CivE 205 Assignment 1

May 13, 2020

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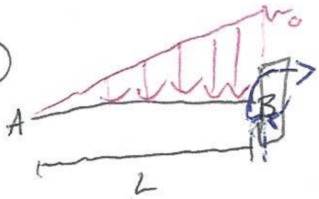


$\Sigma F_y = 0, \quad \frac{-w_0 L}{2} + F_B = 0,$

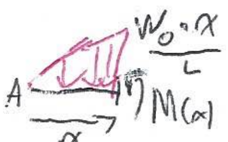
$F_{B_y} = 0$

$\Sigma M_B = 0, \quad \frac{w_0 L}{2} \cdot \frac{L}{3} - M_B = 0$

$M_B = \frac{w_0 L^2}{6}$

$M_A = 0$ since free end

$\therefore$



$M(x) = \frac{-w_0 \cdot x}{2L} \cdot \frac{x}{3} \cdot x = \frac{-w_0 x^3}{6L}$

$EI \frac{d^2 y}{dx^2} = M(x)$

$EI \frac{d^2 y}{dx^2} = \frac{-w_0 x^3}{6L}$

$EI\,\theta(x) = \frac{-w_0}{6L} \int x^3\,dx$

$= \frac{-w_0 x^4}{24L} + C_1$

Since $\theta(L) = 0$,

$0 = \frac{-w_0 L^3}{24} + C_1, \quad C_1 = \frac{w_0 L^3}{24}$

$\therefore EI\,\theta(x) = \frac{-w_0 x^4}{24L} + \frac{w_0 L^3}{24}$

To find $\theta(x)$ @ free end

$EI\,\theta(0) = \frac{-w_0 (0)^4}{24L} + \frac{w_0 L^3}{24} = \frac{-w_0}{24}\left[\frac{x^4}{L} - L^3\right]$

(c) $\theta(0) = \frac{w_0 L^3}{24EI}$

$EI\,y(x) = \frac{-w_0}{24} \int \frac{x^4}{L} - L^3\,dx$

$= \frac{-w_0}{24}\left[\frac{x^5}{5L} - L^3 x\right] + C_2$

Since $y(L) = 0$,

$0 = \frac{-w_0}{24}\left[\frac{L^4}{5} - L^4\right] + C_2$

$C_2 = \frac{-w_0 L^4}{30}$

$EI\,y(x) = \frac{-w_0 x^5}{120L} + \frac{w_0 L^3 x}{24} - \frac{w_0 L^4}{30}$

(a) $y(x) = \frac{-w_0}{120EIL}\left[x^5 - 5L^4 x + 4L^5\right]$

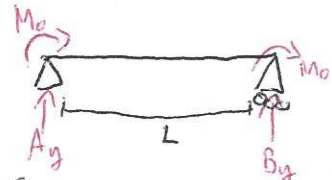
To find deflection @ A

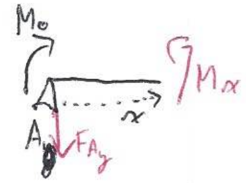
$y(0) = \frac{-w_0}{120EI}\left[4L^4\right]$, since $x = 0$

$= \frac{-4w_0 L^4}{120EI} \rightarrow \frac{-w_0 L^4}{30EI}$

(2)

a)





$\sum M_A = 0,\ B_y L - 2M_o = 0$

$F_{B_y} = \frac{2M_o}{L}$

$\sum F_y = 0,\ \therefore F_{A_y} = \frac{-2M_o}{L}\uparrow;\ \frac{2M_o}{L}\downarrow$

$\sum M = 0,\ M_x - M_o + F_{A_y} x = 0$

$M_x = M_o - F_{A_y} x$

$= M_o - \frac{2M_o}{L} x$

$= M_o(1 - 2x/L)$

$EI\theta(x) = \int M_o(1 - \frac{2x}{L})$

$= M_o\left[x - \frac{x^2}{L}\right] + C_1$

$EIy(x) = M_o \int x - \frac{x^2}{L} + \int C_1$

$= M_o\left[\frac{x^2}{2} - \frac{x^3}{3L}\right] + C_1 x + C_2$

Since $y(0) = 0$,

$0 = M_o[0 - 0] + C_1(0) + C_2$,

$C_2 = 0$

Since $y(L) = 0$

$0 = M_o\left[\frac{L^2}{2} - \frac{L^3}{3L}\right] + C_1 L$

$= \frac{M_o L^2}{2} - \frac{M_o L^2}{3} + C_1 L$

$\frac{M_o L}{3} - \frac{M_o L}{2} = C_1$

$C_1 = \frac{-1}{6} M_o L$

$\therefore EI\theta(x) = M_o\left[x - \frac{x^2}{L} - \frac{1}{6}L\right]$

Since $\theta(x) = 0$ @ max deflection

$0 = \frac{-M_o}{L} x^2 + M_o x - \frac{M_o L}{6}$

$x = \frac{-M_o \pm \sqrt{M_o^2 - 4(\frac{-M_o}{L})(\frac{-M_o L}{6})}}{2(\frac{-M_o}{L})}$

$x = \frac{-M_o \pm \sqrt{M_o^2 - \frac{4M_o^2}{6}}}{\frac{-2M_o}{L}}$

$x = \frac{-M_o \pm \sqrt{\frac{1}{3}M_o^2}}{-2M_o/L}$

$= \frac{-M_o \pm 0.577350269 M_o}{-2M_o/L}$

$x_1 = 0.788675134L$

$x_2 = 0.211324865L$

Since question specifies between A & center of beam, max deflection occurs @ x_2

$y(0.211324865L) = \frac{1}{EI}\left[M_o\left(\frac{x_2^2}{2} - \frac{x_2^3}{3L} - \frac{L}{6}x_2\right)\right]$

$= \frac{-0.016037507 L^2 M_o}{EI}$

b) I_{xx} for W460x113 = 554E6 mm^4

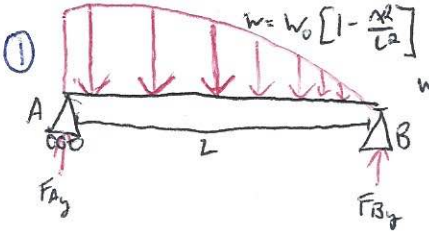
$E = 200$ GPA $= 200 \frac{kN}{mm^2}$

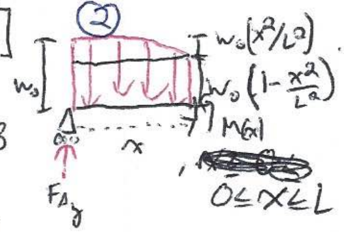
$M_o = 224000$ kN·mm

$-1.2 = \frac{-0.016037507 L^2 \cdot 224000}{200 \cdot 554E6}$

$L = 6083.7$ mm, or 6.0837 m

③





$$\sum M_A = 0, \left[\frac{-2}{3} L w_0\right] \cdot \left[\frac{2}{5} L\right] + F_{By} \cdot L = 0$$

$$F_{By} = \frac{4 L w_0}{15}$$

$$\sum F_y = 0, \ \frac{-2}{3} L w_0 + \frac{4}{15} L w_0 + F_{Ay}$$

$$F_{Ay} = \frac{2}{5} L w_0$$

Given ②

$$\sum M)= 0, \ M(x) + w_0\left(1 - \frac{x^2}{L^2}\right) \cdot \frac{x^2}{2} + \frac{2}{3} w_0 \frac{x^3}{L^2} \cdot \frac{3}{5} x - \frac{2}{5} L w_0 x = 0$$

$$M(x) = \frac{2}{5} L w_0 x - \frac{w_0 x^2}{2}\left(1 - \frac{x^2}{L^2}\right) - \frac{2}{5} w_0 \frac{x^4}{L^2}$$

$$= \frac{2}{5} L w_0 x - \frac{1}{2} w_0 x^2 + \frac{w_0 x^4}{2 L^2} - \frac{2}{5} w_0 \frac{x^4}{L^2}$$

$$M(x) = \frac{2}{5} L w_0 x - \frac{1}{2} w_0 x^2 + \frac{w_0 x^4}{10 L^2} = EI \frac{d^2 y}{dx^2}$$

$$EI\theta(x) = \int \frac{2}{5} L w_0 x - \frac{1}{2} w_0 x^2 + \frac{w_0 x^4}{10 L^2}$$

$$= \frac{1}{5} L w_0 x^2 - \frac{1}{6} w_0 x^3 + \frac{w_0 x^5}{50 L^2} + C_1$$

$$EI y(x) = \int \frac{1}{5} L w_0 x^2 - \frac{1}{6} w_0 x^3 + \frac{w_0 x^5}{50 L^2} + C_1$$

$$= \frac{1}{15} L w_0 x^3 - \frac{1}{24} w_0 x^4 + \frac{w_0 x^6}{300 L^2} + C_1 x + C_2$$

Since $y(0) = 0$, $C_2 = 0$.

$\therefore$, since $y(L) = 0$

$$0 = \frac{1}{15} L^4 w_0 - \frac{1}{24} w_0 L^4 + \frac{w_0}{300} L^4 + C_1 L$$

$$C_1 = \frac{-17}{600} L^3 w_0$$

To find slope @ A

b) $\therefore \theta(0) = \dfrac{-17 L^3 w_0}{600 EI}$ since x-factors cancel out

(a) $y(x) = \dfrac{1}{EI}\left[\dfrac{1}{15} L w_0 x^3 - \dfrac{1}{24} w_0 x^4 + \dfrac{w_0 x^6}{300 L^2} - \dfrac{17}{600} L^3\right.$

$$y\left(\frac{L}{2}\right) = \frac{1}{EI}\left[\frac{L^4 w_0}{120} - \frac{1}{384} w_0 L^4 + \frac{w_0 L^4}{19200} - \frac{17}{1200} L^4 w_0\right]$$

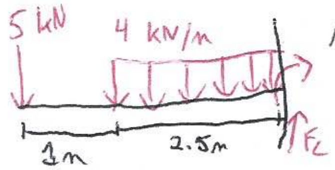
$$= \frac{L^4 w_0}{EI}\left[\frac{1}{200} - \frac{1}{384} + \frac{1}{19200} - \frac{17}{1200}\right]$$

(c) $= \dfrac{-3 L^4 w_0}{256 EI}$

$\therefore$ deflection @ midspan $= \dfrac{-3 L^4 w_0}{256 EI}$

4. This method is interesting because it demonstrates the direct relationship between area loading, shear, moment, slope, and deflection. If you have one, you can likely calculate the others, which is remarkable. In the field, it could be applied to design to any of the above properties for a variety of materials given their respective material property (E). Surely mathematical computer programs could run according to this method since it is entirely equation based. This makes it useful for quick, precise calculations using a computer. However, I think that also makes it impractical for hand calculations, as a simple integration mistake could result in flawed design of a structure. By using this method, I have learned how to find the deflection at any position along the beam. I have also learned that calculus is not only useful, but fundamental in engineering design. Lastly, I have learned about the close relationship between moment, slope and deflection.

(5)



I for $W250 \times 22.3 = 28.7E6\ mm^4$

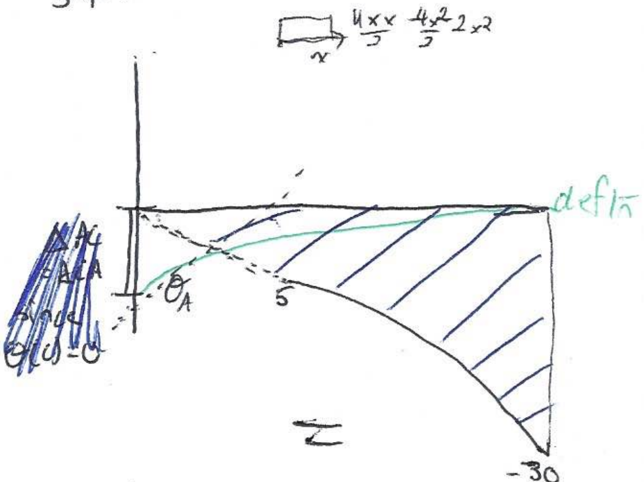
$\Sigma F_y = 0, \; -5 - 4(2.5) + F_C = 0$

$F_C = 15\ kN$

$\Sigma M_C = 0, \; 5(3.5) + 4(2.5)(1.25) - M_C = 0$

$M_C = 30\ kN \cdot m$

$5 \cdot 1 = 5\ kN \cdot m$



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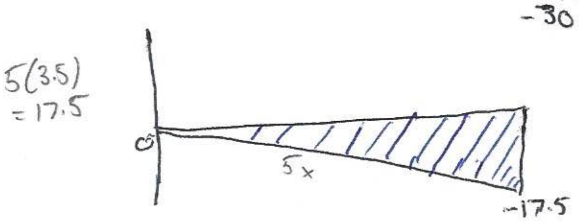

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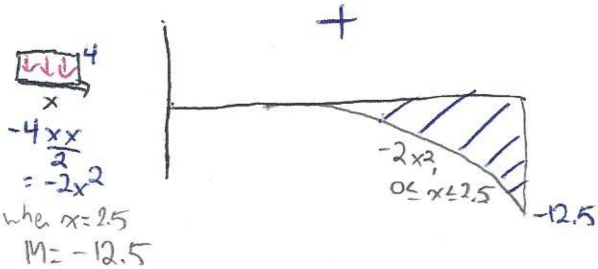


~~Since θ(C)=0, ΔCA=ΔAC~~

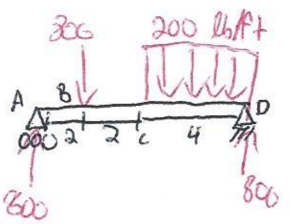
$$\Delta AC = \int_A^C \frac{Mx}{EI} = \left\{ \frac{-17500}{2} \cdot 3500 \cdot \frac{2 \cdot 3500}{3} - \frac{12500 \cdot 2500}{3} \cdot \left[1000 + 3/4 \cdot 2500\right] \right\} / (250 * 28.7E6)$$

$= 22.08\ mm$

$$\theta(A) = \int \frac{M}{EI}, \left[\frac{17500 \cdot 3500}{2} + \frac{12500 \cdot 2500}{3} \right] / [250 \cdot 28.7E6]$$

$= 5.72E-3$ rads

(6)



$I = \frac{1}{12} \cdot 2 \cdot 6^3 = 36\ in^4$

$= \frac{1}{12} \cdot \left[2 \cdot \frac{1\ ft}{12\ in}\right] \cdot \left[6 \cdot \frac{1\ ft}{12\ in}\right]^3$

$= 1/576\ ft^4$

$E = 1.7E6\ \frac{lb}{in^2} \cdot \left[\frac{12\ in}{1\ ft}\right]^2$

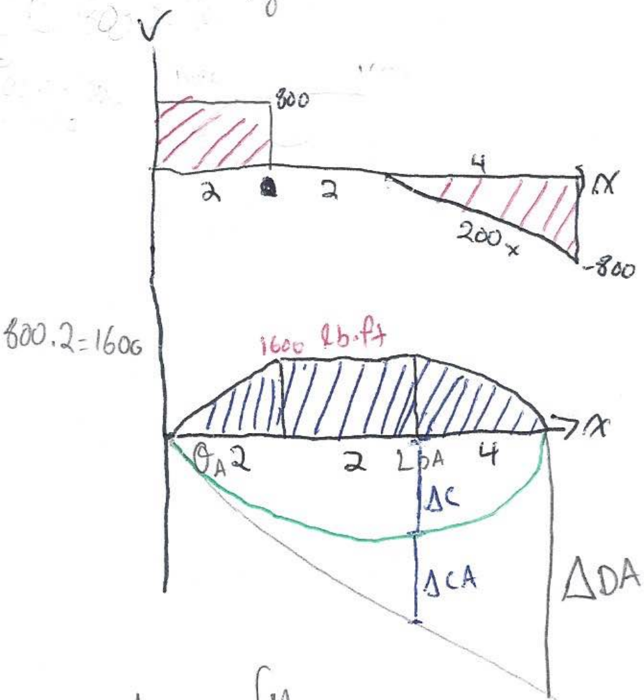
$= 2.448E8\ lb/ft^2$

$\Sigma M_A = 0,\ -800(2) - 200(4)\cdot(6) + F_{Dy}(8) = 0$

$F_{Dy} = 800\ lb$

$\Sigma F_y = 0,\ F_{Ay} - 800 + 800 - 200(4) = 0$

$F_{Ay} = 800$



$\Delta DA = \int \frac{M\bar{x}}{EI}$

$\frac{1}{EI}\left[\left(\frac{2}{3} \cdot 4 \cdot 1600\right)\left(\frac{3}{5} \cdot 4\right) + (1600 \cdot 2)(5) + \left(\frac{1600}{2} \cdot 2\right)\left(6 + 2/3\right)\right]$

$= \frac{1}{EI}\left[10240 + 16000 + \frac{32000}{3}\right]$

$= 0.086839215\ ft$

$\theta_A = \frac{\Delta DA}{L_{DA}},\ \frac{0.086839215}{8} = 0.010854901\ rads$

$\Delta CA = \left[1600(2)(1) + \frac{1600}{2}(2)(2 + 2/3)\right] / EI$

$= 0.017568627$

$\theta_A = \frac{\Delta C + \Delta CA}{4}$

$0.010854901(4) = 0.017568627 + \Delta C$

$\Delta C = 0.02585098\ ft$

$= 0.3102\ in$

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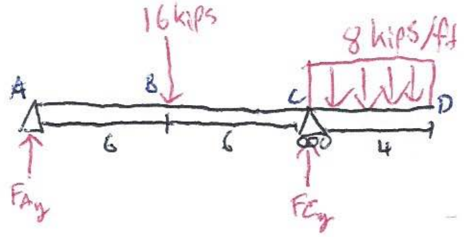


$I = 238 \text{ in}^4$

$= [238]\left[\frac{1 \text{ ft}}{12 \text{ in}}\right]^4$

$= 0.011477623 \text{ ft}^4$

$= \frac{119}{10368} \text{ ft}^4$

$E = 29E6 \frac{lb}{in^2} \cdot \frac{1 \text{ kip}}{1000 lb} \cdot \left[\frac{12 \text{ in}}{ft}\right]^2$

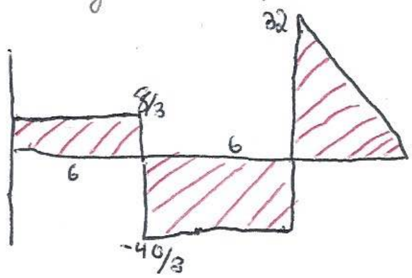
$= 4176000 \frac{kip}{ft^2}$

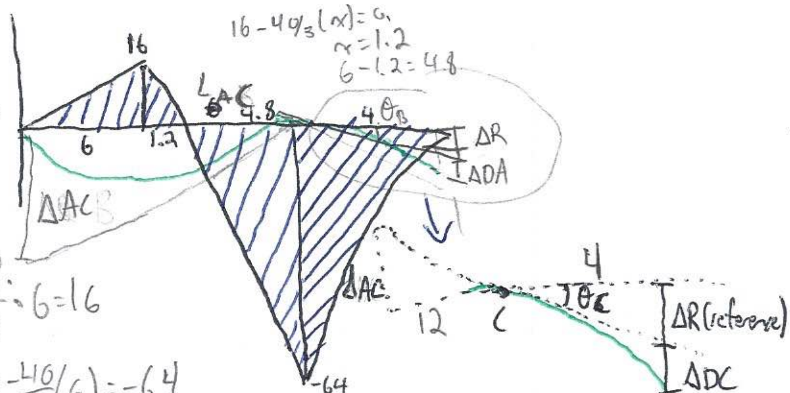
$\Sigma M_A = 0, -16 \cdot 6 - 8 \cdot 4 \cdot 14 + F_{C_y} \cdot 12 = 0$

$F_{C_y} = \frac{136}{3} \text{ kip}$

$\Sigma F_y = 0, F_{A_y} + \frac{136}{3} - 16 - 32 = 0$

$F_{A_y} = 8/3 \text{ kip}$





$\frac{8}{3} \cdot 6 = 16$

$16 - \frac{40}{3}(6) = -64$

free end @ $M_D = 0$

$\theta_C = \frac{\Delta AC}{L_{AC}}$, $\Delta AC = \left[\frac{16 \cdot 6}{2}\left(\frac{2}{3} \cdot 6\right) + \frac{16(1.2)}{2}\left(6 + \frac{1}{3} \cdot 1.2\right) - \frac{64}{2}(4.8)\left(7.2 + \frac{2}{3} \cdot 4.8\right)\right] / EI$

$= \frac{-1344 * 10368}{119 \cdot 4176000}$

$= -0.028040567 \cdot \left[\frac{12 \text{ in}}{ft}\right]$

$= -0.336 \text{ in}$

$\theta_C = \frac{-0.028040507}{12} = \underline{-0.002336713 \text{ rads}}$

$\Delta DA = \frac{-1}{3} \cdot 64 \cdot 4 \left[\frac{7}{10} \cdot 4\right]$

$= -238.93333 / EI$

$= \frac{-3584}{15} / EI = -0.004984989 \text{ ft}$

$= -0.059819878 \text{ in}$

7 (cont.)

Similar triangles provide means to calculate ΔR based on ΔAC

$$\Delta AC = \left[\frac{16\cdot 6}{2}\cdot\frac{2}{3}(6)\right] + \left[\frac{16\cdot 1.2}{2}\cdot\left(6+\tfrac{1}{3}\{1.2\}\right)\right] + \left[\frac{-64\cdot 4.8}{2}\cdot\left(6+1.2+\tfrac{2}{3}\{4.8\}\right)\right] / EI$$

$$= \frac{-1344\cdot 10368}{119\cdot 4176000}$$

$$= -0.028 \text{ ft}$$

$$= -0.336487 \text{ in}$$

$$\Delta R = 4/12 \cdot \Delta AC, \quad 4/12(-0.028040567)$$

$$= -0.009346855 \text{ ft}$$

$$\Delta DC = \left[\frac{-1}{3}\cdot 64\cdot 4\cdot\left(\frac{3}{4}\cdot 4\right)\right] / EI$$

$$= \frac{-256\cdot 10368}{119\cdot 4176000}$$

$$= -0.00534106 \text{ ft}$$

$$\Delta D = \Delta R + \Delta DC$$

$$= -0.014687916 \text{ ft}$$

$$= -0.1763 \text{ in}$$

8. The two methods find the same conclusion using approaches that are based on the exact same theory but using different procedures. Both are based on the relationship between E, I, V(x) and M(x) to solve for deflection. In a sense, calculating the integration is the same as finding the area. However, the integration does not multiply by the centroid of areas as the moment-area method does, but instead relies on the engineer solving C_1, C_2 et cetera. I personally prefer using the moment-area method. The calculations are simpler, which makes it less likely for me to make a mistake. Hence, I believe is more practical for real-world applications. It is also true that integration often unnecessary. According to **Fig. 9.31** of Mechanics of Materials 7th ed., the area and centroid can be found for any spandrel, even those of a high order. Thus, even for complicated loadings, the moment-area method could work just fine.