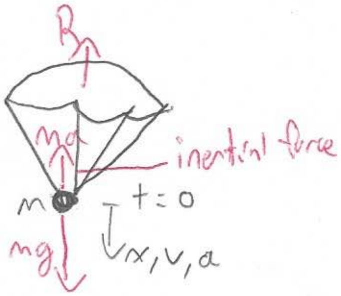
Eg. A = area of parachute projected in horizontal plane

Air Resistance is related to A and v, $R = kAv^2$



From Newton's 2nd Law, $\Sigma F = ma$, $mg - kAv^2 = m\frac{dV}{dt}$

From D'Alembert's, $mg - kAv^2 - ma = 0$

- not first order due to v^2
- variable separable

Case 1 $mg - kAv^2 = 0$, $V = \sqrt{\frac{mg}{kA}}$ (Terminal Velocity)

Case 2, $mg - kAv^2 \neq 0$

$\frac{m}{mg - kAv^2} dV = dt$, may be easier if $\int \frac{m}{kAv^2 - mg} dV = -\int dt + C$

Next, simplify constants (coefficients)

$\int \frac{\frac{m}{kA}}{v^2 - \frac{mg}{kA}} dV = -\int dt + C$, $\frac{mg}{kA} = \mu^2$

$\times\frac{kA}{m}$ $\int \frac{1}{V^2 - \mu^2} dV = \int -\frac{g}{\mu^2} dt + C$

↳ Table of partial fraction

From table

$\frac{1}{2\mu} \ln\left|\frac{v-\mu}{v+\mu}\right| = \frac{-g}{\mu^2} t + C$

$e^{\ln\left|\frac{v-\mu}{v+\mu}\right|} = e^{-\frac{2g}{\mu}t + C}$ e^C

From initial condition, $t=0$, $v=0$, $-1=1\cdot C$, $C=-1$

$$\frac{v-\mu}{v+\mu}=-e^{-\frac{2g}{\mu}t}$$

$$v=\mu\,\frac{1-e^{-\frac{2g}{\mu}t}}{1+e^{-\frac{2g}{\mu}t}}$$, hyperbolic function

*pg. 145 of textbook, $\sinh x=\frac{e^x-e^{-x}}{2}$, $\cosh x=\frac{e^x+e^{-x}}{2}$, formula 33

$$\frac{d}{dx}(\sinh x)=\frac{e^x+e^{-x}}{2}=\cosh x$$

$$\frac{d}{dx}\cosh x=\frac{e^x-e^{-x}}{2}=\sinh x$$

$$\int\cosh x\,dx=\sinh x,\quad \int\sinh x\,dx=\cosh x$$

$$\tanh x=\frac{\sinh x}{\cosh x}=\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{1-e^{-2x}}{1+e^{-2x}},\quad \frac{v-\mu}{v+\mu}=z$$

$$v-\mu=-vz-\mu z$$

$$(1+z)v=\mu(1-z)$$

$$v=\mu\,\frac{1-z}{1+z}$$

$$v=\mu\tanh\left(\frac{g}{\mu}t\right)$$

$\therefore$ Terminal velocity $t\to+\infty$, $\frac{1-e^{-2t}}{1+e^{-2t}}=\frac{1}{1}=1$

$v\to\mu$, $v=V_T=\mu=\sqrt{\frac{mg}{kA}}$, $v=\frac{dx}{dt}$ $\quad\frac{dy}{dt}=V_T\tanh\left(\frac{g}{\mu}t\right)$,

1st order separable, immediately integrable

$$x=V_T\int\tanh\left(\frac{g}{\mu}t\right)dt$$

$$\int\tanh x=\int\frac{\sinh x}{\cosh x}\,dx=\int\frac{1}{\cosh x}\,d(\cosh x)$$, $x=V_T\,\frac{\mu}{g}\ln\cosh\left(\frac{g}{\mu}t\right)+D$ (always (+))

when $t=0$, $\cosh(0)=1$

$$\therefore\ x=\frac{V_T\mu}{g}\ln\cosh\left(\frac{g}{\mu}t\right)$$