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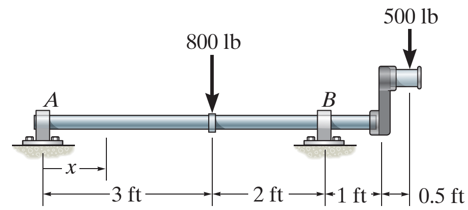
11–1.

Draw the shear and moment diagrams for the shaft and determine the shear and moment throughout the shaft as a function of x for $0 \le x < 3$ ft, $3\text{ ft} < x < 5$ ft, and $5\text{ ft} < x < 6$ ft. The bearings at A and B exert only vertical reactions on the shaft.



SOLUTION

For $0 \le x < 3$ ft,

$+\uparrow\Sigma F_y = 0;\qquad 170 - V = 0\qquad V = 170\text{ lb}$ **Ans.**

$\circlearrowleft+\Sigma M_{NA} = 0;\qquad M - 170x = 0$

$M = (170x)\text{ lb}\cdot\text{ft}$ **Ans.**

For $3\text{ ft} < x < 5$ ft,

$+\uparrow\Sigma F_y = 0;\qquad 170 - 800 - V = 0$

$V = -630\text{ lb}$ **Ans.**

$\circlearrowleft+\Sigma M_{NA} = 0;\qquad M + 800(x - 3) - 170x = 0$

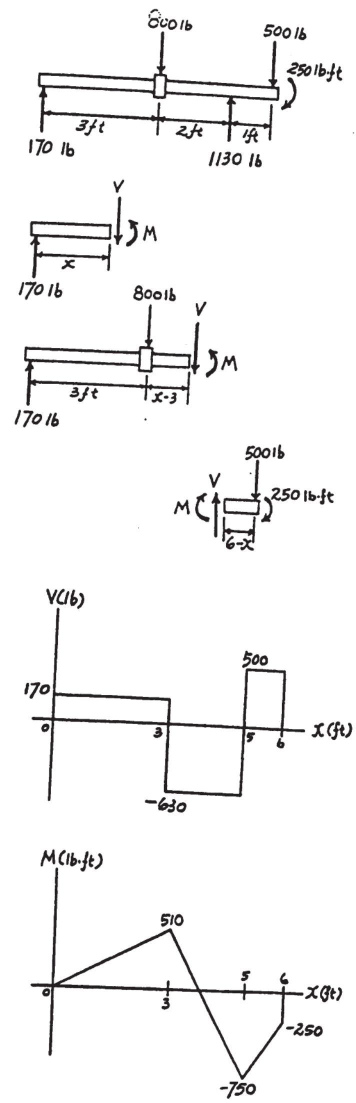
$M = \{-630x + 2400\}\text{ lb}\cdot\text{ft}$ **Ans.**

For $5\text{ ft} < x \le 6$ ft,

$+\uparrow\Sigma F_y = 0;\qquad V - 500 = 0\qquad V = 500\text{ lb}$ **Ans.**

$\circlearrowleft+\Sigma M_{NA} = 0;\qquad -M - 500(6 - x) - 250 = 0$

$M = \{500x - 3250\}\text{ lb}\cdot\text{ft}$ **Ans.**



Ans:
For $0 \le x < 3$ ft,
$V = 170$ lb,
$M = (170x)\text{ lb}\cdot\text{ft}$,
For $3\text{ ft} < x < 5$ ft,
$V = -630$ lb,
$M = \{-630x + 2400\}\text{ lb}\cdot\text{ft}$,
For $5\text{ ft} < x \le 6$ ft,
$V = 500$ lb,
$M = \{500x - 3250\}\text{ lb}\cdot\text{ft}$

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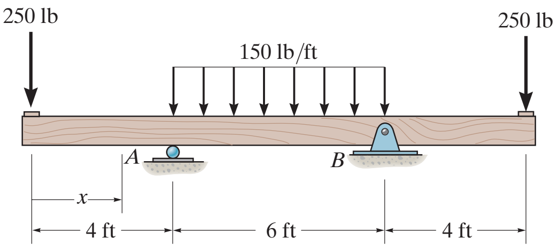
11–2.

Draw the shear and moment diagrams for the beam, and determine the shear and moment in the beam as functions of x for $0 \le x < 4$ ft, 4 ft $< x < 10$ ft, and 10 ft $< x < 14$ ft.



SOLUTION

Support Reactions: As shown on FBD.

Shear and Moment Functions:

For $0 \le x < 4$ ft:

$$+\uparrow \Sigma F_y = 0; \quad -250 - V = 0 \qquad V = -250 \text{ lb} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad M + 250\,x = 0$$

$$M = (-250\,x) \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

For 4 ft $< x < 10$ ft:

$$+\uparrow \Sigma F_y = 0; \quad -250 + 700 - 150(x - 4) - V = 0$$

$$V = \{1050 - 150\,x\} \text{ lb} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad M + 150(x - 4)\left(\frac{x - 4}{2}\right) + 250x - 700(x - 4) = 0$$

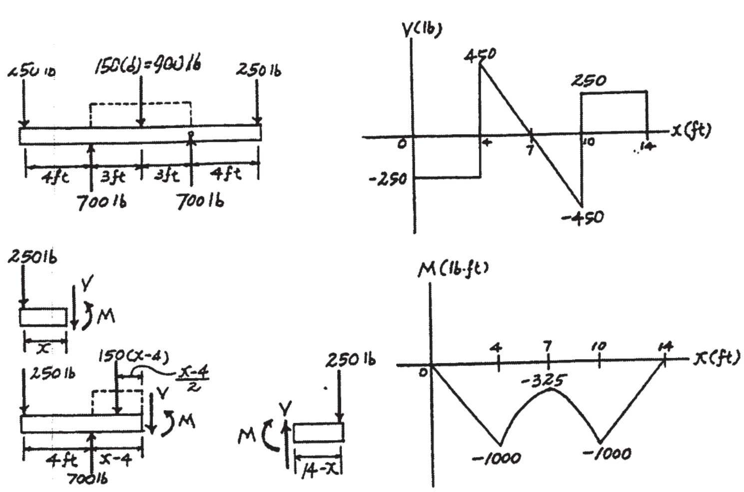
$$M = \{-75x^2 + 1050x - 4000\} \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

For 10 ft $< x \le 14$ ft:

$$+\uparrow \Sigma F_y = 0; \quad V - 250 = 0 \qquad V = 250 \text{ lb} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad -M - 250(14 - x) = 0$$

$$M = \{250x - 3500\} \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$



Ans:
For $0 \le x < 4$ ft,
$V = -250$ lb,
$M = (-250\,x)$ lb · ft,
For 4 ft $< x < 10$ ft,
$V = \{1050 - 150\,x\}$ lb,
$M = \{-75x^2 + 1050x - 4000\}$ lb · ft,
For 10 ft $< x \le 14$ ft,
$V = 250$ lb,
$M = \{250x - 3500\}$ lb · ft

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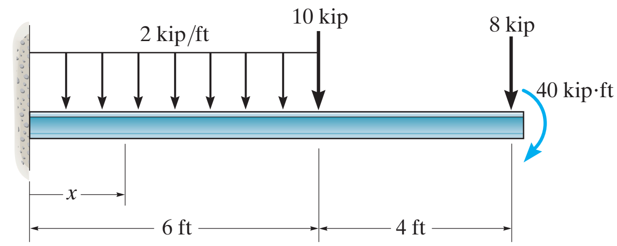
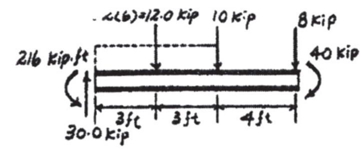
11–3.

Draw the shear and moment diagrams for the beam, and determine the shear and moment throughout the beam as functions of x for $0 \le x \le 6$ ft and $6 \text{ ft} \le x \le 10$ ft.



SOLUTION

Support Reactions: As shown on FBD.

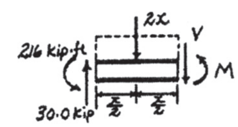


Shear and Moment Function:

For $0 \le x < 6$ ft:

$$+\uparrow \Sigma F_y = 0; \qquad 30.0 - 2x - V = 0$$

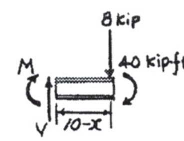
$$V = \{30.0 - 2x\} \text{ kip} \qquad \textbf{Ans.}$$



$$\circlearrowleft + \Sigma M_{NA} = 0; \quad M + 216 + 2x\left(\frac{x}{2}\right) - 30.0x = 0$$

$$M = \{-x^2 + 30.0x - 216\} \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

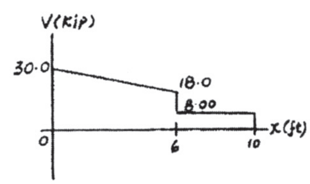
For $6 \text{ ft} < x \le 10$ ft:

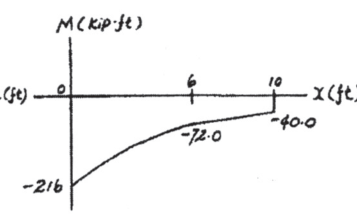


$$+\uparrow \Sigma F_y = 0; \qquad V - 8 = 0 \qquad V = 8.00 \text{ kip} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad -M - 8(10 - x) - 40 = 0$$

$$M = \{8.00x - 120\} \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$





Ans:
For $0 \le x < 6$ ft,
$V = \{30.0 - 2x\}$ kip,
$M = \{-x^2 + 30.0x - 216\}$ kip · ft,
For $6 \text{ ft} < x \le 10$ ft,
$V = 8.00$ kip,
$M = \{8.00x - 120\}$ kip · ft

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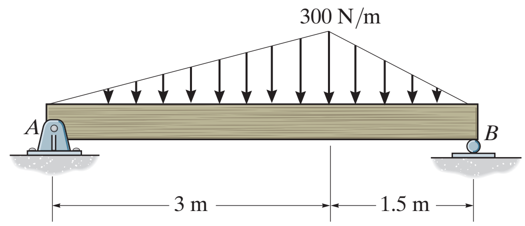
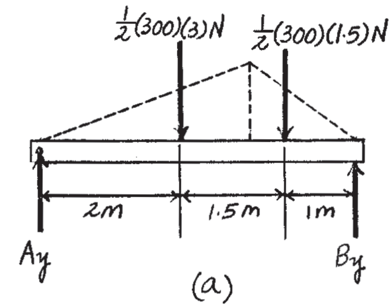
***11–4.**

Express the shear and moment in terms of x for $0 < x < 3$ m and 3 m $< x < 4.5$ m, and then draw the shear and moment diagrams for the simply supported beam.



SOLUTION



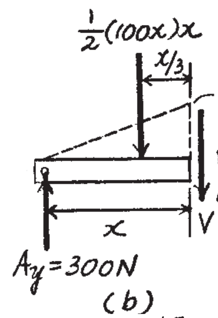
Support Reactions: Referring to the free-body diagram of the entire beam shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(4.5) - \frac{1}{2}(300)(3)(2) - \frac{1}{2}(300)(1.5)(3.5) = 0$$

$$B_y = 375 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + 375 - \frac{1}{2}(300)(3) - \frac{1}{2}(300)(1.5) = 0$$

$$A_y = 300 \text{ N}$$



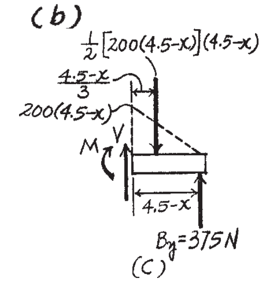
Shear and Moment Function: For $0 \le x < 3$ m, we refer to the free-body diagram of the beam segment shown in Fig. *b*.

$$+\uparrow \Sigma F_y = 0; \qquad 300 - \frac{1}{2}(100x)x - V = 0$$

$$V = \{300 - 50x^2\} \text{ N} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}(100x)x\left(\frac{x}{3}\right) - 300x = 0$$

$$M = \left\{300x - \frac{50}{3}x^3\right\} \text{N} \cdot \text{m} \qquad \textbf{Ans.}$$

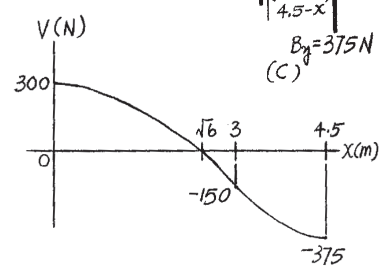


When $V = 0$, from the shear function,

$$0 = 300 - 50x^2 \qquad x = \sqrt{6} \text{ m}$$

Substituting this result into the moment equation,

$$M|_{x = \sqrt{6}\text{ m}} = 489.90 \text{ N} \cdot \text{m}$$



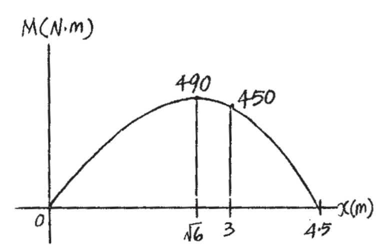
For 3 m $< x \le 4.5$ m, we refer to the free-body diagram of the beam segment shown in Fig. *c*.

$$+\uparrow \Sigma F_y = 0; \qquad V + 375 - \frac{1}{2}[200(4.5 - x)](4.5 - x) = 0$$

$$V = \left\{100(4.5 - x)^2 - 375\right\} \text{N} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \quad 375(4.5 - x) - \frac{1}{2}[200(4.5 - x)](4.5 - x)\left(\frac{4.5 - x}{3}\right) - M = 0$$

$$M = \left\{375(4.5 - x) - \frac{100}{3}(4.5 - x)^3\right\} \text{N} \cdot \text{m} \qquad \textbf{Ans.}$$



Shear and Moment Diagrams: As shown in Figs. *d* and *e*.

Ans:
For $0 \le x < 3$ m, $V = \{300 - 50x^2\}$ N,
$M = \left\{300x - \frac{50}{3}x^3\right\}$ N·m,
For 3 m $< x \le 4.5$ m, $V = \left\{100(4.5 - x)^2 - 375\right\}$ N,
$M = \left\{375(4.5 - x) - \frac{100}{3}(4.5 - x)^3\right\}$ N·m

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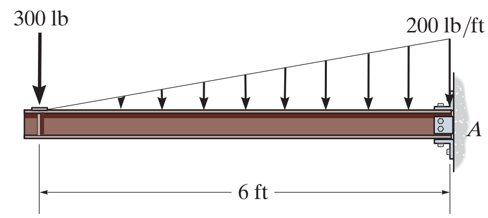
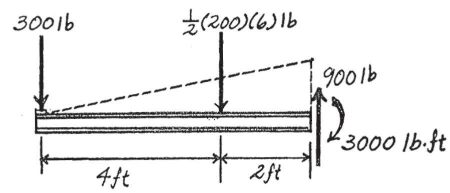
11–5.

Express the internal shear and moment in the cantilevered beam as a function of x and then draw the shear and moment diagrams.



SOLUTION

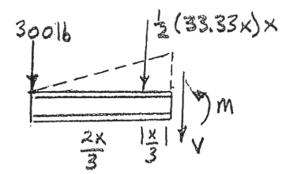
The free-body diagram of the beam's left segment sectioned through an arbitrary point shown in Fig. b will be used to write the shear and moment equations. The intensity of the triangular distributed load at the point of sectioning is



(a)

$$w = 200\left(\frac{x}{6}\right) = 33.33x$$

Referring to Fig. b,



(b)

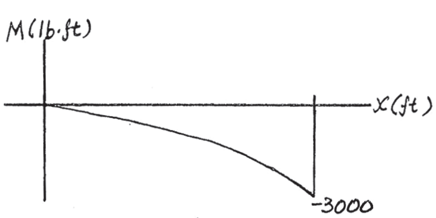
$$+\uparrow \Sigma F_y = 0; \quad -300 - \frac{1}{2}(33.33x)(x) - V = 0 \quad V = \{-300 - 16.67x^2\}\text{ lb} \quad \textbf{(1)} \quad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}(33.33x)(x)\left(\frac{x}{3}\right) + 300x = 0 \quad M = \{-300x - 5.556x^3\}\text{ lb}\cdot\text{ft}$$

(2) Ans.

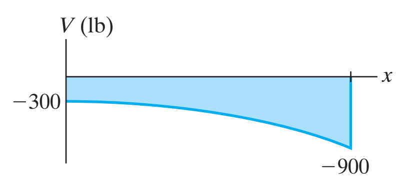
The shear and moment diagrams shown in Figs. c and d are plotted using Eqs. (1) and (2), respectively.

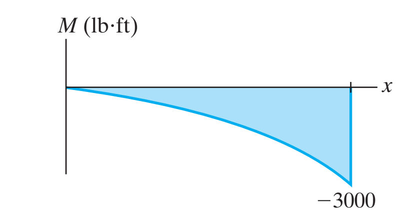
Shear diagram
(c)



Moment diagram
(d)

Ans:
$V = \{-300 - 16.67x^2\}$ lb,
$M = \{-300x - 5.556x^3\}$ lb · ft





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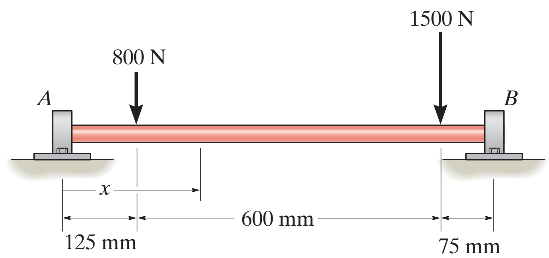
11–6.

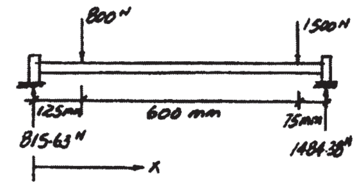
Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft. Also, express the shear and moment in the shaft as a function of x within the region 125 mm $< x <$ 725 mm.



SOLUTION

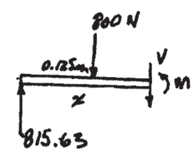
$+\uparrow \Sigma F_y = 0;$ $\quad 815.63 - 800 - V = 0$

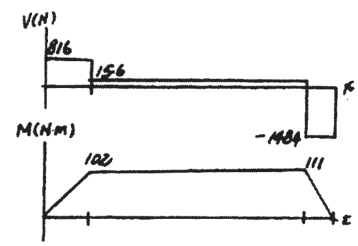
$V = 15.6 \text{ N}$ **Ans.**



$\circlearrowleft + \Sigma M = 0;$ $\quad M + 800(x - 0.125) - 815.63\,x = 0$

$M = (15.6x + 100) \text{ N} \cdot \text{m}$ **Ans.**





Ans:
$V = 15.6$ N,
$M = (15.6x + 100)$ N·m

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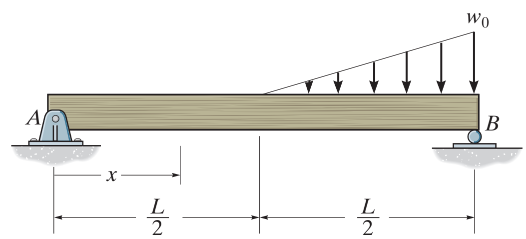
11–7.

Express the internal shear and moment in terms of x for $0 \le x < L/2$, and $L/2 < x < L$, and then draw the shear and moment diagrams.



SOLUTION

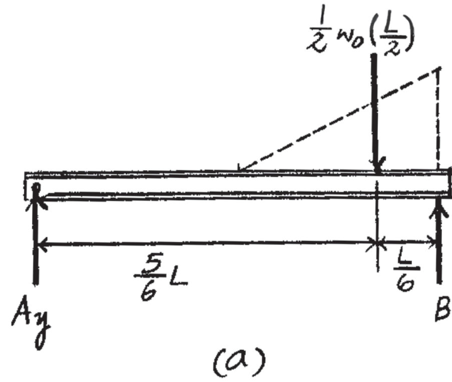
Support Reactions: Referring to the free-body diagram of the entire beam shown in Fig. *a*,

$$\zeta+\Sigma M_A = 0; \qquad B_y(L) - \frac{1}{2}w_0\left(\frac{L}{2}\right)\left(\frac{5}{6}L\right) = 0$$

$$B_y = \frac{5}{24}w_0 L$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y + \frac{5}{24}w_0 L - \frac{1}{2}w_0\left(\frac{L}{2}\right) = 0$$

$$A_y = \frac{w_0 L}{24}$$



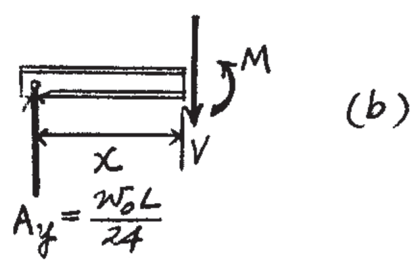
Shear and Moment Function: For $0 \le x < \frac{L}{2}$, we refer to the free-body diagram of the beam segment shown in Fig. *b*.

$$+\uparrow\Sigma F_y = 0; \qquad \frac{w_0 L}{24} - V = 0$$

$$V = \frac{w_0 L}{24} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M = 0; \qquad M - \frac{w_0 L}{24}x = 0$$

$$M = \frac{w_0 L}{24}x \qquad \textbf{Ans.}$$



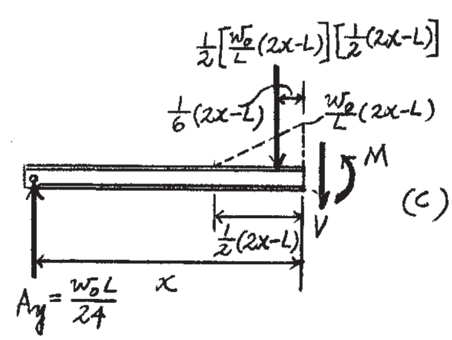
For $\frac{L}{2} < x \le L$, we refer to the free-body diagram of the beam segment shown in Fig. *c*.

$$+\uparrow\Sigma F_y = 0; \qquad \frac{w_0 L}{24} - \frac{1}{2}\left[\frac{w_0}{L}(2x - L)\right]\left[\frac{1}{2}(2x - L)\right] - V = 0$$

$$V = \frac{w_0}{24L}\left[L^2 - 6(2x - L)^2\right] \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M = 0; \qquad M + \frac{1}{2}\left[\frac{w_0}{L}(2x - L)\right]\left[\frac{1}{2}(2x - L)\right]\left[\frac{1}{6}(2x - L)\right] - \frac{w_0}{24L}x = 0$$

$$M = \frac{w_0}{24L}\left[L^2 x - (2x - L)^3\right] \qquad \textbf{Ans.}$$



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11–7. Continued

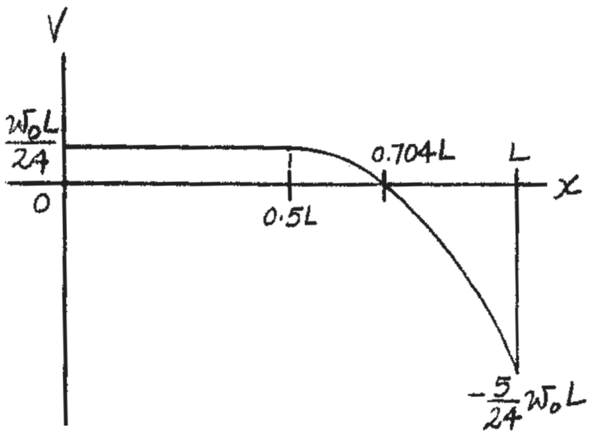
When $V = 0$, the shear function gives

$$0 = L^2 - 6(2x - L)^2 \qquad x = 0.7041L$$

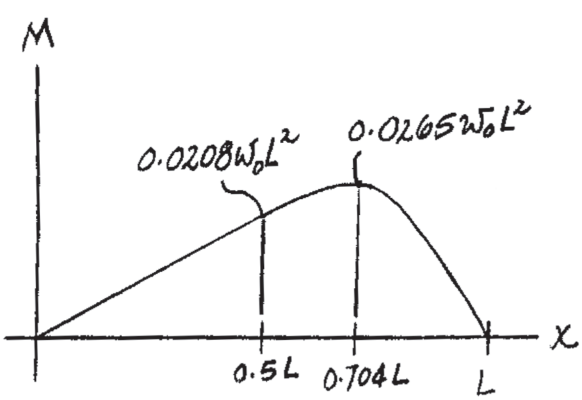
Substituting this result into the moment equation,

$$M|_{x=0.7041L} = 0.0265 w_0 L^2$$

Shear and Moment Diagrams: As shown in Figs. *d* and *e*.



(d)



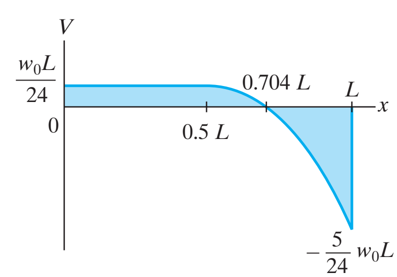
(e)

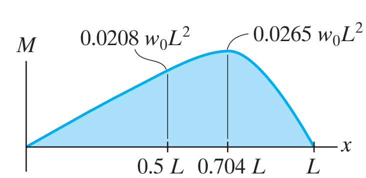
Ans:

For $0 \le x < \dfrac{L}{2}$: $V = \dfrac{w_0 L}{24}$, $M = \dfrac{w_0 L}{24} x$,

For $\dfrac{L}{2} < x \le L$: $V = \dfrac{w_0}{24L}\left[L^2 - 6(2x - L)^2\right]$,

$M = \dfrac{w_0}{24L}\left[L^2 x - (2x - L)^3\right]$





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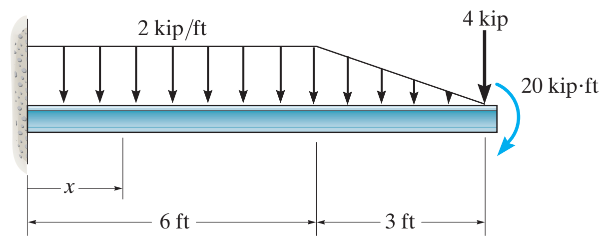
***11–8.**

Draw the shear and moment diagrams for the beam, and determine the shear and moment throughout the beam as functions of x for $0 \le x \le 6$ ft and $6 \text{ ft} \le x \le 9$ ft.



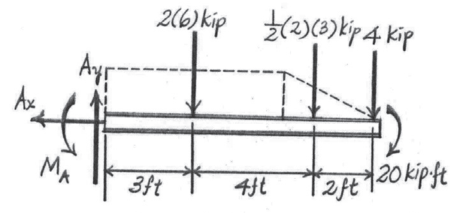
SOLUTION

Support Reactions: Referring to the FBD of the beam, Fig. *a*,

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 2(6) - \frac{1}{2}(2)(3) - 4 = 0 \quad A_y = 19.0 \text{ kip}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad M_A - 2(6)(3) - \left[\frac{1}{2}(2)(3)\right](7) - 4(9) - 20 = 0 \quad M_A = 113 \text{ kip} \cdot \text{ft}$$



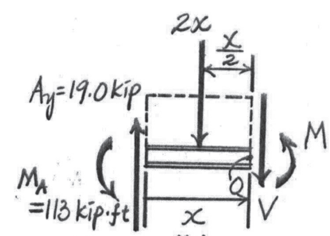
(a)

Shear and Moment Functions: For $0 \le x < 6$ ft, referring to the FBD of the beam segment in Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \quad 19.0 - 2x - V = 0 \quad V = \{19.0 - 2x\} \text{ kip} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_0 = 0; \quad M + 2x\left(\frac{x}{2}\right) + 113 - 19.0x = 0 \quad M = \left\{-x^2 + 19.0x - 113\right\} \text{kip} \cdot \text{ft}$$

Ans.



(b)

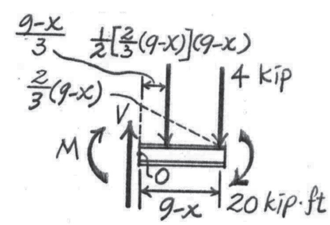
For $6 \text{ ft} < x \le 9$ ft, referring to the FBD of the beam segment in Fig. *c*,

$$+\uparrow \Sigma F_y = 0; \quad V - \frac{1}{2}\left[\frac{2}{3}(9 - x)\right](9 - x) - 4 = 0 \quad V = \left\{\frac{1}{3}x^2 - 6x + 31\right\} \text{kip}$$

Ans.

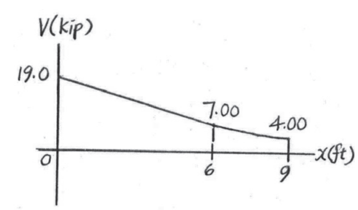
$$\Sigma M_0 = 0; \qquad -M - \left\{\frac{1}{2}\left[\frac{2}{3}(9 - x)\right](9 - x)\right\}\left(\frac{9 - x}{3}\right) - 4(9 - x) - 20 = 0$$

$$M = \left\{\frac{1}{9}x^3 - 3x^2 + 31x - 137\right\} \text{kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

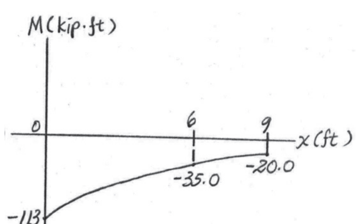


(c)

Using these functions, the shear and moment shown in Fig. *d* and *e* can be plotted.



(d)



(e)

Ans:
For $0 \le x < 6$ ft,
$V = \{19.0 - 2x\}$ kip,
$M = \left\{-x^2 + 19.0x - 113\right\}$ kip·ft,
For $6 \text{ ft} < x \le 9$ ft,
$V = \left\{\frac{1}{3}x^2 - 6x + 31\right\}$kip,
$M = \left\{\frac{1}{9}x^3 - 3x^2 + 31x - 137\right\}$ kip·ft

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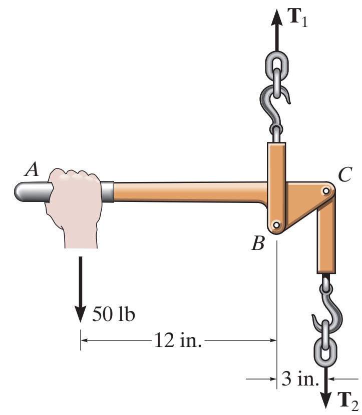
11–9.

If the force applied to the handle of the load binder is 50 lb, determine the tensions T_1 and T_2 in each end of the chain and then draw the shear and moment diagrams for the arm ABC.



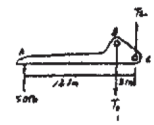
SOLUTION

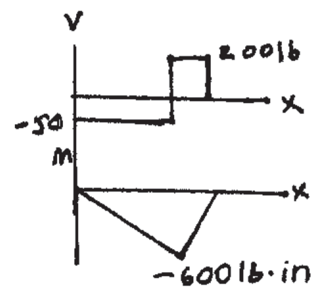
$\circlearrowleft + \Sigma M_C = 0; \quad 50(15) - T_1(3) = 0$

$T_1 = 250 \text{ lb}$ **Ans.**

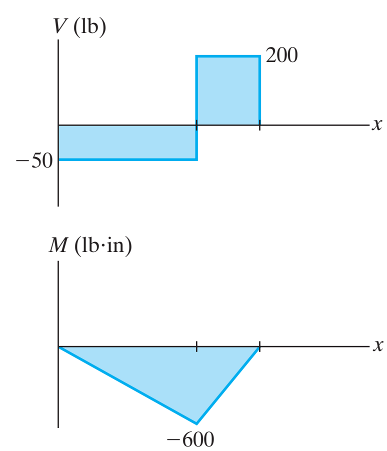
$+\downarrow \Sigma F_y = 0; \quad 50 - 250 + T_2 = 0$

$T_2 = 200 \text{ lb}$ **Ans.**





Ans:
$T_1 = 250 \text{ lb}, T_2 = 200 \text{ lb}$



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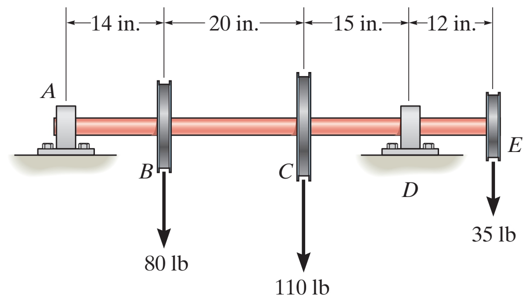
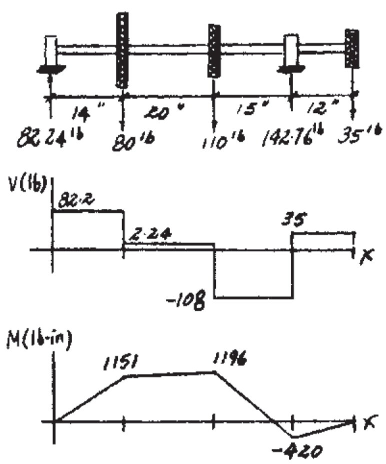
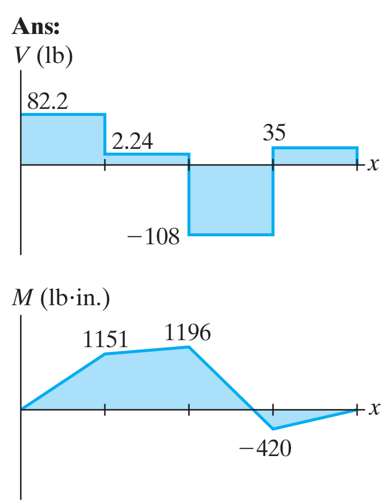
11–10.

Draw the shear and moment diagrams for the shaft. The bearings at A and D exert only vertical reactions on the shaft.



SOLUTION





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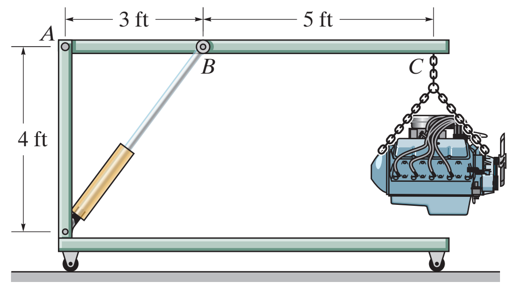
11–11.

The crane is used to support the engine, which has a weight of 1200 lb. Draw the shear and moment diagrams of the boom ABC when it is in the horizontal position.

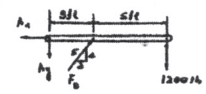


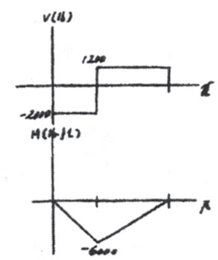
SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \quad \frac{4}{5} F_B(3) - 1200(8) = 0; \qquad F_B = 4000 \text{ lb}$

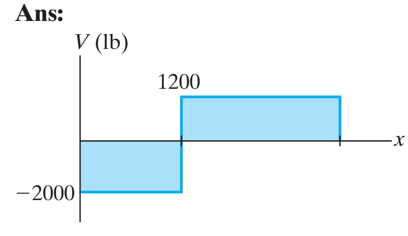
$+\uparrow \Sigma F_y = 0; \quad -A_y + \frac{4}{5}(4000) - 1200 = 0; \qquad A_y = 2000 \text{ lb}$

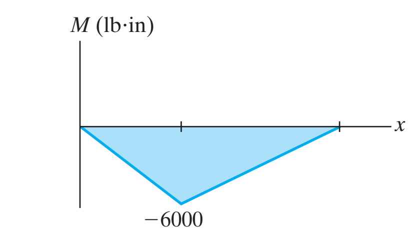
$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad A_x - \frac{3}{5}(4000) = 0; \qquad A_x = 2400 \text{ lb}$





Ans:





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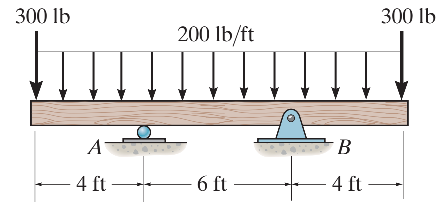
***11–12.**

Draw the shear and moment diagrams for the beam.



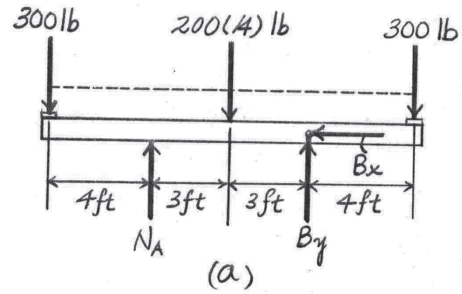
SOLUTION

Support Reactions: Referring to the FBD of the beam Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;\quad B_y(6) + 300(4) - 200(14)(3) - 300(10) = 0 \qquad B_y = 1700 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0;\quad 200(14)(3) + 300(10) - 300(4) - N_A(6) = 0 \qquad N_A = 1700 \text{ lb}$

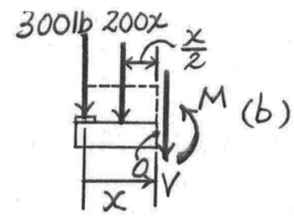
$\xrightarrow{+} \Sigma F_x = 0;\quad B_x = 0$



Shear and Moment Functions: For $0 \le x < 4$ ft, referring to the FBD of beam segment shown in Fig. *b*,

$+\uparrow \Sigma F_y = 0;\quad -V - 200x - 300 = 0 \qquad V = \{-200x - 300\} \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_0 = 0;\quad M + 200x\left(\frac{x}{2}\right) + 300x = 0 \qquad M = \left\{-100x^2 - 300x\right\} \text{ lb} \cdot \text{ft}$ **Ans.**

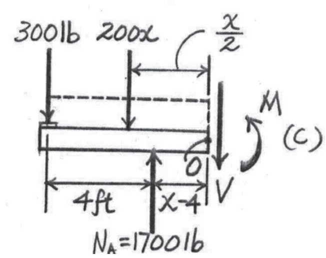


For 4 ft $< x <$ 10 ft, referring to the FBD of beam segment shown in Fig. *c*,

$+\uparrow \Sigma F_y = 0;\quad 1700 - 300 - 200x - V = 0 \qquad V = \{1400 - 200x\} \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_0 = 0;\quad M + 200x\left(\frac{x}{2}\right) + 300x - 1700(x - 4) = 0$

$M = \{-100x^2 + 1400x - 6800\} \text{ lb} \cdot \text{ft}$ **Ans.**

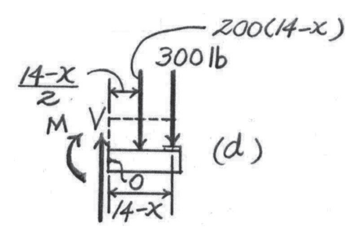


For 10 ft $< x \le$ 14 ft, referring to the FBD of beam's segment shown in Fig. *d*,

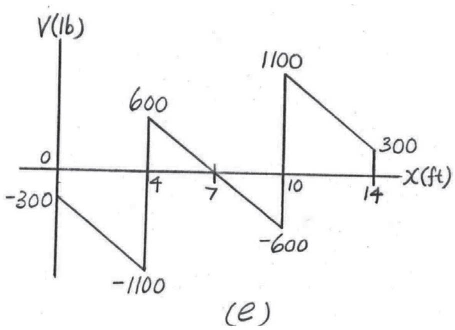
$+\uparrow \Sigma F_y = 0;\quad V - 200(14 - x) - 300 = 0 \qquad V = \{3100 - 200x\} \text{ lb}$ **Ans.**

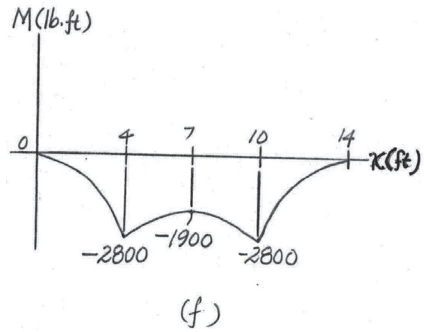
$\circlearrowleft + \Sigma M_0 = 0;\quad -M - 200(14 - x)\left(\frac{14 - x}{2}\right) - 300(14 - x) = 0$

$M = \left\{-100x^2 + 3100x - 23{,}800\right\} \text{ lb} \cdot \text{ft}$ **Ans.**



Using these functions, the shear and moment diagrams shown in Fig. *e* and *f*, respectively, can be plotted.





Ans:

For $0 \le x < 4$ ft,
$V = \{-200x - 300\}$ lb
$M = \left\{-100x^2 - 300x\right\}$ lb·ft

For 4 ft $< x <$ 10 ft,
$V = \{1400 - 200x\}$ lb
$M = \left\{-100x^2 + 1400x - 6800\right\}$ lb·ft

For 10 ft $< x \le$ 14 ft,
$V = \{3100 - 200x\}$ lb
$M = \left\{-100x^2 + 3100x - 23{,}800\right\}$ lb·ft

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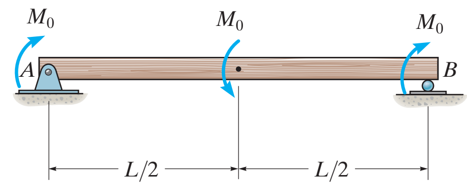
11–13.

Draw the shear and moment diagrams for the beam.



SOLUTION

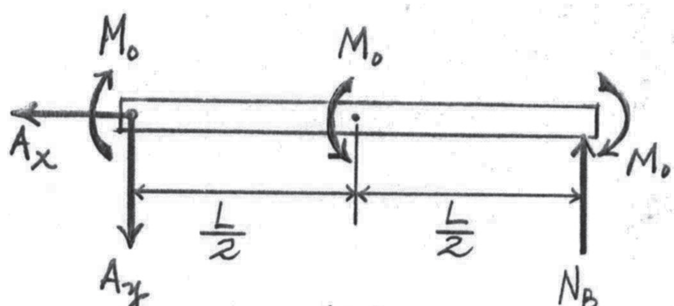
Support Reactions: Referring to the FBD of the beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(L) + M_0 - M_0 - M_0 = 0 \quad N_B = \frac{M_0}{L}$$

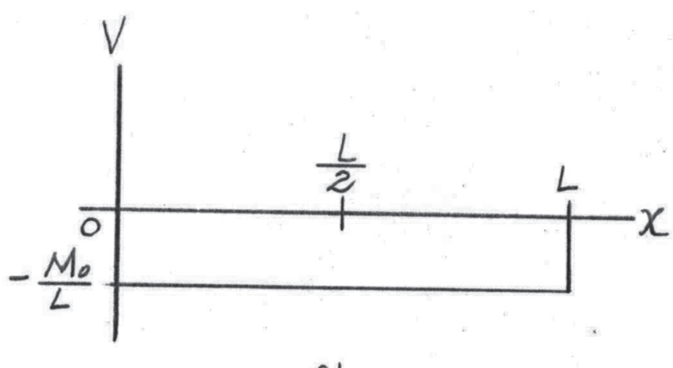
$$\circlearrowleft + \Sigma M_B = 0; \qquad A_y(L) + M_0 - M_0 - M_0 = 0 \quad A_y = \frac{M_0}{L}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad A_x = 0.$$

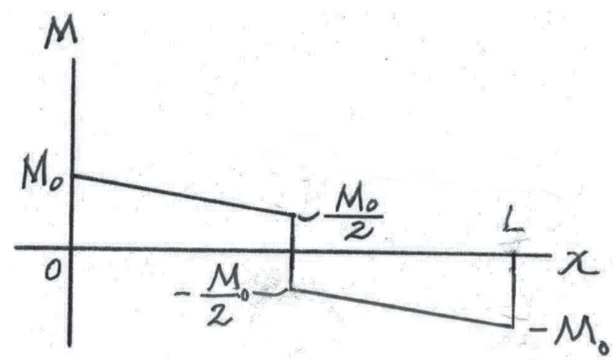
Shear and Moment Diagram: Using the results of the support reaction, the shear and moment diagrams shown in Fig. *b* and *c*, respectively, can be plotted.



(a)



(b)



(c)

Ans:

$V = -\frac{M_0}{L}$,

For $0 \le x < \frac{L}{2}$, $M = M_0 - \left(\frac{M_0}{L}\right)x$,

For $\frac{L}{2} < x \le L$, $M = -\left(\frac{M_0}{L}\right)x$

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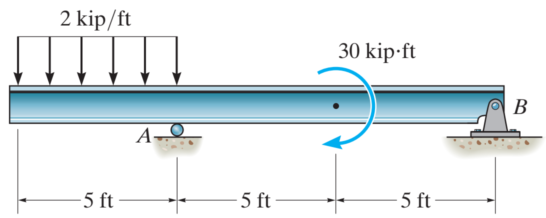
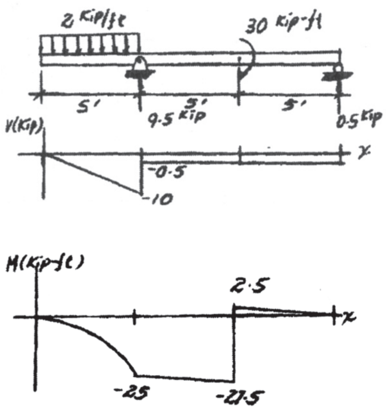
11–14.

Draw the shear and moment diagrams for the beam.



SOLUTION



Ans:
For $0 \leq x < 5$ ft:
$V = \{-2x\}$ kip,
$M = \{-x^2\}$ kip·ft,
For 5 ft $< x <$ 10 ft:
$V = -0.5$ kip,
$M = \{-22.5 - 0.5x\}$ kip·ft,
For 10 ft $< x \leq$ 15 ft:
$V = -0.5$ kip,
$M = \{7.5 - 0.5x\}$ kip·ft

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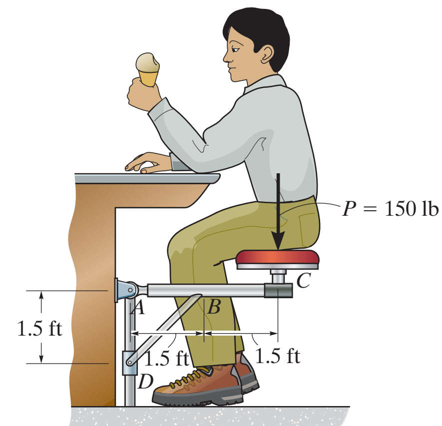
11–15.

Members *ABC* and *BD* of the counter chair are rigidly connected at *B* and the smooth collar at *D* is allowed to move freely along the vertical post. Draw the shear and moment diagrams for member *ABC*.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the frame shown in Fig. *a*,

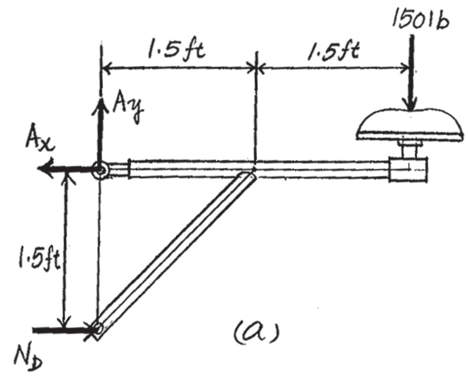
$+\uparrow \Sigma F_y = 0;\qquad A_y - 150 = 0$

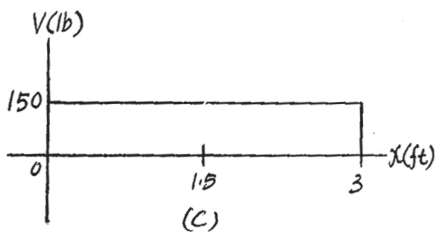
$A_y = 150 \text{ lb}$

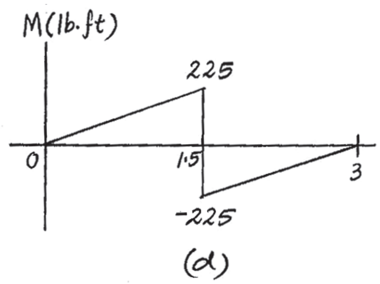
$\circlearrowleft + \Sigma M_A = 0;\qquad N_D(1.5) - 150(3) = 0$

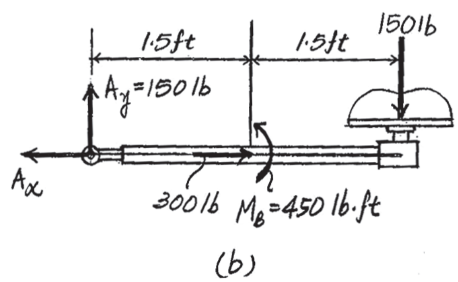
$N_D = 300 \text{ lb}$

Shear and Moment Diagram: The couple moment acting on *B* due to N_D is $M_B = 300(1.5) = 450 \text{ lb}\cdot\text{ft}$. The loading acting on member *ABC* is shown in Fig. *b* and the shear and moment diagrams are shown in Figs. *c* and *d*.

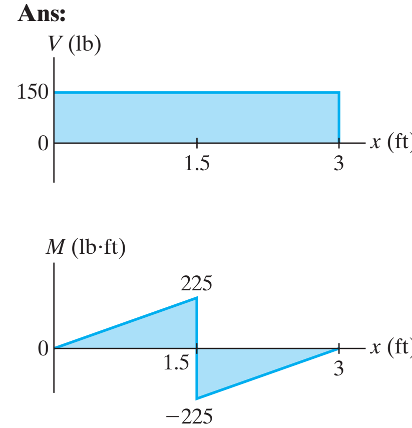








Ans:



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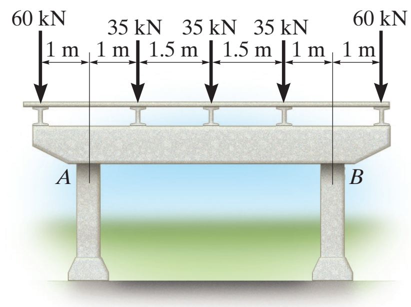
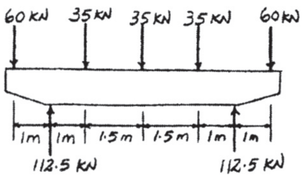
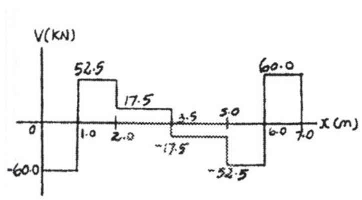
***11–16.**

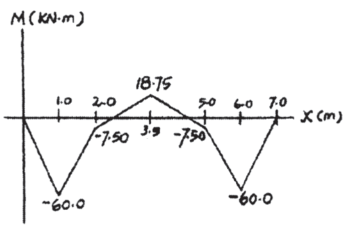
A reinforced concrete pier is used to support the stringers for a bridge deck. Draw the shear and moment diagrams for the pier. Assume the columns at *A* and *B* exert only vertical reactions on the pier.



SOLUTION







Ans:
N/A

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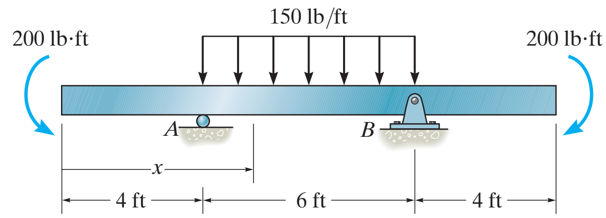
11–17.

Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where $4 \text{ ft} < x < 10 \text{ ft}$.



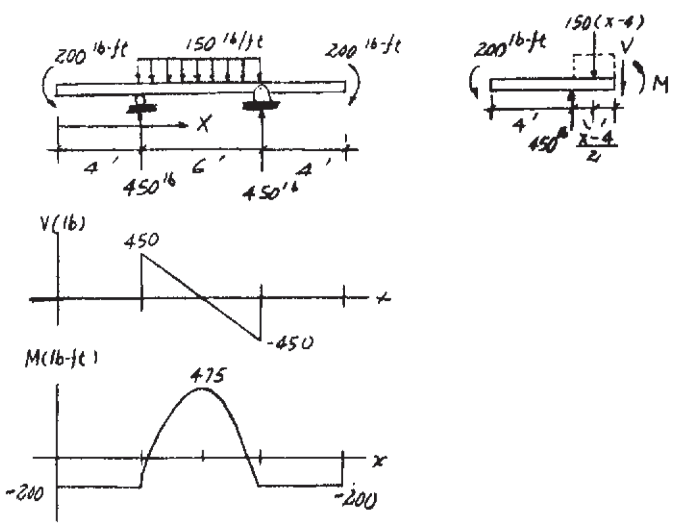
SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad -150(x - 4) - V + 450 = 0$

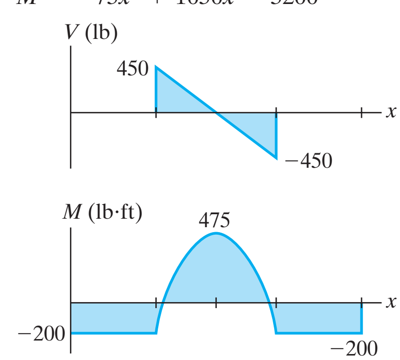
$$V = 1050 - 150x \qquad \textbf{Ans.}$$

$\circlearrowleft + \Sigma M = 0; \qquad -200 - 150(x - 4)\dfrac{(x - 4)}{2} - M + 450(x - 4) = 0$

$$M = -75x^2 + 1050x - 3200 \qquad \textbf{Ans.}$$



Ans:
$V = 1050 - 150x,$
$M = -75x^2 + 1050x - 3200$



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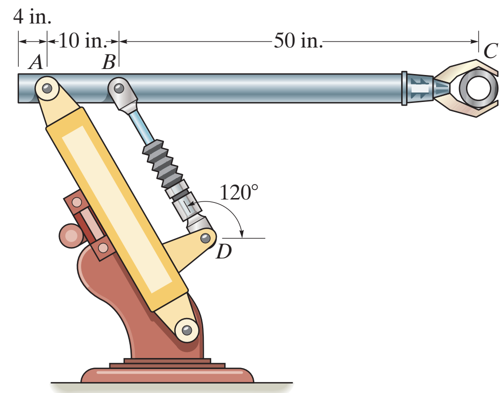
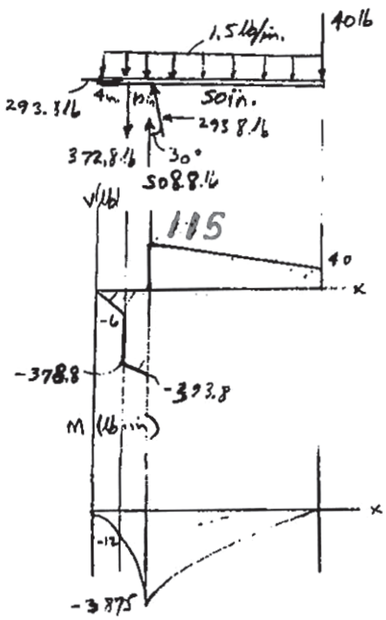
11–18.

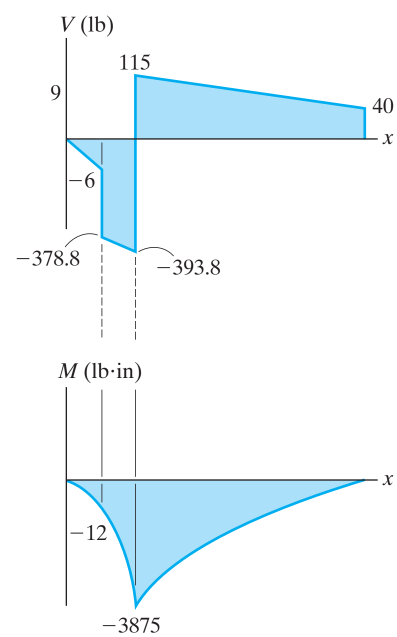
The industrial robot is held in the stationary position shown. Draw the shear and moment diagrams of the arm ABC if it is pin connected at A and connected to a hydraulic cylinder (two-force member) BD. Assume the arm and grip have a uniform weight of 1.5 lb/in. and support the load of 40 lb at C.



SOLUTION



Ans:



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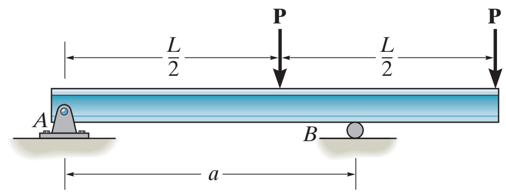
11–19.

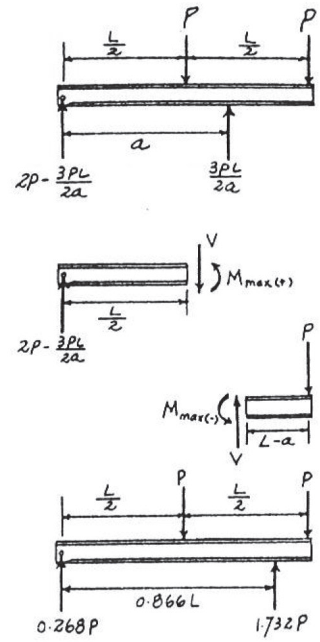
Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



SOLUTION

Support Reactions: As shown on FBD.

Absolute Minimum Moment: In order to get the absolute minimum moment, the maximum positive and maximum negative moment must be equal, that is, $M_{\max(+)} = M_{\max(-)}$.



For the positive moment:

$$\zeta+\Sigma M_{NA} = 0; \qquad M_{\max(+)} - \left(2P - \frac{3PL}{2a}\right)\left(\frac{L}{2}\right) = 0$$

$$M_{\max(+)} = PL - \frac{3PL^2}{4a}$$

For the negative moment:

$$\zeta+\Sigma M_{NA} = 0; \qquad M_{\max(-)} - P(L - a) = 0$$

$$M_{\max(-)} = P(L - a)$$

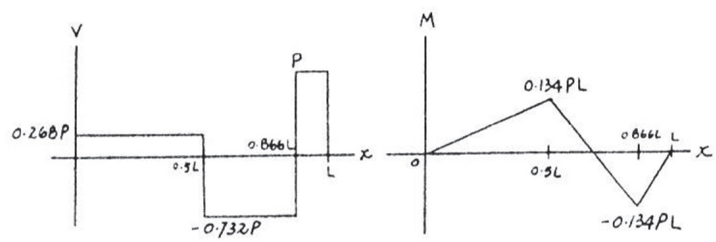
$$M_{\max(+)} = M_{\max(-)}$$

$$PL - \frac{3PL^2}{4a} = P(L - a)$$

$$4a\,L - 3L^2 = 4a\,L - 4a^2$$

$$a = \frac{\sqrt{3}}{2}L = 0.866L \qquad \textbf{Ans.}$$

Shear and Moment Diagram:



Ans:
$a = 0.866L$,
$M_{\max} = 0.134\,PL$

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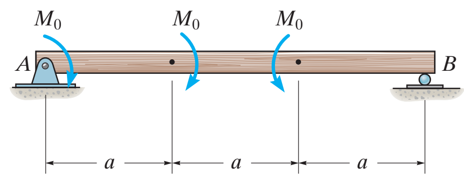
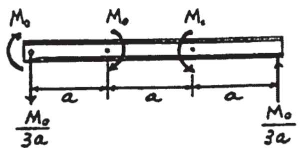
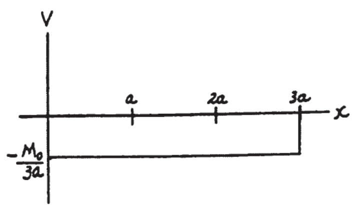
***11–20.**

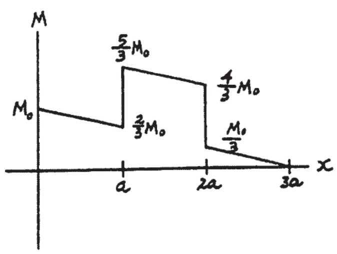
Draw the shear and moment diagrams for the beam.



SOLUTION







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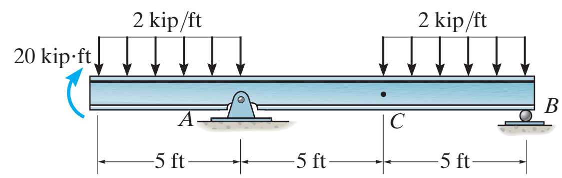
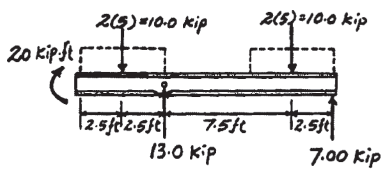
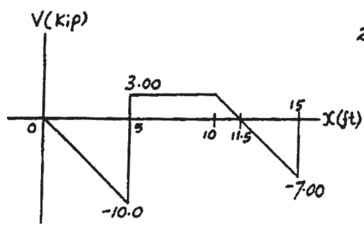
11–21.

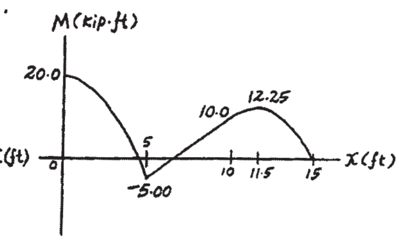
Draw the shear and moment diagrams for the beam.



SOLUTION







Ans:
For $0 \leq x < 5$ ft:
$V = \{-2x\}$ kip,
$M = \{-20.0 - x^2\}$ kip·ft,
For 5 ft $< x < 10$ ft:
$V = 3.00$ kip,
$M = \{-20.0 + 3x\}$ kip·ft,
For 10 ft $< x \leq 15$ ft:
$V = \{23 - 2x\}$ kip,
$M = \{-120 + 23x - x^2\}$ kip·ft

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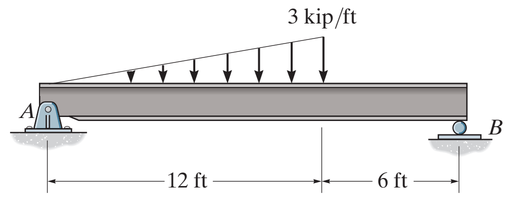
11–22.

Draw the shear and moment diagrams for the overhanging beam.



SOLUTION

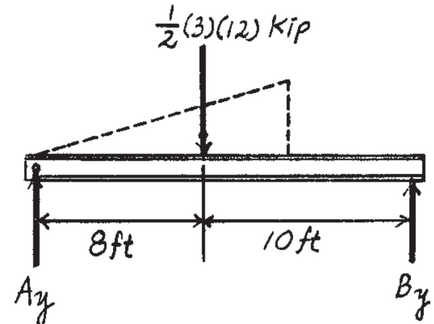
Support Reactions: Referring to the free-body diagram of the beam shown in Fig. *a*,

$\zeta + \Sigma M_A = 0;\qquad B_y(18) - \frac{1}{2}(3)(12)(8) = 0$

$B_y = 8 \text{ kip}$

$+\uparrow \Sigma F_y = 0;\qquad A_y + 8 - \frac{1}{2}(3)(12) = 0$

$A_y = 10 \text{ kip}$



(a)

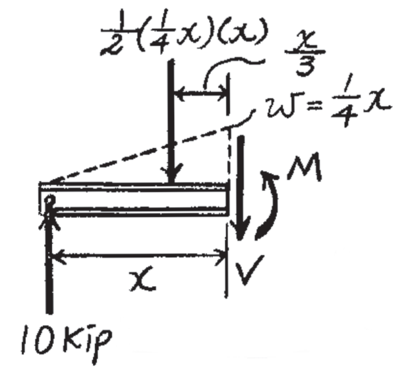
Shear and Moment Functions: For $0 \le x < 12$ ft, we refer to the free-body diagram of the beam segment shown in Fig. *b*.

$+\uparrow \Sigma F_y = 0;\qquad 10 - \frac{1}{2}\left(\frac{1}{4}x\right)(x) - V = 0$

$$V = \left\{10 - \frac{1}{8}x^2\right\} \text{kip} \qquad \textbf{Ans.}$$

$\zeta + \Sigma M = 0;\quad M + \frac{1}{2}\left(\frac{1}{4}x\right)(x)\left(\frac{x}{3}\right) - 10x = 0$

$$M = \left\{10x - \frac{1}{24}x^3\right\} \text{kip} \cdot \text{ft} \qquad \textbf{Ans.}$$



(b)

When $V = 0$, from the shear function,

$0 = 10 - \frac{1}{8}x^2 \qquad x = 8.944 \text{ ft}$

Substituting this result into the moment function,

$M|_{x=8.944\text{ ft}} = 59.6 \text{ kip} \cdot \text{ft}$

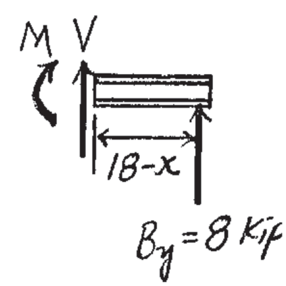
For 12 ft $< x \le 18$ ft, we refer to the free-body diagram of the beam segment shown in Fig. *c*.

$+\uparrow \Sigma F_y = 0;\qquad V + 8 = 0$

$$V = -8 \text{ kip} \qquad \textbf{Ans.}$$

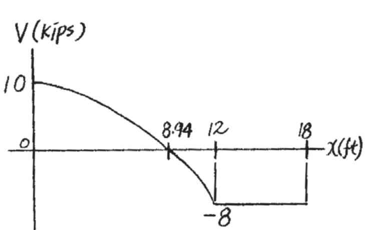
$\zeta + \Sigma M = 0;\quad 8(18 - x) - M = 0$

$$M = \{8(18 - x)\} \text{kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

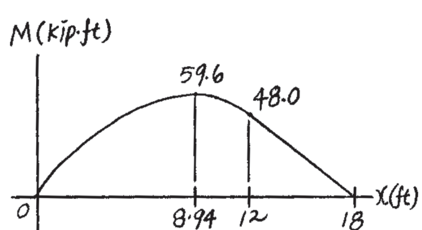


(c)

Shear and Moment Diagrams: As shown in Figs. *d* and *e*.



(d)



(e)

Ans:
For $0 \le x < 12$ ft:
$V = \left\{10 - \frac{1}{8}x^2\right\}$ kip,
$M = \left\{10x - \frac{1}{24}x^3\right\}$ kip · ft,
For 12 ft $< x \le 18$ ft:
$V = -8$ kip,
$M = [8(18 - x)]$ kip · ft

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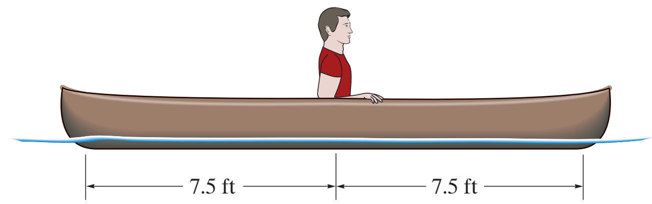
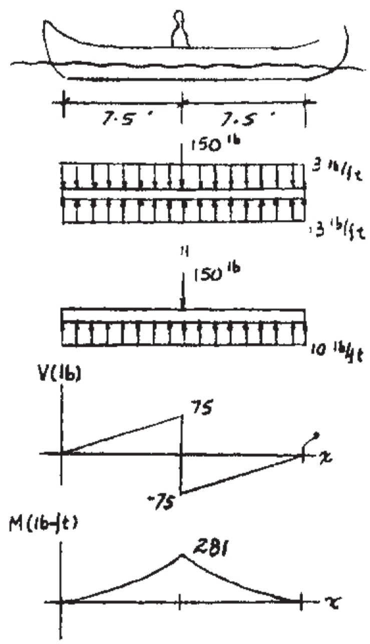
11–23.

The 150-lb man sits in the center of the boat, which has a uniform width and a weight per linear foot of 3 lb/ft. Determine the maximum internal bending moment. Assume that the water exerts a uniform distributed load upward on the bottom of the boat.

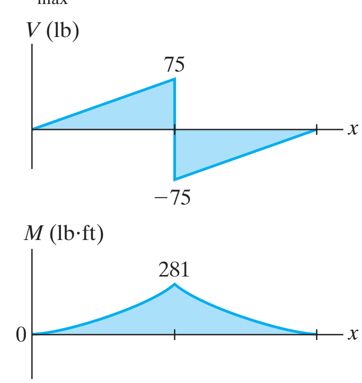


SOLUTION



$M_{max} = 281 \text{ lb} \cdot \text{ft}$ **Ans.**

Ans:
$M_{max} = 281 \text{ lb} \cdot \text{ft}$



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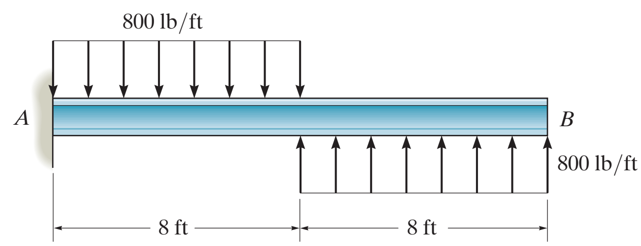
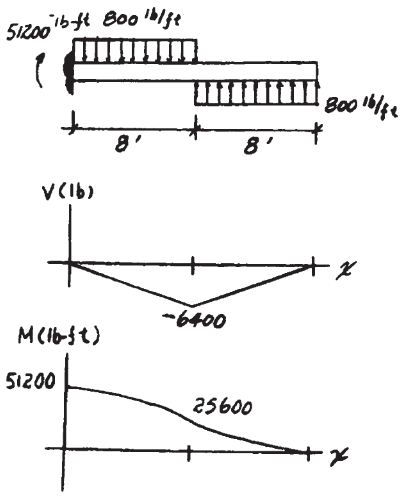
***11–24.**

Draw the shear and moment diagrams for the beam.



SOLUTION



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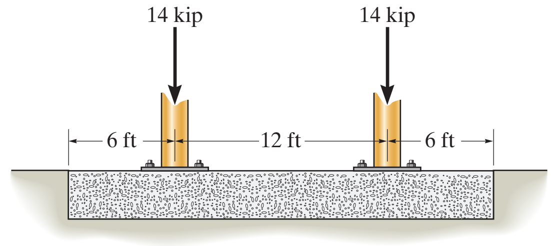
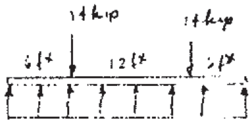
11–25.

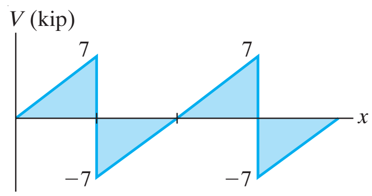
The footing supports the load transmitted by the two columns. Draw the shear and moment diagrams for the footing if the soil pressure on the footing is assumed to be uniform.

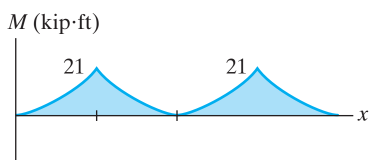


SOLUTION

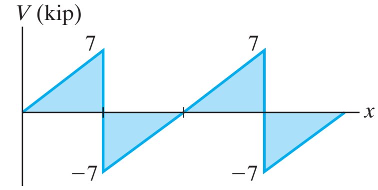


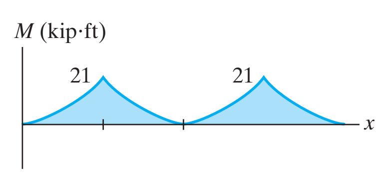
$w = \frac{28}{24} = 1.167$ kip/ft





Ans:





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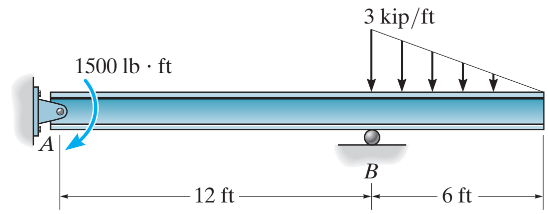
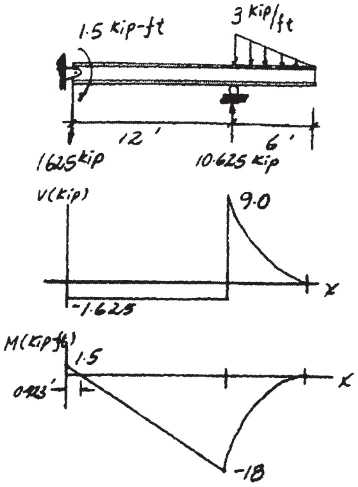
11–26.

Draw the shear and moment diagrams for the beam.



SOLUTION



Ans:
$V_{AB} = -1.625$ kip,
$M_B = -18$ kip · ft

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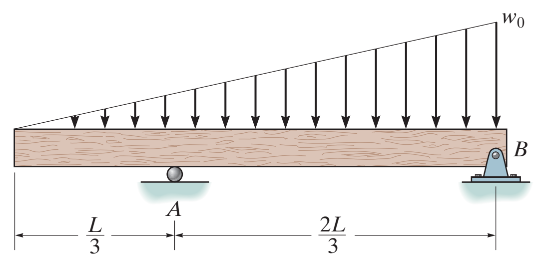
11–27.

Draw the shear and moment diagrams for the beam.



SOLUTION

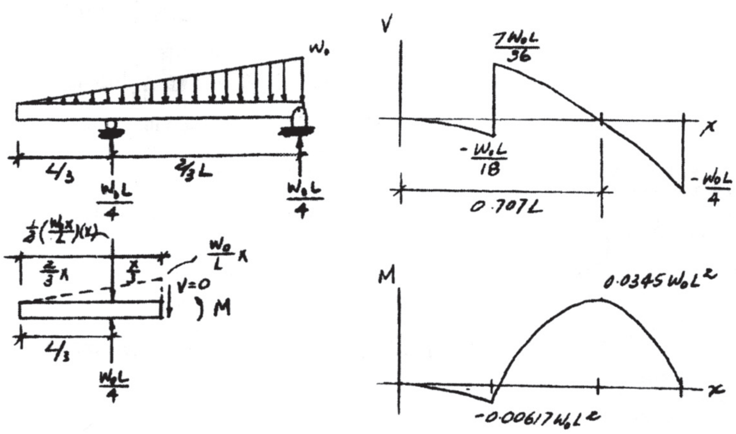
$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{4} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x) = 0$$

$$x = 0.7071\ L$$

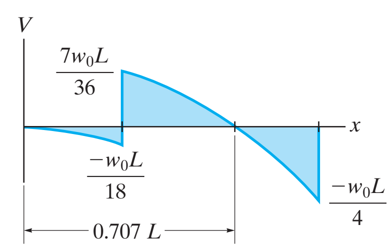
$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M + \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x)\left(\frac{x}{3}\right) - \frac{w_0 L}{4}\left(x - \frac{L}{3}\right) = 0$$

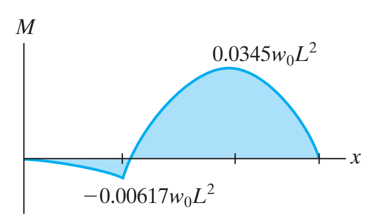
Substitute $x = 0.7071L$,

$$M = 0.0345\ w_0 L^2$$



Ans:





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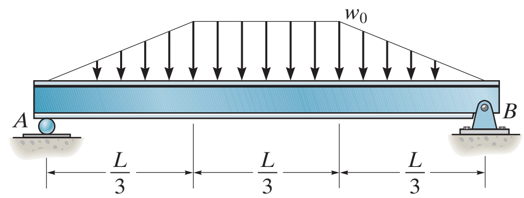
***11–28.**

Draw the shear and moment diagrams for the beam.



SOLUTION

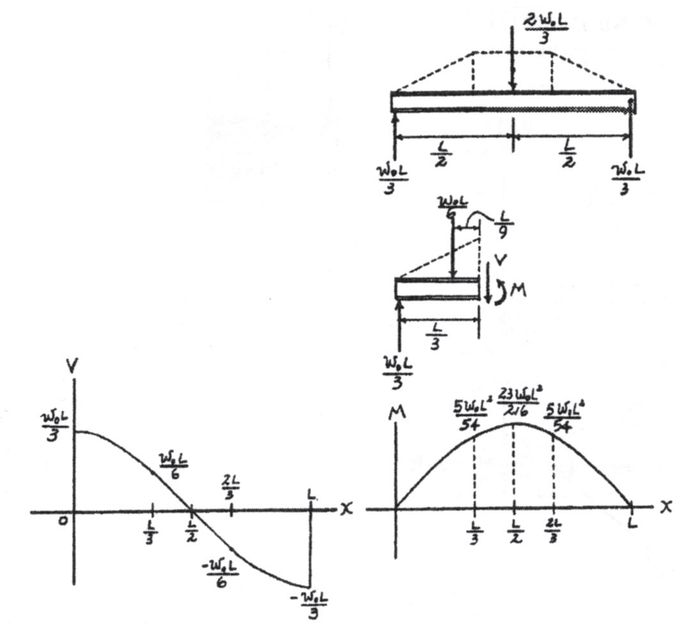
Support Reactions: As shown on FBD.

Shear and Moment Diagram: Shear and moment at $x = L/3$ can be determined using the method of sections.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{3} - \frac{w_0 L}{6} - V = 0 \qquad V = \frac{w_0 L}{6}$$

$$\zeta + \Sigma M_{NA} = 0; \qquad M + \frac{w_0 L}{6}\left(\frac{L}{9}\right) - \frac{w_0 L}{3}\left(\frac{L}{3}\right) = 0$$

$$M = \frac{5 w_0 L^2}{54}$$



Ans:

$$V_A = \frac{w_0 L}{3},$$

$$M_{max} = \frac{23\, w_0 L^2}{216}$$

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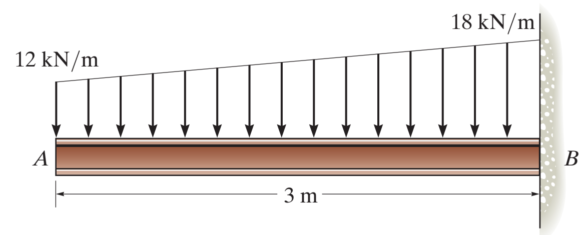
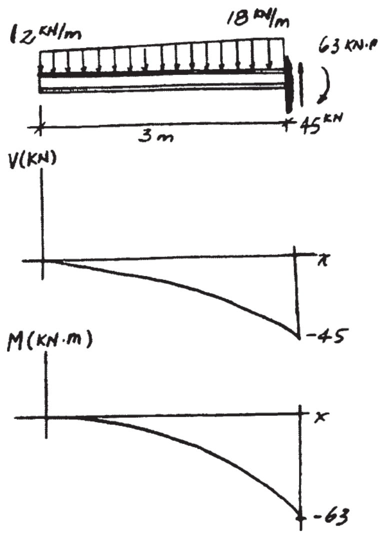
11–29.

Draw the shear and moment diagrams for the beam.



SOLUTION



Ans:
$V_B = -45$ kN,
$M_B = -63$ kN·m

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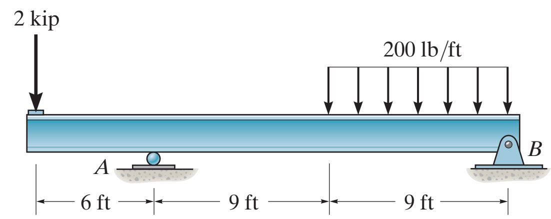
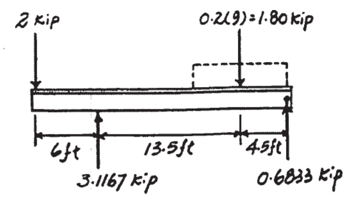
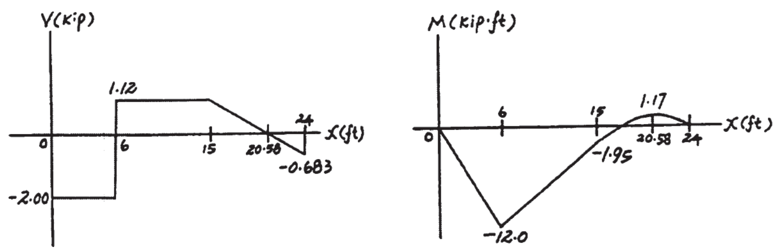
11–30.

Draw the shear and moment diagrams for the beam.



SOLUTION





Ans:
$V|_{x=15\text{ ft}} = 1.12$ kip,
$M|_{x=15} = -1.95$ kip · ft

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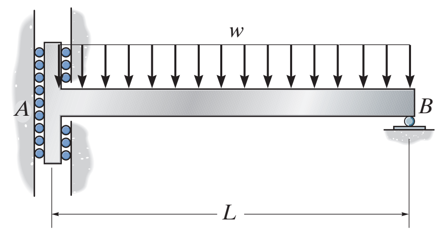
11–31.

The support at A allows the beam to slide freely along the vertical guide so that it cannot support a vertical force. Draw the shear and moment diagrams for the beam.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. a,

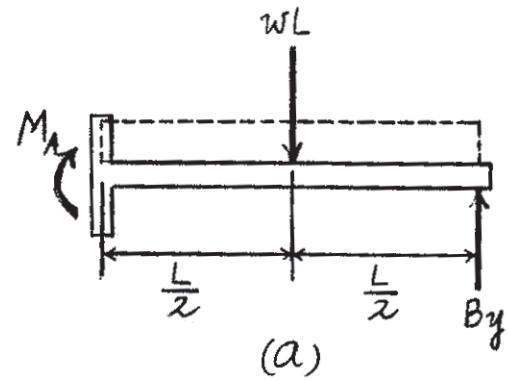
$$\circlearrowleft + \Sigma M_B = 0; \qquad wL\left(\frac{L}{2}\right) - M_A = 0$$

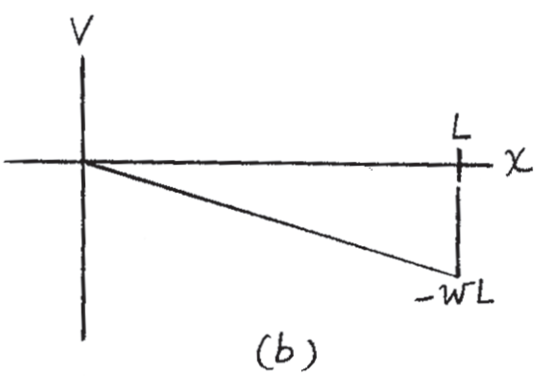
$$M_A = \frac{wL^2}{2}$$

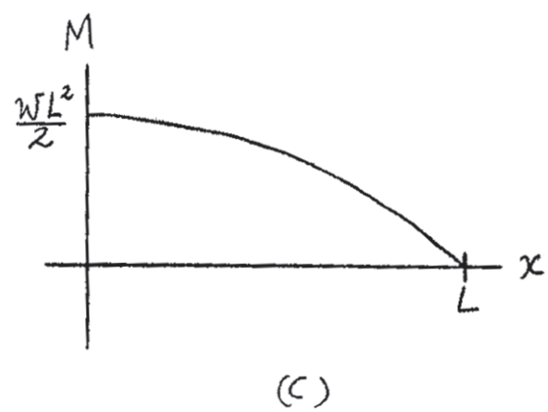
$$+\uparrow \Sigma F_y = 0; \qquad B_y - wL = 0$$

$$B_y = wL$$

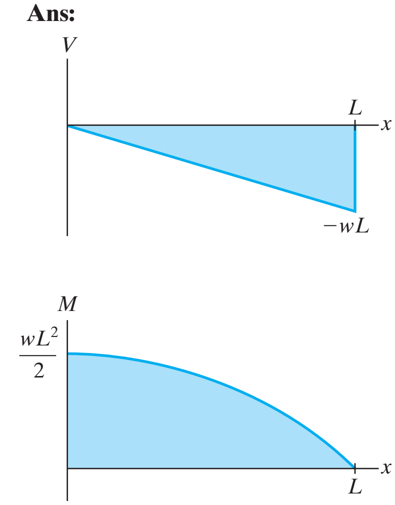
Shear and Moment Diagram: As shown in Figs. b and c.







Ans:



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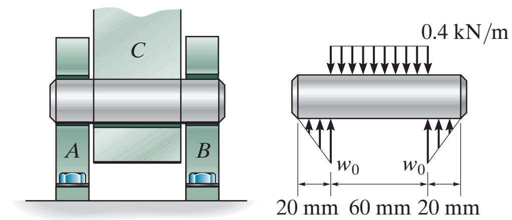
***11–32.**

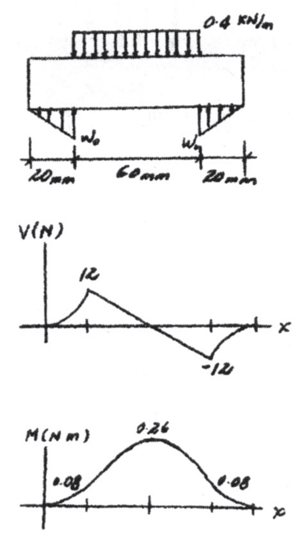
The smooth pin is supported by two leaves A and B and subjected to a compressive load of 0.4 kN/m caused by bar C. Determine the intensity of the distributed load w_0 of the leaves on the pin and draw the shear and moment diagram for the pin.



SOLUTION

$$+\uparrow\Sigma F_y = 0; \qquad 2(w_0)(20)\left(\frac{1}{2}\right) - 60(0.4) = 0$$

$$w_0 = 1.2 \text{ kN/m} \qquad \textbf{Ans.}$$



Ans:
$w_0 = 1.2$ kN/m

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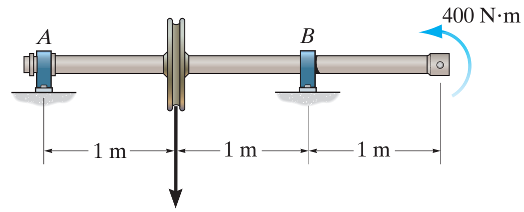
11–33.

The shaft is supported by a smooth thrust bearing at A and smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the shaft shown in Fig. a,

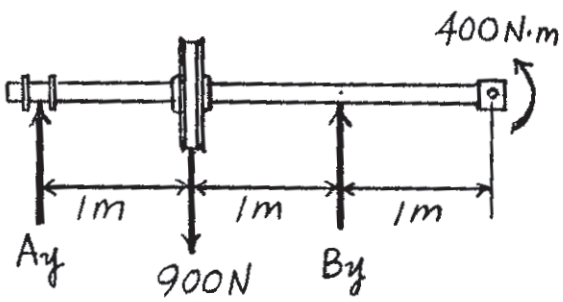
$\circlearrowleft + \Sigma M_A = 0;$ $\quad B_y(2) + 400 - 900(1) = 0$

$$B_y = 250 \text{ N}$$

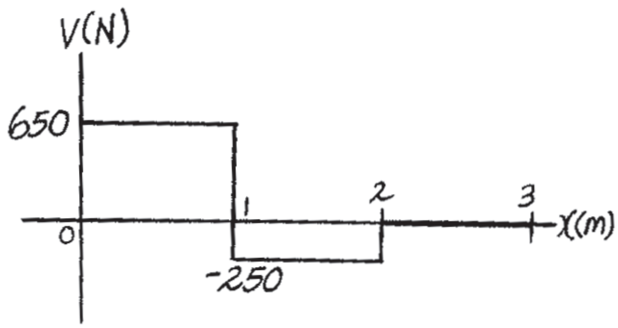
$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 250 - 900 = 0$

$$A_y = 650 \text{ N}$$

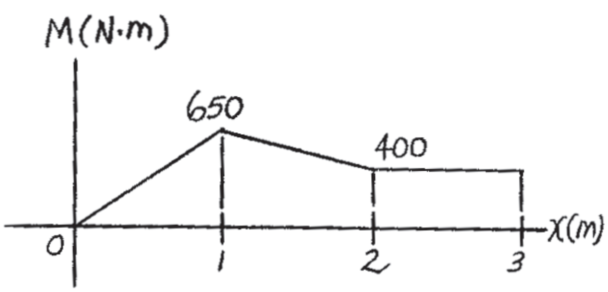
Shear and Moment Diagram: As shown in Figs. b and c.



(a)

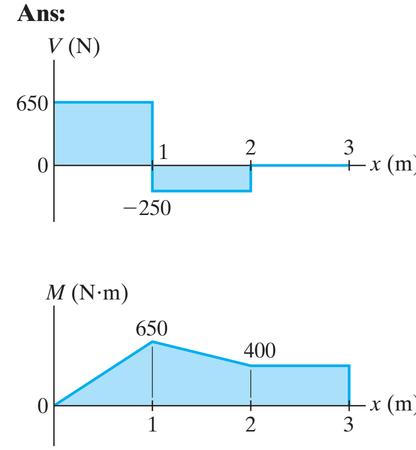


(b)



(c)

Ans:



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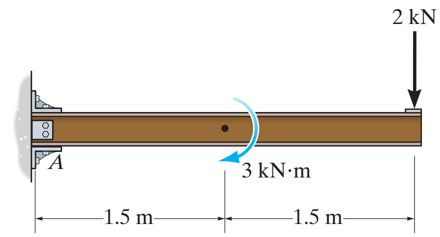
11–34.

Draw the shear and moment diagrams for the cantilever beam.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

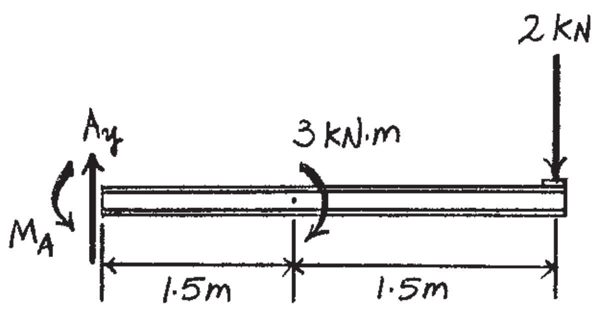
$+\uparrow\Sigma F_y = 0;\qquad A_y - 2 = 0$

$A_y = 2\text{ kN}$

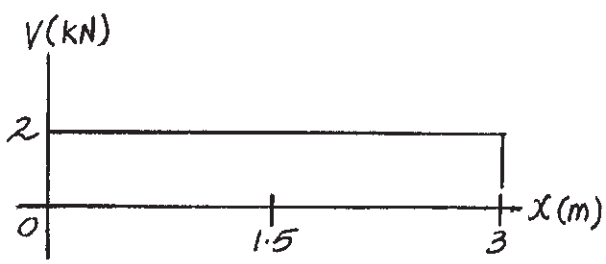
$\circlearrowleft + \Sigma M_A = 0;\qquad M_A - 3 - 2(3) = 0$

$M_A = 9\text{ kN}\cdot\text{m}$

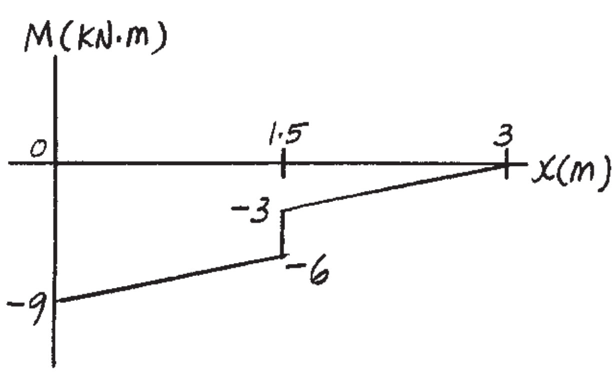
Shear and Moment Diagram: As shown in Figs. *b* and *c*.



(a)

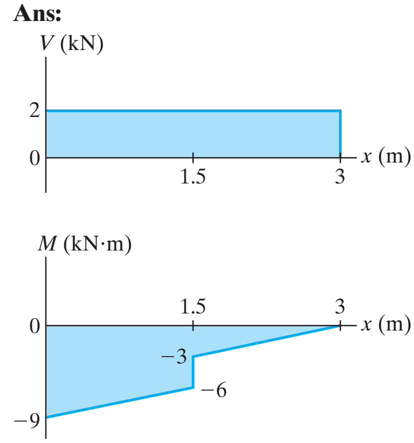


(b)



(c)

Ans:



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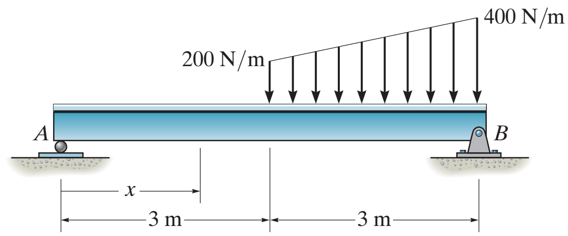
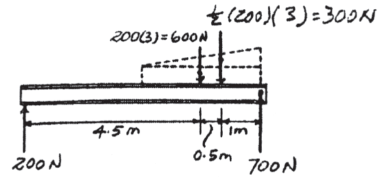
11–35.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: As shown on FBD.



Shear and Moment Functions:

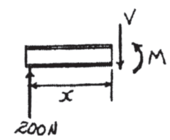
For $0 \le x < 3$ m:

$$+\uparrow \Sigma F_y = 0; \qquad 200 - V = 0 \qquad V = 200 \text{ N}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M - 200\,x = 0$$

$$M = \{200\,x\}\ \text{N}\cdot\text{m}$$

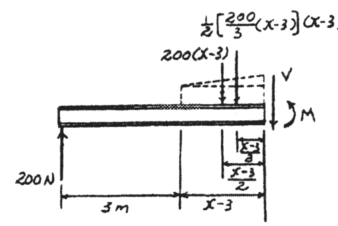
For $3 \text{ m} < x \le 6$ m:



$$+\uparrow \Sigma F_y = 0; \qquad 200 - 200(x-3) - \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3) - V = 0$$

$$V = \left\{-\frac{100}{3}x^2 + 500\right\} \text{N}$$

Set $V = 0, x = 3.873$ m.

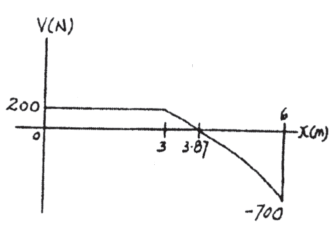


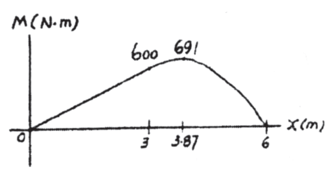
$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M + \frac{1}{2}\left[\frac{200}{3}(x-3)\right](x-3)\left(\frac{x-3}{3}\right)$$

$$+\, 200(x-3)\left(\frac{x-3}{2}\right) - 200x = 0$$

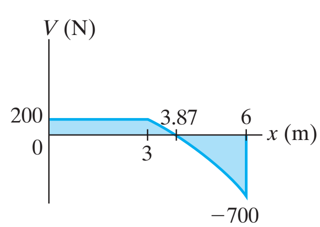
$$M = \left\{-\frac{100}{9}x^3 + 500x - 600\right\} \text{N}\cdot\text{m}$$

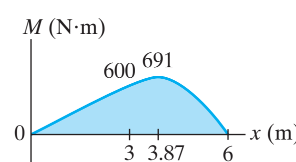
Substitute $x = 3.87$ m, $M = 691$ N·m.





Ans:





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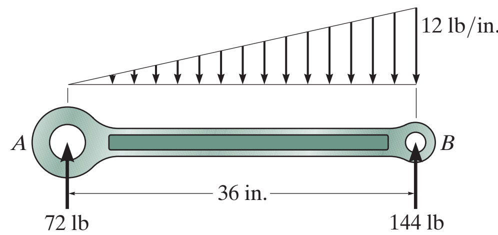
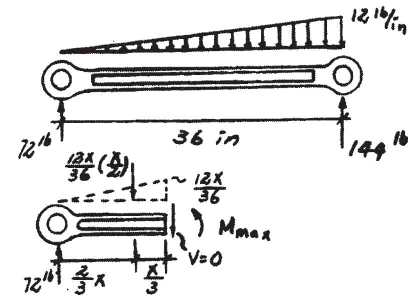
***11–36.**

Draw the shear and moment diagrams for the rod. Only vertical reactions occur at its ends A and B.



SOLUTION



$+\uparrow\Sigma F_y = 0;\qquad 72 - \frac{12x}{36}\left(\frac{x}{2}\right) = 0$

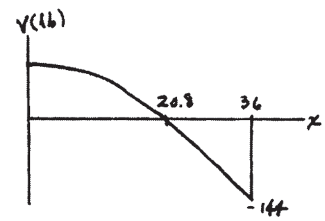
$x = 20.784$ in.

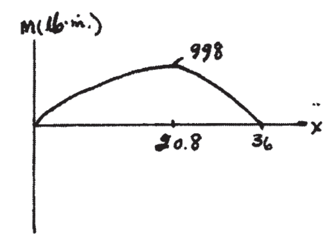
$\zeta + \Sigma M = 0;\qquad M_{max} + \frac{12x}{36}\left(\frac{x}{2}\right)\left(\frac{x}{3}\right) - 72x = 0$

$M_{max} = -\frac{x^3}{18} + 72x$

Substitute $x = 20.784$ in.

$M_{max} = 997.66\ \text{lb}\cdot\text{in.}$





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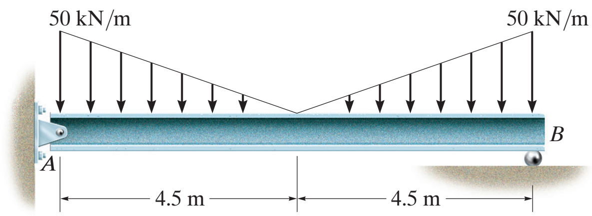
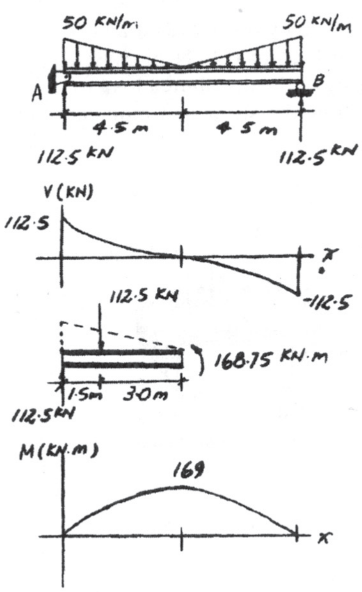
11–37.

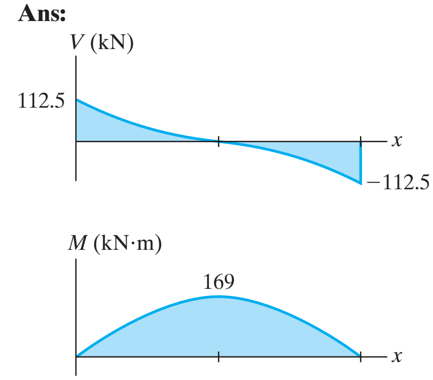
Draw the shear and moment diagrams for the beam.



SOLUTION



Ans:



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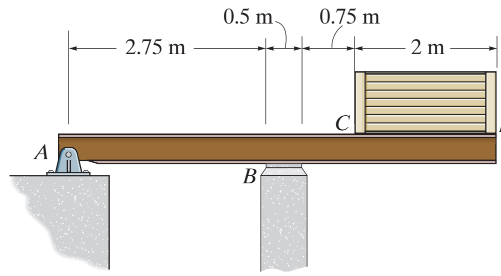
11–38.

The beam is used to support a uniform load along CD due to the 6-kN weight of the crate. Also, the reaction at the bearing support B can be assumed uniformly distributed along its width. Draw the shear and moment diagrams for the beam.



SOLUTION

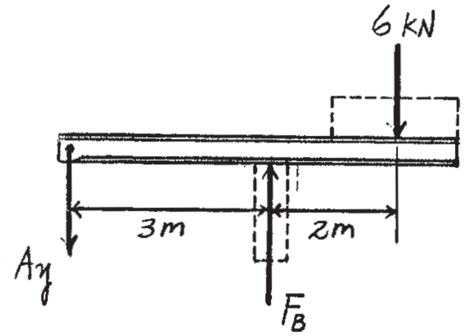
Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;\qquad F_B(3) - 6(5) = 0$

$F_B = 10 \text{ kN}$

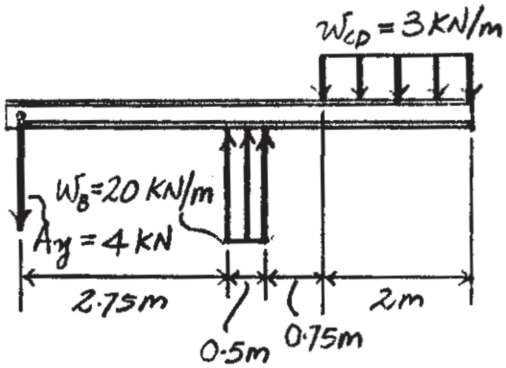
$+\uparrow \Sigma F_y = 0;\qquad 10 - 6 - A_y = 0$

$A_y = 4 \text{ kN}$

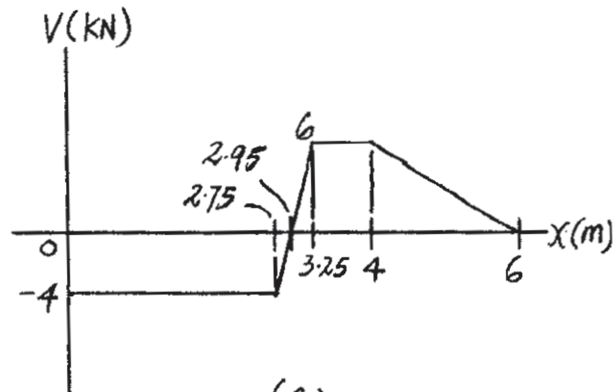


(a)

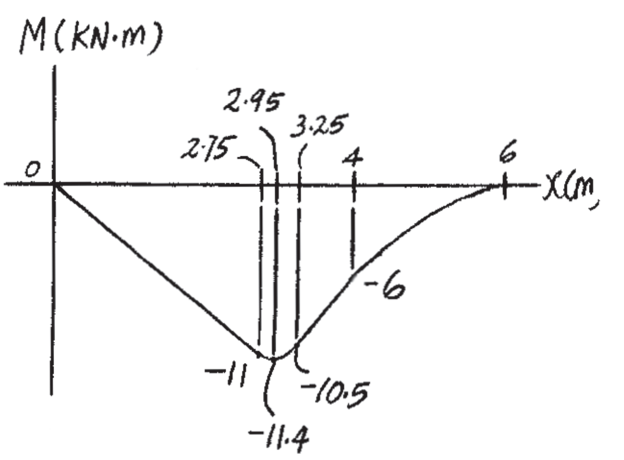
Shear and Moment Diagram: The intensity of the distributed load at support B and portion CD of the beam are $w_B = \dfrac{F_B}{0.5} = \dfrac{10}{0.5} = 20 \text{ kN/m}$ and $w_{CD} = \dfrac{6}{2} = 3 \text{ kN/m}$, Fig. *b*. The shear and moment diagrams are shown in Figs. *c* and *d*.



(b)

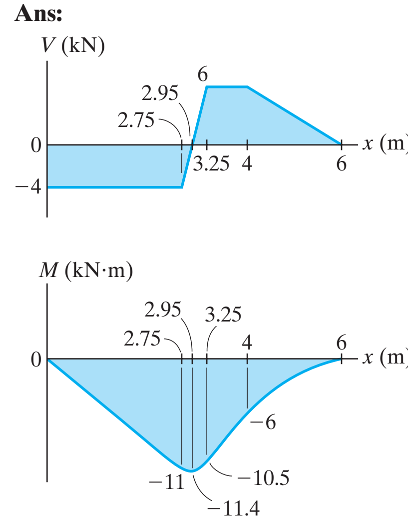


(c)



(d)

Ans:



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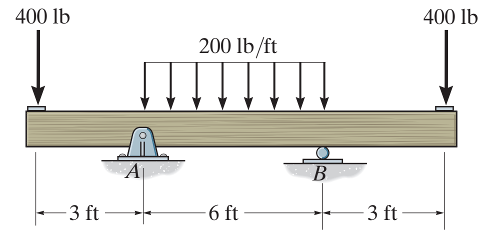
11–39.

Draw the shear and moment diagrams for the double overhanging beam.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the beam shown in Fig. *a*,

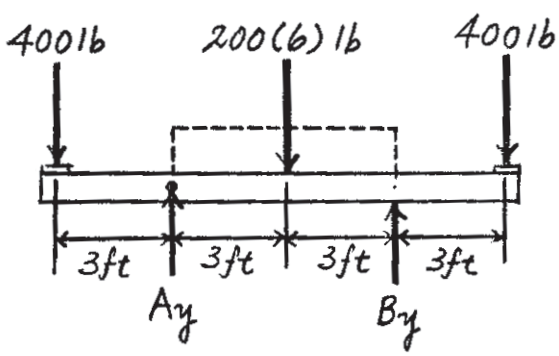
$\circlearrowleft + \Sigma M_A = 0;\qquad B_y(6) + 400(3) - 200(6)(3) - 400(9) = 0$

$B_y = 1000 \text{ lb}$

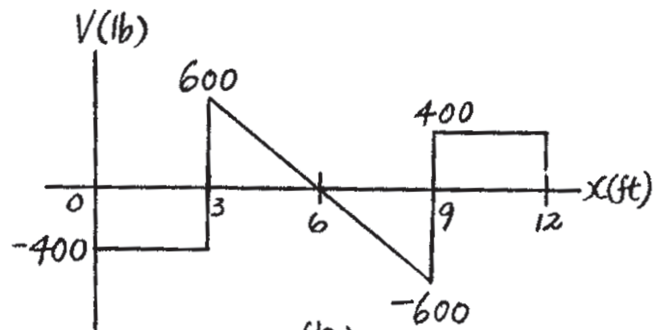
$+\uparrow \Sigma F_y = 0;\qquad A_y + 1000 - 400 - 200(6) - 400 = 0$

$A_y = 1000 \text{ lb}$

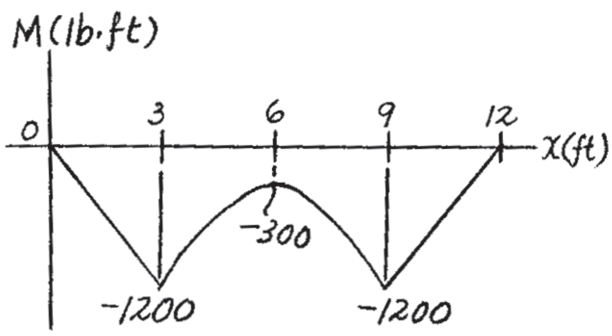
Shear and Moment Diagram: As shown in Figs. *b* and *c*.



(a)

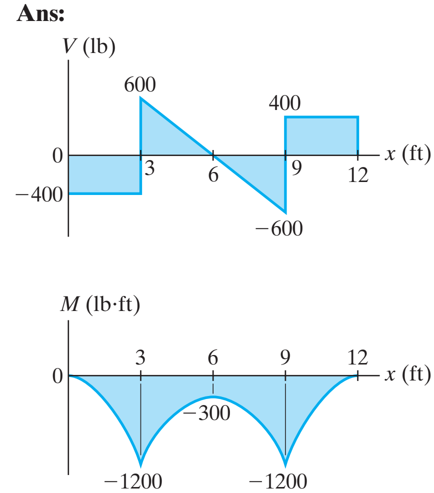


(b)



(c)

Ans:



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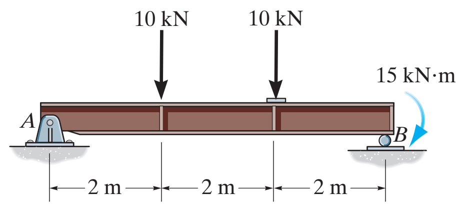
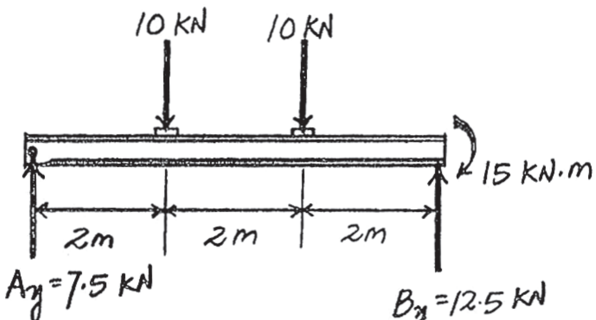
***11–40.**

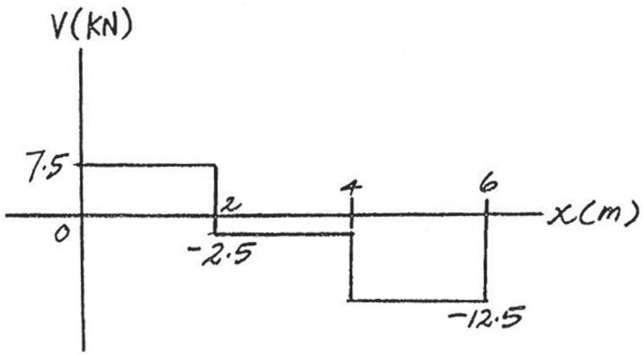
Draw the shear and moment diagrams for the simply supported beam.



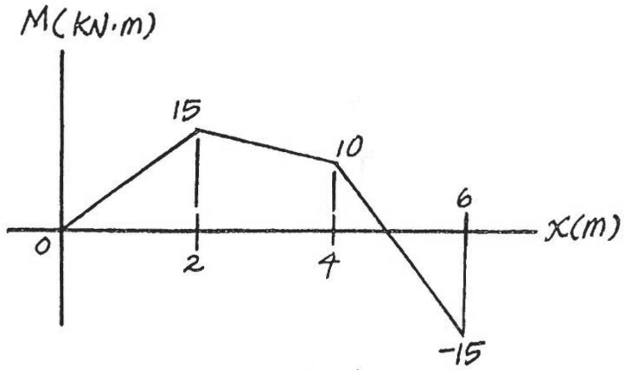
SOLUTION



(a)



shear diagram
(b)



moment diagram
(c)

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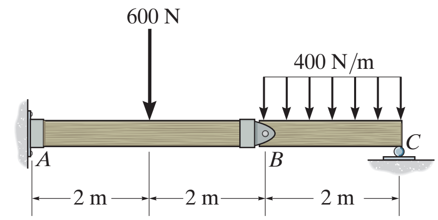
11–41.

The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,

$\circlearrowleft + \Sigma M_B = 0;$ $\quad C_y(2) - 400(2)(1) = 0$

$$C_y = 400 \text{ N}$$

$+\uparrow \Sigma F_y = 0;$ $\quad B_y + 400 - 400(2) = 0$

$$B_y = 400 \text{ N}$$

Using the result of $\mathbf{B}_y$ and referring to the free-body diagram of segment AB, Fig. b,

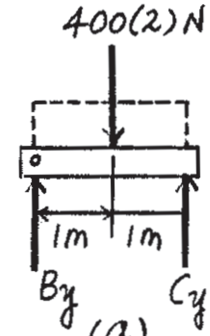
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 600 - 400 = 0$

$$A_y = 1000 \text{ N}$$

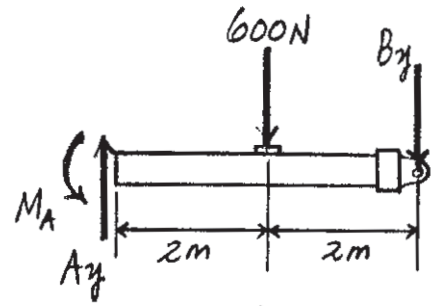
$\circlearrowleft + \Sigma M_A = 0;$ $\quad M_A - 600(2) - 400(4) = 0$

$$M_A = 2800 \text{ N}$$

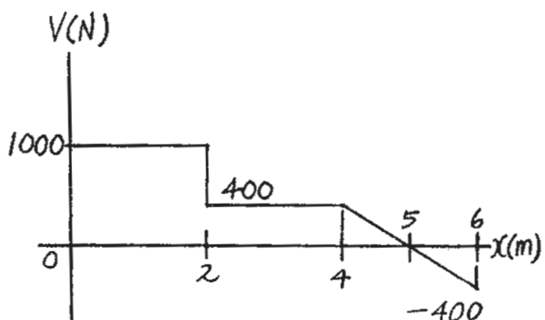
Shear and Moment Diagrams: As shown in Figs. c and d.



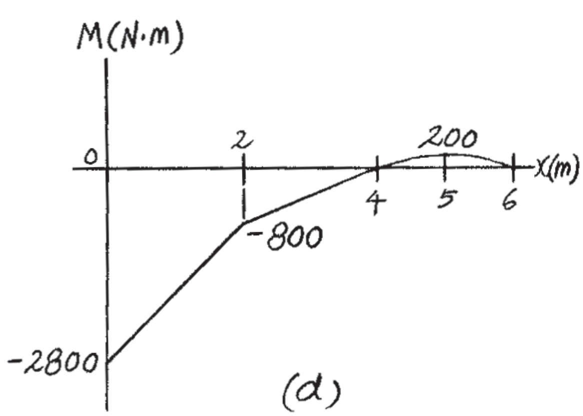
(a)



(b)

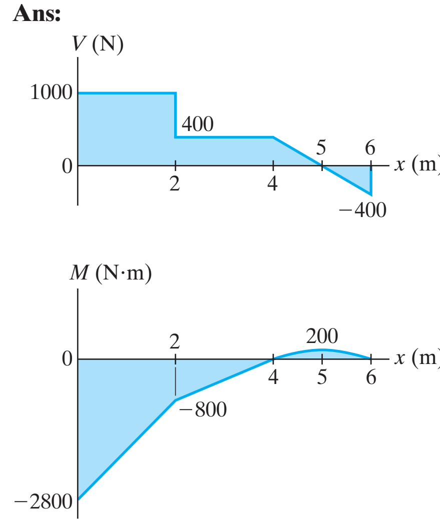


(c)



(d)

Ans:



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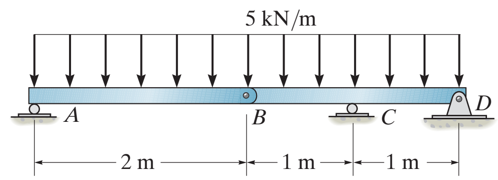
11–42.

Draw the shear and moment diagrams for the compound beam.



SOLUTION

Support Reactions:

From the FBD of segment AB,

$\circlearrowleft+\Sigma M_A = 0; \quad B_y(2) - 10.0(1) = 0 \quad B_y = 5.00 \text{ kN}$

$+\uparrow\Sigma F_y = 0; \quad A_y - 10.0 + 5.00 = 0 \quad A_y = 5.00 \text{ kN}$

From the FBD of segment BD,

$\circlearrowleft+\Sigma M_C = 0; \quad 5.00(1) + 10.0(0) - D_y(1) = 0$

$D_y = 5.00 \text{ kN}$

$+\uparrow\Sigma F_y = 0; \quad C_y - 5.00 - 5.00 - 10.0 = 0$

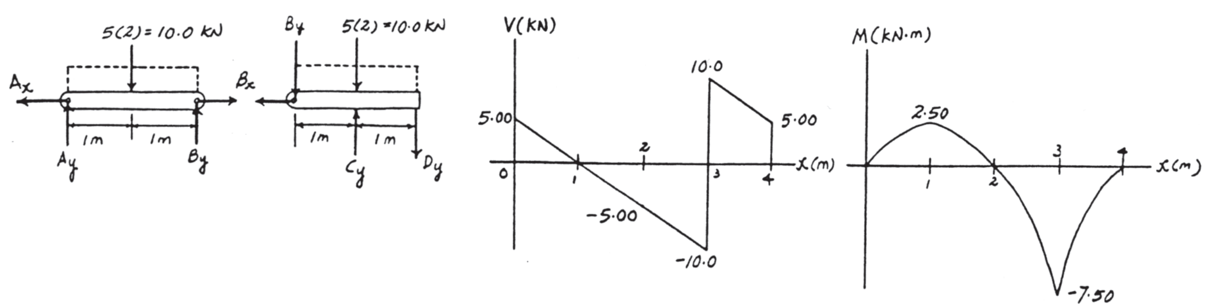
$C_y = 20.0 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = 0$

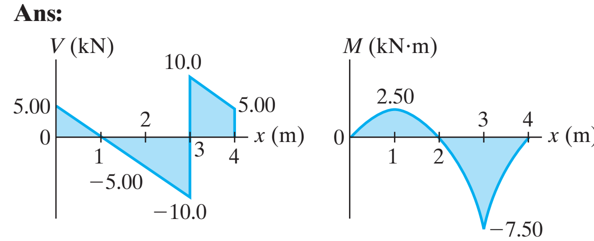
From the FBD of segment AB,

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$

Shear and Moment Diagram:



Ans:



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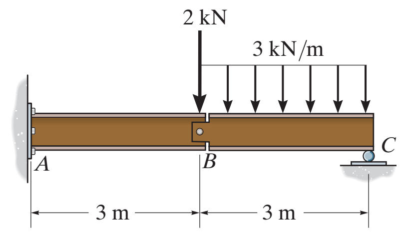
11–43.

The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear and moment diagrams for the beam.



SOLUTION

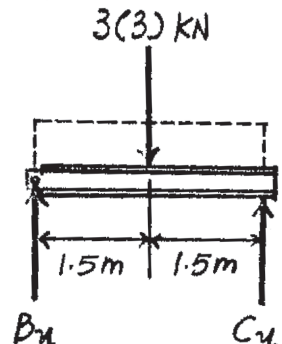
Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,

$\circlearrowleft + \Sigma M_B = 0;$ $\quad C_y(3) - 3(3)(1.5) = 0$

$C_y = 4.5 \text{ kN}$

$\uparrow \Sigma F_y = 0;$ $\quad B_y + 4.5 - 3(3) = 0$

$B_y = 4.5 \text{ kN}$



(a)

Using the result of $\mathbf{B}_y$ and referring to the free-body diagram of segment AB, Fig. b,

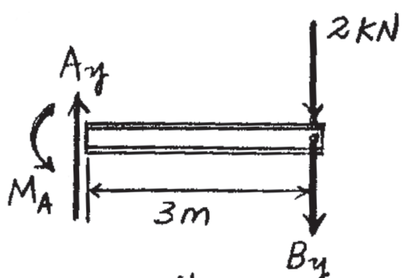
$\uparrow \Sigma F_y = 0;$ $\quad A_y - 2 - 4.5 = 0$

$A_y = 6.5 \text{ kN}$

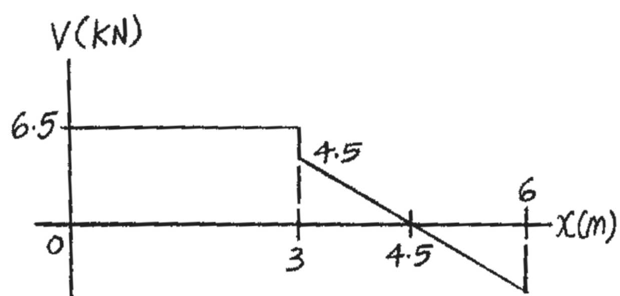
$\circlearrowleft + \Sigma M_A = 0;$ $\quad M_A - 2(3) - 4.5(3) = 0$

$M_A = 19.5 \text{ kN} \cdot \text{m}$

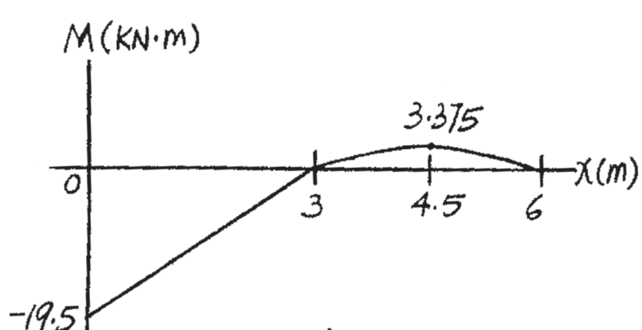
Shear and Moment Diagrams: As shown in Figs. c and d.



(b)

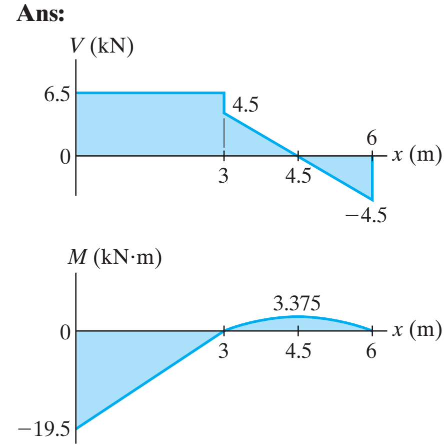


(c)



(d)

Ans:



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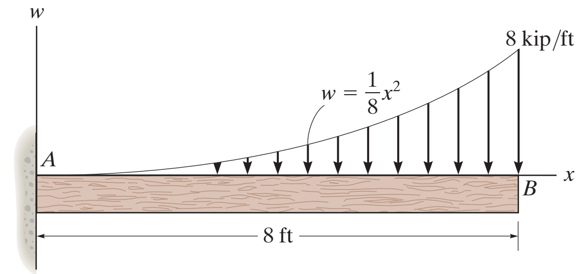
***11–44.**

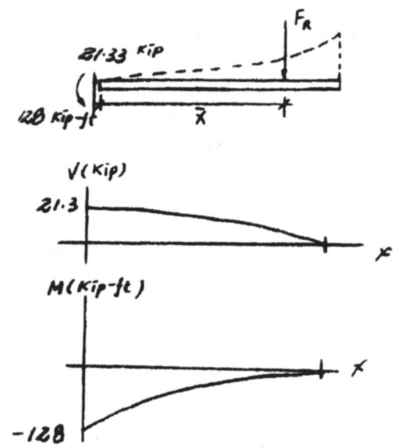
Draw the shear and moment diagrams for the beam.



SOLUTION

$$F_R = \frac{1}{8}\int_0^8 x^2 dx = 21.33 \text{ kip}$$

$$\overline{x} = \frac{\frac{1}{8}\int_0^8 x^3 dx}{21.33} = 6.0 \text{ ft}$$



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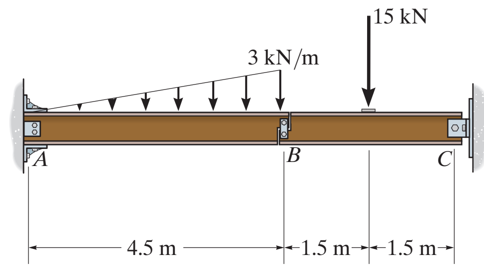
11–45.

A short link at B is used to connect beams AB and BC to form the compound beam. Draw the shear and moment diagrams for the beam if the supports at A and C are considered fixed and pinned, respectively.



SOLUTION

Support Reactions: Referring to the free-body diagram of segment BC shown in Fig. a,

$\circlearrowleft + \Sigma M_C = 0;$ $\quad 15(1.5) - F_B(3) = 0$

$F_B = 7.5 \text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $\quad C_y + 7.5 - 15 = 0$

$C_y = 7.5 \text{ kN}$

Using the result of $\mathbf{F}_B$ and referring to the free-body diagram of segment AB, Fig. b,

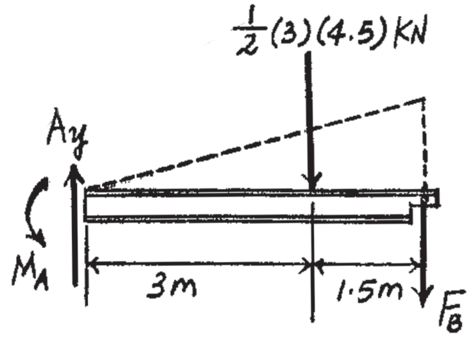
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - \frac{1}{2}(3)(4.5) - 7.5 = 0$

$A_y = 14.25 \text{ kN}$

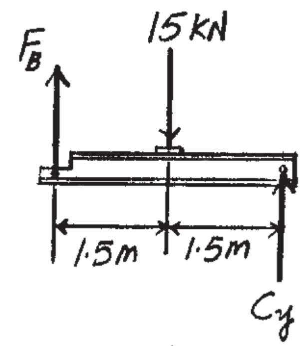
$\circlearrowleft + \Sigma M_A = 0;$ $\quad M_A - \frac{1}{2}(3)(4.5)(3) - 7.5(4.5) = 0$

$M_A = 54 \text{ kN} \cdot \text{m}$

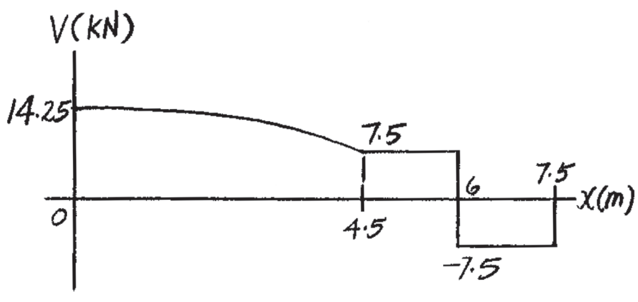
Shear and Moment Diagrams: As shown in Figs. c and d.



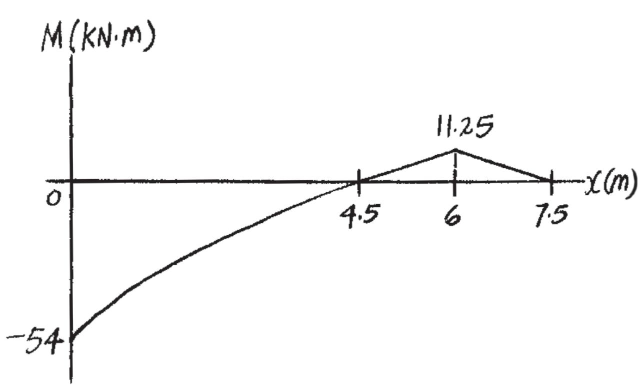
(a)



(b)

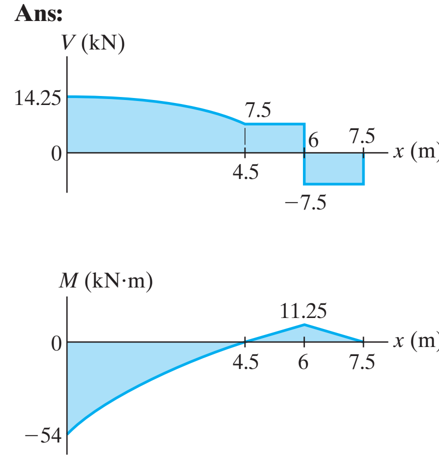


(c)



(d)

Ans:



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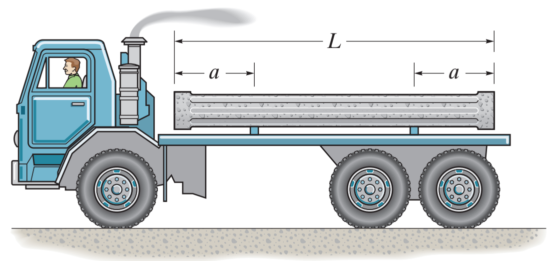
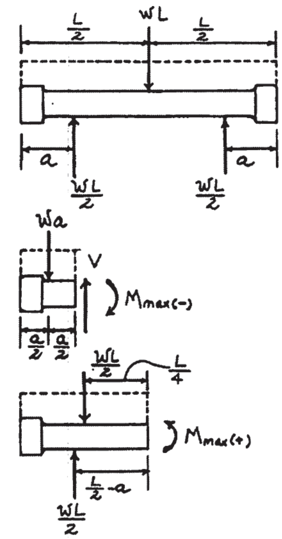
11–46.

The truck is to be used to transport the concrete column. If the column has a uniform weight of w (force/length), determine the equal placement a of the supports from the ends so that the absolute maximum bending moment in the column is as small as possible. Also, draw the shear and moment diagrams for the column.



SOLUTION

Support Reactions: As shown on FBD.



Absolute Minimum Moment: In order to get the absolute minimum moment, the maximum positive and maximum negative moment must be equal, that is, $M_{\max(+)} = M_{\min(-)}$.

For the positive moment:

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad M_{\max(+)} + \frac{wL}{2}\left(\frac{L}{4}\right) - \frac{wL}{2}\left(\frac{L}{2} - a\right) = 0$$

$$M_{\max(+)} = \frac{wL^2}{8} - \frac{waL}{2}$$

For the negative moment:

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad wa\left(\frac{a}{2}\right) - M_{\max(-)} = 0$$

$$M_{\max(-)} = \frac{wa^2}{2}$$

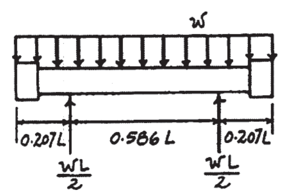
$$M_{\max(+)} = M_{\max(-)}$$

$$\frac{wL^2}{8} - \frac{wL}{2}a = \frac{wa^2}{2}$$

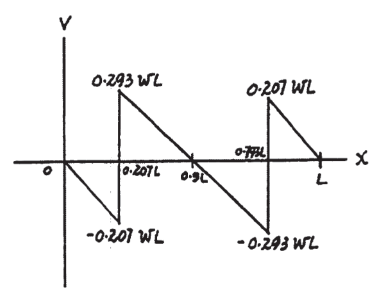
$$4a^2 + 4La - L^2 = 0$$

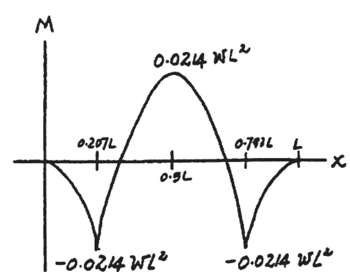
$$a = \frac{-4L \pm \sqrt{16L^2 - 4(4)(-L^2)}}{2(4)}$$

$$a = 0.207L \qquad \textbf{Ans.}$$



Shear and Moment Diagram:





Ans:
$a = 0.207L$

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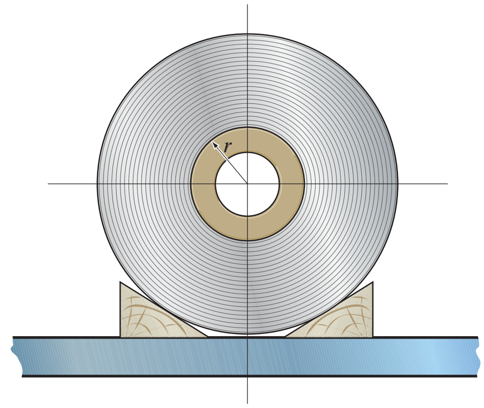
11–47.

An A-36 steel strip has an allowable bending stress of 165 MPa. If it is rolled up, determine the smallest radius r of the spool if the strip has a width of 10 mm and a thickness of 1.5 mm. Also, find the corresponding maximum internal moment developed in the strip.



SOLUTION

Bending Stress-Curvature Relation:

$$\sigma_{\text{allow}} = \frac{Ec}{\rho}; \qquad 165(10^6) = \frac{200(10^9)[0.75(10^{-3})]}{r}$$

$$r = 0.9091 \text{ m} = 909 \text{ mm}$$ **Ans.**

Moment Curvature Relation:

$$\frac{1}{\rho} = \frac{M}{EI}; \qquad \frac{1}{0.9091} = \frac{M}{200(10^9)\left[\frac{1}{12}(1)(0.0015^3)\right]}$$

$$M = 61.875 \text{ N}\cdot\text{m} = 61.9 \text{ N}\cdot\text{m}$$ **Ans.**

Ans:
$r = 909$ mm,
$M = 61.9$ N·m

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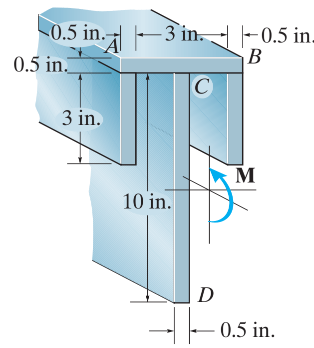
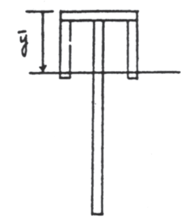
11–48.

Determine the moment M that will produce a maximum stress of 10 ksi on the cross section.



SOLUTION

Section Properties:



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$= \frac{0.25(4)(0.5) + 2[2(3)(0.5)] + 5.5(10)(0.5)}{4(0.5) + 2[(3)(0.5)] + 10(0.5)} = 3.40 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(4)(0.5^3) + 4(0.5)(3.40 - 0.25)^2$$

$$+ 2\left[\frac{1}{12}(0.5)(3^3) + 0.5(3)(3.40 - 2)^2\right]$$

$$+ \frac{1}{12}(0.5)(10^3) + 0.5(10)(5.5 - 3.40)^2$$

$$= 91.73 \text{ in}^4$$

Maximum Bending Stress: Applying the flexure formula,

$$\sigma_{\max} = \frac{Mc}{I}$$

$$10 = \frac{M(10.5 - 3.4)}{91.73}$$

$M = 129.2 \text{ kip} \cdot \text{in} = 10.8 \text{ kip} \cdot \text{ft}$ **Ans.**

Ans:
$M = 10.8 \text{ kip} \cdot \text{ft}$

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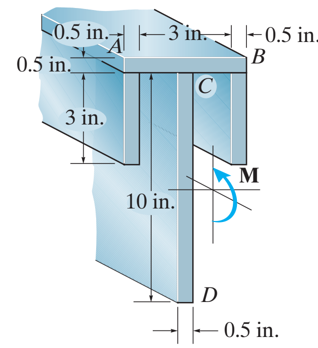
11–49.

Determine the maximum tensile and compressive bending stress in the beam if it is subjected to a moment of $M = 4\ \text{kip} \cdot \text{ft}$.

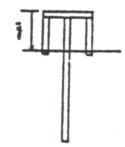


SOLUTION

Section Properties:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$= \frac{0.25(4)(0.5) + 2[2(3)(0.5)] + 5.5(10)(0.5)}{4(0.5) + 2[(3)(0.5)] + 10(0.5)} = 3.40 \text{ in.}$$



$$I_{NA} = \frac{1}{12}(4)(0.5^3) + 4(0.5)(3.40 - 0.25)^2$$

$$+ 2\left[\frac{1}{12}(0.5)(3^3) + 0.5(3)(3.40 - 2)^2\right]$$

$$+ \frac{1}{12}(0.5)(10^3) + 0.5(10)(5.5 - 3.40)^2$$

$$= 91.73 \text{ in}^4$$

Maximum Bending Stress: Applying the flexure formula $\sigma_{max} = \dfrac{Mc}{I}$

$$(\sigma_t)_{max} = \frac{4(10^3)(12)(10.5 - 3.40)}{91.73} = 3715.12 \text{ psi} = 3.72 \text{ ksi}$$ **Ans.**

$$(\sigma_c)_{max} = \frac{4(10^3)(12)(3.40)}{91.73} = 1779.07 \text{ psi} = 1.78 \text{ ksi}$$ **Ans.**

Ans:
$(\sigma_t)_{max} = 3.72$ ksi,
$(\sigma_c)_{max} = 1.78$ ksi

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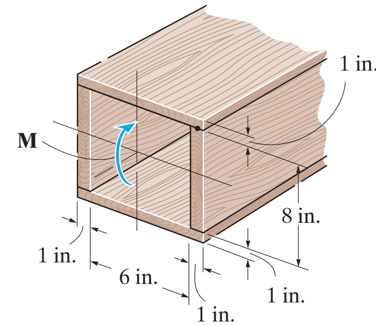
11–50.

The beam is constructed from four pieces of wood, glued together as shown. If $M = 10$ kip·ft, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.



SOLUTION

Section Properties: The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(8)\left(10^3\right) - \frac{1}{12}(6)\left(8^3\right) = 410.67 \text{ in}^4$$

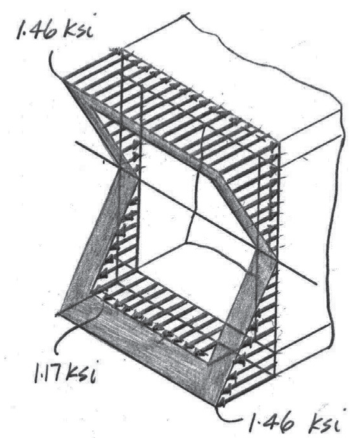
Maximum Bending Stress: Applying the flexure formula,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{10(12)(5)}{410.67} = 1.4610 \text{ ksi} = 1.46 \text{ ksi} \qquad \textbf{Ans.}$$

The bending stress at $y = 4$ in. is

$$\sigma = \frac{My}{I} = \frac{10(12)(4)}{410.67} = 1.1688 \text{ ksi} = 1.17 \text{ ksi}$$

Using these results, the bending stress distribution on the cross-section shown in Fig. *a* can be sketched.



(a)

Ans:
$\sigma_{\max} = 1.46$ ksi

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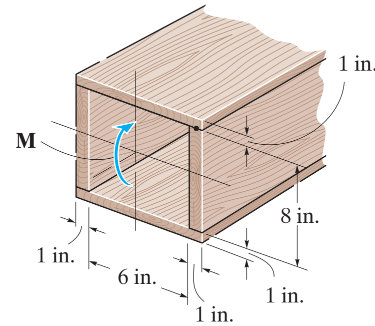
11–51.

The beam is constructed from four pieces of wood, glued together as shown. If $M = 10$ kip · ft, determine the resultant force this moment exerts on the top and bottom boards of the beam.



SOLUTION

Section Properties: The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(8)\left(10^3\right) - \frac{1}{12}(6)\left(8^3\right) = 410.67 \text{ in}^4$$

Bending Stress: Applying the flexure formula, the maximum bending stress is

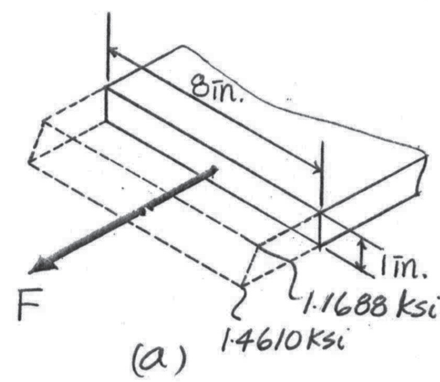
$$\sigma_{\max} = \frac{Mc}{I} = \frac{10(12)(5)}{410.67} = 1.4610 \text{ ksi}$$

The bending stress at $y = 4$ in. is

$$\sigma = \frac{My}{I} = \frac{10(12)(4)}{410.67} = 1.1688 \text{ ksi}$$

The Resultant Force: The resultant force of the stress block on bottom/top board, Fig. a, is

$$F = \frac{1}{2}(1.4610 + 1.1688)(8)(1) = 10.52 \text{ kip} = 10.5 \text{ kip} \qquad \textbf{Ans.}$$



(a)

Ans:
$F = 10.5$ kip

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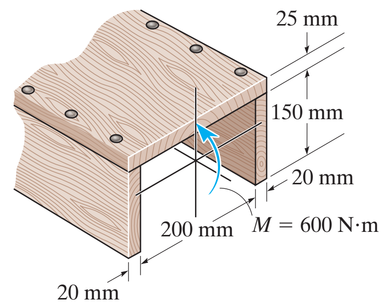
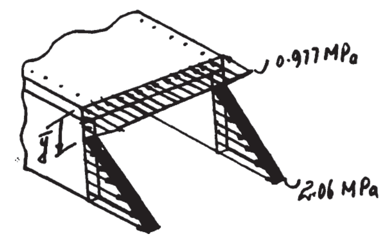
***11–52.**

The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution and cover the cross section.



SOLUTION

$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2(0.1)(0.15)(0.02)}{0.24(0.025) + 2(0.15)(0.02)} = 0.05625\ \text{m}$$



$$I = \frac{1}{12}(0.24)\left(0.025^3\right) + (0.24)(0.025)\left(0.04375^2\right)$$

$$+2\left(\frac{1}{12}\right)(0.02)\left(0.15^3\right) + 2(0.15)(0.02)\left(0.04375^2\right)$$

$$= 34.53125\left(10^{-6}\right)\ \text{m}^4$$

$$\sigma_{\max} = \sigma_B = \frac{Mc}{I}$$

$$= \frac{600(0.175 - 0.05625)}{34.53125\left(10^{-6}\right)}$$

$$= 2.06\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_c = \frac{My}{I} = \frac{600(0.05625)}{34.53125\left(10^{-6}\right)} = 0.977\ \text{MPa}$$

Ans:
$\sigma_{\max} = 2.06\ \text{MPa}$

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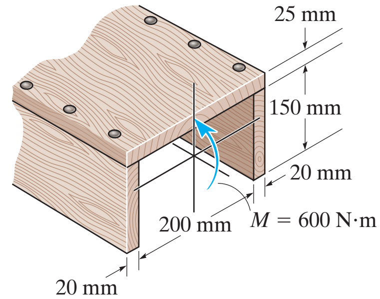
11–53.

The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 600\ \text{N}\cdot\text{m}$, determine the resultant force the bending stress produces on the top board.



SOLUTION

$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2(0.15)(0.1)(0.02)}{0.24(0.025) + 2(0.15)(0.02)} = 0.05625\ \text{m}$$

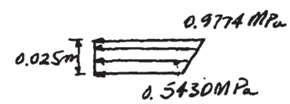
$$I = \frac{1}{12}(0.24)\left(0.025^3\right) + (0.24)(0.025)\left(0.04375^2\right)$$

$$+ 2\left(\frac{1}{12}\right)(0.02)\left(0.15^3\right) + 2(0.15)(0.02)\left(0.04375^2\right)$$

$$= 34.53125\left(10^{-6}\right)\ \text{m}^4$$

$$\sigma_t = \frac{My}{I} = \frac{600(0.05625)}{34.53125\left(10^{-6}\right)} = 0.9774\ \text{MPa}$$

$$\sigma_b = \frac{My}{I} = \frac{600(0.05625 - 0.025)}{34.53125\left(10^{-6}\right)} = 0.5430\ \text{MPa}$$



$$F = \frac{1}{2}(0.025)(0.9774 + 0.5430)\left(10^6\right)(0.240) = 4.56\ \text{kN}$$ **Ans.**

Ans:
$F = 4.56$ kN

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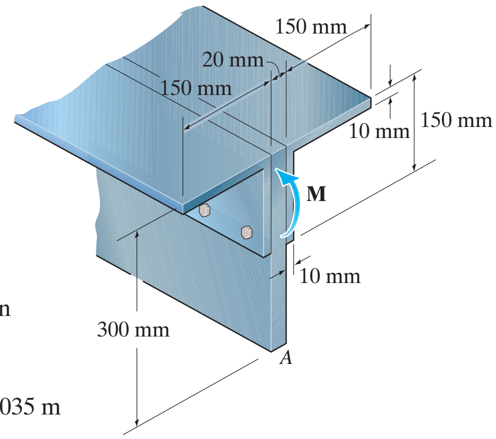
***11–54.**

If the built-up beam is subjected to an internal moment of $M = 75\ \text{kN}\cdot\text{m}$, determine the maximum tensile and compressive stress acting in the beam.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{0.15(0.3)(0.02) + 2[0.225(0.15)(0.01)] + 2[0.295(0.01)(0.14)]}{0.3(0.02) + 2(0.15)(0.01) + 2(0.01)(0.14)} = 0.2035\text{ m}$$

Thus, the moment of inertia of the cross section about the neutral axis is

$$I = \Sigma\bar{I} + Ad^2$$

$$= \frac{1}{12}(0.02)(0.3^3) + 0.02(0.3)(0.2035 - 0.15)^2$$

$$+ 2\left[\frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.225 - 0.2035)^2\right]$$

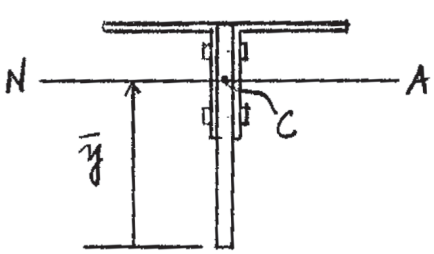
$$+ 2\left[\frac{1}{12}(0.14)(0.01^3) + 0.14(0.01)(0.295 - 0.2035)^2\right]$$

$$= 92.6509(10^{-6})\text{ m}^4$$

Maximum Bending Stress: The maximum compressive and tensile stress occurs at the top and bottom-most fiber of the cross section.

$$(\sigma_{\max})_c = \frac{My}{I} = \frac{75(10^3)(0.3 - 0.2035)}{92.6509(10^{-6})} = 78.1\text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_t = \frac{Mc}{I} = \frac{75(10^3)(0.2035)}{92.6509(10^{-6})} = 165\text{ MPa} \qquad \textbf{Ans.}$$



(a)

Ans:
$(\sigma_{\max})_c = 78.1\text{ MPa}$,
$(\sigma_{\max})_t = 165\text{ MPa}$

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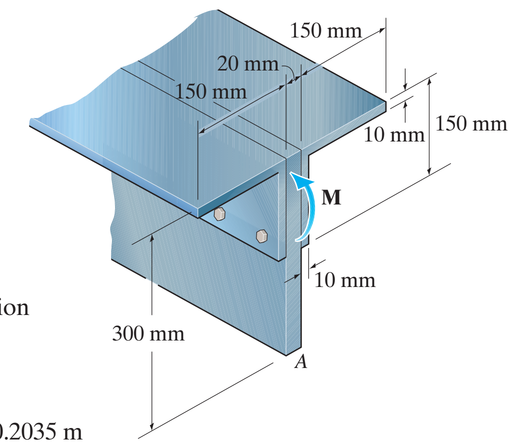
11–55.

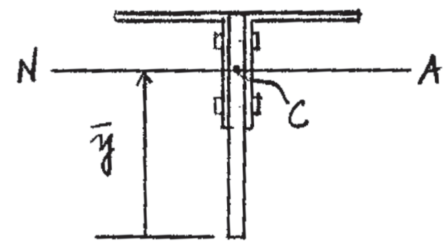
If the built-up beam is subjected to an internal moment of $M = 75\ \text{kN}\cdot\text{m}$, determine the amount of this internal moment resisted by plate A.



SOLUTION

Section Properties: The neutral axis passes through centroid C of the cross section as shown in Fig. a. The location of C is

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.15(0.3)(0.02) + 2[0.225(0.15)(0.01)] + 2[0.295(0.01)(0.14)]}{0.3(0.02) + 2(0.15)(0.01) + 2(0.01)(0.14)} = 0.2035\ \text{m}$$



(a)

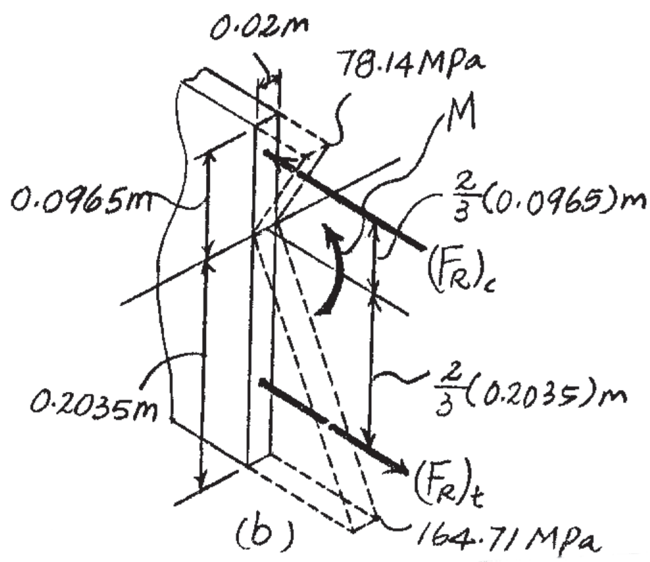
Thus, the moment of inertia of the cross section about the neutral axis is

$$\begin{aligned} I &= \bar{I} + Ad^2 \\ &= \frac{1}{12}(0.02)(0.3^3) + 0.02(0.3)(0.2035 - 0.15)^2 \\ &\quad + 2\left[\frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.225 - 0.2035)^2\right] \\ &\quad + 2\left[\frac{1}{12}(0.14)(0.01^3) + 0.14(0.01)(0.295 - 0.2035)^2\right] \\ &= 92.6509(10^{-6})\ \text{m}^4 \end{aligned}$$

Bending Stress: The distance from the neutral axis to the top and bottom of plate A is $y_t = 0.3 - 0.2035 = 0.0965$ m and $y_b = 0.2035$ m.

$$\sigma_t = \frac{My_t}{I} = \frac{75(10^3)(0.0965)}{92.6509(10^{-6})} = 78.14\ \text{MPa (C)}$$

$$\sigma_b = \frac{My_b}{I} = \frac{75(10^3)(0.2035)}{92.6509(10^{-6})} = 164.71\ \text{MPa (T)}$$



The bending stress distribution across the cross section of plate A is shown in Fig. b. The resultant forces of the tensile and compressive triangular stress blocks are

$$(F_R)_t = \frac{1}{2}(164.71)(10^6)(0.2035)(0.02) = 335\,144.46\ \text{N}$$

$$(F_R)_c = \frac{1}{2}(78.14)(10^6)(0.0965)(0.02) = 75\,421.50\ \text{N}$$

Thus, the amount of internal moment resisted by plate A is

$$\begin{aligned} M &= 335144.46\left[\frac{2}{3}(0.2035)\right] + 75421.50\left[\frac{2}{3}(0.0965)\right] \\ &= 50315.65\ \text{N}\cdot\text{m} = 50.3\ \text{kN}\cdot\text{m} \end{aligned}$$ **Ans.**

Ans:
$M = 50.3\ \text{kN}\cdot\text{m}$

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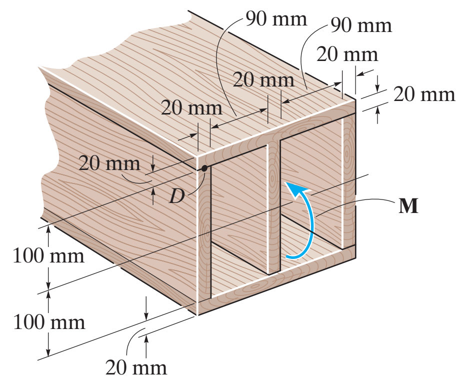
***11–56.**

The beam is subjected to a moment M. Determine the percentage of this moment that is resisted by the stresses acting on both the top and bottom boards of the beam.



SOLUTION

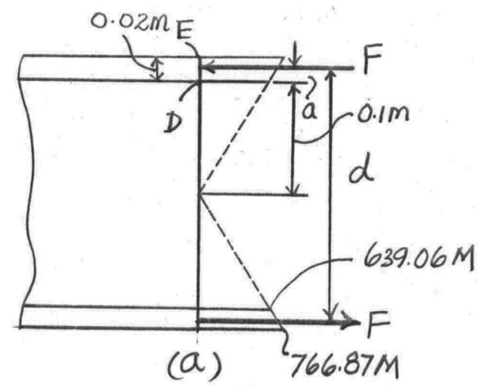
Section Properties: The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \frac{1}{12}(0.24)\left(0.24^3\right) - \frac{1}{12}(0.18)\left(0.2^3\right) = 0.15648\left(10^{-3}\right)\ \text{m}^4$$

Bending Stress: Applying the flexure formula, $\sigma = \dfrac{My}{I}$, the bending stress on points D and E, Fig. a, is

$$\sigma_D = \frac{M(0.1)}{0.15648\left(10^{-3}\right)} = 639.06M$$

$$\sigma_E = \frac{M(0.12)}{0.15648\left(10^{-3}\right)} = 766.87M$$



Resultant Force and Moment: The resultant of the stress block acting on boards A and B, Fig. a, is

$$F = \frac{1}{2}(639.06M + 766.87M)(0.02)(0.24) = 3.3742M$$

The location of line of action of F is

$$a = \frac{1}{3}\left[\frac{2(766.87M) + 639.06M}{639.06M + 766.87M}\right](0.02) = 0.010303\ \text{m}$$

Thus, the moment arm of F is

$$a' = 2(0.1 + 0.010303) = 0.22061\ \text{m}$$

Then

$$M' = Fd = 3.3742M(0.22061) = 0.74438M$$

$$\%\left(\frac{M'}{M}\right) = \left(\frac{0.74438M}{M}\right)(100) = 74.4\% \qquad \textbf{Ans.}$$

Ans:
$\dfrac{M'}{M} = 74.4\%$

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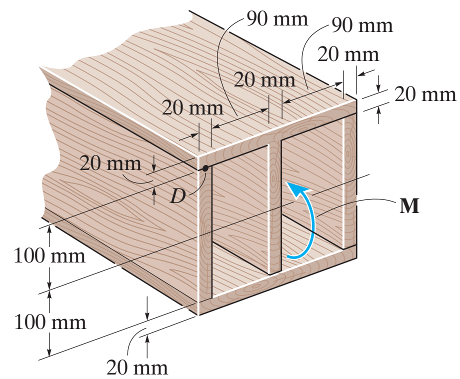
11–57.

Determine the moment M that should be applied to the beam in order to create a compressive stress at point D of $\sigma_D = 10$ MPa. Also sketch the stress distribution acting over the cross section and calculate the maximum stress developed in the beam.



SOLUTION

Section Properties: The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \frac{1}{12}(0.24)\left(0.24^3\right) - \frac{1}{12}(0.18)\left(0.2^3\right) = 0.15648\left(10^{-3}\right)\ M^4$$

Bending Stress: Applying the flexure formula,

$$\sigma_D = \frac{My_D}{I}$$

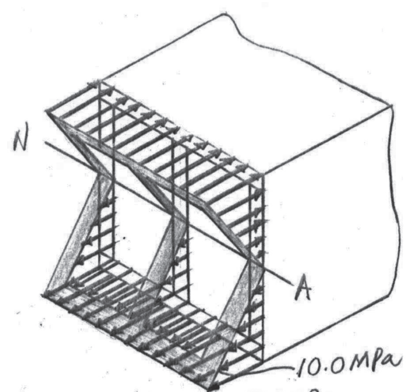
$$10\left(10^6\right) = \frac{M(0.1)}{0.15648\left(10^{-3}\right)}$$

$$M = 15.648\left(10^{-3}\right)\ \text{N}\cdot\text{m} = 15.6\ \text{kN}\cdot\text{m}$$ **Ans.**

The maximum bending stress is

$$\sigma_{\max} = \frac{MC}{I} = \frac{[15.648\left(10^3\right)](0.12)}{0.15648\left(10^{-3}\right)} = 12.0\left(10^6\right)\ \text{Pa} = 12.0\ \text{MPa}$$ **Ans.**

The sketch of bending stress distribution on the beam's cross-section is shown in Fig. *a*.



(a)

Ans:
$M = 15.6\ \text{kN}\cdot\text{m}$,
$\sigma_{\max} = 12.0$ MPa

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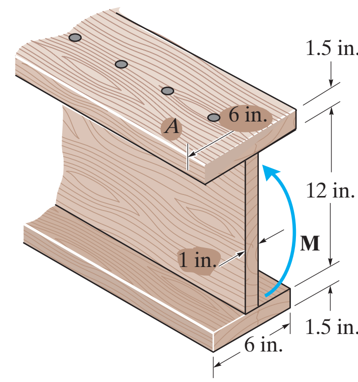
11–58.

The beam is made from three boards nailed together as shown. If the moment acting on the cross section is $M = 1$ kip·ft, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.



SOLUTION

Section Properties: The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \frac{1}{12}(6)\left(15^3\right) - \frac{1}{12}(5)\left(12^3\right) = 967.5 \text{ in}^4$$

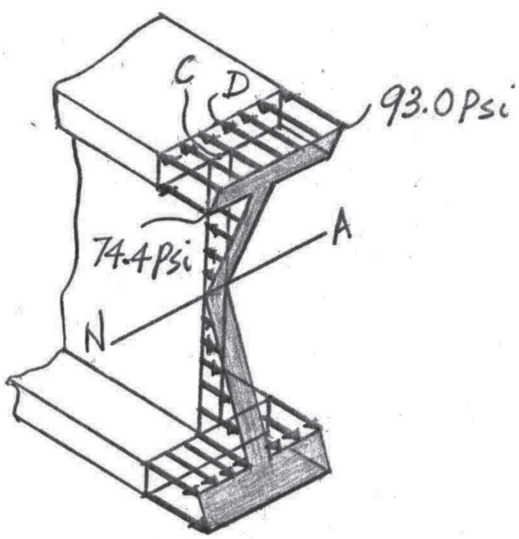
Bending Stress: Maximum bending stress occurs at point D, where $C = 7.5$ in., Fig. *a*. Applying the flexure formula,

$$\sigma_{\max} = \sigma_D = \frac{Mc}{I} = \frac{1000(12)(7.5)}{967.5} = 93.0 \text{ psi} \qquad \textbf{Ans.}$$

The bending stress at point C, where $y_c = 6$ in., is

$$\sigma_C = \frac{My_c}{I} = \frac{1000(12)(6)}{967.5} = 74.4 \text{ psi}$$

Using these results, the bending stress distribution on the beam's cross-section is shown in Fig. *a*.



(a)

Ans:
$\sigma_{\max} = 93.0$ psi

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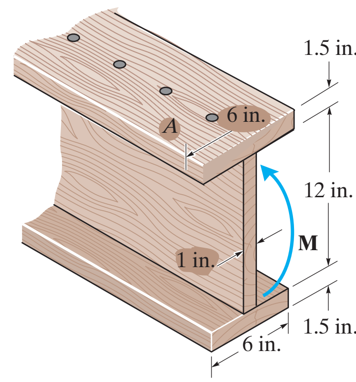
11–59.

If $M = 1$ kip · ft, determine the resultant force the bending stresses produce on the top board A of the beam.



SOLUTION

Section Properties: The moment of inertia of the beam's cross-section about the neutral axis is

$$I = \frac{1}{12}(6)(15^3) - \frac{1}{12}(5)(12^3) = 967.5 \text{ in}^4$$

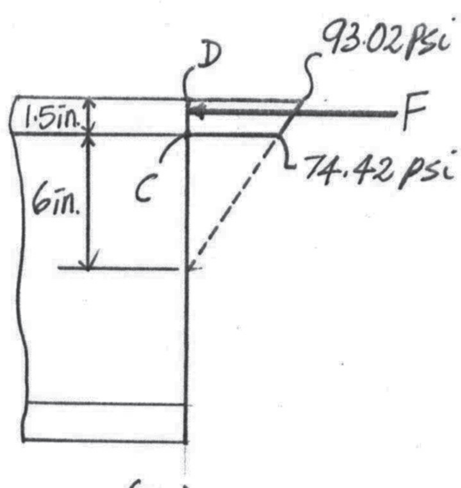
Bending Stress: The bending stresses at points C and D, Fig. a, where $y_C = 6$ in. and $y_D = C = 7.5$ in., can be determined using the flexure formula.

$$\sigma_C = \frac{My_C}{I} = \frac{1000(12)(6)}{967.5} = 74.42 \text{ psi};$$

$$\sigma_D = \frac{Mc}{I} = \frac{1000(12)(7.5)}{967.5} = 93.02 \text{ psi};$$

The Resultant Forces: The resultant force of the stress block on board A, Fig. a, is

$$F = \frac{1}{2}(93.02 + 74.42)(6)(1.5) = 753.48 \text{ lb} = 753 \text{ lb}$$ **Ans.**



(a)

Ans:
$F = 753$ lb

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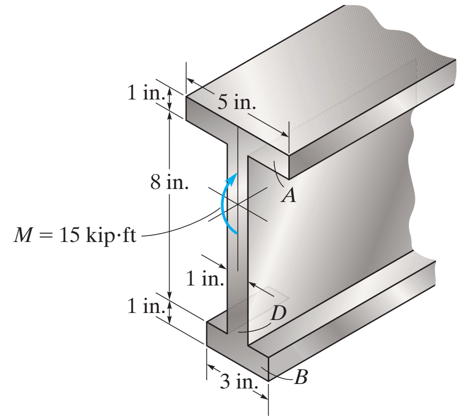
***11–60.**

The beam is subjected to a moment of 15 kip · ft. Determine the resultant force the bending stress produces on the top flange A and bottom flange B. Also calculate the maximum bending stress developed in the beam.



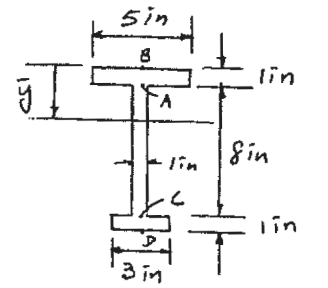
SOLUTION

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(1)(5) + 5(8)(1) + 9.5(3)(1)}{1(5) + 8(1) + 3(1)} = 4.4375 \text{ in.}$$

$$I = \frac{1}{12}(5)(1^3) + 5(1)(4.4375 - 0.5)^2 + \frac{1}{12}(1)(8^3) + 8(1)(5 - 4.4375)^2$$

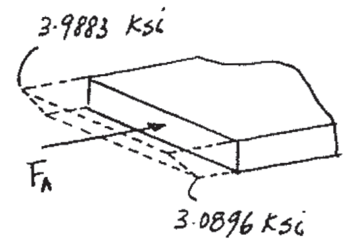
$$+ \frac{1}{12}(3)(1^3) + 3(1)(9.5 - 4.4375)^2$$

$$= 200.27 \text{ in}^4$$



Using flexure formula $\sigma = \dfrac{My}{I}$

$$\sigma_A = \frac{15(12)(4.4375 - 1)}{200.27} = 3.0896 \text{ ksi}$$



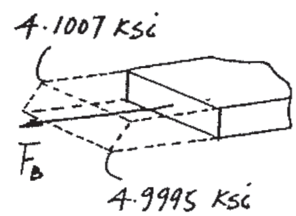
$$\sigma_B = \frac{15(12)(4.4375)}{200.27} = 3.9883 \text{ ksi}$$

$$\sigma_C = \frac{15(12)(9 - 4.4375)}{200.27} = 4.1007 \text{ ksi}$$

$$\sigma_{max} = \frac{15(12)(10 - 4.4375)}{200.27} = 4.9995 \text{ ksi} = 5.00 \text{ ksi (Max)}$$ **Ans.**

$$F_A = \frac{1}{2}(3.0896 + 3.9883)(1)(5) = 17.7 \text{ kip}$$ **Ans.**

$$F_B = \frac{1}{2}(4.9995 + 4.1007)(1)(3) = 13.7 \text{ kip}$$ **Ans.**



Ans:
$\sigma_{max} = 5.00$ ksi,
$F_A = 17.7$ kip,
$F_B = 13.7$ kip

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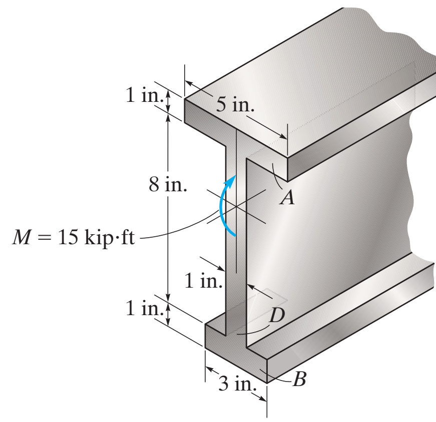
11–61.

The beam is subjected to a moment of 15 kip · ft. Determine the percentage of this moment that is resisted by the web D of the beam.



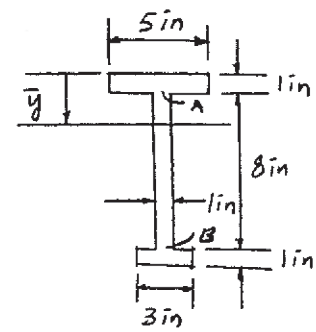
SOLUTION

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(1)(5) + 5(8)(1) + 9.5(3)(1)}{1(5) + 8(1) + 3(1)} = 4.4375 \text{ in.}$$

$$I = \frac{1}{12}(5)(1^3) + 5(1)(4.4375 - 0.5)^2 + \frac{1}{12}(1)(8^3) + 8(1)(5 - 4.4375)^2$$

$$+ \frac{1}{12}(3)(1^3) + 3(1)(9.5 - 4.4375)^2$$

$$= 200.27 \text{ in}^4$$



Using flexure formula $\sigma = \dfrac{My}{I}$

$$\sigma_A = \frac{15(12)(4.4375 - 1)}{200.27} = 3.0896 \text{ ksi}$$

$$\sigma_B = \frac{15(12)(9 - 4.4375)}{200.27} = 4.1007 \text{ ksi}$$

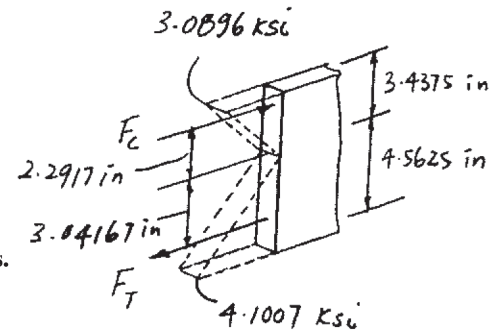
$$F_C = \frac{1}{2}(3.0896)(3.4375)(1) = 5.3102 \text{ kip}$$

$$F_T = \frac{1}{2}(4.1007)(4.5625)(1) = 9.3547 \text{ kip}$$

$$M = 5.3102(2.2917) + 9.3547(3.0417)$$

$$= 40.623 \text{ kip} \cdot \text{in.} = 3.3852 \text{ kip} \cdot \text{ft}$$

$$\% \text{ of moment carried by web} = \frac{3.3852}{15} \times 100 = 22.6\ \%$$ **Ans.**



Ans:
% of moment carried by web = 22.6%

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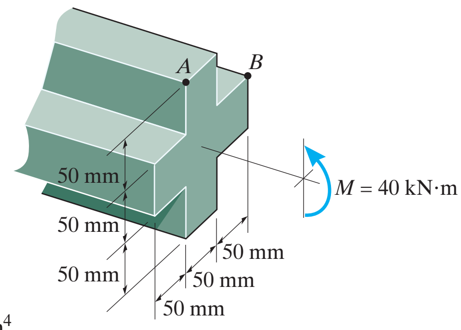
11–62.

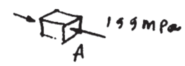
The beam is subjected to a moment of $M = 40$ kN·m. Determine the bending stress at points A and B. Sketch the results on a volume element acting at each of these points.



SOLUTION

$$I = \frac{1}{12}(0.150)(0.05)^3 + 2\left[\frac{1}{12}(0.05)(0.05)^3 + (0.05)(0.05)(0.05)^2\right] = 15.1042\left(10^{-6}\right)\text{ m}^4$$

$$\sigma_A = \frac{Mc}{I} = \frac{40\left(10^3\right)(0.075)}{15.1042\left(10^{-6}\right)} = 199\text{ MPa}$$ **Ans.**



$$\sigma_B = \frac{My}{I} = \frac{40\left(10^3\right)(0.025)}{15.1042\left(10^{-6}\right)} = 66.2\text{ MPa}$$ **Ans.**

Ans:
$\sigma_A = 199$ MPa,
$\sigma_B = 66.2$ MPa

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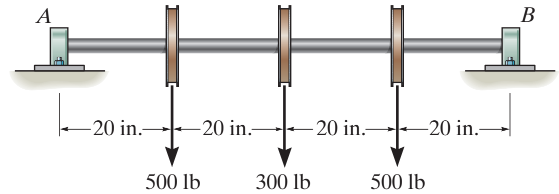
11–63.

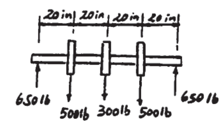
The steel shaft has a diameter of 2 in. It is supported on smooth journal bearings A and B, which exert only vertical reactions on the shaft. Determine the absolute maximum bending stress in the shaft if it is subjected to the pulley loadings shown.

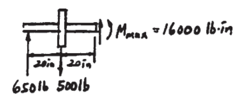


SOLUTION

$$I = \frac{1}{4}\pi\left(1^4\right) = 0.7854 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{16000\ (1)}{0.7854} = 20.4 \text{ ksi} \qquad \textbf{Ans.}$$





Ans:
$\sigma_{\max} = 20.4$ ksi

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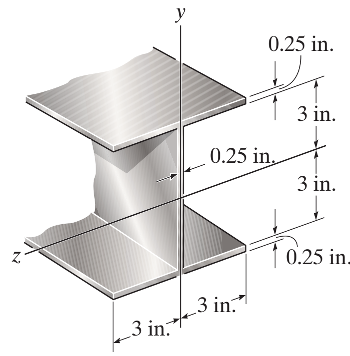
***11–64.**

The beam is made of steel that has an allowable stress of $\sigma_{allow} = 24$ ksi. Determine the largest internal moment the beam can resist if the moment is applied (a) about the z axis, (b) about the y axis.



SOLUTION

$$I_z = \frac{1}{12}(6)(6.5^3) - \frac{1}{12}(5.75)(6^3) = 33.8125 \text{ in}^4$$

$$I_y = 2\left[\frac{1}{12}(0.25)(6^3)\right] + \frac{1}{12}(6)(0.25^3) = 9.0078 \text{ in}^4$$

(a) $$(M_{allow})_z = \frac{\sigma_{allow} I_z}{c} = \frac{24(33.8125)}{3.25}$$

$$= 249.7 \text{ kip} \cdot \text{in.} = 20.8 \text{ kip} \cdot \text{ft}$$ **Ans.**

(b) $$(M_{allow})_y = \frac{\sigma_{allow} I_y}{c} = \frac{24(9.0078)}{3}$$

$$= 72.0625 \text{ kip} \cdot \text{in.} = 6.00 \text{ kip} \cdot \text{ft}$$ **Ans.**

Ans:
$(M_{allow})_z = 20.8$ kip · ft,
$(M_{allow})_y = 6.00$ kip · ft

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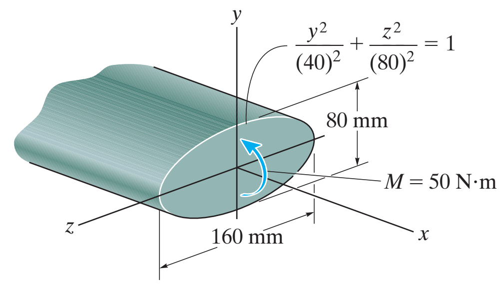
11–65.

A shaft is made of a polymer having an elliptical cross section. If it resists an internal moment of $M = 50\text{ N}\cdot\text{m}$, determine the maximum bending stress in the material (a) using the flexure formula, where $I_z = \frac{1}{4}\pi(0.08\text{ m})(0.04\text{ m})^3$, (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area. Here $I_x = \frac{1}{4}\pi(0.08\text{ m})(0.04\text{ m})^3$.

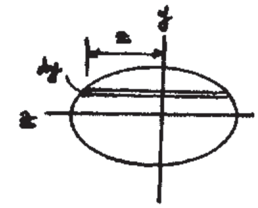


SOLUTION

(a)

$$I = \frac{1}{4}\pi\, ab^3 = \frac{1}{4}\pi(0.08)(0.04)^3 = 4.021238(10^{-6})\text{ m}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\text{ kPa}$$ **Ans.**



(b)

$$M = \frac{\sigma_{\max}}{c}\int_A y^2 dA$$

$$= \frac{\sigma_{\max}}{c}\int y^2 2z\,dy$$

$$z = \sqrt{0.0064 - 4y^2} = 2\sqrt{(0.04)^2 - y^2}$$

$$2\int_{-0.04}^{0.04} y^2 z\,dy = 4\int_{-0.04}^{0.04} y^2\sqrt{(0.04)^2 - y^2}\,dy$$

$$= 4\left[\frac{(0.04)^4}{8}\sin^{-1}\left(\frac{y}{0.04}\right) - \frac{1}{8}y\sqrt{(0.04)^2 - y^2}(0.04^2 - 2y^2)\right]\Bigg|_{-0.04}^{0.04}$$

$$= \frac{(0.04)^4}{2}\sin^{-1}\left(\frac{y}{0.04}\right)\Bigg|_{-0.04}^{0.04}$$

$$= 4.021238(10^{-6})\text{ m}^4$$

$$\sigma_{\max} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\text{ kPa}$$ **Ans.**

Ans:
(a) $\sigma_{\max} = 497$ kPa,
(b) $\sigma_{\max} = 497$ kPa

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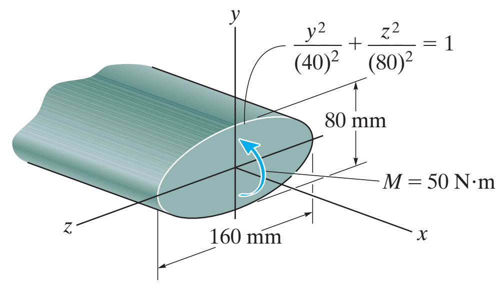
11–66.

Solve Prob. 11–65 if the moment $M = 50\ \text{N}\cdot\text{m}$ is applied about the y axis instead of the x axis. Here $I_y = \frac{1}{4}\pi(0.04\ \text{m})(0.08\ \text{m})^3$.



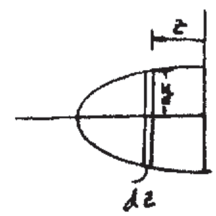
SOLUTION

(a)

$$I = \frac{1}{4}\pi\, ab^3 = \frac{1}{4}\pi(0.04)(0.08)^3 = 16.085(10^{-6})\ \text{m}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{50(0.08)}{16.085(10^{-6})} = 249\ \text{kPa}$$ **Ans.**

(b)



$$M = \int_A z(\sigma\, dA) = \int_A z\left(\frac{\sigma_{\max}}{0.08}\right)(z)(2y)dz$$

$$50 = 2\left(\frac{\sigma_{\max}}{0.04}\right)\int_0^{0.08} z^2\left(1 - \frac{z^2}{(0.08)^2}\right)^{1/2}(0.04)dz$$

$$50 = 201.06(10^{-6})\sigma_{\max}$$

$$\sigma_{\max} = 249\ \text{kPa}$$ **Ans.**

Ans:
(a) $\sigma_{\max} = 249\ \text{kPa}$,
(b) $\sigma_{\max} = 249\ \text{kPa}$

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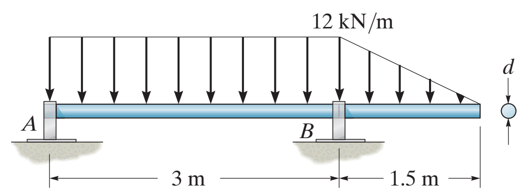
11–67.

The shaft is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. If $d = 90$ mm, determine the absolute maximum bending stress in the beam, and sketch the stress distribution acting over the cross section.



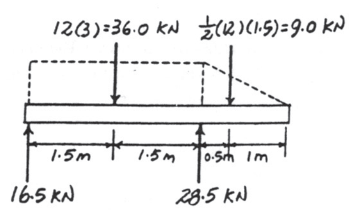
SOLUTION

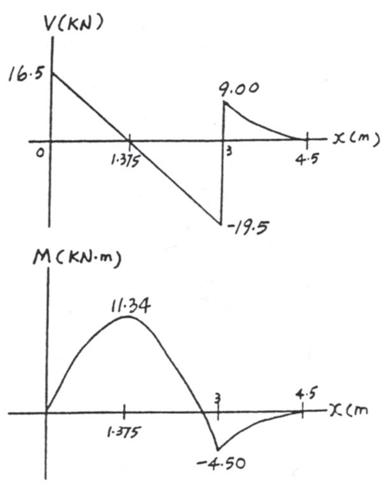
Absolute Maximum Bending Stress: The maximum moment is $M_{max} = 11.34\ \text{kN}\cdot\text{m}$ as indicated on the moment diagram. Applying the flexure formula,

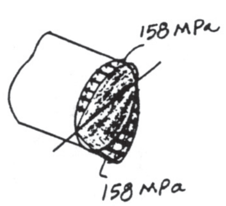
$$\sigma_{max} = \frac{M_{max}\,c}{I}$$

$$= \frac{11.34(10^3)(0.045)}{\frac{\pi}{4}(0.045^4)}$$

$$= 158\ \text{MPa} \qquad \textbf{Ans.}$$







Ans:
$\sigma_{max} = 158\ \text{MPa}$

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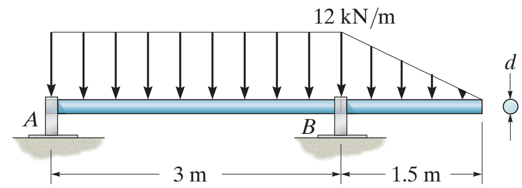
***11–68.**

The shaft is supported by smooth journal bearings at A and B that only exert vertical reactions on the shaft. Determine its smallest diameter d if the allowable bending stress is $\sigma_{allow} = 180$ MPa.



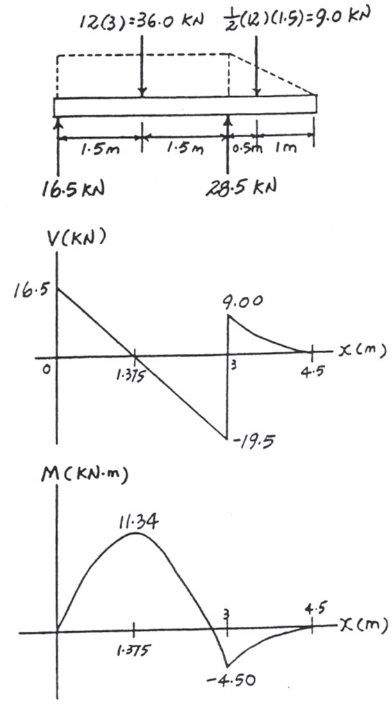
SOLUTION

Allowable Bending Stress: The maximum moment is $M_{max} = 11.34$ kN·m as indicated on the moment diagram. Applying the flexure formula,

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$180\left(10^6\right) = \frac{11.34(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 0.08626 \text{ m} = 86.3 \text{ mm}$$ **Ans.**



Ans:
$d = 86.3$ mm

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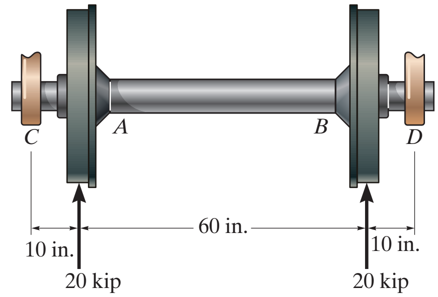
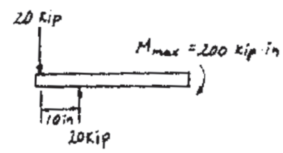
11–69.

The axle of the freight car is subjected to a wheel loading of 20 kip. If it is supported by two journal bearings at C and D, determine the maximum bending stress developed at the center of the axle, where the diameter is 5.5 in.



SOLUTION

$$\sigma_{max} = \frac{Mc}{I} = \frac{200(2.75)}{\frac{1}{4}\pi(2.75)^4} = 12.2 \text{ ksi}$$ **Ans.**



Ans:
$\sigma_{max} = 12.2$ ksi

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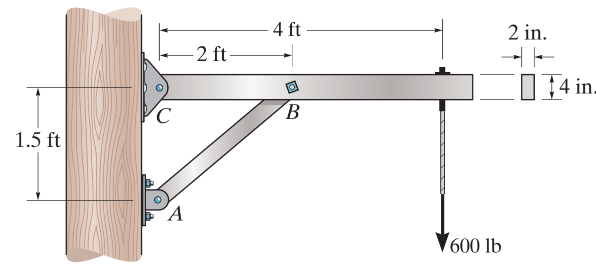
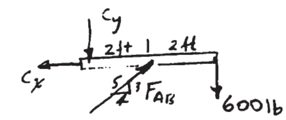
11–70.

The strut on the utility pole supports the cable having a weight of 600 lb. Determine the absolute maximum bending stress in the strut if A, B, and C are assumed to be pinned.



SOLUTION



$$\zeta+\Sigma M_C = 0; \qquad F_{AB}\left(\frac{3}{5}\right)(2) - 600(4) = 0$$

$$F_{AB} = 2000 \text{ lb}$$

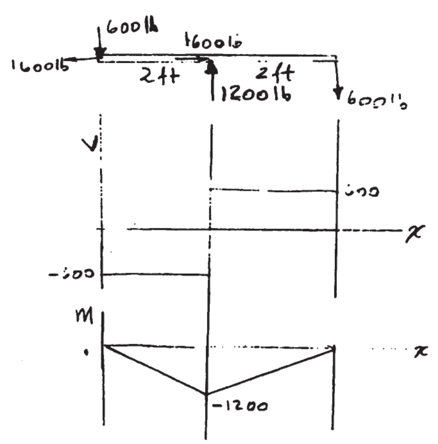
$$+\uparrow\Sigma F_y = 0; \qquad -C_y + 2000\left(\frac{3}{5}\right) - 60 = 0$$

$$C_y = 600 \text{ lb}$$

$$\xrightarrow{+}\ \Sigma F_x = 0; \qquad 2000\left(\frac{4}{5}\right) - C_x = 0$$

$$C_x = 1600 \text{ lb}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{1200(12)(2)}{\frac{1}{12}(2)(4)^3} = 2.70 \text{ ksi} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{max} = 2.70$ ksi

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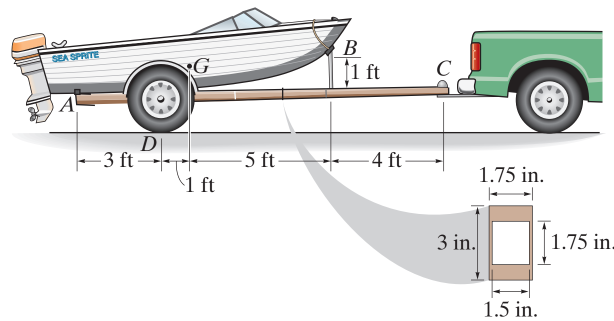
11–71.

The boat has a weight of 2300 lb and a center of gravity at G. If it rests on the trailer at the smooth contact A and can be considered pinned at B, determine the absolute maximum bending stress developed in the main strut of the trailer which is pinned at C. Consider the strut to be a box-beam having the dimensions shown.



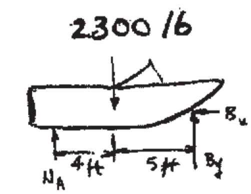
SOLUTION

Boat:

$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad B_x = 0$

$\circlearrowleft + \Sigma M_B = 0;$ $\qquad -N_A(9) + 2300(5) = 0$

$$N_A = 1277.78 \text{ lb}$$



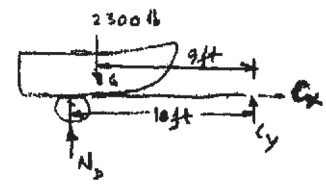
$+\uparrow \Sigma F_y = 0;$ $\qquad 1277.78 - 2300 + B_y = 0$

$$B_y = 1022.22 \text{ lb}$$

Assembly:

$\circlearrowleft + \Sigma M_C = 0;$ $\qquad -N_D(10) + 2300(9) = 0$

$$N_D = 2070 \text{ lb}$$

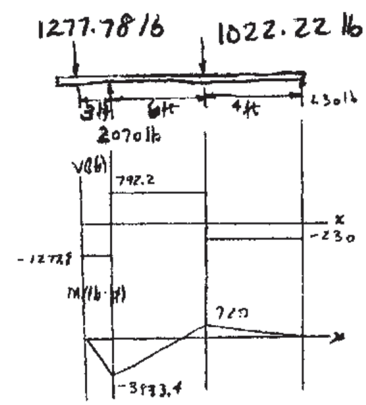


$+\uparrow \Sigma F_y = 0;$ $\qquad C_y + 2070 - 2300 = 0$

$$C_y = 230 \text{ lb}$$

$$I = \frac{1}{12}(1.75)(3)^3 - \frac{1}{12}(1.5)(1.75)^3 = 3.2676 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{3833.3(12)(1.5)}{3.2676} = 21.1 \text{ ksi} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{\max} = 21.1$ ksi

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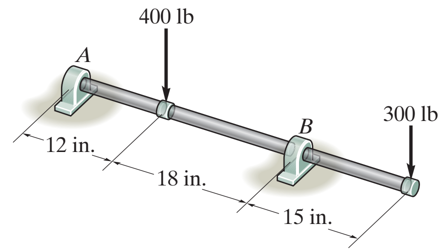
***11–72.**

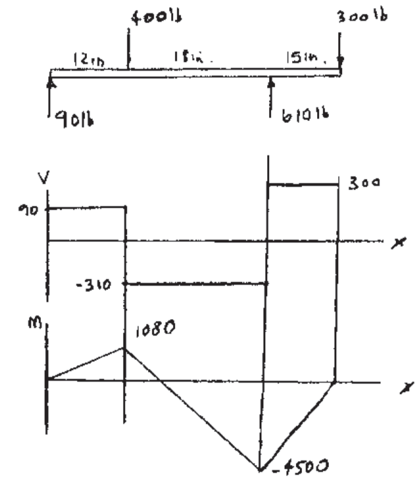
Determine the absolute maximum bending stress in the 1.5-in.-diameter shaft. The shaft is supported by a thrust bearing at A and a journal bearing at B.



SOLUTION

$M_{max} = 4500 \text{ lb} \cdot \text{in.}$

$$\sigma = \frac{Mc}{I} = \frac{4500(0.75)}{\frac{1}{4}\pi(0.75)^4} = 13.6 \text{ ksi}$$ **Ans.**



Ans:
$\sigma = 13.6$ ksi

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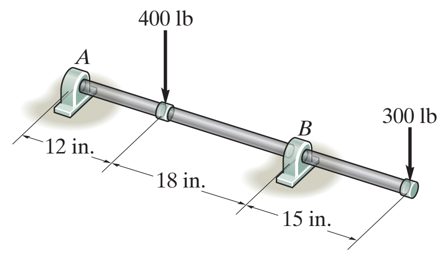
11–73.

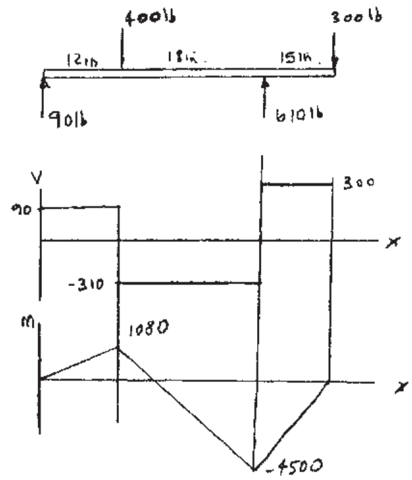
Determine the smallest allowable diameter of the shaft. The shaft is supported by a thrust bearing at A and a journal bearing at B. The allowable bending stress is $\sigma_{\text{allow}} = 22$ ksi.



SOLUTION

$M_{\max} = 4500 \text{ lb} \cdot \text{in.}$

$$\sigma = \frac{Mc}{I}; \qquad 22(10^3) = \frac{4500c}{\frac{1}{4}\pi c^4}$$



$c = 0.639$ in.

$d = 1.28$ in. **Ans.**

Ans:
$d = 1.28$ in.

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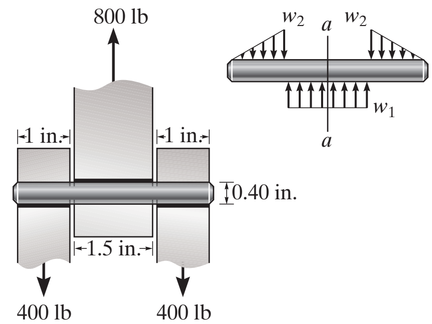
11–74.

The pin is used to connect the three links together. Due to wear, the load is distributed over the top and bottom of the pin as shown on the free-body diagram. If the diameter of the pin is 0.40 in., determine the maximum bending stress on the cross-sectional area at the center section *a–a*. For the solution it is first necessary to determine the load intensities w_1 and w_2.

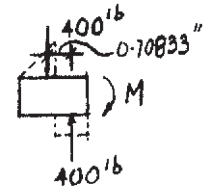


SOLUTION

$$\frac{1}{2}w_2(1) = 400; \qquad w_2 = 800 \text{ lb/in.}$$

$$w_1(1.5) = 800; \qquad w_1 = 533 \text{ lb/in.}$$

$$M = 400(0.70833) = 283.33 \text{ lb} \cdot \text{in}$$



$$I = \frac{1}{4}\pi(0.2^4) = 0.0012566 \text{ in}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{283.33\,(0.2)}{0.0012566}$$

$$= 45.1 \text{ ksi}$$ **Ans.**

Ans:
$\sigma_{max} = 45.1$ ksi

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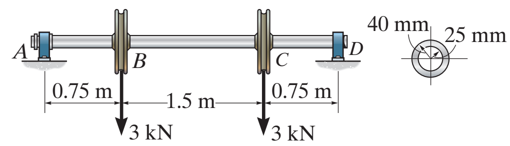
11–75.

The shaft is supported by a thrust bearing at A and journal bearing at D. If the shaft has the cross section shown, determine the absolute maximum bending stress in the shaft.



SOLUTION

Shear and Moment Diagrams: As shown in Fig. *a*.

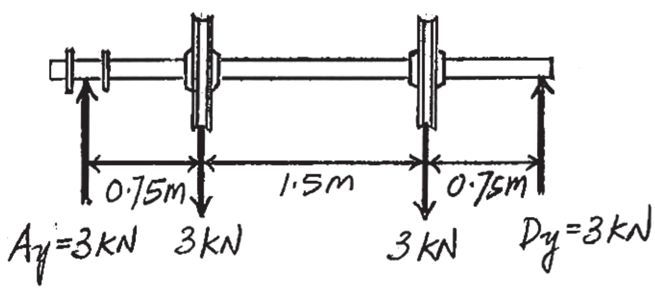
Maximum Moment: Due to symmetry, the maximum moment occurs in region BC of the shaft. This refers to the free-body diagram of the segment shown in Fig. *b*.

Section Properties: The moment of inertia of the cross section about the neutral axis is

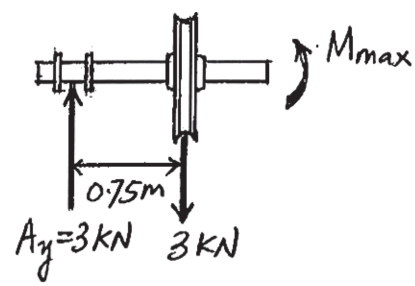
$$I = \frac{\pi}{4}(0.04^4 - 0.025^4) = 1.7038(10^{-6})\text{ m}^4$$

Absolute Maximum Bending Stress:

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{2.25(10^3)(0.04)}{1.7038(10^{-6})} = 52.8\text{ MPa} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\sigma_{\max} = 52.8\text{ MPa}$

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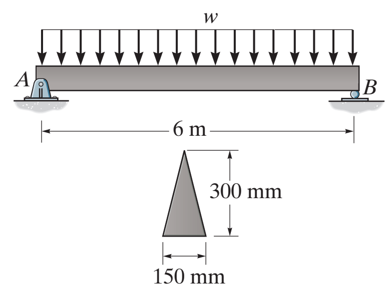
***11–76.**

If the intensity of the load $w = 15$ kN/m, determine the absolute maximum tensile and compressive stress in the beam.



SOLUTION

Support Reactions: Shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The maximum moment occurs when $V = 0$. Referring to the free-body diagram of the beam segment shown in Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad 45 - 15x = 0 \qquad x = 3 \text{ m}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M_{\max} + 15(3)\left(\frac{3}{2}\right) - 45(3) = 0 \qquad M_{\max} = 67.5 \text{ kN} \cdot \text{m}$$

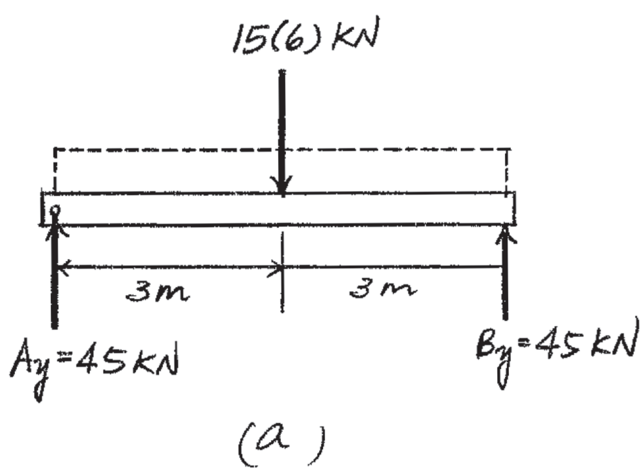
Section Properties: The moment of inertia of the cross section about the neutral axis is

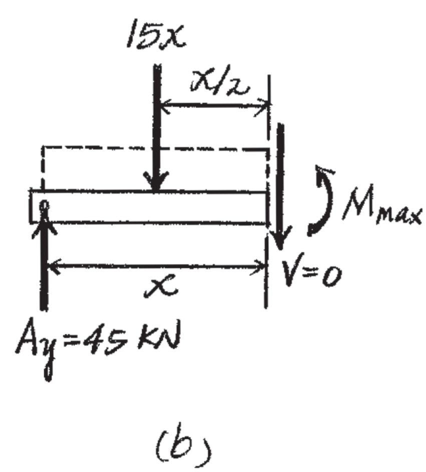
$$I = \frac{1}{36}(0.15)(0.3^3) = 0.1125(10^{-3}) \text{ m}^4$$

Absolute Maximum Bending Stress: The maximum compressive and tensile stresses occur at the top and bottom-most fibers of the cross section.

$$(\sigma_{\max})_c = \frac{Mc}{I} = \frac{67.5(10^3)(0.2)}{0.1125(10^{-3})} = 120 \text{ MPa (C)} \qquad \textbf{Ans.}$$

$$(\sigma_{\max})_t = \frac{My}{I} = \frac{67.5(10^3)(0.1)}{0.1125(10^{-3})} = 60 \text{ MPa (T)} \qquad \textbf{Ans.}$$





Ans:
$(\sigma_{\max})_c = 120$ MPa (C),
$(\sigma_{\max})_t = 60$ MPa (T)

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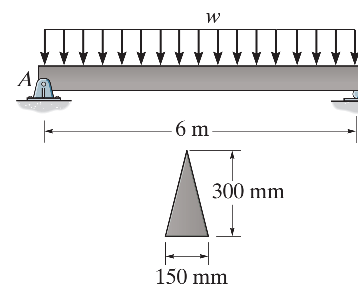
11–77.

If the allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa, determine the maximum intensity w of the uniform distributed load.



SOLUTION

Support Reactions: As shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The maximum moment occurs when $V = 0$. Referring to the free-body diagram of the beam segment shown in Fig. *b*,

$+\uparrow \Sigma F_y = 0;$ $\quad 3w - wx = 0 \quad\quad x = 3\text{ m}$

$\circlearrowleft + \Sigma M = 0;$ $\quad M_{\max} + w(3)\left(\dfrac{3}{2}\right) - 3w(3) = 0 \quad\quad M_{\max} = \dfrac{9}{2}w$

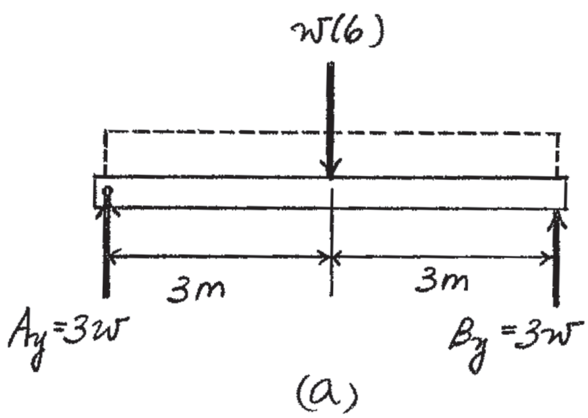
Section Properties: The moment of inertia of the cross section about the neutral axis is

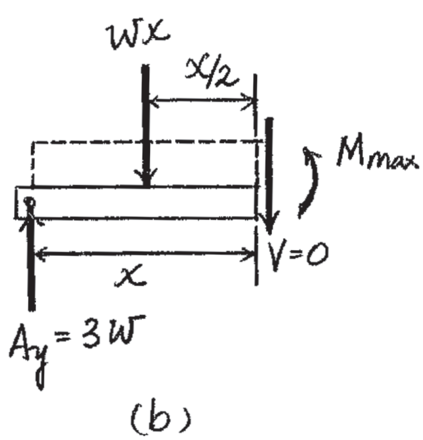
$$I = \frac{1}{36}(0.15)(0.3^3) = 0.1125(10^{-3})\,\text{m}^4$$

Absolute Maximum Bending Stress: Here, $c = \dfrac{2}{3}(0.3) = 0.2$ m.

$$\sigma_{\text{allow}} = \frac{Mc}{I}; \qquad 150(10^6) = \frac{\frac{9}{2}w(0.2)}{0.1125\left(10^{-3}\right)}$$

$$w = 18750\text{ N/m} = 18.75\text{ kN/m} \qquad \textbf{Ans.}$$





Ans:
$w = 18.75\text{ kN/m}$

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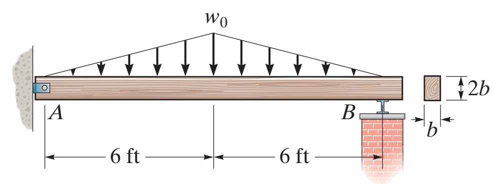
11–78.

The beam is subjected to the triangular distributed load with a maximum intensity of $w_0 = 300$ lb/ft. If the allowable bending stress is $\sigma_{allow} = 1.40$ ksi, determine the required dimension b of its cross section to the nearest $\frac{1}{8}$ in. Assume the support at A is a pin and B is a roller.



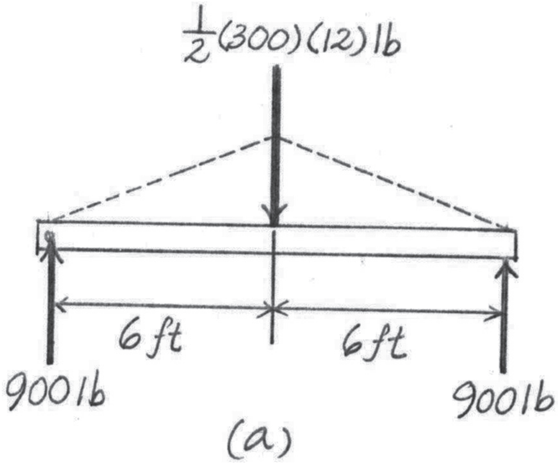
SOLUTION

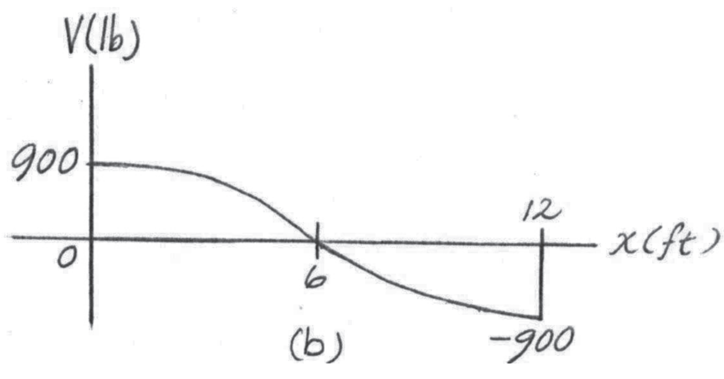
Absolute Maximum Bending Stress: The support reactions, shear diagram and moment diagram are shown in Figs. *a*, *b*, and *c* respectively. From the moment diagram, the maximum moment is $M_{max} = 3600\ \text{lb}\cdot\text{ft}$, which occurs at $x = 6$ ft. Applying the flexure formula,

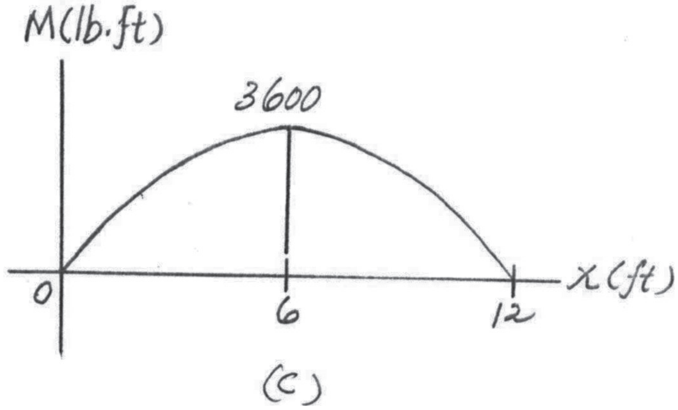
$$\sigma_{\substack{abs\\max}} = \frac{M_{max}C}{I}; \qquad 1.40\left(10^3\right) = \frac{3600(12)b}{\frac{1}{12}(b)(2b)^3}$$

$$b = 3.590 \text{ in.}$$

$$\text{Use } b = 3\frac{5}{8} \text{ in.}$$ **Ans.**







Ans:

Use $b = 3\frac{5}{8}$ in.

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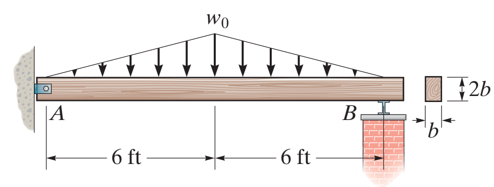
11–79.

The beam has a rectangular cross section with $b = 4$ in. Determine the largest maximum intensity w_0 of the triangular distributed loads that can be supported if the allowable bending stress is $\sigma_{allow} = 1.40$ ksi.

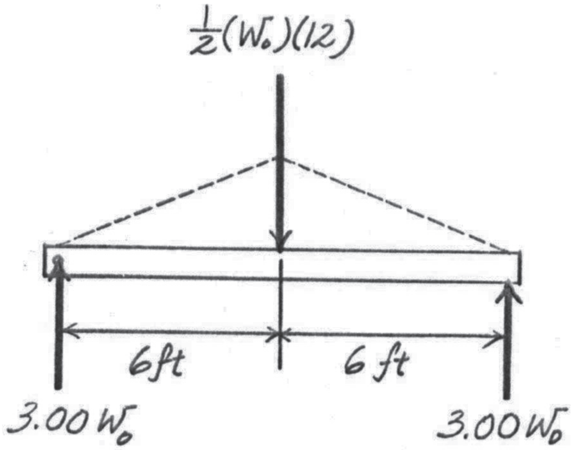


SOLUTION

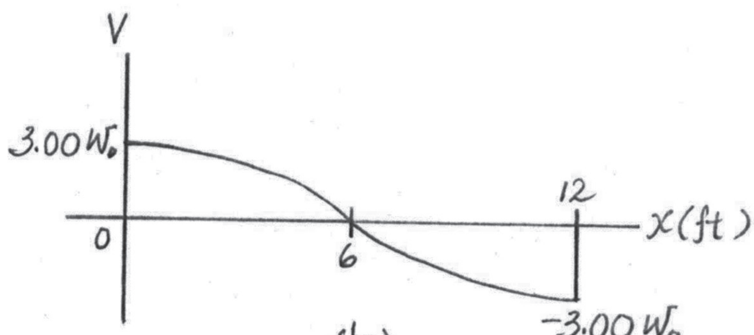
Absolute Maximum Bending Stress: The support reactions, shear diagram, and moment diagram are shown in Figs. *a*, *b* and *c*, respectively. From the moment diagram, the maximum moment is $M_{max} = 12.0\, w_0$, which occurs at $x = 6$ ft. Applying the flexure formula,

$$\sigma_{\substack{abs\\max}} = \frac{M_{max}C}{I}; \qquad 1.40\left(10^3\right) = \frac{(12.0w_0)(12)(4)}{\frac{1}{12}(4)\left(8^3\right)}$$

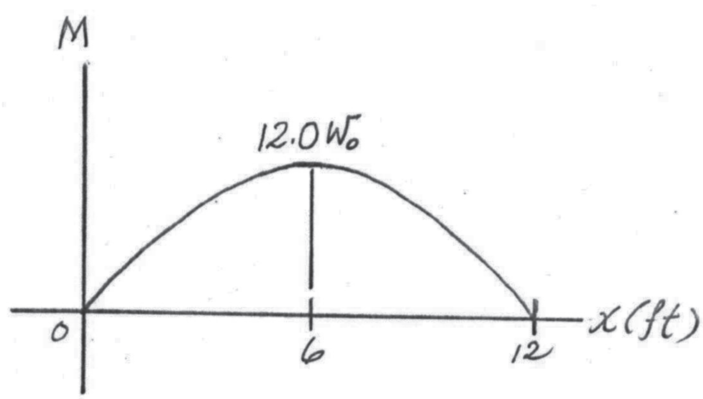
$$w_0 = 414.81 \text{ lb/ft} = 415 \text{ lb/ft} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$w_0 = 415$ lb/ft

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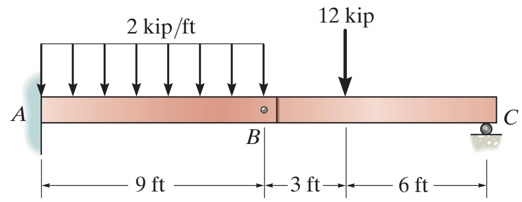
***11–80.**

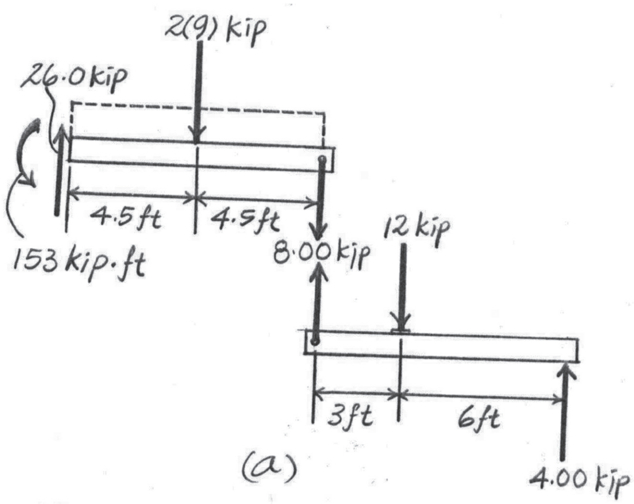
Determine the absolute maximum bending stress in the beam. Each segment has a rectangular cross section with a base of 4 in. and height of 12 in.

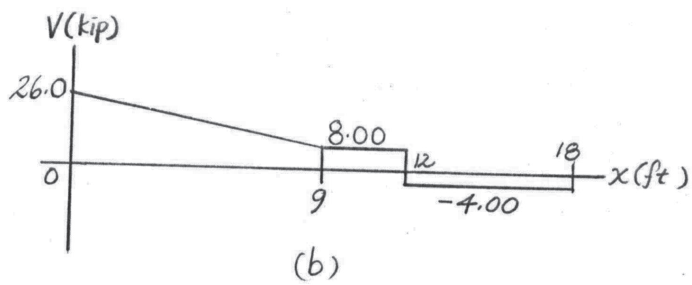


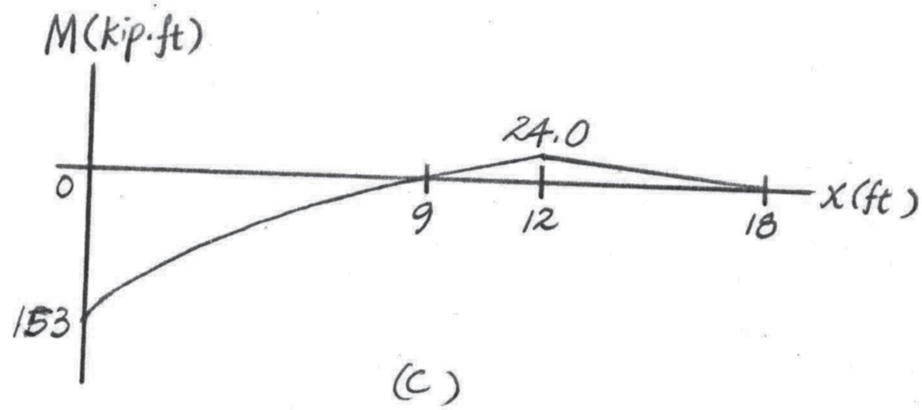
SOLUTION

Absolute Maximum Bending Stress: The support reactions, shear diagram and moment diagram are shown in Figs. *a*, *b* and *c*, respectively. From the moment diagram, the maximum moment is $M_{\text{max}} = 153\ \text{kip}\cdot\text{ft}$, which occurs at fixed support A. Applying the flexure formula,

$$\sigma_{\substack{\text{abs}\\ \text{max}}} = \frac{M_{\text{max}}\,C}{I} = \frac{153(12)(6)}{\frac{1}{12}(4)\left(12^3\right)} = 19.1\ \text{ksi} \qquad \textbf{Ans.}$$







Ans:
$\sigma_{\substack{\text{abs}\\ \text{max}}} = 19.1\ \text{ksi}$

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11–81.

If the compound beam in Prob. 11–42 has a square cross section of side length a, determine the minimum value of a if the allowable bending stress is $\sigma_{\text{allow}} = 150$ MPa.

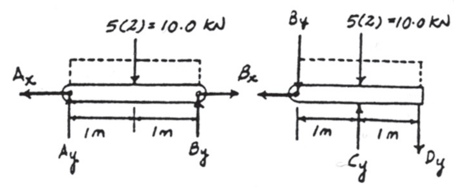
SOLUTION

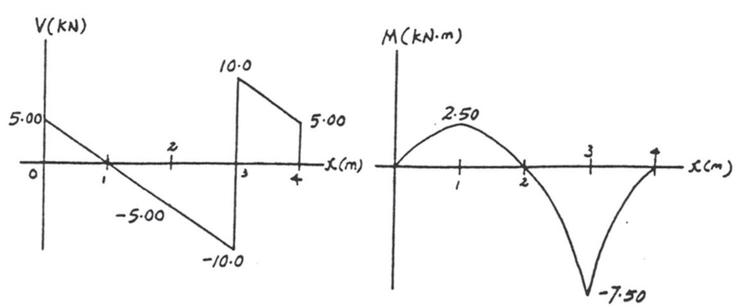
Allowable Bending Stress: The maximum moment is $M_{\text{max}} = 7.50 \text{ kN} \cdot \text{m}$ as indicated on the moment diagram. Applying the flexure formula,

$$\sigma_{\text{max}} = \sigma_{\text{allow}} = \frac{M_{\text{max}}\, c}{I}$$

$$150\left(10^6\right) = \frac{7.50(10^3)\left(\frac{a}{2}\right)}{\frac{1}{12}a^4}$$

$$a = 0.06694 \text{ m} = 66.9 \text{ mm}$$ **Ans.**





Ans:
$a = 66.9$ mm

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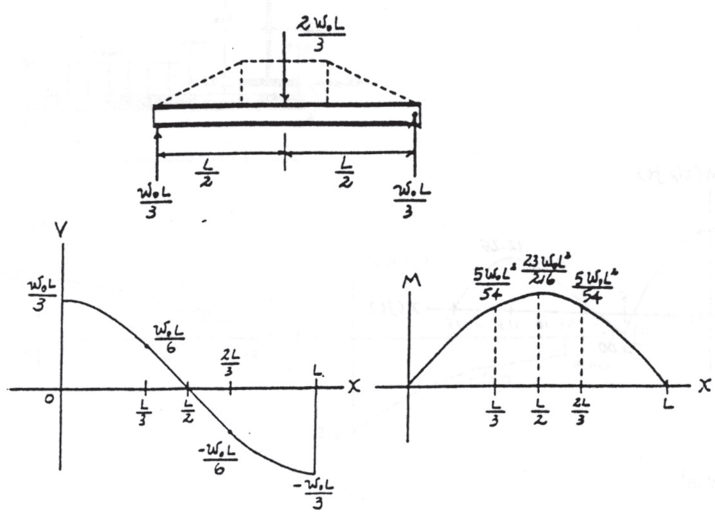
11–82.

If the beam in Prob. 11–28 has a rectangular cross section with a width b and a height h, determine the absolute maximum bending stress in the beam.

SOLUTION

Absolute Maximum Bending Stress: The maximum moment is $M_{\max} = \dfrac{23w_0 L^2}{216}$ as indicated on the moment diagram. Applying the flexure formula,

$$\sigma_{\max} = \frac{M_{\max}\, c}{I} = \frac{\frac{23w_0 L^2}{216}\left(\frac{h}{2}\right)}{\frac{1}{12}\, bh^3} = \frac{23w_0 L^2}{36bh^2} \qquad \textbf{Ans.}$$



Ans:

$$\sigma_{\max} = \frac{23w_0 L^2}{36\, bh^2}$$

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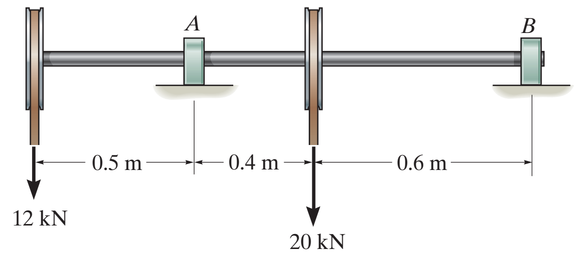
11–83.

Determine the absolute maximum bending stress in the 80-mm-diameter shaft which is subjected to the concentrated forces. There is a journal bearing at A and a thrust bearing at B.



SOLUTION

The FBD of the shaft is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $\left| M_{\max} \right| = 6\ \text{kN} \cdot \text{m}$.

The moment of inertia of the cross section about the neutral axis is

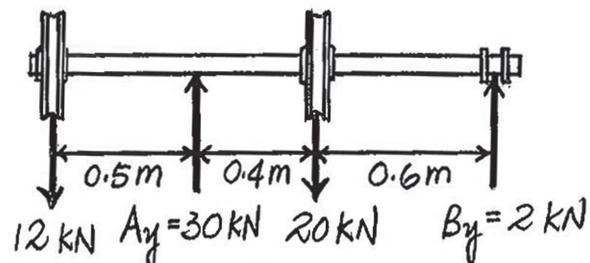
$$I = \frac{\pi}{4}(0.04^4) = 0.64(10^{-6})\pi\ \text{m}^4$$

Here, $c = 0.04$ m. Thus,

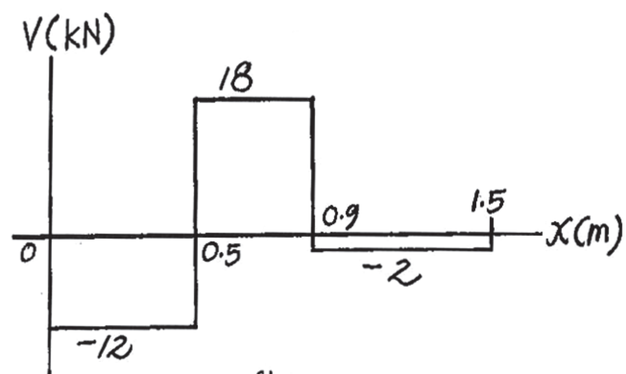
$$\sigma_{\max} = \frac{M_{\max}\,c}{I} = \frac{6(10^3)(0.04)}{0.64(10^{-6})\pi}$$

$$= 119.37(10^6)\ \text{Pa}$$

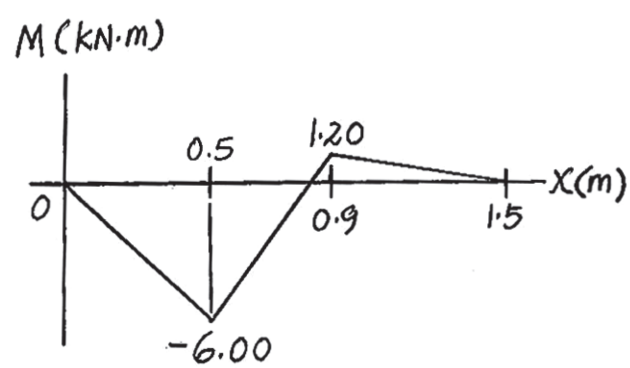
$$= 119\ \text{MPa} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$\sigma_{\max} = 119$ MPa

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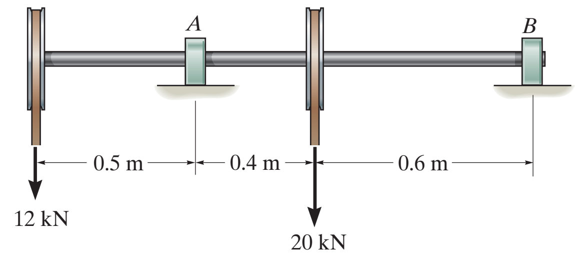
***11–84.**

Determine, to the nearest millimeter, the smallest allowable diameter of the shaft which is subjected to the concentrated forces. There is a journal bearing at A and a thrust bearing at B. The allowable bending stress is $\sigma_{allow} = 150$ MPa.



SOLUTION

The FBD of the shaft is shown in Fig. *a*.

The shear and moment diagrams are shown in Fig. *b* and *c*, respectively. As indicated on the moment diagram, $|M_{max}| = 6\ \text{kN} \cdot \text{m}$.

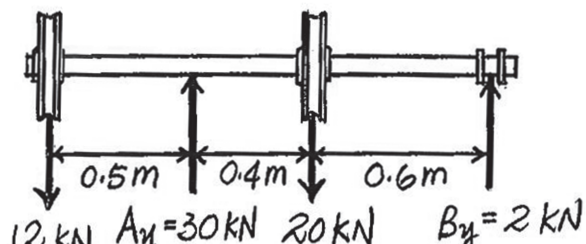
The moment of inertia of the cross section about the neutral axis is

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

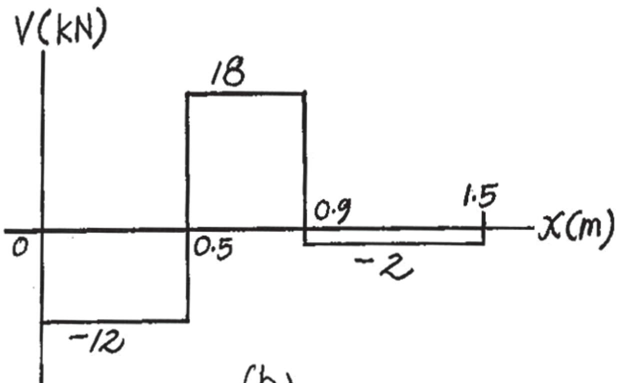
Here, $c = d/2$. Thus,

$$\sigma_{allow} = \frac{M_{max}\, c}{I}; \qquad 150(10^6) = \frac{6(10^3)(d/2)}{\pi d^4/64}$$

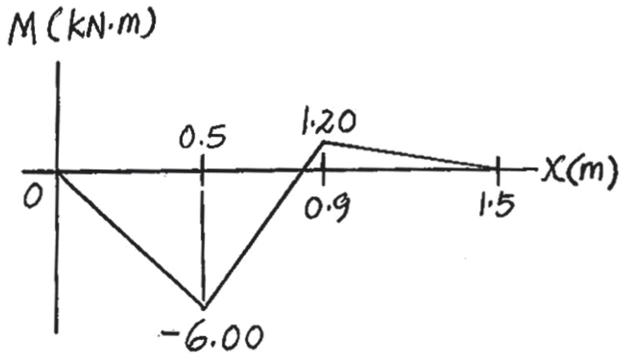
$$d = 0.07413 \text{ m} = 74.13 \text{ mm} = 75 \text{ mm} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$d = 75$ mm

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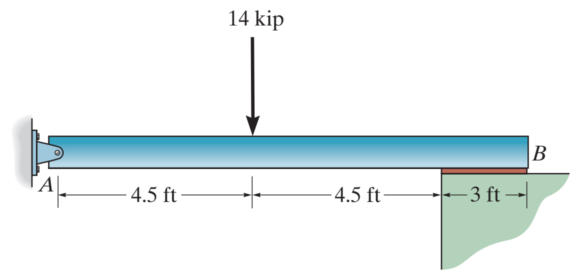
11–85.

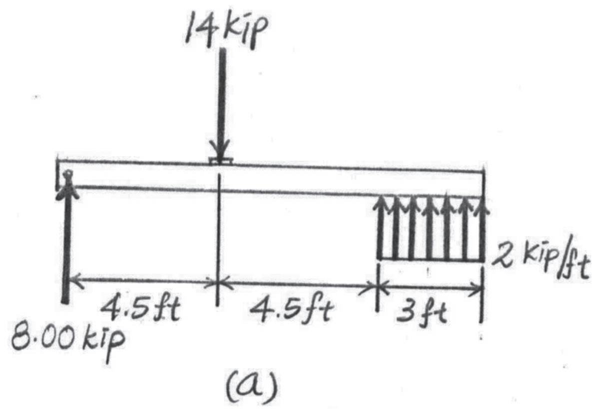
Determine the absolute maximum bending stress in the beam, assuming that the support at B exerts a uniformly distributed reaction on the beam. The cross section is rectangular with a base of 3 in. and height of 6 in.

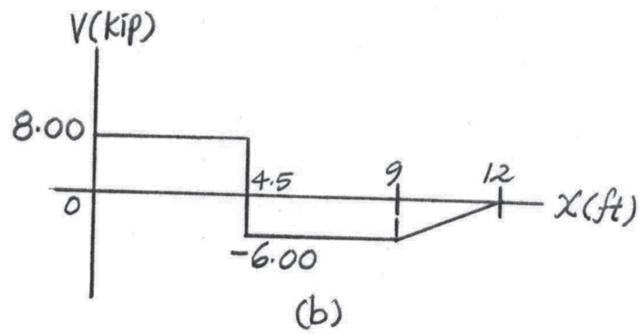


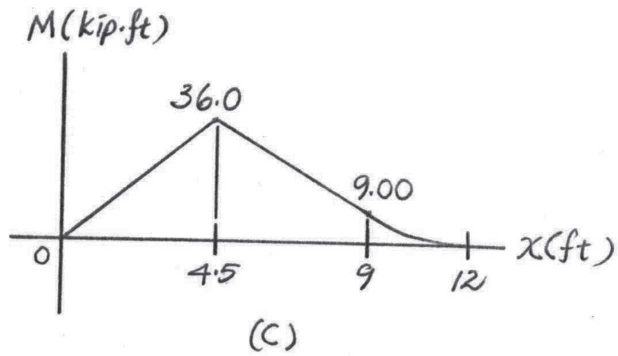
SOLUTION

Absolute Maximum Bending Stress: The support reactions, shear diagram, and moment diagram are shown in Figs. *a*, *b*, and *c*, respectively. From the moment diagram, the maximum moment is $M_{max} = 360$ kip · ft, which occurs at $x = 4.5$ ft. Applying the flexure formula,

$$\sigma_{\substack{abs\\max}} = \frac{M_{max}C}{I} = \frac{36.0(12)(3)}{\frac{1}{12}(3)(6^3)} = 24.0 \text{ ksi} \qquad \textbf{Ans.}$$







Ans:
$\sigma_{\substack{abs\\max}} = 24.0$ ksi

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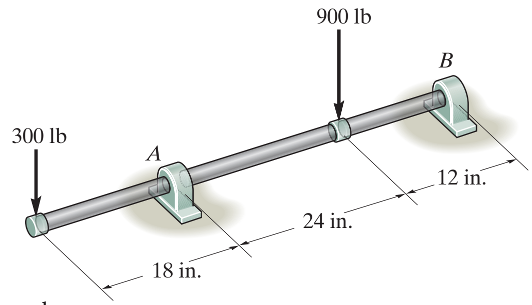
11–86.

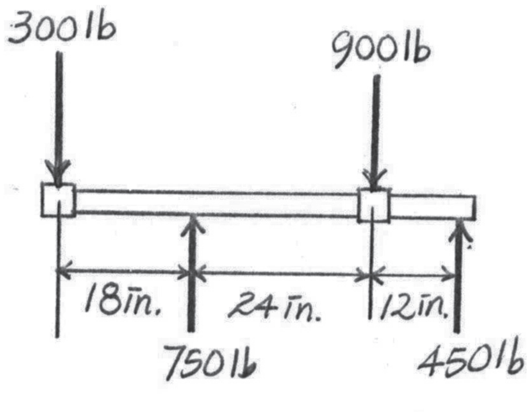
Determine the absolute maximum bending stress in the 2-in.-diameter shaft. There is a journal bearing at A and a thrust bearing at B.



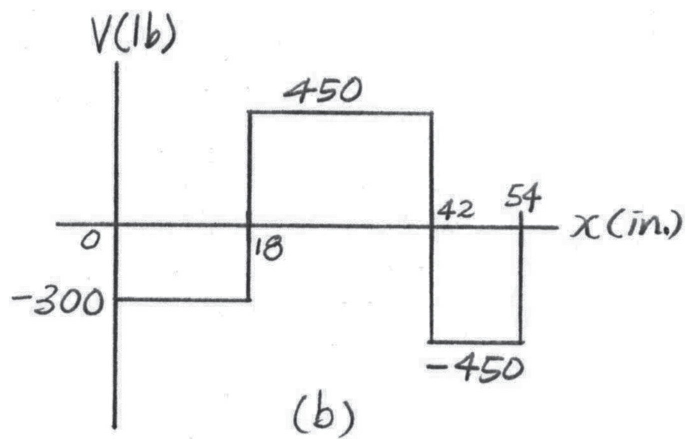
SOLUTION

Absolute Maximum Bending Stress: The support reactions, shear diagram, and moment diagram are shown in Figs. *a*, *b*, and *c*, respectively. From the moment diagram, the maximum moment is $M_{max} = 5400$ lb · in., which occurs at $x = 18$ in. and $x = 42$ in. Applying the flexure formula,

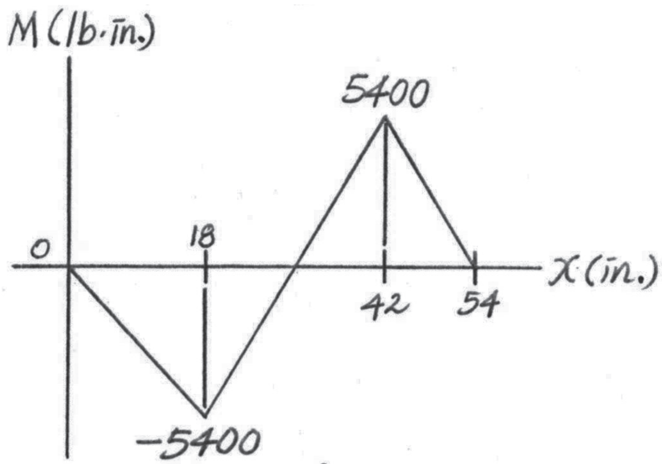
$$\sigma_{\substack{abs \\ max}} = \frac{M_{max}C}{I} = \frac{5400(1)}{\frac{\pi}{4}(1^4)} = 6875 \text{ psi} = 6.88 \text{ ksi} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$\sigma_{\substack{abs \\ max}} = 6.88$ ksi

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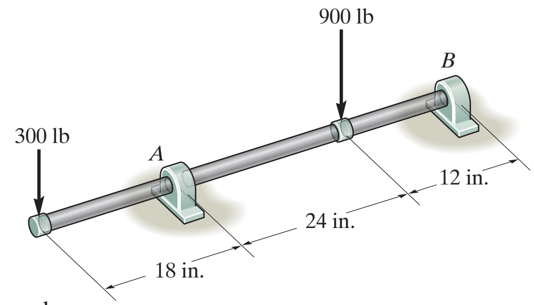
11–87.

Determine the smallest diameter of the shaft to the nearest $\frac{1}{8}$ in. There is a journal bearing at A and a thrust bearing at B. The allowable bending stress is $\sigma_{\text{allow}} = 22$ ksi.



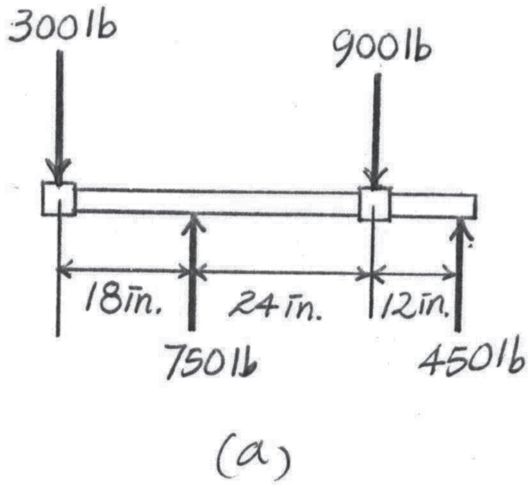
SOLUTION

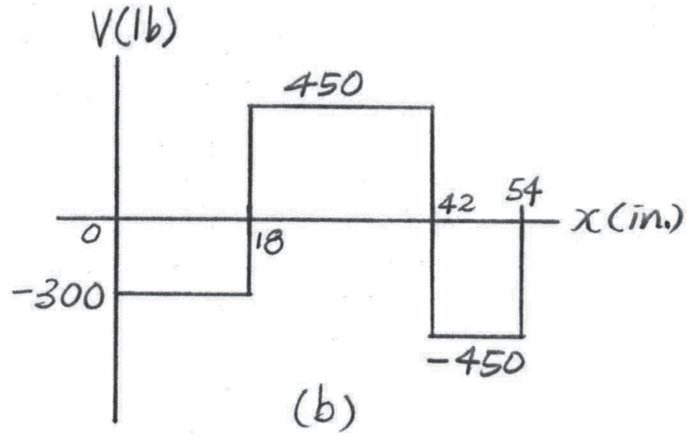
Absolute Maximum Bending Stress: The support reactions, shear diagram and moment diagram are shown in Figs. *a*, *b* and *c*, respectively. From the moment diagram, the maximum moment is $M_{\max} = 5400$ lb·in, which occurs at $x = 18$ in. and $x = 42$ in. Applying the flexure formula,

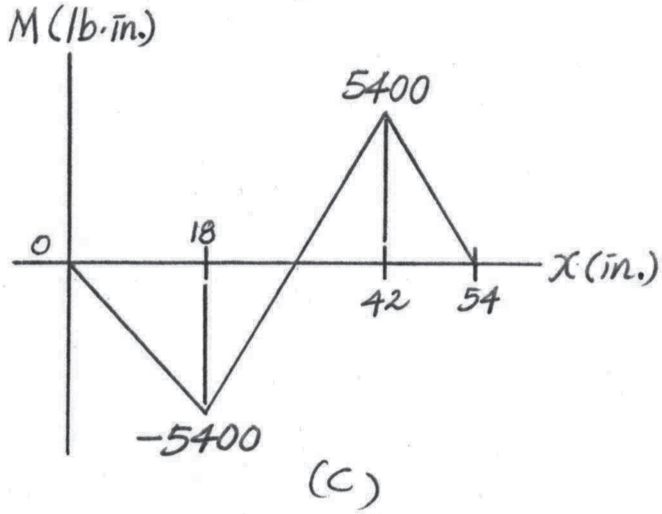
$$\sigma_{\text{allow}} = \frac{M_{\max} C}{I}; \qquad 22(10^3) = \frac{5400\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 1.357 \text{ in.}$$

Use $d = 1\frac{3}{8}$ in. **Ans.**







Ans:
Use $d = 1\frac{3}{8}$ in.

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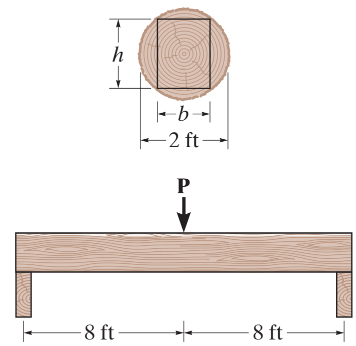
***11–88.**

A log that is 2 ft in diameter is to be cut into a rectangular section for use as a simply supported beam. If the allowable bending stress is $\sigma_{allow} = 8$ ksi, determine the required width b and height h of the beam that will support the largest load possible. What is this load?

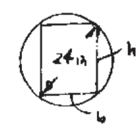


SOLUTION

$$(24)^2 = b^2 + h^2$$

$$M_{max} = \frac{P}{2}(8)(12) = 48P$$

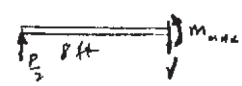
$$\sigma_{allow} = \frac{Mc}{I} = \frac{M_{max}(\frac{h}{2})}{\frac{1}{12}(b)(h)^3}$$



$$\sigma_{allow} = \frac{6\,M_{max}}{bh^2}$$

$$bh^2 = \frac{6}{8000}(48\,P)$$

$$b(24)^2 - b^3 = 0.036\,P$$



$$(24)^2 - 3b^2 = 0.036\,\frac{dP}{db} = 0$$

$b = 13.856$ in.

Thus, from the above equations,

$b = 13.9$ in. **Ans.**

$h = 19.6$ in. **Ans.**

$P = 148$ kip **Ans.**

Ans:
$b = 13.9$ in.,
$h = 19.6$ in.,
$P = 148$ kip

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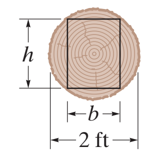
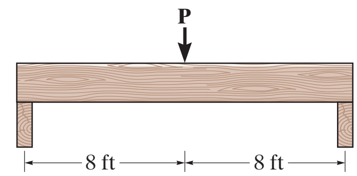
11–89.

A log that is 2 ft in diameter is to be cut into a rectangular section for use as a simply supported beam. If the allowable bending stress is $\sigma_{allow} = 8$ ksi, determine the largest load P that can be supported if the width of the beam is $b = 8$ in.





SOLUTION

$$24^2 = h^2 + 8^2$$

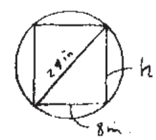
$$h = 22.63 \text{ in.}$$

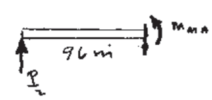
$$M_{max} = \frac{P}{2}(96) = 48\,P$$

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$8(10^3) = \frac{48\,P\left(\frac{22.63}{2}\right)}{\frac{1}{12}(8)(22.63)^3}$$

$P = 114$ kip **Ans.**





Ans:
$P = 114$ kip

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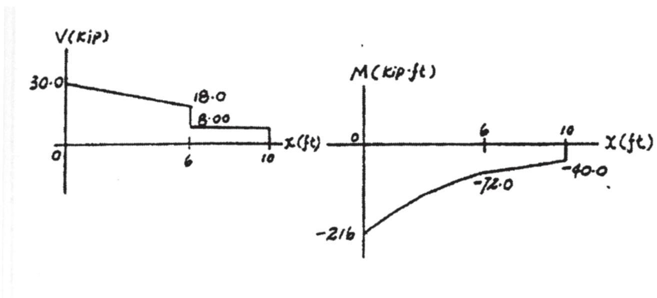
11–90.

If the beam in Prob. 11–19 has a rectangular cross section with a width of 8 in. and a height of 16 in., determine the absolute maximum bending stress in the beam.

SOLUTION

Absolute Maximum Bending Stress: The maximum moment is $M_{\max} = 216\ \text{kip}\cdot\text{ft}$ as indicated on the moment diagram. Applying the flexure formula,

$$\sigma_{\max} = \frac{M_{\max}\,c}{I} = \frac{216(12)(8)}{\frac{1}{12}(8)(16^3)} = 7.59\ \text{ksi} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{\max} = 7.59$ ksi

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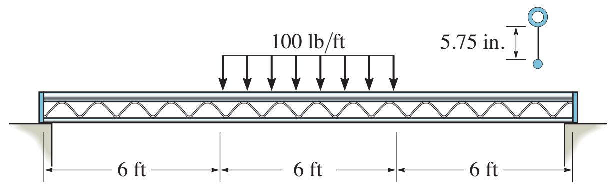
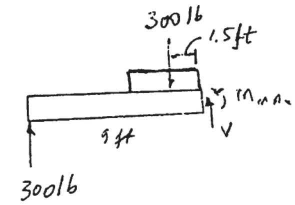
11–91.

The simply supported truss is subjected to the central distributed load. Neglect the effect of the diagonal lacing and determine the absolute maximum bending stress in the truss. The top member is a pipe having an outer diameter of 1 in. and thickness of $\frac{3}{16}$ in., and the bottom member is a solid rod having a diameter of $\frac{1}{2}$ in.



SOLUTION



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0 + (6.50)(0.4786)}{0.4786 + 0.19635} = 4.6091 \text{ in.}$$

$$I = \left[\frac{1}{4}\pi(0.5)^4 - \frac{1}{4}\pi(0.3125)^4\right] + 0.4786(6.50 - 4.6091)^2 + \frac{1}{4}\pi(0.25)^4 + 0.19635(4.6091)^2 = 5.9271 \text{ in}^4$$

$$M_{\max} = 300(9 - 1.5)(12) = 27\,000 \text{ lb} \cdot \text{in.}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{27\,000(4.6091 + 0.25)}{5.9271}$$

$$= 22.1 \text{ ksi}$$ **Ans.**

Ans:
$\sigma_{\max} = 22.1$ ksi

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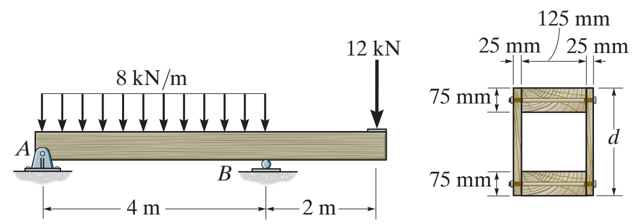
***11–92.**

If $d = 450$ mm, determine the absolute maximum bending stress in the overhanging beam.



SOLUTION

Support Reactions: Shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, $M_{max} = 24\,\text{kN}\cdot\text{m}$.

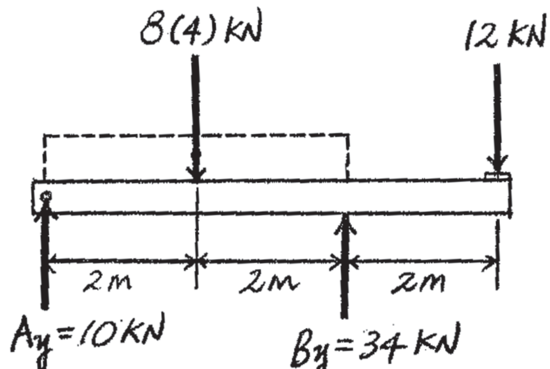
Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.175)(0.45^3) - \frac{1}{12}(0.125)(0.3^3)$$

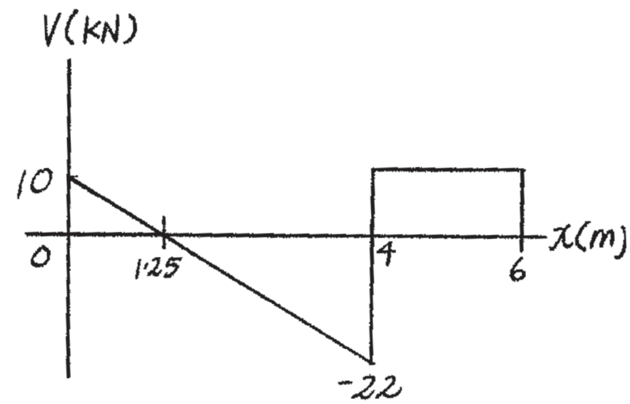
$$= 1.0477(10^{-3})\ \text{m}^4$$

Absolute Maximum Bending Stress: Here, $c = \dfrac{0.45}{2} = 0.225$ m.

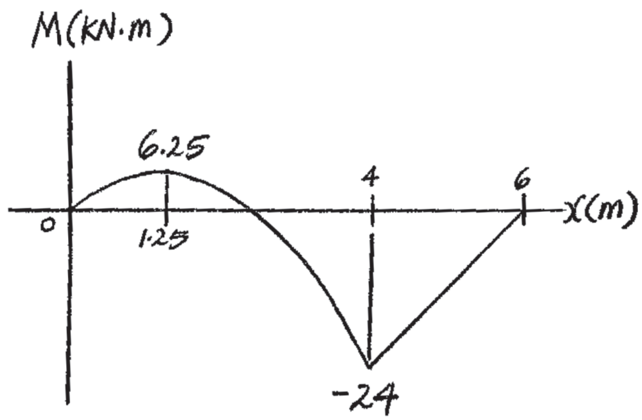
$$\sigma_{max} = \frac{M_{max}\,c}{I} = \frac{24(10^3)(0.225)}{1.0477(10^{-3})} = 5.15\ \text{MPa} \qquad \textbf{Ans.}$$



(a)



(b)



(C)

Ans:
$\sigma_{max} = 5.15$ MPa

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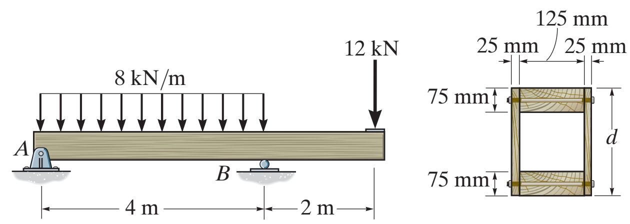
11–93.

If the allowable bending stress is $\sigma_{allow} = 6$ MPa, determine the minimum dimension d of the beam's cross-sectional area to the nearest mm.



SOLUTION

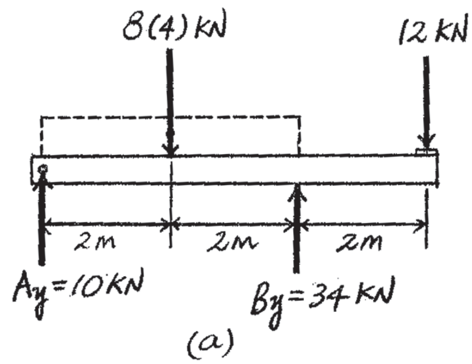
Support Reactions: Shown on the free-body diagram of the beam, Fig. *a*.

Maximum Moment: The shear and moment diagrams are shown in Figs. *b* and *c*. As indicated on the moment diagram, $M_{max} = 24\,\text{kN}\cdot\text{m}$.

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.175)d^3 - \frac{1}{12}(0.125)(d - 0.15)^3$$

$$= 4.1667(10^{-3})d^3 + 4.6875(10^{-3})d^2 - 0.703125(10^{-3})d + 35.15625(10^{-6})$$



Absolute Maximum Bending Stress: Here, $c = \dfrac{d}{2}$.

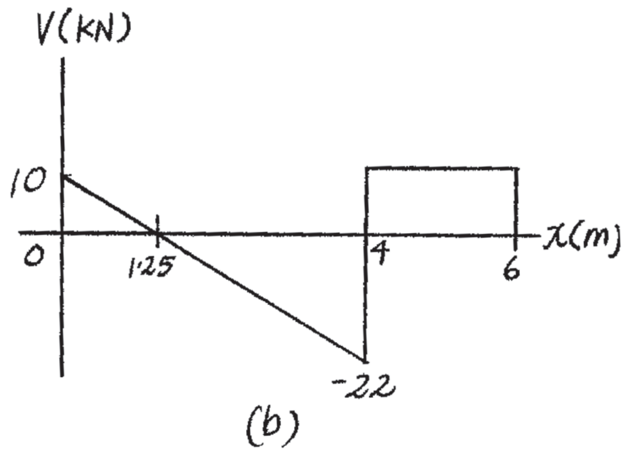
$$\sigma_{allow} = \frac{Mc}{I};$$

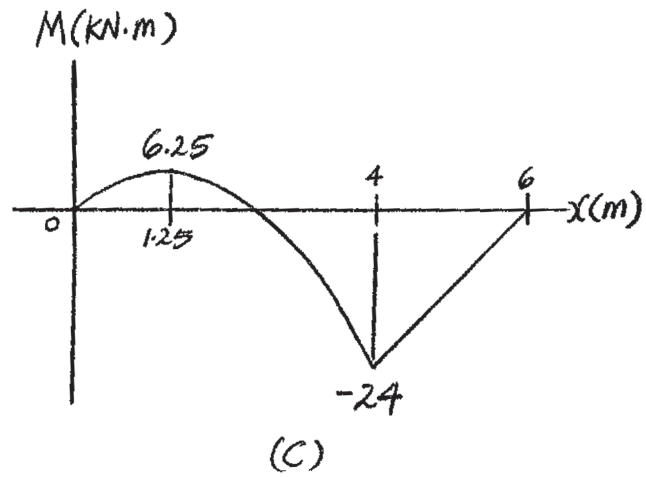
$$6(10^6) = \frac{24(10^3)\frac{d}{2}}{4.1667(10^{-3})d^3 + 4.6875(10^{-3})d^2 - 0.703125(10^{-3})d + 35.15625(10^{-6})}$$

$$4.1667(10^{-3})d^3 + 4.6875(10^{-3})d^2 - 2.703125(10^{-3})d + 35.15625(10^{-6}) = 0$$

Solving,

$$d = 0.4094\text{ m} = 410\text{ mm}$$ **Ans.**





Ans:
$d = 410$ mm

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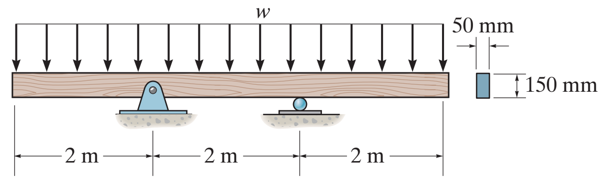
11–94.

The beam has a rectangular cross section as shown. Determine the largest intensity w of the uniform distributed load so that the bending stress in the beam does not exceed $\sigma_{max} = 10$ MPa.

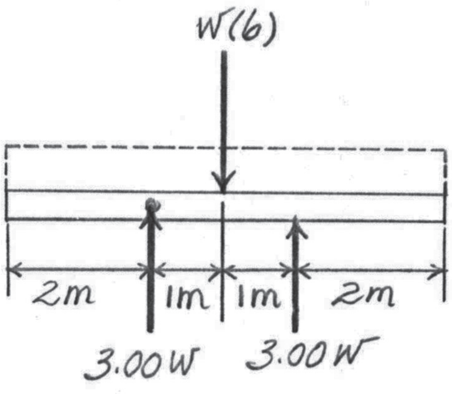


SOLUTION

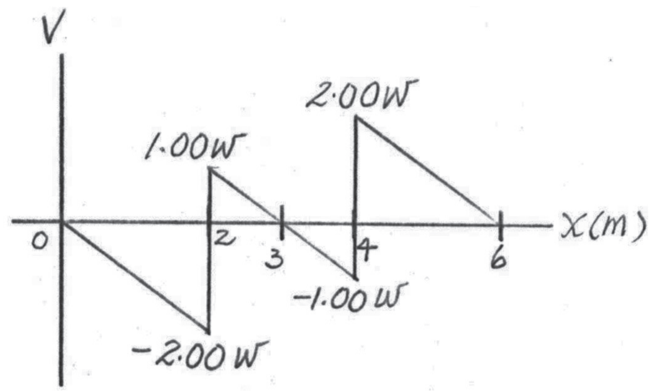
Absolute Maximum Bending Stress: The support reactions, shear diagram, and moment diagrams are shown in Figs. *a*, *b,* and *c*, respectively. From the moment diagram, the maximum moment is $M_{max} = 2.00\,w$, which occurs at the pin support and roller support. Applying the flexure formula,

$$\sigma_{\substack{abs\\max}} = \frac{M_{max}C}{I}; \qquad 10\left(10^6\right) = \frac{(2.00\,w)(0.075)}{\frac{1}{12}(0.05)\left(0.15^3\right)}$$

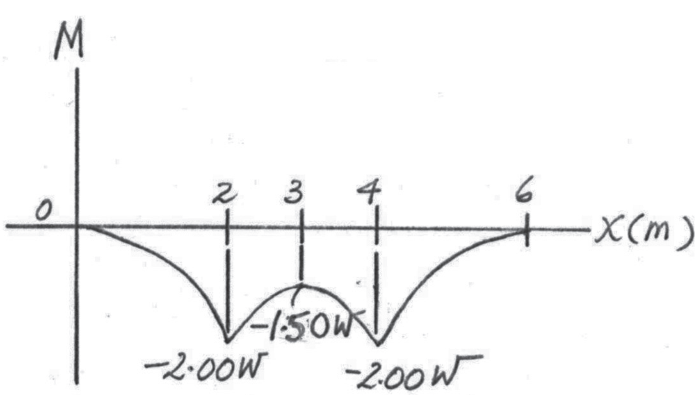
$$w = 937.5 \text{ N/m} \qquad \textbf{Ans.}$$



(a)



(b)



(c)

Ans:
$w = 937.5$ N/m

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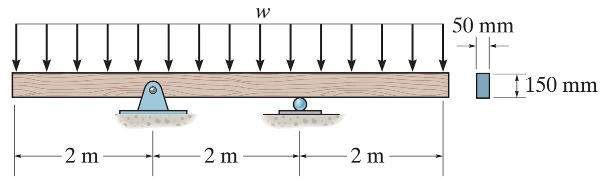
11–95.

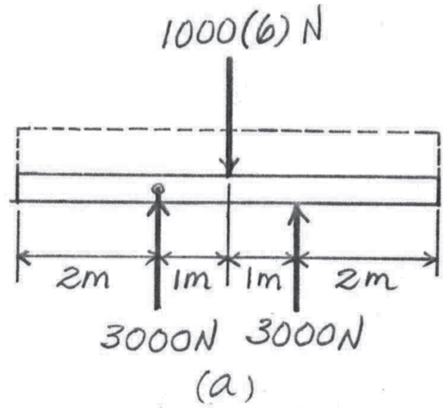
The beam has the rectangular cross section shown. If $w = 1\ \text{kN/m}$, determine the maximum bending stress in the beam. Sketch the stress distribution acting over the cross section.



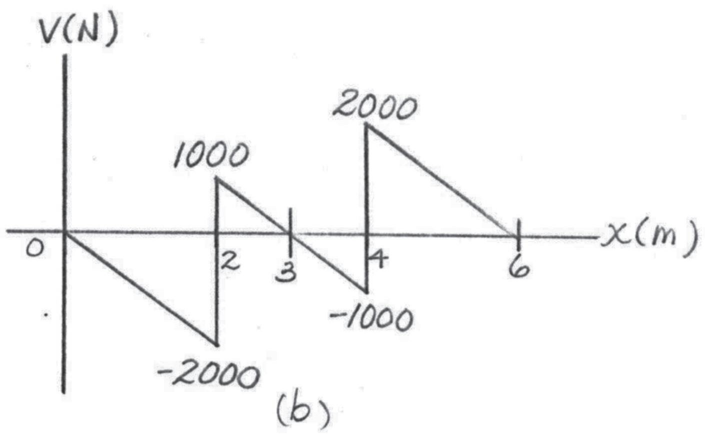
SOLUTION

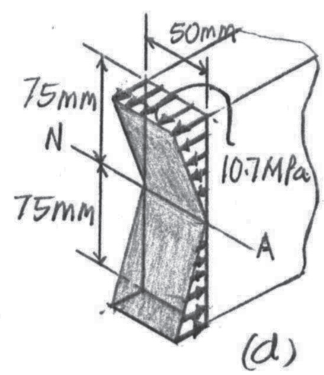
Absolute Maximum Bending Stress: The support reactions, shear diagram, and moment diagram are shown in Figs. *a*, *b,* and *c*, respectively. From the moment diagram, the maximum moment is $M_{max} = 2000\ \text{N}\cdot\text{m}$, which occurs at the supports. Applying the flexure formula,

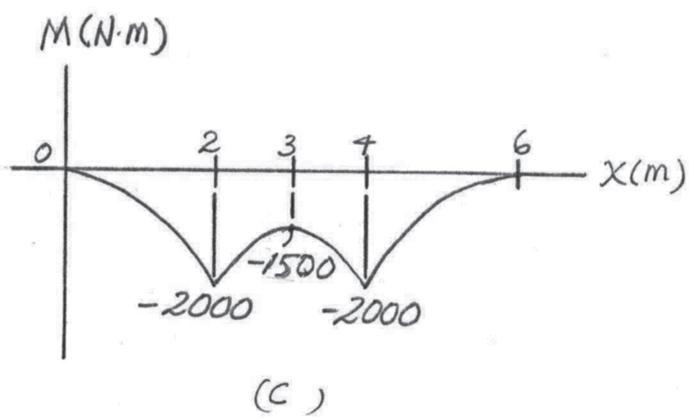
$$\sigma_{\substack{abs\\max}} = \frac{M_{max}C}{I} = \frac{2000(0.075)}{\frac{1}{12}(0.05)(0.15^3)} = 10.67(10^6)\ \text{MPa} = 10.7\ \text{MPa} \qquad \textbf{Ans.}$$



Using this result, the bending stress distribution on the beam's cross-section shown in Fig. *d* can be sketched.







Ans:
$\sigma_{\substack{abs\\max}} = 10.7\ \text{MPa}$

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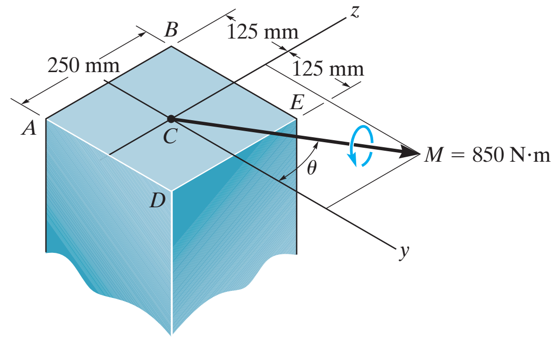
***11–96.**

The member has a square cross section and is subjected to the moment $M = 850\ \text{N}\cdot\text{m}$. Determine the stress at each corner and sketch the stress distribution. Set $\theta = 45°$.

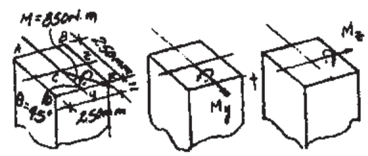


SOLUTION

$M_y = 850 \cos 45° = 601.04\ \text{N}\cdot\text{m}$

$M_z = 850 \sin 45° = 601.04\ \text{N}\cdot\text{m}$

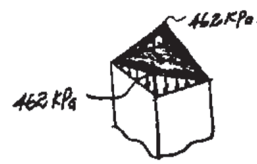
$$I_z = I_y = \frac{1}{12}(0.25)(0.25)^3 = 0.3255208(10^{-3})\ \text{m}^4$$



$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{601.04(-0.125)}{0.3255208(10^{-3})} + \frac{601.04(-0.125)}{0.3255208(10^{-3})} = 0$$ **Ans.**

$$\sigma_B = -\frac{601.04(-0.125)}{0.3255208(10^{-3})} + \frac{601.04(0.125)}{0.3255208(10^{-3})} = 462\ \text{kPa}$$ **Ans.**



$$\sigma_D = -\frac{601.04(0.125)}{0.3255208(10^{-3})} + \frac{601.04(-0.125)}{0.3255208(10^{-3})} = -462\ \text{kPa}$$ **Ans.**

$$\sigma_E = -\frac{601.04(0.125)}{0.3255208(10^{-3})} + \frac{601.04(0.125)}{0.3255208(10^{-3})} = 0$$ **Ans.**

The negative sign indicates compressive stress.

Ans:
$\sigma_A = 0$,
$\sigma_B = 462\ \text{kPa}$,
$\sigma_D = -462\ \text{kPa}$,
$\sigma_E = 0$

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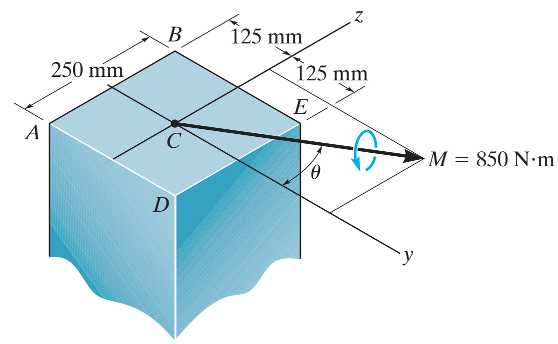
11–97.

The member has a square cross section and is subjected to the moment $M = 850\ \mathrm{N \cdot m}$ as shown. Determine the stress at each corner and sketch the stress distribution. Set $\theta = 30°$.

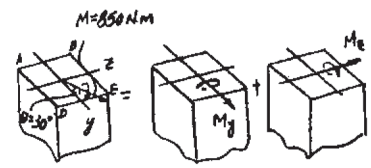


SOLUTION

$$M_y = 850 \cos 30° = 736.12\ \mathrm{N \cdot m}$$

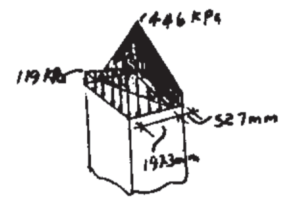
$$M_z = 850 \sin 30° = 425\ \mathrm{N \cdot m}$$

$$I_z = I_y = \frac{1}{12}(0.25)(0.25)^3 = 0.3255208(10^{-3})\ \mathrm{m}^4$$



$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{425(-0.125)}{0.3255208(10^{-3})} + \frac{736.12(-0.125)}{0.3255208(10^{-3})} = -119\ \mathrm{kPa}$$ **Ans.**



$$\sigma_B = -\frac{425(-0.125)}{0.3255208(10^{-3})} + \frac{736.12(0.125)}{0.3255208(10^{-3})} = 446\ \mathrm{kPa}$$ **Ans.**

$$\sigma_D = -\frac{425(0.125)}{0.3255208(10^{-3})} + \frac{736.12(-0.125)}{0.3255208(10^{-3})} = -446\ \mathrm{kPa}$$ **Ans.**

$$\sigma_E = -\frac{425(0.125)}{0.3255208(10^{-3})} + \frac{736.12(0.125)}{0.3255208(10^{-3})} = 119\ \mathrm{kPa}$$ **Ans.**

The negative signs indicate compressive stress.

Ans:
$\sigma_A = -119$ kPa,
$\sigma_B = 446$ kPa,
$\sigma_D = -446$ kPa,
$\sigma_E = 119$ kPa

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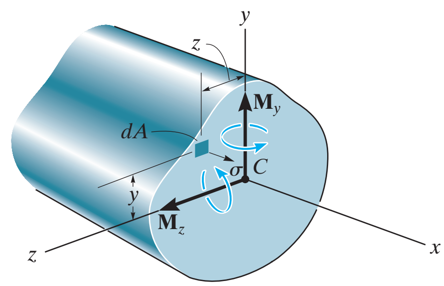
11–98.

Consider the general case of a prismatic beam subjected to bending-moment components $\mathbf{M}_y$ and $\mathbf{M}_z$ as shown, when the x, y, z axes pass through the centroid of the cross section. If the material is linear elastic, the normal stress in the beam is a linear function of position such that $\sigma = a + by + cz$. Using the equilibrium conditions $0 = \int_A \sigma\, dA$, $M_y = \int_A z\sigma\, dA$, $M_z = \int_A -y\sigma\, dA$, determine the constants a, b, and c, and show that the normal stress can be determined from the equation $\sigma = [-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z]/(I_y I_z - I_{yz}{}^2)$, where the moments and products of inertia are defined in Appendix A.



SOLUTION

Equilibrium Condition: $\sigma_x = a + by + cz$

$$0 = \int_A \sigma_x\, dA$$

$$0 = \int_A (a + by + cz)\, dA$$

$$0 = a\int_A dA + b\int_A y\, dA + c\int_A z\, dA \qquad \textbf{(1)}$$

$$M_y = \int_A z\,\sigma_x\, dA$$

$$= \int_A z(a + by + cz)\, dA$$

$$= a\int_A z\, dA + b\int_A yz\, dA + c\int_A z^2\, dA \qquad \textbf{(2)}$$

$$M_z = \int_A -y\,\sigma_x\, dA$$

$$= \int_A -y(a + by + cz)\, dA$$

$$= -a\int_A y dA - b\int_A y^2\, dA - c\int_A yz\, dA \qquad \textbf{(3)}$$

Section Properties: The integrals are defined in Appendix A. Note that $\int_A y\, dA = \int_A z\, dA = 0$. Thus,

From Eq. (1), $\qquad Aa = 0$

From Eq. (2), $\qquad M_y = bI_{yz} + cI_y$

From Eq. (3), $\qquad M_z = -bI_z - cI_{yz}$

Solving for a, b, c:

$a = 0$ (Since $A \neq 0$) **Ans.**

$$b = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right) \qquad c = \frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2} \qquad \textbf{Ans.}$$

Thus, $\sigma_x = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\frac{M_y I_y + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z$ **(Q.E.D.)**

Ans:
$a = 0,$
$b = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right),$
$c = \frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}$

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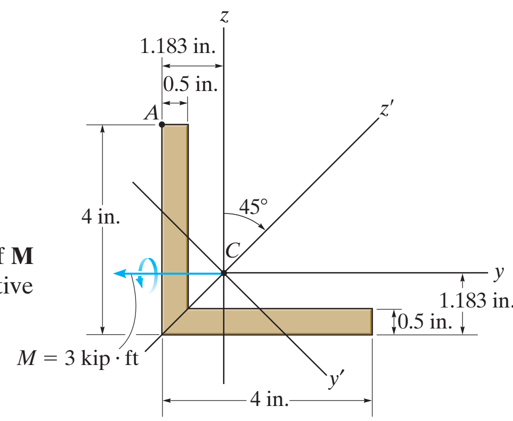
11–99.

Determine the bending stress at point A of the beam, and the orientation of the neutral axis. Using the method in Appendix A, the principal moments of inertia of the cross section are $I_{z'} = 8.828\ \text{in}^4$ and $I_{y'} = 2.295\ \text{in}^4$, where z' and y' are the principal axes. Solve the problem using Eq. 11–17.



SOLUTION

Internal Moment Components: Referring to Fig. *a*, the y' and z' components of **M** are negative since they are directed towards the negative sense of their respective axes. Thus,

Section Properties: Referring to the geometry shown in Fig. *b*,

$$z'_A = 2.817 \cos 45° - 1.183 \sin 45° = 1.155\text{ in.}$$

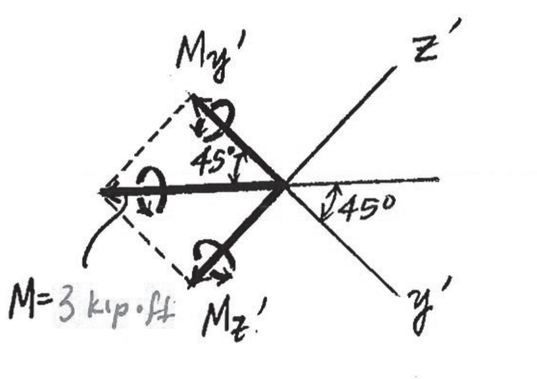
$$y'_A = -(2.817 \sin 45° + 1.183 \cos 45°) = -2.828\text{ in.}$$

Bending Stress:

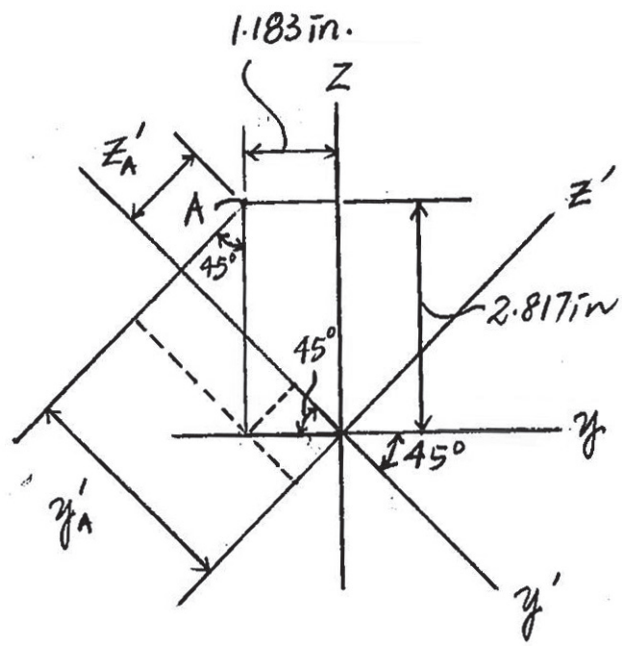
$$\sigma_A = -\frac{M_{z'}y'_A}{I_{z'}} + \frac{M_{y'}z'_A}{I_{y'}}$$

$$= -\frac{(-2.121)(12)(-2.828)}{8.828} + \frac{(-2.121)(12)(1.155)}{2.295}$$

$$= -20.97\text{ ksi} = 21.0\text{ ksi (C)} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:
$\sigma_A = 21.0$ ksi (C)

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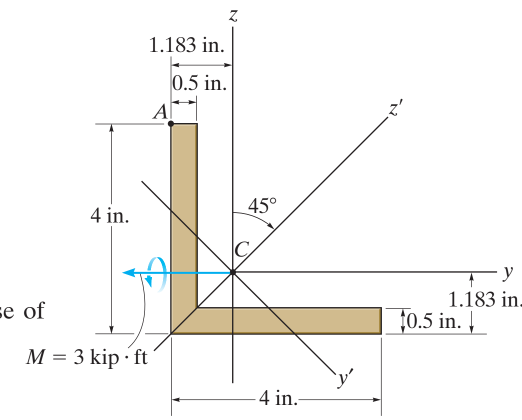
***11–100.**

Determine the bending stress at point A of the beam using the result obtained in Prob. 11–98. The moments of inertia of the cross-sectional area about the z and y axes are $I_z = I_y = 5.561$ in^4 and the product of inertia of the cross sectional area with respect to the z and y axes is $I_{yz} = -3.267$ in^4. (See Appendix A.)



SOLUTION

Internal Moment Components: Since **M** is directed towards the negative sense of the y axis, its y component is negative and it has no z component. Thus,

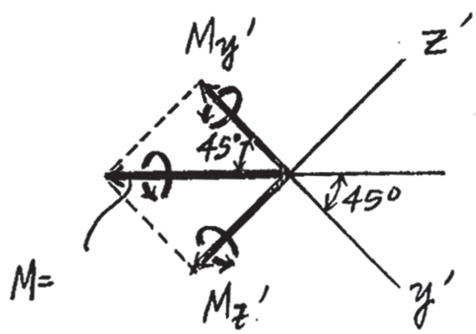
$$M_y = -3 \text{ kip} \cdot \text{ft} \qquad M_z = 0$$

Bending Stress:

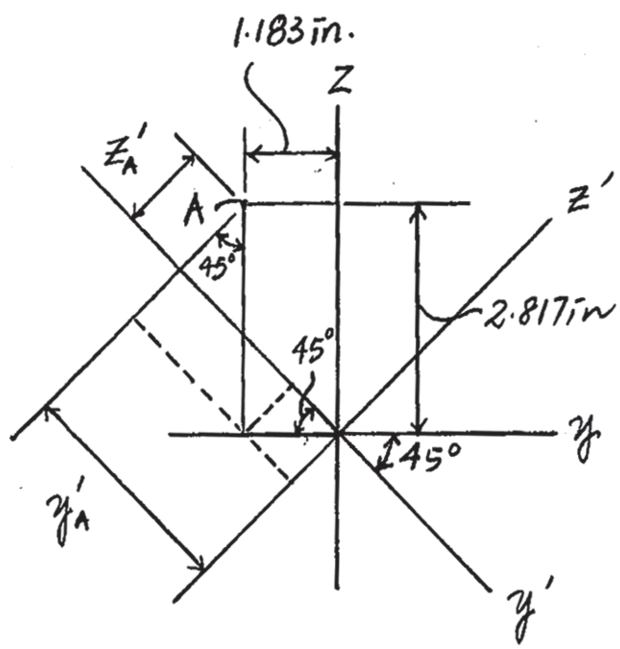
$$\sigma_A = \frac{-(M_z I_y + M_y I_{yz})y_A + (M_y I_z + M_z I_{yz})z_A}{I_y I_z - I_{yz}^2}$$

$$= \frac{-[0(5.561) + (-3)(12)(-3.267)](-1.183) + [-3(12)(5.561) + 0(-3.267)](2.817)}{5.561(5.561) - (-3.267)^2}$$

$$= -20.97 \text{ ksi} = 21.0 \text{ ksi}$$ **Ans.**



(a)



(b)

Ans:
$\sigma_A = 21.0$ ksi

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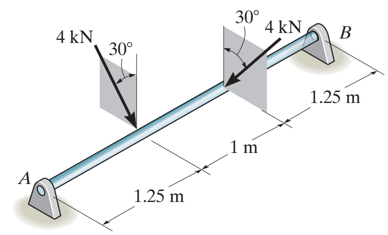
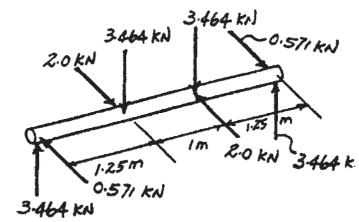
11–101.

The steel shaft is subjected to the two loads. If the journal bearings at A and B do not exert an axial force on the shaft, determine the required diameter of the shaft if the allowable bending stress is $\sigma_{allow} = 180$ MPa.



SOLUTION

Support Reactions: As shown on FBD.



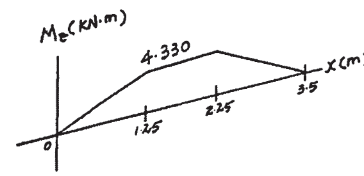
Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

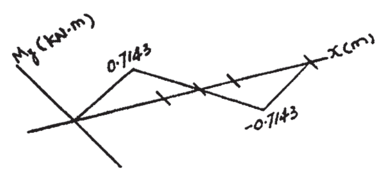
Allowable Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used for the design. The maximum resultant moment is $M_{max} = \sqrt{4.330^2 + 0.7143^2} = 4.389\ \text{kN}\cdot\text{m}$. Applying the flexure formula,

$$\sigma_{max} = \sigma_{allow} = \frac{M_{max}c}{I}$$

$$180(10^6) = \frac{4.389(10^3)\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$d = 0.06286$ m $= 62.9$ mm **Ans.**





Ans:
$d = 62.9$ mm

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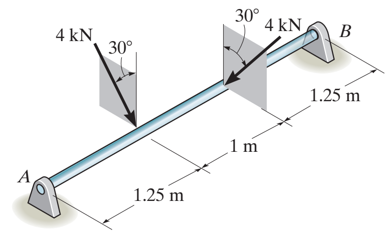
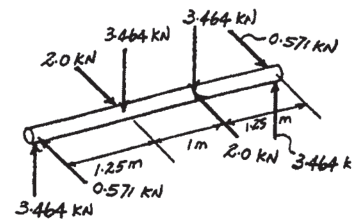
11–102.

The 65-mm-diameter steel shaft is subjected to the two loads If the journal bearings at A and B do not exert an axial force on the shaft, determine the absolute maximum bending stress developed in the shaft.



SOLUTION

Support Reactions: As shown on FBD.



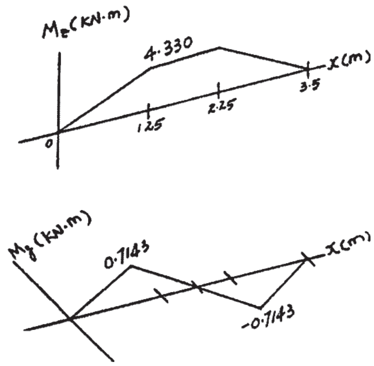
Internal Moment Components: The shaft is subjected to two bending moment components M_y and M_z. The moment diagram for each component is drawn.

Maximum Bending Stress: Since all the axes through the circle's center for a circular shaft are principal axes, then the resultant moment $M = \sqrt{M_y^2 + M_z^2}$ can be used to determine the maximum bending stress. The maximum resultant moment is $M_{\max} = \sqrt{4.330^2 + 0.7143^2} = 4.389\ \text{kN}\cdot\text{m}$. Applying the flexure formula,

$$\sigma_{\max} = \frac{M_{\max}c}{I}$$

$$= \frac{4.389(10^3)(0.0325)}{\frac{\pi}{4}(0.0325^4)}$$

$$= 163\ \text{MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{\max} = 163\ \text{MPa}$

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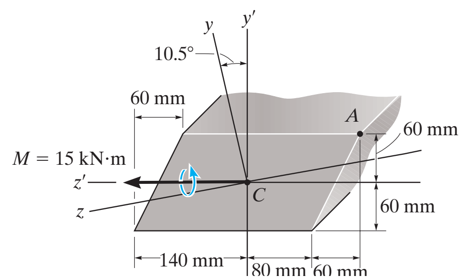
11–103.

For the section, $I_{z'} = 31.7(10^{-6})$ m^4, $I_{y'} = 114(10^{-6})$ m^4, $I_{y'z'} = -15.1(10^{-6})$ m^4. Using the techniques outlined in Appendix A, the member's cross-sectional area has principal moments of inertia of $I_z = 29.0(10^{-6})$ m^4 and $I_y = 117(10^{-6})$ m^4, calculated about the principal axes of inertia y and z, respectively. If the section is subjected to a moment $M = 15$ k · m, determine the stress at point A using Eq. 11–17.



SOLUTION

Internal Moment Components: The y and z components are

$$M_y = 15 \sin 10.5° = 2.7335 \text{ kN}\cdot\text{m} \qquad M_z = 15 \cos 10.5° = 14.7488 \text{ kN}\cdot\text{m}$$

Section Properties: Given that $I_y = 117\left(10^{-6}\right)$ m^4 and $I_z = 28.8\left(10^{-6}\right)$ m^4, the coordinates of point A with respect to the y and z axes are

$$y = 0.06 \cos 10.5° - 0.14 \sin 10.5° = 0.03348 \text{ m}$$

$$z = -(0.06 \sin 10.5° + 0.14 \cos 10.5°) = -0.14859 \text{ m}$$

Bending Stress: Applying the flexure formula for biaxial bending,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{14.7488\left(10^3\right)(0.03348)}{28.8\left(10^{-6}\right)} + \frac{2.7335\left(10^3\right)(-0.14869)}{117\left(10^{-6}\right)}$$

$$= -20.62\left(10^6\right) \text{ Pa}$$

$$= 20.6 \text{ MPa (C)}$$ **Ans.**

Ans:
$\sigma_A = 20.6$ MPa (C)

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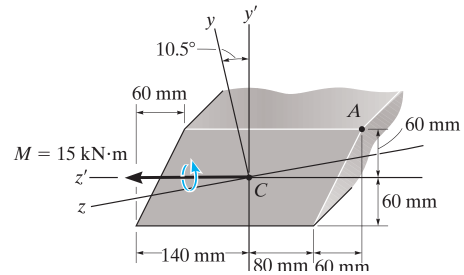
***11–104.**

Solve Prob. 11–103 using the equation developed in Prob. 11–98.



SOLUTION

Internal Moment Components: The y' and z' components are

$$M_{z'} = 15\ \text{kN}\cdot\text{m} \qquad M_{y'} = 0$$

Section Properties: Given that $I_{y'} = 114\left(10^{-6}\right)\ \text{m}^4$, $I_{z'} = 31.7\left(10^{-6}\right)\ \text{m}^4$ and $I_{y'z'} = -15.8\left(10^{-6}\right)\ \text{m}^4$, the coordinates of point A with respect to the y' and z axes are

$$y' = 0.06\ \text{m} \qquad z' = -0.14\ \text{m}$$

Bending Stress: Using the formula,

$$\sigma = \frac{-(M_{z'}I_{y'} + M_{y'}I_{y'z'})y' + (M_{y'}I_{z'} + M_{z'}I_{y'z'})z'}{I_{y'}I_{z'} - I_{y'z'}^2}$$

$$\sigma_A = \frac{-\left[15\left(10^3\right)(114)\left(10^{-6}\right) + 0\right](0.06) + \left[0 + 15\left(10^3\right)(-15.8)\left(10^{-6}\right)\right](-0.14)}{\left[114\left(10^{-6}\right)\right]\left[31.7\left(10^{-6}\right)\right] - \left[-15.8\left(10^{-6}\right)\right]^2}$$

$$= -20.64\left(10^6\right)\ \text{Pa}$$

$$= 20.6\ \text{MPa (C)}$$ **Ans.**

Ans:
$\sigma_A = 20.6$ MPa (C)

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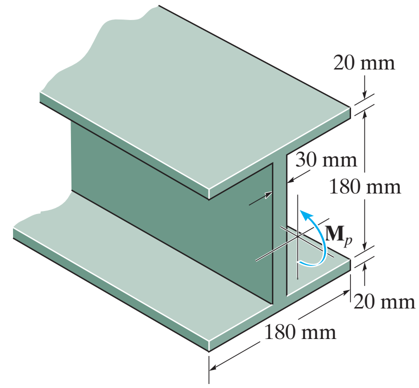
R11–1.

Determine the shape factor for the wide-flange beam.

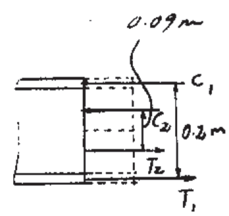


SOLUTION

$$I = \frac{1}{12}(0.18)(0.22^3) - \frac{1}{12}(0.15)(0.18^3)$$

$$= 86.82(10^{-6})\text{ m}^4$$

Plastic moment:



$$M_p = \sigma_Y(0.18)(0.02)(0.2) + \sigma_Y(0.09)(0.03)(0.09)$$

$$= 0.963(10^{-3})\sigma_Y$$

Shape Factor:

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(86.82)(10^{-6})}{0.11} = 0.789273(10^{-3})\sigma_Y$$

$$k = \frac{M_p}{M_Y} = \frac{0.963(10^{-3})\sigma_Y}{0.789273(10^{-3})\sigma_Y} = 1.22$$ **Ans.**

Ans:
$k = 1.22$

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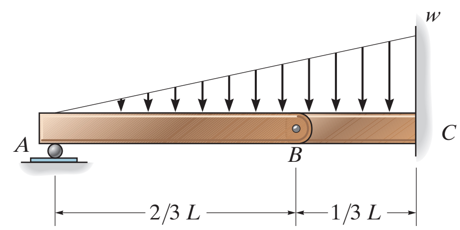
R11–2.

The compound beam consists of two segments that are pinned together at B. Draw the shear and moment diagrams if it supports the distributed loading shown.



SOLUTION

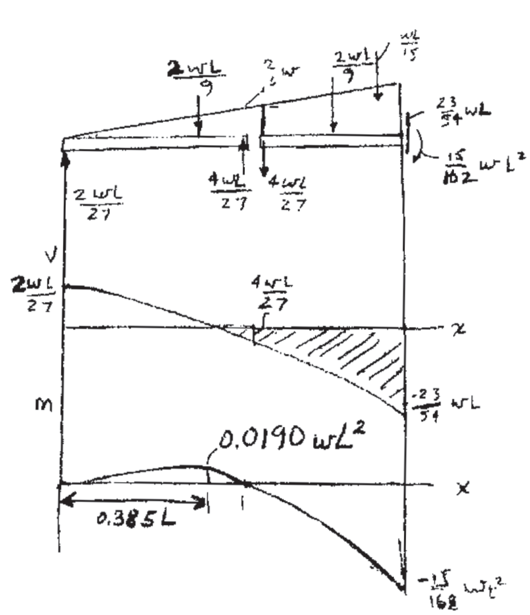
$$+\uparrow\Sigma F_y = 0; \qquad \frac{2wL}{27} - \frac{1}{2}\frac{w}{L}x^2 = 0$$

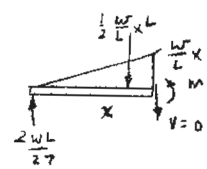
$$x = \sqrt{\frac{4}{27}}\,L = 0.385\ L$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{2}\frac{w}{L}(0.385L)^2\left(\frac{1}{3}\right)(0.385L) - \frac{2wL}{27}(0.385L) = 0$$

$$M = 0.0190\ wL^2$$

$M = 0.0190\ wL^2$





Ans:

$$V = \frac{2wL}{27} - \frac{w}{2L}x^2,$$

$$M = \frac{2wL}{27}x - \frac{w}{6L}x^3$$

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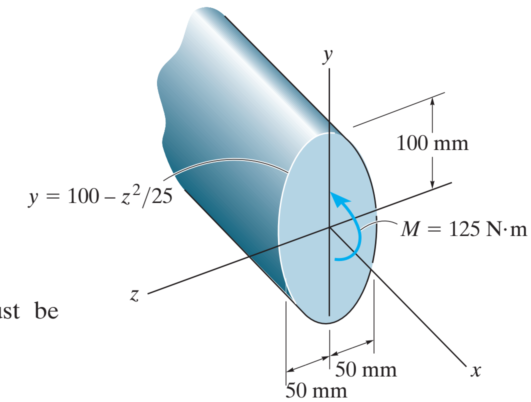
R11–3.

A shaft is made of a polymer having a parabolic upper and lower cross section. If it resists a moment of $M = 125\ \text{N}\cdot\text{m}$, determine the maximum bending stress in the material (a) using the flexure formula and (b) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area. *Hint*: The moment of inertia is determined using Eq. A–3 of Appendix A.



SOLUTION

Maximum Bending Stress: The moment of inertia about the y axis must be determined first in order to use the flexure formula.

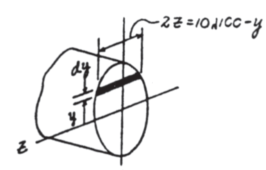
$$I = \int_A y^2\,dA$$

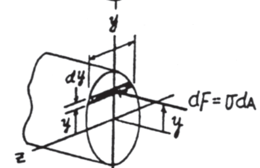
$$= 2\int_0^{100\text{ mm}} y^2\,(2z)\,dy$$

$$= 20\int_0^{100\text{ mm}} y^2\sqrt{100 - y}\,dy$$

$$= 20\left[-\frac{3}{2}y^2(100 - y)^{\frac{3}{2}} - \frac{8}{15}y(100 - y)^{\frac{5}{2}} - \frac{16}{105}(100 - y)^{\frac{7}{2}}\right]\Bigg|_0^{100}\ \text{mm}$$

$$= 30.4762\left(10^6\right)\ \text{mm}^4 = 30.4762\left(10^{-6}\right)\ \text{m}^4$$

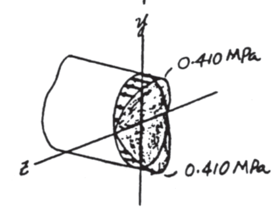




Thus,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{125(0.1)}{30.4762(10^{-6})} = 0.410\ \text{MPa}$$ **Ans.**

Maximum Bending Stress: Using integration,



$$dM = 2[y(\sigma\,dA)] = 2\left\{y\left[\left(\frac{\sigma_{\max}}{100}\right)y\right](2z\,dy)\right\}$$

$$M = \frac{\sigma_{\max}}{5}\int_0^{100\text{ mm}} y^2\sqrt{100 - y}\,dy$$

$$125\left(10^3\right) = \frac{\sigma_{\max}}{5}\left[-\frac{3}{2}y^2(100 - y)^{\frac{3}{2}} - \frac{8}{15}y(100 - y)^{\frac{5}{2}} - \frac{16}{105}(100 - y)^{\frac{7}{2}}\right]\Bigg|_0^{100}\ \text{mm}$$

$$125\left(10^3\right) = \frac{\sigma_{\max}}{5}(1.5238)\left(10^6\right)$$

$$\sigma_{\max} = 0.410\ \text{N/mm}^2 = 0.410\ \text{MPa}$$ **Ans.**

Ans:
$\sigma_{\max} = 0.410\ \text{MPa}$

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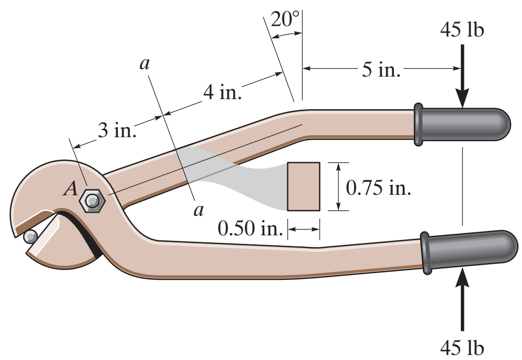
***R11–4.**

Determine the maximum bending stress in the handle of the cable cutter at section *a–a*. A force of 45 lb is applied to the handles.

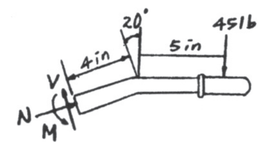


SOLUTION

$\circlearrowleft + \Sigma M = 0; \qquad M - 45(5 + 4\cos 20°) = 0$

$M = 394.14 \text{ lb} \cdot \text{in.}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{394.14(0.375)}{\frac{1}{12}(0.5)(0.75^3)} = 8.41 \text{ ksi}$$ **Ans.**



Ans:
$\sigma_{max} = 8.41$ ksi

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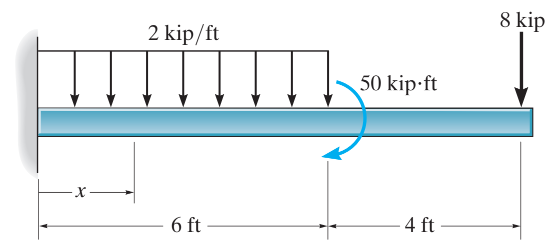
R11–5.

Determine the shear and moment in the beam as functions of x, where $0 \leq x < 6$ ft, then draw the shear and moment diagrams for the beam.



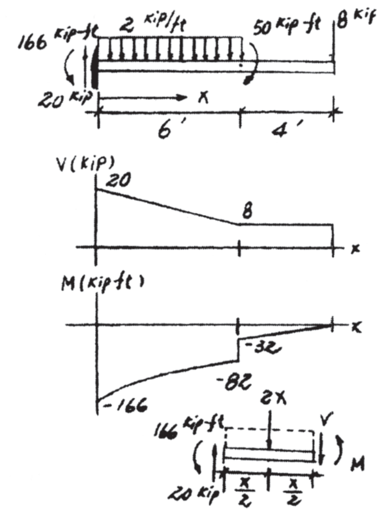
SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad 20 - 2x - V = 0$

$$V = 20 - 2x \qquad \textbf{Ans.}$$

$\circlearrowleft + \Sigma M_{NA} = 0; \qquad 20x - 166 - 2x\left(\frac{x}{2}\right) - M = 0$

$$M = -x^2 + 20x - 166 \qquad \textbf{Ans.}$$



Ans:
$V = 20 - 2x$,
$M = -x^2 + 20x - 166$

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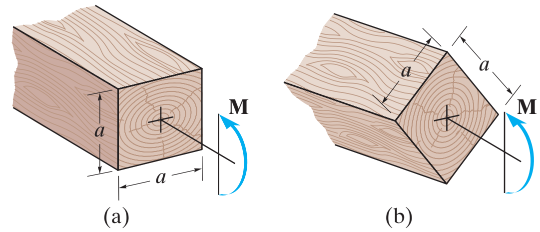
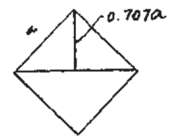
R11–6.

A wooden beam has a square cross section as shown. Determine which orientation of the beam provides the greatest strength at resisting the moment **M**. What is the difference in the resulting maximum stress in both cases?



SOLUTION



Case (a):

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M(a/2)}{\frac{1}{12}(a)^4} = \frac{6M}{a^3}$$

Case (b):

$$I = 2\left[\frac{1}{36}\left(\frac{2}{\sqrt{2}}a\right)\left(\frac{1}{\sqrt{2}}a\right)^3 + \frac{1}{2}\left(\frac{2}{\sqrt{2}}a\right)\left(\frac{1}{\sqrt{2}}a\right)\left[\left(\frac{1}{\sqrt{2}}a\right)\left(\frac{1}{3}\right)\right]^2\right] = 0.08333\,a^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M\left(\frac{1}{\sqrt{2}}a\right)}{0.08333\,a^4} = \frac{8.4853\,M}{a^3}$$

Case (a) provides higher strength, since the resulting maximum stress is less for a given M and a.

$$\Delta\sigma_{\max} = \frac{8.4853\,M}{a^3} - \frac{6M}{a^3} = 2.49\left(\frac{M}{a^3}\right)$$ **Ans.**

Ans:
Case (a),
$\Delta\sigma_{\max} = 2.49\left(\frac{M}{a^3}\right)$

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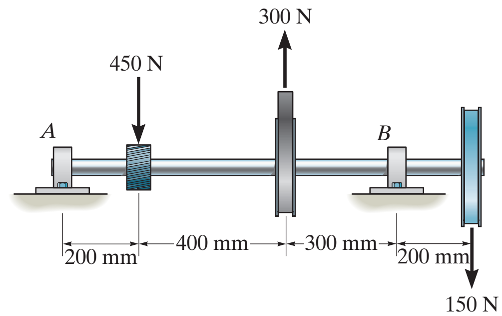
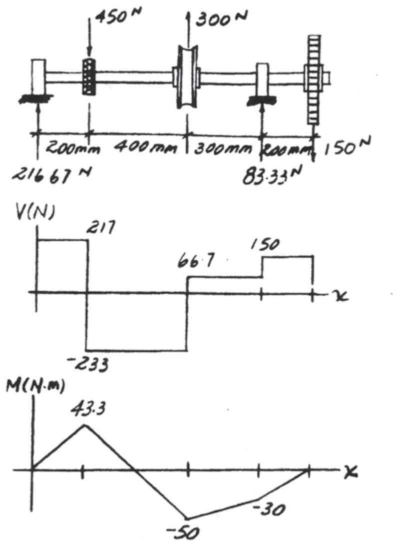
R11–7.

Draw the shear and moment diagrams for the shaft if it is subjected to the vertical loadings. The bearings at A and B exert only vertical reactions on the shaft.



SOLUTION



Ans:
$V|_{x=600\text{ mm}^-} = -233\text{ N},$
$M|_{x=600\text{ mm}} = -50\text{ N}\cdot\text{m}$

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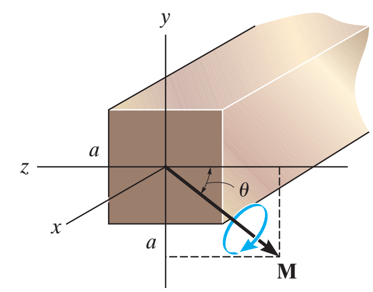
***R11–8.**

The strut has a square cross section *a* by *a* and is subjected to the bending moment **M** applied at an angle θ as shown. Determine the maximum bending stress in terms of *a*, *M*, and θ. What angle θ will give the largest bending stress in the strut? Specify the orientation of the neutral axis for this case.



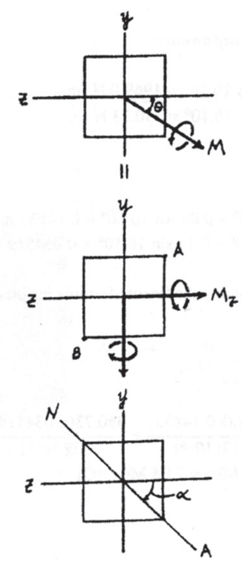
SOLUTION

Internal Moment Components:

$$M_z = -M\cos\theta \qquad M_y = -M\sin\theta$$

Section Property:

$$I_y = I_z = \frac{1}{12}a^4$$



Maximum Bending Stress: By inspection, maximum bending stress occurs at *A* and *B*. Applying the flexure formula for biaxial bending at point *A*,

$$\sigma_{\max} = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= -\frac{-M\cos\theta\left(\frac{a}{2}\right)}{\frac{1}{12}a^4} + \frac{-M\sin\theta\left(-\frac{a}{2}\right)}{\frac{1}{12}a^4}$$

$$= \frac{6M}{a^3}(\cos\theta + \sin\theta) \qquad \textbf{Ans.}$$

$$\frac{d\sigma}{d\theta} = \frac{6M}{a^3}(-\sin\theta + \cos\theta) = 0$$

$$\cos\theta - \sin\theta = 0$$

$$\theta = 45° \qquad \textbf{Ans.}$$

Orientation of Neutral Axis:

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\tan\alpha = (1)\tan(45°)$$

$$\alpha = 45° \qquad \textbf{Ans.}$$

Ans:
$\sigma_{\max} = \dfrac{6M}{a^3}(\cos\theta + \sin\theta)$,
$\theta = 45°$, $\alpha = 45°$