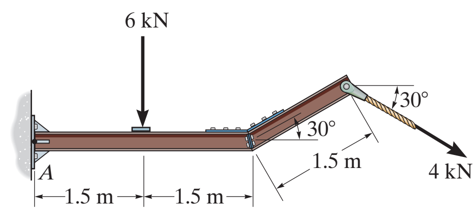
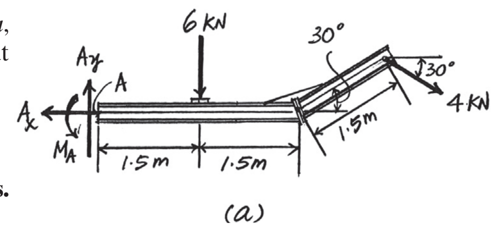
**4–1.**

Determine the components of the support reactions at the fixed support $A$ on the cantilevered beam.




## SOLUTION

***Equations of Equilibrium:*** From the free-body diagram of the cantilever beam, Fig. *a*, $A_x$, $A_y$, and $M_A$ can be obtained by writing the moment equation of equilibrium about point $A$.




$\xrightarrow{+} \Sigma F_x = 0; \qquad 4 \cos 30° - A_x = 0$

$A_x = 3.46 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 6 - 4 \sin 30° = 0$

$A_y = 8 \text{ kN}$ **Ans.**

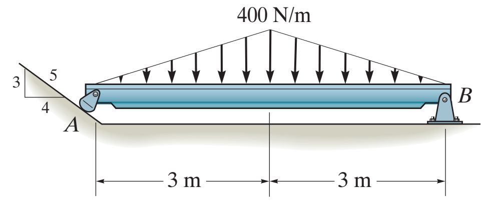
$\circlearrowleft + \Sigma M_A = 0; M_A - 6(1.5) - 4 \cos 30° (1.5 \sin 30°) - 4 \sin 30°(3 + 1.5 \cos 30°) = 0$

$M_A = 20.2 \text{ kN} \cdot \text{m}$ **Ans.**

**Ans:**
$A_x = 3.46 \text{ kN}$
$A_y = 8 \text{ kN}$
$M_A = 20.2 \text{ kN} \cdot \text{m}$

**4–2.**

Determine the reactions at the supports.




## SOLUTION

**Equations of Equilibrium.** $\mathbf{N}_A$ and $\mathbf{B}_y$ can be determined directly by writing the moment equations of equilibrium about points $B$ and $A$, respectively, by referring to the beam's *FBD* shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{1}{2}(400)(6)(3) - N_A\left(\frac{4}{5}\right)(6) = 0$$

$$N_A = 750 \text{ N} \qquad \textbf{Ans.}$$

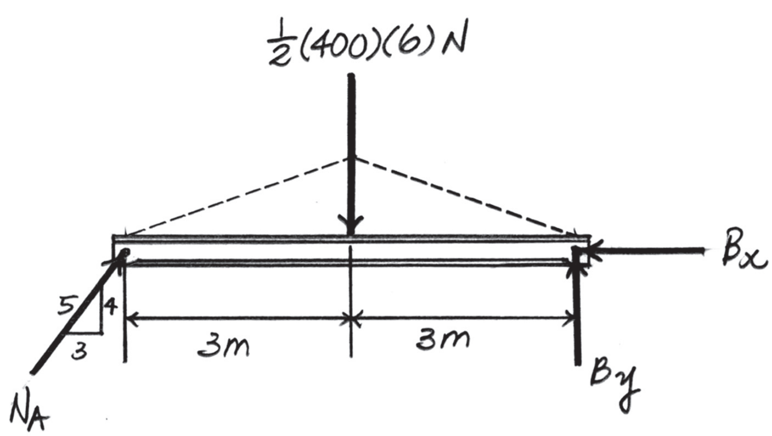
$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(400)(6)(3) = 0$$

$$B_y = 600 \text{ N} \qquad \textbf{Ans.}$$

Using the result of $\mathbf{N}_A$ to write the force equation of equilibrium along the $x$ axis,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 750\left(\frac{3}{5}\right) - B_x = 0$$

$$B_x = 450 \text{ N} \qquad \textbf{Ans.}$$

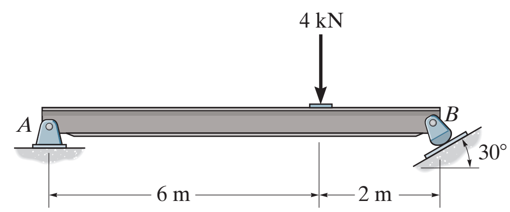



(a)

**Ans:**
$N_A = 750$ N
$B_y = 600$ N
$B_x = 450$ N

**4–3.**

Determine the horizontal and vertical components of reaction of the pin $A$ and the reaction of the rocker $B$ on the beam.




## SOLUTION

***Equations of Equilibrium:*** From the free-body diagram of the beam, Fig. $a$, $N_B$ can be obtained by writing the moment equation of equilibrium about point $A$.

$\circlearrowleft + \Sigma M_A = 0;\qquad N_B \cos 30°(8) - 4(6) = 0$

$N_B = 3.464\text{ kN} = 3.46\text{ kN}$ **Ans.**

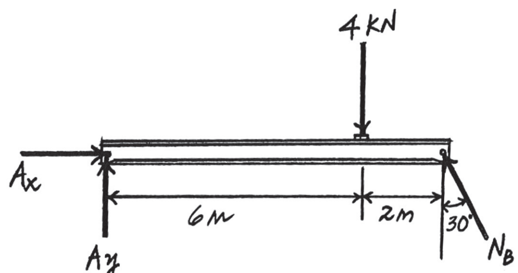
Using this result and writing the force equations of equilibrium along the $x$ and $y$ axes, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad A_x - 3.464 \sin 30° = 0$

$A_x = 1.73\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad A_y + 3.464 \cos 30° - 4 = 0$

$A_y = 1.00\text{ kN}$ **Ans.**

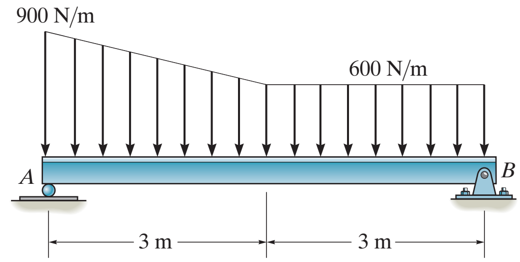



(a)

**Ans:**
$N_B = 3.46\text{ kN}$
$A_x = 1.73\text{ kN}$
$A_y = 1.00\text{ kN}$

***4–4.**

Determine the reactions at the supports.




## SOLUTION

**Equations of Equilibrium.** $\mathbf{N}_A$ and $\mathbf{B}_y$ can be determined directly by writing the moment equations of equilibrium about points $B$ and $A$, respectively, by referring to the *FBD* of the beam shown in Fig. *a*.

$$\zeta+\Sigma M_B = 0; \qquad 600(6)(3) + \frac{1}{2}(300)(3)(5) - N_A(6) = 0$$

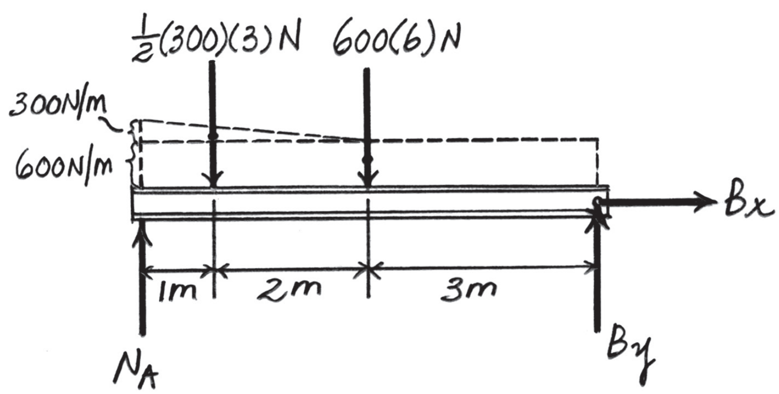
$$N_A = 2175 \text{ N} = 2.175 \text{ kN} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(300)(3)(1) - 600(6)(3) = 0$$

$$B_y = 1875 \text{ N} = 1.875 \text{ kN} \qquad \textbf{Ans.}$$

Also, $\mathbf{B}_x$ can be determined directly by writing the force equation of equilibrium along the $x$ axis.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad B_x = 0 \qquad \textbf{Ans.}$$

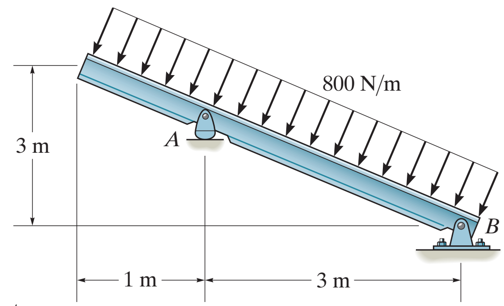



(a)

**Ans:**
$N_A = 2.175$ kN
$B_y = 1.875$ kN
$B_x = 0$

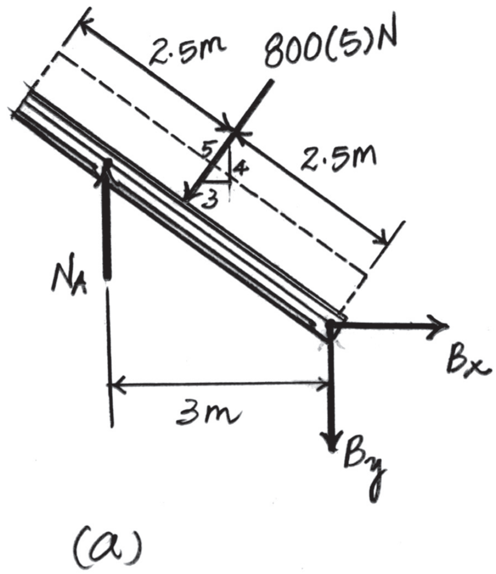
**4–5.**

Determine the reactions at the supports.




## SOLUTION

**Equations of Equilibrium.** $\mathbf{N}_A$ can be determined directly by writing the moment equation of equilibrium about point $B$ by referring to the *FBD* of the beam shown in Fig. *a*.




$$\circlearrowleft + \Sigma M_B = 0; \qquad 800(5)(2.5) - N_A(3) = 0$$

$$N_A = 3333.33 \text{ N} = 3.33 \text{ kN} \qquad \textbf{Ans.}$$

Using this result to write the force equations of equilibrium along the $x$ and $y$ axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x - 800(5)\left(\frac{3}{5}\right) = 0$$

$$B_x = 2400 \text{ N} = 2.40 \text{ kN} \qquad \textbf{Ans.}$$

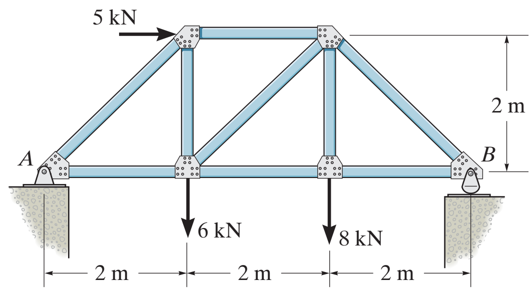
$$+\uparrow \Sigma F_y = 0; \qquad 3333.33 - 800(5)\left(\frac{4}{5}\right) - B_y = 0$$

$$B_y = 133.33 \text{ N} = 133 \text{ N} \qquad \textbf{Ans.}$$

**Ans:**
$N_A = 3.33$ kN
$B_x = 2.40$ kN
$B_y = 133$ N

**4–6.**

Determine the reactions at the supports.




## SOLUTION

**Equations of Equilibrium.** $\mathbf{A}_y$ and $\mathbf{N}_B$ can be determined by writing the moment equations of equilibrium about points $B$ and $A$, respectively, by referring to the *FBD* of the truss shown in Fig. *a*.

$\circlearrowleft + \Sigma M_B = 0; \quad 8(2) + 6(4) - 5(2) - A_y(6) = 0$

$A_y = 5.00 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; \quad N_B(6) - 8(4) - 6(2) - 5(2) = 0$

$N_B = 9.00 \text{ kN}$ **Ans.**

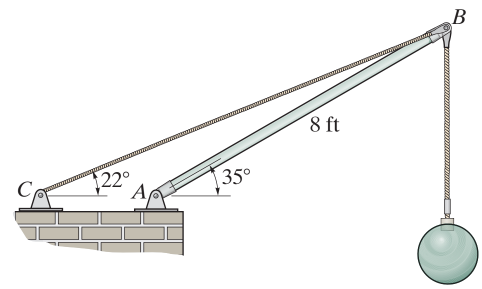
Also, $\mathbf{A}_x$ can be determined directly by writing the force equation of equilibrium along $x$ axis.

$\xrightarrow{+} \Sigma F_x = 0; \quad 5 - A_x = 0 \quad A_x = 5.00 \text{ kN}$ **Ans.**

**Ans:**
$A_y = 5.00 \text{ kN}$
$N_B = 9.00 \text{ kN}$
$A_x = 5.00 \text{ kN}$

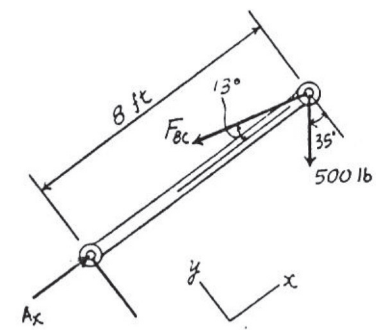
**4–7.**

Determine the magnitude of force at the pin $A$ and in the cable $BC$ needed to support the 500-lb load. Neglect the weight of the boom $AB$.




## SOLUTION

***Equations of Equilibrium:*** The force in cable $BC$ can be obtained directly by summing moments about point $A$.




$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_{BC} \sin 13°(8) - 500 \cos 35°(8) = 0$

$F_{BC} = 1820.7 \text{ lb} = 1.82 \text{ kip}$ **Ans.**

$\overset{+}{\nearrow} \Sigma F_x = 0;$ $\quad A_x - 1820.7 \cos 13° - 500 \sin 35° = 0$

$A_x = 2060.9 \text{ lb}$

$\nwarrow + \Sigma F_y = 0;$ $\quad A_y + 1820.7 \sin 13° - 500 \cos 35° = 0$

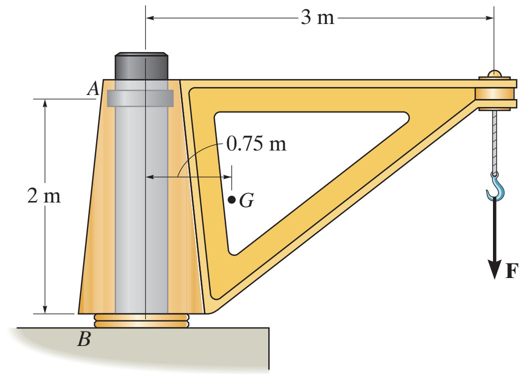
$A_y = 0$

Thus, $\quad F_A = A_x = 2060.9 \text{ lb} = 2.06 \text{ kip}$ **Ans.**

**Ans:**
$F_{BC} = 1.82$ kip
$F_A = 2.06$ kip

***4–8.**

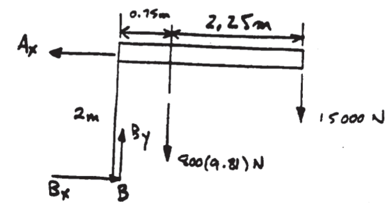
The dimensions of a jib crane are given in the figure. If the crane has a mass of 800 kg and a center of mass at $G$, and the maximum rated force at its end is $F = 15$ kN, determine the reactions at its bearings. The bearing at $A$ is a journal bearing and supports only a horizontal force, whereas the bearing at $B$ is a thrust bearing that supports both horizontal and vertical components.




## SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \quad A_x(2) - 800(9.81)(0.75) - 15\,000(3) = 0$

$A_x = 25.4$ kN **Ans.**




$+\uparrow \Sigma F_y = 0; \quad B_y - 800(9.81) - 15\,000 = 0$

$B_y = 22.8$ kN **Ans.**

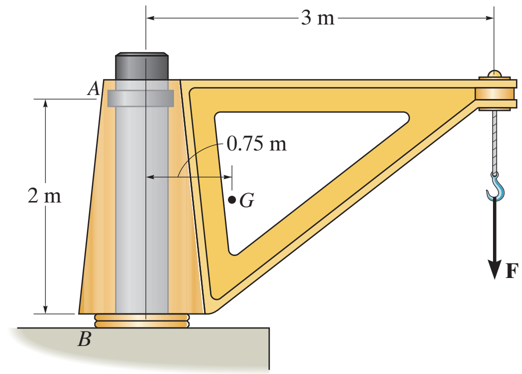
$\xrightarrow{+} \Sigma F_x = 0; \quad B_x - 25.4 = 0$

$B_x = 25.4$ kN **Ans.**

**Ans:**
$A_x = 25.4$ kN
$B_y = 22.8$ kN
$B_x = 25.4$ kN

**4–9.**

The dimensions of a jib crane are given in the figure. The crane has a mass of 800 kg and a center of mass at $G$. The bearing at $A$ is a journal bearing and can support a horizontal force, whereas the bearing at $B$ is a thrust bearing that supports both horizontal and vertical components. Determine the maximum load $F$ that can be suspended from the end of the crane if the bearings at $A$ and $B$ can sustain a maximum resultant load of 24 kN and 34 kN, respectively.



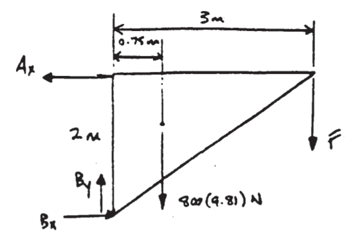

## SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \qquad A_x(2) - 800(9.81)(0.75) - F(3) = 0$

$+\uparrow \Sigma F_y = 0; \qquad B_y - 800(9.81) - F = 0$

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x - A_x = 0$

Assume $A_x = 24\,000$ N.




Solving,

$B_x = 24$ kN

$B_y = 21.9$ kN

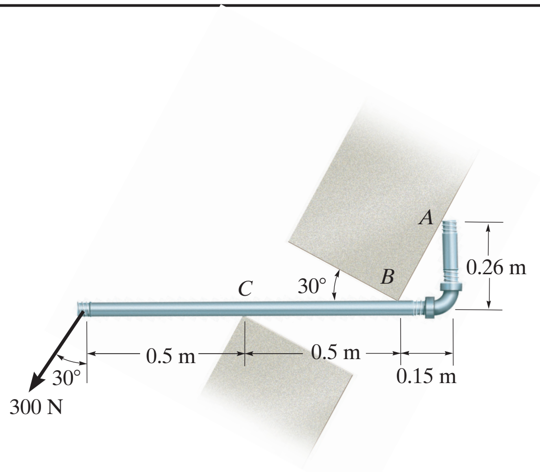
$F = 14.0$ kN **Ans.**

$F_B = \sqrt{(24)^2 + (21.9)^2} = 32.5 \text{ kN} < 34 \text{ kN}$ **OK**

**Ans:**
$F = 14.0$ kN

**4–10.**

The smooth pipe rests against the opening at the points of contact $A$, $B$, and $C$. Determine the reactions at these points needed to support the force of 300 N. Neglect the pipe's thickness.




## SOLUTION

**Equations of Equilibrium.** $\mathbf{N}_A$ can be determined directly by writing the force equation of equilibrium along the $x$ axis by referring to the *FBD* of the pipe shown in Fig. *a*.

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad N_A \cos 30° - 300 \sin 30° = 0 \quad N_A = 173.21 \text{ N} = 173 \text{ N}$ **Ans.**

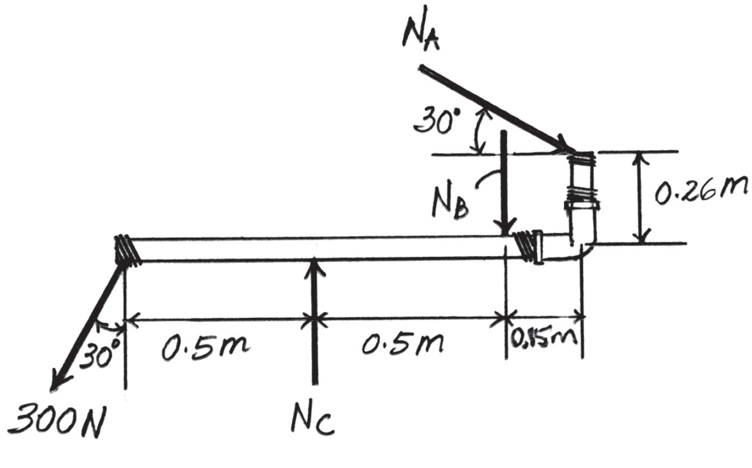
Using this result to write the moment equations of equilibrium about points $B$ and $C$,

$\circlearrowleft + \Sigma M_B = 0; \quad 300 \cos 30°(1) - 173.21 \cos 30°(0.26) - 173.21 \sin 30°(0.15) - N_C(0.5) = 0$

$N_C = 415.63 \text{ N} = 416 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad 300 \cos 30°(0.5) - 173.21 \cos 30°(0.26) - 173.21 \sin 30°(0.65) - N_B(0.5) = 0$

$N_B = 69.22 \text{ N} = 69.2 \text{ N}$ **Ans.**

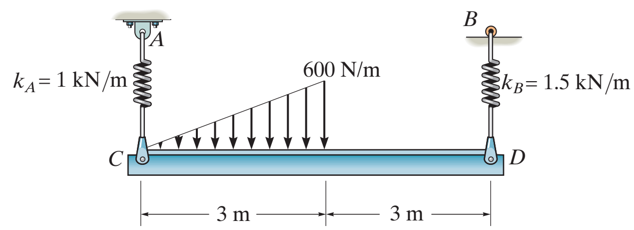



(a)

**Ans:**
$N_A = 173$ N
$N_C = 416$ N
$N_B = 69.2$ N

**4–11.**

The beam is horizontal and the springs are unstretched when there is no load on the beam. Determine the angle of tilt of the beam when the load is applied.




## SOLUTION

**Equations of Equilibrium.** $\mathbf{F}_A$ and $\mathbf{F}_B$ can be determined directly by writing the moment equations of equilibrium about points $B$ and $A$, respectively, by referring to the *FBD* of the beam shown in Fig. *a*.

Assuming that the angle of tilt is small,

$$\zeta+\Sigma M_A = 0; \qquad F_B(6) - \frac{1}{2}(600)(3)(2) = 0 \quad F_B = 300 \text{ N}$$

$$\zeta+\Sigma M_B = 0; \qquad \frac{1}{2}(600)(3)(4) - F_A(6) = 0 \quad F_A = 600 \text{ N}$$

Thus, the stretches of springs $A$ and $B$ can be determined from

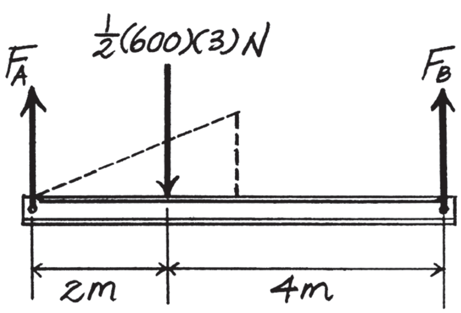
$$F_A = k_A x_A; \qquad 600 = 1000 x_A \qquad x_A = 0.6 \text{ m}$$

$$F_B = k_B x_B; \qquad 300 = 1500 x_B \qquad x_B = 0.2 \text{ m}$$

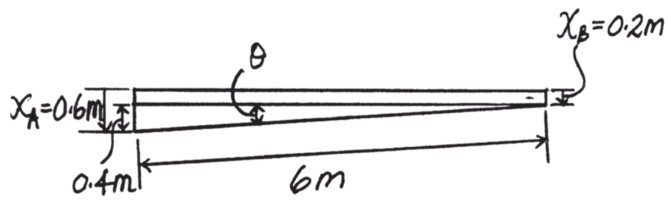
From the geometry shown in Fig. $b$,

$$\theta = \sin^{-1}\left(\frac{0.4}{6}\right) = 3.82° \qquad \textbf{Ans.}$$

The assumption of small $\theta$ is confirmed.




(a)

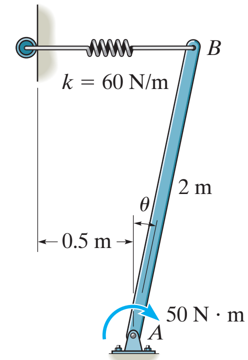



(b)

**Ans:**
$\theta = 3.82°$

***4–12.**

The 10-kg uniform rod is pinned at end $A$. If it is subjected to a couple moment of 50 N · m, determine the smallest angle $\theta$ for equilibrium. The spring is unstretched when $\theta = 0°$, and has a stiffness of $k = 60$ N/m.




## SOLUTION

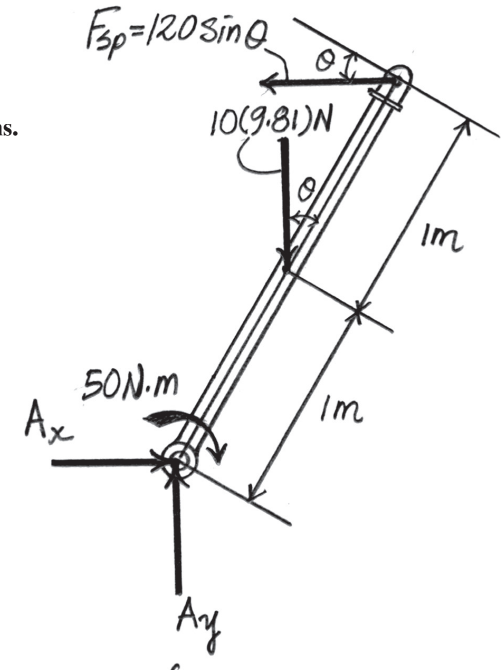
**Equations of Equilibrium.** Here the spring stretches $x = 2 \sin \theta$. The force in the spring is $F_{sp} = kx = 60\,(2 \sin \theta) = 120 \sin \theta$. Write the moment equation of equilibrium about point $A$ by referring to the *FBD* of the rod shown in Fig. *a*.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 120 \sin \theta \cos \theta\,(2) - 10(9.81) \sin \theta\,(1) - 50 = 0$

$$240 \sin \theta \cos \theta - 98.1 \sin \theta - 50 = 0$$

Solve numerically.

$$\theta = 24.598° = 24.6° \qquad \textbf{Ans.}$$

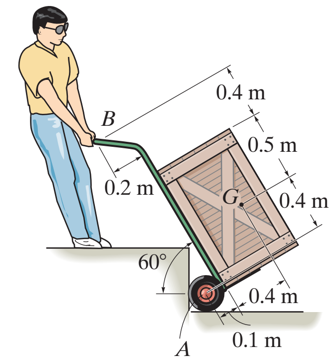


(a)

**Ans:**
$\theta = 24.6°$

**4–13.**

The man uses the hand truck to move material up the step. If the truck and its contents have a mass of 50 kg with center of gravity at $G$, determine the normal reaction on both wheels and the magnitude and direction of the minimum force required at the grip $B$ needed to lift the load.




## SOLUTION

**Equations of Equilibriums.** $\mathbf{P}_y$ can be determined directly by writing the force equation of equilibrium along the $y$ axis by referring to the *FBD* of the hand truck shown in Fig. *a*.

$+\uparrow \Sigma F_y = 0; \qquad P_y - 50(9.81) = 0 \qquad P_y = 490.5 \text{ N}$

Using this result to write the moment equation of equilibrium about point $A$,

$$\circlearrowleft + \Sigma M_A = 0; \qquad P_x \sin 60°(1.3) - P_x \cos 60°(0.1) - 490.5 \cos 30°(0.1)$$
$$-490.5 \sin 30°(1.3) - 50(9.81) \sin 60°(0.5)$$
$$+50(9.81) \cos 60°(0.4) = 0$$
$$P_x = 442.07 \text{ N}$$

Thus, the magnitude of minimum force $\boldsymbol{P}$, Fig. *b*, is

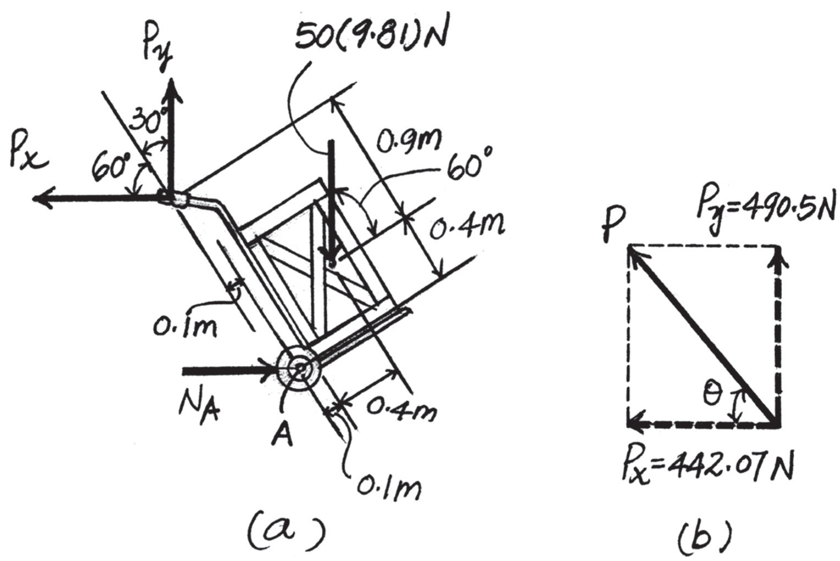
$P = \sqrt{P_x^2 + P_y^2} = \sqrt{442.07^2 + 490.5^2} = 660.32 \text{ N} = 660 \text{ N}$ **Ans.**

and the angle is

$\theta = \tan^{-1}\left(\dfrac{490.5}{442.07}\right) = 47.97° = 48.0°$ ◺ **Ans.**

Write the force equation of equilibrium along the $x$ axis.

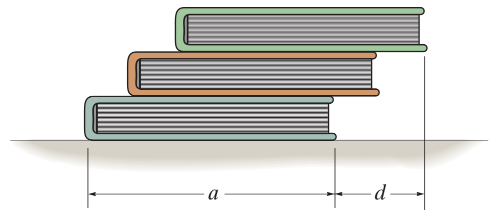
$\xrightarrow{+} \Sigma F_x = 0; \qquad N_A - 442.07 = 0 \qquad N_A = 442.07 \text{ N} = 442 \text{ N}$ **Ans.**




**Ans:**
$P = 660$ N
$N_A = 442$ N
$\theta = 48.0°$ ◺

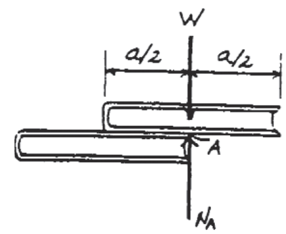
**4–14.**

Three uniform books, each having a weight $W$ and length $a$, are stacked as shown. Determine the maximum distance $d$ that the top book can extend out from the bottom one so the stack does not topple over.




## SOLUTION

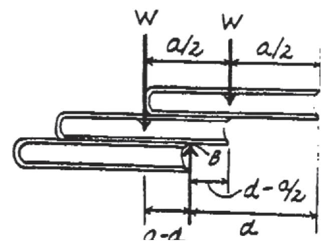
***Equilibrium:*** For top two books, the upper book will topple when the center of gravity of this book is to the right of point $A$. Therefore, the maximum distance from the right edge of this book to point $A$ is $a/2$.




***Equation of Equilibrium:*** For the entire three books, the top two books will topple about point $B$.

$$\circlearrowleft +\Sigma M_B = 0; \qquad W(a-d)-W\left(d-\frac{a}{2}\right) = 0$$

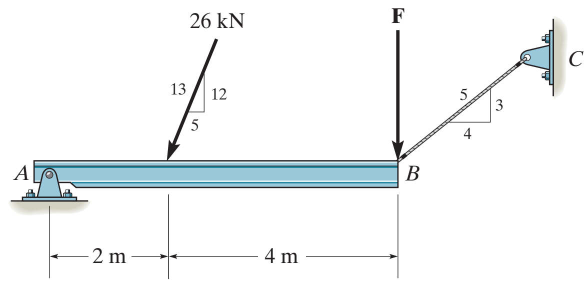
$$d = \frac{3a}{4} \qquad \textbf{Ans.}$$




**Ans:**
$d = \dfrac{3a}{4}$

**4–15.**

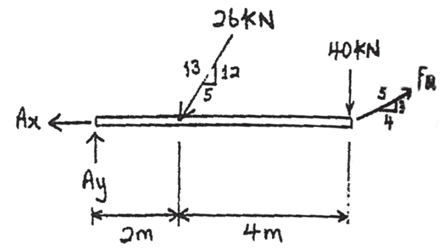
Determine the reactions at the pin $A$ and the tension in cord $BC$. Set $F = 40$ kN. Neglect the thickness of the beam.




## SOLUTION

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -26\left(\frac{12}{13}\right)(2) - 40(6) + \frac{3}{5}F_{BC}(6) = 0$

$F_{BC} = 80$ kN **Ans.**




$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 80\left(\frac{4}{5}\right) - A_x - 26\left(\frac{5}{13}\right) = 0$

$A_x = 54$ kN **Ans.**

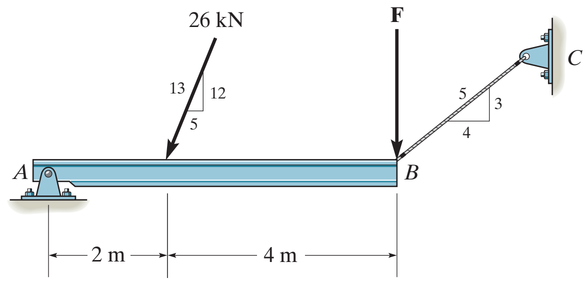
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 26\left(\frac{12}{13}\right) - 40 + 80\left(\frac{3}{5}\right) = 0$

$A_y = 16$ kN **Ans.**

**Ans:**
$F_{BC} = 80$ kN
$A_x = 54$ kN
$A_y = 16$ kN

***4–16.**

If rope $BC$ will fail when the tension becomes 50 kN, determine the greatest vertical load $F$ that can be applied to the beam at $B$. What is the magnitude of the reaction at $A$ for this loading? Neglect the thickness of the beam.




## SOLUTION

$\circlearrowleft + \Sigma M_A = 0;\quad -26\left(\frac{12}{13}\right)(2) - F(6) + \frac{3}{5}(50)(6) = 0$

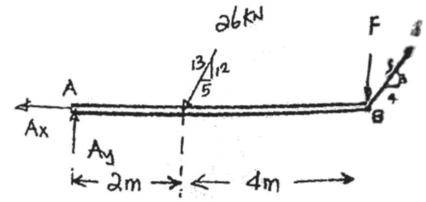
$F = 22 \text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;\quad 50\left(\frac{4}{5}\right) - A_x - 26\left(\frac{5}{13}\right) = 0$

$A_x = 30 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad A_y - 26\left(\frac{12}{13}\right) - 22 + 50\left(\frac{3}{5}\right) = 0$

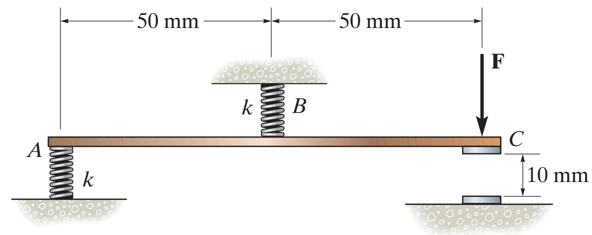
$A_y = 16 \text{ kN}$ **Ans.**




**Ans:**
$F = 22$ kN
$A_x = 30$ kN
$A_y = 16$ kN

**4–17.**

The rigid metal strip of negligible weight is used as part of an electromagnetic switch. If the stiffness of the springs at $A$ and $B$ is $k = 5$ N/m and the strip is originally horizontal when the springs are unstretched, determine the smallest force $F$ needed to close the contact gap at $C$.

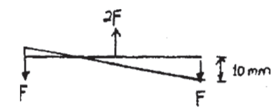



## SOLUTION

$\Sigma M_B = 0; \qquad F_A = F_C = F$

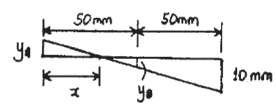
$\Sigma F_y = 0; \qquad F_B = 2F$

$$\frac{x}{y_A} = \frac{50 - x}{y_B} \qquad \textbf{(1)}$$




$$\frac{2F}{F} = \frac{ky_B}{ky_A}$$

$$2y_A = y_B \qquad \textbf{(2)}$$




Substituting into Eq.(1):

$$\frac{x}{y_A} = \frac{50 - x}{2y_A}$$

$$2x = 50 - x$$

$$x = \frac{50}{3} = 16.67 \text{ mm}$$

$$\frac{x}{y_A} = \frac{100 - x}{10}$$

Set $x = 16.67$, then

$$y_A = 2 \text{ mm}$$

From Eq.(2),

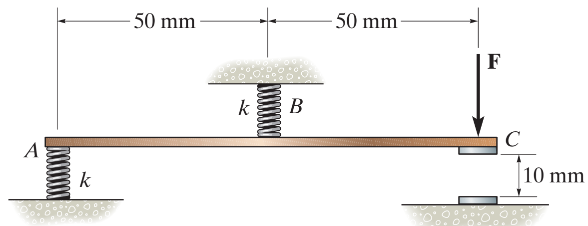
$$y_B = 4 \text{ mm}$$

$$F_C = F_A = ky_A = (5)(0.002) = 10 \text{ mN} \qquad \textbf{Ans.}$$

**Ans:**
$F_C = 10$ mN

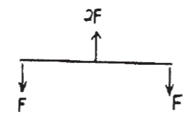
**4–18.**

The rigid metal strip of negligible weight is used as part of an electromagnetic switch. Determine the maximum stiffness $k$ of the springs at $A$ and $B$ so that the contact at $C$ closes when the vertical force developed there is $F = 0.5$ N. Originally the strip is horizontal as shown.




## SOLUTION

$\Sigma M_B = 0; \qquad F_A = F_C = F$



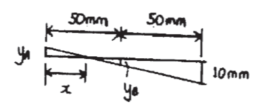

$\Sigma F_y = 0; \qquad F_B = 2F$

$$\frac{x}{y_A} = \frac{50 - x}{y_B} \qquad (1)$$

$$\frac{2F}{F} = \frac{ky_B}{ky_A}$$

$$2y_A = y_B \qquad (2)$$

Substituting into Eq.(1):




$$\frac{x}{y_A} = \frac{50 - x}{2y_A}$$

$$2x = 50 - x$$

$$x = \frac{50}{3} = 16.67 \text{ mm}$$

$$\frac{x}{y_A} = \frac{100 - x}{10}$$

Set $x = 16.67$, then

$y_A = 2$ mm

From Eq.(2),

$y_B = 4$ mm

$F_C = F_A = ky_A$

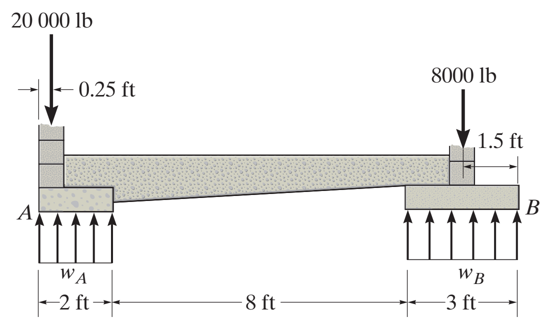
$0.5 = k(0.002)$

$k = 250$ N/m **Ans.**

**Ans:**
$k = 250$ N/m

**4–19.**

The cantilever footing is used to support a wall near its edge $A$ so that it causes a uniform soil pressure under the footing. Determine the uniform distribution loads, $w_A$ and $w_B$, measured in lb/ft at pads $A$ and $B$, necessary to support the wall forces of 8000 lb and 20 000 lb.



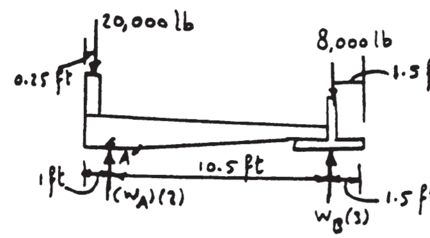

## SOLUTION

$\zeta + \Sigma M_A = 0; \qquad -8000\,(10.5) + w_B\,(3)(10.5) + 20\,000\,(0.75) = 0$

$w_B = 2190.5 \text{ lb/ft} = 2.19 \text{ kip/ft}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 2190.5\,(3) - 28\,000 + w_A\,(2) = 0$

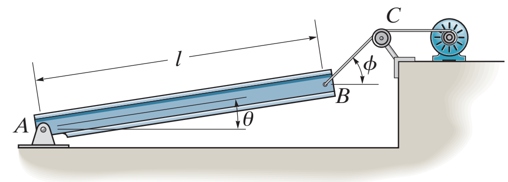
$w_A = 10.7 \text{ kip/ft}$ **Ans.**




**Ans:**
$w_B = 2.19 \text{ kip/ft}$
$w_A = 10.7 \text{ kip/ft}$

***4–20.**

The uniform beam has a weight $W$ and length $l$ and is supported by a pin at $A$ and a cable $BC$. Determine the horizontal and vertical components of reaction at $A$ and the tension in the cable necessary to hold the beam in the position shown.

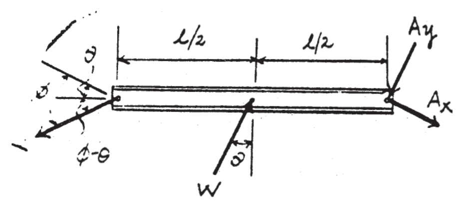



## SOLUTION

***Equations of Equilibrium:*** The tension in the cable can be obtained directly by summing moments about point $A$.

$\circlearrowleft + \Sigma M_A = 0;\qquad T\sin(\phi-\theta)l - W\cos\theta\left(\dfrac{l}{2}\right) = 0$

$$T = \frac{W\cos\theta}{2\sin(\phi-\theta)} \qquad \textbf{Ans.}$$




Using the result $T = \dfrac{W\cos\theta}{2\sin(\phi-\theta)}$

$\xrightarrow{+} \Sigma F_x = 0;\qquad \left(\dfrac{W\cos\theta}{2\sin(\phi-\theta)}\right)\cos\phi - A_x = 0$

$$A_x = \frac{W\cos\phi\cos\theta}{2\sin(\phi-\theta)} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0;\qquad A_y + \left(\dfrac{W\cos\theta}{2\sin(\phi-\theta)}\right)\sin\phi - W = 0$

$$A_y = \frac{W(\sin\phi\cos\theta - 2\cos\phi\sin\theta)}{2\sin(\phi-\theta)} \qquad \textbf{Ans.}$$

**Ans:**

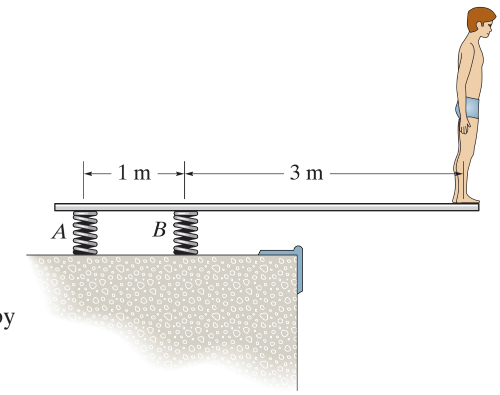
$$T = \frac{W\cos\theta}{2\sin(\phi-\theta)}$$

$$A_x = \frac{W\cos\phi\cos\theta}{2\sin(\phi-\theta)}$$

$$A_y = \frac{W(\sin\phi\cos\theta - 2\cos\phi\sin\theta)}{2\sin(\phi-\theta)}$$

**4–21.**

A boy stands out at the end of the diving board, which is supported by two springs $A$ and $B$, each having a stiffness of $k = 15$ kN/m. In the position shown the board is horizontal. If the boy has a mass of 40 kg, determine the angle of tilt which the board makes with the horizontal after he jumps off. Neglect the weight of the board and assume it is rigid.

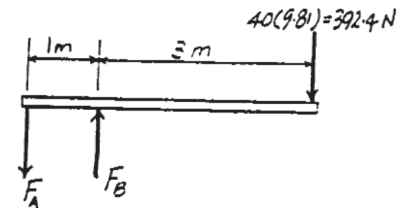



## SOLUTION

***Equations of Equilibrium:*** The spring force at $A$ and $B$ can be obtained directly by summing moments about points $B$ and $A$, respectively.

$\circlearrowleft + \Sigma M_B = 0; \quad F_A(1) - 392.4(3) = 0 \quad F_A = 1177.2\text{ N}$

$\circlearrowleft + \Sigma M_A = 0; \quad F_B(1) - 392.4(4) = 0 \quad F_B = 1569.6\text{ N}$

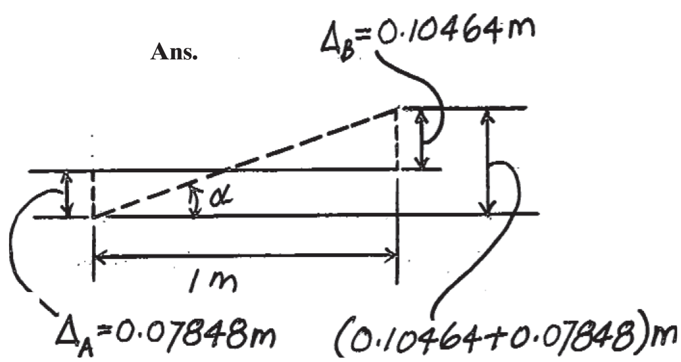



***Spring Formula:*** Applying $\Delta = \dfrac{F}{k}$, we have

$$\Delta_A = \frac{1177.2}{15(10^3)} = 0.07848\text{ m} \qquad \Delta_B = \frac{1569.6}{15(10^3)} = 0.10464\text{ m}$$

***Geometry:*** The angle of tilt $\alpha$ is

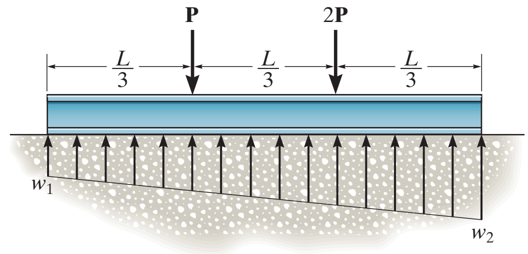
$$\alpha = \tan^{-1}\left(\frac{0.10464 + 0.07848}{1}\right) = 10.4° \qquad \textbf{Ans.}$$




**Ans:**
$\alpha = 10.4°$

**4–22.**

The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities $w_1$ and $w_2$ for equilibrium in terms of the parameters shown.

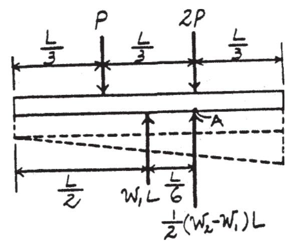



## SOLUTION

***Equations of Equilibrium:*** The load intensity $w_1$ can be determined directly by summing moments about point $A$.

$$\circlearrowleft + \Sigma M_A = 0; \qquad P\left(\frac{L}{3}\right) - w_1 L\left(\frac{L}{6}\right) = 0$$

$$w_1 = \frac{2P}{L} \qquad \textbf{Ans.}$$




$$+\uparrow \Sigma F_y = 0; \qquad \frac{1}{2}\left(w_2 - \frac{2P}{L}\right)L + \frac{2P}{L}(L) - 3P = 0$$

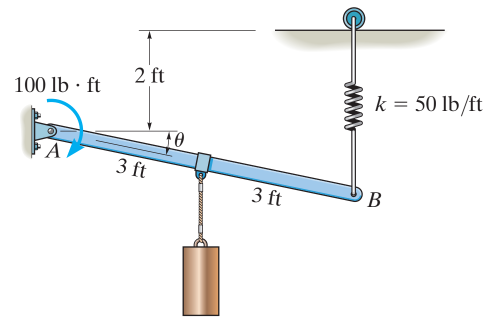
$$w_2 = \frac{4P}{L} \qquad \textbf{Ans.}$$

**Ans:**

$w_1 = \dfrac{2P}{L}, w_2 = \dfrac{4P}{L}$

**4–23.**

The rod supports a weight of 200 lb and is pinned at its end $A$. If it is subjected to a couple moment of 100 lb·ft, determine the angle $\theta$ for equilibrium. The spring has an unstretched length of 2 ft and a stiffness of $k = 50$ lb/ft.



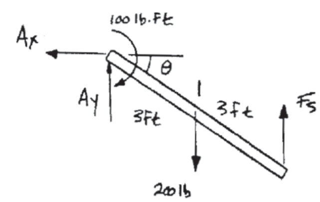

## SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad 100 + 200\,(3\cos\theta) - F_s\,(6\cos\theta) = 0$

$F_s = kx; \qquad F_s = 50\,(6\sin\theta)$

$$100 + 600\cos\theta - 1800\sin\theta\cos\theta = 0$$

$$\cos\theta - 1.5\sin 2\theta + 0.1667 = 0$$



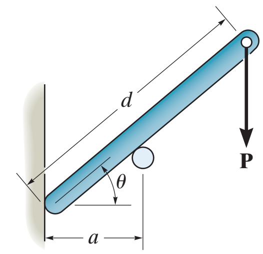

Solving by trial and error,

$$\theta = 23.2^\circ \text{ and } \theta = 85.2^\circ$$ **Ans.**

**Ans:**
$\theta = 23.2^\circ$,
$85.2^\circ$

***4–24.**

Determine the distance $d$ for placement of the load **P** for equilibrium of the smooth bar when it is held in the position $\theta$ as shown. Neglect the weight of the bar.



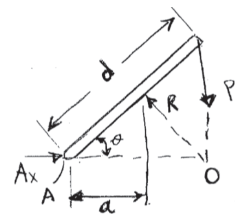

## SOLUTION

$+\uparrow \Sigma F_y = 0;$ $\quad R\cos\theta - P = 0$

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -P(d\cos\theta) + R\left(\dfrac{a}{\cos\theta}\right) = 0$

$$Rd\cos^2\theta = R\left(\frac{a}{\cos\theta}\right)$$

$$d = \frac{a}{\cos^3\theta}$$ **Ans.**




Also,

require forces to be concurrent at point $O$.

$$AO = d\cos\theta = \frac{a/\cos\theta}{\cos\theta}$$

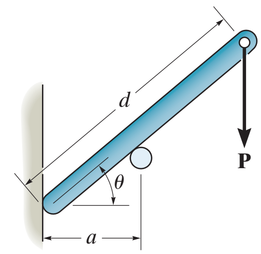
Thus,

$$d = \frac{a}{\cos^3\theta}$$ **Ans.**

**Ans:**
$d = \dfrac{a}{\cos^3\theta}$

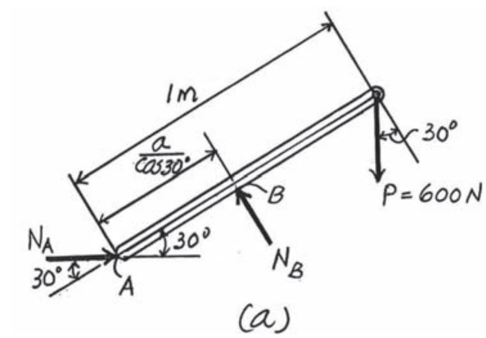
**4–25.**

If $d = 1$ m, and $\theta = 30°$, determine the normal reaction at the smooth supports and the required distance $a$ for the placement of the roller if $P = 600$ N. Neglect the weight of the bar.




## SOLUTION

***Equations of Equilibrium:*** Referring to the FBD of the rod shown in Fig. *a*,




$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_B = \left(\dfrac{a}{\cos 30°}\right) - 600 \cos 30°(1) = 0$

$$N_B = \frac{450}{a} \qquad \textbf{(1)}$$

$\nwarrow + \Sigma F_{y'} = 0;$ $\quad N_B - N_A \sin 30° - 600 \cos 30° = 0$

$$N_B - 0.5N_A = 600 \cos 30° \qquad \textbf{(2)}$$

$+\nearrow \Sigma F_{x'} = 0;$ $\quad N_A \cos 30° - 600 \sin 30° = 0$

$$N_A = 346.41 \text{ N} = 346 \text{ N} \qquad \textbf{Ans.}$$

Substitute this result into Eq (2).

$$N_B - 0.5(346.41) = 600 \cos 30°$$

$$N_B = 692.82 \qquad N_B = 693 \text{ N} \qquad \textbf{Ans.}$$

Substitute this result into Eq (1).

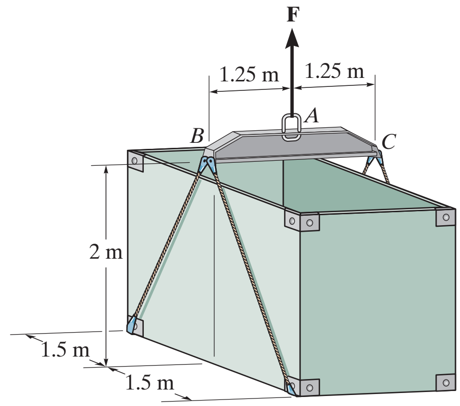
$$692.82 = \frac{450}{a}$$

$$a = 0.6495 \text{ m} \qquad a = 0.650 \text{ m} \qquad \textbf{Ans.}$$

**Ans:**
$N_A = 346$ N
$N_B = 693$ N
$a = 0.650$ m

**4–26.**

The uniform load has a mass of 600 kg and is lifted using a uniform 30-kg strongback beam *BAC* and the four ropes as shown. Determine the tension in each rope and the force that must be applied at *A*.




## SOLUTION

***Equations of Equilibrium:*** Due to symmetry, all wires are subjected to the same tension. This condition statisfies moment equilibrium about the $x$ and $y$ axes and force equilibrium along the $y$ axis.

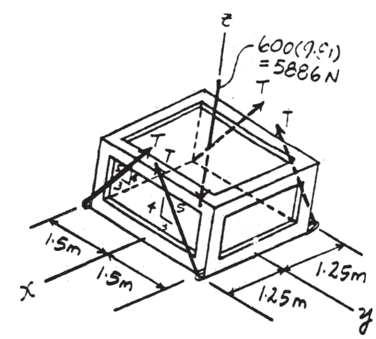
$\Sigma F_z = 0;$ $$4T\left(\frac{4}{5}\right) - 5886 = 0$$

$$T = 1839.375 \text{ N} = 1.84 \text{ kN}$$ **Ans.**

The force **F** applied to the sling $A$ must support the weight of the load and strongback beam. Hence,

$\Sigma F_z = 0;$ $$F - 600(9.81) - 30(9.81) = 0$$

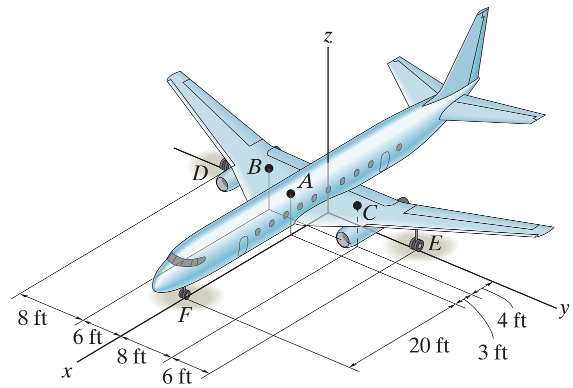
$$F = 6180.3 \text{ N} = 6.18 \text{ kN}$$ **Ans.**




**Ans:**
$T = 1.84$ kN
$F = 6.18$ kN

**4–27.**

Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage $A$ and wings $B$ and $C$ are located as shown. If these components have weights $W_A = 45\,000$ lb, $W_B = 8000$ lb, and $W_C = 6000$ lb, determine the normal reactions of the wheels $D$, $E$, and $F$ on the ground.

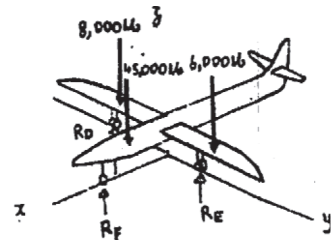



## SOLUTION

$\Sigma M_x = 0;\qquad 8000(6) - R_D(14) - 6000(8) + R_E(14) = 0$

$\Sigma M_y = 0;\qquad 8000(4) + 45\,000(7) + 6000(4) - R_F(27) = 0$

$\Sigma F_z = 0;\qquad R_D + R_E + R_F - 8000 - 6000 - 45\,000 = 0$




Solving,

$R_D = 22.6$ kip **Ans.**

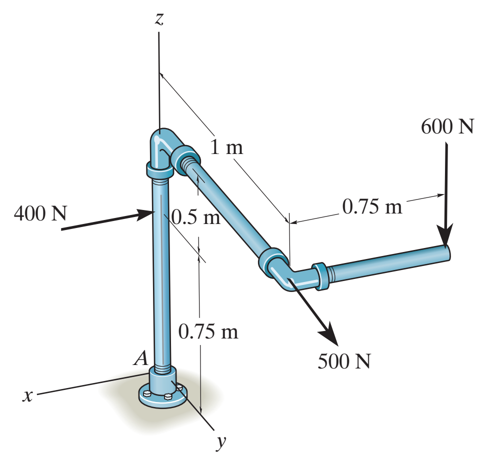
$R_E = 22.6$ kip **Ans.**

$R_F = 13.7$ kip **Ans.**

**Ans:**
$R_D = 22.6$ kip
$R_E = 22.6$ kip
$R_F = 13.7$ kip

***4–28.**

Determine the components of reaction at the fixed support $A$. The 400 N, 500 N, and 600 N forces are parallel to the $x$, $y$, and $z$ axes, respectively.




## SOLUTION

**Equations of Equilibrium.** Referring to the *FBD* of the rod shown in Fig. *a*,

$\Sigma F_x = 0; \quad A_x - 400 = 0 \quad A_x = 400\text{ N}$ **Ans.**

$\Sigma F_y = 0; \quad 500 - A_y = 0 \quad A_y = 500\text{ N}$ **Ans.**

$\Sigma F_z = 0; \quad A_z - 600 = 0 \quad A_z = 600\text{ N}$ **Ans.**

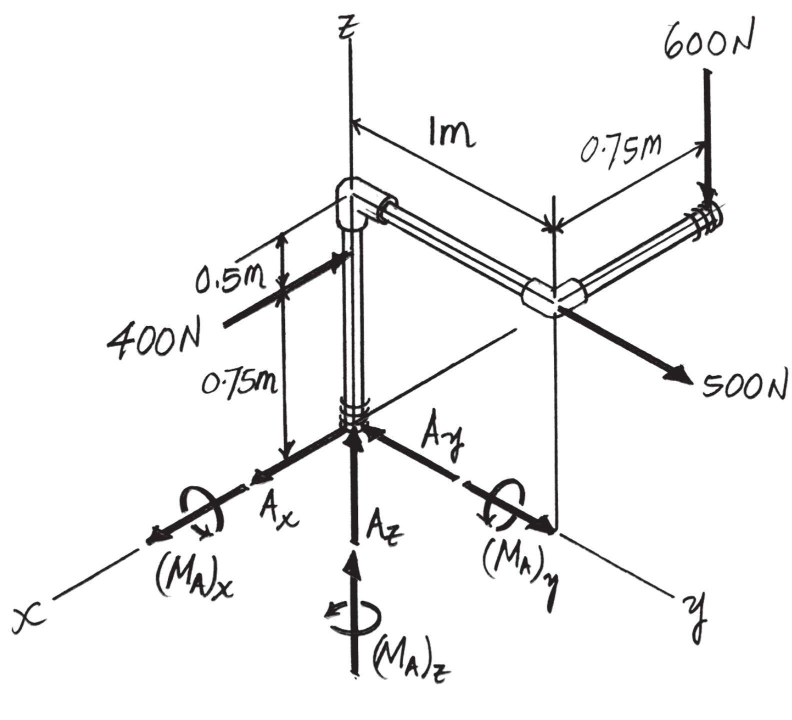
$\Sigma M_x = 0; \quad (M_A)_x - 500(1.25) - 600(1) = 0$

$(M_A)_x = 1225\text{ N}\cdot\text{m} = 1.225\text{ kN}\cdot\text{m}$ **Ans.**

$\Sigma M_y = 0; \quad (M_A)_y - 400(0.75) - 600(0.75) = 0$

$(M_A)_y = 750\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_z = 0; \quad (M_A)_z = 0$ **Ans.**

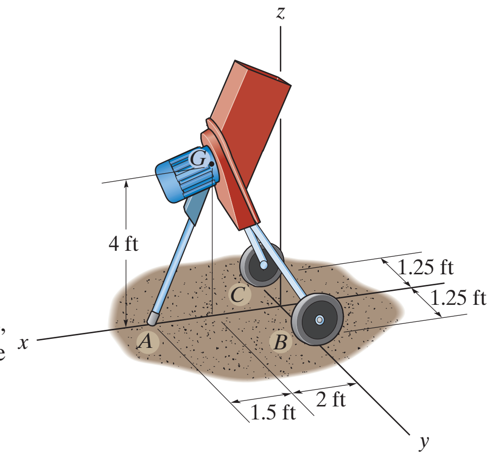



(a)

**Ans:**
$A_x = 400\text{ N}$
$A_y = 500\text{ N}$
$A_z = 600\text{ N}$
$(M_A)_x = 1.225\text{ kN}\cdot\text{m}$
$(M_A)_y = 750\text{ N}\cdot\text{m}$
$(M_A)_z = 0$

**4–29.**

The 50-lb mulching machine has a center of gravity at $G$. Determine the vertical reactions at the wheels $C$ and $B$ and the smooth contact point $A$.




## SOLUTION

***Equations of Equilibrium:*** From the free-body diagram of the mulching machine, Fig. *a*, $N_A$ can be obtained by writing the moment equation of equilibrium about the $y$ axis.

$\Sigma M_y = 0;\quad 50(2) - N_A(1.5 + 2) = 0$

$$N_A = 28.57 \text{ lb} = 28.6 \text{ lb} \qquad \textbf{Ans.}$$

Using the above result and writing the moment equation of equilibrium about the $x$ axis and the force equation of equilibrium along the $z$ axis, we have

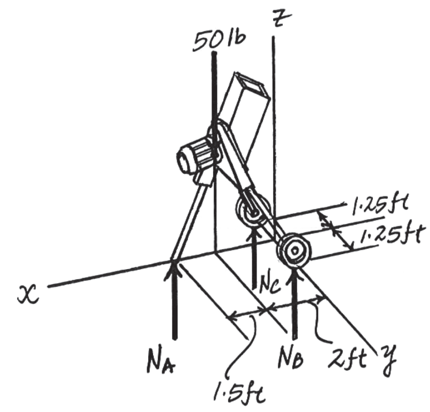
$\Sigma M_x = 0;\quad N_B(1.25) - N_C(1.25) = 0 \qquad \textbf{(1)}$

$\Sigma F_z = 0;\quad N_B + N_C + 28.57 - 50 = 0 \qquad \textbf{(2)}$

Solving Eqs. (1) and (2) yields

$$N_B = N_C = 10.71 \text{ lb} = 10.7 \text{ lb} \qquad \textbf{Ans.}$$

*Note*: If we write the force equation of equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$ and the moment equation of equilibrium $\Sigma M_z = 0$, this indicates that equilibrium is satisfied.

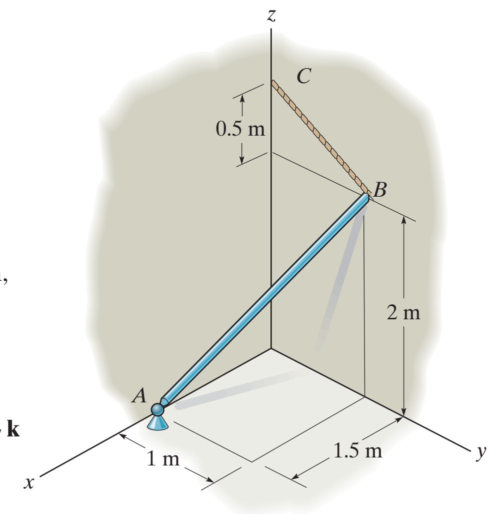



(a)

**Ans:**
$N_A = 28.6$ lb
$N_B = 10.7$ lb, $N_C = 10.7$ lb

**4–30.**

The smooth uniform rod $AB$ is supported by a ball-and-socket joint at $A$, the wall at $B$, and cable $BC$. Determine the components of reaction at $A$, the tension in the cable, and the normal reaction at $B$ if the rod has a mass of 20 kg.




## SOLUTION

**Force And Position Vectors.** The coordinates of points $A$, $B$ and $G$ are $A(1.5, 0, 0)$ m, $B(0, 1, 2)$ m, $C(0, 0, 2.5)$ m and $G(0.75, 0.5, 1)$ m.

$$F_A = -A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{T}_{BC} = T_{BC}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = T_{BC}\left[\frac{(0-1)\mathbf{j} + (2.5-2)k}{\sqrt{(0-1)^2 + (2.5-2)^2}}\right] = -\frac{1}{\sqrt{1.25}}T_{BC}\mathbf{j} + \frac{0.5}{\sqrt{1.25}}T_{BC}\,\mathbf{k}$$

$$\mathbf{N}_B = N_B\mathbf{i}$$

$$\mathbf{W} = \{-20(9.81)\mathbf{k}\}\text{ N}$$

$$\mathbf{r}_{AG} = (0.75 - 1.5)\mathbf{i} + (0.5 - 0)\mathbf{j} + (1 - 0)\mathbf{k} = \{-0.75\mathbf{i} + 0.5\mathbf{j} + \mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{AB} = (0 - 1.5)\mathbf{i} + (1 - 0)\mathbf{j} + (2 - 0)\mathbf{k} = \{-1.5\mathbf{i} + \mathbf{j} + 2\mathbf{k}\}\text{ m}$$

**Equations of Equilibrium.** Referring to the *FBD* of the rod shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_A + \mathbf{T}_{BC} + \mathbf{N}_B + \mathbf{W} = 0$$

$$(-A_x + N_B)\mathbf{i} + \left(A_y - \frac{1}{\sqrt{1.25}}T_{BC}\right)\mathbf{j} + \left[A_z + \frac{0.5}{\sqrt{1.25}}T_{BC} - 20\,(9.81)\right]\mathbf{k} = 0$$

Equating $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ components,

$$-A_x + N_B = 0 \tag{1}$$

$$A_y - \frac{1}{\sqrt{1.25}}T_{BC} = 0 \tag{2}$$

$$A_z + \frac{0.5}{\sqrt{1.25}}T_{BC} - 20(9.81) = 0 \tag{3}$$

The moment equation of equilibrium gives

$$\Sigma\mathbf{M}_A = 0; \qquad \mathbf{r}_{AG} \times \mathbf{W} + \mathbf{r}_{AB} \times (\mathbf{T}_{BC} + \mathbf{N}_B) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.75 & 0.5 & 1 \\ 0 & 0 & -20(9.81) \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.5 & 1 & 2 \\ N_B & -\frac{1}{\sqrt{1.25}}T_{BC} & \frac{0.5}{\sqrt{1.25}}T_{BC} \end{vmatrix} = 0$$

$$\left(\frac{0.5}{\sqrt{1.25}}T_{BC} + \frac{2}{\sqrt{1.25}}T_{BC} - 98.1\right)\mathbf{i} + \left(\frac{0.75}{\sqrt{1.25}}T_{BC} + 2N_B - 147.15\right)\mathbf{j} + \left(\frac{1.5}{\sqrt{1.25}}T_{BC} - N_B\right)\mathbf{k} = 0$$

**4–30. Continued**

Equating **i**, **j** and **k** components,

$$\frac{0.5}{\sqrt{1.25}}T_{BC} + \frac{2}{\sqrt{1.25}}T_{BC} - 98.1 = 0 \qquad \textbf{(4)}$$

$$\frac{0.75}{\sqrt{1.25}}T_{BC} + 2N_B - 147.15 = 0 \qquad \textbf{(5)}$$

$$\frac{1.5}{\sqrt{1.25}}T_{BC} - N_B = 0 \qquad \textbf{(6)}$$

Solving Eqs. (1) to (6),

$T_{BC} = 43.87\text{ N} = 43.9\text{ N}$ **Ans.**

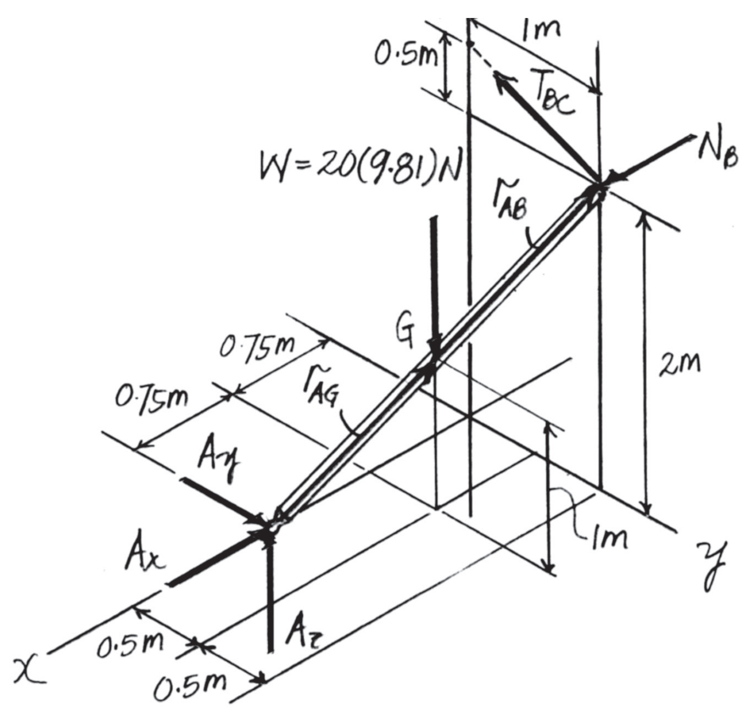
$N_B = 58.86\text{ N} = 58.9\text{ N}$ **Ans.**

$A_x = 58.86\text{ N} = 58.9\text{ N}$ **Ans.**

$A_y = 39.24\text{ N} = 39.2\text{ N}$ **Ans.**

$A_z = 176.58\text{ N} = 177\text{ N}$ **Ans.**

Note: One of the Equations (4), (5) and (6) is redundant, which will be satisfied automatically.

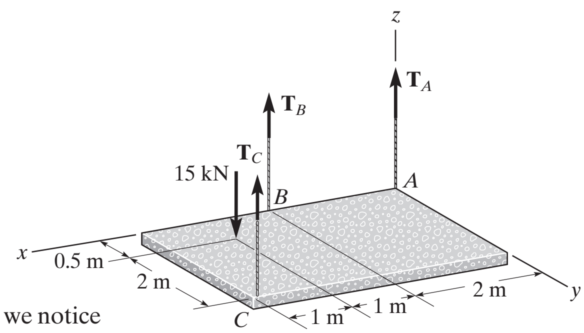



(a)

**Ans:**
$T_{BC} = 43.9\text{ N}$
$N_B = 58.9\text{ N}$
$A_x = 58.9\text{ N}$
$A_y = 39.2\text{ N}$
$A_z = 177\text{ N}$

**4–31.**

The uniform concrete slab has a mass of 2400 kg. Determine the tension in each of the three parallel supporting cables.




## SOLUTION

**Equations of Equilibrium.** Referring to the *FBD* of the slab shown in Fig. *a*, we notice that $T_C$ can be obtained directly by writing the moment equation of equilibrium about the $x$ axis.

$\Sigma M_x = 0;\quad T_C(2.5) - 2400(9.81)(1.25) - 15(10^3)(0.5) = 0$

$T_C = 14{,}772\text{ N} = 14.8\text{ kN}$ **Ans.**

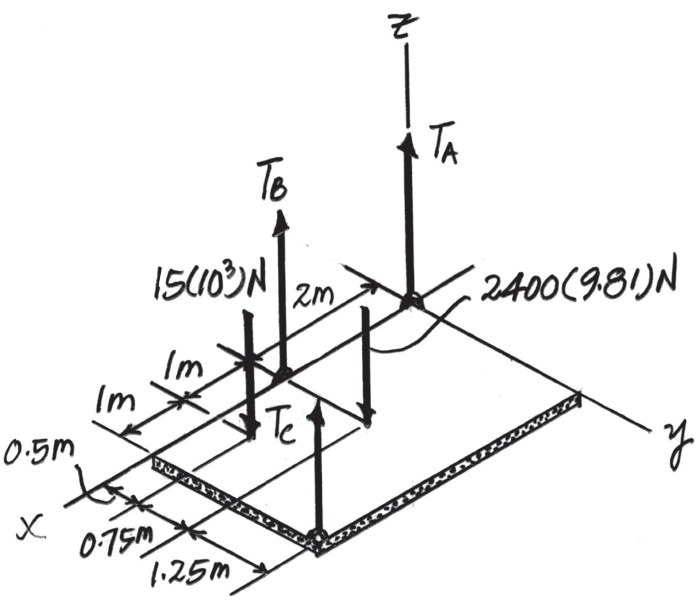
Using this result to write moment equation of equilibrium about the $y$ axis and force equation of equilibrium along the $z$ axis,

$\Sigma M_y = 0;\quad T_B(2) + 14{,}772(4) - 2400(9.81)(2) - 15(10^3)(3) = 0$

$T_B = 16{,}500\text{ N} = 16.5\text{ kN}$ **Ans.**

$\Sigma F_z = 0;\quad T_A + 16{,}500 + 14{,}772 - 2400(9.81) - 15(10^3) = 0$

$T_A = 7272\text{ N} = 7.27\text{ kN}$ **Ans.**

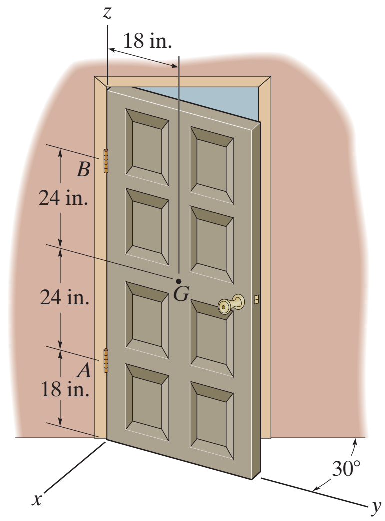



(a)

**Ans:**
$T_C = 14.8\text{ kN}$
$T_B = 16.5\text{ kN}$
$T_A = 7.27\text{ kN}$

***4–32.**

The 100-lb door has its center of gravity at $G$. Determine the components of reaction at hinges $A$ and $B$ if hinge $B$ resists only forces in the $x$ and $y$ directions and $A$ resists forces in the $x, y, z$ directions.




## SOLUTION

***Equations of Equilibrium:*** From the free-body diagram of the door, Fig. *a*, $B_y$, $B_x$, and $A_z$ can be obtained by writing the moment equation of equilibrium about the $x'$ and $y'$ axes and the force equation of equilibrium along the $z$ axis.

$\Sigma M_{x'} = 0;$ $\quad -B_y(48) - 100(18) = 0$

$B_y = -37.5 \text{ lb}$ **Ans.**

$\Sigma M_{y'} = 0;$ $\quad B_x = 0$ **Ans.**

$\Sigma F_z = 0;$ $\quad -100 + A_z = 0;$ $\quad A_z = 100 \text{ lb}$ **Ans.**

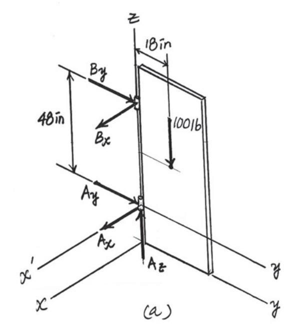
Using the above result and writing the force equations of equilibrium along the $x$ and $y$ axes, we have

$\Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$\Sigma F_y = 0;$ $\quad A_y + (-37.5) = 0$

$A_y = 37.5 \text{ lb}$ **Ans.**

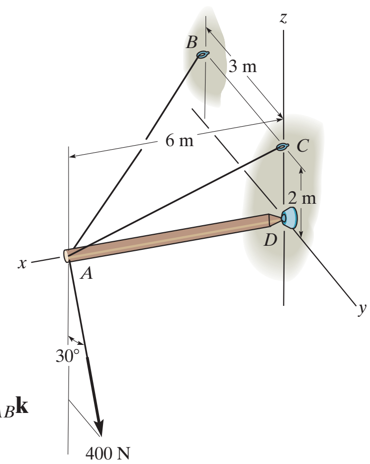
The negative sign indicates that $\mathbf{B}_y$ acts in the opposite sense to that shown on the free-body diagram. If we write the moment equation of equilibrium $\Sigma M_z = 0$, it shows that equilibrium is satisfied.




**Ans:**
$B_y = -37.5 \text{ lb}$
$B_x = 0$
$A_z = 100 \text{ lb}$
$A_x = 0$
$A_y = 37.5 \text{ lb}$

**4–33.**

Determine the tension in each cable and the components of reaction at $D$ needed to support the load.




## SOLUTION

**Force And Position Vectors.** The coordinates of points $A$, $B$, and $C$ are $A(6, 0, 0)$ m, $B(0, -3, 2)$ m and $C(0, 0, 2)$ m, respectively.

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = F_{AB}\left[\frac{(0-6)\mathbf{i} + (-3-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(0-6)^2 + (-3-0)^2 + (2-0)^2}}\right] = -\frac{6}{7}F_{AB}\mathbf{i} - \frac{3}{7}F_{AB}\mathbf{j} + \frac{2}{7}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = F_{AC}\left[\frac{(0-6)\mathbf{i} + (2-0)\mathbf{k}}{\sqrt{(0-6)^2 + (2-0)^2}}\right] = -\frac{6}{\sqrt{40}}F_{AC}\mathbf{i} + \frac{2}{\sqrt{40}}F_{AC}\mathbf{k}$$

$$\mathbf{F} = 400\,(\sin 30°\mathbf{j} - \cos 30°\mathbf{k}) = \{200\mathbf{j} - 346.41\mathbf{k}\}\text{N}$$

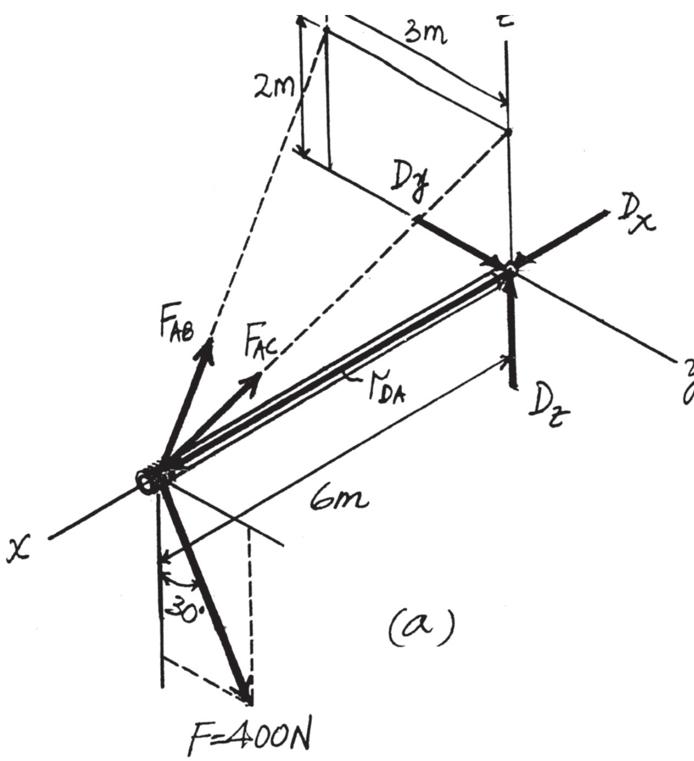
$$F_D = D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}$$

$$\mathbf{r}_{DA} = \{6\mathbf{i}\}\text{ m}$$

Referring to the $FBD$ of the rod shown in Fig. $a$, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F} + \mathbf{F}_D = 0$$

$$\left(-\frac{6}{7}F_{AB} - \frac{6}{\sqrt{40}}F_{AC} + D_x\right)\mathbf{i} + \left(-\frac{3}{7}F_{AB} + D_y + 200\right)\mathbf{j} + \left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} + D_z - 346.41\right)\mathbf{k} = 0$$




(a)

**4–33. Continued**

Equating **i**, **j** and **k** components,

$$-\frac{6}{7}F_{AB} - \frac{6}{\sqrt{40}}F_{AC} + D_x = 0 \qquad \textbf{(1)}$$

$$-\frac{3}{7}F_{AB} + D_y + 200 = 0 \qquad \textbf{(2)}$$

$$\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} + D_z - 346.41 = 0 \qquad \textbf{(3)}$$

Moment equation of equilibrium gives

$\Sigma \mathbf{M}_D = 0;\qquad \mathbf{r}_{DA} \times (\mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ \left(-\frac{6}{7}F_{AB} - \frac{6}{\sqrt{40}}F_{AC}\right) & \left(-\frac{3}{7}F_{AB} + 200\right) & \left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} - 346.41\right) \end{vmatrix} = 0$$

$$-6\left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} - 346.41\right)\mathbf{j} + 6\left(-\frac{3}{7}F_{AB} + 200\right)\mathbf{k} = 0$$

Equating **j** and **k** components,

$$-6\left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} - 346.41\right) = 0 \qquad \textbf{(4)}$$

$$6\left(-\frac{3}{7}F_{AB} + 200\right) = 0 \qquad \textbf{(5)}$$

Solving Eqs. (1) to (5),

$F_{AB} = 466.67\text{ N} = 467\text{ N}$ **Ans.**

$F_{AC} = 673.81\text{ N} = 674\text{ N}$ **Ans.**

$D_x = 1039.23\text{ N} = 1.04\text{ kN}$ **Ans.**

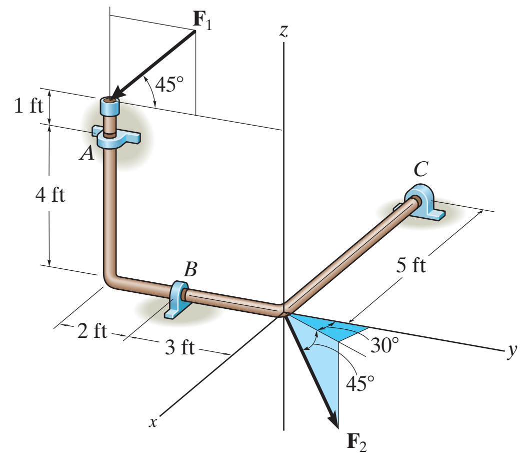
$D_y = 0$ **Ans.**

$D_z = 0$ **Ans.**

**Ans:**
$F_{AB} = 467\text{ N}$
$F_{AC} = 674\text{ N}$
$D_x = 1.04\text{ kN}$
$D_y = 0$
$D_z = 0$

**4–34.**

The bent rod is supported at $A$, $B$, and $C$ by smooth journal bearings. Calculate the $x$, $y$, $z$ components of reaction at the bearings if the rod is subjected to forces $F_1 = 300$ lb and $F_2 = 250$ lb. $\mathbf{F}_1$ lies in the $y$–$z$ plane. The bearings are in proper alignment and exert only force reactions on the rod.



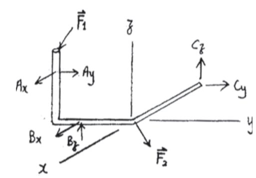

## SOLUTION

$$\mathbf{F}_1 = (-300 \cos 45°\mathbf{j} - 300 \sin 45°\mathbf{k})$$

$$= \{-212.1\mathbf{j} - 212.1\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = (250 \cos 45° \sin 30°\mathbf{i} + 250 \cos 45° \cos 30°\mathbf{j} - 250 \sin 45°\mathbf{k})$$

$$= \{88.39\mathbf{i} + 153.1\mathbf{j} - 176.8\mathbf{k}\}\text{ lb}$$




$\Sigma F_x = 0;\quad A_x + B_x + 88.39 = 0$

$\Sigma F_y = 0;\quad A_y + C_y - 212.1 + 153.1 = 0$

$\Sigma F_z = 0;\quad B_z + C_z - 212.1 - 176.8 = 0$

$\Sigma M_x = 0;\quad -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\Sigma M_y = 0;\quad C_z(5) + A_x(4) = 0$

$\Sigma M_z = 0;\quad A_x(5) + B_x(3) - C_y(5) = 0$

$A_x = 633$ lb **Ans.**

$A_y = -141$ lb **Ans.**

$B_x = -721$ lb **Ans.**

$B_z = 895$ lb **Ans.**

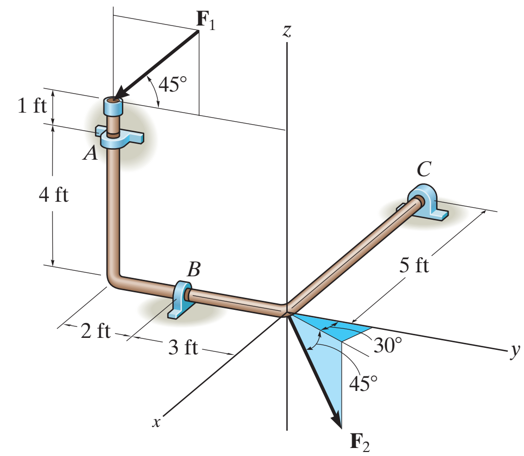
$C_y = 200$ lb **Ans.**

$C_z = -506$ lb **Ans.**

**Ans:**
$A_x = 633$ lb
$A_y = -141$ lb
$B_x = -721$ lb
$B_z = 895$ lb
$C_y = 200$ lb
$C_z = -506$ lb

**4–35.**

The bent rod is supported at $A$, $B$, and $C$ by smooth journal bearings. Determine the magnitude of $\mathbf{F}_2$ which will cause the reaction $\mathbf{C}_y$ at the bearing $C$ to be equal to zero. The bearings are in proper alignment and exert only force reactions on the rod. Set $F_1 = 300$ lb.




## SOLUTION

$\mathbf{F}_1 = (-300\cos 45°\mathbf{j} - 300\sin 45°\mathbf{k})$

$= \{-212.1\mathbf{j} - 212.1\mathbf{k}\}$ lb

$\mathbf{F}_2 = (F_2\cos 45°\sin 30°\mathbf{i} + F_2\cos 45°\cos 30°\mathbf{j} - F_2\sin 45°\mathbf{k})$

$= \{0.3536F_2\mathbf{i} + 0.6124F_2\mathbf{j} - 0.7071F_2\mathbf{k}\}$ lb

$\Sigma F_x = 0;\quad A_x + B_x + 0.3536F_2 = 0$

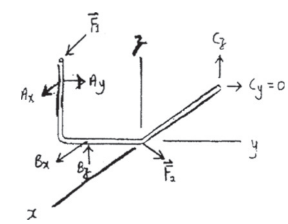
$\Sigma F_y = 0;\quad A_y + 0.6124F_2 - 212.1 = 0$

$\Sigma F_z = 0;\quad B_z + C_z - 0.7071F_2 - 212.1 = 0$

$\Sigma M_x = 0;\quad -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\Sigma M_y = 0;\quad C_z(5) + A_x(4) = 0$

$\Sigma M_z = 0;\quad A_x(5) + B_x(3) = 0$




$A_x = 357$ lb

$A_y = -200$ lb

$B_x = -595$ lb

$B_z = 974$ lb

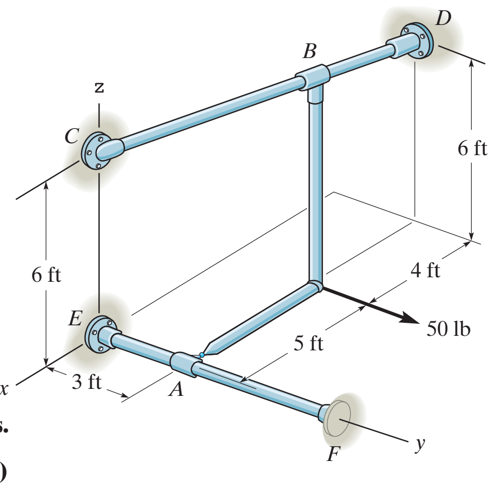
$C_z = -286$ lb

$F_2 = 674$ lb **Ans.**

**Ans:**
$F_2 = 674$ lb

***4–36.**

The bar $AB$ is supported by two smooth collars. At $A$ the connection is a ball-and-socket joint and at $B$ it is a rigid attachment. If a 50-lb load is applied to the bar, determine the $x$, $y$, $z$ components of reaction at $A$ and $B$.




## SOLUTION

$A_x + B_x = 0$ **(1)**

$B_y + 50 = 0$

$B_y = -50$ lb **Ans.**

$A_z + B_z = 0$ **(2)**

$M_{B_z} = 0$ **Ans.**

$M_{B_x} + 50(6) = 0$

$M_{B_x} = -300 \text{ lb} \cdot \text{ft}$ **Ans.**

$B_{C_D} = -9\mathbf{i} + 3\mathbf{j}$

$B_{C_D} = -0.94868\mathbf{i} + 0.316228\mathbf{j}$

Require

$\mathbf{F}_B \cdot \mathbf{u}_{CD} = 0$

$(B_x\mathbf{i} - 50\mathbf{j} + B_z\mathbf{k}) \cdot (-0.94868\mathbf{i} + 0.316228\mathbf{j}) = 0$

$-0.94868B_x - 50(0.316228) = 0$

$B_x = -16.667 = -16.7$ lb **Ans.**

From Eq. (1),

$A_x = 16.7$ lb **Ans.**

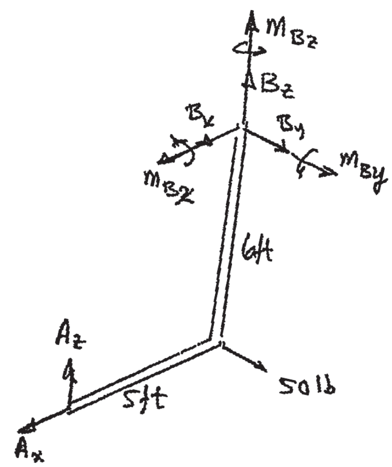
Require

$\mathbf{M}_B \cdot \mathbf{u}_{CD} = 0$

$(-300\mathbf{i} + M_{By}\mathbf{j}) \cdot (-0.94868\mathbf{i} + 0.316228\mathbf{j}) = 0$

$300(0.94868) + M_{By}(0.316228) = 0$

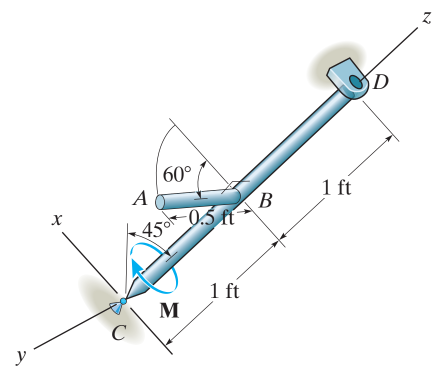
$M_{B_y} = -900 \text{ lb} \cdot \text{ft}$ **Ans.**




**Ans:**
$B_y = -50$ lb
$M_{B_z} = 0$
$M_{B_x} = -300 \text{ lb} \cdot \text{ft}$
$B_x = -16.7$ lb
$A_x = 16.7$ lb
$M_{B_y} = -900 \text{ lb} \cdot \text{ft}$

**4–37.**

The rod has a weight of 6 lb/ft. If it is supported by a ball-and-socket joint at $C$ and a journal bearing at $D$, determine the $x$, $y$, $z$ components of reaction at these supports and the moment $M$ that must be applied along the axis of the rod to hold it in the position shown.




## SOLUTION

$\Sigma F_x = 0;$ $\quad C_x + D_x - 15 \sin 45° = 0$ **(1)**

$\Sigma F_y = 0;$ $\quad C_y + D_y = 0$ **(2)**

$\Sigma F_z = 0;$ $\quad C_z - 15 \cos 45° = 0$

$C_z = 10.6 \text{ lb}$ **Ans.**

$\Sigma M_x = 0;$ $\quad -3 \cos 45°(0.25 \sin 60°) - D_y(2) = 0$

$D_y = -0.230 \text{ lb}$ **Ans.**

From Eq. (2);

$C_y = 0.230 \text{ lb}$ **Ans.**

$\Sigma M_y = 0;$ $\quad -(12 \sin 45°)(1) - (3 \sin 45°)(1) + (3 \cos 45°)(0.25 \cos 60°) + D_x(2) = 0$

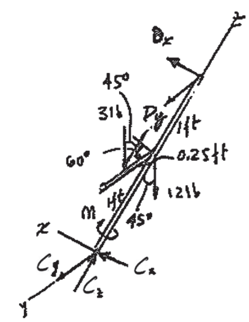
$D_x = 5.17 \text{ lb}$ **Ans.**

From Eq. (1);

$C_x = 5.44 \text{ lb}$ **Ans.**

$\Sigma M_z = 0;$ $\quad -M + (3 \sin 45°)(0.25 \sin 60°) = 0$

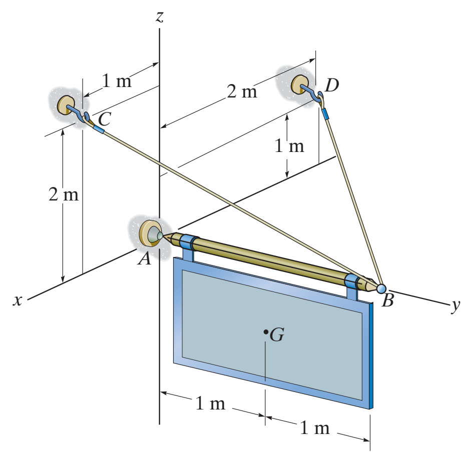
$M = 0.459 \text{ lb} \cdot \text{ft}$ **Ans.**




**Ans:**
$C_z = 10.6 \text{ lb}$
$D_y = -0.230 \text{ lb}$
$C_y = 0.230 \text{ lb}$
$D_x = 5.17 \text{ lb}$
$C_x = 5.44 \text{ lb}$
$M = 0.459 \text{ lb} \cdot \text{ft}$

**4–38.**

The sign has a mass of 100 kg with center of mass at $G$. Determine the $x$, $y$, $z$ components of reaction at the ball-and-socket joint $A$ and the tension in wires $BC$ and $BD$.




## SOLUTION

**Equations of Equilibrium:** Expressing the forces indicated on the free-body diagram, Fig. $a$, in Cartesian vector form, we have

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{W} = \{-100(9.81)\mathbf{k}\}\ \text{N} = \{-981\text{k}\}\ \text{N}$$

$$\mathbf{F}_{BD} = F_{BD}\mathbf{u}_{BD} = F_{BD}\left[\frac{(-2-0)\mathbf{i} + (0-2)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (0-2)^2 + (1-0)^2}}\right] = \left(-\frac{2}{3}F_{BD}\mathbf{i} - \frac{2}{3}F_{BD}\mathbf{j} + \frac{1}{3}F_{BD}\mathbf{k}\right)$$

$$\mathbf{F}_{BC} = F_{BC}\mathbf{u}_{BC} = F_{BC}\left[\frac{(1-0)\mathbf{i} + (0-2)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(1-0)^2 + (0-2)^2 + (2-0)^2}}\right] = \left(\frac{1}{3}F_{BC}\mathbf{i} - \frac{2}{3}F_{BC}\mathbf{j} + \frac{2}{3}F_{BC}\mathbf{k}\right)$$

Applying the forces equation of equilibrium, we have

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_A + \mathbf{F}_{BD} + \mathbf{F}_{BC} + \mathbf{W} = 0$$

$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) + \left(-\frac{2}{3}F_{BD}\mathbf{i} - \frac{2}{3}F_{BD}\mathbf{j} + \frac{1}{3}F_{BD}\mathbf{k}\right) + \left(\frac{1}{3}F_{BC}\mathbf{i} - \frac{2}{3}F_{BC}\mathbf{j} + \frac{2}{3}F_{BC}\mathbf{k}\right) + (-981\,k) = 0$$

$$\left(A_x - \frac{2}{3}F_{BD} + \frac{1}{3}F_{BC}\right)\mathbf{i} + \left(A_y - \frac{2}{3}F_{BD} - \frac{2}{3}F_{BC}\right)\mathbf{j} + \left(A_z + \frac{1}{3}F_{BD} + \frac{2}{3}F_{BC} - 981\right)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components, we have

$$A_x - \frac{2}{3}F_{BD} + \frac{1}{3}F_{BC} = 0 \qquad \textbf{(1)}$$

$$A_y - \frac{2}{3}F_{BD} - \frac{2}{3}F_{BC} = 0 \qquad \textbf{(2)}$$

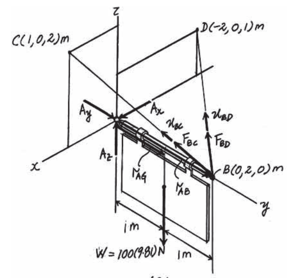
$$A_z + \frac{1}{3}F_{BD} + \frac{2}{3}F_{BC} - 981 = 0 \qquad \textbf{(3)}$$

In order to write the moment equation of equilibrium about point $A$, the position vectors $\mathbf{r}_{AG}$ and $\mathbf{r}_{AB}$ must be determined first.

$$\mathbf{r}_{AG} = \{1\mathbf{j}\}\ \text{m}$$

$$\mathbf{r}_{AB} = \{2\mathbf{j}\}\ \text{m}$$

**4–38. Continued**



(a)

Thus,

$$\Sigma \mathbf{M}_A = 0;\ \mathbf{r}_{AB} \times (\mathbf{F}_{BC} + \mathbf{F}_{BD}) + (\mathbf{r}_{AG} \times \mathbf{W}) = 0$$

$$(2\mathbf{j}) \times \left[\left(\frac{1}{3}F_{BC} - \frac{2}{3}F_{BD}\right)\mathbf{i} - \left(\frac{2}{3}F_{BC} + \frac{2}{3}F_{BD}\right)\mathbf{j} + \left(\frac{2}{3}F_{BC} + \frac{1}{3}F_{BD}\right)\mathbf{k}\right] + (1\mathbf{j}) \times (-981\mathbf{k}) = 0$$

$$\left(\frac{4}{3}F_{BC} + \frac{2}{3}F_{BD} - 981\right)\mathbf{i} + \left(\frac{4}{3}F_{BD} - \frac{2}{3}F_{BC}\right)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components, we have

$$\frac{4}{3}F_{BC} + \frac{2}{3}F_{BC} - 981 = 0 \qquad \textbf{(4)}$$

$$\frac{4}{3}F_{BC} - \frac{2}{3}F_{BC} = 0 \qquad \textbf{(5)}$$

Solving Eqs. (1) through (5), yields

$F_{BD} = 294.3\text{ N} = 294\text{ N}$ **Ans.**

$F_{BC} = 588.6\text{ N} = 589\text{ N}$ **Ans.**

$A_x = 0$ **Ans.**

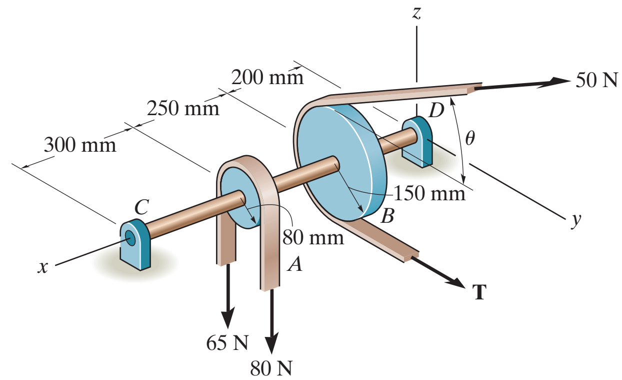
$A_y = 588.6\text{ N} = 589\text{ N}$ **Ans.**

$A_z = 490.5\text{ N}$ **Ans.**

**Ans:**
$F_{BD} = 294\text{ N}$
$F_{BC} = 589\text{ N}$
$A_x = 0$
$A_y = 589\text{ N}$
$A_z = 490.5\text{ N}$

**4–39.**

Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley $A$ is transmitted to pulley $B$. Determine the horizontal tension $\mathbf{T}$ in the belt on pulley $B$ and the $x, y, z$ components of reaction at the journal bearing $C$ and thrust bearing $D$ if $\theta = 0°$. The bearings are in proper alignment and exert only force reactions on the shaft.




## SOLUTION

***Equations of Equilibrium:***

$\Sigma M_x = 0;$ $\quad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

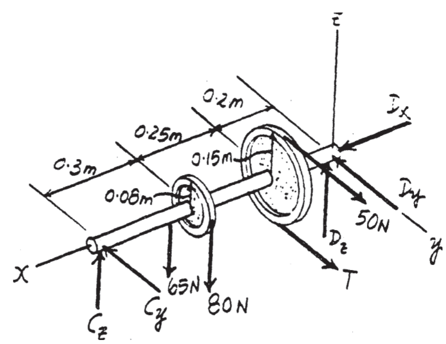
$T = 58.0\text{ N}$ **Ans.**

$\Sigma M_y = 0;$ $\quad (65 + 80)(0.45) - C_z(0.75) = 0$

$C_z = 87.0\text{ N}$ **Ans.**

$\Sigma M_z = 0;$ $\quad (50 + 58.0)(0.2) - C_y(0.75) = 0$

$C_y = 28.8\text{ N}$ **Ans.**




$\Sigma F_x = 0;$ $\quad D_x = 0$ **Ans.**

$\Sigma F_y = 0;$ $\quad D_y + 28.8 - 50 - 58.0 = 0$

$D_y = 79.2\text{ N}$ **Ans.**

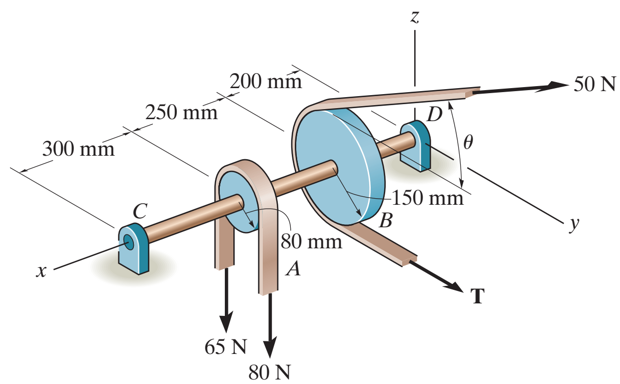
$\Sigma F_z = 0;$ $\quad D_z + 87.0 - 80 - 65 = 0$

$D_z = 58.0\text{ N}$ **Ans.**

**Ans:**
$T = 58.0\text{ N}$
$C_z = 87.0\text{ N}$
$C_y = 28.8\text{ N}$
$D_x = 0$
$D_y = 79.2\text{ N}$
$D_z = 58.0\text{ N}$

***4–40.**

Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley $A$ is transmitted to pulley $B$. Determine the horizontal tension $\mathbf{T}$ in the belt on pulley $B$ and the $x, y, z$ components of reaction at the journal bearing $C$ and thrust bearing $D$ if $\theta = 45°$. The bearings are in proper alignment and exert only force reactions on the shaft.




## SOLUTION

***Equations of Equilibrium:***

$\Sigma M_x = 0;\quad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

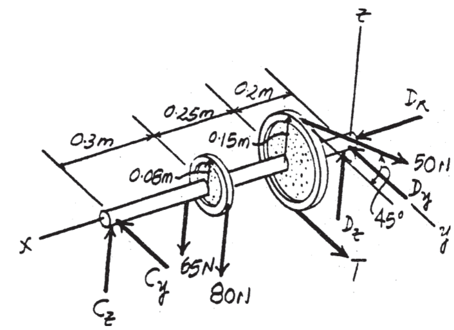
$T = 58.0\text{ N}$ **Ans.**

$\Sigma M_y = 0;\quad (65 + 80)(0.45) - 50\sin 45°(0.2) - C_z(0.75) = 0$

$C_z = 77.57\text{ N} = 77.6\text{ N}$ **Ans.**

$\Sigma M_z = 0;\quad 58.0(0.2) + 50\cos 45°(0.2) - C_y(0.75) = 0$

$C_y = 24.89\text{ N} = 24.9\text{ N}$ **Ans.**




$\Sigma F_x = 0;\quad D_x = 0$ **Ans.**

$\Sigma F_y = 0;\quad D_y + 24.89 - 50\cos 45° - 58.0 = 0$

$D_y = 68.5\text{ N}$ **Ans.**

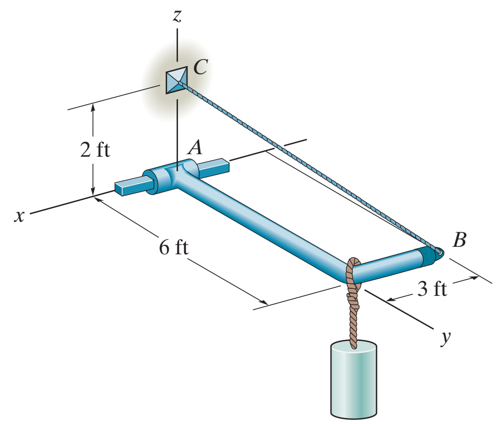
$\Sigma F_z = 0;\quad D_z + 77.57 + 50\sin 45° - 80 - 65 = 0$

$D_z = 32.1\text{ N}$ **Ans.**

**Ans:**
$T = 58.0\text{ N}$
$C_z = 77.6\text{ N}$
$C_y = 24.9\text{ N}$
$D_x = 0$
$D_y = 68.5\text{ N}$
$D_z = 32.1\text{ N}$

**4–41.**

Member $AB$ is supported by a cable $BC$ and at $A$ by a *square* rod which fits loosely through the square hole at the end collar of the member as shown. Determine the $x, y, z$ components of reaction at $A$ and the tension in the cable needed to hold the 800-lb cylinder in equilibrium.




## SOLUTION

$$\mathbf{F}_{BC} = F_{BC}\left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$\Sigma F_x = 0;$ $\quad F_{BC}\left(\frac{3}{7}\right) = 0$

$F_{BC} = 0$ **Ans.**

$\Sigma F_y = 0;$ $\quad A_y = 0$ **Ans.**

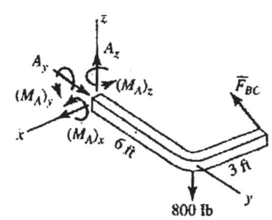
$\Sigma F_z = 0;$ $\quad A_z = 800 \text{ lb}$ **Ans.**

$\Sigma M_x = 0;$ $\quad (M_A)_x - 800(6) = 0$

$(M_A)_x = 4.80 \text{ kip} \cdot \text{ft}$ **Ans.**

$\Sigma M_y = 0;$ $\quad (M_A)_y = 0$ **Ans.**

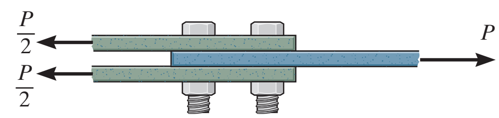
$\Sigma M_z = 0;$ $\quad (M_A)_z = 0$ **Ans.**




**Ans:**
$F_{BC} = 0$
$A_y = 0$
$A_z = 800 \text{ lb}$
$(M_A)_x = 4.80 \text{ kip} \cdot \text{ft}$
$(M_A)_y = 0$
$(M_A)_z = 0$

**4–42.**

Determine the maximum force $P$ the connection can support so that no slipping occurs between the plates. There are four bolts used for the connection and each is tightened so that it is subjected to a tension of 4 kN. The coefficient of static friction between the plates is $\mu_s = 0.4$.

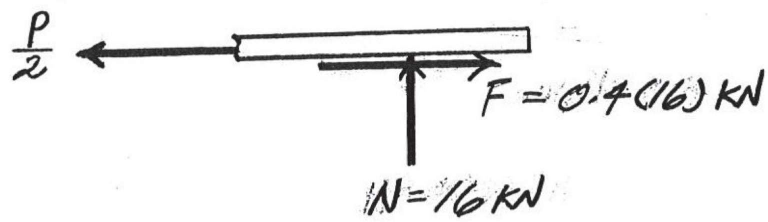



## SOLUTION

***Free-Body Diagram:*** The normal reaction acting on the contacting surface is equal to the sum total tension of the bolts. Thus, $N = 4(4)\text{ kN} = 16\text{ kN}$. When the plate is on the verge of slipping, the magnitude of the friction force acting on each contact surface can be computed using the friction formula $F = \mu_s N = 0.4(16)\text{ kN}$. As indicated on the free-body diagram of the upper plate, **F** acts to the right since the plate has a tendency to move to the left.

***Equations of Equilibrium:***

$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad 0.4(16) - \dfrac{P}{2} = 0 \qquad P = 12.8\text{ kN}$ **Ans.**

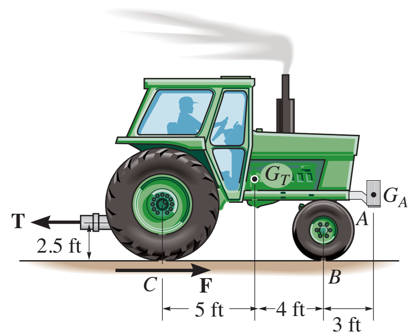



(a)

**Ans:**
$P = 12.8\text{kN}$

**4–43.**

The tractor exerts a towing force $T = 400$ lb. Determine the normal reactions at each of the two front and two rear tires and the tractive frictional force **F** on each rear tire needed to pull the load forward at constant velocity. The tractor has a weight of 7500 lb and a center of gravity located at $G_T$. An additional weight of 600 lb is added to its front having a center of gravity at $G_A$. Take $\mu_s = 0.4$. The front wheels are free to roll.




## SOLUTION

***Equations of Equilibrium:***

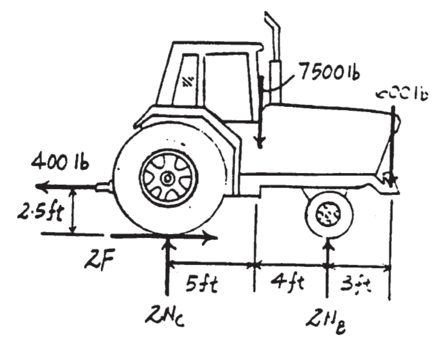
$\circlearrowleft + \Sigma M_C = 0 \qquad 2N_B(9) + 400(2.5) - 7500(5) - 600(12) = 0$

$N_B = 2427.78 \text{ lb} = 2.43 \text{ kip}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 2N_C + 2(2427.78) - 7500 - 600 = 0$

$N_C = 1622.22 \text{ lb} = 1.62 \text{ kip}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F - 400 = 0 \qquad F = 200 \text{ lb}$ **Ans.**

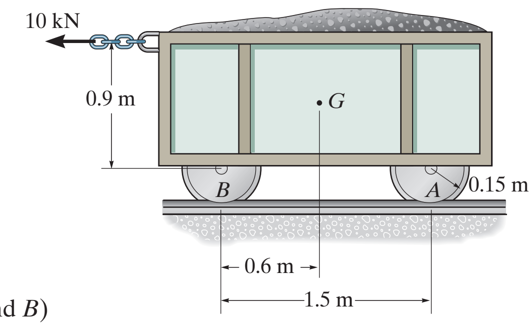



***Friction:*** The maximum friction force that can be developed between each of the rear tires and the ground is $F_{max} = \mu_s N_C = 0.4(1622.22) = 648.89$ lb. Since $F_{max} > F = 200$ lb, the rear tires will not slip. Hence, the tractor is capable of towing the 400-lb load.

**Ans:**
$N_B = 2.43$ kip
$N_C = 1.62$ kip
$F = 200$ lb

***4–44.**

The mine car and its contents have a total mass of 6 Mg and a center of gravity at $G$. If the coefficient of static friction between the wheels and the tracks is $\mu_s = 0.4$ when the wheels are locked, find the normal force acting on the front wheels at $B$ and the rear wheels at $A$ when the brakes at both $A$ and $B$ are locked. Does the car move?




## SOLUTION

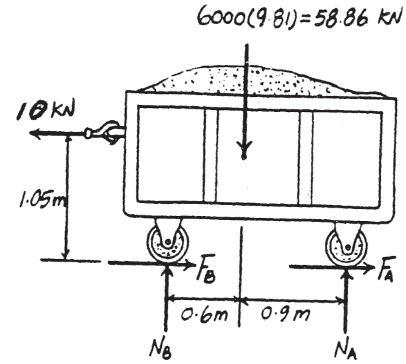
***Equations of Equilibrium:*** The normal reactions acting on the wheels at ($A$ and $B$) are independent as to whether the wheels are locked or not. Hence, the normal reactions acting on the wheels are the same for both cases.

$\zeta + \Sigma M_B = 0; \qquad N_A(1.5) + 10(1.05) - 58.86(0.6) = 0$

$N_A = 16.544 \text{ kN} = 16.5 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad N_B + 16.544 - 58.86 = 0$

$N_B = 42.316 \text{ kN} = 42.3 \text{ kN}$ **Ans.**

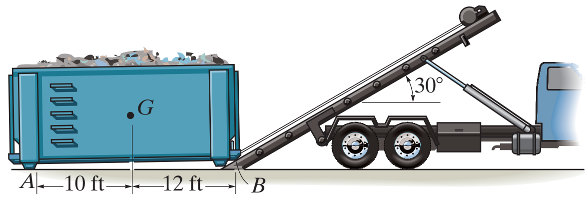



When both wheels at $A$ and $B$ are locked, then $(F_A)_{max} = \mu_s N_A = 0.4(16.544) = 6.6176$ kN and $(F_B)_{max} = \mu_s N_B = 0.4(42.316) = 16.9264$ kN. Since $(F_A)_{max} + (F_B)_{max} = 23.544 \text{ kN} > 10$ kN, the wheels do not slip. Thus, **the mine car does not move**. **Ans.**

**Ans:**
$N_A = 16.5$ kN
$N_B = 42.3$ kN
It does not move.

**4–45.**

The winch on the truck is used to hoist the garbage bin onto the bed of the truck. If the loaded bin has a weight of 8500 lb and center of gravity at $G$, determine the force in the cable needed to begin the lift. The coefficients of static friction at $A$ and $B$ are $\mu_A = 0.3$ and $\mu_B = 0.2$, respectively. Neglect the height of the support at $A$.




## SOLUTION

$\zeta + \Sigma M_B = 0; \quad 8500(12) - N_A(22) = 0$

$N_A = 4636.364 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad T \cos 30^\circ$

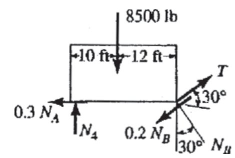
$- 0.2N_B \cos 30^\circ - N_B \sin 30^\circ - 0.3(4636.364) = 0$

$T(0.86603) - 0.67321\, N_B = 1390.91$

$+\uparrow \Sigma F_y = 0; \quad 4636.364 - 8500 + T \sin 30^\circ + N_B \cos 30^\circ$

$- 0.2N_B \sin 30^\circ = 0$

$T(0.5) + 0.766025\, N_B = 3863.636$




Solving:

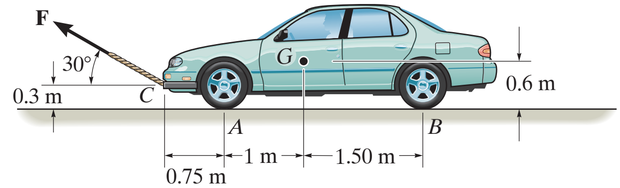
$T = 3666.5 \text{ lb} = 3.67 \text{ kip}$ **Ans.**

$N_B = 2650.6 \text{ lb}$

**Ans:**
$T = 3.67 \text{ kip}$

**4–46.**

The automobile has a mass of 2 Mg and center of mass at $G$. Determine the towing force $\mathbf{F}$ required to move the car if the back brakes are locked, and the front wheels are free to roll. Take $\mu_s = 0.3$.




## SOLUTION

***Equations of Equilibrium.*** Referring to the *FBD* of the car shown in Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0; \quad F_B - F\cos 30° = 0$ **(1)**

$+\uparrow \Sigma F_y = 0; \quad N_A + N_B + F\sin 30° - 2000(9.81) = 0$ **(2)**

$\circlearrowleft + \Sigma M_A = 0; \quad F\cos 30°(0.3) - F\sin 30°(0.75) + N_B(2.5) - 2000(9.81)(1) = 0$ **(3)**

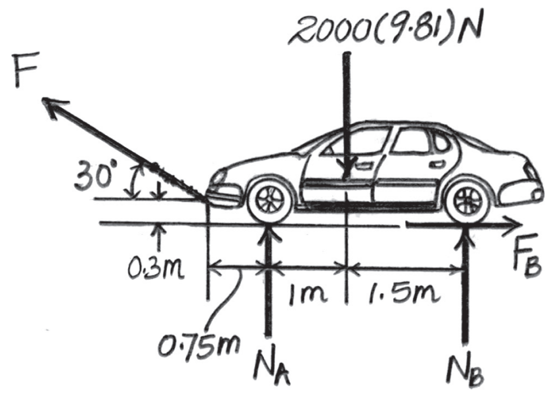
***Friction.*** It is required that the rear wheels are on the verge to slip. Thus,

$F_B = \mu_s N_B = 0.3\,N_B$ **(4)**

Solving Eqs. (1) to (4),

$F = 2{,}762.72\text{ N} = 2.76\text{ kN}$ **Ans.**

$N_B = 7975.30\text{ N} \quad N_A = 10{,}263.34\text{ N} \quad F_B = 2392.59\text{ N}$

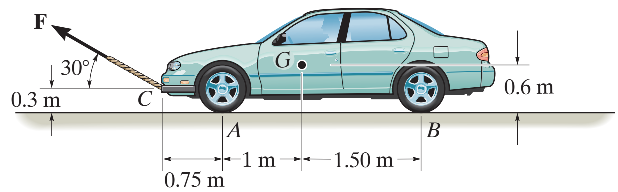



(a)

**Ans:**
$F = 2.76$ kN

**4–47.**

The automobile has a mass of 2 Mg and center of mass at $G$. Determine the towing force $\mathbf{F}$ required to move the car. Both the front and rear brakes are locked. Take $\mu_s = 0.3$.




## SOLUTION

***Equations of Equilibrium.*** Referring to the *FBD* of the car shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - F\cos 30^\circ = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F\sin 30^\circ + N_A + N_B - 2000(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F\cos 30^\circ(0.3) - F\sin 30^\circ(0.75) + N_B(2.5) - 200(9.81)(1) = 0 \qquad \textbf{(3)}$$

***Friction.*** It is required that both the front and rear wheels are on the verge to slip. Thus,

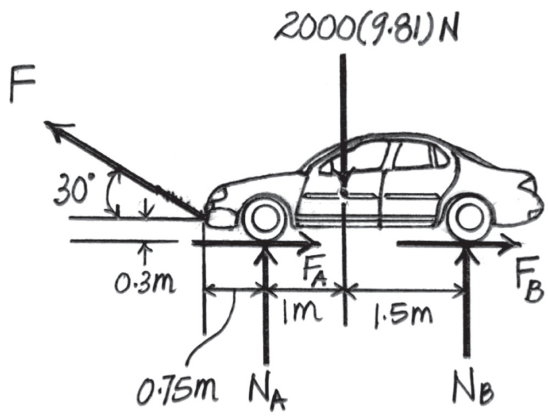
$$F_A = \mu_s N_A = 0.3\,N_A \qquad \textbf{(4)}$$

$$F_B = \mu_s N_B = 0.3\,N_B \qquad \textbf{(5)}$$

Solving Eqs. (1) to (5),

$$F = 5793.16\text{ N} = 5.79\text{ kN} \qquad \textbf{Ans.}$$

$$N_B = 8114.93\text{ N} \quad N_A = 8608.49\text{ N} \quad F_A = 2582.55\text{ N} \quad F_B = 2434.48\text{ N}$$

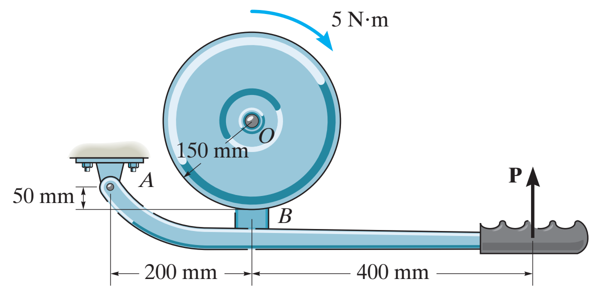



(a)

**Ans:**
$F = 5.79$ kN

***4–48.**

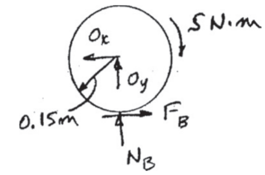
The block brake consists of a pin-connected lever and friction block at $B$. The coefficient of static friction between the wheel and the lever is $\mu_s = 0.3$, and a torque of $5\ \text{N}\cdot\text{m}$ is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) $P = 30$ N, (b) $P = 70$ N.




## SOLUTION

To hold lever,

$\zeta + \Sigma M_O = 0; \qquad F_B(0.15) - 5 = 0; \qquad F_B = 33.333\ \text{N}$

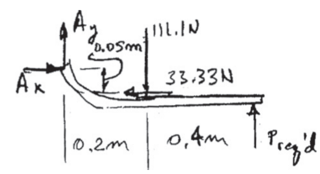



Require

$$N_B = \frac{33.333\ \text{N}}{0.3} = 111.1\ \text{N}$$

Lever:

$\zeta + \Sigma M_A = 0; \qquad P_{\text{Reqd.}}(0.6) - 111.1(0.2) - 33.333(0.05) = 0$




$P_{\text{Reqd.}} = 39.8\ \text{N}$

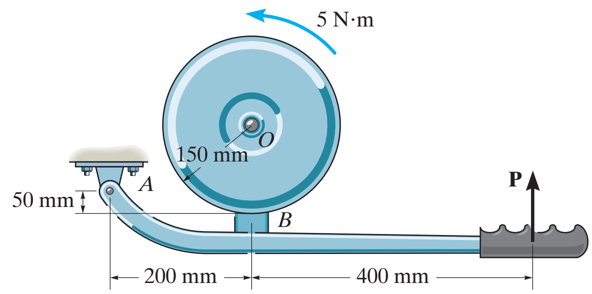
(a) $P = 30\ \text{N} < 39.8\ \text{N}$ No **Ans.**

(b) $P = 70\ \text{N} > 39.8\ \text{N}$ Yes **Ans.**

**Ans:**
a) No
b) Yes

**4–49.**

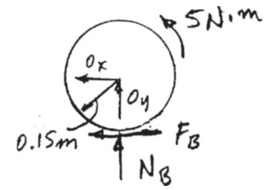
The block brake consists of a pin-connected lever and friction block at $B$. The coefficient of static friction between the wheel and the lever is $\mu_s = 0.3$, and a torque of $5\ \text{N}\cdot\text{m}$ is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) $P = 30$ N, (b) $P = 70$ N.




## SOLUTION

To hold lever,

$$\circlearrowleft + \Sigma M_O = 0; \qquad -F_B(0.15) + 5 = 0; \qquad F_B = 33.333\ \text{N}$$



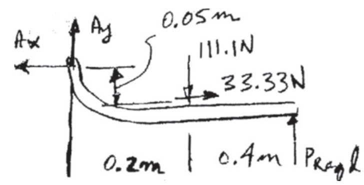

Require

$$N_B = \frac{33.333\ \text{N}}{0.3} = 111.1\ \text{N}$$

Lever:

$$\circlearrowleft + \Sigma M_A = 0; \qquad P_{\text{Reqd.}}(0.6) - 111.1(0.2) + 33.333(0.05) = 0$$

$$P_{\text{Reqd.}} = 34.26\ \text{N}$$



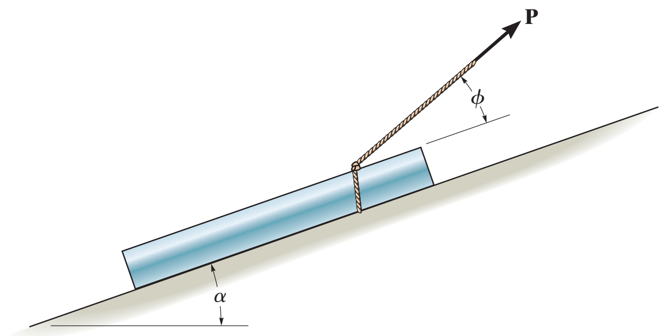

(a) $P = 30\ \text{N} < 34.26\ \text{N}$ No **Ans.**

(b) $P = 70\ \text{N} > 34.26\ \text{N}$ Yes **Ans.**

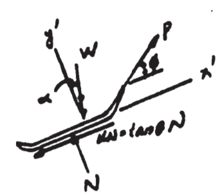
**Ans:**
a) No
b) Yes

**4–50.**

The pipe of weight $W$ is to be pulled up the inclined plane of slope $\alpha$ using a force $\mathbf{P}$. If $\mathbf{P}$ acts at an angle $\phi$, show that for slipping $P = W\ \sin(\alpha + \theta)/\cos(\phi - \theta)$, where $\theta$ is the angle of static friction; $\theta = \tan^{-1}\mu_s$.




## SOLUTION




$+\nwarrow\Sigma F_{y'} = 0;\qquad N + P\sin\phi - W\cos\alpha = 0\qquad N = W\cos\alpha - P\sin\phi$

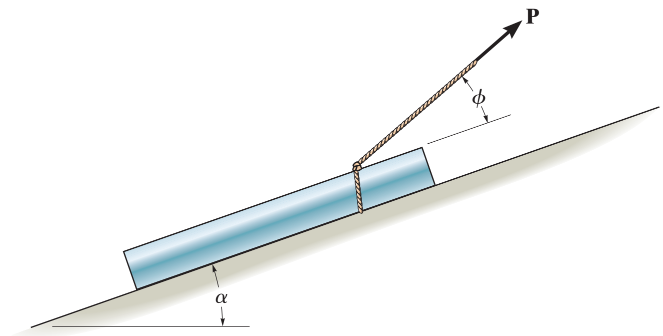
$+\nearrow\Sigma F_{x'} = 0;\qquad P\cos\phi - W\sin\alpha - \tan\theta(W\cos\alpha - P\sin\phi) = 0$

$$P = \frac{W(\sin\alpha + \tan\theta\cos\alpha)}{\cos\phi + \tan\theta\sin\phi}$$

$$= \frac{W(\cos\theta\sin\alpha + \sin\theta\cos\alpha)}{\cos\phi\cos\theta + \sin\phi\sin\theta} = \frac{W\sin(\alpha + \theta)}{\cos(\phi - \theta)}\qquad \textbf{Q.E.D.}$$

**4–51.**

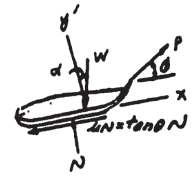
Determine the angle $\phi$ at which the applied force **P** should act on the pipe so that the magnitude of **P** is as small as possible for pulling the pipe up the incline. What is the corresponding value of $P$? The pipe weighs $W$ and the slope $\alpha$ is known. Express the answer in terms of the angle of kinetic friction, $\theta = \tan^{-1} \mu_k$.




## SOLUTION

$+\nwarrow\Sigma F_{y'} = 0; \qquad N + P\sin\phi - W\cos\alpha = 0 \qquad N = W\cos\alpha - P\sin\phi$

$+\nearrow\Sigma F_{x'} = 0; \qquad P\cos\phi - W\sin\alpha - \tan\theta(W\cos\alpha - P\sin\phi) = 0$




$$P = \frac{W(\sin\alpha + \tan\theta\cos\alpha)}{\cos\phi + \tan\theta\sin\phi}$$

$$= \frac{W(\cos\theta\sin\alpha + \sin\theta\cos\alpha)}{\cos\phi\cos\theta + \sin\phi\sin\theta}$$

$$= \frac{W\sin(\alpha + \theta)}{\cos(\phi - \theta)}$$

$$\frac{dP}{d\phi} = \frac{W\sin(\alpha + \theta)\sin(\phi - \theta)}{\cos^2(\phi - \theta)} = 0$$

$W\sin(\alpha + \theta)\sin(\phi - \theta) = 0 \qquad W\sin(\alpha + \theta) = 0$

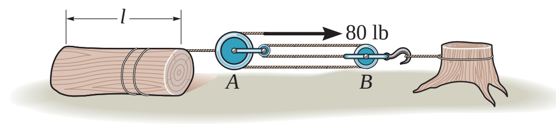
$\sin(\phi - \theta) = 0 \qquad \phi - \theta = 0 \qquad \phi = \theta$ **Ans.**

$P = \dfrac{W\sin(\alpha + \theta)}{\cos(\theta - \theta)} = W\sin(\alpha + \theta)$ **Ans.**

**Ans:**
$\phi = \theta$
$P = W\sin(\alpha + \theta)$

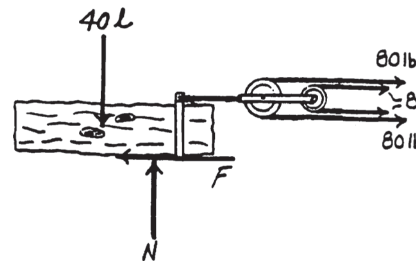
***4–52.**

The log has a coefficient of static friction of $\mu_s = 0.3$ with the ground and a weight of 40 lb/ft. If a man can pull on the rope with a maximum force of 80 lb, determine the greatest length $l$ of log he can drag.




## SOLUTION

***Equations of Equilibrium:***




$+\uparrow \Sigma F_y = 0; \quad N - 40l = 0 \quad N = 40l$

$\xrightarrow{+} \Sigma F_x = 0; \quad 4(80) - F = 0 \quad F = 320 \text{ lb}$

***Friction:*** Since the log slides,

$$F = (F)_{\max} = \mu_s N$$

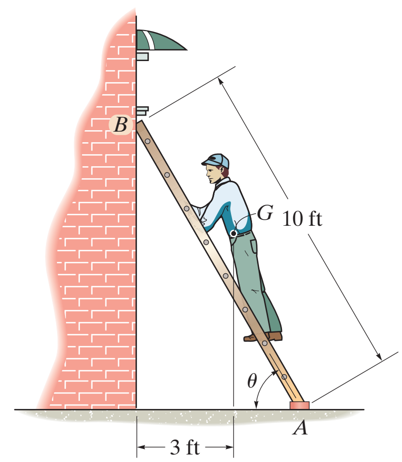
$$320 = 0.3(40l)$$

$$l = 26.7 \text{ ft}$$ **Ans.**

**Ans:**
$l = 26.7$ ft

**4–53.**

The 180-lb man climbs up the ladder and stops at the position shown after he senses that the ladder is on the verge of slipping. Determine the inclination $\theta$ of the ladder if the coefficient of static friction between the friction pad $A$ and the ground is $\mu_s = 0.4$. Assume the wall at $B$ is smooth. The center of gravity for the man is at $G$. Neglect the weight of the ladder.




## SOLUTION

**Free-Body Diagram.** Since the weight of the man tends to cause the friction pad $A$ to slide to the right, the frictional force $\mathbf{F}_A$ must act to the left as indicated on the free-body diagram of the ladder, Fig. *a*. Here, the ladder is on the verge of slipping. Thus, $F_A = \mu_s N_A$.

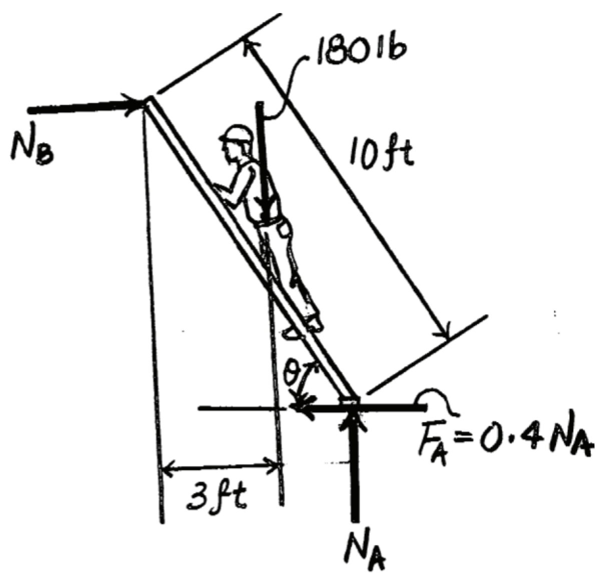
**Equations of Equilibrium.**

$+\uparrow \Sigma F_y = 0;\qquad N_A - 180 = 0\qquad N_A = 180 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0;\qquad 180(10\cos\theta°) - 0.4(180)(10\sin\theta°) - 180(3) = 0$

$\cos\theta - 0.4\sin\theta = 0.3$

$\theta = 52.0°$ **Ans.**

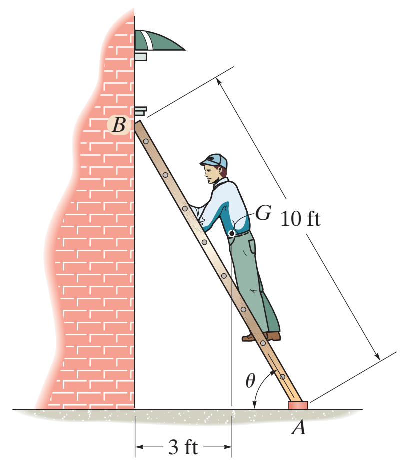



(a)

**Ans:**
$\theta = 52.0°$

**4–54.**

The 180-lb man climbs up the ladder and stops at the position shown after he senses that the ladder is on the verge of slipping. Determine the coefficient of static friction between the friction pad at $A$ and ground if the inclination of the ladder is $\theta = 60°$ and the wall at $B$ is smooth. The center of gravity for the man is at $G$. Neglect the weight of the ladder.




## SOLUTION

**Free-Body Diagram.** Since the weight of the man tends ot cause the friction pad $A$ to slide to the right, the frictional force $\mathbf{F}_A$ must act to the left as indicated on the free-body diagram of the ladder, Fig. *a*. Here, the ladder is on the verge of slipping. Thus, $F_A = \mu_s N_A$.

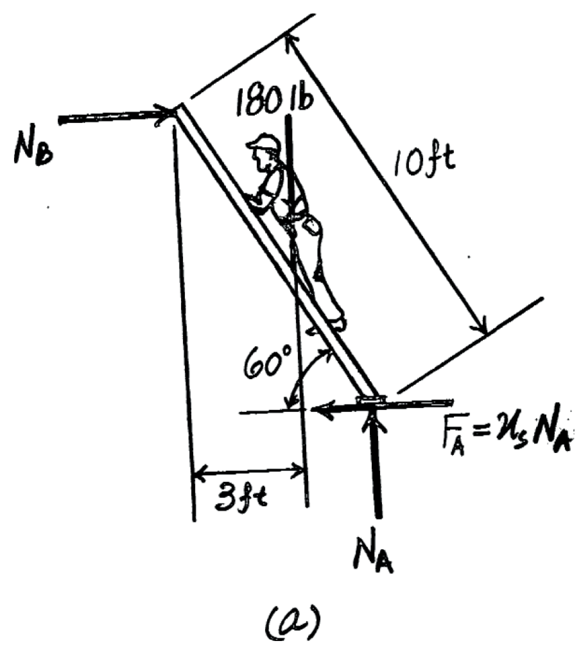
**Equations of Equilibrium.**

$+\uparrow \Sigma F_y = 0;$ $\quad N_A - 180 = 0 \quad N_A = 180 \text{ lb}$

$\zeta + \Sigma M_B = 0;$ $\quad 180(10 \cos 60°) - \mu_s(180)(10 \sin 60°) - 180(3) = 0$

$$180 \cos \theta - 72 \sin \theta = 54$$

$\mu_s = 0.231$ **Ans.**

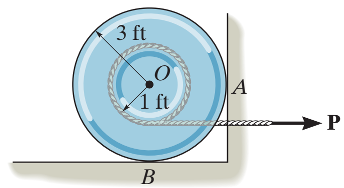


(a)

**Ans:**
$\mu_s = 0.231$

**4–55.**

The spool of wire having a weight of 300 lb rests on the ground at $B$ and against the wall at $A$. Determine the force $P$ required to begin pulling the wire horizontally off the spool. The coefficient of static friction between the spool and its points of contact is $\mu_s = 0.25$.




## SOLUTION

***Equations of Equilibrium.*** Referring to the *FBD* of the spool shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad P - N_A - F_B = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - F_A - 300 = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad P(1) - F_B(3) - F_A(3) = 0 \qquad \textbf{(3)}$$

***Frictions.*** It is required that slipping occurs at $A$ and $B$. Thus,

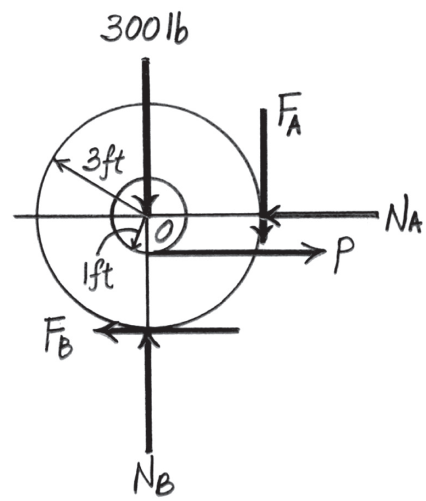
$$F_A = \mu N_A = 0.25\, N_A \qquad \textbf{(4)}$$

$$F_B = \mu N_B = 0.25\, N_B \qquad \textbf{(5)}$$

Solving Eqs. (1) to (5),

$$P = 1350 \text{ lb} \qquad \textbf{Ans.}$$

$$N_A = 1200 \text{ lb} \quad N_B = 600 \text{ lb} \quad F_A = 300 \text{ lb} \quad F_B = 150 \text{ lb}$$

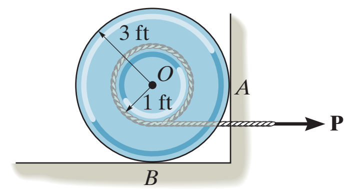



(a)

**Ans:**
$P = 1350$ lb

***4–56.**

The spool of wire having a weight of 300 lb rests on the ground at $B$ and against the wall at $A$. Determine the normal force acting on the spool at $A$ if $P = 300$ lb. The coefficient of static friction between the spool and the ground at $B$ is $\mu_s = 0.35$. The wall at $A$ is smooth.




## SOLUTION

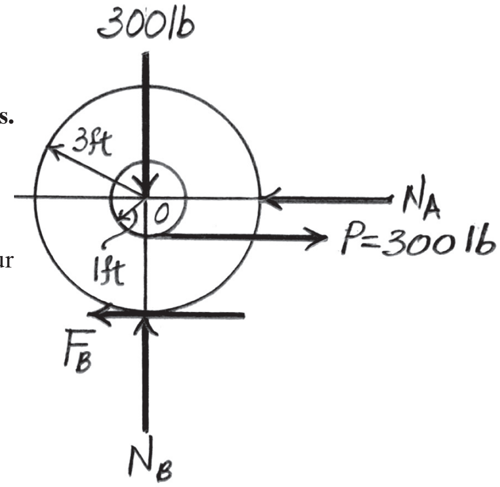
***Equations of Equilibrium.*** Referring to the *FBD* of the spool shown in Fig. *a*,

$\curvearrowleft + \Sigma M_B = 0; \quad N_A(3) - 300(2) = 0 \quad N_A = 200 \text{ lb}$ **Ans.**

$\curvearrowleft + \Sigma M_O = 0; \quad 300(1) - F_B(3) = 0 \quad F_B = 100 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad N_B - 300 = 0 \quad N_B = 300 \text{ lb}$

***Friction.*** Since $F_B < (F_B)_{\max} = \mu_s N_B = 0.35(300) = 105$ lb, slipping will not occur at $B$. Thus, the spool will remain at rest.

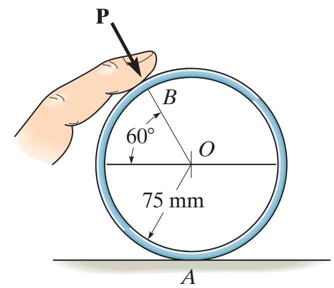



(a)

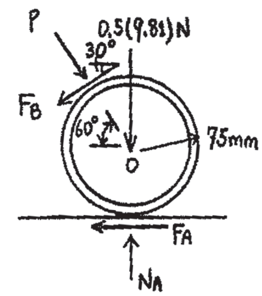
**Ans:**
$N_A = 200$ lb

**4–57.**

The ring has a mass of 0.5 kg and is resting on the surface of the table. To move the ring a normal force **P** from the finger is exerted on it as shown. Determine its magnitude when the ring is on the verge of slipping at $A$. The coefficient of static friction at $A$ is $\mu_A = 0.2$ and at $B$, $\mu_B = 0.3$.




## SOLUTION




$F_A = F_B$

$P\cos 60° - F_B\cos 30° - F_A = 0$

$N_A - 0.5(9.81) - P\sin 60° - F_B\sin 30° = 0$

$F_A = 0.2\,N_A$

$N_A = 19.34\text{ N}$

$F_A = F_B = 3.868\text{ N}$

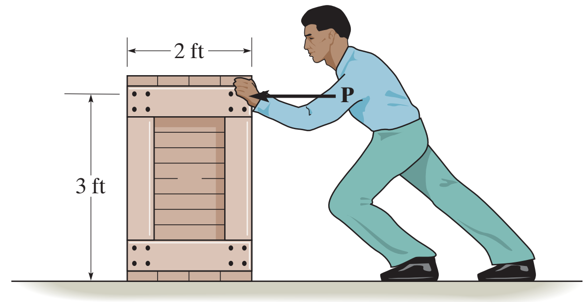
$P = 14.4\text{ N}$ **Ans.**

$(F_B)_{\max} = 0.3(14.44) = 4.33\text{ N} > 3.868\text{ N}$ **(O.K!)**

**Ans:**
$P = 14.4\text{ N}$

**4–58.**

Determine the smallest force $P$ that must be applied in order to cause the 150-lb uniform crate to move. The coefficent of static friction between the crate and the floor is $\mu_s = 0.5$.




## SOLUTION

***Equations of Equilibrium.*** Referring to the *FBD* of the crate shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F - P = 0 \qquad (1)$$

$$+\uparrow\Sigma F_y = 0; \qquad N - 150 = 0 \quad N = 150 \text{ lb}$$

$$\circlearrowleft+\Sigma M_O = 0; \qquad P(3) - 150x = 0 \qquad (2)$$

***Friction.*** Assuming that the crate slides before tipping. Thus,

$$F = \mu N = 0.5(150) = 75 \text{ lb}$$

Substitute this value into Eq. (1).

$$P = 75 \text{ lb}$$

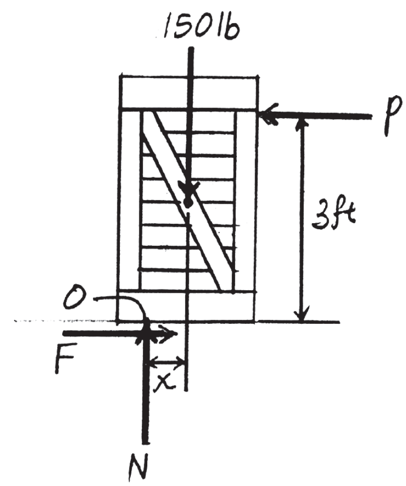
Then Eq. (2) gives

$$75(3) - 150x = 0 \quad x = 1.5 \text{ ft}$$

Since $x > 1$ ft, the crate tips before sliding. Thus, the assumption was wrong. Substitute $x = 1$ ft into Eq. (2).

$$P(3) - 150(1) = 0$$

$$P = 50 \text{ lb} \qquad \textbf{Ans.}$$

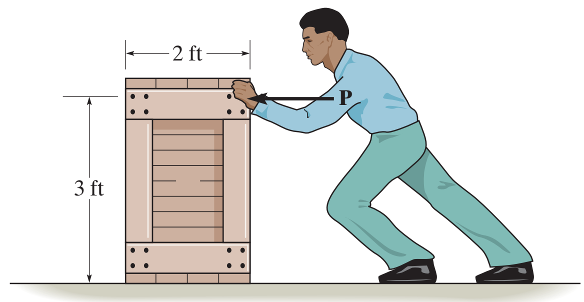


(a)

**Ans:**
$P = 50$ lb

**4–59.**

The man having a weight of 200 lb pushes horizontally on the crate. If the coefficient of static friction between the 450-lb crate and the floor is $\mu_s = 0.3$ and between his shoes and the floor is $\mu'_s = 0.6$, determine if he can move the crate.




## SOLUTION

***Equations of Equilibrium.*** Referring to the *FBD* of the crate shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_C - P = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_C - 450 = 0 \quad N_C = 450 \text{ lb}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad P(3) - 450(x) = 0 \qquad \textbf{(2)}$$

Also, from the *FBD* of the man, Fig. *b*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_m = 0 \qquad \textbf{(3)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_m - 200 = 0 \quad N_m = 200 \text{ lb}$$

***Friction.*** Assuming that the crate slides before tipping. Thus,

$$F_C = \mu_s N_C = 0.3(450) = 135 \text{ lb}$$

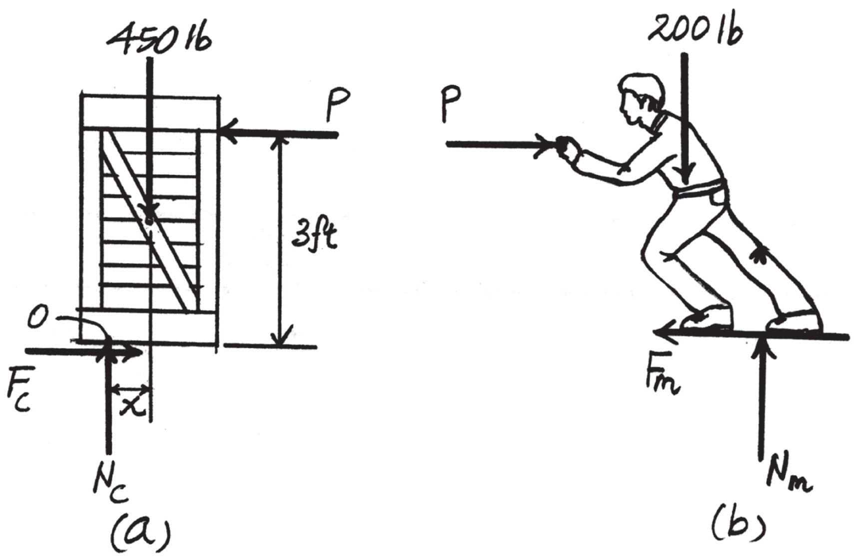
Using this result to solve Eqs. (1), (2) and (3),

$$F_m = P = 135 \text{ lb} \quad x = 0.9 \text{ ft}$$

Since $x < 1$ ft, the crate indeed slides before tipping as assumed.

Also, since $F_m > (F_m)_{\max} = \mu_s' N_C = 0.6(200) = 120$ lb, the man slips.

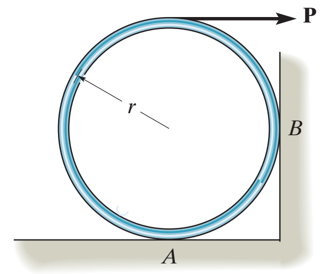
Thus **he is not able to move the crate.**




**Ans:**
No

***4–60.**

The uniform hoop of weight $W$ is subjected to the horizontal force $P$. Determine the coefficient of static friction between the hoop and the surface at $A$ and $B$ if the hoop is on the verge of rotating.



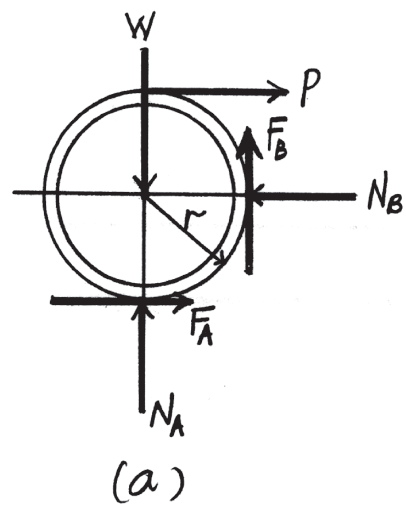

## SOLUTION

***Equations of Equilibrium.*** Referring to the *FBD* of the hoop shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P + F_A - N_B = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A + F_B - W = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(r) + F_B\,(r) - P(2r) = 0 \qquad \textbf{(3)}$$




***Friction.*** It is required that slipping occurs at point $A$ and $B$. Thus,

$$F_A = \mu_s N_A \qquad \textbf{(4)}$$

$$F_B = \mu_s N_B \qquad \textbf{(5)}$$

Substituting Eq. (5) into (3),

$$N_B\,r + \mu_s N_B\,r = 2Pr \quad N_B = \frac{2P}{1 + \mu_s} \qquad \textbf{(6)}$$

Substituting Eq. (4) into (1) and Eq. (5) into (2), we obtain

$$N_B - \mu_s N_A = P \qquad \textbf{(7)}$$

$$N_A + \mu_s N_B = W \qquad \textbf{(8)}$$

Eliminate $\mathbf{N}_A$ from Eqs. (7) and (8),

$$N_B = \frac{P + \mu_s W}{1 + \mu_s^2} \qquad \textbf{(9)}$$

Equating Eq. (6) and (9),

$$\frac{2P}{1 + \mu_s} = \frac{P + \mu_s W}{1 + \mu_s^2}$$

$$2P(1 + \mu_s^2) = (P + \mu_s W)(1 + \mu_s)$$

$$2P + 2\mu_s^2 P = P + P\mu_s + \mu_s W + \mu_s^2 W$$

$$(2P - W)\mu_s^2 - (P + W)\mu_s + P = 0$$

If $P = \dfrac{1}{2}W$, the quadratic term drops out, and then

$$\mu_s = \frac{P}{P + W}$$

$$= \frac{\frac{1}{2}W}{\frac{1}{2}W + W}$$

$$= \frac{1}{3} \qquad \textbf{Ans.}$$

***4–60. Continued**

If $P \neq \frac{1}{2}W$, then

$$\mu_s = \frac{(P + W) \pm \sqrt{[-(P + W)]^2 - 4(2P - W)P}}{2(2P - W)}$$

$$\mu_s = \frac{(P + W) \pm \sqrt{W^2 + 6PW - 7P^2}}{2(2P - W)}$$

$$\mu_s = \frac{(P + W) \pm \sqrt{(W + 7P)(W - P)}}{2(2P - W)}$$

In order to have a solution,

$$(W + 7P)(W - P) > 0$$

Since $W + 7P > 0$, then

$$W - P > 0 \quad W > P$$

Also, $P > 0$. Thus,

$$0 < P < W$$

Choosing the smaller value of $\mu_s$,

$$\mu_s = \frac{(P + W) - \sqrt{(W + 7P)(W - P)}}{2(2P - W)} \text{ for } 0 < P < W \text{ and } P \neq \frac{W}{2} \qquad \textbf{Ans.}$$

The two solutions, for $P = \frac{1}{2}W$ and $P \neq \frac{1}{2}W$, are continuous.

Note: Choosing the larger value of $\mu_s$ in the quadratic solution leads to $N_A, F_A < 0$, which is nonphysical. Also, $(\mu_s)_{\max} = 1$. For $\mu_s > 1$, the hoop will tend to climb the wall rather than rotate in place.

**Ans:**

If $P = \frac{1}{2}W$,

$\mu_s = \frac{1}{3}$

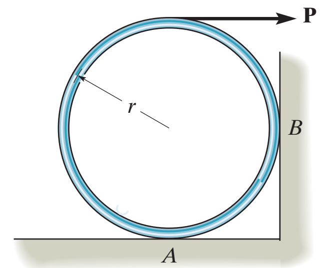
If $P \neq \frac{1}{2}W$,

$$\mu_s = \frac{(P + W) - \sqrt{(W + 7P)(W - P)}}{2(2P - W)}$$

for $0 < P < W$

**4–61.**

Determine the maximum horizontal force **P** that can be applied to the 30-lb hoop without causing it to rotate. The coefficient of static friction between the hoop and the surfaces $A$ and $B$ is $\mu_s = 0.2$. Take $r = 300$ mm.




## SOLUTION

***Equations of Equilibrium.*** Referring to the *FBD* of the hoop shown in Fig. *a*,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad P + F_A - N_B = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0; \quad N_A + F_B - 30 = 0 \qquad$ **(2)**

$\circlearrowleft + \Sigma M_A = 0; \quad F_B(0.3) + N_B(0.3) - P(0.6) = 0 \qquad$ **(3)**

***Friction.*** Assuming that the hoop is on the verge to rotate due to the slipping occur at $A$ and $B$. Then

$$F_A = \mu_s N_A = 0.2\, N_A \qquad \textbf{(4)}$$

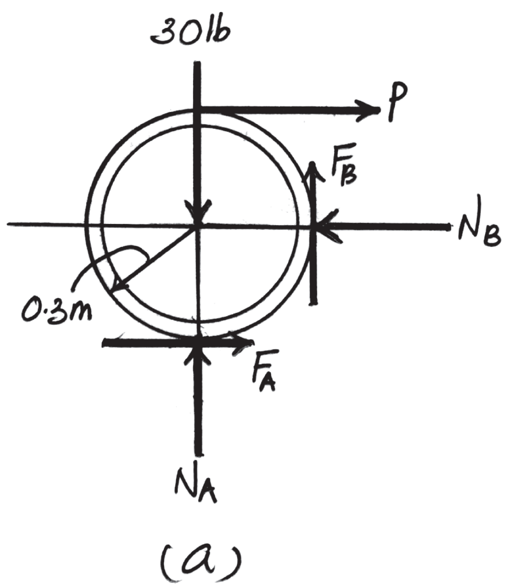
$$F_B = \mu_s N_B = 0.2\, N_B \qquad \textbf{(5)}$$

Solving Eq. (1) to (5),

$$N_A = 27.27 \text{ lb} \quad N_B = 13.64 \text{ lb} \quad F_A = 5.455 \text{ lb} \quad F_B = 2.727 \text{ lb}$$

$$P = 8.182 \text{ lb} = 8.18 \text{ lb}$$ **Ans.**

Since $N_A$ is positive, the hoop will be in contact with the floor. Thus, the assumption was correct.

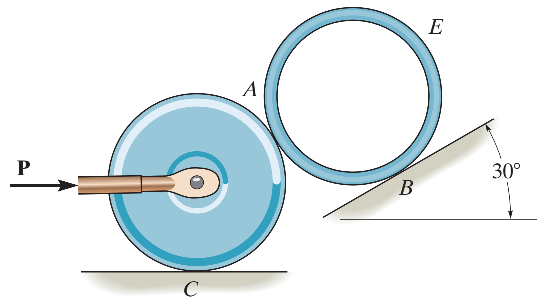



(a)

**Ans:**
$P = 8.18$ lb

**4–62.**

Determine the minimum force $P$ needed to push the tube $E$ up the incline. The coefficients of static friction at the contacting surfaces are $\mu_A = 0.2$, $\mu_B = 0.3$, and $\mu_C = 0.4$. The 100-kg roller and 40-kg tube each have a radius of 150 mm.




## SOLUTION

***Equations of Equilibrium.*** Referring to the *FBD* of the roller, Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad P - N_A\cos 30^\circ - F_A\sin 30^\circ - F_C = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad N_C + F_A\cos 30^\circ - N_A\sin 30^\circ - 100(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft +\Sigma M_D = 0; \qquad F_A(0.15) - F_C(0.15) = 0 \qquad \textbf{(3)}$$

Also, for the *FBD* of the tube, Fig. *b*,

$$+\nearrow\Sigma F_x = 0; \qquad N_A - F_B - 40(9.81)\sin 30^\circ = 0 \qquad \textbf{(4)}$$

$$+\nwarrow\Sigma F_y = 0; \qquad N_B - F_A - 40(9.81)\cos 30^\circ = 0 \qquad \textbf{(5)}$$

$$\circlearrowleft +\Sigma M_E = 0; \qquad F_A(0.15) - F_B(0.15) = 0 \qquad \textbf{(6)}$$

***Friction.*** Assuming that slipping is about to occur at $A$. Thus,

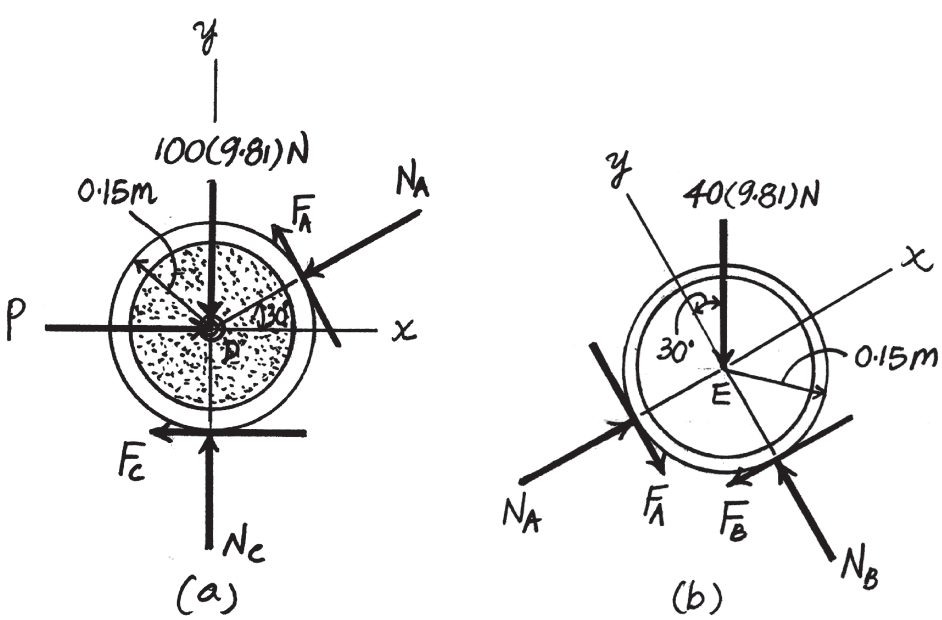
$$F_A = \mu_A N_A = 0.2\,N_A \qquad \textbf{(7)}$$

Solving Eqs. (1) to (7),

$$P = 285.97\text{ N} = 286\text{ N} \qquad \textbf{Ans.}$$

$N_A = 245.25$ N $\quad N_B = 388.88$ N $\quad N_C = 1061.15$ N $\quad F_A = F_B = F_C = 49.05$ N

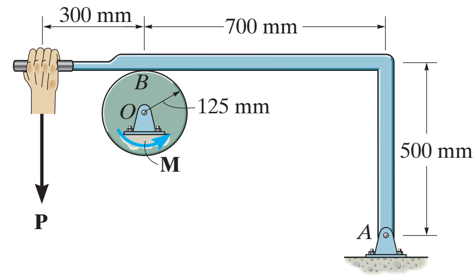
Since $F_B < (F_B)_{max} = \mu_B N_B = 0.3(388.88) = 116.66$ N and $F_C < (F_C)_{max} = \mu_C N_C = 0.4(1061.15) = 424.46$ N, slipping indeed will not occur at $B$ and $C$. Thus, the assumption was correct.




**Ans:**
$P = 286$ N

**4–63.**

The coefficients of static and kinetic friction between the drum and brake bar are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. If $M = 50\ \text{N}\cdot\text{m}$ and $P = 85$ N, determine the horizontal and vertical components of reaction at the pin $O$. Neglect the weight and thickness of the brake. The drum has a mass of 25 kg.




## SOLUTION

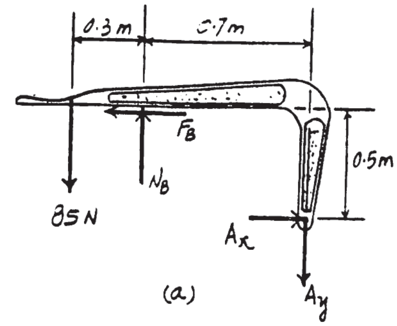
***Equations of Equilibrium:*** From FBD (b),

$\circlearrowleft + \Sigma M_O = 0 \qquad 50 - F_B(0.125) = 0 \qquad F_B = 400\text{ N}$

From FBD (a),

$\circlearrowleft + \Sigma M_A = 0; \qquad 85(1.00) + 400(0.5) - N_B(0.7) = 0$

$$N_B = 407.14\text{ N}$$

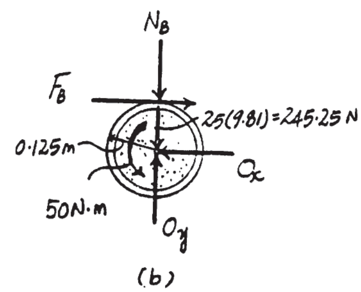



***Friction:*** Since $F_B > (F_B)_{\max} = \mu_s N_B = 0.4(407.14) = 162.86$ N, the drum slips at point $B$ and rotates. Therefore, the coefficient of kinetic friction should be used. Thus, $F_B = \mu_k N_B = 0.3N_B$.

***Equations of Equilibrium:*** From FBD (b),

$\circlearrowleft + \Sigma M_A = 0; \qquad 85(1.00) + 0.3N_B(0.5) - N_B(0.7) = 0$

$$N_B = 154.54\text{ N}$$




From FBD ($b$),

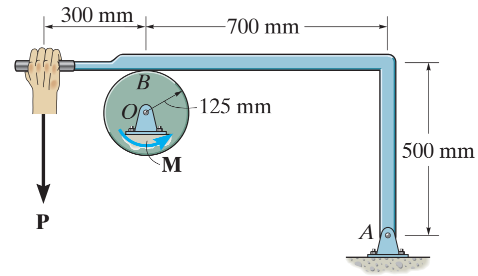
$+\uparrow \Sigma F_y = 0; \qquad O_y - 245.25 - 154.54 = 0 \qquad O_y = 400\text{ N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3(154.54) - O_x = 0 \qquad O_x = 46.4\text{ N}$ **Ans.**

**Ans:**
$O_y = 400$ N
$O_x = 46.4$ N

***4–64.**

The coefficient of static friction between the drum and brake bar is $\mu_s = 0.4$. If the moment $M = 35\ \text{N}\cdot\text{m}$, determine the smallest force $P$ that needs to be applied to the brake bar in order to prevent the drum from rotating. Also determine the corresponding horizontal and vertical components of reaction at pin $O$. Neglect the weight and thickness of the brake bar. The drum has a mass of 25 kg.




## SOLUTION

***Equations of Equilibrium:*** From FBD (b),

$$\circlearrowleft +\Sigma M_O = 0 \qquad 35 - F_B(0.125) = 0 \qquad F_B = 280\text{ N}$$

From FBD (a),

$$\circlearrowleft +\Sigma M_A = 0; \qquad P(1.00) + 280(0.5) - N_B(0.7) = 0$$

***Friction:*** When the drum is on the verge of rotating,

$$F_B = \mu_S N_B$$

$$280 = 0.4 N_B$$

$$N_B = 700\text{ N}$$

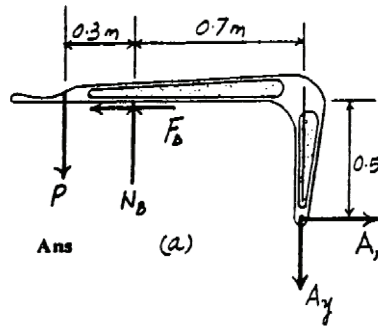
Substituting $N_B = 700$ N into Eq.[1] yields

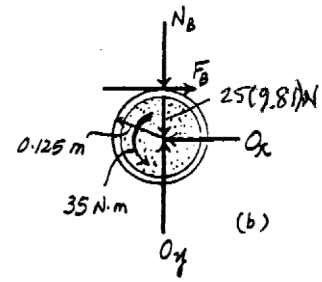
$$P = 350\text{ N} \qquad \textbf{Ans.}$$

***Equations of Equilibrium:*** From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad O_y - 245.25 - 700 = 0 \qquad O_y = 945\text{ N} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 280 - O_x = 0 \qquad O_x = 280\text{ N} \qquad \textbf{Ans.}$$

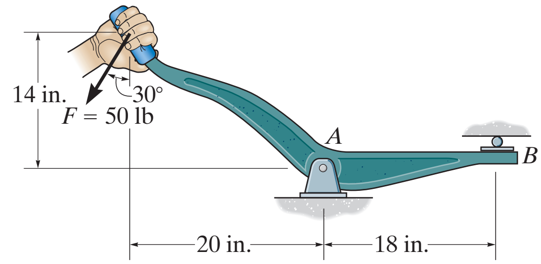



**Ans:**
$P = 350$ N
$O_y = 945$ N
$O_x = 280$ N

***R4–4.**

Determine the horizontal and vertical components of reaction at the pin $A$ and the reaction at the roller $B$ on the lever.




## SOLUTION

***Equations of Equilibrium:*** From the free-body diagram, $F_B$ and $A_x$ can be obtained by writing the moment equation of equilibrium about point $A$ and the force equation of equilibrium along the $x$ axis, respectively.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 50 \cos 30°(20) + 50 \sin 30°(14) - F_B(18) = 0$

$F_B = 67.56 \text{ lb} = 67.6 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 50 \sin 30° = 0$

$A_x = 25 \text{ lb}$ **Ans.**

Using the result $F_B = 67.56$ lb and writing the force equation of equilibrium along the $y$ axis, we have

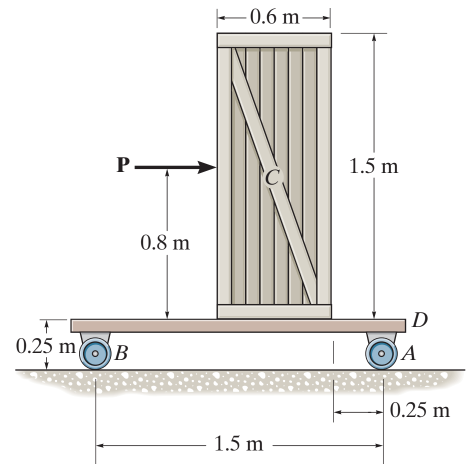
$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 50 \cos 30° - 67.56 = 0$

$A_y = 110.86 \text{ lb} = 111 \text{ lb}$ **Ans.**

**Ans:**
$F_B = 67.6$ lb
$A_x = 25$ lb
$A_y = 111$ lb

***R4–8.**

The uniform 60-kg crate $C$ rests uniformly on a 10-kg dolly $D$. If the front wheels of the dolly at $A$ are locked to prevent rolling while the wheels at $B$ are free to roll, determine the maximum force **P** that may be applied without causing motion of the crate. The coefficient of static friction between the wheels and the floor is $\mu_f = 0.35$ and between the dolly and the crate, $\mu_d = 0.5$.




## SOLUTION

***Equations of Equilibrium:*** From FBD (a),

$+\uparrow \Sigma F_y = 0; \qquad N_d - 588.6 = 0 \qquad N_d = 588.6 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_d = 0 \qquad$ **(1)**

$\circlearrowleft + \Sigma M_A = 0; \qquad 588.6(x) - P(0.8) = 0 \qquad$ **(2)**

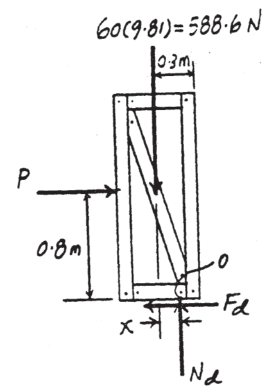
From FBD (b),

$+\uparrow \Sigma F_y = 0 \qquad N_B + N_A - 588.6 - 98.1 = 0 \qquad$ **(3)**

$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_A = 0 \qquad$ **(4)**

$\circlearrowleft + \Sigma M_B = 0; \qquad N_A(1.5) - P(1.05)$

$\qquad - 588.6(0.95) - 98.1(0.75) = 0 \qquad$ **(5)**




(a)

***Friction:*** Assuming the crate slips on dolly, then $F_d = \mu_{s\,d} N_d = 0.5(588.6) = 294.3$ N. Substituting this value into Eqs. (1) and (2) and solving, we have

$$P = 294.3 \text{ N} \qquad x = 0.400 \text{ m}$$

Since $x > 0.3$ m, the crate tips on the dolly. If this is the case $x = 0.3$ m. Solving Eqs. (1) and (2) with $x = 0.3$ m yields

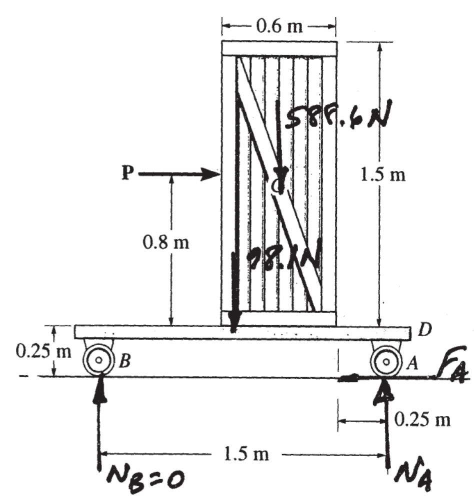
$$P = 220.725 \text{ N}$$

$$F_d = 220.725 \text{ N}$$

Assuming the dolly slips at $A$, then $F_A = \mu_{s\,f} N_A = 0.35 N_A$. Substituting this value into Eqs. (3), (4), and (5) and solving, we have

$$N_A = 559 \text{ N} \qquad N_B = 128 \text{ N}$$

$$P = 195.6 \text{ N} = 196 \text{ N} \quad (\textbf{\textit{Control!}})$$ **Ans.**




**Ans:**
$P = 196$ N