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DESIGN OF REINFORCED CONCRETE CIRCULAR SLABS BY TWO-WAY ISOTROPIC STRAIGHT JOISTS

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ABSTRACT

This research introduces a new simple, efficient, and practical procedure to design the ***r****einforced* ***c****oncrete (RC) circular slabs which have large diameters. The principal idea of this paper concerns to use the isotropic perpendicular RC straight joists to resist the external load. The yield-line theory was adapted to analysis the circular waffle slabs. The steps of design were according to the ACI Code provisions. Fixed and simply supported circular slabs were presented. Closed form equations have been driven by author for the purposes of analysis and design this type of slabs by the present procedure. Uniformly distributed load was considered, that represent almost practical cases. Useful illustration example is presented in this study according to the available materials in Iraq to facilitate the job of designers. The good performance of RC circular slab which design by the present procedure proved clearly the efficiency of this technique.*

Keywords: Circular Slab, Two-Way Joist Slabs, Ribbed Slabs, Waffle Floors, Grid Floors, Yield-Line Theory.

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1. INTRODUCTION

Circular slabs are extensively used during the last decade in many engineering applications due to architectural and/or structural needs. They may be floor and/or roof of building, halls, water tanks, silos, bunkers, etc., hence, many researchers devoted their studies for this field. The simply supported and fixed edge circular floors under uniformly distributed loads represent the most popular practical cases, thus, they are considered here. The deflection shape of circular slabs likes a saucer which produces stresses in radial and circumferential directions that necessitates steel bars in these directions. Practically provision of reinforcements in these directions is difficult due to the difficulty of bending the steel bars, as well as the congestion in the central region of the floor, where maximum moments occurred.

Therefore, practically the reinforcements of circular slabs are usually provided in two straight orthogonal directions, as provided for two-way rectangular slabs, from this constructional view the idea of present paper was appeared.

Two-way joist floor systems, always consist joists spaced at regular intervals in two perpendicular directions, are generally used for architectural reasons for large rooms or halls. The joist slabs is known as, ribbed slabs, waffle floors, grid slabs, and coffered floors. The structural engineers have preferred the ribbed floors, from side of economy for buildings that have a small live loads and long spans such as hotels, hospitals, and apartment houses [1]. Three major advantages draw attention of designers to use the joist floor; economy of formwork, exemption of joists from the requirement that stirrups must be provided whenever the applied shear exceeds 0.5 V_C, and the ability of joists to distribute the local over loads to the adjacent joists batter than the typical beams. The waffle slab can be treated as solid slab, i.e. all procedures used to analysis the solid slabs can be used to analysis the waffle slabs [1, 2, 3].

In the last three decades the methods which used to analysis the RC structures have been advanced and based on inelastic considerations that obtained just prior to failure. In the present study the yield-line theory adopted to analysis the circular slabs. The yield-line theory is classified as one of the limit design methods that can be used for all types of slabs [4]. The experimental tests have closely verified that the results of yield-line analysis; it appears that structural designers may use this method with confidence, where the calculated load usually underestimates the actual results [2]. Also the slabs always designed under reinforced section, thus the steel reaches its yield-stress before the slab reaches its ultimate strength which gives adequate warning before failure.

2. REVIEW OF LITERATURE

Timoshenko and Woinowsky [5] have driven equations of deflection, radial moments, and tangential moments of circular plates, clamp and simply supported edges, under axisymmetric uniformly distributed load. Their work was according to the Small Deflection Theory. Wang et al. [6] devoted their study to investigate the behavior of circular plates. The analysis had been done utilizing direct displacement method. Several examples have been performed by authors to test the proposed method. Hayder [7] used the yield line theory to obtain the optimal design parameters of orthotropic rectangular RC simply supported slabs. Uniform distributed load considered in this study. The results of study showed a linear relationship between the perimeter and the optimal thickness of square slabs. Alaa and Zainab [8] presented a computer program to analysis and design the rectangular RC two-way waffle slabs. Their work focused to obtain the optimal design of RC two-way joist slabs. The research includes two cases; two-way joist slabs with solid heads, and two-way joist slab with band beams among columns centerlines. Hayder [9] presented analytical study includes the effects of compressive strength of concrete, yield strength of steel, and thickness of two-way solid slabs on the required area of reinforcements. Akinyele and Alade [10] introduce a new procedure to analysis the pre-cast waffle slab utilizing yield line theory. Their study focused on the effect of slab portion on the overall strength of pre-cast waffle slabs. The results proved that the presented program, which depended on the yield line theory, can be used to analyze this type of slabs. Naziya and Chitra [11] investigated three available methods (Rankine-Grashoff method, plate theory, and stiffness method) for analyzing the joist floor system. A comparison was presented among the results of these methods, such as bending moments and shear forces. The results have proven that the stiffness method represents the more accurate and suitable method than the other studied methods. Reviewing of the previous works appeared clearly that the researchers, designers, and engineers don't deal or perform the straight joist in the field of circular slabs that proves the novelty of this research.

3. ANALYSIS AND DESIGN STEPS OF PRESENT METHOD

3.1. General

Due to difficulty of providing tangential and circumference reinforcements for circular slabs, the engineers tends to perform straight perpendicular reinforcements to facility the technical staff job and to reduce the work hours of construction this type of slabs. Hence, the major idea of this paper is focused for providing straight perpendicular ribs.

3.2. Assumptions

The following assumptions are required to perform the derivations of equations.

- Edge ring beam must be provided, and the effect of its torsional strength on the discontinuous RC circular slab neglected.
- The strength of ribs represents the total strength of circular slab, i.e. the strength of slab that generated due to shrinkage reinforcements neglected.
- Isotropic ribs will be used and uniform distributed load is considered, thus the value of resisting moments will be constant at any angle.
- Constant bottom reinforcement must be provided along the span.
- The negative reinforcements at clamped edge should be uniformly distributed.
- It is useful to imagine the ribs as reinforcements distributed uniformly in two perpendicular directions.
- The negative moment is equal to the positive moment.

3.3. Moment-Load Relationship

When apply the principles of virtual work of yield line theory for RC circular two-way ribbed slab of discontinuous and continuous edge, which shown in Fig.1 and Fig.2, the following equations can be obtained respectively:

$$\boldsymbol{m^{+} = \frac{Wu\,R^2}{6}} \quad (1)$$

$$\boldsymbol{m^{+} + m^{-} = \frac{Wu\,R^2}{6}} \quad (2)$$

3.4. Design Equations

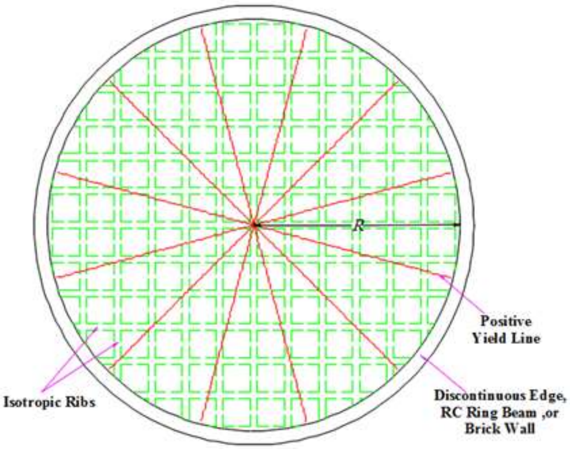
3.4.1. General Variables

The following variables are required in design the positive and negative reinforcements.Eq.4 selected here, from ACI-Code [12], because it is usually controlled according to the popular loads that used in building.

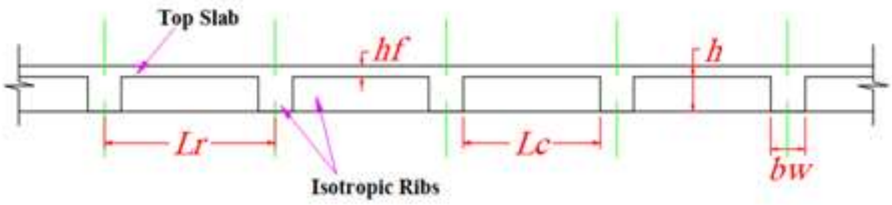
$$\boldsymbol{Wd_1 = \left[hf\,Lr^2 + (2Lr - bw)bw\,(h - hf)\right] \times 10^{-3}\,\frac{\gamma_c}{Lr^2}} \quad (3)$$

$$\boldsymbol{Wu = 1.2(Wd_1 + Wd_2) + 1.6\,Wl} \quad (4)$$

$$\boldsymbol{d = h - 20 - 0.5d_b} \quad (5)$$



(a) Top View



(b) Section in Two-Way Ribbed Slab

Figure 1 Discontinuous Edge RC Circular Two-Way Ribbed Slab

3.4.2. Equations of Bottom Reinforcement of Ribs (Positive Reinforcement)

The bottom reinforcement must be provided for discontinuous or continuous two-way RC ribbed circular slabs. The section of rib at positive region (mid-span) analyzes as a rectangular section or T-section according to the geometry of ribs and the applied moment.

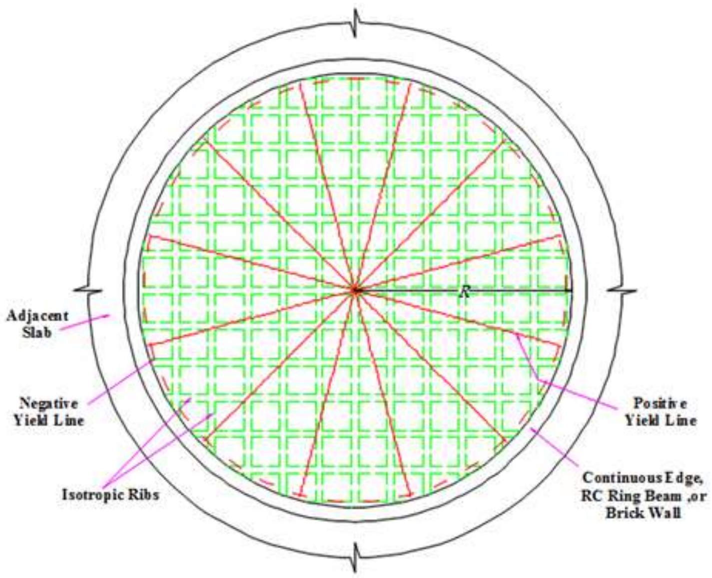


Figure 2 Top View of Continuous RC Circular Two-Way Ribbed Slab

3.4.2.1. Equations of Rectangular Section

Here the depth of stress block is less than or equal to the depth of flange, this is achieved in RC circular two-way ribbed slabs if the radius of slab **R** is less or equal to the variable $\mathbf{H_1}$, as shown in Eq.6, where H_1 can be calculated by Eq.7. The positive ratio of reinforcement which required for computing the positive reinforcement as detailed in Eq.8, while the positive area of reinforcement (bottom reinforcement) for one rib can be calculated by Eq.9.

$$\boldsymbol{R \leq H_1} \quad \text{(Rectangular Section)} \tag{6}$$

$$\boldsymbol{H_1 = \left[\emptyset\, H_2 fc'\ \ hf\ (d - 0.5hf) Wu^{-1} 10^{-3}\right]^{0.5}} \tag{7}$$

$$\rho^{+}_{\boldsymbol{req}} = \boldsymbol{\left[0.85 - \left(0.7225 - \ H_3\ Wu(\emptyset fc')^{-1} (\tfrac{R}{d})^2\right)^{0.5}\right] \frac{fc'}{fy}} \tag{8}$$

$$\boldsymbol{As^{+}} = \rho^{+}_{\boldsymbol{req}}\ \boldsymbol{Lr\ d} \tag{9}$$

The parameter $\mathbf{H_2}$ in Eq.7 equals to **5.1** and **10.2,** while the parameter $\mathbf{H_3}$ in Eq.8 equals to **283.3** and **141.7** for discontinuous and continuous RC circular two-way ribbed slabs respectively.

3.4.2.2. Equations of T-Section

The survey analysis proved that when the radius of slab **R** is larger than the variable $\mathbf{H_1}$ the depth of stress block will be larger than the depth of flange and the section must be analyzed as T-section. The following equations are required to design the positive region of ribs in this type of slabs.

$$\boldsymbol{R > H_1} \quad \text{(T- Section)} \tag{10}$$

$$\boldsymbol{Asf = 0.85\ \frac{fc'}{fy}(Lr - bw)hf} \tag{11}$$

$$\boldsymbol{a = d - \left[d^2 - \frac{2.353}{bw\, fc'}\left(\frac{H_4\ Lr\ Wu\ R^2}{\emptyset} - Asf\ fy\ (d - 0.5hf)\right)\right]^{0.5}} \tag{12}$$

$$\boldsymbol{As^{+} = 0.85\ a\ bw\ \frac{fc'}{fy}\ + Asf} \tag{13}$$

The parameter $\mathbf{H_4}$ in Eq.12 equals to **166.7** and **83.8** for discontinuous and continuous RC circular two-way ribbed slabs respectively.

3.4.3. Equations of Top Reinforcement of Ribs (Negative Reinforcement)

The top reinforcement must be provided for continuous two-way RC ribbed circular slabs. The section of rib at negative region (support) analyzes as a rectangular section of width equal to the width of rib (**bw**).

$$\rho^{-}_{\boldsymbol{req}} = \boldsymbol{\left[0.85 - \left(0.7225 - \frac{141.7\ Wu\ Lr}{\emptyset\ fc'\ bw}\ (\tfrac{R}{d})^2\right)^{0.5}\right] \frac{fc'}{fy}} \tag{14}$$

$$\boldsymbol{As^{-}} = \rho^{-}_{\boldsymbol{req}}\ \boldsymbol{bw\ d} \tag{15}$$

3.4.4. Equation of Shear Force

To avoid providing shear reinforcement in the ribs the shear strength Eq.16, should be larger than the applied shear, Eq.17, thus Eq.18 has been derived depending on this concept. Therefore if Eq.18 satisfied the shear reinforcement will not be provided, [12].

$$\emptyset Vc = \emptyset\, 0.187\sqrt{fc'}\, bw\, d10^{-3} \quad (16)$$

$$Vu_d = 5Wu\, Lr\, (1000R - d)10^{-7} \quad (17)$$

$$R \leq \frac{d}{10^3}\left(1 + \frac{280.5\ bw\sqrt{fc'}}{Wu\, Lr}\right) \quad (18)$$

3.4.5. Deflection Equations

This paragraph conducts to compute the maximum deflection of RC circular two-way ribbed slabs that occurred usually at the center of slab when uniformly distributed load has been applied. The deflection can be computed by using Eq.19 and Eq.20 for simply supported (discontinuous) and clamped (continuous) circular slab respectively, [13]. The flexural rigidity (D_r) in Eq.19 and Eq.20 is derived according to the geometry of ribbed slab, as defined in Eq.21. The effective moment of inertia and the cracking moment are defined by Eq.22 and Eq.23 respectively [12].

The gross moment of inertia of one rib (Ig) at mid-span has been derived, as defined in Eq.24. The cracking moment of inertia (Icr) at mid-span can be calculated either by Eq.26 or Eq.29 depending on the location of **n**eutral **a**xis (NA); if the location of NA is within flange (M1≥M2) then Eq.26 controlled else Eq.29 will be controlled. While Icr of ribs at edges of continuous slab, are defined in Eq.35.

$$\delta_{max} = \frac{W\, R^4\, (5+\upsilon)}{64\, D_r\, (1+\upsilon)} \times 10^3 \quad (19)$$

$$\delta_{max} = \frac{W\, R^4}{64\, D_r} \times 10^3 \quad (20)$$

$$D_r = \frac{Ec\, Ie}{(1-\upsilon^2)Lr} \times 10^{-6} \quad (21)$$

$$Ie = \left(\frac{M_{cr}}{M_a}\right)^3 Ig + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] Icr \ \leq\ Ig \quad (22)$$

$$M_{cr} = \frac{f_r\, Ig}{y_t} 10^{-6} \quad (23)$$

$$Ig = \frac{1}{3}\left[Lr\, \bar{y}^3 - (Lr - bw)(\bar{y} - hf)^3 + bw\, (h - \bar{y})^3\right] \quad (24)$$

$$\bar{y} = \frac{hf^2(Lr-bw) + h^2\, bw}{2[hf\, (Lr-bw) + h\, bw]} \quad (25)$$

$$Icr = \frac{Lr\, y^3}{3} + nAs^+_{pro}(d - y)^2\ , Rectangular - section\ if\ M_1 \geq M_2. \quad (26)$$

$$y = -H_5 + \sqrt{{H_5}^2 + 2H_5\, d} \quad (27)$$

$$H_5 = n\, As^+_{pro}/Lr \quad (28)$$

$$Icr = \frac{Lr\, y_1^3}{3} - \frac{Lc(y_1-hf)^3}{3} + nAs^+_{pro}(d - y_1)^2, T - section\ if\ M_1 < M_2 \ldots \quad (29)$$

$$y_1 = 0.5(-H_6 + \sqrt{{H_6}^2 + 4H_7}\,) \quad (30)$$

$$H_6 = 2(Lc\, hf +\ n\, As^+_{pro})/bw \quad (31)$$

$$H_7 = (Lc\, hf^2 + 2\, n\, As^+_{pro} d)/bw \quad (32)$$

$$\boldsymbol{M1 = 0.5\,Lr\,hf^2} \tag{33}$$

$$\boldsymbol{M2 = nAs^{+}_{pro}(d - hf)} \tag{34}$$

$$\boldsymbol{Icr = \frac{bw\,y_2^3}{3} + (n-1)As^{+}_{pro}(y_2 - 20 - d_b)^2 + nAs^{-}_{pro}(d - y_2)^2} \tag{35}$$

$$\boldsymbol{y_2 = 0.5(-H_8 + \sqrt{H_8{}^2 + 4H_9}\,)} \tag{36}$$

$$\boldsymbol{H_8 = 2[(\,n-1)As^{+}_{pro} + n\,As^{-}_{pro}]/bw\,\ldots} \tag{37}$$

$$\boldsymbol{H_9 = 2[(\,n-1)As^{+}_{pro}(20 + 0.5d_b) + n\,As^{-}_{pro}d]/bw} \tag{38}$$

3.5. Illustration Example

Design RC circular continuous roof by two-way perpendicular straight ribs, the radius of slab is equal to 7.65m, take fc'=20 MPa, fy=400 MPa, d_b=16mm, Lc=750mm, bw=200mm, h=550mm, hf=70mm, Wd_2=3.6kN/m^2, Wl=2kN/m^2. Check the deflection of roof if it is supporting to non-structural elements not likely to be damaged by large deflections, assume 30% of live load sustained, $\zeta = 2$. This example represents an actual case designed by author in 2012, as shown in Fig.3.

Solution:

- Check the geometry of ribs, [12]:

bw ≥ 100mm (o.k.), h ≤ 3.5bw (o.k.), Lc ≤ 750mm (o.k.), and hf ≥ max.(Lc/12,50mm)→70mm > 62.5mm (o.k.), all conditions are satisfied.

- Compute the total load and moments:

Wd_1 = 6.15 kN/m^2 (Eq.3), Wd_2 = 3.6 kN/m^2 [9], Wu = 14.9 kN/m^2 (Eq.4), $m^+ = m^-$ → $m^+ = m^-$ = $Wu\,R^2/12$ = 72.67 kN.m/m (Eq.2).

- Computation of bottom reinforcement:

$d = h - 20 - 0.5d_b = 550 - 20 - 8 = 522mm$(Eq.5)

Assume $\emptyset = 0.9$, from Eq.7$H_1 = 22.91m$> $R = 7.65m\,,\therefore\,Rect.Section$ (Eq.6)

From(Eq.8), $\rho^{+}_{req} = 7.475\times10^{-4} > \rho_{min} = \left(\frac{1.4}{400}\times\frac{200}{950}\right) = 7.368\times10^{-4}$ ∴ o.k.

$\rho^{+}_{req} = 7.475\times10^{-4} < \rho_{max} = \left(0.85^2\frac{20}{400}\times\frac{0.003}{0.003+0.004}\right) = 154.8\times10^{-4}$ ∴ o.k.

$\rho^{+}_{req} = 7.475\times10^{-4} < \left(0.85^2\frac{20}{400}\times\frac{0.003}{0.003+0.005}\right) = 135.5\times10^{-4} \therefore \emptyset = 0.9$

$As^{+}_{req} = 7.475\times10^{-4}\times950\times522 = 371\,mm^2/rib$,(Eq.9)→(Use 2bars of 16mm)

- Computation of top reinforcement:

$\rho^{-}_{req} = \left[0.85 - \left(0.7225 - \frac{141.7\,Wu\,Lr}{\emptyset\,fc'\,bw}\,(\frac{R}{d})^2\right)^{0.5}\right]\frac{fc'}{fy} = 3.679\times10^{-3}$(Eq.14)→

$\rho_{max} = 15.48\times10^{-3} > \rho^{-}_{req} = 3.679\times10^{-3} > \rho_{min} = \frac{1.4}{400} = 3.5\times10^{-3}$ ∴ o.k.

$\rho^{-}_{req} = 3.679\times10^{-3} < \left(0.85^2\frac{20}{400}\times\frac{0.003}{0.003+0.005}\right) = 135.5\times10^{-4} \therefore \emptyset = 0.9$

$As^{-}_{req} = 3.679\times10^{-3}\times200\times522 = 384\,mm^2/rib$,(Eq.15)→(Use 2bars of 16mm)

- Check the shear requirements:

  $R \leq \frac{d}{10^3}\left(1+\frac{280.5\ bw\sqrt{fc'}}{Wu\ Lr}\right)$, 7.65 ≤ 9.77(Eq.18)→ shear reinforcement not required

- Check deflection:

a-Ie at mid-span; $(M1 = 2327500mm^3 > M2 = 1728914mm^3) \rightarrow from\ Eq.26$

$Icr = 884776241\ mm^4,\ \ y = 60.93mm\ Eq.27\ , and\ H_5 = 4.03\ mm\ Eq.28$

$From\ Eq.24\ \ Ig = 4841369551\ mm^4, \qquad where\ \bar{y} = 197.46mm\ Eq.25$

$Mcr = 0.62\sqrt{20} \times 4841369551 \times (550 - 197.46)^{-1} \times 10^{-6} = 38.08\ kN.m\ /rib\ Eq.23,$

$Ma = (6.15 + 3.6 + 2) \times (7.65)^2 \times 950 \times 10^{-3}/12 = 54.44\ kN.m\ /rib\ Eq.2,$

$Ie\ (mid - span) = 2238898515\ mm^4, Eq.22.$

b-Ie at support; $from\ Eq.35\ Icr = 760778983\ mm^4,\ \ y_2 = 112.9mm\ Eq.36\ ,$

$H_8 = 72.5\ mm\ Eq.37\ and\ H_9 = 20925.1\ mm\ Eq.38$

$Ig = bw\ h^3/12 = 2772916667\ mm^4,\ \ \bar{y} = h/2 = 275mm$

$Mcr = 0.62\sqrt{20} \times 2772916667 \times (275)^{-1} \times 10^{-6} = 27.96\ kN.m\ /rib\ Eq.23,$

$Ma = (6.15 + 3.6 + 2) \times (7.65)^2 \times 950 \times 10^{-3}/12 = 54.44\ kN.m\ /rib\ Eq.2,$

$Ie\ (support) = 1033372075\ mm^4, Eq.22.$

c-$Ie(contiuous\ rib) = 0.5(2238898515 + 1033372075) = 1636135295mm^4$

d-$D_r = \frac{4700\sqrt{20}\times 1636135295}{(1-\upsilon^2)\times 950\times 10^{+6}}, \upsilon = 0.15\ [13], \rightarrow D_r = 37033.2kN.m, from\ Eq.21$

e-$\delta_{i(D+L)} = \frac{W\,R^4}{64\ D_r} \times 10^3 = \frac{(6.15+3.6+2)\times 7.65^4}{64\times 37033.2} \times 10^3 = 16.98mm$

$$\delta_{i(D+0.3L)} = \frac{(9.75 + 0.3 \times 2)}{11.75} \times 16.98 = 14.96mm, \lambda = \frac{\zeta}{1 + 50\bar{\rho}} = 2\ \ [12], \rightarrow$$

$$\delta_{LT} = 14.96 \times 2 = 29.92mm, \rightarrow \delta_{i(0.7L)} = \frac{(0.7 \times 2)}{11.75} \times 16.98 = 2.02mm$$

$$\delta_{LT} + \delta_{i(0.7L)} = 29.92 + 2.02 = 31.94mm < (\frac{L}{240} = 63.75mm \therefore o.k.)$$

4. CONCLUSIONS

According to the results of analytical investigation and practical observation one can be concluded the following points:

- The present new procedure represents an efficient, simple, and applicable method can be used to design the circular RC slabs.
- Under the popular live loads the cross-section at mid-span of the ribs of continuous RC circular slabs worked as rectangular section (i.e. the NA located within the flange) even when the radius of slab is large.

- The practical application is proved an excellent performance of circular RC slab which designed by this procedure (perpendicular straight ribs).

Fig. 3. Continuous RC Circular Two-Way Ribbed Slab, College of Dentist/University of Babylon, Designed by Author in 2012

APPENDIX A: NOTATION

a	Depth of stress block, mm
As^+	Bottom area of reinforcement of rib (required), mm^2/rib
As^-	Top area of reinforcement of rib (required), mm^2/rib
As_{pro}^{+}	Bottom area of reinforcement of rib (provided), mm^2/rib
As_{pro}^{-}	Top area of reinforcement of rib (provided), mm^2/rib
bw	Width of rib, mm
D_r	Flexural rigidity, kN.m/m
d	Distance from extreme compression fiber to centroid of tension reinforcement, mm
d_b	Nominal diameter bar, mm
Ec	Modulus of elasticity of concrete, $4700\sqrt{fc'}$ (ACI-Code), N/mm^2
Es	Modulus of elasticity of steel, N/mm^2
fc'	Specified compressive strength of concrete, cylinder test, N/mm^2
f_r	Modulus of rupture of concrete, $0.62\sqrt{fc'}$(ACI-Code), N/mm^2
fy	Specified yield strength of nonprestressed reinforcement, N/mm^2
h	Total depth of rib, mm
hf	Thickness of top slab, mm
Icr	Moment of inertia of cracked transformed section of rib, mm^4
Ie	Effective moment of inertia of one rib, mm^4
Ig	Moment of inertia of gross uncrackedconcrete section of rib, mm^4
Lc	Clear dimension between ribs, mm
Lr	Dimension between center lines of ribs, mm
Ma	Maximum moment due to service loads at stage deflection is calculated, kN.m/rib
Mcr	Cracking moment, kN.m/rib
M1	Moment of compression area, cracked section, mm^3
M2	Moment of tension area, cracked section, mm^3
m^+	Internal ultimate resisting moment (positive), kN.m/m
m^-	Internal ultimate resisting moment (negative), kN.m/m

n	Modular ration Es/Ec
R	Radius of circular two-way ribbed slab, m
Vc	Shear strength of concrete, kN
y, y_1, y_2	Distance from centriodal axis of cracked section to compression face, mm
$\bar{y}$	Distance from centriodal axis of gross section to compression face, mm
y_t	Distance from centriodal axis of gross section to tension face, neglecting reinforcement, mm
W	Service uniformly distributed total load, kN/m^2
Wd_1	Service self-weight dead load, kN/m^2
Wd_2	Service super imposed dead load, kN/m^2
Wl	Service live load, kN/m^2
Wu	Factored uniformly distributed total load, kN/m^2
γ_c	Density of reinforced concrete, taken as 24.5 kN/m^3
δ_i	Immediate deflection, mm
δ_{max}	Maximum deflection at center of circular slab, mm
δ_{LT}	Long term deflection, mm
υ	Poisson's ratio
ρ^{+}_{req}	Ratio of As^+ to Lrd (or to bwd)
ρ^{-}_{req}	Ratio of As^- to bwd
$\bar{\rho}$	Ratio of compression reinforcement at mid-span
ζ	Time dependet factor sustained losd
ϕ	Strength reduction factor
λ	Multiplier for additional long term deflection, ACI-Code

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