Reliability of Service-Proven Structures

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Abstract: The successful past performance of an existing structure is evidence of its reliability and safety. A proof load test, for example, provides direct information on strength: the resistance of a surviving structure is shown by the test to be greater than the proof load. Accordingly, the estimate of reliability increases after a successful test. Other kinds of information can be shown to have an effect on reliability similar to proof load testing, including correlation of test and design failure modes, successful resistance to past service loading, and survival age of a structure. In the analyses, statistical techniques are used to combine a first estimate of strength with new information on the past performance of the structure, thereby obtaining an improved estimate of structural resistance. The new estimate of strength is then used in the subsequent evaluation of the structure. Increasing reliability estimates and decreasing failure rates are observed for older structures, and the likelihood of gross errors in strength is found to reduce with survival age.

Introduction

Reliability assessment of structures in service is a problem area that is often associated with deterioration effects such as corrosion or fatigue; in short, the problem of structural wearout. However, not all effects of the passage of time and service on a structure are negative. This paper is concerned with a collection of positive effects on safety and reliability that result from the successful past performance of an existing structure. The used structure is a proven structure and, if undamaged, is in many respects better than new. The illustrations herein do not consider wearout, but apply to many structures in early and useful life (i.e., when deterioration is controlled or not yet significant) and develop several important features of the reliability of existing structures. These include the relationship of time-dependent reliability to successful past service of a structure and the effects of gross errors.

In the analysis, a first estimate of the strength of a structure is combined statistically with new information on past performance, giving a revised estimate of strength. This improved estimate of strength is then used in the evaluation of reliability for subsequent service by the structure. A simple example is the successful resistance of a structure to a proof load. Therefore, the paper begins with a brief review of structural reliability and proof load testing. This is followed by a reliability analysis of structures that have survived a past service load, which can be likened to a random proof load. Finally, the analysis is tied to the survival age of a structure, and its influence on time-dependent reliability and the role of gross errors in strength.

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Structural Reliability and Proof Load Testing

Let the strength of a structure be represented by the random variable R and the load effect by the random variable Q. The performance of the structure can be described by

$$Z = R - Q \quad \text{(1)}$$

Failure occurs if the load exceeds the strength; thus, the probability of failure is

$$P_F = P(Z < 0) \quad \text{(2)}$$

which can be calculated as

$$P_F = \int_{-\infty}^{\infty} F_R(q) f_Q(q) \, dq \quad \text{(3)}$$

in which $F_R(r)$ = the cumulative probability distribution function of the strength estimate R; and $f_Q(q)$ = the probability density function of the load effect Q.

The probability of failure is related to (but is not equal to) the overlap of the two distributions as shown in Fig. 1 (solid lines). A measure of safety frequently used is the safety index:

$$\beta = -\Phi^{-1}(P_F) \quad \text{(4)}$$

in which Φ^{-1} = the inverse of the standard normal distribution function. Typical design values of the safety index lie between 3.0–4.0 and correspond approximately to nominal probabilities of failure in the range of 10^{-3}–10^{-4}.

It is not always necessary to perform the integration in Eq. 3. If R and Q are normal

$$\beta = \frac{\bar{Z}}{\sigma_Z} = \frac{\bar{R} - \bar{Q}}{(\sigma_R^2 + \sigma_Q^2)^{1/2}} \quad \text{(5)}$$

i.e., β = the margin between mean strength and mean load measured in standard deviations σ_Z. For other distributions of strength and load,

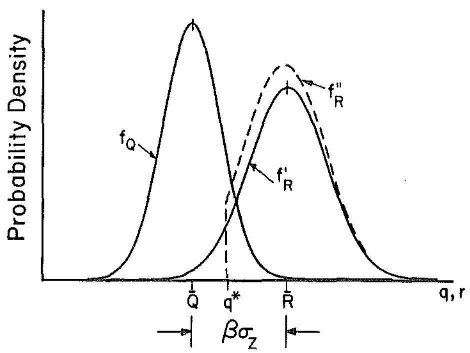

FIG. 1. Random Load Q versus Random Strength R before and after Proof Test at Load q^*

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first-order (normal) approximations of the distributions of R and Q can be made at the so-called "design point," a point at or near the point of maximum probability density on the failure boundary $R - Q = 0$ [e.g., see Hasofer and Lind (1974) and Rackwitz and Fiessler (1978)]. The design point is found iteratively, and the approximating normal distributions (*) are then used to define the safety index:

$$\beta^* = \frac{\bar{R}^* - \bar{Q}^*}{(\sigma_{R^*}^2 + \sigma_{Q^*}^2)^{1/2}} \qquad (6)$$

Design-point methods can also be applied to more complicated performance functions than Eq. 1, including nonlinear multivariate functions and functions of correlated random variables, and to multiple failure mode and system reliability problems. These and other advanced first-order second-moment methods are well known in the structural reliability field and have been developed by many contributors. A good overview of the literature can be found in the paper by Shinozuka (1983), a CIRIA report (1977), and some recent books including those by Thoft-Christensen and Baker (1982), and Ang and Tang (1984).

Proof Load Testing

One of the objectives in reliability-based design is to provide a structure with strength estimate R that does not overlap too much with the load estimate Q, as measured by the safety index, for example. Knowledge of the strength can be improved by testing, either destructively or nondestructively, and depending on the test results, may provide an increased estimate of reliability.

The strength of a structure can sometimes be demonstrated nondestructively by proof load testing, in which a known proof load is applied to the structure (Barnett and Hermann 1965; Shinozuka 1969; Fujino and Lind 1977). A structure that survives the test belongs to that part of the population with strength greater than the proof load. Thus, the revised distribution of strength for a successful structure, $f_R'(r)$, is obtained from a (normalized) truncation of the distribution prior to the test, $f_R(r)$ at the proof load q^* (shown in dashed lines in Fig. 1):

$$f_R''(r) = \frac{f'_R(r)}{1 - F'_R(q^*)}; \qquad r \geq q^* \qquad (7a)$$

$$f_R''(r) = 0; \qquad r \leq q^* \qquad (7b)$$

This improved estimate of strength can then be used in the evaluation of reliability

$$\text{Rel} = 1 - P_F = \int_{-\infty}^{\infty} F_Q(r) f_R''(r)\, dr \qquad (8)$$

or probability of failure using Eq. 3 or other safety measure. A proof load test screens out low-strength members of the population and, by removing the lower tail of the strength distribution, can produce a significant increase in the reliability estimate for a successful structure.

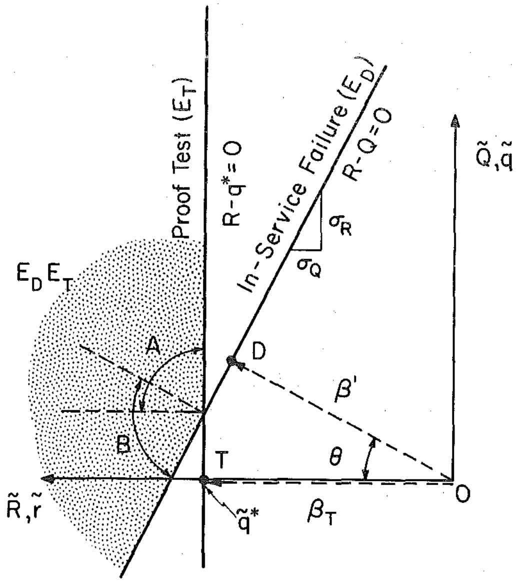

FIG. 2. Intersecting Proof Test and In-Service Modes in Load-Strength Plane (Reduced Variates)

Another way to illustrate the effects of proof testing is shown in Fig. 2, in which $\tilde{R}$ and $\tilde{Q}$ are reduced variates of strength and load

$$\tilde{R} = \frac{R - \bar{R}}{\sigma_R} \quad \text{(9a)}$$

$$\tilde{Q} = \frac{Q - \bar{Q}}{\sigma_Q} \quad \text{(9b)}$$

The before-test safety index β' is the (minimum) distance from the origin O to the design point D on the in-service failure boundary. The effect of the proof load test on strength is to establish a second boundary, this time a proven-strength boundary at the (reduced) proof load:

$$\tilde{r} \geq \tilde{q}^* = \frac{q^* - \bar{R}}{\sigma_R} \quad \text{(10)}$$

Thus, the test event also has a safety index β_T, represented by the vector **OT** in the figure. If the test is successful, β_T represents a survival mode rather than a failure mode.

It is evident from the representation in Fig. 2 that the reliability problem for a proof tested structure is similar to the problem of multiple failure modes. The difference is that, for the proof-test problem, it is the conditional (rather than unconditional) probability of failure that is to be estimated, given a successful test result. In calculating this, the main problem (since both β' and β_T are easily obtained) is to find the probability of intersection between the in-service or "design" failure event E_D and the

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proof test event E_T. Here, the techniques developed for multiple failure mode problems can be applied with minor modifications. For example, bounds by Ditlevson (1979) can be found for the intersection $E_D E_T$. Accordingly, an upper bound on the after-test probability of failure is given by (assuming positive correlation)

$$P_F'' \leq \frac{\Phi(-\beta') - P(A)}{\Phi(\beta_T)} \quad \text{(11)}$$

in which

$$P(A) = \Phi(-\beta_T)\Phi\left(-\frac{\beta' - \rho\beta_T}{\sqrt{1-\rho^2}}\right) \quad \text{(12)}$$

A safe lower bound on the after-test safety index β'' can then be found using Eq. 4 (it should be noted here, however, that Ditlevson bounds such as Eq. 11 can be wide for this problem, owing to the low safety indexes and high correlation likely to be encountered). In Eq. 12, ρ = the correlation coefficient between the test and design modes:

$$\rho = \cos\theta = \frac{\sigma_R}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad \text{(13)}$$

The greater the correlation between the proof test and the design failure modes, the greater will be the influence of the proof test. In this example, as expected, the proof test will be most effective when the design uncertainty lies predominantly in the strength rather than the load. Also, for the same loads and coefficients of variation of strength, a less conservative design (lower β') will experience a greater improvement in after-test safety estimates than a more conservative design, if the test is successful.

One should note in passing that for this simple two-dimensional case (load versus strength), it is not too inconvenient to perform the numerical integration of Eq. 8. More complicated problems, however, will favor the application of design-point methods to proof loading and other conditional reliability problems.

Differing Test and Service Modes

Survival of a structural member in one failure mode affects survival and failure probabilities for other failure modes, if random variables are shared between modes (or they are otherwise correlated). For a proof test result in one failure mode

$$Z_1^p = Z_1(X_2, X_2, \ldots, x_p, \ldots, X_n) \geq 0 \quad \text{(14)}$$

in which $\{X_i\}$ = the basic random variables with x_p representing the deterministic proof load. The conditional probability of failure for failure mode j becomes

$$P_{Fj} = P[Z_j(X_1, X_2, \ldots, X_n) \leq 0 \,|\, Z_1^p \geq 0] \quad \text{(15)}$$

Consider the example of a square bar in which the first failure mode, tension, is given by

$$Z_1(S, Y, P) = S^2 Y - P \leq 0 \quad \text{(16)}$$

and the second failure mode, bending, is

$$Z_2(S, Y, L, W) = \frac{4S^3Y}{6L} - W \leq 0 \quad (17)$$

in which S = the side of the square cross section; Y = the yield strength; P = the tension load; L = the simply supported span length; and W = the transverse bending load at midspan. For simplicity, it is assumed that simultaneous tension and bending is precluded, so that these two failure modes are all that need to be considered.

Now suppose that it is observed that the bar survives a known load $p*$ in tension. The failure probabilities for each mode then become

$$P''_{F1} = P(S^2Y - P \leq 0 \mid S^2Y > p^*) \quad (18)$$

$$P''_{F2} = P\left(\frac{4S^3Y}{6L} - W \leq 0 \mid S^2Y > p^*\right) \quad (19)$$

The effect of the proof load condition from p^* is to truncate the joint probability density function of S and Y along the boundary $S^2Y = p^*$ in the S-Y plane (the posterior joint density function would then be renormalized to have a volume of unity). The probability of failure of both modes is thus affected by the survival in tension, which is to be expected since the modes are correlated through S and Y. It is interesting to note that the success of a proof test in one mode cannot be attributed, in general, to any one random variable. For example, the success of the tension test might have been owed to a high strength, or a large bar size, or a combination of these events.

In this small example, the failure and test boundaries are nonlinear, and the problem has four random variables, making a solution by integration difficult. However, it would be straightforward to obtain design-point approximations for the safety indexes of the test and two service modes, their respective correlation coefficients, and bounds on after-test reliability. As in the previous example, Fig. 2, the situation is analogous to the correlated multimode or system failure problem. It is apparent that many conditional reliability problems, once defined in terms of unions and intersections of failure and survival events, can be handled by first-order techniques. This has been noticed by Rackwitz and Schrupp (1985) who demonstrated the compatibility of proof loading, acceptance testing, and quality control procedures with first-order reliability methodology. Other related work on the subject of proof load testing includes applications to series-produced (thus correlated) structures, in which the proof tests of a small example can be used to infer the safety of an entire lot (Grigoriu and Hall 1984), and to correlation between an early damage state and the failure state [and other extensions of proof testing (Veneziano et al. 1978)].

Service-Proven Structures

Successful past resistance of a structure to a service load acts as a proof test on its strength, except that there is often uncertainty in the value of the load. In some applications it may be possible that a successful service load and its variance can be estimated from data on existing or past loads, inspections, or other information. Successful past service in one mode can

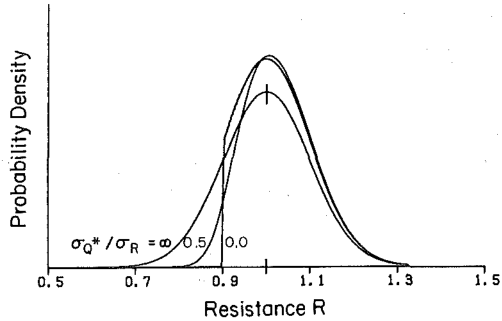

FIG. 3. Strength Distribution after Successful Service Loading; Effect of Proof Load Variance on Strength Estimate ($\overline{R} = 1.0$; $\overline{Q}^* = 0.9$; $\sigma_R = 0.1$)

then be represented by Eq. 14, with the proof (service) load X_p now treated as a random variable. The conditional probabilities of failure are determined as before, except for the increase in dimensionality resulting from the randomness of the proof load (e.g., p^* becomes a random variable in Eqs. 18 and 19). One might also point out here that, in principle, the proof load need not be a load at all, but could be any variable(s) on which new information has been gained.

Returning to the basic problem of load versus strength, if the value of the proof load has uncertainty, i.e., the proof load is a random variable Q^*, then the revised distribution of strength becomes

$$f''_R(r) = \frac{F_{Q^*}(r) f'_R(r)}{\int_{-\infty}^{\infty} F_{Q^*}(r) f'_R(r)\, dr} \quad \text{(20)}$$

This result reduces to Eq. 7 when the proof load has zero variance, i.e., when it is known. Eq. 20 can be used to revise the initial strength estimate without a formal proof load test (Hall and Nowak 1984). The improved strength estimate is then used to evaluate the structural reliability of the service-proven system for the anticipated design load.

The effect of variance in the proof load is shown in Fig. 3, assuming a normal prior strength distribution. For lower variance in the proof load, the probability of low strength components is much reduced, similarly to the screening action of a classical proof load test (zero variance, known proof load). If the proof load has high variance, then little change in the strength distribution occurs.

Conceptually, this analysis would apply to a structure that has experienced a maximum load Q^* in service; given the probability distributions of the initial strength estimate and service load, the reliability of the same structure under a different load Q is required. From a practical viewpoint, this situation could arise in many different ways. For example, it may be that the design load must be revised because of a change in use of the structure, or it may be discovered that a different grade of material than intended has been used, successfully, in service. Whatever the reason, it is required to take a second look at the safety of the structure, taking into account its successful past resistance to service loads.

TABLE 1. Effect of Successful Service Loads on Reliability

Mean service proof load, $\overline{Q}^*$ (1)	Strength uncertainty, σ_R (2)	Design load uncertainty, σ_Q (3)	Nominal probability of failure, P''_F (4)	Safety index, β'' (5)
100	50	0	0.00198	2.88
	49	10	0.00564	2.54
	40	30	0.0187	2.38
	10	49	0.0228	2.00
	0	50	0.0228	2.00
150	50	0	$<10^{-7}$	>4.00
	49	10	3.39×10^{-7}	>4.00
	40	30	0.00436	2.62
	10	49	0.0228	2.00
	0	50	0.0228	2.00

Note: $\overline{R} = 200$; $\overline{Q} = 100$; $\sigma_Z = 50$; $\beta' = 2.0$; $V_{Q*} = 0.05$.

As illustration, consider the case in which the design load Q and strength estimate R are normally distributed with mean values of 100 and 200 units, respectively, and the apparent safety index is 2.0. The observed service load Q^* is normally distributed with mean $\overline{Q}^*$ and an assumed coefficient of variation of 5%. Eq. 20 is used to revise the strength distribution and Eqs. 8 and 4 are then used to evaluate safety for various levels of the mean service load $\overline{Q}^*$ and strength variance σ_R . Table 1 summarizes the results. As expected, the probability of failure and safety index, after service, shows significant improvements when the design uncertainty comes primarily from strength (i.e., when σ_R is high relative to σ_Q). Conversely, past service has little impact on safety when the design uncertainty is predominantly in the load. In this case components already have small probabilities of low strength, and little is gained from the screening effect of past loadings. However, while its impact may not always be large (e.g., it is not unrealistic for a design to have a high uncertainty in loads rather than strength), the use of service load information to verify safety will often have much lower costs than other methods of evaluation, such as direct testing. Of course, the higher the prior service load, the greater is the improvement in safety levels.

Example: Electrical Cable Support Systems

Two spot-welded cold-formed steel cross sections, used to support electrical cables and conduits in nuclear power plants, are shown in Fig. 4. Also given in the figure are the results of strength tests on two types of welded systems: back-to-back and side-to-side welded channel sections. In an earlier study (Nowak and Regupathy 1984), these systems were found to have low safety levels. The fact that these systems were apparently performing well in service, and that the service loads could be estimated from cable weights, suggested the following analysis.

The strength of single spot welds for the back-to-back case is represented by a normal distribution with $\overline{R}$ = 3,400 lb (15.1 kN) and σ_R = 1,100 lb (4.90 kN). A hypothetical design load is assumed to be normal with $\overline{Q}$ = 1,200 (5.34 kN) and σ_Q = 600 lb (2.67 kN). This approximates the design

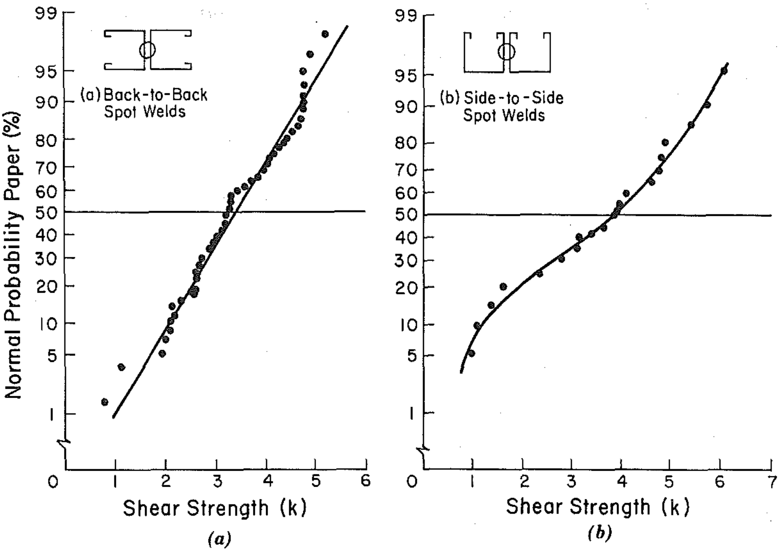

FIG. 4. Test Results and CDF for: (*a*) Back-to-Back Spot Welds; (*b*) Side-to-Side Spot Welds [from Nowak and Regupathy (1984)]

situation of the systems, giving a safety index, before service, of $\beta' = 1.76$ ($P'_F = 0.0396$). The actual design load is a combination of the sustained load from self-weight and cable loads D and earthquake loads E and is not normally distributed. The successful service load is assumed to be equal to the previously mentioned sustained load, giving $Q^* = 900$ lb (4.01 kN) in this example, with a coefficient of variation of 5%. Using Eq. 20, the resulting after-service safety index was found to be $\beta'' = 1.88$ ($P''_F = 0.0302$), a modest increase in the safety estimate. The estimate of fifth percentile strength increased from 1,590 lb (7.08 kN) to 1,670 lb (7.43 kN).

An analysis of the side-to-side case was made with $\overline{R} = 3{,}800$ lb (16.9 kN), $\sigma_R = 2{,}000$ lb (8.90 kN), $\overline{Q} = 1{,}200$ lb (5.34 kN), and $\sigma_Q = 600$ lb (2.67 kN), giving $\beta' = 1.25$ ($P'_F = 0.107$). For a service load of $Q^* = 1{,}000$ lb (4.45 kN) with 5% coefficient of variation, the safety index, after service, increased to $\beta'' = 1.68$ ($P''_F = 0.0398$). The estimate of fifth percentile strength increased from 510 lb (2.27 kN) to 1,460 lb (6.50 kN), and the failure probability improved by a factor of almost three.

In both of these cases, although improvements in failure probabilities were obtained, and obtained without the costs of formal proof testing, the revised safety levels still appear to be low. The reason for the modest levels of improvement appears to be that the design load (earthquake) is not well represented by the service loads (cable and element weights). In other design situations with more similarity between working loads and design loads, the proof load effect of service loads on reliability is likely to be more pronounced. Also, the service proof load is still very effective in screening out welds with the lowest strengths, i.e., abnormal or defective welds. (Defective welds with small fused areas were observed for the side-to-side case, which was more difficult to weld than the back-to-back

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case.) Screening of gross errors in strength is not reflected in the calculated safety levels and is considered later.

Options that can be used to further improve the reliability estimates for these systems include the use of system reliability calculations for weld groups, the possible use of lognormal distributions to model strength, and the use of a superimposed proof load q^* in addition to the sustained load already acting on the structure. In the side-to-side case, for example, a superimposed load of about 1,100 lb (4.90 kN), in combination with the existing cable loads, would verify the safety level at approximately $\beta'' = 3.0$.

Time-Dependent Reliability

Although not explicitly stated in the previous section, the analysis of service-proven structures is inherently a time-dependent reliability problem. That is, the analysis attempts to find the reliability of a structure for some future service, after a period of successful service has passed. In fact, the subsequent reliability of the structure is conditional upon its previous service life, i.e., its age. It is natural, therefore, to take a brief look at time-dependent structural reliability and to examine how it might be related to service-proven structures.

Classical time-dependent reliability is based on a failure rate approach in which three stages of component lifetime can be identified: (1) Early life, in which defective components fail in early service with an initially high but rapidly decreasing failure rate; (2) useful life, in which good (nondefective and as yet undamaged) components fail more or less randomly with constant failure rate; and (3) wearout, in which accumulating damage results in deteriorating component performance and an increasing failure rate. Time-dependent reliability is expressed as an exponential function of the time-dependent failure rate, called the hazard function. References on lifetime reliability include the books by Dhillon and Singh (1980) and Smith (1983).

When time-dependent structural reliability is formulated by the failure rate approach, the three stages of lifetime show similar characteristics to the classical results. For example, a structure with known constant strength r and random load Q has probability of survival in t years

$$\mathrm{Rel}(t) = F_Q^t(r) \quad (21)$$

in which $F_Q^t(q)$ = the cumulative probability distribution of the t-year maximum load.

If the t-year maximum load is assumed to be the maximum of t statistically independent and identically distributed annual maximum loads, with distribution $F_Q(q)$, the superscript t in Eq. 21 can be treated as an exponent. The reliability function can then be written:

$$\mathrm{Rel}(t) = e^{-\lambda t} \quad (22)$$

in which the nominal annual failure rate is

$$\lambda = -\ln F_Q(r) \quad (23)$$

In this case the failure rate is constant, as in the classical model of useful life reliability. A feature of Eq. 22 is that the conditional reliability of a structure with known survival age T

$$\mathrm{Rel}(t \mid T) = \frac{\mathrm{Rel}(T + t)}{\mathrm{Rel}(T)} \qquad (24)$$

equals Rel(t). That is, the reliability of a surviving structure is unaffected by its age. In this model, reliability is a function only of random overload, and an unfailed structure is as good as new.

Unknown but Maintained Strength

This result (constant failure rate) is only to be expected from the simple model assumed, namely known, invariant strength. What is perhaps more interesting is that a structure with unknown but maintained strength has a decreasing failure rate during useful life. For this more general case, the reliability of an older, surviving structure is found to be greater than that of a new structure from the same population (Hall 1984). In a sense, an older undamaged structure is better than new.

To show this, let the probability density function of the unknown strength R be $f_R(r)$. Then the reliability for t years is

$$\mathrm{Rel}(t) = \int_{-\infty}^{\infty} F_Q^t(r) f_R(r)\, dr \qquad (25)$$

after Eq. 8. In this case the reliability for an additional service period t of a surviving structure of age T years is, from Eq. 24,

$$\mathrm{Rel}(t \mid T) = \frac{\int_{-\infty}^{\infty} F_Q^{T+t}(r) f_r(r)\, dr}{\int_{-\infty}^{\infty} F_Q^T(r) f_R(r)\, dr} \qquad (26)$$

which is greater than Rel(t), Eq. 25. In terms of the hazard function $\lambda(t)$

$$\mathrm{Rel}(t) = \exp\left[-\int_0^t \lambda(t)\, dt\right] \qquad (27)$$

in which $\lambda(t)$ = a decreasing failure rate with service life. For structural reliability, it is simpler to use Eq. 25; $\lambda(t)$, if desired, is the negative of the slope of the reliability function, divided by the reliability at time t.

At first it may seem unreasonable to suggest that an older structure should have a higher reliability. However, it is shown in the next section that this result can be attributed to a beneficial effect of survival age similar to proof load testing.

Proof Load Effect of Survival Age

The maximum load that a structure has survived acts as a proof test on its strength and is a random variable that depends on the survival age of the structure. If a structure has survived to an age T under the random load Q, then the revised strength estimate, given T, is obtained from Eq. 20 with $F_{Q*}(q) = F_Q^T(q)$, the T-year maximum load distribution:

$$f''_R(r) = \frac{F_Q^T(r) f'_R(r)}{\int_{-\infty}^{\infty} F_Q^T(r) f'_R(r)} \qquad (28)$$

Successful service for T years has a beneficial effect on the knowledge of strength. Specifically, the longer a structure has been in service, the more likely it is to have survived an extreme load. This is shown in Fig. 5 for the case of a structure having a normally distributed strength with a mean of 200 units and a coefficient of variation of 20%, and experiencing a sequence of annual maximum loads, normally distributed with a mean of 100 units and a coefficient of variation of 30%. As the survival age T increases, the lower tail of the strength distribution (Eq. 28) is reduced much as it would be in a proof load test with uncertainty in the value of the proof load.

The new estimate of strength for a structure "proven" in service, Eq. 28, can be used to calculate reliability for a subsequent service of t years by Eq. 8 giving

$$\mathrm{Rel}(t \mid T) = \frac{\int_{-\infty}^{\infty} F_Q^t(r) F_Q^T(r) f'(r)\, dr}{\int_{-\infty}^{\infty} F_Q^T(r) f'_R(r)\, dr} \quad \text{(29)}$$

This result is the same as Eq. 26, since

$$F_Q^{T+t}(q) = F_Q^T(q) F_Q^t(q) \quad \text{(30)}$$

in which the load distribution for $F_Q^t(q)$ is defined for the period t immediately following T in the load process. If annual maximum loads are statistically independent and identically distributed, then the superscripts T and t are simply treated as exponents in the preceding equations. (For simplicity in design, it might be reasonable to model some kinds of stationary load processes in this way.) By this interpretation, then, the improving conditional reliability observed in Eq. 26 can be explained by the proof load effect of successful past performance implied in the survival age T.

Fig. 6 shows the effect of survival age on time-dependent reliability. The initial strength R is normal with a mean of 200 units and coefficient of variation of 20%. The annual maximum loads are independent and normal with a mean of 100 units and a coefficient of variation of 30%. As shown in the figure, the reliability of an existing structure for an additional period of service t is higher for longer survival histories T. The reduced likelihood of lower strengths, as shown in Fig. 5, has a strong effect on reliability in early years of service. This screening out of low strengths can be especially significant for gross errors, as will be discussed later. On the other hand, this screening effect may not be very significant if the load levels experienced in T years of service are not representative of the design load, as when, for example, a design governed by earthquake experiences only ordinary service loads in early life.

For a structure with known survival age, the proof load effect of successful past service results in increasing strength and reliability estimates the longer the structure survives. The used structure is better than new. This interesting result holds for the useful life of a structure, i.e., for the period of service life in which deterioration is negligible. Reliability models for wearout (e.g., corrosion or fatigue) are not considered here. Early life reliability, however, is strongly affected by the screening effect

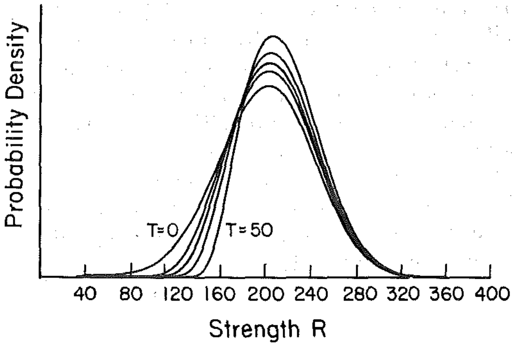

FIG. 5. Proof Load Effect of Survival Age T on Revised Strength R (T = 0, 5, 10, 20, and 50 yr)

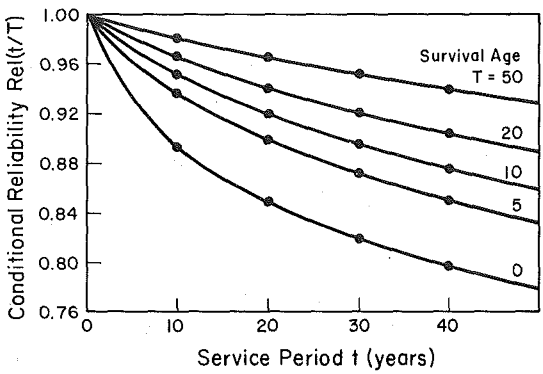

FIG. 6. Improvement in Time-Dependent Reliability from Proof Load Effect of Survival Age T (see Fig. 5)

of survival age on low strength and, as shown in the next section, especially on gross errors in strength.

Gross Error

A simple model of gross error in strength can be used to show that the effect of this type of error on reliability diminishes markedly with survival age. Let the population of structures be divided into two parts, one having reduced strength gR with probability $p(g)$ and the other having full strength R with probability $1 - p(g)$. g is a factor between zero and one representing the effect of a gross error on strength, and $p(g)$ is the probability of gross error.

If R is deterministic at the value r, then the reliability of a structure from this population, conditioned on survival for T years, is

$$\mathrm{Rel}(t \mid T) = \frac{p(g)F_Q^{T+t}(gr) + [1 - p(g)]F_Q^{T+t}(r)}{p(g)F_Q^{T}(gr) + [1 - p(g)]F_Q^{T}(r)} \quad (31)$$

The probability of gross error, for a successful structure in service for T years, is then

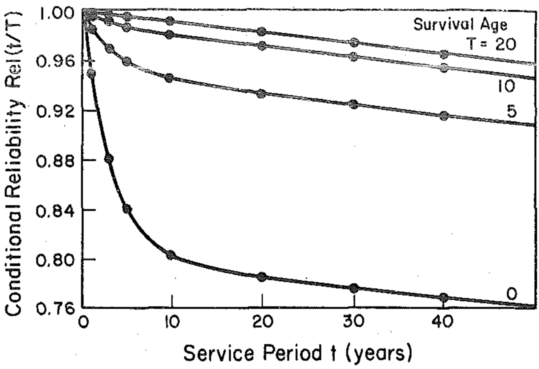

FIG. 7. Effect of Gross Error on Time-Dependent Reliability (g = 0.6; $P(g)$ = 0.10; R = 200; $\overline{Q}$ = 100; V_Q = 0.3)

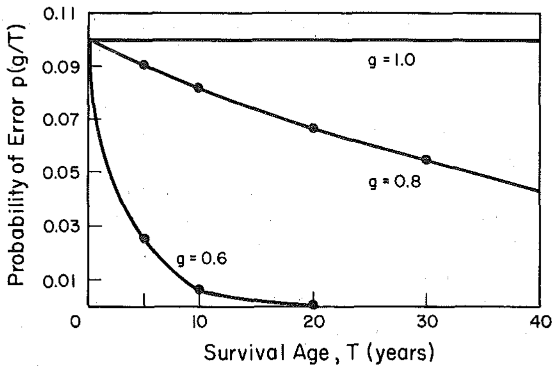

FIG. 8. Effect of Survival Age on Probability of Gross Error for a Successful Structure (Same Conditions as Fig. 7)

$$p(g \mid T) = \frac{p(g)F_Q^T(gr)}{p(g)F_Q^T(gr) + [1 - p(g)]F_Q^T(r)} \quad (32)$$

Eq. 31 approaches Eq. 21 as survival age T increases. The larger the gross error (smaller g), the more rapid is this approach. If the unreduced strength R is nondeterministic, the results are qualitatively the same.

As illustration, Fig. 7 plots Eq. 31 for a gross error that reduces strength by 20% (g = 0.8) and has a 10% rate of occurrence [$p(g)$ = 0.10]. The unreduced strength r is 200 units, and, as in the previous two examples, the loadings are independent and normal with a mean of 100 units and a coefficient of variation of 30%. As before, the figure shows improving reliability for older (proven) structures, but perhaps more interesting is the effect on gross error, shown in Fig. 8. The probability of gross error, Eq. 32, reduces with survival age for successful structures.

The effects of gross error on time-dependent reliability are characteristic of early life reliability; namely, a high initial failure rate that rapidly decreases with component lifetime, if defects are large. In turn, the effect of past reliability is to reduce the likelihood of gross errors in older structures. One may expect that, after some time in service, surviving structures are unlikely to contain many types of gross errors that significantly affect strength.

Conclusion

A variety of beneficial effects on the reliability of existing structures has been described that apply to nondeteriorating structures with successful service histories.

The reliability of existing structures that have survived past service loads can be analyzed by treating the service load as a proof load with uncertainty. Structures in service can be expected to have smaller probabilities of low strength than untried structures, owing to the proof load effect of successful past performance. The greatest improvements in reliability are found in structures that have a high uncertainty in initial strength estimates and that have survived relatively high service loads.

Estimates of time-dependent reliability increase and failure rates decrease with the survival age of a structure. The effect of gross error on the reliability estimate diminishes, and the likelihood of a gross error in strength reduces, for older structures. In early and useful life, before fatigue or other wearout effects become significant, a used structure is better than new.

Reliability analysis using service loads, survival age, and other in-service performance evidence can be combined with conventional proof testing, destructive and nondestructive testing, and other methods of analysis, as part of a safety verification program for existing structures. First-order reliability methods can be applied to these and many other conditional reliability problems similar to proof load testing.

Appendix I. References

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Appendix II. Notation

The following symbols are used in this paper:

D = sustained load;
E = earthquake load;
F = cumulative distribution function;
f = probability density function;
g = gross error factor on R;
P = tension load;
P_F = probability of failure;
P^* = tension proof load;
$p(g)$ = probability of gross error;
Q = design load;
Q^* = service load;
R = resistance;
$\mathrm{Rel}(t)$ = time-dependent reliability;
S = side dimension of square bar;
T = survival to age T;
t = service period after survival age T;
v = coefficient of variation;
W = bending load;
X = random variable in performance function Z;
Y = yield strength;
Z = performance function, e.g., $R - Q$;
β = reliability index;
λ = time-dependent failure rate (hazard function);
σ = standard deviation; and
Φ = standard normal distribution function.

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Superscripts and Subscripts

p = proof load variable or test mode;
$\sim$ = normalized or reduced variate;
$\bar{}$ = mean value;
$*$ = design point estimate or proof load variable;
$'$ = before-test or before-service estimate; and
$''$ = after-test or after-service estimate.