1. W310 x 143 , $r_x = 138$ mm , $A = 18200$ mm^2

a) K_x? Column B , RL = 350 kN (Dead), 550 kN (Live)
FL = 600 kN (dead), 1000 kN (Live)

For lower column

$G_e = 10$ (pinned)

$G_u = \dfrac{\Sigma I/L_c}{\Sigma I/L_g}$, $I_{cB} = 348E6$ mm^4 , $I_{410\times60} = 216E6$ mm^4 , $I_{410\times74} = 275E6$ mm^4 , $I_{410\times54} = 186E6$, $I_{410\times67} = 245E6$

$\therefore G_u = \dfrac{348/4200 + \frac{348}{3600}}{216/9300 + 275/11000} = 3.72256728 8$

$K_x \simeq 0.935$ (Ance 6) 1-219

For upper column

$G_e = \dfrac{348/3600 + \frac{348}{4200}}{216/9300 + \frac{275}{11000}} = 3.722567288$

$G_u = \dfrac{348/3600}{\frac{186}{9300} + \frac{245}{11000}} = 2.1553$

$K \simeq 0.87$ (1-219)

b) Check bottom

$\dfrac{K_x L_x}{r_x} = \dfrac{0.935 \cdot 4200}{138} = 28.3049 < 200$, OK ← governs

Check top

$\dfrac{0.87 \cdot 3600}{138} = 22.696 < 200$, OK

Section classification

$h/w = \frac{323-2(22.9)}{14} = 19.8 \leq \frac{1900}{\sqrt{345}} = 102.3$, $\therefore$ not class 4

$\frac{b_{el}}{t} = \frac{309/2}{22.9} = 6.746 \leq \frac{200}{\sqrt{345}} = 10.7676$, $\therefore$ not class 4

$$\lambda = \sqrt{\frac{345}{\pi^2 \cdot 2E5 / 28.3049^2}} = 0.374202$$

$$C_r = 0.9 \cdot 345 \cdot 18200 \cdot \left(1 + 0.37420^{2 \cdot 1.34}\right)^{-1/1.34}$$
$$= 5366.24 \text{ kN}$$

$1.25(350+600) + 1.5(550+1000) = 3512.5$ kN

$C_r > 3512.5$, $\therefore$ OK

② $l = 6m$, W410x54 ; spacing = 1.6m of 15kN DL, 25kN LL

Loading = $15 \cdot 1.25 + 25 \cdot 1.5 = 56.25$ kN

From 10 in 5-135, $M_x = R_1 x - P(x-a)$, $R_1 = P/l\,(l - a + b)$, $b = l - a - 1.6$
$= P/l\,[2l - 2a - 1.6]$

$\therefore M_x = P/l\,[2l - 2a - 1.6]\,x - Px + Pa$

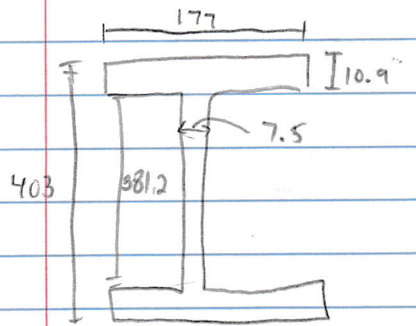
when M_x is maximized, $\frac{dM}{dx} = 0$

$\frac{dM}{dx} = P - \frac{2Pa}{l} - \frac{1.6P}{l} = 0$, $a = ?$ $P = 56.25$, $l = 6m$

$\therefore a = 2.2$, $\therefore b = 6 - 2.2 - 1.6 = 2.2$

Since case for symmetric loading satisfied, $M_{max} = Pa$

$M = 56.25 \cdot 2.2 = 123.75$ kNm



$\frac{b_{el}}{t} = \frac{177/2}{10.9} = 8.11927 < \frac{200}{\sqrt{345}} = 10.76$, $\therefore$ Not class 4

$< \frac{170}{\sqrt{345}} = 9.152$ $\therefore$ Not class 3

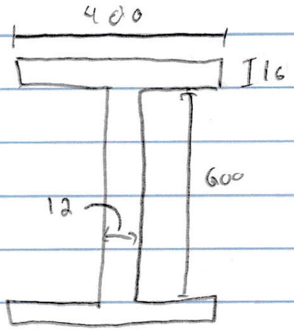
$\frac{h}{w} = \frac{403 - 2(10.9)}{7.5} = 50.827 < \frac{1900}{\sqrt{345}} = 102.29$, $\therefore$ not class 4

$< \frac{1700}{\sqrt{345}}$, $\therefore$ not class 3

$Z = \frac{381.2}{2} \cdot \frac{381.2}{4} \cdot 7.5 \cdot 2 + 177 \cdot 10.9 \cdot 2 \cdot \left(\frac{381.2}{2} + \frac{10.9}{2}\right)$
$= 1028941.23$ mm^3

$M_r = 0.9 \cdot 1028941.23 \cdot 345 = 319.49$ kNm > 123.75, $\therefore$ OK

(3)



$$J = 2\cdot\frac{1}{3}\cdot 400\cdot 16^3 + \frac{1}{3}\cdot 600\cdot 12^3$$
$$= 1437866667 \text{ mm}^4$$

$$I_y = \frac{1}{12}\left(2\cdot 400\cdot 16^3 + 600\cdot 12^3\right)$$
$$= 359466.6667$$

Section classification

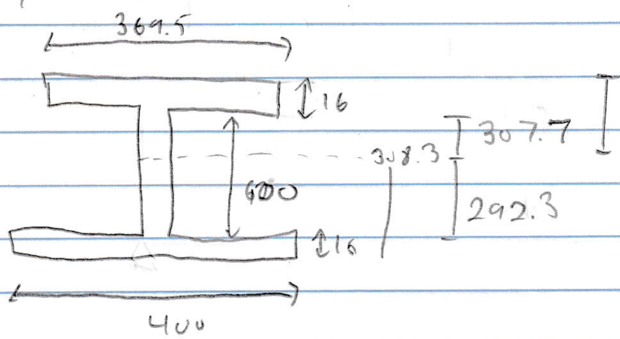
$$C_w = I_y\left(\frac{600-16}{2}\right)^2 = 3.064956587$$

$$\frac{b_{el}}{t} = \frac{400/2}{16} = 12.5 \nleq \frac{200}{\sqrt{300}} = 11.547 \quad , \therefore \text{ class } 4$$

$$\frac{h}{w} = \frac{600}{12} \leq \frac{1900}{\sqrt{300}} \quad , 50 \leq 109.6966 \quad \therefore \text{ at class } 3$$

Reduce flange area - Local buckling check

$$\frac{b_{el}}{16} = \frac{200}{\sqrt{300}}, \quad b_{el} = 184.752, \quad \Delta b = 30.5 \text{ mm}$$



$$\bar{y} = \frac{400\cdot 16\cdot 8 + 600\cdot 12\cdot 316 + 369.5\cdot 16\cdot 624}{400\cdot 16 + 600\cdot 12 + 369.5\cdot 16}$$

$$= 308.30 \text{ mm}$$

$$I_{xe} = 12\int_{-292.3}^{307.7} y^2\,dy + 369.5\int_{307.7}^{323.7} y^2\,dy + 400\int_{292.3}^{308.3} y^2\,dy$$

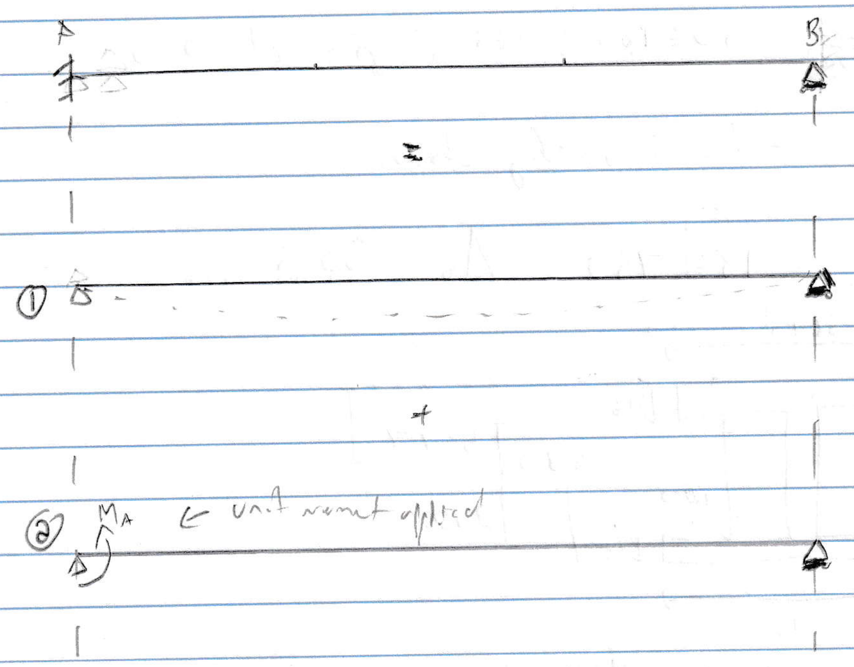
$$= \frac{12}{3}\left[307.7^3 - (-292.3)^3\right] + \frac{369.5}{3}\left[323.7^3 - 307.7^3\right] + \frac{400}{3}\left[308.3^3 - 292.3^3\right]$$
$$= 1383070409 \text{ mm}^4$$

$S_e = \frac{I_{xc}}{\bar{y}_{max}} = \frac{1383070409}{323.7} = 4272692.026\ mm^3$

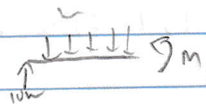
$M_r = \phi S_e F_y = 0.9\ S_e \cdot 300$
$= 1153.63$ kNm

To check FT-buckling

1. Determine moment envelope



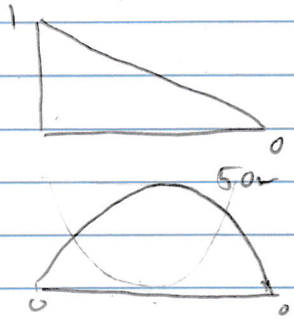
① $\Sigma F_y = 0$, $A_y = B_y = \frac{w \cdot 20}{2} = 10w$,



$M = 10wx - \frac{wx^2}{2}$

② $\Sigma M_B = 0$, $1 + A_y(20)$, $A_y = -1/20$, $B_y = 1/20$, $m = 1 - \frac{x}{20}$

$m_{max} = 1$, $M_{max} = \frac{wL^2}{8} = \frac{w \cdot 20^2}{8} = 50w$



From product integrals,

$$\int \frac{Mm}{EI} dx = \frac{1}{3} \cdot 1 \cdot 50w \cdot 20 = \frac{1000}{3EI} w$$

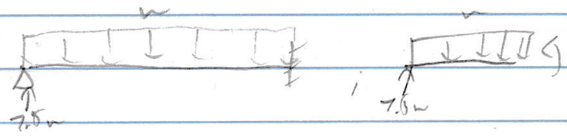
$$\int \frac{mm}{EI} dx = \frac{1}{3} \cdot 1 \cdot 1 \cdot 20 = \frac{20}{3EI}$$

$$\frac{1000}{3EI} w - M_A \left(\frac{20}{3EI}\right) = 0, \quad M_A = 50w$$

$\Sigma M_A = 0$, $50w - w \cdot 20 \cdot \frac{20}{2} + B_y \cdot 20 = 0$

$B_y = 7.5w$

$\Sigma F_y = 0$, $20 \cdot w - 7.5w = A_y$, $A_y = 12.5w$



$M = 7.5wx - \frac{wx^2}{2}$

M_{max}, $\frac{dM}{dx} = 0$, $7.5w - \frac{2wx}{2}$, $x = 7.5$

After looking at all 3 unbraced segments (see excel), segment 3 governed w/ $M_u = 97.1402w$ kNm). Next, determine M_y since class 4

$M_y = S F_y$, $S =$

$$I_x = 12\int_{-300}^{300} y^2 dy + 2 \cdot 400 \int_{300}^{316} y^2 dy$$

$$I_x = \frac{12}{3}\left[300^3 + 300^3\right] + \frac{800}{3}\left[316^3 - 300^3\right]$$

$= 1430532267 \ mm^4$

$$S = \frac{1430532267}{316} = 4527000.844$$

$$M_y = SF_y = S \cdot 300$$
$= 1358.1$ kNm

$0.67M_y = 909.93$ kNm

$M_u = 97.1402w$ ∴ if $w > 9.367$ kN/m,

$$M_r = 1.15 \cdot 0.9 \cdot 1358.1 \cdot \left[1 - \frac{0.28 \cdot 1358.1}{97.1402 \cdot w}\right] \leq 0.9 \cdot 1358.1$$

if $w < 9.367$,

$$M_r = 0.9 \cdot 97.1402w = 87.4262w \text{ kNm}$$

Constants

J	1437866,667
Iy	359466,6667
Cw	30649565870
E	200000
G	77000
L	6666,67

Section 1

x_{start}	0					
x_{end}	6,666666667					
	At Start	At 1/4x	At 2/4x	At 3/4x	At End	Max
x	0	1,66666667	3,33333333	5	6,66666667	6,66666667
M (in terms of w)	0	11,1111111	19,4444444	25	27,7777778	27,7777778
w2	1,387584626	<2.5				
M_u	58,69524369					

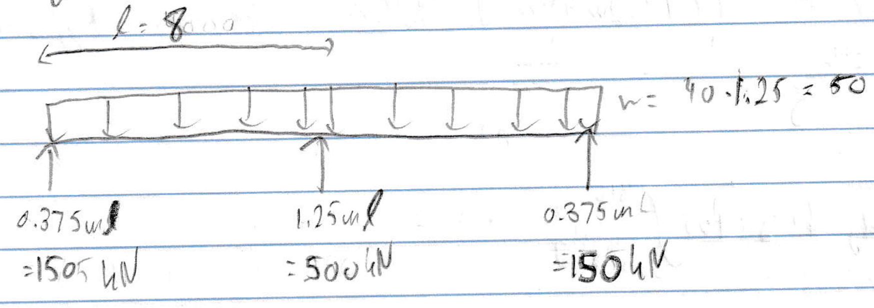
Section 2

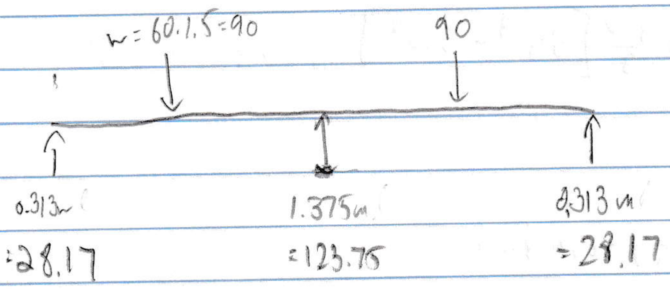
x_{start}	6,666666667					
x_{end}	13,33333333					
	At Start	At 1/4x	At 2/4x	At 3/4x	At End	Max
x	6,666666667	8,33333333	10	11,6666667	13,3333333	7,5
M (in terms of w)	27,77777778	27,7777778	25	19,4444444	11,1111111	28,125
w2	1,138469152	<2.5				
M_u	48,15758479					

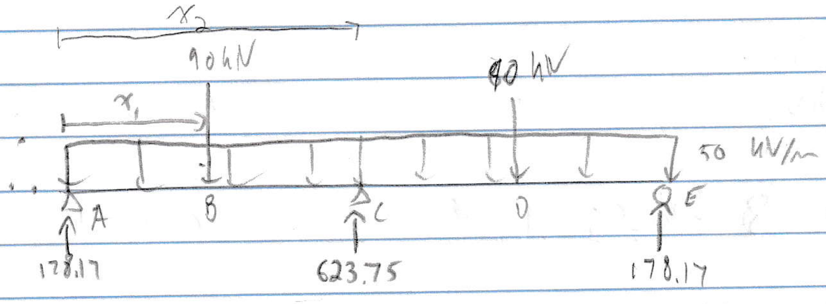
Section 3

x_{start}	13,33333333					
x_{end}	20					
	At Start	At 1/4x	At 2/4x	At 3/4x	At End	Max
x	13,33333333	15	16,6666667	18,3333333	20	20
M (in terms of w)	11,11111111	0	-13,888889	-30,555556	-50	-50
w2	2,296443356	<2.5				
M_u	97,1402391					

(4) Using beam table on 5-145, we have the following







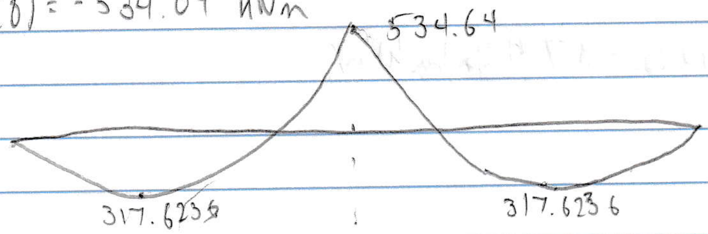
$M(x_1) = 178.17x_1 - \frac{50x^2}{2}$, $\frac{dM}{dx} = 178.17 - 50x$, $x = 3.5634$

$M(3.5634) = 317.6236$

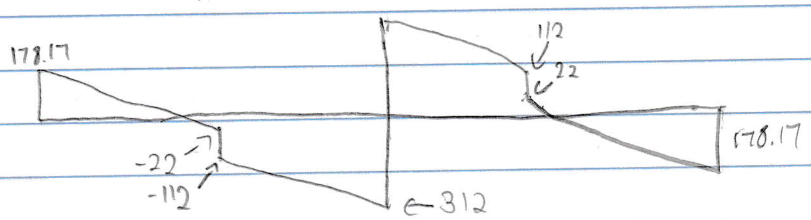
$M(x_2) = 178.17x - 25x^2 - 90(x-4)$

$M(8) = -534.64$ kNm

BMD



SFD



$\therefore V_{f\,max} = 312\ kN, \quad M_{f\,max} = 534.64$

From beam selection table, we have

W530x85, $M_r' = 446\ kNm$
W530x92, $M_r' = 621\ kNm$ ← likely to govern

To verify

1. find moments @ 1/4 x, 2/4 x, 3/4 x

For x_1

$M(1) = 178.17 \cdot 1 - 25(1)^2 = 153.17$, $M(2) = 178.17(2) - 25(2)^2 = 256.34$

$M(3) = 178.17 \cdot 3 - 25(3)^2 = 309.51$ $\quad M_{max} = 317.6236$

$$\therefore \omega_2 = \frac{4 \cdot 317.6236}{\sqrt{4M_{max}^2 + 4M(1)^2 + 7M(2)^2 + 4M(3)^2}} \neq \omega_2 = 1.0973$$

For x_2

$M(5) = 175.85\ kNm$
$M(6) = -10.98\ kNm$
$M(7) = -247.81\ kNm$
$M(8) = -534.64\ kNm$

$$\omega_2 = \frac{4 \cdot 534.64}{\sqrt{4M_{max}^2 + 4M(5)^2 + 7M(6)^2 + 4M(7)^2}} = 1.7383$$

W530x92

$I_y = 23.8E6\ mm^4$
$G = 77000$
$J = 762E3\ mm^4$
$C_w = 1590E9\ mm^6$

$\frac{h}{w} = \frac{502}{10.2} = 49.21 < \frac{1700}{\sqrt{345}}$, Not class 3

$\frac{b_{el}}{t} = \frac{209/2}{15.6} = 6.7 \leq \frac{170}{\sqrt{345}} = 9.1524$
$\therefore$ Not class 3

$$M_u = \frac{1.0973 \cdot \pi}{4000}\sqrt{200000 \cdot 23.8E6 \cdot 77E3 \cdot 762E3 + \left(\frac{\pi \cdot 200000}{4000}\right)^2 \cdot 23.8E6 \cdot 1590E9}$$
$= 3331.058\ kN \cdot m$

$M_p;\ Z_x = 2360E3\ mm^3$

$M_p = 2360E3 \cdot 345 \cdot 0.9 \cdot (1/1000)^2$
$= 732.78\ kNm$

$M_u > 0.67\ M_p\ \therefore$ inelastic buckling

$M_r = 1.15 \cdot 0.9 \cdot 732.78 \cdot \left(1 - \frac{0.28 \cdot 732.78}{3331.058}\right)$

$= 711.704\ kNm \nleq 0.9 \cdot 732.78 = 659.502$

$\therefore M_r = 659.502 > 534.64$, OK

Check Shear

$F_s = 0.66 \cdot 345$
$= 227.7\ MPa$, $A_w = 533 \cdot 10.2 = 5436.6\ mm^2$

$V_r = 0.9 \cdot A_w \cdot F_s \cdot [2.2 - 1.6\ M_f/M_r]$
$= 0.9 \cdot 5436.6 \cdot 227.7 \cdot \left[2.2 - 1.6 \cdot \frac{534.64}{659.502}\right]$
$= 1005.97 > 312$, OK