Limit states design of steel structures—performance factors

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A detailed statistical analysis to give ratios of mean to nominal values and associated coefficients of variation (based on raw data collected from Canadian mills on the strength and geometric properties of rolled W shapes, welded W shapes, and class H hollow structural sections) is presented. By relating the tested capacity (based on physical tests performed by others) to the predicted capacity (based on the design equations in CSA standard S16.1-1974, *Steel Structures for Buildings—Limit States Design*), the professional ratio and its associated coefficient of variation were determined for steel columns as a function of the slenderness ratio, as well as for laterally supported and laterally unsupported steel beams, enabling the performance factor to be determined for these members over the entire range of behaviour. A serviceability criterion for steel bridges is presented.

Une analyse détaillée de données est présentée pour fournir les rapports de valeurs moyennes en fonction des valeurs nominales et les coefficients de variation associés, fondée sur des données brutes recuillies des aciéries canadiennes, sur la résistance et les propriétés géométriques de profilés laminés en W, profilés soudés en W et fers structuraux évidés, classe H. En comparant la charge analysée (fondée sur des essais physiques effectués par d'autres) à la charge prévue (fondée sur les calculs de conception dans la norme S16.1-1974 de l'ACNOR *Steel Structures for Buildings—Limit States Design*), le rapport entre la résistance déterminée par essais sur la résistance prévue à l'aide de formules mathématiques et son coefficient de variation associé ont été établis pour les colonnes d'acier comme une fonction du rapport d'élancement, ainsi que pour les poutres d'acier soutenues latérallement et celles sans support latéral, permettant d'établir le facteur de rendement devant être déterminé pour ces piéces, sur toute la gamme de comportement. Les critéres d'utilisation pour les ponts d'acier sont présentés.

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1. Introduction

The development of CSA (Canadian Standards Association) standard S16.1-1974 *Steel Structures for Buildings—Limit States Design* and the introduction in part 4, *Structural Loads and Procedures*, of the National Building Code 1975 of a section on limit states design have provided to Canadian engineers the necessary formulation to design steel structures for buildings using limit states design procedures. Some revisions to S16.1-1974 have taken place since, as reflected in CAN 3 S16.1-M78, the new SI standard and the 1977 edition of the National Building Code reflect some changes in the loads formulation dealing with limit states design. The probabilistic study that formed the basis for the development of both the load factors, load combination factors, and importance factors (as given in section 4.1 of the National Building Code) and the performance factor for steel, generally taken as 0.90 (as given in CSA standard S16.1-1974), has been reported by Allen (1975). The Ontario Highway Bridge Design Code, introduced in January 1979, provides designers with a code to design bridges in different structural materials using limit states design procedures. In this new code, the general performance factor for steel members is 0.90 although, as will be discussed, the performance factor on one side of the design inequality and the load factors on the other are not independent. Thus, unless the statistical characteristics of the loads and materials are the same in the two cases of buildings and bridges, it would not be expected, even if all statistical factors relating to the material were unchanged, that the performance factor would be the same.

The name "limit states design," with a letter "s" on "state," relates to the fact that designers try to ensure the structures will be satisfactory for a number of limiting conditions or states. Generally, under service conditions at specified load levels, the structure must provide satisfactory service. A number of serviceability limit states need to be met: for example, fatigue cracks should not grow; frequency and amplitude of vibrations must not be such as to make people uncomfortable; deflections may need to be limited because of possible damage to other materials; inelastic deflections or distortions may be limited.

Much of the effort of structural design is directed toward the consideration of ultimate limit states

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in which the safety of the structure is of concern and the attempt is to minimize, consistent with economic considerations, the probability that a structure will collapse. Because the information that must be used in assessing the safety is not known absolutely, *is not deterministic*, but can only be assessed on a probabilistic basis, it follows then that (1) sound statistical information must be available for the loading on the structure, the effects of the loads, and for the resistance of members; and (2) the more reliable the data on a statistical basis, other factors being equal, the less is the separation required between the resistance of a member and the effect of loads to give a uniform probability against collapse.

The main portion of this paper, following a brief discussion of one serviceability criterion, deals with the development of statistical data for the resistance of steel members fabricated in Canada.

2. A Serviceability Criterion for Steel Bridges

Whereas some serviceability criteria are common to structures made from different materials (for example, the psychological effect on people of floor vibrations), other criteria are unique to a particular material because of its properties. For reinforced concrete members a serviceability limitation is placed on the width of cracks.

For steel flexural members, when the collapse mode of the structure entails partial or complete yielding of the cross section under the factored loading, the question arises as to what serviceability limit is needed for inelastic deformation or deflection. Is no inelastic deflection ever allowed and, if not, at what load levels is the deflection calculated? As well, which frequency of occurrence of this limiting load level should be used—once in a year, once in 10 years, or once in the 50 year lifetime of the structure? Following much debate in the development of the Ontario Highway Bridge Design Code, it was accepted that a serviceability limit state "type II" loading corresponding to the maximum allowed vehicle weights should be used. The frequency of occurrence of the loading is in fact not a question of substance. Given that the ultimate limit state of complete yielding is obtained under the corresponding factored loads, at any smaller load the recovery on unloading is elastic, and, on subsequent reloading to the same level, is essentially elastic as shown in Fig. 1, where a moment curvature diagram is plotted for a steel beam. Corresponding to this moment curvature diagram, deflections can be calculated for the bridge based on the bending moment diagrams. Thus, a limiting permanent deflection, as a fraction of the span, can be set for this working condition. In Russia, Belski (1978) has made

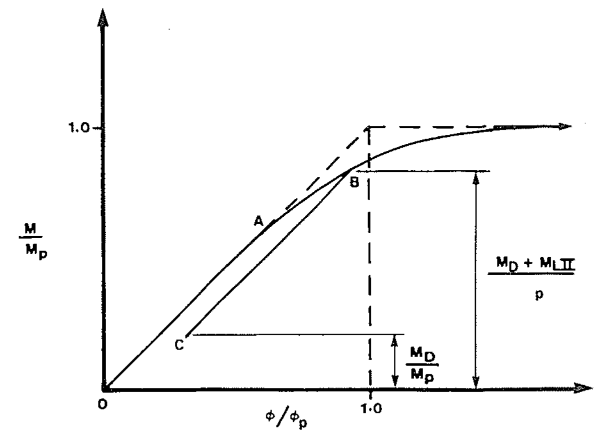

FIG. 1. Nondimensional moment–curvature diagram for a steel beam.

a similar proposal in establishing quantitative criteria for residual strains. In the Ontario Highway Bridge Design Code, 1/1000 of the span has been chosen for the limiting permanent deflection. The loads causing this deflection could occur on the first day of service—or never—and the designer may wish to provide camber for all or part of this deflection.

3. Ultimate Limit States

As given in part 4 of the National Building Code the limit states design criterion is: the factored resistance is greater than or equal to the effect of factored loads, or

$$[1] \quad \phi R \geq \alpha_D D + \gamma\psi(\alpha_L L + \alpha_Q Q + \alpha_T T)$$

If S represents the effect of some particular combination of loads, then the expression "factored resistance is greater than or equal to the effect of factored loads" can be written succinctly as:

$$[2] \quad \phi R \geq \alpha' S$$

where α' is the corresponding "overall" load factor and depends, of course, on the individual load factors as well as the proportions of the loads.

Figure 2 shows possible distribution curves for the resistance of a member, R, and the effect of loads, S, on that member. In the figure as depicted: the mean value of the resistance, $\bar{R}$, is greater than the nominal value, R; the mean value of the effect of loads, $\bar{S}$, is less than the nominal value, S; the dispersion of S is broader than that for R; the value ϕR is slightly greater than $\alpha' S$, indicating that the code requirement has been met; and some of the larger values of S are greater than some of the smaller values of R and therefore under these circumstances failure would occur. Obviously by making the value

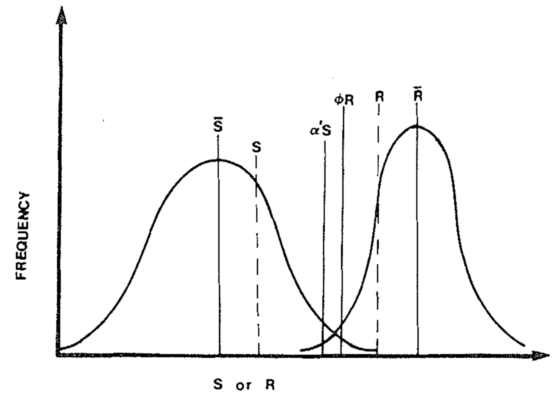

FIG. 2. Frequency distributions for S and R.

of R larger ($R > \alpha' S/\phi$), the intersection of the two distribution curves can be brought to any small value desired but at the corresponding expense.

Rather than plot distribution curves for resistance, R, and the effect of loads, S, a distribution curve as given in Fig. 3 for X (the excess of the resistance over the effect of loads) can be plotted as $X = R - S$.

From Fig. 3 it is noted that if $X > 0$ a safe condition exists and that if $X < 0$ failure occurs. Figure 4 shows the frequency distribution of $\ln X = \ln R - \ln S = \ln R/S$, which is equivalent to Fig. 3. Now $\overline{\ln R/S} \simeq \ln \bar{R}/\bar{S}$ (Galambos and Ravindra 1973) and setting the number of standard deviations of $\sigma_{\ln R/S}$ from 0 to the mean value of $\ln \overline{R/S}$ equal to β gives $\beta\sigma_{\ln R/S} = \ln \overline{R/S}$ where β is termed the safety index, for which values of 3–4 have been commonly used.

Because

$$[3] \quad \sigma_{\ln X}{}^2 \simeq \overline{\frac{\partial \ln (R/S)^2}{\partial R}}\sigma_R{}^2 + \overline{\frac{\partial \ln (R/S)^2}{\partial S}}\sigma_S{}^2$$

$$= \frac{\sigma_R{}^2}{\bar{R}^2} + \frac{\sigma_S{}^2}{\bar{S}^2} = V_R{}^2 + V_S{}^2$$

where V_R = coefficient of variation of the resistance and V_S = coefficient of variation of the effect of loads, then

$$[4] \quad \beta(V_R{}^2 + V_S{}^2)^{1/2} = \ln \overline{R/S}$$

$$[5] \quad \overline{R/S} = \exp \beta(V_R{}^2 + V_S{}^2)^{1/2}$$

Lind (1971) proposed an approximation for $(V_R{}^2 + V_S{}^2)^{1/2}$ as follows:

$$[6] \quad (V_R{}^2 + V_S{}^2)^{1/2} = \alpha(V_R + V_S)$$

and for the range $1/3 \leq V_R/V_S \leq 3$, with $\alpha = 0.75$, the approximation is within 6%. Galambos and Ravindra (1973) extended this concept further by introducing two separation factors α_R and α_S such that:

$$[7] \quad (V_R{}^2 + V_S{}^2)^{1/2} = \alpha_R V_R + \alpha_S V_S$$

Thus,

$$\overline{R/S} = \exp \beta(V_R{}^2 + V_S{}^2)$$
$$[8] \quad \bar{R} = \bar{S} \exp \beta(\alpha_R V_R + \alpha_S V_S)$$
$$\bar{R} \exp(-\beta\alpha_R V_R) = \bar{S} \exp(\beta\alpha_S V_S)$$

They went on to show, with the further introduction of separation factors on the load effect side of the equation, when the load effect S is written as $S = E(D + L)$ where E accounts for uncertainties in structural analysis and D and L are dead and live loads respectively, that a value of $\alpha_R = 0.55$ gave a near zero error for

$$[9] \quad R/S = \exp \beta(V_R{}^2 + V_S{}^2)^{1/2}$$

and a standard deviation of 3% for the following range of variables: $V_R = 0.10$–0.15; $V_E = 0.05$–0.15; $V_D = 0.02$–0.10; $V_L = 0.10$–0.40; $D/L = 0.10$–4.00; and $\beta = 3.00$–4.00.

The ranges for the coefficients of variation above encompass the values determined by Allan (1975) and as well the D/L ratio represents the central load range he used. When considering combinations of dead, live, and wind load the study of Galambos and Ravindra (1977) indicated that a single constant

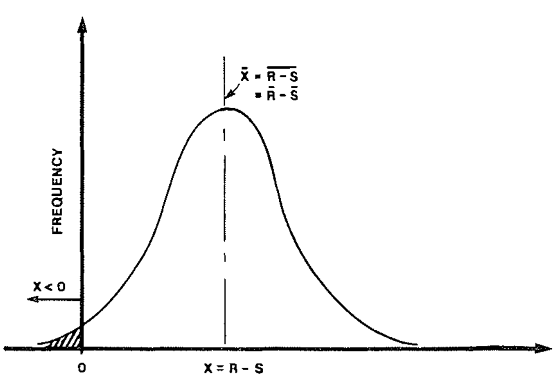

FIG. 3. Frequency distribution for $X = R - S$.

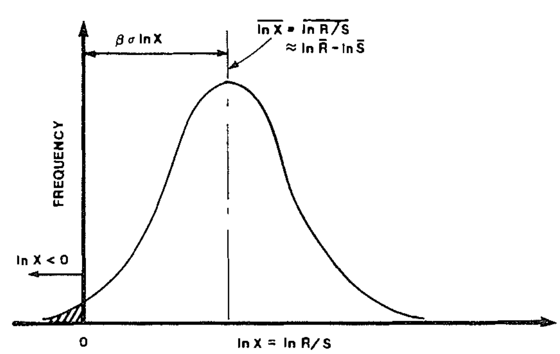

FIG. 4. Frequency distribution for $\ln X$.

value of $\alpha = 0.55$ could be used. This value will be used in this paper.

The equation

$$[10] \quad \bar{R} \exp(-\beta\alpha_R V_R) = \bar{S} \exp(\beta\alpha_S V_S)$$

relates mean values of the resistance and the effect of loads, and the equation

$$[11] \quad \phi R = \alpha' S$$

relates the nominal values. Setting $\rho_R = \bar{R}/R$ and $\rho_S = \bar{S}/S$ then,

$$[12] \quad \rho_R \exp(-\beta\alpha_R V_R)R = \rho_S \exp(\beta\alpha_S V_S)S$$

from which

$$[13] \quad \phi = \rho_R \exp(\beta\alpha_R V_R)$$

and

$$[14] \quad \alpha' = \rho_S \exp(\beta\alpha_S V_S)$$

which gives, in particular, the necessary equation to calculate the performance factor ϕ.

The ratio of the mean to the nominal resistance $\bar{R}/R = \rho_R$ of a member can be considered to comprise three ratios itself:

$$[15] \quad \rho_R = \rho_M \rho_G \rho_P$$

where ρ_M is a "material" ratio, i.e., the ratio of the mean to the nominal of the relevant material property such as the yield strength; ρ_G is a "geometric" or fabrication ratio, i.e., the ratio of the mean to the nominal relevant geometric property such as area or section modulus; and ρ_P is a "professional" ratio, i.e., the ratio of observed capacity in tests to predicted capacity based on observed values of the relevant material and geometric properties.

The above ratios are assumed to be independent random variables and therefore the square of the coefficient of variation of the resistance $V_R^2 = V_M^2 + V_G^2 + V_P^2$ where V_M, V_G, and V_P are the coefficients of variation associated with ρ_M, ρ_G, and ρ_P respectively.

Knowing, then, the values for the ratios, ρ, and the associated coefficients of variation, V, for given values of the coefficient of separation, α_R, and the safety index, β, the performance factors, ϕ, can be established.

This approach applies directly for members whose resistance is written simply as the product of a geometric property and a material property as, for example, the maximum capacity of a laterally supported class 1 beam where $M_p = ZF_y$. However, for members for which the resistance as a function (say of the slenderness ratio of the member) depends on several geometric properties and material properties (such as area, moment of inertia, yield strength, modulus of elasticity), the relative contribution or participation of the several properties over the domain of the independent variable must be assessed. Thus, for short columns the geometric and material properties of concern are the area and yield strength, whereas for long columns they are respectively the moment of inertia and the modulus of elasticity; for columns of intermediate length there will be a gradual transition from one set to the other. The relative contribution of the different properties to the ratios, ρ, and associated coefficients of variation, V, are obtained by mathematical manipulation of the equations describing the resistance of the member. Where the equation has been derived on a theoretical basis solely, only one set of participation factors will be obtained at a given value of the independent variable; but where the resistance has been determined semi-empirically or by curve-fitting, another curve, fitting the data points equally well over a short range (say a linear or second-degree curve rather than a third-degree curve), would yield somewhat different participation factors.

The equations generally used to determine strengths and therefore participation factors are those from CSA standard S16.1-1974, *Steel Structures for Buildings—Limit States Design.*

The performance factors derived in the following are based on a statistical study of raw data on geometric and strength properties obtained from Canadian sources.

The information on professional parameters is based chiefly on the work of others as referenced.

The results of a study of raw data chiefly from Canadian sources are reported in the following sections.

4. Statistical Parameters for Rolled W Shapes

W shapes are produced by passing hot blooms, billets, or slabs of steel through a series of grooved rolls. Wear on the rolls may cause the dimensions of the finished product to vary slightly from the theoretical dimensions. Standard rolling tolerances to allow for roll wear and other factors are contained in CSA standard G40.20-1974 for shapes supplied in grades of steel covered by CSA material standards.

4.1 Material Parameters, ρ_M, V_M

The most significant strength property used in determining the capacity of a structural member is the yield strength. Figure 5 shows a stress–strain curve for steel with various terms defined. Canadian mills (rolling W shapes) in mill tests report currently the lower yield point F_{yl}, which lies between the

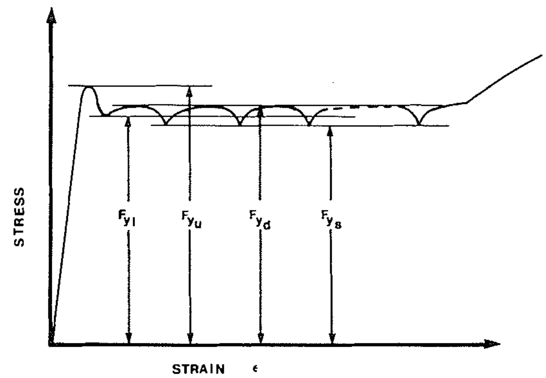

FIG. 5. Tensile stress–strain curve and definition of terms.

dynamic yield point F_{yd} and the really significant value valid for static loading, the static yield point F_{ys}. Rao *et al.* (1966) report that in the inelastic range, strain rates $\dot{\varepsilon}$ of up to 800 μin. · in.$^{-1}$ · s^{-1} are used and proposed the relationship $F_{yd} - F_{ys} = 3.2 + 0.001\dot{\varepsilon}$, where stress is expressed in ksi and strain rate in μ/s, which (for the maximum value of strain allowed in CSA standard G40.20-1974 of 1/16 in. · in.$^{-1}$ · min^{-1} = 1024 μin. · in.$^{-1}$ · s^{-1}) gives a difference of 4.2 ksi (47.6 MPa). The difference between the lower yield point and the static yield point in this study will be taken as 2 ksi (13.8 MPa).

In addition to the speed of testing, the yield strength of steel varies with the thickness of the member (for a given chemical composition); through the thickness of heavy plates and shapes; and with the location on the cross section.

The variation with thickness has been included in the raw data collected and thus has been taken into account. For W shapes of the sizes rolled in Canada the through-thickness variation is not considered significant.

Mill test coupons for rolled shapes are taken from the quarter depth of the web where the static yield stress will be greater than that of the flanges due to the fact that the thinner web is work-hardened to a greater extent in the rolling process. The results of tests (Beedle and Tall 1960) depicted in Fig. 6 show that the difference between the static yield stress of the flange and that of the web is from 4 to 7%. For this study the difference will be taken as 5%.

Histograms are plotted in Figs. 7 and 8, giving the lower yield point for rolled W shapes produced in Canada to CSA standard grade 44W and 44T as noted for different web thicknesses. Note that steel not meeting the specified minimum is rejected. For the 4796 measurements, the mean value of the web lower yield point, F_{ylw}, is 50.74 ksi (350.0 MPa) and the coefficient of variation V_{F_y} is 0.065.

This value is reduced by 2 ksi (13.8 MPa) to ob-

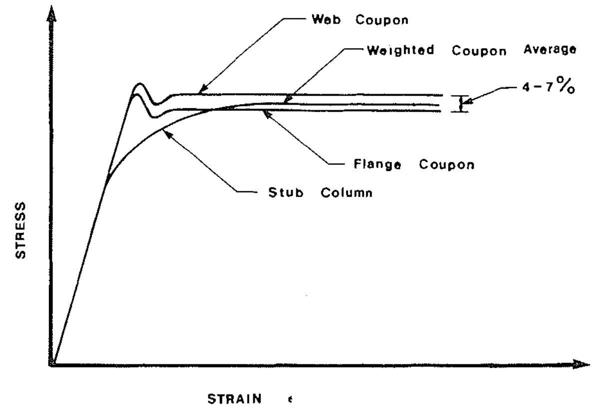

FIG. 6. Various stress–strain curves for steel (Beedle and Tall 1960).

tain the web static yield stress of 48.74 ksi (336.0 MPa), which results in a ratio of the mean to the nominal for the static yield value of the web (the material strength ratio for the web) of

$$\rho_{M_w} = \bar{F}_{ysw}/F_{ysw} = 48.74/44.0 = 1.11$$

For flanges, reducing this value by 5% gives

$$\rho_{M_f} = 1.11(0.95) = 1.05$$

For W sections rolled in Canadian mills, the mean ratio of the web area to the total area is 0.33. Thus, for axial load design conditions when the entire area contributes,

$$\rho_M = 1.11(0.33) + 1.05(0.67) = 1.07$$

For flexural loading when the strength of flanges predominates,

$$\rho_M = \rho_{M_f} = 1.05$$

The other material property of significance when buckling occurs is the modulus of elasticity, E. From Galambos and Ravindra (1978) $\rho_E = 1.00$, and $V_E = 0.019$ is obtained as the weighted average value of the North American data listed in Table 1 of that reference.

The statistical parameters of the variation of material properties for W shapes, both rolled and welded, are given in Table 1.

4.2 Geometric Parameters, ρ_G, V_G

Data available from the mills giving the statistical variation of geometric properties are sometimes incomplete and can be supplemented by considering the tolerances within which the products are manufactured.

The ratio of the mean to nominal flange width can be estimated as:

$$[16] \quad \rho_b = \bar{b}/b = 1 + (\Delta_1 - \Delta_2)/2b$$

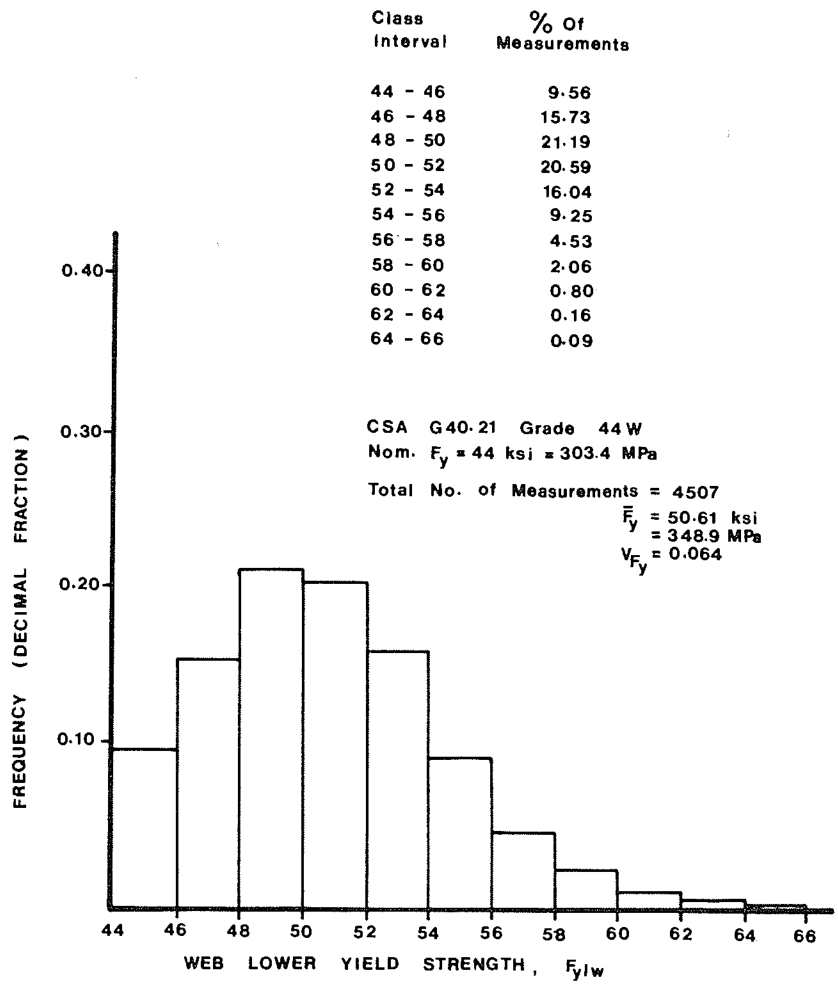

FIG. 7. Yield strength variation for W sections, different web thicknesses.

where $\bar{b}$ = mean flange width; b = nominal flange width; and Δ_1, Δ_2 = over and under tolerance limits.

By assuming that ρ_b is normally distributed, and that three standard deviations each side of the mean are included within the tolerance limits, the standard deviation is found to be $\sigma_b = (\Delta_1 + \Delta_2)/6$ from which the coefficient of variation $V_b = \sigma_b/\bar{b}$.

For other geometrical quantities (such as the cross-sectional area) when raw data giving the statistical variations are not available, the required properties can be developed as follows:

$$[17] \quad A = 2bt_f + hw$$

$$[18] \quad \bar{A} = \overline{2bt_f} + \overline{hw}$$

Hence,

$$[19] \quad \rho_A = \bar{A}/A = (\overline{2bt_f} + \overline{hw})/(2bt_f + hw)$$

and

$$[20] \quad V_A^2 = \frac{1}{\bar{A}^2}\left[\left(\frac{\partial A}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{\partial A}{\partial t_f}\right)^2 \sigma_{t_f}^2 + \left(\frac{\partial A}{\partial h}\right)^2 \sigma_h^2 + \left(\frac{\partial A}{\partial w}\right)^2 \sigma_w^2\right]$$

The ratio ρ_A to represent all the cross sections available from Canadian mills can be taken as

$$[21] \quad \bar{\rho}_A = \frac{1}{n}(\rho_{A_1} + \rho_{A_2} + \dots + \rho_{A_n}) = \frac{1}{n}\sum^{i=n} \rho_A$$

and the coefficient of variation using moment algebra as given by Benjamin and Cornell (1970) is

$$[22] \quad V_A = \frac{1}{\bar{\rho}_A}\frac{1}{n^{1/2}}\left[\sum^{i=n} (V_{Ai}\rho_i)^2 + \sum (\rho_A - \rho_{Ai})^2\right]$$

Data from Canadian mills for the variation of

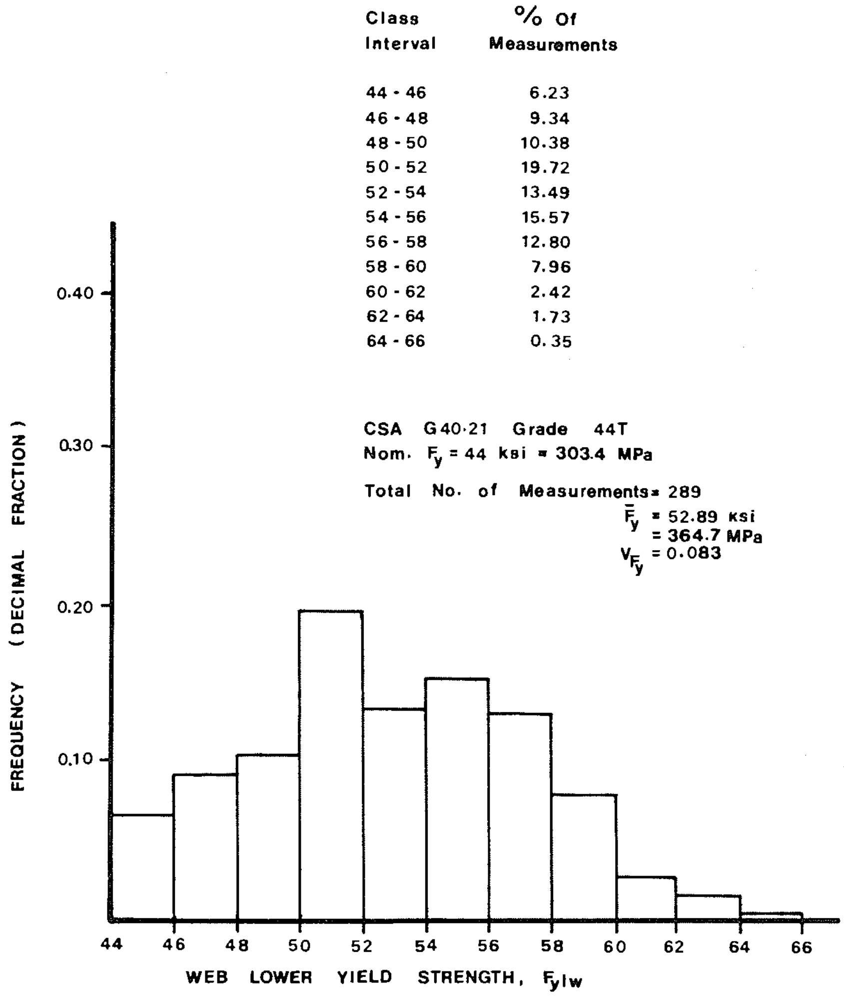

FIG. 8. Yield strength variation for W sections, different web thicknesses.

TABLE 1. Statistical parameters of the variation of material properties for rolled and welded W shapes

	Statistical parameters	
Property	Mean/nominal, ρ_M	Coefficient of variation, V_M
Yield strength of cross section rolled W shape, F_y	1.07	0.065
Yield strength of flanges rolled W shape, F_{yf}	1.05	0.065
Yield strength of cross section welded W shapes, F_y	1.10	0.11
Modulus of elasticity, E	1.00	0.019

flange width, b, flange thickness, t_f, and web thickness, w, are given in Figs. 9–11. By calculating the variation in the web depth using the equations given the statistical parameters for various cross-sectional properties are calculated for all Canadian W shapes as summarized in Table 2. To two significant figures $\rho_G = 0.99$ and V_G varies from 0.021 to 0.058.

4.3 Professional Parameters, ρ_P, V_P

4.3.1 General

Allen (1975) proposed a general value of ρ_P (the ratio of the tested capacity to the measured capacity) of 1.05 with a coefficient of variation $V_P = 0.07$ based on the earlier work of Galambos and Ravindra (1973). Information is now available from others for different types of structural members as discussed below.

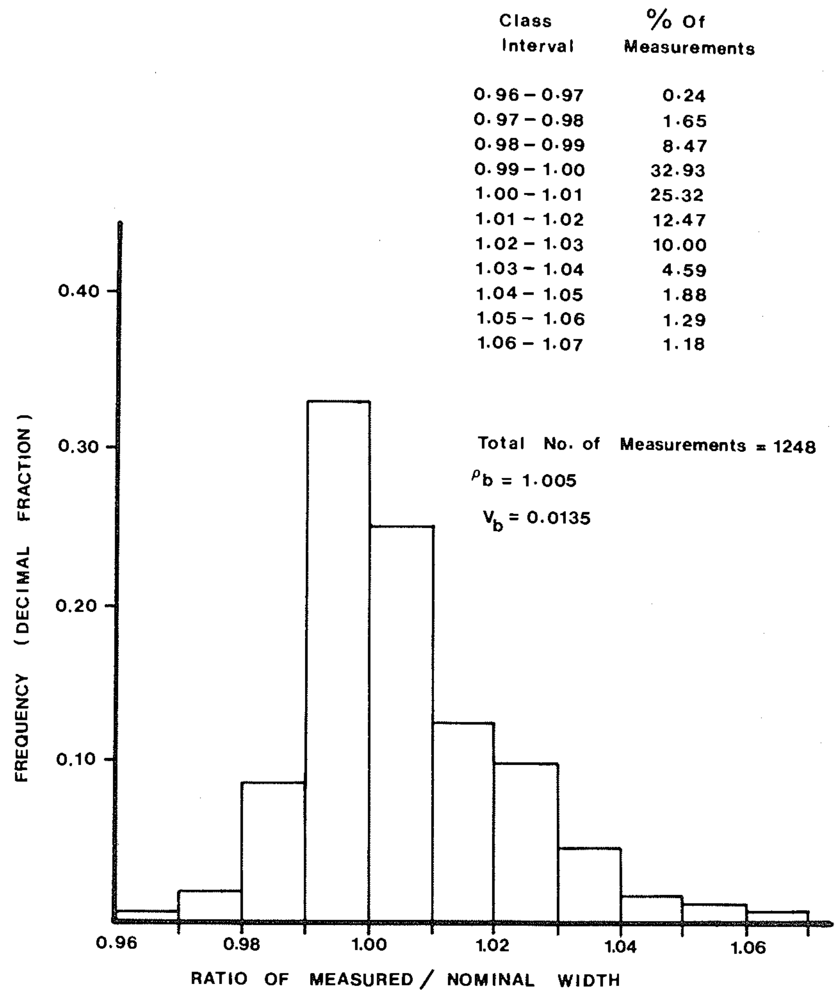

FIG. 9. Flange width variation for W sections.

4.3.2 Columns

Bjorhovde (1972) established 112 column curves, that is, curves relating the nondimensional ultimate capacity C_r/C_y to the slenderness parameter λ from experimentally measured data on the yield strength, residual stress, and cross-sectional dimensions, and based on an initial out-of-straightness of 1/1000. Included in this study were: different steel grades with yield strengths varying from 33 to 130 ksi (227–896 MPa); different sizes ranging from 13 to 774 lb/ft (19–1152 kg/m); and different cross-sectional shapes (rolled and welded).

Tests on 26 representative samples, when compared to Bjorhovde's ultimate strength theory including the effects of actual out-of-straightness, gave

$$[23] \quad \rho_{P_1} = \frac{C_r/C_y \text{ experiment}}{C_r/C_y \text{ ultimate strength theory}} = 1.03$$

and a coefficient of variation $V_{P_1} = 0.05$.

For rolled W shapes consisting of low-strength, medium-strength, and high-strength steels, Bjorhovde established 40 column curves and lower and upper envelope curves as well. The CSA standard S16.1-1974 design curve used in this paper to predict the mean column strength does not fit the mean curve of the 40 curves for rolled shapes established by Bjorhovde and therefore a second professional factor, ρ_{P_2}, which is a function of the slenderness parameter, must be introduced. The coefficient of variation, V_{P_2}, can be conservatively estimated from the band width of the 40 curves. When 95% or more of the curves fall within 95% of the band width, this width (assuming a normal distribution) represents ± 1.96 standard deviations. This is the case. The overall professional factor is therefore given by

$$[24] \quad \rho_P = \rho_{P_1}\rho_{P_2} = \frac{C_r/C_y \text{ experiment}}{C_r/C_y \text{ ultimate strength theory}} \times \frac{C_r/C_y \text{ ultimate strength theory}}{C_r/C_y \text{ predicted by CSA S16.1-1974}}$$

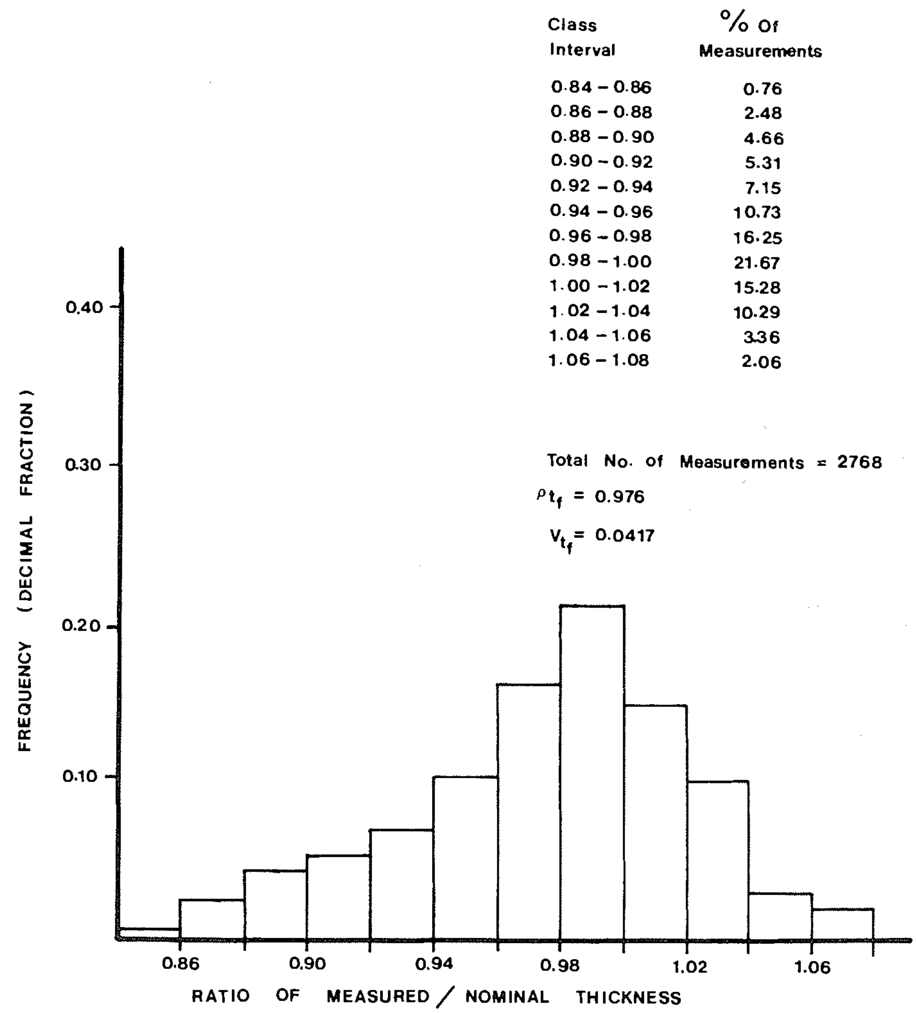

FIG. 10. Flange thickness variation for W sections.

Values of ρ_{P_2} given in Table 3 for rolled shapes lie between 1.00 and 1.08, and the coefficient of variation V_{P_2} given in the same table ranges from 0 to a maximum of 0.11 for $\lambda = 1.0$. V_{P_2} is relatively larger for values of λ in the range 0.8–1.4 because of the marked and varied effect on the ultimate strengths of the variation in residual stresses and residual stress patterns. For λ greater than 2, the statistical parameters have been estimated. Also,

$$[25] \quad V_P{}^2 = V_{P_1}{}^2 + V_{P_2}{}^2$$

4.3.3 Beams

Galambos and Ravindra (1976) report the following values for the professional factor and its coefficient of variation for rolled W beams depending on the mode of failure (by reaching the fully plastic moment, by inelastic buckling, or by elastic buckling).

Mode of failure	Professional ratio	Coefficient of variation
M_p	1.10	0.11
Inelastic buckling	1.06	0.09
Elastic buckling	1.03	0.09

It is necessary as well to distinguish among the classes of beams given in CSA S16.1-1974, as follows.

Beams of class 1 and 2 sections have sufficiently low width-to-thickness ratios that the maximum moment capacity of the cross section of M_p can be attained. Class 3 section beams with higher width-to-thickness ratios have a maximum cross-sectional moment capacity of M_y, and class 4 sections will fail by local buckling of the plate elements.

Class Interval	% Of Measurements
0.92 – 0.94	1.42
0.94 – 0.96	5.40
0.96 – 0.98	9.38
0.98 – 1.00	15.62
1.00 – 1.02	24.15
1.02 – 1.04	20.45
1.04 – 1.06	12.78
1.06 – 1.08	4.55
1.08 – 1.10	3.69
1.10 – 1.12	1.14
1.12 – 1.14	0.57
1.14 – 1.16	0.85

Total No. of Measurements = 352

$\rho_W = 1.0167$

$V_W = 0.0384$

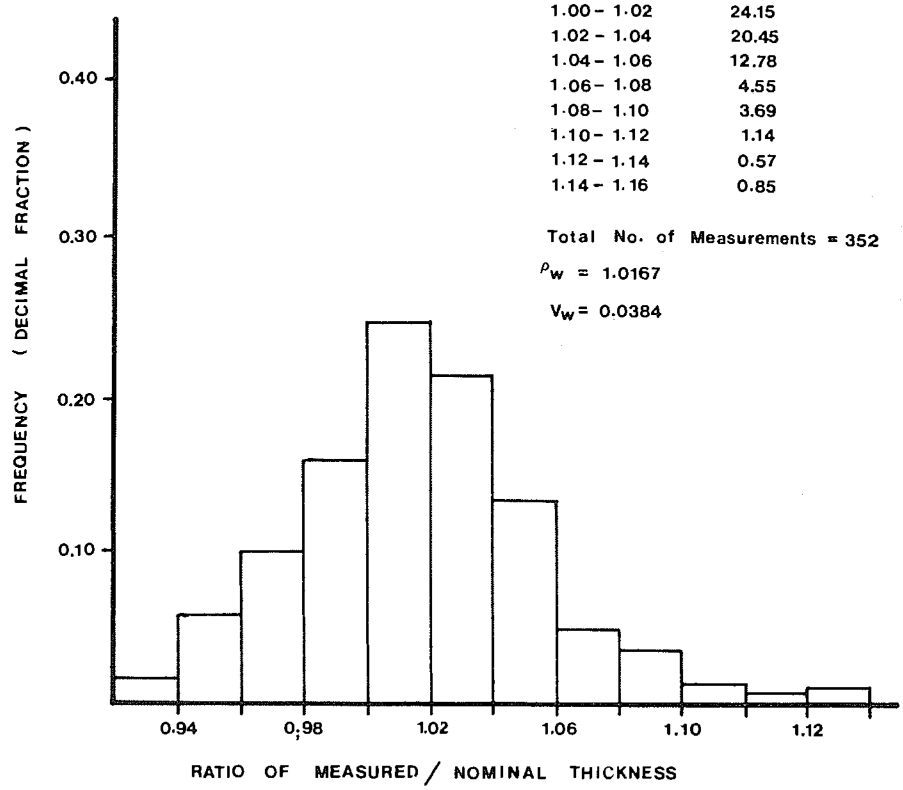

FIG. 11. Web thickness variation for W sections.

The results for beams reaching the plastic moment, M_p, as reported by Galambos and Ravindra (1976) are for a total of 117 tests: 33 on statically determinate beams under uniform moment, 43 on such beams under moment gradient, and 41 on statically indeterminate beams and simple frames.

For beams with width-to-thickness ratios that limit the maximum moment attainable to the yield moment, M_y, no data were found to determine the professional ratio, ρ_P.

A conservative estimate, however, can be made based on the professional ratio for reaching M_p of 1.10 and its coefficient of variation. The range of b/t_f ratios for these sections (designated as class 3 in CSA S16.1-1974) varies from that which just allows M_y to be reached to that which falls short of the attainment of $M_p = 1.14M_y$. Thus, for the range of sections, the moment reached will in fact be about $(M_y + M_p)/2$ on the average, that is, about $1.07M_y$.

The coefficient of variation associated with this range (± 2 standard deviations) would be 0.03 and, allowing for the effect of variation in b/t_f, V_P is estimated as $V_P{}^2 = (0.03^2 + 0.05^2)^{1/2} = 0.06$.

In the inelastic range a detailed analysis of Dibley's (1968) results, when the CSA S16.1-1974 straight line equation is used to predict this failure mode, gives $\rho_P = 1.05$ and a coefficient of variation of 0.078. These results are slightly different than those reported by Galambos and Ravindra (1976) of -1.06 and 0.09 because the CSA S16.1-1974 straight line intersecting the elastic buckling curve at either $2/3M_p$ or $2/3M_y$, depending on the class of the section, lies below that used by Galambos and Ravindra, which places the intersection at a moment corresponding to a maximum stress less than yield stress by the maximum compressive residual stress. The values $\rho_P = 1.05$ and $V_P = 0.078$ will be used herein.

In the elastic buckling range, the values are based on 185 tests as reported by Hechtman *et al.* (1957),

TABLE 2. Statistical parameters of the variation of geometric properties for rolled W shapes

	Statistical parameters	
Property	Mean/nominal, ρ_G	Coefficient of variation, V_G
b	1.005	0.0135
t_f	0.979	0.0417
w	1.017	0.0384
d	1.000	0.0009–0.0030*
A	0.99	0.033
I_x	0.99	0.043
I_y	1.00	0.058
S_x	0.99	0.021
S_y	0.99	0.049
Z_x	0.99	0.038
Z_y	0.99	0.048
r	1.00	0.023
J	0.96	0.100
C_w	0.99	0.090

*Depending on range in depths of rolled sections.

TABLE 3. Statistical parameters for professional factors ρ_{P_2}, V_{P_2} for rolled W shape columns

λ	Mean/nominal, ρ_{P_2}	Coefficient of variation, V_{P_2}
0.0	1.000	0.000
0.2	0.986	0.014
0.4	1.000	0.034
0.6	1.010	0.066
0.8	1.025	0.100
1.0	1.062	0.111
1.2	1.083	0.102
1.4	1.031	0.084
1.6	1.019	0.064
1.8	1.000	0.056
2.0	1.000	0.052
>2.0	1.000	0.050

Hartmann (1969), and Trahair (1969) where the elastic capacity was predicted by the equation

$$[26] \quad M_u = \frac{\pi}{\omega L}\sqrt{EI_yGJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$$

which is valid for doubly symmetric beams and as used in CSA S16.1-1974.

The professional factors for rolled W shapes used as beams are given in Table 4.

5. Performance Factors for Rolled W Shapes

5.1 General

In the establishment of performance factors, the statistical variations of all parameters *beyond the designer's control* (such as the variations in material properties; yield stress, modulus of elasticity; geometric properties; area, moment of inertia, etc.; predicted strengths versus strengths determined by test (when other parameters are known)) are taken into account.

TABLE 4. Statistical parameters for professional factors for rolled W and welded W shape beams

	Statistical parameter	
Mode of failure	Mean/nominal, ρ_P	Coefficient of variation, V_P
Maximum strength		
M_p	1.10	0.110
M_y	1.07	0.060
M_{cr}	1.05	0.050
Inelastic buckling	1.05	0.078
Elastic buckling	1.03	0.093

Some equations are necessary to predict the strength; where instability is a problem, these equations will reflect (as a function of the increased length of the member) the gradual transition from a capacity determined solely on the basis of cross-sectional capacity through that determined with inelastic action, occurring to the purely elastic failure mode.

In the design process, the designer must estimate or determine the "effective length" of the member. Errors would directly affect the member strength prediction. However, the determination of the effective length, to whatever precision the designer desires, is within his control. He either determines a correct (or conservative) value or he does not. Therefore the effective length is not a parameter subject to statistical variation beyond his control and properly should not be taken into consideration in the establishment of performance factors. In the establishment of the latter only those factors over which the designer has no control are considered. The same argument applies when the designer assesses the unsupported length of beams.

The ratio of the mean value of the resistance to the nominal value and the coefficient of variation of the resistance having been determined, performance factors can be calculated for selected values of the coefficient of separation, α_R, and the safety index, β.

As determined by Galambos and Ravindra (1977) a single value of $\alpha_R = 0.55$ will be used throughout. As well, a single value of 3.0 for the safety index, β, will be used.

Although it has been suggested that a higher value for slender columns than for laterally supported class 1 and 2 beams is appropriate, Galambos and Ravindra (1978) recommend a value of 3.0 for routine design situations. They point out, furthermore, that $\beta = 3.0$ represents approximately the reliability of presently designed simple beams *and* axially loaded columns.

The use of unique values for α_R and β also allows the variation in the performance factor due to differences in ρ_R and in V_R to be readily apparent.

5.2 Columns

The factored unit compressive resistance, F, in ksi, applying to class 1, 2, or 3 sections in CSA S16.1-1974, is given as

$$\begin{aligned} &0 \le \lambda \le 1.0 \\ &\qquad F = \phi F_y(1.035 - 0.202\lambda - 0.222\lambda^2) \\ &1.0 < \lambda \le 2.0 \\ [27] \quad &\qquad F = \phi F_y(-0.111 + 0.636\lambda^{-1} + 0.087\lambda^{-2}) \\ &2.0 < \lambda \le 3.6 \\ &\qquad F = \phi F_y(0.009 + 0.877\lambda^{-2}) \\ &3.6 < \lambda \qquad F = \phi F_y \lambda^{-2} \end{aligned}$$

in which

[28] $\lambda = (KL/r)(F_y/\pi^2 E)^{1/2}$

from which the compressive resistance C_r is then

[29] $C_r = AF$

The performance factor is given by

$$[30] \quad \phi = (\bar{C}_r/C_r)\exp(-\alpha\beta V_{C_r}) = \rho_{C_r}\exp(-\alpha\beta V_{C_r})$$

in which $C_r = AF$; $\bar{C}_r = \overline{AF}$; $\alpha = 0.55$; and $\beta = 3$.

Working with the first of the equations for F above for the material strength, the mean unit compressive resistance, $\bar{F}$, can be written as

$$[31] \quad \bar{F} = F_y\rho_{F_y}\left[1.035 - 0.202\lambda\left(\frac{\rho_{F_y}}{\rho_r 2\rho_E}\right)^{1/2} - 0.22\lambda^2\left(\frac{\rho_{F_y}}{\rho_r 2\rho_E}\right)\right]$$

in which

[32] $\rho_{F_y} = \bar{F}_y/F_y$

and the other subscripted ratios, ρ, correspond, or

[33] $\bar{F} = F_y\rho_{F_y}(1.035 - 0.202\lambda\rho_\lambda - 0.222\lambda^2\rho_\lambda^2)$

in which

$$[34] \quad \rho_\lambda = \frac{\bar{\lambda}}{\lambda} = \sqrt{\frac{\rho_{F_y}}{\rho_r^2\rho_E}}$$

The ratio $\bar{F}/F = \rho_F$ is then

$$[35] \quad \rho_F = \rho_{F_y}\frac{(1.035 - 0.202\lambda\rho_\lambda - 0.222\lambda^2\rho_\lambda^2)}{(1.035 - 0.202\lambda - 0.222\lambda^2)}$$

The coefficient of variation V_F is given by

$$[36] \quad V_F^2 = \frac{1}{\bar{F}^2}\left[\left(\frac{\overline{\partial F}}{\partial F_y}\right)^2\sigma_{F_y}^2 + \left(\frac{\overline{\partial F}}{\partial r}\right)^2\sigma_r^2 + \left(\frac{\overline{\partial F}}{\partial E}\right)^2\sigma_E^2\right]$$

when the column length KL is considered deterministic.

From the equation for F,

$$[37] \quad V_F = \left(\frac{T_1^2 V_{F_y}^2 + T_2^2 V_r^2 + T_3^2 V_E^2}{(1.035 - 0.202\lambda\rho_\lambda - 0.222\lambda^2\rho_\lambda^2)^2}\right)^{1/2}$$

in which V_{F_y}, V_r, V_E are the coefficients of variation of F_y, r, and E respectively. T_1, T_2, T_3, participation factors reflecting the relative contribution of the several respective V's, are

[38] $T_1 = 1.035 - 0.303\lambda\rho_\lambda - 0.444\lambda^2\rho_\lambda^2$

[39] $T_2 = 0.202\lambda\rho_\lambda^2 - 0.444\lambda^2\rho_\lambda^2$

[40] $T_3 = 0.101\lambda\rho_\lambda^2 + 0.222\lambda^2\rho_\lambda^2$

The ratio $\rho_{cr} = \rho_F\rho_A\rho_P$, and $V_{cr}^2 = V_F^2 + V_A^2 + V_P^2$ (where ρ_F and V_F are as given above (Table 1 gives values of material parameters); ρ_A, V_A are the geometric (area) parameters (Table 2); ρ_P, V_P are the professional parameters (Table 3) now calculated over the range $0 \le \lambda \le 1.0$) lead to the determination of the performance factor, ϕ, as a function of λ. In a similar manner, equations are developed for the other ranges of λ for ρ_F and V_F to give ϕ as a function of λ over the entire range.

It is recognized in the formulation of V_{cr} that V_F and V_A are treated as independent variables when, in fact, F depends on the radius of gyration and, therefore, A. The approximation involved is conservative as the interdependence of the two variables would result in a correlation coefficient applied to both V_F and V_A of less than 1.0.

In the elastic range $3.6 < \lambda$ the compressive resistance can be written as

[41] $C_r = \phi\pi^2 EI/(KL)^2$

with

[42] $\phi = \rho_{cr}\exp(-\alpha\beta V_{cr})$

[43] $\rho_{cr} = \rho_P\rho_E\rho_I$

and

[44] $V_{cr}^2 = V_P^2 + V_E^2 + V_I^2$

In the inelastic range the performance factor ϕ depends on the slenderness parameter λ and on the equation used for the column strength curve as the participation factors are determined from the partial derivatives of the equation. Where the equation has been derived on a theoretical basis to give a closed form solution, only one set of participation factors will be obtained at a given value of the independent variable; but where the resistance has been determined by curve-fitting, another curve fitting the data points equally well over a short range (say a linear or second-degree curve rather than a third-degree curve) would yield somewhat different participation factors.

Although based on exact theories that take into account both the effects of residual stresses and out-of-straightness, Bjorhovde's (1972) analysis does not yield a closed form solution and therefore the problem exists of anomolies in the values determined for performance factors for columns failing by inelastic buckling. To study the possible magnitude of the anomaly, Bjorhovde's curve in the range $0.99 \leq \lambda \leq 1.0$ was approximated by a series of straight lines over small intervals. In this range of the slenderness parameter, where the maximum differences by using the two different curves (Bjorhovde's and the straight line approximations) were obtained, the maximum difference in the performance factor was found to be less than 0.4%. This is considered negligible.

The same circumstances apply mutatis mutandi for laterally unsupported beams failing by inelastic buckling.

Table 5 gives the performance factor for rolled W shape columns as a function of λ based on the statistical parameters (ρ's and V's) and the derived participation factors. The tabulated contribution factors, T', are the participation factors, T, divided by $\bar{F}$.

For rolled W shape columns, based on SSRC curve 2 as used in CSA S16.1-1974, Table 5 shows that the performance factor lies generally above 0.90 for $0 \leq \lambda \leq 2$. For $\lambda > 2$, where the stiffness of the column predominates (the contribution factors $V_r(T_2')$ and $V_E(T_3')$ are within 10% of their limiting values of 2 and 1.00; ρ_F approaches a value of 1.0), the performance factor has a minimum value of 0.88. The blanket value of 0.90 used in CSA S16.1-1974 is therefore on the conservative side, up to 5% for low slenderness parameters and about 2% too liberal for $\lambda = 2$.

5.3 Beams

5.3.1 Laterally Supported

For class 1 and class 2 sections the maximum capacity is given by $M_r = \phi Z F_y$. Hence,

$$[45] \quad \phi = \rho_R \exp(-\alpha\beta V_R) = \rho_P \rho_G \rho_M \exp[-\alpha\beta(V_P^2 + V_G^2 + V_M^2)^{1/2}]$$

where from Table 4, $\rho_P = 1.10$ and $V_P = 0.110$; from Table 2, $\rho_G = \rho_Z = 0.99$ and $V_G = V_Z = 0.038$; from Table 1, $\rho_M = \rho_{F_{yr}} = 1.05$ and $V_M = V_{F_y} = 0.065$. Therefore, $\phi = 1.14 \exp[-0.55(3)(0.133)] = 0.92$.

For class 3 sections the maximum capacity is given by $M_r = \phi S F_y$ and the values of the parameters, different from those above, from the same tables are: $\rho_P = 1.07$; $V_P = 0.06$; $\rho_S = 0.99$; and $V_S = 0.021$; to give $\phi = 1.12 \exp[-0.55(3)(0.091)] = 0.96$.

For class 4 sections the maximum capacity is given by $M_r = \phi S F_{cr}$ where F_{cr} is the critical stress corresponding to local buckling of the compression element, which depends on its b/t ratio. For example, in CSA S16.1-1974 when $b/t_f \leq 198(k/F_y)^{1/2}$

$$[46] \quad F_{cr} = F_y[1.46 - 0.004(F_y/k)^{1/2}(b/t_f)]$$

To make the problem more tractable, express

$$[47] \quad S_x = (2[bt_f^3/12 + bt_f(d/2)^2] + wd^3/12)/(d/2)$$

as $S_x \simeq d(bt_f + wd/6) \simeq 7bdt_f/6$.

The moment resistance is given by

$$[48] \quad M_r = (7bdt_f/6)F_y[1.46 - 0.004(F/k)(b/t_f)]$$

From this expression the necessary mean to nominal ratios, ρ, their coefficients of variation, V, and the participation factors, designated R, can be isolated. Thus,

$$[49] \quad \bar{M}_r/M_r = \rho_{M_r} = \rho_P \rho_b \rho_d \rho_{t_f} \rho_{F_y} \rho_F$$

where

$$[50] \quad \rho_F = \bar{F}/F = [1.46 - 0.004(\bar{F}_y/\bar{k})^{1/2}(\bar{b}/\bar{t}_f)] \div [1.46 - 0.004(F_y/k)^{1/2}(b/t_f)]$$

The coefficient of variation is determined from

$$[51] \quad V_{M_r} = \left\{\frac{1}{\bar{M}_r^2}\left[\left(\frac{\partial M_r}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{\partial M_r}{\partial d}\right)^2 \sigma_d^2 + \left(\frac{\partial M_r}{\partial t_f}\right)^2 \sigma_{t_f}^2 + \left(\frac{\partial M_r}{\partial F_y}\right)^2 \sigma_{F_y}^2 + \left(\frac{\partial M_r}{\partial k}\right)^2 \sigma_k^2\right] + V_P^2\right\}^{1/2}$$

For class 4 sections rolled in Canada, b/t_f ratios applicable to this design equation range from 15 to 25 for $F_y = 44$ ksi (303 MPa) and will affect the performance factor. Galambos and Ravindra (1976) give the mean value of the plate buckling coefficient to be $k = 0.821$, and $V_k = 0.13$. Because CSA S16.1-1974 uses $k = 0.70$, therefore $\rho_k = 1.17$. Using values for ρ_{F_y}, ρ_P, ρ_k, V_{F_y}, V_P, and V_k equal to 1.05, 1.05, 1.17, 0.065, 0.05, and 0.13 respectively, performance factors ϕ ranging from 0.93 to 0.90 are obtained for b/t_f ratios from 15 to 25.

When $b/t_f > 198(k/F_y)^{1/2}$, the critical buckling

TABLE 5. Performance factors for rolled W shape columns

					Contribution factors			Coefficients of variation								
λ	ρ_F*	ρ_{P_1}	ρ_{P_2}	ρ_A†	T_1'	T_2'	T_3'	V_{F_y}	V_r	V_E	V_A	V_{P_1}	V_{P_2}	ρ_{C_r}	V_{C_r}	ϕ
0.00	1.071	1.030	1.000	0.990	1.000	0.000	0.000	0.065	0.023	0.019	0.033	0.50	0.000	1.092	0.088	0.944
0.20	1.069	1.030	0.986	0.990	0.969	0.062	0.031	0.065	0.023	0.019	0.033	0.05	0.014	1.075	0.088	0.930
0.40	1.065	1.030	1.000	0.990	0.913	0.175	0.087	0.065	0.023	0.019	0.033	0.05	0.034	1.086	0.091	0.935
0.60	1.058	1.030	1.010	0.990	0.820	0.360	0.180	0.065	0.023	0.019	0.033	0.05	0.066	1.090	0.104	0.918
0.80	1.048	1.030	1.025	0.990	0.670	0.659	0.330	0.065	0.023	0.019	0.033	0.05	0.100	1.095	0.125	0.891
1.00	1.031	1.030	1.062	0.990	0.418	1.164	0.582	0.065	0.023	0.019	0.033	0.05	0.111	1.116	0.132	0.898
1.20	1.022	1.030	1.083	0.990	0.317	1.366	0.683	0.065	0.023	0.019	0.033	0.05	0.102	1.129	0.125	0.919
1.40	1.020	1.030	1.031	0.990	0.294	1.413	0.706	0.065	0.023	0.019	0.033	0.05	0.084	1.072	0.110	0.894
1.60	1.019	1.030	1.019	0.990	0.262	1.468	0.734	0.065	0.023	0.019	0.033	0.05	0.064	1.059	0.096	0.904
1.80	1.016	1.030	1.000	0.990	0.234	1.533	0.766	0.065	0.023	0.019	0.033	0.05	0.056	1.036	0.091	0.892
2.00	1.014	1.030	1.000	0.990	0.197	1.606	0.803	0.065	0.023	0.019	0.033	0.05	0.052	1.034	0.089	0.893
2.20	1.003	1.030	1.000	0.990	0.051	1.899	0.949	0.065	0.023	0.019	0.033	0.05	0.050	1.023	0.091	0.880
2.40	1.004	1.030	1.000	0.990	0.060	1.881	0.940	0.065	0.023	0.019	0.033	0.05	0.050	1.024	0.091	0.881
2.60	1.004	1.030	1.000	0.990	0.069	1.862	0.931	0.065	0.023	0.019	0.033	0.05	0.050	1.024	0.091	0.881
2.80	1.005	1.030	1.000	0.990	0.079	1.841	0.921	0.065	0.023	0.019	0.033	0.05	0.050	1.025	0.090	0.884
3.00	1.006	1.030	1.000	0.990	0.090	1.820	0.910	0.065	0.023	0.019	0.033	0.05	0.050	1.026	0.090	0.884
3.20	1.007	1.030	1.000	0.990	0.101	1.798	0.899	0.065	0.023	0.019	0.033	0.05	0.050	1.027	0.090	0.885
3.40	1.007	1.030	1.000	0.990	0.113	1.774	0.887	0.065	0.023	0.019	0.033	0.05	0.050	1.027	0.090	0.885
3.60	1.008	1.030	1.000	0.990	0.125	1.751	0.875	0.065	0.023	0.019	0.033	0.05	0.050	1.028	0.089	0.886
>3.60		1.030	1.000	0.999				0.065	0.023	0.019	0.033	0.05	0.050	1.029	0.079	0.093

*Includes $\rho F_y = 1.071$, $\rho_\lambda = 1.035$.
†For $\lambda \geq 3.6$, ρ_A is replaced by ρ_1 and V_A by V_1.

stress is given by $F_{cr} = 26200k/(b/t_f)^2$ and, using the approximate expressions for the section modulus, the resisting moment is

$$[52] \quad M_r = (7bdt_f/6)(26200k)/(b/t_f)^2$$

from which

$$[53] \quad \overline{M}_r/M_r = \rho M_r = \rho_P\rho_k\rho_d\rho_{t_f}{}^3\rho_b{}^{-1}$$

and

$$[54] \quad V_{M_r} = (V_k{}^2 + V_d{}^2 + 9V_{t_f}{}^2 + V_b{}^2 + V_P{}^2)^{1/2}$$

to give a performance factor of ϕ based on the same statistical parameters of 0.84.

5.3.2 Laterally Unsupported, Elastic Range

The design equation is

$$[55] \quad M_u = \frac{\pi}{\omega L}\left[EI_yGJ + \left(\frac{\pi E}{L}\right)^2 I_yC_w\right]^{1/2} = \frac{\pi}{\omega L}\left[E^2I_y\left(\frac{GJ}{E} + \frac{\pi^2}{L^2}C_w\right)\right]^{1/2}$$

The mean value of M_u, $\overline{M}_u$, is obtained by using mean values of the variables in the above expression and by multiplying by ρ_P, the tested to predicted capacity ratio.

To make the statistical analysis tractable, simplifications and approximations are introduced as follows:

$$[56] \quad J = (1/3)\sum l_it_i{}^3$$

$$[57] \quad C_w = I_fh^2/4 = t_fb^3h^2/24$$

Introducing B_1 such that

$$[58] \quad \overline{B}_1 = (\overline{J}/J + \overline{C}_w/C_w)/2 = (0.96 + 0.99)/2 = 0.975$$

and approximates both $\overline{J}/J$ and $\overline{C}_w/C_w$, and considering that G/E = a constant = 0.384 and that L is deterministic, then

$$[59] \quad \overline{M}_u = \rho_P\frac{\pi}{\omega L}\left[\overline{E}^2\overline{I}_y\overline{B}_1\left(\frac{G}{E}J + \frac{\pi^2}{L^2}C_w\right)\right]^{1/2}$$

Hence,

$$[60] \quad \overline{M}_u/M_u = \rho_P\rho_E(\rho_{I_y}\overline{B}_1)^{1/2} = (1.03) \times 1.00[1.00(0.975)]^{1/2} = 1.017$$

with a further substitution B_2 such that

$$[61] \quad \overline{B}_2{}^2 = \rho_{I_y}\overline{B}_1$$

because I_y, J, and C_w are principally functions of b and t_f. Then

$$[62] \quad V_{B_2} = [(V_{I_y}{}^2 + V_{B_1}{}^2)/2]^{1/2}$$

Hence,

$$[63] \quad \overline{M}_u/M_u = \rho_{M_u} = \rho_P\rho_E\overline{B}_2$$

$$[64] \quad V_R = V_{M_u} = V_{\rho_{M_u}}$$

and

$$[65] \quad V_{\rho_{M_u}}{}^2 = \left(\frac{1}{\rho_{M_u}{}^2}\left[\left(\frac{\partial\rho_{M_u}}{\partial\rho_E}\right)^2\sigma_E{}^2 + \left(\frac{\partial\rho_{M_u}}{\partial\overline{B}_2}\right)^2\sigma_{B_2}{}^2\right]^2 + V_P{}^2\right)$$

$$[66] \quad V_{M_u} = (V_E{}^2 + V_{B_2}{}^2 + V_P{}^2)^{1/2} = \left[V_E{}^2 + \frac{(V_{I_y}{}^2 + V_{B_1}{}^2)}{2} + V_P{}^2\right]^{1/2}$$

Letting $\overline{J}/J = C_1$ and $\overline{C}/C_w = C_2$,

$$[67] \quad B_1 = (C_1 + C_2)/2$$

Gad Aly (1978) establishes that

$$[68] \quad V_{B_1}{}^2 = \frac{\overline{C}_1{}^2}{(\overline{C}_1 + \overline{C}_2)^2}V_{C_1}{}^2 + \frac{\overline{C}_2{}^2}{(\overline{C}_1 + \overline{C}_2)^2}V_{C_2}{}^2 + \hat{\Delta}f^2$$

where

$$[69] \quad \hat{\Delta}f = \frac{1}{\overline{B}_1}\frac{[(\overline{C}_1 - \overline{B}_1)^2 + (\overline{C}_2 - \overline{B}_1)^2]^{1/2}}{2}$$

to give $V_{B_1} = 0.070$, from which $V_{B_2} = 0.065$ and $V_{M_u} = 0.114$. Thus

$$[70] \quad \phi = 1.017\exp[-0.55(3)(0.114)] = 0.84$$

5.3.3 Laterally Unsupported, Inelastic Range

The basic design equations given in CSA S16.1-1974 (depending on whether or not the cross-sectional capacity is M_p or M_y) are respectively, when $M_u > \frac{2}{3}M_p$

$$[71] \quad M_r = 1.15\phi M_p\left(1 - 0.28\frac{M_p}{M_u}\right), \quad \not> \phi M_p$$

and when $M_u > \frac{2}{3}M_y$

$$[72] \quad M_r = 1.15\phi M_y\left(1 - 0.28\frac{M_y}{M_u}\right), \quad \not> \phi M_y$$

in which M_u is as previously defined. In the first of these equations

$$[73] \quad \overline{M}_r = \rho_P 1.15\overline{M}_p\left(1 - 0.28\frac{\overline{M}_p}{\overline{M}_u}\right)$$

and the performance factor is given by

$$[74]\quad \phi = \rho_{\mathrm{P}}\frac{\overline{M}_{\mathrm{p}}}{M_{\mathrm{p}}}\frac{1-0.28\dfrac{\overline{M}_{\mathrm{p}}}{\overline{M}_{\mathrm{u}}}}{1-0.28\dfrac{M_{\mathrm{p}}}{M_{\mathrm{u}}}}\exp(-\alpha\beta V_{M_{\mathrm{r}}})$$

Setting

$$[75]\quad \overline{M}_{\mathrm{p}}/\overline{M}_{\mathrm{u}} = a_1 M_{\mathrm{p}}/M_{\mathrm{u}}$$

$$[76]\quad a_1 = (\overline{M}_{\mathrm{p}}/M_{\mathrm{p}})/(\overline{M}_{\mathrm{u}}/M_{\mathrm{u}})$$

with $\overline{M}_{\mathrm{p}}/M_{\mathrm{p}} = \rho_{M_{\mathrm{p}}} = \rho_Z\rho_{Fy} = 0.99(1.05) = 1.04$ and

$$V_{M_{\mathrm{p}}} = (V_Z{}^2 + V_{F_y}{}^2)^{1/2} = (0.038^2 + 0.065^2)^{1/2} = 0.076$$

Previously, $\overline{M}_{\mathrm{u}}/M_{\mathrm{u}} = 0.985$, with $V_{M_{\mathrm{u}}} = 0.067$. Therefore, from [76], $a_1 = 1.056$ and using [74]

$$[77]\quad \phi = 1.048(1.04)\frac{1-(0.28)1.056\dfrac{M_{\mathrm{p}}}{M_{\mathrm{u}}}}{1-0.28\dfrac{M_{\mathrm{p}}}{M_{\mathrm{u}}}} \times \exp(-\alpha\beta V_{M_{\mathrm{u}}})$$

By taking partial derivatives of the expression for M_{r}, $V_{M_{\mathrm{r}}}{}^2$ is obtained as

$$[78]\quad V_{M_{\mathrm{r}}}{}^2 = \frac{\left(1-0.56a_1\dfrac{M_{\mathrm{p}}}{M_{\mathrm{u}}}\right)^2}{\left(1-0.28a_1\dfrac{M_{\mathrm{p}}}{M_{\mathrm{u}}}\right)^2}V_{M_{\mathrm{p}}}{}^2 + \frac{\left(0.28a_1\dfrac{M_{\mathrm{p}}}{M_{\mathrm{u}}}\right)^2}{\left(1-0.28a_1\dfrac{M_{\mathrm{p}}}{M_{\mathrm{u}}}\right)^2}V_{M_{\mathrm{u}}}{}^2 + V_{\mathrm{p}}{}^2$$

Thus the coefficient of variation, $V_{M_{\mathrm{r}}}$, and therefore the performance factor ϕ depend on the ratio of $M_{\mathrm{p}}/M_{\mathrm{u}}$, which because M_{u} lies between M_{p} and $\frac{2}{3}M_{\mathrm{p}}$ in the inelastic range, varies from 1.0 to 1.5, giving a variation in ϕ of 0.91–0.89 in this range.

For class 3 rolled W sections, procedures paralleling the above, based on the equation $M_{\mathrm{r}} = 1.15\phi M_{\mathrm{y}}(1 - 0.28M_{\mathrm{y}}/M_{\mathrm{u}})$, yield corresponding expressions and, because the statistical parameters are virtually identical, the performance factors obtained are also the same over the range

$$[79]\quad 1.0 \le (M_{\mathrm{y}}/M_{\mathrm{u}}) \le 1.5$$

For class 4 sections the only difference is that the maximum moment obtainable is $M_{\mathrm{cr}} = SF_{\mathrm{cr}} \le SF_{\mathrm{y}} = M_{\mathrm{y}}$ and therefore the performance factors are valid here as well.

Performance factors for rolled W shape sections used as beams are summarized in Table 6. From this table, performance factors of 0.92 for maximum strength, 0.90 for inelastic buckling, and 0.84 for elastic buckling for all classes of sections would be appropriate.

6. Statistical Parameters for Welded W Shapes

Welded W shapes are produced in accordance with the requirements of CSA standard G40.20-1974 from plates meeting the physical and chemical requirements of CSA standard G40.21. Most shapes produced in Canada from plate up to 1.57 in. (40 mm) in thickness meet the requirements of G40.21 grade 44W and those from thicker plates meet the requirements of G40.21 grade 44T.

The welded W shape is manufactured using automated production methods. Hot-rolled plates are automatically flame-cut to the required width. The assembly machine holds a web plate horizontally and two flange plates vertically in the correct relative position and, by means of rolls, forces the plates into intimate contact and propels them at uniform speed under the automated welding heads, which are adjacent to the pressure rolls.

6.1 Material Parameters, ρ_M, V_M

The American Iron and Steel Institute (AISI) survey (1974) showed that, in addition to the variation in yield strength that would be expected from heat to heat, a variation in yield strength is exhibited within one heat from coupons taken at different locations across the width and along the length. This study showed that the mean difference from the reference test was only 0.03 ksi (0.2 MPa), which is considered negligible, but the coefficient of variation of the test strengths from the reference test strength was 0.073, which must be taken into account when considering the variation in the strength of plates.

Figures 12–14 give the variation among heats of the reference test measurements for CSA G40.20-1974 grades 44W and 44T steels. The mean yield strength, $\overline{F}_{\mathrm{y}}$, for 3370 readings is 50.54 ksi (384.5 MPa) and, correcting for the speed of testing as for rolled W shapes, the static yield stress is 48.54 ksi (334.7 MPa), giving $\rho_{F_y} = 48.54/44 = 1.10$.

The coefficient of variation for the three samples of data, using moment algebra, is found to be 0.034 and incorporating the within-heat variation as determined in the AISI survey $V_{F_y} = (0.083^2 + 0.073^2)^{1/2} = 0.11$, as given in Table 1.

6.2 Geometric Parameters, ρ_G, V_G

Figures 15 and 16 give statistical parameters for representative 2 in. (50.8 mm) and 0.375 in. (9.5 mm)

TABLE 6. Performance factors for rolled W shape beams

Classification	Performance factors: Maximum strength	Inelastic	Elastic
Classes 1 and 2	0.92	0.89–0.91	0.84
Class 3	0.96	0.89–0.91	0.84
Class 4	0.90–0.93* 0.84†	0.89–0.91	0.84

*If $b/t_f \le 198(k/F_y)^{1/2}$.
†If $b/t_f > 198(k/F_y)^{1/2}$.

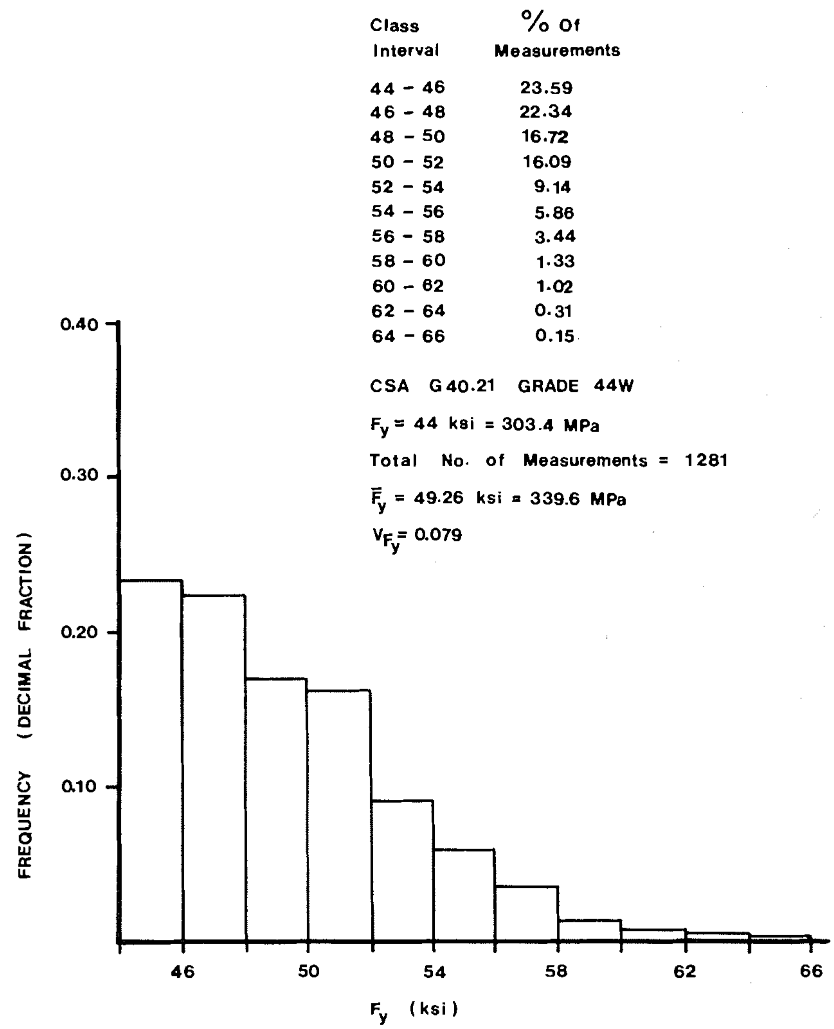

FIG. 12. Variation of yield strength, different plate thicknesses.

thick plate. For the total of 1223 measurements, $\rho_t = 1.015$ and $V_t = 0.013$. For the variation in plate width, that is, flange width and web depth, of the welded W shapes recourse must be made to specification limits as discussed under geometric parameters for rolled W shapes. CSA standard G40.20-1974 gives the following limits for welded shapes, which are identical to those for W shapes: variation in depth, d, $\pm 1/8$ in. (3.175 mm); and variation in width, b, $+1/4$, $-3/16$ in. (4.763 mm).

Using the expression previously derived, $\rho_d = 1.00$ and $\rho_b = 1.003$–1.007 for the range in flange widths of 9–22 in. (22.86–55.88 cm). Standard deviations are calculated to be $\sigma_d = 0.042$ and $\sigma_b =$

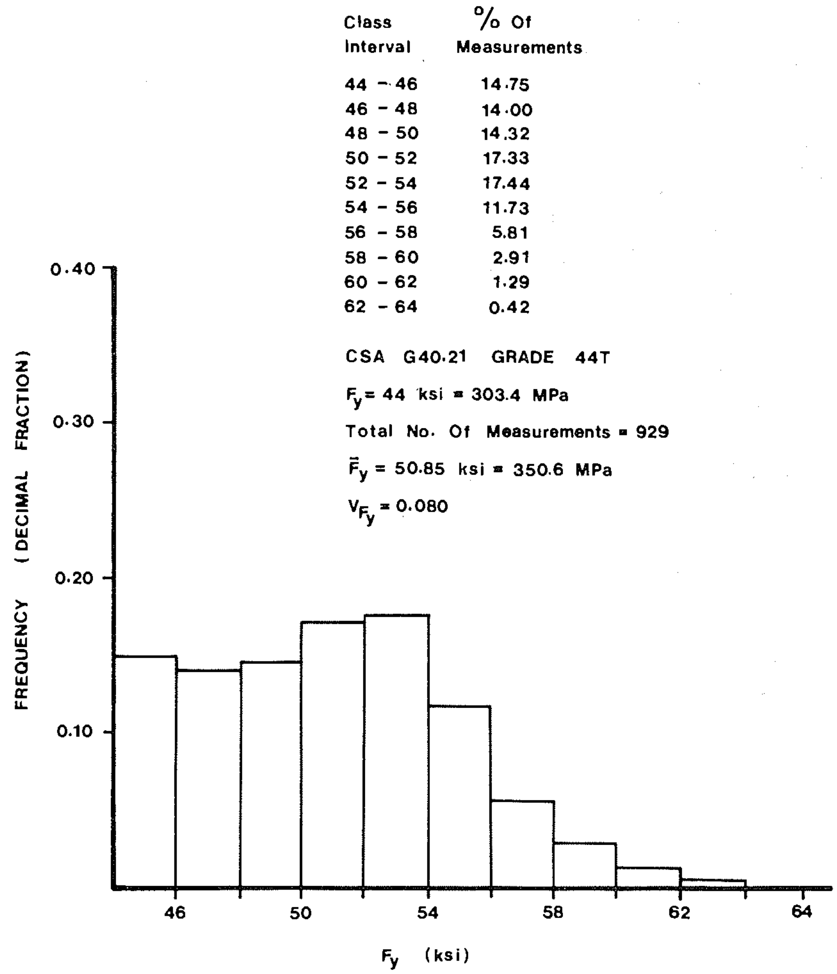

FIG. 13. Variation of yield strength, different plate thicknesses.

0.073 from which coefficients of variation for the range of depths of 14–48 in. (35.56–121.92 cm) and flange widths of 9–22 in. (22.86–55.88 cm) are computed to be: V_d = 0.0009–0.0030 and V_b = 0.0033–0.0081.

Using these statistical data and those for the variation in plate thickness and the algebra previously given, geometric properties for welded W shapes are developed as given in Table 7 for the total population of shapes produced in Canada.

Because the variation of the geometrical properties is chiefly controlled by the statistical parameters relating to plate thickness, which, as Figs. 15 and 16 show, give ratios of mean to nominal greater than 1.0 with small coefficients of variation, the geometric properties of welded W shapes generally have mean/nominal ratios greater than 1.0 with small coefficients of variation.

6.3 Professional Parameters, ρ_P, V_P

The professional parameters for basic strength equations, based on the geometric properties of the cross section and the yield strength of the steel, are not different for welded W shapes and rolled W shapes. Similarly, for elastic stability equations—long columns or beams failing by elastic lateral–torsional buckling—based on the geometric properties of the cross section and the modulus of elasticity no difference would be expected.

In the inelastic range, the capacity of a member is appreciably affected by the pattern and magnitude of residual stresses. Welded W shapes produced in Canada have flanges with flame-cut edges and thus there develop favourable (tensile) stresses. The welding at the flange–web junction produces residual stresses of the same pattern as that due to cooling, though probably somewhat more severe. For inelastic buckling of beams, the professional factor has been taken to be the same as for rolled W shapes. The professional factors given in Table 4 for rolled W shape beams are therefore considered applicable to welded W shapes.

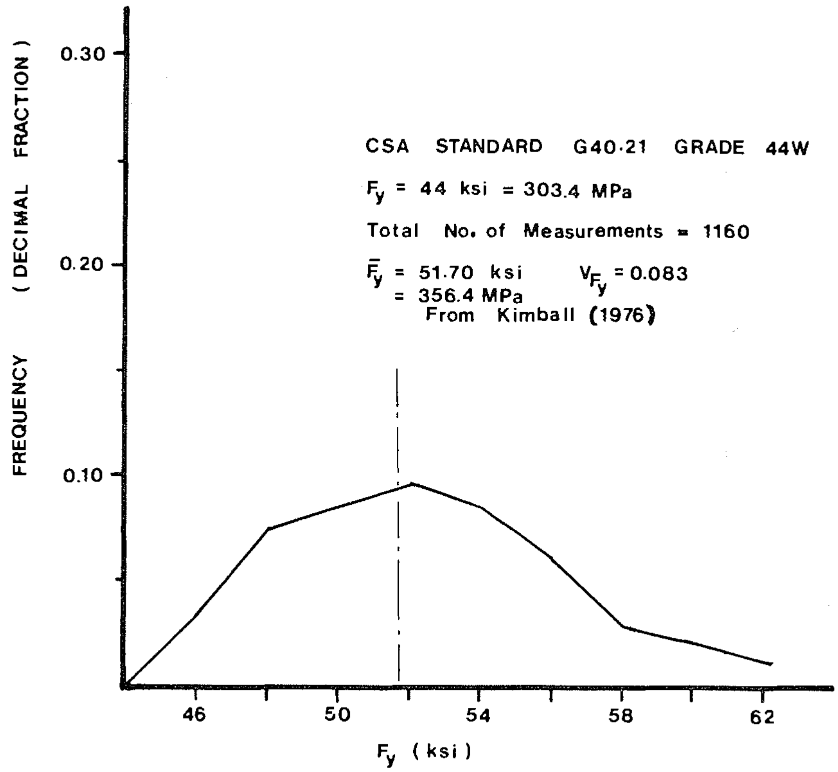

FIG. 14. Variation of yield strength, different plate thicknesses.

TABLE 7. Statistical parameters of the variation of the geometric properties for welded W shapes

	Statistical parameters	
Property	Mean/nominal, ρ	Coefficient of variation, V
b	1.003–1.007	0.0033–0.0081
t_f, w	1.015	0.013
d	1.000	0.0009–0.0030
A	1.019	0.012
I_x	1.021	0.017
I_y	1.030	0.020
S_x	1.020	0.010
S_y	1.025	0.018
Z_x	1.020	0.015
Z_y	1.025	0.017
r	1.005	0.009
J	1.049	0.043
C_w	1.029	0.035

TABLE 8. Statistical parameters for professional factors ρ_{P_2} and V_{P_2} for rolled W shape columns

λ	Mean/nominal, ρ_2	Coefficient of variation, V_{P_2}
0.0	1.000	0.000
0.2	0.977	0.011
0.4	0.974	0.022
0.6	0.942	0.025
0.8	0.925	0.034
1.0	0.941	0.049
1.2	0.984	0.059
1.4	0.995	0.053
1.6	0.997	0.048
1.8	0.955	0.046
2.0	0.960	0.042
>2.0	0.980	0.040

For columns, the professional factor is determined as the product of two factors, ρ_{P_1} and ρ_{P_2}, as for rolled W shapes. As before,

$$[80] \quad \rho_{P_1} = \frac{C_r/C_y \text{ experiment}}{C_r/C_y \text{ ultimate strength theory}} = 1.03$$

based on Bjorhovde's work, with $V_{P_1} = 0.05$, giving the ratio of the 26 representative test strengths to that predicted by the ultimate strength theory. The second professional factor, ρ_{P_2}, for welded W shapes compares the mean of 24 column curves derived by Bjorhovde (1972, Fig. 17) for welded W shapes with flanges flame-cut to the column curve given in CSA S16.1-1974. These values are given in Table 8 as a function of λ together with the coefficient of variation V_{P_2}. Although the values of ρ_{P_2} are somewhat below the corresponding values for rolled W shapes, the values of V_{P_2} for the welded W shapes are considerably less and therefore compensate.

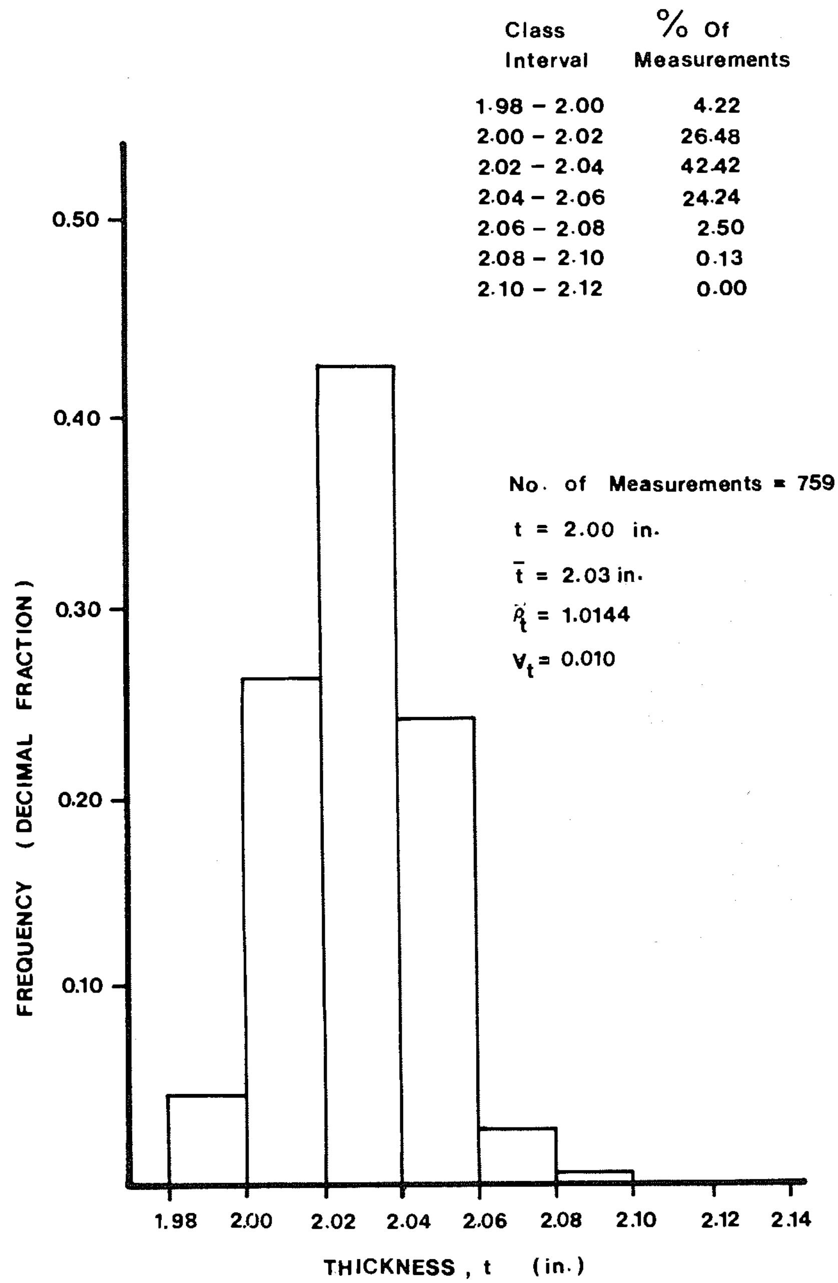

FIG. 15. Variation of plate thickness.

7. Performance Factors for Welded W Shapes

7.1 General

The same procedures and equations used to develop performance factors for rolled W shapes are used.

7.2 Columns

Using the basic material parameters given in Table 1 for welded W shapes, the relevant geometric parameters from Table 7, the professional parameters from Table 8, and the participation factors derived from the unit compressive resistance curve for CSA standard S16.1-1974, performance factors as given in Table 9 are determined for welded W shape columns.

For welded W shape columns based on SSRC curve 2 as used in CSA S16.1-1974, Table 9 shows that the performance factor lies generally above 0.90.

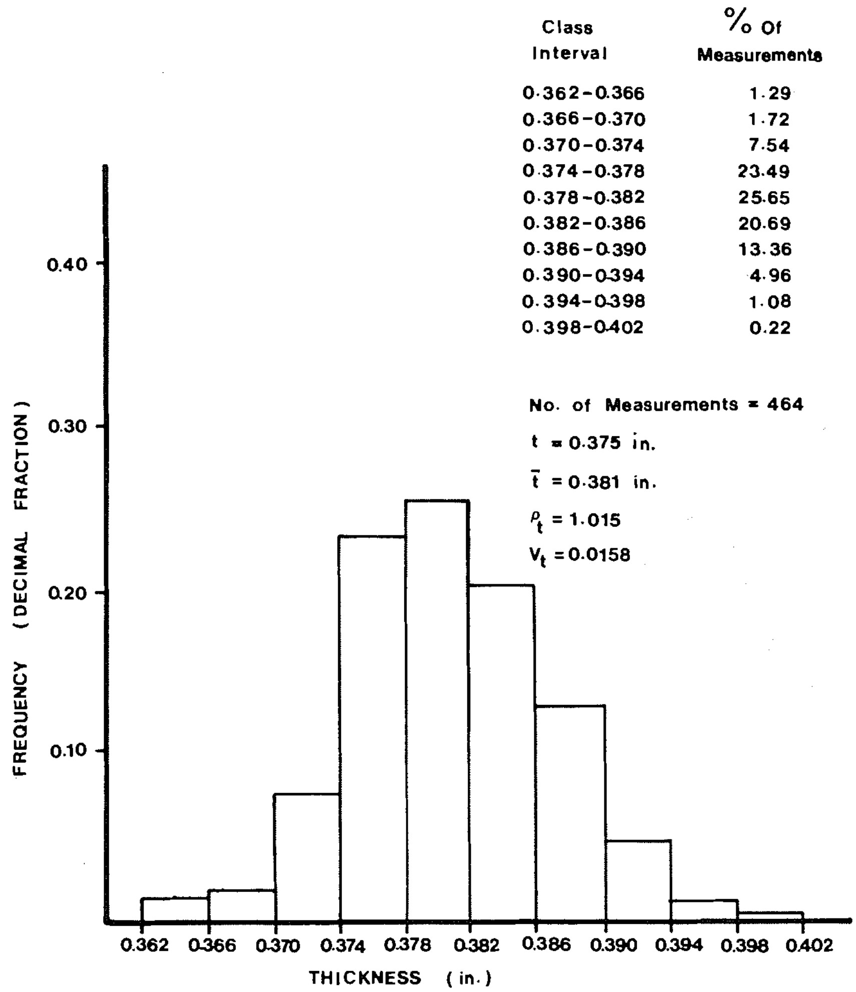

FIG. 16. Variation of plate thickness.

By using a mean value of 0.92 throughout for the discrete points for which performance factors were calculated a maximum error of 3% is introduced and a coefficient of variation of 0.003 is obtained.

7.3 Beams

7.3.1 Laterally Supported

For class 1 and 2 sections, using the appropriate statistical parameters for welded W shapes given in Tables 1, 4, and 7 for material, professional, and geometric properties, respectively, the coefficient of variation of resistance V_{M_p} was obtained as

[81] $$V_{M_p} = (0.015^2 + 0.11 + 0.11^2)^{1/2} = 0.156$$

and

[82] $$\phi = 1.10(1.02)(1.10) \times \exp[-0.55(3)(0.156)] = 0.957$$

For class 3 sections where $M_{max} = M_y$

[83] $$V_{M_y} = (0.018^2 + 0.11^2 + 0.06^2)^{1/2} = 0.126$$

and

[84] $$\phi = 1.07(1.025)(1.10) \times \exp[-0.55(3)(0.126)] = 0.979$$

For class 4 sections where $M_{max} = M_{cr} = S_x F_{cr}$ when $b/t_f \leq 198(k/F_y)^{1/2}$ for b/t_f ratios from 15 to 25, by following the same analytical procedures as for rolled W shapes, ϕ is determined to range from 1.02 to 1.04. When $b/t_f > 198(k/F_y)^{1/2}$ the procedures developed for rolled W shapes, by applying the appropriate statistical parameters, give $\phi = 1.01$.

7.3.2 Laterally Unsupported, Elastic Range

The different statistical parameters for welded W sections, when compared with rolled W sections using the procedures developed previously, yield: $\bar{B}_1 = 1.039$; $\rho_{M_u} = 1.07$; $\hat{\Delta}f = 0.01$; $V_{B_2} = 0.024$;

TABLE 9. Performance factors for welded W shape columns

					Contribution factors			Coefficients of variation								
λ	ρ_F*	ρ_{P_1}	ρ_{P_2}	ρ_A†	T_1'	T_2'	T_3'	V_{F_y}	V_r	V_E	V_A	V_{P_1}	V_{P_2}	ρ_{C_r}	V_{C_r}	ϕ
0.00	1.110	1.030	1.000	1.020	1.000	0.000	0.000	0.110	0.010	0.020	0.015	0.05	0.000	1.166	0.122	0.953
0.20	1.107	1.030	0.977	1.020	0.968	0.063	0.032	0.110	0.010	0.020	0.015	0.05	0.011	1.136	0.119	0.933
0.40	1.100	1.030	0.975	1.020	0.909	0.179	0.090	0.110	0.010	0.020	0.015	0.05	0.022	1.127	0.115	0.932
0.60	1.091	1.030	0.942	1.020	0.817	0.371	0.186	0.110	0.010	0.020	0.015	0.05	0.025	1.080	0.107	0.905
0.80	1.075	1.030	0.925	1.020	0.658	0.684	0.342	0.110	0.010	0.020	0.015	0.05	0.034	1.045	0.096	0.892
1.00	1.049	1.030	0.941	1.020	0.392	1.220	0.608	0.110	0.010	0.020	0.015	0.05	0.049	1.037	0.086	0.899
1.20	1.037	1.030	0.984	1.020	0.315	1.370	0.685	0.110	0.010	0.020	0.015	0.05	0.059	1.072	0.090	0.924
1.40	1.035	1.030	0.995	1.020	0.291	1.421	0.709	0.110	0.010	0.020	0.015	0.05	0.053	1.082	0.083	0.942
1.60	1.032	1.030	0.997	1.020	0.262	1.477	0.738	0.110	0.010	0.020	0.015	0.05	0.048	1.081	0.081	0.946
1.80	1.029	1.030	0.955	1.020	0.228	1.540	0.768	0.110	0.010	0.020	0.015	0.05	0.046	1.032	0.078	0.910
2.00	1.025	1.030	0.960	1.020	0.191	1.621	0.810	0.110	0.010	0.020	0.015	0.05	0.042	1.033	0.075	0.914
2.20	1.010	1.030	0.980	1.020	0.052	1.896	0.948	0.110	0.010	0.020	0.015	0.05	0.040	1.040	0.072	0.923
2.40	1.011	1.030	0.980	1.020	0.061	1.877	0.939	0.110	0.010	0.020	0.015	0.05	0.040	1.041	0.072	0.924
2.60	1.012	1.030	0.980	1.020	0.971	1.861	0.931	0.110	0.010	0.020	0.015	0.05	0.040	1.042	0.072	0.925
2.80	1.013	1.030	0.980	1.020	0.082	1.837	0.918	0.110	0.010	0.020	0.015	0.05	0.040	1.043	0.072	0.926
3.00	1.014	1.030	0.980	1.020	0.093	1.805	0.903	0.110	0.010	0.020	0.015	0.05	0.040	1.044	0.072	0.927
3.20	1.015	1.030	0.980	1.020	0.105	1.792	0.896	0.110	0.010	0.020	0.015	0.05	0.040	1.045	0.072	0.928
3.40	1.016	1.030	0.980	1.020	0.116	1.768	0.884	0.110	0.010	0.020	0.015	0.05	0.040	1.046	0.072	0.929
3.60	1.017	1.030	0.980	1.020	0.125	1.750	0.875	0.110	0.010	0.020	0.015	0.05	0.040	1.047	0.072	0.930
>3.60		1.030	0.980	1.020				0.110	0.010	0.020	0.015	0.05	0.040	1.040	0.070	0.927

*Includes $\rho F_y = 1.10$ and $\rho\lambda = 1.043$.
†For $\lambda \geq 3.6$, ρ_A is replaced by ρ_I and V_A by V_I.

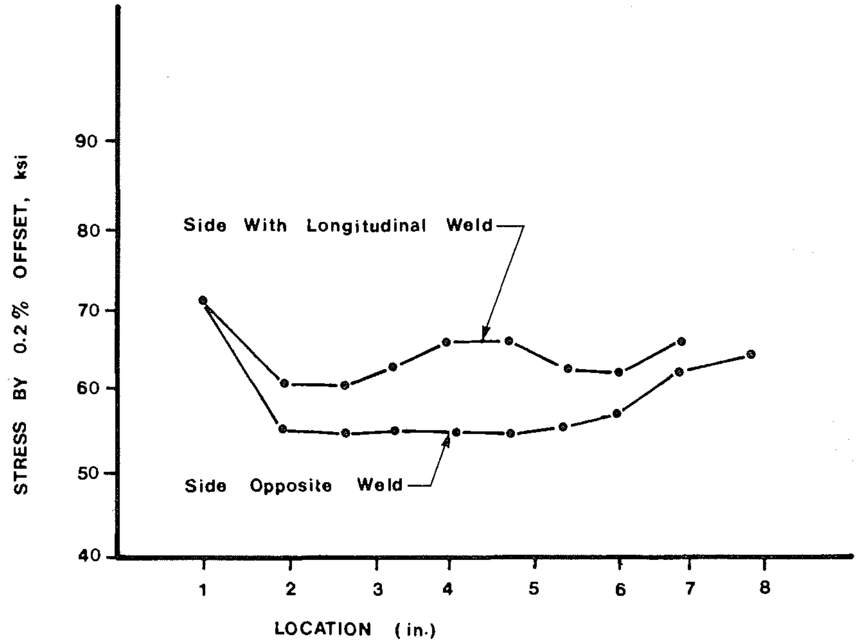

FIG. 17. Variation of yield strength around perimeter of 8 in. × 8 in. × 0.375 in. (203 mm × 203 mm × 9.525 mm) HSS.

TABLE 10. Performance factors for welded W shape beams

	Performance factors		
Classification	Maximum strength	Inelastic	Elastic
Classes 1 and 2	0.96	0.96	0.91
Class 3	0.98	0.96	0.91
Class 4	1.02–1.04	0.96	0.91

and $V_{M_u} = 0.098$, from which $\phi = \rho_{M_u} \exp(-V_{M_u}) = 1.07 \exp[-0.55(3)(0.098)] = 0.907$.

7.3.3 Laterally Unsupported, Inelastic Range

For class 1 and 2 welded W sections the following are established: $\overline{M}_p/M_p = 1.125$; $V_{M_p} = (0.110^2 + 0.015^2)^{1/2} = 0.111$; $\overline{M}_u/M_u = 1.035$; $V_{M_u} = (V_E^2 + V_B^2) = 0.039$; and $a_1 = 1.125/1.035 = 1.087$, to give $\phi = 0.96 + 0.5\% - 0\%$ over the range $1.00 < M_p/M_u < 1.50$.

For class 3 and 4 welded W sections the data are almost identical to those for class 1 and 2 sections and therefore a value of $\phi = 0.96$ is appropriate.

Performance factors for welded W sections used as beams are summarized in Table 10. From this table, conservative performance factors of 0.96 for maximum strength and inelastic buckling and 0.91 for elastic buckling would be appropriate.

8. Statistical Parameters for Hollow Structural Sections (HSS)

The data presented here are for hollow structural sections (HSS) manufactured in Canada in accordance with CSA standard G40.20-1976 class "H", which requires the sections after a seamless or continuous welding process to be either hot formed or cold formed and subsequently stress-relieved by heating to a temperature of 850°F (454°C) or higher and then cooled in the air.

8.1 Material Parameters, ρ_M, V_M

From Fig. 17 (Birkemoe 1977), it may be noted that there exists a significant variation in the yield strength around the perimeter of an HSS, with higher values in the corners due to cold working and at the weld due to metallurgical changes. The standard requires that mill tension tests be taken from the mid-width of the side 90° to the weld, and thus the average yield strength will exceed the mill test value—a situation opposite to that for W shapes where the mill test coupon is taken from the stronger web. The two other sides of the HSS could, of course, be expected to have yield strengths similar to that for the side opposite the weld.

By carrying out stub column tests the average yield strength of the cross section is determined experimentally. Birkemoe (1977) confirmed that the static yield strength as determined by stub column tests is the weighted average of the yield strength around the perimeter. From 1189 measurements at the reference location (as represented by Fig. 18 for CSA standard G40.21 grade 50W steel of three thicknesses, 0.250, 0.375, and 0.450 in. (6.3, 9.5, 11.4 mm), with a nominal yield point of 50 ksi

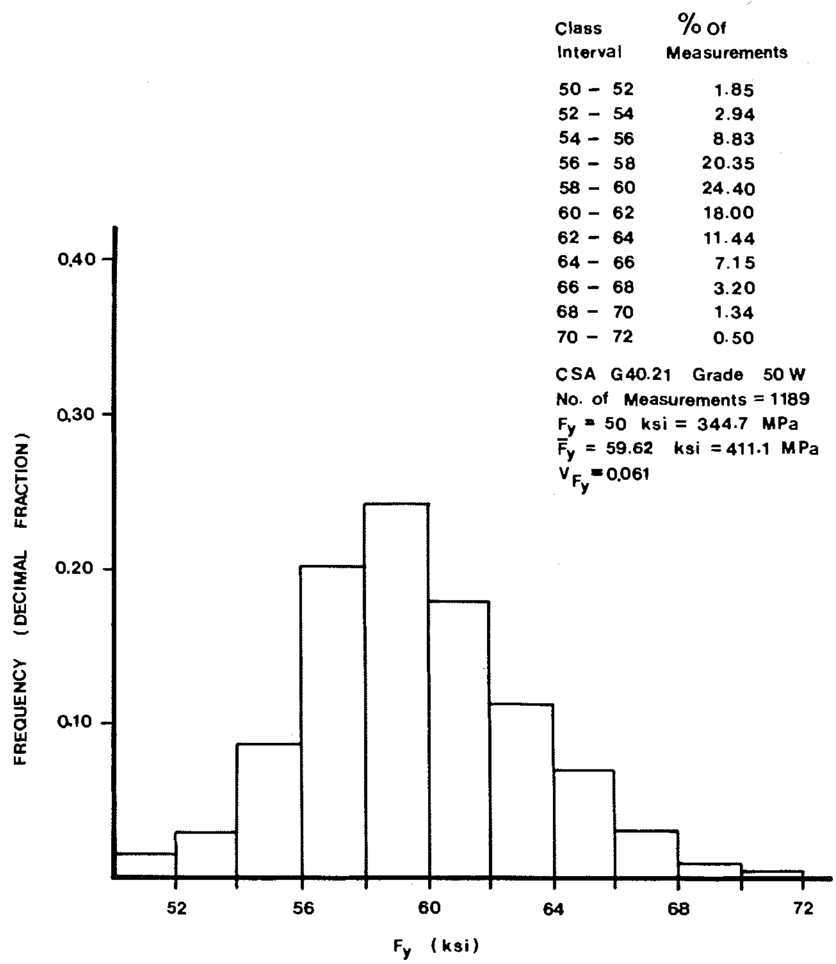

FIG. 18. Variation of yield strength, HSS, all wall thicknesses.

(344.7 MPa)) the mean mill yield strength, $\bar{F}_y$, was determined to be 59.62 ksi (411.1 MPa), with a coefficient of variation V_{F_1} of 0.061.

Table 11, based on Birkemoe (1976), gives a comparison between stub column strengths and mill yield strengths from which the ratio of the stub column strength, which represents the static yield strength, to the mill yield strength is determined to be 1.038. The coefficients of variation of the two independent series of measurements are 0.056 and 0.041. Therefore for all mill tests $\bar{F}_{ys} = 59.62(62.25/59.98) = 61.88$ ksi (426.6 MPa); $\rho_M = F_{ys}/F_y = 61.88/50 = 1.24$; and the coefficient of variation $V_M = (0.061^2 + 0.041^2 + 0.056^2)^{1/2} = 0.092$.

8.2 *Geometric Parameters*, ρ_G, V_G

Figures 19–21 give the statistical variations in cross-sectional depth and width, wall thickness, and area for HSS. Based on the actual measurements of the width and depth—each at two locations—diagonal dimensions and corner radii on 200 cross sections, including square cross sections of dimensions from 4 in. × 4 in. × 0.375 in. (101.6 mm × 101.6 mm × 9.5 mm) to 12 in. × 12 in. × 0.500 in. (304.8 mm × 304.8 mm × 12.7 mm) and rectangular cross sections of dimensions 6 in. × 4 in. × 0.188 in. (152.4 mm × 101.6 mm × 4.8 mm) to 12 in. × 8 in. × 0.450 in. (304.8 mm × 203.2 mm × 11.4 mm), histograms giving the variation in cross-sectional properties were developed from which the ratio of mean to nominal values and the coefficients of variation were determined.

Figure 20 shows that the ratio of mean to nominal wall thickness is 0.975 with $V_t = 0.025$. From Fig. 21 the ratio of mean/nominal area is 0.985 with $V_A = 0.034$.

The statistical parameters for various cross-sectional properties are summarized in Table 12. There are not significant differences in these parameters between rectangular sections, about the major and minor axes, and square sections.

Figure 22 gives a ratio of mean/nominal weight

TABLE 11. Comparison of stub column yield stresses and mill yield stresses

	Mill tests			Stub column tests			
HSS size (in. × in. × in.)	No. of tests	Mean yield stress $\bar{F}_{ym}$ (ksi)	$V_{F_{ym}}$	No. of tests	Mean yield stress $\bar{F}_{ys}$ (ksi)	$\bar{F}_{ys}/\bar{F}_{ym}$	$V_{F_{ys}}$
6 × 4 × 0.250	11	58.04	0.056	1	59.8	1.030	—
6 × 4 × 0.375	31	63.55	0.034	1	68.7	1.081	—
8 × 8 × 0.375	49	59.12	0.042	3	60.3	1.020	—
8 × 8 × 0.450	24	59.15	0.063	3	62.5	1.057	—
12 × 12 × 0.450	25	58.83	0.044	3	63.0	1.071	—
All sizes	140	59.98	0.056	11	62.25	1.038	0.041

NOTES: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa.

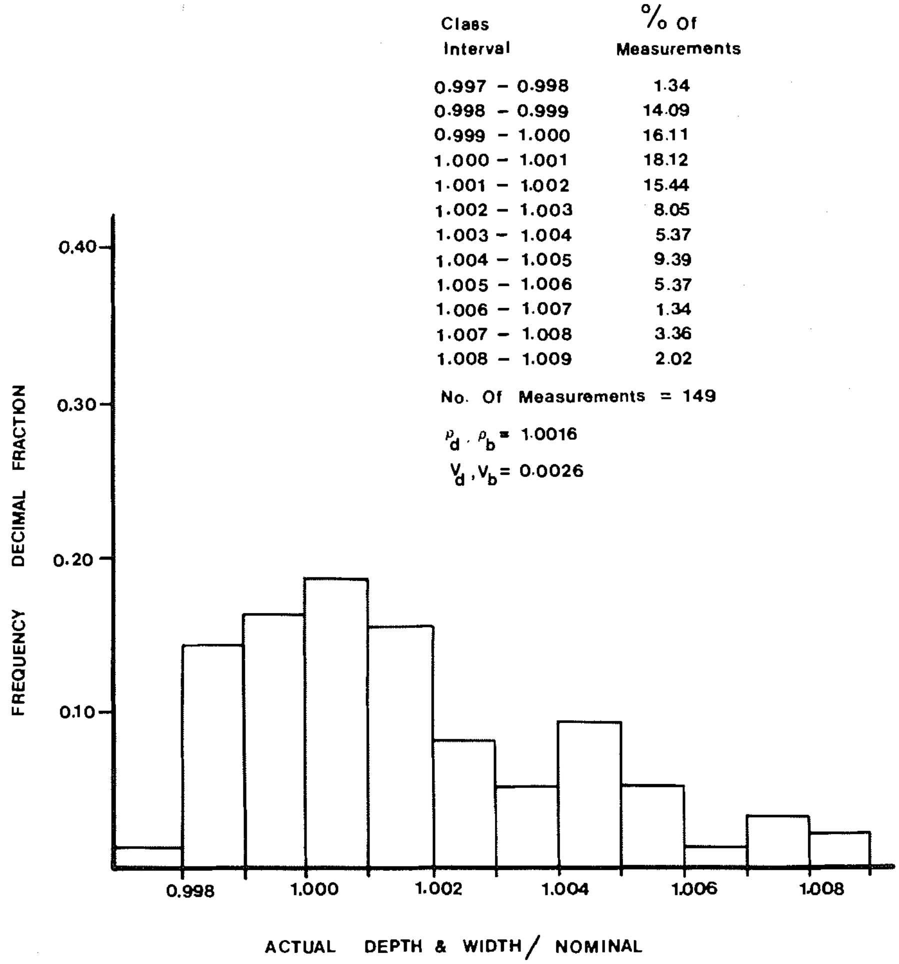

FIG. 19. Cross-sectional depth and width variation for HSS.

per foot of 0.985 for the variation in member weight per foot—the same as that for the cross-sectional area given in Fig. 21.

Out-of-straightness measurements were made of 172 HSS ranging in length from 20 to 60 ft (6.10–18.29 m). For sections whose out-of-straightness approximates a circular arc, the deviation is proportional to the length. By reducing all measurements to equivalent 20 ft (6.10 m) lengths the histogram of Fig. 23 was obtained from which the mean value

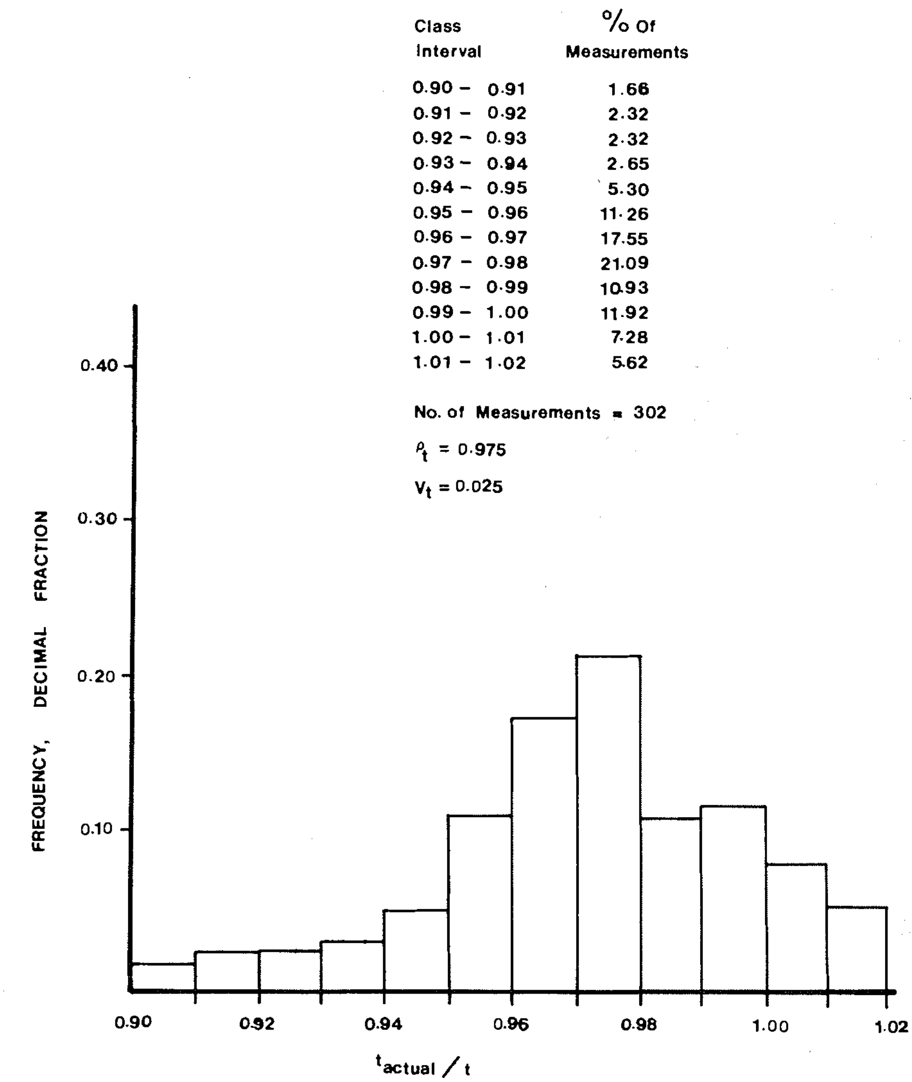

FIG. 20. Variation in wall thickness for HSS.

of the out-of-straightness is 0.00033, about 1/3000 of the length, and the coefficient of variation is 0.48.

8.3 Professional Parameters, ρ_P, V_P

8.3.1 General

The professional parameter ρ_P (the ratio of the tested capacity to the predicted capacity) and its coefficient of variation V_P depend on the equation used to describe the capacity of the member. The equation selected should be developed rationally and be the one that best fits the experimental data.

8.3.2 Columns

An examination of the available test data and predictor curves indicated that SSRC curve 1 (Bjorhovde 1972) was appropriate and it has therefore been used rather than the lower SSRC curve 2.

Table 13, based on Birkemoe (1976), for slenderness parameters λ of 0.64–0.98 gives $\rho_P = 0.965$ and $V_P = 0.040$.

The results of 158 tests on hot-formed HSS, as reported by Sherman (1974), were also analyzed to determine professional factors. The tests were performed in Europe under CIDECT (Comité international pour le développement et l'étude de la construction tubulaire) program 2A. As reported, the tests are based on measured cross-sectional area, measured modulus of elasticity, and nominal radius of gyration. The yield stress had been determined by stub column test on one section of each size only and statistical data were not available. Therefore, in determining the predicted capacity to find ρ_P, the statistical variation of the area and modulus of

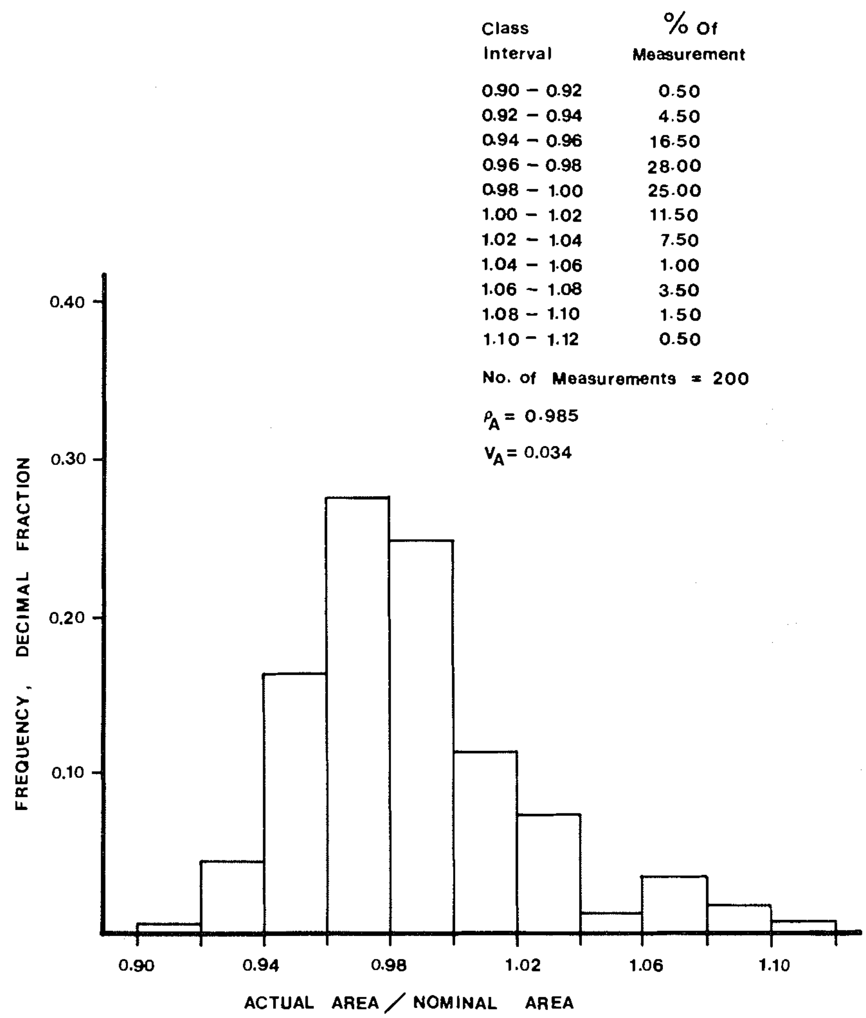

FIG. 21. Variation of cross-section area for HSS.

elasticity were taken into account but, because statistical data were lacking on the other quantities, the predicted capacity was based on the nominal values of F_y and r. The professional parameters ρ_P and V_P would therefore include the variation in the yield stress and the radius of gyration.

In Table 14, the results of this study give a tested to predicted ratio of about 1.08 and a coefficient of variation of 0.033–0.088 for different slenderness ratios. Because of the different factors included in the professional factor, performance factors were calculated separately for these data, as discussed later. They are listed in Table 14.

8.3.3 Beams

Lacking other data, the professional factors ρ_P V_P for HSS beams will be taken the same as for class 1 and 2 W sections, that is: for maximum strength, $\rho_P = 1.10$ and $V_P = 0.11$; for the inelastic range, $\rho_P = 1.048$ and $V_P = 0.078$; and for the elastic range, $\rho_P = 1.030$ and $V_P = 0.093$.

9. Performance Factors for Class H Hollow Structural Sections

9.1 General

Following the same procedures as used for rolled and welded W shapes and using appropriate design equations, generally from CSA standard S16.1-1974, performance factors are developed for class H hollow structural sections again using a coefficient of separation, α, equal to 0.55 and a safety index, β, equal to 3.0.

9.2 Columns

Because of the relatively high value of the material ratio, ρ_M, of 1.24 it would be expected that the

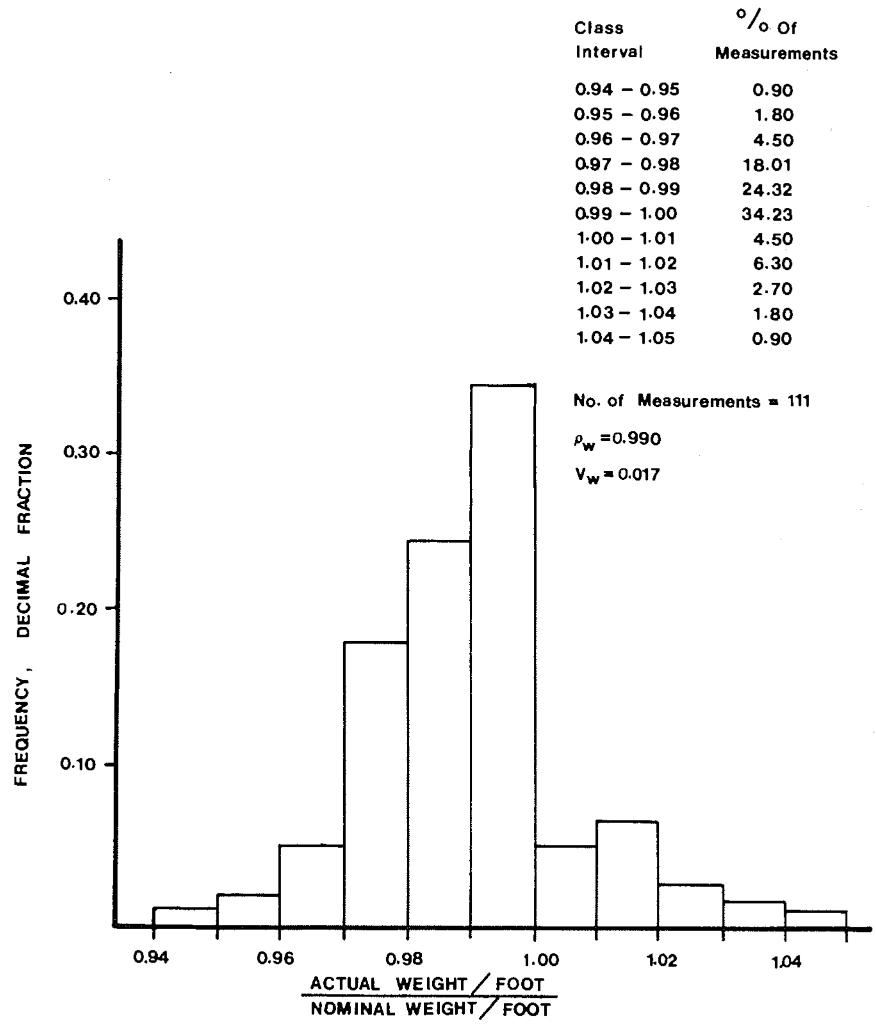

FIG. 22. Variation of member weight for HSS.

performance factor for class H hollow structural section columns would be high for those slenderness ratios where the strength is influenced by the yield strength.

The curve used to predict the strength of columns is Bjorhovde's (1972) curve 1, which is almost identical to SSRC curve 1, and is given by

For $0.15 \le \lambda \le 1.2$

$$C_r/C_y = (0.990 + 0.122\lambda - 0.38\lambda^2)$$

For $1.2 < \lambda \le 1.8$

[85] $$C_r/C_y = (0.051 + 0.778\lambda^{-2})$$

For $1.8 < \lambda \le 2.8$

$$C_r/C_y = (0.013 + 0.895\lambda^{-2})$$

For $2.8 < \lambda$ $\quad C_r/C_y = \lambda^{-2}$ (= Euler curve)

By algebraic manipulation of the above equation for each range of the slenderness parameter, λ, expressions can be derived for the performance factor, as for example for $0.15 \le \lambda \le 1.20$ given below. The compressive unit resistance F is equal to

[86] $$F = \phi F_y(0.99 + 0.122\lambda - 0.38\lambda^2)$$

in which

[87] $$\phi = \rho_P\rho_A\rho_{F_y}\frac{(0.99 + 0.122\lambda\rho_\lambda - 0.38\lambda^2\rho_\lambda^2)}{(0.99 + 0.122\lambda - 0.38\lambda^2)} \times \exp\left(-0.55(3)\left[\frac{H_{a1}^2V_{F_y}^2 + H_{a2}^2V_r^2 + H_{a3}^2V_E^2}{(0.99 + 0.122\lambda\rho_\lambda - 0.38\lambda^2\rho_\lambda^2)^2} + V_A^2 + V_P^2\right]^{1/2}\right)$$

TABLE 12. Statistical parameters of the variation of the cross-sectional dimensions and properties of area for HSS

Property	Statistical parameters ρ	Statistical parameters V
Wall thickness, t	0.975	0.025
Depth and width	1.001	0.002
A	0.985	0.034
Rectangular sections		
I_x	1.002	0.031
S_x	1.002	0.030
Z_x	0.980	0.032
I_y	1.000	0.032
S_y	1.001	0.031
Z_y	0.981	0.034
Square sections		
I_x	1.003	0.033
S_x	1.003	0.034
Z_x	0.989	0.036
Radius of gyration r, both sections	1.010	0.015

where

[88] $H_{a1} = (0.99 + 0.183\lambda\rho_\lambda - 0.760\lambda^2\rho_\lambda^2)$

[89] $H_{a2} = (0.122\lambda\rho_\lambda - 0.760\lambda^2\rho_\lambda^2)$

[90] $H_{a3} = (-0.061\lambda\rho_\lambda + 0.380\lambda^2\rho_\lambda^2)$

Performance factors for class H hollow structural sections as a function of λ, given in Table 15, are seen to range from 0.95 to 0.99 for values of λ less than 1.0, in which the effect of the yield strength is high. For λ greater than 1.0 the performance factor is in the range 0.88–0.89. A value of 0.89 would not be inappropriate.

An analysis of the CIDECT data in which the professional ratio ρ_P includes the variation in the yield stress and the radius of gyration was also made. The performance factor is

[91] $$\phi = \rho_P\rho_A\rho_F \exp(-\alpha\beta V_R)$$

where ρ_P includes the variation in F_y and r; ρ_A includes the variation in A only and was taken as 0.98; $\rho_F = 1.00$ because it is based on the nominal values of F_y and r and because $\rho_E = 1.00$; and

[92] $$V_R = (V_P^2 + V_A^2 + H_3^2V_E^2)^{1/2}$$

where $V_A = 0.034$; $V_E = 0.019$; and H_3 = contribution factor applied to V_E for the appropriate portion of curve 2.

Values calculated for the performance factor on the above basis are given in Table 14. A comparison of these with those given in Table 15 indicates that for λ less 1.0, where ϕ is high, those given in Table 14 are about 2% less and that for λ = 1.44 the Table 14 value of 0.98 is higher. Thus the values given in Table 15 are substantiated.

TABLE 13. Comparison of tested capacity and predicted capacity for HSS

Entry	Size (in. × in. × in.)	KL/r Nominal	KL/r Measured	Stub column* yield stress (ksi)	λ†	C_r Tested (kips)	C_r‡ Predicted (kips)	ρ_P
1	8 × 8 × 0.375	69.50	68.62	59.97	0.977	450	497	0.910
2	8 × 8 × 0.375	69.50	68.24	59.97	0.971	477	500	0.956
3	8 × 8 × 0.375	69.50	68.49	59.97	0.975	460	498	0.924
4	8 × 8 × 0.375	69.50	66.45	59.97	0.946	505	510	0.991
5	8 × 8 × 0.375	69.50	65.95	59.97	0.939	500	512	0.976
6	8 × 8 × 0.450	45.40	45.00	62.63	0.654	750	739	1.015
7	8 × 8 × 0.450	45.40	44.45	62.63	0.647	725	741	0.980
8	8 × 8 × 0.450	45.40	45.24	62.63	0.658	735	737	0.997
9	8 × 8 × 0.450	49.62	45.24	62.63	0.721	710	717	0.991
10	8 × 8 × 0.450	45.40	48.58	62.63	0.707	670	721	0.930
11	8 × 8 × 0.450	69.50	64.70	62.63	0.940	605	626	0.970
12	8 × 8 × 0.450	69.50	66.58	62.63	0.968	565	612	0.923
13	8 × 8 × 0.450	69.50	65.80	62.63	0.957	555	618	0.900
14	8 × 8 × 0.450	69.50	56.36	62.63	0.820	630	679	0.930
15	8 × 8 × 0.450	69.50	61.30	62.63	0.892	600	648	0.930
16	12 × 12 × 0.375	45.20	44.50	63.00	0.649	940	951	1.091
17	12 × 12 × 0.375	45.20	45.47	63.00	0.663	900	945	0.950
18	11 × 12 × 0.375	45.20	44.97	63.00	0.656	960	948	1.013
19	12 × 12 × 0.375	45.20	46.16	63.00	0.673	930	941	0.990
							$\rho_P = 0.965$ and $V_P = 0.04$	

NOTES: 1 in. = 25.4 mm; 1 ksi = 6.895 MPa; 1 kip = 4.448 kN.
*Based on three tests for 8 in. × 8 in. × 0.375 in., 8 in. × 8 in. × 0.450 in. and on two tests for 12 in. × 12 in. × 0.375 in.
†Calculated by using the measured values of KL/r the stub column yield stress, and assumed elastic modulus E = 30 000 ksi.
‡Calculated by using Bjorhovde's (1972) curve 1, where $C_r = AF_y(0.99 - 0.122\lambda - 0.38\lambda^2)$.

TABLE 14. Comparison of tested to predicted capacity ratios for hot-formed HSS

Slenderness parameter λ	No. of tests	Tested capacity / predicted capacity ρ_P	Coefficient of variation V_P	Performance factor ϕ
0.59–0.65	37	1.089	0.070	0.95
0.70–0.83	40	1.083	0.077	0.93
0.95–1.15	69	1.091	0.088	0.92
. 1.44	12	1.078	0.033	0.98

TABLE 15. Performance factors for HSS class H columns

				Contribution factors			Coefficients of variation							
λ	ρ_F*	ρ_P	ρ_A†	H_1	H_2	H_3	V_{F_y}	V_r	V_E	V_A	V_P	ρ_{C_r}	V_{C_r}	ϕ
0.00	1.240	0.965	0.985	1.000	0.000	0.000	0.092	0.015	0.019	0.034	0.040	1.179	0.106	0.990
0.20	1.239	0.965	0.985	0.993	0.010	0.005	0.092	0.015	0.019	0.034	0.040	1.178	0.105	0.991
0.40	1.229	0.965	0.985	0.950	0.099	0.048	0.092	0.015	0.019	0.034	0.040	1.168	0.102	0.987
0.60	1.209	0.965	0.985	0.858	0.283	0.128	0.092	0.015	0.019	0.034	0.040	1.149	0.095	0.982
0.80	1.174	0.965	0.985	0.692	0.615	0.306	0.092	0.015	0.019	0.034	0.040	1.116	0.083	0.973
1.00	1.113	0.965	0.985	0.391	1.217	0.609	0.092	0.015	0.019	0.034	0.040	1.058	0.067	0.947
1.20	1.025	0.965	0.985	0.050	1.790	0.898	0.092	0.015	0.019	0.034	0.040	0.974	0.062	0.880
1.40	1.032	0.965	0.985	0.050	1.732	0.866	0.092	0.015	0.019	0.034	0.040	0.981	0.061	0.887
1.60	1.040	0.965	0.985	0.050	1.661	0.830	0.092	0.015	0.019	0.034	0.040	0.986	0.060	0.892
1.80	1.048	0.965	0.985	0.05	1.591	0.795	0.092	0.015	0.019	0.034	0.040	0.996	0.060	0.902
2.00	1.021	0.965	0.985	0.013	1.863	0.931	0.092	0.015	0.019	0.034	0.040	0.970	0.062	0.875
2.20	1.021	0.965	0.985	0.013	1.840	0.921	0.092	0.015	0.019	0.034	0.040	0.970	0.062	0.876
2.40	1.025	0.965	0.985	0.013	1.815	0.907	0.092	0.015	0.019	0.034	0.040	0.974	0.062	0.880
2.60	1.027	0.965	0.985	0.013	1.791	0.895	0.092	0.015	0.019	0.034	0.040	0.976	0.061	0.882
2.80	1.035	0.965	0.985	0.013	1.750	0.874	0.092	0.015	0.019	0.034	0.040	0.984	0.061	0.890
>2.80	1.000	0.965	1.000	0.000	0.000	1.000	0.092	0.015	0.019	0.034	0.040	0.965	0.056	0.880

*Includes $\rho_{F_y} = 1.24$ and $\rho_\lambda = 1.11$.
†For $\lambda \geq 2.80$, ρ_A is replaced by ρ_I and V_A by V_I.

9.3 Beams

9.3.1 Laterally Supported

Using the professional, geometric, and material parameters given previously, and based on the design equation $M_r = \phi Z F_y$ (because all sections manufactured to CSA standard G40.21 in grade 50W steel are class 1 or class 2), ϕ is determined to be 1.05.

9.3.2 Laterally Unsupported, Elastic Range

In the elastic range the ultimate moment is given by

$$[93] \quad M_u = (\pi/L)(EI_y GJ)^{1/2}$$

Now,

$$[94] \quad I_y = \frac{2b^3t}{12} + 2\left(\frac{dt^3}{12} + \frac{dtb^2}{4}\right) \simeq \frac{b^3t}{6} + \frac{b^2dt}{2}$$

$$= \frac{b^2t}{6}(b + 3d)$$

TABLE 16. Performance factors for HSS beams

Classes 1 and 2	Maximum strength	Inelastic	Elastic
HSS	1.05	0.92–0.98	0.86

Statistically the parameters for b and d will be similar and therefore an equivalent statistical expression, I_{yes}, will be used.

$$[95] \quad I_{yes} = (b^2t/6)(4b) = 2b^3t/3$$

The St. Venant torsional constant is

$$[96] \quad J = 2b^2d^2t/bd$$

and an equivalent statistical expression is

TABLE 17. Suggested performance factors for columns

Type	Predictor equation	Performance factor, ϕ
Rolled W section	CSA standard S16.1-1974 (SSRC curve 2)	0.90
Welded W section	CSA standard S16.1-1974 (SSRC curve 2)	0.92
HSS class H	(SSRC curve 1)	0.98–0.075λ* 0.89†

*If $0 \leq \lambda \leq 1.2$.
†If $1.2 < \lambda$.

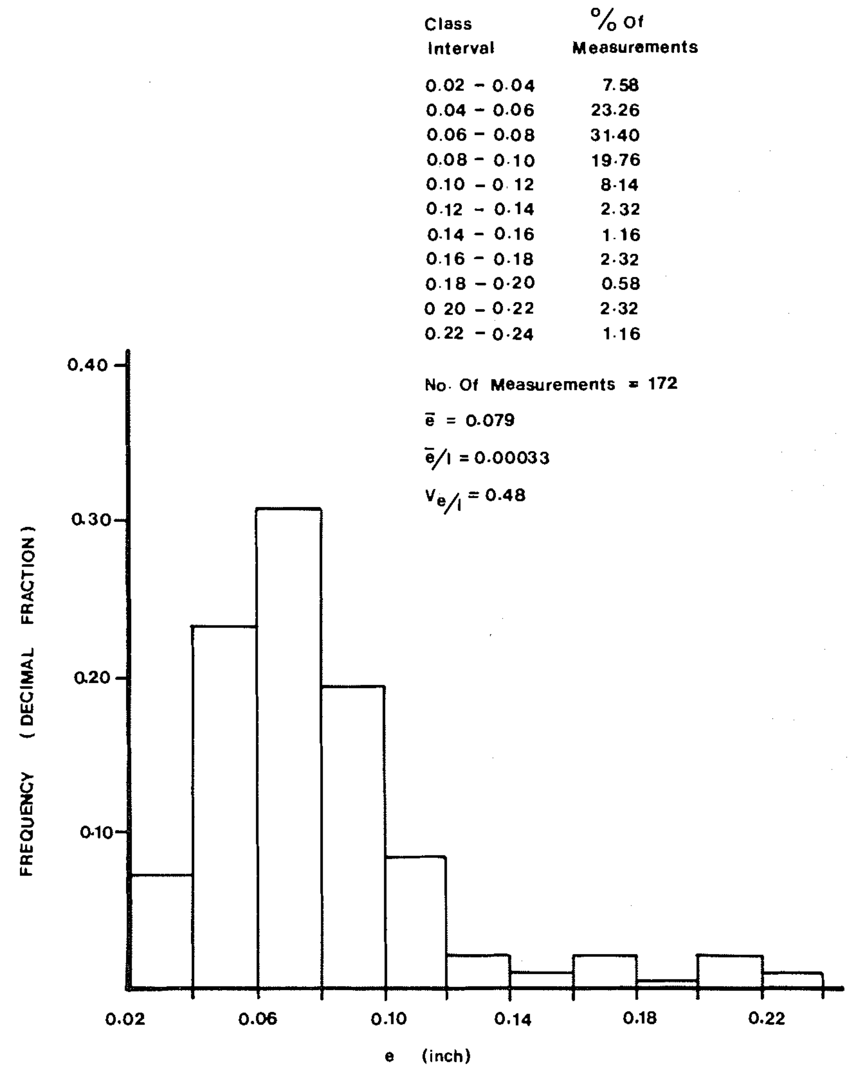

FIG. 23. Variation of initial out-of-straightness for 20 ft (6.09 m) of column length for HSS.

[97] $J_{es} = b^3 t$

to give an equivalent statistical expression for M_u of

[98] $$M_{ues} = \pi/L[(E^2 G/E)(2b^6 t^2/3)]^{1/2} = (\pi/L)Eb^3 t(2G/3E)^{1/2}$$

$\rho_{(G/E)}$ is taken as 1.00 and $V_{(G/E)}$ as zero. Therefore,

[99] $$\rho_{M_u} = \rho_P \rho_E \rho_b{}^3 \rho_t$$

and

[100] $$V_{M_u} = (V_E{}^2 + 9V_b{}^2 + V_t{}^2 + V_P{}^2)^{1/2}$$

to give, using the statistical data developed, $\phi =$ 0.856.

9.3.3 Laterally Unsupported, Inelastic Range

Following the analyses and procedures for class 1 and 2 W sections, but with the appropriate statistical data for HSS, performance factors for $1.0 \leq M_p/M_u \leq 1.5$ are obtained that vary from 0.92 to 0.98.

9.3.4 Summary

Performance factors for HSS used as beams are summarized in Table 16. Except where elastic buckling controls, and where the modulus of elasticity is the significant material property, the performance factor ranges from 0.92 to over 1.0.

10. Summary and Conclusions

Performance factors have been determined for steel building columns and beams made from rolled W, welded W, and class H hollow structural sections as produced in Canada, based on raw data relating

TABLE 18. Suggested performance factors for beams

Classifications	Laterally supported Maximum strength M_p', M_y', M_{cr}	Laterally unsupported Inelastic buckling	Elastic buckling
Rolled W shapes			
Classes 1, 2	0.92	0.90	0.84
Class 3	0.96	0.90	0.84
Class 4	0.92*	0.90	0.84
	0.84†		
Welded W shapes			
Classes 1, 2	0.96	0.96	0.91
Class 3	0.98	0.96	0.91
Class 4	1.02	0.96	0.91
HSS			
Classes 1, 2	1.05	0.93	0.86

NOTE: predicted strength based on CSA standard S16.1–1974.
*If $b/t_f \leq 198(k/F_y)^{1/2}$.
†If $b/t_f \geq 198(k/F_y)^{1/2}$.

to material and geometric properties obtained from Canadian mills, and on tested to predicted capacity ratios reported by others. The equations of CSA standard S16.1-1974 were used to predict strengths except for HSS class H columns where the higher SSRC curve 1 was considered more appropriate.

The exponential expression for the performance factor $\phi = \rho_R \exp(-\alpha\beta V_R)$, with $\alpha = 9.55$ and $\beta = 3.0$, was used. Suggested performance factors for those sections used as columns are given in Table 17 and for those used as beams are given in Table 18.

From these tables it is seen that the general performance factor of 0.90 used in CSA standard S16.1-1974 and in its companion standard CAN3 S16.1-78M tends to be conservative except for the cases of elastic buckling of rolled W and HSS beams where performance factors of 0.84 and 0.86—7 and 5% less than that currently used—are appropriate.

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Nomenclature

NOTE: a bar over a symbol denotes the mean value of that quantity. Thus, $\bar{A}$ denotes the mean area.

A	= cross-sectional area
B_1, B_2	= geometric factors
b	= flange width
C_r	= maximum compressive force of a member
C_w	= warping torsional constant
C_y	= axial compressive load at yield stress
d	= depth of rolled or welded W sections: distance between center lines of two flanges
E	= elastic modulus of steel (29 000 ksi assumed)
e	= initial out-of-straightness
F	= unit compressive resistance or nondimensional unit resistance
F_{cr}	= critical plate buckling stress
F_y	= specified yield stress of steel
F_{ym}	= mill yield stress
F_{ys}	= stub column yield stress or static yield stress
F_{ysf}	= flange static yield stress
$\hat{\Delta}f$	= coefficient of variation to calculate error in any assumption
$H_{1,2,3}$ or $H_{a1,b1,c1}$	= contribution factors for HSS
h	= clear depth of web between flanges
J	= St. Venant torsion constant
K	= effective length factor
k	= place buckling coefficient
L	= unbraced length of beams
l_i	= length of any element of the section
M_p	= plastic moment = ZF_y
M_r	= maximum moment resistance of a member
M_u	= moment resistance of a member subject to lateral buckling
M_y	= yield moment = SF_y
Q	= wind or earthquake loads
R	= member resistance, contribution factor for class 4 section
r	= radius of gyration of the cross section
S	= load effect
S	= elastic section modulus of a steel section
T	= temperature load
$T_{1,2}$	= contribution factors for compression members
t	= thickness of any element in cross section
t_f	= flange thickness
V_A	= coefficient of variation of the random variable denoted by the subscript
w	= web thickness
x	= subscript relating to strong axis of a member
y	= subscript relating to weak axis of a member
Z	= plastic section modulus of a steel section
α	= separation factor = 0.55
$\alpha_k = \alpha_L, \alpha_Q, \alpha_T$	= load factors of L, Q, T
β	= safety index = 3.0
γ	= importance factor
λ	= nondimensional slenderness ratio in column formula
ϕ	= performance factor
ψ	= load combination factor
ω	= constant in equation for M_u depending on loading conditions, position of loads, and shape of moment diagram
ρ_A	= the ratio of the mean to nominal value of the subscripted quantity