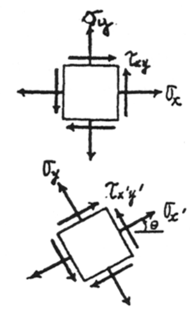
14–1.

Prove that the sum of the normal stresses $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$ is constant. See Figs. 14–2*a* and 14–2*b*.

SOLUTION

Stress Transformation Equations: Applying Eqs. 9–1 and 9–3 of the text,



$$\sigma_{x'} + \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

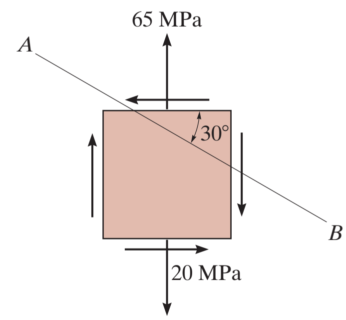
$$+ \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \qquad \textbf{(Q. E. D.)}$$

Ans:
N/A

14–2.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 14.1.



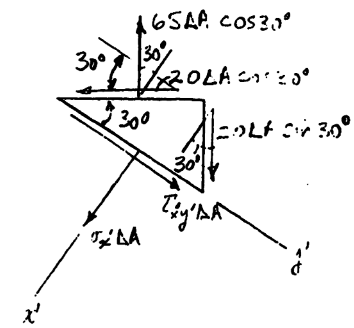
SOLUTION

$$\swarrow + \Sigma F_{x'} = 0; \quad \sigma_{x'}\Delta A + 20\Delta A \sin 30° \cos 30° + 20\Delta A \cos 30° \cos 60° - 65\Delta A \cos 30° \cos 30° = 0$$

$$\sigma_{x'} = 31.4 \text{ MPa} \qquad \textbf{Ans.}$$

$$\searrow + \Sigma F_{y'} = 0; \quad \tau_{x'y'}\Delta A + 20\Delta A \sin 30° \sin 30° - 20\Delta A \cos 30° \sin 60° - 65\Delta A \cos 30° \sin 30° = 0$$

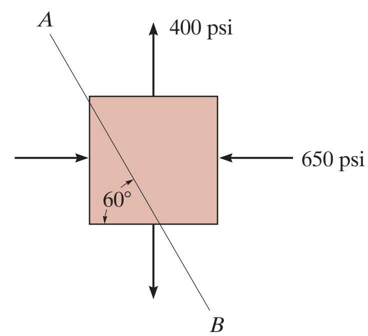
$$\tau_{x'y'} = 38.1 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{x'} = 31.4$ MPa,
$\tau_{x'y'} = 38.1$ MPa

14–3.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 14.1.



SOLUTION

$\nearrow + \Sigma F_{x'} = 0; \qquad \Delta F_{x'} - 400(\Delta A \cos 60°) \cos 60° + 650(\Delta A \sin 60°) \cos 30° = 0$

$$\Delta F_{x'} = -387.5\Delta A$$

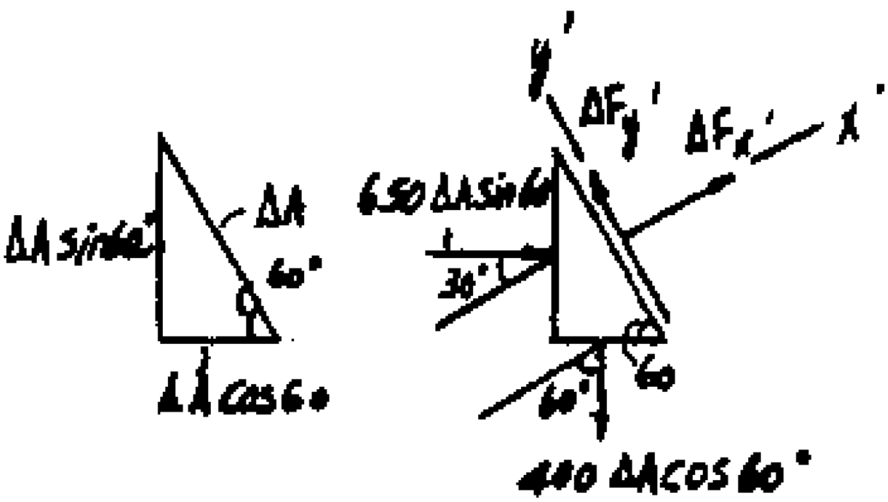
$\nwarrow + \Sigma F_{y'} = 0; \qquad \Delta F_{y'} - 650(\Delta A \sin 60°) \sin 30° - 400(\Delta A \cos 60°) \sin 60° = 0$

$$\Delta F_{y'} = 455\Delta A$$

$\sigma_{x'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{x'}}{\Delta A} = -388 \text{ psi}$ **Ans.**

$\tau_{x'y'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{y'}}{\Delta A} = 455 \text{ psi}$ **Ans.**

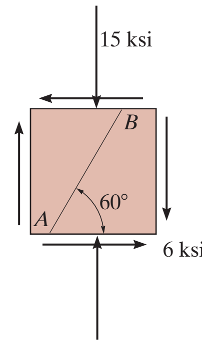
The negative sign indicates that the sense of $\sigma_{x'}$ is opposite to that shown on the FBD.



Ans:
$\sigma_{x'} = -388$ psi,
$\tau_{x'y'} = 455$ psi

***14–4.**

Determine the normal stress and shear stress acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 14.1.



SOLUTION

Force Equilibrium: Referring to Fig. *a*, if we assume that the area of the inclined plane AB is ΔA, then the area of the vertical and horizontal faces of the triangular sectioned element are $\Delta A \sin 60°$ and $\Delta A \cos 60°$, respectively. The forces acting on the free-body diagram of the triangular sectioned element, Fig. *b*, are

$\Sigma F_{x'} = 0;\quad \Delta F_{x'} - (6\Delta A \sin 60°)\cos 60° - (6\Delta A \cos 60°)\sin 60° + (15\Delta A \cos 60°)\cos 60° = 0$

$$\Delta F_{x'} = 1.4461\Delta A$$

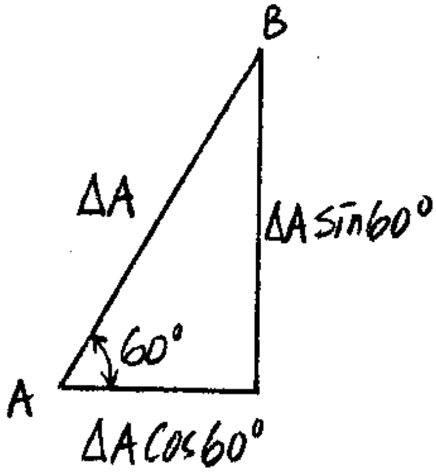
$\Sigma F_{y'} = 0;\quad \Delta F_{y'} + (6\Delta A \sin 60°)\sin 60° - (6\Delta A \cos 60°)\cos 60° - (15\Delta A \cos 60°)\sin 60° = 0$

$$\Delta F_{y'} = 3.4952\Delta A$$

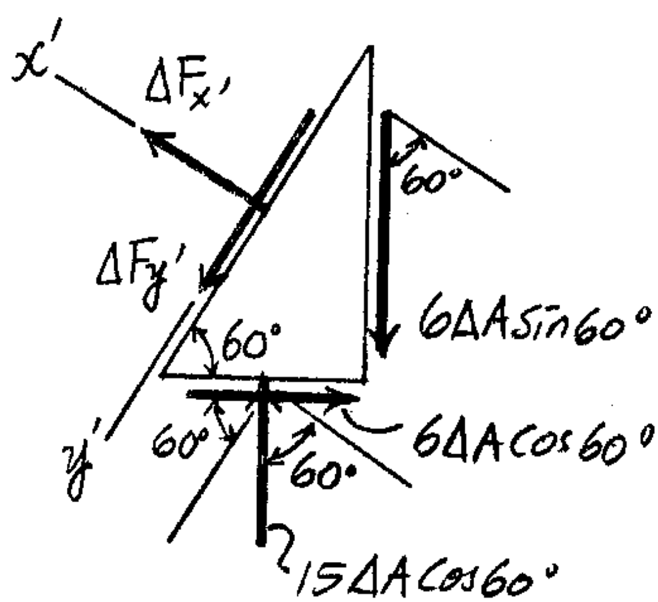
Normal and Shear Stress: From the definition of normal and shear stress,

$$\sigma_{x'} = \lim_{\Delta A \to 0}\frac{\Delta F_{x'}}{\Delta A} = 1.45 \text{ ksi}$$ **Ans.**

$$\tau_{x'y'} = \lim_{\Delta A \to 0}\frac{\Delta F_{y'}}{\Delta A} = 3.50 \text{ ksi}$$ **Ans.**



(a)

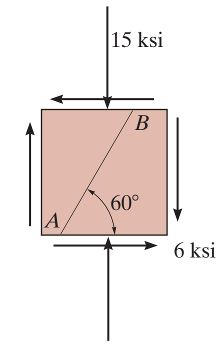


(b)

Ans:
$\sigma_{x'} = 1.45$ ksi,
$\tau_{x'y'} = 3.50$ ksi

14–5.

Determine the normal stress and shear stress acting on the inclined plane AB. Solve the problem using the stress transformation equations. Show the results on the sectional element.



SOLUTION

Stress Transformation Equations:

$$\theta = +150° \text{ (Fig. } a) \qquad \sigma_x = 0 \qquad \sigma_y = -15 \text{ ksi} \qquad \tau_{xy} = -6 \text{ ksi}$$

We obtain

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{0 + (-15)}{2} + \frac{0 - (-15)}{2} \cos 300° + (-6) \sin 300°$$

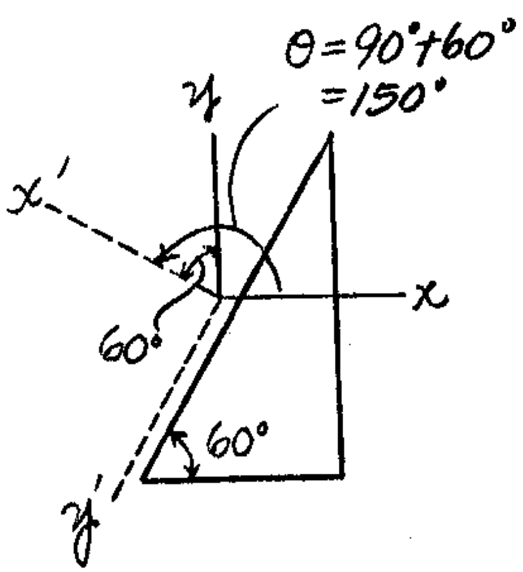
$$= 1.45 \text{ ksi}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

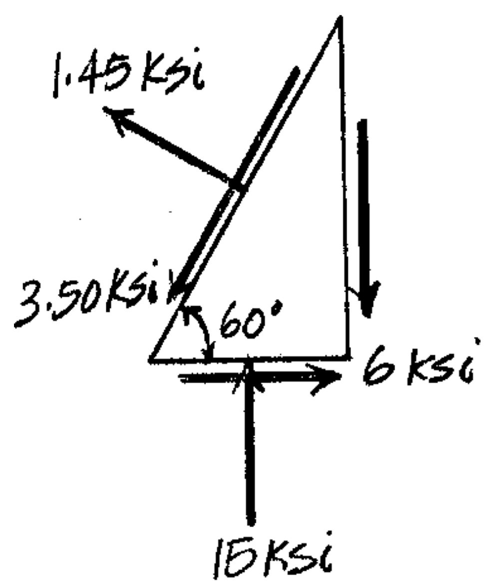
$$= -\frac{0 - (-15)}{2} \sin 300° + (-6) \cos 300°$$

$$= 3.50 \text{ ksi}$$ **Ans.**

The results are indicated on the triangular sectioned element shown in Fig. b.



(a)

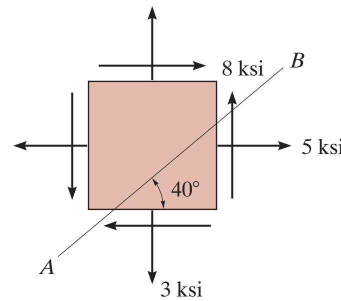


(b)

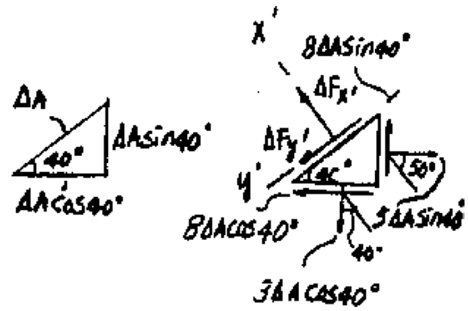
Ans:
$\sigma_{x'} = 1.45$ ksi,
$\tau_{x'y'} = 3.50$ ksi

14–6.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 14.1.



SOLUTION



$\nwarrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} + (8\Delta A \sin 40°)\cos 40° - (5\Delta A \sin 40°)\cos 50°$

$$- (3\Delta A \cos 40°)\cos 40° + (8\Delta A \cos 40°)\cos 50° = 0$$

$$\Delta F_{x'} = -4.052\Delta A$$

$\swarrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - (8\Delta A \sin 40°)\sin 40° - (5\Delta A \sin 40°)\sin 50°$

$$+ (3\Delta A \cos 40°)\sin 40° + (8\Delta A \cos 40°)\sin 50° = 0$$

$$\Delta F_{y'} = -0.4044\Delta A$$

$\sigma_{x'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{x'}}{\Delta A} = -4.05 \text{ ksi}$ **Ans.**

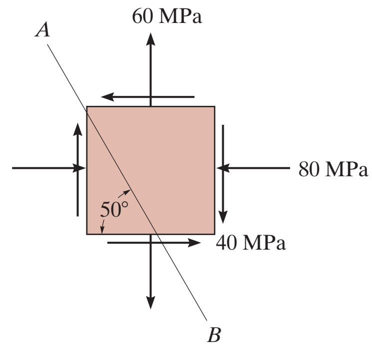
$\tau_{x'y'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{y'}}{\Delta A} = -0.404 \text{ ksi}$ **Ans.**

The negative signs indicate that the sense of $\sigma_{x'}$ and $\tau_{x'y'}$ are opposite to that shown on the FBD.

Ans:
$\sigma_{x'} = -4.05$ ksi,
$\tau_{x'y'} = -0.404$ ksi

14–7.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 14.1.



SOLUTION

Force Equilibrium: The areas, thus the forces acting on the faces of the lower segment of the sectioned element, are shown in Fig. *a*.

$$\nwarrow + \Sigma F_{y'} = 0; \quad \Delta F_{y'} + (40\,\Delta A \sin 50^\circ)\sin 50^\circ - (80\,\Delta A \sin 50^\circ)\cos 50^\circ$$
$$- (40\,\Delta A \cos 50^\circ)\cos 50^\circ - (60\,\Delta A \cos 50^\circ)\sin 50^\circ = 0$$
$$\Delta F_{y'} = 61.99\,\Delta A$$

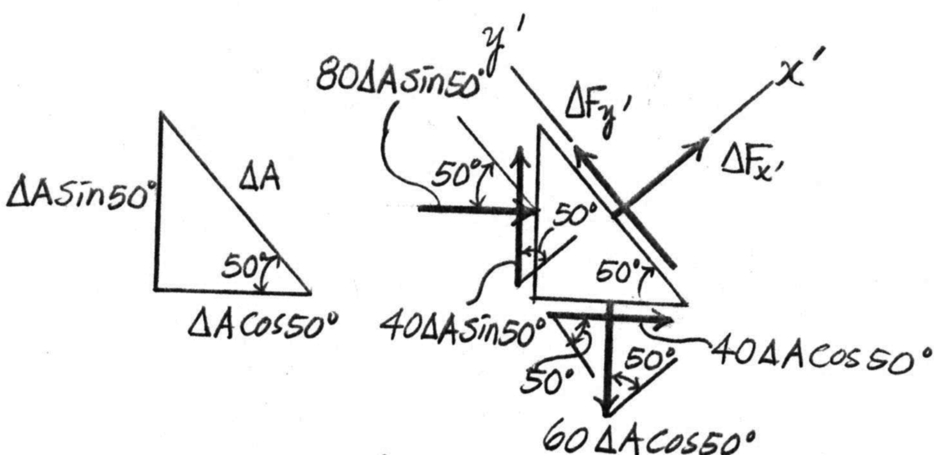
$$+\nearrow \Sigma F_{x'} = 0; \quad \Delta F_{x'} + (40\,\Delta A \sin 50^\circ)\cos 50^\circ + (80\,\Delta A \sin 50^\circ)\sin 50^\circ$$
$$+ (40\,\Delta A \cos 50^\circ)\sin 50^\circ - (60\,\Delta A \cos 50^\circ)\cos 50^\circ = 0$$
$$\Delta F_{x'} = -61.54\,\Delta A$$

Normal And Shear Stresses: For the inclined plane that has an area of ΔA,

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -61.54 \text{ MPa} = -61.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 61.99 \text{ MPa} = 62.0 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$ is a compressive normal stress.

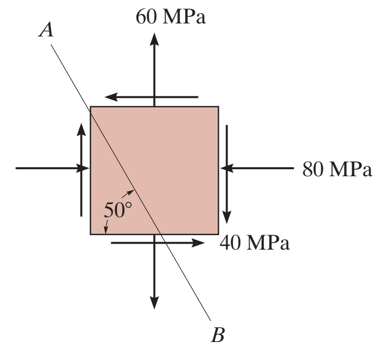


(a)

Ans:
$\sigma_{x'} = -61.5$ MPa,
$\tau_{x'y'} = 62.0$ MPa

***14–8.**

Solve Prob. 14–7 using the stress-transformation equations developed in Sec. 14.2.



SOLUTION

Normal And Shear Stress: In accordance with the established sign conventions,

$\theta = +40°$ (Fig. *a*) $\quad \sigma_x = -80 \text{ MPa} \quad \sigma_y = 60 \text{ MPa} \quad \tau_{xy} = -40 \text{ MPa}$

Stress Transformation Equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-80 + 60}{2} + \frac{-80 - 60}{2}\cos 80° + (-40)\sin 80°$$

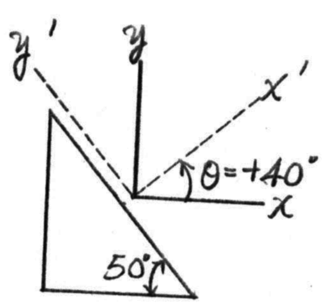
$$= -61.54 \text{ MPa} = -61.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

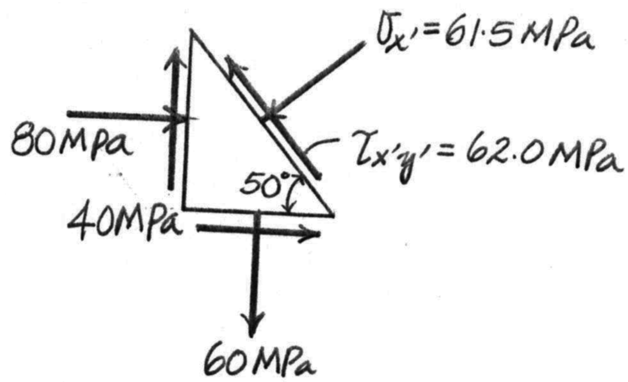
$$= -\left(\frac{-80 - 60}{2}\right)\sin 80° + (-40)\cos 80°$$

$$= 61.99 \text{ MPa} = 62.0 \text{ MPa} \qquad \textbf{Ans.}$$

These results are indicated on the sectioned element shown in Fig. *b*.



(a)

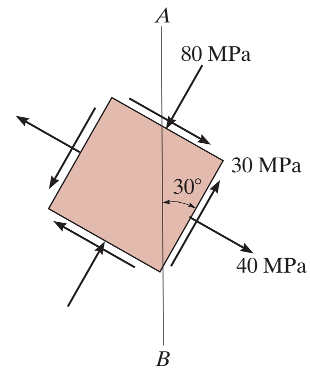


(b)

Ans:
$\sigma_{x'} = -61.5$ MPa,
$\tau_{x'y'} = 62.0$ MPa

14–9.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 14.1.



SOLUTION

Force Equilibrium: The areas, thus the forces acting on the faces of the upper segment of the sectioned element, are shown in Fig. *a*.

$+\downarrow \Sigma F_{y'} = 0; \quad \Delta F_{y'} + (80\,\Delta A \sin 30°)\cos 30° + (30\,\Delta A \sin 30°)\sin 30°$

$\qquad - (30\,\Delta A \cos 30°)\cos 30° + (40\,\Delta A \cos 30°)\sin 30° = 0$

$$\Delta F_{y'} = -36.96\,\Delta A$$

$\overset{+}{\leftarrow} \Sigma F_{x'} = 0; \quad \Delta F_{x'} - (30\,\Delta A \sin 30°)\cos 30° + (80\,\Delta A \sin 30°)\sin 30°$

$\qquad - (30\,\Delta A \cos 30°)\sin 30° - (40\,\Delta A \cos 30°)\cos 30° = 0$

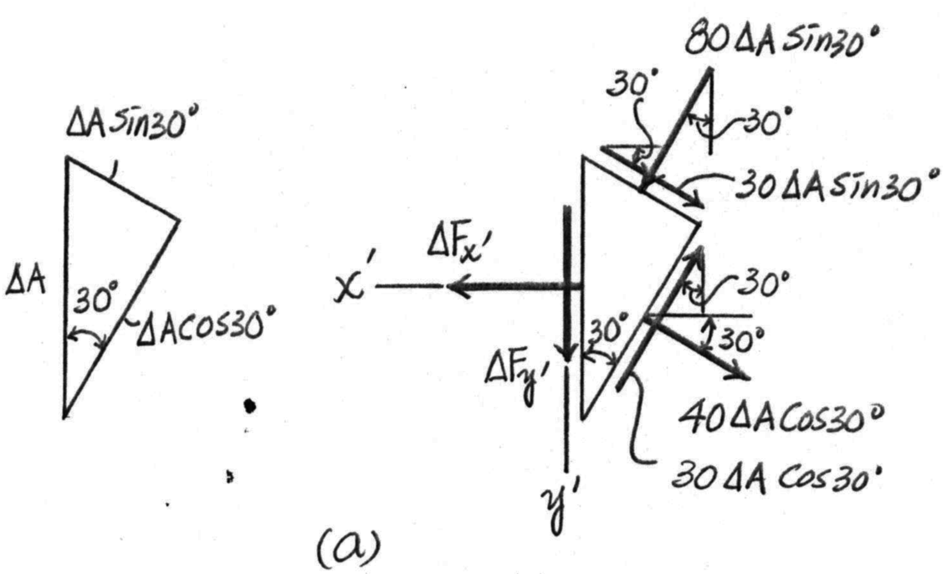
$$\Delta F_{x'} = 35.98\,\Delta A$$

Normal And Shear Stress: For the inclined plane that has an area of ΔA,

$$\sigma_{x'} = \lim_{\Delta A \to 0} = \frac{\Delta F_{x'}}{\Delta A} = 35.98\text{ MPa} = 36.0\text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} = \frac{\Delta F_{y'}}{\Delta A} = -36.96\text{ MPa} = -37.0\text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\tau_{x'y'}$ acts in the sense opposite to that shown in the FBD.

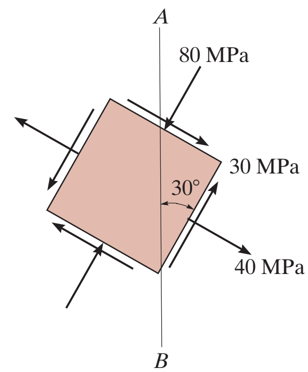


(a)

Ans:
$\sigma_{x'} = 36.0$ MPa,
$\tau_{x'y'} = -37.0$ MPa

14–10.

Solve Prob. 14–9 using the stress-transformation equation developed in Sec. 14.2.



SOLUTION

Normal And Shear Stress: In accordance with the established sign conventions,

$\theta = +120°$ (Fig. *a*) $\quad \sigma_x = -80$ MPa $\quad \sigma_y = 40$ MPa $\quad \tau_{xy} = -30$ MPa

Stress Transformation Equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-80 + 40}{2} + \left(\frac{-80 - 40}{2}\right)\cos 240° + (-30)\sin 240°$$

$$= 35.98 \text{ MPa} = 36.0 \text{ MPa}$$ **Ans.**

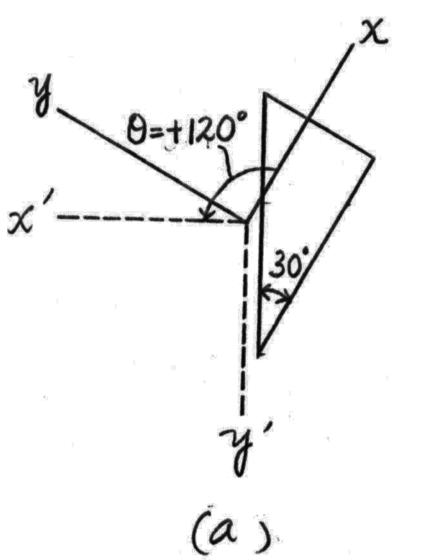
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

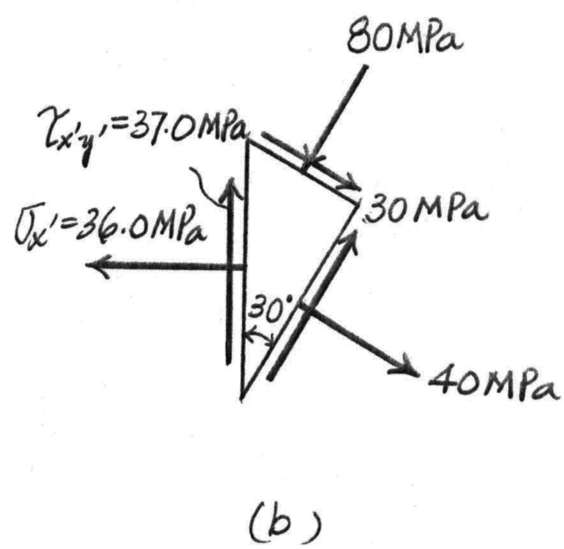
$$= -\left(\frac{-80 - 40}{2}\right)\sin 240° + (-30)\cos 240°$$

$$= -36.96 \text{ MPa} = -37.0 \text{ MPa}$$ **Ans.**

The negative sign indicates that $\tau_{x'y'}$ acts in the negative y' direction.

These results are indicated on the sectioned element shown in Fig. *b*.

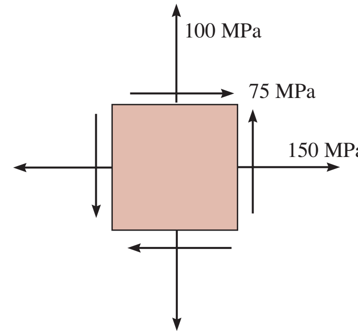




Ans:
$\sigma_{x'} = 36.0$ MPa,
$\tau_{x'y'} = -37.0$ MPa

14–11.

Determine the equivalent state of stress on an element at the same point oriented 60° clockwise with respect to the element shown. Sketch the results on the element.



SOLUTION

Stress Transformation Equations:

$\theta = -60°$ (Fig. *a*) $\quad \sigma_x = 150 \text{ MPa} \quad \sigma_y = 100 \text{ MPa} \quad \tau_{xy} = 75 \text{ MPa}$

We obtain

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} + \frac{150 - 100}{2}\cos(-120°) + 75\sin(-120°)$$

$$= 47.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} - \frac{150 - 100}{2}\cos(-120°) - 75\sin(-120°)$$

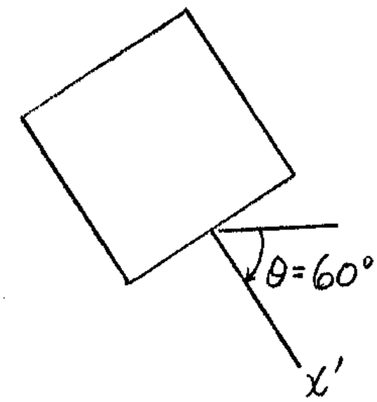
$$= 202 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

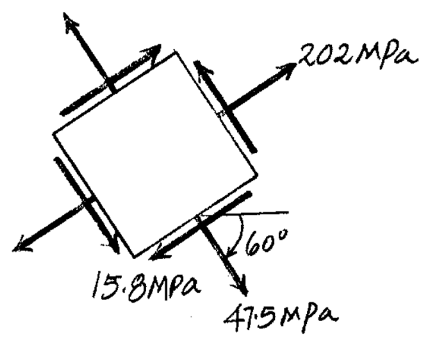
$$= -\frac{150 - 100}{2}\sin(-120°) + 75\cos(-120°)$$

$$= -15.8 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\tau_{x'y'}$ is directed towards the negative sense of the y' axis. These results are indicated on the element shown in Fig. *b*.



(a)

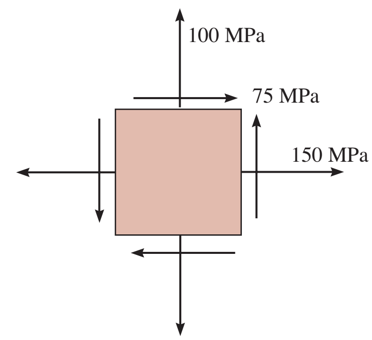


(b)

Ans:
$\sigma_{x'} = 47.5$ MPa,
$\sigma_{y'} = 202$ MPa,
$\tau_{x'y'} = -15.8$ MPa

***14–12.**

Determine the equivalent state of stress on an element at the same point oriented 60° counterclockwise with respect to the element shown. Sketch the results on the element.



SOLUTION

Stress Transformation Equations:

$\theta = +60°$ (Fig. *a*) $\quad \sigma_x = 150$ MPa $\quad \sigma_y = 100$ MPa $\quad \tau_{xy} = 75$ MPa

We obtain

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{150 + 100}{2} + \frac{150 - 100}{2}\cos 120° + 75 \sin 120°$$

$$= 177 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

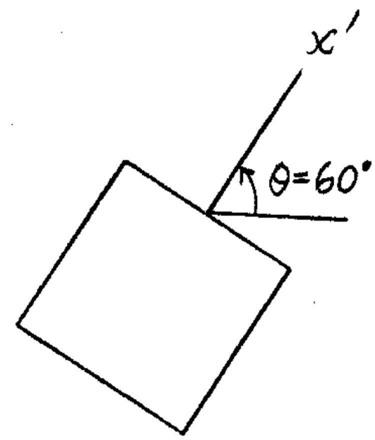
$$= \frac{150 + 100}{2} - \frac{150 - 100}{2}\cos 120° - 75 \sin 120°$$

$$= 72.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

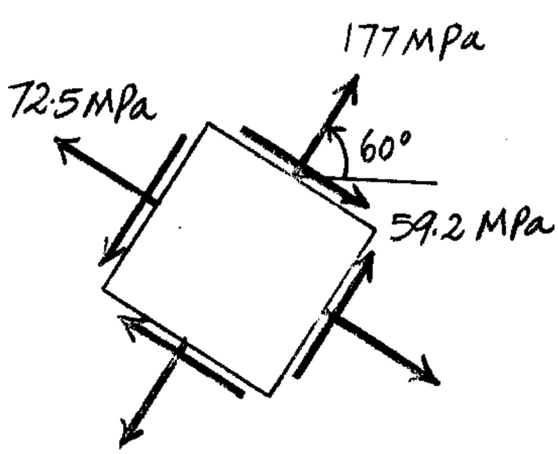
$$= -\frac{150 - 100}{2}\sin 120° + 75 \cos 120°$$

$$= -59.2 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

The negative sign indicates that $\tau_{x'y'}$ is directed towards the negative sense of the y' axis. These results are indicated on the element shown in Fig. *b*.

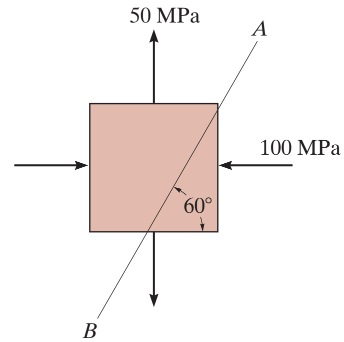


(b)

Ans:
$\sigma_{x'} = 177$ MPa,
$\sigma_{y'} = 72.5$ MPa,
$\tau_{x'y'} = -59.2$ MPa

14–13.

Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 14.1.



SOLUTION

Force Equilibrium: The areas, thus the forces acting on the faces of the lower segment of the sectioned element, are shown in Fig. *a*.

$$+\swarrow\Sigma F_{y'} = 0; \quad \Delta F_{y'} + (100\,\Delta A \sin 60^\circ)\cos 60^\circ + (50\,\Delta A \cos 60^\circ)\sin 60^\circ = 0$$

$$\Delta F_{y'} = -64.95\,\Delta A$$

$$\nwarrow+\Sigma F_{x'} = 0; \quad \Delta F_{x'} + (100\,\Delta A \sin 60^\circ)\sin 60^\circ - (50\,\Delta A \cos 60^\circ)\cos 60^\circ = 0$$

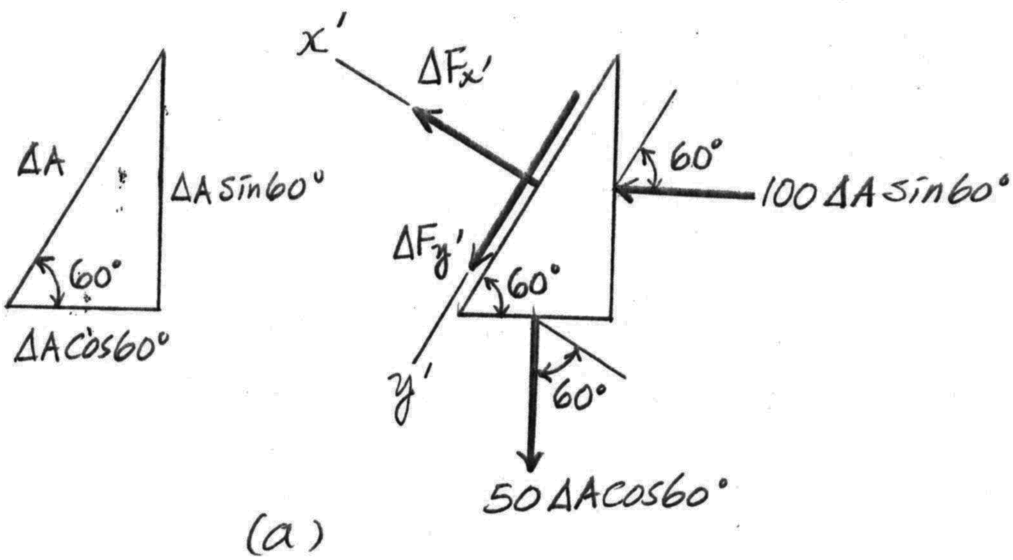
$$\Delta F_{x'} = -62.5\,\Delta A$$

Normal And Shear Stress: For the inclined plane that has an area of ΔA,

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -62.5\text{ MPa} = -62.5\text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = -64.95\text{ MPa} = -65.0\text{ MPa} \qquad \textbf{Ans.}$$

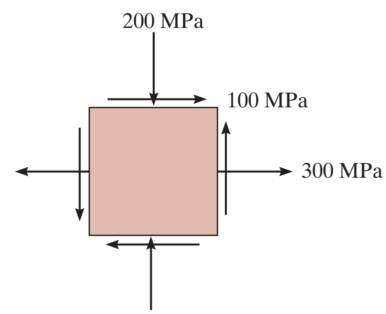
The negative signs indicate that $\sigma_{x'}$ is a compressive normal stress and $\tau_{x'y'}$ acts in the sense opposite to that shown in the FBD.



Ans:
$\sigma_{x'} = -62.5$ MPa,
$\tau_{x'y'} = -65.0$ MPa

14–14.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



SOLUTION

Normal And Shear Stress: In accordance with the established sign conventions,

$\sigma_x = 300 \text{ MPa} \qquad \sigma_y = -200 \text{ MPa} \qquad \tau_{xy} = 100 \text{ MPa}$

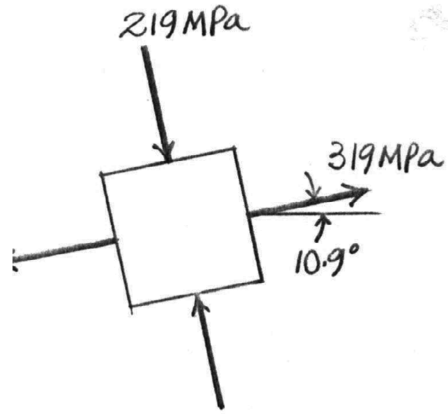
(a) In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{300 + (-200)}{2} \pm \sqrt{\left[\frac{300 - (-200)}{2}\right]^2 + 100^2}$$

$$= 50 \pm 269.26$$

$\sigma_1 = 319.26 \text{ MPa} = 319 \text{ MPa} \qquad \sigma_2 = -219.26 \text{ MPa} = -219 \text{ MPa}$ **Ans.**



(a)

Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{100}{[300 - (-200)]/2} = 0.4$$

$$\theta_p = 10.90° \text{ and } -79.10°$$

Substitute the result of $\theta = 10.90°$.

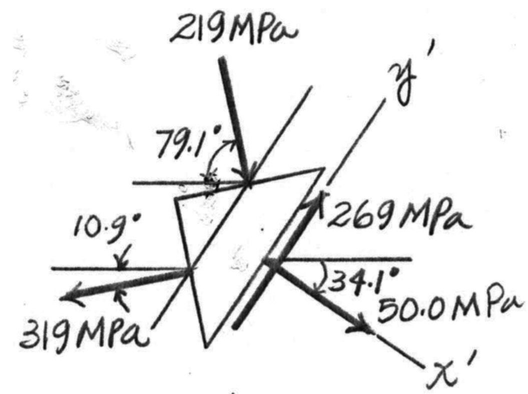
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{300 + (-200)}{2} + \frac{300 - (-200)}{2}\cos 21.80° + 100 \sin 21.80°$$

$$= 319.26 \text{ MPa} = \sigma_1$$

Hence,

$\theta_{p1} = 10.90° = 10.9° \qquad \theta_{p2} = -79.10° = -79.1°$ **Ans.**



(b)

Using these results, the state of in-plane principal stress can be represented by the differential element shown in Fig. *a*.

(b) Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left[\frac{300 - (-200)}{2}\right]^2 + 100^2} = 269.26 \text{ MPa} = 269 \text{ MPa}$$ **Ans.**

14–14. Continued

Orientation of the Plane for Maximum In-Plane Shear Stress:

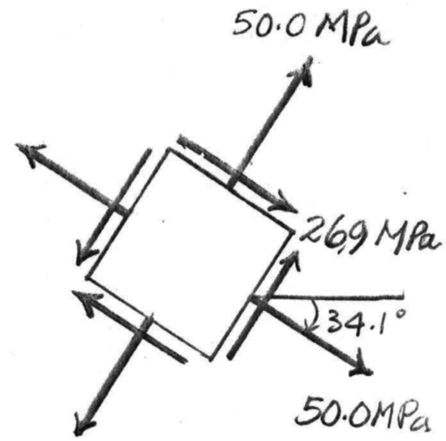
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-[300 - (-200)]/2}{100} = -2.5$$

$\theta_s = -34.10° = -34.1°$ and $55.90° = 55.9°$ **Ans.**

Average Normal Stress:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{300 + (-200)}{2} = 50.0 \text{ MPa}$$ **Ans.**

By observing the segment of the element of principal stresses sectioned through the diagonal, Fig. *b*, equilibrium along the y' axis requires that $\tau_{\substack{max \\ in\text{-}plane}}$ to act in the direction shown. Thus, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. *c*.

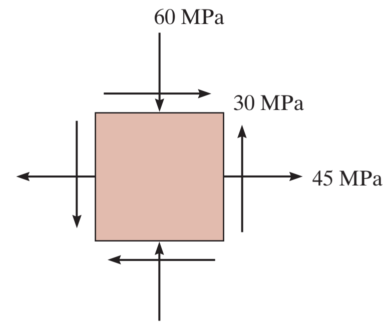


(C)

Ans:
$\sigma_1 = 319$ MPa,
$\sigma_2 = -219$ MPa,
$\theta_{p1} = 10.9°$,
$\theta_{p2} = -79.1°$,
$\tau_{\substack{max \\ in\text{-}plane}} = 269$ MPa,
$\theta_s = -34.1°$ and $55.9°$,
$\sigma_{avg} = 50.0$ MPa

14–15.

The state of stress at a point is shown on the element. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



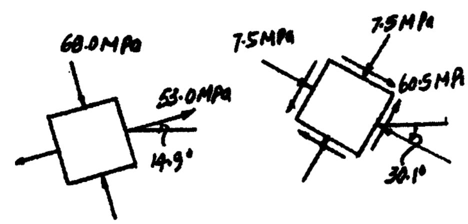
SOLUTION

$\sigma_x = 45 \text{ MPa} \qquad \sigma_y = -60 \text{ MPa} \qquad \tau_{xy} = 30 \text{ MPa}$

(a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2}$$

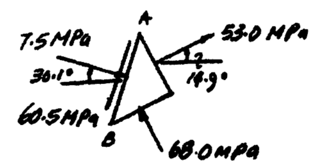
$\sigma_1 = 53.0 \text{ MPa} \qquad \sigma_2 = -68.0 \text{ MPa}$ **Ans.**



Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{30}{(45 - (-60))/2} = 0.5714$$

$\theta_p = 14.87, \quad -75.13$



Use Eq. 9–1 to determine the principal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 14.87°$$

$$= \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74° + 30 \sin 29.74° = 53.0 \text{ MPa}$$

Therefore, $\theta_{p1} = 14.9°$ and $\theta_{p2} = -75.1°$. **Ans.**

(b)

$$\tau_{\substack{\max\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2}$$

$= 60.5 \text{ MPa}$ **Ans.**

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50 \text{ MPa}$$ **Ans.**

Orientation of maximum in-plane shear stress:

$$\tan 2\theta_s = \frac{-\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = \frac{-(45 - (-60))/2}{30} = -1.75$$

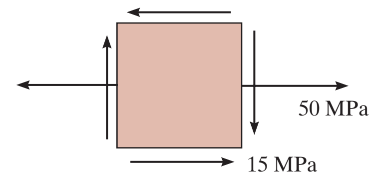
$\theta_s = -30.1°$ and $\theta_s = 59.9°$ **Ans.**

By observation, in order to preserve equilibrium along AB, $\tau_{\max}$ has to act in the direction shown.

Ans:
$\sigma_1 = 53.0$ MPa,
$\sigma_2 = -68.0$ MPa,
$\theta_{p1} = 14.9°$,
$\theta_{p2} = -75.1°$,
$\tau_{\substack{\max\\ \text{in-plane}}} = 60.5$ MPa,
$\sigma_{\text{avg}} = -7.50$ MPa,
$\theta_s = -30.1°$, $\theta_s = 59.9°$

***14–16.**

Determine the equivalent state of stress on an element at the point which represents (a) the principal stresses and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$$\sigma_x = 50\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -15\text{ MPa}$$

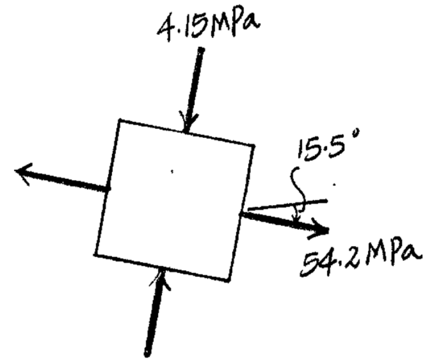
In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{50 + 0}{2} \pm \sqrt{\left(\frac{50 - 0}{2}\right)^2 + (-15)^2}$$

$$= 25 \pm \sqrt{850}$$

$$\sigma_1 = 54.2\text{ MPa} \qquad \sigma_2 = -4.15\text{ MPa} \qquad \textbf{Ans.}$$



(a)

Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-15}{(50 - 0)/2} = -0.6$$

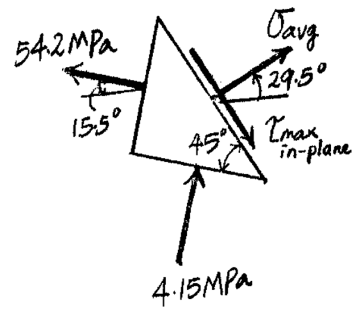
$$\theta_p = -15.48° \text{ and } 74.52°$$

Substitute $\theta = -15.48°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{50 + 0}{2} + \frac{50 - 0}{2}\cos(-30.96°) + (-15)\sin(-30.96°)$$

$$= 54.2\text{ MPa} = \sigma_1$$



(b)

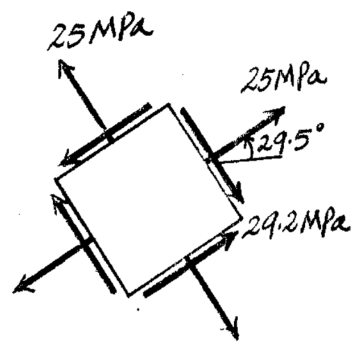
Thus,

$$(\theta_p)_1 = -15.5° \text{ and } (\theta_p)_2 = 74.5° \qquad \textbf{Ans.}$$

The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{50 - 0}{2}\right)^2 + (-15)^2} = 29.2\text{ MPa} \qquad \textbf{Ans.}$$



(c)

***14–16. Continued**

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(50 - 0)/2}{-15} = 1.667$$

$\theta_s = 29.5°$ and $120°$ **Ans.**

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

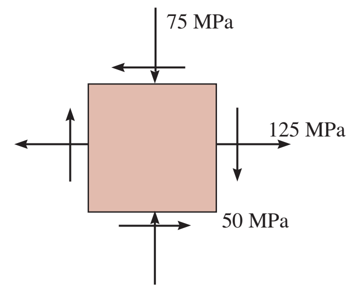
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 0}{2} = 25 \text{ MPa}$$ **Ans.**

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 54.2$ MPa,
$\sigma_2 = -4.15$ MPa,
$(\theta_p)_1 = -15.5°$,
$(\theta_p)_2 = 74.5°$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 29.2$ MPa,
$\theta_s = 29.5°$ and $120°$,
$\sigma_{avg} = 25$ MPa

14–17.

Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on each element.



SOLUTION

Normal and Shear Stress:

$\sigma_x = 125\text{ MPa} \qquad \sigma_y = -75\text{ MPa} \qquad \tau_{xy} = -50\text{ MPa}$

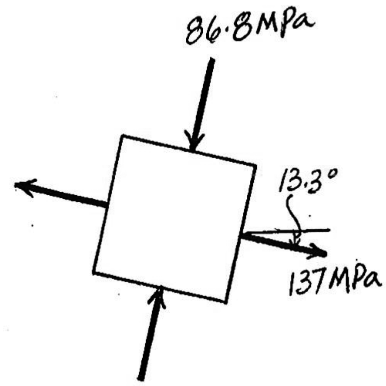
In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x - \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{125 + (-75)}{2} \pm \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + (-50)^2}$$

$$= 25 \pm \sqrt{12\,500}$$

$\sigma_1 = 137\text{ MPa} \qquad \sigma_2 = -86.8\text{ MPa}$ **Ans.**



(a)

Orientation of Principal Plane:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-50}{(125-(-75))/2} = -0.5$$

$$\theta_p = -13.28° \text{ and } 76.72°$$

Substitute $\theta = -13.28°$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

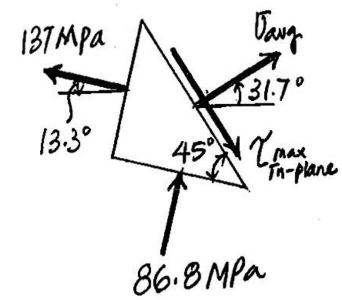
$$= \frac{125 + (-75)}{2} + \frac{125 - (-75)}{2}\cos(-26.57°) + (-50)\sin(-26.57°)$$

$$= 137\text{ MPa} = \sigma_1$$

Thus,

$$(\theta_p)_1 = -13.3° \text{ and } (\theta_p)_2 = 76.7°$$ **Ans.**

$$125 - (-75)/(-50)$$

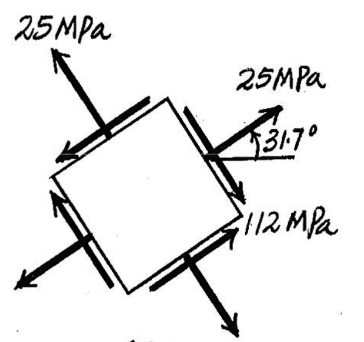


(b)

The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + 50^2} = 112\text{ MPa}$$ **Ans.**



(c)

14–17. Continued

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(125 - (-75))/2}{-50} = 2$$

$\theta_s = 31.7°$ and $122°$ **Ans.**

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

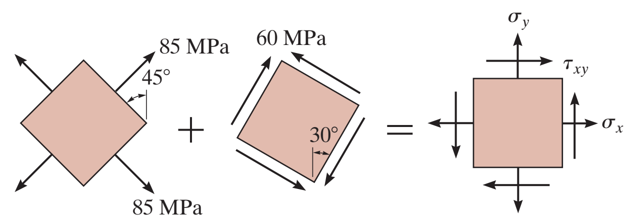
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{125 + (-75)}{2} = 25 \text{ MPa}$$ **Ans.**

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

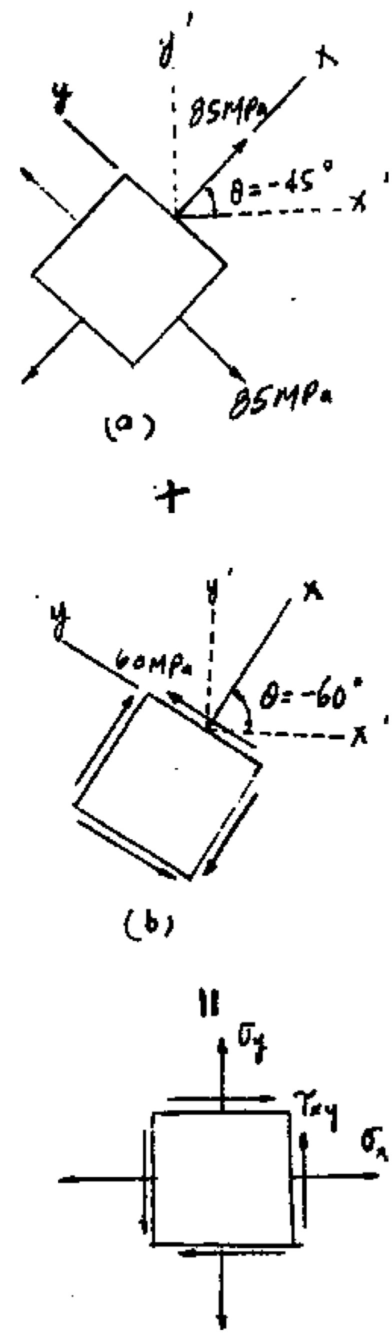
Ans:
$\sigma_1 = 137$ MPa, $\sigma_2 = -86.8$ MPa,
$\theta_{p1} = -13.3°$, $\theta_{p2} = 76.7°$, $\tau_{\substack{\max \\ \text{in-plane}}} = 112$ MPa,
$\theta_s = 31.7°$ and $122°$, $\sigma_{avg} = 25$ MPa

14–18.

A point on a thin plate is subjected to the two stress components. Determine the resultant state of stress represented on the element oriented as shown on the right.



SOLUTION



For element *a*:

$$\sigma_x = \sigma_y = 85\text{ MPa} \qquad \tau_{xy} = 0 \qquad \theta = -45°$$

$$(\sigma_{x'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{85 + 85}{2} + \frac{85 - 85}{2}\cos(-90°) + 0 = 85\text{ MPa}$$

$$(\sigma_{y'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{85 + 85}{2} - \frac{85 - 85}{2}\cos(-90°) - 0 = 85\text{ MPa}$$

$$(\tau_{x'y'})_a = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{85 - 85}{2}\sin(-90°) + 0 = 0$$

For element *b*:

$$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = 60\text{ MPa} \qquad \theta = -60°$$

$$(\sigma_{x'})_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + 60\sin(-120°) = -51.96\text{ MPa}$$

$$(\sigma_{y'})_b = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= 0 - 0 - 60\sin(-120°) = 51.96\text{ MPa}$$

$$(\tau_{x'y'})_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= -\frac{85 - 85}{2}\sin(-120°) + 60\cos(-120°) = -30\text{ MPa}$$

$\sigma_x = (\sigma_{x'})_a + (\sigma_{x'})_b = 85 + (-51.96) = 33.0\text{ MPa}$ **Ans.**

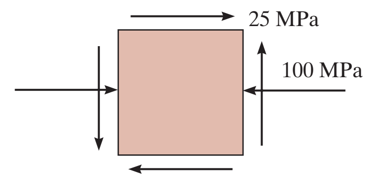
$\sigma_y = (\sigma_{y'})_a + (\sigma_{y'})_b = 85 + 51.96 = 137\text{ MPa}$ **Ans.**

$\tau_{xy} = (\tau_{x'y'})_a + (\tau_{x'y'})_b = 0 + (-30) = -30\text{ MPa}$ **Ans.**

Ans:
$\sigma_x = 33.0\text{ MPa}$,
$\sigma_y = 137\text{ MPa}$,
$\tau_{xy} = -30\text{ MPa}$

14–19.

Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown and sketch the results on the element.



SOLUTION

Normal and Shear Stress:

$$\sigma_x = -100 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 25 \text{ MPa}$$

In-Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-100 + 0}{2} \pm \sqrt{\left(\frac{-100 - 0}{2}\right)^2 + 25^2}$$

$$= -50 \pm \sqrt{3125}$$

$$\sigma_1 = 5.90 \text{ MPa} \qquad \sigma_2 = -106 \text{ MPa}$$ **Ans.**

Orientation of Principal Plane:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{25}{(-100 - 0)/2} = -0.5$$

$$\theta_p = -13.28^\circ \text{ and } 76.72^\circ$$

Substitute $\theta = -13.28^\circ$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-100 + 0}{2} + \frac{-100 - 0}{2} \cos(-26.57^\circ) + 25 \sin(-26.57^\circ)$$

$$= -106 \text{ MPa} = \sigma_2$$

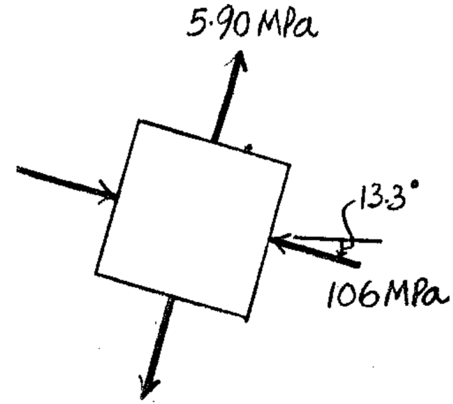
Thus,

$$(\theta_p)_1 = 76.7^\circ \text{ and } (\theta_p)_2 = -13.3^\circ$$ **Ans.**

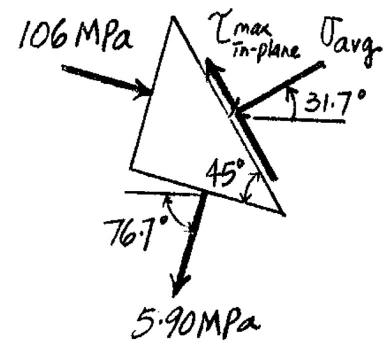
The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In-Plane Shear Stress:

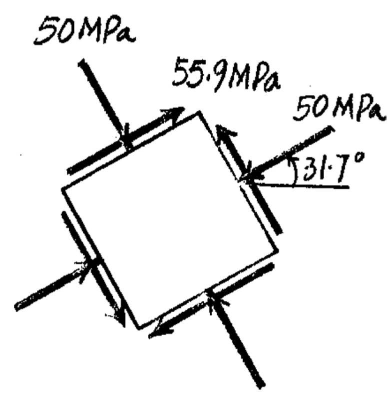
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-100 - 0}{2}\right)^2 + 25^2} = 55.9 \text{ MPa}$$ **Ans.**



(a)



(b)



(c)

14–19. Continued

Orientation of the Plane of Maximum In-Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-100 - 0)/2}{25} = 2$$

$\theta_s = 31.7°$ and $122°$ **Ans.**

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

Average Normal Stress:

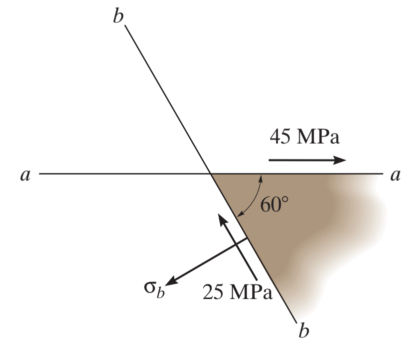
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-100 + 0}{2} = -50 \text{ MPa}$$ **Ans.**

The element that represents the state of maximum in-plane shear stress is shown in Fig. *c*.

Ans:
$\sigma_1 = 5.90$ MPa, $\sigma_2 = -106$ MPa,
$\theta_{p1} = 76.7°$ and $\theta_{p2} = -13.3°$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 55.9$ MPa, $\sigma_{avg} = -50$ MPa,
$\theta_s = 31.7°$ and $122°$

***14–20.**

The stress along two planes at a point is indicated. Determine the normal stresses on plane *b–b* and the principal stresses.

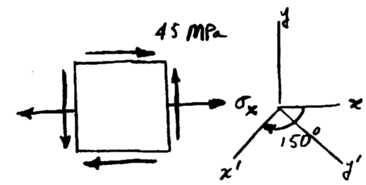


SOLUTION

$$\tau_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$-25 = -\frac{(\sigma_x - 0)}{2}\sin(-300°) + 45\cos(-300°)$$

$$\sigma_x = 109.70 \text{ MPa}$$



$$\sigma_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{109.70 + 0}{2} + \frac{109.70 - 0}{2}\cos(-300°) + 45\sin(-300°)$$

$$\sigma_b = 121 \text{ MPa}$$ **Ans.**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{109.70 + 0}{2} \pm \sqrt{\left(\frac{109.70 - 0}{2}\right)^2 + (45)^2}$$

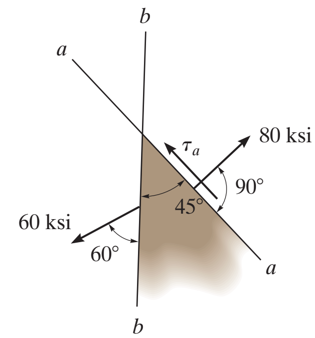
$$\sigma_1 = 126 \text{ MPa}$$ **Ans.**

$$\sigma_2 = -16.1 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_b = 121$ MPa,
$\sigma_1 = 126$ MPa,
$\sigma_2 = -16.1$ MPa

14–21.

The stress acting on two planes at a point is indicated. Determine the shear stress on plane *a–a* and the principal stresses at the point.



SOLUTION

$$\sigma_x = 60 \sin 60° = 51.962 \text{ ksi}$$

$$\tau_{xy} = 60 \cos 60° = 30 \text{ ksi}$$

$$\sigma_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$80 = \frac{51.962 + \sigma_y}{2} + \frac{51.962 - \sigma_y}{2} \cos (90°) + 30 \sin (90°)$$

$$\sigma_y = 48.038 \text{ ksi}$$

$$\tau_a = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos \theta$$

$$= -\left(\frac{51.962 - 48.038}{2}\right) \sin (90°) + 30 \cos (90°)$$

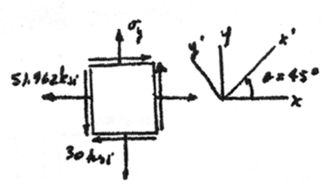
$$\tau_a = -1.96 \text{ ksi}$$ **Ans.**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{51.962 + 48.038}{2} \pm \sqrt{\left(\frac{51.962 - 48.038}{2}\right)^2 + (30)^2}$$

$$\sigma_1 = 80.1 \text{ ksi}$$ **Ans.**

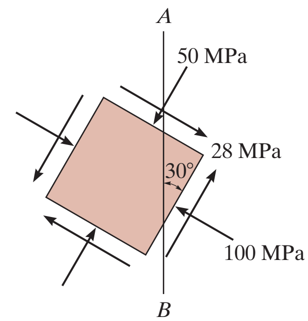
$$\sigma_2 = 19.9 \text{ ksi}$$ **Ans.**



Ans:
$\tau_a = -1.96$ ksi,
$\sigma_1 = 80.1$ ksi,
$\sigma_2 = 19.9$ ksi

14–22.

The state of stress at a point in a member is shown on the element. Determine the stress components acting on the plane AB.



SOLUTION

Construction of the Circle: In accordance with the sign convention, $\sigma_x = -50$ MPa, $\sigma_y = -100$ MPa, and $\tau_{xy} = -28$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-50 + (-100)}{2} = -75.0 \text{ MPa}$$

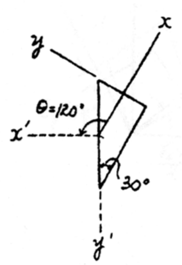
The coordinates for reference points A and C are $A(-50, -28)$ and $C(-75.0, 0)$.

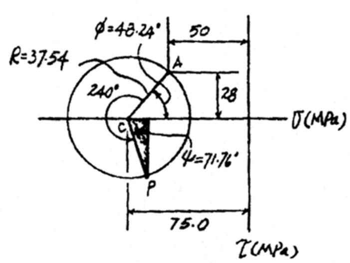
The radius of the circle is $R = \sqrt{(75.0 - 50)^2 + 28^2} = 37.54$ MPa.

Stress on the Rotated Element: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinates of point P on the circle

$$\sigma_{x'} = -75.0 + 37.54 \cos 71.76° = -63.3 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 37.54 \sin 71.76° = 35.7 \text{ MPa} \qquad \textbf{Ans.}$$

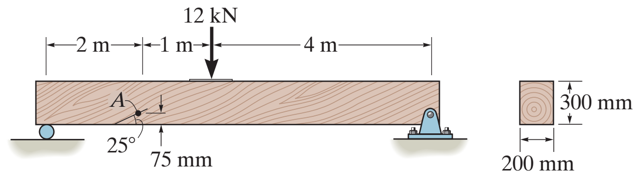




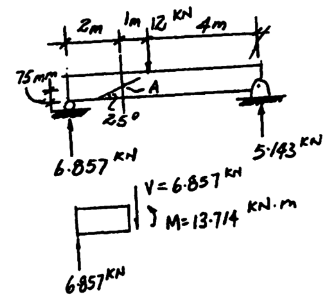
Ans:
$\sigma_{x'} = -63.3$ MPa,
$\tau_{x'y'} = 35.7$ MPa

14–23.

The wood beam is subjected to a load of 12 kN. If grains of wood in the beam at point A make an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular to the grains due to the loading.



SOLUTION



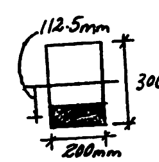
$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45\left(10^{-3}\right)\text{ m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875\left(10^{-3}\right)\text{ m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714\left(10^3\right)(0.075)}{0.45\left(10^{-3}\right)} = 2.2857\text{ MPa (T)}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{6.875\left(10^3\right)(1.6875)\left(10^{-3}\right)}{0.45\left(10^{-3}\right)(0.2)} = 0.1286\text{ MPa}$$

$$\sigma_x = 2.2857\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -0.1286\text{ MPa}$$

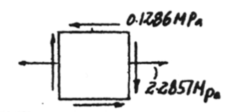


$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{2.2857 + 0}{2} \pm \sqrt{\left(\frac{2.2857 - 0}{2}\right)^2 + (-0.1286)^2}$$

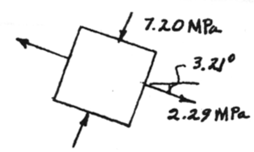
$\sigma_1 = 2.29\text{ MPa}$ **Ans.**

$\sigma_2 = -7.20\text{ kPa}$ **Ans.**



$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.1286}{(2.2857 - 0)/2}$$

$\theta_p = -3.21°$ **Ans.**



Check direction of principal stress:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

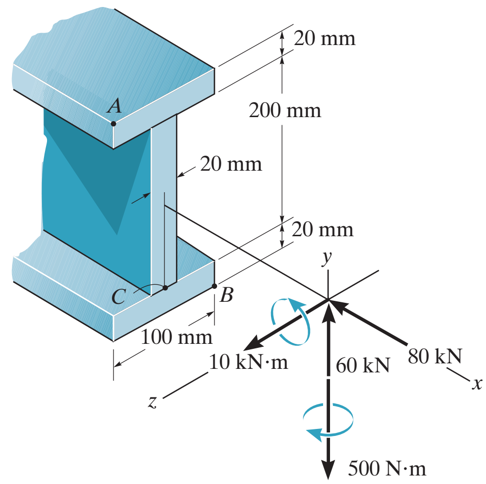
$$= \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos(-6.42°) - 0.1285\sin(-6.42)$$

$$= 2.29\text{ MPa}$$

Ans:
$\sigma_1 = 2.29\text{ MPa}$,
$\sigma_2 = -7.20\text{ kPa}$,
$\theta_p = -3.21°$

***14–24.**

The internal loadings at a section of the beam are shown. Determine the in-plane principal stresses at point A. Also compute the maximum in-plane shear stress at this point.



SOLUTION

Section Properties: For the wide flange section, Fig. *a*,

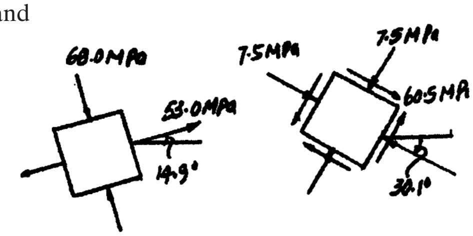
$$A = 0.1(0.24) - 0.08(0.2) = 0.008 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.24^3) - \frac{1}{12}(0.08)(0.2^3) = 61.8667(10^{-6}) \text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.02)(0.1^3)\right] + \frac{1}{12}(0.2)(0.02^3) = 3.4667(10^{-6}) \text{ m}^4$$

$$(Q_A)_y = 0$$

Internal Loadings: Here, $N_x = -80$ kN, $V_y = 60$ kN, $M_y = -0.5$ kN·m and $M_z = 10$ kN·m.



Normal Stress: For the combined loadings,

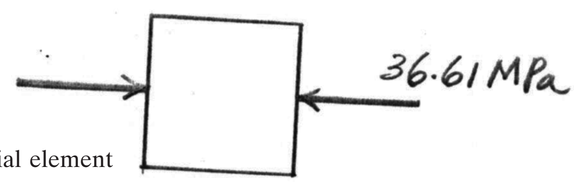
$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-80(10^3)}{0.008} - \frac{10(10^3)(0.12)}{61.8667(10^{-6})} + \frac{\left[-0.5(10^3)\right](0.05)}{3.4667(10^{-6})}$$

$$= -36.6080(10^6) \text{ Pa} = 36.61 \text{ MPa (C)}$$

Shear Stress: Since $(Q_A)_y = 0$,

$$\tau_A = 0$$



(b)

Thus, the state of stress at point A can be represented by the differential element shown in Fig. *b*.

In-Plane Principal Stresses: In accordance to the sign convention $\sigma_x = -36.61$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point A, Fig. *a*. Since no shear stress is acting on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_x = -36.61 \text{ MPa} = -36.6 \text{ MPa} \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

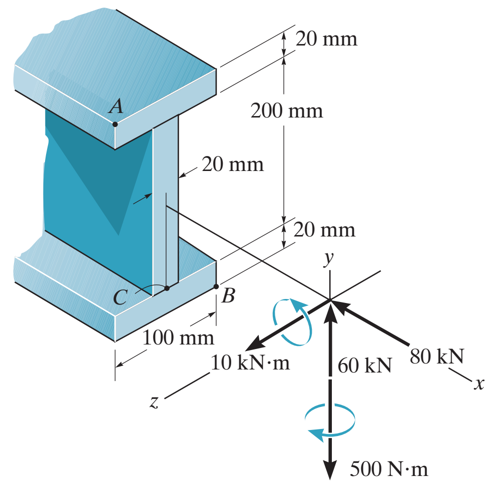
$$= \sqrt{\left(\frac{-36.61 - 0}{2}\right)^2 + 0}$$

$$= 18.30 \text{ MPa} = 18.3 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 0$,
$\sigma_2 = -36.6$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = 18.3$ MPa

14–25.

Solve Prob. 14–24 for point *B*.



SOLUTION

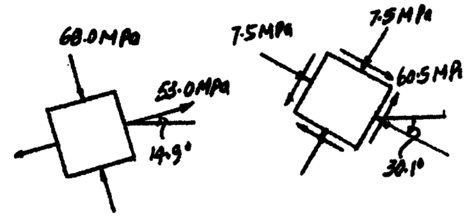
Section Properties: For the wide flange section, Fig. *a*,

$$A = 0.1(0.24) - 0.08(0.2) = 0.008 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.24^3) - \frac{1}{12}(0.08)(0.2^3) = 61.8667(10^{-6}) \text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.02)(0.1^3)\right] + \frac{1}{12}(0.2)(0.02^3) = 3.4667(10^{-6}) \text{ m}^4$$

$$(Q_B)_y = 0$$



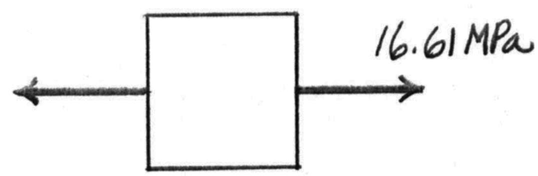
Internal Loadings: Here, $N_x = -80$ kN, $V_y = 60$ kN, $M_y = -0.5$ kN·m and $M_z = 10$ kN·m.

Normal Stress: For the combined loadings,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = \frac{-80(10^3)}{0.008} - \frac{10(10^3)(-0.12)}{61.8667(10^{-6})} + \frac{[-0.5(10^3)](-0.05)}{3.4667(10^{-6})}$$

$$= 16.6081(10^6) \text{ Pa} = 16.61 \text{ MPa (T)}$$



(b)

Shear Stress: Since $(Q_B)_y = 0$, $\tau_B = 0$.

Thus, the state of stress at point *B* can be represented by the differential element shown in Fig. *b*.

In-Plane Principal Stresses: In accordance to the sign convention $\sigma_x = 16.61$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point *B*, Fig. *a*. Since no shear stress is acting on the element,

$$\sigma_1 = \sigma_x = 16.61 \text{ MPa} = 16.6 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

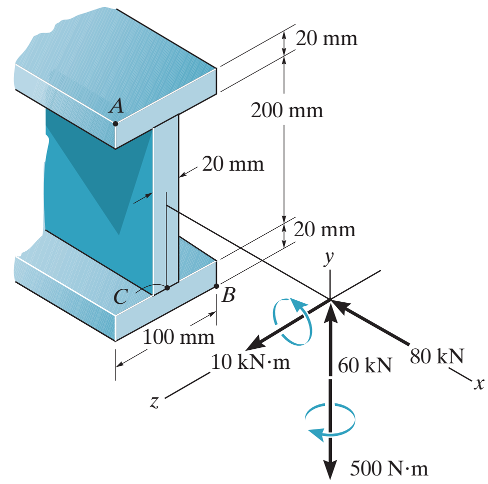
$$= \sqrt{\left(\frac{16.61 - 0}{2}\right)^2 + 0}$$

$$= 8.304 \text{ MPa} = 8.30 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 16.6$ MPa,
$\sigma_2 = 0$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 8.30$ MPa

14–26.

Solve Prob. 14–24 for point C.



SOLUTION

Section Properties: For the wide-flange section, Fig. *a*,

$$A = 0.1(0.24) - 0.08(0.2) = 0.008 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.1)(0.24^3) - \frac{1}{12}(0.08)(0.2^3) = 61.8667(10^{-6}) \text{ m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.02)(0.1^3)\right] + \frac{1}{12}(0.2)(0.02^3) = 3.4667(10^{-6}) \text{ m}^4$$

For the area shown shaded in Fig. *a*,

$$(Q_C)_y = \bar{y}'A' = 0.11[0.1(0.02)] = 0.22(10^{-3}) \text{ m}^3$$

Internal Loadings: Here, $N_x = -80$ kN, $V_y = 60$ kN, $M_y = -0.5$ kN·m and $M_z = 10$ kN·m.

Normal Stress: For the combined loadings,

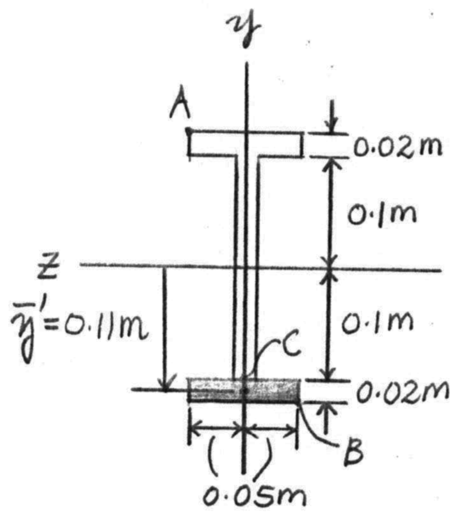
$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_C = \frac{-80(10^3)}{0.008} - \frac{10(10^3)(-0.1)}{61.8667(10^{-6})} + \frac{[-0.5(10^3)](0)}{3.4667(10^{-6})}$$

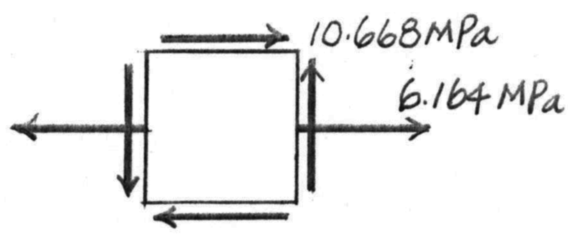
$$= 6.164(10^6) \text{ Pa} = 6.164 \text{ MPa (T)}$$

Shear Stress: Applying the shear formula,

$$\tau_c = \frac{V_y(Q_C)_y}{I_z t} = \frac{60(10^3)[0.22(10^{-3})]}{61.8667(10^{-6})(0.02)} = 10.668(10^6) \text{ Pa} = 10.668 \text{ MPa}$$



(a)



(b)

14–26. Continued

Thus, the state of stress at point C can be represented by the differential element shown in Fig. b.

In-Plane Principal Stress: In accordance to the sign convention, $\sigma_x = 6.164$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 10.668$ MPa for point C, Fig. a.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{6.164 + 0}{2} \pm \sqrt{\left(\frac{6.164 - 0}{2}\right)^2 + 10.668^2}$$

$$= 3.082 \pm 11.104$$

$\sigma_1 = 14.19 \text{ MPa} = 14.2 \text{ MPa} \qquad \sigma_2 = -8.022 \text{ MPa} = -8.02 \text{ MPa}$ **Ans.**

Maximum In-Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

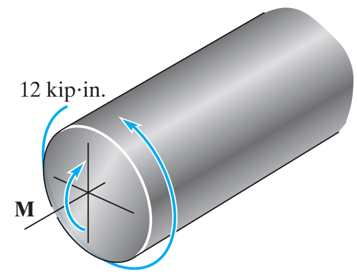
$$= \sqrt{\left(\frac{6.164 - 0}{2}\right)^2 + 10.668^2}$$

$= 11.104 \text{ MPa} = 11.1 \text{ MPa}$ **Ans.**

Ans:
$\sigma_1 = 14.2$ MPa,
$\sigma_2 = -8.02$ MPa,
$\tau_{\substack{\max \\ \text{in-plane}}} = 11.1$ MPa

14–27.

A rod has a circular cross section with a diameter of 2 in. It is subjected to a torque of 12 kip · in. and a bending moment **M**. The greater principal stress at the point of maximum flexural stress is 15 ksi. Determine the magnitude of the bending moment.



SOLUTION

$$J = \frac{\pi}{2}(1)^4 = 1.5708 \text{ in}^4$$

$$I = \frac{\pi}{4}(1)^2 = 0.7854 \text{ in}^4$$

$$\tau = \frac{Tc}{J} = \frac{12(1)}{1.5708} = 7.639 \text{ ksi}$$

$$\sigma_x = \frac{Mc}{I} = \frac{M(1)}{0.7854} = 1.2732M$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

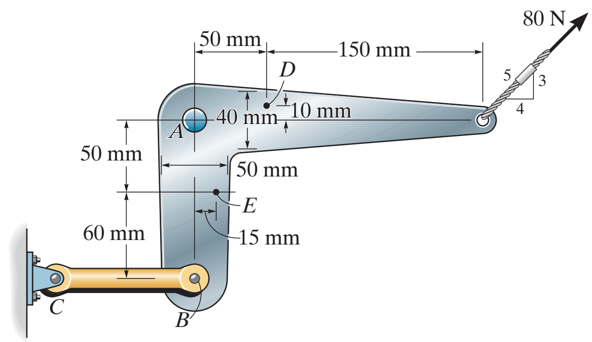
$$15 = \frac{1.2732M}{2} + \sqrt{\left(\frac{-1.2732M}{2}\right)^2 + 7.639^2}$$

$M = 8.73$ kip · in. **Ans.**

Ans:
$M = 8.73$ kip · in.

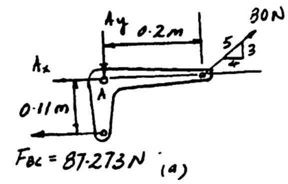
***14–28.**

The bell crank is pinned at A and supported by a short link BC. If it is subjected to the force of 80 N, determine the principal stresses at (a) point D and (b) point E. The crank is constructed from an aluminum plate having a thickness of 20 mm.



SOLUTION

Point D:



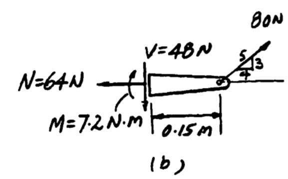
$$A = 0.04(0.02) = 0.8\left(10^{-3}\right)\text{ m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.04^3\right) = 0.1067\left(10^{-6}\right)\text{ m}^4$$

$$Q_D = \bar{y}'A' = 0.015(0.02)(0.01) = 3\left(10^{-6}\right)\text{ m}^3$$

Normal stress:

$$\sigma_D = \frac{P}{A} + \frac{My}{I} = \frac{64}{0.8\left(10^{-3}\right)} - \frac{7.2(0.01)}{0.1067\left(10^{-6}\right)} = -0.595\text{ MPa}$$



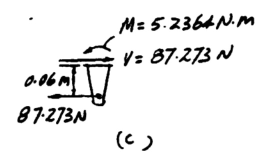
Shear stress:

$$\tau_D = \frac{VQ}{It} = \frac{48(3)\left(10^{-6}\right)}{0.1067\left(10^{-6}\right)(0.02)} = 0.0675\text{ MPa}$$

Principal stress: $\sigma_x = -0.595\text{ MPa} \quad \sigma_y = 0 \quad \tau_{xy} = 0.0675\text{ MPa}$

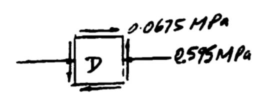
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-0.595 + 0}{2} \pm \sqrt{\left(\frac{-0.595 - 0}{2}\right)^2 + 0.0675^2}$$



$\sigma_1 = 7.56\text{ kPa}$ **Ans.**

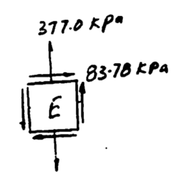
$\sigma_2 = -603\text{ kPa}$ **Ans.**



Point E:

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) = 0.2083\left(10^{-6}\right)\text{ m}^4$$

$$Q_E = \bar{y}'A' = 0.02(0.01)(0.02) = 4.0\left(10^{-6}\right)\text{ m}^3$$



Normal stress:

$$\sigma_E = \frac{My}{I} = \frac{5.2364(0.015)}{0.2083\left(10^{-6}\right)} = 377.0\text{ kPa}$$

Shear stress:

$$\tau_E = \frac{VQ}{It} = \frac{87.273(4.0)\left(10^{-6}\right)}{0.2083\left(10^{-6}\right)(0.02)} = 83.78\text{ kPa}$$

14–28. Continued

Principal stress: $\sigma_x = 0 \quad \sigma_y = 377.0\text{ kPa} \quad \tau_{xy} = 83.78\text{ kPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{0 + 377.0}{2} \pm \sqrt{\left(\frac{0 - 377.0}{2}\right)^2 + 83.78^2}$$

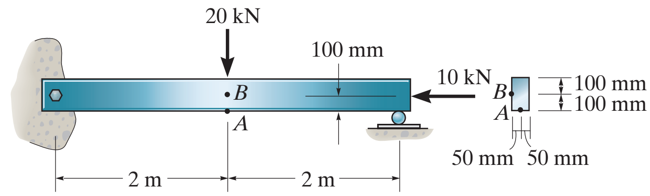
$\sigma_1 = 395\text{ kPa}$ **Ans.**

$\sigma_2 = -17.8\text{ kPa}$ **Ans.**

Ans:
Point *D*:
$\sigma_1 = 7.56\text{ kPa}$,
$\sigma_2 = -603\text{ kPa}$,
Point *E*:
$\sigma_1 = 395\text{ kPa}$,
$\sigma_2 = -17.8\text{ kPa}$

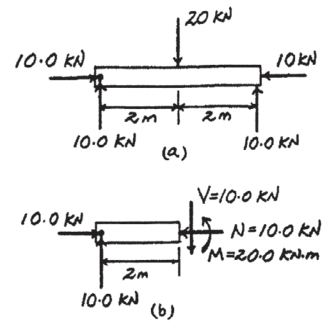
14–29.

The beam has a rectangular cross section and is subjected to the loadings shown. Determine the principal stresses at point A and point B, which are located just to the left of the 20-kN load. Show the results on elements located at these points.



SOLUTION

Internal Forces and Moment: As shown on FBD(b).



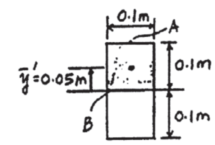
Section Properties:

$$A = 0.1\,(0.2) = 0.020 \text{ m}^2$$

$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.667\left(10^{-6}\right) \text{ m}^4$$

$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.05(0.1)(0.1) = 0.50\left(10^{-3}\right) \text{ m}^3$$



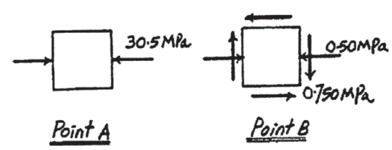
Normal Stresses:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{-10.0\left(10^3\right)}{0.020} - \frac{20.0\left(10^3\right)(0.1)}{66.667\left(10^{-6}\right)} = -30.5 \text{ MPa}$$

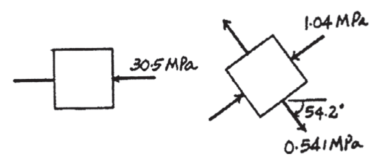
$$\sigma_B = \frac{-10.0\left(10^3\right)}{0.020} - \frac{20.0\left(10^3\right)(0)}{66.667\left(10^{-6}\right)} = -0.500 \text{ MPa}$$

Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$,



$$\tau_A = 0$$

$$\tau_B = \frac{10.0\left(10^3\right)\left[0.50\left(10^{-3}\right)\right]}{66.667\left(10^{-6}\right)(0.1)} = 0.750 \text{ MPa}$$



In-Plane Principal Stresses: $\sigma_x = -30.5$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_x = -30.5 \text{ MPa} \qquad \textbf{Ans.}$$

$\sigma_x = -0.500$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -0.750$ MPa for point B. Applying Eq. 9–5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-0.500 + 0}{2} \pm \sqrt{\left(\frac{-0.500 - 0}{2}\right)^2 + (-0.750)^2}$$

$$= -0.250 \pm 0.7906$$

$$\sigma_1 = 0.541 \text{ MPa} \qquad \sigma_2 = -1.04 \text{ MPa} \qquad \textbf{Ans.}$$

14–29. Continued

Orientation of Principal Plane: Applying Eq. 9–4 for point B,

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.750}{(-0.500 - 0)/2} = 3.000$$

$$\theta_P = 35.78^\circ \quad \text{and} \quad -54.22^\circ$$

Substituting the results into Eq. 9–1 with $\theta = 35.78^\circ$ yields

$$\begin{aligned}\sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ &= \frac{-0.500 + 0}{2} + \frac{-0.500 - 0}{2}\cos 71.56^\circ + (-0.750 \sin 71.56^\circ) \\ &= -1.04 \text{ MPa} = \sigma_2\end{aligned}$$

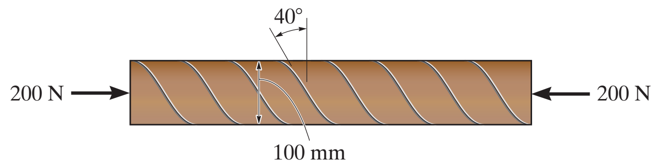
Hence,

$$\theta_{p1} = -54.2^\circ \qquad \theta_{p2} = 35.8^\circ$$ **Ans.**

Ans:
Point A:
$\sigma_1 = \sigma_y = 0$,
$\sigma_2 = \sigma_x = -30.5$ MPa,
Point B:
$\sigma_1 = 0.541$ MPa,
$\sigma_2 = -1.04$ MPa,
$\theta_{p1} = -54.2^\circ$,
$\theta_{p2} = 35.8^\circ$

14–30.

A paper tube is formed by rolling a cardboard strip in a spiral and then gluing the edges together as shown. Determine the shear stress acting along the seam, which is at 50° from the horizontal, when the tube is subjected to an axial compressive force of 200 N. The paper is 2 mm thick and the tube has an outer diameter of 100 mm.



SOLUTION

Normal And Shear Stresses: The normal stress is caused by the axial force only. Thus,

$$\sigma = \frac{N}{A} = \frac{-200}{\frac{\pi}{4}\left(0.1^2 - 0.096^2\right)} = -324.81\left(10^3\right)\text{ Pa} = 324.81\text{ kPa (C)}$$

Since there is no shear force on the cross-section,

$$\tau = 0$$

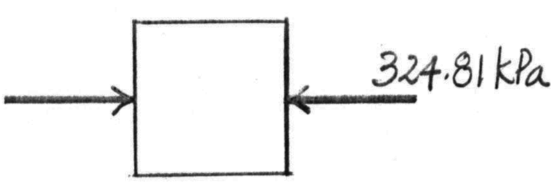
The state of stress of a point on the cross-section can be represented by the element shown in Fig. *a*.

Stress Transformation Equations: With $\sigma_x = -324.81$ kPa, $\sigma_y = 0$, $\tau_{xy} = 0$ and $\theta = +40°$ (Fig. *b*),

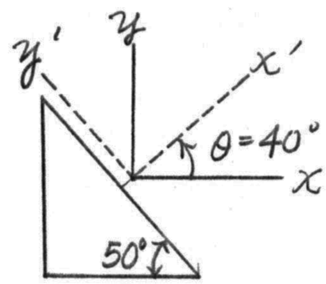
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{(-324.81 - 0)}{2}\sin 80° + 0$$

$$= 159.94\text{ kPa} = 160\text{ kPa}$$ **Ans.**



(a)

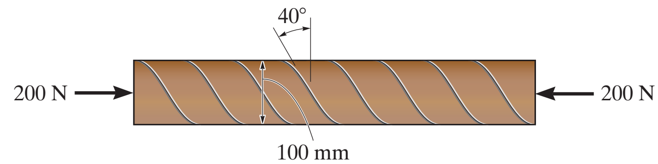


(b)

Ans:
$\tau_{x'y'} = 160$ kPa

14–31.

Solve Prob. 14–30 for the normal stress acting perpendicular to the seam.



SOLUTION

Normal and Shear Stresses: The normal stress is caused by the axial force only. Thus,

$$\sigma = \frac{N}{A} = \frac{-200}{\frac{\pi}{4}\left(0.1^2 - 0.096^2\right)} = -324.81\left(10^3\right)\text{ Pa} = 324.81\text{ kPa (C)}$$

Since there is no shear force on the cross-section,

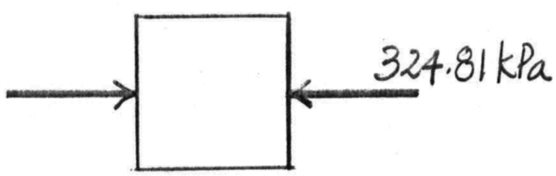
$$\tau = 0$$

The state of stress of a point on the cross-section can be represented by the element shown in Fig. *a*.

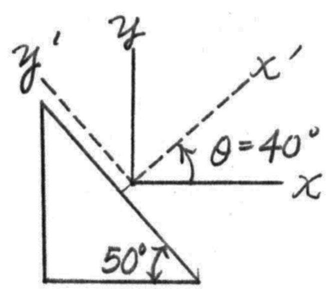
Stress Transformation Equations: With $\sigma_x = -324.81$ kPa, $\sigma_y = 0$, $\tau_{xy} = 0$ and $\theta = +40°$ (Fig. *b*),

$$\begin{aligned}\sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \\ &= \frac{-324.81 + 0}{2} + \frac{(-324.81 - 0)}{2}\cos 80° + 0 \\ &= -190.60\text{ kPa} = -191\text{ kPa}\end{aligned}$$

Ans.



(a)



(b)

Ans:
$\sigma_{x'} = -191$ kPa

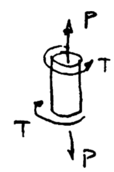
***14–32.**

The 2-in.-diameter drive shaft AB on the helicopter is subjected to an axial tension of 10 000 lb and a torque of 300 lb·ft. Determine the principal stresses and the maximum in-plane shear stress that act at a point on the surface of the shaft.

SOLUTION

$$\sigma = \frac{P}{A} = \frac{10\,000}{\pi(1)^2} = 3.183 \text{ ksi}$$

$$\tau = \frac{Tc}{J} = \frac{300(12)(1)}{\frac{\pi}{2}(1)^4} = 2.292 \text{ ksi}$$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{3.183 + 0}{2} \pm \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

$\sigma_1 = 4.38$ ksi **Ans.**

$\sigma_2 = -1.20$ ksi **Ans.**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

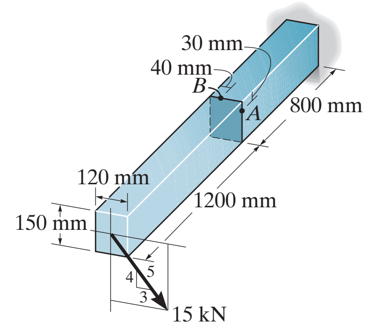
$$= \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

$= 2.79$ ksi **Ans.**

Ans:
$\sigma_1 = 4.38$ ksi,
$\sigma_2 = -1.20$ ksi,
$\tau_{\substack{\max \\ \text{in-plane}}} = 2.79$ ksi

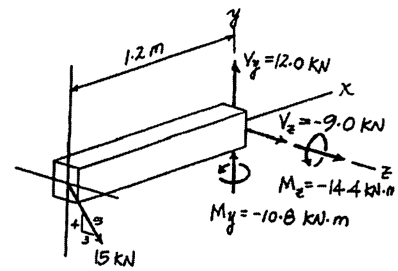
14–33.

Determine the principal stresses in the cantilevered beam at points A and B.



SOLUTION

Internal Forces and Moment: As shown on FBD.



Section Properties:

$$I_z = \frac{1}{12}(0.12)(0.15^3) = 33.75(10^{-6})\text{ m}^4$$

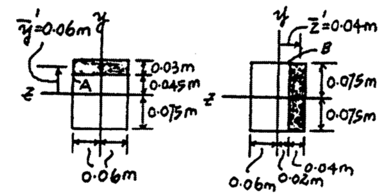
$$I_y = \frac{1}{12}(0.15)(0.12^3) = 21.6(10^{-6})\text{ m}^4$$

$$(Q_A)_y = \bar{y}'A' = 0.06(0.03)(0.12) = 0.216(10^{-3})\text{ m}^3$$

$$(Q_A)_z = 0$$

$$(Q_B)_z = \bar{z}'A' = 0.04(0.04)(0.15) = 0.240(10^{-3})\text{ m}^3$$

$$(Q_B)_y = 0$$



Normal Stress:

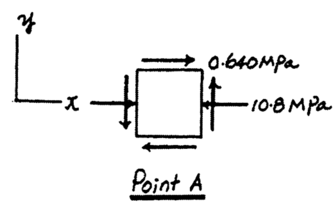
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-14.4(10^3)(0.045)}{33.75(10^{-6})} + \frac{-10.8(10^3)(0.06)}{21.6(10^{-6})}$$

$$= -10.8\text{ MPa}$$

$$\sigma_B = -\frac{-14.4(10^3)(0.075)}{33.75(10^{-6})} + \frac{-10.8(10^3)(-0.02)}{21.6(10^{-6})}$$

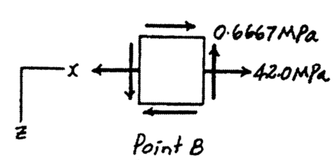
$$= 42.0\text{ MPa}$$



Shear Stress: Applying the shear formula,

$$\tau_A = \frac{V_y (Q_A)_y}{I_z t} = \frac{12.0(10^3)\left[0.216(10^{-3})\right]}{33.75(10^{-6})(0.12)} = 0.640\text{ MPa}$$

$$\tau_B = \frac{V_z (Q_B)_z}{I_y t} = \frac{-9.00(10^3)\left[0.240(10^{-3})\right]}{21.6(10^{-6})(0.15)} = -0.6667\text{ MPa}$$



In-Plane Principal Stress: $\sigma_x = -10.8$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0.640$ MPa for point A.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-10.8 + 0}{2} \pm \sqrt{\left(\frac{-10.8 - 0}{2}\right)^2 + 0.640^2}$$

$$= -5.40 \pm 5.4378$$

$$\sigma_1 = 37.8\text{ kPa} \qquad \sigma_2 = -10.8\text{ MPa}$$ **Ans.**

14–33. Continued

$\sigma_x = 42.0$ MPa, $\sigma_z = 0$, and $\tau_{xz} = 0.6667$ MPa for point B.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{42.0 + 0}{2} \pm \sqrt{\left(\frac{42.0 - 0}{2}\right)^2 + 0.6667^2}$$

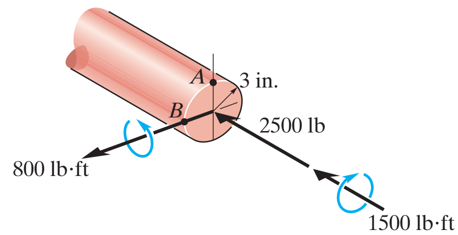
$$= 21.0 \pm 21.0105$$

$\sigma_1 = 42.0$ MPa $\qquad \sigma_2 = -10.6$ kPa **Ans.**

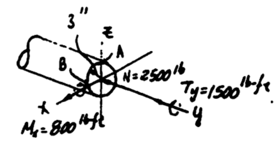
Ans:
Point A:
$\sigma_1 = 37.8$ kPa,
$\sigma_2 = -10.8$ MPa,
Point B:
$\sigma_1 = 42.0$ MPa,
$\sigma_2 = -10.6$ kPa

14–34.

The internal loadings at a cross section through the 6-in.-diameter drive shaft of a turbine consist of an axial force of 2500 lb, a bending moment of 800 lb · ft, and a torsional moment of 1500 lb · ft. Determine the principal stresses at point A. Also calculate the maximum in-plane shear stress at this point.



SOLUTION

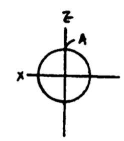


$$A = \pi(3)^2 = 28.274 \text{ in}^2$$

$$J = \frac{\pi}{2}(3^4) = 127.23 \text{ in}^4$$

$$I = \frac{\pi}{4}(3^4) = 63.62 \text{ in}^4$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x c}{I} = -\frac{2500}{28.274} - \frac{800(12)(3)}{63.62} = -541.1 \text{ psi}$$

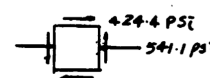


$$\tau_A = \frac{T_y c}{J} = \frac{1500(12)(3)}{127.23} = 424.4 \text{ psi}$$

$$\sigma_x = -541.1 \text{ psi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 424.4 \text{ psi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-541.1 + 0}{2} \pm \sqrt{\left(\frac{-541.1 - 0}{2}\right)^2 + (424.4)^2}$$



$$\sigma_1 = 233 \text{ psi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -774 \text{ psi} \qquad \textbf{Ans.}$$

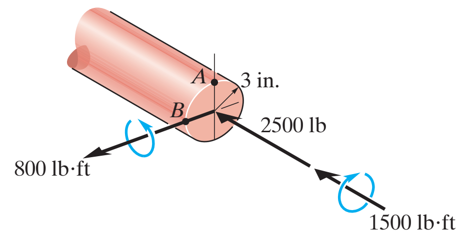
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{-541.1 - 0}{2}\right)^2 + (424.4)^2} = 503 \text{ psi} \qquad \textbf{Ans.}$$

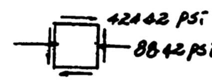
Ans:
$\sigma_1 = 233$ psi,
$\sigma_2 = -774$ psi,
$\tau_{\substack{\max \\ \text{in-plane}}} = 503$ psi

14–35.

The internal loadings at a cross section through the 6-in.-diameter drive shaft of a turbine consist of an axial force of 2500 lb, a bending moment of 800 lb·ft, and a torsional moment of 1500 lb·ft. Determine the principal stresses at point B. Also calculate the maximum in-plane shear stress at this point.



SOLUTION



$$A = \pi(3)^2 = 28.274 \text{ in}^2$$

$$J = \frac{\pi}{2}\left(3^4\right) = 127.23 \text{ in}^4$$

$$I = \frac{\pi}{4}\left(3^4\right) = 63.62 \text{ in}^4$$

$$\sigma_B = -\frac{P}{A} = -\frac{2500}{28.274} = -88.42 \text{ psi}$$

$$\tau_B = \frac{T_y c}{J} = \frac{1500(12)(3)}{127.23} = 424.42$$

$$\sigma_x = -88.42 \text{ psi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 424.4 \text{ psi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-88.42 + 0}{2} \pm \sqrt{\left(\frac{-88.42 - 0}{2}\right)^2 + (424.4)^2}$$

$$\sigma_1 = 382 \text{ psi}$$ **Ans.**

$$\sigma_2 = -471 \text{ psi}$$ **Ans.**

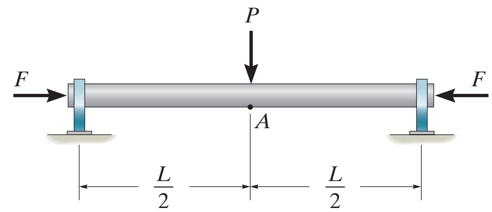
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{-88.42 - 0}{2}\right)^2 + (424.42)^2} = 427 \text{ psi}$$ **Ans.**

Ans:
$\sigma_1 = 382$ psi,
$\sigma_2 = -471$ psi,
$\tau_{\substack{\max \\ \text{in-plane}}} = 427$ psi

***14–36.**

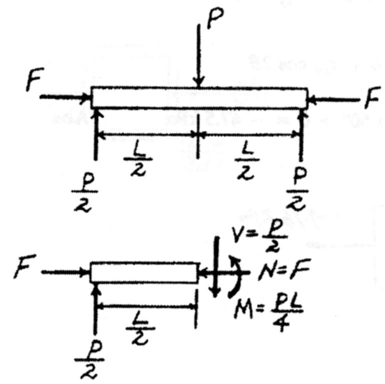
The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stresses and the maximum in-plane shear stress at point A. The bearings only support vertical reactions.



SOLUTION

Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).



Section Properties:

$$A = \frac{\pi}{4}d^2 \quad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4 \quad Q_A = 0$$

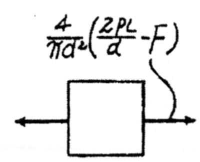
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-F}{\frac{\pi}{4}d^2} \pm \frac{\frac{PL}{4}\left(\frac{d}{2}\right)}{\frac{\pi}{64}d^4}$$

$$\sigma_A = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right)$$

Shear Stress: Since $Q_A = 0, \tau_A = 0$.



In-Plane Principal Stress: $\sigma_x = \dfrac{4}{\pi d^2}\left(\dfrac{2PL}{d} - F\right)$.

$\sigma_y = 0$ and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_x = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress: Applying Eq. 9–7 for point A,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{\frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) - 0}{2}\right)^2 + 0}$$

$$= \frac{2}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans.}$$

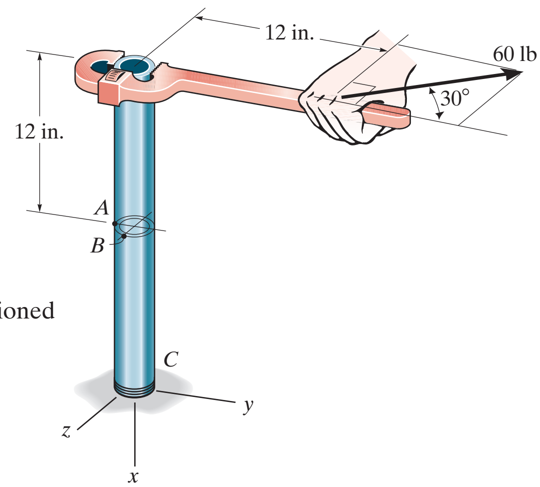
Ans:

$\sigma_1 = \dfrac{4}{\pi d^2}\left(\dfrac{2PL}{d} - F\right), \sigma_2 = 0,$

$\tau_{\substack{\max \\ \text{in-plane}}} = \dfrac{2}{\pi d^2}\left(\dfrac{2PL}{d} - F\right)$

14–37.

The steel pipe has an inner diameter of 2.75 in. and an outer diameter of 3 in. If it is fixed at C and subjected to the horizontal 60-lb force acting on the handle of the pipe wrench at its end, determine the principal stresses in the pipe at point A, which is located on the outer surface of the pipe.



SOLUTION

Internal Loadings: Consider the equilibrium of the upper segment of the sectioned pipe, Fig. *a*.

$\Sigma F_x = 0;$	$N_x = 0$	
$\Sigma F_y = 0;$	$V_y + 60 \cos 30° = 0$	$V_y = -51.96 \text{ lb}$
$\Sigma F_z = 0;$	$V_z - 60 \sin 30° = 0$	$V_z = 30.0 \text{ lb}$
$\Sigma M_x = 0;$	$T_x - 60 \sin 30°(12) = 0$	$T_x = 360 \text{ lb} \cdot \text{in.}$
$\Sigma M_y = 0;$	$M_y - 60 \sin 30°(12) = 0$	$M_y = 360 \text{ lb} \cdot \text{in.}$
$\Sigma M_z = 0;$	$M_z - 60 \cos 30°(12) = 0$	$M_z = 623.54 \text{ lb} \cdot \text{in.}$

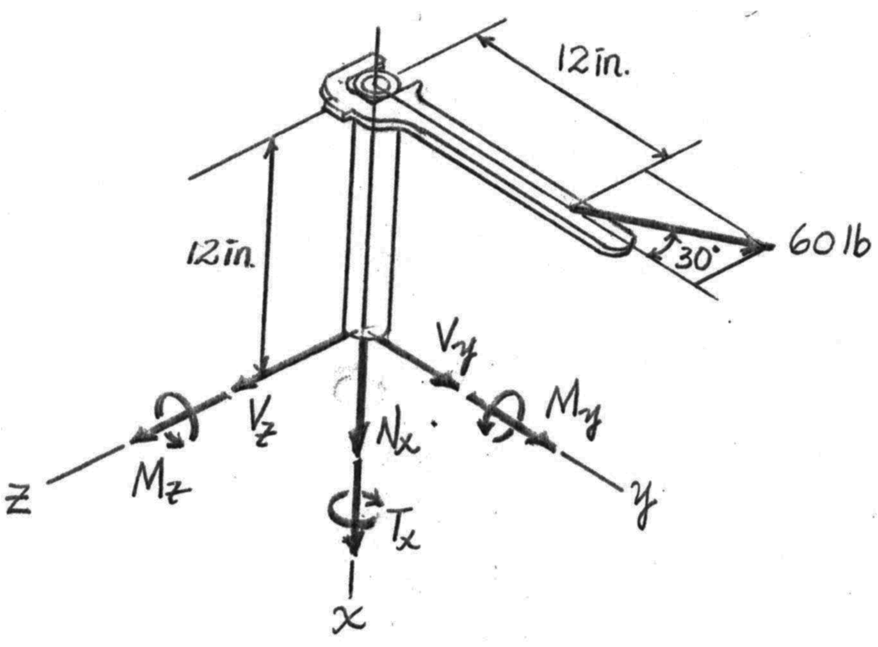
Section Properties: For the circular tubular cross section, Fig. *b,*

$$I_y = I_z = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687 \text{ in}^4$$

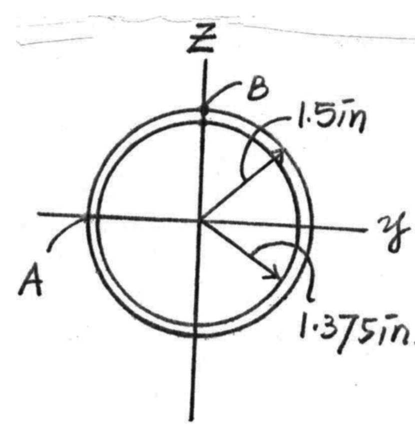
$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374 \text{ in}^4$$

$$(Q_A)_z = \Sigma \bar{y}'A' = \frac{4(1.5)}{3\pi}\left[\frac{\pi}{2}\left(1.5^2\right)\right] - \frac{4(1.375)}{3\pi}\left[\frac{\pi}{2}\left(1.375^2\right)\right] = 0.51693 \text{ in}^3$$

$$(Q_A)_y = 0$$



(a)



(b)

14–37. Continued

Normal Stress: For the combined loadings, the normal stress at point A can be determined from

$$\sigma_A = \frac{N_x}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$= 0 - \frac{623.54(-1.5)}{1.1687} + \frac{360(0)}{1.1687}$$

$$= 800.30 \text{ psi (T)}$$

Shear Stress: The transverse shear stress in z and y directions and torsional shear stress can be obtained using the shear formula $\tau_V = \dfrac{VQ}{It}$ and the torsion formula $\tau_T = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xz})_A = (\tau_V)_z - \tau_T$$

$$= \frac{30.0(0.51693)}{1.1687[2(0.125)]} - \frac{360(1.5)}{2.3374}$$

$$= -177.95 \text{ psi}$$

$$(\tau_{xy})_A = 0$$

The state of stress at point A can be represented by the differential element shown in Fig. c.

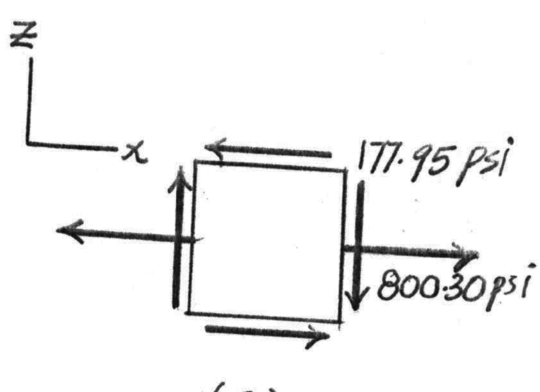
In-Plane Principal Stress: In accordance to the sign convention, the stresses on the element, Fig. c, are $\sigma_x = 800.30$ psi, $\sigma_z = 0$ and $\tau_{xz} = -800.30$ psi.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}{}^2}$$

$$= \frac{800.30 + 0}{2} \pm \sqrt{\left(\frac{800.30 - 0}{2}\right)^2 + (-177.95)^2}$$

$$= 400.15 \pm 437.93$$

$\sigma_1 = 838.08 \text{ psi} = 838 \text{ psi} \qquad \sigma_2 = -37.78 \text{ psi} = -37.8 \text{ psi}$ **Ans.**

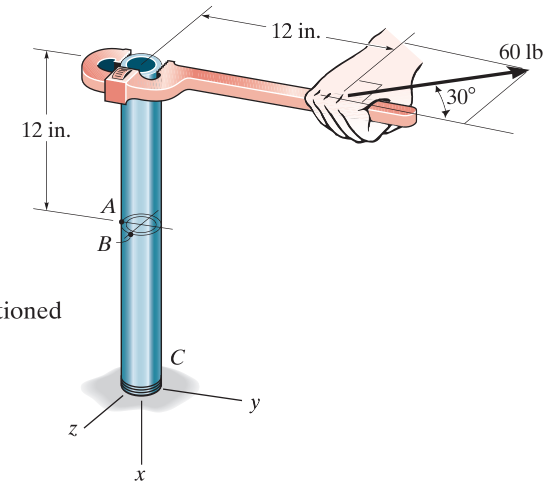


(c)

Ans:
$\sigma_1 = 838$ psi,
$\sigma_2 = -37.8$ psi

14–38.

Solve Prob. 14–37 for point B, which is located on the outer surface of the pipe.



SOLUTION

Internal Loadings: Consider the equilibrium of the upper segment of the sectioned pipe, Fig. *a*.

$\Sigma F_x = 0;\quad N_x = 0$

$\Sigma F_y = 0;\quad V_y + 60\cos 30° = 0\quad V_y = -51.96\text{ lb}$

$\Sigma F_z = 0;\quad V_z - 60\sin 30° = 0\quad V_z = 30.0\text{ lb}$

$\Sigma M_x = 0;\quad T_x - 60\sin 30°(12) = 0\quad T_x = 360\text{ lb}\cdot\text{in}$

$\Sigma M_y = 0;\quad M_y - 60\sin 30°(12) = 0\quad M_y = 360\text{ lb}\cdot\text{in}$

$\Sigma M_z = 0;\quad M_z - 60\cos 30°(12) = 0\quad M_z = 623.54\text{ lb}\cdot\text{in}$

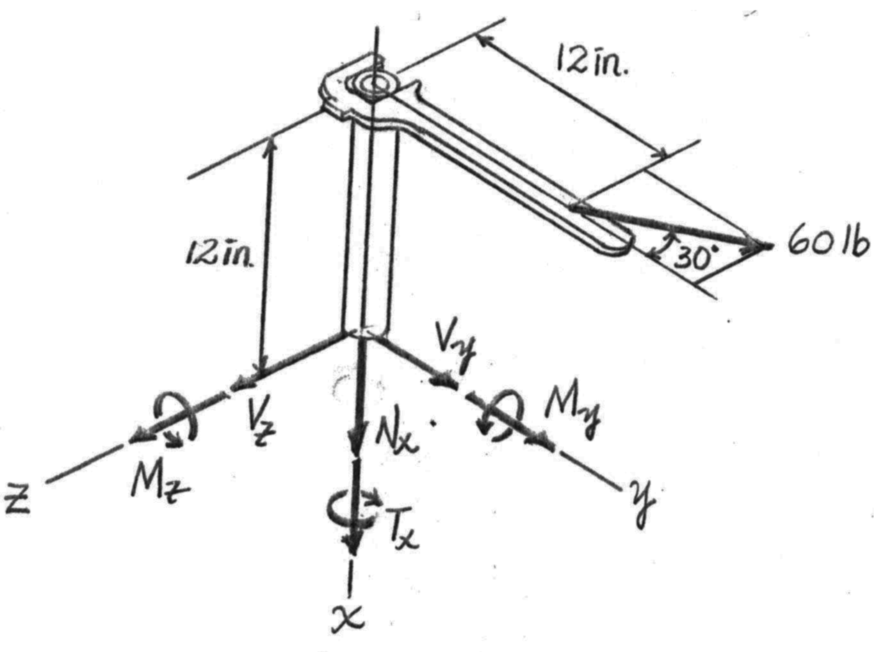
Section Properties: For the circular tubular cross-section, Fig. *b*,

$$I_y = I_z = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687\text{ in}^4$$

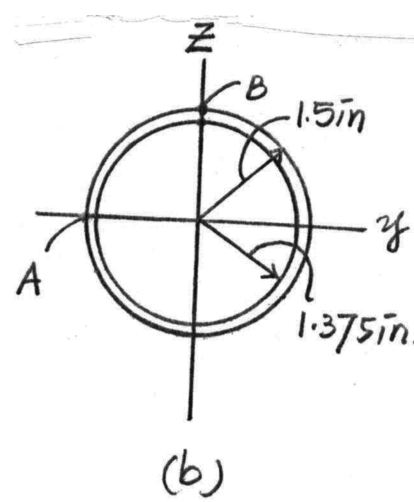
$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374\text{ in}^4$$

$$(Q_B)_y = \Sigma\bar{y}'A' = \frac{4(1.5)}{3\pi}\left[\frac{\pi}{2}\left(1.5^2\right)\right] - \frac{4(1.375)}{3\pi}\left[\frac{\pi}{2}\left(1.375^2\right)\right]$$

$$= 0.51693\text{ in}^3$$



(a)



(b)

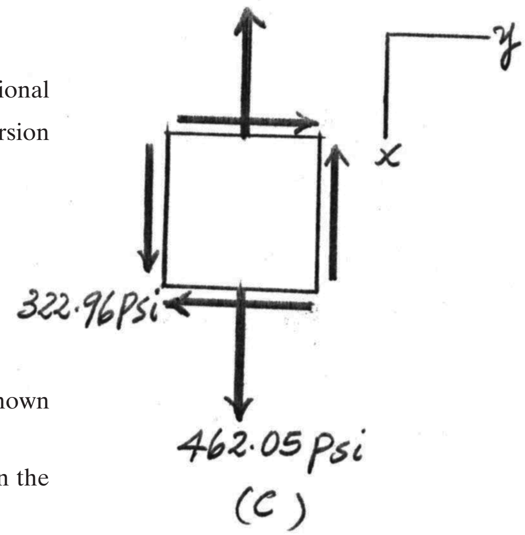
14–38. Continued

Normal Stress: For the combined loadings, the normal stress at point B can be determined from

$$\sigma_B = \frac{N_x}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$= 0 - \frac{623.54(0)}{1.1687} + \frac{360(1.5)}{1.1687}$$

$$= 462.05 \text{ psi (T)}$$



Shear Stress: The transverse shear stress in the z and y directions, and the torsional shear stress can be obtained using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$, respectively.

$$(\tau_{xy})_B = (\tau_V)_y - \tau_T$$

$$= \frac{(-51.96)(0.51693)}{1.1687[2(0.125)]} - \frac{360(1.5)}{2.3374}$$

$$= -322.96 \text{ psi}$$

The state of stress at point B can be represented by the differential element shown in Fig. c.

In-Plane Principal Stress: In accordance to the sign convention, the stresses on the element, Fig. c, are $\sigma_x = 462.05$ psi, $\sigma_y = 0$ and $\tau_{xy} = -322.96$ psi.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{462.05 + 0}{2} \pm \sqrt{\left(\frac{462.05 - 0}{2}\right)^2 + (-322.96)^2}$$

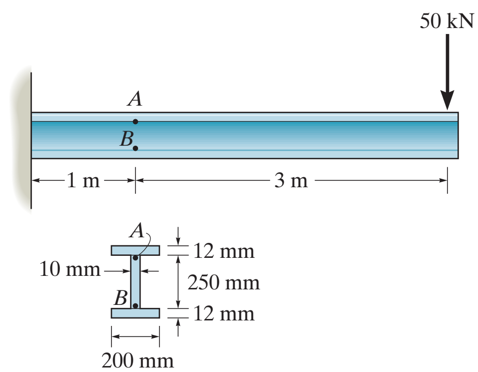
$$= 231.03 \pm 397.08$$

$\sigma_1 = 628.11 \text{ psi} = 628 \text{ psi} \qquad \sigma_2 = -166.06 \text{ psi} = -166 \text{ psi}$ **Ans.**

Ans:
$\sigma_1 = 628$ psi,
$\sigma_2 = -166$ psi

14–39.

The wide-flange beam is subjected to the 50-kN force. Determine the principal stresses in the beam at point *A* located on the *web* at the bottom of the upper flange. Although it is not very accurate, use the shear formula to calculate the shear stress.



SOLUTION

$$I = \frac{1}{12}(0.2)(0.274)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_A = (0.131)(0.012)(0.2) = 0.3144(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My}{I} = \frac{150(10^3)(0.125)}{95.451233(10^{-6})} = 196.43\ \text{MPa}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$

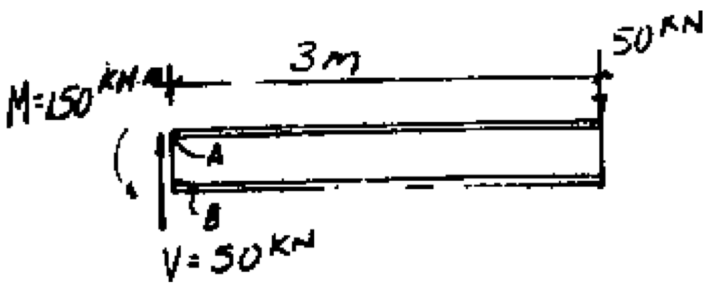
$$\sigma_x = 196.43\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47\ \text{MPa}$$

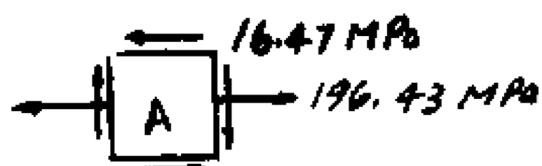
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{196.43 + 0}{2} \pm \sqrt{\left(\frac{196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 198\ \text{MPa}$ **Ans.**

$\sigma_2 = -1.37\ \text{MPa}$ **Ans.**

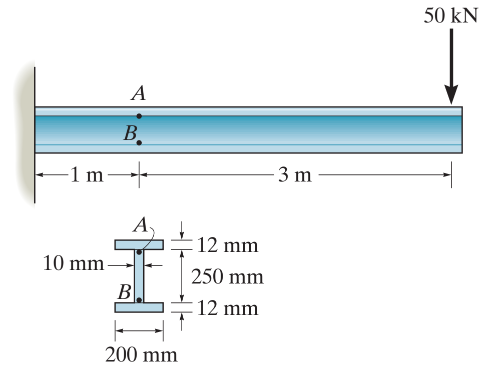




Ans:
$\sigma_1 = 198\ \text{MPa}$,
$\sigma_2 = -1.37\ \text{MPa}$

***14–40.**

Solve Prob. 14–39 for point *B* located on the *web* at the top of the bottom flange.



SOLUTION

$$I = \frac{1}{12}(0.2)(0.247)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\text{ m}^4$$

$$Q_B = (0.131)(0.012)(0.2) = 0.3144(10^{-3})$$

$$\sigma_B = -\frac{My}{I} = -\frac{150(10^3)(0.125)}{95.451233(10^{-6})} = -196.43\text{ MPa}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\text{ MPa}$$

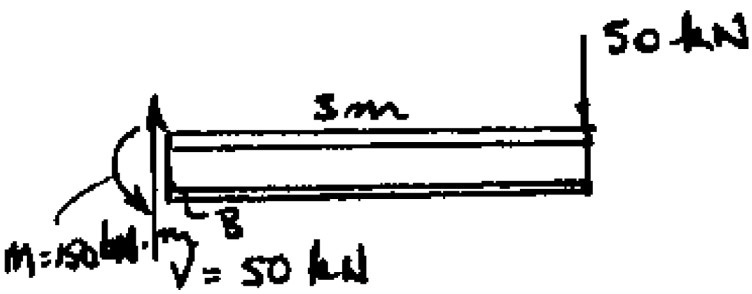
$\sigma_x = -196.43$ MPa $\qquad \sigma_y = 0 \qquad \tau_{xy} = -16.47$ MPa

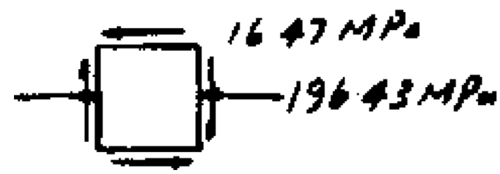
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-196.43 + 0}{2} \pm \sqrt{\left(\frac{-196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 1.37$ MPa **Ans.**

$\sigma_2 = -198$ MPa **Ans.**

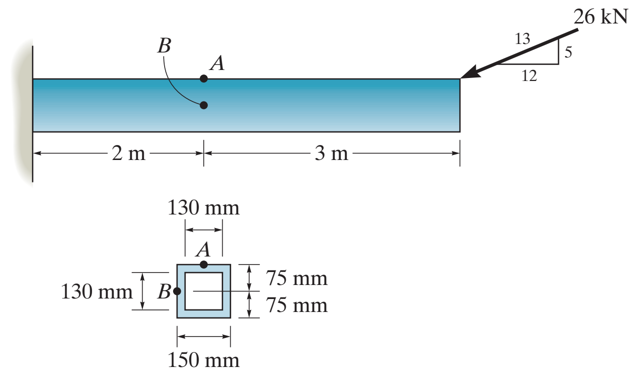




Ans:
$\sigma_1 = 1.37$ MPa,
$\sigma_2 = -198$ MPa

14–41.

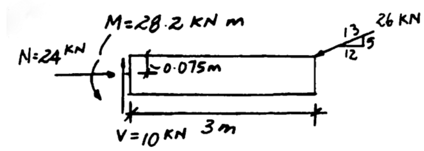
The box beam is subjected to the 26-kN force that is applied at the center of its width, 75 mm from each side. Determine the principal stresses at point A and show the results in an element located at this point. Use the shear formula to calculate the shear stress.



SOLUTION

$$I = \frac{1}{12}(0.15)\left(0.15^3\right) - \frac{1}{12}(0.13)\left(0.13^3\right) = 18.38667\left(10^{-6}\right)\text{ m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6\left(10^{-3}\right)\text{ m}^2$$



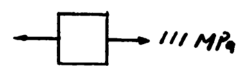
$$Q_A = 0$$

$$\tau_A = 0$$

$$\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = \frac{-24\left(10^3\right)}{5.6\left(10^{-3}\right)} + \frac{28.2\left(10^3\right)(0.075)}{18.38667\left(10^{-6}\right)} = 111\text{ MPa}$$

$\sigma_1 = 111\text{ MPa}$ **Ans.**

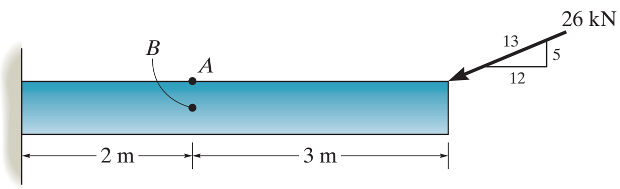
$\sigma_2 = 0$ **Ans.**

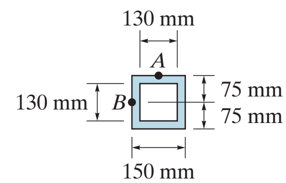


Ans:
$\sigma_1 = 111\text{ MPa}$,
$\sigma_2 = 0$

14–42.

Solve Prob. 14–41 for point B.





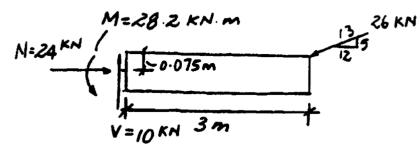
SOLUTION

$$I = \frac{1}{12}(0.15)\left(0.15^3\right) - \frac{1}{12}(0.13)\left(0.13^3\right) = 18.38667\left(10^{-6}\right)\text{ m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6\left(10^{-3}\right)\text{ m}^2$$

$$Q_B = (0.07)(0.15)(0.01) + 2(0.0325)(0.065)(0.01) = 0.14725\left(10^{-3}\right)\text{ m}^3$$

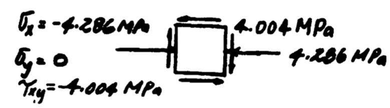
$$\sigma_B = -\frac{P}{A} = -\frac{24\left(10^3\right)}{5.6\left(10^{-3}\right)} = -4.286\text{ MPa}$$



$$\tau_B = \frac{VQ_B}{It} = \frac{10\left(10^3\right)(0.14725)\left(10^{-3}\right)}{18.38667\left(10^{-6}\right)(0.02)} = 4.004\text{ MPa}$$

$$\sigma_x = -4.286\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -4.004\text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

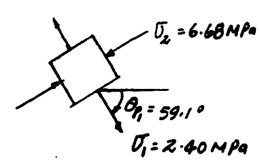


$$= \frac{-4.286 + 0}{2} \pm \sqrt{\left(\frac{-4.286 - 0}{2}\right)^2 + \left(-4.004^2\right)}$$

$$\sigma_1 = 2.40\text{ MPa} \qquad \sigma_2 = -6.68\text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-4.004}{(-4.286 - 0)/2}$$

$$\theta_P = 30.9^\circ \quad \text{or} \quad -59.1^\circ$$



Use Eq. 9–1.

$$\theta_{P1} = -59.1^\circ \qquad \theta_{P2} = 30.9^\circ$$

Ans:
$\sigma_1 = 2.40$ MPa,
$\sigma_2 = -6.68$ MPa

14–43.

Solve Prob. 14–2 using Mohr's circle.

SOLUTION

$$\frac{\sigma_x + \sigma_y}{2} = \frac{0 + 65}{2} = 32.5 \text{ MPa}$$

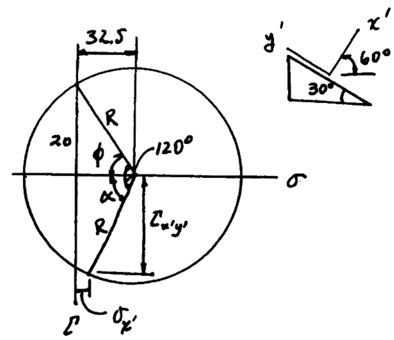
$$R = \sqrt{(32.5)^2 + (20)^2} = 38.1608$$

$$\phi = \tan^{-1}\frac{20}{32.5} = 31.6075°$$

$$\alpha = 120° - 31.6075° = 88.392°$$

$$\sigma_{x'} = 32.5 - 38.1608 \cos 88.392° = 31.4 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 38.1608 \sin 88.392° = 38.1 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{x'} = 31.4$ MPa,
$\tau_{x'y'} = 38.1$ MPa

***14–44.**

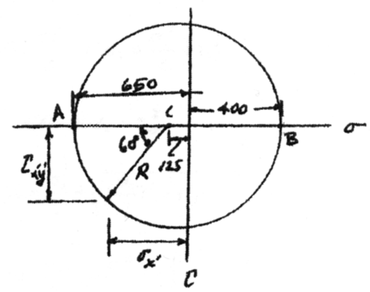
Solve Prob. 14–3 using Mohr's circle.

SOLUTION

$$\frac{\sigma_x + \sigma_y}{2} = \frac{-650 + 400}{2} = -125$$

$A(-650, 0)$ $\qquad$ $B(400, 0)$ $\qquad$ $C(-125, 0)$

$R = CA = \ = 650 - 125 = 525$



$\sigma_{x'} = -125 - 525 \cos 60° = -388 \text{ psi}$ **Ans.**

$\tau_{x'y'} = 525 \sin 60° = 455 \text{ psi}$ **Ans.**

Ans:
$\sigma_{x'} = -388$ psi,
$\tau_{x'y'} = 455$ psi

14–45.

Solve Prob. 14–6 using Mohr's circle.

SOLUTION

$$\frac{\sigma_x + \sigma_y}{2} = \frac{5 + 3}{2} = 4 \text{ ksi}$$

$$R = \sqrt{(5 - 4)^2 + 8^2} = 8.0623$$

$$\phi = \tan^{-1}\frac{8}{(5 - 4)} = 82.875°$$

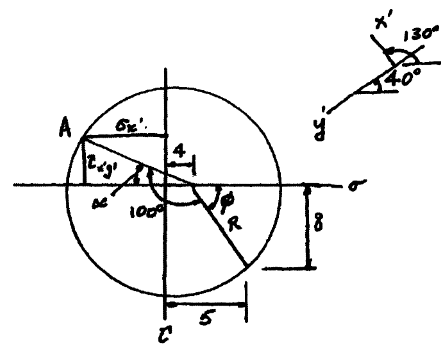
$$2\theta = 2(130°) = 260°$$

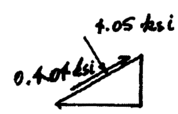
$$360° - 260° = 100°$$

$$\alpha = 100° + 82.875° - 180° = 2.875°$$

$$\sigma_{x'} = 8.0623 \cos 2.875° - 4 = -4.05 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -8.0623 \sin 2.875° = -0.404 \text{ ksi} \qquad \textbf{Ans.}$$





Ans:
$\sigma_{x'} = -4.05$ ksi,
$\tau_{x'y'} = -0.404$ ksi

14–46.

Solve Prob. 14–10 using Mohr's circle.

SOLUTION

Construction of the Circle: $\theta = -60°$, $\sigma_x = 150$ MPa, $\sigma_y = 100$ MPa, $\tau_{xy} = 75$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{150 + 100}{2} = 125 \text{ MPa}$$

The coordinates of the reference point A and center C of the circle are

$$A(150, 75) \qquad C(125, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(150 - 125)^2 + (75)^2} = 79.06 \text{ MPa}$$

Normal and Shear Stress on Rotated Element: Here $\theta = 60°$ clockwise. By rotating the radial line CA clockwise $2\theta = 120°$, it coincides with the radial line OP and the coordinates of reference point $P(\sigma_{x'}, \tau_{x'y'})$ represent the normal and shear stresses on the face of the element defined by $\theta = -60°$. $\sigma_{y'}$ can be determined by calculating the coordinates of point Q. From the geometry of the circle, Fig. (a),

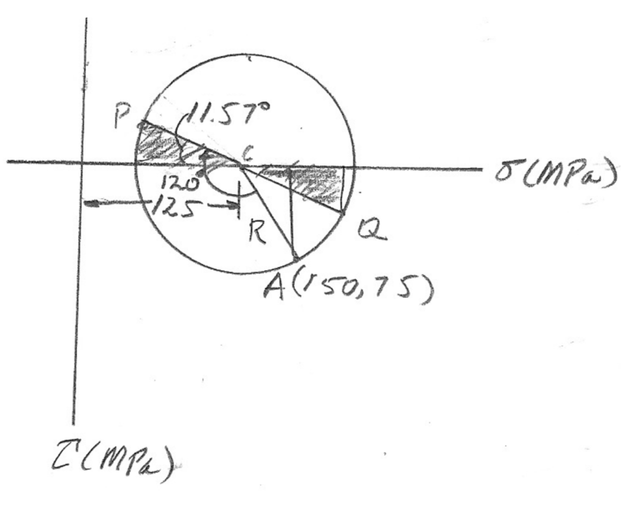
$$\sin\alpha = \frac{75}{79.06}, \ \alpha = 71.57°, \ \beta = 120° + 71.57° - 180° = 11.57°$$

$\sigma_{x'} = 125 - 79.06 \cos 11.57° = 47.5$ MPa **Ans.**

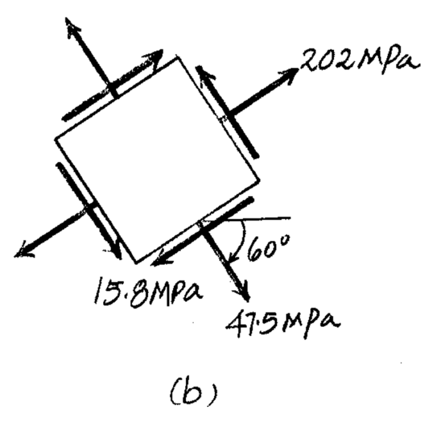
$\tau_{x'y'} = -79.06 \sin 11.57° = -15.8$ MPa **Ans.**

$\sigma_{y'} = 125 + 79.06 \cos 11.57° = 202$ MPa **Ans.**

The results are shown in Fig. (b).



(a)



(b)

Ans:
$\sigma_{x'} = 47.5$ MPa,
$\tau_{x'y'} = -15.8$ MPa,
$\sigma_{y'} = 202$ MPa

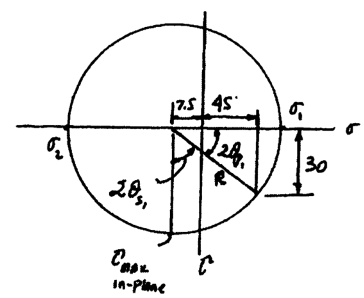
14–47.

Solve Prob. 14–15 using Mohr's circle.

SOLUTION

$$\frac{\sigma_x + \sigma_y}{2} = \frac{45 - 60}{2} = -7.5 \text{ MPa}$$

$$R = \sqrt{(45 + 7.5)^2 + (30)^2} = 60.467 \text{ MPa}$$



$\sigma_1 = 60.467 - 7.5 = 53.0 \text{ MPa}$ **Ans.**

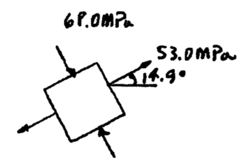
$\sigma_2 = -60.467 - 7.5 = -68.0 \text{ MPa}$ **Ans.**

$$2\theta_{p1} = \tan^{-1}\frac{30}{(45 + 7.5)}$$

$\theta_{p1} = 14.9°$ counterclockwise **Ans.**

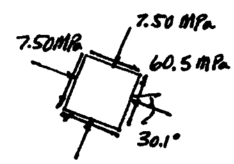
$\tau_{\substack{\max \\ \text{in-plane}}} = 60.5 \text{ MPa}$ **Ans.**

$\sigma_{avg} = -7.50 \text{ MPa}$ **Ans.**



$$2\theta_{s1} = 90° - \tan^{-1}\frac{30}{(45 + 7.5)}$$

$\theta_{s1} = 30.1°$ clockwise **Ans.**



Ans:
$\sigma_1 = 53.0 \text{ MPa}$,
$\sigma_2 = -68.0 \text{ MPa}$,
$\theta_{p1} = 14.9°$ counterclockwise,
$\tau_{\substack{\max \\ \text{in-plane}}} = 60.5 \text{ MPa}$,
$\sigma_{avg} = -7.50 \text{ MPa}$,
$\theta_{s1} = 30.1°$ clockwise

***14–48.**

Solve Prob. 14–16 using Mohr's circle.

SOLUTION

Construction of Circle: $\sigma_x = 50$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -15$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 0}{2} = 25 \text{ MPa} \qquad \textbf{Ans.}$$

The coordinates of reference point A and center C of the circle are

$$A(50, -15) \qquad C(25, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(50 - 25)^2 + (-15)^2} = \tau_{\substack{\max \\ \text{in-plane}}} = 29.15 \text{ MPa} = 29.2 \text{ MPa} \qquad \textbf{Ans.}$$

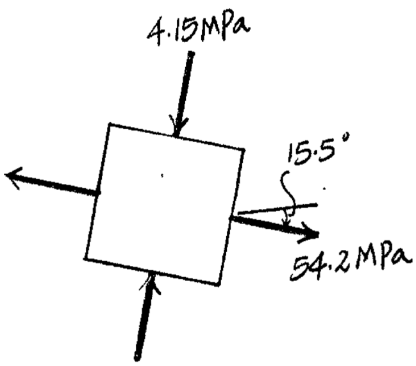
See Fig. (a).

(a) **Principal Stress:**

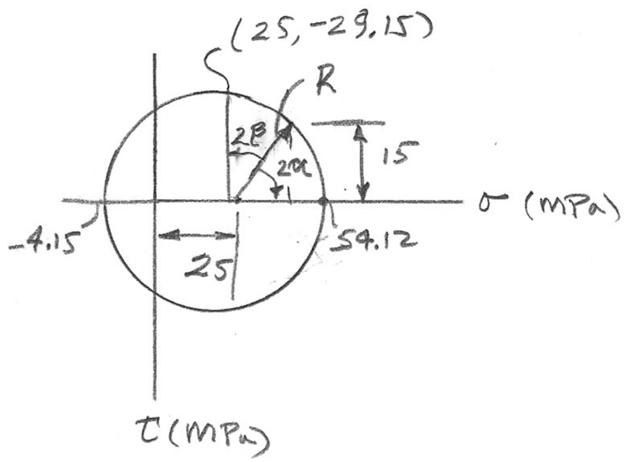
$$\sigma_1 = 54.2 \text{ MPa}, \qquad \sigma_2 = -4.15 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sin 2\alpha = \frac{15}{29.15}, \qquad \alpha = 15.5^\circ \qquad \textbf{Ans.}$$

See Fig. (b).



(a)



(a)

Ans:
$\sigma_1 = 54.2$ MPa, $\sigma_2 = -4.15$ MPa, $\theta_p = -15.5^\circ$,
$\sigma_{avg} = 25$ MPa, $\tau_{\substack{\max \\ \text{in-plane}}} = 29.2$ MPa, $\theta_s = 29.5^\circ$,
$\alpha = 15.5^\circ$

14–49.

Mohr's circle for the state of stress is shown in Fig. 14–17*a*. Show that finding the coordinates of point $P\,(\sigma_{x'}, \tau_{x'y'})$ on the circle gives the same value as the stress transformation Eqs. 14–1 and 14–2.

SOLUTION

$A(\sigma_x, \tau_{xy}) \qquad B(\sigma_y, -\tau_{xy}) \qquad C\left(\left(\frac{\sigma_x + \sigma_y}{2}\right), 0\right)$

$$R = \sqrt{\left[\sigma_x - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\cos\theta' \qquad \textbf{(1)}$$

$$\theta' = 2\theta_P - 2\theta$$

$$\cos(2\theta_P - 2\theta) = \cos 2\theta_P \cos 2\theta + \sin 2\theta_p \sin 2\theta \qquad \textbf{(2)}$$

From the circle:

$$\cos 2\theta_P = \frac{\sigma_x - \frac{\sigma_x + \sigma_y}{2}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(3)}$$

$$\sin 2\theta_P = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(4)}$$

Substitute Eq. (2) and (3) into Eq. (1).

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{QED}$$

$$\tau_{x'y'} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\,\sin\theta' \qquad \textbf{(5)}$$

$$\sin\theta' = \sin(2\theta_P - 2\theta)$$
$$= \sin 2\theta_P \cos 2\theta - \sin 2\theta \cos 2\theta_P \qquad \textbf{(6)}$$

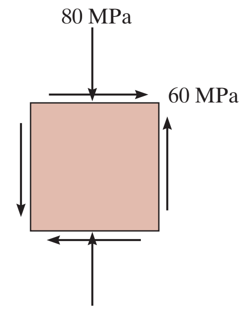
Substitute Eq. (3), (4) and (6) into Eq. (5).

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{QED}$$

Ans:
N/A

14–50.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 0$, $\sigma_y = -80$ MPa and $\tau_{xy} = 60$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-80)}{2} = -40.0 \text{ MPa} \qquad \textbf{Ans.}$$

The coordinates for reference point A and center of circle C are

$$A(0, 60) \qquad C(-40.0, 0)$$

Thus, the radius of the circle is

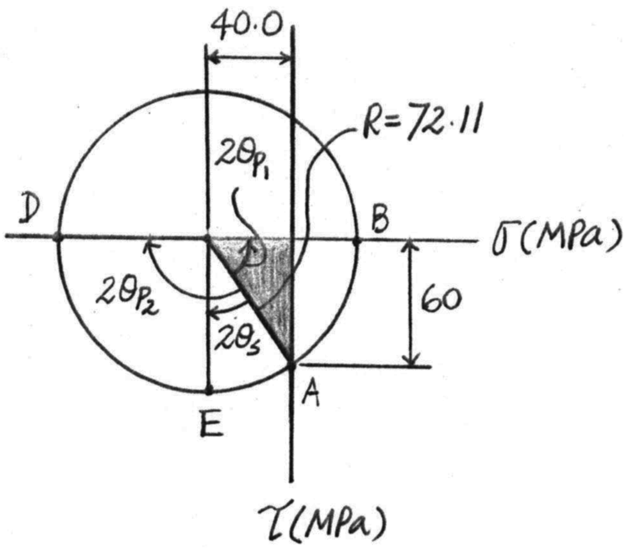
$$R = CA = \sqrt{[0 - (-40.0)]^2 + (60 - 0)^2} = 72.11 \text{ MPa}$$

Using these results, the circle shown in Fig. a can be constructed.

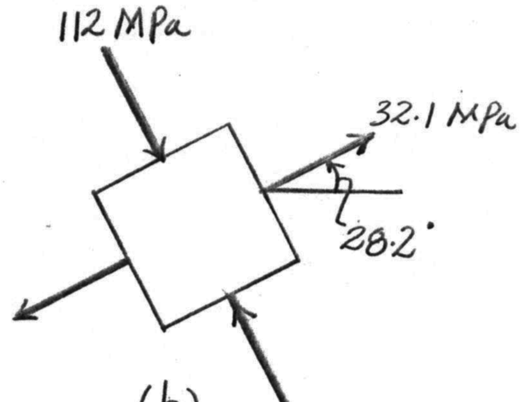
(a) In-Plane Principal Stresses: The coordinates of points B and D on the circle represent σ_1 and σ_2, respectively.

$$\sigma_1 = -40.0 + 72.11 = 32.11 \text{ MPa} = 32.1 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -40.0 - 72.11 = -112.11 \text{ MPa} = -112 \text{ MPa} \qquad \textbf{Ans.}$$



(a)



(b)

14–50. Continued

Orientation of Principal Plane: From the shaded triangle on the circle,

$$\tan 2\theta_{p1} = \frac{60}{40.0} = 1.5$$

$$2\theta_{p1} = 56.31°$$

$$\theta_{p1} = 28.15° = 28.2° \textbf{ (counterclockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of in-plane principal stresses can be represented by the differential element shown in Fig. *b*.

(b) Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 72.11 \text{ MPa} = 72.1 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of The Plane For Maximum In-Plane Shear Stress: From the circle,

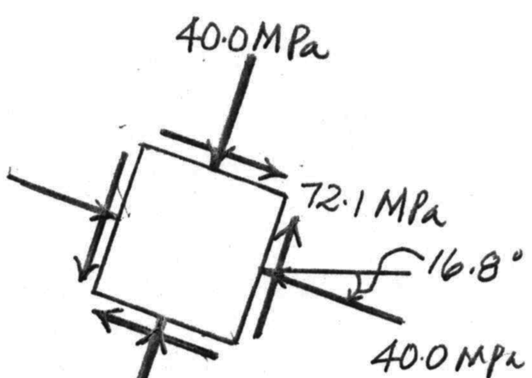
$$\tan 2\theta_s = \frac{40.0}{60} = 0.6667$$

$$2\theta_s = 33.69°$$

$$\theta_s = 16.845° = 16.8° \textbf{ (clockwise)}$$

$$\theta_s = -16.8° \qquad \textbf{Ans.}$$

Using these results, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. *c*.

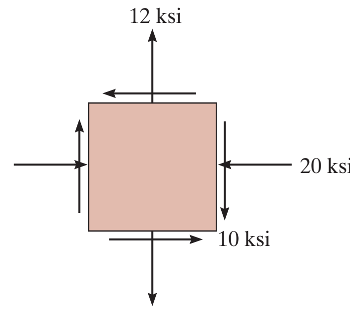


(c)

Ans:
$\sigma_{avg} = -40.0$ MPa,
$\sigma_1 = 32.1$ MPa,
$\sigma_2 = -112$ MPa,
$\theta_{p1} = 28.2°$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 72.1$ MPa,
$\theta_s = -16.8°$

14–51.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

Construction of the Circle: In accordance to the sign convention, $\sigma_x = -20$ ksi, $\sigma_y = 12$ ksi and $\tau_{xy} = -10$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-20 + 12}{2} = -4.00 \text{ ksi} \qquad \textbf{Ans.}$$

The coordinates for reference point A and center of circle C are

$$A(-20, -10) \qquad C(-4.00, 0)$$

Thus, the radius of the circle is

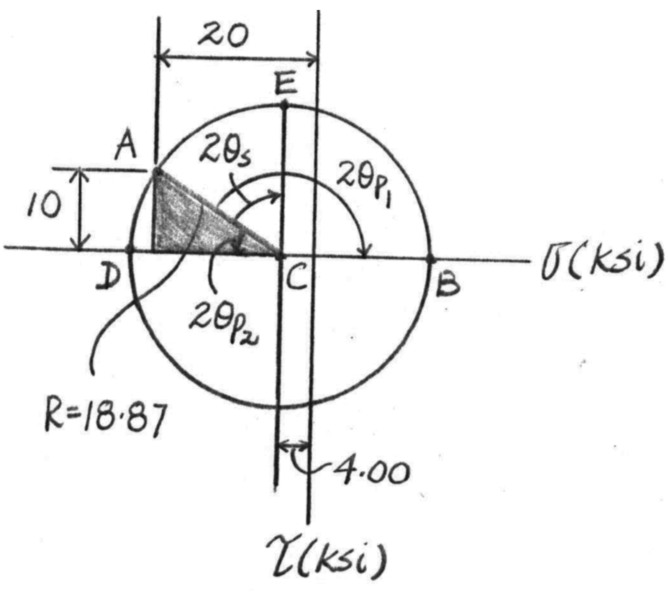
$$R = CA = \sqrt{[-20 - (-4.00)]^2 + (-10 - 0)^2} = 18.87 \text{ ksi}$$

Using these results, the circle shown in Fig. *a* can be constructed.

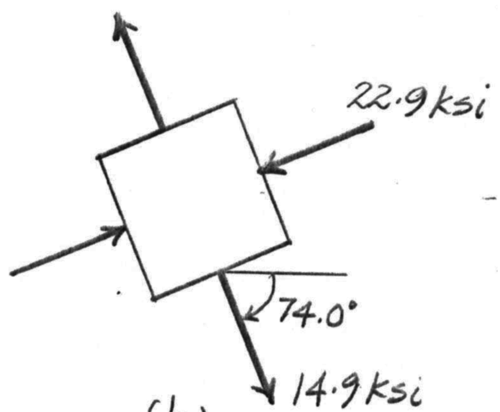
(a) In-Plane Principal Stresses: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -4.00 + 18.87 = 14.87 \text{ ksi} = 14.9 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -4.00 - 18.87 = -22.87 \text{ ksi} = -22.9 \text{ ksi} \qquad \textbf{Ans.}$$



(a)



(b)

14–51. Continued

Orientation of Principal Plane: From the shaded triangle on the circle,

$$\tan 2\theta_{p2} = \frac{10}{20 - 4} = 0.625 \qquad 2\theta_{p2} = 32.01^\circ$$

$$2\theta_{p1} = 180^\circ - 2\theta_{p2} = 180^\circ - 32.01^\circ = 147.99^\circ$$

$$\theta_{p1} = 74.00^\circ = 74.0^\circ \textbf{ (clockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of stress of in-plane principal stresses can be represented by the differential element shown in Fig. *b*.

(b) Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

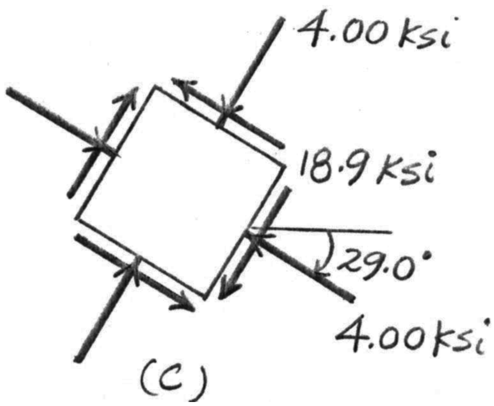
$$\tau_{\substack{\max \\ \text{in-plane}}} = -R = -18.87 \text{ ksi} = -18.9 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of the Plane For Maximum In-Plane Shear Stress: From the circle,

$$\tan 2\theta_s = \frac{20 - 4}{10}$$

$$2\theta_s = 57.99^\circ$$

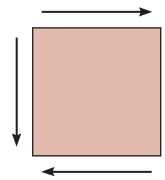
$$\theta_s = 29.00^\circ = 29.0^\circ \textbf{ (clockwise)} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{avg} = -4.00$ ksi,
$\sigma_1 = 14.9$ ksi,
$\sigma_2 = -22.9$ ksi,
$\theta_{p1} = -74.0^\circ$,
$\tau_{\substack{\max \\ \text{in-plane}}} = -18.9$ ksi,
$\theta_s = -29.0^\circ$

***14–52.**

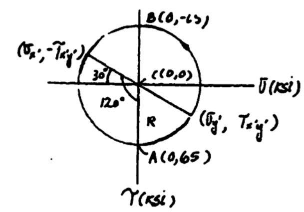
Determine the equivalent state of stress if an element is oriented 60° clockwise from the element shown.



SOLUTION

$A(0, 65) \qquad B(0, -65) \qquad C(0, 0)$

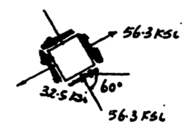
$R = 65$



$\sigma_{x'} = 0 - 65 \cos 30° = -56.3 \text{ ksi}$ **Ans.**

$\sigma_{y'} = 0 + 65 \cos 30° = 56.3 \text{ ksi}$ **Ans.**

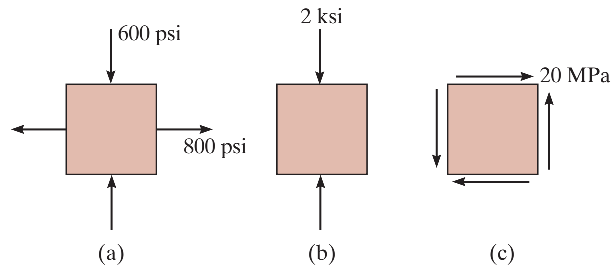
$\tau_{x'y'} = -65 \sin 30° = -32.5 \text{ ksi}$ **Ans.**



Ans:
$\sigma_{x'} = -56.3 \text{ ksi},$
$\sigma_{y'} = 56.3 \text{ ksi},$
$\tau_{x'y'} = -32.5 \text{ ksi}$

14–53.

Draw Mohr's circle that describes each of the following states of stress.



SOLUTION

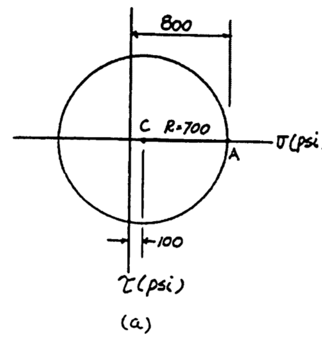
(a) Construction of the Circle: In accordance with the sign convention, $\sigma_x = 800$ psi, $\sigma_y = -600$ psi, and $\tau_{xy} = 0$. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{800 + (-600)}{2} = 100 \text{ psi}$$

The coordinates for reference point A and C are

$$A(800, 0) \qquad C(100, 0)$$

The radius of the circle is $\quad R = 800 - 100 = 700$ psi



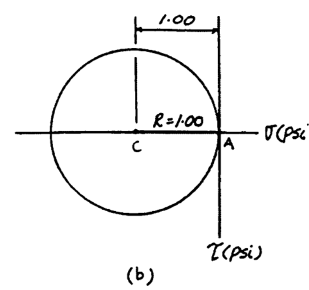
(b) Construction of the Circle: In accordance with the sign convention, $\sigma_x = 0$, $\sigma_y = -2$ ksi and $\tau_{xy} = 0$. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-2)}{2} = -1.00 \text{ ksi}$$

The coordinates for reference point A and C are

$$A(0, 0) \qquad C(-1.00, 0)$$

The radius of the circle is $\quad R = 1.00 - 0 = 1.00$ ksi



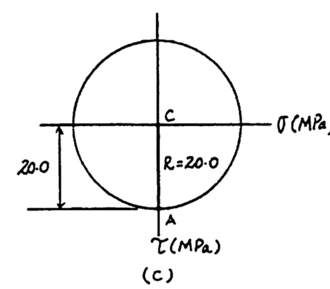
(c) Construction of the Circle: In accordance with the sign convention, $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 20$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 0$$

The coordinates for reference point A and C are

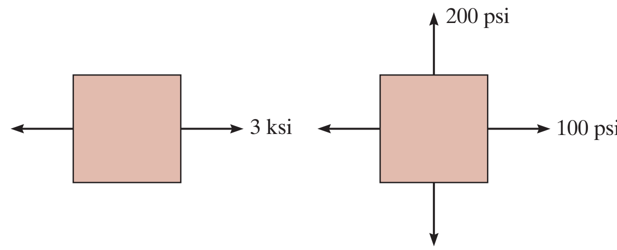
$$A(0, 20) \qquad C(0, 0)$$

The radius of the circle is $\quad R = 20.0$ MPa



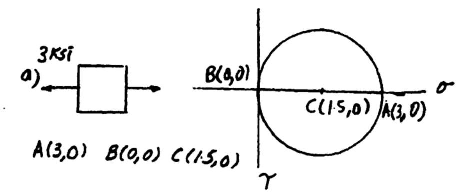
14–54.

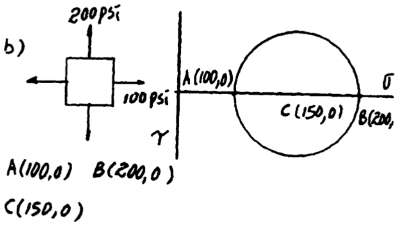
Draw Mohr's circle that describes each of the following states of stress.



(a) (b)

SOLUTION

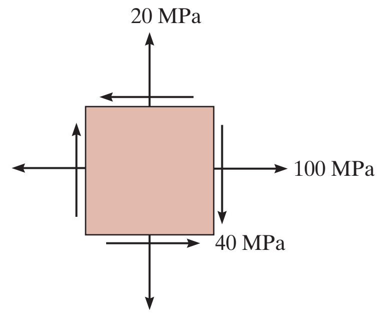




Ans:
N/A

14–55.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 100$ MPa, $\sigma_y = 20$ MPa, and $\tau_{xy} = -40$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{100 + 20}{2} = 60.0 \text{ MPa} \qquad \textbf{Ans.}$$

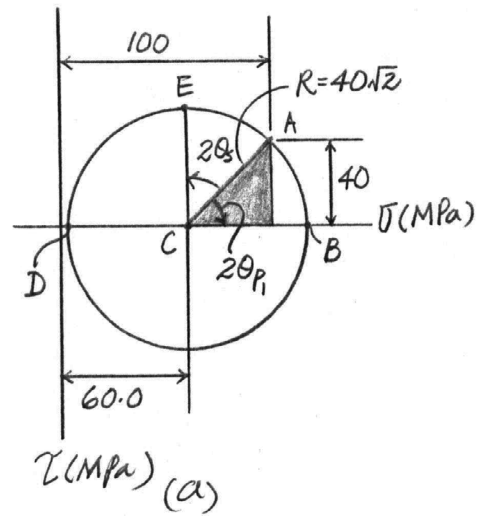
The coordinates for reference point A and center of circle C are

$$A(100, -40) \qquad C(60.0, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(100 - 60.0)^2 + (-40 - 0)^2} = 40\sqrt{2} \text{ MPa}$$

Using these results, the circle shown in Fig. a can be constructed.



(a) In-Plane Principal Stresses: The coordinates of points B and D on the circle represent σ_1 and σ_2, respectively.

$$\sigma_1 = 60.0 + 40\sqrt{2} = 116.57 \text{ MPa} = 117 \text{ MPa} \qquad \textbf{Ans.}$$

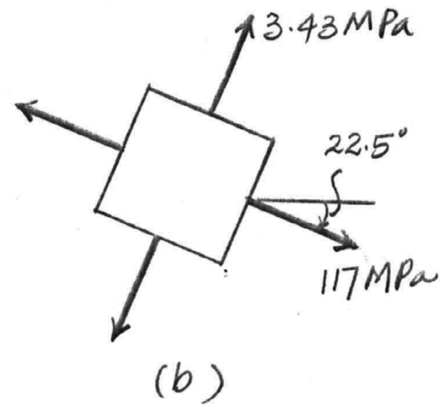
$$\sigma_2 = 60.0 - 40\sqrt{2} = 3.4315 \text{ MPa} = 3.43 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of Principal Plane: From the shaded triangle on the circle,

$$\tan 2\theta_{p1} = \frac{40}{100 - 60} = 1 \qquad 2\theta_{p1} = 45.0°$$

$$\theta_{p1} = 22.5° \textbf{ (Clockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of in-plane principal stress can be represented by the differential element shown in Fig. b.



(b) Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

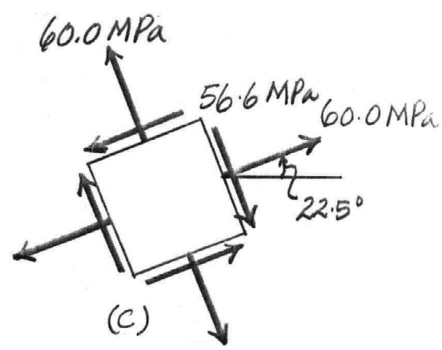
$$\tau_{\substack{max \\ \text{in-plane}}} = -R = -40\sqrt{2} \text{ MPa} = -56.6 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of the Plane for Maximum In-Plane Shear Stress: From the circle,

$$\tan 2\theta_s = \frac{100 - 60.0}{40} = 1 \qquad 2\theta_s = 45.0°$$

$$\theta_s = 22.5° \textbf{ (Counterclockwise)} \qquad \textbf{Ans.}$$

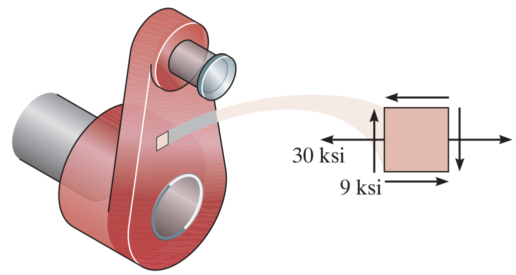
Using these results, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. c.



Ans:
$\sigma_{avg} = 60.0$ MPa, $\sigma_1 = 117$ MPa, $\sigma_2 = 3.43$ MPa,
$\tau_{\substack{max \\ \text{in-plane}}} = -56.6$ MPa, $\theta_s = 22.5°$,
$\theta_{p1} = 22.5°$ (Clockwise)

***14–56.**

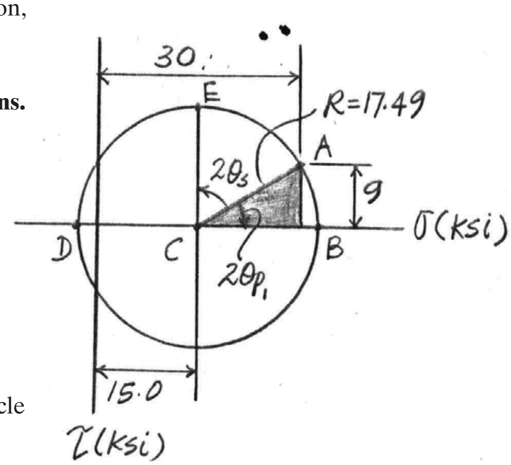
Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 30$ ksi, $\sigma_y = 0$ and $\tau_{xy} = -9$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{30 + 0}{2} = 15.0 \text{ ksi} \qquad \textbf{Ans.}$$



(a)

The coordinates for reference point A and center of circle C are

$$A(30, -9) \qquad C(15.0, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(30 - 15.0)^2 + (-9 - 0)^2} = 17.49 \text{ ksi}$$

Using these results, the circle shown in Fig. *a* can be constructed.

(a) In-Plane Principal Stresses: The coordinates of points B and D on the circle represent σ_1 and σ_2, respectively.

$$\sigma_1 = 15.0 + 17.4928 = 32.49 \text{ ksi} = 32.5 \text{ ksi} \qquad \textbf{Ans.}$$

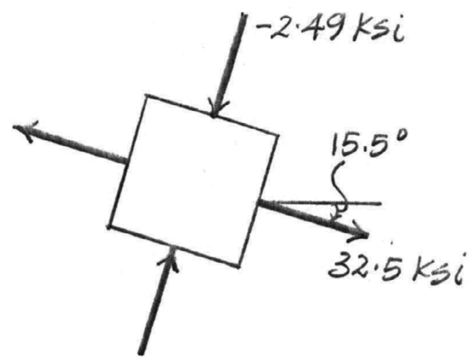
$$\sigma_2 = 15.0 - 17.4928 = -2.493 \text{ ksi} = -2.49 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of Principal Plane: From the shaded triangle on the circle,

$$\tan 2\theta_{p1} = \frac{9}{30 - 15.0} = 0.6 \qquad 2\theta_{p1} = 30.96^\circ$$

$$\theta_{p1} = 15.48^\circ = 15.5^\circ \textbf{ (Clockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of in-plane principal stress can be represented by the differential element shown in Fig. *b*.



(b)

(b) Maximum In-Plane Shear Stress: Represented by the coordinates of point E on the circle.

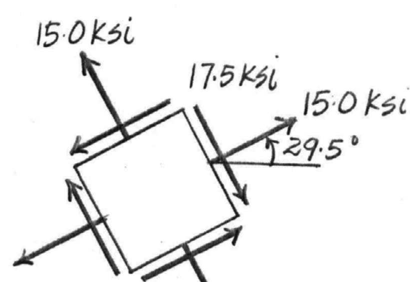
$$\tau_{\substack{\max \\ \text{in-plane}}} = -R = -17.4928 \text{ ksi} = -17.5 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of the Plane for Maximum In-Plane Shear Stress: From the circle,

$$\tan 2\theta_s = \frac{30 - 15.0}{9} = 1.6667 \qquad 2\theta_s = 59.04^\circ$$

$$\theta_s = 29.52^\circ = 29.5^\circ \textbf{ (Counterclockwise)} \qquad \textbf{Ans.}$$

Using these results, the state of maximum in-plane shear stress can be represented by the differential element shown in Fig. *c*.

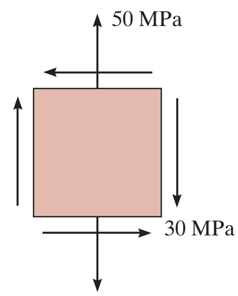


(c)

Ans:
$\sigma_{avg} = 15.0$ ksi, $\sigma_1 = 32.5$ ksi,
$\sigma_2 = -2.49$ ksi, $\theta_{p1} = -15.5^\circ$,
$\tau_{\substack{\max \\ \text{in-plane}}} = -17.5$ ksi, $\theta_s = 29.5^\circ$

14–57.

Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

$A(0, -30) \qquad B(50, 30) \qquad C(25, 0)$

$$R = CA = \sqrt{(25-0)^2 + 30^2} = 39.05$$

$$\sigma_1 = 25 + 39.05 = 64.1 \text{ MPa}$$ **Ans.**

$$\sigma_2 = 25 - 39.05 = -14.1 \text{ MPa}$$ **Ans.**

$$\tan 2\theta_P = \frac{30}{25-0} = 1.2$$

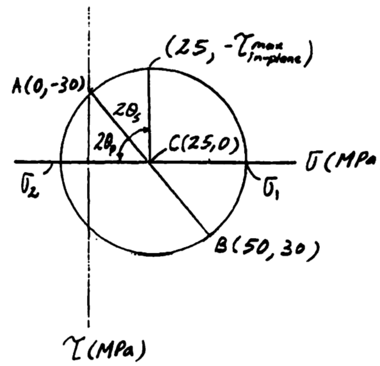
$$\theta_P = 25.1^\circ$$ **Ans.**

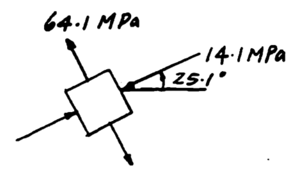
$$\sigma_{avg} = 25.0 \text{ MPa}$$ **Ans.**

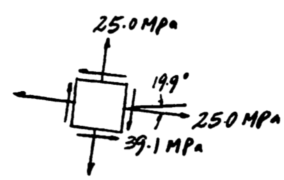
$$\tau_{\substack{max \\ \text{in-plane}}} = R = 39.1 \text{ MPa}$$ **Ans.**

$$\tan 2\theta_s = \frac{25-0}{30} = 0.8333$$

$$\theta_s = -19.9^\circ$$ **Ans.**



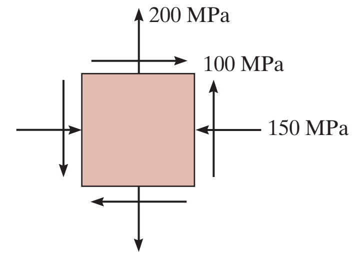




Ans:
$\sigma_1 = 64.1$ MPa,
$\sigma_2 = -14.1$ MPa,
$\theta_P = 25.1^\circ$,
$\sigma_{avg} = 25.0$ MPa,
$\tau_{\substack{max \\ \text{in-plane}}} = 39.1$ MPa,
$\theta_s = -19.9^\circ$

14–58.

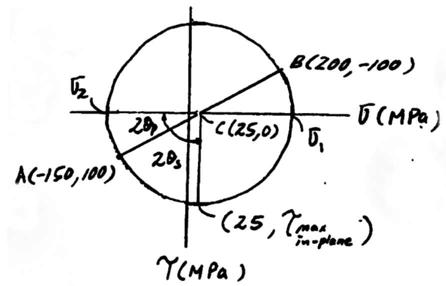
Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

$A(-150, 100) \qquad B(200, -100) \qquad C(25, 0)$

$$R = CA = \sqrt{(150 + 25)^2 + 100^2} = 201.556$$



$$\tan 2\theta_P = \frac{100}{150 + 25} = 0.5714$$

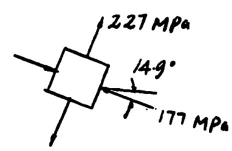
$\theta_P = -14.9°$ **Ans.**

$\sigma_1 = 25 + 201.556 = 227\text{ MPa}$ **Ans.**

$\sigma_2 = 25 - 201.556 = -177\text{ MPa}$ **Ans.**

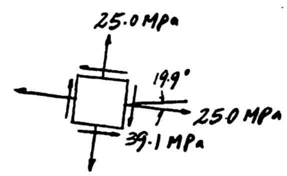
$\tau_{\substack{\max \\ \text{in-plane}}} = R = 202\text{ MPa}$ **Ans.**

$\sigma_{avg} = 25\text{ MPa}$ **Ans.**



$$\tan 2\theta_s = \frac{150 + 25}{100} = 1.75$$

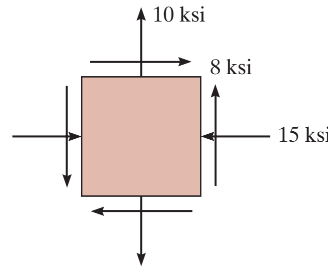
$\theta_s = 30.1°$ **Ans.**



Ans:
$\theta_P = -14.9°$,
$\sigma_1 = 227\text{ MPa}$,
$\sigma_2 = -177\text{ MPa}$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 202\text{ MPa}$,
$\sigma_{avg} = 25\text{ MPa}$,
$\theta_s = 30.1°$

14–59.

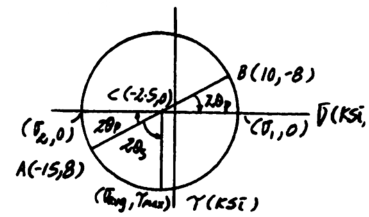
Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



SOLUTION

$A(-15, 8), \quad B(10, -8), \quad C(-2.5, 0)$

$R = CA = CB = \sqrt{12.5^2 + 8^2} = 14.84$

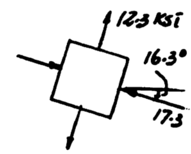


(a)

$\sigma_1 = -2.5 + 14.84 = 12.3 \text{ ksi}$ **Ans.**

$\sigma_2 = -2.5 - 14.84 = -17.3 \text{ ksi}$ **Ans.**

$\tan 2\theta_P = \dfrac{8}{12.5} \qquad 2\theta_P = 32.62° \qquad \theta_P = -16.3°$ **Ans.**



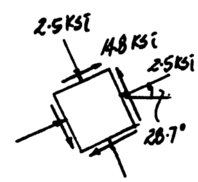
(b)

$\tau_{\substack{\max \\ \text{in-plane}}} = R = 14.8 \text{ ksi}$ **Ans.**

$\sigma_{avg} = -2.5 \text{ ksi}$ **Ans.**

$2\theta_s = 90 - 2\theta_p$

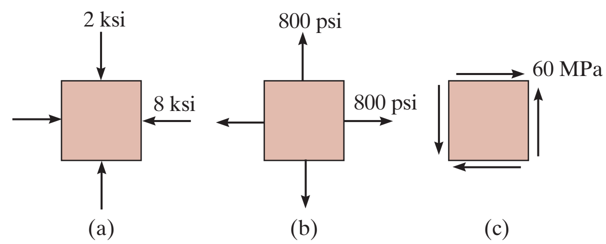
$\theta_s = 28.7°$ **Ans.**



Ans:
$\sigma_1 = 12.3$ ksi,
$\sigma_2 = -17.3$ ksi,
$\theta_P = -16.3°$,
$\tau_{\substack{\max \\ \text{in-plane}}} = 14.8$ ksi,
$\sigma_{avg} = -2.5$ ksi,
$\theta_s = 28.7°$

***14–60.**

Draw Mohr's circle that describes each of the following states of stress.



SOLUTION

Construction of the Circle: In accordance to the sign convention, for the element in (a), $\sigma_x = -8$ ksi, $\sigma_y = -2$ ksi and $\tau_{xy} = 0$. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-8 + (-2)}{2} = -5.00 \text{ ksi}$$

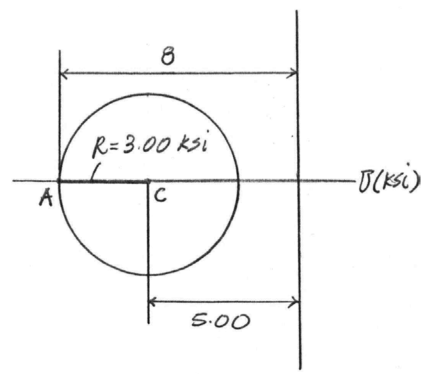
The coordinates for reference point A and center of circle C are

$$A(-8, 0) \qquad C(-5.00, 0)$$

Thus, the radius of the circle is

$$R = CA = 8 - 5.00 = 3.00 \text{ ksi}$$

Using these results, the circle shown in Fig. *a* can be constructed.



(a)

For the element in (b), $\sigma_x = 800$ psi, $\sigma_y = 800$ psi and $\tau_{xy} = 0$. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{800 + 800}{2} = 800 \text{ psi}$$

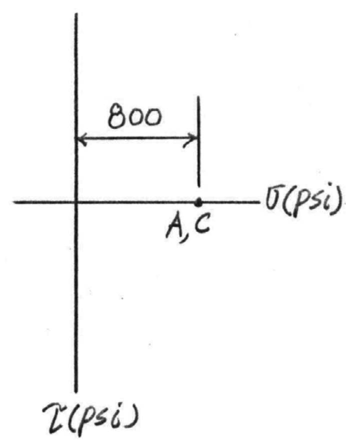
The coordinates for reference point A and center of circle C are

$$A(800, 0) \qquad C(800, 0)$$

Thus, the radius of the circle is

$$R = CA = 0$$

Using this result, the circle is just a point as shown in Fig. *b*. For the element in (c), $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 60$ MPa. Hence,



(b)

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = 0$$

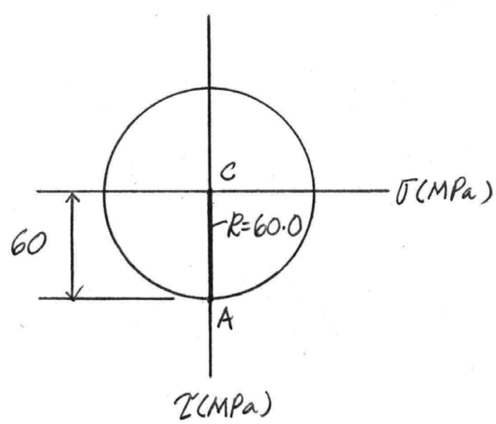
The coordinates for reference point A and center of circle C are

$$A(0, 60) \qquad C(0, 0)$$

Thus, the radius of the circle is

$$R = CA = 60.0 \text{ MPa}$$

Using these results, the circle shown in Fig. *c* can be constructed.

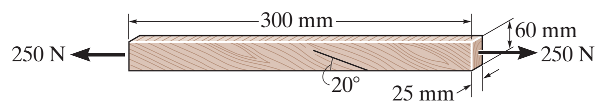


(c)

Ans:
N/A

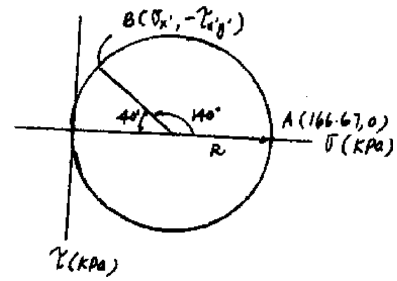
14–61.

The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stresses that act perpendicular and parallel to the grains if the board is subjected to an axial load of 250 N.



SOLUTION

$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67 \text{ kPa}$$



$R = 83.33$

Coordinates of point *B*:

$\sigma_{x'} = 83.33 - 83.33 \cos 40°$

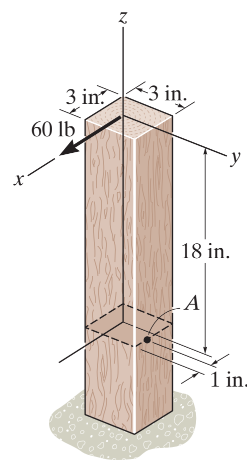
$\sigma_{x'} = 19.5 \text{ kPa}$ **Ans.**

$\tau_{x'y'} = -83.33 \sin 40° = -53.6 \text{ kPa}$ **Ans.**

Ans:
$\sigma_{x'} = 19.5$ kPa,
$\tau_{x'y'} = -53.6$ kPa

14–62.

The post is fixed supported at its base and a horizontal force is applied at its end as shown, determine (a) the maximum in-plane shear stress developed at A and (b) the principal stresses at A.

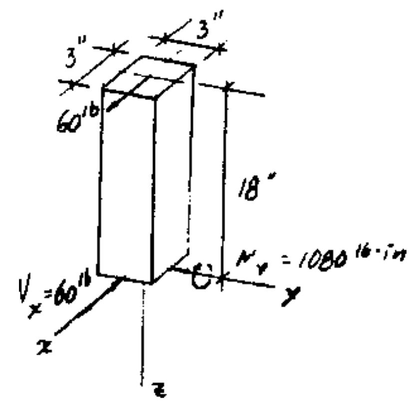


SOLUTION

$I = \frac{1}{12}(3)(3^3) = 6.75 \text{ in}^4 \qquad Q_A = (1)(1)(3) = 3 \text{ in}^3$

$$\sigma_A = \frac{M_y x}{I} = \frac{1080(0.5)}{6.75} = -80 \text{ psi}$$

$$\tau_A = \frac{V_y Q_A}{It} = \frac{60(3)}{6.75(3)} = 8.889 \text{ psi}$$

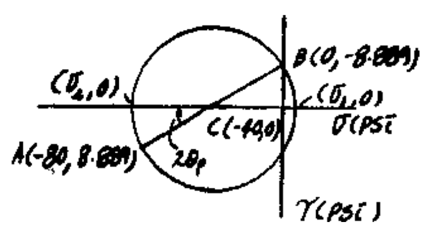


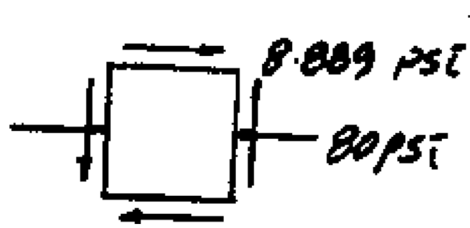
$A(-80, 8.889) \qquad B(0, -8.889) \qquad C(-40, 0)$

$\tau_{\substack{\max \\ \text{in-plane}}} = R = \sqrt{40^2 + 8.889^2} = 41.0 \text{ psi}$ **Ans.**

$\sigma_1 = -40 + 40.9757 = 0.976 \text{ psi}$ **Ans.**

$\sigma_2 = -40 - 40.9757 = -81.0 \text{ psi}$ **Ans.**

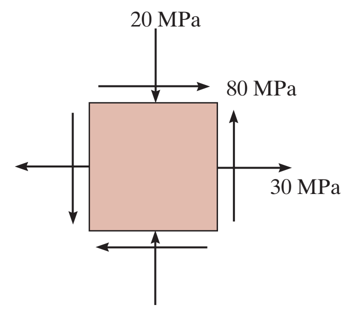




Ans:
$\tau_{\substack{\max \\ \text{in-plane}}} = 41.0 \text{ psi},$
$\sigma_1 = 0.976 \text{ psi},$
$\sigma_2 = -81.0 \text{ psi}$

14–63.

Determine the principal stresses, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



SOLUTION

In accordance to the established sign convention, $\sigma_x = 30$ MPa, $\sigma_y = -20$ MPa and $\tau_{xy} = 80$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{30 + (-20)}{2} = 5 \text{ MPa} \qquad \textbf{Ans.}$$

Then, the coordinates of reference point A and the center C of the circle is

$$A(30, 80) \qquad C(5, 0)$$

Thus, the radius of circle is given by

$$R = CA = \sqrt{(30 - 5)^2 + (80 - 0)^2} = 83.815 \text{ MPa}$$

Using these results, the circle shown in Fig. a, can be constructed.

The coordinates of points B and D represent σ_1 and σ_2, respectively. Thus,

$$\sigma_1 = 5 + 83.815 = 88.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 5 - 83.815 = -78.8 \text{ MPa} \qquad \textbf{Ans.}$$

Referring to the geometry of the circle, Fig. a,

$$\tan 2(\theta_P)_1 = \frac{80}{30 - 5} = 3.20$$

$$\theta_P = 36.3° \ (\textit{Counterclockwise}) \qquad \textbf{Ans.}$$

The state of maximum in-plane shear stress is represented by the coordinate of point E. Thus,

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 83.8 \text{ MPa} \qquad \textbf{Ans.}$$

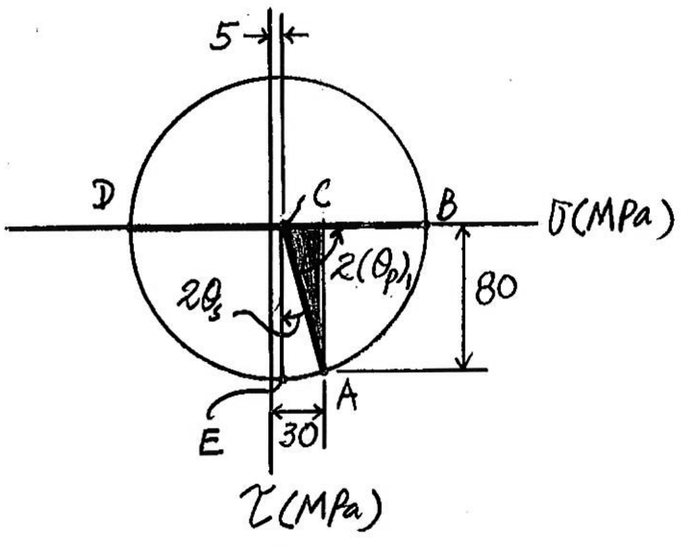
From the geometry of the circle, Fig. a,

$$\tan 2\theta_s = \frac{30 - 5}{80} = 0.3125$$

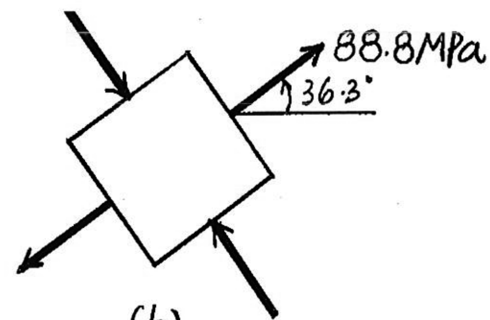
$$\theta_s = 8.68° \quad (\textit{Clockwise}) \qquad \textbf{Ans.}$$

The state of maximum in-plane shear stress is represented by the element in Fig. c.

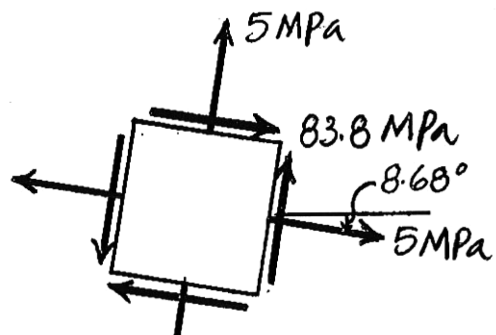
14–63. Continued



(a)



(b)

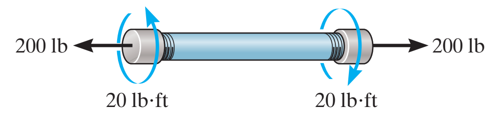


(c)

Ans:
$\sigma_{avg} = 5$ MPa,
$\sigma_1 = 88.8$ MPa,
$\sigma_2 = -78.8$ MPa,
$\theta_P = 36.3°$ (*Counterclockwise*),
$\tau_{\substack{max \\ in\text{-}plane}} = 83.8$ MPa,
$\theta_s = 8.68°$ (*Clockwise*)

***14–64.**

The thin-walled pipe has an inner diameter of 0.5 in. and a thickness of 0.025 in. If it is subjected to an internal pressure of 500 psi and the axial tension and torsional loadings shown, determine the principal stress at a point on the surface of the pipe.

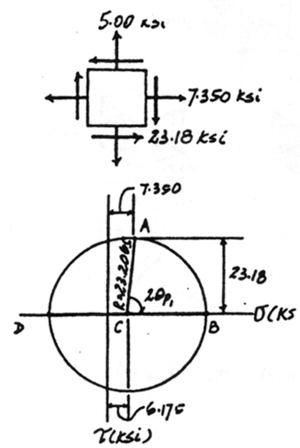


SOLUTION

Section Properties:

$$A = \pi\left(0.275^2 - 0.25^2\right) = 0.013125\pi \text{ in}^2$$

$$J = \frac{\pi}{2}\left(0.275^4 - 0.25^4\right) = 2.84768\left(10^{-3}\right) \text{ in}^4$$



Normal Stress: Since $\frac{r}{t} = \frac{0.25}{0.025} = 10$, thin-wall analysis is valid.

$$\sigma_{\text{long}} = \frac{N}{A} + \frac{pr}{2t} = \frac{200}{0.013125\pi} + \frac{500(0.25)}{2(0.025)} = 7.350 \text{ ksi}$$

$$\sigma_{\text{hoop}} = \frac{pr}{t} = \frac{500(0.25)}{0.025} = 5.00 \text{ ksi}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{20(12)(0.275)}{2.84768(10^{-3})} = 23.18 \text{ ksi}$$

Construction of the Circle: In accordance with the sign convention $\sigma_x = 7.350$ ksi, $\sigma_y = 5.00$ ksi, and $\tau_{xy} = -23.18$ ksi. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{7.350 + 5.00}{2} = 6.175 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(7.350, -23.18) \qquad C(6.175, 0)$$

The radius of the circle is

$$R = \sqrt{(7.350 - 6.175)^2 + 23.18^2} = 23.2065 \text{ ksi}$$

In-Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

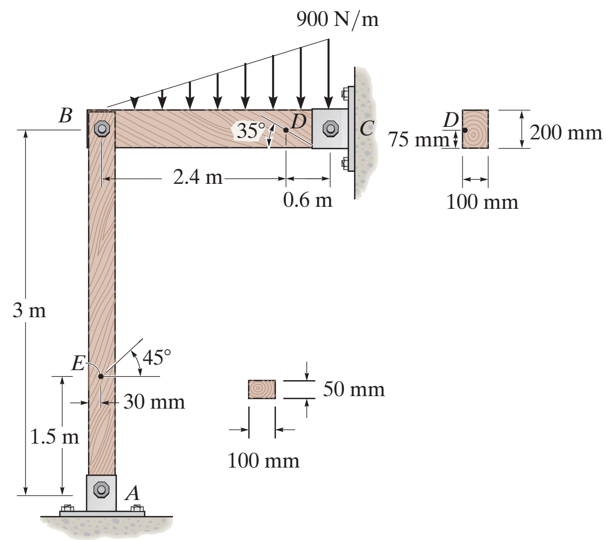
$$\sigma_1 = 6.175 + 23.2065 = 29.4 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 6.175 - 23.2065 = -17.0 \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 29.4$ ksi,
$\sigma_2 = -17.0$ ksi

14–65.

The frame supports the triangular distributed load shown. Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 35° with the horizontal as shown.



SOLUTION

Support Reactions and Internal Loadings: Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_C = 0; \qquad \left[\frac{1}{2}(900)(3)\right](1) - B_y(3) = 0 \qquad B_y = 450\text{ N}$$

Referring to the FBD of the left segment of the sectioned beam, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad 450 - \frac{1}{2}(720)(2.4) - V_D = 0 \qquad V_D = -414\text{ N}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad M_D + \left[\frac{1}{2}(720)(2.4)\right](0.8) - 450(2.4) = 0 \qquad M_D = 388.8\text{ N}\cdot\text{m}$$

Section Properties: For the rectangular cross section, Fig. *c*,

$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.6667\left(10^{-6}\right)\text{ m}^4$$

$$Q_D = \widetilde{y}'A' = 0.0625[0.1(0.075)] = 0.46875\left(10^{-3}\right)\text{ m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Applying the flexure formula,

$$\sigma_D = \frac{M_D y_D}{I} = \frac{388.8(0.025)}{66.6667\left(10^{-6}\right)} = 145.8\left(10^3\right)\text{ Pa} = 145.8\text{ kPa (T)}$$

Shear Stress: The shear stress is contributed by the transverse shear stress only. Applying the shear formula,

$$\tau_D = \frac{VQ}{It} = \frac{414\left[0.46875\left(10^{-3}\right)\right]}{66.6667\left(10^{-6}\right)(0.1)} = 29.11\left(10^3\right)\text{ Pa} = 29.11\text{ kPa}$$

Using these results, the state of stress at point D can be represented by the differential element shown in Fig. *d*.

Construction of The Circle: In accordance to the sign convention, $\sigma_x = 145.8$ kPa, $\sigma_y = 0$ $\tau_{xy} = 29.11$ kPa and $\theta = 55°$ (counterclockwise, Fig. *e*). Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{145.8 + 0}{2} = 72.9\text{ kPa}$$

The coordinates of reference point A and center of circle C are

$$A(145.8, 29.11) \qquad C(72.9, 0)$$

The radius of the circle is

$$R = CA = \sqrt{(145.8 - 72.9)^2 + (29.11 - 0)^2} = 78.497\text{ kPa}$$

14–65. Continued

Using these results, the circle shown in Fig. *f* can be constructed.

Stress on the Inclined Plane: The normal stress and shear stress on the inclined plane are represented by the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle which can be established by rotating radial line CA $2\theta = 110°$ counterclockwise to coincide with radial line CP, Fig. *f*. Here,

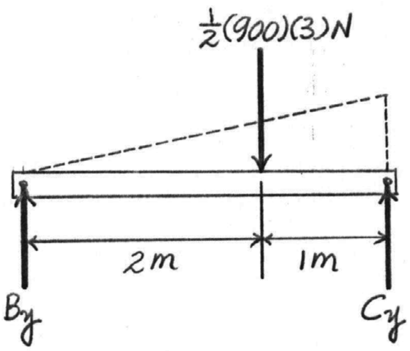
$$\alpha = \tan^{-1}\left(\frac{29.11}{145.8 - 72.9}\right) = 21.77°$$

$$\phi = 110° - 21.77° = 88.23°$$

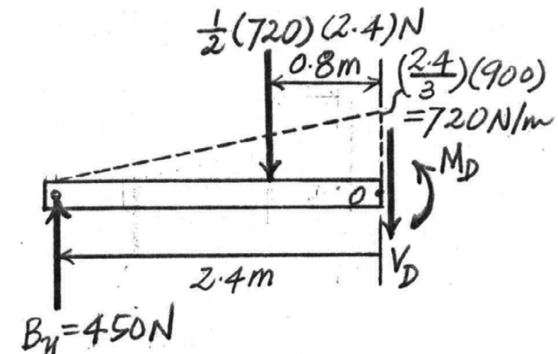
Then

$$\sigma_{x'} = 72.9 + 78.497 \cos 88.23° = 75.32 \text{ kPa} = 75.3 \text{ kPa} \qquad \textbf{Ans.}$$

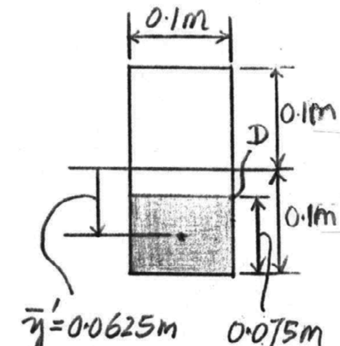
$$\tau_{x'y'} = -78.497 \sin 88.23° = -78.46 \text{ kPa} = -78.5 \text{ kPa} \qquad \textbf{Ans.}$$



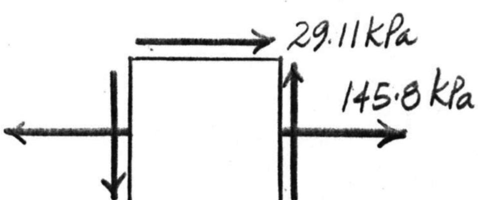
(a)



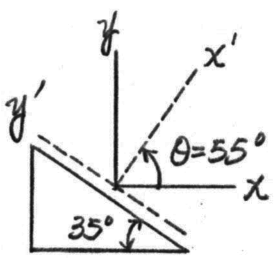
(b)



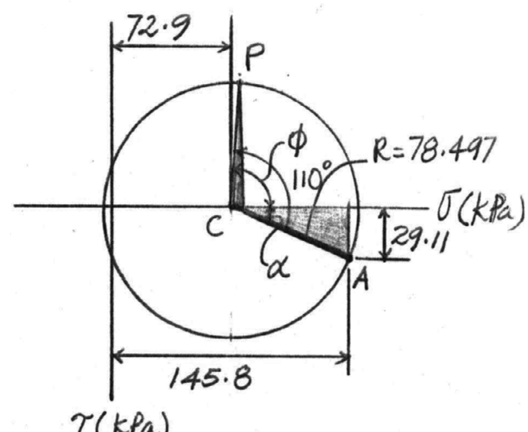
(c)



(d)



(e)

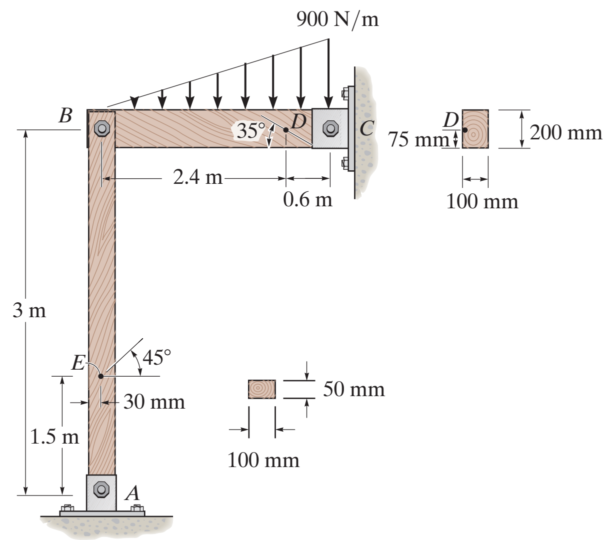


(f)

Ans:
$\sigma_{x'} = 75.3$ kPa,
$\tau_{x'y'} = -78.5$ kPa

14–66.

The frame supports the triangular distributed load shown. Determine the normal and shear stresses at point E that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 45° with the horizontal as shown.



SOLUTION

Support Reactions and Internal Loadings: Referring to the FBD of the entire beam, Fig. a,

$$\circlearrowleft + \Sigma M_C = 0; \qquad \left[\frac{1}{2}(900)(3)\right](1) - B_y(3) = 0 \qquad B_y = 450 \text{ N}$$

Referring to the FBD of the upper segment of the sectioned column, Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad -450 - N_E = 0 \qquad N_E = -450 \text{ N}$$

Section Properties: For the rectangular cross-section,

$$A = 0.05(0.1) = 5.00\left(10^{-3}\right) \text{ m}^2$$

Normal Stress: The normal stress is contributed by axial load only.

$$\sigma_E = \frac{N_E}{A} = \frac{-450}{5.00\left(10^{-3}\right)} = -90.0 \text{ kPa}$$

Using this result, the state of stress at point E can be represented by the differential element shown in Fig. c.

Construction of the Circle: In accordance to the sign convention, $\sigma_x = 0$, $\sigma_y = -90.0$ kPa, $\tau_{xy} = 0$ and $\theta = 135°$ (counterclockwise, Fig. d). Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-90.0)}{2} = -45.0 \text{ kPa}$$

The coordinates of reference point A and center of circle C are

$$A(0, 0) \qquad C(-45.0, 0)$$

The radius of the circle is

$$R = CA = 0 - (-45.0) = 45.0 \text{ kPa}$$

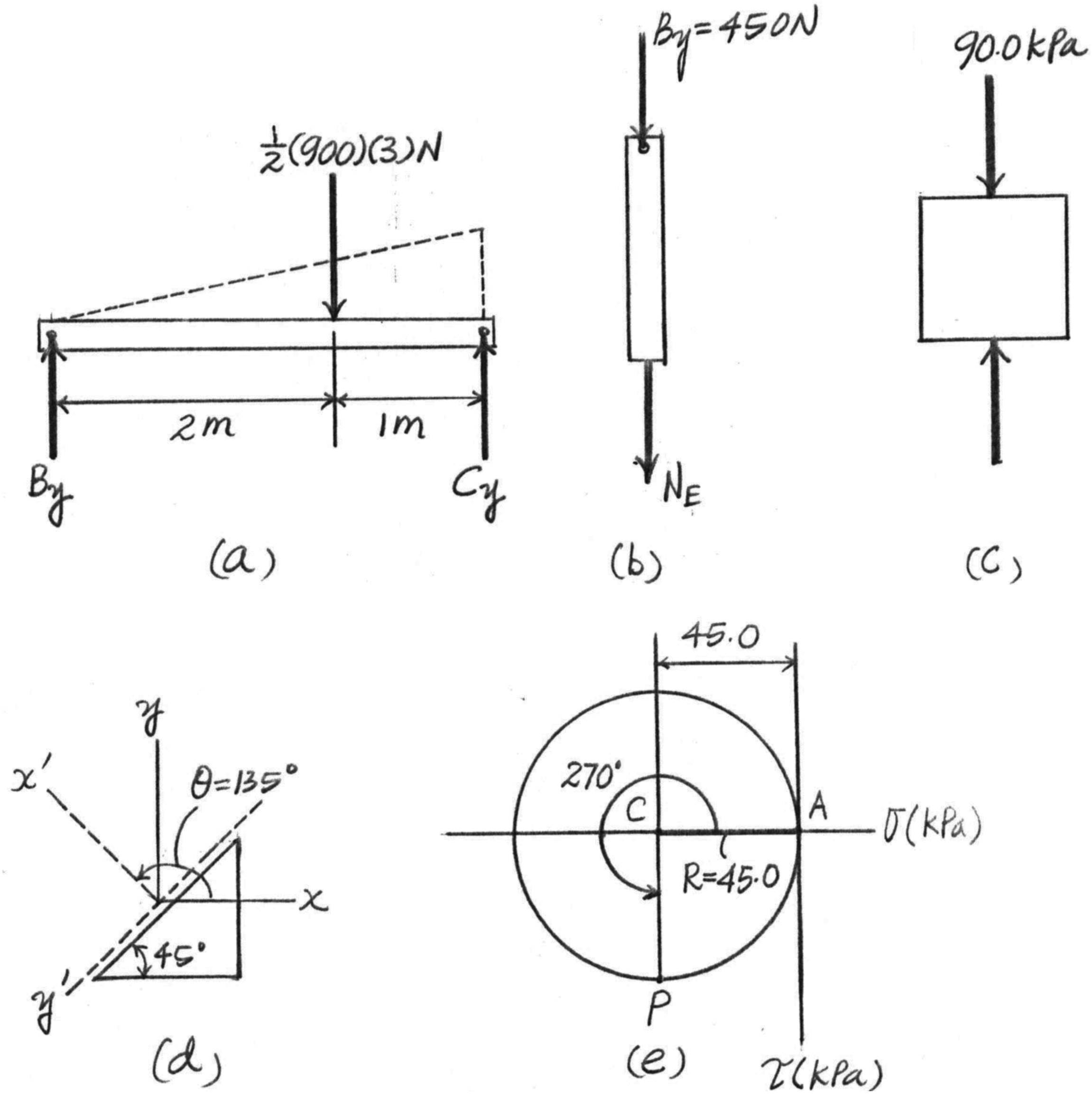
Using the results, the circle shown in Fig. e can be constructed.

Stress on the Inclined Plane: The normal stress and shear stress on the inclined plane are represented by the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle which can be established by rotating the radial line CA $2\theta = 270°$ counterclockwise to coincide with radial line CP, Fig. e. Thus,

$$\sigma_{x'} = -45.0 \text{ kPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 45.0 \text{ kPa} \qquad \textbf{Ans.}$$

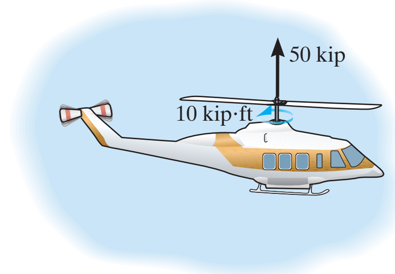
14–66. Continued



Ans:
$\sigma_{x'} = -45.0 \text{ kPa}$,
$\tau_{x'y'} = 45.0 \text{ kPa}$

14–67.

The rotor shaft of the helicopter is subjected to the tensile force and torque shown when the rotor blades provide the lifting force to suspend the helicopter at midair. If the shaft has a diameter of 6 in., determine the principal stresses and maximum in-plane shear stress at a point located on the surface of the shaft.

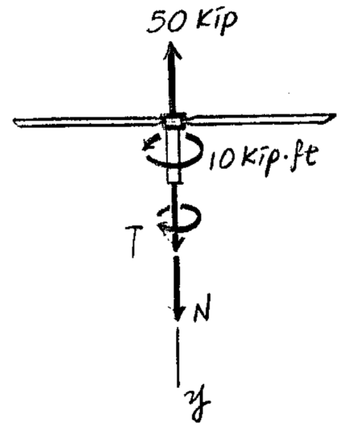


SOLUTION

Internal Loadings: Considering the equilibrium of the free-body diagram of the rotor shaft's upper segment, Fig. *a*,

$\Sigma F_y = 0;\quad N - 50 = 0\quad N = 50\text{ kip}$

$\Sigma M_y = 0;\quad T - 10 = 0\quad T = 10\text{ kip}\cdot\text{ft}$



(a)

Section Properties: The cross-sectional area and the polar moment of inertia of the rotor shaft's cross section are

$$A = \pi(3^2) = 9\pi\text{ m}^2$$

$$J = \frac{\pi}{2}(3^4) = 40.5\pi\text{ in}^4$$

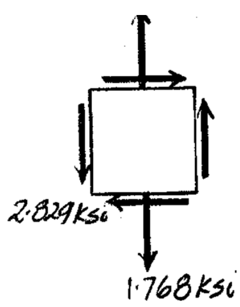
Normal and Shear Stress: The normal stress is contributed by axial stress only.

$$\sigma_A = \frac{N}{A} = \frac{50}{9\pi} = 1.768\text{ ksi}$$

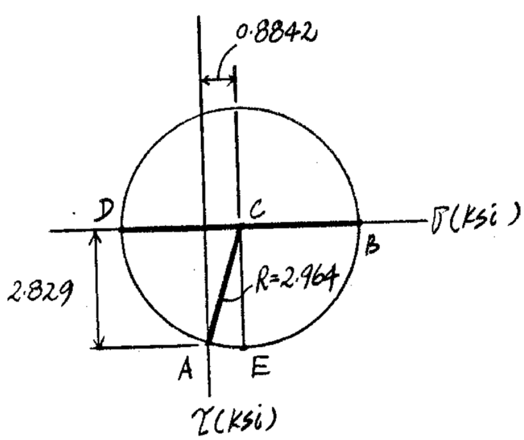
The shear stress is contributed by the torsional shear stress only.

$$\tau_A = \frac{Tc}{J} = \frac{10(12)(3)}{40.5\pi} = 2.829\text{ ksi}$$

The state of stress at point A is represented by the element shown in Fig. *b*.



(b)



(c)

14–67. Continued

Construction of the Circle: $\sigma_x = 0$, $\sigma_y = 1.768$ ksi, and $\tau_{xy} = 2.829$ ksi. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 1.768}{2} = 0.8842 \text{ ksi}$$

The coordinates of reference point A and the center C of the circle are

$$A(0, 2.829) \qquad C(0.8842, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(0 - 0.8842)^2 + 2.829^2} = 2.964 \text{ ksi}$$

Using these results, the circle is shown in Fig. *c*.

In-Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 0.8842 + 2.964 = 3.85 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0.8842 - 2.964 = -2.08 \text{ ksi} \qquad \textbf{Ans.}$$

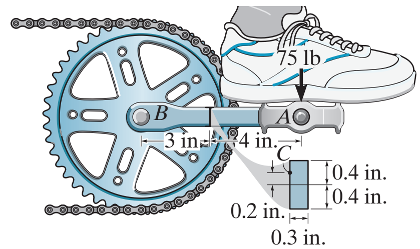
Maximum In-Plane Shear Stress: The state of maximum shear stress is represented by the coordinates of point E, Fig. *a*.

$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 2.96 \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 3.85$ ksi,
$\sigma_2 = -2.08$ ksi,
$\tau_{\substack{\max \\ \text{in-plane}}} = 2.96$ ksi

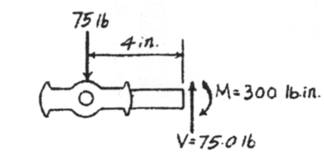
***14–68.**

The pedal crank for a bicycle has the cross section shown. If it is fixed to the gear at B and does not rotate while subjected to a force of 75 lb, determine the principal stresses on the cross section at point C.



SOLUTION

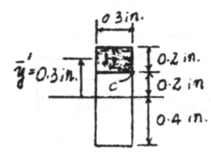
Internal Forces and Moment: As shown on FBD.



Section Properties:

$$I = \frac{1}{12}(0.3)\left(0.8^3\right) = 0.0128 \text{ in}^3$$

$$Q_C = \bar{y}'A' = 0.3(0.2)(0.3) = 0.0180 \text{ in}^3$$

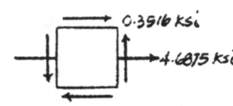


Normal Stress: Applying the flexure formula,

$$\sigma_C = -\frac{My}{I} = -\frac{-300(0.2)}{0.0128} = 4687.5 \text{ psi} = 4.6875 \text{ ksi}$$

Shear Stress: Applying the shear formula,

$$\tau_C = \frac{VQ_C}{It} = \frac{75.0(0.0180)}{0.0128(0.3)} = 351.6 \text{ psi} = 0.3516 \text{ ksi}$$

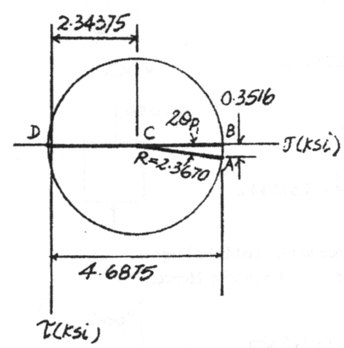


Construction of the Circle: In accordance with the sign convention, $\sigma_x = 4.6875$ ksi, $\sigma_y = 0$, and $\tau_{xy} = 0.3516$ ksi. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{4.6875 + 0}{2} = 2.34375 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(4.6875, 0.3516) \qquad C(2.34375, 0)$$



The radius of the circle is

$$R = \sqrt{(4.6875 - 2.34375)^2 + 0.3516^2} = 2.370 \text{ ksi}$$

In-Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 2.34375 + 2.370 = 4.71 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 2.34375 - 2.370 = -0.0262 \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 4.71$ ksi,
$\sigma_2 = -0.0262$ ksi

14–69.

A spherical pressure vessel has an inner radius of 5 ft and a wall thickness of 0.5 in. Draw Mohr's circle for the state of stress at a point on the vessel and explain the significance of the result. The vessel is subjected to an internal pressure of 80 psi.

SOLUTION

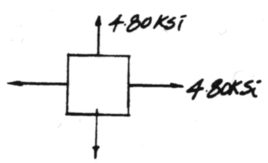
Normal Stress:

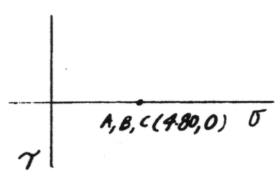
$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{80(5)(12)}{2(0.5)} = 4.80 \text{ ksi}$$

Mohr's circle:

$A(4.80, 0)$ $\quad B(4.80, 0)$ $\quad C(4.80, 0)$

Regardless of the orientation of the element, the shear stress is zero and the state of stress is represented by the same two normal stress components. **Ans.**

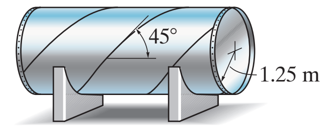




Ans:
Mohr's circle is a point located at (4.80, 0)

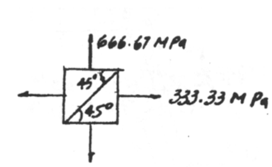
14–70.

The cylindrical pressure vessel has an inner radius of 1.25 m and a wall thickness of 15 mm. It is made from steel plates that are welded along the 45° seam. Determine the normal and shear stress components along this seam if the vessel is subjected to an internal pressure of 8 MPa.



SOLUTION

$$\sigma_x = \frac{pr}{2t} = \frac{8(1.25)}{2(0.015)} = 333.33 \text{ MPa}$$

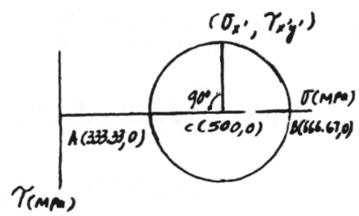


$$\sigma_y = 2\sigma_x = 666.67 \text{ MPa}$$

$A(333.33, 0) \qquad B(666.67, 0) \qquad C(500, 0)$

$$\sigma_{x'} = \frac{333.33 + 666.67}{2} = 500 \text{ MPa} \qquad \textbf{Ans.}$$

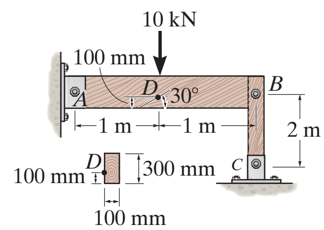
$$\tau_{x'y'} = -R = 500 - 666.67 = -167 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{x'} = 500$ MPa,
$\tau_{x'y'} = -167$ MPa

14–71.

Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 30° with the horizontal as shown. Point D is located just to the left of the 10-kN force.

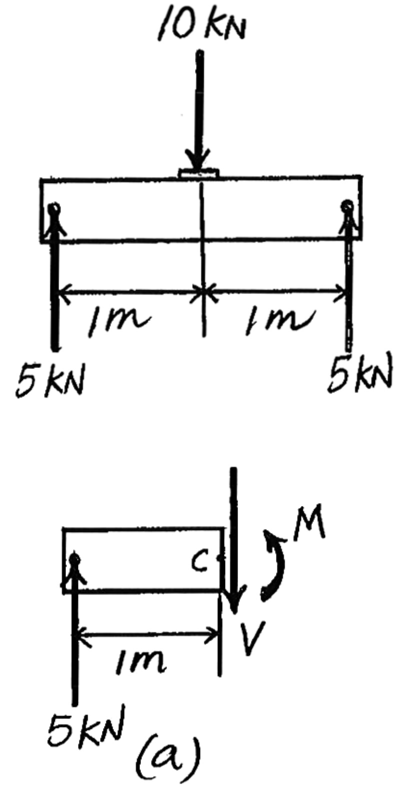


SOLUTION

Using the method of section and considering the FBD of the left cut segment, Fig. *a,*

$+\uparrow \Sigma F_y = 0; \qquad 5 - V = 0 \qquad\qquad V = 5\text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \qquad M - 5(1) = 0 \qquad M = 5\text{ kN}\cdot\text{m}$



The moment of inertia of the rectangular cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3})\text{ m}^4$$

Referring to Fig. *b*,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001\text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m. Then

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111\text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222\text{ MPa}$$

The state of stress at point D can be represented by the element shown in Fig. *c*.

In accordance with the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -0.2222$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556\text{ MPa}$$

Then, the coordinate of reference point A and the center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

Thus, the radius of the circle is given by

$$R = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984\text{ MPa}$$

Using these results, the circle shown in Fig. *d* can be constructed.

Referring to the geometry of the circle, Fig. *d*,

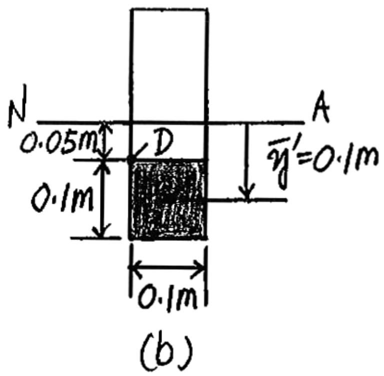
$$\alpha = \tan^{-1}\left(\frac{0.2222}{1.111 - 0.5556}\right) = 21.80° \qquad \beta = 180° - (120° - 21.80°) = 81.80°$$

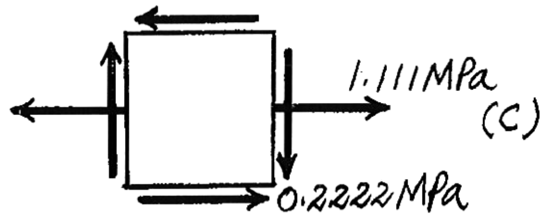
14–71. Continued

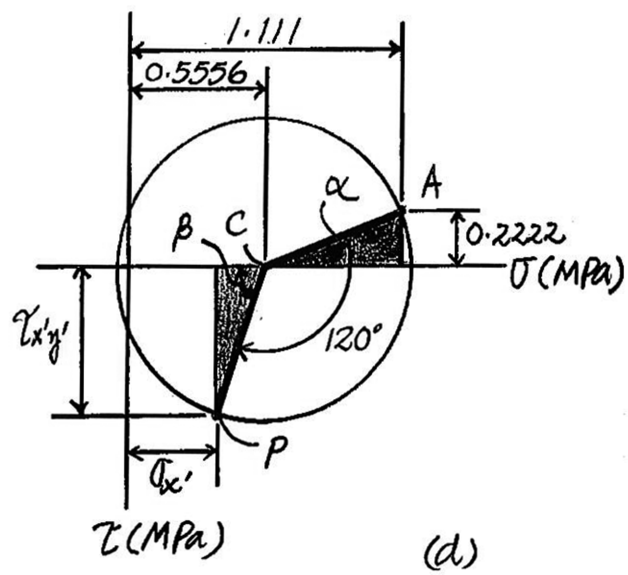
Then

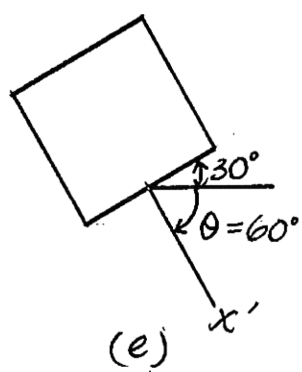
$\sigma_{x'} = 0.5556 - 0.5984 \cos 81.80° = 0.4702 \text{ MPa} = 470 \text{ kPa}$ **Ans.**

$\tau_{x'y'} = 0.5984 \sin 81.80° = 0.5922 \text{ MPa} = 592 \text{ kPa}$ **Ans.**





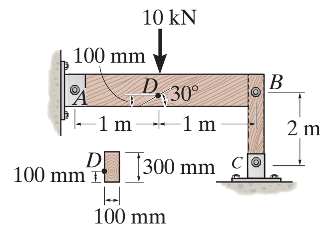




Ans:
$\sigma_{x'} = 470 \text{ kPa}$,
$\tau_{x'y'} = 592 \text{ kPa}$

***14–72.**

Determine the principal stress at point D, which is located just to the left of the 10-kN force.



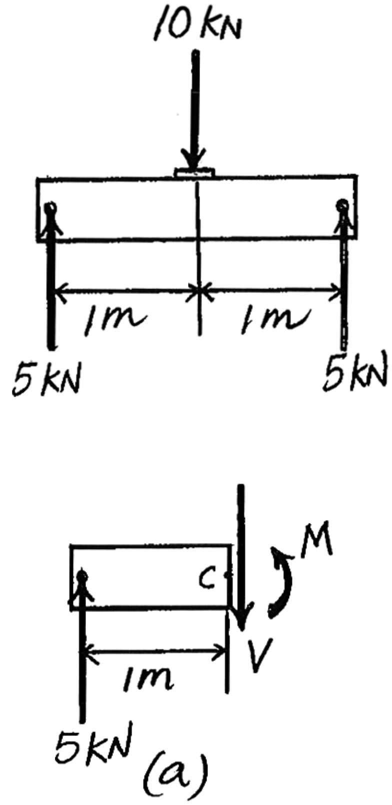
SOLUTION

Using the method of section and considering the FBD of the left cut segment, Fig. a,

$+\uparrow \Sigma F_y = 0; \qquad 5 - V = 0 \qquad\qquad V = 5 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \qquad M - 5(1) = 0 \qquad M = 5 \text{ kN} \cdot \text{m}$

$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3}) \text{ m}^4$$



(a)

Referring to Fig. b,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001 \text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m.

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111 \text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222 \text{ MPa}$$

The state of stress at point D can be represented by the element shown in Fig. c.

In accordance with the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -0.2222$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556 \text{ MPa}$$

Then, the coordinate of reference point A and center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984 \text{ MPa}$$

Using these results, the circle shown in Fig. d can be constructed.

In-Plane Principal Stresses: The coordinates of points B and D represent σ_1 and σ_2, respectively. Thus,

$\sigma_1 = 0.5556 + 0.5984 = 1.15 \text{ MPa}$ **Ans.**

$\sigma_2 = 0.5556 - 0.5984 = -0.0428 \text{ MPa}$ **Ans.**

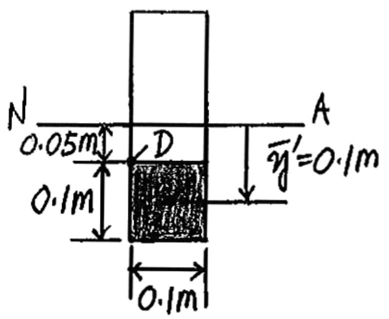
14–72. Continued

Referring to the geometry of the circle, Fig. *d*,

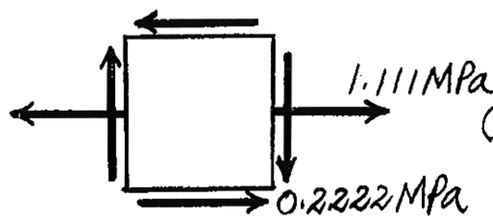
$$\tan\,(2\theta_P)_1 = \frac{0.2222}{1.111 - 0.5556} = 0.4$$

$$(\theta_P)_1 = 10.9^\circ\ (\textit{Clockwise})$$

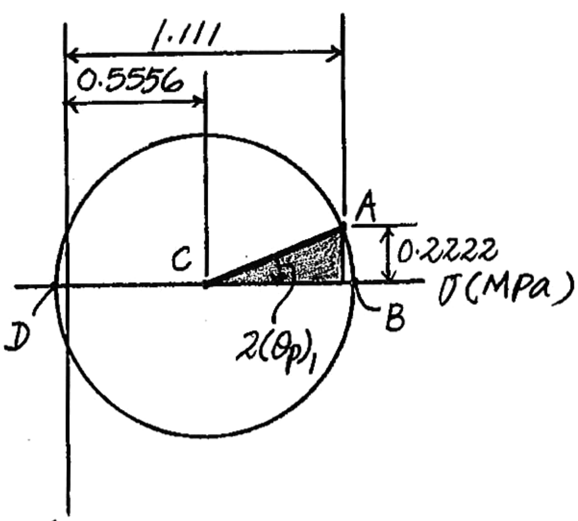
The state of principal stresses is represented by the element shown in Fig. *e*.



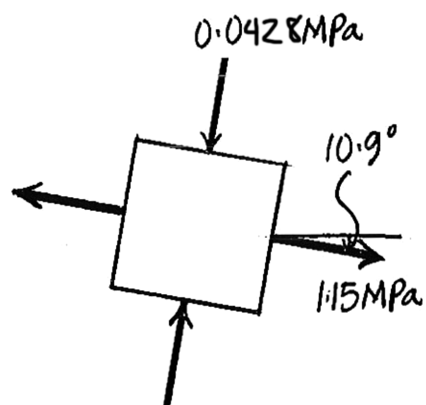
(b)



(c)



(d)

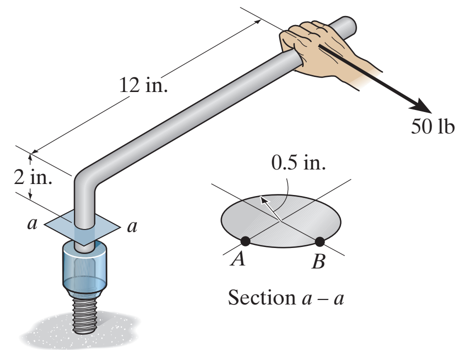


(e)

Ans:
$\sigma_1 = 1.15$ MPa,
$\sigma_2 = -0.0428$ MPa

14–73.

If the box wrench is subjected to the 50 lb force, determine the principal stresses and maximum in-plane shear stress at point A on the cross section of the wrench at section a–a. Specify the orientation of these states of stress and indicate the results on elements at the point.



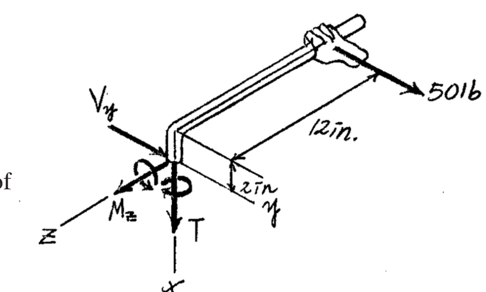
SOLUTION

Internal Loadings: Considering the equilibrium of the free-body diagram of the wrench's segment, Fig. a,

$$\Sigma F_y = 0; \qquad V_y + 50 = 0 \qquad V_y = -50 \text{ lb}$$

$$\Sigma M_x = 0; \qquad T + 50(12) = 0 \qquad T = -600 \text{ lb} \cdot \text{in}$$

$$\Sigma M_z = 0; \qquad M_z - 50(2) = 0 \qquad M_z = 100 \text{ lb} \cdot \text{in}$$



(a)

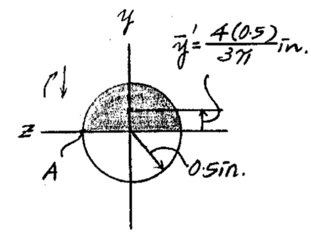
Section Properties: The moment of inertia about the z axis and the polar moment of inertia of the wrench's cross section are

$$I_z = \frac{\pi}{4}(0.5^4) = 0.015625\pi \text{ in}^4$$

$$J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$$

Referring to Fig. b,

$$(Q_y)_A = \bar{y}'A' = \frac{4(0.5)}{3\pi}\left[\frac{\pi}{2}(0.5^2)\right] = 0.08333 \text{ in}^3$$



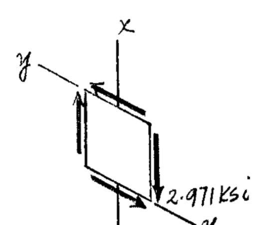
(b)

Normal and Shear Stress: The shear stress of point A along the z axis is $(\tau_{xz})_A = 0$; however, the shear stress along the y axis is a combination of torsional and transverse shear stress.

$$(\tau_{xy})_A = [(\tau_{xy})_T]_A - [(\tau_{xy})_V]_A$$

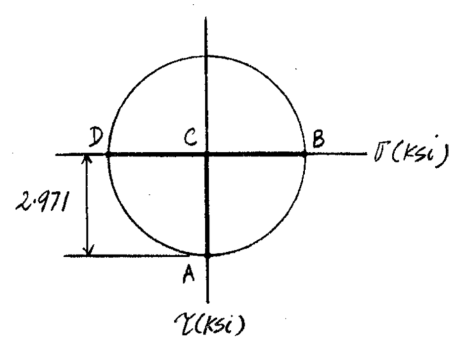
$$= \frac{Tc}{J} + \frac{V_y(Q_y)_A}{I_z t}$$

$$= \frac{600(0.5)}{0.03125\pi} + \frac{-50(0.08333)}{0.015625\pi(1)} = 2.971 \text{ ksi}$$



(C)

The state of stress at point A is represented by the two-dimensional element shown in Fig. c.



(d)

14–73. Continued

Construction of the Circle: $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 2.971$ ksi. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 0}{2} = 0$$

The coordinates of reference point A and the center C of the circle are

$$A(0, 2.971) \qquad C(0, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(0 - 0)^2 + 2.971^2} = 2.971 \text{ ksi}$$

Using these results, the circle is shown in Fig. *d*.

In-Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 0 + 2.971 = 2.97 \text{ ksi}$$ **Ans.**

$$\sigma_2 = 0 - 2.971 = -2.97 \text{ ksi}$$ **Ans.**

$$\theta_{p1} = 45.0°, \theta_{p2} = -45.0°$$ **Ans.**

Maximum In-Plane Shear Stress: Since there is no normal stress acting on the element,

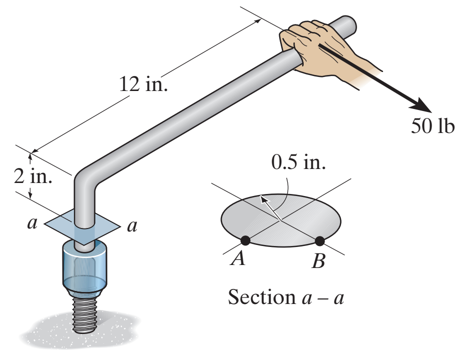
$$\tau_{\substack{\max \\ \text{in-plane}}} = (\tau_{xy})_A = 2.97 \text{ ksi}$$ **Ans.**

$$\theta_s = 0°$$ **Ans**

Ans:
$\sigma_1 = 2.97$ ksi, $\sigma_2 = -2.97$ ksi, $\theta_{p1} = 45.0°$,
$\theta_{p2} = -45.0°$, $\tau_{\substack{\max \\ \text{in-plane}}} = 2.97$ ksi, $\theta_s = 0°$

14–74.

If the box wrench is subjected to the 50-lb force, determine the principal stresses and maximum in-plane shear stress at point B on the cross section of the wrench at section a–a. Specify the orientation of these states of stress and indicate the results on elements at the point.



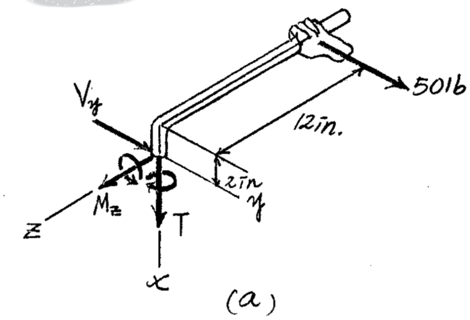
SOLUTION

Internal Loadings: Considering the equilibrium of the free-body diagram of the wrench's cut segment, Fig. a,

$\Sigma F_y = 0;\qquad V_y + 50 = 0\qquad V_y = -50 \text{ lb}$

$\Sigma M_x = 0;\qquad T + 50(12) = 0\qquad T = -600 \text{ lb}\cdot\text{in}$

$\Sigma M_z = 0;\qquad M_z - 50(2) = 0\qquad M_z = 100 \text{ lb}\cdot\text{in}$



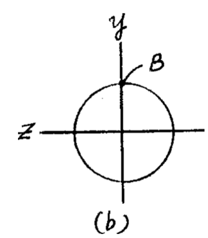
Section Properties: The moment of inertia about the z axis and the polar moment of inertia of the wrench's cross section are

$$I_z = \frac{\pi}{4}(0.5^4) = 0.015625\pi \text{ in}^4$$

$$J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$$

Referring to Fig. b,

$$(Q_y)_B = 0$$



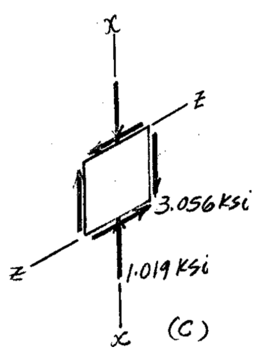
Normal and Shear Stress: The normal stress is caused by the bending stress due to $\mathbf{M}_z$.

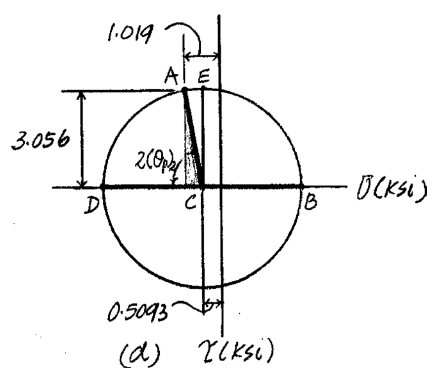
$$(\sigma_x)_B = -\frac{M_z y_B}{I_z} = -\frac{100(0.5)}{0.015625\pi} = -1.019 \text{ ksi}$$

The shear stress at point B along the y axis is $(\tau_{xy})_B = 0$ since $(Q_y)_B$; however, the shear stress along the z axis is caused by torsion.

$$(\tau_{xz})_B = \frac{Tc}{J} = \frac{600(0.5)}{0.03125\pi} = 3.056 \text{ ksi}$$

The state of stress at point B is represented by the two-dimensional element shown in Fig. c.





14–74. Continued

Construction of the Circle: $\sigma_x = -1.019$ ksi, $\sigma_z = 0$, and $\tau_{xz} = -3.056$ ksi. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-1.019 + 0}{2} = -0.5093 \text{ ksi}$$

The coordinates of reference point A and the center C of the circle are

$$A(-1.019, -3.056) \qquad C(-0.5093, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[-1.019 - (-0.5093)]^2 + (-3.056)^2} = 3.0979 \text{ ksi}$$

Using these results, the circle is shown in Fig. *d*.

In-Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -0.5093 + 3.0979 = 2.59 \text{ ksi}$$ **Ans.**

$$\sigma_2 = -0.5093 - 3.0979 = -3.61 \text{ ksi}$$ **Ans.**

$$\theta_{p1} = -40.3^\circ \qquad \theta_{p2} = 49.7^\circ$$ **Ans.**

Maximum In-Plane Shear Stress: The coordinates of point E represent the maximum in-plane stress, Fig. *a*.

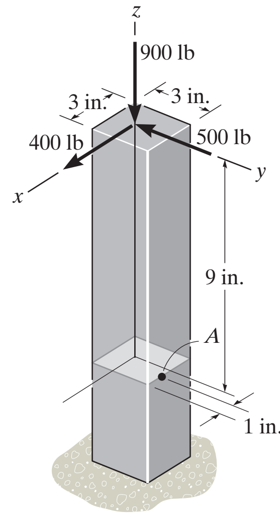
$$\tau_{\substack{max \\ in\text{-}plane}} = R = 3.10 \text{ ksi}$$ **Ans.**

$$\theta_s = 4.73^\circ$$ **Ans.**

Ans:
$\sigma_1 = 2.59$ ksi, $\sigma_2 = -3.61$ ksi, $\theta_{p1} = -40.3^\circ$, $\theta_{p2} = 49.7^\circ$, $\tau_{\substack{max \\ in\text{-}plane}} = 3.10$ ksi, $\theta_s = 4.73^\circ$

14–75.

The post is fixed supported at its base and the loadings are applied at its end as shown. Determine (a) the maximum in-plane shear stress developed at A and (b) the principal stresses at A.



SOLUTION

Section properties:

$$I_x = I_y = \frac{1}{12}(3)\left(3^3\right) = 6.75 \text{ in}^4$$

$$A = 3(3) = 9 \text{ in}^2$$

$$(Q_A)_x = \bar{y}'A' = (1)(1)(3) = 3 \text{ in}^3$$

$$(Q_A)_y = 0$$

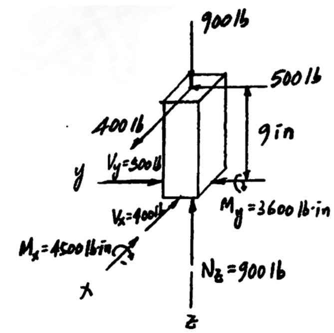
Normal stress: Applying $\sigma = \dfrac{P}{A} + \dfrac{M_x y}{I_x} + \dfrac{M_y x}{I_y}$

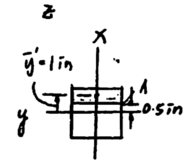
$$\sigma_A = -\frac{900}{9} + \frac{4500(1.5)}{6.75} - \frac{3600(0.5)}{6.75} = 633.33 \text{ psi}$$

Shear stress: Applying $\tau = \dfrac{VQ}{It}$

$$\tau_{zx} = \frac{400(3)}{6.75(3)} = 59.259 \text{ psi}$$

$$\tau_{zy} = 0$$





Mohr's circle:

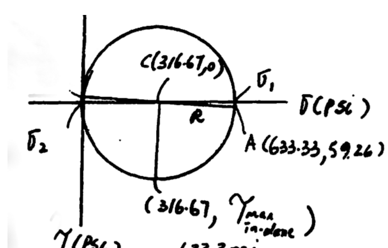
$A(633.33, 59.26)$ $\quad$ $C(316.67, 0)$

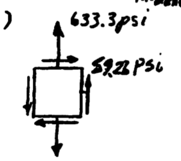
$$R = CA = \sqrt{(633.33 - 316.67)^2 + 59.26^2} = 322.16$$

(a) $\tau_{\substack{\max \\ \text{in-plane}}} = R = 322 \text{ psi}$ **Ans.**

(b) $\sigma_1 = 316.67 + 322.16 = 639 \text{ psi}$ **Ans.**

$\sigma_2 = 316.67 - 322.16 = -5.50 \text{ psi}$ **Ans.**

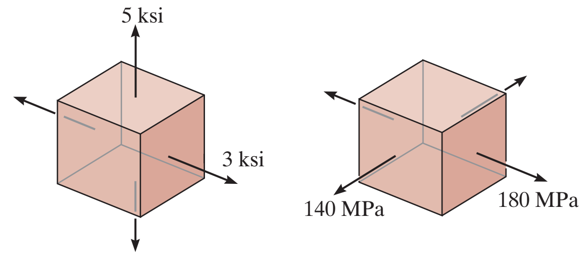




Ans:
$\tau_{\substack{\max \\ \text{in-plane}}} = 322$ psi,
$\sigma_1 = 639$ psi,
$\sigma_2 = -5.50$ psi

***14–76.**

Draw the three Mohr's circles that describe each of the following states of stress.

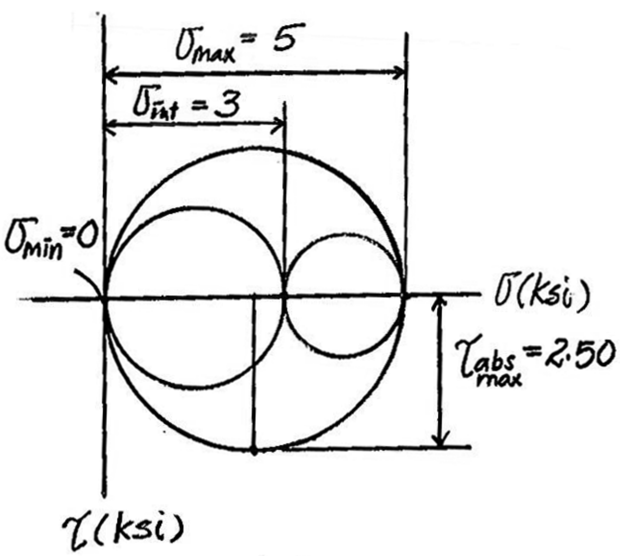


(a) (b)

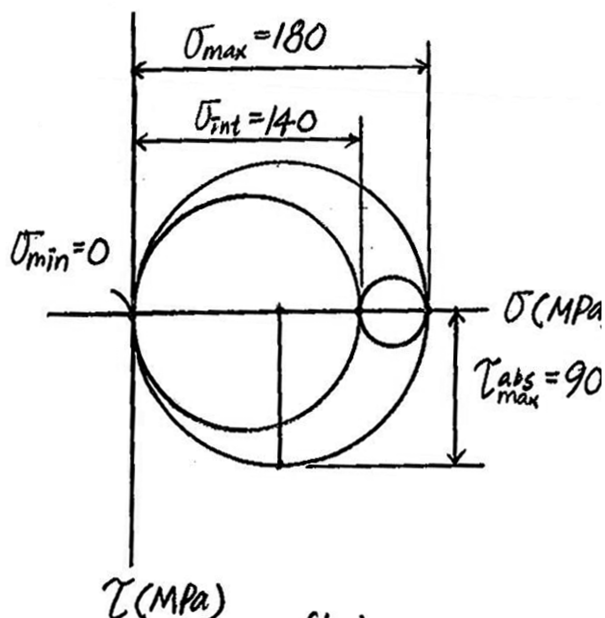
SOLUTION

(a) Here, $\sigma_{min} = 0$, $\sigma_{int} = 3$ ksi and $\sigma_{max} = 5$ ksi. The three Mohr's circles of this state of stress are shown in Fig. *a*.

(b) Here, $\sigma_{min} = 0$, $\sigma_{int} = 140$ MPa and $\sigma_{max} = 180$ MPa. The three Mohr's circles of this state of stress are shown in Fig. *b*.



(a)

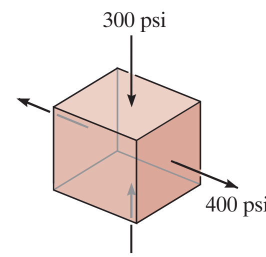


(b)

Ans:
N/A

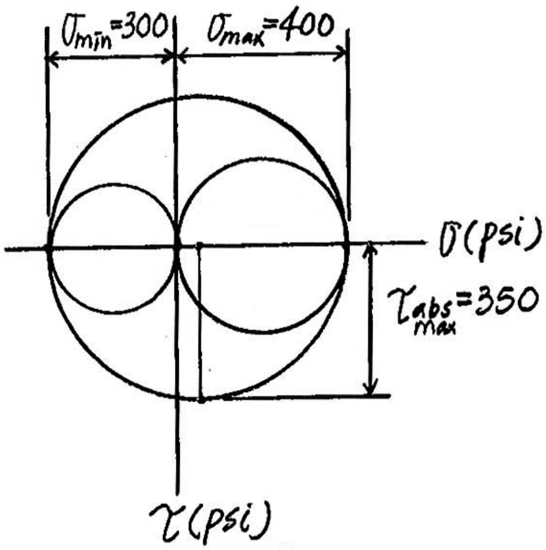
14–77.

Draw the three Mohr's circles that describe the following state of stress.



SOLUTION

Here, $\sigma_{min} = -300$ psi and $\sigma_{max} = 400$ psi. The three Mohr's circles for this state of stress are shown in Fig. *a*.

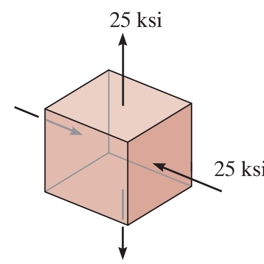


(a)

Ans:
N/A

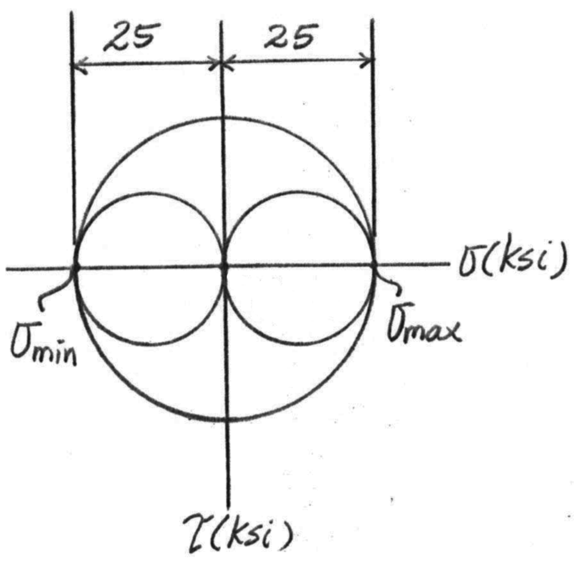
14–78.

Draw the three Mohr's circles that describe the following state of stress.



SOLUTION

Three Mohr's Circles: For $\sigma_{max} = 25$ ksi and $\sigma_{min} = -25$ ksi. Then, the three Mohr's circles represent this state of stress as shown in Fig. *a*.

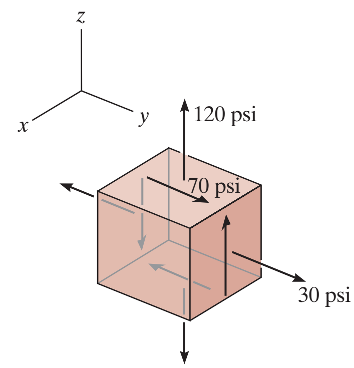


(a)

Ans:
N/A

14–79.

Determine the principal stresses and the absolute maximum shear stress.



SOLUTION

Mohr's circle for the element in the y–z plane, Fig. *a*, will be drawn first. In accordance with the established sign convention, $\sigma_y = 30$ psi, $\sigma_z = 120$ psi and $\tau_{yz} = 70$ psi. Thus,

$$\sigma_{avg} = \frac{\sigma_y + \sigma_z}{2} = \frac{30 + 120}{2} = 75 \text{ psi}$$

Thus, the coordinates of reference point A and the center C of the circle are

$$A(30, 70) \qquad C(75, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(75 - 30)^2 + 70^2} = 83.217 \text{ psi}$$

Using these results, the circle is shown in Fig. *b*.

The coordinates of point B and D represent the principal stresses.

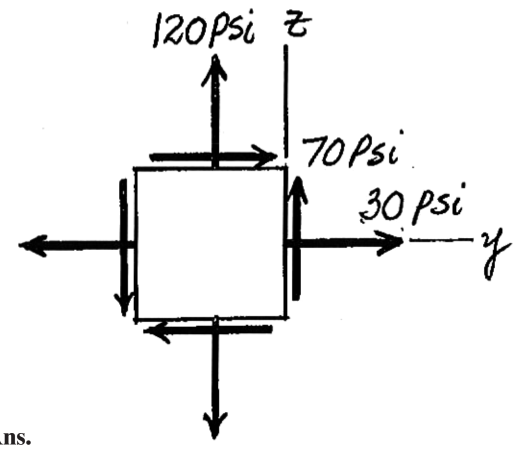
From the results,

$$\sigma_{max} = 158 \text{ psi} \qquad \sigma_{min} = -8.22 \text{ psi} \qquad \sigma_{int} = 0 \text{ psi} \qquad \textbf{Ans.}$$

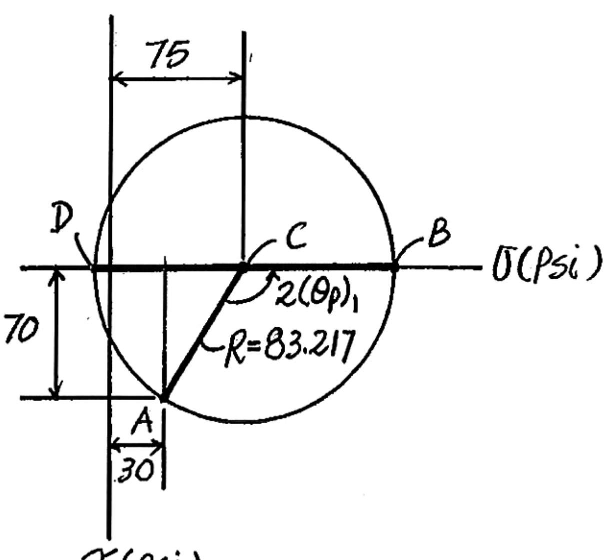
Using these results, the three Mohr's circles are shown in Fig. *c*.

From the geometry of the three circles,

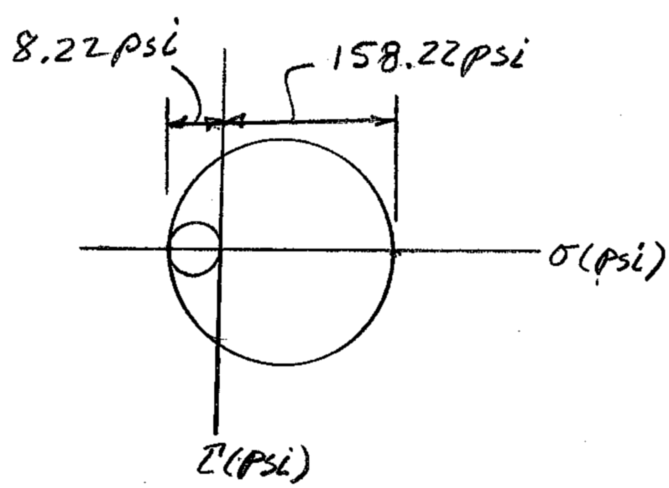
$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{158.22 - (-8.22)}{2} = 83.2 \text{ psi} \qquad \textbf{Ans.}$$



(a)



(b)

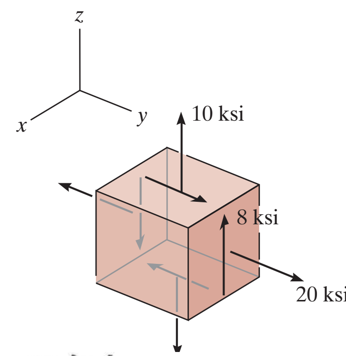


(c)

Ans:
$\sigma_{max} = 158$ psi, $\sigma_{min} = -8.22$ psi, $\sigma_{int} = 0$ psi,
$\tau_{\substack{abs\\max}} = 83.2$ psi

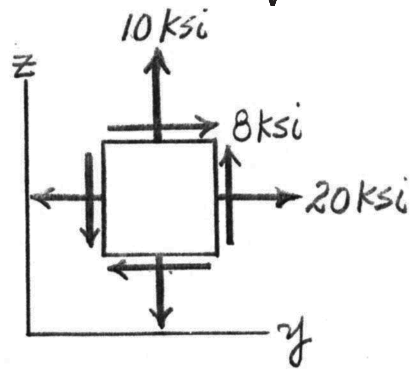
***14–80.**

Determine the principal stresses and the absolute maximum shear stress.



SOLUTION

Construction of the Circle: The stresses acting on the differential element in the y–z plane are shown in Fig. *a*. The circle for this element will be drawn first. In accordance with sign conventions, $\sigma_y = 20$ ksi, $\sigma_z = 10$ ksi and $\tau_{yz} = 8$ ksi. Hence,



(a)

$$\sigma_{avg} = \frac{\sigma_y + \sigma_z}{2} = \frac{20 + 10}{2} = 15.0 \text{ ksi}$$

The coordinates for reference point A and center of circle C are

$A(20, 8)$ $C(15.0, 0)$

The radius of the circle is $R = CA = \sqrt{(20 - 15.0)^2 + (8 - 0)^2} = \sqrt{89}$ ksi. The circle shown in Fig. *b* can be constructed based on these results.

In-Plane Principal Stresses: The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 15.0 + \sqrt{89} = 24.43 \text{ ksi} \qquad \sigma_2 = 15.0 - \sqrt{89} = 5.566 \text{ ksi}$$

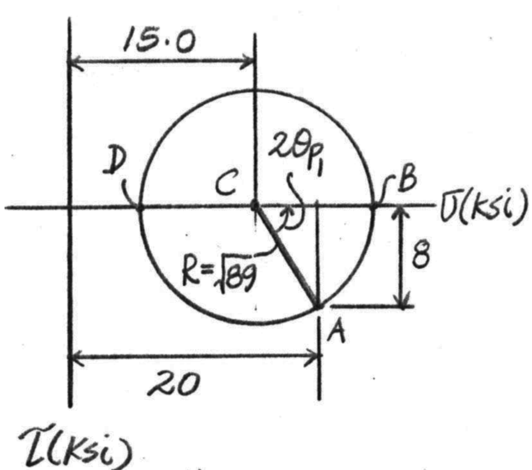
Construction of Three Mohr's Circles: Using the results obtained above,

$$\sigma_1 = 24.43 \text{ ksi} = 24.4 \text{ ksi} \qquad \sigma_2 = 5.566 \text{ ksi} = 5.57 \text{ ksi}$$ **Ans.**

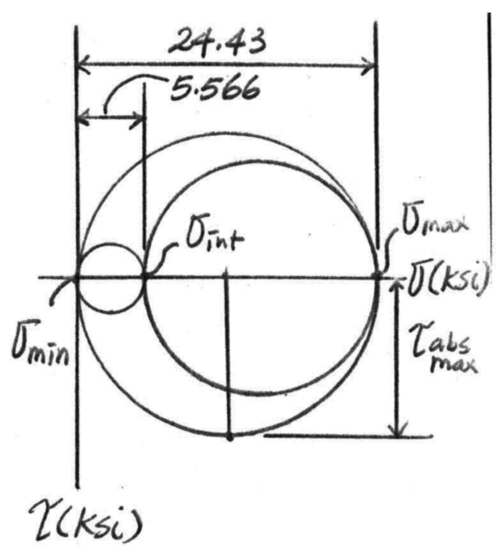
Then, the three Mohr's circles shown in Fig. *c* can be constructed.

Absolute Maximum Shear Stress: From the three Mohr's circles,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max}}{2} = \frac{24.43}{2} = 12.22 \text{ ksi} = 12.2 \text{ ksi}$$ **Ans.**



(b)

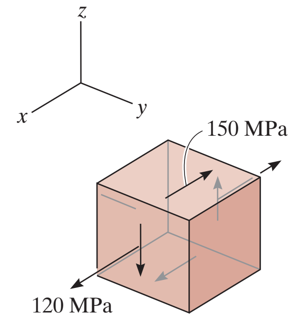


(c)

Ans:
$\sigma_1 = 24.4$ ksi,
$\sigma_2 = 5.57$ ksi,
$\tau_{\substack{abs\\max}} = 12.2$ ksi

14–81.

Determine the principal stresses and the absolute maximum shear stress.



SOLUTION

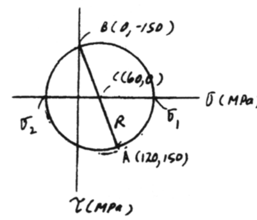
For *x*-*z* plane:

$$R = CA = \sqrt{(120 - 60)^2 + 150^2} = 161.55$$

$$\sigma_1 = 60 + 161.55 = 221.55 \text{ MPa}$$

$$\sigma_2 = 60 - 161.55 = -101.55 \text{ MPa}$$

$\sigma_1 = 222 \text{ MPa} \qquad \sigma_2 = -102 \text{ MPa}$ **Ans.**

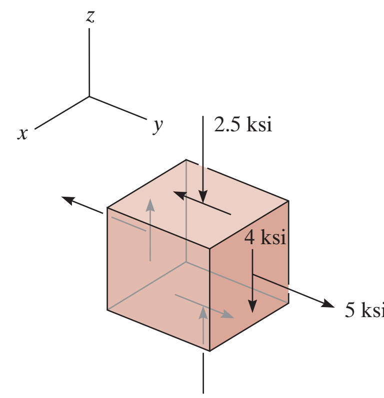


$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{221.55 - (-101.55)}{2} = 162 \text{ MPa}$$ **Ans.**

Ans:
$\sigma_1 = 222$ MPa,
$\sigma_2 = -102$ MPa,
$\tau_{\substack{\text{abs}\\\text{max}}} = 162$ MPa

14–82.

Determine the principal stresses and the absolute maximum shear stress.



SOLUTION

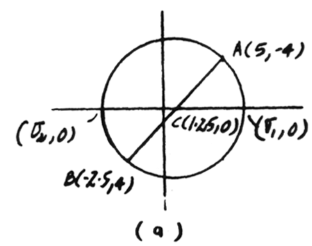
For *y*-*z* plane:

$$A(5, -4) \qquad B(-2.5, 4) \qquad C(1.25, 0)$$

$$R = \sqrt{3.75^2 + 4^2} = 5.483$$

$$\sigma_1 = 1.25 + 5.483 = 6.733 \text{ ksi}$$

$$\sigma_2 = 1.25 - 5.483 = -4.233 \text{ ksi}$$



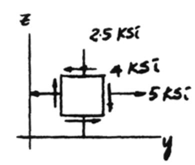
Thus,

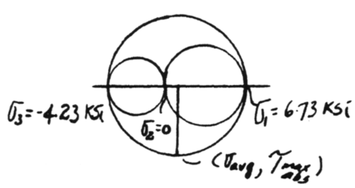
$$\sigma_1 = 6.73 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -4.23 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{avg} = \frac{6.73 + (-4.23)}{2} = 1.25 \text{ ksi}$$

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{6.73 - (-4.23)}{2} = 5.48 \text{ ksi} \qquad \textbf{Ans.}$$

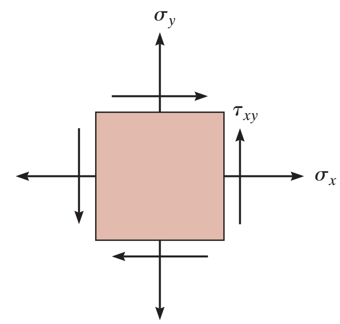




Ans:
$\sigma_1 = 6.73$ ksi,
$\sigma_2 = -4.23$ ksi,
$\tau_{\substack{abs\\max}} = 5.48$ ksi

14–83.

Consider the general case of plane stress as shown. Write a computer program that will show a plot of the three Mohr's circles for the element, and will also determine the maximum in-plane shear stress and the absolute maximum shear stress.



***14–84.**

Prove that the sum of the normal strains in perpendicular directions is constant, i.e., $\epsilon_x + \epsilon_y = \epsilon_{x'} + \epsilon_{y'}$.

SOLUTION

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta \qquad \textbf{(1)}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta \qquad \textbf{(2)}$$

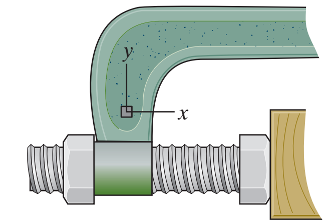
Adding Eq. (1) and Eq. (2) yields:

$$\epsilon_{x'} + \epsilon_{y'} = \epsilon_x + \epsilon_y = \text{constant} \qquad \textbf{(Q.E.D.)}$$

Ans:
N/A

14–85.

The state of strain at the point on the arm has components of $\epsilon_x = 200(10^{-6})$, $\epsilon_y = -300(10^{-6})$, and $\gamma_{xy} = 400(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of 30° counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



SOLUTION

In accordance with the established sign convention,

$$\epsilon_x = 200(10^{-6}), \qquad \epsilon_y = -300(10^{-6}) \qquad \gamma_{xy} = 400(10^{-6}) \qquad \theta = 30°$$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + (-300)}{2} + \frac{200 - (-300)}{2}\cos 60° + \frac{400}{2}\sin 60°\right](10^{-6})$$

$$= 248\,(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \left\{-\left[200 - (-300)\right]\sin 60° + 400\cos 60°\right\}(10^{-6})$$

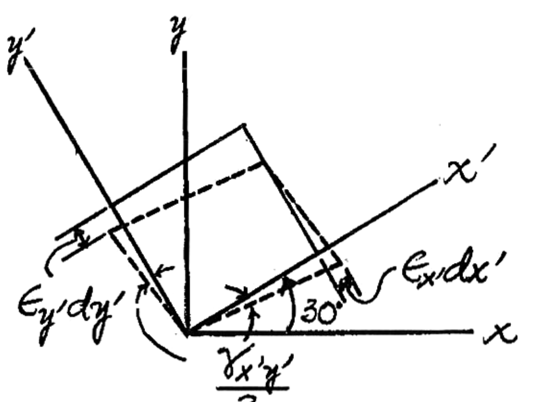
$$= -233(10^{-6})$$ **Ans.**

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + (-300)}{2} - \frac{200 - (-300)}{2}\cos 60° - \frac{400}{2}\sin 60°\right](10^{-6})$$

$$= -348(10^{-6})$$ **Ans.**

The deformed element of this equivalent state of strain is shown in Fig. *a*.

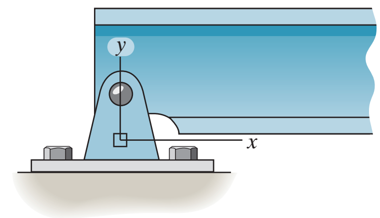


(a)

Ans:
$\epsilon_{x'} = 248(10^{-6})$,
$\gamma_{x'y'} = -233(10^{-6})$,
$\epsilon_{y'} = -348(10^{-6})$

14–86.

The state of strain at the point on the pin leaf has components of $\epsilon_x = 200(10^{-6})$, $\epsilon_y = 180(10^{-6})$, and $\gamma_{xy} = -300(10^{-6})$. Use the strain transformation equations and determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 60°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.

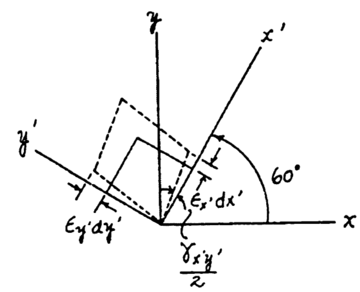


SOLUTION

Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 200\left(10^{-6}\right) \qquad \epsilon_y = 180\left(10^{-6}\right) \qquad \gamma_{xy} = -300\left(10^{-6}\right)$$

$$\theta = +60°$$



Strain Transformation Equations: Applying Eqs. 10–5, 10–6, and 10–7,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{200 + 180}{2} + \frac{200 - 180}{2}\cos 120° + \frac{-300}{2}\sin 120°\right)\left(10^{-6}\right)$$

$$= 55.1\left(10^{-6}\right) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(200 - 180)\sin 120° + (-300)\cos 120°]\left(10^{-6}\right)$$

$$= 133\left(10^{-6}\right) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

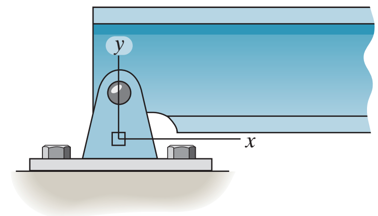
$$= \left(\frac{200 + 180}{2} - \frac{200 - 180}{2}\cos 120° - \frac{-300}{2}\sin 120°\right)\left(10^{-6}\right)$$

$$= 325\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 55.1\left(10^{-6}\right)$,
$\gamma_{x'y'} = 133\left(10^{-6}\right)$,
$\epsilon_{y'} = 325\left(10^{-6}\right)$

14–87.

Solve Prob. 14–86 for an element oriented $\theta = 30°$ clockwise.

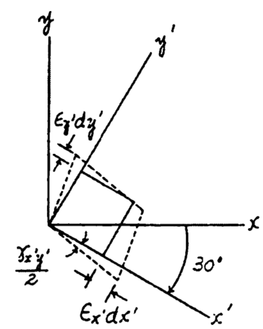


SOLUTION

Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 200(10^{-6}) \qquad \epsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -300(10^{-6})$$

$$\theta = -30°$$



Strain Transformation Equations: Applying Eqs. 10–5, 10–6, and 10–7,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + 180}{2} + \frac{200 - 180}{2}\cos(-60°) + \frac{-300}{2}\sin(-60°)\right](10^{-6})$$

$$= 325(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(200 - 180)\sin(-60°) + (-300)\cos(-60°)](10^{-6})$$

$$= -133(10^{-6})$$ **Ans.**

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

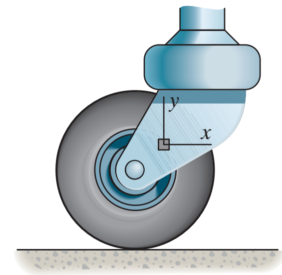
$$= \left(\frac{200 + 180}{2} - \frac{200 - 180}{2}\cos(-60°) - \frac{-300}{2}\sin(-60°)\right)(10^{-6})$$

$$= 55.1(10^{-6})$$ **Ans.**

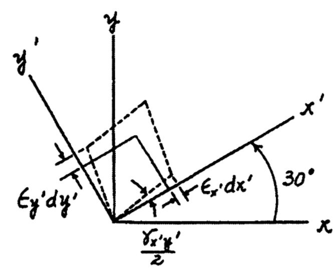
Ans:
$\epsilon_{x'} = 325(10^{-6})$,
$\gamma_{x'y'} = -133(10^{-6})$,
$\epsilon_{y'} = 55.1(10^{-6})$

***14–88.**

The state of strain at the point on the leaf of the caster assembly has components of $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 860(10^{-6})$, and $\gamma_{xy} = 375(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



SOLUTION



Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = -400(10^{-6}) \qquad \epsilon_y = 860(10^{-6}) \qquad \gamma_{xy} = 375(10^{-6})$$

$$\theta = +30°$$

Strain Transformation Equations: Applying Eqs. 10–5, 10–6, and 10–7,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left(\frac{-400 + 860}{2} + \frac{-400 - 860}{2}\cos 60° + \frac{375}{2}\sin 60°\right)(10^{-6})$$

$$= 77.4(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-400 - 860)\sin 60° + 375\cos 60°](10^{-6})$$

$$= 1279(10^{-6})$$ **Ans.**

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

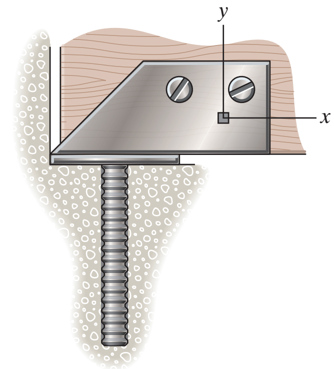
$$= \left(\frac{-400 + 860}{2} - \frac{-400 - 860}{2}\cos 60° - \frac{375}{2}\sin 60°\right)(10^{-6})$$

$$= 383(10^{-6})$$ **Ans.**

Ans:
$\epsilon_{x'} = 77.4(10^{-6})$,
$\gamma_{x'y'} = 1279(10^{-6})$,
$\epsilon_{y'} = 383(10^{-6})$

14–89.

The state of strain at a point on the bracket has component: $\epsilon_x = 150(10^{-6})$, $\epsilon_y = 200(10^{-6})$, $\gamma_{xy} = -700(10^{-6})$. Use the strain transformation equations and determine the equivalent in plane strains on an element oriented at an angle of $\theta = 60°$ counterclockwise from the original position. Sketch the deformed element within the x–y plane due to these strains.



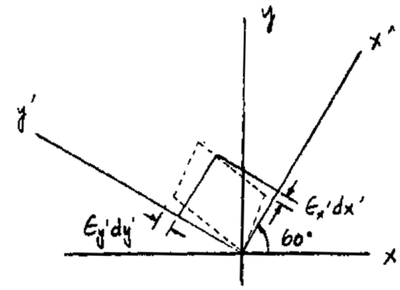
SOLUTION

$\epsilon_x = 150(10^{-6}) \qquad \epsilon_y = 200(10^{-6}) \qquad \gamma_{xy} = -700(10^{-6}) \qquad \theta = 60°$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos 120° + \left(\frac{-700}{2}\right)\sin 120°\right]10^{-6}$$

$$= -116(10^{-6})$$ **Ans.**



$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos 120° - \left(\frac{-700}{2}\right)\sin 120°\right]10^{-6}$$

$$= 466(10^{-6})$$ **Ans.**

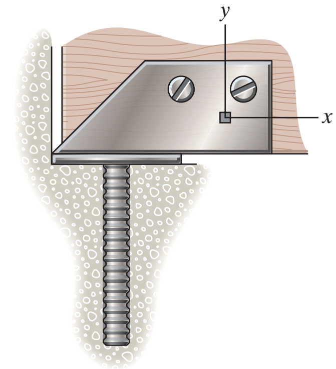
$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2\left[-\frac{150 - 200}{2}\sin 120° + \left(\frac{-700}{2}\right)\cos 120°\right]10^{-6} = 393(10^{-6})$$ **Ans.**

Ans:
$\epsilon_{x'} = -116(10^{-6})$,
$\epsilon_{y'} = 466(10^{-6})$,
$\gamma_{x'y'} = 393(10^{-6})$

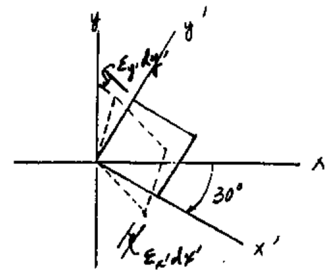
14–90.

Solve Prob. 14–89 for an element oriented $\theta = 30°$ clockwise.



SOLUTION

$\epsilon_x = 150(10^{-6}) \qquad \epsilon_y = 200(10^{-6}) \qquad \gamma_{xy} = -700(10^{-6}) \qquad \theta = -30°$



$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos(-60°) + \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6}$$

$$= 466(10^{-6})$$ **Ans.**

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos(-60°) - \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6}$$

$$= -116(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

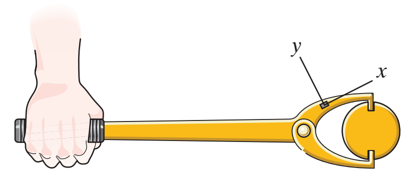
$$\gamma_{x'y'} = 2\left[-\frac{150 - 200}{2}\sin(-60°) + \frac{-700}{2}\cos(-60°)\right]10^{-6}$$

$$= -393(10^{-6})$$ **Ans.**

Ans:
$\epsilon_{x'} = 466(10^{-6})$,
$\epsilon_{y'} = -116(10^{-6})$,
$\gamma_{x'y'} = -393(10^{-6})$

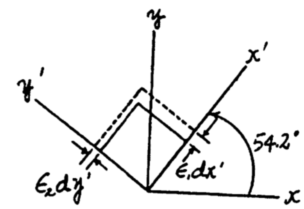
14–91.

The state of strain at the point on the spanner wrench has components of $\epsilon_x = 260(10^{-6})$, $\epsilon_y = 320(10^{-6})$, and $\gamma_{xy} = 180(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

(a)

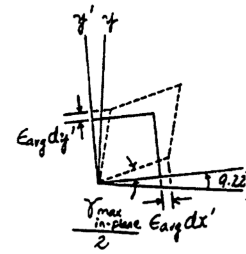


In-Plane Principal Strain: Applying Eq. 10–9,

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{260 + 320}{2} \pm \sqrt{\left(\frac{260 - 320}{2}\right)^2 + \left(\frac{180}{2}\right)^2}\right](10^{-6})$$

$$= 290 \pm 94.87$$



$$\epsilon_1 = 385(10^{-6}) \qquad \epsilon_2 = 195(10^{-6})$$ **Ans.**

Orientation of Principal Strain: Applying Eq. 10–8,

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{180(10^{-6})}{(260 - 320)(10^{-6})} = -3.000$$

$$\theta_p = -35.78° \qquad \text{and} \qquad 54.22°$$

Use Eq. 10–5 to determine which principal strain deforms the element in the x' direction with $\theta = -35.78°$.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{260 + 320}{2} + \frac{260 - 320}{2}\cos(-71.56°) + \frac{180}{2}\sin(-71.56°)\right](10^{-6})$$

$$= 195(10^{-6}) = \epsilon_2$$

Hence, $\theta_{p1} = 54.2°$ and $\theta_{p1} = -35.8°$ **Ans.**

(b)

Maximum In-Plane Shear Strain: Applying Eq. 10–11,

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{260 - 320}{2}\right)^2 + \left(\frac{180}{2}\right)^2}\right](10^{-6})$$

$$= 190(10^{-6})$$ **Ans.**

14–91. Continued

Orientation of Maximum In-Plane Shear Strain: Applying Eq. 10–10,

$$\tan 2\theta_s = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}} = -\frac{260 - 320}{180} = 0.3333$$

$$\theta_s = 9.22° \qquad \text{and} \qquad -80.8° \qquad \textbf{Ans.}$$

Normal Strain and Shear Strain: In accordance with the sign convention,

$\epsilon_x = 260\left(10^{-6}\right) \qquad \epsilon_y = 320\left(10^{-6}\right) \qquad \gamma_{xy} = 180\left(10^{-6}\right)$

The proper sign of $\gamma_{\substack{\text{max} \\ \text{in-plane}}}$ can be determined by substituting $\theta = 9.22°$ into Eq. 10–6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(260 - 320)\sin 18.44° + 180\cos 18.44°]\left(10^{-6}\right)$$

$$= 190\left(10^{-6}\right)$$

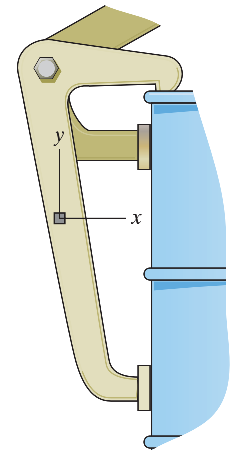
Average Normal Strain: Applying Eq. 10–12,

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left[\frac{260 + 320}{2}\right]\left(10^{-6}\right) = 290\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 385\left(10^{-6}\right), \epsilon_2 = 195\left(10^{-6}\right),$
$\theta_{p1} = 54.2°, \theta_{p1} = -35.8°,$
$\gamma_{\substack{\text{max} \\ \text{in-plane}}} = 190\left(10^{-6}\right),$
$\theta_s = 9.22° \quad \text{and} \quad -80.8°,$
$\epsilon_{\text{avg}} = 290\left(10^{-6}\right)$

***14–92.**

The state of strain at the point on the member has components of $\epsilon_x = 180(10^{-6})$, $\epsilon_y = -120(10^{-6})$, and $\gamma_{xy} = -100(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

(a) In accordance with the established sign convention, $\epsilon_x = 180(10^{-6})$, $\epsilon_y = -120(10^{-6})$ and $\gamma_{xy} = -100(10^{-6})$.

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left\{\frac{180 + (-120)}{2} \pm \sqrt{\left[\frac{180 - (-120)}{2}\right]^2 + \left(\frac{-100}{2}\right)^2}\right\}(10^{-6})$$

$$= \left(30 \pm 158.11\right)(10^{-6})$$

$\epsilon_1 = 188(10^{-6}) \qquad \epsilon_2 = -128(10^{-6})$ **Ans.**

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-100(10^{-6})}{\left[180 - (-120)\right](10^{-6})} = -0.3333$$

$$\theta_P = -9.217^\circ \quad \text{and} \quad 80.78^\circ$$

Substitute $\theta = -9.217^\circ$.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{180 + (-120)}{2} + \frac{180 - (-120)}{2}\cos(-18.43^\circ) + \frac{-100}{2}\sin(-18.43)\right](10^{-6})$$

$$= 188(10^{-6}) = \epsilon_1$$

Thus,

$(\theta_P)_1 = -9.22^\circ \qquad (\theta_P)_2 = 80.8^\circ$ **Ans.**

The deformed element is shown in Fig (a).

(b) $\epsilon_{avg} = \dfrac{180(10^{-6}) + (-120)(10^{-6})}{2} = 30(10^{-6})$ **Ans.**

$$\frac{\gamma_{\substack{max\\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{\substack{max\\ \text{in-plane}}} = \left\{2\sqrt{\left[\dfrac{180 - (-120)}{2}\right]^2 + \left(\dfrac{-100}{2}\right)^2}\right\}(10^{-6}) = 316\left(10^{-6}\right)$ **Ans.**

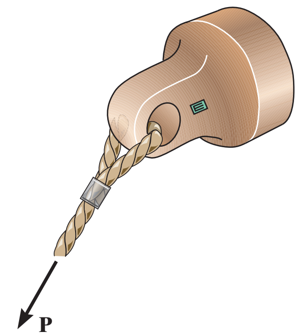
$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\left\{\frac{\left[180 - (-120)\right](10^{-6})}{-100(10^{-6})}\right\} = 3$$

$\theta_s = 35.78^\circ = 35.8^\circ \quad \text{and} \quad -54.22^\circ = -54.2^\circ$ **Ans.**

Ans:
$\epsilon_1 = 188(10^{-6})$, $\epsilon_2 = -128(10^{-6})$,
$(\theta_P)_1 = -9.22^\circ$, $(\theta_P)_2 = 80.8^\circ$,
$\epsilon_{avg} = 30(10^{-6})$,
$\gamma_{\substack{max\\ \text{in-plane}}} = 316\left(10^{-6}\right)$,
$\theta_s = 35.8^\circ$ and -54.2°

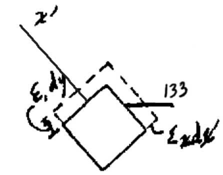
14–93.

The state of strain at the point on the support has components of $\epsilon_x = 350(10^{-6})$, $\epsilon_y = 400(10^{-6})$, $\gamma_{xy} = -675(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

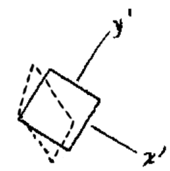
(a)



$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{350 + 400}{2} \pm \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$\epsilon_1 = 713(10^{-6})$ **Ans.** $\quad\epsilon_2 = 36.6(10^{-6})$ **Ans.**



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-675}{(350 - 400)}$$

$\theta_{p1} = 133°$ **Ans.**

(b)

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$(\gamma_{x'y'})_{\max} = 677(10^{-6})$ **Ans.**

$\epsilon_{avg} = \dfrac{\epsilon_x + \epsilon_y}{2} = \dfrac{350 + 400}{2} = 375(10^{-6})$ **Ans.**

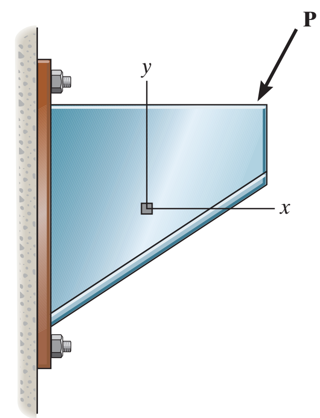
$$\tan 2\theta_s = \frac{-(\epsilon_x - \epsilon_y)}{\gamma_{xy}} = \frac{350 - 400}{675}$$

$\theta_s = -2.12°$ **Ans.**

Ans:
(a) $\epsilon_1 = 713(10^{-6})$, $\epsilon_2 = 36.6(10^{-6})$, $\theta_{p1} = 133°$,
(b) $\gamma_{\substack{\max \\ \text{in-plane}}} = 677(10^{-6})$, $\epsilon_{avg} = 375(10^{-6})$,
$\theta_s = -2.12°$

14–94.

Due to the load **P**, the state of strain at the point on the bracket has components of $\epsilon_x = 500(10^{-6})$, $\epsilon_y = 350(10^{-6})$, and $\gamma_{xy} = -430(10^{-6})$. Use the strain transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ clockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



SOLUTION

Normal Strain and Shear Strain: In accordance with the sign convention,

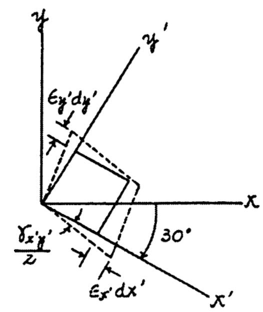
$$\epsilon_x = 500\left(10^{-6}\right) \qquad \epsilon_y = 350\left(10^{-6}\right) \qquad \gamma_{xy} = -430\left(10^{-6}\right)$$
$$\theta = -30°$$

Strain Transformation Equations:

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{500 + 350}{2} + \frac{500 - 350}{2}\cos(-60°) + \frac{-430}{2}\sin(-60°)\right]\left(10^{-6}\right)$$

$$= 649\left(10^{-6}\right) \qquad \textbf{Ans.}$$



$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(500 - 350)\sin(-60°) + (-430)\cos(-60°)]\left(10^{-6}\right)$$

$$= -85.1\left(10^{-6}\right) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

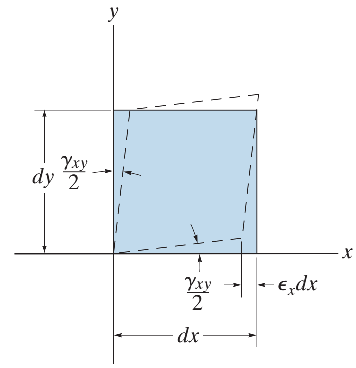
$$= \left(\frac{500 + 350}{2} - \frac{500 - 350}{2}\cos(-60°) - \frac{-430}{2}\sin(-60°)\right)\left(10^{-6}\right)$$

$$= 201\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 649\left(10^{-6}\right)$,
$\gamma_{x'y'} = -85.1\left(10^{-6}\right)$,
$\epsilon_{y'} = 201\left(10^{-6}\right)$

14–95.

The state of strain on an element has components $\epsilon_x = -400(10^{-6})$, $\epsilon_y = 0$, $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain on an element at the same point oriented 30° clockwise with respect to the original element. Sketch the results on this element.



SOLUTION

Strain Transformation Equations:

$$\epsilon_x = -400(10^{-6}) \qquad \epsilon_y = 0 \qquad \gamma_{xy} = 150(10^{-6}) \qquad \theta = -30^\circ$$

We obtain

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-400 + 0}{2} + \frac{-400 - 0}{2}\cos(-60^\circ) + \frac{150}{2}\sin(-60^\circ)\right](10^{-6})$$

$$= -365(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-400 - 0)\sin(-60^\circ) + 150\cos(-60^\circ)](10^{-6})$$

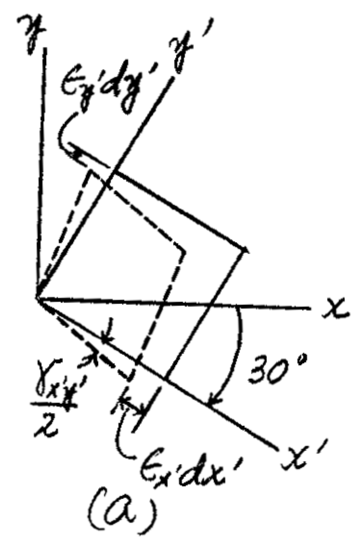
$$= -271(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_{y'} = \frac{\epsilon_x + \epsilon_y}{2} - \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-400 + 0}{2} - \frac{-400 - 0}{2}\cos(-60^\circ) - \frac{150}{2}\sin(-60^\circ)\right](10^{-6})$$

$$= -35.0(10^{-6}) \qquad \textbf{Ans.}$$

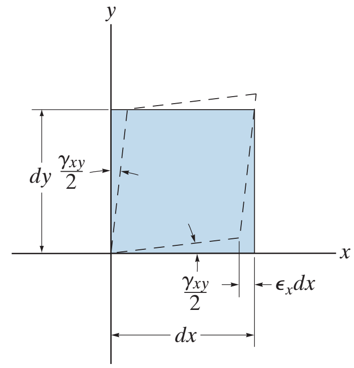
The deformed element for this state of strain is shown in Fig. *a*.



Ans:
$\epsilon_{x'} = -365(10^{-6})$,
$\gamma_{x'y'} = -271(10^{-6})$,
$\epsilon_{y'} = -35.0(10^{-6})$

***14–96.**

The state of plane strain on the element is $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 0$, and $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding elements for these states of strain with respect to the original element.



SOLUTION

In-Plane Principal Strains: $\epsilon_x = -300\left(10^{-6}\right)$, $\epsilon_y = 0$, and $\gamma_{xy} = 150\left(10^{-6}\right)$. We obtain

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-300 + 0}{2} \pm \sqrt{\left(\frac{-300 - 0}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right]\left(10^{-6}\right)$$

$$= (-150 \pm 167.71)\left(10^{-6}\right)$$

$\epsilon_1 = 17.7\left(10^{-6}\right) \qquad \epsilon_2 = -318\left(10^{-6}\right)$ **Ans.**

Orientation of Principal Strain:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{150\left(10^{-6}\right)}{(-300 - 0)\left(10^{-6}\right)} = -0.5$$

$\theta_P = -13.28°$ and $76.72°$

Substituting $\theta = -13.28°$ into Eq. 9–1,

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-300 + 0}{2} + \frac{-300 - 0}{2}\cos(-26.57°) + \frac{150}{2}\sin(-26.57°)\right]\left(10^{-6}\right)$$

$$= -318\left(10^{-6}\right) = \epsilon_2$$

Thus,

$\left(\theta_P\right)_1 = 76.7°$ and $\left(\theta_P\right)_2 = -13.3°$ **Ans.**

The deformed element of this state of strain is shown in Fig. *a*.

Maximum In-Plane Shear Strain:

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{\substack{\max \\ \text{in-plane}}} = \left[2\sqrt{\left(\frac{-300 - 0}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right]\left(10^{-6}\right) = 335\left(10^{-6}\right)$ **Ans.**

Orientation of the Maximum In-Plane Shear Strain:

$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\left[\frac{(-300 - 0)\left(10^{-6}\right)}{150\left(10^{-6}\right)}\right] = 2$$

$\theta_s = 31.7°$ and $122°$ **Ans.**

14–96. Continued

The algebraic sign for $\gamma_{\substack{\max \\ \text{in-plane}}}$ when $\theta = \theta_s = 31.7°$ can be obtained using

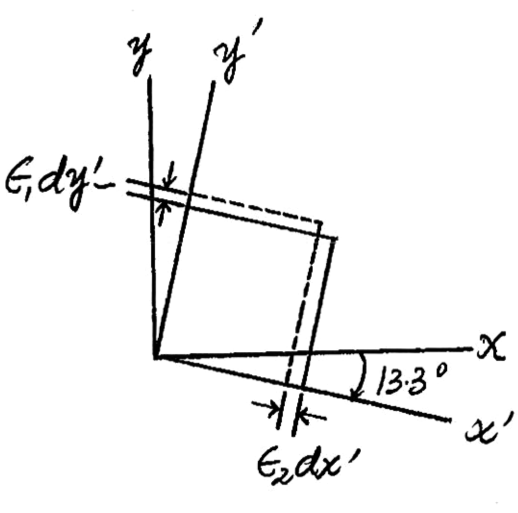
$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-300 - 0)\sin 63.43° + 150\cos 63.43°]\left(10^{-6}\right)$$
$$= 335\left(10^{-6}\right)$$

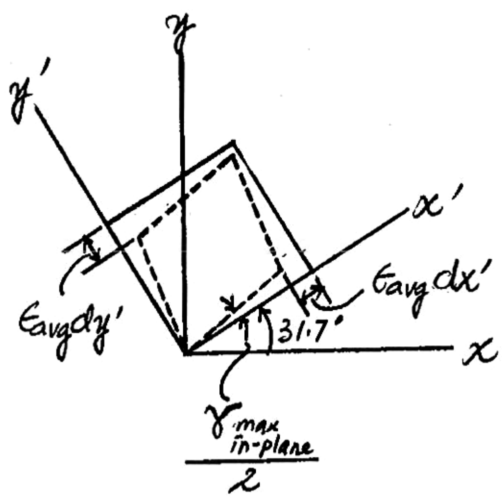
Average Normal Strain:

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-300 + 0}{2}\right)\left(10^{-6}\right) = -150\left(10^{-6}\right)$$ **Ans.**

The deformed element for this state of strain is shown in Fig. *b*.



(a)

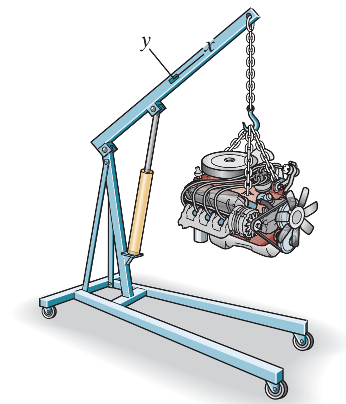


(b)

Ans:
$\epsilon_1 = 17.7(10^{-6})$, $\epsilon_2 = -318(10^{-6})$,
$\theta_{p1} = 76.7°$ and $\theta_{p2} = -13.3°$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 335(10^{-6})$, $\theta_s = 31.7°$ and $122°$,
$\epsilon_{\text{avg}} = -150(10^{-6})$

14–97.

The state of strain at the point on a boom of a shop crane has components of $\epsilon_x = 250(10^{-6})$, $\epsilon_y = 300(10^{-6})$, $\gamma_{xy} = -180(10^{-6})$. Use the strain transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case, specify the orientation of the element and show how the strains deform the element within the x–y plane.



SOLUTION

(a)

In-Plane Principal Strain: Applying Eq. 10–9,

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{250 + 300}{2} \pm \sqrt{\left(\frac{250 - 300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right]\left(10^{-6}\right)$$

$$= 275 \pm 93.41$$

$\epsilon_1 = 368\left(10^{-6}\right)$ $\qquad$ $\epsilon_2 = 182\left(10^{-6}\right)$ **Ans.**

Orientation of Principal Strain: Applying Eq. 10–8,

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-180(10^{-6})}{(250 - 300)(10^{-6})} = 3.600$$

$$\theta_P = 37.24^\circ \quad \text{and} \quad -52.76^\circ$$

Use Eq. 10–5 to determine which principal strain deforms the element in the x' direction with $\theta = 37.24°$.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{250 + 300}{2} + \frac{250 - 300}{2}\cos 74.48^\circ + \frac{-180}{2}\sin 74.48^\circ\right]\left(10^{-6}\right)$$

$$= 182\left(10^{-6}\right) = \epsilon_2$$

Hence,

$\theta_{p1} = -52.8^\circ$ and $\theta_{p2} = 37.2^\circ$ **Ans.**

(b)

Maximum In-Plane Shear Strain: Applying Eq. 10–11,

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{250 - 300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right]\left(10^{-6}\right)$$

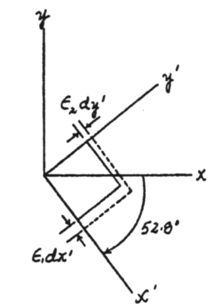
$$= 187\left(10^{-6}\right)$$ **Ans.**

14–97. Continued

Orientation of the Maximum In-Plane Shear Strain: Applying Eq. 10–10,

$$\tan 2\theta_s = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}} = -\frac{250 - 300}{-180} = -0.2778$$

$\theta_s = -7.76°$ and $82.2°$ **Ans.**

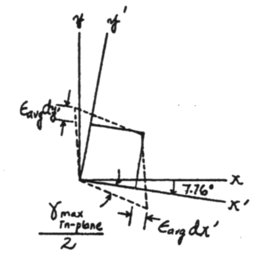


The proper sign of $\gamma_{\substack{\max \\ \text{in-plane}}}$ can be determined by substituting $\theta = -7.76°$ into Eq. 10–6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \{-[250 - 300]\sin(-15.52°) + (-180)\cos(-15.52°)\}\left(10^{-6}\right)$$

$$= -187\left(10^{-6}\right)$$



Normal Strain and Shear Strain: In accordance with the sign convention,

$$\epsilon_x = 250\left(10^{-6}\right) \quad \epsilon_y = 300\left(10^{-6}\right) \quad \gamma_{xy} = -180\left(10^{-6}\right)$$

Average Normal Strain: Applying Eq. 10–12,

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left[\frac{250 + 300}{2}\right]\left(10^{-6}\right) = 275\left(10^{-6}\right)$$ **Ans.**

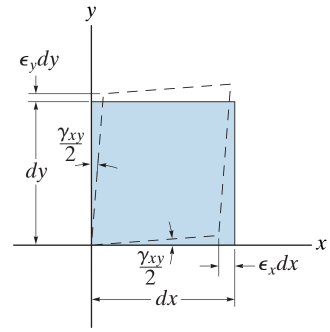
Ans:
$\epsilon_1 = 368(10^{-6})$, $\epsilon_2 = 182(10^{-6})$,
$\theta_{p1} = -52.8°$ and $\theta_{p2} = 37.2°$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 187(10^{-6})$, $\theta_s = -7.76°$ and $82.2°$,
$\epsilon_{\text{avg}} = 275(10^{-6})$

14–98.

Consider the general case of plane strain where ϵ_x, ϵ_y, and γ_{xy} are known. Write a computer program that can be used to determine the normal and shear strain, $\epsilon_{x'}$ and $\gamma_{x'y'}$, on the plane of an element oriented θ from the horizontal. Also, include the principal strains and the element's orientation, and the maximum in-plane shear strain, the average normal strain, and the element's orientation.

14–99.

The state of strain on the element has components $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 100(10^{-6})$, $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain, which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding elements for these states of strain with respect to the original element.



SOLUTION

In-Plane Principal Strains: $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 100(10^{-6})$, and $\gamma_{xy} = 150(10^{-6})$. We obtain

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-300 + 100}{2} \pm \sqrt{\left(\frac{-300 - 100}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right](10^{-6})$$

$$= (-100 \pm 213.60)(10^{-6})$$

$$\epsilon_1 = 114(10^{-6}) \qquad \epsilon_2 = -314(10^{-6})$$ **Ans.**

Orientation of Principal Strains:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{150(10^{-6})}{(-300 - 100)(10^{-6})} = -0.375$$

$$\theta_p = -10.28° \text{ and } 79.72°$$

Substituting $\theta = -10.28°$ into

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-300 + 100}{2} + \frac{-300 - 100}{2}\cos(-20.56°) + \frac{150}{2}\sin(-20.56°)\right](10^{-6})$$

$$= -314(10^{-6}) = \epsilon_2$$

Thus,

$$(\theta_p)_1 = 79.7° \text{ and } (\theta_p)_2 = -10.3°$$ **Ans.**

The deformed element for the state of principal strain is shown in Fig. *a*.

Maximum In-Plane Shear Strain:

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = \left[2\sqrt{\left(\frac{-300 - 100}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right](10^{-6}) = 427(10^{-6})$$ **Ans.**

14–99. Continued

Orientation of Maximum In-Plane Shear Strain:

$$\tan 2\theta_s = -\left(\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}}\right) = -\left[\frac{(-300 - 100)(10^{-6})}{150(10^{-6})}\right](10^{-6}) = 2.6667$$

$$\theta_s = 34.7° \text{ and } 125° \qquad \textbf{Ans.}$$

The algebraic sign for $\gamma_{\substack{\max \\ \text{in-plane}}}$ when $\theta = \theta_s = 34.7°$ can be determined using

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x - \epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

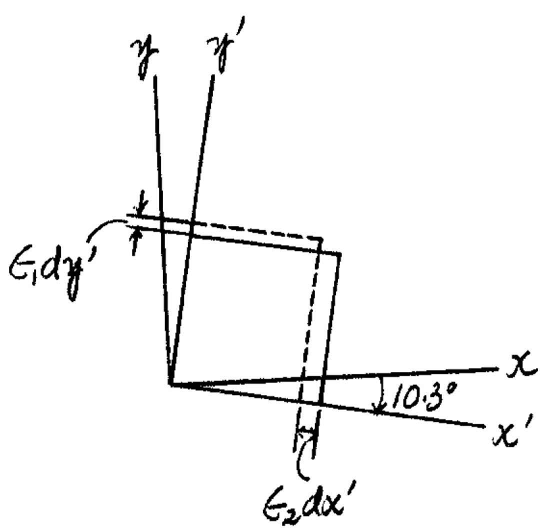
$$\gamma_{x'y'} = [-(-300 - 100)\sin 69.44° + 150\cos 69.44°](10^{-6})$$

$$= 427(10^{-6})$$

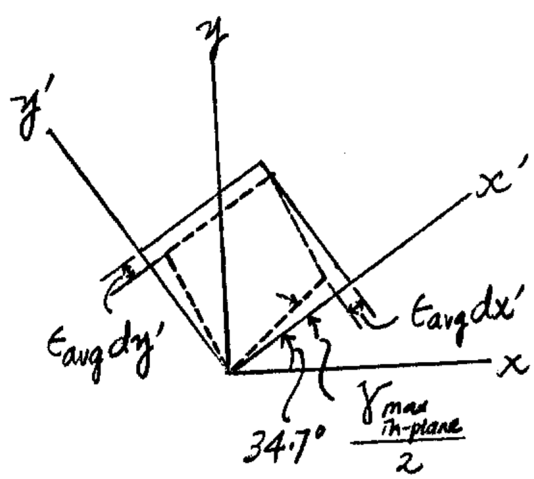
Average Normal Strain:

$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-300 + 100}{2}\right)(10^{-6}) = -100(10^{-6}) \qquad \textbf{Ans.}$$

The deformed element for this state of maximum in-plane shear strain is shown in Fig. *b*.



(a)



(b)

Ans:
$\epsilon_1 = 114(10^{-6})$, $\epsilon_2 = -314(10^{-6})$,
$(\theta_p)_1 = 79.7°$, $(\theta_p)_2 = -10.3°$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 427(10^{-6})$, $\theta_s = 34.7°$ and $125°$,
$\epsilon_{\text{avg}} = -100(10^{-6})$

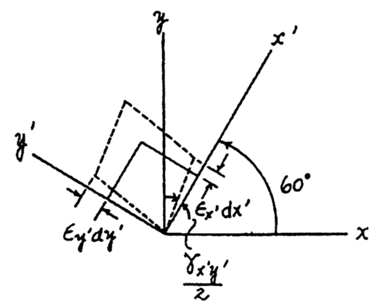
***14–100.**

Solve Prob. 14–86 using Mohr's circle.

SOLUTION

Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 200(10^{-6})$, $\varepsilon_y = 180(10^{-6})$, and $\dfrac{\gamma_{xy}}{2} = -150(10^{-6})$. Hence,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{200 + 180}{2}\right)(10^{-6}) = 190(10^{-6})$$



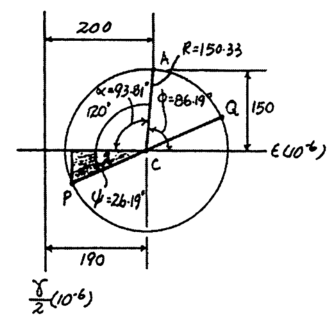
The coordinates for reference points A and C are

$A(200, -150)(10^{-6})$ $\quad C(190, 0)(10^{-6})$

The radius of the circle is

$$R = \left(\sqrt{(200 - 190)^2 + 150^2}\right)(10^{-6}) = 150.33(10^{-6})$$

Strain on the Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \dfrac{\gamma_{x'y'}}{2}\right)$ are represented by coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.



$$\varepsilon_{x'} = (190 - 150.33\cos 26.19°)(10^{-6}) = 55.1(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = (150.33\sin 26.19°)(10^{-6})$$

$$\gamma_{x'y'} = 133(10^{-6}) \qquad \textbf{Ans.}$$

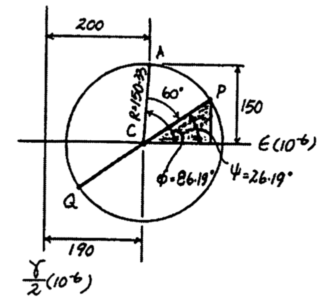
$$\varepsilon_{y'} = (190 + 150.33\cos 26.19°)(10^{-6}) = 325(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{x'} = 55.1(10^{-6})$,
$\gamma_{x'y'} = 133(10^{-6})$,
$\epsilon_{y'} = 325(10^{-6})$

14–101.

Solve Prob. 14–87 using Mohr's circle.

SOLUTION



Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 200(10^{-6})$, $\varepsilon_y = 180(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -150(10^{-6})$. Hence,

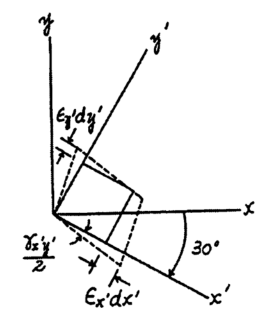
$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{200 + 180}{2}\right)(10^{-6}) = 190(10^{-6})$$

The coordinates for reference points A and C are

$$A(200, -150)(10^{-6}) \qquad C(190, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(200 - 190)^2 + 150^2}\right)(10^{-6}) = 150.33(10^{-6})$$



Strain on the Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

$$\varepsilon_{x'} = (190 + 150.33\cos 26.19°)(10^{-6}) = 325(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -(150.33\sin 26.19°)(10^{-6})$$

$$\gamma_{x'y'} = -133(10^{-6})$$ **Ans.**

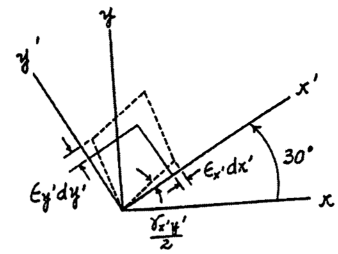
$$\varepsilon_{y'} = (190 - 150.33\cos 26.19°)(10^{-6}) = 55.1(10^{-6})$$ **Ans.**

Ans:
$\epsilon_{x'} = 325(10^{-6})$,
$\gamma_{x'y'} = -133(10^{-6})$,
$\epsilon_{y'} = 55.1(10^{-6})$

14–102.

Solve Prob. 14–88 using Mohr's circle.

SOLUTION



Construction of the Circle: In accordance with the sign convention $\varepsilon_x = -400(10^{-6})$, $\varepsilon_y = 860(10^{-6})$ and

$$\frac{\gamma_{xy}}{2} = 187.5(10^{-6}). \text{ Hence,}$$

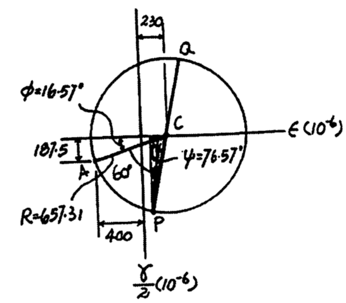
$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-400 + 860}{2}\right)(10^{-6}) = 230(10^{-6})$$

The coordinates for reference points A and C are

$$A(-400, 187.5)(10^{-6}) \qquad C(230, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(400 + 230)^2 + 187.5^2}\right)(10^{-6}) = 657.31(10^{-6})$$



Strain on the Inclined Element: The normal and shear strain $\left(\varepsilon_{x'} \text{ and } \frac{\gamma_{x'y'}}{2}\right)$ are represented by the coordinates of point P on the circle. $\varepsilon_{y'}$ can be determined by calculating the coordinates of point Q on the circle.

$$\varepsilon_{x'} = (230 - 657.31\cos 76.57°)(10^{-6}) = 77.4(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = (657.31\sin 76.57°)(10^{-6})$$

$$\gamma_{x'y'} = 1279(10^{-6})$$ **Ans.**

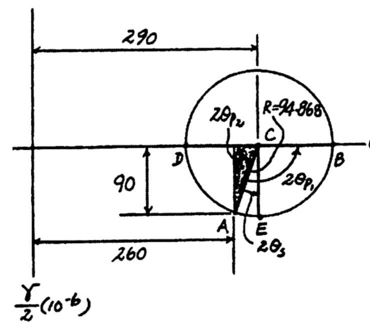
$$\varepsilon_{y'} = (230 + 657.31\cos 76.57°)(10^{-6}) = 383(10^{-6})$$ **Ans.**

Ans:
$\epsilon_{x'} = 77.4(10^{-6})$,
$\gamma_{x'y'} = 1279(10^{-6})$,
$\epsilon_{y'} = 383(10^{-6})$

14–103.

Solve Prob. 14–91 using Mohr's circle.

SOLUTION



Construction of the Circle: In accordance with the sign convention $\varepsilon_x = 260(10^{-6})$, $\varepsilon_y = 320(10^{-6})$, and

$\dfrac{\gamma_{xy}}{2} = 90(10^{-6})$. Hence,

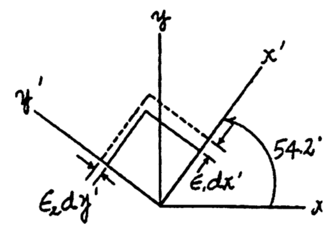
$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{260 + 320}{2}\right)(10^{-6}) = 290(10^{-6}) \qquad \textbf{Ans.}$$

The coordinates for reference points A and C are

$$A(260, 90)(10^{-6}) \qquad C(290, 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(290 - 260)^2 + 90^2}\right)(10^{-6}) = 94.868(10^{-6})$$



In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (290 + 94.868)(10^{-6}) = 385(10^{-6}) \qquad \textbf{Ans.}$$

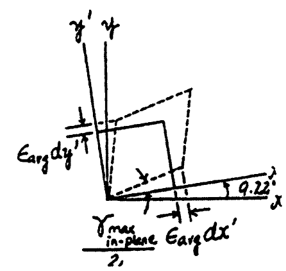
$$\varepsilon_2 = (290 - 94.868)(10^{-6}) = 195(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{P_2} = \frac{90}{290 - 260} = 3.000 \qquad 2\theta_{P_2} = 71.57°$$

$$2\theta_{P_1} = 180° - 2\theta_{P_2}$$

$$\theta_{P_1} = \frac{180° - 71.57°}{2} = 54.2° \qquad \textbf{(Counterclockwise)} \qquad \textbf{Ans.}$$



Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = R = 94.868(10^{-6})$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 190(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{290 - 260}{90} = 0.3333$$

$$\theta_s = 9.22° \qquad \textbf{(Counterclockwise)} \qquad \textbf{Ans.}$$

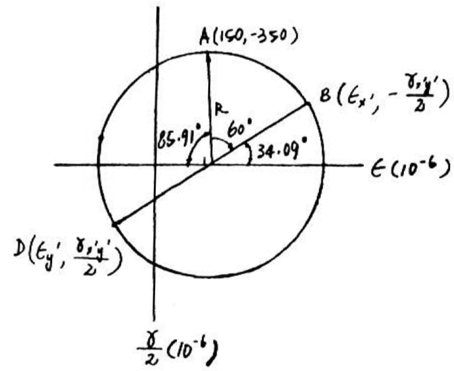
Ans:

$\varepsilon_{avg} = 290(10^{-6})$,
$\varepsilon_1 = 385(10^{-6})$,
$\varepsilon_2 = 195(10^{-6})$,
$\theta_{P_1} = 54.2°$ **(Counterclockwise)**,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 190(10^{-6})$,
$\theta_s = 9.22°$ **(Counterclockwise)**

***14–104.**

Solve Prob. 14–90 using Mohr's circle.

SOLUTION



$\epsilon_x = 150(10^{-6}) \qquad \epsilon_y = 200(10^{-6}) \qquad \gamma_{xy} = -700(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = -350(10^{-6})$

$\theta = -30^\circ \qquad 2\theta = -60^\circ$

$A(150, -350); \qquad C(175, 0)$

$R = \sqrt{(175 - 150)^2 + (-350)^2} = 350.89$

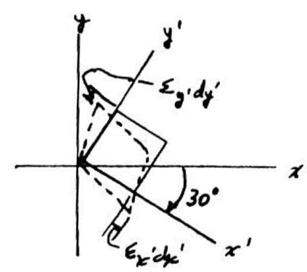
Coordinates of point B:

$\epsilon_{x'} = 350.89 \cos 34.09^\circ + 175$

$= 466(10^{-6})$ **Ans.**

$\dfrac{\gamma_{x'y'}}{2} = -350.89 \sin 34.09^\circ$

$\gamma_{x'y'} = -393(10^{-6})$ **Ans.**



Coordinates of point D:

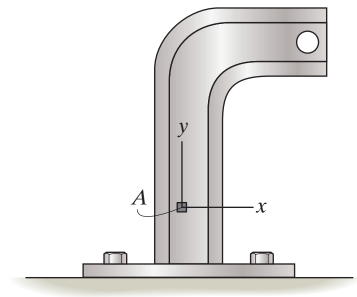
$\epsilon_{y'} = 175 - 350.89 \cos 34.09^\circ$

$= -116(10^{-6})$ **Ans.**

Ans:
$\epsilon_{x'} = 466(10^{-6})$,
$\gamma_{x'y'} = -393(10^{-6})$,
$\epsilon_{y'} = -116(10^{-6})$

14–105.

The strain at point A on the bracket has components $\epsilon_x = 300(10^{-6})$, $\epsilon_y = 550(10^{-6})$, $\gamma_{xy} = -650(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A in the x–y plane, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.

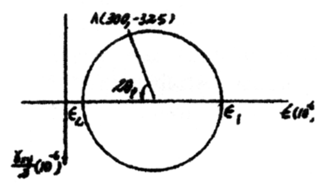


SOLUTION

$$\epsilon_x = 300(10^{-6}) \quad \epsilon_y = 550(10^{-6}) \quad \gamma_{xy} = -650(10^{-6}) \quad \frac{\gamma_{xy}}{2} = -325(10^{-6})$$

$A(300, -325)10^{-6} \quad C(425, 0)10^{-6}$

$$R = \left[\sqrt{(425 - 300)^2 + (-325)^2}\right]10^{-6} = 348.2(10^{-6})$$



(a)

$\epsilon_1 = (425 + 348.2)(10^{-6}) = 773(10^{-6})$ **Ans.**

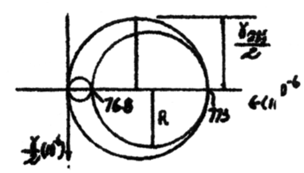
$\epsilon_2 = (425 - 348.2)(10^{-6}) = 76.8(10^{-6})$ **Ans.**

(b)

$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(348.2)(10^{-6}) = 696(10^{-6})$ **Ans.**

(c)

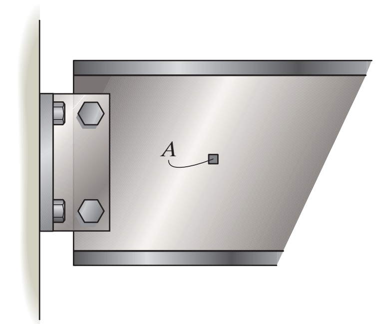
$\dfrac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \dfrac{773(10^{-6})}{2}; \quad \gamma_{\substack{\text{abs} \\ \max}} = 773(10^{-6})$ **Ans.**



Ans:
(a) $\epsilon_1 = 773(10^{-6})$, $\epsilon_2 = 76.8(10^{-6})$,
(b) $\gamma_{\substack{\max \\ \text{in-plane}}} = 696(10^{-6})$,
(c) $\gamma_{\substack{\text{abs} \\ \max}} = 773(10^{-6})$

14–106.

The strain at point A on a beam has components $\epsilon_x = 450(10^{-6})$, $\epsilon_y = 825(10^{-6})$, $\gamma_{xy} = 275(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.

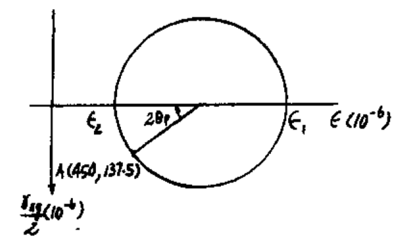


SOLUTION

$\epsilon_x = 450(10^{-6}) \qquad \epsilon_y = 825(10^{-6}) \qquad \gamma_{xy} = 275(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = 137.5(10^{-6})$

$A(450, 137.5)10^{-6} \qquad C(637.5, 0)10^{-6}$

$R = [\sqrt{(637.5 - 450)^2 + 137.5^2}]10^{-6} = 232.51(10^{-6})$



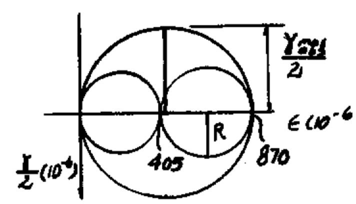
(a)

$\epsilon_1 = (637.5 + 232.51)(10^{-6}) = 870(10^{-6})$ **Ans.**

$\epsilon_2 = (637.5 - 232.51)(10^{-6}) = 405(10^{-6})$ **Ans.**

(b)

$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(232.51)(10^{-6}) = 465(10^{-6})$ **Ans.**



(c)

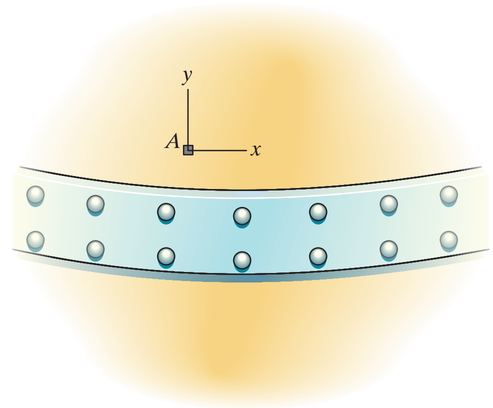
$\dfrac{\gamma_{\substack{\text{abs} \\ \max}}}{2} = \dfrac{870(10^{-6})}{2}; \quad \gamma_{\substack{\text{abs} \\ \max}} = 870(10^{-6})$ **Ans.**

Ans:

$\epsilon_1 = 870(10^{-6})$, $\epsilon_2 = 405(10^{-6})$,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 465(10^{-6})$,
$\gamma_{\substack{\text{abs} \\ \max}} = 870(10^{-6})$

14–107.

The strain at point A on the pressure-vessel wall has components $\epsilon_x = 480(10^{-6})$, $\epsilon_y = 720(10^{-6})$, $\gamma_{xy} = 650(10^{-6})$. Determine (a) the principal strains at A, in the x–y plane, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



SOLUTION

$\epsilon_x = 480(10^{-6}) \qquad \epsilon_y = 720(10^{-6}) \qquad \gamma_{xy} = 650(10^{-6}) \qquad \frac{\gamma_{xy}}{2} = 325(10^{-6})$

$A(480, 325)10^{-6} \qquad C(600, 0)10^{-6}$

$R = \left(\sqrt{(600 - 480)^2 + 325^2}\right)10^{-6} = 346.44(10^{-6})$

(a)

$\epsilon_1 = (600 + 346.44)10^{-6} = 946(10^{-6})$ **Ans.**

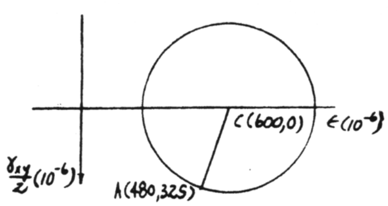
$\epsilon_2 = (600 - 346.44)10^{-6} = 254(10^{-6})$ **Ans.**

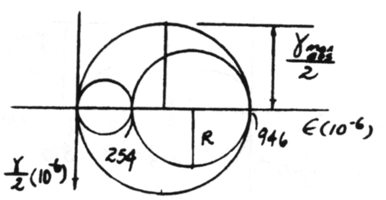
(b)

$\gamma_{\substack{\text{max}\\ \text{in-plane}}} = 2R = 2(346.44)10^{-6} = 693(10^{-6})$ **Ans.**

(c)

$\frac{\gamma_{\substack{\text{abs}\\ \text{max}}}}{2} = \frac{946(10^{-6})}{2}; \qquad \gamma_{\substack{\text{abs}\\ \text{max}}} = 946(10^{-6})$ **Ans.**

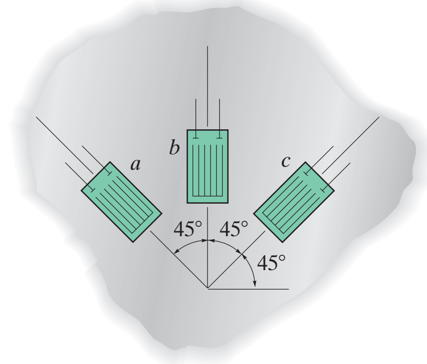




Ans:
$\epsilon_1 = 946(10^{-6})$,
$\epsilon_2 = 254(10^{-6})$,
$\gamma_{\substack{\text{max}\\ \text{in-plane}}} = 693(10^{-6})$,
$\gamma_{\substack{\text{abs}\\ \text{max}}} = 946(10^{-6})$

***14–108.**

The 45° strain rosette is mounted on the surface of a shell. The following readings are obtained for each gage: $\epsilon_a = -200(10^{-6})$, $\epsilon_b = 300(10^{-6})$, and $\epsilon_c = 250(10^{-6})$. Determine the in-plane principal strains.



SOLUTION

Strain Rosettes (45°): Applying the equation in the text with $\epsilon_a = -200(10^{-6})$, $\epsilon_b = 300(10^{-6})$, $\epsilon_c = 250(10^{-6})$, $\theta_a = 135°$, $\theta_b = 90°$ and $\theta_c = 45°$.

$$300(10^{-6}) = \epsilon_x \cos^2 90° + \epsilon_y \sin^2 90° + \gamma_{xy} \sin 90° \cos 90°$$

$$\epsilon_y = 300(10^{-6})$$

$$-200(10^{-6}) = \epsilon_x \cos^2 135° + 300(10^{-6}) \sin^2 135° + \gamma_{xy} \sin 135° \cos 135°$$

$$\epsilon_x - \gamma_{xy} = -700(10^{-6}) \qquad \textbf{(1)}$$

$$250(10^{-6}) = \epsilon_x \cos^2 45° + 300(10^{-6}) \sin^2 45° + \gamma_{xy} \sin 45° \cos 45°$$

$$\epsilon_x + \gamma_{xy} = 200(10^{-6}) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$\epsilon_x = -250(10^{-6}) \qquad \gamma_{xy} = 450(10^{-6})$$

Construction of The Circle: With $\epsilon_x = -250(10^{-6})$, $\epsilon_y = 300(10^{-6})$ and $\frac{\gamma_{xy}}{2} = 225(10^{-6})$,

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-250 + 300}{2}\right)(10^{-6}) = 25.0(10^{-6})$$

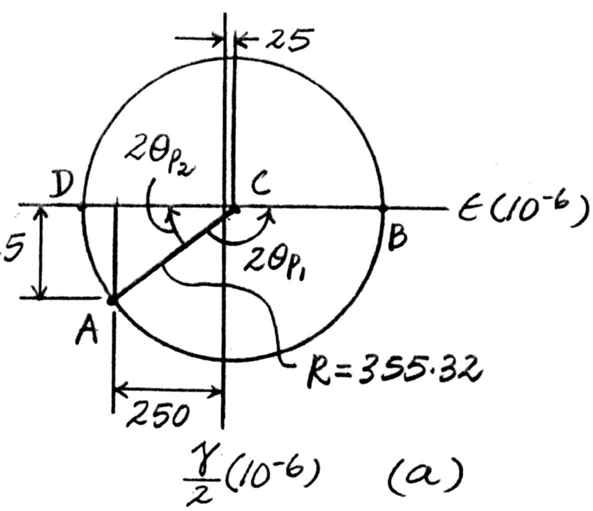
The coordinates of reference point A and center of circle C are

$$A(-250, 225)(10^{-6}) \qquad C(25.0, 0)(10^{-6})$$

The radius of the circle is

$$R = CA = \left[\sqrt{(-250 - 25)^2 + (225 - 0)^2}\right](10^{-6}) = 355.32(10^{-6})$$

Using these results, the circle shown in Fig. *a* can be constructed.



In-Plane Principal Strain: The coordinates of points B and D represent ϵ_1 and ϵ_2, respectively.

$$\epsilon_1 = (25.0 + 355.32)(10^{-6}) = 380.32(10^{-6}) = 380(10^{-6}) \qquad \textbf{Ans.}$$

$$\epsilon_2 = (25.0 - 355.32)(10^{-6}) = -330.32(10^{-6}) = -330(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_1 = 380(10^{-6})$,
$\epsilon_2 = -330(10^{-6})$

14–109.

For the case of plane stress, show that Hooke's law can be written as

$$\sigma_x = \frac{E}{(1-\nu^2)}(\epsilon_x + \nu\epsilon_y), \quad \sigma_y = \frac{E}{(1-\nu^2)}(\epsilon_y + \nu\epsilon_x)$$

SOLUTION

Generalized Hooke's Law: For plane stress, $\sigma_z = 0$. Applying Eq. 10–18,

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$

$$\nu E\epsilon_x = \left(\sigma_x - \nu\,\sigma_y\right)\nu$$

$$\nu E\epsilon_x = \nu\,\sigma_x - \nu^2\,\sigma_y \qquad \textbf{(1)}$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\,\sigma_x)$$

$$E\,\epsilon_y = -\nu\,\sigma_x + \sigma_y \qquad \textbf{(2)}$$

Adding Eq. (1) and Eq. (2) yields

$$\nu E\,\epsilon_x + E\,\epsilon_y = \sigma_y - \nu^2\,\sigma_y$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\nu\epsilon_x + \epsilon_y\right) \qquad \textit{(Q.E.D.)}$$

Substituting σ_y into Eq. (2),

$$E\,\epsilon_y = -\nu\sigma_x + \frac{E}{1-\nu^2}\left(\nu\,\epsilon_x + \epsilon_y\right)$$

$$\sigma_x = \frac{E\left(\nu\,\epsilon_x + \epsilon_y\right)}{\nu\,(1-\nu^2)} - \frac{E\epsilon_y}{\nu}$$

$$= \frac{E\nu\epsilon_x + E\epsilon_y - E\epsilon_y + E\epsilon_y\,\nu^2}{\nu(1-\nu^2)}$$

$$= \frac{E}{1-\nu^2}\,(\epsilon_x + \nu\,\epsilon_y) \qquad \textit{(Q.E.D.)}$$

Ans:
N/A

14–110.

Use Hooke's law, Eq. 14–32, to develop the strain tranformation equations, Eqs. 14–19 and 14–20, from the stress tranformation equations, Eqs. 14–1 and 14–2.

SOLUTION

Stress Transformation Equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{(1)}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{(2)}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad \textbf{(3)}$$

Hooke's Law:

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu\,\sigma_y}{E} \qquad \textbf{(4)}$$

$$\epsilon_y = \frac{-\nu\,\sigma_x}{E} + \frac{\sigma_y}{E} \qquad \textbf{(5)}$$

$$\tau_{xy} = G\,\gamma_{xy} \qquad \textbf{(6)}$$

$$G = \frac{E}{2(1+\nu)} \qquad \textbf{(7)}$$

From Eqs. (4) and (5),

$$\epsilon_x + \epsilon_y = \frac{(1-\nu)(\sigma_x + \sigma_y)}{E} \qquad \textbf{(8)}$$

$$\epsilon_x - \epsilon_y = \frac{(1+\nu)(\sigma_x - \sigma_y)}{E} \qquad \textbf{(9)}$$

From Eqs. (6) and (7),

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} \qquad \textbf{(10)}$$

From Eq. (4),

$$\epsilon_{x'} = \frac{\sigma_{x'}}{E} - \frac{\nu\,\sigma_{y'}}{E} \qquad \textbf{(11)}$$

Substitute Eqs. (1) and (3) into Eq. (11).

$$\epsilon_{x'} = \frac{(1-\nu)(\sigma_x + \sigma_y)}{2\,E} + \frac{(1+\nu)(\sigma_x - \sigma_y)}{2\,E}\cos 2\theta + \frac{(1+\nu)\tau_{xy}\sin 2\theta}{E} \qquad \textbf{(12)}$$

By using Eqs. (8), (9) and (10) and substituting into Eq. (12),

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{(\textit{Q.E.D.})}$$

14–110. Continued

From Eq. (6),

$$\tau_{x'y'} = G\gamma_{x'y'} = \frac{E}{2(1+\nu)}\gamma_{x'y'} \qquad \textbf{(13)}$$

Substitute Eqs. (13), (6) and (9) into Eq. (2).

$$\frac{E}{2(1+\nu)}\gamma_{x'y'} = -\frac{E(\epsilon_x - \epsilon_y)}{2(1+\nu)}\sin 2\theta + \frac{E}{2(1+\nu)}\gamma_{xy}\cos 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{(\epsilon_x - \epsilon_y)}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta \qquad \textit{\textbf{(Q.E.D.)}}$$

Ans:
N/A

14–111.

The principal plane stresses and associated strains in a plane at a point are $\sigma_1 = 36$ ksi, $\sigma_2 = 16$ ksi, $\epsilon_1 = 1.02(10^{-3})$, $\epsilon_2 = 0.180(10^{-3})$. Determine the modulus of elasticity and Poisson's ratio.

SOLUTION

$\sigma_3 = 0$

$$\epsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$1.02(10^{-3}) = \frac{1}{E}[36 - \nu(16)]$$

$$1.02(10^{-3})E = 36 - 16\nu \qquad \textbf{(1)}$$

$$\epsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$

$$0.180(10^{-3}) = \frac{1}{E}[16 - \nu(36)]$$

$$0.180(10^{-3})E = 16 - 36\nu \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$E = 30.7(10^3)$ ksi **Ans.**

$\nu = 0.291$ **Ans.**

Ans:
$E = 30.7(10^3)$ ksi,
$\nu = 0.291$

***14–112.**

A rod has a radius of 10 mm. If it is subjected to an axial load of 15 N such that the axial strain in the rod is $\epsilon_x = 2.75(10^{-6})$, determine the modulus of elasticity E and the change in the rod's diameter. $\nu = 0.23$.

SOLUTION

$$\sigma_x = \frac{15}{\pi(0.01)^2} = 47.746 \text{ kPa}, \qquad \sigma_y = 0, \qquad \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$2.75(10^{-6}) = \frac{1}{E}[47.746(10^3) - 0.23(0 + 0)]$$

$$E = 17.4 \text{ GPa}$$ **Ans.**

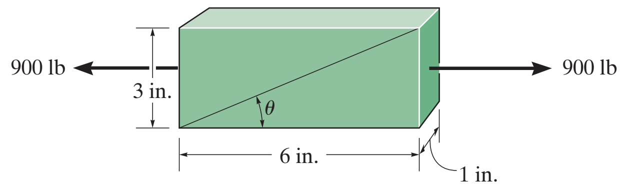
$$\epsilon_y = \epsilon_z = -\nu\epsilon_x = -0.23(2.75)(10^{-6}) = -0.632(10^{-6})$$

$$\Delta d = 20(-0.632(10^{-6})) = -12.6(10^{-6}) \text{ mm}$$ **Ans.**

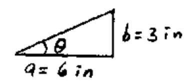
Ans:
$E = 17.4$ GPa,
$\Delta d = -12.6(10^{-6})$ mm

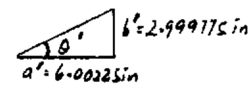
14–113.

The polyvinyl chloride bar is subjected to an axial force of 900 lb. If it has the original dimensions shown, determine the change in the angle θ after the load is applied. $E_{pvc} = 800(10^3)$ psi, $\nu_{pvc} = 0.20$.



SOLUTION





$$\sigma_x = \frac{900}{3(1)} = 300 \text{ psi}$$

$$\sigma_y = 0 \qquad \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$= \frac{1}{800(10^3)}[300 - 0] = 0.375(10^{-3})$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$= \frac{1}{800(10^3)}[0 - 0.2(300 + 0)] = -75(10^{-6})$$

$$a' = 6 + 6(0.375)(10^{-3}) = 6.00225 \text{ in.}$$

$$b' = 3 + 3(-75)(10^{-6}) = 2.999775 \text{ in.}$$

$$\theta = \tan^{-1}\left(\frac{3}{6}\right) = 26.56505118°$$

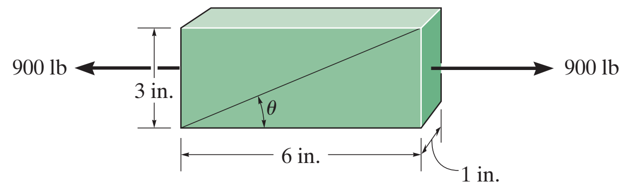
$$\theta' = \tan^{-1}\left(\frac{2.999775}{6.00225}\right) = 26.55474088°$$

$$\Delta\theta = \theta' - \theta = 26.55474088° - 26.56505118° = -0.0103°$$ **Ans.**

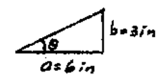
Ans:
$\Delta\theta = -0.0103°$

14–114.

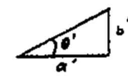
The polyvinyl chloride bar is subjected to an axial force of 900 lb. If it has the original dimensions shown, determine the value of Poisson's ratio if the angle θ decreases by $\Delta\theta = 0.01°$ after the load is applied. $E_{\text{pvc}} = 800(10^3)$ psi.



SOLUTION



$$\sigma_x = \frac{900}{3(1)} = 300 \text{ psi} \qquad \sigma_y = 0 \qquad \sigma_z = 0$$



$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu_{\text{pvc}}(\sigma_y + \sigma_z)]$$

$$= \frac{1}{800(10^3)}[300 - 0] = 0.375(10^{-3})$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu_{\text{pvc}}(\sigma_x + \sigma_z)]$$

$$= \frac{1}{800(10^3)}[0 - \nu_{\text{pvc}}(300 + 0)] = -0.375(10^{-3})\nu_{\text{pvc}}$$

$$a' = 6 + 6(0.375)(10^{-3}) = 6.00225 \text{ in.}$$

$$b' = 3 + 3(-0.375)(10^{-3})\nu_{\text{pvc}} = 3 - 1.125(10^{-3})\nu_{\text{pvc}}$$

$$\theta = \tan^{-1}\left(\frac{3}{6}\right) = 26.56505118°$$

$$\theta' = 26.56505118° - 0.01° = 26.55505118°$$

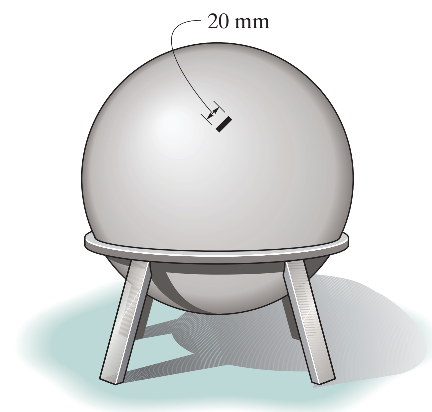
$$\tan\theta' = 0.49978185 = \frac{3 - 1.125(10^{-3})\nu_{\text{pvc}}}{6.00225}$$

$\nu_{\text{pvc}} = 0.164$ **Ans.**

Ans:
$\nu_{\text{pvc}} = 0.164$

14–115.

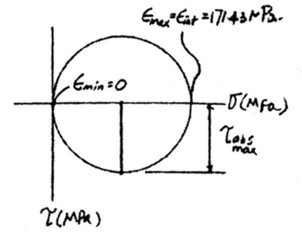
The spherical pressure vessel has an inner diameter of 2 m and a thickness of 10 mm. A strain gage having a length of 20 mm is attached to it, and it is observed to increase in length by 0.012 mm when the vessel is pressurized. Determine the pressure causing this deformation, and find the maximum in-plane shear stress, and the absolute maximum shear stress at a point on the outer surface of the vessel. The material is steel, for which $E_{st} = 200$ GPa and $\nu_{st} = 0.3$.



SOLUTION

Normal Stresses: Since $\frac{r}{t} = \frac{1000}{10} = 100 > 10$, the *thin-wall* analysis is valid to determine the normal stress in the wall of the spherical vessel. This is a plane stress problem where $\sigma_{min} = 0$ since there is no load acting on the outer surface of the wall.

$$\sigma_{max} = \sigma_{lat} = \frac{pr}{2t} = \frac{p(1000)}{2(10)} = 50.0p \qquad \textbf{(1)}$$



Normal Strains: Applying the generalized Hooke's Law with

$$\epsilon_{max} = \epsilon_{lat} = \frac{0.012}{20} = 0.600\left(10^{-3}\right) \text{ mm/mm}$$

$$\epsilon_{max} = \frac{1}{E}\left[\sigma_{max} - \nu\left(\sigma_{lat} + \sigma_{min}\right)\right]$$

$$0.600\left(10^{-3}\right) = \frac{1}{200(10^9)}\left[50.0p - 0.3(50.0p + 0)\right]$$

$$p = 3.4286 \text{ MPa} = 3.43 \text{ MPa} \qquad \textbf{Ans.}$$

From Eq. (1), $\sigma_{max} = \sigma_{lat} = 50.0(3.4286) = 171.43$ MPa

Maximum In-Plane Shear (Sphere's Surface): Mohr's circle is simply a dot. As the result, the state of stress is the same, consisting of two normal stresses with zero shear stress regardless of the orientation of the element.

$$\tau_{\substack{max\\ \text{in-plane}}} = 0 \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{171.43 - 0}{2} = 85.7 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$p = 3.43$ MPa,
$\tau_{\substack{max\\ \text{in-plane}}} = 0$,
$\tau_{\substack{abs\\max}} = 85.7$ MPa

***14–116.**

Determine the bulk modulus for each of the following materials: (a) rubber, $E_r = 0.4$ ksi, $\nu_r = 0.48$, and (b) glass, $E_g = 8(10^3)$ ksi, $\nu_g = 0.24$.

SOLUTION

(a) For rubber:

$$k_r = \frac{E_r}{3\,(1 - 2\,\nu_r)} = \frac{0.4}{3[1 - 2(0.48)]} = 3.33 \text{ ksi}$$ **Ans.**

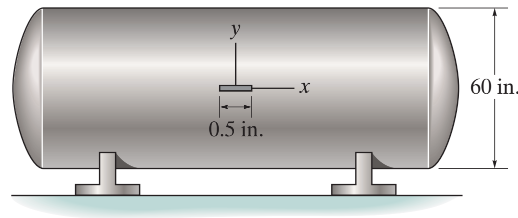
(b) For glass:

$$k_g = \frac{E_g}{3(1 - 2\,\nu_g)} = \frac{8(10^3)}{3[1 - 2(0.24)]} = 5.13(10^3) \text{ ksi}$$ **Ans.**

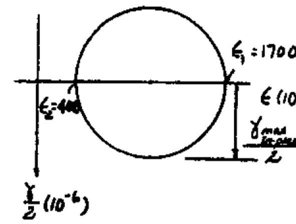
Ans:
(a) $k_r = 3.33$ ksi,
(b) $k_g = 5.13(10^3)$ ksi

14–117.

The strain gage is placed on the surface of the steel boiler as shown. If it is 0.5 in. long, determine the pressure in the boiler when the gage elongates $0.2(10^{-3})$ in. The boiler has a thickness of 0.5 in. and inner diameter of 60 in. Also, determine the maximum x, y in-plane shear strain in the material. $E_{st} = 29(10^3)$ ksi, $\nu_{st} = 0.3$.



SOLUTION



$$\epsilon_2 = \frac{0.2(10^{-3})}{0.5} = 400(10^{-6})$$

$$\epsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$

where $\sigma_2 = \dfrac{1}{2}\sigma_1 \qquad \sigma_3 = 0$

$$400(10^{-6}) = \frac{1}{29(10^3)}\left[\frac{1}{2}\sigma_1 - 0.3\sigma_1\right]$$

$$\sigma_1 = 58 \text{ ksi}$$

Thus,

$$p = \frac{\sigma_1 t}{r} = \frac{58(0.5)}{30} = 0.967 \text{ ksi}$$ **Ans.**

$$\epsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

where $\sigma_3 = 0 \text{ and } \sigma_2 = \dfrac{58}{2} = 29 \text{ ksi}$

$$\epsilon_1 = \frac{1}{29(10^3)}[58 - 0.3(29 + 0)] = 1700(10^{-6})$$

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \frac{\epsilon_1 - \epsilon_2}{2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = (1700 - 400)(10^{-6}) = 1.30(10^{-3})$$ **Ans.**

Ans:
$p = 0.967$ ksi,
$\gamma_{\substack{\max \\ \text{in-plane}}} = 1.30(10^{-3})$

14–118.

The principal strains at a point on the aluminum fuselage of a jet aircraft are $\epsilon_1 = 780(10^{-6})$ and $\epsilon_2 = 400(10^{-6})$. Determine the associated principal stresses at the point in the same plane. $E_{al} = 10(10^3)$ ksi, $\nu_{al} = 0.33$. *Hint:* See Prob. 14–109.

SOLUTION

Plane stress: $\sigma_3 = 0$

$$\sigma_1 = \frac{E}{1 - \nu^2}(\epsilon_1 + \nu\epsilon_2)$$

$$= \frac{10(10^3)}{1 - 0.33^2}\left(780(10^{-6}) + 0.33(400)(10^{-6})\right) = 10.2 \text{ ksi} \qquad \textbf{Ans.}$$

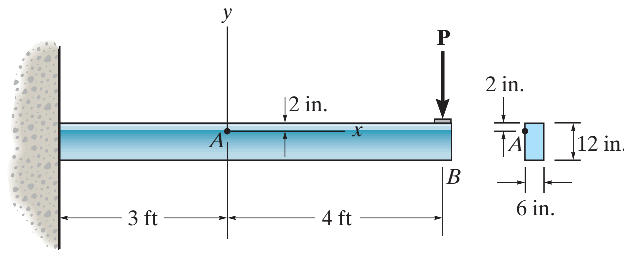
$$\sigma_2 = \frac{E}{1 - \nu^2}(\epsilon_2 + \nu\epsilon_1)$$

$$= \frac{10(10^3)}{1 - 0.33^2}\left(400(10^{-6}) + 0.33(780)(10^{-6})\right) = 7.38 \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 10.2$ ksi,
$\sigma_2 = 7.38$ ksi

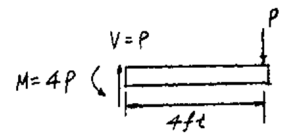
14–119.

The strain in the x direction at point A on the A-36 structural-steel beam is measured and found to be $\epsilon_x = 200(10^{-6})$. Determine the applied load P. What is the shear strain γ_{xy} at point A?



SOLUTION

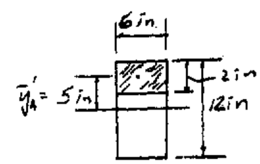
Section Properties:



$$Q_A = \bar{y}'A' = 5(6)(2) = 60 \text{ in}^3$$

$$I = \frac{1}{12}(6)(12^3) = 864 \text{ in}^4$$

Normal Stress:



$$\sigma = E\,\epsilon_x = 29(10^3)(200)(10^{-6}) = 5.80 \text{ ksi}$$

$$\sigma = \frac{My}{I}; \qquad 5.80(10^3) = \frac{4P(12)(4)}{864}$$

$$P = 26.1 \text{ kip}$$ **Ans.**

Shear Stress and Shear Strain:

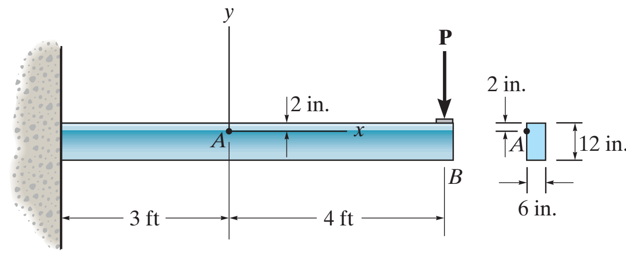
$$\tau_A = \frac{VQ}{It} = \frac{26.1(60)}{864(6)} = 0.302 \text{ ksi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{0.302}{11.0(10^3)} = -27.5(10^{-6}) \text{ rad}$$ **Ans.**

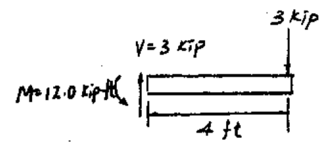
Ans:
$P = 26.1$ kip,
$\gamma_{xy} = -27.5(10^{-6})$ rad

***14–120.**

If a load of $P = 3$ kip is applied to the A-36 structural-steel beam, determine the strain ϵ_x and γ_{xy} at point A.



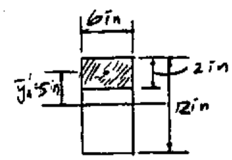
SOLUTION



Section Properties:

$$Q_A = \bar{y}'A' = 2(6)(5) = 60 \text{ in}^3$$

$$I = \frac{1}{12}(6)(12^3) = 864 \text{ in}^4$$



Normal Stress and Strain:

$$\sigma_A = \frac{My}{I} = \frac{12.0(10^3)(12)(4)}{864} = 666.7 \text{ psi}$$

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{666.7}{29(10^6)} = 23.0(10^{-6}) \qquad \textbf{Ans.}$$

Shear Stress and Shear Strain:

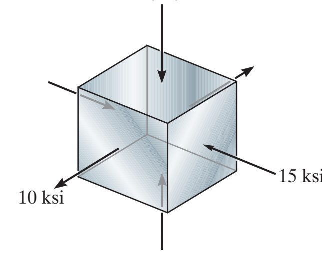
$$\tau_A = \frac{VQ}{It} = \frac{3(10^3)(60)}{864(6)} = 34.72 \text{ psi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{34.72}{11.0(10^6)} = -3.16(10^{-6}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_x = 23.0(10^{-6})$,
$\gamma_{xy} = -3.16(10^{-6})$

14–121.

The cube of aluminum is subjected to the three stresses shown. Determine the principal strains. Take $E_{al} = 10(10^3)$ ksi and $\nu_{al} = 0.33$.



SOLUTION

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \frac{1}{10(10^3)}(10 - 0.33(-15 - 26)) = 2.35(10^{-3}) \qquad \textbf{Ans.}$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) = \frac{1}{10(10^3)}(-15 - 0.33)(10 - 26)) = -0.972(10^{-3}) \qquad \textbf{Ans.}$$

$$\epsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) = \frac{1}{10(10^3)}(-26 - 0.33(10 - 15)) = -2.44(10^{-3}) \qquad \textbf{Ans.}$$

Ans:
$\epsilon_x = 2.35(10^{-3})$,
$\epsilon_y = -0.972(10^{-3})$,
$\epsilon_z = -2.44(10^{-3})$

14–122.

The principal strains at a point on the aluminum surface of a tank are $\epsilon_1 = 630(10^{-6})$ and $\epsilon_2 = 350(10^{-6})$. If this is a case of plane stress, determine the associated principal stresses at the point in the same plane. $E_{al} = 10(10^3)$ ksi, $\nu_{al} = 0.33$. *Hint:* See Prob. 14–109.

SOLUTION

For plane stress $\sigma_3 = 0$.

$$\sigma_1 = \frac{E}{1 - \nu^2}(\epsilon_1 + \nu\epsilon_2)$$

$$= \frac{10(10^3)}{1 - 0.33^2}\left[630(10^{-6}) + 0.33(350)(10^{-6})\right]$$

$$= 8.37 \text{ ksi}$$ **Ans.**

$$\sigma_2 = \frac{E}{1 - \nu^2}(\epsilon_2 + \nu\epsilon_1)$$

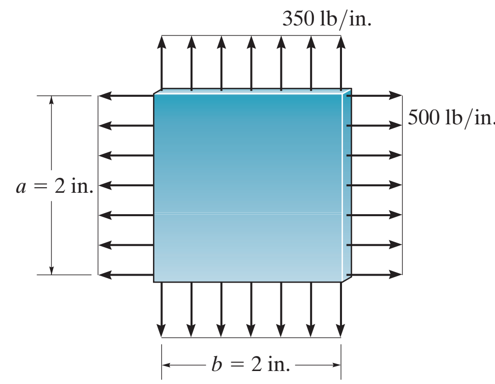
$$= \frac{10(10^3)}{1 - 0.33^2}\left[350(10^{-6}) + 0.33(630)(10^{-6})\right]$$

$$= 6.26 \text{ ksi}$$ **Ans.**

Ans:
$\sigma_1 = 8.37$ ksi,
$\sigma_2 = 6.26$ ksi

14–123.

A uniform edge load of 500 lb/in. and 350 lb/in. is applied to the polystyrene specimen. If the specimen is originally square and has dimensions of $a = 2$ in., $b = 2$ in., and a thickness of $t = 0.25$ in., determine its new dimensions a', b', and t' after the load is applied. $E_p = 597(10^3)$ psi and $\nu_p = 0.25$.



SOLUTION

Normal Stresses: For plane stress, $\sigma_z = 0$.

$$\sigma_x = \frac{500}{0.25} = 2000 \text{ psi} \qquad \sigma_y = \frac{350}{0.25} = 1400 \text{ psi}$$

Normal Strains: Applying the generalized Hooke's Law,

$$\begin{aligned} \epsilon_x &= \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \\ &= \frac{1}{597\left(10^3\right)}[2000 - 0.25(1400 + 0)] \\ &= 2.7638\left(10^{-3}\right) \end{aligned}$$

$$\begin{aligned} \epsilon_y &= \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] \\ &= \frac{1}{597\left(10^3\right)}[1400 - 0.25(2000 + 0)] \\ &= 1.5075\left(10^{-3}\right) \end{aligned}$$

$$\begin{aligned} \epsilon_z &= \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] \\ &= \frac{1}{597\left(10^3\right)}[0 - 0.25(2000 + 1400)] \\ &= -1.4238\left(10^{-3}\right) \end{aligned}$$

The new dimensions for the new specimen are

$$a' = 2 + 2\left[1.5075\left(10^{-3}\right)\right] = 2.00302 \text{ in.}$$ **Ans.**

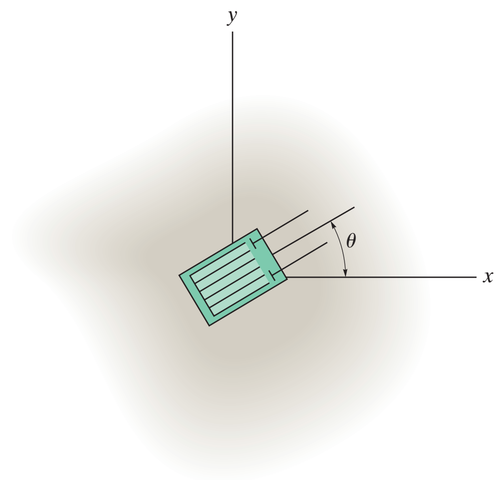
$$b' = 2 + 2\left[2.7638\left(10^{-3}\right)\right] = 2.00553 \text{ in.}$$ **Ans.**

$$t' = 0.25 + 0.25\left[-1.4238\left(10^{-3}\right)\right] = 0.24964 \text{ in.}$$ **Ans.**

Ans:
$a' = 2.00302$ in.,
$b' = 2.00553$ in.,
$t' = 0.24964$ in.

***14–124.**

A material is subjected to principal stresses σ_x and σ_y. Determine the orientation θ of the strain gage so that its reading of normal strain responds only to σ_y and not σ_x. The material constants are E and ν.



SOLUTION

$$\sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Since $\tau_{xy} = 0$,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\sigma_n = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} + (\sigma_x - \sigma_y)\cos^2\theta - \frac{\sigma_x}{2} + \frac{\sigma_y}{2}$$

$$= \sigma_y(1 - \cos^2\theta) + \sigma_x\cos^2\theta$$

$$= \sigma_x\cos^2\theta + \sigma_y\sin^2\theta$$

$$\sigma_{n+90^\circ} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)(2\cos^2\theta - 1)$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - (\sigma_x - \sigma_y)\cos^2\theta + \frac{\sigma_x}{2} - \frac{\sigma_y}{2}$$

$$= \sigma_x(1 - \cos^2\theta) + \sigma_y\cos^2\theta$$

$$= \sigma_x\sin^2\theta + \sigma_y\cos^2\theta$$

$$\epsilon_n = \frac{1}{E}(\sigma_n - \nu\,\sigma_{n+90^\circ})$$

$$= \frac{1}{E}(\sigma_x\cos^2\theta + \sigma_y\sin^2\theta - \nu\,\sigma_x\sin^2\theta - \nu\,\sigma_y\cos^2\theta)$$

If ϵ_n is to be independent of σ_x, then

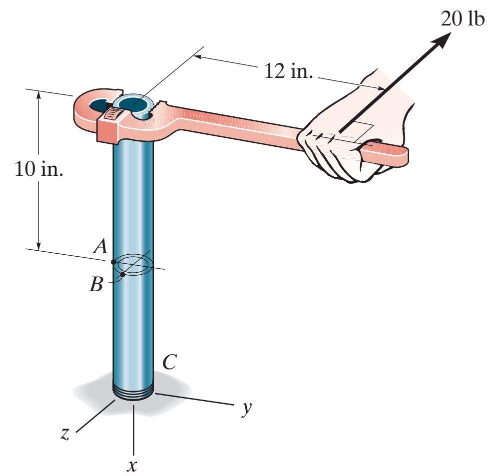
$$\cos^2\theta - \nu\sin^2\theta = 0 \qquad \text{or} \qquad \tan^2\theta = 1/\nu$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{\nu}}\right)$$ **Ans.**

Ans:
$\theta = \tan^{-1}\left(\frac{1}{\sqrt{\nu}}\right)$

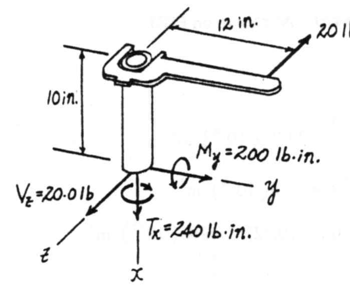
R14–1.

The steel pipe has an inner diameter of 2.75 in. and an outer diameter of 3 in. If it is fixed at C and subjected to the horizontal 20-lb force acting on the handle of the pipe wrench, determine the principal stresses in the pipe at point A, which is located on the surface of the pipe.



SOLUTION

Internal Forces, Torque and Moment: As shown on FBD.



Section Properties:

$$I = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687 \text{ in}^4$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374 \text{ in}^4$$

$$(Q_A)_z = \Sigma \bar{y}'A'$$

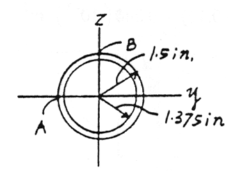
$$= \frac{4(1.5)}{3\pi}\left[\frac{1}{2}\pi\left(1.5^2\right)\right] - \frac{4(1.375)}{3\pi}\left[\frac{1}{2}\pi\left(1.375^2\right)\right]$$

$$= 0.51693 \text{ in}^3$$

Normal Stress: Applying the flexure formula $\sigma = \dfrac{M_y z}{I_y}$,

$$\sigma_A = \frac{200(0)}{1.1687} = 0$$

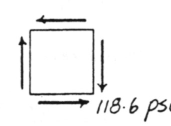
Shear Stress: The transverse shear stress in the z direction and the torsional shear stress can be obtained using shear formula and torsion formula, $\tau_v = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.



$$\tau_A = (\tau_v)_z - \tau_{\text{twist}}$$

$$= \frac{20.0(0.51693)}{1.1687(2)(0.125)} - \frac{240(1.5)}{2.3374}$$

$$= -118.6 \text{ psi}$$



In-Plane Principal Stress: $\sigma_x = 0, \sigma_z = 0$ and $\tau_{xz} = -118.6$ psi for point A.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

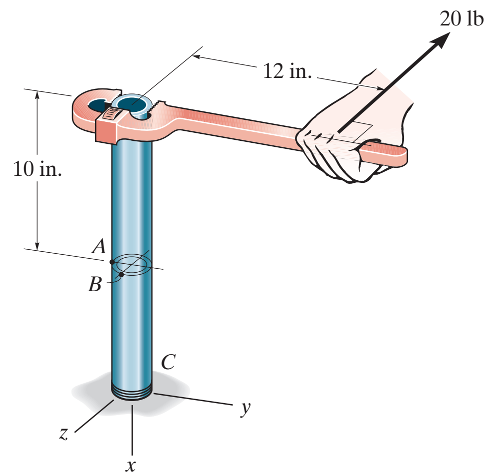
$$= 0 \pm \sqrt{0 + (-118.6)^2}$$

$\sigma_1 = 119$ psi $\qquad \sigma_2 = -119$ psi **Ans.**

Ans:
$\sigma_1 = 119$ psi,
$\sigma_2 = -119$ psi

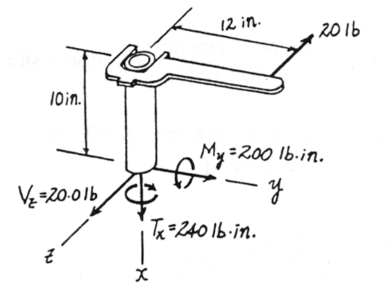
R14–2.

The steel pipe has an inner diameter of 2.75 in. and an outer diameter of 3 in. If it is fixed at C and subjected to the horizontal 20-lb force acting on the handle of the pipe wrench, determine the principal stresses in the pipe at point B, which is located on the surface of the pipe.



SOLUTION

Internal Forces, Torque and Moment: As shown on FBD.



Section Properties:

$$I = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687 \text{ in}^4$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374 \text{ in}^4$$

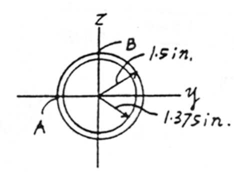
$$(Q_B)_z = 0$$

Normal Stress: Applying the flexure formula $\sigma = \dfrac{M_y\, z}{I_y}$,

$$\sigma_B = \frac{200(1.5)}{1.1687} = 256.7 \text{ psi}$$

Shear Stress: Torsional shear stress can be obtained using torsion formula, $\tau_{\text{twist}} = \dfrac{T\rho}{J}$.

$$\tau_B = \tau_{\text{twist}} = \frac{240(1.5)}{2.3374} = 154.0 \text{ psi}$$

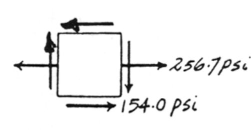


In-Plane Prinicipal Stress: $\sigma_x = 256.7$ psi, $\sigma_y = 0$, and $\tau_{xy} = -154.0$ psi for point B.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{256.7 + 0}{2} \pm \sqrt{\left(\frac{256.7 - 0}{2}\right)^2 + (-154.0)^2}$$

$$= 128.35 \pm 200.49$$

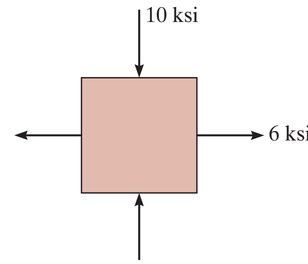


$\sigma_1 = 329$ psi $\qquad \sigma_2 = -72.1$ psi **Ans.**

Ans:
$\sigma_1 = 329$ psi,
$\sigma_2 = -72.1$ psi

R14–3.

Determine the equivalent state of stress if an element is oriented 40° clockwise from the element shown. Use Mohr's circle.



SOLUTION

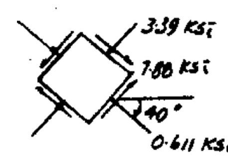
$A(6, 0) \qquad B(-10, 0) \qquad C(-2, 0)$

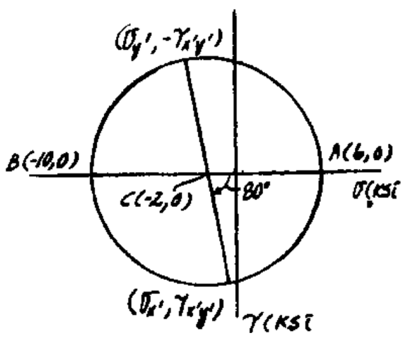
$R = CA = CB = 8$

$\sigma_{x'} = -2 + 8 \cos 80° = -0.611 \text{ ksi}$ **Ans.**

$\tau_{x'y'} = 8 \sin 80° = 7.88 \text{ ksi}$ **Ans.**

$\sigma_{y'} = -2 - 8 \cos 80° = -3.39 \text{ ksi}$ **Ans.**

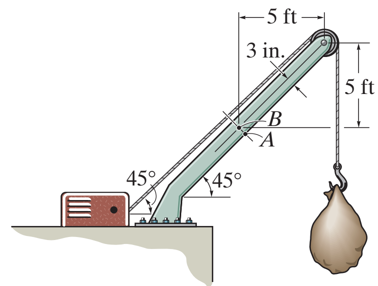




Ans:
$\sigma_{x'} = -0.611\,\text{ksi}$,
$\tau_{x'y'} = 7.88\ \text{ksi}$,
$\sigma_{y'} = -3.39\ \text{ksi}$

***R14–4.**

The crane is used to support the 350-lb load. Determine the principal stresses acting in the boom at points A and B. The cross section is rectangular and has a width of 6 in. and a thickness of 3 in. Use Mohr's circle.

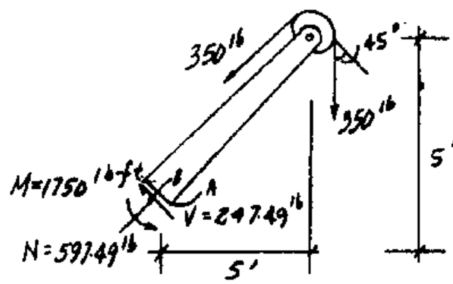


SOLUTION

$A = 6(3) = 18 \text{ in}^2 \qquad I = \frac{1}{12}(3)(6^3) = 54 \text{ in}^4$

$Q_B = (1.5)(3)(3) = 13.5 \text{ in}^3$

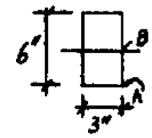
$Q_A = 0$



For point A:

$$\sigma_A = -\frac{P}{A} - \frac{My}{I} = \frac{597.49}{18} - \frac{1750(12)(3)}{54} = -1200 \text{ psi}$$

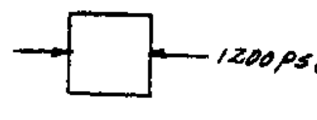
$\tau_A = 0$



$\sigma_1 = 0$ **Ans.**

$\sigma_2 = -1200 \text{ psi} = -1.20 \text{ ksi}$ **Ans.**

For point B:



$$\sigma_B = -\frac{P}{A} = -\frac{597.49}{18} = -33.19 \text{ psi}$$

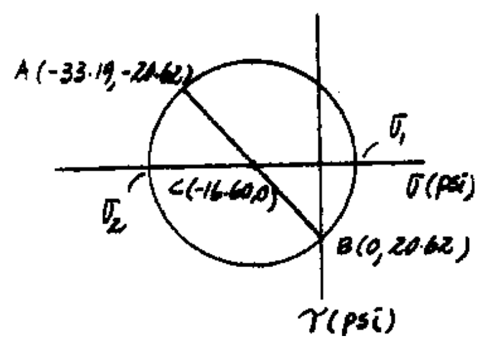
$$\tau_B = \frac{VQ_B}{It} = \frac{247.49(13.5)}{54(3)} = 20.62 \text{ psi}$$

$A(-33.19, -20.62) \qquad B(0, 20.62) \qquad C(-16.60, 0)$

$R = \sqrt{16.60^2 + 20.62^2} = 26.47$

$\sigma_1 = -16.60 + 26.47 = 9.88 \text{ psi}$ **Ans.**

$\sigma_2 = -16.60 - 26.47 = -43.1 \text{ psi}$ **Ans.**



Ans:
Point A: $\sigma_1 = 0$, $\sigma_2 = -1.20$ ksi,
Point B: $\sigma_1 = 9.88$ psi, $\sigma_2 = -43.1$ psi

R14–5.

In the case of plane stress, where the in-plane principal strains are given by ϵ_1 and ϵ_2, show that the third principal strain can be obtained from

$$\epsilon_3 = \frac{-\nu(\epsilon_1 + \epsilon_2)}{(1 - \nu)}$$

where ν is Poisson's ratio for the material.

SOLUTION

Generalized Hooke's Law: In the case of plane stress, $\sigma_3 = 0$. Thus,

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad \textbf{(1)}$$

$$\epsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad \textbf{(2)}$$

$$\epsilon_3 = -\frac{\nu}{E}(\sigma_1 + \sigma_2) \qquad \textbf{(3)}$$

Solving for σ_1 and σ_2 using Eqs. (1) and (2), we obtain

$$\sigma_1 = \frac{E(\epsilon_1 + \nu\epsilon_2)}{1 - \nu^2} \qquad \sigma_2 = \frac{E(\epsilon_2 + \nu\epsilon_1)}{1 - \nu^2}$$

Substituting these results into Eq. (3),

$$\epsilon_3 = -\frac{\nu}{E}\left[\frac{E(\epsilon_1 + \nu\epsilon_2)}{1 - \nu^2} + \frac{E(\epsilon_2 + \nu\epsilon_1)}{1 - \nu^2}\right]$$

$$\epsilon_3 = -\frac{\nu}{1 - \nu}\left[\frac{(\epsilon_1 + \epsilon_2) + \nu(\epsilon_1 + \epsilon_2)}{1 + \nu}\right]$$

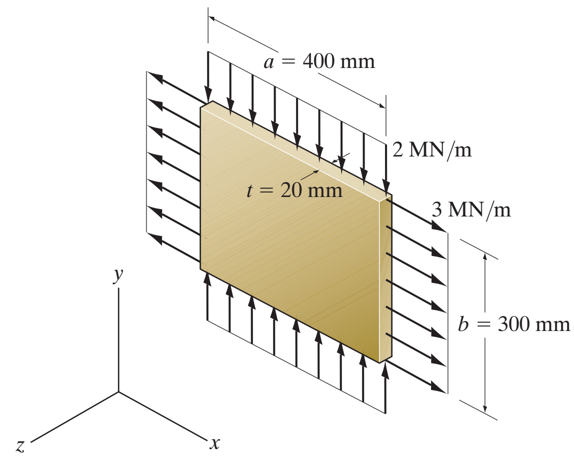
$$\epsilon_3 = -\frac{\nu}{1 - \nu}\left[\frac{(\epsilon_1 + \epsilon_2)(1 + \nu)}{1 + \nu}\right]$$

$$\epsilon_3 = -\frac{\nu}{1 - \nu}(\epsilon_1 + \epsilon_2) \qquad \textbf{(Q.E.D.)}$$

Ans:
N/A

R14–6.

The plate is made of material having a modulus of elasticity $E = 200$ GPa and Poisson's ratio $\nu = \frac{1}{3}$. Determine the change in width a, height b, and thickness t when it is subjected to the uniform distributed loading shown.



SOLUTION

Normal Stress: The normal stresses along the x, y, and z axes are

$$\sigma_x = \frac{3(10^6)}{0.02} = 150 \text{ MPa}$$

$$\sigma_y = -\frac{2(10^6)}{0.02} = -100 \text{ MPa}$$

$$\sigma_z = 0$$

Generalized Hooke's Law:

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$= \frac{1}{200(10^9)}\left\{150(10^6) - \frac{1}{3}\left[-100(10^6) + 0\right]\right\}$$

$$= 0.9167(10^{-3})$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{200(10^9)}\left\{-100(10^6) - \frac{1}{3}\left[150(10^6) + 0\right]\right\}$$

$$= -0.75(10^{-3})$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$= \frac{1}{200(10^9)}\left\{0 - \frac{1}{3}\left[150(10^6) + (-100)(10^6)\right]\right\}$$

$$= -83.33(10^{-6})$$

Thus, the changes in dimensions of the plate are

$$\delta_a = \epsilon_x a = 0.9167(10^{-3})(400) = 0.367 \text{ mm} \qquad \textbf{Ans.}$$

$$\delta_b = \epsilon_y b = -0.75(10^{-3})(300) = -0.225 \text{ mm} \qquad \textbf{Ans.}$$

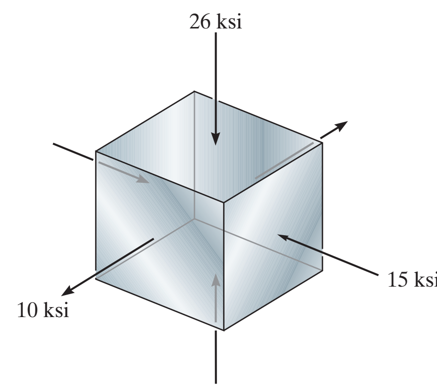
$$\delta_t = \epsilon_z t = -83.33(10^{-6})(20) = -0.00167 \text{ mm} \qquad \textbf{Ans.}$$

The negative signs indicate that b and t contract.

Ans:
$\delta_a = 0.367$ mm,
$\delta_b = -0.255$ mm,
$\delta_t = -0.00167$ mm

R14–7.

If the material is graphite for which $E_g = 800$ ksi and $\nu_g = 0.23$, determine the principal strains.



SOLUTION

Normal Strains: Applying the generalized Hooke's Law with $\sigma_x = 10$ ksi, $\sigma_y = -15$ ksi, and $\sigma_z = -26$ ksi.

$$\begin{aligned}\epsilon_x &= \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]\\ &= \frac{1}{800}[10 - 0.23(-15 - 26)]\\ &= 0.0242875\end{aligned}$$

$$\begin{aligned}\epsilon_y &= \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]\\ &= \frac{1}{800}[-15 - 0.23(10 - 26)]\\ &= -0.01415\end{aligned}$$

$$\begin{aligned}\epsilon_z &= \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]\\ &= \frac{1}{800}[-26 - 0.23(10 - 15)]\\ &= -0.0310625\end{aligned}$$

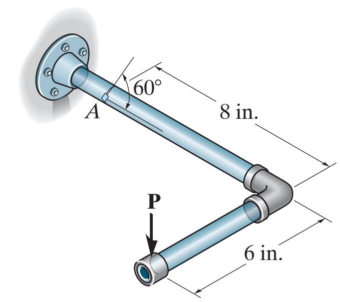
Principal Strains: From the results obtained above,

$\epsilon_1 = 0.0243 \qquad \epsilon_{\text{int}} = -0.01415 \qquad \epsilon_2 = -0.0311$ **Ans.**

Ans:
$\epsilon_1 = 0.0243$,
$\epsilon_{\text{int}} = -0.01415$,
$\epsilon_2 = -0.0311$

***R14–8.**

A single strain gage, placed in the vertical plane on the outer surface and at an angle 60° to the axis of the pipe, gives a reading at point A of $\epsilon_A = -250(10^{-6})$. Determine the principal strains in the pipe at this point. The pipe has an outer diameter of 1 in. and an inner diameter of 0.6 in. and is made of C86100 bronze.



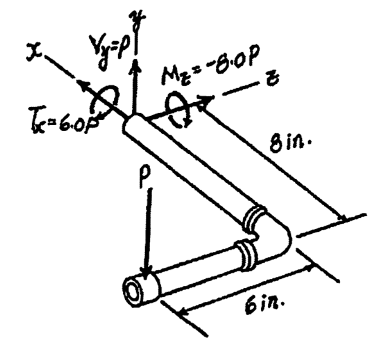
SOLUTION

Internal Forces, Torque and Moments: As shown on FBD.
By observation, this is a pure shear problem.

Strain Rosettes: For pure shear, $\epsilon_x = \epsilon_y = 0$. Applying Eq. 10–15 with $\epsilon_b = 250\left(10^{-6}\right)$ and $\theta_b = 60°$,

$$-250\left(10^{-6}\right) = 0 + 0 + \gamma_{xy} \sin 60° \cos 60°$$

$$\gamma_{xy} = -577.35\left(10^{-6}\right)$$



Construction of the Circle: In accordance with the sign convention,

$\epsilon_x = \epsilon_y = 0$ and $\dfrac{\gamma_{xy}}{2} = -288.675\left(10^{-6}\right)$.

Hence,

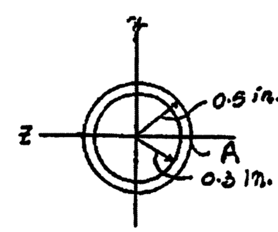
$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = 0$$

The coordinates for reference points A and C are

$$A(0, -288.675)\left(10^{-6}\right) \qquad C(0, 0)\left(10^{-6}\right)$$

The radius of the circle is

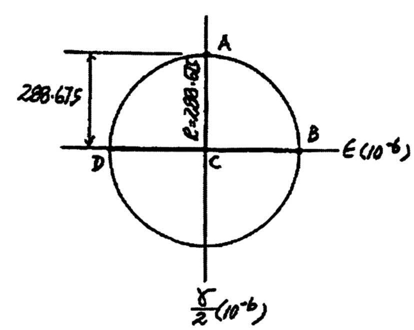
$$R = \left(\sqrt{(0 - 0)^2 + 288.675^2}\right)\left(10^{-6}\right) = 288.675\left(10^{-6}\right)$$



In-Plane Principal Strain: The coordinates of points B and D represent ϵ_1 and ϵ_2, respectively.

$$\epsilon_1 = (0 + 288.675)\left(10^{-6}\right) = 288.675\left(10^{-6}\right)$$

$$\epsilon_2 = (0 - 288.675)\left(10^{-6}\right) = -288.675\left(10^{-6}\right)$$



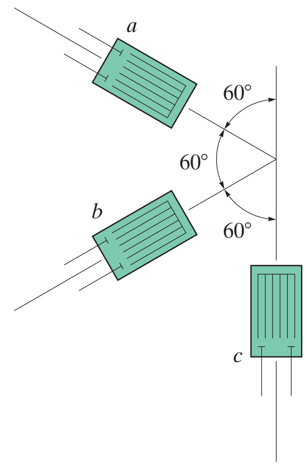
Principal Stress: Since $\sigma_x = \sigma_y = \sigma_z = 0$, then from the generalized Hooke's Law, $\epsilon_z = 0$. From the results obtained above, we have

$\epsilon_1 = 289\left(10^{-6}\right) \qquad \epsilon_2 = -289\left(10^{-6}\right)$ **Ans.**

Ans:
$\epsilon_1 = 289\left(10^{-6}\right)$,
$\epsilon_2 = -289\left(10^{-6}\right)$

R14–9.

The 60° strain rosette is mounted on a beam. The following readings are obtained for each gage: $\epsilon_a = 600(10^{-6})$, $\epsilon_b = -700(10^{-6})$, and $\epsilon_c = 350(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.



SOLUTION

Strain Rosettes (60º): Applying Eq. 10–15 with $\epsilon_x = 600\left(10^{-6}\right)$,

$\epsilon_b = -700\left(10^{-6}\right), \epsilon_c = 350\left(10^{-6}\right), \theta_a = 150°, \theta_b = -150°$ and $\theta_c = -90°$,

$$350\left(10^{-6}\right) = \epsilon_x \cos^2(-90°) + \epsilon_y \sin^2(-90°) + \gamma_{xy} \sin(-90°) \cos(-90°)$$

$$\epsilon_y = 350\left(10^{-6}\right)$$

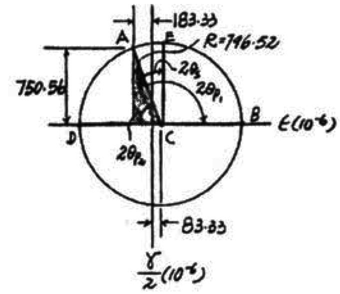
$$600\left(10^{-6}\right) = \epsilon_x \cos^2 150° + 350\left(10^{-6}\right) \sin^2 150° + \gamma_{xy} \sin 150° \cos 150°$$

$$512.5\left(10^{-6}\right) = 0.75\,\epsilon_x - 0.4330\,\gamma_{xy} \qquad \textbf{(1)}$$

$$-700\left(10^{-6}\right) = \epsilon_x \cos^2(-150°) + 350\left(10^{-6}\right) \sin^2(-150°) + \gamma_{xy} \sin(-150°) \cos(-150°)$$

$$-787.5\left(10^{-6}\right) = 0.75\epsilon_x + 0.4330\,\gamma_{xy} \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields $\epsilon_x = -183.33\left(10^{-6}\right)$ $\gamma_{xy} = -1501.11\left(10^{-6}\right)$



Construction of the Circle: With $\epsilon_x = -183.33\left(10^{-6}\right)$, $\epsilon_y = 350\left(10^{-6}\right)$, and $\dfrac{\gamma_{xy}}{2} = -750.56\left(10^{-6}\right)$,

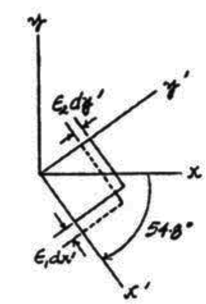
$$\epsilon_{\text{avg}} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-183.33 + 350}{2}\right)\left(10^{-6}\right) = 83.3\left(10^{-6}\right) \qquad \textbf{Ans.}$$

The coordinates for reference points A and C are

$A(-183.33, -750.56)\left(10^{-6}\right)$ $\quad C(83.33, 0)\left(10^{-6}\right)$

The radius of the circle is

$$R = \left(\sqrt{(183.33 + 83.33)^2 + 750.56^2}\right)\left(10^{-6}\right) = 796.52\left(10^{-6}\right)$$

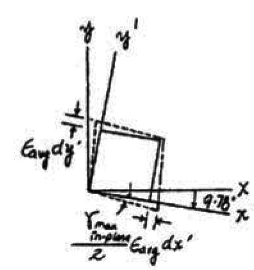


(a)

In-Plane Principal Strain: The coordinates of points B and D represent ϵ_1 and ϵ_2, respectively.

$$\epsilon_1 = (83.33 + 796.52)\left(10^{-6}\right) = 880\left(10^{-6}\right) \qquad \textbf{Ans.}$$

$$\epsilon_2 = (83.33 - 796.52)\left(10^{-6}\right) = -713\left(10^{-6}\right) \qquad \textbf{Ans.}$$



Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{P1} = \frac{750.56}{183.33 + 83.33} = 2.8145 \quad 2\theta_{P2} = 70.44°$$

$$2\theta_{P1} = 180° - 2\theta_{P2}$$

$$\theta_P = \frac{180° - 70.44°}{2} = 54.8° \quad \text{(clockwise)} \qquad \textbf{Ans.}$$

R14–9. Continued

(b)

Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = -R = -796.52\left(10^{-6}\right)$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = -1593\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Orientation of Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{183.33 + 83.33}{750.56} = 0.3553$$

$$\theta_s = 9.78^\circ \text{ (clockwise)} \qquad \textbf{Ans.}$$

Ans:
$\epsilon_{avg} = 83.3(10^{-6})$, $\epsilon_1 = 880(10^{-6})$,
$\epsilon_2 = -713(10^{-6})$,
$\theta_p = 54.8^\circ$ (clockwise),
$\gamma_{\substack{\max \\ \text{in-plane}}} = -1593(10^{-6})$,
$\theta_s = 9.78^\circ$ (clockwise)