Chapter 5. Plate Girders

Plate girders are custom fabricated beam members utilised to carry large loads over long spans. They are used in buildings and industrial structures for long span floor girders to create large open space. Plate girders are very commonly used in bridge construction, too.

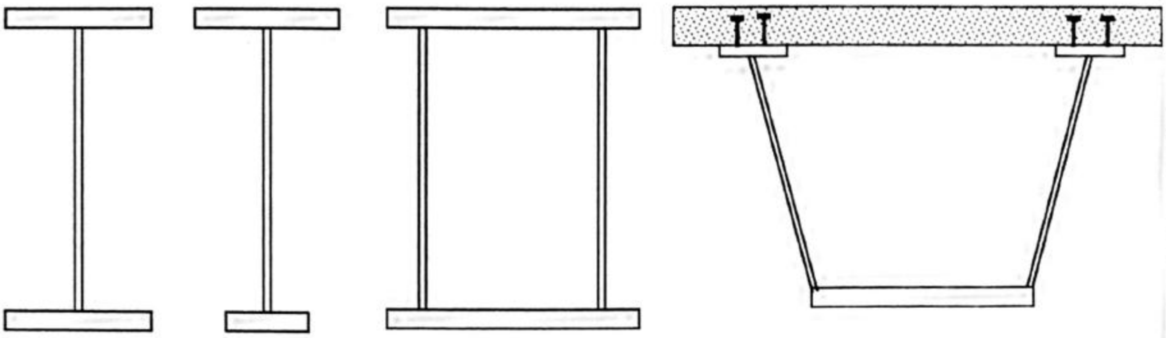
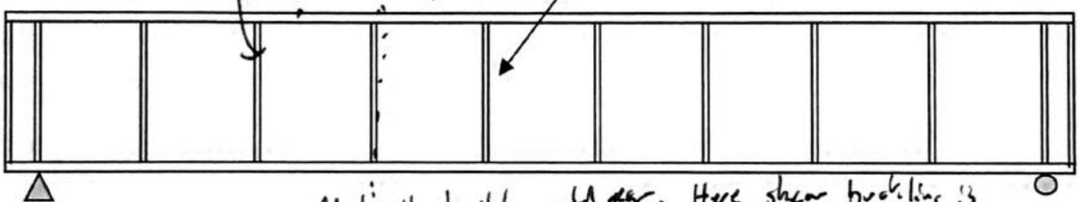


Plate Girder Box Girder Composite Box Girder

$M = \frac{wL^2}{8}$

$F = M/d$

Thin web for cost efficiency

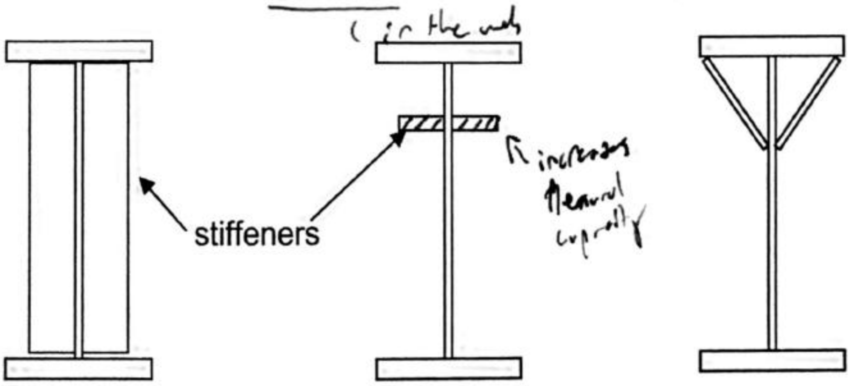


If thin, buckling would occur. Here, shear buckling is dominant. Stiffeners are used

A plate girder is generally defined as a built-up flexural member having a slender web. Special attention should be paid on the presence of slender web when designing a plate girder. Web stiffeners are generally required.

Web Stiffeners: functions

1. ♦ prevent buckling (in the web) due to compression from bending and shear
2. ♦ promote tension field action (additional resistance to increase capacity) to increase shear strength
3. ♦ prevent web local failure (in the web) at concentrated load locations



F_y: M_f: $\phi S F_y$, $M_f/\phi S$

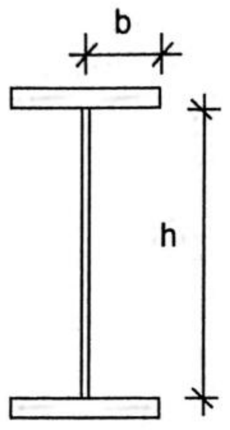


Plate Girder

Normally, for a plate girder,

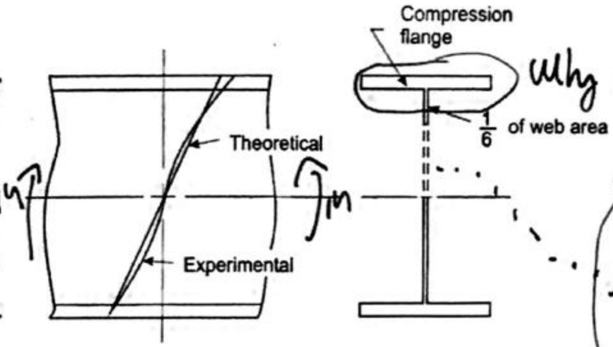
Flange: $b/t \le 200/\sqrt{F_y}$ (at least Calss 3)

Web: $h/w > 1900/\sqrt{F_y}$ (usually Class 4)

It is economical to use thin web. However, it leads to many problems which must be addressed in design.

Compression zone

Tension zone



Why local buckling is not located at this location

local buckling

Effect of Thin Webs on Moment Resistance

14.3.4

flange provides stiffness

- Web is thin and not perfectly flat, therefore, it tends to buckle or deflect.
- Part of the longitudinal shortening of the compression zone is associated with the above geometry changes and only the remainder of the shortening is associated with theoretical strain/stress relationship.
- "Soft web": the web will be less effective than expected and the flange will receive the higher stress.

Moment resistance reduction is applied since local buckling of web decreases the moment resistance. It is a linear reduction that is a function of web slenderness, the ratio of the area of flange to the area of web, and buckling load of the web.

Must be greater than zero

$$M_r' = M_r\left[1.0 - 0.0005\frac{A_w}{A_f}\left(\frac{h}{w} - \frac{1900}{\sqrt{F_y}}\right)\right]$$

2019

$$\cancel{M_f' = M_f\left[1.0 - 0.0005\frac{A_w}{A_f}\left(\frac{h}{w} - \frac{1900}{\sqrt{M_f/\varphi S}}\right)\right]}$$

2014

Max factored $M/\phi S$

A_w =web area; A_f = compression flange area; S = elastic section modulus

M_r ($M_r \le \phi M_y$) is the factored moment resistance based on Cl. 13.5 or 13.6. No reduction in moment resistance if

$$h/w \le 1900/\sqrt{F_y}$$

With the presence of axial force C_f in addition to the bending, the resistance moment for a plate girder is make ≥ 0

$$M_r' = M_r\left[1.0 - 0.0005\frac{A_w}{A_f}\left(\frac{h}{w} - \left(1 - 0.65\,{}^{C_f}\!/_{\varphi C_y}\right)\frac{1900}{\sqrt{F_y}}\right)\right]$$

$\frac{C_f}{\varphi C_y} \le 1$

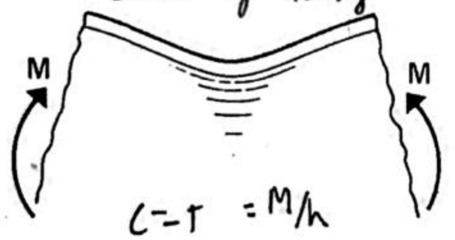
$C_y = F_y A$

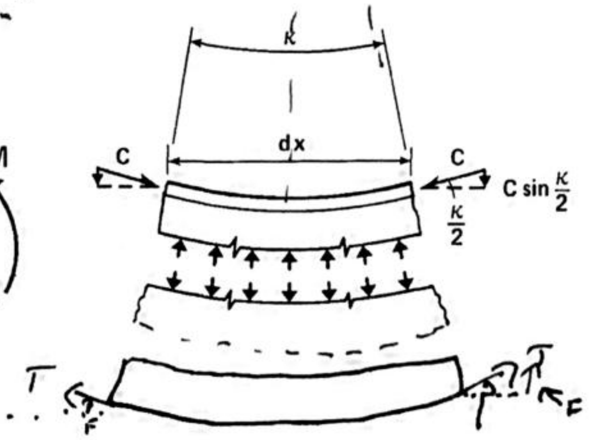
Vertical Buckling of Web

$K/2$

As shown in below, the curved (deformed) flange exerts vertical pressure on which may cause web buckling.

Web buckling due to compression induced by bending





where, κ is the bending curvature. The similar situation exists on tension flange, however, buckling will not occur on tension side.

Applied force of the web ***F***:

The vertical force transmitted to web from compressive flange over the length ***dx*** is

$$\boldsymbol{F = 2\,C \sin(\kappa/2)} \approx 2C\,\kappa/2 = C\kappa = A_f\,\sigma_f\,\kappa$$

where,

$$\because\ \kappa = \frac{\varepsilon_f dx}{h/2} \qquad \therefore\ \ F = {}^{2A_f\sigma_f\varepsilon_f dx}\!\Big/_{h}$$

A_f and σ_f is the area and normal stress of compressive flange, respectively.

Resisting force of the web ***R***:

Based on elastic buckling, the vertical resisting force over the length ***dx*** is

$$R = \sigma_{cr} A$$

where,

A is the web area in compression over the length ***dx***, $A = w\,dx$, and σ_{cr} is the elastic buckling stress for web plate:

Plate eqn

$$\sigma_{cr} = \frac{\pi^2 E}{(1-\mu^2)\left(\frac{KL}{r}\right)^2} = \frac{k\pi^2 E}{(1-\mu^2)\left(\frac{L}{r}\right)^2};\ \text{where, } k = \frac{1}{K^2}$$

based on boundary conditions of the plate

$$\because \quad r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{1}{12}w^3 dx}{wdx}} = \frac{w}{\sqrt{12}} \quad \text{and} \quad L = h$$

$$\therefore \quad \sigma_{cr} = \frac{k\pi^2 E}{12(1-\mu^2)\left(\frac{h}{w}\right)^2}$$

Let $k = 1$ (conservative), therefore,

$$R = \sigma_{cr} A = \frac{\pi^2 E}{12(1-\mu^2)\left(\frac{h}{w}\right)^2}\underbrace{w\,dx}_{A}$$

Considering the equilibrium between applied and resisting forces,

$$\frac{2A_f \sigma_f \varepsilon_f dx}{h} = \frac{\pi^2 E}{12(1-\mu^2)\left(\frac{h}{w}\right)^2} w\,dx$$

F=R

solve for *h/w* and note that $A_w = hw$, therefore

μ: poisson ratio

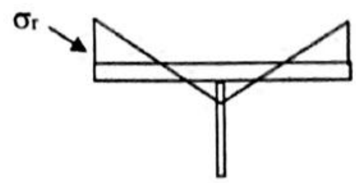
$$\frac{h}{w} = \sqrt{\frac{\pi^2 E A_w}{24(1-\mu^2)A_f \sigma_f \varepsilon_f}}$$

Benjamin K

By considering,

- Residual stress:

conservative



$$\varepsilon_f = (\sigma_y + \sigma_r) / E = \frac{F_y + \sigma_r}{8} \cdot \frac{1}{E}$$

- $A_f / A_w = 0.5$ (conservative)

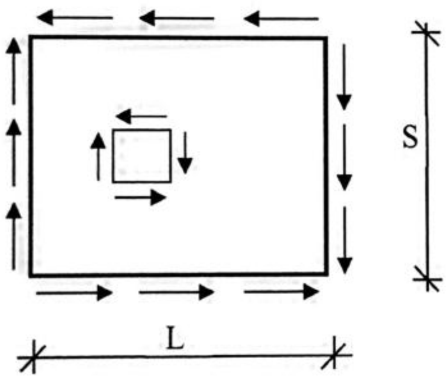
- Poission's ratio $\mu = 0.3$ and Young's modulus $E = 2\times10^5$ MPa

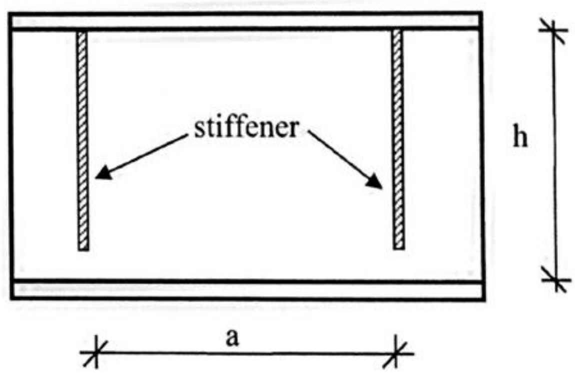
then, $h/w = \frac{83000}{F_y}$ SI limit only 14.3.1

This is the maximum permissible web slenderness h/w. F_y is associated with compression flange steel. This limit may be waived if analysis indicates that buckling of the compression flange into the web will not occur at factored load levels. $F_y = 300$ MPa, $h/w = 276.6$; if $w = 26$ mm, $h = 6415$ mm
$F_y = 350$ MPa, $h/w = 237.1$; if $w = 25$ mm, $h = 5925$ mm

Web Buckling under Shear 13.4.1.1

Although bending can be presented unaccompanied by shear, a transversely loaded plate girder will always have shear combined with moment. The thin web of the plate girder could buckle under the shear stress.





S = Smaller panel dimension; L = Larger panel dimension

$$\because \quad \tau_{cr} = \frac{k\pi^2 E}{12(1-\mu^2)\left(\frac{S}{w}\right)^2}; \qquad \text{and} \quad k = 5.34 + \frac{4}{\left(L/S\right)^2}$$

$$\therefore \quad \tau_{cr} = \left(5.34 + \frac{4}{\left(L/S\right)^2}\right)\frac{\pi^2 E}{12(1-\mu^2)\left(\frac{S}{w}\right)^2} = \left(\frac{5.34}{\left(S/L\right)^2} + 4\right)\frac{\pi^2 E}{12(1-\mu^2)\left(\frac{L}{w}\right)^2}$$

Using plate girder nomenclature, **a**, **h**, F_{cre}, etc. The elastic critical plate buckling stress in shear

$$F_{cre} = \frac{k_v \pi^2 E}{12(1-\mu^2)\left(\frac{h}{w}\right)^2}$$

kv - Plate shear buckling coefficient

If h is smaller panel dimension, i.e. **a/h ≥ 1**, then

$$k_v = 5.34 + \frac{4}{\left(a/h\right)^2}$$

Web:
- hinged edge, kv = 5.34
- fixed edge, kv = 8.94
- fixed edge, if the compression flange of the girder is continuously contact with concrete slab

otherwise, if **a/h < 1**

$$k_v = \frac{5.34}{\left(a/h\right)^2} + 4$$

However, S16 doesn't acknowledge it

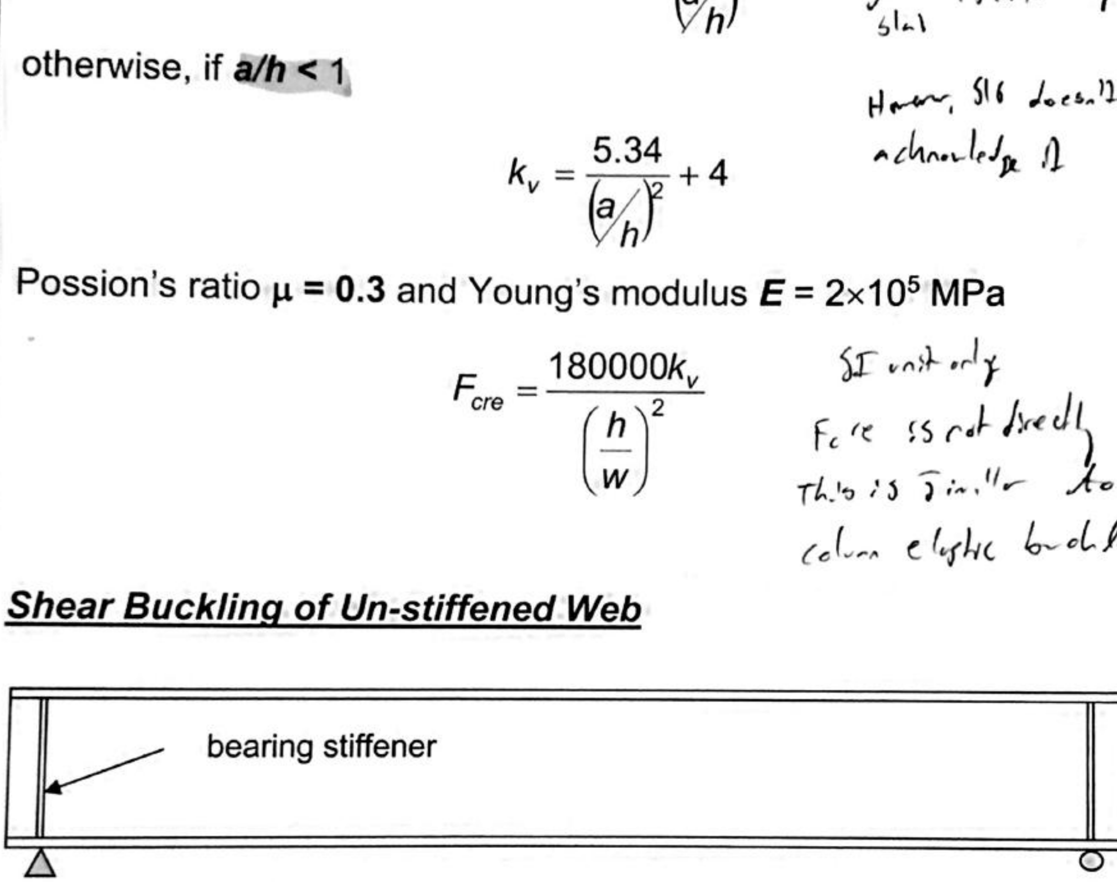
Possion's ratio **μ = 0.3** and Young's modulus ***E*** = 2×10^5 MPa — 13.4.1.1(b)

$$F_{cre} = \frac{180000 k_v}{\left(\frac{h}{w}\right)^2}$$

SI unit only

Fcre is not directly related to Fy. This is similar to the case of column elastic buckling strength

Shear Buckling of Un-stiffened Web



Let *a/h* = 20, then $k_v = 5.34 + 4 / (a/h)^2 = 5.34 + 0.01$

It is almost as the same as *a/h* = ∞, therefore $k_v = 5.34$

Elastic critical buckling stress,

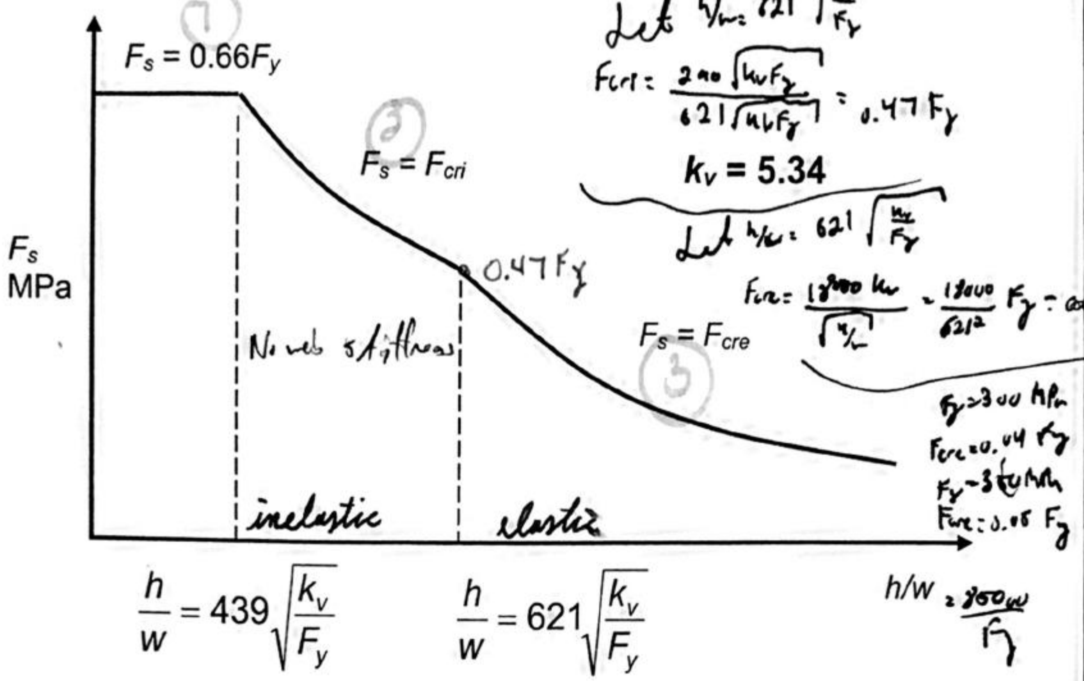
$$F_{cre} = \frac{180000 \times 5.34}{\left(\frac{h}{w}\right)^2} = \frac{961200}{\left(\frac{h}{w}\right)^2}$$

If AND(h/w = 83000/Fy, Fy = 300 MPa)
Fcre = 12.1 MPa

If Fy = 300 MPa
Fcre = 17 MPa

Inelastic critical buckling stress (residual stress, test results)

$$F_{cri} = \frac{290\sqrt{k_v F_y}}{\left(\frac{h}{w}\right)} = \frac{670F_y}{\left(\frac{h}{w}\right)}$$



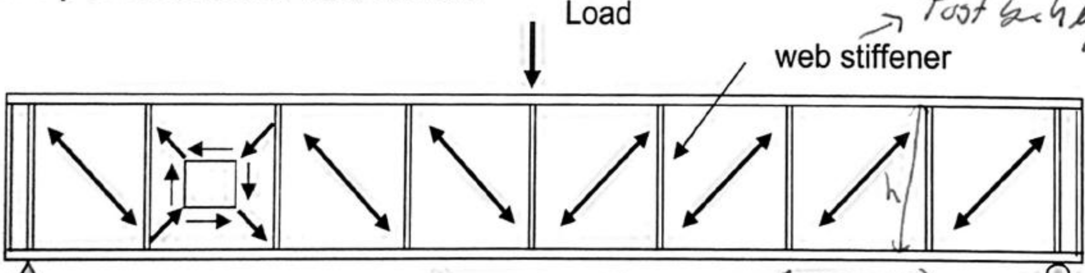
Shear Buckling of Stiffened Web

Tension Field Action: shear capacity > theoretical value? Post buckling strength

Web stiffeners improve shear strength of web – increase buckling stress and permit tension field action.

Post buckling strength

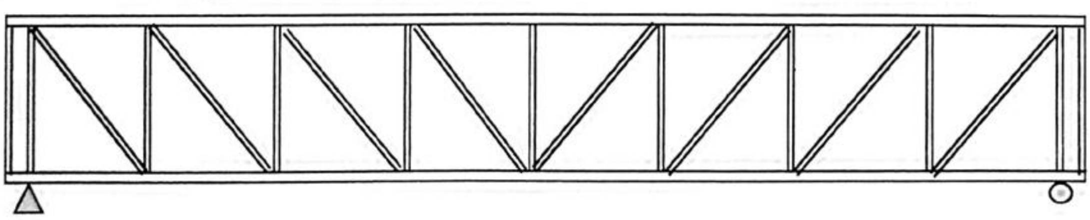
aspect ratio a/h



Girder may become like truss after buckle:

Similar to diagonal members of a Pratt Truss

Truss more economical



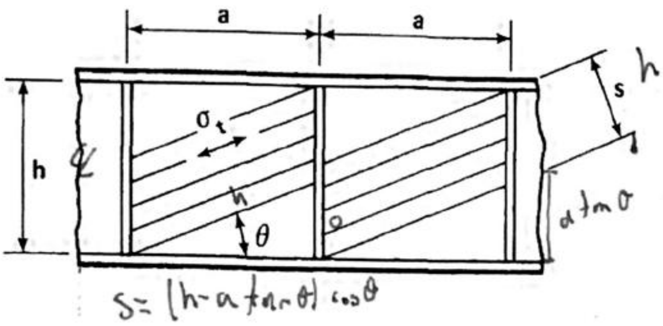
V_b – shear capacity of web contributed by normal beam capacity (capacity of theoretical buckling)

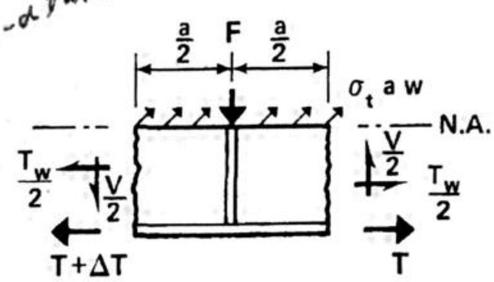
V_t – additional shear capacity of web due to tension field action

V_u – the ultimate shear capacity of web ← Post buckling strength (only develops after web buckles)

$$V_u = V_b + V_t$$

$$V_b = \tau_{cr}\, h w \qquad A_w \cdot h_w$$





$$s = h\cos\theta - a\sin\theta$$

$$\therefore \qquad V_t = \underline{\sigma_t\, w \sin\theta\, s} = \sigma_t\, w \sin\theta\,(h\cos\theta - a\sin\theta)$$

$\sigma_t \sin\theta\,(w \cdot s)$

to obtain maximum V_t,

$$Let\ \frac{\partial V_t}{\partial \theta} = 0; \qquad \Rightarrow\ \theta = \tan^{-1}\left(\sqrt{1+(a/h)^2} - a/h\right)$$

$$V_t = \sigma_t w h \frac{1}{2\sqrt{1+(a/h)^2}}$$

Therefore, the ultimate shear strength is

$$V_u = V_b + V_t = \tau_{cr} h w + \sigma_t \frac{wh}{2}\frac{1}{\sqrt{1+(a/h)^2}}$$

By assuming the linear relationship between tension and shear for element subjected to pure shear or pure tension, with $\theta = 45^\circ$ (approximately), the yield condition of combining tension and shear is

$$\frac{\sigma_t}{\sigma_y} = 1 - \frac{\tau_{cr}}{\tau_y} \qquad \sigma_t = \left(1 - \frac{\tau_{cr}}{\tau_y}\right)\sigma_y$$

Then, the ultimate shear strength can be rewritten as

$$V_u = \tau_{cr} hw + \frac{\sigma_y wh}{2\sqrt{1+(a/h)^2}} - \frac{\tau_{cr}}{\tau_y}\frac{\sigma_y}{2}\frac{wh}{\sqrt{1+(a/h)^2}} \quad (A)$$

The critical plate buckling stress in shear

$$\tau_{cr} = \frac{k_v \pi^2 E}{12(1-\mu^2)(h/w)^2} \quad \frac{180760\, k_v}{(h/w)^2}$$

If $a/h \geq 1$, then

$$k_v = 5.34 + \frac{4}{(a/h)^2} \quad \frac{\pi^2 E}{12(1-\mu^2)} = 180\,760\ SE$$

otherwise, if $a/h < 1$

Substituting following eqn (A) $$k_v = \frac{5.34}{(a/h)^2} + 4$$

$\mu = 0.3$; $E = 2\times10^5$ MPa; $\tau_y = F_y/\sqrt{3}$ and $\sigma_y = F_y$

The ultimate shear stress, $F_s = V_u / A_w = V_u / (hw)$

$$F_s = \frac{180760 k_v}{(h/w)^2} + \frac{F_y}{2\sqrt{1+(a/h)^2}} - \frac{156544 k_v}{(h/w)^2\sqrt{1+(a/h)^2}} \quad (AA)$$
$$\approx F_{cre} + k_a\left(0.5F_y - 0.866F_{cre}\right)$$

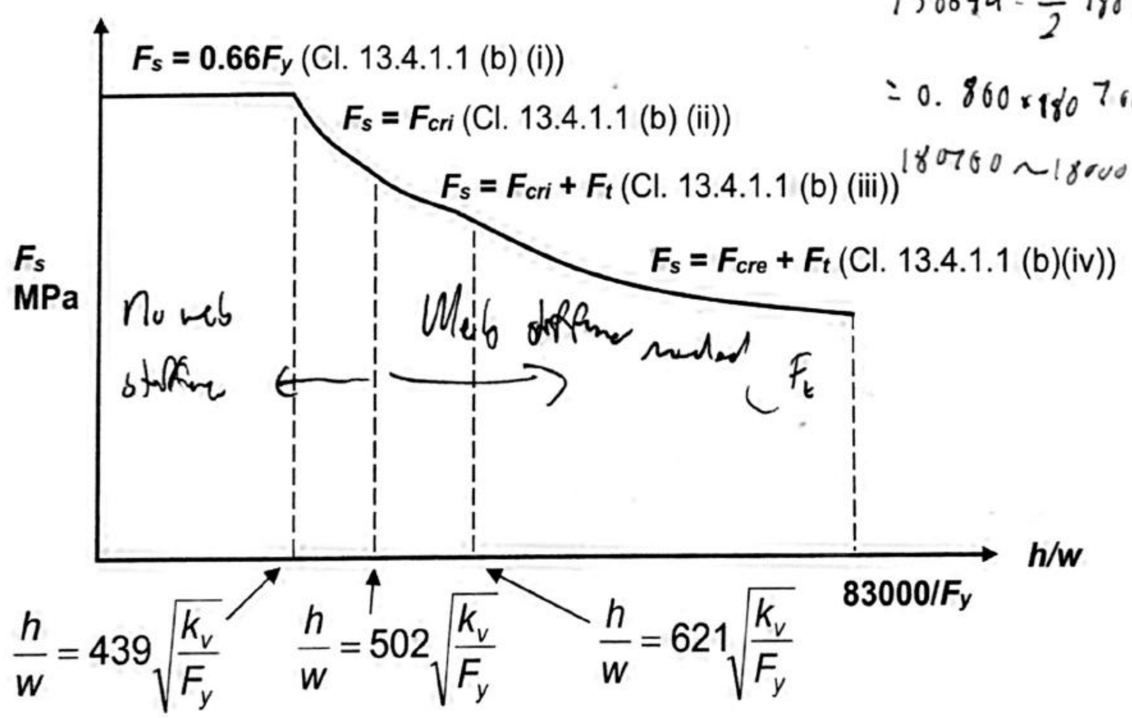
Eqn (AA)

This is a more fundamental form of Clause 13.4.1.1 (b) (iv)

$156544 = \frac{\sqrt{3}}{2}\, 180\,760$

$= 0.866 \times 180\,760$

$180760 \sim 18000$



Shear Resistance :

Stiffened Webs of Flexural Members with Two Flanges 13.4.1

Elastic Analysis 13.4.1.1

The factored shear resistance, V_r, developed by the web of a flexural member shall be taken as

$$V_r = \phi A_w F_s$$

where, $A_w = d \times w$ for rolled shapes and $h \times w$ for girders; F_s is taken as follows:

(a) when $\dfrac{h}{w} \le 439\sqrt{\dfrac{k_v}{F_y}}$; $F_s = 0.66F_y$ Web shear yielding

(b) when $439\sqrt{\dfrac{k_v}{F_y}} < \dfrac{h}{w} \le 502\sqrt{\dfrac{k_v}{F_y}}$; $F_s = F_{cri}$ Web inelastic shear buckling

No web stiffener ↑

↓ w/ web stiffener

(c) when $502\sqrt{\dfrac{k_v}{F_y}} < \dfrac{h}{w} \le 621\sqrt{\dfrac{k_v}{F_y}}$; Web inelastic shear buckling strength + post inelastic shear buckling strength

$$F_s = F_{cri} + k_a(0.50F_y - 0.866F_{cri})$$

(d) when $621\sqrt{\dfrac{k_v}{F_y}} < \dfrac{h}{w}$; $F_s = F_{cre} + k_a(0.50F_y - 0.866F_{cre})$

where, k_v = shear bucking coefficient

Web elastic shear buckling strength + post elastic shear buckling

(i) when $\dfrac{a}{h} < 1$; $k_v = 4 + \dfrac{5.34}{(a/h)^2}$

(ii) when $\dfrac{a}{h} \ge 1$; $k_v = 5.34 + \dfrac{4}{(a/h)^2}$

a/h =aspect ratio, the ratio of the distance between stiffeners to web depth

$$F_{cri} = 290\frac{\sqrt{F_y k_v}}{(h/w)}$$

k_a = aspect coefficient $= \dfrac{1}{\sqrt{1+(a/h)^2}}$ $F_{cre} = \dfrac{180000k_v}{(h/w)^2}$