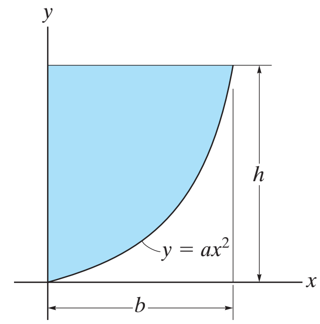
**6–1.**

Locate the centroid $\bar{x}$ of the area.




## SOLUTION

***Differential Element:*** The area element parallel to the $x$ axis shown shaded in Fig. $a$ will be considered. The area of the element is

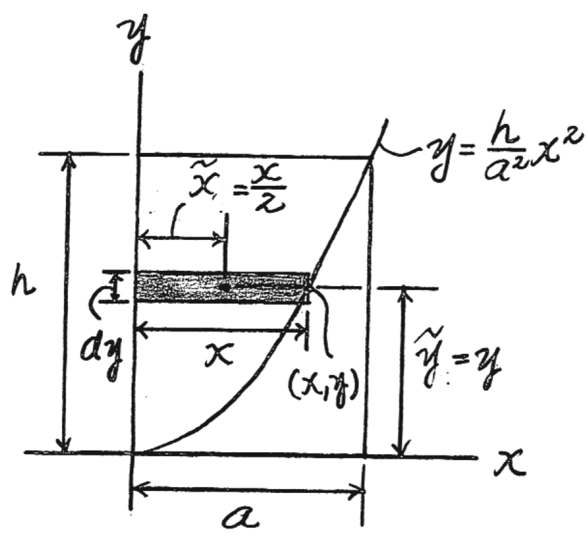
$$dA = x\,dy = \frac{a}{h^{1/2}}\,y^{1/2}\,dy$$

***Centroid:*** The centroid of the element is located at $\tilde{x} = \dfrac{x}{2} = \dfrac{a}{2h^{1/2}}\,y^{1/2}$ and $\tilde{y} = y$.

***Area:*** Integrating,

$$A = \int_A dA = \int_0^h \frac{a}{h^{1/2}}\,y^{1/2}\,dy = \frac{2a}{3h^{1/2}}\left(y^{3/2}\right)\Big|_0^h = \frac{2}{3}ah$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^h \left(\frac{a}{2h^{1/2}}y^{1/2}\right)\left(\frac{a}{h^{1/2}}\,y^{1/2}\,dy\right)}{\frac{2}{3}ah} = \frac{\int_0^h \frac{a^2}{2h}\,y\,dy}{\frac{2}{3}ah} = \frac{\frac{a^2}{2h}\left(\frac{y^2}{2}\right)\Big|_0^h}{\frac{2}{3}ah} = \frac{3}{8}a \quad \textbf{Ans.}$$



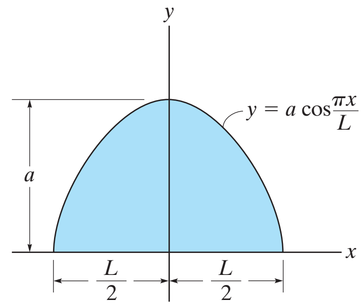

(a)

**Ans:**

$\bar{x} = \dfrac{3}{8}a$

**6–2.**

Locate the centroid of the area.




## SOLUTION

***Area And Moment Arm.*** The area of the differential element shown shaded in Fig. $a$ is $dA = ydx = a\cos\frac{\pi}{L}x\,dx$ and its centroid is at $\tilde{y} = \frac{y}{2} = \frac{a}{2}\cos\frac{\pi}{2}x.$

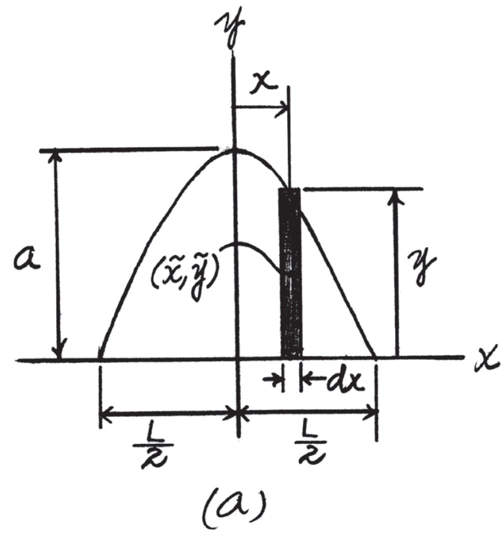
***Centroid.*** Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{-L/2}^{L/2}\left(\frac{a}{2}\cos\frac{\pi}{L}x\right)\left(a\cos\frac{\pi}{L}x\,dx\right)}{\int_{-L/2}^{L/2} a\cos\frac{\pi}{L}x\,dx}$$

$$= \frac{\int_{-L/2}^{L/2}\frac{a^2}{4}\left(\cos\frac{2\pi}{L}x + 1\right)dx}{\int_{-L/2}^{L/2} a\cos\frac{\pi}{L}x\,dx}$$

$$= \frac{\frac{a^2}{4}\left(\frac{L}{2\pi}\sin\frac{2\pi}{L}x + x\right)\Big|_{-L/2}^{L/2}}{\left(\frac{aL}{\pi}\sin\frac{\pi}{L}x\right)\Big|_{-L/2}^{L/2}}$$

$$= \frac{a^2L/4}{2aL/\pi} = \frac{\pi}{8}a \qquad \textbf{Ans.}$$



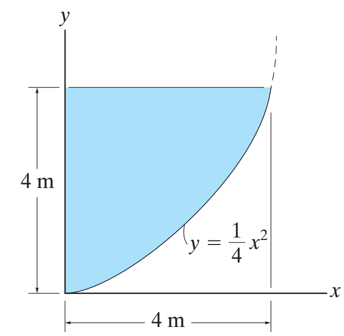

Due to symmetry,

$$\bar{x} = 0 \qquad \textbf{Ans.}$$

**Ans:**
$\bar{y} = \frac{\pi}{8}a$
$\bar{x} = 0$

**6–3.**

Locate the centroid $\bar{x}$ of the area.



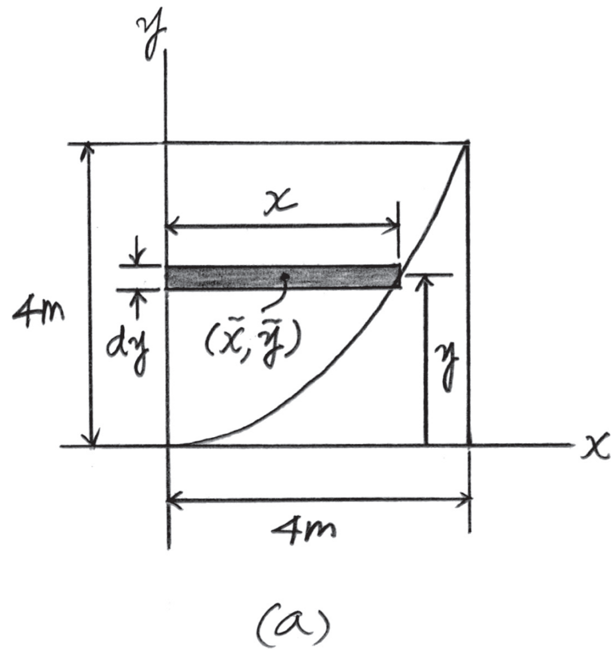

## SOLUTION

***Area And Moment Arm.*** The area of the differential element shown shaded in Fig. *a* is $dA = x\,dy$ and its centroid is at $\tilde{x} = \frac{1}{2}x$. Here, $x = 2y^{1/2}$.

***Centroid.*** Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} \frac{1}{2}\left(2y^{1/2}\right)\left(2y^{1/2}\,dy\right)}{\int_0^{4\text{ m}} 2y^{1/2}\,dy}$$

$$= \frac{3}{2}\text{ m} \qquad \textbf{Ans.}$$

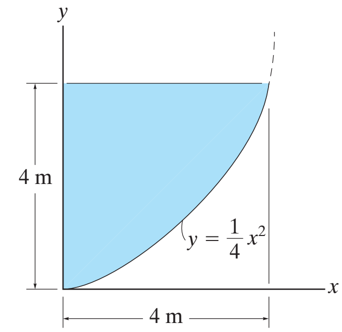



(a)

**Ans:**
$\bar{x} = \frac{3}{2}$ m

***6–4.**

Locate the centroid $\bar{y}$ of the area.




## SOLUTION

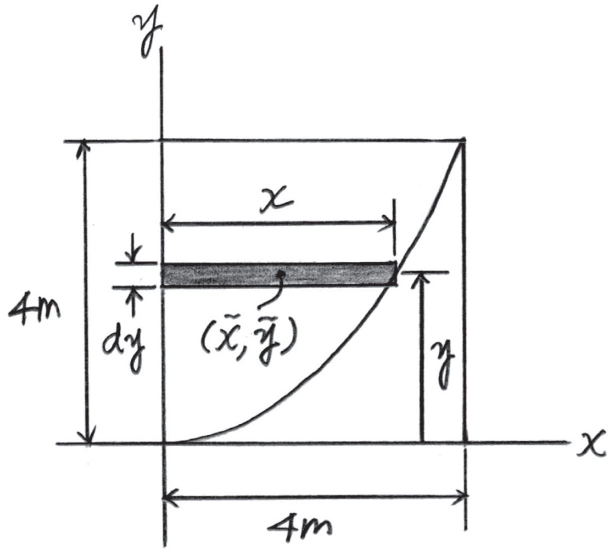
***Area And Moment Arm.*** The area of the differential element shown shaded in Fig. *a* is $dA = x\,dy$ and its centroid is at $\tilde{y} = y$. Here, $x = 2y^{1/2}$.

***Centroid.*** Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} y\left(2y^{1/2}\,dy\right)}{\int_0^{4\text{ m}} 2y^{1/2}\,dy}$$

$$= \frac{\left(\frac{4}{5}y^{5/2}\right)\Big|_0^{4\text{ m}}}{\left(\frac{4}{3}y^{3/2}\right)\Big|_0^{4\text{ m}}}$$

$$= \frac{12}{5}\text{ m}$$ **Ans.**

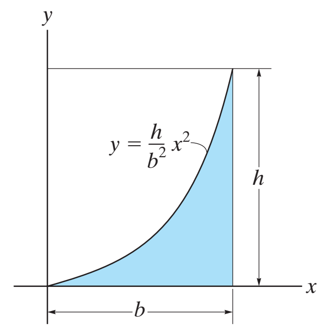



(a)

**Ans:**
$\bar{y} = \frac{12}{5}$ m

**6–5.**

Locate the centroid $\bar{x}$ of the area.

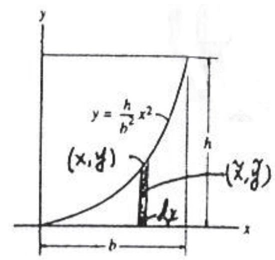



## SOLUTION

$dA = y\,dx$

$\tilde{x} = x$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^b \frac{h}{b^2}x^3\,dx}{\int_0^b \frac{h}{b^2}x^2\,dx} = \frac{\left[\frac{h}{4b^2}x^4\right]_0^b}{\left[\frac{h}{3b^2}x^3\right]_0^b} = \frac{3}{4}b$$ **Ans.**

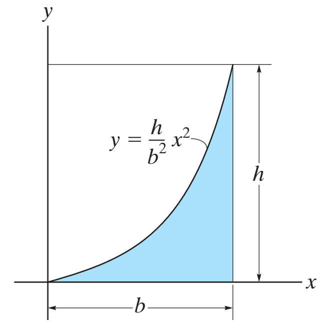



**Ans:**

$\bar{x} = \frac{3}{4}b$

**6–6.**

Locate the centroid $\overline{y}$ of the area.

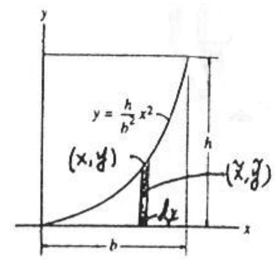



## SOLUTION

$$dA = y\,dx$$

$$\widetilde{y} = \frac{y}{2}$$

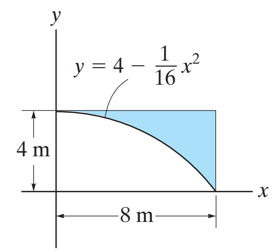
$$\overline{y} = \frac{\int_A \widetilde{y}\,dA}{\int_A dA} = \frac{\int_0^b \frac{h^2}{2b^4}x^4\,dx}{\int_0^b \frac{h}{b^2}x^2\,dx} = \frac{\left[\frac{h^2}{10b^4}x^5\right]_0^b}{\left[\frac{h}{3b^2}x^3\right]_0^b} = \frac{3}{10}h$$ **Ans.**




**Ans:**
$\overline{y} = \frac{3}{10}h$

**6–7.**

Locate the centroid $\bar{x}$ of the area.



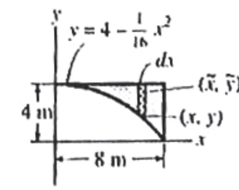

## SOLUTION

$$dA = (4 - y)dx = \left(\frac{1}{16}x^2\right)dx$$

$$\tilde{x} = x$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^8 x\left(\frac{x^2}{16}\right)dx}{\int_0^8 \left(\frac{1}{16}x^2\right)dx}$$

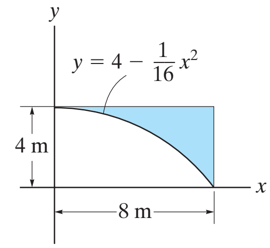
$$\bar{x} = 6 \text{ m}$$ **Ans.**




**Ans:**
$\bar{x} = 6$ m

***6–8.**

Locate the centroid $\overline{y}$ of the area.



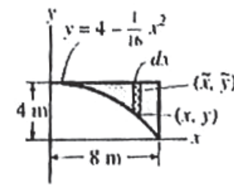

## SOLUTION

$$dA = (4 - y)dx = \left(\frac{1}{16}x^2\right)dx$$

$$\tilde{y} = \frac{4 + y}{2}$$

$$\overline{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^8 \left(8 - \frac{x^2}{16}\right)\left(\frac{x^2}{16}\right)dx}{\int_0^8 \left(\frac{1}{16}x^2\right)dx}$$

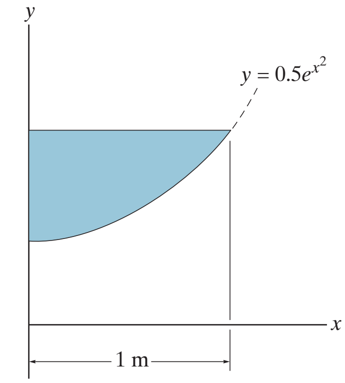
$$\overline{y} = 2.8 \text{ m}$$ **Ans.**



**Ans:**
$\overline{y} = 2.8$ m

**6–9.**

Locate the centroid $\bar{x}$ of the area. Solve the problem by evaluating the integrals using Simpson's rule.




## SOLUTION

At $x = 1$ m,

$$y = 0.5e^{1^2} = 1.359 \text{ m}$$

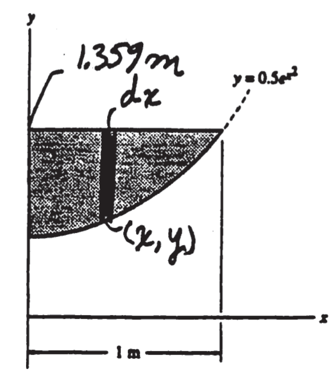
$$\int_A dA = \int_0^1 (1.359 - y)\,dx = \int_0^1 \left(1.359 = 0.5\,e^{x^2}\right) dx = 0.6278 \text{ m}^2$$

$$\bar{x} = x$$

$$\int_A \bar{x}\,dA = \int_0^1 x\left(1.359 - 0.5\,e^{x^2}\right) dx$$

$$= 0.25 \text{ m}^3$$

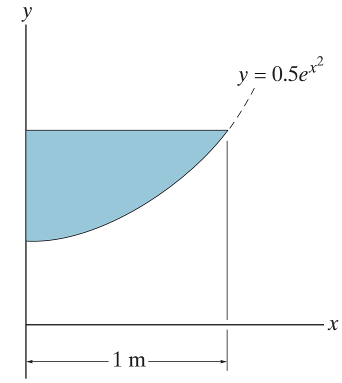
$$\bar{x} = \frac{\int_A \bar{x}\,dA}{\int_A dA} = \frac{0.25}{0.6278} = 0.398 \text{ m}$$ **Ans.**




**Ans:**
$\bar{x} = 0.398$ m

**6–10.**

Locate the centroid $\bar{y}$ of the area. Solve the problem by evaluating the integrals using Simpson's rule.




## SOLUTION

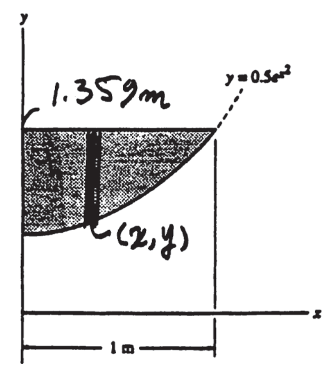
$$\int_A dA = \int_0^1 (1.359 - y)\,dx = \int_0^1 \left(1.359 - 0.5e^{x^2}\right) dx = 0.6278 \text{ m}^2$$

$$\bar{y} = \frac{1.359 + y}{2}$$

$$\int_A \bar{y}\,dA = \int_0^1 \left(\frac{1.359 + 0.5\,e^{x^2}}{2}\right)\left(1.359 - 0.5\,e^{x^2}\right) dx$$

$$= \frac{1}{2}\int_0^1 \left(1.847 - 0.25\,e^{2x^2}\right) dx = 0.6278 \text{ m}^3$$

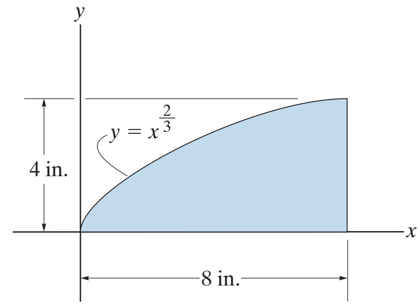
$$\bar{y} = \frac{\int_A \bar{y}\,dA}{\int_A dA} = \frac{0.6278}{0.6278} = 1.00 \text{ m}$$ **Ans.**




**Ans:**
$\bar{y} = 1.00$ m

**6–11.**

Locate the centroid $\bar{y}$ of the area.




## SOLUTION

***Area:*** Integrating the area of the differential element gives

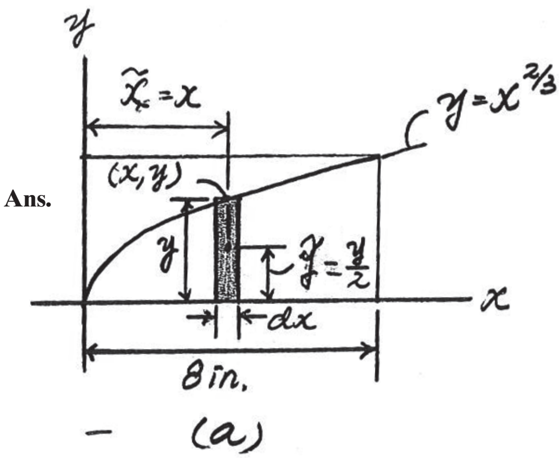
$$A = \int_A dA = \int_0^{8\text{ in.}} x^{2/3}\,dx = \left[\frac{3}{5}x^{5/3}\right]\Bigg|_0^{8\text{ in.}} = 19.2\text{ in.}^2$$

***Centroid:*** The centroid of the element is located at $\tilde{y} = y/2 = \frac{1}{2}x^{2/3}$. Applying Eq. 9–4, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{8\text{ in.}} \frac{1}{2}x^{2/3}\left(x^{2/3}\right)dx}{19.2} = \frac{\int_0^{8\text{ in.}} \frac{1}{2}x^{4/3}dx}{19.2}$$

$$= \frac{\left[\frac{3}{14}x^{7/3}\right]\Big|_0^{8\text{ in.}}}{19.2} = 1.43\text{ in.}$$

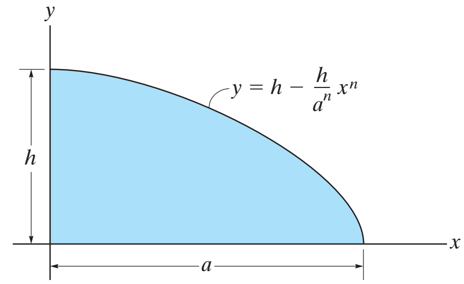
**Ans.**




**Ans:**
$\bar{y} = 1.43$ in.

***6–12.**

Locate the centroid $\bar{x}$ of the area.




## SOLUTION

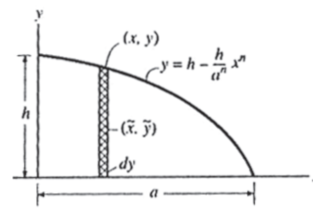
$$dA = y\,dx$$

$$\tilde{x} = x$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a \left(hx - \frac{h}{a^n}x^{n+1}\right)dx}{\int_0^a \left(h - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\left[\frac{h}{2}x^2 - \frac{h(x^{n+2})}{a^n(n+2)}\right]_0^a}{\left[hx - \frac{h(x^{n+1})}{a^n(n+1)}\right]_0^a}$$

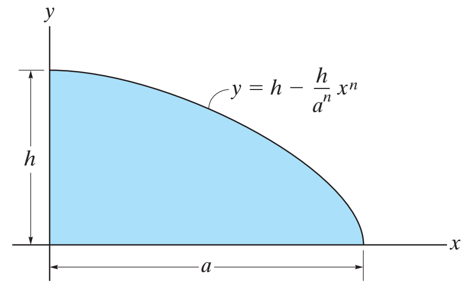
$$\bar{x} = \frac{\left(\frac{h}{2} - \frac{h}{n+2}\right)a^2}{\left(h - \frac{h}{n+1}\right)a} = \frac{a(1+n)}{2(2+n)}$$ **Ans.**




**Ans:**
$$\bar{x} = \frac{a(1+n)}{2(2+n)}$$

**6–13.**

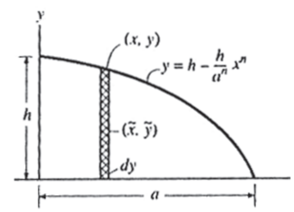
Locate the centroid $\bar{y}$ of the area.




## SOLUTION

$$dA = y\,dx$$

$$\tilde{y} = \frac{y}{2}$$




$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \left(h^2 - 2\frac{h^2}{a^n}x^n + \frac{h^2}{a^{2n}}x^{2n}\right)dx}{\int_0^a \left(h - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\frac{1}{2}\left[h^2x - \frac{2h^2(x^{n+1})}{a^n(n+1)} + \frac{h^2(x^{2n+1})}{a^{2n}(2n+1)}\right]_0^a}{\left[hx - \frac{h(x^{n+1})}{a^n(n+1)}\right]_0^a}$$

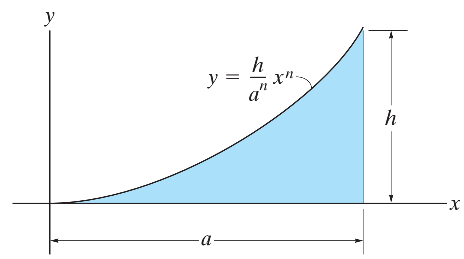
$$\bar{y} = \frac{\frac{2n^2}{2(n+1)(2n+1)}h}{\frac{n}{n+1}} = \frac{hn}{2n+1}$$ **Ans.**

**Ans:**

$$\bar{y} = \frac{hn}{2n+1}$$

**6–14.**

Locate the centroid $\bar{y}$ of the area.

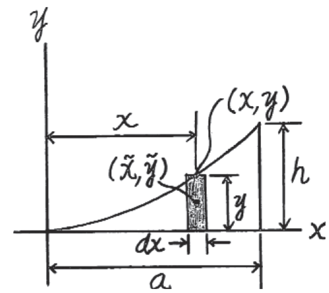



## SOLUTION

$$dA = y\,dx$$

$$\tilde{y} = \frac{y}{2}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \frac{h^2}{a^{2n}}x^{2n}\,dx}{\int_0^a \frac{h}{a^n}x^n\,dx} = \frac{\frac{h^2(a^{2n+1})}{2a^{2n}(2n+1)}}{\frac{h(a^{n+1})}{a^n(n+1)}} = \frac{hn+1}{2(2n+1)}$$ **Ans.**

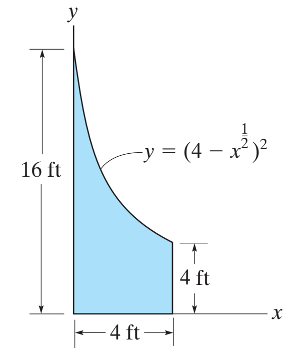



**Ans:**

$$\bar{y} = \frac{hn+1}{2(2n+1)}$$

**6–15.**

Locate the centroid $\bar{x}$ of the area.




## SOLUTION

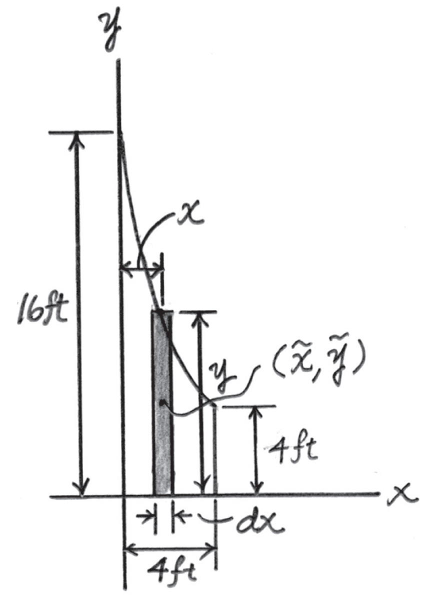
***Area And Moment Arm.*** The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(4 - x^{1/2}\right)^2 dx = (x - 8x^{1/2} + 16)dx$ and its centroid is at $\tilde{x} = x$.

***Centroid.*** Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ ft}} x(x - 8x^{1/2} + 16)dx}{\int_0^{4\text{ ft}} \left(x - 8x^{1/2} + 16\right) dx}$$

$$= \frac{\left.\left(\frac{x^3}{3} - \frac{16}{5}x^{5/2} + 8x^2\right)\right|_0^{4\text{ ft}}}{\left.\left(\frac{x^2}{2} - \frac{16}{3}x^{3/2} + 16x\right)\right|_0^{4\text{ ft}}}$$

$$= 1\frac{3}{5}\text{ ft} \qquad \textbf{Ans.}$$

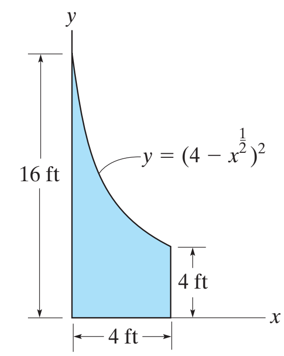



(a)

**Ans:**
$\bar{x} = 1\frac{3}{5}$ ft

***6–16.**

Locate the centroid $\bar{y}$ of the area.




## SOLUTION

***Area And Moment Arm.*** The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(4 - x^{\frac{1}{2}}\right)^2 dx = (x - 8x^{1/2} + 16)dx$ and its centroid is at $\tilde{y} = \dfrac{y}{2} = \dfrac{1}{2}\left(4 - x^{1/2}\right)^2$.

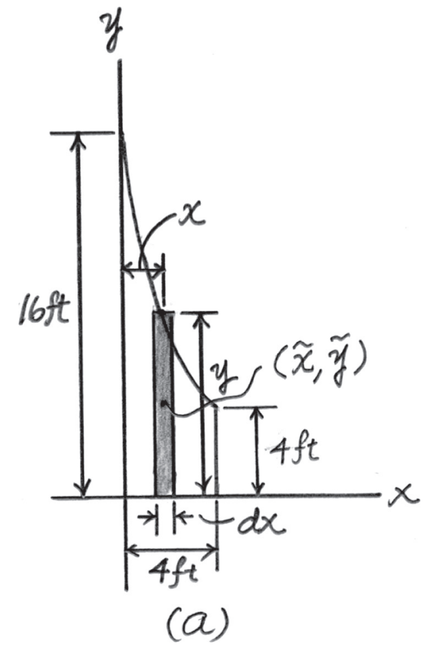
***Centroid.*** Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\displaystyle\int_0^{4\text{ ft}} \frac{1}{2}\left(4 - x^{1/2}\right)^2\left(x - 8x^{1/2} + 16\right)dx}{\displaystyle\int_0^{4\text{ ft}} (x - 8x^{1/2} + 16)dx}$$

$$= \frac{\displaystyle\int_0^{4\text{ ft}} \left(\frac{1}{2}x^2 - 8x^{3/2} + 48x - 128x^{1/2} + 128\right)dx}{\displaystyle\int_0^{4\text{ ft}} \left(x - 8x^{1/2} + 16\right)dx}$$

$$= \frac{\left(\dfrac{x^3}{6} - \dfrac{16}{5}x^{5/2} + 24x^2 - \dfrac{256}{3}x^{3/2} + 128x\right)\Bigg|_0^{4\text{ ft}}}{\left(\dfrac{x^2}{2} - \dfrac{16}{3}x^{3/2} + 16x\right)\Bigg|_0^{4\text{ ft}}}$$

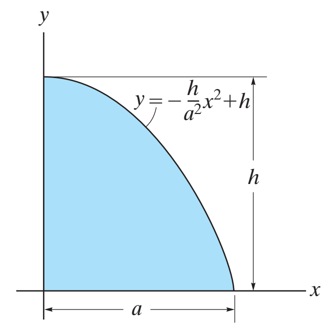
$$= 4\frac{8}{55}\text{ ft}$$ **Ans.**




**Ans:**
$\bar{y} = 4\dfrac{8}{55}$ ft

**6–17.**

Locate the centroid $\bar{x}$ of the area.




## SOLUTION

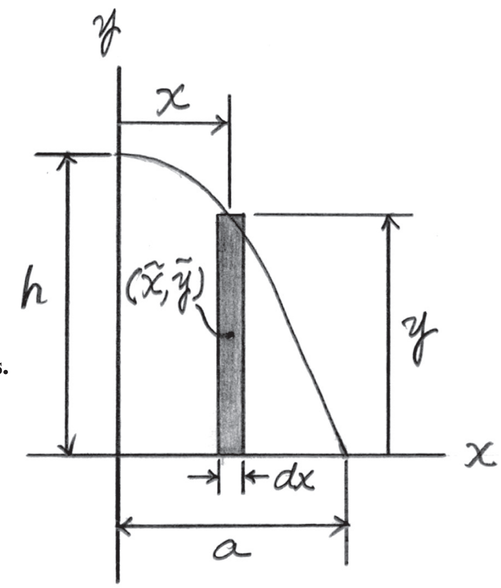
***Area And Moment Arm.*** The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(-\frac{h}{a^2}x^2 + h\right)dx$ and its centroid is at $\tilde{x} = x$.

***Centroid.*** Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left(-\frac{h}{a^2}x^2 + h\right)dx}{\int_0^a \left(-\frac{h}{a^2}x^2 + h\right)dx}$$

$$= \frac{\left.\left(-\frac{h}{4a^2}x^4 + \frac{h}{2}x^2\right)\right|_0^a}{\left.\left(-\frac{h}{3a^2}x^3 + hx\right)\right|_0^a}$$

$$= \frac{3}{8}a \qquad \textbf{Ans.}$$

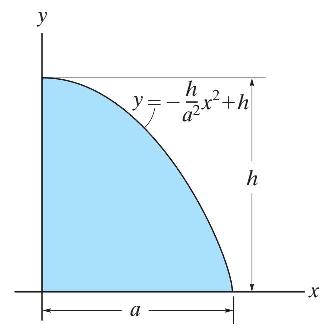



(a)

**Ans:**
$\bar{x} = \frac{3}{8}a$

**6–18.**

Locate the centroid $\bar{y}$ of the area.




## SOLUTION

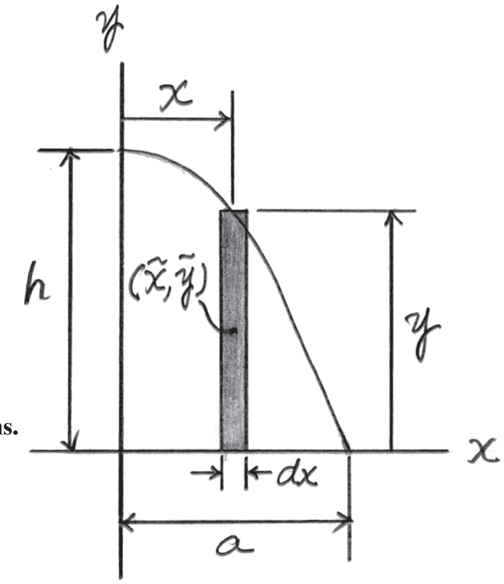
***Area And Moment Arm.*** The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(-\frac{h}{a^2}x^2 + h\right)dx$ and its centroid is at $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(-\frac{h^2}{a}x^2 + h\right)$.

***Centroid.*** Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^a \frac{1}{2}\left(-\frac{h}{a^2}x^2 + h\right)\left(-\frac{h}{a^2}x^2 + h\right)dx}{\int_0^a \left(-\frac{h}{a^2}x^2 + h\right)dx}$$

$$= \frac{\frac{1}{2}\left(\frac{h^2}{5a^4}x^5 - \frac{2h^2}{3a^2}x^3 + h^2x\right)\Big|_0^a}{\left(-\frac{h}{3a^2}x^3 + hx\right)\Big|_0^a}$$

$$= \frac{2}{5}h$$ **Ans.**



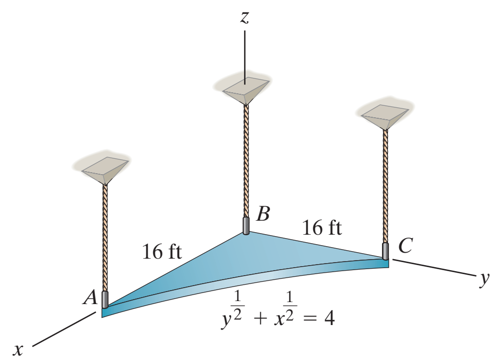

(a)

**Ans:**

$\bar{y} = \frac{2}{5}h$

**6–19.**

The plate has a thickness of 0.25 ft and a specific weight of $\gamma = 180\ \mathrm{lb/ft^3}$. Determine the location of its center of gravity. Also, find the tension in each of the cords used to support it.

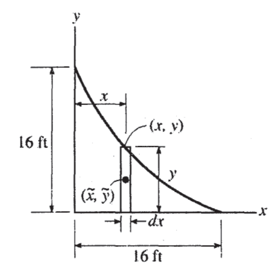



## SOLUTION

***Area and Moment Arm:*** Here, $y = x - 8x^{\frac{1}{2}} + 16$. The area of the differential element is $dA = ydx = (x - 8x^{\frac{1}{2}} + 16)dx$ and its centroid is $\widetilde{x} = x$ and $\widetilde{y} = \dfrac{1}{2}(x - 8x^{\frac{1}{2}} + 16)$. Evaluating the integrals, we have

$$A = \int_A dA = \int_0^{16\ \mathrm{ft}} (x - 8x^{\frac{1}{2}} + 16)dx$$

$$= \left(\frac{1}{2}x^2 - \frac{16}{3}x^{\frac{3}{2}} + 16x\right)\Bigg|_0^{16\ \mathrm{ft}} = 42.67\ \mathrm{ft}^2$$




$$\int_A \widetilde{x}dA = \int_0^{16\ \mathrm{ft}} x[(x - 8x^{\frac{1}{2}} + 16)dx]$$

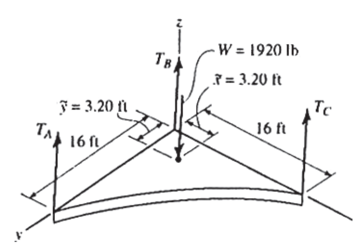
$$= \left(\frac{1}{3}x^3 - \frac{16}{5}x^{\frac{5}{2}} + 8x^2\right)\Bigg|_0^{16\ \mathrm{ft}} = 136.53\ \mathrm{ft}^3$$

$$\int_A \widetilde{y}dA = \int_0^{16\ \mathrm{ft}} \frac{1}{2}(x - 8x^{\frac{1}{2}} + 16)[(x - 8x^{\frac{1}{2}} + 16)dx]$$

$$= \frac{1}{2}\left(\frac{1}{3}x^3 - \frac{32}{5}x^{\frac{5}{2}} + 48x^2 - \frac{512}{3}x^{\frac{3}{2}} + 256x\right)\Bigg|_0^{16\ \mathrm{ft}}$$

$$= 136.53\ \mathrm{ft}^3$$

***Centroid:*** Applying Eq. 9–6, we have




$$\bar{x} = \frac{\int_A \widetilde{x}dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \mathrm{ft} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \widetilde{y}dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \mathrm{ft} \qquad \textbf{Ans.}$$

***Equations of Equilibrium:*** The weight of the plate is $W = 42.67(0.25)(180) = 1920$ lb.

$$\Sigma M_x = 0; \qquad 1920(3.20) - T_A(16) = 0 \quad T_A = 384\ \mathrm{lb} \qquad \textbf{Ans.}$$

$$\Sigma M_y = 0; \qquad T_C(16) - 1920(3.20) = 0 \quad T_C = 384\ \mathrm{lb} \qquad \textbf{Ans.}$$

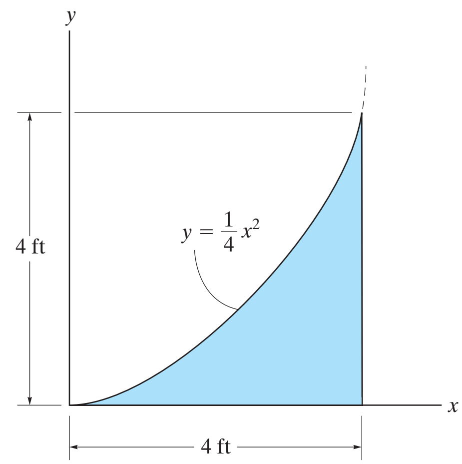
$$\Sigma F_z = 0; \qquad T_B + 384 + 384 - 1920 = 0$$

$$T_B = 1152\ \mathrm{lb} = 1.15\ \mathrm{kip} \qquad \textbf{Ans.}$$

**Ans:**
$\bar{x} = 3.20$ ft
$\bar{y} = 3.20$ ft
$T_A = 384$ lb
$T_C = 384$ lb
$T_B = 1.15$ kip

***6–20.**

Locate the centroid $\bar{x}$ of the area.




## SOLUTION

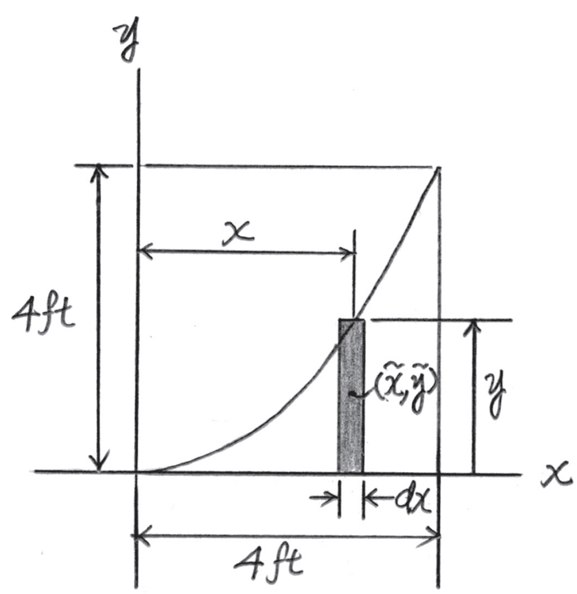
***Area And Moment Arm.*** The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \frac{1}{4}x^2\,dx$ and its centroid is at $\tilde{x} = x$.

***Centroid.*** Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ ft}} x\left(\frac{1}{4}x^2\,dx\right)}{\int_0^{4\text{ ft}} \frac{1}{4}x^2 dx}$$

$$= \frac{\left(\frac{1}{16}x^4\right)\Big|_0^{4\text{ ft}}}{\left(\frac{1}{12}x^3\right)\Big|_0^{4\text{ ft}}}$$

$$= 3\text{ ft}$$ **Ans.**

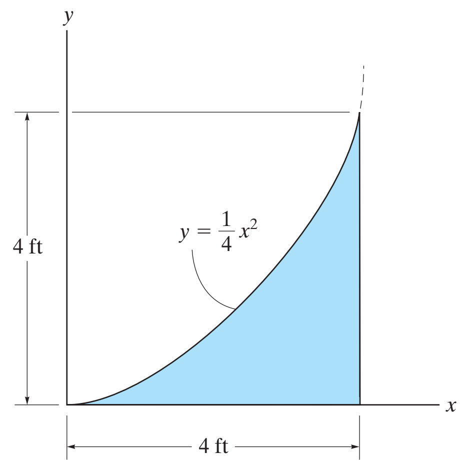



(a)

**Ans:**
$\bar{x} = 3$ ft

**6–21.**

Locate the centroid $\bar{y}$ of the area.



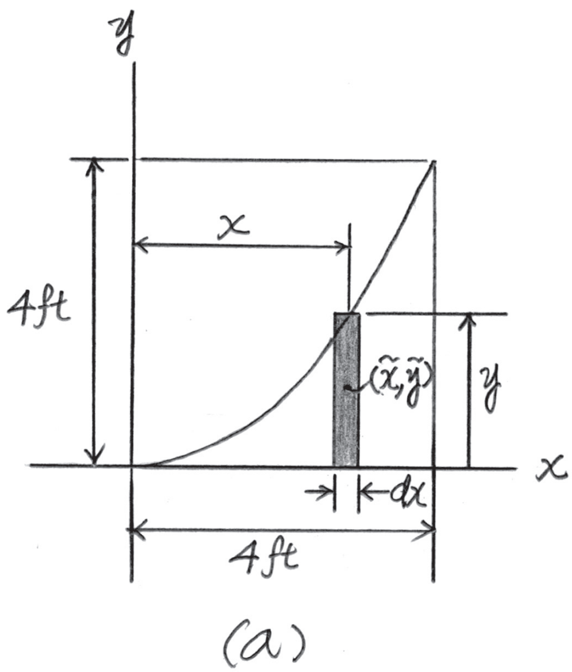

## SOLUTION

***Area And Moment Arm.*** The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \frac{1}{4}x^2\,dx$ and its centroid is located at $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(\frac{1}{4}x^2\right) = \frac{1}{8}x^2$.

***Centroid.*** Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\displaystyle\int_0^{4\text{ ft}} \frac{1}{8}x^2\left(\frac{1}{4}x^2\,dx\right)}{\displaystyle\int_0^{4\text{ ft}} \frac{1}{4}x^2 dx}$$

$$= \frac{6}{5}\text{ ft} \qquad \textbf{Ans.}$$

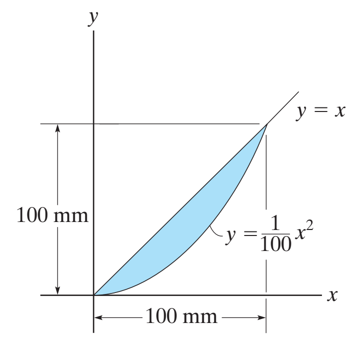



(a)

**Ans:**
$\bar{y} = \frac{6}{5}$ ft

**6–22.**

Locate the centroid $\bar{x}$ of the area.




## SOLUTION

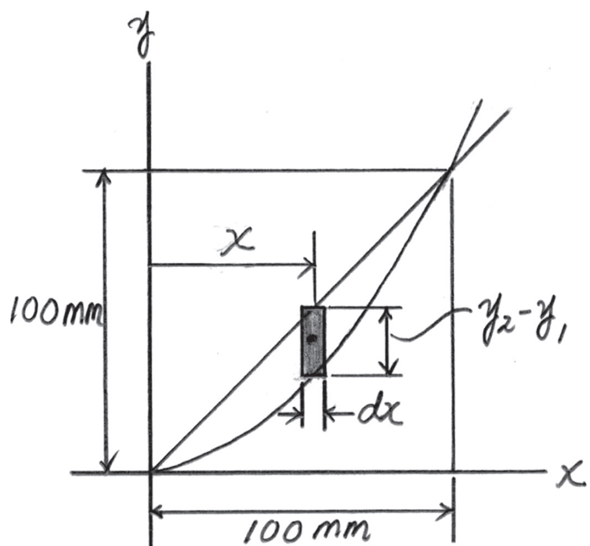
***Area And Moment Arm.*** Here, $y_2 = x$ and $y_1 = \frac{1}{100}x^2$. Thus, the area of the differential element shown shaded in Fig. *a* is $dA = (y_2 - y_1)\,dx = \left(x - \frac{1}{100}x^2\right)dx$ and its centroid is at $\tilde{x} = x$.

***Centroid.*** Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{100\text{ mm}} x\left(x - \frac{1}{100}x^2\right)dx}{\int_0^{100\text{ mm}}\left(x - \frac{1}{100}x^2\right)dx}$$

$$= \frac{\left(\frac{x^3}{3} - \frac{1}{400}x^4\right)\Big|_0^{100\text{ mm}}}{\left(\frac{x^2}{2} - \frac{1}{300}x^3\right)\Big|_0^{100\text{ mm}}}$$

$$= 50.0\text{ mm}$$ **Ans.**

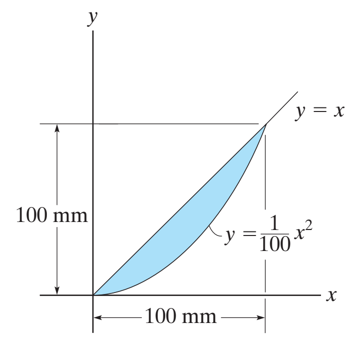



(a)

**Ans:**
$\bar{x} = 50.0$ mm

**6–23.**

Locate the centroid $\bar{y}$ of the area.




## SOLUTION

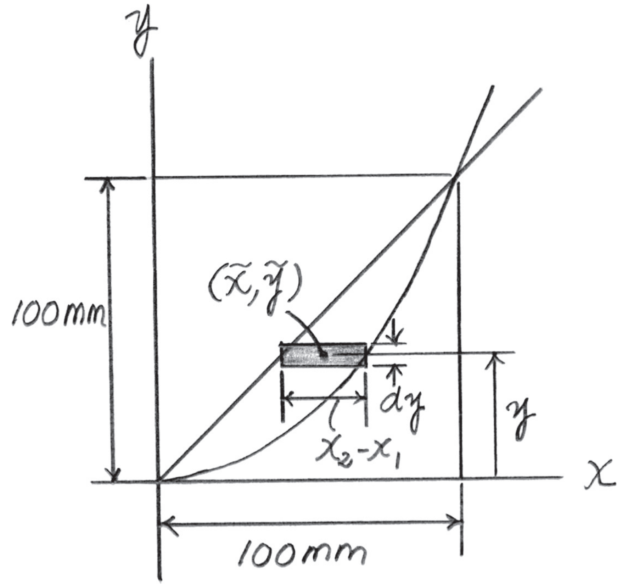
***Area And Moment Arm.*** Here, $x_2 = 10y^{1/2}$ and $x_1 = y$. Thus, the area of the differential element shown shaded in Fig. *a* is $dA = (x_2 - x_1)\,dy = (10y^{1/2} - y)dy$ and its centroid is at $\tilde{y} = y$.

***Centroid.*** Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{100\text{ mm}} y\left(10y^{1/2} - y\right)dy}{\int_0^{100\text{ mm}}\left(10y^{1/2} - y\right)dy}$$

$$= \frac{\left(4y^{5/2} - \frac{y^3}{3}\right)\Big|_0^{100\text{ mm}}}{\left(\frac{20}{3}y^{3/2} - \frac{y^2}{2}\right)\Big|_0^{100\text{ mm}}}$$

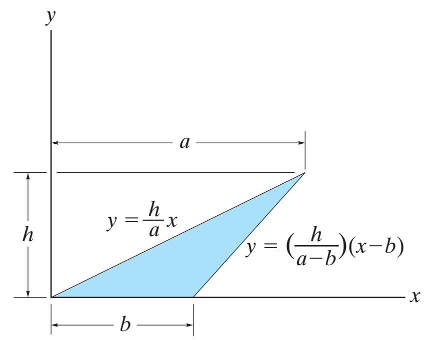
$$= 40.0\text{ mm}$$ **Ans.**




**Ans:**
$\bar{y} = 40.0$ mm

***6–24.**

Locate the centroid $\bar{x}$ of the area.




## SOLUTION

***Area And Moment Arm.*** Here $x_1 = \frac{a}{h}y$ and $x_2 = \left(\frac{a-b}{h}\right)y + b$. Thus, the area of the differential element is $dA = (x_2 - x_1)\,dy = \left[\left(\frac{a-b}{h}\right)y + b - \frac{a}{h}y\right]dy = \left(b - \frac{b}{h}y\right)dy$ and its centroid is at $\tilde{x} = x_1 + \frac{x_2 - x_1}{2} = \frac{1}{2}(x_2 + x_1) = \frac{a}{h}y - \frac{b}{2h}y + \frac{b}{2}$.

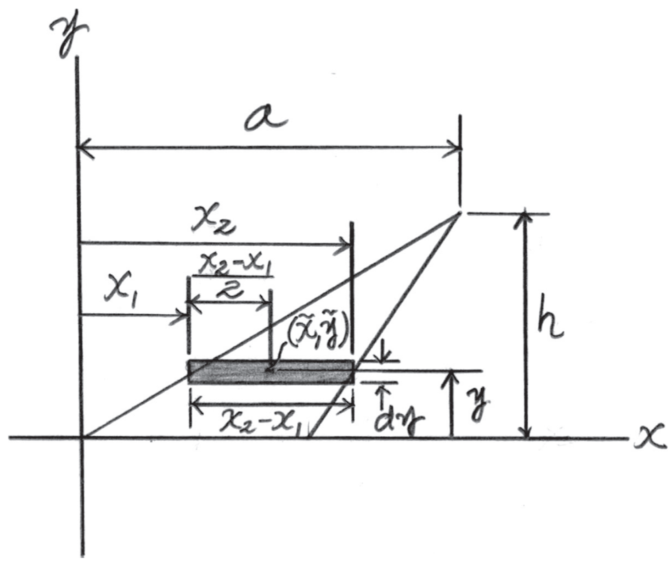
***Centroid.*** Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^h \left(\frac{a}{h}y - \frac{b}{2h}y + \frac{b}{2}\right)\left[\left(b - \frac{b}{h}y\right)dy\right]}{\int_0^h \left(b - \frac{b}{h}y\right)dy}$$

$$= \frac{\left[\frac{b}{2h}(a-b)y^2 + \frac{b}{6h^2}(b-2a)y^3 + \frac{b^2}{2}y\right]\Big|_0^h}{\left(by - \frac{b}{2h}y^2\right)\Big|_0^h}$$

$$= \frac{\frac{bh}{6}(a+b)}{\frac{bh}{2}}$$

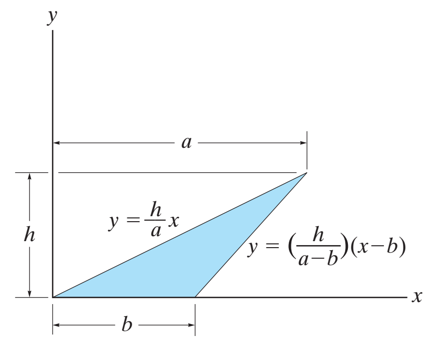
$$= \frac{1}{3}(a+b) \qquad \textbf{Ans.}$$




**Ans:**
$\bar{x} = \frac{1}{3}(a+b)$

**6–25.**

Locate the centroid $\bar{y}$ of the area.




## SOLUTION

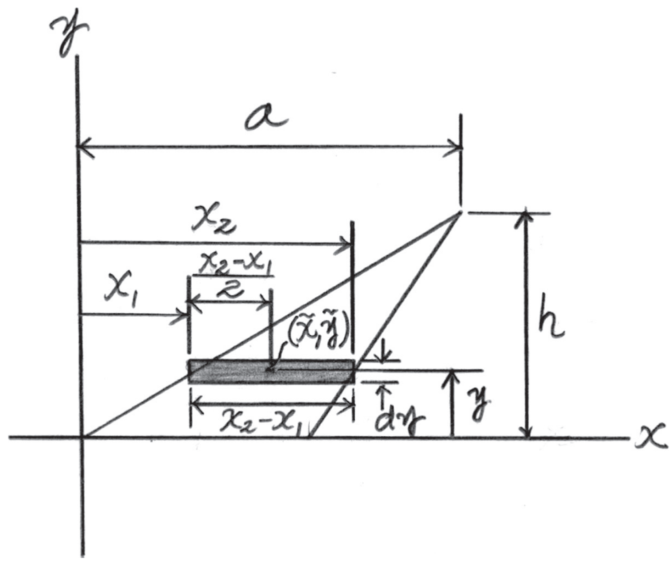
***Area And Moment Arm.*** Here, $x_1 = \frac{a}{h}y$ and $x_2 = \left(\frac{a-b}{h}\right)y + b$. Thus, the area of the differential element is $dA = (x_2 - x_1)\,dy = \left[\left(\frac{a-b}{h}\right)y + b - \frac{a}{h}y\right]dy = \left(b - \frac{b}{h}y\right)dy$ and its centroid is at $\tilde{y} = y$.

***Centroid.*** Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^h y\left(b - \frac{b}{h}y\right)dy}{\int_0^h \left(b - \frac{b}{h}y\right)dy}$$

$$= \frac{\left(\frac{b}{2}y^2 - \frac{b}{3h}y^3\right)\Big|_0^h}{\left(by - \frac{b}{2h}y^2\right)\Big|_0^h}$$

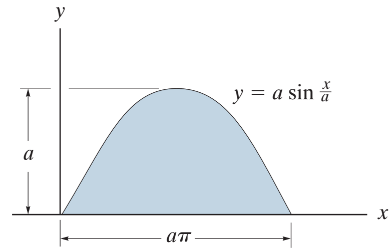
$$= \frac{\frac{1}{6}bh^2}{\frac{1}{2}bh} = \frac{h}{3}$$ **Ans.**




**Ans:**
$\bar{y} = \frac{h}{3}$

**6–26.**

Locate the centroid $\bar{x}$ of the area.




## SOLUTION

***Area and Moment Arm:*** The area of the differential element is $dA = y\,dx = a \sin \dfrac{x}{a} dx$ and its centroid are $\bar{x} = x$.

$$\bar{x} = \frac{\int_A \tilde{x}\, dA}{\int_A dA} = \frac{\int_0^{\pi a} x\left(a \sin \dfrac{x}{a} dx\right)}{\int_0^{\pi a} a \sin \dfrac{x}{a} dx}$$

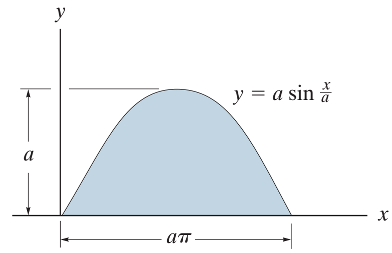
$$= \frac{\left[a^3 \sin \dfrac{x}{a} - x\left(a^2 \cos \dfrac{x}{a}\right)\right]\Bigg|_0^{\pi a}}{\left(-a^2 \cos \dfrac{x}{a}\right)\Bigg|_0^{\pi a}}$$

$$= \frac{\pi}{2} a$$ **Ans.**

**Ans:**
$\bar{x} = \dfrac{\pi}{2} a$

**6–27.**

Locate the centroid $\overline{y}$ of the area.




## SOLUTION

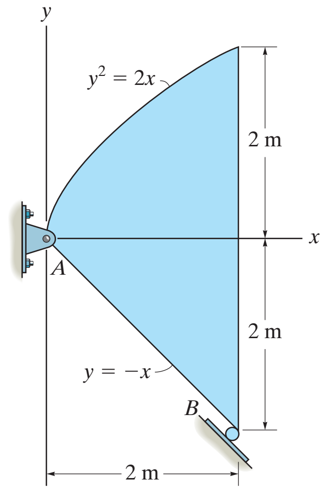
***Area and Moment Arm:*** The area of the differential element is $dA = y\,dx = a\sin\frac{x}{a}dx$ and its centroid are $\overline{y} = \frac{y}{2} = \frac{a}{2}\sin\frac{x}{a}$.

$$\overline{y} = \frac{\int_A \overline{y}\,dA}{\int_A dA} = \frac{\int_0^{\pi a} \frac{a}{2}\sin\frac{x}{a}\left(a\sin\frac{x}{a}dx\right)}{\int_0^{\pi a} a\sin\frac{x}{a}dx} = \frac{\left[\frac{1}{4}a^2\left(x - \frac{1}{2}a\sin\frac{2x}{a}\right)\right]\Big|_0^{\pi a}}{\left(-a^2\cos\frac{x}{a}\right)\Big|_0^{\pi a}} = \frac{\pi a}{8}$$ **Ans.**

**Ans:**
$\overline{y} = \dfrac{\pi a}{8}$

***6–28.**

The steel plate is 0.3 m thick and has a density of 7850 kg/m$^3$. Determine the location of its center of gravity. Also find the reactions at the pin and roller support.




## SOLUTION

$$y_1 = -x_1$$

$$y_2^2 = 2x_2$$

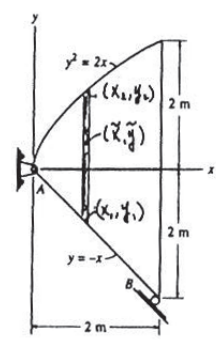
$$dA = (y_2 - y_1)\,dx = \left(\sqrt{2x} + x\right)dx$$

$$\widetilde{x} = x$$

$$\widetilde{y} = \frac{y_2 + y_1}{2} = \frac{\sqrt{2x} - x}{2}$$

$$\overline{x} = \frac{\int_A \widetilde{x}\,dA}{\int_A dA} = \frac{\int_0^2 x\left(\sqrt{2x} + x\right)dx}{\int_0^2 \left(\sqrt{2x} + x\right)dx} = \frac{\left[\frac{2\sqrt{2}}{5}x^{5/2} + \frac{1}{3}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^2} = 1.2571 = 1.26 \text{ m} \qquad \textbf{Ans.}$$

$$\overline{y} = \frac{\int_A \widetilde{y}\,dA}{\int_A dA} = \frac{\int_0^2 \frac{\sqrt{2x} - x}{2}\left(\sqrt{2x} + x\right)dx}{\int_0^2 \left(\sqrt{2x} + x\right)dx} = \frac{\left[\frac{x^2}{2} - \frac{1}{6}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^2} = 0.143 \text{ m} \qquad \textbf{Ans.}$$




$$A = 4.667 \text{ m}^2$$

$$W = 7850(9.81)(4.667)(0.3) = 107.81 \text{ kN}$$

$\circlearrowleft + \Sigma M_A = 0;$ $\qquad -1.2571(107.81) + N_B\left(2\sqrt{2}\right) = 0$

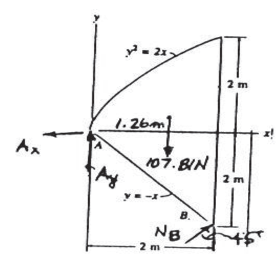
$N_B = 47.92 = 47.9 \text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad -A_x + 47.92 \sin 45° = 0$

$A_x = 33.9 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\qquad A_y + 47.92 \cos 45° - 107.81 = 0$

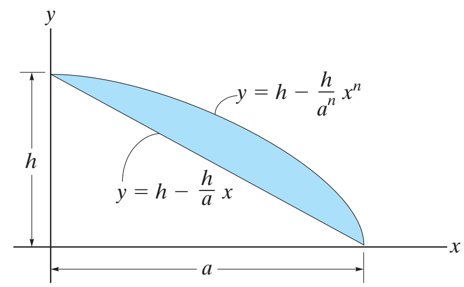
$A_y = 73.9 \text{ kN}$ **Ans.**




**Ans:**
$\overline{x} = 1.26$ m
$\overline{y} = 0.143$ m
$N_B = 47.9$ kN
$A_x = 33.9$ kN
$A_y = 73.9$ kN

**6–29.**

Locate the centroid $\bar{x}$ of the area.




## SOLUTION

***Area And Moment Arm.*** Here, $y_2 = h - \frac{h}{a^n}x^n$ and $y_1 = h - \frac{h}{a}x$. Thus, the area of the differential element shown shaded in Fig. $a$ is $dA = (y_2 - y_1)\,dx$ $= \left[h - \frac{h}{a^n}x^n - \left(h - \frac{h}{a}x\right)\right]dx = \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx$ and its centroid is $\tilde{x} = x$.

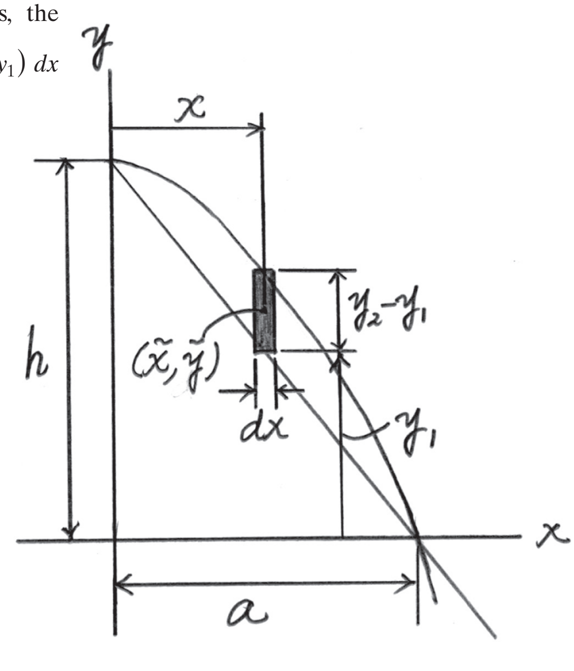
***Centroid.*** Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\displaystyle\int_0^a x\left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}{\displaystyle\int_0^a \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\left[\frac{h}{3a}x^3 - \frac{h}{a^n(n+2)}x^{n+2}\right]\Big|_0^a}{\left[\frac{h}{2a}x^2 - \frac{h}{a^n(n+1)}x^{n+1}\right)\Big|_0^a}$$

$$= \frac{\dfrac{ha^2(n-1)}{3(n+2)}}{\dfrac{ha(n-1)}{2(n+1)}}$$

$$= \left[\frac{2(n+1)}{3(n+2)}\right]a \qquad \textbf{Ans.}$$



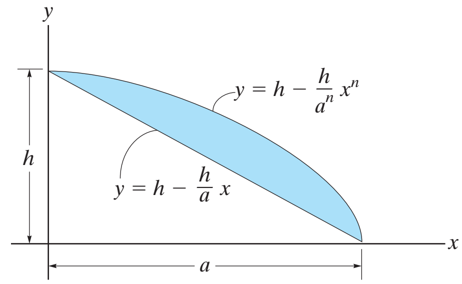
(a)

**Ans:**

$\bar{x} = \left[\frac{2(n+1)}{3(n+2)}\right]a$

**6–30.**

Locate the centroid $\bar{y}$ of the area.




## SOLUTION

***Area And Moment Arm.*** Here, $y_2 = h - \frac{h}{a^n}x^n$ and $y_1 = h - \frac{h}{a}x$. Thus, the area of the differential element shown shaded in Fig. $a$ is $dA = (y_2 - y_1)\,dx = \left[h - \frac{h}{a^n}x^n - \left(h - \frac{h}{a}x\right)\right]dx = \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx$ and its centroid is at $\tilde{y} = y_1 + \left(\frac{y_2 - y_1}{2}\right) = \frac{1}{2}(y_2 + y_1) = \frac{1}{2}\left(h - \frac{h}{a^n}x^n + h - \frac{h}{a}x\right) = \frac{1}{2}\left(2h - \frac{h}{a^n}x^n - \frac{h}{a}x\right)$.

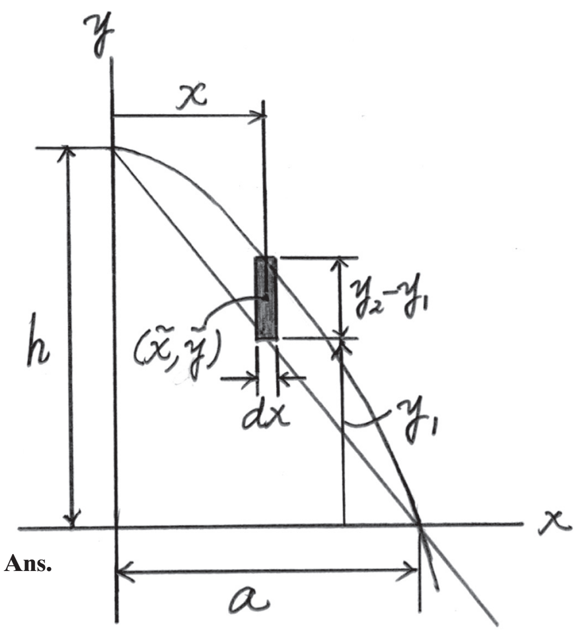
***Centroid.*** Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^a \frac{1}{2}\left(2h - \frac{h}{a^n}x^n - \frac{h}{a}x\right)\left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}{\int_0^a \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\frac{1}{2}\left[\frac{h^2}{a}x^2 - \frac{h^2}{3a^2}x^3 - \frac{2h^2}{a^n(n+1)}x^{n+1} + \frac{h^2}{a^{2n}(2n+1)}x^{2n+1}\right]\Big|_0^a}{\left[\frac{h}{2a}x^2 - \frac{h}{a^n(n+1)}x^{n+1}\right)\Big|_0^a}$$

$$= \frac{h^2a\left[\frac{(4n+1)(n-1)}{6(n+1)(2n+1)}\right]}{ha\left[\frac{n-1}{2(n+1)}\right]}$$

$$= \left[\frac{(4n+1)}{3(2n+1)}\right]h \qquad \textbf{Ans.}$$

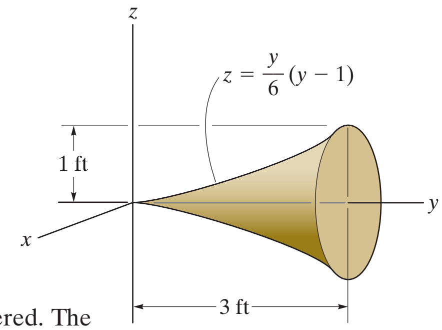



(a)

**Ans:**
$\bar{y} = \left[\frac{(4n+1)}{3(2n+1)}\right]h$

**6–31.**

Locate the centroid $\bar{y}$ of the solid.




## SOLUTION

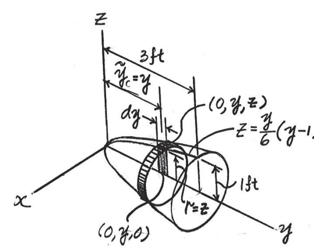
***Differential Element:*** The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi z^2\,dy = \pi\left[\frac{y}{6}(y-1)\right]^2 dy = \frac{\pi}{36}(y^4 - 2y^3 + y^2)dy$$

***Centroid:*** The centroid of the element is located at $y_c = y$. We have

$$\bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\int_0^{3\text{ ft}} y\left[\frac{\pi}{36}(y^4 - 2y^3 + y^2)dy\right]}{\int_0^{3\text{ ft}} \frac{\pi}{36}(y^4 - 2y^3 + y^2)dy} = \frac{\int_0^{3\text{ ft}} \frac{\pi}{36}(y^5 - 2y^4 + y^3)dy}{\int_0^{3\text{ ft}} \frac{\pi}{36}(y^4 - 2y^3 + y^2)dy} = \frac{\frac{\pi}{36}\left[\frac{y^6}{6} - \frac{2}{5}y^5 + \frac{y^4}{4}\right]\Bigg|_0^{3\text{ ft}}}{\frac{\pi}{36}\left[\frac{y^5}{5} - \frac{y^4}{2} + \frac{y^3}{3}\right]\Bigg|_0^{3\text{ ft}}}$$

$$= 2.61\text{ ft} \qquad \textbf{Ans.}$$

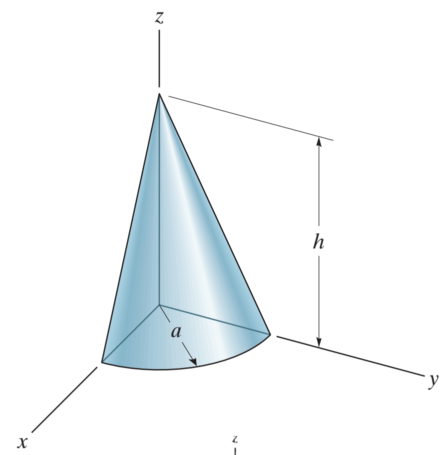


(a)

**Ans:**
$\bar{y} = 2.61$ ft

***6–32.**

Locate the centroid of the quarter-cone.

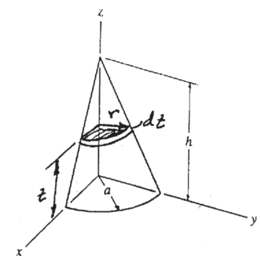



## SOLUTION

$$\tilde{z} = z$$

$$r = \frac{a}{h}(h - z)$$

$$dV = \frac{\pi}{4} r^2\, dz = \frac{\pi\, a^2}{4\, h^2}(h - z)^2\, dz$$




$$\int dV = \frac{\pi\, a^2}{4\, h^2}\int_0^h (h^2 - 2hz + z^2)\, dz = \frac{\pi\, a^2}{4\, h^2}\left[h^2 z - hz^2 + \frac{z^3}{3}\right]_0^h$$

$$= \frac{\pi\, a^2}{4\, h^2}\left(\frac{h^3}{3}\right) = \frac{\pi a^2\, h}{12}$$

$$\int \tilde{z} dV = \frac{\pi\, a^2}{4\, h^2}\int_0^h (h^2 - 2hz + z^2)\, z\, dz = \frac{\pi\, a^2}{4\, h^2}\left[h^2\frac{z^2}{2} - 2h\frac{z^3}{3} + \frac{z^4}{4}\right]_0^h$$

$$= \frac{\pi\, a^2}{4\, h^2}\left(\frac{h^4}{12}\right) = \frac{\pi a^2 h^2}{48}$$

$$\bar{z} = \frac{\int \tilde{z} dV}{\int dV} = \frac{\dfrac{\pi\, a^2\, h^2}{48}}{\dfrac{\pi\, a^2 h}{12}} = \frac{h}{4}$$ **Ans.**

$$\int \tilde{x} dV = \frac{\pi a^2}{4\, h^2}\int_0^h \frac{4\, r}{3\pi}(h - z)^2\, dz = \frac{\pi a^2}{4\, h^2}\int_0^h \frac{4a}{3\, \pi\, h}(h^3 - 3h^2\, z + 3hz^2 - z^3)\, dz$$

$$= \frac{\pi\, a^2}{4\, h^2}\,\frac{4a}{3\pi h}\left(h^4 - \frac{3h^4}{2} + h^4 - \frac{h^4}{4}\right)$$

$$= \frac{\pi\, a^2}{4\, h^2}\left(\frac{a\, h^3}{3\pi}\right) = \frac{a^3\, h}{12}$$

$$\bar{x} = \bar{y} = \frac{\int \tilde{x} dV}{\int dV} = \frac{\dfrac{a^3\, h}{12}}{\dfrac{\pi\, a^2 h}{12}} = \frac{a}{\pi}$$ **Ans.**

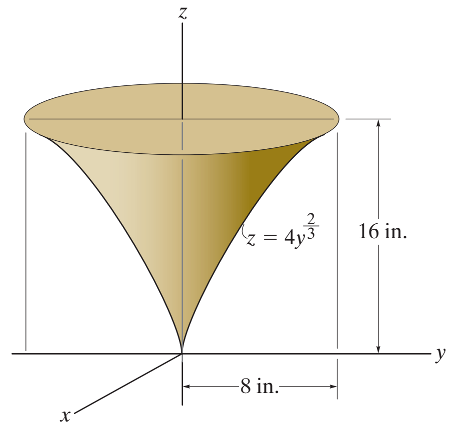
**Ans:**

$\bar{z} = \dfrac{h}{4}$

$\bar{x} = \bar{y} = \dfrac{a}{\pi}$

**6–33.**

Locate the centroid $\bar{z}$ of the solid.



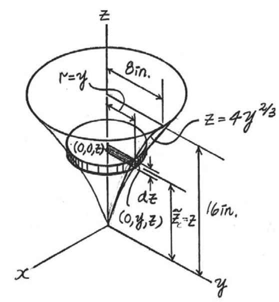

## SOLUTION

***Differential Element:*** The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi y^2\,dz = \pi\left[\frac{1}{8}z^{3/2}\right]^2 dz = \frac{\pi}{64}z^3\,dz$$

***Centroid:*** The centroid of the element is located at $z_c = z$. We have

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^{16\text{ in.}} z\left[\frac{\pi}{64}z^3\,dz\right]}{\int_0^{16\text{ in.}} \frac{\pi}{64}z^3\,dz} = \frac{\int_0^{16\text{ in.}} \frac{\pi}{64}z^4\,dz}{\int_0^{16\text{ in.}} \frac{\pi}{64}z^3\,dz} = \frac{\frac{\pi}{64}\left(\frac{z^5}{5}\right)\Big|_0^{16\text{ in.}}}{\frac{\pi}{64}\left(\frac{z^4}{4}\right)\Big|_0^{16\text{ in.}}} = 12.8\text{ in.} \quad \textbf{Ans.}$$

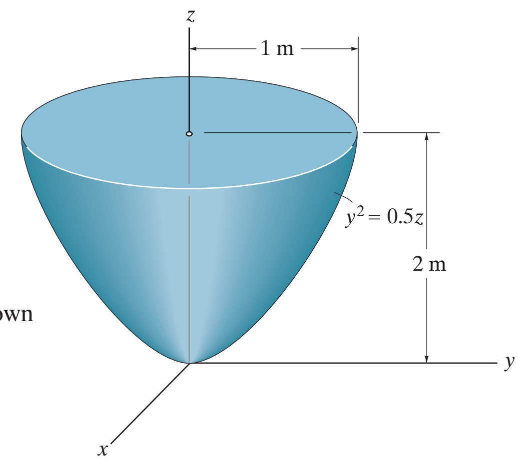


(a)

**Ans:**
$\bar{z} = 12.8$ in.

**6–34.**

Locate the centroid $\bar{z}$ of the volume.




## SOLUTION

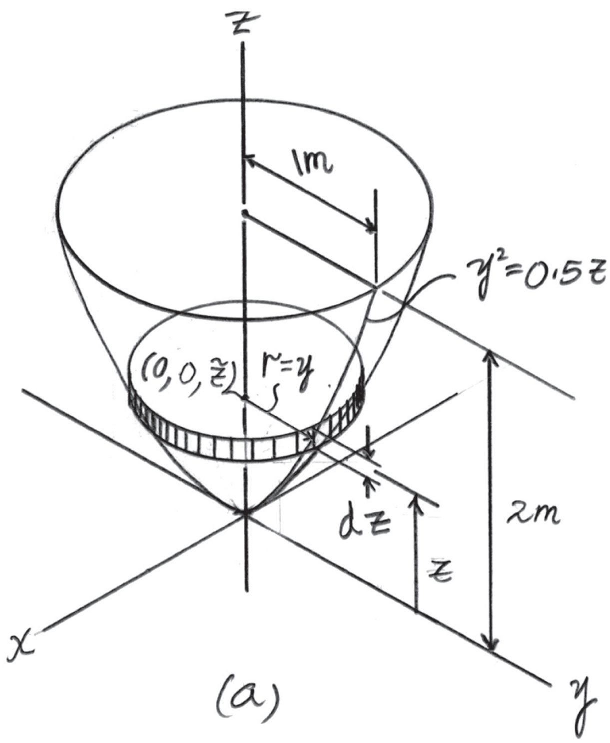
***Volume And Moment Arm.*** The volume of the thin disk differential element shown shaded in Fig. *a* is $dV = \pi y^2\, dz = \pi(0.5z)dz$ and its centroid is at $\tilde{z} = z$.

***Centroid.*** Perform the integration

$$\bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV} = \frac{\int_0^{2\text{ m}} z[\pi(0.5z)dz]}{\int_0^{2\text{ m}} \pi(0.5z)dz}$$

$$= \frac{\frac{0.57}{3} z^3 \Big|_0^{2\text{ m}}}{\frac{0.5\pi}{2} z^2 \Big|_0^{2\text{ m}}}$$

$$= \frac{4}{3}\text{ m} \qquad \textbf{Ans.}$$

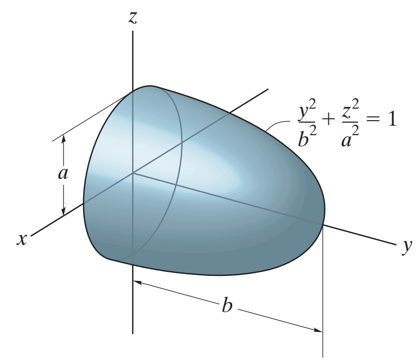



(a)

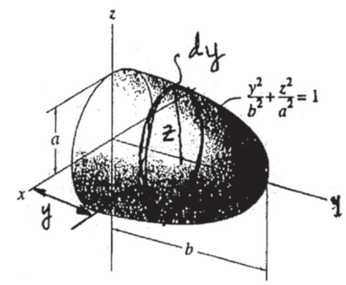
**Ans:**
$\bar{z} = \frac{4}{3}$ m

**6–35.**

Locate the centroid of the ellipsoid of revolution.




## SOLUTION




$$dV = \pi\, z^2\, dy$$

$$\int dV = \int_0^b \pi\, a^2\left(1 - \frac{y^2}{b^2}\right) dy = \pi\, a^2\left[y - \frac{y^3}{3b^2}\right]_0^b = \frac{2\pi a^2 b}{3}$$

$$\int \tilde{y} dV = \int_0^b \pi\, a^2 y\left(1 - \frac{y^2}{b^2}\right) dy = \pi\, a^2\left[\frac{y^2}{2} - \frac{y^4}{4b^2}\right]_0^b = \frac{\pi\, a^2 b^2}{4}$$

$$\bar{y} = \frac{\int_V \tilde{y} dV}{\int_V dV} = \frac{\frac{\pi a^2 b^2}{4}}{\frac{2\pi a^2 b}{3}} = \frac{3}{8}b \qquad \textbf{Ans.}$$

$$\bar{x} = \bar{z} = 0 \qquad \text{(By symmetry)} \qquad \textbf{Ans.}$$

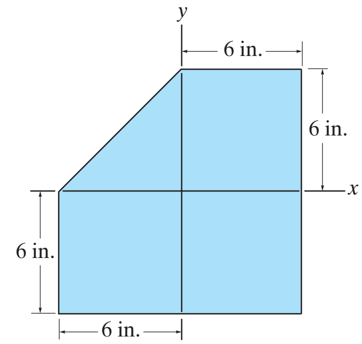
**Ans:**

$\bar{y} = \frac{3}{8}b$

$\bar{x} = \bar{z} = 0$

***6–36.**

Locate the centroid ($\bar{x}$, $\bar{y}$) of the area.




## SOLUTION

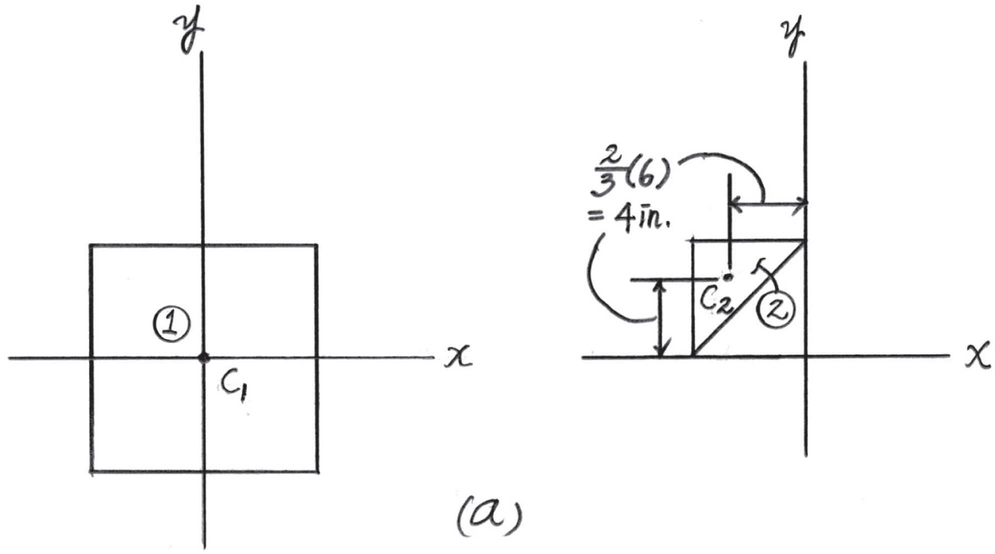
***Centroid.*** Referring to Fiq. *a*, the areas of the segments and the locations of their respective centroids are tabulated below.

| Segment | $A$ (in.$^2$) | $\tilde{x}$ (in.) | $\tilde{y}$ (in.) | $\tilde{x}A$ (in.$^3$) | $\tilde{y}A$ (in.$^3$) |
|---|---|---|---|---|---|
| 1 | 12(12) | 0 | 0 | 0 | 0 |
| 2 | $-\frac{1}{2}(6)(6)$ | −4 | 4 | 72.0 | −72.0 |
| Σ | 126 | | | 72.0 | −72.0 |

Thus,

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{72.0 \text{ in.}^3}{126 \text{ in.}^2} = 0.5714 \text{ in.} = 0.571 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{-72.0 \text{ in.}^3}{126 \text{ in.}^2} = -0.5714 \text{ in.} = -0.571 \text{ in.} \qquad \textbf{Ans.}$$

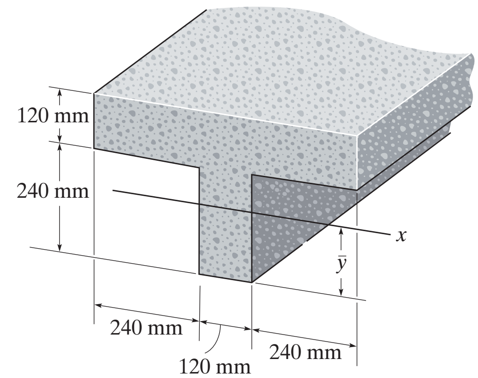



(a)

**Ans:**
$\bar{x} = 0.571$ in.
$\bar{y} = -0.571$ in.

**6–37.**

Locate the centroid $\bar{y}$ for the beam's cross-sectional area.

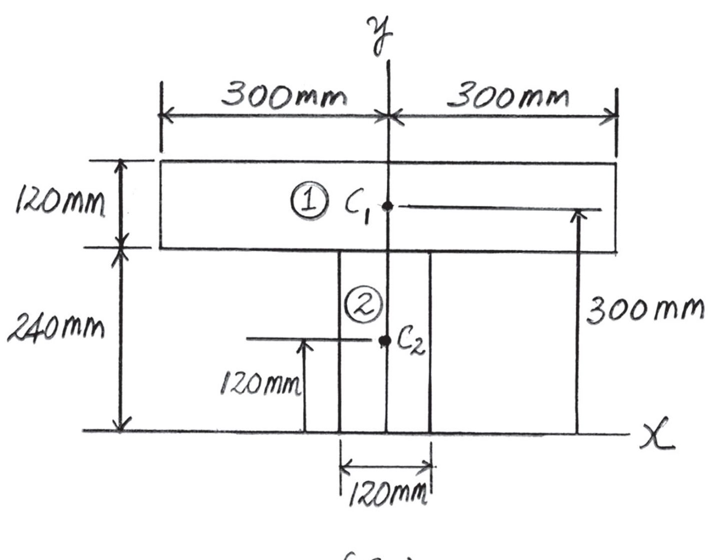



## SOLUTION

***Centroid.*** The locations of the centroids measuring from the $x$ axis for segments ① and ② are indicated in Fig. $a$. Thus,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{300(120)(600) + 120(240)(120)}{120(600) + 240(120)}$$

$$= 248.57 \text{ mm} = 249 \text{ mm}$$

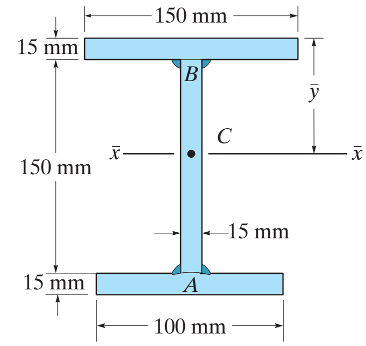



(a)

**Ans:**
$\bar{y} = 249$ mm

**6–38.**

Locate the centroid $\bar{y}$ of the beam having the cross-sectional area shown.

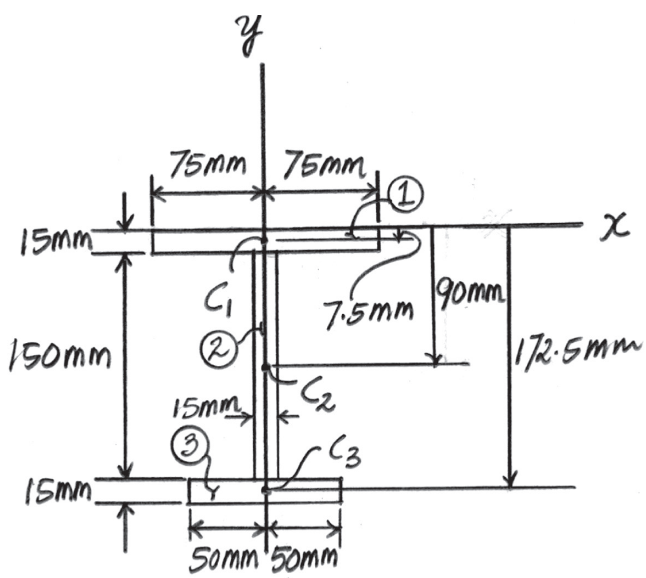



## SOLUTION

***Centroid.*** The locations of the centroids measuring from the $x$ axis for segments ①, ② and ③ are indicated in Fig. $a$. Thus,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{7.5(15)(150) + 90(150)(15) + 172.5(15)(100)}{15(150) + 150(15) + 15(100)}$$

$$= 79.6875 \text{ mm} = 79.7 \text{ mm} \qquad \textbf{Ans.}$$

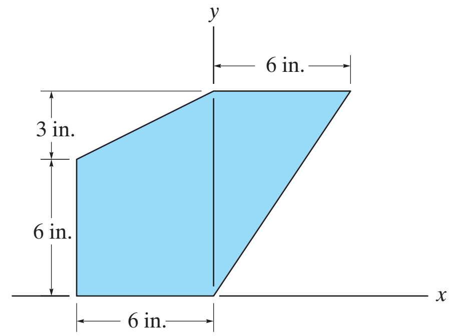



(a)

**Ans:**
$\bar{y} = 79.7$ mm

**6–39.**

Locate the centroid $(\bar{x}, \bar{y})$ of the area.




## SOLUTION

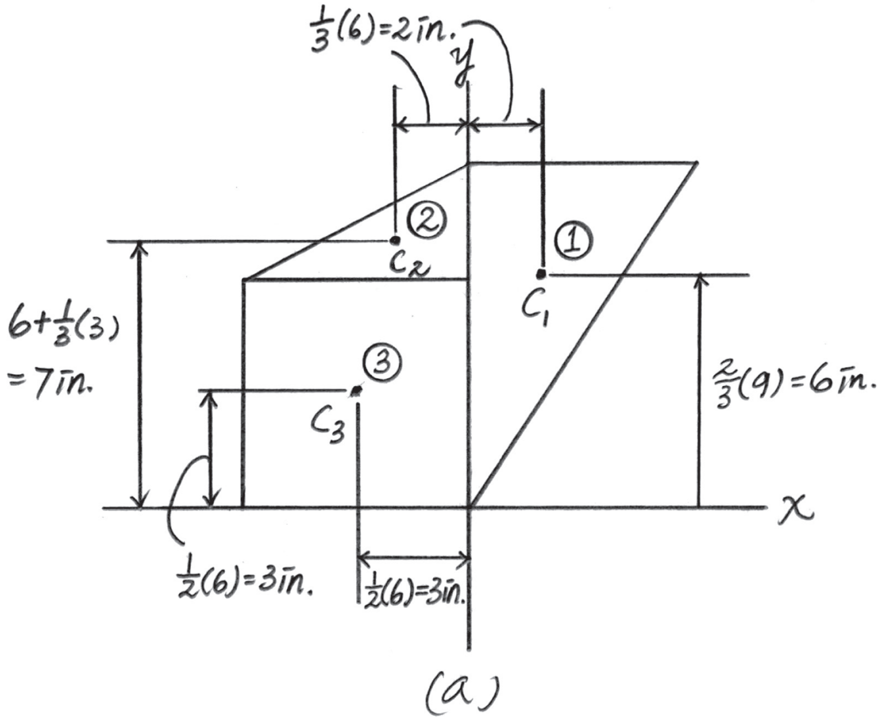
***Centroid.*** Referring to Fig. *a*, the areas of the segments and the locations of their respective centroids are tabulated below.

| Segment | $A$ (in.$^2$) | $\tilde{x}$ (in.) | $\tilde{y}$ (in.) | $\tilde{x}A$ (in.$^3$) | $\tilde{y}A$ (in.$^3$) |
|---|---|---|---|---|---|
| 1 | $\frac{1}{2}(6)(9)$ | 2 | 6 | 54.0 | 162.0 |
| 2 | $\frac{1}{2}(6)(3)$ | −2 | 7 | −18.0 | 63.0 |
| 3 | 6(6) | −3 | 3 | −108.0 | 108.00 |
| Σ | 72.0 | | | −72.0 | 333.0 |

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-72.0 \text{ in.}^3}{72.0 \text{ in.}^2} = -1.00 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{333.0 \text{ in.}^3}{72.0 \text{ in.}^2} = 4.625 \text{ in.} \qquad \textbf{Ans.}$$

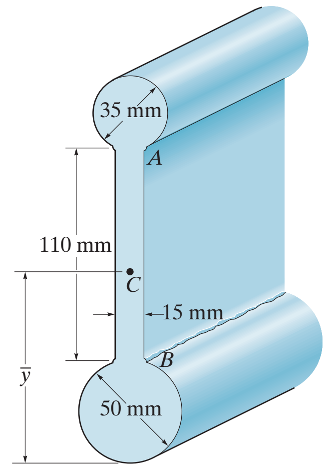



(a)

**Ans:**
$\bar{x} = -1.00$ in.
$\bar{y} = 4.625$ in.

***6–40.**

Locate the centroid $\overline{y}$ of the beam's cross-sectional area. Neglect the size of the corner welds at $A$ and $B$ for the calculation.




## SOLUTION

$$\Sigma \widetilde{y} A = \pi(25)^2(25) + 15(110)(50 + 55) + \pi\left(\frac{35}{2}\right)^2\left(50 + 110 + \frac{35}{2}\right) = 393\,112 \text{ mm}^3$$

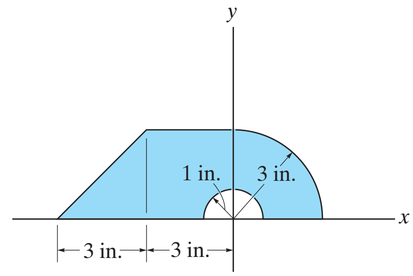
$$\Sigma A = \pi(25)^2 + 15(110) + \pi\left(\frac{35}{2}\right)^2 = 4575.6 \text{ mm}^2$$

$$\overline{y} = \frac{\Sigma \widetilde{y} A}{\Sigma A} = \frac{393\,112}{4575.6} = 85.9 \text{ mm}$$ **Ans.**

**Ans:**
$\overline{y} = 85.9$ mm

**6–41.**

Locate the centroid $(\bar{x}, \bar{y})$ of the area.




## SOLUTION

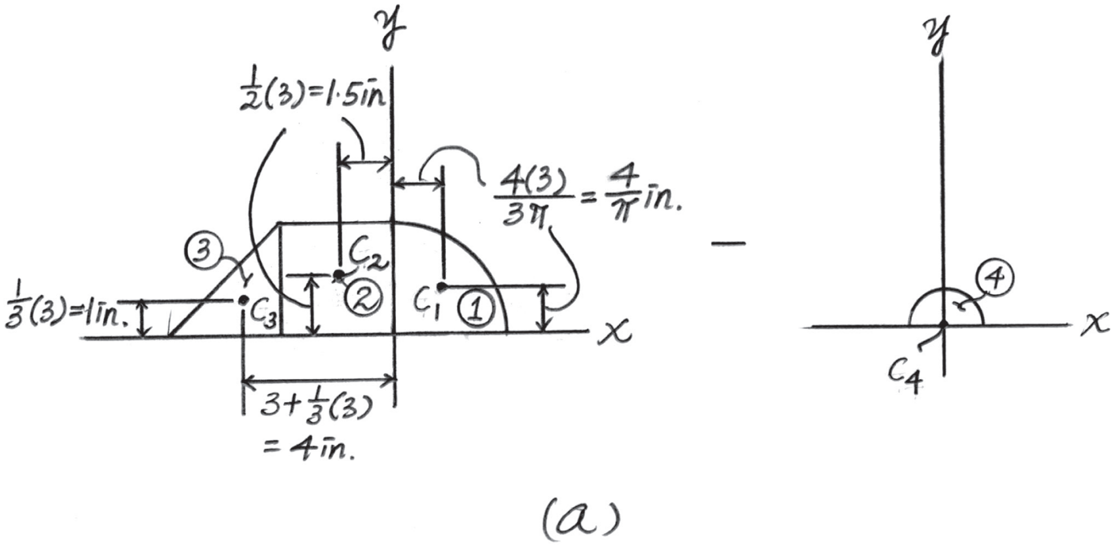
***Centroid.*** Referring to Fig. *a*, the areas of the segments and the locations of their respective centroids are tabulated below.

| Segment | $A$(in.$^2$) | $\tilde{x}$ (in.) | $\tilde{y}$ (in.) | $\tilde{x}A$(in.$^3$) | $\tilde{y}A$(in.$^3$) |
|---|---|---|---|---|---|
| 1 | $\frac{\pi}{4}(3^2)$ | $\frac{4}{\pi}$ | $\frac{4}{\pi}$ | 9.00 | 9.00 |
| 2 | 3(3) | −1.5 | 1.5 | −13.50 | 13.50 |
| 3 | $\frac{1}{2}(3)(3)$ | −4 | 1 | −18.00 | 4.50 |
| 4 | $-\frac{\pi}{2}(1^2)$ | 0 | $\frac{4}{3\pi}$ | 0 | −0.67 |
| $\Sigma$ | 18.9978 | | | −22.50 | 26.33 |

Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{-22.50 \text{ in.}^3}{18.9978 \text{ in.}^2} = -1.1843 \text{ in.} = -1.18 \text{ in.} \qquad \textbf{Ans.}$$

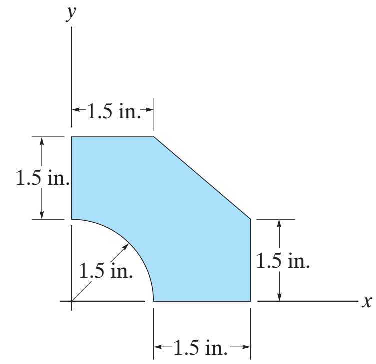
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{26.33 \text{ in.}^3}{18.9978 \text{ in.}^2} = 1.3861 \text{ in.} = 1.39 \text{ in.} \qquad \textbf{Ans.}$$




**Ans:**
$\bar{x} = -1.18$ in.
$\bar{y} = 1.39$ in.

**6–42.**

Locate the centroid ($\bar{x}, \bar{y}$) of the area.




## SOLUTION

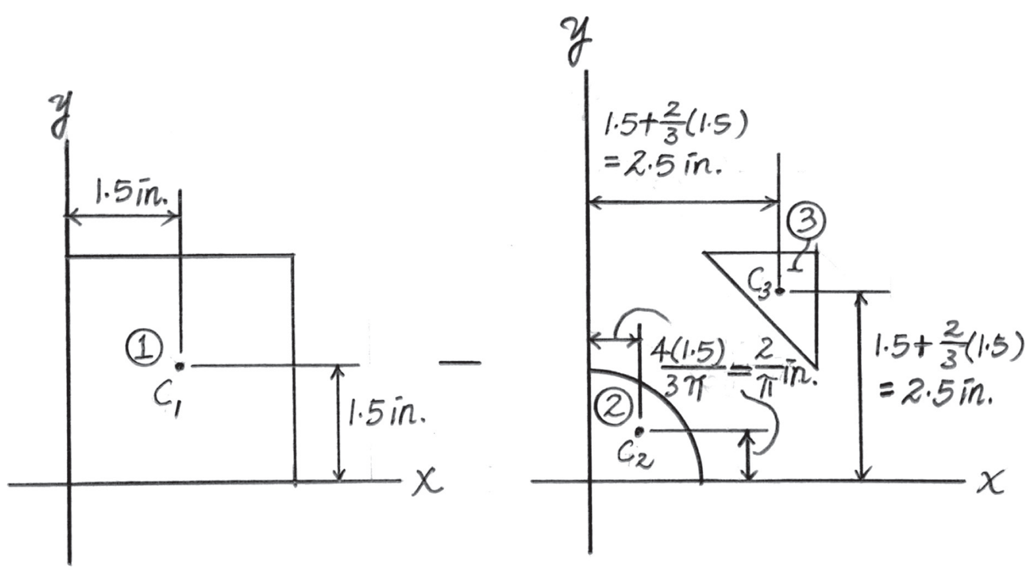
***Centroid.*** Referring to Fig. *a*, the areas of the segments and the locations of their respective centroids are tabulated below.

| Segment | $A$ (in.$^2$) | $\tilde{x}$ (in.) | $\tilde{y}$ (in.) | $\tilde{x}A$ (in.$^3$) | $\tilde{y}A$ (in.$^3$) |
|---|---|---|---|---|---|
| 1 | $3(3)$ | 1.5 | 1.5 | 13.5 | 13.5 |
| 2 | $-\frac{\pi}{4}(1.5^2)$ | $\frac{2}{\pi}$ | $\frac{2}{\pi}$ | $-1.125$ | $-1.125$ |
| 3 | $-\frac{1}{2}(1.5)(1.5)$ | 2.5 | 2.5 | $-2.8125$ | $-2.8125$ |
| $\Sigma$ | 6.1079 | | | 9.5625 | 9.5625 |

Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{9.5625 \text{ in.}^3}{6.1079 \text{ in.}^2} = 1.5656 \text{ in.} = 1.57 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{9.5625 \text{ in.}^3}{6.1079 \text{ in.}^2} = 1.5656 \text{ in.} = 1.57 \text{ in.} \qquad \textbf{Ans.}$$

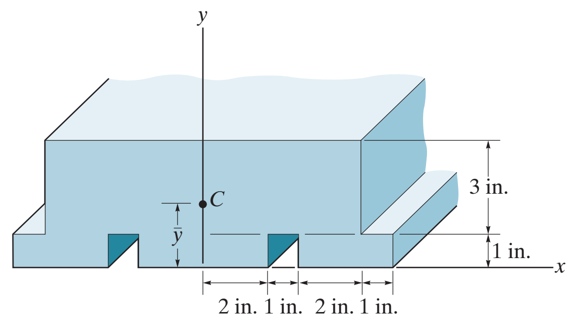



(a)

**Ans:**
$\bar{x} = 1.57$ in.
$\bar{y} = 1.57$ in.

**6–43.**

Locate the centroid $\bar{y}$ of the cross-sectional area of the beam. The beam is symmetric with respect to the $y$ axis.




## SOLUTION

$$\Sigma \tilde{y} A = 6(4)(2) - 1(1)(0.5) - 3(1)(2.5) = 40 \text{ in}^3$$

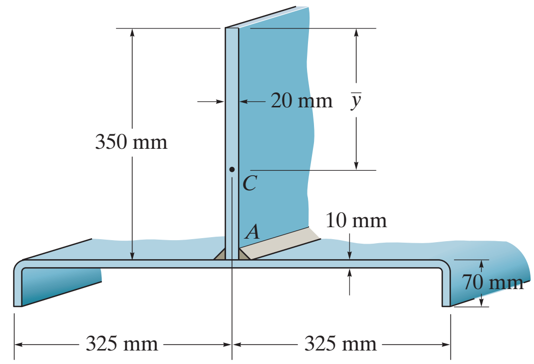
$$\Sigma A = 6(4) - 1(1) - 3(1) = 20 \text{ in}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{40}{20} = 2 \text{ in.}$$ **Ans.**

**Ans:**
$\bar{y} = 2$ in.

***6–44.**

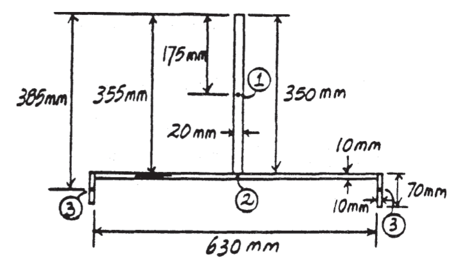
Locate the centroid $\bar{y}$ of the cross-sectional area of the beam constructed from a channel and a plate. Assume all corners are square and neglect the size of the weld at $A$.




## SOLUTION

***Centroid:*** The area of each segment and its respective centroid are tabulated below.

| **Segment** | ***A* (mm²)** | $\tilde{y}$ **(mm)** | $\tilde{y}A$ **(mm³)** |
|---|---|---|---|
| 1 | 350(20) | 175 | 1 225 000 |
| 2 | 630(10) | 355 | 2 236 500 |
| 3 | 70(20) | 385 | 539 000 |
| Σ | 14 700 | | 4 000 500 |



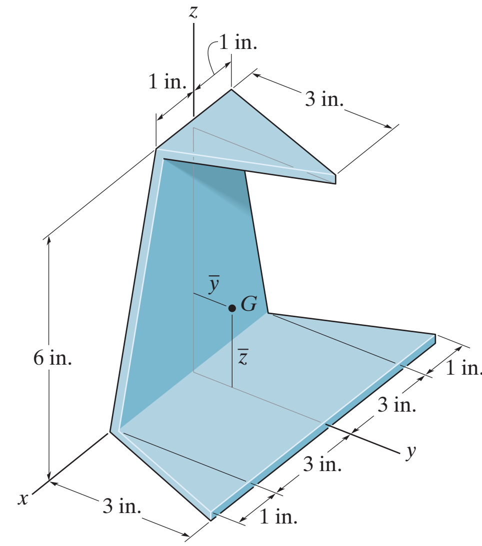

Thus,

$$\tilde{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{4\,000\,500}{14\,700} = 272.14 \text{ mm} = 272 \text{ mm}$$ **Ans.**

**Ans:**
$\bar{y} = 272\,\text{mm}$

**6–45.**

A triangular plate made of homogeneous material has a constant thickness that is very small. If it is folded over as shown, determine the location $\bar{y}$ of the plate's center of gravity $G$.




## SOLUTION

$$\Sigma A = \frac{1}{2}(8)(12) = 48 \text{ in}^2$$

$$\Sigma \tilde{y}A = 2(1)\left(\frac{1}{2}\right)(1)(3) + 1.5(6)(3) + 2(2)\left(\frac{1}{2}\right)(1)(3)$$

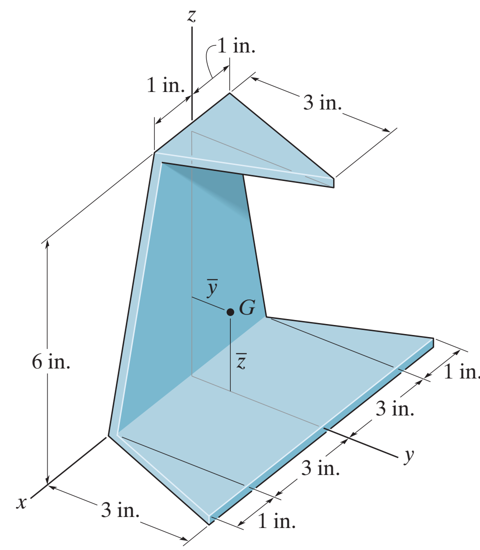
$$= 36 \text{ in}^3$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{36}{48} = 0.75 \text{ in.}$$ **Ans.**

**Ans:**
$\bar{y} = 0.75$ in.

**6–46.**

A triangular plate made of homogeneous material has a constant thickness that is very small. If it is folded over as shown, determine the location $\bar{z}$ of the plate's center of gravity $G$.




## SOLUTION

$$\Sigma A = \frac{1}{2}(8)(12) = 48 \text{ in}^2$$

$$\Sigma \tilde{z} A = 2(2)\left(\frac{1}{2}\right)(2)(6) + 3(6)(2) + 6\left(\frac{1}{2}\right)(2)(3)$$

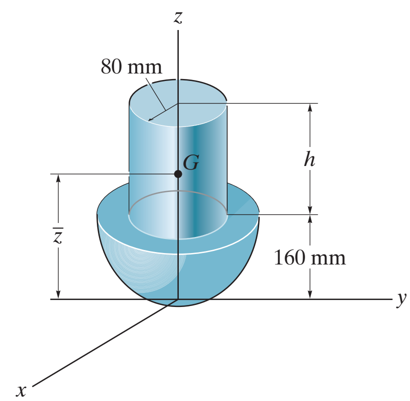
$$= 78 \text{ in}^3$$

$$\bar{z} = \frac{\Sigma \tilde{z} A}{\Sigma A} = \frac{78}{48} = 1.625 \text{ in.}$$ **Ans.**

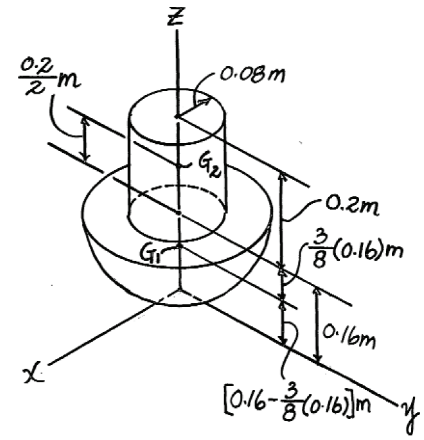
**Ans:**
$\bar{z} = 1.625$ in.

**6–47.**

The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the center of gravity of the assembly if the height of the cylinder is $h = 200$ mm.




## SOLUTION




$$\Sigma\,\bar{z}m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{0.2}{2}\right)\pi(0.2)(0.08)^2(2.70)$$

$$= 9.51425(10^{-3})\ \text{Mg}\cdot\text{m}$$

$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(0.2)(0.08)^2(2.70)$$

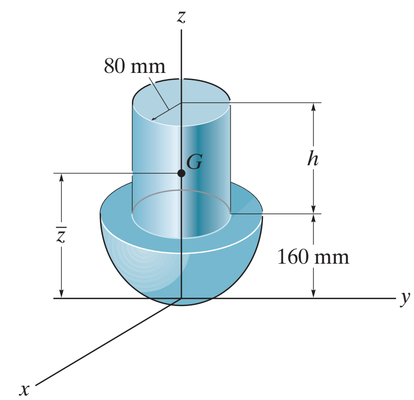
$$= 77.7706(10^{-3})\ \text{Mg}$$

$$\bar{z} = \frac{\Sigma\,\bar{z}m}{\Sigma m} = \frac{9.51425(10^{-3})}{77.7706(10^{-3})} = 0.122\ \text{m} = 122\ \text{mm}$$ **Ans.**

**Ans:**
$\bar{z} = 122$ mm

***6–48.**

The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the height $h$ of the cylinder so that the center of gravity of the assembly is located at $\bar{z} = 160$ mm.




## SOLUTION

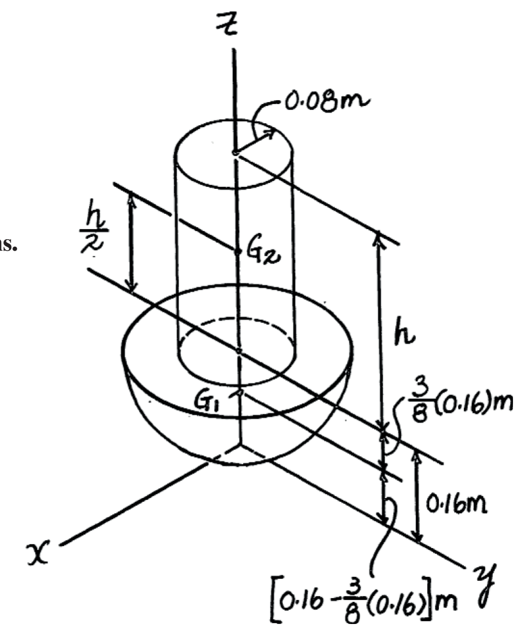
$$\Sigma\,\tilde{z}m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{h}{2}\right)\pi(h)(0.08)^2(2.70)$$

$$= 6.691(10^{-3}) + 8.686(10^{-3})\,h + 27.143(10^{-3})\,h^2$$

$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(h)(0.08)^2(2.70)$$

$$= 66.91(10^{-3}) + 54.29(10^{-3})\,h$$

$$\bar{z} = \frac{\Sigma\,\tilde{z}m}{\Sigma m} = \frac{6.691(10^{-3}) + 8.686(10^{-3})h + 27.143(10^{-3})h^2}{66.91(10^{-3}) + 54.29(10^{-3})h} = 0.160$$



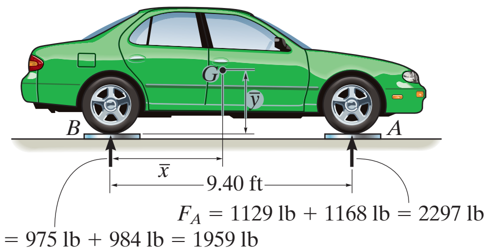

Solving,

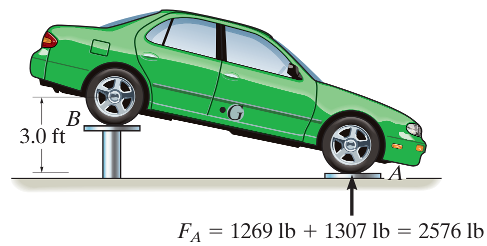
$$h = 0.385\ \text{m} = 385\ \text{mm} \qquad \textbf{Ans.}$$

**Ans:**
$h = 385$ mm

**6–49.**

The car rests on four scales and in this position the scale readings of both the front and rear tires are shown by $F_A$ and $F_B$. When the rear wheels are elevated to a height of 3 ft above the front scales, the new readings of the front wheels are also recorded. Use this data to calculate the location $\bar{x}$ and $\bar{y}$ to the center of gravity $G$ of the car. The tires each have a diameter of 1.98 ft.




## SOLUTION

In horizontal position,

$$W = 1959 + 2297 = 4256 \text{ lb}$$

$$\zeta + \Sigma M_B = 0; \qquad 2297(9.40) - 4256\bar{x} = 0$$

$$\bar{x} = 5.0733 = 5.07 \text{ ft} \qquad \textbf{Ans.}$$

$$\theta = \sin^{-1}\left(\frac{3 - 0.990}{9.40}\right) = 12.347^\circ$$

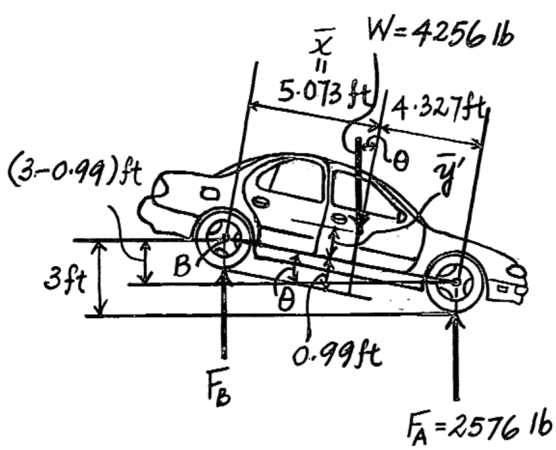
With rear whells elevated

$$\zeta + \Sigma M_B = 0; \qquad 2576(9.40 \cos 12.347^\circ) - 4256 \cos 12.347^\circ(5.0733)$$

$$- 4256 \sin 12.347^\circ\, \bar{y}' = 0$$

$$\bar{y}' = 2.86 \text{ ft}$$

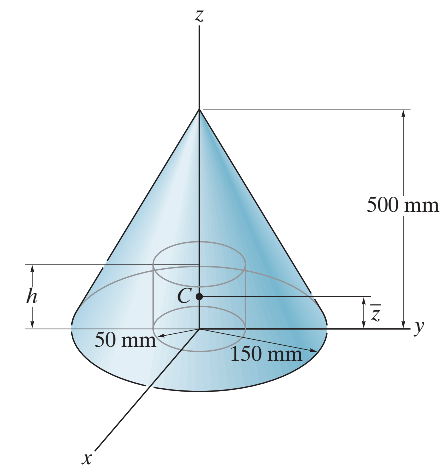
$$\bar{y} = 2.815 + 0.990 = 3.80 \text{ ft} \qquad \textbf{Ans.}$$




**Ans:**
$\bar{x}$ = 5.07 ft
$\bar{y}$ = 3.80 ft

**6–50.**

Determine the distance $h$ to which a 100-mm-diameter hole must be bored into the base of the cone so that the center of gravity of the resulting shape is located at $\bar{z} = 115$ mm. The material has a density of 8 Mg/m$^3$.




## SOLUTION

$$\frac{\frac{1}{3}\pi(0.15)^2(0.5)\left(\frac{0.5}{4}\right) - \pi(0.05)^2(h)\left(\frac{h}{2}\right)}{\frac{1}{3}\pi(0.15)^2(0.5) - \pi(0.05)^2(h)} = 0.115$$

$$0.4313 - 0.2875\,h = 0.4688 - 1.25\,h^2$$

$$h^2 - 0.230\,h - 0.0300 = 0$$

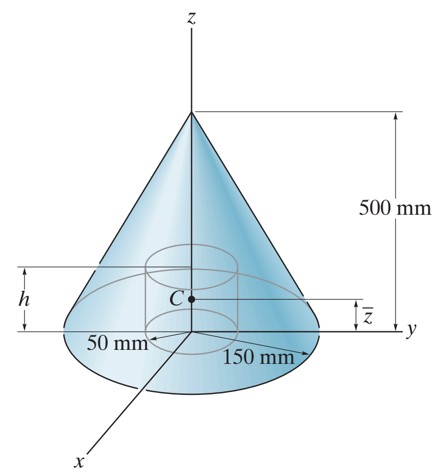
Choosing the positive root,

$$h = 323 \text{ mm}$$ **Ans.**

**Ans:**
$h = 323$ mm

**6–51.**

Determine the distance $\bar{z}$ to the centroid of the shape that consists of a cone with a hole of height $h = 50$ mm bored into its base.




## SOLUTION

$$\Sigma\tilde{z}V = \frac{1}{3}\pi\,(0.15)^2(0.5)\left(\frac{0.5}{4}\right) - \pi\,(0.05)^2(0.05)\left(\frac{0.05}{2}\right)$$

$$= 1.463(10^{-3})\ \text{m}^4$$

$$\Sigma V = \frac{1}{3}\pi\,(0.15)^2(0.5) - \pi\,(0.05)^2(0.05)$$

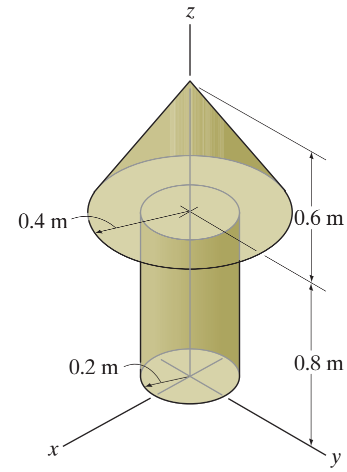
$$= 0.01139\ \text{m}^3$$

$$\bar{z} = \frac{\Sigma\tilde{z}\,V}{\Sigma V} = \frac{1.463\,(10^{-3})}{0.01139} = 0.12845\ \text{m} = 128\ \text{mm}$$ **Ans.**

**Ans:**
$\bar{z} = 128$ mm

***6–52.**

Locate the center of gravity $\bar{z}$ of the assembly. The cylinder and the cone are made from materials having densities of 5 Mg/m$^3$ and 9 Mg/m$^3$, respectively.

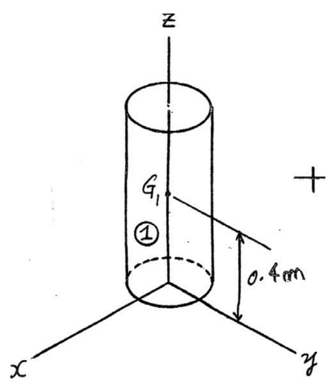



## SOLUTION

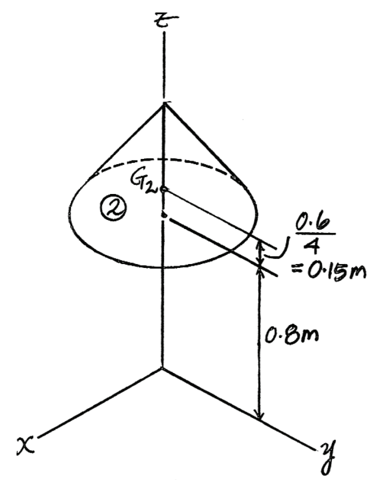
***Center of mass:*** The assembly is broken into two composite segments, as shown in Figs. *a* and *b*.

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{5000(0.4)\left[\pi(0.2^2)(0.8)\right] + 9000(0.8 + 0.15)\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}{5000\left[\pi(0.2^2)(0.8)\right] + 9000\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}$$

$$= \frac{1060.60}{1407.4} = 0.754 \text{ m} = 754 \text{ mm}$$ **Ans.**




(a)

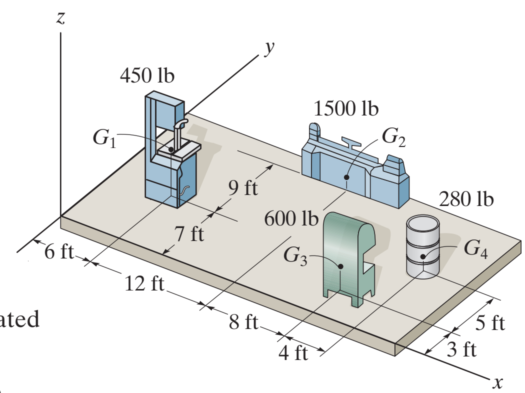



(b)

**Ans:**
$\bar{z} = 754$ mm

**6–53.**

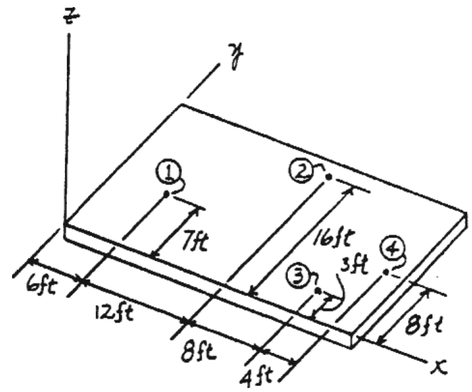
Major floor loadings in a shop are caused by the weights of the objects shown. Each force acts through its respective center of gravity $G$. Locate the center of gravity $(\bar{x}, \bar{y})$ of all these components.




## SOLUTION

***Centroid:*** The floor loadings on the floor and its respective centroid are tabulated below.

| **Loading** | ***W* (lb)** | $\bar{x}$ **(ft)** | $\bar{y}$ **(ft)** | $\bar{x}W$**(lb · ft)** | $\bar{y}W$**(lb · ft)** |
|---|---|---|---|---|---|
| 1 | 450 | 6 | 7 | 2700 | 3150 |
| 2 | 1500 | 18 | 16 | 27000 | 24000 |
| 3 | 600 | 26 | 3 | 15600 | 1800 |
| 4 | 280 | 30 | 8 | 8400 | 2240 |
| Σ | 2830 | | | 53700 | 31190 |




Thus,

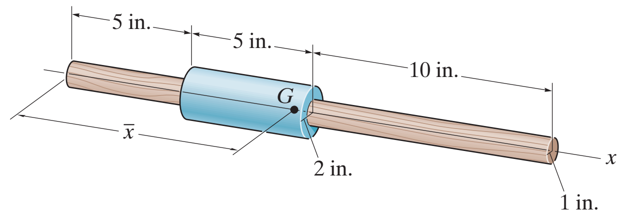
$$\bar{x} = \frac{\Sigma \bar{x}W}{\Sigma W} = \frac{53700}{2830} = 18.98 \text{ ft} = 19.0 \text{ ft} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \bar{y}W}{\Sigma W} = \frac{31190}{2830} = 11.02 \text{ ft} = 11.0 \text{ ft} \qquad \textbf{Ans.}$$

**Ans:**
$\bar{x} = 19.0$ ft
$\bar{y} = 11.0$ ft

**6–54.**

The assembly consists of a 20-in. wooden dowel rod and a tight-fitting steel collar. Determine the distance $\bar{x}$ to its center of gravity if the specific weights of the materials are $\gamma_w = 150\ \text{lb/ft}^3$ and $\gamma_{st} = 490\ \text{lb/ft}^3$. The radii of the dowel and collar are shown.




## SOLUTION

$$\Sigma \bar{x}W = \left\{10\pi(1)^2(20)(150) + 7.5\pi(5)(2^2 - 1^2)(490)\right\}\frac{1}{(12)^3}$$

$$= 154.8\ \text{lb} \cdot \text{in.}$$

$$\Sigma W = \left\{\pi(1)^2(20)(150) + \pi(5)(2^2 - 1^2)(490)\right\}\frac{1}{(12)^3}$$

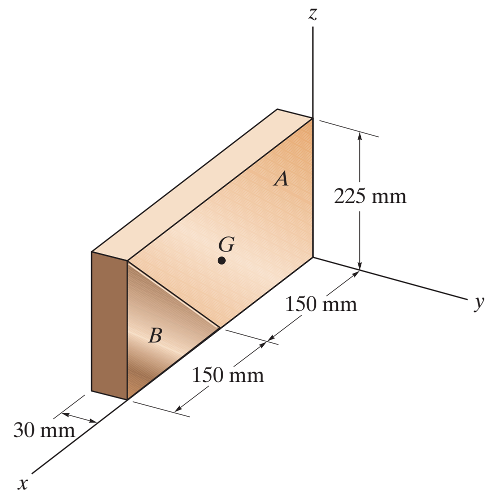
$$= 18.82\ \text{lb}$$

$$\bar{x} = \frac{\Sigma \bar{x}W}{\Sigma W} = \frac{154.8}{18.82} = 8.22\ \text{in.}$$ **Ans.**

**Ans:**
$\bar{x} = 8.22$ in.

**6–55.**

The composite plate is made from both steel ($A$) and brass ($B$) segments. Determine the weight and location $(\bar{x}, \bar{y}, \bar{z})$ of its center of gravity $G$. Take $\rho_{st} = 7.85\ \text{Mg/m}^3$, and $\rho_{br} = 8.74\ \text{Mg/m}^3$.




## SOLUTION

$$\Sigma m = \Sigma \rho V = \left[8.74\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right] + \left[7.85\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right]$$

$$+ [7.85(0.15)(0.225)(0.03)]$$

$$= \left[4.4246\left(10^{-3}\right)\right] + \left[3.9741\left(10^{-3}\right)\right] + \left[7.9481\left(10^{-3}\right)\right]$$

$$= 16.347\left(10^{-3}\right) = 16.4\ \text{kg} \qquad \textbf{Ans.}$$

$$\Sigma \bar{x} m = \left(0.150 + \frac{2}{3}(0.150)\right)(4.4246)\left(10^{-3}\right) + \left(0.150 + \frac{1}{3}(0.150)\right)(3.9741)\left(10^{-3}\right)$$

$$+ \frac{1}{2}(0.150)(7.9481)(10^{-3}) = 2.4971(10^{-3})\ \text{kg}\cdot\text{m}$$

$$\Sigma \bar{z} m = \left(\frac{1}{3}(0.225)\right)(4.4246)\left(10^{-3}\right) + \left(\frac{2}{3}(0.225)\right)(3.9471)\left(10^{-3}\right) + \left(\frac{0.225}{2}\right)(7.9481)\left(10^{-3}\right)$$

$$= 1.8221\left(10^{-3}\right)\text{kg}\cdot\ \text{m}$$

$$\bar{x} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{2.4971(10^{-3})}{16.347(10^{-3})} = 0.153\ \text{m} = 153\ \text{mm} \qquad \textbf{Ans.}$$

Due to symmetry:

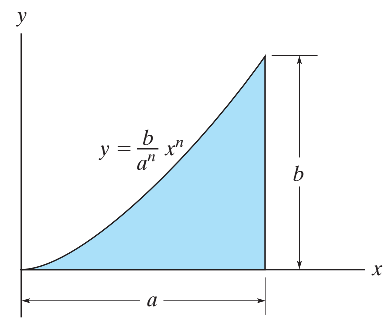
$$\bar{y} = -15\ \text{mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \bar{z} m}{\Sigma m} = \frac{1.8221(10^{-3})}{16.347(10^{-3})} = 0.1115\ \text{m} = 111\ \text{mm} \qquad \textbf{Ans.}$$

**Ans:**
$\Sigma m = 16.4$ kg
$\bar{x} = 153$ mm
$\bar{y} = -15$ mm
$\bar{z} = 111$ mm

***6–56.**

Determine the moment of inertia of the area about the $x$ axis.




## SOLUTION

**Differential Element.** Here $x = \frac{a}{b^{\frac{1}{n}}} y^{\frac{1}{n}}$. The area of the differential element parallel to the $x$ axis shown shaded in Fig. $a$ is $dA = (a - x)dy = \left(a - \frac{a}{b^{\frac{1}{n}}} y^{\frac{1}{n}}\right)dy$.

**Moment of Inertia.** Perform the integration

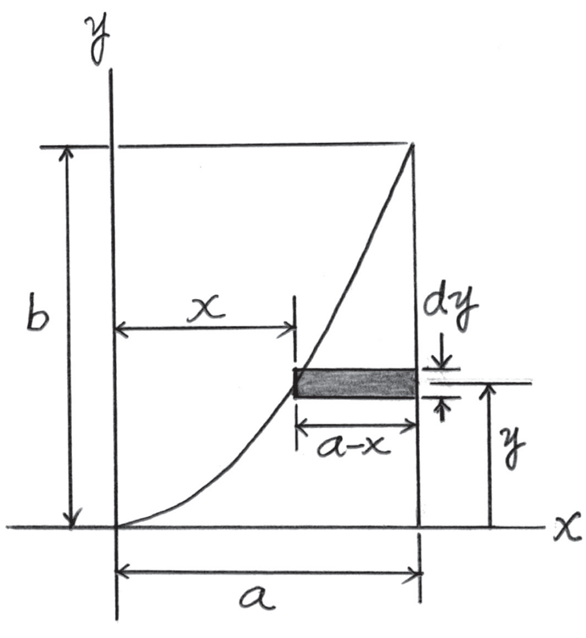
$$I_x = \int_A y^2 dA = \int_0^b y^2\left(a - \frac{a}{b^{\frac{1}{n}}} y^{\frac{1}{n}}\right)dy$$

$$= \int_0^b \left(ay^2 - \frac{a}{b^{\frac{1}{n}}} y^{\frac{1}{n}+2}\right)dy$$

$$= \left[\frac{a}{3}y^3 - \left(\frac{a}{b^{\frac{1}{n}}}\right)\left(\frac{n}{3n+1}\right)y^{\frac{3n+1}{n}}\right]\Bigg|_0^b$$

$$= \frac{1}{3}ab^3 - \left(\frac{n}{3n+1}\right)ab^3$$

$$= \frac{ab^3}{3(3n+1)}$$ **Ans.**



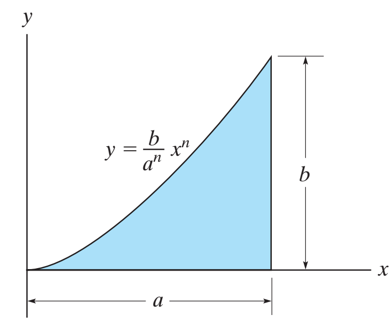

(a)

**Ans:**

$I_x = \frac{ab^3}{3(3n+1)}$

**6–57.**

Determine the moment of inertia of the area about the $y$ axis.




## SOLUTION

**Differential Element.** The area of the differential element parallel to the $y$ axis shown shaded in Fig. $a$ is $dA = y\,dx = \dfrac{b}{a^n}x^n dx$.

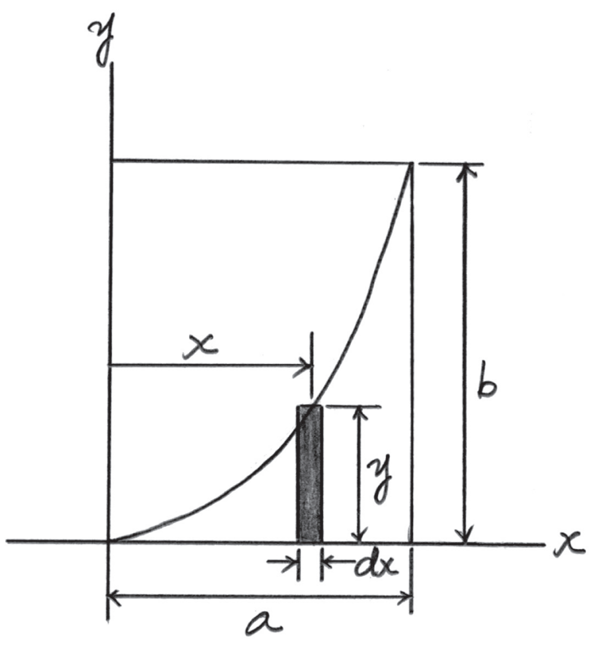
**Moment of Inertia.** Perform the integration

$$I_y = \int_A x^2 dA = \int_0^a x^2\left(\frac{b}{a^n}x^n dx\right)$$

$$= \int_0^a \frac{b}{a^n}x^{n+2}dx$$

$$= \frac{b}{a^n}\left(\frac{1}{n+3}\right)\left(x^{n+3}\right)\Big|_0^a$$

$$= \frac{a^3 b}{n+3} \qquad \textbf{Ans.}$$



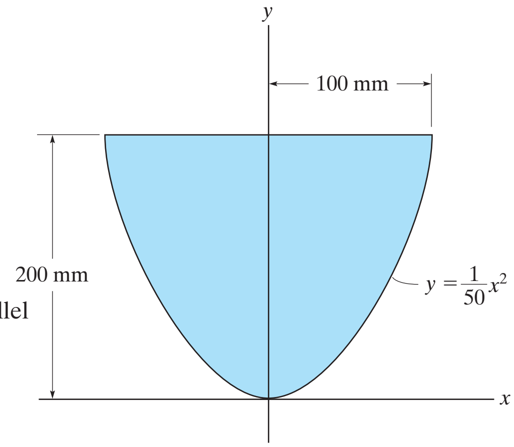

(a)

**Ans:**

$$I_y = \frac{a^3 b}{n+3}$$

**6–58.**

Determine the moment of inertia for the area about the $x$ axis.




## SOLUTION

**Differential Element.** Here $x = \sqrt{50}y^{\frac{1}{2}}$. The area of the differential element parallel to the $x$ axis shown shaded in Fig. $a$ is $dA = 2x\ dy = 2\sqrt{50}y^{\frac{1}{2}}dy$.

**Moment of Inertia.** Perform the integration

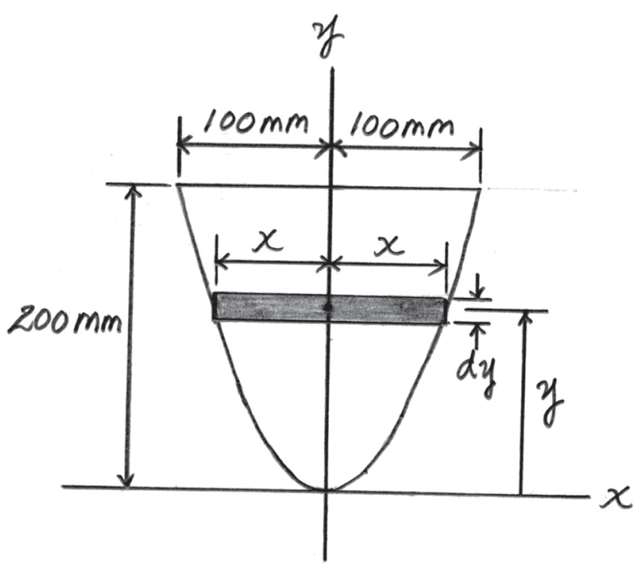
$$I_x = \int_A y^2 dA = \int_0^{200\text{ mm}} y^2\left[2\sqrt{50}y^{\frac{1}{2}}dy\right]$$

$$= 2\sqrt{50}\int_0^{200\text{ mm}} y^{\frac{5}{2}}dy$$

$$= 2\sqrt{50}\left(\frac{2}{7}y^{\frac{7}{2}}\right)\Bigg|_0^{200\text{ mm}}$$

$$= 457.14(10^6)\text{ mm}^4$$

$$= 457(10^6)\text{ mm}^4$$ **Ans.**

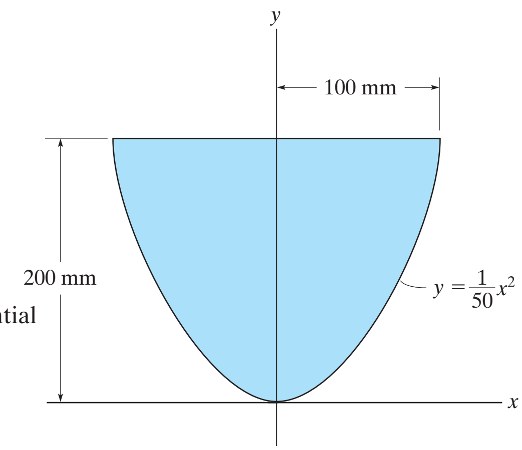


(a)

**Ans:**
$I_x = 457(10^6)\text{ mm}^4$

**6–59.**

Determine the moment of inertia for the area about the $y$ axis.




## SOLUTION

**Differential Element.** Here, $x = \sqrt{50}\,y^{\frac{1}{2}}$. The moment of inertia of the differential element parallel to the $x$ axis shown in Fig. $a$ about the $y$ axis is

$$dI_y = \frac{1}{12}(dy)(2x)^3 = \frac{2}{3}x^3 dy = \frac{2}{3}\left(\sqrt{50}y^{\frac{1}{2}}\right)^3 dy = \frac{100\sqrt{50}}{3}y^{\frac{3}{2}}dy.$$

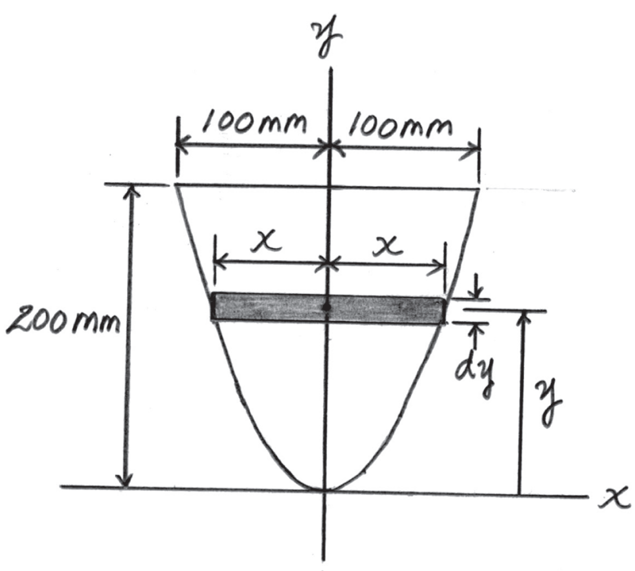
**Moment of Inertia.** Perform the integration

$$I_y = \int dI_y = \int_0^{200\text{ mm}} \frac{100\sqrt{50}}{3}y^{\frac{3}{2}}dy$$

$$= \frac{100\sqrt{50}}{3}\left(\frac{2}{5}y^{\frac{5}{2}}\right)\Bigg|_0^{200\text{ mm}}$$

$$= 53.33(10^6)\text{ mm}^4$$

$$= 53.3(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

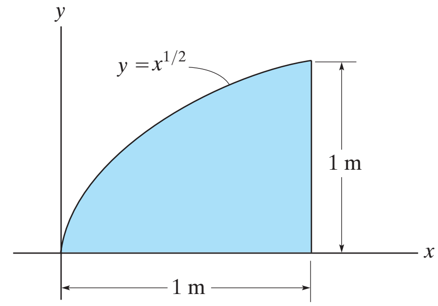



(a)

**Ans:**
$I_y = 53.3(10^6)\text{ mm}^4$

***6–60.**

Determine the moment of inertia for the area about the $x$ axis.




## SOLUTION

**Differential Element.** The area of the differential element parallel to the $y$ axis shown shaded in Fig. *a* is $dA = ydx$. The moment of inertia of this element about the $x$ axis is

$$dI_x = d\bar{I}_{x'} + dA\,\tilde{y}^2$$

$$= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2$$

$$= \frac{1}{3}y^3dx$$

$$= \frac{1}{3}(x^{\frac{1}{2}})^3dx$$

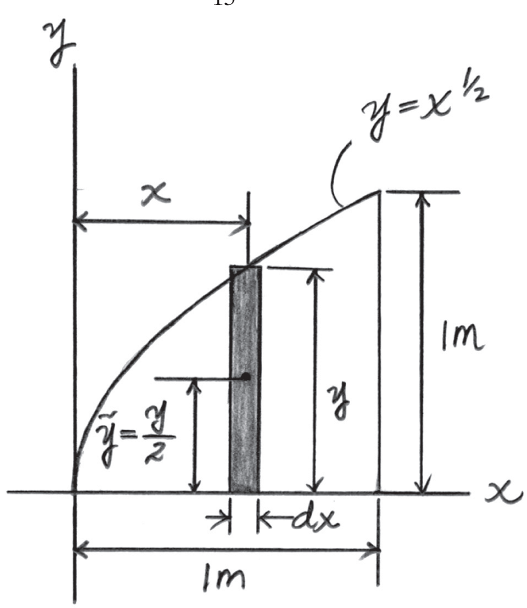
$$= \frac{1}{3}x^{\frac{3}{2}}dx$$

**Moment of Inertia.** Perform the integration.

$$I_x = \int dI_x = \int_0^{1\text{ m}} \frac{1}{3}x^{\frac{3}{2}}dx$$

$$= \frac{2}{15}x^{\frac{5}{2}}\Big|_0^{1\text{ m}}$$

$$= \frac{2}{15}\text{m}^4 = 0.133\,\text{m}^4 \qquad \textbf{Ans.}$$

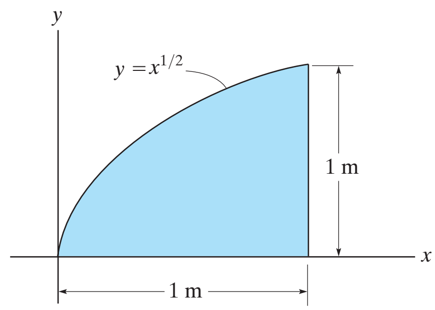



(a)

**Ans:**
$I_x = 0.133\text{ m}^4$

**6–61.**

Determine the moment of inertia for the area about the $y$ axis.




## SOLUTION

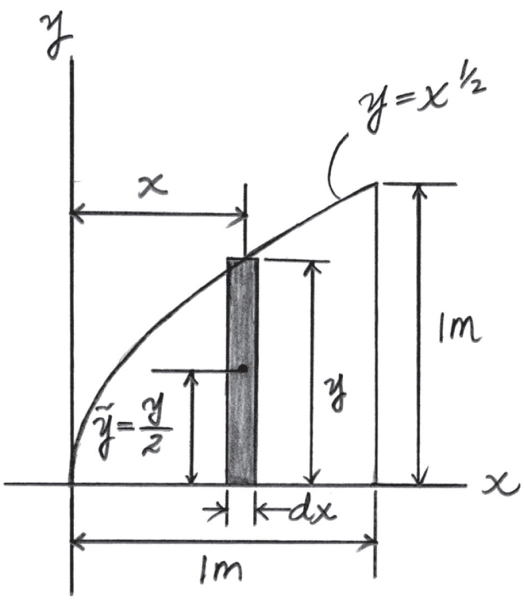
**Differential Element.** The area of the differential element parallel to the $y$ axis shown shaded in Fig. $a$ is $dA = ydx = x^{\frac{1}{2}}dx$.

**Moment of Inertia.** Perform the integration

$$I_y = \int_A x^2 dA = \int_0^{1\text{ m}} x^2\left(x^{\frac{1}{2}}dx\right)$$

$$= \frac{2}{7}x^{\frac{7}{2}}\Big|_0^{1\text{ m}}$$

$$= \frac{2}{7}\text{ m}^4 = 0.286\text{ m}^4 \qquad \textbf{Ans.}$$

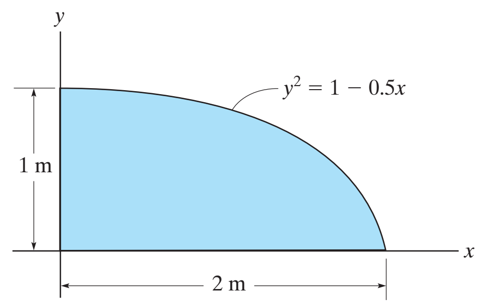



(a)

**Ans:**
$I_y = 0.286\text{ m}^4$

**6–62.**

Determine the moment of inertia for the area about the $x$ axis.




## SOLUTION

**Differential Element.** Here, $x = 2(1 - y^2)$. The area of the differential element parallel to the $x$ axis shown shaded in Fig. *a* is $dA = xdy = 2(1 - y^2)dy$.

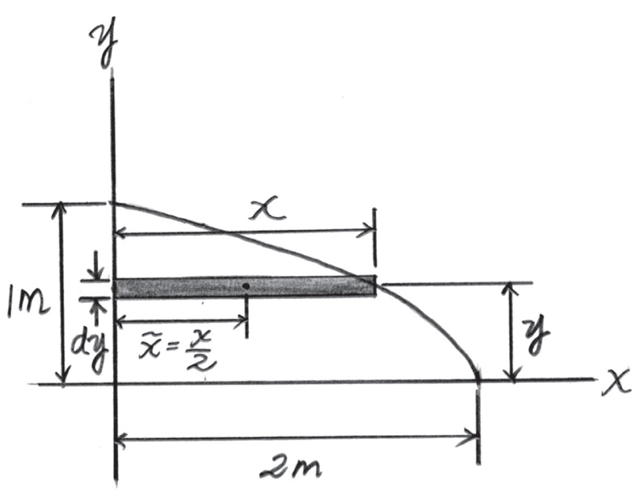
**Moment of Inertia.** Perform the integration

$$I_x = \int_A y^2 dA = \int_0^{1\text{ m}} y^2[2(1 - y^2)dy]$$

$$= 2\int_0^{1\text{ m}} (y^2 - y^4)dy$$

$$= 2\left(\frac{y^3}{3} - \frac{y^5}{5}\right)\Bigg|_0^{1\text{ m}}$$

$$= \frac{4}{15}\text{ m}^4 = 0.267\text{ m}^4 \qquad \textbf{Ans.}$$

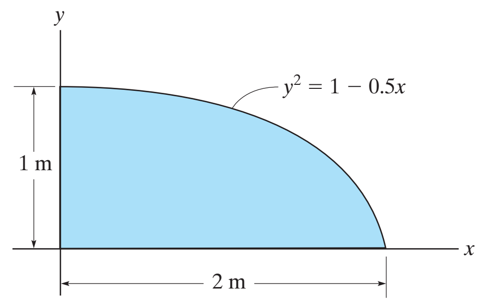



(a)

**Ans:**
$I_x = 0.267\text{ m}^4$

**6–63.**

Determine the moment of inertia for the area about the $y$ axis.




## SOLUTION

**Differential Element.** Here, $x = 2(1 - y^2)$. The moment of inertia of the differential element parallel to the $x$ axis shown shaded in Fig. $a$ about the $y$ axis is

$$dI_y = d\bar{I}_{y'} + dA\tilde{x}^2$$

$$= \frac{1}{12}(dy)x^3 + xdy\left(\frac{x}{2}\right)^2$$

$$= \frac{1}{3}x^3 dy$$

$$= \frac{1}{3}\left[2(1 - y^2)\right]^3 dy$$

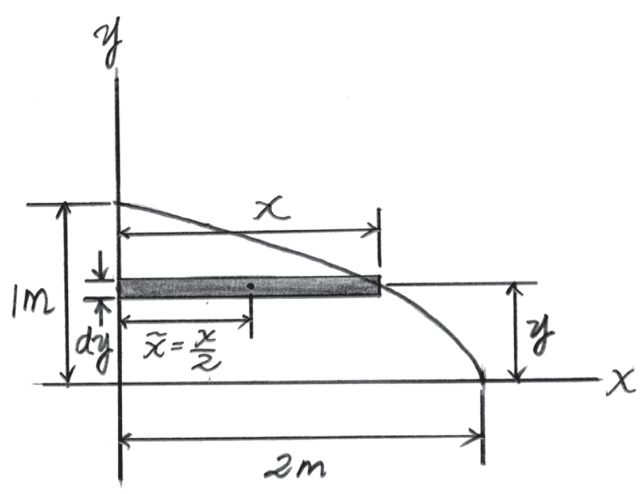
$$= \frac{8}{3}(-y^6 + 3y^4 - 3y^2 + 1)dy$$

**Moment of Inertia.** Perform the integration

$$I_y = \int dI_y = \frac{8}{3}\int_0^{1\text{ m}} (-y^6 + 3y^4 - 3y^2 + 1)dy$$

$$= \frac{8}{3}\left(-\frac{y^7}{7} + \frac{3}{5}y^5 - y^3 + y\right)\Bigg|_0^{1\text{ m}}$$

$$= \frac{128}{105}\text{ m}^4 = 1.22\text{ m}^4$$ **Ans.**

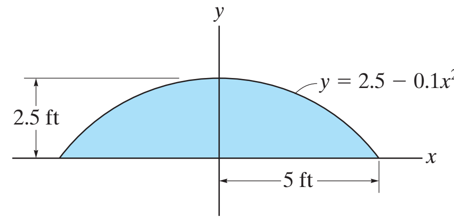



(a)

**Ans:**
$I_y = 1.22\text{ m}^4$

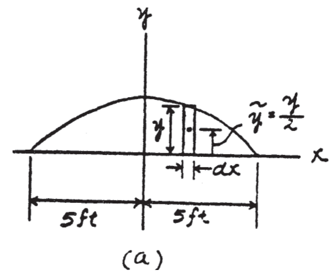
***6–64.**

Determine the moment of inertia of the area about the $x$ axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness $dx$ and (b) having a thickness of $dy$.




## SOLUTION

(a) ***Differential Element:*** The area of the differential element parallel to the $y$ axis is $dA = ydx$. The moment of inertia of this element about the $x$ axis is



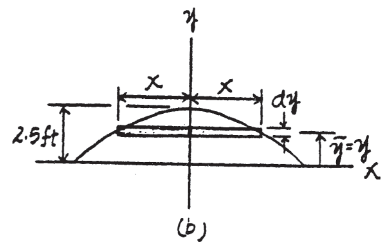

$$dI_x = d\bar{I}_{x'} + dA\tilde{y}^2$$

$$= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2$$

$$= \frac{1}{3}(2.5 - 0.1x^2)^3\,dx$$

$$= \frac{1}{3}(-0.001x^6 + 0.075x^4 - 1.875x^2 + 15.625)\,dx$$

***Moment of Inertia:*** Performing the integration, we have




$$I_x = \int dI_x = \frac{1}{3}\int_{-5\text{ ft}}^{5\text{ ft}}(-0.001x^6 + 0.075x^4 - 1.875x^2 + 15.625)\,dx$$

$$= \frac{1}{3}\left(-\frac{0.001}{7}x^7 + \frac{0.075}{5}x^5 - \frac{1.875}{3}x^3 + 15.625x\right)\Bigg|_{-5\text{ ft}}^{5\text{ ft}}$$

$$= 23.8\text{ ft}^4$$ **Ans.**

(b) ***Differential Element:*** Here, $x = \sqrt{25 - 10y}$. The area of the differential element parallel to the $x$ axis is $dA = 2xdy = 2\sqrt{25 - 10y}\,dy$.

***Moment of Inertia:*** Applying Eq. 10–1 and performing the integration, we have

$$I_x = \int_A y^2 dA$$

$$= 2\int_0^{2.5\text{ ft}} y^2\sqrt{25 - 10y}\,dy$$

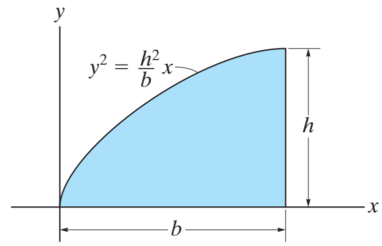
$$= 2\left[-\frac{y^2}{15}(25 - 10y)^{\frac{3}{2}} - \frac{2y}{375}(25 - 10y)^{\frac{3}{2}} - \frac{2}{13125}(25 - 10y)^{\frac{7}{2}}\right]\Bigg|_0^{2.5\text{ ft}}$$

$$= 23.8\text{ ft}^4$$ **Ans.**

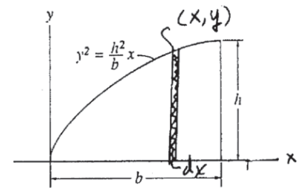
**Ans:**
$I_x = 23.8\text{ ft}^4$

**6–65.**

Determine the moment of inertia of the area about the $x$ axis.




## SOLUTION



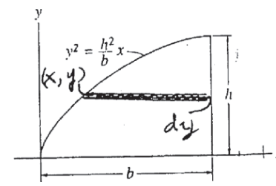

$$d\,I_x = \frac{1}{3}y^3\,dx$$

$$I_x = \int d\,I_x$$

$$= \int_0^b \frac{y^3}{3}\;dx = \int_0^b \frac{1}{3}\left(\frac{h^2}{b}\right)^{3/2} x^{3/2}\,dx$$

$$= \frac{1}{3}\left(\frac{h^2}{b}\right)^{3/2}\left(\frac{2}{5}\right)x^{5/2}\Big]_0^b$$

$$= \frac{2}{15}bh^3 \qquad \textbf{Ans.}$$




Also,

$$dA = (b-x)\,dy = (b - \frac{b}{h^2}y^2)\,dy$$

$$I_x = \int y^2\,dA$$

$$= \int_0^h y^2\,(b - \frac{b}{h^2}y^2)\,dy$$

$$= \left[\frac{b}{3}y^3 - \frac{b}{5h^2}y^5\right]_0^h$$

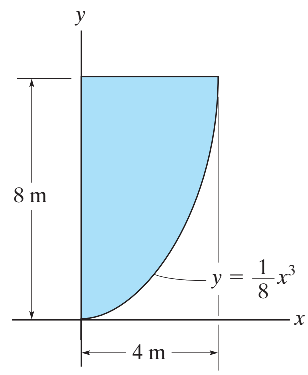
$$= \frac{2}{15}bh^3 \qquad \textbf{Ans.}$$

**Ans:**

$$I_x = \frac{2}{15}bh^3$$

**6–66.**

Determine the moment of inertia for the area about the $x$ axis.




## SOLUTION

**Differential Element.** Here, $x = 2y^{\frac{1}{3}}$. The area of the differential element parallel to the $x$ axis shown shaded in Fig. $a$ is $dA = xdy = 2y^{\frac{1}{3}}\,dy$.

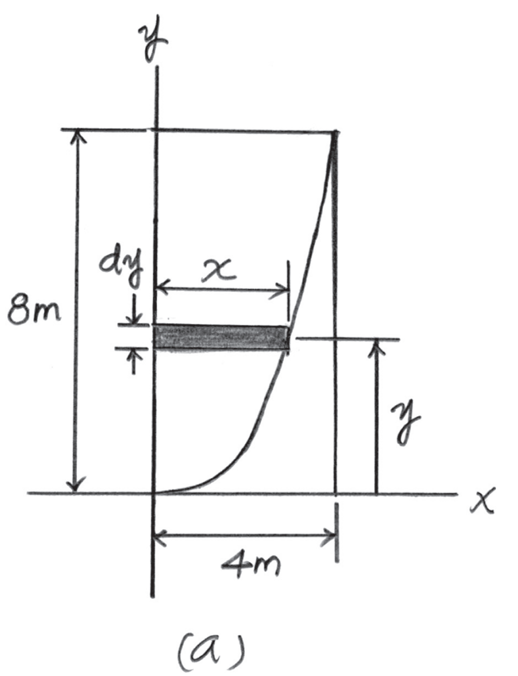
**Moment of Inertia.** Perform the integration

$$I_x = \int_A y^2 dA = \int_0^{8\text{ m}} y^2\,(2y^{\frac{1}{3}}\,dy)$$

$$= 2\int_0^{8\text{ m}} y^{\frac{7}{3}}\,dy$$

$$= 2\left(\frac{3}{10}y^{\frac{10}{3}}\right)\Bigg|_0^{8\text{ m}}$$

$$= 614.4\text{ m}^4 = 614\text{ m}^4$$ **Ans.**

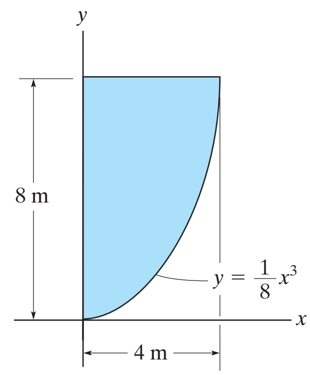


(a)

**Ans:**
$I_x = 614\text{ m}^4$

**6–67.**

Determine the moment of inertia for the area about the $y$ axis.




## SOLUTION

**Differential Element.** The area of the differential element parallel to the $y$ axis, shown shaded in Fig. $a$, is $dA = (8 - y)d_x = \left(8 - \frac{1}{8}x^3\right)dx$.

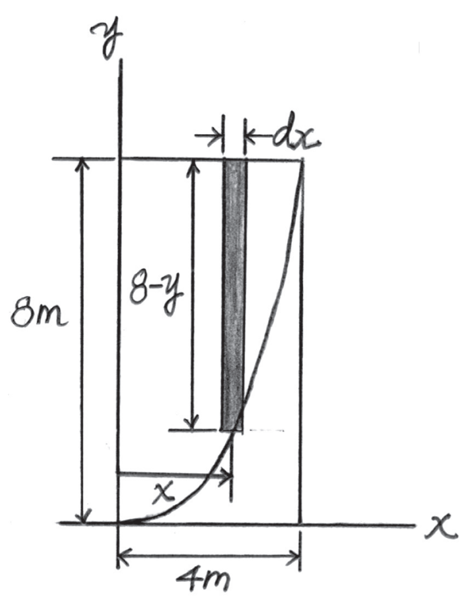
**Moment of Inertia.** Perform the integration

$$I_y = \int_A x^2 dA = \int_0^{4\text{ m}} x^2\left(8 - \frac{1}{8}x^3\right) dx$$

$$= \int_0^{4\text{ m}} \left(8x^2 - \frac{1}{8}x^5\right) dx$$

$$= \left(\frac{8}{3}x^3 - \frac{1}{48}x^6\right)\Bigg|_0^{4\text{ m}}$$

$$= 85.33\text{ m}^4 = 85.3\text{ m}^4 \qquad \textbf{Ans.}$$

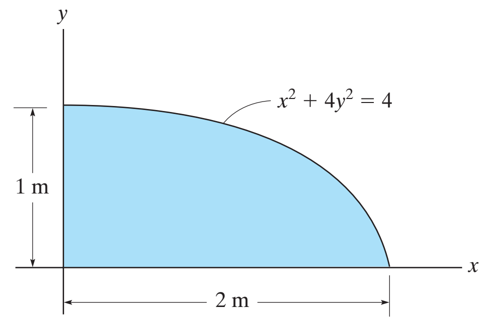



(a)

**Ans:**
$I_y = 85.3\text{ m}^4$

***6–68.**

Determine the moment of inertia about the $x$ axis.




## SOLUTION

**Differential Element.** Here, $y = \frac{1}{2}\sqrt{4 - x^2}$. The moment of inertia of the differential element parallel to the $y$ axis shown shaded in Fig. $a$ about the $x$ axis is

$$dI_x = d\bar{I}_{x'} + dA\tilde{y}^2$$

$$= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2$$

$$= \frac{1}{3}y^3 dx$$

$$= \frac{1}{3}\left(\frac{1}{2}\sqrt{4 - x^2}\right)^3 dx$$

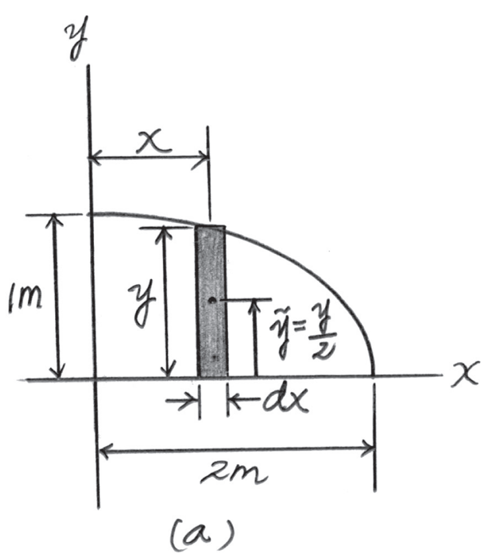
$$= \frac{1}{24}\sqrt{(4 - x^2)^3}\, dx$$

**Moment of Inertia.** Perform the integration.

$$I_x = \int dI_x = \int_0^{2\text{ m}} \frac{1}{24}\sqrt{(4 - x^2)^3}\, dx$$

$$= \frac{1}{96}\left[x\sqrt{(4 - x^2)^3} + 6x\sqrt{4 - x^2} + 24 \sin^{-1}\frac{x}{2}\right]\Bigg|_0^{2\text{ m}}$$

$$= \frac{\pi}{8}\text{ m}^4 \qquad \textbf{Ans.}$$



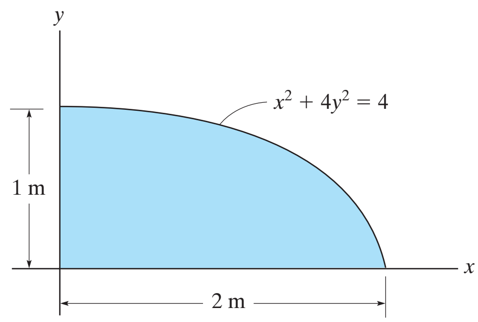

(a)

**Ans:**

$I_x = \frac{\pi}{8}\text{ m}^4$

**6–69.**

Determine the moment of inertia about the $y$ axis.




## SOLUTION

**Differential Element.** Here, $y = \frac{1}{2}\sqrt{4 - x^2}$. The area of the differential element parallel to the $y$ axis shown shaded in Fig. $a$ is $dA = ydx = \frac{1}{2}\sqrt{4 - x^2}dx$.

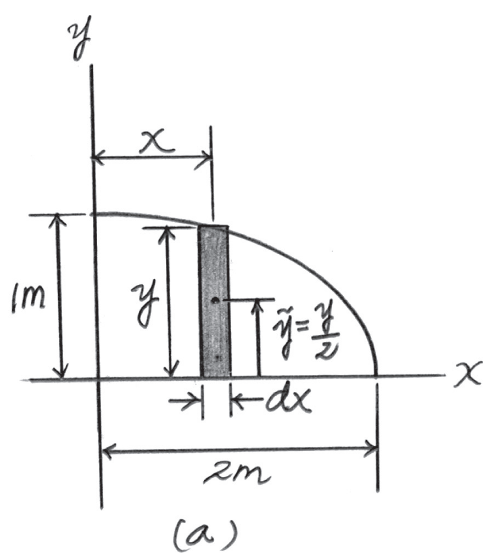
**Moment of Inertia.** Perform the integration

$$I_y = \int_A x^2 dA = \int_0^{2\text{ m}} x^2\left[\frac{1}{2}\sqrt{4 - x^2}dx\right]$$

$$= \frac{1}{2}\int_0^{2\text{ m}} x^2\sqrt{4 - x^2}dx$$

$$= \frac{1}{2}\left[-\frac{x}{4}\sqrt{(4 - x^2)^3} + \frac{1}{2}\left(x\sqrt{4 - x^2} + 4\sin^{-1}\frac{x}{2}\right)\right]\Bigg|_0^{2\text{ m}}$$

$$= \frac{\pi}{2}\text{ m}^4$$ **Ans.**



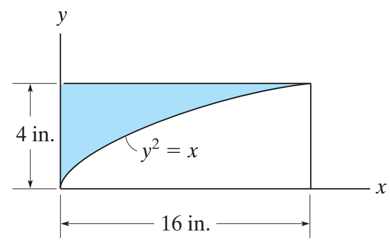

(a)

**Ans:**

$I_y = \frac{\pi}{2}\text{ m}^4$

**6–70.**

Determine the moment of inertia for the area about the $x$ axis.




## SOLUTION

**Differential Element.** The area of the differential element parallel with the $x$ axis shown shaded in Fig. $a$ is $dA = x\,dy = y^2\,dy$.

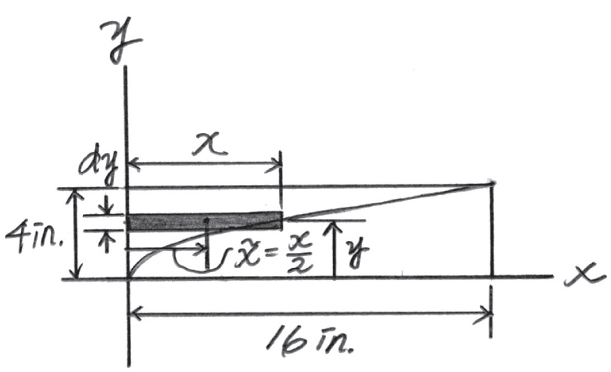
**Moment of Inertia.** Perform the integration

$$I_x = \int_A y^2 dA = \int_0^{4\text{ in.}} y^2(y^2 dy)$$

$$= \int_0^{4\text{ in.}} y^4 dy$$

$$= \left.\frac{y^5}{5}\right|_0^{4\text{ in.}}$$

$$= 204.8 \text{ in}^4 = 205 \text{ in}^4$$ **Ans.**

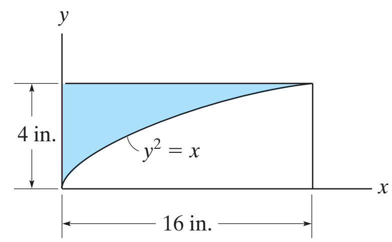



(a)

**Ans:**
$I_x = 205 \text{ in}^4$

**6–71.**

Determine the moment of inertia for the area about the $y$ axis.



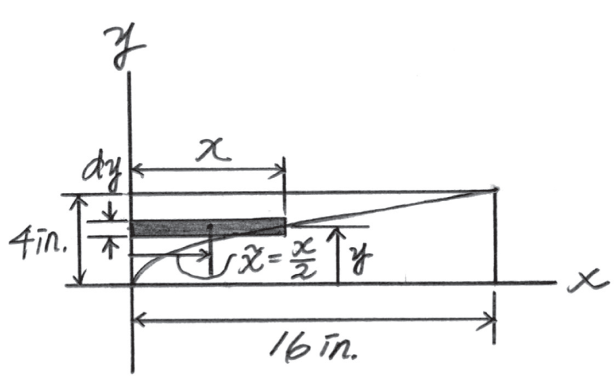

## SOLUTION

**Differential Element.** The moment of inertia of the differential element parallel to the $x$ axis shown shaded in Fig. $a$ about the $y$ axis is

$$dI_y = d\bar{I}_y + dA\tilde{x}^2$$
$$= \frac{1}{12}(dy)x^3 + (xdy)\left(\frac{x}{2}\right)^2$$
$$= \frac{1}{3}x^3dy$$
$$= \frac{1}{3}(y^2)^3dy$$
$$= \frac{1}{3}y^6dy$$

**Moment of Inertia.** Perform the integration

$$I_y = \int dI_y = \int_0^{4\text{ in.}} \frac{1}{3}y^6dy$$
$$= \frac{1}{3}\left(\frac{y^7}{7}\right)\Bigg|_0^{4\text{ in.}}$$
$$= 780.19\text{ in}^4 = 780\text{ in}^4$$ **Ans.**

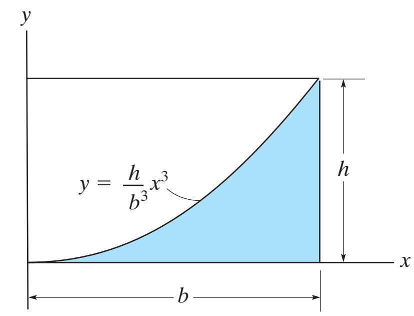



(a)

**Ans:**
$I_y = 780\text{ in}^4$

***6–72.**

Determine the moment of inertia for the area about the $x$ axis.



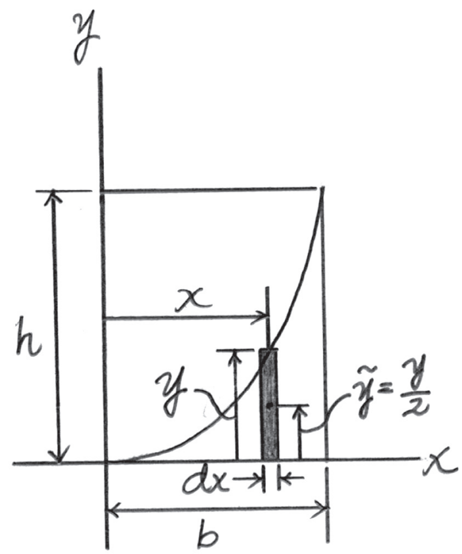

## SOLUTION

**Differential Element.** The moment of inertia of the differential element parallel to the $y$ axis shown shaded in Fig. $a$ about the $x$ axis is

$$dI_x = d\bar{I}_{x'} + dA\tilde{y}^2$$
$$= \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2$$
$$= \frac{1}{3}y^3dx$$
$$= \frac{1}{3}\left(\frac{h}{b^3}x^3\right)^3 dx$$
$$= \frac{h^3}{3b^9}x^9dx$$

**Moment of Inertia.** Perform the integration

$$I_x = \int dI_x = \int_0^b \frac{h^3}{3b^9}x^9dx$$
$$= \frac{h^3}{3b^9}\left(\frac{x^{10}}{10}\right)\Bigg|_0^b$$
$$= \frac{1}{30}bh^3$$ **Ans.**

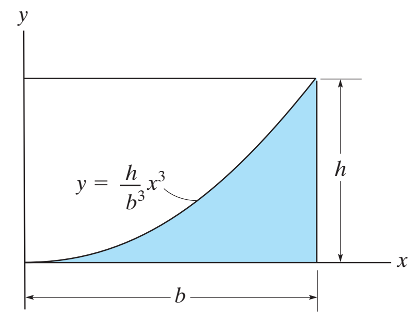



(a)

**Ans:**
$I_x = \frac{1}{30}bh^3$

**6–73.**

Determine the moment of inertia for the area about the $y$ axis.




## SOLUTION

**Differential Element.** The area of the differential element parallel to the $y$ axis shown shaded in Fig. $a$ is $dA = ydx = \frac{h}{b^3}x^3dx$.

**Moment of Inertia.** Perform the integration

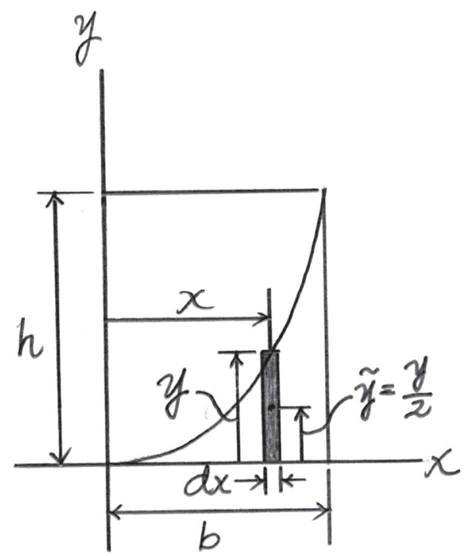
$$I_y = \int_A x^2 dA = \int_0^b x^2\left(\frac{h}{b^3}x^3\right)dx$$

$$= \frac{h}{b^3}\int_0^b x^5 dx$$

$$= \frac{h}{b^3}\left(\frac{x^6}{6}\right)\Bigg|_0^6$$

$$= \frac{b^3h}{6}$$

**Ans.**



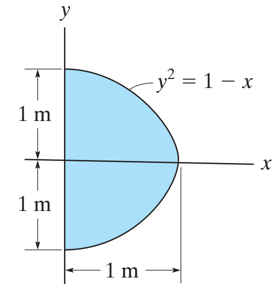

(a)

**Ans:**

$I_y = \frac{b^3h}{6}$

**6–74.**

Determine the moment of inertia for the area about the $x$ axis.




## SOLUTION

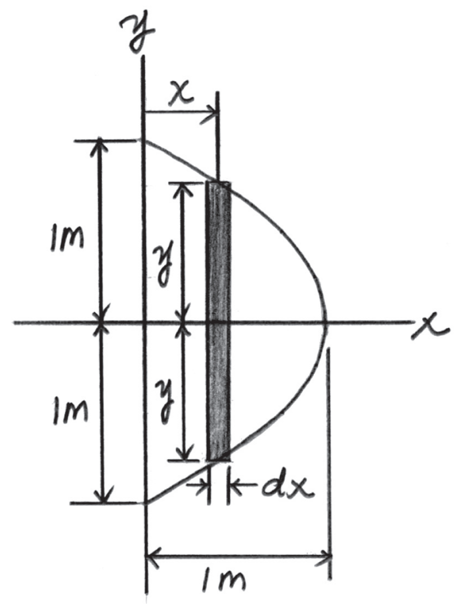
**Differential Element.** Here, $y = (1 - x)^{\frac{1}{2}}$. The moment of inertia of the differential element parallel to the $y$ axis shown shaded in Fig. $a$ about the $x$ axis is $dI_x = \frac{1}{12}(dx)(2y)^3 = \frac{2}{3}y^3 dx = \frac{2}{3}[(1 - x)^{\frac{1}{2}}]^3 dx = \frac{2}{3}(1 - x)^{\frac{3}{2}} dx$.

**Moment of Inertia.** Perform the integration

$$I_x = \int dI_x = \int_0^{1\text{ m}} \frac{2}{3}(1 - x)^{\frac{3}{2}} dx$$

$$= \frac{2}{3}\left[-\frac{2}{5}(1 - x)^{\frac{5}{2}}\right]\Bigg|_0^{1\text{ m}}$$

$$= \frac{4}{15}\text{ m}^4 = 0.267\text{ m}^4$$ **Ans.**

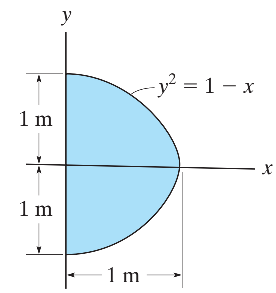


(a)

**Ans:**
$I_x = 0.267\text{ m}^4$

**6–75.**

Determine the moment of inertia for the area about the $y$ axis.




## SOLUTION

**Differential Element.** Here $x = 1 - y^2$. The moment of inertia of the differential element parallel to the $x$ axis shown shaded in Fig. $a$ about the $y$ axis is

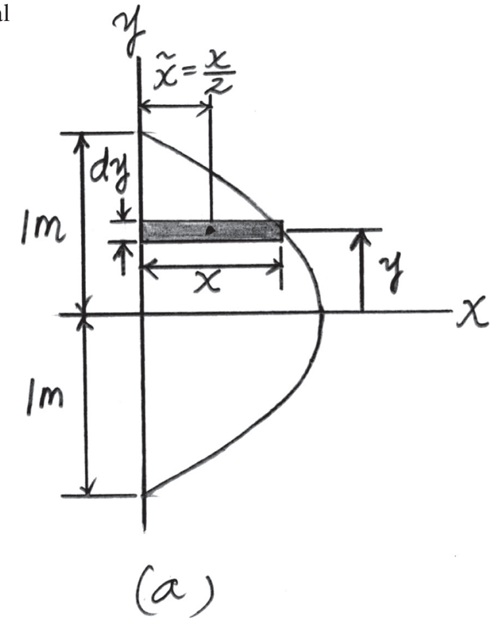
$$dI_y = d\bar{I}_{y'} + dA\tilde{x}^2$$

$$= \frac{1}{12}(dy)x^3 + xdy\left(\frac{x}{2}\right)^2$$

$$= \frac{1}{3}x^3dy$$

$$= \frac{1}{3}(1 - y^2)^3dy$$

$$= \frac{1}{3}(-y^6 + 3y^4 - 3y^2 + 1)dy$$




**Moment of Inertia.** Perform the integration

$$I_y = \int dI_y = \int_{-1\text{ m}}^{1\text{ m}} \frac{1}{3}(-y^6 + 3y^4 - 3y^2 + 1)dy$$

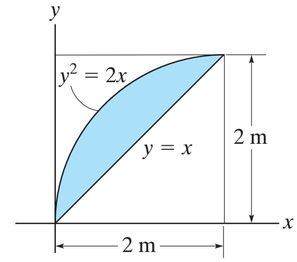
$$= \frac{1}{3}\left(-\frac{y^7}{7} + \frac{3}{5}y^5 - y^3 + y\right)\Bigg|_{-1\text{ m}}^{1\text{ m}}$$

$$= \frac{32}{105}\text{ m}^4 = 0.305\text{ m}^4 \qquad \textbf{Ans.}$$

**Ans:**
$I_y = 0.305\text{ m}^4$

***6–76.**

Determine the moment of inertia for the area about the $x$ axis.




## SOLUTION

**Differential Element.** Here, $x_2 = y$ and $x_1 = \frac{1}{2}y^2$. The area of the differential element parallel to the $x$ axis shown shaded in Fig. $a$ is $dA = (x_2 - x_1)dy = \left(y - \frac{1}{2}y^2\right)dy$.

**Moment of Inertia.** Perform the integration

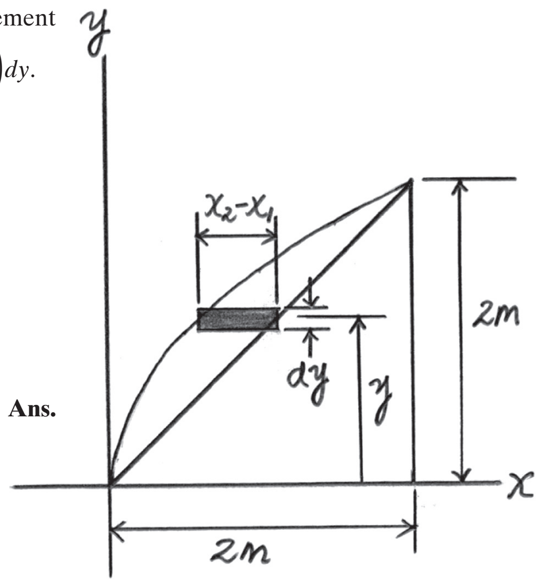
$$I_x = \int_A y^2 dA = \int_0^{2\text{ m}} y^2\left(y - \frac{1}{2}y^2\right)dy$$

$$= \int_0^{2\text{ m}} \left(y^3 - \frac{1}{2}y^4\right)dy$$

$$= \left(\frac{y^4}{4} - \frac{y^5}{10}\right)\Bigg|_0^{2\text{ m}}$$

$$= 0.8\text{ m}^4$$

**Ans.**

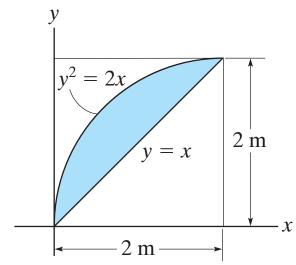



(a)

**Ans:**
$I_x = 0.8\text{ m}^4$

**6–77.**

Determine the moment of inertia for the area about the $y$ axis.




## SOLUTION

**Differential Element.** Here, $y_2 = \sqrt{2}x^{\frac{1}{2}}$ and $y_1 = x$. The area of the differential element parallel to the $y$ axis shown shaded in Fig. $a$ is $dA = (y_2 - y_1)\,dx = \left(\sqrt{2}x^{\frac{1}{2}} - x\right)dx$.

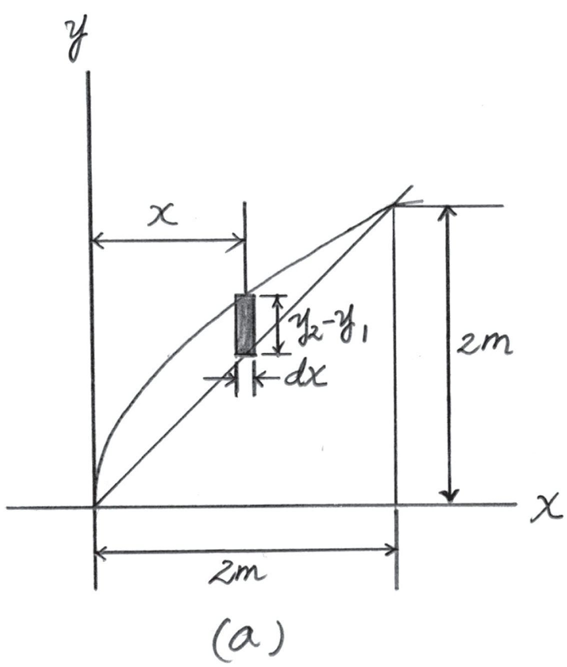
**Moment of Inertia.** Perform the integration

$$I_y = \int_A x^2 dA = \int_0^{2\text{ m}} x^2\left(\sqrt{2}x^{\frac{1}{2}} - x\right)dx$$

$$= \int_0^{2\text{ m}} \left(\sqrt{2}x^{\frac{5}{2}} - x^3\right)dx$$

$$= \left(\frac{2\sqrt{2}}{7}x^{\frac{7}{2}} - \frac{x^4}{4}\right)\Bigg|_0^{2\text{ m}}$$

$$= \frac{4}{7}\text{ m}^4 = 0.571\text{ m}^4 \qquad \textbf{Ans.}$$

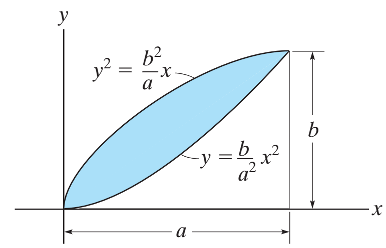



(a)

**Ans:**
$I_y = 0.571\text{ m}^4$

**6–78.**

Determine the moment of inertia for the area about the $x$ axis.




## SOLUTION

**Differential Element.** Here, $x_2 = \frac{a}{b^{\frac{1}{2}}}y^{\frac{1}{2}}$ and $x_1 = \frac{a}{b^2}y^2$. Thus, the area of the differential element parallel to the $x$ axis shown shaded in Fig. $a$ is $dA = (x_2 - x_1)\,dy = \left(\frac{a}{b^{\frac{1}{2}}}y^{\frac{1}{2}} - \frac{a}{b^2}y^2\right)dy$.

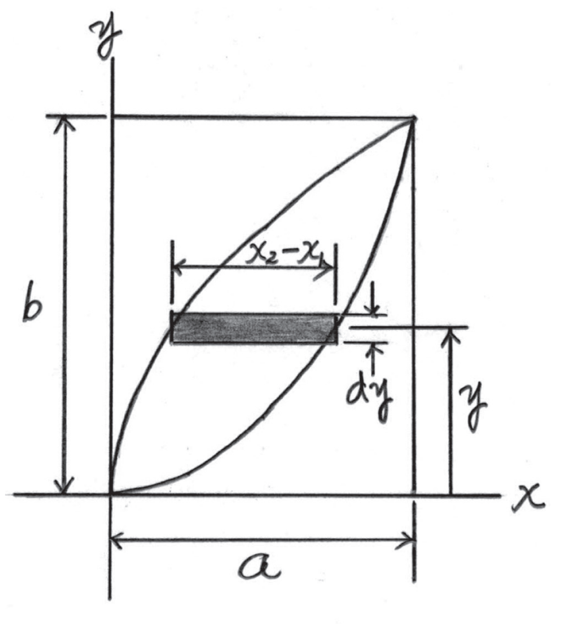
**Moment of Inertia.** Perform the integration

$$I_x = \int_A y^2 dA = \int_0^b y^2\left(\frac{a}{b^{\frac{1}{2}}}y^{\frac{1}{2}} - \frac{a}{b^2}y^2\right)dy$$

$$= \int_0^b \left(\frac{a}{b^{\frac{1}{2}}}y^{\frac{5}{2}} - \frac{a}{b^2}y^4\right)dy$$

$$= \left(\frac{2a}{7b^{\frac{1}{2}}}y^{\frac{7}{2}} - \frac{a}{5b^2}y^5\right)\Bigg|_0^b$$

$$= \frac{3ab^3}{35}$$ **Ans.**



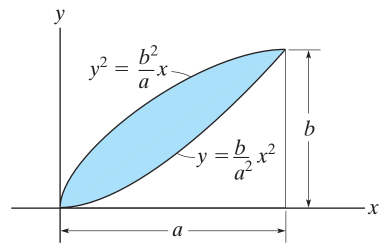

(a)

**Ans:**

$I_x = \frac{3ab^3}{35}$

**6–79.**

Determine the moment of inertia for the area about the $y$ axis.




## SOLUTION

**Differential Element.** Here, $y_2 = \frac{b}{a^{\frac{1}{2}}}x^{\frac{1}{2}}$ and $y_1 = \frac{b}{a^2}x^2$. Thus, the area of the differential element parallel to the $y$ axis shown shaded in Fig. $a$ is $dA = (y_2 - y_1)dx$ $= \left(\frac{b}{a^{\frac{1}{2}}}x^{\frac{1}{2}} - \frac{b}{a^2}x^2\right)dx$.

**Moment of Inertia.** Perform the integration

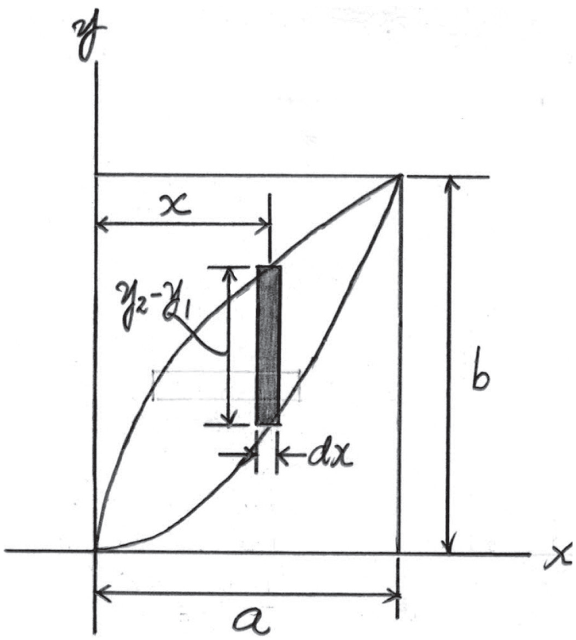
$$I_y = \int_A x^2 dA = \int_0^a x^2\left(\frac{b}{a^{\frac{1}{2}}}x^{\frac{1}{2}} - \frac{b}{a^2}x^2\right)dx$$

$$= \int_0^a \left(\frac{b}{a^{\frac{1}{2}}}x^{\frac{5}{2}} - \frac{b}{a^2}x^4\right)dx$$

$$= \left(\frac{2b}{7a^{\frac{1}{2}}}x^{\frac{7}{2}} - \frac{b}{5a^2}x^5\right)\Bigg|_0^a$$

$$= \frac{2}{7}a^3b - \frac{1}{5}a^3b$$

$$= \frac{3a^3b}{35} \qquad \textbf{Ans.}$$



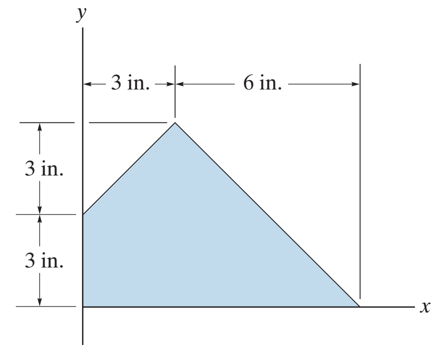

(a)

**Ans:**

$I_y = \frac{3a^3b}{35}$

***6–80.**

Determine the moment of inertia of the composite area about the $x$ axis.




## SOLUTION

***Composite Parts:*** The composite area can be subdivided into three segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the $x$ axis is also indicated.

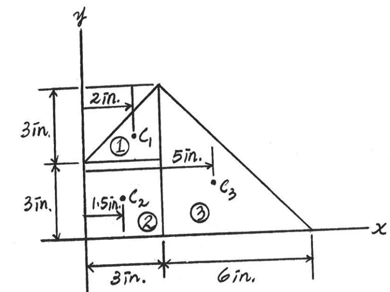
***Moment of Inertia:*** The moment of inertia of each segment about the $x$ axis can be determined using the parallel-axis theorem. Thus,

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

$$= \left[\frac{1}{36}(3)(3^3) + \frac{1}{2}(3)(3)(4)^2\right] + \left[\frac{1}{12}(3)(3^3) + 3(3)(1.5)^2\right]$$

$$+ \left[\frac{1}{36}(6)(6^3) + \frac{1}{2}(6)(6)(2)^2\right]$$

$$= 209 \text{ in}^4$$ **Ans.**

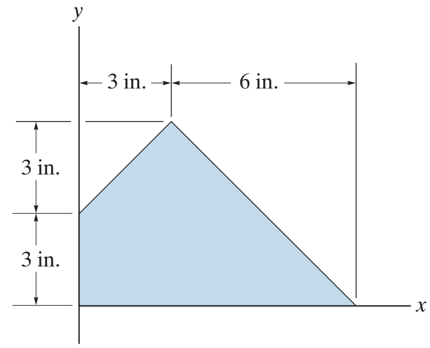


(a)

**Ans:**
$I_x = 209 \text{ in}^4$

**6–81.**

Determine the moment of inertia of the composite area about the $y$ axis.




## SOLUTION

***Composite Parts:*** The composite area can be subdivided into three segments as shown in Fig. *a*. The perpendicular distance measured from the centroid of each segment to the $x$ axis is also indicated.

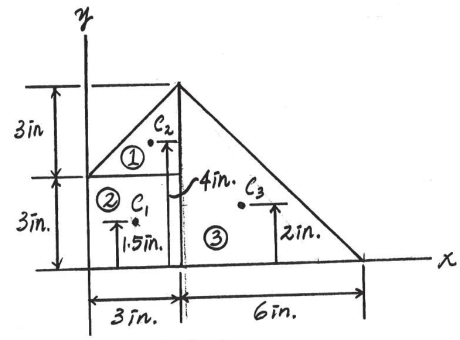
***Moment of Inertia:*** The moment of inertia of each segment about the $y$ axis can be determined using the parallel-axis theorem. Thus,

$$I_y = \bar{I}_{y'} + A(d_x)^2$$

$$= \left[\frac{1}{36}(3)(3^3) + \frac{1}{2}(3)(3)(2)^2\right] + \left[\frac{1}{12}(3)(3^3) + 3(3)(1.5)^2\right]$$

$$+ \left[\frac{1}{36}(6)(6^3) + \frac{1}{2}(6)(6)(5)^2\right]$$

$$= 533 \text{ in}^4$$ **Ans.**

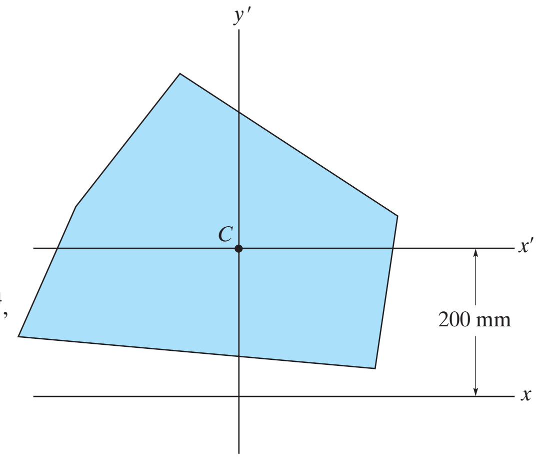


(a)

**Ans:**
$I_y = 533 \text{ in}^4$

**6–82.**

The polar moment of inertia for the area is $\bar{J}_C = 642\ (10^6)\ \text{mm}^4$, about the $z'$ axis passing through the centroid $C$. The moment of inertia about the $y'$ axis is $264\ (10^6)\ \text{mm}^4$, and the moment of inertia about the $x$ axis is $938\ (10^6)\ \text{mm}^4$. Determine the area $A$.




## SOLUTION

Applying the parallel-axis theorem with $d_y = 200$ mm and $I_x = 938(10^6)\ \text{mm}^4$,

$$I_x = \bar{I}_{x'} + Ad_y^2$$

$$938(10^6) = \bar{I}_{x'} + A(200^2)$$

$$\bar{I}_{x'} = 938(10^6) - 40(10^3)A$$

with known polar moment of inertia about $C$,

$$\bar{J}_C = \bar{I}_{x'} + \bar{I}_{y'}$$

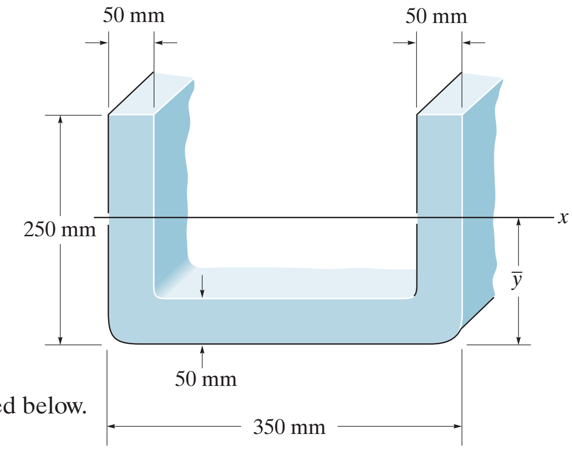
$$642(10^6) = 938(10^6) - 40(10^3)A + 264(10^6)$$

$$A = 14.0(10^3)\ \text{mm}^2 \qquad \textbf{Ans.}$$

**Ans:**
$A = 14.0(10^3)\ \text{mm}^2$

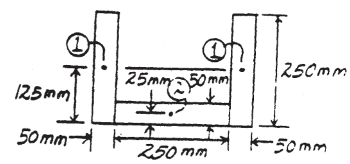
**6–83.**

Determine the location $\bar{y}$ of the centroid of the cross-sectional area of the channel, and then calculate the moment of inertia of this area about this axis.




## SOLUTION

***Centroid:*** The area of each segment and its respective centroid are tabulated below.

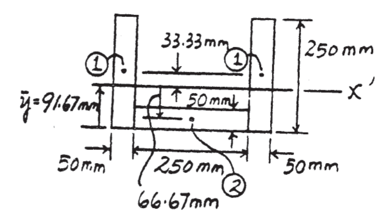



| Segment | $A$ (mm$^2$) | $\tilde{y}$ (mm) | $\tilde{y}A$ (mm$^3$) |
|---|---|---|---|
| 1 | 100(250) | 125 | $3.125(10^6)$ |
| 2 | 250(50) | 25 | $0.3125(10^6)$ |
| $\Sigma$ | $37.5(10^3)$ | | $3.4375(10^6)$ |

Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{3.4375(10^6)}{37.5(10^3)} = 91.67 \text{ mm} = 91.7 \text{ mm} \qquad \textbf{Ans.}$$

***Moment of Inertia:*** The moment of inertia about the $x'$ axis for each segment can be determined using the parallel-axis theorem $I_{x'} = \bar{I}_{x'} + Ad_y^2$.




| Segment | $A_i$ (mm$^2$) | $(d_y)_i$ (mm) | $(\bar{I}_{x'})_i$ (mm$^4$) | $(Ad_y^2)_i$ (mm$^4$) | $(I_{x'})_i$ (mm$^4$) |
|---|---|---|---|---|---|
| 1 | 100(250) | 33.33 | $\frac{1}{12}(100)(250^3)$ | $27.778(10^6)$ | $157.99(10^6)$ |
| 2 | 250(50) | 66.67 | $\frac{1}{12}(250)(50^3)$ | $55.556(10^6)$ | $58.16(10^6)$ |

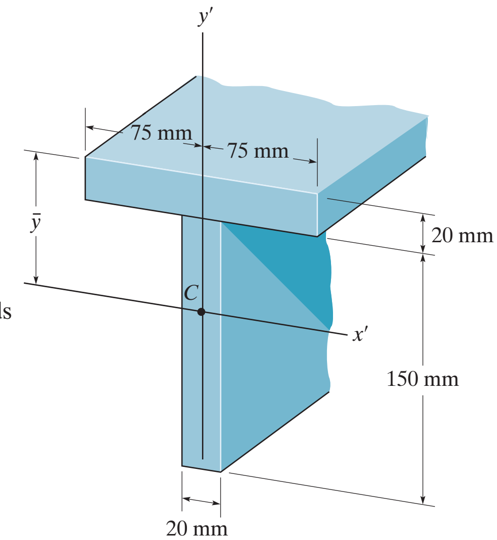
Thus,

$$I_{x'} = \Sigma(I_{x'})_i = 216.15(10^6) \text{ mm}^4 = 216(10^6) \text{ mm}^4 \qquad \textbf{Ans.}$$

**Ans:**
$\bar{y} = 91.7$ mm
$I_{x'} = 216(10^6)$ mm$^4$

***6–84.**

Determine $\bar{y}$, which locates the centroidal axis $x'$ for the cross-sectional area of the T-beam, and then find the moments of inertia $I_{x'}$ and $I_{y'}$.




## SOLUTION

**Centroid.** Referring to Fig. *a*, the areas of the segments and their respective centroids are tabulated below.

| Segment | $A(\text{mm}^2)$ | $\tilde{y}(\text{mm})$ | $\tilde{y}A(\text{mm}^3)$ |
|---|---|---|---|
| 1 | 150(20) | 10 | $30(10^3)$ |
| 2 | 20(150) | 95 | $285(10^3)$ |
| $\Sigma$ | $6(10^3)$ | | $315(10^3)$ |

Thus, $\bar{y} = \dfrac{\Sigma\tilde{y}^2 A}{\Sigma A} = \dfrac{315(10^3)}{6(10^3)} = 52.5\text{ mm}$ **Ans.**

**Moment of Inertia.** The moment of inertia about the $x'$ axis for each segment can be determined using the parallel axis theorem, $I_{x'} = \bar{I}_{x'} + Ad_y^2$. Referring to Fig. *b*,

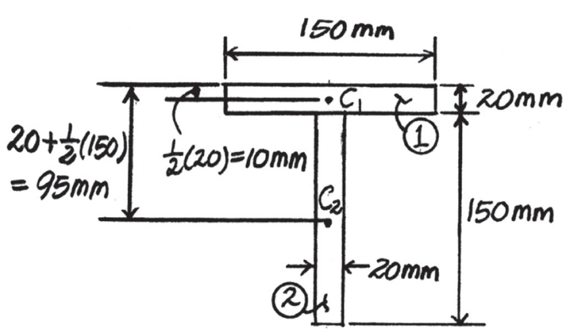
| Segment | $A_i(\text{mm}^2)$ | $(d_y)_i\,(\text{mm})$ | $(\bar{I}_{x'})_i\,(\text{mm}^4)$ | $(Ad_y^2)_i\,(\text{mm}^4)$ | $(I_{x'})_i\,(\text{mm}^4)$ |
|---|---|---|---|---|---|
| 1 | 150(20) | 42.5 | $\frac{1}{12}(150)(20^3)$ | $5.41875(10^6)$ | $5.51875(10^6)$ |
| 2 | 20(150) | 42.5 | $\frac{1}{12}(20)(150^3)$ | $5.41875(10^6)$ | $11.04375(10^6)$ |

Thus,

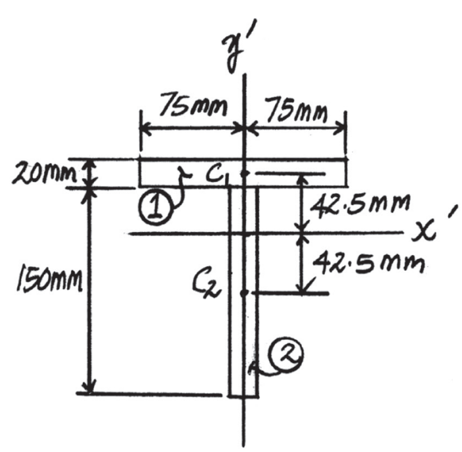
$$I_{x'} = \Sigma(I_{x'})_i = 16.5625(10^6)\text{ mm}^4 = 16.6(10^6)\text{ mm}^4$$ **Ans.**

Since the $y'$ axis passes through the centroids of segments 1 and 2,

$$I_{y'} = \frac{1}{12}(20)(150^3) + \frac{1}{12}(150)(20^3)$$
$$= 5.725(10^6)\text{ mm}^4$$ **Ans.**



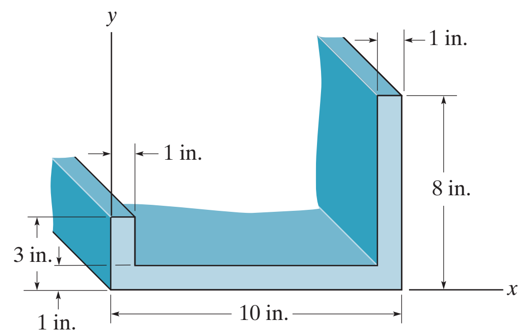

(a)




**Ans:**
$\bar{y} = 52.5\text{ mm}$
$I_{x'} = 16.6(10^6)\text{ mm}^4$
$I_{y'} = 5.725(10^6)\text{ mm}^4$

**6–85.**

Determine the moment of inertia of the cross-sectional area of the beam about the $x$ axis.



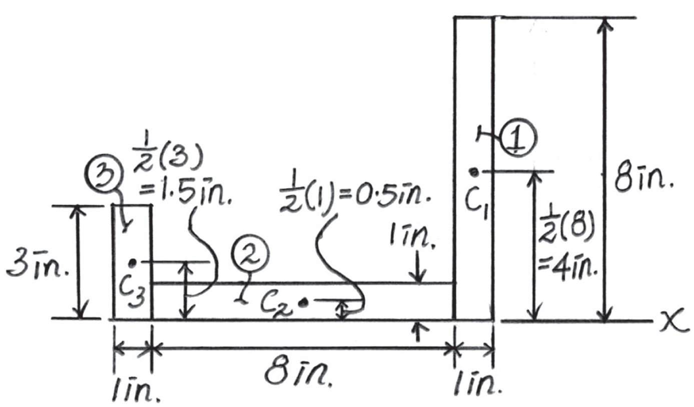

## SOLUTION

**Moment of Inertia.** The moment of inertia about the $x$ axis for each segment can be determined using the parallel axis theorem, $I_x = \bar{I}_{x'} + Ad_y^2$. Referring to Fig. $a$,

| Segment | $A_i(\text{in}^2)$ | $(d_y)_i\,(\text{in.})$ | $(\bar{I}_{x'})_i\,(\text{in}^4)$ | $(Ad_y^2)_i\,(\text{in}^4)$ | $(\bar{I}_x)_i\,(\text{in}^4)$ |
|---|---|---|---|---|---|
| 1 | 1(8) | 4 | $\frac{1}{12}(1)(8^3)$ | 128 | 170.67 |
| 2 | 8(1) | 0.5 | $\frac{1}{12}(8)(1^3)$ | 2 | 2.67 |
| 3 | 1(3) | 1.5 | $\frac{1}{12}(1)(3^3)$ | 6.75 | 9.00 |

Thus,

$$I_x = \Sigma(I_x)_i = 182.33 \text{ in}^4 = 182 \text{ in}^4$$ **Ans.**

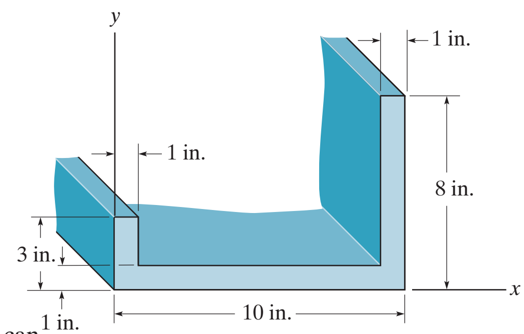



(a)

**Ans:**
$I_x = 182 \text{ in}^4$

**6–86.**

Determine the moment of inertia of the cross-sectional area of the beam about the $y$ axis.



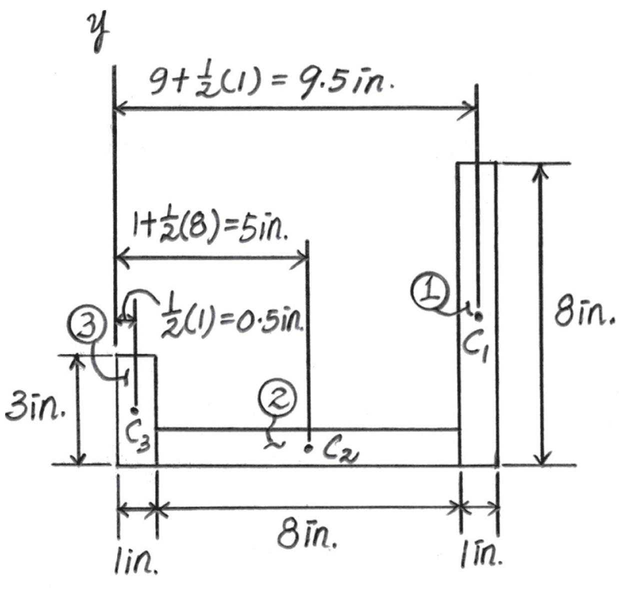

## SOLUTION

**Moment of Inertia.** The moments of inertia about the $y$ axis for each segment can be determined using the parallel axis theorem, $I_x = \bar{I}_{x'} + Ad_y^2$. Referring to Fig. $a$,

| Segment | $A_i$(in$^2$) | $(dx)_i$ (in.) | $(\bar{I}_{y'})_i$ (in$^4$) | $(Adx^2)_i$ (in$^4$) | $(\bar{I}_y)_i$ (in$^4$) |
|---|---|---|---|---|---|
| 1 | 8(1) | 9.5 | $\frac{1}{12}(8)(1^3)$ | 722 | 722.67 |
| 2 | 1(8) | 5 | $\frac{1}{12}(1)(8^3)$ | 200 | 242.67 |
| 3 | 3(1) | 0.5 | $\frac{1}{12}(3)(1^3)$ | 0.75 | 1.00 |

Thus,

$$I_y = \Sigma(I_y)_i = 966.33 \text{ in}^4 = 966 \text{ in}^4 \qquad \textbf{Ans.}$$

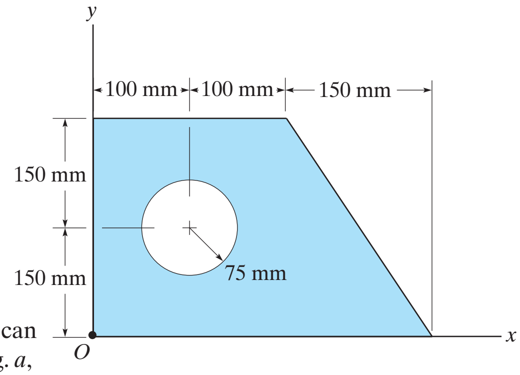


(a)

**Ans:**
$I_y = 966 \text{ in}^4$

**6–87.**

Determine the moment of inertia $I_x$ of the area about the $x$ axis.



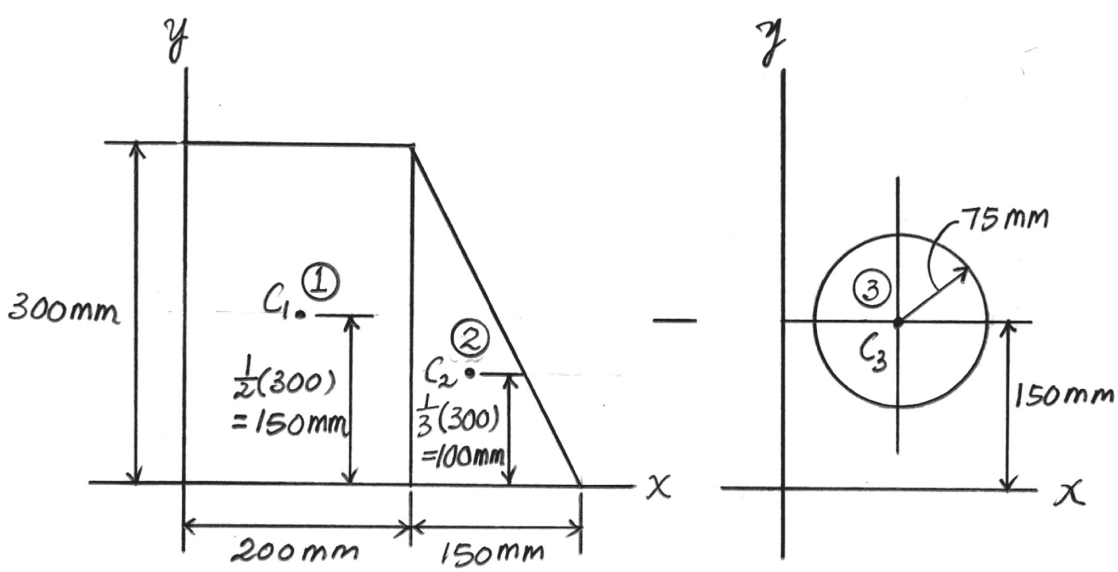

## SOLUTION

**Moment of Inertia.** The moment of inertia about the $x$ axis for each segment can be determined using the parallel axis theorem, $I_x = \bar{I}_{x'} + Ad^2y$. Referring to Fig. $a$,

| Segment | $A_i(\text{mm}^2)$ | $(d_y)_i(\text{mm})$ | $(\bar{I}_{x'})_i(\text{mm}^4)$ | $(Ad_y)_i^2(\text{mm}^4)$ | $(\bar{I}_x)_i(\text{mm}^4)$ |
|---|---|---|---|---|---|
| 1 | $200(300)$ | 150 | $\frac{1}{12}(200)(300^3)$ | $1.35(10^9)$ | $1.80(10^9)$ |
| 2 | $\frac{1}{2}(150)(300)$ | 100 | $\frac{1}{36}(150)(300^3)$ | $0.225(10^9)$ | $0.3375(10^9)$ |
| 3 | $-\pi(75^2)$ | 150 | $-\frac{\pi(75^4)}{4}$ | $-0.3976(10^9)$ | $-0.4225(10^9)$ |

Thus,

$$I_x = \Sigma(I_x)_i = 1.715(10^9)\text{ mm}^4 = 1.72(10^9)\text{ mm}^4 \qquad \textbf{Ans.}$$

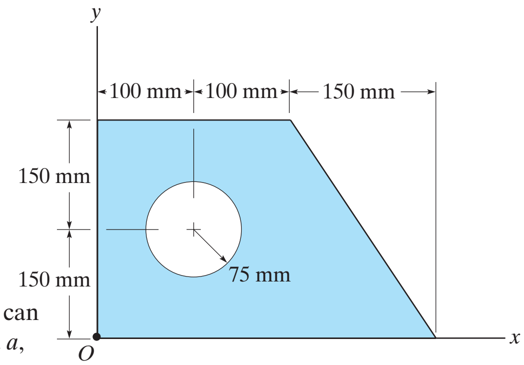



(a)

**Ans:**
$I_x = 1.72(10^9)\text{ mm}^4$

***6–88.**

Determine the moment of inertia $I_x$ of the area about the $y$ axis.



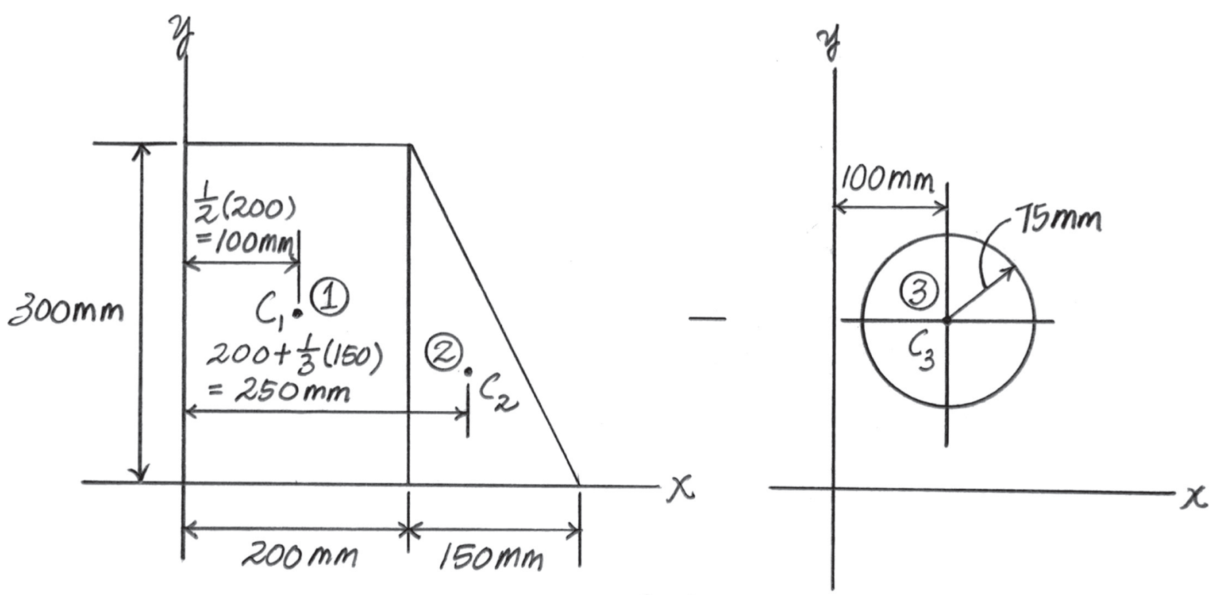

## SOLUTION

**Moment of Inertia.** The moment of inertia about the $y$ axis for each segment can be determined using the parallel-axis theorem, $I_y = \bar{I}_{y'} + Ad_x^2$. Referring to Fig. $a$,

| Segment | $A_i(\text{mm}^2)$ | $(d_x)_i\,(\text{mm})$ | $\bar{I}_{y'}(\text{mm}^4)$ | $(Ad_x^2)_i\,(\text{mm}^4)$ | $(\bar{I}_y)_i\,(\text{mm}^4)$ |
|---|---|---|---|---|---|
| 1 | 200(300) | 100 | $\frac{1}{12}(300)(200^3)$ | $0.6(10^9)$ | $0.800(10^9)$ |
| 2 | $\frac{1}{2}(150)(300)$ | 250 | $\frac{1}{36}(300)(150^3)$ | $1.40625(10^9)$ | $1.434375(10^9)$ |
| 3 | $-\pi(75^2)$ | 100 | $-\frac{\pi(75^4)}{4}$ | $-0.1767(10^9)$ | $-0.20157(10^9)$ |

Thus,

$$I_y = \Sigma(I_y)_i = 2.033(10^9)\text{ mm}^4 = 2.03(10^9)\text{ mm}^4 \qquad \textbf{Ans.}$$

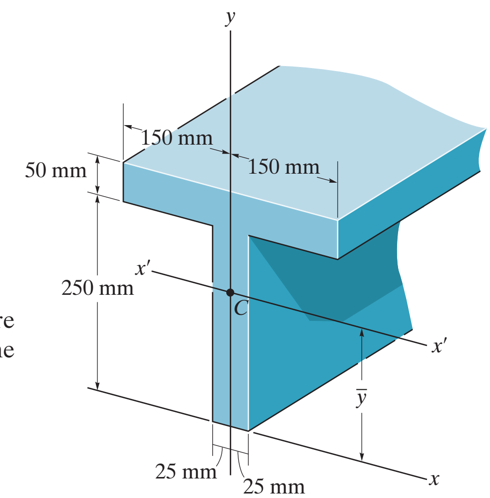



(a)

**Ans:**
$I_y = 2.03\left(10^9\right)\text{mm}^4$

**6–89.**

Determine the moment of inertia of the cross-sectional area of the beam about the $y$ axis.

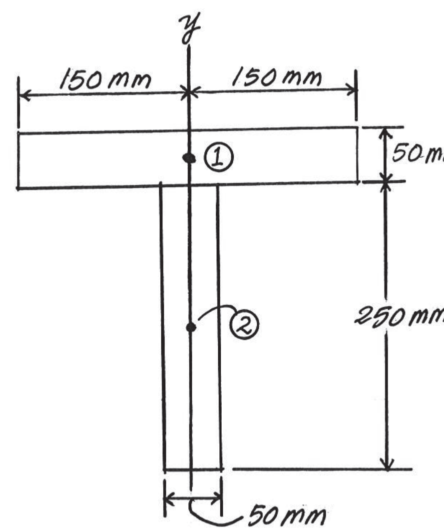



## SOLUTION

***Moment of Inertia:*** The dimensions and location of centroid of each segment are shown in Fig. $a$. Since the $y$ axis passes through the centroid of both segments, the moment of inertia about $y$ axis for each segment is simply $(I_y)_i = (I_{y'})_i$.

$$I_y = \Sigma(I_y)_i = \frac{1}{12}(50)(300^3) + \frac{1}{12}(250)(50^3)$$

$$= 115.10(10^6)\text{ mm}^4 = 115(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

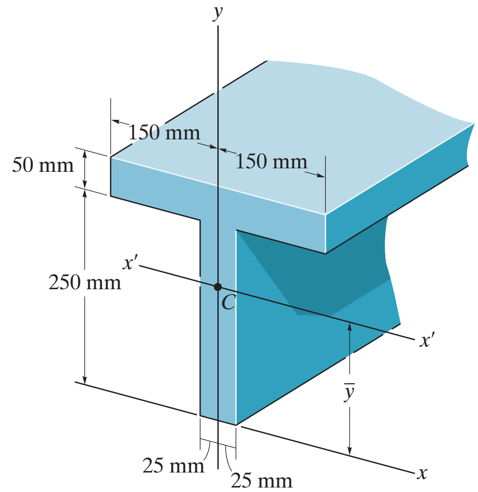



(a)

**Ans:**
$I_y = 115\left(10^6\right)\text{ mm}^4$

**6–90.**

Determine $\bar{y}$, which locates the centroidal axis $x'$ for the cross-sectional area of the T-beam, and then find the moment of inertia about the $x'$ axis.




## SOLUTION

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{125(250)(50) + (275)(50)(300)}{250(50) + 50(300)}$$

$$= 206.818 \text{ mm}$$

$$\bar{y} = 207 \text{ mm}$$ **Ans.**

$$\bar{I}_{x'} = \left[\frac{1}{12}(50)(250)^3 + 50(250)(206.818 - 125)^2\right]$$

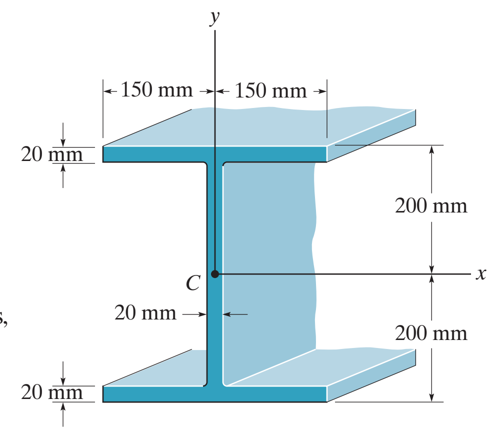
$$+\left[\frac{1}{12}(300)(50)^3 + 50(300)(275 - 206.818)^2\right]$$

$$\bar{I}_{x'} = 222(10^6) \text{ mm}^4$$ **Ans.**

**Ans:**
$\bar{y} = 207 \text{ mm}$
$\bar{I}_{x'} = 222\left(10^6\right) \text{mm}^4$

**6–91.**

Determine the moment of inertia of the cross-sectional area of the beam about the $x$ axis.



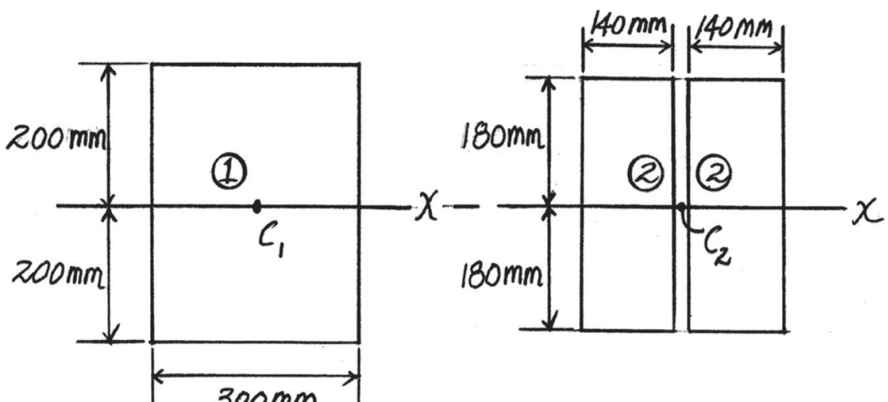

## SOLUTION

**Moment of Inertia.** Since the $x$ axis passes through the centroids of the two segments, Fig. $a$,

$$I_x = \frac{1}{12}(300)(400^3) - \frac{1}{12}(280)(360^3)$$

$$= 511.36(10^6)\text{ mm}^4$$

$$= 511(10^6)\text{ mm}^4 \qquad \textbf{Ans.}$$

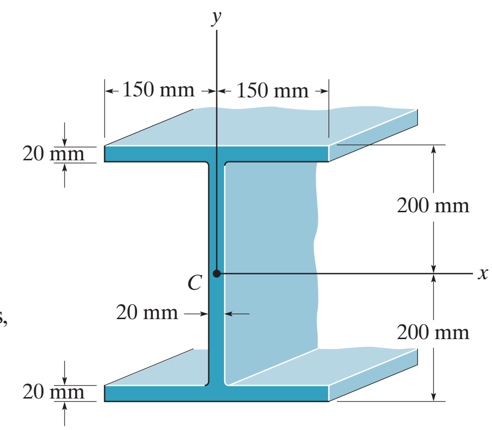



(a)

**Ans:**
$I_x = 511(10^6)\text{ mm}^4$

***6–92.**

Determine the moment of inertia of the cross-sectional area of the beam about the $y$ axis.



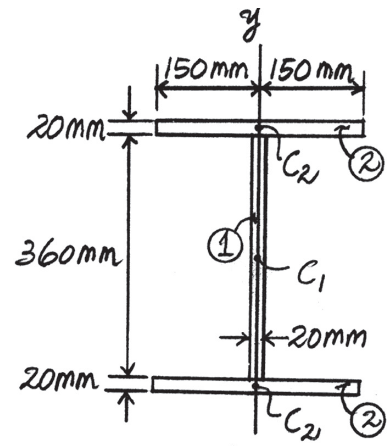

## SOLUTION

**Moment of Inertia.** Since the $y$ axis passes through the centroid of the two segments, Fig. $a$,

$$I_y = \frac{1}{12}(360)(20^3) + \frac{1}{12}(40)(300^3)$$

$$= 90.24(10^6)\ \text{mm}^4$$

$$= 90.2(10^6)\ \text{mm}^4 \qquad \textbf{Ans.}$$

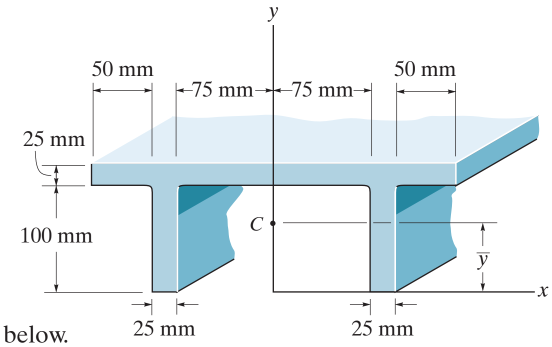


(a)

**Ans:**
$I_y = 90.2\left(10^6\right)\ \text{mm}^4$

***R6–4.**

Locate the centroid $\bar{y}$ of the cross-sectional area of the beam.



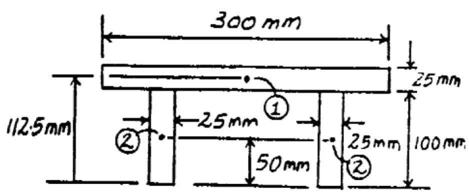

## SOLUTION

***Centroid:*** The area of each segment and its respective centroid are tabulated below.

| Segment | $A$ (mm$^2$) | $\tilde{y}$ (mm) | $\tilde{y}A$ (mm$^3$) |
|---|---|---|---|
| 1 | 300(25) | 112.5 | 843 750 |
| 2 | 100(50) | 50 | 250 000 |
| $\Sigma$ | 12 500 | | 1 093 750 |

Thus,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{1\ 093\ 750}{12\ 500} = 87.5 \text{ mm} \qquad \textbf{Ans.}$$




**Ans:**
$\bar{y} = 87.5$ mm