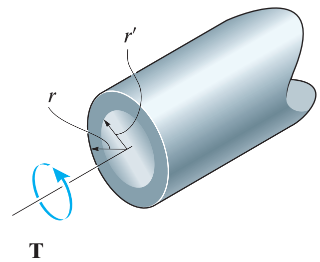
10–1.

The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-half of the applied torque $(T/2)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



SOLUTION

(a) $\tau_{\max} = \dfrac{Tc}{J} = \dfrac{Tr}{\frac{\pi}{2} r^4} = \dfrac{2T}{\pi r^3}$

$$\tau = \frac{\left(\frac{T}{2}\right) r'}{\frac{\pi}{2}(r')^4} = \frac{T}{\pi (r')^3}$$

Since $\tau = \dfrac{r'}{r}\tau_{\max}$, $\quad \dfrac{T}{\pi (r')^3} = \dfrac{r'}{r}\left(\dfrac{2T}{\pi r^3}\right)$

$r' = \dfrac{r}{2^{\frac{1}{4}}} = 0.841r$ **Ans.**

(b) $\displaystyle\int_0^{\frac{r}{2}} dT = 2\pi \int_0^{r'} \tau \rho^2 \, d\rho$

$$\int_0^{\frac{r}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r} \tau_{\max} \rho^2 \, d\rho$$

$$\int_0^{\frac{r}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r}\left(\frac{2T}{\pi r^3}\right) \rho^2 \, d\rho$$

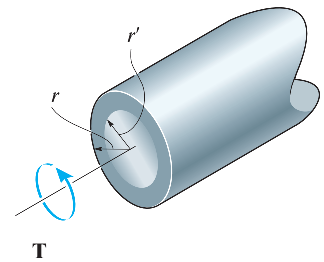
$$\frac{T}{2} = \frac{4T}{r^4} \int_0^{r'} \rho^3 \, d\rho$$

$r' = \dfrac{r'}{2^{\frac{1}{4}}} = 0.841r$ **Ans.**

Ans:
$r' = 0.841r$

10–2.

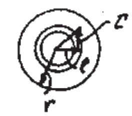
The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-quarter of the applied torque $(T/4)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



SOLUTION

(a) $\tau_{\max} = \dfrac{Tc}{J} = \dfrac{T(r)}{\frac{\pi}{2}(r^4)} = \dfrac{2T}{\pi r^3}$

Since $\tau = \dfrac{r'}{r}\tau_{\max} = \dfrac{2Tr'}{\pi r^4}$



$\tau' = \dfrac{T'c'}{J'}; \qquad \dfrac{2Tr'}{\pi r^4} = \dfrac{(\frac{T}{4})r'}{\frac{\pi}{2}(r')^4}$

$r' = \dfrac{r}{4^{\frac{1}{4}}} = 0.707\, r$ **Ans.**

(b) $\tau = \dfrac{\rho}{c}\tau_{\max} = \dfrac{\rho}{r}\left(\dfrac{2T}{\pi r^3}\right) = \dfrac{2T}{\pi r^4}\rho;$ $\qquad dA = 2\pi\rho\, d\rho$

$dT = \rho\tau\, dA = \rho\left[\dfrac{2T}{\pi r^4}\rho\right](2\pi\rho\, d\rho) = \dfrac{4T}{r^4}\rho^3 d\rho$

$$\int_0^{\frac{T}{4}} dT = \frac{4T}{r^4}\int^{r'} \rho^3 d\rho$$

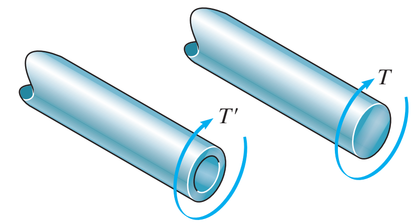
$\dfrac{T}{4} = \dfrac{4T}{r^4}\dfrac{\rho^4}{4}\Big|_0^{r'}; \qquad \dfrac{1}{4} = \dfrac{(r')^4}{r^4}$

$r' = 0.707\, r$ **Ans.**

Ans:
$r' = 0.707\, r$

10–3.

A shaft is made of an aluminum alloy having an allowable shear stress of $\tau_{allow} = 100$ MPa. If the diameter of the shaft is 100 mm, determine the maximum torque **T** that can be transmitted. What would be the maximum torque **T′** if a 75-mm-diameter hole were bored through the shaft? Sketch the shear-stress distribution along a radial line in each case.



SOLUTION

Allowable Shear Stress: Torsion formula can be applied. For the solid shaft,

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J};\ 100(10^6) = \frac{T(0.05)}{\frac{\pi}{2}(0.05^4)}$$

$$T = 19.63\left(10^3\right) \text{ N}\cdot\text{m} = 19.6 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

For the hollow shaft,

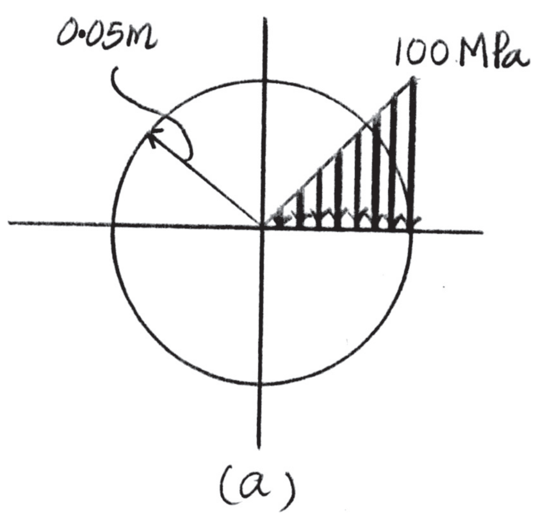
$$\tau_{max} = \tau_{allow} = \frac{Tc}{J};\ 100(10^6) = \frac{T'(0.05)}{\frac{\pi}{2}(0.05^4 - 0.0375^4)}$$

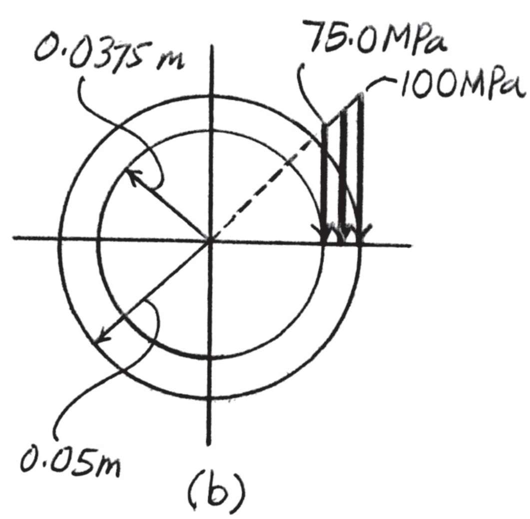
$$T' = 13.42\left(10^3\right) \text{ N}\cdot\text{m} = 13.4 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The shear stress at the inner surface where $\rho = 0.0375$ m is

$$\tau_{\rho} = 0.0375 \text{ m} = \frac{T'\rho}{\text{J}} = \frac{13.42(10^3)\,(0.0375)}{\frac{\pi}{2}(0.05^4 - 0.0375^4)} = 75.0(10^6) \text{ Pa} = 75.0 \text{ MPa}$$

The shear-stress distribution along the radius of the cross-section of the solid and hollow shafts are shown in Figs. *a* and *b*, respectively.

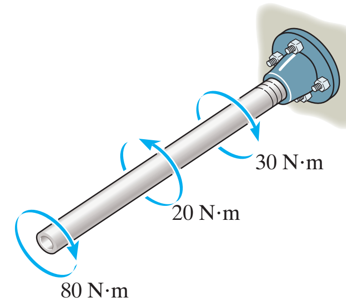




Ans:
$T = 19.6$ kN·m,
$T' = 13.4$ kN·m

***10–4.**

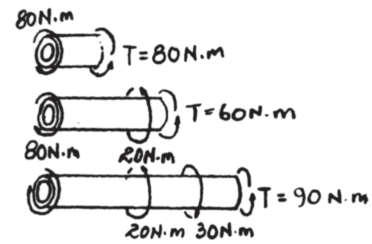
The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall and three torques are applied to it, determine the absolute maximum shear stress developed in the pipe.



SOLUTION

$$\tau_{max} = \frac{T_{max}\,c}{J} = \frac{90(0.02)}{\frac{\pi}{2}(0.02^4 - 0.0185^4)}$$

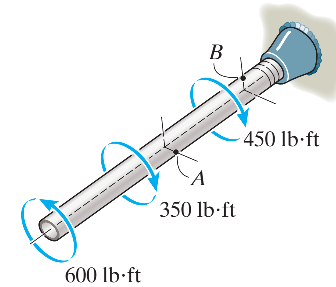
$$= 26.7 \text{ MPa}$$ **Ans.**



Ans:
$\tau_{max} = 26.7$ MPa

10–5.

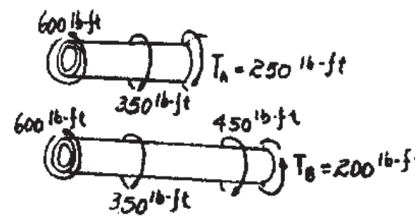
The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall and three torques are applied to it, determine the shear stress developed at points A and B. These points lie on the pipe's outer surface. Sketch the shear stress on volume elements located at A and B.



SOLUTION

$$\tau_A = \frac{Tc}{J} = \frac{250(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 3.45 \text{ ksi}$$ **Ans.**

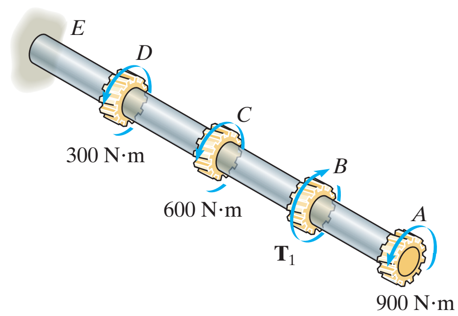
$$\tau_B = \frac{Tc}{J} = \frac{200(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 2.76 \text{ ksi}$$ **Ans.**



Ans:
$\tau_A = 3.45$ ksi, $\tau_B = 2.76$ ksi

10–6.

The solid aluminum shaft has a diameter of 50 mm and an allowable shear stress of $\tau_{\text{allow}} = 60$ MPa. Determine the largest torque T_1 that can be applied to the shaft if it is also subjected to the other torsional loadings. It is required that $\mathbf{T}_1$ act in the direction shown. Also, determine the maximum shear stress within regions CD and DE.



SOLUTION

Internal Torque: Assuming that failure occurs at region BC of the shaft, where the torque will be greatest. Referring to the FBD of the right segment of the shaft sectioned through region BC, Fig. a,

$$\Sigma M_x = 0; \qquad T_1 - 900 - T_{BC} = 0 \qquad T_{BC} = T_1 - 900$$

Maximum Shear Stress: Applying the torsion formula,

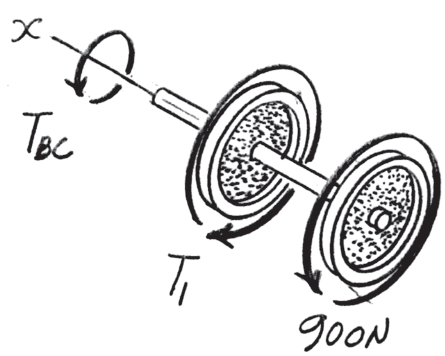
$$\tau_{\max} = \tau_{\text{allow}} = \frac{T_{BC}\,c}{J}; \qquad 60\left(10^6\right) = \frac{(T_1 - 900)(0.025)}{\frac{\pi}{2}\left(0.025^4\right)}$$

$$(T_1)_{\max} = T_1 = 2372.62\ \text{N}\cdot\text{m} = 2.37\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

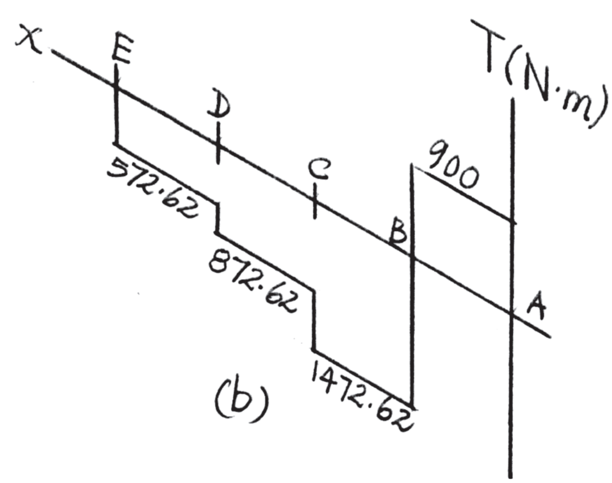
Using this result, the torque diagram shown in Fig. b can be plotted. This indicates that region BC indeed is subjected to maximum internal torque; thus, it is the critical region. From the torque diagram, the internal torques in regions CD and DE are $T_{CD} = 872.62\ \text{N}\cdot\text{m}$ and $T_{DE} = 572.62\ \text{N}\cdot\text{m}$, respectively.

$$(\tau_{\max})_{CD} = \frac{T_{CD}\,C}{J} = \frac{872.62\,(0.025)}{\frac{\pi}{2}\left(0.025^4\right)} = 35.55\left(10^6\right)\text{ Pa} = 35.6\text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\max})_{DE} = \frac{T_{DC}\,C}{J} = \frac{572.62\,(0.025)}{\frac{\pi}{2}\left(0.025^4\right)} = 23.33\left(10^6\right)\text{ Pa} = 23.3\text{ MPa} \qquad \textbf{Ans.}$$



(a)

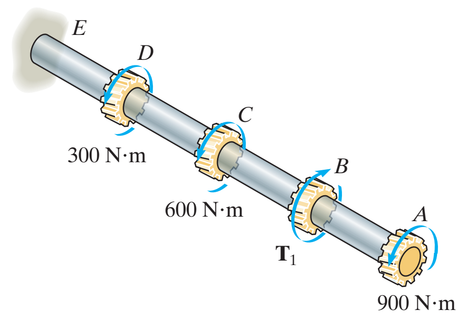


(b)

Ans:
$(T_1)_{\max} = 2.37\ \text{kN}\cdot\text{m}$,
$(\tau_{\max})_{CD} = 35.6$ MPa,
$(\tau_{\max})_{DE} = 23.3$ MPa

10–7.

The solid aluminum shaft has a diameter of 50 mm. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line of the shaft where the shear stress is maximum. Set $T_1 = 2000\ \text{N} \cdot \text{m}$.

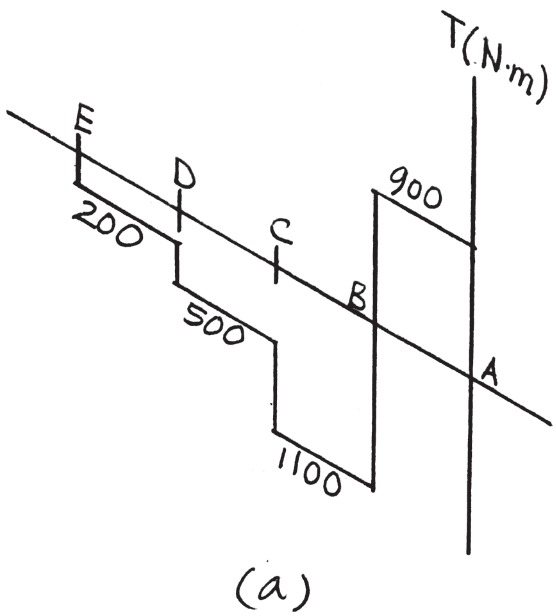


SOLUTION

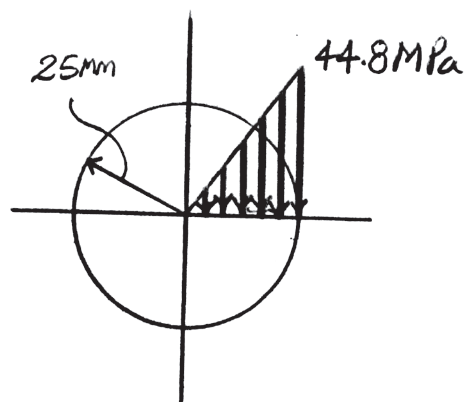
Internal Torque: The torque diagram plotted in Fig. *a* indicates that region *BC* of the shaft is subjected to the greatest internal torque; thus, it is the critical region where the absolute maximum shear stress occurs. Here, $T_{BC} = 1100\ \text{N} \cdot \text{m}$. Applying the torsion formula,

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{T_{BC}\,c}{J} = \frac{1100\,(0.025)}{\frac{\pi}{2}\left(0.025^4\right)} = 44.82\left(10^6\right)\ \text{Pa} = 44.8\ \text{MPa} \qquad \textbf{Ans.}$$

The shear-stress distribution along the radius of the cross-section of the shaft is shown in Fig. *b*.



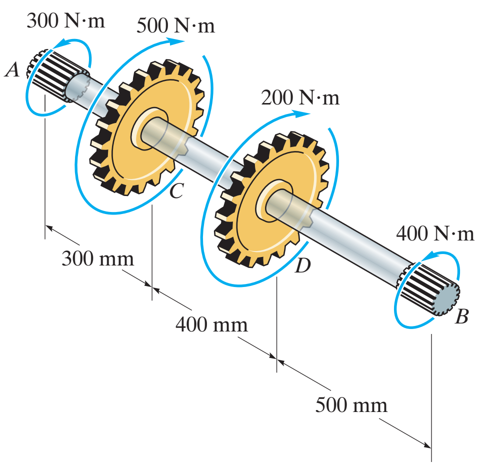
(a)



Ans:
$\tau_{\substack{\text{abs}\\ \text{max}}} = 44.8\ \text{MPa}$

***10–8.**

The solid 30-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress in the shaft.



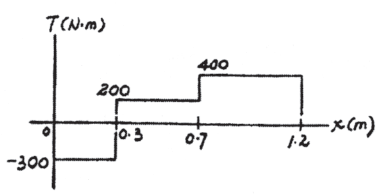
SOLUTION

Internal Torque: As shown on torque diagram.

Maximum Shear Stress: From the torque diagram, $T_{max} = 400\text{ N}\cdot\text{m}$. Then, applying the torsion formula,

$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,c}{J}$$

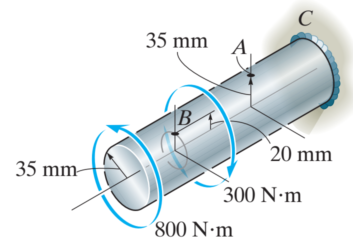
$$= \frac{400(0.015)}{\frac{\pi}{2}(0.015^4)} = 75.5\text{ MPa}$$ **Ans.**



Ans:
$\tau_{\substack{abs\\max}} = 75.5\text{ MPa}$

10–9.

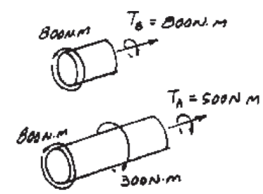
The solid shaft is fixed to the support at C and subjected to the torsional loadings. Determine the shear stress at points A and B on the surface, and sketch the shear stress on volume elements located at these points.

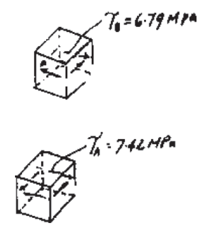


SOLUTION

$$\tau_B = \frac{T_B\,\rho}{J} = \frac{800(0.02)}{\frac{\pi}{2}(0.035^4)} = 6.79 \text{ MPa}$$ **Ans.**

$$\tau_A = \frac{T_A\,c}{J} = \frac{500(0.035)}{\frac{\pi}{2}(0.035^4)} = 7.42 \text{ MPa}$$ **Ans.**

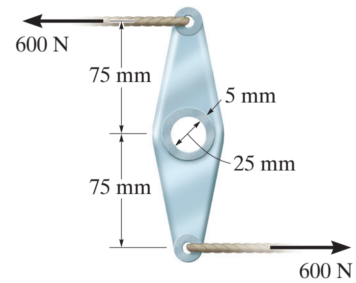




Ans:
$\tau_B = 6.79$ MPa, $\tau_A = 7.42$ MPa

10–10.

The link acts as part of the elevator control for a small airplane. If the attached aluminum tube has an inner diameter of 25 mm and a wall thickness of 5 mm, determine the maximum shear stress in the tube when the cable force of 600 N is applied to the cables. Also, sketch the shear-stress distribution over the cross section.

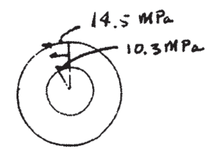


SOLUTION

$T = 600(0.15) = 90\ \text{N} \cdot \text{m}$

$$\tau_{\max} = \frac{Tc}{J} = \frac{90(0.0175)}{\frac{\pi}{2}\left[(0.0175)^4 - (0.0125)^4\right]} = 14.5\ \text{MPa}$$ **Ans.**

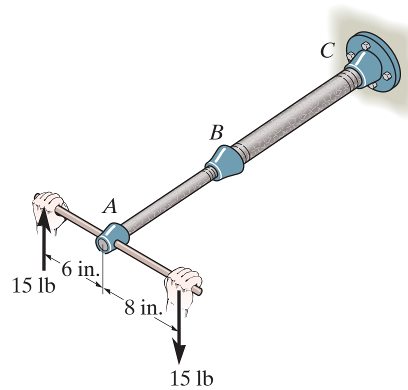
$$\tau_i = \frac{T\rho}{J} = \frac{90(0.0125)}{\frac{\pi}{2}\left[(0.0175)^4 - (0.0125)^4\right]} = 10.3\ \text{MPa}$$



Ans:
$\tau_{\max} = 14.5\ \text{MPa}$

10–11.

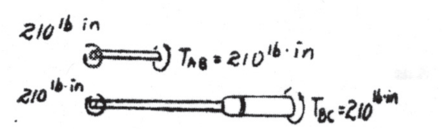
The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at B. The smaller pipe has an outer diameter of 0.75 in. and an inner diameter of 0.68 in., whereas the larger pipe has an outer diameter of 1 in. and an inner diameter of 0.86 in. If the pipe is tightly secured to the wall at C, determine the maximum shear stress in each section of the pipe when the couple is applied to the handles of the wrench.



SOLUTION

$$\tau_{AB} = \frac{Tc}{J} = \frac{210(0.375)}{\frac{\pi}{2}(0.375^4 - 0.34^4)} = 7.82 \text{ ksi}$$ **Ans.**

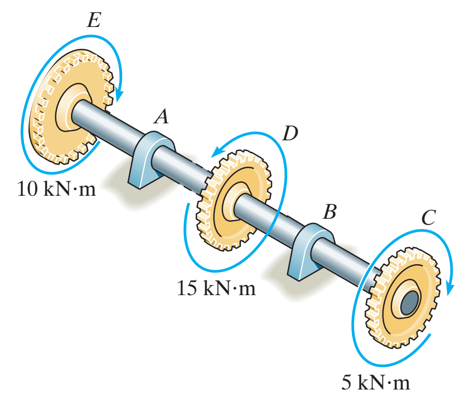
$$\tau_{BC} = \frac{Tc}{J} = \frac{210(0.5)}{\frac{\pi}{2}(0.5^4 - 0.43^4)} = 2.36 \text{ ksi}$$ **Ans.**



Ans:
$\tau_{AB} = 7.82$ ksi, $\tau_{BC} = 2.36$ ksi

***10–12.**

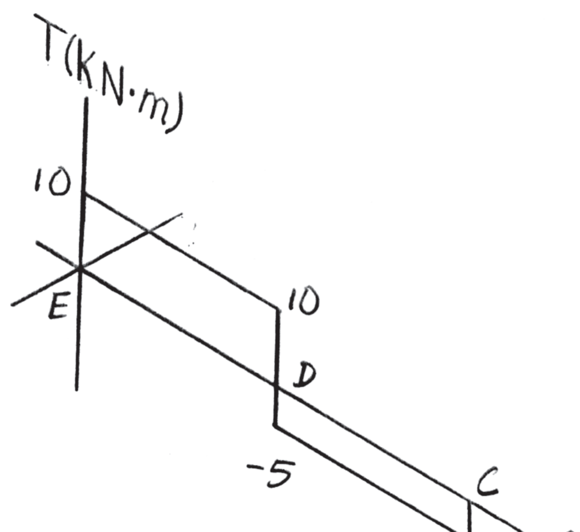
The shaft has an outer diameter of 100 mm and an inner diameter of 80 mm. If it is subjected to the three torques, determine the absolute maximum shear stress in the shaft. The smooth bearings *A* and *B* do not resist torque.



SOLUTION

Maximum Shear Stress: Referring to the torque diagram shown in Fig. *a*, region *ED* is subjected to the largest internal torque, which is $T_{max} = 10\ \text{kN}\cdot\text{m}$. Thus, the absolute maximum shear stress occurs in this region. Applying the torsion formula,

$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,C}{J} = \frac{10\left(10^3\right)(0.05)}{\frac{\pi}{2}\left(0.05^4 - 0.04^4\right)} = 86.26\left(10^6\right)\ \text{Pa} = 86.3\ \text{MPa} \qquad \textbf{Ans.}$$

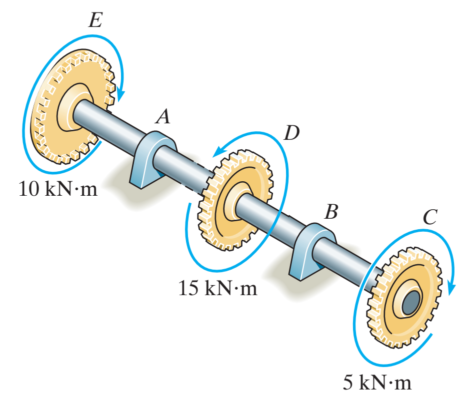


(a)

Ans:
$\tau_{\substack{abs\\max}} = 86.3\ \text{MPa}$

10–13.

The shaft has an outer diameter of 100 mm and an inner diameter of 80 mm. If it is subjected to the three torques, plot the shear stress distribution along a radial line for the cross section within region *CD* of the shaft. The smooth bearings at *A* and *B* do not resist torque.



SOLUTION

Shear Stress: Referring to the torque diagram shown in Fig. *a*, region *CD* of the shaft is subjected to an internal torque of $T_{CD} = 5\ \text{kN}\cdot\text{m}$. The torsion formula will be applied. The maximum shear stress is

$$(\tau_{\max})_{CD} = \frac{T_{CD}\,c}{J} = \frac{5(10^3)(0.05)}{\frac{\pi}{2}\left(0.05^4 - 0.04^4\right)} = 43.13\left(10^6\right)\ \text{Pa} = 43.1\ \text{MPa}$$

The shear stress at the inner surface of the hollow shaft is

$$(\tau_\rho = 0.04\ \text{m})_{CD} = \frac{T_{CD}\rho}{J} = \frac{5(10^3)(0.04)}{\frac{\pi}{2}\left(0.05^4 - 0.04^4\right)} = 34.51\left(10^6\right)\ \text{Pa} = 34.5\ \text{MPa}$$

Also,

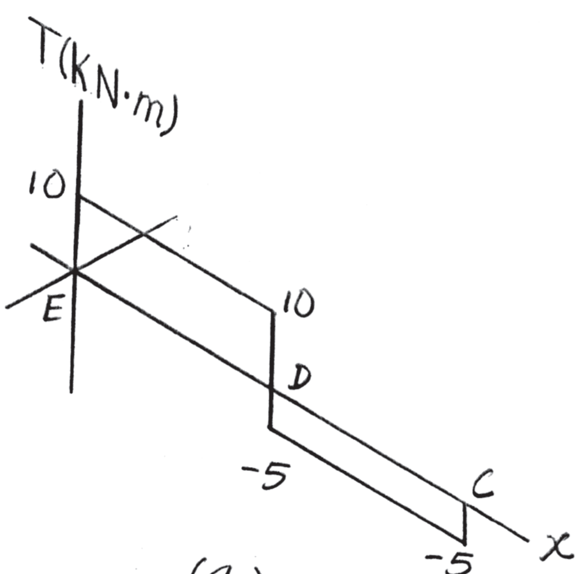
$$\frac{\tau_{\max}}{c} = \frac{\tau_\rho}{\rho}; \quad (\tau_{\rho=0.04\ \text{m}})_{CD} = \left(\frac{\rho}{c}\right)\tau_{\max}$$

$$= \left(\frac{0.04}{0.05}\right)(43\cdot 13)$$

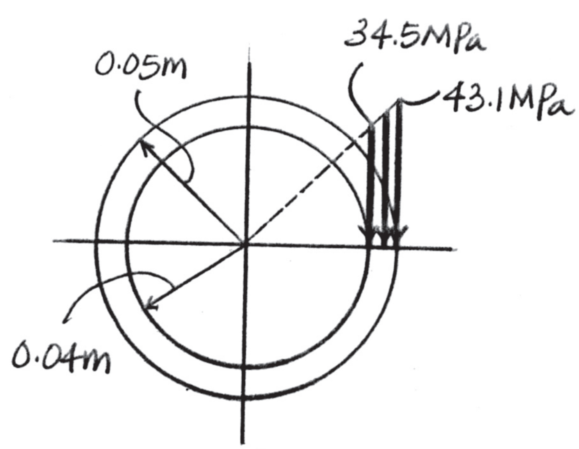
$$= 34.51\ \text{MPa}$$

$$= 34.5\ \text{MPa}$$

The shear stress distribution along the radius of the cross-section of the shaft is shown in Fig. *b*.



(a)

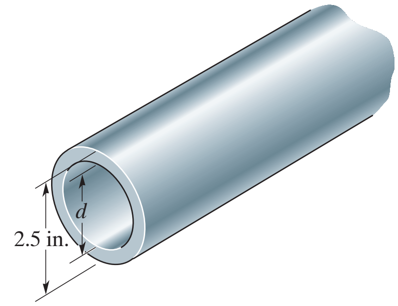


(b)

Ans:
$\tau_i = 34.5$ MPa,
$\tau_o = 43.1$ MPa

10–14.

A steel tube having an outer diameter of 2.5 in. is used to transmit 9 hp when turning at 27 rev/min. Determine the inner diameter d of the tube to the nearest $\frac{1}{8}$ in. if the allowable shear stress is $\tau_{\text{allow}} = 10$ ksi.



SOLUTION

$$\omega = \frac{27(2\pi)}{60} = 2.8274 \text{ rad/s}$$

$$P = T\omega$$

$$9(550) = T(2.8274)$$

$$T = 1750.7 \text{ ft} \cdot \text{lb}$$

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$10(10^3) = \frac{1750.7(12)(1.25)}{\frac{\pi}{2}(1.25^4 - c_i^{\,4})}$$

$$c_i = 0.9366 \text{ in.}$$

$$d = 1.873 \text{ in.}$$

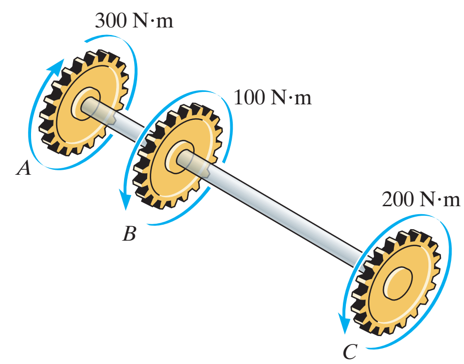
Use $d = 1\frac{3}{4}$ in. **Ans.**

Ans:

Use $d = 1\frac{3}{4}$ in.

10–15.

If the gears are subjected to the torques shown, determine the maximum shear stress in the segments AB and BC of the A-36 steel shaft. The shaft has a diameter of 40 mm.



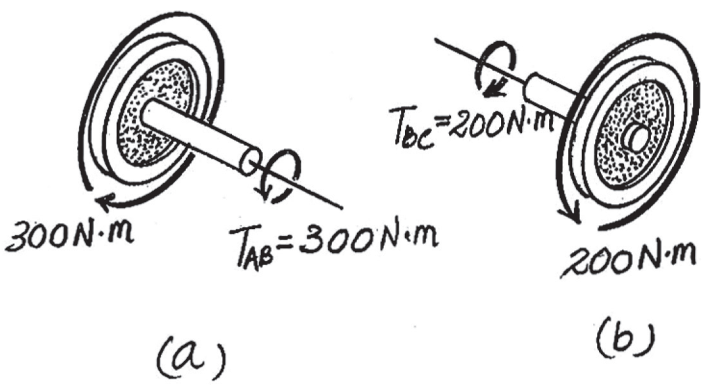
SOLUTION

The internal torque developed in segments AB and BC are shown in their respective FBDs, Figs. a and b.

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.02^4) = 80(10^{-9})\pi \text{ m}^4$. Thus,

$$(\tau_{AB})_{\max} = \frac{T_{AB}\,c}{J} = \frac{300(0.02)}{80(10^{-9})\pi} = 23.87(10^6)\text{Pa} = 23.9 \text{ MPa} \qquad \textbf{Ans.}$$

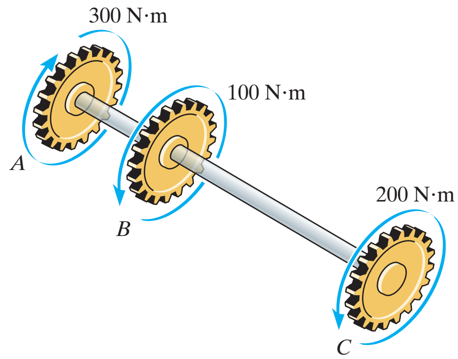
$$(\tau_{BC})_{\max} = \frac{T_{BC}\,c}{J} = \frac{200(0.02)}{80(10^{-9})\pi} = 15.92(10^6)\text{Pa} = 15.9 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$(\tau_{AB})_{\max} = 23.9$ MPa,
$(\tau_{BC})_{\max} = 15.9$ MPa

***10–16.**

If the gears are subjected to the torques shown, determine the required diameter of the A-36 steel shaft to the nearest mm if $\tau_{\text{allow}} = 60$ MPa.



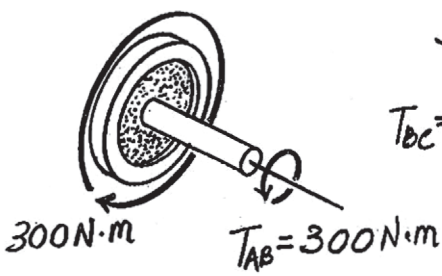
SOLUTION

The internal torque developed in segments AB and BC are shown in their respective FBDs, Fig. a and b.

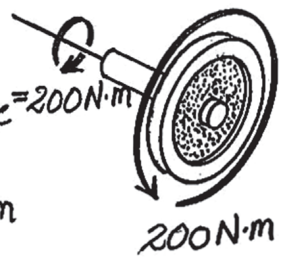
Here, segment AB is critical since its internal torque is the greatest. The polar moment of inertia of the shaft is $J = \dfrac{\pi}{2}\left(\dfrac{d}{2}\right)^4 = \dfrac{\pi d^4}{32}$. Thus,

$$\tau_{\text{allow}} = \frac{Tc}{J}; \quad 60(10^6) = \frac{300(d/2)}{\pi d^4/32}$$

$$d = 0.02942 \text{ m} = 30 \text{ mm} \qquad \textbf{Ans.}$$



(a)

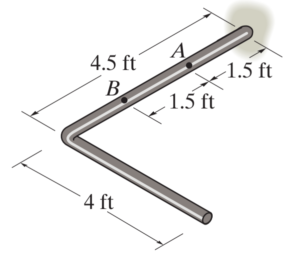


(b)

Ans:
$d = 30$ mm

10–17.

The rod has a diameter of 1 in. and a weight of 10 lb/ft. Determine the maximum torsional stress in the rod at a section located at A due to the rod's weight.



SOLUTION

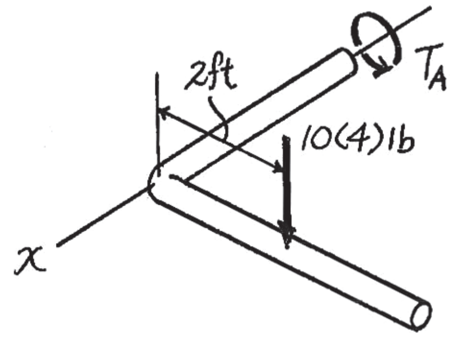
Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. *a*.

$$\Sigma M_x = 0; \qquad T_A - 10(4)(2) = 0 \qquad T_A = 80 \text{ lb}\cdot\text{ft}\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = 960 \text{ lb}\cdot\text{in.}$$

The polar moment of inertia of the cross section at A is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$.

Thus,

$$\tau_{max} = \frac{T_A\, c}{J} = \frac{960\,(0.5)}{0.03125\pi} = 4889.24 \text{ psi} = 4.89 \text{ ksi} \qquad \textbf{Ans.}$$

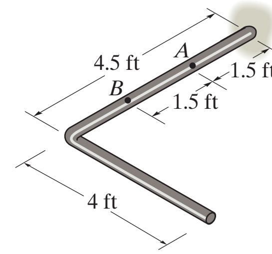


(a)

Ans:
$\tau_{max} = 4.89$ ksi

10–18.

The rod has a diameter of 1 in. and a weight of 15 lb/ft. Determine the maximum torsional stress in the rod at a section located at B due to the rod's weight.



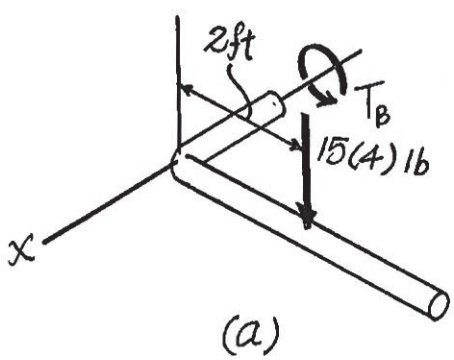
SOLUTION

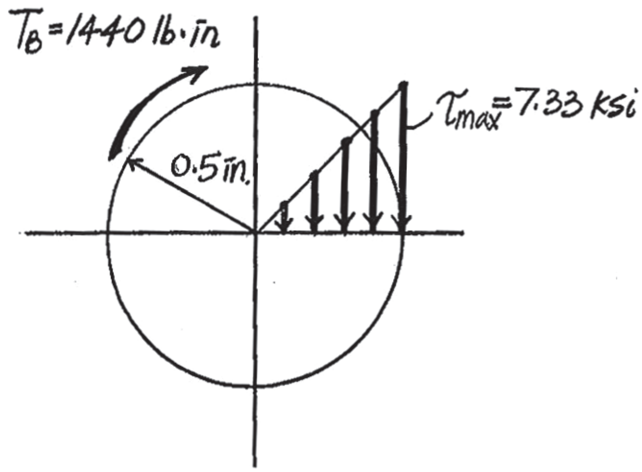
Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. *a*.

$$\Sigma M_x = 0; \qquad T_B - 15(4)(2) = 0 \qquad T_B = 120\ \text{lb}\cdot\text{ft}\left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = 1440\ \text{lb}\cdot\text{in}.$$

The polar moment of inertia of the cross-section at B is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi\ \text{in}^4$. Thus,

$$\tau_{\max} = \frac{T_B\, c}{J} = \frac{1440(0.5)}{0.03125\pi} = 7333.86\ \text{psi} = 7.33\ \text{ksi} \qquad \textbf{Ans.}$$

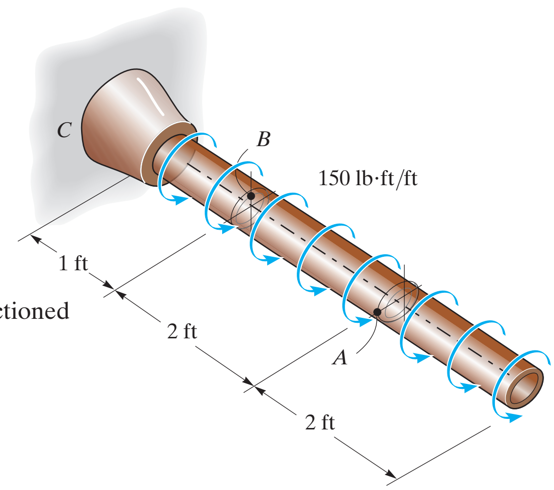




Ans:
$\tau_{\max} = 7.33$ ksi

10–19.

The copper pipe has an outer diameter of 3 in. and an inner diameter of 2.5 in. If it is tightly secured to the wall at C and a uniformly distributed torque is applied to it as shown, determine the shear stress at points A and B. These points lie on the pipe's outer surface. Sketch the shear stress on volume elements located at A and B.



SOLUTION

Internal Torque: Referring to the FBD of the right segment of the shaft sectioned through points A and B shown in Figs. a and b, respectively,

$$\Sigma M_x = 0; \qquad T_A - 150\,(2) = 0 \qquad T_A = 300\ \text{lb}\cdot\text{ft}$$

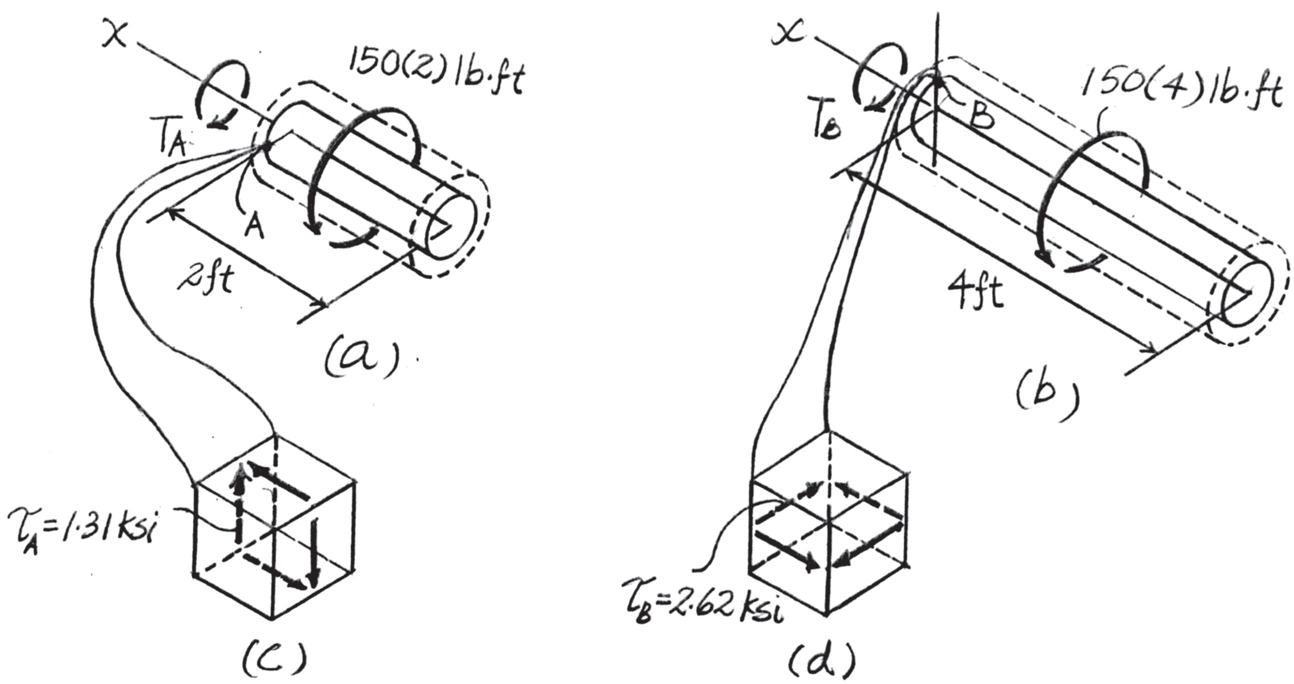
$$\Sigma M_x = 0; \qquad T_B - 150\,(4) = 0 \qquad T_B = 600\ \text{lb}\cdot\text{ft}$$

Maximum Shear Stress: Applying the torsion formula,

$$\tau_A = \frac{T_A\,c}{J} = \frac{300\,(12)(1.5)}{\frac{\pi}{2}\left(1.5^4 - 1.25^4\right)} = 1.3121\left(10^3\right)\ \text{psi} = 1.31\ \text{ksi} \qquad \textbf{Ans.}$$

$$\tau_B = \frac{T_B\,c}{J} = \frac{600\,(12)\,(1.5)}{\frac{\pi}{2}\left(1.5^4 - 1.25^4\right)} = 2.6231\left(10^3\right)\ \text{psi} = 2.62\ \text{ksi} \qquad \textbf{Ans.}$$

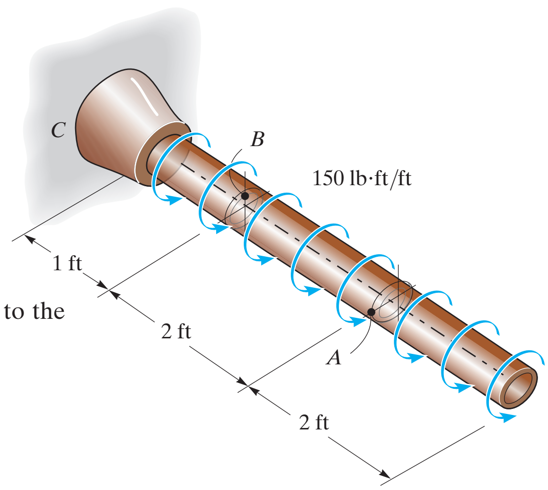
The shear stresses on the volume elements at points A and B are shown in Figs. c and d, respectively.



Ans:
$\tau_A = 1.31$ ksi,
$\tau_B = 2.62$ ksi

***10–20.**

The copper pipe has an outer diameter of 3 in. and an inner diameter of 2.50 in. If it is tightly secured to the wall at C and it is subjected to the uniformly distributed torque along its entire length, determine the absolute maximum shear stress in the pipe. Discuss the validity of this result.



SOLUTION

Internal Torque: The maximum torque occurs at fixed support C. Referring to the FBD of the entire shaft, Fig. a,

$$\Sigma M_x = 0; \qquad T_{\max} - 150\,(5) = 0 \qquad T_{\max} = 750 \text{ lb}\cdot\text{ft}$$

Maximum Shear Stress: Applying the torsion formula,

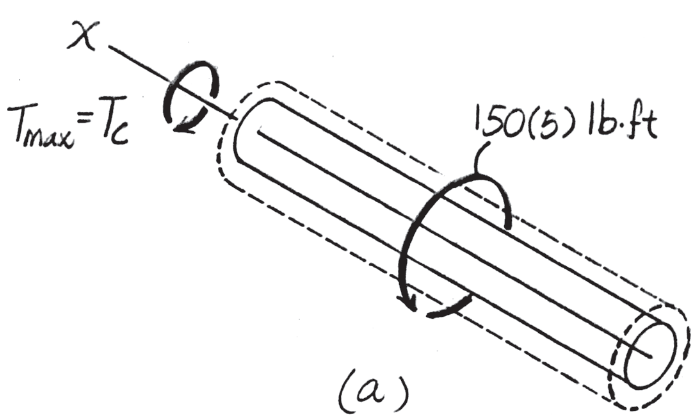
$$\tau_{\max}^{\text{abs}} = \frac{T_{\max}\,c}{J}$$

$$= \frac{750\,(12)\,(1.5)}{\frac{\pi}{2}\left(1.5^4 - 1.25^4\right)}$$

$$= 3.279\left(10^3\right) \text{ psi}$$

$$= 3.28 \text{ ksi}$$ **Ans.**

According to Saint-Venant's principle, application of the torsion formula should be at a section sufficiently removed from the supports or points of concentrated loading.

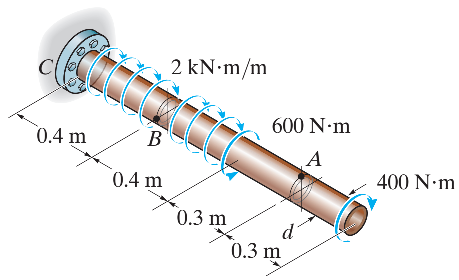


(a)

Ans:
$\tau_{\max}^{\text{abs}} = 3.28$ ksi

10–21.

The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the shear stress at points A and B, and sketch the shear stress on volume elements located at these points.



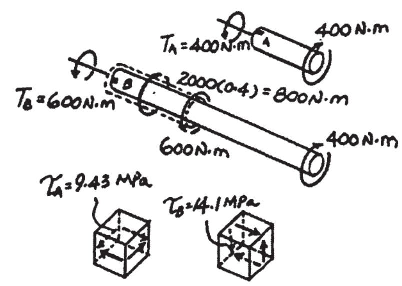
SOLUTION

Internal Torque: As shown on FBD.

Maximum Shear Stress: Applying the torsion formula,

$$\tau_A = \frac{T_A\, c}{J} = \frac{400(0.03)}{\frac{\pi}{2}(0.03^4)} = 9.43 \text{ MPa} \qquad \textbf{Ans.}$$

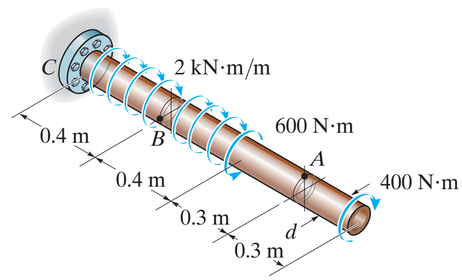
$$\tau_B = \frac{T_B\, c}{J} = \frac{600(0.03)}{\frac{\pi}{2}(0.03^4)} = 14.1 \text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\tau_A = 9.43$ MPa,
$\tau_B = 14.1$ MPa

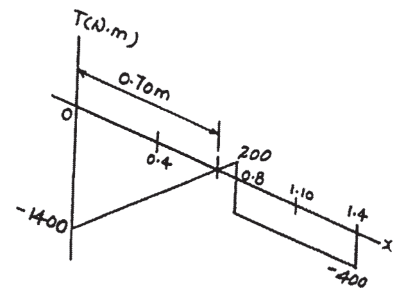
10–22.

The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the shaft's surface, and specify their locations, measured from the fixed end *C*.



SOLUTION

Internal Torque: From the torque diagram, the maximum torque $T_{max} = 1400\ \text{N}\cdot\text{m}$ occurs at the fixed support and the minimum torque $T_{min} = 0$ occurs at $x = 0.700$ m.



Shear Stress: Applying the torsion formula,

$$\tau_{\substack{abs\\min}} = \frac{T_{min}\, c}{J} = 0 \qquad \text{occurs at} \qquad x = 0.700\ \text{m} \qquad \textbf{Ans.}$$

$$\tau_{\substack{abs\\max}} = \frac{T_{max}\, c}{J} = \frac{1400(0.03)}{\frac{\pi}{2}\left(0.03^4\right)} = 33.0\ \text{MPa} \qquad \textbf{Ans.}$$

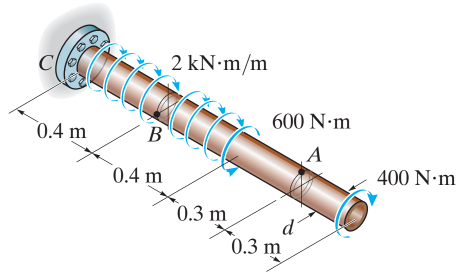
$$\text{occurs at} \qquad x = 0 \qquad \textbf{Ans.}$$

According to Saint-Venant's principle, application of the torsion formula should be at points sufficiently removed from the supports or points of concentrated loading. Therefore, $\tau_{\substack{abs\\max}}$ obtained *is not valid*.

Ans:
$\tau_{\substack{abs\\max}} = 0$ occurs at $x = 0.700$ m,
$\tau_{\substack{abs\\max}} = 33.0$ MPa occurs at $x = 0$

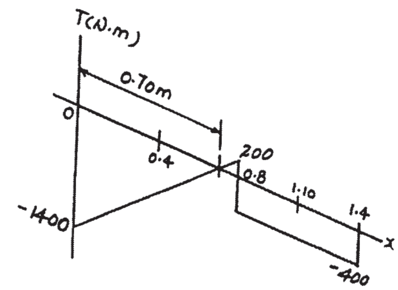
10–23.

The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft if the allowable shear stress for the material is $\tau_{\text{allow}} = 1.6$ MPa.



SOLUTION

Internal Torque: From the torque diagram, the maximum torque $T_{\max} = 1400\text{ N}\cdot\text{m}$ occurs at the fixed support.



Allowable Shear Stress: Applying the torsion formula,

$$\tau_{\underset{\max}{\text{abs}}} = \tau_{\text{allow}} = \frac{T_{\max}\, c}{J}$$

$$175(10^6) = \frac{1400\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

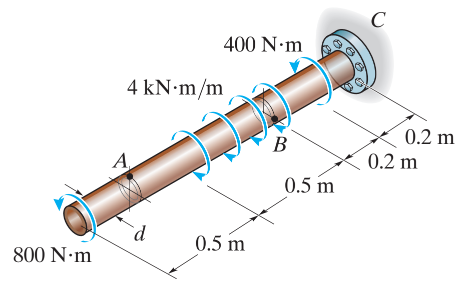
$d = 0.03441\text{ m} = 34.4\text{ mm}$ **Ans.**

According to Saint-Venant's principle, application of the torsion formula should be at points sufficiently removed from the supports or points of concentrated loading. Therefore, the above analysis *is not valid*.

Ans:
$d = 34.4$ mm

***10–24.**

The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses in the shaft's surface and specify their locations, measured from the free end.



SOLUTION

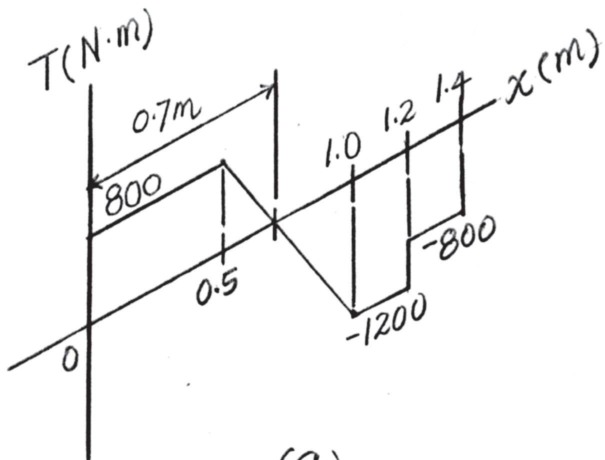
Internal Torque: The torque diagram plotted in Fig. *a* indicates that region $1.0\text{ m} < x < 1.2\text{ m}$ of the shaft is subjected to the greatest internal torque where $T_{max} = 1200\text{ N}\cdot\text{m}$, whereas the minimum internal torque, $T_{min} = 0$, occurs at $x = 0.7\text{ m}$.

Shear Stress: Applying the torsion formula,

$$\tau_{\substack{abs\\max}} = \frac{T_{max}\,c}{J} = \frac{1200\,(0.03)}{\frac{\pi}{2}\left(0.03^4\right)} = 28.29\left(10^6\right)\text{ Pa} = 28.3\text{ MPa} \qquad \textbf{Ans.}$$

occurs within the region $1.0\text{ m} < x < 1.2\text{ m}$ **Ans.**

$$\tau_{\substack{abs\\min}} = \frac{T_{min}\,c}{J} = 0 \qquad \text{occurs at } x = 0.700\text{ m} \qquad \textbf{Ans.}$$

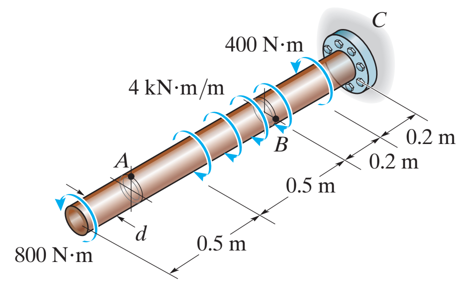


(a)

Ans:
$\tau_{\substack{abs\\max}} = 28.3\text{ MPa}$,
for $1.0\text{ m} < x < 1.2\text{ m}$,
$\tau_{\substack{abs\\min}} = 0$,
at $x = 0.700\text{ m}$

10–25.

The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft if the allowable shear stress for the material is $\tau_{allow} = 60$ MPa.



SOLUTION

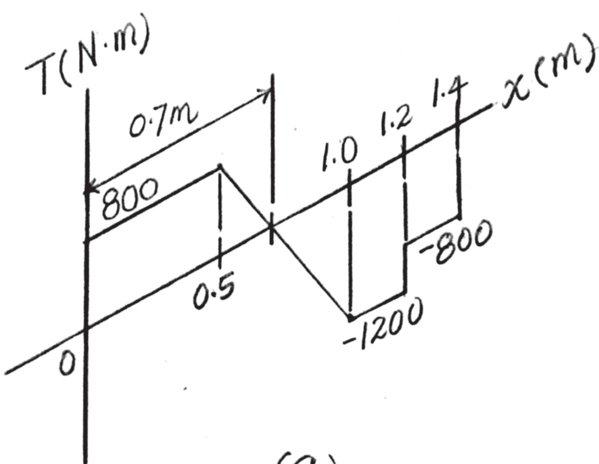
Internal Torque: The torque diagram plotted in Fig. *a* indicates that region $1.0 \text{ m} < x < 1.2 \text{ m}$ of the shaft is subjected to the greatest internal torque $T_{max} = 1200 \text{ N} \cdot \text{m}$; thus, that is the critical region where the absolute maximum shear stress occurs.

Allowable Shear Stress: Applying the torsion formula,

$$\tau_{\substack{abs \\ max}} = \tau_{allow} = \frac{T_{max}\,C}{J}$$

$$60\left(10^6\right) = \frac{1200\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.04670 \text{ m} = 46.7 \text{ mm} \qquad \textbf{Ans.}$$

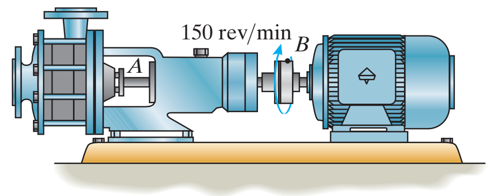


(a)

Ans:
$d = 46.7$ mm

10–26.

The pump operates using the motor that has a power of 85 W. If the impeller at B is turning at 150 rev/min, determine the maximum shear stress in the 20-mm-diameter transmission shaft at A.



SOLUTION

Internal Torque:

$$\omega = 150 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 5.00\pi \text{ rad/s}$$

$$P = 85 \text{ W} = 85 \text{ N} \cdot \text{m/s}$$

$$T = \frac{P}{\omega} = \frac{85}{5.00\pi} = 5.411 \text{ N} \cdot \text{m}$$

Maximum Shear Stress: Applying the torsion formula,

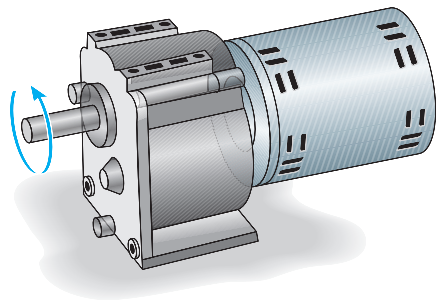
$$\tau_{\max} = \frac{T\,c}{J}$$

$$= \frac{5.411\,(0.01)}{\frac{\pi}{2}(0.01^4)} = 3.44 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 3.44$ MPa

10–27.

The gear motor can develop $\frac{1}{10}$ hp when it turns at 300 rev/min. If the shaft has a diameter of $\frac{1}{2}$ in., determine the maximum shear stress in the shaft.



SOLUTION

Internal Torque:

$$\omega = 300\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\frac{1\text{ min}}{60\text{ s}} = 10.0\pi\text{ rad/s}$$

$$P = \frac{1}{10}\text{hp}\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 55.0\text{ ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{55.0}{10.0\pi} = 1.751\text{ lb}\cdot\text{ft}$$

Maximum Shear Stress: Applying the torsion formula,

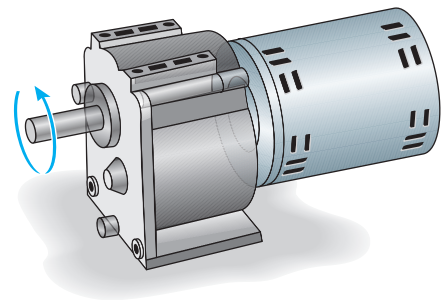
$$\tau_{\max} = \frac{T\,c}{J}$$

$$= \frac{1.751(12)(0.25)}{\frac{\pi}{2}(0.25^4)} = 856\text{ psi}$$ **Ans.**

Ans:
$\tau_{\max} = 856$ psi

***10–28.**

The gear motor can develop $\frac{1}{10}$ hp when it turns at 80 rev/min. If the allowable shear stress for the shaft is $\tau_{\text{allow}} = 4$ ksi, determine the smallest diameter of the shaft to the nearest $\frac{1}{8}$ in. that can be used.



SOLUTION

Internal Torque:

$$\omega = 80\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\frac{1\text{ min}}{60\text{ s}} = 2.667\pi\text{ rad/s}$$

$$P = \frac{1}{10}\text{ hp}\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 55.0\text{ ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{55.0}{2.667\pi} = 6.565\text{ lb}\cdot\text{ft}$$

Allowable Shear Stress: Applying the torsion formula,

$$\tau_{\max} = \tau_{\text{allow}} = \frac{T\,c}{J}$$

$$4(10^3) = \frac{6.565(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

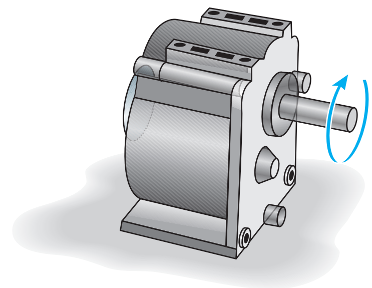
$$d = 0.4646\text{ in.}$$

Use $d = \frac{1}{2}$ in. diameter of shaft. **Ans.**

Ans:
Use $d = \frac{1}{2}$ in.

10–29.

The gear motor can develop $\frac{1}{4}$ hp when it turns at 600 rev/min. If the shaft has a diameter of $\frac{1}{2}$ in., determine the maximum shear stress in the shaft.



SOLUTION

Internal Torque: The angular velocity of the shaft is

$$\omega = \left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 20\,\pi\text{ rad/s}$$

And the power is

$$P = \left(\frac{1}{4}\text{ hp}\right)\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 137.5\text{ ft}\cdot\text{lb/s}$$

Then, the torque can be determined from

$$T = \frac{P}{\omega} = \frac{137.5}{20\,\pi} = 2.188\text{ lb}\cdot\text{ft}$$

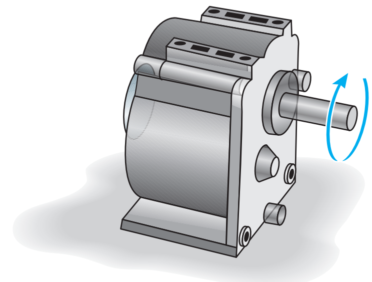
Maximum Shear Stress: Applying the torsion formula,

$$\tau_{\max} = \frac{T_C}{J} = \frac{2.188\,(12)\,(0.25)}{\frac{\pi}{2}(0.25^4)} = 1069.95\text{ psi} = 1.07\text{ ksi}$$ **Ans.**

Ans:
$\tau_{\max} = 1.07$ ksi

10–30.

The gear motor can develop 2 hp when it turns at 150 rev/min. If the allowable shear stress for the shaft is $\tau_{\text{allow}} = 8$ ksi, determine the smallest diameter of the shaft to the nearest $\frac{1}{8}$ in. that can be used.



SOLUTION

Internal Torque: The angular velocity of the shaft is

$$\omega = \left(150\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 5\,\pi\text{ rad/s}$$

And the power is

$$P = (2\text{ hp})\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 1100\text{ ft}\cdot\text{lb/s}$$

Then, the torque can be determined from

$$T = \frac{P}{\omega} = \frac{1100}{5\,\pi} = 70.028\text{ lb}\cdot\text{ft}$$

Allowable Shear Stress: Applying the torsion formula,

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$8(10^3) = \frac{(70.028)(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.8118\text{ in.}$$

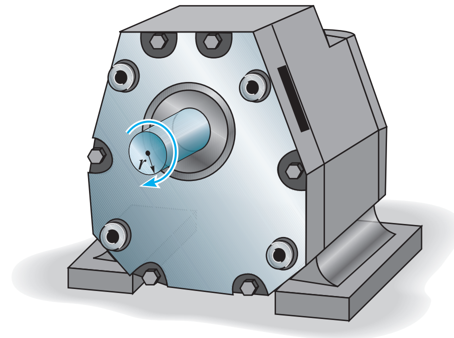
$$\text{Use } d = \frac{7}{8}\text{ in.}$$ **Ans.**

Ans:

Use $d = \frac{7}{8}$ in.

10–31.

The 6-hp reducer motor can turn at 1200 rev/min. If the allowable shear stress for the shaft is $\tau_{allow} = 6$ ksi, determine the smallest diameter of the shaft to the nearest $\frac{1}{16}$ in. that can be used.



SOLUTION

Internal Torque: The angular velocity of the shaft is

$$\omega = \left(1200\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 40\,\pi\text{ rad/s}$$

And the power is

$$P = (6\text{ hp})\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 3300\text{ ft}\cdot\text{lb/s}$$

Then, the torque can be determined from

$$T = \frac{P}{\omega} = \frac{3300}{40\,\pi} = 26.26\text{ lb}\cdot\text{ft}$$

Allowable Shear Stress: Applying the torsion formula,

$$\tau_{max} = \tau_{allow} = \frac{T_C}{J}$$

$$6(10^3) = \frac{26.26\,(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.6443\text{ in.}$$

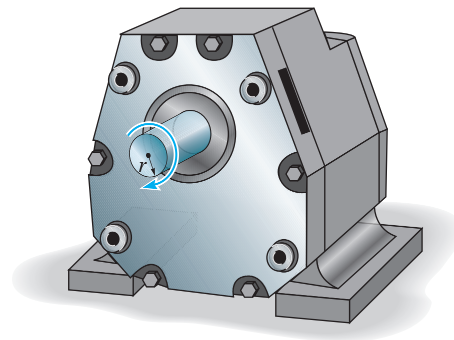
$$\text{Use } d = \frac{11}{16}\text{ in.}$$ **Ans.**

Ans:

Use $d = \frac{11}{16}$ in.

***10–32.**

The 6-hp reducer motor can turn at 1200 rev/min. If the shaft has a diameter of $\frac{5}{8}$ in., determine the maximum shear stress in the shaft.



SOLUTION

Internal Torque: The angular velocity of the shaft is

$$\omega = \left(1200\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 40\,\pi\text{ rad/s}$$

And the power is

$$P = (6\text{ hp})\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 3300\text{ ft}\cdot\text{lb/s}$$

Then, the torque can be determined from

$$T = \frac{P}{\omega} = \frac{3300}{40\,\pi} = 26.26\text{ lb}\cdot\text{ft}$$

Maximum Shear Stress: Applying the torsion formula,

$$\tau_{\max} = \frac{Tc}{J}$$

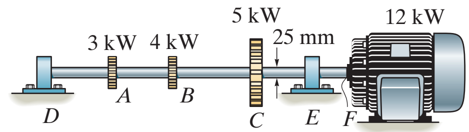
$$= \frac{26.26\,(12)\left(\frac{5}{16}\right)}{\frac{\pi}{2}\left(\frac{5}{16}\right)^4}$$

$$= 6.573\left(10^3\right)\text{ psi} = 6.57\text{ ksi}$$ **Ans.**

Ans:
$\tau_{\max} = 6.57$ ksi

10–33.

The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress developed in the shaft within regions CF and BC. The shaft is free to turn in its support bearings D and E.



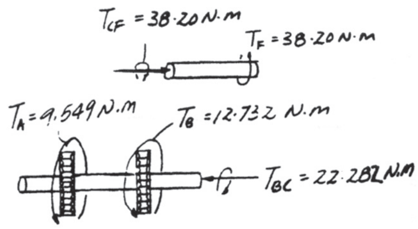
SOLUTION

$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\,\pi} = 38.20\ \text{N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\,\pi} = 9.549\ \text{N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\,\pi} = 12.73\ \text{N}\cdot\text{m}$$



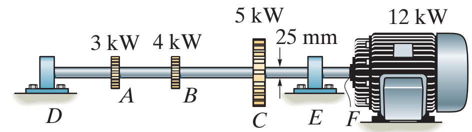
$$(\tau_{\max})_{CF} = \frac{T_{CF}\,c}{J} = \frac{38.20(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans.**

$$(\tau_{\max})_{BC} = \frac{T_{BC}\,c}{J} = \frac{22.282(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 7.26\ \text{MPa}$$ **Ans.**

Ans:
$(\tau_{\max})_{CF} = 12.5$ MPa, $(\tau_{\max})_{BC} = 7.26$ MPa

10–34.

The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the absolute maximum shear stress in the shaft.



SOLUTION

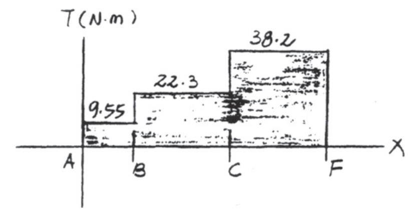
$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\text{ rad}}{\text{rev}}\right] = 100\,\pi\text{ rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\pi} = 38.20\text{ N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\pi} = 9.549\text{ N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\pi} = 12.73\text{ N}\cdot\text{m}$$

From the torque diagram,



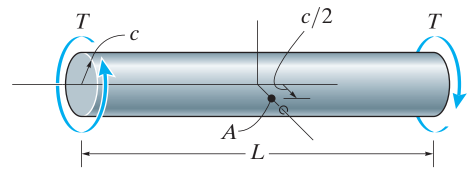
$$T_{\max} = 38.2\text{ N}\cdot\text{m}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{Tc}{J} = \frac{38.2(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\text{ MPa}$$ **Ans.**

Ans:
$\tau_{\substack{\text{abs}\\\text{max}}} = 12.5\text{ MPa}$

10–35.

The propellers of a ship are connected to an A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist.



SOLUTION

$$T = \frac{P}{\omega} = \frac{4.5(10^6)}{20} = 225(10^3)\ \text{N}\cdot\text{m}$$

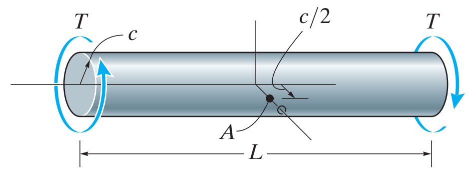
$$\tau_{\max} = \frac{Tc}{J} = \frac{225(10^3)(0.170)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4]} = 44.3\ \text{MPa}$$ **Ans.**

$$\phi = \frac{TL}{JG} = \frac{225\left(10^3\right)(60)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4)75(10^9)} = 0.2085\ \text{rad} = 11.9^\circ$$ **Ans.**

Ans:
$\tau_{\max} = 44.3$ MPa, $\phi = 11.9^\circ$

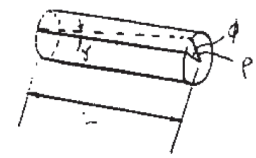
***10–36.**

The solid shaft of radius c is subjected to a torque **T** at its ends. Show that the maximum shear strain in the shaft is $\gamma_{max} = Tc/JG$. What is the shear strain on an element located at point A, $c/2$ from the center of the shaft? Sketch the shear strain distortion of this element.



SOLUTION

From the geometry:



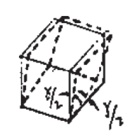
$$\gamma L = \rho\,\phi; \qquad \gamma = \frac{\rho\,\phi}{L}$$

Since $\phi = \dfrac{TL}{JG}$, then

$$\gamma = \frac{T\rho}{JG} \qquad \textbf{(1)}$$

However, the maximum shear strain occurs when $\rho = c$.

$$\gamma_{max} = \frac{Tc}{JG} \qquad \textbf{QED}$$



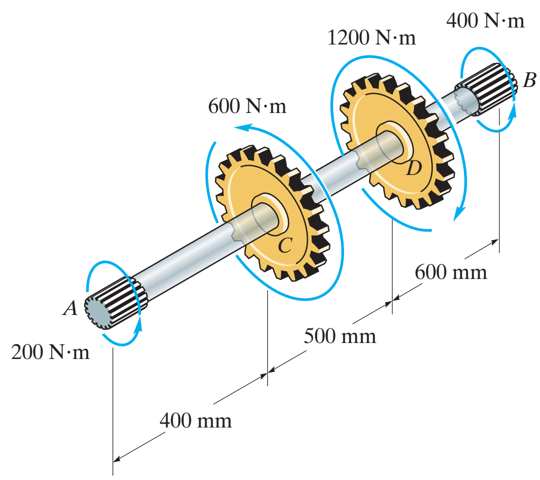
Shear strain when $\rho = \dfrac{c}{2}$ is from Eq. (1).

$$\gamma = \frac{T(c/2)}{JG} = \frac{Tc}{2\,JG} \qquad \textbf{Ans.}$$

Ans:
$\gamma = \dfrac{Tc}{2JG}$

10–37.

The splined ends and gears attached to the A992 steel shaft are subjected to the torques shown. Determine the angle of twist of end B with respect to end A. The shaft has a diameter of 40 mm.



SOLUTION

Internal Torque: The torque diagram shown in Fig. *a* can be plotted. From this diagram, $T_{AC} = 200\text{ N}\cdot\text{m}$, $T_{CD} = 800\text{ N}\cdot\text{m}$ and $T_{DB} = 400\text{ N}\cdot\text{m}$.

Angle of Twist:

$$\phi_{B/A} = \Sigma\frac{TL}{JG}$$

$$= \frac{1}{JG}\left(T_{AC}L_{AC} + T_{CD}L_{CD} + T_{DB}L_{DB}\right)$$

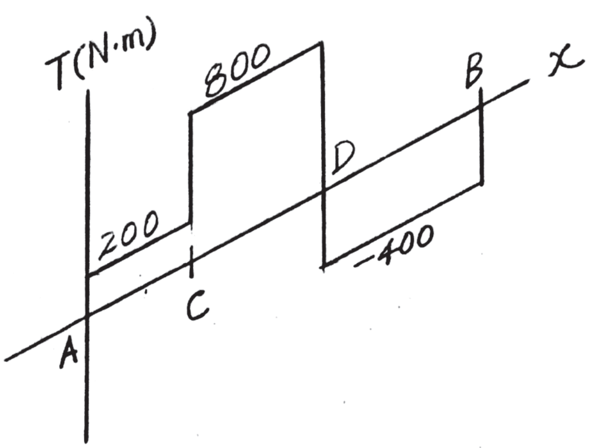
$$= \frac{1}{JG}[200(0.4) + 800(0.5) + (-400)(0.6)]$$

$$= \frac{240\text{ N}\cdot\text{m}^2}{JG}$$

For A992 steel, $G = 75$ GPa. Then

$$\phi_{B/A} = \frac{240}{\frac{\pi}{2}\left(0.02^4\right)\left[75\left(10^9\right)\right]}$$

$$= (0.01273\text{ rad})\left(\frac{180^\circ}{\pi\text{ rad}}\right) = 0.730^\circ$$ **Ans.**

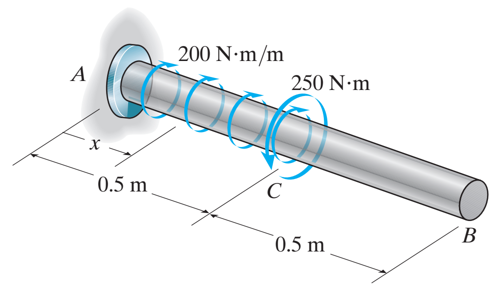


(a)

Ans:
$\phi_{B/A} = 0.730^\circ$

10–38.

The A-36 steel shaft has a diameter of 50 mm and is subjected to the distributed and concentrated loadings shown. Determine the absolute maximum shear stress in the shaft and plot a graph of the angle of twist of the shaft in radians versus x.

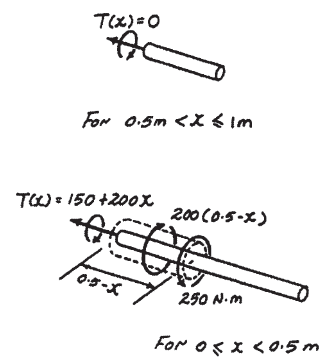


SOLUTION

Internal Torque: As shown on FBD.

Maximum Shear Stress: The maximum torque occurs at $x = 0.5\text{ m}$ where $T_{max} = 150 + 200(0.5) = 250\text{ N}\cdot\text{m}$.

$$\tau_{\substack{max\\ABS}} = \frac{T_{max}\,c}{J} = \frac{250(0.025)}{\frac{\pi}{2}(0.025^4)} = 10.2\text{ MPa}$$ **Ans.**



Angle of Twist:

For $0 \le x < 0.5\text{ m}$,

$$\phi(x) = \int_0^L \frac{T(x)\,dx}{JG}$$

$$= \int_0^x \frac{(150 + 200x)\,dx}{JG}$$

$$= \frac{150x + 100x^2}{JG}$$

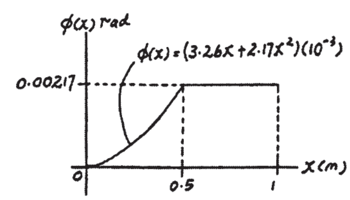
$$= \frac{150x + 100x^2}{\frac{\pi}{2}\left(0.025^4\right)75.0\left(10^9\right)}$$

$$= \left[3.26x + 2.17x^2\right](10^{-3})\text{ rad}$$

At $x = 0.5\text{ m}$, $\quad \phi = \phi_C = 0.00217\text{ rad}$

For $0.5\text{ m} < x < 1\text{ m}$, $\quad$ since $T(x) = 0$, then

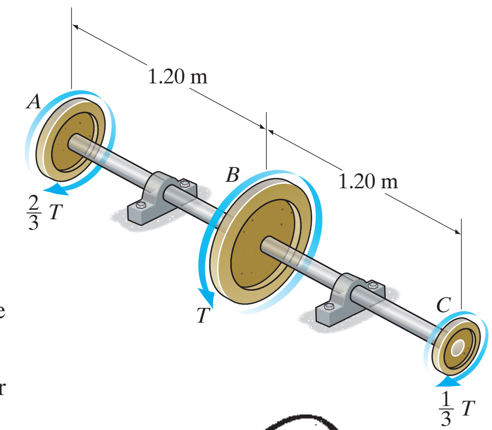
$$\phi(x) = \phi_C = 0.00217\text{ rad}$$



Ans:
$\tau_{\substack{abs\\max}} = 10.2\text{ MPa}$

10–39.

The 60-mm-diameter shaft is made of 6061-T6 aluminum having an allowable shear stress of $\tau_{\text{allow}} = 80$ MPa. Determine the maximum allowable torque **T**. Also, find the corresponding angle of twist of disk A relative to disk C.



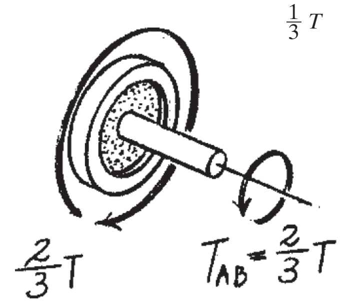
SOLUTION

Internal Loading: The internal torques developed in segments AB and BC of the shaft are shown in Figs. a and b, respectively.

Allowable Shear Stress: Segment AB is critical since it is subjected to a greater internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405\left(10^{-6}\right)\pi\ \text{m}^4$. We have

$$\tau_{\text{allow}} = \frac{T_{AB}\,c}{J}; \qquad 80(10^6) = \frac{\left(\frac{2}{3}T\right)(0.03)}{0.405(10^{-6})\pi}$$

$$T = 5089.38\ \text{N}\cdot\text{m} = 5.09\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



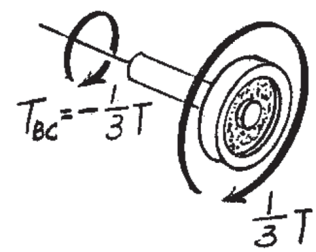
(a)

Angle of Twist: The internal torques developed in segments AB and BC of the shaft are $T_{AB} = \frac{2}{3}(5089.38) = 3392.92\ \text{N}\cdot\text{m}$ and $T_{BC} = -\frac{1}{3}(5089.38) = -1696.46\ \text{N}\cdot\text{m}$. We have

$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \frac{T_{BC}L_{BC}}{JG_{al}}$$

$$\phi_{A/C} = \frac{3392.92(1.20)}{0.405(10^{-6})\pi(26)(10^9)} + \frac{-1696.46(1.20)}{0.405(10^{-6})\pi(26)(10^9)}$$

$$= 0.06154\ \text{rad} = 3.53^\circ \qquad \textbf{Ans.}$$

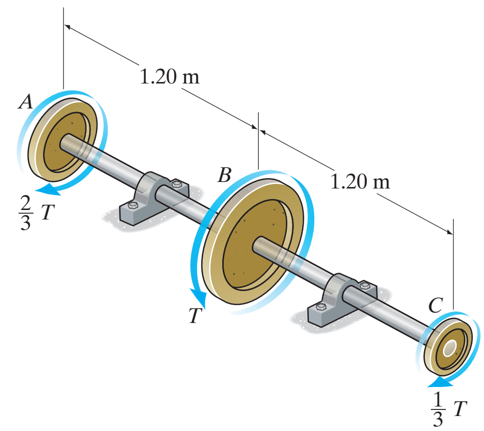


(b)

Ans:
$T = 5.09\ \text{kN}\cdot\text{m}$, $\phi_{A/C} = 3.53^\circ$

***10–40.**

The 60-mm-diameter shaft is made of 6061-T6 aluminum. If the allowable shear stress is $\tau_{allow} = 80$ MPa, and the angle of twist of disk A relative to disk C is limited so that it does not exceed 0.06 rad, determine the maximum allowable torque **T**.



SOLUTION

Internal Loading: The internal torques developed in segments AB and BC of the shaft are shown in Figs. a and b, respectively.

Allowable Shear Stress: Segment AB is critical since it is subjected to a greater internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi$ m^4. We have

$$\tau_{allow} = \frac{T_{AB}\,c}{J}; \qquad 80(10^3) = \frac{(\frac{2}{3}T)(0.03)}{0.405(10^{-6})\pi}$$

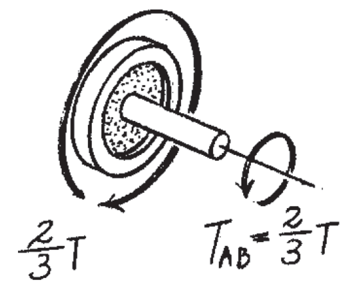
$$T = 5089.38 \text{ N}\cdot\text{m} = 5.089 \text{ kN}\cdot\text{m}$$

Angle of Twist: It is required that $\phi_{A/C} = 0.06$ rad. We have

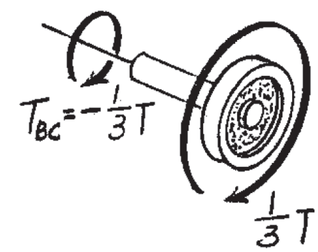
$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \frac{T_{BC}L_{BC}}{JG_{al}}$$

$$0.06 = \frac{(\frac{2}{3}T)(1.2)}{0.405(10^{-6})\pi(26)(10^9)} + \frac{(-\frac{1}{3}T)(1.2)}{0.405(10^{-6})\pi(26)(10^9)}$$

$$T = 4962.14 \text{ N}\cdot\text{m} = 4.96 \text{ kN}\cdot\text{m (controls)}$$ **Ans.**



(a)

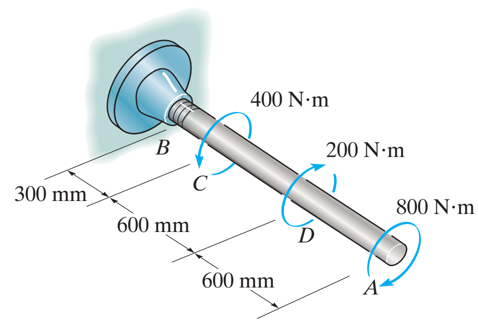


(b)

Ans:
$T = 4.96$ kN·m

10–41.

The 50-mm-diameter A992 steel shaft is subjected to the torques shown. Determine the angle of twist of the end *A*.



SOLUTION

Internal Torque: The torque diagram shown in Fig. *a* can be plotted. From this diagram, $T_{AD} = -800\text{ N}\cdot\text{m}$, $T_{DC} = -1000\text{ N}\cdot\text{m}$ and $T_{CB} = -600\text{ N}\cdot\text{m}$.

Angle of Twist:

$$\phi_{B/A} = \Sigma \frac{TL}{JG}$$

$$= \frac{1}{JG}\left(T_{AD}\,L_{AD} + T_{DC}\,L_{DC} + T_{CB}\,L_{CB}\right)$$

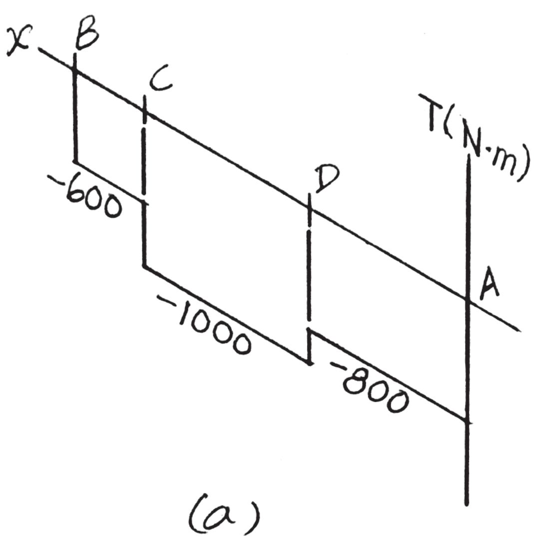
$$= \frac{1}{JG}[(-800)(0.6) + (-1000)(0.6) + (-600)(0.3)]$$

$$= -\frac{1260\text{ N}\cdot\text{m}^2}{JG}$$

For A992 steel, $G = 75$ GPa. Then

$$\phi_A = \frac{1260}{\frac{\pi}{2}\left(0.025^4\right)\left[75\left(10^9\right)\right]}$$

$$= (-0.02738\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right) = -1.569° = 1.57°$$ **Ans.**

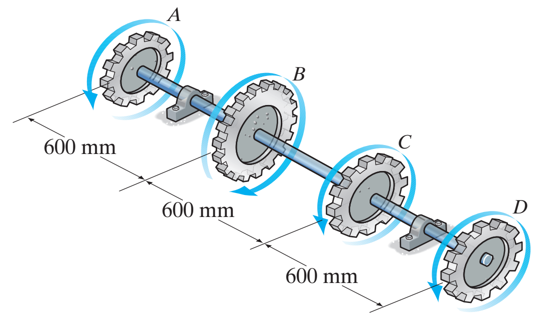


(a)

Ans:
$\phi_A = 1.57°$

10–42.

The shaft is made of A992 steel with the allowable shear stress of $\tau_{\text{allow}} = 75$ MPa. If gear B supplies 15 kW of power, while gears A, C and D withdraw 6 kW, 4 kW and 5 kW, respectively, determine the required minimum diameter d of the shaft to the nearest millimeter. Also, find the corresponding angle of twist of gear A relative to gear D. The shaft is rotating at 600 rpm.



SOLUTION

Internal Loading: The angular velocity of the shaft is

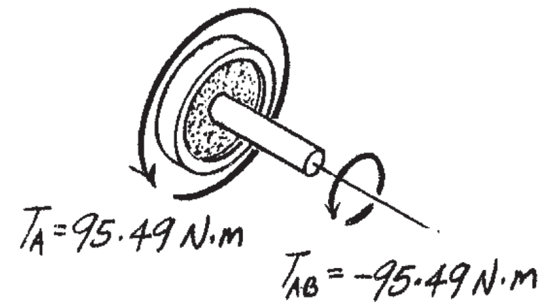
$$\omega = \left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 20\pi\text{ rad/s}$$

Thus, the torque exerted on gears A, C, and D are

$$T_A = \frac{P_A}{\omega} = \frac{6(10^3)}{20\pi} = 95.49\text{ N}\cdot\text{m}$$

$$T_C = \frac{P_C}{\omega} = \frac{4(10^3)}{20\pi} = 63.66\text{ N}\cdot\text{m}$$

$$T_D = \frac{P_D}{\omega} = \frac{5(10^3)}{20\pi} = 79.58\text{ N}\cdot\text{m}$$



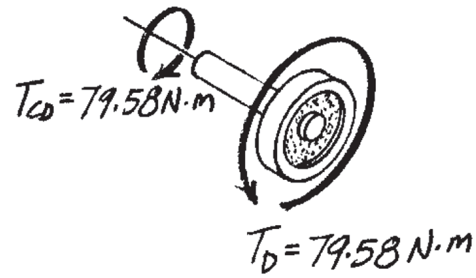
(a)

The internal torque developed in segments AB, CD, and BC of the shaft are shown in Figs. a, b, and c, respectively.

Allowable Shear Stress: Segment BC of the shaft is critical since it is subjected to a greater internal torque.

$$\tau_{\text{allow}} = \frac{T_{BC}\,c}{J}; \qquad 75(10^6) = \frac{143.24\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02135\text{ m} = 21.35\text{ mm}$$



(b)

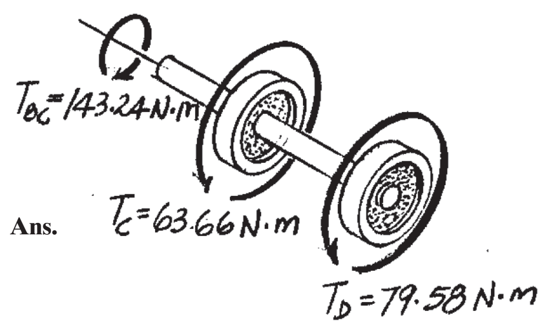
Use $\quad d = 22$ mm **Ans.**

Angle of Twist: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.011^4) = 7.3205(10^{-9})\pi\text{ m}^4$. We have

$$\phi_{A/D} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG_{st}} + \frac{T_{BC}L_{BC}}{JG_{st}} + \frac{T_{CD}L_{CD}}{JG_{st}}$$

$$\phi_{A/D} = \frac{0.6}{7.3205(10^{-9})\pi(75)(10^9)}(-95.49 + 143.24 + 79.58)$$

$$= 0.04429\text{ rad} = 2.54^\circ$$ **Ans.**

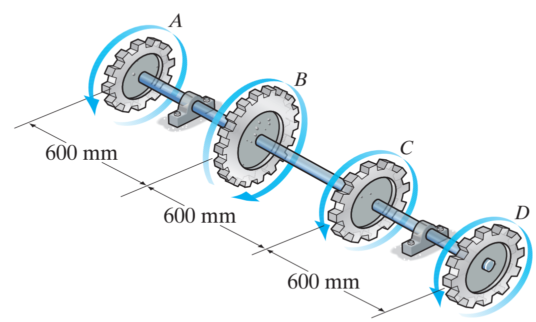


(c)

Ans:
Use $d = 22$ mm, $\phi_{A/D} = 2.54^\circ$

10–43.

Gear B supplies 15 kW of power, while gears A, C, and D withdraw 6 kW, 4 kW and 5 kW, respectively. If the shaft is made of steel with the allowable shear stress of $\tau_{\text{allow}} = 75$ MPa, and the relative angle of twist between any two gears cannot exceed 0.05 rad, determine the required minimum diameter d of the shaft to the nearest millimeter. The shaft is rotating at 600 rpm.



SOLUTION

Internal Loading: The angular velocity of the shaft is

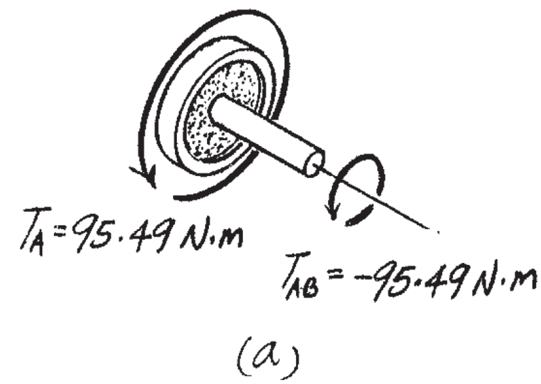
$$\omega = \left(600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 20\pi\text{ rad/s}$$

Thus, the torque exerted on gears A, C, and D are

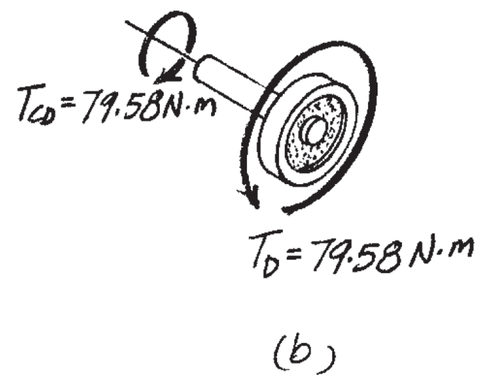
$$T_A = \frac{P_A}{\omega} = \frac{6(10^3)}{20\pi} = 95.49\text{ N}\cdot\text{m}$$

$$T_C = \frac{P_C}{\omega} = \frac{4(10^3)}{20\pi} = 63.66\text{ N}\cdot\text{m}$$

$$T_D = \frac{P_D}{\omega} = \frac{5(10^3)}{20\pi} = 79.58\text{ N}\cdot\text{m}$$



The internal torque developed in segments AB, CD, and BC of the shaft are shown in Figs. a, b, and c, respectively.



Allowable Shear Stress: Segment BC of the shaft is critical since it is subjected to a greater internal torque.

$$\tau_{\text{allow}} = \frac{T_{BC}\,c}{J}; \qquad 75(10^6) = \frac{143.24\left(\dfrac{d}{2}\right)}{\dfrac{\pi}{2}\left(\dfrac{d}{2}\right)^4}$$

$$d = 0.02135\text{ m} = 21.35\text{ mm}$$

Angle of Twist: By observation, the relative angle of twist of gear D with respect to gear B is the greatest.

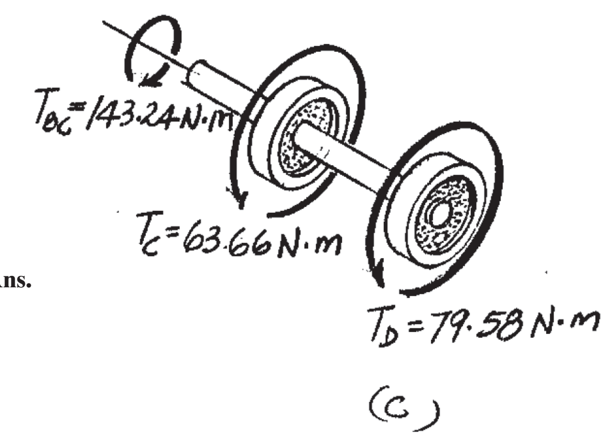
Thus, the requirement is $\phi_{D/B} = 0.05$ rad.

$$\phi_{D/B} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{BC}L_{BC}}{JG_{st}} + \frac{T_{CD}L_{CD}}{JG_{st}} = 0.05$$

$$\frac{0.6}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4(75)(10^9)}(143.24 + 79.58) = 0.05$$

$$d = 0.02455\text{ m} = 24.55\text{ mm} = 25\text{ mm (controls!)}$$

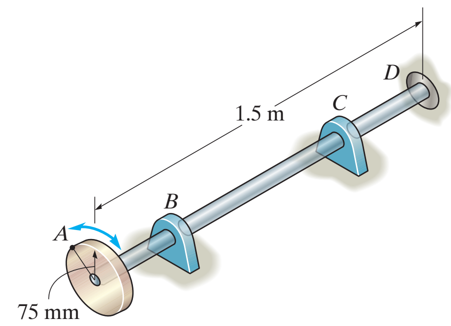
Use $\quad d = 25$ mm **Ans.**



Ans:
Use $d = 25$ mm

***10–44.**

The rotating flywheel-and-shaft, when brought to a sudden stop at D, begins to oscillate clockwise-counterclockwise such that a point A on the outer edge of the flywheel is displaced through a 6-mm arc. Determine the maximum shear stress developed in the tubular A-36 steel shaft due to this oscillation. The shaft has an inner diameter of 24 mm and an outer diameter of 32 mm. The bearings at B and C allow the shaft to rotate freely, whereas the support at D holds the shaft fixed.



SOLUTION

$s = r\,\theta$

$6 = 75\,\phi \qquad \phi = 0.08 \text{ rad}$

$$\phi = \frac{TL}{JG}$$

$$0.08 = \frac{T(1.5)}{J(75)\left(10^9\right)}$$

$$T = 4\left(10^9\right) J$$

$$\tau_{\max} = \frac{T_C}{J}$$

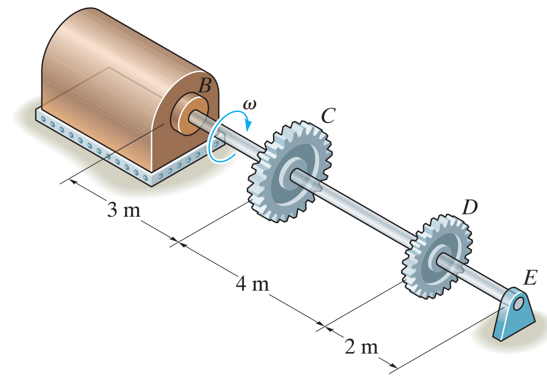
$$= \frac{4(10^9)(J)(0.016)}{J}$$

$$= 64.0 \text{ MPa}$$ **Ans.**

Ans:
$\tau_{\max} = 64.0 \text{ MPa}$

10–45.

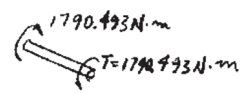
The turbine develops 150 kW of power, which is transmitted to the gears such that C receives 70% and D receives 30%. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 800$ rev/min., determine the absolute maximum shear stress in the shaft and the angle of twist of end E of the shaft relative to B. The journal bearing at E allows the shaft to turn freely about its axis.



SOLUTION

$$P = T\omega; \qquad 150(10^3)\text{ W} = T\left(800\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)$$

$$T = 1790.493\text{ N}\cdot\text{m}$$

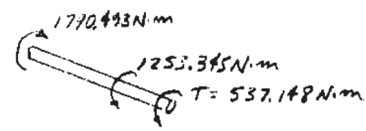


$$T_C = 1790.493(0.7) = 1253.345\text{ N}\cdot\text{m}$$

$$T_D = 1790.493(0.3) = 537.148\text{ N}\cdot\text{m}$$

Maximum torque is in region BC.

$$\tau_{\max} = \frac{T_C}{J} = \frac{1790.493(0.05)}{\frac{\pi}{2}(0.05)^4} = 9.12\text{ MPa} \qquad \textbf{Ans.}$$



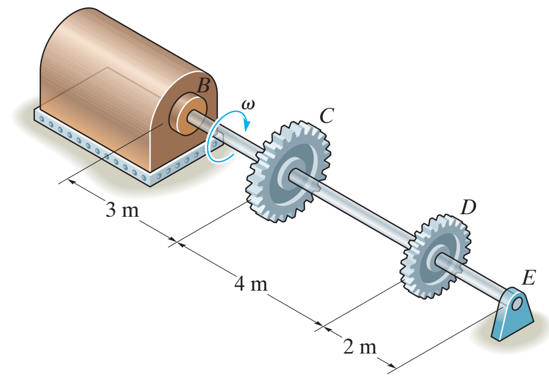
$$\phi_{E/B} = \Sigma\left(\frac{TL}{JG}\right) = \frac{1}{JG}[1790.493(3) + 537.148(4) + 0]$$

$$= \frac{7520.171}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0102\text{ rad} = 0.585° \qquad \textbf{Ans.}$$

Ans:
$\tau_{\max} = 9.12$ MPa, $\phi_{E/B} = 0.585°$

10–46.

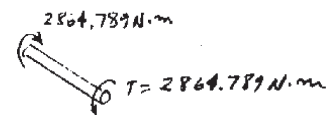
The turbine develops 150 kW of power, which is transmitted to the gears such that both C and D receive an equal amount. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 500$ rev/min., determine the absolute maximum shear stress in the shaft and the rotation of end B of the shaft relative to E. The journal bearing at E allows the shaft to turn freely about its axis.



SOLUTION

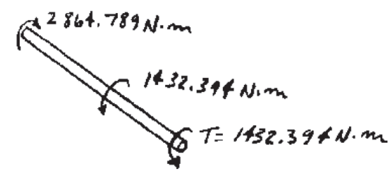
$$P = T\omega; \qquad 150(10^3)\text{ W} = T\left(500\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)$$

$$T = 2864.789\text{ N}\cdot\text{m}$$



$$T_C = T_D = \frac{T}{2} = 1432.394\text{ N}\cdot\text{m}$$

Maximum torque is in region BC.



$$\tau_{\max} = \frac{T_C}{J} = \frac{2864.789(0.05)}{\frac{\pi}{2}(0.05)^4} = 14.6\text{ MPa}$$ **Ans.**

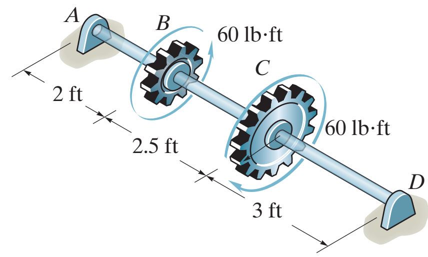
$$\phi_{B/E} = \Sigma\left(\frac{TL}{JG}\right) = \frac{1}{JG}[2864.789(3) + 1432.394(4) + 0]$$

$$= \frac{14323.945}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0195\text{ rad} = 1.11°$$ **Ans.**

Ans:
$\tau_{\max} = 14.6$ MPa, $\phi_{B/E} = 1.11°$

10–47.

The shaft is made of A992 steel. It has a diameter of 1 in. and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of B with respect to D.



SOLUTION

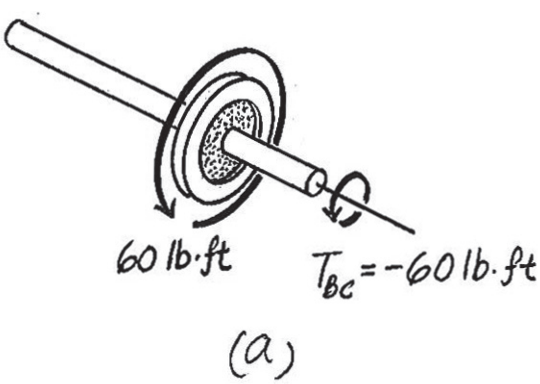
The internal torques developed in segments BC and CD are shown in Figs. a and b.

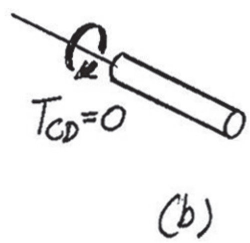
The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi\ \text{in}^4$. Thus,

$$\phi_{B/D} = \sum\frac{T_iL_i}{J_iG_i} = \frac{T_{BC}L_{BC}}{J\,G_{st}} + \frac{T_{CD}L_{CD}}{J\,G_{st}}$$

$$= \frac{-60(12)(2.5)(12)}{(0.03125\pi)[11.0(10^6)]} + 0$$

$$= -0.02000\ \text{rad} = 1.15° \qquad \textbf{Ans.}$$

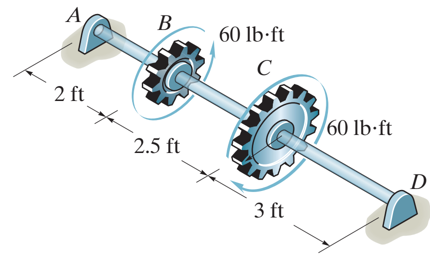




Ans:
$\phi_{B/D} = 1.15°$

***10–48.**

The shaft is made of A-36 steel. It has a diameter of 1 in. and is supported by bearings at *A* and *D*, which allow free rotation. Determine the angle of twist of gear *C* with respect to *B*.



SOLUTION

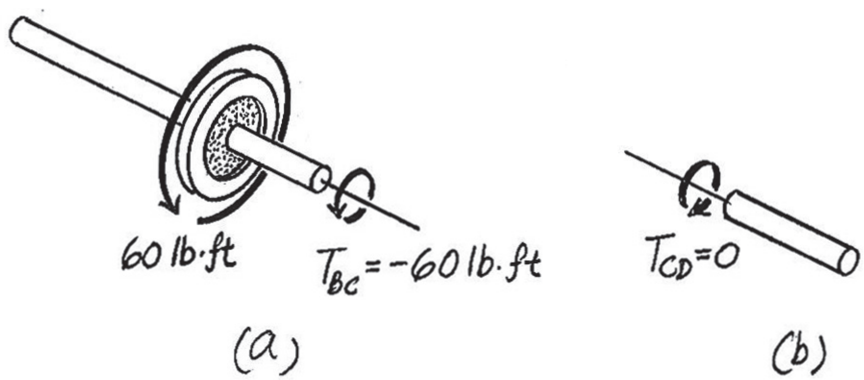
The internal torque developed in segment BC is shown in Fig. *a*.

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi\ \text{in}^4$. Thus,

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{J\,G_{st}} = \frac{-60(12)(2.5)(12)}{(0.03125\pi)[11.0(10^6)]}$$

$$= -0.02000\ \text{rad}$$

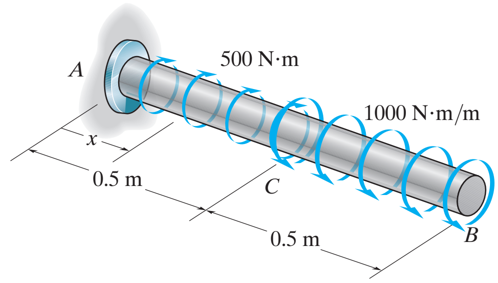
$$= 1.15° \qquad \textbf{Ans.}$$



Ans:
$\phi_{C/B} = 1.15°$

10–49.

The A992 steel shaft has a diameter of 50 mm and is subjected to the distributed loadings shown. Determine the absolute maximum shear stress in the shaft and plot a graph of the angle of twist of the shaft in radians versus x.



SOLUTION

Internal Torque: Referring to the FBD of the right segment of the shaft shown in Fig. *a* ($0.5\text{ m} < x \leq 1\text{ m}$),

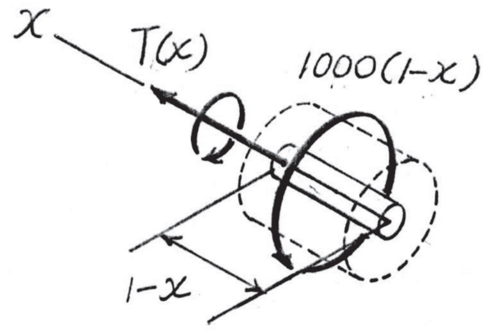
$$\Sigma M_x = 0; \quad T(x) - 1000\,(1-x) = 0 \qquad T(x) = \{1000 - 1000x\}\ \text{N}\cdot\text{m}$$

And Fig. *b* ($0 \leq x < 0.5$ m),

$$\Sigma M_x = 0; \quad T(x) + 500\,(0.5-x) - 1000\,(0.5) = 0 \quad T_{Ac} = \{500x + 250\}\ \text{N}\cdot\text{m}$$

Maximum Shear Stress: The maximum torque is at $x = 0.5$ m, where $T_{\max} = 1000 - 1000(0.5) = 500\ \text{N}\cdot\text{m}$. Applying the torsion formula,

$$\tau_{\substack{\text{abs}\\ \max}} = \frac{T_{\max}c}{J} = \frac{500\,(0.025)}{\frac{\pi}{2}(0.025^4)} = 20.37\left(10^6\right)P_a = 20.4\text{ MPa} \qquad \textbf{Ans.}$$

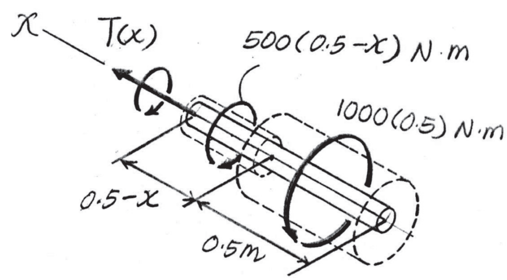


(a)

Angle of Twist: For A992 steel, $G = 75$ GPa. For region $0 \leq x < 0.5$ m (between B and C),

$$\phi(x) = \int \frac{T(x)dx}{JG} = \frac{1}{\left[\frac{\pi}{2}(0.025^4)\right]\left[75\left(10^9\right)\right]}\int_o^x (500\,x + 250)\,dx$$

$$= \left\{0.005432\left(x^2 + x\right)\right\}\text{rad} \qquad \textbf{Ans.}$$



(b)

At $x = 0.5$ m, $\phi = \phi_C = 0.004074$ rad. For region $0.5\text{ m} < x \leq 1$ m,

$$\phi\,(x) = \phi_C + \int \frac{T(x)\,dx}{JG}$$

$$= 0.004074 + \frac{1}{\left[\frac{\pi}{2}(0.025^4)\left[75\left(10^9\right)\right]\right.}\int_{0.5\text{ m}}^{x}(1000 - 1000x)\,dx$$

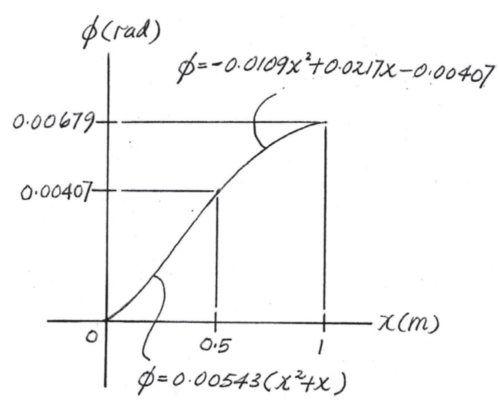
$$= \{-0.01086x^2 + 0.02173\,x - 0.004074\}\text{ rad} \qquad \textbf{Ans.}$$

The maximum ϕ occurs at where $\dfrac{d\phi}{dx} = 0$.

$$\frac{d\phi}{dx} = -0.02173x + 0.02173 = 0$$

$$x = 1\text{ m}$$

Thus, $\phi_{\max} = \phi_B = -0.01086\left(1^2\right) + 0.02173(1) - 0.004074 = 0.006791$ rad. The plot of ϕ vs x is as shown in Fig. *c*.

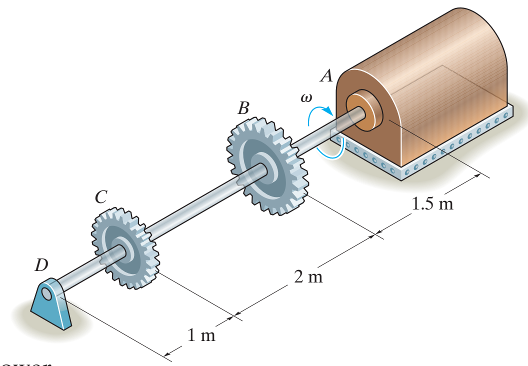


(c)

Ans:
$\tau_{\substack{\text{abs}\\ \max}} = 20.4$ MPa
For $0 \leq x < 0.5$ m,
$\phi(x) = \left\{0.005432\left(x^2 + x\right)\right\}$ rad
For $0.5\text{ m} < x \leq 1$ m,
$\phi(x) = \{-0.01086x^2 + 0.02173\,x - 0.004074\}$ rad

10–50.

The turbine develops 300 kW of power, which is transmitted to the gears such that both B and C receive an equal amount. If the rotation of the 100-mm-diameter A992 steel shaft is $\omega = 600$ rev/min., determine the absolute maximum shear stress in the shaft and the rotation of end D of the shaft relative to A. The journal bearing at D allows the shaft to turn freely about its axis.



SOLUTION

External Applied Torque: Gears B and C withdraw equal amount of power, $P_B = P_C = P = \dfrac{300}{2} = 150$ kW. Here, the angular velocity of the shaft is

$\omega = \left(600\,\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right) = 20\pi$ rad/s. Then, the torque exerted on gears B and C can be determined from

$$T_B = T_C\frac{P}{\omega} = \frac{150(10^3)}{20\pi} = \frac{7500}{\pi}\ \text{N}\cdot\text{m}$$

Using this result, the torque diagram shown in Fig. *a* can be plotted.

Maximum Shear Stress: From the torque diagram, we notice that the maximum torque $T_{\max} = \dfrac{15000}{\pi}$ N·m occurs at region BA. Thus, it is the critical region where the absolute maximim shear stress occurs. Applying the torsion formula,

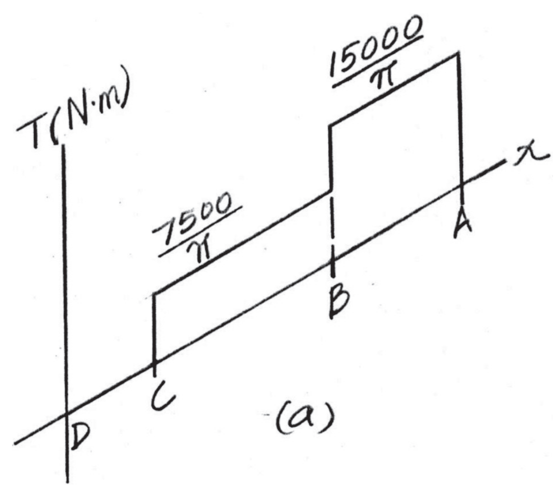
$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{T_{\max}\,c}{J} = \frac{\frac{15000}{\pi}(0.05)}{\frac{\pi}{2}(0.05^4)} = 24.32\,(10^6)\ \text{Pa} = 24.3\ \text{MPa} \qquad \textbf{Ans.}$$

Angle of twist: From the torque diagram, $T_{DC} = 0$, $T_{CB} = \dfrac{7500}{\pi}$ N·m and $T_{BA} = \dfrac{15000}{\pi}$ N·m.

$$\phi_{D/A} = \Sigma\frac{TL}{JG} = \frac{1}{JG}(T_{DC}L_{DC} + T_{CB}L_{CB} + T_{BA}L_{BA})$$

$$= \frac{1}{JG}\left[0 + \left(\frac{7500}{\pi}\right)(2) + \left(\frac{15000}{\pi}\right)(1.5)\right]$$

$$= \frac{37500\ \text{N}\cdot\text{m}^2}{\pi JG}$$



For A992 steel, $G = 75$ GPa. Thus,

$$\phi_{D/A} = \frac{37500}{\pi\left[\frac{\pi}{2}(0.05^4)\right][75(10^9)]}$$

$$= (0.016211\ \text{rad})\left(\frac{180°}{\pi\ \text{rad}}\right)$$

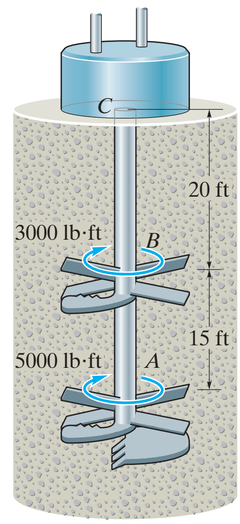
$$= 0.9288° \qquad \textbf{Ans.}$$

$$= 0.929°$$

Ans:
$\tau_{\substack{\text{abs}\\ \text{max}}} = 24.3$ MPa
$\phi_{D/A} = 0.929°$

10–51.

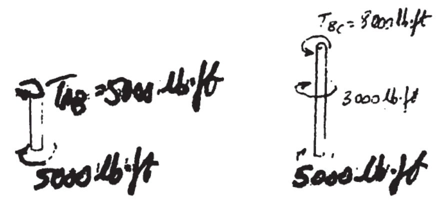
The device shown is used to mix soils in order to provide in-situ stabilization. If the mixer is connected to an A-36 steel tubular shaft that has an inner diameter of 3 in. and an outer diameter of 4.5 in., determine the angle of twist of the shaft at A relative to C if each mixing blade is subjected to the torques shown.



SOLUTION

$$\phi_{A/C} = \Sigma\left(\frac{TL}{JG}\right) = \frac{5000(12)(15)(12)}{\frac{\pi}{2}\left((2.25)^4 - (1.5)^4\right)(11)\left(10^6\right)} + \frac{8000(12)(20)(12)}{\frac{\pi}{2}\left((2.25)^4 - (1.5)^4\right)(11)\left(10^6\right)}$$

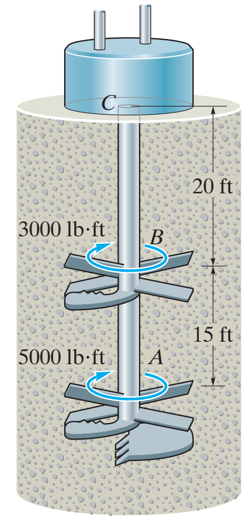
$$= 0.0952 \text{ rad} = 5.45° \qquad \textbf{Ans.}$$



Ans:
$\phi_{A/C} = 5.45°$

***10–52.**

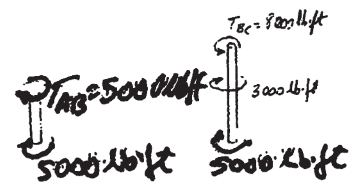
The device shown is used to mix soils in order to provide in-situ stabilization. If the mixer is connected to an A-36 steel tubular shaft that has an inner diameter of 3 in. and an outer diameter of 4.5 in, determine the angle of twist of the shaft at A relative to B and the absolute maximum shear stress in the shaft if each mixing blade is subjected to the torques shown.



SOLUTION

$$\phi_{A/B} = \frac{TL}{JG} = \frac{5000(12)(15)(12)}{\frac{\pi}{2}\left[(2.25)^4 - (1.5)^4\right]11\left(10^6\right)} = 0.03039 \text{ rad} = 1.74^\circ \qquad \textbf{Ans.}$$

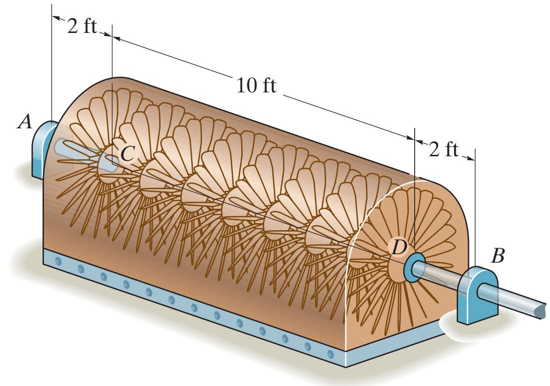
$$\tau_{\max} = \frac{T_C}{J} = \frac{8000(12)(2.25)}{\frac{\pi}{2}\left[(2.25)^4 - (1.50)^4)\right]} = 6.69 \text{ ksi} \qquad \textbf{Ans.}$$



Ans:
$\phi_{A/B} = 1.74^\circ$,
$\tau_{\max} = 6.69$ ksi

10–53.

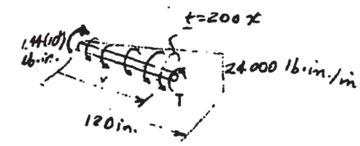
The 6-in.-diameter L-2 steel shaft on the turbine is supported on journal bearings at A and B. If C is held fixed and the turbine blades create a torque on the shaft that increases linearly from zero at C to 2000 lb · ft at D, determine the angle of twist of the shaft at D relative to C. Also, calculate the absolute maximum shear stress in the shaft. Neglect the size of the blades.



SOLUTION

$$T_{max} = \frac{1}{2}(120)(200(120)) = 1.44(10^6)$$

$$T = 1.44(10^6) - \frac{1}{2}(x)(200x) = 1.44(10^6) - 100x^2$$



$$\phi_{D/C} = \int \frac{T\,dx}{JG} = \frac{1}{JG}\int_0^{120} 1.44\left(10^6\right)dx - 100x^2\,dx$$

$$= \frac{1.44\left(10^6\right)(120)}{\pi\frac{(3)^4}{2}\left(11.0\left(10^6\right)\right)} - \frac{100\,(120)^3}{3\left(\pi\frac{(3)^4}{2}\right)\left(11.0\left(10^6\right)\right)} = 0.0823 \text{ rad}$$ **Ans.**

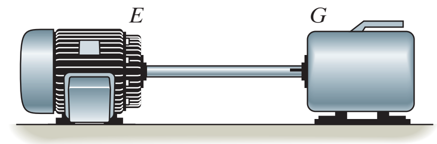
Maximum torque occurs at $x = 0$.

$$\tau_{\substack{abs\\max}} = \frac{Tc}{J} = \frac{1.44\left(10^6\right)(3)}{\pi\frac{(3)^4}{2}} = 34.0 \text{ ksi}$$ **Ans.**

Ans:
$\phi_{D/C} = 0.0823$ rad,
$\tau_{\substack{abs\\max}} = 34.0$ ksi

10–54.

The A-36 hollow steel shaft is 2 m long and has an outer diameter of 40 mm. When it is rotating at 80 rad/s, it transmits 32 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{\text{allow}} = 140$ MPa and the shaft is restricted not to twist more than 0.05 rad.



SOLUTION

$P = T\omega$

$32(10^3) = T(80)$

$T = 400\ \text{N}\cdot\text{m}$

Shear stress failure:

$$\tau = \frac{Tc}{J}$$

$$\tau_{\text{allow}} = 140(10^6) = \frac{400(0.02)}{\frac{\pi}{2}(0.02^4 - r_i^{\,4})}$$

$r_i = 0.01875$ m

Angle of twist limitation:

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{400(2)}{\frac{\pi}{2}(0.02^4 - r_i^{\,4})(75)(10^9)}$$

$r_i = 0.01247$ m (controls)

$t = r_o - r_i = 0.02 - 0.01247$

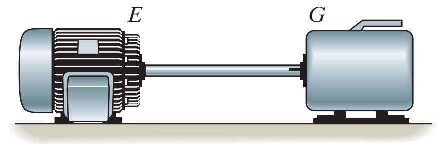
$= 0.00753$ m

$= 7.53$ mm **Ans.**

Ans:
$t = 7.53$ mm

10–55.

The A-36 solid steel shaft is 3 m long and has a diameter of 50 mm. It is required to transmit 35 kW of power from the engine E to the generator G. Determine the smallest angular velocity of the shaft if it is restricted not to twist more than 1°.



SOLUTION

$$\phi = \frac{TL}{JG}$$

$$\frac{1°(\pi)}{180°} = \frac{T(3)}{\frac{\pi}{2}(0.025^4)(75)(10^9)}$$

$$T = 267.73 \text{ N} \cdot \text{m}$$

$$P = T\omega$$

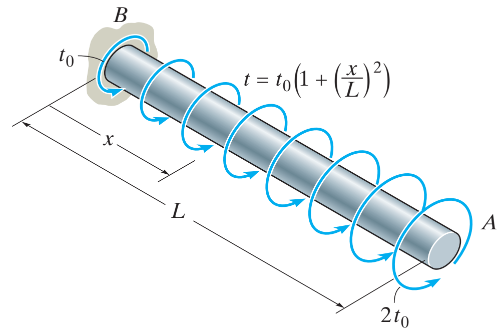
$$35(10^3) = 267.73(\omega)$$

$$\omega = 131 \text{ rad/s} \qquad \textbf{Ans.}$$

Ans:
$\omega = 131$ rad/s

***10–56.**

The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the angle of twist at end A. The shear modulus is G.

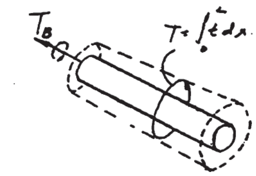


SOLUTION

$$T_B - \int t\,dx = 0$$

$$T_B = \int t\,dx = t_0 \int \left(1 + \frac{x^2}{L^2}\right) dx$$

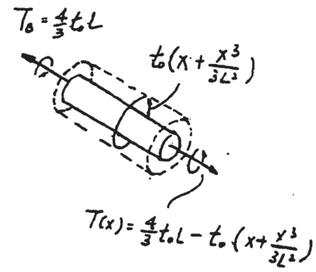
$$= t_0\left[x + \frac{x^3}{3L^2}\right]\Bigg|_0^L = t_0\left(L + \frac{L}{3}\right) = \frac{4}{3} t_0 L$$



$$\phi = \int \frac{T(x)\,dx}{JG}$$

$$= \frac{1}{JG} \int_0^L \left[\frac{4}{3} t_0 L - t_0\left(x + \frac{x^3}{3L^2}\right)\right] dx$$

$$= \frac{t_0}{JG}\left[\frac{4}{3} Lx - \left(\frac{x^2}{2} + \frac{x^4}{12L^2}\right)\right]\Bigg|_0^L = \frac{7\,t_0\,L^2}{12\,JG}$$



However, $J = \frac{\pi}{2} c^4$.

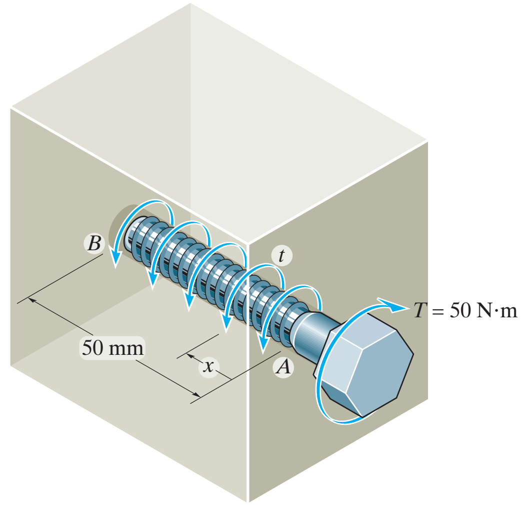
$$\phi = \frac{7\,t_0\,L^2}{6\,\pi\,c^4 G}$$ **Ans.**

Ans:

$$\phi = \frac{7\,t_0\,L^2}{6\,\pi\,c^4 G}$$

10–57.

The A-36 steel bolt is tightened within a hole so that the reactive torque on the shank AB can be expressed by the equation $t = (kx^2)\ \text{N}\cdot\text{m/m}$, where x is in meters. If a torque of $T = 50\ \text{N}\cdot\text{m}$ is applied to the bolt head, determine the constant k and the amount of twist in the 50-mm length of the shank. Assume the shank has a constant radius of 4 mm.



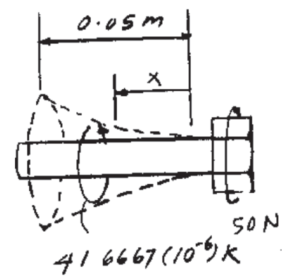
SOLUTION

$$dT = t\,dx$$

$$T = \int_0^{0.05\text{ m}} kx^2 dx = k\frac{x^3}{3}\Bigg|_0^{0.05} = 41.667(10^{-6})k$$

$$50 - 41.6667(10^{-6})\,k = 0$$

$$k = 1.20(10^6)\ \text{N/m}^2$$ **Ans.**



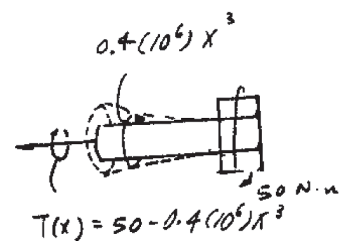
In the general position, $T = \int_0^x 1.20(10^6)x^2 dx = 0.4(10^6)x^3$

$$\phi = \int \frac{T(x)dx}{JG} = \frac{1}{JG}\int_0^{0.05\text{ m}} [50 - 0.4(10^6)x^3]dx$$

$$= \frac{1}{JG}\left[50x - \frac{0.4(10^6)x^4}{4}\right]\Bigg|_0^{0.05\text{ m}}$$

$$= \frac{1.875}{JG} = \frac{1.875}{\frac{\pi}{2}(0.004^4)(75)(10^9)}$$

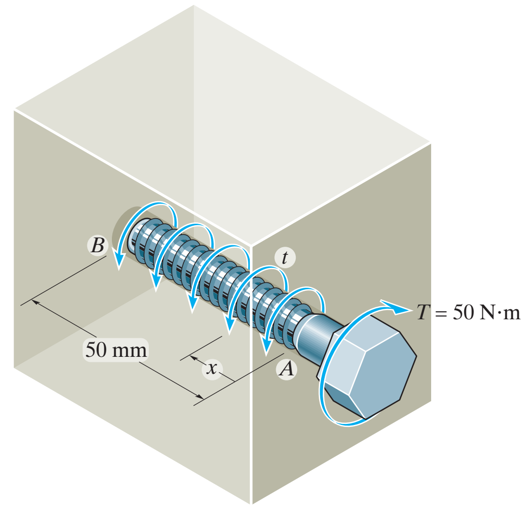
$$= 0.06217\ \text{rad} = 3.56°$$ **Ans.**



Ans:
$k = 1.20(10^6)\ \text{N/m}^2$, $\phi = 3.56°$

10–58.

Solve Prob. 10–57 if the distributed torque is $t = (kx^{2/3})\ \mathrm{N \cdot m/m}$.



SOLUTION

$dT = t\,dx$

$$T = \int_0^{0.05} kx^{\frac{2}{3}}\,dx = k\frac{3}{5}x^{\frac{5}{3}}\Big|_0^{0.05} = (4.0716)(10^{-3})\,k$$

$$50 - 4.0716\,(10^{-3})\,k = 0$$

$$k = 12.3(10^3)\ \mathrm{N/m}^{\left(\frac{2}{3}\right)}$$ **Ans.**

In the general position,

$$T = \int_0^x 12.28(10^3)x^{\frac{2}{3}}\,dx = 7.368(10^3)\,x^{\frac{5}{3}}$$

Angle of twist:

$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{0.05\text{ m}} [50 - 7.3681(10^3)x^{\frac{8}{3}}]dx$$

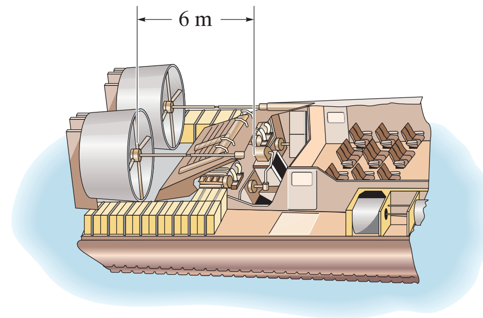
$$= \frac{1}{JG}\left[50x - 7.3681(10^3)\left(\frac{3}{8}\right)x^{\frac{8}{3}}\right]\Big|_0^{0.05\text{ m}}$$

$$= \frac{1.5625}{\frac{\pi}{2}(0.004^4)(75)(10^9)} = 0.0518\text{ rad} = 2.97°$$ **Ans.**

Ans:
$k = 12.3(10^3)\ \mathrm{N/m}^{2/3}$, $\phi = 2.97°$

10–59.

The tubular drive shaft for the propeller of a hovercraft is 6 m long. If the motor delivers 4 MW of power to the shaft when the propellers rotate at 25 rad/s, determine the required inner diameter of the shaft if the outer diameter is 250 mm. What is the angle of twist of the shaft when it is operating? Take $\tau_{\text{allow}} = 90$ MPa and $G = 75$ GPa.



SOLUTION

Internal Torque:

$$P = 4\left(10^6\right)\text{ W} = 4\left(10^6\right)\text{ N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{4\left(10^6\right)}{25} = 160(10^3)\text{ N}\cdot\text{m}$$

Maximum Shear Stress: Applying torsion formula,

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$90\left(10^6\right) = \frac{160\left(10^3\right)(0.125)}{\frac{\pi}{2}\left[0.125^4 - \left(\frac{d_t}{2}\right)^4\right]}$$

$$d_t = 0.2013\text{ m} = 201\text{ mm}$$ **Ans.**

Angle of Twist:

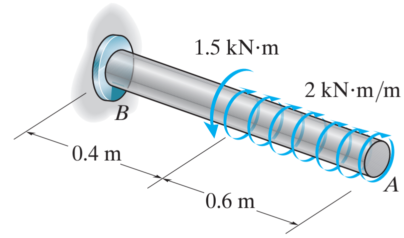
$$\phi = \frac{TL}{JG} = \frac{160\left(10^3\right)(6)}{\frac{\pi}{2}\left(0.125^4 - 0.10065^4\right)75\left(10^9\right)}$$

$$= 0.0576\text{ rad} = 3.30^\circ$$ **Ans.**

Ans:
$d_t = 201$ mm,
$\phi = 3.30^\circ$

***10–60.**

The 60-mm diameter solid shaft is made of 2014-T6 aluminum and is subjected to the distributed and concentrated torsional loadings shown. Determine the angle of twist at the free end A of the shaft.



SOLUTION

Internal Torque: Referring to the FBD of the right segments of the shaft shown in Fig. *a* and *b*,

$\Sigma M_x = 0;\qquad T_{AC} + 2000x = 0\qquad T_{AC} = (-2000x)\ \text{N}\cdot\text{m}$

And

$\Sigma M_x = 0;\qquad T_{CB} + 2000\,(0.6) - 1500 = 0\qquad T_{CB} = 300\ \text{N}\cdot\text{m}$

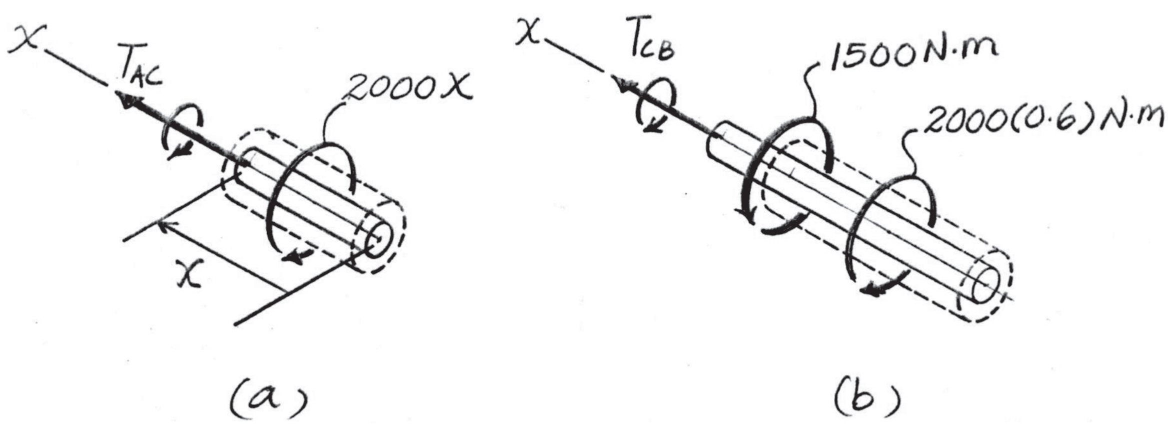
Angle of Twist: For 2014 - T6 aluminum, $G = 27$ GPa.

$$\phi_A = \Sigma\frac{TL}{JG} = \frac{1}{JG}\left[\int_0^x T_{AC}\,dx + T_{CB}\,L_{CB}\right]$$

$$= \frac{1}{\left[\frac{\pi}{2}(0.03)^4\right]\left[27(10^9)\right]}\left[\int_0^{0.6\text{ m}}(-2000\,x)dx + 300\,(0.4)\right]$$

$$= (-0.006986\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right)$$

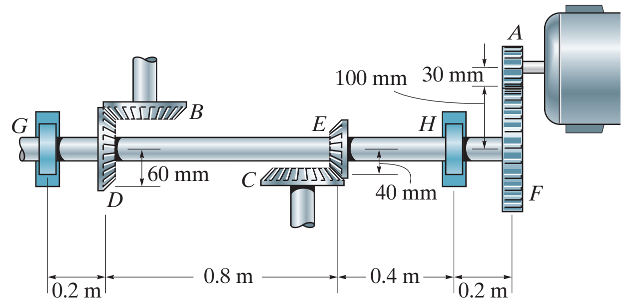
$$= -0.400° = 0.400°$$ **Ans.**



Ans:
$\phi_A = 0.400°$

10–61.

The motor produces a torque of $T = 20\ \text{N}\cdot\text{m}$ on gear A. If gear C is suddenly locked so it does not turn, yet B can freely turn, determine the angle of twist of F with respect to E and F with respect to D of the L2-steel shaft, which has an inner diameter of 30 mm and an outer diameter of 50 mm. Also, calculate the absolute maximum shear stress in the shaft. The shaft is supported on journal bearings at G at H.

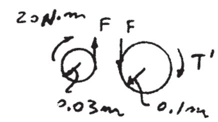


SOLUTION

$F(0.03) = 20$

$F = 666.67\ \text{N}$

$T' = (666.67)(0.1) = 66.67\ \text{N}\cdot\text{m}$



Since shaft is held fixed at C, the torque is only in region EF of the shaft.

$$\phi_{F/E} = \frac{TL}{JG} = \frac{66.67(0.6)}{\frac{\pi}{2}\left[(0.025)^4 - (0.015)^4\right]75\left(10^9\right)} = 0.999\left(10\right)^{-3}\ \text{rad}$$ **Ans.**

Since the torque in region ED is zero,

$\phi_{F/D} = 0.999(10)^{-3}\ \text{rad}$ **Ans.**

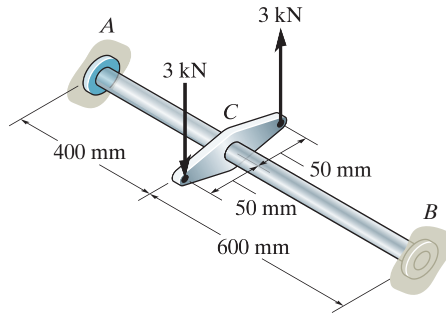
$$\tau_{\max} = \frac{Tc}{J} = \frac{66.67(0.025)}{\frac{\pi}{2}\left((0.025)^4 - (0.015)^4\right)}$$

$= 3.12\ \text{MPa}$ **Ans.**

Ans:
$\phi_{F/E} = 0.999(10)^{-3}$ rad,
$\phi_{F/D} = 0.999(10)^{-3}$ rad,
$\tau_{\max} = 3.12$ MPa

10–62.

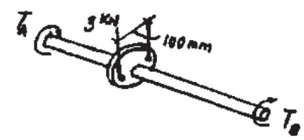
The steel shaft has a diameter of 40 mm and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB of the shaft. $G_{st} = 75$ GPa.



SOLUTION

Equilibrium:

$$T_A + T_B - 3000(0.1) = 0 \qquad \textbf{(1)}$$



Compatibility condition:

$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A\,(400)}{JG} = \frac{T_B(600)}{JG}$$

$$T_A = 1.5\ T_B \qquad \textbf{(2)}$$

Solving Eqs (1) and (2) yields:

$$T_B = 120\ \mathrm{N \cdot m}$$

$$T_A = 180\ \mathrm{N \cdot m}$$

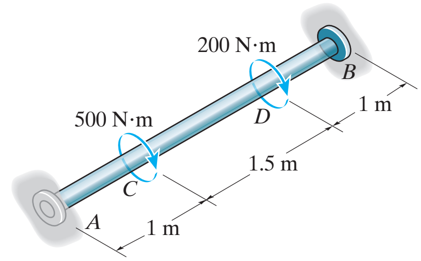
$$(\tau_{AC})_{\max} = \frac{Tc}{J} = \frac{180(0.02)}{\frac{\pi}{2}(0.02^4)} = 14.3\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CB})_{\max} = \frac{Tc}{J} = \frac{120(0.02)}{\frac{\pi}{2}(0.02^4)} = 9.55\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$(\tau_{AC})_{\max} = 14.3$ MPa, $(\tau_{CB})_{\max} = 9.55$ MPa

10–63.

The A992 steel shaft has a diameter of 60 mm and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft.



SOLUTION

Referring to the FBD of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_A + T_B - 500 - 200 = 0 \qquad \textbf{(1)}$$

Using the method of superposition, Fig. *b*,

$$\phi_A = (\phi_A)_{T_A} - (\phi_A)_T$$

$$0 = \frac{T_A\,(3.5)}{JG} - \left[\frac{500\,(1.5)}{JG} + \frac{700\,(1)}{JG}\right]$$

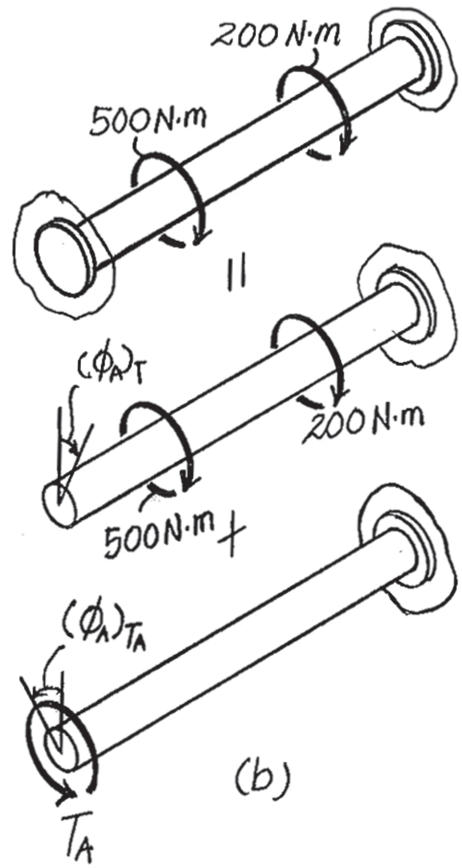
$$T_A = 414.29\ \text{N}\cdot\text{m}$$

Substitute this result into Eq (1).

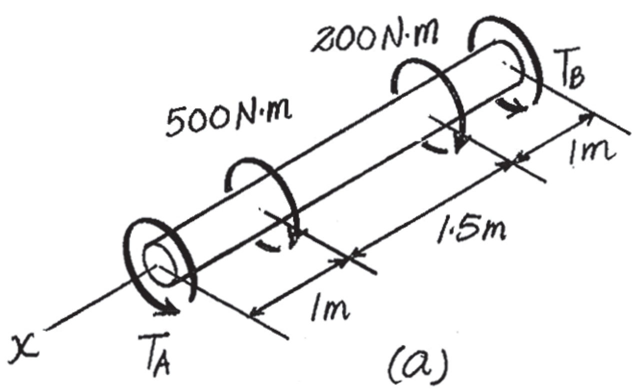
$$T_B = 285.71\ \text{N}\cdot\text{m}$$

Referring to the torque diagram shown in Fig. *c*, segment AC is subjected to maximum internal torque. Thus, the absolute maximum shear stress occurs here.

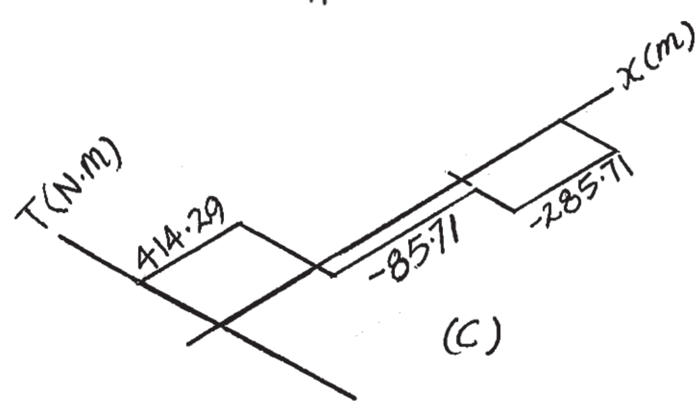
$$\tau_{\substack{abs\\max}} = \frac{T_{AC}\,c}{J} = \frac{414.29\,(0.03)}{\frac{\pi}{2}(0.03)^4} = 9.77\ \text{MPa} \qquad \textbf{Ans.}$$



(b)



(a)



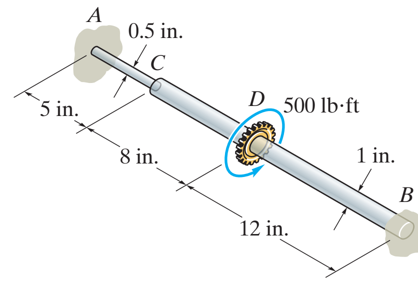
(c)

Ans:

$\tau_{\substack{abs\\max}} = 9.77\ \text{MPa}$

***10–64.**

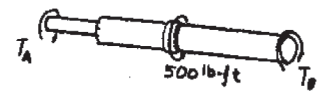
The steel shaft is made from two segments: AC has a diameter of 0.5 in., and CB has a diameter of 1 in. If the shaft is fixed at its ends A and B and subjected to a torque of 500 lb · ft, determine the maximum shear stress in the shaft. $G_{st} = 10.8(10^3)$ ksi.



SOLUTION

Equilibrium:

$$T_A + T_B - 500 = 0 \qquad \textbf{(1)}$$



Compatibility condition:

$$\phi_{D/A} = \phi_{D/B}$$

$$\frac{T_A(5)}{\frac{\pi}{2}(0.25^4)G} + \frac{T_A(8)}{\frac{\pi}{2}(0.5^4)G} = \frac{T_B(12)}{\frac{\pi}{2}(0.5^4)G}$$

$$1408\, T_A = 192\, T_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$T_A = 60 \text{ lb} \cdot \text{ft} \qquad T_B = 440 \text{ lb} \cdot \text{ft}$$

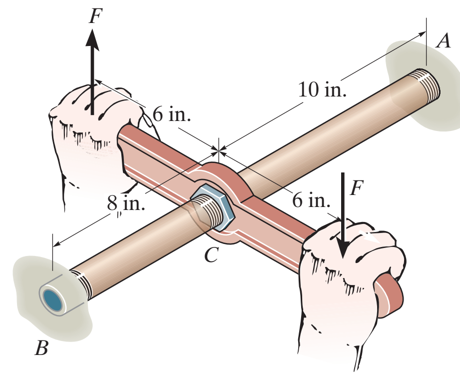
$$\tau_{AC} = \frac{Tc}{J} = \frac{60(12)(0.25)}{\frac{\pi}{2}(0.25^4)} = 29.3 \text{ ksi} \qquad \text{(max)} \qquad \textbf{Ans.}$$

$$\tau_{DB} = \frac{Tc}{J} = \frac{440(12)(0.5)}{\frac{\pi}{2}(0.5^4)} = 26.9 \text{ ksi}$$

Ans:
$\tau_{max} = 29.3$ ksi

10–65.

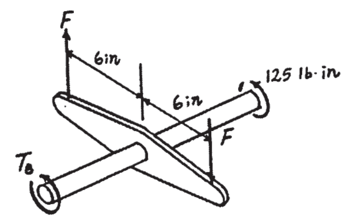
The bronze C86100 pipe has an outer diameter of 1.5 in. and a thickness of 0.125 in. The coupling on it at C is being tightened using a wrench. If the torque developed at A is 125 lb · in., determine the magnitude F of the couple forces. The pipe is fixed supported at end B.



SOLUTION

Equilibrium:

$$F(12) - T_B - 125 = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\phi_{C/B} = \phi_{C/A}$$

$$\frac{T_B(8)}{JG} = \frac{125(10)}{JG}$$

$$T_B = 156.25 \text{ lb} \cdot \text{in.}$$

From Eq (1),

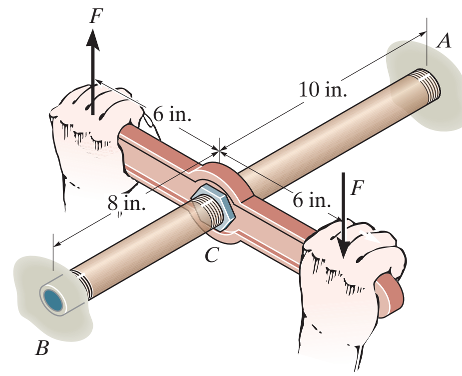
$$F(12) - 156.25 - 125 = 0$$

$$F = 23.4 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$F = 23.4$ lb

10–66.

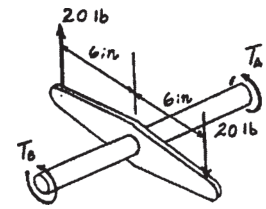
The bronze C86100 pipe has an outer diameter of 1.5 in. and a thickness of 0.125 in. The coupling on it at C is being tightened using a wrench. If the applied force is $F = 20$ lb, determine the maximum shear stress in the pipe.



SOLUTION

Equilibrium:

$$T_A + T_B - 20(12) = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\phi_{C/B} = \phi_{C/A}$$

$$\frac{T_B(8)}{JG} = \frac{T_A(10)}{JG}$$

$$T_B = 1.25T_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$T_A = 106.67 \text{ lb} \cdot \text{in.} \qquad T_B = 133.33 \text{ lb} \cdot \text{in.}$$

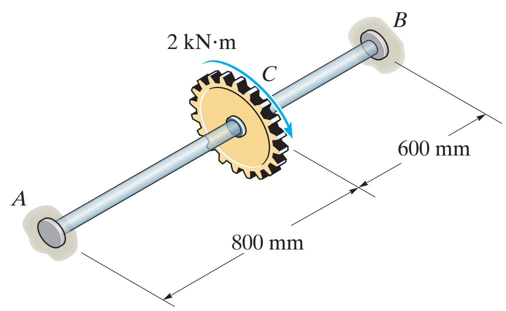
Maximum shear stress:

$$\tau_{max} = \frac{T_B\, c}{J} = \frac{133.33(0.75)}{\frac{\pi}{2}\left(0.75^4 - 0.625^4\right)} = 389 \text{ psi} \qquad \textbf{Ans.}$$

Ans:
$\tau_{max} = 389$ psi

10–67.

The shaft is made of L2 tool steel, has a diameter of 40 mm, and is fixed at its ends A and B. If it is subjected to the torque, determine the maximum shear stress in regions AC and CB.



SOLUTION

Equilibrium: Referring to the FBD of the shaft, Fig. a,

$$\Sigma M_x = 0; \qquad T_A + T_B - 2000 = 0 \qquad \textbf{(1)}$$

Compatibility: It is required that

$$\phi_{CA} = \phi_{CB}$$

$$\frac{T_A\, L_{CA}}{JG} = \frac{T_B\, L_{CB}}{JG}$$

$$\frac{T_A\,(0.8)}{JG} = \frac{T_B\,(0.6)}{JG}$$

$$T_A = 0.75\, T_B \qquad \textbf{(2)}$$

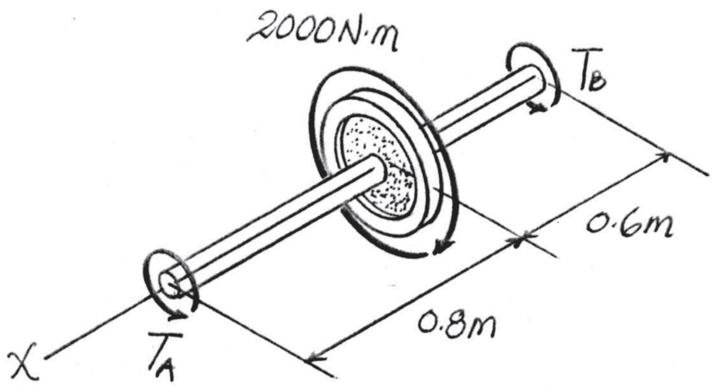
Solving Eqs (1) and (2),

$$T_B = 1142.86\ \text{N}\cdot\text{m} \qquad T_A = 857.14\ \text{N}\cdot\text{m}$$

Maximum Shear Stress: Applying the torsion formula,

$$(\tau_{\max})_{AC} = \frac{T_A\, c}{J} = \frac{857.14\,(0.02)}{\frac{\pi}{2}\left(0.02^4\right)} = 68.21\left(10^6\right)\ \text{Pa} = 68.2\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\max})_{BC} = \frac{T_B\, c}{J} = \frac{1142.86\,(0.02)}{\frac{\pi}{2}\left(0.02^4\right)} = 90.946\left(10^6\right)\ \text{Pa} = 90.9\ \text{MPa} \qquad \textbf{Ans.}$$

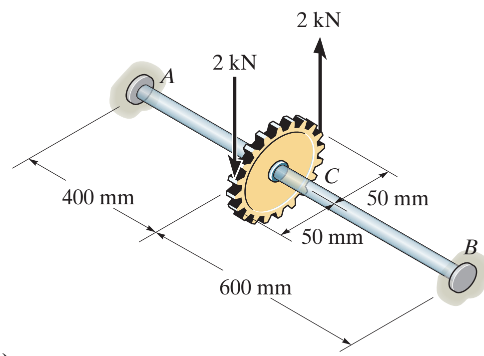


(a)

Ans:
$(\tau_{\max})_{AC} = 68.2$ MPa,
$(\tau_{\max})_{BC} = 90.9$ MPa

***10–68.**

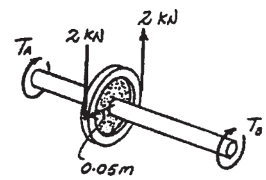
The shaft is made of L2 tool steel, has a diameter of 40 mm, and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB.



SOLUTION

Equilibrium:

$$T_A + T_B - 2(0.1) = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A(0.4)}{JG} = \frac{T_B(0.6)}{JG}$$

$$T_A = 1.50 T_B \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$T_B = 0.080 \text{ kN}\cdot\text{m} \qquad T_A = 0.120 \text{ kN}\cdot\text{m}$$

Maximum Shear Stress:

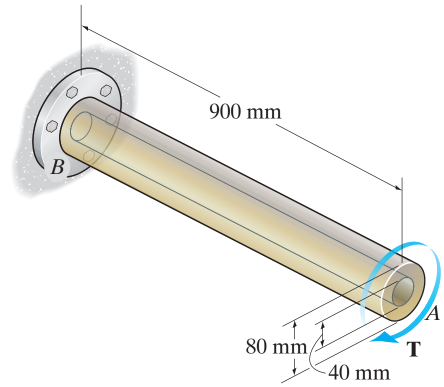
$$(\tau_{AC})_{\max} = \frac{T_A\, c}{J} = \frac{0.12(10^3)(0.02)}{\frac{\pi}{2}(0.02^4)} = 9.55 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CB})_{\max} = \frac{T_B\, c}{J} = \frac{0.08(10^3)(0.02)}{\frac{\pi}{2}(0.02^4)} = 6.37 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$(\tau_{AC})_{\max} = 9.55$ MPa,
$(\tau_{CB})_{\max} = 6.37$ MPa

10–69.

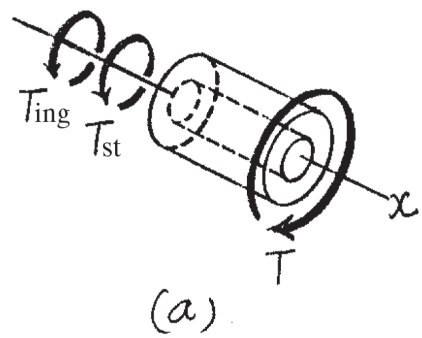
The Am1004-T61 magnesium tube is bonded to the A-36 steel rod. If the allowable shear stresses for the magnesium and steel are $(\tau_{allow})_{mg} = 45$ MPa and $(\tau_{allow})_{st} = 75$ MPa, respectively, determine the maximum allowable torque that can be applied at A. Also, find the corresponding angle of twist of end A.



SOLUTION

Equilibrium: Referring to the free-body diagram of the cut part of the assembly shown in Fig. a,

$$\Sigma M_x = 0; \quad T_{mg} + T_{st} - T = 0 \qquad \textbf{(1)}$$



Compatibility Equation: Since the steel rod is bonded firmly to the magnesium tube, the angle of twist of the rod and the tube must be the same. Thus,

$$(\phi_{st})_A = (\phi_{mg})_A$$

$$\frac{T_{st}L}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = \frac{T_{mg}L}{\frac{\pi}{2}(0.04^4 - 0.02^4)(18)(10^9)}$$

$$T_{st} = 0.2778T_{mg} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_{mg} = 0.7826T \qquad T_{st} = 0.2174T$$

Allowable Shear Stress:

$$(\tau_{allow})_{mg} = \frac{T_{mg}\,c}{J}; \quad 45(10^6) = \frac{0.7826T(0.04)}{\frac{\pi}{2}(0.04^4 - 0.02^4)}$$

$$T = 5419.25 \text{ N} \cdot \text{m}$$

$$(\tau_{allow})_{st} = \frac{T_{st}c}{J}; \quad 75(10^6) = \frac{0.2174T(0.02)}{\frac{\pi}{2}(0.02^4)}$$

$$T = 4335.40 \text{ N} \cdot \text{m} = 4.34 \text{ kN} \cdot \text{m (control!)} \qquad \textbf{Ans.}$$

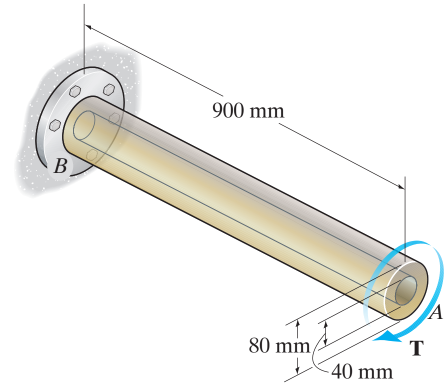
Angle of Twist: Using the result of T, $T_{st} = 942.48$ N · m. We have

$$\phi_A = \frac{T_{st}L}{J_{st}G_{st}} = \frac{942.48(0.9)}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = 0.045 \text{ rad} = 2.58^\circ \qquad \textbf{Ans.}$$

Ans:
$T = 4.34$ kN · m,
$\phi_A = 2.58°$

10–70.

The Am1004-T61 magnesium tube is bonded to the A-36 steel rod. If a torque of $T = 5\ \text{kN}\cdot\text{m}$ is applied to end A, determine the maximum shear stress in each material. Sketch the shear stress distribution.



SOLUTION

Equilibrium: Referring to the free-body diagram of the cut part of the assembly shown in Fig. *a*,

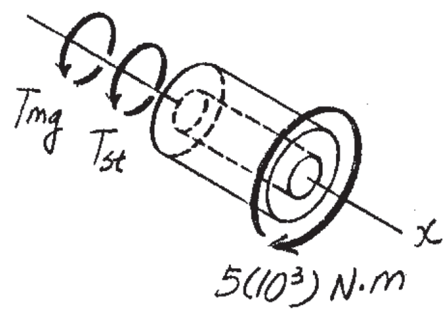
$$\Sigma M_x = 0; \quad T_{mg} + T_{st} - 5(10^3) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Since the steel rod is bonded firmly to the magnesium tube, the angle of twist of the rod and the tube must be the same. Thus,

$$(\phi_{st})_A = (\phi_{mg})_A$$

$$\frac{T_{st}L}{\frac{\pi}{2}(0.02^4)(75)(10^9)} = \frac{T_{mg}L}{\frac{\pi}{2}(0.04^4 - 0.02^4)(18)(10^9)}$$

$$T_{st} = 0.2778T_{mg} \qquad \textbf{(2)}$$



(a)

Solving Eqs. (1) and (2),

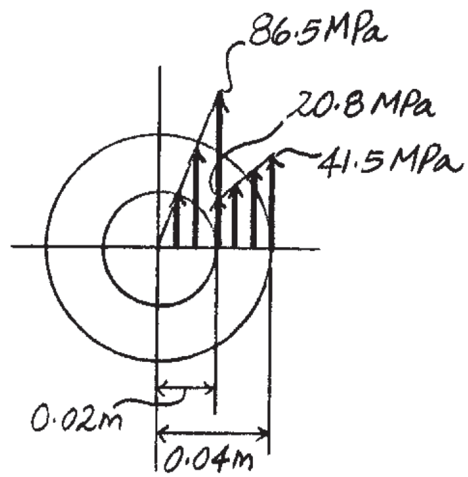
$$T_{mg} = 3913.04\ \text{N}\cdot\text{m} \qquad T_{st} = 1086.96\ \text{N}\cdot\text{m}$$

Maximum Shear Stress:

$$(\tau_{st})_{max} = \frac{T_{st}c_{st}}{J_{st}} = \frac{1086.96(0.02)}{\frac{\pi}{2}(0.02^4)} = 86.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{mg})_{max} = \frac{T_{mg}c_{mg}}{J_{mg}} = \frac{3913.04(0.04)}{\frac{\pi}{2}(0.04^4 - 0.02^4)} = 41.5\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{mg})\big|_{\rho=0.02\ \text{m}} = \frac{T_{mg}\,\rho}{J_{mg}} = \frac{3913.04(0.02)}{\frac{\pi}{2}(0.04^4 - 0.02^4)} = 20.8\ \text{MPa} \qquad \textbf{Ans.}$$

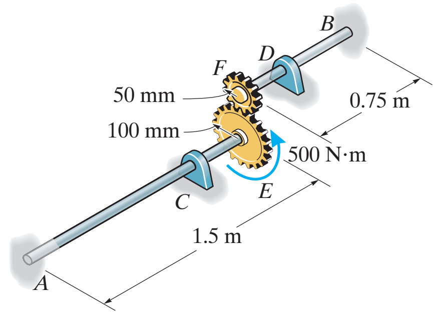


(b)

Ans:
$(\tau_{st})_{max} = 86.5\ \text{MPa}$, $(\tau_{mg})_{max} = 41.5\ \text{MPa}$, $(\tau_{mg})\big|_{\rho=0.02\,\text{m}} = 20.8\ \text{MPa}$

10–71.

The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by journal bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of $500\ \mathrm{N \cdot m}$ is applied to the gear at E, determine the reactions at A and B.

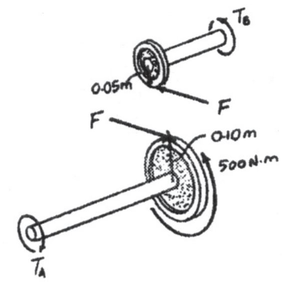


SOLUTION

Equilibrium:

$$T_A + F(0.1) - 500 = 0 \qquad \textbf{(1)}$$

$$T_B - F(0.05) = 0 \qquad \textbf{(2)}$$



From Eqs. (1) and (2),

$$T_A + 2T_B - 500 = 0 \qquad \textbf{(3)}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

$$T_A = 0.250T_B \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

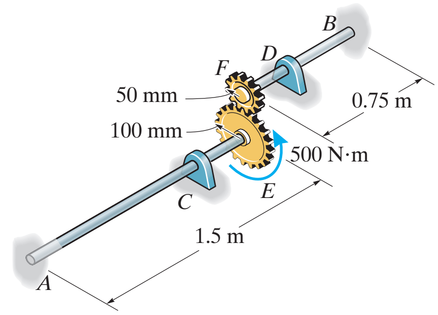
$$T_B = 22.2\ \mathrm{N \cdot m} \qquad \textbf{Ans.}$$

$$T_A = 55.6\ \mathrm{N \cdot m} \qquad \textbf{Ans.}$$

Ans:
$T_B = 22.2\ \mathrm{N \cdot m}$, $T_A = 55.6\ \mathrm{N \cdot m}$

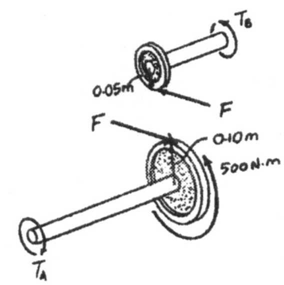
***10–72.**

The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by journal bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 500 N · m is applied to the gear at E, determine the rotation of this gear.



SOLUTION

Equilibrium:



$$T_A + F(0.1) - 500 = 0 \qquad \textbf{(1)}$$

$$T_B - F(0.05) = 0 \qquad \textbf{(2)}$$

From Eqs. (1) and (2),

$$T_A + 2T_B - 500 = 0 \qquad \textbf{(3)}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

$$T_A = 0.250T_B \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$$T_B = 222.22\ \text{N} \cdot \text{m} \qquad T_A = 55.56\ \text{N} \cdot \text{m}$$

Angle of Twist:

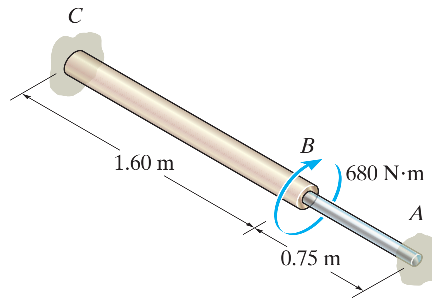
$$\phi_E = \frac{T_A L}{JG} = \frac{55.56(1.5)}{\frac{\pi}{2}(0.0125^4)(75.0)(10^9)}$$

$$= 0.02897\ \text{rad} = 1.66° \qquad \textbf{Ans.}$$

Ans:
$\phi_E = 1.66°$

10–73.

A rod is made from two segments: AB is steel and BC is brass. It is fixed at its ends and subjected to a torque of $T = 680\ \text{N}\cdot\text{m}$. If the steel portion has a diameter of 30 mm, determine the required diameter of the brass portion so the reactions at the walls will be the same. $G_{st} = 75$ GPa, $G_{br} = 39$ GPa.



SOLUTION

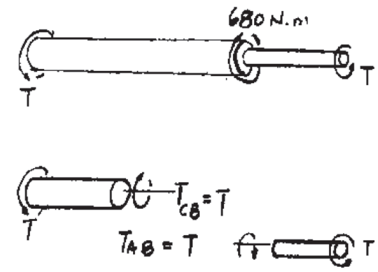
Compatibility Condition:

$$\phi_{B/C} = \phi_{B/A}$$

$$\frac{T(1.60)}{\frac{\pi}{2}(c^4)(39)(10^9)} = \frac{T(0.75)}{\frac{\pi}{2}(0.015^4)(75)(10^9)}$$

$c = 0.02134$ m

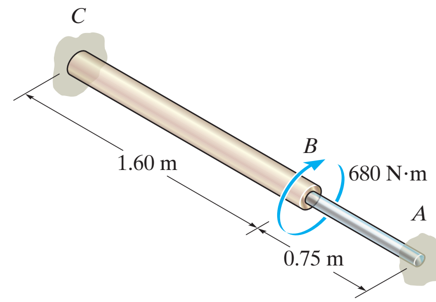
$d = 2c = 0.04269\text{ m} = 42.7\text{ mm}$ **Ans.**



Ans:
$d = 42.7$ mm

10–74.

Determine the absolute maximum shear stress in the shaft of Prob. 10–73.



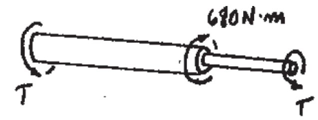
SOLUTION

Equilibrium,

$$2T = 680$$

$$T = 340\ \text{N}\cdot\text{m}$$

$\tau_{\substack{\text{abs}\\ \text{max}}}$ occurs in the steel. See solution to Prob. 5–88.



$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{Tc}{J} = \frac{340(0.015)}{\frac{\pi}{2}(0.015)^4}$$

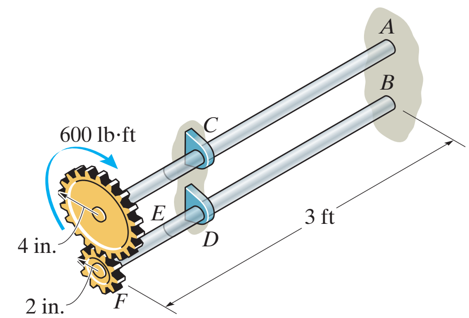
$$= 64.1\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:

$\tau_{\substack{\text{abs}\\ \text{max}}} = 64.1\ \text{MPa}$

10–75.

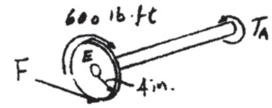
The two 3-ft-long shafts are made of 2014-T6 aluminum. Each has a diameter of 1.5 in. and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 600 lb·ft is applied to the top gear as shown, determine the maximum shear stress in each shaft.

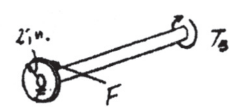


SOLUTION

$$T_A + F\left(\frac{4}{12}\right) - 600 = 0 \qquad (1)$$

$$T_B - F\left(\frac{2}{12}\right) = 0 \qquad (2)$$





From Eqs. (1) and (2),

$$T_A + 2T_B - 600 = 0 \qquad (3)$$

$$4(\phi_E) = 2(\phi_F); \qquad \phi_E = 0.5\phi_F$$

$$\frac{T_A L}{JG} = 0.5\left(\frac{T_B L}{JG}\right); \qquad T_A = 0.5T_B \qquad (4)$$

Solving Eqs. (3) and (4) yields:

$$T_B = 240 \text{ lb} \cdot \text{ft}; \qquad T_A = 120 \text{ lb} \cdot \text{ft}$$

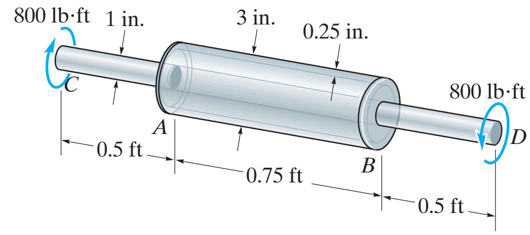
$$(\tau_{BD})_{max} = \frac{T_B\, c}{J} = \frac{240(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 4.35 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{AC})_{max} = \frac{T_A\, c}{J} = \frac{120(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 2.17 \text{ ksi} \qquad \textbf{Ans.}$$

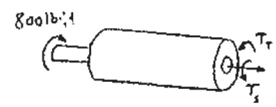
Ans:
$(\tau_{BD})_{max} = 4.35$ ksi,
$(\tau_{AC})_{max} = 2.17$ ksi

***10–76.**

The composite shaft consists of a mid-section that includes the 1-in.-diameter solid shaft and a tube that is welded to the rigid flanges at A and B. Neglect the thickness of the flanges and determine the angle of twist of end C of the shaft relative to end D. The shaft is subjected to a torque of 800 lb · ft. The material is A-36 steel.



SOLUTION



Equilibrium:

$$800(12) - T_\gamma - T_s = 0$$

Compatibility Condition:

$$\phi_T = \phi_S; \qquad \frac{T_T(0.75)(12)}{\frac{\pi}{2}((1.5)^4 - (1.25)^4)G} = \frac{T_S(0.75)(12)}{\frac{\pi}{2}(0.5)^4 G}$$

$$T_T = 9376.42 \text{ lb} \cdot \text{in.}$$

$$T_S = 223.58 \text{ lb} \cdot \text{in.}$$

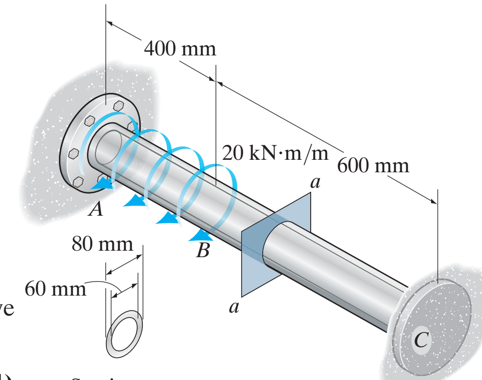
$$\phi_{C/D} = \Sigma\frac{TL}{JG} = \frac{800(12)(1)(12)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} + \frac{223.58(0.75)(12)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)}$$

$$= 0.1085 \text{ rad} = 6.22°$$ **Ans.**

Ans:
$\phi_{C/D} = 6.22°$

10–77.

If the shaft is subjected to a uniform distributed torque of $t = 20\ \text{kN}\cdot\text{m/m}$, determine the maximum shear stress developed in the shaft. The shaft is made of 2014-T6 aluminum alloy and is fixed at A and C.



Section a–a

SOLUTION

Equilibrium: Referring to the free-body diagram of the shaft shown in Fig. a, we have

$$\Sigma M_x = 0; \quad T_A + T_C - 20(10^3)(0.4) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: The resultant torque of the distributed torque within the region x of the shaft is $T_R = 20(10^3)x\ \text{N}\cdot\text{m}$. Thus, the internal torque developed in the shaft as a function of x when end C is free is $T(x) = 20(10^3)x\ \text{N}\cdot\text{m}$, Fig. b. Using the method of superposition, Fig. c,

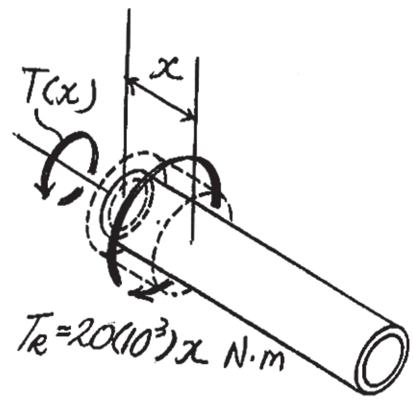
$$\phi_C = (\phi_C)_t - (\phi_C)_{T_C}$$

$$0 = \int_0^{0.4\text{ m}} \frac{T(x)dx}{JG} - \frac{T_C L}{JG}$$

$$0 = \int_0^{0.4\text{ m}} \frac{20(10^3)x dx}{JG} - \frac{T_C(1)}{JG}$$

$$0 = 20(10^3)\left(\frac{x^2}{2}\right)\Bigg|_0^{0.4\text{ m}} - T_C$$

$$T_C = 1600\ \text{N}\cdot\text{m}$$



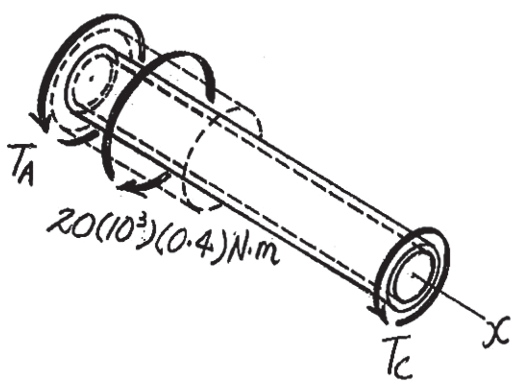
(b)

Substituting this result into Eq. (1),

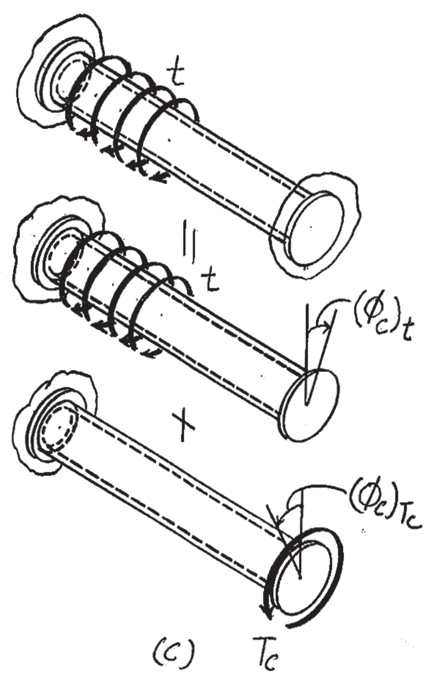
$$T_A = 6400\ \text{N}\cdot\text{m}$$

Maximum Shear Stress: By inspection, the maximum internal torque occurs at support A. Thus,

$$\left(\tau_{\max}\right)_{\text{abs}} = \frac{T_A\, c}{J} = \frac{6400(0.04)}{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)} = 93.1\ \text{MPa} \qquad \textbf{Ans.}$$



(a)

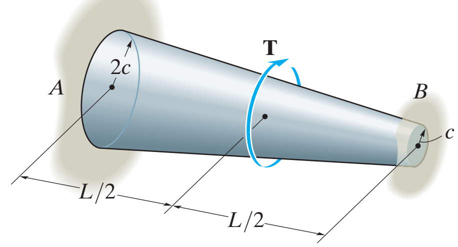


(c)

Ans:
$\tau_{\substack{\text{abs}\\ \text{max}}} = 93.1$ MPa

10–78.

The tapered shaft is confined by the fixed supports at A and B. If a torque **T** is applied at its mid-point, determine the reactions at the supports.



SOLUTION

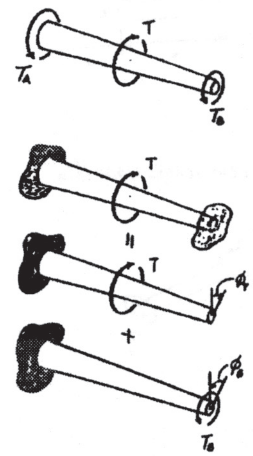
Equilibrium:

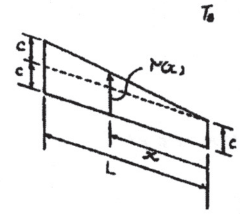
$$T_A + T_B - T = 0 \qquad \textbf{(1)}$$

Section Properties:

$$r(x) = c + \frac{c}{L}x = \frac{c}{L}(L + x)$$

$$J(x) = \frac{\pi}{2}\left[\frac{c}{L}(L + x)\right]^4 = \frac{\pi c^4}{2L^4}(L + x)^4$$





Angle of Twist:

$$\phi_T = \int \frac{Tdx}{J(x)G} = \int_{\frac{L}{2}}^{L} \frac{Tdx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2TL^4}{\pi c^4 G}\int_{\frac{L}{2}}^{L} \frac{dx}{(L + x)^4}$$

$$= -\frac{2TL^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_{\frac{L}{2}}^{L}$$

$$= \frac{37TL}{324\,\pi c^4 G}$$

$$\phi_B = \int \frac{Tdx}{J(x)G} = \int_0^L \frac{T_B dx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2T_B L^4}{\pi c^4 G}\int_0^L \frac{dx}{(L + x)^4}$$

$$= -\frac{2T_B L^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_0^L$$

$$= \frac{7T_B L}{12\pi c^4 G}$$

Compatibility:

$$0 = \phi_T - \phi_B$$

$$0 = \frac{37TL}{324\pi c^4 G} - \frac{7T_B L}{12\pi c^4 G}$$

$$T_B = \frac{37}{189}T \qquad \textbf{Ans.}$$

Substituting the result into Eq. (1) yields:

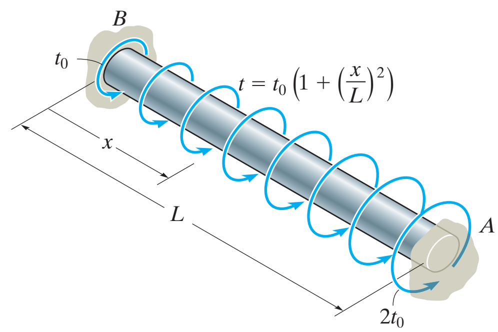
$$T_A = \frac{152}{189}T \qquad \textbf{Ans.}$$

Ans:

$T_B = \frac{37}{189}T,\ T_A = \frac{152}{189}T$

10–79.

The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the reactions at the fixed supports A and B.



SOLUTION

$$T(x) = \int_0^x t_0\left(1 + \frac{x^2}{L^2}\right) dx = t_0\left(x + \frac{x^3}{3L^2}\right) \qquad \textbf{(1)}$$

By superposition:

$$0 = \phi - \phi_B$$

$$0 = \int_0^L \frac{t_0\left(x + \frac{x^3}{3L^2}\right) dx}{JG} - \frac{T_B(L)}{JG} = \frac{7t_0L^2}{12} - T_B(L)$$

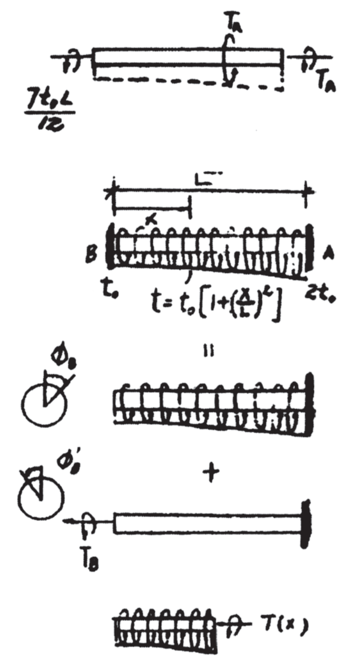
$$T_B = \frac{7t_0L}{12} \qquad \textbf{Ans.}$$

From Eq. (1),

$$T = t_0\left(L + \frac{L^3}{3L^2}\right) = \frac{4t_0L}{3}$$

$$T_A + \frac{7t_0L}{12} - \frac{4t_0L}{3} = 0$$

$$T_A = \frac{3t_0L}{4} \qquad \textbf{Ans.}$$

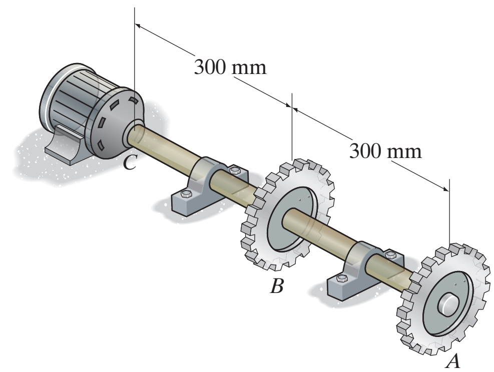


Ans:

$T_B = \dfrac{7t_0L}{12}, T_A = \dfrac{3t_0L}{4}$

R10–1.

The shaft is made of A992 steel and has an allowable shear stress of $\tau_{allow} = 75$ MPa. when the shaft is rotating at 300 rpm, the motor supplies 8 kW of power, while gears A and B withdraw 5 kW and 3 kW, respectively. Determine the required minimum diameter of the shaft to the nearest millimeter. Also, find the rotation of gear A relative to C.



SOLUTION

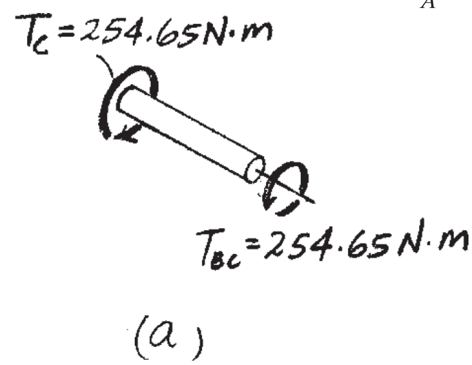
Applied Torque: The angular velocity of the shaft is

$$\omega = \left(300\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{rad}}{1\text{ rev}}\right) = 10\pi\text{ rad/s}$$

Thus, the torque at C and gear A are

$$T_C = \frac{P_C}{\omega} = \frac{8(10^3)}{10\pi} = 254.65\text{ N}\cdot\text{m}$$

$$T_A = \frac{P_A}{\omega} = \frac{5(10^3)}{10\pi} = 159.15\text{ N}\cdot\text{m}$$



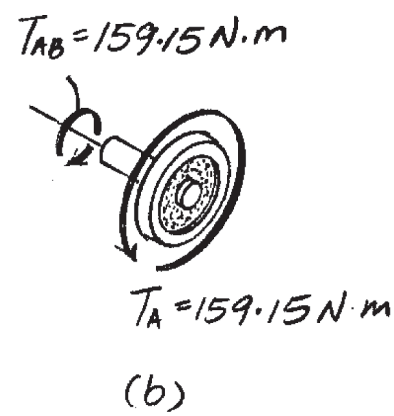
Internal Loading: The internal torque developed in segment BC and AB of the shaft are shown in Figs. a and b, respectively.

Allowable Shear Stress: By inspection, segment BC is critical.

$$\tau_{allow} = \frac{T_{BC}c}{J}; \qquad 75(10^6) = \frac{254.65\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 0.02586\text{ m}$$

Use $\quad d = 26$ mm **Ans.**



Angle of Twist: Using $d = 26$ mm,

$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG} + \frac{T_{BC}L_{BC}}{JG}$$

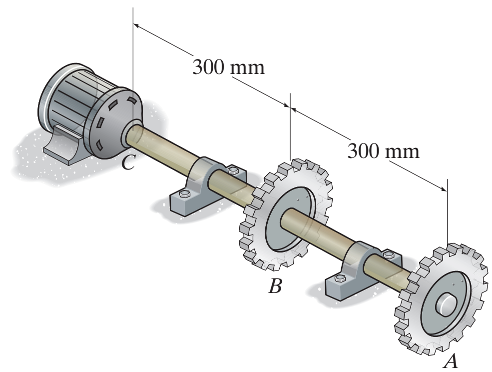
$$= \frac{0.3}{\frac{\pi}{2}(0.013^4)(75)(10^9)}(159.15 + 254.65)$$

$$= 0.03689\text{ rad} = 2.11°$$ **Ans.**

Ans:
Use $d = 26$ mm, $\phi_{A/C} = 2.11°$

R10–2.

The shaft is made of A992 steel and has an allowable shear stress of $\tau_{allow} = 75$ MPa. when the shaft is rotating at 300 rpm, the motor supplies 8 kW of power, while gears A and B withdraw 5 kW and 3 kW, respectively. If the angle of twist of gear A relative to C is not allowed to exceed 0.03 rad, determine the required minimum diameter of the shaft to the nearest millimeter.



SOLUTION

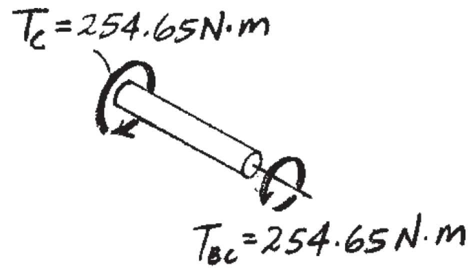
Applied Torque: The angular velocity of the shaft is

$$\omega = \left(300\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{rad}}{1\text{ rev}}\right) = 10\pi\text{ rad/s}$$

Thus, the torque at C and gear A are

$$T_C = \frac{P_C}{\omega} = \frac{8(10^3)}{10\pi} = 254.65\text{ N}\cdot\text{m}$$

$$T_A = \frac{P_A}{\omega} = \frac{5(10^3)}{10\pi} = 159.15\text{ N}\cdot\text{m}$$



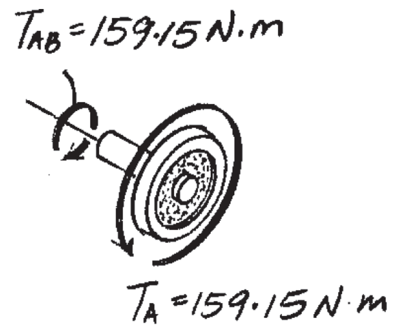
(a)

Internal Loading: The internal torque developed in segment BC and AB of the shaft are shown in Figs. a and b, respectively.

Allowable Shear Stress: By inspection, segment BC is critical.

$$\tau_{allow} = \frac{T_{BC}\,c}{J}; \qquad 75(10^3) = \frac{254.65(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4}$$

$$d = 0.02586$$



(b)

Angle of Twist:

$$\phi_{A/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG} + \frac{T_{BC}L_{BC}}{JG}$$

$$0.03 = \frac{0.3}{\frac{\pi}{2}(\frac{d}{2})^4(75)(10^9)}(159.15 + 254.65)$$

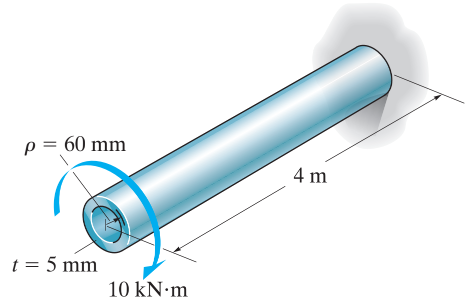
$$d = 0.02738\text{ m(controls)}$$

Use $d = 28$ mm. **Ans.**

Ans:
Use $d = 28$ mm.

R10–3.

The A-36 steel circular tube is subjected to a torque of 10 kN·m. Determine the shear stress at the mean radius $\rho = 60$ mm and calculate the angle of twist of the tube if it is 4 m long and fixed at its far end. Solve the problem using Eqs. 10–7 and 10–15 and using Eqs. 10–18 and 10–20.



SOLUTION

We show that two different methods give similar results:

Shear Stress:

Applying Eq. 5-7,

$$r_o = 0.06 + \frac{0.005}{2} = 0.0625 \text{ m} \qquad r_i = 0.06 - \frac{0.005}{2} = 0.0575 \text{ m}$$

$$\tau_{\rho=0.06\text{ m}} = \frac{T\rho}{J} = \frac{10(10^3)(0.06)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)} = 88.27 \text{ MPa}$$

Applying Eq. 5-18,

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{10(10^3)}{2(0.005)(\pi)(0.06^2)} = 88.42 \text{ MPa}$$

Angle of Twist:

Applying Eq. 5-15,

$$\phi = \frac{TL}{JG}$$

$$= \frac{10(10^3)(4)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)(75.0)(10^9)}$$

$$= 0.0785 \text{ rad} = 4.495°$$

Applying Eq. 5-20,

$$\phi = \frac{TL}{4A_m^2 G}\int \frac{ds}{t}$$

$$= \frac{TL}{4A_m^2 G\,t}\int ds \qquad \text{Where} \qquad \int ds = 2\pi\rho$$

$$= \frac{2\pi TL\rho}{4A_m^2 G\,t}$$

$$= \frac{2\pi(10)(10^3)(4)(0.06)}{4[(\pi)(0.06^2)]^2(75.0)(10^9)(0.005)}$$

$$= 0.0786 \text{ rad} = 4.503°$$

Rounding to three significant figures, we find

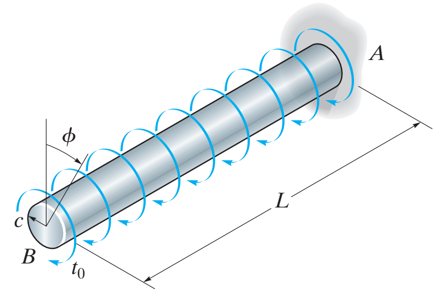
$\tau = 88.3$ MPa **Ans.**

$\phi = 4.50°$ **Ans.**

Ans:
$\tau = 88.3$ MPa, $\phi = 4.50°$

***R10–4.**

The shaft has a radius c and is subjected to a torque per unit length of t_0, which is distributed uniformly over the shaft's entire length L. If it is fixed at its far end A, determine the angle of twist ϕ of end B. The shear modulus is G.

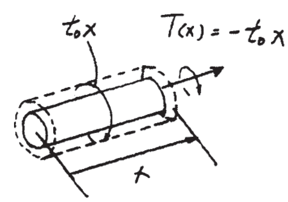


SOLUTION

$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{-t_0}{JG}\int_0^L x\,dx$$

$$= \frac{-t_0}{JG}\left[\frac{x^2}{2}\right]\Bigg|_0^L = \frac{-t_0}{JG}\,\frac{L^2}{2}$$

$$= \frac{-t_0L^2}{2JG}$$



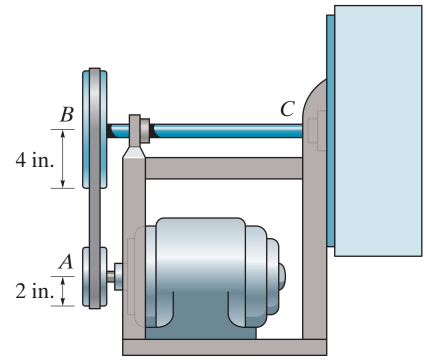
However, $J = \dfrac{\pi}{2}c^4$

$$\phi = \frac{-t_0L^2}{\pi c^4 G} = \frac{t_0L^2}{\pi c^4 G}$$ **Ans.**

Ans:
$\phi = \dfrac{t_0L^2}{\pi c^4 G}$

R10–5.

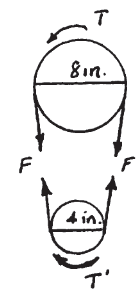
The motor delivers 50 hp while turning at a constant rate of 1350 rpm at A. Using the belt and pulley system this loading is delivered to the steel blower shaft BC. Determine to the nearest $\frac{1}{8}$ in. the smallest diameter of this shaft if the allowable shear stress for steel is $\tau_{allow} = 12$ ksi.



SOLUTION

$P = T\omega$

$50(550) = T'(1350 \text{ rev/min})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$



$T' = 194.52 \text{ lb} \cdot \text{ft}$

$4(F' - F) = T'$

$4(F' - F) = (194.52)(12)$

$(F' - F) = 583.57 \text{ lb}$

$T = 8(F' - F)$

$\quad = 8(583.57) = 4668.5 \text{ lb} \cdot \text{in}$

$\tau_{allow} = \frac{Tc}{J}; \qquad 12(10^3) = \frac{4668.5c}{\frac{\pi}{2}(c)^4}$

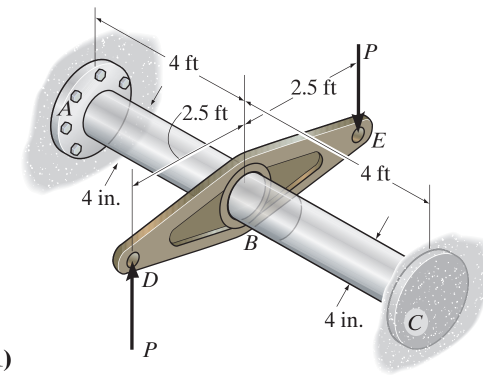
$c = 0.628 \text{ in.}$

Use $1\frac{3}{8}$ -in. -diameter shaft. **Ans.**

Ans:
Use $d = 1\frac{3}{8}$ in.

R10–6.

Segments AB and BC of the assembly are made from 6061-T6 aluminum and A992 steel, respectively. If couple forces $P = 3$ kip are applied to the lever arm, determine the maximum shear stress developed in each segment. The assembly is fixed at A and C.



SOLUTION

Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$\Sigma M_x = 0;\qquad T_A + T_C - 3(5) = 0$ **(1)**

Compatibility Equation: It is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG_{al}} = \frac{T_C L_{BC}}{JG_{st}}$$

$$\frac{T_A L}{J(3.7)(10^3)} = \frac{T_C L}{J(11)(10^3)}$$

$$T_A = 0.3364\, T_C \qquad \textbf{(2)}$$

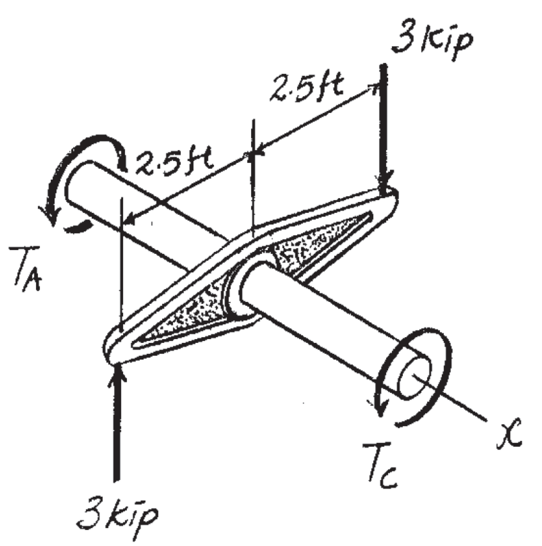
Solving Eqs. (1) and (2),

$$T_C = 11.224 \text{ kip}\cdot\text{ft} \qquad T_A = 3.775 \text{ kip}\cdot\text{ft}$$

Maximum Shear Stress:

$$(\tau_{\max})_{AB} = \frac{T_A c}{J} = \frac{3.775(12)(2)}{\frac{\pi}{2}(2^4)} = 3.60 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{\max})_{BC} = \frac{T_C c}{J} = \frac{11.224(12)(2)}{\frac{\pi}{2}(2^4)} = 10.7 \text{ ksi} \qquad \textbf{Ans.}$$

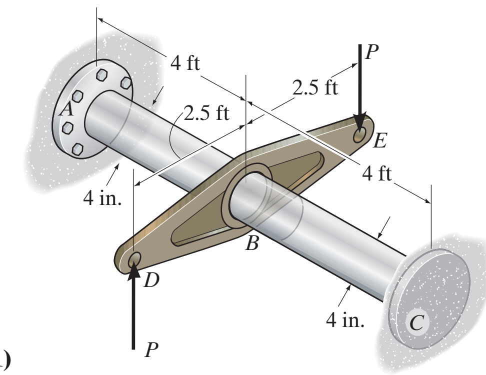


(a)

Ans:
$(\tau_{\max})_{AB} = 3.60$ ksi,
$(\tau_{\max})_{BC} = 10.7$ ksi

R10–7.

Segments AB and BC of the assembly are made from 6061-T6 aluminum and A992 steel, respectively. If the allowable shear stress for the aluminum is $(\tau_{allow})_{al} = 12$ ksi and for the steel $(\tau_{allow})_{st} = 10$ ksi, determine the maximum allowable couple forces P that can be applied to the lever arm. The assembly is fixed at A and C.



SOLUTION

Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$\Sigma M_x = 0; \qquad T_A + T_C - P(5) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: It is required that

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG_{al}} = \frac{T_C L_{BC}}{JG_{st}}$$

$$\frac{T_A L}{J(3.7)(10^3)} = \frac{T_C L}{J(11)(10^3)}$$

$$T_A = 0.3364\, T_C \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_C = 3.7415P \qquad T_A = 1.259P$$

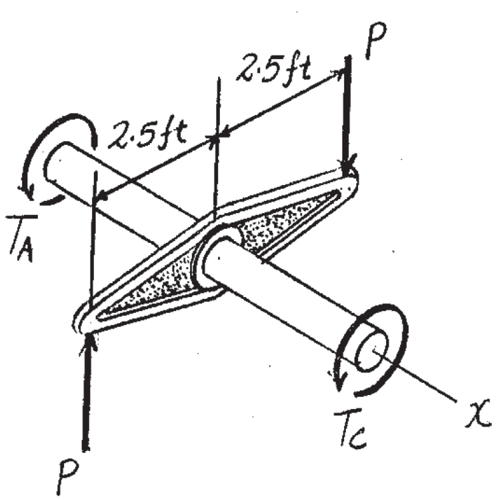
Allowable Shear Stress:

$$(\tau_{allow})_{AB} = \frac{T_A c}{J}; \qquad 12 = \frac{1.259P(12)(2)}{\frac{\pi}{2}(2^4)}$$

$$P = 9.98 \text{ kip}$$

$$(\tau_{allow})_{BC} = \frac{T_C c}{J}; \qquad 10 = \frac{3.7415P(12)(2)}{\frac{\pi}{2}(2^4)}$$

$$P = 2.80 \text{ kip (controls)} \qquad \textbf{Ans.}$$

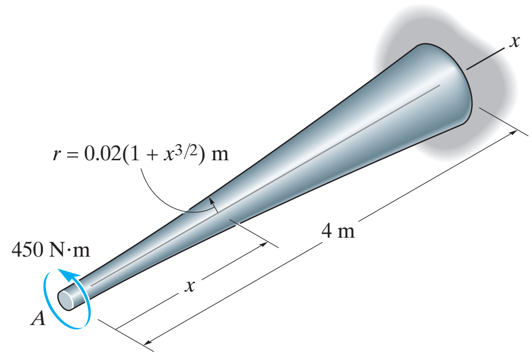


(a)

Ans:
$P = 2.80$ kip

***R10–8.**

The tapered shaft is made from 2014-T6 aluminum alloy, and has a radius which can be described by the equation $r = 0.02(1 + x^{3/2})$ m, where x is in meters. Determine the angle of twist of its end A if it is subjected to a torque of 450 N · m.



SOLUTION

$T = 450\ \text{N}\cdot\text{m}$

$$\phi_A = \int \frac{T dx}{JG} = \int_0^4 \frac{450\, dx}{\frac{\pi}{2}(0.02)^4(1 + x^{\frac{3}{2}})^4(27)(10^9)} = 0.066315 \int_0^4 \frac{dx}{(1 + x^{\frac{3}{2}})^4}$$

Evaluating the integral numerically, we have

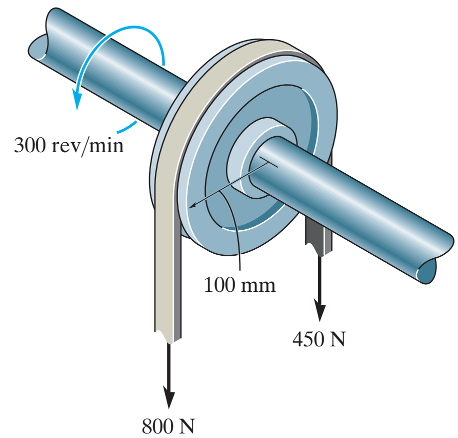
$\phi_A = 0.066315\ [0.4179]$ rad

$= 0.0277\ \text{rad} = 1.59°$ **Ans.**

Ans:
$\phi_A = 1.59°$

R10–9.

The 60-mm-diameter shaft rotates at 300 rev/min. This motion is caused by the unequal belt tensions on the pulley of 800 N and 450 N. Determine the power transmitted and the maximum shear stress developed in the shaft.



SOLUTION

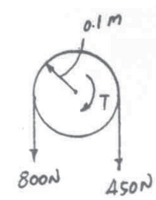
$$\omega = 300\frac{\text{rev}}{\text{min}}\left[\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right]\frac{1\ \text{min}}{60\ \text{s}} = 10\ \pi\ \text{rad/s}$$

$$T + 450(0.1) - 800(0.1) = 0$$

$$T = 35.0\ \text{N}\cdot\text{m}$$

$$P = T\omega = 35.0(10\pi) = 1100\ \text{W} = 1.10\ \text{kW}$$ **Ans.**

$$\tau_{\text{max}} = \frac{Tc}{J} = \frac{35.0(0.03)}{\frac{\pi}{2}(0.03^4)} = 825\ \text{kPa}$$ **Ans.**



Ans:
$P = 1.10\ \text{kW}$,
$\tau_{\text{max}} = 825\ \text{kPa}$