Lecture 2A – Key Messages

- Load and resistance factors in structural design codes are calibrated to achieve a target probability of failure or uniform risk level, taking into account the various sources of uncertainty in the design equation.
- For design problem with one load and one resistance parameter, probability of failure (P_f) can be calculated using equation.
- P_f is also the product of the load PDF and resistance CDF ordinates integrated for all values of load/resistance parameter.

Limit states design

The various functional requirements for civil structures can generally be grouped into those related to the safety of the structure from collapse and those related to the proper function or serviceability of the structure over the course of its useful life. The onsets of various types of failure are often referred to as limit states.

-NBCC Structural Commentary 1995



Limit states design

The limit states design process includes the following steps:

- identifying significant potential failure modes or "limit states"
- determining acceptable levels of safety
- verifying the structure with respect to the identified limit states

The various limit states are normally divided into two (2) types:

- Ultimate Limit States — i.e. rupture, instability, overturning, progressive collapse
- Serviceability Limit States — i.e. excessive deflections, vibrations, crack widths

Limit states design

Verification of ultimate limit states:

– Factored "Resistance" ≥ Factored "Solicitation" (or "Load Effect").

– $\phi \cdot R_n \geq \alpha_1 \cdot S_1 + \alpha_2 \cdot S_2 + \ldots$

where: R_n is nominal resistance

S_i are load effects from various sources

ϕ_i is a resistance factor (normally < 1)

α_i are load factors (normally > 1)

Limit states design

Uncertainties in resistance:

Resistance factors (ϕ) handle these...

Inherent variability in material strength

Simplified resistance models

Geometric variability

Uncertainties in loads or "solicitations":

Simplified structural analysis models

Simplified load models

Inherent variability in loads

Load factors (α) handle these...

The National Building Code of Canada ultimate limit state verification (see *[NBCC 2005]* or *[CSA A23.3 Annex C]*) includes the following five load combinations: (for building structures)

	Load Combinations	
Load Case	Principal Loads	Companion Loads
1	Load factor (α) 1.4·D	–
2	(1.25 or 0.9)·D + 1.5·L	0.5·S or 0.4·W
3	(1.25 or 0.9)·D + 1.5·S	0.5·L or 0.4·W
4	(1.25 or 0.9)·D + 1.4·W	0.5·L or 0.5·S
5	1.0·D + 1.0·E	0.5·L + 0.25·S

where: D = "dead", L = "live", "S" = snow, W = "wind", and E = "earthquake"

α_D = 1.25 if dead load effect is detrimental or 0.9 if it is beneficial

Load effects from the following additional sources should also be considered when applicable:

– thermal strain, shrinkage, creep (T, α_T = 1.25)

– lateral earth pressures (H, α_H = 1. 5)

– prestressing (P, α_P = 1.0)

Table 3.1
Load factors and load combinations

(See Clauses 3.5.1, 3.10.1.1, 3.10.5.2, 3.13, 3.16.3, 4.10.7, 4.10.10.1, 7.6.3.1.1, 7.7.3.1.1, 9.4.2, and 15.6.2.4.)

(for bridges)

	Permanent loads			Transitory loads					Exceptional loads			
Loads	*D*	*E*	*P*	*L**	*K*	*W*	*V*	*S*	*EQ*	*F*	*A*	*H*
Fatigue limit state												
FLS Combination 1	1.00	1.00	1.00	1.00	0	0	0	0	0	0	0	0
Serviceability limit states												
SLS Combination 1	1.00	1.00	1.00	0.90	0.80	0	0	1.00	0	0	0	0
SLS Combination 2†	0	0	0	0.90	0	0	0	0	0	0	0	0
Ultimate limit states‡												
ULS Combination 1	α_D	α_E	α_P	1.70	0	0	0	0	0	0	0	0
ULS Combination 2	α_D	α_E	α_P	1.60	1.15	0	0	0	0	0	0	0
ULS Combination 3	α_D	α_E	α_P	1.40	1.00	0.50§	0.50	0	0	0	0	0
ULS Combination 4	α_D	α_E	α_P	0	1.25	1.65§	0	0	0	0	0	0
ULS Combination 5	α_D	α_E	α_P	0	0	0	0	0	1.00	0	0	0
ULS Combination 6**	α_D	α_E	α_P	0	0	0	0	0	0	1.30	0	0
ULS Combination 7	α_D	α_E	α_P	0	0	0.90§	0	0	0	0	1.30	0
ULS Combination 8	α_D	α_E	α_P	0	0	0	0	0	0	0	0	1.00
ULS Combination 9	1.35	α_E	α_P	0	0	0	0	0	0	0	0	0

[CSA S6] ← Canadian Highway Bridge Design Code

*For the construction live load factor, see Clause 3.16.3.

†For superstructure vibration only.

‡For ultimate limit states, the maximum or minimum values of α_D, α_E, and α_P specified in Table 3.2 shall be used.

§For wind loads determined from wind tunnel tests, the load factors shall be as specified in Clause 3.10.5.2.

**For long spans, it is possible that a combination of ice load F and wind load W will require investigation.

Legend:

A	=	ice accretion load
D	=	dead load
E	=	loads due to earth pressure and hydrostatic pressure, including surcharges but excluding dead load
EQ	=	earthquake load
F	=	loads due to stream pressure and ice forces or to debris torrents
H	=	collision load arising from highway vehicles or vessels
K	=	all strains, deformations, and displacements and their effects, including the effects of their restraint and the effects of friction or stiffness in bearings. Strains and deformations include strains and deformations due to temperature change and temperature differential, concrete shrinkage, differential shrinkage, and creep, but not elastic strains
L	=	live load (including the dynamic load allowance, when applicable)
P	=	secondary prestress effects
S	=	load due to differential settlement and/or movement of the foundation
V	=	wind load on traffic
W	=	wind load on structure

[CSA S6]

Table 3.2
Permanent loads — Maximum and minimum values of load factors for ULS

(See Clauses 3.5.1, 3.5.2.1, 4.4.1, 4.4.9.3, and 7.8.7.1 and Table 3.1.)

Dead load	**Maximum α_D**	**Minimum α_D**
Factory-produced components, excluding wood	1.10	0.95
Cast-in-place concrete, wood, and all non-structural components	1.20	0.90
Wearing surfaces, based on nominal or specified thickness	1.50	0.65
Earth fill, negative skin friction on piles	1.25	0.80
Water	1.10	0.90
Dead load in combination with earthquakes	**Maximum α_D**	**Minimum α_D**
All dead loads for ULS Combination 5 (see Table 3.1)	1.25	0.80
Earth pressure and hydrostatic pressure	**Maximum α_E**	**Minimum α_E**
Passive earth pressure, considered as a load*	1.25	0.50
At-rest earth pressure	1.25	0.80
Active earth pressure	1.25	0.80
Backfill pressure	1.25	0.80
Hydrostatic pressure	1.10	0.90
Prestress	**Maximum α_P**	**Minimum α_P**
Secondary prestress effects	1.05	0.95

**When passive earth pressure is considered as a resistance, it is factored in accordance with Section 6.*

[CSA S6]

Resistance factors (concrete)

The Canadian Concrete Code resistance factors *[CSA A23.3 §8.4]* can be summarized as follows:

– For the concrete, $\phi_c = 0.65$ Resistance factor (ϕ)

– For the reinforcing steel, $\phi_s = 0.85$

– For steel-concrete composite structures or pretensioned structures, $\phi_a = \phi_p = 0.90$.

Resistance factors (steel)

The Canadian Steel Structures Design Code resistance factors *[CSA S16 §13.1]* can be summarized as follows:

- structural steel, $\phi = 0.90$ and $\phi_u = 0.75$
- bolts, $\phi_b = 0.80$
- weld metal, $\phi_w = 0.67$
- steel reinforcing bars (rebar), $\phi_r = 0.85$
- concrete, $\phi_c = 0.65$
- shear connectors, $\phi_{sc} = 0.80$
- beam web bearing, interior, $\phi_{bi} = 0.80$
- beam web bearing, end, $\phi_{be} = 0.75$
- bearing of bolts on steel, $\phi_{br} = 0.80$
- anchor rods, $\phi_{ar} = 0.67$

Resistance factors (bridges)

Table 8.1
Material resistance factors
(See Clause 8.4.6.)

Concrete

Material	Material resistance factor
Concrete	$\phi_c = 0.75$
Reinforcement	
Reinforcing bars, wire, and wire fabric	$\phi_s = 0.90$
Prestressing strands	$\phi_p = 0.95$
High-strength bars	$\phi_p = 0.90$
Anchor bolts and studs	In accordance with Section 10

[CSA S6]

10.5.7 Resistance factors

Resistance factors shall be taken as follows:

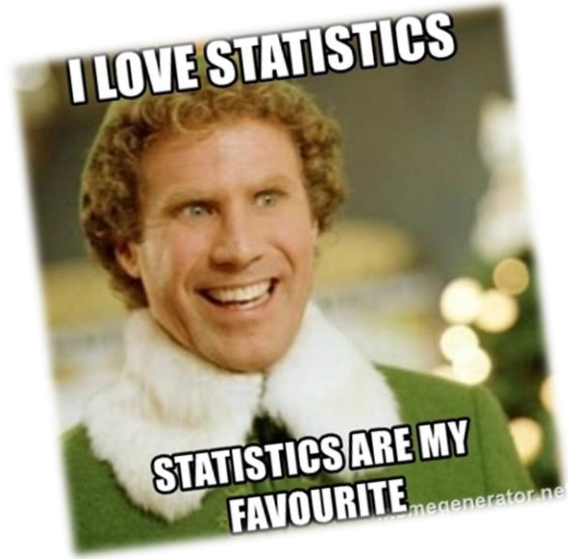
(a) flexure: $\phi_s = 0.95$;
(b) shear: $\phi_s = 0.95$;
(c) compression: $\phi_s = 0.90$;
(d) tension: $\phi_s = 0.95$;
(e) torsion: $\phi_s = 0.90$;
(f) tension in cables: $\phi_{tc} = 0.55$;
(g) reinforcing steel in composite construction: $\phi_r = 0.90$;
(h) concrete in composite construction: ϕ_c as specified in Section 8;
(i) bolts: $\phi_b = 0.80$;
(j) load bearing in bolted connections: $\phi_{br} = 0.67$;
(k) welds: $\phi_w = 0.67$;
(l) shear connectors: $\phi_{sc} = 0.85$;
(m) beam web bearing, interior: $\phi_{bi} = 0.80$; and
(n) beam web bearing, end: $\phi_{be} = 0.75$.

Steel

[CSA S6]

Where do load and resistance factors come from?

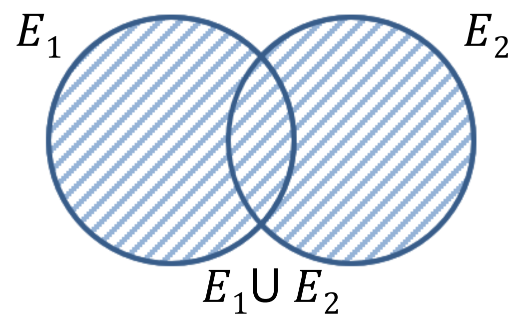
Statistics Review



P(E) is the probability of an event.

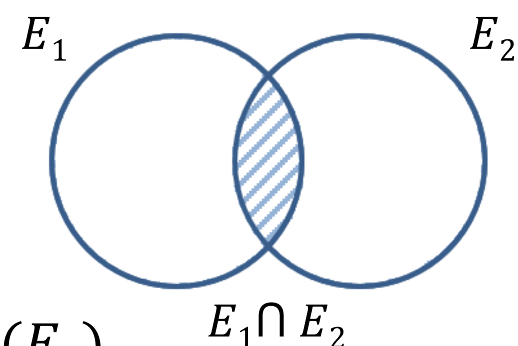
$$0 \leq P(E) \leq 1$$

The probability of 2 events, E_1 and E_2



The probability that either E_1 or E_2 or both occur:

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$



The probability that both E_1 or E_2 occur together:

$$P(E_1 \cap E_2) = P(E_1 | E_2) \cdot P(E_2)$$

If E_1 and E_2 are independent, then: $P(E_1 | E_2) = P(E_1)$

The probability of an event not occurring:

$$P(\overline{E_1}) = 1 - P(E_1)$$

The conditional probability:

$$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

Example: 6-sided dice.

$$P(rolling\ a\ 6) = \frac{1}{6}$$

$$P(rolling\ something\ else) = 1 - \frac{1}{6} = \frac{5}{6}$$

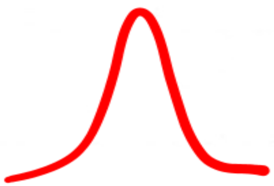
$$P(rolling\ 2 - 6s\ in\ a\ row) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

Continuous random variables

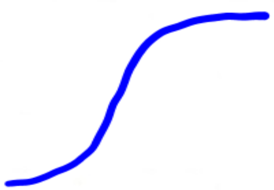
- Strength of material, dead load, live load, etc.

- Can be described by various statistical distributions.

- Statistical distributions can be depicted graphically with PDFs and CDFs:



Probability density function, $PDF = f_X(x)$

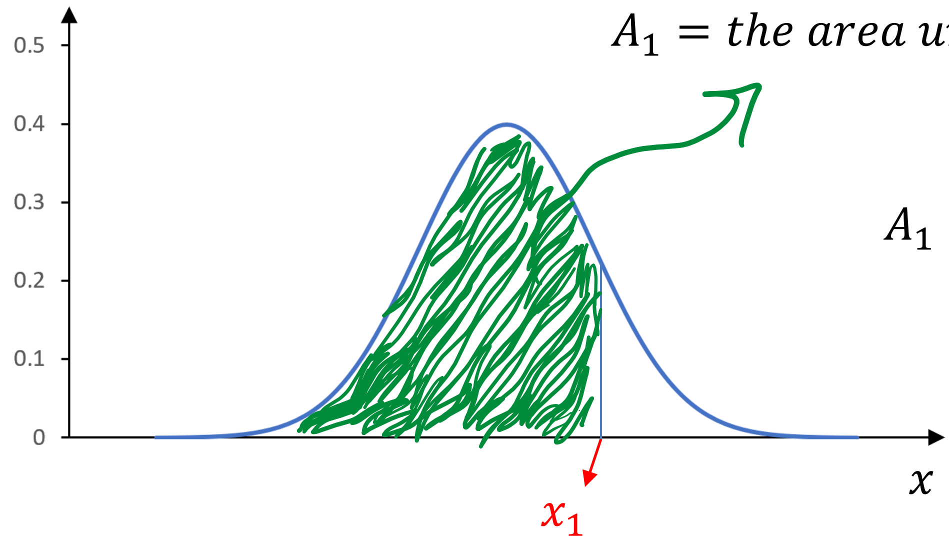


Cumulative density function, $CDF = P(X \leq x) = F_X(x)$

-The two curves are related: $F_X(x) = \int_{-\infty}^{x} f_X(u)du$

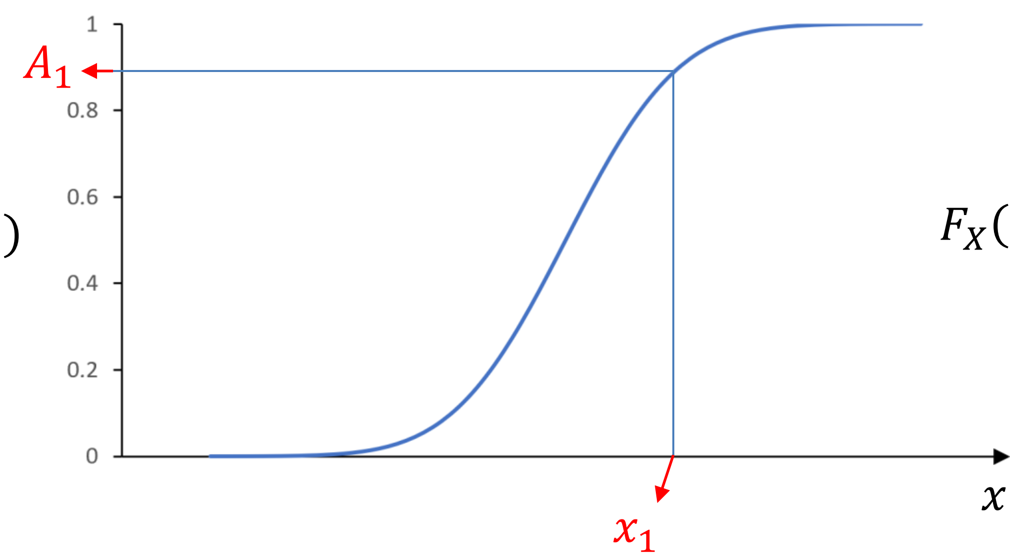
$A_1 = the\ area\ under\ PDF\ from\ -\infty\ to\ x_1$

PDF
(or $f_X(x)$)



$$A_1 = \int_{-\infty}^{x_1} f_X(x)dx$$

CDF
(or $F_X(x)$)



$$F_X(x) = \int_{-\infty}^{x} f_X(u)du$$

Moments of distributions

- Statistical distributions can also be described by their "moments":

1) Mean $, E(x) = \mu_x = \int_{-\infty}^{+\infty} x \cdot f_X(x) dx$

2) Variance $, Var(x) = \int_{-\infty}^{+\infty} (x - \mu_x)^2 \cdot f_X(x) dx$

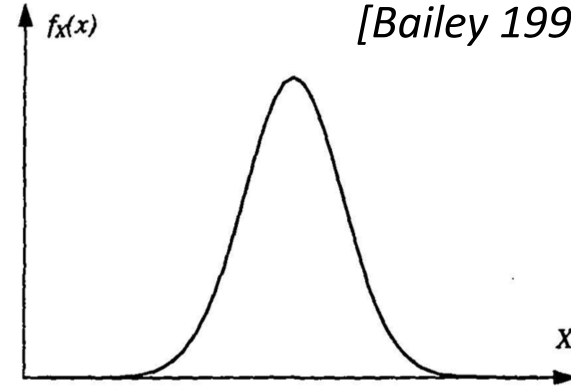
   Standard deviation $, \sigma_x = \sqrt{Var(x)}$

   Coefficient of Variation $, V_x = \frac{\sigma_x}{\mu_x}$

3) Skewness, …

Common statistical distributions:

[Bailey 1996]



(a) Normal distribution

1) Normal Distribution

- $PDF = f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$

- Commonly used and describes many naturally reoccurring phenomena very well.

- Complicated formulations for PDF and CDF make closed form solutions difficult. Tables and functions in programs such as Excel are helpful:

$$PDF = f_X(x) = \phi(x) = normdist(x, \mu, \sigma, 0)$$
$$CDF = F_X(x) = \underbrace{\Phi(x)}_{\text{Common notation}} = \underbrace{normdist(x, \mu, \sigma, 1)}_{\text{Excel}}$$

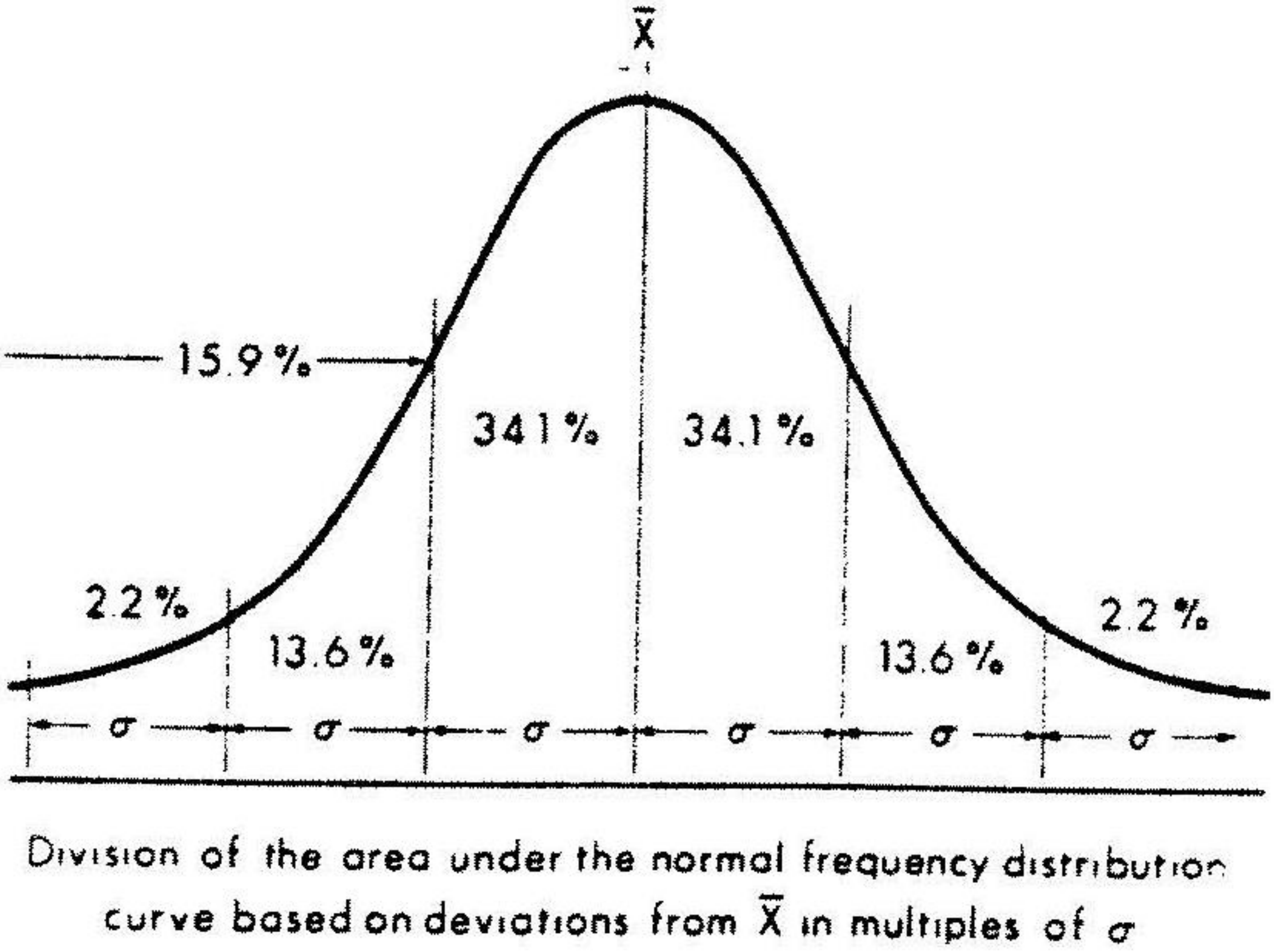


FIG. 2. Properties of a normal distribution.

[MacGregor 1976]

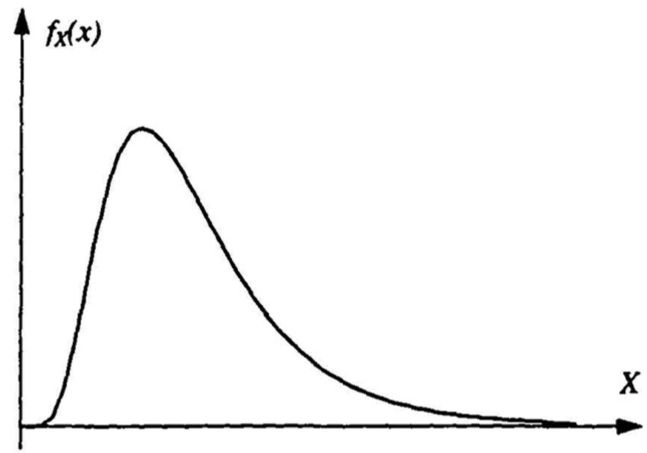
Common statistical distributions:

2) Lognormal Distribution

- $ln(x)$ is normally distributed.

- $x < 0$ is not possible.

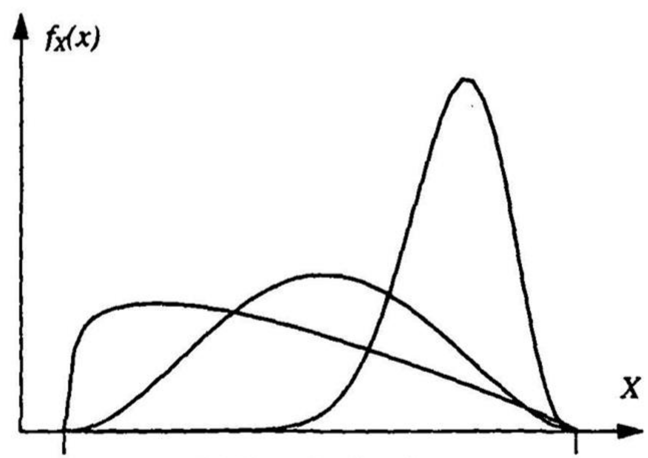
- Good for strength properties.



(b) Lognormal distribution

3) Beta Distribution

- Flexible, and therefore handy for fitting to the measured data that doesn't conform to Normal distribution.



(d) Beta distribution

[Bailey 1996]

Structural Reliability

Def. “ the probability that a product will function within specified limits for at least a specified period of time under specified environmental conditions.” [Melchers 1999]

Limit states: the onsets of various types of failures

Limit state functions= $G(\bar{Z})$

$\bar{Z}$ is a vector containing all random variables in the limit state function.

$G(\bar{Z}) < 0 \longrightarrow$ failure

$G(\bar{Z}) > 0 \longrightarrow$ acceptable

$G(\bar{Z}) = 0 \longrightarrow$ “failure surface”

The basic reliability problem:

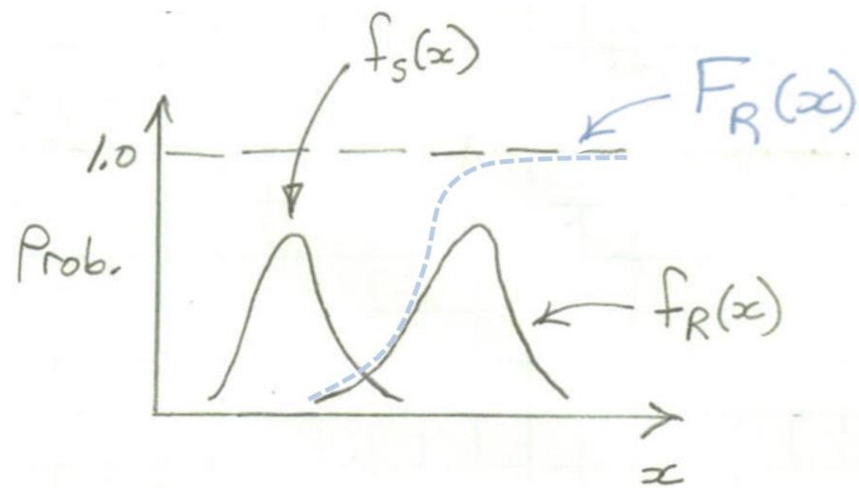
$$\bar{Z} = \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} R \\ S \end{Bmatrix}$$

Two random variables in Vector '$\bar{Z}$' one representing resistance and one representing load or "solicitation"

$$G(\bar{Z}) = R - S$$

Probability of failure:

$$P_f = \int_{-\infty}^{+\infty} F_R(x) \cdot f_S(x) dx$$



$f(x)$ = pdf = $P(x = x_i)$

$F(x)$ = cdf = $P(x \leq x_i)$

$\beta = reliability\ index$

$p_f = \Phi(-\beta) = \mathrm{normsdist}(-\beta)$ in Excel

e.g., $\beta = 2 \rightarrow p_f$ = normsdist(-2) = 0.02275 in Excel

(using table: $p_f = \Phi(z = -2) = 1 - 0.9772 = 0.0228$)

If S and R are both normally distributed:

$$\beta = \frac{\bar{R} - \bar{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}}$$

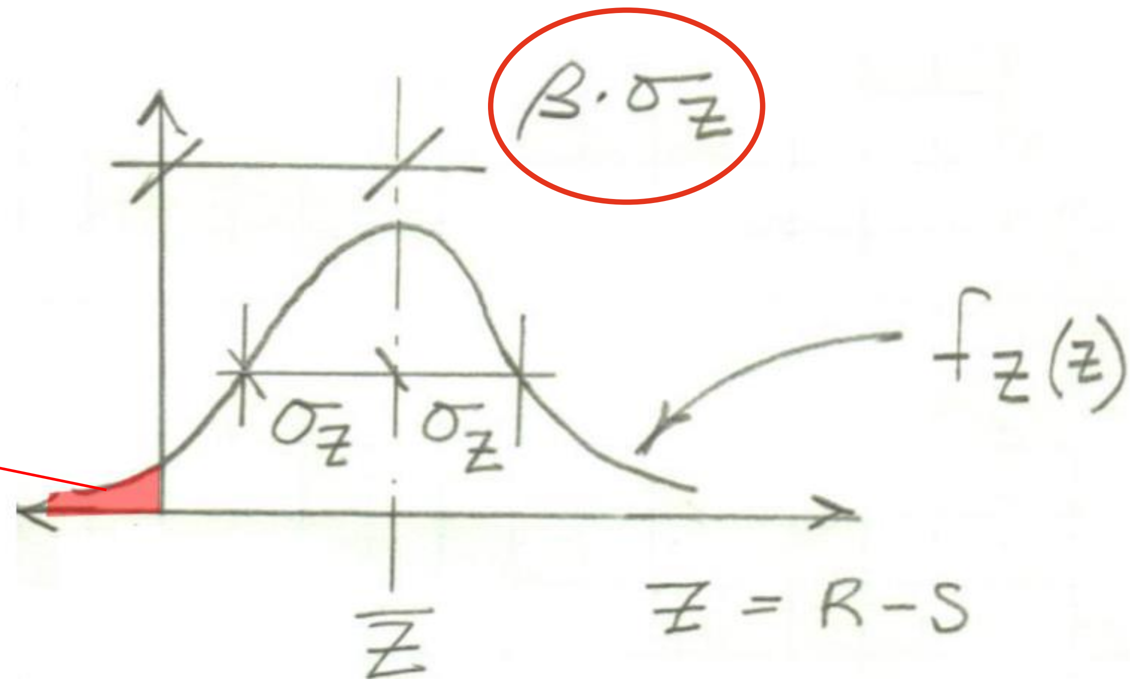
Target β: 3.0 - 3.5 for ductile
3.5 - 4.0 for brittle

Codes are calibrated for a target level of risk
Risk = (probability of failure) · (consequence)

When both R and S are Normally distributed.

$$Z = R - S$$

$$P_f = P(R - S < 0)$$

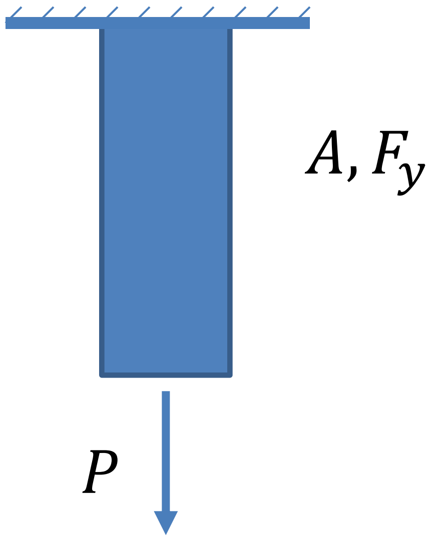


$$P_f = \int_{-\infty}^{0} f_Z(z)dz$$

Reliability index, β	Notional probability of failure, P_f
2.0	2.3×10^{-2} or 1:44
2.25	1.2×10^{-2} or 1:81
2.50	6.2×10^{-3} or 1:160
2.75	2.8×10^{-3} or 1:360
3.00	1.4×10^{-3} or 1:740
3.25	5.6×10^{-4} or 1:1 800
3.50	2.3×10^{-4} or 1:4 300
3.75	8.8×10^{-5} or 1:11 000
4.00	3.2×10^{-5} or 1:31 500
4.25	1.1×10^{-5} or 1:93 500
4.50	3.4×10^{-6} or 1:294 000

[CSA S6]

Example 1.



Assume F_y and P are normally distributed and the variability in A can be ignored.

$$\bar{P} = 900\ kN, \sigma_P = 200\ kN$$

$$\bar{F_y} = 330\ MPa, \sigma_{F_y} = 22\ MPa$$

Select P_{design} associated with 1% probability of exceedance.

Select $F_{y-design}$ associated with 1% probability $(F_y < F_{y-design})$.

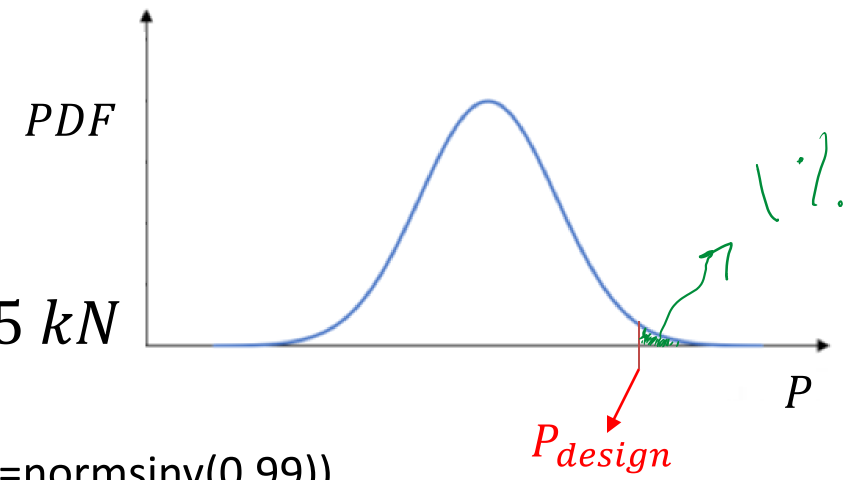
Calculate A, and p_f.

Select P_{design} associated with 1% probability of exceedance.

$$\bar{P} = 900\ kN; \sigma_P = 200\ kN$$

$$P_{design} = 900 + 200 \cdot \underbrace{2.326}_{} = 1365\ kN$$

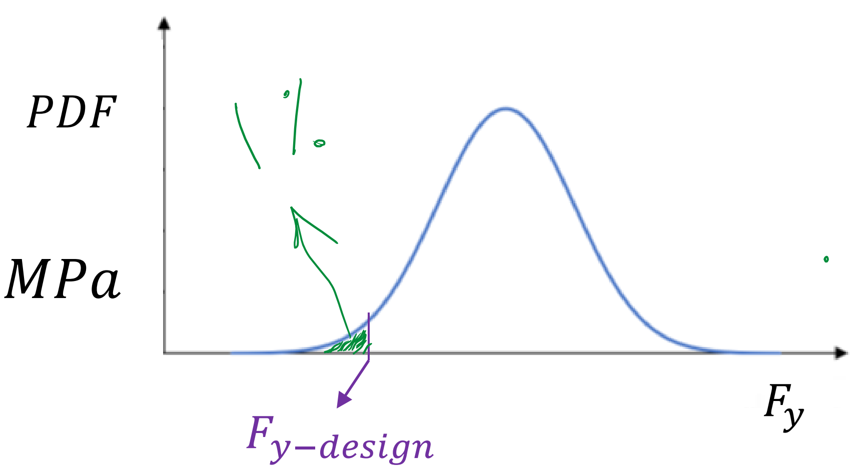
From standard normal table or Excel (=normsinv(0.99))



Select $F_{y-design}$ associated with 1% probability $(F_y < F_{y-design})$.

$$\bar{F_y} = 330\ MPa; \sigma_{F_y} = 22\ MPa$$

$$F_{y-design} = 330 - 22 \cdot 2.326 = 278.8\ MPa$$



$$A_{required} = \frac{P_{design}}{F_{y-design}} = \frac{1365 \cdot 1000}{278.8} = 4897\ mm^2$$

$$R = A \cdot F_y$$

$$\left\{ \begin{array}{l} \bar{R} = A \cdot \bar{F_y} = 4897 \cdot 330 = 1616010\ N = 1616\ kN \\ \\ \sigma_R = A \cdot \sigma_{F_y} = 4897 \cdot 22 = 107734\ N = 107kN \end{array} \right.$$

$$\beta = \frac{\bar{R} - \bar{P}}{\sqrt{\sigma_R^2 + \sigma_P^2}} = \frac{1616 - 900}{\sqrt{107^2 + 200^2}} = 3.15$$

$$p_f = \Phi(-3.15) = normsdist(-3.15) = 0.0008$$

(Note: In design code calibration, we actually run this problem in the opposite direction. We define an acceptable probability of failure and then back calculate load and resistance factors to achieve it.)

CIVE 512 - Reliability Aspects - 2017.xls [Compatibility Mode] - Microsoft Excel

CIVE 512 - Example Calculation (pf and Beta)

Sbar	900.00
SigmaS	200.00

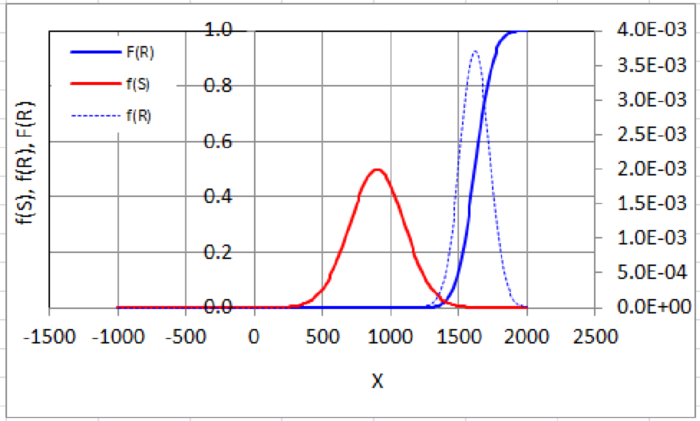
Rbar	1616.01
SigmaR	107.73

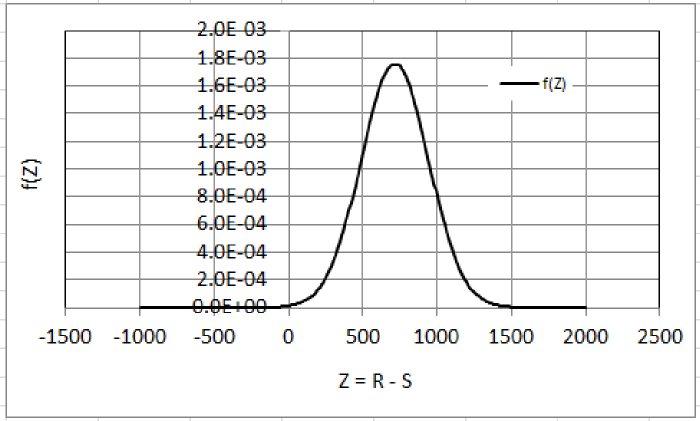
Zbar	716.01
SigmaZ	227.17

pf	8.13E-04	<-- based on area under f(Z) for Z<=0
beta	3.15	
pf	8.11E-04	<-- based on calculated beta

A	4897.0

X	f(S)	f(R)	F(R)	f(Z)	Area under f(Z) for Z <=0
-1000.00	5.04E-23	3.42E-131	1.51E-130	7.15E-16	-
-990.00	8.09E-23	3.24E-130	1.44E-129	9.96E-16	8.55E-15
-980.00	1.30E-22	3.05E-129	1.36E-128	1.38E-15	1.19E-14
-970.00	2.07E-22	2.84E-128	1.27E-127	1.92E-15	1.65E-14
-960.00	3.30E-22	2.63E-127	1.18E-126	2.66E-15	2.29E-14
-950.00	5.25E-22	2.41E-126	1.09E-125	3.68E-15	3.17E-14
-940.00	8.33E-22	2.19E-125	9.91E-125	5.07E-15	4.38E-14
-930.00	1.32E-21	1.97E-124	8.96E-124	6.99E-15	6.03E-14
-920.00	2.08E-21	1.76E-123	8.03E-123	9.60E-15	8.30E-14
-910.00	3.27E-21	1.56E-122	7.14E-122	1.32E-14	1.14E-13
-900.00	5.14E-21	1.37E-121	6.29E-121	1.80E-14	1.56E-13
-890.00	8.05E-21	1.19E-120	5.49E-120	2.46E-14	2.13E-13
-880.00	1.26E-20	1.03E-119	4.76E-119	3.36E-14	2.91E-13
-870.00	1.96E-20	8.77E-119	4.09E-118	4.57E-14	3.97E-13
-860.00	3.05E-20	7.43E-118	3.48E-117	6.21E-14	5.39E-13
-850.00	4.73E-20	6.25E-117	2.94E-116	8.42E-14	7.32E-13
-840.00	7.31E-20	5.21E-116	2.46E-115	1.14E-13	9.91E-13
-830.00	1.13E-19	4.30E-115	2.04E-114	1.54E-13	1.34E-12
-820.00	1.74E-19	3.52E-114	1.68E-113	2.08E-13	1.81E-12
-810.00	2.67E-19	2.86E-113	1.37E-112	2.79E-13	2.43E-12
-800.00	4.08E-19	2.30E-112	1.11E-111	3.75E-13	3.27E-12
-790.00	6.24E-19	1.84E-111	8.86E-111	5.02E-13	4.39E-12
-780.00	9.51E-19	1.46E-110	7.04E-110	6.72E-13	5.87E-12
-770.00	1.44E-18	1.14E-109	5.55E-109	8.97E-13	7.85E-12
-760.00	2.19E-18	8.89E-109	4.33E-108	1.20E-12	1.05E-11
-750.00	3.31E-18	6.85E-108	3.36E-107	1.59E-12	1.39E-11
-740.00	5.00E-18	5.24E-107	2.58E-106	2.11E-12	1.85E-11
-730.00	7.52E-18	3.97E-106	1.96E-105	2.80E-12	2.45E-11
-720.00	1.13E-17	2.99E-105	1.48E-104	3.70E-12	3.25E-11
-710.00	1.69E-17	2.22E-104	1.11E-103	4.88E-12	4.29E-11
-700.00	2.53E-17	1.64E-103	8.22E-103	6.42E-12	5.65E-11
-690.00	3.76E-17	1.20E-102	6.04E-102	8.44E-12	7.43E-11





Example 1 | Example 1 - MCS | Example 2 - MCS

Target reliability index

- Can be determined by:
    1)” back calculation ” using old codes
    2) “ Judgement ”
    3) determining acceptable p_f and using reliability theory
    4) setting for a constant level of risk ($= p_f \cdot failure\ cost$)

- CSA-S6-06 sets target β @ 3.75 (annual) or 3.5 (for 75-year service life)

- CSA target β is for new bridges. Ch14 lets you calculate different target βs for the assessment of existing bridges

TABLE 3. Death rate per year per million people

Cause	Death rate
Disease	726*
Accidents	550*
Smoking	500†
Automobile	360†
Construction work	200†
Swimming	60†
Building fires	35†
Collapse during construction	20†
Lightning	0.5†
Collapse of finished structures	0.2†
Venomous bite	0.05†

*Average rate for Canadians aged 20 to 44.
†Projected rate.

[Allen 1992]

TABLE 2. Risk of death for various activities*

Activity	Yearly death rate per person per year: For those concerned	Yearly death rate per person per year: For the total population
Motorcycle racing	5×10^{-3}	
Mountain climbing	5×10^{-3}	
Mining	7×10^{-4}	
Swimming	1×10^{-4}	2×10^{-5}
Automobile travel		3.6×10^{-4}
Airplane travel	1×10^{-4}	
Fire in buildings		2×10^{-5}
Poisoning		1.1×10^{-5}
Lightning		5×10^{-7}
Vaccinations and innoculations		1×10^{-8}
Structural collapse		
During construction	3×10^{-5}	
All others		2×10^{-7}

*Data from Allen (1968), Otway *et al.* (1970), and Rüsch and Rackwitz (1972).

[MacGregor 1976]

Table C2 - Target reliability index β for Class RC2 structural members [1)]

Limit state	Target reliability index	
	1 year	50 years
Ultimate	4,7	3,8
Fatigue		1,5 to 3,8 [2)]
Serviceability (irreversible)	2,9	1,5

[1)] See Annex B

[2)] Depends on degree of inspectability, reparability and damage tolerance.

When prescribing target reliability index, the reference period must also be specified. The Eurocode for structures provides a formula for converting to a different reference period. It assumes no dependence between the failure probabilities from one year to the next:

$$\Phi(\beta_n) = [\Phi(\beta_1)]^n$$

where :

β_n is the reliability index for a reference period of n years,

β_1 is the reliability index for one year.

[Eurocode 1990]