⑤ $y\,dx + (y^2 e^y - x)\,dy = 0$

$$\frac{dx}{dy} = -\frac{(y^2 e^y - x)}{y}$$

$$= -ye^y + \frac{x}{y}$$

$$\frac{dx}{dy} - \frac{1}{y}x = -ye^y$$

$$\therefore x = e^{\int \frac{1}{y}dy}\left[-\int ye^y e^{-\int \frac{1}{y}dy}\,dy + C\right]$$

$$= y\left[-\int e^y + C\right]$$

$$x = -ye^y + Cy$$

⑥ $\dfrac{dy}{dx} - \dfrac{1}{1+x}\, y = x + x^2$

$$e^{-\ln|(1+x)|}, \quad e^{\ln|(1+x)|}$$

$$y = e^{\int \frac{1}{1+x}dx}\left[\int (x+x^2)\, e^{-\int \frac{1}{1+x}dx}\, dx + C \right]$$

$$= (1+x)\left[\int \frac{x(1+x)}{(1+x)}\, dx + C \right]$$

$$y = (1+x)\left[\left(\frac{x^2}{2}\right) + C \right]$$

$$y = \frac{x^2}{2} + \frac{x^3}{2} + C\,(1+x)$$