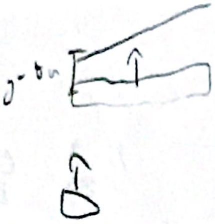
$\frac{ab}{2} - a$

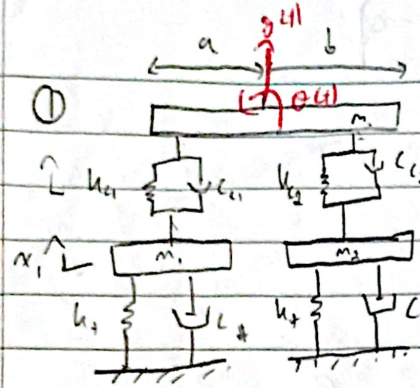


$(b/2 - a/2)^2 \; (b/2 - a/2)$

$b^2/4 - \frac{ab}{2} + a^2/4$

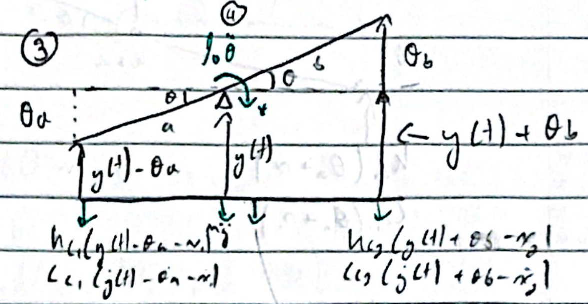
$(a+b)^2 \quad (a+b$

$a^2 + 2ab + b^2$



$\sin\theta = z/b, \quad z \simeq \theta b$

assume top bar rotates as follows.



*For rigid rotation $\Sigma M_o = 0$

$I_o = \frac{mL^2}{12} + m\left(\frac{a+b}{2} - a\right)^2$

$= m\left[\frac{a^2}{12} + \frac{ab}{6} + \frac{b^2}{12} + \frac{b^2}{4} - \frac{ab}{2} + \frac{a^2}{4}\right]$

$= \frac{m}{3}\left[a^2 - ab + b^2\right]$

①

FBD m_1 and m_2

① $c_{c_1}(\dot{y}(t) - \dot{\theta} a - \dot{x}_1)$

$k_{c_1}(y(t) - \theta a - x_1)$

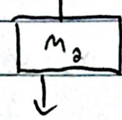
$k_t x_1$

$c_t \dot{x}_1$

$m_1 \ddot{x}_1$

② $c_{c_2}(\dot{y}(t) + \dot{\theta} b - \dot{x}_2)$

$k_{c_2}(y(t) + \theta b - x_2)$



$k_t x_2$

$c_t \dot{x}_2$

$m_2 \ddot{x}_2$

① $m_1\ddot{x}_1 + (c_t + c_{c_1})\dot{x}_1 + c_{c_1}\dot{\theta} a - c_{c_1}\dot{y}(t) + (k_t + k_{c_1})x_1 + k_{c_1}\theta a - k_{c_1} y(t) = 0$

② $m_2\ddot{x}_2 + (c_t + c_{c_2})\dot{x}_2 - c_{c_2}\dot{\theta} b - c_{c_2}\dot{y}(t) + (k_t + k_{c_2})x_2 - k_{c_2}\theta b - k_{c_2} y(t) = 0$

ΣF ③ $m\ddot{y} + (c_{c_1}a - c_{c_2}b)\dot{\theta} + (c_{c_1} + c_{c_2})\dot{y}(t) - c_{c_1}\dot{x}_1 - c_{c_2}\dot{x}_2 + (k_{c_1} + k_{c_2})y(t) - (k_{c_1}a - k_{c_2}b)\theta - k_{c_1}x_1 - k_{c_2}x_2 = 0$

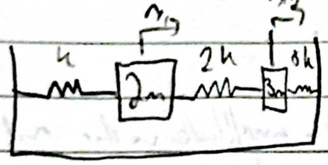
ΣM ④ $\frac{m}{3}\left[a^2 - ab + b^2\right]\ddot{\theta} + c_{c_2}\dot{y}(t)b + c_{c_2}\dot{\theta}b^2 - c_{c_2}\dot{x}_2 b + k_{c_2}y(t)b + k_{c_2}\theta b^2 - k_{c_2}x_2 b - k_{c_1}y(t)a + k_{c_1}\theta a^2 + k_{c_1}x_1 a$

$- c_{c_1}\dot{y}(t)a + c_{c_1}\dot{\theta}a^2 + c_{c_1}\dot{x}_1 a = 0$

∴ See the following Matrix

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & \frac{m(a^2-ab+b^2)}{3} \end{bmatrix} \begin{bmatrix} \ddot{x_1} \\ \ddot{x_2} \\ \ddot{y} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} C_t + C_{c_1} & 0 & -C_{c_1} & C_{c_1}a \\ 0 & C_t + C_{c_2} & -C_{c_2} & -C_{c_2}b \\ -C_{c_1} & -C_{c_2} & C_{c_1} + C_{c_2} & C_{c_2}b - C_{c_1}a \\ C_{c_1}a & -C_{c_2}b & C_{c_2}b - C_{c_1}a & C_{c_1}a^2 + C_{c_2}b^2 \end{bmatrix} \begin{bmatrix} \dot{x_2} \\ \dot{x_2} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} k_t + k_{c_1} & 0 & -k_{c_1} & k_{c_1}a \\ 0 & k_t + k_{c_2} & -k_{c_2} & -k_{c_2}b \\ -k_{c_1} & -k_{c_2} & k_{c_1} + k_{c_2} & k_{c_2}b - k_{c_1}a \\ k_{c_1}a & -k_{c_2}b & k_{c_2}b - k_{c_1}a & k_{c_1}a^2 + k_{c_2}b^2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_2 \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Question 2

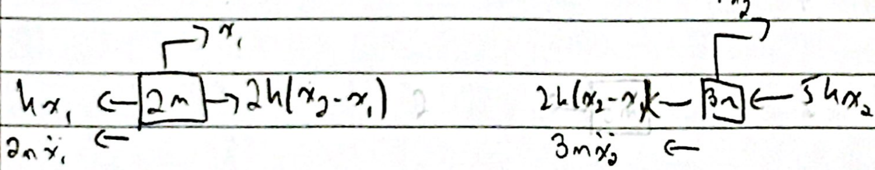


, $m = 175\ kg$, $k = 175\ N/m$

$x(0) = [-0.2\ \ 0]^T m$

$\dot{x}(0) = [0\ \ 0]^T m/s$

1. FBD



2. Equations of motion

$2m\ddot{x}_1 + 3kx_1 - 2kx_2 = 0$, $3m\ddot{x}_2 + 7kx_2 - 2kx_1 = 0$, plugging in m and k,

$$\begin{bmatrix} 350 & 0 \\ 0 & 525 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 525 & -350 \\ -350 & 1225 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

3. Since we have $\det[-\omega_n^2 M + k] = 0$ for free vibration, we have

$$\begin{vmatrix} 525 - 350\omega_n^2 & -350 \\ -350 & 1225 - 525\omega_n^2 \end{vmatrix}, \quad \text{let } \omega_n^2 = n$$

$(525 - 350n)(1225 - 525n) - (-350)(-350) = 0$

$643125 - (275625 + 428750)n + 183750n^2 - 122500 = 0$

$183750n^2 - 704375n + 520625 = 0$, $n = 1, 2.833$,

$\omega_{n,1}^2 = 1\ (rad/s)^2$

$\omega_{n,2}^2 = 2.833\ (rad/s)^2$

$$\begin{bmatrix} 525-350(1) & -350 \\ -350 & 1225-525(1) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{let } X_1 = 1$$

$$175(1) - 350 X_2 = 0, \quad X_2 = 1/2$$

Oth node, $[525 - 350(2.8333)]1 - 350 X_2 = 0, \quad X_2 = -1.3333$

$$\phi_1 = \alpha \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}, \quad \phi_2 = \beta \begin{bmatrix} 1 \\ -1.3333 \end{bmatrix}$$

For ϕ_1, let $\phi_1^T M \phi_1 = 1$ (mass normalized)

$$\begin{bmatrix} \alpha & \alpha/2 \end{bmatrix} \begin{bmatrix} 350 & 0 \\ 0 & 525 \end{bmatrix} \begin{bmatrix} \alpha \\ \alpha/2 \end{bmatrix} = 1$$

$$\begin{bmatrix} \alpha & \alpha/2 \end{bmatrix} \begin{bmatrix} 350\alpha \\ 525/2\,\alpha \end{bmatrix} = 1 = \begin{bmatrix} 350\alpha^2 \\ \frac{525}{4}\alpha^2 \end{bmatrix}$$

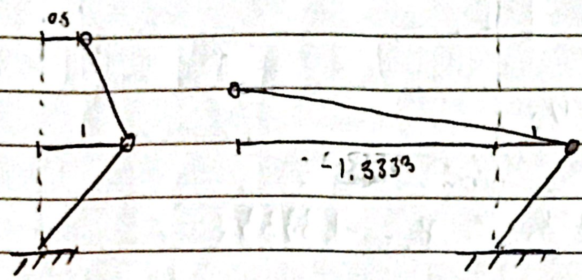
$$\therefore \left(350 + \frac{525}{4}\right)\alpha^2 = 1, \quad \alpha = \frac{2}{\sqrt{1925}} =$$

For ϕ_2, $\begin{bmatrix} \beta & -1.3333\beta \end{bmatrix} \begin{bmatrix} 350 & 0 \\ 0 & 525 \end{bmatrix} \begin{bmatrix} \beta \\ -1.3333\beta \end{bmatrix} = 1 = \begin{bmatrix} \beta & -1.3333\beta \end{bmatrix} \begin{bmatrix} 350\beta \\ = 1 \\ -699.9825 \end{bmatrix}$

$$\beta = \frac{1}{\sqrt{1283.287}}$$

$$\therefore \phi = \begin{bmatrix} \frac{2}{\sqrt{1925}} & \frac{1}{\sqrt{1283.287}} \\ \frac{1}{\sqrt{1925}} & \frac{-1.3333}{\sqrt{1283.287}} \end{bmatrix}$$

Mode i Mode j



Displacement response

since $\dot{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\dot{y} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$y_0 = \phi^T M x_0$$

$$= \begin{bmatrix} \frac{2}{\sqrt{1925}} & \frac{1}{\sqrt{1925}} \\ \frac{1}{\sqrt{1283.287}} & \frac{-1.3333}{\sqrt{1283.287}} \end{bmatrix} \begin{bmatrix} 350 & 0 \\ 0 & 525 \end{bmatrix} \begin{bmatrix} -0.2 \\ 0 \end{bmatrix}$$

$$= \quad '' \quad \begin{bmatrix} -70 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3.1909 \\ -1.5954 \end{bmatrix}$$

$\therefore$ from 3.99,

$$y_1 = -3.1909 \cos t$$
$$y_2 = -1.5954 \cos(1.6833t)$$

Now, $x = \phi y$

$$\therefore x = \begin{bmatrix} \frac{2}{\sqrt{1925}} & \frac{1}{\sqrt{1283.287}} \\ \frac{1}{\sqrt{1925}} & \frac{-1.3333}{\sqrt{1283.287}} \end{bmatrix} \begin{bmatrix} -3.1909 \cos t \\ -1.5954 \cos(1.6833t) \end{bmatrix}$$

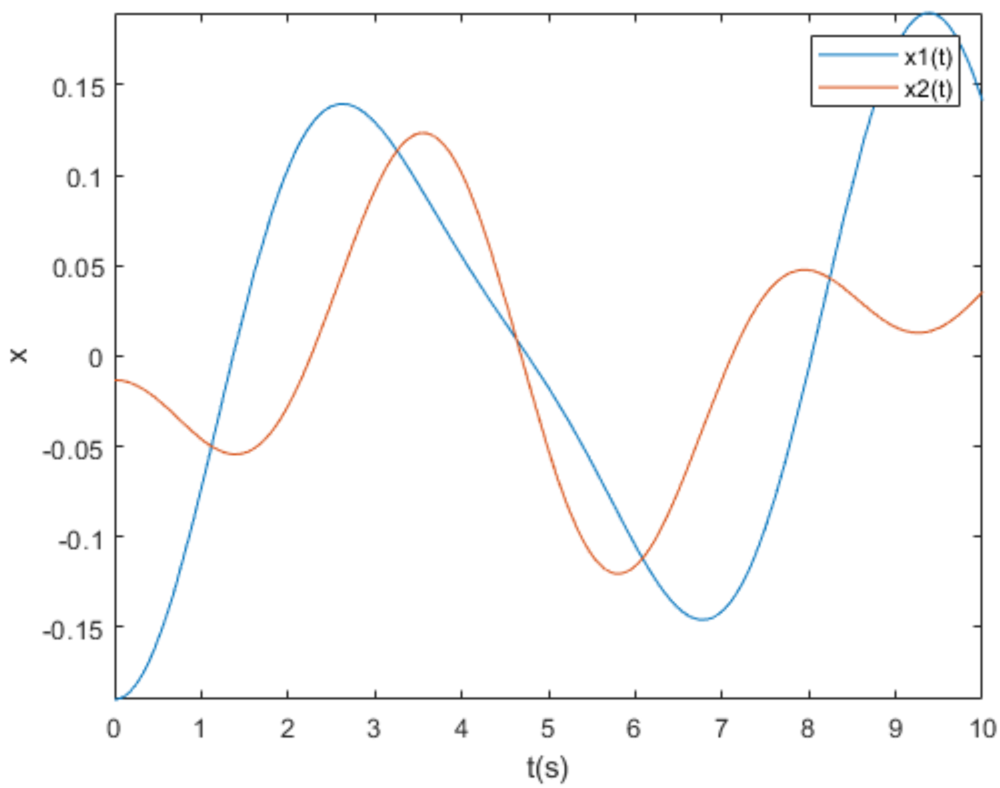
$$x_1 = -0.1455 \cos t - 0.0445 \cos(1.6833t)$$

$$x_2 = -0.07277 \cos t + 0.0594 \cos(1.6833t)$$

See the following graph on MATLAB

Question 2

```
clc,clear
M=[350,0;0,525];
K=[525,-350;-350,1225];

[V1,wn1]=eig(K,M);
[wn1_sorted,ind]=sort(diag(wn1));
wn1=sqrt(wn1_sorted)
V1_sorted=V1(:,ind);

x2=(525-350*2.8333)/350
x=vpa(350+-1.3333*525*-1.3333);

phiT=[2/sqrt(1925),1/sqrt(1925);1/sqrt(x),x2/sqrt(x)];
phi=transpose(phiT)
MX=[-70;0];

y0=vpa(phiT.*MX);

syms t
X=vpa(phi*[-3.1909*cos(t);-1.5954*cos(1.6833*t)]);
fplot(X(1),[0,10])
hold on
fplot(X(2),[0,10])
legend('x1(t)','x2(t)')
xlabel('t(s)')
ylabel('x')
```



x1(t)
x2(t)
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
x
0
1
2
3
4
5
6
7
8
9
10
t(s)

3. $\zeta_1 = 0.05$, $\zeta_2 = 0.05$, $\zeta_3 = ?$ $\quad \omega_{n,1} = \frac{2\pi}{0.5} = 12.5664$

$\omega_{n,2} = 2\pi/0.183 = 34.3343$

$I\ddot{y} + \Phi^T C \Phi \dot{y} + \Lambda y = 0 \quad \omega_{n,3} = 2\pi/0.134 = 46.8894$

1. Since $C = \alpha M + \beta K$, determine α and β

$$\beta = \frac{2\left[\zeta_1 \omega_{n,1} - \zeta_2 \omega_{n,2}\right]}{\omega_{n,1}^2 - \omega_{n,2}^2} = \frac{2\left[0.05 \cdot 12.5664 - 0.05(34.3343)\right]}{12.5664^2 - 34.3343^2} = 0.002039729$$

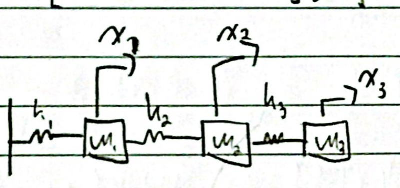
$$\alpha = 2 \cdot 0.05 \cdot 12.5664 - 0.002039729 \cdot 12.5664^2$$
$$= 0.9345374$$

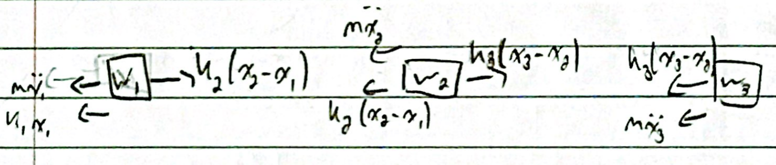
$\Phi^T C \Phi = \alpha \Phi^T M \Phi + \beta \Phi^T K \Phi$

$$\begin{bmatrix} 2\zeta_1 \omega_{n,1} & & \\ & 2\zeta_2 \omega_{n,2} & \\ & & 2\zeta_3 \omega_{n,3} \end{bmatrix} = \begin{bmatrix} \alpha & & \\ & \alpha & \\ & & \alpha \end{bmatrix} + \begin{bmatrix} \beta \omega_{n,1}^2 & & \\ & \beta \omega_{n,2}^2 & \\ & & \beta \omega_{n,3}^2 \end{bmatrix}$$

$2\zeta_3 \cdot 46.8894 = 0.9345374 + 0.002039729 \cdot 46.8894^2$, $\zeta_3 = 0.05778$

$$M = \begin{bmatrix} 400 & & \\ & 400 & \\ & & 200 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} \qquad K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} = \begin{bmatrix} 1220 & -610 & 0 \\ -610 & 1220 & -610 \\ 0 & -610 & 610 \end{bmatrix}$$





$C = \alpha M + \beta K$

$$= \begin{bmatrix} \alpha 400 + \beta 1220 & \beta \text{-} 610 & 0 \\ \beta \text{-} 610 & \alpha 400 + \beta 1220 & \beta \text{-} 610 \\ 0 & \beta \text{-} 610 & \alpha 200 + \beta 610 \end{bmatrix} = \begin{bmatrix} 376.3 & -1.24 & 0 \\ -1.24 & 376.3 & -1.24 \\ 0 & -1.24 & 188.15 \end{bmatrix}$$

5487 Raaf

4.

$$M = \begin{bmatrix} 3 & & & \\ & 2 & & \\ & & 2 & \\ & & & 1 \end{bmatrix} \qquad K = \begin{bmatrix} 7 & -3 & 0 & 0 \\ -3 & 5 & -2 & 0 \\ & & 3 & -1 \\ & & & 1 \end{bmatrix}$$

a) For free vibration, we know $\det\left[-\omega_n^2 M + K\right] = 0$

From MATLAB $\omega_n = \begin{bmatrix} 13.2935 \\ 29.6597 \\ 41.0787 \\ 55.882 \end{bmatrix}$ and $\Phi_{\text{mass normalized}} = \begin{bmatrix} -0.5774 & 1.2347 & 0.5774 & -0.5774 \\ -1.2196 & 1.5232 & 0.1293 & 0.9065 \\ -1.9136 & 0.2811 & -0.8155 & -0.4063 \\ -2.4562 & -2.8213 & 0.7351 & 0.1399 \end{bmatrix}$

b) See attached sketch

c) No initial conditions

From 3.167, we can say $\ddot{y}_N + 2\zeta_N \omega_{n,N} \dot{y}_N + \omega_{n,N}^2 y_N = \phi_N^T F(t)$, since $\zeta = 0$ (assumed)

$$\ddot{y}_N + \omega_{n,N}^2 y_N = \phi_N^T F(t)$$

① $\ddot{y}_1 + 13.2935^2 y_N = \begin{bmatrix} -0.5774 & -1.2196 & -1.9136 & -2.4562 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ P_4 \sin \omega t \end{bmatrix}$

$\therefore \ddot{y}_1 + 13.2935^2 y_1 = -2.4562\, P_4 \sin \omega t$

Similarly,

② $\ddot{y}_2 + 29.6597^2 y_2 = -2.8213\, P_4 \sin \omega t$

③ $\ddot{y}_3 + 41.0787^2 y_3 = 0.7351\, P_4 \sin \omega t$

④ $\ddot{y}_4 + 55.882^2 y_4 = 0.1399\, P_4 \sin \omega t$

From 2.136, P the undamped SDOF response to a sinusoidal forcing frequency is

$$x = \frac{P_0}{k} \cdot \frac{1}{1-\frac{\omega}{\omega_n}} \left(\sin \omega t \right)$$

$$\therefore y_1(t) = \frac{-2.4562 P_4}{k_{11}} \cdot \frac{1}{1-\frac{\omega}{13.2935}} \left(\sin \omega t \right)$$

$$y_2(t) = \frac{-2.8213 P_4}{k_{22}} \cdot \frac{1}{1-\frac{\omega}{24.6547}} \left(\sin \omega t \right)$$

$$y_3(t) = \frac{0.7351 P_4}{k_{33}} \cdot \frac{1}{1-\frac{\omega}{41.0787}} \left(\sin \omega t \right)$$

$$y_4(t) = \frac{0.1344 P_4}{k_{44}} \cdot \frac{1}{1-\frac{\omega}{55.862}} \left(\sin \omega t \right)$$

$$x = \phi y$$

$$x_4 = \begin{bmatrix} -2.4562 & -2.8213 & 0.7351 & 0.1344 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix}$$

See result from MATLAB:

D)

From MATLAB, the following responses have been found:

w=0.5wn

One mode: x411 = 0.0001243*sin(6.647*t)

Two modes: x412 = 0.0001243*sin(6.647*t) + 0.0002296*sin(14.83*t)

Three modes: x413 = 0.0001551*sin(20.54*t) + 0.0001243*sin(6.647*t) + 0.0002296*sin(14.83*t)

All modes: x414 = 0.0001551*sin(20.54*t) + 0.0001243*sin(6.647*t) + 1.635e-5*sin(27.94*t) + 0.0002296*sin(14.83*t)

w=4wn

One mode: x421 = -2.072e-5*sin(53.17*t)

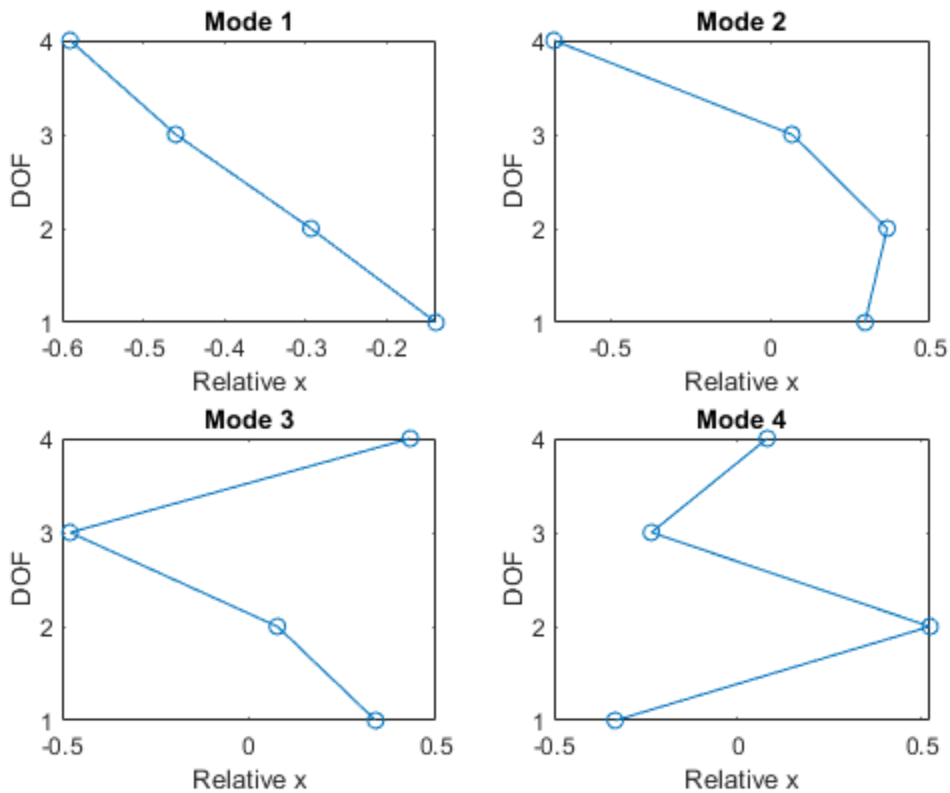
Two modes: x422 = - 2.072e-5*sin(53.17*t) - 3.827e-5*sin(118.6*t)

Three modes: x423 = - 2.585e-5*sin(164.3*t) - 2.072e-5*sin(53.17*t) - 3.827e-5*sin(118.6*t)

All modes: x424 = - 2.585e-5*sin(164.3*t) - 2.072e-5*sin(53.17*t) - 2.726e-6*sin(223.5*t) - 3.827e-5*sin(118.6*t)

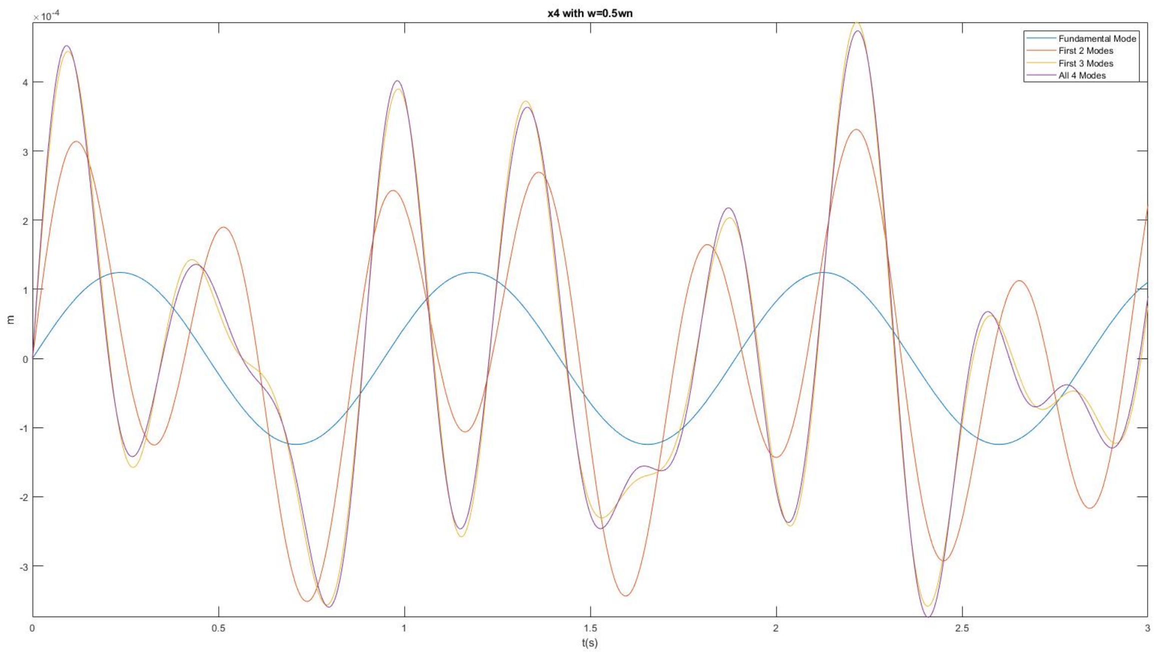
Answer to d:

It is clear that the first three terms are in terms of E-5, while the fourth term is in terms of E-6. This makes it clear that the fourth mode is less significant than the first three. It also shows that the third mode should not be ignored. This is further exemplified by the following graphs, which visually demonstrate this phenomenon.

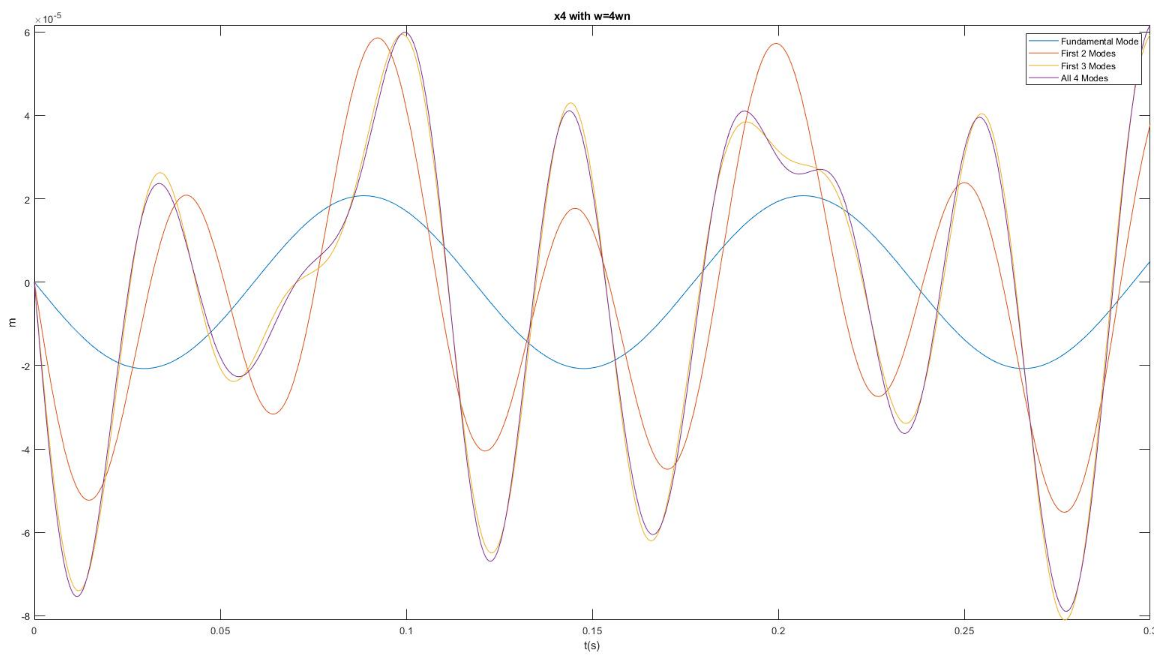

Mode 1
Mode 2
Mode 3
Mode 4
DOF
Relative x
4
3
2
1
-0.6
-0.5
-0.4
-0.3
-0.2
0
0.5

Formula for x

$$-(0.00198*P*sin(t*w))/(0.03371*w-1.0)-(0.00002*P*sin(t*w))/(0.01789*w-1.0)-(0.00107*P*sin(t*w))/(0.07522*w-1.0)-(0.00022*P*sin(t*w))/(0.02434*w-1.0)$$



x4 with w=0.5wn
× 10^-4
Fundamental Mode
First 2 Modes
First 3 Modes
All 4 Modes
m
t(s)
4
3
2
1
0
-1
-2
-3
0
0.5
1
1.5
2
2.5
3



x4 with w=4wn
Fundamental Mode
First 2 Modes
First 3 Modes
All 4 Modes
×10^-5
6
4
2
0
-2
-4
-6
-8
m
0
0.05
0.1
0.15
0.2
0.25
0.3
t(s)

Question 4

d)

```
clc,clear all

% Matrices
M=[3,0,0,0;0,2,0,0;0,0,2,0;0,0,0,1]
K=800.*[7,-3,0,0;-3,5,-2,0;0,-2,3,-1;0,0,-1,1]

% Eigenvalue determination
[V1,wn1]=eig(K,M)
[wn1_sorted,ind]=sort(diag(wn1))
wn1=sqrt(wn1_sorted)
V1_sorted=V1(:,ind)

% Plot
%figure(1)
%subplot(221)
%plot(V1_sorted(:,1),[1,2,3,4],'-o')
%xlabel('Relative x')
%ylabel('DOF')
%title('Mode 1')
%subplot(222)
%plot(V1_sorted(:,2),[1,2,3,4],'-o')
%xlabel('Relative x')
%ylabel('DOF')
%title('Mode 2')
%subplot(223)
%plot(V1_sorted(:,3),[1,2,3,4],'-o')
%xlabel('Relative x')
%ylabel('DOF')
%title('Mode 3')
%subplot(224)
%plot(V1_sorted(:,4),[1,2,3,4],'-o')
%xlabel('Relative x')
%ylabel('DOF')
%title('Mode 4')

% Mass normalization
syms a b c d

V11=V1(:,1)*a;
phiMphi=transpose(V11)*(M)*V11;
expression=phiMphi(1,1)-1==0;
Answer=double(solve(expression,a));
Alpha=Answer(2);
MNV11=Alpha.*V1(:,1);

V12=V1(:,2)*b;
phiMphi=transpose(V12)*(M)*V12;
expression=phiMphi(1,1)-1==0;
Answer=double(solve(expression,b));
Alpha=Answer(2);
MNV12=Alpha.*V1(:,2);
```

```
V13=V1(:,3)*c;
phiMphi=transpose(V13)*(M)*V13;
expression=phiMphi(1,1)-1==0;
Answer=double(solve(expression,c));
Alpha=Answer(2);
MNV13=Alpha.*V1(:,3);

V14=V1(:,4)*d;
phiMphi=transpose(V14)*(M)*V14;
expression=phiMphi(1,1)-1==0;
Answer=double(solve(expression,d));
Alpha=Answer(2);
MNV14=Alpha.*V1(:,4);
MNV1=[MNV11,MNV12,MNV13,MNV14]

% x4 function
phiT4=MNV1(4,:);
syms P w t
y1=phiT4(1)*P/K(1,1)*1/(1-w/wn1(1))*sin(w*t);
y2=phiT4(2)*P/K(2,2)*1/(1-w/wn1(2))*sin(w*t);
y3=phiT4(3)*P/K(3,3)*1/(1-w/wn1(3))*sin(w*t);
y4=phiT4(4)*P/K(4,4)*1/(1-w/wn1(4))*sin(w*t);
y=[y1;y2;y3;y4];
x4=vpa(phiT4*y);

% x4=- (0.00198*P*sin(t*w))/(0.03371*w - 1.0) -
(0.00002*P*sin(t*w))/(0.01789*w - 1.0) - (0.00107*P*sin(t*w))/(0.07522*w -
1.0) - (0.00022*P*sin(t*w))/(0.02434*w - 1.0);

% Letting P=1
y1=subs(y1,P,1);
y2=subs(y2,P,1);
y3=subs(y3,P,1);
y4=subs(y4,P,1);

% Letting w=0.5wn;
y11=subs(y1,w,wn1(1).*0.5);
y21=subs(y2,w,wn1(2).*0.5);
y31=subs(y3,w,wn1(3).*0.5);
y41=subs(y4,w,wn1(4).*0.5);
x411=vpa(phiT4(1)*y11,4)
x412=vpa(x411+phiT4(2)*y21,4)
x413=vpa(x412+phiT4(3)*y31,4)
x414=vpa(x413+phiT4(4)*y41,4)

figure(2)
fplot(x411,[0,3])
hold on
fplot(x412,[0,3])
fplot(x413,[0,3])
fplot(x414,[0,3])
legend('Fundamental Mode','First 2 Modes','First 3 Modes','All 4 Modes')
xlabel('t(s)')
ylabel('m')
title('x4 with w=0.5wn')
hold off
```

```
% Letting w=4wn;
y12=subs(y1,w,wn1(1).*4);
y22=subs(y2,w,wn1(2).*4);
y32=subs(y3,w,wn1(3).*4);
y42=subs(y4,w,wn1(4).*4);
x421=vpa(phiT4(1)*y12,4)
x422=vpa(x421+phiT4(2)*y22,4)
x423=vpa(x422+phiT4(3)*y32,4)
x424=vpa(x423+phiT4(4)*y42,4)

figure(3)
fplot(x421,[0,0.3])
hold on
fplot(x422,[0,0.3])
fplot(x423,[0,0.3])
fplot(x424,[0,0.3])
legend('Fundamental Mode','First 2 Modes','First 3 Modes','All 4 Modes')
xlabel('t(s)')
ylabel('m')
title('x4 with w=4wn')
hold off
```

5. a) $\omega_{n,1} = 13.2935$ $\omega_{n,3} = 41.0787$

$\omega_{n,2} = 29.6597$ $\omega_{n,4} = 55.8820$

$\beta = 5.5304E-4$ (See Matlab)

$\alpha = 0.6999$ (See Matlab)

b) $2\zeta_3 \omega_{n,3} = \alpha + \beta \omega_{n,3}^2$

$\zeta_3 = 0.0199$ (See MATLAB)

Similarly, $\zeta_4 = 0.0217$ (See Matlab)

c) $x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}$, from MATLAB, $y_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.6996 \end{bmatrix}$

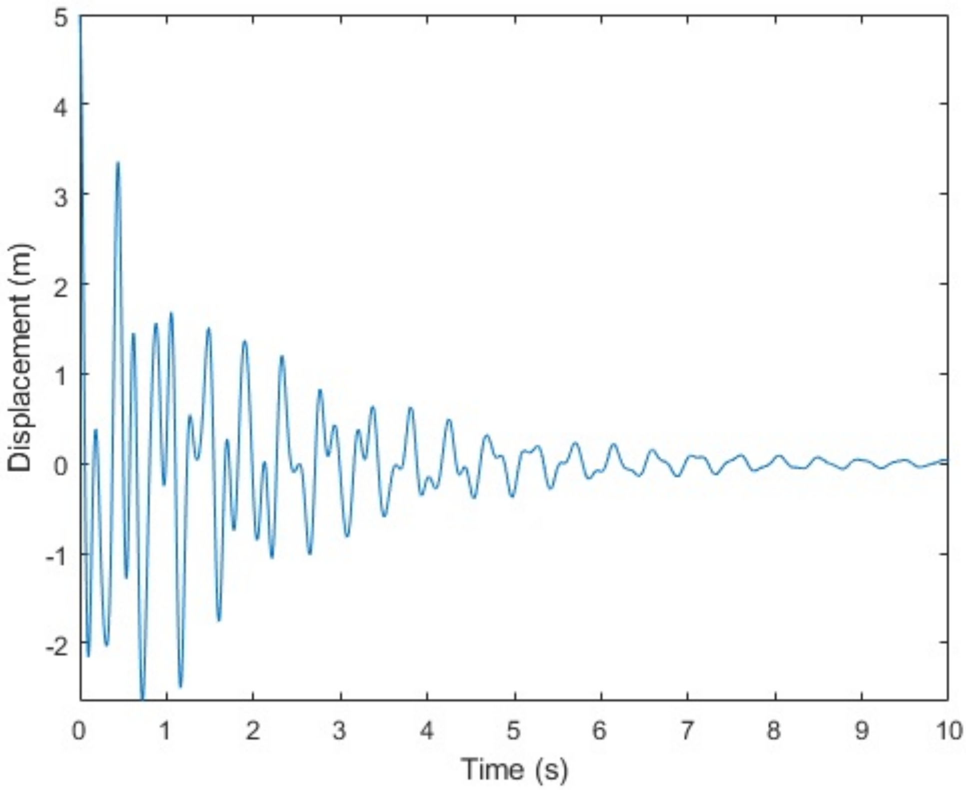
$\therefore y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{-\zeta_4 \omega_{n,4} t}\left(y_{0,4}\cos(\omega_{D,4} t) + \frac{0 + y_{0,4}\zeta_4 \omega_{n,4}}{\omega_{D,4}} \sin \omega_{D,4} t\right) \end{bmatrix}$

$x = \phi y$

$x_4 = [\phi_{1,4} \ \phi_{2,4} \ \phi_{3,4} \ \phi_{4,4}] \begin{bmatrix} 0 \\ 0 \\ 0 \\ y_4(t) \end{bmatrix} = \phi_{4,4} y_4(t)$

From MATLAB $\phi_{4,4} = 0.1399$, $\omega_{D,4} = 55.8688$

$\therefore x_4(t) = 0.1399 e^{-0.0217 \cdot 55.8688 t}\left[0.6996\cos(55.8688t) + \frac{55.8688 \cdot 0.0217 \cdot 55.8820}{55.8688} \sin(55.8688t)\right]$



Displacement (m)
Time (s)
5
4
3
2
1
0
-1
-2
0
1
2
3
4
5
6
7
8
9
10

Question 5

```
clc,clear
% Givens
M=[3,0,0,0;0,2,0,0;0,0,2,0;0,0,0,1];
K=800*[7,-3,0,0;-3,5,-2,0;0,-2,3,-1;0,0,-1,1];
x0=[0;0;0;5];
z1=0.03;
z2=0.02;
% Eigenvalue determination
[V1,wn1]=eig(K,M);
[wn1_sorted,ind]=sort(diag(wn1));
wn1=sqrt(wn1_sorted)
V1_sorted=V1(:,ind);
% Rayleigh Coefficients
B=2*(z1.*wn1(1)-z2.*wn1(2))/(wn1(1)^2-wn1(2)^2);
alpha=2.*z1.*wn1(1)-B.*wn1(1)^2;
% Damping ratios
z3=(alpha+B.*wn1(3)^2)/(2.*wn1(3));
z4=(alpha+B.*wn1(4)^2)/(2.*wn1(4));
% Mass normalization
syms a b c d
V11=V1(:,1)*a;
phiMphi=transpose(V11)*(M)*V11;
expression=phiMphi(1,1)-1==0;
Answer=double(solve(expression,a));
Alpha=Answer(2);
MNV11=Alpha.*V1(:,1);
V12=V1(:,2)*b;
phiMphi=transpose(V12)*(M)*V12;
expression=phiMphi(1,1)-1==0;
Answer=double(solve(expression,b));
Alpha=Answer(2);
MNV12=Alpha.*V1(:,2);
V13=V1(:,3)*c;
phiMphi=transpose(V13)*(M)*V13;
expression=phiMphi(1,1)-1==0;
Answer=double(solve(expression,c));
Alpha=Answer(2);
MNV13=Alpha.*V1(:,3);
V14=V1(:,4)*d;
phiMphi=transpose(V14)*(M)*V14;
expression=phiMphi(1,1)-1==0;
Answer=double(solve(expression,d));
Alpha=Answer(2);
MNV14=Alpha.*V1(:,4);
MNV1=[MNV11,MNV12,MNV13,MNV14]
% y response
syms t
y0=transpose(MNV1)*M*x0;
y10=y0(1)
y20=y0(2)
y30=y0(3)
y40=y0(4)
omegan1=wn1(1)
omegad1=omegan1*sqrt(1-z1^2)
wn2=wn1(2)
wd2=wn2*sqrt(1-z2^2)
```

```
wn3=wn1(3)
wd3=wn3*sqrt(1-z3^2)
wn4=wn1(4)
wd4=wn1(4)*sqrt(1-z4^2)
x4=MNV1(4,1)*exp(-
z1*omegan1.*t)*(y10*cos(omegad1*t)+y10*z1*omegan1/omegad1*sin(omegad1*t))+MNV
1(4,2)*exp(-
z2*wn2.*t)*(y20*cos(wd2*t)+y20*z2*wn2/wd2*sin(wd2*t))+MNV1(4,3)*exp(-
z3*wn3.*t)*(y30*cos(wd3*t)+y30*z3*wn3/wd3*sin(wd3*t))+MNV1(4,4)*exp(-
z4*wn4.*t)*(y40*cos(wd4*t)+y40*z4*wn4/wd4*sin(wd4*t))
fplot(x4,[0,10]);
xlabel('Time (s)')
ylabel('Displacement (m)')
```