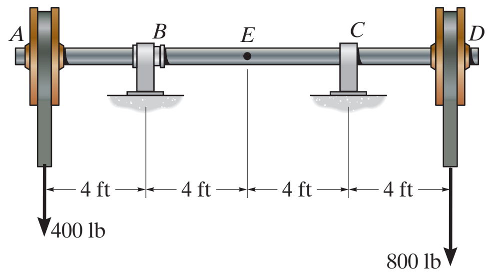
7–1.

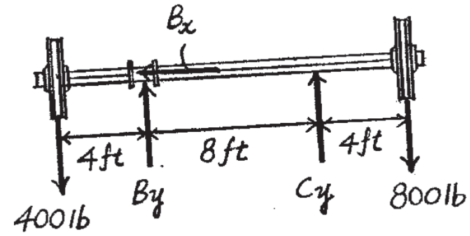
The shaft is supported by a smooth thrust bearing at B and a journal bearing at C. Determine the resultant internal loadings acting on the cross section at E.



SOLUTION

Support Reactions: We will only need to compute $\mathbf{C}_y$ by writing the moment equation of equilibrium about B with reference to the free-body diagram of the entire shaft, Fig. *a*.

$\circlearrowleft + \Sigma M_B = 0; \qquad C_y(8) + 400(4) - 800(12) = 0 \qquad C_y = 1000 \text{ lb}$



(a)

Internal Loadings: Using the result for $\mathbf{C}_y$, section DE of the shaft will be considered. Referring to the free-body diagram, Fig. *b*,

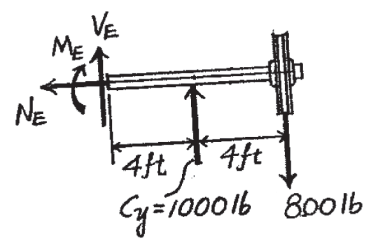
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_E = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_E + 1000 - 800 = 0 \qquad V_E = -200 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \; 1000(4) - 800(8) - M_E = 0$

$M_E = -2400 \text{ lb} \cdot \text{ft} = -2.40 \text{ kip} \cdot \text{ft}$ **Ans.**

The negative signs indicate that $\mathbf{V}_E$ and $\mathbf{M}_E$ act in the opposite sense to that shown on the free-body diagram.

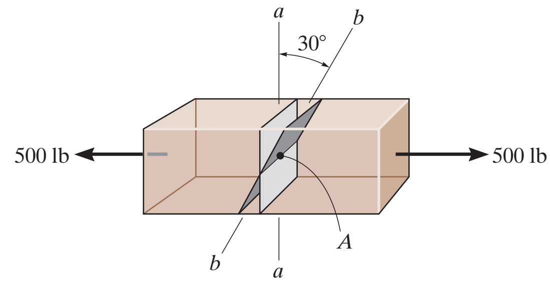


(b)

Ans:
$N_E = 0, V_E = -200 \text{ lb}, M_E = -2.40 \text{ kip} \cdot \text{ft}$

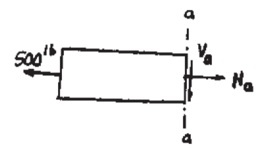
7–2.

Determine the resultant internal normal and shear force in the member at (a) section a–a and (b) section b–b, each of which passes through point A. The 500-lb load is applied along the centroidal axis of the member.



SOLUTION

(a)

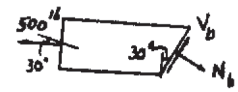


$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_a - 500 = 0$

$N_a = 500 \text{ lb}$ **Ans.**

$+\downarrow \Sigma F_y = 0; \qquad V_a = 0$ **Ans.**

(b)



$\searrow^{+} \Sigma F_x = 0; \qquad N_b - 500 \cos 30^\circ = 0$

$N_b = 433 \text{ lb}$ **Ans.**

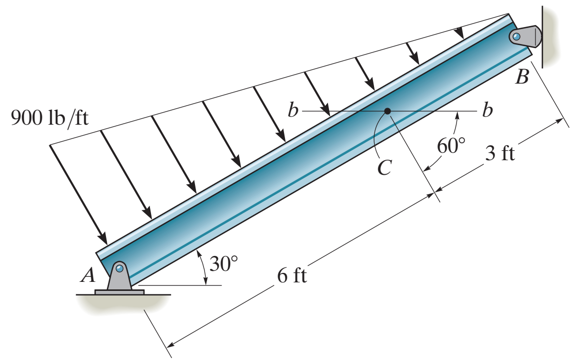
$+\nearrow \Sigma F_y = 0; \qquad V_b - 500 \sin 30^\circ = 0$

$V_b = 250 \text{ lb}$ **Ans.**

Ans:
(a) $N_a = 500 \text{ lb}, V_a = 0,$
(b) $N_b = 433 \text{ lb}, V_b = 250 \text{ lb}$

7–3.

Determine the resultant internal loadings acting on section b–b through the centroid, point C on the beam.

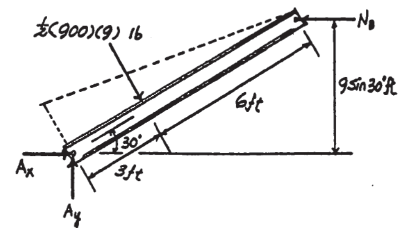


SOLUTION

Support Reaction:

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(9 \sin 30°) - \frac{1}{2}(900)(9)(3) = 0$$

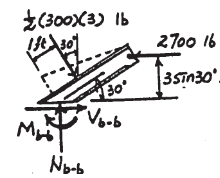
$$N_B = 2700 \text{ lb}$$



Equations of Equilibrium: For section b–b

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad V_{b-b} + \frac{1}{2}(300)(3) \sin 30° - 2700 = 0$$

$$V_{b-b} = 2475 \text{ lb} = 2.475 \text{ kip} \qquad \textbf{Ans.}$$



$$+\uparrow \Sigma F_y = 0; \quad N_{b-b} - \frac{1}{2}(300)(3) \cos 30° = 0$$

$$N_{b-b} = 389.7 \text{ lb} = 0.390 \text{ kip} \qquad \textbf{Ans.}$$

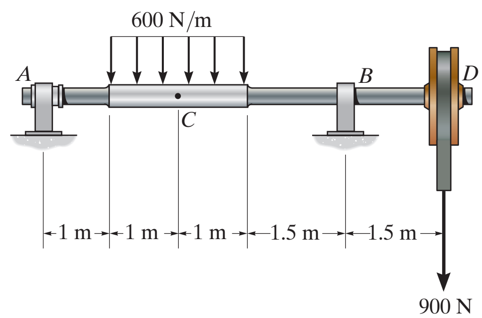
$$\circlearrowleft + \Sigma M_C = 0; \quad 2700(3 \sin 30°) - \frac{1}{2}(300)(3)(1) - M_{b-b} = 0$$

$$M_{b-b} = 3600 \text{ lb} \cdot \text{ft} = 3.60 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

Ans:
$V_{b-b} = 2.475$ kip,
$N_{b-b} = 0.390$ kip,
$M_{b-b} = 3.60$ kip · ft

***7–4.**

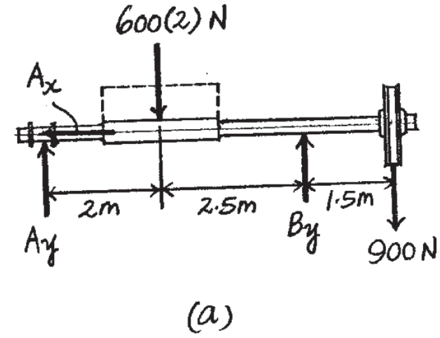
The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Determine the resultant internal loadings acting on the cross section at C.



SOLUTION

Support Reactions: We will only need to compute $\mathbf{B}_y$ by writing the moment equation of equilibrium about A with reference to the free-body diagram of the entire shaft, Fig. *a*.

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(4.5) - 600(2)(2) - 900(6) = 0 \quad B_y = 1733.33 \text{ N}$



Internal Loadings: Using the result of $\mathbf{B}_y$, section CD of the shaft will be considered. Referring to the free-body diagram of this part, Fig. *b*,

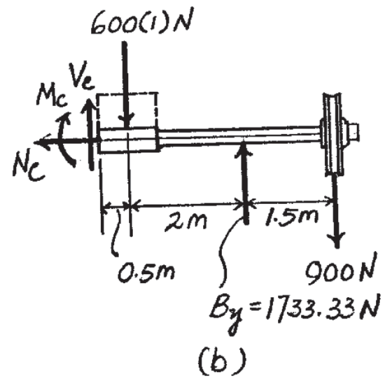
$\xrightarrow{+} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_C - 600(1) + 1733.33 - 900 = 0 \quad V_C = -233 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad 1733.33(2.5) - 600(1)(0.5) - 900(4) - M_C = 0$

$M_C = 433 \text{ N} \cdot \text{m}$ **Ans.**

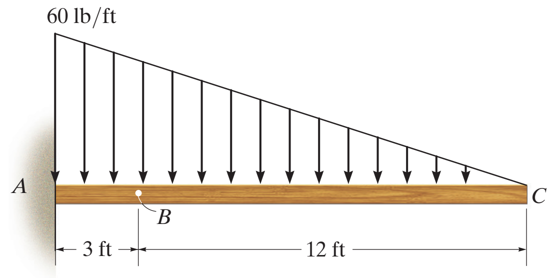
The negative sign indicates that $\mathbf{V}_C$ acts in the opposite sense to that shown on the free-body diagram.



Ans:
$N_C = 0$,
$V_C = -233 \text{ N}$,
$M_C = 433 \text{ N} \cdot \text{m}$

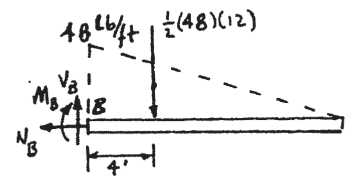
7–5.

Determine the resultant internal loadings acting on the cross section at point B.



SOLUTION

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_B = 0$ **Ans.**



$+\uparrow \Sigma F_y = 0;$ $\quad V_B - \frac{1}{2}(48)(12) = 0$

$V_B = 288 \text{ lb}$ **Ans.**

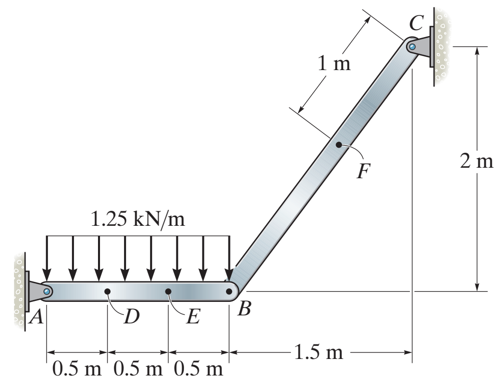
$\circlearrowleft + \Sigma M_B = 0;$ $\quad -M_B - \frac{1}{2}(48)(12)(4) = 0$

$M_B = -1152 \text{ lb} \cdot \text{ft} = -1.15 \text{ kip} \cdot \text{ft}$ **Ans.**

Ans:
$N_B = 0,$
$V_B = 288 \text{ lb},$
$M_B = -1.15 \text{ kip} \cdot \text{ft}$

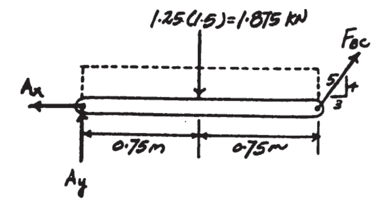
7–6.

Determine the resultant internal loadings on the cross section at point D.



SOLUTION

Support Reactions: Member BC is the two-force member.



$$\circlearrowleft+\Sigma M_A = 0; \quad \frac{4}{5}F_{BC}(1.5) - 1.875(0.75) = 0$$

$$F_{BC} = 1.1719 \text{ kN}$$

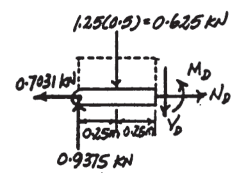
$$+\uparrow\Sigma F_y = 0; \quad A_y + \frac{4}{5}(1.1719) - 1.875 = 0$$

$$A_y = 0.9375 \text{ kN}$$

$$\xrightarrow{+}\Sigma F_x = 0; \quad \frac{3}{5}(1.1719) - A_x = 0$$

$$A_x = 0.7031 \text{ kN}$$

Equations of Equilibrium: For point D,



$$\xrightarrow{+}\Sigma F_x = 0; \quad N_D - 0.7031 = 0$$
$$N_D = 0.703 \text{ kN} \qquad \textbf{Ans.}$$

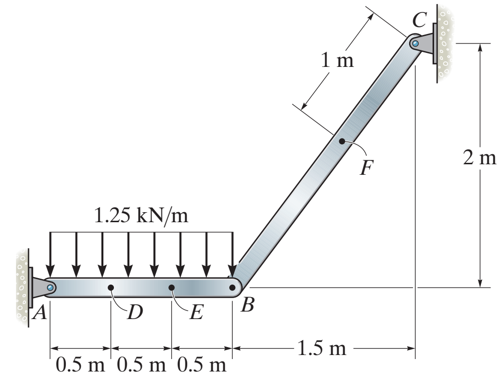
$$+\uparrow\Sigma F_y = 0; \quad 0.9375 - 0.625 - V_D = 0$$
$$V_D = 0.3125 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft+\Sigma M_D = 0; \quad M_D + 0.625(0.25) - 0.9375(0.5) = 0$$
$$M_D = 0.3125 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$N_D = 0.703$ kN,
$V_D = 0.3125$ kN,
$M_D = 0.3125$ kN·m

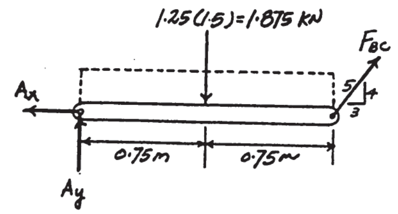
7–7.

Determine the resultant internal loadings at cross sections at points E and F on the assembly.



SOLUTION

Support Reactions: Member BC is the two-force member.



$$\circlearrowleft + \Sigma M_A = 0; \quad \frac{4}{5}F_{BC}(1.5) - 1.875(0.75) = 0$$

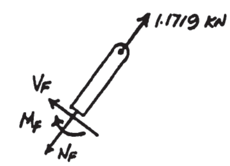
$$F_{BC} = 1.1719 \text{ kN}$$

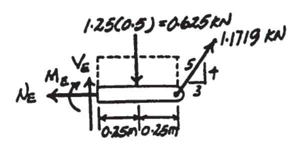
$$+\uparrow \Sigma F_y = 0; \quad A_y + \frac{4}{5}(1.1719) - 1.875 = 0$$

$$A_y = 0.9375 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{3}{5}(1.1719) - A_x = 0$$

$$A_x = 0.7031 \text{ kN}$$





Equations of Equilibrium: For point F,

$$+\swarrow \Sigma F_{x'} = 0; \quad N_F - 1.1719 = 0$$

$$N_F = 1.17 \text{ kN} \qquad \textbf{Ans.}$$

$$\nwarrow + \Sigma F_{y'} = 0; \quad V_F = 0 \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_F = 0; \quad M_F = 0 \qquad \textbf{Ans.}$$

Equations of Equilibrium: For point E,

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad N_E - \frac{3}{5}(1.1719) = 0$$

$$N_E = 0.703 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad V_E - 0.625 + \frac{4}{5}(1.1719) = 0$$

$$V_E = -0.3125 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_E = 0; \quad -M_E - 0.625(0.25) + \frac{4}{5}(1.1719)(0.5) = 0$$

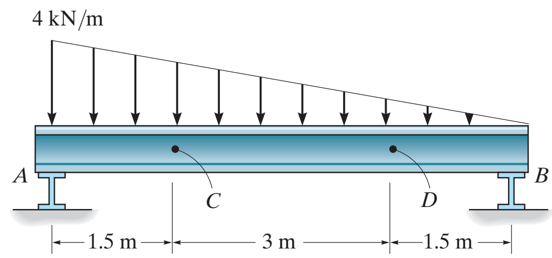
$$M_E = 0.3125 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Negative sign indicates that $\mathbf{V}_E$ acts in the opposite direction to that shown on the FBD.

Ans:
$N_F = 1.17$ kN,
$V_F = 0$,
$M_F = 0$,
$N_E = 0.703$ kN,
$V_E = -0.3125$ kN,
$M_E = 0.3125$ kN·m

***7–8.**

The beam supports the distributed load shown. Determine the resultant internal loadings acting on the cross section at point C. Assume the reactions at the supports A and B are vertical.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. a,

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(6) - \frac{1}{2}(4)(6)(2) = 0 \quad B_y = 4.00 \text{ kN}$

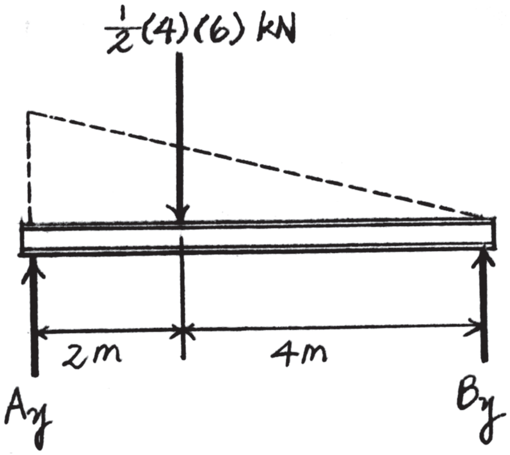
Internal Loadings: Referring to the FBD of the right segment of the beam sectioned through C, Fig. b,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

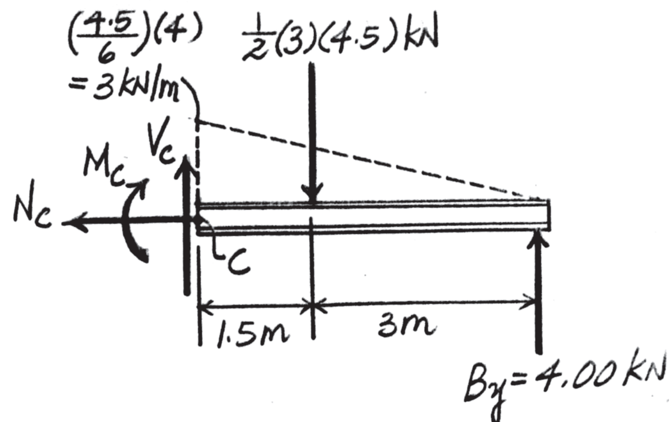
$+\uparrow \Sigma F_y = 0; \quad V_C + 4.00 - \frac{1}{2}(3)(4.5) = 0 \quad V_C = 2.75 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad 4.00(4.5) - \frac{1}{2}(3)(4.5)(1.5) - M_C = 0$

$M_C = 7.875 \text{ kN} \cdot \text{m}$ **Ans.**



(a)

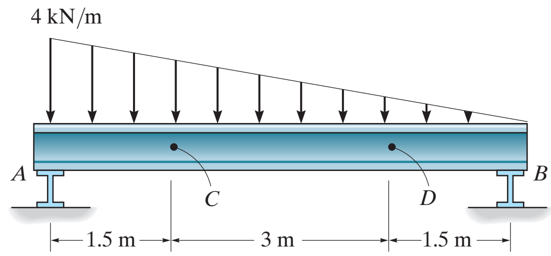


(b)

Ans:
$N_C = 0,$
$V_C = 2.75 \text{ kN},$
$M_C = 7.875 \text{ kN} \cdot \text{m}$

7–9.

The beam supports the distributed load shown. Determine the resultant internal loadings acting on the cross section at point *D*. Assume the reactions at the supports *A* and *B* are vertical.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft+\Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(4)(6)(2) = 0 \qquad B_y = 4.00 \text{ kN}$$

Internal Loadings: Referring to the FBD of the right segment of the beam sectioned through *D*, Fig. *b*,

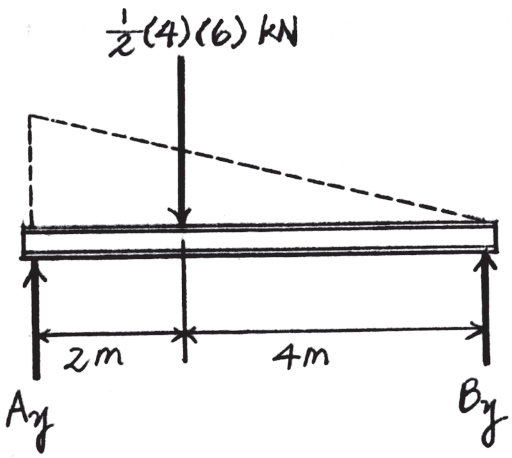
$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_D = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_D + 4.00 - \frac{1}{2}(1.00)(1.5) = 0 \qquad V_D = -3.25 \text{ kN} \qquad \textbf{Ans.}$$

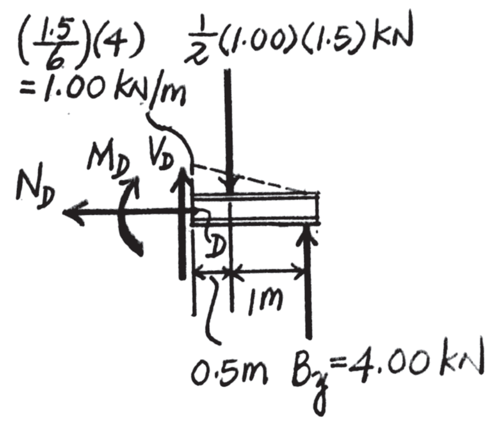
$$\circlearrowleft+\Sigma M_D = 0; \qquad 4.00(1.5) - \frac{1}{2}(1.00)(1.5)(0.5) - M_D = 0$$

$$M_D = 5.625 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{V}_D$ acts in the sense opposite to that shown on the FBD.



(a)

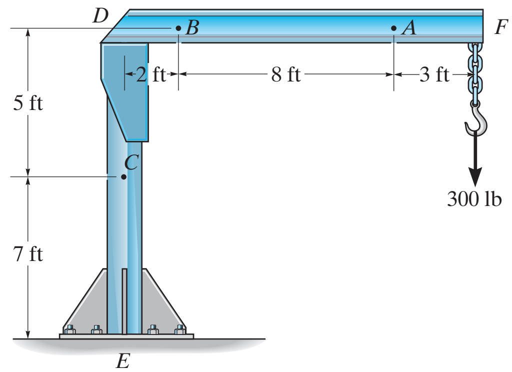


(b)

Ans:
$N_D = 0$,
$V_D = -3.25$ kN,
$M_D = 5.625$ kN·m

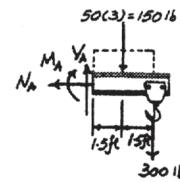
7–10.

The boom DF of the jib crane and the column DE have a uniform weight of 50 lb/ft. If the hoist and load weigh 300 lb, determine the resultant internal loadings in the crane on cross sections at points A, B, and C.



SOLUTION

Equations of Equilibrium: For point A,



$\xleftarrow{+}\ \Sigma F_x = 0;\qquad N_A = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad V_A - 150 - 300 = 0$

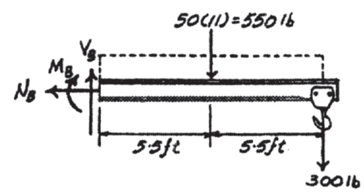
$V_A = 450\text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;\qquad -M_A - 150(1.5) - 300(3) = 0$

$M_A = -1125\text{ lb}\cdot\text{ft} = -1.125\text{ kip}\cdot\text{ft}$ **Ans.**

Negative sign indicates that M_A acts in the opposite direction to that shown on the FBD.

Equations of Equilibrium: For point B,



$\xleftarrow{+}\ \Sigma F_x = 0;\qquad N_B = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad V_B - 550 - 300 = 0$

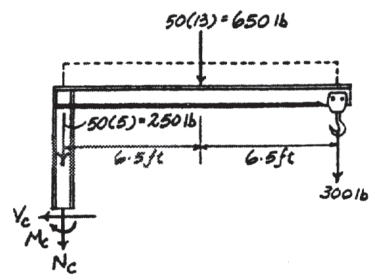
$V_B = 850\text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0;\qquad -M_B - 550(5.5) - 300(11) = 0$

$M_B = -6325\text{ lb}\cdot\text{ft} = -6.325\text{ kip}\cdot\text{ft}$ **Ans.**

Negative sign indicates that M_B acts in the opposite direction to that shown on the FBD.

Equations of Equilibrium: For point C,



$\xleftarrow{+}\ \Sigma F_x = 0;\qquad V_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad -N_C - 250 - 650 - 300 = 0$

$N_C = -1200\text{ lb} = -1.20\text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\qquad -M_C - 650(6.5) - 300(13) = 0$

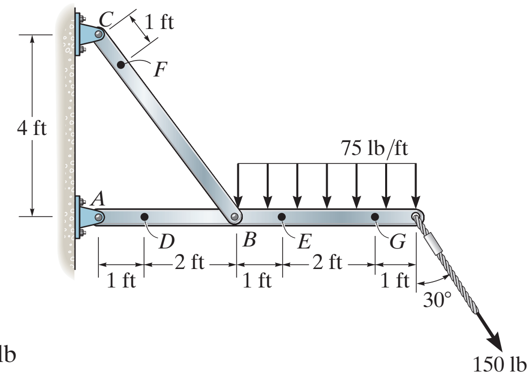
$M_C = -8125\text{ lb}\cdot\text{ft} = -8.125\text{ kip}\cdot\text{ft}$ **Ans.**

Negative signs indicate that N_C and M_C act in the opposite direction to that shown on the FBD.

Ans:
$N_A = 0, V_A = 450\text{ lb}, M_A = -1.125\text{ kip}\cdot\text{ft},$
$N_B = 0, V_B = 850\text{ lb}, M_B = -6.325\text{ kip}\cdot\text{ft},$
$V_C = 0, N_C = -1.20\text{ kip}, M_C = -8.125\text{ kip}\cdot\text{ft}$

7–11.

Determine the resultant internal loadings acting on the cross sections at points D and E of the frame.



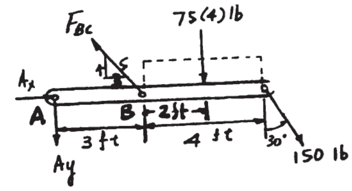
SOLUTION

Member AG:

$\circlearrowleft + \Sigma M_A = 0;\quad \frac{4}{5}F_{BC}(3) - 75(4)(5) - 150\cos 30°(7) = 0;\quad F_{BC} = 1003.89 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0;\quad A_y(3) - 75(4)(2) - 150\cos 30°(4) = 0;\quad A_y = 373.20 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - \frac{3}{5}(1003.89) + 150\sin 30° = 0;\quad A_x = 527.33 \text{ lb}$



For point D,

$\xrightarrow{+} \Sigma F_x = 0;\quad N_D + 527.33 = 0$

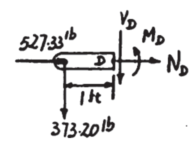
$N_D = -527 \text{ lb}$ **Ans.**

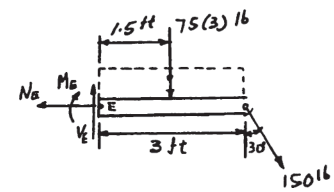
$+\uparrow \Sigma F_y = 0;\quad -373.20 - V_D = 0$

$V_D = -373 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;\quad M_D + 373.20(1) = 0$

$M_D = -373 \text{ lb} \cdot \text{ft}$ **Ans.**





For point E,

$\xrightarrow{+} \Sigma F_x = 0;\quad 150\sin 30° - N_E = 0$

$N_E = 75.0 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad V_E - 75(3) - 150\cos 30° = 0$

$V_E = 355 \text{ lb}$ **Ans.**

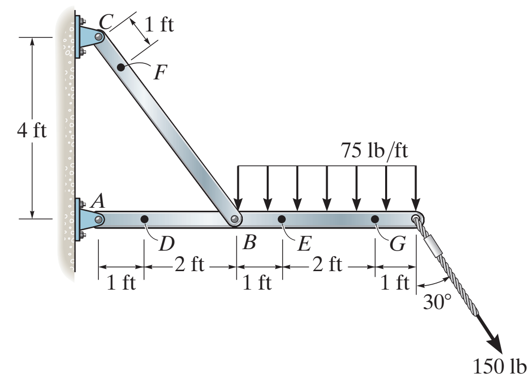
$\circlearrowleft + \Sigma M_E = 0;\quad -M_E - 75(3)(1.5) - 150\cos 30°(3) = 0;$

$M_E = -727 \text{ lb} \cdot \text{ft}$ **Ans.**

Ans:
$N_D = -527$ lb,
$V_D = -373$ lb,
$M_D = -373$ lb · ft,
$N_E = 75.0$ lb,
$V_E = 355$ lb,
$M_E = -727$ lb · ft

***7–12.**

Determine the resultant internal loadings acting on the cross sections at points F and G of the frame.

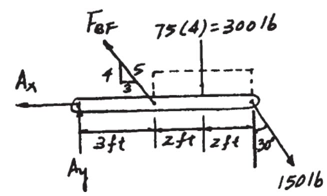


SOLUTION

Member AG:

$\circlearrowleft + \Sigma M_A = 0; \quad \frac{4}{5}F_{BF}(3) - 300(5) - 150\cos 30°(7) = 0$

$F_{BF} = 1003.9 \text{ lb}$



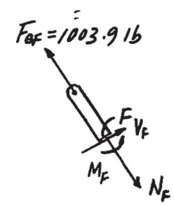
For point F,

$\overset{+}{\nearrow} \Sigma F_{x'} = 0; \quad V_F = 0$ **Ans.**

$\nwarrow + \Sigma F_{y'} = 0; \quad N_F - 1003.9 = 0$

$N_F = 1004 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_F = 0; \quad M_F = 0$ **Ans.**



For point G,

$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad N_G - 150\sin 30° = 0$

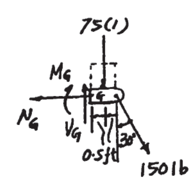
$N_G = 75.0 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_G - 75(1) - 150\cos 30° = 0$

$V_G = 205 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_G = 0; \quad -M_G - 75(1)(0.5) - 150\cos 30°(1) = 0$

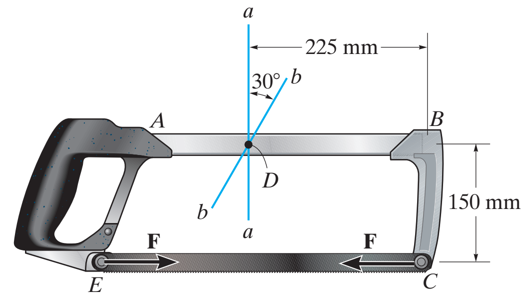
$M_G = -167 \text{ lb} \cdot \text{ft}$ **Ans.**



Ans:
$V_F = 0$,
$N_F = 1004$ lb,
$M_F = 0$,
$N_G = 75.0$ lb,
$V_G = 205$ lb,
$M_G = -167 \text{ lb} \cdot \text{ft}$

7–13.

The blade of the hacksaw is subjected to a pretension force of $F = 100$ N. Determine the resultant internal loadings acting on section a–a at point D.



SOLUTION

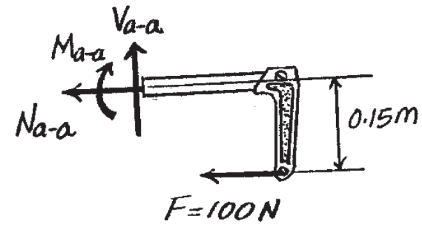
Internal Loadings: Referring to the free-body diagram of the section of the hacksaw shown in Fig. a,

$\overset{+}{\leftarrow} \Sigma F_x = 0;$ $\quad N_{a-a} + 100 = 0$ $\quad N_{a-a} = -100$ N **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_{a-a} = 0$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $\quad -M_{a-a} - 100(0.15) = 0$ $\quad M_{a-a} = -15\ \text{N}\cdot\text{m}$ **Ans.**

The negative sign indicates that $\mathbf{N}_{a-a}$ and $\mathbf{M}_{a-a}$ act in the opposite sense to that shown on the free-body diagram.

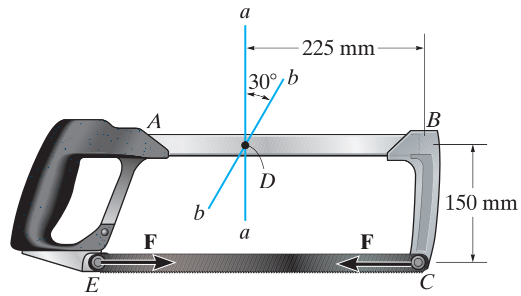


(a)

Ans:
$N_{a-a} = -100$ N, $V_{a-a} = 0$, $M_{a-a} = -15\ \text{N}\cdot\text{m}$

7–14.

The blade of the hacksaw is subjected to a pretension force of $F = 100$ N. Determine the resultant internal loadings acting on section b–b at point D.



SOLUTION

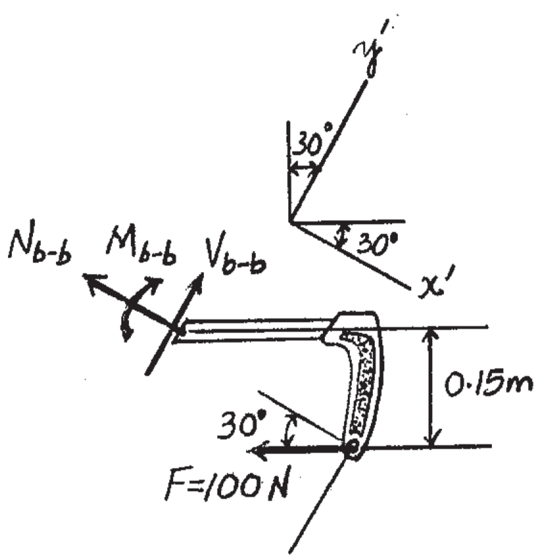
Internal Loadings: Referring to the free-body diagram of the section of the hacksaw shown in Fig. *a*,

$\Sigma F_{x'} = 0;$ $\quad N_{b-b} + 100 \cos 30° = 0 \quad N_{b-b} = -86.6 \text{ N}$ **Ans.**

$\Sigma F_{y'} = 0;$ $\quad V_{b-b} - 100 \sin 30° = 0 \quad V_{b-b} = 50 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $\quad -M_{b-b} - 100(0.15) = 0 \quad M_{b-b} = -15 \text{ N} \cdot \text{m}$ **Ans.**

The negative sign indicates that $\mathbf{N}_{b-b}$ and $\mathbf{M}_{b-b}$ act in the opposite sense to that shown on the free-body diagram.

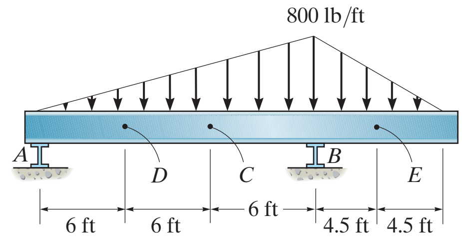


(a)

Ans:
$N_{b-b} = -86.6$ N, $V_{b-b} = 50$ N,
$M_{b-b} = -15$ N·m

7–15.

The beam supports the triangular distributed load shown. Determine the resultant internal loadings on the cross section at point C. Assume the reactions at the supports A and B are vertical.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \quad \frac{1}{2}(0.8)(18)(6) - \frac{1}{2}(0.8)(9)(3) - A_y(18) = 0 \qquad A_y = 1.80 \text{ kip}$$

Internal Loadings: Referring to the FBD of the left beam segment sectioned through point C, Fig. *b*,

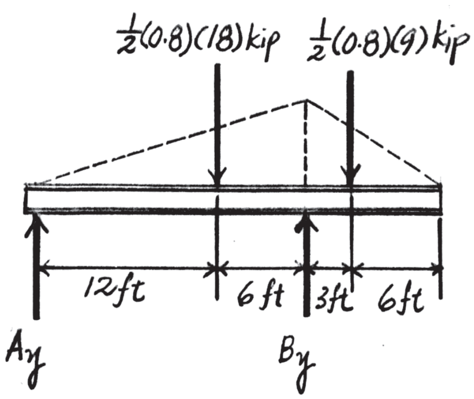
$$\xrightarrow{+} \Sigma F_x = 0; \quad N_C = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 1.80 - \frac{1}{2}(0.5333)(12) - V_C = 0 \qquad V_C = -1.40 \text{ kip} \qquad \textbf{Ans.}$$

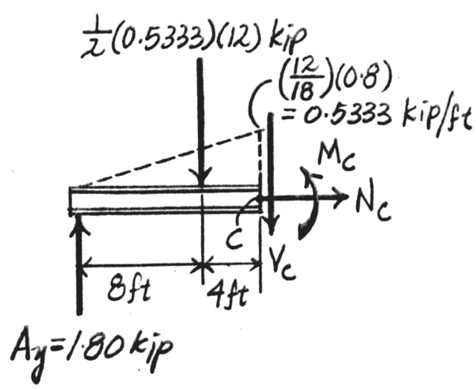
$$\circlearrowleft + \Sigma M_C = 0; \quad M_C + \frac{1}{2}(0.5333)(12)(4) - 1.80(12) = 0$$

$$M_C = 8.80 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

The negative sign indicates that $\mathbf{V}_C$ acts in the sense opposite to that shown on the FBD.



(a)

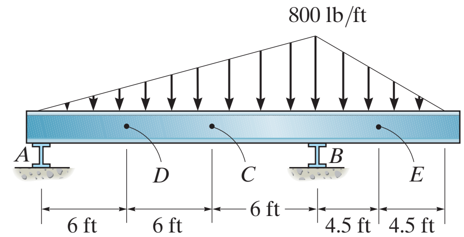


(b)

Ans:
$N_C = 0,$
$V_C = -1.40 \text{ kip},$
$M_C = 8.80 \text{ kip} \cdot \text{ft}$

***7–16.**

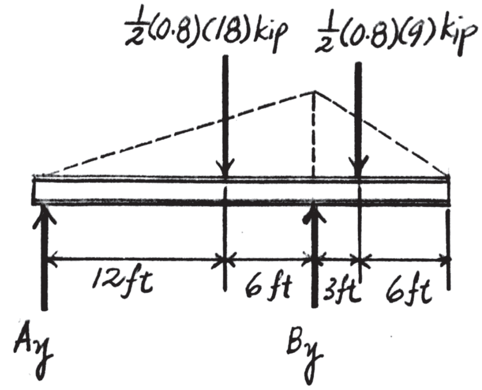
The beam supports the distributed load shown. Determine the resultant internal loadings on the cross sections at points D and E. Assume the reactions at the supports A and B are vertical.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$\circlearrowleft + \Sigma M_B = 0;\quad \frac{1}{2}(0.8)(18)(6) - \frac{1}{2}(0.8)(9)(3) - A_y(18) = 0 \qquad A_y = 1.80 \text{ kip}$



(a)

Internal Loadings: Referring to the FBD of the left segment of the beam section through D, Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad 1.80 - \frac{1}{2}(0.2667)(6) - V_D = 0 \qquad V_D = 1.00 \text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;\qquad M_D + \frac{1}{2}(0.2667)(6)(2) - 1.80(6) = 0$

$M_D = 9.20 \text{ kip} \cdot \text{ft}$ **Ans.**

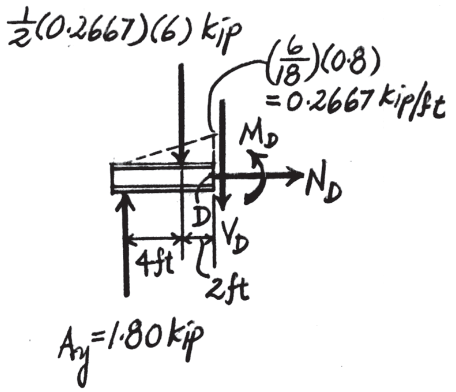
Referring to the FBD of the right segment of the beam sectioned through E, Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad N_E = 0$ **Ans.**

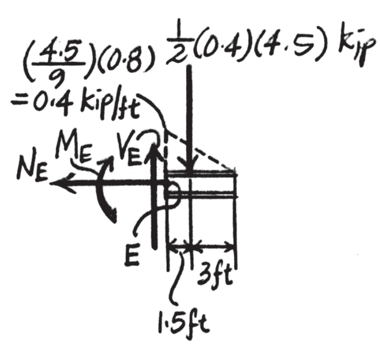
$+\uparrow \Sigma F_y = 0;\qquad V_E - \frac{1}{2}(0.4)(4.5) = 0 \qquad V_E = 0.900 \text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;\qquad -M_E - \frac{1}{2}(0.4)(4.5)(1.5) = 0 \qquad M_E = -1.35 \text{ kip} \cdot \text{ft}$ **Ans.**

The negative sign indicates that $\mathbf{M}_E$ act in the sense opposite to that shown in Fig. *c*.



(b)

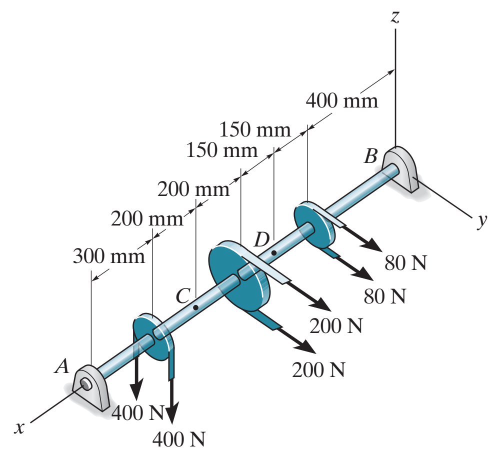


(c)

Ans:
$N_D = 0,$
$V_D = 1.00 \text{ kip},$
$M_D = 9.20 \text{ kip} \cdot \text{ft},$
$N_E = 0,$
$V_E = 0.900 \text{ kip},$
$M_E = -1.35 \text{ kip} \cdot \text{ft}$

7–17.

The shaft is supported at its ends by two bearings A and B and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section at point D. The 400-N forces act in the $-z$ direction and the 200-N and 80-N forces act in the $+y$ direction. The journal bearings at A and B exert only y and z components of force on the shaft.



SOLUTION

Support Reactions:

$\Sigma M_z = 0; \qquad 160(0.4) + 400(0.7) - A_y(1.4) = 0$

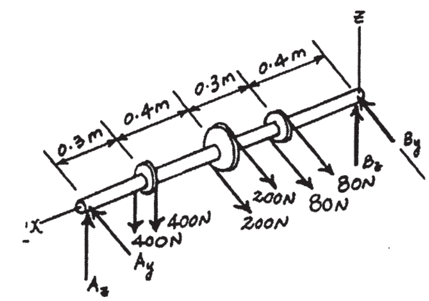
$A_y = 245.71\text{ N}$

$\Sigma F_y = 0; \qquad -245.71 - B_y + 400 + 160 = 0$

$B_y = 314.29\text{ N}$

$\Sigma M_y = 0; \qquad 800(1.1) - A_z(1.4) = 0 \qquad A_z = 628.57\text{ N}$

$\Sigma F_z = 0; \qquad B_z + 628.57 - 800 = 0 \qquad B_z = 171.43\text{ N}$



Equations of Equilibrium: For point D,

$\Sigma F_x = 0; \qquad (N_D)_x = 0$ **Ans.**

$\Sigma F_y = 0; \qquad (V_D)_y - 314.29 + 160 = 0$

$(V_D)_y = 154\text{ N}$ **Ans.**

$\Sigma F_z = 0; \qquad 171.43 + (V_D)_z = 0$

$(V_D)_z = -171\text{ N}$ **Ans.**

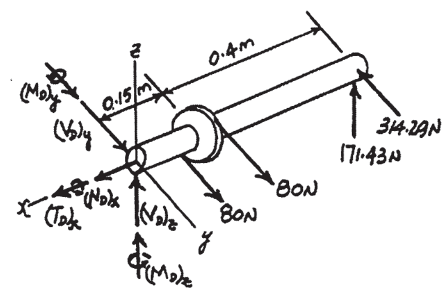
$\Sigma M_x = 0; \qquad (T_D)_x = 0$ **Ans.**

$\Sigma M_y = 0; \qquad 171.43(0.55) + (M_D)_y = 0$

$(M_D)_y = -94.3\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_z = 0; \qquad 314.29(0.55) - 160(0.15) + (M_D)_z = 0$

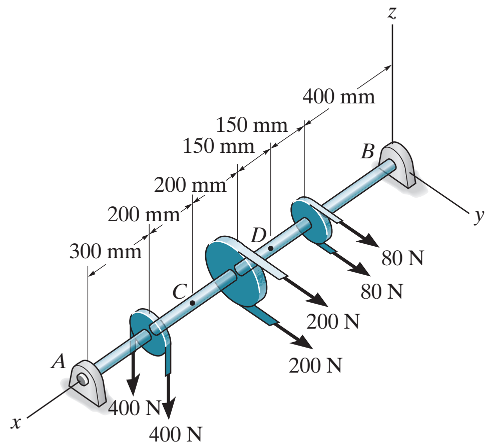
$(M_D)_z = -149\text{ N}\cdot\text{m}$ **Ans.**



Ans:
$(N_D)_x = 0$,
$(V_D)_y = 154\text{ N}$,
$(V_D)_z = -171\text{ N}$,
$(T_D)_x = 0$,
$(M_D)_y = -94.3\text{ N}\cdot\text{m}$,
$(M_D)_z = -149\text{ N}\cdot\text{m}$

7–18.

The shaft is supported at its ends by two bearings A and B and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section at point C. The 400-N forces act in the $-z$ direction and the 200-N and 80-N forces act in the $+y$ direction. The journal bearings at A and B exert only y and z components of force on the shaft.



SOLUTION

Support Reactions:

$\Sigma M_z = 0; \qquad 160(0.4) + 400(0.7) - A_y(1.4) = 0$

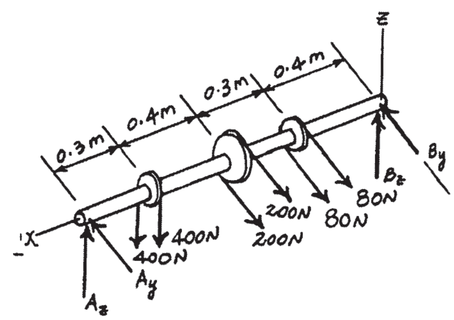
$A_y = 245.71 \text{ N}$

$\Sigma F_y = 0; \qquad -245.71 - B_y + 400 + 160 = 0$

$B_y = 314.29 \text{ N}$

$\Sigma M_y = 0; \qquad 800(1.1) - A_z(1.4) = 0 \qquad A_z = 628.57 \text{ N}$

$\Sigma F_z = 0; \qquad B_z + 628.57 - 800 = 0 \qquad B_z = 171.43 \text{ N}$



Equations of Equilibrium: For point C

$\Sigma F_x = 0; \qquad (N_C)_x = 0$ **Ans.**

$\Sigma F_y = 0; \qquad -245.71 + (V_C)_y = 0$

$(V_C)_y = -246 \text{ N}$ **Ans.**

$\Sigma F_z = 0; \qquad 628.57 - 800 + (V_C)_z = 0$

$(V_C)_z = -171 \text{ N}$ **Ans.**

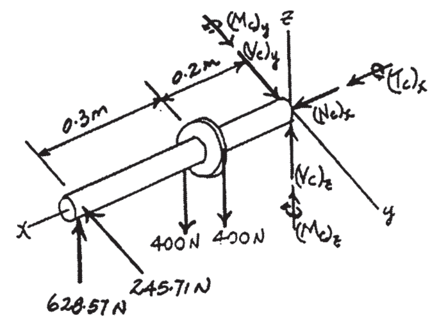
$\Sigma M_x = 0; \qquad (T_C)_x = 0$ **Ans.**

$\Sigma M_y = 0; \qquad (M_C)_y - 628.57(0.5) + 800(0.2) = 0$

$(M_C)_y = -154 \text{ N} \cdot \text{m}$ **Ans.**

$\Sigma M_z = 0; \qquad (M_C)_z - 245.71(0.5) = 0$

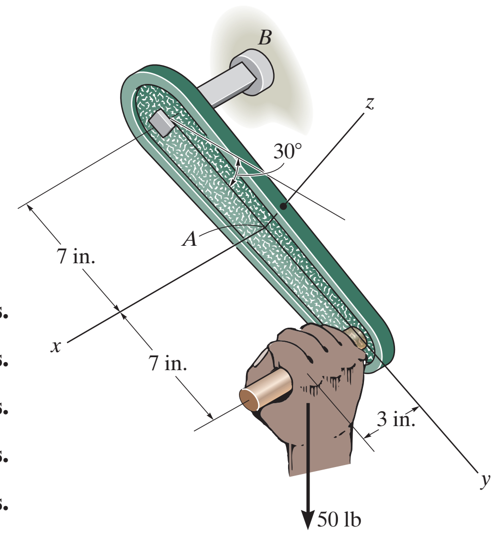
$(M_C)_z = -123 \text{ N} \cdot \text{m}$ **Ans.**



Ans:
$(N_C)_x = 0,$
$(V_C)_y = -246 \text{ N},$
$(V_C)_z = -171 \text{ N},$
$(T_C)_x = 0,$
$(M_C)_y = -154 \text{ N} \cdot \text{m},$
$(M_C)_z = -123 \text{ N} \cdot \text{m}$

7–19.

The hand crank that is used in a press has the dimensions shown. Determine the resultant internal loadings acting on the cross section at point A if a vertical force of 50 lb is applied to the handle as shown. Assume the crank is fixed to the shaft at B.



SOLUTION

$\Sigma F_x = 0;\quad (V_A)_x = 0$ **Ans.**

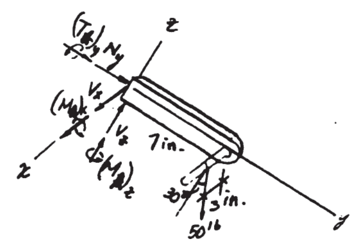
$\Sigma F_y = 0;\quad (N_A)_y + 50 \sin 30° = 0;\quad (N_A)_y = -25 \text{ lb}$ **Ans.**

$\Sigma F_z = 0;\quad (V_A)_z - 50 \cos 30° = 0;\quad (V_A)_z = 43.3 \text{ lb}$ **Ans.**

$\Sigma M_x = 0;\quad (M_A)_x - 50 \cos 30°(7) = 0;\quad (M_A)_x = 303 \text{ lb} \cdot \text{in.}$ **Ans.**

$\Sigma M_y = 0;\quad (T_A)_y + 50 \cos 30°(3) = 0;\quad (T_A)_y = -130 \text{ lb} \cdot \text{in.}$ **Ans.**

$\Sigma M_z = 0;\quad (M_A)_z + 50 \sin 30°(3) = 0;\quad (M_A)_z = -75 \text{ lb} \cdot \text{in.}$ **Ans.**

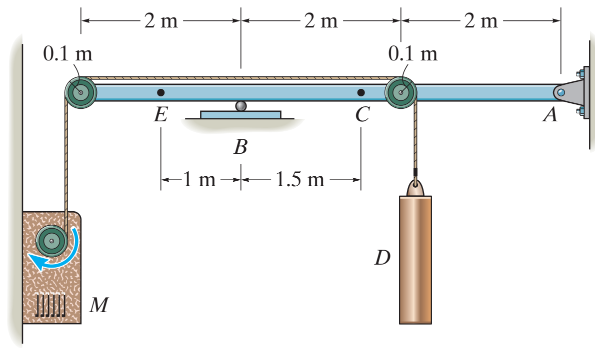


Ans:

$(V_A)_x = 0,$
$(N_A)_y = -25 \text{ lb},$
$(V_A)_z = 43.3 \text{ lb},$
$(M_A)_x = 303 \text{ lb} \cdot \text{in.},$
$(T_A)_y = -130 \text{ lb} \cdot \text{in.},$
$(M_A)_z = -75 \text{ lb} \cdot \text{in.}$

***7–20.**

Determine the resultant internal loadings acting on the cross section at point *C* in the beam. The load *D* has a mass of 300 kg and is being hoisted by the motor *M* with constant velocity.



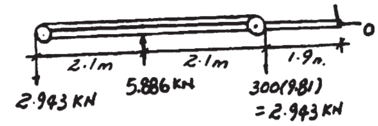
SOLUTION

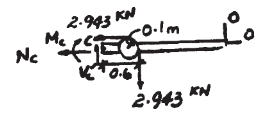
$\xleftarrow{+} \Sigma F_x = 0; \qquad N_C + 2.943 = 0; \qquad N_C = -2.94 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_C - 2.943 = 0; \qquad V_C = 2.94 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \qquad -M_C - 2.943(0.6) + 2.943(0.1) = 0$

$M_C = -1.47 \text{ kN} \cdot \text{m}$ **Ans.**



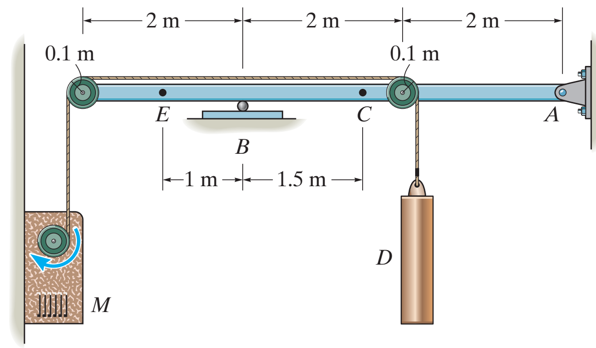


Ans:

$N_C = -2.94$ kN,
$V_C = 2.94$ kN,
$M_C = -1.47$ kN · m

7–21.

Determine the resultant internal loadings acting on the cross section at point E. The load D has a mass of 300 kg and is being hoisted by the motor M with constant velocity.



SOLUTION

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad N_E + 2943 = 0$

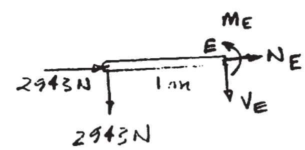
$N_E = -2.94\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad -2943 - V_E = 0$

$V_E = -2.94\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;$ $\quad M_E + 2943(1) = 0$

$M_E = -2.94\text{ kN}\cdot\text{m}$ **Ans.**

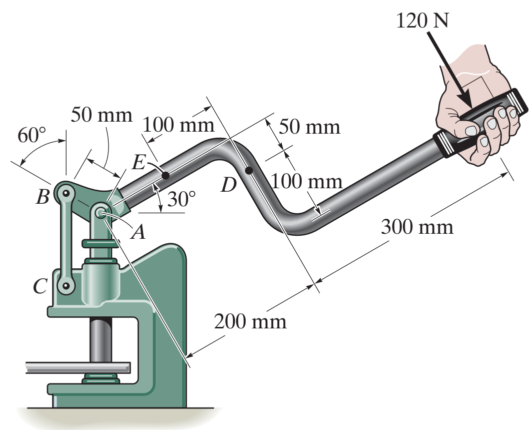


Ans:

$N_E = -2.94\text{ kN},$
$V_E = -2.94\text{ kN},$
$M_E = -2.94\text{ kN}\cdot\text{m}$

7–22.

The metal stud punch is subjected to a force of 120 N on the handle. Determine the magnitude of the reactive force at the pin A and in the short link BC. Also, determine the resultant internal loadings acting on the cross section at point D.



SOLUTION

Member:

$\circlearrowleft + \Sigma M_A = 0;\qquad F_{BC}\cos 30°(50) - 120(500) = 0$

$F_{BC} = 1385.6\text{ N} = 1.39\text{ kN}$ **Ans.**

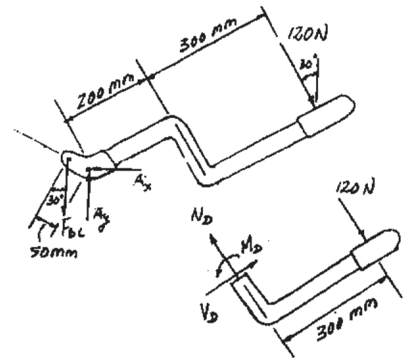
$+\uparrow \Sigma F_y = 0;\qquad A_y - 1385.6 - 120\cos 30° = 0$

$A_y = 1489.56\text{ N}$

$\overset{+}{\leftarrow} \Sigma F_x = 0;\qquad A_x - 120\sin 30° = 0;\qquad A_x = 60\text{ N}$

$F_A = \sqrt{1489.56^2 + 60^2}$

$= 1491\text{ N} = 1.49\text{ kN}$ **Ans.**



Segment:

$\nwarrow^{+} \Sigma F_{x'} = 0;\qquad N_D - 120 = 0$

$N_D = 120\text{ N}$ **Ans.**

$^{+}\nearrow \Sigma F_{y'} = 0;\qquad V_D = 0$ **Ans.**

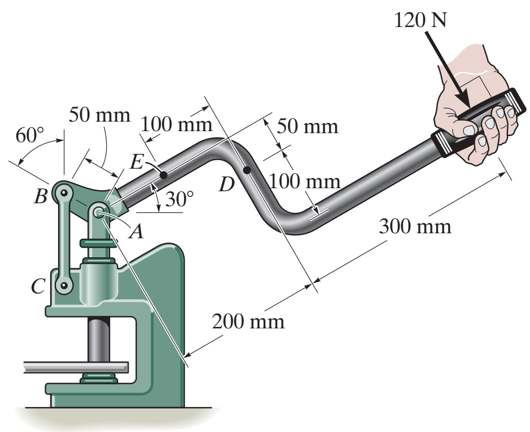
$\circlearrowleft + \Sigma M_D = 0;\qquad M_D - 120(0.3) = 0$

$M_D = 36.0\text{ N}\cdot\text{m}$ **Ans.**

Ans:
$F_{BC} = 1.39\text{ kN}, F_A = 1.49\text{ kN}, N_D = 120\text{ N},$
$V_D = 0, M_D = 36.0\text{ N}\cdot\text{m}$

7–23.

Determine the resultant internal loadings acting on the cross section at point E of the handle arm, and on the cross section of the short link BC.



SOLUTION

Member:

$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC} \cos 30°(50) - 120(500) = 0$

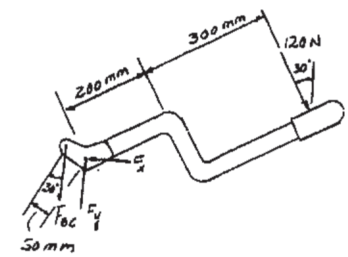
$F_{BC} = 1385.6 \text{ N} = 1.3856 \text{ kN}$

Segment:

$\swarrow + \Sigma F_{x'} = 0; \qquad N_E = 0$ **Ans.**

$\nwarrow + \Sigma F_{y'} = 0; \qquad V_E - 120 = 0; \qquad V_E = 120 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \qquad M_E - 120(0.4) = 0; \qquad M_E = 48.0 \text{ N} \cdot \text{m}$ **Ans.**

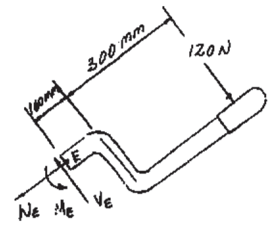


Short link:

$\xleftarrow{+} \Sigma F_x = 0; \qquad V = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 1.3856 - N = 0; \qquad N = 1.39 \text{ kN}$ **Ans.**

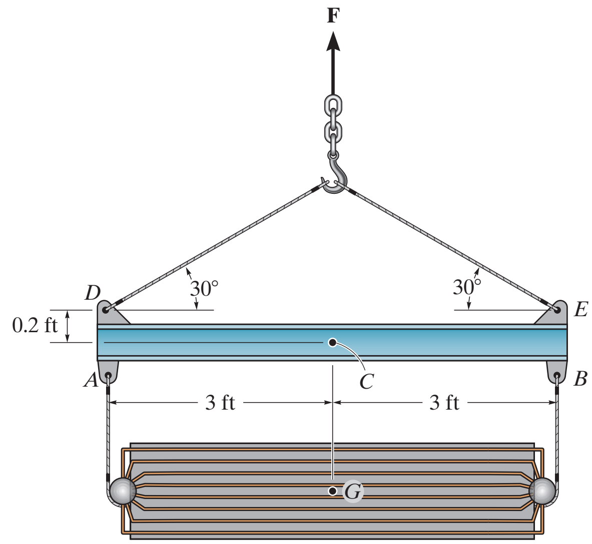
$\circlearrowleft + \Sigma M_H = 0; \qquad M = 0$ **Ans.**



Ans:
$N_E = 0, V_E = 120 \text{ N}, M_E = 48.0 \text{ N} \cdot \text{m},$
Short link: $V = 0, N = 1.39 \text{ kN}, M = 0$

***7–24.**

Determine the resultant internal loadings acting on the cross section at point C. The cooling unit has a total weight of 52 kip and a center of gravity at G.



SOLUTION

From FBD (a)

$\circlearrowleft + \Sigma M_A = 0; \quad T_B(6) - 52(3) = 0; \quad T_B = 26 \text{ kip}$

From FBD (b)

$\circlearrowleft + \Sigma M_D = 0; \quad T_E \sin 30°(6) - 26(6) = 0; \quad T_E = 52 \text{ kip}$

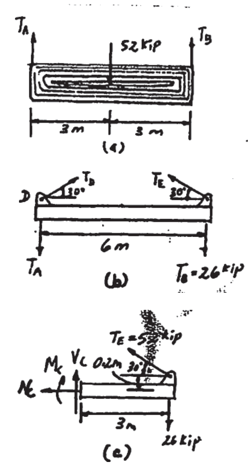
From FBD (c)

$\xrightarrow{+} \Sigma F_x = 0; \quad -N_C - 52 \cos 30° = 0; \quad N_C = -45.0 \text{ kip}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_C + 52 \sin 30° - 26 = 0; \quad V_C = 0$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad 52 \cos 30°(0.2) + 52 \sin 30°(3) - 26(3) - M_C = 0$

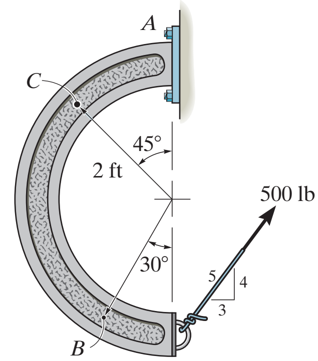
$M_C = 9.00 \text{ kip} \cdot \text{ft}$ **Ans.**



Ans:
$N_C = -45.0$ kip,
$V_C = 0$,
$M_C = 9.00$ kip · ft

7–25.

Determine the resultant internal loadings acting on the cross section at points B and C of the curved member.



SOLUTION

From FBD (a),

$\nearrow + \Sigma F_{x'} = 0;$ $\quad 400 \cos 30° + 300 \cos 60° - V_B = 0$

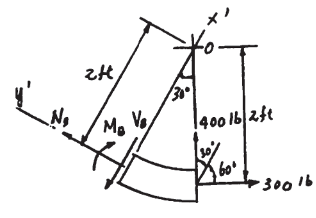
$V_B = 496 \text{ lb}$ **Ans.**

$\nwarrow + \Sigma F_{y'} = 0;$ $\quad N_B + 400 \sin 30° - 300 \sin 60° = 0$

$N_B = 59.80 = 59.8 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_O = 0;$ $\quad 300(2) - 59.80(2) - M_B = 0$

$M_B = 480 \text{ lb} \cdot \text{ft}$ **Ans.**



(a)

From FBD (b),

$\nearrow + \Sigma F_{x'} = 0;$ $\quad 400 \cos 45° + 300 \cos 45° - N_C = 0$

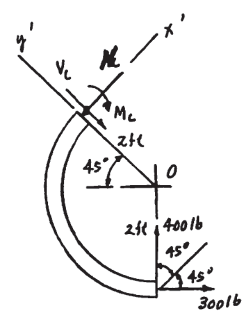
$N_C = 495 \text{ lb}$ **Ans.**

$\nwarrow + \Sigma F_{y'} = 0;$ $\quad -V_C + 400 \sin 45° - 300 \sin 45° = 0$

$V_C = 70.7 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_O = 0;$ $\quad 300(2) + 495(2) - M_C = 0$

$M_C = 1590 \text{ lb} \cdot \text{ft} = 1.59 \text{ kip} \cdot \text{ft}$ **Ans.**

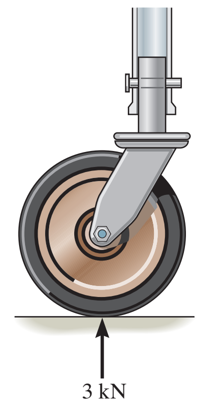


(b)

Ans:
$V_B = 496$ lb,
$N_B = 59.8$ lb,
$M_B = 480$ lb·ft,
$N_C = 495$ lb,
$V_C = 70.7$ lb,
$M_C = 1.59$ kip·ft

7–26.

The supporting wheel on a scaffold is held in place on the leg using a 4-mm-diameter pin. If the wheel is subjected to a normal force of 3 kN, determine the average shear stress in the pin. Assume the pin only supports the vertical 3-kN load.



SOLUTION

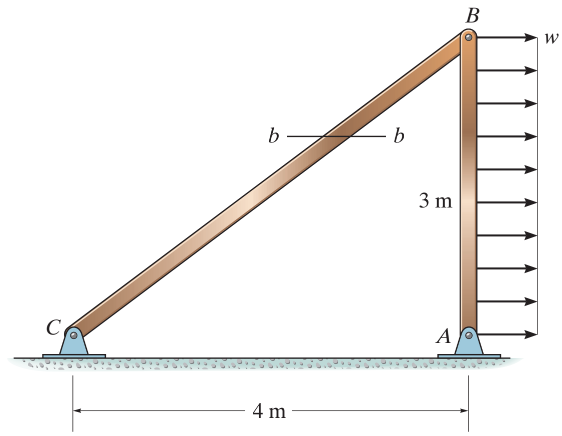
$+\uparrow \Sigma F_y = 0;$ $\quad 3\text{ kN} \cdot 2V = 0;$ $\quad V = 1.5\text{ kN}$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{1.5(10^3)}{\frac{\pi}{4}(0.004)^2} = 119\text{ MPa}$$ **Ans.**

Ans:
$\tau_{\text{avg}} = 119\text{ MPa}$

7–27.

Determine the largest intensity w of the uniform loading that can be applied to the frame without causing either the average normal stress or the average shear stress at section b–b to exceed $\sigma = 15$ MPa and $\tau = 16$ MPa, respectively. Member CB has a square cross section of 30 mm on each side.

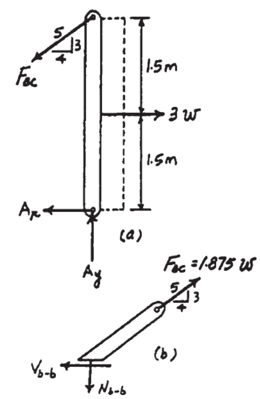


SOLUTION

Support Reactions: FBD(a)

$\circlearrowleft + \Sigma M_A = 0; \qquad \frac{4}{5}F_{BC}(3) - 3w(1.5) = 0 \qquad F_{BC} = 1.875w$

Equations of Equilibrium: For section b–b, FBD(b)



$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{4}{5}(1.875w) - V_{b-b} = 0 \qquad V_{b-b} = 1.50w$

$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5}(1.875w) - N_{b-b} = 0 \qquad N_{b-b} = 1.125w$

Average Normal Stress and Shear Stress: The cross-sectional area of section b–b, $A' = \dfrac{5A}{3}$, where $A = (0.03)(0.03) = 0.90\left(10^{-3}\right)\text{ m}^2$.

Then $A' = \dfrac{5}{3}(0.90)\left(10^{-3}\right) = 1.50\left(10^{-3}\right)\text{ m}^2$.

Assume failure due to normal stress.

$$(\sigma_{b-b})_{\text{Allow}} = \frac{N_{b-b}}{A'}; \qquad 15\left(10^6\right) = \frac{1.125w}{1.50\left(10^{-3}\right)}$$

$$w = 20000\text{ N/m} = 20.0\text{ kN/m}$$

Assume failure due to shear stress.

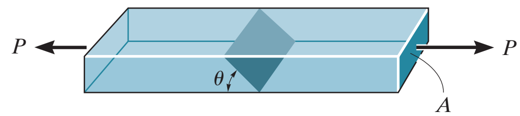
$$(\tau_{b-b})_{\text{Allow}} = \frac{V_{b-b}}{A'}; \qquad 16(10^6) = \frac{1.50w}{1.50(10^{-3})}$$

$w = 16000\text{ N/m} = 16.0\text{ kN/m}$ ***(Controls !)*** **Ans.**

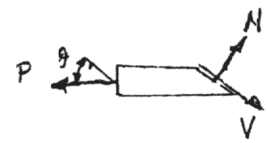
Ans:
$w = 16.0\text{ kN/m}$

***7–28.**

The bar has a cross-sectional area A and is subjected to the axial load P. Determine the average normal and average shear stresses acting over the shaded section, which is oriented at θ from the horizontal. Plot the variation of these stresses as a function of θ $(0 \leq \theta \leq 90°)$.



SOLUTION



Equations of Equilibrium:

$\searrow + \Sigma F_x = 0; \qquad V - P\cos\theta = 0 \qquad V = P\cos\theta$

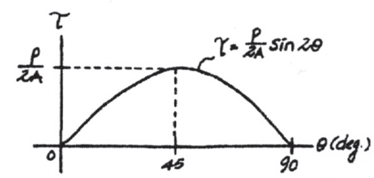
$\nearrow + \Sigma F_y = 0; \qquad N - P\sin\theta = 0 \qquad N = P\sin\theta$

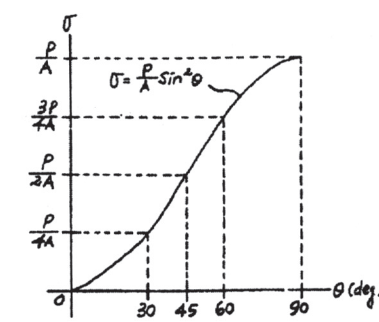
Average Normal Stress and Shear Stress: Area at θ plane, $A' = \dfrac{A}{\sin\theta}$.

$$\sigma_{\text{avg}} = \frac{N}{A'} = \frac{P\sin\theta}{\frac{A}{\sin\theta}} = \frac{P}{A}\sin^2\theta \qquad \textbf{Ans.}$$

$$\tau_{\text{avg}} = \frac{V}{A'} = \frac{P\cos\theta}{\frac{A}{\sin\theta}}$$

$$= \frac{P}{A}\sin\theta\cos\theta = \frac{P}{2A}\sin 2\theta \qquad \textbf{Ans.}$$

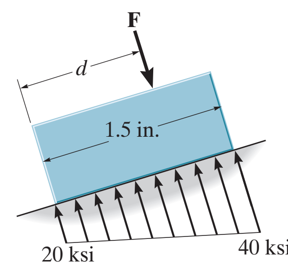




Ans:
$\sigma_{\text{avg}} = \dfrac{P}{A}\sin^2\theta, \ \tau_{\text{avg}} = \dfrac{P}{2A}\sin 2\theta$

7–29.

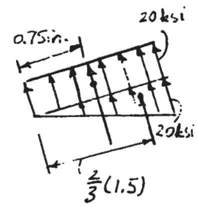
The small block has a thickness of 0.5 in. If the stress distribution at the support developed by the load varies as shown, determine the force **F** applied to the block, and the distance d to where it is applied.



SOLUTION

$$F = \int \sigma dA = \text{volume under load diagram}$$

$$F = 20(1.5)(0.5) + \frac{1}{2}(20)(1.5)(0.5) = 22.5 \text{ kip}$$ **Ans.**



$$Fd = \int x(\sigma\, dA)$$

$$(22.5)d = (0.75)(20)(1.5)(0.5) + \frac{2}{3}(1.5)\left(\frac{1}{2}\right)(20)(1.5)(0.5)$$

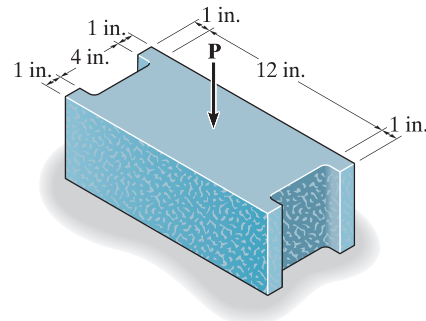
$$(22.5)d = 18.75$$

$$d = 0.833 \text{ in.}$$ **Ans.**

Ans:
$F = 22.5$ kip,
$d = 0.833$ in.

7–30.

If the material fails when the average normal stress reaches 120 psi, determine the largest centrally applied vertical load **P** the block can support.



SOLUTION

Average Normal Stress: The cross-sectional area of the block is

$$A = 14(6) - 2\left[4(1)\right] = 76 \text{ in}^2$$

Thus,

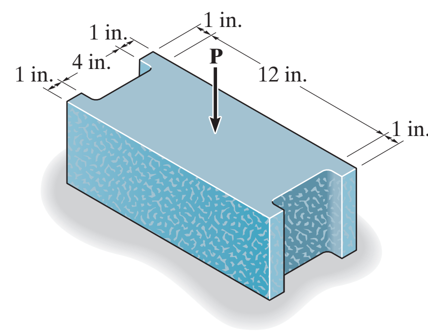
$$\sigma_{\text{allow}} = \frac{N_{\text{allow}}}{A}; \qquad 120 = \frac{P_{\text{allow}}}{76}$$

$$P_{\text{allow}} = 9120 \text{ lb} = 9.12 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 9.12$ kip

7–31.

If the block is subjected to a centrally applied force of $P = 6$ kip, determine the average normal stress in the material. Show the stress acting on a differential volume element of the material.



SOLUTION

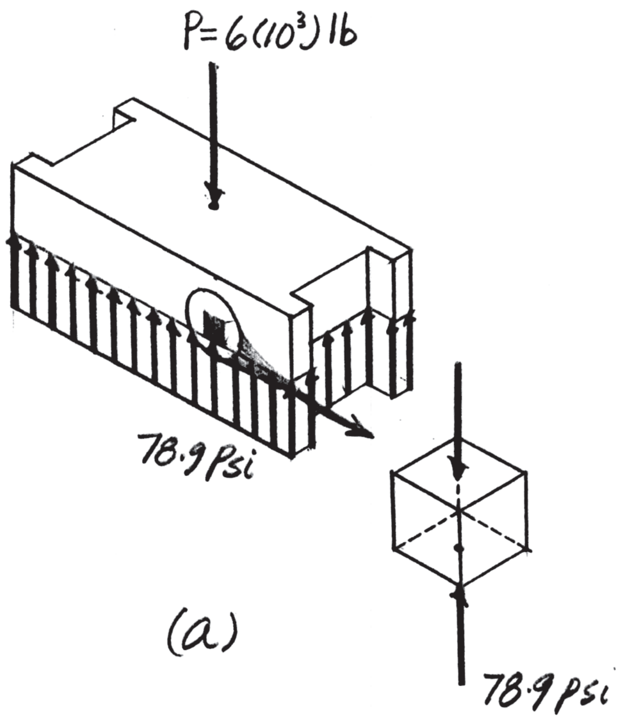
Average Normal Stress: The cross-sectional area of the block is

$$A = 14(6) - 2\left[4(1)\right] = 76 \text{ in}^2$$

Thus,

$$\sigma = \frac{N}{A} = \frac{6\left(10^3\right)}{76} = 78.947 \text{ psi} = 78.9 \text{ psi} \qquad \textbf{Ans.}$$

The average normal stress acting on the differential volume element is shown in Fig. *a*.

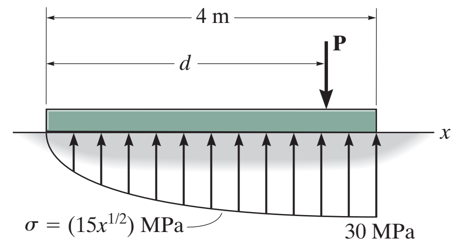


(a)

Ans:
$\sigma = 78.9$ psi

***7–32.**

The plate has a width of 0.5 m. If the stress distribution at the support varies as shown, determine the force **P** applied to the plate and the distance d to where it is applied.



SOLUTION

The resultant force dF of the bearing pressure acting on the plate of area $dA = b\,dx = 0.5\,dx$, Fig. a,

$$dF = \sigma_b\,dA = (15x^{\frac{1}{2}})(10^6)(0.5dx) = 7.5(10^6)x^{\frac{1}{2}}\,dx$$

$$+\uparrow\Sigma F_y = 0; \qquad \int dF - P = 0$$

$$\int_0^{4\,\text{m}} 7.5(10^6)x^{\frac{1}{2}}\,dx - P = 0$$

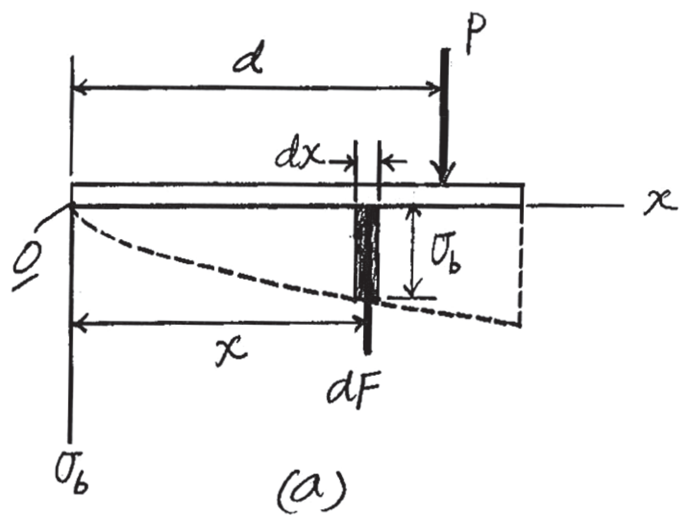
$$P = 40(10^6)\text{ N} = 40\text{ MN} \qquad \textbf{Ans.}$$

Equilibrium requires

$$\circlearrowleft+\Sigma M_O = 0; \qquad \int x\,dF - Pd = 0$$

$$\int_0^{4\,\text{m}} x[7.5(10^6)x^{\frac{1}{2}}\,dx] - 40(10^6)\,d = 0$$

$$d = 2.40\text{ m} \qquad \textbf{Ans.}$$

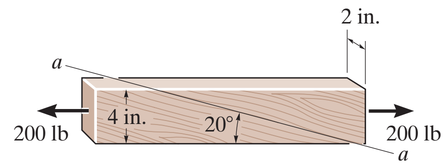


(a)

Ans:
$P = 40$ MN, $d = 2.40$ m

7–33.

The board is subjected to a tensile force of 200 lb. Determine the average normal and average shear stress in the wood fibers, which are oriented along plane *a–a* at 20° with the axis of the board.



SOLUTION

Internal Loadings: Referring to the FBD of the lower segment of the board sectioned through plane *a–a*, Fig. *a*,

$\Sigma F_x = 0;\qquad N - 200 \sin 20° = 0 \qquad N = 68.40 \text{ lb}$

$\Sigma F_y = 0;\qquad 200 \cos 20° - V = 0 \qquad V = 187.94 \text{ lb}$

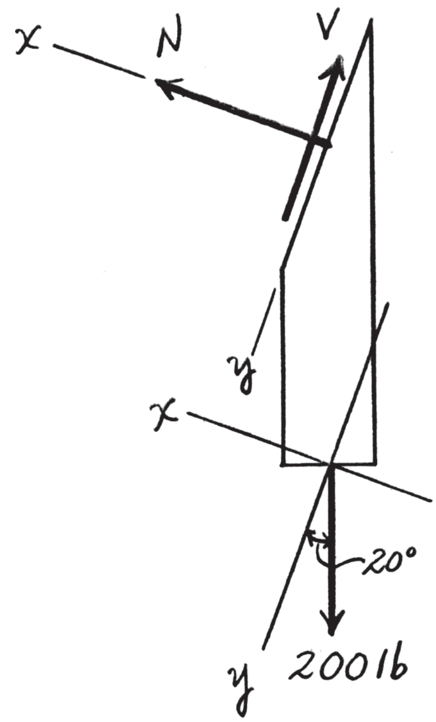
Average Normal and Shear Stress: The area of plane *a–a* is

$$A = 2\left(\frac{4}{\sin 20°}\right) = 23.39 \text{ in}^2$$

Then,

$$\sigma = \frac{N}{A} = \frac{68.40}{23.39} = 2.92 \text{ psi} \qquad \textbf{Ans.}$$

$$\tau = \frac{V}{A} = \frac{187.94}{23.39} = 8.03 \text{ psi} \qquad \textbf{Ans.}$$

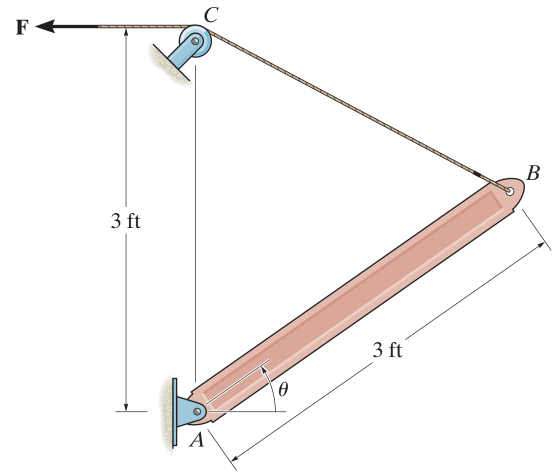


(a)

Ans:
$\sigma = 2.92$ psi,
$\tau = 8.03$ psi

7–34.

The boom has a uniform weight of 600 lb and is hoisted into position using the cable BC. If the cable has a diameter of 0.5 in., plot the average normal stress in the cable as a function of the boom position θ for $0° \leq \theta \leq 90°$.

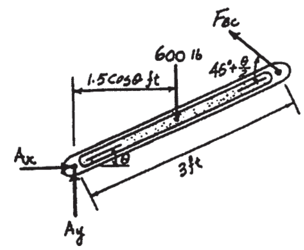


SOLUTION

Support Reactions:

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC} \sin\left(45° + \frac{\theta}{2}\right)(3) - 600(1.5 \cos\theta) = 0$$

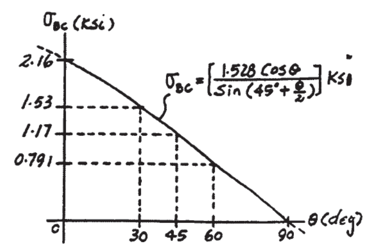
$$F_{BC} = \frac{300 \cos\theta}{\sin\left(45° + \frac{\theta}{2}\right)}$$



Average Normal Stress:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{\dfrac{300 \cos\theta}{\sin\left(45° + \frac{\theta}{2}\right)}}{\frac{\pi}{4}\left(0.5^2\right)}$$

$$= \left\{ \frac{1.528 \cos\theta}{\sin\left(45° + \frac{\theta}{2}\right)} \right\} \text{ksi} \qquad \textbf{Ans.}$$

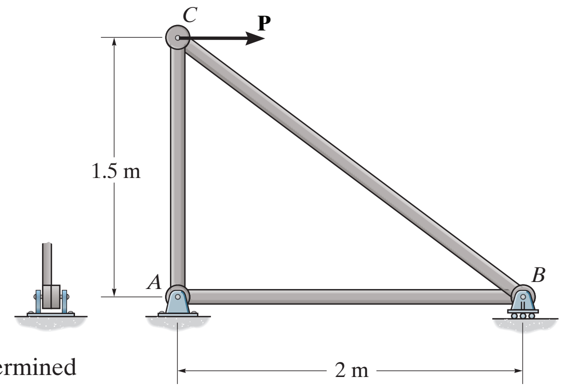


Ans:

$$\sigma_{BC} = \left\{ \frac{1.528 \cos\theta}{\sin\left(45° + \frac{\theta}{2}\right)} \right\} \text{ksi}$$

7–35.

Determine the average normal stress in each of the 20-mm-diameter bars of the truss. Set $P = 40$ kN.



SOLUTION

Internal Loadings: The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint C, Fig. a.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 40 - F_{BC}\left(\frac{4}{5}\right) = 0 \qquad F_{BC} = 50 \text{ kN (C)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 50\left(\frac{3}{5}\right) - F_{AC} = 0 \qquad F_{AC} = 30 \text{ kN (T)}$$

Subsequently, the equilibrium of joint B, Fig. b, is considered.

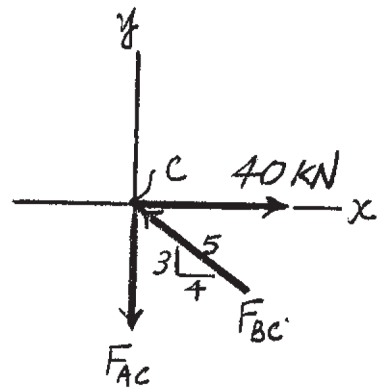
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 50\left(\frac{4}{5}\right) - F_{AB} = 0 \qquad F_{AB} = 40 \text{ kN (T)}$$

Average Normal Stress: The cross-sectional area of each of the bars is $A = \frac{\pi}{4}(0.02^2) = 0.3142(10^{-3})\text{ m}^2$. We obtain

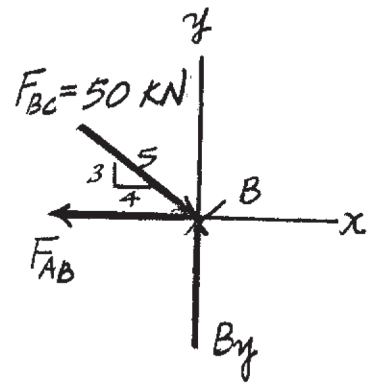
$$(\sigma_{\text{avg}})_{BC} = \frac{F_{BC}}{A} = \frac{50(10^3)}{0.3142(10^{-3})} = 159 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\text{avg}})_{AC} = \frac{F_{AC}}{A} = \frac{30(10^3)}{0.3142(10^{-3})} = 95.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_{\text{avg}})_{AB} = \frac{F_{AB}}{A} = \frac{40(10^3)}{0.3142(10^{-3})} = 127 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

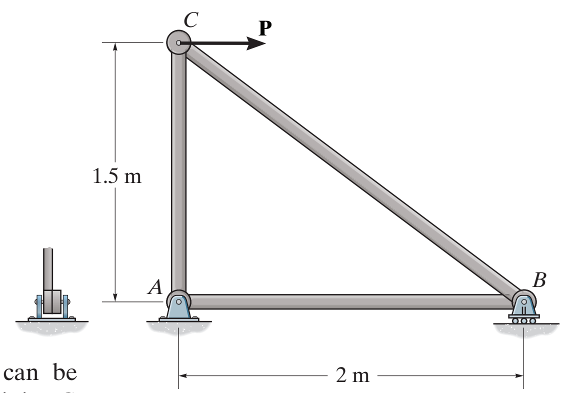


(b)

Ans:
$(\sigma_{\text{avg}})_{BC} = 159$ MPa,
$(\sigma_{\text{avg}})_{AC} = 95.5$ MPa,
$(\sigma_{\text{avg}})_{AB} = 127$ MPa

***7–36.**

If the average normal stress in each of the 20-mm-diameter bars is not allowed to exceed 150 MPa, determine the maximum force **P** that can be applied to joint *C*.



SOLUTION

Internal Loadings: The force developed in each member of the truss can be determined by using the method of joints. First, consider the equilibrium of joint *C*, Fig. *a*.

$\xrightarrow{+} \Sigma F_x = 0; \quad P - F_{BC}\left(\frac{4}{5}\right) = 0 \quad F_{BC} = 1.25P \text{ (C)}$

$+\uparrow \Sigma F_y = 0; \quad 1.25P\left(\frac{3}{5}\right) - F_{AC} = 0 \quad F_{AC} = 0.75P \text{ (T)}$

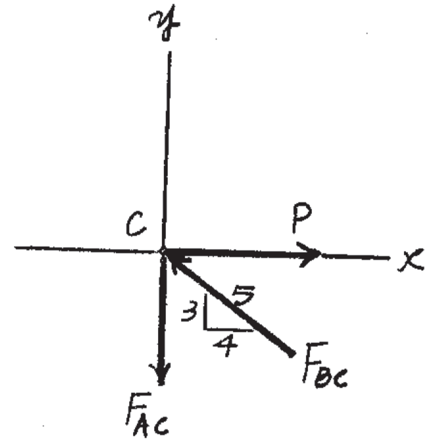
Subsequently, the equilibrium of joint *B*, Fig. *b*, is considered.

$\xrightarrow{+} \Sigma F_x = 0; \quad 1.25P\left(\frac{4}{5}\right) - F_{AB} = 0 \quad F_{AB} = P \text{ (T)}$

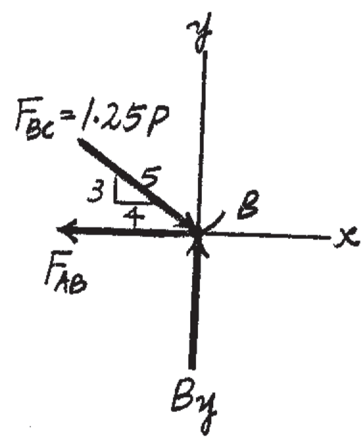
Average Normal Stress: Since the cross-sectional area and the allowable normal stress of each bar are the same, member *BC*, which is subjected to the maximum normal force, is the critical member. The cross-sectional area of each of the bars is $A = \frac{\pi}{4}(0.02^2) = 0.3142(10^{-3})\text{ m}^2$. We have

$$(\sigma_{avg})_{allow} = \frac{F_{BC}}{A}; \quad 150(10^6) = \frac{1.25P}{0.3142(10^{-3})}$$

$$P = 37\,699 \text{ N} = 37.7 \text{ kN}$$ **Ans.**



(a)

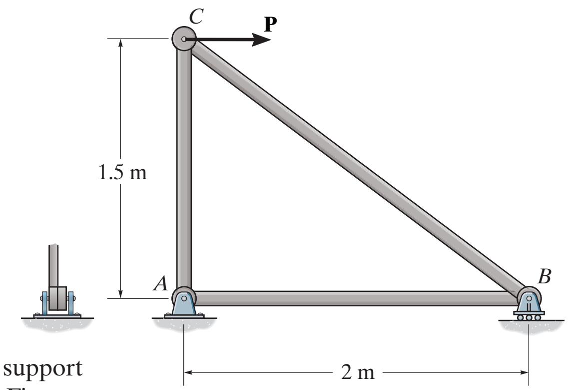


(b)

Ans:
$P = 37.7$ kN

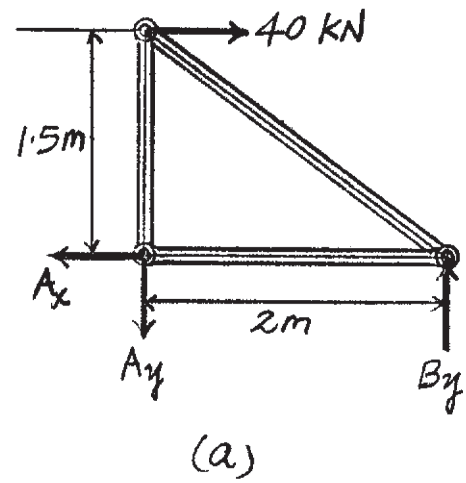
7–37.

Determine the maximum average shear stress in pin A of the truss. A horizontal force of $P = 40$ kN is applied to joint C. Each pin has a diameter of 25 mm and is subjected to double shear.



SOLUTION

Internal Loadings: The forces acting on pins A and B are equal to the support reactions at A and B. Referring to the free-body diagram of the entire truss, Fig. a,



$\Sigma M_A = 0;\qquad B_y(2) - 40(1.5) = 0\qquad B_y = 30$ kN

$\xrightarrow{+} \Sigma F_x = 0;\qquad 40 - A_x = 0\qquad A_x = 40$ kN

$+\uparrow \Sigma F_y = 0;\qquad 30 - A_y = 0\qquad A_y = 30$ kN

Thus,

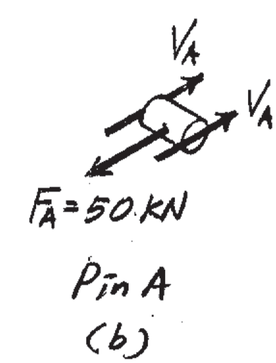
$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{40^2 + 30^2} = 50 \text{ kN}$$

Since pin A is in double shear, Fig. b, the shear forces developed on the shear planes of pin A are

$$V_A = \frac{F_A}{2} = \frac{50}{2} = 25 \text{ kN}$$

Average Shear Stress: The area of the shear plane for pin A is $A_A = \frac{\pi}{4}(0.025^2) = 0.4909(10^{-3})\text{ m}^2$. We have

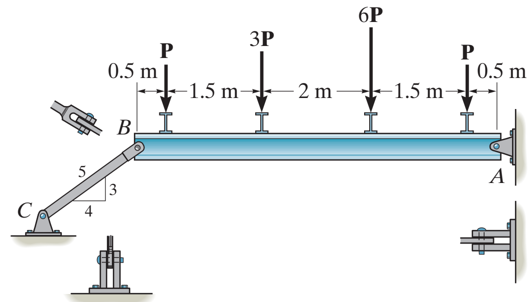
$$(\tau_{avg})_A = \frac{V_A}{A_A} = \frac{25(10^3)}{0.4909(10^{-3})} = 50.9 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$(\tau_{avg})_A = 50.9$ MPa

7–38.

If $P = 5$ kN, determine the average shear stress in the pins at A, B, and C. All pins are in double shear, and each has a diameter of 18 mm.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 5(0.5) + 30(2) + 15(4) + 5(5.5) - F_{BC}\left(\frac{3}{5}\right)(6) = 0$$

$$F_{BC} = 41.67 \text{ kN}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad A_y(6) - 5(0.5) - 15(2) - 30(4) - 5(5.5) = 0 \quad A_y = 30.0 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 41.67\left(\frac{4}{5}\right) - A_x = 0 \qquad A_x = 33.33 \text{ kN}$$

Thus,

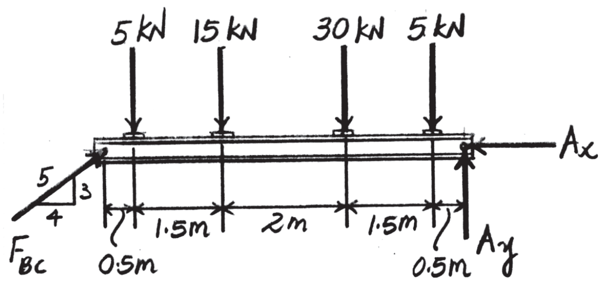
$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{33.33^2 + 30.0^2} = 44.85 \text{ kN}$$

Average Shear Stress: Since all the pins are subjected to double shear, $V_B = V_C = \dfrac{F_{BC}}{2} = \dfrac{41.67}{2}$ kN $= 20.83$ kN (Fig. *b*) and $V_A = 22.42$ kN (Fig. *c*)

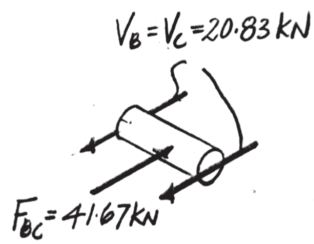
For pins B and C,

$$\tau_B = \tau_C = \frac{V_C}{A} = \frac{20.83(10^3)}{\frac{\pi}{4}(0.018^2)} = 81.87 \text{ MPa} = 81.9 \text{ MPa} \qquad \textbf{Ans.}$$

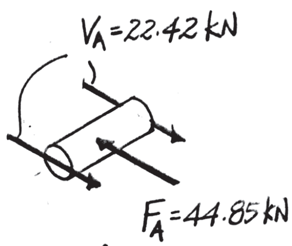
$$\tau_A = \frac{V_A}{A} = \frac{22.42(10^3)}{\frac{\pi}{4}(0.018^2)} = 88.12 \text{ MPa} = 88.1 \text{ MPa} \qquad \textbf{Ans.}$$



(a)



(b)

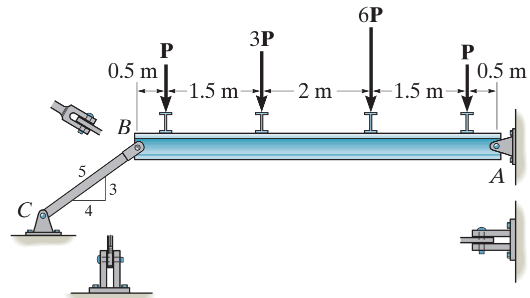


(c)

Ans:
$\tau_B = \tau_C = 81.9$ MPa,
$\tau_A = 88.1$ MPa

7–39.

Determine the maximum magnitude P of the loads the beam can support if the average shear stress in each pin is not to exceed 80 MPa. All pins are in double shear, and each has a diameter of 18 mm.



SOLUTION

Support Reactions: Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad P(0.5) + 6P(2) + 3P(4) + P(5.5) - F_{BC}\left(\frac{3}{5}\right)(6) = 0$$

$$F_{BC} = 8.3333P$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad A_y(6) - P(0.5) - 3P(2) - 6P(4) - P(5.5) = 0 \quad A_y = 6.00P$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 8.3333P\left(\frac{4}{5}\right) - A_x = 0 \qquad A_x = 6.6667P$$

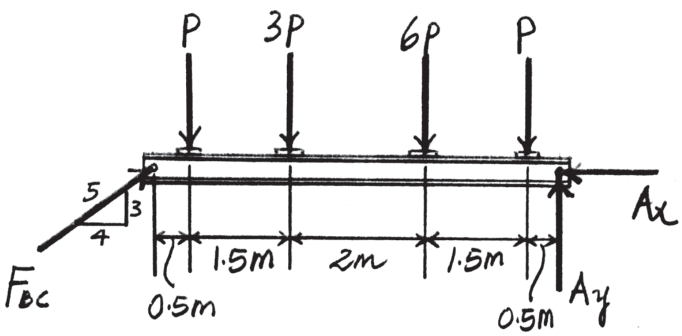
Thus,

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(6.6667P)^2 + (6.00P)^2} = 8.9691P$$

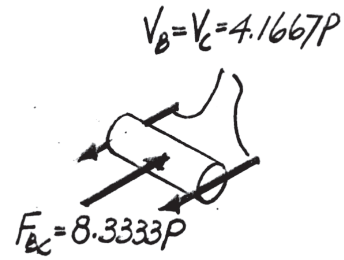
Average Shear Stress: Since all the pins are subjected to double shear, $V_B = V_C = \dfrac{F_{BC}}{2} = \dfrac{8.3333P}{2} = 4.1667P$ (Fig. *b*) and $V_A = 4.4845P$ (Fig. *c*). Since pin A is subjected to a larger shear force, it is critical. Thus,

$$\tau_{\text{allow}} = \frac{V_A}{A}; \qquad 80\left(10^6\right) = \frac{4.4845P}{\frac{\pi}{4}\left(0.018^2\right)}$$

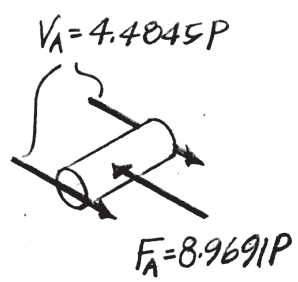
$$P = 4.539\left(10^3\right)\text{ N} = 4.54\text{ kN} \qquad \textbf{Ans.}$$



(a)



(b)

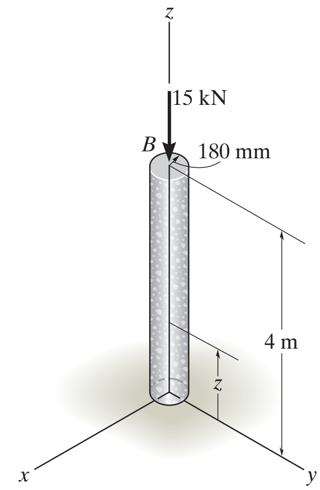


(c)

Ans:
$P = 4.54$ kN

***7–40.**

The column is made of concrete having a density of 2.30 $\mathrm{Mg/m^3}$. At its top B it is subjected to an axial compressive force of 15 kN. Determine the average normal stress in the column as a function of the distance z measured from its base.

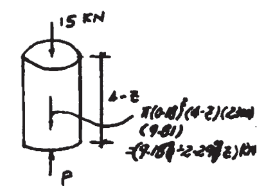


SOLUTION

$+\uparrow\Sigma F_y = 0 \qquad P - 15 - 9.187 + 2.297z = 0$

$P = 24.187 - 2.297z$

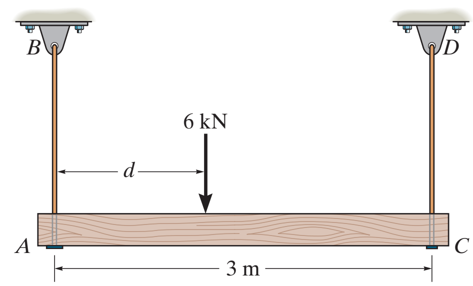
$$\sigma = \frac{P}{A} = \frac{24.187 - 2.297z}{\pi(0.18)^2} = (238 - 22.6z)\ \text{kPa}$$ **Ans.**



Ans:
$\sigma = (238 - 22.6z)$ kPa

7–41.

The beam is supported by two rods AB and CD that have cross-sectional areas of 12 mm^2 and 8 mm^2, respectively. If $d = 1$ m, determine the average normal stress in each rod.



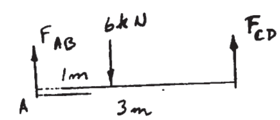
SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD}(3) - 6(1) = 0$

$$F_{CD} = 2 \text{ kN}$$

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - 6 + 2 = 0$

$$F_{AB} = 4 \text{ kN}$$



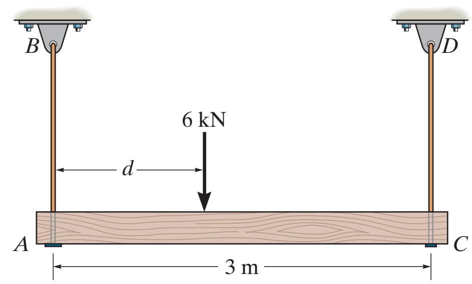
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{4(10^3)}{12(10^{-6})} = 333 \text{ MPa}$$ **Ans.**

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{2(10^3)}{8(10^{-6})} = 250 \text{ MPa}$$ **Ans.**

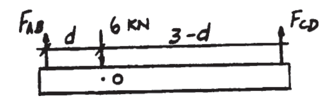
Ans:
$\sigma_{AB} = 333$ MPa,
$\sigma_{CD} = 250$ MPa

7–42.

The beam is supported by two rods AB and CD that have cross-sectional areas of 12 mm^2 and 8 mm^2, respectively. Determine the position d of the 6-kN load so that the average normal stress in each rod is the same.



SOLUTION



$\circlearrowleft + \Sigma M_O = 0; \qquad F_{CD}(3 - d) - F_{AB}(d) = 0$ **(1)**

$$\sigma = \frac{F_{AB}}{12} = \frac{F_{CD}}{8}$$

$F_{AB} = 1.5\,F_{CD}$ **(2)**

From Eqs. (1) and (2),

$$F_{CD}(3 - d) - 1.5\,F_{CD}(d) = 0$$

$$F_{CD}(3 - d - 1.5\,d) = 0$$

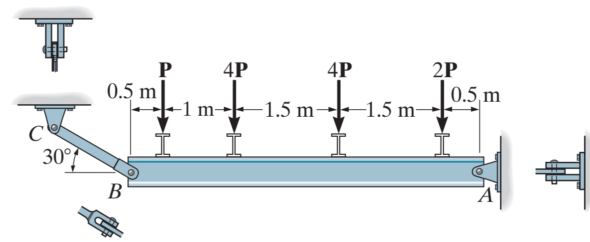
$$3 - 2.5\,d = 0$$

$$d = 1.20\text{ m}$$ **Ans.**

Ans:
$d = 1.20$ m

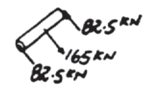
7–43.

If $P = 15$ kN, determine the average shear stress in the pins at A, B, and C. All pins are in double shear, and each has a diameter of 18 mm.



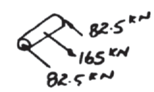
SOLUTION

For pins B and C,



$$\tau_B = \tau_C = \frac{V}{A} = \frac{82.5\left(10^3\right)}{\frac{\pi}{4}\left(\frac{18}{1000}\right)^2} = 324 \text{ MPa}$$ **Ans.**

For pin A,



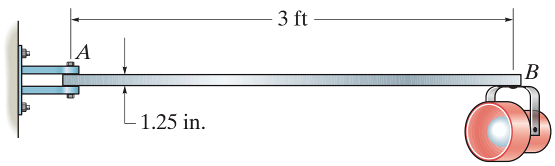
$$F_A = \sqrt{(82.5)^2 + (142.9)^2} = 165 \text{ kN}$$

$$\tau_A = \frac{V}{A} = \frac{82.5\left(10^3\right)}{\frac{\pi}{4}\left(\frac{18}{1000}\right)^2} = 324 \text{ MPa}$$ **Ans.**

Ans:
$\tau_B = 324$ MPa,
$\tau_A = 324$ MPa

***7–44.**

The railcar docklight is supported by the $\frac{1}{8}$-in.-diameter pin at A. If the lamp weighs 4 lb, and the extension arm AB has a weight of 0.5 lb/ft, determine the average shear stress in the pin needed to support the lamp. *Hint:* The shear force in the pin is caused by the couple moment required for equilibrium at A.

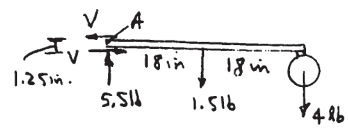


SOLUTION

$\zeta\!+\Sigma M_A = 0; \qquad V(1.25) - 1.5(18) - 4(36) = 0$

$$V = 136.8\text{ lb}$$

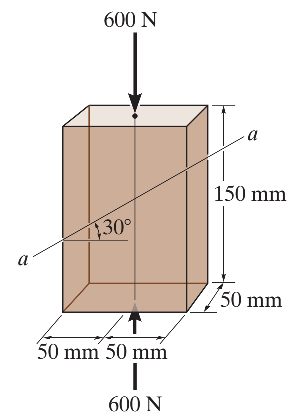
$$\tau_{avg} = \frac{V}{A} = \frac{136.8}{\frac{\pi}{4}\left(\frac{1}{8}\right)^2} = 11.1\text{ ksi}$$ **Ans.**



Ans:
$\tau_{avg} = 11.1$ ksi

7–45.

The plastic block is subjected to an axial compressive force of 600 N. Assuming that the caps at the top and bottom distribute the load uniformly throughout the block, determine the average normal and average shear stress acting along section *a–a*.



SOLUTION

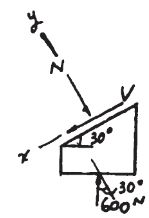
Along *a–a*:

$+\swarrow \Sigma F_x = 0; \qquad V - 600 \sin 30° = 0$

$V = 300\text{ N}$

$+\searrow \Sigma F_y = 0; \qquad -N + 600 \cos 30° = 0$

$N = 519.6\text{ N}$



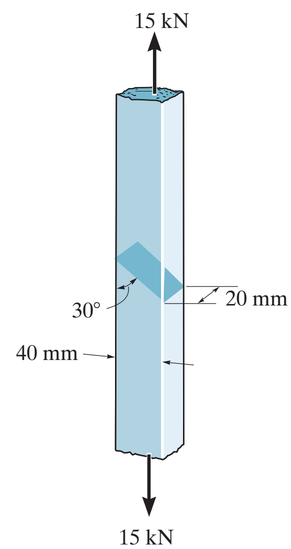
$$\sigma_{a-a} = \frac{519.6}{(0.05)\left(\frac{0.1}{\cos 30°}\right)} = 90.0\text{ kPa}$$ **Ans.**

$$\tau_{a-a} = \frac{300}{(0.05)\left(\frac{0.1}{\cos 30°}\right)} = 52.0\text{ kPa}$$ **Ans.**

Ans:
$\sigma_{a-a} = 90.0\text{ kPa}$,
$\tau_{a-a} = 52.0\text{ kPa}$

7–46.

The two steel members are joined together using a 30° scarf weld. Determine the average normal and average shear stress resisted in the plane of the weld.



SOLUTION

Internal Loadings: Referring to the FBD of the upper segment of the member sectioned through the scarf weld, Fig. *a*,

$\Sigma F_x = 0;\qquad N - 15\sin 30° = 0\qquad N = 7.50\text{ kN}$

$\Sigma F_y = 0;\qquad V - 15\cos 30° = 0\qquad V = 12.99\text{ kN}$

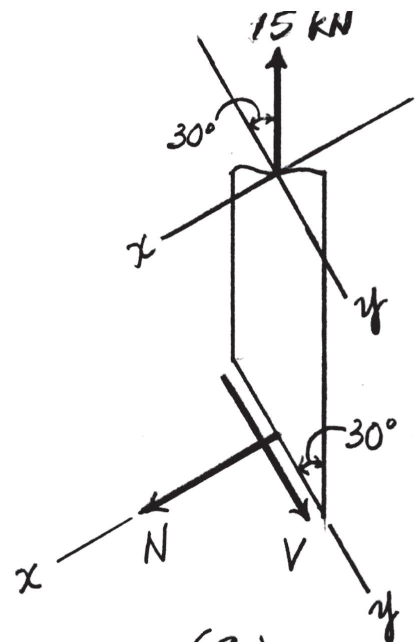
Average Normal and Shear Stress: The area of the scarf weld is

$$A = 0.02\left(\frac{0.04}{\sin 30°}\right) = 1.6\left(10^{-3}\right)\text{ m}^2$$

Thus,

$$\sigma = \frac{N}{A_n} = \frac{7.50\left(10^3\right)}{1.6\left(10^{-3}\right)} = 4.6875\left(10^6\right)\text{ Pa} = 4.69\text{ MPa}\qquad\textbf{Ans.}$$

$$\tau = \frac{V}{A_v} = \frac{12.99\left(10^3\right)}{1.6\left(10^{-3}\right)} = 8.119\left(10^6\right)\text{ Pa} = 8.12\text{ MPa}\qquad\textbf{Ans.}$$

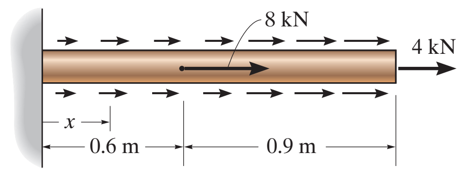


(a)

Ans:
$\sigma = 4.69$ MPa,
$\tau = 8.12$ MPa

7–47.

The bar has a cross-sectional area of $400(10^{-6})\ \text{m}^2$. If it is subjected to a triangular axial distributed loading along its length which is 0 at $x = 0$ and 9 kN/m at $x = 1.5$ m, and to two concentrated loads as shown, determine the average normal stress in the bar as a function of x for $0 \leq x < 0.6$ m.



SOLUTION

Internal Loading: Referring to the FBD of the right segment of the bar sectioned at x, Fig. a,

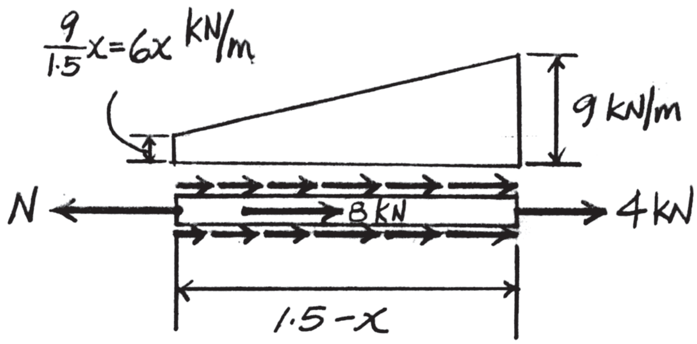
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 8 + 4 + \frac{1}{2}(6x + 9)(1.5 - x) = 0$$

$$N = \{18.75 - 3x^2\}\ \text{kN}$$

Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{(18.75 - 3x^2)(10^3)}{400(10^{-6})}$$

$$= \{46.9 - 7.50x^2\}\ \text{MPa} \qquad \textbf{Ans.}$$

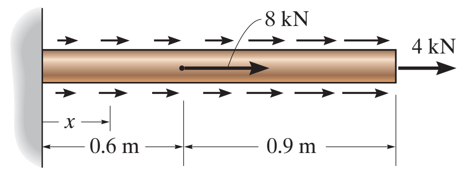


(a)

Ans:
$\sigma = \{46.9 - 7.50x^2\}$ MPa

***7–48.**

The bar has a cross-sectional area of $400(10^{-6})$ m^2. If it is subjected to a uniform axial distributed loading along its length of 9 kN/m, and to two concentrated loads as shown, determine the average normal stress in the bar as a function of x for $0.6 \text{ m} < x \leq 1.5 \text{ m}$.



SOLUTION

Internal Loading: Referring to a FBD of the right segment of the bar sectioned at x,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4 + 9(1.5 - x) - N = 0$$

$$N = \{17.5 - 9x\} \text{ kN}$$

Average Normal Stress:

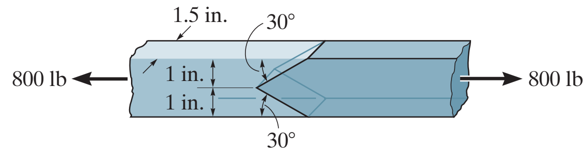
$$\sigma = \frac{N}{A} = \frac{(17.5 - 9x)(10^3)}{400(10^{-6})}$$

$$= \{43.75 - 22.5x\} \text{ MPa} \qquad \textbf{Ans.}$$

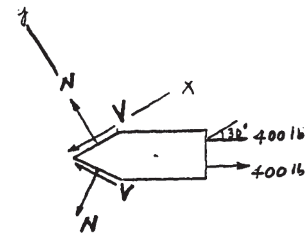
Ans:
$\sigma = \{43.75 - 22.5x\}$ MPa

7–49.

The two members used in the construction of an aircraft fuselage are joined together using a 30° fish-mouth weld. Determine the average normal and average shear stress on the plane of each weld. Assume each inclined plane supports a horizontal force of 400 lb.



SOLUTION



$N - 400 \sin 30° = 0; \qquad N = 200 \text{ lb}$

$400 \cos 30° - V = 0; \qquad V = 346.41 \text{ lb}$

$$A' = \frac{1.5(1)}{\sin 30°} = 3 \text{ in}^2$$

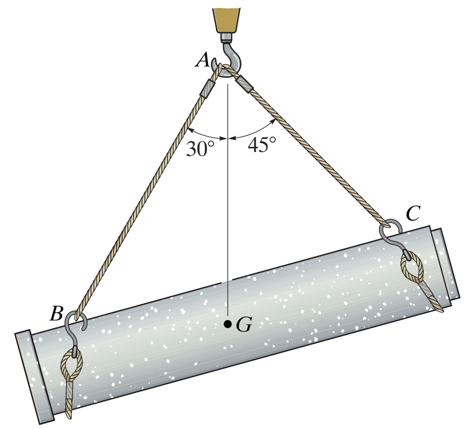
$$\sigma = \frac{N}{A'} = \frac{200}{3} = 66.7 \text{ psi}$$ **Ans.**

$$\tau = \frac{V}{A'} = \frac{346.41}{3} = 115 \text{ psi}$$ **Ans.**

Ans:
$\sigma = 66.7$ psi,
$\tau = 115$ psi

7–50.

The 2-Mg concrete pipe has a center of mass at point G. If it is suspended from cables AB and AC, determine the average normal stress in the cables. The diameters of AB and AC are 12 mm and 10 mm, respectively.



SOLUTION

Internal Loadings: The normal force developed in cables AB and AC can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. a.

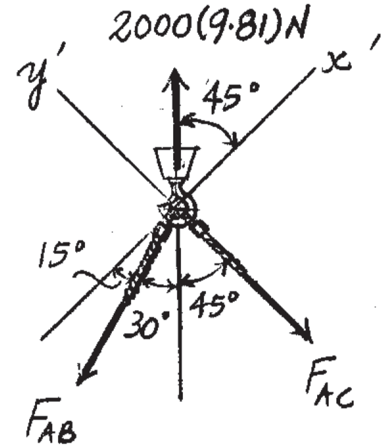
$\Sigma F_{x'} = 0; \quad 2000(9.81)\cos 45° - F_{AB}\cos 15° = 0 \quad F_{AB} = 14\,362.83 \text{ N (T)}$

$\Sigma F_{y'} = 0; \quad 2000(9.81)\sin 45° - 14\,362.83 \sin 15° - F_{AC} = 0 \quad F_{AC} = 10\,156.06 \text{ N (T)}$

Average Normal Stress: The cross-sectional areas of cables AB and AC are $A_{AB} = \frac{\pi}{4}(0.012^2) = 0.1131(10^{-3}) \text{ m}^2$ and $A_{AC} = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6}) \text{ m}^2$. We have

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{14\,362.83}{0.1131(10^{-3})} = 127 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{10\,156.06}{78.540(10^{-6})} = 129 \text{ MPa} \qquad \textbf{Ans.}$$

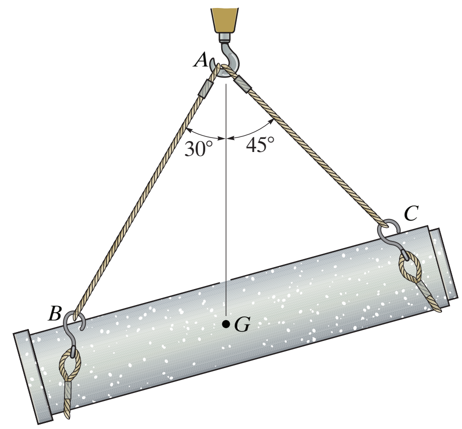


(a)

Ans:
$\sigma_{AB} = 127$ MPa, $\sigma_{AC} = 129$ MPa

7–51.

The 2-Mg concrete pipe has a center of mass at point G. If it is suspended from cables AB and AC, determine the diameter of cable AB so that the average normal stress in this cable is the same as in the 10-mm-diameter cable AC.

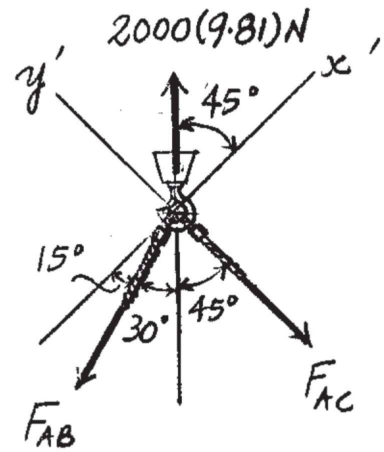


SOLUTION

Internal Loadings: The normal force in cables AB and AC can be determined by considering the equilibrium of the hook for which the free-body diagram is shown in Fig. *a*.

$\Sigma F_{x'} = 0;\ 2000(9.81)\cos 45° - F_{AB}\cos 15° = 0 \qquad F_{AB} = 14\,362.83\text{ N (T)}$

$\Sigma F_{y'} = 0;\ 2000(9.81)\sin 45° - 14\,362.83\sin 15° - F_{AC} = 0 \quad F_{AC} = 10\,156.06\text{ N (T)}$



(a)

Average Normal Stress: The cross-sectional areas of cables AB and AC are $A_{AB} = \frac{\pi}{4}d_{AB}^{\,2}$ and $A_{AC} = \frac{\pi}{4}(0.01^2) = 78.540(10^{-6})\text{ m}^2$.

Here, we require

$$\sigma_{AB} = \sigma_{AC}$$

$$\frac{F_{AB}}{A_{AB}} = \frac{F_{AC}}{A_{AC}}$$

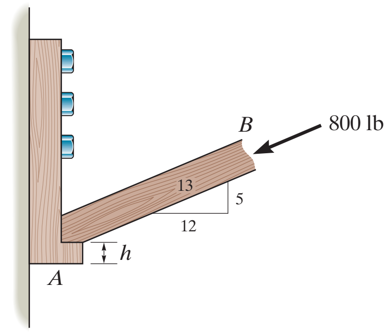
$$\frac{14\,362.83}{\frac{\pi}{4}d_{AB}^{\,2}} = \frac{10\,156.06}{78.540(10^{-6})}$$

$$d_{AB} = 0.01189\text{ m} = 11.9\text{ mm}$$ **Ans.**

Ans:
$d_{AB} = 11.9\text{ mm}$

***7–52.**

If A and B are both made of wood and are $\frac{3}{8}$ in. thick, determine to the nearest $\frac{1}{4}$ in. the smallest dimension h of the vertical segment so that it does not fail in shear. The allowable shear stress for the segment is $\tau_{\text{allow}} = 300$ psi.

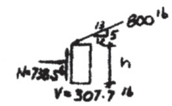


SOLUTION

$$\tau_{\text{allow}} = 300 = \frac{307.7}{\left(\frac{3}{8}\right) h}$$

$$h = 2.74 \text{ in.}$$

Use $h = 2\dfrac{3}{4}$ in. **Ans.**

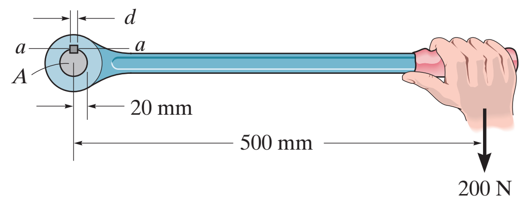


Ans:

Use $h = 2\dfrac{3}{4}$ in.

7–53.

The lever is attached to the shaft A using a key that has a width d and length of 25 mm. If the shaft is fixed and a vertical force of 200 N is applied perpendicular to the handle, determine the dimension d if the allowable shear stress for the key is $\tau_{\text{allow}} = 35$ MPa.



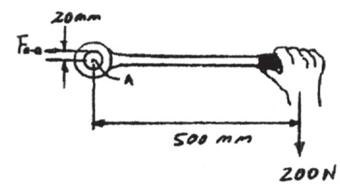
SOLUTION

$\zeta + \Sigma M_A = 0; \qquad F_{a-a}(20) - 200(500) = 0$

$$F_{a-a} = 5000 \text{ N}$$

$$\tau_{\text{allow}} = \frac{F_{a-a}}{A_{a-a}}; \qquad 35(10^6) = \frac{5000}{d(0.025)}$$

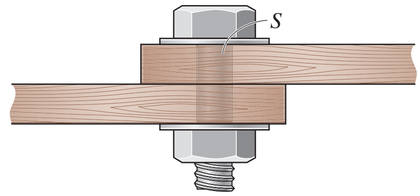
$$d = 0.00571 \text{ m} = 5.71 \text{ mm}$$ **Ans.**



Ans:
$d = 5.71$ mm

7–54.

The connection is made using a bolt and nut and two washers. If the allowable bearing stress of the washers on the boards is $(\sigma_b)_{\text{allow}} = 2$ ksi, and the allowable tensile stress within the bolt shank S is $(\sigma_t)_{\text{allow}} = 18$ ksi, determine the maximum allowable tension in the bolt shank. The bolt shank has a diameter of 0.31 in., and the washers have an outer diameter of 0.75 in. and inner diameter (hole) of 0.50 in.



SOLUTION

Allowable Normal Stress: Assume tension failure.

$$\sigma_{\text{allow}} = \frac{P}{A}; \qquad 18 = \frac{P}{\frac{\pi}{4}(0.31^2)}$$

$$P = 1.36 \text{ kip}$$

Allowable Bearing Stress: Assume bearing failure.

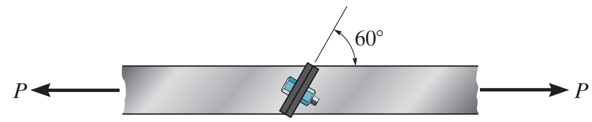
$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \qquad 2 = \frac{P}{\frac{\pi}{4}(0.75^2 - 0.50^2)}$$

$$P = 0.491 \text{ kip } (\textbf{\textit{controls!}})$$ **Ans.**

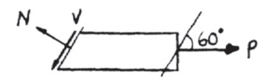
Ans:
$P = 0.491$ kip

7–55.

The tension member is fastened together using *two* bolts, one on each side of the member as shown. Each bolt has a diameter of 0.3 in. Determine the maximum load P that can be applied to the member if the allowable shear stress for the bolts is $\tau_{\text{allow}} = 12$ ksi and the allowable average normal stress is $\sigma_{\text{allow}} = 20$ ksi.



SOLUTION



$\nwarrow + \Sigma F_y = 0; \qquad N - P\sin 60° = 0$

$P = 1.1547\, N \qquad$ **(1)**

$\swarrow + \Sigma F_x = 0; \qquad V - P\cos 60° = 0$

$P = 2\, V \qquad$ **(2)**

Assume failure due to shear:

$$\tau_{\text{allow}} = 12 = \frac{V}{(2)\,\frac{\pi}{4}\,(0.3)^2}$$

$V = 1.696$ kip

From Eq. (2),

$P = 3.39$ kip

Assume failure due to normal force:

$$\sigma_{\text{allow}} = 20 = \frac{N}{(2)\,\frac{\pi}{4}\,(0.3)^2}$$

$N = 2.827$ kip

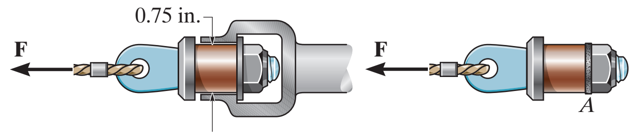
From Eq. (1),

$P = 3.26$ kip (controls) **Ans.**

Ans:
$P = 3.26$ kip

***7–56.**

The steel swivel bushing in the elevator control of an airplane is held in place using a nut and washer as shown in Fig. (*a*). Failure of the washer *A* can cause the push rod to separate as shown in Fig. (*b*). If the maximum average shear stress is $\tau_{max} = 21$ ksi, determine the force **F** that must be applied to the bushing. The washer is $\frac{1}{16}$ in. thick.



(a) (b)

SOLUTION

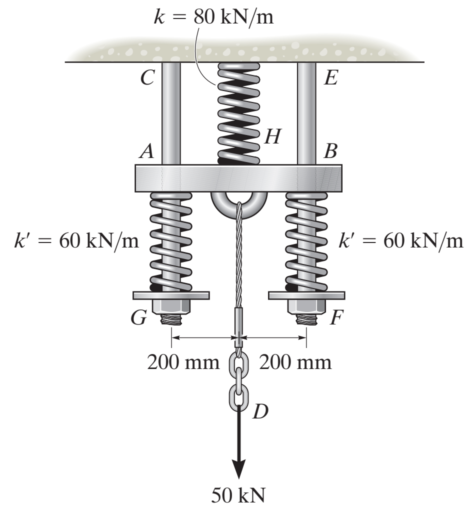
$$\tau_{avg} = \frac{V}{A}; \qquad 21(10^3) = \frac{F}{2\pi(0.375)(\frac{1}{16})}$$

$$F = 3092.5 \text{ lb} = 3.09 \text{ kip}$$ **Ans.**

Ans:
$F = 3.09$ kip

7–57.

The spring mechanism is used as a shock absorber for a load applied to the drawbar AB. Determine the force in each spring when the 50-kN force is applied. Each spring is originally unstretched and the drawbar slides along the smooth guide posts CG and EF. The ends of all springs are attached to their respective members. Also, what is the required diameter of the shank of bolts CG and EF if the allowable stress for the bolts is $\sigma_{\text{allow}} = 150$ MPa?



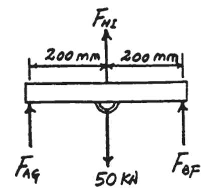
SOLUTION

Equations of Equilibrium:

$\circlearrowleft + \Sigma M_H = 0; \qquad -F_{BF}(200) + F_{AG}(200) = 0$

$$F_{BF} = F_{AG} = F$$

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_H - 50 = 0 \qquad \textbf{(1)}$



Required:

$\Delta_H = \Delta_B; \qquad \dfrac{F_H}{80} = \dfrac{F}{60}$

$$F = 0.75\,F_H \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$F_H = 20.0 \text{ kN} \qquad \textbf{Ans.}$$

$$F_{BF} = F_{AG} = F = 15.0 \text{ kN} \qquad \textbf{Ans.}$$

Allowable Normal Stress: Design of bolt shank size.

$\sigma_{\text{allow}} = \dfrac{P}{A}; \qquad 150\left(10^6\right) = \dfrac{15.0\left(10^3\right)}{\frac{\pi}{4}d^2}$

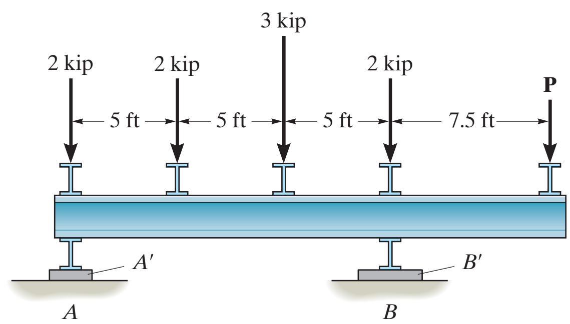
$$d = 0.01128 \text{ m} = 11.3 \text{ mm}$$

$$d_{EF} = d_{CG} = 11.3 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$F_H = 20.0$ kN,
$F_{BF} = F_{AG} = 15.0$ kN,
$d_{EF} = d_{CG} = 11.3$ mm

7–58.

Determine the size of *square* bearing plates A' and B' required to support the loading. Take $P = 1.5$ kip. Dimension the plates to the nearest $\frac{1}{2}$ in. The reactions at the supports are vertical and the allowable bearing stress for the plates is $(\sigma_b)_{\text{allow}} = 400$ psi.



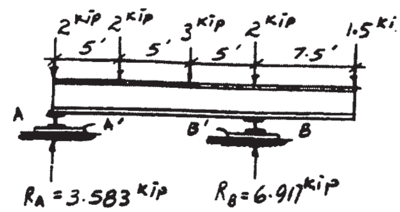
SOLUTION

For Plate A:

$$\sigma_{\text{allow}} = 400 = \frac{3.583(10^3)}{a_{A'}^2}$$

$$a_{A'} = 2.99 \text{ in.}$$

Use a 3 in. × 3 in. plate. **Ans.**



For Plate B':

$$\sigma_{\text{allow}} = 400 = \frac{6.917(10^3)}{a_{B'}^2}$$

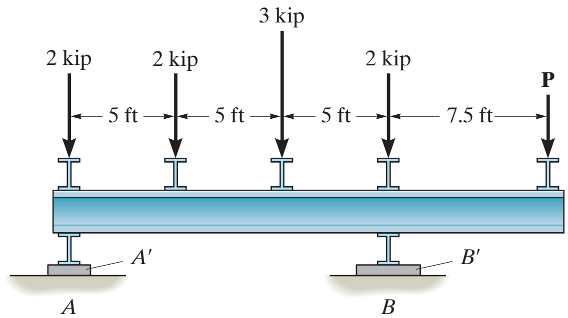
$$a_{B'} = 4.16 \text{ in.}$$

Use a $4\frac{1}{2}$ in. × $4\frac{1}{2}$ in. plate. **Ans.**

Ans:
For A':
Use a 3 in. × 3 in. plate,
For B':
Use a $4\frac{1}{2}$ in. × $4\frac{1}{2}$ in. plate

7–59.

Determine the maximum load **P** that can be applied to the beam if the bearing plates A' and B' have square cross sections of 2 in. $\times$ 2 in. and 4 in. $\times$ 4 in., respectively, and the allowable bearing stress for the material is $(\sigma_b)_{\text{allow}} = 400$ psi.



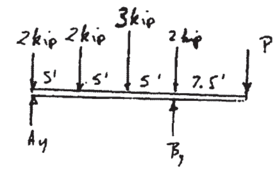
SOLUTION

$\zeta + \Sigma M_A = 0;\qquad B_y(15) - 2(5) - 3(10) - 2(15) - P(225) = 0$

$$B_y = 1.5P + 4.667$$

$+\uparrow \Sigma F_y = 0;\qquad A_y + 1.5P + 4.667 - 9 - P = 0$

$$A_y = 4.333 - 0.5P$$



At A:

$$0.400 = \frac{4.333 - 0.5P}{2(2)}$$

$$P = 5.47 \text{ kip}$$

At B:

$$0.400 = \frac{1.5P + 4.667}{4(4)}$$

$$P = 1.16 \text{ kip}$$

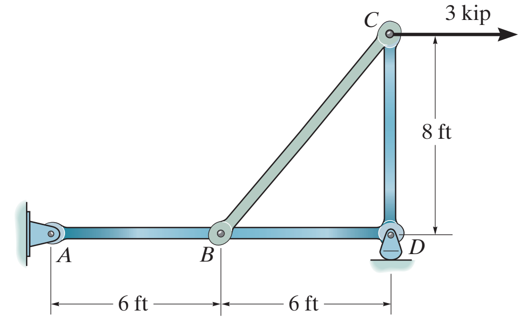
Thus,

$$P_{\text{allow}} = 1.16 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{allow}} = 1.16$ kip

***7–60.**

Determine the required diameter of the pins at A and B to the nearest $\frac{1}{16}$ in. if the allowable shear stress for the material is $\tau_{\text{allow}} = 6$ ksi. Pin A is subjected to double shear, whereas pin B is subjected to single shear.



SOLUTION

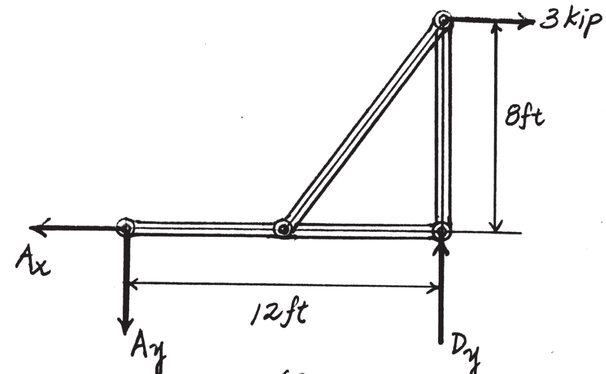
Support Reaction: Referring to the FBD of the entire frame, Fig. a,

$$\circlearrowleft + \Sigma M_D = 0; \qquad A_y(12) - 3(18) = 0 \qquad A_y = 2.00 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3 - A_x = 0 \qquad A_x = 3.00 \text{ kip}$$

Thus,

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{3.00^2 + 2.00^2} = 3.6056 \text{ kip}$$



(a)

Consider the equilibrium of joint C, Fig. b.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3 - F_{BC}\left(\frac{3}{5}\right) = 0 \qquad F_{BC} = 5.00 \text{ kip}$$

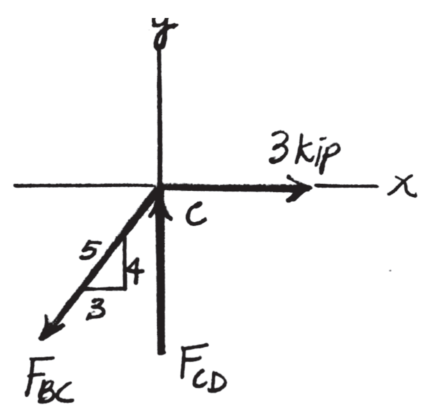
Average Shear Stress: Pin A is subjected to double shear, Fig. c.

$$\text{Thus, } V_A = \frac{F_A}{2} = \frac{3.6056}{2} = 1.8028 \text{ kip}$$

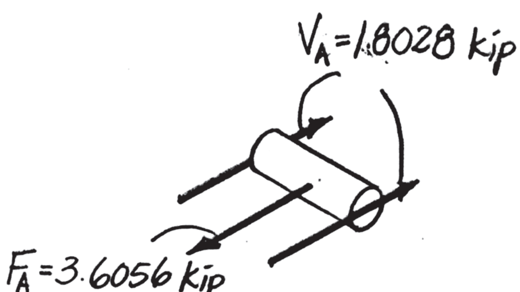
$$\tau_{\text{allow}} = \frac{V_A}{A_A}; \qquad 6 = \frac{1.8028}{\frac{\pi}{4} d_A^{\,2}} \qquad d_A = 0.6185 \text{ in.} \quad \text{Use } d_A = \frac{5}{8} \text{ in.} \qquad \textbf{Ans.}$$

Since pin B is subjected to single shear, Fig. d, $V_B = F_{BC} = 5.00$ kip.

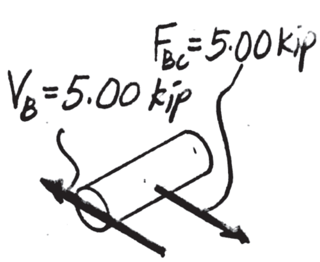
$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \qquad 6 = \frac{5.00}{\frac{\pi}{4} d_B^{\,2}} \qquad d_B = 1.0301 \text{ in.} \quad \text{Use } d_B = 1\frac{1}{16} \text{ in.} \qquad \textbf{Ans.}$$



(b)



(c)



(d)

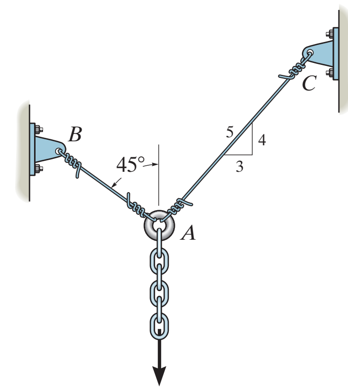
Ans:

Use $d_A = \frac{5}{8}$ in.,

Use $d_B = 1\frac{1}{16}$ in.

7–61.

If the allowable tensile stress for wires AB and AC is $\sigma_{\text{allow}} = 200$ MPa, determine the required diameter of each wire if the applied load is $P = 6$ kN.



SOLUTION

Normal Forces: Analyzing the equilibrium of joint A, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\sin 45^\circ = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{4}{5}\right) + F_{AB}\cos 45^\circ - 6 = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$F_{AC} = 4.2857 \text{ kN} \qquad F_{AB} = 3.6365 \text{ kN}$$

Average Normal Stress: For wire AB,

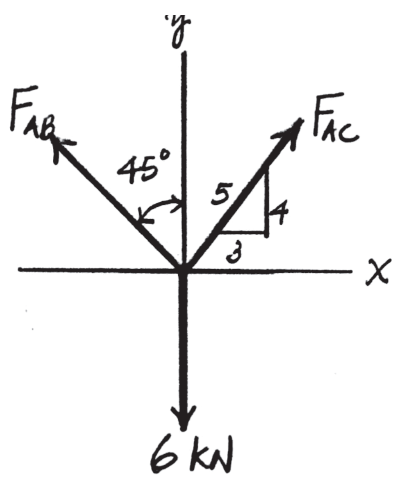
$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \qquad 200\left(10^6\right) = \frac{3.6365\left(10^3\right)}{\frac{\pi}{4}d_{AB}^2}$$

$$d_{AB} = 0.004812 \text{ m} = 4.81 \text{ mm} \qquad \textbf{Ans.}$$

For wire AC,

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \qquad 200\left(10^6\right) = \frac{4.2857\left(10^3\right)}{\frac{\pi}{4}d_{AC}^2}$$

$$d_{AC} = 0.005223 \text{ m} = 5.22 \text{ mm} \qquad \textbf{Ans.}$$

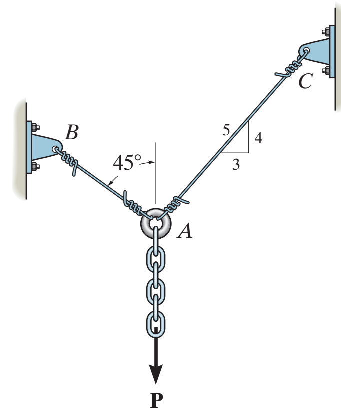


(a)

Ans:
$d_{AB} = 4.81$ mm,
$d_{AC} = 5.22$ mm

7–62.

If the allowable tensile stress for wires AB and AC is $\sigma_{allow} = 180$ MPa, and wire AB has a diameter of 5 mm and AC has a diameter of 6 mm, determine the greatest force P that can be applied to the chain.



SOLUTION

Normal Forces: Analyzing the equilibrium of joint A, Fig. a,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\sin 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{4}{5}\right) + F_{AB}\cos 45° - P = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$F_{AC} = 0.7143P \qquad F_{AB} = 0.6061P$$

Average Normal Stress: Assuming failure of wire AB,

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \qquad 180\left(10^6\right) = \frac{0.6061P}{\frac{\pi}{4}\left(0.005^2\right)}$$

$$P = 5.831\left(10^3\right)\text{ N} = 5.83\text{ kN}$$

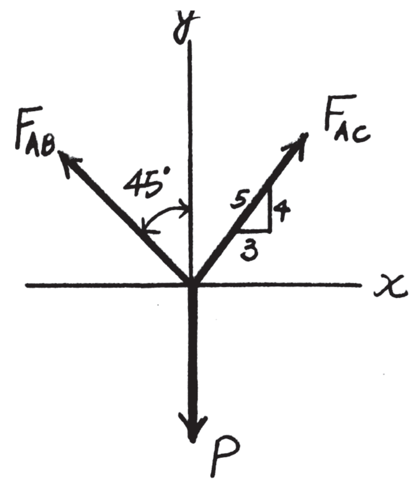
Assume the failure of wire AC.

$$\sigma_{allow} = \frac{F_{AC}}{A_{AC}}; \qquad 180\left(10^6\right) = \frac{0.7143P}{\frac{\pi}{4}\left(0.006^2\right)}$$

$$P = 7.125\left(10^3\right)\text{ N} = 7.13\text{ kN}$$

Choose the smaller of the two values of P.

$$P = 5.83\text{ kN} \qquad \textbf{Ans.}$$

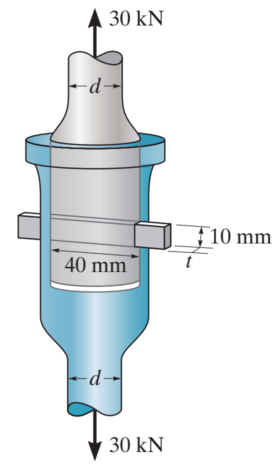


(a)

Ans:
$P = 5.83$ kN

7–63.

The cotter is used to hold the two rods together. Determine the smallest thickness t of the cotter and the smallest diameter d of the rods. All parts are made of steel for which the failure normal stress is $\sigma_{\text{fail}} = 500$ MPa and the failure shear stress is $\tau_{\text{fail}} = 375$ MPa. Use a factor of safety of $(\text{F.S.})_t = 2.50$ in tension and $(\text{F.S.})_s = 1.75$ in shear.



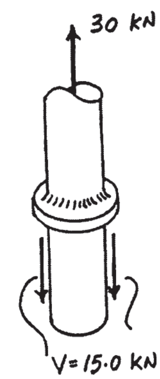
SOLUTION

Allowable Normal Stress: Design of rod size.

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S}} = \frac{P}{A}; \qquad \frac{500(10^6)}{2.5} = \frac{30(10^3)}{\frac{\pi}{4}d^2}$$

$$d = 0.01382 \text{ m} = 13.8 \text{ mm}$$ **Ans.**

Allowable Shear Stress: Design of cotter size.



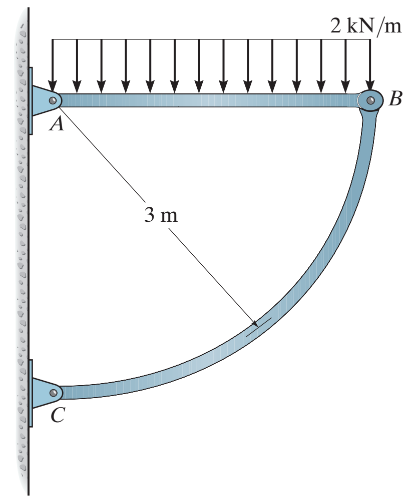
$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S}} = \frac{V}{A}; \qquad \frac{375(10^6)}{1.75} = \frac{15.0(10^3)}{(0.01)t}$$

$$t = 0.0070 \text{ m} = 7.00 \text{ mm}$$ **Ans.**

Ans:
$d = 13.8$ mm,
$t = 7.00$ mm

***7–64.**

Determine the required diameter of the pins at A and B if the allowable shear stress for the material is $\tau_{\text{allow}} = 100$ MPa. Both pins are subjected to double shear.



SOLUTION

Support Reactions: Member BC is a two-force member.

$\circlearrowleft + \Sigma M_A = 0;\qquad F_{BC} \sin 45°(3) - 6(1.5) = 0$

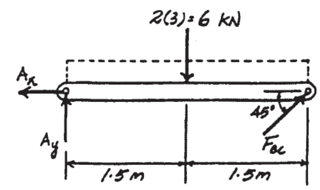
$$F_{BC} = 4.243 \text{ kN}$$

$+\uparrow \Sigma F_y = 0;\qquad A_y + 4.243 \sin 45° - 6 = 0$

$$A_y = 3.00 \text{ kN}$$

$\underset{\leftarrow}{+} \Sigma F_x = 0;\qquad A_x - 4.243 \cos 45° = 0$

$$A_x = 3.00 \text{ kN}$$

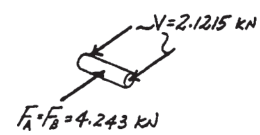


Allowable Shear Stress: Pin A and pin B are subjected to double shear.

$F_A = \sqrt{3.00^2 + 3.00^2} = 4.243$ kN and

$F_B = F_{BC} = 4.243$ kN.

Therefore,



$$V_A = V_B = \frac{4.243}{2} = 2.1215 \text{ kN}$$

$$\tau_{\text{allow}} = \frac{V}{A};\qquad 100\left(10^6\right) = \frac{2.1215\left(10^3\right)}{\frac{\pi}{4}d^2}$$

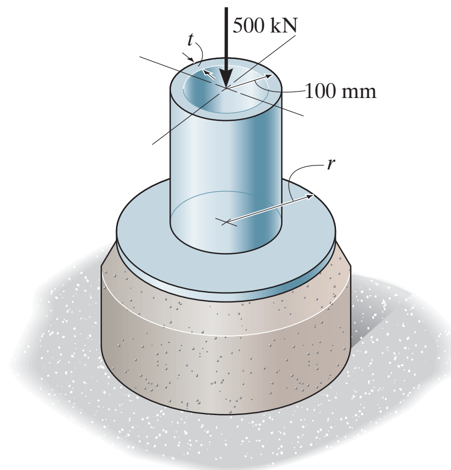
$$d = 0.005197 \text{ m} = 5.20 \text{ mm}$$

$$d_A = d_B = d = 5.20 \text{ mm}$$ **Ans.**

Ans:
$d_A = d_B = 5.20$ mm

7–65.

The steel pipe is supported on the circular base plate and concrete pedestal. If the thickness of the pipe is $t = 5$ mm and the base plate has a radius of 150 mm, determine the factors of safety against failure of the steel and concrete. The applied force is 500 kN, and the normal failure stresses for steel and concrete are $(\sigma_{\text{fail}})_{\text{st}} = 350$ MPa and $(\sigma_{\text{fail}})_{\text{con}} = 25$ MPa, respectively.



SOLUTION

Average Normal and Bearing Stress: The cross-sectional area of the steel pipe and the bearing area of the concrete pedestal are $A_{\text{st}} = \pi(0.1^2 - 0.095^2) = 0.975(10^{-3})\pi$ m^2 and $(A_{\text{con}})_{\text{b}} = \pi(0.15^2) = 0.0225\pi$ m^2. We have

$$(\sigma_{\text{avg}})_{\text{st}} = \frac{P}{A_{\text{st}}} = \frac{500(10^3)}{0.975(10^{-3})\pi} = 163.24 \text{ MPa}$$

$$(\sigma_{\text{avg}})_{\text{con}} = \frac{P}{(A_{\text{con}})_{\text{b}}} = \frac{500(10^3)}{0.0225\pi} = 7.074 \text{ MPa}$$

Thus, the factor of safety against failure of the steel pipe and concrete pedestal are

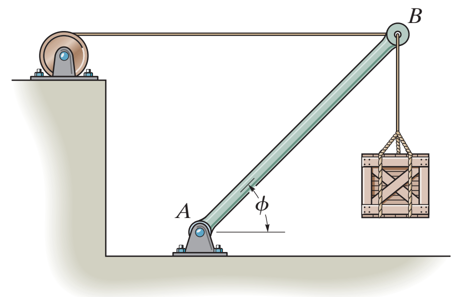
$$(\text{F.S.})_{\text{st}} = \frac{(\sigma_{\text{fail}})_{\text{st}}}{(\sigma_{\text{avg}})_{\text{st}}} = \frac{350}{163.24} = 2.14 \qquad \textbf{Ans.}$$

$$(\text{F.S.})_{\text{con}} = \frac{(\sigma_{\text{fail}})_{\text{con}}}{(\sigma_{\text{avg}})_{\text{con}}} = \frac{25}{7.074} = 3.53 \qquad \textbf{Ans.}$$

Ans:
$(\text{F.S.})_{\text{st}} = 2.14$, $(\text{F.S.})_{\text{con}} = 3.53$

7–66.

The boom is supported by the winch cable that has a diameter of 0.25 in. and an allowable normal stress of $\sigma_{allow} = 24$ ksi. Determine the greatest weight of the crate that can be supported without causing the cable to fail if $\phi = 30°$. Neglect the size of the winch.



SOLUTION

Normal Force: Analyzing the equilibrium of joint B, Fig. a,

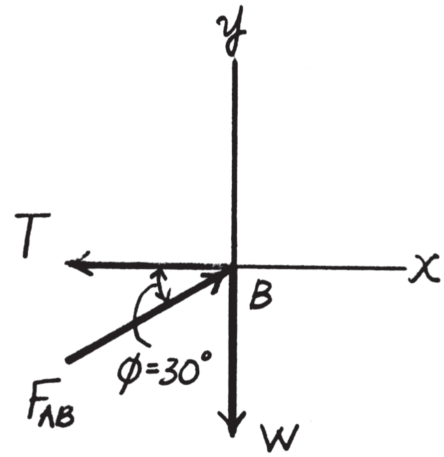
$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin 30° - W = 0 \qquad F_{AB} = 2.00W$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 2.00W \cos 30° - T = 0 \qquad T = 1.7321W$

Average Normal Stress:

$$\sigma_{allow} = \frac{T}{A}; \qquad 24(10^3) = \frac{1.7321W}{\frac{\pi}{4}(0.25^2)}$$

$$W = 680.17 \text{ lb} = 680 \text{ lb}$$ **Ans.**

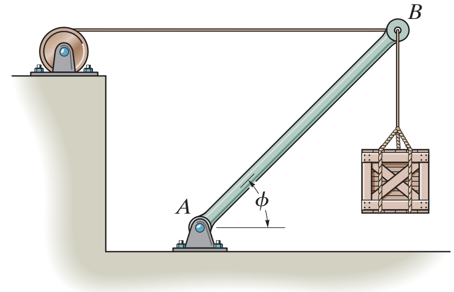


(a)

Ans:
$W = 680$ lb

7–67.

The boom is supported by the winch cable that has an allowable normal stress of $\sigma_{\text{allow}} = 24$ ksi. If it supports the 5000 lb crate when $\phi = 20°$, determine the smallest diameter of the cable to the nearest $\frac{1}{16}$ in.



SOLUTION

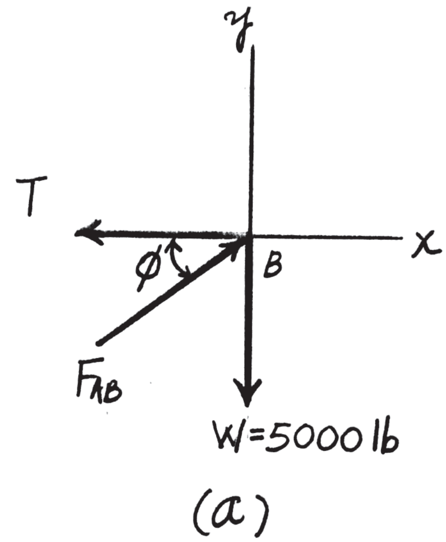
Normal Force: Consider the equilibrium of joint B, Fig. a.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin \phi - 5000 = 0 \qquad F_{AB} = \frac{5000}{\sin \phi}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \left(\frac{5000}{\sin \phi}\right) \cos \phi - T = 0 \qquad T = 5000 \cot \phi$$

When $\phi = 20°$, the design value for T is

$$T = 5000 \cot 20° = 13.737\left(10^3\right) \text{ lb} = 13.737 \text{ kip}$$



Average Normal Stress:

$$\sigma_{\text{allow}} = \frac{T}{A}; \qquad 24 = \frac{13.737}{\frac{\pi}{4} d^2}$$

$$d = 0.8537 \text{ in.}$$

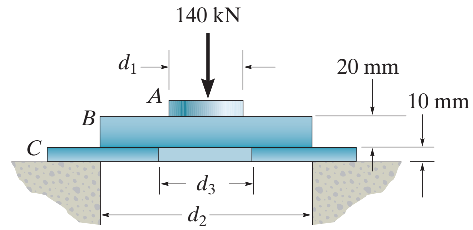
$$\text{Use } d = \frac{7}{8} \text{ in.}$$ **Ans.**

Ans:

Use $d = \frac{7}{8}$ in.

***7–68.**

The assembly consists of three disks A, B, and C that are used to support the load of 140 kN. Determine the smallest diameter d_1 of the top disk, the diameter d_2 within the support space, and the diameter d_3 of the hole in the bottom disk. The allowable bearing stress for the material is $(\sigma_b)_{\text{allow}} = 350$ MPa and allowable shear stress is $\tau_{\text{allow}} = 125$ MPa.



SOLUTION

Allowable Shear Stress: Assume shear failure for disk C.

$$\tau_{\text{allow}} = \frac{V}{A}; \quad 125\left(10^6\right) = \frac{140\left(10^3\right)}{\pi d_2(0.01)}$$

$$d_2 = 0.03565 \text{ m} = 35.7 \text{ mm} \qquad \textbf{Ans.}$$

Allowable Bearing Stress: Assume bearing failure for disk C.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \quad 350\left(10^6\right) = \frac{140\left(10^3\right)}{\frac{\pi}{4}\left(0.03565^2 - d_3^2\right)}$$

$$d_3 = 0.02760 \text{ m} = 27.6 \text{ mm} \qquad \textbf{Ans.}$$

Allowable Bearing Stress: Assume bearing failure for disk B.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \quad 350\left(10^6\right) = \frac{140\left(10^3\right)}{\frac{\pi}{4} d_1^2}$$

$$d_1 = 0.02257 \text{ m} = 22.6 \text{ mm}$$

Since $d_3 = 27.6 \text{ mm} > d_1 = 22.6$ mm, disk B might fail due to shear.

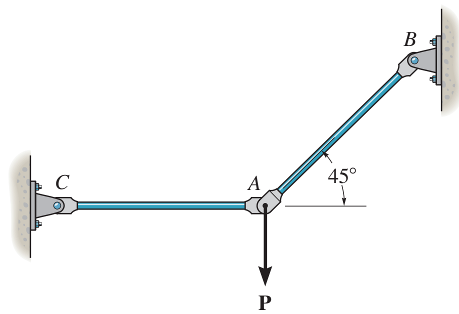
$$\tau = \frac{V}{A} = \frac{140\left(10^3\right)}{\pi(0.02257)(0.02)} = 98.7 \text{ MPa} < \tau_{\text{allow}} = 125 \text{ MPa } (\textbf{\textit{O.K!}})$$

Therefore, $\quad d_1 = 22.6$ mm **Ans.**

Ans:
$d_2 = 35.7$ mm,
$d_3 = 27.6$ mm,
$d_1 = 22.6$ mm

7–69.

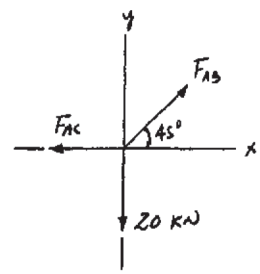
The two aluminum rods support the vertical force of $P = 20$ kN. Determine their required diameters if the allowable tensile stress for the aluminum is $\sigma_{\text{allow}} = 150$ MPa.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin 45° - 20 = 0; \qquad F_{AB} = 28.284 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 28.284 \cos 45° - F_{AC} = 0; \qquad F_{AC} = 20.0 \text{ kN}$



For rod AB,

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \qquad 150(10^6) = \frac{28.284(10^3)}{\frac{\pi}{4} d_{AB}^2}$$

$d_{AB} = 0.0155 \text{ m} = 15.5 \text{ mm}$ **Ans.**

For rod AC,

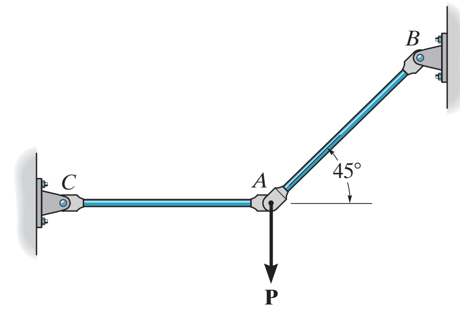
$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \qquad 150(10^6) = \frac{20.0(10^3)}{\frac{\pi}{4} d_{AC}^2}$$

$d_{AC} = 0.0130 \text{ m} = 13.0 \text{ mm}$ **Ans.**

Ans:
$d_{AB} = 15.5$ mm, $d_{AC} = 13.0$ mm

7–70.

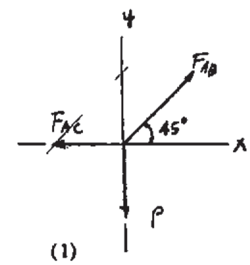
The two aluminum rods AB and AC have diameters of 10 mm and 8 mm, respectively. Determine the largest vertical force **P** that can be supported. The allowable tensile stress for the aluminum is $\sigma_{\text{allow}} = 150$ MPa.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin 45° - P = 0; \qquad P = F_{AB} \sin 45° \qquad$ **(1)**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB} \cos 45° - F_{AC} = 0 \qquad$ **(2)**



Assume failure of rod AB:

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \qquad 150(10^6) = \frac{F_{AB}}{\frac{\pi}{4}(0.01)^2}$$

$F_{AB} = 11.78$ kN

From Eq. (1),

$P = 8.33$ kN

Assume failure of rod AC:

$$\sigma_{\text{allow}} = \frac{F_{AC}}{A_{AC}}; \qquad 150(10^6) = \frac{F_{AC}}{\frac{\pi}{4}(0.008)^2}$$

$F_{AC} = 7.540$ kN

Solving Eqs. (1) and (2) yields:

$F_{AB} = 10.66$ kN; $\quad P = 7.54$ kN

Choose the smallest value.

$P = 7.54$ kN **Ans.**

Ans:
$P = 7.54$ kN

7–71.

An air-filled rubber ball has a diameter of 6 in. If the air pressure within the ball is increased until the diameter becomes 7 in., determine the average normal strain in the rubber.

SOLUTION

$d_0 = 6$ in.

$d = 7$ in.

$$\epsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7 - 6}{6} = 0.167 \text{ in./in.}$$ **Ans.**

Ans:
$\epsilon = 0.167$ in./in.

***7–72.**

A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

SOLUTION

$L_0 = 15 \text{ in.}$

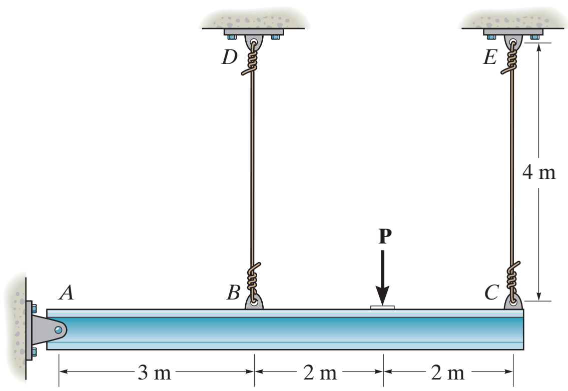
$L = \pi(5 \text{ in.})$

$$\epsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.}$$ **Ans.**

Ans:
$\epsilon = 0.0472 \text{ in./in.}$

7–73.

If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain in wires CE and BD.



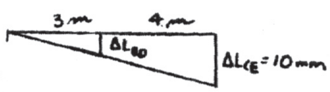
SOLUTION

$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\epsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

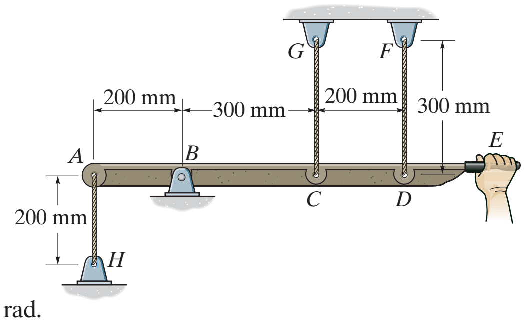
$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_{CE} = 0.00250$ mm/mm, $\epsilon_{BD} = 0.00107$ mm/mm

7–74.

The force applied at the handle of the rigid lever causes the lever to rotate clockwise about the pin B through an angle of 2°. Determine the average normal strain in each wire. The wires are unstretched when the lever is in the horizontal position.



SOLUTION

Geometry: The lever arm rotates through an angle of $\theta = \left(\frac{2°}{180}\right)\pi \text{ rad} = 0.03491 \text{ rad}$.

Since θ is small, the displacements of points A, C, and D can be approximated by

$$\delta_A = 200(0.03491) = 6.9813 \text{ mm}$$

$$\delta_C = 300(0.03491) = 10.4720 \text{ mm}$$

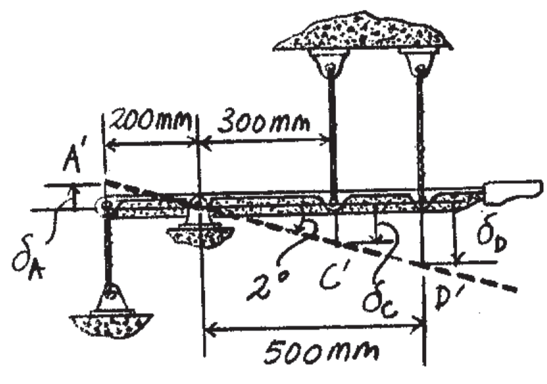
$$\delta_D = 500(0.03491) = 17.4533 \text{ mm}$$

Average Normal Strain: The unstretched length of wires AH, CG, and DF are $L_{AH} = 200$ mm, $L_{CG} = 300$ mm, and $L_{DF} = 300$ mm. We obtain

$$(\epsilon_{\text{avg}})_{AH} = \frac{\delta_A}{L_{AH}} = \frac{6.9813}{200} = 0.0349 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$(\epsilon_{\text{avg}})_{CG} = \frac{\delta_C}{L_{CG}} = \frac{10.4720}{300} = 0.0349 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$(\epsilon_{\text{avg}})_{DF} = \frac{\delta_D}{L_{DF}} = \frac{17.4533}{300} = 0.0582 \text{ mm/mm} \qquad \textbf{Ans.}$$

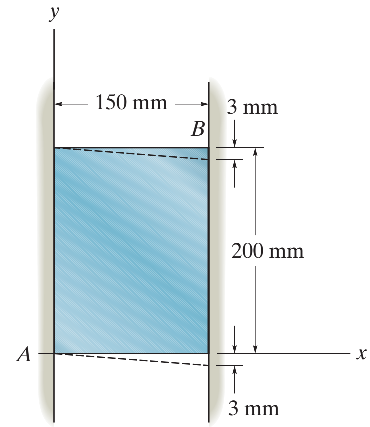


(a)

Ans:
$(\epsilon_{\text{avg}})_{AH} = 0.0349$ mm/mm
$(\epsilon_{\text{avg}})_{CG} = 0.0349$ mm/mm
$(\epsilon_{\text{avg}})_{DF} = 0.0582$ mm/mm

7–75.

The rectangular plate is subjected to the deformation shown by the dashed lines. Determine the average shear strain γ_{xy} in the plate.



SOLUTION

Geometry:

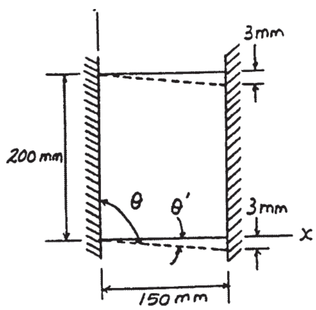
$$\theta' = \tan^{-1}\frac{3}{150} = 0.0200 \text{ rad}$$

$$\theta = \left(\frac{\pi}{2} + 0.0200\right) \text{rad}$$

Shear Strain:

$$\gamma_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \left(\frac{\pi}{2} + 0.0200\right)$$

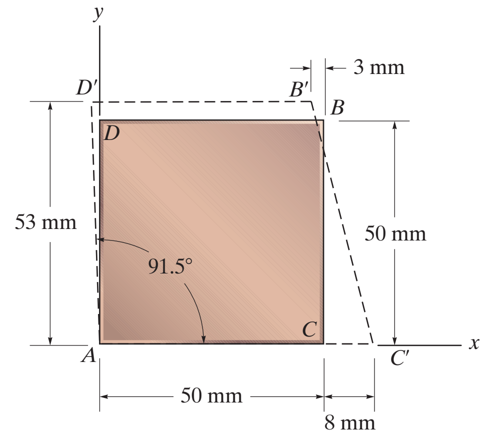
$$= -0.0200 \text{ rad} \qquad \textbf{Ans.}$$



Ans:
$\gamma_{xy} = -0.0200$ rad

***7–76.**

The square deforms into the position shown by the dashed lines. Determine the shear strain at each of its corners, A, B, C, and D, relative to the x, y axes. Side $D'B'$ remains horizontal.



SOLUTION

Geometry:

$$B'C' = \sqrt{(8 + 3)^2 + (53 \sin 88.5°)^2} = 54.1117 \text{ mm}$$

$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58) \cos 91.5°}$$

$$= 79.5860 \text{ mm}$$

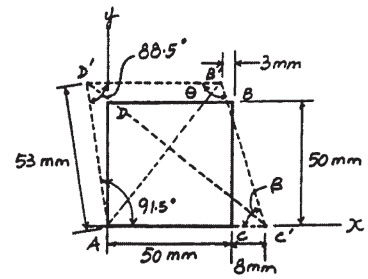
$$B'D' = 50 + 53 \sin 1.5° - 3 = 48.3874 \text{ mm}$$

$$\cos \theta = \frac{(B'D')^2 + (B'C')^2 - (C'D')^2}{2(B'D')(B'C')}$$

$$= \frac{48.3874^2 + 54.1117^2 - 79.5860^2}{2(48.3874)(54.1117)} = -0.20328$$

$$\theta = 101.73°$$

$$\beta = 180° - \theta = 78.27°$$



Shear Strain:

$$(\gamma_A)_{xy} = \frac{\pi}{2} - \pi\left(\frac{91.5°}{180°}\right) = -0.0262 \text{ rad}$$ **Ans.**

$$(\gamma_B)_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \pi\left(\frac{101.73°}{180°}\right) = -0.205 \text{ rad}$$ **Ans.**

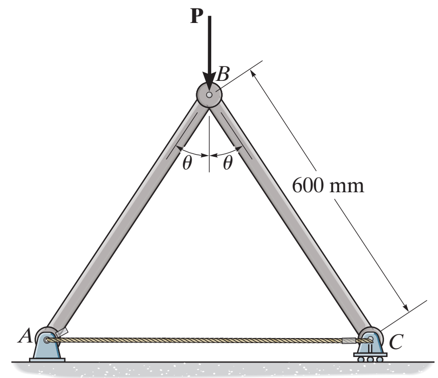
$$(\gamma_C)_{xy} = \beta - \frac{\pi}{2} = \pi\left(\frac{78.27°}{180°}\right) - \frac{\pi}{2} = -0.205 \text{ rad}$$ **Ans.**

$$(\gamma_D)_{xy} = \pi\left(\frac{88.5°}{180°}\right) - \frac{\pi}{2} = -0.0262 \text{ rad}$$ **Ans.**

Ans:
$(\gamma_A)_{xy} = -0.0262$ rad
$(\gamma_B)_{xy} = -0.205$ rad
$(\gamma_C)_{xy} = -0.205$ rad
$(\gamma_D)_{xy} = -0.0262$ rad

7–77.

The pin-connected rigid rods AB and BC are inclined at $\theta = 30°$ when they are unloaded. When the force **P** is applied θ becomes 30.2°. Determine the average normal strain in wire AC.



SOLUTION

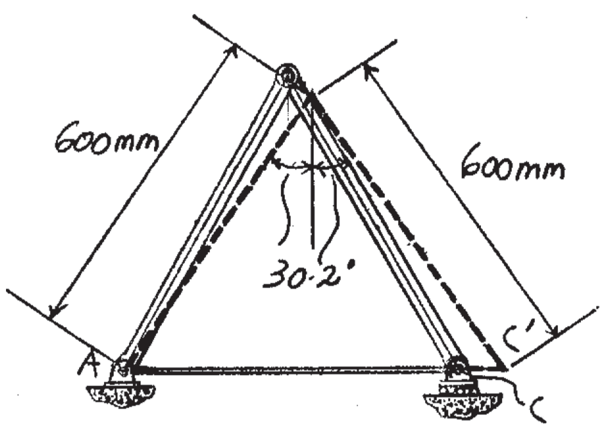
Geometry: Referring to Fig. *a*, the unstretched and stretched lengths of wire AD are

$$L_{AC} = 2(600 \sin 30°) = 600 \text{ mm}$$

$$L_{AC'} = 2(600 \sin 30.2°) = 603.6239 \text{ mm}$$

Average Normal Strain:

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{603.6239 - 600}{600} = 6.04(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$

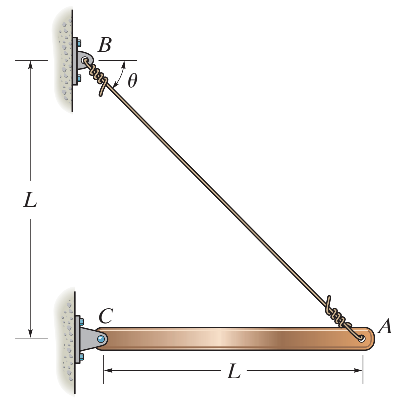


(a)

Ans:
$(\epsilon_{\text{avg}})_{AC} = 6.04(10^{-3}) \text{ mm/mm}$

7–78.

The wire AB is unstretched when $\theta = 45°$. If a load is applied to the bar AC, which causes θ to become 47°, determine the normal strain in the wire.



SOLUTION

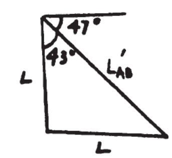
$$L^2 = L^2 + L'^2_{AB} - 2LL'_{AB}\cos 43°$$

$$L'_{AB} = 2L\cos 43°$$

$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{2L\cos 43° - \sqrt{2}L}{\sqrt{2}L}$$

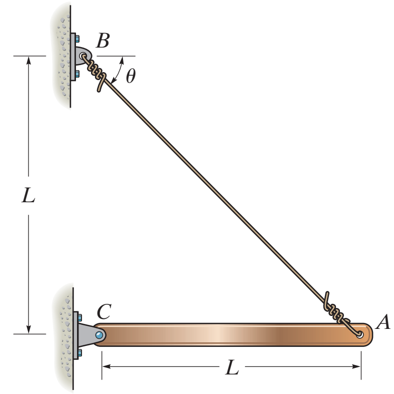
$$= 0.0343$$ **Ans.**



Ans:
$\epsilon_{AB} = 0.0343$

7–79.

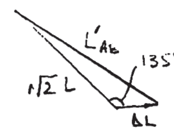
If a load applied to the bar AC causes point A to be displaced to the right by an amount ΔL, determine the normal strain in the wire AB. Originally, $\theta = 45°$.



SOLUTION

$$L'_{AB} = \sqrt{(\sqrt{2}L)^2 + \Delta L^2 - 2(\sqrt{2}L)(\Delta L)\cos 135°}$$

$$= \sqrt{2L^2 + \Delta L^2 + 2L\Delta L}$$



$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{\sqrt{2L^2 + \Delta L^2 + 2L\Delta L} - \sqrt{2}L}{\sqrt{2}L}$$

$$= \sqrt{1 + \frac{\Delta L^2}{2L^2} + \frac{\Delta L}{L}} - 1$$

Neglecting the higher-order terms,

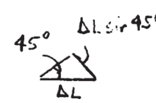
$$\epsilon_{AB} = \left(1 + \frac{\Delta L}{L}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + \ldots.. - 1 \qquad \text{(binomial theorem)}$$

$$= \frac{0.5\Delta L}{L} \qquad \textbf{Ans.}$$

Also,

$$\epsilon_{AB} = \frac{\Delta L \sin 45°}{\sqrt{2}L} = \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans.}$$

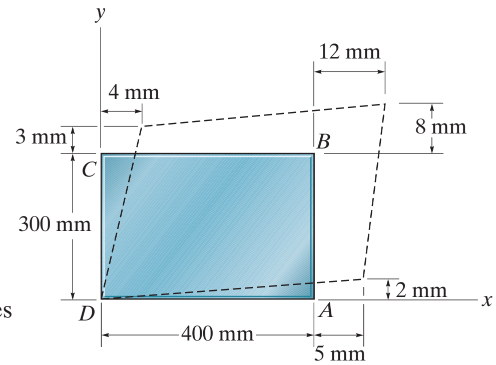


Ans:

$$\epsilon_{AB} = \frac{0.5\Delta L}{L}$$

***7–80.**

Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{7}{306} = 0.022876 \text{ rad}$$

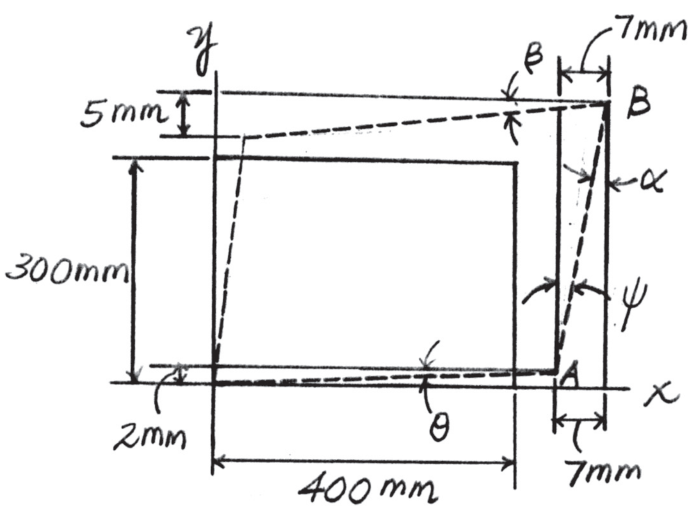
$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

Shear Strain: By definition,

$$(\gamma_A)_{xy} = \theta + \psi = 0.02781 \text{ rad} = 27.8(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_B)_{xy} = \alpha + \beta = 0.03513 \text{ rad} = 35.1(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

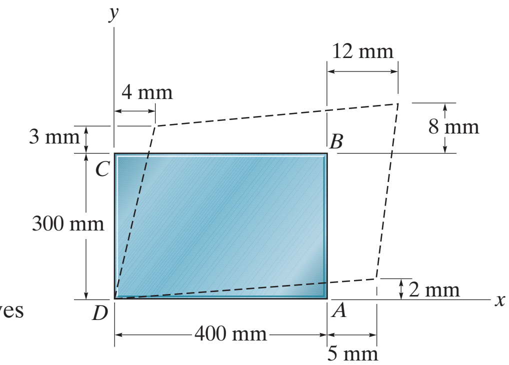


(a)

Ans:
$(\gamma_A)_{xy} = 27.8(10^{-3})$ rad
$(\gamma_B)_{xy} = 35.1(10^{-3})$ rad

7–81.

Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*, the small-angle analysis gives

$$\alpha = \psi = \frac{4}{303} = 0.013201 \text{ rad}$$

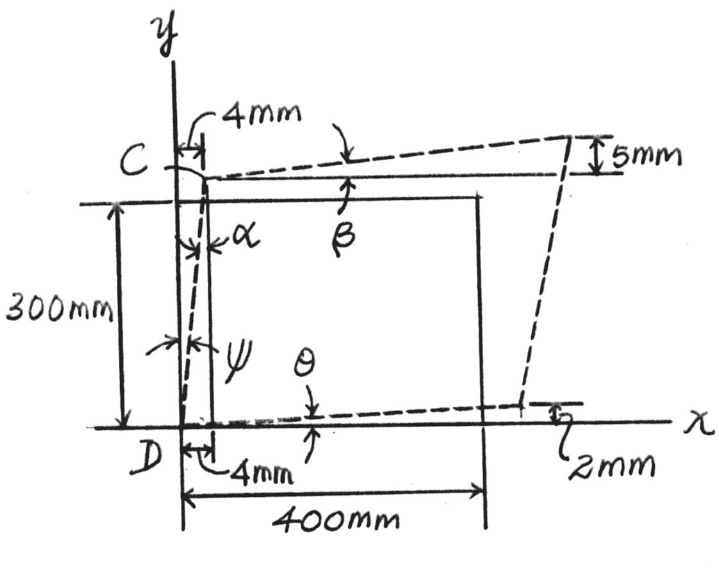
$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

Shear Strain: By definition,

$$(\gamma_{xy})_C = \alpha + \beta = 0.02546 \text{ rad} = 25.5(10^{-3}) \text{ rad}$$ **Ans.**

$$(\gamma_{xy})_D = \theta + \psi = 0.01814 \text{ rad} = 18.1(10^{-3}) \text{ rad}$$ **Ans.**

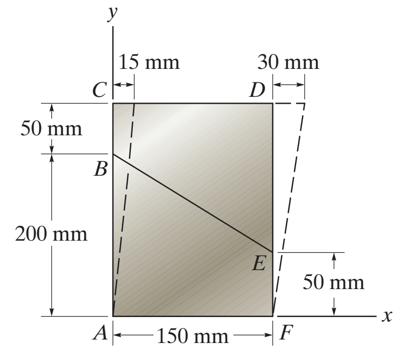


(a)

Ans:
$(\gamma_{xy})_C = 25.5(10^{-3})$ rad
$(\gamma_{xy})_D = 18.1(10^{-3})$ rad

7–82.

The material distorts into the dashed position shown. Determine the average normal strains ϵ_x, ϵ_y and the shear strain γ_{xy} at A, and the average normal strain along line BE.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

$$\tan\theta = \frac{15}{250}; \qquad \theta = (3.4336°)\left(\frac{\pi}{180°}\text{ rad}\right) = 0.05993 \text{ rad}$$

$$L_{AC}' = \sqrt{15^2 + 150^2} = \sqrt{62725} \text{ mm}$$

$$\frac{BB'}{15} = \frac{200}{250}; \qquad BB' = 12 \text{ mm} \qquad \frac{EE'}{30} = \frac{50}{250}; \qquad EE' = 6 \text{ mm}$$

$$x' = 150 + EE' - BB' = 150 + 6 - 12 = 144 \text{ mm}$$

$$L_{BE} = \sqrt{150^2 + 150^2} = 150\sqrt{2} \text{ mm} \quad L_{B'E'} = \sqrt{144^2 + 150^2} = \sqrt{43236} \text{ mm}$$

Average Normal and Shear Strain: Since no deformation occurs along the *x* axis,

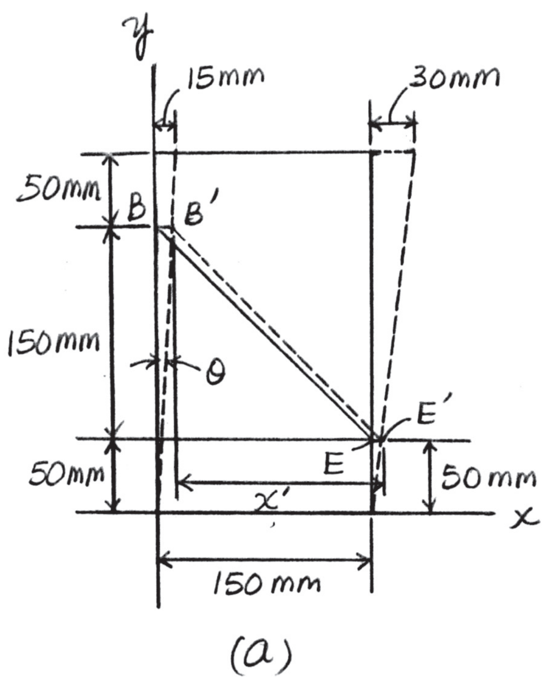
$$(\epsilon_x)_A = 0 \qquad \textbf{Ans.}$$

$$(\epsilon_y)_A = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{\sqrt{62725} - 250}{250} = 1.80(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$

By definition,

$$(\gamma_{xy})_A = \theta = 0.0599 \text{ rad} \qquad \textbf{Ans.}$$

$$\epsilon_{BE} = \frac{L_{B'E'} - L_{BE}}{L_{BE}} = \frac{\sqrt{43236} - 150\sqrt{2}}{150\sqrt{2}} = -0.0198 \text{ mm/mm} \qquad \textbf{Ans.}$$

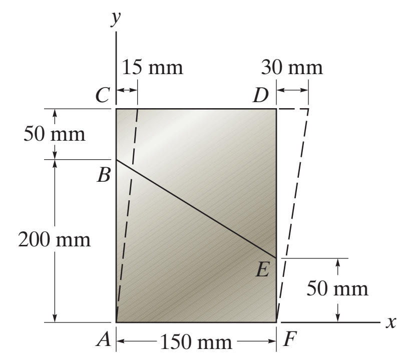


(a)

Ans:
$(\epsilon_x)_A = 0$
$(\epsilon_y)_A = 1.80(10^{-3})$ mm/mm
$(\gamma_{xy})_A = 0.0599$ rad
$\epsilon_{BE} = -0.0198$ mm/mm

7–83.

The material distorts into the dashed position shown. Determine the average normal strains along the diagonals *AD* and *CF*.



SOLUTION

Geometry: Referring to the geometry shown in Fig. *a*,

$$L_{AD} = L_{CF} = \sqrt{150^2 + 250^2} = \sqrt{85000}\text{ mm}$$

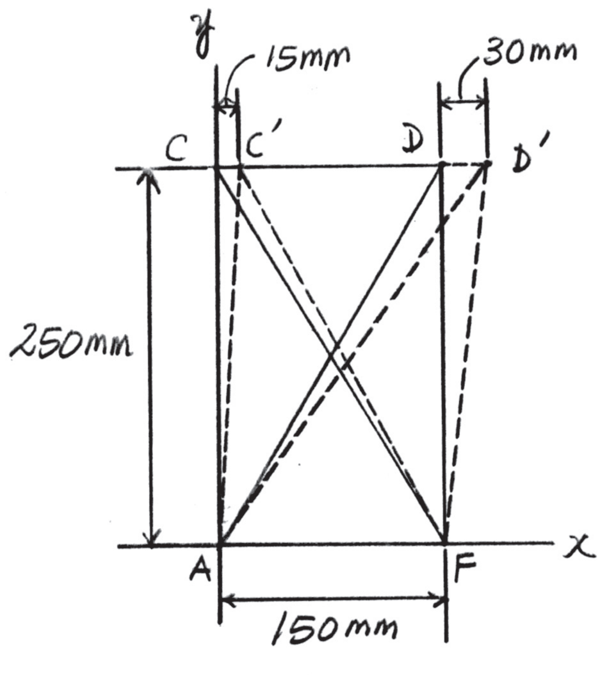
$$L_{AD'} = \sqrt{(150 + 30)^2 + 250^2} = \sqrt{94900}\text{ mm}$$

$$L_{C'F} = \sqrt{(150 - 15)^2 + 250^2} = \sqrt{80725}\text{ mm}$$

Average Normal Strain:

$$\epsilon_{AD} = \frac{L_{AD'} - L_{AD}}{L_{AD}} = \frac{\sqrt{94900} - \sqrt{85000}}{\sqrt{85000}} = 0.0566\text{ mm/mm} \qquad \textbf{Ans.}$$

$$\epsilon_{CF} = \frac{L_{C'F} - L_{CF}}{L_{CF}} = \frac{\sqrt{80725} - \sqrt{85000}}{\sqrt{85000}} = -0.0255\text{ mm/mm} \qquad \textbf{Ans.}$$

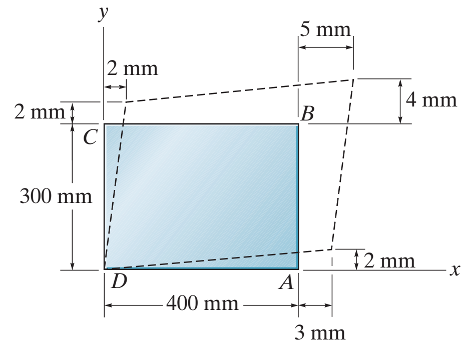


(a)

Ans:
$\epsilon_{AD} = 0.0566\text{ mm/mm}$
$\epsilon_{CF} = -0.0255\text{ mm/mm}$

***7–84.**

Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.

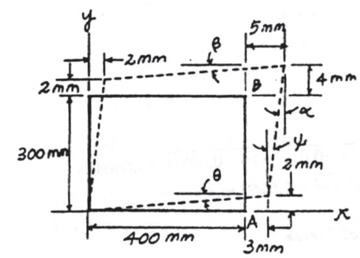


SOLUTION

Geometry: For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$



Shear Strain:

$$(\gamma_B)_{xy} = \alpha + \beta$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

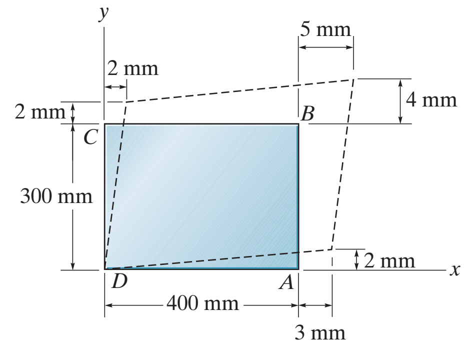
$$(\gamma_A)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_B)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_A)_{xy} = 11.6(10^{-3})$ rad

7–85.

Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.

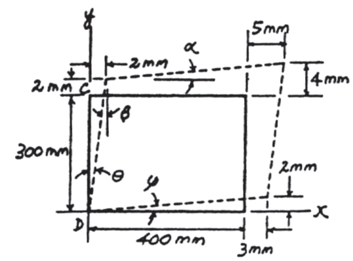


SOLUTION

Geometry: For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$



Shear Strain:

$$(\gamma_C)_{xy} = \alpha + \beta$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

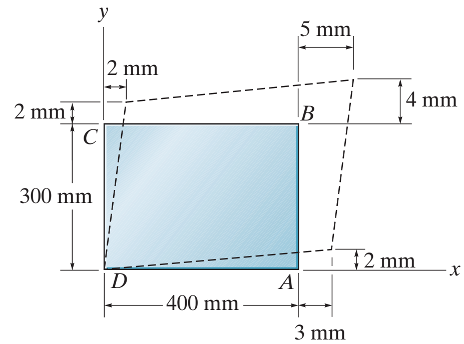
$$(\gamma_D)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

Ans:
$(\gamma_C)_{xy} = 11.6(10^{-3})$ rad,
$(\gamma_D)_{xy} = 11.6(10^{-3})$ rad

7–86.

Determine the average normal strain that occurs along the diagonals AC and DB.



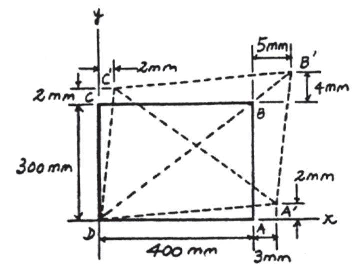
SOLUTION

Geometry:

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$

$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$



Average Normal Strain:

$$\epsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$

$$= 0.00160 \text{ mm/mm} = 1.60\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

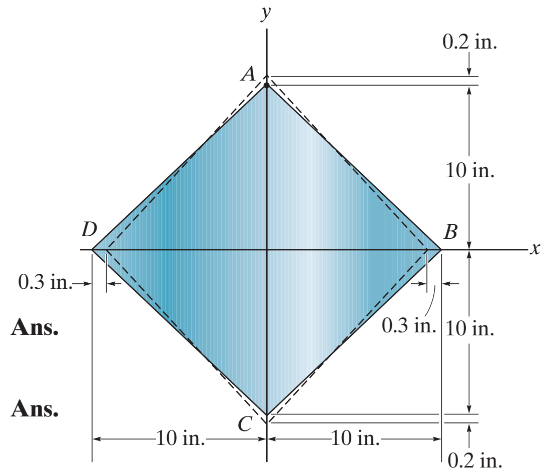
$$\epsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$

$$= 0.0128 \text{ mm/mm} = 12.8\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

Ans:
$\epsilon_{AC} = 1.60\left(10^{-3}\right)$ mm/mm
$\epsilon_{DB} = 12.8\left(10^{-3}\right)$ mm/mm

7–87.

The corners of the square plate are given the displacements indicated. Determine the average normal strains ϵ_x and ϵ_y along the x and y axes.



SOLUTION

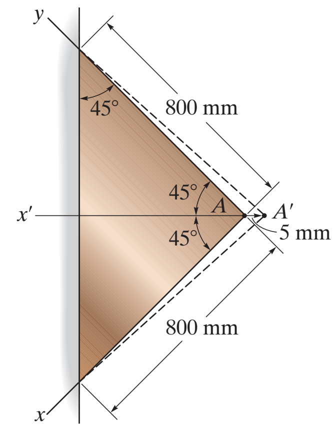
$$\epsilon_x = \frac{-0.3}{10} = -0.03 \text{ in./in.}$$ **Ans.**

$$\epsilon_y = \frac{0.2}{10} = 0.02 \text{ in./in.}$$ **Ans.**

Ans:
$\epsilon_x = -0.03$ in./in.
$\epsilon_y = 0.02$ in./in.

***7–88.**

The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the shear strain, γ_{xy}, at A.



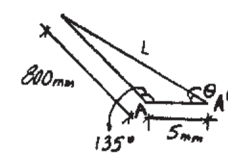
SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\frac{\sin 135^\circ}{803.54} = \frac{\sin\theta}{800}; \qquad \theta = 44.75^\circ = 0.7810 \text{ rad}$$

$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

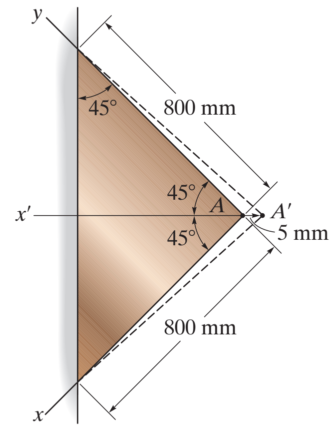
$$= 0.00880 \text{ rad}$$ **Ans.**



Ans:
$\gamma_{xy} = 0.00880$ rad

7–89.

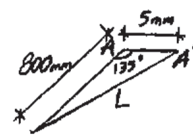
The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain ϵ_x along the x axis.



SOLUTION

$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

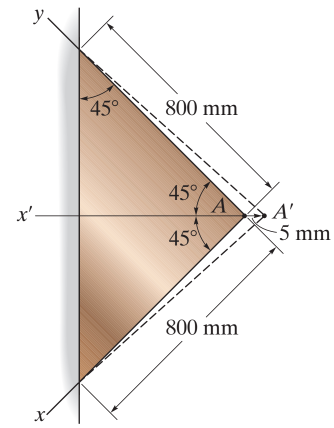
$$\epsilon_x = \frac{803.54 - 800}{800} = 0.00443 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_x = 0.00443$ mm/mm

7–90.

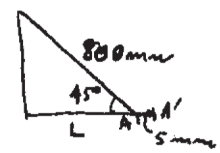
The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_{x'}$ along the x' axis.



SOLUTION

$L = 800 \cos 45° = 565.69 \text{ mm}$

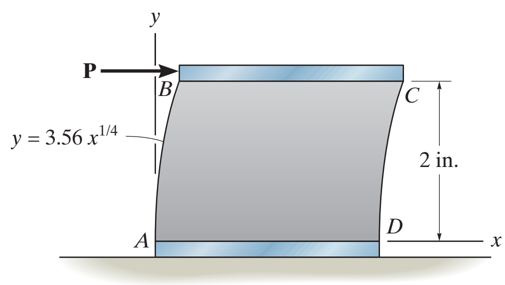
$$\epsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm}$$ **Ans.**



Ans:
$\epsilon_{x'} = 0.00884 \text{ mm/mm}$

7–91.

The polysulfone block is glued at its top and bottom to the rigid plates. If a tangential force, applied to the top plate, causes the material to deform so that its sides are described by the equation $y = 3.56x^{1/4}$, determine the shear strain at the corners A and B.



SOLUTION

$$y = 3.56\,x^{1/4}$$

$$\frac{dy}{dx} = 0.890\,x^{-3/4}$$

$$\frac{dx}{dy} = 1.123\,x^{3/4}$$

At A, $x = 0$

$$\gamma_A = \frac{dx}{dy} = 0 \qquad \textbf{Ans.}$$

At B,

$$2 = 3.56\,x^{1/4}$$

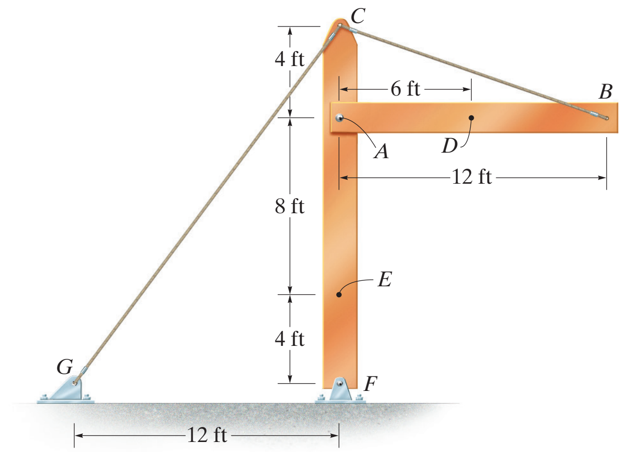
$$x = 0.0996\text{ in.}$$

$$\gamma_B = \frac{dx}{dy} = 1.123(0.0996)^{3/4} = 0.199\text{ rad} \qquad \textbf{Ans.}$$

Ans:
$\gamma_A = 0$
$\gamma_B = 0.199$ rad

R7–1.

The beam AB is pin supported at A and supported by a cable BC. A *separate* cable CG is used to hold up the frame. If AB weighs 120 lb/ft and the column FC has a weight of 180 lb/ft, determine the resultant internal loadings acting on cross sections located at points D and E. Neglect the thickness of both the beam and column in the calculation.



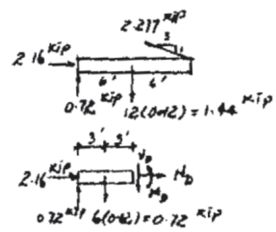
SOLUTION

Segment AD:

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad N_D + 2.16 = 0;\quad N_D = -2.16 \text{ kip}$ **Ans.**

$+\downarrow\Sigma F_y = 0;\quad V_D + 0.72 - 0.72 = 0;\quad V_D = 0$ **Ans.**

$\circlearrowleft +\Sigma M_D = 0;\quad M_D - 0.72(3) = 0;\quad M_D = 2.16 \text{ kip}\cdot\text{ft}$ **Ans.**

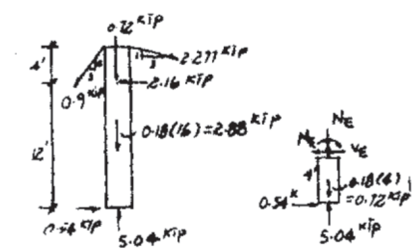


Segment FE:

$\overset{+}{\leftarrow}\Sigma F_x = 0;\quad V_E - 0.54 = 0;\quad V_E = 0.540 \text{ kip}$ **Ans.**

$+\downarrow\Sigma F_y = 0;\quad N_E + 0.72 - 5.04 = 0;\quad N_E = 4.32 \text{ kip}$ **Ans.**

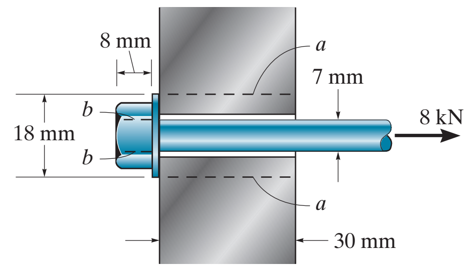
$\circlearrowleft +\Sigma M_E = 0;\quad -M_E + 0.54(4) = 0;\quad M_E = 2.16 \text{ kip}\cdot\text{ft}$ **Ans.**



Ans:
$N_D = -2.16 \text{ kip},\ V_D = 0,\ M_D = 2.16 \text{ kip}\cdot\text{ft}$
$V_E = 0.540 \text{ kip},\ N_E = 4.32 \text{ kip},\ M_E = 2.16 \text{ kip}\cdot\text{ft}$

R7–2.

The long bolt passes through the 30-mm-thick plate. If the force in the bolt shank is 8 kN, determine the average normal stress in the shank, the average shear stress along the cylindrical area of the plate defined by the section lines *a–a*, and the average shear stress in the bolt head along the cylindrical area defined by the section lines *b–b*.



SOLUTION

$$\sigma_s = \frac{P}{A} = \frac{8\,(10^3)}{\frac{\pi}{4}\,(0.007)^2} = 208\text{ MPa}$$ **Ans.**

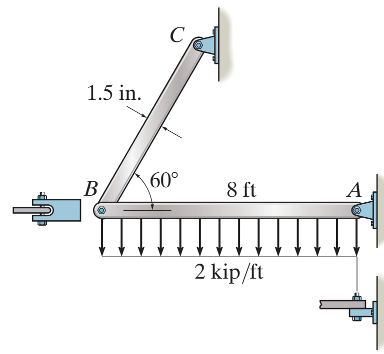
$$(\tau_{\text{avg}})_a = \frac{V}{A} = \frac{8\,(10^3)}{\pi\,(0.018)(0.030)} = 4.72\text{ MPa}$$ **Ans.**

$$(\tau_{\text{avg}})_b = \frac{V}{A} = \frac{8\,(10^3)}{\pi\,(0.007)(0.008)} = 45.5\text{ MPa}$$ **Ans.**

Ans:
$\sigma_s = 208\text{ MPa}$, $(\tau_{\text{avg}})_a = 4.72\text{ MPa}$,
$(\tau_{\text{avg}})_b = 45.5\text{ MPa}$

R7–3.

Determine the required thickness of member BC and the diameter of the pins at A and B if the allowable normal stress for member BC is $\sigma_{\text{allow}} = 29$ ksi, and the allowable shear stress for the pins is $\tau_{\text{allow}} = 10$ ksi.



SOLUTION

Referring to the FBD of member AB, Fig. a,

$$\zeta + \Sigma M_A = 0; \quad 2(8)(4) - F_{BC} \sin 60^\circ (8) = 0 \quad F_{BC} = 9.238 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 9.238 \cos 60^\circ - A_x = 0 \quad A_x = 4.619 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0; \quad 9.238 \sin 60^\circ - 2(8) + A_y = 0 \quad A_y = 8.00 \text{ kip}$$

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{4.619^2 + 8.00^2} = 9.238 \text{ kip}$$

Pin A is subjected to single shear, Fig. c, while pin B is subjected to double shear, Fig. b.

$$V_A = F_A = 9.238 \text{ kip} \qquad V_B = \frac{F_{BC}}{2} = \frac{9.238}{2} = 4.619 \text{ kip}$$

For member BC,

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \quad 29 = \frac{9.238}{1.5(t)} \quad t = 0.2124 \text{ in.}$$

$$\text{Use } t = \frac{1}{4} \text{ in.} \qquad \textbf{Ans.}$$

For pin A,

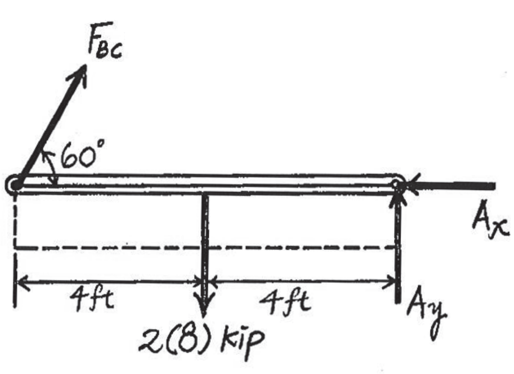
$$\tau_{\text{allow}} = \frac{V_A}{A_A}; \quad 10 = \frac{9.238}{\frac{\pi}{4} d_A^2} \quad d_A = 1.085 \text{ in.}$$

$$\text{Use } d_A = 1\frac{1}{8} \text{ in.} \qquad \textbf{Ans.}$$

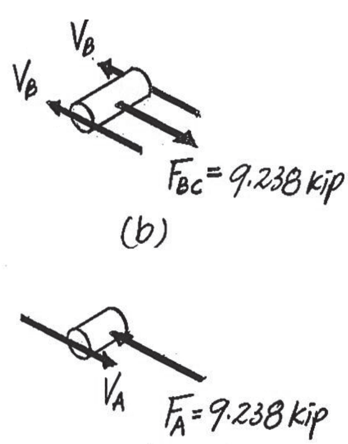
For pin B,

$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \quad 10 = \frac{4.619}{\frac{\pi}{4} d_B^2} \quad d_B = 0.7669 \text{ in.}$$

$$\text{Use } d_B = \frac{13}{16} \text{ in.} \qquad \textbf{Ans.}$$



(a)



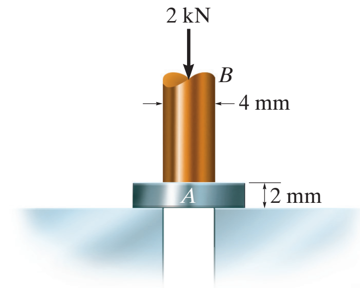
(b)

(c)

Ans:
$t = \frac{1}{4}$ in., $d_A = 1\frac{1}{8}$ in., $d_B = \frac{13}{16}$ in.

***R7–4.**

The circular punch B exerts a force of 2 kN on the top of the plate A. Determine the average shear stress in the plate due to this loading.



SOLUTION

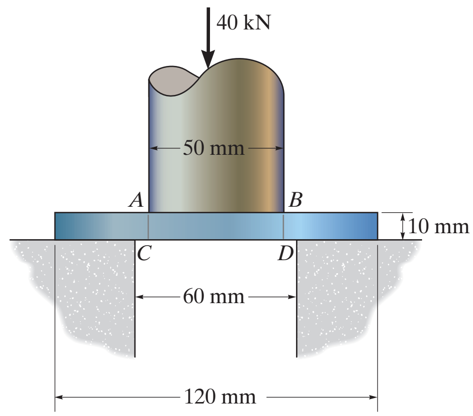
Average Shear Stress: The shear area $A = \pi(0.004)(0.002) = 8.00(10^{-6})\pi \text{ m}^2$.

$$\tau_{avg} = \frac{V}{A} = \frac{2(10^3)}{8.00(10^{-6})\pi} = 79.6 \text{ MPa}$$ **Ans.**

Ans:
$\tau_{avg} = 79.6$ MPa

R7–5.

Determine the average punching shear stress the circular shaft creates in the metal plate through section AC and BD. Also, what is the bearing stress developed on the surface of the plate under the shaft?



SOLUTION

Average Shear and Bearing Stress: The area of the shear plane and the bearing area on the punch are $A_V = \pi(0.05)(0.01) = 0.5(10^{-3})\pi \text{ m}^2$ and $A_b = \frac{\pi}{4}\left(0.12^2 - 0.06^2\right) = 2.7(10^{-3})\pi \text{ m}^2$. We obtain

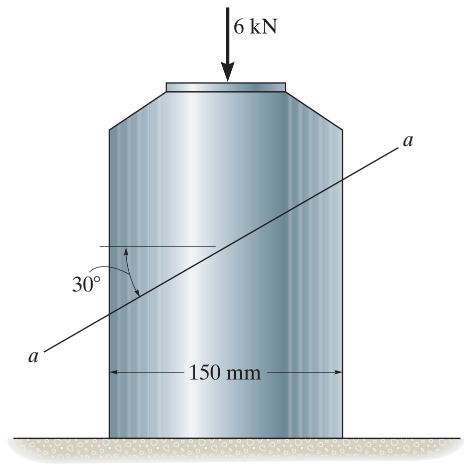
$$\tau_{avg} = \frac{P}{A_V} = \frac{40(10^3)}{0.5(10^{-3})\pi} = 25.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_b = \frac{P}{A_b} = \frac{40(10^3)}{2.7(10^{-3})\pi} = 4.72 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{avg} = 25.5 \text{ MPa}, \sigma_b = 4.72 \text{ MPa}$

R7–6.

The bearing pad consists of a 150 mm by 150 mm block of aluminum that supports a compressive load of 6 kN. Determine the average normal and shear stress acting on the plane through section *a–a*. Show the results on a differential volume element located on the plane.



SOLUTION

Equation of Equilibrium:

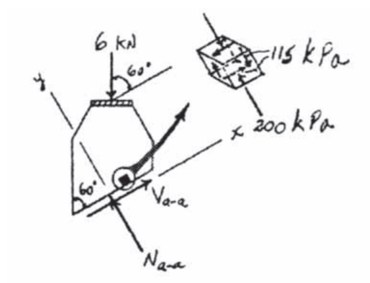
$+\nearrow\Sigma F_x = 0; \quad V_{a-a} - 6\cos 60^\circ = 0 \quad V_{a-a} = 3.00\text{ kN}$

$\nwarrow + \Sigma F_y = 0; \quad N_{a-a} - 6\sin 60^\circ = 0 \quad N_{a-a} = 5.196\text{ kN}$

Averge Normal Stress And Shear Stress: The cross-sectional area at section *a–a* is $A = \left(\dfrac{0.15}{\sin 60^\circ}\right)(0.15) = 0.02598\text{ m}^2$.

$$\sigma_{a-a} = \frac{N_{a-a}}{A} = \frac{5.196(10^3)}{0.02598} = 200\text{ kPa}$$ **Ans.**

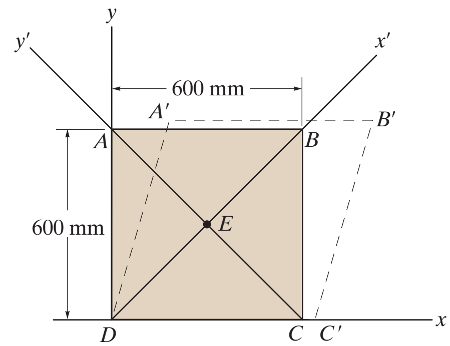
$$\tau_{a-a} = \frac{V_{a-a}}{A} = \frac{3.00(10^3)}{0.02598} = 115\text{ kPa}$$ **Ans.**



Ans:
$\sigma_{a-a} = 200\text{ kPa}, \tau_{a-a} = 115\text{ kPa}$

R7–7.

The square plate is deformed into the shape shown by the dashed lines. If DC has a normal strain $\epsilon_x = 0.004$, DA has a normal strain $\epsilon_y = 0.005$ and at D, $\gamma_{xy} = 0.02$ rad, determine the average normal strain along diagonal CA.



SOLUTION

Average Normal Strain: The stretched length of sides DA and DC are

$$L_{DC'} = (1 + \epsilon_x)L_{DC} = (1 + 0.004)(600) = 602.4 \text{ mm}$$

$$L_{DA'} = (1 + \epsilon_y)L_{DA} = (1 + 0.005)(600) = 603 \text{ mm}$$

Also,

$$\alpha = \frac{\pi}{2} - 0.02 = 1.5508 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 88.854°$$

Thus, the length of $C'A'$ can be determined using the cosine law with reference to Fig. *a*.

$$L_{C'A'} = \sqrt{602.4^2 + 603^2 - 2(602.4)(603)\cos 88.854°}$$

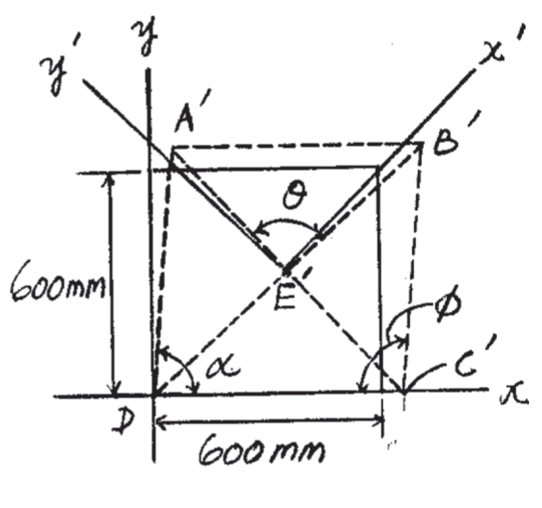
$$= 843.7807 \text{ mm}$$

The original length of diagonal CA can be determined using the Pythagorean theorem.

$$L_{CA} = \sqrt{600^2 + 600^2} = 848.5281 \text{ mm}$$

Thus,

$$(\epsilon_{avg})_{CA} = \frac{L_{C'A'} - L_{CA}}{L_{CA}} = \frac{843.7807 - 848.5281}{848.5281} = -5.59(10^{-3}) \text{ mm/mm} \quad \textbf{Ans.}$$

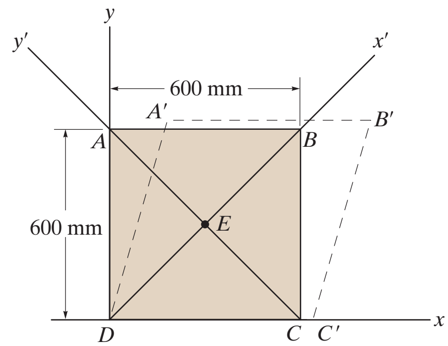


(a)

Ans:
$(\epsilon_{avg})_{CA} = -5.59(10^{-3})$ mm/mm

***R7–8.**

The square plate is deformed into the shape shown by the dashed lines. If DC has a normal strain $\epsilon_x = 0.004$, DA has a normal strain $\epsilon_y = 0.005$ and at D, $\gamma_{xy} = 0.02$ rad, determine the shear strain at point E with respect to the x' and y' axes.



SOLUTION

Average Normal Strain: The stretched length of sides DC and BC are

$$L_{DC'} = (1 + \epsilon_x)L_{DC} = (1 + 0.004)(600) = 602.4 \text{ mm}$$

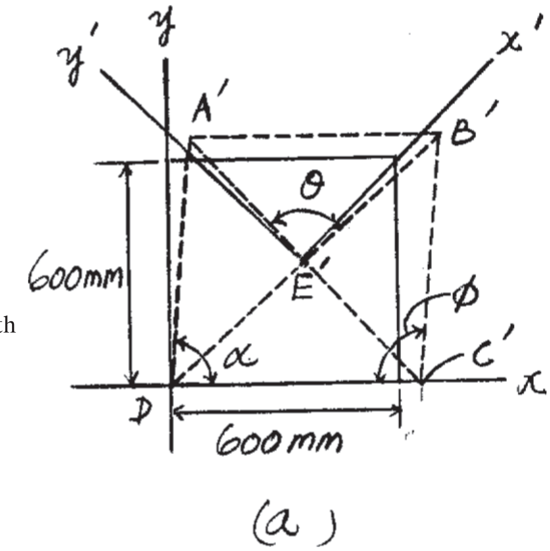
$$L_{B'C'} = (1 + \epsilon_y)L_{BC} = (1 + 0.005)(600) = 603 \text{ mm}$$

Also,

$$\alpha = \frac{\pi}{2} - 0.02 = 1.5508 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 88.854°$$

$$\phi = \frac{\pi}{2} + 0.02 = 1.5908 \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = 91.146°$$

Thus, the length of $C'A'$ and DB' can be determined using the cosine law with reference to Fig. a.



$$L_{C'A'} = \sqrt{602.4^2 + 603^2 - 2(602.4)(603)\cos 88.854°} = 843.7807 \text{ mm}$$

$$L_{DB'} = \sqrt{602.4^2 + 603^2 - 2(602.4)(603)\cos 91.146°} = 860.8273 \text{ mm}$$

Thus,

$$L_{E'A'} = \frac{L_{C'A'}}{2} = 421.8903 \text{ mm} \qquad L_{E'B'} = \frac{L_{DB'}}{2} = 430.4137 \text{ mm}$$

Using this result and applying the cosine law to the triangle $A'E'B'$, Fig. a,

$$602.4^2 = 421.8903^2 + 430.4137^2 - 2(421.8903)(430.4137)\cos\theta$$

$$\theta = 89.9429°\left(\frac{\pi \text{ rad}}{180°}\right) = 1.5698 \text{ rad}$$

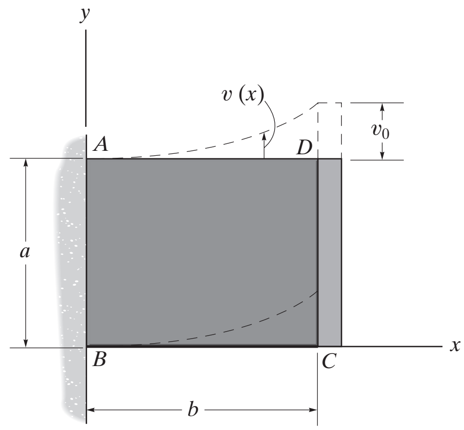
Shear Strain:

$$(\gamma_E)_{x'y'} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 1.5698 = 0.996(10^{-3}) \text{ rad}$$ **Ans.**

Ans:
$(\gamma_E)_{x'y'} = 0.996(10^{-3})$ rad

R7–9.

The rubber block is fixed along edge AB, and edge CD is moved so that the vertical displacement of any point in the block is given by $v(x) = (v_0/b^3)\,x^3$. Determine the shear strain γ_{xy} at points $(b/2, a/2)$ and (b, a).



SOLUTION

Shear Strain: From Fig. *a*,

$$\frac{dv}{dx} = \tan\gamma_{xy}$$

$$\frac{3v_0}{b^3}x^2 = \tan\gamma_{xy}$$

$$\gamma_{xy} = \tan^{-1}\left(\frac{3v_0}{b^3}x^2\right)$$

Thus, at point $(b/2, a/2)$,

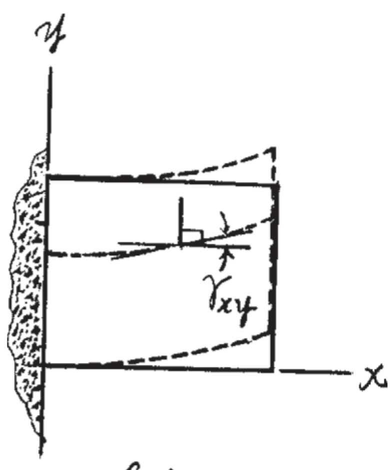
$$\gamma_{xy} = \tan^{-1}\left[\frac{3v_0}{b^3}\left(\frac{b}{2}\right)^2\right]$$

$$= \tan^{-1}\left[\frac{3}{4}\left(\frac{v_0}{b}\right)\right] \qquad \textbf{Ans.}$$

and at point (b, a),

$$\gamma_{xy} = \tan^{-1}\left[\frac{3v_0}{b^3}(b^2)\right]$$

$$= \tan^{-1}\left[3\left(\frac{v_0}{b}\right)\right] \qquad \textbf{Ans.}$$



(a)

Ans:

At $(b/2, a/2)$: $\gamma_{xy} = \tan^{-1}\left[\frac{3}{4}\left(\frac{v_0}{b}\right)\right]$,

At (b, a): $\gamma_{xy} = \tan^{-1}\left[3\left(\frac{v_0}{b}\right)\right]$