CivE 414
Structural Concrete Design

Design for Serviceability: Deflection and Cracking

DEFLECTIONS

➢ Excessive deflections will:

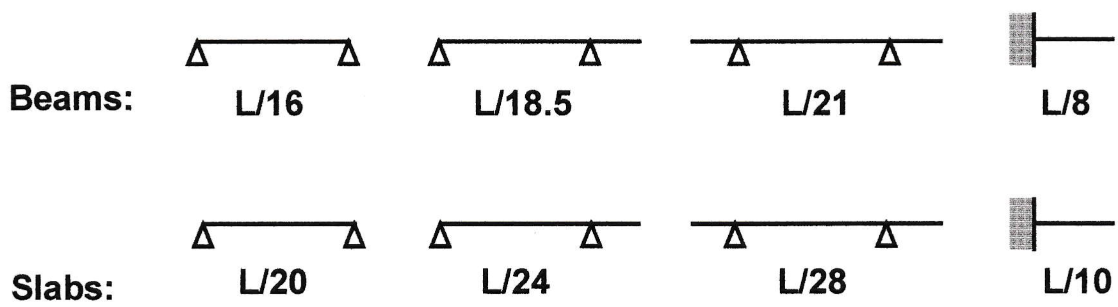
- Cause aesthetic problems
- Raise concerns about the safety of the structure
- Cause cracking of partitions, ceiling, etc.
- Alter the basic geometry of the structure → may lead to unexpected second-order effects

DESIGN FOR DEFLECTION

Two approaches:

1. Choose minimum thickness for slabs and beams to satisfy deflection requirements

   CSA A23.3 *Clause 9.8.2.1*



2. Compute deflections of the structure and compare to permissible limits

$$\Delta \leq \Delta_{lim}$$

CSA A23.3 DEFLECTION LIMITS

Clause 9.8.2.6 → see Table 9.3 (A23.3-04)

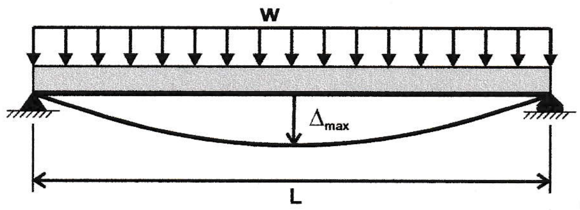
➢ **Deflection limits are placed on live load deflections**

Table 9.3
Maximum permissible computed deflections
(See Clauses 9.8.2.6, 9.8.4.5, 9.8.5.3, 13.2.2, and 13.2.7.)

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to non-structural elements likely to be damaged by large deflections	Immediate deflection due to specified live load, L, or snow load, S	$\ell_n/180$*
Floors not supporting or attached to non-structural elements likely to be damaged by large deflections	Immediate deflection due to specified live load, L	$\ell_n/360$
Roof or floor construction supporting or attached to non-structural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of non-structural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell_n/480$‡
Roof or floor construction supporting or attached to non-structural elements not likely to be damaged by large deflections	That part of the total deflection occurring after attachment of non-structural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell_n/240$§

**This limit is not intended to guard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and the long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage should be taken into consideration.*
†Long-term deflections shall be determined in accordance with Clause 9.8.2.5 or 9.8.4.4, but may be reduced by the amount of deflection calculated to occur before the attachment of non-structural elements.
‡This limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.
§This limit shall not be greater than the tolerance provided for non-structural elements. It may be exceeded if camber is provided so that total deflection minus camber does not exceed the limit.
Note: *For two-way slab construction, ℓ_n shall be taken as the clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.*

DEFLECTION CALCULATIONS

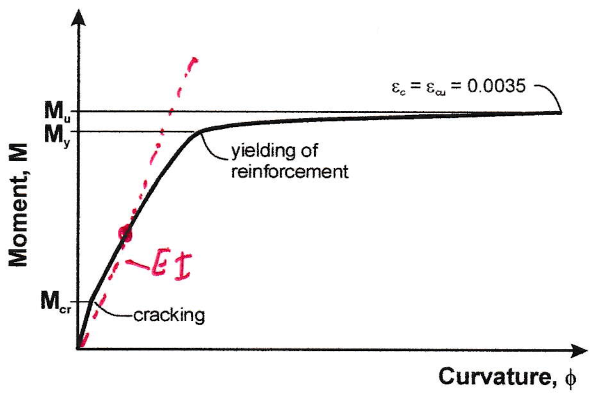


$$\Delta_{max} = \frac{5wL^4}{384EI}$$

➢ The fundamental assumption used to derive the deflection equation above is that behaviour is linear elastic:

- EI (flexural stiffness) is constant
- $\phi = \dfrac{M}{EI}$

➢ For reinforced concrete, the moment-curvature relationship is not linear:



$$\phi \neq \frac{M}{EI}$$

, in reality, E changes, not I. Beam is less stiff! Concrete is still there after cracking! However, in code, I and Δed

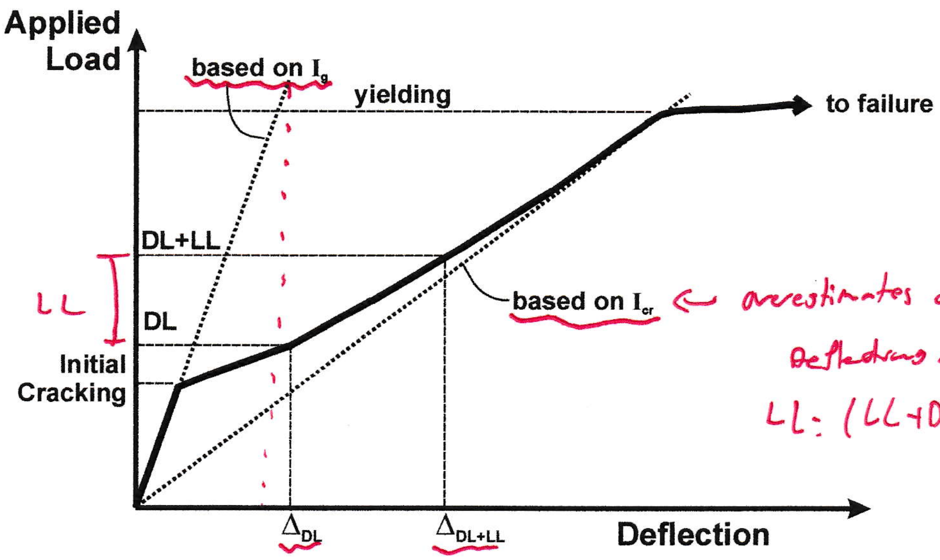
➢ **Options:**

1. **Calculate deflections from curvatures** → for non linear analysis

$$\Delta = \int x\phi dx$$

2. **Use an effective moment of inertia, I_e**
   - **function of load level → calculate for each load level**

Load-Deflection Behaviour of RC Beams

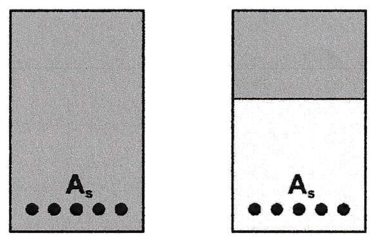


← overestimates deflections

Deflections due to LL only

LL: (LL+DL) - DL

- The load-deflection behaviour is non-linear since the moment-curvature behaviour is non-linear
- The flexural stiffness, EI, is a function of applied moment (load level)
- The actual moment of inertia for a given load level lies between I_g and I_{cr}



Section Before Cracking | Section After Cracking

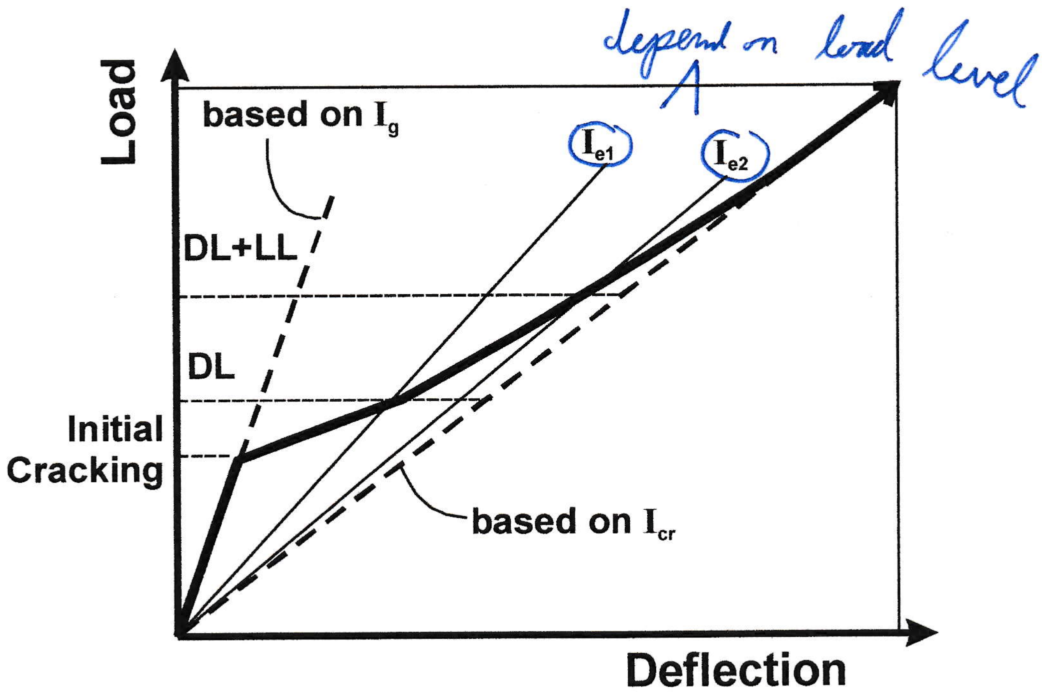
- Since load-deflection behaviour is non-linear, incremental deflections, such as live load deflection, can not be computed directly.

  e.g., can not compute Δ_{LL} directly from w_L

$$\boxed{\Delta_{LL} = \Delta_{DL+LL} - \Delta_{DL}}$$

EFFECTIVE MOMENT OF INERTIA

- The concept of an effective moment of inertia, I_e, was introduced to simplify deflection calculations where EI is not constant
- The effective moment of inertia is computed for a given load level (moment) to compute deflections at that load level. I_e is recalculated for other load levels (moment)



$$* \quad I_g \geq I_e \geq I_{cr}$$

$$EI_g \geq EI_e \geq EI_{cr}$$

E_C AND I_E FOR DEFLECTION CALCULATION

CSA A23.3 *Clause 9.8.2.3*

$$E_c = \left(3300\sqrt{f'_c} + 6900\right)\left(\frac{\gamma_c}{2300}\right)^{1.5}$$ *(Clause 8.6.2.2)*

for $1500 \le \gamma_c \le 2500$ kg/m^3

or

$$E_c = 4500\sqrt{f'_c}$$ *(Clause 8.6.2.3)*

for *normal density concrete* and $20 \le f'_c \le 40$ MPa

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \le I_g$$

Where, Key

I_e = effective moment of inertia for $M = M_a$

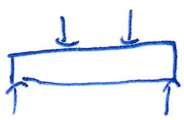
M_a = maximum moment (critical section) at the load level for which deflections are being computed

I_{cr} = cracked section moment of inertia

I_g = gross section moment of inertia (neglect A_s)

M_{cr} = cracking moment

$$= \frac{f_r I_g}{y_t}$$



f_r determined by 4 point bending test

$$f_r = \frac{0.6\lambda\sqrt{f'_c}}{2}$$ divided by 2 for deflections only! ***new requirement in CSA A23.3-14**

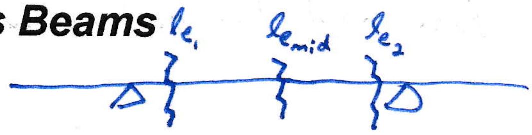
y_t = distance from centroid to extreme tensile fibre

Tensile strength of concrete $= 0.33\sqrt{f'_c}$, determined by direct tension test $\therefore$ less than f_r

It's okay, farm

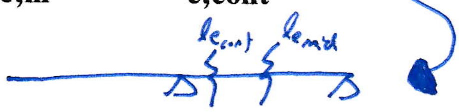
Clause 9.8.2.4 – I_e for Continuous Beams

- Use an average I_e:



Two ends continuous: $$I_{e,avg} = 0.7 I_{e,m} + 0.15\left(I_{e1} + I_{e2}\right)$$

One end continuous: $$I_{e,avg} = 0.85 I_{e,m} + 0.15 I_{e,cont}$$

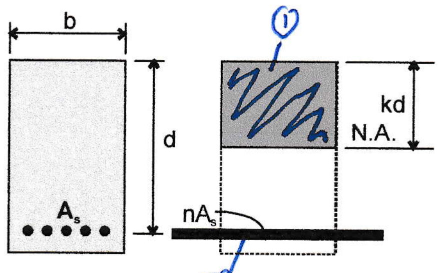


CRACKED SECTION MOMENT OF INERTIA

Singly-reinforced Sections:

$$bkd\left(\frac{kd}{2}\right) = nA_s(d-kd)$$

$n = \frac{E_s}{E_c}$



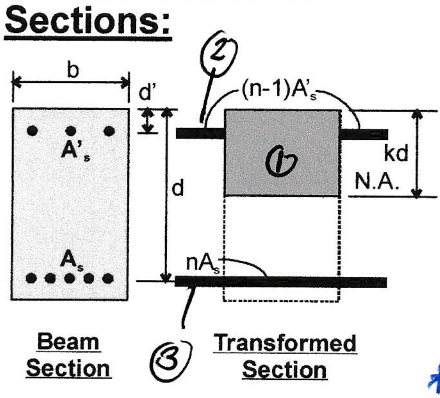
Beam Section — Transformed Section

$$\rho = \frac{A_s}{bd} \quad \Rightarrow \quad \frac{k^2}{2} + \rho n k - \rho n = 0$$

Solving: $k = \sqrt{2\rho n + (\rho n)^2} - \rho n$

$$* \quad I_{cr} = \underset{①}{\frac{b(kd)^3}{3}} + \underset{②}{nA_s(d-kd)^2}$$

Doubly-reinforced Sections:



Beam Section — Transformed Section

$$\frac{b(kd)^2}{2} + (n-1)A'_s(kd-d') = nA_s(d-kd)$$

$$\frac{b(kd)^2}{2} + \left[(n-1)A'_s + nA_s\right]kd$$
$$-(n-1)A'_s d' - nA_s d = 0$$

$\Rightarrow$ **Solve for "kd"**

$$* \quad I_{cr} = \underset{①}{\frac{b(kd)^3}{3}} + \underset{③}{nA_s(d-kd)^2} + \underset{②}{(n-1)A'_s(kd-d')^2}$$

DEFLECTIONS UNDER SUSTAINED LOAD

- Deflections increase under sustained loads due to long-term creep and shrinkage of concrete
- Deflections under sustained loads may be treated as having 2 components:
    1. Immediate deflection, Δ_i, due to applied loads
    2. Long-term deflection, Δ_t, due to creep and shrinkage

$$\Delta_{total} = \Delta_i + \Delta_t$$

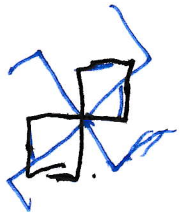
- Time dependent deflections are reduced by the presence of compression steel

<u>CSA A23.3 *Clause 9.8.2.5*</u>

Define: $$\zeta_s = \frac{\Delta_{total}}{\Delta_i} = \frac{\Delta_i + \Delta_t}{\Delta_i} = \left(1 + \frac{s}{1+50\rho'}\right)$$

compression reinforcement ratio

Clash of clans barge



Thus: $$\Delta_{total} = \Delta_i + \Delta_t = \left(1 + \frac{s}{1+50\rho'}\right)\Delta_i$$

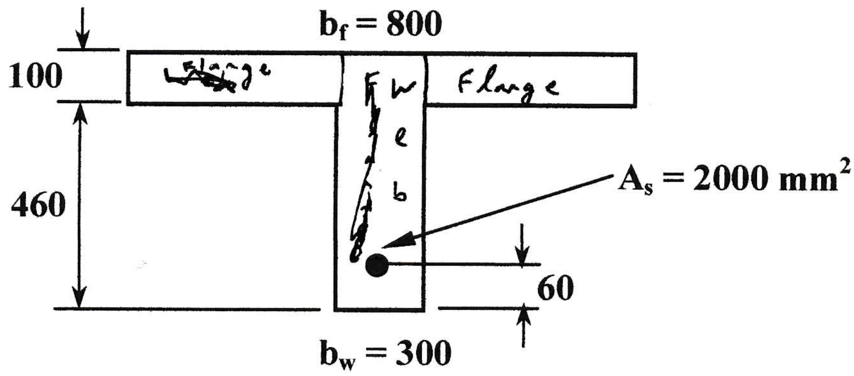
$$\Delta_t = \left(\frac{s}{1+50\rho'}\right)\Delta_i$$

where,

$\rho' = \dfrac{A'_s}{bd}$ at midspan for simple and continuous spans and at the supports for cantilevers *(compression reinforcement)*

s = time dependent factor
= 1.0 for loads sustained for 3 months
= 1.2 for loads sustained for 6 months
= 1.4 for loads sustained for 12 months
= 2.0 for loads sustained for 5 years or more

Example 1: ***Determine the mid-span dead load and live load deflections for the simply-supported beam shown. Assume f'_c = 30 MPa and f_y = 400 MPa.***



L = 8 m

w_D = 6 kN/m

w_L = 10 kN/m

$$E_s = 200{,}000\,\text{MPa}$$

$$E_c = 4500\sqrt{f'_c} = 24{,}650\,\text{MPa}$$

$$n = E_s/E_c = 8.1$$

From Table 1.14 CAC Handbook

(Linear elastic solution for s.s. beam):

$$\Delta_{max} = \frac{5\,w\,L^4}{384\,E_c I_e}$$

Gross Moment of Inertia, I_g

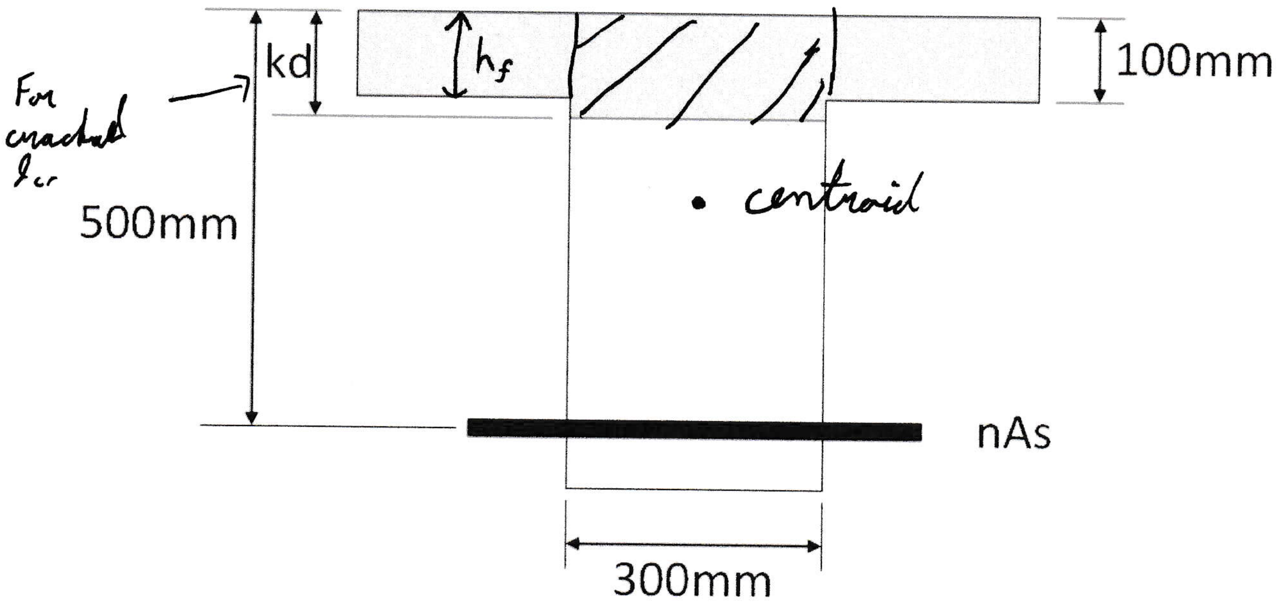
Item	Dimensions (mm)	Area (mm^2)	$\bar{y}_{top}$ (mm)	$A \times \bar{y}_{top}$ (mm^3)
Flange	500 x 100	50,000	50	2,500,000
Web	300 x 560	168,000	280	47,040,000
Totals:	--	218,000	--	49,540,000

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{49{,}540{,}000}{218{,}000} = 227\,\text{mm}$$ *(centroid)*

$$I_g = \sum \bar{I}_i + \sum A_i d_i^2$$

$$= \frac{500(100)^3}{12} + (50{,}000)(227-50)^2 + \frac{300(560)^3}{12} + (168{,}000)(227-280)^2$$

$$I_g = 6{,}470 \times 10^6 \text{ mm}^4$$



Assume $kd > h_f$

Sum Moments of area about neutral axis

$$(800-300)\cdot 100 \cdot \left(kd - \frac{100}{2}\right) + \frac{300(kd)kd}{2}$$
$$= 8.1 \cdot 2000 \cdot (500 - kd)$$

$$150 \cdot kd^2 + 66200 \cdot kd - 10{,}600{,}000 = 0$$

$$kd = 124.8mm > h_f \text{ ok.}$$ ← okay since outside of flange

$$I_{cr} = \frac{(800-300)100^3}{12} + (800-300)\cdot 100 \cdot \left(\frac{100}{2} - 124.8\right)^2$$ flange
$$+ \frac{300(124.8)^3}{3} + 8.1 \cdot 2000 \cdot (500-124.8)^2$$

web reinforcement

$$I_{cr} = 2796 \cdot 10^6 mm^4$$

Solve for cracking moment

$$M_{cr} = \frac{f_r I_g}{y_{tension}}$$

$$f_r = \frac{0.6\sqrt{f'_c}}{2} = \frac{0.6\sqrt{30}}{2} = 1.64\ MPa$$

For gross sections

$$y_{tension} = h - \bar{y} \rightarrow 560 - 227 = 333mm$$

$$M_{cr} = \frac{1.64MPa \cdot 6470 \cdot 10^6 mm^4}{333mm \cdot 10^6} = 31.9\ kN \cdot m$$

Dead Load Deflection, Δ_D

$$M_D = \frac{w_D L^2}{8} \rightarrow \frac{6 \cdot 8^2}{8}\ (without\ factors) = 48\ kN \cdot m > M_{cr}$$

Therefore, section is cracked under dead load use I_e

Clause 9.8.2.3

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g \rightarrow \left[2796 + (6470 - 2796)\left(\frac{31.9}{48}\right)^3\right] \cdot 10^6$$

$$I_e = 3874 \cdot 10^6$$

$$\Delta_D = \frac{5 w_D L^4}{384 E_c I_e} = \frac{5 \cdot \frac{6N}{mm} \cdot 8000mm^4}{384 \cdot 24650MPa \cdot 3874 \cdot 10^6 mm^4} \rightarrow \Delta_D = 3.35mm$$

Live Load Deflection – behaviour is non-linear *(after cracking)*

$$\Delta_L \neq \frac{5 w_L L^4}{384 E_c I_g} \rightarrow \Delta_L = \Delta_{D+L} - \Delta_D$$

$$M_{D+L} = \frac{(6+10) \cdot 8^2}{8} = 128\ kN \cdot m > M_{cr} \rightarrow use\ I_e$$

Clause 9.8.2.3

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3 \le I_g \rightarrow \left[2796 + (6470 - 2796)\left(\frac{31.9}{128}\right)^3\right] \cdot 10^6$$

$$I_e = 2852 \cdot 10^6 mm^4$$

$$\Delta_{D+L} = \frac{5 \cdot \frac{16N}{mm} \cdot 8000mm^4}{384 \cdot 24650MPa \cdot 2852 \cdot 10^6 mm^4} \rightarrow \Delta_{D+L} = 12.1mm$$

$$\Delta_L = \Delta_{D+L} - \Delta_D \rightarrow 12.1mm - 3.35mm \rightarrow \Delta_L = 8.8mm$$

Check deflection limit → Table 9.3 clause 9.8.2.6

Assume floor does not support elements likely to be damaged by deflections:

$$Floor\ \Delta_{max} = \frac{l_n}{360} = \frac{8000}{360} = 22.2mm > \Delta_L = 8.8mm\ O.K.$$

What if deflection limits are not met?

Increase section (b, h)	Increases I_y, I_{cr}, M_{cr}
Increase A_s	Increase I_{cr}
Add A_s'	Increases I_{cr}