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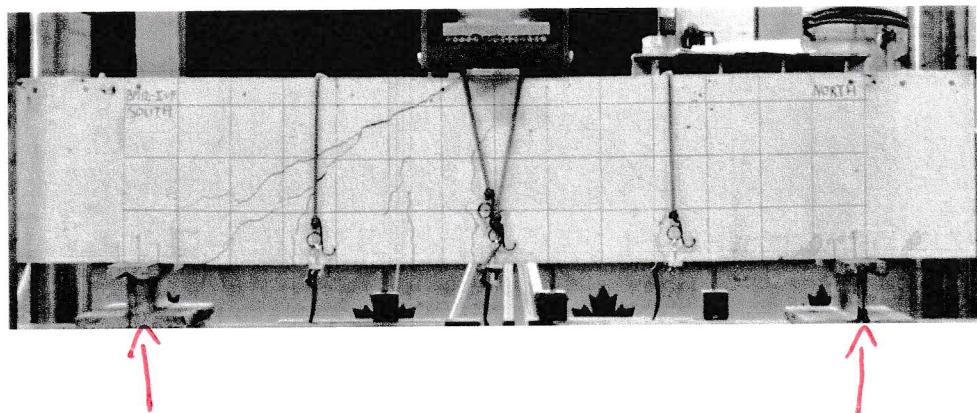
# CIVE 414

## STRUCTURAL CONCRETE DESIGN

### SHEAR

in REINFORCED

CONCRETE MEMBERS

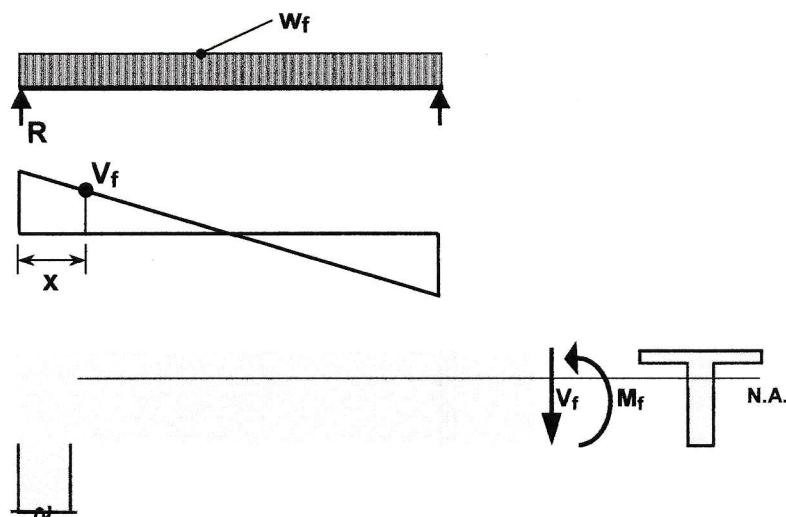


*Majority of failures are due to shear or development length*

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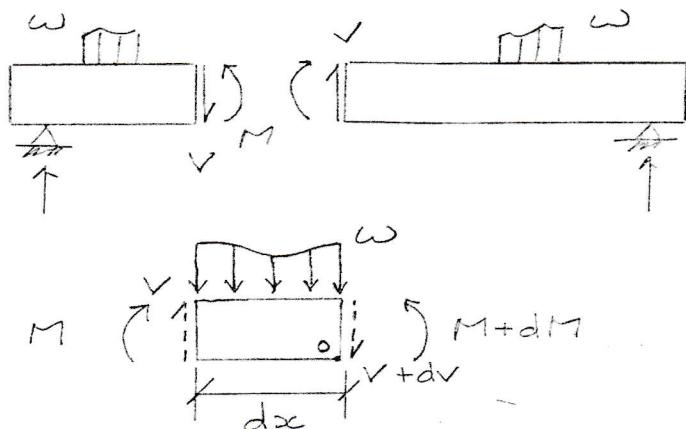
## SHEAR IN REINFORCED CONCRETE

- Brittle failure mode → very undesirable
- Shear in reinforced concrete is complex:
  - non-linear material
  - non-homogeneous material
  - cracking
  - presence of reinforcement



## MECHANICS OF SHEAR IN BEAMS

In beams, the external load is resisted by moment and shear, i.e.:



*From moment equilibrium of a differential element around point O*

$$Vdx = dM$$

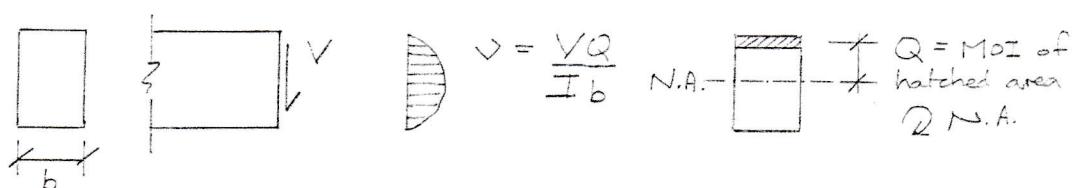
$$V = \frac{dM}{dx}$$

*Shear exists where moment is changing*

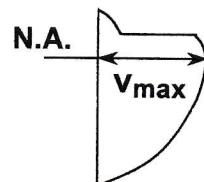
For uncracked beams in shear we have homogenous, linear elastic material:

$$v_c = \frac{V_f Q}{I_b}$$

**For Rectangular Section:**



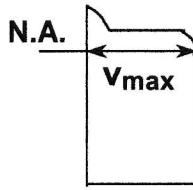
**For T-Section:**



**After cracking:**

$$(nominal) v_c = \frac{V_f}{b_w d}$$

*shear stress ↑*  
*web width*



**non-homogeneous concrete member**

*Plane stress for beam*

**Flexural and shear stresses:**

*Along principal directions,  
Shear stress is zero*

**Principal Stresses:**

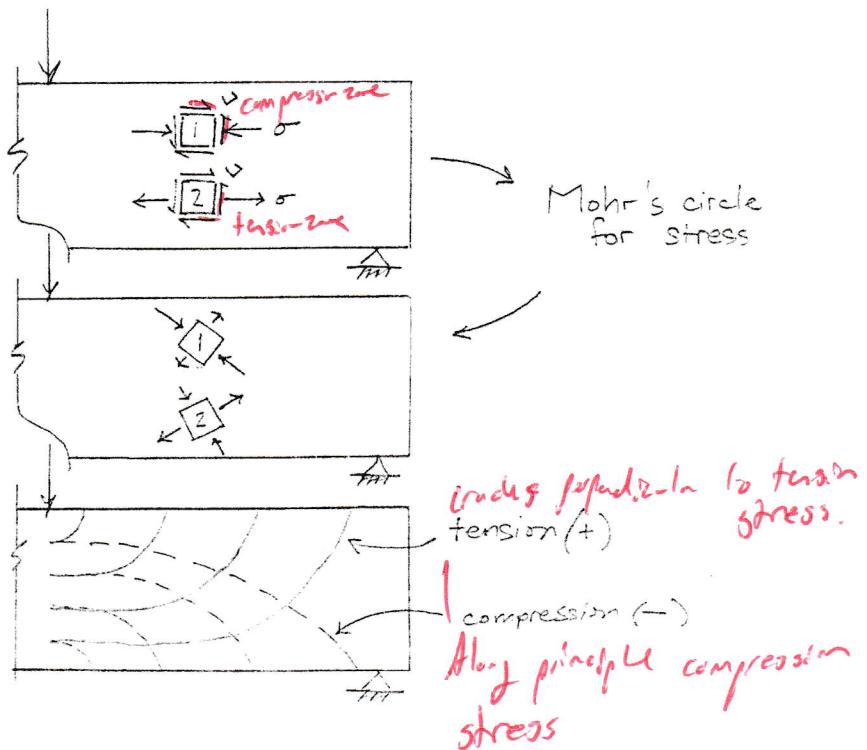
**Principal stress**

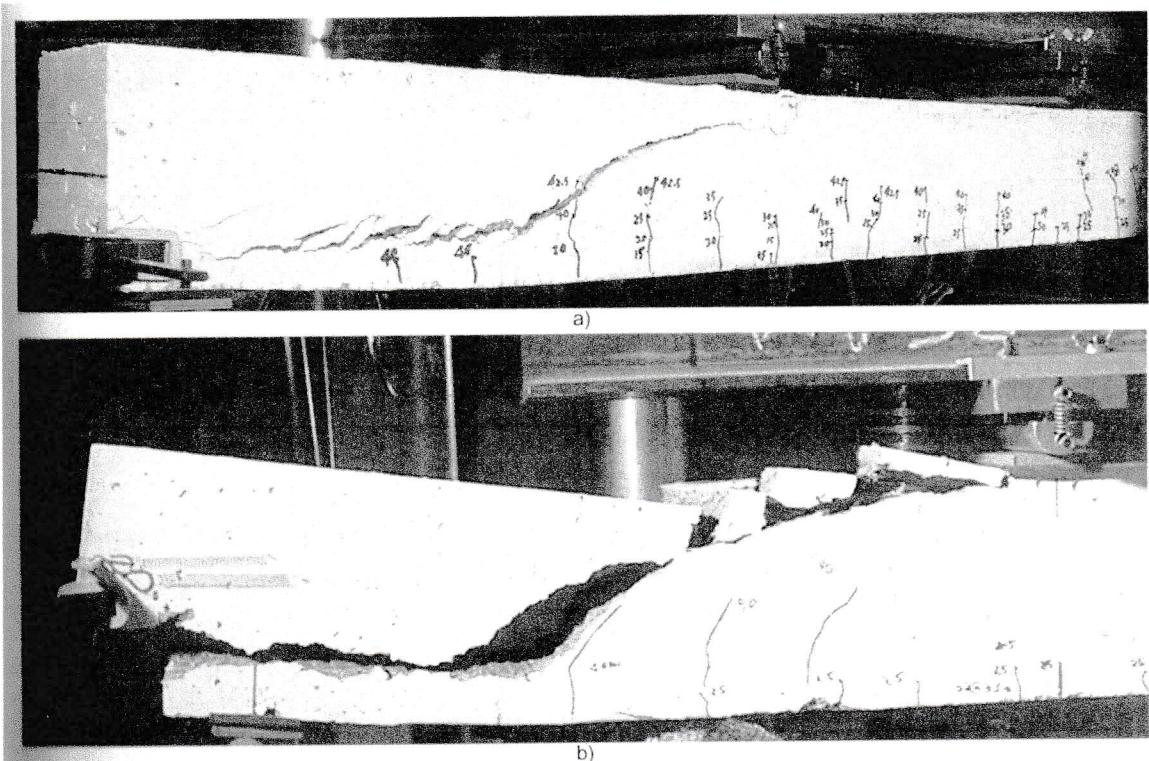
**trajectories:**

*Note: (principal stresses)*

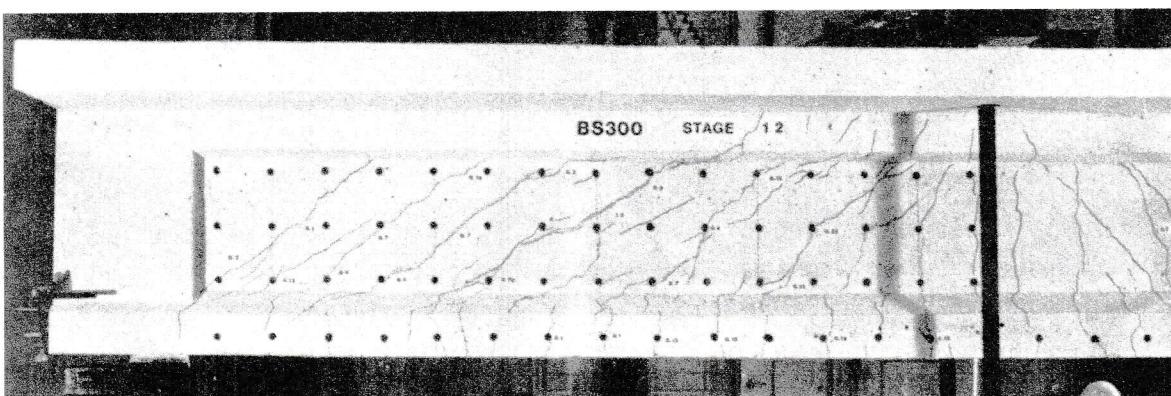
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$





→ beam with no shear reinforcement [Brzev & Pao]

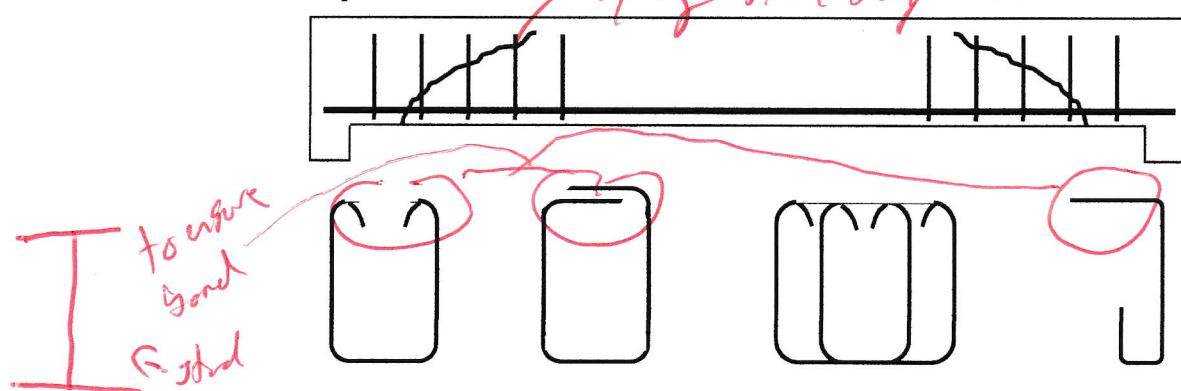


Prestressed! ⚡ Built cracks

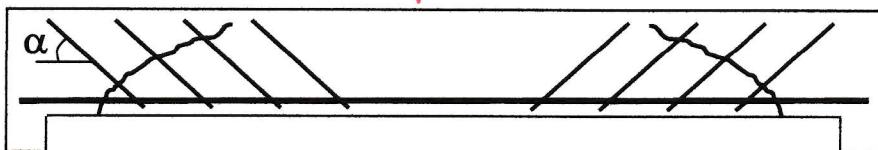
# SHEAR REINFORCEMENT

## CSA A23.3 Clause 11.2.4

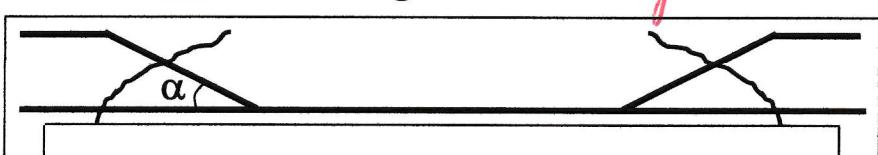
### 1. Stirrups



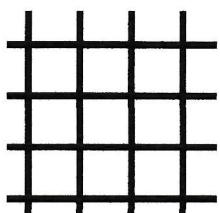
### 2. Stirrups with $\alpha \geq 45$ deg. - practical, but not constructable



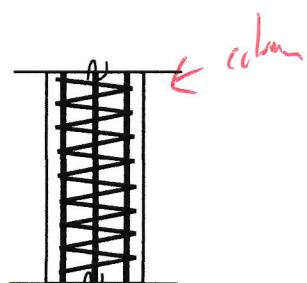
### 3. Bent Bars with $\alpha \geq 30$ deg. - not used anymore



### 4. Welded-wire Fabric



### 5. Spirals and Ties



In the design of reinforced concrete structures, so-called 'B' and 'D' regions are distinguished [CSA A23.3 §11.1.1-2]:

B-regions	D-regions
Plane sections remain plane, Linear axial strain distribution	Nonlinear axial strain distribution through the thickness
Slender beams, Bernoulli beams <b>B Regions</b>	Deep Beams <b>D Regions</b>
"sectional analysis" Based on <u>truss model</u>	<u>Strut and Tie Analysis</u>

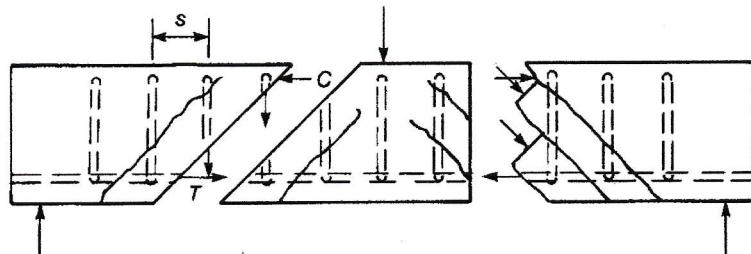
Shear reinforcement in beams The purpose of shear reinforcement is to restrain diagonal cracks and ensure that full flexural capacity can be developed.

- Note: Shear reinforcement does not prevent diagonal shear cracks from occurring altogether.

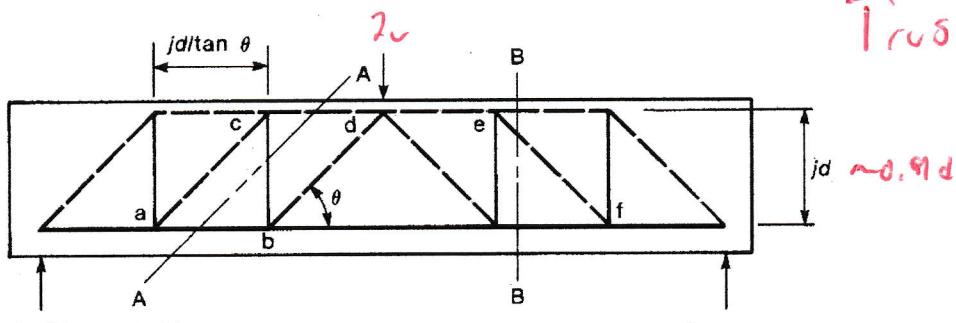
# Analysis for Shear

## Truss Models

Given a reinforced concrete beam with shear cracks:



(a) Internal forces in a cracked beam.



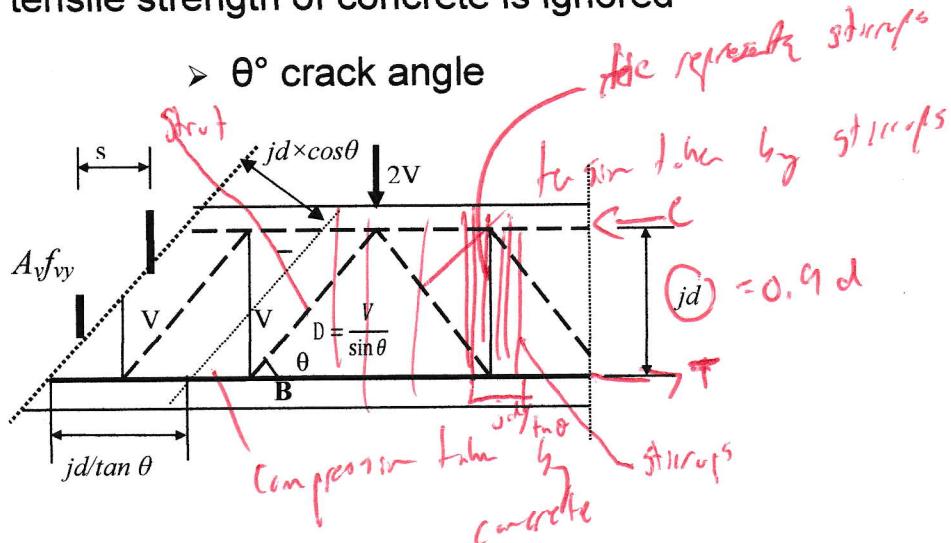
(b) Pin-jointed truss.

MacGregor, J.G., Bartlett, M. "Reinforced Concrete Mechanics and Design", Prentice Hall, 2000

# VARIABLE ANGLE TRUSS MODEL

Assumes:

- tensile strength of concrete is ignored



## Equilibrium in vertical direction (vertical tie)

$$\left(\frac{jd}{\tan \theta}\right) \times A_v f_{vy} = V \quad \frac{(jd)}{s} \text{ is number of stirrups within } jd/\tan \theta$$

used to calculate required cross-sectional area of a stirrup (knowing the spacing):

$$A_v = \frac{V \times s}{f_{vy} \times (jd / \tan \theta)}$$

or spacing if stirrups (knowing the cross-sectional area):

$$s = \frac{A_v \times f_{vy} \times (jd / \tan \theta)}{V}$$

Or the stirrup contribution:

$$V = \frac{A_v \times f_{vy} \times (jd / \tan \theta)}{s}$$

## Equilibrium along the inclined strut

Used to dimension the overall dimensions of the beam to avoid concrete crushing

$$f_2 = \frac{V}{b \times jd \times \sin \theta \cos \theta} \leq \text{compressive strength of concrete}$$

width of beam  
 ,  $jd \cot \theta$  = width strut

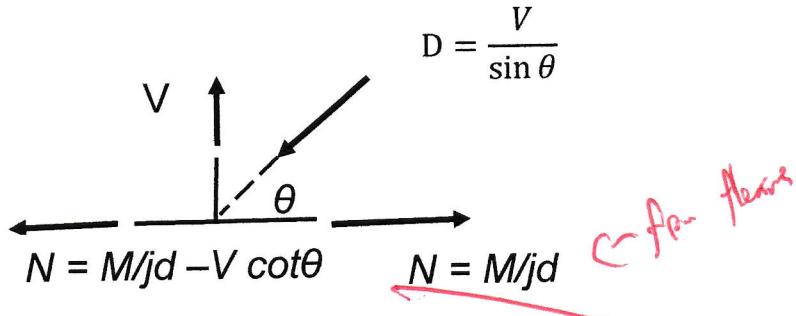
D (force in strut)  
 As  $\theta \downarrow$ , compression T,  
 less stirrups req'd

Note: The truss models predict that a beam with no stirrups will have no shear capacity:

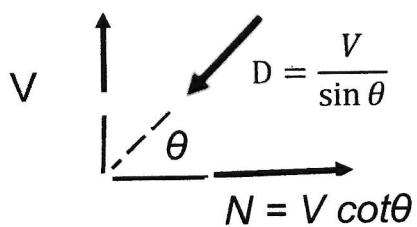
- conservative!
- basis for traditional " $V_c + V_s$ " approach

## Effect of Shear on tensile forces in flexural reinforcement

### Equilibrium of Joint B



Equilibrium at support of simply supported beam:



## 45° Truss Model by Ritter and Mörsch

[Ritter 1899, Mörsch 1902]: angle  $\theta = 45^\circ$

$$\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$$

$$\frac{jd}{s} \times A_v f_{vy} = V$$

$$A_v = \frac{V \times s}{f_{vy} \times jd}$$

$$s = \frac{A_v \times f_{vy} \times jd}{V}$$

### Traditional “ $V_c + V_s$ ” Approach

This approach was the basis for the “simplified method” in the old Canadian Concrete Design Code [CSA A23.3 1994].

It can be summarized as follows:

- $V_r \geq V_f$
- $V_r = V_c + V_s$
- $V_c = 0.2 \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \cdot b_w \cdot d$

(for beams with > min. shear reinf. or  $d < 300$  mm)

$$= \left( \frac{260}{1000 + d} \right) \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \cdot b_w \cdot d \text{ but } \geq 0.1 \cdot \lambda \cdot \phi_c \cdot \sqrt{f'_c} \cdot b_w \cdot d$$

(all other cases)

$$- V_s = \frac{\phi_s \cdot A_v \cdot f_y \cdot d}{s}$$

$\checkmark D = \frac{1}{\sin \theta}$  More economical to use lower angle,  $j d \approx 0.9d$  12

## CSA A23.3 -2014 DESIGN FOR SHEAR

$\checkmark$   $\triangleright$  Required Shear Resistance Cl 11.3.1  
shear capacity

$\text{ls } 30^\circ, \text{ conservative?}$

$$V_r \geq V_f$$

### METHOD 1: SIMPLIFIED DESIGN METHOD – CLAUSE 11.3.6.3 (up to 60 MPa f'c)

#### FACTORED SHEAR RESISTANCE

CSA A23.3-14 Clause 11.3.3

$$V_r = V_c + V_s$$

$V_c$  = shear resistance provided by concrete, for my gear,

$V_s$  = shear resistance provided by steel

(provide stirrups if necessary!)

$f'_c$  tensile strength  
 $V_c = 0.2 \lambda \phi_c f'_c b v$   
See Cl. 11.

#### Maximum Factored Shear Resistance (Clause 11.3.3)

$\triangleright$  Upper limit to prevent web crushing failure

CSA A23.3-14 Clause 11.3.3

$$V_{r,\max} = 0.25 \phi_c f'_c b_w d_v$$

where,

$$\phi_c = 0.65$$

$d_v$  = effective shear depth

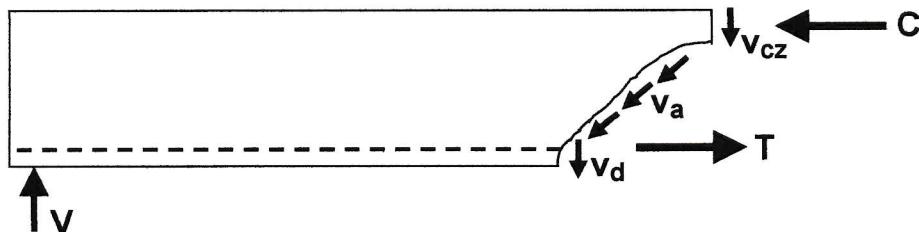
= 0.9d or 0.72h, whichever is greater

$b_w$  = web width

$$d_v = j d = 0.9d, D = \frac{1}{\sin \theta}$$

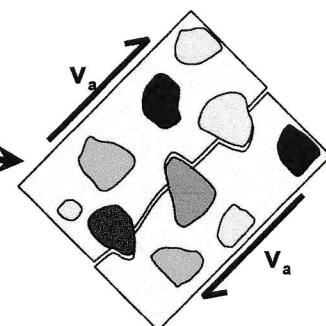
## CONCRETE RESISTANCE IN SHEAR, $V_c$

CSA A23.3-14 Clause 11.3.4



Three contributions:

- $v_{cz}$  shear in compression zone
  - $v_a$  aggregate interlock
  - $v_d$  dowel action (longitudinal bar)
- $v_a$  is the largest contribution to concrete shear strength



$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v$$

In Eurocode depends on flexural, ~~reinforced~~ longitudinal reinforcement

Where,

$$\phi_c = 0.65$$

$\lambda$  = factor to account for low-density concrete  
= 1 for normal density concrete

$\beta$  = factor accounting for shear resistance of cracked concrete,  
determined in Clause 11.3.6

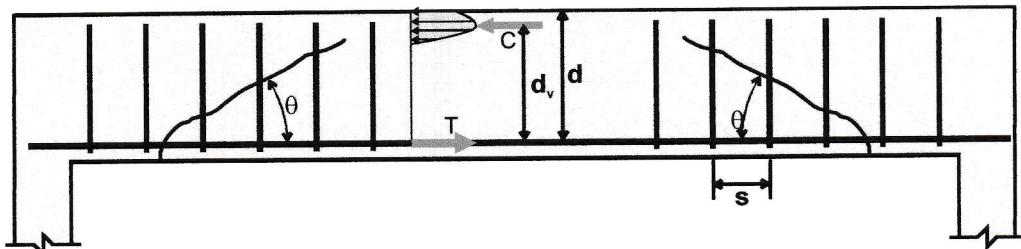
Note that  $\sqrt{f'_c} \leq 8 \text{ MPa}$  when computing  $V_c$ .

- For  $f'_c > 64 \text{ MPa}$ , use  $\sqrt{f'_c} = 8 \text{ MPa}$
- Accounts for reduced aggregate interlock in high strength concrete

If hpc cracks go through aggregate. Not necessarily word  
like here of aggregate interlock

## STEEL RESISTANCE IN SHEAR, $V_s$

CSA A23.3-14 Clause 11.3.5



Force in one stirrup:  $\phi_s A_v f_y$

number of stirrups crossing a crack:  $n = \frac{d_v \cot \theta}{s}$

$$V_s = \frac{\phi_s A_v f_y d_v \cot \theta}{s} \quad (\text{from truss model})$$

Where,

$A_v$  = area of shear reinforcement within distance "s"

=  $A_b \times$  no. of legs in stirrup

$s$  = stirrup spacing

$\theta$  = angle of inclination of compression stresses, determined in Clause 11.3.6

≈ angle of inclined cracks due to shear

**For design:**

$$(V_s)_{\text{req'd}} = V_f - V_c = \frac{\phi_s A_v f_y d_v \cot \theta}{s}$$

*Req'd # stirrups based on  
V<sub>f</sub> - V<sub>c</sub>  
Tribal concrete can carry*

Thus:  $s_{\text{req'd}} = \frac{\phi_s A_v f_y d_v \cot \theta}{V_f - V_c}$

## DETERMINATION OF $\beta$ AND $\theta$

CSA A23.3-14 Clause 11.3.6

### Clause 11.3.6.2 - Special Member Types

$$\beta = 0.21 \quad \theta = 42 \text{ deg.}$$

*$\beta \uparrow, \theta \downarrow \text{ slightly.} \therefore \text{not much } \Delta \text{ from } 0.2848$*

For

- slabs with thickness  $\leq 350$  mm
- beams with overall thickness  $\leq 250$  mm
- concrete joist construction (Clause 10.4)
- beams cast integrally with slabs where the depth of the beam below the slab is not greater than one-half of the web width or 350 mm

### Clause 11.3.6.3 – Simplified Method

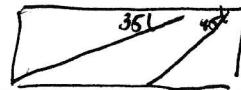
- Applicable to cases other than Clause 11.3.6.2 and members not subject to significant axial tension
- Limitations:  $f_c \leq 60$  MPa  
 $f_y \leq 400$  MPa

$$\theta = 35 \text{ deg.}$$

$$\beta = 0.18 \quad \text{for sections containing at least the minimum transverse reinforcement (Clause 11.2.8.2)}$$

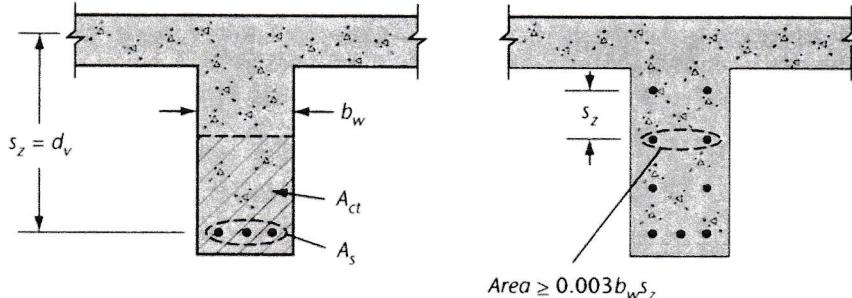
$$\beta = \frac{230}{1000 + d_v} \quad \text{for sections containing no transverse reinforcement and having maximum aggregate size} \geq 20 \text{ mm}$$

$$\beta = \frac{230}{1000 + s_{ze}} \quad \text{for sections containing no transverse reinforcement and all aggregate sizes}$$



$s_{ze}$  = equivalent crack spacing parameter

$$= \frac{35s_z}{15 + a_g} \geq 0.85s_z$$



### Additional requirements:

1) Must provide stirrups if:  $V_f \geq V_c$  or  $h > 750$  mm

2)  $A_{v,min} = 0.06 \cdot \frac{\sqrt{f'_c} \cdot b_w \cdot s}{f_y}$  *(so strength maintained after cracking)*

3)  $s_{max} = 600$  mm or  $0.7 \cdot d_v$   *$\downarrow V_{r,max}$*   
 $= 300$  mm or  $0.35 \cdot d_v$  if  $V_f > 0.125\lambda\phi_c f'_c b_w d_v$

4)  $V_{r,max} = 0.25 \cdot \phi_c f'_c b_w d_v$

5) Extend tension reinforcement  $d_v \cot \theta$  past location it is required for flexure alone (cl 11.3.9.1) or

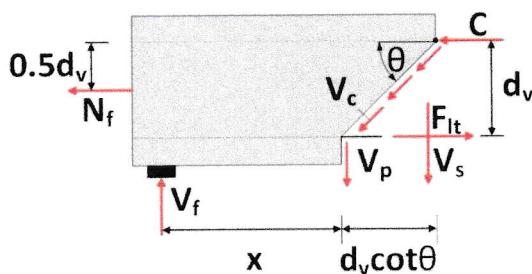
5a) provide tension reinforcement such as: (cl 11.3.9.2)

$$F_{lt} = \frac{M_f}{d_v} + 0.5N_f + \underbrace{(V_f - 0.5V_s - V_p) \cot \theta}_{\text{prestressing shear}}$$

*flexure*

$$F_{lt} \times d_v = V_f(x + d_v \cot \theta) + N_f 0.5d_v - V_s 0.5d_v \cot \theta - V_p d_v \cot \theta$$

$$F_{lt} = \frac{M_f}{d_v} + V_f \cot \theta + N_f 0.5 - V_s 0.5 \cot \theta - V_p \cot \theta$$

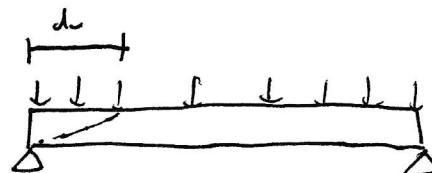


6) At exterior direct bearing supports, the longitudinal reinforcement on the flexural tension side of the member shall be capable of resisting a tensile force of  $(V_f - 0.5 \cdot V_s) \cot \theta$

7) Sections near supports (*clause 11.3.2*)

➤ **Sections located less than a distance  $d_v$  from the face of the support may be designed for the same shear,  $V_f$ , as that computed at a distance  $d_v$ , provided that**

- a) the action force in the direction of applied shear introduces compression into the member;
- b) no concentrated load that causes a shear force greater than is applied within the distance  $d_v$  from the face of the support; and
- c) loads applied within distance  $d_v$  from the face of the support do not increase the absolute magnitude of the shear at the face by more than 20%.



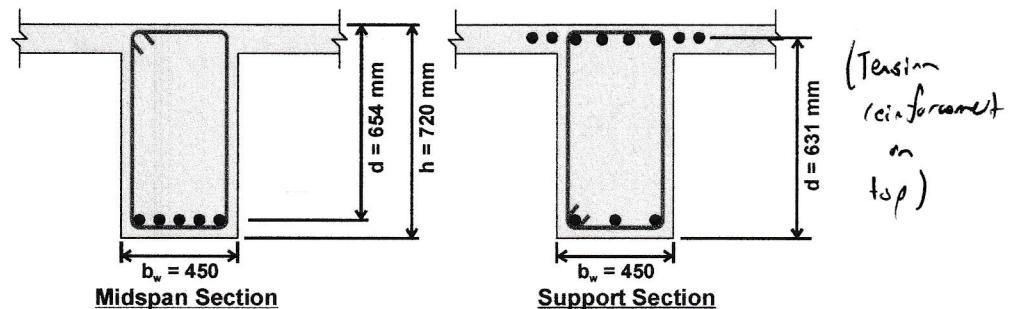
**Example 1:** The factored shear force envelope for a continuous interior beam is shown below. Design the shear reinforcement for the beam.

$$f'_c = 25 \text{ MPa}$$

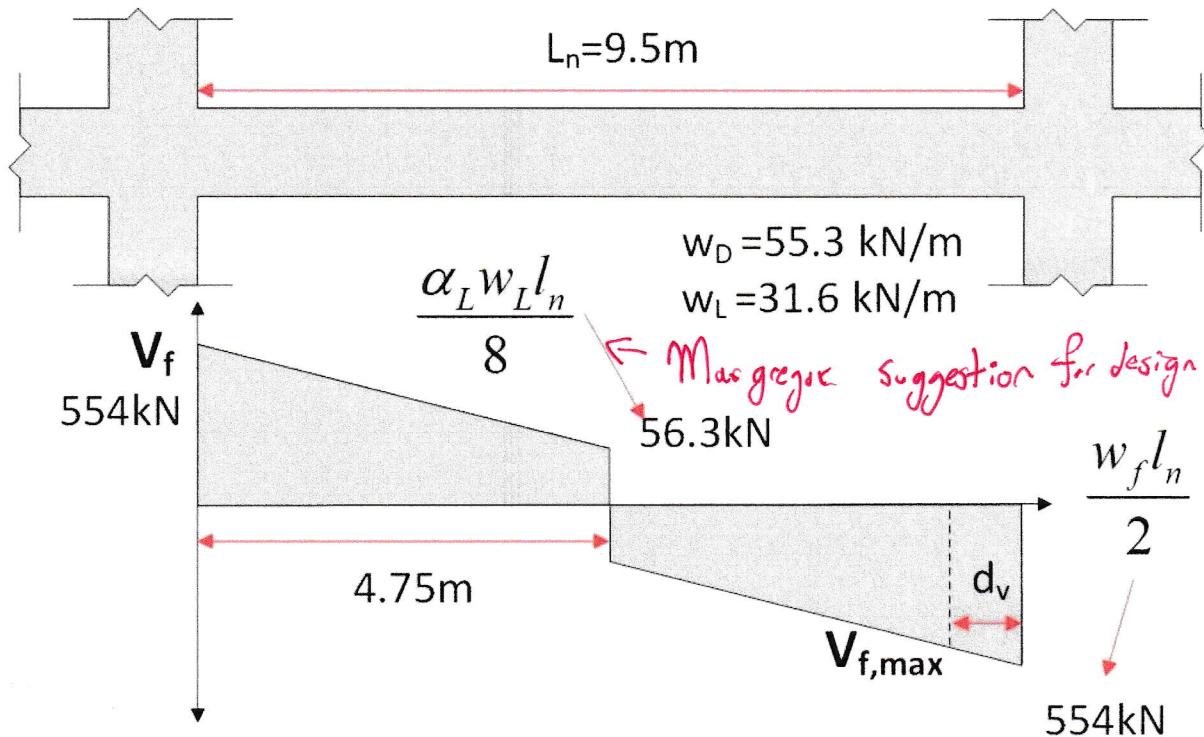
$$f_y = 400 \text{ MPa}$$

Max. C.A.

size = 20 mm



$$V_f(x) = 554 - \left( \frac{554 - 56.3}{4750} \right) x = 554 - 0.104x$$



$$\alpha_L = 1.5, \quad \alpha_D = 1.25$$

$$w_f = 116.6 \text{ kN/m}$$

$$w_D = 55.3 \text{ kN/m}$$

$$w_L = 31.6 \text{ kN/m}$$

Determine  $d_v$ :

$$d_v \geq \begin{cases} 0.9d = 0.9(631\text{mm}) = 568\text{mm} \rightarrow \text{governs} \\ 0.72h = 0.72(720\text{mm}) = 518\text{mm} \end{cases}$$

Since the maximum shear occurs at the support, use  $d=631\text{mm}$  when determining  $d_v$  for the critical section (at support). Although we could consider increasing  $d_v$  based on  $d=654\text{mm}$  in the positive moment region, it is conservative to use the ~~smaller~~ value through-out. However, use bigger anyway!!  $\ell, l$  <sup>bigger</sup>

1) Determine if section is adequate

Check  $V_{f,max} \leq V_{r,max}$  <sup>568</sup>

$V_{f,max}$  → calculate at critical section (at  $d_v$  from support, CL 11.3.2)

$V_{r,max}$  → Clause 11.3.3

$V_{f,max}$  occurs at  $d_v = 568\text{mm}$

$$V_f(568) = 554 - 0.104(568\text{mm}) = 494\text{kN}$$

$$V_{r,max} = 0.25\phi_c f'_c b_w d_v$$

<sup>tie, horizontal tie (strong)</sup>  
<sup>crash or diagonal (concrete failure)</sup>

$$= 0.25(0.65)(25\text{MPa})(450\text{mm})(568\text{mm}) \div 10^3 = 1038\text{kN} > V_{f,max} \therefore \text{max}$$

∴ section is adequate

\*If  $V_{f,max} > V_{r,max}$  → must increase  $b_w$ ,  $d_v$  or  $f'_c$  to make section adequate

2)  $\theta$  and  $\beta$ , Clause 11.3.6

Simplified method,  $\theta = 35^\circ$

With stirrups,  $\beta = 0.18$

$$\text{Without stirrups, } \beta = \frac{230}{1000+d_v} = \frac{230}{1000+568} = 0.14$$

∴ concrete contribution larger w/ stirrups, as concrete-resistance is maintained after cracks

### 3) Compute $V_c \rightarrow 2$ values

$$V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v \quad \lambda = 1, \text{ normal density concrete}$$

With stirrups:  $V_c = (0.65)(0.18)(\sqrt{25})(450)(568) \div 10^3 = 150kN$

Without stirrups:  $V_c = (0.65)(0.14)(\sqrt{25})(450)(568) \div 10^3 = 122kN$

### 4) Design Stirrups Near Support ( $V_{f,max} = 494kN$ )

Check whether stirrups are needed

$$V_f = V_{f,max} = 494kN > V_c = 122kN \text{ (without stirrups)}$$

→ stirrups are required

$$\text{Thus } V_{s,req'd} = V_{f,max} - V_c \text{ [with stirrups]} = 494 - 150 = 344kN$$

Use: 10M stirrup, double-legged  $\leftarrow 2 \text{ legs}$

$$A_v = 2(100mm^2) = 200mm^2$$

$$\text{Area of single leg of 10M stirrup} = 100mm^2$$

$$V_s = \frac{\phi_s A_v f_y d_v \cot \theta}{s}$$

$$s_{req'd} = \frac{\phi_s A_v f_y d_v \cot \theta}{V_f - V_c}$$

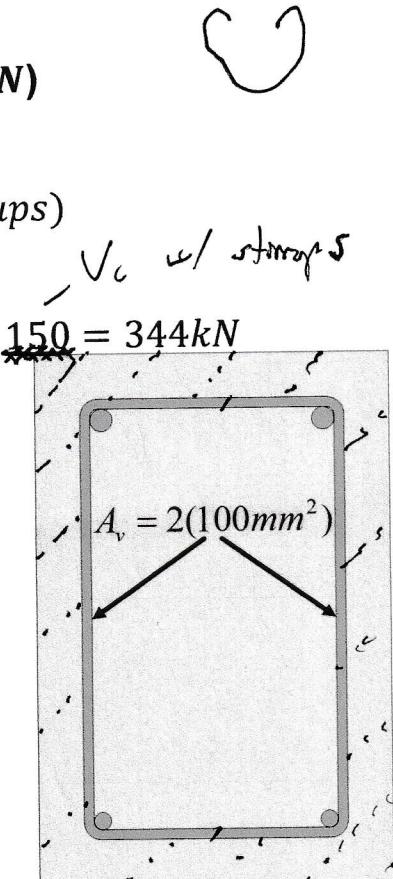
$$= \frac{(0.85)(200mm^2)(400MPa)(568mm)\cot(35^\circ)}{344 \times 10^3 N} = 160mm$$

∴ use  $s = 150mm$

spacing

$$\text{Check: } A_{v,min} = \frac{0.06 \sqrt{f'_c} b_w s}{f_y} = \frac{0.06 \sqrt{25}(450)(150)}{400} \text{ mm} = \text{spacing}$$

$$= 51mm^2 < A_v = 200mm^2 \rightarrow OK$$



### Check max spacing (Clause 11.3.8)

Two limits based on  $V_f$  at section of interest

check if  $V_f >$  or  $\leq \frac{1}{2} V_{r,max}$

$$\frac{1}{2}(V_{r,max}) = \frac{1}{2}(1038kN) = 519kN$$

Since  $V_f = V_{f,max} = 494kN < 519kN$   $\checkmark$

Then  $s_{max} \leq \begin{cases} 0.7d_v = 0.7(568) = 398mm \rightarrow governs \\ 600mm \end{cases}$

$\therefore s = 150mm < s_{max} = 398mm \rightarrow OK$

$$\text{For } s = 150mm: V_r = V_c + V_s = 150kN + \frac{\phi_s A_v f_y d_v \cot \theta}{s} \quad [s=150mm]$$

$$= 150kN + 368kN = 518kN$$

Use  $s=150mm$  near support,  $V_r = 518kN$

### 5) Design Stirrups for Other Regions of Beam

Since  $V_f < V_{f,max}$  along length of beam,  $V_{s,req'd}$  will decrease as we move away from support

- $s$  increases
- Fewer stirrups are required

Balance computational effort, construction considerations and material/labour savings

Consider:

A) Regions where no stirrups are required or where minimum stirrups are required  $\rightarrow V_{r,min}$

If  $h \leq 750mm$ , look for regions where  $V_f \leq V_c$  [no stirrups  $\beta = 0.14$  0.14] (applies to this example since  $h=720mm$ )

If  $h > 750mm$ , look for regions where  $s_{max}$  governs

$V_f \leq V_r = V_c$  [with stirrups,  $\beta = 0.18$ ] +  $V_s$  [based on  $s_{max}$ ]

$s_{max}$  based on Clause 11.3.8 or 11.2.8.2

### B) One or two other regions

$$V_{f,max} > V_f > V_{r,min} [V_{r,min} \text{ from Region A}]$$

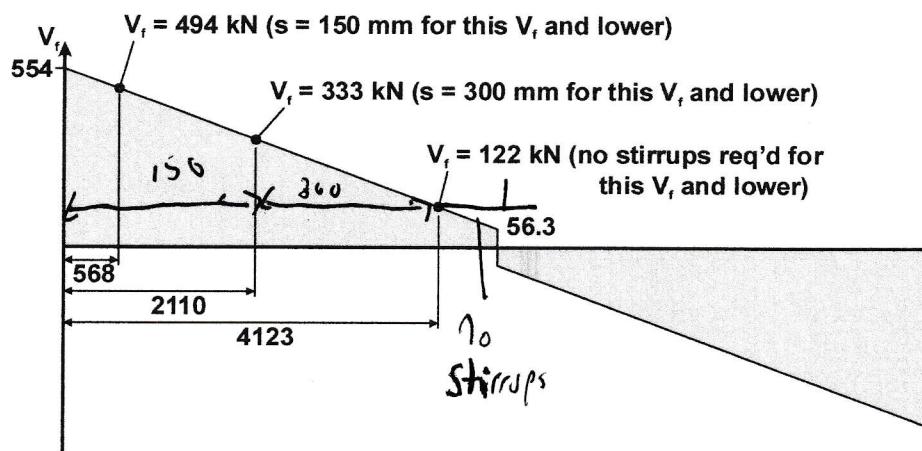
→ choose an intermediate spacing and solve for corresponding

$$V_r = 333 \text{ kN} \text{ for } s = 300 \text{ mm} \quad (\approx \text{not})$$

### A) Region where no stirrups are required since $h = 720 \text{ mm} < 750 \text{ mm}$

$$V_{r,min} = V_c = 122 \text{ kN} \text{ (no stirrups)}$$

Where  $V_f \leq 122 \text{ kN} \rightarrow \text{no stirrups required}$



### B) Intermediate Region (spacing)

Consider:

- 1) maximum spacing allowed in regions of low shear (Clause 11.3.8)

For  $V_f < 519 \text{ kN}$  [from step 4]  $\rightarrow s_{max} = 398 \text{ mm}$

**398mm is largest spacing we can use for this beam**

- 2) For  $V_f = V_{f,max} = 494 \text{ kN} \rightarrow \text{use } s = 150 \text{ mm}$

**150mm is smallest spacing we can use in this problem**

→ choose spacing between these values

→ choose  $s = 300 \text{ mm} \rightarrow \text{now calculate } V_r \text{ for } s = 300 \text{ mm}$

$$V_r = V_c + V_s = 150kN + \frac{\phi_s A_v f_y d_v \cot \theta}{s}$$

$$= 150kN + \frac{(0.85)(200)(400)(568) \cot(35^\circ)}{300mm} \div 10^3$$

$V_r = 333kN$  for  $s=300mm$

**For  $127kN \leq V_f \leq 333kN$  use  $s = 300mm \rightarrow V_r = 333kN$**

## 6) Determine Stirrup Layout and $V_r$ Diagram

\* Determine the location,  $x$ , where  $V_f = V_r$  for the various design spacings and for  $V_f = V_{r,min} = V_{c,no stirrups}$  (see next page)

Start stirrups at 50mm from face of column (must not exceed  $s/2$ )

**Region 1:**  $s=150mm$

$X=50mm$  to at least  $x=2110mm$

Use 14 spaces @  $s=150mm$ ,  $x = 50+14(150) = 2150mm$

**Region 2:**  $s=300mm$

$X=2150mm$  to at least  $x=4123mm$

Use 7 spaces @  $s=300mm$ ,  $x = 2150+7(300)=4250mm$

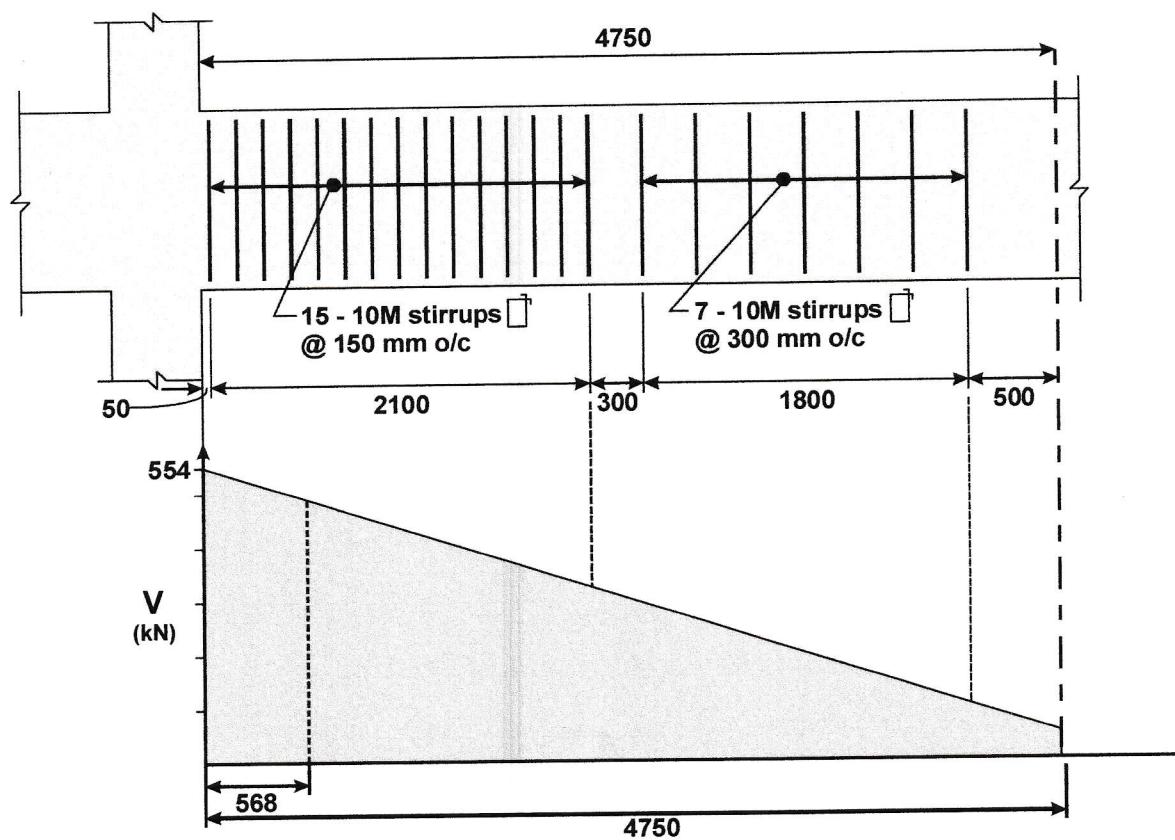
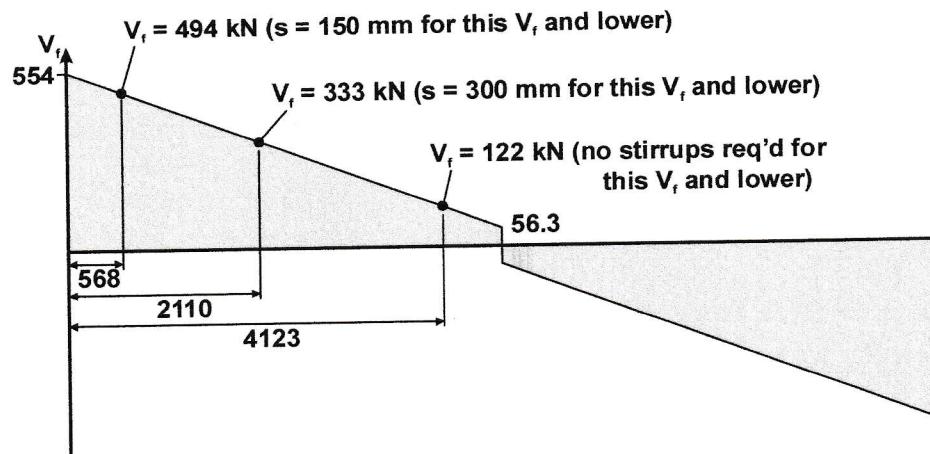
**Region 3:** no stirrups

$X=4250mm$  to beam centerline (CL)

Draw  $V_r$  Diagram, Plot  $V_r$  for 3 regions

Clause 11.3.7 – where spacing,  $s$ , changes,  $V_s$  is assumed to vary linearly over a length of “ $h$ ” centered on the location of the spacing change

## Develop stirrup layout and shear resistance envelope:



## CSA A23.3-2014 General Method: clause 11. 3.6.4

➤ Use for:

- $f'_c > 60 \text{ MPa}$
- Members subject to significant tension
- Prestressed concrete elements
- Situations where designer wants a more rigorous approach  
→ non-typical members/structures

➤ The general method is essentially the same as the simplified method (in fact it is the basis for the simplified method) in all respects except in the calculation of  $\beta$  and  $\theta$ .

➤ The general method can be summarized as follows:

$$\circ \quad \beta = \left( \frac{0.40}{1 + 1500 \cdot \varepsilon_x} \right) \cdot \left[ \frac{1300}{1000 + s_{ze}} \right] = 1 \quad (\text{for beams w/ shear reinforcement})$$

- "strain effect" → less aggregate interlock as cracks get wider
- "size effect" → increase in beam size w.r.t. aggregate size

- $s_{ze} = 300 \text{ mm if } A_v \geq A_{v,min}$  for beams w/ shear reinforcement

- otherwise,  $s_{ze}$  can be determined using the same formula given for the simplified method

→ for beams w/o shear reinforcement

*Same location*

- $\varepsilon_x = \frac{(M_f/d_v + V_f) + 0.5 \cdot N_f}{2 \cdot E_s \cdot A_s} = \text{estimate of long. strain at mid-height}$
- $\theta = 29 + 7000 \cdot \varepsilon_x$

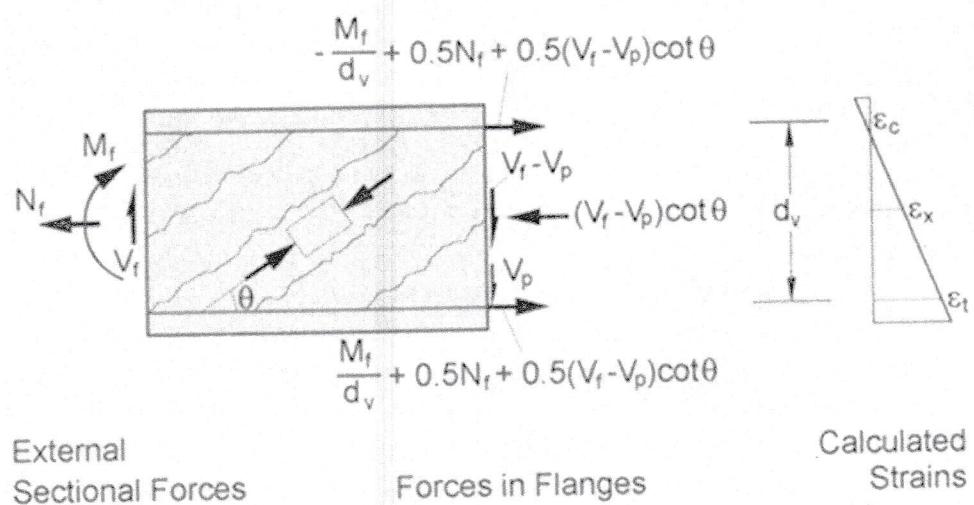
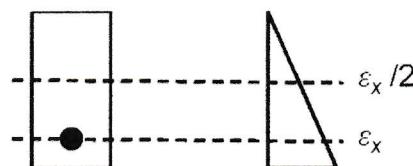
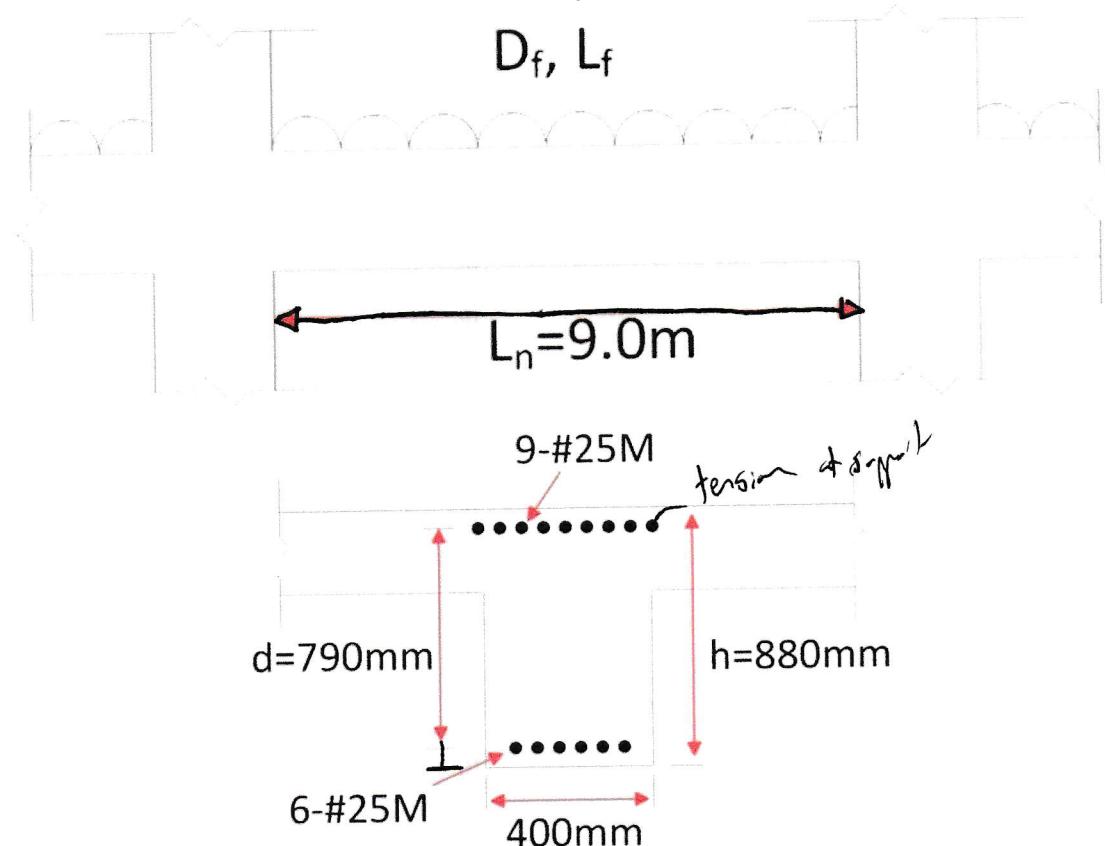


Fig. N11.3.6.4(a)  
More accurate procedure for determining  $\varepsilon_x$ .

for shear  
**EXAMPLE 2** Design the beam below using General Method-  
 – from Macgregor and Bartlett (example 6.5)

↗ factored ,  $\lambda_u = 1.5$   
 $\lambda_o = 1.25$



"My as well use smaller one for shear"

$$D_f = 60 \frac{kN}{m},$$

$$L_f = 75 \frac{kN}{m}, \quad \rightarrow \quad w_f = 135 \frac{kN}{m}$$

$$f'_c = 25\text{MPa}, \quad f_y = 300\text{MPa},$$

## Solution

### 1) Calculate the factored Shear force diagram and Bending Moment Diagram

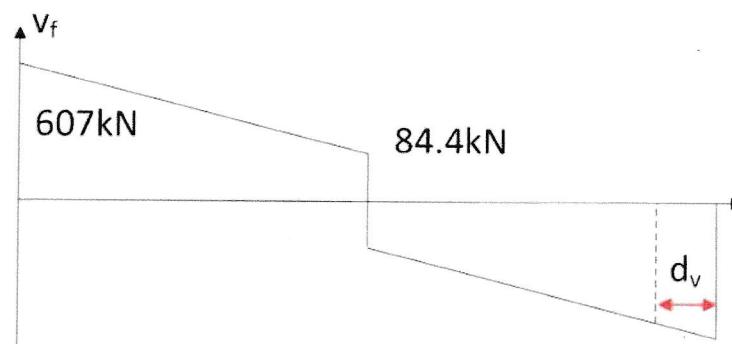
- From fig N9.3.3 (approximate Frame analysis, CSA A 23.3-04)

At the support

$$V_f = \frac{w_f \times l_n}{2} = \frac{135 \times 9}{2} = 607 \text{ kN}$$

To consider pattern loading loading, MacGregor and Bartlett suggest the following at midspan:

$$V_f = \frac{L_f \times l_n}{8} = \frac{75 \times 9}{8} = 84.4 \text{ kN}$$



$$d_v = \text{MAX}(0.9d, 0.72h) = \text{MAX}(711, 634) = 711 \text{ mm}$$

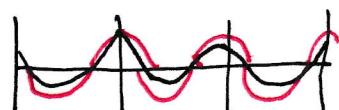
$$V_f' = 84.4 + \frac{607 - 84.4}{4500} (4500 - 711) = 525 \text{ kN} \quad \text{Q } d_v \text{ from support}$$

At the support

$$M_f = \frac{-w_f \cdot l_n^2}{11} = -994 \text{ kN} \cdot \text{m}$$

From MacGregor and Bartlett Figure A1:

$$x = \frac{711}{9000} = 0.079 \rightarrow M_f' = 0.6 \cdot M_f = -596 \text{ kN} \cdot \text{m}$$



Note: The major difference between the simplified method and the general method is that the shear resistance depends on the applied moment.

*Necessary for exam*

2522  
Is the section large enough?

Avoiding Cracking of diagonal slabs

$$V_{r\ max} = 0.25 \cdot \phi_c \cdot f'_c \cdot b_w \cdot d_v$$

$$V_{r\ max} = 0.25 \times 0.65 \times 25 \times 400 \times 711 = 1155 \text{ kN} > 525 \text{ kN}$$

## 2) Are stirrups required?

$$V_c = \lambda \cdot \phi_c \cdot \beta \cdot \sqrt{f'_c} \cdot b_w \cdot d_v$$

$$\beta = \left( \frac{0.40}{1 + 1500 \cdot \varepsilon_x} \right) \left( \frac{1300}{1000 + s_{ze}} \right)$$

Ex: large, larger  
arcs, smaller  $\beta$

$$\varepsilon_x = \frac{\left( \frac{M_f}{d_v} + V_f \right)}{2 \cdot E_s \cdot A_s} = 0.000758$$

$$s_{ze} = \frac{35 \cdot s_z}{15 + a_g} = s_z = d_v, \text{ with no stirrups, and } 20\text{mm aggregate}$$

$$\beta = 0.142$$

$$V_c = 1 \cdot 0.65 \cdot 0.142 \cdot \sqrt{25} \cdot 400 \cdot 711 = 131 \text{ kN} < 525 \text{ kN}$$

**Stirrups required**

$V_c < V_f'$   
 $\therefore$  Need stirrups

## 3) Determine the max stirrup spacing

$$0.125 \cdot \lambda \cdot \phi_c \cdot f'_c \cdot b_w \cdot d_v = 577 \text{ kN} \rightarrow \frac{1}{2} V_{r\ max}$$

$$\lambda = 1 \text{ for normal density} \quad 525 < 577$$

therefore

$$s \leq \text{MIN}(600, 0.7 \cdot d_v) = (600, 498)$$

Based on  $A_v$  min

$$A_{v\ min} = \frac{0.06 \cdot \sqrt{f'_c} \cdot b_w \cdot s}{f_y}$$

$$s \leq \frac{f_y \cdot A_v}{0.06 \cdot \sqrt{f'_c} \cdot b_w}$$

Assuming 10M stirrups

$\hookrightarrow$  Always go with 10M stirrups

$$s \leq \frac{300 \cdot 200}{0.06 \cdot \sqrt{25} \cdot 400} \leq 500\text{mm}$$

Maximum spacing is 500 mm

#### 4) Determine $s$ to resist $V_f'$

$$S_{req'd} = \frac{\phi_s \cdot A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{V_f' - V_c}$$

$$V_c = \lambda \cdot \phi_c \cdot \beta \cdot \sqrt{f_c'} \cdot b_w \cdot d_v$$

from P.25 → If  $A_v > A_{v \min}$  then  $s_{ze} = 300$

$$\beta = \left( \frac{0.40}{1 + 1500 \cdot \varepsilon_x} \right) \left( \frac{1300}{1000 + 300} \right)$$

$$\varepsilon_x = 0.000758 \rightarrow \text{doesn't change from step 2.}$$

$$\beta = 0.187$$

$$V_c = 173.1 \text{ kN}$$

$$\theta = 29 + 7000 \cdot 0.000758 = 34.3^\circ$$

$$S_{req'd} = \frac{0.85 \cdot 200 \cdot 300 \cdot 711 \cdot \cot(34.3)}{525 - 173.1} = 151\text{mm}$$

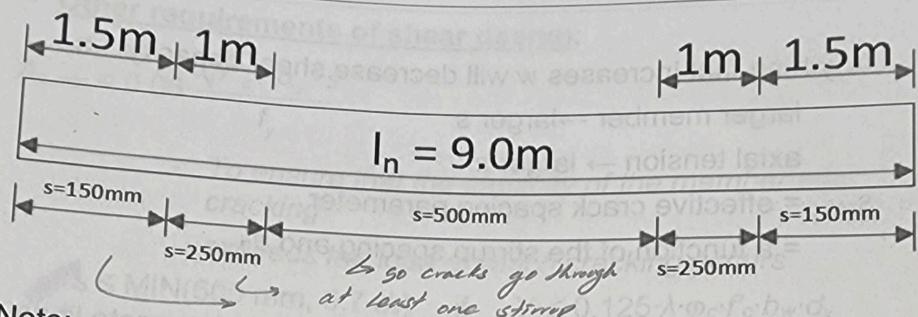
$$\frac{1}{\tan(34.3)}$$

#### 5) Repeat steps 1-4 @ several other locations

X	711	1350	2250	3600	mm
$V_f'$	525	451	346	189	kN
$M_f'$	-596	-298	342	615	$\text{kN} \cdot \text{m}$
$A_s$	4500	4500	3000	3000	$\text{mm}^2$
$S_{req'd}$	151	242	330	1747	mm

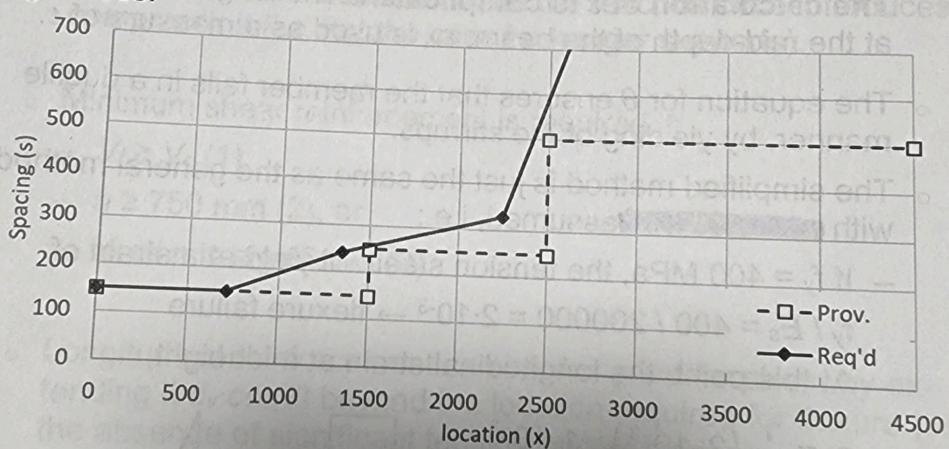
One possible stirrup layout

↳ max spacing  
is 500mm so at 3600mm  
distance,  $1747 \rightarrow 500$ .



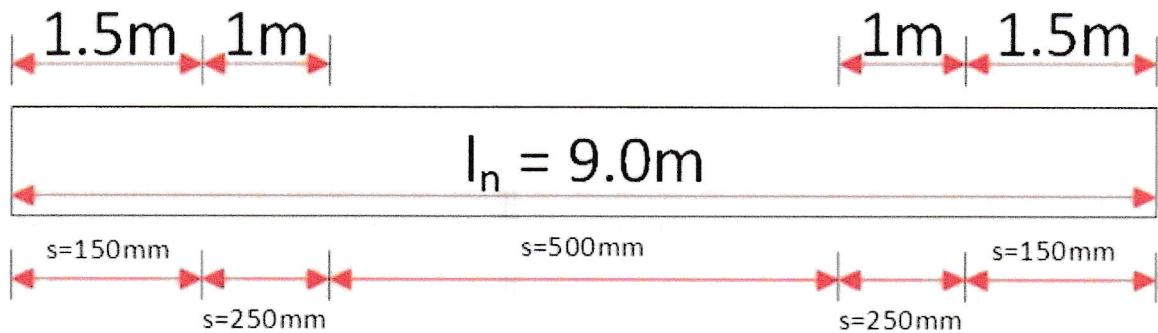
Note:

- Further check of spacing at 2500 should be done to ensure it is 500 mm
- no stirrups required when  $V_f$  drops below  $V_c$  (if  $h < 750\text{mm}$ ), however some engineers would provide minimum stirrups regardless.



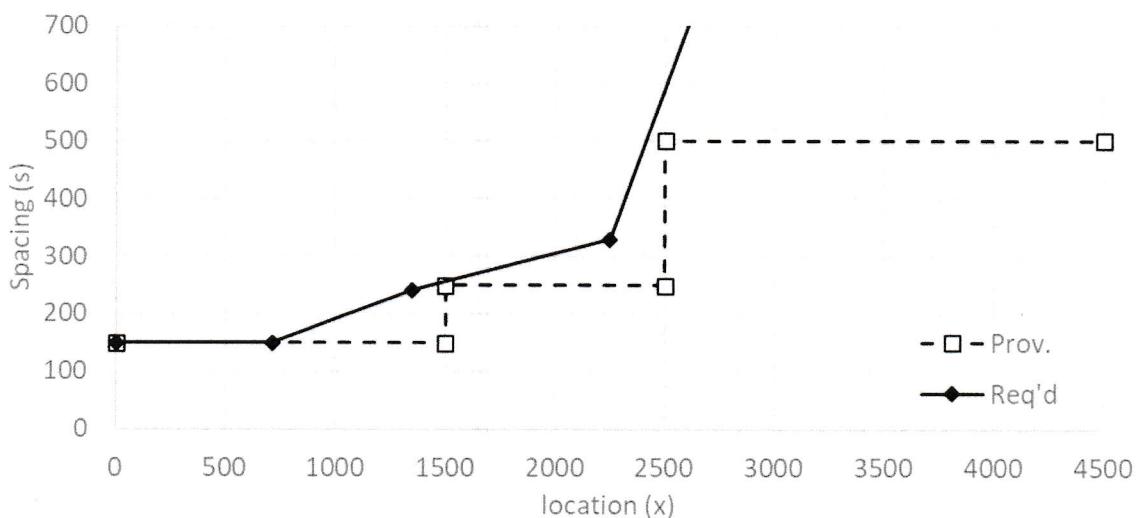
### Final Comments and Review:

- o
- o Aggregate interlock depends on the concrete strength, aggregate size, and crack width.
- o  $w = \varepsilon \cdot s$ , where  $\varepsilon$  = average strain perpendicular to crack, and  $s$  = crack spacing



Note:

- Further check of spacing at 2500 should be done to ensure it is 500 mm
- no stirrups required when  $V_f$  drops below  $V_c$  (if  $h < 750\text{mm}$ ), however some engineers would provide minimum stirrups regardless.



### Final Comments and Review:

- o Aggregate interlock depends on the concrete strength, aggregate size, and crack width.
- o  $w = \varepsilon \cdot s$ , where  $\varepsilon$  = average strain perpendicular to crack, and  $s$  = crack spacing

- Anything that increases  $w$  will decrease shear capacity, i.e.:
  - larger member  $\rightarrow$  larger  $s$
  - axial tension  $\rightarrow$  larger  $\varepsilon$

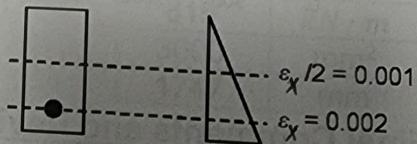
- $s_{ze}$  = effective crack spacing parameter  
= a function of the stirrup spacing and  $a_g$

- For high strength concrete ( $f'_c \geq 60$  MPa), the aggregate is assumed to fracture (i.e. the crack runs through the aggregate), and therefore the roughness parameter,  $a_g$ , is reduced:  
 $a_g = 0$  for  $f'_c \geq 70$  MPa

- **The calculation of  $\varepsilon$  is complicated.** The longitudinal strain at the mid-depth of the beam,  $\varepsilon_x$ , is used as a measure of  $\varepsilon$ .

- The equation for  $\theta$  ensures that the member fails in a ductile manner, by yielding of the stirrups.
- The simplified method is just the same as the general method with  $\varepsilon_x = 0.85 \cdot 10^{-3}$  assumed, i.e.:
  - If  $f_y = 400$  MPa, the tension steel will yield at a strain of:  
 $f_y / E_s = 400 / 200000 = 2 \cdot 10^{-3} \rightarrow$  flexure failure
  - At this point, the longitudinal strain at midheight,

$$\varepsilon_x = \frac{1}{2} \cdot (2 \cdot 10^{-3}) = 1 \cdot 10^{-3}$$



{ – The assumed  $\varepsilon_x = 0.85 \cdot 10^{-3}$  limit ensures that shear failure does not occur prior to flexural failure.

–  $\varepsilon_x = 0.85 \cdot 10^{-3} \rightarrow \theta = 35^\circ, \beta = 0.18$  (with stirrups)

$$= \frac{230}{1000 + d_v(\text{or } s_{ze})} \text{ (without stirrups)}$$

Simplified method

*W5L2*

- Other requirements of shear design:

$$A_{v,min} = 0.06 \cdot \frac{\sqrt{f'_c} \cdot b_w \cdot s}{f_y}$$

- To ensure that the capacity of the member after cracking

exceeds the load at which cracking occurs

  $s \leq \text{MIN}(600 \text{ mm}, 0.7 \cdot d_v)$  if  $V_f \leq 0.125 \cdot \lambda \cdot \varphi_c \cdot f'_c \cdot b_w \cdot d_v$

  $s \leq \text{MIN}(300 \text{ mm}, 0.35 \cdot d_v)$  if  $V_f > 0.125 \cdot \lambda \cdot \varphi_c \cdot f'_c \cdot b_w \cdot d_v$

*Not at →  
the support*

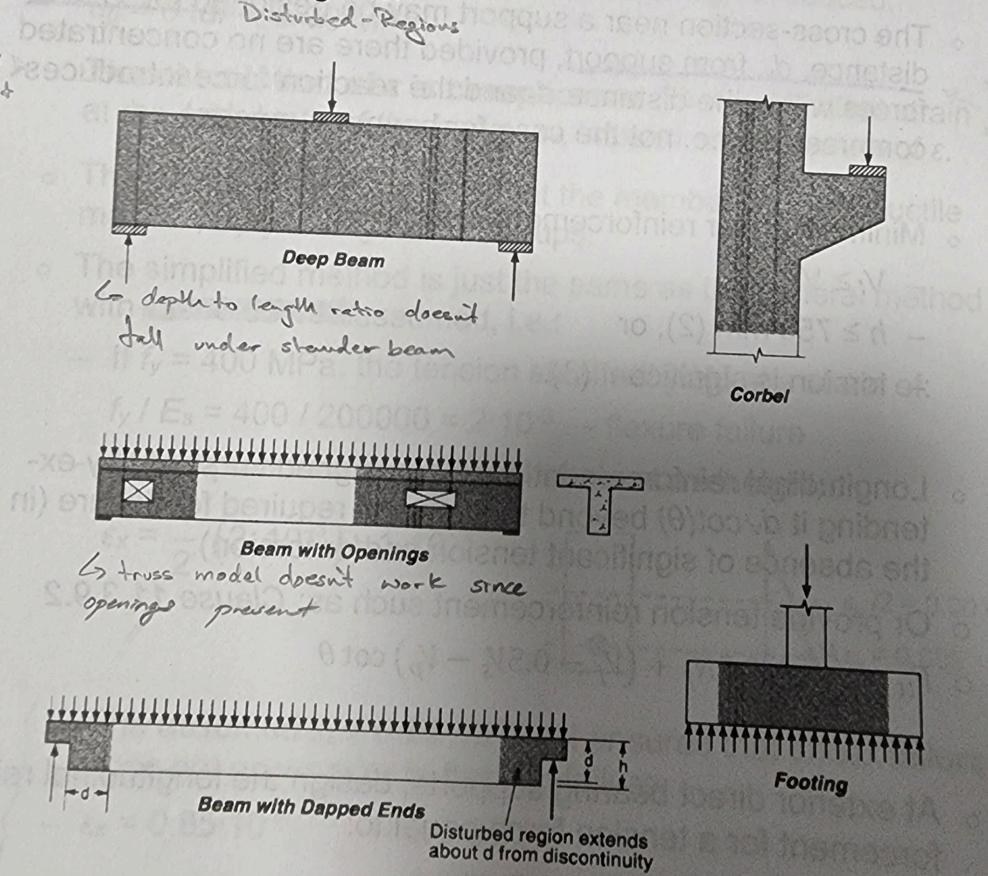
- The cross-section near a support may be designed for  $V_f$  at a distance,  $d_v$ , from support, provided there are no concentrated forces within the distance  $d_v$  and the reaction force introduces compression (i.e. not the case for hanger supports).
- Minimum shear reinforcement is required if:
  - $V_f \geq V_c$  (1)
  - $h \geq 750 \text{ mm}$  (2), or
  - torsion is significant (3).
- Longitudinal reinforcement can be designed for shear by extending it  $d_v \cdot \cot(\theta)$  beyond the location required for flexure (in the absence of significant tension and/or torsion).
- Or provide tension reinforcement such as: Clause 11.3.9.2
- $F_{lt} = \frac{M_f}{d_v} + 0.5N_f + (V_f - 0.5V_s - V_p) \cot \theta$
- At exterior direct bearing supports, design the longitudinal reinforcement for a tension force equal to:  

$$(V_f - 0.5 \cdot V_s) \cdot \text{COT}(\theta) + 0.5 \cdot N_f$$

## SHEAR DESIGN IN D-REGIONS USING STRUT-AND-TIE METHOD

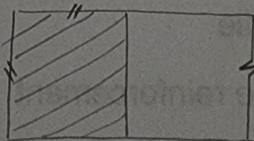
- Simple truss models and the modified compression field theory were developed for B-regions, which are characterized by a linear strain distribution (i.e. "plane sections remain plane").
- In D-regions, these approaches are not directly applicable.
- In D-regions, a significant portion of the shear is carried by in-plane forces (i.e. "arch action").
- Examples of D-regions:

for beams  
that are not  
stender

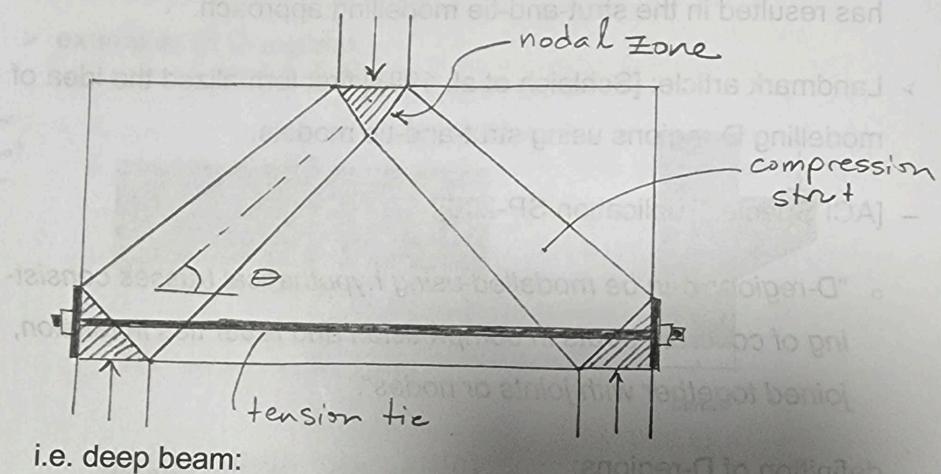


**Fig. N11.1.2(a)**  
**Examples of disturbed regions** [CSA A23.3 2004]

- CivE 414*
- D-regions are characterized by abrupt changes in geometry and/or concentrated loads.
  - Historically, D-regions designed using rules of thumb. These led to inconsistent performance and failures in some cases.
  - Work to replace rules of thumb with a rational design procedure has resulted in the strut-and-tie modelling approach.
  - Landmark article: [Schlaich et al. 1987] first formalized the idea of modelling D-regions using strut-and-tie models.
  - [ACI Special Publication SP-208]:
    - “D-regions can be modelled using hypothetical trusses consisting of concrete struts in compression and rebar ties in tension, joined together with joints or nodes”.
  - definition of D-regions:
    - based on St. Venant’s principle: “Stresses caused by a self-equilibrating stress system tend to die out over a distance equal to the extent of the self-equilibrating system.”



- 10572
- Prior to cracking, elastic (i.e. finite element) analysis can be a useful tool for predicting the behaviour of D-regions.
  - After cracking, D-region can be modeled with strut-and-tie model consisting of: 1) concrete compression struts, 2) steel tension ties, and 3) joints or nodal zones.



i.e. deep beam:

- D-region can fail by:
  1. compression failure of strut
  2. yielding of tie
  3. pullout of tie reinforcement
  4. crushing of concrete in nodal region