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13–1.

A spherical gas tank has an inner radius of $r = 1.5$ m. If it is subjected to an internal pressure of $p = 300$ kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

SOLUTION

$$\sigma_{\text{allow}} = \frac{p\,r}{2\,t}; \qquad 12(10^6) = \frac{300(10^3)(1.5)}{2\,t}$$

$t = 0.0188 \text{ m} = 18.8 \text{ mm}$ **Ans.**

Ans:
$t = 18.8$ mm

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13–2.

A pressurized spherical tank is made of 0.5-in.-thick steel. If it is subjected to an internal pressure of $p = 200$ psi, determine its outer radius if the maximum normal stress is not to exceed 15 ksi.

SOLUTION

$$\sigma_{\text{allow}} = \frac{p\,r}{2\,t}; \qquad 15(10^3) = \frac{200\,r_i}{2(0.5)}$$

$r_i = 75$ in.

$r_o = 75 \text{ in.} + 0.5 \text{ in.} = 75.5 \text{ in.}$ **Ans.**

Ans:
$r_o = 75.5$ in.

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13–3.

The thin-walled cylinder can be supported in one of two ways as shown. Determine the state of stress in the wall of the cylinder for both cases if the piston P causes the internal pressure to be 65 psi. The wall has a thickness of 0.25 in., and the inner diameter of the cylinder is 8 in.

(a) (b)

SOLUTION

Case (a):

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{65(4)}{0.25} = 1.04 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0 \qquad \textbf{Ans.}$$

Case (b):

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{65(4)}{0.25} = 1.04 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = \frac{pr}{2t}; \qquad \sigma_2 = \frac{65(4)}{2(0.25)} = 520 \text{ psi} \qquad \textbf{Ans.}$$

Ans:
(a) $\sigma_1 = 1.04$ ksi, $\sigma_2 = 0$,
(b) $\sigma_1 = 1.04$ ksi, $\sigma_2 = 520$ psi

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***13–4.**

The tank of the air compressor is subjected to an internal pressure of 90 psi. If the inner diameter of the tank is 22 in., and the wall thickness is 0.25 in., determine the stress components acting at point A. Draw a volume element of the material at this point, and show the results on the element.

SOLUTION

Hoop Stress for Cylindrical Vessels: Since $\frac{r}{t} = \frac{11}{0.25} = 44 > 10$, then *thin wall* analysis can be used. Applying Eq. 13–1

$$\sigma_1 = \frac{pr}{t} = \frac{90(11)}{0.25} = 3960 \text{ psi} = 3.96 \text{ ksi} \qquad \textbf{Ans.}$$

Longitudinal Stress for Cylindrical Vessels: Applying Eq. 13–2,

$$\sigma_2 = \frac{pr}{2t} = \frac{90(11)}{2(0.25)} = 1980 \text{ psi} = 1.98 \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$\sigma_1 = 3.96$ ksi,
$\sigma_2 = 1.98$ ksi

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13–5.

Air pressure in the cylinder is increased by exerting forces $P = 2$ kN on the two pistons, each having a radius of 45 mm. If the cylinder has a wall thickness of 2 mm, determine the state of stress in the wall of the cylinder.

SOLUTION

$$p = \frac{P}{A} = \frac{2(10^3)}{\pi(0.045^2)} = 314\,380.13 \text{ Pa}$$

$$\sigma_1 = \frac{p\,r}{t} = \frac{314\,380.13(0.045)}{0.002} = 7.07 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0 \qquad \textbf{Ans.}$$

The pressure P is supported by the surface of the pistons in the longitudinal direction.

Ans:
$\sigma_1 = 7.07$ MPa, $\sigma_2 = 0$

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13–6.

Determine the maximum force P that can be exerted on each of the two pistons so that the circumferential stress in the cylinder does not exceed 3 MPa. Each piston has a radius of 45 mm and the cylinder has a wall thickness of 2 mm.

SOLUTION

$$\sigma = \frac{p\,r}{t}; \qquad 3(10^6) = \frac{p(0.045)}{0.002}$$

$$p = 133.3 \text{ kPa}$$

$$P = pA = 133.3\left(10^3\right)(\pi)(0.045)^2 = 848 \text{ N}$$ **Ans.**

Ans:
$P = 848$ N

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13–7.

A boiler is constructed of 8-mm-thick steel plates that are fastened together at their ends using a butt joint consisting of two 8-mm cover plates and rivets having a diameter of 10 mm and spaced 50 mm apart as shown. If the steam pressure in the boiler is 1.35 MPa, determine (a) the circumferential stress in the boiler's plate away from the seam, (b) the circumferential stress in the outer cover plate along the rivet line *a–a*, and (c) the shear stress in the rivets.

SOLUTION

(a) $$\sigma_1 = \frac{pr}{t} = \frac{1.35(10^6)(0.75)}{0.008} = 126.56(10^6) = 127 \text{ MPa}$$ **Ans.**

(b) $$126.56\,(10^6)(0.05)(0.008) = \sigma_1'(2)(0.04)(0.008)$$

$$\sigma_1' = 79.1 \text{ MPa}$$ **Ans.**

(c) From FBD(a),

$$+\uparrow \Sigma F_y = 0; \qquad F_b - 79.1(10^6)[(0.008)(0.04)] = 0$$

$$F_b = 25.3 \text{ kN}$$

$$(\tau_{avg})_b = \frac{F_b}{A} = \frac{25312.5}{\frac{\pi}{4}(0.01)^2} = 322 \text{ MPa}$$ **Ans.**

Ans:
(a) $\sigma_1 = 127$ MPa,
(b) $\sigma_1' = 79.1$ MPa,
(c) $(\tau_{avg})_b = 322$ MPa

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***13–8.**

The steel water pipe has an inner diameter of 12 in. and a wall thickness of 0.25 in. If the valve A is opened and the flowing water has a pressure of 250 psi as it passes point B, determine the longitudinal and hoop stress developed in the wall of the pipe at point B.

SOLUTION

Normal Stress: Since the pipe has two open ends,

$$\sigma_{\text{long}} = \sigma_2 = 0 \qquad \textbf{Ans.}$$

Since $\dfrac{r}{t} = \dfrac{6}{0.25} = 24 > 10$, thin-wall analysis can be used.

$$\sigma_h = \sigma_1 = \frac{pr}{t} = \frac{250(6)}{0.25} = 6000 \text{ psi} = 6 \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{\text{long}} = 0,$
$\sigma_h = 6$ ksi

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13–9.

The steel water pipe has an inner diameter of 12 in. and a wall thickness of 0.25 in. If the valve A is closed and the water pressure is 300 psi, determine the longitudinal and hoop stress developed in the wall of the pipe at point B. Draw the state of stress on a volume element located on the wall.

SOLUTION

Normal Stress: Since $\frac{r}{t} = \frac{6}{0.25} = 24 > 10$, thin-wall analysis can be used.

$$\sigma_{\text{hoop}} = \sigma_1 = \frac{pr}{t} = \frac{300(6)}{0.25} = 7200 \text{ psi} = 7.20 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{\text{long}} = \sigma_2 = \frac{pr}{2t} = \frac{300(6)}{2(0.25)} = 3600 \text{ psi} = 3.60 \text{ ksi} \qquad \textbf{Ans.}$$

The state of stress on an element in the pipe wall is shown in Fig. a.

(a)

Ans:
$\sigma_{\text{hoop}} = 7.20$ ksi, $\sigma_{\text{long}} = 3.60$ ksi

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13–10.

The A-36-steel band is 2 in. wide and is secured around the smooth rigid cylinder. If the bolts are tightened so that the tension in them is 400 lb, determine the normal stress in the band, the pressure exerted on the cylinder, and the distance half the band stretches.

SOLUTION

$$\sigma_1 = \frac{400}{2(1/8)(1)} = 1600 \text{ psi} = 1.60 \text{ ksi}$$ **Ans.**

$$\sigma_1 = \frac{pr}{t}; \quad 1600 = \frac{p(8)}{(1/8)}$$

$$p = 25 \text{ psi}$$ **Ans.**

$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{1600}{29(10^6)} = 55.1724(10^{-6})$$

$$\delta = \epsilon_1 L = 55.1724(10^{-6})(\pi)\left(8 + \frac{1}{16}\right) = 0.00140 \text{ in.}$$ **Ans.**

Ans:
$\sigma_1 = 1.60$ ksi, $p = 25$ psi, $\delta = 0.00140$ in.

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13–11.

The gas pipe line is supported every 20 ft by concrete piers and also lays on the ground. If there are rigid retainers at the piers that hold the pipe fixed, determine the longitudinal and hoop stress in the pipe if the temperature rises 60° F from the temperature at which it was installed. The gas within the pipe is at a pressure of 600 lb/in^2. The pipe has an inner diameter of 20 in. and thickness of 0.25 in. The material is A-36 steel.

SOLUTION

Require

$$\delta_F = \delta_T; \qquad \delta_F = \frac{PL}{AE} = \frac{\sigma L}{E}, \qquad \delta_T = \alpha \Delta T L$$

$$\frac{\sigma_2(20)(12)}{29(10^6)} = (6.60)(10^{-6})(60)(20)(12)$$

$\sigma_2 = 11.5$ ksi **Ans.**

$$\sigma_1 = \frac{pr}{t} = \frac{600(10)}{0.25} = 24 \text{ ksi}$$ **Ans.**

Ans:
$\sigma_2 = 11.5$ ksi,
$\sigma_1 = 24$ ksi

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***13–12.**

A pressure-vessel head is fabricated by welding the circular plate to the end of the vessel as shown. If the vessel sustains an internal pressure of 450 kPa, determine the average shear stress in the weld and the state of stress in the wall of the vessel.

SOLUTION

$+\uparrow \Sigma F_y = 0; \quad \pi(0.225)^2 450(10^3) - \tau_{avg}(2\pi)(0.225)(0.01) = 0;$

$\tau_{avg} = 5.06 \text{ MPa}$ **Ans.**

$\sigma_1 = \dfrac{p\,r}{t} = \dfrac{450(10^3)(0.225)}{0.02} = 5.06 \text{ MPa}$ **Ans.**

$\sigma_2 = \dfrac{p\,r}{2\,t} = \dfrac{450(10^3)(0.225)}{2(0.02)} = 2.53 \text{ MPa}$ **Ans.**

Ans:
$\tau_{avg} = 5.06$ MPa,
$\sigma_1 = 5.06$ MPa,
$\sigma_2 = 2.53$ MPa

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13–13.

An A-36-steel hoop has an inner diameter of 23.99 in., thickness of 0.25 in., and width of 1 in. If it and the 24-in.-diameter rigid cylinder have a temperature of 65° F, determine the temperature to which the hoop should be heated in order for it to just slip over the cylinder. What is the pressure the hoop exerts on the cylinder, and the tensile stress in the ring when it cools back down to 65° F?

SOLUTION

$\delta_T = \alpha \, \Delta \, TL$

$\pi(24) - \pi(23.99) = 6.60(10^{-6})(T_1 - 65)(\pi)(23.99)$

$T_1 = 128.16°\text{F} = 128°\text{F}$ **Ans.**

Cool down:

$\delta_F = \delta_T$

$$\frac{FL}{AE} = \alpha \, \Delta \, TL$$

$$\frac{F(\pi)(24)}{(1)(0.25)(29)(10^6)} = 6.60(10^{-6})(128.16 - 65)(\pi)(24)$$

$F = 3022.21 \text{ lb}$

$\sigma_1 = \dfrac{F}{A}; \quad \sigma_1 = \dfrac{3022.21}{(1)(0.25)} = 12\,088 \text{ psi} = 12.1 \text{ ksi}$ **Ans.**

$\sigma_1 = \dfrac{pr}{t}; \quad 12\,088 = \dfrac{p(12)}{(0.25)}$

$p = 252 \text{ psi}$ **Ans.**

Ans:
$T_1 = 128°\text{F}, \sigma_1 = 12.1 \text{ ksi}, p = 252 \text{ psi}$

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13–14.

The ring, having the dimensions shown, is placed over a flexible membrane which is pumped up with a pressure p. Determine the change in the inner radius of the ring after this pressure is applied. The modulus of elasticity for the ring is E.

SOLUTION

Equilibrium for the Ring: From the FBD,

$$\xrightarrow{+} \Sigma F_x = 0; 2P - 2pr_i w = 0 \qquad P = pr_i w$$

Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_i w}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$

Using Hooke's Law,

$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad \textbf{(1)}$$

However, $\epsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r_i} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}.$

Then, from Eq. (1),

$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)} \qquad \textbf{Ans.}$$

Ans:
$\delta r_i = \dfrac{pr_i^2}{E(r_o - r_i)}$

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13–15.

The inner ring A has an inner radius r_1 and outer radius r_2. The outer ring B has an inner radius r_3 and an outer radius r_4, and $r_2 > r_3$. If the outer ring is heated and then fitted over the inner ring, determine the pressure between the two rings when ring B reaches the temperature of the inner ring. The material has a modulus of elasticity of E and a coefficient of thermal expansion of α.

SOLUTION

Equilibrium for the Ring: From the FBD,

$$\xrightarrow{+} \Sigma F_x = 0; 2P - 2pr_i w = 0 \qquad P = pr_i w$$

Hoop Stress and Strain for the Ring:

$$\sigma_1 = \frac{P}{A} = \frac{pr_i w}{(r_o - r_i)w} = \frac{pr_i}{r_o - r_i}$$

Using Hooke's Law,

$$\epsilon_1 = \frac{\sigma_1}{E} = \frac{pr_i}{E(r_o - r_i)} \qquad \textbf{(1)}$$

However, $\epsilon_1 = \dfrac{2\pi(r_i)_1 - 2\pi r_i}{2\pi r_i} = \dfrac{(r_i)_1 - r_i}{r_i} = \dfrac{\delta r_i}{r_i}.$

Then, from Eq. (1),

$$\frac{\delta r_i}{r_i} = \frac{pr_i}{E(r_o - r_i)}$$

$$\delta r_i = \frac{pr_i^2}{E(r_o - r_i)}$$

Compatibility: The pressure between the rings requires

$$\delta r_2 + \delta r_3 = r_2 - r_3 \qquad \textbf{(2)}$$

From the result obtained above,

$$\delta r_2 = \frac{pr_2^2}{E(r_2 - r_1)} \qquad \delta r_3 = \frac{pr_3^2}{E(r_4 - r_3)}$$

Substitute into Eq. (2).

$$\frac{pr_2^2}{E(r_2 - r_1)} + \frac{pr_3^2}{E(r_4 - r_3)} = r_2 - r_3$$

$$p = \frac{E(r_2 - r_3)}{\dfrac{r_2^2}{r_2 - r_1} + \dfrac{r_3^2}{r_4 - r_3}} \qquad \textbf{Ans.}$$

Ans:

$$p = \frac{E(r_2 - r_3)}{\dfrac{r_2^2}{r_2 - r_1} + \dfrac{r_3^2}{r_4 - r_3}}$$

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***13–16.**

Two hemispheres having an inner radius of 2 ft and wall thickness of 0.25 in. are fitted together, and the inside pressure is reduced to -10 psi. If the coefficient of static friction is $\mu_s = 0.5$ between the hemispheres, determine (a) the torque T needed to initiate the rotation of the top hemisphere relative to the bottom one, (b) the vertical force needed to pull the top hemisphere off the bottom one, and (c) the horizontal force needed to slide the top hemisphere off the bottom one.

SOLUTION

Normal Pressure: Vertical force equilibrium for FBD(a):

$$+\uparrow \Sigma F_y = 0; \qquad 10\left[\pi(24^2)\right] - N = 0 \qquad N = 5760\pi \text{ lb}$$

The Friction Force: Applying friction formula,

$$F_f = \mu_s N = 0.5(5760\pi) = 2880\pi \text{ lb}$$

(a) **The Required Torque:** In order to initiate rotation of the two hemispheres relative to each other, the torque must overcome the moment produced by the friction force about the center of the sphere.

$$T = F_f r = 2880\pi(2 + 0.125/12) = 18190 \text{ lb}\cdot\text{ft} = 18.2 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

(b) **The Required Vertical Force:** In order to just pull the two hemispheres apart, the vertical force P must overcome the normal force.

$$P = N = 5760\pi = 18096 \text{ lb} = 18.1 \text{ kip} \qquad \textbf{Ans.}$$

(c) **The Required Horizontal Force:** In order to just cause the two hemispheres to slide relative to each other, the horizontal force F must overcome the friction force.

$$F = F_f = 2880\pi = 9048 \text{ lb} = 9.05 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
(a) $T = 18.2$ kip · ft,
(b) $P = 18.1$ kip,
(c) $F = 9.05$ kip

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13–17.

In order to increase the strength of the pressure vessel, filament winding of the same material is wrapped around the circumference of the vessel as shown. If the pretension in the filament is T and the vessel is subjected to an internal pressure p, determine the hoop stresses in the filament and in the wall of the vessel. Use the free-body diagram shown, and assume the filament winding has a thickness t' and width w for a corresponding length L of the vessel.

SOLUTION

Normal Stress in the Wall and Filament Before the Internal Pressure is Applied: The entire length L of the wall is subjected to pretension filament force T. Hence, from equilibrium, the normal stress in the wall at this state is

$$2T - (\sigma')_w(2Lt) = 0 \qquad (\sigma')_w = \frac{T}{Lt}$$

and for the filament the normal stress is

$$(\sigma')_{fil} = \frac{T}{wt'}$$

Normal Stress in the Wall and Filament After the Internal Pressure is Applied: In order to use $\sigma_1 = pr/t$, developed for a vessel of uniform thickness, we redistribute the filament's cross-section as if it were thinner and wider, to cover the vessel with no gaps. The modified filament has width L and thickness $t'w/L$, still with cross-sectional area wt' subjected to tension T. Then the stress in the filament becomes

$$\sigma_{fil} = \sigma + (\sigma')_{fil} = \frac{pr}{(t + t'w/L)} + \frac{T}{wt'} \qquad \textbf{Ans.}$$

And for the wall,

$$\sigma_w = \sigma - (\sigma')_w = \frac{pr}{(t + t'w/L)} - \frac{T}{Lt} \qquad \textbf{Ans.}$$

Check: $2wt'\sigma_{fil} + 2Lt\sigma_w = 2rLp$ OK

Ans:
$\sigma_{fil} = \dfrac{pr}{t + t'w/L} + \dfrac{T}{wt'}$,
$\sigma_w = \dfrac{pr}{t + t'w/L} - \dfrac{T}{Lt}$

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13–18.

Determine the shortest distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses in the plate at section a–a. The plate has a thickness of 10 mm and **P** acts along the centerline of this thickness.

SOLUTION

$$\sigma_A = 0 = \sigma_a - \sigma_b$$

$$0 = \frac{P}{A} - \frac{M\,c}{I}$$

$$0 = \frac{P}{(0.2)(0.01)} - \frac{P(0.1 - d)(0.1)}{\frac{1}{12}(0.01)(0.2^3)}$$

$$P(-1000 + 15000\,d) = 0$$

$$d = 0.0667 \text{ m} = 66.7 \text{ mm}$$ **Ans.**

Ans:
$d = 66.7$ mm

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13–19.

Determine the maximum distance d to the edge of the plate at which the force **P** can be applied so that it produces no compressive stresses on the plate at section a–a. The plate has a thickness of 20 mm and **P** acts along the centerline of this thickness.

SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the plate sectioned through section a–a, Fig. a,

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad N - P = 0 \qquad N = P$

$\circlearrowleft + \Sigma M_o = 0; \qquad P(d - 0.1) - M = 0 \qquad M = P(d - 0.1)$

Section Properties: For the rectangular cross section,

$$A = 0.2(0.02) = 0.004 \text{ m}^2$$

$$I = \frac{1}{12}(0.02)(0.2^3) = 13.3333(10^{-6}) \text{ m}^4$$

Normal Stress: It is required that $\sigma_A = 0$. For the combined loadings,

$$\sigma_A = \frac{N}{A} - \frac{Mc}{I}$$

$$0 = \frac{P}{0.004} - \frac{P(d - 0.1)(0.1)}{13.3333(10^{-6})}$$

$$d = 0.1333 \text{ m} = 133 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$d = 133$ mm

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***13–20.**

The plate has a thickness of 20 mm and the force $P = 3$ kN acts along the centerline of this thickness such that $d = 150$ mm. Plot the distribution of normal stress acting along section *a–a*.

SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the plate sectioned through section *a–a*, Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N - 3 = 0 \qquad N = 3.00 \text{ kN}$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad 3(0.05) - M = 0 \qquad M = 0.150 \text{ kN} \cdot \text{m}$$

Section Properties: For the rectangular cross section,

$$A = 0.2(0.02) = 0.004 \text{ m}^2$$

$$I = \frac{1}{12}(0.02)(0.2^3) = 13.3333(10^{-6}) \text{ m}^4$$

Normal Stress: For the combined loadings, the normal stress at points A and B can be determined from

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I} = \frac{3.00(10^3)}{0.004} \pm \frac{0.150(10^3)(0.1)}{13.3333(10^{-6})}$$

$$\sigma_A = 750(10^3) - 1.125(10^6) = -0.375(10^6) \text{ Pa} = 0.375 \text{ MPa (C)} \qquad \textbf{Ans.}$$

$$\sigma_B = 750(10^3) + 1.125(10^6) = 1.875(10^6) \text{ Pa} = 1.875 \text{ MPa (T)} \qquad \textbf{Ans.}$$

Using similar triangles, the location of the neutral axis can be determined

$$\frac{y}{1.875} = \frac{0.2 - y}{0.375}; \qquad y = 0.1667 \text{ m}$$

(a)

(b)

Ans:
$\sigma_A = 0.375$ MPa (C),
$\sigma_B = 1.875$ MPa (T)

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13–21.

If the load has a weight of 600 lb, determine the maximum normal stress on the cross section of the supporting member at section *a–a*. Also, plot the normal-stress distribution over the cross section.

SOLUTION

Internal Loadings: Consider the equilibrium of the free-body diagram of the bottom cut segment shown in Fig. *a*.

$\circlearrowleft + \uparrow \Sigma F_y = 0; \qquad N - 600 = 0 \qquad N = 600 \text{ lb}$

$\circlearrowleft + \Sigma M_C = 0; \qquad 600(1.5) - M = 0 \qquad M = 900 \text{ lb} \cdot \text{ft}$

Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the member are

$$A = \pi(1^2) = \pi \text{ in}^2 \qquad I = \frac{\pi}{4}(1^4) = \frac{\pi}{4} \text{ in}^4$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

By observation, the maximum normal stress occurs at point *B*, Fig. *b*. Thus,

$$\sigma_{\max} = \sigma_B = \frac{600}{\pi} + \frac{900(12)(1)}{\pi/4} = 13.9 \text{ ksi (T)} \qquad \textbf{Ans.}$$

For Point *A*,

$$\sigma_A = \frac{600}{\pi} + -\frac{900(12)(1)}{\pi/4} = -13.6 \text{ ksi} = 13.6 \text{ ksi (C)} \qquad \textbf{Ans.}$$

Using these results, the normal stress distribution over the cross section is shown in Fig. *b*. The location of the neutral axis can be determined from

$$\frac{\pi}{13.9} = \frac{2 - x}{13.6}; \qquad x = 1.01 \text{ in.}$$

(a)

(b)

Ans:
$\sigma_{\max} = \sigma_B = 13.9 \text{ ksi (T)}, \sigma_A = 13.6 \text{ ksi (C)}$

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13–22.

The steel bracket is used to connect the ends of two cables. If the allowable normal stress for the steel is $\sigma_{\text{allow}} = 30$ ksi, determine the largest tensile force P that can be applied to the cables. Assume the bracket is a rod having a diameter of 1.5 in.

SOLUTION

Internal Loading: Consider the equilibrium of the left segment of the bracket sectioned through an arbitrary cross section, Fig. *a*.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N - P = 0 \qquad N = P$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M - P(4.75) = 0 \qquad M = 4.75P$$

Section Properties: For the circular cross section,

$$A = \pi c^2 = \pi\left(0.75^2\right) = 0.5625\pi \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}\left(0.75^4\right) = 0.24850 \text{ in}^4$$

Allowable Normal Stress: The maximum normal stress occurs at the bottom of the circular cross section. For the combined loadings,

$$\sigma_{\max} = \sigma_{\text{allow}} = \frac{N}{A} + \frac{Mc}{I}$$

$$30\left(10^3\right) = \frac{P}{0.5625\pi} + \frac{4.75P(0.75)}{0.24850}$$

$$P = 2.01320\left(10^3\right) \text{ lb}$$

$$= 2.01 \text{ kip} \qquad \textbf{Ans.}$$

(a)

Ans:
$P_{\max} = 2.01$ kip

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13–23.

The steel bracket is used to connect the ends of two cables. If the applied force $P = 1.50$ kip, determine the maximum normal stress in the bracket. Assume the bracket is a rod having a diameter of 1.5 in.

SOLUTION

Internal Loading: Consider the equilibrium of the left segment of the bracket sectioned through an arbitrary cross section, Fig. *a*.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N - 1.50 = 0 \qquad N = 1.50 \text{ kip}$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M - 1.50(4.75) = 0 \qquad M = 7.125 \text{ kip} \cdot \text{in.}$$

Section Properties: For the circular cross section,

$$A = \pi c^2 = \pi\left(0.75^2\right) = 0.5625\pi \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}\left(0.75^4\right) = 0.24850 \text{ in}^4$$

Maximum Normal Stress: The maximum normal stress occurs at the bottom of the circular cross section. For the combined loadings,

$$\sigma_{\max} = \frac{N}{A} + \frac{Mc}{I}$$

$$= \frac{1.50}{0.5625\pi} + \frac{7.125(0.75)}{0.24850}$$

$$= 22.35 \text{ ksi} = 22.4 \text{ ksi (T)} \qquad \textbf{Ans.}$$

(a)

Ans:
$\sigma_{\max} = 22.4$ ksi (T)

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***13–24.**

The column is built up by gluing the two boards together. Determine the maximum normal stress on the cross section when the eccentric force of $P = 50$ kN is applied.

SOLUTION

Section Properties: The location of the centroid of the cross section, Fig. *a*, is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.075(0.15)(0.3) + 0.3(0.3)(0.15)}{0.15(0.3) + 0.3(0.15)} = 0.1875 \text{ m}$$

The cross-sectional area and the moment of inertia about the z axis of the cross section are

$$A = 0.15(0.3) + 0.3(0.15) = 0.09 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.3)(0.15^3) + 0.3(0.15)(0.1875 - 0.075)^2 + \frac{1}{12}(0.15)(0.3^3) + 0.15(0.3)(0.3 - 0.1875)^2$$

$$= 1.5609(10^{-3}) \text{ m}^4$$

Equivalent Force System: Referring to Fig. *b*,

$+\uparrow \Sigma F_x = (F_R)_x;$ $\quad -50 = -F \quad F = 50$ kN

$\Sigma M_z = (M_R)_z;$ $\quad -50(0.2125) = -M \quad M = 10.625$ kN · m

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress occurs at points along the edge where $y = 0.45 - 0.1875 = 0.2625$ m, such as at point A. Thus,

$$\sigma_{\max} = \frac{-50(10^3)}{0.09} - \frac{10.625(10^3)(0.2625)}{1.5609(10^{-3})}$$

$$= -2.342 \text{ MPa} = 2.34 \text{ MPa (C)}$$ **Ans.**

(a)

(b)

Ans:
$\sigma_{\max} = 2.34$ MPa (C)

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13–25.

The column is built up by gluing the two boards together. If the wood has an allowable normal stress of $\sigma_{\text{allow}} = 6$ MPa, determine the maximum allowable eccentric force **P** that can be applied to the column.

SOLUTION

Section Properties: The location of the centroid c of the cross section, Fig. a, is

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.075(0.15)(0.3) + 0.3(0.3)(0.15)}{0.15(0.3) + 0.3(0.15)} = 0.1875 \text{ m}$$

The cross-sectional area and the moment of inertia about the z axis of the cross section are

$$A = 0.15(0.3) + 0.3(0.15) = 0.09 \text{ m}^2$$

$$I_z = \frac{1}{12}(0.3)(0.15^3) + 0.3(0.15)(0.1875 - 0.075)^2 + \frac{1}{12}(0.15)(0.3^3) + 0.15(0.3)(0.3 - 0.1875)^2$$

$$= 1.5609(10^{-3}) \text{ m}^4$$

Equivalent Force System: Referring to Fig. b,

$+\uparrow \Sigma F_x = (F_R)_x;$ $\quad -P = -F \quad F = P$

$\Sigma M_z = (M_R)_z;$ $\quad -P(0.2125) = -M \quad M = 0.2125P$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$F = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress, which is compression, occurs at points along the edge where $y = 0.45 - 0.1875 = 0.2625$ m, such as at point A. Thus,

$$-6(10^6) = \frac{-P}{0.09} - \frac{0.2125P(0.2625)}{1.5609(10^{-3})}$$

$$P = 128\,076.92 \text{ N} = 128 \text{ kN}$$ **Ans.**

Ans:
$P_{\max} = 128$ kN

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13–26.

The screw of the clamp exerts a compressive force of 500 lb on the wood blocks. Determine the maximum normal stress along section *a–a*. The cross section is rectangular, 0.75 in. by 0.50 in.

SOLUTION

$A = 0.75(0.5) = 0.375 \text{ in}^2$

$$I = \frac{1}{12}(0.5)(0.75^3) = 0.017578 \text{ in}^4$$

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}$$

$$= \frac{500}{0.375} + \frac{2000(0.375)}{0.017578} = 44.0 \text{ ksi (T)}$$ **Ans.**

Ans:
$\sigma_{\max} = 44.0$ ksi (T)

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13–27.

The screw of the clamp exerts a compressive force of 500 lb on the wood blocks. Sketch the stress distribution along section *a–a* of the clamp. The cross section is rectangular, 0.75 in. by 0.50 in.

SOLUTION

$A = 0.75(0.5) = 0.375 \text{ in}^2$

$$I = \frac{1}{12}(0.5)(0.75^3) = 0.017578 \text{ in}^4$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{500}{0.375} + \frac{2000(0.375)}{0.017578} = 44.0 \text{ ksi (T)}$$ **Ans.**

$$\sigma_{min} = \frac{P}{A} - \frac{Mc}{I} = \frac{500}{0.375} - \frac{2000(0.375)}{0.017578} = -41.3 \text{ ksi (C)}$$ **Ans.**

$$\frac{y}{41.33} = \frac{(0.75 - y)}{44.0}$$

$y = 0.363$ in.

Ans:
$\sigma_{max} = 44.0$ ksi (T),
$\sigma_{min} = -41.3$ ksi (C)

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***13–28.**

The joint is subjected to the force system shown. Sketch the normal-stress distribution acting over section *a–a* if the member has a rectangular cross section of width 0.5 in. and thickness 1 in.

SOLUTION

Internal Loadings: Consider the equilibrium of the lower segment of the joint sectioned through *a–a*, Fig. *a*.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad V - 250 - 500\left(\frac{3}{5}\right) = 0 \qquad V = 550 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad N - 500\left(\frac{4}{5}\right) = 0 \qquad N = 400 \text{ lb}$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M + 500\left(\frac{4}{5}\right)(1.5) - 500\left(\frac{3}{5}\right)(2) - 250(2) = 0$$

$$M = 500 \text{ lb} \cdot \text{in.}$$

(a)

Section Properties: For the rectangular cross section,

$$A = 0.5(1) = 0.5 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)(1^3) = 0.04167 \text{ in}^4$$

Normal Stress: For the combined loadings, the normal stress at points *B* and *C*, Fig. *b*, can be determined from

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I} = \frac{400}{0.5} \pm \frac{500(0.5)}{0.04167}$$

$$\sigma_C = 800 + 6000 = 6800 \text{ psi} = 6.80 \text{ ksi (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = 800 - 6000 = -5200 \text{ psi} = 5.20 \text{ ksi (C)} \qquad \textbf{Ans.}$$

Using similar triangles,

$$\frac{1 - y}{5.20} = \frac{y}{6.80}; \qquad y = 0.5667 \text{ in.}$$

Using these results, the normal stress distribution over section *a–a*, shown in Fig. *b*, can be sketched.

(b)

Ans:
$\sigma_C = 6.80$ ksi (T),
$\sigma_B = 5.20$ ksi (C)

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13–29.

The joint is subjected to the force system shown. Determine the state of stress at points A and B, and sketch the results on differential elements located at these points. The member has a rectangular cross-sectional area of width 0.5 in. and thickness 1 in.

SOLUTION

Internal Loadings: Consider the equilibrium of the lower segment of the joint sectioned through a–a, Fig. a,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad V - 250 - 500\left(\frac{3}{5}\right) = 0 \qquad V = 550 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad N - 500\left(\frac{4}{5}\right) = 0 \qquad N = 400 \text{ lb}$$

$$\circlearrowleft + \Sigma M_o = 0; \qquad M + 500\left(\frac{4}{5}\right)(1.5) - 500\left(\frac{3}{5}\right)(2) - 250(2) = 0$$

$$M = 500 \text{ lb}\cdot\text{in.}$$

(a)

Section Properties: For the rectangular cross section,

$$A = 0.5(1) = 0.5 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)(1^3) = 0.04167 \text{ in}^4$$

For points A and B shown in Fig. b,

$$Q_B = 0 \qquad Q_A = \bar{y}'A' = 0.25[0.5(0.5)] = 0.0625 \text{ in}^3$$

(b)

Normal Stress: For the combined loadings, the normal stress at points A and B, Fig. b, can be determined from

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I} = \frac{400}{0.5} + \frac{500(0)}{0.04167} = 800 \text{ psi} = 0.800 \text{ ksi (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{N}{A} - \frac{Mc}{I} = \frac{400}{0.5} - \frac{500(0.5)}{0.04167} = -5200 \text{ psi} = 5.20 \text{ ksi (C)} \qquad \textbf{Ans.}$$

Shear Stress: Applying the shear formula,

$$\tau_A = \frac{VQ_A}{It} = \frac{550(0.0625)}{0.04167(0.5)} = 1650 \text{ psi} = 1.65 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_B = \frac{VQ_B}{It} = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress acting on the differential elements at points A and B shown in Fig. c can be sketched.

(c)

Ans:
$\sigma_A = 0.800$ ksi (T),
$\sigma_B = 5.20$ ksi (C),
$\tau_A = 1.65$ ksi,
$\tau_B = 0$

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13–30.

The rib-joint pliers are used to grip the smooth pipe C. If the force of 100 N is applied to the handles, determine the state of stress at points A and B on the cross section of the jaw at section a–a. Indicate the results on an element at each point.

SOLUTION

Support Reactions: Referring to the free-body diagram of the handle shown in Fig. a,

$$\circlearrowleft + \Sigma M_D = 0; \qquad 100(0.25) - F_C(0.05) = 0 \qquad F_C = 500 \text{ N}$$

Internal Loadings: Consider the equilibrium of the free-body diagram of the segment shown in Fig. b.

$$\Sigma F_{y'} = 0; \qquad 500 - V = 0 \qquad V = 500 \text{ N}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad M - 500(0.025) = 0 \qquad M = 12.5 \text{ N} \cdot \text{m}$$

Section Properties: The moment of inertia of the cross section about the centroidal axis is

$$I = \frac{1}{12}(0.0075)(0.02^3) = 5(10^{-9}) \text{ m}^4$$

Referring to Fig. c, Q_A and Q_B are

$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.005(0.01)(0.0075) = 0.375(10^{-6}) \text{ m}^3$$

Normal Stress: The normal stress is contributed by bending stress only. Thus,

$$\sigma = \frac{My}{I}$$

For point A, $y = 0.01$ m. Then

$$\sigma_A = -\frac{12.5(0.01)}{5(10^{-9})} = -25 \text{ MPa} = 25 \text{ MPa (C)} \qquad \textbf{Ans.}$$

For point B, $y = 0$. Then

$$\sigma_B = 0 \qquad \textbf{Ans.}$$

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13–30. Continued

Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

$$\tau_A = \frac{VQ_A}{It} = 0 \qquad \textbf{Ans.}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{500[0.375(10^{-6})]}{5(10^{-9})(0.0075)} = 5 \text{ MPa} \qquad \textbf{Ans.}$$

The state of stress of points A and B are represented by the elements shown in Figs. d and e, respectively.

Ans:
$\sigma_A = 25$ MPa (C), $\sigma_B = 0$,
$\tau_A = 0$, $\tau_B = 5$ MPa

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13–31.

The $\frac{1}{2}$-in.-diameter bolt hook is subjected to the load of $F = 150$ lb. Determine the stress components at point A on the shank. Show the result on a volume element located at this point.

SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_A - 150 \cos 30° = 0$

$$N_A = 129.9038 \text{ lb}$$

$+\uparrow \Sigma F_y = 0; \qquad V_A - 150 \sin 30° = 0$

$$V_A = 75 \text{ lb}$$

$\circlearrowleft + \Sigma M_A = 0; \qquad 150 \cos 30°(1.5) + 150 \sin 30°(2) - M_A = 0$

$$M_A = 344.8557 \text{ lb} \cdot \text{in.}$$

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{129.9038}{\pi\left(\frac{1}{4}\right)^2} + \frac{344.8557\left(\frac{1}{4}\right)}{\frac{\pi}{4}\left(\frac{1}{4}\right)^4} = 28.8 \text{ ksi}$$ **Ans.**

$\tau_A = 0$ (Since $Q_A = 0$) **Ans.**

Ans:
$\sigma_A = 28.8$ ksi,
$\tau_A = 0$

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***13–32.**

The $\frac{1}{2}$-in.-diameter bolt hook is subjected to the load of $F = 150$ lb. Determine the stress components at point B on the shank. Show the result on a volume element located at this point.

SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_B - 150\cos 30° = 0; \qquad N_B = 129.9038$

$+\uparrow \Sigma F_y = 0; \qquad V_B - 150\sin 30° = 0; \qquad V_B = 75 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0; \qquad 150\cos 30°(1.5) + 150\sin 30°(4) - M_B = 0$

$$M_B = 494.8557 \text{ lb}\cdot\text{in.}$$

$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{129.9038}{\pi\left(\frac{1}{4}\right)^2} - \frac{494.8557\left(\frac{1}{4}\right)}{\frac{\pi}{4}\left(\frac{1}{4}\right)^4} = -39.7 \text{ ksi}$$ **Ans.**

Ans:
$\sigma_B = -39.7$ ksi

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13–33.

The block is subjected to the eccentric load shown. Determine the normal stress developed at points A and B. Neglect the weight of the block.

SOLUTION

Internal Loadings: Consider the equilibrium of the upper segment of the block sectioned through a–a, Fig. a.

$$\Sigma F_x = 0; \qquad N + 150 = 0 \qquad N = -150 \text{ kN}$$

$$\Sigma M_y = 0; \qquad M_y + 150(0.05) = 0 \qquad M_y = -7.50 \text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z - 150(0.075) = 0 \qquad M_z = 11.25 \text{ kN}\cdot\text{m}$$

Section Properties: For the rectangular cross section,

$$A = 0.1(0.15) = 0.015 \text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.1^3) = 12.5(10^{-6}) \text{ m}^4$$

$$I_z = \frac{1}{12}(0.1)(0.15^3) = 28.125(10^{-6}) \text{ m}^4$$

Normal Stresses: For the combined loadings, the normal stress can be determined from

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y_A = 0.075$ m and $z_A = 0.05$ m.

$$\sigma_A = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(0.075)}{28.125(10^{-6})} + \frac{[-7.50(10^3)](0.05)}{12.5(10^{-6})}$$

$$= -70.0(10^6) \text{ Pa} = 70.0 \text{ MPa (C)} \qquad \textbf{Ans.}$$

For point B, $y_B = 0.075$ m and $z_B = -0.05$ m.

$$\sigma_B = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(0.075)}{28.125(10^{-6})} + \frac{[-7.50(10^3)](-0.05)}{12.5(10^{-6})}$$

$$= -10.0(10^6) \text{ Pa} = 10.0 \text{ MPa (C)} \qquad \textbf{Ans.}$$

Ans:
$\sigma_A = 70.0$ MPa (C),
$\sigma_B = 10.0$ MPa (C)

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13–34.

The block is subjected to the eccentric load shown. Sketch the normal-stress distribution acting over the cross section at section *a–a*. Neglect the weight of the block.

SOLUTION

Internal Loadings: Consider the equilibrium of the upper segment of the block sectioned through *a–a*, Fig. *a*.

$$\Sigma F_x = 0; \qquad N + 150 = 0 \qquad N = -150\text{ kN}$$

$$\Sigma M_y = 0; \qquad M_y + 150(0.05) = 0 \qquad M_y = -7.50\text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z - 150(0.075) = 0 \qquad M_z = 11.25\text{ kN}\cdot\text{m}$$

Section Properties: For the rectangular cross section,

$$A = 0.1(0.15) = 0.015\text{ m}^2$$

$$I_y = \frac{1}{12}(0.15)(0.1^3) = 12.5(10^{-6})\text{ m}^4$$

$$I_z = \frac{1}{12}(0.1)(0.15^3) = 28.125(10^{-6})\text{ m}^4$$

Normal Stress: For the combined loadings, the normal stress can be determined from

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y_A = 0.075$ m and $z_A = 0.05$ m.

$$\sigma_A = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(0.075)}{28.125(10^{-6})} + \frac{(-7.50)(10^3)(0.05)}{12.5(10^{-6})}$$

$$= -70.0(10^6)\text{ Pa} = 70.0\text{ MPa (C)} \qquad \textbf{Ans.}$$

For point B, $y_B = 0.075$ m and $z_B = -0.05$ m.

$$\sigma_B = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(0.075)}{28.125(10^{-6})} + \frac{(-7.50)(10^3)(-0.05)}{12.5(10^{-6})}$$

$$= -10.0(10^6)\text{ Pa} = 10.0\text{ MPa (C)} \qquad \textbf{Ans.}$$

For point C, $y_C = -0.075$ m and $z_C = -0.05$ m.

$$\sigma_C = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(-0.075)}{28.125(10^{-6})} + \frac{(-7.50)(10^3)(-0.05)}{12.5(10^{-6})}$$

$$= 50.0(10^6)\text{ Pa} = 50.0\text{ MPa (T)} \qquad \textbf{Ans.}$$

For point D, $y_D = -0.075$ m and $z_D = 0.05$ m.

$$\sigma_D = \frac{-150(10^3)}{0.015} - \frac{11.25(10^3)(-0.075)}{28.125(10^{-6})} + \frac{(-7.50)(10^3)(0.05)}{12.5(10^{-6})}$$

$$= -10.0(10^6)\text{ Pa} = 10.0\text{ MPa (C)} \qquad \textbf{Ans.}$$

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13–34. Continued

The location of neutral axis can be found using similar triangles.

$$\frac{y}{10.0} = \frac{0.15 - y}{50.0}; \qquad y = 0.025 \text{ m}$$

$$\frac{z}{10.0} = \frac{0.1 - z}{50.0}; \qquad z = 0.01667 \text{ m}$$

Using these result, the normal stress distribution over the cross section shown in Fig. *b* can be sketched.

Ans:
$\sigma_A = 70.0$ MPa (C),
$\sigma_B = 10.0$ MPa (C),
$\sigma_C = 50.0$ MPa (T),
$\sigma_D = 10.0$ MPa (C)

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13–35.

The spreader bar is used to lift the 2000-lb tank. Determine the state of stress at points A and B, and indicate the results on a differential volume element.

SOLUTION

Support Reactions:

$+\uparrow \Sigma F_y = 0; \qquad 2000 - 2F\cos 30° = 0 \qquad F = 1154.70 \text{ lb}$

Internal Forces and Moment:

$\xrightarrow{+} \Sigma F_x = 0; \qquad 1154.70 \sin 30° - N = 0 \qquad N = 577.35 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad V - 1154.70 \cos 30° = 0 \qquad V = 1000 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0; \qquad M - 1154.70 \cos 30°(1.5) = 0$

$M = 1500 \text{ lb} \cdot \text{ft}$

Section Properties:

$$A = 1(2) = 2.00 \text{ in}^2$$

$$I = \frac{1}{12}(1)(2^3) = 0.6666 \text{ in}^4$$

$$Q_B = \bar{y}'A' = 0.5(1)(1) = 0.500 \text{ in}^3$$

$$Q_A = 0$$

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{577.35}{2.00} + \frac{1500(12)(1)}{0.6666}$$

$$= 27\,300 \text{ psi} = 27.3 \text{ ksi (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{577.35}{2.00} + \frac{1500(12)(0)}{0.6666}$$

$$= 289 \text{ psi} = 0.289 \text{ ksi (T)} \qquad \textbf{Ans.}$$

Shear Stress: Applying the shear formula,

$$\tau = \frac{VQ}{It}$$

$$\tau_A = 0 \qquad \textbf{Ans.}$$

$$\tau_B = \frac{1000(0.500)}{0.6666(1)}$$

$$= 750 \text{ psi} = 0.750 \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$\sigma_A = 27.3$ ksi (T),
$\sigma_B = 0.289$ ksi (T),
$\tau_A = 0$,
$\tau_B = 0.750$ ksi

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***13–36.**

The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point A on the cross section of the drill bit at section a–a.

SOLUTION

Internal Loadings: Consider the equilibrium of the free-body diagram of the drill's right cut segment, Fig. *a*.

$$\Sigma F_x = 0; \quad N - 150\left(\frac{4}{5}\right) = 0 \qquad N = 120\text{ N}$$

$$\Sigma F_y = 0; \quad 150\left(\frac{3}{5}\right) - V_y = 0 \qquad V_y = 90\text{ N}$$

$$\Sigma M_x = 0; \; 20 - T = 0 \qquad T = 20\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \; -150\left(\frac{3}{5}\right)(0.4) + 150\left(\frac{4}{5}\right)(0.125) + M_z = 0$$

$$M_z = 21\text{ N}\cdot\text{m}$$

Section Properties: The cross-sectional area, the moment of inertia about the z axis, and the polar moment of inertia of the drill's cross section are

$$A = \pi\left(0.005^2\right) = 25\pi\left(10^{-6}\right)\text{ m}^2$$

$$I_z = \frac{\pi}{4}\left(0.005^4\right) = 0.15625\pi\left(10^{-9}\right)\text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.005^4\right) = 0.3125\pi\left(10^{-9}\right)\text{ m}^4$$

Referring to Fig. *b*, Q_A is

$$Q_A = 0$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point A, $y = 0.005$ m. Then

$$\sigma_A = \frac{-120}{25\pi\left(10^{-6}\right)} - \frac{21(0.005)}{0.15625\pi\left(10^{-9}\right)} = -215.43\text{ MPa} = 215\text{ MPa (C)} \qquad \textbf{Ans.}$$

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***13–36. Continued**

Shear Stress: The transverse shear stress developed at point A is

$$\left[\left(\tau_{xy}\right)_V\right]_A = \frac{V_y Q_A}{I_z t} = 0$$

The torsional shear stress developed at point A is

$$\left[\left(\tau_{xz}\right)_T\right]_A = \frac{Tc}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86 \text{ MPa}$$

Thus,

$$\left(\tau_{xy}\right)_A = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xz}\right)_A = \left[\left(\tau_{xz}\right)_T\right]_A = 102 \text{ MPa} \qquad \textbf{Ans.}$$

The state of stress at point A is represented on the element shown in Fig. c.

(a)

(b)

(C)

Ans:
$\sigma_A = 215$ MPa (C),
$\left(\tau_{xy}\right)_A = 0$,
$\left(\tau_{xz}\right)_A = 102$ MPa

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13–37.

The drill is jammed in the wall and is subjected to the torque and force shown. Determine the state of stress at point B on the cross section of the drill bit at section a–a.

SOLUTION

Internal Loadings: Consider the equilibrium of the free-body diagram of the drill's right cut segment, Fig. *a*.

$$\Sigma F_x = 0; \quad N - 150\left(\frac{4}{5}\right) = 0 \qquad N = 120 \text{ N}$$

$$\Sigma F_y = 0; \quad 150\left(\frac{3}{5}\right) - V_y = 0 \qquad V_y = 90 \text{ N}$$

$$\Sigma M_x = 0; \quad 20 - T = 0 \qquad T = 20 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \; -150\left(\frac{3}{5}\right)(0.4) + 150\left(\frac{4}{5}\right)(0.125) + M_z = 0$$

$$M_z = 21 \text{ N}\cdot\text{m}$$

Section Properties: The cross-sectional area, the moment of inertia about the z axis, and the polar moment of inertia of the drill's cross section are

$$A = \pi\left(0.005^2\right) = 25\pi\left(10^{-6}\right) \text{m}^2$$

$$I_z = \frac{\pi}{2}\left(0.005^4\right) = 0.15625\pi\left(10^{-9}\right) \text{m}^4$$

$$J = \frac{\pi}{2}\left(0.005^4\right) = 0.3125\pi\left(10^{-9}\right) \text{m}^4$$

Referring to Fig. *b*, Q_B is

$$Q_B = \bar{y}'A' = \frac{4(0.005)}{3\pi}\left[\frac{\pi}{2}\left(0.005^2\right)\right] = 83.333\left(10^{-9}\right) \text{m}^3$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z}$$

For point B, $y = 0$. Then

$$\sigma_B = \frac{-120}{25\pi\left(10^{-6}\right)} - 0 = -1.528 \text{ MPa} = 1.53 \text{ MPa (C)} \qquad \textbf{Ans.}$$

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13–37. Continued

Shear Stress: The transverse shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_V\right]_B = \frac{V_y Q_B}{I_z t} = \frac{90\left[83.333\left(10^{-9}\right)\right]}{0.15625\pi\left(10^{-9}\right)(0.01)} = 1.528 \text{ MPa}$$

The torsional shear stress developed at point B is

$$\left[\left(\tau_{xy}\right)_T\right]_B = \frac{Tc}{J} = \frac{20(0.005)}{0.3125\pi\left(10^{-9}\right)} = 101.86 \text{ MPa}$$

Thus,

$$\left(\tau_{xz}\right)_B = 0 \qquad \textbf{Ans.}$$

$$\left(\tau_{xy}\right)_B = \left[\left(\tau_{xy}\right)_T\right]_B - \left[\left(\tau_{xy}\right)_V\right]_B$$

$$= 101.86 - 1.528 = 100.33 \text{ MPa} = 100 \text{ MPa} \qquad \textbf{Ans.}$$

The state of stress at point B is represented on the element shown in Fig. d.

Ans:
$\sigma_B = 1.53$ MPa (C), $\left(\tau_{xz}\right)_B = 0$, $\left(\tau_{xy}\right)_B = 100$ MPa

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13–38.

The frame supports the distributed load shown. Determine the state of stress acting at point D. Show the results on a differential element at this point.

SOLUTION

$$\sigma_D = -\frac{P}{A} - \frac{My}{I} = -\frac{8(10^3)}{(0.1)(0.05)} - \frac{12(10^3)(0.03)}{\frac{1}{12}(0.05)(0.1)^3}$$

$\sigma_D = -88.0$ MPa **Ans.**

$\tau_D = 0$ **Ans.**

Ans:
$\sigma_D = -88.0$ MPa, $\tau_D = 0$

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13–39.

The frame supports the distributed load shown. Determine the state of stress acting at point E. Show the results on a differential element at this point.

SOLUTION

$$\sigma_E = -\frac{P}{A} - \frac{My}{I} = \frac{8(10^3)}{(0.1)(0.05)} + \frac{8.25(10^3)(0.03)}{\frac{1}{12}(0.05)(0.1)^3} = 57.8 \text{ MPa}$$ **Ans.**

$$\tau_E = \frac{VQ}{It} = \frac{4.5(10^3)(0.04)(0.02)(0.05)}{\frac{1}{12}(0.05)(0.1)^3(0.05)} = 864 \text{ kPa}$$ **Ans.**

Ans:
$\sigma_E = 57.8$ MPa, $\tau_E = 864$ kPa

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***13–40.**

The rod has a diameter of 40 mm. If it is subjected to the force system shown, determine the stress components that act at point A, and show the results on a volume element located at this point.

SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. a.

$$\Sigma F_x = 0; \qquad N_x - 1500 = 0 \qquad N_x = 1500\text{ N}$$

$$\Sigma F_y = 0; \qquad V_y - 600 = 0 \qquad V_y = 600\text{ N}$$

$$\Sigma F_z = 0; \qquad V_z + 800 = 0 \qquad V_z = -800\text{ N}$$

$$\Sigma M_x = 0; \qquad T_x - 100 = 0 \qquad T_x = 100\text{ N}\cdot\text{m}$$

$$\Sigma M_y = 0; \qquad M_y + 800(0.3) = 0 \qquad M_y = -240\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z + 600(0.3) = 0 \qquad M_z = -180\text{ N}\cdot\text{m}$$

Section Properties: For the circular cross section, Fig. b,

$$A = \pi c^2 = \pi(0.02^2) = 0.400(10^{-3})\pi\text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.02^4) = 40.0(10^{-9})\pi\text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.02^4) = 80.0(10^{-9})\pi\text{ m}^4$$

$$(Q_A)_z = 0 \qquad (Q_A)_y = \bar{y}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}(0.02^2)\right] = 5.3333(10^{-6})\text{ m}^3$$

Normal Stress: For the combined loadings, the normal stress at point A can be determined from

$$\sigma_x = \sigma_A = \frac{N_x}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$= \frac{1500}{0.400(10^{-3})\pi} - \frac{(-180)(0)}{40.0(10^{-9})\pi} + \frac{(-240)(0.02)}{40.0(10^{-9})\pi}$$

$$= -37.00(10^6)\text{ Pa} = 37.0\text{ MPa (C)} \qquad \textbf{Ans.}$$

$\sigma_A = 37.0$ MPa

Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be obtained using the shear formula $\tau_V = \dfrac{VQ}{It}$ and the torsion formula $\tau_T = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xy})_A = (\tau_V)_y - \tau_T$$

$$= \frac{600\left[5.3333(10^{-6})\right]}{40.0(10^{-9})\pi(0.04)} - \frac{100(0.02)}{80.0(10^{-9})\pi}$$

$$= -7.321(10^6)\text{ Pa} = -7.32\text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xz})_A = (\tau_V)_z = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point A can be represented by the differential volume element shown in Fig. c.

Ans:
$\sigma_A = 37.0$ MPa (C),
$(\tau_{xy})_A = -7.32$ MPa,
$(\tau_{xz})_A = 0$

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13–41.

Solve Prob. 13–40 for point B.

SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \quad N_x - 1500 = 0 \quad N_x = 1500 \text{ N}$$

$$\Sigma F_y = 0; \quad V_y - 600 = 0 \quad V_y = 600 \text{ N}$$

$$\Sigma F_z = 0; \quad V_z + 800 = 0 \quad V_z = -800 \text{ N}$$

$$\Sigma M_x = 0; \quad T_x - 100 = 0 \quad T_x = 100 \text{ N}\cdot\text{m}$$

$$\Sigma M_y = 0; \quad M_y + 800(0.3) = 0 \quad M_y = -240 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0; \quad M_z + 600(0.3) = 0 \quad M_z = -180 \text{ N}\cdot\text{m}$$

(a)

Section Properties: For the circular cross section,

$$A = \pi c^2 = \pi(0.02^2) = 0.400(10^{-3})\pi \text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.02^4) = 40.0(10^{-9})\pi \text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.02^4) = 80.0(10^{-9})\pi \text{ m}^4$$

$$(Q_B)_z = \bar{z}'A' = \frac{4(0.02)}{3\pi}\left[\frac{\pi}{2}(0.02^2)\right] = 5.3333(10^{-6}) \text{ m}^3 \quad (Q_B)_y = 0$$

(b)

Normal Stress: For the combined loadings, the normal stress at point B can be determined from

$$\sigma_x = \sigma_B = \frac{N_x}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$= \frac{1500}{0.400(10^{-3})\pi} - \frac{(-180)(-0.02)}{40.0(10^{-9})\pi} + \frac{(-240)(0)}{40.0(10^{-9})\pi}$$

$$= -27.45(10^6) = 27.5 \text{ MPa (C)} \qquad \textbf{Ans.}$$

Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be obtained using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$, respectively.

$$(\tau_{xz})_B = (\tau_V)_z - \tau_T$$

$$= \frac{-800[5.3333(10^{-6})]}{40.0(10^{-9})\pi(0.04)} - \frac{100(0.02)}{80.0(10^{-9})\pi}$$

$$= -8.807(10^6) \text{ Pa} = -8.81 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_B = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$

(c)

Using these results, the state of stress at point B, can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_B = 27.5$ MPa (C),
$(\tau_{xz})_B = -8.81$ MPa,
$(\tau_{xy})_B = 0$

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13–42.

The beveled gear is subjected to the loads shown. Determine the stress components acting on the shaft at point A, and show the results on a volume element located at this point. The shaft has a diameter of 1 in. and is fixed to the wall at C.

SOLUTION

$\Sigma F_x = 0; \quad V_x - 125 = 0; \quad V_x = 125 \text{ lb}$

$\Sigma F_y = 0; \quad 75 - N_y = 0; \quad N_y = 75 \text{ lb}$

$\Sigma F_z = 0; \quad V_z - 200 = 0; \quad V_z = 200 \text{ lb}$

$\Sigma M_x = 0; \quad 200(8) - M_x = 0; \quad M_x = 1600 \text{ lb} \cdot \text{in.}$

$\Sigma M_y = 0; \quad 200(3) - T_y = 0; \quad T_y = 600 \text{ lb} \cdot \text{in.}$

$\Sigma M_z = 0; \quad M_z + 75(3) - 125(8) = 0; \quad M_z = 775 \text{ lb} \cdot \text{in.}$

$A = \pi(0.5^2) = 0.7854 \text{ in}^2$

$J = \frac{\pi}{2}(0.5^4) = 0.098175 \text{ in}^4$

$I = \frac{\pi}{2}(0.5^4) = 0.049087 \text{ in}^4$

$(Q_A)_x = 0$

$(Q_A)_z = \frac{4(0.5)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.5^2) = 0.08333 \text{ in}^3$

$(\sigma_A)_y = -\frac{N_y}{A} + \frac{M_x c}{I}$

$= -\frac{75}{0.7854} + \frac{1600(0.5)}{0.049087}$

$= 16202 \text{ psi} = 16.2 \text{ ksi (T)}$ **Ans.**

$(\tau_A)_{yx} = (\tau_A)_V - (\tau_A)_{\text{twist}}$

$= \frac{V_x(Q_A)_Z}{I\,t} - \frac{T_y c}{J}$

$= \frac{125(0.08333)}{0.049087(1)} - \frac{600(0.5)}{0.098175}$

$= -2843 \text{ psi} = -2.84 \text{ ksi}$ **Ans.**

$(\tau_A)_{yz} = \frac{V_z(Q_A)_x}{It} = 0$ **Ans.**

Ans:
$(\sigma_A)_y = 16.2 \text{ ksi (T)}$,
$(\tau_A)_{yx} = -2.84 \text{ ksi}$,
$(\tau_A)_{yz} = 0$

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13–43.

The beveled gear is subjected to the loads shown. Determine the stress components acting on the shaft at point B, and show the results on a volume element located at this point. The shaft has a diameter of 1 in. and is fixed to the wall at C.

SOLUTION

$\Sigma F_x = 0; \quad V_x - 125 = 0; \quad V_x = 125 \text{ lb}$

$\Sigma F_y = 0; \quad 75 - N_y = 0; \quad N_y = 75 \text{ lb}$

$\Sigma F_z = 0; \quad V_z - 200 = 0; \quad V_z = 200 \text{ lb}$

$\Sigma M_x = 0; \quad 200(8) - M_x = 0; \quad M_x = 1600 \text{ lb} \cdot \text{in.}$

$\Sigma M_y = 0; \quad 200(3) - T_y = 0; \quad T_y = 600 \text{ lb} \cdot \text{in.}$

$\Sigma M_z = 0; \quad M_z + 75(3) - 125(8) = 0; \quad M_z = 775 \text{ lb} \cdot \text{in.}$

$A = \pi(0.5^2) = 0.7854 \text{ in}^2$

$J = \frac{\pi}{2}(0.5^4) = 0.098175 \text{ in}^4$

$I = \frac{\pi}{2}(0.5^4) = 0.049087 \text{ in}^4$

$(Q_B)_z = 0$

$(Q_B)_x = \frac{4(0.5)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.5^2) = 0.08333 \text{ in}^3$

$(\sigma_B)_y = -\frac{P_y}{A} + \frac{M_z c}{I}$

$= -\frac{75}{0.7854} + \frac{775(0.5)}{0.049087}$

$= 7.80 \text{ ksi (T)}$ **Ans.**

$(\tau_B)_{yz} = (\tau_B)_V + (\tau_B)_{\text{twist}}$

$= \frac{V_z(Q_B)_x}{It} - \frac{T_y c}{J}$

$= \frac{200(0.08333)}{0.049087\,(1)} + \frac{600(0.5)}{0.098175}$

$= 3395 \text{ psi} = 3.40 \text{ ksi}$ **Ans.**

$(\tau_B)_{yx} = \frac{V_x(Q_B)_z}{It} = 0$ **Ans.**

Ans:
$(\sigma_B)_y = 7.80 \text{ ksi (T)}$,
$(\tau_B)_{yz} = 3.40 \text{ ksi}$,
$(\tau_B)_{yx} = 0$

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***13–44.**

Determine the normal-stress developed at points A and B. Neglect the weight of the block.

SOLUTION

Referring to Fig. a,

$\Sigma F_x = (F_R)_x;\qquad -6 - 12 = F \qquad F = -18.0 \text{ kip}$

$\Sigma M_y = (M_R)_y;\qquad 6(1.5) - 12(1.5) = M_y \qquad M_y = -9.00 \text{ kip}\cdot\text{in}$

$\Sigma M_z = (M_R)_z;\qquad 12(3) - 6(3) = M_z \qquad M_z = 18.0 \text{ kip}\cdot\text{in}$

The cross-sectional area and moments of inertia about the y and z axes of the cross section are

$$A = 6(3) = 18 \text{ in}^2$$
$$I_y = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$$
$$I_z = \frac{1}{12}(3)(6^3) = 54.0 \text{ in}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{F}{A} - \frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

For point A, $y = 3$ in. and $z = -1.5$ in.

$$\sigma_A = \frac{-18.0}{18.0} - \frac{18.0(3)}{54.0} + \frac{-9.00(-1.5)}{13.5}$$
$$= -1.00 \text{ ksi} = 1.00 \text{ ksi (C)} \qquad \textbf{Ans.}$$

For point B, $y = 3$ in and $z = 1.5$ in.

$$\sigma_B = \frac{-18.0}{18.0} - \frac{18.0(3)}{54} + \frac{-9.00(1.5)}{13.5}$$
$$= -3.00 \text{ ksi} = 3.00 \text{ ksi (C)} \qquad \textbf{Ans.}$$

(a)

Ans:
$\sigma_A = 1.00$ ksi (C),
$\sigma_B = 3.00$ ksi (C)

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13–45.

Sketch the normal-stress distribution acting over the cross section at section *a–a.* Neglect the weight of the block.

SOLUTION

Referring to Fig. *a*,

$\Sigma F_x = (F_R)_x; \quad -6 - 12 = F \qquad F = -18.0 \text{ kip}$

$\Sigma M_y = (M_R)_y; \quad 6(1.5) - 12(1.5) = M_y \qquad M_y = -9.00 \text{ kip} \cdot \text{in}$

$\Sigma M_z = (M_R)_z; \quad 12(3) - 6(3) = M_z \qquad M_z = 18.0 \text{ kip} \cdot \text{in}$

The cross-sectional area and the moment of inertia about the y and z axes of the cross section are

$$A = 3\,(6) = 18.0 \text{ in}^2$$

$$I_y = \frac{1}{12}(6)(3^3) = 13.5 \text{ in}^4$$

$$I_z = \frac{1}{12}(3)(6^3) = 54.0 \text{ in}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{F}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

For point A, $y = 3$ in. and $z = -1.5$ in.

$$\sigma_A = \frac{-18.0}{18.0} - \frac{18.0(3)}{54.0} + \frac{-9.00(-1.5)}{13.5}$$

$$= -1.00 \text{ ksi} = 1.00 \text{ ksi (C)} \qquad \textbf{Ans.}$$

For point B, $y = 3$ in. and $z = 1.5$ in.

$$\sigma_B = \frac{-18.0}{18.0} - \frac{18.0(3)}{54.0} + \frac{-9.00(1.5)}{13.5}$$

$$= -3.00 \text{ ksi} = 3.00 \text{ ksi (C)} \qquad \textbf{Ans.}$$

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13–45. Continued

For point C, $y = -3$ in. and $z = 1.5$ in.

$$\sigma_C = \frac{-18.0}{18.0} - \frac{18.0(-3)}{54.0} + \frac{-9.00(1.5)}{13.5}$$

$$= -1.00 \text{ ksi} = 1.00 \text{ ksi (C)} \qquad \textbf{Ans.}$$

For point D, $y = -3$ in. and $z = -1.5$ in.

$$\sigma_D = \frac{-18.0}{18.0} - \frac{18.0(-3)}{54.0} + \frac{-9.00(-1.5)}{13.5}$$

$$= 1.00 \text{ ksi (T)} \qquad \textbf{Ans.}$$

The normal-stress distribution over the cross section is shown in Fig. b.

(a)

(b)

Ans:
$\sigma_A = 1.00$ ksi (C), $\sigma_B = 3.00$ ksi (C),
$\sigma_C = 1.00$ ksi (C), $\sigma_D = 1.00$ ksi (T)

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13–46.

The vertebra of the spinal column can support a maximum compressive stress of σ_{max}, before undergoing a compression fracture. Determine the smallest force P that can be applied to a vertebra if we assume this load is applied at an eccentric distance e from the centerline of the bone, and the bone remains elastic. Model the vertebra as a hollow cylinder with an inner radius r_i and outer radius r_o.

SOLUTION

$$\sigma_{max} = \frac{P}{A} + \frac{Per_o}{\frac{\pi}{4}(r_o^4 - r_i^4)}$$

$$\sigma_{max} = P\left[\frac{1}{\pi(r_o^2 - r_i^2)} + \frac{4er_o}{\pi(r_o^4 - r_i^4)}\right]$$

$$\sigma_{max} = \frac{P}{\pi(r_o^2 - r_i^2)}\left[1 + \frac{4er_o}{(r_o^2 + r_i^2)}\right]$$

$$\sigma_{max} = \frac{P(r_o^2 + r_i^2 + 4er_o)}{\pi(r_o^2 - r_i^2)(r_o^2 + r_i^2)}$$

$$\sigma_{max} = \frac{P(r_o^2 + r_i^2 + 4er_o)}{\pi(r_o^4 - r_i^4)}$$

$$P = \frac{\delta_{max}\pi(r_o^4 - r_i^4)}{r_o^2 + r_i^2 + 4er_o}$$ **Ans.**

Ans:
$$P = \frac{\delta_{max}\pi(r_o^4 - r_i^4)}{r_o^2 + r_i^2 + 4er_o}$$

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13–47.

The solid rod is subjected to the loading shown. Determine the state of stress at point A, and show the results on a differential volume element located at this point.

SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \qquad N_x - 100 = 0 \qquad N_x = 100 \text{ kN}$$

$$\Sigma F_y = 0; \qquad V_y - 10 = 0 \qquad V_y = 10 \text{ kN}$$

$$\Sigma F_z = 0; \qquad V_z + 20 = 0 \qquad V_z = -20 \text{ kN}$$

$$\Sigma M_x = 0; \qquad T_x + 20(0.03) + 10(0.03) = 0 \qquad T_x = -0.9 \text{ kN}\cdot\text{m}$$

$$\Sigma M_y = 0; \qquad M_y + 20(0.2) + 100(0.03) = 0 \qquad M_y = -7.00 \text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z + 10(0.4) = 0 \qquad M_z = -4.00 \text{ kN}\cdot\text{m}$$

(a)

Section Properties: For the circular cross section, Fig. *b*,

$$A = \pi c^2 = \pi(0.03^2) = 0.9(10^{-3})\pi \text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\pi \text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi \text{ m}^4$$

$$(Q_A)_z = \bar{z}'A' = \frac{4(0.03)}{3\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 18.0(10^{-6}) \text{ m}^3$$

$$(Q_A)_y = 0$$

(b)

Normal Stress: For the combined loadings, the normal stress at point A can be determined from

$$\sigma_x = \sigma_A = \frac{N_x}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$= \frac{100(10^3)}{0.9(10^{-3})\pi} - \frac{[-4.00(10^3)](0.03)}{0.2025(10^{-6})\pi} + \frac{[-7.00(10^3)](0)}{0.2025(10^{-6})\pi}$$

$$= 224.00(10^6) \text{ Pa} = 224 \text{ MPa (T)} \qquad \textbf{Ans.}$$

(c)

Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be obtained using the shear formula $\tau_V = \dfrac{VQ}{It}$ and the torsion formula $\tau_T = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xz})_A = (\tau_V)_z + \tau_T$$

$$= \frac{-20(10^3)[18.0(10^{-6})]}{0.2025(10^{-6})\pi(0.06)} + \frac{-0.9(10^3)(0.03)}{0.405(10^{-6})\pi}$$

$$= -30.65(10^6) \text{ Pa} = -30.7 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_A = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point A can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_A = 224$ MPa (T),
$(\tau_{xz})_A = -30.7$ MPa,
$(\tau_{xy})_A = 0$

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***13–48.**

The solid rod is subjected to the loading shown. Determine the state of stress at point B, and show the results on a differential volume element at this point.

SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \qquad N_x - 100 = 0 \qquad N_x = 100 \text{ kN}$$

$$\Sigma F_y = 0; \qquad V_y - 10 = 0 \qquad V_y = 10 \text{ kN}$$

$$\Sigma F_z = 0; \qquad V_z + 20 = 0 \qquad V_z = -20 \text{ kN}$$

$$\Sigma M_x = 0; \qquad T_x + 20(0.03) + 10(0.03) = 0 \qquad T_x = -0.9 \text{ kN}\cdot\text{m}$$

$$\Sigma M_y = 0; \qquad M_y + 20(0.2) + 100(0.03) = 0 \qquad M_y = -7.00 \text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z + 10(0.4) = 0 \qquad M_z = -4.00 \text{ kN}\cdot\text{m}$$

(a)

Section Properties: For the circular cross section, Fig. *b*,

$$A = \pi c^2 = \pi(0.03^2) = 0.9(10^{-3})\pi \text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\pi \text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi \text{ m}^4$$

$$(Q_B)_z = \bar{z}'A' = \frac{4(0.03)}{3\pi}\left[\frac{\pi}{2}(0.03^2)\right] = 18.0(10^{-6}) \text{ m}^3$$

$$(Q_B)_y = 0$$

(b)

Normal Stress: For the combined loadings, the normal stress at point B can be determined from

$$\sigma_x = \sigma_B = \frac{N_x}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y}$$

$$= \frac{100(10^3)}{0.9(10^{-3})\pi} - \frac{[-4.00(10^3)](-0.03)}{0.2025(10^{-6})\pi} + \frac{[-7.00(10^3)](0)}{0.2025(10^{-6})\pi}$$

$$= -153.26(10^6) \text{ Pa} = 153 \text{ MPa (C)} \qquad \textbf{Ans.}$$

(c)

Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be determined using the shear formula $\tau_V = \frac{VQ}{It}$ and the torsion formula $\tau_T = \frac{T\rho}{J}$, respectively.

$$(\tau_{xz})_B = -\tau_T + (\tau_V)_z$$

$$= \frac{0.9(10^3)(0.03)}{0.405(10^{-6})\pi} + \frac{-20(10^3)[18.0(10^{-6})]}{0.2025(10^{-6})\pi(0.06)}$$

$$= 11.79(10^6) \text{ Pa} = 11.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_B = (\tau_V)_y = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point B can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_B = 153$ MPa (C),
$(\tau_{xz})_B = 11.8$ MPa,
$(\tau_{xy})_B = 0$

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13–49.

The solid rod is subjected to the loading shown. Determine the state of stress at point C, and show the results on a differential volume element at this point.

SOLUTION

Internal Loadings: Consider the equilibrium of the left segment of the rod being sectioned, Fig. *a*.

$$\Sigma F_x = 0; \qquad N_x - 100 = 0 \qquad N_x = 100\text{ kN}$$

$$\Sigma F_y = 0; \qquad V_y - 10 = 0 \qquad V_y = 10\text{ kN}$$

$$\Sigma F_z = 0; \qquad V_z + 20 = 0 \qquad V_z = -20\text{ kN}$$

$$\Sigma M_x = 0; \qquad T_x + 20(0.03) + 10(0.03) = 0 \qquad T_x = -0.9\text{ kN}\cdot\text{m}$$

$$\Sigma M_y = 0; \qquad M_y + 20(0.2) + 100(0.03) = 0 \qquad M_y = -7.00\text{ kN}\cdot\text{m}$$

$$\Sigma M_z = 0; \qquad M_z + 10(0.4) = 0 \qquad M_z = -4.00\text{ kN}\cdot\text{m}$$

(a)

Section Properties: For the circular cross section, Fig. *b*,

$$A = \pi c^2 = \pi(0.03^2) = 0.9(10^{-3})\pi\text{ m}^2$$

$$I_y = I_z = \frac{\pi}{4}c^4 = \frac{\pi}{4}(0.03^4) = 0.2025(10^{-6})\pi\text{ m}^4$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi\text{ m}^4$$

$$(Q_C)_y = \bar{y}'A' = \left[\frac{4(0.03)}{3\pi}\right]\left[\frac{\pi}{2}(0.03^2)\right] = 18.0(10^{-6})\text{ m}^3$$

$$(Q_C)_z = 0$$

(b)

Normal Stress: For the combined loadings, the normal stress at point C can be determined from

$$\sigma_x = \sigma_C = \frac{N_x}{A} - \frac{M_z y_C}{I_z} + \frac{M_y z_C}{I_y}$$

$$= \frac{100(10^3)}{0.9(10^{-3})\pi} - \frac{[-4.00(10^3)](0)}{0.2025(10^{-6})\pi} + \frac{[-7.00(10^3)](0.03)}{0.2025(10^{-6})\pi}$$

$$= -294.73(10^6)\text{ Pa} = 295\text{ MPa (C)} \qquad \textbf{Ans.}$$

(c)

Shear Stress: The transverse shear stress in z and y directions and the torsional shear stress can be determined using the shear formula $\tau_V = \dfrac{VQ}{It}$ and the torsion formula $\tau_T = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xy})_C = (\tau_V)_y - \tau_T$$

$$= \frac{10(10^3)[18.0(10^{-6})]}{0.2025(10^{-6})\pi(0.06)} - \frac{-0.9(10^3)(0.03)}{0.405(10^{-6})\pi}$$

$$= 25.94(10^6)\text{ Pa} = 25.9\text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{xz})_C = (\tau_V)_z = 0 \qquad \textbf{Ans.}$$

Using these results, the state of stress at point C can be represented by the differential volume element shown in Fig. *c*.

Ans:
$\sigma_C = 295$ MPa (C),
$(\tau_{xy})_C = 25.9$ MPa,
$(\tau_{xz})_C = 0$

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13–50.

The C-frame is used in a riveting machine. If the force at the ram on the clamp at D is $P = 8$ kN, sketch the stress distribution acting over the section a–a.

SOLUTION

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{(0.005)(0.04)(0.01) + 0.04(0.06)(0.01)}{0.04(0.01) + 0.06(0.01)} = 0.026 \text{ m}$$

$$A = 0.04(0.01) + 0.06(0.01) = 0.001 \text{ m}^2$$

$$I = \frac{1}{12}(0.04)(0.01^3) + (0.04)(0.01)(0.026 - 0.005)^2 + \frac{1}{12}(0.01)(0.06^3) + 0.01(0.06)(0.040 - 0.026)^2 = 0.4773(10^{-6}) \text{ m}^4$$

$$(\sigma_{\max})_t = \frac{P}{A} + \frac{Mx}{I} = \frac{8(10^3)}{0.001} + \frac{1.808(10^3)0.026}{0.4773(10^{-6})}$$

$$= 106.48 \text{ MPa} = 106 \text{ MPa}$$ **Ans.**

$$(\sigma_{\max})_c = \frac{P}{A} - \frac{Mc}{I} = \frac{8(10^3)}{0.001} - \frac{1.808(10^3)(0.070 - 0.026)}{0.4773(10^{-6})}$$

$$= -158.66 \text{ MPa} = -159 \text{ MPa}$$ **Ans.**

$$\frac{x}{158.66} = \frac{70 - x}{106.48}; \qquad x = 41.9 \text{ mm}$$

Ans:
$(\sigma_{\max})_t = 106$ MPa, $(\sigma_{\max})_c = -159$ MPa

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13–51.

Determine the maximum ram force P that can be applied to the clamp at D if the allowable normal stress for the material is $\sigma_{allow} = 180$ MPa.

SOLUTION

$$\bar{x} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{(0.005)(0.04)(0.01) + 0.04(0.06)(0.01)}{0.04(0.01) + 0.06(0.01)} = 0.026 \text{ m}$$

$$A = 0.04(0.01) + 0.06(0.01) = 0.001 \text{ m}^2$$

$$I = \frac{1}{12}(0.04)(0.01^3) + (0.04)(0.01)(0.026 - 0.005)^2 + \frac{1}{12}(0.01)(0.06^3) + 0.01(0.06)(0.040 - 0.026)^2 = 0.4773(10^{-6}) \text{ m}^4$$

$$\sigma = \frac{P}{A} \pm \frac{Mx}{I}$$

Assume tension failure.

$$180(10^6) = \frac{P}{0.001} + \frac{0.226P(0.026)}{0.4773(10^{-6})}$$

$$P = 13524 \text{ N} = 13.5 \text{ kN}$$

Assume compression failure.

$$-180(10^6) = \frac{P}{0.001} - \frac{0.226P(0.070 - 0.026)}{0.4773(10^{-6})}$$

$$P = 9076 \text{ N} = 9.08 \text{ kN (controls)}$$ **Ans.**

Ans:
$P_{max} = 9.08$ kN

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***13–52.**

The uniform sign has a weight of 1500 lb and is supported by the pipe *AB*, which has an inner radius of 2.75 in. and an outer radius of 3.00 in. If the face of the sign is subjected to a uniform wind pressure of $p = 150\ \text{lb/ft}^2$, determine the state of stress at points *C* and *D*. Show the results on a differential volume element located at each of these points. Neglect the thickness of the sign, and assume that it is supported along the outside edge of the pipe.

SOLUTION

Internal Forces and Moments: As shown on FBD.

$\Sigma F_x = 0; \quad 1.50 + N_x = 0 \quad N_x = -15.0\ \text{kip}$

$\Sigma F_y = 0; \quad V_y - 10.8 = 0 \quad V_y = 10.8\ \text{kip}$

$\Sigma F_z = 0; \quad V_z = 0$

$\Sigma M_x = 0; \quad T_x - 10.8(6) = 0 \quad T_x = 64.8\ \text{kip}\cdot\text{ft}$

$\Sigma M_y = 0; \quad M_y - 1.50(6) = 0 \quad M_y = 9.00\ \text{kip}\cdot\text{ft}$

$\Sigma M_z = 0; \quad 10.8(6) + M_z = 0 \quad M_z = -64.8\ \text{kip}\cdot\text{ft}$

Section Properties:

$$A = \pi\left(3^2 - 2.75^2\right) = 1.4375\pi\ \text{in}^2$$

$$I_y = I_z = \frac{\pi}{4}\left(3^4 - 2.75^4\right) = 18.6992\ \text{in}^4$$

$$(Q_C)_z = (Q_D)_y = 0$$

$$(Q_C)_y = (Q_D)_z = \frac{4(3)}{3\pi}\left[\frac{1}{2}(\pi)\left(3^2\right)\right] - \frac{4(2.75)}{3\pi}\left[\frac{1}{2}(\pi)\left(2.75^2\right)\right]$$

$$= 4.13542\ \text{in}^3$$

$$J = \frac{\pi}{2}\left(3^4 - 2.75^4\right) = 37.3984\ \text{in}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_C = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(0)}{18.6992} + \frac{9.00(12)(2.75)}{18.6992}$$

$$= 15.6\ \text{ksi (T)}$$ **Ans.**

$$\sigma_D = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(3)}{18.6992} + \frac{9.00(12)(0)}{18.6992}$$

$$= 124\ \text{ksi (T)}$$ **Ans.**

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***13–52. Continued**

Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula.

$\tau_V = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xz})_D = \tau_{\text{twist}} = \frac{64.8(12)(3)}{37.3984} = 62.4 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_D = \tau_{V_y} = 0 \qquad \textbf{Ans.}$$

$$(\tau_{xy})_C = \tau_{V_y} - \tau_{\text{twist}}$$

$$= \frac{10.8(4.13542)}{18.6992(2)(0.25)} - \frac{64.8(12)(2.75)}{37.3984}$$

$$= -52.4 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{xz})_C = \tau_{V_z} = 0 \qquad \textbf{Ans.}$$

Ans:
$\sigma_C = 15.6$ ksi (T), $\sigma_D = 124$ ksi (T),
$(\tau_{xz})_D = 62.4$ ksi, $(\tau_{xy})_D = 0$,
$(\tau_{xy})_C = -52.4$ ksi, $(\tau_{xz})_C = 0$

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13–53.

Solve Prob. 13–52 for points E and F.

SOLUTION

Internal Forces and Moments: As shown on FBD.

$$\Sigma F_x = 0; \quad 1.50 + N_x = 0 \qquad N_x = -1.50 \text{ kip}$$

$$\Sigma F_y = 0; \quad V_y - 10.8 = 0 \qquad V_y = 10.8 \text{ kip}$$

$$\Sigma F_z = 0; \quad V_z = 0$$

$$\Sigma M_x = 0; \quad T_x - 10.8(6) = 0 \qquad T_x = 64.8 \text{ kip}\cdot\text{ft}$$

$$\Sigma M_y = 0; \quad M_y - 1.50(6) = 0 \qquad M_y = 9.00 \text{ kip}\cdot\text{ft}$$

$$\Sigma M_z = 0; \quad 10.8(6) + M_z = 0 \qquad M_z = -64.8 \text{ kip}\cdot\text{ft}$$

Section Properties:

$$A = \pi\left(3^2 - 2.75^2\right) = 1.4375\pi \text{ in}^2$$

$$I_y = I_z = \frac{\pi}{4}\left(3^4 - 2.75^4\right) = 18.6992 \text{ in}^4$$

$$(Q_F)_z = (Q_E)_y = 0$$

$$(Q_F)_y = (Q_E)_z = \frac{4(3)}{3\pi}\left[\frac{1}{2}(\pi)\left(3^2\right)\right] - \frac{4(2.75)}{3\pi}\left[\frac{1}{2}(\pi)\left(2.75^2\right)\right]$$

$$= 4.13542 \text{ in}^3$$

$$J = \frac{\pi}{2}\left(3^4 - 2.75^4\right) = 37.3984 \text{ in}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_F = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(0)}{18.6992} + \frac{9.00(12)(-3)}{18.6992}$$

$$= -17.7 \text{ ksi} = 17.7 \text{ ksi (C)}$$ **Ans.**

$$\sigma_E = \frac{-1.50}{1.4375\pi} - \frac{(-64.8)(12)(-3)}{18.6992} + \frac{9.00(12)(0)}{18.6992}$$

$$= -125 \text{ ksi} = 125 \text{ ksi (C)}$$ **Ans.**

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13–53. Continued

Shear Stress: The tranverse shear stress in the z and y directions and the torsional shear stress can be obtained using the shear formula and the torsion formula, $\tau_V = \dfrac{VQ}{It}$ and $\tau_{\text{twist}} = \dfrac{T\rho}{J}$, respectively.

$$(\tau_{xz})_E = -\tau_{\text{twist}} = -\frac{64.8(12)(3)}{37.3984} = -62.4 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{xy})_E = \tau_{V_y} = 0 \qquad \textbf{Ans.}$$

$$(\tau_{xy})_F = \tau_{V_y} + \tau_{\text{twist}}$$

$$= \frac{10.8(4.13542)}{18.6992(2)(0.25)} + \frac{64.8(12)(3)}{37.3984}$$

$$= 67.2 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{xz})_F = \tau_{V_y} = 0 \qquad \textbf{Ans.}$$

Ans:
$\sigma_F = 17.7$ ksi (C),
$\sigma_E = 125$ ksi (C),
$(\tau_{xz})_E = -62.4$ ksi,
$(\tau_{xy})_E = 0$,
$(\tau_{xy})_F = 67.2$ ksi,
$(\tau_{xz})_F = 0$

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R13–1.

The post has a circular cross section of radius c. Determine the maximum radius e at which the load **P** can be applied so that no part of the post experiences a tensile stress. Neglect the weight of the post.

SOLUTION

$$\int \frac{dA}{r} = 2\pi\left(3.125 - \sqrt{(3.125)^2 - (0.625)^2}\right) = 0.395707$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\pi(0.625)^2}{0.396707} = 3.09343 \text{ in.}$$

$$M = 800(3.09343) = 2.475(10^3)$$

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)} + \frac{P}{A}$$

$$(\sigma_t)_{\max} = \frac{2.475(10^3)(3.09343 - 2.5)}{\pi(0.625)^2(2.5)(3.125 - 3.09343)} + \frac{800}{\pi(0.625)^2} = 15.8 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\sigma_c)_{\max} = \frac{2.475(10^3)(3.09343 - 3.75)}{\pi(0.625)^2(3.75)(3.125 - 3.09343)} + \frac{800}{\pi(0.625)^2} = -10.5 \text{ ksi} \qquad \textbf{Ans.}$$

Ans:
$(\sigma_t)_{\max} = 15.8$ ksi, $(\sigma_c)_{\max} = -10.5$ ksi

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R13–2.

The 20-kg drum is suspended from the hook mounted on the wooden frame. Determine the state of stress at point E on the cross section of the frame at section a–a. Indicate the results on an element.

SOLUTION

Support Reactions: Referring to the free-body diagram of member BC shown in Fig. a,

$\circlearrowleft + \Sigma M_B = 0;\qquad F\sin 45°(1) - 20(9.81)(2) = 0 \qquad F = 554.94\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0;\qquad 554.94\cos 45° - B_x = 0 \qquad B_x = 392.4\text{ N}$

$+\uparrow \Sigma F_y = 0;\qquad 554.94\sin 45° - 20(9.81) - B_y = 0 \qquad B_y = 196.2\text{ N}$

Internal Loadings: Consider the equilibrium of the free-body diagram of the right segment shown in Fig. b.

$\xrightarrow{+} \Sigma F_x = 0;\qquad N - 392.4 = 0 \qquad N = 392.4\text{ N}$

$+\uparrow \Sigma F_y = 0;\qquad V - 196.2 = 0 \qquad V = 196.2\text{ N}$

$\circlearrowleft + \Sigma M_C = 0;\qquad 196.2(0.5) - M = 0 \qquad M = 98.1\text{ N}\cdot\text{m}$

Section Properties: The cross-sectional area and the moment of inertia of the cross section are

$$A = 0.05(0.075) = 3.75\left(10^{-3}\right)\text{ m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.075^3\right) = 1.7578\left(10^{-6}\right)\text{ m}^4$$

Referring to Fig. c, Q_E is

$$Q_E = \bar{y}'A' = 0.025(0.025)(0.05) = 31.25\left(10^{-6}\right)\text{ m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point E, $y = 0.0375 - 0.025 = 0.0125\text{ m}$. Then

$$\sigma_E = \frac{392.4}{3.75\left(10^{-3}\right)} + \frac{98.1(0.0125)}{1.7578\left(10^{-6}\right)} = 802\text{ kPa} \qquad \textbf{Ans.}$$

Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

$$\tau_E = \frac{VQ_A}{It} = \frac{196.2\left[31.25\left(10^{-6}\right)\right]}{1.7578\left(10^{-6}\right)(0.05)} = 69.8\text{ kPa} \qquad \textbf{Ans.}$$

The state of stress at point E is represented on the element shown in Fig. d.

Ans:
$\sigma_E = 802\text{ kPa},\ \tau_E = 69.8\text{ kPa}$

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R13–3.

The 20-kg drum is suspended from the hook mounted on the wooden frame. Determine the state of stress at point F on the cross section of the frame at section b–b. Indicate the results on an element.

SOLUTION

Support Reactions: Referring to the free-body diagram of the entire frame shown in Fig. a,

$\circlearrowleft + \Sigma M_A = 0;\quad F_{BD}\sin 30°(3) - 20(9.81)(2) = 0 \qquad F_{BD} = 261.6\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad A_y - 261.6\cos 30° - 20(9.81) = 0 \qquad A_y = 422.75\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 261.6\sin 30° = 0 \qquad A_x = 130.8\text{ N}$

Internal Loadings: Consider the equilibrium of the free-body diagram of the lower cut segment, Fig. b.

$\xrightarrow{+} \Sigma F_x = 0;\quad 130.8 - V = 0 \qquad V = 130.8\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad 422.75 - N = 0 \qquad N = 422.75\text{ N}$

$\circlearrowleft + \Sigma M_C = 0;\quad 130.8(1) - M = 0 \qquad M = 130.8\text{ N}\cdot\text{m}$

Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the cross section are

$$A = 0.075(0.075) = 5.625\left(10^{-3}\right)\text{ m}^2$$

$$I = \frac{1}{12}(0.075)\left(0.075^3\right) = 2.6367\left(10^{-6}\right)\text{ m}^4$$

Referring to Fig. c, Q_F is

$$Q_F = \bar{y}'A' = 0.025(0.025)(0.075) = 46.875\left(10^{-6}\right)\text{ m}^3$$

Normal Stress: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point F, $y = 0.0375 - 0.025 = 0.0125$ m. Then

$$\sigma_F = \frac{-422.75}{5.625\left(10^{-3}\right)} - \frac{130.8(0.0125)}{2.6367\left(10^{-6}\right)}$$

$$= -695.24\text{ kPa} = 695\text{ kPa (C)} \qquad \textbf{Ans.}$$

Shear Stress: The shear stress is contributed by transverse shear stress only. Thus,

$$\tau_F = \frac{VQ_F}{It} = \frac{130.8\left[46.875\left(10^{-6}\right)\right]}{2.6367\left(10^{-6}\right)(0.075)} = 31.0\text{ kPa} \qquad \textbf{Ans.}$$

The state of stress at point F is represented on the element shown in Fig. d.

Ans:
$\sigma_F = 695$ kPa (C),
$\tau_F = 31.0$ kPa

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***R13–4.**

The gondola and passengers have a weight of 1500 lb and center of gravity at G. The suspender arm AE has a square cross-sectional area of 1.5 in. by 1.5 in., and is pin connected at its ends A and E. Determine the largest tensile stress developed in regions AB and DC of the arm.

SOLUTION

Segment AB:

$$(\sigma_{\max})_{AB} = \frac{P_{AB}}{A} = \frac{1500}{(1.5)(1.5)} = 667 \text{ psi}$$ **Ans.**

Segment CD:

$$\sigma_a = \frac{P_{CD}}{A} = \frac{1500}{(1.5)(1.5)} = 666.67 \text{ psi}$$

$$\sigma_b = \frac{Mc}{I} = \frac{1875(12)(0.75)}{\frac{1}{12}(1.5)(1.5^3)} = 40\,000 \text{ psi}$$

$$(\sigma_{\max})_{CD} = \sigma_a + \sigma_b = 666.67 + 40\,000$$

$$= 40\,666.67 \text{ psi} = 40.7 \text{ ksi}$$ **Ans.**

Ans:
$(\sigma_{\max})_{AB} = 667$ psi, $(\sigma_{\max})_{CD} = 40.7$ ksi

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R13–5.

If the cross section of the femur at section *a–a* can be approximated as a circular tube as shown, determine the maximum normal stress developed on the cross section at section *a–a* due to the load of 75 lb.

SOLUTION

Internal Loadings: Considering the equilibrium for the free-body diagram of the femur's upper segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad N - 75 = 0 \qquad N = 75\,\text{lb}$

$\circlearrowleft + \Sigma M_O = 0; \qquad M - 75(2) = 0 \qquad M = 150\,\text{lb}\cdot\text{in}$

Section Properties: The cross-sectional area and the moment of inertia about the centroidal axis of the femur's cross section are

$$A = \pi\left(1^2 - 0.5^2\right) = 0.75\pi\ \text{in}^2$$

$$I = \frac{\pi}{4}\left(1^4 - 0.5^4\right) = 0.234375\pi\ \text{in}^4$$

Normal Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

By inspection, the maximum normal stress is in compression.

$$\sigma_{\max} = \frac{-75}{0.75\pi} - \frac{150(1)}{0.234375\pi} = -236\ \text{psi} = 236\ \text{psi (C)} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{\max} = 236$ psi (C)

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R13–6.

A bar having a square cross section of 30 mm by 30 mm is 2 m long and is held upward. If it has a mass of 5 kg/m, determine the largest angle θ, measured from the vertical, at which it can be supported before it is subjected to a tensile stress along its axis near the grip.

SOLUTION

$A = 0.03(0.03) = 0.9(10^{-3})\ \text{m}^2$

$I = \frac{1}{12}(0.03)(0.03^3) = 67.5(10^{-9})\ \text{m}^4$

Require $\sigma_A = 0$.

$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}$

$0 = \frac{-98.1\cos\theta}{0.9(10^{-3})} + \frac{98.1\sin\theta(0.015)}{67.5(10^{-9})}$

$0 = -1111.11\cos\theta + 222222.22\sin\theta$

$\tan\theta = 0.005; \qquad \theta = 0.286°$ **Ans.**

Ans:
$\theta = 0.286°$

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R13–7.

The wall hanger has a thickness of 0.25 in. and is used to support the vertical reactions of the beam that is loaded as shown. If the load is transferred uniformly to each strap of the hanger, determine the state of stress at points C and D on the strap at A. Assume the vertical reaction **F** at this end acts in the center and on the edge of the bracket as shown.

SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \qquad 12(3) + 10(8) - F_A(10) = 0$

$$F_A = 11.60 \text{ kip}$$

$$I = 2\left[\frac{1}{12}(0.25)(2)^3\right] = 0.333 \text{ in}^4$$

$$A = 2(0.25)(2) = 1 \text{ in}^2$$

At point C,

$$\sigma_C = \frac{P}{A} = \frac{2(5.80)}{1} = 11.6 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_C = 0 \qquad \textbf{Ans.}$$

At point D,

$$\sigma_D = \frac{P}{A} - \frac{Mc}{I} = \frac{2(5.80)}{1} - \frac{[2(5.80)](1)}{0.333} = -23.2 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_D = 0 \qquad \textbf{Ans.}$$

Ans:
$\sigma_C = 11.6$ ksi, $\tau_C = 0$,
$\sigma_D = -23.2$ ksi, $\tau_D = 0$

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***R13–8.**

The wall hanger has a thickness of 0.25 in. and is used to support the vertical reactions of the beam that is loaded as shown. If the load is transferred uniformly to each strap of the hanger, determine the state of stress at points C and D on the strap at B. Assume the vertical reaction **F** at this end acts in the center and on the edge of the bracket as shown.

SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \quad F_B(10) - 10(2) - 12(7) = 0; \quad F_B = 10.40 \text{ kip}$

$$I = 2\left[\frac{1}{12}(0.25)(2)^3\right] = 0.333 \text{ in}^4; \quad A = 2(0.25)(2) = 1 \text{ in}^2$$

At point C:

$$\sigma_C = \frac{P}{A} = \frac{2(5.20)}{1} = 10.4 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_C = 0 \qquad \textbf{Ans.}$$

At point D:

$$\sigma_D = \frac{P}{A} - \frac{Mc}{I} = \frac{2(5.20)}{1} - \frac{[2(5.20)](1)}{0.333} = -20.8 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_D = 0 \qquad \textbf{Ans.}$$

Ans:
$\sigma_C = 10.4$ ksi, $\tau_C = 0$,
$\sigma_D = -20.8$ ksi, $\tau_D = 0$