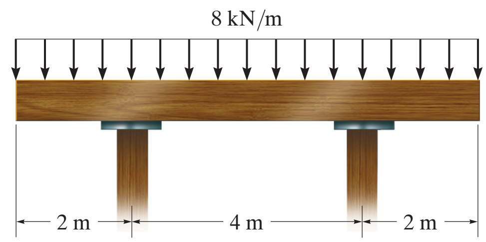
**15–1.**

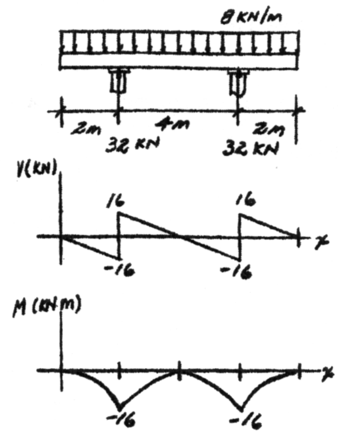
The beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 6.5$ MPa and an allowable shear stress of $\tau_{allow} = 500$ kPa. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of 1.25. Assume the beam rests on smooth supports.




## SOLUTION

$$I_x = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$Q_{max} = \bar{y}'A' = (0.3125b)(0.625b)(b) = 0.1953125b^3$$




**Assume Bending Moment Controls:**

$$M_{max} = 16 \text{ kN} \cdot \text{m}$$

$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$6.5(10^6) = \frac{16(10^3)(0.625b)}{0.16276b^4}$$

$b = 0.21143$ m $= 211$ mm **Ans.**

$h = 1.25b = 264$ mm **Ans.**

**Check Shear:**

$$Q_{max} = 1.846159(10^{-3}) \text{ m}^3$$

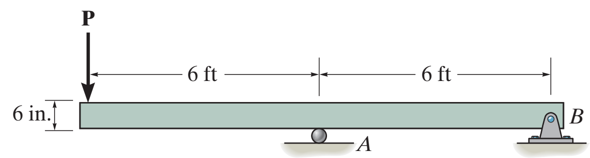
$$I = 0.325248(10^{-3}) \text{ m}^4$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{16(10^3)(1.846159)(10^{-3})}{0.325248(10^{-3})(0.21143)} = 429 \text{ kPa} < 500 \text{ kPa},$$ **OK**

**Ans:**
$b = 211$ mm, $h = 264$ mm

**15–2.**

Determine the minimum width of the beam to the nearest $\frac{1}{4}$ in. that will safely support the loading of $P = 8$ kip. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 15$ ksi.




## SOLUTION

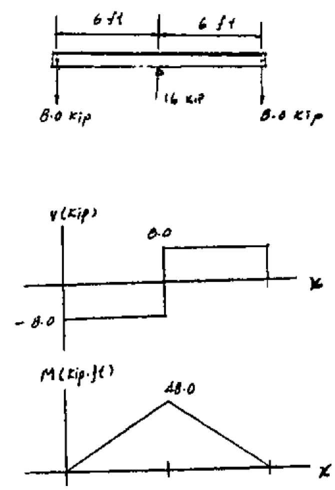
**Beam Design:** Assume moment controls.

$$\sigma_{allow} = \frac{Mc}{I}; \qquad 24 = \frac{48.0(12)(3)}{\frac{1}{12}(b)(6^3)}$$

$b = 4$ in. **Ans.**

**Check Shear:**

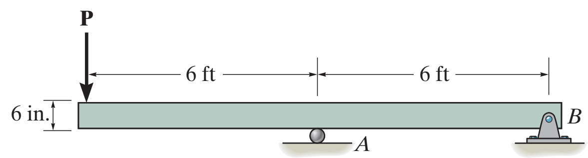
$$\tau_{max} = \frac{VQ}{It} = \frac{8(1.5)(3)(4)}{\frac{1}{12}(4)(6)^3(4)} = 0.5 \text{ ksi} < 15 \text{ ksi}$$ **OK**




**Ans:**
Use $b = 4$ in.

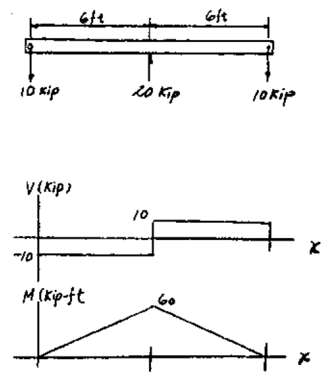
**15–3.**

Solve Prob. 15–2 if $P = 10$ kip.




## SOLUTION

**Beam Design:** Assume moment controls.




$$\sigma_{\text{allow}} = \frac{Mc}{I}; \qquad 24 = \frac{60(12)(3)}{\frac{1}{12}(b)(6^3)}$$

$b = 5$ in. **Ans.**

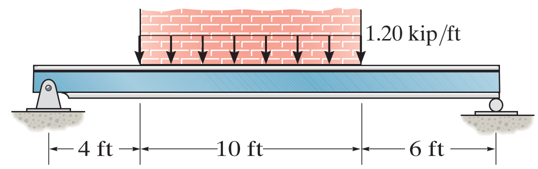
**Check Shear:**

$$\tau_{\max} = \frac{VQ}{It} = \frac{10(1.5)(3)(5)}{\frac{1}{12}(5)(6)^3(5)} = 0.5 \text{ ksi} < 15 \text{ ksi} \qquad \textbf{OK}$$

**Ans:**
Use $b = 5$ in.

***15–4.**

The brick wall exerts a uniform distributed load of 1.20 kip/ft on the beam. If the allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi, select the lightest wide-flange section with the shortest depth from Appendix B that will safely support the load. If there are several choices of equal weight, choose the one with the shortest height.

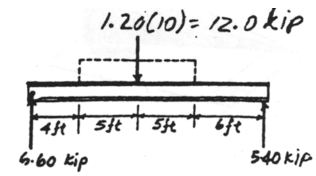



## SOLUTION

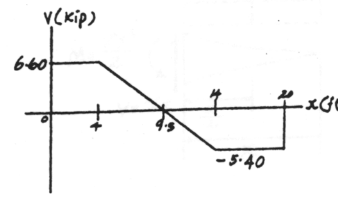
**Bending Stress:** From the moment diagram, $M_{max} = 44.55$ kip · ft. Assuming bending controls the design and applying the flexure formula,

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{44.55\,(12)}{22} = 24.3 \text{ in}^3$$

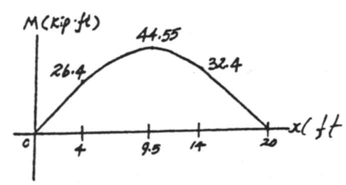



Two choices of wide-flange section having the weight 22 lb/ft can be made. They are W12 × 22 and W14 × 22. However, W12 × 22 is the shortest.




**Select** W12 × 22 $\left(S_x = 25.4 \text{ in}^3,\ d = 12.31 \text{ in.},\ t_w = 0.260 \text{ in.}\right)$

**Shear Stress:** Provide a shear stress check using $\tau = \dfrac{V}{t_w\, d}$ for the W12 × 22 wide-flange section. From the shear diagram, $V_{max} = 6.60$ kip.




$$\tau_{max} = \frac{V_{max}}{t_w\, d}$$

$$= \frac{6.60}{0.260(12.31)}$$

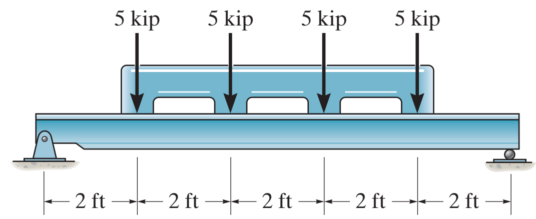
$$= 2.06 \text{ ksi} < \tau_{allow} = 12 \text{ ksi} \qquad \textbf{(O.K!)}$$

Hence, Use W12 × 22 **Ans.**

**Ans:**
Use W12 × 22

**15–5.**

Select the lightest-weight wide-flange beam from Appendix B that will safely support the machine loading shown. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.




## SOLUTION

**Bending Stress:** From the moment diagram, $M_{max} = 30.0$ kip · ft. Assume bending controls the design. Applying the flexure formula,

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{30.0(12)}{24} = 15.0 \text{ in}^3$$

**Select** W12 × 16 $\left(S_x = 17.1 \text{ in}^3,\ d = 11.99 \text{ in.},\ t_w = 0.220 \text{ in.}\right)$

**Shear Stress:** Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W12 × 16 wide-flange section. From the shear diagram, $V_{max} = 10.0$ kip.

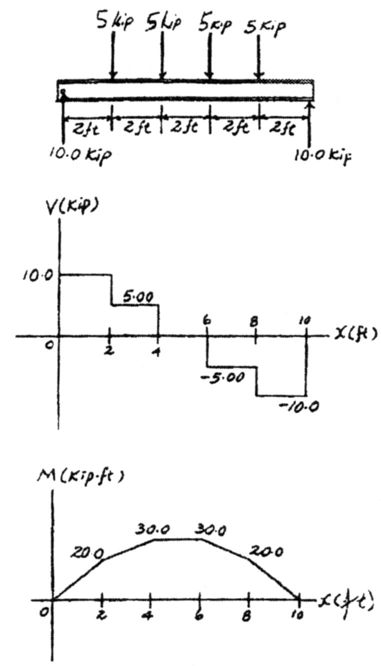
$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{10.0}{0.220(11.99)}$$

$$= 3.79 \text{ ksi} < \tau_{allow} = 14 \text{ ksi} \qquad \textbf{(O.K!)}$$

Hence,

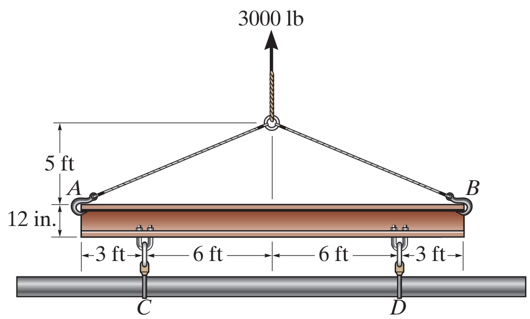
Use W12 × 16 **Ans.**




**Ans:**
Use W12 × 16.

**15–6.**

The spreader beam $AB$ is used to slowly lift the 3000-lb pipe that is centrally located on the straps at $C$ and $D$. If the beam is a W12 × 45, determine if it can safely support the load. The allowable bending stress is $\sigma_{\text{allow}} = 22$ ksi and the allowable shear stress is $\tau_{\text{allow}} = 12$ ksi.



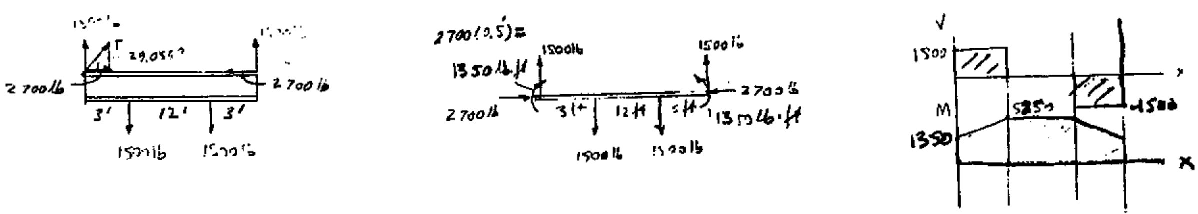

## SOLUTION

$$h = \frac{1500}{\tan 29.055^\circ} = 2700 \text{ lb}$$

$$\sigma = \frac{M}{S}; \qquad \sigma = \frac{5850(12)}{58.1} = 1.21 \text{ ksi} < 22 \text{ ksi} \qquad \textbf{OK}$$

$$\tau = \frac{V}{A_{web}}; \qquad \tau = \frac{1500}{(12.06)(0.335)} = 371 \text{ psi} < 12 \text{ ksi} \qquad \textbf{OK}$$

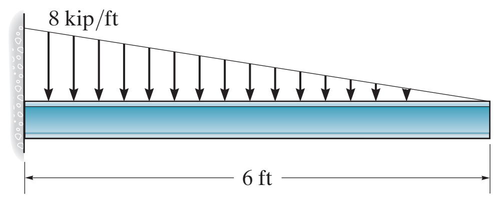
**Yes.** **Ans.**



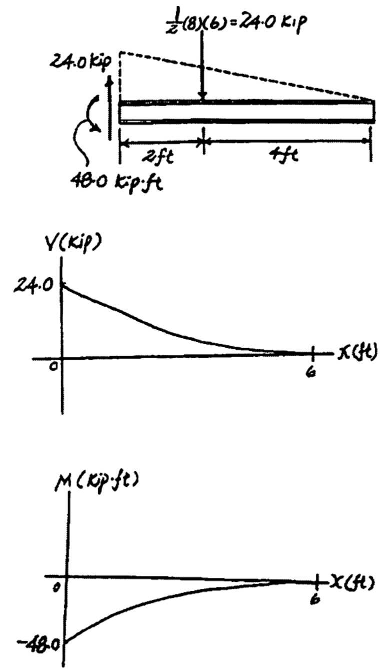
**Ans:**
Yes

**15–7.**

Select the lightest-weight wide-flange beam with the shortest depth from Appendix B that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress of $\tau_{allow} = 14$ ksi.




## SOLUTION




**Bending Stress:** From the moment diagram, $M_{max} = 48.0$ kip · ft. Assume bending controls the design. Applying the flexure formula,

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$= \frac{48.0(12)}{24} = 24.0 \text{ in}^3$$

Two choices of wide-flange section having the weight 22 lb/ft can be made. They are a W12 × 22 and W14 × 22. However, the W12 × 22 is the shortest.

**Select** W12 × 22 $\left(S_x = 25.4 \text{ in}^3, d = 12.31 \text{ in.}, t_w = 0.260 \text{ in.}\right)$

**Shear Stress:** Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W12 × 22 wide-flange section. From the shear diagram, $V_{max} = 24.0$ kip.

$$\tau_{max} = \frac{V_{max}}{t_w d}$$

$$= \frac{24.0}{0.260(12.31)}$$

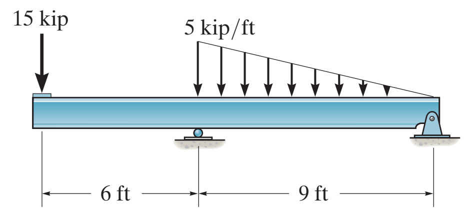
$$= 7.50 \text{ ksi} < \tau_{allow} = 14 \text{ ksi} \qquad \textbf{(O.K!)}$$

Hence, Use W12 × 22 **Ans.**

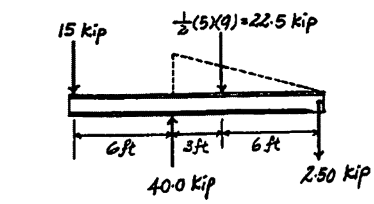
**Ans:**
Use W12 × 22.

***15–8.**

Select the lightest-weight wide-flange beam from Appendix B that will safely support the loading shown. The allowable bending stress $\sigma_{\text{allow}} = 24$ ksi and the allowable shear stress of $\tau_{\text{allow}} = 14$ ksi.




## SOLUTION

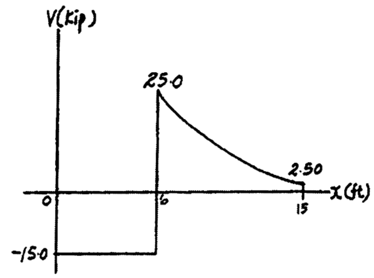



**Bending Stress:** From the moment diagram, $M_{\max} = 90.0$ kip · ft. Assume bending controls the design. Applying the flexure formula,

$$S_{\text{req'd}} = \frac{M_{\max}}{\sigma_{\text{allow}}}$$

$$= \frac{90.0(12)}{24} = 45.0 \text{ in}^3$$

**Select** W16 × 31 $\left(S_x = 47.2 \text{ in}^3,\ d = 15.88 \text{ in.},\ t_w = 0.275 \text{ in.}\right)$



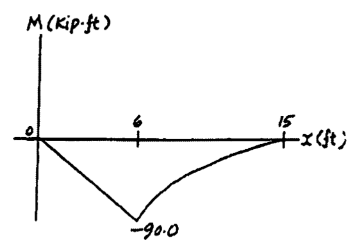

**Shear Stress:** Provide a shear stress check using $\tau = \dfrac{V}{t_w d}$ for the W16 × 31 wide-flange section. From the shear diagram, $V_{\max} = 25.0$ kip.

$$\tau_{\max} = \frac{V_{\max}}{t_w\, d}$$

$$= \frac{25.0}{0.275(15.88)}$$

$$= 5.72 \text{ ksi} < \tau_{\text{allow}} = 14 \text{ ksi} \qquad \textbf{(O.K!)}$$

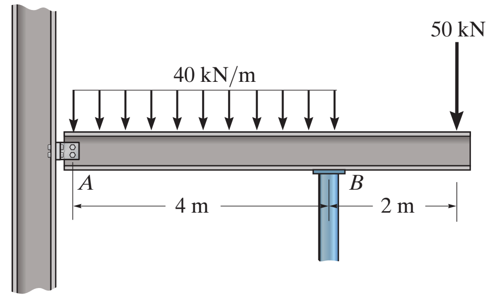
Hence, Use W16 × 31 **Ans.**




**Ans:**
Use W16 × 31.

**15–9.**

Select the lightest W360 wide-flange beam from Appendix B that can safely support the loading. The beam has an allowable normal stress of $\sigma_{\text{allow}} = 150$ MPa and an allowable shear stress of $\tau_{\text{allow}} = 80$ MPa. Assume there is a pin at $A$ and a roller support at $B$.




## SOLUTION

**Shear and Moment Diagram:** As shown in Fig. *a*.

**Bending Stress:** Referring to the moment diagram, Fig. *a*, $M_{\max} = 100\ \text{kN}\cdot\text{m}$. Applying the flexure formula,

$$S_{\text{required}} = \frac{M_{\max}}{\sigma_{\text{allow}}} = \frac{100(10^3)}{150(10^6)}$$

$$= 0.6667(10^{-3})\ \text{m}^3 = 666.67(10^3)\ \text{mm}^3$$

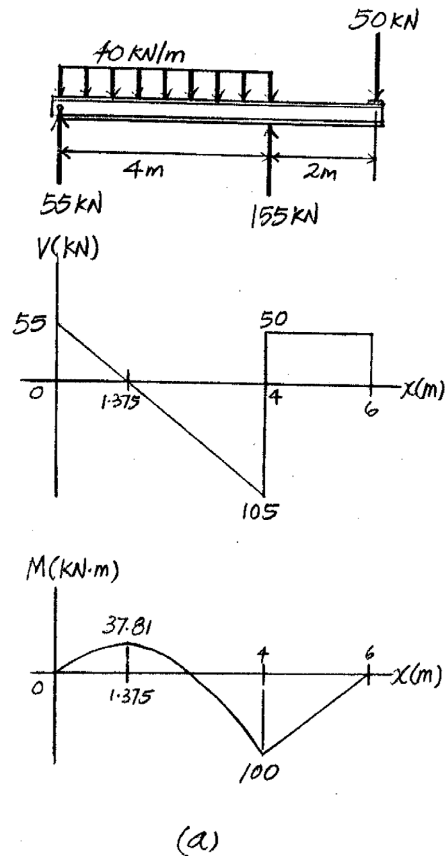
Select W360 $\times$ 45 ($S_x = 688(10^3)\ \text{mm}^3$, $d = 352$ mm and $t_w = 6.86$ mm)

**Shear Stress:** Referring to the shear diagram, Fig. *a*, $V_{\max} = 105$ kN. We have

$$\tau_{\max} = \frac{V_{\max}}{t_w d} = \frac{105(10^3)}{6.86(10^{-3})(0.352)}$$

$$= 43.48\ \text{MPa} < \tau_{\text{allow}} = 80\ \text{MPa} \qquad \textbf{(OK)}$$

Hence, use W360 $\times$ 45 **Ans.**

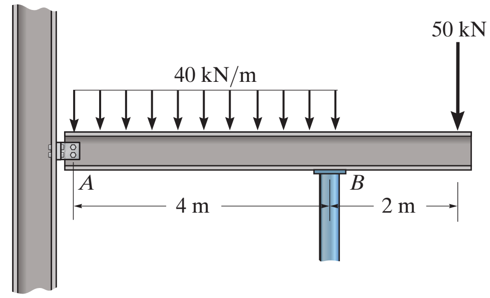



(a)

**Ans:**
Use W360 $\times$ 45.

**15–10.**

Investigate if the W250 × 58 beam can safely support the loading. The beam has an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 80$ MPa. Assume there is a pin at $A$ and a roller support at $B$.




## SOLUTION

**Shear and Moment Diagram:** As shown in Fig. *a*.

**Bending Stress:** Referring to the moment diagram, Fig. *a*, $M_{max} = 100\ \text{kN}\cdot\text{m}$. For a W250 × 58 section, $S_x = 693(10^3)\ \text{mm}^3 = 0.693(10^{-3})\ \text{m}^4$. Applying the flexure formula,

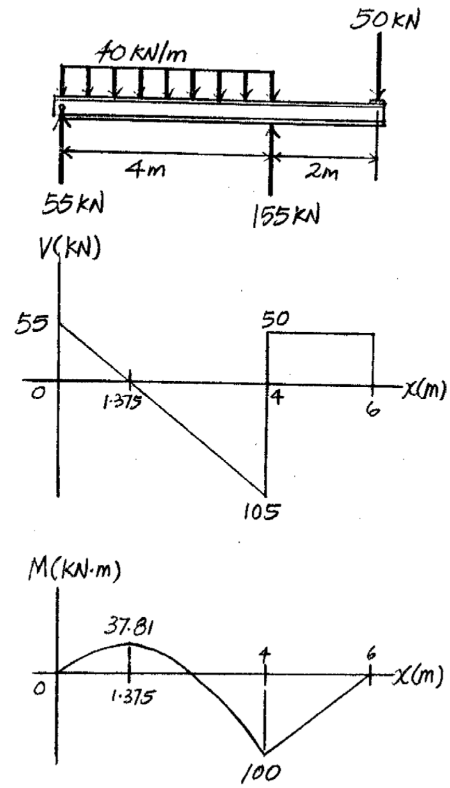
$$\sigma_{max} = \frac{M_{max}}{S_x} = \frac{100(10^3)}{0.693(10^{-3})} = 144.30\ \text{MPa} < \sigma_{allow} = 150\ \text{MPa} \qquad \textbf{(OK)}$$

**Shear Stress:** Referring to the shear diagram, Fig. *a*, $V_{max} = 105$ kN. For a W250 × 58 section, $d = 252$ mm and $t_w = 8.00$ mm. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{105(10^3)}{8.00(10^{-3})(0.252)}$$

$$= 52.08\ \text{MPa} < \tau_{allow} = 80\ \text{MPa} \qquad \textbf{(OK)}$$

The W250 × 58 can safely support the loading. **Ans.**

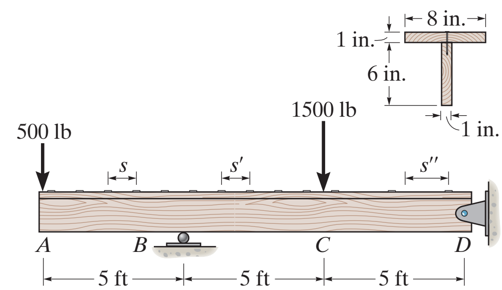



(a)

**Ans:**
Yes, it can.

**15–11.**

The beam is constructed from two boards. If each nail can support a shear force of 200 lb, determine the maximum spacing of the nails, $s$, $s'$, and $s''$, to the nearest $\frac{1}{8}$ inch for regions $AB$, $BC$, and $CD$, respectively.




## SOLUTION

**Section Properties:**

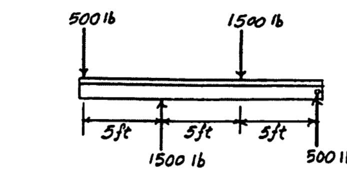
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(8)(1) + 4(6)(1)}{8(1) + 6(1)} = 2.00 \text{ in.}$$

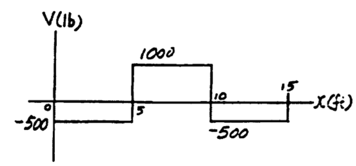
$$I = \frac{1}{12}(8)(1^3) + 8(1)(2 - 0.5)^2$$

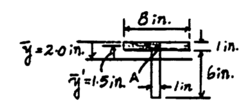
$$\frac{1}{12}(1)(6^3) + 1(6)(4 - 2)^2$$

$$= 60.667 \text{ in}^4$$

$$Q_A = \bar{y}'A' = 1.5(1)(8) = 12.0 \text{ in}^3$$




**Shear Flow:**

For $0 \le x < 5$ ft and 10 ft $< x \le 15$ ft (region $AB$ and $CD$), the design shear force is $V = 500$ lb and the allowable shear flow is $q = \dfrac{200}{s}$.

$$q = \frac{VQ_A}{I}$$

$$\frac{200}{s} = \frac{500(12.0)}{60.667}$$

$$s'' = s = 2.02 \text{ in.}$$

Use $s = s'' = 2$ in. **Ans.**

For 5 ft $< x < 10$ ft (region $BC$), the design shear force is $V = 1000$ lb and the allowable shear flow is $q = \dfrac{200}{s'}$.

$$q = \frac{VQ_A}{I}$$

$$\frac{200}{s'} = \frac{1000(12.0)}{60.667}$$

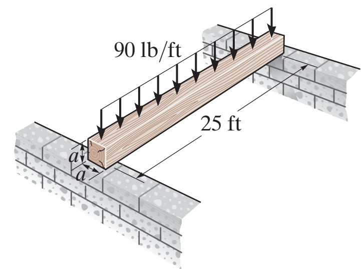
$$s' = 1.01 \text{ in}$$

Use $s' = 1$ in. **Ans.**

**Ans:**
Use $s = s'' = 2$ in.,
Use $s' = 1$ in.

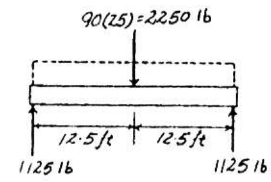
***15–12.**

The joists of a floor in a warehouse are to be selected using square timber beams made of oak. If each beam is to be designed to carry 90 lb/ft over a simply supported span of 25 ft, determine the dimension $a$ of its square cross section to the nearest $\frac{1}{4}$ in. The allowable bending stress is $\sigma_{\text{allow}} = 4.5$ ksi and the allowable shear stress is $\tau_{\text{allow}} = 125$ psi.




## SOLUTION

**Bending Stress:** From the moment diagram, $M_{\max} = 7031.25 \text{ lb}\cdot\text{ft}$. Assume bending controls the design. Applying the flexure formula,

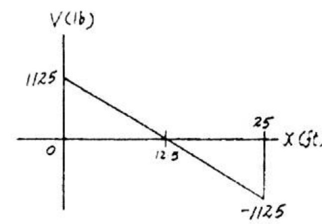



$$\sigma_{\text{allow}} = \frac{M_{\max}\, c}{I}$$

$$4.5(10^3) = \frac{7031.25(12)(\frac{a}{2})}{\frac{1}{12}a^4}$$

$$a = 4.827 \text{ in.}$$

Use $\qquad a = 5 \text{ in.}$ **Ans.**

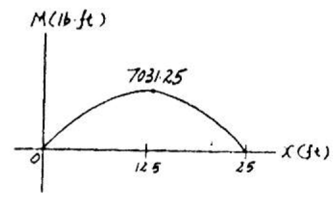



**Shear Stress:** Provide a shear stress check using the shear formula with $I = \frac{1}{12}(5^4) = 52.083 \text{ in}^4$ and $Q_{\max} = 1.25(2.5)(5) = 15.625 \text{ in}^3$. From the shear diagram, $V_{\max} = 1125$ lb.

$$\tau_{\max} = \frac{V_{\max}\, Q_{\max}}{I\,t}$$

$$= \frac{1125(15.625)}{52.083(5)}$$

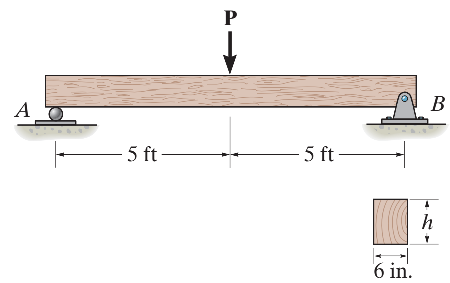
$$= 67.5 \text{ psi} < \tau_{\text{allow}} = 125 \text{ psi} \qquad \textbf{(OK!)}$$



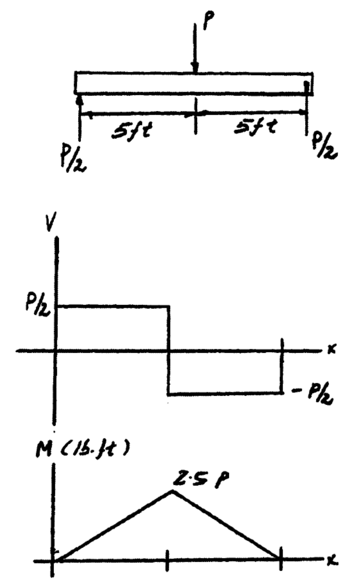

**Ans:**
Use $a = 5$ in.

**15–13.**

The timber beam has a width of 6 in. Determine its height $h$ so that it simultaneously reaches its allowable bending stress $\sigma_{\text{allow}} = 1.50$ ksi and an allowable shear stress of $\tau_{\text{allow}} = 50$ psi. Also, what is the maximum load $P$ that the beam can then support?




## SOLUTION




**Section Properties:**

$$I = \frac{1}{12}(6)\left(h^3\right) = 0.5\,h^3$$

$$S = \frac{I}{c} = \frac{0.5h^3}{0.5h} = h^2$$

$$Q_{\text{max}} = 0.25h(0.5h)(6) = 0.75\,h^2$$

**If Shear Controls:**

$$\tau_{\text{allow}} = \frac{V_{\text{max}}Q_{\text{max}}}{It}; \qquad 50 = \frac{\left(\frac{P}{2}\right)\left(0.75h^2\right)}{0.5h^3(6)}$$

$$150h = 0.375\,P \qquad \textbf{(1)}$$

**If Bending Controls:**

$$\sigma_{\text{allow}} = \frac{M_{\text{max}}}{S}$$

$$S = \frac{I}{c} = \frac{0.5\left(h^3\right)}{\frac{h}{2}} = h^2$$

$$1.50\left(10^3\right) = \frac{2.5P(12)}{h^2}$$

$$1.50\left(10^3\right)h^2 = 30P \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

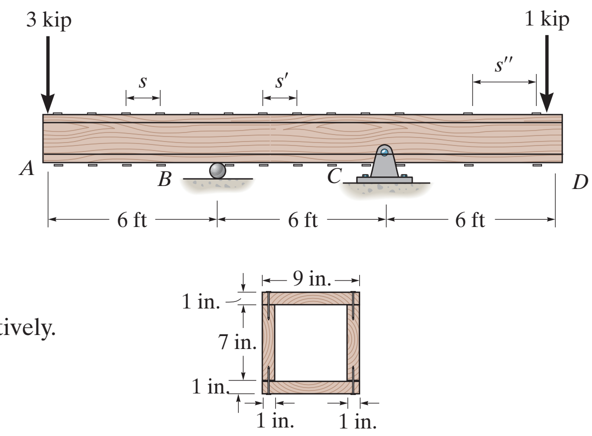
$h = 8.0$ in. **Ans.**

$P = 3.20$ kip **Ans.**

**Ans:**
$h = 8.0$ in.,
$P = 3.20$ kip

**15–14.**

The beam is constructed from four boards. If each nail can support a shear force of 300 lb, determine the maximum spacing of the nails, $s$, $s'$ and $s''$, for regions $AB$, $BC$, and $CD$, respectively.




## SOLUTION

**Support Reactions and Shear Diagram:** As shown in Figs. $a$ and $b$, respectively.

**Section Properties:** For the cross-section shown in Fig. $c$,

$$I = \frac{1}{12}(9)(9^3) - \frac{1}{12}(7)(7^3) = 346.67 \text{ in}^4$$

$$Q = \bar{y}'A' = 4[9(1)] = 36.0 \text{ in}^3$$

**Shear Flow:** Since there are two rows of nails, the allowable shear flow is $\sigma_{\text{allow}} = \frac{2(300)}{s} = \frac{600}{s}$ for $0 \leq x < 6$ ft (region $AB$), the shear force is $V = 3.00$ kip (from shear diagram). Then

$$\sigma_{\text{allow}} = \frac{VQ}{I}; \qquad \frac{600}{s} = \frac{3.00(10^3)(36.0)}{346.67}$$

$$s = 1.926 \text{ in.} = 1.93 \text{ in.} \qquad \textbf{Ans.}$$

For $6 \text{ ft} < x < 12 \text{ ft}$ (region $BC$), $V = 2.00$ kip. Then

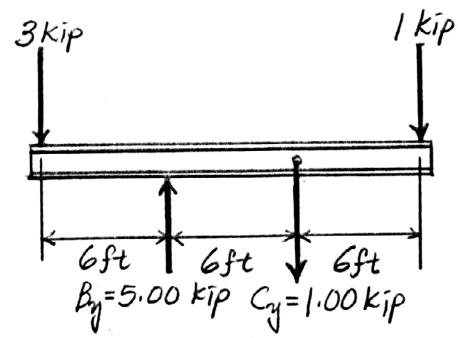
$$\sigma_{\text{allow}} = \frac{VQ}{I}; \qquad \frac{600}{s'} = \frac{2.00(10^3)(36.0)}{346.67}$$

$$s' = 2.889 \text{ in.} = 2.89 \text{ in.} \qquad \textbf{Ans.}$$

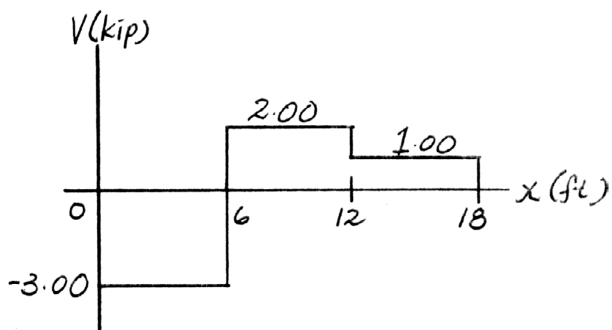
For $12 \text{ ft} < x \leq 18 \text{ ft}$ (region $CD$), $V = 1.00$ kip. Then

$$\sigma_{\text{allow}} = \frac{VQ}{I}; \qquad \frac{600}{s''} = \frac{1.00(10^3)(36.0)}{346.67}$$

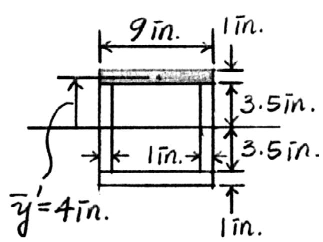
$$s'' = 5.778 \text{ in.} = 5.78 \text{ in.} \qquad \textbf{Ans.}$$




(a)




(b)

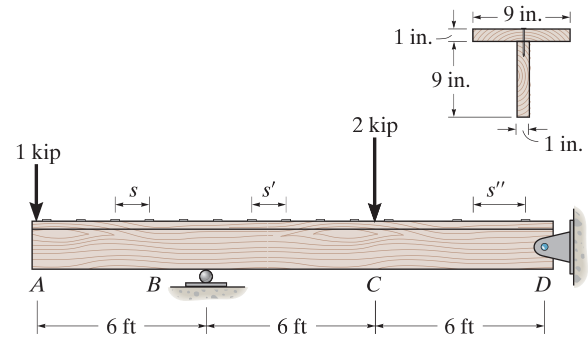



(c)

**Ans:**
$s = 1.93$ in.,
$s' = 2.89$ in.,
$s'' = 5.78$ in.

**15–15.**

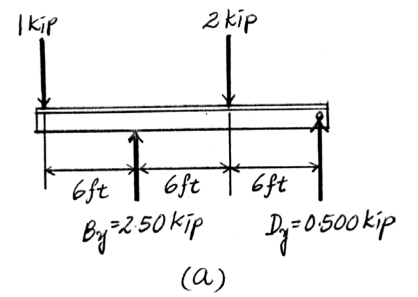
The beam is constructed from two boards. If each nail can support a shear force of 200 lb, determine the maximum spacing of the nails, $s$, $s'$, and $s''$, to the nearest $\frac{1}{8}$ in. for regions $AB$, $BC$, and $CD$, respectively.




## SOLUTION

**Support Reactions and Shear Diagram:** As shown in Figs. $a$ and $b$, respectively.

**Section Properties:** For the cross-section shown in Fig. $c$,



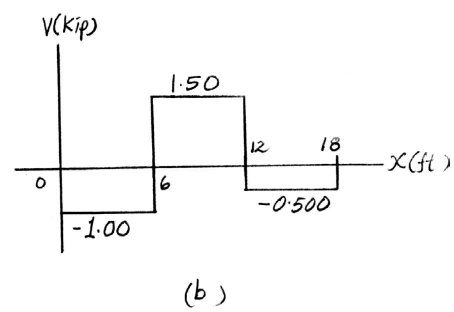

$$\bar{y} = \frac{4.5(9)(1) + 9.5(1)(9)}{9(1) + (1)(9)} = 7.00 \text{ in.}$$

$$I = \frac{1}{12}(1)(9^3) + 1(9)(7 - 4.50)^2 + \frac{1}{12}(9)(1^3) + 9(1)(9.50 - 7.00)^2$$

$$= 174 \text{ in.}^4$$

$$Q = \bar{y}'A' = 2.50[9(1)] = 22.5 \text{ in}^3$$

**Shear Flow:** The allowable shear flow is $\sigma_{\text{allow}} = \frac{200}{s}$ for $0 \le x < 6$ ft (region $AB$), $V = 1.00$ kip (from shear diagram). Then

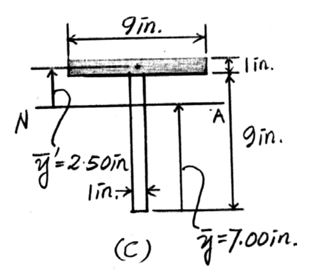



$$\sigma_{\text{allow}} = \frac{VQ}{I}; \qquad \frac{200}{s} = \frac{1.00(10^3)(22.5)}{174}$$

$$s = 1.547 \text{ in.}$$

Use $\quad s = 1\frac{5}{8}$ in. **Ans.**

For 6 ft $< x <$ 12 ft (region $BC$), $V = 1.50$ kip (from the shear diagram). Then




$$\sigma_{\text{allow}} = \frac{VQ}{I}; \qquad \frac{200}{s'} = \frac{1.50(10^3)(22.5)}{174}$$

$$s' = 1.031 \text{ in.}$$

Use $\quad s' = 1\frac{1}{8}$ in. **Ans.**

For 12 ft $< x \le$ 18 ft (region $CD$), $V = 0.500$ kip (from the shear diagram). Then

$$\sigma_{\text{allow}} = \frac{VQ}{I}; \qquad \frac{200}{s''} = \frac{0.500(10^3)(22.5)}{174}$$

$$s'' = 3.093 \text{ in.}$$

Use $\quad s'' = 3\frac{1}{8}$ in. **Ans.**

**Ans:**

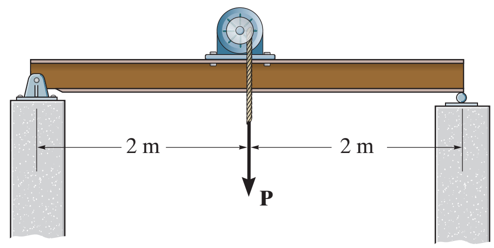
Use $s = 1\frac{5}{8}$ in.,

$s' = 1\frac{1}{8}$ in.,

$s'' = 3\frac{1}{8}$ in.

***15–16.**

If the cable is subjected to a maximum force of $P = 50$ kN, select the lightest W310 wide-flange beam that can safely support the load. The beam has an allowable normal stress of $\sigma_{\text{allow}} = 150$ MPa and an allowable shear stress of $\tau_{\text{allow}} = 85$ MPa.




## SOLUTION

**Shear and Moment Diagram:** As shown in Fig. *a*.

**Bending Stress:** From the moment diagram, Fig. *a*, $M_{\text{max}} = 50$ kN·m. Applying the flexure formula,

$$S_{\text{required}} = \frac{M_{\text{max}}}{\sigma_{\text{allow}}} = \frac{50(10^3)}{150(10^6)}$$

$$= 0.3333(10^{-3})\ \text{m}^3 = 333.33(10^3)\ \text{mm}^3$$

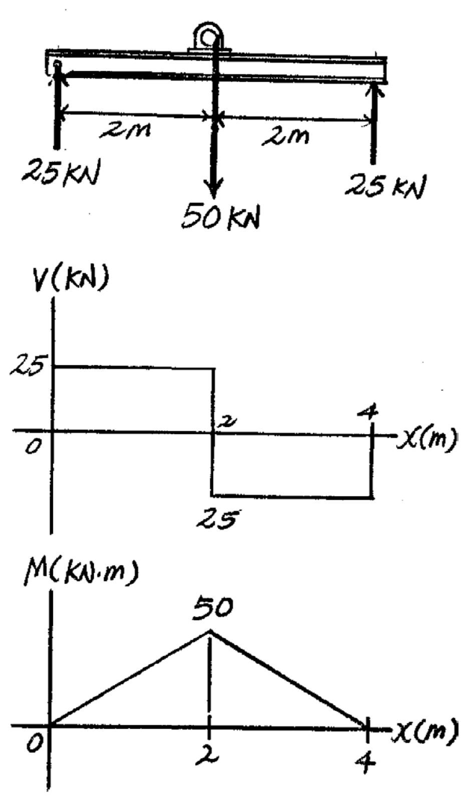
Select a W310 × 33 [$S_x = 415(10^3)\ \text{mm}^3$, $d = 313$ mm, and $t_w = 6.60$ mm]

**Shear Stress:** From the shear diagram, Fig. *a*, $V_{\text{max}} = 25$ kN. We have

$$\tau_{\text{max}} = \frac{V_{\text{max}}}{t_w\, d} = \frac{25(10^3)}{6.60(10^{-3})(0.313)}$$

$$= 12.10\ \text{MPa} < \tau_{\text{allow}} = 85\ \text{MPa} \qquad \textbf{OK}$$

Hence, use W310 × 33. **Ans.**

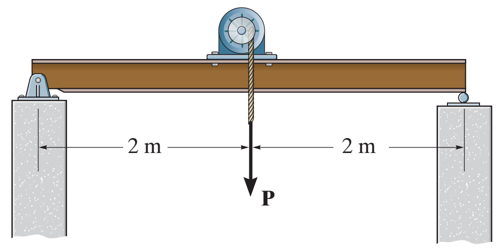



(a)

**Ans:**
Use W310 × 33.

**15–17.**

If the W360 × 45 wide-flange beam has an allowable normal stress of $\sigma_{allow} = 150$ MPa and an allowable shear stress of $\tau_{allow} = 85$ MPa, determine the maximum cable force **P** that can safely be supported by the beam.




## SOLUTION

**Shear and Moment Diagram:** As shown in Fig. *a*.

**Bending Stress:** From the moment diagram, Fig. *a*, $M_{max} = P$. For W360 × 45 section, $S_x = 688(10^3)\ \text{mm}^3 = 0.688(10^{-3})\ \text{m}^3$.

Applying the flexure formula,

$$M_{max} = S_x \sigma_{allow}$$

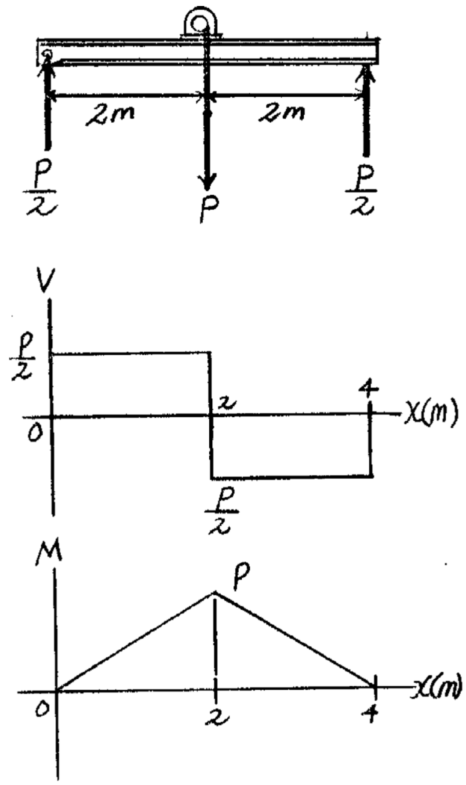
$$P = 0.688(10^{-3})[150(10^6)]$$

$$P = 103\ 200\ \text{N} = 103\ \text{kN}$$ **Ans.**

**Shear Stress:** From the shear diagram, Fig. *a*, $V_{max} = \dfrac{P}{2} = \dfrac{103\ 200}{2} = 51\ 600$ N. For W360 × 45 section, $d = 352$ mm and $t_w = 6.86$ mm. We have

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{51\ 600}{6.86(10^{-3})(0.352)}$$

$$= 21.37\ \text{MPa} < \tau_{allow} = 85\ \text{MPa}$$ **(OK!)**

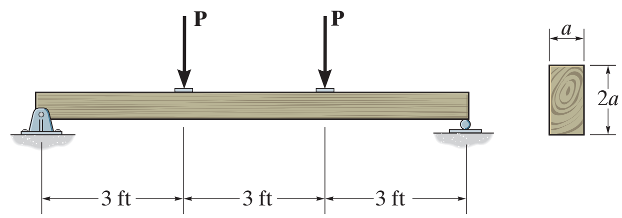



(a)

**Ans:**
$P = 103$ kN

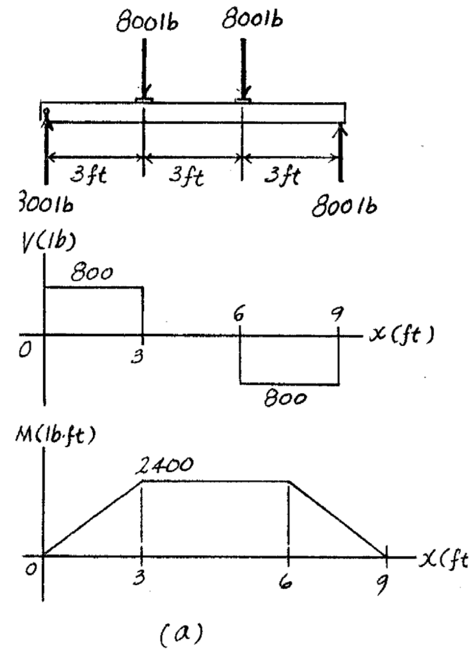
**15–18.**

If $P = 800$ lb, determine the minimum dimension $a$ of the beam's cross section to the nearest $\frac{1}{8}$ in. to safely support the load. The wood has an allowable normal stress of $\sigma_{\text{allow}} = 1.5$ ksi and an allowable shear stress of $\tau_{\text{allow}} = 150$ psi.




## SOLUTION

**Shear and Moment Diagram:** As shown in Fig. *a*.




**Bending Stress:** The moment of inertia of the beam's cross section about the bending axis is $I = \frac{1}{12}(a)(2a)^3 = \frac{2}{3}a^4$. Referring to the moment diagram in Fig. *a*, $M_{\max} = 2400$ lb · ft. Applying the flexure formula,

$$\sigma_{\text{allow}} = \frac{M_{\max}c}{I}$$

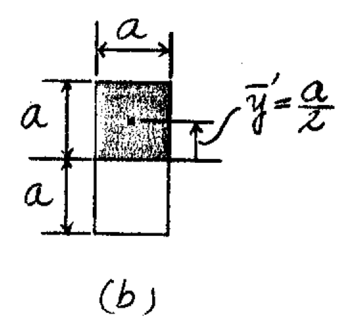
$$1.5(10^3) = \frac{2400(12)(a)}{\frac{2}{3}a^4}$$

$$a = 3.065 \text{ in.}$$

Use $a = 3\frac{1}{8}$ in. **Ans.**

**Shear Stress:** Using this result, $I = \frac{1}{12}(3.125)(6.25^3) = 63.578 \text{ in}^4$ and $Q_{\max} = \bar{y}'A' = \left(\frac{3.125}{2}\right)(3.125)(3.125) = 15.259 \text{ in}^3$, Fig. *b*. Referring to the shear diagram, Fig. *a*, $V_{\max} = 800$ lb. Using the shear formula,

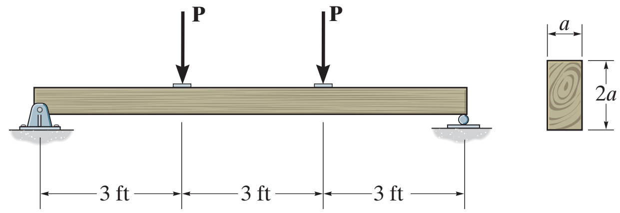
$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{800(15.259)}{63.578(3.125)} = 61.44 \text{ psi} < \tau_{\text{allow}} = 150 \text{ psi} \qquad \textbf{(OK!)}$$




**Ans:**
Use $a = 3\frac{1}{8}$ in.

**15–19.**

If $a = 3$ in. and the wood has an allowable normal stress of $\sigma_{allow} = 1.5$ ksi, and an allowable shear stress of $\tau_{allow} = 150$ psi, determine the maximum allowable value of $P$ that can act on the beam.




## SOLUTION

**Shear and Moment Diagram:** As shown in Fig. *a*.

**Bending Stress:** The moment of inertia of the beam's cross section about the bending axis is $I = \frac{1}{12}(3)(6^3) = 54 \text{ in}^4$. Referring to the moment diagram in Fig. *a*, $M_{max} = 3P$. Applying the flexure formula,

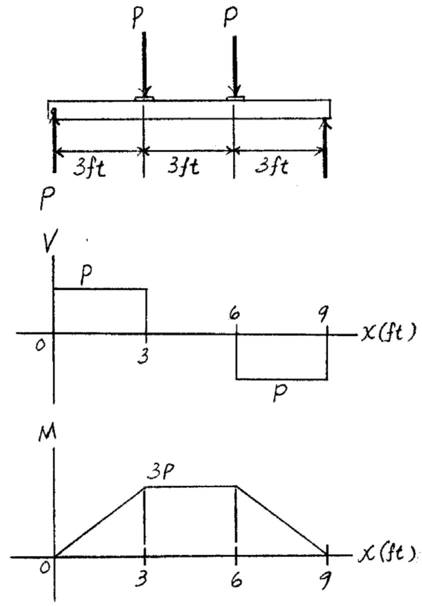
$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$1.5(10^3) = \frac{3P(12)(3)}{54}$$

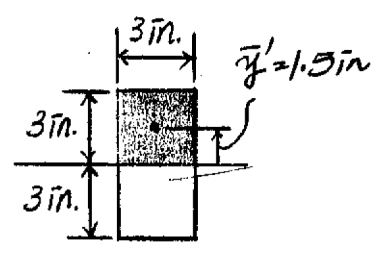
$$P = 750 \text{ lb}$$ **Ans.**

**Shear Stress:** Referring to Fig. *b*, $Q_{max} = \bar{y}'A' = 1.5(3)(3) = 13.5 \text{ in}^3$, Fig. *b*. Referring to the shear diagram, Fig. *a*, $V_{max} = 750$ lb. Using the shear formula,

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{750(13.5)}{54(3)} = 62.5 \text{ psi} < \tau_{allow} = 150 \text{ psi}$$ **(OK!)**



(a)

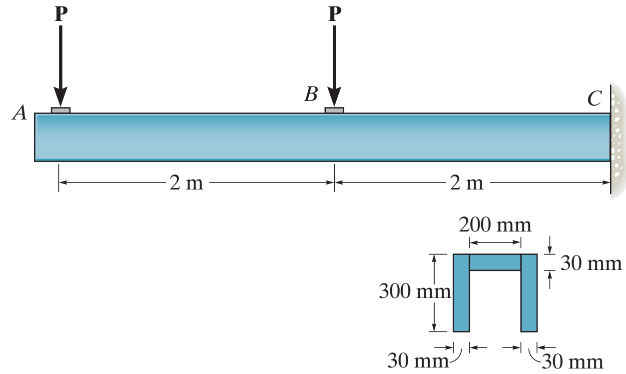


(b)

**Ans:**
$P = 750$ lb

***15–20.**

The beam is constructed from three plastic strips. If the glue can support a shear stress of $\tau_{\text{allow}} = 8$ kPa, determine the largest magnitude of the loads **P** that the beam can support.




## SOLUTION

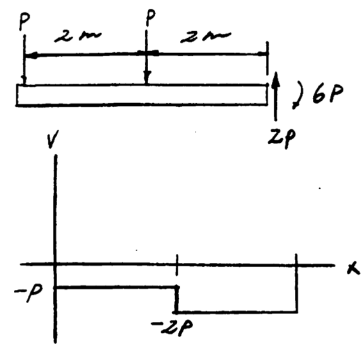
**Section Properties:**

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.015(0.2)(0.03) + 2[0.15(0.3)(0.03)]}{0.2(0.03) + 2(0.3)(0.03)} = 0.11625 \text{ m}$$

$$I = \frac{1}{12}(0.2)(0.03)^3 + 0.2(0.03)(0.11625 - 0.015)^2 + 2\left[\frac{1}{12}(0.03)(0.3)^3 + 0.03(0.3)(0.15 - 0.11625)^2\right]$$

$$= 0.2174625\left(10^{-3}\right) \text{ m}^4$$

$$Q_A = \bar{y}'A' = (0.11625 - 0.015)(0.2)(0.03) = 0.6075\left(10^{-3}\right) \text{ m}^3$$




**Maximum Load:**

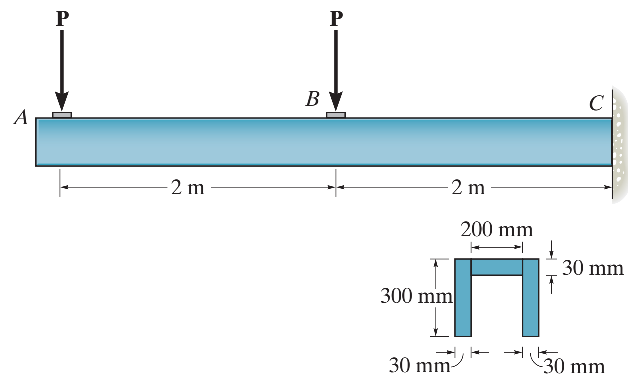
$$\tau_{\text{allow}} = \frac{V_{\max} Q_A}{It}; \qquad 8000 = \frac{2P(0.6075)\left(10^{-3}\right)}{0.2174625\left(10^{-3}\right)(2)(0.03)}$$

$P = 85.9$ N **Ans.**

**Ans:**
$P = 85.9$ N

**15–21.**

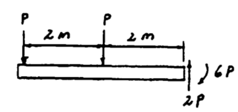
If the allowable bending stress is $\sigma_{\text{allow}} = 6$ MPa, and the glue can support a shear stress of $\tau_{\text{allow}} = 8$ kPa, determine the largest magnitude of the loads **P** that can be applied to the beam.




## SOLUTION

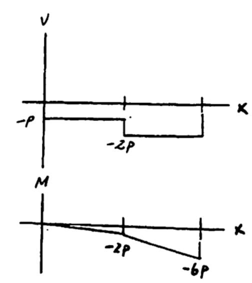
**Section Properties:**

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.015(0.2)(0.03) + 2[0.15(0.3)(0.03)]}{0.2(0.03) + 2(0.3)(0.03)} = 0.1162 \text{ m}$$




$$I = \frac{1}{12}(0.2)(0.03)^3 + 0.2(0.03)(0.11625 - 0.015)^2$$

$$+ 2\left[\frac{1}{12}(0.03)(0.3)^3 + 0.03(0.3)(0.15 - 0.11625)^2\right] = 0.2174625\left(10^{-3}\right) \text{ m}^4$$




**Maximum Load:**

$$\sigma_{\text{allow}} = \frac{M_{\max} c}{I}; \qquad 6\left(10^6\right) = \frac{6P(0.3 - 0.11625)}{0.2174625\left(10^{-3}\right)}$$

$P = 1183 \text{ N} = 1.18 \text{ kN}$

**Check Glue:** $Q_A = \bar{y}' A' = (0.11625 - 0.015)(0.2)(0.03) = 0.6075\left(10^{-3}\right) \text{ m}^3$

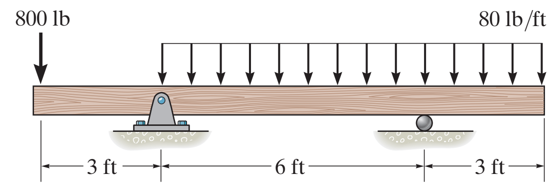
$$\tau_{\text{allow}} = \frac{V_{\max} Q_A}{It}; \qquad 8000 = \frac{2P(0.6075)\left(10^{-3}\right)}{0.2174625\left(10^{-3}\right)(2)(0.03)}$$

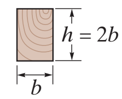
$P = 85.9$ N **Ans.**

**Ans:**
$P = 85.9$ N

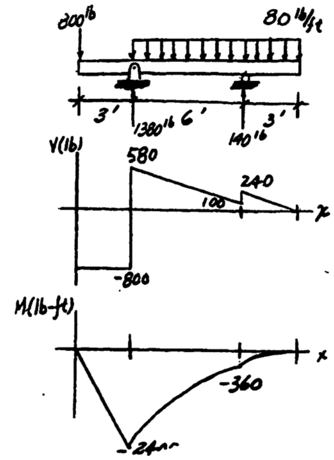
**15–22.**

The beam is made of Douglas fir having an allowable bending stress of $\sigma_{\text{allow}} = 1.1$ ksi and an allowable shear stress of $\tau_{\text{allow}} = 0.70$ ksi. Determine the width $b$ if the height $h = 2b$.




## SOLUTION




$$I_x = \frac{1}{12}(b)(2b)^3 = 0.6667\,b^4$$

$$Q_{\max} = \bar{y}'A' = (0.5b)(b)(b) = 0.5b^3$$

**Assume Bending Moment Controls:**

$$M_{\max} = 2400 \text{ lb} \cdot \text{ft}$$

$$\sigma_{\text{allow}} = \frac{M_{\max}c}{I}$$

$$1100 = \frac{2400(12)(b)}{0.6667b^4}$$

$$b = 3.40 \text{ in.}$$ **Ans.**

**Check Shear:**

$$Q_{\max} = 19.65 \text{ in}^3$$

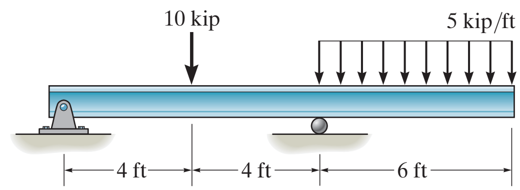
$$I = 89.09 \text{ in}^4$$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{800(19.65)}{89.09(3.40)} = 51.9 \text{ psi} < 700 \text{ psi}$$ **OK**

**Ans:**
$b = 3.40$ in.

**15–23.**

Select the lightest-weight wide-flange beam from Appendix B that will safely support the loading. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.




## SOLUTION

Assume bending moment controls.

$$M_{max} = 90 \text{ kip} \cdot \text{ft}$$

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{90(12)}{24} = 45 \text{ in}^3$$

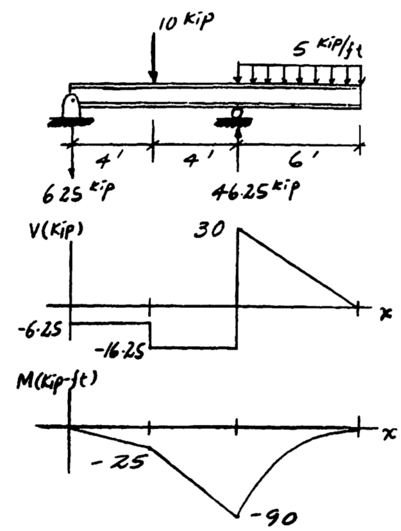
Select a W16 × 31

$$S_x = 47.5 \text{ in}^3 \qquad d = 15.88 \text{ in.} \qquad t_w = 0.275 \text{ in.}$$

**Check Shear:**

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{30}{(15.88)(0.275)} = 6.87 \text{ ksi} < 14 \text{ ksi}$$ **OK**

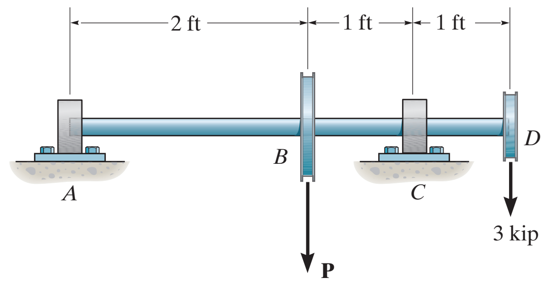
Use W16 × 31. **Ans.**




**Ans:**
Use W16 × 31.

***15–24.**

Draw the shear and moment diagrams for the shaft, and determine its required diameter to the nearest $\frac{1}{8}$ in. if $\sigma_{allow} = 30$ ksi and $\tau_{allow} = 15$ ksi. The journal bearings at $A$ and $C$ exert only vertical reactions on the shaft. Take $P = 6$ kip.




## SOLUTION

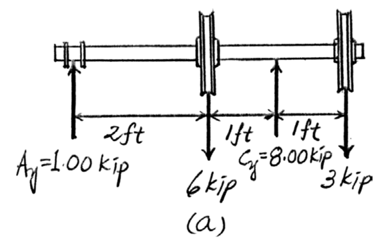
**Support Reactions, Shear and Moment Diagram:** As shown in Figs. *a, b* and *c*, respectively.

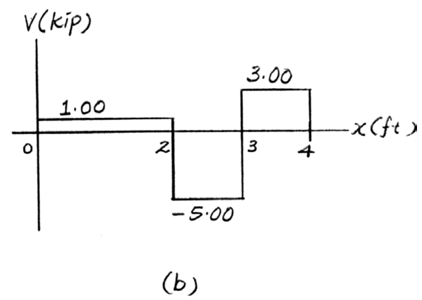
**Bending Stress:** From the moment diagram, $M_{max} = 3.00$ kip·ft. Assume that bending controls the design. Applying the flexure formula,

$$\sigma_{allow} = \frac{Mc}{I}; \qquad 30\left(10^3\right) = \frac{\left[3.00\left(10^3\right)(12)\right]\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 2.304 \text{ in.}$$

$$\text{Use} \quad d = 2\frac{3}{8} \text{ in.}$$ **Ans.**




**Shear Stress:** Provide a shear stress check using the shear formula with

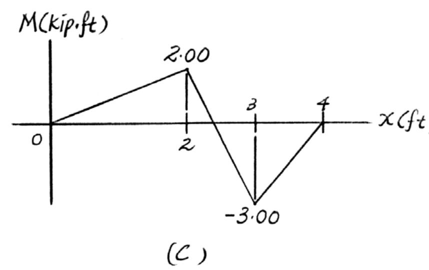
$$I = \frac{\pi}{4}\left(1.1875^4\right) = 1.5618 \text{ in}^4$$

$$Q_{max} = \frac{4(1.1875)}{3\pi}\left[\frac{\pi}{2}\left(1.1875^2\right)\right] = 1.1164 \text{ in}^3$$

From the shear diagram, $V_{max} = 5.00$ kip. Then

$$\tau_{max} = \frac{V_{max}\,Q_{max}}{It} = \frac{5.00\left(10^3\right)(1.1164)}{1.5618(2.375)}$$

$$= 1.505\left(10^3\right)\text{psi} = 1.505 \text{ ksi} < \tau_{allow} = 15 \text{ ksi}$$ **(O.K!)**

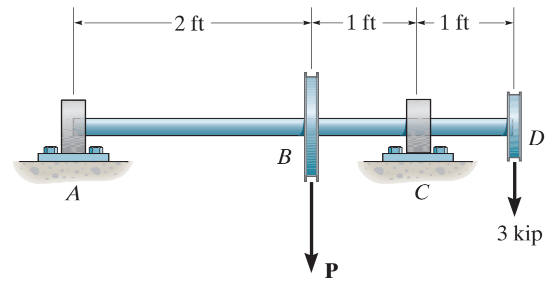



**Ans:**

Use $d = 2\frac{3}{8}$ in.

**15–25.**

Draw the shear and moment diagrams for the shaft, and determine its required diameter to the nearest $\frac{1}{4}$ in. if $\sigma_{allow} = 30$ ksi and $\tau_{allow} = 15$ ksi. The journal bearings at $A$ and $C$ exert only vertical reactions on the shaft. Take $P = 12$ kip.




## SOLUTION

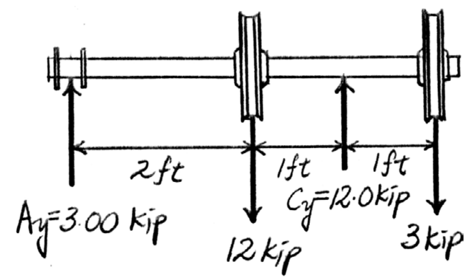
**Support Reactions, Shear and Moment Diagrams:** As shown in Figs. *a, b* and *c,* respectively.

**Bending Stress:** From the moment diagram, $M_{max} = 6.00$ kip·ft. Assume that bending controls the design. Applying flexure formula,

$$\sigma_{allow} = \frac{Mc}{I}; \qquad 30(10^3) = \frac{\left[6(10^3)(12)\right]\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4}$$

$$d = 2.902 \text{ in.}$$

Use $\quad d = 3$ in. **Ans.**




(a)

**Shear Stress:** Provide a shear stress check using shear formula with

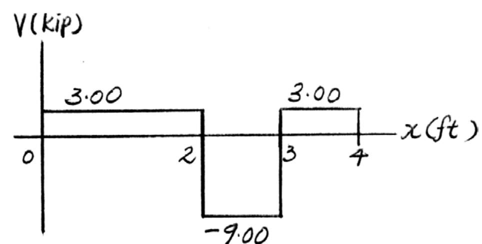
$$I = \frac{\pi}{4}(1.5^4) = 3.9761 \text{ in}^4$$

$$Q_{max} = \frac{4(1.5)}{3\pi}\left[\frac{\pi}{2}(1.5^2)\right] = 2.25 \text{ in}^3$$

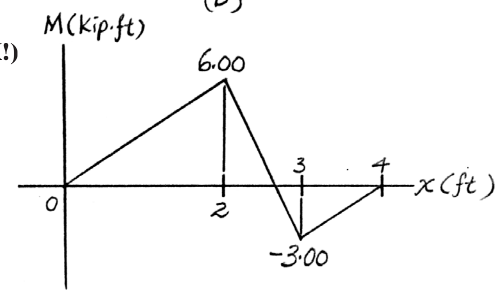
From the shear diagram, $V_{max} = 9.00$ kip. Then

$$\tau_{max} = \frac{V_{max}Q_{max}}{It} = \frac{9(10^3)(2.25)}{3.9761(3)}$$

$$= 1.698(10^3)\text{psi} = 1.698 \text{ ksi} < \tau_{allow} = 15 \text{ ksi} \qquad \textbf{(O.K!)}$$




(b)




(c)

**Ans:**
Use $\quad d = 3$ in.