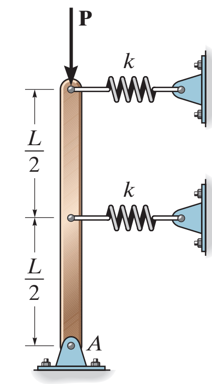
17–1.

Determine the critical buckling load for the column. The material can be assumed rigid.

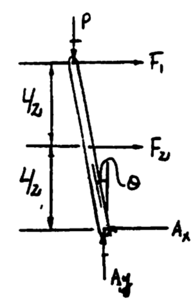


SOLUTION

$$F_1 = k(L\,\theta); \qquad F_2 = k\left(\frac{L}{2}\,\theta\right)$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad P(\theta)(L) - (F_1 L) - F_2\left(\frac{L}{2}\right) = 0$$

$$P(\theta)(L) - kL^2\,\theta - k\left(\frac{L}{2}\right)^2\theta = 0$$



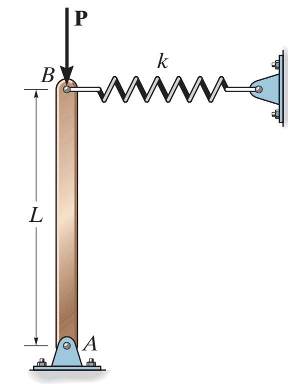
Require:

$$P_{cr} = kL + \frac{kL}{4} = \frac{5kL}{4}$$ **Ans.**

Ans:
$$P_{cr} = \frac{5KL}{4}$$

17–2.

The column consists of a rigid member that is pinned at its bottom and attached to a spring at its top. If the spring is unstretched when the column is in the vertical position, determine the critical load that can be placed on the column.



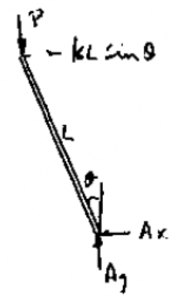
SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad PL \sin\theta - (kL\sin\theta)(L\cos\theta) = 0$

$$P = kL\cos\theta$$

Since θ is small, $\quad \cos\theta = 1$

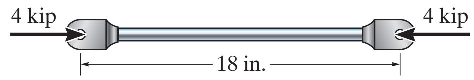
$$P_{cr} = kL \qquad \textbf{Ans.}$$



Ans:
$P_{cr} = kL$

17–3.

The aircraft link is made from an A992 steel rod. Determine the smallest diameter of the rod, to the nearest $\frac{1}{16}$ in., that will support the load of 4 kip without buckling. The ends are pin connected.



SOLUTION

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi\, d^4}{64}$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2}$$

$$4 = \frac{\pi^2(29)(10^3)\left(\frac{\pi d^4}{64}\right)}{((1.0)(18))^2}$$

$$d = 0.551 \text{ in.}$$

Use $d = \dfrac{9}{16}$ in. **Ans.**

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4}{\frac{\pi}{4}(0.562^2)} = 16.2 \text{ ksi} < \sigma_Y$$

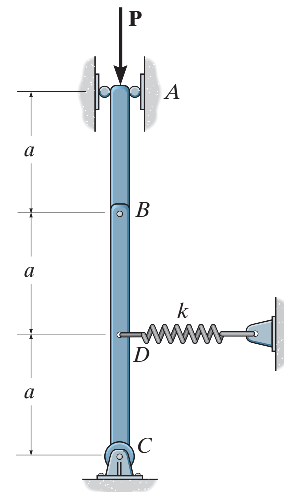
Therefore, Euler's formula is valid.

Ans:

Use $d = \dfrac{9}{16}$ in.

***17–4.**

Rigid bars AB and BC are pin connected at B. If the spring at D has a stiffness k, determine the critical load P_{cr} that can be applied to the bars.



SOLUTION

Equilibrium: The disturbing force F can be related to P by considering the equilibrium of joint A and then the equilibrium of member BC.

Joint A: (Fig. b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\cos\phi - P = 0 \qquad\qquad F_{AB} = \frac{P}{\cos\phi}$$

Member BC: (Fig. c)

$$\Sigma M_C = 0;\ F(a\cos\theta) - \frac{P}{\cos\phi}\cos\phi\,(2a\sin\theta) - \frac{P}{\cos\phi}\sin\phi(2a\cos\theta) = 0$$

$$F = 2P(\tan\theta + \tan\phi)$$

Since θ and ϕ are small, $\tan\theta \cong \theta$ and $\tan\phi \cong \phi$. Thus,

$$F = 2P(\theta + \phi) \qquad \textbf{(1)}$$

Also, from the geometry shown in Fig. a,

$$2a\theta = a\phi \qquad\qquad \phi = 2\theta$$

Thus Eq. (1) becomes

$$F = 2P(\theta + 2\theta) = 6P\theta$$

Spring Force: The restoring spring force F_{sp} can be determined using the spring formula, $F_{sp} = kx$, where $x = a\theta$, Fig. a. Thus,

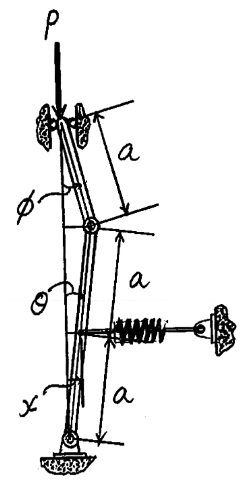
$$F_{sp} = kx = ka\theta$$

Critical Buckling Load: When the mechanism is on the verge of buckling, the disturbing force F must be equal to the restoring spring force F_{sp}.

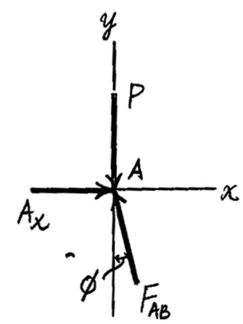
$$F = F_{sp}$$

$$6P_{cr}\theta = ka\theta$$

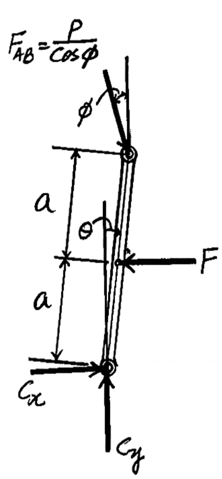
$$P_{cr} = \frac{ka}{6} \qquad \textbf{Ans.}$$



(a)



(b)



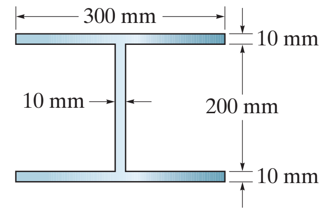
(c)

Ans:

$P_{cr} = \dfrac{ka}{6}$

17–5.

A 2014-T6 aluminum alloy column has a length of 6 m and is fixed at one end and pinned at the other. If the cross-sectional area has the dimensions shown, determine the critical load. $\sigma_Y = 250$ MPa.



SOLUTION

Section Properties: For the cross-section shown,

$$A = 0.3(0.22) - 0.29(0.2) = 8.00\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.3)\left(0.22^3\right) - \frac{1}{12}(0.29)\left(0.2^3\right) = 72.8667\left(10^{-6}\right)\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.3^3\right)\right] + \frac{1}{12}(0.2)\left(0.01^3\right) = 45.0167\left(10^{-6}\right)\text{ m}^4 \text{ (controls)}$$

Critical Buckling Load: $K = 0.7$ for the column with one end fixed and the other end pinned. For 2014-T6 aluminium alloy, $E = 73.1$ GPa. Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[45.0167\left(10^{-6}\right)\right]}{[0.7(6)]^2}$$

$$= 1.8412\left(10^6\right)\text{ N} = 1.84\text{ MN} \qquad \textbf{Ans.}$$

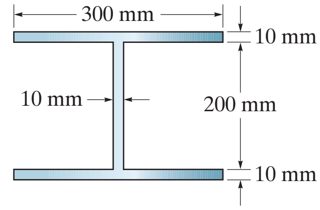
Critical Stress: Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$. For 2014-T6 aluminium alloy, $\sigma_Y = 414$ MPa.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{1.8412\left(10^6\right)}{8.00\left(10^{-3}\right)} = 230.15\text{ MPa} < \sigma_Y. \qquad \textbf{(O.K!)}$$

Ans:
$P_{\text{cr}} = 1.84$ MN

17–6.

Solve Prob. 17–5 if the column is pinned at its top and bottom.



SOLUTION

Section Properties: For the cross-section shown,

$$A = 0.3(0.22) - 0.29(0.2) = 8.00\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.3)\left(0.32^3\right) - \frac{1}{12}(0.29)\left(0.2^3\right) = 72.8667\left(10^{-6}\right)\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.3^3\right)\right] + \frac{1}{12}(0.2)\left(0.01^3\right) = 45.0167\left(10^{-6}\right)\text{ m}^4 \qquad \text{(controls!)}$$

Critical Buckling Load: $K = 1.0$ for the column pinned at both ends. For 2014-T6 aluminium alloy, $E = 73.1$ GPa. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[45.0167\left(10^{-6}\right)\right]}{[1.0(6)]^2}$$

$$= 902.17\left(10^3\right)\text{ N} = 902\text{ kN} \qquad \textbf{Ans.}$$

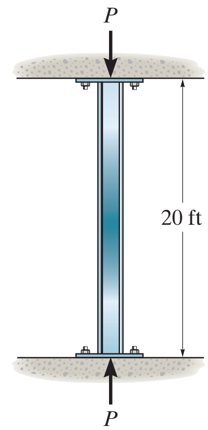
Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$. For 2014-T6 aluminium alloy, $\sigma_Y = 414$ MPa.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{902.17\left(10^3\right)}{8.00\left(10^{-3}\right)} = 112.77\text{ MPa} < \sigma_Y \qquad \textbf{(O.K!)}$$

Ans:
$P_{cr} = 902$ kN

17–7.

The W12 × 50 is made of A992 steel and is used as a column that has a length of 20 ft. If its ends are assumed pin supported, and it is subjected to an axial load of 150 kip, determine the factor of safety with respect to buckling.



SOLUTION

Critical Buckling Load: $I_y = 56.3\text{ in}^4$ for W12 × 50 wide-flange section and $E = 29(10^3)$ ksi for A992 steel. Also, $K = 1$ for pin supported ends column. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2\left[29(10^3)\right](56.3)}{[1(20)(12)]^2}$$

$$= 279.76\text{ kip}$$

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$. $A = 14.7\text{ in}^2$ for W12 × 50 wide-flange section and $\sigma_Y = 50$ ksi for A992 steel.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{279.76}{14.7} = 19.03\text{ ksi} < \sigma_Y \qquad \textbf{(O.K!)}$$

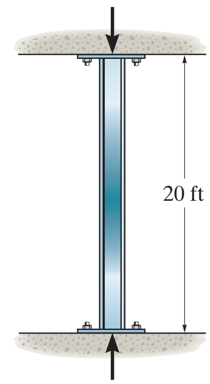
Then the factor of safety against buckling is

$$\text{F.S.} = \frac{P_{cr}}{P} = \frac{279.76}{150} = 1.865 = 1.87 \qquad \textbf{Ans.}$$

Ans:
F.S. = 1.87

***17–8.**

The W12 × 50 is made of A992 steel and is used as a column that has a length of 20 ft. If the ends of the column are fixed supported, can the column support the critical load without yielding?



SOLUTION

Critical Buckling Load: $I_y = 56.3\ \text{in}^4$ for W12 × 50 wide-flange section and $E = 29(10^3)$ ksi for A992 steel. Also, $K = 0.5$ for column with both ends fixed. Applying the Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2\left[29(10^3)\right](56.3)}{[0.5(20)(12)]^2}$$

$$= 1119.03\ \text{kip}$$

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$. $A = 14.7\ \text{in}^2$ for W12 × 50 wide-flange section and $\sigma_Y = 50$ ksi for A992 steel.

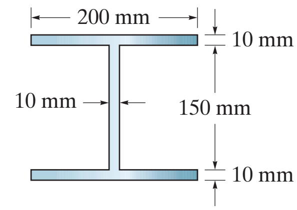
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1119.03}{14.7} = 76.12\ \text{ksi} > \sigma_Y \quad \text{(No Good!)}$$

The column will **yield** before the axial force P achieves the critical load P_{cr}. Thus, Euler's formula is not valid. **Ans.**

Ans:
No

17–9.

A steel column has a length of 9 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

Section Properties:

$$A = 0.2(0.17) - 0.19(0.15) = 5.50\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.2)\left(0.17^3\right) - \frac{1}{12}(0.19)\left(0.15^3\right) = 28.44583\left(10^{-6}\right)\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.2^3\right)\right] + \frac{1}{12}(0.15)\left(0.01^3\right) = 13.34583\left(10^{-6}\right)\text{ m}^4 \quad \text{(Controls!)}$$

Critical Buckling Load: $K = 0.5$ for fixed support ends column.
Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(200)\left(10^9\right)(13.34583)\left(10^{-6}\right)}{[0.5(9)]^2}$$

$$= 1300919\text{ N} = 1.30\text{ MN} \qquad \textbf{Ans.}$$

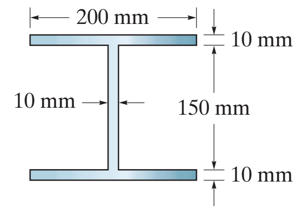
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1300919}{5.50\left(10^{-3}\right)} = 236.53\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \textbf{(O.K!)}$$

Ans:
$P_{cr} = 1.30$ MN

17–10.

A steel column has a length of 9 m and is pinned at its top and bottom. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

Section Properties:

$$A = 0.2(0.17) - 0.19(0.15) = 5.50\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.2)\left(0.17^3\right) - \frac{1}{12}(0.19)\left(0.15^3\right) = 28.44583\left(10^{-6}\right)\text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)\left(0.2^3\right)\right] + \frac{1}{12}(0.15)\left(0.01^3\right) = 13.34583\left(10^{-6}\right)\text{ m}^4 \text{ (Controls!)}$$

Critical Buckling Load: $K = 1$ for pin supported ends column.
Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(200)\left(10^9\right)(13.34583)\left(10^{-6}\right)}{[1(9)]^2}$$

$$= 325229.87\text{ N} = 325\text{ kN} \qquad \textbf{Ans.}$$

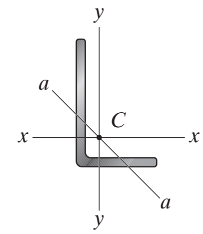
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{325229.87}{5.50\left(10^{-3}\right)} = 59.13\text{ MPa} < \sigma_Y = 250\text{ MPa} \qquad \textbf{(O.K!)}$$

Ans:
$P_{cr} = 325$ kN

17–11.

The A992 steel angle has a cross-sectional area of $A = 2.48\ \text{in}^2$ and a radius of gyration about the x axis of $r_x = 1.26$ in. and about the y axis of $r_y = 0.879$ in. The smallest radius of gyration occurs about the a–a axis and is $r_a = 0.644$ in. If the angle is to be used as a pin-connected 10-ft-long column, determine the largest axial load that can be applied through its centroid C without causing it to buckle.



SOLUTION

The Least Radius of Gyration:

$r_2 = 0.644$ in. controls.

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}; \qquad K = 1.0$$

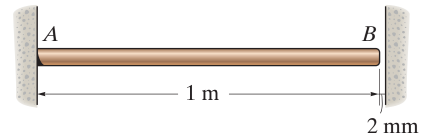
$$= \frac{\pi^2\,(29)(10^3)}{\left[\frac{1.0\,(120)}{0.644}\right]^2} = 8.243\ \text{ksi} < \sigma_\gamma \qquad \textbf{O.K.}$$

$$P_{cr} = \sigma_{cr}\,A = 8.243\,(2.48) = 20.4\ \text{kip} \qquad \textbf{Ans.}$$

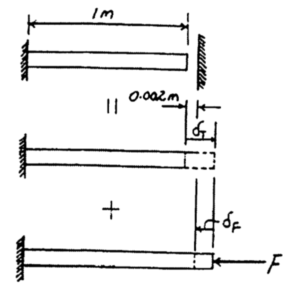
Ans:
$P_{cr} = 20.4$ kip

***17–12.**

The 50-mm-diameter C86100 bronze rod is fixed supported at A and has a gap of 2 mm from the wall at B. Determine the increase in temperature ΔT that will cause the rod to buckle. Assume that the contact at B acts as a pin.



SOLUTION



Section Properties:

$$A = \frac{\pi}{4}\left(0.05^2\right) = 0.625\pi\left(10^{-3}\right)\text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.025^4\right) = 97.65625\pi\left(10^{-9}\right)\text{ m}^4$$

Compatibility Condition: This requires

$$(\overset{+}{\rightarrow}) \qquad 0.002 = \delta_T + \delta_F$$

$$0.002 = 17\left(10^{-6}\right)(\Delta T)(1) - \frac{F(1)}{0.625\pi\left(10^{-3}\right)(103)\left(10^9\right)}$$

$$F = 3438.08\Delta T - 404480.05$$

Critical Buckling Load: $K = 0.7$ for a column with one end fixed and the other end pinned. Applying *Euler's* formula,

$$P_{\text{cr}} = F = \frac{\pi^2 EI}{(KL)^2}$$

$$3438.08\Delta T - 404\,480.05 = \frac{\pi^2\,(103)\left(10^9\right)\left[97.65625\pi\left(10^{-9}\right)\right]}{[0.7(1)]^2}$$

$$\Delta T = 302.78°\text{C} = 303°\text{C}$$ **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{\text{cr}} < \sigma_Y$.

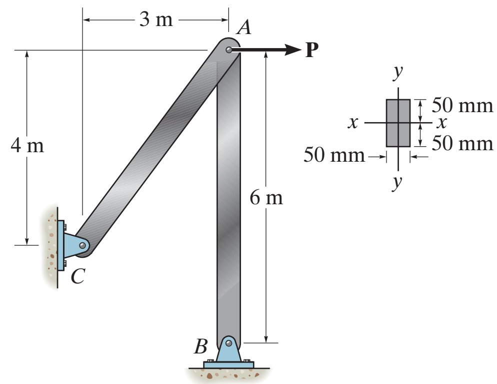
$$P_{\text{cr}} = 3438.08(302.78) - 404\,480.05 = 636\,488.86\text{ N}$$

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{636\,488.86}{0.625\pi\left(10^{-3}\right)} = 324.2\text{ MPa} < \sigma_Y = 345\text{ MPa}$$ **(O.K!)**

Ans:
$\Delta T = 303°\text{C}$

17–13.

Determine the maximum load P the frame can support without buckling member AB. Assume that AB is made of steel and is pinned at its ends for y–y axis buckling and fixed at its ends for x–x axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



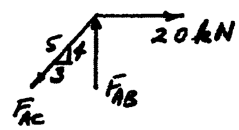
SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{AC}\left(\frac{3}{5}\right) + P = 0$

$$F_{AC} = \frac{5}{3}P$$

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{5}{3}P\left(\frac{4}{5}\right) = 0$

$$F_{AB} = \frac{4}{3}P$$



$$I_y = \frac{1}{12}(0.10)(0.05)^3 = 1.04167\left(10^{-6}\right)\text{m}^4$$

$$I_x = \frac{1}{12}(0.05)(0.10)^3 = 4.16667\left(10^{-6}\right)\text{m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

x–x axis buckling:

$$P_{cr} = \frac{\pi^2(200)\left(10^9\right)(4.16667)\left(10^{-6}\right)}{(0.5(6))^2} = 914 \text{ kN}$$

y–y axis buckling:

$$P_{cr} = \frac{\pi^2(200)\left(10^9\right)(1.04167)\left(10^{-6}\right)}{(1(6))^2} = 57.12 \text{ kN}$$

y–y axis buckling controls.

$$\frac{4}{3}P = 57.12$$

$P = 42.8$ kN **Ans.**

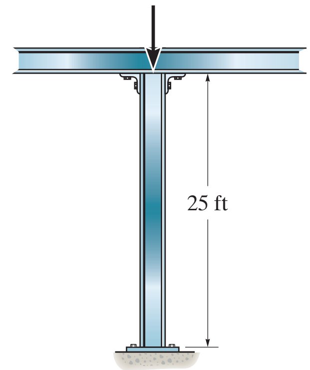
Check:

$\sigma_{cr} = \frac{P}{A} = \frac{57.12\left(10^3\right)}{(0.1)(0.05)} = 11.4 \text{ MPa} < \sigma_Y$ **OK**

Ans:
$P = 42.8$ kN

17–14.

The W8 × 67 wide-flange A-36 steel column can be assumed fixed at its base and pinned at its top. Determine the largest axial force P that can be applied without causing it to buckle.



SOLUTION

Critical Buckling Load: $I_y = 88.6\ \text{in}^4$ for a W8 × 67 wide-flange section and $K = 0.7$ for one end fixed and the other end pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (29)\left(10^3\right)(88.6)}{[0.7(25)(12)]^2}$$

$$= 575\ \text{kip} \qquad \textbf{Ans.}$$

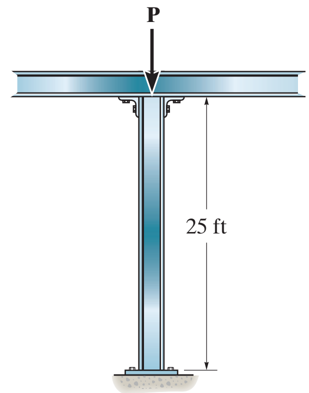
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$. $A = 19.7\ \text{in}^2$ for W8 × 67 wide-flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{575.03}{19.7} = 29.19\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \qquad \textbf{(O.K!)}$$

Ans:
$P_{cr} = 575\ \text{kip}$

17–15.

Solve Prob. 17–14 if the column is assumed fixed at its bottom and free at its top.



SOLUTION

Critical Buckling Load: $I_y = 88.6 \text{ in}^4$ for a W8 $\times$ 67 wide-flange section and $K = 2$ for one end fixed and the other end free. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (29)(10^3)(88.6)}{[2(25)(12)]^2}$$

$$= 70.4 \text{ kip}$$ **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$. $A = 19.7 \text{ in}^2$ for a W8 $\times$ 67 wide-flange section.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{70.44}{19.7} = 3.58 \text{ ksi} < \sigma_Y = 36 \text{ ksi}$$ **(O.K!)**

Ans:
$P_{cr} = 70.4$ kip

***17–16.**

An A992 steel W200 × 46 column of length 9 m is fixed at one end and free at its other end. Determine the allowable axial load the column can support if F.S. = 2 against buckling.

SOLUTION

Section Properties: From the table listed in the appendix, the cross-sectional area and moment of inertia about the y axis for a W200 × 46 are

$$A = 5890 \text{ mm}^2 = 5.89(10^{-3}) \text{ m}^2$$

$$I_y = 15.3(10^6) \text{ mm}^4 = 15.3(10^{-6}) \text{ m}^4$$

Critical Buckling Load: The column will buckle about the weak (y) axis. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2[200(10^9)][15.3(10^{-6})]}{[2(9)]^2} = 93.21 \text{ kN}$$

Thus, the allowable centric load is

$$P_{allow} = \frac{P_{cr}}{\text{F.S.}} = \frac{93.21}{2} = 46.61 \text{ kN} = 46.6 \text{ kN} \qquad \textbf{Ans.}$$

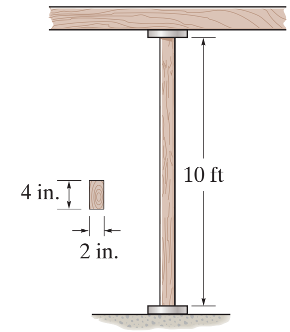
Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{93.21(10^3)}{5.89(10^{-3})} = 15.83 \text{ MPa} < \sigma_Y = 345 \text{ MPa} \qquad \textbf{(O.K.)}$$

Ans:
$P_{allow} = 46.6$ kN

17–17.

The 10-ft wooden rectangular column has the dimensions shown. Determine the critical load if the ends are assumed to be pin connected. $E_w = 1.6(10^3)$ ksi, $\sigma_Y = 5$ ksi.



SOLUTION

Section Properties:

$$A = 4(2) = 8.00 \text{ in}^2$$

$$I_x = \frac{1}{12}(2)\left(4^3\right) = 10.667 \text{ in}^4$$

$$I_y = \frac{1}{12}(4)\left(2^3\right) = 2.6667 \text{ in}^4 \text{ (Controls !)}$$

Critical Buckling Load: $K = 1$ for pin supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(1.6)(10^3)(2.6667)}{[1(10)(12)]^2}$$

$$= 2.924 \text{ kip} = 2.92 \text{ kip}$$ **Ans.**

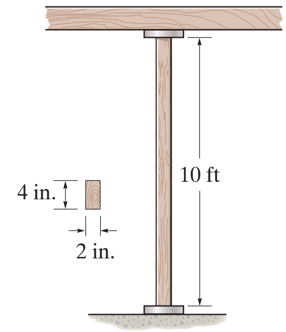
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{2.924}{8.00} = 0.3655 \text{ ksi} < \sigma_Y = 5 \text{ ksi}$$ **O.K.**

Ans:
$P_{cr} = 2.92$ kip

17–18.

The 10-ft wooden column has the dimensions shown. Determine the critical load if the bottom is fixed and the top is pinned. $E_w = 1.6(10^3)$ ksi, $\sigma_Y = 5$ ksi.



SOLUTION

Section Properties:

$$A = 4(2) = 8.00 \text{ in}^2$$

$$I_x = \frac{1}{12}(2)\left(4^3\right) = 10.667 \text{ in}^4$$

$$I_y = \frac{1}{12}(4)\left(2^3\right) = 2.6667 \text{ in}^4 \text{ (Controls!)}$$

Critical Buckling Load: $K = 0.7$ for column with one end fixed and the other end pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2 (1.6)(10^3)(2.6667)}{[0.7(10)(12)]^2}$$

$$= 5.968 \text{ kip} = 5.97 \text{ kip} \qquad \textbf{Ans.}$$

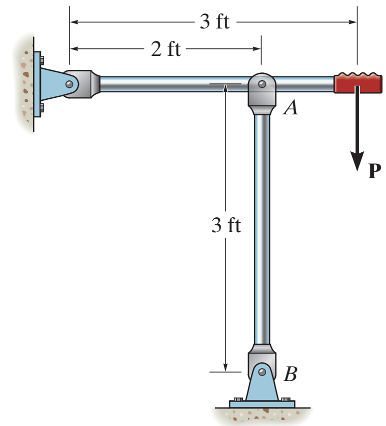
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{5.968}{8.00} = 0.7460 \text{ ksi} < \sigma_Y = 5 \text{ ksi} \qquad \textbf{O.K.}$$

Ans:
$P_{cr} = 5.97$ kip

17–19.

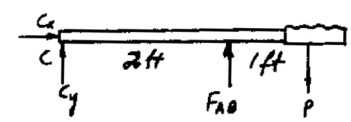
Determine the maximum force P that can be applied to the handle so that the A992 steel control rod AB does not buckle. The rod has a diameter of 1.25 in. It is pin connected at its ends.



SOLUTION

$\zeta + \Sigma M_C = 0; \qquad F_{AB}(2) - P(3) = 0$

$$P = \frac{2}{3} F_{AB} \qquad \textbf{(1)}$$



Bucking Load for Rod ***AB*****:**

$$I = \frac{\pi}{4}(0.625^4) = 0.1198 \text{ in}^4$$

$$A = \pi(0.625^2) = 1.2272 \text{ in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = P_{cr} = \frac{\pi^2(29)(10^3)(0.1198)}{[1.0(3)(12)]^2} = 26.47 \text{ kip}$$

From Eq. (1),

$$P = \frac{2}{3}(26.47) = 17.6 \text{ kip} \qquad \textbf{Ans.}$$

Check:

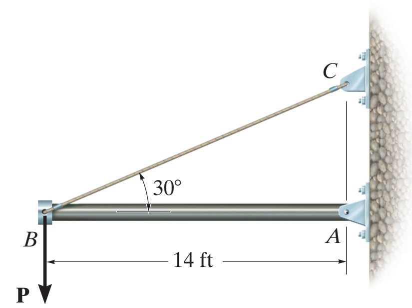
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{26.47}{1.2272} = 21.6 \text{ ksi} < \sigma_Y \qquad \textbf{OK}$$

Therefore, Euler's formula is valid.

Ans:
$P = 17.6$ kip

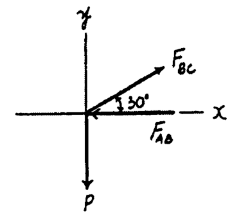
***17–20.**

The A-36 steel pipe has an outer diameter of 2 in. and a thickness of 0.5 in. If it is held in place by a guywire, determine the largest vertical force P that can be applied without causing the pipe to buckle. Assume that the ends of the pipe are pin connected.



SOLUTION

Member Forces: Use the method of joints.



$+\uparrow \Sigma F_y = 0; \quad F_{BC} \sin 30° - P = 0 \quad F_{BC} = 2.00P$

$\xrightarrow{+} \Sigma F_x = 0; \quad 2.00P \cos 30° - F_{AB} = 0 \quad F_{AB} = 1.7321P$

Section Properties:

$$A = \frac{\pi}{4}\left(2^2 - 1^2\right) = 0.750\pi \text{ in}^2$$

$$I_x = I_y = \frac{\pi}{4}\left(1^4 - 0.5^4\right) = 0.234375\pi \text{ in}^4$$

Critical Buckling Load: $K = 1$ for column with both ends pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$1.7321P = \frac{\pi^2(29)(10^3)(0.234375\pi)}{[1(14)(12)]^2}$$

$$P = 4.31 \text{ kip} \qquad \textbf{Ans.}$$

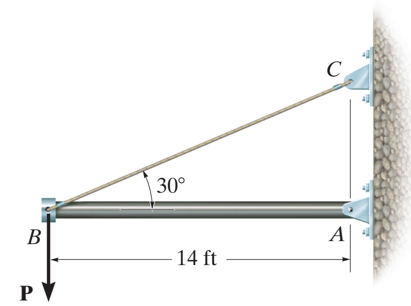
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1.7321(4.311)}{0.750\pi} = 3.169 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \textbf{(O.K!)}$$

Ans:
$P = 4.31$ kip

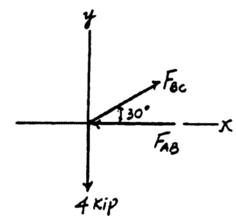
17–21.

The A-36 steel pipe has an outer diameter of 2 in. If it is held in place by a guywire, determine its required inner diameter to the nearest $\frac{1}{8}$ in., so that it can support a maximum vertical load of $P = 4$ kip without causing the pipe to buckle. Assume the ends of the pipe are pin connected.



SOLUTION

Member Forces: Use the method of joints.



$+\uparrow \Sigma F_y = 0;\quad F_{BC} \sin 30^\circ - 4 = 0 \qquad F_{BC} = 8.00 \text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0;\quad 8.00 \cos 30^\circ - F_{AB} = 0 \qquad F_{AB} = 6.928 \text{ kip}$

Section Properties:

$$A = \frac{\pi}{4}(2^2 - d_i^2) = \frac{\pi}{4}(4 - d_i^2)$$

$$I_x = I_y = \frac{\pi}{4}\left[1^4 - \left(\frac{d_i}{2}\right)^4\right] = \frac{\pi}{64}(16 - d_i^4)$$

Critical Buckling Load: $K = 1$ for column with both ends pinned. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$6.928 = \frac{\pi^2(29)(10^3)\left[\frac{\pi}{64}(16 - d_i^4)\right]}{[1(14)(12)]^2}$$

$$d_i = 1.201 \text{ in.}$$

Use $\quad d_i = 1\frac{1}{8}$ in. **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

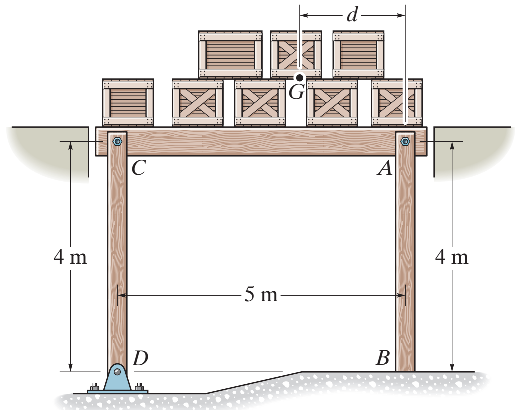
$$A = \frac{\pi}{4}\left[4 - \left(\frac{9}{8}\right)^2\right] = 2.1475 \text{ in}^2$$

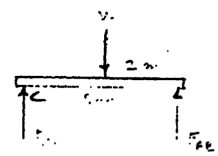
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{6.928}{2.1475} = 3.226 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \textbf{(O.K!)}$$

Ans:
Use $d_i = 1\frac{1}{8}$ in.

17–22.

The deck is supported by the two 40-mm-square columns. Column AB is pinned at A and fixed at B, whereas CD is pinned at C and D. If the deck is prevented from sidesway, determine the greatest weight of the load that can be applied without causing the deck to collapse. The center of gravity of the load is located at $d = 2$ m. Both columns are made from Douglas Fir.





SOLUTION

$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(5) - W(3) = 0$

$$F_{AB} = 0.6\,W$$

$+\uparrow \Sigma F_y = 0; \qquad F_{CD} + 0.6\,W - W = 0$

$$F_{CD} = 0.4\,W$$

Column CD:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(13.1)\left(10^9\right)\left(\frac{1}{12}\right)(0.04)^4}{(1(4))^2} = 0.4\,W$$

$$W = 4.31 \text{ kN}$$

Column AB:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(13.1)\left(10^9\right)\left(\frac{1}{12}\right)(0.04)^4}{(0.7(4))^2} = 0.6\,W$$

$$W = 5.86 \text{ kN}$$

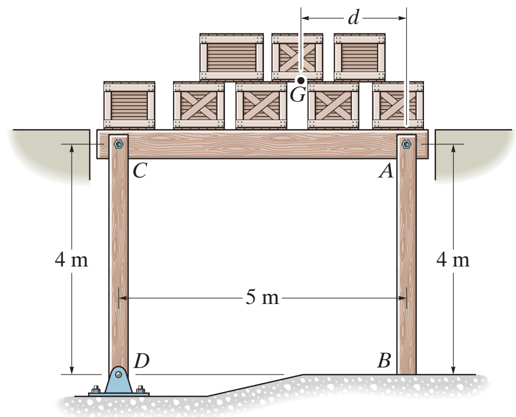
Thus,

$W = 4.31$ kN **Ans.**

Ans:
$W = 4.31$ kN

17–23.

The deck is supported by the two 40-mm-square columns. Column AB is pinned at A and fixed at B, whereas CD is pinned at C and D. If the deck is prevented from sidesway, determine the position d of the center of gravity of the load and the load's greatest magnitude without causing the deck to collapse. Both columns are made from Douglas Fir.

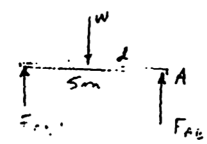


SOLUTION

Column CD:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{CD} = \frac{\pi^2(13.1)(10^9)\left(\frac{1}{12}\right)(0.04)^4}{(1.0(4))^2} = 1.7239(10^3)\ \text{N}$$



Column AB:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = \frac{\pi^2(13.1)(10^9)\left(\frac{1}{12}\right)(0.04)^4}{(0.7(4))^2} = 3.5181(10^3)\ \text{N}$$

Thus,

$$+\uparrow\Sigma F_y = 0; \qquad 1.7239(10^3) + 3.5181(10^3) - W = 0$$

$$W = 5.2420(10^3)\ \text{N} = 5.24\ \text{kN} \qquad \textbf{Ans.}$$

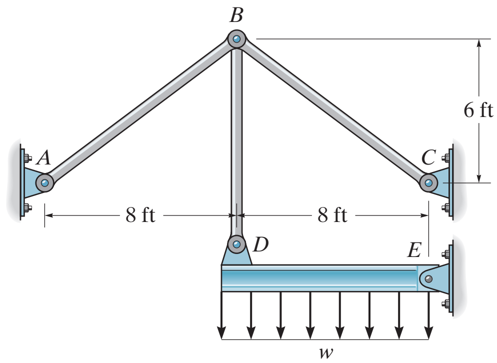
$$\circlearrowleft + \Sigma M_A = 0; \qquad 5.2420(10^3)(d) - 1.7239(10^3)(5) = 0$$

$$d = 1.64\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$W = 5.24$ kN,
$d = 1.64$ m

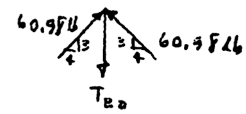
***17–24.**

The beam is supported by the three pin-connected suspender bars, each having a diameter of 0.5 in. and made from A-36 steel. Determine the greatest uniform load w that can be applied to the beam without causing AB or CB to buckle.



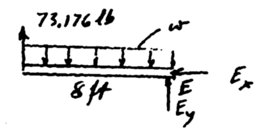
SOLUTION

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^6)\left(\frac{\pi}{4}\right)(0.25)^4}{(1.0(10)(12))^2} = 60.98 \text{ lb}$$



$$+\uparrow\Sigma F_y = 0; \qquad 2\left((60.98)\left(\frac{3}{5}\right)\right) - T_{BD} = 0$$

$$T_{BD} = 73.176 \text{ lb}$$



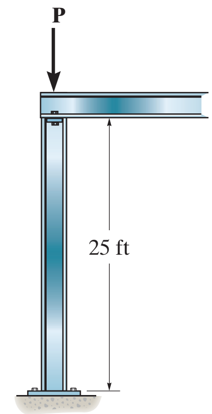
$$\circlearrowleft + \Sigma M_E = 0; \qquad -73.176(8) + w(8)(4) = 0$$

$$w = 18.3 \text{ lb/ft}$$ **Ans.**

Ans:
$w = 18.3$ lb/ft

17–25.

The W14 × 30 A992 steel column is assumed pinned at both of its ends. Determine the largest axial force P that can be applied without causing it to buckle.



SOLUTION

From the table in appendix, the cross-sectional area and the moment of inertia about weak axis (y-axis) for W14 × 30 are

$$A = 8.85 \text{ in}^2 \qquad I_y = 19.6 \text{ in}^4$$

Critical Buckling Load: Since the column is pinned at its base and top, $K = 1$. For A992 steel, $E = 29.0(10^3)$ ksi and $\sigma_Y = 50$ ksi. Here, the buckling occurs about the weak axis (y-axis).

$$P = P_{cr} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2 \left[29.0(10^3)\right](19.6)}{\left[1(25)(12)\right]^2}$$

$$= 62.33 \text{ kip} = 62.3 \text{ kip} \qquad \textbf{Ans.}$$

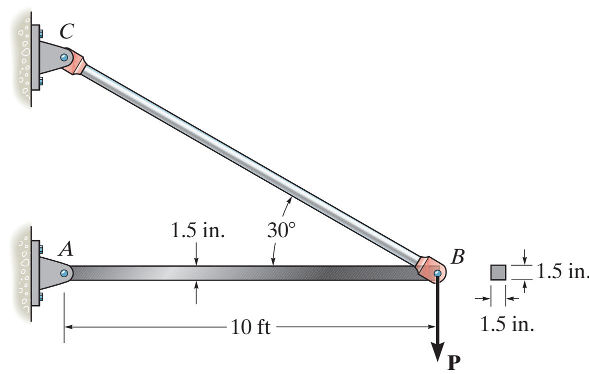
Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{62.33}{8.85} = 7.04 \text{ ksi} < \sigma_Y = 50 \text{ ksi} \qquad \textbf{O.K.}$$

Ans:
$P = 62.3$ kip

17–26.

The A992 steel bar AB has a square cross section. If it is pin connected at its ends, determine the maximum allowable load P that can be applied to the frame. Use a factor of safety with respect to buckling of 2.



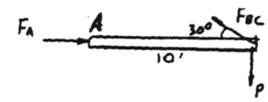
SOLUTION

$\zeta\!+\Sigma M_A = 0;\quad F_{BC}\sin 30°(10) - P(10) = 0$

$$F_{BC} = 2\,P$$

$\xrightarrow{+}\Sigma F_x = 0;\quad F_A - 2P\cos 30° = 0$

$$F_A = 1.732\,P$$



Buckling Load:

$$P_{cr} = F_A(\text{F.S.}) = 1.732\,P(2) = 3.464\,P$$

$$L = 10(12) = 120\text{ in.}$$

$$I = \frac{1}{12}(1.5)(1.5)^3 = 0.421875\text{ in}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$3.464\,P = \frac{\pi^2(29)(10^3)(0.421875)}{[(1.0)(120)]^2}$$

$P = 2.42\text{ kip}$ **Ans.**

$$P_{cr} = F_A(\text{F.S.}) = 1.732(2.42)(2) = 8.38\text{ kip}$$

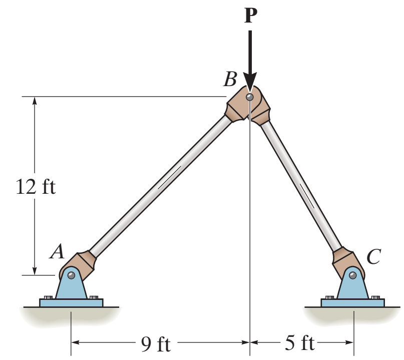
Check:

$\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{8.38}{1.5\,(1.5)} = 3.72\text{ ksi} < \sigma_Y$ **O.K.**

Ans:
$P = 2.42$ kip

17–27.

The linkage is made using two A992 steel rods, each having a circular cross section. Determine the diameter of each rod to the nearest $\frac{1}{8}$ in. that will support a load of $P = 10$ kip. Assume that the rods are pin connected at their ends. Use a factor of safety with respect to buckling of 1.8.

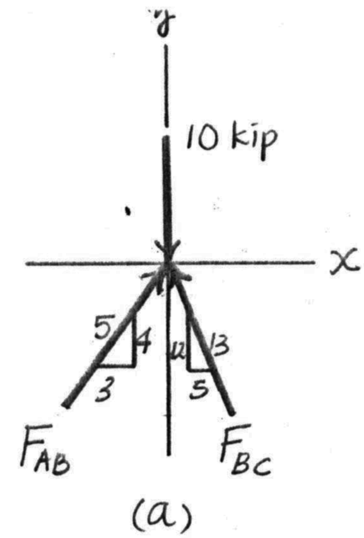


SOLUTION

Equilibrium: Referring to the FBD of joint B, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB}\left(\frac{3}{5}\right) - F_{BC}\left(\frac{5}{13}\right) = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB}\left(\frac{4}{5}\right) + F_{BC}\left(\frac{12}{13}\right) - 10 = 0 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2),

$$F_{AB} = 4.4643 \text{ kip} \qquad F_{BC} = 6.9643 \text{ kip}$$

Critical Buckling Load: For the required factor of safety,

$$F.S. = \frac{(P_{cr})_{AB}}{F_{AB}}; \quad 1.8 = \frac{(P_{cr})_{AB}}{4.4643} \quad (P_{cr})_{AB} = 8.0357 \text{ kip}$$

$$F.S. = \frac{(P_{cr})_{BC}}{F_{BC}}; \quad 1.8 = \frac{(P_{cr})_{BC}}{6.9643} \quad (P_{cr})_{BC} = 12.5357 \text{ kip}$$

Applying the Euler's formula with $L_{AB} = \sqrt{9^2 + 12^2} = 15$ ft and $L_{BC} = \sqrt{5^2 + 12^2} = 13$ ft, $E = 29(10^3)$ ksi for A992 steel, $K = 1$ and $I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$,

$$(P_{cr})_{AB} = \frac{\pi^2 E\, I_{AB}}{(K\, L_{AB})^2}; \quad 8.0357 = \frac{\pi^2\left[29(10^3)\right]\left(\pi d_{AB}^4/64\right)}{[1(15)(12)]^2}$$

$$d_{AB} = 2.075 \text{ in}$$

$$\text{Use } d_{AB} = 2\frac{1}{8} \text{ in.} \qquad \textbf{Ans.}$$

$$(P_{cr})_{BC} = \frac{\pi^2 E\, I_{BC}}{(K\, L_{BC})^2}; \quad 12.5357 = \frac{\pi^2\left[29(10^3)\right]\left(\pi d_{BC}^4/64\right)}{[1(13)(12)]^2}$$

$$d_{BC} = 2.159 \text{ in.}$$

$$\text{Use } d_{BC} = 2\frac{1}{4} \text{ in.} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula valid only if $\sigma_{cr} < \sigma_y$. For A992 steel, $\sigma_y = 50$ ksi.

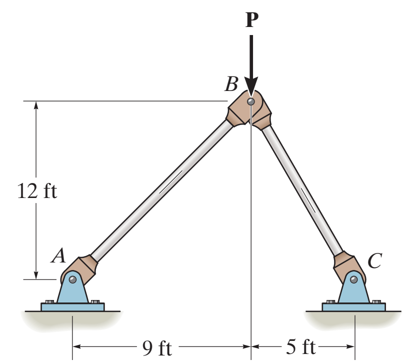
$$(\sigma_{cr})_{AB} = \frac{(P_{cr})_{AB}}{A_{AB}} = \frac{8.0357}{\frac{\pi}{4}(2.125^2)} = 2.266 \text{ ksi} < \sigma_Y \qquad \textbf{(O.K!)}$$

$$(\sigma_{cr})_{BC} = \frac{(P_{cr})_{BC}}{A_{BC}} = \frac{12.5357}{\frac{\pi}{4}(2.25^2)} = 3.153 \text{ ksi} < \sigma_Y \qquad \textbf{(O.K!)}$$

Ans:
Use $d_{AB} = 2\frac{1}{8}$ in.,
$d_{BC} = 2\frac{1}{4}$ in.

***17–28.**

The linkage is made using two A992 steel rods, each having a circular cross section. If each rod has a diameter of 2 in., determine the largest load it can support without causing any rod to buckle if the factor of safety against buckling is 1.8. Assume that the rods are pin connected at their ends.



SOLUTION

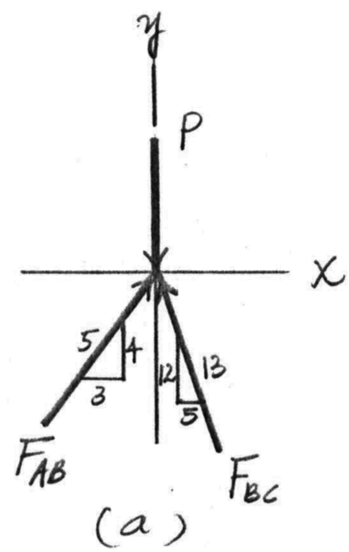
Equilibrium. Referring to the FBD of joint B, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB}\left(\frac{3}{5}\right) - F_{BC}\left(\frac{5}{13}\right) = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\left(\frac{4}{5}\right) + F_{BC}\left(\frac{12}{13}\right) - P = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$F_{BC} = \frac{39}{56}P \qquad F_{AB} = \frac{25}{26}P$$



Critical Buckling Load: For the required factor of safety,

$$F.S. = \frac{(P_{cr})_{AB}}{F_{AB}}; \quad 1.8 = \frac{(P_{cr})_{AB}}{\frac{25}{56}P} \qquad (P_{cr})_{AB} = 0.8036\text{ P}$$

$$F.S. = \frac{(P_{cr})_{BC}}{F_{BC}}; \quad 1.8 = \frac{(P_{cr})_{BC}}{\frac{39}{56}P} \qquad (P_{cr})_{BC} = 1.2536\text{ P}$$

Applying the Euler's formula with $L_{AB} = \sqrt{9^2 + 12^2} = 15$ ft and $L_{BC} = \sqrt{5^2 + 12^2} = 13$ ft, $E = 29(10^3)$ ksi for A992 steel, $K = 1$ and $I_{AB} = I_{BC} = \frac{\pi}{4}(1^4) = \frac{\pi}{4}\text{ in}^4$,

$$(P_{cr})_{AB} = \frac{\pi^2 E\, I_{AB}}{(KL_{AB})^2}; \qquad 0.8036P = \frac{\pi^2[29(10^3)]\,(\pi/4)}{[1(15)(12)]^2}$$

$$P = 8.634\text{ kip}$$

$$(P_{cr})_{BC} = \frac{\pi^2 E\, I_{BC}}{(KL_{BC})^2}; \qquad 1.2536\,P = \frac{\pi^2[29(10^3)](\pi/4)}{[1(12)(13)]^2}$$

$P = 7.369\text{ kip} = 7.37\text{ kip}$ (Control!) **Ans.**

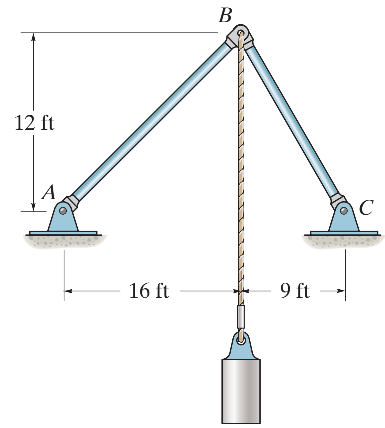
Critical Stress: Euler's formula valid only if $\sigma_{cr} < \sigma_Y$. For A992 steel, $\sigma_y = 50$ ksi.

$$(\sigma_{cr})_{BC} = \frac{(P_{cr})_{BC}}{A_{BC}} = \frac{1.2536(7.369)}{\pi\,(1^2)} = 2.94\text{ ksi} < \sigma_Y \qquad \textbf{(O.K!)}$$

Ans:
$P = 7.37$ kip

17–29.

The linkage is made using two A-36 steel rods, each having a circular cross section. Determine the diameter of each rod to the nearest $\frac{1}{8}$ in. that will support the 900-lb load. Assume that the rods are pin connected at their ends. Use a factor of safety with respect to buckling of F.S. = 1.8.



SOLUTION

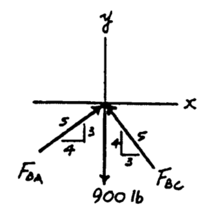
Member Forces: Use the method of joints.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{4}{5}F_{BA} - \frac{3}{5}F_{BC} = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5}F_{BA} + \frac{4}{5}F_{BC} - 900 = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$F_{BC} = 720 \text{ lb} \qquad F_{BA} = 540 \text{ lb}$$



Critical Buckling Load: $K = 1$ for column with both ends pinned. Applying *Euler's* formula to member AB,

$$P_{cr} = 1.8F_{BA} = \frac{\pi^2 EI}{(KL_{AB})^2}$$

$$1.8(540) = \frac{\pi^2(29)(10^6)\left(\frac{\pi}{64}d_{AB}^4\right)}{[1(20)(12)]^2}$$

$$d_{AB} = 1.413 \text{ in.}$$

Use $\quad d_{AB} = 1\frac{1}{2}$ in. **Ans.**

For member BC,

$$P_{cr} = 1.8F_{BC} = \frac{\pi^2 EI}{(KL_{BC})^2}$$

$$1.8(720) = \frac{\pi^2(29)(10^6)\left(\frac{\pi}{64}d_{BC}^4\right)}{[1(15)(12)]^2}$$

$$d_{BC} = 1.315 \text{ in.}$$

Use $\quad d_{BC} = 1\frac{3}{8}$ in. **Ans.**

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$(\sigma_{cr})_{AB} = \frac{P_{cr}}{A} = \frac{1.8(540)}{\frac{\pi}{4}(1.50^2)} = 550.0 \text{ psi} < \sigma_Y = 36 \text{ ksi} \qquad \textbf{(O.K!)}$$

$$(\sigma_{cr})_{BC} = \frac{P_{cr}}{A} = \frac{1.8(720)}{\frac{\pi}{4}(1.375^2)} = 872.8 \text{ psi} < \sigma_Y = 36 \text{ ksi} \qquad \textbf{(O.K!)}$$

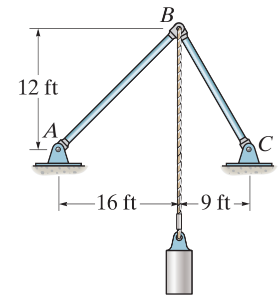
Ans:

Use $d_{AB} = 1\frac{1}{2}$ in.,

$d_{BC} = 1\frac{3}{8}$ in.

17–30.

The linkage is made using two A-36 steel rods, each having a circular cross section. If each rod has a diameter of $\frac{3}{4}$ in., determine the largest load it can support without causing any rod to buckle. Assume that the rods are pin connected at their ends.

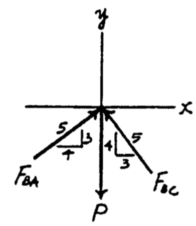


SOLUTION

Member Forces: Use the method of joints.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{4}{5} F_{BA} - \frac{3}{5} F_{BC} = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5} F_{BA} + \frac{4}{5} F_{BC} - P = 0 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$F_{BC} = 0.800P \qquad F_{BA} = 0.600P$$

Critical Buckling Load: $K = 1$ for column with both ends pinned. Assume member AB buckles. Applying *Euler's* formula,

$$P_{cr} = F_{BA} = \frac{\pi^2 EI}{(KL_{AB})^2}$$

$$0.600P = \frac{\pi^2(29)(10^6)\left[\frac{\pi}{4}(0.375^4)\right]}{[1(20)(12)]^2}$$

$$P = 128.6 \text{ lb} = 129 \text{ lb (Controls!)} \qquad \textbf{Ans.}$$

Assume member BC buckles.

$$P_{cr} = F_{BC} = \frac{\pi^2 EI}{(KL_{BC})^2}$$

$$0.800P = \frac{\pi^2(29)(10^6)\left[\frac{\pi}{4}(0.375^4)\right]}{[1(15)(12)]^2}$$

$$P = 171.5 \text{ lb}$$

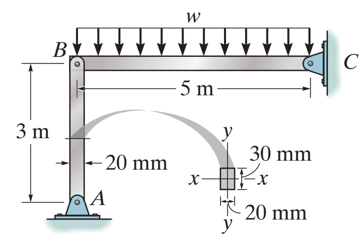
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$(\sigma_{cr})_{BC} = \frac{P_{cr}}{A} = \frac{0.8(128.6)}{\frac{\pi}{4}(0.75^2)} = 232.9 \text{ psi} < \sigma_Y = 36 \text{ ksi} \qquad \textbf{(O.K!)}$$

Ans:
$P = 129$ lb

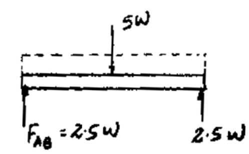
17–31.

The steel bar AB has a rectangular cross section. If it is pin connected at its ends, determine the maximum allowable intensity w of the distributed load that can be applied to BC without causing AB to buckle. Use a factor of safety with respect to buckling of 1.5. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Buckling Load:



$$P_{cr} = F_{AB}(\text{F.S.}) = 2.5\,w(1.5) = 3.75\,w$$

$$I = \frac{1}{12}(0.03)(0.02)^3 = 20\,(10^{-9})\text{ m}^4$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$3.75\,w = \frac{\pi^2(200)(10^9)(20)(10^{-9})}{[(1.0)(3)]^2}$$

$$w = 1170\text{ N/m} = 1.17\text{ kN/m}$$ **Ans.**

$$P_{cr} = 4.39\text{ kN}$$

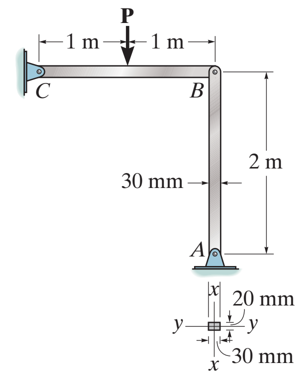
Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4.39\,(10^3)}{0.02\,(0.03)} = 7.31\text{ MPa} < \sigma_Y$$ **OK**

Ans:
$w = 1.17$ kN/m

***17–32.**

Determine if the frame can support a load of $P = 20$ kN if the factor of safety with respect to buckling of member AB is F.S. $= 3$. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Support Reactions:

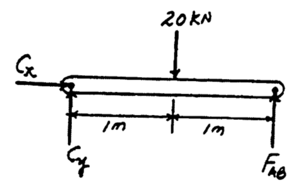
$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(2) - 20(1) = 0 \qquad F_{AB} = 10.0 \text{ kN}$$

Section Properties:

$$A = 0.02(0.03) = 0.600\left(10^{-3}\right) \text{ m}^2$$

$$I_x = \frac{1}{12}(0.02)\left(0.03^3\right) = 45.0\left(10^{-9}\right) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.03)\left(0.02^2\right) = 20.0\left(10^{-9}\right) \text{ m}^4$$



Critical Buckling Load: With respect to x–x axis, $K = 1$ (column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(200)\left(10^9\right)\left[45.0\left(10^{-9}\right)\right]}{[1(2)]^2}$$

$$= 22\,206.61 = 22.207 \text{ kN (Controls!)}$$

With respect to y–y axis, $K = 0.5$ (column with both ends fixed).

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(200)\left(10^9\right)\left[20.0\left(10^{-9}\right)\right]}{[0.5(2)]^2}$$

$$= 39\,478.42 \text{ N}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{32\,127.6}{5.00\left(10^{-3}\right)} = 6.426 \text{ MPa} < \sigma_Y = 360 \text{ MPa} \qquad \textbf{(O.K!)}$$

Factor of Safety: The required factor of safety is 3.

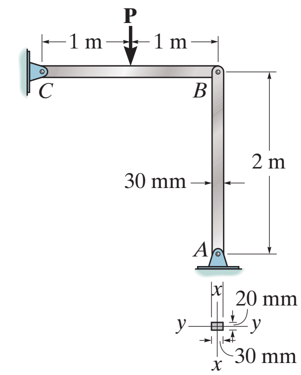
$$\text{F.S} = \frac{P_{cr}}{F_{AB}} = \frac{22.207}{10.0} = 2.22 < 3 \text{ (No Good!)}$$

Hence, the frame cannot support the load with the required F.S. **Ans.**

Ans:
No

17–33.

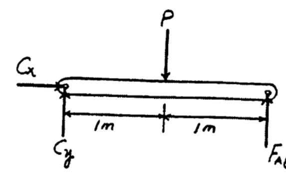
Determine the maximum allowable load P that can be applied to member BC without causing member AB to buckle. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. Use a factor of safety with respect to buckling of F.S. $= 3$. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Support Reactions:

$$\zeta + \Sigma M_C = 0; \qquad F_{AB}(2) - P(1) = 0 \qquad F_{AB} = 0.500P$$



Section Properties:

$$A = 0.02(0.03) = 0.600\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.02)\left(0.03^3\right) = 45.0\left(10^{-9}\right)\text{ m}^4$$

$$I_y = \frac{1}{12}(0.03)\left(0.02^2\right) = 20.0\left(10^{-9}\right)\text{ m}^4$$

Critical Buckling Load: With respect to x–x axis, $K = 1$ (column with both ends pinned). Applying *Euler's* formula,

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

$$3(0.500P) = \frac{\pi^2(200)\left(10^9\right)\left[45.0\left(10^{-9}\right)\right]}{[1(2)]^2}$$

$$P = 14804.4\text{ N} = 14.8\text{ kN (Controls!)}$$ **Ans.**

With respect to y–y axis, $K = 0.5$ (column with both ends fixed).

$$P_{cr} = 3F_{AB} = \frac{\pi^2 EI}{(KL)^2}$$

$$3(0.500P) = \frac{\pi^2(200)\left(10^9\right)\left[20.0\left(10^{-9}\right)\right]}{[0.5(2)]^2}$$

$$P = 26\,318.9\text{ N}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{3(0.5)(14\,804.4)}{0.600\left(10^{-3}\right)} = 37.01\text{ MPa} < \sigma_Y = 360\text{ MPa (O.K!)}$$

Ans:
$P = 14.8$ kN

17–34.

A 6061-T6 aluminum alloy solid circular rod of length 4 m is pinned at both of its ends. If it is subjected to an axial load of 15 kN and F.S. = 2 against buckling, determine the minimum required diameter of the rod to the nearest mm.

SOLUTION

Section Properties: The cross-sectional area and moment of inertia of the solid rod are

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load: The critical buckling load is

$$P_{cr} = P_{allow}(\text{F.S.}) = 15(2) = 30 \text{ kN}$$

Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)^2}$$

$$30(10^3) = \frac{\pi^2\left[68.9(10^9)\right]\left[\frac{\pi}{64}d^4\right]}{[1(4)]^2}$$

$$d = 0.06158 \text{ m} = 61.58 \text{ mm}$$

Use $d = 62$ mm. **Ans.**

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{30(10^3)}{\frac{\pi}{4}(0.062^2)} = 9.94 \text{ MPa} < \sigma_Y = 255 \text{ MPa} \qquad \textbf{(O.K.)}$$

Ans:
Use $d = 62$ mm.

17–35.

A 6061-T6 aluminum alloy solid circular rod of length 4 m is pinned at one end while fixed at the other end. If it is subjected to an axial load of 15 kN and F.S. = 2 against buckling, determine the minimum required diameter of the rod to the nearest mm.

SOLUTION

Section Properties: The cross-sectional area and moment of inertia of the solid rod are

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load: The critical buckling load is

$$P_{cr} = P_{allow}(\text{F.S.}) = 15(2) = 30\text{ kN}$$

Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)^2}$$

$$30(10^3) = \frac{\pi^2\left[68.9(10^9)\right]\left[\frac{\pi}{64}d^4\right]}{[0.7(4)]^2}$$

$$d = 0.05152\text{ m} = 51.52\text{ mm}$$

Use $d = 52$ mm. **Ans.**

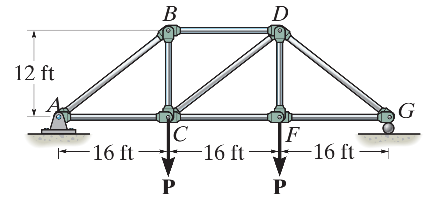
Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{30(10^3)}{\frac{\pi}{4}(0.052^2)} = 14.13\text{ MPa} < \sigma_Y = 255\text{ MPa} \qquad \text{(O.K.)}$$

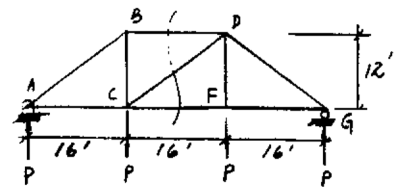
Ans:
Use $d = 52$ mm.

***17–36.**

The members of the truss are assumed to be pin connected. If member BD is an A992 steel rod of radius 2 in., determine the maximum load P that can be supported by the truss without causing the member to buckle.

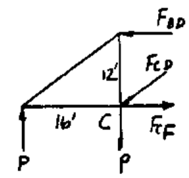


SOLUTION



$\zeta + \Sigma M_C = 0; \qquad F_{BD}(12) - P(16) = 0$

$$F_{BD} = \frac{4}{3}P$$



Buckling Load:

$$A = \pi(2^2) = 4\pi \text{ in}^2$$

$$I = \frac{\pi}{4}(2^4) = 4\pi \text{ in}^4$$

$$L = 16(12) = 192 \text{ in.}$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{BD} = \frac{4}{3}P = \frac{\pi^2(29)(10^3)(4\pi)}{[(1.0)(192)]^2}$$

$P = 73.2$ kip **Ans.**

$$P_{cr} = F_{BD} = 97.56 \text{ kip}$$

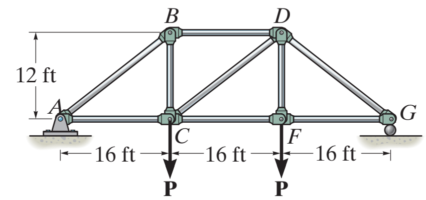
Check:

$\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{97.56}{4\pi} = 7.76 \text{ ksi} < \sigma_Y$ **OK**

Ans:
$P = 73.2$ kip

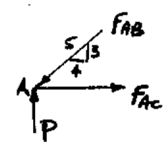
17–37.

Solve Prob. 17–36 for member AB, which has a radius of 2 in.



SOLUTION

$+\uparrow \Sigma F_y = 0;$ $\quad P - \frac{3}{5}F_{AB} = 0$



$$F_{AB} = 1.667\,P$$

Buckling Load:

$$A = \pi(2)^2 = 4\pi\ \text{in}^2$$

$$I = \frac{\pi}{4}(2)^4 = 4\pi\ \text{in}^4$$

$$L = 20(12) = 240\ \text{in.}$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(4\pi)}{(1.0(240))^2} = 62.443\ \text{kip}$$

$$P_{cr} = F_{AB} = 1.667\,P = 62.443$$

$$P = 37.5\ \text{kip}$$ **Ans.**

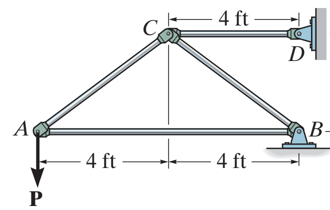
Check:

$$\sigma_{cr} = \frac{P}{A} = \frac{62.443}{4\pi} = 4.97\ \text{ksi} < \sigma_Y$$ **OK**

Ans:
$P = 37.5$ kip

17–38.

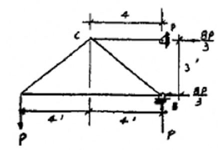
The truss is made from A992 steel bars, each of which has a circular cross section with a diameter of 1.5 in. Determine the maximum force P that can be applied without causing any of the members to buckle. The members are pin connected at their ends.



SOLUTION

$$I = \frac{\pi}{4}(0.75^4) = 0.2485 \text{ in}^4$$

$$A = \pi(0.75^2) = 1.7671 \text{ in}^2$$



Members *AB* and *BC* are in compression:

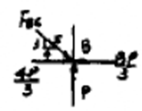
Joint *A*:

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5}F_{AC} - P = 0$$

$$F_{AC} = \frac{5P}{3}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad F_{AB} - \frac{4}{5}\left(\frac{5P}{3}\right) = 0$$

$$F_{AB} = \frac{4P}{3}$$



Joint *B*:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{4}{5}F_{BC} + \frac{4P}{3} - \frac{8P}{3} = 0$$

$$F_{BC} = \frac{5P}{3}$$

Failure of rod *AB*:

$$K = 1.0 \qquad L = 8(12) = 96 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = \frac{4P}{3} = \frac{\pi^2(29)(10^3)(0.2485)}{((1.0)(96))^2}$$

$$P = 5.79 \text{ kip (controls)}$$ **Ans.**

Check:

$$P_{cr} = F_{AB} = 7.72 \text{ kip}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{7.72}{1.7671} = 4.37 \text{ ksi} < \sigma_Y$$ **OK**

Failure of rod *BC*:

$$K = 1.0 \qquad L = 5(12) = 60 \text{ in.}$$

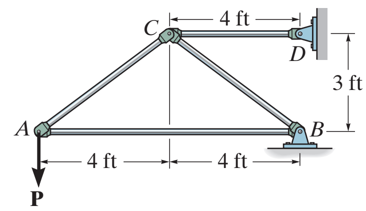
$$F_{BC} = \frac{5P}{3} = \frac{\pi^2(29)(10^3)(0.2485)}{[(1.0)(60)]^2}$$

$$P = 11.9 \text{ kip}$$

Ans:
$P = 5.79$ kip

17–39.

The truss is made from A992 steel bars, each of which has a circular cross section. If the applied load $P = 10$ kip, determine the diameter of member AB to the nearest $\frac{1}{8}$ in. that will prevent this member from buckling. The members are pin connected at their ends.



SOLUTION

Joint *A*:

$$+\uparrow \Sigma F_y = 0; \quad -10 + F_{AC}\left(\frac{3}{5}\right) = 0; \qquad F_{AC} = 16.667 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{AB} + 16.667\left(\frac{4}{5}\right) = 0; \qquad F_{AB} = 13.33 \text{ kip}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$13.33 = \frac{\pi^2(29)(10^3)\left(\frac{\pi}{4}\right)(r)^4}{(1.0(8)(12))^2}$$

$$r = 0.8599 \text{ in.}$$

$$d = 2r = 1.72 \text{ in.}$$

Use:

$$d = 1\frac{3}{4} \text{ in.}$$ **Ans.**

Check:

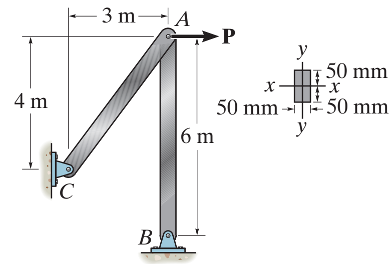
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{13.33}{\frac{\pi}{4}(1.75)^2} = 5.54 \text{ ksi} < \sigma_Y$$ **OK**

Ans:

Use $d = 1\frac{3}{4}$ in.

***17–40.**

The steel bar AB of the frame is assumed to be pin connected at its ends for y–y axis buckling. If $P = 18$ kN, determine the factor of safety with respect to buckling about the y–y axis. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

$$I_y = \frac{1}{12}(0.10)(0.05^3) = 1.04167(10^{-6})\text{ m}^4$$

Joint A:

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad \frac{3}{5}F_{AC} - 18 = 0$$

$$F_{AC} = 30\text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{4}{5}(30) = 0$$

$$F_{AB} = 24\text{ kN}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(1.04167)(10^{-6})}{[(1.0)(6)]^2} = 57116\text{ N} = 57.12\text{ kN}$$

$$\text{F.S.} = \frac{P_{cr}}{F_{AB}} = \frac{57.12}{24} = 2.38 \qquad \textbf{Ans.}$$

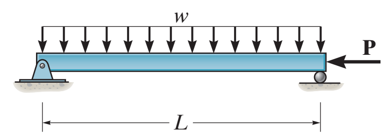
Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{57.12(10^3)}{0.1(0.05)} = 11.4\text{ MPa} < \sigma_Y \qquad \textbf{OK}$$

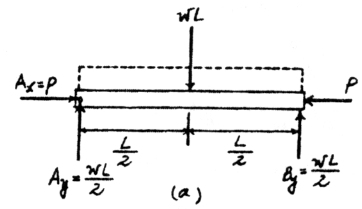
Ans:
F.S. $= 2.38$

17–41.

The ideal column has a weight w (force/length) and is subjected to the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint:* Establish the differential equation for deflection, Eq. 17–1, with the origin at the midspan. The general solution is $v = C_1 \sin kx + C_2 \cos kx + (w/(2P))x^2 - (wL/(2P))x - (wEI/P^2)$ where $k^2 = P/EI$.



SOLUTION

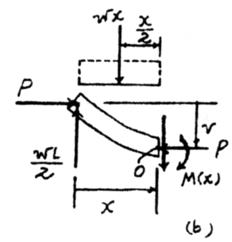


Moment Functions: FBD(b).

$$\circlearrowleft + \Sigma M_o = 0; \qquad wx\left(\frac{x}{2}\right) - M(x) - \left(\frac{wL}{2}\right)x - Pv = 0$$

$$M(x) = \frac{w}{2}\left(x^2 - Lx\right) - Pv \qquad \textbf{(1)}$$

Differential Equation of The Elastic Curve:



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w}{2}\left(x^2 - Lx\right) - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{w}{2EI}\left(x^2 - Lx\right)$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{2P}x^2 - \frac{wL}{2P}x - \frac{wEI}{P^2} \qquad \textbf{(2)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{P}x - \frac{wL}{2P} \qquad \textbf{(3)}$$

The integration constants can be determined from the boundary conditions.

Boundary Condition:

At $x = 0$, $v = 0$. From Eq. (2),

$$0 = C_2 - \frac{wEI}{P^2} \qquad\qquad C_2 = \frac{wEI}{P^2}$$

At $x = \frac{L}{2}, \frac{dv}{dx} = 0$. From Eq. (3),

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{wEI}{P^2}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{w}{P}\left(\frac{L}{2}\right) - \frac{wL}{2P}$$

$$C_1 = \frac{wEI}{P^2}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

17–41. Continued

Elastic Curve:

$$v = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{x^2}{2} - \frac{L}{2}x - \frac{EI}{P}\right]$$

However, $v = v_{\max}$ at $x = \dfrac{L}{2}$. Then,

$$v_{\max} = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L^2}{8} - \frac{EI}{P}\right]$$

$$= \frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]$$

Maximum Moment: The maximum moment occurs at $x = \dfrac{L}{2}$. From Eq. (1),

$$M_{\max} = \frac{w}{2}\left[\frac{L^2}{4} - L\left(\frac{L}{2}\right)\right] - Pv_{\max}$$

$$= -\frac{wL^2}{8} - P\left\{\frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]\right\}$$

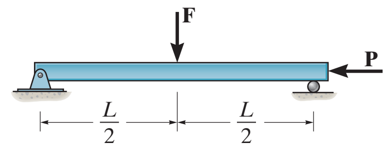
$$= -\frac{wEI}{P}\left[\sec\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right) - 1\right] \qquad \textbf{Ans.}$$

Ans:

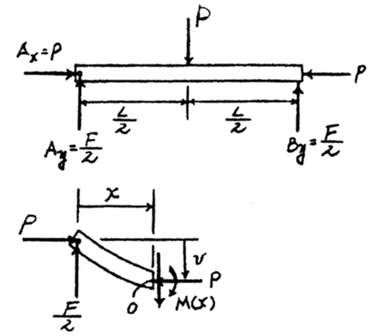
$M_{\max} = -\dfrac{wEI}{P}\left[\sec\left(\dfrac{L}{2}\sqrt{\dfrac{P}{EI}}\right) - 1\right]$

17–42.

The ideal column is subjected to the force **F** at its midpoint and the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint*: Establish the differential equation for deflection, Eq. 17–1. The general solution is $v = C_1 \sin kx + C_2 \cos kx - c^2 x/k^2$, where $c^2 = F/2EI, k^2 = P/EI$.



SOLUTION



Moment Functions: FBD(b).

$$\circlearrowleft +\Sigma M_o = 0; \qquad M(x) + \frac{F}{2}x + P(v) = 0$$

$$M(x) = -\frac{F}{2}x - Pv \tag{1}$$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -\frac{F}{2}x - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = -\frac{F}{2EI}x$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) - \frac{F}{2P}x \tag{2}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P} \tag{3}$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq. (2), $C_2 = 0$

At $x = \frac{L}{2}, \frac{dv}{dx} = 0$. From Eq. (3),

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{F}{2P}$$

$$C_1 = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

Elastic Curve:

$$v = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P}x$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - x\right]$$

17–42. Continued

However, $v = v_{max}$ at $x = \frac{L}{2}$. Then,

$$v_{max} = \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

Maximum Moment: The maximum moment occurs at $x = \frac{L}{2}$. From Eq. (1),

$$M_{max} = -\frac{F}{2}\left(\frac{L}{2}\right) - Pv_{max}$$

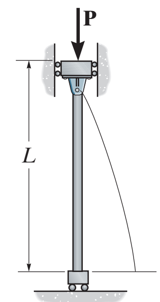
$$= -\frac{FL}{4} - P\left\{\frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]\right\}$$

$$= -\frac{F}{2}\sqrt{\frac{EI}{P}}\tan\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right)$$ **Ans.**

Ans:
$M_{max} = -\frac{F}{2}\sqrt{\frac{EI}{P}}\tan\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right)$

17–43.

The column with constant EI has the end constraints shown. Determine the critical load for the column.



SOLUTION

Moment Function. Referring to the free-body diagram of the upper part of the deflected column, Fig. *a*,

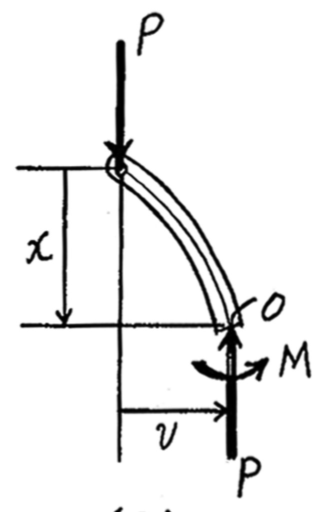
$\circlearrowleft + \Sigma M_O = 0; \qquad M + Pv = 0 \qquad M = -Pv$

Differential Equation of the Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$



(a)

The solution is in the form of

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) \qquad (1)$$

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) \qquad (2)$$

Boundary Conditions. At $x = 0$, $v = 0$. Then Eq. (1) gives

$$0 = 0 + C_2 \qquad C_2 = 0$$

At $x = L$, $\frac{dv}{dx} = 0$. Then Eq. (2) gives

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}L\right)$$

$C_1 = 0$ is the trivial solution, where $v = 0$. This means that the column will remain straight and buckling will not occur regardless of the load P. Another possible solution is

$$\cos\left(\sqrt{\frac{P}{EI}}L\right) = 0$$

$$\sqrt{\frac{P}{EI}}L = \frac{n\pi}{2} \qquad n = 1, 3, 5$$

The smallest critical load occurs when $n = 1$, then

$$\sqrt{\frac{P_{cr}}{EI}}L = \frac{\pi}{2}$$

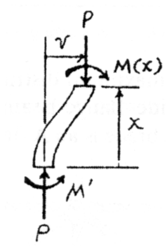
$$P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad \textbf{Ans.}$$

Ans:
$P_{cr} = \frac{\pi^2 EI}{4L^2}$

***17–44.**

Consider an ideal column as in Fig. 17–10*c*, having both ends fixed. Show that the critical load on the column is $P_{cr} = 4\pi^2 EI/L^2$. *Hint*: Due to the vertical deflection of the top of the column, a constant moment $\mathbf{M}'$ will be developed at the supports. Show that $d^2v/dx^2 + (P/EI)v = M'/EI$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + M'/P$.

SOLUTION



Moment Functions:

$M(x) = M' - Pv$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M' - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{M'}{EI} \qquad \textbf{(Q.E.D.)}$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{M'}{P} \qquad \textbf{(1)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) \qquad \textbf{(2)}$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq. (1), $C_2 = -\frac{M'}{P}$

At $x = 0, \frac{dv}{dx} = 0$. From Eq. (2), $C_1 = 0$

Elastic Curve:

$$v = \frac{M'}{P}\left[1 - \cos\left(\sqrt{\frac{P}{EI}}x\right)\right]$$

and

$$\frac{dv}{dx} = \frac{M'}{P}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

However, due to symmetry $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then,

$$\sin\left[\sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right)\right] = 0 \qquad \text{or} \qquad \sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right) = n\pi \qquad \text{where } n = 1, 2, 3, \ldots$$

The smallest critical load occurs when $n = 1$.

$$P_{cr} = \frac{4\pi^2 EI}{L^2} \qquad \textbf{(Q.E.D.)}$$

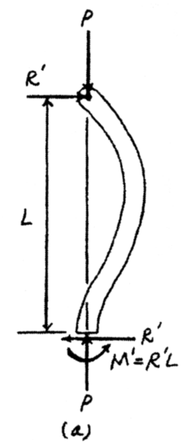
Ans:
N/A

17–45.

Consider an ideal column as in Fig. 17–10*d*, having one end fixed and the other pinned. Show that the critical load on the column is $P_{cr} = 20.19EI/L^2$. *Hint*: Due to the vertical deflection at the top of the column, a constant moment $\mathbf{M}'$ will be developed at the fixed support and horizontal reactive forces $\mathbf{R}'$ will be developed at both supports. Show that $d^2v/dx^2 + (P/EI)v = (R'/EI)(L - x)$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + (R'/P)(L - x)$. After application of the boundary conditions show that $\tan(\sqrt{P/EI}L) = \sqrt{P/EI}\,L$. Solve numerically for the smallest nonzero root.

SOLUTION

Equilibrium. FBD(a).



Moment Functions: FBD(b).

$M(x) = R'(L - x) - Pv$

Differential Equation of The Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = R'(L - x) - Pv$$

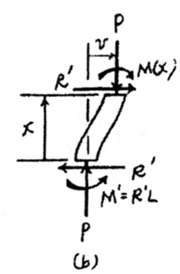
$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{R'}{EI}(L - x) \qquad \textbf{(Q.E.D.)}$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{R'}{P}(L - x) \qquad \textbf{(1)}$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{R'}{P} \qquad \textbf{(2)}$$



The integration constants can be determined from the boundary conditions.

Boundary Conditions:

At $x = 0, v = 0$. From Eq. (1), $C_2 = -\dfrac{R'L}{P}$

At $x = 0, \dfrac{dv}{dx} = 0$. From Eq. (2), $C_1 = \dfrac{R'}{P}\sqrt{\dfrac{EI}{P}}$

Elastic Curve:

$$v = \frac{R'}{P}\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{R'L}{P}\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{R'}{P}(L - x)$$

$$= \frac{R'}{P}\left[\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - L\cos\left(\sqrt{\frac{P}{EI}}x\right) + (L - x)\right]$$

17–45. Continued

However, $v = 0$ at $x = L$. Then,

$$0 = \sqrt{\frac{EI}{P}} \sin\left(\sqrt{\frac{P}{EI}}L\right) - L\cos\left(\sqrt{\frac{P}{EI}}L\right)$$

$$\tan\left(\sqrt{\frac{P}{EI}}L\right) = \sqrt{\frac{P}{EI}}L \qquad \textbf{(Q.E.D.)}$$

By trial and error and choosing the smallest root, we have

$$\sqrt{\frac{P}{EI}}L = 4.49341$$

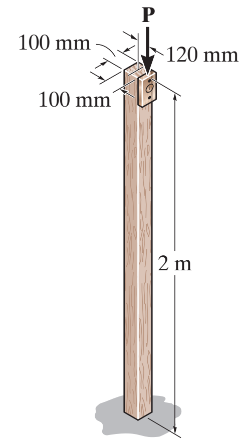
Then,

$$P_{cr} = \frac{20.19EI}{L^2} \qquad \textbf{(Q.E.D.)}$$

Ans:
N/A

17–46.

The wood column is fixed at its base and free at its top. Determine the load P that can be applied to the edge of the column without causing the column to fail either by buckling or by yielding. $E_w = 12$ GPa, $\sigma_Y = 55$ MPa.



SOLUTION

Section Properties:

$$A = 0.1(0.1) = 0.01\ \text{m}^2 \qquad I = \frac{1}{12}(0.1)(0.1)^3 = 8.333(10^{-6})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{8.333(10^{-6})}{0.01}} = 0.02887\ \text{m}$$

Buckling:

$$P_{\text{cr}} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(12)(10^9)(8.333)(10^{-6})}{[2.0(2)]^2} = 61.7\ \text{kN}$$

Check: $\sigma_{\text{cr}} = \dfrac{P_{\text{cr}}}{A} = \dfrac{61.7(10^3)}{0.01} = 6.17\ \text{MPa} < \sigma_Y$ **OK**

Yielding:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{ec}{r^2} = \frac{0.12(0.05)}{(0.02887)^2} = 7.20$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(2)}{2(0.02887)}\sqrt{\frac{P}{12(10^9)(0.01)}} = 0.006324\sqrt{P}$$

$$55(10^6)(0.01) = P[1 + 7.20\sec(0.006324\sqrt{P})]$$

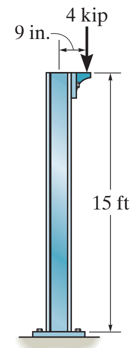
By trial and error:

$P = 31400\ \text{N} = 31.4\ \text{kN}$ controls **Ans.**

Ans:
$P = 31.4$ kN

17–47.

The W10 × 12 structural A-36 steel column is used to support a load of 4 kip. If the column is fixed at the base and free at the top, determine the sidesway deflection at the top of the column due to the loading.



SOLUTION

Section Properties for W10 × 12

$A = 3.54 \text{ in}^2 \qquad I_x = 53.8 \text{ in}^4 \qquad r_x = 3.90 \text{ in.} \qquad d = 9.89 \text{ in.}$

Maximum Deflection:

$$v_{\max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right] \qquad K = 2.0$$

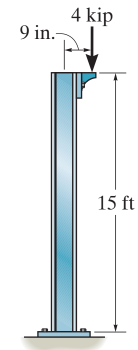
$$\sqrt{\frac{P}{EI}}\frac{KL}{2} = \sqrt{\frac{4}{29(10^3)53.8}}\left(\frac{2.0(15)(12)}{2}\right) = 0.2882$$

$$v_{\max} = 9[\sec(0.2882) - 1] = 0.387 \text{ in.}$$ **Ans.**

Ans:
$v_{\max} = 0.387$ in.

***17–48.**

The W10 × 12 structural A-36 steel column is used to support a load of 4 kip. If the column is fixed at its base and free at its top, determine the maximum stress in the column due to this loading.



SOLUTION

Section Properties for W10 × 12:

$A = 3.54\text{ in}^2 \qquad I_x = 53.8\text{ in}^4 \qquad r_x = 3.90\text{ in.} \qquad d = 9.89\text{ in.}$

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{P}{A} = \frac{4}{3.54} = 1.13\text{ ksi}$$

$$\frac{e\,c}{r^2} = \frac{9\left(\frac{9.89}{2}\right)}{3.90^2} = 2.926$$

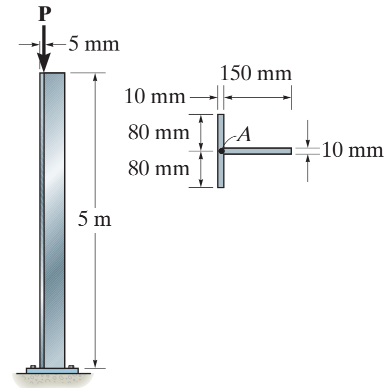
$$\frac{KL}{2\,r}\sqrt{\frac{P}{EA}} = \frac{2.0(15)(12)}{2(3.90)}\sqrt{\frac{4}{29\left(10^3\right)(3.54)}} = 0.2881$$

$$\sigma_{\max} = 1.13[1 + 2.926\sec(0.2881)] = 4.57\text{ ksi} < \sigma_Y \quad \textbf{OK} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{\max} = 4.57\text{ ksi}$

17–49.

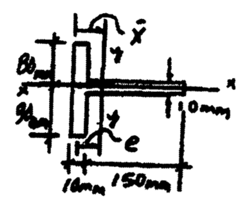
The aluminum column is fixed at the bottom and free at the top. Determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding. $E_{al} = 70$ GPa, $\sigma_Y = 95$ MPa.



SOLUTION

$$\bar{x} = \frac{(0.005)(0.16)(0.01) + (0.085)(0.15)(0.01)}{0.16(0.01) + 0.15(0.01)} = 0.04371 \text{ m}$$

$$I_y = \frac{1}{12}(0.16)(0.01)^3 + (0.16)(0.01)(0.04371 - 0.005)^2 + \frac{1}{12}(0.01)(0.15)^3$$
$$+ (0.15)(0.01)(0.085 - 0.04371)^2 = 7.7807\left(10^{-6}\right) \text{m}^4$$



$$I_x = \frac{1}{12}(0.01)\left(0.16^3\right) + \frac{1}{12}(0.15)\left(0.01^3\right) = 3.42583\left(10^{-6}\right) \text{m}^4$$

$$A = (0.16)(0.01) + (0.15)(0.01) = 3.1\left(10^{-3}\right) \text{m}^2$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{7.7807\left(10^{-6}\right)}{3.1\left(10^{-3}\right)}} = 0.0501 \text{ m}$$

Buckling about x–x axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(70)\left(10^9\right)(3.42583)\left(10^{-6}\right)}{[(2.0)(5)]^2} = 23668 \text{ N}$$

$$P_{allow} = \frac{P_{cr}}{3} = 7.89 \text{ kN} \qquad \textbf{(controls)} \qquad \textbf{Ans.}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{23668 \text{ N}}{3.1\left(10^{-3}\right)} = 7.63 \text{ MPa} < \sigma_Y$ **OK**

Yielding about y–y axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}}\right)$$

$$\frac{ec}{r^2} = \frac{(0.03871)(0.04371)}{0.0501^2} = 0.6741$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2.0(5)}{2(0.0501)}\sqrt{\frac{P}{70\left(10^8\right)(3.1)\left(10^{-3}\right)}} = 6.7749\left(10^{-3}\right)\sqrt{P}$$

$$95\left(10^6\right)(3.1)\left(10^{-3}\right) = P[1 + 0.6741 \sec\left(6.7749\left(10^{-3}\right)\sqrt{P}\right]$$

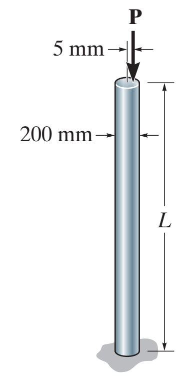
By trial and error:

$$P = 45.61 \text{ kN} \qquad P_{allow} = \frac{45.61}{3} = 15.2 \text{ kN}$$

Ans:
$P_{allow} = 7.89$ kN

17–50.

The aluminum rod is fixed at its base and free at its top. If the eccentric load $P = 200\text{ kN}$ is applied, determine the greatest allowable length L of the rod so that it does not buckle or yield. $E_{al} = 72\text{ GPa}$, $\sigma_Y = 410\text{ MPa}$.



SOLUTION

Section Properties:

$$A = \pi(0.1^2) = 0.031416\text{ m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6})\text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05\text{ m}$$

Yielding:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{P}{A} = \frac{200(10^3)}{0.031416} = 6.3662(10^4)\text{ Pa}$$

$$\frac{ec}{r^2} = \frac{0.005(0.1)}{(0.05)^2} = 0.2$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2.0(L)}{2(0.05)}\sqrt{\frac{200(10^3)}{72(10^9)(0.031416)}} = 0.188063L$$

$$410(10^4) = 6.3662(10^6)[1 + 0.2\sec(0.188063\text{ L})]$$

$$L = 8.34\text{ m} \qquad \text{(controls)}$$ **Ans.**

Buckling about *x–x* axis:

$$\frac{P}{A} = 6.36\text{ MPa} < \sigma_Y \quad \text{Euler formula is valid.}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

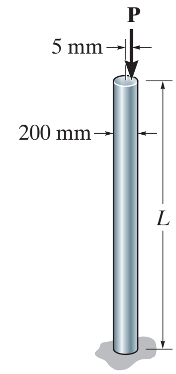
$$200(10^3) = \frac{\pi^2(72)(10^9)(78.54)(10^{-4})}{[(2.0)(L)]^2}$$

$$L = 8.34\text{ m}$$

Ans:
$L = 8.34\text{ m}$

17–51.

The aluminum rod is fixed at its base and free and at its top. If the length of the rod is $L = 2$ m, determine the greatest allowable load P that can be applied so that the rod does not buckle or yield. $E_{al} = 72$ GPa, $\sigma_Y = 410$ MPa.



SOLUTION

Section Properties:

$$A = \pi(0.1^2) = 0.031416 \text{ m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6}) \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05 \text{ m}$$

Yielding:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2} \sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{ec}{r^2} = \frac{(0.005)(0.1)}{0.05^2} = 0.2$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2(2)}{2(0.05)}\sqrt{\frac{P}{72(10^9)(0.031416)}} = 0.8410\,(10^{-3})\sqrt{P}$$

$$410(10^4)(0.031416) = P[(1 + 0.2 \sec (0.8410(10^{-3})\sqrt{P}\,)]$$

By Trial and Error:

$P = 3.20$ MN (controls) **Ans.**

Buckling:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(72)(10^9)(78.54)(10^{-6})}{[(2.0)(2)]^2} = 3488 \text{ kN}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{3488(10^3)}{0.031416} = 111 \text{ MPa} < \sigma_Y$ OK

Maximum Deflection:

$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

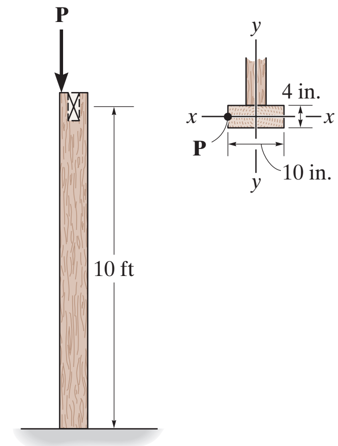
$$\sqrt{\frac{P}{EI}}\frac{KL}{2} = \sqrt{\frac{3.20(10^6)}{72(10^9)(78.54)(10^{-6})}}\left(\frac{2.0(2)}{2}\right) = 1.5045$$

$v_{max} = 5[\sec (1.5045) - 1] = 70.5$ mm **Ans.**

Ans:
$P = 3.20$ MN,
$v_{max} = 70.5$ mm

***17–52.**

Assume that the wood column is pin connected at its base and top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



SOLUTION

Section Properties:

$$A = 10(4) = 40 \text{ in}^2 \quad I_y = \frac{1}{12}(4)(10^3) = 333.33 \text{ in}^4 \quad I_x = \frac{1}{12}(10)(4^3) = 53.33 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{333.33}{40}} = 2.8868 \text{ in.}$$

Buckling about $y-y$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(333.33)}{[(2)(10)(12)]^2} = 102.8 \text{ kip}$$

Buckling about $x-x$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(53.33)}{[(1)(10)(12)]^2} = 65.8 \text{ kip (controls)} \qquad \textbf{Ans.}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{65.8}{40} = 1.64 \text{ ksi} < \sigma_Y$ O.K.

Yielding about $y - y$ axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}}\right)$$

$$\frac{ec}{r^2} = \frac{5(5)}{2.8868^2} = 3.0$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{(1)(10)(12)}{2(2.8868)}\sqrt{\frac{P}{1.8(10^3)(40)}} = 0.077460\sqrt{P}$$

$$8(40) = P[1 + 3.0\sec(0.077460\sqrt{P})]$$

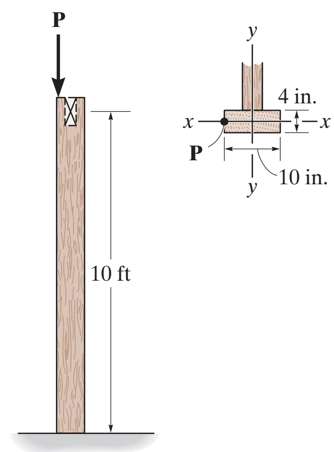
By trial and error:

$P = 67.6$ kip

Ans:
$P = 65.8$ kip

17–53.

Assume that the wood column is pinned top and bottom for movement about the $x-y$ axis, and fixed at the bottom and free at the top for movement about the $y-y$ axis. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



SOLUTION

Section Properties:

$$A = 10(4) = 40 \text{ in}^2 \quad I_y = \frac{1}{12}(4)(10^3) = 333.33 \text{ in}^4 \quad I_x = \frac{1}{12}(10)(4^3) = 53.33 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{333.33}{40}} = 2.8868 \text{ in.}$$

Buckling about $x-x$ axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(53.33)}{[(1)(10)(12)]^2} = 65.8 \text{ kip}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{65.8}{40} = 1.64 \text{ ksi} < \sigma_Y$ O.K.

Yielding about $y-y$ axis:

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right)$$

$$\frac{ec}{r^2} = \frac{5(5)}{2.8868^2} = 3.0$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{(2)(10)(12)}{2(2.8868)}\sqrt{\frac{P}{1.8(10^3)(40)}} = 0.15492\sqrt{P}$$

$$8(40) = P[1 + 3.0 \sec(0.15492\sqrt{P})]$$

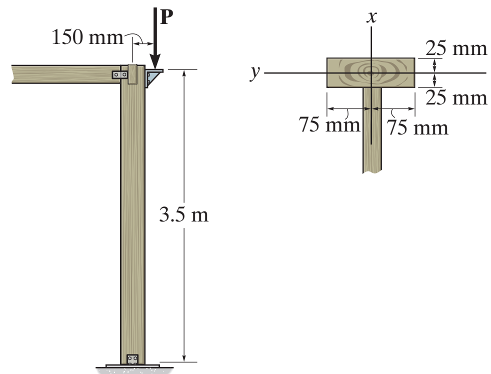
By trial and error:

$P = 45.7$ kip (controls) **Ans.**

Ans:
$P = 45.7$ kip

17–54.

The wood column is pinned at its base and top. If the eccentric force $P = 10\text{ kN}$ is applied to the column, investigate whether the column is adequate to support this loading without buckling or yielding. Take $E = 10\text{ GPa}$ and $\sigma_Y = 15\text{ MPa}$.



SOLUTION

Section Properties:

$$A = 0.05(0.15) = 7.5\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.05)\left(0.15^3\right) = 14.0625\left(10^{-6}\right)\text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{14.0625\left(10^{-6}\right)}{7.5\left(10^{-3}\right)}} = 0.04330\text{ m}$$

$$I_y = \frac{1}{12}(0.15)\left(0.05^3\right) = 1.5625\left(10^{-6}\right)\text{m}^4$$

$e = 0.15\text{ m}$ $\qquad$ $c = 0.075\text{ m}$

For a column that is pinned at both ends, $K = 1$. Then,

$$(KL)_x = (KL)_y = 1(3.5) = 3.5\text{ m}$$

Buckling About the Weak Axis: Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[10\left(10^9\right)\right]\left[1.5625\left(10^{-6}\right)\right]}{3.5^2} = 12.59\text{ kN}$$

Euler's formula is valid if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)} = 1.68\text{ MPa} < \sigma_Y = 15\text{ MPa}$$ **O.K.**

Since $P_{\text{cr}} > P = 10\text{ kN}$, the column *will not buckle*.

Yielding About Strong Axis: Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$= \frac{10\left(10^3\right)}{7.5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.075)}{0.04330^2}\sec\left[\frac{3.5}{2(0.04330)}\sqrt{\frac{10\left(10^3\right)}{10\left(10^9\right)\left[7.5\left(10^{-3}\right)\right]}}\right]\right]$$

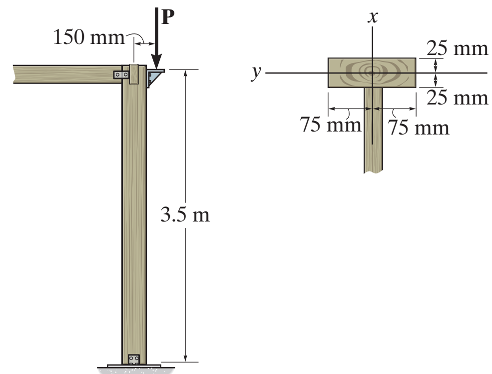
$$= 10.29\text{ MPa}$$

Since $\sigma_{\max} < \sigma_Y = 15\text{ MPa}$, the column *will not yield.* **Ans.**

Ans:
Yes

17–55.

The wood column is pinned at its base and top. Determine the maximum eccentric force P the column can support without causing it to either buckle or yield. Take $E = 10$ GPa and $\sigma_Y = 15$ MPa.



SOLUTION

Section Properties:

$$A = 0.05(0.15) = 7.5\left(10^{-3}\right)\text{ m}^2$$

$$I_x = \frac{1}{12}(0.05)\left(0.15^3\right) = 14.0625\left(10^{-6}\right)\text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{14.0625\left(10^{-6}\right)}{7.5\left(10^{-3}\right)}} = 0.04330\text{ m}$$

$$I_y = \frac{1}{12}(0.15)\left(0.05^3\right) = 1.5625\left(10^{-6}\right)\text{m}^4$$

$e = 0.15$ m $\qquad c = 0.075$ m

For a column that is pinned at both ends, $K = 1$. Then,

$$(KL)_x = (KL)_y = 1(3.5) = 3.5\text{ m}$$

Buckling About the Weak Axis: Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[10\left(10^9\right)\right]\left[1.5625\left(10^{-6}\right)\right]}{3.5^2} = 12.59\text{ kN} = 12.6\text{ kN} \qquad \textbf{Ans.}$$

Euler's formula is valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)} = 1.68\text{ MPa} < \sigma_Y = 15\text{ MPa} \qquad \textbf{O.K.}$$

Yielding About Strong Axis: Applying the secant formula with $P = P_{cr} = 12.59$ kN,

$$\sigma_{max} = \frac{P}{A}\left[\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]\right]$$

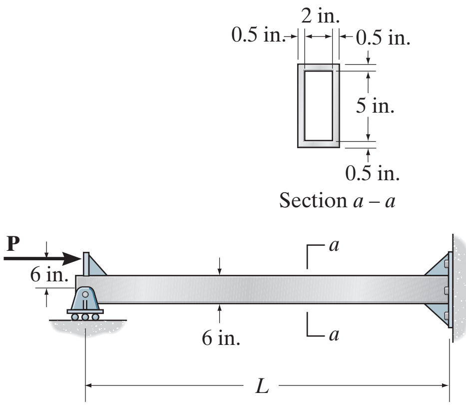
$$= \frac{12.59\left(10^3\right)}{7.5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.075)}{0.04330^2}\sec\left[\frac{3.5}{2(0.04330)}\sqrt{\frac{12.59\left(10^3\right)}{10\left(10^9\right)\left[7.5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 13.31\text{ MPa} < \sigma_Y = 15\text{ MPa} \qquad \textbf{O.K.}$$

Ans:
$P_{cr} = 12.6$ kN

***17–56.**

The A992 steel rectangular hollow section column is pinned at both ends. If it has a length of $L = 14$ ft, determine the maximum allowable eccentric force **P** it can support without causing it to either buckle or yield.



SOLUTION

Section Properties.

$$A = 3(6) - 2(5) = 8 \text{ in}^2$$

$$I_x = \frac{1}{12}(3)(6^3) - \frac{1}{12}(2)(5^3) = 33.167 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{33.167}{8}} = 2.0361 \text{ in.}$$

$$I_y = \frac{1}{12}(6)(3^3) - \frac{1}{12}(5)(2^3) = 10.167 \text{ in}^4$$

For a column that is pinned at both ends, $K = 1$. Then,

$$(KL)_x = (KL)_y = 1(14)(12) = 168 \text{ in.}$$

Buckling About the Weak Axis. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 E I_y}{(KL)_y{}^2} = \frac{\pi^2[29(10^3)](10.167)}{168^2} = 103.10 \text{ kip}$$

Euler's formula is valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{103.10}{8} = 12.89 \text{ ksi} < \sigma_Y = 50 \text{ ksi} \qquad \textbf{O.K.}$$

Yielding About Strong Axis. Applying the secant formula,

$$\sigma_{max} = \frac{P_{max}}{A}\left[1 + \frac{ec}{r_x{}^2} \sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P_{max}}{EA}}\right]\right]$$

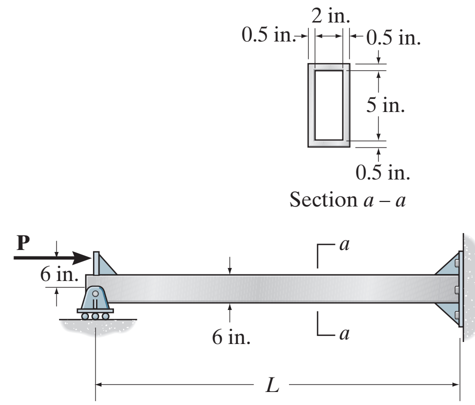
$$50 = \frac{P_{max}}{8}\left[1 + \frac{6(3)}{2.0361^2} \sec\left[\frac{168}{2(2.0361)}\sqrt{\frac{P_{max}}{29(10^3)(8)}}\right]\right]$$

$$P_{max} = 61.174 = 61.2 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$P_{max} = 61.2$ kip

17–57.

The A992 steel rectangular hollow section column is pinned at both ends. If it is subjected to the eccentric force $P = 45$ kip, determine its maximum allowable length L without causing it to either buckle or yield.



SOLUTION

Section Properties.

$$A = 3(6) - 2(5) = 8 \text{ in}^2$$

$$I_x = \frac{1}{12}(3)(6^3) - \frac{1}{2}(2)(5^3) = 33.167 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{33.167}{8}} = 2.0361 \text{ in.}$$

$$I_y = \frac{1}{12}(6)(3^3) - \frac{1}{12}(5)(2^3) = 10.167 \text{ in}^4$$

For a column that is pinned at both ends, $K = 1$. Then,

$$(KL)_x = (KL)_y = 1L$$

Buckling About the Weak Axis. The critical load is

$$P_{\text{cr}} = 45 \text{ kip}$$

Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)_y^2}$$

$$45 = \frac{\pi^2[29(10^3)](10.167)}{L^2}$$

$$L = 254.3 \text{ in.} = 21.2 \text{ ft} \qquad \text{(controls)} \qquad \textbf{Ans.}$$

Euler's formula is valid if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{45}{8} = 5.625 \text{ ksi} < \sigma_Y = 50 \text{ ksi} \qquad \textbf{O.K.}$$

Yielding About Strong Axis. Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x^{\,2}} \sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

$$50 = \frac{45}{8}\left[1 + \frac{6(3)}{2.0361^2} \sec\left[\frac{L}{2(2.0361)}\sqrt{\frac{45}{29(10^3)(8)}}\right]\right]$$

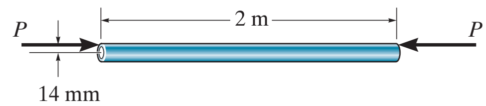
$$\sec\left[3.420(10^{-3})L\right] = 1.817$$

$$L = 288.89 \text{ in.} = 24.07 \text{ ft}$$

Ans:
$L = 21.2$ ft

17–58.

The tube is made of copper and has an outer diameter of 35 mm and a wall thickness of 7 mm. Determine the eccentric load P that it can support without failure. The tube is pin supported at its ends. $E_{cu} = 120$ GPa, $\sigma_Y = 750$ MPa.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.035^2 - 0.021^2) = 0.61575(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.0175^4 - 0.0105^4) = 64.1152(10^{-9})\ \text{m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{64.1152(10^{-9})}{0.61575(10^{-3})}} = 0.010204\ \text{m}$$

For a column pinned at both ends, $K = 1$. Then $KL = 1(2) = 2$ m.

Buckling: Applying *Euler's* formula,

$$P_{max} = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(120)(10^9)\left[64.1152(10^{-9})\right]}{2^2} = 18983.7\ \text{N} = 18.98\ \text{kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_{\gamma}$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{18983.7}{0.61575(10^{-3})} = 30.83\ \text{MPa} < \sigma_{\gamma} = 750\ \text{MPa} \qquad \text{O.K.}$$

Yielding: Applying the secant formula,

$$\sigma_{max} = \frac{P_{max}}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P_{max}}{EA}}\right)\right]$$

$$750\left(10^6\right) = \frac{P_{max}}{0.61575(10^{-3})}\left[1 + \frac{0.014(0.0175)}{0.010204^2}\sec\left(\frac{2}{2(0.010204)}\sqrt{\frac{P_{max}}{120(10^9)[0.61575(10^{-3})]}}\right)\right]$$

$$750\left(10^6\right) = \frac{P_{max}}{0.61575(10^{-3})}\left(1 + 2.35294\sec 0.0114006\sqrt{P_{max}}\right)$$

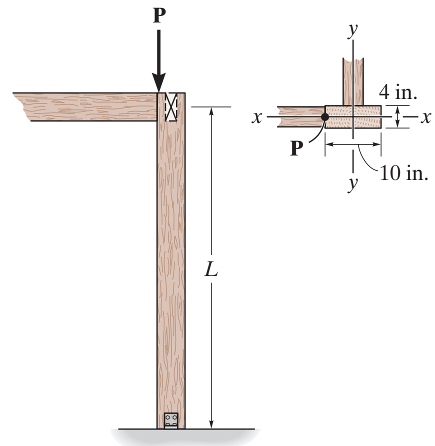
Solving by trial and error,

$$P_{max} = 16\,884\ \text{N} = 16.9\ \text{kN} \quad (\textit{Controls!}) \qquad \textbf{Ans.}$$

Ans:
$P_{max} = 16.9$ kN

17–59.

The wood column is pinned at its base and top. If $L = 5$ ft, determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



SOLUTION

Section Properties:

$$A = 10(4) = 40 \text{ in}^2 \quad I_y = \frac{1}{12}(4)(10^3) = 333.33 \text{ in}^4 \quad I_x = \frac{1}{2}(10)(4^3) = 53.33 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{333.33}{40}} = 2.8868 \text{ in.}$$

Buckling about x–x axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(53.33)}{[1(5)(12)]^2} = 263 \text{ kip}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{263}{40} = 6.58 \text{ ksi} < \sigma_Y$ **OK**

Yielding about y–y axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{ec}{r^2} = \frac{5(5)}{2.8868^2} = 3.0$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{1(5)(12)}{2(2.8868)}\sqrt{\frac{P}{1.8(10^3)(40)}} = 0.038729\sqrt{P}$$

$$8(40) = P[1 + 3.0\sec(0.038729\sqrt{P})]$$

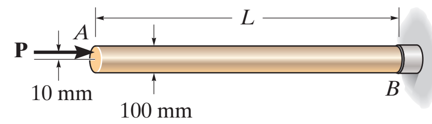
By trial and error:

$P = 76.6$ kip **(controls)** **Ans.**

Ans:
$P = 76.6$ kip

***17–60.**

The brass rod is fixed at one end and free at the other end. If the eccentric load $P = 200$ kN is applied, determine the greatest allowable length L of the rod so that it does not buckle or yield. $E_{br} = 101$ GPa, $\sigma_Y = 69$ MPa.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}\left(0.1^2\right) = 2.50\left(10^{-3}\right)\pi \text{ m}^2$$

$$I = \frac{\pi}{4}\left(0.05^4\right) = 1.5625\left(10^{-6}\right)\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625\left(10^{-6}\right)\pi}{2.50\left(10^{-3}\right)\pi}} = 0.025 \text{ m}$$

For a column fixed at one end and free at the other end, $K = 2$. Then $KL = 2(L) = 2L$.

Buckling: Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$200\left(10^3\right) = \frac{\pi^2(101)\left(10^9\right)\left[1.5625\left(10^{-6}\right)\pi\right]}{(2L)^2}$$

$$L = 2.473 \text{ m}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{200\left(10^3\right)}{2.50\left(10^{-3}\right)\pi} = 25.46 \text{ MPa} < \sigma_Y = 69 \text{ MPa} \qquad \textbf{(O.K!)}$$

Yielding: Applying the secant formula,

$$\sigma_Y = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$69\left(10^6\right) = \frac{200\left(10^3\right)}{2.50\left(10^{-3}\right)\pi}\left[1 + \frac{0.01(0.05)}{0.025^2}\sec\left(\frac{2L}{2(0.025)}\sqrt{\frac{200\left(10^3\right)}{101\left(10^9\right)\left[2.50\left(10^{-3}\right)\pi\right]}}\right)\right]$$

$$69 = \frac{80}{\pi}(1 + 0.800 \sec 0.635140L)$$

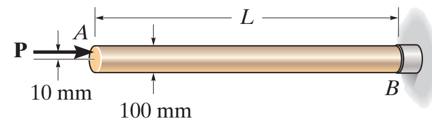
Solving by trial and error,

$$L = 1.7065 \text{ m} = 1.71 \text{ m} \qquad \textbf{(Controls!)} \qquad \textbf{Ans.}$$

Ans:
$L = 1.71$ m

17–61.

The brass rod is fixed at one end and free at the other end. If the length of the rod is $L = 2$ m, determine the greatest allowable load P that can be applied so that the rod does not buckle or yield. Also, determine the largest sidesway deflection of the rod due to the loading. $E_{br} = 101$ GPa, $\sigma_Y = 69$ MPa.



SOLUTION

Section Properties:

$$A = \frac{\pi}{4}(0.1^2) = 2.50(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.05^4) = 1.5625(10^{-6})\pi \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{1.5625(10^{-6})\pi}{2.50(10^{-3})\pi}} = 0.025 \text{ m}$$

For a column fixed at one end and free at the other end, $K = 2$. Then $KL = 2(2) = 4$ m.

Buckling: Applying *Euler's* formula,

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$= \frac{\pi^2(101)(10^9)[1.5625(10^{-6})\pi]}{4^2}$$

$$= 305823.6 \text{ N} = 305.8 \text{ kN}$$

Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{305823.6}{2.50(10^{-3})\pi} = 38.94 \text{ MPa} < \sigma_Y = 69 \text{ MPa} \qquad \textbf{(O.K!)}$$

Yielding: Applying the secant formula,

$$\sigma_Y = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{(KL)}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$69(10^6) = \frac{P}{2.50(10^{-3})\pi}\left[1 + \frac{0.01(0.05)}{0.025^2}\sec\left(\frac{4}{2(0.025)}\sqrt{\frac{P}{101(10^9)[2.50(10^{-3})\pi]}}\right)\right]$$

$$69(10^6) = \frac{400P}{\pi}\left[1 + 0.800\sec 2.84043(10^{-3})\sqrt{P}\right]$$

Solving by trial and error,

$$P = 173700 \text{ N} = 174 \text{ kN} \qquad \textbf{(Controls!)} \qquad \textbf{Ans.}$$

Maximum Displacement:

$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

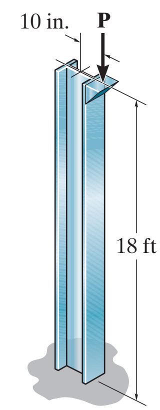
$$= 0.01\left[\sec\left(\sqrt{\frac{173700}{101(10^9)[1.5625(10^{-6})\pi]}}\left(\frac{4}{2}\right)\right) - 1\right]$$

$$= 0.01650 \text{ m} = 16.5 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$P = 174$ kN
$v_{max} = 16.5$ mm

17–62.

The W14 × 53 structural A992 steel column is fixed at its base and free at its top. If $P = 75$ kip, determine the sidesway deflection at its top and the maximum stress in the column.



SOLUTION

Section Properties for a W14 × 53:

$$A = 15.6 \text{ in}^2 \qquad I_x = 541 \text{ in}^4 \qquad I_y = 57.7 \text{ in}^4$$

$$r_x = 5.89 \text{ in.} \qquad d = 13.92 \text{ in.}$$

Maximum Deflection:

$$v_{\max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{KL}{2}\right) - 1\right]$$

$$\sqrt{\frac{P}{EI}}\frac{KL}{2} = \sqrt{\frac{75}{29(10^3)541}}\left(\frac{2.0(18)(12)}{2}\right) = 0.472267$$

$$v_{\max} = 10[\sec(0.472267) - 1] = 1.23 \text{ in.}$$ **Ans.**

Maximum Stress:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{P}{A} = \frac{75}{15.6} = 4.808 \text{ ksi}$$

$$\frac{ec}{r^2} = \frac{10\left(\frac{13.92}{2}\right)}{5.89^2} = 2.0062$$

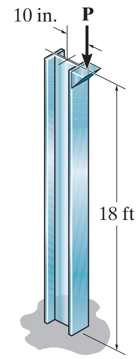
$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(18)(12)}{2(5.89)}\sqrt{\frac{75}{29(10^3)(15.6)}} = 0.47218$$

$$\sigma_{\max} = 4.808[1 + 2.0062\sec(0.47218)] = 15.6 \text{ ksi} < \sigma_Y$$ **Ans.**

Ans:
$v_{\max} = 1.23$ in.,
$\sigma_{\max} = 15.6$ ksi

17–63.

The W14 × 53 column is fixed at its base and free at its top. Determine the maximum eccentric load P that it can support without causing it to buckle or yield. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



SOLUTION

Section properties for a W14 × 53:

$A = 15.6\text{ in}^2 \qquad I_x = 541\text{ in}^4 \qquad I_y = 57.7\text{ in}^4$

$r_x = 5.89\text{ in.} \qquad d = 13.92\text{ in.}$

Buckling about *y–y* axis:

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(57.7)}{[(2.0)(18)(12)]^2} = 88.5\text{ kip} \qquad \text{controls} \qquad \textbf{Ans.}$$

Check: $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{88.5}{15.6} = 5.67\text{ ksi} < \sigma_Y$ **OK**

Yielding about *x–x* axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

$$\frac{ec}{r^2} = \frac{10\left(\frac{13.92}{2}\right)}{5.89^2} = 2.0062$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{2.0(18)(12)}{2(5.89)}\sqrt{\frac{P}{29(10^3)(15.6)}} = 0.054523\sqrt{P}$$

$$50(15.6) = P[1 + 2.0062\sec(0.054523\sqrt{P})]$$

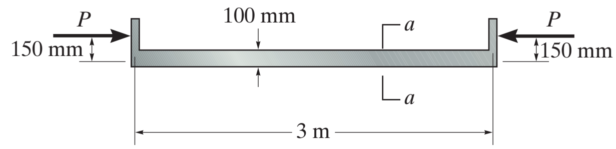
By trial and error:

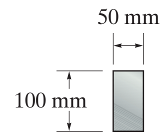
$P = 204$ kip

Ans:
$P = 88.5$ kip

***17–64.**

Determine the maximum eccentric load P the 2014-T6-aluminum-alloy strut can support without causing it either to buckle or yield. The ends of the strut are pin connected.





Section $a-a$

SOLUTION

Section Properties: The necessary section properties are

$$A = 0.05(0.1) = 5\left(10^{-3}\right)\text{m}^2$$

$$I_y = \frac{1}{12}(0.1)\left(0.05^3\right) = 1.04167\left(10^{-6}\right)\text{ m}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{4.1667\left(10^{-6}\right)}{5\left(10^{-3}\right)}} = 0.02887\text{ m}$$

For a column that is pinned at both of its ends, $K = 1$. Thus,

$$(KL)_x = (KL)_y = 1(3) = 3\text{ m}$$

Buckling About the Weak Axis: Applying Euler's formula,

$$P_{\text{cr}} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[73.1\left(10^9\right)\right]\left[1.04167\left(10^{-6}\right)\right]}{3^2} = 83.50\text{ kN} = 83.5\text{ kN} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula is valid only if $\sigma_{\text{cr}} < \sigma_Y$.

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{83.50\left(10^3\right)}{5\left(10^{-3}\right)} = 16.70\text{ MPa} < \sigma_Y = 414\text{ MPa} \qquad \textbf{O.K.}$$

Yielding About Strong Axis: Applying the secant formula,

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r_x{}^2}\sec\left[\frac{(KL)_x}{2r_x}\sqrt{\frac{P}{EA}}\right]\right]$$

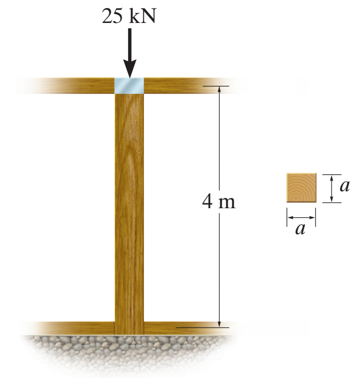
$$= \frac{83.50\left(10^3\right)}{5\left(10^{-3}\right)}\left[1 + \frac{0.15(0.05)}{0.02887^2}\sec\left[\frac{3}{2(0.02887)}\sqrt{\frac{83.50\left(10^3\right)}{73.1\left(10^9\right)\left[5\left(10^{-3}\right)\right]}}\right]\right]$$

$$= 229.27\text{ MPa} < \sigma_Y = 414\text{ MPa} \qquad \textbf{O.K.}$$

Ans:
$P_{\text{cr}} = 83.5$ kN

R17–1.

The wood column is 4 m long and is required to support the axial load of 25 kN. If the cross section is square, determine the dimension a of each of its sides using a factor of safety against buckling of F.S. $= 2.5$. The column is assumed to be pinned at its top and bottom. Use the Euler equation. $E_w = 11$ GPa, and $\sigma_Y = 10$ MPa.



SOLUTION

Critical Buckling Load: $I = \frac{1}{12}(a)(a^3) = \frac{a^4}{12}$, $P_{cr} = (2.5)25 = 62.5$ kN and $K = 1$ for pin supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$62.5(10^3) = \frac{\pi^2(11)(10^9)\left(\frac{a^4}{12}\right)}{[1(4)]^2}$$

$$a = 0.1025 \text{ m} = 103 \text{ mm}$$ **Ans.**

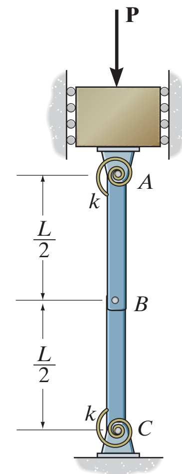
Critical Stress: *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{62.5(10^3)}{0.1025(0.1025)} = 5.94 \text{ MPa} < \sigma_Y = 10 \text{ MPa}$$ O.K.

Ans:
$a = 103$ mm

R17–2.

If the torsional springs attached to ends A and C of the rigid members AB and BC have a stiffness k, determine the critical load P_{cr}.



SOLUTION

Equilibrium. When the system is given a slight lateral disturbance, the configuration shown in Fig. *a* is formed. The couple moment M can be related to P by considering the equilibrium of members AB and BC.

Member *AB*.

$$+\uparrow \Sigma F_y = 0; \qquad B_y - P = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y\left(\frac{L}{2}\sin\theta\right) + B_x\left(\frac{L}{2}\cos\theta\right) - M = 0 \qquad (2)$$

Member *BC*.

$$\circlearrowleft + \Sigma M_C = 0; \; -B_y\left(\frac{L}{2}\sin\theta\right) + B_x\left(\frac{L}{2}\cos\theta\right) + M = 0 \qquad (3)$$

Solving Eqs. (1), (2), and (3), we obtain

$$B_x = 0 \qquad B_y = \frac{2M}{L\sin\theta} \qquad M = \frac{PL}{2}\sin\theta$$

Since θ is very small, the small angle analysis gives $\sin\theta \cong \theta$. Thus,

$$M = \frac{PL}{2}\theta \qquad (4)$$

Torsional Spring Moment. The restoring couple moment M_{sp} can be determined using the torsional spring formula, $M = k\theta$. Thus,

$$M_{sp} = k\theta$$

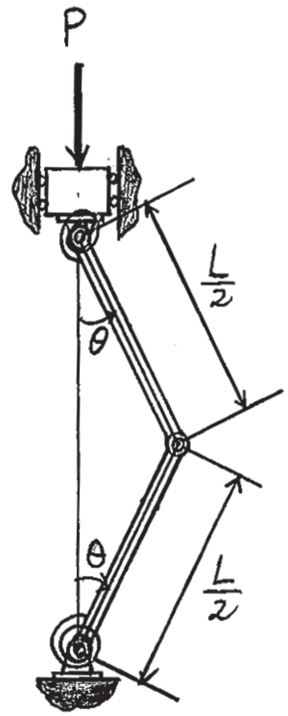
Critical Buckling Load. When the mechanism is on the verge of buckling, M must equal M_{sp}.

$$M = M_{sp}$$

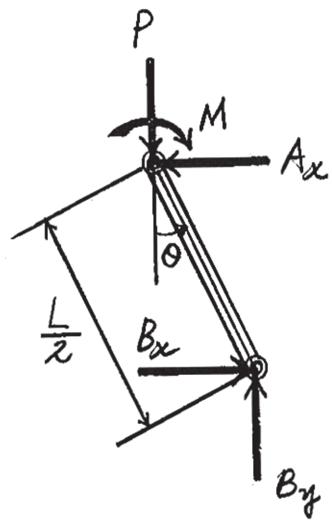
$$\frac{P_{cr}\,L}{2}\theta = k\theta$$

$$P_{cr} = \frac{2k}{L} \qquad \textbf{Ans.}$$

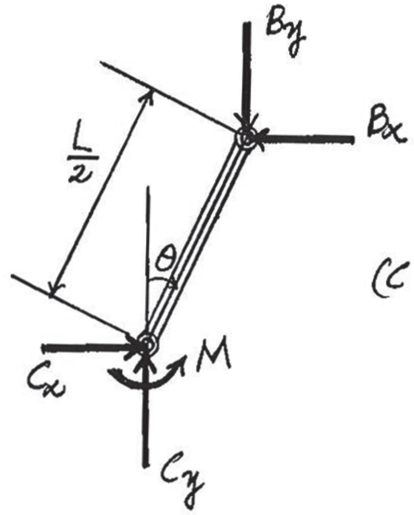
R17–2. Continued



(a)



(b)



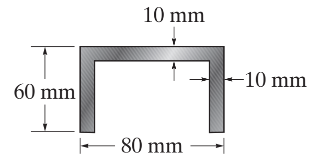
(c)

Ans:

$P_{cr} = \dfrac{2k}{L}$

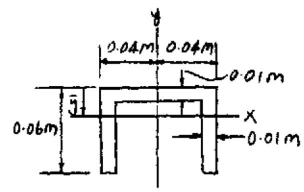
R17–3.

A steel column has a length of 5 m and is free at one end and fixed at the other end. If the cross-sectional area has the dimensions shown, determine the critical load. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



SOLUTION

Section Properties:



$$A = 0.06\,(0.01) + 2\,(0.06)(0.01) = 1.80(10^{-3})\text{ m}^2$$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.005\,(0.06)(0.01) + 2[0.03\,(0.06)(0.01)]}{0.06\,(0.01) + 2\,(0.06)(0.01)} = 0.02167\text{ m}$$

$$I_x = \frac{1}{12}(0.06)(0.01)^3 + 0.06\,(0.01)(0.02167 - 0.005)^2$$

$$+ [\frac{1}{12}(0.01)(0.06)^3 + 0.01\,(0.06)(0.03 - 0.02167)^2] = 0.615\,(10^{-6})\text{ m}^4 \quad \text{(controls)}$$

$$I_y = \frac{1}{12}(0.06)(0.08)^3 - \frac{1}{12}(0.05)(0.06)^3 = 1.66\,(10^{-6})\text{ m}^4$$

Critical Load:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad K = 2.0$$

$$= \frac{\pi^2\,(200)(10^9)(0.615)(10^{-6})}{[2.0\,(5)]^2}$$

$$= 12140\text{ N} = 12.1\text{ kN}$$ **Ans.**

Check Stress:

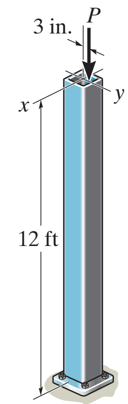
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12140}{1.80\,(10^{-3})} = 6.74\text{ MPa} < \sigma_Y = 360\text{ MPa}$$

Hence, Euler's equation is still valid.

Ans:
$P_{cr} = 12.1$ kN

***R17–4.**

The square structural A992 steel tubing has outer dimensions of 8 in. by 8 in. Its cross-sectional area is 14.40 in^2 and its moments of inertia are $I_x = I_y = 131$ in^4. Determine the maximum load P it can support. The column can be assumed fixed at its base and free at its top.



SOLUTION

Section Properties:

$$A = 14.4 \text{ in}^2; \qquad I_x = I_y = 131 \text{ in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{131}{14.4}} = 3.01616 \text{ in.}$$

Yielding:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2\,r}\sqrt{\frac{P}{E\,A}}\right)\right]; \qquad K = 2.0$$

$$\frac{ec}{r^2} = \frac{3\,(4)}{(3.01616)^2} = 1.319084$$

$$\frac{K\,L}{2\,r}\sqrt{\frac{P}{E\,A}} = \frac{2(12)(12)}{2(3.01616)}\sqrt{\frac{P}{29\,(10^3)(14.40)}} = 0.073880\sqrt{P}$$

$$50\,(14.4) = P[1 + 1.319084\sec(0.073880\sqrt{P})]$$

By Trial and Error:

$P = 199$ kip (controls) **Ans.**

Buckling:

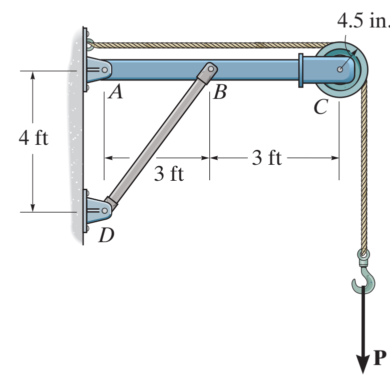
$$P = P_{cr} = \frac{\pi^2 E\,I}{(K\,L)^2} = \frac{\pi^2(29)(10^3)(131)}{[2(12)(12)]^2} = 452 \text{ kip}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{452}{14.4} = 31.4 \text{ ksi} < \sigma_Y = 50 \text{ ksi}$$ **(OK)**

Ans:
$P = 199$ kip

R17–5.

If the A-36 steel solid circular rod BD has a diameter of 2 in., determine the allowable maximum force **P** that can be supported by the frame without causing the rod to buckle. Use F.S. = 2 against buckling.



SOLUTION

Equilibrium: The compressive force developed in BD can be determined by considering the equilibrium of the free-body diagram of member ABC, Fig. a.

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BD}\left(\frac{4}{5}\right)(3) + P(0.375) - P(6.375) = 0 \qquad F_{BD} = 2.5P$$

Section Properties: The cross-sectional area and moment of inertia of BD are

$$A = \pi(1^2) = \pi \text{ in}^2 \qquad I = \frac{\pi}{4}(1^4) = 0.25\pi \text{ in}^4$$

Critical Buckling Load: Since BD is pinned at both of its ends, $K = 1$. The critical buckling load is

$$P_{cr} = F_{BD}(\text{F.S.}) = 2.5P(2) = 5P$$

The length of BD is $L = \sqrt{3^2 + 4^2} = 5$ ft. Applying Euler's formula,

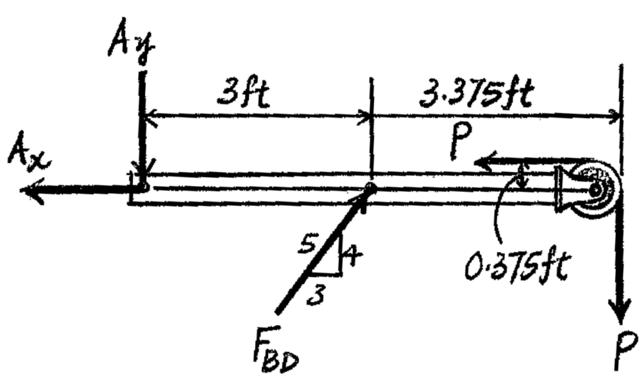
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$5P = \frac{\pi^2[29(10^3)](0.25\pi)}{[1(5)(12)]^2}$$

$$P = 12.49 \text{ kip} = 12.5 \text{ kip} \qquad \textbf{Ans.}$$

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{5(12.49)}{\pi} = 19.88 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \textbf{(OK)}$$

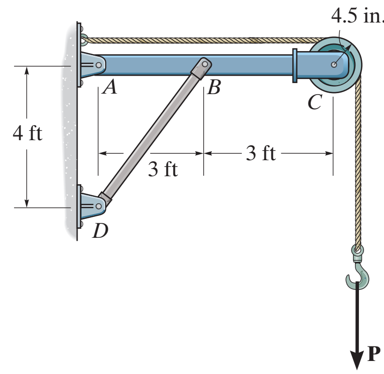


(a)

Ans:
$P = 12.5$ kip

R17–6.

If $P = 15$ kip, determine the required minimum diameter of the A992 steel solid circular rod BD to the nearest $\frac{1}{16}$ in. Use F.S. $= 2$ against buckling.



SOLUTION

Equilibrium: The compressive force developed in BD can be determined by considering the equilibrium of the free-body diagram of member ABC, Fig. *a*.

$$\zeta + \Sigma M_A = 0; \qquad F_{BD}\left(\frac{4}{5}\right)(3) + 15(0.375) - 15(6.375) = 0 \qquad F_{BC} = 37.5 \text{ kip}$$

Section Properties: The cross-sectional area and moment of inertia of BD are

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4$$

Critical Buckling Load: Since BD is pinned at both of its ends, $K = 1$. The critical buckling load is

$$P_{cr} = F_{BD}(\text{F.S.}) = 37.5(2) = 75 \text{ kip}$$

The length of BD is $L = \sqrt{3^2 + 4^2} = 5$ ft. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

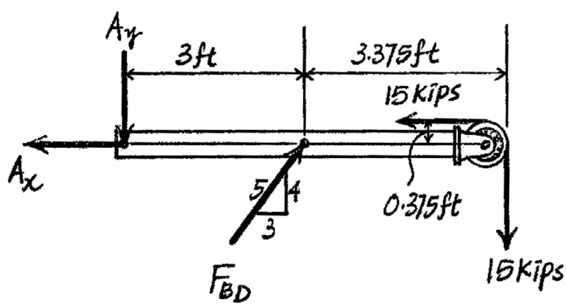
$$75 = \frac{\pi^2\left[29(10^3)\right]\left(\frac{\pi}{64}d^4\right)}{[1(5)(12)]^2}$$

$$d = 2.094 \text{ in.}$$

Use $d = 2\frac{1}{8}$ in. **Ans.**

Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{75}{\frac{\pi}{4}(2.125^2)} = 21.15 \text{ ksi} < \sigma_Y = 50 \text{ ksi}$$ **(O.K.)**

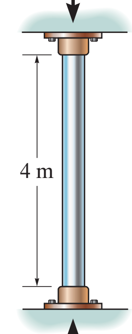


(a)

Ans:
Use $d = 2\frac{1}{8}$ in.

R17–7.

The steel pipe is fixed supported at its ends. If it is 4 m long and has an outer diameter of 50 mm, determine its required thickness so that it can support an axial load of $P = 100$ kN without buckling. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

$$I = \frac{\pi}{4}(0.025^4 - r_i^{\,4})$$

Critical Load:

$$P_{cr} = \frac{\pi^2 EI}{(K\,L)^2}; \qquad K = 0.5$$

$$100(10^3) = \frac{\pi^2(200)(10^9)[\frac{\pi}{4}(0.025^4 - r_i^{\,4})]}{[0.5(4)]^2}$$

$r_i = 0.01908\text{ m} = 19.1\text{ mm}$

$t = 25\text{ mm} - 19.1\text{ mm} = 5.92\text{ mm}$ **Ans.**

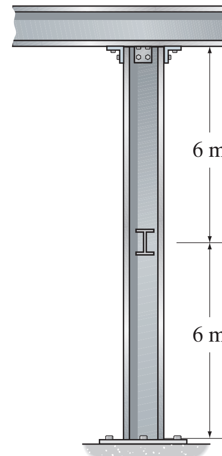
Check Stress:

$$\sigma = \frac{P_{cr}}{A} = \frac{100(10^3)}{\pi(0.025^2 - 0.0191^2)} = 122\text{ MPa} < \sigma_Y = 345\text{ MPa}$$ **(OK)**

Ans:
$t = 5.92$ mm

***R17–8.**

The W200 × 46 wide-flange A992-steel column can be considered pinned at its top and fixed at its base. Also, the column is braced at its mid-height against weak axis buckling. Determine the maximum axial load the column can support without causing it to buckle.



SOLUTION

Section Properties: From the table listed in the appendix, the section properties for a W200 × 46 are

$A = 5890\text{ mm}^2 = 5.89\left(10^{-3}\right)\text{ m}^2 \qquad I_x = 45.5\left(10^6\right)\text{ mm}^4 = 45.5\left(10^{-6}\right)\text{ m}^4$

$I_y = 15.3\left(10^6\right)\text{ mm}^4 = 15.3\left(10^{-6}\right)\text{ m}^4$

Critical Buckling Load: For buckling about the strong axis, $K_x = 0.7$ and $L_x = 12$ m. Since the column is fixed at its base and pinned at its top,

$$P_{cr} = \frac{\pi^2 EI_x}{(KL)_x{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[45.5\left(10^{-6}\right)\right]}{[0.7(12)]^2} = 1.273\left(10^6\right)\text{N} = 1.27\text{ MN}$$

For buckling about the weak axis, $K_y = 1$ and $L_y = 6$ m since the bracing provides a support equivalent to a pin. Applying Euler's formula,

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)_y{}^2} = \frac{\pi^2\left[200\left(10^9\right)\right]\left[15.3\left(10^{-6}\right)\right]}{[1(6)]^2} = 838.92\text{ kN} = 839\text{ kN (controls)}$$

Ans.

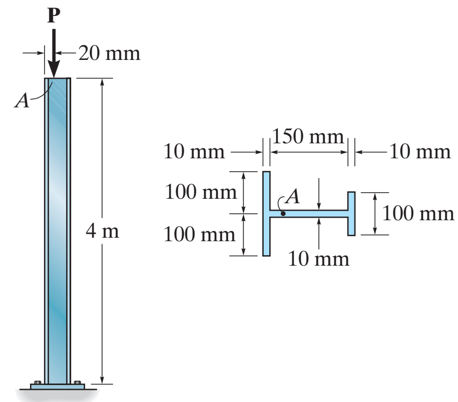
Critical Stress: Euler's formula is valid only if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{838.92\left(10^3\right)}{5.89\left(10^{-3}\right)} = 142.43\text{ MPa} < \sigma_Y = 345\text{ MPa} \qquad \textbf{(O.K.)}$$

Ans:
$P_{cr} = 839$ kN

R17–9.

The wide-flange A992 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding.



SOLUTION

Section Properties:

$$\Sigma A = 0.2(0.01) + 0.15\,(0.01) + 0.1(0.01) = 4.5(10^{-3})\ \text{m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{0.005\,(0.2)(0.01) + 0.085\,(0.15)(0.01) + 0.165\,(0.1)(0.01)}{4.5(10^{-3})}$$

$$= 0.06722\ \text{m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2\,(0.01)(0.06722 - 0.005)^2$$

$$+ \frac{1}{12}(0.01)(0.15^3) + 0.01\,(0.15)(0.085 - 0.06722)^2$$

$$+ \frac{1}{12}(0.1)(0.01^3) + 0.1\,(0.01)(0.165 - 0.06722)^2$$

$$= 20.615278\,(10^{-6})\ \text{m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3)$$

$$= 7.5125\,(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278(10^{-6})}{4.5\,(10^{-3})}} = 0.0676844$$

Buckling about x–x axis:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0(4)]^2}$$

$$= 231.70\ \text{kN} \qquad \text{(controls)}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{231.7\,(10^3)}{4.5\,(10^{-3})} = 51.5\ \text{MPa} < \sigma_Y = 345\ \text{MPa}$$

Yielding about y–y axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]; \qquad e = 0.06722 - 0.02 = 0.04722\ \text{m}$$

$$\frac{e\,c}{r^2} = \frac{0.04722\,(0.06722)}{0.0676844^2} = 0.692919$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0\,(4)}{2(0.0676844)}\sqrt{\frac{P}{200\,(10^9)(4.5)(10^{-3})}} = 1.96992\,(10^{-3})\sqrt{P}$$

$$345(10^6)(4.5)(10^{-3}) = P[1 + 0.692919\sec(1.96992\,(10^{-3})\sqrt{P})]$$

By trial and error:

$$P = 434.342\ \text{kN}$$

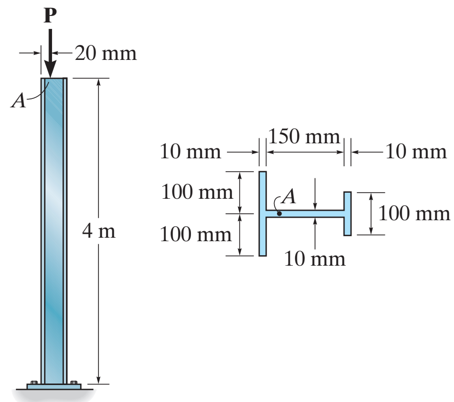
Hence,

$$P_{allow} = \frac{231.70}{3} = 77.2\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$P_{allow} = 77.2$ kN

R17–10.

The wide-flange A992 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine if the column will buckle or yield when the load $P = 10$ kN is applied at A. Use a factor of safety of 3 with respect to buckling and yielding.



SOLUTION

Section Properties:

$$\Sigma A = 0.2\,(0.01) + 0.15\,(0.01) + 0.1\,(0.01) = 4.5\,(10^{-3})\text{ m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{0.005\,(0.2)(0.01) + 0.085\,(0.15)(0.01) + 0.165\,(0.1)(0.01)}{4.5\,(10^{-3})} = 0.06722\text{ m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2\,(0.01)(0.06722 - 0.005)^2$$

$$+ \frac{1}{12}(0.01)(0.15^3) + 0.01\,(0.15)(0.085 - 0.06722)^2$$

$$+ \frac{1}{12}(0.1)(0.01^3) + 0.1\,(0.01)(0.165 - 0.06722)^2 = 20.615278\,(10^{-6})\text{ m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3) = 7.5125\,(10^{-6})\text{ m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278\,(10^{-6})}{4.5\,(10^{-3})}} = 0.0676844\text{ m}$$

Buckling about *x–x* axis:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0(4)]^2} = 231.70\text{ kN}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{231.7\,(10^3)}{4.5\,(10^{-3})} = 51.5\text{ MPa} < \sigma_Y = 345\text{ MPa} \qquad \textbf{(O.K.)}$$

$$P_{allow} = \frac{P_{cr}}{\text{FS}} = \frac{231.7}{3} = 77.2\text{ kN} > P = 10\text{ kN}$$

Hence, the column does not buckle.

Yielding about *y–y* axis:

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad e = 0.06722 - 0.02 = 0.04722\text{ m}$$

$$P = \frac{10}{3} = 3.333\text{ kN}$$

$$\frac{P}{A} = \frac{3.333\,(10^3)}{4.5\,(10^{-3})} = 0.7407\text{ MPa}$$

$$\frac{e\,c}{r^2} = \frac{0.04722\,(0.06722)}{(0.067844)^2} = 0.692919$$

$$\frac{K\,L}{2\,r}\sqrt{\frac{P}{E\,A}} = \frac{2.0\,(4)}{2(0.0676844)}\sqrt{\frac{3.333\,(10^3)}{200\,(10^9)(4.5)(10^{-3})}} = 0.113734$$

$$\sigma_{max} = 0.7407\,[1 + 0.692919\sec(0.113734)] = 1.26\text{ MPa} < \sigma_Y = 345\text{ MPa}$$

Hence, the column does not yield!

No. **Ans.**

Ans:
It does not buckle or yield.