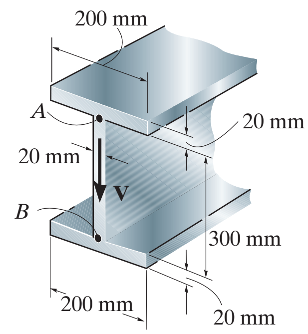
12–1.

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the shear stress on the web at A. Indicate the shear-stress components on a volume element located at this point.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.34^3) - \frac{1}{12}(0.18)(0.3^3) = 0.2501(10^{-3})\ \text{m}^4$$

From Fig. a,

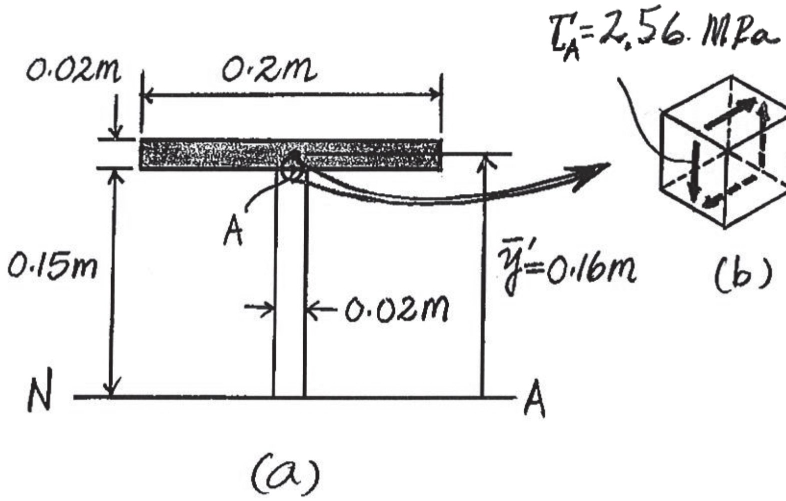
$$Q_A = \bar{y}'A' = 0.16\,(0.02)(0.2) = 0.64(10^{-3})\ \text{m}^3$$

Applying the shear formula,

$$\tau_A = \frac{VQ_A}{It} = \frac{20(10^3)[0.64(10^{-3})]}{0.2501(10^{-3})(0.02)}$$

$$= 2.559(10^6)\ \text{Pa} = 2.56\ \text{MPa} \qquad \textbf{Ans.}$$

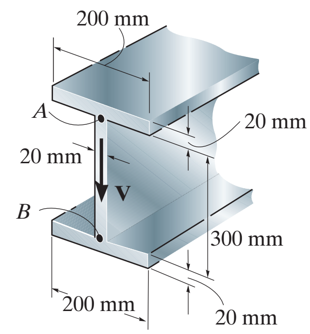
The shear stress component at A is represented by the volume element shown in Fig. b.



Ans:
$\tau_A = 2.56$ MPa

12–2.

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the maximum shear stress in the beam.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.34^3) - \frac{1}{12}(0.18)(0.3^3) = 0.2501(10^{-3})\ \text{m}^4$$

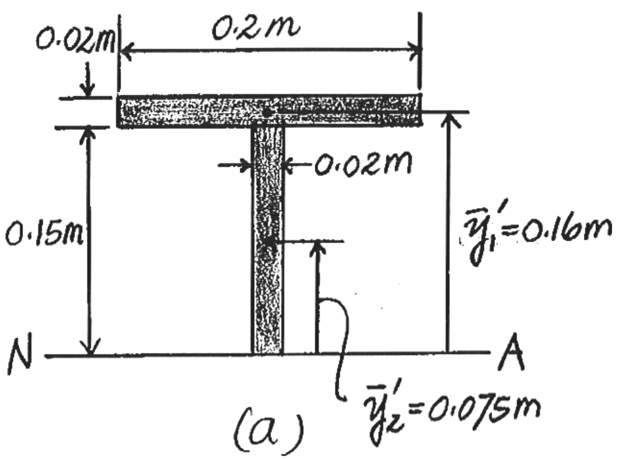
From Fig. *a*,

$$Q_{\max} = \Sigma \bar{y}'A' = 0.16\,(0.02)(0.2) + 0.075\,(0.15)(0.02) = 0.865(10^{-3})\ \text{m}^3$$

The maximum shear stress occurs at the points along neutral axis since Q is maximum and thickness t is the smallest.

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{20(10^3)\,[0.865(10^{-3})]}{0.2501(10^{-3})\,(0.02)}$$

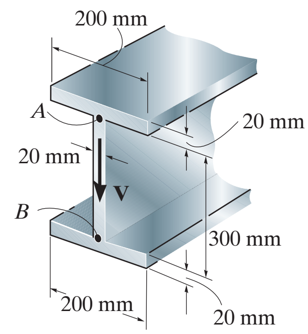
$$= 3.459(10^6)\ \text{Pa} = 3.46\ \text{MPa}$$ **Ans.**



Ans:
$\tau_{\max} = 3.46$ MPa

12–3.

If the wide-flange beam is subjected to a shear of $V = 20$ kN, determine the shear force resisted by the web of the beam.



SOLUTION

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.34^3) - \frac{1}{12}(0.18)(0.3^3) = 0.2501(10^{-3})\text{ m}^4$$

For $0 \leq y < 0.15$ m, Fig. a, Q as a function of y is

$$Q = \Sigma \bar{y}'A' = 0.16\,(0.02)(0.2) + \frac{1}{2}(y + 0.15)(0.15 - y)(0.02)$$

$$= 0.865(10^{-3}) - 0.01y^2$$

For $0 \leq y < 0.15$ m, $t = 0.02$ m. Thus,

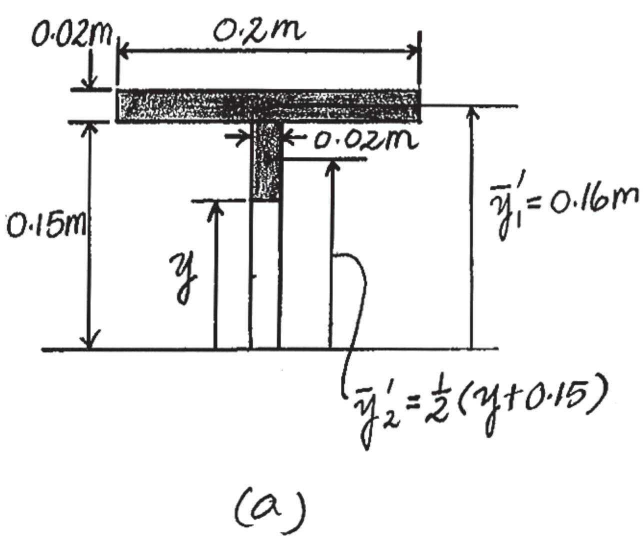
$$\tau = \frac{VQ}{It} = \frac{20(10^3)\left[0.865(10^{-3}) - 0.01y^2\right]}{0.2501(10^{-3})\,(0.02)}$$

$$= \left\{3.459(10^6) - 39.99(10^6)\,y^2\right\}\text{ Pa}$$

The sheer force resisted by the web is

$$V_w = 2\int_0^{0.15\text{ m}} \tau dA = 2\int_0^{0.15\text{ m}} \left[3.459(10^6) - 39.99(10^6)\,y^2\right](0.02\,dy)$$

$$= 18.95\,(10^3)\text{ N} = 19.0\text{ kN}$$ **Ans.**

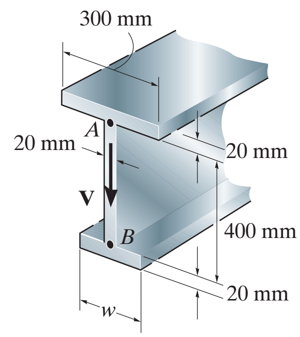


(a)

Ans:
$V_w = 19.0$ kN

***12–4.**

If the beam is subjected to a shear of $V = 30$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 200$ mm. Show that the neutral axis is located at $\bar{y} = 0.2433$ m from the bottom and $I = 0.5382(10^{-3})$ m^4.

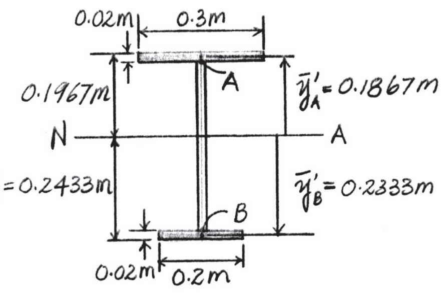


SOLUTION

Section Properties: The location of the centroid measured from the bottom is

$$\bar{y} = \frac{(0.01)(0.2)(0.02) + 0.22(0.02)(0.4) + 0.430(0.3)(0.02)}{0.2(0.02) + 0.02(0.4) + 0.3(0.02)}$$

$$= 0.2433 \text{ m}$$



(a)

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.02^3) + 0.2(0.02)(0.2433 - 0.01)^2$$

$$+ \frac{1}{12}(0.02)(0.4^3) + 0.02(0.4)(0.2433 - 0.22)^2$$

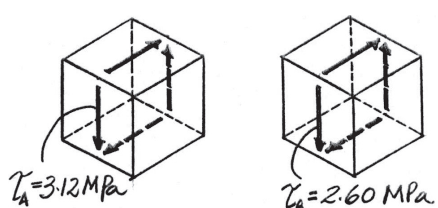
$$+ \frac{1}{12}(0.3)(0.02^3) + 0.3(0.02)(0.43 - 0.2433)^2$$

$$= 0.5382(10^{-3}) \text{ m}^4$$

Referring to Fig. *a*,

$$Q_A = \bar{y}'_A A'_A = 0.1867[0.3(0.02)] = 1.12(10^{-3}) \text{ m}^3$$

$$Q_B = \bar{y}'_B A'_B = 0.2333[0.2(0.02)] = 0.9333(10^{-3}) \text{ m}^3$$



(b)

Shear Stress: Applying the shear formula,

$$\tau_A = \frac{VQ_A}{It_A} = \frac{30(10^3)\left[1.12(10^{-3})\right]}{0.5382(10^{-3})(0.02)} = 3.122(10^6)\text{Pa} = 3.12 \text{ MPa} \qquad \textbf{Ans.}$$

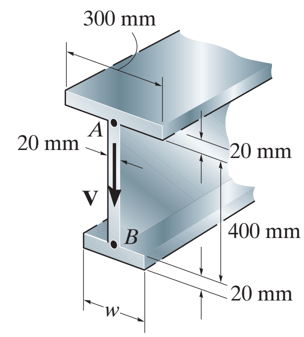
$$\tau_B = \frac{VQ_B}{It_B} = \frac{30(10^3)\left[0.9333(10^{-3})\right]}{0.5382(10^{-3})(0.02)} = 2.601(10^6)\text{Pa} = 2.60 \text{ MPa} \qquad \textbf{Ans.}$$

These shear stresses on the volume element at points A and B are shown in Fig. *b*.

Ans:
$\tau_A = 3.12$ MPa,
$\tau_B = 2.60$ MPa

12–5.

If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam. Set $w = 300$ mm.



SOLUTION

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.3)\left(0.44^3\right) - \frac{1}{12}(0.28)\left(0.4^3\right) = 0.63627\left(10^{-3}\right)\text{ m}^4$$

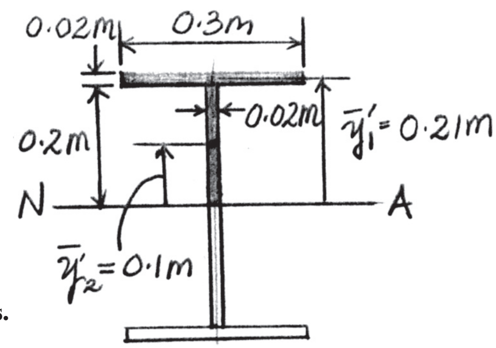
Maximum shear stress occurs at the neutral axis. Referring to Fig. *a*,

$$Q_{\max} = \Sigma\bar{y}'A' = 0.21[0.3(0.02)] + 0.1[0.2(0.02)]$$
$$= 1.66\left(10^{-3}\right)\text{ m}^3$$

Maximum Shear Stress: Applying the shear formula,

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{30\left(10^3\right)\left[1.66\left(10^{-3}\right)\right]}{0.63627\left(10^{-3}\right)(0.02)}$$

$$= 3.913\left(10^6\right)\text{Pa} = 3.91\text{ MPa}$$ **Ans.**

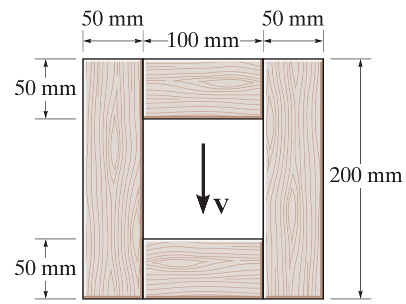


(a)

Ans:
$\tau_{\max} = 3.91$ MPa

12–6.

The wood beam has an allowable shear stress of $\tau_{\text{allow}} = 7$ MPa. Determine the maximum shear force V that can be applied to the cross section.



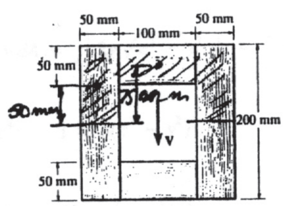
SOLUTION

$$I = \frac{1}{12}(0.2)(0.2)^3 - \frac{1}{12}(0.1)(0.1)^3 = 125(10^{-6})\ \text{m}^4$$

$$\tau_{\text{allow}} = \frac{VQ_{\text{max}}}{It}$$

$$7(10^6) = \frac{V[(0.075)(0.1)(0.05) + 2(0.05)(0.1)(0.05)]}{125(10^{-6})(0.1)}$$

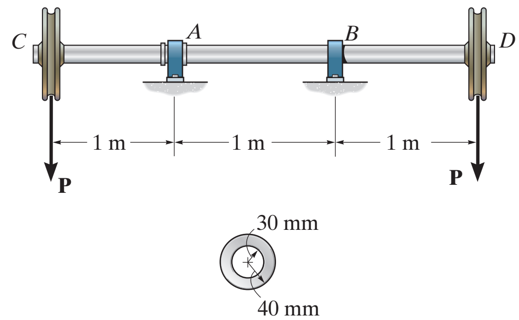
$V = 100$ kN **Ans.**



Ans:
$V_{\text{max}} = 100$ kN

12–7.

The shaft is supported by a thrust bearing at A and a journal bearing at B. If $P = 20$ kN, determine the absolute maximum shear stress in the shaft.



SOLUTION

Support Reactions: As shown on the free-body diagram of the beam, Fig. *a*.

Maximum Shear: The shear diagram is shown in Fig. *b*. As indicated, $V_{max} = 20$ kN.

Section Properties: The moment of inertia of the hollow circular shaft about the neutral axis is

$$I = \frac{\pi}{4}(0.04^4 - 0.03^4) = 0.4375(10^{-6})\pi \text{ m}^4$$

Q_{max} can be computed by taking the first moment of the shaded area in Fig. *c* about the neutral axis.

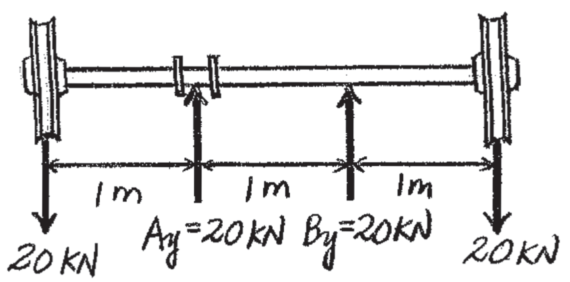
Here, $\bar{y}_1' = \dfrac{4(0.04)}{3\pi} = \dfrac{4}{75\pi}$ m and $\bar{y}_2' = \dfrac{4(0.03)}{3\pi} = \dfrac{1}{25\pi}$ m. Thus,

$$Q_{max} = \bar{y}_1'A_1' - \bar{y}_2'A_2'$$

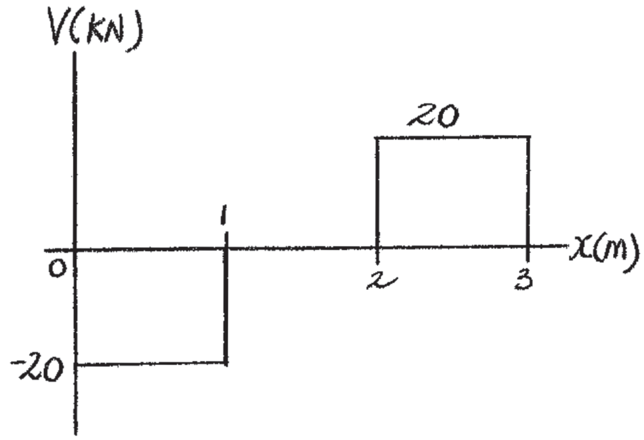
$$= \frac{4}{75\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{1}{25\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 24.667(10^{-6})\text{ m}^3$$

Shear Stress: The maximum shear stress occurs at points on the neutral axis since Q is maximum and the thickness $t = 2(0.04 - 0.03) = 0.02$ m is the smallest.

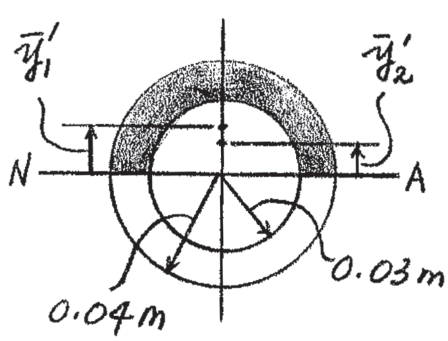
$$\tau_{max} = \frac{V_{max}Q_{max}}{It} = \frac{20(10^3)(24.667)(10^{-6})}{0.4375(10^{-6})\pi(0.02)} = 17.9 \text{ MPa}$$ **Ans.**



(a)



(b)

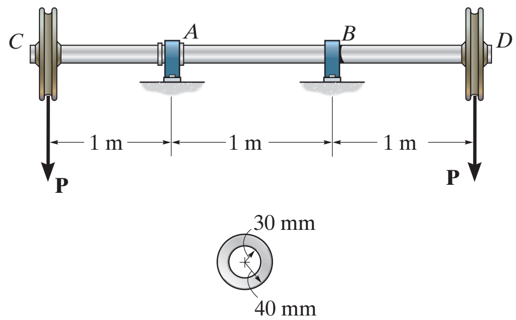


(c)

Ans:
$\tau_{max} = 17.9$ MPa

***12–8.**

The shaft is supported by a thrust bearing at A and a journal bearing at B. If the shaft is made from a material having an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa, determine the maximum value for P.



SOLUTION

Support Reactions: As shown on the free-body diagram of the shaft, Fig. *a*.

Maximum Shear: The shear diagram is shown in Fig. *b*. As indicated, $V_{\max} = P$.

Section Properties: The moment of inertia of the hollow circular shaft about the neutral axis is

$$I = \frac{\pi}{4}(0.04^4 - 0.03^4) = 0.4375(10^{-6})\pi \text{ m}^4$$

$Q_{\max}$ can be computed by taking the first moment of the shaded area in Fig. *c* about the neutral axis.

Here, $\bar{y}_1' = \dfrac{4(0.04)}{3\pi} = \dfrac{4}{75\pi}$ m and $\bar{y}_2' = \dfrac{4(0.03)}{3\pi} = \dfrac{1}{25\pi}$ m. Thus,

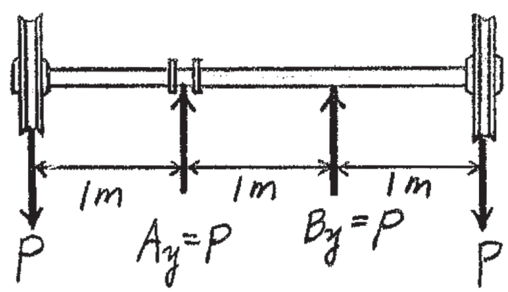
$$Q_{\max} = \bar{y}_1'A_1' - \bar{y}_2'A_2'$$

$$= \frac{4}{75\pi}\left[\frac{\pi}{2}(0.04^2)\right] - \frac{1}{25\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 24.667(10^{-6}) \text{ m}^3$$

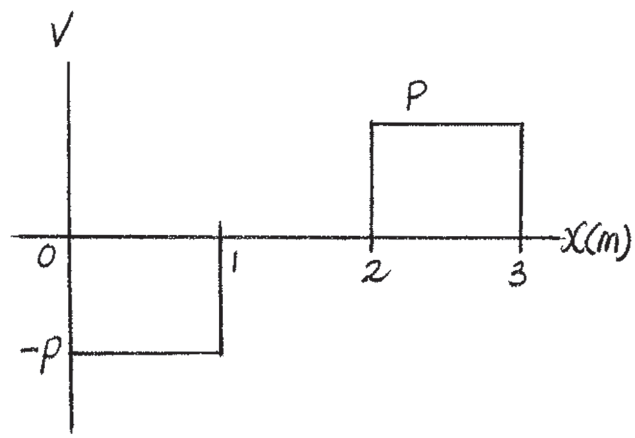
Shear Stress: The maximum shear stress occurs at points on the neutral axis since Q is maximum and the thickness $t = 2(0.04 - 0.03) = 0.02$ m.

$$\tau_{\text{allow}} = \frac{V_{\max} Q_{\max}}{It}; \qquad 75(10^6) = \frac{P(24.667)(10^{-6})}{0.4375(10^{-6})\pi(0.02)}$$

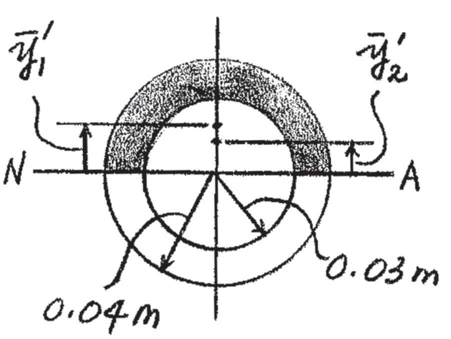
$$P = 83\,581.22 \text{ N} = 83.6 \text{ kN} \qquad \textbf{Ans.}$$



(a)



(b)

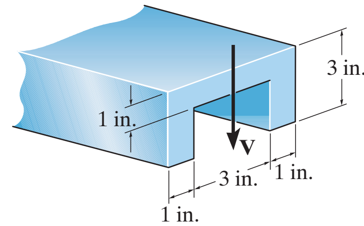


(c)

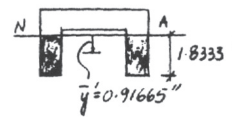
Ans:
$P = 83.6$ kN

12–9.

Determine the largest shear force V that the member can sustain if the allowable shear stress is $\tau_{allow} = 8$ ksi.



SOLUTION



$$\bar{y} = \frac{(0.5)(1)(5) + 2\,[(2)(1)(2)]}{1\,(5) + 2\,(1)(2)} = 1.1667 \text{ in.}$$

$$I = \frac{1}{12}(5)(1^3) + 5\,(1)(1.1667 - 0.5)^2$$

$$+ 2\left(\frac{1}{12}\right)(1)(2^3) + 2\,(1)(2)(2 - 1.1667)^2 = 6.75 \text{ in}^4$$

$$Q_{max} = \Sigma\bar{y}'A' = 2\,(0.91665)(1.8333)(1) = 3.3611 \text{ in}^3$$

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{I\,t}$$

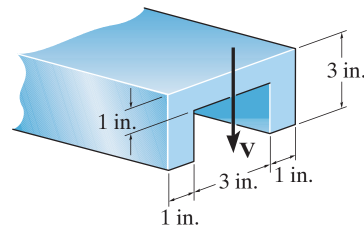
$$8\,(10^3) = \frac{V\,(3.3611)}{6.75\,(2)(1)}$$

$V = 32132$ lb $= 32.1$ kip **Ans.**

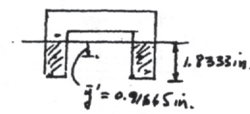
Ans:
$V_{max} = 32.1$ kip

12–10.

If the applied shear force $V = 18$ kip, determine the maximum shear stress in the member.



SOLUTION



$$\bar{y} = \frac{(0.5)(1)(5) + 2\,[(2)(1)(2)]}{1\,(5) + 2\,(1)(2)} = 1.1667 \text{ in.}$$

$$I = \frac{1}{12}(5)(1^3) + 5\,(1)(1.1667 - 0.5)^2$$

$$+ 2\left(\frac{1}{12}\right)(1)(2^3) + 2\,(1)(2)(2 - 1.1667) = 6.75 \text{ in}^4$$

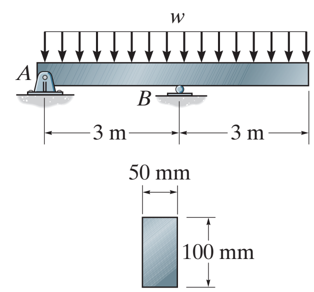
$$Q_{max} = \Sigma\bar{y}'A' = 2\,(0.91665)(1.8333)(1) = 3.3611 \text{ in}^3$$

$$\tau_{max} = \frac{VQ_{max}}{I\,t} = \frac{18(3.3611)}{6.75\,(2)(1)} = 4.48 \text{ ksi}$$ **Ans.**

Ans:
$\tau_{max} = 4.48$ ksi

12–11.

The overhang beam is subjected to the uniform distributed load having an intensity of $w = 50$ kN/m. Determine the maximum shear stress in the beam.



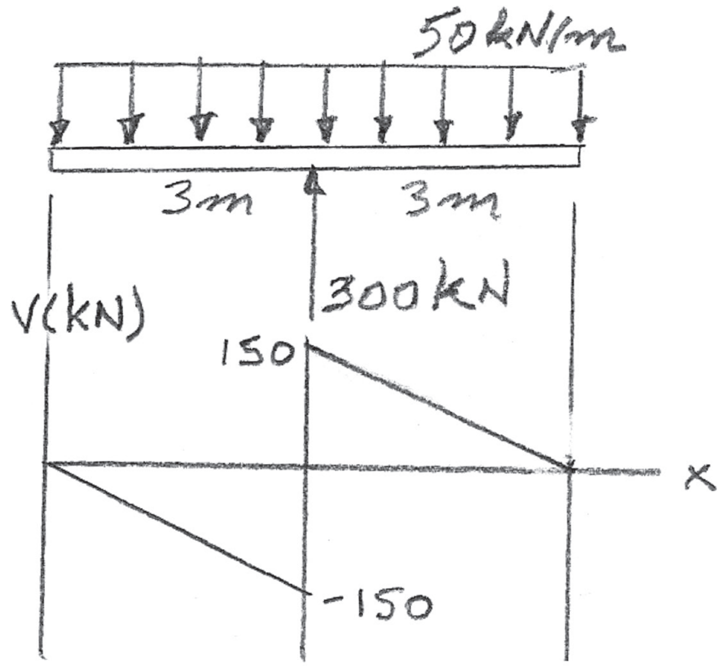
SOLUTION

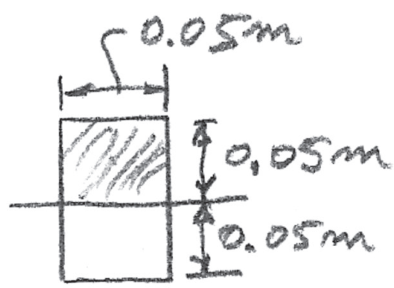
$$\tau_{max} = \frac{VQ}{It} = \frac{150(10^3)\text{ N }(0.025\text{ m})(0.05\text{ m})(0.05\text{ m})}{\frac{1}{12}(0.05\text{ m})(0.1\text{ m})^3(0.05\text{ m})}$$

$$\tau_{max} = 45.0\text{ MPa}$$ **Ans.**

Because the cross section is a rectangle, then also,

$$\tau_{max} = 1.5\frac{V}{A} = 1.5\frac{150(10^3)\text{ N}}{(0.05\text{ m})(0.1\text{ m})} = 45.0\text{ MPa}$$ **Ans.**

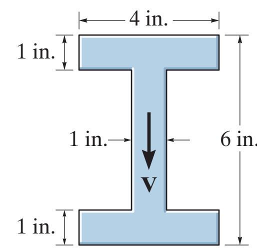




Ans:
$\tau_{max} = 45.0$ MPa

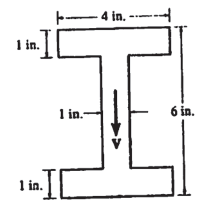
***12–12.**

The beam is made from a polymer and is subjected to a shear of $V = 7$ kip. Determine the maximum shear stress in the beam and plot the shear-stress distribution over the cross section. Report the values of the shear stress every 0.5 in. of beam depth.



SOLUTION

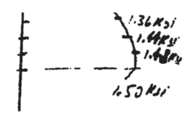
$$I = \frac{1}{12}(1)(4)^3 + 2\left[\frac{1}{12}(4)(1)^3 + 4(1)(2.5)^2\right] = 56 \text{ in}^4$$



$$\tau_1 = \frac{VQ}{It} = \frac{7(2.75)(4)(0.5)}{56(4)} = 0.172 \text{ ksi}$$

$$\tau_{2^+} = \frac{VQ}{It} = \frac{7(2.5)(4)(1)}{56(4)} = 0.3125 \text{ ksi}$$

$$\tau_{2^-} = \frac{VQ}{It} = \frac{7(2.5)(4)(1)}{56(1)} = 1.25 \text{ ksi}$$



$$\tau_3 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1.75)(1)(0.5)]}{56(1)} = 1.36 \text{ ksi}$$

$$\tau_4 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1.5)(1)(1)]}{56(1)} = 1.44 \text{ ksi}$$

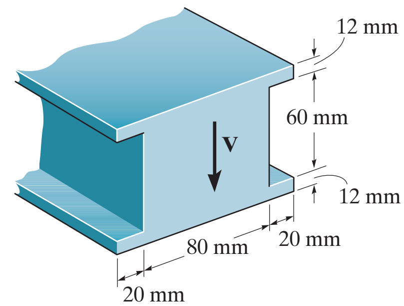
$$\tau_4 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1.25)(1)(1.5)]}{56(1)} = 1.48 \text{ ksi}$$

$$\tau_{max} = \tau_5 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1)(1)(2)]}{56(1)} = 1.50 \text{ ksi}$$ **Ans.**

Ans:
$\tau_{max} = 1.50$ ksi

12–13.

Determine the maximum shear stress in the strut if it is subjected to a shear force of $V = 20$ kN.

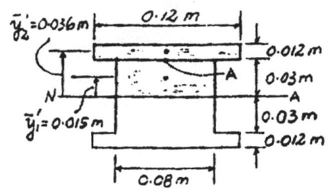


SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(0.12)\left(0.084^3\right) - \frac{1}{12}(0.04)\left(0.06^3\right)$$

$$= 5.20704\left(10^{-6}\right)\text{ m}^4$$



$$Q_{\max} = \Sigma \bar{y}' A'$$

$$= 0.015(0.08)(0.03) + 0.036(0.012)(0.12)$$

$$= 87.84\left(10^{-6}\right)\text{ m}^3$$

Maximum Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula,

$$\tau_{\max} = \frac{VQ_{\max}}{It}$$

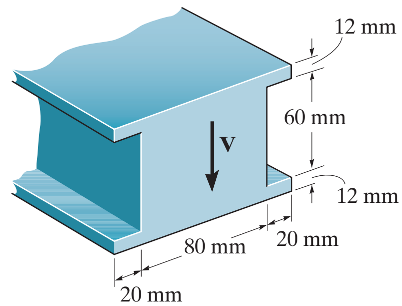
$$= \frac{20(10^3)(87.84)(10^{-6})}{5.20704(10^{-6})(0.08)}$$

$$= 4.22\text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 4.22$ MPa

12–14.

Determine the maximum shear force V that the strut can support if the allowable shear stress for the material is $\tau_{allow} = 40$ MPa.



SOLUTION

Section Properties:

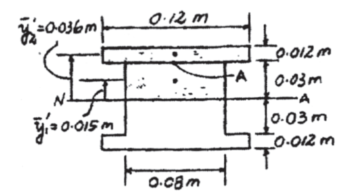
$$I_{NA} = \frac{1}{12}(0.12)\left(0.084^3\right) - \frac{1}{12}(0.04)\left(0.06^3\right)$$

$$= 5.20704\left(10^{-6}\right) \text{ m}^4$$

$$Q_{max} = \Sigma \bar{y}'A'$$

$$= 0.015(0.08)(0.03) + 0.036(0.012)(0.12)$$

$$= 87.84\left(10^{-6}\right) \text{ m}^3$$



Allowable Shear Stress: Maximum shear stress occurs at the point where the neutral axis passes through the section.

Applying the shear formula,

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

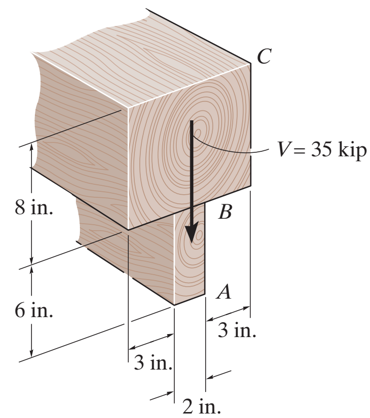
$$40\left(10^6\right) = \frac{V(87.84)(10^{-6})}{5.20704(10^{-6})(0.08)}$$

$$V = 189\,692 \text{ N} = 190 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$V_{max} = 190$ kN

12–15.

Sketch the intensity of the shear-stress distribution acting over the beam's cross-sectional area, and determine the resultant shear force acting on the segment AB. The shear force acting at the section is $V = 35$ kip. Show that $I_{NA} = 872.49\text{ in}^4$.

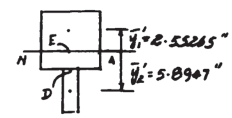


SOLUTION

$$\bar{y} = \frac{(4)(8)(8) + (11)(6)(2)}{8(8) + 6(2)} = 5.1053\text{ in.}$$

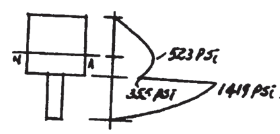
$$I = \frac{1}{12}(8)(8^3) + 8(8)(5.1053 - 4)^2$$

$$+ \frac{1}{12}(2)(6^3) + 2(6)(11 - 5.1053)^2 = 872.49\text{ in}^4$$



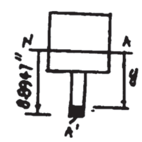
$$Q_E = \bar{y}'_1 A' = (2.55265)(5.1053)(8) = 104.26\text{ in}^3$$

$$Q_D = \bar{y}' A' = (5.8947)(6)(2) = 70.74\text{ in}^3$$



$$\tau = \frac{VQ}{It}$$

$$\tau_E = \frac{35(10^3)(104.26)}{872.49(8)} = 523\text{ psi}$$



$$(\tau_D)_{t = 2\text{ in.}} = \frac{35(10^3)(70.74)}{872.49(2)} = 1419\text{ psi}$$

$$(\tau_D)_{t = 8\text{ in.}} = \frac{35(10^3)(70.74)}{872.49(8)} = 355\text{ psi}$$

$$A' = 2(8.8947 - y)$$

$$\bar{y}' = y + \frac{(8.8947 - y)}{2} = \frac{(8.8947 + y)}{2}$$

$$Q = \bar{y}' A' = 79.1157 - y^2$$

$$\tau = \frac{VQ}{It} = \frac{35(79.1157 - y^2)}{872.49(2)} = 1.586866 - 0.0200575\, y^2$$

$$V = \int \tau\, dA \quad dA = 2\, dy$$

$$V = \int (1.586866 - 0.0200575\, y^2) 2\, dy$$

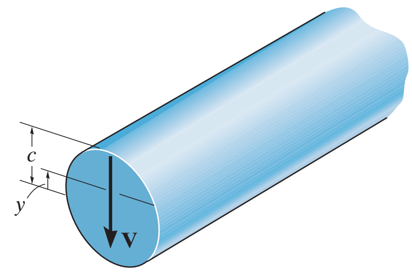
$$= \int_{2.8947}^{8.8947} (3.173732 - 0.040115\, y^2)\, dy$$

$$= 9.96\text{ kip}$$ **Ans.**

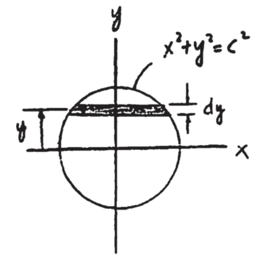
Ans:
$V = 9.96$ kip

***12–16.**

Plot the shear-stress distribution over the cross section of a rod that has a radius c. By what factor is the maximum shear stress greater than the average shear stress acting over the cross section?



SOLUTION



$$x = \sqrt{c^2 - y^2}; \qquad I = \frac{\pi}{4}c^4$$

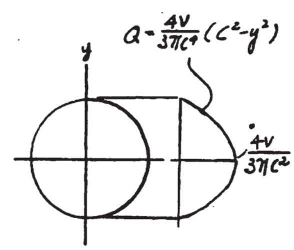
$$t = 2x = 2\sqrt{c^2 - y^2}$$

$$dA = 2x\,dy = 2\sqrt{c^2 - y^2}\,dy$$

$$dQ = y\,dA = 2y\sqrt{c^2 - y^2}\,dy$$

$$Q = \int_y^c 2y\sqrt{c^2 - y^2}\,dy = -\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}\Big|_y^c = \frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}$$

$$\tau = \frac{VQ}{It} = \frac{V\left[\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}\right]}{\left(\frac{\pi}{4}c^4\right)\left(2\sqrt{c^2 - y^2}\right)} = \frac{4V}{3\pi c^4}(c^2 - y^2)$$



The maximum shear stress occur when $y = 0$.

$$\tau_{\max} = \frac{4V}{3\pi c^2}$$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{V}{\pi c^2}$$

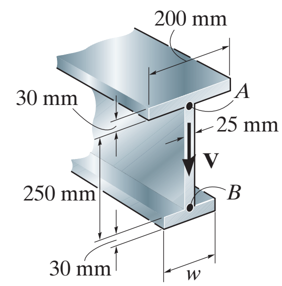
$$\text{The factor} = \frac{\tau_{\max}}{\tau_{\text{avg}}} = \frac{\frac{4V}{3\pi c^2}}{\frac{V}{\pi c^2}} = \frac{4}{3}$$ **Ans.**

Ans:

The factor $= \dfrac{4}{3}$

12–17.

If the beam is subjected to a shear of $V = 15$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 125$ mm. Show that the neutral axis is located at $\bar{y} = 0.1747$ m from the bottom and $I_{NA} = 0.2182(10^{-3})$ m^4.



SOLUTION

$$\bar{y} = \frac{(0.015)(0.125)(0.03) + (0.155)(0.025)(0.25) + (0.295)(0.2)(0.03)}{0.125(0.03) + (0.025)(0.25) + (0.2)(0.03)} = 0.1747 \text{ m}$$

$$I = \frac{1}{12}(0.125)(0.03^3) + 0.125(0.03)(0.1747 - 0.015)^2$$

$$+ \frac{1}{12}(0.025)(0.25^3) + 0.25(0.025)(0.1747 - 0.155)^2$$

$$+ \frac{1}{12}(0.2)(0.03^3) + 0.2(0.03)(0.295 - 0.1747)^2 = 0.218182(10^{-3}) \text{ m}^4$$

$$Q_A = \bar{y}A'_A = (0.310 - 0.015 - 0.1747)(0.2)(0.03) = 0.7219(10^{-3}) \text{ m}^3$$

$$Q_B = \bar{y}A'_B = (0.1747 - 0.015)(0.125)(0.03) = 0.59883(10^{-3}) \text{ m}^3$$

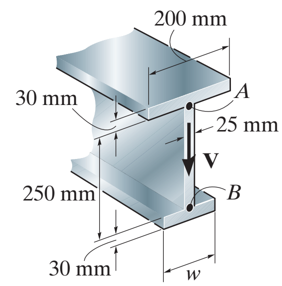
$$\tau_A = \frac{VQ_A}{It} = \frac{15(10^3)(0.7219)(10^{-3})}{0.218182(10^{-3})0.025} = 1.99 \text{ MPa}$$ **Ans.**

$$\tau_B = \frac{VQ_B}{It} = \frac{15(10^3)(0.59883)(10^{-3})}{0.218182(10^{-3})0.025} = 1.65 \text{ MPa}$$ **Ans.**

Ans:
$\tau_A = 1.99$ MPa, $\tau_B = 1.65$ MPa

12–18.

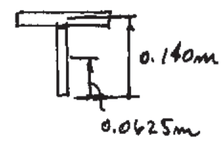
If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam. Set $w = 200$ mm.



SOLUTION

Section Properties:

$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$



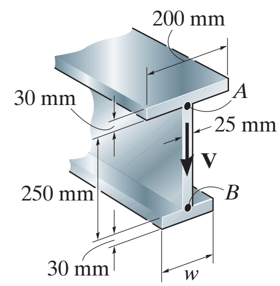
$$Q_{\max} = \Sigma \bar{y}A = 0.0625(0.125)(0.025) + 0.140(0.2)(0.030) = 1.0353(10)^{-3}\ \text{m}^3$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{30(10)^3(1.0353)(10)^{-3}}{268.652(10)^{-6}(0.025)} = 4.62\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 4.62$ MPa

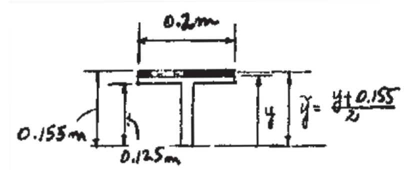
12–19.

If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the shear force resisted by the web of the beam. Set $w = 200$ mm.



SOLUTION

$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$



$$Q = \left(\frac{0.155 + y}{2}\right)(0.155 - y)(0.2) = 0.1(0.024025 - y^2)$$

$$\tau_f = \frac{30(10)^3(0.1)(0.024025 - y^2)}{268.652(10)^{-6}(0.2)}$$

$$V_f = \int \tau_f\, dA = 55.8343(10)^6 \int_{0.125}^{0.155} (0.024025 - y^2)(0.2\, dy)$$

$$= 11.1669(10)^6 \left[0.024025y - \frac{1}{3}y^3 \right]_{0.125}^{0.155}$$

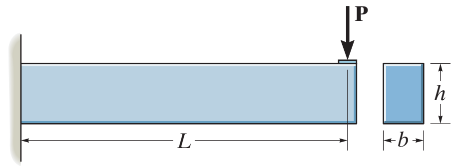
$$V_f = 1.457\ \text{kN}$$

$$V_w = 30 - 2(1.457) = 27.1\ \text{kN}$$ **Ans.**

Ans:
$V_w = 27.1$ kN

***12–20.**

Determine the length of the cantilevered beam so that the maximum bending stress in the beam is equivalent to the maximum shear stress.



SOLUTION

$V_{\max} = P$

$M_{\max} = PL$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{PL(h/2)}{I} = \frac{PLh}{2I}$$

$$\tau_{\max} = \frac{VQ}{It} = \frac{P(h/2)(b)(h/4)}{Ib} = \frac{Ph^2}{8I}$$

Require

$$\sigma_{\max} = \tau_{\max}$$

$$\frac{PLh}{2I} = \frac{Ph^2}{8I}$$

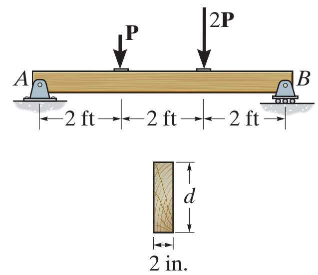
$$L = \frac{h}{4}$$ **Ans.**

Ans:

$L = \dfrac{h}{4}$

12–21.

If the beam is made from wood having an allowable shear stress $\tau_{\text{allow}} = 400$ psi, determine the maximum magnitude of **P**. Set $d = 4$ in.



SOLUTION

Support Reactions: As shown on the free-body diagram of the beam, Fig. *a*.

Maximum Shear: The shear diagram is shown in Fig. *b*. As indicated, $V_{\max} = 1.667P$.

Section Properties: The moment of inertia of the rectangular beam is

$$I = \frac{1}{12}(2)(4^3) = 10.667\ \text{in}^4$$

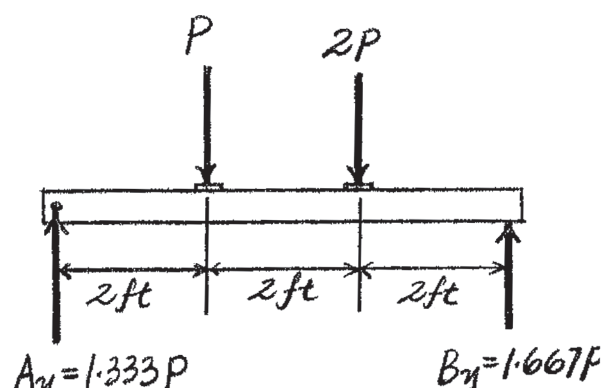
$Q_{\max}$ can be computed by taking the first moment of the shaded area in Fig. *c* about the neutral axis.

$$Q_{\max} = \bar{y}'A' = 1(2)(2) = 4\ \text{in}^3$$

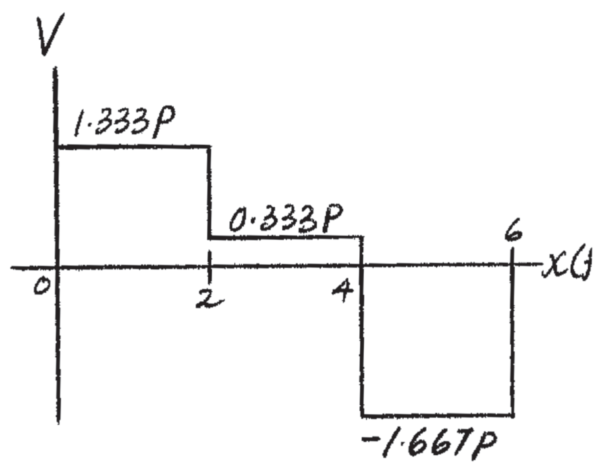
Shear Stress: The maximum shear stress occurs at points on the neutral axis since Q is maximum and the thickness $t = 2$ in. is constant.

$$\tau_{\text{allow}} = \frac{V_{\max} Q_{\max}}{It}; \qquad 400 = \frac{1.667P(4)}{10.667(2)}$$

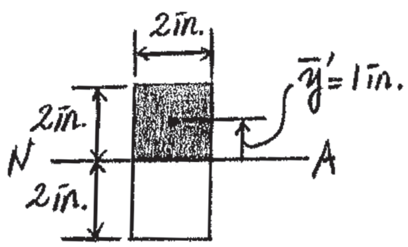
$$P_{\max} = 1280\ \text{lb} = 1.28\ \text{kip} \qquad \textbf{Ans.}$$



(a)



(b)

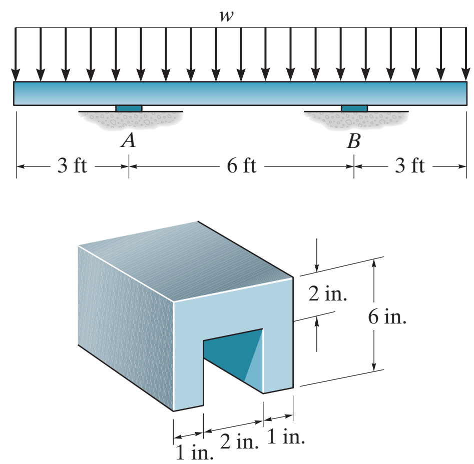


(C)

Ans:
$P_{\max} = 1.28$ kip

12–22.

Determine the largest intensity w of the distributed load that the member can support if the allowable shear stress is $\tau_{\text{allow}} = 800$ psi. The supports at A and B are smooth.



SOLUTION

Internal Shear Force: As indicated on the shear diagram, Fig. *a*, $V_{\max} = 3.00\,w$.

Section Properties: The location of centroid of the cross-section must be determined first:

$$\bar{y} = \frac{2(4)(2) + 5(2)(4)}{4(2) + 2(4)} = 3.50 \text{ in.}$$

The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(2)(4^3) + 2(4)(3.5 - 2)^2 + \frac{1}{12}(4)(2^3) + 4(2)(5 - 3.50)^2$$
$$= 49.33 \text{ in}^4$$

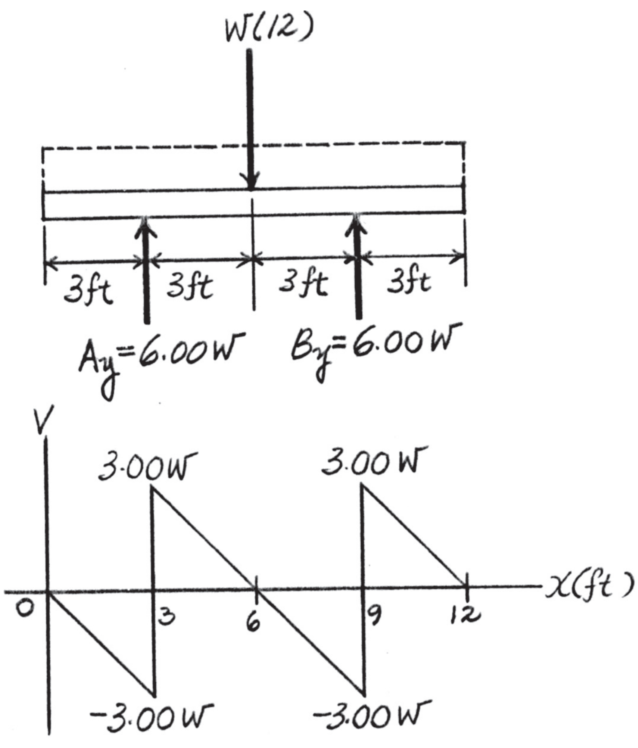
The maximum shear stress occurs at the points where the neutral axis passes through the cross-section. Referring to Fig. *b*,

$$Q_{\max} = \bar{y}'A' = 1.75[2(3.50)] = 12.25 \text{ in}^3$$

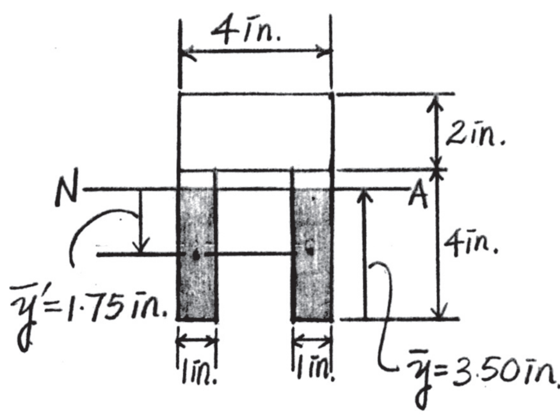
Allowable Shear Stress: Applying the shear formula,

$$\tau_{\max} = \tau_{\text{allow}} = \frac{V_{\max}Q_{\max}}{It}; \qquad 800 = \frac{3.00\,w\,(12.25)}{49.33\,(2)}$$

$$w_{\max} = 2147.85 \text{ lb/ft} = 2.15 \text{ kip/ft} \qquad \textbf{Ans.}$$



(a)

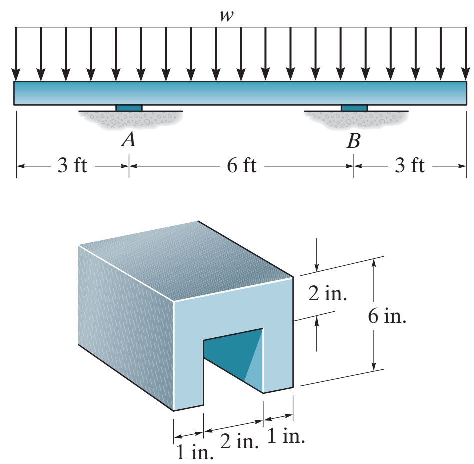


(b)

Ans:
$w_{\max} = 2.15$ kip/ft

12–23.

If $w = 800$ lb/ft, determine the absolute maximum shear stress in the beam. The supports at A and B are smooth.



SOLUTION

Internal Shear Force: As indicated on the shear diagram, Fig. *a*, $V_{max} = 2400$ lb.

Section Properties: The location of centroid of the cross-section must be determined first:

$$\bar{y} = \frac{2(4)(2) + 5(2)(4)}{4(2) + 2(4)} = 3.50 \text{ in.}$$

The moment of inertia of the cross-section about the neutral axis is

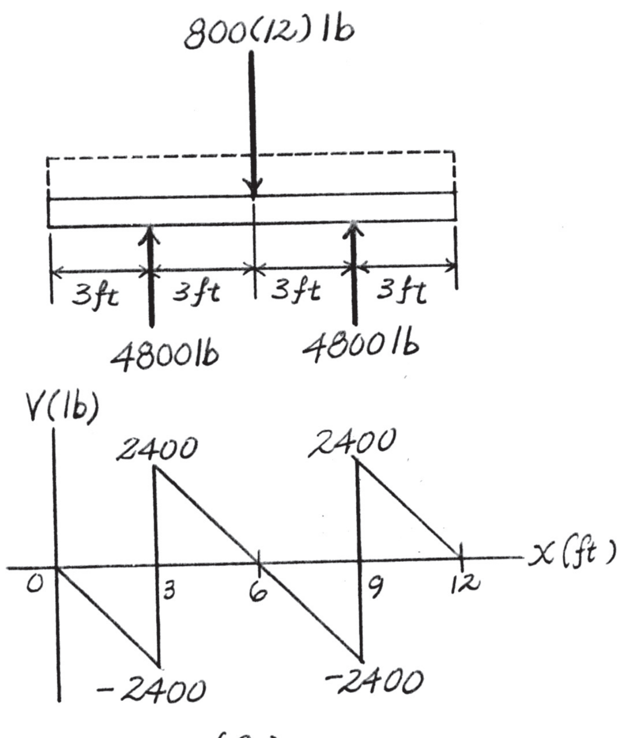
$$I = \frac{1}{12}(2)(4^3) + 2(4)(3.50 - 2)^2 + \frac{1}{12}(4)(2^3) + 4(2)(5 - 3.50)^2$$
$$= 49.33 \text{ in}^4$$

The maximum shear stress occurs at the points on the neutral plane where it passes through the cross-section. Referring to Fig. *b*,

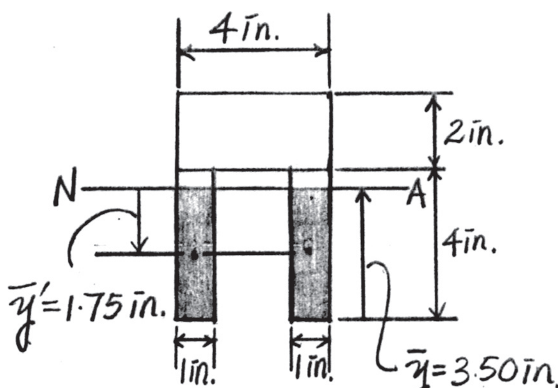
$$Q_{max} = \bar{y}'A' = 1.75[2(3.50)] = 12.25 \text{ in}^3$$

Maximum Shear Stress: Applying the shear formula,

$$\tau_{max} = \frac{V_{max}Q_{max}}{It} = \frac{2400(12.25)}{49.33(2)} = 297.97 \text{ psi} = 298 \text{ psi} \qquad \textbf{Ans.}$$



(a)

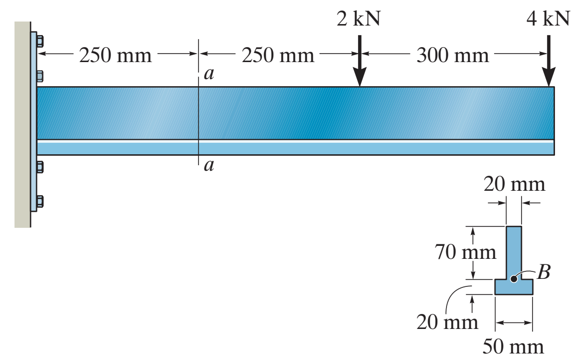


(b)

Ans:
$\tau_{max} = 298$ psi

***12–24.**

Determine the shear stress at point B on the web of the cantilevered strut at section a–a.

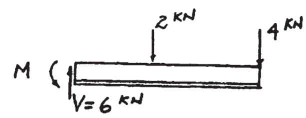


SOLUTION

$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$

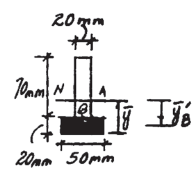
$$I = \frac{1}{12}(0.05)\left(0.02^3\right) + (0.05)(0.02)(0.03625 - 0.01)^2$$

$$+ \frac{1}{12}(0.02)\left(0.07^3\right) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625\left(10^{-6}\right) \text{m}^4$$



$$\bar{y}'_B = 0.03625 - 0.01 = 0.02625 \text{ m}$$

$$Q_B = (0.02)(0.05)(0.02625) = 26.25\left(10^{-6}\right) \text{m}^3$$



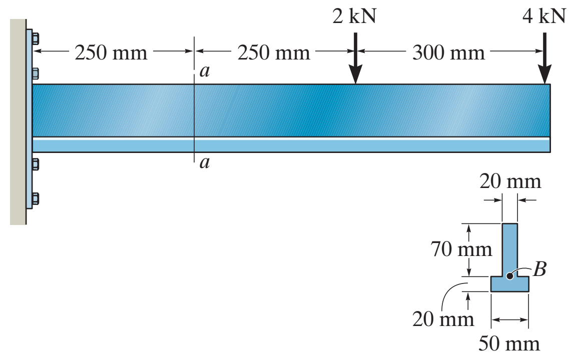
$$\tau_B = \frac{VQ_B}{It} = \frac{6\left(10^3\right)(26.25)\left(10^{-6}\right)}{1.78622\left(10^{-6}\right)(0.02)}$$

$$= 4.41 \text{ MPa}$$ **Ans.**

Ans:
$\tau_B = 4.41$ MPa

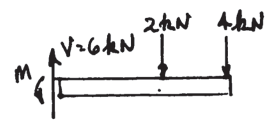
12–25.

Determine the maximum shear stress acting at section *a–a* of the cantilevered strut.



SOLUTION

$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$



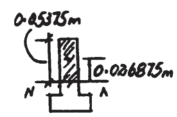
$$I = \frac{1}{12}(0.05)\left(0.02^3\right) + (0.05)(0.02)(0.03625 - 0.01)^2$$

$$+ \frac{1}{12}(0.02)\left(0.07^3\right) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625\left(10^{-6}\right) \text{m}^4$$

$$Q_{\max} = \bar{y}'A' = (0.026875)(0.05375)(0.02) = 28.8906\left(10^{-6}\right) \text{m}^3$$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{6\left(10^3\right)(28.8906)\left(10^{-6}\right)}{1.78625\left(10^{-6}\right)(0.02)}$$

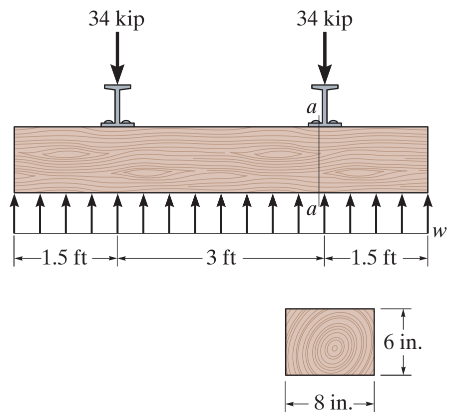
$$= 4.85 \text{ MPa}$$ **Ans.**



Ans:
$\tau_{\max} = 4.85$ MPa

12–26.

Railroad ties must be designed to resist large shear loadings. If the tie is subjected to the 34-kip rail loadings and an assumed uniformly distributed ground reaction, determine the intensity w for equilibrium, and calculate the maximum shear stress in the tie at section a–a, which is located just to the left of the rail.



SOLUTION

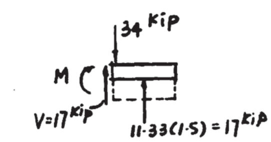
$+\uparrow \Sigma F_y = 0; \quad 6w - 2(34) = 0$

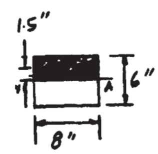
$w = 11.3 \text{ kip/ft}$ **Ans.**

$$I = \frac{1}{12}(8)\left(6^3\right) = 144 \text{ in}^4$$

$$Q_{\max} = \bar{y}'A' = 1.5(3)(8) = 36 \text{ in}^3$$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{17\left(10^3\right)(36)}{144(8)} = 531 \text{ psi}$$ **Ans.**

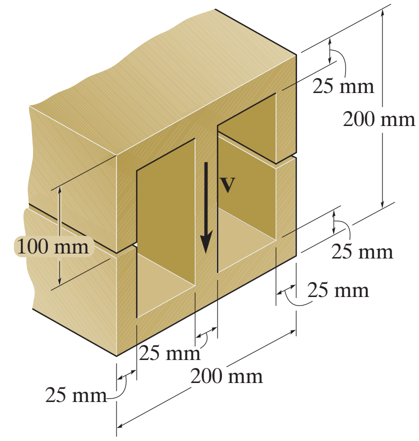




Ans:
$w = 11.3 \text{ kip/ft},$
$\tau_{\max} = 531 \text{ psi}$

12–27.

The beam is slit longitudinally along both sides. If it is subjected to a shear of $V = 250$ kN, compare the maximum shear stress in the beam before and after the cuts were made.



SOLUTION

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.125)(0.15^3) = 98.1771(10^{-6})\ \text{m}^4$$

$Q_{\max}$ is the first moment of the shaded area shown in Fig. *a* about the neutral axis. Thus,

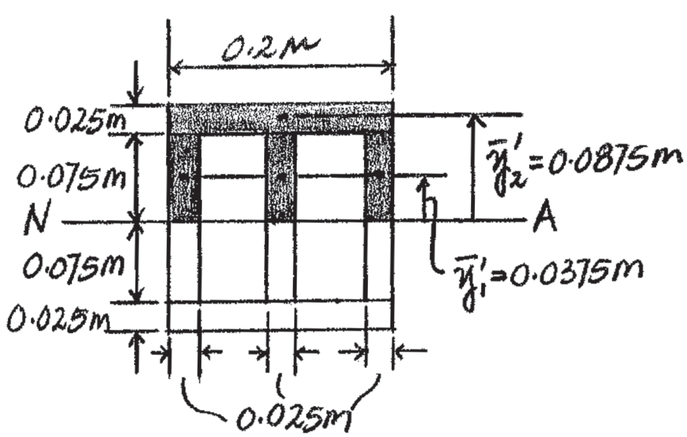
$$\begin{aligned} Q_{\max} &= 3\bar{y}'_1 A'_1 + \bar{y}'_2 A'_2 \\ &= 3\left[0.0375(0.075)(0.025)\right] + 0.0875(0.025)(0.2) \\ &= 0.6484375(10^{-3})\ \text{m}^3 \end{aligned}$$

Maximum Shear Stress: The maximum shear stress occurs at the points on the neutral axis since Q is maximum and t is minimum. Before the cross section is slit, $t = 3(0.025) = 0.075$ m.

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{250(10^3)(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.075)} = 22.0\ \text{MPa} \qquad \textbf{Ans.}$$

After the cross section is slit, $t = 0.025$ m.

$$(\tau_{\max})_s = \frac{VQ_{\max}}{It} = \frac{250(10^3)(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.025)} = 66.0\ \text{MPa} \qquad \textbf{Ans.}$$

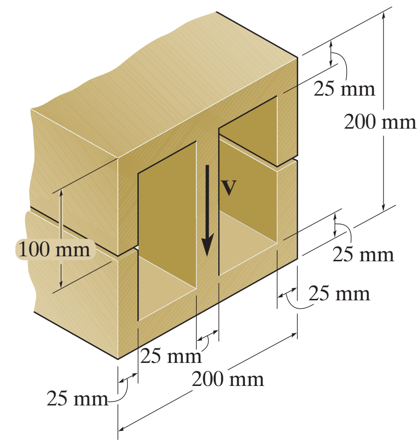


(a)

Ans:
$\tau_{\max} = 22.0$ MPa, $(\tau_{\max})_s = 66.0$ MPa

***12–28.**

The beam is to be cut longitudinally along both sides as shown. If it is made from a material having an allowable shear stress of $\tau_{allow} = 75$ MPa, determine the maximum allowable shear force **V** that can be applied before and after the cut is made.



SOLUTION

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.2)(0.2^3) - \frac{1}{12}(0.125)(0.15^3) = 98.1771(10^{-6})\text{ m}^4$$

Q_{max} is the first moment of the shaded area shown in Fig. *a* about the neutral axis. Thus,

$$\begin{aligned} Q_{max} &= 3\bar{y}_1'A_1' + \bar{y}_2'A_2' \\ &= 3(0.0375)(0.075)(0.025) + 0.0875(0.025)(0.2) \\ &= 0.6484375(10^{-3})\text{ m}^3 \end{aligned}$$

Shear Stress: The maximum shear stress occurs at the points on the neutral axis since Q is maximum and thickness t is minimum. Before the cross section is slit, $t = 3(0.025) = 0.075$ m.

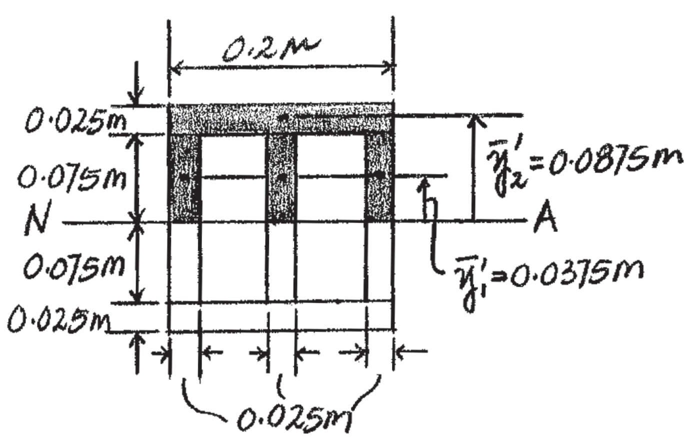
$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 75(10^6) = \frac{V(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.075)}$$

$$V = 851\,656.63\text{ N} = 852\text{ kN} \qquad \textbf{Ans.}$$

After the cross section is slit, $t = 0.025$ m.

$$\tau_{allow} = \frac{VQ_{max}}{It}; \qquad 75(10^6) = \frac{V_s(0.6484375)(10^{-3})}{98.1771(10^{-6})(0.025)}$$

$$V_s = 283\,885.54\text{ N} = 284\text{ kN} \qquad \textbf{Ans.}$$

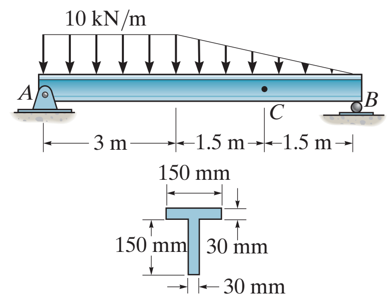


(a)

Ans:
$V = 852$ kN,
$V_s = 284$ kN

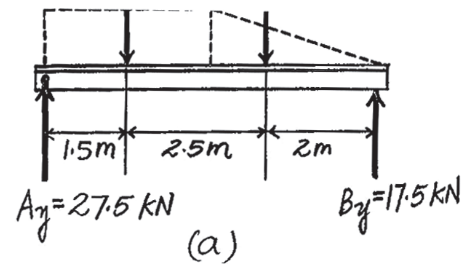
12–29.

Determine the maximum shear stress in the T-beam at the critical section where the internal shear force is maximum.

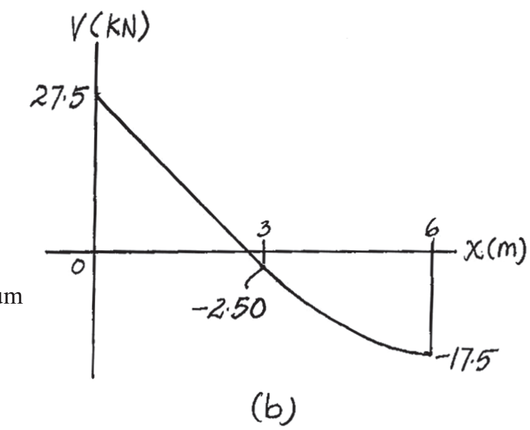


SOLUTION

The FBD of the beam is shown in Fig. *a*.



The shear diagram is shown in Fig. *b*. As indicated, $V_{max} = 27.5$ kN.



The neutral axis passes through centroid *c* of the cross section, Fig. *c*.

$$\bar{y} = \frac{\Sigma\, \tilde{y}\, A}{\Sigma A} = \frac{0.075(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

$$I = \frac{1}{12}(0.03)(0.15^3) + 0.03(0.15)(0.12 - 0.075)^2$$

$$+ \frac{1}{12}(0.15)(0.03^3) + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0(10^{-6}) \text{ m}^4$$

From Fig. *d*,

$$Q_{max} = \bar{y}'A' = 0.06(0.12)(0.03)$$

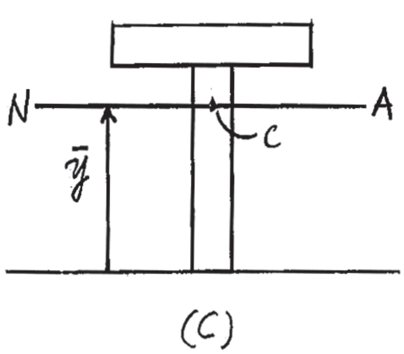
$$= 0.216\,(10^{-3}) \text{ m}^3$$

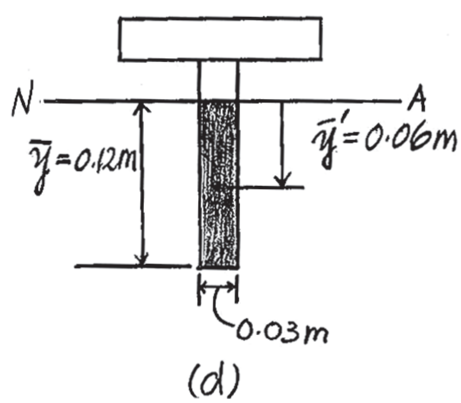
The maximum shear stress occurs at points on the neutral axis since Q is maximum and thickness $t = 0.03$ m is the smallest.

$$\tau_{max} = \frac{V_{max}Q_{max}}{It} = \frac{27.5(10^3)\left[0.216(10^{-3})\right]}{27.0(10^{-6})(0.03)}$$

$$= 7.333(10^6) \text{ Pa}$$

$$= 7.33 \text{ MPa}$$ **Ans.**

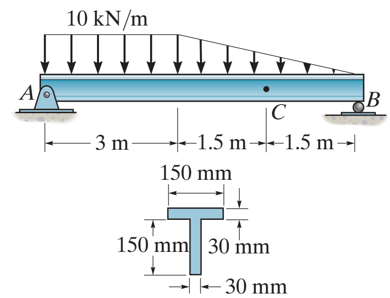




Ans:
$\tau_{max} = 7.33$ MPa

12–30.

Determine the maximum shear stress in the T-beam at section C. Show the result on a volume element at this point.

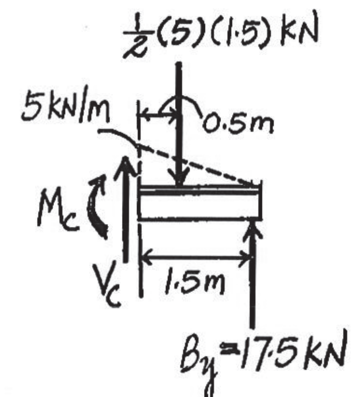


SOLUTION

Using the method of sections (Fig. *a*),

$$+\uparrow \Sigma F_y = 0; \qquad V_C + 17.5 - \frac{1}{2}(5)(1.5) = 0$$

$$V_C = -13.75 \text{ kN}$$



(a)

The neutral axis passes through centroid C of the cross section.

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.075\,(0.15)(0.03) + 0.165(0.03)(0.15)}{0.15(0.03) + 0.03(0.15)}$$

$$= 0.12 \text{ m}$$

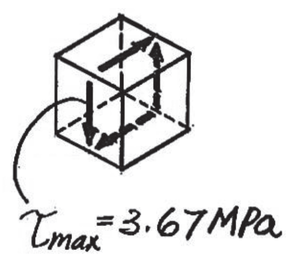
$$I = \frac{1}{12}(0.03)(0.15^3) + 0.03(0.15)(0.12 - 0.075)^2$$

$$+ \frac{1}{12}(0.15)(0.03^3) + 0.15(0.03)(0.165 - 0.12)^2$$

$$= 27.0\,(10^{-6}) \text{ m}^4$$

$$Q_{\max} = \bar{y}'A' = 0.06\,(0.12)(0.03)$$

$$= 0.216\,(10^{-3}) \text{ m}^3 \; 490$$



(b)

The maximum shear stress occurs at points on the neutral axis since Q is maximum and thickness $t = 0.03$ m is the smallest (Fig. *b*).

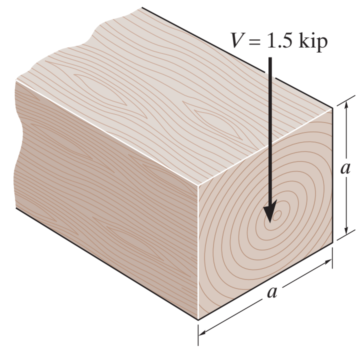
$$\tau_{\max} = \frac{V_C\,Q_{\max}}{It} = \frac{13.75(10^3)\left[0.216(10^{-3})\right]}{27.0(10^{-6})\,(0.03)}$$

$$= 3.667(10^6) \text{ Pa} = 3.67 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 3.67$ MPa

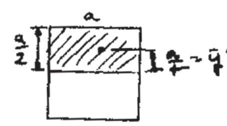
12–31.

The beam has a square cross section and is made of wood having an allowable shear stress of $\tau_{\text{allow}} = 1.4$ ksi. If it is subjected to a shear of $V = 1.5$ kip, determine the smallest dimension a of its sides.



SOLUTION

$$I = \frac{1}{12}a^4$$



$$Q_{\max} = \bar{y}'A' = \left(\frac{a}{4}\right)\left(\frac{a}{2}\right)a = \frac{a^3}{8}$$

$$\tau_{\max} = \tau_{\text{allow}} = \frac{VQ_{\max}}{It}$$

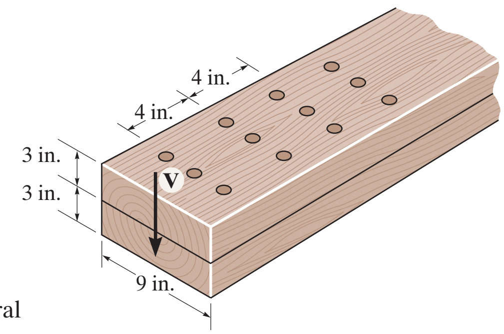
$$1.4 = \frac{1.5\left(\frac{a^3}{8}\right)}{\frac{1}{12}(a^4)(a)}$$

$a = 1.27$ in. **Ans.**

Ans:
$\alpha = 1.27$ in.

***12–32.**

The beam is constructed from two boards fastened together at the top and bottom with three rows of nails spaced every 4 in. If each nail can support a 400-lb shear force, determine the maximum shear force V that can be applied to the beam.



SOLUTION

Section Properties: The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(9)(6^3) = 162 \text{ in}^4$$

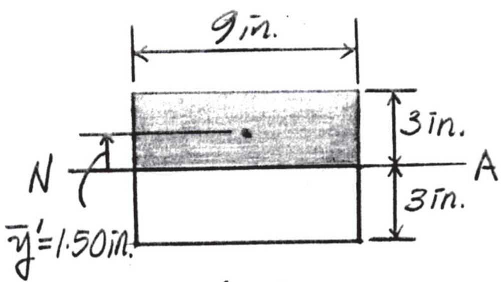
For the area shown shaded in Fig. a,

$$Q = \bar{y}'A' = 1.50[3(9)] = 40.5 \text{ in}^3$$

Shear Flow: There are three rows of nails. Hence, the allowable shear flow is $q_{\text{allow}} = 3\left(\frac{F}{s}\right) = 3\left(\frac{400}{4}\right) = 300 \text{ lb/in.}$ Applying the shear flow formula,

$$q_{\text{allow}} = \frac{VQ}{I}; \qquad 300 = \frac{V(40.5)}{162}$$

$$V = 1200 \text{ lb} \qquad \textbf{Ans.}$$

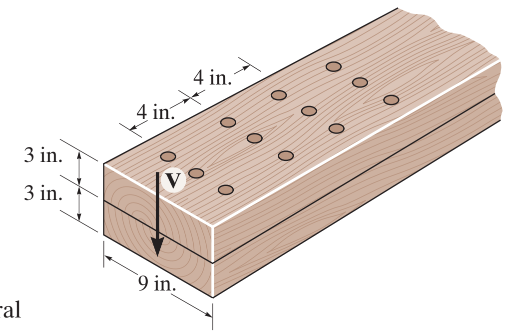


(a)

Ans:
$V = 1200$ lb

12–33.

The beam is constructed from two boards fastened together at the top and bottom with three rows of nails spaced every 4 in. If a shear force of $V = 900$ lb is applied to the boards, determine the shear force resisted by each nail.



SOLUTION

Section Properties: The moment of inertia of the cross-section about the neutral axis is

$$I = \frac{1}{12}(9)(6^3) = 162 \text{ in}^4$$

For the area shown shaded in Fig. *a*,

$$Q = \bar{y}'A' = 1.50[3(9)] = 40.5 \text{ in}^3$$

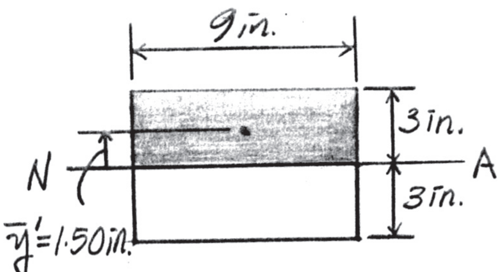
Shear Flow: There are three rows of nails. Hence, the total shear flow that can be resisted by the nails is

$$q = 3\left(\frac{F}{s}\right) = \frac{3F}{4}$$

Applying the shear flow formula,

$$q = \frac{VQ}{I}; \qquad \frac{3F}{4} = \frac{900(40.5)}{162}$$

$$F = 300 \text{ lb}$$ **Ans.**

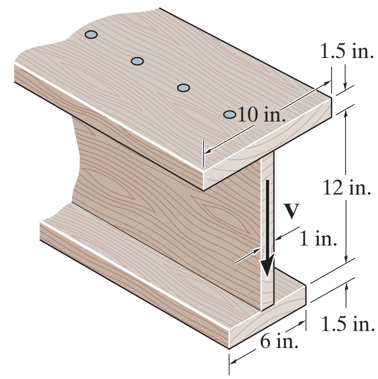


(a)

Ans:
$F = 300$ lb

12–34.

The beam is constructed from three boards. If it is subjected to a shear of $V = 5$ kip, determine the maximum allowable spacing s of the nails used to hold the top and bottom flanges to the web. Each nail can support a shear force of 500 lb.

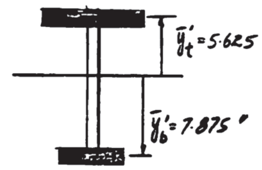


SOLUTION

$$\bar{y} = \frac{0.75(10)(1.5) + 7.5(12)(1) + 14.25(6)(1.5)}{10(1.5) + 12(1) + 6(1.5)} = 6.375 \text{ in.}$$

$$I = \frac{1}{12}(10)(1.5^3) + 10(1.5)(6.375 - 0.75)^2 + \frac{1}{12}(1)(12^3) + (1)(12)(7.5 - 6.375)^2$$

$$+ \frac{1}{12}(6)(1.5^3) + (1.5)(6)(14.25 - 6.375)^2 = 1196.4375 \text{ in}^4$$



$$Q_t = \bar{y}_t'A' = 5.625(10)(1.5) = 84.375 \text{ in}^3$$

$$Q_b = \bar{y}_b'A' = 7.875(6)(1.5) = 70.875 \text{ in}^3$$

$$q_t = \frac{VQ_t}{I} = \frac{5(10^3)(84.375)}{1196.4375} = 352.61 \text{ lb/in.}$$

$$q_b = \frac{VQ_b}{I} = \frac{5(10^3)(70.875)}{1196.4375} = 296.19 \text{ lb/in.}$$

$$F = q\,s; \quad s = \frac{F}{q}$$

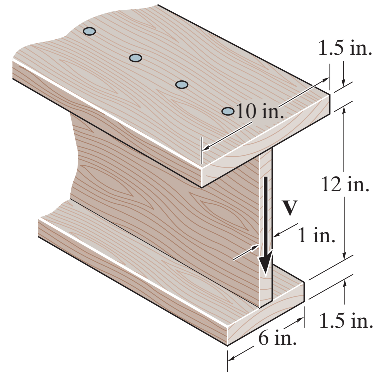
$$s_t = \frac{500}{352.61} = 1.42 \text{ in.}$$ **Ans.**

$$s_b = \frac{500}{296.19} = 1.69 \text{ in.}$$ **Ans.**

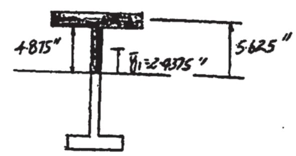
Ans:
$s_t = 1.42$ in.,
$s_b = 1.69$ in.

12–35.

The beam is constructed from three boards. Determine the maximum shear V that it can support if the allowable shear stress for the wood is $\tau_{\text{allow}} = 400$ psi. What is the maximum allowable spacing s of the nails if each nail can resist a shear force of 400 lb?



SOLUTION



$$\bar{y} = \frac{0.75(10)(1.5) + 7.5(12)(1) + 14.25(6)(1.5)}{10(1.5) + 12(1) + 6(1.5)} = 6.375 \text{ in.}$$

$$I = \frac{1}{12}(10)\left(1.5^3\right) + 10(1.5)(6.375 - 0.75)^2 + \frac{1}{12}(1)\left(12^3\right) + (1)(12)(7.5 - 6.375)^2$$

$$+ \frac{1}{12}(6)\left(1.5^3\right) + (1.5)(6)(14.25 - 6.375)^2 = 1196.4375 \text{ in}^4$$

$$Q_{\max} = \Sigma \bar{y}'A' = 5.625(10)(1.5) + 2.4375(4.875)(1) = 96.258 \text{ in}^3$$

$$\tau_{\max} = \tau_{\text{allow}} = \frac{VQ_{\max}}{It}$$

$$0.4 = \frac{V(96.258)}{1196.4375(1)}$$

$$V = 4.97 \text{ kip} \qquad \textbf{Ans.}$$

$$Q_t = \bar{y}_t'A' = 5.625(10)(1.5) = 84.375 \text{ in}^3$$

$$Q_b = \bar{y}_b'A' = 7.875(6)(1.5) = 70.875 \text{ in}^3$$

$$q_t = \frac{4.9718\left(10^3\right)(84.375)}{1196.4375} = 350.62 \text{ lb/in.}$$

$$q_b = \frac{4.9718\left(10^3\right)(70.875)}{1196.4375} = 294.52 \text{ lb/in.}$$

$$s = \frac{F}{q}$$

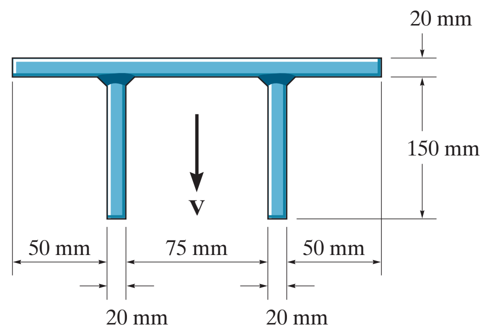
$$s_t = \frac{400}{350.62} = 1.14 \text{ in.} \qquad \textbf{Ans.}$$

$$s_b = \frac{400}{294.52} = 1.36 \text{ in.} \qquad \textbf{Ans.}$$

Ans:
$V = 4.97$ kip,
$s_t = 1.14$ in.,
$s_b = 1.36$ in.

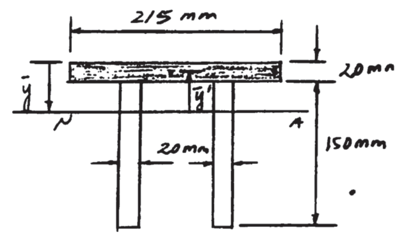
***12–36.**

The double T-beam is fabricated by welding the three plates together as shown. Determine the shear stress in the weld necessary to support a shear force of $V = 80$ kN.



SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.01(0.215)(0.02) + 2[0.095(0.15)(0.02)]}{0.215(0.02) + 2(0.15)(0.02)} = 0.059515 \text{ m}$$



$$I = \frac{1}{12}(0.215)\left(0.02^3\right) + 0.215(0.02)(0.059515 - 0.01)^2$$

$$+ 2\left[\frac{1}{12}(0.02)\left(0.15^3\right) + 0.02(0.15)(0.095 - 0.059515)^2\right] = 29.4909\left(10^{-6}\right) \text{ m}^4$$

$$\bar{y}' = 0.059515 - 0.01 = 0.049515 \text{ m}$$

$$Q = \bar{y}'A' = 0.049515(0.215)(0.02) = 0.2129\left(10^{-3}\right) \text{ m}^3$$

Shear stress:

$$\tau = \frac{VQ}{It} = \frac{80\left(10^3\right)(0.2129)\left(10^{-3}\right)}{29.4909\left(10^{-6}\right)(2)(0.02)}$$

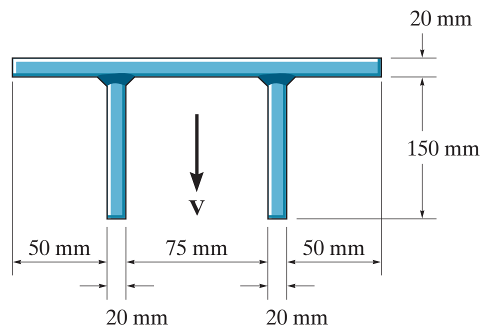
$$= 14.4 \text{ MPa}$$

Ans.

Ans:
$\tau = 14.4$ MPa

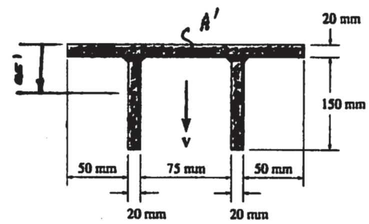
12–37.

The double T-beam is fabricated by welding the three plates together as shown. If the weld can resist a shear stress $\tau_{allow} = 90$ MPa, determine the maximum shear V that can be applied to the beam.



SOLUTION

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{0.01(0.215)(0.02) + 2[0.095(0.15)(0.02)]}{0.215(0.02) + 2(0.15)(0.02)} = 0.059515 \text{ m}$$



$$I = \frac{1}{12}(0.215)\left(0.02^3\right) + 0.215(0.02)(0.059515 - 0.01)^2$$

$$+ 2\left[\frac{1}{12}(0.02)\left(0.15^3\right) + 0.02(0.15)(0.095 - 0.059515)^2\right] = 29.4909\left(10^{-6}\right) \text{ m}^4$$

$$\bar{y}' = 0.059515 - 0.01 = 0.049515 \text{ m}$$

$$Q = \bar{y}'A' = 0.049515(0.215)(0.02) = 0.2129\left(10^{-3}\right) \text{ m}^3$$

$$\tau = \frac{VQ}{It}$$

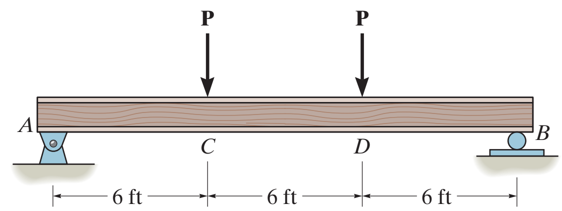
$$90\left(10^6\right) = \frac{V(0.2129)\left(10^{-3}\right)}{29.491\left(10^{-6}\right)(2)(0.02)}$$

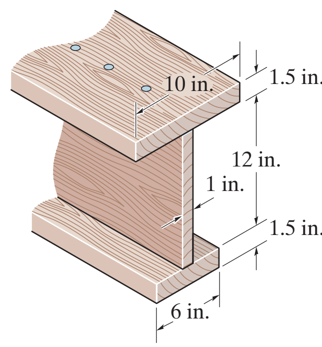
$$V = 499 \text{ kN}$$ **Ans.**

Ans:
$V = 499$ kN

12–38.

The beam is constructed from three boards. Determine the maximum loads P that it can support if the allowable shear stress for the wood is $\tau_{\text{allow}} = 400$ psi. What is the maximum allowable spacing s of the nails used to hold the top and bottom flanges to the web if each nail can resist a shear force of 400 lb?





SOLUTION

As shown on FDB,

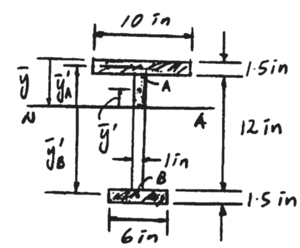
$$V_{\max} = P, V_{AC} = V_{DB} = P, V_{CD} = 0$$

$$\bar{y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{0.75(10)(1.5) + 7.5(12)(1) + 14.25(6)(1.5)}{10(1.5) + 12(1) + 6(1.5)} = 6.375 \text{ in.}$$

$$I = \frac{1}{12}(10)\left(1.5^3\right) + 10(1.5)(6.375 - 0.75)^2 + \frac{1}{12}(1)\left(12^3\right)$$

$$+ 1(12)(7.5 - 6.375)^2 + \frac{1}{12}(6)\left(1.5^3\right) + 6(1.5)\left(7.875^2\right)$$

$$= 1196.4375 \text{ in}^4$$

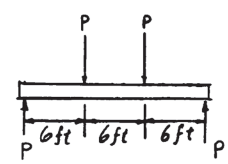


$$Q_A = \bar{y}'_A A' = 5.625(10)(1.5) = 84.375 \text{ in}^3$$

$$Q_B = \bar{y}'_B A' = 7.875(6)(1.5) = 70.875 \text{ in}^3$$

$$Q_{\max} = \Sigma\bar{y}'A' = 5.625(10)(1.5) + \frac{4.875}{2}(4.875)(1) = 96.2578 \text{ in}^3$$

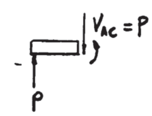
Maximum shear stress:



$$\tau_{\text{allow}} = \frac{VQ_{\max}}{It}; \qquad 400 = \frac{P(96.2578)}{1196.4375(1)}$$

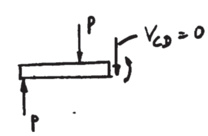
$P = 4971.8 \text{ lb} = 4.97 \text{ kip}$ **Ans.**

For region AC and BD,



$$q_A = \frac{VQ_A}{I} = \frac{4971.8(84.375)}{1196.4375} = 350.62 \text{ lb/in.}$$

$$q_B = \frac{VQ_B}{I} = \frac{4971.8(70.875)}{1196.4375} = 294.52 \text{ lb/in.}$$



Nail spacing at top and bottom flange for regions AC and BD,

$s_{\text{top}} = \dfrac{F}{q_A} = \dfrac{400}{350.62} = 1.14$ in. (Regions AC and BD) **Ans.**

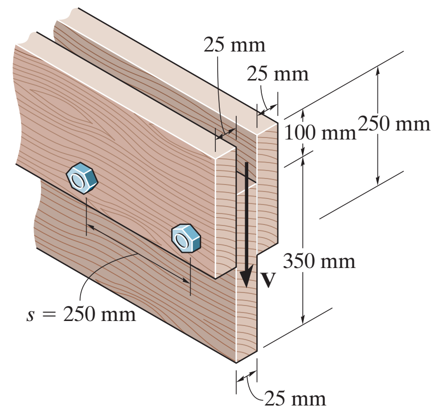
$s_{\text{bottom}} = \dfrac{F}{q_B} = \dfrac{400}{294.52} = 1.36$ in. (Regions AC and BD) **Ans.**

For region CD, theoretically no nails are required to hold the flange and web together since $V_{CD} = 0$; however, it is advisible to provide some nails within this region.

Ans:
$P = 4.97$ kip.
For regions AC and BD,
$s_{\text{top}} = 1.14$ in., $s_{\text{bottom}} = 1.36$ in.
For region CD, theoretically no nails are required.

12–39.

A beam is constructed from three boards bolted together as shown. Determine the shear force in each bolt if the bolts are spaced $s = 250$ mm apart and the shear is $V = 35$ kN.



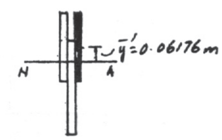
SOLUTION

$$\bar{y} = \frac{2\,(0.125)(0.25)(0.025) + 0.275\,(0.35)(0.025)}{2\,(0.25)(0.025) + 0.35\,(0.025)} = 0.18676 \text{ m}$$

$$I = (2)\left(\frac{1}{12}\right)(0.025)(0.25^3) + 2\,(0.025)(0.25)(0.18676 - 0.125)^2$$

$$+ \frac{1}{12}(0.025)(0.35)^3 + (0.025)(0.35)(0.275 - 0.18676)^2$$

$$= 0.270236\,(10^{-3})\text{ m}^4$$



$$Q = \bar{y}'A' = 0.06176(0.025)(0.25) = 0.386(10^{-3})\text{ m}^3$$

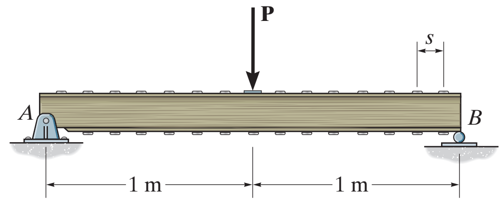
$$q = \frac{VQ}{I} = \frac{35(0.386)(10^{-3})}{0.270236(10^{-3})} = 49.997 \text{ kN/m}$$

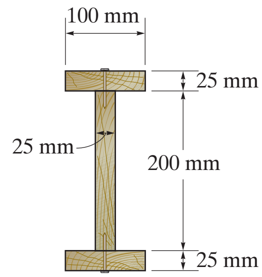
$$F = q(s) = 49.997\,(0.25) = 12.5 \text{ kN}$$ **Ans.**

Ans:
$F = 12.5$ kN

***12–40.**

The simply supported beam is built up from three boards by nailing them together as shown. The wood has an allowable shear stress of $\tau_{\text{allow}} = 1.5$ MPa, and an allowable bending stress of $\sigma_{\text{allow}} = 9$ MPa. The nails are spaced at $s = 75$ mm, and each has a shear strength of 1.5 kN. Determine the maximum allowable force **P** that can be applied to the beam.





SOLUTION

Support Reactions: As shown on the free-body diagram of the beam shown in Fig. *a*.

Maximum Shear and Moment: The shear diagram is shown in Fig. *b*. As indicated, $V_{\max} = \dfrac{P}{2}$.

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.25^3) - \frac{1}{12}(0.075)(0.2^3)$$

$$= 80.2083(10^{-6})\text{ m}^4$$

Referring to Fig. *d*,

$$Q_B = \bar{y}_2' A_2' = 0.1125(0.025)(0.1) = 0.28125(10^{-3})\text{ m}^3$$

Shear Flow: Since there is only one row of nails, $q_{\text{allow}} = \dfrac{F}{s} = \dfrac{1.5(10^3)}{0.075} = 20(10^3)\text{ N/m}$.

$$q_{\text{allow}} = \frac{V_{\max} Q_B}{I}; \qquad 20(10^3) = \frac{\frac{P}{2}\left[0.28125(10^{-3})\right]}{80.2083(10^{-6})}$$

$$P = 11417.41\text{ N} = 11.4\text{ kN (controls)} \qquad \textbf{Ans.}$$

Bending,

$$\sigma_{\max} = \frac{Mc}{I}$$

$$\sigma(10^6)\text{ N/m}^2 = \frac{\left(\frac{P}{2}\right)(0.125\text{ m})}{80.2083(10^{-6})\text{ m}^4}$$

$$P = 11.550\text{ N} = 11.55\text{ kN}$$

Shear,

$$\tau_{\max} = \frac{VQ}{It}$$

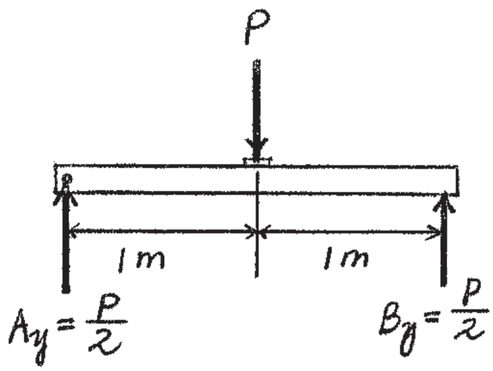
$$Q = (0.1125)(0.025)(0.1) + (0.05)(0.1)(0.025)$$

$$= 0.40625(10^{-3})\text{ m}^3$$

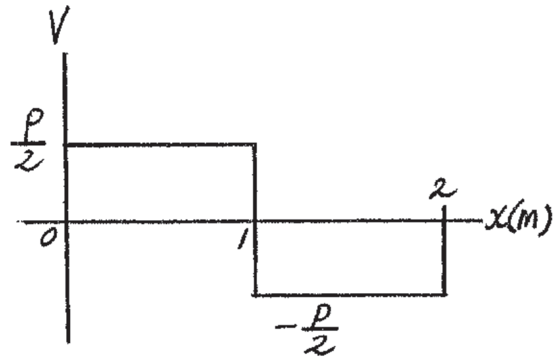
$$1.5(10^6) = \frac{\left(\frac{P}{2}\right)(0.40625)(10^{-3})\text{ m}^3}{80.2083(10^{-6})\text{ m}^4(0.025\text{ m})}$$

$$P = 14.808\text{ N} = 14.8\text{ kN}$$

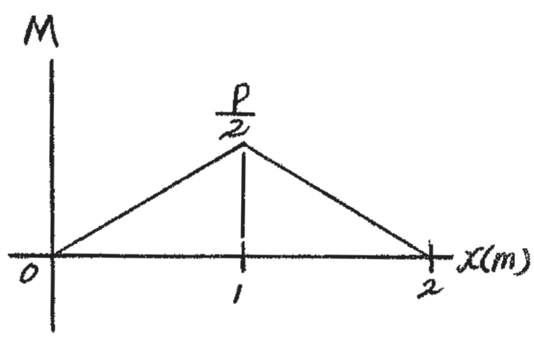
***12–40. Continued**



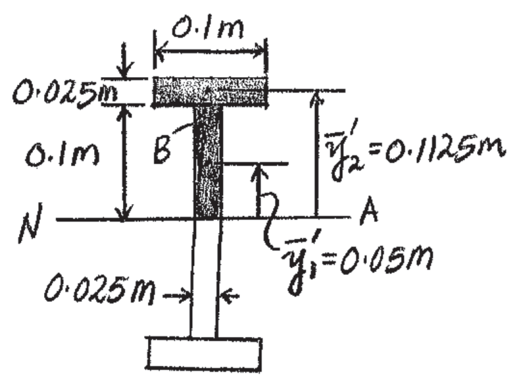
(a)



(b)



(c)

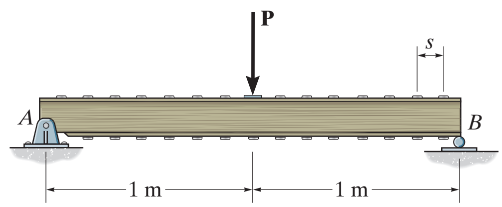


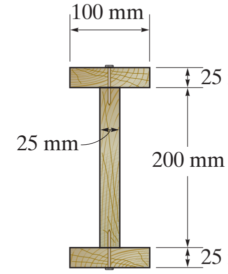
(d)

Ans:
$P = 11.4$ kN

12–41.

The simply supported beam is built up from three boards by nailing them together as shown. If $P = 12$ kN, determine the maximum allowable spacing s of the nails to support that load, if each nail can resist a shear force of 1.5 kN.





SOLUTION

Support Reactions: As shown on the free-body diagram of the beam shown in Fig. a.

Maximum Shear and Moment: The shear diagram is shown in Fig. b. As indicated,

$$V_{\max} = \frac{P}{2} = \frac{12}{2} = 6 \text{ kN.}$$

Section Properties: The moment of inertia of the cross section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.25^3) - \frac{1}{12}(0.075)(0.2^3)$$
$$= 80.2083(10^{-6}) \text{ m}^4$$

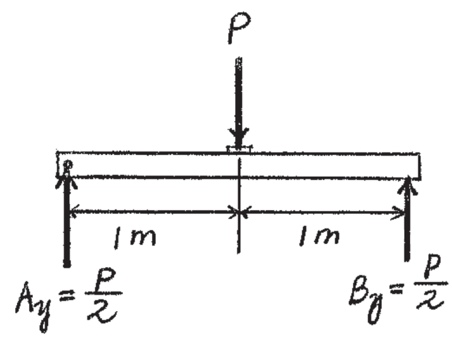
Referring to Fig. d,

$$Q_B = \bar{y}'_2 A'_2 = 0.1125(0.025)(0.1) = 0.28125(10^{-3}) \text{ m}^3$$

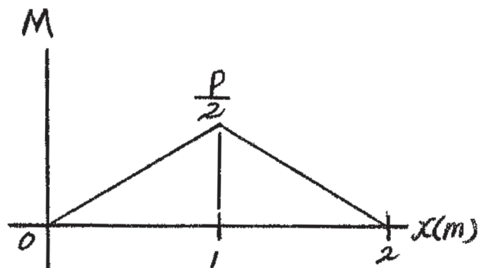
Shear Flow: Since there is only one row of nails, $q_{\text{allow}} = \dfrac{F}{s} = \dfrac{1.5(10^{3}1)}{s}$.

$$q_{\text{allow}} = \frac{V_{\max} Q_B}{I}; \qquad \frac{1.5(10^{3}1)}{s} = \frac{6000\left[0.28125(10^{-3})\right]}{80.2083(10^{-6}1)}$$

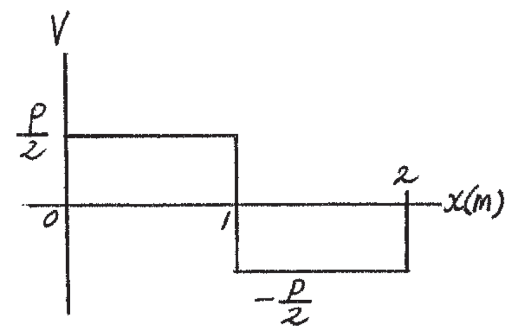
$$s = 0.07130 \text{ m} = 71.3 \text{ mm} \qquad \textbf{Ans.}$$



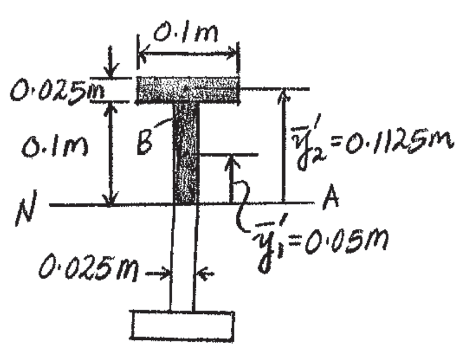
(a)



(c)



(b)

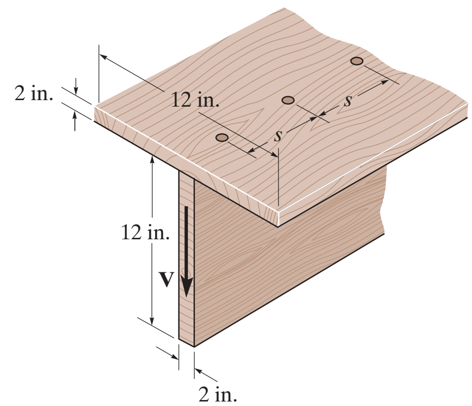


(d)

Ans:
$s = 71.3$ mm

12–42.

The T-beam is constructed as shown. If each nail can support a shear force of 950 lb, determine the maximum shear force V that the beam can support and the corresponding maximum nail spacing s to the nearest $\frac{1}{8}$ in. The allowable shear stress for the wood is $\tau_{\text{allow}} = 450$ psi.



SOLUTION

The neutral axis passes through the centroid c of the cross section as shown in Fig. a.

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{13(2)(12) + 6(12)(2)}{2(12) + 12(2)} = 9.5 \text{ in.}$$

$$I = \frac{1}{12}(2)(12^3) + 2(12)(9.5 - 6)^2$$

$$+ \frac{1}{12}(12)(2^3) + 12(2)(13 - 9.5)^2$$

$$= 884 \text{ in}^4$$

Referring to Fig. a, $Q_{\max}$ and Q_A are

$$Q_{\max} = \bar{y}_1' A_1' = 4.75(9.5)(2) = 90.25 \text{ in}^3$$

$$Q_A = \bar{y}_2' A_2' = 3.5\,(2)(12) = 84 \text{ in}^3$$

The maximum shear stress occurs at the points on the neutral axis where Q is maximum and $t = 2$ in.

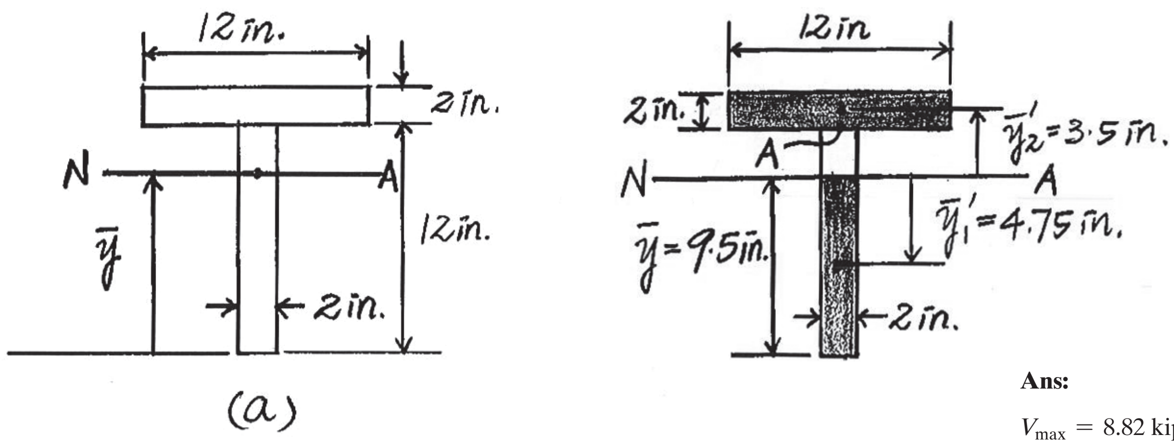
$$\tau_{\text{allow}} = \frac{VQ_{\max}}{It}; \qquad 450 = \frac{V\,(90.25)}{884\,(2)}$$

$$V = 8815.51 \text{ lb} = 8.82 \text{ kip} \qquad \textbf{Ans.}$$

Here, $q_{\text{allow}} = \frac{F}{s} = \frac{950}{s}$ lb/in. Then

$$q_{\text{allow}} = \frac{VQ_A}{I}; \qquad \frac{950}{s} = \frac{8815.51(84)}{884}$$

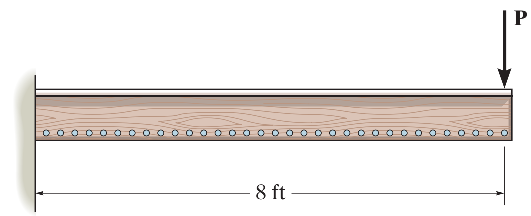
$$s = 1.134 \text{ in.} = 1\frac{1}{8} \text{ in.} \qquad \textbf{Ans.}$$



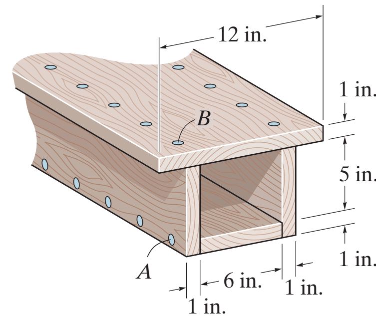
Ans:
$V_{\max} = 8.82$ kip, use $s = 1\frac{1}{8}$ in.

12–43.

The box beam is constructed from four boards that are fastened together using nails spaced along the beam every 2 in. If each nail can resist a shear force of 50 lb, determine the largest force P that can be applied to the beam without causing failure of the nails.



SOLUTION



Shear Force: $V = P$

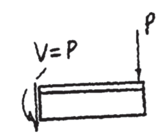
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.5(12)(1) + 2[4(6)(1)] + 6.5(6)(1)}{12(1) + 2(6)(1) + 6(1)} = 3.10 \text{ in.}$$

$$I = \frac{1}{12}(12)(1^3) + 12(1)(3.10 - 0.5)^2 + 2\left[\frac{1}{12}(1)(6^3) + (1)(6)(4 - 3.10)^2\right]$$

$$+ \frac{1}{12}(6)(1^3) + 6(1)(6.5 - 3.10)^2 = 197.7 \text{ in}^4$$

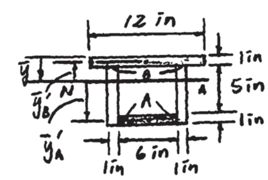
$$Q_B = \bar{y}_B{}'A' = (3.10 - 0.5)(12)(1) = 31.2 \text{ in}^3$$

$$Q_A = \bar{y}_A{}'A' = (6.5 - 3.10)(6)(1) = 20.4 \text{ in}^3$$



For *B*:

$$q_B = \frac{VQ_B}{I} = \frac{P(31.2)}{197.7} = 0.1578\,P, \qquad \text{however } q = \frac{2(50)}{2} = 50 \text{ lb/in.}$$



$50 = 0.1578\,P$

$P = 317$ lb (controls) **Ans.**

For *A*:

$$q_A = \frac{VQ_A}{I} = \frac{P(20.4)}{197.7} = 0.1032\,P \qquad \text{However } q = \frac{2(50)}{2} = 50 \text{ lb/in.}$$

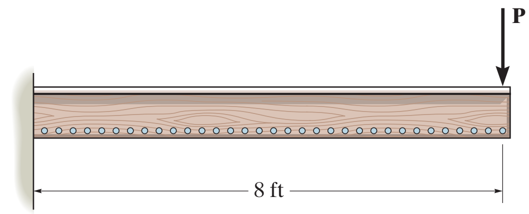
$50 = 0.1032\,P$

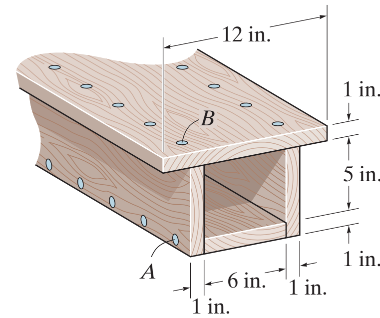
$P = 485$ lb

Ans:
$P_{\max} = 317$ lb

***12–44.**

The box beam is constructed from four boards that are fastened together using nails spaced along the beam every 2 in. If a force $P = 2$ kip is applied to the beam, determine the shear force resisted by each nail at A and B.





SOLUTION

As shown on FBD, $V_{\max} = 2$ kip.

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.5(12)(1) + 2[4(6)(1)] + 6.5(6)(1)}{12(1) + 2(6)(1) + 6(1)} = 3.10 \text{ in.}$$

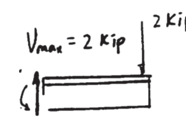
$$I = \frac{1}{12}(12)\left(1^3\right) + 12(1)(3.10 - 0.5)^2 + 2\left[\frac{1}{12}(1)\left(6^3\right) + (1)(6)(4 - 3.10)^2\right]$$

$$+ \frac{1}{12}(6)\left(1^3\right) + 6(1)(6.5 - 3.10)^2 = 197.7 \text{ in}^4$$

$$Q_B = \bar{y}_B{}'A' = (3.10 - 0.5)(12)(1) = 31.2 \text{ in}^3$$

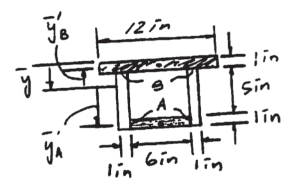
$$Q_A = \bar{y}_A{}'A' = (6.5 - 3.10)(6)(1) = 20.4 \text{ in}^3$$

$$V = P = 2 \text{ kip}$$



$$q_B = \frac{1}{2}\left(\frac{VQ_B}{I}\right) = \frac{1}{2}\left[\frac{2(31.2)}{197.7}\right] = 157.81 \text{ lb/in.}$$

$$q_A = \frac{1}{2}\left(\frac{VQ_A}{I}\right) = \frac{1}{2}\left[\frac{2(20.4)}{197.7}\right] = 103.19 \text{ lb/in.}$$



Shear force in nail:

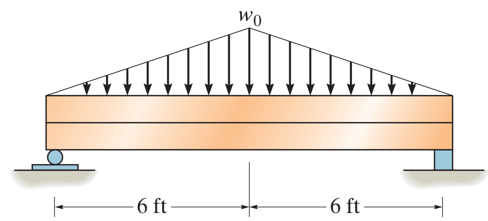
$F_B = q_B s = 157.81(2) = 316$ lb **Ans.**

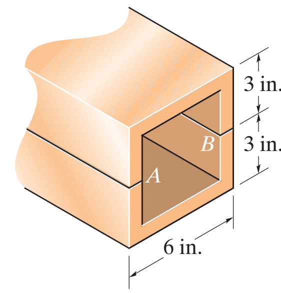
$F_A = q_A s = 103.19(2) = 206$ lb **Ans.**

Ans:
$F_B = 316$ lb,
$F_A = 206$ lb

12–45.

The member consists of two plastic channel strips 0.5 in. thick, glued together at A and B. If the distributed load has a maximum intensity of $w_0 = 3$ kip/ft, determine the maximum shear stress resisted by the glue.





SOLUTION

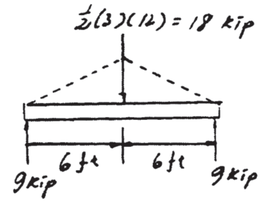
$V_{max} = 9$ kip

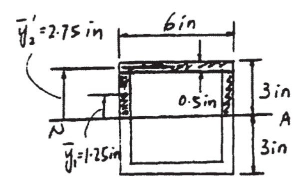
$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(5)(5^3) = 55.916 \text{ in}^4$$

$$Q = \Sigma \bar{y}'A' = 2[1.25(2.5)(0.5)] + 2.75(6)(0.5)$$

$$= 11.375 \text{ in}^3$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{9(11.375)}{55.916(1)} = 1.83 \text{ ksi}$$ **Ans.**

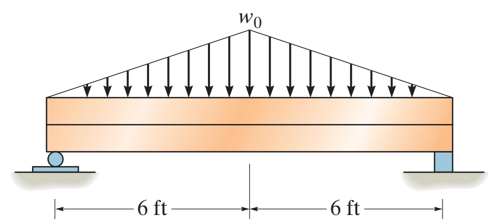


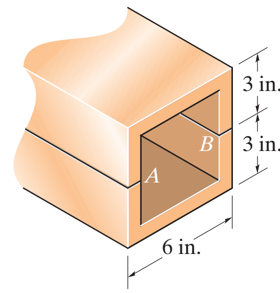


Ans:
$\tau_{max} = 1.83$ ksi

12–46.

The member consists of two plastic channel strips 0.5 in. thick, glued together at A and B. If the glue can support an allowable shear stress of $\tau_{\text{allow}} = 600$ psi, determine the maximum intensity w_0 of the triangular distributed loading that can be applied to the member based on the strength of the glue.





SOLUTION

Maximum shear force: $V_{\max} = 3w_0$

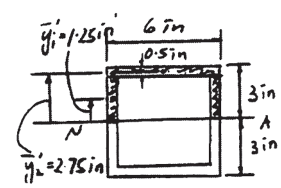
$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(5)(5^3) = 55.916 \text{ in}^4$$

$$Q = \Sigma \bar{y}'A' = 2[1.25(2.5)(0.5)] + 2.75(6)(0.5) = 11.375 \text{ in}^3$$

$$q = \tau_{\text{allow}}\, t = \frac{VQ}{I}$$

$$600(2)(0.5) = \frac{3w_0(11.375)}{55.916}$$

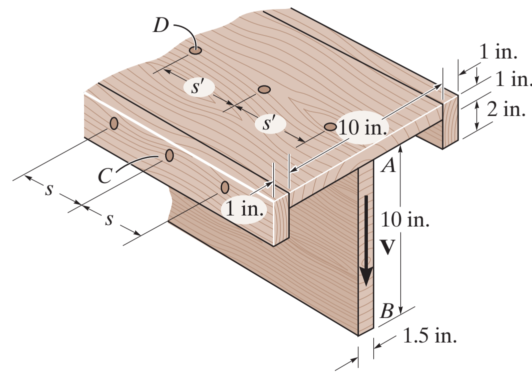
$$w_0 = 983 \text{ lb/ft}$$ **Ans.**



Ans:
$w_0 = 983$ lb/ft

12–47.

The beam is made from four boards nailed together as shown. If the nails can each support a shear force of 100 lb., determine their required spacing s' and s if the beam is subjected to a shear of $V = 700$ lb.



SOLUTION

Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.5(10)(1) + 1.5(2)(3) + 6(1.5)(10)}{10(1) + 2(3) + 1.5(10)}$$

$$= 3.3548 \text{ in}$$

$$I_{NA} = \frac{1}{12}(10)(1^3) + 10(1)(3.3548 - 0.5)^2$$

$$+ \frac{1}{12}(2)(3^3) + 2(3)(3.3548 - 1.5)^2$$

$$+ \frac{1}{12}(1.5)(10^3) + (1.5)(10)(6 - 3.3548)^2$$

$$= 337.43 \text{ in}^4$$

$$Q_C = \bar{y}_1' A' = 1.8548(3)(1) = 5.5645 \text{ in}^3$$

$$Q_D = \bar{y}_2' A' = (3.3548 - 0.5)(10)(1) + 2\left[(3.3548 - 1.5)(3)(1)\right] = 39.6774 \text{ in}^3$$

Shear Flow: The allowable shear flow at points C and D is $q_C = \dfrac{100}{s}$ and $q_B = \dfrac{100}{s'}$, respectively.

$$q_C = \frac{VQ_C}{I}$$

$$\frac{100}{s} = \frac{700(5.5645)}{337.43}$$

$$s = 8.66 \text{ in.}$$ **Ans.**

$$q_D = \frac{VQ_D}{I}$$

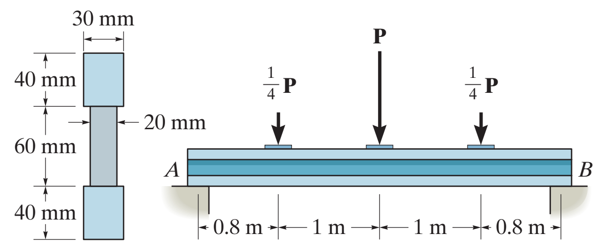
$$\frac{100}{s'} = \frac{700(39.6774)}{337.43}$$

$$s' = 1.21 \text{ in.}$$ **Ans.**

Ans:
$s = 8.66$ in., $s' = 1.21$ in.

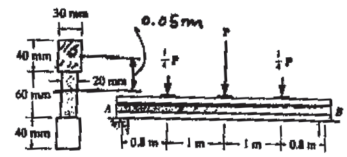
***12–48.**

The beam is made from three polystyrene strips that are glued together as shown. If the glue has a shear strength of 80 kPa, determine the maximum load P that can be applied without causing the glue to lose its bond.



SOLUTION

Maximum shear is at supports.



$$V_{\max} = \frac{3P}{4}$$

$$I = \frac{1}{12}(0.02)(0.06)^3 + 2\left[\frac{1}{12}(0.03)(0.04)^3 + (0.03)(0.04)(0.05)^2\right] = 6.68(10^{-6})\ \mathrm{m}^4$$

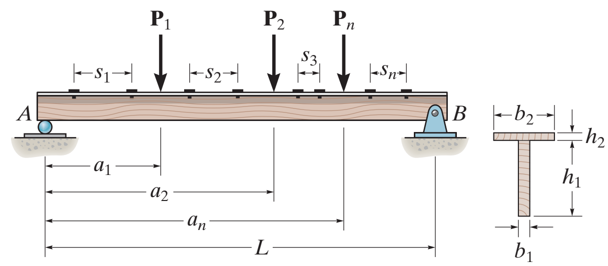
$$\tau = \frac{VQ}{It}; \qquad 80(10^3) = \frac{(3P/4)(0.05)(0.04)(0.03)}{6.68(10^{-6})(0.02)}$$

$$P = 238\ \mathrm{N} \qquad \textbf{Ans.}$$

Ans:
$P = 238$ N

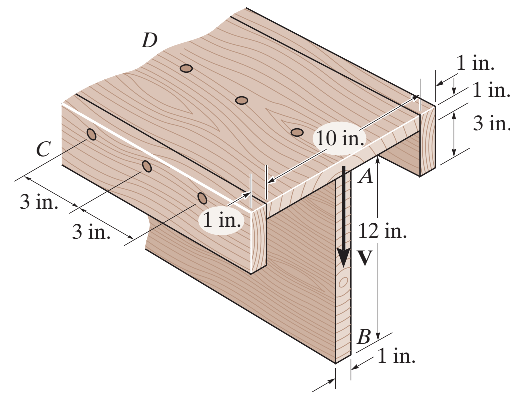
12–49.

The timber T-beam is subjected to a load consisting of n concentrated forces, P_n. If the allowable shear V_{nail} for each of the nails is known, write a computer program that will specify the nail spacing between each load. Show an application of the program using the values $L = 15$ ft, $a_1 = 4$ ft, $P_1 = 600$ lb, $a_2 = 8$ ft, $P_2 = 1500$ lb, $b_1 = 1.5$ in., $h_1 = 10$ in., $b_2 = 8$ in., $h_2 = 1$ in., and $V_{nail} = 200$ lb.



R12–1.

The beam is fabricated from four boards nailed together as shown. Determine the shear force each nail along the sides C and the top D must resist if the nails are uniformly spaced at $s = 3$ in. The beam is subjected to a shear of $V = 4.5$ kip.



SOLUTION

Section Properties:

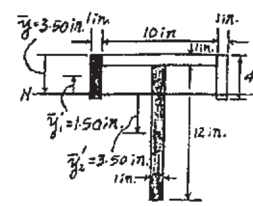
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(10)(1) + 2(4)(2) + 7(12)(1)}{10(1) + 4(2) + 12(1)} = 3.50 \text{ in.}$$

$$I_{NA} = \frac{1}{12}(10)(1^3) + (10)(1)(3.50 - 0.5)^2$$

$$+ \frac{1}{12}(2)(4^3) + 2(4)(3.50 - 2)^2$$

$$+ \frac{1}{12}(1)(12^3) + 1(12)(7-3.50)^2$$

$$= 410.5 \text{ in}^4$$



$$Q_C = \bar{y}_1' A' = 1.5(4)(1) = 6.00 \text{ in}^3$$

$$Q_D = \bar{y}_2' A' = 3.50(12)(1) = 42.0 \text{ in}^3$$

Shear Flow:

$$q_C = \frac{VQ_C}{I} = \frac{4.5(10^3)(6.00)}{410.5} = 65.773 \text{ lb/in.}$$

$$q_D = \frac{VQ_D}{I} = \frac{4.5(10^3)(42.0)}{410.5} = 460.41 \text{ lb/in.}$$

Hence, the shear force resisted by each nail is

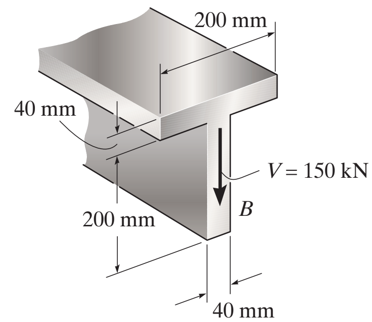
$F_C = q_C s = (65.773 \text{ lb/in.})(3 \text{ in.}) = 197 \text{ lb}$ **Ans.**

$F_D = q_D s = (460.41 \text{ lb/in.})(3 \text{ in.}) = 1.38 \text{ kip}$ **Ans.**

Ans:
$F_C = 197$ lb,
$F_D = 1.38$ kip

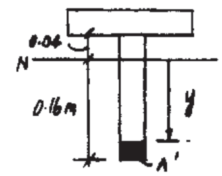
R12–2.

The T-beam is subjected to a shear of $V = 150$ kN. Determine the amount of this force that is supported by the web B.



SOLUTION

$$\bar{y} = \frac{(0.02)(0.2)(0.04) + (0.14)(0.2)(0.04)}{0.2(0.04) + 0.2(0.04)} = 0.08 \text{ m}$$



$$I = \frac{1}{12}(0.2)(0.04^3) + 0.2(0.04)(0.08 - 0.02)^2$$

$$+ \frac{1}{12}(0.04)(0.2^3) + 0.2(0.04)(0.14 - 0.08)^2 = 85.3333(10^{-6}) \text{ m}^4$$

$$A' = 0.04(0.16 - y)$$

$$\bar{y}' = y + \frac{(0.16 - y)}{2} = \frac{(0.16 + y)}{2}$$

$$Q = \bar{y}'A' = 0.02(0.0256 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{150(10^3)(0.02)(0.0256 - y^2)}{85.3333(10^{-6})(0.04)} = 22.5(10^6) - 878.9(10^6)y^2$$

$$V = \int \tau \, dA, \qquad dA = 0.04\, dy$$

$$V = \int_{-0.04}^{0.16} (22.5(10^6) - 878.9(10^6)y^2)\, 0.04\, dy$$

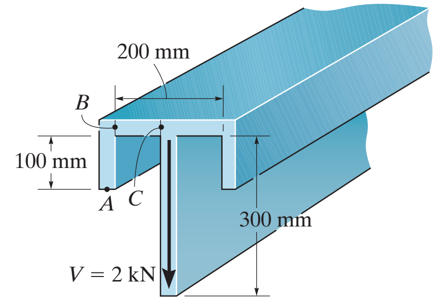
$$= \int_{-0.04}^{0.16} (900(10^3) - 35.156(10^6)y^2)\, dy$$

$$= 131\,250 \text{ N} = 131 \text{ kN}$$ **Ans.**

Ans:
$V = 131$ kN

R12–3.

The member is subjected to a shear force of $V = 2$ kN. Determine the shear flow at points A, B, and C. The thickness of each thin-walled segment is 15 mm.



SOLUTION

Section Properties:

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{0.0075(0.2)(0.015) + 0.0575(0.115)(0.03) + 0.165(0.3)(0.015)}{0.2(0.015) + 0.115(0.03) + 0.3(0.015)}$$

$$= 0.08798 \text{ m}$$

$$I_{NA} = \frac{1}{12}(0.2)\left(0.015^3\right) + 0.2(0.015)(0.08798 - 0.0075)^2$$

$$+ \frac{1}{12}(0.03)\left(0.115^3\right) + 0.03(0.115)(0.08798 - 0.0575)^2$$

$$+ \frac{1}{12}(0.015)\left(0.3^3\right) + 0.015(0.3)(0.165 - 0.08798)^2$$

$$= 86.93913\left(10^{-6}\right) \text{ m}^4$$

$$Q_A = 0$$

$$Q_B = \tilde{y}'_1 A' = 0.03048(0.115)(0.015) = 52.57705\left(10^{-6}\right) \text{ m}^3$$

$$Q_C = \Sigma \bar{y}' A'$$

$$= 0.03048(0.115)(0.015) + 0.08048(0.0925)(0.015)$$

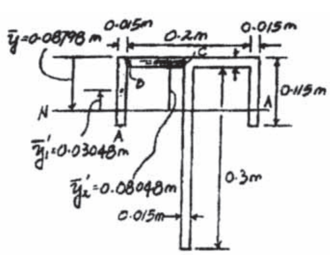
$$= 0.16424\left(10^{-3}\right) \text{ m}^3$$

Shear Flow:

$$q_A = \frac{VQ_A}{I} = 0 \qquad \textbf{Ans.}$$

$$q_B = \frac{VQ_B}{I} = \frac{2(10^3)(52.57705)(10^{-6})}{86.93913(10^{-6})} = 1.21 \text{ kN/m} \qquad \textbf{Ans.}$$

$$q_C = \frac{VQ_C}{I} = \frac{2(10^3)(0.16424)(10^{-3})}{86.93913(10^{-6})} = 3.78 \text{ kN/m} \qquad \textbf{Ans.}$$

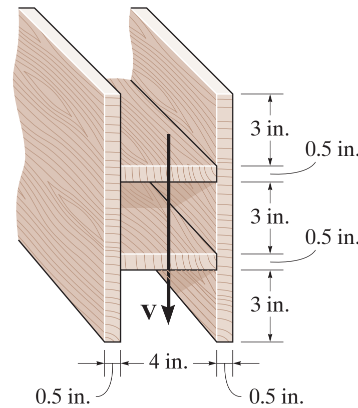


Ans

Ans:
$q_A = 0,$
$q_B = 1.21$ kN/m,
$q_C = 3.78$ kN/m

***R12–4.**

The beam is constructed from four boards glued together at their seams. If the glue can withstand 75 lb/in., what is the maximum vertical shear V that the beam can support? What is the maximum vertical shear V that the beam can support if it is rotated 90° from the position shown?



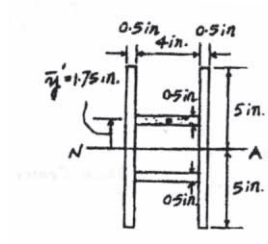
SOLUTION

Section Properties:

$$I_{NA} = \frac{1}{12}(1)\left(10^3\right) + 2\left[\frac{1}{12}(4)\left(0.5^3\right) + 4(0.5)\left(1.75^2\right)\right]$$

$$= 95.667 \text{ in}^4$$

$$Q = \bar{y}'A' = 1.75(4)(0.5) = 3.50 \text{ in}^3$$



Shear Flow: There are two glue joints in this case, hence the allowable shear flow is $2(75) = 150$ lb/in.

$$q = \frac{VQ}{I}$$

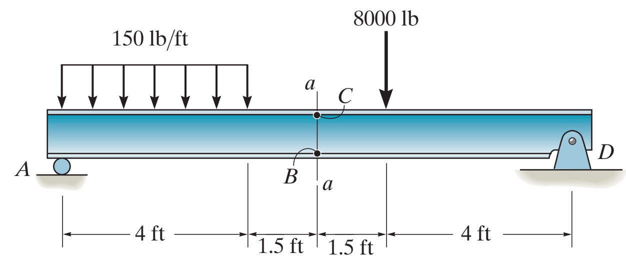
$$150 = \frac{V(3.50)}{95.667}$$

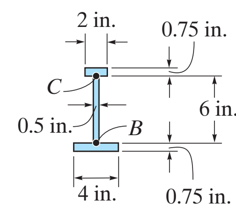
$$V = 4100 \text{ lb} = 4.10 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$V = 4.10$ kip

R12–5.

Determine the shear stress at points B and C on the web of the beam located at section a–a.





SOLUTION

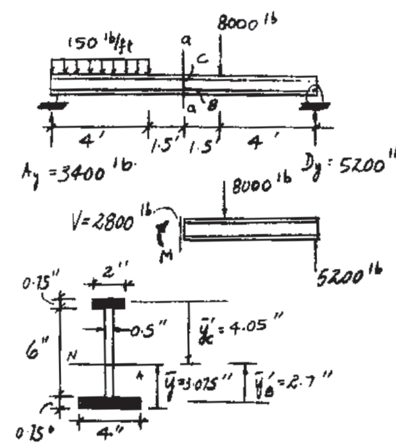
$$\bar{y} = \frac{(0.375)(4)(0.75) + (3.75)(6)(0.5) + (7.125)(2)(0.75)}{4(0.75) + 6(0.5) + 2(0.75)} = 3.075 \text{ in.}$$

$$I = \frac{1}{12}(4)(0.75^3) + 4(0.75)(3.075 - 0.375)^2$$

$$+ \frac{1}{12}(0.5)(6^3) + 0.5(6)(3.75 - 3.075)^2$$

$$+ \frac{1}{12}(2)(0.75^3) + 2(0.75)(7.125 - 3.075)^2$$

$$= 57.05 \text{ in}^4$$



$$Q_B = \bar{y}'_B A' = 2.7(4)(0.75) = 8.1 \text{ in}^3$$

$$Q_C = \bar{y}'_C A' = 4.05(2)(0.75) = 6.075 \text{ in}^3$$

$$\tau = \frac{VQ}{It}$$

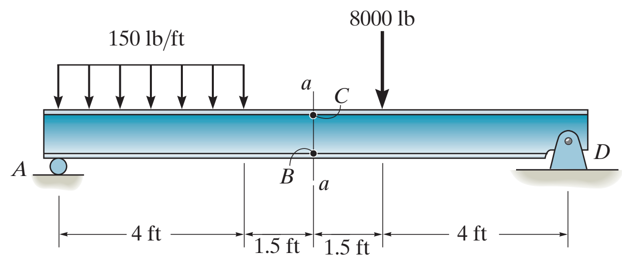
$$\tau_B = \frac{2800(8.1)}{57.05(0.5)} = 795 \text{ psi}$$ **Ans.**

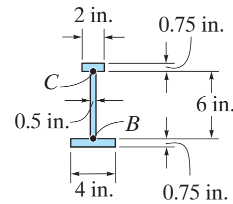
$$\tau_C = \frac{2800(6.075)}{57.05(0.5)} = 596 \text{ psi}$$ **Ans.**

Ans:
$\tau_B = 795$ psi,
$\tau_C = 596$ psi

R12–6.

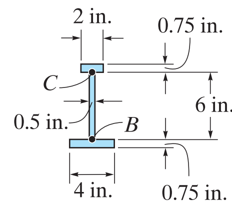
Determine the maximum shear stress acting at section *a–a* in the beam.





SOLUTION

$$\bar{y} = \frac{(0.375)(4)(0.75) + (3.75)(6)(0.5) + (7.125)(2)(0.75)}{4(0.75) + 6(0.5) + 2(0.75)} = 3.075 \text{ in.}$$

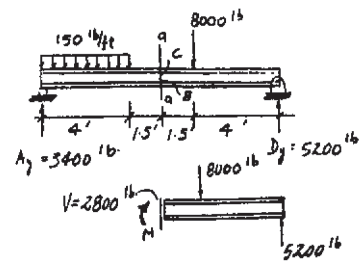


$$I = \frac{1}{12}(4)(0.75^3) + 4(0.75)(3.075 - 0.375)^2$$

$$+ \frac{1}{12}(0.5)(6^3) + 0.5(6)(3.75 - 3.075)^2$$

$$+ \frac{1}{12}(2)(0.75^3) + 2(0.75)(7.125 - 3.075)^2$$

$$= 57.05 \text{ in}^4$$

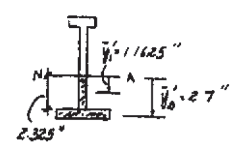


$$Q_{\max} = \Sigma \bar{y}' A'$$

$$= 2.7(4)(0.75) + 2.325(0.5)(1.1625)$$

$$= 9.4514 \text{ in}^3$$

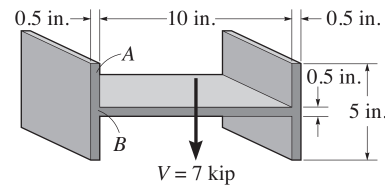
$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{2800(9.4514)}{57.05(0.5)} = 928 \text{ psi}$$ **Ans.**



Ans:
$\tau_{\max} = 928$ psi

R12–7.

The beam supports a vertical shear of $V = 7$ kip. Determine the resultant force this develops in segment AB of the beam.



SOLUTION

$$I = 2\left(\frac{1}{12}\right)(0.5)(5^3) + \frac{1}{12}(10)\left(0.5^3\right)$$

$$= 10.52083 \text{ in}^4$$

$$\tilde{y} = y + \frac{2.5 - y}{2} = \frac{2.5 + y}{2}$$

$$Q = \tilde{y}'A' = \frac{1}{2}(2.5 + y)\left(2.5 - y\right)(0.5)$$

$$= 0.25\left(6.25 - y^2\right)$$

$$= 1.5625 - 0.25y^2$$

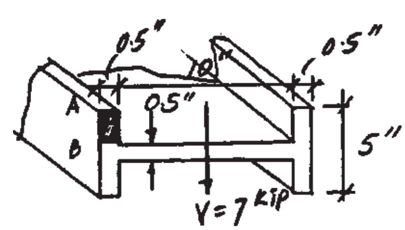
$$q = \frac{VQ}{I} = \frac{(V)(1.5625 - 0.25y^2)}{10.52083}$$

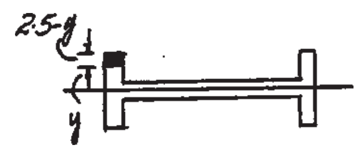
$$V_{AB} = \int_{0.25}^{2.5} q\,dy = 0.09505V \int_{0.25}^{2.5} (1.5625 - 0.25y^2)dy$$

$$= 0.2105V$$

$$= 0.2105(7)$$

$$= 1.47 \text{ Kip}$$

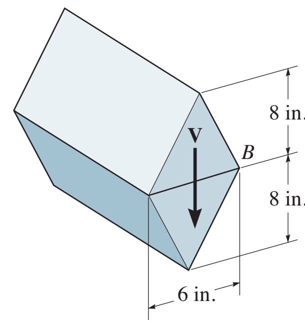




Ans:
$V_{AB} = 1.47$ kip

***R12–8.**

The member consists of two triangular plastic strips bonded together along AB. If the glue can support an allowable shear stress of $\tau_{\text{allow}} = 600$ psi, determine the maximum vertical shear V that can be applied to the member based on the strength of the glue.

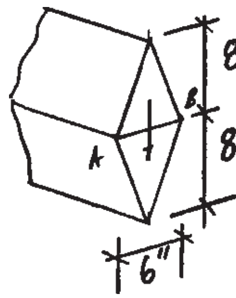


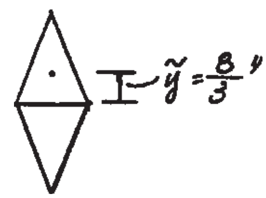
SOLUTION

$$I_{NA} = 2\left(\frac{1}{12}\right)(6)(8^3) = 512 \text{ in}^4$$

$$Q = \tilde{y}A' = \frac{8}{3}\left(\tfrac{1}{2}\right)(8)(6)$$

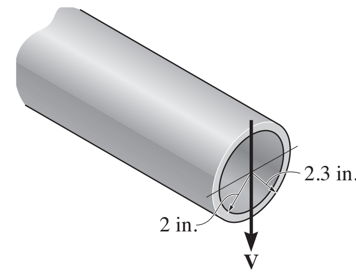
$$= 64 \text{ in}^3$$





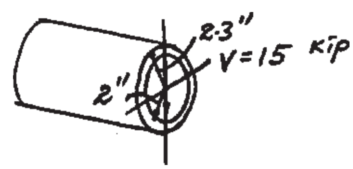
R12–9.

If the pipe is subjected to a shear of $V = 15$ kip, determine the maximum shear stress in the pipe.



SOLUTION

$$I_{NA} = \frac{1}{4}\pi\,(2.3^4 - 2^4) = 9.4123 \text{ in}^4$$



$$\begin{aligned} Q_{\max} &= \Sigma \tilde{y}' A' \\ &= \frac{4\,(2.3)}{3\pi}\left[\frac{\pi(2.3^2)}{2}\right] \\ &= \frac{4\,(2)}{3\pi}\left[\frac{\pi(2^2)}{2}\right] \\ &= 2.\underline{778} \text{ in}^3 \end{aligned}$$

$$\begin{aligned} \tau_{\max} &= \frac{VQ_{\max}}{It} \\ &= \frac{15\,(2.778)}{9.4123\,(2)(0.3)} \\ &= \underline{7.38} \text{ ksi} \end{aligned}$$

Ans:
$\tau_{\max} = 7.38$ ksi