What are some tools to solve more complicated reliability problems?

Lecture 2B – Key Messages

- Monte Carlo Simulation (MCS) is a statistical simulation modelling approach, which can be applied easily (e.g. in MS Excel) to determine P_f for more complex design equations.
- Why do we care about this stuff in a course on assessment / rehabilitation of structures?

Answer: understanding these concepts, we can identify parameters to measure to reduce uncertainty and "gain" capacity to possibly avoid rehabilitation.

Monte Carlo Simulation (MCS)

- R and S are not necessarily normally distributed, and much more complicated limit state functions also exist. MCS is a method to solve these more difficult problems.

- The basic idea:

$$p_f = \frac{n(G(\bar{Z}) < 0)}{N}$$

Number of failures

Number of trials

Monte Carlo Simulation (MCS) Steps

- Step 1: Generate random trial values for the input parameters (e.g., Strength of material, live load, dead load, etc.)

- Step 2: Check if $G(\bar{Z}) < 0$ (i.e., check if the trial values lead to a failure.)

- Step 3: Steps 1 and 2 should be repeated N times. (required number of trials (N) will be discussed in the following slides)

- Step 4: The probability of failure is equal to the number of failures (n_f) divided by the total number of trials (N).

$$p_f = \frac{n_f}{N}$$

Generating a random trial value for each input parameter

- A random trial value for each input parameter can be determined using a random number between 0 and 1 ($0 \leq r_i \leq 1$) and the inverse CDF of the input parameter:

$$\hat{x}_1 = F_X^{-1}(r_i)$$

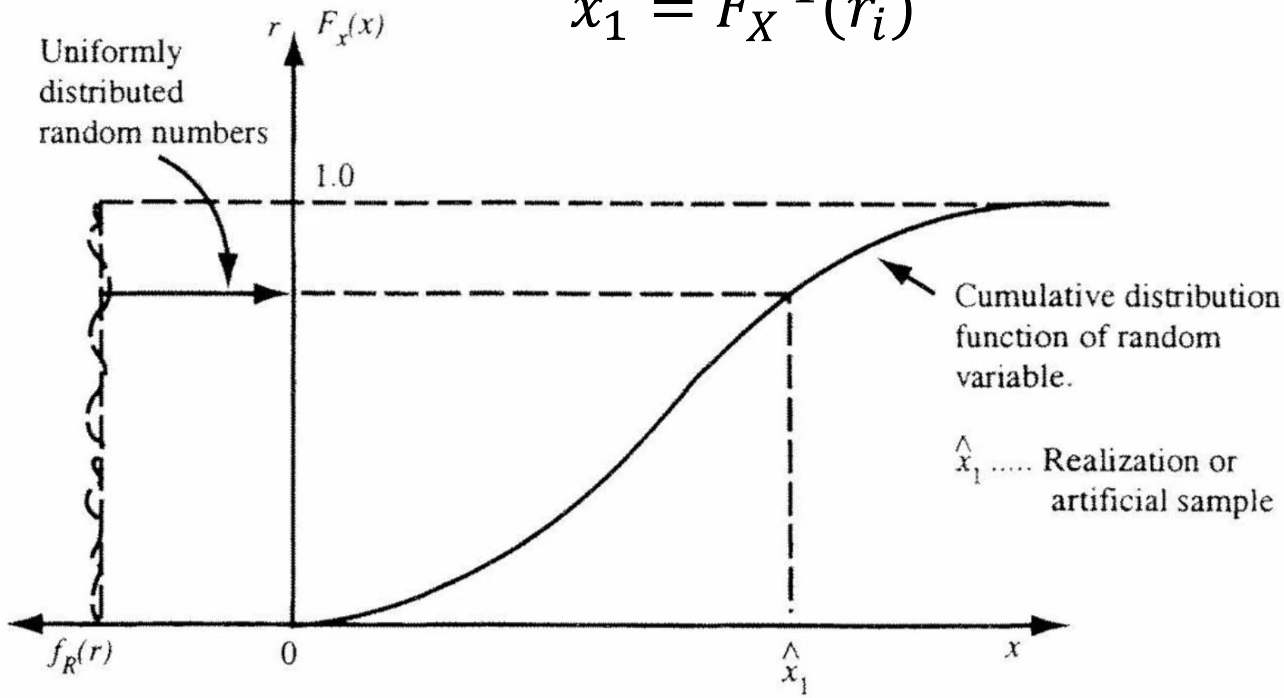
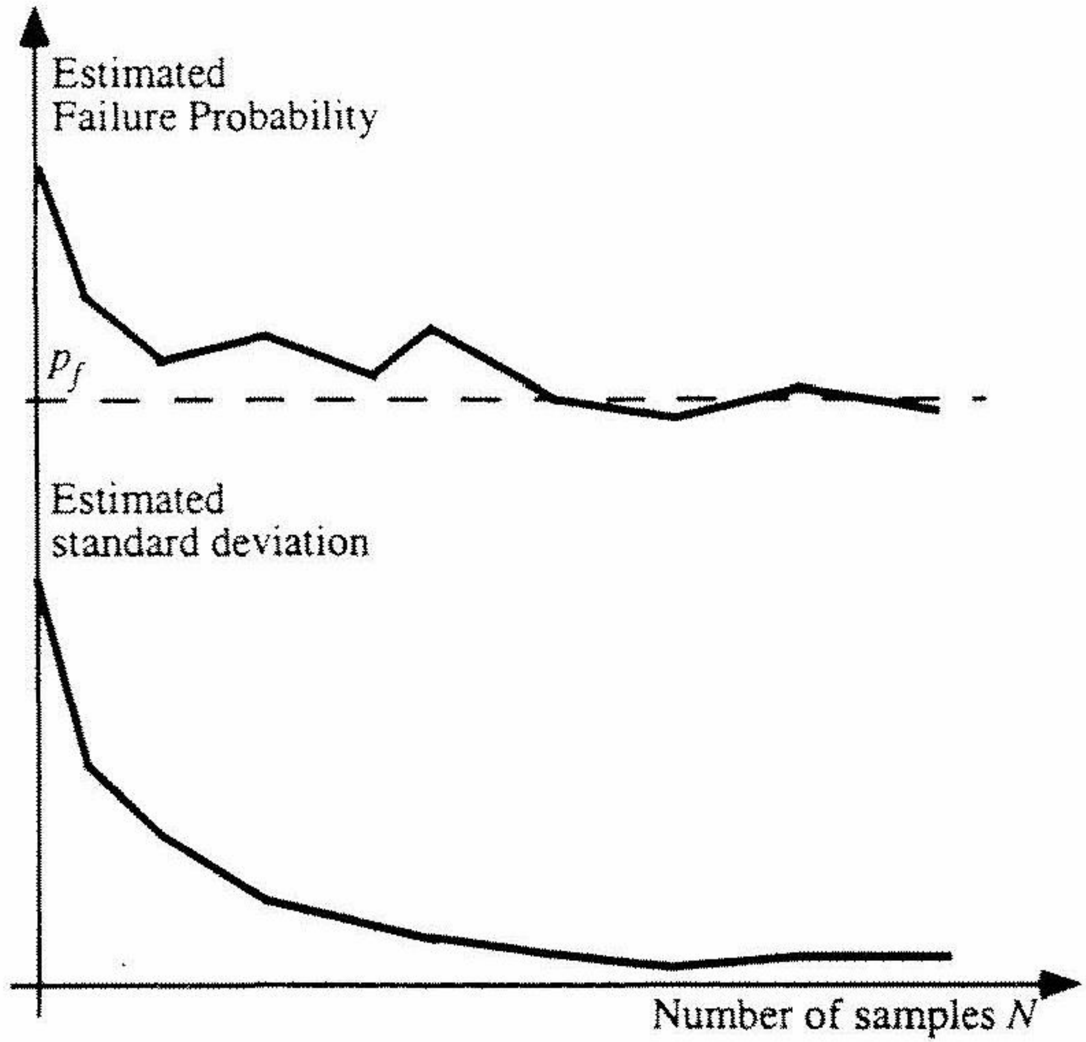


Figure 3.1 Inverse transform method for generation of random variates.

[Melchers 1999]

The required number of trials, N



[Melchers 1999]

The required number of trials, N

The number of trials, N, depends on how small p_f is and the level of confidence desired in the results. Various researchers have proposed formulas.

$$N > \frac{-\ln(1 - C)}{p_f}$$

Level of Confidence

For example: $p_f = 10^{-3}, C = 95\% \rightarrow N \approx 3000$

[Broding 1964]

Bias factors

- characterize difference between design parameter & actual parameter value, ie.

$$\lambda = \frac{Z}{Z_n}$$

- can also be treated as statistical input parameters in limit state functions, ie:

$$\mu_\lambda = \frac{\bar{Z}}{Z_n}, \quad V_\lambda = \frac{\sigma_Z}{\bar{Z}}$$

- various references including the CSA-S6 Commentary & Calibration Report give statistics for bias factors for various parameters associated with the loading & resistance of structures

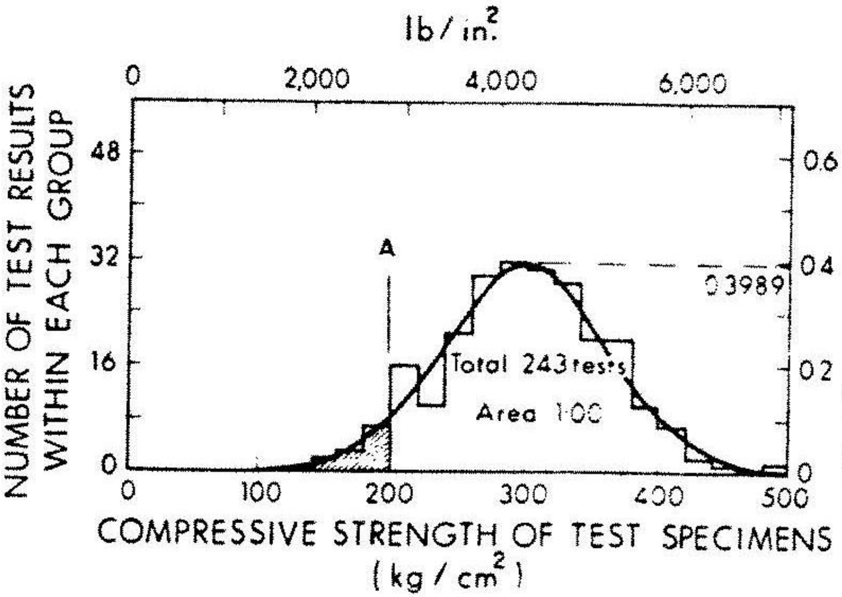


FIG. 1. Distribution of concrete compressive strength.

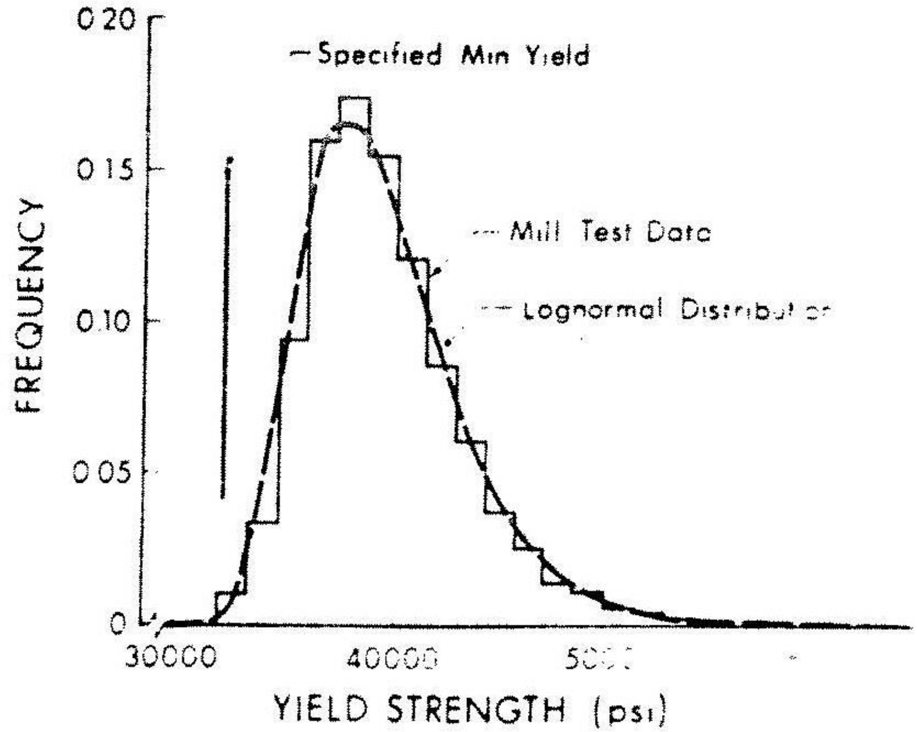


FIG. 3. Distribution of structural steel yield strengths (Alpsten 1972).

[MacGregor 1976]

Table 3.1 Statistical Data for Resistance of Steel Components

Category	Source	Bias Factor, δ	Coefficient of Variation, V
Welded Sections:	Kennedy &		
Fy	Baker (1984)	1.101	0.0915
S		1.020	0.010
Test/predicted		1.090	0.045
Rolled Sections:	Kennedy &		
Fy	Baker (1984)	1.060	0.051
S		0.990	0.021
Test/predicted		1.090	0.045
Welded Sections:	Kennedy &		
plastic moment	Baker (1984),	1.133	0.096
yield moment		1.221	0.100
inelastic moment		1.155	0.084
elastic moment		1.090	0.092
composite, fully plastic moment		1.098	0.096
composite negative yield moment*	*Kennedy	1.221	0.100
compression, $\lambda = 0.8$	(1996)	1.058	0.079
compression, $\lambda = 1.0$		1.048	0.068
compression, $\lambda = 1.2$		1.096	0.075
shear		1.178	0.103

[CSA S6-06 Calibration Report]

Rolled Sections: plastic moment yield moment composite, fully plastic moment* composite negative yield moment* compression, $\lambda = 0.8$ compression, $\lambda = 1.0$ compression, $\lambda = 1.2$ shear	Kennedy & Baker (1984), *Kennedy (1996)	 1.126 1.210 1.095 1.210 1.143 1.187 1.185 1.102	 0.081 0.077 0.075 0.077 0.116 0.132 0.117 0.071
WWF, with n=2.24 in.: $C_r = \phi A F_y[1+2n]^{-1/n}$ compression, $\lambda = 0.34$ compression, $\lambda = 0.67$ compression, $\lambda = 1.01$	Chernenko & Kennedy (1991)	 1.132 1.087 1.010	 0.063 0.065 0.082
A325 Bolts: Tension Shear	Fisher et al. (1978)	 1.12 1.16	 0.090 0.100
Welds, fillet	Fisher et al. (1978)	1.22	0.172
Welds, fillet, for use with equation: $V_r = 0.67\ \phi_w A_w X_u[1.00+0.50\sin^{1.5}\theta]$	Lesik & Kennedy (1990)	1.137	0.189

Table 3.2 *Statistical Data for Resistance of Prestressed Concrete Components*

Component Type	Response	Bias Factor δ	Coefficient of Variation, V
Simple Span Bridges:			
PC Girder	Moment Shear	1.05 1.15	0.075 0.140
Continuous Bridges:			
PC Girder continuous for live load only	Positive Moment Negative Moment Shear	1.05 1.14 1.15	0.075 0.130 0.140
PSC Post-tensioned Voided Slab	Positive Moment Negative Moment Shear	1.07 1.07 1.15	0.065 0.065 0.140
PSC Post-tensioned Box Girder	Positive Moment Negative Moment Shear	1.07 1.07 1.15	0.065 0.065 0.140

[CSA S6-06 Calibration Report]

Table 4.1/ Statistical Parameters for Dead Loads

DEAD LOAD TYPE	DESCRIPTION	BIAS FACTOR δ_D	COEFFICIENT OF VARIATION, V_D
D1	factory-produced components, excluding wood	1.03	0.08
D2	cast-in-place concrete, wood and all non-structural components	1.05	0.10
D3	asphalt wearing surface	1.03	0.30

Table 4.2.3 *Statistical Parameters for Live Load Analysis*

ANALYSIS TYPE	BIAS FACTOR, δ_{AL}	COV, V_{AL}
Simplified Code Methods	0.93	0.12
Sophisticated Analysis	0.98	0.07

Table 4.3 *Statistical Parameters for Dynamic Loads*

	MEAN VALUE μ_I	STANDARD DEVIATION σ_I	COV V_I	DYNAMIC AMPLIFICATION FACTOR	COV FOR DYNAMIC AMPLIFICATION
Span ≤ 10 m, and Local Components	$0.20\ \mu_L$	$0.12\ \mu_L$	0.60	1.20	0.100
Span > 10 m:					
One Lane Loaded	$0.15\ \mu_L$	$0.12\ \mu_L$	0.80	1.15	0.104
Two Lanes Loaded	$0.10\ \mu_L$	$0.08\ \mu_L$	0.80	1.10	0.072

Example 2

“dynamic load allowance”
(aka “impact factor”)

$$M_r \geq M_f$$

$$\phi M_p \geq \alpha_L \cdot M_L \cdot (1+DLA) + \alpha_D \cdot M_D$$

Let:

- M_r = 1000 kN·m
- $\phi = 0.95$
- $\alpha_L = 1.7$
- $DLA = 0.25$
- $\alpha_D = 1.1$
- M_L = 286 kN·m
- M_D = 357 kN·m

Note: To create this example, M_r, M_L, and M_D were selected to create a problem where M_r and M_f are exactly equal, to show what probability of failure results in this case.

*annual extreme load event

Bias factors:

factor	$\bar{Z}$	COV_Z	Uncertainty in:
Z_1	1.06	0.051	F_y
Z_2	0.99	0.021	S
Z_3	1.09	0.045	resistance model
Z_4	1.03	0.08	M_D
Z_5	1.367	0.0388	M_L*
Z_6	0.93	0.12	analysis model
Z_7	0.15	0.8	DLA

CIVE 512 - Reliability Aspects - 2020 [Compatibility Mode] - Excel

A1 Z

Z	Modifies	Zbar	COV	STDEV	Dist.
Z1	Mp	1.06000	0.05100	0.05406	Normal
Z2	Mp	0.99000	0.02100	0.02079	Normal
Z3	Mp	1.09000	0.04500	0.04905	Normal
Z4	MD	1.03000	0.08000	0.08240	Normal
Z5	ML	1.36700	0.03880	0.05304	Normal
Z6	ML	0.93000	0.12000	0.11160	Normal
Z7	DLA	0.15000	0.80000	0.12000	Normal

Mr	1000
Mp	1053
MD	357
ML	286

n	2
N	10000
pf	2.00E-04
beta	3.54008

These values are close to target values used in the code calibration!!

U1	U2	U3	U4	U5	U6	U7	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Mr	Ms	G	Failure?
0.238028	0.448929	0.322636	0.765755	0.747585	0.054781	0.759809	1.021474	0.987331	1.06742	1.089735	1.402373	0.751422	0.234683	1133.188	761.143	372.0448	0
0.776279	0.19442	0.181199	0.271464	0.84995	0.889367	0.439176	1.101069	0.972085	1.045325	0.979869	1.421961	1.066506	0.131633	1177.732	840.6333	337.0983	0
0.24783	0.768972	0.750787	0.357258	0.569626	0.620076	0.986144	1.023167	1.00529	1.123205	0.999858	1.376304	0.964114	0.41416	1216.112	893.6195	322.4926	0
0.137406	0.83985	0.627272	0.896812	0.172838	0.553154	0.707758	1.000964	1.010662	1.105923	1.13412	1.316983	0.944914	0.215622	1177.676	837.5309	340.145	0
0.014152	0.838369	0.904395	0.762157	0.245102	0.223605	0.297825	0.941443	1.010536	1.154109	1.088772	1.330404	0.845176	0.08632	1155.763	738.0366	417.7268	0
0.609655	0.278367	0.897607	0.830902	0.757707	0.899785	0.866186	1.075051	0.977782	1.152197	1.108917	1.404072	1.072885	0.283025	1274.895	948.6523	326.2423	0
0.299455	0.363509	0.005126	0.988188	0.662463	0.273546	0.231259	1.031566	0.982742	0.964077	1.216487	1.389234	0.862803	0.061835	1028.785	798.2933	230.492	0
0.460548	0.175681	0.236123	0.679537	0.618996	0.82997	0.846642	1.054645	0.970625	1.054741	1.068432	1.383063	1.036472	0.272657	1136.528	903.1972	233.3308	0
0.277788	0.809814	0.039168	0.648404	0.022125	0.000548	0.463908	1.028136	1.008237	1.003651	1.061396	1.2603	0.56568	0.139129	1095.147	611.1832	483.9635	0
0.703944	0.640883	0.814107	0.018391	0.555223	0.365862	0.464242	1.088964	0.997501	1.133808	0.857935	1.374366	0.89174	0.13923	1296.412	705.5996	590.8126	0
0.130832	0.496375	0.662767	0.058531	0.564855	0.855033	0.226546	0.99932	0.989811	1.110602	0.90086	1.375661	1.048102	0.059968	1156.356	758.7001	397.6562	0
0.867951	0.100496	0.376127	0.104181	0.004979	0.830442	0.021612	1.120372	0.963415	1.074516	0.926334	1.230302	1.03668	-0.09259	1220.858	661.7014	559.1566	0
0.595155	0.173169	0.023821	0.293952	0.854828	0.970262	0.051369	1.073019	0.970422	0.992854	0.98535	1.423082	1.140328	-0.04581	1088.253	794.6257	293.627	0
0.603306	0.22518	0.441386	0.598237	0.206094	0.310083	0.730568	1.074159	0.974307	1.082767	1.0505	1.323505	0.874689	0.223744	1192.823	780.1972	412.626	0
0.251116	0.447581	0.75338	0.317068	0.723769	0.145761	0.801391	1.023727	0.98726	1.123607	0.990785	1.398509	0.812286	0.251592	1195.382	760.3436	435.0385	0
0.68474	0.983807	0.669666	0.11918	0.844155	0.844877	0.096823	1.086003	1.034483	1.111533	0.932843	1.420659	1.043241	-0.00598	1314.476	754.366	560.1099	0

Example 1 | Example 1 - MCS | Example 2 - MCS

What has all of this got to do with assessment of structures?

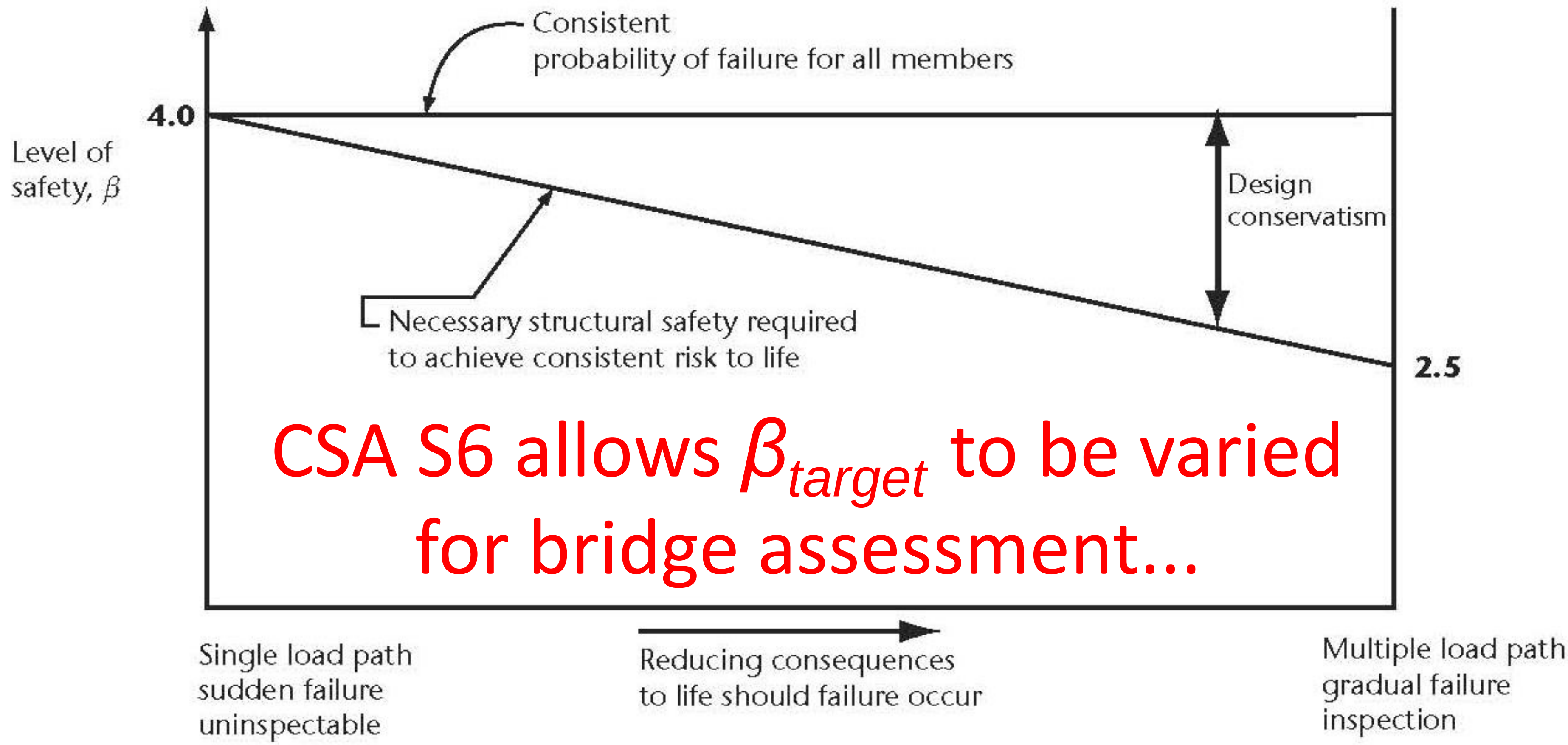


Figure C14.1
Relationship between risk and probability of failure
(See Clause C14.12.1.)

[CSA S6]

…this means load factors can be adjusted favourably in some cases:

Table 14.7
Maximum dead load factors, α_D
(See Clause 14.13.2.1.)

	Target Reliability Index, β								
Dead load category	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00
D1	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10	1.11
D2	1.06	1.08	1.10	1.12	1.14	1.16	1.18	1.20	1.22
D3	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55

Table 14.8
Live load factors, α_L, for normal traffic (Evaluation Levels 1, 2, and 3) for all types of analysis
(See Clause 14.13.3.1.)

	Target reliability index, β						
Spans	2.50	2.75	3.00	3.25	3.50	3.75	4.00
All Spans	1.35	1.42	1.49	1.56	1.63	1.70	1.77

[CSA S6]