

# CivE 414 Structural Concrete Design

---

## **Bond between Concrete and Reinforcement**

**Development of bars**

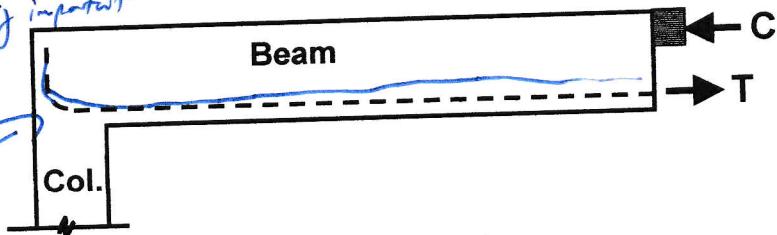
**Splicing of bars**

**Detailing**

## ANCHORAGE OF REINFORCEMENT

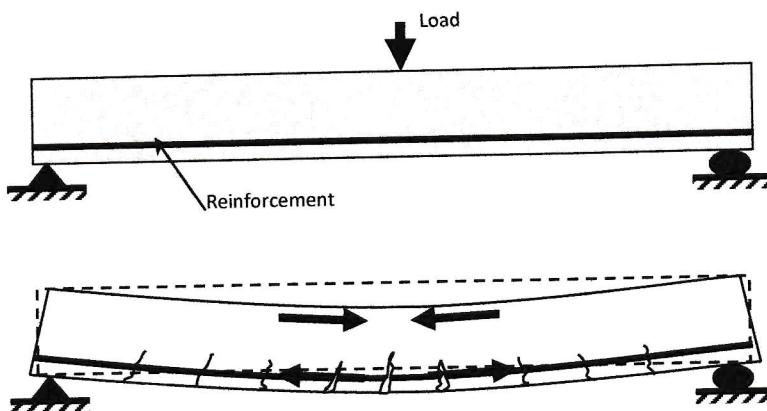
- Reinforcement must be "anchored" in the concrete to prevent relative slip between the materials. This may also be described as bond between the steel and concrete.

*Bond is how rebar yields at failure  
Very important*



- If bond does not exist between the steel and concrete, the bars will slip and not develop strain. Then the tension force,  $T$  cannot be developed.

→ If  $T = 0$  then there is no internal moment resistance

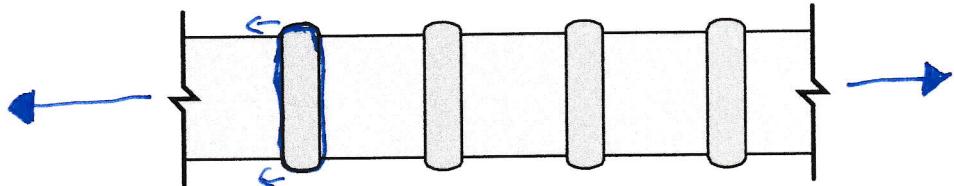


*In code, simplified to developed length.*

## BOND MECHANISMS

Bond has 3 components:

1. Adhesion
2. Friction
3. Bearing  
→ most significant



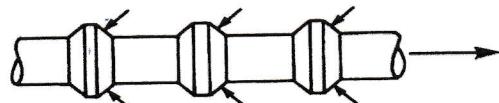
- Adhesion and friction are quickly lost with deformations of the bar, especially since bar's cross section decreases slightly due to Poisson' effect
- When smooth bars are used for reinforcing, mechanical anchorage like hooks, nuts, washers, should be used

## BOND SPLITTING CRACK PATTERNS

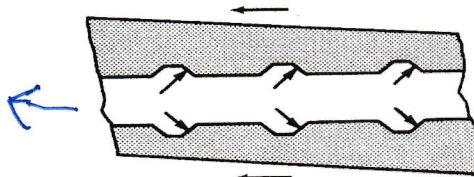
- Bearing of bar deformations on concrete generates radial stresses in concrete around bar → may result in splitting cracks.

### Bond Transfer Mechanism

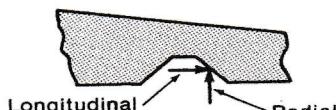
"hoop" tensile stresses are generated as a result of wedging action (or bearing) of the bar deformations on the surrounding concrete



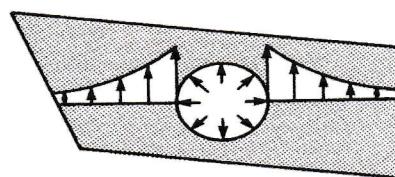
(a) Forces on bar.



(b) Forces on concrete.



(c) Components of force on concrete.

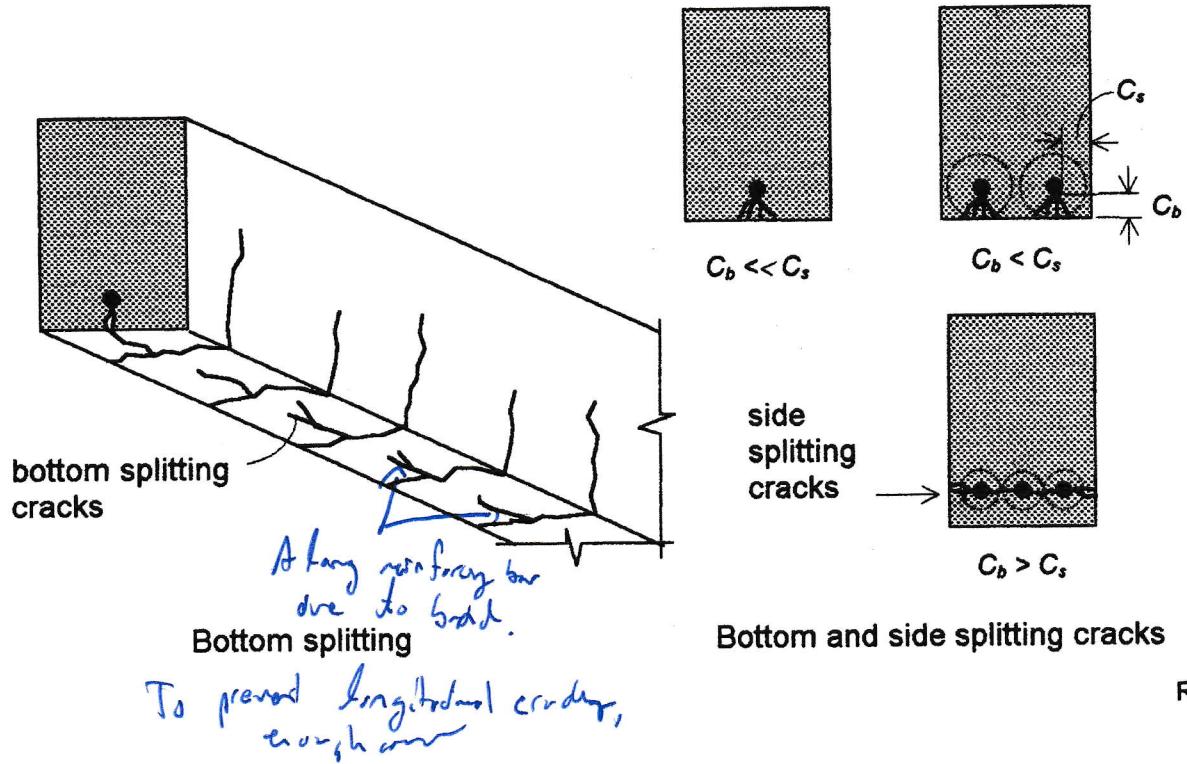


(d) Radial forces on concrete and splitting stresses shown on a section through the bar.

*bond cracks  
are along the  
bars*

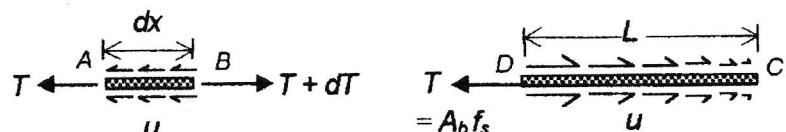
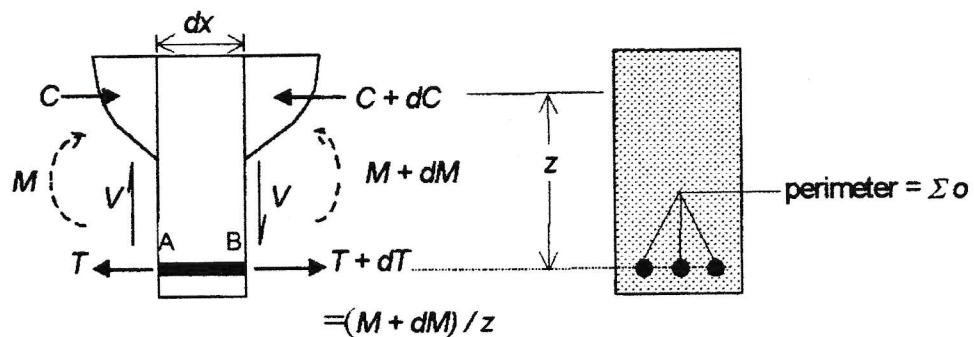
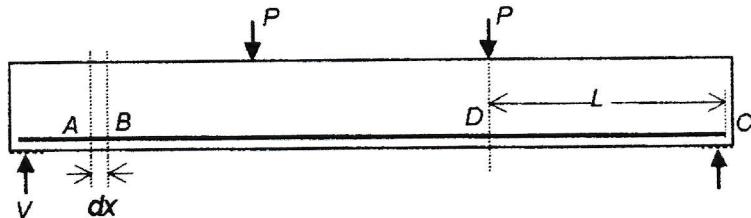
*Forces  $\perp$  to  
bar*

MacGregor and Bartlett



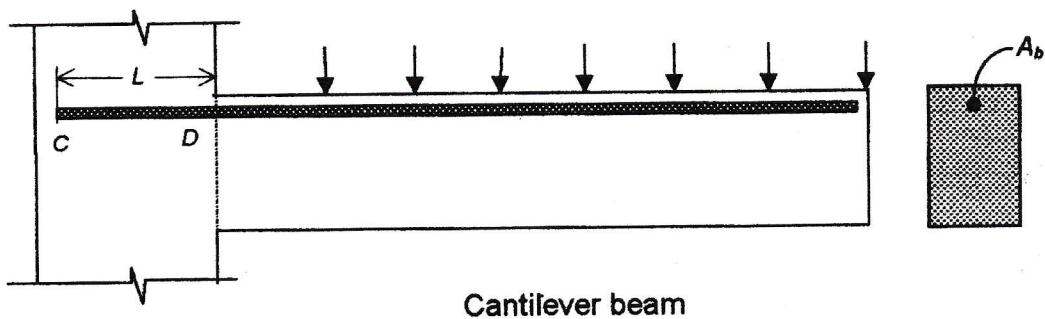
Ref. 1

## BOND STRESSES

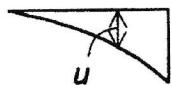


Flexural bond

Development bond

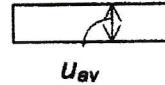


$$C \xrightarrow{u} D \rightarrow T = A_b f_s$$



Anchorage bond stress  $u$   
(probable variation)

$$C \xrightarrow{u_{av}} D \rightarrow T = A_b f_s$$



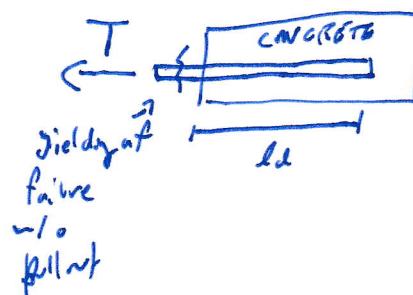
Assumed uniform  
average bond stress  $u$

Pillai S.U., Kirk, D.W., and Erki, M.A

- The relationships between bond, shear and moment are very complicated
- Most Code provisions are based on tests (experiments) of splitting failure, not bond.

**Code Philosophy:** Provide sufficient development length to ensure that bar can yield  $\rightarrow$  no need to compute bond stress

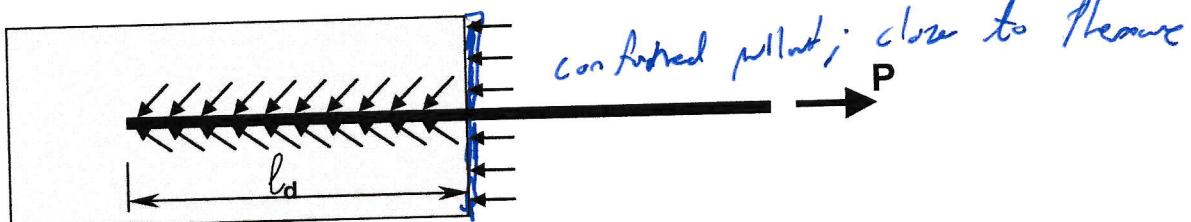
Development = length between end of bar and



## DEVELOPMENT LENGTH

Development length:

anchorage length required to develop the yield force of the bar without slip or splitting



## DEVELOPMENT LENGTH IN TENSION

CSA A23.3 Clause 12.2

$$l_d \geq 300\text{mm} \quad \text{minimum development length}$$

*calibration from testing*

$$l_d = 1.15 \frac{k_1 k_2 k_3 k_4}{(d_{cs} + K_{tr})} \frac{f_y}{\sqrt{f'_c}} A_b$$

Note that  $(d_{cs} + K_{tr}) \leq 2.5 d_b$

Clause 12.2.2

Where

$k_i$  = modification factors (defined later)

$$K_{tr} = \frac{A_{tr} f_{yt}}{10.5 s n} \quad (\text{stirrup strength factor})$$

$A_{tr}$  = area of stirrups (transverse reinforcement)

$f_{yt}$  = yield strength of stirrups

$s$  = stirrup spacing

$n$  = number of bars developed

$d_{cs}$  = smaller of: minimum cover to centre of bar being developed or 2/3 of centre to centre spacing of bar

*In continuous beam, always have to splice bar. Don't do this in tension zone, any where.*

## Simplified Equations

### CSA A23.3 Clause 12.2.3

The following equations may be used if:

- clear cover  $\geq d_b$
- clear spacing  $\geq 1.4d_b$

Cases	Minimum development length
Member containing minimum transverse reinforcement Slabs & walls having clear spacing $> 2d_b$	$l_d = 0.45k_1k_2k_3k_4 \frac{f_y}{\sqrt{f'_c}} d_b$
Other cases	$l_d = 0.6k_1k_2k_3k_4 \frac{f_y}{\sqrt{f'_c}} d_b$

Stirrups keep concrete confined around the bar.  $\therefore$  stirrups formula is less conservative.

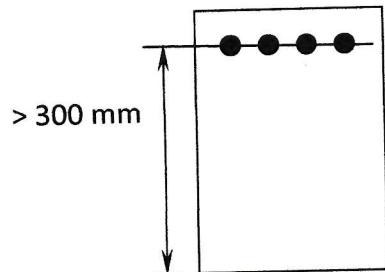
## MODIFICATION FACTORS

CSA A23.3 Clause 12.2.4

### $k_1$ bar location factor

$k_1 = 1.3$  for top bars

$k_1 = 1.0$  for other cases



### $k_2$ bar coating factor

$k_2 = 1.5$  for Epoxy-coated bars  
with clear cover  $< 3 d_b$   
or with clear spacing  $< 6 d_b$

$k_2 = 1.2$  for all other Epoxy-coated bars

$k_2 = 1.0$  for uncoated bars

*↓ wall of no cracks in epoxy.  
∴ abandoned  
↑ epoxy helps w/ corrosion*

### $k_3$ concrete density factor

$k_3 = 1.3$  for structural low density concrete

$k_3 = 1.2$  for structural semi-low density concrete

$k_3 = 1.0$  for normal density concrete

### $k_4$ bar size factor

$k_4 = 0.8$  for No. 20 and smaller bars and def. wires

$k_4 = 1.0$  for No. 25 and larger bars

$$k_1 k_2 \leq 1.7$$

Or use Handbook Table 3.1

## DEVELOPMENT LENGTH IN COMPRESSION

CSA A23.3 Clause 12.3

basic development length:  $\ell_{db} = 0.24d_b \frac{f_y}{\sqrt{f_c}} \geq 0.044d_b f_y$

development length in compression:

$$\ell_d = \text{modification factor} \times \ell_{db}$$

$$\ell_d \geq 200 \text{ mm}$$

modification factor = 0.75

for longitudinal bars in a spiral column

## DEVELOPMENT LENGTH FOR BUNDLED BARS

CSA A23.3 Clause 12.4

- For bundled bars in tension or compression, increase development length of a single bar by:

1.1 for 2 bars

1.2 for 3 bars 00%

1.33 for 4 bars

## DEVELOPMENT OF STANDARD HOOKS

CSA A23.3 Clause 12.5

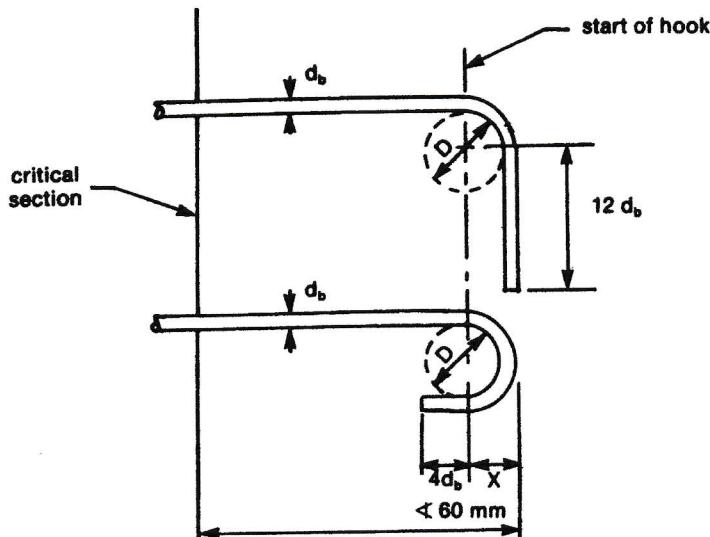
- Used to reduce the required development length in tension.

$$l_{dh} = l_{hb} \times \text{modification factor(s)}$$

$\geq 8d_b$  or 150 mm, whichever is greater

Basic development length  
for a hooked bar  
with  $f_y = 400$  MPa:

$$l_{hb} = \frac{100d_b}{\sqrt{f'_c}}$$



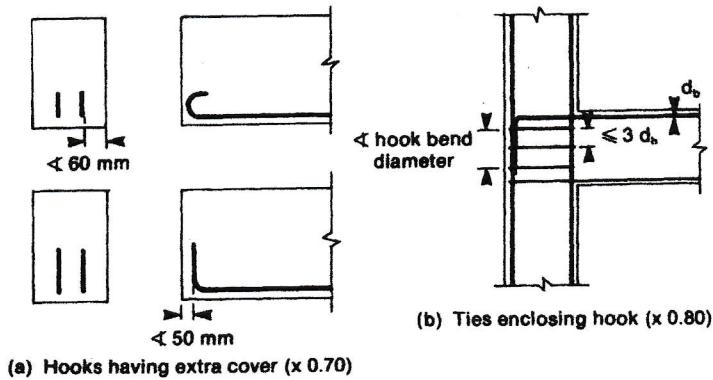
$$l_{dh} = l_{hb} \times \text{modification factor}$$

Fig. N12.5.2  
Hook Geometry Details

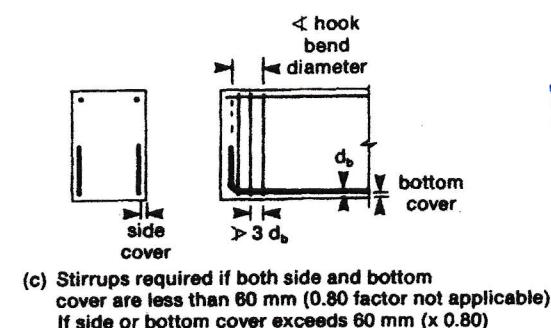
## Modification factors for hooks:

## Clause 12.5.3

(a) for bars with $f_y$ other than 400 MPa	$f_y/400$
(b) for No. 35 bars or smaller, where the side cover (normal to the plane of the hook) is not less than 60 mm and for 90° hooks where the cover of the bar extension beyond the hook is not less than 50 mm	0.7
(c) for No. 35 bars or smaller, where the hook is enclosed vertically or horizontally within at least three ties or stirrups spaced along a length at least equal to the inside diameter of the hook, at a spacing not greater than $3d_b$	0.8
(d) where anchorage or development for $f_y$ is not specifically required, for reinforcement in excess of that required by analysis	$A_s(\text{req'd.})$ $A_s(\text{prov.})$
(e) for structural low density concrete	1.3
(f) for epoxy-coated reinforcement	1.2



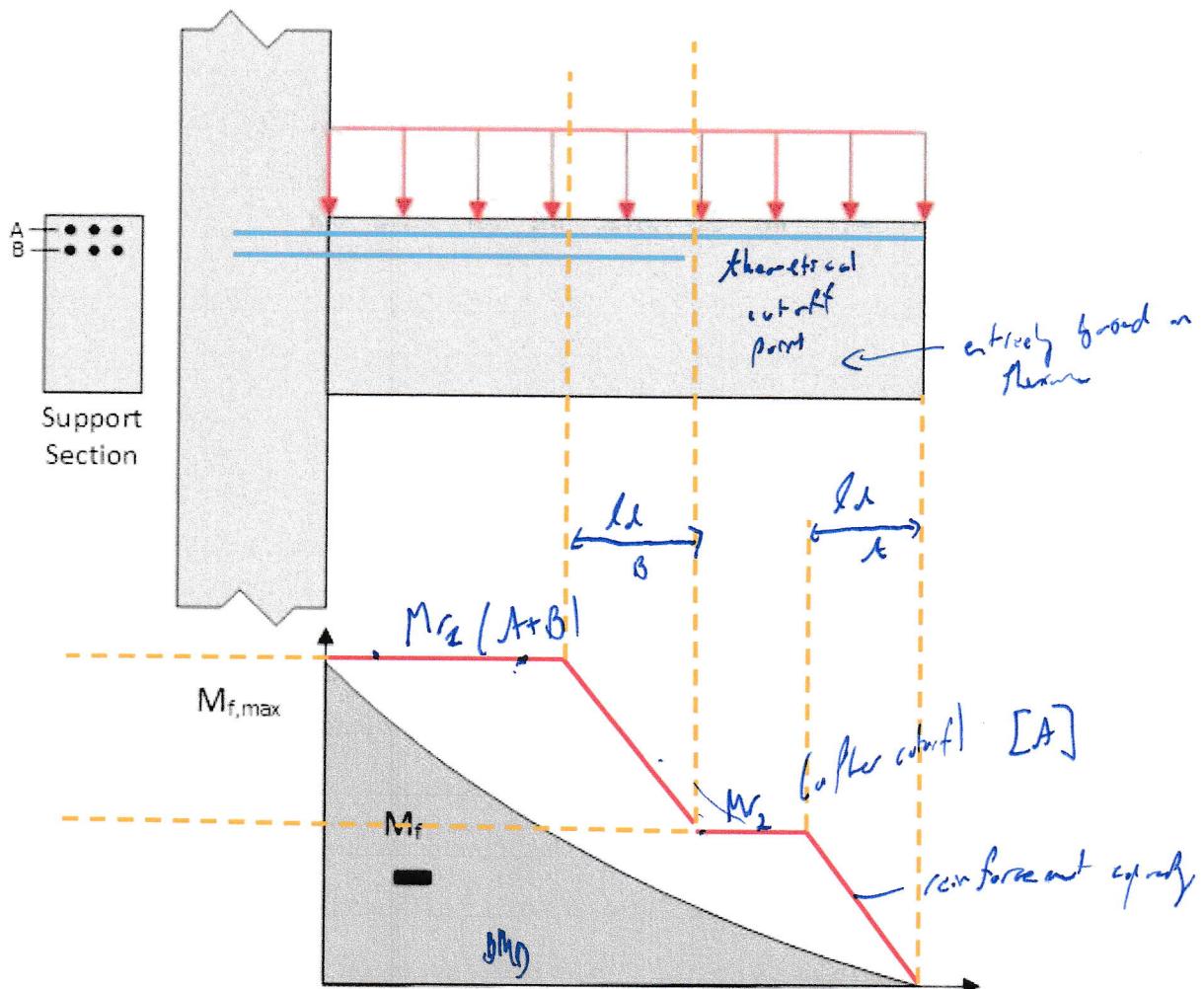
(b) Ties enclosing hook (x 0.80)



end plates  
 mechanical  
 anchorage

Fig. N12.5.3  
Factors Modifying Hook Development Length for No. 35 or Smaller Bars

# DEVELOPMENT OF FLEXURAL REINFORCEMENT



- $A_s$  is designed for  $M_{f,\max}$  at critical sections
- Between critical sections, stress in steel varies according to  $M_f$  diagram  $\rightarrow f_s$  may be less than  $f_y$ .
  - ➔ If  $A_s$  is kept constant, the beam is over designed.
- Practical Solution: “cut-off” bars at suitable points
  - Choose number of bars to terminate
  - Compute  $M_r$  for continuing bars

- Locate where  $M_r$  for continuing steel =  $M_f$

→ **theoretical cut-off point**

➤ **Theoretical cut-off point:** Location where terminated steel is no longer needed for flexure ( $M_r$ )

- Not more than 50% of steel should be cut-off at a given section to minimize stress concentrations and cracking
- Cut-off bars must extend beyond the theoretical cut-off point to account for:
  - Tensile forces in longitudinal steel due to shear
  - Stress concentrations due to transfer of stress from terminated bars to continuing bars
  - Variations in bending moment diagram due to assumptions and approximations in structural analysis

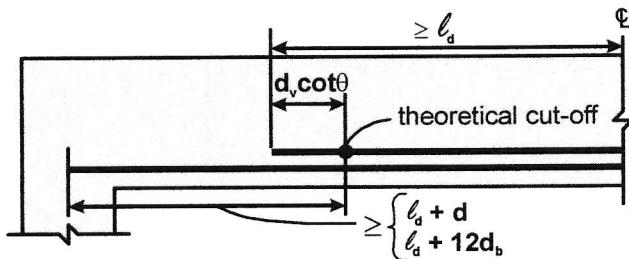
## **CSA A23.3 DEVELOPMENT & DETAILING**

### **REQUIREMENTS**

- CSA A23.3 provides detailing requirements for flexural reinforcement to provide necessary bar extensions for development and anchorage
- *Clauses 12.10, 12.11 and 12.12*
- Specific requirements are provided for positive and negative moment reinforcement

# GENERAL DEVELOPMENT REQUIREMENTS

<b>Code Requirement</b>	<b>Reason</b>
<u>Clause 12.1.1</u> General Anchorage All bars must have an embedment length beyond the point of maximum moment of at least $\ell_d$ .	<i>Ensures that <math>f_y</math> can be developed in bars at location of maximum moment.</i>
<u>Clause 12.10.3</u> Actual Cut-off Point All bars must extend beyond the point where they are <u>theoretically no longer required</u> for flexure by a distance of (also see Clause 11.3.9): $d_v \cot \theta$ <p style="text-align: center;"><i>for effective = actual point</i></p> where, $d_v$ = effective shear depth = 0.9d or 0.72h, whichever is greater $\theta$ = see Cl. 11.3.6	<i>Accounts for additional tension force in bars due to shear.</i> <i>Also provides contingency for unexpected loads, assumptions in analysis, etc., that could shift moment diagram.</i>
<u>Clause 12.10.4</u> Continuing Reinforcement Continuing reinforcement must have an embedment length beyond the point where the terminated reinforcement is no longer theoretically required in flexure of: $\text{larger of } \begin{cases} \ell_d + d \\ \ell_d + 12d_b \end{cases}$	<i>Ensures continuing bars can develop <math>f_y</math> at section where other bars are cut off.</i> <i>Includes contingency for additional tension due to shear, and for factors that could shift moment diagram.</i>



## POSITIVE MOMENT REINFORCEMENT REQUIREMENTS

Code Requirement	Reason
<p><b>Clause 12.11.1 At Supports</b></p> <ul style="list-style-type: none"> <li>For simply supported beams at least 1/3 of <math>A_{s\text{ total}}</math> must continue into the support</li> <li>For continuous beams at least 1/4 of <math>A_{s\text{ total}}</math> must continue into the support</li> <li>Bars must extend at least 150 mm into the support</li> <li>Also check Cl. 11.3.9.5 (see Ex. 1)</li> </ul>	<p>Provides contingency for factors that could shift moment diagram.</p> <p>Also provides minimum continuous reinforcement for structural integrity.</p>
<p><b>Clause 12.11.3 Development at Zero Moment Locations</b></p> <ul style="list-style-type: none"> <li>Protect against bond failure at simple supports and at points of inflection in continuous beams (low (zero) moment and high shear):</li> </ul>	<ul style="list-style-type: none"> <li>bending moment increases rapidly → force in bars increases rapidly</li> <li>if <math>l_d</math> is too large, bond failure is possible since bar force and <math>M_f</math> do not develop rapidly enough</li> </ul>

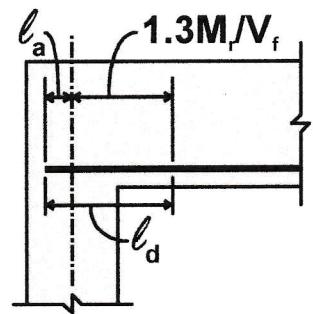
Cl. 12.11.3 - At simple supports:

$$1.3 \frac{M_r}{V_f} + l_a \geq l_d$$

$l_a$  = additional embedment length beyond centre line of support

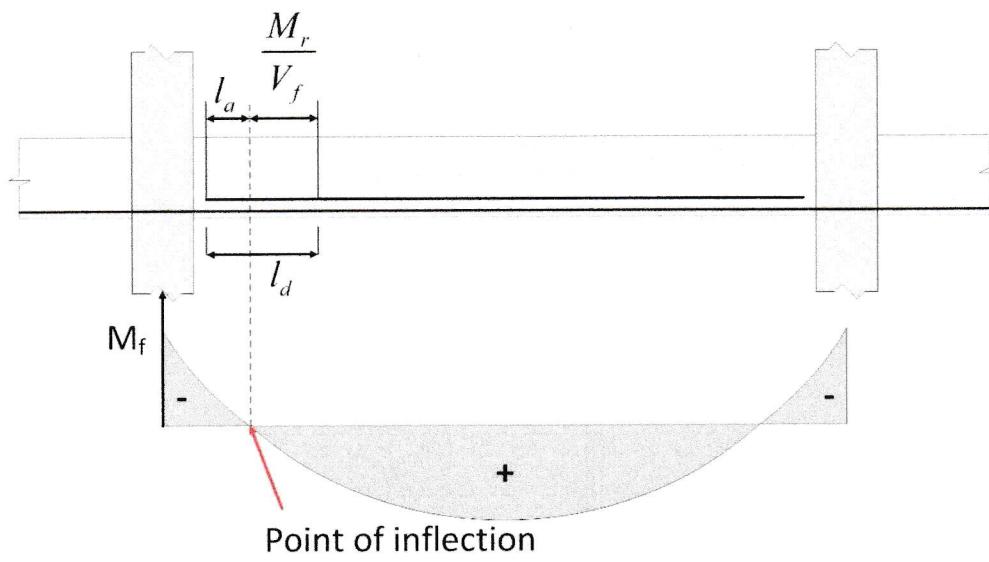
$M_r$  = moment resistance based on bars continuing into support

$V_f$  = factored shear force at support



Cl. 12.11.3 - At points of inflection:

$$\frac{M_r}{V_f} + l_a \geq l_d$$



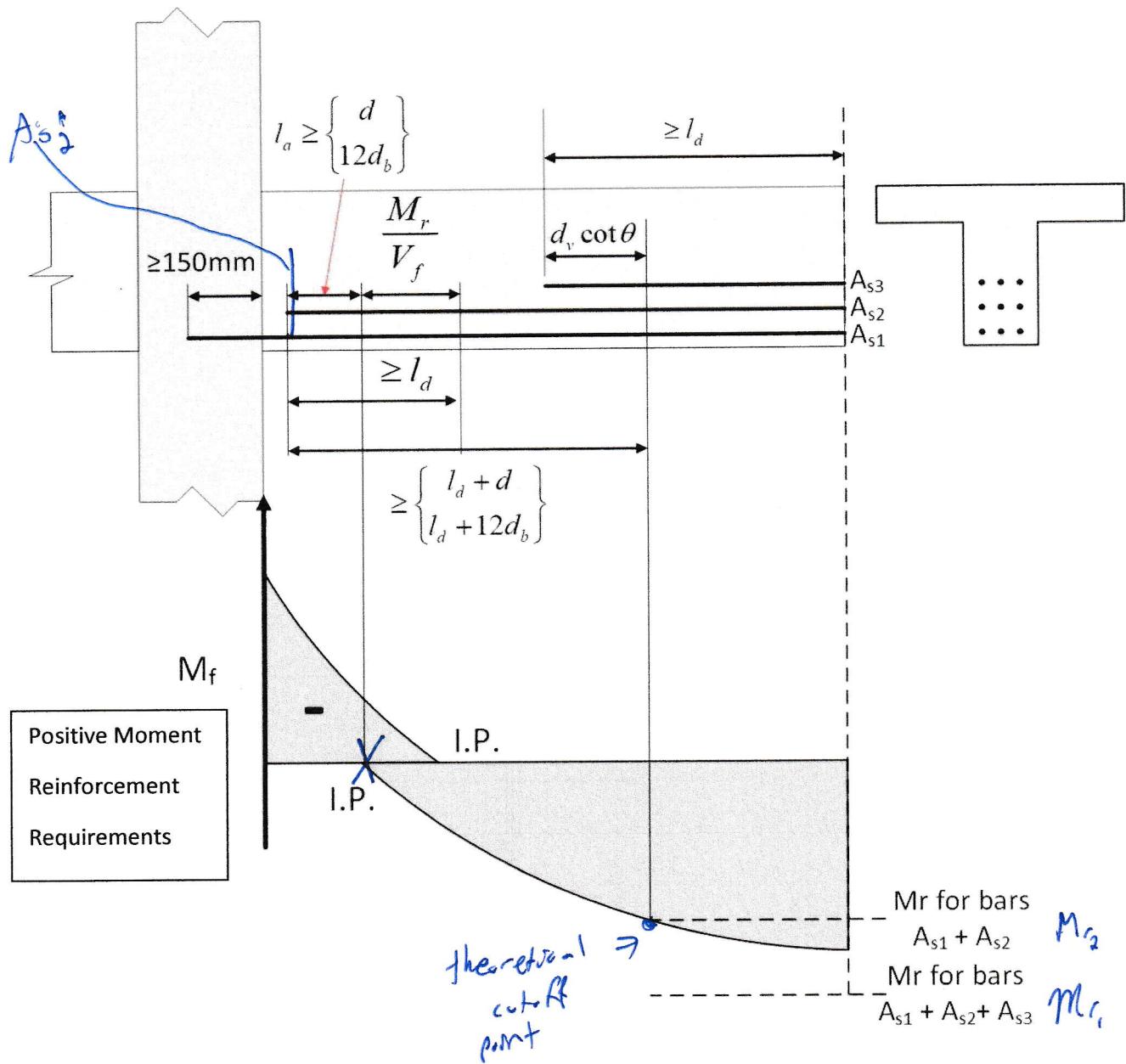
$l_a$  = greater of  $d$  or  $12d_b$

$M_r$  = moment resistance based on bars continuing through point of inflection

$V_f$  = factored shear force at point of inflection

If Cl. 12.11.3 inequality is not satisfied, decrease  $d_b$  to decrease  $l_d$

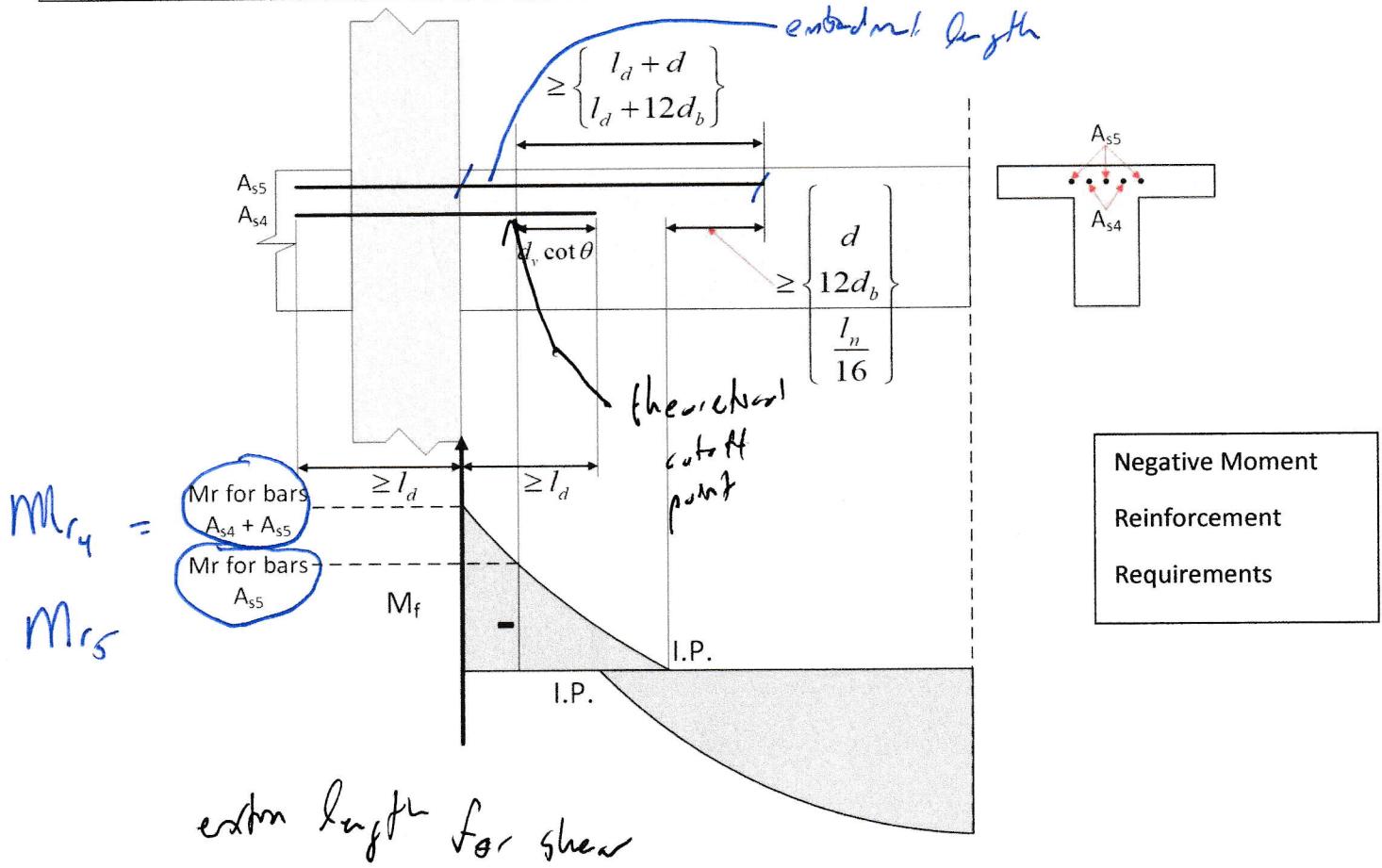
## Consider an interior span of a continuous beam...



Adapted from Ref. 2

## NEGATIVE MOMENT REINFORCEMENT REQUIREMENTS

Code Requirement	Reason
<u>Clause 12.12.1</u> Anchorage Into Supporting Members Reinforcement must be anchored in or through the supporting member by a length of at least $l_d$ .	Ensures that $f_y$ can be developed in bars at location of maximum moment (face of support).
<u>Clause 12.12.2</u> Inflection Points At least 1/3 of $A_s$ total must have an embedment length beyond the point of inflection which is: $\text{larger of } \begin{cases} d \\ 12d_b \\ l_n/16 \end{cases}$	Provides contingency for unexpected loads, assumptions in analysis, etc., that could shift moment diagram.



## DESIGN PROCEDURE – REINFORCEMENT DETAILING

1. Design Flexural Reinforcement for maximum moments

2. Select number of bars to cut-off. Consider:

- Should not terminate more than 50% of  $A_s$  at one location
- Cl. 12.11.1: Positive moment steel continued to support
- Cl. 12.12.2: Negative moment steel continued to POI
- Practical number of cut-offs

3. Calculate  $M_r$  for cut-off locations



*Based on flange alone*

4. Locate theoretical cut-offs

- Where  $M_f = M_r$  for continuing steel
- Points of inflection

5. Determine actual cut-off locations:

Positive Moment Bars	Negative Moment Bars
12.10.3: Extension of terminated bars past theoretical cut-off	12.10.3: Extension of terminated bars past theoretical cut-off
12.11.1: Extension of continuing bars into support	12.12.2: Extension of continuing bars past POI

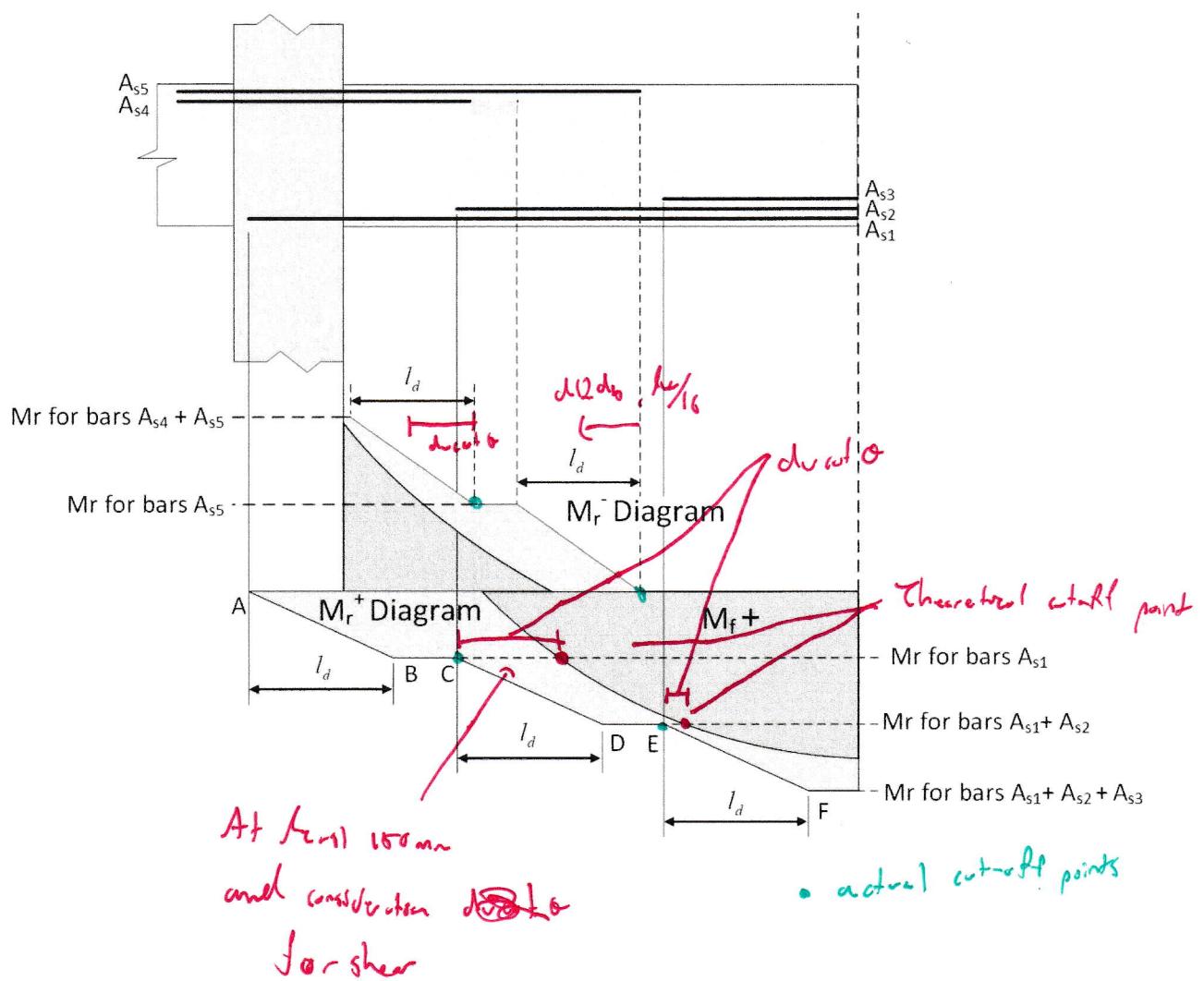


6. Check remaining detailing requirements:

Positive Moment Bars	Negative Moment Bars
12.1.1: General anchorage	12.1.1: General anchorage
12.10.4: Extension of continuing bars past terminated bars	12.10.4: Extension of continuing bars past terminated bars
12.11.3: Development at zero moment locations (supports and POI)	12.12.1: Anchorage into supporting members

## MOMENT RESISTANCE DIAGRAMS

- Envelope of moment resistance
- Construct from actual reinforcement layout
- Include development of reinforcement over  $l_d$



## Consider development of positive moment reinforcement:

A to B	$f_{s1}$ varies from 0 to $f_y$ over $l_d$ in $A_{s1}$ $M_r = 0$ to $M_{r1}$
B to C	$M_r = M_{r1} = (\phi_s A_{s1} f_y) z_1$ (constant)
C to D	$f_{s1} = f_y$ $f_{s2}$ varies from 0 to $f_y$ over $l_d$ in $A_{s2}$ $M_r = M_{r1} + M_{r2} = (\phi_s A_{s1} f_y) z_1 + (\phi_s A_{s2} f_{s2}) z_2$
D to E	$M_r = [\phi_s (A_{s1} + A_{s2}) f_y] z_{1,2}$ (constant)
E to F	$f_{s1} = f_y$ $f_{s2} = f_y$ $f_{s3}$ varies from 0 to $f_y$ over $l_d$ in $A_{s3}$ $M_r = M_{r1} + M_{r2} + M_{r3}$ $= (\phi_s A_{s1} f_y) z_1 + (\phi_s A_{s2} f_y) z_2 + (\phi_s A_{s3} f_{s3}) z_3$
Beyond F	$M_r = [\phi_s (A_{s1} + A_{s2} + A_{s3}) f_y] z_{1,2,3}$ (constant)

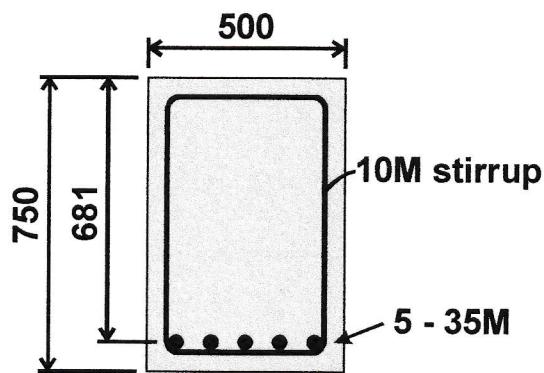
➤ Use same approach for negative moment reinforcement

Ensure  $M_r \geq M_f$  everywhere along beam length

**Example 1:** The simply-supported beam shown has been designed for the maximum factored moment at midspan.

A) Complete the flexural design by making practical bar cut-offs and checking all development requirements.

B) Construct the  $M_r$  diagram.



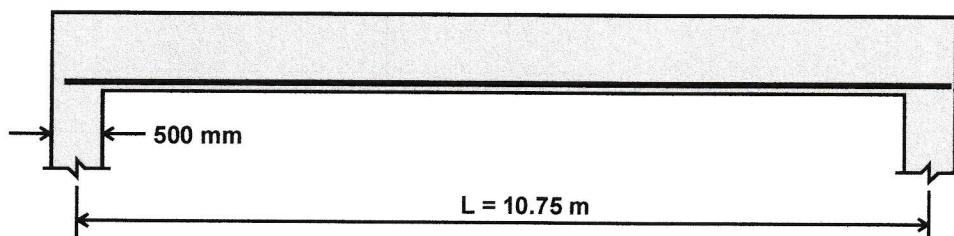
$$f_c = 30 \text{ MPa}$$

$$f_y = 400 \text{ MPa}$$

$$w_f = 65 \text{ kN/m}$$

$$M_f = 939 \text{ kNm (midspan)}$$

$$M_r = 974 \text{ kNm (5-35M)}$$



### 1. Select number of bars to cut-off

- Should not terminate more than 50% of  $A_s$  at one location
- Must continue 1/3  $A_s$  into support
  - stop 2 bars
  - continue 3 bars to support

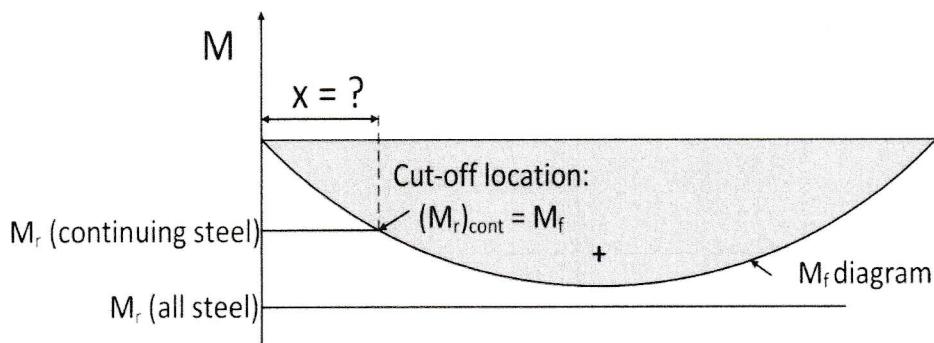
### 2. Calculate $M_r$ for cut-off locations: find $M_r$ for 3 35M bars → Use Table 2.1

$$\rho = \frac{3 \times 1000 \text{ mm}^2}{(500 \text{ mm})(681 \text{ mm})} = 0.881\% \\ \Rightarrow K_r = 2.71 \text{ MPa}$$

$$M_r = (2.71 \text{ MPa})(500 \text{ mm})(681 \text{ mm})^2 \div 10^6 = 628 \text{ kNm}$$



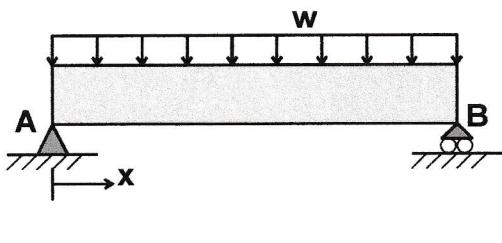
**3. Find theoretical cut-off point:** → where does  $M_f = M_r = 628 \text{ kNm}$



Options:

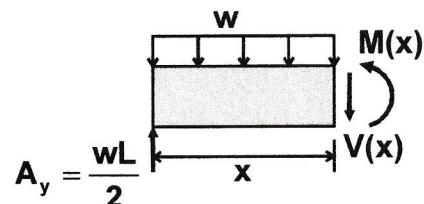
1. Statics – Method of Sections
2. Statics – Differential Relationships
3. Graphical solution

**Method of Sections:**



$$\begin{aligned}\sum M_{\text{cut}} &= 0 = M(x) + wx\left(\frac{x}{2}\right) - \frac{wL}{2}x \\ \Rightarrow M(x) &= \frac{wLx}{2} - \frac{wx^2}{2}\end{aligned}$$

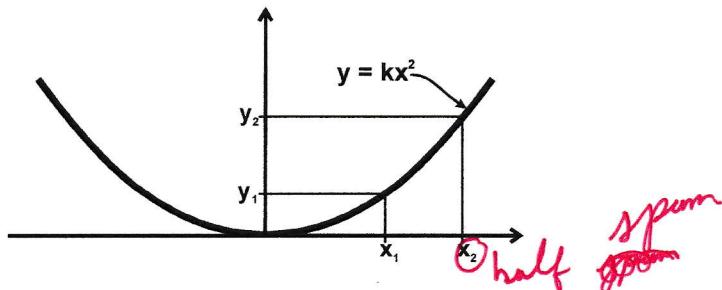
**Section beam at x from support A:**



**Solve for x when  $M(x) = (M_r)_{\text{cont}}$**

## Proportional Parabolas:

Consider function  $y = kx^2$ :



For a parabolic bending moment diagram:

$x_1$  = distance from midspan to cut-off (unknown)

$$y_1 = \Delta M = (M_f)_{\max} - (M_r)_{\text{cont}}$$

$$x_2 = L/2$$

$$y_2 = (M_f)_{\max}$$

$$y_1 = kx_1^2 \quad \text{and} \quad y_2 = kx_2^2$$

$$\frac{x_1^2}{y_1} = \frac{1}{k} = \frac{x_2^2}{y_2}$$

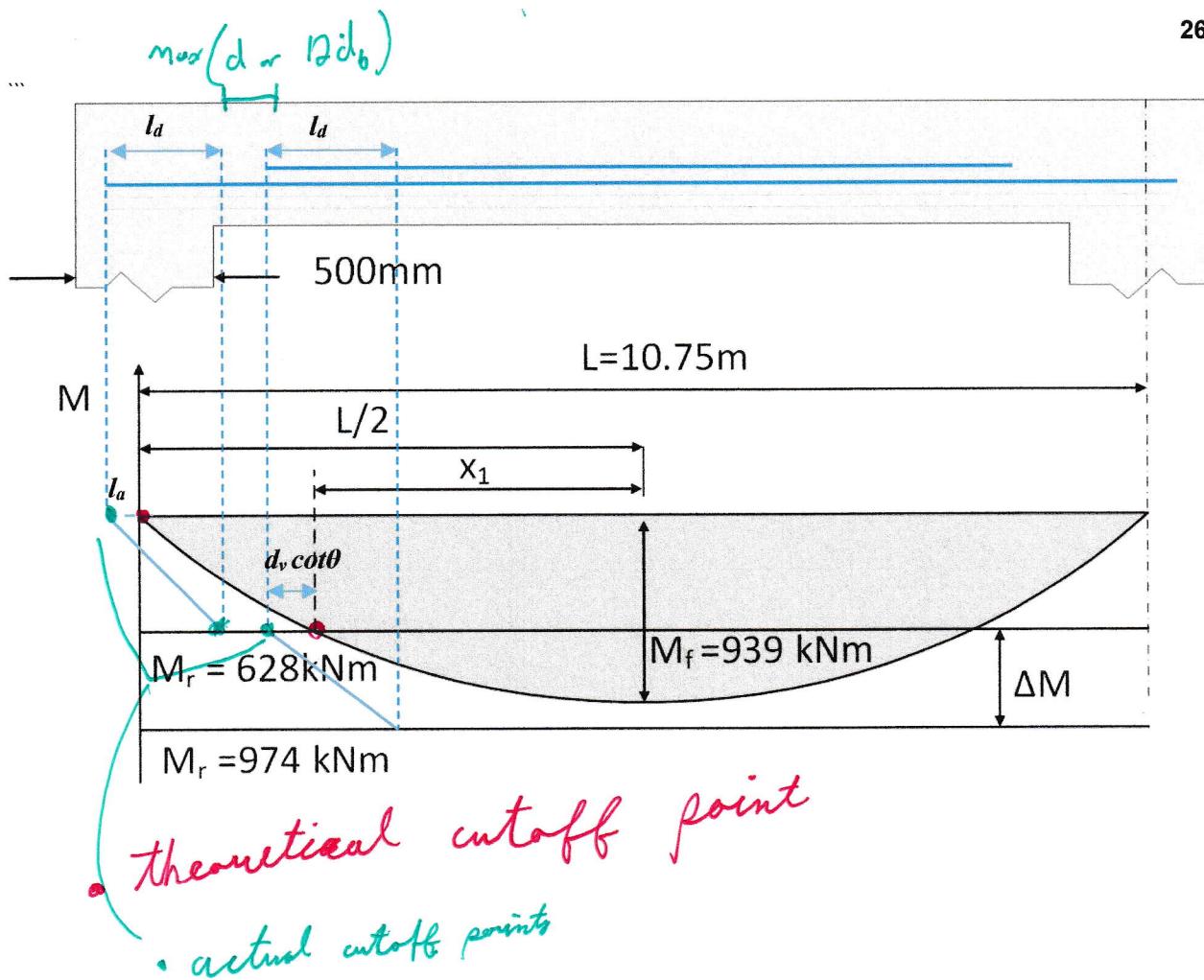
$$\Rightarrow x_1^2 = x_2^2 \left( \frac{y_1}{y_2} \right)$$

Thus:

$$x_1^2 = \left( \frac{L}{2} \right)^2 \left( \frac{(M_f)_{\max} - (M_r)_{\text{cont}}}{(M_f)_{\max}} \right)$$

Theoretical cut-off point for  $M_f = M_r = 628 \text{ kNm}$

$$(x_1)^2 = \left( \frac{L}{2} \right)^2 \left( \frac{\Delta M}{(M_f)_{\max}} \right) = \left( \frac{10750}{2} \right)^2 \left( \frac{939 - 628}{939} \right) \Rightarrow x_1 = 3093 \text{ mm}$$



#### 4. Determine actual cut-off locations

Check Requirements: Clauses 12.10.3, 12.11.1

##### Compute $l_d$ :

- Assume:
- minimum transverse reinforcement is provided
  - cover >  $d_b$
  - spacing > 1.4  $d_b$

$$\rightarrow l_d = 0.45 k_1 k_2 k_3 k_4 \frac{f_y}{\sqrt{f'_c}} d_b \quad \begin{array}{ll} k_1 = 1.0 & \text{not a top bar} \\ k_2 = 1.0 & \text{not epoxy-coated} \\ k_3 = 1.0 & \text{normal density concrete} \\ k_4 = 1.0 & 35M \text{ bar} \end{array}$$

$$l_d = 0.45(1.0)(1.0)(1.0)(1.0) \frac{400 \text{ MPa}}{\sqrt{30 \text{ MPa}}} (35.7 \text{ mm}) = \underline{1173 \text{ mm}}$$

12.10.3 Actual cut-off point: cut-off bars must extend past the theoretical cut-off by  $d_v \cot \theta$

$$d_v = \text{larger of } \begin{cases} 0.9d = 0.9(681) = 613 \text{ mm} \\ 0.72h = 0.72(750) = 540 \text{ mm} \end{cases}$$

$$\theta = 35^\circ \quad (\text{Clause 11.3.6})$$

$$\boxed{d_v \cot \theta = 875 \text{ mm}}$$

12.11.1 At support: at least 1/3 of  $(A_s)_{total}$  must continue into the support:

$$\frac{\text{continuing}}{\text{total}} = \frac{A_{s(A)}}{A_{stot}} = \frac{3000 \text{ mm}^2}{5000 \text{ mm}^2} = \frac{3}{5} > \frac{1}{3} \rightarrow \text{O.K.}$$

12.11.1 Bars must extend at least 150 mm into support (also check Cl. 11.3.9.5):

Assume continuing reinforcement, bars (A), are extended as far as possible into support.

Total embedment  $\ell_{\text{embed}} = (\text{column width}) - (\text{end cover}) = 500 - 40$   
into support:  $= 460 \text{ mm} > 150 \text{ mm} \Rightarrow \text{O.K.}$

11.3.9.5 At support, longitudinal reinforcement must be capable of resisting a tensile force,  $T_f$ , due to shear:

where  $T_f = (V_f - 0.5V_s) \cot \theta$   $V_f = \frac{w_f L}{2} = \frac{(65 \text{ kN/m})(10.75 \text{ m})}{2}$

$$= 349 \text{ kN}$$

Assume:  $V_s = 140.0 \text{ kN}$

$$T_f = (349 - 0.5(140)) \cot(35) = 398.4 \text{ kN}$$

*done symbol*

Required embedment into support to develop  $T_f$ :

$$\begin{aligned} (\ell_{\text{embed}})_{\text{req'd}} &= \frac{T_f}{\phi_s A_{s(A)} f_y} \times l_d \\ &= \frac{398.4 \times 10^3 \text{ N}}{(0.85)(3000 \text{ mm}^2)(400 \text{ MPa})} (1173 \text{ mm}) \\ &= 458.2 \text{ mm} < \ell_{\text{embed}} = 460 \text{ mm} \Rightarrow \text{O.K.} \end{aligned}$$

## 5. Check remaining detailing requirements

Check Requirements: Clauses 12.1.1, 12.10.4, 12.11.3

12.1.1 General anchorage: all bars must extend  $l_d$  past  $M_{\max}$

$$\begin{aligned} \text{Extension} &= x_1 + d_v \cot \theta \\ &= 3093 \text{ mm} + 875 \text{ mm} > l_d = 1173 \text{ mm} \rightarrow \text{O.K.} \end{aligned}$$

12.10.4 Continuing reinforcement: must extend past the theoretical cut-off point by:

larger of  $\begin{cases} l_d + d = 1173 + 681 = 1854 \text{ mm} \\ l_d + 12d_b = 1173 + 12(35.7) = 1601 \text{ mm} \end{cases}$

*= O.K.*

Determine length of "continuing" reinforcement  $\rightarrow$  assume continuing bars (A) are extended as far as possible into support:

$$\begin{aligned} \text{Total length of bars (A), } L_A &= L_{\text{span}} + b_{\text{col}} - 2(\text{end cover}) \\ &= 10750 + 500 - 2(40) = 11170 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Extension of bars (A) past theoretical cut-off for bars (B):} &= \frac{1}{2}(L_A) - x_1 = (11170/2) - 3093 \\ &= 2492 \text{ mm} > 1854 \text{ mm} \rightarrow \text{O.K.} \end{aligned}$$

### 12.11.3 Development at zero moment locations: development at support:

$$1.3 \frac{M_r}{V_f} + l_a \geq l_d$$

$$\begin{aligned} \text{Extend continuing bars (A) as far as possible into support:} &l_a = \frac{1}{2}(\text{column width}) - \text{cover} \\ &= \frac{1}{2}(500 \text{ mm}) - 40 \text{ mm} = 210 \text{ mm} \end{aligned}$$

$$M_r = 628 \text{ kNm (3 bars)}$$

$$V_f = 349 \text{ kN}$$

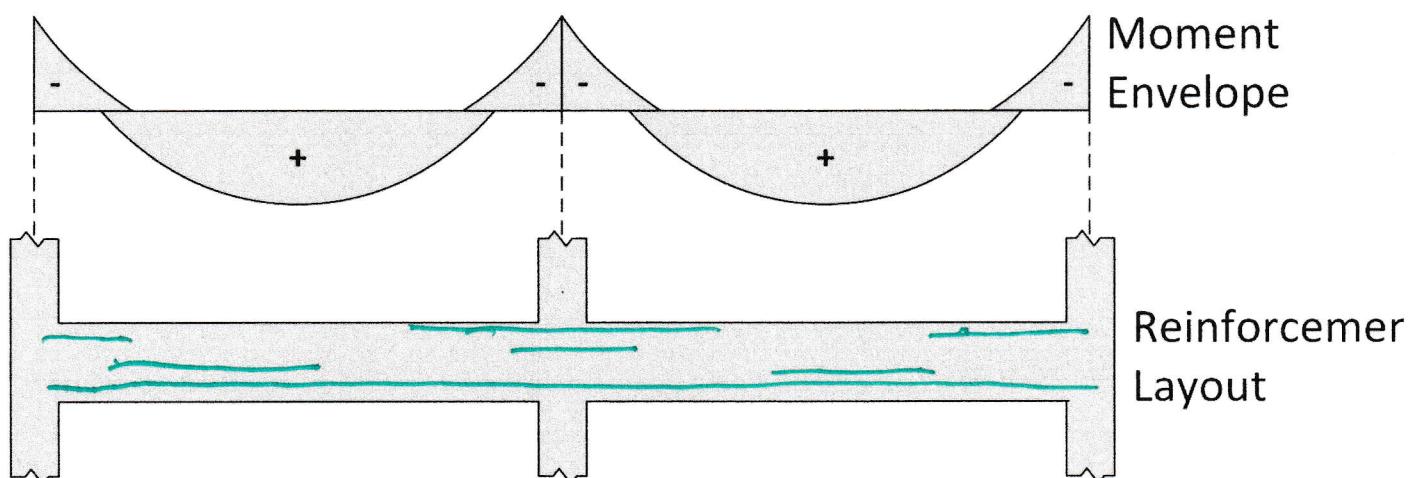
$$1.3 \left( \frac{628 \times 10^3 \text{ kNm}}{349 \text{ kN}} \right) + 210 \text{ mm} = 2549 \text{ mm} > l_d = 1173 \text{ mm} \rightarrow \text{O.K.}$$

### 6. Construct $M_r$ Diagram

$\rightarrow$  Ensure  $M_r \geq M_f$  everywhere along length of beam.

## **DETAILING FOR CONTINUOUS BEAMS AND SLABS**

- Determining bar cut-offs and detailing requirements must also consider:
- Moment envelope (positive and negative)
  - Both positive and negative moment reinforcement are present → one may be compression reinforcement for the other
  - At least 1/4 of positive moment steel must continue into support (Cl. 12.11.1)
  - At least 1/3 of negative moment steel must continue past point of inflection (Cl. 12.12.2)

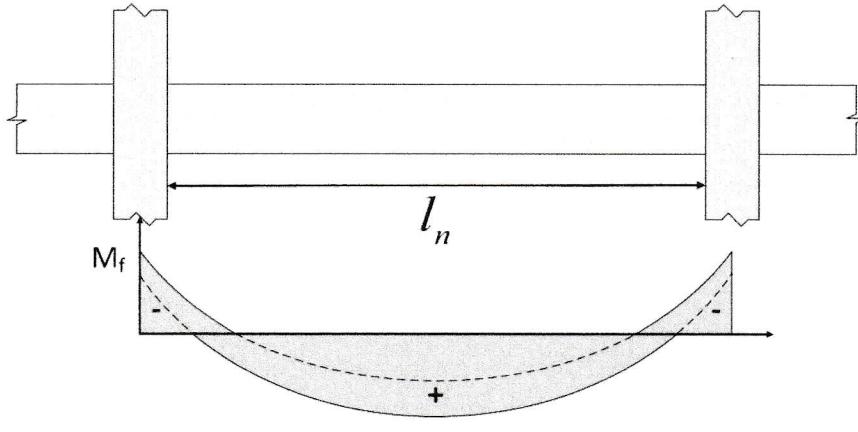


## Determining Cut-Off Locations in Continuous Beams

- Use same approach as for simply-supported beams (Ex. 1)

1. Statics – Method of Sections
2. Statics – Differential Relationships
3. Graphical solution

- Bending moment envelope can be constructed using the moment coefficients from Cl. 9.3.3.



- Defines the maximum factored moments due to dead and live loads at critical sections
- Accounts for pattern live loading → moment coefficients represent an envelope of maximum moments that may not occur simultaneously and may be produced by different load patterns

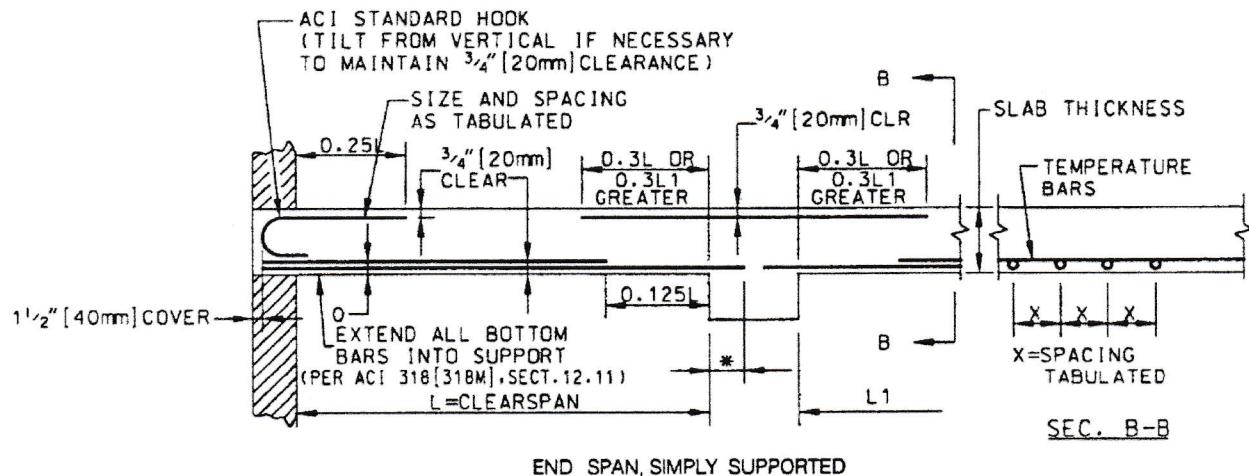
- Proportional parabolas can be used to locate the points of inflection (i.e., where  $M_f = 0$ ) and other theoretical cut-off locations by working from the parabolic portion of the  $M_f$  diagram with the baseline taken as the line joining  $M_A$  and  $M_B$ .
- If  $M_A \neq M_B$ , then the vertex of the parabola must be located (not at  $L/2$ ). In this case, it is normally easier to derive a moment function  $M(x)$ .

## Use of Typical Details for One-way Slabs and Beams

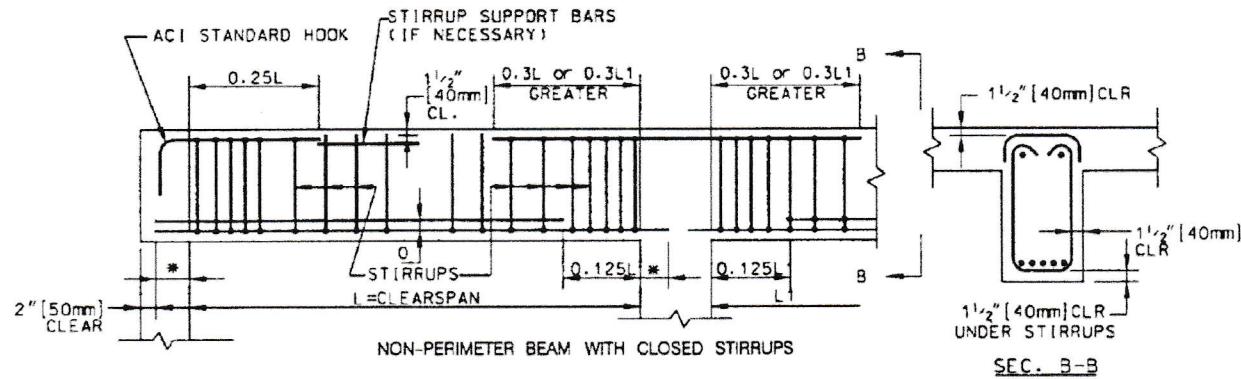
➤ ACI 315-99<sup>3</sup> provides typical details for continuous beams and slabs, including locations for actual bar cut-offs.

- No need to compute theoretical cut-off
- May be conservative
- Engineer is still responsible for ensuring that all detailing requirements in CSA A23.3-04 Chp 12 are met.

### Slab



### Beam

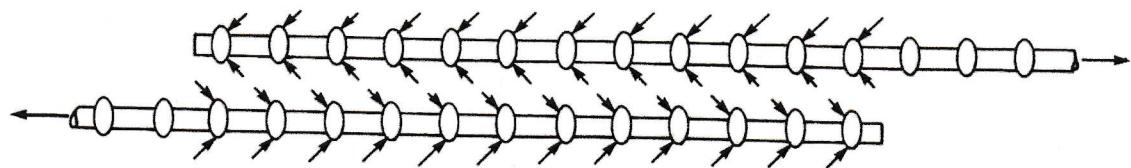


## DETAILING

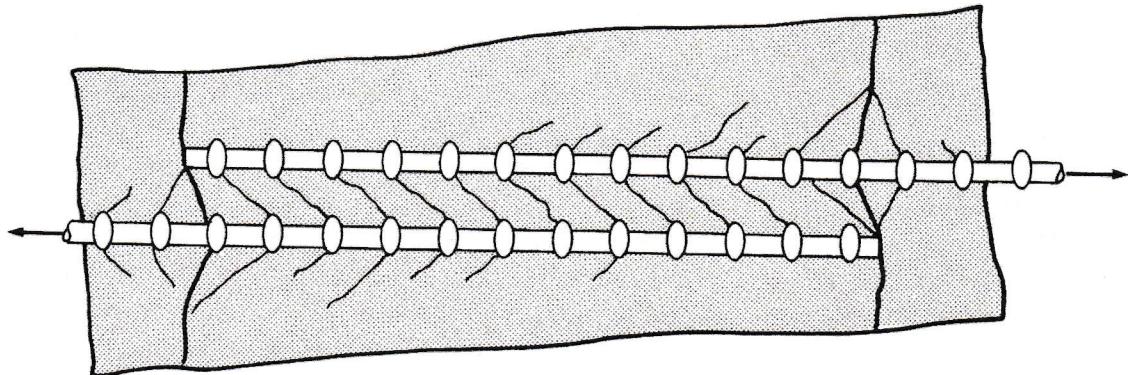
- The code provides guidance on :
- Minimum and maximum spacing of the bars
- Placement of stirrups in certain areas: e.g within the column beam joints

***These aspects will be discussed in relevant sections on the course. Following detailing rules is essential for reinforced concrete to work as a COMPOSITE, and for all strength and serviceability calculations to be valid.***

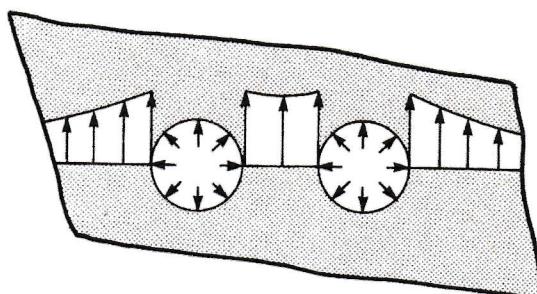
## SPICES OF REINFORCEMENT



(a) Forces on bars at splice.



(b) Internal cracks at splice.



(c) Radial forces on concrete and splitting stresses shown on a section through the splice.

*Macgregor and Bartlet*

The minimum lap splice is 300mm

Bars 35M and larger cannot be spliced (Clause 12.14.2.1)

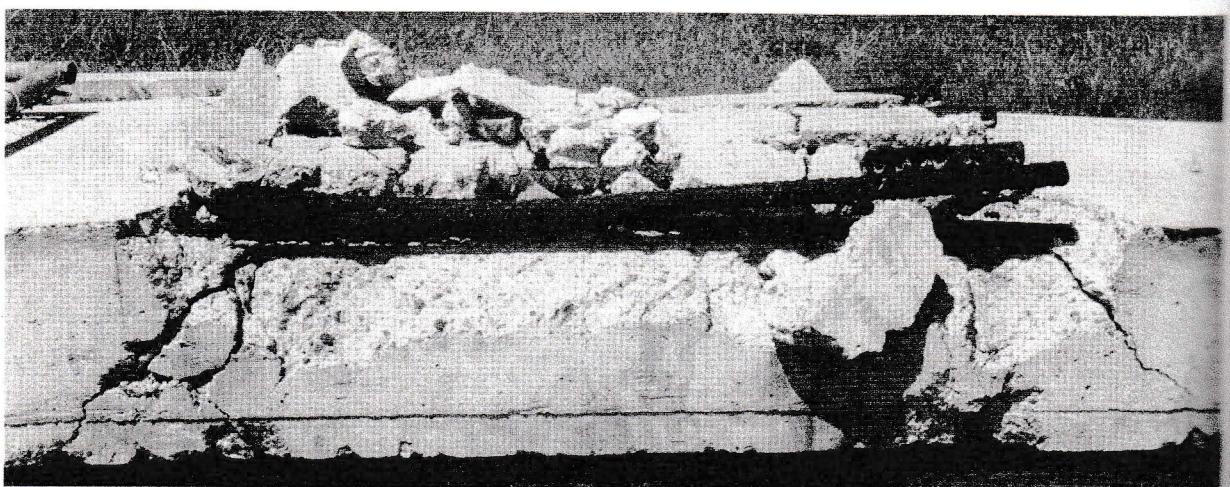
Class A lap splice :  $1.0 l_d$

Class B splice :  $1.3 l_d$

Class A splices:

- a) the area of reinforcement provided is at least twice that required by analysis at the splice location; and
- b) less than one-half of the total reinforcement is spliced within the required lap length.

Class B splices – all other cases



**Figure 8-35**  
Failure of a tension lap splice. (Photograph courtesy of J. G. MacGregor.)