Simplified

$V_c = \phi_c \lambda \beta \sqrt{f'_c}\, b_w d_v$

- $\lambda = 1$ (normal), 0.85 (semi-low), 0.75 (low)
- $\sqrt{f'_c} \le 8$ MPa
- $d_v = \max[0.9d, 0.72h]$

$V_s = [\phi_s A_v f_y d_v \cot\theta] \cdot 1/s$

- $A_v = A_b \cdot$ # of legs in stirrup

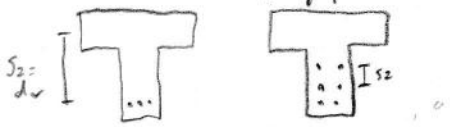
$V_{s\,req'd} = V_f - V_c = \phi_s A_v f_y \frac{d_v \cot\theta}{s}$

- $d_v \cot\theta / s$ is # stirrups crossing crack
- $\therefore s_{req'd} = \frac{\phi_s A_v f_y d_v \cot\theta}{V_f - V_c}$

- If special member (i.e. below), $\beta = 0.21$, $\theta = 42$
  - Slab w/ $t \ge 350$
  - Beam w/ $t \ge 250$
  - Concrete joist
  - Beam cast integrally w/ slab and depth of beam below slab < [1/2 web width or 350]
- If at least the min transverse reinforcement, $\beta = 0.18$, $\theta = 35$
- If no transverse reinforcement & CA > 20mm, $\beta = \frac{230}{1000 + d_v}$, $\theta = 35$
- If no transverse reinforcement & all CA, $\beta = \frac{200}{1000 + s_{ze}}$, $\theta = 35$

↳ $s_{ze} = \frac{35 s_z}{15 + CA_{max}} > 0.85 s_z$ $= 300$ if $A > A_{v\,min}$,

↳ s_z is crack spacing parameter



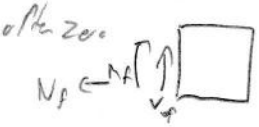
Note:

- $V_{max} = 0.25 \phi_c f'_c b_w d_v$ (crushing)
- $A_{v\,min} = \frac{0.06\sqrt{f'_c}\, b_w s}{f_y}$ (strength maintained after crack) } Either could define max spacing
- $s_{max} = \min[600, 0.7 d_v]$ ($V_f < 0.125 \lambda \phi_c f'_c b_w d_v$)
- $s_{max} = \min[300, 0.35 d_v]$ ($V_f >$ "")
- Take shear and moment w/ clear span
  - $V_{f\,end} = [w_f l_n]/2$, $V_{f\,int} = 1.1 l_n/8$
  - M at support dependent on attached graphs

General

$\beta = \left[\frac{0.4}{1+1500\varepsilon_x}\right] \cdot \left[\frac{1300}{1000 + s_{ze}}\right]$

- $\varepsilon_x = \frac{\frac{M_f}{d_v} + V_f + 0.5 N_f}{2 E_s A_s}$ ← often zero



- $\theta = 29 + 7000\varepsilon_x$

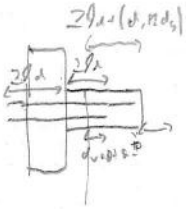
Procedure

1. Shear force and BMD
2. Check if stirrups req'd (and check $V_{f\,max}$)
3. Check max spacing (Av not req'd)
4. Determine s to resist V_f
5. Check at several locations
6. Final check

X	X_1	X_2	X_3
V_f			
M_f			
A_s			
$s_{req'd}$			

Reinforcing Detailing Procedures

1. Design flexural reinforcement for max moments
2. Select # of bars to cutoff
   - Not more than 50% of A_s @ one location
   - Positive moment steel to support,
   - Negative moment steel to POI
3. Calculate M_r for cutoff locations
4. Find theoretical cutoff points (flexure diag)
   - Where $M_f = M_r$ for continuing steel
   - POI
5. Determine actual cut-off locations
   - All bars must extend past theoretical point by $d_v \cot\theta$ (12.10.3),
   - Supports (12.11.1)
     - → If simply supported, 1/3 of $A_{s\,total}$ must continue into support
     - → If continuous, 1/4 of $A_{s\,total}$ must continue into support
     - → Bars must extend @ least 150 mm into support, $l_{embed} = $ column width − end cover
     - → Longitudinal reinforcement must be capable of resisting T_f due to shear (11.3.9.5)
       - $T_f = (V_f - V_s/2)\cot\theta$
         - ↳ $V_f = wsL/2$, $V_s = $ stirrups
       - $l_{embed,req'd} = \frac{T_f}{\phi_s A_s f_y} l_d$
     - → For negative moment (12.12.1), must be anchored at least l_d



   - Points of inflection (12.12.2)
     - → At least 1/3 of A_{steel} must be embedded beyond POI by max[d, 12d_b, l_n/16]
6. Additional requirements
   - General anchorage: all bars must extend l_d past M_{max} (12.1.1)
   - Continuing reinforcement must pass theoretical cutoff point by max[l_d + d, l_d + 12d_b]
     - → Length of continuing bars = L_{span} + b_{col} − 2 cover
     - → Extension of A bars past B = 1/2 L_A − x,
   - For positive moment bars, develop as follows at zero-moment locations:
     - → $1.3 M_r/V_f + l_a \geq l_d$
     - → l_a = 1/2 cut cover
   - Lap splice
     - → Min lap splice is 300 mm
     - → 35M + cannot be spliced
     - → Class A splice: 1.0 l_d
     - → Class B splice: 1.3 l_d
     - → Class A if area of reinforcement provided is at least 2x req'd by analysis and splice location, and less than 1/2 of total reinforcement is spliced within required lap length

$$l_d = \frac{1.15\, k_1 k_2 k_3 k_4}{d_{cs} + K_{tr}} \cdot \frac{f_y}{\sqrt{f'_c}} A_b \quad \text{(Development Length Tension)}$$

- $d_{cs} + K_{tr} \leq 2.5 d_b$
- $K_{tr} = \frac{A_{tr} f_{yt}}{10.5 s n}$ (Stirrup Strength Factor)
  - → A_{tr} = area of stirrups (transverse reinforcement)
  - → f_{yt} = yield strength stirrup
  - → n = # bars developed
  - → s = stirrup spacing
  - → d_{cs} = min[min cover → center of developed bar, 2/3 c/c bar spacing]
- Note: $l_d \geq 300$

If clear cover ≥ d_b, clear spacing ≥ 1.4 d_b

$l_d = 0.45 k_1 k_2 k_3 k_4 \frac{f_y}{\sqrt{f'_c}} d_b$ (Min transverse reinforcement, slab or wall w/ clear spacing ≥ 2 d_b)

$l_d = 0.6 k_1 k_2 k_3 k_4 \frac{f_y}{\sqrt{f'_c}} d_b$ (Other)

- k_1 = 1.3 (top bar), 1.0 (else) (Bar location factor)
- k_2 = 1.0 (not coated), 1.2 (Bar coating factor)
  - → 1.5 if epoxy coated w/ clear cover < 3d_b or clear spacing < 6d_b
  - → 1.8 for other epoxy
- k_3 = 1 (normal), 1.2 (semi-low), 1.3 (low) (Density factor)
- k_4 = 0.8 (No. 20 and lower), 1 (25+) (Bar size factor)
- $k_1 k_2 \leq 1.7$

$l_{db} = 0.24 d_b f_y/\sqrt{f'_c} \geq 0.044 d_b f_y$ (Basic Development Length (compression))

l_d = Mod factor × l_{db} ≥ 200 mm (Development length (compression))

↳ Mod factor = 0.75 (longitudinal bar in spiral column)

If bundled bars, ↑ l_d of single bar by 1.1 (2 bars), 1.2 (3 bars), 1.33 (4 bars)

If hooked bar in tension, ↓ development length. Basic $l_{hb} = \frac{100 d_b}{\sqrt{f'_c}}$ (f_y = 400 MPa)

↳ $l_{dh} = l_{hb}$ × Mod factor ≥ max[8d_b, 150] (Development length Tension Hook)

f_y 1000, f_y 400

ACI

- All Negative moment terminated @ 0.3L or L,
- 1/2 Positive moment terminated @ 0.125L

Columns

To determine forces and moment from strain diagram:

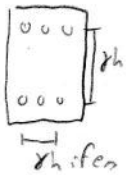
1. Find α and β
2. Solve for a
3. Using similar triangles,
   - $C_c = \alpha \phi_c f'_c \cdot a \cdot b$ (concrete compression force)
   - $F_s = \phi_s f_s A_s$ (steel tension force)
   - $F_s = (\phi_s f_s - \alpha \phi_c f'_c) A_s$ (steel compression force)
4. Axial $= \Sigma F$
   Moment $= \Sigma F$(distance from side)

To solve column load if given e (interaction)

$P_r e = M_r$, $e = M_r/P_r$, $1/e = P_r/M_r$

Slope of line $= [P_r/A_g] / [M_r/A_g h] = P_r h / M_r = h/e$

1. Draw straight line w/ slope h/e from origin for interaction diagram for proper ρ and γ
   - $\gamma = \gamma h / h$, γh = distance between bars
   - $\rho_g = A_s / A_g$



To solve column if given P (interaction)

1. Set $P_r = P_f$, calculate P_r/A_g
2. Draw horizontal line at P_r/A_g to find the moment $M_r/A_g h$
3. Find $e = M_r/P_r$

If given D, L, and Wind

1. Check different LCs
2. Calculate P_f/A_g and $M_f/A_g h$ for each
3. Verify each w/ interaction diagram

Short column - pure compression

1. Check if short. If short:

$$\frac{k l_u}{r} \le \frac{25 - 10\, M_1/M_2}{\sqrt{P_f/(f'_c A_g)}}, \quad M_1/M_2 \ge -1/2$$

- M_1 = absolute min end moment
  M_2 = absolute max end moment

M_1/M_2 + single curvature, − double

- $r = \sqrt{I/A}$, ~0.3h (rectangular column)
  0.25d (circular column)

l_u = unsupported length

- $k = 1$ conservative
- has to be laterally supported at both ends to be considered short

Short column - pure compression cont.

2. $P_o = \alpha_1 f'_c (A_g - A_{st}) + f_y A_{st}$ (if not factored, remove factors)
3. Account for eccentricity
   - $P_n = 0.8 P_o$ (Tied)
   - $P_n = 0.85 P_o$ (Spiral)

Detailing requirements

- $0.01 A_g \le A_{st} \le 0.08 A_g$ (recommended upper bound $= 0.04 A_g$)
  ↳ If $0.005 A_g < A_{st} < 0.01 A_g$, $P_r = \frac{P_r}{2}\left[1 + \frac{\rho_t}{0.01}\right]$
- 4 bars req'd (rect), 6 req'd (spiral), 3 req'd (triangular)
- Tie spacing = min[16·d smallest longitudinal bar, 48 d tie, smallest x-section dim, 300 mm] (for bundled bars)
- Each corner & alternating longitudinal bar is restrained by a tie
- Max longitudinal spacing = 150 (across), 500 mm (end to end)
- Max angle of tie corner = 135° (7.6.4)

Biaxial bending

$$\frac{1}{P_r} = \frac{1}{P_{rx}} + \frac{1}{P_{ry}} - \frac{1}{P_{ro}}, \quad \text{(check if } P_r \ge P_f\text{)}$$

- P_{rx} = load applied @ e_x, $e_y = 0$
- P_{ry} = load applied @ e_y, $e_x = 0$
- P_{ro}, $e_y = e_x = 0$

$$P_c = \frac{\pi^2 E I}{(k l)^2}$$

Hinged (+), Elastic ($\psi = 3$), Elastic ($\psi = 1.6$), Fixed (T → Fixed)

k			
0.71	0.91	0.95	1
0.77	0.86	0.9	
0.74	0.83		
0.67			
F	E	E	H

(Table 8.5)

Moment Magnification Factor, $\delta_b = C_m / \left[1 - \frac{P}{\phi_m P_c}\right]$ (Braced)

- $\phi_m = 0.75$
- $C_m = 0.6 + 0.4\, M_1/M_2 \ge 0.4$
  ↳ $M_2 \ge P_f(15 + 0.03h)$
  ↳ $C_m = 1$ if transverse loads present between column ends

Sustained Loads

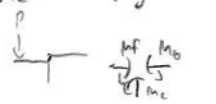
$$EI = [0.2 E_c I_g + E_s I_{se}]/(1 + \beta_d) = EI_{eff} \text{ or } \frac{0.4}{1+\beta_d} E_c I_g$$

- $\beta_d = \frac{\text{factored DL moment}}{\text{total factored moment}}$ (Nonsway (Braced))
- $\beta_d = \frac{\text{factored sustained shear}}{\text{total factored shear}}$

Brace frame vs. Sway frame.

- If $Q = \frac{\Sigma P_f \Delta_o}{V_f l_c} \le 0.05$, braced
  - ΣP_f = all vertical loads
  - Δ_o = deflection of story relative to bottom due to shear
  - V_f = shear in that story due to factored lateral loads
  - l_c = story height
  - If $Q > 0.2$, use more rigid structure

$M_c = \frac{K_c}{K_c + K_b} M_f$



$\psi = \frac{\Sigma E_c I_c / l_c}{\Sigma E_b I_b / l_b}$ → 0 (fully fixed), → ∞ (fully hinged)

- $E_c I_c = 0.7 I_g E_c$
- $E_b I_b = 0.35 I_g E_b$

Design Slender Column (Non-sway)

1. Find factored loads
   - P_f, M_f top and bottom
2. choose trial section
   $A_{g\,trial} \ge \frac{P_f}{0.35 f'_c + 0.65 f_y \rho_t}$
   - ρ_t between 0.01 → 0.08, choose 0.03 or <u>0.04</u>
3. Check if sway or not
4. Check if slender (use k from table)
5. Find ψ top and bottom
6. Find k from nomogram nomograph
7. Find P_c
8. Find moment-magnified M_c
9. Design column for P_f and M_c

Design sway frame

$M = M_{ns} + \delta_s M_s$

- $\delta_s = \frac{1}{1 - \frac{\Sigma P_f}{\phi_m \Sigma P_c}}$
  - ΣP_c = Euler buckling loads for all columns on the story
  - $\delta_s \le 2.5$ for gravity loads only

1. Repeat 5 steps
2. Find δ_s
3. Check that max moment occurs at one end
   $\frac{l_u}{r} \le \frac{35}{\sqrt{P_f / f'_c \cdot A_g}}$
4. Calculate M_1 and M_2 for each LC ($\min[M_{1,2}, P_f(15 + 0.03h)]$)
5. Design column for P_f, M_2 for each LC

Slender Column Design

column dim

1. $C_m = 0.6 + 0.4 M_1/M_2 \ge 0.4$, $M_2 \ge P_f(15 + 0.03h)$
   = 1 if transverse loads present
2. $\beta = \frac{DL_f}{DL_f + LL_f}$
3. $E_c = 4500\sqrt{f'_c}$
4. $I_g = \pi r^4 / 4$
5. $EI = \frac{0.4}{1 + \beta_d} E_c I_g$
6. $P_c = \pi^2 EI / (kl)^2$
7. $\delta_b = \frac{C_m}{1 - P_f / \phi_m P_c}$
8. $M_c = \delta_b M_2$

Flexure design - Normal Beam

1. Check if doubly-reinforced req'd
   - ↳ $d = h - 60$ (one layer), $d' = 60$
   - ↳ $d = h - 90$ (2 layer), $d' = 90$

   $K_r = \frac{M_r}{bd^2} \times 10^6$

2. If singly reinforced, use K_r to find ρ

   $A_s = \rho b d$

   $\rho_b = \alpha_1 \beta_1 \frac{\phi_c}{\phi_s}\left[\frac{700}{700+f_y}\right] f'_c/f_y$, $c_b = \frac{700}{700+f_y}$

   $M_r = A_s \phi_s f_y (d - a/2) = K_r b d^2$, $A_s = \frac{M_r}{\phi_s f_y jd}$, $jd = 0.9d$

   $K_r = \phi_s \rho f_y \left[1 - \frac{\phi_s \rho f_y}{2\phi_c \alpha_1 f'_c}\right]$

3. If doubly reinforced, determine tensile steel

   $\rho = 0.6\rho_b$, find K_r

   $M_r = K_r b d^2$

   $A_s = \rho b d$

4. Find compression steel to resist remaining moment

   $M_r = M_r - M_{r_1}$

   $A_{s_2} = \frac{M_{r_2}}{\phi_s f_y (d - d')}$

   $A'_{s_{req}} = A_{s_2}$, $A_{s_{req}} = A_s + A_{s_2}$

5. Choose reinforcement
6. Choose cover, check spacing

   $S_{max} = \min\begin{bmatrix} 1.4d \\ 1.4\,CA \\ 30 \end{bmatrix}$, assume CA = 19

7. Find actual d and d'
8. Check yielding - assuming yielding, we have:

   $a = \frac{\phi_s (A_s - A'_s) f_y}{\phi_c \alpha_1 f'_c b}$

   $\rho < 0.8\rho_b + \rho'$, $\rho' = \frac{A'_s}{bd}$ (tension), or use c/d check!

   $\varepsilon'_s = (1 - d'/c)\varepsilon_{cu}$ (compression)

9. If both yield, as analysis
10. Check $A_{s_{min}} = 0.2\sqrt{f'_c}\, bh / f_y$

Design - T beam

Positive

1. $d = h - 90$ (2 layer)
2. $A_s = M_f / 0.85 f_y jd$, $jd = 0.9d$
3. Find spacing
4. Find α and β and new d
5. Calculate a
6. Find moment r at max, and for cutoff steel
7. Checked yielding c/d
8. Check if $A_{s_{min}} = \frac{0.2\sqrt{f'_c}}{f_y} b_w h$

Negative

1. Determine d (1 layer)
2. Calculate b_f; $jd = 0.8d$
3. Find a
4. Find moments
5. Check yielding
6. Check $A_{s_{min}} = \frac{0.2\sqrt{f'_c}}{f_y} b_w h$, $b_f \le \begin{Bmatrix} b_f \\ 2.5 b_w \end{Bmatrix}$

   $A_{flange_{min}} = 0.004 b' h_f$, $b' \le \begin{Bmatrix} l_b/2 \\ 10.3.3 \end{Bmatrix}$

9.3.3 for approximate analysis, 10.5.1, 10.5.2, and 10.5.3 satisfied

Slabs

1. Find slab depth w/ 9.8.2.1
2. 9.3.3 for moment
3. Choose cover (Annex A)
4. Using 7.8, 10.5.1, and 10.5.2 satisfied

ARUP	Job Title	
www.arup.com	Job #	
	Reference	Sheet #
	Made by	Date

T-beam

$b_f = b'_{T_L} + b'_{T_R} + b_w$ (T section)

$b_f = b'_L + b_w$ (L section)

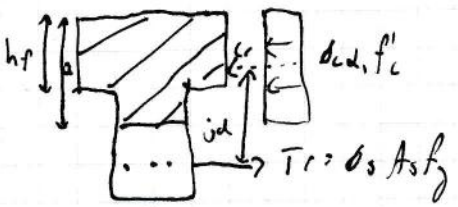
$b'_T = \min[0.2L_b \text{ (simple) or } 0.1L_b \text{ (continuous)}, 12h_f, 0.5L_s]$

$b'_L = \min[L/12, 6h_f, 0.5L_s]$

If positive moment, compression zone into web

$$A_{sf} = \frac{\phi_c \alpha_1 f'_c (b-b_w) h_f}{\phi_s f_y}$$

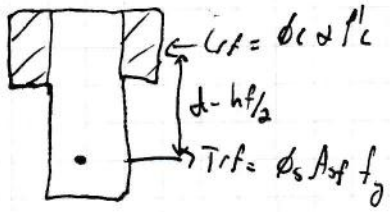
$$M_{cf} = \phi_c \alpha_1 f'_c (b-b_w) h_f (d - h_f/2) = \phi_s A_{sf} f_y (d - h_f/2)$$

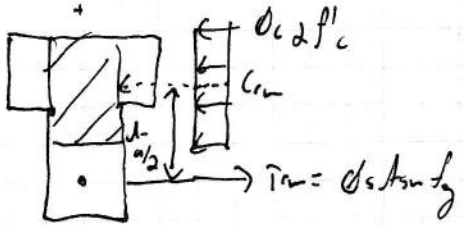


$A_{sw} = A_s - A_{sf}$

$$M_{rw} = \phi_c \alpha_1 f'_c b_w a (d - a/2) = \phi_s A_{sw} f_y (d - a/2)$$

$$\hookrightarrow a = \frac{\phi_s A_{sw} f_y}{\phi_c \alpha_1 f'_c b_w}$$





$C_{cf} = \alpha_1 \phi_c f'_c (b_f - b_w) h_f$
$C_{cw} = \alpha_1 \phi_c f'_c b_w a$
$T_r = \phi_s A_s f_y$

$$M_r = C_{cw}(d - a/2) + C_{cf}(d - h_f/2) + \underline{C'_s (d - d')}$$

↑ if doubly reinforced

$$A_{s_{min}} = \frac{0.2\sqrt{f'_c}}{f_y} b_w h$$

$\gamma = 24\ kN/m^3$

T-beam Analysis (Positive Moment)

1. Determine effective flange width (use clear spans)

$b'_T = \min[0.2L_b\ (\text{Simple}),\ 0.1L_b\ (\text{Continuous});\ 12h_f;\ 0.5L_s]$

$b'_L = \min[L_b/12,\ 6h_f,\ 0.5L_s]$

- L_s: span length, L_b: beam length
- $b_T \le 2.5b_w$ (2 flanges) $\le 1.5b_w$ (1 flange)

2. Calculate effective depths
3. Calculate a - Check if $a > h_f$
4. Check if tension steel has yielded
5. Check that reinforcement exceeds minimum requirement
6. Check spacing s_{min}: $[1.4d_b, 1.4d_A, 30mm]$
7. Calculate M_r

i) If $a < h_f$, $M_r = \phi_s A_s f_y (d - a/2)$

ii) If $a > h_f$, split beam into F and W

Beam F (flange resistance) - $C_{rf} = \phi_c \alpha_1 f'_c (b - b_w) h_f$, $C_{rf} = \phi_s A_{sf} f_y$, $M_{rf} = C_{rf}(d - h_f/2)$

Beam W (web resistance) - $A_{sw} = A_s - A_{sf}$, $a = \dfrac{\phi_s A_{sw} f_y}{\phi_c \alpha_1 f'_c b_w}$, $C_{rf} = \phi_s A_{sw} f_y$, $M_{rw} = C_{rf}(d - a/2)$

$M_r = M_{rf} + M_{rw}$

Note:

- Negative beams, treat as rectangle section
  - overhang width: $\min[L_s/20$, effective flange width$]$
- $T\ A_{s,min} = 0.004\ A$ of overhang (T-beam flange area)
- $A_{s,min} = \dfrac{0.2\sqrt{f'_c}\, b_w h}{f_y}$ (Beam), $0.002 A_g$ (Slab)
- $a = \dfrac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b}$ - Alt, force equilibrium

$C'_r = \phi_s A'_s f_y$
$C_c = \alpha_1 \phi_c f'_c b\, a$
$T_r = \phi_s A_s f_y$

Assume yielding; if compression not yield

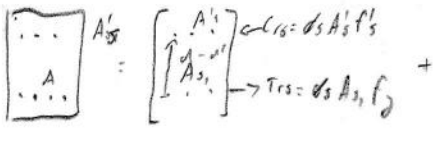
$\rightarrow (\phi_c \alpha_1 f'_c b)a^2 + \phi_s(0.0035E_s A'_s - A_s f_y)a - (0.0035\phi_s E_s A'_s \beta d') = 0$

- $\alpha_1 = 0.85 - 0.0015 f'_c \ge 0.67$, $\beta = 0.97 - 0.0025 f'_c \ge 0.67$
- $c/d < 0.8 \dfrac{700}{700 + f_y}$, $c = a/\beta$
- If assembled occupancy w/ LL < 4.8 kPa, LLRF = 1
- If assembled occupancy w/ LL > 4.8, or storage, manufacturing, retail, footbridge, or garage w/ $T_u > 280 m^2$,

$LLRF = 0.5 + \sqrt{\dfrac{20}{A}}$

- All else, $LLRF = 0.3 + \sqrt{\dfrac{9.3}{A}}$ w/ $T_u > 20$,

Doubly Reinforced



$a = \dfrac{\phi_s A_{s2} f_y}{\phi_c \alpha_1 f'_c b}$

$M_r = \phi_s A'_s f_y (d - d') + \phi_s A_{s2} f_y (d - a/2)$

assuming compressive steel yields.

Compressive $\rho = \dfrac{A_s}{bd} \le \rho_b + \rho'$, $\rho' = \dfrac{A'_s}{bd}$, $\varepsilon'_s = \varepsilon_{cu}(1 - d'/c) > \varepsilon_y$

Tensile $c/d < 0.8 \dfrac{700}{700 + f_y}$

If compressive steel does yield, use $C_{rs} = \phi_s (E_s \varepsilon'_s) A'_s$

10	11.3	100
15	16	200
20	19.5	300
25	25.2	500
30	29.9	700
35	35.7	1000
45	43.7	1500
55	56.4	2500

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Singly Reinforced

$M_r = A_s \phi_s f_y (d - a/2) = \alpha_1 \phi_c f'_c a b (d - a/2)$

- $a = \dfrac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b}$

Also, $M_r = K_r b d^2$

- $K_r = \phi_s \rho f_y \left[1 - \dfrac{\phi_s \rho f_y}{2 \phi_c \alpha_1 f'_c} \right]$

$\rho < 0.8 \rho_b$

- $\rho_b = \dfrac{A_{sb}}{bd} = \alpha_1 \beta_1 \dfrac{\phi_c}{\phi_s} \left[\dfrac{700}{700 + f_y} \right] \dfrac{f'_c}{f_y}$
- $\rho = \dfrac{A_s}{bd}$

$\dfrac{c}{d} < 0.8 \dfrac{c_b}{d}$

- $\dfrac{c_b}{d} = \dfrac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} = \dfrac{0.0035}{0.0035 + \underbrace{f_y}_{E_s = 200000}} = \dfrac{700}{700 + f_y}$
- $c = \dfrac{a}{\beta}$

$\alpha = 0.85 - 0.0015 f'_c \geq 0.67$

$\beta = 0.97 - 0.0025 f'_c \geq 0.67$

Analysis

1. Find α, β, a, c, d, and c_b/d
2. Check if underreinforced
3. Plug into formula

Design - Beam

Potential, 9.8.2.1 for h, 1/2 for b

1. Check if doubly-reinforced req'd

$\rightarrow d = h - 60$ (One layer), $d' = 60$

$d = h - 90$ (Two layers), $d' = 90$

$K_r = \frac{M_r}{bd^2} \times 10^6$

2. If singly reinforced, use K_r to find ρ

$A_s = \rho b d$

$\rho_b = \alpha_1 \beta_1 \frac{\phi_c}{\phi_s} \left[\frac{700}{700+f_y}\right] \frac{f'_c}{f_y}$

$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b}$, $c = a/\beta$

$\alpha = 0.85 - 0.0015 f'_c \geq 0.67$

$\beta = 0.97 - 0.0025 f'_c \geq 0.67$

$c/d < 0.8 \frac{700}{700+f_y}$

$M_r = A_s \phi_s f_y (d - a/2)$

$= K_r b d^2$, $K_r = \phi_s \rho f_y \left[1 - \frac{\phi_s \rho f_y}{2\phi_c \alpha f'_c}\right]$

3. If doubly reinforced, determine tensile steel.

$\rho = 0.6\rho_b$, find K_r

$M_r = K_r b d^2$

$A_s = \rho b d$

4. Find compression steel to resist remaining moment

$M_r = M_f - M_{r_1}$

$A_{s_2} = \frac{M_{r_2}}{\phi_s f_y (d - d')}$

$\therefore A'_{s_{req}} = A_{s_2}$, $A_{s_{req}} = A_{s_1} + A_{s_2}$

5. Determine reinforcement

6. Choose cover and check spacing

$S_{min} = \max \begin{Bmatrix} 1.4d \\ 1.4(A) \\ 30 \end{Bmatrix}$

7. Find actual d and d'

8. Check yielding

- Assuming yielding, we have:

$a = \frac{\phi_s (A_s - A'_s) f_y}{\phi_c \alpha_1 f'_c b}$

$\rho < 0.75\rho_b + \rho'$, $\rho' = \frac{A'_s}{bd}$ (Tension), or c/d check

$\varepsilon'_s = (1 - d'/c)\varepsilon_{cu}$ (Compression)

9. If both yield:

$M_{r_1} = \phi_s A'_s f_y (d - d')$

$M_{r_2} = \phi_s (A_s - A'_s) f_y (d - a/2)$

If compression doesn't yield:

$(\phi_c \alpha f'_c b) a^2 + \phi_s (0.0035 E_s A'_s - A_s f_y) a - (0.0035 \phi_s E_s A'_s \beta d') = 0$

- $E_s = 200000$

$C_{rs} = \phi_s (E_s \varepsilon'_s) A'_s$

$C_{rc} = \phi_c \alpha f'_c b a$

$M_r = C_{rs}(d - d') + C_{rc}(d - a/2)$

10. Check $A_{s_{min}} = \frac{0.2\sqrt{f'_c}}{f_y} bh$

Design - T beam

1. Assume $d = h - 90$ (2 layers)

2. Determine flange width

$b'_T = \min[0.2 L_0 \text{ (Simple)}, 0.1 L_0 \text{ (Continuous)}, 12h_f, 0.5 L_s]$

3. Check if T-beam section analysis req'd by subbing $h_f = a$

$M_r = \alpha_1 \phi_c f'_c a h_f (d - h_f/2)$ If not, treat as rectangular beam

4. Find contribution of flange

$$A_{sw} = \frac{\phi_c \alpha_1 f'_c (b - b_w) h_f}{\phi_s f_y}$$

$$M_{rf} = \phi_c \alpha_1 f'_c (b - b_w) h_f (d - h_f/2) = \phi_s A_{sf} f_y (d - h_f/2)$$

5. Check if doubly-reinforced req'd in web

$M_{rw} = M_f - M_{rf}$

$K_{rw} = \frac{M_{rw}}{b_w d^2}$, check if exceeds max

6. Determine tension steel web area

$A_s = \rho b d$

or

$a = \frac{\phi_s A_{sw} f_y}{\phi_c \alpha f'_c b_w}$, $M_{rw} = \phi_s A_{sw} f_y (d - a/2)$

7. Select steel bars

8. Choose concrete cover and check spacing

9. Find a through force equilibrium

$C_{cf} = \alpha_1 \phi_c f'_c (b_f - b_w) h_f$

$C_{cw} = \alpha_1 \phi_c f'_c b_w a$

$T_r = \phi_s A_s f_y$

10. Check $c/d < 0.8 \frac{700}{700 + f_y}$

11. Find $M_r = C_{cw}(d - a/2) + C_{cf}(d - h_f/2)$

12. Check $A_{s,min} = \frac{0.2\sqrt{f'_c}}{f_y} b_w h$

13. NEGATIVE MOMENT

Find $K_r = \frac{M_f}{bd^2}$ assuming $d = d_{original}$,

check if $\rho < 0.8\rho_b$

14. If doubly-reinforce not req'd,

$A_s = \rho b d$

15. Determine bar size and quantity

Recheck ρ

16. Check w/ minimum values

$A_{s,flange\,min} = 0.004 b' h_f$

$b' \le \begin{cases} b_w/20 \\ \text{10.3.3 requirement} \end{cases}$

$A_{s,min} = \frac{0.2\sqrt{f'_c} b_w h}{f_y}$

$b_w \le \begin{cases} b_f \\ 2.5 b_w \end{cases}$

17. If $\rho > 0.8\rho_b$ double reinforce.

Find d' and M_{r2}

$M_{r2} = \phi_s A_{s2} f_y (d - d')$

18. Find A_s' M_{r1}

$M_{r1} = M_f - M_{r2}$

$K_r = \frac{M_{r2}}{bd^2}$, ρ from table

$A_s = \rho b d$

19. Check min reinforcement

20. check given $\rho < 0.4\rho + \rho'$ (Tension)

$a = \frac{\phi_s A_s f_y - \phi_s A'_s f'_c}{\alpha_1 \phi_c f'_c b_w}$, $c = a/\beta$

$\varepsilon'_s = (1 - d'/c)\varepsilon_{cu}$

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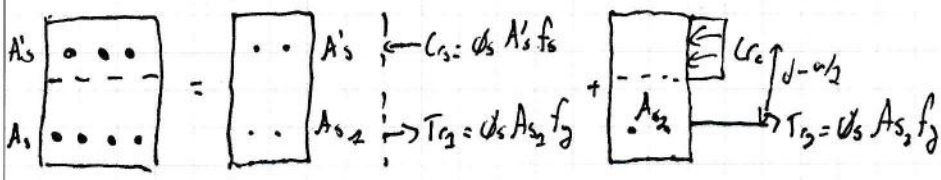
Doubly Reinforced



Assuming compression steel yields

$$a = \frac{\phi_s (A_s - A'_s) f_y}{\phi_c \alpha_1 f'_c b}$$

$$M_{r1} = \phi_s A'_s f_y (d - d')$$

$$M_{r2} = \phi_s (A_s - A'_s) f_y (d - a/2)$$

$$\rho < 0.8\rho_b + \rho', \quad \rho' = \frac{A'_s}{bd} \text{ (Ductile generally)}$$

$$\varepsilon'_s = \left(1 - \frac{d'}{c}\right)\varepsilon_{cu} \quad \text{(Compression steel yielding)}$$

If compression steel doesn't yield

$$(\phi_c \alpha_1 f'_c b)\, a^2 + \phi_s (0.0035 E_s A'_s - A_s f_y) a - (0.0035 \phi_s E_s A'_s \beta_1 d') = 0$$

- if compression steel doesn't yield, OK as long as tension steel yields
- $T_r = C_{rc} + C_{rs}$ ← new
  - ↳ $C_{rs} = \phi_s f'_s A'_s = \phi_s (E_s \varepsilon'_s) A'_s$
  - ↳ $C_{rc} = \phi_c \alpha_1 f'_c b a$

$$M_r = C_{rs}(d - d') + C_{rc}(d - a/2)$$

In concrete handbook, assuming compression reinforcement yields,

$$M_r = M_{r1} + M_{r2} = (K_r + K'_r) b d^2$$

- $K'_r = \rho' \phi_s f_y \left[1 - \frac{d'}{d}\right]$

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Shear

1. Determine factored shear-force envelope
2. Check if cross-section is large enough

$V_{r_{max}} = 0.25 \phi_c f'_c b_w d_v$

- d_v = effective shear depth = max$[0.9d, 0.72h]$

3. Check if stirrups req'd

- $V_f \geq V_c$
- $V_c = \phi_c \lambda \beta \sqrt{f'_c} b_w d_v$
- $h > 750mm$

4. Determine spacing requirements at several points

$|V_s|$ req'd $= V_f - V_c = \dfrac{\phi_s A_v f_y d_v \cot\theta}{s}$

$V_s \rightarrow$

$s_{req'd} = \dfrac{\phi_s A_v f_y d_v \cot\theta}{V_f - V_c}$

A_v = double that of single leg

5. Find maximum stirrup spacing

$A_{v_{min}} = \dfrac{0.06 \sqrt{f'_c} b_w s}{f_y}$, $s_{max} = \dfrac{A_v f_y}{0.06 \sqrt{f'_c} b_w}$

$s_{max} = \min[600, 0.7d_v]$

$\min[300, 0.35 d_v]$ if $V_f > 0.125 \lambda \phi_c f'_c b_w d_v$

For one-way slab design

1. Use 9.8.2.1 to find slab depth based on clear span
2. Estimate effective depth
   - ~~Slab spacing~~ Slabs = 10M bars
   - Cover = A23.3 Annex A (230)
   - $c \geq$ {Table 17 requirements} (232)
3. Determine loading
   - $1.4w_D$
   - $1.25w_D + 1.5w_L$
4. Determine moment at required location (9.3.3) (57)
5. Determine required A_s
6. 1. $K_{r_{req'd}} = \frac{M_f}{bd^2}$
   2. Linearly interpolate to find ρ
   3. $A_s = \rho bd$
   4. Check $\rho < 0.8\rho_b$, $\rho_b = \alpha_1 \beta_1 \frac{\phi_c}{\phi_s}\left(\frac{700}{700+f_y}\right)\frac{f'_c}{f_y}$ ← might be unnecessary
6. Determine spacing requirement
   $s = \frac{100mm^2}{A_{s_{req'd}}} \times 1000$ ← mm slab width
7. Check $A_{s_{min}}$ and s_{max} (7.8) (41)

   - $A_{s_{min}} = 0.002A_g$, $A_g = bh$
   - $s_{max} = \min[3h, 500]$ (slabs)

8. Temperature and shrinkage

   - $A = A_{s_{min}}$
   - $s_{max} = \min[5h, 500]$
   - $s = \frac{100}{A_{s_{min}}} \cdot 1000$

9. Check adequacy (10.5.1 & 10.5.2), (

   - ~~$A_s = 4.2M_{ac}$~~, $A_s = \frac{100}{s}1000$, $\rho = \frac{A_s}{bd}$
   - $K_r = \rho\phi_s f_y\left(1 - \frac{\rho\phi_s f_y}{2\alpha\phi_c f'_c}\right)$, $M_r = K_r bd^2 > M_f$, OK

$$\cdot\ a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b}$$

$$\cdot\ \frac{c}{d} = \frac{a/\beta}{d} \le 0.8\,\frac{700}{700+f_y}$$

For beam cross-section (T-beam)

1. 9.8.2.1 to find beam depth using clear span
2. Assume beam width = $h/3 \to h$. $h/2$ good
3. $d = h - \text{cover} - \text{stirrup dia} - \phi/2$.
   - ↳ if d_b unknown..
     - One layer of steel, $d = h - 60$
     - 2 layers of steel, $d = h - 90$
   - ↳ Check cover requirements
4. Determine loading
   - Check if LLRF

$T_u > 80m^2$ ↳ $0.5 + \sqrt{\frac{20}{A}}$ if assembly occupancy w/ LL > 4.8 or storage, manufacturing, retail, garage, or footbridge

$20m^2$ $0.3 + \sqrt{\frac{7.8}{B}}$, non-assembly occupancy w/ any LL

All 1 Assembly occupancy w/ LL < 4.8 kPa

   - Check both load cases
5. Determine moment (9.5.3) (57)
6. Find effective flange width [L or T-beam]

   ↳ $b'_T = \min\left[0.2L_{b\,clear}\ (\text{simple}) \text{ or } 0.1L_{b\,clear}\ (\text{continuous}),\ 12h_f,\ 0.5L_s\right]$ (clear span)

   $b'_L = \min\left[L_{b\,clear}/12,\ 6h_f,\ 0.5L_s\right]$

7. Assuming a is within flange, such that $M_r = \alpha\phi_c f'_c h_f b_f (d - h_f/2)$, and letting $M_f = M_r$,

   check $K_r = \frac{M_f}{b_f d^2}$

   Using K_r, find ρ and check that $\rho < 0.8\rho_b$

   Find $A_{req'd} = \rho b_f d$. Determine number of bars
8. Now that b_d known, confirm bar diameter

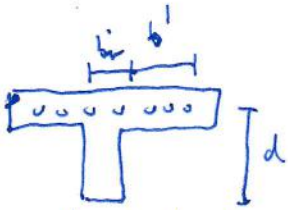
9. Determine spacing

$$s = \frac{b - 2c - 2d_{st} - nd_b}{n-1}, \quad s \geq \begin{Bmatrix} 1.4 d_b \\ 1.4 CA \\ 30 \end{Bmatrix}$$

Also ensure min steel area $A_{s_{min}} = \frac{0.2\sqrt{f'_c}}{f_y} bh$ (10.6.1.2)

10. Check adequacy

Find real d
Find real a, check if within flange
Check c/d for yielding
Check $M_r = \phi_b A_s f_y (d - a/2) > M_f$



11. NEGATIVE MOMENT

Assume d = d, find $k_r = \frac{|M_f|}{bd^2}$ and check if $\rho < 0.8\rho_b$.
↳ If not, either Δ cross-section dims or analyze as doubly reinforced

If not doubly reinforced, find As knowing $A_s = \rho bd$. ~~Then find~~ and determine ~~check if~~ bar size and quantity. Then recheck ρ

Check w/ minimum values, $b \geq \begin{Bmatrix} L/20 \\ 0.1L \end{Bmatrix}$, $A_{s_{min}} = 0.004 b' h \rho$

10.5.3.1 $\quad A_{s_{min}} = \frac{0.2\sqrt{f'_c}\, b_t h}{f_y}$

$b_t \leq \begin{Bmatrix} b_f \\ 2 \cdot b_w \end{Bmatrix}$ 10.5.3.2

$b_t \geq \begin{cases} 1.5 b_w & (L) \\ 2.5 b_w & (T) \end{cases}$

If $\rho \geq 0.8\rho_b$, double reinforced

Deflection

Either
1. Check min t for slab and beam (9.8.2.1)
2. Compute deflections of the structure and compare to permissible limits $\Delta \le \Delta_{lim}$ (9.8.2.6)

$$\Delta = \frac{5wL^4}{384EI}$$

$\Delta_{LL} = \Delta_{DL+LL} - \Delta_{DL}$

↳ To simplify, use EI_e

$$E_c = (3300\sqrt{f'_c} + 6900)\left(\frac{\gamma_c}{2300}\right)^{1.5} \quad (1500 \le \gamma_c \le 2500)$$

$$= 4500\sqrt{f'_c} \quad (\text{Normal}, 20 \le f'_c \le 40)$$

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3 \le I_g$$

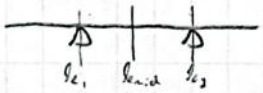
- M_a = max moment @ load level
- I_{cr} = cracked section MoI
- I_g = gross I (neglect A_s)

$$M_{cr} = \frac{f_r I_g}{y_t}, \quad f_r = \frac{0.6\lambda\sqrt{f'_c}}{2} \text{ for deflections only}$$

• y_t = distance from centroid to extreme tensile fibre

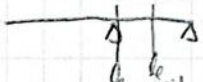
Continuous - use average I_e

• 2 ends continuous,



$$I_{e_{avg}} = 0.7 I_{e_{mid}} + 0.15(I_{e_1} + I_{e_2})$$

• 1 end continuous



$$I_{e_{avg}} = 0.85 I_{e_{mid}} + 0.15 I_{e_{cont}}$$

Cracked Mat

Single Reinforced

$$bkd\left(\frac{kd}{2}\right) = nA_s(d - kd), \quad \rho = \frac{A_s}{bd}$$

$$\frac{k^2}{2} + \rho n k - \rho n = 0 \rightarrow k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$I_{cr} = b(kd)^3/3 + nA_s(d - kd)^2, \quad n = E_s/E_c$$

Doubly Reinforced

$$\frac{b(kd)^2}{2} + (n-1)A'_s(kd - d') = nA_s(d - kd)$$

$$\frac{b(kd)^2}{2} + [(n-1)A'_s + nA_s]kd - [(n-1)A'_s d' + nA_s d] = 0$$

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2 + (n-1)A'_s(kd - d')^2$$

Procedure

1. Determine I_{cr}
2. Find f_r and M_{cr} and $\bar{y}$
3. Find M_{cr}
4. Check if DL (unfactored) moment > M_{cr}. If so, use I_e
5. Find dead load deflection
6. Check if $M_{DL+LL} > M_{cr}$; if so, use I_e
7. Find Δ_{DL+LL}
8. $\Delta_L = \Delta_{DL+LL} - \Delta_D$
9. Check deflection limit in 9.8.2.6
10. If deflection limits not met
    a) ↑ section (b, h) to ↑ I_g, I_{cr}, M_{cr}
    b) ↑ A_s or A'_s to ↑ I_{cr}, ↓ ξ_s

Sustained load

Immediate deflection Δ_i
Long-term deflection Δ_t, $\xi_s = \frac{\Delta_i + \Delta_t}{\Delta_i} = \left(1 + \frac{s}{1+50\rho'}\right)$

$\Delta_t = \frac{s}{1+50\rho'}\Delta_i$, $\rho' = \frac{A'_s}{bd}$ @ midspan for simple and continuous spans and at support for cantilevers

time	s
3 month	1
6 month	1.2
12 month	1.4
5 years	2

$$z = f_s(d_c A)^{1/3}$$

- ≤ 30000 N/mm for interior exposure
- ≤ 25000 N/mm for external exposure

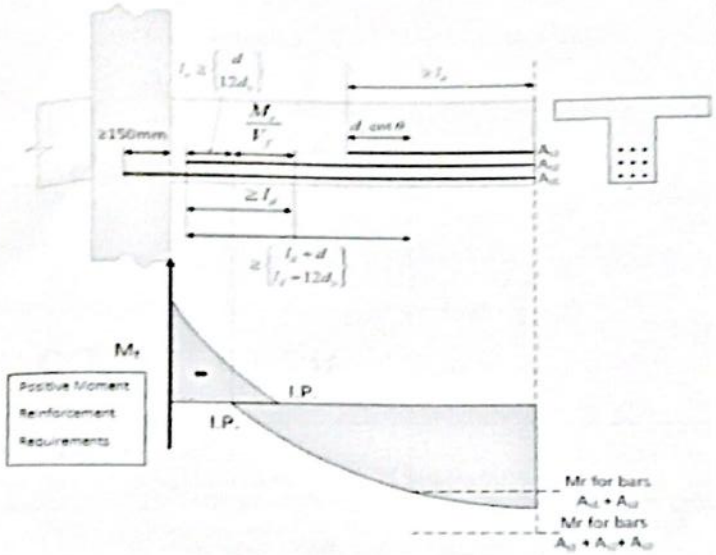
f_s = tensile stress in reinforce under service load (0.6 f_y)
d_c = distance from extreme layer to tension surface of concrete
A = effective tension area of concrete supporting the main tension reinforcement, divided by # of bars

bhd, $\left(\frac{h-d}{2}\right)^2$

T-beam

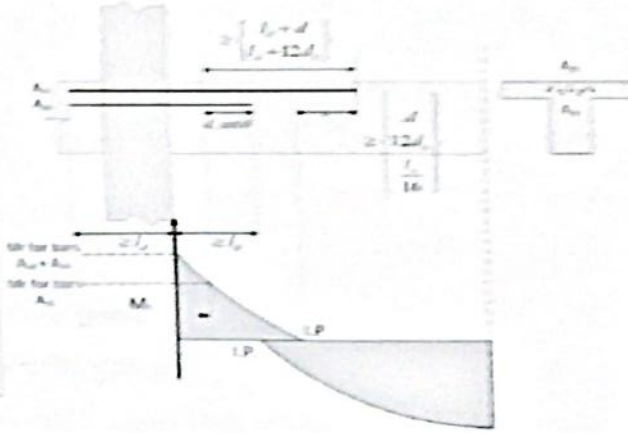
Assume kd is outside flange

Skin reinforcement



NEGATIVE MOMENT REINFORCEMENT REQUIREMENTS

Code Requirement	Reason
Clause 12.12.1 Anchorage Into Supporting Members Reinforcement must be anchored in or through the supporting member by a length of at least ℓ_d	*Ensures that f_y can be developed in bars at location of maximum moment (face of support)*
Clause 12.12.2 Inflection Points At least 1/3 of $A_{s\,total}$ must have an embedment length beyond the point of inflection which is larger of $\begin{cases} d \\ 12d_b \\ \ell_n/16 \end{cases}$	*Provides contingency for unexpected loads, assumptions in analysis, etc. that could shift moment diagram*



-VE MOMENT

11.3.9.5 At support, longitudinal reinforcement must be capable of resisting a tensile force, T_c due to shear:

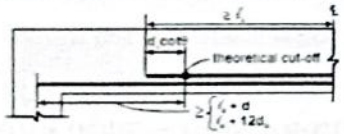
where $T_c = (V_f - 0.5V_c)\cot\theta$

Required embedment into support to develop T_c

$$(\ell_{embed})_{req'd} = \frac{T_c}{\phi_s A_{s\,total} f_y} = \ell_d$$

+VE MOMENT

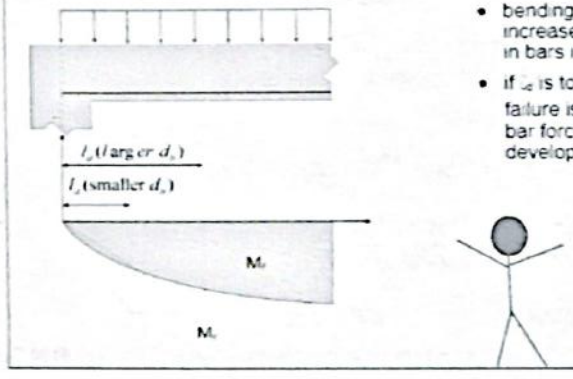
Code Requirement	Reason
Clause 12.1.1 General Anchorage All bars must have an embedment length beyond the point of maximum moment of at least ℓ_d	*Ensures that f_y can be developed in bars at location of maximum moment*
Clause 12.10.3 Actual Cut-off Point All bars must extend beyond the point where they are theoretically no longer required for flexure by a distance of *(also see Clause 11.3.9)* $d_v \cot\theta$ where d_v = effective shear depth = 0.9d or 0.72h, whichever is greater θ = see Cl. 11.3.6	*Accounts for additional tension force in bars due to shear* *Also provides contingency for unexpected loads, assumptions in analysis, etc. that could shift moment diagram*
Clause 12.10.4 Continuing Reinforcement Continuing reinforcement must have an embedment length beyond the point where the terminated reinforcement is no longer theoretically required in flexure of larger of $\begin{cases} \ell_d - d \\ \ell_d - 12d_b \end{cases}$	*Ensures continuing bars can develop f_y at section where other bars are cut off* *Includes contingency for additional tension due to shear, and for factors that could shift moment diagram*



Code Requirement	Reason
Clause 12.11.1 At Supports • For simply supported beams at least 1/3 of $A_{s\,total}$ must continue into the support • For continuous beams at least 1/4 of $A_{s\,total}$ must continue into the support • Bars must extend at least 150 mm into the support • Also check Cl. 11.3.9.5 (see Ex. 1)	*Provides contingency for factors that could shift moment diagram.* *Also provides minimum continuous reinforcement for structural integrity.*

Clause 12.11.3 Development at Zero Moment Locations

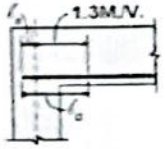
- Protect against bond failure at simple supports and at points of inflection in continuous beams (low (zero) moment and high shear).



- bending moment increases rapidly → force in bars increases rapidly
- if ℓ_d is too large, bond failure is possible since bar force and M_r do not develop rapidly enough

Cl. 12.11.3 - At simple supports

$$1.3\frac{M_r}{V_f} + \ell_a \geq \ell_d$$



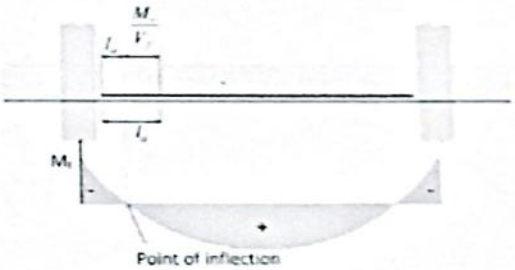
ℓ_a = additional embedment length beyond centre line of support

M_r = moment resistance based on bars continuing into support

V_f = factored shear force at support

Cl. 12.11.3 - At points of inflection

$$\frac{M_r}{V_f} + \ell_a \geq \ell_d$$



ℓ_a = greater of d_v, $12d_b$

M_r = moment resistance based on bars continuing through the inflection

V_f = factored shear force at inflection

If Cl. 12.11.3 inequality is not satisfied, decrease d_b to decrease

DESIGN PROCEDURE – REINFORCEMENT DETAILING

1. Design Flexural Reinforcement for maximum moments
2. Select number of bars to cut-off. Consider:
   - Should not terminate more than 50% of A_s at one location
   - Cl. 12.11.1: Positive moment steel continued to support
   - Cl. 12.12.2: Negative moment steel continued to POI
   - Practical number of cut-offs
3. Calculate M_r for cut-off locations
4. Locate theoretical cut-offs
   - Where $M_r = M_r$ for continuing steel
   - Points of inflection
5. Determine actual cut-off locations:

Positive Moment Bars		Negative Moment Bars	
12.10.3:	Extension of terminated bars past theoretical cut-off	12.10.3:	Extension of terminated bars past theoretical cut-off
12.11.1:	Extension of continuing bars into support	12.12.2:	Extension of continuing bars past POI

6. Check remaining detailing requirements:

Positive Moment Bars		Negative Moment Bars	
12.1.1:	General anchorage	12.1.1:	General anchorage
12.10.4:	Extension of continuing bars past terminated bars	12.10.4:	Extension of continuing bars past terminated bars
12.11.3:	Development at zero moment locations (supports and POI)	12.12.1:	Anchorage into supporting members

COMPRESSION

basic development length: $\ell_{db} = 0.24 d_b \frac{f_y}{\sqrt{f'_c}}$

$\geq 0.044 d_b f_y$

development length in compression:

ℓ_d = modification factor x ℓ_{db}

$\ell_d \geq 200$ mm

modification factor = 0.75 for longitudinal bars in a spiral column

CSA A23.3 *Clause 12.2*

$\ell_d \geq 300$mm minimum development length

$$\ell_d = 1.15 \frac{k_1 k_2 k_3 k_4}{(d_{cs} + K_{tr})} \frac{f_y}{\sqrt{f'_c}} A_b$$ *Clause 12.2.2*

Note that $(d_{cs} + K_{tr}) \leq 2.5 d_b$

Where

k_i = modification factors (defined later)

$$K_{tr} = \frac{A_{tr} f_{yt}}{10.5 s n}$$

A_{tr} = area of stirrups (transverse reinforcement)

f_{yt} = yield strength of stirrups

s = stirrup spacing

n = number of bars developed

d_{cs} = smaller of: minimum cover to centre of bar being developed or 2/3 of centre to centre spacing of bar

K1	1.3	Top bars (C300mm from bot)
K1	1.0	Other
K2	1.5	Epoxy Cov<3db S<6db
K2	1.2	Other Epoxy
K2	1.0	No epoxy
K3	1.3	Low dense
K3	1.2	Semi low
K3	1.0	normal
K4	0.8	20 & Smaller
K4	1.0	25 & Larger

$k_1 k_2 \leq 1.7$

TENSION DEV-LENGTH

Hooks next Pages

DEVELOPMENT LENGTH FOR BUNDLED BARS

CSA A23.3 *Clause 12.4*

- For bundled bars in tension or compression, increase development length of a single bar by:
  - 1.1 for 2 bars
  - 1.2 for 3 bars
  - 1.33 for 4 bars

DEVELOPMENT OF STANDARD HOOKS

CSA A23.3 *Clause 12.5*

➢ Used to reduce the required development length in tension.

$l_{dh} = l_{hb}$ x modification factor(s)

$\geq 8d_b$ or 150 mm, whichever is greater

Basic development length for a hooked bar with f_y = 400 MPa:

$$l_{hb} = \frac{100d_b}{\sqrt{f'_c}}$$

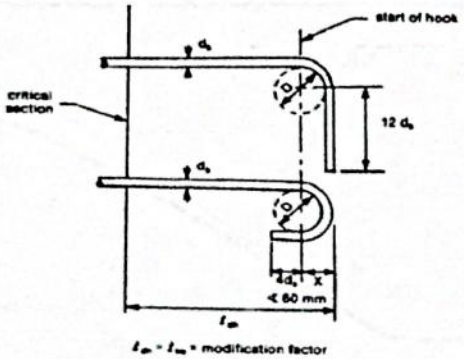


Fig. N12.5.2
Hook Geometry Details

The minimum lap splice is 300mm

Bars 35M and larger cannot be spliced (Clause 12.14.2.1)

Class A lap splice : 1.0 l_d

Class B splice : 1.3 l_d

Class A splices:

a) the area of reinforcement provided is at least twice that required by analysis at the splice location; and

b) less than one-half of the total reinforcement is spliced within the required lap length.

Class B splices – all other cases

Modification factors for hooks: *Clause 12.5.3*

(a)	for bars with f_y other than 400 MPa	$f_y/400$
(b)	for No. 35 bars or smaller, where the side cover (normal to the plane of the hook) is not less than 60 mm and for 90° hooks where the cover of the bar extension beyond the hook is not less than 50 mm	0.7
(c)	for No. 35 bars or smaller, where the hook is enclosed vertically or horizontally within at least three ties or stirrups spaced along a length at least equal to the inside diameter of the hook, at a spacing not greater than $3d_b$	0.8
(d)	where anchorage or development for f_y is not specifically required, for reinforcement in excess of that required by analysis	$\frac{A_s(req'd.)}{A_s(prov.)}$
(e)	for structural low density concrete	1.3
(f)	for epoxy-coated reinforcement	1.2

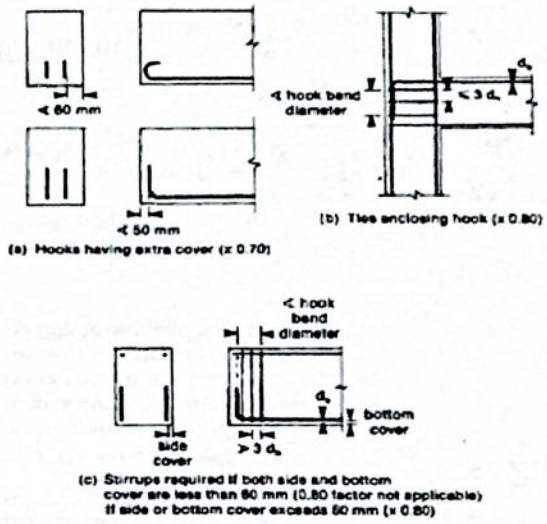


Fig. N12.5.3
Factors Modifying Hook Development Length for No. 35 or Smaller Bars

1. Choose minimum thickness for slabs and beams to satisfy deflection requirements
CSA A23.3 Clause 9.8.2.1

Beams: L/16, L/18.5, L/21, L/8

Slabs: L/20, L/24, L/28, L/10

2. Compute deflections of the structure and compare to permissible limits

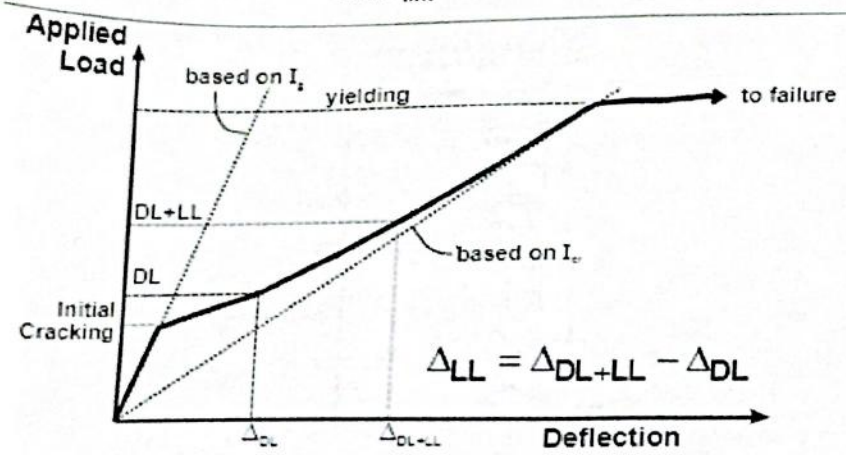
$$\Delta \le \Delta_{lim}$$

attached to non-structural elements likely to be damaged by large deflections	specified live load, L, or snow load, S	
Floors not supporting or attached to non-structural elements likely to be damaged by large deflections	Immediate deflection due to specified live load, L	$\ell_n/360$
Roof or floor construction supporting or attached to non-structural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of non-structural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell_n/480$‡
Roof or floor construction supporting or attached to non-structural elements not likely to be damaged by large deflections	That part of the total deflection occurring after attachment of non-structural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell_n/240$§

$$\Delta_{total} = \Delta_i + \Delta_t = \left(1 + \frac{s}{1+50\rho'}\right)\Delta$$

$$\Delta_t = \left(\frac{s}{1+50\rho'}\right)\Delta_i$$

s = time dependent factor
= 1.0 3 months
= 1.2 6 months
= 1.4 12 months
= 2.0 5 years or more



$$\Delta_{LL} = \Delta_{DL+LL} - \Delta_{DL}$$

$$E_c = \left(3300\sqrt{f'_c} + 6900\right)\left(\frac{\gamma_c}{2300}\right)^{1.5}$$

$$E_c = 4500\sqrt{f'_c} \quad \text{20-40MPa norm}$$

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \le I_g$$

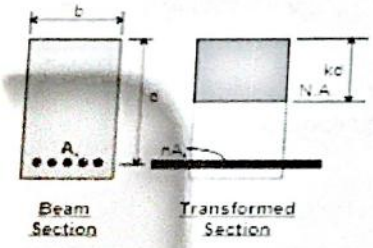
M_{cr} = cracking moment

$$= \frac{f_r I_g}{y_t} \qquad f_r = \frac{0.6\lambda\sqrt{f'_c}}{2}$$

$$I_{e,avg} = 0.7I_{e,m} + 0.15\left(I_{e1} + I_{e2}\right) \quad \text{2-end cont}$$

$$I_{e,avg} = 0.85I_{e,m} + 0.15I_{e,cont} \quad \text{1-end cont}$$

Singly-reinforced Sections:



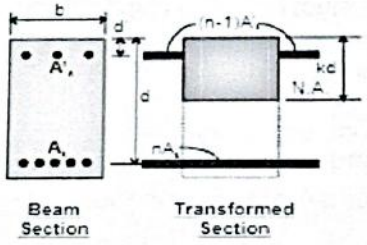
$$bkd\left(\frac{kd}{2}\right) = nA_s(d - kd)$$

$$\rho = \frac{A_s}{bd} \quad \Rightarrow \quad \frac{k^2}{2} + \rho nk - \rho n = 0$$

Solving: $k = \sqrt{2\rho n + (\rho n)^2} - \rho n$

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2$$

Doubly-reinforced Sections:



$$\frac{b(kd)^2}{2} + (n-1)A'_s(kd - d') = nA_s(d - kd)$$

$$\frac{b(kd)^2}{2} + \left[(n-1)A'_s + nA_s\right]kd - (n-1)A'_s d' - nA_s d = 0$$

$\Rightarrow$ Solve for "kd"

$$I_{cr} = \frac{b(kd)^3}{3} + nA_s(d - kd)^2 + (n-1)A'_s(kd - d')^2$$

CSA A23.3 - CRACK CONTROL PARAMETER:

Clause 10.6.1

$$z = f_s (d_c A)^{1/3}$$

$\leq$ **30,000 N/mm for Interior exposure**

$\leq$ **25,000 N/mm for Exterior exposure**

where:

f_s = tensile stress in reinforcement under service load

$\approx 0.6 f_y$ (can be used in lieu of more detailed analysis)

d_c = distance from c.g. of extreme layer of reinforcement to tension surface of concrete *(Note: clear cover need not be taken larger than 50 mm when determining d_c)*

A = effective tension area of concrete surrounding the main tension reinforcement, divided by the number of bars

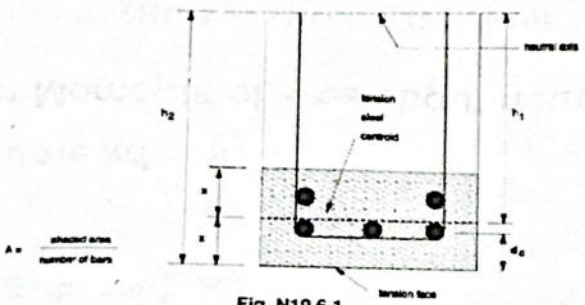
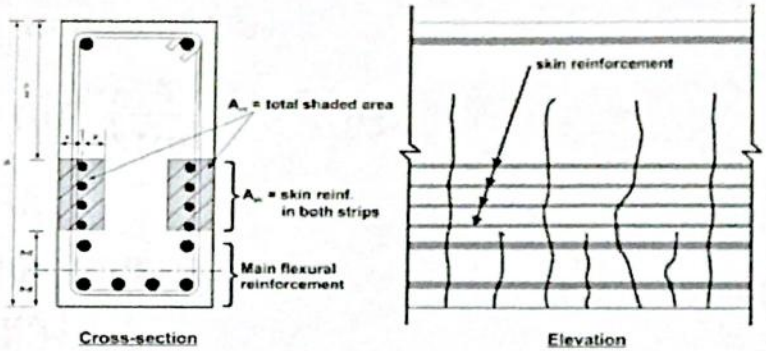


Fig. N10.6.1
Crack Width Parameters

- In deep beams, crack widths may be larger on the webs of the beam between the neutral axis and the main tension reinforcement → side-face cracks
- "Skin reinforcement" is used to control these cracks



CSA A23.3 *Clause 10.6.2* - Skin Reinforcement

Longitudinal skin reinforcement, A_{sk}, shall be used for beams with h > 750 mm

$$A_{sk} = \rho_{sk} A_{cs}$$

$\rho_{sk} = 0.008$ **for interior exposure**

$= 0.010$ **for exterior exposure**

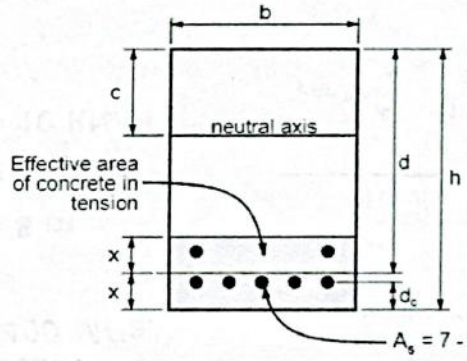
$$A_{cs} = 2 \times [0.5h - 2(h - d)] \times [2x]$$

Skin reinforcement shall be uniformly distributed along the exposed side faces of the member over the height of A_{cs}.

For reinforced members with an overall depth, *h*, exceeding 750 mm, longitudinal skin reinforcement shall be uniformly distributed along the exposed side faces of the member for a distance 0.5*h* – 2(*h* – *d*) nearest the principal reinforcement

Maximum spacing = 200 mm

Example 2: ***Check crack control for an exterior exposure.***



b = 400 mm
h = 600 mm
d = 515 mm

d_c = 70 mm

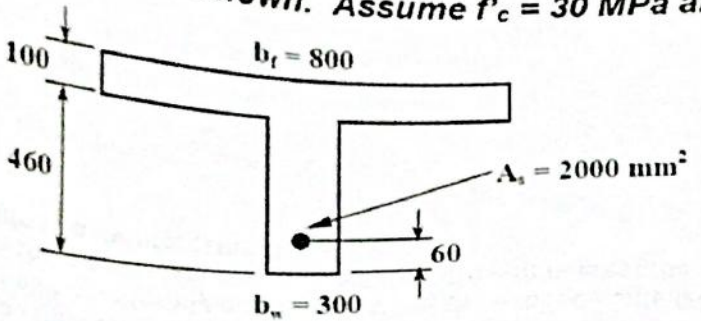
f_y = 400 MPa

$$x = h - d = 600 - 515$$
$$= 85\,mm$$

$$A = \frac{(2x)b}{n_{bars}} = \frac{(2 \times 85\,mm)(400\,mm)}{7}$$
$$= 9714\ mm^2 / bar$$

$$z = (0.6 \times 400\,MPa)\left[(70\,mm)(9714\,mm^2)\right]^{1/3}$$
$$= 21{,}104\ N/mm$$
$$< 25{,}000\ N/mm \quad \Rightarrow \quad O.K.$$

Example 1: *Determine the mid-span dead load and live load deflections for the simply-supported beam shown. Assume $f'_c = 30$ MPa and $f_y = 400$ MPa.*



$L = 8$ m

$w_D = 6$ kN/m

$w_L = 10$ kN/m

$E_s = 200{,}000$ MPa

$E_c = 4500\sqrt{f'_c} = 24{,}650$ MPa

$n = E_s/E_c = 8.1$

From Table 1.14 CAC Handbook

(Linear elastic solution for s.s. beam):

$$\Delta_{max} = \frac{5wL^4}{384E_cI_e}$$

Gross Moment of Inertia, I_g

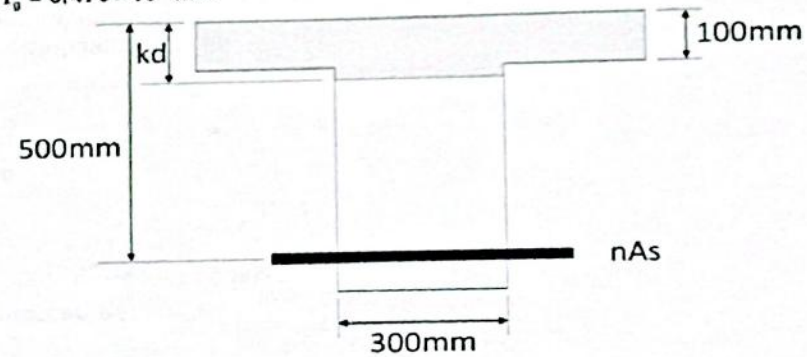
Item	Dimensions (mm)	Area (mm^2)	$\bar{y}_{top}$ (mm)	$A \times \bar{y}_{top}$ (mm^3)
Flange	500 x 100	50,000	50	2,500,000
Web	300 x 560	168,000	280	47,040,000
Totals:	--	218,000	--	49,540,000

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{49{,}540{,}000}{218{,}000} = 227\text{mm} \quad \textit{(centroid)}$$

$$I_g = \sum \bar{I}_i + \sum A_i d_i^2$$

$$= \frac{500(100)^3}{12} + (50{,}000)(227-50)^2 + \frac{300(560)^3}{12} + (168{,}000)(227-280)^2$$

$$I_g = 6{,}470 \times 10^6\ mm^4$$



Assume $kd > h_f$

Sum Moments of area about neutral axis

$$(800-300)\cdot 100\cdot\left(kd - \frac{100}{2}\right) + \frac{300(kd)kd}{2} = 8.1\cdot 2000\cdot(500-kd)$$

$$150\cdot kd^2 + 66200\cdot kd - 10{,}600{,}000 = 0$$

$$kd = 124.8mm > h_f\ ok.$$

$$I_{cr} = \frac{(800-300)100^3}{12} + (800-300)\cdot 100\cdot\left(\frac{100}{2} - 124.8\right)^2 + \frac{300(124.8)^3}{3} + 8.1\cdot 2000\cdot(500-124.8)^2$$

$$I_{cr} = 2796\cdot 10^6 mm^4$$

Solve for cracking moment

$$M_{cr} = \frac{f_r I_g}{y_{tension}}$$

$$f_r = \frac{0.6\sqrt{f'_c}}{2} = \frac{0.6\sqrt{30}}{2} = 1.64\ MPa$$

For gross sections

$$y_{tension} = h - \bar{y} \rightarrow 560 - 227 = 333mm$$

$$M_{cr} = \frac{1.64MPa \cdot 6470 \cdot 10^6 mm^4}{333mm \cdot 10^6} = 31.9\ kN \cdot m$$

Dead Load Deflection, Δ_D

$$M_D = \frac{w_D L^2}{8} \rightarrow \frac{6 \cdot 8^2}{8}\ (without\ factors) = 48\ kN \cdot m > M_{cr}$$

Therefore, section is cracked under dead load use I_e

Clause 9.8.2.3

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g \rightarrow \left[2796 + (6470 - 2796)\left(\frac{31.9}{48}\right)^3\right] \cdot 10^6$$

$$I_e = 3874 \cdot 10^6$$

$$\Delta_D = \frac{5 w_D L^4}{384 E_c I_e} = \frac{5 \cdot \frac{6N}{mm} \cdot 8000mm^4}{384 \cdot 24650MPa \cdot 3874 \cdot 10^6 mm^4} \rightarrow \Delta_D = 3.35mm$$

Live Load Deflection – behaviour is non-linear *(after cracking)*

$$\Delta_L \neq \frac{5 w_L L^4}{384 E_c I_g} \rightarrow \Delta_L = \Delta_{D+L} - \Delta_D$$

$$M_{D+L} = \frac{(6 + 10) \cdot 8^2}{8} = 128\ kN \cdot m > M_{cr} \rightarrow use\ I_e$$

Clause 9.8.2.3

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g \rightarrow \left[2796 + (6470 - 2796)\left(\frac{31.9}{128}\right)^3\right] \cdot 10^6$$

$$I_e = 2852 \cdot 10^6 mm^4$$

$$\Delta_{D+L} = \frac{5 \cdot \frac{16N}{mm} \cdot 8000mm^4}{384 \cdot 24650MPa \cdot 2852 \cdot 10^6 mm^4} \rightarrow \Delta_{D+L} = 12.1mm$$

$$\Delta_L = \Delta_{D+L} - \Delta_D \rightarrow 12.1mm - 3.35mm \rightarrow \Delta_L = 8.8mm$$

Check deflection limit → Table 9.3 clause 9.8.2.6

Assume floor does not support elements likely to be damaged by deflections:

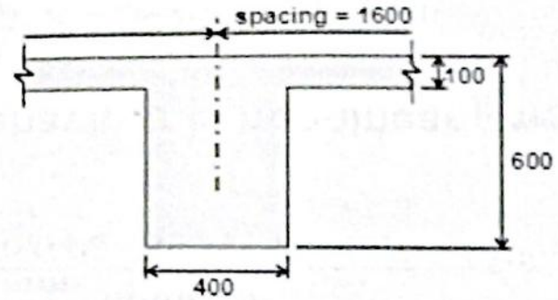
$$Floor\ \Delta_{max} = \frac{l_n}{360} = \frac{8000}{360} = 22.2mm > \Delta_L = 8.8mm\ O.K.$$

What if deflection limits are not met?

Increase section (b, h)	Increases I_y, I_{cr}, M_{cr}
Increase A_s	Increase I_{cr}
Add A'_s	Increases I_{cr}

Example 11: ***Design reinforcement for the simply-supported T-beam shown for M_f = 600 kNm.***

Beam span, L = 10 m



Use:

- f'_c = 30 MPa
- f_y = 400 MPa
- 10M stirrup
- Cover = 40 mm
- Max. C.A. = 19 mm

1. Assume section dimensions:

h, h_f, b_w → given

b_f → Based on Clause 10.3.3

$$b_f \le \begin{cases} b_w - 0.4L \\ b_w - 24h_f \\ b_w + L_s \end{cases} \le \begin{cases} 400 + 0.4(10{,}000) & = 4400\text{mm} \\ 400 + 24(100) & = 2800\text{mm} \\ 400 + (1600 - 400) & = 1600\text{mm} \Rightarrow \text{governs} \end{cases}$$

d → assume two equal layers of 25M bars

$$d = 600 - 40 - 11.3 - 25.2 - \frac{1.4(25.2)}{2} = 506\text{mm}$$

2. Determine whether a T-section design is needed:

Assume rectangular section with b = b_f and a = h_f

$$M_r^* = \alpha_1\phi_c f'_c h_f b_f \left(d - \frac{h_f}{2}\right)$$

$$= (0.81)(0.65)(30\text{MPa})(100\text{mm})(1600\text{mm})\left(506 - \frac{100\text{mm}}{2}\right) \div 10^6$$

$$= 1152 \text{ kNm}$$

Since M^*_r > M_f = 600 kNm, we know that "a" will actually be less than h_f to provide M_r = 600 kNm, and thus we can design the beam as a rectangular section with b = b_f.

3. Calculate required A_s using rectangular section design

$$K_r = \frac{M_f}{b_f d^2} = \frac{600 \times 10^6 \text{ Nmm}}{(1600\text{mm})(506\text{mm})^2} = 1.46\text{MPa}$$

Using CAC Handbook Table 2.1 → ρ = 0.453% (required)

$$A_s = (0.00453)(1600\text{mm})(506\text{mm})$$

$$= 3676\text{mm}^2$$

4. Choose bar size and determine number required

Use 8 – 25M A_s = 4000 mm² (provided in 2 layers)

Check spacing:

$$s = \frac{b - 2c - 2d_{st} - nd_b}{n-1} = \frac{400 - 2(40) - 2(11.3) - 4(25.2)}{4-1} = 65.53\text{mm}$$

$$s \ge \begin{cases} 1.4d_b = 35.3\text{mm} \Rightarrow \text{governs} \\ 1.4\text{max.C.A.} = 26.6\text{mm} \\ 30\text{mm} \end{cases}$$

Since:

s = 65.53 mm > 35.3 mm

→ spacing is OK

Check $A_{s\,min}$: $$A_{s\,min} = \frac{0.2\sqrt{f'_c}}{f_y} b_t h = \frac{0.2\sqrt{30}}{400}(400)(600) = 657\,mm^2$$

$A_s > A_{s\,min} \rightarrow$ OK

5. Check adequacy of section (i.e., $M_r > M_f$)

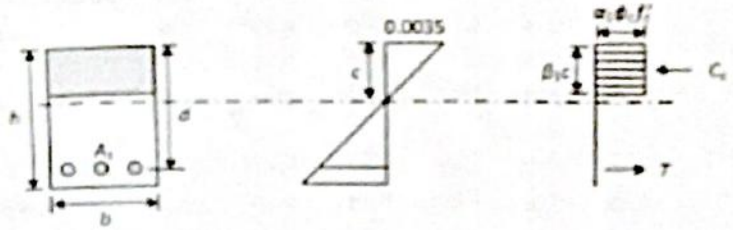
$$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b_f} = \frac{0.85(4000)(400)}{0.81(0.65)(30)(1600)} = 53.8\ mm$$

$$\frac{c}{d} = \frac{53.8/0.9}{506} = 0.118 < \frac{700}{700 + f_y} 0.8 = 0.51$$

Steel is yielding

$$M_r = \phi_s A_s f_y (d - a/2)$$
$$= 0.85(4000)(400)(506 - 53.8/2) \div 10^6$$
$$= 651.6\ kNm \quad > M_f \Rightarrow OK$$

$M_r = K_r b d^2 \times 10^{-6}$ kN·m; $K_r = \left[1 - \frac{\rho \phi_s f_y}{2\alpha_1 \phi_c f'_c}\right] \rho \phi_s f_y$; $\rho = \frac{A_s}{bd}$



f'_c (MPa)	20	25	30	35	40	45	50	55	60
α_1 :	0.82	0.81	0.81	0.80	0.79	0.78	0.78	0.77	0.76
β_1 :	0.92	0.91	0.90	0.88	0.87	0.86	0.85	0.83	0.82
ρ_{bal} :	1.83	2.24	2.63	3.00	3.34	3.67	3.98	4.27	4.55
K_r					ρ (%)				
0.5	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
0.6	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
0.7	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
0.8	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
0.9	0.28	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
1.0	0.31	0.31	0.30	0.30	0.30	0.30	0.30	0.30	0.30
1.1	0.34	0.34	0.34	0.33	0.33	0.33	0.33	0.33	0.33
1.2	0.38	0.37	0.37	0.37	0.36	0.36	0.36	0.36	0.36
1.3	0.41	0.40	0.40	0.40	0.40	0.39	0.39	0.39	0.39
1.4	0.44	0.44	0.43	0.43	0.43	0.43	0.42	0.42	0.42
1.5	0.48	0.47	0.46	0.46	0.46	0.46	0.46	0.45	0.45
1.6	0.51	0.50	0.50	0.49	0.49	0.49	0.49	0.49	0.48
1.7	0.55	0.54	0.53	0.53	0.52	0.52	0.52	0.52	0.52
1.8	0.58	0.57	0.56	0.56	0.55	0.55	0.55	0.55	0.55
1.9	0.62	0.61	0.60	0.59	0.59	0.58	0.58	0.58	0.58

Example 12: *Repeat Example 11 with b_f = 800 mm.*

1. Assume section dimensions

$h, h_f, b_w, b_f \rightarrow$ Given

$d \rightarrow$ assume two equal layers of 25M bars

$$d = 600 - 40 - 11.3 - 25.2 - \frac{1.4(25.2)}{2} = 506\,mm$$

2. Determine whether a T-section design is needed

Assume rectangular section with $b = b_f$ and $a = h_f$

$$M_f^* = \alpha_1\phi_c f_c' h_f b_f \left(d - \frac{h_f}{2}\right)$$

$$= (0.81)(0.65)(30\,MPa)(100\,mm)(800\,mm)\left(506 - \frac{100\,mm}{2}\right) \cdot 10^6$$

$$= 576.2\,kNm$$

Since $M_f^* < M_r = 600$ kNm, we know that "a" will have to be greater than h_f to provide $M_r \geq 600$ kNm, and thus we will need to do a T-section design.

3. Determine flange contribution

$$A_{sf} = \frac{\alpha_1\phi_c f_c'(b_f - b_w)h_f}{\phi_s f_y} = \frac{(0.81)(0.65)(30)(800 - 400)(100)}{(0.85)(400)}$$

$$= 1858\,mm^2$$

$$M_{rf} = \phi_s A_{sf} f_y (d - h_f/2) = (0.85)(1858)(400)\left(506 - \frac{100}{2}\right) \cdot 10^6$$

$$= 288.0\,kNm$$

4. Determine web contribution

$$M_{rw} = M_r - M_{rf} = 600 - 288$$

$$= 312\,kNm \quad \text{(required)}$$

$$K_{rw} = \frac{M_{rw}}{b_w d^2} = \frac{312 \times 10^6\,Nmm}{(400\,mm)(506\,mm)^2} = 3.05\,MPa$$

Using CAC Handbook Table 2.1 $\rightarrow \rho_w = 1.01\%$ (required)

$$A_{sw} = (0.0101)(400\,mm)(506\,mm)$$

$$= 2044\,mm^2$$

5. Choose bar size and determine number required

$$A_s = A_{sf} - A_{sw} = 1858 - 2044$$

$$= 3902\,mm^2 \quad \text{(required)}$$

Use 8 – 25M $\rightarrow A_s = 4000\,mm^2$ (provided in 2 equal layers)

Check spacing: $$s = \frac{400 - 2(40) - 2(11.3) - 4(25.2)}{4 - 1}$$

$$= 65.53\,mm$$

$$s \geq \begin{cases} 1.4d_b = 35.3\,mm & \Rightarrow \text{governs} \\ 1.4\,max.\,C.A. = 26.6\,mm \\ 30\,mm \end{cases}$$

Since $s = 65.53\,mm > 35.3\,mm$ $\rightarrow$ spacing is OK

Check $A_{s\,min}$: $$A_{s\,min} = \frac{0.2\sqrt{f_c'}}{f_y} b_t h = \frac{0.2\sqrt{30}}{400}(400)(600) = 657\,mm^2$$

$A_s > A_{s\,min} \rightarrow$ OK

6. Check adequacy of section (i.e., $M_r > M_f$)

T-section Analysis will be required (we already know $a > h_f$ from Step 2.):

M_{rf} The flange contribution is unchanged from Step 3 of the design process

$A_{sf} = 1858\ mm^2$

$M_{rf} = 288\ kNm$

M_{rw} The web contribution is determined based on the analysis of a rectangular section with A_{sw} and width b_w:

$$a = \frac{\phi_c A_{sw} f_y}{\alpha_1 \phi_c f'_c b_w} = \frac{0.85(4000-1858)(400)}{0.81(0.65)(30)(400)} = 115.3\ mm$$

$$\frac{c}{d} = \frac{115.3/0.9}{506} = 0.253 < \frac{700}{700+f_y}0.8 = 0.51$$

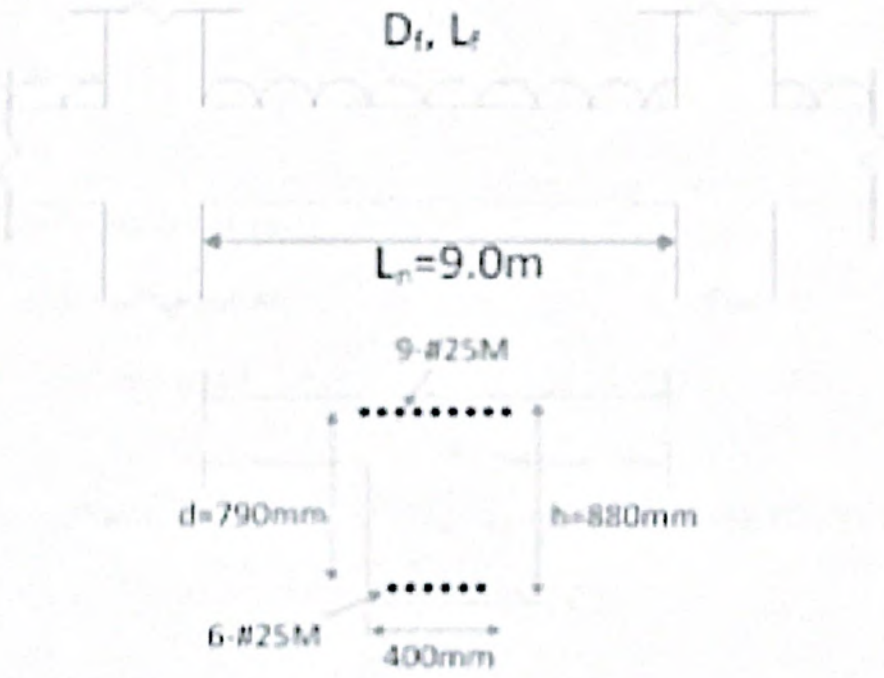
Steel is yielding

$$M_{rw} = \phi_c A_{sw} f_y (d - a/2)$$
$$= 0.85(4000-1858)(400)(506-115.3/2)\cdot 10^6$$
$$= 326.5\ kNm$$

M_r $M_r = M_{rf} + M_{rw} = 288 + 326.5$

$= 614.5\ kNm > M_f = 600\ kNm \Rightarrow$ OK

EXAMPLE 2 Design the beam below using General Method – from Macgregor and Bartlett (example 6.6)



$D_f = 60\ \frac{kN}{m}$

$L_f = 75\ \frac{kN}{m}$ -> $w_f = 135\ \frac{kN}{m}$

$f'_c = 25MPa$, $f_y = 300MPa$

Solution

1) Calculate the factored Shear force diagram and Bending Moment Diagram

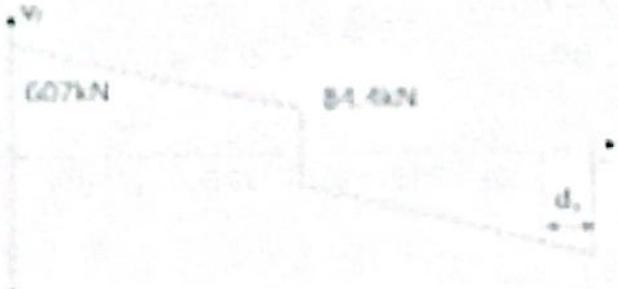
- From fig Nº 3.3 (approximate Frame analysis, CSA A 23.3-04

At the support

$$V_f = \frac{w_f \times L_n}{2} = \frac{135 \times 9}{2} = 607kN$$

To consider pattern loading loading, MacGregor and Bartlett suggest the following at midspan

$$V_f = \frac{L_f \times L_n}{8} = \frac{75 \times 9}{8} = 84.4kN$$



$$d_v = MAX(0.9d, 0.72h) = MAX(711, 634) = 711mm$$

$$V_f' = 84.4 + \frac{607 - 84.4}{4500}(4500 - 711) = 525\ kN$$

At the support

$$M_f = \frac{-w_f \cdot L_n^2}{12} = -594\ kN \cdot m$$

From MacGregor and Bartlett Figure A1:

$$x = \frac{711}{9000} = 0.079 \rightarrow M_f = 0.6 \cdot M_f = -596kN \cdot m$$

Note: The major difference between the simplified method and the general method is that the shear resistance depends on the applied moment.

Is the section large enough?

$$V_{r\,max} = 0.25 \cdot \phi_c \cdot f_c' \cdot b_w \cdot d_v$$

$$V_{r\,max} = 0.25 \times 0.65 \times 25 \times 400 \times 711 = 1155\ kN > 525kN$$

2) Are stirrups required?

$$V_c = \lambda \cdot \phi_c \cdot \beta \cdot \sqrt{f_c'} \cdot b_w \cdot d_v$$

$$\beta = \left(\frac{0.40}{1 + 1500 \cdot \varepsilon_x}\right)\left(\frac{1300}{1000 + s_{xe}}\right)$$

$$\varepsilon_x = \frac{\left(\frac{M_f}{d_v} + V_f\right)}{2 \cdot E_s \cdot A_s} = 0.000758$$

$$s_{xe} = \frac{35 \cdot s_z}{15 + a_g} = s_z = d_v \text{, with no stirrups, and 20mm aggregate}$$

$$\beta = 0.142$$

$$V_c = 1 \cdot 0.65 \cdot 0.142 \cdot \sqrt{25} \cdot 400 \cdot 711 = 131\ kN < 525kN$$

Stirrups required

3) Determine the max stirrup spacing

$$0.125 \cdot \lambda \cdot \phi_c \cdot f_c' \cdot b_w \cdot d_v = 577\ kN$$

$$525 < 577$$

therefore

$$s \le MIN(600, 0.7 \cdot d_v) = (600, 498)$$

Based on A_v min

$$A_{v\,min} = \frac{0.06 \cdot \sqrt{f_c'} \cdot b_w \cdot s}{f_y}$$

$$s \le \frac{f_y \cdot A_v}{0.06 \cdot \sqrt{f_c'} \cdot b_w}$$

Assuming 10M stirrups

$$s \le \frac{300 \cdot 200}{0.06 \cdot \sqrt{25} \cdot 400} \le 500mm$$

Maximum spacing is 500 mm

4) Determine s to resist V_f'

$$S_{req'd} = \frac{\varphi_s \cdot A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{V_f' - V_c}$$

$$V_c = \lambda \cdot \varphi_c \cdot \beta \cdot \sqrt{f_c'} \cdot b_w \cdot d_v$$

If $A_v > A_{v\,min}$ then $s_{ze} = 300$

$$\beta = \left(\frac{0.40}{1 + 1500 \cdot \varepsilon_x}\right)\left(\frac{1300}{1000 + 300}\right)$$

$$\varepsilon_x = 0.000758$$

$$\beta = 0.187$$

$$V_c = 173.1\ kN$$

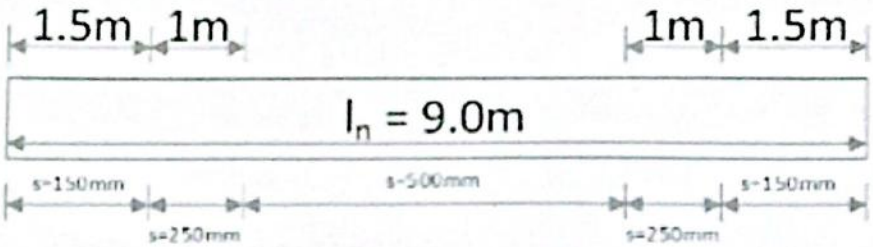
$$\theta = 29 + 7000 \cdot 0.000758 = 34.3°$$

$$S_{req'd} = \frac{0.85 \cdot 200 \cdot 300 \cdot 711 \cdot \cot(34.3)}{525 - 173.1} = 151mm$$

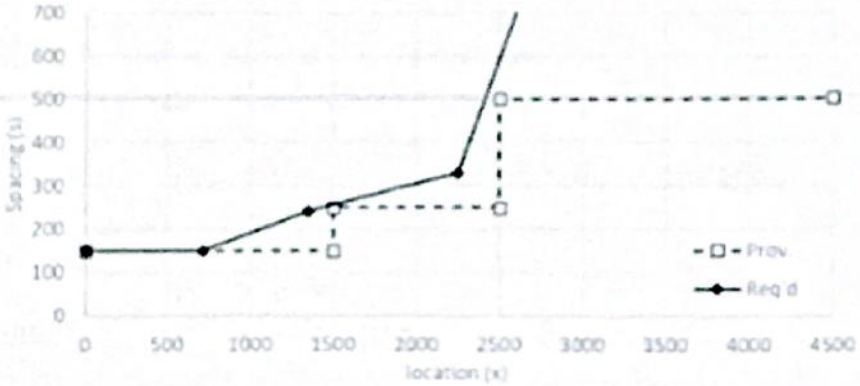
5) Repeat steps 1-4 @ several other locations

X	711	1350	2250	3600	mm
V_f'	525	451	346	189	kN
M_f'	-596	-298	342	615	$kN \cdot m$
A_s	4500	4500	3000	3000	mm^2
$S_{req'd}$	151	242	330	1747	mm

One possible stirrup layout



Note:
- Further check of spacing at 2500 should be done to ensure it is 500 mm
- no stirrups required when V_f drops below V_c (if h < 750mm), however some engineers would provide minimum stirrups regardless.



Final Comments and Review:

-
- Aggregate interlock depends on the concrete strength, aggregate size, and crack width.

APPROACH 1 – ANALYSIS FOR A GIVEN ECCENTRICITY

Given: $e = \frac{M_f}{P_f}$ $\quad e' = e + (d - h/2)$

➢ Looking for P_r & M_r

Equilibrium: $P = C_c + C_s - T$

Moment about T: $Pe' = C_c(d - a/2) + C_s(d - d')$

Unknowns: a, P

Solution: $P_r = P$ for given "e"

$M_r = P\,e$

Verify: $P_r \geq P_f$ for column in question

$M_r \geq M_f$

APPROACH 2 – ANALYSIS FOR A GIVEN AXIAL LOAD

Given: $P_r = P_f$ find M_r *(and e)*

Equations:

Equilibrium: $P_f = C_c + C_s - T$

Moment about T: $P_f e' = C_c(d - a/2) + C_s(d - d')$

Unknowns: a, e'

Solution: $e = e' - (d - h/2)$ for given "P_f"

$M_r = P_f\,e$

Verify: $M_r \geq M_f$ for column in question

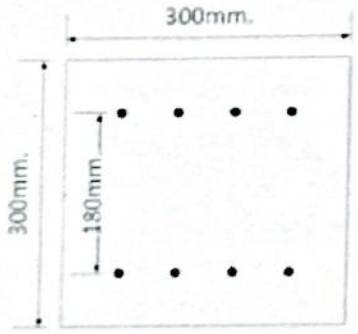
Example 1

- Calculate factored balanced failure point for the column in the Figure

$f_c' = 40\ MPa$

$8 - 20M\ bars$

$f_y = 400\ MPa$



Solution *(compression positive in this example)*

$C = 240 \cdot \frac{0.0035}{0.002} = 152.7mm$

$\varepsilon_{s1} = -0.002 \quad \varepsilon_c = -0.0035$ *(given)*

$\varepsilon_{s2} = 0.0035\left(\frac{152.7-60}{152.7}\right) = 0.00213$

$f_{s1} = -400\ MPa\ (tension)$

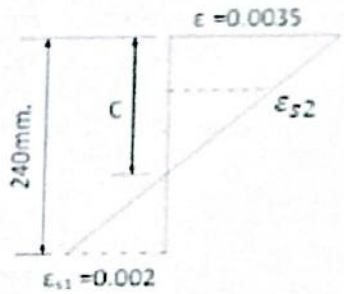
$f_{s2} = MIN(0.00213 \cdot 200GPA, 400MPa)$

$f_{s2} = 400\ MPa$

$\beta_1 = 0.97 - 0.0025(40) = 0.87$

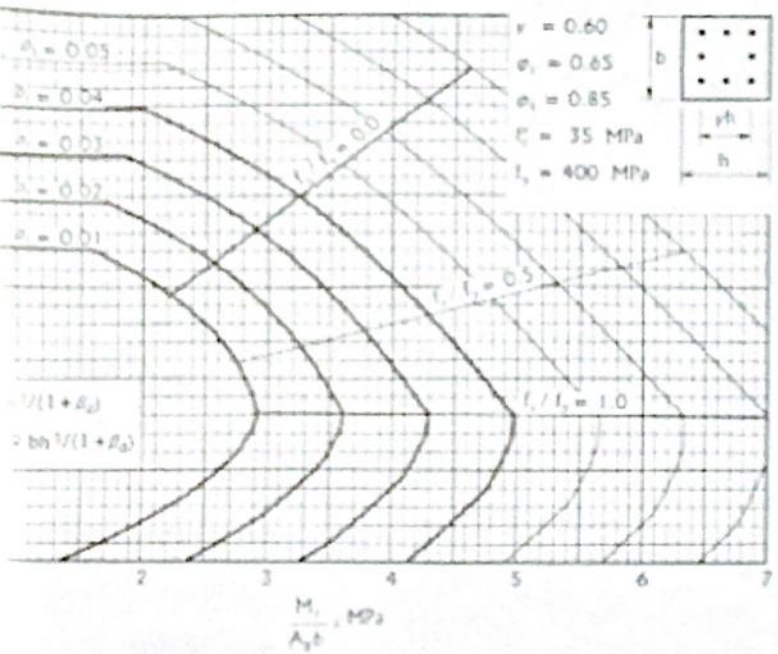
$a = \beta_1 \cdot c = 0.87 \cdot 152.7 = 132.9mm$

$\alpha_1 = 0.85 - 0.0015(40) = 0.79 > 0.67\ ok$



for rectangular columns

for circular columns



ANALYSIS FOR GIVEN ECCENTRICITY, e (FIND P_R AND M_R)

- $P_r e = M_r \quad \Rightarrow \quad e = \frac{M_r}{P_r}$ and $\frac{1}{e} = \frac{P_r}{M_r}$
- Slope of a line in the chart $= \frac{P_r/A_g}{M_r/A_g h}$

$$= \frac{P_r h}{M_r} = \frac{h}{e}$$

- Draw a straight line from the origin with a slope of h/e, and where it crosses the interaction curve for a given value of ρ_g determines the corresponding values of P_r/A_g and $M_r/A_g h$

ANALYSIS FOR GIVEN FACTORED AXIAL LOAD, P_F

- Set $P_r = P_f$ and calculate P_r/A_g
- Draw horizontal line at calculated value of P_r/A_g, and where it crosses the interaction curve for a given value of ρ_g determines the corresponding value of $M_r/A_g h$.
- Compute $e = M_r/P_r$.

e

$\phi_c \cdot f_c' \cdot a \cdot b = 0.79 \cdot 0.65 \cdot 40 \cdot 132.9 \cdot 300 = 818.8\ kN$

$\cdot f_{s1} \cdot A_{s1} = 0.85 \cdot 400 \cdot 1200 \times 10^{-3} = -408\ kN$

$\cdot f_s - \alpha_1 \cdot \phi_c \cdot f_c')A_{s2} = (0.85 \cdot 400 - 0.79 \cdot 0.65 \cdot 40) \cdot 1200 \times 10^{-3}$

already accounted for when calculating C_{rc} where area replaced
sion steel was not accounted for

3.4 kN

.8 + 383.4 − 408 = 794 kN

around centroidal axis

$$\frac{h}{2} - \frac{a}{2}) + \sum_{i=1}^{n} F_{rsi}\left(\frac{h}{2} - d_i\right)$$

$$\left(150 - \frac{132.9}{2}\right) - 408(150 - 240) + 383.4(150 - 60)$$

$$M_r = 139.6\ kN \cdot m$$

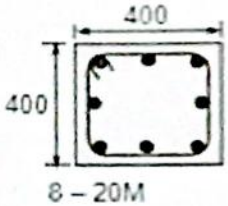
Example 2: ***Calculate the factored axial resistance of the tied column shown.***

Assume:

f_c = 30 MPa

f_y = 400 Mpa

α_1 = 0.81 (α_1 = 0.85 − 0.0015f_c) ≥ 0.67



$$A_g = 400 \times 400 = 160{,}000\,mm^2$$

$$A_s = 8 \times 300\,mm^2 = 2{,}400\,mm^2$$

$$P_{ro} = \alpha_1 \phi_c f_c'(A_g - A_s) + \phi_s A_s f_y$$

$$= (0.81)(0.65)(30MPa)(160{,}000 - 2400\,mm^2)$$

$$+ (0.85)(400MPa)(2{,}400\,mm^2)$$

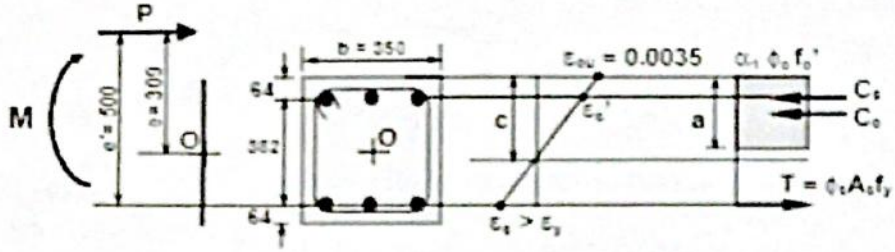
$$P_{ro} = 3305\,kN$$

Tied Column: k = 0.80 *(unintended eccentricities)*

$$P_{rmax} = 0.80(3305\,kN)$$

$$P_{rmax} = 2644\,kN$$

Example 3: *Calculate P_r and M_r when e = 309 mm for the cross-section shown below using Handbook Interaction Diagram.*
f'_c = 30 MPa, f_y = 400 MPa, A_s = 6 – 25M bars.



Compute section properties, γ, A_g, ρ_g

$\gamma h = 382mm$

$h = 382 + 2(64) = 510mm$

$\therefore \gamma = \frac{\gamma h}{h} = \frac{382}{510} = 0.75$

$A_g = bh = 350mm(510mm) = 178500mm^2$

$\rho_g = \frac{A_s}{A_g} = \frac{6(500mm^2)}{178500mm^2} = 1.68\%$

Use interaction diagrams to estimate P_r and M_r

To do this need to plot line with slope of h/e

e = 309mm (given)

$\therefore \frac{h}{e} = \frac{510}{309} = 1.65$

Since interaction diagrams for $\gamma = 0.75$ are not provided, we need to interpolate for $\gamma = 0.7$ and $\gamma = 0.8$

Use Table 7.11.7 and 7.11.8 provided by CAC

$\gamma = 0.7 \rightarrow Table\ 7.11.7$

$\rho = 1\% \rightarrow$ "x" = 2.8 $\therefore$ "y" = mx = 1.65(2.8) = 4.62

$\rho = 2\% \rightarrow$ "x" = 4.24, "y" = 6.996

Use linear interpolation to find values for $\rho = 1.68\%$

$$\frac{x - 2.8}{1.68 - 1} = \frac{4.24 - 2.8}{2 - 1} \rightarrow x = 3.78$$

$$\frac{y - 4.62}{1.68 - 1} = \frac{6.996 - 4.62}{2 - 1} \rightarrow y = 6.235$$

$\gamma = 0.8 \rightarrow Table\ 7.11.8$

$\rho = 1\% \rightarrow$ "x" = 3.05, "y" = 5.0325

$\rho = 2\% \rightarrow$ "x" = 4.6, "y" = 7.59

Use linear interpolation to find values for $\rho = 1.68\%$

$$\frac{x - 3.05}{1.68 - 1} = \frac{4.6 - 3.05}{2 - 1} \rightarrow x = 4.104$$

$$\frac{y - 5.0325}{1.68 - 1} = \frac{7.59 - 5.0325}{2 - 1} \rightarrow y = 6.7716$$

Since $\gamma = 0.75$ is the midpoint of 0.7 and 0.8 take the average of the points found for $\rho = 1.68\%$

$$x_{avg} = \frac{3.7792 + 4.104}{2} = 3.94 = \frac{M_r}{A_g h}$$

$$y_{avg} = \frac{6.235 + 6.7716}{2} = 6.5 = P_r/A_g$$

$$\therefore M_r = 3.94(178500mm^2)(510mm) \div 1000^2 = 358.6kNm$$

$$P_r = 6.5(178500mm^2) \div 1000 = 1160.2kN$$

***

Table 7.11.7 Rectangular Columns with Bars on End Faces Only

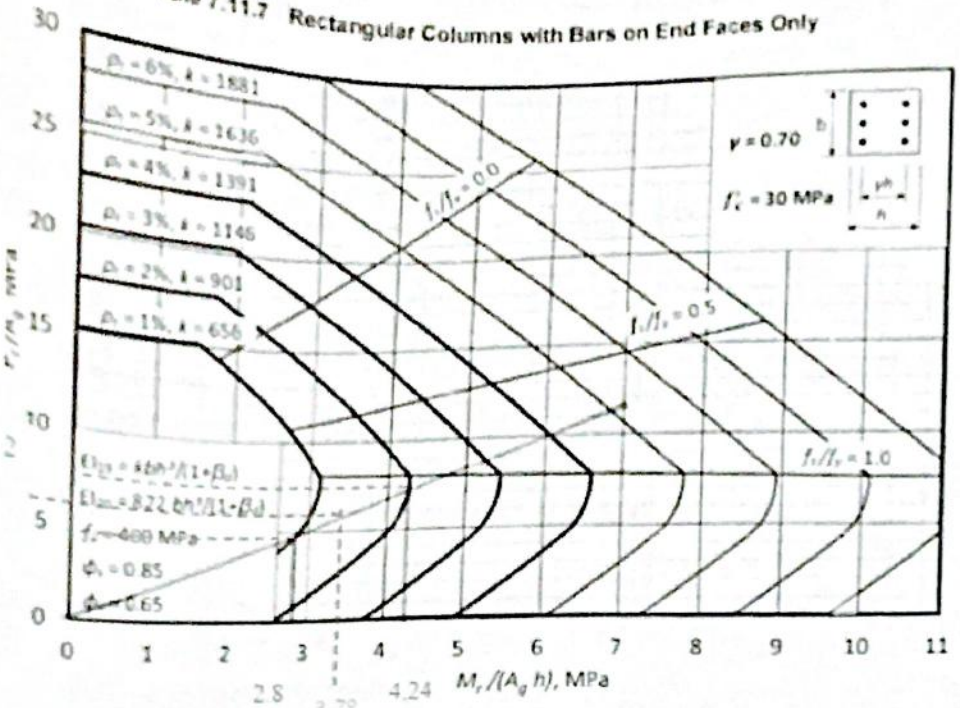
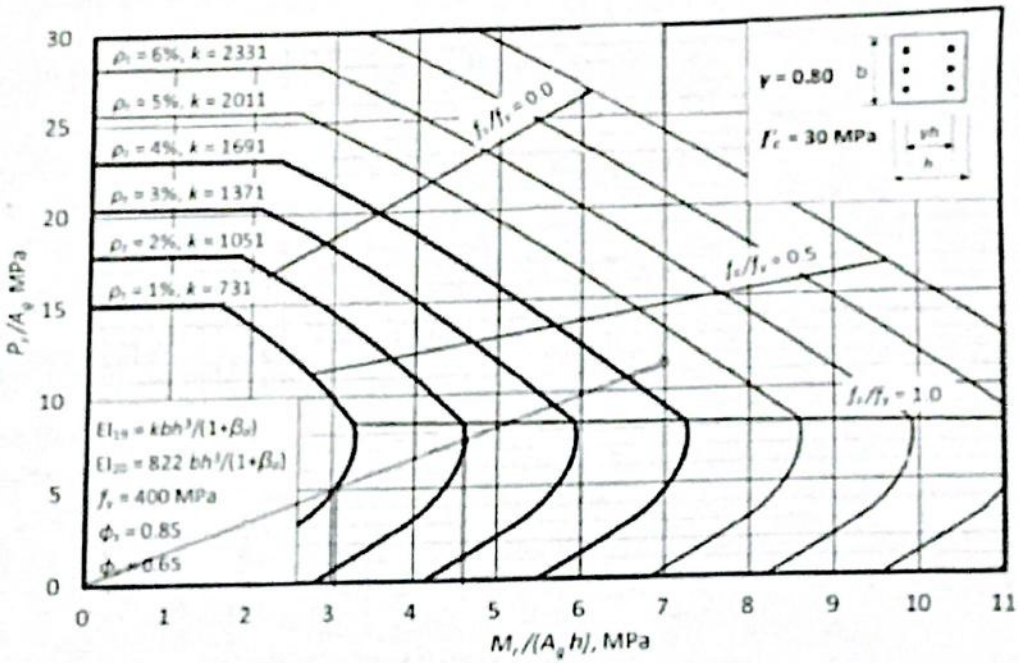


Table 7.11.8 Rectangular Columns with Bars on End Faces Only



		k			
TOP	Hinged	0.81	0.91	0.95	1.00
	Elastic Ψ = 3.1	0.77	0.86	0.90	0.95
	Elastic Ψ = 1.6	0.74	0.83	0.86	0.91
	Fixed	0.67	0.74	0.77	0.81
		Fixed	Elastic	Elastic	Hinged
		BOTTOM			

Effective length factors *[CAC Handbook Table 8.5]*:

5. Tie details

Use 10M ties (ok up to 55M long. bar Cl. 7.6.5.1)

- tie spacing: MIN of:

$16 \cdot 25.2 = 403$ or

$48 \cdot 11.3 = 542$ or

$350 \leftarrow governs$

- tie configuration: See Fig. N7.6.5.5

$$x = (350 - (40 \cdot 2 + 11.3 \cdot 2 + 25.2)/\,2 \;- 25.2$$

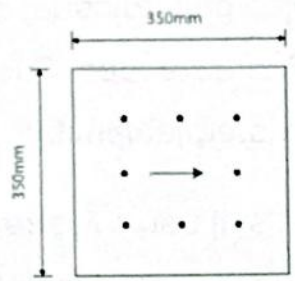
$$= 85.9mm < 150mm$$

$$\rightarrow single\ tie\ ok!$$

Example 6:

Verify column design in Example 4 assuming $l_u = 3.5m$ and moments are equal at column ends and acting in opposite directions for LC #2:

$LC\#$	$\frac{P_f}{A_g}$	$\frac{M_f}{A_g \cdot h}$
2	15.0	2.2



25 M long. bars

$f'_c = 35\ MPa$

LC# 2: $P_f = 1.25 \cdot 750 + 1.5 \cdot 600 = 1837.5kN$

$M_f = 1.25 \cdot 25 + 1.5 \cdot 20 + 0.4 \cdot 85$
$= 95.25\ kNm$

Solution:

1. Is column slender?

$$\frac{k \cdot l_u}{r} > \frac{25 - 10 \cdot M_1/M_2}{\sqrt{P_f/(f'_c \cdot A_g)}}$$

$\frac{3500}{03 \cdot 350} = 33.3$ (assuming k=1 is most conservative)

$$\frac{25 - 10 \cdot \left(\frac{95.25}{95.25}\right)}{\sqrt{1837.5 \cdot 1000/(35 \cdot 350^2)}} = 22.9 < 33.3$$

$\rightarrow$ *column is slender*

2. Compute EI, P_c

$$EI = \left(\frac{0.4}{1 + \beta_d}\right) E_c \cdot I_g$$

$E_c = 4500\sqrt{35} = 26622.4\ MPA$

$I_g = 350^4/12 = 1.25 \cdot 10^9 mm^4$

$\beta_d = 1.25 \cdot 750 / 1837.5 = 0.51$

$\rightarrow E \cdot I = 8.81 \cdot 10^{12}\ N \cdot mm^2$

$P_c = \pi^2 \cdot EI / (k \cdot l_u)^2$
$= 7104\ kN\ (> P_f \rightarrow ok!)$

3. Compute magnified moment:

$$M_c = M_2 \cdot C_m/(1 - P_f/(\phi_m \cdot P_c))$$

$$M_2 = 95.25\ kNM$$

$$C_m = 0.6 + 0.4 \cdot (M_1/M_2) \geq 0.4$$
$$= 0.6 + 0.4(1) = 1.0$$

$$M_c = 95.25 \cdot (1)/\left(1 - \frac{1837.5}{0.75 \cdot 7104}\right) = 1.53 \cdot 95.25$$
$$= 145.4\ kNm$$

- A minimum amount of longitudinal reinforcement of $A_{st} \geq 0.01 \cdot A_g$ (or $\rho_t \geq 0.01$) is normally called for to limit the effects of creep *[CSA A23.3 §10.9.1]*.

- In the current code, $0.005 \cdot A_g \leq A_{st} < 0.01 \cdot A_g$ may be used, but the column capacity must be limited in this case as follows: $P_r = P_r \cdot 0.5 \cdot (1 + \rho_t / 0.01)$ *[CSA A23.3 §10.10.5]*.

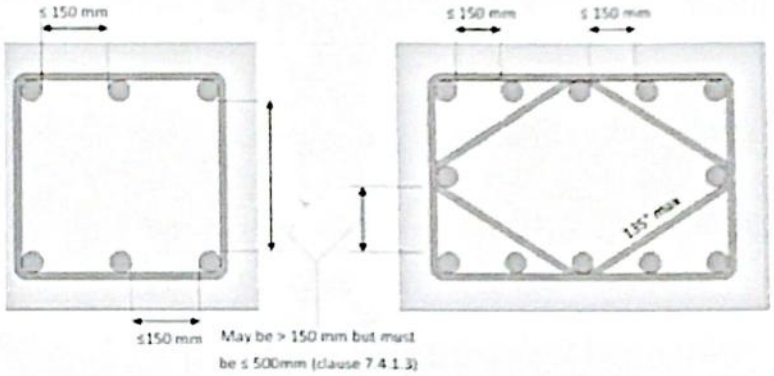
- A maximum amount of longitudinal reinforcement of $A_{st} \leq 0.08 \cdot A_g$ (or $\rho_t \leq 0.08$) is specified *[CSA A23.3 §10.9.2]* for practical reasons. However, even this is a lot of reinforcement. It is therefore recommended that $A_{st} \leq 0.04 \cdot A_g$, so that the longitudinal bars can be easily placed, lap splices can be accommodated, and the concrete will flow easily when it is poured.

- Normally, a minimum of four (4) longitudinal bars are required. (exceptions: minimum three (3) long. bars for triangular columns, minimum six (6) long. bars for spiral tied columns).

- The main purpose of the column ties is to provide lateral support for the longitudinal reinforcement. They also provide a small amount of confinement (much more in spiral columns).

  - 16 · (diameter of smallest longitudinal bar)
  - 48 · (tie diameter)
  - the smallest column x-section dimension
  - 300 mm, in the case of columns with "bundled" bars

- For rectangular columns, each corner and alternating longitudinal bar around the perimeter shall be laterally restrained by a tie corner (max. angle: 135°). No longitudinal bar can be more than 150 mm away on either side from a laterally restrained neighbouring bar.



Max. longitudinal bar spacing is 500 mm [*CSA A23.3 §7.4.1.3*]

– for determining P_c

$$E \cdot I = \frac{0.2 \cdot E_c \cdot I_g + E_s \cdot I_{se}}{1 + \beta_d} \quad \text{or} \quad E \cdot I = \left(\frac{0.4}{1 + \beta_d}\right) \cdot E_c \cdot I_g$$

$$\beta_d = \frac{\text{factored dead vertical load}}{\text{total factored vertical load}}$$

$$P_c = \frac{\pi^2 \cdot E \cdot I}{(k \cdot l_u)^2}$$

– Note: If $P_c < P_f$, then the column is unstable

→ increase the x-section dimensions

6. Compute the magnified moment, M_c:

$$- M_c = \frac{C_m \cdot M_2}{1 - \frac{P_f}{\phi_m \cdot P_c}} \geq M_2$$

where $\phi_m = 0.75$

$$C_m = 0.6 + 0.4\left(\frac{M_1}{M_2}\right) \geq 0.4$$

$M_2 \geq P_f(15 + 0.03\,h)$

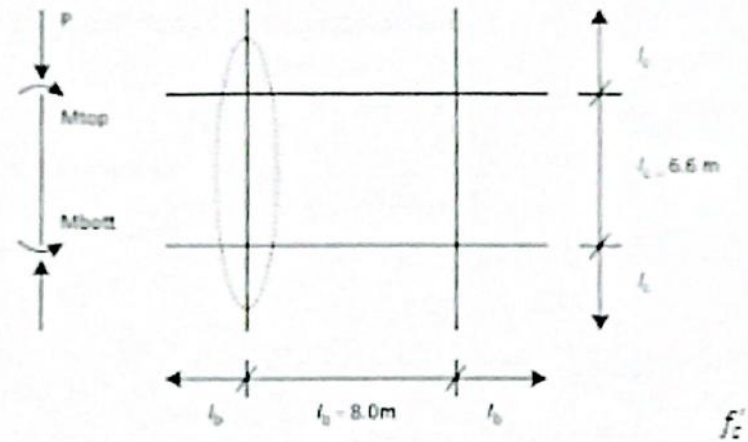
ign column for P_f, M_c:

e interaction diagrams, calculate $\frac{P_f}{A_g}$ and $\frac{M_c}{A_g \cdot h}$ – ρ_t

d required, consider increasing column size

$\rho_{t\,max}$ = 0.08 (≤ 0.04 recommended)

Example 7: ***Design the column in a non sway frame***



$f_c' = 40\,MPa$

$f_y = 400\,MPa$

Columns are 400 x 400 mm

- Beams are 300 x 600 mm
- P_f = 2000 kN (factored) (P_d = 800 kN -unfactored)
- M_f = 200 kNm (top), 200 kNm (bott)

Step 1 Calculate load effects
Step 2 Choose trial section
Step 3 Sway or non-sway?
} **already done**

Step 4 Is column slender?

$k \cong 0.9$ (Table 8.5) *Can be approximated as 1*

$l_u = 6.0m$ $(6.6 - 0.6)$

$r \cong 0.3 \cdot 400 = 120mm$

1. **Calculate the factored loads:**
   - P_f, M_f – top
   - P_f, M_f – bottom
2. **Choose a trial section:**
   - $A_g(trial) \geq \dfrac{P_f}{0.35 \cdot f'_c + 0.65 \cdot f_y \cdot \rho_t}$ is recommended, where:

     ρ_t = longitudinal reinforcement ratio ($0.01 \leq \rho_t \leq 0.08$)
3. **Sway or non-sway?**
   - this can often be determined by inspection
   - otherwise, calculate stability index, Q:

$$Q = \frac{\sum P_f \cdot \Delta_o}{V_f \cdot \ell_c} \leq 0.05$$

where: $\sum P_f$ = Σ all factored vertical loads on all columns and walls on storey

Δ_o = 1st order storey relative deflection due to V_f

V_f = storey shear

l_c = centre-to-centre storey height

note: $Q \leq 0.05$ → non-sway (otherwise, assume sway frame)

note: If $Q > 0.2$, a more rigid structure might be needed

4. **Is column slender?**
   - yes, if: $\dfrac{k \cdot \ell_u}{r} > \dfrac{25 - 10 \cdot (M_1/M_2)}{\sqrt{P_f/(f'_c \cdot A_g)}}$

     where: $\dfrac{k \cdot \ell_u}{r}$ is the "slenderness ratio"

     l_u = unsupported length

     r = radius of gyration = $\sqrt{\dfrac{I}{A}}$

     $k \cdot l_u$ = effective length

     M_1 = absolute minimum end moment

     M_2 = absolute maximum end moment
   - at this stage, estimate k using *[CAC Handbook Table 8.5]*
5. **Compute $E \cdot I$, k, P_c:**
   - for determining k: $I_c = 0.70 \cdot I_g$ ← columns

     $I_b = 0.35 \cdot I_g$ ← beams
   - determine $\psi = \dfrac{\sum (E_c \cdot I_c / \ell_c)}{\sum (E_b \cdot I_b / \ell_b)}$ for top and bottom of column
   - $l_{b,c}$ = centre-to-centre beam and column lengths
   - $E_{b,c} = 4500 \cdot \sqrt{f'_c}$
   - k is obtained from the non-sway monograph ($0.5 \leq k \leq 1.0$)

$$C_m = 0.6 + 0.4\left(\frac{200}{200}\right) = 1.0$$

$$(\geq 0.4 \checkmark)$$

$$\phi_m = 0.75$$

$$\rightarrow M_c = \frac{200}{1 - \left(\frac{2000}{0.75 \cdot 7296}\right)} = 315.2\ KNm$$

$$> M_2 \checkmark$$

$M_2 > 200\,(15 + 0.03 \cdot 400) = 54\ KNm$ (unintended eccentricities)

Step 7 Design for P_f, M_c

$$\frac{P_f}{A_g} = 12.5$$

$$\frac{M_c}{A_g \cdot h} = 4.93$$

$$try\ 35M\ bar \rightarrow \gamma = \frac{400 - 2 \cdot 40 + 2 \cdot 11.2 - 35.7}{400}$$

$= 0.65$ (estimate as 0.6)

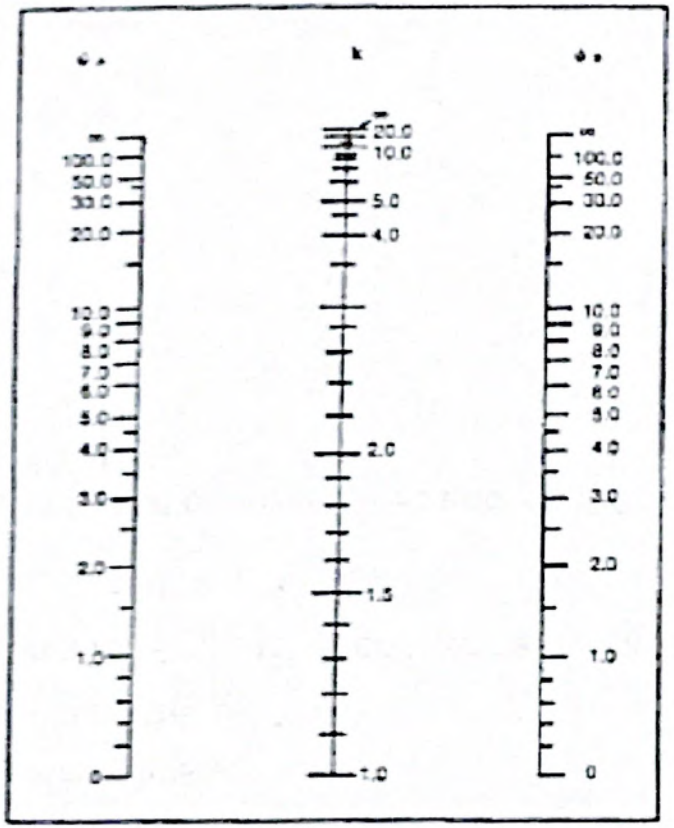
$$\rightarrow \rho_t = 0.04$$

$$As\ (req'd) = 6400\ mm^2$$

$$8 - 35M \rightarrow As = 8000\ mm^2$$

Try 30 M bars and estimate γ

Nomograph for columns in unbraced (sway) frames:



Design of Slender Columns in Sway Frames

1. Calculate the factored loads. Consider the various possible load combinations (LCs).

2. Calculate the corresponding load effects. Separate them into non-sway (ns) and sway (s).

3. Choose a trial section (same formula as for non-sway).

4. Sway or non-sway?

5. Compute $E \cdot I$, k, P_c:

   – same as for non-sway, except:

   - k obtained from sway nomograph ($1.0 \le k \le \infty$)
   - $\beta_d = \dfrac{\text{factored sustained shear}}{\text{total factored shear}}$
   - need to compute for each column in the storey

6. Calculate δ_s:

   – $\delta_s = \dfrac{1}{1 - \dfrac{\sum P_f}{\phi_m \cdot \sum P_c}}$ where:

   $\sum P_c$ = Σ Euler buckling loads for all columns on the storey

   – $\delta_s \le 2.5$ for gravity load only LCs to preclude sway buckling under these LCs *[CSA A23.3 2004 §10.16.5]*

7. Check if maximum moment occurs at one of column ends:

   – to do this, verify that: $\dfrac{\ell_u}{r} \le \dfrac{35}{\sqrt{P_f / (f'_c \cdot A_g)}}$

8. Calculate M_1, M_2 for each LC:

   – $M_1 = M_{1,ns} + \delta_s \cdot M_{1,s}$

   – $M_2 = M_{2,ns} + \delta_s \cdot M_{2,s}$

   – For gravity load only LCs: $M_{1,2} = \text{MIN}\{M_{1,2}, P_f(15 + 0.03 \cdot h)\}$

9. Design column for P_f, M_2 for each LC:

   – note: may be able to reduce the number of LCs by inspection

   – interaction diagrams used in the same way as before

Note: If column or beam size changes in the final design, it may necessary to do a second iteration with revised stiffness values.

Note: If verification in Step 7 fails:

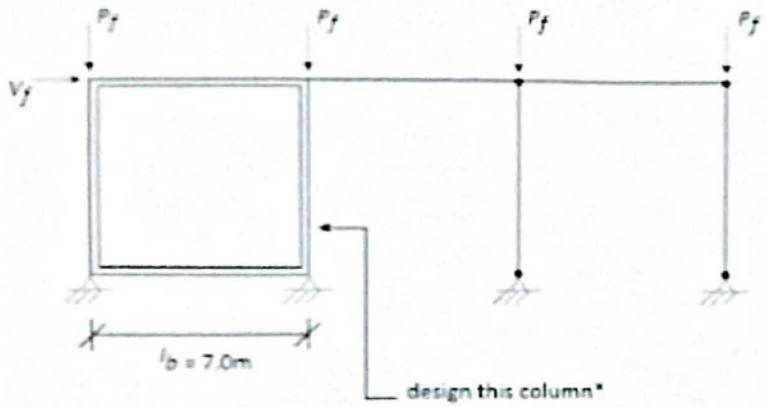
– increase column size, or

– design column for P_f, M_c for each load case, where:

$$M_c = (C_m \cdot M_2) / (1 - P_f / (\phi_m \cdot P_c)) \ge M_2$$

– assume: β_d is based on axial forces and $k \le 1.0$ ($k = 1.0$ can be conservatively used).

Example 8: Design the column in a sway frame



$$l_c = 6.6m, l_u = 6.0m$$

$$f'_c = 35\,MPa, f_y = 400\,MPa$$

For the column to be designed:

Load case	M top kNm	M bottom kNm	P kN
D	-60	60	210
L	-40	40	140
W	-41	41	12

From structural analysis of frame

3. Choose trial section:

$$A_{g\,trial} \geq \frac{P_f}{0.35\,f'_c + 0.65 \cdot f_y \cdot \rho_t}$$

$$\geq \frac{477.3 \cdot 1000}{0.35 \cdot 35 + 0.65 \cdot 400 \cdot 0.015} \geq 29554\,mm^2$$

$$0.015\ was\ assumed$$

$$\rightarrow h \geq 171.9mm$$

$$\rightarrow try\ 400 \cdot 400mm\ column$$

* design column for LC#2 only (1.25D + 1.5L + 0.4W)

1 & 2. Calculate loads, load effects:

$P_f = 1.25 \cdot 210 + 1.5 \cdot 140 + 0.4 \cdot 12 = 477.3$ KN

$$M_{f\,top}\,(sway) = 0.4\,(-41) = -16.4\,kNm$$

$$M_{f\,top}\,(non-sway) = 1.25\,(-60) + 1.5\,(-40)$$

$$= -135\,kNm$$

$$M_{f\,bott}(sway) = 0.4\,(41) = 16.4\,kNm$$

$$Mf_{f\,bott}\,(non-sway) = 125\,(60) + 1.5\,(40)$$

$$= 135\,kNm$$

3. Choose trial section:

$$A_{g\,trial} \geq \frac{P_f}{0.35\,f'_c + 0.65 \cdot f_y \cdot \rho_t}$$

$$\geq \frac{477.3 \cdot 1000}{0.35 \cdot 35 + 0.65 \cdot 400 \cdot 0.015} \geq 29554\,mm^2$$

$$0.015\ was\ assumed$$

$$\rightarrow h \geq 171.9mm$$

$$\rightarrow try\ 400 \cdot 400mm\ column$$

4. Sway or non-sway?

$$\therefore Q, \Delta_0\ not\ provided, assume\ sway\ frame\ (by\ inspection)\ *$$

*since column is in sway frame, design procedure is the same whether it is short or slender

5. Compute EI, k, P_c:

$$EI = \frac{0.4 \cdot E_c \cdot I_g}{1 + \beta_d}$$

$$E_c = 4500\sqrt{f'_c} = 4500\sqrt{35} = 26622\ \text{MPa}$$

$$I_g = \frac{bh^3}{12} = \frac{400^4}{12} = 2.13 \cdot 10^9\,mm^4$$

$$\beta_d = 0 \text{ (no sustained shear)}$$

$$\rightarrow EI = 0.4 \cdot 26622 \cdot 2.13 \cdot 10^9 = 2.27 \cdot 10^{13}\ Nmm^2$$

Calculation of k:

$$I_c = 0.7 \cdot I_g \text{ (columns)}$$

$$I_b = 0.35 \cdot I_g \text{ (beams)}$$

$$E_c I_c = 26622 \cdot 0.7 \cdot 400^4 / 12 = 4.0 \cdot 10^{13}\ Nmm^2$$

$$E_b I_b = 26622 \cdot 0.35 \cdot 400 \cdot 600^3 / 12 = 6.7 \cdot 10^{13}\ Nmm^2$$

$$\psi_{top} = \psi_{bott} = \frac{4.0 \cdot 10^{13} / 6600}{6.7 \cdot 10^{13} / 7000} = 0.63$$

From sway monograph: $k \simeq 1.2$

From this....

$$\delta_s = 1.42$$

6. Calculate M_1, M_2:

$$M_f, top = -135 + 1.42\,(-16.4) = -158.2\ kNm$$
$$M_f, bott = 135 + 1.42\,(16.4) = 158.2\ kNm$$
$$M_1 = M_2 = 158.2\ kNm$$

7. Design column:

$$\frac{P_f}{A_g} = 477.3 \cdot 1000/400^2 = 2.98$$

$$\frac{M_f}{(A_g \cdot h)} = 158.2 \cdot 10^6/400^3 = 2.47$$

Assume: 25M longitudinal bars

10M ties

40mm cover

$$\rightarrow \gamma = \frac{400 - 2(25/2 + 10 + 40)}{400} = 0.69 \cong 0.7$$

$$\rightarrow \rho_t \cong 0.012\ (Table\ 7.4.10)$$

$$A_s = 1920 mm^2$$

$$\rightarrow use\ 8 - 20M\ bars \quad \rightarrow A_s = 2400\ mm^2$$

Metric bar size	Linear Mass Density (kg/m)	Nominal diameter (mm)	Cross-sectional Area (mm²)
10M	0.785	11.3	100
15M	1.570	16.0	200
20M	2.355	19.5	300
25M	3.925	25.2	500
30M	5.495	29.9	700
35M	7.850	35.7	1000
45M	11.775	43.7	1500
55M	19.625	56.4	2500