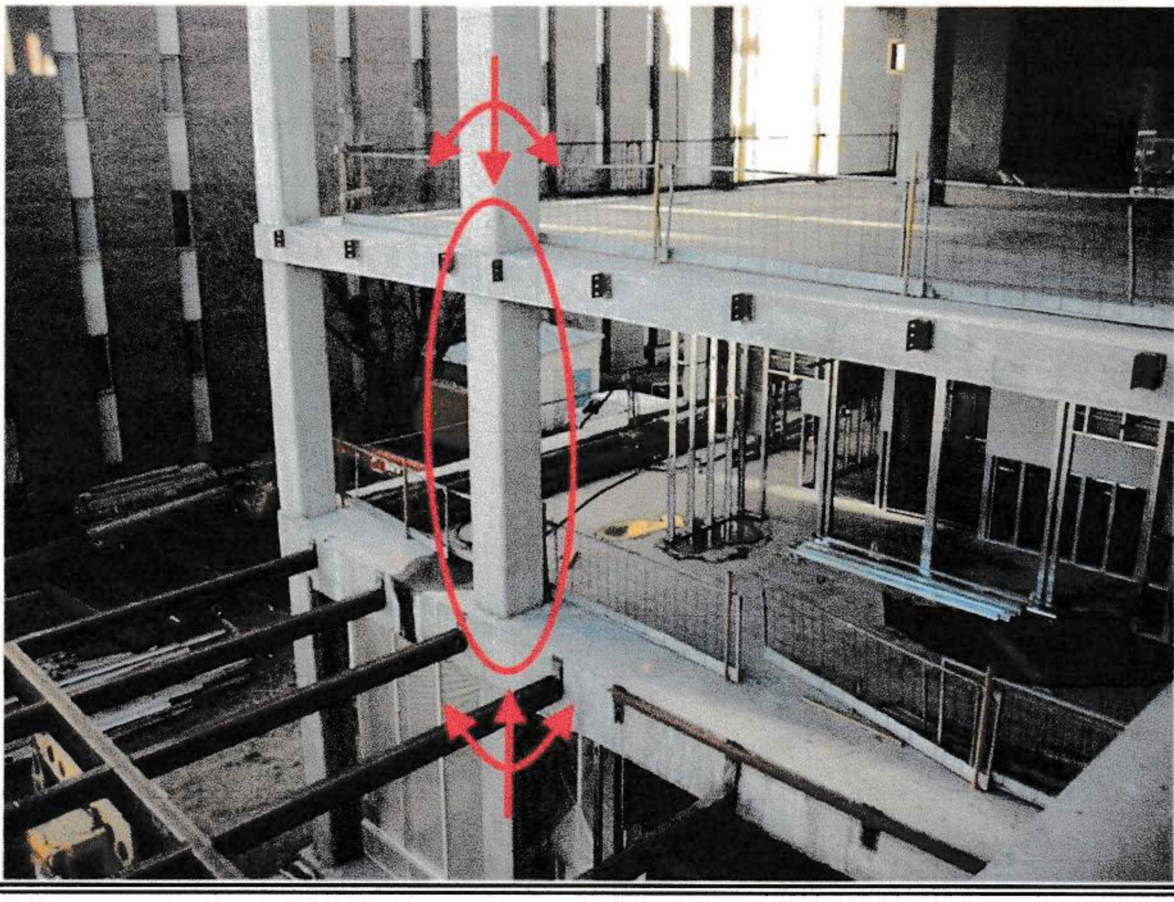
CivE 414
Structural Concrete Design

Topic 6
COLUMNS

Axial Compression and Bending



CONCRETE COLUMNS

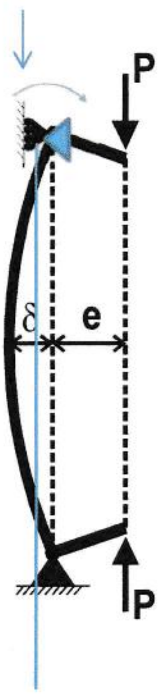
- Primarily compression members
- In general, must design for combined axial load and bending
- Usually vertical, but may be inclined or horizontal in trusses and frames

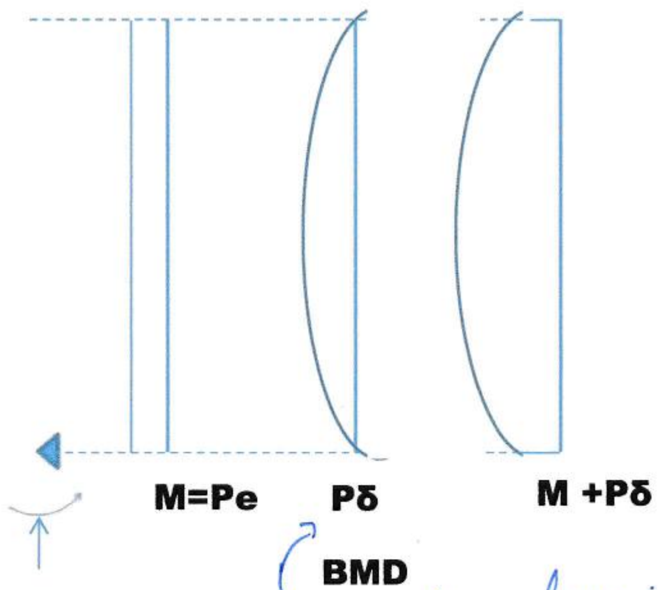
Because columns are subjected primarily to compression loading, stability effects must be considered. This is done as follows:

- If these effects have little or no impact on the column capacity, we have what we call a "short" column. In this case, stability effects can be safely ignored.
- If stability effects significantly reduce the column capacity, we have a "slender" column. In this case, stability effects must be considered explicitly in the design.

- **Short columns:** no second order effects or buckling
- **Slender columns**: influenced by second order effects (buckling may occur)

$$M_{max} = P(e + \delta)$$



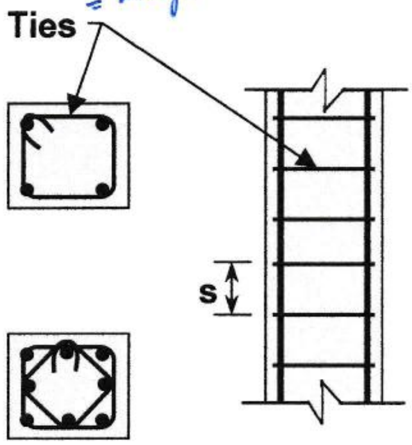


Question: how large is δ

TYPES OF COLUMNS

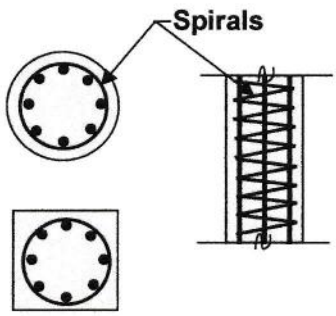
1. Tied Columns

= always a closed loop, unlike U-bars



- Ties are used mainly to prevent **buckling** of the longitudinal bars and consequently, to prevent the concrete cover from spalling off.
- For a large number of bars, other tie arrangements may be required.

2. Spiral Columns

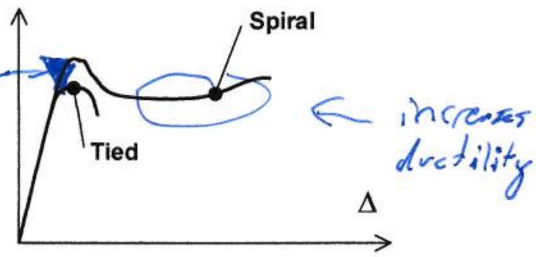


- Spirals are helical ties (continuous) which contain the concrete and prevent local buckling.
- Spirals may be used in square columns.
- Ductility and ultimate strength are increased with the use of spirals.

! good for seismic

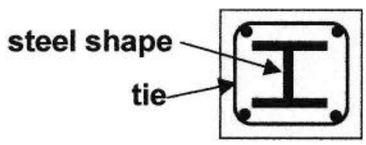
confinement increases compressive strength

Load vs Deflection for Tied & Spiral Columns



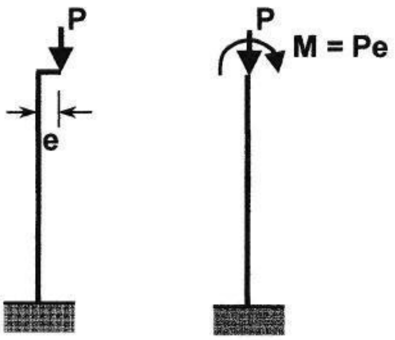
3. Composite Columns

Combination of structural steel shape and reinforced concrete



COLUMNS IN BENDING

- Very rare for a column to be subjected to pure axial load
- Both vertical loads and lateral loads produce moments in frame columns

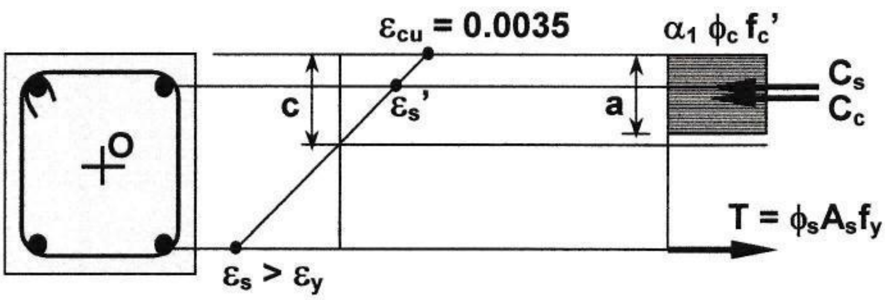


For $e = 0$ → pure axial load ($M = 0$)
$e = \infty$ → pure bending ($P = 0$)

PURE BENDING

$M > 0$, $P = 0$ → $e = \infty$

Short column to sketch



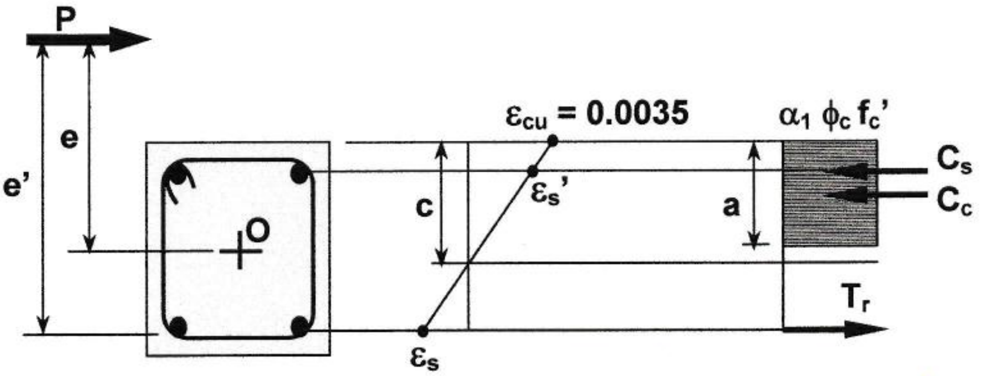
Summation of forces: $$C_c + C_s - T = 0$$

Summation of moments: $$M_r = C_c\left(d - a/2\right) + C_s\left(d - d'\right)$$

(no axial load, any point can be used for summation of moments)

Moment and Axial Load

$M > 0$, $P > 0$ → $0 < e < \infty$



1. Summation of forces: $\mathbf{P = C_c + C_s - T}$ Axial load

2 Summation of moments:

a) about O *(centroid)*: ← prefered as this corresponds to structural analysis results

M $$\mathbf{Pe = C_c(h/2 - a/2) + C_s(h/2 - d') + T_r(d - h/2)}$$

or

b) about T_r: location of tensile reinforcement $$\mathbf{Pe' = C_c(d - a/2) + C_s(d - d')}$$

where $\mathbf{e' = e + (d - h/2)}$

Two unknowns: **P and a** (for a given e)
e and a (for a given P)

(a is related to strain in tension reinforcement)

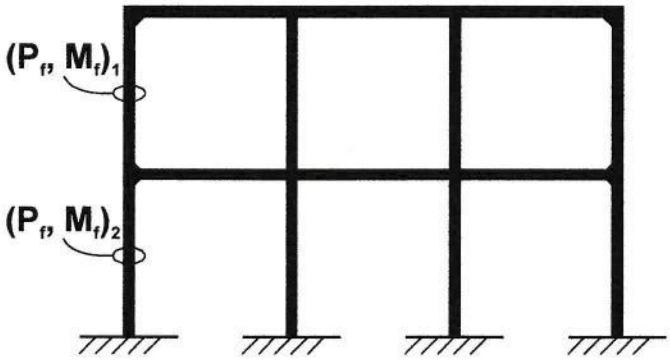
Two equations: $\mathbf{\Sigma F = 0}$

$\mathbf{\Sigma M = 0}$

- **of P_r & M_r**

ANALYSIS OF COLUMN CAPACITY

- Frame analysis results give P_f and M_f for each column



- Check adequacy of a given column section and reinforcement

APPROACH 1 – ANALYSIS FOR A GIVEN ECCENTRICITY

Given: $\mathbf{e = \dfrac{M_f}{P_f}}$ $\mathbf{e' = e + (d - h/2)}$

- Looking for $\mathbf{P_r}$ & $\mathbf{M_r}$

Equilibrium: $\mathbf{P = C_c + C_s - T}$

Moment about T: $\mathbf{Pe' = C_c(d - a/2) + C_s(d - d')}$

Unknowns: a, P

Solution: $P_r = P$ for given "e"

$M_r = P\,e$

Verify: $P_r \geq P_f$ for column in question
$M_r \geq M_f$

APPROACH 2 – ANALYSIS FOR A GIVEN AXIAL LOAD

Given: $P_r = P_f$ find M_r *(and e)*

Equations:

Equilibrium: $$\mathbf{P_f = C_c + C_s - T}$$

Moment about T: $$\mathbf{P_f\, e' = C_c\left(d - a/2\right) + C_s\left(d - d'\right)}$$

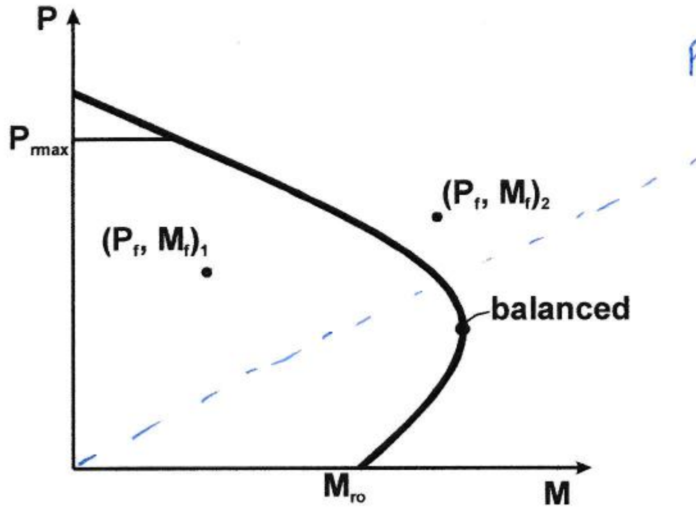
Unknowns: a, e'

Solution: $\mathbf{e = e' - \left(d - h/2\right)}$ for given "P_f"
$M_r = P_f\, e$

Verify: $M_r \geq M_f$ for column in question

APPROACH 3 – GENERATE P-M INTERACTION DIAGRAM

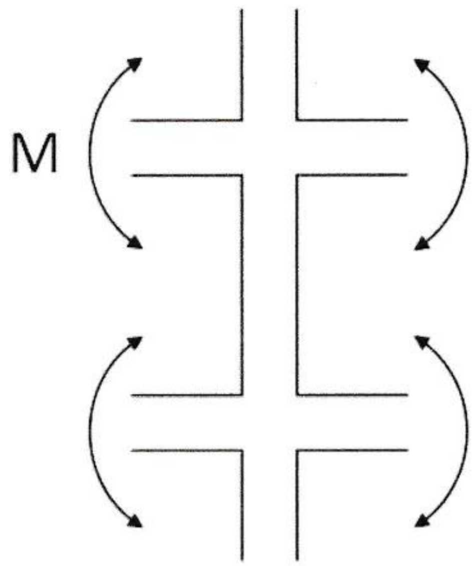
- Useful for several combinations of P_f and M_f

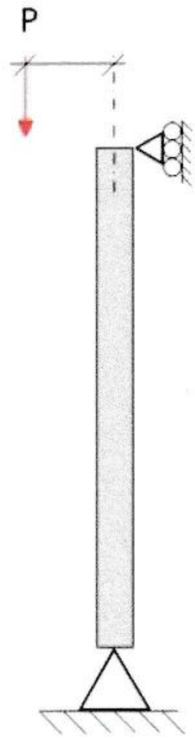


Pf, Mf must be within interaction diagram

Interaction Diagrams

- Columns are usually loaded by a compressive axial load and a moment. Combination
- Bending moments can be introduced at column ends via monolithically cast beams or slabs or due to eccentricity of the point of axial load application, with respect to the column centroid.





Under combined axial and bending loads, the strength of a column is generally lower than in the pure axial load case, because the axial and bending stresses add together, reaching the concrete compressive strength at a lower load level. This is accounted for in design through the use of "interaction diagrams".

For an ideal elastic, brittle material, the theoretical interaction diagram can be determined as follows:

- Failure occurs when the strength, $f_{cu} = f_{tu} = f$, is reached.
- Failure of the x-section is thus defined by the locus of points:

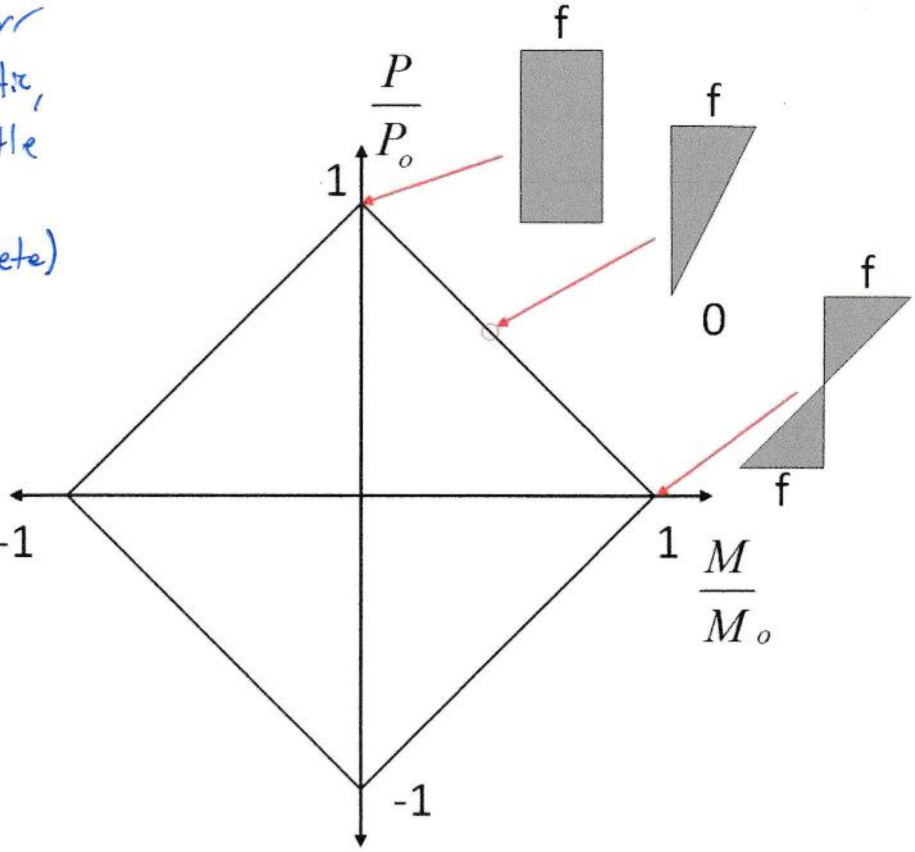
$$f = \frac{P}{A} \pm \frac{M \cdot y}{I}$$ *(Stress is a combination of stress due to axial load and moment)*

- This expression can be rewritten (normalized) :

$$1=\frac{P}{A\cdot f}\pm\frac{M\cdot y}{I\cdot f}=\frac{P}{P_0}\pm\frac{M}{M_0}$$

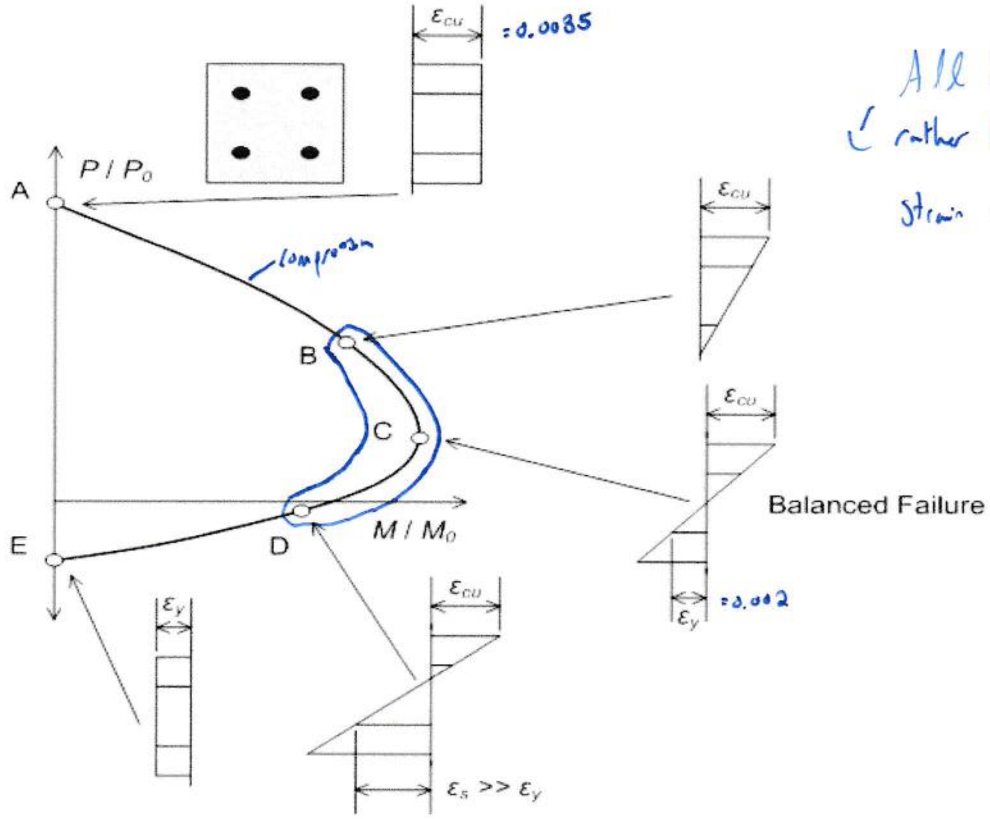
- The resulting “interaction diagram” takes the following form:

linear elastic, brittle (Not concrete)



Benjamin Klassen

For reinforced concrete columns, the interaction diagram can be determined by analyzing the column x-section under various strain distributions, corresponding with different P-M combinations:



All based on strain distribution, & rather than stress, as strain easier to predict.

Each point on the interaction diagram represents the maximum axial load that can be applied at a given eccentricity.

Three Types of Failure:

1. Tension failure $\varepsilon_s > \varepsilon_y$
   $e > e_b$
2. Balanced failure $\varepsilon_s = \varepsilon_y$
   $e = e_b$
3. Compression failure $\varepsilon_s < \varepsilon_y$
   $e < e_b$

- To determine points A-E indicated in the preceding diagram (and others), the following assumptions are made:

– ε_{cu} = 0.0035

– $\varepsilon_y = f_y / E_s$ = 400 MPa / 200 000 MPa = 0.002

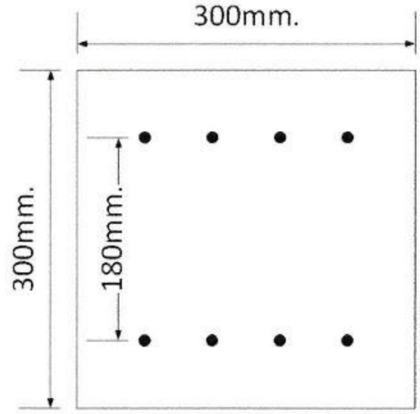
Example 1

- Calculate factored balanced failure point for the column in the Figure

$$f_c' = 40\ MPa$$

$$8 - 20M\ bars$$

$$f_y = 400\ MPa$$



Solution *(compression positive in this example)*

$$C = 240 \cdot \frac{0.0035}{0.002} = 152.7mm$$

$$\varepsilon_{s1} = -0.002 \quad \varepsilon_c = -0.0035 \quad \textit{(given)}$$

$$\varepsilon_{s2} = 0.0035\left(\frac{152.7-60}{152.7}\right) = 0.00213$$

$$f_{s1} = -400\ MPa\ (tension)$$ ε

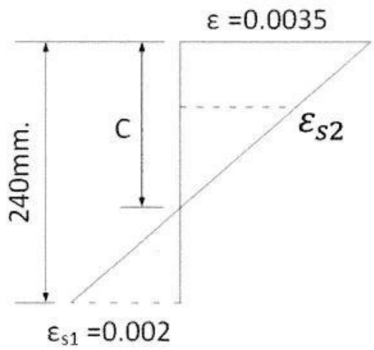
$$f_{s2} = MIN(0.00213 \cdot 200GPA, 400MPa)$$

$$f_{s2} = 400\ MPa$$

$$\beta_1 = 0.97 - 0.0025(40) = 0.87$$

$$a = \beta_1 \cdot c = 0.87 \cdot 152.7 = 132.9mm$$

$$\alpha_1 = 0.85 - 0.0015(40) = 0.79 > 0.67\ ok$$



Concrete

$$C_{rc} = \alpha_1 \cdot \phi_c \cdot f_c' \cdot a \cdot b = 0.79 \cdot 0.65 \cdot 40 \cdot 132.9 \cdot 300 = 818.8\,kN$$

Steel

$$F_{rs1} = \phi_s \cdot f_{s1} \cdot A_{s1} = 0.85 \cdot 400 \cdot 1200 \times 10^{-3} = -408\,kN$$

$$F_{rs} = (\phi_s \cdot f_s - \alpha_1 \cdot \phi_c \cdot f_c')A_{s2} = (0.85 \cdot 400 - 0.79 \cdot 0.65 \cdot 40) \cdot 1200 \times 10^{-3}$$

$\alpha_1 \cdot \phi_c \cdot f_c'$ already accounted for when calculating C_{rc} where area replaced by compression steel was not accounted for.

$$F_{rs2} = 383.4\,kN$$

$$P_r = +818.8 + 383.4 - 408 = 794\,kN$$

Moment around centroidal axis

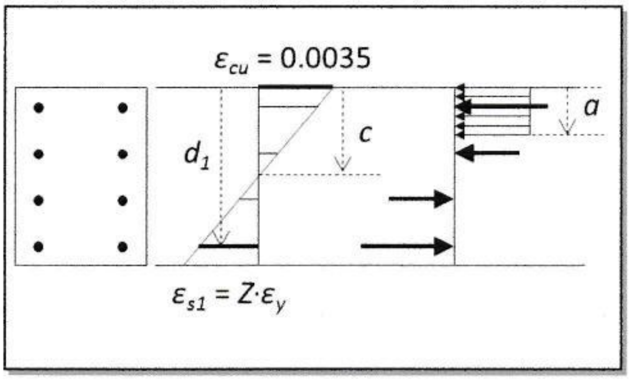
$$M_r = C_{rc}\left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n} F_{rsi}\left(\frac{h}{2} - d_i\right)$$

$$M_r = 818.8\left(150 - \frac{132.9}{2}\right) - 408(150 - 240) + 383.4(150 - 60)$$

$$M_r = 139.6\,kN \cdot m$$

Steps for calculating points on the interaction diagram:

1. Choose the strain distribution (through the depth of the column)



2. Determine depth to neutral axis, *c*, by inspection
3. Calculate depth of rectangular stress block, $a = \beta_1 \cdot c$ where $\beta_1 = 0.97 - 0.0025 \cdot f_c' \geq 0.67$
4. Calculate compression force in the concrete, $C_{rc} = \alpha_1 \cdot \phi_c \cdot f_c' \cdot a \cdot b$ where *b* is the column width, where

$\alpha_1 = 0.85 - 0.0015 f'_c \geq 0.67$

5. Calculate forces in compression and tension reinforcement:
   a. Tension reinforcement force, $F_r = \phi_s \cdot f_s \cdot A_s$
   b. Compression reinforcement force, $F_r = (\phi_s \cdot f_s - \alpha_1 \cdot \phi_c \cdot f_c') \cdot A_{st}$
6. Calculate the axial force and moment, *P* and *M*, by assuming force and moment equilibrium of the section

Short Columns

Under purely compressive loading, the unfactored resistance of a column is a function of: the x-sectional areas and strengths of the concrete and longitudinal reinforcement present, i.e.:

- $P_0 = \overset{\text{Concrete}}{\alpha_1 \cdot f'_c \cdot \left(A_g - A_{st}\right)} + \overset{\text{steel}}{f_y \cdot A_{st}}$
- where: A_g = gross x-section area of column
  A_{st} = area of reinforcing steel
  f'_c = specified compressive strength of concrete
  f_y = specified yield strength of reinforcing steel

- $\alpha_1 = 0.85 - 0.0015 \cdot f'_c \geq 0.67$ *[CSA A23.3 §10.1.7]. (equivalent stress block factor)*

Short Column Analysis and Design

In order to account for unintended eccentricities, the maximum load that a column can carry is limited *[CSA A23.3 §10.10.4]*:

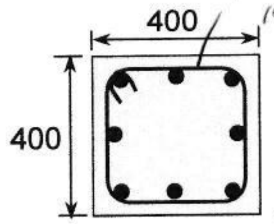
- $P_{r,max} = 0.80 \cdot P_{r0}$ for tied columns (80% theoretical strength
- $P_{r,max} = 0.85 \cdot P_{r0}$ for spiral columns (85%)

Where: $P_{r0} = \alpha_1 \cdot \phi_c \cdot f'_c \cdot \left(A_g - A_{st}\right) + \phi_s \cdot f_y \cdot A_{st}$

Example 2: ***Calculate the factored axial resistance of the tied column shown.***

ties so longitudinal reinforcement doesn't buckle

Also provide some confinement



8 – 20M

Assume:

f'_c = 30 MPa

f_y = 400 Mpa

α_1 =0.81 (α_1 =0.85 – 0.0015f'c) ≥ 0.67

$$\mathbf{A_g = 400 \times 400 = 160{,}000\,mm^2}$$

$$\mathbf{A_s = 8 \times 300\,mm^2 = 2{,}400\,mm^2}$$

$$\mathbf{P_{ro} = \alpha_1 \phi_c f'_c \left(A_g - A_s\right) + \phi_s A_s f_y}$$

$$\mathbf{= (0.81)(0.65)(30MPa)\left(160{,}000 - 2400mm^2\right) + (0.85)(400MPa)\left(2{,}400mm^2\right)}$$

$$\mathbf{P_{ro} = 3305\,kN}$$

Tied Column: k = 0.80 *(unintended eccentricities)*

$$\mathbf{P_{r\,max} = 0.80(3305\,kN)}$$

$$\mathbf{P_{r\,max} = 2644\,kN}$$

Interaction diagrams for design

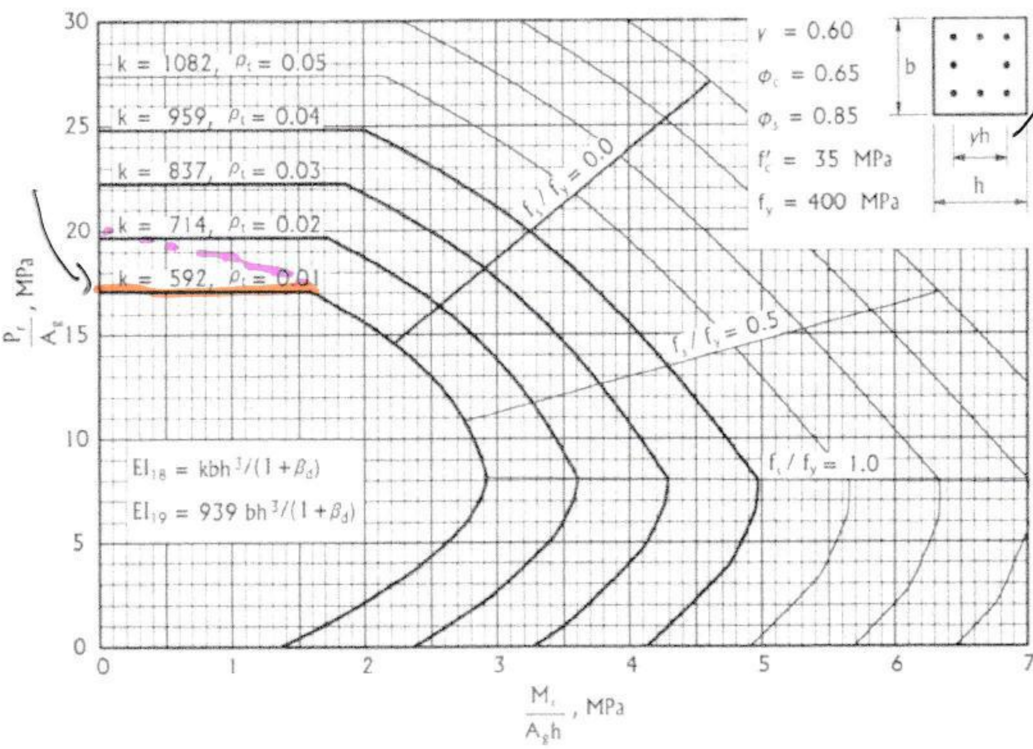
- Can be generated and then compared with the calculated factored axial load and moment, or
- non-dimensional diagrams provided in *[CAC Handbook 2006]* can be used.
- Currently available for download and included in course material

$\left(\frac{P_r}{A_g}, \frac{M_r}{A_g h}\right)$ **for rectangular columns**

$\left(\frac{P_r}{h^2}, \frac{M_r}{h^3}\right)$ **for circular columns**

γ defines distance between reinforcement

Cutoff at 0.8



ANALYSIS FOR GIVEN ECCENTRICITY, e (FIND P_R AND M_R)

- $P_r e = M_r \quad \Rightarrow \quad e = \dfrac{M_r}{P_r}$ and $\dfrac{1}{e} = \dfrac{P_r}{M_r}$
- Slope of a line in the chart $= \dfrac{P_r/A_g}{M_r/A_g h}$

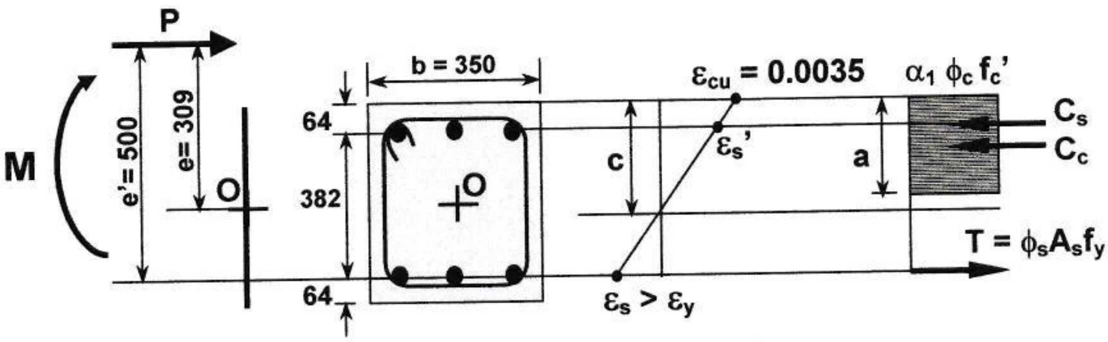
$$= \frac{P_r h}{M_r} = \frac{h}{e}$$

- Draw a straight line from the origin with a slope of h/e, and where it crosses the interaction curve for a given value of ρ_g determines the corresponding values of P_r/A_g and $M_r/A_g h$

ANALYSIS FOR GIVEN FACTORED AXIAL LOAD, P_F

- Set $P_r = P_f$ and calculate P_r/A_g
- Draw horizontal line at calculated value of P_r/A_g, and where it crosses the interaction curve for a given value of ρ_g determines the corresponding value of $M_r/A_g h$.
- Compute $e = M_r/P_r$.

Example 3: ***Calculate P_r and M_r when e = 309 mm for the cross-section shown below using Handbook Interaction Diagram.***
f'_c = 30 MPa, f_y = 400 MPa, A_s = 6 – 25M bars.



Compute section properties, γ, A_g, ρ_g

$\gamma h = 382mm$

$h = 382 + 2(64) = 510mm$

$$\therefore \gamma = \frac{\gamma h}{h} = \frac{382}{510} = 0.75$$

$A_g = bh = 350mm(510mm) = 178500mm^2$

$$\rho_g = \frac{A_s}{A_g} = \frac{6(500mm^2)}{178500mm^2} = 1.68\%$$

$= P_r e$

Use interaction diagrams to estimate P_r and M_r

To do this need to plot line with slope of h/e

e = 309mm (given)

$$\therefore \frac{h}{e} = \frac{510}{309} = 1.65$$

Since interaction diagrams for $\gamma = 0.75$ are not provided, we need to interpolate for $\gamma = 0.7$ *and* $\gamma = 0.8$

Use Table 7.11.7 and 7.11.8 provided by CAC

$\gamma = 0.7 \rightarrow Table\ 7.11.7$

$\rho = 1\% \rightarrow$ "x" $= 2.8 \therefore$ "y" $=$ mx $= 1.65(2.8) = 4.62$

$\rho = 2\% \rightarrow$ "x" $= 4.24$, "y" $= 6.996$

Use linear interpolation to find values for $\rho = 1.68\%$

$$\frac{x - 2.8}{1.68 - 1} = \frac{4.24 - 2.8}{2 - 1} \rightarrow x = 3.78$$

$$\frac{y - 4.62}{1.68 - 1} = \frac{6.996 - 4.62}{2 - 1} \rightarrow y = 6.235$$

$\gamma = 0.8 \rightarrow Table\ 7.11.8$

$\rho = 1\% \rightarrow$ "x" $= 3.05$, "y" $= 5.0325$

$\rho = 2\% \rightarrow$ "x" $= 4.6$, "y" $= 7.59$

Use linear interpolation to find values for $\rho = 1.68\%$

$$\frac{x - 3.05}{1.68 - 1} = \frac{4.6 - 3.05}{2 - 1} \rightarrow x = 4.104$$

$$\frac{y - 5.0325}{1.68 - 1} = \frac{7.59 - 5.0325}{2 - 1} \rightarrow y = 6.7716$$

Since $\gamma = 0.75$ is the midpoint of 0.7 and 0.8 take the average of the points found for $\rho = 1.68\%$

$$x_{avg} = \frac{3.7792 + 4.104}{2} = 3.94 = \frac{M_r}{A_g h}$$

$$y_{avg} = \frac{6.235 + 6.7716}{2} = 6.5 = P_r/A_g$$

$$\therefore M_r = 3.94(178500mm^2)(510mm) \div 1000^2 = 358.6kNm$$

$$P_r = 6.5(178500mm^2) \div 1000 = 1160.2kN$$

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Actual values computed using Approach #1

$$M_r = 370kNm, P_r = 1196kN$$

Table 7.11.7 Rectangular Columns with Bars on End Faces Only

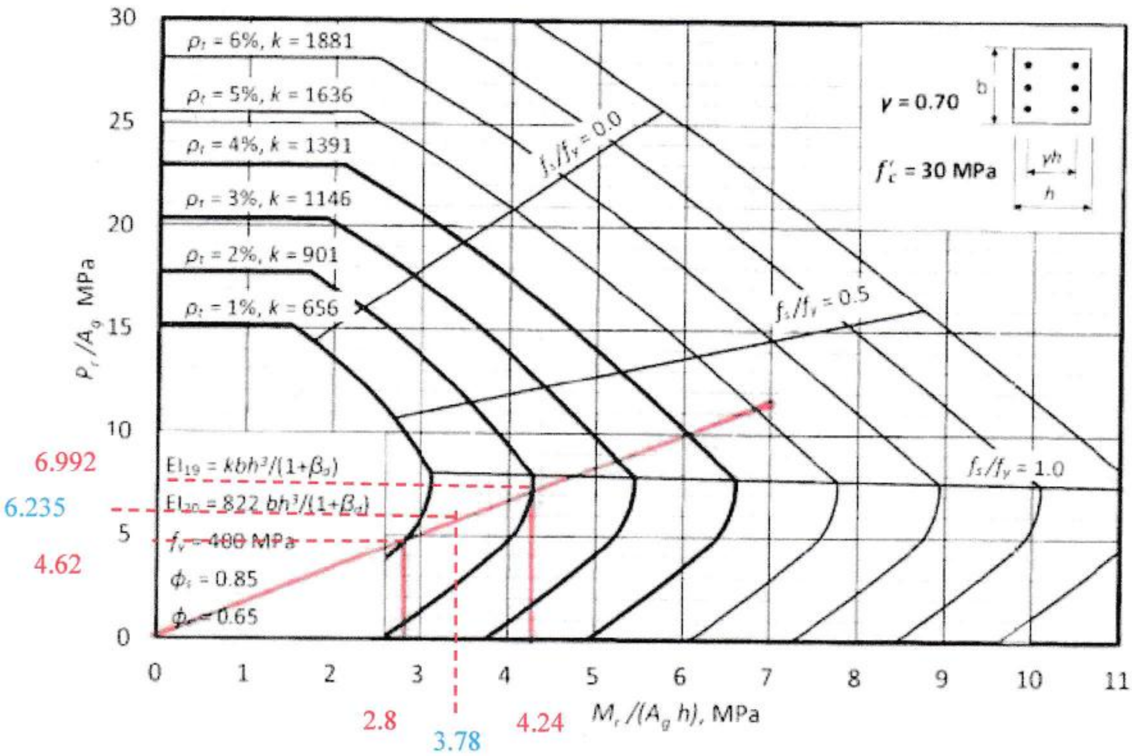
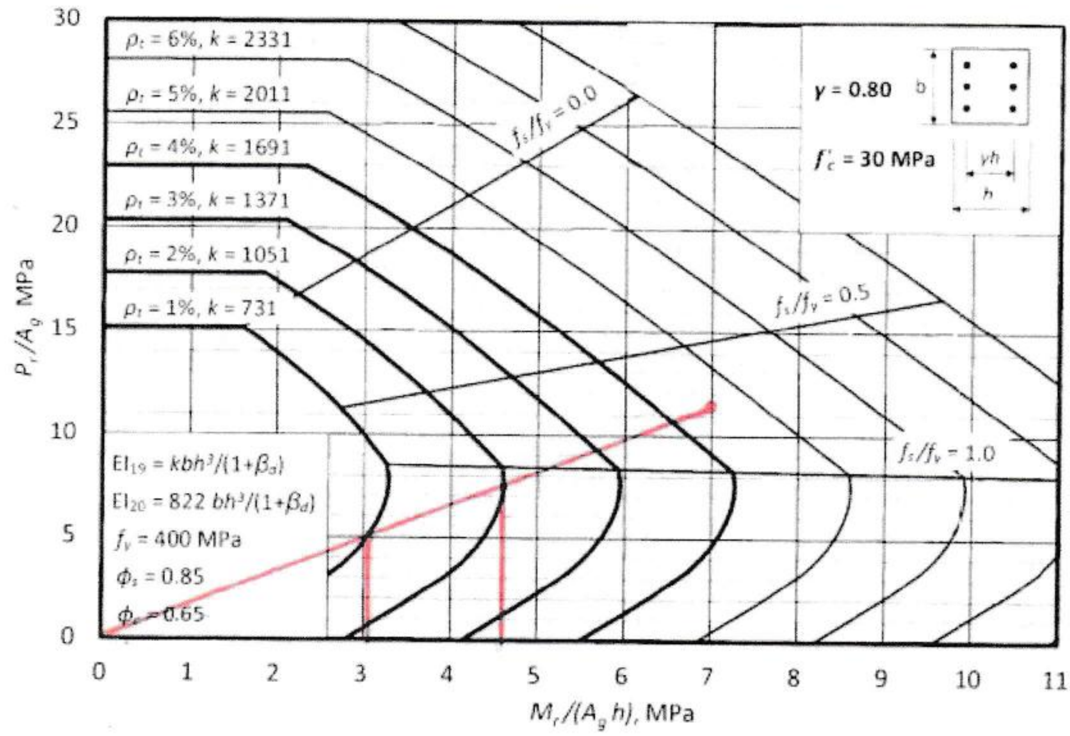


Table 7.11.8 Rectangular Columns with Bars on End Faces Only



Example 4 ***Design a column subject to the following factored loads***

P (kN), M(kNm)

	Dead	Live	Wind
P	750	600	-
M	25	20	85

$f_c' = 35\ MP_a$

Solution

1. Calculate factored load effects (see annex C , Table C.11)

$$\text{LC\# 1: } P_f = 1.4 \cdot 750 = 1050kN$$

$$M_f = 1.4 \cdot 25 = 35\ kNM$$

$$\text{LC\# 2: } P_f = 1.25 \cdot 750 + 1.5 \cdot 600 = 1837.5kN$$

$$M_f = 1.25 \cdot 25 + 1.5 \cdot 20 + 0.4 \cdot 85 = 95.25\ kNm$$

$$LC\#4: P_f = 1.25 \cdot 750 + 0.5 \cdot 600 = 1237.5kN$$

$$M_f = 1.25 \cdot 25 + 1.4 \cdot 85 + 0.5 \cdot 20 = 160.25kNm$$

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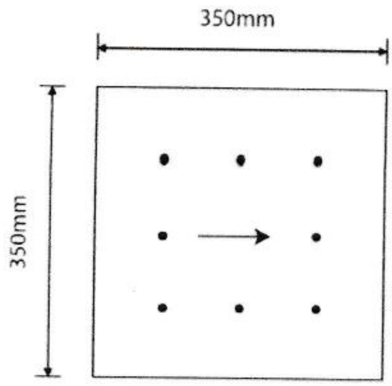
2. Trial Section

$$given \quad or \quad A_g(trial) = \frac{P_f}{0.35 \cdot f_c' + \rho_t \cdot f_y}$$

$$\uparrow pick\ something\ large$$

3. Calculate

$\frac{P_f}{A_g}, \frac{M_f}{A_g \cdot h}$



$A_g = 350^2 = 122500\ mm^2$

$h = 350mm$

$LC\#$	$\frac{P_f}{A_g}$	$\frac{M_f}{A_g \cdot h}$
1	8.6	0.82
2	15.0	2.2
4	10.1	3.74

4. Estimate γ

Try 30M bars, 10M ties, 40mm cover

$$\gamma = \frac{350-(40+40+11.3+11.3+29.9)}{350} = 0.62 \qquad \textit{(use 0.6)}$$

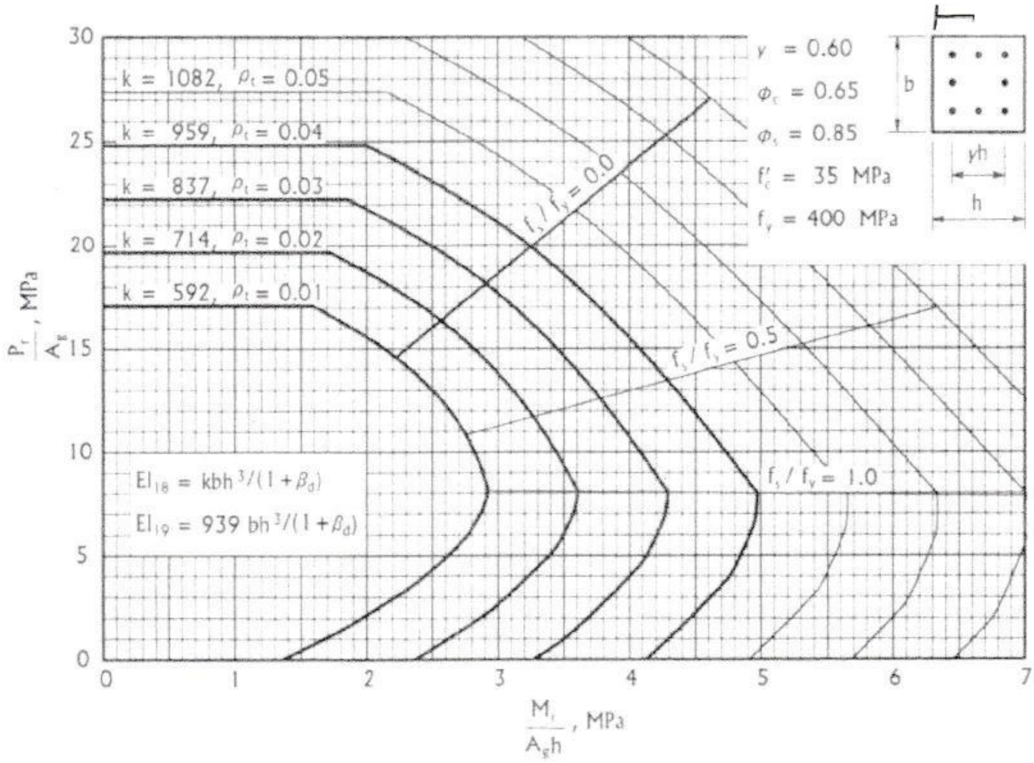
$$\rho_t = 0.03$$

(see interaction diagram take the largest from load combinations)

$$\rightarrow A_{st} = 0.03 \cdot 122500 = 3675mm^2$$

$$8 - 25M\ bars \rightarrow A_s = 4000mm^2 > 3675\ mm^2 \checkmark$$

recalculate w/ 25M bars to be same



k = 1082, ρ_t = 0.05
k = 959, ρ_t = 0.04
k = 837, ρ_t = 0.03
k = 714, ρ_t = 0.02
k = 592, ρ_t = 0.01
$f_s / f_y = 0.0$
$f_s / f_y = 0.5$
$f_s / f_y = 1.0$
$\gamma = 0.60$
$\phi_c = 0.65$
$\phi_s = 0.85$
$f'_c = 35$ MPa
$f_y = 400$ MPa
b
γh
h
$EI_{18} = kbh^3/(1+\beta_d)$
$EI_{19} = 939\ bh^3/(1+\beta_d)$
$\frac{P_r}{A_g}$, MPa
$\frac{M_r}{A_g h}$, MPa

Column Reinforcement Detailing

The following requirements should be followed when selecting longitudinal column reinforcement *[CSA A23.3 §10.9]*:

1%

- A minimum amount of longitudinal reinforcement of $A_{st} \geq 0.01 \cdot A_g$ (or $\rho_t \geq 0.01$) is normally called for to limit the effects of creep *[CSA A23.3 §10.9.1]*. Continuous 0.5%

- In the current code, $0.005 \cdot A_g \leq A_{st} < 0.01 \cdot A_g$ may be used, but the column capacity must be limited in this case as follows: $P_r = P_r \cdot 0.5 \cdot (1 + \rho_t / 0.01)$ *[CSA A23.3 §10.10.5]*.

8%

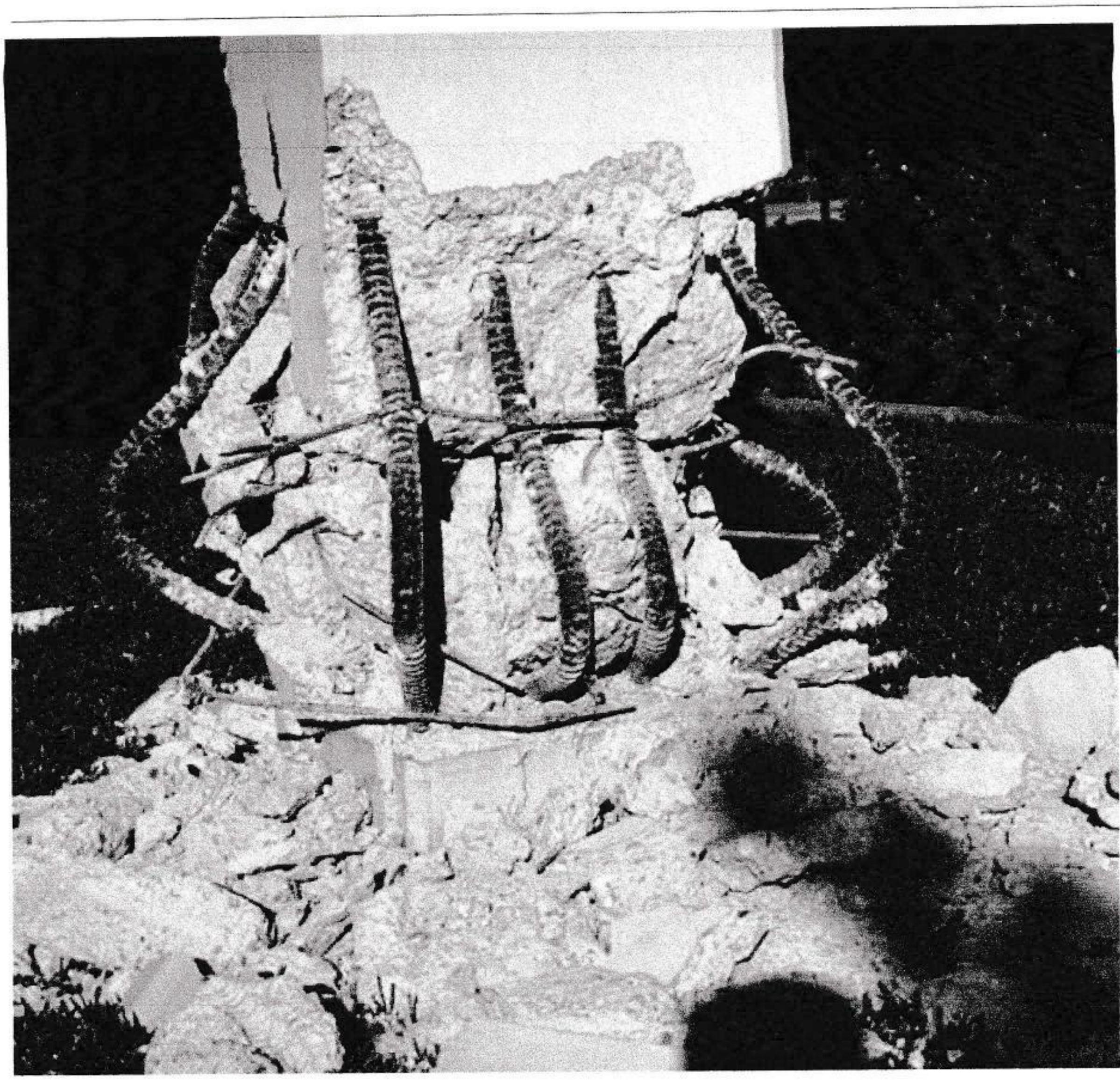
4%

try As to less than 4

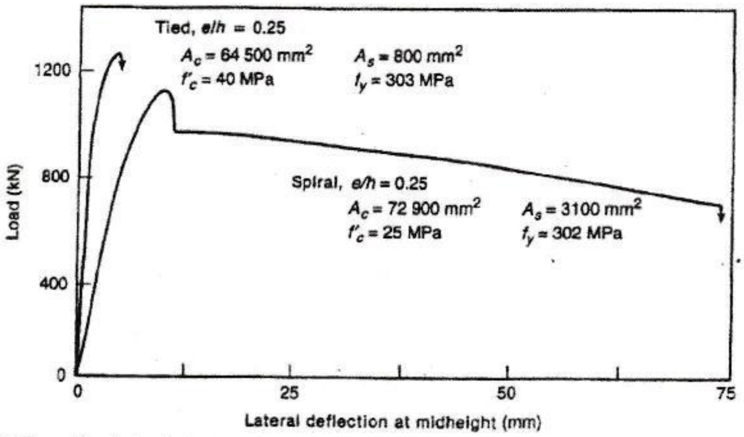
- A maximum amount of longitudinal reinforcement of $A_{st} \leq 0.08 \cdot A_g$ (or $\rho_t \leq 0.08$) is specified *[CSA A23.3 §10.9.2]* for practical reasons. However, even this is a lot of reinforcement. It is therefore recommended that $A_{st} \leq 0.04 \cdot A_g$, so that the longitudinal bars can be easily placed, lap splices can be accommodated, and the concrete will flow easily when it is poured.

- Normally, a minimum of four (4) longitudinal bars are required. (exceptions: minimum three (3) long. bars for triangular columns, minimum six (6) long. bars for spiral tied columns).

- The main purpose of the column ties is to provide lateral support for the longitudinal reinforcement. They also provide a small amount of confinement (much more in spiral columns).



No proper confinement



(b) Eccentrically loaded columns.

The following requirements apply to the selection of the column ties *[CSA A23.3 §7.6.5]*:

➢ Tie spacing shall not exceed:

- 16 · (diameter of smallest longitudinal bar)
- 48 · (tie diameter)
- the smallest column x-section dimension
- 300 mm, in the case of columns with "bundled" bars

➢ For rectangular columns, each corner and alternating longitudinal bar around the perimeter shall be laterally restrained by a tie corner (max. angle: 135°). No longitudinal bar can be more than 150 mm away on either side from a laterally restrained neighbouring bar.

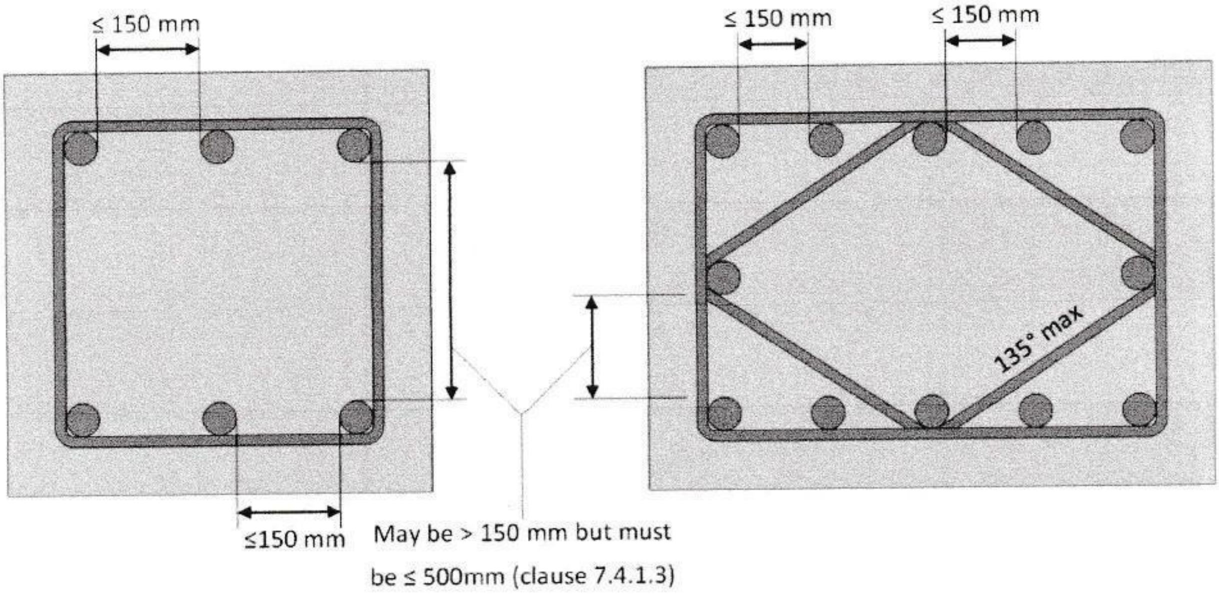
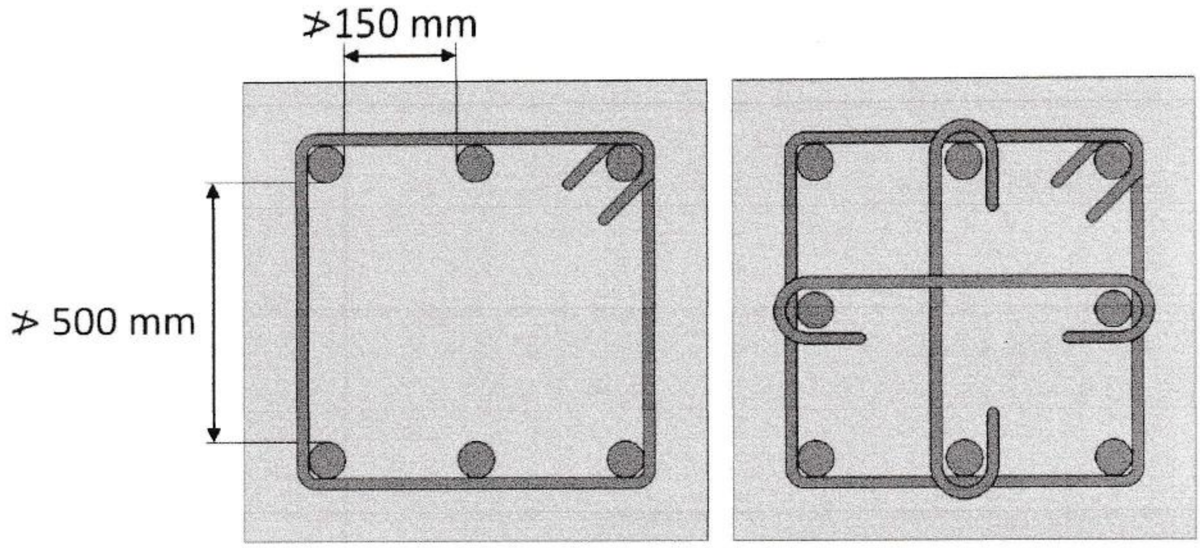


Fig. N7.6.5.5 Requirements for Lateral Support of Column Bars

Max. longitudinal bar spacing is 500 mm [*CSA A23.3 §7.4.1.3*].



See [*CSA A23.3 §7.5.1*] for tie requirements near offset bars.

Example 4 continued…

5. Tie details

Use 10M ties (ok up to 55M long. bar Cl. 7.6.5.1)

- tie spacing: MIN of:

$16 \cdot \overset{25M}{25.2} = 403$ or

$48 \cdot 11.3 = 542$ or

$350 \leftarrow governs$

- tie configuration: See Fig. N7.6.5.5

$$x = (350 - (40 \cdot 2 + 11.3 \cdot 2 + 25.2)/\,2\; - 25.2$$

$$= 85.9mm < 150mm \leftarrow \text{Requirement}$$

$$\rightarrow single\ tie\ ok!$$

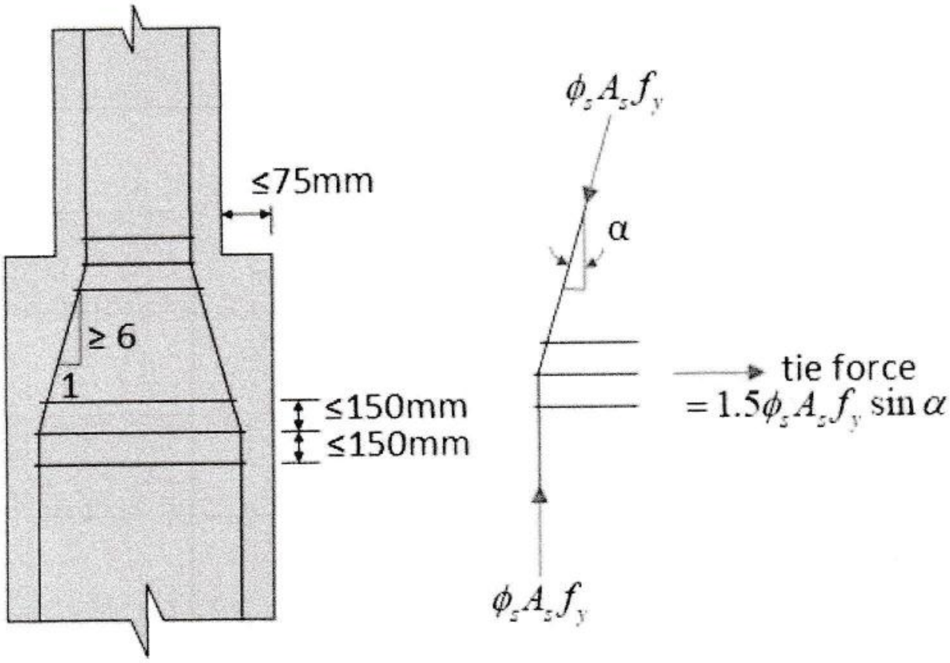
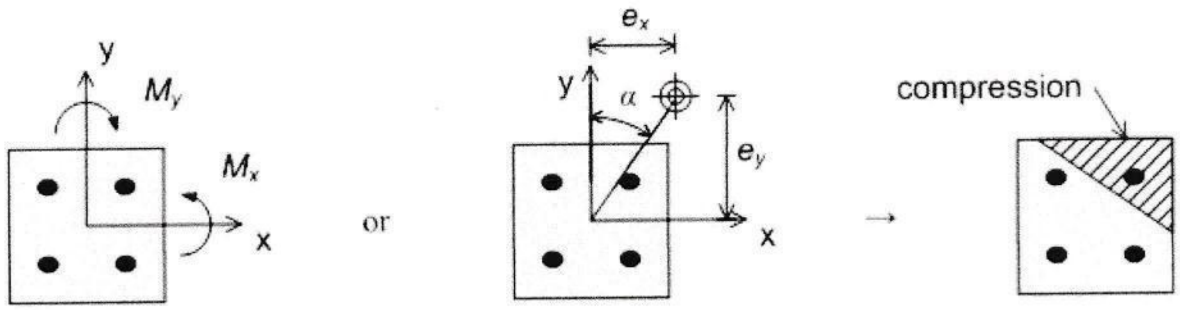


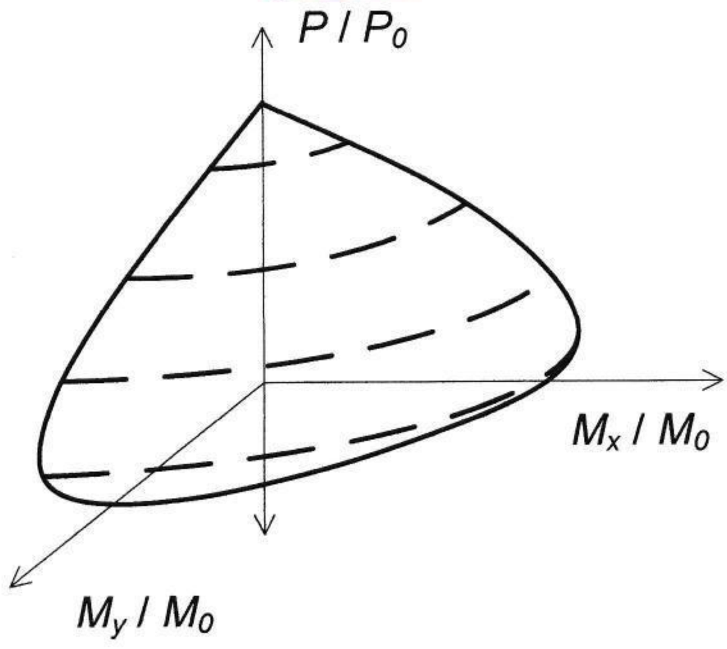
Fig. N7.5.1 Tie Requirement Near Offset Bars

Biaxial Bending

Biaxial bending is when moments are present about both axes, i.e.:



- Analysis of the resulting x-section can be rather involved, but if it were carried out (by assuming different strain distributions, as before), a 3D interaction diagram would result:



We approximate this diagram in design by considering the bending moments in the x and y directions separately, using the "Bresler" or "reciprocal load" method, i.e.:

$$\frac{1}{P_r} = \frac{1}{P_{rx}} + \frac{1}{P_{ry}} - \frac{1}{P_{r0}}$$

where: P_{rx} is the factored load resistance if the load is applied at the eccentricity e_x with $e_y = 0$

P_{ry} is the factored load resistance if the load is applied at the eccentricity e_y with $e_x = 0$

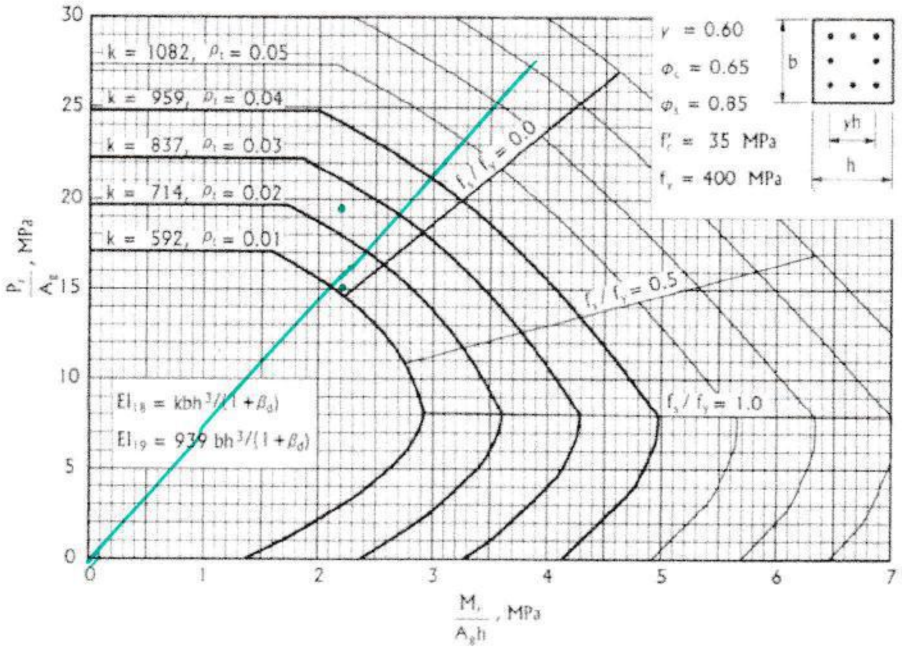
P_{r0} is the factored load resistance if $e_y = e_x = 0$

If $P_r \geq P_f$, then the column is adequate.

Bresler, Boris, „Design Criteria for Reinforced Columns under Axial Loads and Biaxial Bending" ACI Journal, Proceedings V, 57, No. 11, Nov 1960, pp. 481-490

Example 5 Verify column design in Example 4, assuming $M_y = M_x$ for LC#2

$LC\#$	$\frac{P_f}{A_g}$	$\frac{M_f}{A_g \cdot h}$
2	15.0	2.2


k = 1082, ρ_t = 0.05
k = 959, ρ_t = 0.04
k = 837, ρ_t = 0.03
k = 714, ρ_t = 0.02
k = 592, ρ_t = 0.01
γ = 0.60
ϕ_c = 0.65
ϕ_s = 0.85
f'_c = 35 MPa
f_y = 400 MPa
b
γh
h
f_s / f_y = 0.0
f_s / f_y = 0.5
f_s / f_y = 1.0
$EI_{18} = kbh^3/(1+\beta_d)$
$EI_{19} = 939\ bh^3/(1+\beta_d)$
$\frac{P_r}{A_g}$, MPa
$\frac{M_r}{A_g h}$, MPa
0
5
10
15
20
25
30
1
2
3
4
5
6
7

Solution

$$\rho_{prov} = \frac{4000}{122500} = 0.033 \text{ reinf. ratio}$$

$$\frac{P_{rx}}{A_g} = 19.5 = \frac{P_{ry}}{A_g}$$

$$\frac{P_{ro}}{A_g} = \frac{P_{rmax}}{A_g 0.8} = 23 \cdot 1.25 = 28.75$$

$$P_{rx} = 19.5 \cdot 122500 = 2389\, kN$$

$$P_{ro} = 28.75 \cdot 122500 = 3522\, kN$$

$$\frac{1}{P_r} = \frac{1}{2389} + \frac{1}{2389} - \frac{1}{3522}$$

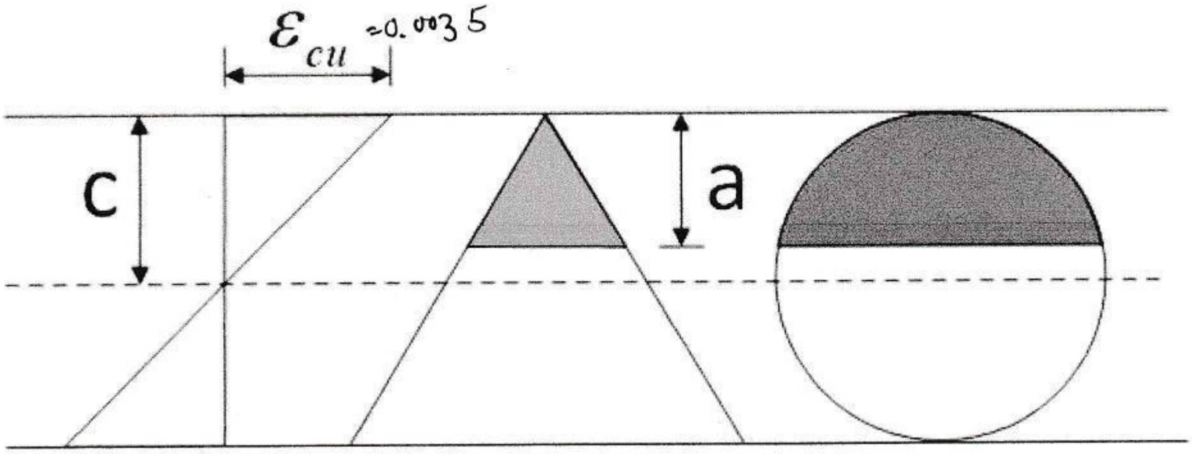
$$\rightarrow P_r = 1807kN < 1837.5kN \rightarrow Not\ Good *$$

*might be able to make column work by recalculating $\gamma\ with\ 20\ M\ bars$

*1.6% underdesign

Non-Rectangular and Non-Symmetrical Columns

Interaction diagrams can be produced in the same way as for rectangular columns. However, calculation of resultant force in concrete compression block is more complicated.



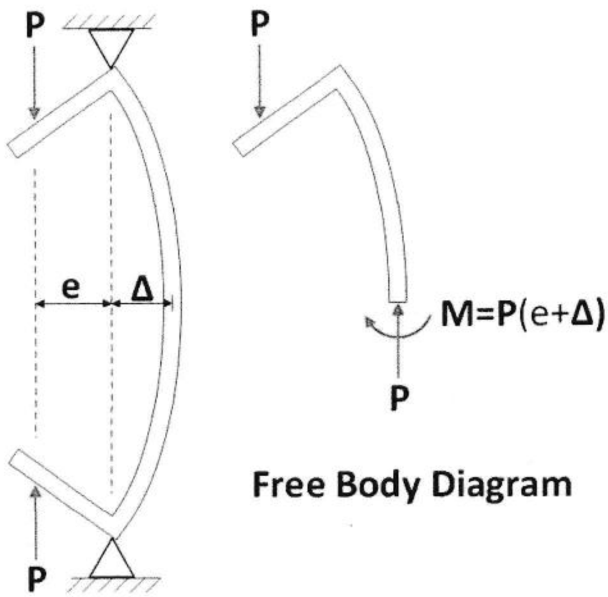
Compression block not rectangular
→ software available

When column short, assume same moment through whole section
When column slender account for $P\Delta$.

SLENDER COLUMNS

- Slender columns are columns with capacities that are significantly reduced due to second order effects.

- Second order effects can be explained using the following example of a column with an axial load, *P*, applied at both ends with an eccentricity, *e*:



- The first order moment is uniform along the column length, and can be calculated as: $\boldsymbol{M = P \cdot e}$.
- As *P* increases, the column will deflect at the mid-height by an amount, Δ.
- The second order moment at the mid-height is calculated based on the geometry of the deformed structure. The resulting moment, $\boldsymbol{M = P \cdot (e + \Delta)}$, is typically larger than the first order moment.

- The deflection, Δ, increases the moment for which the column must be designed. This will decrease the axial load capacity of the column. When the so-called "second order effects" are significant (~5%), the column is defined as a slender column, and must be designed as such.
- In practice, 90% of columns are short.
- The question is: how do we design the other 10%?
- According to *[CSA A23.3 §10.15.2]*, columns that **are laterally supported at the top and bottom** are short columns if:

CSA Short

$$\frac{k \cdot \ell_u}{r} \leq \frac{25 - 10 \cdot \dfrac{M_1}{M_2}}{\sqrt{P_f / \left(f'_c \cdot A_g\right)}} \text{ with } \frac{M_1}{M_2} \geq -0.5 \text{ where:}$$

M_1 = absolute minimum end moment

M_2 = absolute maximum end moment

Note: $\dfrac{M_1}{M_2}$ is positive (+) for single curvature,

$\dfrac{M_1}{M_2}$ is negative (–) for double curvature.

$r = \sqrt{\dfrac{I}{A}}$ is the "radius of gyration"

$r \cong 0.3 \cdot h \rightarrow$ rectangular columns

$r \cong 0.25 \cdot d \rightarrow$ circular columns

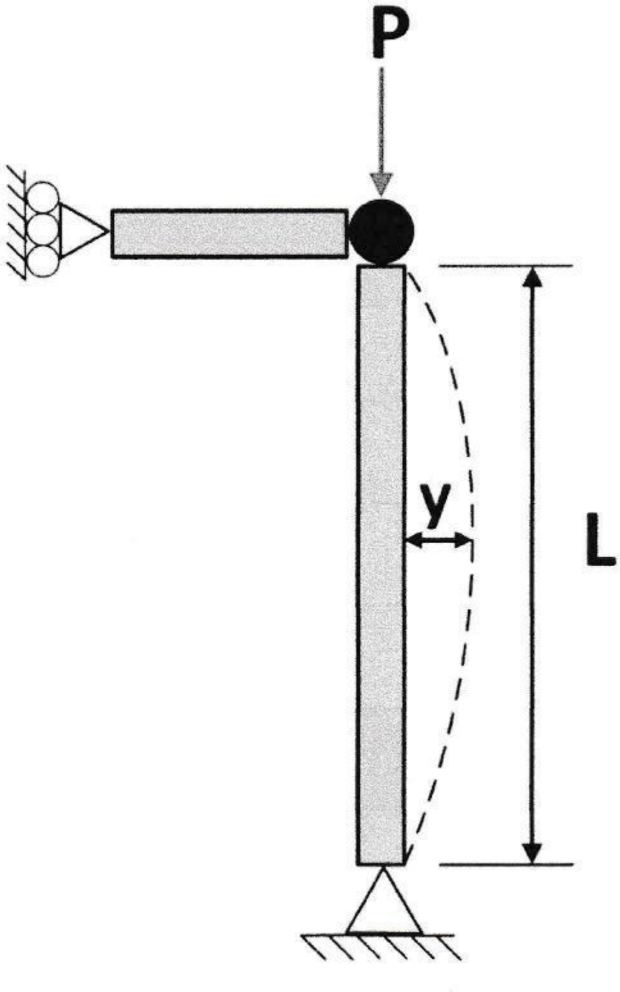
l_u = unsupported length of the column

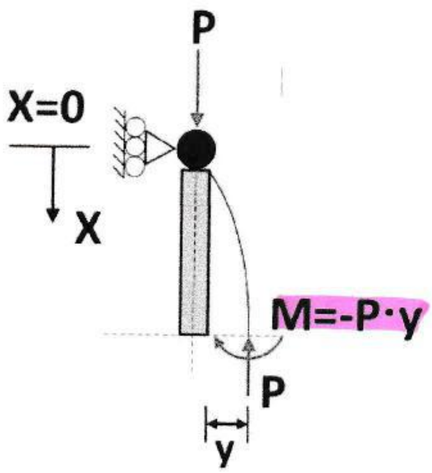
note: taking k= 1 is conservative first trial

Buckling of Columns

When a column is pushed laterally at mid-height and then released and does not return to its original, undeformed shape, bifurcation or "buckling" is said to have occurred

Pin-ended column





$$M = EI \, \frac{d^2y}{dx^2}$$

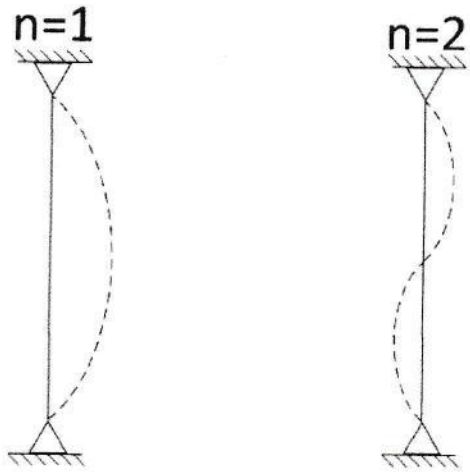
- Cutting the column and summing the moments for the resulting free body leads to the following expression:

$$E \cdot I \cdot \frac{d^2y}{dx^2} = -P \cdot y$$

- Euler's solution for the buckling load, P_c:

$$P_c = \frac{n^2 \cdot \pi^2 \cdot E \cdot I}{\ell^2}$$ (for pin-ended columns)

where n is the number of half-sine waves within the column length:



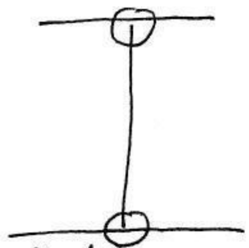
EI = stiffness (address material non-linearity)

for n = 1, the Euler buckling load is $P_e = \dfrac{\pi^2 \cdot E \cdot I}{\ell^2}$ ← pin-ended column

Effective Length Concept

- True pin-ended columns are rare in real construction. Columns in frames can be idealized as pin-ended columns, however, through the effective length concept. For the various possible end support conditions, the Euler buckling load is calculated as:

$$P_c = \frac{\pi^2 \cdot E \cdot I}{(k \cdot \ell)^2}$$ where:

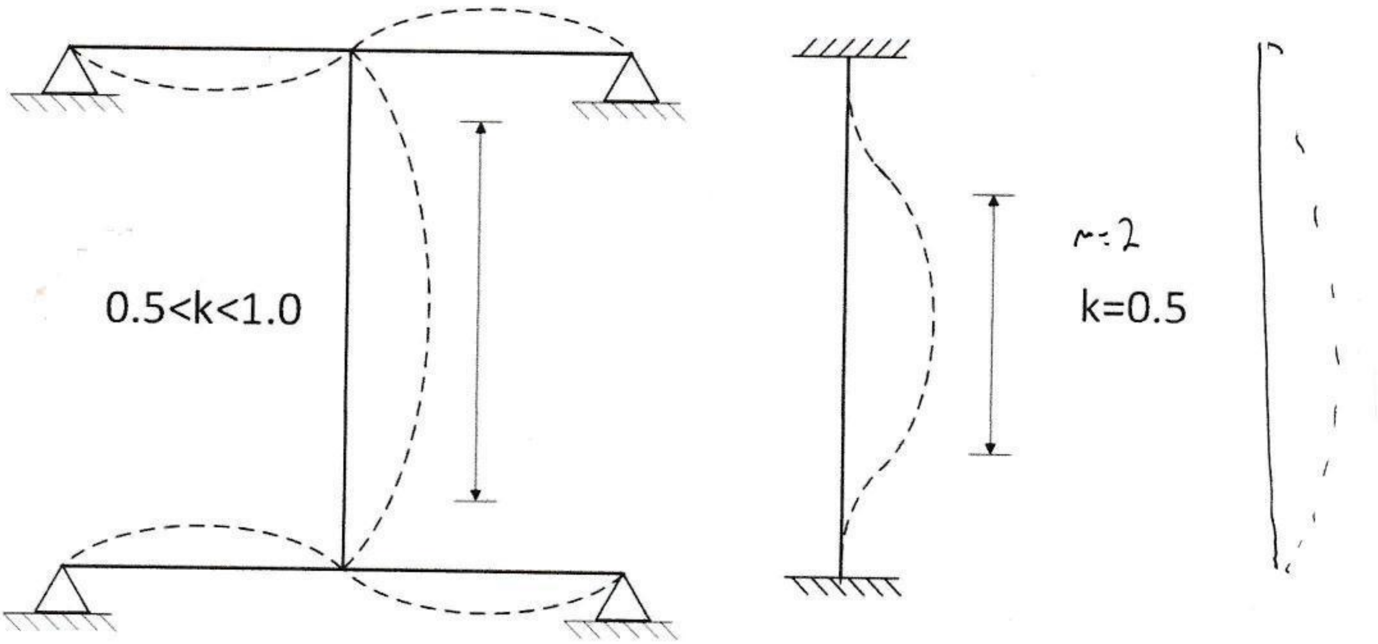


$k \cdot \ell$ is the so-called "effective length", and

k is the "effective length factor", $k = \frac{1}{n}$.

number of half sine waves w/in the column length

Examples:



Ψ

TOP		k (Bottom: Fixed)	k (Bottom: Elastic)	k (Bottom: Elastic)	k (Bottom: Hinged)
TOP	Hinged	0.81	0.91	0.95	1.00
TOP	Elastic Ψ = 3.1	0.77	0.86	0.90	0.95
TOP	Elastic Ψ = 1.6	0.74	0.83	0.86	0.91
TOP	Fixed	0.67	0.74	0.77	0.81
		Fixed	Elastic	Elastic	Hinged
		BOTTOM			

Effective length factors *[CAC Handbook Table 8.5]*:

or braced:

$$\frac{kl_u}{r} \le \frac{25 - 10\, M_1/M_2}{\sqrt{\frac{P_f}{f'_c A_g}}}, \quad M_1/M_2 \ge -0.5$$

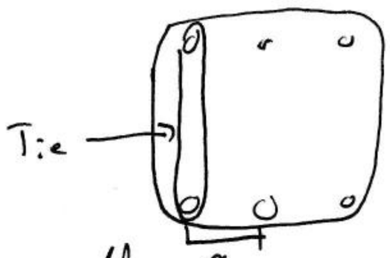
· k is effective length factor
↳ If k=1 and short, ok

↳ Almost always braced

Note: use first
1. Create interaction diagrams of cross-sections
2. Design using interaction diagrams from code
3. Short columns and long columns

Note: buckling never desirable. Failure by crushing ok

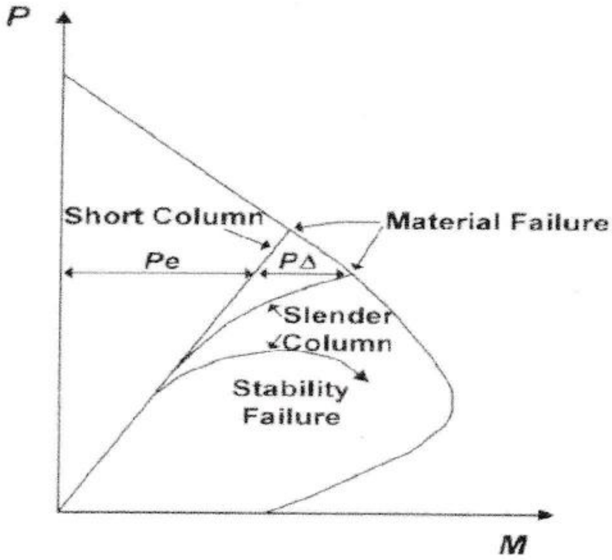
· $P_c = \frac{n^2 \pi^2 EI}{\ell_e^2}$ (Euler Buckling load), n depends on buckling mode. Usually



If x > 150, tie req'd

Material vs. Stability Failures

- The increase in moment due to second order effects can be considered by superimposing the P-M curve on the interaction diagram.
- For a short column, the slope of the curve is constant and can be related to the eccentricity of the load application.

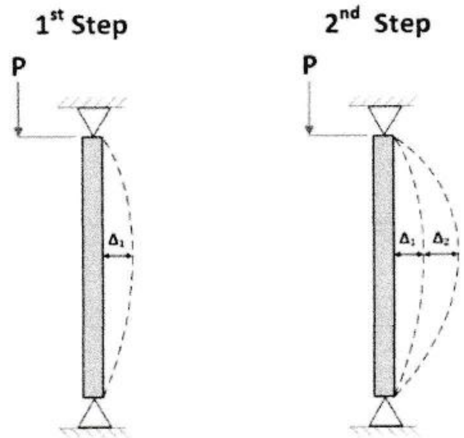


[CAC Handbook 2014]

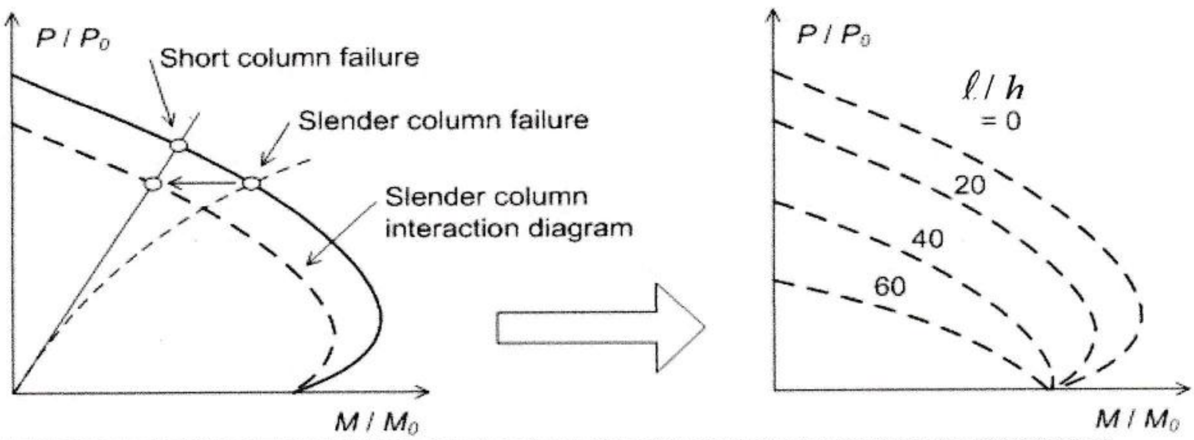
- As the column becomes more and more slender, the axial force vs. moment curve becomes increasingly nonlinear. Depending on the slenderness, material or stability failures may result:
- If the axial force – moment curve intercepts the interaction diagram at the peak axial load level, a material failure occurs.
- If the peak axial load occurs prior to this, then a stability failure is said to have occurred.
- Material failures can be expected for most practical cases.

There are three ways to consider second order effects in design:

1. Second order analysis: this means carrying out a stepwise analysis of the column that considers the change in column geometry (i.e. out-of-plane displacement) for each step.



2. Through the use of slender column interaction diagrams, i.e.:



3. Through the use of a **moment magnification factor.**

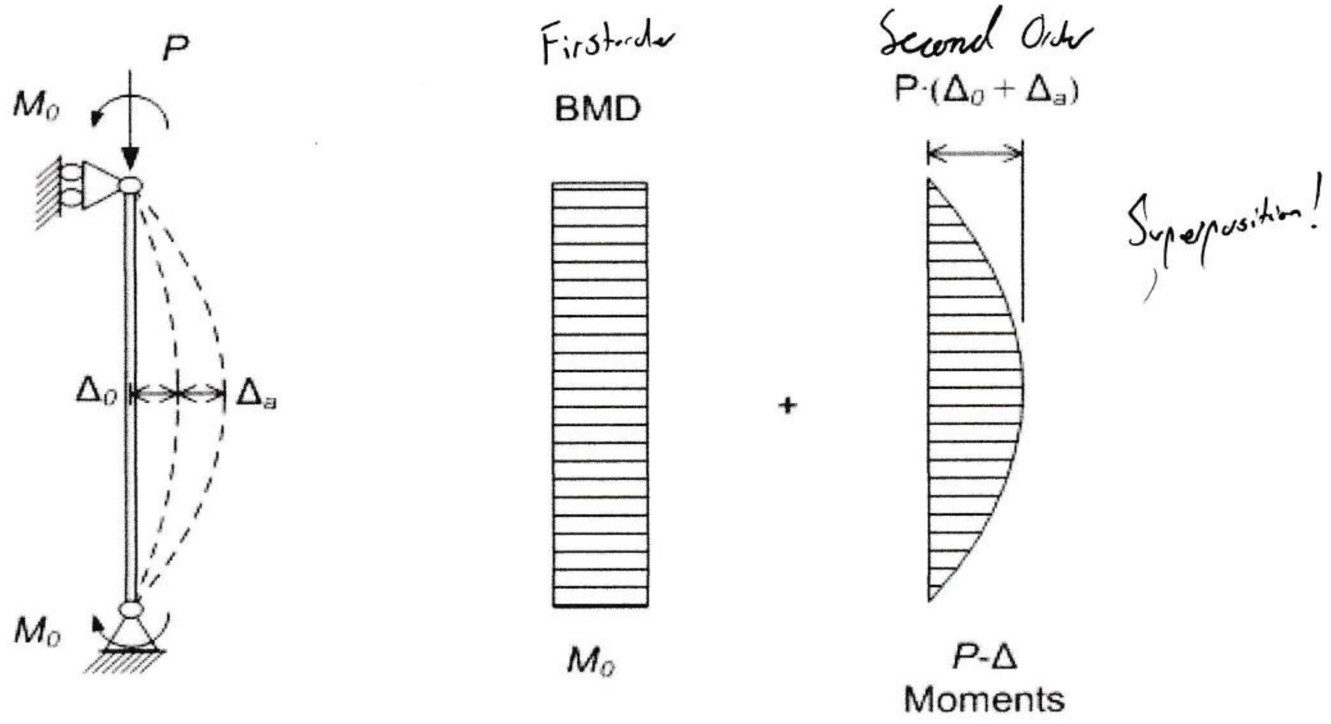
Moment Magnifier Concept

- Recall that the moment in an eccentrically loaded, pin-ended column considering second order effects (we'll now call this M_c) can be calculated as follows: $M_c = P \cdot (e + \Delta)$.

- Alternatively, we can write: $M_c = M_0 + P \cdot \Delta$ where M_0 is the first order moment, or $M_c = \delta \cdot M_0$.

- $\boldsymbol{\delta}$ is a correction factor, which increases M_0 to account for second order effects. It is a **moment magnifier**.

- Consider the following column subjected to first order end moments, M_0, and an axial force, P.



$$\Delta_0 = first\ order\ deflection$$

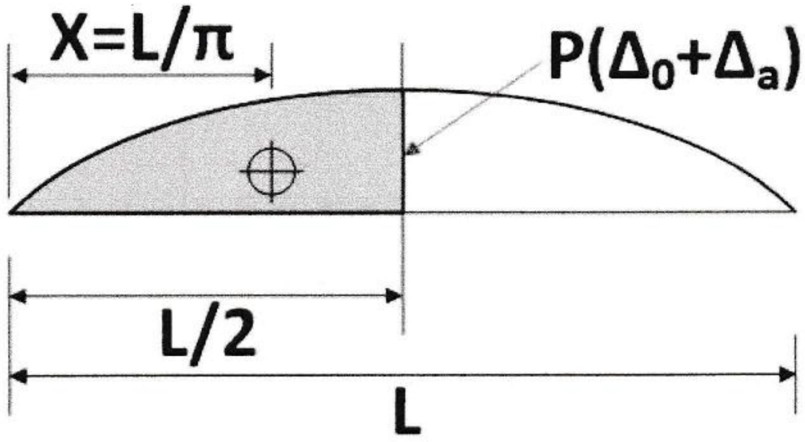
$$\Delta_a = second\ order\ deflection$$

Using the moment-area method we can calculate Δ:

Recall: $\Delta = \frac{A \cdot x}{E \cdot I}$

Where: A - area under the bending moment diagram

X - distance to the centroid of bending moment diagram, from support



$$A = P(\Delta_0 + \Delta_a)\frac{L}{2}\cdot\frac{2}{\pi}$$

$$\Delta_a = \left(\frac{P}{EI}\cdot(\Delta_0 + \Delta_a)\frac{\ell}{2}\cdot\frac{2}{\pi}\right)\cdot\frac{\ell}{\pi}$$

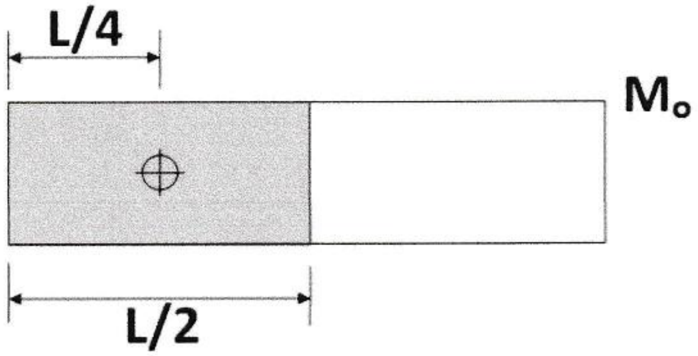
$$\Delta_a = \frac{P \cdot \ell^2}{\pi^2 \cdot E \cdot I}\cdot(\Delta_0 + \Delta_a) \rightarrow \frac{P}{P_e}\cdot(\Delta_0 + \Delta_a)$$

Rearranging $\rightarrow$ $\Delta_a = \Delta_0 \cdot \left(\frac{P/P_e}{1 - P/P_e}\right)$

Total deflection, $\Delta = \Delta_0 + \Delta_a = \Delta_0 \cdot \left(\frac{1}{1 - P/P_e}\right)$

Therefore: $M_c = M_0 + P \cdot \Delta_0 \cdot \left(\dfrac{1}{1 - P/P_e} \right)$

Again, using the moment-area method we can calculate Δ_0:



A=M₀ L/2

$$\Delta_0 = \frac{M_0 \cdot \ell^2}{8 \cdot E \cdot I}$$

2 reasons for nonlinearity ≈
1. Material non linearity
2. Geometric non linearity
· Deflections impact

first order moment

$$\rightarrow \quad M_c = M_0 \cdot \left(\frac{1 + 0.23 \cdot P/P_e}{1 - P/P_e} \right) \approx M_0 \cdot \left(\frac{1}{1 - P/P_c} \right)$$

(code approximation *[CSA A23.3]*) → $\boxed{\delta = \left(\dfrac{1}{1 - P/P_c} \right)}$

Moment magnifier factor

Unequal End Moments – Pin-Ended Columns

For column with unequal end moments, the maximum M_0 and $P\text{-}\Delta$ moments do not occur at the same cross-section. To design columns in this case, the real column is replaced by a similar column subjected to equal end moments $C_m \cdot M_2$. These end moments are chosen so that the maximum magnified moment is the same in both (the real and the "effective") columns.

$$C_m = 0.6 + 0.4 \cdot \frac{M_1}{M_2} \geq 0.4$$ *[CSA A23.3 §10.15.3.2]*

where: M_1 and M_2 are the smaller and larger end moments

$\frac{M_1}{M_2}$ is positive for single, negative for double curvature

Conditions:

- $\frac{M_1}{M_2} \geq -0.5$ *(implied because $C_m \geq 0.4$)*
- M_2 should not be less than the moment associated with assumed minimum eccentricity, i.e.:

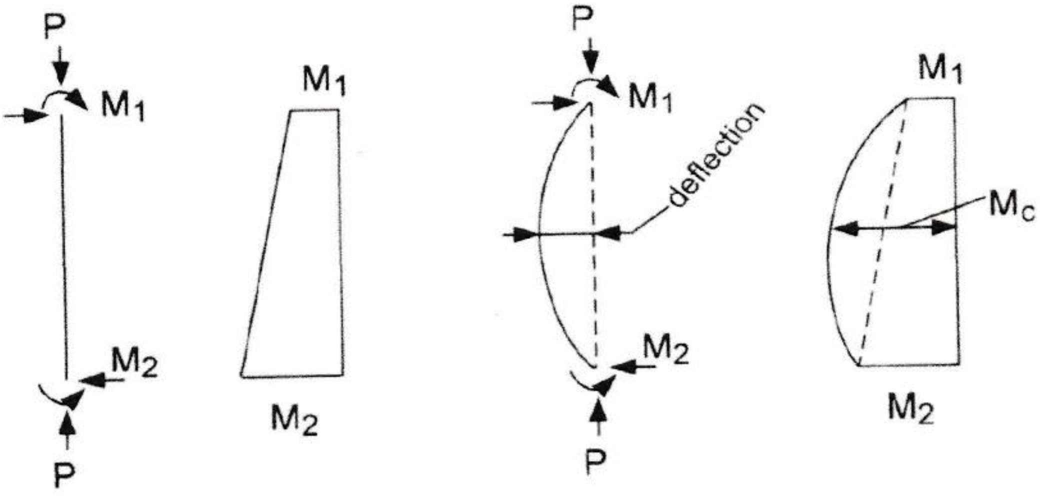
  $M_2 \geq P_f(15 + 0.03 \cdot h)$
- $C_m = 1.0$ if transverse loads are present between column ends *[CSA A23.3 §10.15.3.3]*

The resulting moment magnifier is expressed as follows:

$$\delta_b = \frac{C_m}{1 - \dfrac{P}{\phi_m \cdot P_c}}$$

where: $\phi_m = 0.75$ is a member resistance factor

The '*b*' subscript refers to "braced" columns (i.e. columns in braced or "non-sway" frames).

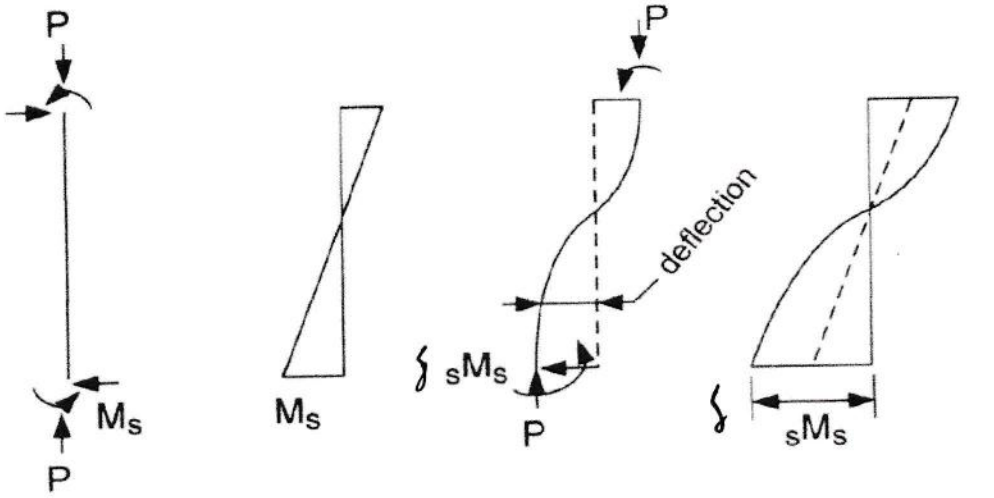


(i) 1 st Order Moments Due to Gravity Loads

$\boxed{\delta_b}$

(ii) Amplified Moments

(a) Members Stability Effect - Amplification of Gravity Load Moments Due to Deflections Between Member Ends



(i) 1 st Order Moments Due to Gravity Loads

(ii) Amplified Moments

(a) Lateral Drift Effect - Amplification of lateral Load Moments Due to Relative End Deflections

Column Stiffness

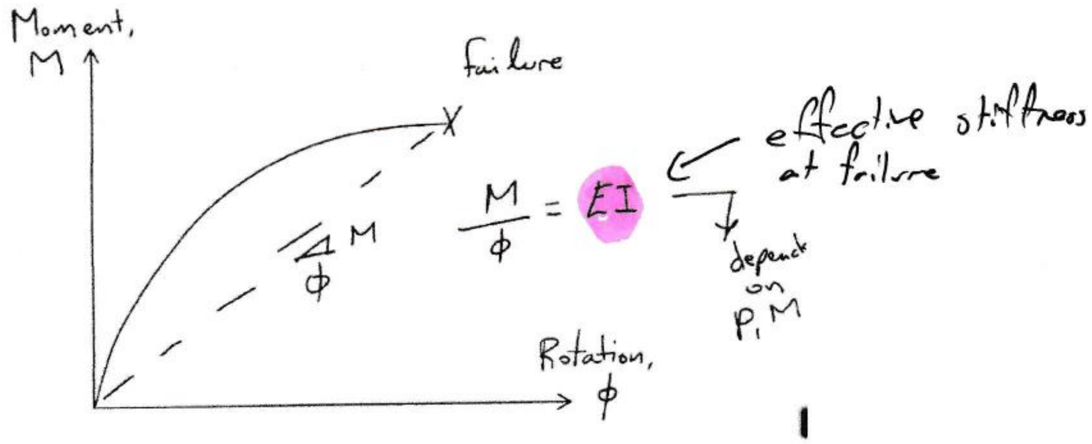
- The column stiffness ($E{\cdot}I$) used in the calculation of the Euler buckling load should take into account:

– cracking (material non-linearity)

– creep [long-term effect (material non-linearity)]

– the true, nonlinear stress-strain relationship

- The column stiffness chosen for the analysis of the structure must approximate the $E{\cdot}I$ of the column at failure.



[CSA A23.3 §10.15.3.1] suggests the following empirical formulas for calculating $E{\cdot}I$ for columns under sustained loads:

$$E \cdot I = \frac{0.2 \cdot E_c \cdot I_g + E_s \cdot I_{se}}{1+\beta_d} = EI_{eff}$$ ← cracking (numerator), ← creep (denominator)

or

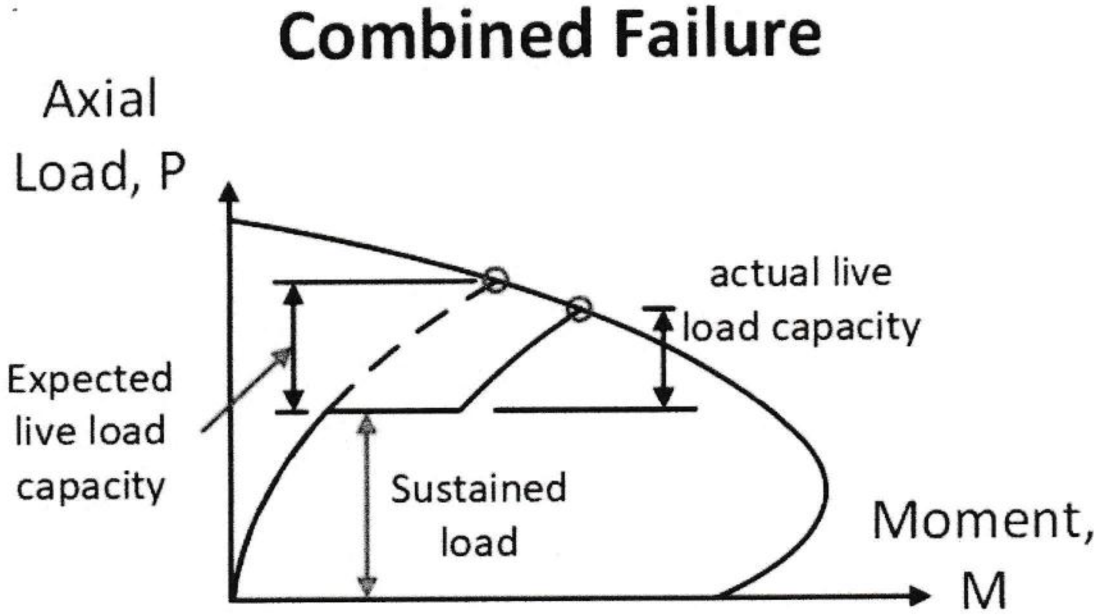
$$E \cdot I = \left(\frac{0.4}{1+\beta_d}\right) \cdot E_c \cdot I_g = EI_{eff}$$ ← cracking, ← creep

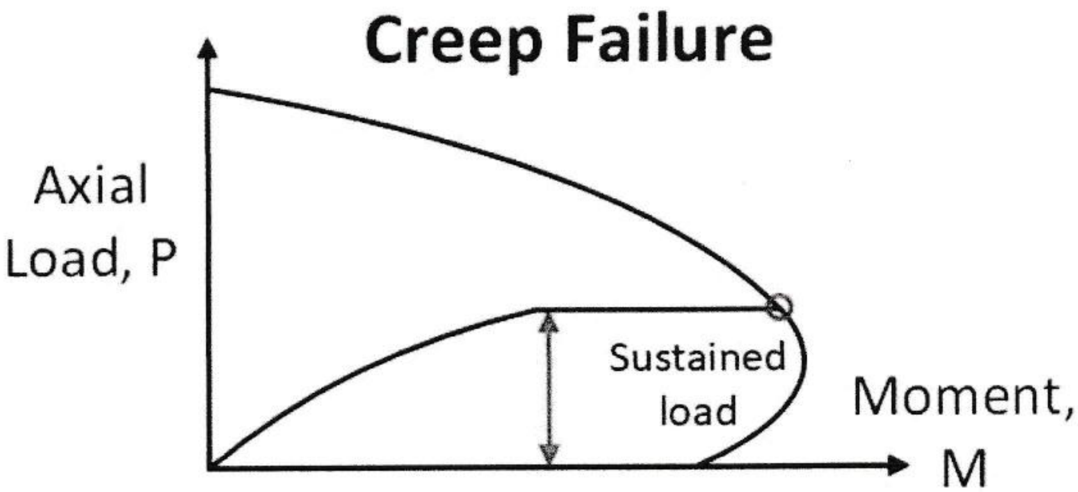
I_{se} = moment of inertia of the reinforcing steel around centroidal axis of the gross cross section

I_g = gross moment of inertia of concrete x-section
β_d = effect of sustained loads

Effect of Sustained Loads

Creep increases the deflections under sustained loading, thus increasing $P{\cdot}\Delta$ effects and thereby weakening the column:





To address this concern:

- reduced modulus approach adopted by *[CSA A23.3 2004]*
- $E \cdot I$ is divided by $(1 + \beta_d)$ where:
  - for columns in "non-sway" or "braced" frames:

$$\beta_d = \frac{\text{factored dead vertical load}}{\text{total factored vertical load}}$$

  - for columns in "sway" or "unbraced" frames (next topic in this course):

$$\beta_d = \frac{\text{factored sustained shear}}{\text{total factored shear}}$$ ← Not wind

Example 6:

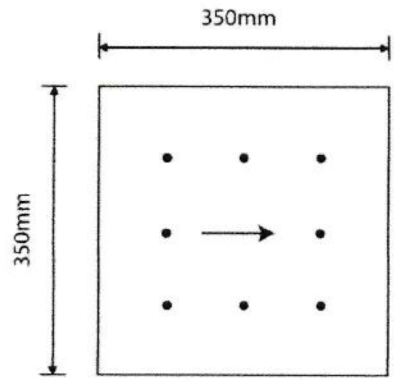
Verify column design in Example 4 assuming $l_u = 3.5m$ and moments are equal at column ends and acting in opposite directions for LC #2:

4 Steps:
1. Check slenderness
2. Determine Dead EI, P_c
3. Find magnification
4. Design for P_f and M_c w/ interaction diagram

LC#	$\frac{P_f}{A_g}$	$\frac{M_f}{A_g \cdot h}$
2	15.0	2.2

25 M long. bars

$f'_c = 35\ MPa$



$$\text{LC\# 2}: P_f = 1.25 \cdot 750 + 1.5 \cdot 600 = 1837.5kN$$

$$M_f = 1.25 \cdot 25 + 1.5 \cdot 20 + 0.4 \cdot 85$$
$$= 95.25\ kNm$$

Solution:

1. Is column slender?

$$\frac{k \cdot l_u}{r} > \frac{25 - 10 \cdot M_1/M_2}{\sqrt{P_f/(f'_c \cdot A_g)}}$$

$\frac{3500}{0.3 \cdot 350} = 33.3$ (assuming k=1 is most conservative)
← from code. 0.3 x shorter length

$$\frac{25 - 10 \cdot \left(\frac{95.25}{95.25}\right)}{\sqrt{1837.5 \cdot 1000/(35 \cdot 350^2)}} = 22.9 < 33.3$$

$$\rightarrow column\ is\ slender$$

2. Compute EI, P_c

$$EI = \left(\frac{0.4}{1 + \beta_d}\right) E_c \cdot I_g$$
$$E_c = 4500\sqrt{35} = 26622.4\ MPA$$
$$I_g = 350^4/12 = 1.25 \cdot 10^9 mm^4$$
$$\beta_d = 1.25 \cdot 750\ /\ 1837.5 = 0.51$$
$$\rightarrow E \cdot I = 8.81 \cdot 10^{12}\ N \cdot mm^2$$

$$P_c = \pi^2 \cdot EI\ /\ (k \cdot l_u)^2$$
$$= 7104\ kN\ (> P_f \rightarrow ok!)$$, $P_f = 1837.5$ kN

3. Compute magnified moment:

$$M_c = M_2 \cdot C_m/(1 - P_f/(\phi_m \cdot P_c))$$, $C_m = 1$ bc end moments are the same

$$M_2 = 95.25\ kNM$$

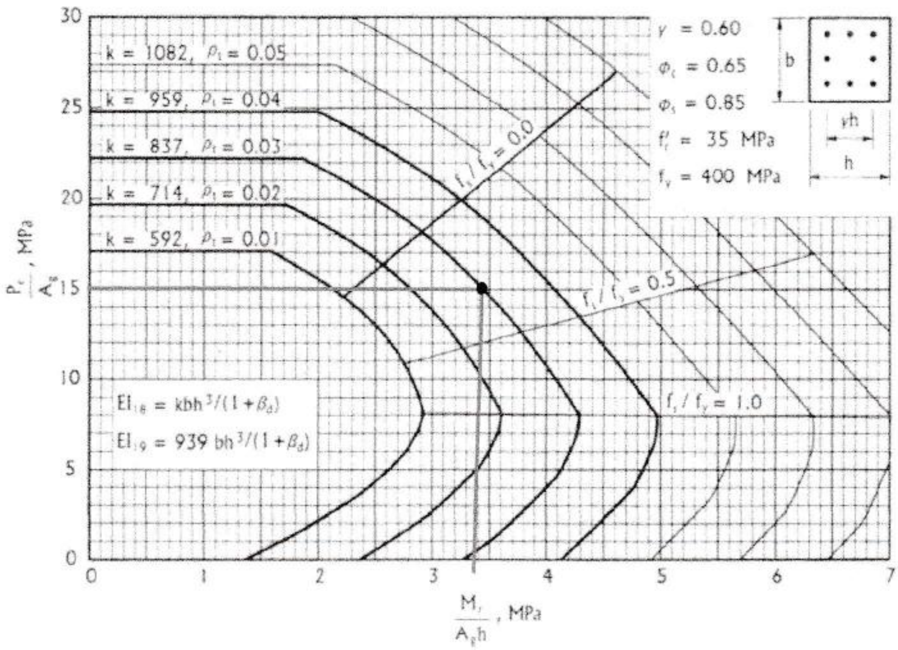
$$C_m = 0.6 + 0.4 \cdot (M_1/M_2) \geq 0.4$$
$$= 0.6 + 0.4(1) = 1.0$$
$$M_c = 95.25 \cdot (1)/\left(1 - \frac{1837.5}{0.75 \cdot 7104}\right) = 1.53 \cdot 95.25$$
$$= 145.4\ kNm$$

4. Design column for P_f, M_c

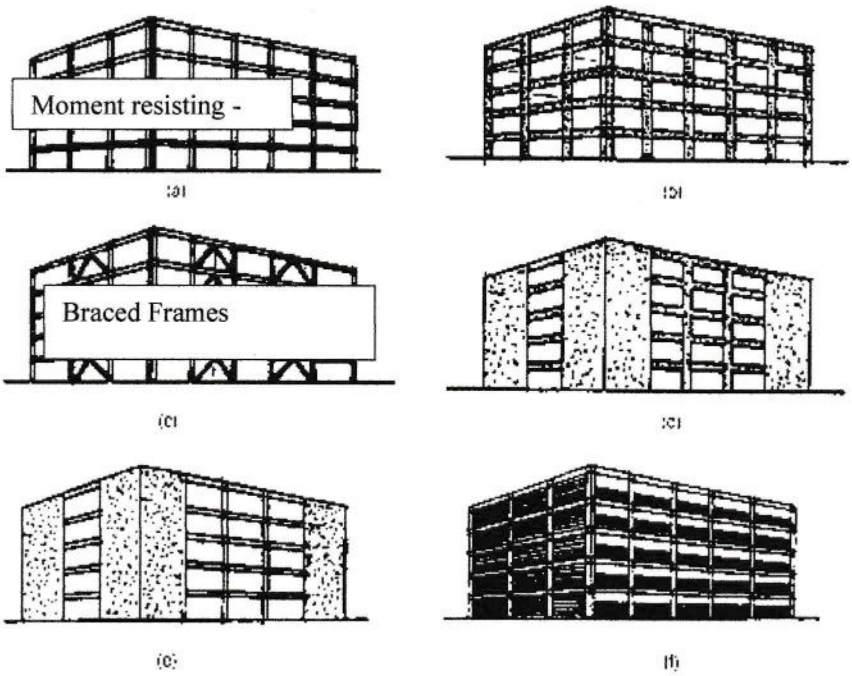
$$P_f/A_g = 15$$

$$M_c/(A_g \cdot h) = 3.4$$

Reinforcement ratio = 3%

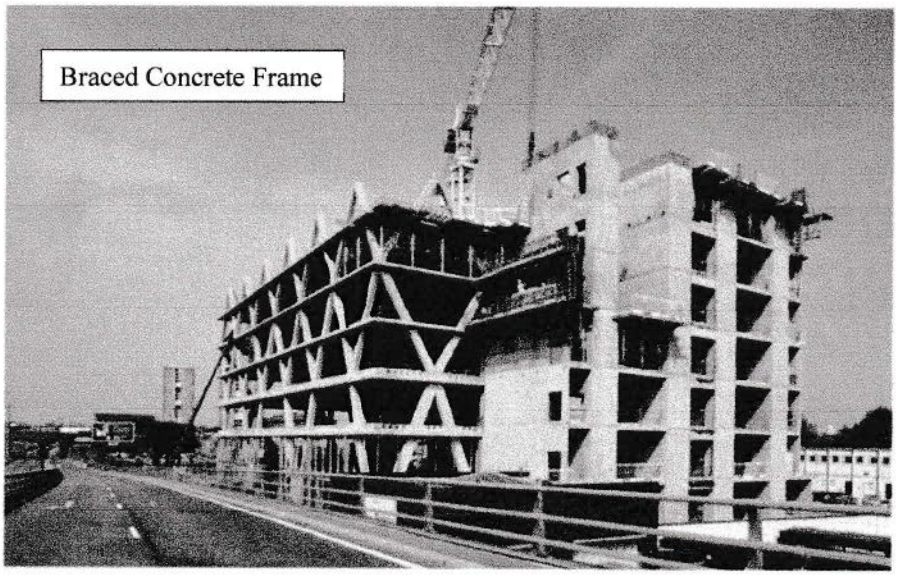


Columns in Braced (Non-Sway) Frames



(a) steel moment-resisting frame; (b) reinforced concrete moment-resisting frame; (c) braced steel frame; (d) reinforced concrete shear walls; (e) steel frame building with cast-in-place concrete shear walls; (f) steel frame building with in-filled walls of nonreinforced masonry.

https://www.northernarchitecture.us/resisting-system/lateralforceresisting-systems.html

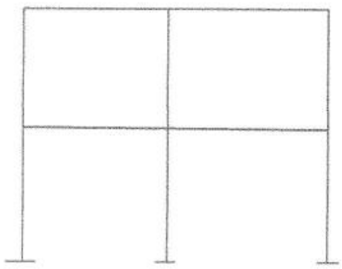


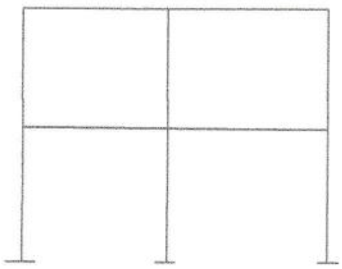
https://www.quora.com/Is-bracing-system-used-only-in-steel-structures

Completely braced (non-sway) and/or completely unbraced (sway) frames do not exist.

- A frame is "braced" in a given plane if the lateral stability of the structure as a whole is provided by walls, bracing, or buttresses.
- A frame is "unbraced" if all resistance to lateral loads comes from bending in the columns.

Shearwall, wall core





According to *[CSA A23.3 2004]*, a storey is considered **braced** if:

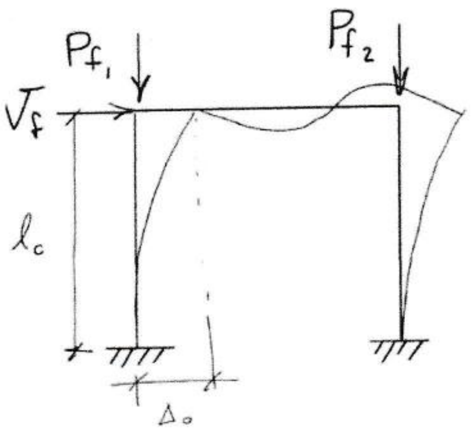
$$Q = \frac{\sum P_f \cdot \Delta_0}{V_f \cdot \ell_c} \leq 0.05 \text{ where:}$$

$\sum P_f$ = total vertical load on all columns in a storey

Δ_0 = first order deflection of the top of the storey relative to the bottom (due to V_f)

V_f = shear in that storey due to factored lateral loads

l_c = storey height



$\Sigma P_f = P_{f_1} + P_{f_2}$

10.14.1.2

The following properties may be used to determine the section properties specified in Clause 10.14.1.1:

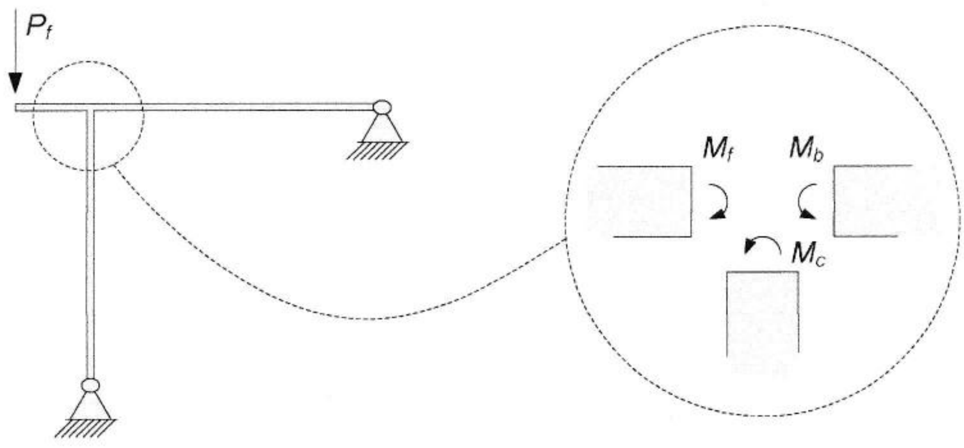
Modulus of elasticity		E_c from Clause 8.6.2
Moment of inertia: (beam weaker than column)		
	Beams	$0.35 I_g$
	Columns	$0.70 I_g$
	Walls — uncracked	$0.70 I_g$
	Walls — cracked	$0.35 I_g$
	Flat plates and flat slabs	$0.25 I_g$
Area		A_g

Note: If Q > 0.2, a more rigid structure might be needed.

too flimsy

Moment Distribution from Beams to Columns

- End restraints cause unbalanced moments at the column ends. Each joint in the frame is subjected to moments and forces, i.e.:



"Moment redistribution": $M_c = \left(\dfrac{K_c}{K_c + K_b}\right) \cdot M_f$ where:

M_f = unbalanced moment at the joint

K_b, K_c = flexural stiffnesses of beam, column, respectively

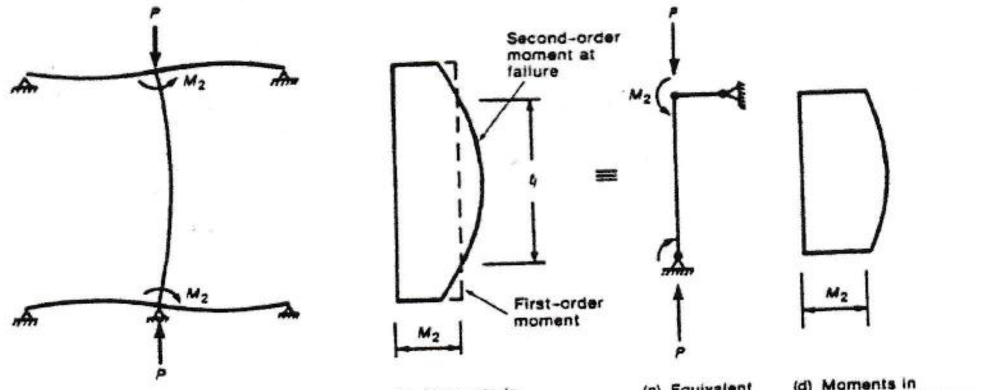
Higher K_c means Higher M_c

Note: This is the case for both short and slender columns.

- In reinforced concrete frames, the stiffnesses of the beams and columns depend on the amount of cracking under a given load.
- As the beam cracks (reducing K_b), the column attracts more load.
- More load in the column means more lateral deflection, more cracking → reduced K_c – so the beam takes more load, etc., until equilibrium and compatibility are achieved.

Moment Magnifier for Columns in Non-Sway Frames (braced)

Approximation of the end restraints:

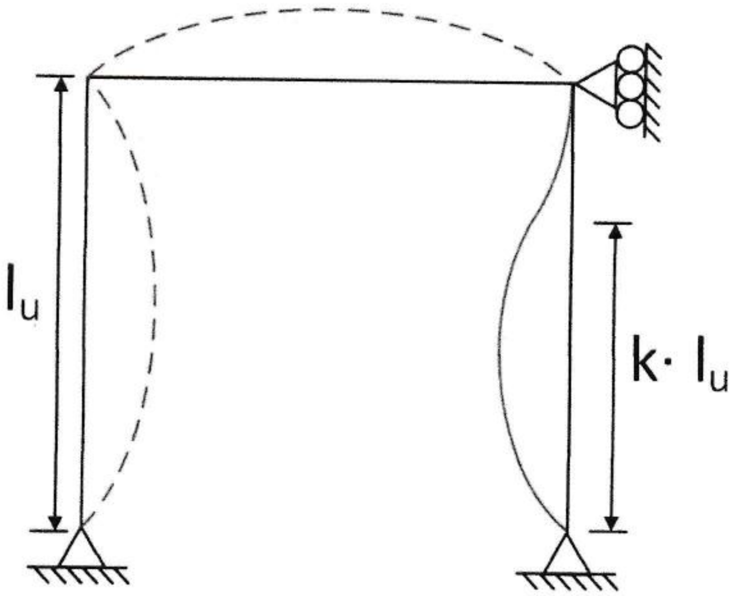


(a) Restrained column. (b) Moments in restrained column. (c) Equivalent hinged column. (d) Moments in equivalent column.

The idea is as follows:

- From the first order analysis we get the moment M_2.
- At failure, the end moments in the column tend to decrease while the midspan moment increases.
- In design, the restrained column is replaced by an equivalent hinged column of length ℓ_i.
- In the code, the length ℓ_i is based on the elastic buckling effective length and taken as being equal to $k \cdot \ell$.
- The effective length = length of an equivalent pin-ended column having the same buckling load.
- When a pin-ended column buckles, it assumes a half-sine shape.
  - Thus, the effective length of a partially-restrained column is taken as the length of one half-sine wave in the deflected shape.

Effective lengths of columns in frames:

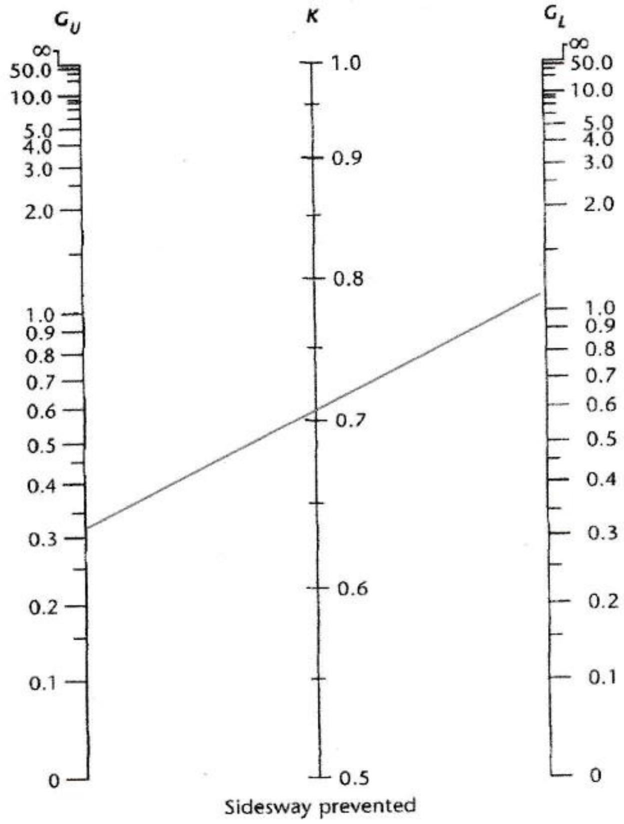


Comments: ·

- Truly fixed ends do not exist.
- Beams and footings always allow some rotations.
- Therefore, effective length will always be greater than for fully fixed ends (for fully fixed $k = 0.5$).
- k is a function of ψ (psi), where:
- $\psi = \dfrac{\sum(E_c \cdot I_c/\ell_c)}{\sum(E_b \cdot I_b/\ell_b)}$ for all of the members framing into a joint

  ← real k
- $c \rightarrow$ column, $b \rightarrow$ beam
- $\psi = 0 \rightarrow$ fully fixed, $\psi = \infty \rightarrow$ fully hinged

- Nomograph for columns in braced (non-sway) frames:



Calculation of ψ:

- Values of $E_b \cdot I_b$ and $E_c \cdot I_c$ should be realistic for the state of loading immediately prior to failure, i.e.: assuming the beam is extensively cracked and the columns are uncracked.
- However, at the stage of design when ψ is calculated, information on the degree of cracking is not known.
- Therefore, we can take *[CSA A23.3 §10.14.1.2]*:

  $E_c \cdot I_c = 0.7 \cdot I_g \cdot E_c$

  $E_b \cdot I_b = 0.35 \cdot I_g \cdot E_b$

Main reference: *[CAC Handbook Chapter 8]*
Additional reading: *[M&B Chapter 12]*

Design of Slender Columns in Non-Sway Frames

Do the following:

1. **Calculate the factored loads:**
   - P_f, M_f – top
   - P_f, M_f – bottom
2. **Choose a trial section:**
   - $A_g\left(trial\right) \geq \dfrac{P_f}{0.35 \cdot f'_c + 0.65 \cdot f_y \cdot \rho_t}$ is recommended, where:

     ρ_t = longitudinal reinforcement ratio ($0.01 \leq \rho_t \leq 0.08$)
3. **Sway or non-sway?**
   - this can often be determined by inspection
   - otherwise, calculate stability index, Q:

$$Q = \frac{\sum P_f \cdot \Delta_0}{V_f \cdot \ell_c} \leq 0.05$$

where: $\sum P_f$ = Σ all factored vertical loads on all columns and walls on storey

Δ_0 = 1st order storey relative deflection due to V_f

V_f = storey shear

l_c = centre-to-centre storey height

note: $Q \leq 0.05 \rightarrow$ non-sway (otherwise, assume sway frame)

note: If $Q > 0.2$, a more rigid structure might be needed

4. <u>Is column slender</u>?

– yes, if : $\dfrac{k \cdot \ell_u}{r} > \dfrac{25 - 10 \cdot (M_1/M_2)}{\sqrt{P_f/(f'_c \cdot A_g)}}$

where: $\dfrac{k \cdot \ell_u}{r}$ is the "slenderness ratio"

ℓ_u = unsupported length

r = radius of gyration = $\sqrt{\dfrac{I}{A}}$

$k \cdot \ell_u$ = effective length

M_1 = absolute minimum end moment

M_2 = absolute maximum end moment

– at this stage, estimate k using *[CAC Handbook Table 8.5]*

5. <u>Compute *E·I, k, P*$_c$</u>:

– for determining k: $I_c = 0.70 \cdot I_g$ ← columns

$I_b = 0.35 \cdot I_g$ ← beams

– determine $\psi = \dfrac{\sum(E_c \cdot I_c/\ell_c)}{\sum(E_b \cdot I_b/\ell_b)}$ for top and bottom of column

– $\ell_{b,c}$ = centre-to-centre beam and column lengths

– $E_{b,c} = 4500 \cdot \sqrt{f'_c}$

– k is obtained from the non-sway monograph ($0.5 \le k \le 1.0$)

– for determining P_c:

$$E \cdot I = \frac{0.2 \cdot E_c \cdot I_g + E_s \cdot I_{se}}{1+\beta_d} \quad \text{or} \quad E \cdot I = \left(\frac{0.4}{1+\beta_d}\right) \cdot E_c \cdot I_g$$

$$\beta_d = \frac{\text{factored dead vertical load}}{\text{total factored vertical load}}$$

$$P_c = \frac{\pi^2 \cdot E \cdot I}{(k \cdot \ell_u)^2}$$

– Note: If $P_c < P_f$, then the column is unstable

→ increase the x-section dimensions.

6. <u>Compute the magnified moment, M_c</u>:

– $M_c = \dfrac{C_m \cdot M_2}{1 - \dfrac{P_f}{\phi_m \cdot P_c}} \geq M_2$

where: $\phi_m = 0.75$

$$C_m = 0.6 + 0.4 \cdot \left(\frac{M_1}{M_2}\right) \geq 0.4$$

– $M_2 \geq P_f(15 + 0.03 \cdot h)$

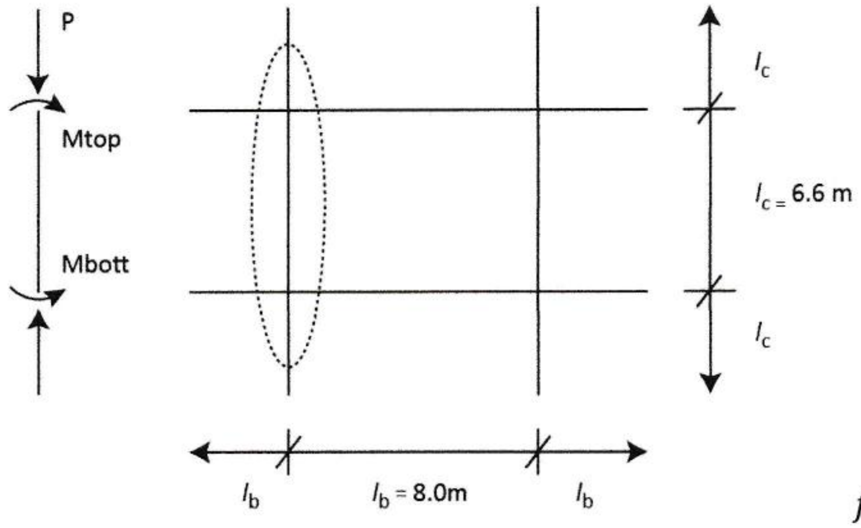
7. <u>Design column for P_f, M_c</u>:

– to use interaction diagrams, calculate $\dfrac{p_f}{A_g}$ and $\dfrac{M_c}{A_g \cdot h} \rightarrow \rho_t$

– if $\rho_t > 0.04$ required, consider increasing column size

– $\rho_{t,min} = 0.01$, $\rho_{t,max} = 0.08$ (≤ 0.04 recommended)

Example 7: *Design the column in a non sway frame*



$$f_c' = 40\ MPa$$
$$f_y = 400\ MPa$$

Columns are 400 x 400 mm

- Beams are 300 x 600 mm
- P_f = 2000 kN (factored) (P_d = 800 kN -unfactored)
- M_f = 200 kNm (top), 200 kNm (bott)

Step 1 Calculate load effects
Step 2 Choose trial section — **already done**
Step 3 Sway or non-sway?

Step 4 Is column slender?

$k \cong 0.9 \quad (Table\ 8.5)$ *Can be approximated as 1*

$$l_u = 6.0m \qquad (6.6 - 0.6)$$
$$r \cong 0.3 \cdot 400 = 120mm$$

$$\rightarrow k\frac{l_u}{r} = 45$$

$$> \frac{25 - 10 \cdot 200 \,/\, 200}{\sqrt{2000 \ \cdot 10^3 \,/\, (40 \ \cdot 400^2)}} = 26.83 < 45 \rightarrow slender\ column$$

Step 5 Compute $EI \ \cdot k \ \cdot P_c$:

1. <u>For determining k: *(more exactly)*</u>

$$I_c = 0.7 \ \cdot \ I_{g,c} = 0.7 \ \cdot \frac{400^4}{12}$$

$$I_b = 0.35 \ \cdot \ I_{g,b} = 0.35 \ \cdot 300 \ \cdot \ \frac{600^3}{12}$$

$$E_c = 4500 \ \cdot \ \sqrt{40} = 28460\ MPa$$

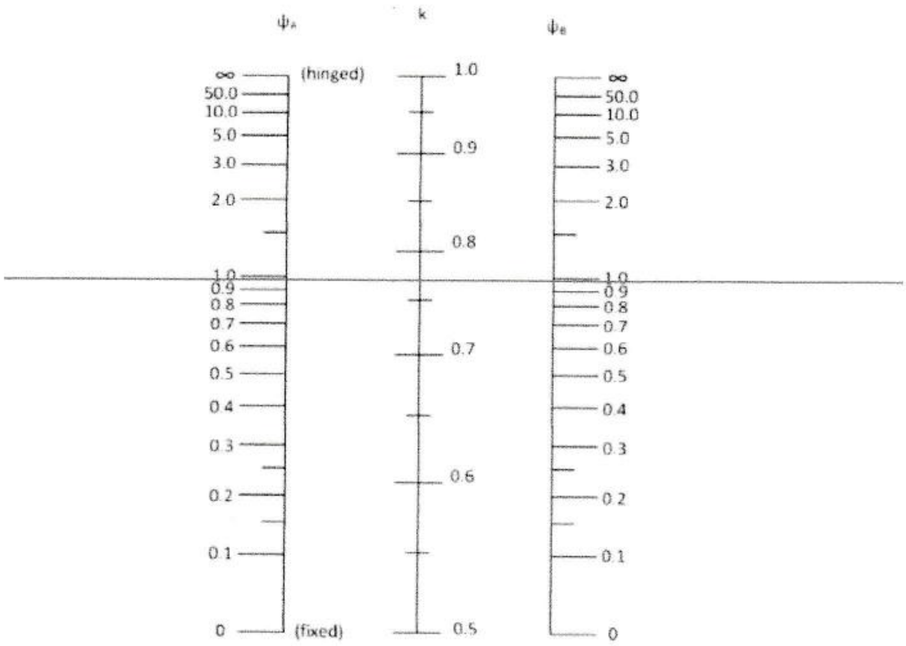
$$\rightarrow \ E_c I_c = 4.25 \ \cdot \ 10^{13}\ MPa \cdot mm^4$$

$$E_b I_b = 5.38 \ \cdot \ 10^{13}\ MPa \cdot mm^4$$

$$\psi = \frac{2 \cdot 4.25 \cdot 10^{13} \,/\, 6600}{2 \cdot 5.38 \cdot 10^{13} \,/\, 8000}$$

= 0.96 (same, top & bottom)

$$\rightarrow k = 0.78$$



2. <u>For determining P_c</u> :

$$I = \frac{0.4}{1+\ \beta_d}\ I_g$$

$$\beta_d = \frac{1.25\ \cdot 800}{2000} = 0.5$$

$$E = 28460\ MPa$$

$$\rightarrow E \cdot I = 1.62\ \cdot\ 10^{13}\ MPa\ \cdot mm^4$$

$$P_c = \frac{\pi^2 EI}{(k\ \cdot l_u)^2}$$

$$= \frac{\pi^2\ \cdot 1.62\ \cdot 10^{13}}{(0.78\ \cdot 6000)^2}$$

$$= 7296\ KN\ (> P_f\ \checkmark)$$

Step 6: Compute magnified moment

$$M_c = \frac{C_m\ \cdot M_2}{1 - P_f\ /\ (\phi_m \cdot\ P_c)}$$

$$C_m = 0.6 + 0.4\left(\frac{200}{200}\right) = 1.0$$

$$(\geq 0.4 \checkmark)$$

$$\phi_m = 0.75$$

$$\rightarrow M_c = \frac{200}{1 - \left(\frac{2000}{0.75 \cdot 7296}\right)} = 315.2\ KNm$$

$$> M_2 \checkmark$$

$M_2 > 200(15 + 0.03 \cdot 400) = 54\ KNm$ (unintended eccentricities)

Step 7 Design for P_f, M_c

$$\frac{P_f}{A_g} = 12.5$$

$$\frac{M_c}{A_g \cdot h} = 4.93$$

$$try\ 35M\ bar \rightarrow \gamma = \frac{400 - 2 \cdot 40 + 2 \cdot 11.2 - 35.7}{400}$$

$= 0.65$ (estimate as 0.6)

$$\rightarrow \rho_t = 0.04$$

$$As\ (req'd) = 6400\ mm^2$$

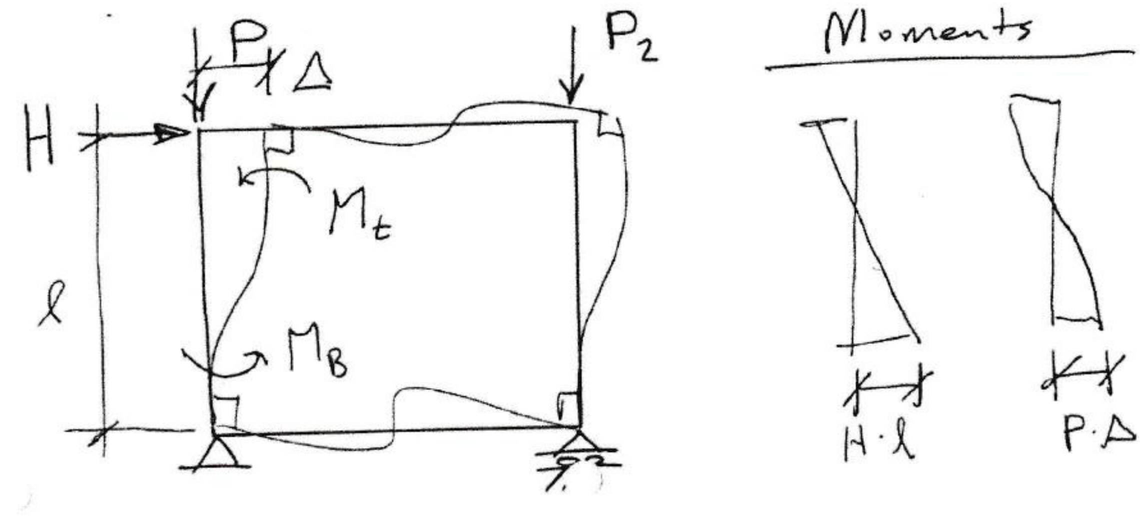
$$8 - 35M \rightarrow As = 8000\ mm^2$$

Try 30 M bars and estimate γ

COLUMNS IN SWAY (UNBRACED) FRAMES

BEHAVIOUR OF COLUMNS IN SWAY FRAMES

- Unbraced frames depend on moments in columns to resist lateral loads and deflections. In general, it is recommended that braced frames be used whenever possible, since the effectiveness of unbraced frames can be highly reduced by creep and rotation of the supports at the foundation.
- In unbraced frames, $\sum(M_t + M_b) = H \cdot \ell + \sum P \cdot \Delta$ where M_t and M_b are the top and bottom moments, respectively:



- This expression assumes that Δ is approximately the same for all columns in the frame.
- For the first order ($H \cdot \ell$) moment distribution shown above, the maximum moment considering second order effects, M_c, can be calculated as:

$$M_c = M_0 \cdot \left(\frac{1 - 0.18 \cdot P/P_e}{1 - P/P_e} \right)$$

This expression can be derived using the moment-area method.

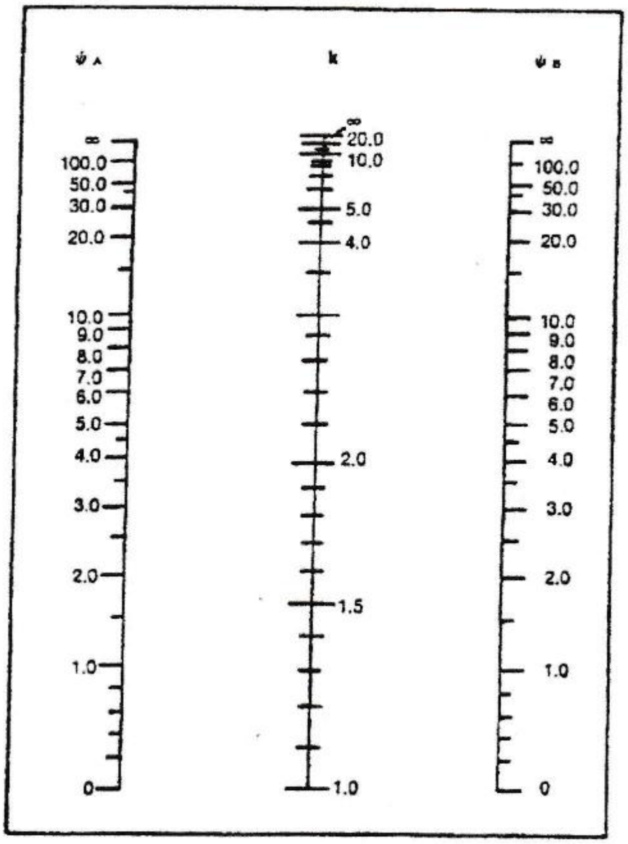
In sway frames, the column moments can be divided into components due to loads causing sway (M_s) and those due to loads not causing sway (M_{ns}). The corresponding deflections are typically much greater in the case of the sway deflections. Since the deflections due to non-sway moments generally increase the moment near the midspan of the column, and not at the column ends, normally only the second order effects due to the sway moments are considered in the design of columns in sway frames.

A moment magnifier for sway frame columns, δ_s, is applied to moments causing sway only, i.e.:

$$M = M_{ns} + \delta_s \cdot M_s \text{ where: } \delta_s = \frac{1}{1 - \dfrac{\sum P_f}{\phi_m \cdot \sum P_c}}$$

In the design of sway frames with "lean on" (pin-ended) columns, consider the axial loads in these columns in the calculation of $\sum P_f$ and assume that their contribution to $\sum P_c$ is zero.

Nomograph for columns in unbraced (sway) frames:



Design of Slender Columns in Sway Frames

1. Calculate the factored loads. Consider the various possible load combinations (LCs).
2. Calculate the corresponding load effects. Separate them into non-sway (*ns*) and sway (*s*). P and M
3. Choose a trial section (same formula as for non-sway).
4. Sway or non-sway?
5. Compute $E \cdot I$, k, P_c:
   - same as for non-sway, except:

- k obtained from sway nomograph ($1.0 \le k \le \infty$)
- $\beta_d = \dfrac{\text{factored sustained shear}}{\text{total factored shear}}$ (like Earth pressure)
- need to compute for each column in the storey

6. Calculate δ_s:

– $\delta_s = \dfrac{1}{1 - \dfrac{\sum P_f}{\phi_m \cdot \sum P_c}}$ where:

$\sum P_c$ = Σ Euler buckling loads for all columns on the storey

– $\delta_s \le 2.5$ for gravity load only LCs to preclude sway buckling under these LCs *[CSA A23.3 2004 §10.16.5]*

7. Check if maximum moment occurs at one of column ends:

– to do this, verify that: $\dfrac{\ell_u}{r} \le \dfrac{35}{\sqrt{P_f/(f'_c \cdot A_g)}}$ (Ensure max moment is due to sway effect, not non-sway effect)

8. Calculate M_1, M_2 for each LC:

– $M_1 = M_{1,ns} + \delta_s \cdot M_{1,s}$

– $M_2 = M_{2,ns} + \delta_s \cdot M_{2,s}$

– For gravity load only LCs: $M_{1,2} = \text{MIN}\{M_{1,2}, P_f(15 + 0.03 \cdot h)\}$

9. Design column for P_f, M_2 for each LC:

– note: may be able to reduce the number of LCs by inspection

– interaction diagrams used in the same way as before

Note: If column or beam size changes in the final design, it may necessary to do a second iteration with revised stiffness values.

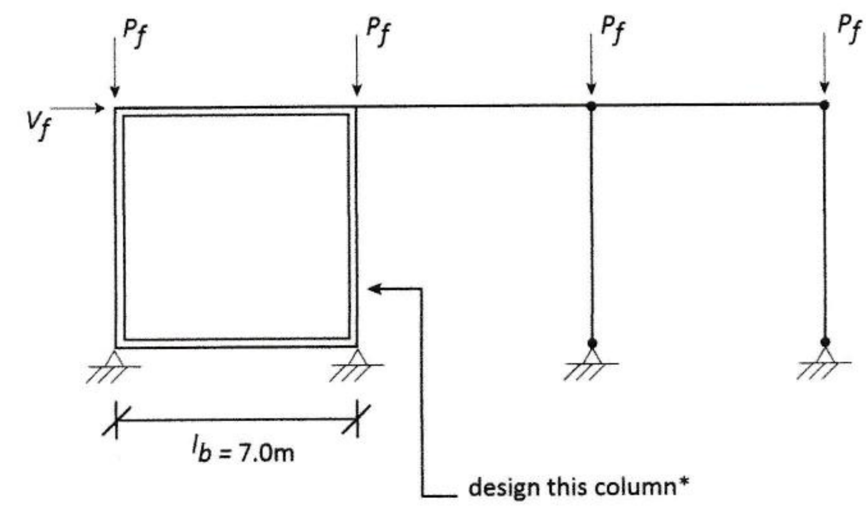
Note: If verification in Step 7 fails:

- increase column size, or
- design column for P_f, M_c for each load case, where:

  $M_c = (C_m \cdot M_2)/(1 - P_f/(\phi_m \cdot P_c)) \geq M_2$

- assume: β_d is based on axial forces and $k \leq 1.0$ ($k = 1.0$ can be conservatively used).

Example 8: Design the column in a sway frame



$$l_c = 6.6m, l_u = 6.0m$$

$$f'_c = 35\ MPa, f_y = 400\ MPa$$

For the column to be designed:

Load case	M top kNm	M bottom kNm	P kN
D	-60	60	210
L	-40	40	140
W	-41	41	12

From structural analysis of frame

* design column for LC#2 only (1.25D + 1.5L + 0.4W)

1 & 2. Calculate loads, load effects:

$P_f = 1.25 \cdot 210 + 1.5 \cdot 140 + 0.4 \cdot 12$=**477.3 KN**

$$M_{f,top}\,(sway) = 0.4\,(-41) = -16.4\ kNm$$

$$M_{f,top}\,(non-sway) = 1.25\,(-60) + 1.5\,(-40)$$

$$= -135\ kNm$$

$$M_{\ f,bott}(sway) = 0.4\,(41) = 16.4\ kNm$$

$$Mf_{f,bott}\,(non-sway) = 125\,(60) + 1.5\,(40)$$

$$= 135\ kNm$$

3. Choose trial section:

$$A_{g,trial} \geq \frac{P_f}{0.35\,f'_c + 0.65 \cdot f_y \cdot \rho_t}$$

$$\geq \frac{477.3 \cdot 1000}{0.35 \cdot 35 + 0.65 \cdot 400 \cdot 0.015} \geq 29554\ mm^2$$

$$0.015\ was\ assumed$$

$$\rightarrow h \geq 171.9mm$$

$$\rightarrow try\ 400 \cdot 400mm\ column$$

4. Sway or non-sway?

$$\therefore Q, \Delta_0\ not\ provided, assume\ sway\ frame\ (by\ inspection)\ *$$

*since column is in sway frame, design procedure is the same whether it is short or slender

5. Compute EI, k, P_c:

$$EI = \frac{0.4 \cdot E_c \cdot I_g}{1 + \beta_d}$$

$$E_c = 4500\sqrt{f'_c} = 4500\sqrt{35} = 26622\ \text{MPa}$$

$$I_g = \frac{bh^3}{12} = \frac{400^4}{12} = 2.13 \cdot 10^9\ mm^4$$

$$\beta_d = 0\ (no\ sustained\ shear)$$

$$\rightarrow\ EI = 0.4\ \cdot 26622 \cdot 2.13\ \cdot 10^9 = 2.27 \cdot 10^{13}\ Nmm^2$$

Calculation of k:

$$I_C = 0.7 \cdot I_g \text{ (columns)}$$

$$I_b = 0.35 \cdot I_g \text{ (beams)}$$

$$E_c I_c = 26622\ \cdot\ 0.7 \cdot 400^4\ /12 = 4.0 \cdot 10^{13}\ Nmm^2$$

$$E_b I_b = 26622\ \cdot\ 0.35 \cdot 400 \cdot 600^3\ /12 = 6.7 \cdot 10^{13}\ Nmm^2$$

$$\psi_{top} = \psi_{bott} = \frac{4.0 \cdot 10^{13}\ /\ 6600}{6.7 \cdot 10^{13}\ /\ 7000} = \mathbf{0.63}$$

From sway monograph: $k \simeq 1.2$

From this....

$$\delta_s = 1.42$$

6. Calculate M_1, M_2:

$$M_f, top = \ -135 + 1.42\ (-16.4) = \ -158.2\ kNm$$
$$M_f, bott = \ 135 \pm 1.42\ (16.4) = 158.2\ kNm$$
$$M_1 = M_2 = 158.2\ kNm$$

7. Design column:

$$\frac{P_f}{A_g} = 477.3 \cdot 1000/400^2 = 2.98$$
$$\frac{M_f}{(A_g \cdot h)} = \ 158.2\ \cdot 10^6/400^3 = 2.47$$

Assume: 25M longitudinal bars

10M ties

40mm cover

$$\rightarrow\ \gamma = \frac{400 - 2(25/2 + 10 + 40)}{400} = 0.69 \cong 0.7$$

$$\rightarrow \rho_t \cong 0.012\ (Table\ 7.4.10)$$

$$A_s = 1920mm^2$$

$$\rightarrow use\ 8 - 20M\ bars \quad \rightarrow A_s = 2400\ mm^2$$