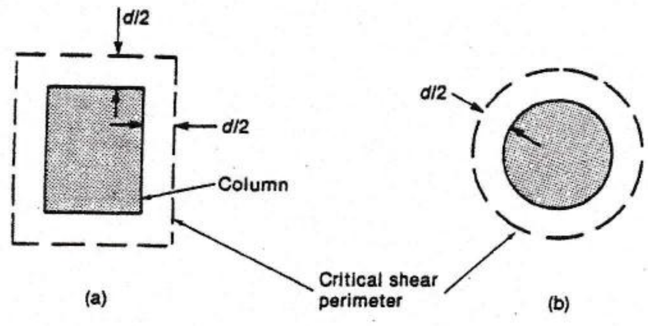
Behaviour

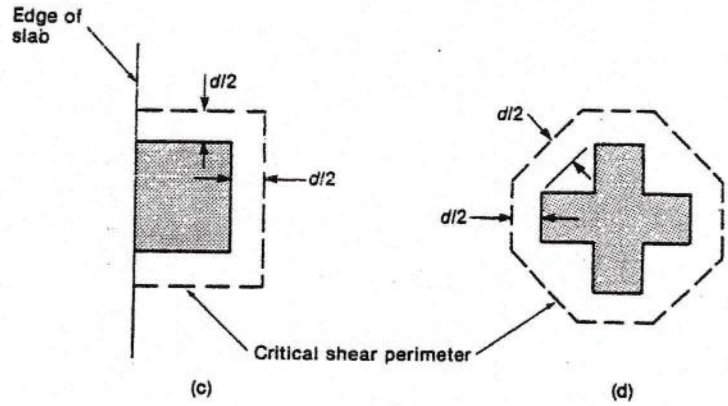
- The maximum moment occurs around the column.
- Flexural cracking occurs first, at the top of the slab (tension side).
- Shear cracking follows, on a pyramid-shaped surface around the column perimeter.

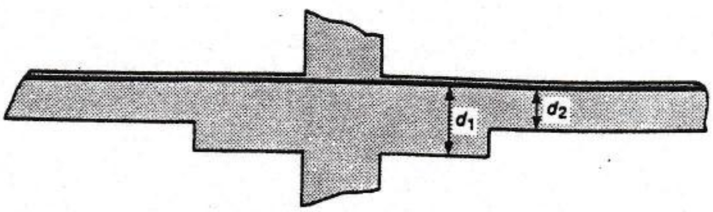
- The ensuing "failure" essentially consists of the slab sliding down the column; the negative moment steel rips out of top of the slab.
- The shear in two-way slabs comes from the vertical loads and unbalanced moments.

Two-Way Shear Design

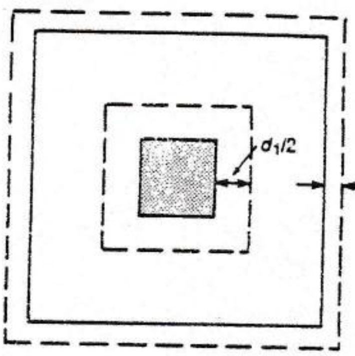
- Test indicates that critical section for shear is at the column face.
- To simplify the design, the code adopts the concept of a "critical perimeter", b_o:
  - Two-way shear is assumed to occur on a vertical surface along b_o. b_o is the minimum perimeter around the column, but need never approach closer than $d / 2$ to column face.





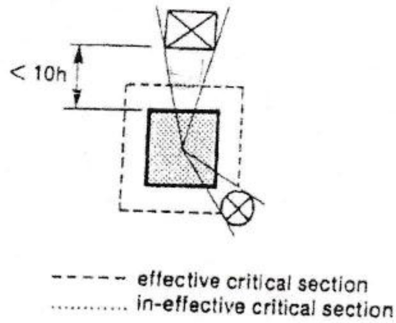


(a) Section through drop panel.

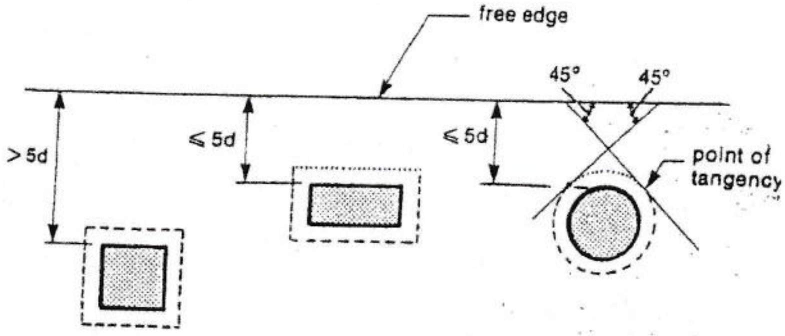


(b) Critical sections.

- b_0 is reduced if slab openings or slab edge present.

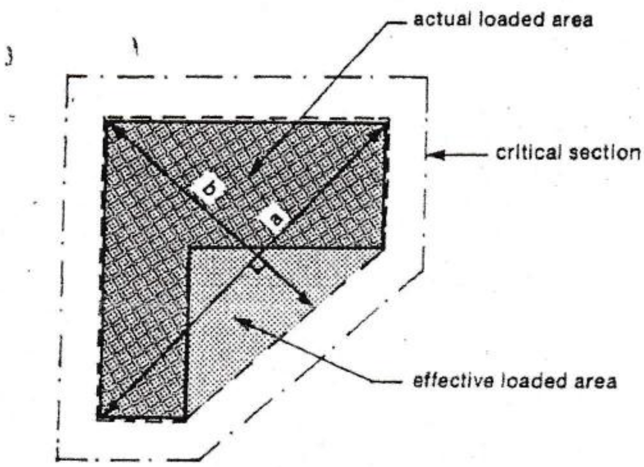


(a) Proximity to Openings



(b) Proximity to Free Edge

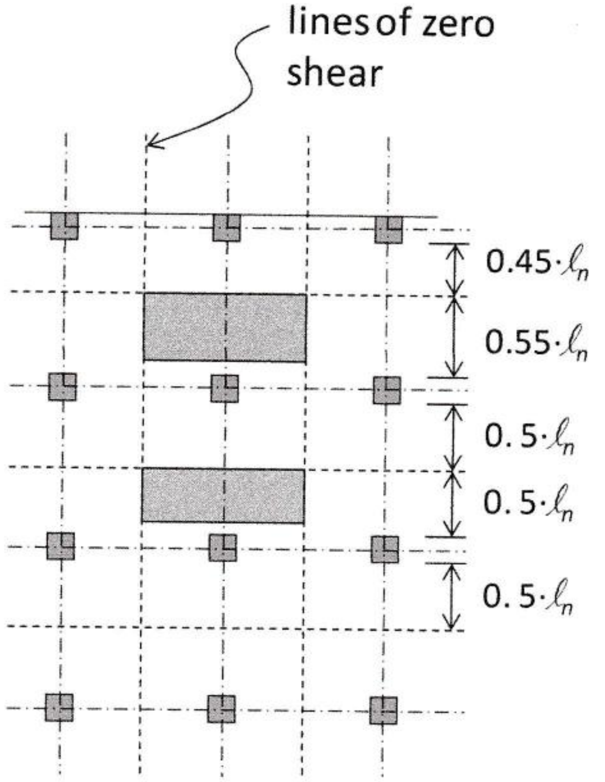
Fig. N11.27:
Effects of Openings and Free Edges on Effective Critical Section



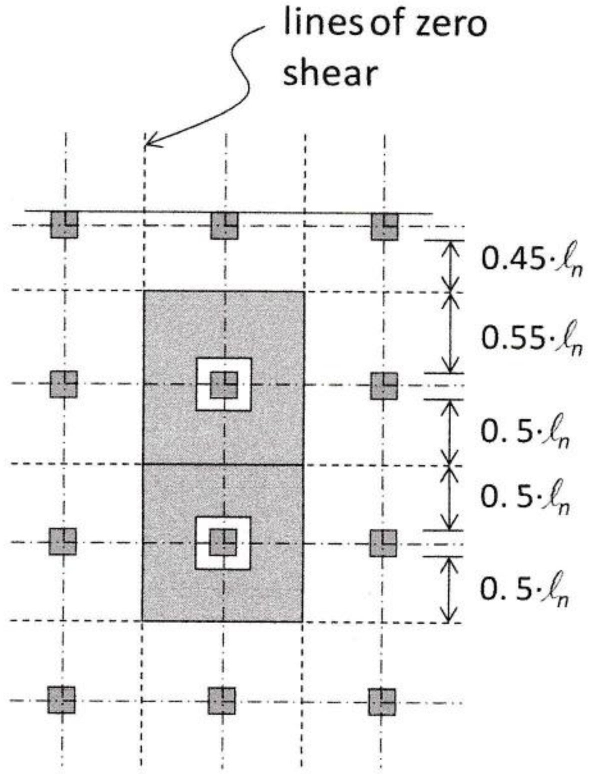
[CSA A23.3 2004]

Fig. N11.26:
β_c for a Non-Rectangular Column

Critical sections and tributary areas: 1-way shear



Critical sections and tributary areas: 2-way shear



Punching Shear Design

The factored shear stress resistance, v_r, must satisfy:

$$v_f \leq v_r$$

where:

v_f = factored nominal shear acting on critical perimeter

The factored shear resistance is:

$$v_r = v_c + v_s$$

v_s = shear reinforcement contribution
v_c = concrete contribution.

Slabs without shear reinforcement: *clause 13.3.4*

$$v_f \leq v_c$$

Factored shear resistance

$$\left.\begin{aligned} v_c &= \left(1 + \frac{2}{\beta_c}\right) 0.2 \lambda \phi_c \sqrt{f_c'} \\ v_c &= \left(\frac{\alpha_s d}{b_o} + 0.19\right) \lambda \phi_c \sqrt{f_c'} \\ v_c &= 0.38 \lambda \phi_c \sqrt{f_c'} \end{aligned}\right\}$$

v_c decreases with increasing rectangularity of the loaded area. Therefore v_c depends on:

$$\beta_c = A/B$$

where A - long side of the column, B - short side of the column

(For irregular columns see previous page to determine β_c)

also $\alpha_s = 4$ for interior column

$\alpha_s = 3$ for edge column

If v_c is less than v_f, the shear capacity can be increased by:

1. Thickening of a slab over the entire panel
2. Using drop panels
3. Adding a capital to increase b_o
4. Adding shear reinforcement
5. Increasing the column size (increases b_o)

Slabs with shear reinforcement

$$v_f \le v_r = v_c + v_s$$

NOTE: v_c is now different than for slabs without shear reinforcement

- shear resistance must be investigated at
  - critical perimeter at d/2 from support (measured at d/2 from column), and
  - at the end of shear reinforcement (measured at d/2 from the end of shear reinforcement).
- shear reinforcement must be extended to the section where $\boxed{\frac{V_f}{b_o d} \le 0.19 \times \phi_c \sqrt{f'_c}}$ but at least a distance 2d from the face of the column.

Factored shear resistance from shear reinforcement:

$$v_s = \frac{\phi_s A_{vs} f_{yv}}{b_0 s}$$ *clause 13.3.8.5*

A_{vs} - cross-sectional area of the headed shear reinforcement on a concentric line parallel to the perimeter of the column.
s - spacing of headed shear reinforcement or stirrups measured perpendicular to b_o

Spacings Headed shear reinforcement : *clause 13.3.8.6*

- The distance between the column face and the first line of headed shear reinforcement shall be 0.35d to 0.4d.

Further spacing:

- s $\leq$ 0.75 d when $v_f \leq 0.56\lambda\phi_c\sqrt{f_c'}$
- s $\leq$ 0.5 d when $v_f > 0.56\,\lambda\phi_c\sqrt{f_c'}$

Spacings Stirrup reinforcement *clause 13.3.9.5*

- First stirrup from column: d/4
- Subsequent stirrups d/2

.

Factored shear resistance from concrete for:

<u>Headed Shear Reinforcement</u> *clause13.3.8.3*

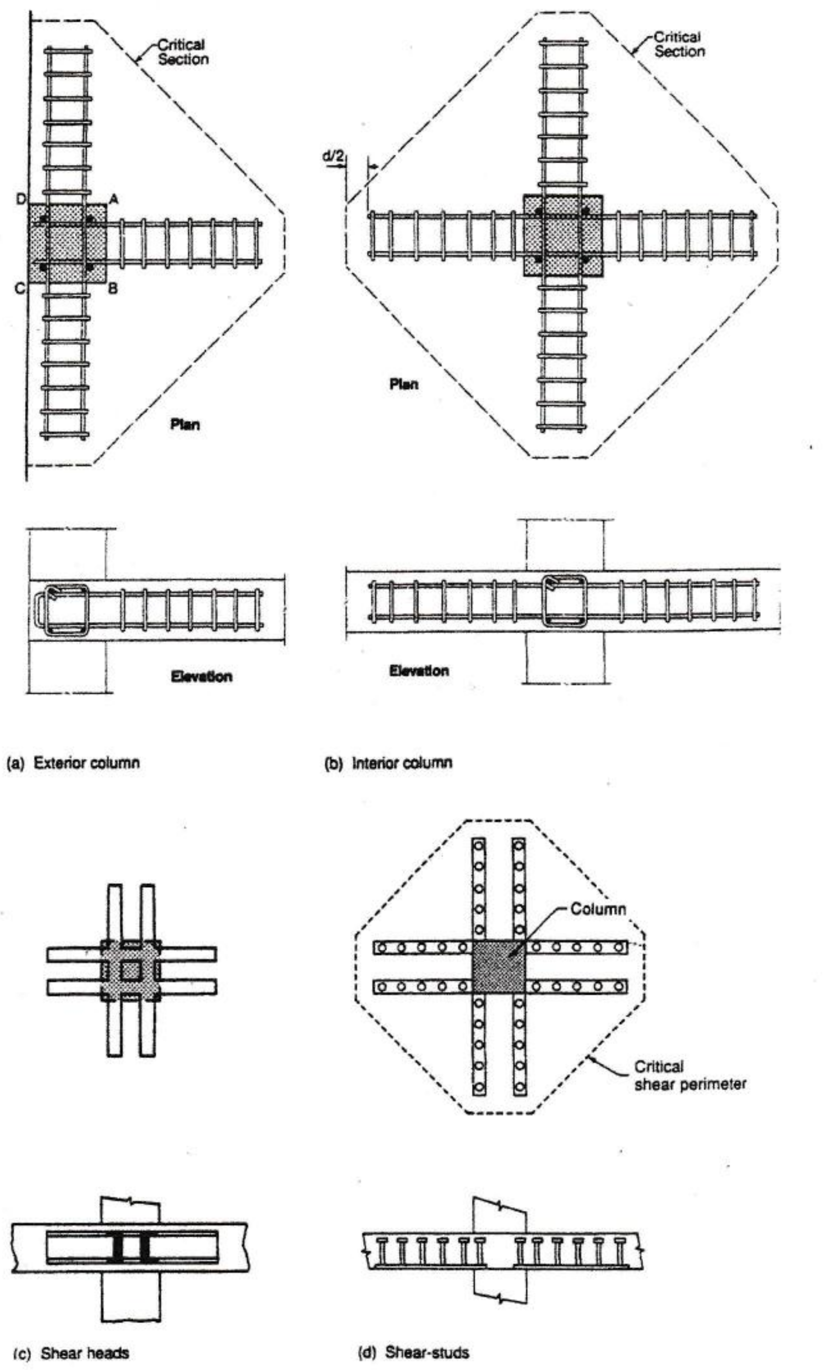
- $v_c = 0.28\lambda\phi_c\sqrt{f_c'}$, and $v_f \leq 0.75\,\lambda\phi_c\sqrt{f_c'}$

<u>Stirrup Reinforcement</u> *clause 13.3.9.3*

- Only in slabs 300 mm deep or greater.
- $v_c = 0.19\lambda\phi_c\sqrt{f_c'}$, and $v_f \leq 0.55\,\lambda\sqrt{f_c'}$

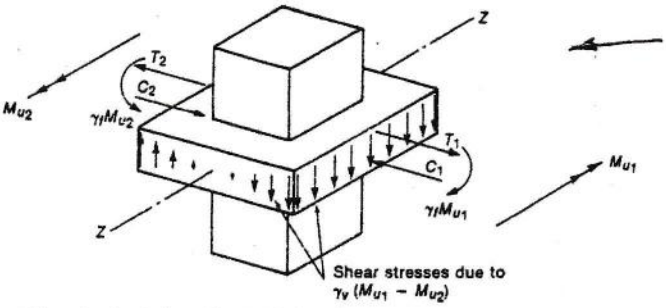
- Note: Anchorage is very important.

TYPES OF SHEAR REINFORCEMENT

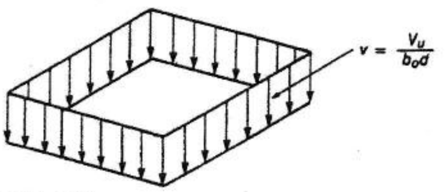


SHEAR AND MOMENT TRANSFER

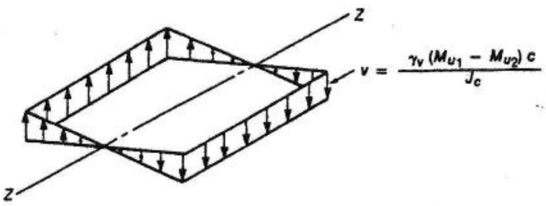
Interior Column



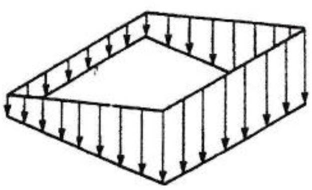
(a) Transfer of unbalanced moments to column.

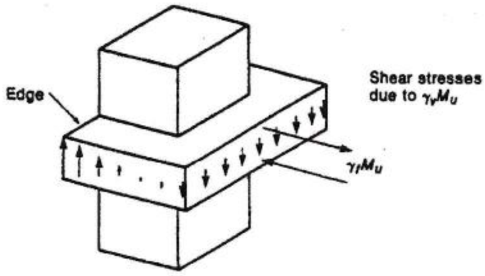


(b) Shear stresses due to V_u.

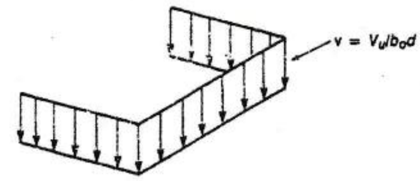


(c) Shear due to unbalanced moment.

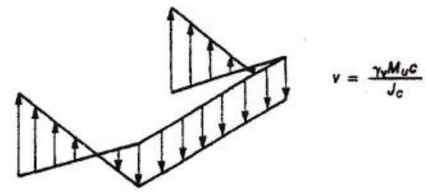




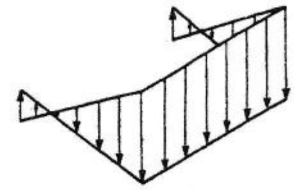
(a) Transfer of moment at edge column.



(b) Shear stresses due to V_u.

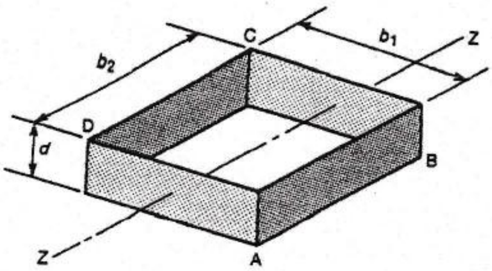


(c) Shear stresses due to M_u.



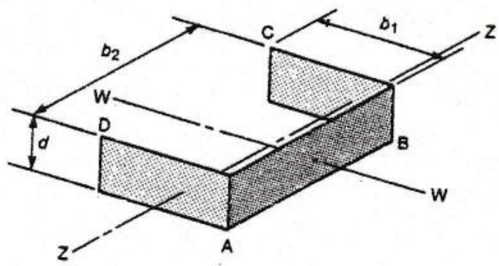
Edge Column

(d) Total shear stresses.

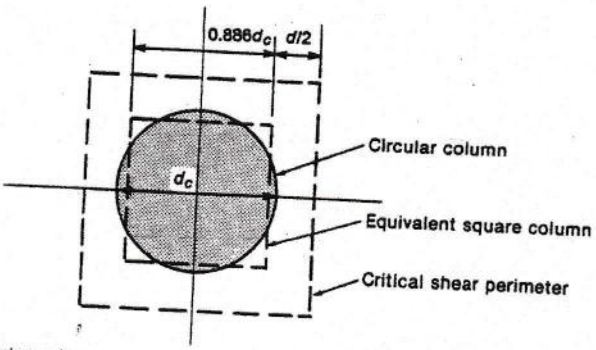


(a) Critical perimeter of an interior column.

Critical shear perimeters

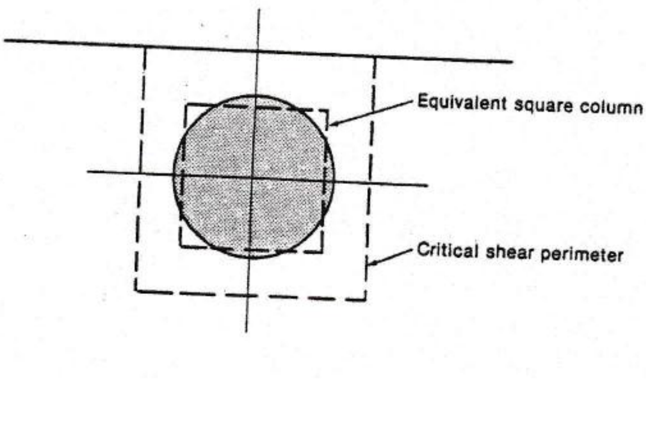


(b) Critical perimeter of edge column.

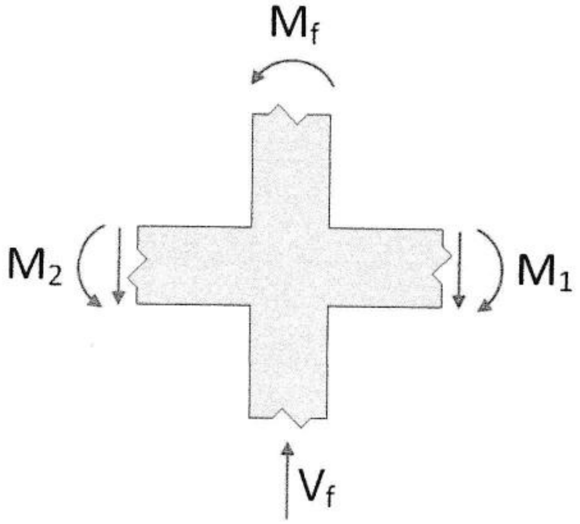


(a) Interior column.

Critical shear perimeter for circular column



Shear and Moment Transfer in Two-Way Slabs



When $M_1 \neq M_2$ there is an "unbalanced" moment M_f

$M_f = M_1 - M_2$

- This "unbalanced" moment will be resisted by columns (above and below, in proportion to their stiffnesses) but it must be able to somehow "get" to the column.
- The behaviour involves flexure, shear and torsion in the part of the slab attached to the column.

In the design of shear and moment transfer is is assumed that approx 60% of M_f is transferred by flexural reinforcement. For this, sufficient reinforcement must be provided within of 1.5 x slab thickness (or drop panel(from the column. The reinfcement already there designed for flexure can be used for that purpose (Generally this unblanaced moment flexural reinforcement can only be a problem at the edge column with large unbalanced moments

The reaining fortion of the unbalanced moment (approx 40%) is transferred by shear in slabs.

Portion of unbalanced moment carried by FLEXURE (clause 13.3.5.3)

$$\gamma_f = \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}}$$

b_1 width of the critical section masured in the direction of for which moments are determined $b_1 = c_1+d$
b_2 – perpendicular direction

Moment transferred by shear:

$$M_{fv} = (1-\gamma_f)M_f$$

This moment is transferred through shearing stresses

$$v_M = \frac{M_{fv}c}{J_C}$$

where:

c = distance from the centroidal axis of the shear perimeter to the point where
the shear stress is being computed

J_c = polar moment of inertia about the centroidal axis

total shear stresses are equal to:

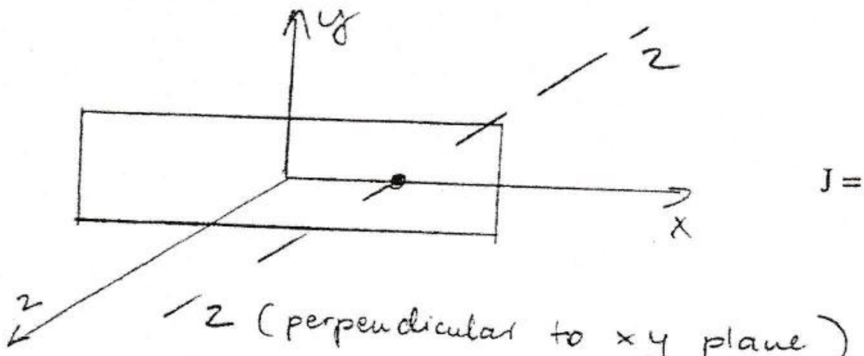
$$v = \frac{V_f}{b_o d} \pm \frac{M_{fv}c}{J_c}$$ (Edge columns!)

Moment M_f must be calculated about the centroid of the shear perimeter.

PROPERTIES OF THE SHEAR PERIMETER

- length of the perimeter
- location of the neutral axis z-z
- polar moment of inertia J

Polar moment of inertia of a rectangle about the axis z-z, perpendicular to plane of the rectangle and displaced by a distance $\overline{x}$.

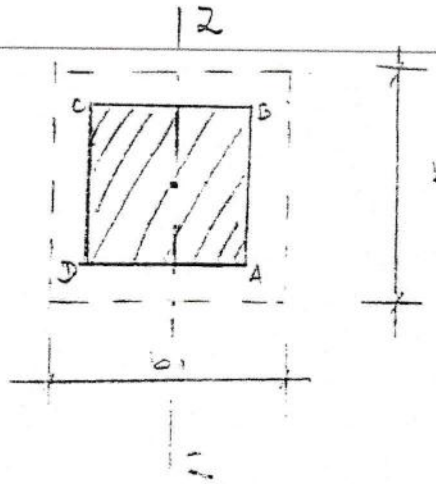


$$J = I_x + I_y + A\overline{x}^2$$

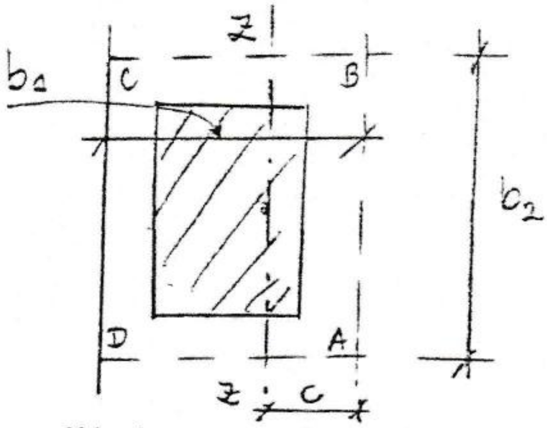
INTERIOR COLUMN

$$\underbrace{J = 2\times\frac{b_1 d^3}{12} + 2\times\frac{d b_1{}^3}{12}}_{\text{faces CB, DA}} + \underbrace{2\times b_2 d\times\left(\frac{b_1}{2}\right)^2}_{\text{faces AB, DC}}$$

polar moment of inertia of faces parallel to Z–Z



EDGE COLUMN: moments about the axis parallel to the edge



$b_o = b_2 + 2b_1$

if b_1 is perpendicular to the edge

$$\text{location of } z-z \begin{cases} c = \dfrac{\text{moment of area of the sides about AB}}{\text{area of the sides}} \\ c = \dfrac{2\times b_1 d \dfrac{b_1}{2}}{2\times b_1 \times d + b_2 \times d} \end{cases}$$

$$J = \underbrace{2\times\frac{b_1 d^3}{12} + 2\times\frac{d b_1^{\,3}}{12} + 2\times b_1 d\left(\frac{b_1}{2} - c\right)^2}_{\text{faces DA \& CB}} + \underbrace{b_2 d c^2}_{\text{face AB}}$$

EDGE COLUMN: moments about the axis normal to the edge

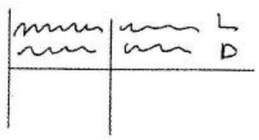
$$C = \frac{b_2}{2}$$

$$J = \underbrace{\frac{b_2 d^3}{12} + \frac{d b_2^{\,3}}{12}}_{\text{face AB}} + \underbrace{2b_1 d\, c^2}_{\text{faces DA \& CB}}$$

Moments Transfer to Columns

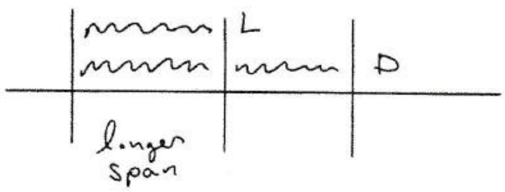
For edge columns:

- Critical load case is full live load on both adjacent panels, i.e.:



For interior columns:

- Pattern live loading case may be critical, i.e.:



- Use the following equation to account for moments due to uneven distribution of live loads:

  $M_f = 0.07 \cdot \left\{ \left(w_{df} + 0.5 \cdot w_{lf} \right) \cdot l_{2a} \cdot l_n^2 - w'_{df} \cdot l'_{2a} \cdot \left(l'_n \right)^2 \right\}$, where:

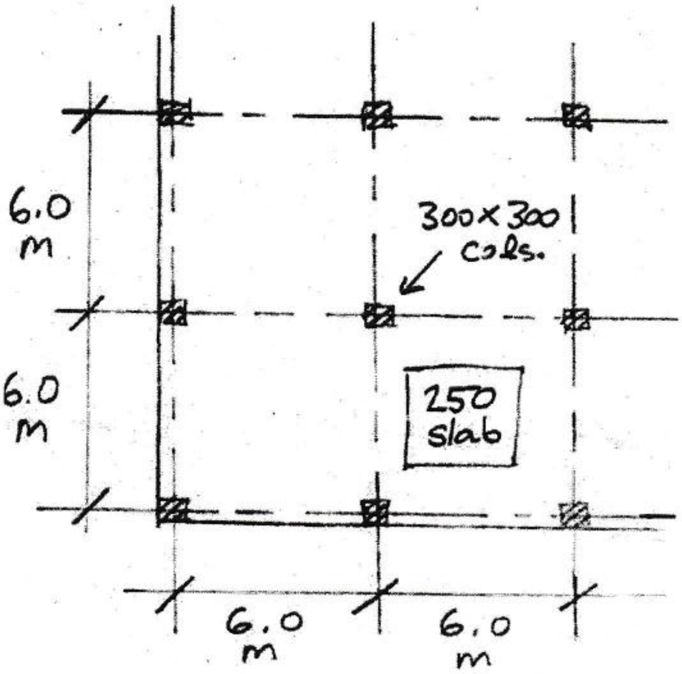
  $w_{df}, w_{lf}, l_{2a}, l_n$ refer to longer span adjacent to column, and

  w'_{df}, l'_{2a}, l'_n refer to shorter span adjacent to column.

- Must design columns or walls to resist this moment.
- The total column moment is distributed between columns above and below the joint in proportion to their flexural stiffnesses.

– Must place enough reinforcement to resist this moment times a distribution factor, γ_f, in a band of width, b_b, where b_b = column width + $1.5 \cdot h_s$ on either side of column.

Example 1: Shear Design of 2-Way Slabs



Check slab if adequate for shear capacity

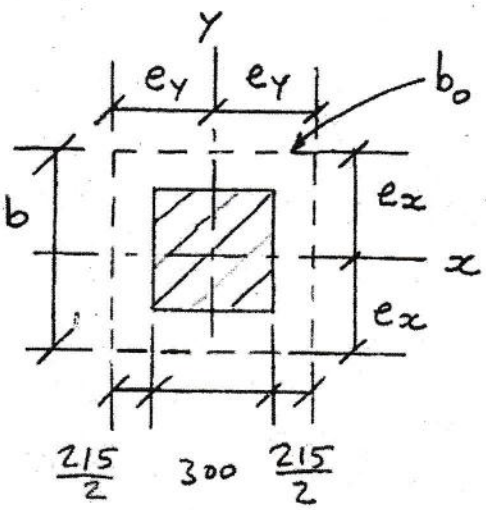
$f_c' = 40MPa,\ f_y = 400MPa$

$SDL = 1.5kPa, LL = 2.4kPa$

Part 1: 2-way shear, interior columns:

$\rightarrow d = 250 - 20 - 15 = 215mm\ (average)$

$20mm \rightarrow concrete\ cover\ (see\ CSA\ A23.1), 15mm \rightarrow bar\ diameter$



$$e_x = e_y = 257.5mm$$

$$b_1 = b_2 = 515mm$$

$$\gamma_v = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = 0.4$$

$$\rightarrow J = I_x + I_y + Ax^2 \qquad = 2 \cdot \frac{b_1 d^3}{12} + \frac{2db_1^3}{12} + 2b_2 d\left(\frac{b_1}{2}\right)^2$$

$$= 2(515)\left(\frac{215^3}{12}\right) + 2(215)\left(\frac{515^3}{12}\right) + 2(515)(215)\left(\frac{515}{2}\right)^2$$

$$= 2.043x10^{10}mm^4$$

Calculate V_f, M_f:

According to CSA A23.3 Commentary N13.3.5.5, we only need to consider $V_{f,max}$ and maximum of M_{fx} and M_{fy}

$$w_f = 1.25\big(1.5 + 24(0.25)\big) + 1.5(2.4) = 13.0kPa$$

$$V_{f,max} = 13.0(6^2 - 0.515^2) = 463.7kN$$

$$M_{fx} = 0.07\{(w_{df} + 0.5w_{lf})l_{2a}(l_n)^2 - w_{df}l'_{2a}(l'_n)^2\}$$

$$= 0.07\{\big(9.38 + 0.5(3.6)\big) \times 6 \times 5.7^2 - 9.38(6)(5.7)^2\}$$

$$= 0.07\{0.5(3.6)(6)(5.7)^2\} = 24.56kNm$$

(Note: Since panels are square, $M_{fy} = M_{fx}$

Now calculate v_c:

$$v_c = min\begin{Bmatrix} \left(1 + \frac{2}{\beta_c}\right)0.19 \\ \frac{\alpha_s d}{b_o} + 0.19 \\ 0.38 \end{Bmatrix}\lambda\phi_c\sqrt{f'_c}$$

$$= min\begin{Bmatrix} (3)0.19 = 0.57 \\ \frac{4(215)}{4(515)} + 0.19 = 0.61 \\ 0.38 \end{Bmatrix}0.65\sqrt{40} = 1.56MPa$$

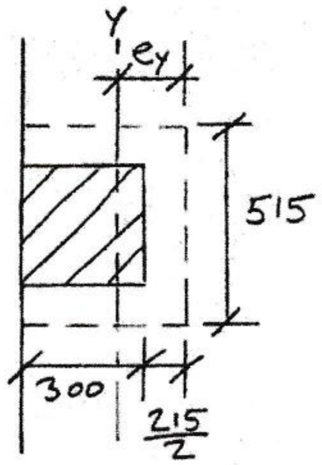
Confirm that $v_c \geq v_f$

$$v_f = \frac{V_f}{b_o d} + \left(\frac{\gamma_v M_f e}{J}\right)_x$$

$$= \frac{463.7 \times 10^3}{4(515)(215)} + \left(\frac{0.4(24.56 \times 10^6)(257.5)}{2.043 \times 10^4}\right)_x$$

$$= 1.17MPa < 1.56MPa \rightarrow ok!$$

Part 2: 2-way shear, edge column:



$$e_y = \frac{407.5^2}{2(407.5) + 515} = 125mm$$

$$b_o = 2(407.5) + 515 = 1330mm$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{407.5}{515}}} = 0.37$$

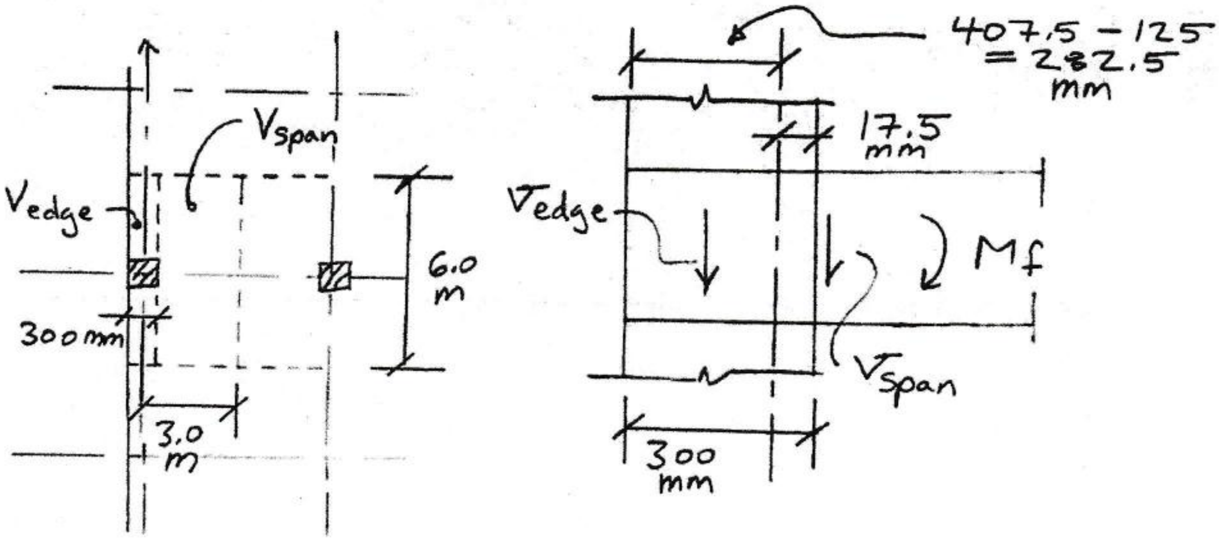
$$J = \frac{2b_1 d^3}{12} + \frac{2d(b_1)^3}{12} + 2b_1 d\left(\frac{b_1}{2} - e_y\right)^2 + b_2 d(e_y)^2$$

$b_1 = 407.5, b_2 = 515, e_y = 125, d = 215mm$

$\rightarrow J = 5.916 \times 10^9 mm^4$

Calculate V_f, M_f:

According to CSA A23.3 Commentary N13.3.5.5, we only need to consider $V_{f,max}$ and M_{fy} when adjacent spans along slab edge are equal



$V_{span} = 13(2.85)(6) = 221.9kN$

$V_{edge} = 13(0.3)(5.7) = 22.2kN$

$M_{fy} = 0.26M_o$

$= 0.26(13)(6.0)\left(\frac{5.7^2}{8}\right) = 82.2kNm$

$V_f = 221.9 + 22.2 = 244.1kN$

$M_{fy} = 82.2 + 221.9(0.0175) - 22.2(0.15 - 0.0175) = 83.1kNm$

Now calculate v_c:

$$v_c = min\begin{Bmatrix} 0.57 \\ \frac{3(215)}{1330} + 0.19 = 0.67 \\ 0.38 \end{Bmatrix} 0.65\sqrt{40} = 1.56MPa$$

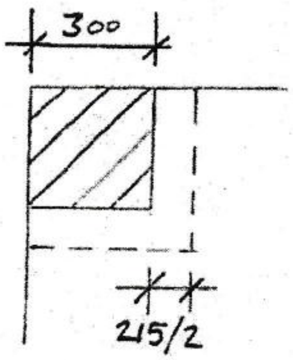
Same v_c as for interior column

Confirm that $v_c \geq v_f$

$$v_f = \frac{244.1 \times 10^3}{1330(215)} + \left(\frac{0.37(83.1 \times 10^6)(125)}{5.916 \times 10^4}\right)_x$$

$= 1.51 < 1.56MPa \rightarrow ok!$

Part 3: 1-way shear check, corner column



$$b_o = 2\left(300 + \frac{215}{2}\right) = 815mm$$

$$v_c = \beta\lambda\phi_c\sqrt{f_c'}$$

$$= 0.21(1.0)(0.65)(\sqrt{40})$$

$$= 0.86MPa$$

$$V_f = \left\{(3.0 + 0.15)^2 - \left(0.3 + \frac{0.215}{2}\right)^2\right\}(13.0) = 126.8kN$$

$$v_f = 126.8 \times \frac{10^3}{815 \cdot 215} = 0.72MPa < v_c \rightarrow ok! *$$

*Since checking corner columns for two-way shear not required