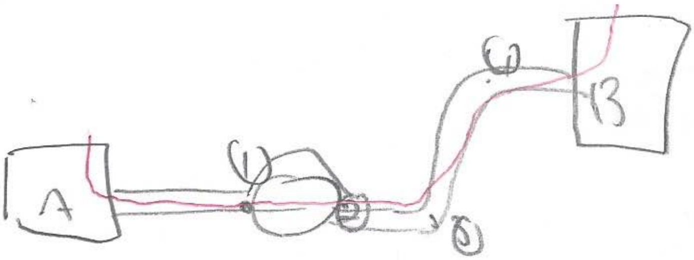
Pump Curve, $h_p = 100 - 8000Q^2$, $f = 0.02$, $\phi = 15\text{ cm}$, $l = 100\text{ m}$




$K_{entrance} = 0.5$
$K_{bend} = 0.25$
$K_{exit} = 1$ $\leftarrow K_d$

a) Find $Q$, $\therefore$ find operating point. Determine system curve

System Curve: E eqn from $A \to B$

$$\cancel{\frac{V_A^2}{2g}} + \cancel{\frac{P_A}{\gamma}} + z_A + h_p = \cancel{\frac{V_B^2}{2g}} + \cancel{\frac{P_B}{\gamma}} + z_B + \sum h_L$$

$$h_p = 50 + \left[K_e + 2K_B + K_d + \frac{fL}{d}\right]\frac{V^2}{2g}, \quad V^2 = \frac{Q^2}{A^2} = \frac{16Q^2}{\pi^2 d^4}$$

$$= 50 + \left[0.5 + 2(0.25) + 1 + \frac{0.02(100)}{0.15}\right]\frac{8Q^2}{g\pi^2(0.15)^4}$$

$$= 50 + 2503\,Q^2$$

$$100 - 8000Q^2 = 50 + 2503Q^2$$

$$Q = 0.0690\ \text{m}^3/\text{s}$$

$$\therefore 50 + 2503(0.069)^2 = 61.92\text{ m} \quad \text{(operating point)}$$

b) Find hp provided by pump to the water $\to Po_{hyd}$

$$Po_{hyd} = \gamma Q h_p = 9790(0.0690)(61.92) = 41828\text{ W} \times \frac{1\text{ hp}}{745.7\text{ W}} = 56.1\text{ hp}$$

c) Location of lowest and highest pipe pressure

$P_{min}$ just before the pump ($P_1$)

$P_{max}$ just after the pump ($P_2$)

E equation from A → ①

datum

$$z_A = \cancel{z_1} + \frac{P_1}{\gamma} + \sum h_L + \frac{V_1^2}{2g}$$

$$\frac{P_1}{\gamma} = z_A - \frac{V^2}{2g} - \sum h_L$$

$$\frac{P_1}{\gamma} = 4 - \frac{V^2}{2g} - \left[K_e + \frac{fL}{d}\right]\frac{V^2}{2g}$$

Q at operating point known = 0.064 $m^3/s$

∴ V = 3.905 m/s

$$\frac{V^2}{2g} = 0.7772$$

$$= 4 - \left[0.5 + 1 + \frac{0.02(25)}{0.15}\right]0.7772$$

$$\frac{P_1}{\gamma} = 0.244$$

From ① → ②

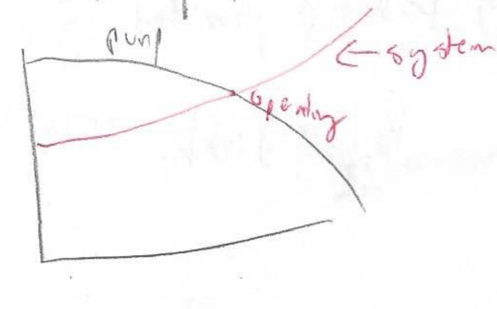
pump curve includes this

$$\cancel{\frac{V_1^2}{2g}} + \frac{P_1}{\gamma} + \cancel{z_1} + h_p = \cancel{\frac{V_2^2}{2g}} + \frac{P_2}{\gamma} + \cancel{z_2} + h_L$$

- V before and after pump are equal

$$\frac{P_2}{\gamma} = \frac{P_1}{\gamma} + h_p = 0.244 + 61.92 = 62.16\text{ m}$$

d)



1. System Curve

a) Apply Energy Equation from (A) $\rightarrow$ (B)

$$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + z_A + h_p = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + z_B + \sum h_{L_{A\to B}}$$

$$h_p = z_B - z_A + \sum h_L$$

$$= 30 + \left(K_E + K_B + K_O + f\frac{L}{D}\right)\frac{1}{2g}\left(\frac{Q}{A}\right)^2$$

$$= 30 + \left(0.5 + 0.35 + 1 + 0.015 \cdot \frac{1000}{0.4}\right)\frac{1}{2(9.81)} \cdot \frac{Q^2}{(0.1257)^2}$$

$$= 30 + \frac{(1.85 + 37.5)Q^2}{0.31} = 30 + 126.9Q^2$$

= System curve

Pump Curve, $h_p = 65 - 400Q^2$ (2)

Operating Point, set (1) = 2

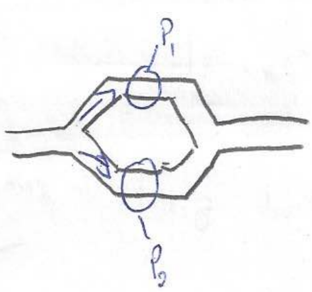
$$30 + 126.9Q^2 = 65 - 400Q^2$$

$$Q = \sqrt{\frac{35}{526.9}} = 0.2577 \, m^3/s$$

$$\therefore h_p = 65 - 400(0.2577)^2 = 38.44$$

b) What if 2 identical pipes in parallel, same system curve,

$\Rightarrow$ so no new losses




$$h_p = h_{p_1} = h_{p_2}$$

$$Q = Q_1 + Q_2$$

Pump Curve for 1 pump, $h_p = 65 - 400Q^2$, $Q = \sqrt{\frac{65 - h_p}{400}}$

2 Pumps in parallel: $Q = Q_1 + Q_2 = \sqrt{\frac{65-h_p}{400}} + \sqrt{\frac{65-h_p}{400}} = 2\sqrt{\frac{65-h_p}{400}}$

Alternatively, $h_p = 65 - 100Q^2$ (2b)

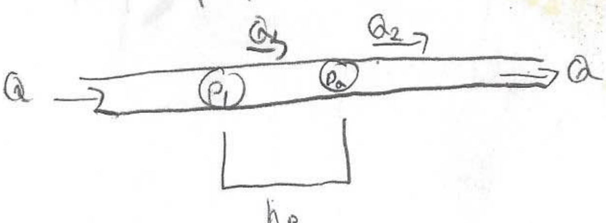
From before, system curve $= 30 + 126.9Q^2$

$\therefore 30 + 126.9Q^2 = 65 - 100Q^2$

$Q = \sqrt{\frac{35}{226.9}} = 0.3928 \ m^3/s$ $\rightarrow$ $h_p = 65 - 100(0.3928)^2 = 49.57m$

(Operating Point

2 identical pumps in series $\rightarrow$ Same system curve, $h_p = 65 - 400Q^2$




Single pump curve, $h_p = 65 - 400Q^2$

2 pumps in series, $h_p = h_{p_1} + h_{p_2} = (65 - 400Q^2) + (65 - 400Q^2)$

$h_p = 130 - 800Q^2$ (2c)

$30 + 126.9Q^2 = 130 - 800Q^2$

$Q = \sqrt{\frac{100}{926.9}} = 0.3285 \ m^3/s$ $\rightarrow$ $h_p = 130 - 800(0.3285)^2 = 43.67m$

$\therefore$ higher flow w/ parallel

hydraulic power $P_o = \gamma Q h_p$ $\therefore$ Power lower in series, $\Delta$ head greater in parallel

Eg. Pump 10 ft above tank, $Q = 0.5\ ft^3/s$

$NPSH = 15\ ft$

$K_L = 20$

$\phi = 4\ in$

Determine whether $NPSH_A \geq NPSH_R = 15\ ft$

Water @ 80°F

$\gamma_w = 62.22\ \frac{lb}{ft^3}$

$P_v = 0.5069$ psia

1). Apply E eqn from 1 → 2

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_s^2}{2g} + \frac{P_2}{\gamma} + z_s + \sum h_{L,1-2}$$

$$\frac{P_{atm}}{\gamma} + z_1 = \frac{V_s^2}{2g} + \frac{P_s}{\gamma} + K_L \frac{V^2}{2g}$$

$$\frac{V_s^2}{2g} + \frac{P_s}{\gamma} = \frac{P_{atm}}{\gamma} + z_1 - \frac{K_L Q^2}{2gA^2}$$

$$\frac{V_s^2}{2g} + \frac{P_s}{\gamma} - \frac{P_{vap}}{\gamma} = \frac{P_{atm}}{\gamma} - \frac{P_{vap}}{\gamma} + z_1 - \frac{K_L Q^2}{2gA^2}$$

$$NPSH_A = \frac{(14.7 - 0.5069)\frac{lb}{in^2}}{62.22\ lb/ft^3} \times \frac{12\ in^2}{1\ ft^2} - 10\ ft - \frac{20(0.5\ ft^3/s)^2}{2(32.2\ ft/s^2)\left[\frac{\pi}{4}\left(4\ in \cdot \frac{1\ ft}{12\ in}\right)^2\right]^2}$$

$$= 32.8\ ft - 10\ ft - 10.2\ ft = 12.6\ ft$$

$\therefore NPSH_A < NPSH_R$, Pump may not perform as expected

If not given, suction pressure assumed = vapor pressure

$P_s < P_{vap}$

# Lec 23 – Pump Coeff

Eg 1. A pump w/ $D = 10''$ ops @ 3560 rpm

↳ Pumps water under head of 300 ft

a) what speed operated @ for max efficiency?

b) Discharge?

i) Use pump-scaling relationships to determine the 2nd pump characteristics

| Pump 1 | Pump 2 |
|---|---|
| $D = 10''$ | $D = 10''$ |
| $N = 3560$ rpm | $h_p = 300$ ft |
| max efficiency (BEP) | max efficiency (BEP) |

↳ Using performance curve, $Q = 400$ GPM @ $\eta_p = 60$, $h_p = 390$ ft (falls in 65% efficient center)

Assume geometrically similar pumps

$$C_{H_1} = C_{H_2}$$

$$C_{Q_1} = C_{Q_2}$$

$$C_{H_1} = C_{H_2},\quad \left(\frac{g h_1}{\omega^2 \cancel{D^2}}\right)_1 = \left(\frac{g h_2}{\omega^2 \cancel{D^2}}\right)_2 \quad \frac{\omega_2^2}{\omega_1^2} = \frac{h_2}{h_1},\quad \frac{\omega_2}{\omega_1} = \sqrt{\frac{h_2}{h_1}}$$

$$\omega = N \cdot \frac{2\pi}{\text{rotation}} \times \frac{1\text{ min}}{60\text{ sec}}$$

$$\therefore \frac{\omega_2}{\omega_1} = \frac{N_2 \cdot \frac{2\pi}{60}}{N_1 \cdot \frac{2\pi}{60}} = \sqrt{\frac{h_2}{h_1}}$$

$$N_2 = N_1 \sqrt{\frac{h_2}{h_1}} = 3560 \sqrt{\frac{300}{390}} = 3122 \text{ rpm}$$

b) $C_{Q_1} = C_{Q_2}$, $\left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2$

$$Q_2 = Q_1 \frac{\omega_2}{\omega_1} = Q_1 \frac{N_2}{N_1} = 400 \text{ GPM} \cdot \frac{3122}{3560}$$

$$Q_2 = 350.8 \text{ gal/min}$$

∴ Pump 2 should run @ N=3122 rpm

↳ Check w/ figure 9.18 on p. 8-8

pt. Q = 351 gpm & $h_p$ = 300 ft lies on max efficiency line just below N = 3200 rpm

Eg. 2. 500 L/s, under head 10m, $N_s$ = 2000 rpm

Type of pump?

1. Convert rpm to $\omega$
2. Calculate specific speed
3. Check whether radial, axial, or mixed one best

$$\omega = N \cdot \frac{\pi}{30} = \frac{200}{3}\pi$$

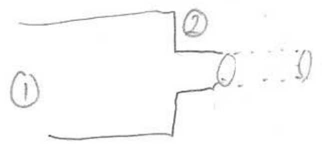
$$\text{Specific Speed, } N_s = \frac{\omega Q^{0.5}}{(g h_p)^{0.75}} = \frac{209.4 (0.5)^{0.5}}{[9.81 \times 10]^{0.75}} = 4.75$$

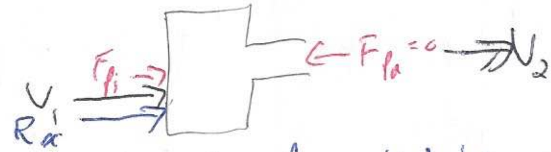
∴ Axial $N_s$ > 3.7. Axial flow pump required most efficient

1

# Momentum Conservation

Assume steady-state flow
↳ liquid = water, ∴ incompressible




① $\phi = 3$ in
$V_1 = 25$ ft/s
$P_1 = 75$ psi

② $V_2 = 100$ ft/s
$P_2 = 0$ psi

Flow rate: $Q = V_1 A_1 = 25\,\frac{ft}{s}\cdot\frac{\pi}{4}\left(\frac{3}{12}\,ft\right)^2 = 25\,\frac{ft}{s}(0.04909\,ft^2)$

$Q = 1.227\,\frac{ft^3}{s}$

$\Sigma F_x = (\rho Q V_x)_{out} - (\rho Q V_x)_{in}$ momentum out/in signs

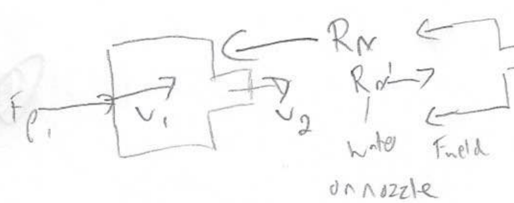
$R_x + F_{P_1} - \overset{0}{F_{P_2}} = \rho Q[+V_2 - V_1]$ velocity signs wrt x axis

$R_x + \underbrace{75\,\frac{lb}{in^2}\times 144\,\frac{in^2}{ft^2}}_{P}\times\underbrace{0.04909}_{A} - 0 = 1.94\,\frac{slug}{ft^3}\left(1.227\,\frac{ft^3}{s}\right)\left[+100 - (+25)\right]\,ft/s$

$R_x + 530.14\,lb = 178.5\,lb$

$R_x = -351.6\,lb \rightarrow$

$\therefore R_x = 351.6\,lb \leftarrow$, force acting on water to satisfy momentum balance due to the nozzle



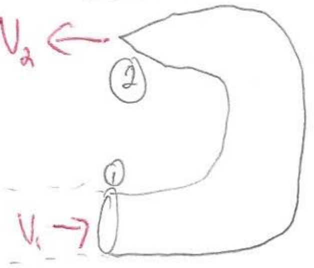

Fweld = F to keep nozzle attached to the pipe

Fweld = $R_x = 351.6$ lb

b) is flow ideal? Check using E eqn from ① → ②

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + \cancel{z_1} = \frac{V_2^2}{2g} + \cancel{\frac{P_2}{\gamma}} + \cancel{z_2} + h_L$, $z_1 = z_2$ ∴ $h_L = 27.5$ ft ∴ flow is not ideal

# Momentum Conservation




Assume steady - state water flow
- incompressible
- constant density

Is flow ideal? If ideal, no friction along walls
↳ $h_L$ calc after

$P_2 = 0$
$P_1 = 100$ kPa
$V_1 = 2$ m/s
$A_1 = 300$ mm $\phi$
$A_2 = 160$ mm $\phi$

1. Continuity. $Q = V_1 A_1 = 2 \cdot \frac{\pi}{4}(0.3)^2$
$= 0.1414$ m³/s

$$V_2 = \frac{Q}{A_2} = \frac{0.1414}{\pi/4(0.16)^2} = 7.033 \text{ m/s}$$

$$\Sigma F_x = (\rho Q V_x)_{out} - (\rho Q V_x)_{in}$$

$$R_x + F_{P_1} = \rho Q (V_2 - V_1)$$

- direction

$$R_x + 100000 \text{kPa} \left(\frac{\pi}{4} \cdot 0.3^2\right) = 998 \left(0.1414 \tfrac{m^3}{s}\right)\left((-7.033) - (+2)\right)$$

$$R_x + 7068.6N = -1274.7 \text{ N}$$

$$R_x = -8343 N \rightarrow$$

$$R_x = 8343 N \leftarrow$$

Is flow ideal? $\rightarrow h_L = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 7.9 \neq 0$, ∴ not ideal

Can we solve if $P_1$ not given?

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + \cancel{z_1} = \frac{V_2^2}{2g} + \cancel{\frac{P_2}{\gamma}} + \cancel{z_2} + \cancel{h_L}$$

→ 0 (assumed ideal flow)

$$\frac{P_1}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

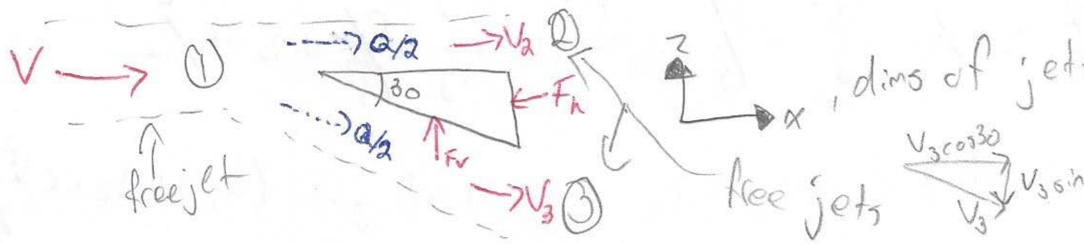
Alternatively... info to calculate $h_L$ given, then $\frac{P_1}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L$

E.g. free jet of ideal fluid (→ inviscid, density constant)

$Q = 10\ L/s$ ← steady-state flow

$V = 10\ m/s$

half flow one way, half other way. Wedge weighs 5 N




, dims of jets & wedge not given

E balance from ① → ② & ① → ③

$$\frac{V_1^2}{2g} + \cancel{\frac{P_1}{\gamma}} + \cancel{z_1} = \frac{V_2^2}{2g} + \cancel{\frac{P_2}{\gamma}} + \cancel{z_2} + h_{L_{1-2}} = \frac{V_3^2}{2g} + \cancel{\frac{P_3}{\gamma}} + \cancel{z_3} + h_{L_{1-3}}, \quad \text{Assume } z_1 \sim z_2 \sim z_3$$

$$\therefore |V_1| = |V_2| = |V_3| = \underline{|V|}$$

Momentum Balance

x-axis: $\sum F_x = \sum(\rho Q V_x)_{out} - \sum(\rho Q V_x)_{in}$

$$-F_H = \rho\frac{Q}{2}\left[V_2\right] + \rho\frac{Q}{2}\left[V_3\cos30\right] - \rho Q\left[V_1\right]$$

$$= \rho Q V\left[\frac{1}{2} + \frac{\cos30}{2} - 1\right] = 998(0.01)(10)\left[+0.067\right]$$

$$= 6.69\,N \leftarrow$$

y-axis: $\sum F_y = \sum(\rho Q V_y)_{out} - \sum(\rho Q V_y)_{in}$

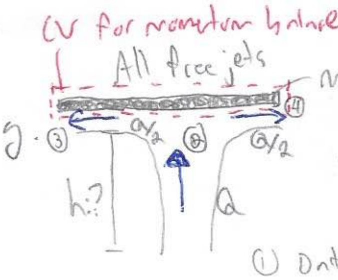
assume small since dims not given; neg dir

$$F_V - W_{wedge} + \cancel{W_{water}} = \rho\frac{Q}{2}\left[(-V_3\sin30) - 0\right]$$

$$F_V - 5 = 998\left(\frac{0.01}{2}\right)(-10\sin30) = -24.95\,N$$

$$F_V = -19.95 \uparrow, \quad F_V = 30\,N \downarrow$$

Eg.




$m_{plate} = 1.5$ kg

Assume steady-state flow

$Q = V_1 A_1 = 10 \cdot \frac{\pi}{4}(0.02)^2$
$= 3.14 \times 10^{-3}\ m^3/s$

① Datum
$d_1 = 20$ mm
$V_1 = 10$ m/s

* assume jet splits into 2 equal sub-jets

Energy Balance from ①→② or ①→③ or ①→④

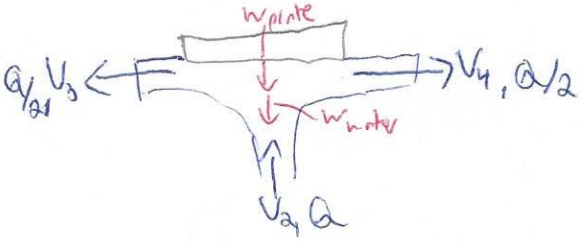
$$\frac{V_1^2}{2g} + \cancel{\frac{P_1}{\gamma}} + \cancel{z_1}^{0}_{\text{Datum}} = \frac{V_2^2}{2g} + \cancel{\frac{P_2}{\gamma}} + z_2 + \cancel{h_{L,1-2}}^{0} = \frac{V_3^2}{2g} + \cancel{\frac{P_3}{\gamma}} + z_3 + \cancel{h_{L,1-3}}^{0} = \frac{V_4^2}{2g} + \cancel{\frac{P_4}{\gamma}} + z_4 + \cancel{h_{L,1-4}}^{0}$$

↳ Assume $h_L$ are small & $z_2 \sim z_3 \sim z_4$ since no info given ($h_L$ and $\Delta z << \frac{V^2}{2g}$)

$$\frac{10^2}{2g} = \frac{V_2^2}{2g} + h = \frac{V_3^2}{2g} + h = \frac{V_4^2}{2g} + h$$

$$\therefore h = \frac{10^2}{2g} - \frac{V_2^2}{2g} = 5.1 - \frac{V_2^2}{2g} \quad (1)$$

Vertical Momentum Balance @ CV drawn on last slide




$$\Sigma F_z = (\rho Q V_z)_{out} - (\rho Q V_z)_{in}$$

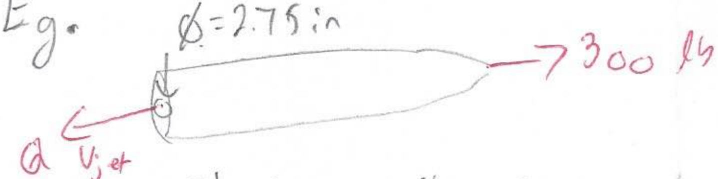
$$-W_{plate} - \underbrace{\cancel{W_{water}}}_{\text{no dims}}^{\text{small}} = 0 - \rho Q V_2$$

$$V_2 = \frac{W_{plate}}{\rho Q} = \frac{mg}{\rho Q} = \frac{1.5\text{kg}\,(9.81\ m/s^2)}{998\ \frac{kg}{m^3}(3.14E{-3}\ \frac{m^3}{s})}$$

$$= 4.696\ m/s$$

$$\therefore h = 5.1\,m - \frac{(4.696\ \frac{m}{s})^2}{2(9.81\ m/s^2)} = 3.98\ m$$

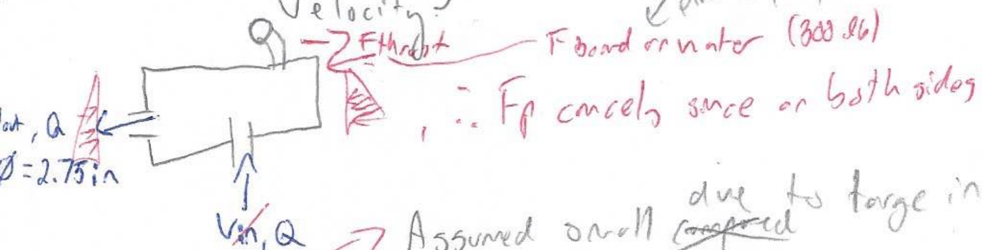
Eg.




Thrust = 300 lb = F water on board

Jet flow rate = ?

Velocity?




$\vdash\!\rightarrow x$, hor momentum balance:

$$\sum F_x = (\rho Q V_x)_{out} - (\rho Q V_x)_{in}$$

$$-300 = \rho (V_{jet} A_{jet})(-V_{jet}) - 0$$

$$300\ lb = \rho A_{jet} V_{jet}^2$$

$$V_{jet} = \sqrt{\frac{300}{1.94 \frac{slug}{ft^3} \times \frac{\pi}{4}\left(2.75\,in \times \frac{1\,ft}{12\,in}\right)^2}} = 61.24\ ft/s$$

$$Q = V_{jet} A_{jet} = 61.24 \cdot \frac{\pi}{4} \cdot \left(2.75\,in \times \frac{1\,ft}{12\,in}\right)^2 = 2.526\ \frac{ft^3}{s}$$

Momentum Correction Factor

$\rho Q V = \rho V A V = \rho A V^2 \neq \rho \int v^2 dA$. If not uniform, $\beta \rho A V^2$ used.

$\beta$ is correction factor