Mechanics Assignment ~~#~~ 4

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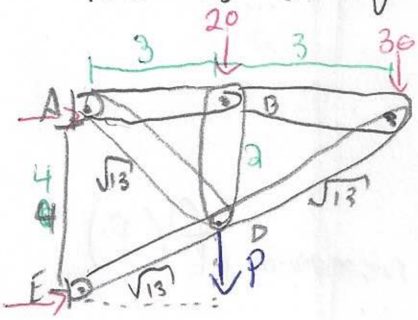
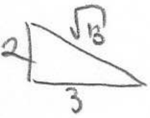
① $E = 200E3$ MPa
$A = 300\ mm^2$





$\sum M_A = 0,\ 20(3) + 30(6) + P(3) - E_x(4) = 0$

$240 + P(3) = E_x(4)$

E_x

$E_x = 60 + \frac{3}{4}P$

$E_x = F_{ED}\cos\theta = F_{ED}\frac{3}{\sqrt{13}},\quad T_{ED} = \frac{\sqrt{13}}{3}\left(60 + \frac{3}{4}P\right)$ [C]

$\sum M_E = 0,\ 20(3) + 30(6) + P(3) + A_x(4) = 0$

$A_x = -\left(60 + \frac{3}{4}P\right)$

@ Joint C

$\sum F_y = 0,\ -30 + F_{CD}\left(\frac{2}{\sqrt{13}}\right) = 0$

$F_{CD} = \frac{30\sqrt{13}}{2} = 15\sqrt{13}$ [C]

$\sum F_x = 0,\ 15\sqrt{13}\cdot\frac{3}{\sqrt{13}} + F_{CB} = 0$

$F_{CB} = 45$ [T]

@ Joint B

$\sum F_y = 0,\ -20 + F_{BD} = 0$

$F_{BD} = 20$ [C]

@ Joint D

$\sum F_y = 0,\ -P + \left(60 + \frac{3}{4}P\right)\frac{\sqrt{13}}{3}\cdot\frac{2}{\sqrt{13}} - 15\sqrt{13}\cdot\frac{2}{\sqrt{13}} - 20 + \frac{2}{\sqrt{13}}F_{DA} = 0$

$\frac{2}{\sqrt{13}}F_{DA} = P + 50 - 40 - \frac{1}{2}P$
$= \frac{1}{2}(P + 20)$

$F_{DA} = \frac{\sqrt{13}}{4}(P + 20)$ [T]

$$\frac{dF_{DA}}{dP} = \frac{\sqrt{13}}{4}$$

$$\frac{dF_{ED}}{dP} = \frac{-\sqrt{13}}{4}$$

no other members $f(P)$

$$\Delta_i = \sum \frac{P_i L_i \partial P_i}{E A_i \partial F_i}$$ since $P=0$

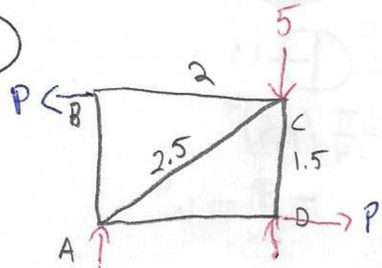
$$= \frac{\left(-\frac{\sqrt{13}}{3}(60)\right)\left(-\frac{\sqrt{13}}{4}\right)\cdot\left[\sqrt{13}(1000)\right]^{mm}}{200\,\frac{kN}{mm^2}\cdot 300\,mm^2} + \frac{5\sqrt{13}\cdot\frac{\sqrt{13}}{4}\cdot\sqrt{13}(1000)}{200\cdot 300}$$

$$= \frac{13\cdot 65\cdot\sqrt{13}\cdot 1000}{200\cdot 300} + 0.9765$$

$$= 3.906 + 0.977$$

$$= 4.883\text{ mm }[\downarrow]$$

②



$\sum M_D = 0,\ P(1.5) - A_y(2) = 0$

$A_y = \frac{3P}{4}$

$\sum M_A = 0,\ P(1.5) - 5(2) + D_y(2) = 0$

$\sum F_y = 0,\ D_y = 5 - \frac{3}{4}P$

@ Joint D

$\sum F_x = 0,\ \therefore F_{AD} = P\ [T]$

$\sum F_y = 0,\ 5 - \frac{3}{4}P + F_{DC} = 0$

$F_{DC} = \frac{3}{4}P - 5\ [T]$

@ Joint C

$\sum F_y = 0,\ -5 - \frac{3}{4}P + 5 - \frac{1.5}{2.5}F_{CA} = 0$

$F_{CA} = -\frac{5}{4}P\ [T]$

@ Joint A

$\sum F_y = 0,\ \frac{3}{4}P - \frac{1.5}{2.5}\left(\frac{5}{4}P\right) + F_{BA}$

$F_{BA} = 0$

@ Joint B

$F_{BC} = P\ [T]$

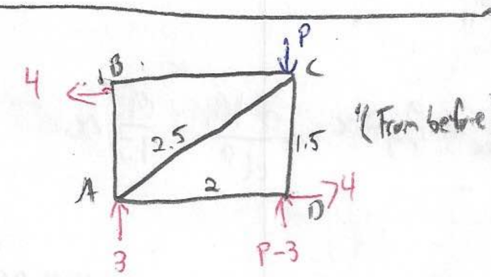
$\frac{dF_{DC}}{dP} = \frac{3}{4},\ \frac{dF_{CA}}{dP} = \frac{-5}{4},\ \frac{dF_{BC}}{dP} = 1,\ \frac{dF_{DA}}{dP}$

$\Delta B = \sum \frac{PL}{EA}\frac{\partial F_i}{\partial P}$

$= \frac{\left[\frac{12}{4} - 5\right][1500]}{200 \cdot 400} \cdot \frac{3}{4} + \frac{\frac{-20}{4}(2500)\left(\frac{-5}{4}\right)}{200 \cdot 400} + \frac{4(2000)}{200 \cdot 400}(2)$

$= -0.028125 + 0.1953125 + 0.2$

$\Delta_B = 0.367$ mm [left]



$F_{BA} = 0$

$F_{BC} = 4\ [T]$

$F_{AD} = 4\ [T]$

F_{CA} not a $f(P)$

@ Joint D

$\sum F_y = 0,\ P - 3 + F_{CD} = 0$

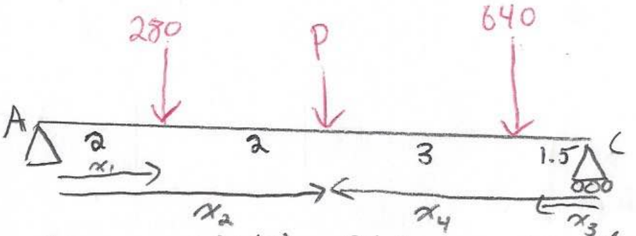
$F_{CD} = 3 - P\ (T),\ \frac{\partial F}{\partial P} = -1$

$\Delta_C = \frac{(3-5)(-1)(1.5)(1000)}{200 \cdot 400}$

$= 0.0375$ mm [↓]

③

$\frac{l}{in^2} \cdot in^4$



$\Sigma M_A = 0,\ 280(2) + P(4) + 640(7) - C_y(8.5) = 0$

$C_y = \frac{8}{17}P + \frac{10080}{17}$

$\Sigma F_y = 0,\ -280 - 640 - P + \frac{10080}{17} + \frac{8}{17}P + A_y = 0$

$A_y = \frac{5560}{17} + \frac{9}{17}P$

$M(x_1) = \frac{5560}{17}x_1,\ \frac{\partial M}{\partial P} = \frac{9}{17}x_1$

$M(x_2) = \frac{5560}{17}x_2 - 280(x_2 - 2) = \frac{800}{17}x_2 + 560,\ \frac{\partial M}{\partial P} = \frac{9}{17}x_2$

$M(x_3) = \frac{10080}{17}x_3,\ \frac{\partial M}{\partial P} = \frac{8}{17}x_3,$

$M(x_4) = \frac{10080}{17}x_4 - 640(x_4 - 1.5)\ \ \frac{\partial M}{\partial P} = \frac{8}{17}x_4$

$= -\frac{800}{17}x_4 + 960$

$$\Sigma\int_0^L M\frac{\partial M}{\partial P} = \int_0^2 \frac{5560}{17}\cdot\frac{9}{17}x^2 + \int_2^4 \frac{800}{17}\cdot\frac{9}{17}x_2^2 + \int_2^4 560\cdot\frac{9}{17}x_2 + \int_0^{1.5} \frac{10080}{17}\cdot\frac{8}{17}x_3^2$$

$$+ \int_{1.5}^{4.5} 960\cdot\frac{8}{17}x_4 - \int_{1.5}^{4.5} \frac{800}{17}\cdot\frac{8}{17}x_4^2$$

$= 461.7301 + 465.0519 + 1778.8235 + 313.9100 + 4065.8824$

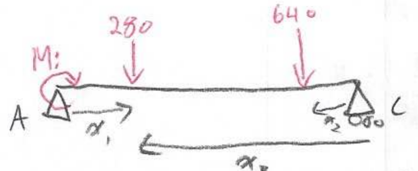
$- 647.7509$

$= 6437.647\ lb\cdot ft^3$

$\Delta = \int_0^L \frac{M}{EI},\ \therefore \Delta = 6437.647 \cdot \frac{12^3}{\frac{\pi}{4}(0.75)^4 \cdot 29E6}$

$= 1.5436$ in [↓]

b)



$\sum M_A = 0, \quad M_i + 280(2) + 640(7) - F_C(8.5) = 0$

$$C_y = \frac{10080}{17} + \frac{2}{17}M_i$$

$\sum F_y = 0, \quad -280 - 640 + \frac{10080}{17} + \frac{2}{17}M_i + A_y = 0$

$$A_y = \frac{5560}{17} - \frac{2}{17}M_i$$

$M(x_1) = \frac{5560}{17}x_1, \quad \frac{\partial M}{\partial M_i} = -\frac{2}{17}x_1 + 1$

$M(x_2) = \frac{10080}{17}x_2, \quad \frac{\partial M}{\partial M_i} = \frac{2}{17}x_2$

$M(x_3) = \frac{10080}{17}x_3 - 640(x_3 - 1.5) = -\frac{800}{17}x_3 + 960, \quad \frac{\partial M}{\partial M_i} = \frac{2}{17}x_3$

$\theta_A = \int \frac{M}{EI}\frac{\partial M}{\partial M_i}, \quad \sum M\frac{\partial M}{\partial M_i} = \int_0^2 \frac{5560}{17}x_1 - \int_0^2 \frac{5560}{17}\cdot\frac{2}{17}x_1^2 + \int_0^{1.5} \frac{10080}{17}\cdot\frac{2}{17}x_2^2 - \int_{1.5}^{6.5} \frac{800}{17}\cdot\frac{2}{17}x_3^2 + \int_{1.5}^{6.5} 960\cdot\frac{2}{17}x_3$

$654.1176 + -102.6067 + 78.4775 - 500.5767 + 2258.8235$

$= 2388.2352$

$\theta_A = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_i}$

$$\frac{2388.2352 \cdot 12^2}{29E6 \cdot (\frac{\pi}{4})(0.75)^4} = 0.0477 \text{ rads}$$

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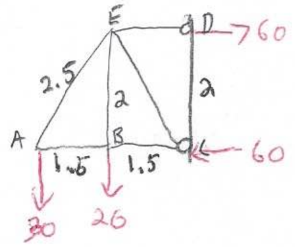


$\Sigma M_C = 0,\ 30(3) + 20(1.5) - D_x(2) = 0$

$D_x = 60\text{ kN}$

$\Sigma F_x = 0,\ \therefore C_x = -60$

@ Joint A

$\Sigma F_y = 0,\ -30 + F_{AE} \cdot \frac{2}{2.5} = 0$

$F_{AE} = 37.5\ [T]$

$\Sigma F_x = 0,\ 37.5 \cdot \frac{1.5}{2.5} + F_{AB}$

$F_{AB} = 22.5\ [C]$

@ Joint B

$F_{BE} = 20\ [T]$

$F_{BC} = 22.5\ [C]$

@ Joint E

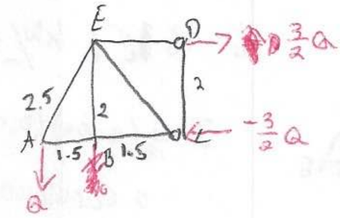
$\Sigma F_y = 0,\ -37.5 \cdot \frac{2}{2.5} - 20 - F_{EC} \cdot \frac{2}{2.5} = 0$

$F_{EC} = 62.5\ [C]$

@ Joint D

$\Sigma F_x = 0,\ F_{DE} = 60\ [T]$

$\frac{\pi}{4}\left(\frac{0.75}{12}\right)^4$



$mm = \frac{1m}{1000mm}$

$\Sigma M_C = 0,\ D_x(2) = 3Q$

$D_x = \frac{3}{2}Q$

$\Sigma F_x = 0,\ C_x = -\frac{3}{2}Q$

@A, F_{EA}

$\Sigma F_y = 0,\ F_{EA} = Q \cdot \frac{2.5}{2}\ [T] = 1.25\,Q\ [T]$

$\Sigma F_x = 0,\ 1.25 \cdot \frac{1.5}{2.5} + F_{AB} = 0$

$F_{AB} = 0.75\ [C]$

$\therefore F_{BC} = 0.75\ [C],$

@E

$F_{BE} = 0$

$\therefore \Sigma F_y = 0,\ F_{EA} = -F_{EC}$

$F_{EC} = 1.25\,Q\ [C]$

@D

$F_{DE} = \frac{3}{2}Q\ [T] = 1.5\,Q$

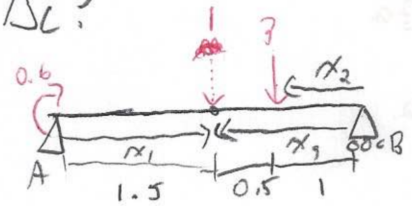
$\Delta = \Sigma \frac{F \cdot f \cdot L}{AE}$

$= \frac{1}{\frac{400}{1000^2} 200E6}\Big[37.5(1.25)(2.5) + (-22.5)(-0.75)(1.5) + (-22.5)(-0.75)(1.5) + (-62.5)(-1.25)(2.5) + 60(1.5)(1.5)\Big]$

$= 0.006226\ m$

$= 6.227\ mm\ [\downarrow]$

⑤ Δ_C?



$E = 13.1E6\ kN/m^2$

$I = \frac{1}{12}(0.075)(0.15)^3$

$= 0.000021093$

$EI = 276.328125\ m^2 kN$

$\Sigma M_A = 0, \ -0.6 - 3(2) + B_y(3)$

$B_y = 2.2$

$\Sigma F_y = 0, \ -3 + 2.2 + A_y = 0$

$A_y = 0.8$

$M(x_1) = 0.6 + 0.8x_1$

$M(x_2) = 2.2x_2$ +

$M(x_3) = 2.2x_3 - 3(x_3 - 1)$

$= -0.8x_3 + 3$

From virtual load due to symmetry

$A_y = B_y = 0.5$

$m(x_1) = 0.5x_1$

$m(x_2) = 0.5x_2$

$m(x_3) = 0.5x_3$

$$\therefore \Sigma \frac{1}{EI}\int_0^2 Mm\,dx = \frac{1}{EI}\left[\int_0^{1.5}(0.6+0.8x_1)(0.5x_1)dx + \int_0^1 2.2x_2 \cdot 0.5x_2\,dx + \int_1^{1.5}(3-0.8x_3)(0.5x_3)dx\right]$$

$$= \frac{1}{EI}\left[0.15x_1^2\Big|_0^{1.5} + \frac{2}{15}x_1^3\Big|_0^{1.5} + \frac{11}{30}x_2^3\Big|_0^1 + 0.75x_3^2\Big|_1^{1.5} - \frac{2}{15}x_3^3\Big|_1^{1.5}\right]$$

$$= \frac{1.775\ m^3 kN}{276.3281\ kNm^2}$$

$= 6.424$ mm [↓]

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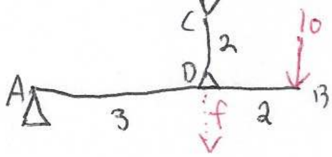


A_2 (0.1)^2 $A_1 = \frac{\pi}{4}(0.01)^2$

$E = 200E6\ kN/m^2$

$I_2 = \frac{1}{12}(0.1)(0.1)^3$

$\Sigma M_A = 0, \quad -10(5) + F_{DC}(3)$

$F_{DC} = \frac{50}{3}$ [T]

$\therefore A_y = \frac{-20}{3}$

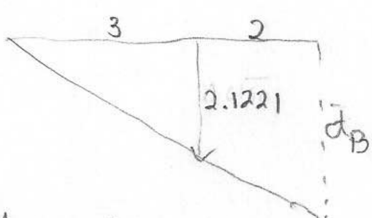
@ Joint D

$\Sigma F_y = 0, \quad -1 + f_{DC} = 0$

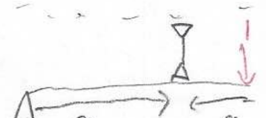
$f_{DC} = 1$

$\therefore \text{defl}_D = \dfrac{\frac{50}{3} \cdot 2}{\frac{\pi}{4}(0.01)^2(200E6)}$

$= 2.1220659098$



$\Delta_B = \dfrac{5(2.1221)}{3} = 3.5368$ mm (uniaxial)



$\Sigma M_A = 0, \quad 1(5) - F_{DC}(3)$

$F_{DC} = 5/3$

$\therefore A_y = \frac{-2}{3}$

$m(x_1) = \frac{-2}{3}x_1$

$m(x_2) = -x_2$

from above,

$M(x_1) = \frac{-20}{3}x_1$

$M(x_2) = -10x_2$

$\therefore \Delta_B = \frac{1}{EI}\left[\int_0^3 \frac{-2}{3} \cdot \frac{-20}{3} x_1^2 + \int_0^2 10x_2^2\right]$

$= \frac{12}{200E6 \cdot (0.1)^4}\left[40 + \frac{80}{3}\right]$

$= 40$ mm

$\therefore 40 + 3.5368 = 43.5$ mm [$\downarrow$]