Civl 222 Assignment 2 Benjamin Klassen

① $(3+y+2y^2\sin^2 x)dx + (x+2xy - y\sin 2x)dy = 0$

$\frac{\partial M}{\partial y} = 1 + 4y\sin^2 x$, $\sin^2 x = \frac{1-\cos 2x}{2}$, $1 + 4y\left(\frac{1}{2} - \frac{\cos 2x}{2}\right)$, $= 1 + 2y - 2y\cos(2x)$

$\frac{\partial N}{\partial x} = 1 + 2y - 2y\cos(2x)$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $\therefore$ exact de

$(3+y+2y^2\sin^2 x)dx + (x+2xy - y\sin 2x)dy = 0$

x $\quad \frac{4y}{2}x - y\sin 2x$

$3x + yx \quad 2y^2\left(\frac{x}{2} - \frac{1}{4}\sin 2x\right)$

$\therefore u(x,y) = 3x + yx + 2y^2\left(\frac{x}{2} - \frac{1}{4}\sin 2x\right) = C$

② $(x^2+2x+y)dx + (3x^2y - x)dy = 0$

$\frac{\partial M}{\partial y} = 1$

$\frac{\partial N}{\partial x} = 6xy - 1$

$\therefore$ not exact

$\frac{2-6xy}{3x^2y-x} = \frac{-2(3xy-1)}{x(3xy-1)} = \frac{-2}{x}$, $\therefore$ Case 1

$\mu(x) = e^{-2\int \frac{1}{x}}$

$\mu(x) = e^{-2\ln|x|} = e^{\ln|x^{-2}|}$

$= x^{-2}$

$(1 + \frac{2}{x} + \frac{y}{x^2})dx + (3y - \frac{1}{x})dy = 0$

$1 \to x$, $\frac{2}{x} \to 2\ln|x|$, $\frac{y}{x^2} \to -y/x$, $-\frac{1}{x}$, $3y \to \frac{3}{2}y^2$

$\therefore x + 2\ln|x| - y/x + 3/2y^2 = C$

(3) $3(x^2+y^2)dx + x(x^2+3y^2+6y)dy = 0$

$$\left.\begin{aligned}\frac{\partial M}{\partial y} &= 6y \\ \frac{\partial N}{\partial x} &= 3x^2+3y^2+6y\end{aligned}\right\} \text{NEQ, } \therefore \text{ not exact}$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -3x^2-3y^2 = \frac{-3(x^2+y^2)}{3(x^2+y^2)} = -1, \ \therefore \text{ Case 2}$$

$$\mu(y) = e^{\int 1 dy} = e^y$$

$$(3x^2e^y + 3y^2e^y)dx + (x^3e^y + 3y^2xe^y + 6xye^y)dy = 0$$

x^3e^y $6yxe^y + 3y^2xe^y$

x^3e^y $3y^2e^yx$

$$\therefore x^3e^y + 3xy^2e^y = C$$

$$= xe^y(x^2+3y^2) = C$$

(4) $(2xy^2 - y)dx + (y^2 + x + y)dy = 0$

$\frac{\partial M}{\partial y} = 4xy - 1$

$\frac{\partial N}{\partial x} = 1$

$\Big\}$ NEQ, $\therefore$ not exact

$4xy - 2 = \frac{2(2xy-1)}{y(2xy-1)} = \frac{2}{y}$ $\therefore$ Case 2

$\mu(y) = e^{\int -2/y \, dy}$

$= e^{-2\ln|y|}, \ e^{\ln|y^2|}$

$= y^{-2} = \frac{1}{y^2}$

$\left(2x - \frac{1}{y}\right)dx + \left(1 + \frac{x}{y^2} + \frac{1}{y}\right)dy = 0$

$2x \to x^2$

$1 \to y$

$\frac{x}{y^2} \to \frac{-x}{y}$ $\left(\frac{-1}{y}\right)$

$\frac{1}{y} \to \ln|y|$

$x^2 + y - \frac{x}{y} + \ln|y| = C$

⑤ $y\,dx + (y^2 e^y - x)\,dy = 0$

$$\frac{dx}{dy} = \frac{-(y^2 e^y - x)}{y}$$

$$= -ye^y + \frac{x}{y}$$

$$\frac{dx}{dy} - \frac{1}{y}x = -ye^y$$

$$\therefore x = e^{\int \frac{1}{y} dy}\left[-\int ye^y e^{-\int \frac{1}{y} dy} dy + C\right]$$

$$= y\left[-\int e^y + C\right]$$

$$x = -ye^y + Cy$$

(6) $\frac{dy}{dx} - \frac{1}{1+x}y = x + x^2$

$e^{-\ln|1+x|}, \; e^{\ln|1+x|}$

$$y = e^{\int \frac{1}{1+x}dx}\left[\int (x+x^2)e^{-\int\frac{1}{1+x}dx}\,dx + C\right]$$

$$= (1+x)\left[\int \frac{x(1+x)}{(1+x)}\,dx + C\right]$$

$$y = (1+x)\left[\left(\frac{x^2}{2}\right) + C\right]$$