Table of Trigonometric Identities

Trigonometric Functions

1. $\sin^2\theta + \cos^2\theta = 1$
   $\implies \quad 1 + \tan^2\theta = \sec^2\theta$
   $\implies \quad 1 + \cot^2\theta = \csc^2\theta$

2. $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

3. $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

4. $\sin 2\theta = 2\sin\theta\cos\theta$

5. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
   $= 2\cos^2\theta - 1$
   $= 1 - 2\sin^2\theta$

6. $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

7. $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

8. $\sin A\sin B = -\dfrac{1}{2}[\cos(A+B) - \cos(A-B)]$

9. $\cos A\cos B = \dfrac{1}{2}[\cos(A+B) + \cos(A-B)]$

10. $\sin A\cos B = \dfrac{1}{2}[\sin(A+B) + \sin(A-B)]$

11. $\cos A\sin B = \dfrac{1}{2}[\sin(A+B) - \sin(A-B)]$

12. $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

13. $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

14. $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

15. $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

16. Euler's Formula: $e^{i\theta} = \cos\theta + i\sin\theta$

17. $z^n = a\big(\cos\theta + i\sin\theta\big)$
    $\implies z = \sqrt[n]{a}\Big(\cos\dfrac{\theta+2k\pi}{n} + i\sin\dfrac{\theta+2k\pi}{n}\Big), \quad k = 0, 1, \ldots, n-1$

Hyperbolic Functions

$\cosh^2 x - \sinh^2 x = 1$
$\implies \quad 1 - \tanh^2 x = \operatorname{sech}^2 x$
$\implies \quad \coth^2 x - 1 = \operatorname{csch}^2 x$

$\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

$\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

$\sinh 2x = 2\sinh x\cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$
$= 2\cosh^2 x - 1$
$= 1 + 2\sinh^2 x$

Table of Derivatives

1. $\dfrac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$

2. $\dfrac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^x = \mathrm{e}^x \implies \dfrac{\mathrm{d}}{\mathrm{d}x}a^x = a^x \ln a, \qquad \because\ a^x = \mathrm{e}^{x\ln a}$

3. $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln x = \dfrac{1}{x} \implies \dfrac{\mathrm{d}}{\mathrm{d}x}\log_a x = \dfrac{1}{x\ln a}, \quad \because\ \log_a x = \dfrac{\ln x}{\ln a}$

Trigonometric Functions

4. $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$

5. $\dfrac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

6. $\dfrac{\mathrm{d}}{\mathrm{d}x}\tan x = \dfrac{1}{\cos^2 x} = \sec^2 x$

7. $\dfrac{\mathrm{d}}{\mathrm{d}x}\cot x = -\dfrac{1}{\sin^2 x} = -\csc^2 x$

8. $\dfrac{\mathrm{d}}{\mathrm{d}x}\sec x = \dfrac{\sin x}{\cos^2 x} = \tan x \sec x$

9. $\dfrac{\mathrm{d}}{\mathrm{d}x}\csc x = -\dfrac{\cos x}{\sin^2 x} = -\cot x \csc x$

Hyperbolic Functions

$\dfrac{\mathrm{d}}{\mathrm{d}x}\sinh x = \cosh x$

$\dfrac{\mathrm{d}}{\mathrm{d}x}\cosh x = \sinh x$

$\dfrac{\mathrm{d}}{\mathrm{d}x}\tanh x = \dfrac{1}{\cosh^2 x} = \operatorname{sech}^2 x$

$\dfrac{\mathrm{d}}{\mathrm{d}x}\coth x = -\dfrac{1}{\sinh^2 x} = -\operatorname{csch}^2 x$

$\dfrac{\mathrm{d}}{\mathrm{d}x}\operatorname{sech} x = -\dfrac{\sinh x}{\cosh^2 x} = -\tanh x \operatorname{sech} x$

$\dfrac{\mathrm{d}}{\mathrm{d}x}\operatorname{csch} x = -\dfrac{\cosh x}{\sinh^2 x} = -\coth x \operatorname{csch} x$

Inverse Trigonometric Functions

10. $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin^{-1} x = \dfrac{1}{\sqrt{1-x^2}}$ $\qquad$ $\dfrac{\mathrm{d}}{\mathrm{d}x}\cos^{-1} x = -\dfrac{1}{\sqrt{1-x^2}}$

11. $\dfrac{\mathrm{d}}{\mathrm{d}x}\tan^{-1} x = \dfrac{1}{1+x^2}$ $\qquad$ $\dfrac{\mathrm{d}}{\mathrm{d}x}\cot^{-1} x = -\dfrac{1}{1+x^2}$

12. $\dfrac{\mathrm{d}}{\mathrm{d}x}\sec^{-1} x = \dfrac{1}{x\sqrt{x^2-1}}$ $\qquad$ $\dfrac{\mathrm{d}}{\mathrm{d}x}\csc^{-1} x = -\dfrac{1}{x\sqrt{x^2-1}}$

Table of Integrals

1. $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$

2. $\displaystyle\int \frac{1}{x}\,dx = \ln|x|, \quad x \neq 0 \qquad \int \ln x\,dx = x\left(\ln x - 1\right), \quad x > 0$

3. $\displaystyle\int e^{ax}\,dx = \frac{1}{a}e^{ax} \qquad \int b^{ax}\,dx = \frac{b^{ax}}{a \ln b}, \quad b > 0$

	Trigonometric Functions	Hyperbolic Functions
4.	$\displaystyle\int \sin x\,dx = -\cos x$	$\displaystyle\int \sinh x\,dx = \cosh x$
5.	$\displaystyle\int \cos x\,dx = \sin x$	$\displaystyle\int \cosh x\,dx = \sinh x$
6.	$\displaystyle\int \tan x\,dx = -\ln\left|\cos x\right|$	$\displaystyle\int \tanh x\,dx = \ln\cosh x$
7.	$\displaystyle\int \cot x\,dx = \ln\left|\sin x\right|$	$\displaystyle\int \coth x\,dx = \ln\left|\sinh x\right|$
8.	$\displaystyle\int \sec x\,dx = \ln\left|\sec x + \tan x\right| = \ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| = \ln\left|\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|$	$\displaystyle\int \operatorname{sech} x\,dx = \tan^{-1}\left(\sinh x\right)$
9.	$\displaystyle\int \csc x\,dx = \ln\left|\csc x - \cot x\right| = \ln\left|\tan\frac{x}{2}\right|$	$\displaystyle\int \operatorname{csch} x\,dx = \ln\left|\tanh\frac{x}{2}\right|$
10.	$\displaystyle\int \sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}\sin 2x$	$\displaystyle\int \sinh^2 x\,dx = \frac{1}{4}\sinh 2x - \frac{x}{2}$
11.	$\displaystyle\int \cos^2 x\,dx = \frac{x}{2} + \frac{1}{4}\sin 2x$	$\displaystyle\int \cosh^2 x\,dx = \frac{1}{4}\sinh 2x + \frac{x}{2}$
12.	$\displaystyle\int \tan^2 x\,dx = \tan x - x$	$\displaystyle\int \tanh^2 x\,dx = -\tanh x + \frac{1}{2}\ln\left\lvert\frac{\tanh x + 1}{\tanh x - 1}\right\rvert$
13.	$\displaystyle\int \cot^2 x\,dx = -\cot x - x$	$\displaystyle\int \coth^2 x\,dx = -\coth x + \frac{1}{2}\ln\left\lvert\frac{\coth x + 1}{\coth x - 1}\right\rvert$
14.	$\displaystyle\int \sec^2 x\,dx = \tan x$	$\displaystyle\int \operatorname{sech}^2 x\,dx = \tanh x$
15.	$\displaystyle\int \csc^2 x\,dx = -\cot x$	$\displaystyle\int \operatorname{csch}^2 x\,dx = -\coth x$

16. $\int x \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax$

17. $\int x^2 \sin ax \, dx = \frac{2x}{a^2} \sin ax - \frac{a^2x^2 - 2}{a^3} \cos ax$

18. $\int x^3 \sin ax \, dx = \frac{3a^2x^2 - 6}{a^4} \sin ax - \frac{a^2x^3 - 6x}{a^3} \cos ax$

19. $\int x^4 \sin ax \, dx = \frac{4a^2x^3 - 24x}{a^4} \sin ax - \frac{a^4x^4 - 12a^2x^2 + 24}{a^5} \cos ax$

20. $\int x \cos ax \, dx = \frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax$

21. $\int x^2 \cos ax \, dx = \frac{a^2x^2 - 2}{a^3} \sin ax + \frac{2x}{a^2} \cos ax$

22. $\int x^3 \cos ax \, dx = \frac{a^2x^3 - 6x}{a^3} \sin ax + \frac{3a^2x^2 - 6}{a^4} \cos ax$

23. $\int x^4 \cos ax \, dx = \frac{a^4x^4 - 12a^2x^2 + 24}{a^5} \sin ax + \frac{4a^2x^3 - 24x}{a^4} \cos ax$

24. $\int e^{bx} \sin ax \, dx = e^{bx} \frac{b \sin ax - a \cos ax}{a^2 + b^2}$

25. $\int e^{bx} \cos ax \, dx = e^{bx} \frac{a \sin ax + b \cos ax}{a^2 + b^2}$

26. $\int x e^{ax} \, dx = \frac{ax - 1}{a^2} e^{ax}$

27. $\int x^2 e^{ax} \, dx = \frac{a^2x^2 - 2ax + 2}{a^3} e^{ax}$

28. $\int x^3 e^{ax} \, dx = \frac{a^3x^3 - 3a^2x^2 + 6ax - 6}{a^4} e^{ax}$

29. $\int x^4 e^{ax} \, dx = \frac{a^4x^4 - 4a^3x^3 + 12a^2x^2 - 24ax + 24}{a^5} e^{ax}$

30. $\int x^5 e^{ax} \, dx = \frac{a^5x^5 - 5a^4x^4 + 20a^3x^3 - 60a^2x^2 + 120ax - 120}{a^6} e^{ax}$

31. $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a > 0$

32. $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \ln \frac{a + x}{a - x} \qquad \text{or} \qquad \frac{1}{a} \tanh^{-1} \frac{x}{a}, \quad |x| < |a|$

33. $\displaystyle\int \frac{1}{x^2-a^2}\,\mathrm{d}x = \frac{1}{2a}\ln\frac{x-a}{x+a} \qquad \text{or} \qquad -\frac{1}{a}\coth^{-1}\frac{x}{a}, \quad |x|>|a|$

34. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,\mathrm{d}x = \sin^{-1}\frac{x}{a} \qquad \text{or} \qquad -\cos^{-1}\frac{x}{a}$

35. $\displaystyle\int \frac{1}{\sqrt{x^2-a^2}}\,\mathrm{d}x = \cosh^{-1}\frac{x}{a} \qquad \text{or} \qquad \ln\left|x+\sqrt{x^2-a^2}\right|$

36. $\displaystyle\int \frac{1}{\sqrt{x^2+a^2}}\,\mathrm{d}x = \sinh^{-1}\frac{x}{a} \qquad \text{or} \qquad \ln\left(x+\sqrt{x^2+a^2}\right)$

37. $\displaystyle\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$

38. $\displaystyle\int \sqrt{x^2+a^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\sinh^{-1}\frac{x}{a}$

$\displaystyle\text{or} \qquad \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right)$

39. $\displaystyle\int \sqrt{x^2-a^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\cosh^{-1}\frac{x}{a}$

$\displaystyle\text{or} \qquad \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|$

40. $\displaystyle\int \frac{1}{x\sqrt{a^2-x^2}}\,\mathrm{d}x = -\frac{1}{a}\cosh^{-1}\frac{a}{x} \qquad \text{or} \qquad -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$

41. $\displaystyle\int \frac{1}{x\sqrt{x^2+a^2}}\,\mathrm{d}x = -\frac{1}{a}\sinh^{-1}\frac{a}{x} \qquad \text{or} \qquad -\frac{1}{a}\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right|$

42. $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,\mathrm{d}x = \frac{1}{a}\cos^{-1}\frac{a}{x} \qquad \text{or} \qquad \frac{1}{a}\sec^{-1}\frac{x}{a}$

43. $\displaystyle\int \frac{1}{\sqrt{2ax-x^2}}\,\mathrm{d}x = \cos^{-1}\left(1-\frac{x}{a}\right) \qquad \text{or} \qquad \sin^{-1}\left(\frac{x}{a}-1\right)$

44. $\displaystyle\int \sqrt{2ax-x^2}\,\mathrm{d}x = \frac{x-a}{2}\sqrt{2ax-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}-1\right)$

45. $\displaystyle\int_0^{\frac{\pi}{2}} \left\{\begin{matrix}\sin^n\theta\\ \cos^n\theta\end{matrix}\right\}\mathrm{d}\theta = \frac{(n-1)!!}{n!!}\times\begin{cases}\frac{1}{2}\pi, & \text{if } n \text{ is an even integer}\\ 1, & \text{if } n \text{ is an odd integer}\end{cases}$

46. $\displaystyle\int_0^{\frac{\pi}{2}} \sin^n\theta\cos^m\theta\,\mathrm{d}\theta = \frac{(n-1)!!\,(m-1)!!}{(n+m)!!}\times\begin{cases}\frac{1}{2}\pi, & n, m \text{ even integers}\\ 1, & \text{otherwise}\end{cases}$

✍ $\displaystyle n!! = \begin{cases} n\cdot(n-2)\cdots 5\cdot 3\cdot 1, & n>0 \text{ odd integer}\\ n\cdot(n-2)\cdots 6\cdot 4\cdot 2, & n>0 \text{ even integer}\\ 1, & n=0\end{cases}$