4

DE Assignment 5

Benjamin Klassen
July 8 2020

① $(D^4 + 18D^2 + 81)y = 0$, let $D^2 = n$

$(n^2 + 18n + 81)y = 0$

$(n+9)(n+9)y = 0$

$\therefore D^2 = -9 \pm i0$

$-3 \pm i4$

$r = 5$

if roots $\alpha \pm i\beta$

$D^2 = 9(\cos\theta \pm i\sin\theta) = 9(-1+0)$

$D = \sqrt{9}\left[\cos\left(\frac{\theta + 2k\pi}{n}\right) \pm i\sin\left(\frac{\theta + 2k\pi}{n}\right)\right]$,

when $k=0$

$D = \sqrt{9}\left[\cos\left(\frac{\theta}{2}\right) \pm i\sin\left(\frac{\theta}{2}\right)\right]$, $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$, since $\cos\theta =$, quadrant 2, $\therefore$ $\cos\frac{\theta}{2} =$ quadrant 1 = +ve

$= 0 \pm 3i$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1-1}{2}} = 0$

$\sin\theta/2 = \pm\sqrt{\frac{2}{2}}$

$= \pm 1$

when $k=1$

$D = 3\left[\cos\left(\frac{\theta}{2} + \pi\right) \pm i\sin\left(\frac{\theta}{2} + \pi\right)\right]$, from trig identity $(\cos(A \pm B), \sin(A \pm B))$

$D = 3\left[-\cos\left(\frac{\theta}{2}\right) \pm i\sin\left(\frac{\theta}{2}\right)\right]$

$= 0 \pm 3i$

$\therefore$ repeated roots, $y_c = e^{0x}[A_0 + A_1x]\cos 3x + e^{0x}[B_0 + B_1x]\sin 3x$

$= [A_0 + A_1x]\cos 3x + [B_0 + B_1x]\sin 3x$

(2) $(D^4-4D^2+16)y=0$, let $D^2=n$

$(n^2-4n+16)y=0$

$n=\dfrac{4\pm\sqrt{16-4(16)}}{2}$

$=\dfrac{4\pm\sqrt{-48}}{2}$

$D^2=2\pm i\sqrt{12}$, $\overset{\alpha}{2}\pm \overset{iB}{2\sqrt{3}i}$

$r=\sqrt{4+4(3)}$
$=4$

$\therefore D=2\left(\cos\left(\frac{\theta}{2}+k\pi\right)\pm i\sin\left(\frac{\theta}{2}+k\pi\right)\right)$

$k=0$, $\cos\left(\frac{\theta}{2}\right)=\pm\sqrt{\dfrac{1+\cos\theta}{2}}$, $\cos\theta=\frac{2}{4}=\frac{1}{2}$ = quadrant 1, $\cos\frac{\theta}{2}$ = quadrant 1

$=+\sqrt{\dfrac{1+0.5}{2}}$

$=\dfrac{\sqrt{3}}{2}$

$\sin\left(\frac{\theta}{2}\right)=\sqrt{\dfrac{1-0.5}{2}}$

$=\dfrac{1}{2}$

$D=2\left(\frac{\sqrt{3}}{2}\pm i\frac{1}{2}\right)$, $\sqrt{3}\pm i1$

$k=1$, By trig identity, $D=2\left(-\frac{\sqrt{3}}{2}\pm i\frac{1}{2}\right)$, $-\sqrt{3}\pm 1i$

$\therefore y_c=e^{\sqrt{3}x}\left(A_1\cos x+B_1\sin x\right)+e^{-\sqrt{3}x}\left(A_2\cos x+B_2\sin(x)\right)$

③ $(D^4 - 5D^3 + 5D^2 + 5D - 6)y = 0$

$\phi(D) = 0$, D=2 is a soln

2	1	-5	5	5	-6
		2	-6	-2	6
	1	-3	-1	3	0

$(D-2)(D^3 - 3D^2 - D + 3) = 0$

D=1 is a soln

-1	1	-3	-1	3
		1	-2	-3
	1	-2	-3	0

$(D-2)(D-1)(D^2 - 2D - 3D) = 0$

$(D-2)(D-1)(D-3)(D+1) = 0$

$\therefore y_c = C_1 e^{-1x} + C_2 e^{x} + C_3 e^{2x} + C_4 e^{3x}$

④ $(D^2 - D - 2)y = 36xe^{2x}$

$\therefore y = \frac{1}{D^2-D-2}\left[36xe^{2x}\right]$, Theorem 2

$(D+2)(D+2)$

D^2+4D+4

$y = e^{2x}\left[\frac{1}{(D+2)^2-(D+2)-2}(36x)\right]$

$y = e^{2x}\left[\frac{1}{D^2+3D}36x\right]$

$y = e^{2x}\frac{1}{D}\frac{1}{(D+3)}36x$

$y = e^{2x}\frac{1}{D}\frac{1}{1+\frac{1}{3}D}12x$

$y = e^{2x}\frac{1}{D}\left(1-\frac{1}{3}D\right)12x$

$y = e^{2x}\frac{1}{D}\left(12x - \frac{12}{3}\right)$

$y = e^{2x}\left[6x^2 - 4x\right]$

$y_p = 2e^{2x}\left[3x^2 - 2x\right]$

(5) $(D^3 + D^2 + 5D + 5)y = 5\cos 2x$

$$y = \frac{1}{D^3 + D^2 + 5D + 5} 5\cos 2x$$

$$\begin{array}{r|rrrr} -1 & 1/5 & 1/5 & 1 & 1 \\ & & -1/5 & 0 & -1 \\ \hline & 1/5 & 0 & 1 & 0 \end{array}$$

$$= \frac{1}{\frac{1}{5}D^3 + \frac{1}{5}D^2 + D + 1} \cos 2x$$

$$= \frac{1}{(D+1)(1/5\, D^2 + 1)(D-1)} (D-1) \cos 2x$$

$$= \frac{D-1}{(D^2-1)(1/5\, D^2 + 1)} \cos 2x$$, Theorum 3

$$y = \frac{D-1}{(-2^2-1)(1/5(-2^2)+1)}$$

$$y_p = \frac{D-1}{(-5)(1/5)} \cos 2x$$

$$= -1\left[-2\sin 2x - \cos 2x\right]$$

$$y_p = 2\sin 2x + \cos 2x$$

⑥ $(D^4 - D^2)y = 2e^x$

$y = \frac{1}{D^4 - D^2} 2e^x$, Try Theorum 1

$y = \frac{2}{0} e^x$, ∴ Theorum 1 fails , ∴ Theorum 4

∴ $y = \frac{1}{D^4 - D^2} 2e^x = \frac{1}{\phi'(0)(1)} 2e^x x^1$

$y = \frac{1}{4D^3 - 2D} 2xe^x$

$= \frac{2}{2} xe^x$

$y_p = xe^x$