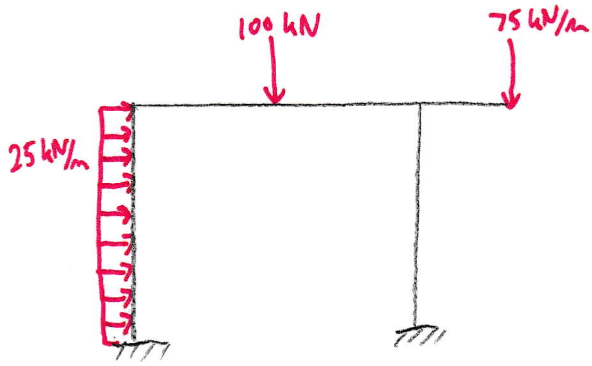
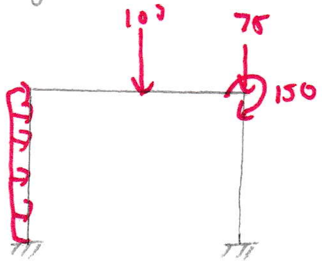


① Simplify structure



② Neglect axial effects to reduce the independent unknowns to 3

$D_1 : \theta_B$

$D_2 : \theta_C$

$D_3 : \Delta_{HC} = \Delta_{AB}$

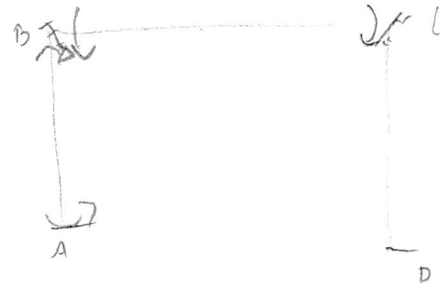
③ Equilibrium Eqns

$D_1 \quad \sum M_B = 0$

$D_2 \quad \sum M_C = -150$

$D_3 . \sum F_x = 0$

④ Fix all degrees of freedom and determine FEMs

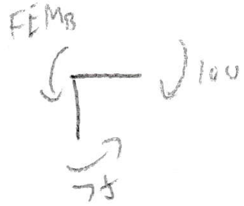


$FEM_{AB} = \frac{wL^2}{12} = 75$

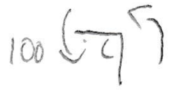
$FEM_{BA} = 75$

$FEM_{BC} = PL/8 = 100$

$FEM_{CB} = 100$

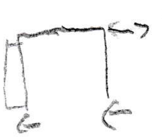


$FEM_B = 25$ kNm



$FEM_C = 100$ kNm

↳ (does not account for moment bc
only fixed w/. span loads

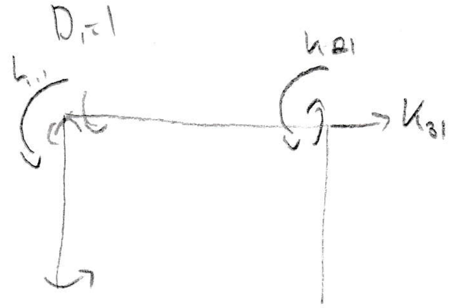


if $V_{AB} = 75$,

$F_{EF} = -75 [\rightarrow]$

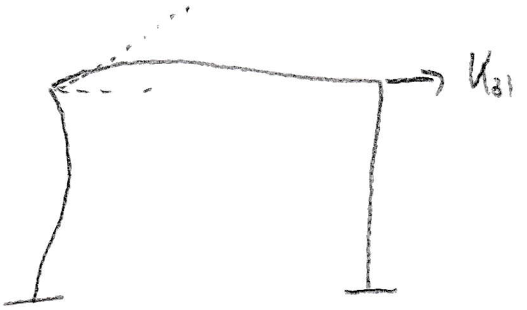
$= 75 [\leftarrow]$

5. Apply unit rotations/displacements of D_1, D_2, D_3 to determine stiffness coefficients



$$K_{11} = \frac{4EI_{BA}}{L_{BA}} + \frac{4EI_{BC}}{L_{BC}}$$
$$= 21667 \text{ kNm}$$

$$K_{21} = \frac{2EI_{BC}}{L_{BC}} = 7500 \text{ kNm}$$



$$K_{31} = \frac{6EI_{BA}}{L_{BA}^2} = \frac{6(10000)}{6^2} = 1667$$

$D_2 = 1$

$$K_{12} = \frac{2EI_{BC}}{L_{BC}} = \frac{2(30000)}{8} = 7500 \text{ kNm}$$

$$K_{22} = \frac{4EI_{BC}}{L_{BC}} + \frac{4EI_{CD}}{L_{CD}} = \frac{4(30000)}{8} + \frac{4(20000)}{6}$$

$$= 28333 \text{ kNm}$$

$$K_{32} = \frac{6EI_{CD}}{L_{CD}^2} = \frac{6(20000)}{6^2} = 3333 \text{ kN}$$

K_{32} is force to resist this

$\underline{D_3 = 1}$

$$K_{13} = \frac{6EI_{BA}}{L_{BA}^2} = 1667 \text{ kN } (= K_{31}) \quad = \frac{\text{kNm}}{\text{m}}$$

$$K_{23} = \frac{6EI_{CD}}{L_{CD}^2} = \frac{6(20000)}{6^2} = 3333 \text{ kN } (K_{32}) \quad = \frac{\text{kNm}}{\text{m}}$$

K_{33} is force req'd to produce $D_3 = 1$

$$K_{33} = V_{AB} + V_{CD}$$

$$= \frac{1}{L_{BA}}\left(\frac{6EI_{BA}}{L_{BA}^2} + \frac{6EI_{BA}}{L_{BA}^2}\right) + \frac{1}{L_{CD}}\left(\frac{6EI_{CD}}{L_{CD}^2} + \frac{6EI_{CD}}{L_{CD}^2}\right)$$

$$= \frac{12EI_{BA}}{L_{BA}^3} + \frac{12EI_{CD}}{L_{CD}^3} = 1667 \text{ kN/m}$$

⑥ Compatibility

$D_1 \rightarrow \sum M_B : FEM_1 + K_{11} D_1 + K_{12} D_2 + K_{13} D_3 = 0$

$D_2 \rightarrow \sum M_c = FEM_2 + K_{21} D_1 + K_{22} D_2 + K_{23} D_3 = M_2 = -150 \text{ kNm}$

$D_3 \rightarrow \sum F_N = \underset{\substack{\uparrow \\ \text{assumption}}}{FEF_3} + K_{31} D_1 + K_{32} D_2 + K_{33} D_3$

$$\begin{array}{l} \quad K_{11} \\ \quad \downarrow \end{array}$$
$$\begin{bmatrix} 21667 & 7500 & 1667 \\ 7500 & 28333 & 3333 \\ 1667 & 3333 & 1667 \end{bmatrix} \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{array}{c} F \quad FEM \\ \begin{bmatrix} 0 - 25 \\ -150 - (-100) \\ 0 - (-75) \end{bmatrix} \end{array}$$

$D_1 = -0.003206$ rad θ_B

$D_2 = -0.008611$ rad θ_c

$D_3 = 0.0654$ m Δ_c

⑦ Plug into slope-deflection equation

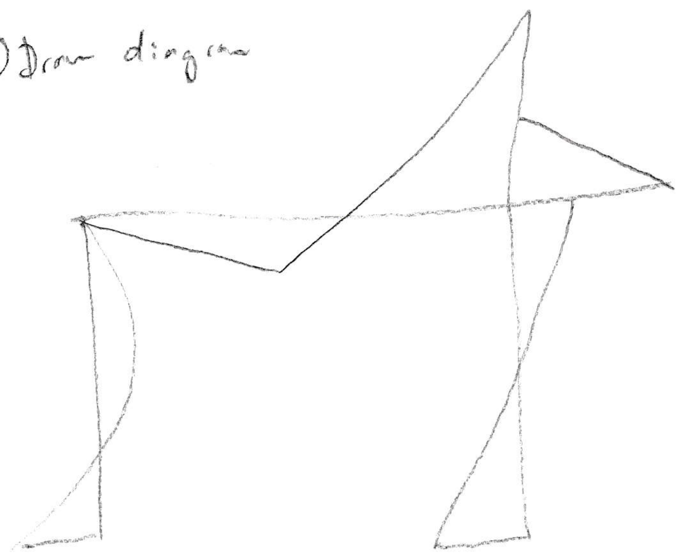
$$M_{AB} = \frac{4EI}{L}(0) + \frac{2EI}{L}(-0.003206) - \frac{6EI}{L^2}(-0.0654) + 75 = 172.3 \text{ kNm}$$

$$M_{BA} = \frac{4EI}{L}(-0.003206) + \frac{2EI}{L}(0) - \frac{6EI}{L^2}(-0.0654) + (-75) = 12.7 \text{ kNm}$$

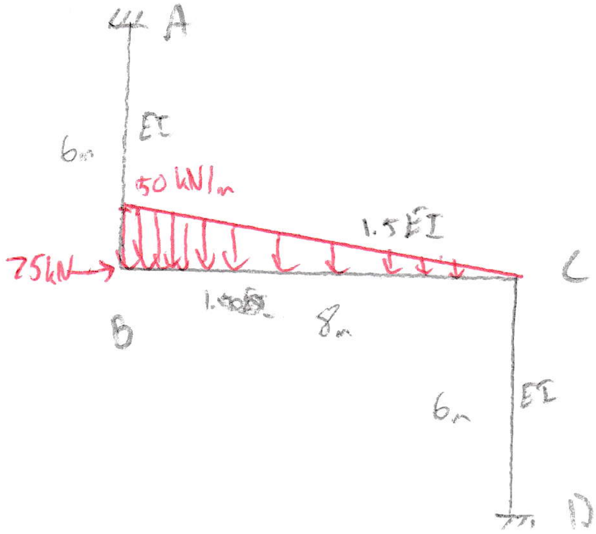
$$M_{BC} = \frac{4EI}{L}(-0.003206) + \frac{2EI}{L}(-0.008611) - \frac{6EI}{L^2}(0) + 110 = -12.7 \text{ kNm}$$

$$M_{CB} = \frac{4EI}{L}(0) + \frac{2EI}{L}(-0.008611) - \frac{6EI}{L^2}(-0.0654) + 6 = 160.6 \text{ kNm}$$

⑧ Draw diagram



Example 2 – Displacement

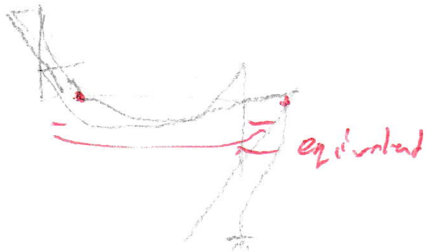


1) Deflected shape

Assumptions

①

②



② Kinematically indeterminate to 3rd degree (ignoring axial effects)

↳ $D_1 = \theta_B$, $\sum M_B = 0$

$D_2 = \theta_C$ $\quad$ $\sum M_C = 0$

$D_3 = \Delta_{HC} (= \Delta_{HB})$ $\quad$ $\sum F_x = 75$ kN ↑

③ Determine FEMs & FEFs

$\frac{wL^2}{20} = \frac{50(8)^2}{20} = 160$

FEM_1

$FEM_1 = 160$ kNm

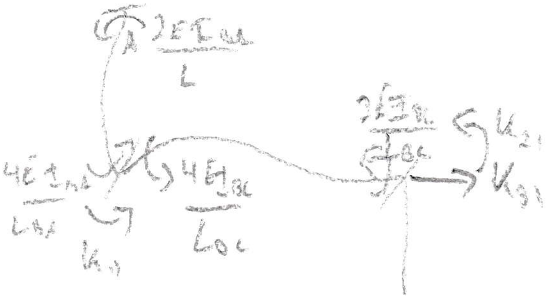
$\frac{50(8)^2}{30} = 106.7$ kNm

FEM_2

$FEM_2 = -106.7$ kNm

④ Apply unit rotations/displacements of D_1, D_2, D_3 to find stiffness coeffs

$D_1 = 1$

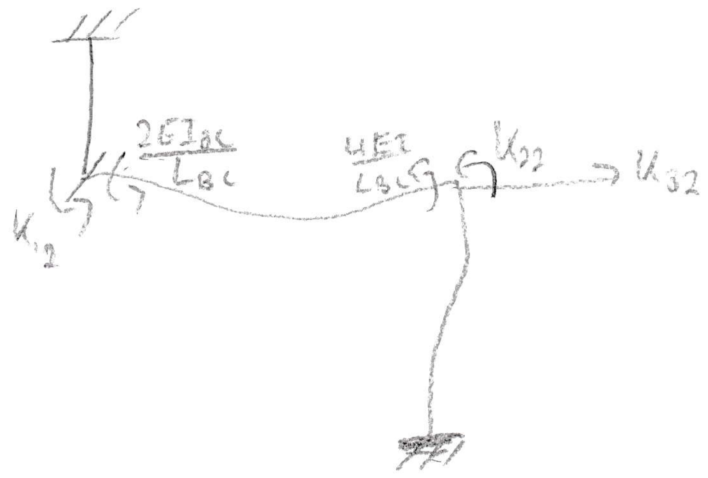


$$K_{11} = \frac{4EI_{BA}}{L_{BA}} + \frac{4EI_{BC}}{L_{BC}} = \frac{4(10000)}{6} + \frac{4(15000)}{8} = 14167 \text{ kNm}$$

$$K_{21} = \frac{2EI_{BC}}{L_{BC}} = 3750 \text{ kNm}$$

$$K_{31} = -\frac{6EI_{BA}}{L_{BA}^2} = \frac{6(10000)}{6^2} = -1667 \text{ kN}$$

$D_2 = 1$

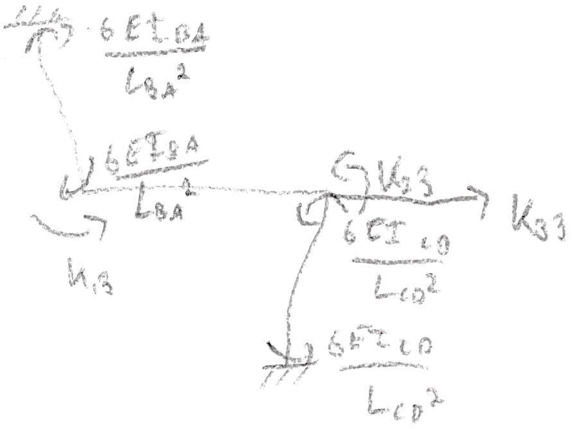


$$K_{12} = \frac{2EI_{BC}}{L_{BC}} = 3750 \text{ kNm}$$

$$K_{22} = \frac{4EI_{BC}}{L_{BC}} + \frac{4EI}{L_{CD}} = 14167 \text{ kNm}$$

$$K_{32} = \frac{6EI_{CD}}{L_{CD}^2} = 1667$$

$D_3 = 1$



$$K_{13} = -\frac{6EI_{BA}}{L_{BA}^2} = -1667\ \frac{kNm}{m}$$

$$K_{23} = \frac{6EI_{CD}}{L_{CD}^2} = 1667\ \frac{kNm}{m}$$

K_{33} is force req'd to produce unit displacement

$$K_{33} = \frac{12EI_{BA}}{L_{BA}^3} + \frac{12EI_{CD}}{L_{CD}^3} = 1111\ \frac{kN}{m}$$

$\sum M_A = 0$

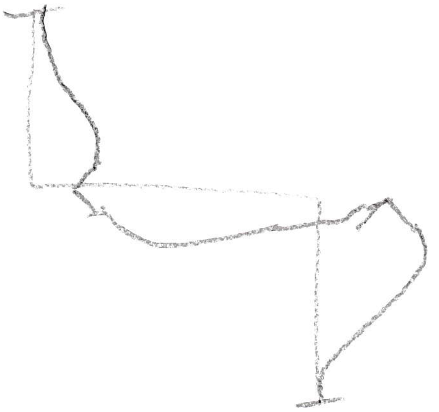
$\sum M_c = 0$

$\sum F_x = 76$

$D_1 = -0.00533$

$D_2 = 0.002357$

$D_3 = 0.05597$



$$M_{NF} = \frac{4EI}{L}\theta_N + \frac{2EI}{L}\theta_F - \frac{6EI}{L^2}\Delta + FEM_{NF}$$

$$M_{AB} = \frac{4EI}{L}(0) + \frac{2EI}{L}(-0.00583) - \frac{6EI}{L^2}(0.05597) + 0 = -1114 \text{ kNm}$$

$$M_{BA} = \frac{4EI}{L}(-0.00583) + \frac{2EI}{L}(0) - \frac{6EI}{L^2}(0.05597) + 0 = -1294 \text{ kNm}$$