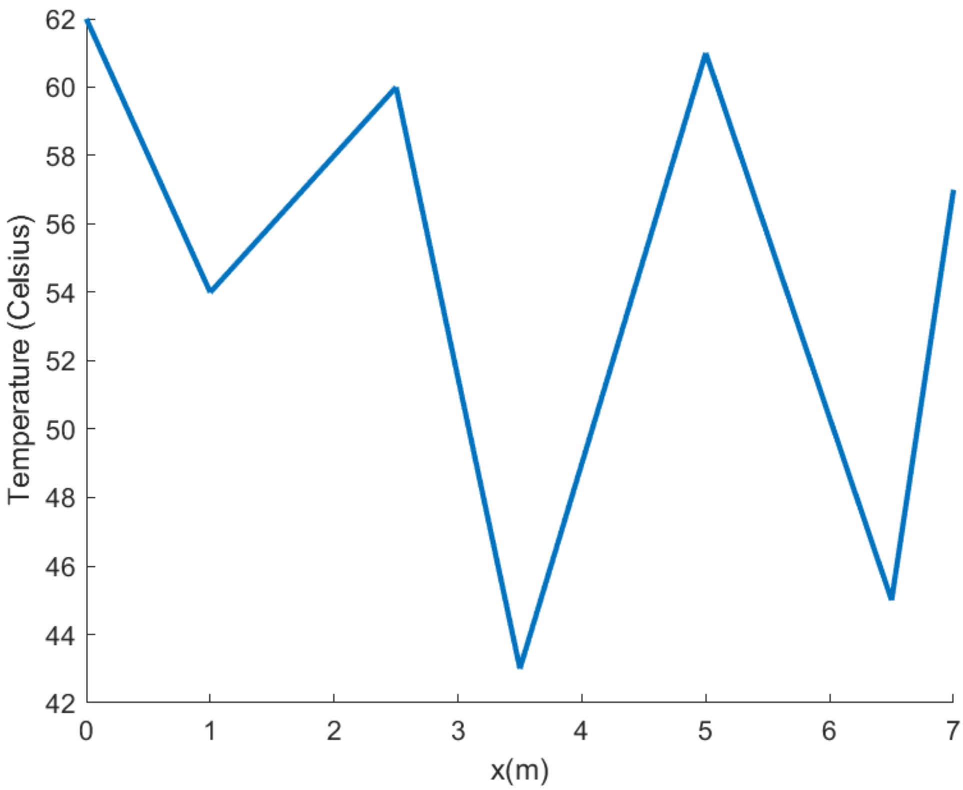
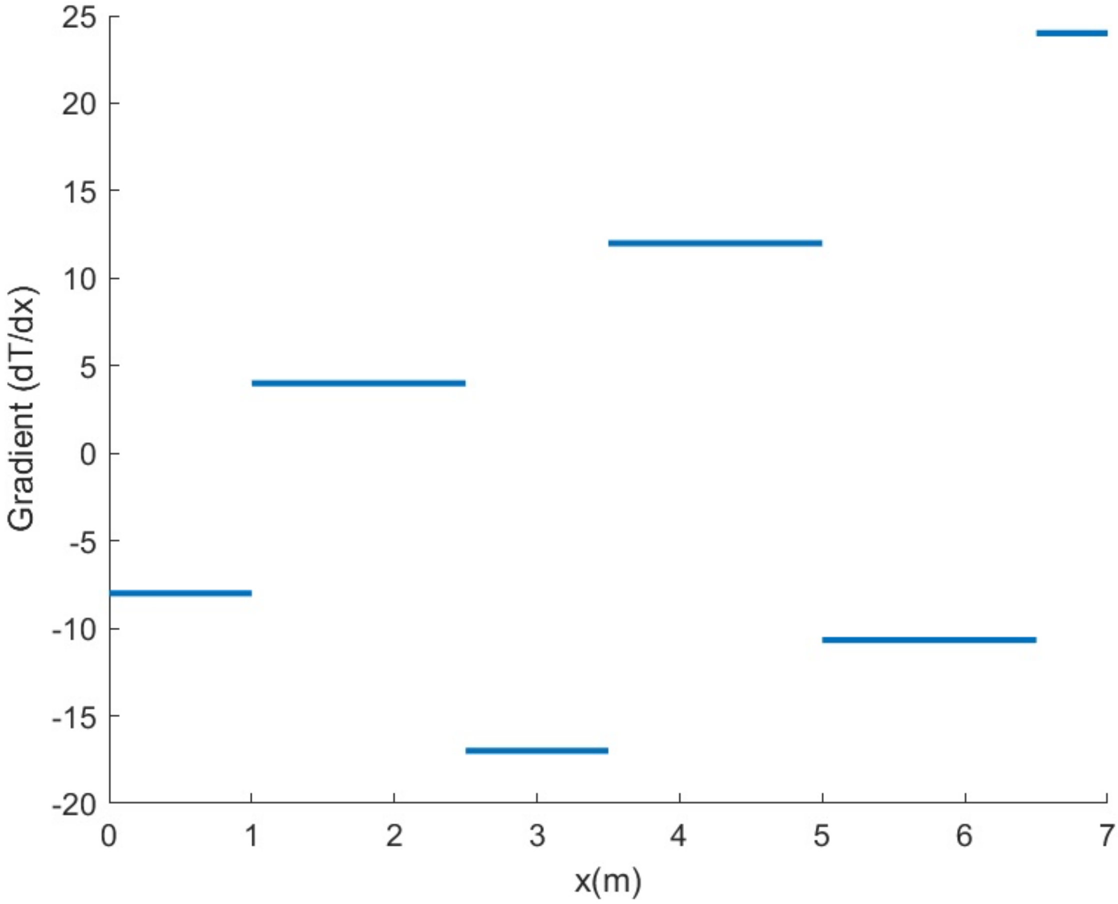
Temperature Approximation (1-Part 1)



Gradient Plot (1-Part 2)




Text

**Question 1 - Parts 3 & 4**

ENGINEERING INC.

Project No.

Page of

Date Project Title

Designer Description

3. $\underset{\sim}{T}^h = \underset{\sim}{N}_e \underset{\sim}{d}_e$

$$\underset{\sim}{N}_4^e(3.5) = \frac{(x-5)}{(3.5-5)} \qquad , \quad \underset{\sim}{N}_7^e(5) = \frac{(x-3.5)}{5-3.5}$$

$$= \frac{-2}{3}x + \frac{10}{3} \qquad\qquad = \frac{2}{3}x - 7/3$$

$$\underset{\sim}{N}_4^e(4.5) = \frac{-2}{3}(4.5) + 10/3 \qquad \underset{\sim}{N}_7^e(4.5) = 2/3(4.5) - 7/3$$

$$= 1/3 \qquad\qquad = 2/3$$

$$T^h = \begin{bmatrix} 1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 43 \\ 61 \end{bmatrix}$$

$$= 55°$$

4. $\frac{d(N_4^e)}{dx} = -2/3 \quad , \quad d(N_7^e) = 2/3$

$$\frac{dT^h}{dx} = \begin{bmatrix} -2/3 & 2/3 \end{bmatrix} \begin{bmatrix} 43 \\ 61 \end{bmatrix}$$

$$= 12$$

# Question 2 - Part a

WitzelDyce ENGINEERING INC.

Project No. ______

Page ______ of ______

Date ______ Project Title ______

Designer ______ Description ______

2. a) $$\underset{\sim}{K}^e = \int_{x_1^e}^{x_2^e} B_e^T A E \underset{\sim}{B}^e \, dx$$

$$= \int_{x_1^e}^{x_2^e} \begin{bmatrix} \frac{dN_1}{dx} \\ \frac{dN_2}{dx} \end{bmatrix} [\alpha_1 + \alpha_2 x] E \left[ \frac{dN_1}{dx} \quad \frac{dN_2}{dx} \right] dx$$

$$= \int_{x_1^e}^{x_2^e} \begin{bmatrix} 1/h_e \\ -1/h_e \end{bmatrix} [\alpha_1 + \alpha_2 x] E \left[ 1/h_e \quad -1/h_e \right] dx$$

$$= \int_{x_1^e}^{x_2^e} \begin{bmatrix} 1/h_e \\ -1/h_e \end{bmatrix} \left[ \frac{E}{h_e}(\alpha_1 + \alpha_2 x) \quad \frac{-E}{h_e}(\alpha_1 + \alpha_2 x) \right] dx$$

$$= \int_{x_1^e}^{x_2^e} \begin{bmatrix} \frac{E}{h_e^2}(\alpha_1 + \alpha_2 x) & \frac{-E}{h_e}(\alpha_1 + \alpha_2 x) \\ \frac{-E}{h_e^2}(\alpha_1 + \alpha_2 x) & \frac{E}{h_e^2}(\alpha_1 + \alpha_2 x) \end{bmatrix} dx$$

$$= \frac{E}{h_e^2} \int_{x_1^e}^{x_2^e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} (\alpha_1 + \alpha_2 x) \, dx$$

$$= \frac{E}{h_e^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \left( \alpha_1 x + \frac{\alpha_2 x^2}{2} \Big|_{x_1^e}^{x_2^e} \right)$$

$$= \frac{E}{h_e^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \left[ \alpha_1 (x_2^e - x_1^e) + \frac{\alpha_2}{2} \left( {x_2^e}^2 - {x_1^e}^2 \right) \right]$$

**Question 2 - End of Part a and Part b-1**



$$\frac{E}{h_e^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\left[d_1(h_e) + \frac{d_2}{2}\left({x_2^e}^2 - {x_1^e}^2\right)\right]$$

b) 1) $f_\Omega^e = \int_{x_1^e}^{x_2^e} \underline{N_e}^T b(x)\,dx$ , $N_1 = \frac{-(x-x_2)}{(x_2-x_1)}$, $N_2(x) = \frac{(x-x_1)}{(x_2-x_1)}$

$$\frac{1}{(x_2-x_1)}\int_{x_1^e}^{x_2^e}\begin{bmatrix} x_2 - x \\ x - x_1 \end{bmatrix} 4E3\,x^2\,dx$$

$$\frac{4E3}{x_2-x_1}\int_{x_1^e}^{x_2^e}\begin{bmatrix} x_2x^2 - x^3 \\ x^3 - x_1x^2 \end{bmatrix}dx$$

$$\frac{4E3}{x_2-x_1}\begin{bmatrix} \frac{x_2x^3}{3} - \frac{x^4}{4} \\ \frac{x^4}{4} - x_1x^3/3 \end{bmatrix}\Bigg|_{x_1^e}^{x_2^e}$$

$$\frac{4E3}{x_2-x_1}\begin{bmatrix} x_2^4/3 - x_2^4/4 - x_2x_1^3/3 + x_1^4/4 \\ \frac{x_2^4}{4} - x_1x_2^3/3 - x_1^4/4 + x_1^4/3 \end{bmatrix},$$ subbing $x_1 = 0.1$, $x_2 = 0.2$

$$\frac{4E3}{0.1}\begin{bmatrix} 0.2^4/3 - 0.2^4/4 - 0.2(0.1)^3/3 + 0.1^4/4 \\ 0.2^4/4 - 0.1\cdot0.2^3/3 - 0.1^4/4 + 0.1^4/3 \end{bmatrix}$$

$$f_\Omega^e = \begin{bmatrix} 3.67 \\ 5.67 \end{bmatrix} N$$

Question 2 b-2

Now, approximating $b(x)$

$2\times1 \times \cancel{4}\ 1\times2$

$$\underset{\sim}{b}^e = \begin{bmatrix} 4000\times(0.1)^2 \\ 4000\times(0.2)^2 \end{bmatrix} = \begin{bmatrix} 40 \\ 160 \end{bmatrix}$$

$$b(x)^h = \frac{x-0.2}{-0.1}\times 40 + \frac{x-0.1}{0.1}\times 160$$

$$\underset{\sim}{f}_\Omega^e = \int_{x_1^e}^{x_2^e} N_e^T\, b(x)\, dx$$

$$= 10\int \underset{\sim}{N}_e^T \left(160(x-0.1) - 40(x-0.2)\right) dx$$

$$= 10\int_{x_1^e}^{x_2^e} \underset{\sim}{N}_e^T \left[160x - 16 - 40x + 8\right] dx$$

$$= 10\int_{x_1^e}^{x_2^e} \underset{\sim}{N}_e^T (120x - 8)\, dx \qquad \underset{\sim}{N}_e^T = 10\left[0.2-x,\ x-0.1\right]^T$$

$$= 100\int_{x_1^e}^{x_2^e} \begin{bmatrix} -120x^2 + 8x + 24x - 1.6 \\ 120x^2 - 12x - 8x + 0.8 \end{bmatrix} dx = 100\int_{x_1^e}^{x_2^e} \begin{matrix} -120x^2 + 32x - 1.6 \\ 120x^2 - 20x + 0.8 \end{matrix}\, dx$$

$$= \begin{bmatrix} 100(-40x^3 + 16x^2 - 1.6x)\Big|_{0.1}^{0.2} \\ 100(40x^3 - 10x^2 + 0.8x)\Big|_{0.1}^{0.2} \end{bmatrix}$$

$$\underset{\sim}{f}^e = \begin{bmatrix} 100\times0.04 \\ 100\times0.06 \end{bmatrix} = \boxed{\begin{bmatrix} 4N \\ 6N \end{bmatrix} = \underset{\sim}{f}_\Omega^e}$$

The answers were close, with 1/3N as error. They ended up being whole numbers rounded up.

3. $D\dfrac{d^2C}{dr^2} - \dfrac{v}{r}\dfrac{dC}{dr} + s(r) = 0, \quad a \le r \le b$

1. Multiply each component by $w(r)$

$$D\,w(r)\frac{d^2C}{dr^2} - w(r)\frac{dC}{dr} + w(r)\,s(r) = 0$$

2. Integrate over domain

$$\underbrace{\int_a^b D\,w(r)\frac{d^2C}{dr^2}\,dr}_{\boxed{A}} - \underbrace{\int_a^b w(r)\frac{v}{r}\frac{dC}{dr}\,dr}_{\boxed{B}} + \underbrace{\int_a^b w(r)s(r)\,dr}_{\boxed{C}} = 0$$

$\boxed{A}$ $\dfrac{d\left[\frac{dC}{dr}\right]}{dr} = \dfrac{d^2C}{dr^2}$

$$d\left(\frac{dC}{dr}\right) = \frac{d^2C}{dr^2}\,dr$$

$$\int_a^b D\,w(r)\,d\left(\frac{dC}{dr}\right)$$

$$= D\,w(r)\frac{dC}{dr}\Big|_a^b - \int_a^b D\frac{dw}{dr}\frac{dC}{dr}\,dr$$

$\therefore$ We have $D\,w(r)\dfrac{dC}{dr}\Big|_a^b - \displaystyle\int_a^b D\frac{dw}{dr}\frac{dC}{dr}\,dr - \int_a^b w\frac{v}{r}\frac{dC}{dr}\,dr + \int_a^b w s\,dr = 0,$

let $w(b) = w(a) = 0$

Weak form $\rightarrow$

$\therefore$ $0 = -\displaystyle\int_a^b D\frac{dw}{dr}\frac{dC}{dr}\,dr - v\int_a^b \frac{1}{r}w\frac{dC}{dr}\,dr + \int_a^b w s\,dr, \quad \forall w,\ w(a)=0,\ w(b)=0$

b) $w(r) = N^e(r)\, w^e$ ; $\frac{dw}{dr} = \frac{dN^e(r)}{dr} w^e = B^e w^e = w^{e^T} B^{e^T}$

$c(r) = N^e(r)\, d$ ; $\frac{dc}{dr} = B^e d$

$$0 = \sum_1^{n_e} \left\{ -\int_{x_1^e}^{x_2^e} D \frac{dw}{dr}\frac{dc}{dr}\, dr - v\int_{x_1^e}^{x_2^e} \frac{1}{r} w \frac{dc}{dr}\, dr + \int_{x_1^e}^{x_2^e} w s\, dr \right.$$

$$0 = \sum_1^{n_e} \left\{ -\int_{x_1^e}^{x_2^e} D\, w^{e^T} B^{e^T} B^e d\, dr - v\int_{x_1^e}^{x_2^e} w^{e^T} \frac{1}{r} N^{e^T} B^e d\, dr + \int_{x_1^e}^{x_2^e} w^{e^T} B^{e^T} s\, dr \right\}$$

$$0 = \sum_1^{n_e} w^{e^T} \left\{ \underbrace{-\int_{x_1^e}^{x_2^e} D B^{e^T} B^e d\, dr}_{K_1} \underbrace{- v\int_{x_1^e}^{x_2^e} \frac{1}{r} N^{e^T} B^e d\, dr}_{K_2} + \underbrace{\int_{x_1^e}^{x_2^e} B^{e^T} s\, dr}_{f} \right\}$$

$$N_e = \left[ \frac{r - x_2}{x_1^e - x_2^e} \;,\; \frac{r - x_1^e}{x_2^e - x_1^e} \right] = \frac{1}{h_e}\left[ x_2^e - r \;,\; r - x_1^e \right]$$

$$B^e = \frac{d}{dr} = \frac{1}{h_e}\begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$B^{e^T} = \frac{1}{h_e}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$N^{e^T} = \frac{1}{h_e}\begin{bmatrix} x_2^e - r \\ r - x_1^e \end{bmatrix}$$

$$0 = \sum_{1}^{ne} w^{eT} \left\{ \frac{-D}{h_e^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} d \Big|_{x_1^e}^{x_2^e} - \frac{v}{h_e^2} \int_{x_1^e}^{x_2^e} \frac{1}{r} \begin{bmatrix} x_2^e - r \\ r - x_1^e \end{bmatrix} [-1 \quad 1] \, d\,dr + \int_{x_1^e}^{x_2^e} B^{eT} s\,dr \right\}$$

$$0 = \sum_{1}^{ne} w^{eT} \left\{ \underbrace{\frac{-D}{h_e^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} d \Big|_{x_1^e}^{x_2^e}}_{K^1} - \underbrace{\frac{v}{h_e^2} \int_{x_1^e}^{x_2^e} \frac{1}{r} \begin{bmatrix} r - x_2^e & x_2^e - r \\ x_1^e - r & r - x_1^e \end{bmatrix} d\,dr}_{K^2} + \underbrace{\int_{x_1^e}^{x_2^e} B^{eT} s\,dr}_{f} \right\}$$

Looking at $K^2$:

$$\frac{-v}{h_e^2} \int_{x_1^e}^{x_2^e} \begin{bmatrix} 1 - x_2^e/r & x_2^e/r - 1 \\ x_1^e/r - 1 & 1 - x_1^e/r \end{bmatrix} d\,dr$$

$$\frac{-v}{h_e^2} \begin{bmatrix} r - x_2^e \ln r & x_2^e \ln r - r \\ x_1^e \ln r - r & r - x_1^e \ln r \end{bmatrix} \Bigg|_{x_1^e}^{x_2^e}$$

$$\frac{-v}{h_e^2} \begin{bmatrix} x_{2e} - x_{2e} \ln x_{2e} - x_{1e} + x_{1e} \ln x_{1e} & x_{2e} \ln x_{2e} - x_{2e} - x_{2e} \ln x_{1e} + x_{1e} \\ x_{1e} \ln x_{2e} - x_{2e} + x_{1e} - x_{1e} \ln x_{1e} & x_{2e} - x_{1e} - x_{1e} \ln x_{2e} + x_{1e} \ln x_{1e} \end{bmatrix}$$

Looking at $K^1$

$$\frac{-D}{h_e^2} \begin{bmatrix} x_{2e} - x_{1e} & x_{1e} - x_{2e} \\ x_{1e} - x_{2e} & x_{2e} - x_{1e} \end{bmatrix}$$

WitzelDyce
ENGINEERING INC.

Project No. ______

Page ______ of ______

Date ______ Project Title ______

Designer ______ Description ______

$$\therefore Kd = f + r = \sum_{e=1}^{ne} \left\{ K^1 + K^2 \right\} = \sum_{e=1}^{ne} \left\{ \int_{x_1^e}^{x_2^e} \frac{1}{h^e} \begin{bmatrix} -1 \\ 1 \end{bmatrix} s(x) dx + r \right\}, \quad r = \begin{bmatrix} r_a \\ 0 \\ 0 \\ \vdots \\ 0 \\ r_b \end{bmatrix}$$

as shown on the first page

only $r_a$ and $r_b$ are unknown

$, \forall w, w(a)=0, w(b)=0$