1. W310x143 , $r_x = 138$ mm , $A = 18200$ mm^2

a) K_x? Column B , RL = 350 kN (Dead), 550 kN (Live)
FL = 600 kN (dead), 1000 kN (Live)

For lower column

$G_e = 10$ (pinned)

$G_u = \frac{\Sigma I/L_c}{\Sigma I/L_g}$, $I_{cs} = 348E6$ mm^4
$I_{410x60} = 216E6$ mm^4 , $I_{410x54} = 186E6$
$I_{410x74} = 275E6$ mm^4 , $I_{410x67} = 245E6$

$\therefore G_u = \frac{348/4200 + \frac{348}{3600}}{216/9300 + 275/11000} = 3.72256728$

$K_x \simeq 0.93$ (Annex G) 1-219

For upper column

$G_e = \frac{348/3600 + \frac{348}{4200}}{216/9300 + \frac{275}{11000}} = 3.72256728$

$G_u = \frac{348/3600}{\frac{186}{9300} + \frac{245}{11000}} = 2.1553$

$K \simeq 0.87$ (1-219)

b) Check bottom

$\frac{K_x L_x}{r_x} = \frac{0.93 \cdot 4200}{138} = 28.3047 < 200$, OK ← governs

Check top

$\frac{0.87 \cdot 3600}{138} = 22.696 < 200$, OK

Section classification

$h/w = \dfrac{323-2(22.9)}{14} = 19.8 \leq \dfrac{1900}{\sqrt{345}} = 102.3$, $\therefore$ not class 4

$\dfrac{b_{el}}{t} = \dfrac{309/2}{22.9} = 6.746 \leq \dfrac{200}{\sqrt{345}} = 10.7676$, $\therefore$ not class 4

$\lambda = \sqrt{\dfrac{345}{\pi^2 \cdot 2E5 / 28.3049^2}} = 0.374202$

$C_r = 0.9 \cdot 345 \cdot 18200 \cdot \left(1 + 0.37420^{2 \cdot 1.34}\right)^{-1/1.34}$
$= 5366.24$ kN

$1.25(350+600) + 1.5(550+1000) = 3512.5$ kN

$C_r > 3512.5$, $\therefore$ OK

(2) $L = 6m$, W410x54 ; spacing = 1.6m of 15kN DL, 25kN LL

Loading = $15 \cdot 1.25 + 25 \cdot 1.5 = 56.25$ kN

From 10 in 5-135, $M_x = R_1 x - P(x-a)$, $R_1 = P/l(l-a+b)$, $b = l - a - 1.6$

$= P/l[2l - 2a - 1.6]$

$\therefore M_x = P/l[2l - 2a - 1.6]x - Px + Pa$

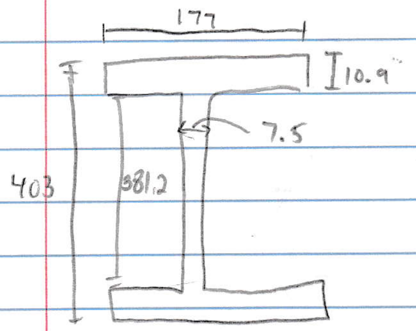
when M_x is maximized, $\frac{dM}{dx} = 0$

$\frac{dM}{dx} = P - \frac{2Pa}{l} - \frac{1.6P}{l} = 0$, $a = ?$ $P = 56.25$, $l = 6m$

$\therefore a = 2.2$, $\therefore b = 6 - 2.2 - 1.6 = 2.2$

Since code for symmetric loading satisfied, $M_{max} = Pa$

$M = 56.25 \cdot 2.2 = 123.75$ kNm



$\frac{b_{el}}{t} = \frac{177/2}{10.9} = 8.11927 < \frac{200}{\sqrt{345}} = 10.76$, ∴ Not class 4

$< \frac{170}{\sqrt{345}} = 9.152$ ∴ Not class 3

$\frac{h}{w} = \frac{403 - 2(10.9)}{7.5} = 50.827 < \frac{1900}{\sqrt{345}} = 102.29$, ∴ not class 4

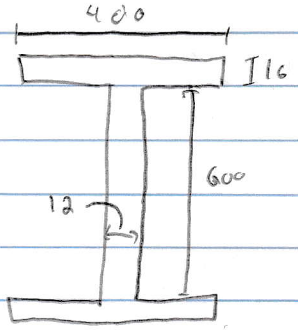
$< \frac{1700}{\sqrt{345}}$, ∴ not class 3

$Z = \frac{381.2}{2} \cdot \frac{381.2}{4} \cdot 7.5 \cdot 2 + 177 \cdot 10.9 \cdot 2 \cdot \left(\frac{381.2}{2} + \frac{10.9}{2}\right)$

$= 1028941.23 \text{ mm}^3$

$M_r = 0.9 \cdot 1028941.23 \cdot 345 = 319.49$ kNm > 123.75, ∴ OK

③



$$J = 2\cdot\frac{1}{3}\cdot 400\cdot 16^3 + \frac{1}{3}\cdot 600\cdot 12^3$$
$$= 1437866667\ mm^4$$

$$I_y = 600\int_{-6}^{6} x^2 dx + 2\cdot 16\cdot\int_{-200}^{200} x^2 dx$$
$$= \frac{600}{3}\left[6^3 - (-6)^3\right] + \frac{2\cdot 16}{3}\cdot\left[200^3 - (-200)^3\right] = 170753066.7$$

Section classification

$$C_w = I_y\left(\frac{600-16}{2}\right)^2$$

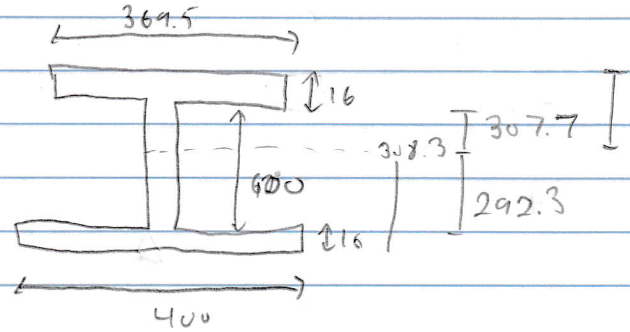
$$\frac{b_{el}}{t} = \frac{400/2}{16} = 12.5 \nleq \frac{200}{\sqrt{300}} = 11.547 \quad , \therefore \text{ class } 4$$

$$\frac{h}{w} = \frac{600}{12} \leq \frac{1900}{\sqrt{300}}, \quad 50 \leq 109.6965 \quad \therefore \text{ at class } 3$$

Reduce flange area - Local buckling check

$$\frac{b_{el}}{16} = \frac{200}{\sqrt{300}}, \quad b_{el} = 184.752, \quad \Delta_b = 30.5\ mm$$

$\therefore$



$$\bar{y} = \frac{400\cdot 16\cdot 8 + 600\cdot 12\cdot 316 + 369.5\cdot 16\cdot 624}{400\cdot 16 + 600\cdot 12 + 369.5\cdot 16}$$

$= 308.30\ mm$

$$I_{xe} = 12\int_{-292.3}^{307.7} y^2 dy + 369.5\int_{307.7}^{323.7} y^2 dy + 400 \int_{292.3}^{308.3} y^2 dy$$

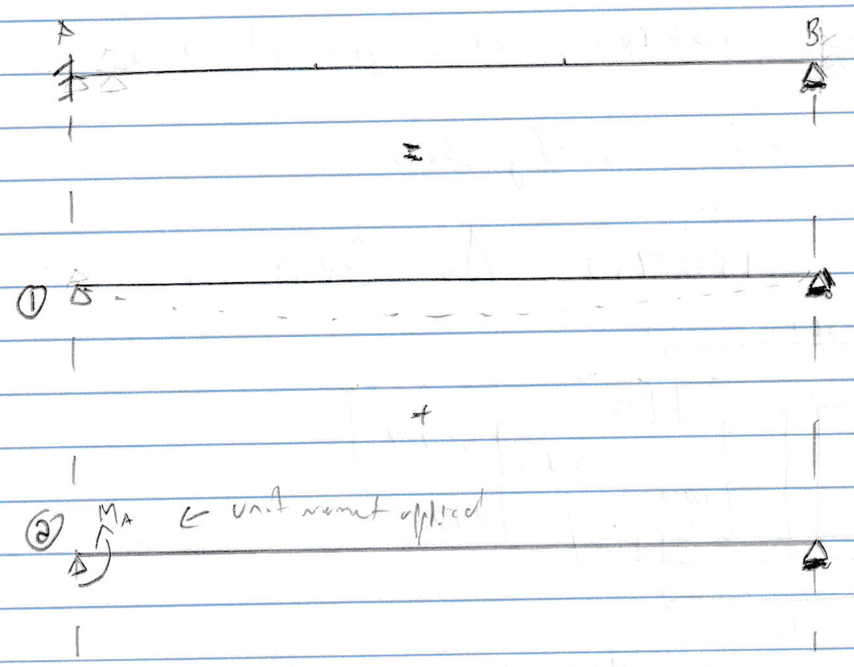
$$= \frac{12}{3}\left[307.7^3 - (-292.3)^3\right] + \frac{369.5}{3}\left[323.7^3 - 307.7^3\right] + \frac{400}{3}\left[308.3^3 - 292.3^3\right]$$
$$= 1383070409\ mm^4$$

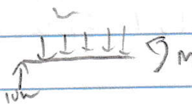
$$S_e = \frac{I_{xe}}{y_{max}} = \frac{1383070409}{323.7} = 4272692.026\ mm^3$$

$$M_r = \phi S_e F_y = 0.9\ S_e \cdot 300$$
$$= 1153.63\ kNm$$

To check FT-buckling

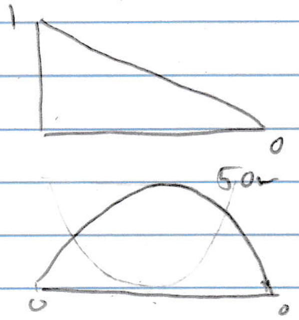
1. Determine moment envelope



① $\Sigma F_y = 0$, $A_y = B_y = \frac{w \cdot 20}{2} = 10w$,  $M = 10wx - \frac{wx^2}{2}$

② $\Sigma M_B = 0$, $1 + A_y(20)$, $A_y = -1/20$, $B_y = 1/20$, $m = 1 - \frac{x}{20}$

$m_{max} = 1$, $M_{max} = \frac{wL^2}{8} = \frac{w \cdot 20^2}{8} = 50w$



From product integrals,

$$\int \frac{Mm}{EI}dx = \frac{1}{3} \cdot 1 \cdot 50w \cdot 20 = \frac{1000}{3EI}w$$

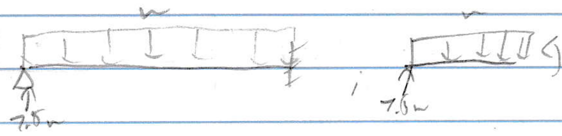
$$\int \frac{mm}{EI}dx = \frac{1}{3} \cdot 1 \cdot 1 \cdot 20 = \frac{20}{3EI}$$

$$\frac{1000}{3EI}w - M_A\left(\frac{20}{3EI}\right) = 0, \quad M_A = 50w$$

$\sum M_A = 0$, $50w - w \cdot 20 \cdot \frac{20}{2} + B_y \cdot 20 = 0$

$B_y = 7.5w$

$\sum F_y = 0$, $20 \cdot w - 7.5w = A_y$, $A_y = 12.5w$



$M = 7.5wx - \frac{wx^2}{2}$

M_{max}, $\frac{dM}{dx} = 0$, $7.5w - \frac{2wx}{2}$, $x = 7.5$

After looking at all 3 unbraced segments (see excel), segment 3 governed w/ $M_u = 2728.37675$ kNm). Next, determine M_y since class 4

$M_y = S F_y$,

$I_x = 12 \int_{-300}^{300} y^2 dy + 2 \cdot 400 \int_{300}^{316} y^2 dy$

$I_x = \frac{12}{3}[300^3 + 300^3] + \frac{800}{3}[316^3 - 300^3]$

$= 1430532267 \text{ mm}^4$

$S = \frac{1430532267}{316} = 4527000.844$

$M_y = SF_y = S \cdot 300$
$= 1358.1$ kNm

$0.67M_y = 909.93$ kNm, $\therefore M_u > 0.67 M_y$

$M_r = 1.15 \cdot 0.9 \cdot 1358.1 \cdot \left[1 - \frac{0.28 \cdot 1358.1}{2728.377}\right] \leq 0.9 \cdot 1358.1$

$= 1209.723 \leq 1222.29$!

$\therefore M_r = 1209.723$ kNm

Constants

J	1437866,667	mm^4
Iy	170753066,7	mm^4
Cw	1,45591E+13	mm^6
E	200000	MPa
G	77000	MPa
L	6666,67	mm

Section 1

x_{start}	0					
x_{end}	6,666666667					
	At Start	At 1/4x	At 2/4x	At 3/4x	At End	Max
x	0	1,66666667	3,33333333	5	6,66666667	6,66666667
M (in terms of w)	0	11,1111111	19,4444444	25	27,7777778	27,7777778
w2	1,387584626	<2.5				
M_u	3325,389735					

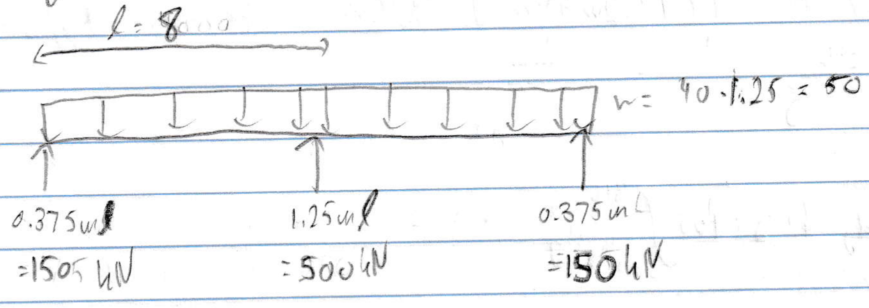
Section 2

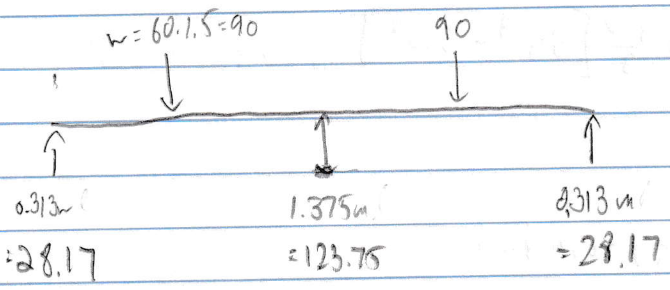
x_{start}	6,666666667					
x_{end}	13,33333333					
	At Start	At 1/4x	At 2/4x	At 3/4x	At End	Max
x	6,666666667	8,33333333	10	11,6666667	13,3333333	7,5
M (in terms of w)	27,77777778	27,7777778	25	19,4444444	11,1111111	28,125
w2	1,138469152	<2.5				
M_u	2728,376749					

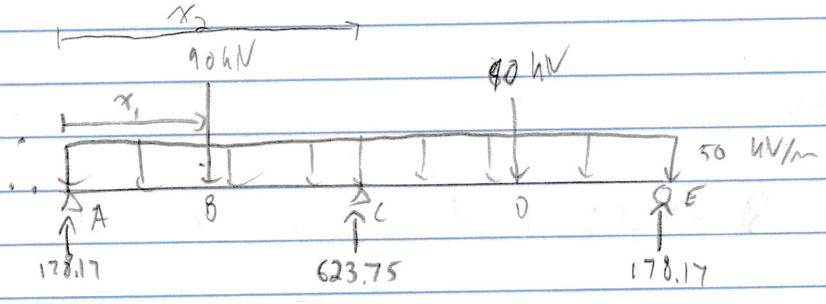
Section 3

x_{start}	13,33333333					
x_{end}	20					
	At Start	At 1/4x	At 2/4x	At 3/4x	At End	Max
x	13,33333333	15	16,6666667	18,3333333	20	20
M (in terms of w)	11,11111111	0	-13,888889	-30,555556	-50	-50
w2	2,296443356	<2.5				
M_u	5503,497961					

(4) Using beam table on 5-145, we have the following







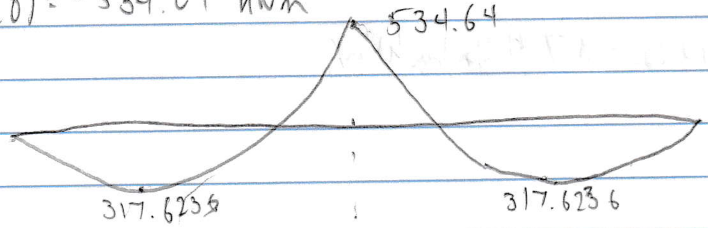
$M(x_1) = 178.17x_1 - \frac{50x^2}{2}$, $\frac{dM}{dx} = 178.17 - 50x$, $x = 3.5634$

$M(3.5634) = 317.6236$

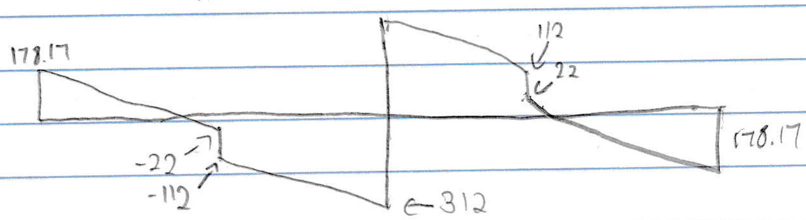
$M(x_2) = 178.17x - 25x^2 - 90(x-4)$

$M(8) = -534.64$ kNm

BMD



SFD



$\therefore$ $V_{f max} = 312$ kN, $M_{f max} = 534.64$

From beam selection table, we have

W530x85, $M_r' = 446$ kNm
W530x92, $M_r' = 621$ kNm ← likely to govern

To verify

1. find moments @ $1/4\,x$, $2/4\,x$, $3/4\,x$

For x_1

$M(1) = 178.17 \cdot 1 - 25(1)^2 = 153.17$, $M(2) = 178.17(2) - 25(2)^2 = 256.34$

$M(3) = 178.17 \cdot 3 - 25(3)^2 = 309.51$ $M_{max} = 317.6236$

$$\therefore \omega_2 = \frac{4 \cdot 317.6236}{\sqrt{4M_{max}^2 + 4M(1)^2 + 7M(2)^2 + 4M(3)^2}} \neq \omega_2 = 1.0973$$

For x_2

$M(5) = 175.85$ kNm
$M(6) = -10.98$ kNm
$M(7) = -247.81$ kNm
$M(8) = -534.64$ kNm

$$\omega_2 = \frac{4 \cdot 534.64}{\sqrt{4M_{max}^2 + 4M(5)^2 + 7M(6)^2 + 4M(7)^2}} = 1.7383$$

From 5-22, 3 potential options:

W530x82 = 531 kNm (M'_r)
W530x74 = 357 kNm (M'_r)
W530x92 = 621 kNm (M'_r)

Try W530x74

$\frac{h}{w} = \frac{502}{9.7} = 51.75 \le \frac{1700}{\sqrt{345}}$, Not class 3

$\frac{b_{el}}{t} = \frac{166/2}{13.6} = 6.103 \le \frac{170}{\sqrt{345}}$, Not class 3

$I_y = 10.4E6\ mm^4$
$G = 77000$
$J = 480E3\ mm^4$
$C_w = 692E9\ mm^6$
$E = 200E3$
$Z_x = 1810E3\ mm^3$

Span 1

$$M_u = \frac{1.0973\pi}{4000}\sqrt{200000 \cdot 10.4E6 \cdot 77E3 \cdot 480E3 + \left(\frac{\pi \cdot 200E3}{4000}\right)^2 \cdot 10.4E6 \cdot 692E9}$$
$$= 434.729\ kNm$$

$$M_p = 1810E3 \cdot 345 \cdot 0.9$$
$$= 624.45$$

$0.67\ M_p = 418.3815\ kNm$, since $M_u > 0.67 M_p$, inelastic buckling

$M_r = 1.15 \cdot 0.9 \cdot 624.455 \left(1 - \frac{0.28 \cdot 624.455}{434.727}\right) < 0.9 M_p = 386.36\ kN$

∴ W530x74 ok for span 1. However, for next span, $M_{f,max} = 534.64\ kN$.

Span 2 $M_{f,max} > 0.9 M_p$, ∴ section cannot be used.

Span 2

$$M_u = \frac{1.7383\,\pi}{4000}\sqrt{200000 \cdot 10.4E6 \cdot 77E3 \cdot 480E3 + \left(\frac{\pi \cdot 200E3}{4000}\right)^2 \cdot 10.4E6 \cdot 642E9}$$

$$M_u = 688.68$$

$$M_r = 1.15 \cdot 0.9 \cdot 624.45\left(1 - \frac{0.28 \cdot 624.45}{689.68}\right)$$

$$= 482.217 < 0.9 M_u$$

$$482217 < 534.64, \therefore \text{ Fails}$$

Try W530x82

$\frac{h}{w} = \frac{501}{9.5} = 52.74 < \frac{1700}{\sqrt{345}}$, Not class 3

$I_y = 20.3E6\ mm^4$

$G = 77000$ MPa

$\frac{b_{el}}{t} = \frac{209/2}{13.3} = 7.86 < \frac{170}{\sqrt{345}}$, Not class 3

$J = 518E3\ mm^4$

$C_w = 1340E9\ mm^6$

$E = 200E3$

$Z_x = 2060E3\ mm^3$

$M_p = 2060E3 \cdot 345 \cdot 0 = 710.7$ kNm

$0.9M_p = 639.63$ kNm > 534.64

$0.67M_p = 476.1697 > 534.64$ kN, ∴ ...

$0.67M_p =$

Note $\sqrt{200E3 \cdot 20.3E6 \cdot 77E3 \cdot 518E3 + \left(\frac{\pi \cdot 200E3}{4000}\right)^2 \cdot 20.3E6 \cdot 1340E9} / 1000^2$

$= 912753.8691$

Span 1

$M_u = \frac{1.0473\pi}{4000} \cdot 912753.8591 = 786.627286$ kNm

$M_u > 0.67M_p$, inelastic buckling

$M_r = 1.15 \cdot 0.9 \cdot 710.7 \left(1 - \frac{0.28 \cdot 710.7}{M_u = 786}\right) \leq 0.9 \cdot 710.7$

$= 689.054 \nleq 0.9 \cdot 710.7 = 639.63$ kNm, ∴ 639.63 governs

$549.493 > 317.62$, ∴ OK

Span Check span 2 to be checked as 639.63 > 534.64 kNm ... is greater in span 2, ... capacity.

∴ W530x82 OK

Regardless, check [illegible]

$$M_u = \frac{1.7383\pi}{4100} \cdot 912753.8591$$

$$= 1246.144 \text{ kNm}$$

$M_u > 0.67 M_p$, inelastic buckling

$$M_r = 1.15 \cdot 0.9 \cdot 710.7 \left(1 - \frac{0.28 \cdot 710.7}{1246.144}\right) \leq 0.9 M_p$$

$$= 618.11 \text{ kNm} > 534.64, \therefore \text{OK}$$

Check Shear

$\frac{1014}{\sqrt{345}} = 54.59 \cdot 1.27 \geq h/w, \therefore$ Shear Yielding

$F_s = 0.66 \cdot 345 = 227.7$ MPa

$A_w = d \cdot w = 528 \cdot 9.5 = 5016\ mm^2$

$V_r = 0.9 A_w F_y [2.2 - 1.6\ M_f/M_r]$

$\uparrow$

≤ 1.0

$= 0.9 \cdot 5016 \cdot 345 \left[2.2 - 1.6 \cdot \frac{534.64}{618.11}\right]$

$= 1271$ kN

$V_r > 312$ kN, OK

Check bearing

Since interior load, $B_r = \min\left[\phi_{bi} w (N + 10t) F_y, 1.45\ \phi_{bi} w^2 \sqrt{F_y E}\right]$

$0.8 \cdot 9.5 \cdot (209 + 10 \cdot 13.3) 345 = 896{,}724$ kN

$1.45 \cdot 0.8 \cdot 9.5^2 \sqrt{345 \cdot 200E3} = 869.62$ kN $\leftarrow$ governs

$869.62 > 312$ kN, OK

$\therefore$ W530x82