16–1.

An L2 steel strap having a thickness of 0.125 in. and a width of 2 in. is bent into a circular arc of radius 600 in. Determine the maximum bending stress in the strap.

SOLUTION

$$\frac{1}{\rho} = \frac{M}{EI}, \qquad M = \frac{EI}{\rho}$$

However,

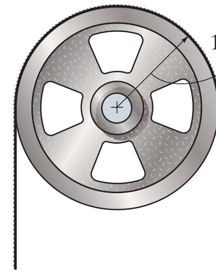
$$\sigma = \frac{Mc}{I} = \frac{(EI/\rho)c}{I} = \left(\frac{c}{\rho}\right)E$$

$$\sigma = \frac{0.0625}{600}(29)(10^3) = 3.02 \text{ ksi}$$ **Ans.**

Ans:
$\sigma = 3.02$ ksi

16–2.

The L2 steel blade of the band saw wraps around the pulley having a radius of 12 in. Determine the maximum normal stress in the blade. The blade has a width of 0.75 in. and a thickness of 0.0625 in.



SOLUTION

$$\frac{1}{\rho} = \frac{M}{EI}; \qquad M = \frac{EI}{\rho}$$

However,

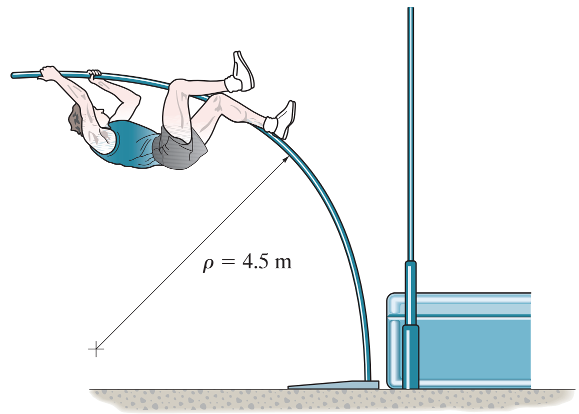
$$\sigma = \frac{Mc}{I} = \frac{(EI/\rho)c}{I} = \left(\frac{c}{\rho}\right)E$$

$$\sigma = \left(\frac{0.03125}{12}\right)(29)(10^3) = 75.5 \text{ ksi}$$ **Ans.**

Ans:
$\sigma = 75.5$ ksi

16–3.

A picture is taken of a man performing a pole vault, and the minimum radius of curvature of the pole is estimated by measurement to be 4.5 m. If the pole is 40 mm in diameter and it is made of a glass-reinforced plastic for which $E_g = 131$ GPa, determine the maximum bending stress in the pole.



SOLUTION

Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M}{EI} \qquad \text{however,} \qquad M = \frac{I}{c}\sigma$$

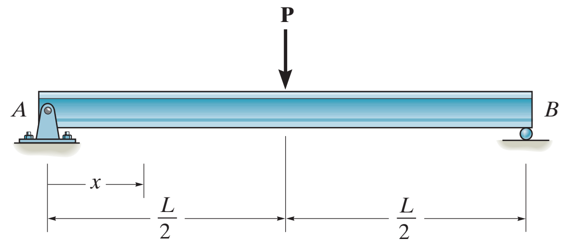
$$\frac{1}{\rho} = \frac{\frac{I}{c}\sigma}{EI}$$

$$\sigma = \frac{c}{\rho}E = \left(\frac{0.02}{4.5}\right)\left[131\left(10^9\right)\right] = 582 \text{ MPa}$$ **Ans.**

Ans:
$\sigma = 582$ MPa

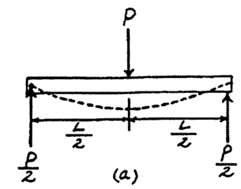
***16–4.**

Determine the equation of the elastic curve for the beam using the x coordinate that is valid for $0 \leq x < L/2$. Specify the slope at A and the beam's maximum deflection. EI is constant.

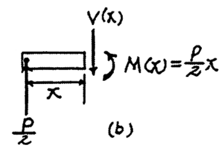


SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).



Moment Function: As shown on FBD(b).



Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad \textbf{(1)}$$

$$EI\,v = \frac{P}{12}x^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$.

Also, $v = 0$ at $x = 0$.

From Eq. (1), $\quad 0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq. (2), $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

The Slope: Substitute the value of C_1 into Eq. (1).

$$\frac{dv}{dx} = \frac{P}{16EI}\left(4x^2 - L^2\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{PL^2}{16EI} \qquad \textbf{Ans.}$$

The negative sign indicates clockwise rotation.

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. (2).

$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad \textbf{Ans.}$$

$v_{\max}$ occurs at $x = \frac{L}{2}$.

$$v_{\max} = -\frac{PL^3}{48EI} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

Ans:

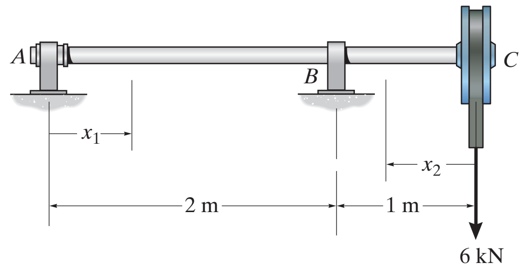
$\theta_A = -\frac{PL^2}{16EI}$,

$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right)$,

$v_{\max} = -\frac{PL^3}{48EI}$

16–5.

Determine the deflection of end C of the 100-mm-diameter solid circular shaft. Take $E = 200$ GPa.



SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the shaft's cut segments, Fig. *b*, $M(x_1)$ is

$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) + 3x_1 = 0 \qquad M(x_1) = -3x_1 \text{ kN}\cdot\text{m}$

and $M(x_2)$ is

$\circlearrowleft + \Sigma M_O = 0; \qquad -M(x_2) - 6x_2 = 0 \qquad M(x_2) = -6x_2 \text{ kN}\cdot\text{m}$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = -3x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{3}{2}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = -\frac{1}{2}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = -6x_2$$

$$EI\frac{dv_2}{dx_2} = -3x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = -x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions. At $x_1 = 0$, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = 2$ m, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}(2^3) + C_1(2) + 0 \qquad C_1 = 2 \text{ kN}\cdot\text{m}^2$$

16–5. Continued

At $x_2 = 1$ m, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -(1^3) + C_3(1) + C_4$$

$$C_3 + C_4 = 1 \qquad \textbf{(5)}$$

Continuity Conditions. At $x_1 = 2$ m and $x_2 = 1$, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give

$$-\frac{3}{2}(2^2) + 2 = -[-3(1^2) + C_3] \qquad C_3 = 7\ \text{kN}\cdot\text{m}^2$$

Substituting the values of C_3 into Eq. (5),

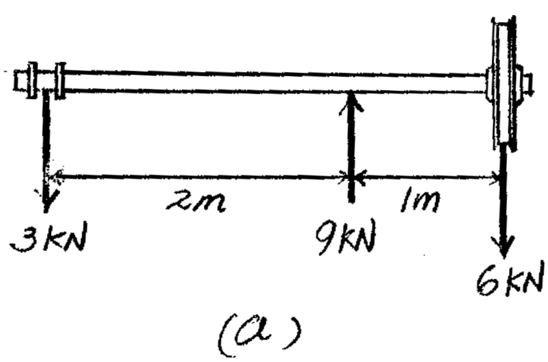
$$C_4 = -6\ \text{kN}\cdot\text{m}^3$$

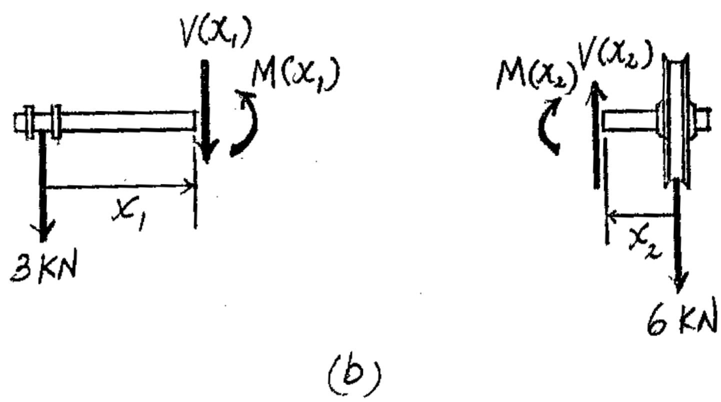
Substituting the values of C_3 and C_4 into Eq. (4),

$$v_2 = \frac{1}{EI}(-x_2^3 + 7x_2 - 6)$$

$$v_C = v_2|_{x_2=0} = -\frac{6\ \text{kN}\cdot\text{m}^3}{EI}$$

$$= -\frac{6(10^3)}{200(10^9)\left[\frac{\pi}{4}\left(0.05^4\right)\right]} = -0.006112\ \text{m} = -6.11\ \text{mm} \qquad \textbf{Ans.}$$

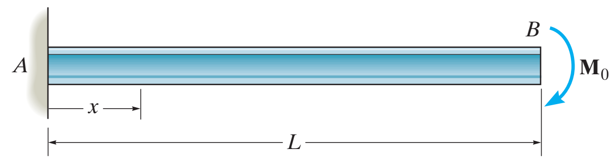




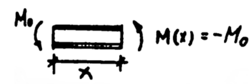
Ans:
$v_C = -6.11$ mm

16–6.

Determine the elastic curve for the cantilevered beam, which is subjected to the couple moment $\mathbf{M}_0$. Also calculate the maximum slope and maximum deflection of the beam. EI is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -M_0$$

$$EI\frac{dv}{dx} = -M_0x + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{-M_0x^2}{2} + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = 0$$

From Eq. (1), $C_1 = 0$.

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2), $C_2 = 0$.

$$\frac{dv}{dx} = -\frac{M_0x}{EI}$$

$$\theta_{\max} = \frac{dv}{dx}\bigg|_{x=L} = -\frac{M_0L}{EI} \qquad \textbf{Ans.}$$

The negative sign indicates clockwise rotation.

$$v = -\frac{M_0x^2}{2EI} \qquad \textbf{Ans.}$$

$$v_{\max} = v\bigg|_{x=L} = -\frac{M_0L^2}{2EI} \qquad \textbf{Ans.}$$

Negative sign indicates downward displacement.

Ans:

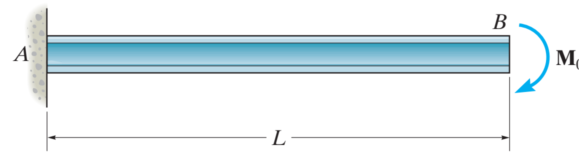
$\theta_{\max} = -\dfrac{M_0L}{EI}$,

$v = -\dfrac{M_0x^2}{2EI}$,

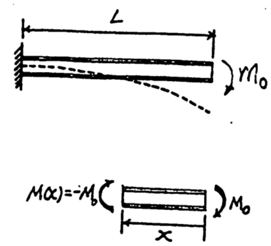
$v_{\max} = -\dfrac{M_0L^2}{2EI}$

16–7.

The A-36 steel beam has a depth of 10 in. and is subjected to a constant moment $\mathbf{M}_0$, which causes the stress at the outer fibers to become $\sigma_Y = 36$ ksi. Determine the radius of curvature of the beam and the beam's maximum slope and deflection.



SOLUTION



Moment-Curvature Relationship:

$$\frac{1}{\rho} = \frac{M_0}{EI} \qquad \text{however,} \qquad M_0 = \frac{I}{c}\sigma_Y$$

$$\frac{1}{\rho} = \frac{\frac{I}{c}\sigma_Y}{EI}$$

$$\rho = \frac{Ec}{\sigma_Y} = \frac{29.0\left(10^3\right)(5)}{36} = 4027.78 \text{ in.} = 336 \text{ ft} \qquad \textbf{Ans.}$$

Elastic Curve: As shown.

Moment Function: As shown on FBD.

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -M_0$$

$$EI\frac{dv}{dx} = -M_0x + C_1 \qquad \textbf{(1)}$$

$$EI\,v = -\frac{M_0}{2}x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: $\dfrac{dv}{dx} = 0$ at $x = L$, and $v = 0$ at $x = L$.

From Eq. (1), $\quad 0 = -M_0L + C_1 \qquad C_1 = M_0L$

From Eq. (2), $\quad 0 = -\dfrac{M_0}{2}\left(L^2\right) + M_0L^2 + C_2 \qquad C_2 = -\dfrac{M_0L^2}{2}$

The Slope: Substitute the value of C_1 into Eq. (1).

$$\frac{dv}{dx} = \frac{M_0}{EI}(-x + L)$$

The maximum slope occurs at $x = 0$.

$$\theta_{max} = \left.\frac{dv}{dx}\right|_{x=0} = \frac{M_0L}{EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. (2).

$$v = \frac{M_0}{2EI}\left(-x^2 + 2Lx - L^2\right)$$

The maximum displacement occurs at $x = 0$.

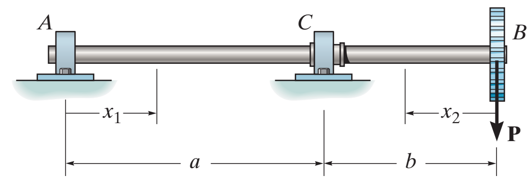
$$v_{max} = -\frac{M_0L^2}{2EI} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

Ans:
$\rho = 336$ ft,
$\theta_{max} = \dfrac{M_0L}{EI}$,
$v_{max} = -\dfrac{M_0L^2}{2EI}$

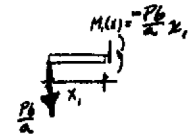
***16–8.**

Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



SOLUTION

Elastic Curve and Slope:



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\dfrac{Pb}{a}x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{Pb}{a}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{Pb}{2a}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = -\frac{Pb}{6a}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = -Px_2$,

$$EI\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = \frac{-Px_2^2}{2} + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = \frac{-Px_2^3}{6} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$$v_1 = 0 \quad \text{at} \quad x = 0$$

From Eq. (2), $\qquad C_2 = 0$

$$v_1 = 0 \quad \text{at} \quad x_1 = a$$

From Eq. (2),

$$0 = -\frac{Pb}{6a}a^3 + C_1a$$

$$C_1 = \frac{Pab}{6}$$

$$v_2 = 0 \quad \text{at} \quad x_2 = b$$

From Eq. (4),

$$0 = \frac{Pb^3}{6} + C_3b + C_4$$

$$C_3b + C_4 = \frac{Pb^3}{6} \qquad \textbf{(5)}$$

***16–8. Continued**

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{-dv_2}{dx_2} \quad \text{at} \quad x_1 = a \qquad x_2 = b$$

From Eqs. (1) and (3),

$$-\frac{Pb}{2a}(a^2) + \frac{Pab}{6} = \frac{Pb^2}{2} - C_3$$

$$C_3 = \frac{Pab}{3} + \frac{Pb^2}{2}$$

Substitute C_3 into Eq. (5).

$$C_4 = \frac{Pb^3}{3} - \frac{Pab^2}{3}$$

$$v_1 = \frac{-Pb}{6aEI}(x_1^3 - a^2x_1)$$ **Ans.**

$$v_2 = \frac{P}{6EI}[-x_2^3 + b(2a + 3b)x_2 - 2b^2(a + b)]$$ **Ans.**

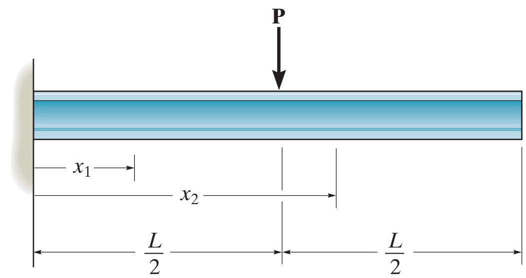
Ans:

$v_1 = \dfrac{-Pb}{6aEI}(x_1^3 - a^2x_1),$

$v_2 = \dfrac{P}{6EI}[-x_2^3 + b(2a + 3b)x_2 - 2b^2(a + b)]$

16–9.

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. EI is constant.



SOLUTION

Referring to the FBDs of the beam's cut segments shown in Fig. *b* and *c*,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) + \frac{PL}{2} - Px_1 = 0 \qquad M(x_1) = Px_1 - \frac{PL}{2}$$

And

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_2) = 0$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = Px_1 - \frac{PL}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{P}{2}x_1{}^2 - \frac{PL}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = \frac{P}{6}x_1{}^3 - \frac{PL}{4}x_1{}^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = 0$$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = C_3x_2 = C_4 \qquad \textbf{(4)}$$

At $x_1 = 0$, $\frac{dv_1}{dx_1} = 0$. Then, Eq. (1) gives

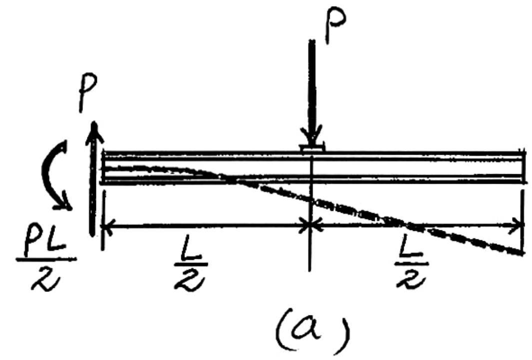
$$EI(0) = \frac{P}{2}(0^2) - \frac{PL}{2}(0) + C_1 \qquad C_1 = 0$$

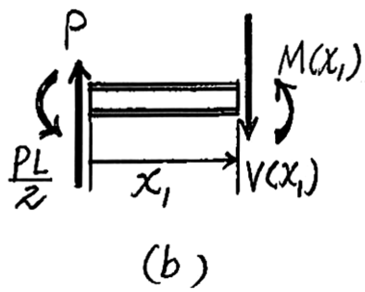
At $x_1 = 0$, $v_1 = 0$. Then, Eq(2) gives

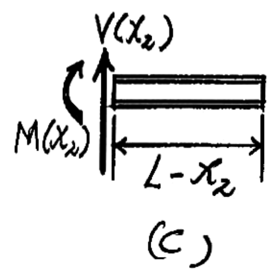
$$EI(0) = \frac{P}{6}(0^3) - \frac{PL}{4}(0^2) + 0 + C_2 \qquad C_2 = 0$$

At $x_1 = x_2 = \frac{L}{2}$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. Thus, Eqs.(1) and (3) gives

$$\frac{P}{2}\left(\frac{L}{2}\right)^2 - \frac{PL}{2}\left(\frac{L}{2}\right) = C_3 \qquad C_3 = -\frac{PL^2}{8}$$







16–9. Continued

Also, at $x_1 = x_2 = \frac{L}{2}$, $v_1 = v_2$. Thus, Eqs, (2) and (4) gives

$$\frac{P}{6}\left(\frac{L}{2}\right)^3 - \frac{PL}{4}\left(\frac{L}{2}\right)^2 = \left(-\frac{PL^2}{8}\right)\left(\frac{L}{2}\right) + C_4 \qquad C_4 = \frac{PL^3}{48}$$

Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4).

$$v_1 = \frac{P}{12EI}\left(2x_1^3 - 3Lx_1^2\right) \qquad \textbf{Ans.}$$

$$v_2 = \frac{PL^2}{48EI}(-6x_2 + L) \qquad \textbf{Ans.}$$

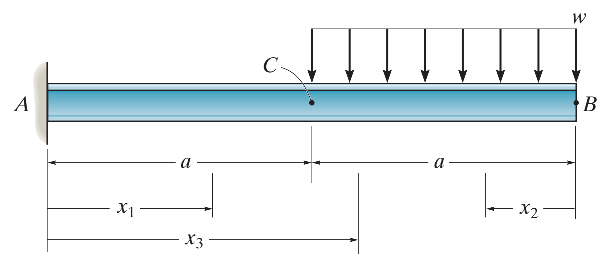
Ans:

$v_1 = \frac{P}{12EI}(2x_1^3 - 3Lx_1^2)$,

$v_2 = \frac{PL^2}{48EI}(-6x_2 + L)$

16–10.

Determine the equations of the elastic curve using the coordinates x_1 and x_2. What is the slope at C and displacement at B? EI is constant.

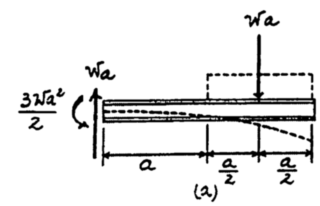


SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

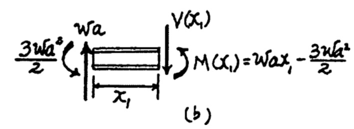
Slope and Elastic Curve:



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = w\,ax_1 = -\dfrac{3wa^2}{2}$,

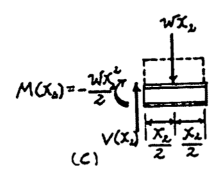
$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$



$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M(x_2) = -\dfrac{w}{2}x_2^2$,



$$EI\frac{d^2v_2}{dx_2^2} = -\frac{w}{2}x_2^2$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}x_2^3 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = -\frac{w}{24}x_2^4 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$\dfrac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq. (1), $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq. (2), $C_2 = 0$

Continuity Conditions:

At $x_1 = a$ and $x_2 = a$, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. From Eqs. (1) and (3),

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = -\left(-\frac{wa^3}{6} + C_3\right) \qquad C_3 = \frac{7wa^3}{6}$$

At $x_1 = a$ and $x_2 = a$, $v_1 = v_2$. From Eqs. (2) and (4),

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = -\frac{wa^4}{24} + \frac{5wa^4}{6} + C_4 \qquad C_4 = -\frac{11wa^4}{8}$$

16–10. Continued

The Slope: Substituting into Eq. (1),

$$\frac{dv_1}{dx_1} = \frac{wax_1}{2EI}(x_1 - 3a)$$

$$\theta_C = \left.\frac{dv_1}{dx_1}\right|_{x_1=a} = -\frac{wa^3}{EI} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively,

$$v_1 = \frac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right) \qquad \textbf{Ans.}$$

$$v_2 = \frac{w}{24EI}\left(-x_2^4 + 28a^3x_2 - 41a^4\right) \qquad \textbf{Ans.}$$

$$v_B = v_2\big|_{x_2=0} = -\frac{41wa^4}{24EI} \qquad \textbf{Ans.}$$

Ans:

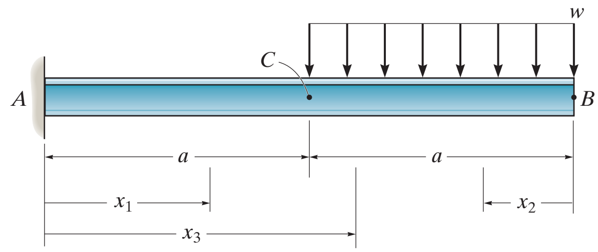
$v_1 = \dfrac{wax_1}{12EI}\left(2x^2 - 9ax_1\right)$,

$v_2 = \dfrac{w}{24EI}\left(-x_2^4 + 28a^3x_2 - 41a^4\right)$,

$\theta_C = -\dfrac{wa^3}{EI}, v_B = -\dfrac{41\,wa^4}{24\,EI}$

16–11.

Determine the equations of the elastic curve using the coordinates x_1 and x_3. What is the slope at B and deflection at C? EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = wax_1 - \dfrac{3wa^2}{2}$,

$$EI\frac{d^2v_1}{dx_1^2} = wax_1 - \frac{3wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{wa}{2}x_1^2 - \frac{3wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = \frac{wa}{6}x_1^3 - \frac{3wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M(x_3) = 2wax_3 - \dfrac{w}{2}x_3^2 - 2wa^2$,

$$EI\frac{d^2v_3}{dx_3^2} = 2wax_3 - \frac{w}{2}x_3^2 - 2wa^2$$

$$EI\frac{dv_3}{dx_3} = wax_3^2 - \frac{w}{6}x_3^3 - 2wa^2x_3 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_3 = \frac{wa}{3}x_3^3 - \frac{w}{24}x_3^4 - wa^2x_3^2 + C_3x_3 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$\dfrac{dv_1}{dx_1} = 0$ at $x_1 = 0$. From Eq. (1), $C_1 = 0$

$v_1 = 0$ at $x_1 = 0$. From Eq. (2), $C_2 = 0$

Continuity Conditions:

At $x_1 = a$ and $x_3 = a$, $\dfrac{dv_1}{dx_1} = \dfrac{dv_3}{dx_3}$. From Eqs. (1) and (3),

$$\frac{wa^3}{2} - \frac{3wa^3}{2} = wa^3 - \frac{wa^3}{6} - 2wa^3 + C_3 \qquad C_3 = \frac{wa^3}{6}$$

At $x_1 = a$ and $x_3 = a$, $v_1 = v_3$. From Eqs. (2) and (4),

$$\frac{wa^4}{6} - \frac{3wa^4}{4} = \frac{wa^4}{3} - \frac{wa^4}{24} - wa^4 + \frac{wa^4}{6} + C_4 \qquad C_4 = -\frac{wa^4}{24}$$

16–11. Continued

The Slope: Substituting the value of C_1 into Eq. (1),

$$\frac{dv_3}{dx_3} = \frac{w}{6EI}(6ax_3^2 - x_3^3 - 12a^2x_3 + a^3)$$

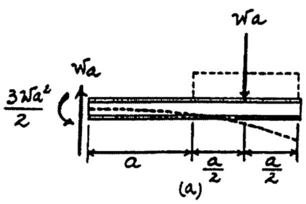
$$\theta_B = \left.\frac{dv_3}{dx_3}\right|_{x_3=2a} = -\frac{7wa^3}{6EI} \qquad \textbf{Ans.}$$

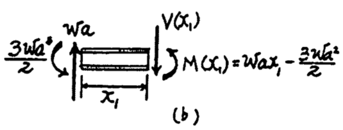
The Elastic Curve: Substituting the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively,

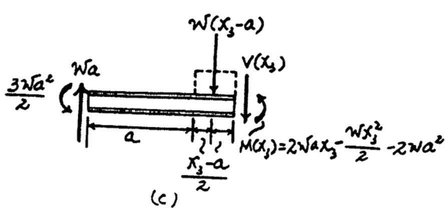
$$v_1 = \frac{wax_1}{12EI}\left(2x_1^2 - 9ax_1\right) \qquad \textbf{Ans.}$$

$$v_C = v_1\big|_{x_1=a} = -\frac{7\,wa^4}{12EI} \qquad \textbf{Ans.}$$

$$v_3 = \frac{w}{24EI}\left(-x_3^4 + 8ax_3^3 - 24a^2x_3^2 + 4a^3x_3 - a^4\right) \qquad \textbf{Ans.}$$







Ans:

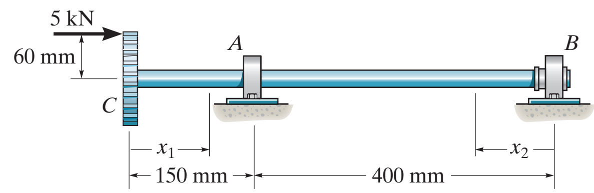
$v_1 = \frac{wax_1}{12EI}\left(2x^2 - 9ax_1\right)$,

$v_3 = \frac{w}{24EI}\left(-x_3^4 + 8ax_3^3 - 24a^2x_3^2 + 4a^3x_3 - a^4\right)$,

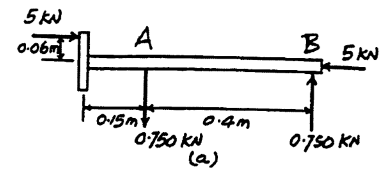
$\theta_B = -\frac{7wa^3}{6EI}, v_C = -\frac{7wa^4}{12EI}$

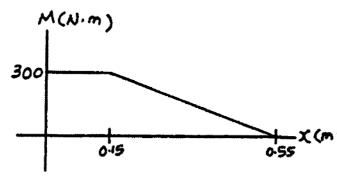
***16–12.**

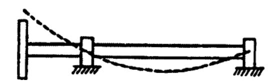
Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.

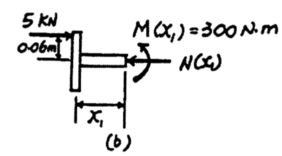


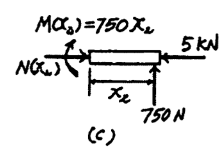
SOLUTION











Elastic Curve: As shown.

Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = 300\text{ N}\cdot\text{m}$,

$$EI\frac{d^2v_1}{dx_1^2} = 300$$

$$EI\frac{dv_1}{dx_1} = 300x_1 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = 150x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M(x_2) = 750x_2$,

$$EI\frac{d^2v_2}{dx_2^2} = 750x_2$$

$$EI\frac{dv_2}{dx_2} = 375x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = 125x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0.15$ m. From Eq. (2),

$$0 = 150(0.15^2) + C_1(0.15) + C_2 \qquad \textbf{(5)}$$

$v_2 = 0$ at $x_2 = 0$. From Eq. (4), $C_4 = 0$

$v_2 = 0$ at $x_2 = 0.4$ m. From Eq. (4),

$$0 = 125(0.4^3) + C_3(0.4) \qquad C_3 = -20.0$$

Continuity Condition:

At $x_1 = 0.15$ m and $x_2 = 0.4$ m, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. From Eqs. (1) and (3),

$$300(0.15) + C_1 = -\left[375(0.4^2) - 20\right] \qquad C_1 = -85.0$$

From Eq. (5), $C_2 = 9.375$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively.

$$v_1 = \frac{1}{EI}\left(150x_1^2 - 85.0x_1 + 9.375\right)\text{ N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

$$v_2 = \frac{1}{EI}\left(125x_2^3 - 20.0x_2\right)\text{ N}\cdot\text{m}^3 \qquad \textbf{Ans.}$$

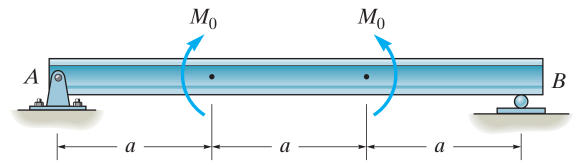
Ans:

$v_1 = \dfrac{1}{EI}\left(150x_1^2 - 85.0x_1 + 9.375\right)\text{ N}\cdot\text{m}^3,$

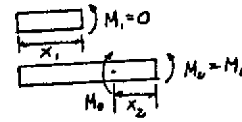
$v_2 = \dfrac{1}{EI}\left(125x_2^3 - 20.0x_2\right)\text{ N}\cdot\text{m}^3$

16–13.

Determine the maximum deflection of the beam and the slope at A. EI is constant.



SOLUTION



$M_1 = 0$

$$EI\frac{d^2v_1}{dx_1^2} = 0; \qquad EI\frac{dv_1}{dx_1} = C_1$$

$$EIv_1 = C_1x_1 + C_2$$

At $x_1 = 0, \quad v_1 = 0; \quad C_2 = 0$

$$M_2 = M_0; \qquad EI\frac{d^2v_1}{dx_2^2} = M_0$$

$$EI\frac{dv_2}{dx_2} = M_0x_2 + C_2$$

$$EI\,v_2 = \frac{1}{2}M_0x_2^2 + C_3x_2 + C_4$$

At $x_2 = \frac{a}{2}, \quad \frac{dv_2}{dx_2} = 0; \qquad C_1 = \frac{-M_0\,a}{2}$

At $x_1 = a, \quad x_2 = 0, \quad v_1 = v_2, \quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

$$C_1a = C_a$$

$$C_1 = \frac{-M_0\,a}{2}, \qquad C_a = \frac{-M_0\,a^2}{2}$$

At $x_1 = 0,$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0\,a}{2}$$

$$\theta_A = -\frac{M_0\,a}{2EI} \qquad \textbf{Ans.}$$

At $x_2 = \frac{a}{2},$

$$EI\,v_{max} = \frac{1}{2}M_0\left(\frac{a^2}{4}\right) - \frac{M_0\,a}{2}\left(\frac{a}{2}\right) - \frac{M_0\,a^2}{2}$$

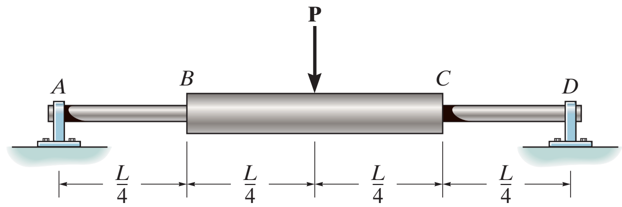
$$v_{max} = -\frac{5M_0\,a^2}{8EI} \qquad \textbf{Ans.}$$

Ans:

$\theta_A = -\frac{M_0a}{2EI}, \ v_{max} = -\frac{5M_0\,a^2}{8EI}$

16–14.

The simply supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the shaft due to the load **P**.



SOLUTION

$$M_1(x) = \frac{P}{2}x_1$$

$$M_2(x) = \frac{P}{2}x_2$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v_1}{dx_1{}^2} = \frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{4} + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{Px_1^3}{12} + C_1x_1 + C_2 \qquad \textbf{(2)}$$

$$2EI\frac{d^2v_2}{dx_2{}^2} = \frac{P}{2}x_2$$

$$2EI\frac{dv_2}{dx_1} = \frac{Px_2^2}{4} + C_3 \qquad \textbf{(3)}$$

$$2EIv_2 = \frac{Px_2^3}{12} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$

From Eq. (2), $C_2 = 0$

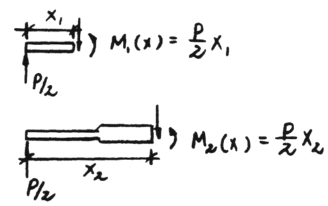
$\frac{dv_2}{dx_2} = 0$ at $x_2 = \frac{L}{2}$

From Eq. (3),

$$0 = \frac{PL^2}{16} + C_3$$

$$C_3 = -\frac{PL^2}{16}$$

16–14. Continued



Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} \text{ at } x_1 = x_2 = \frac{L}{4}$$

From Eqs. (1) and (3),

$$\frac{PL^2}{64} + C_1 = \frac{PL^2}{128} - \frac{1}{2}\left(\frac{PL^2}{16}\right)$$

$$C_1 = \frac{-5PL^2}{128}$$

$$v_1 = v_2 \text{ at } x_1 = x_2 = \frac{L}{4}$$

From Eqs. (2) and (4),

$$\frac{PL^3}{768} - \frac{5PL^2}{128}\left(\frac{L}{4}\right) = \frac{PL^3}{1536} - \frac{1}{2}\left(\frac{PL^2}{16}\right)\left(\frac{L}{4}\right) + \frac{1}{2}C_4$$

$$C_4 = \frac{-PL^3}{384}$$

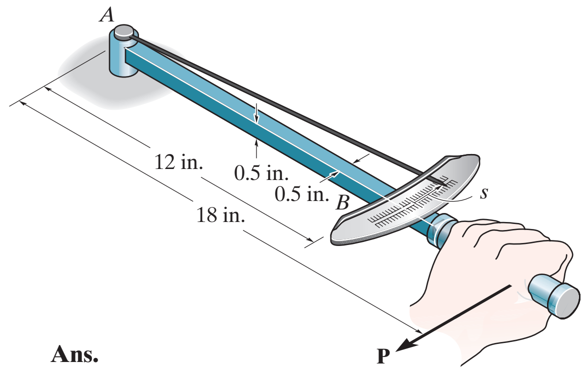
$$v_2 = \frac{P}{768EI}\left(32x_2^3 - 24L^2x_2 - L^3\right)$$

$$v_{max} = v_2\Big|_{x_2=\frac{L}{2}} = \frac{-3PL^3}{256EI}$$ **Ans.**

Ans:
$$v_{max} = -\frac{3PL^3}{256EI}$$

16–15.

A torque wrench is used to tighten the nut on a bolt. If the dial indicates that a torque of 60 lb·ft is applied when the bolt is fully tightened, determine the force P acting at the handle and the distance s the needle moves along the scale. Assume only the portion AB of the beam distorts. The cross section is square having dimensions of 0.5 in. by 0.5 in. $E = 29(10^3)$ ksi.



SOLUTION

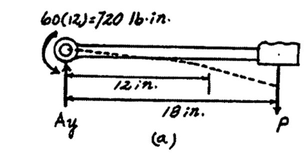
Equations of Equilibrium: From FBD(a),

$\circlearrowleft + \Sigma M_A = 0;\quad 720 - P(18) = 0\quad P = 40.0\text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad A_y - 40.0 = 0\quad A_y = 40.0\text{ lb}$

Moment Function: As shown on FBD(b).

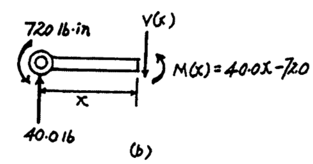
Slope and Elastic Curve:



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = 40.0x - 720$$

$$EI\frac{dv}{dx} = 20.0x^2 - 720x + C_1 \qquad \textbf{(1)}$$



$$EI\,v = 6.667x^3 - 360x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: $\dfrac{dv}{dx} = 0$ at $x = 0$ and $v = 0$ at $x = 0$.

From Eq. (1), $\quad 0 = 0 - 0 + C_1 \qquad C_1 = 0$

From Eq. (2), $\quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

The Elastic Curve: Substitute the values of C_1 and C_2 into Eq. (2).

$$v = \frac{1}{EI}\left(6.667x^3 - 360x^2\right) \qquad \textbf{(1)}$$

At $x = 12$ in., $v = -s$. From Eq. (1),

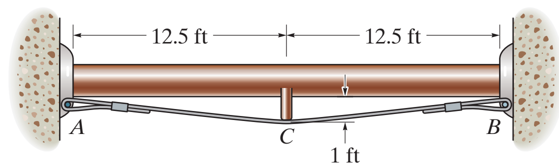
$$-s = \frac{1}{(29)(10^6)\left(\frac{1}{12}\right)(0.5)(0.5^3)}\left[6.667(12^3) - 360(12^2)\right]$$

$$s = 0.267\text{ in.}$$ **Ans.**

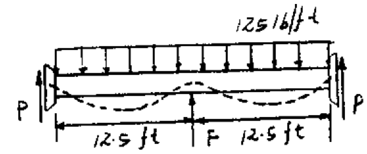
Ans:
$P = 40.0$ lb,
$s = 0.267$ in.

***16–16.**

The pipe can be assumed roller supported at its ends and by a rigid saddle C at its center. The saddle rests on a cable that is connected to the supports. Determine the force that should be developed in the cable if the saddle keeps the pipe from sagging or deflecting at its center. The pipe and fluid within it have a combined weight of 12.5 lb/ft. EI is constant.



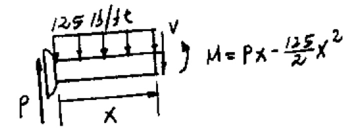
SOLUTION



$$2P + F - 125(25) = 0$$

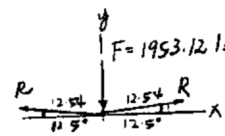
$$2P + F = 3125$$

$$M = Px - \frac{125}{2}x^2$$



$$EI\frac{d^2v}{dx^2} = Px - \frac{125}{2}x^2$$

$$EI\frac{dv}{dx} = \frac{Px^2}{2} - 20.833x^3 + C_1$$



$$EIv = \frac{Px^3}{6} - 5.2083x^4 + C_1 x + C_2$$

At $x = 0$, $v = 0$. Therefore $C_2 = 0$

At $x = 12.5$ ft, $v = 0$.

$$0 = \frac{P(12.5)^3}{6} - 5.2083(12.5)^4 + C_1(12.5) \qquad \textbf{(1)}$$

At $x = 12.5$ ft, $\dfrac{dv}{dx} = 0$.

$$0 = \frac{P(12.5)^2}{2} - 20.833(12.5)^3 + C_1 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) for P,

$P = 585.94 \qquad F = 3125 - 2(585.94) = 1953.12$ lb

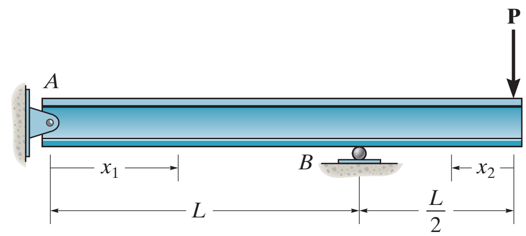
$+\uparrow\Sigma F_y = 0; \qquad 2R\left(\dfrac{1}{12.54}\right) - 1953.12 = 0$

$R = 12\,246$ lb $= 12.2$ kip **Ans.**

Ans:
$R = 12.2$ kip

16–17.

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the beam's maximum deflection. EI is constant.

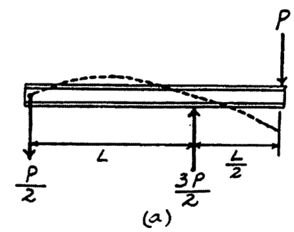


SOLUTION

Support Reactions and Elastic Curve: As shown on FBD(a).

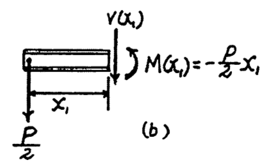
Moment Function: As shown on FBD(b) and (c).

Slope and Elastic Curve:



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M(x_1) = -\dfrac{P}{2}x_1$,

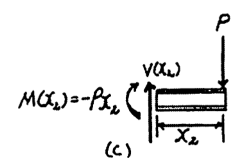


$$EI\frac{d^2v_1}{dx_1^2} = -\frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{P}{4}x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EI\,v_1 = -\frac{P}{12}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M(x_2) = -Px_2$,



$$EI\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = -\frac{P}{2}x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = -\frac{P}{6}x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_1 = 0$ at $x_1 = 0$. From Eq. (2), $C_2 = 0$

$v_1 = 0$ at $x_1 = L$. From Eq. (2),

$$0 = -\frac{PL^3}{12} + C_1L \qquad C_1 = \frac{PL^2}{12}$$

$v_2 = 0$ at $x_2 = \dfrac{L}{2}$. From Eq. (4),

$$0 = -\frac{PL^3}{48} + \frac{L}{2}C_3 + C_4 \qquad \textbf{(5)}$$

16–17. Continued

Continuity Conditions:

At $x_1 = L$ and $x_2 = \dfrac{L}{2}$, $\quad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. $\quad$ From Eqs. (1) and (3),

$$-\frac{PL^2}{4} + \frac{PL^2}{12} = -\left(-\frac{PL^2}{8} + C_3\right) \qquad C_3 = \frac{7PL^2}{24}$$

From Eq. (5), $\qquad C_4 = -\dfrac{PL^3}{8}$

The Slope: Substitute the value of C_1 into Eq. (1).

$$\frac{dv_1}{dx_1} = \frac{P}{12EI}\left(L^2 - 3x_1^2\right)$$

$$\frac{dv_1}{dx_1} = 0 = \frac{P}{12EI}\left(L^2 - 3x_1^2\right) \qquad x_1 = \frac{L}{\sqrt{3}}$$

The Elastic Curve: Substitute the values of C_1, C_2, C_3, and C_4 into Eqs. (2) and (4), respectively.

$$v_1 = \frac{Px_1}{12EI}\left(-x_1^2 + L^2\right) \qquad \textbf{Ans.}$$

$$v_D = v_1\big|_{x_1 = \frac{L}{\sqrt{3}}} = \frac{P\left(\frac{L}{\sqrt{3}}\right)}{12EI}\left(-\frac{L^2}{3} + L^2\right) = \frac{0.0321PL^3}{EI}$$

$$v_2 = \frac{P}{24EI}\left(-4x_2^3 + 7L^2x_2 - 3L^3\right) \qquad \textbf{Ans.}$$

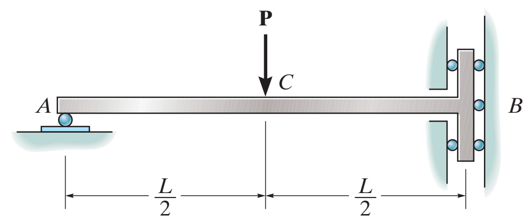
$$v_C = v_2\big|_{x_2 = 0} = -\frac{PL^3}{8EI}$$

Hence, $\qquad v_{max} = v_C = \dfrac{PL^3}{8EI} \qquad$ **Ans.**

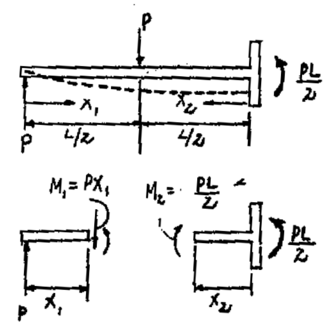
Ans:
$v_1 = \dfrac{Px_1}{12EI}\left(-x_1^2 + L^2\right)$,
$v_2 = \dfrac{P}{24EI}\left(-4x_2^3 + 7L^2x_2 - 3L^3\right)$,
$v_{max} = \dfrac{PL^3}{8EI}$

16–18.

The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.



SOLUTION



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\,v_1 = \frac{Px_1^2}{6} + C_1$$

$$EI\,v_1 = \frac{Px_1^2}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary Conditions:

At $x_1 = 0$, $v_1 = 0$.

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_2 = 0$, $\quad \dfrac{dv_2}{dx_2} = 0$

$0 + C_3 = 0; \quad C_3 = 0$

At $x_1 = \dfrac{L}{2}, \quad x_2 = \dfrac{L}{2}, v_1 = v_2, \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

$$\frac{P\left(\frac{1}{2}\right)^2}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{1}{2}\right)^2}{4} + C_4$$

$$\frac{P\left(\frac{1}{2}\right)^2}{2} + C_1 = -\frac{PL\left(\frac{1}{2}\right)}{2}; \quad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_1 = 0$,

$$\frac{dv_1}{dx_1} = \theta_A = -\frac{3}{8}\frac{PL^2}{EI} \qquad \textbf{Ans.}$$

At $x_1 = \dfrac{L}{2}$,

$$v_C = \frac{P\left(\frac{1}{2}\right)^3}{6EI} - \left(\frac{3}{8EI}PL^3\right)\left(\frac{L}{2}\right) + 0$$

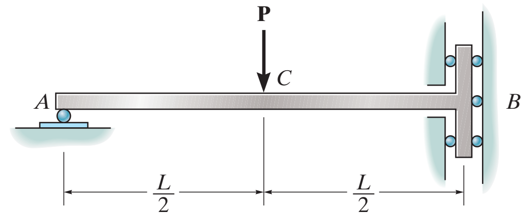
$$v_C = -\frac{PL^3}{6EI} \qquad \textbf{Ans.}$$

Ans:

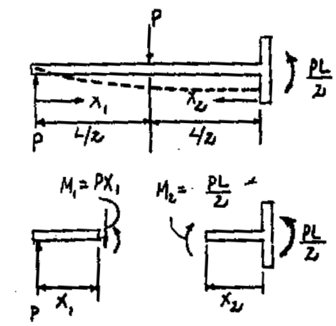
$\theta_A = -\dfrac{3PL^2}{8EI}, v_C = -\dfrac{PL^3}{6EI}$

16–19.

Determine the deflection at B of the bar in Prob. 16–18.



SOLUTION



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$$

$$EI\,v_1 = \frac{Px_1^2}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_2$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary Conditions:

At $x_1 = 0,\ v_1 = 0.$

$$0 = 0 + 0 + C_2; \qquad C_2 = 0$$

At $x_2 = 0, \qquad \dfrac{dv_2}{dx_2} = 0$

$$0 + C_3 = 0; \qquad C_3 = 0$$

At $x_1 = \dfrac{L}{2}, \qquad x_2 = \dfrac{L}{2},\ v_1 = v_2,\ \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

$$\frac{P\left(\frac{1}{2}\right)^2}{6} + C_1\left(\frac{L}{2}\right) = \frac{PL\left(\frac{1}{2}\right)^2}{4} + C_4$$

$$\frac{P\left(\frac{1}{2}\right)^2}{2} + C_1 = -\frac{PL\left(\frac{1}{2}\right)}{2}; \qquad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_2 = 0,$

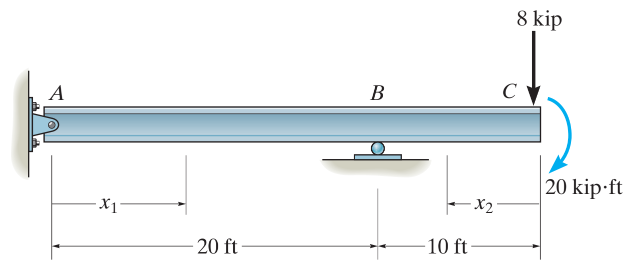
$$v_2 = v_B = -\frac{11PL^3}{48EI} \qquad \textbf{Ans.}$$

Ans:

$$v_B = -\frac{11PL^3}{48EI}$$

***16–20.**

Determine the equations of the elastic curve using the x_1 and x_2 coordinates. What is the slope at A and the deflection at C? EI is constant.



SOLUTION

Referring to the FBDs of the beam's cut segments shown in Fig. *b* and *c*,

$$\circlearrowleft + \Sigma M_o = 0; \qquad M(x_1) + 5x_1 = 0 \qquad M(x_1) = (-5x_1)\ \text{kip}\cdot\text{ft}$$

And

$$\circlearrowleft + \Sigma M_o = 0; \qquad -M(x_2) - 8x_2 - 20 = 0 \quad M(x_2) = (-8x_2 - 20)\ \text{kip}\cdot\text{ft}$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = (-5x_1)\ \text{kip}\cdot\text{ft}$$

$$EI\frac{dv_1}{dx_1} = \left(-\frac{5}{2}x_1^2 + C_1\right)\text{kip}\cdot\text{ft}^2 \qquad \textbf{(1)}$$

$$EI\,v_1 = \left(-\frac{5}{6}x_1{}^3 + C_1x_1 + C_2\right)\text{kip}\cdot\text{ft}^3 \qquad \textbf{(2)}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2{}^2} = (-8x_2 - 20)\ \text{kip}\cdot\text{ft}$$

$$EI\frac{dv_2}{dx_2} = \left(-4x_2{}^2 - 20x_2 + C_3\right)\text{kip}\cdot\text{ft}^2 \qquad \textbf{(3)}$$

$$EI\,v_2 = \left(-\frac{4}{3}x_2{}^3 - 10x_2{}^2 + C_3x_2 + C_4\right)\text{kip}\cdot\text{ft}^3 \qquad \textbf{(4)}$$

At $x_1 = 0$, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{5}{6}\left(0^3\right) + C_1(0) + C_2 \qquad C_2 = 0$$

Also, at $x_1 = 20$ ft, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{5}{6}\left(20^3\right) + C_1(20) + 0 \qquad C_1 = 333.33\ \text{kip}\cdot\text{ft}^2$$

Also, at $x_2 = 10$ ft, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{4}{3}\left(10^3\right) - 10\left(10^2\right) + C_3(10) + C_4$$

$$10C_3 + C_4 = 2333.33 \qquad \textbf{(5)}$$

***16–20. Continued**

At $x_1 = 20$ ft and $x_2 = 10$ ft, $\dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$. Then Eq. (1) and (3) gives

$$-\frac{5}{2}\left(20^2\right) + 333.33 = -\left[-4\left(10^2\right) - 20(10) + C_3\right]$$

$$C_3 = 1266.67 \text{ kip}\cdot\text{ft}^2$$

Substitute the value of C_3 into Eq. (5).

$$C_4 = -10333.33 \text{ kip}\cdot\text{ft}^3$$

Substitute the value of C_1 into Eq. (1).

$$\frac{dv_1}{dx_1} = \frac{1}{EI}\left(-\frac{5}{2}{x_1}^2 + 333.33\right) \text{kip}\cdot\text{ft}^2$$

At A, $x_1 = 0$. Thus,

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{333 \text{ kip}\cdot\text{ft}^2}{EI} \qquad \textbf{Ans.}$$

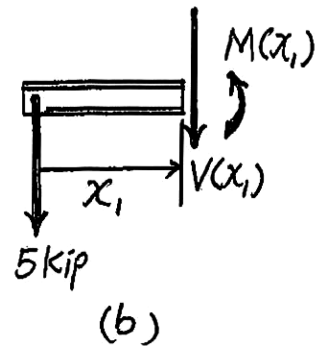
Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq. (4).

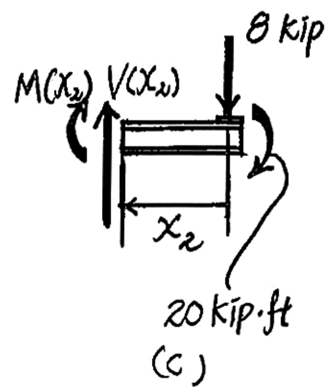
$$v_1 = \frac{1}{EI}\left(-\frac{5}{6}{x_1}^3 + 333\,x_1\right) \text{kip}\cdot\text{ft}^3 \qquad \textbf{Ans.}$$

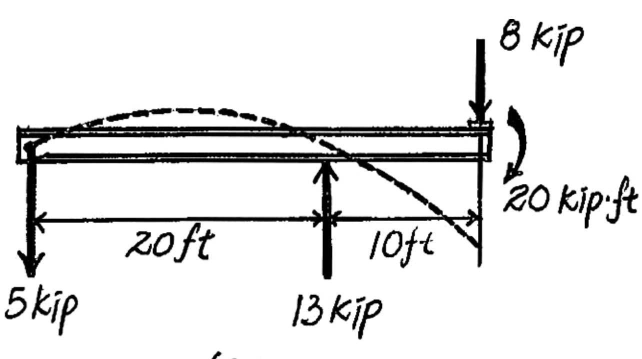
$$v_2 = \frac{1}{EI}\left(-\frac{4}{3}{x_2}^3 - 10{x_2}^2 + 1267x_2 - 10\,333\right) \text{kip}\cdot\text{ft}^3 \qquad \textbf{Ans.}$$

At C, $x_2 = 0$. Thus,

$$v_C = v_2\big|_{x_2=0} = -\frac{10\,333 \text{ kip}\cdot\text{ft}^3}{EI} = \frac{10\,333 \text{ kip}\cdot\text{ft}^3}{EI}\downarrow \qquad \textbf{Ans.}$$







Ans:

$\theta_A = \dfrac{333 \text{ kip}\cdot\text{ft}^2}{EI}$,

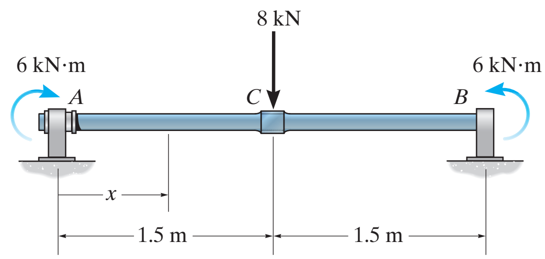
$v_1 = \dfrac{1}{EI}\left(-\dfrac{5}{6}x_1^3 + 333\,x_1\right) \text{kip}\cdot\text{ft}^3$,

$v_2 = \dfrac{1}{EI}\left(-\dfrac{4}{3}x_2^3 - 10x_2^2 + 1267x_2 - 10\,333\right) \text{kip}\cdot\text{ft}^3$,

$v_C = \dfrac{10\,333 \text{ kip}\cdot\text{ft}^3}{EI}\downarrow$

16–21.

Determine the maximum deflection of the solid circular shaft. The shaft is made of steel having $E = 200$ GPa. It has a diameter of 100 mm.



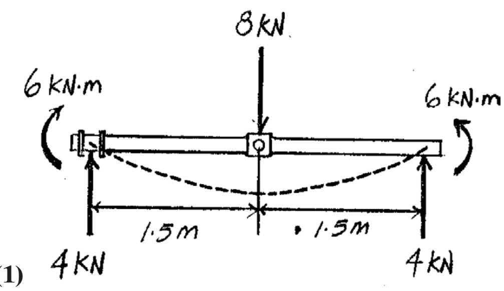
SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. Referring to the free-body diagram of the beam's cut segment, Fig. *b*,

$\circlearrowleft + \Sigma M_O = 0; \qquad M(x) - 4x - 6 = 0 \qquad M(x) = (4x + 6) \text{ kN} \cdot \text{m}$

Equations of Slope and Elastic Curve.



(a)

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = 4x + 6$$

$$EI\frac{dv}{dx} = 2x^2 + 6x + C_1 \qquad \textbf{(1)}$$

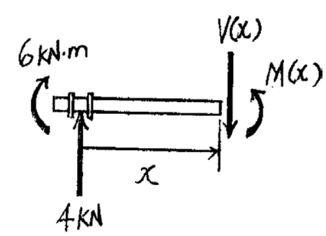
$$EIv = \frac{2}{3}x^3 + 3x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions. Due to symmetry, $\dfrac{dv}{dx} = 0$ at $x = 1.5$ m. Then Eq. (1) gives

$$EI(0) = 2(1.5^2) + 6(1.5) + C_1 \qquad C_1 = -13.5 \text{ kN} \cdot \text{m}^2$$

Also, at $x = 0$, $v = 0$. Then Eq. (2) gives

$$EI(0) = \frac{2}{3}(0^3) + 3(0^2) + C_1(0) + C_2 \qquad C_2 = 0$$



(b)

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left(\frac{2}{3}x^3 + 3x^2 - 13.5x\right)$$

v_{max} occurs at $x = 1.5$ m, where $\dfrac{dv}{dx} = 0$. Thus,

$$v_{max} = v|_{x=1.5\text{ m}} = \frac{1}{EI}\left[\frac{2}{3}(1.5^3) + 3(1.5^2) - 13.5(1.5)\right]$$

$$= -\frac{11.25 \text{ kN} \cdot \text{m}^3}{EI}$$

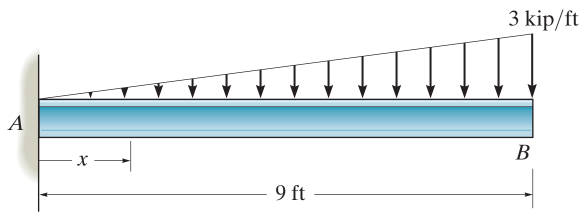
$$= -\frac{11.25(10^3)}{200(10^9)\left[\frac{\pi}{4}(0.05^4)\right]}$$

$$= -0.01146 \text{ m} = -11.5 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$v_{max} = -11.5$ mm

16–22.

Determine the elastic curve for the cantilevered W14 × 30 beam using the x coordinate. Specify the maximum slope and maximum deflection. $E = 29(10^3)$ ksi.



SOLUTION

Referring to the FBD of the beam's cut segment shown in Fig. *b*,

$$\circlearrowleft + \Sigma M_o = 0; \quad M(x) + 81 + \tfrac{1}{2}\left(\tfrac{1}{3}x\right)(x)\left(\tfrac{x}{3}\right) - 13.5x = 0$$

$$M(x) = \left(13.5x - 0.05556x^3 - 81\right) \text{ kip} \cdot \text{ft.}$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \left(13.5x - 0.05556x^3 - 81\right) \text{ kip} \cdot \text{ft}$$

$$EI\frac{dv}{dx} = \left(6.75x^2 - 0.01389x^4 - 81x + C_1\right) \text{ kip} \cdot \text{ft}^2 \qquad \textbf{(1)}$$

$$EI\,v = \left(2.25x^3 - 0.002778x^5 - 40.5x^2 + C_1x + C_2\right) \text{ kip} \cdot \text{ft}^3 \qquad \textbf{(2)}$$

At $x = 0, \dfrac{dv}{dx} = 0$. Then, Eq. (1) gives

$$EI(0) = 6.75\left(0^2\right) - 0.01389\left(0^4\right) - 81(0) + C_1 \qquad C_1 = 0$$

Also, at $x = 0$, $v = 0$. Then Eq. (2) gives

$$EI(0) = 2.25\left(0^3\right) - 0.002778\left(0^5\right) - 40.5\left(0^2\right) + 0 + C_2 \qquad C_2 = 0$$

Substituting the value of C_1 into Eq. (1) gives

$$\frac{dv}{dx} = \frac{1}{EI}\left(6.75x^2 - 0.01389x^4 - 81x\right) \text{ kip} \cdot \text{ft}^2$$

The maximum slope occurs at $x = 9$ ft. Thus,

$$\theta_{\max} = \left.\frac{dv}{dx}\right|_{x=9\text{ft}} = -\frac{273.375 \text{ kip} \cdot \text{ft}^2}{EI}$$

For $W14 \times 30$, $I = 291 \text{ in}^4$. Thus,

$$\theta_{\max} = -\frac{273.375(12^2)}{(29 \times 10^3)(291)} = -0.00466 \text{ rad} \qquad \textbf{Ans.}$$

Substitute the values of C_1 and C_2 into Eq. (2).

$$v = \frac{1}{EI}\left(2.25x^3 - 0.002778x^5 - 40.5x^2\right) \text{ kip} \cdot \text{ft}^3 \qquad \textbf{Ans.}$$

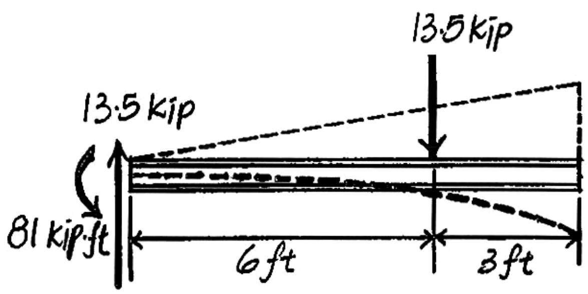
16–22. Continued

The maximum deflection occurs at $x = 9$ ft. Thus,

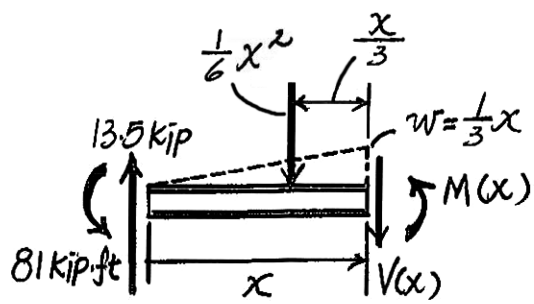
$$v_{max} = v\Big|_{x=9\text{ ft}} = -\frac{1804.275 \text{ kip}\cdot\text{ft}^3}{EI}$$

$$= -\frac{1804.275\left(12^3\right)}{29.0\left(10^3\right)(291)}$$

$$= -0.369 \text{ in.}$$ **Ans.**



(a)

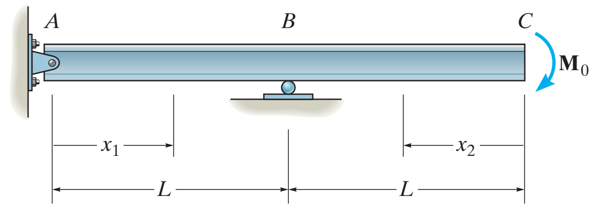


(b)

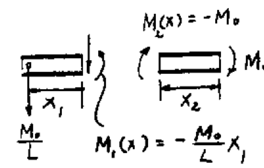
Ans:
$v = \frac{1}{EI}(2.25x^3 - 0.002778x^5 - 40.5x^2)$ kip·ft^3,
$\theta_{max} = -0.00466$ rad, $v_{max} = -0.369$ in.

16–23.

Determine the equations of the elastic curve using the coordinates x_1 and x_2. What is the deflection and slope at C? EI is constant.



SOLUTION



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\dfrac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \quad \textbf{(1)}$$

$$EIv_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \quad \textbf{(2)}$$

For $M_2(x) = -M_0;$ $\qquad EI\dfrac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \quad \textbf{(3)}$$

$$EIv_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \quad \textbf{(4)}$$

Boundary Conditions:

At $x_1 = 0, v_1 = 0.$

From Eq. (2),

$0 = 0 + 0 + C_2;\; C_2 = 0$

At $x_1 = x_2 = L, v_1 = v_2 = 0.$

From Eq. (2),

$$0 = -\frac{M_0L^2}{6} + C_1L; \qquad C_1 = \frac{M_0L}{6}$$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \quad \textbf{(5)}$$

16–23. Continued

Continuity Condition:

At $x_1 = x_2 = L, \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$.

From Eqs. (1) and (3),

$$-\frac{M_0L}{2} + \frac{M_0L}{6} = -(-M_0L + C_3); \qquad C_3 = \frac{4M_0L}{3}$$

Substituting C_3 into Eq. (5) yields

$$C_4 = -\frac{5M_0L^2}{6}$$

The Slope:

$$\frac{dv_2}{dx_2} = \frac{1}{EI}\left[-M_0x_2 + \frac{4M_0L}{3}\right]$$

$$\theta_C = \left.\frac{dv_2}{dx_2}\right|_{x_2=0} = \frac{4M_0L}{3EI} \circlearrowright \qquad \textbf{Ans.}$$

The Elastic Curve:

$$v_1 = \frac{M_0}{6EIL}(-x_1^3 + L^2x_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{M_0}{6EIL}(-3Lx_2^2 + 8L^2x_2 - 5L^3) \qquad \textbf{Ans.}$$

$$v_C = v_2\big|_{x_2=0} = -\frac{5M_0L^2}{6EI} \qquad \textbf{Ans.}$$

The negative sign indicates downward deflection.

Ans:

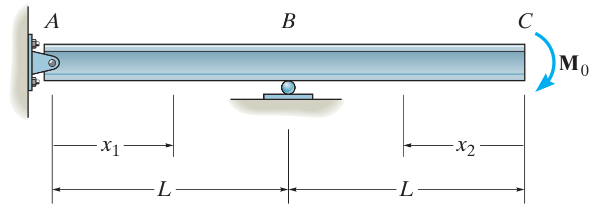
$\theta_C = \dfrac{4M_0L}{3EI} \circlearrowright, v_1 = \dfrac{M_0}{6EIL}(-x_1^3 + L^2x_1),$

$v_2 = \dfrac{M_0}{6EIL}(-3Lx_2^2 + 8L^2x_2 - 5L^3),$

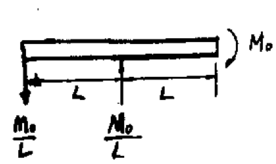
$v_C = -\dfrac{5M_0L^2}{6EI}$

***16–24.**

Determine the equations of the elastic curve using the coordinates x_1 and x_2. What is the slope at A? EI is constant.



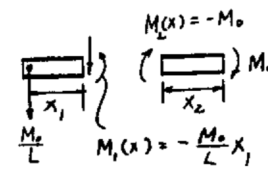
SOLUTION



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\frac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$



$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \quad \textbf{(1)}$$

$$EIv_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \quad \textbf{(2)}$$

For $M_2(x) = -M_0$; $\quad EI\frac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \quad \textbf{(3)}$$

$$EIv_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \quad \textbf{(4)}$$

Boundary Conditions:

At $x_1 = 0$, $v_1 = 0$.

From Eq. (2),

$0 = 0 + 0 + C_2$; $\quad C_2 = 0$

At $x_1 = x_2 = L$, $v_1 = v_2 = 0$.

From Eq. (2),

$$0 = -\frac{M_0L^2}{6} + C_1L; \quad C_1 = \frac{M_0L}{6}$$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \quad \textbf{(5)}$$

***16–24. Continued**

Continuity Condition:

At $x_1 = x_2 = L$, $\qquad \dfrac{dv_1}{dx_1} = -\dfrac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{M_0L}{2} + \frac{M_0L}{6} = -(-M_0L + C_3); \qquad C_3 = \frac{4M_0L}{3}$$

Substituting C_3 into Eq. (5) yields

$$C_4 = -\frac{5M_0L^2}{6}$$

The Elastic Curve:

$$v_1 = \frac{M_0}{6EIL}(-x_1^3 + L^2x_1) \qquad \textbf{Ans.}$$

$$v_2 = \frac{M_0}{6EIL}(-3Lx_2^2 + 8L^2x_2 - 5L^3) \qquad \textbf{Ans.}$$

From Eq. (1),

$$EI\frac{dv_1}{dx_1} = 0 + C_1 = \frac{M_0L}{6}$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=0} = \frac{M_0L}{6EI} \qquad \textbf{Ans.}$$

Ans:

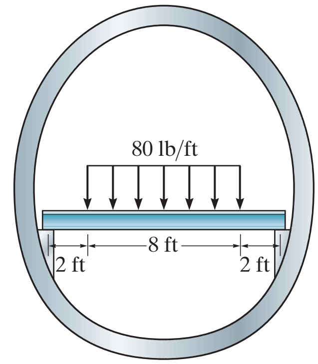
$v_1 = \dfrac{M_0}{6EIL}(-x_1^3 + L^2x_1)$,

$v_2 = \dfrac{M_0}{6EIL}(-3Lx_2^2 + 8L^2x_2 - 5L^3)$,

$\theta_A = \dfrac{M_0L}{6EI}$

16–25.

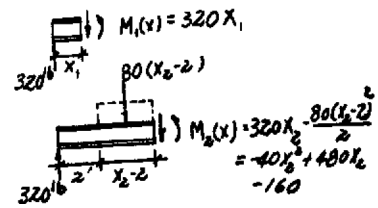
The floor beam of the airplane is subjected to the loading shown. Assuming that the fuselage exerts only vertical reactions on the ends of the beam, determine the maximum deflection of the beam. EI is constant.



SOLUTION

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$



For $M_1(x) = 320x_1$,

$$EI\frac{d^2v_1}{dx_1^2} = 320x_1$$

$$EI\frac{dv_1}{dx_1} = 160x_1^2 + C_1 \qquad (1)$$

$$EIv_1 = 53.33x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -40x_2^2 + 480x_2 - 160$,

$$EI\frac{d^2v_2}{dx_2^3} = -40x_2^2 + 480x_2 - 160$$

$$EI\frac{dv_2}{dx_2} = -13.33x_2^3 + 240x_2^2 - 160x_2 + C_1 \qquad (3)$$

$$EIv_2 = -3.33x_2^4 + 80x_2^3 - 80x_2^2 + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions:

$$v_1 = 0 \quad \text{at} \quad x_1 = 0$$

From Eq. (2), $\quad C_2 = 0$

Due to symmetry,

$$\frac{dv_2}{dx_2} = 0 \quad \text{at} \quad x_2 = 6\text{ ft}$$

From Eq. (3),

$$-2880 + 8640 - 960 + C_3 = 0$$

$$C_3 = -4800$$

16–25. Continued

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = 2\text{ ft}$$

From Eqs. (1) and (3),

$$640 + C_1 = -106.67 + 960 - 320 - 4800$$

$$C_1 = -4906.67$$

$$v_1 = v_2 \quad \text{at} \quad x_1 = x_2 = 2\text{ ft}$$

From Eqs. (2) and (4),

$$426.67 - 9813.33 = -53.33 + 640 - 320 - 9600 + C_4$$

$$C_4 = -53.33$$

$$v_2 = \frac{1}{EI}(-3.33x_2^4 + 80x_2^3 - 80x_2^2 - 4800x_2 - 53.33)$$

$v_{\max}$ occurs at $x_2 = 6\text{ft}$.

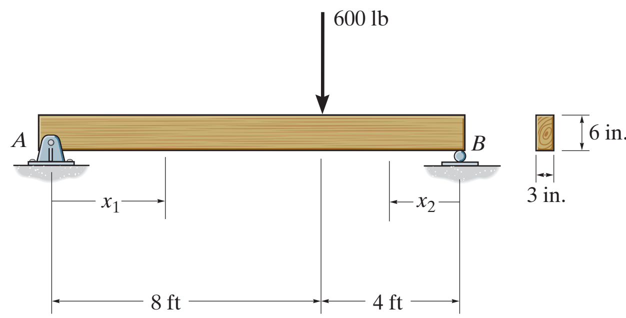
$$v_{\max} = v_2\big|_{x_2=6} = \frac{-18.8\text{ kip}\cdot\text{ft}^3}{EI} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

Ans:
$v_{\max} = \dfrac{-18.8\text{ kip}\cdot\text{ft}^3}{EI}$

16–26.

Determine the maximum deflection of the simply supported beam. The beam is made of wood having a modulus of elasticity of $E = 1.5\ (10^3)$ ksi.



SOLUTION

Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Function. Referring to the free-body diagrams of the beam's cut segments, Fig. *b*, $M(x_1)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) - 200(x_1) = 0 \qquad M(x_1) = 200x_1 \text{ lb} \cdot \text{ft}$$

and $M(x_2)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad 400(x_2) - M(x_2) = 0 \qquad M(x_2) = 400x_2 \text{ lb} \cdot \text{ft}$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = 200x_1$$

$$EI\frac{dv_1}{dx_1} = 100x_1^2 + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{100}{3}x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = 400x_2$$

$$EI\frac{dv_2}{dx_2} = 200x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = \frac{200}{3}x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions. At $x_1 = 0$, $v_1 = 0$. Then, Eq. (2) gives

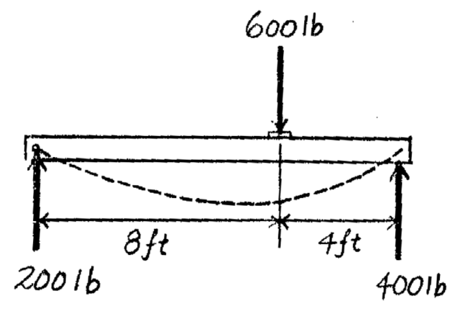
$$EI(0) = \frac{100}{3}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

Also, at $x_2 = 0$, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = \frac{200}{3}(0^3) + C_3(0) + C_4 \qquad C_4 = 0$$

16–26. Continued

Continuity Conditions. At $x_1 = 8$ ft and $x_2 = 4$ ft, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give



(a)

$$100(8^2) + C_1 = -\left[200(4^2) + C_3\right]$$

$$C_1 + C_3 = -9600 \qquad \textbf{(5)}$$

At $x_1 = 8$ ft and $x_2 = 4$ ft, $v_1 = v_2$. Then Eqs. (2) and (4) gives

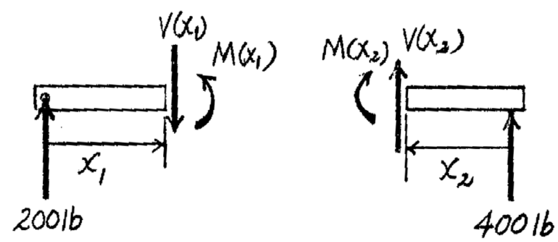
$$\frac{100}{3}(8^3) + C_1(8) = \frac{200}{3}(4^3) + C_3(4)$$

$$4C_3 - 8C_1 = 12800 \qquad \textbf{(6)}$$

Solving Eqs. (5) and (6),

$$C_1 = -4266.67 \text{ lb}\cdot\text{ft}^2 \qquad C_3 = -5333.33 \text{ lb}\cdot\text{ft}^2$$

Substituting the result of C_1 into Eq. (1),



(b)

$$\frac{dv_1}{dx_1} = \frac{1}{EI}(100x_1^2 - 4266.67)$$

$$\frac{dv_1}{dx_1} = 0 = \frac{1}{EI}(100x_1^2 - 4266.67) \qquad x_1 = 6.5320 \text{ ft}$$

Substituting the result of C_1 and C_2 into Eq. (2),

$$v_1 = \frac{1}{EI}\left(\frac{100}{3}x_1^3 - 4266.67x_1\right)$$

v_{max} occurs at $x_1 = 6.5320$ ft, where $\frac{dv_1}{dx_1} = 0$. Thus,

$$v_{max} = v_1\big|_{x_1=6.5320\text{ ft}} = -\frac{18579.83 \text{ lb}\cdot\text{ft}^3}{EI}$$

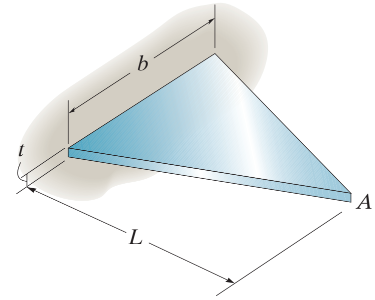
$$= -\frac{18579.83(1728)}{1.5(10^6)\left[\frac{1}{12}(3)(6^3)\right]}$$

$$= -0.396 \text{ in.} \qquad \textbf{Ans.}$$

Ans:
$v_{max} = -0.396$ in.

16–27.

The beam is made of a material having a specific weight γ. Determine the displacement and slope at its end A due to its weight. The modulus of elasticity for the material is E.

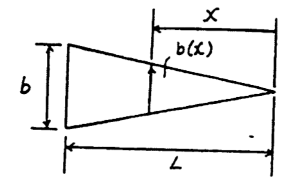


SOLUTION

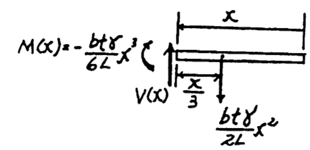
Section Properties:

$$b(x) = \frac{b}{L}x \qquad V(x) = \frac{1}{2}\left(\frac{b}{L}x\right)(x)(t) = \frac{bt}{2L}x^2$$

$$I(x) = \frac{1}{12}\left(\frac{b}{L}x\right)t^3 = \frac{bt^3}{12L}x$$



Moment Function: As shown on FBD.



Slope and Elastic Curve:

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = -\frac{\frac{bt\gamma}{6L}x^3}{\frac{bt^3}{12L}x} = -\frac{2\gamma}{t^2}x^2$$

$$E\frac{dv}{dx} = -\frac{2\gamma}{3t^2}x^3 + C_1 \qquad \textbf{(1)}$$

$$Ev = -\frac{\gamma}{6t^2}x^4 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: $\frac{dv}{dx} = 0$ at $x = L$ and $v = 0$ at $x = L$.

From Eq. (1), $\quad 0 = -\frac{2\gamma}{3t^2}(L^3) + C_1 \qquad C_1 = \frac{2\gamma L^3}{3t^2}$

From Eq. (2), $\quad 0 = -\frac{\gamma}{6t^2}(L^4) + \left(\frac{2\gamma L^3}{3t^2}\right)(L) + C_2$

$$C_2 = -\frac{\gamma L^4}{2^2}$$

The Slope: Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{2\gamma}{3t^2E}\left(-x^3 + L^3\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{2\gamma L^3}{3t^2E} \qquad \textbf{Ans.}$$

The Elastic Curve: Substituting the value of C_1 and C_2 into Eq. (2),

$$v = \frac{\gamma}{6t^2E}\left(-x^4 + 4L^3x - 3L^4\right)$$

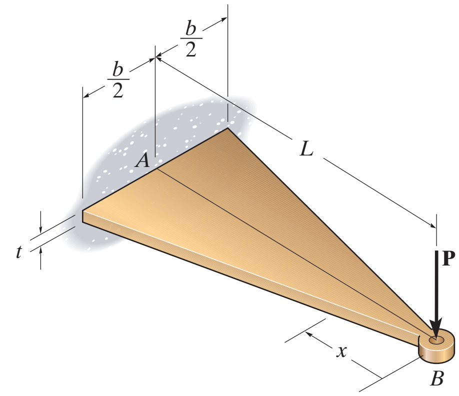
$$v_A = v|_{x=0} = -\frac{\gamma L^4}{2t^2E} \qquad \textbf{Ans.}$$

The negative sign indicates downward displacement.

Ans:
$\theta_A = \frac{2\gamma L^3}{3t^2E}$,
$v_A = -\frac{\gamma L^4}{2t^2E}$

***16–28.**

Determine the slope at end B and the maximum deflection of the cantilever triangular plate of constant thickness t. The plate is made of material having a modulus of elasticity of E.



SOLUTION

Section Properties: Referring to the geometry shown in Fig. a,

$$\frac{b(x)}{x} = \frac{b}{L}; \qquad b(x) = \frac{b}{L}x$$

Thus, the moment of the plate as a function of x is

$$I(x) = \frac{1}{12}[b(x)]t^3 = \frac{bt^3}{12L}x$$

Moment Functions. Referring to the free-body diagram of the plate's cut segment, Fig. b,

$$\circlearrowleft +\Sigma M_O = 0; \qquad -M(x) - Px = 0 \qquad M(x) = -Px$$

Equations of Slope and Elastic Curve.

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = \frac{-Px}{\frac{bt^3}{12L}x} = -\frac{12PL}{bt^3}$$

$$E\frac{dv}{dx} = -\frac{12PL}{bt^3}x + C_1 \qquad \textbf{(1)}$$

$$Ev = -\frac{6PL}{bt^3}x^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions. At $x = L$, $\frac{dv}{dx} = 0$. Then Eq. (1) gives

$$E(0) = -\frac{12PL}{bt^3}(L) + C_1 \qquad C_1 = \frac{12PL^2}{bt^3}$$

At $x = L$, $v = 0$. Then Eq. (2) gives

$$E(0) = -\frac{6PL}{bt^3}(L^2) + C_1(L) + C_2 \qquad C_2 = -\frac{6PL^3}{bt^3}$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{12PL}{bt^3E}(-x + L)$$

At B, $x = 0$. Thus,

$$\theta_B = \left.\frac{dv}{dx}\right|_{x=0} = \frac{12PL^2}{bt^3E} \qquad \textbf{Ans.}$$

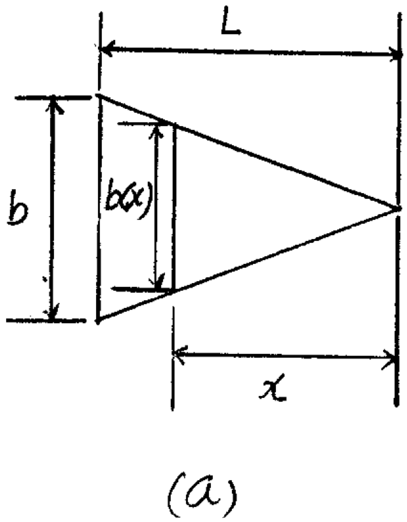
***16–28. Continued**

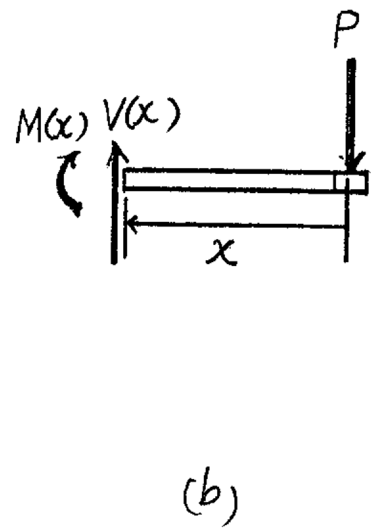
Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{6PL}{Ebt^3}(-x^2 + 12Lx - L^2)$$

v_{max} occurs at $x = 0$. Thus,

$$v_{max} = v|_{x=0} = -\frac{6PL^3}{Ebt^3} = \frac{6PL^3}{Ebt^3} \downarrow$$ **Ans.**

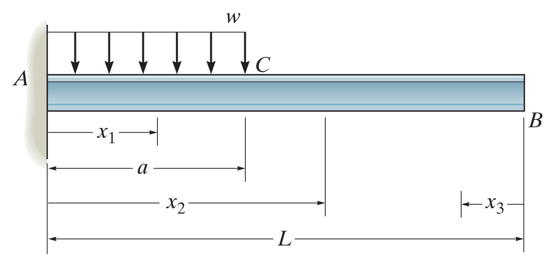




Ans:
$\theta_B = \frac{12PL^2}{bt^3E}$,
$v_{max} = \frac{6PL^3}{Ebt^3} \downarrow$

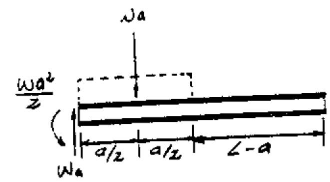
16–29.

Determine the equation of the elastic curve using the coordinates x_1 and x_2. What is the slope and deflection at B? EI is constant.



SOLUTION

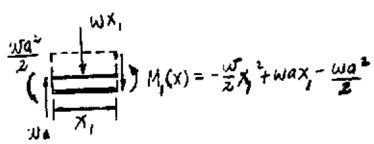
$$EI\frac{d^2v}{dx^2} = M(x)$$



For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$,

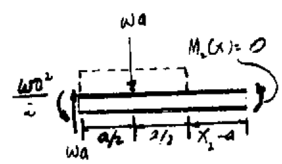
$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$



$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = 0$; $\quad EI\frac{d^2v_2}{dx_2{}^2} = 0$



$$EI\frac{dv_2}{dx_2} = C_3 \qquad \textbf{(3)}$$

$$EI\,v_2 = C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

At $x_1 = 0$, $\quad \frac{dv_1}{dx_1} = 0$

From Eq. (1), $\quad C_1 = 0$

At $x_1 = 0$, $v_1 = 0$.

From Eq. (2); $\quad C_2 = 0$

Continuity Conditions:

At $x_1 = a$, $\quad x_2 = a$; $\quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = C_3; \qquad C_3 = -\frac{wa^3}{6}$$

16–29. Continued

From Eqs. (2) and (4),

At $x_1 = a, \quad x_2 = a \quad v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = -\frac{wa^4}{6} + C_4; \qquad C_4 = \frac{wa^4}{24}$$

The slope, from Eq. (3), is

$$\theta_B = \frac{dv_1}{dx_2} = -\frac{wa^3}{6EI}$$ **Ans.**

The Elastic Curve:

$$v_1 = \frac{w}{24EI}(-x_1^4 + 4ax_1^3 - 6a^2x_1^2)$$ **Ans.**

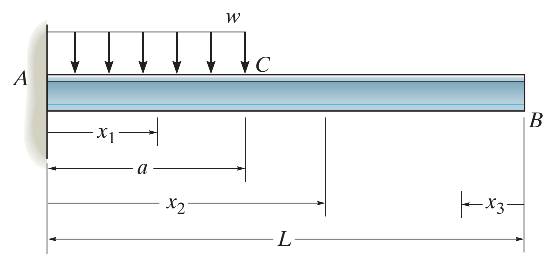
$$v_2 = \frac{wa^3}{24EI}(-4x_2 + a)$$ **Ans.**

$$v_B = v_2\Big|_{x_2=L} = \frac{wa^3}{24EI}(-4L + a)$$ **Ans.**

Ans:
$\theta_B = -\frac{wa^3}{6EI}$,
$v_1 = \frac{w}{24EI}(-x_1^4 + 4ax_1^3 - 6a^2x_1^2)$,
$v_2 = \frac{wa^3}{24EI}(-4x_2 + a)$,
$v_B = \frac{wa^3}{24EI}(-4L + a)$

16–30.

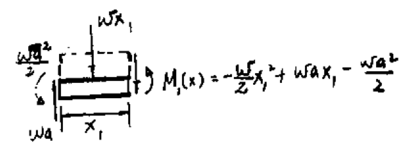
Determine the equations of the elastic curve using the coordinates x_1 and x_3. What is the slope and deflection at point B? EI is constant.



SOLUTION

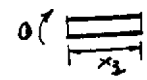
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\dfrac{w}{2}x_1^2 + wax_1 - \dfrac{wa^2}{2}$



$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad \textbf{(1)}$$



$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_3(x) = 0, \qquad EI\dfrac{d^2v_3}{dx_3^2} = 0$ **(3)**

$$EI\frac{dv_3}{dx_3} = C_3 \qquad \textbf{(4)}$$

$$EI\,v_3 = C_3x_3 + C_4$$

Boundary Conditions:

At $x_1 = 0, \dfrac{dv_1}{dx_1} = 0$

From Eq. (1),

$0 = -0 + 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0, \quad v_1 = 0$

From Eq. (2),

$0 = -0 - 0 - 0 + 0 + C_2; \qquad C_2 = 0$

Continuity Conditions:

At $x_1 = a, \qquad x_3 = L - a; \qquad \dfrac{dv_1}{dx_1} = -\dfrac{dv_3}{dx_3}$

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = -C_3; \quad C_3 = +\frac{wa^3}{6}$$

16–30. Continued

At $x_1 = a, x_3 = L - a \qquad v_1 = v_3$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = \frac{wa^3}{6}(L - a) + C_4; \quad C_4 = \frac{wa^4}{24} - \frac{wa^3L}{6}$$

The Slope:

$$\frac{dv_1}{dx_3} = \frac{wa^3}{6EI}$$

$$\theta_B = \frac{dv_1}{dx_3}\bigg|_{x_3=0} = \frac{wa^3}{6EI} \qquad \textbf{Ans.}$$

The Elastic Curve:

$$v_1 = \frac{wx_1^2}{24EI}(-x_1^2 + 4ax_1 - 6a^2) \qquad \textbf{Ans.}$$

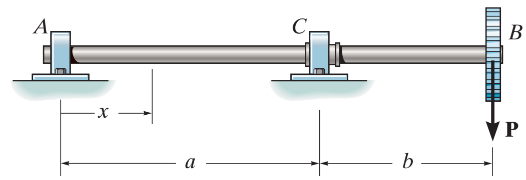
$$v_2 = \frac{wa^3}{24EI}(4x_3 + a - 4L) \qquad \textbf{Ans.}$$

$$v_B = v_3\bigg|_{x_3=a} = \frac{wa^3}{24EI}(a - 4L) \qquad \textbf{Ans.}$$

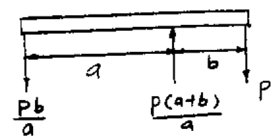
Ans:
$\theta_B = -\frac{wa^3}{6EI}, v_1 = \frac{wx_1^2}{24EI}(-x_1^2 + 4ax_1 - 6a^2),$
$v_2 = \frac{wa^3}{24EI}(4x_3 + a - 4L), v_B = \frac{wa^3}{24EI}(a - 4L)$

16–31.

The shaft is supported at A by a journal bearing and at C by a thrust bearing. Determine the equation of the elastic curve. EI is constant.



SOLUTION



$$M = -\frac{Pb}{a}\langle x - 0\rangle - \left(-\frac{P(a+b)}{a}\langle x - a\rangle\right) = -\frac{Pb}{a}x + \frac{P(a+b)}{a}\langle x - a\rangle$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{Pb}{a}x + \frac{P(a+b)}{a}\langle x - a\rangle$$

$$EI\frac{dv}{dx} = -\frac{Pb}{2a}x^2 + \frac{P(a+b)}{2a}\langle x - a\rangle^2 + C_1 \qquad \textbf{(1)}$$

$$EIv = -\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x - a\rangle^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

At $x = 0$, $v = 0$.

From Eq. (2),

$$0 = -0 + 0 + 0 + C_2; \qquad C_2 = 0$$

At $x = a$, $v = 0$.

From Eq. (2),

$$0 = -\frac{Pb}{6a}(a^3) + 0 + C_1a + 0; \qquad C_1 = \frac{Pab}{6}$$

From Eq. (2),

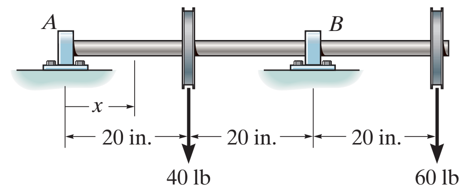
$$v = \frac{1}{EI}\left[-\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x - a\rangle^3 + \frac{Pab}{6}x\right] \qquad \textbf{Ans.}$$

Ans:

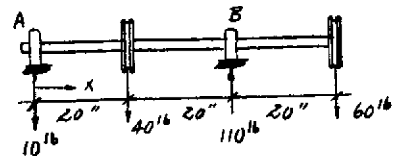
$$v = \frac{1}{EI}\left[-\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x - a\rangle^3 + \frac{Pab}{6}x\right]$$

***16–32.**

The shaft supports the two pulley loads shown. Determine the equation of the elastic curve. EI is constant.



SOLUTION



$M = -10\langle x - 0\rangle - 40\langle x - 20\rangle - (-110)\langle x - 40\rangle$

$M = -10x - 40\langle x - 20\rangle + 110\langle x - 40\rangle$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -10x - 40\langle x - 20\rangle + 110\langle x - 40\rangle$$

$$EI\frac{dv}{dx} = -5x^2 - 20\langle x - 20\rangle^2 + 55\ \langle x - 40\rangle^2 + C_1$$

$$EIv = -1.667x^3 - 6.667\langle x - 20\rangle^3 + 18.33\langle x - 40\rangle^3 + C_1x + C_2 \qquad \textbf{(1)}$$

Boundary Conditions:

$v = 0$ at $x = 0$

From Eq. (1):

$C_2 = 0$

$v = 0$ at $x = 40$ in.

$0 = -106{,}666.67 - 53{,}333.33 + 0 + 40C_1$

$C_1 = 4000$

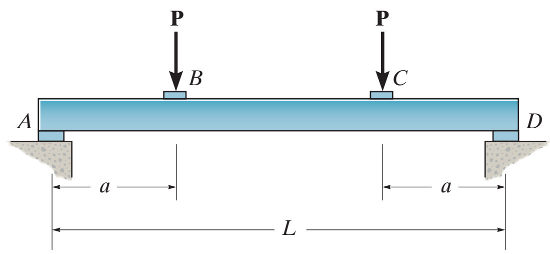
$$v = \frac{1}{EI}[-1.67x^3 - 6.67\langle x - 20\rangle^3 + 18.3\langle x - 40\rangle^3 + 4000x]\ \text{lb}\cdot\text{in}^3 \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{1}{EI}[-1.67x^3 - 6.67\langle x - 20\rangle^3 + 18.3\langle x - 40\rangle^3 + 4000x]\ \text{lb}\cdot\text{in}^3$$

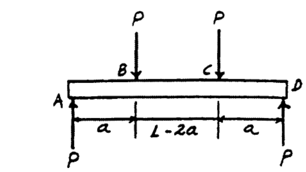
16–33.

The beam is made of a ceramic material. If it is subjected to the elastic loading shown, and the moment of inertia is I and the beam has a measured maximum deflection Δ at its center, determine the modulus of elasticity, E. The supports at A and D exert only vertical reactions on the beam.



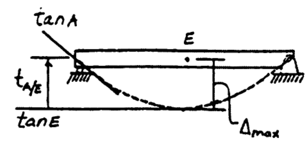
SOLUTION

Moment-Area Theorems: Due to symmetry, the slope at midspan (point E) is zero. Hence, the maximum displacement is



$$\Delta_{\max} = t_{A/E} = \left(\frac{Pa}{EI}\right)\left(\frac{L-2a}{2}\right)\left(a + \frac{L-2a}{4}\right) + \frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(\frac{2}{3}a\right)$$

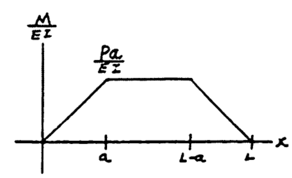
$$= \frac{Pa}{24EI}(3L^2 - 4a^2)$$



Require $\Delta_{\max} = \Delta$, then

$$\Delta = \frac{Pa}{24EI}(3L^2 - 4a^2)$$

$$E = \frac{Pa}{24\Delta I}(3L^2 - 4a^2) \qquad \textbf{Ans.}$$

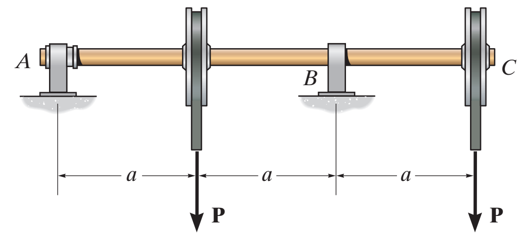


Ans:

$$E = \frac{Pa}{24\Delta I}(3L^2 - 4a^2)$$

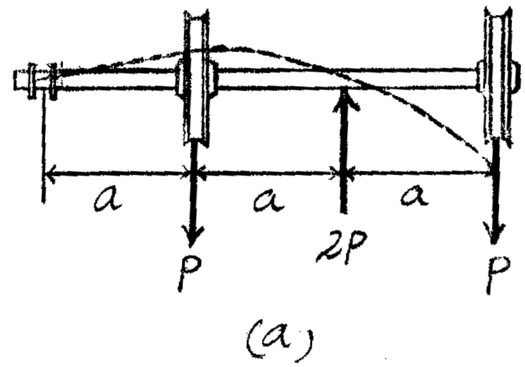
16–34.

Determine the equation of the elastic curve, the maximum deflection in region AB, and the deflection of end C. EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. a.



Moment Function:

$$M = -P\langle x - a\rangle - (-2P)\langle x - 2a\rangle$$
$$= -P\langle x - a\rangle + 2P\langle x - 2a\rangle$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -P\langle x - a\rangle + 2P\langle x - 2a\rangle$$

$$EI\frac{dv}{dx} = \frac{-P}{2}\langle x - a\rangle^2 + P\langle x - 2a\rangle^2 + C_1 \qquad \textbf{(1)}$$

$$EIv = \frac{-P}{6}\langle x - a\rangle^3 + \frac{P}{3}\langle x - 2a\rangle^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: At $x = 0, v = 0$. Then Eq. (2) gives

$$EI(0) = -0 + 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 2a, v = 0$. Then Eq. (2) gives

$$EI(0) = -\frac{P}{6}(2a - a)^3 + \frac{P}{3}(2a - 2a)^3 + C_1(2a) + 0 \qquad C_1 = \frac{Pa^2}{12}$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{P}{12EI}[-6\langle x - a\rangle^2 + 12\langle x - 2a\rangle^2 + a^2]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region $a < x < 2a$,

$$-6(x - a)^2 + 0 + a^2 = 0 \qquad x = 1.4082a \qquad \textbf{(O.K.)}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{P}{12EI}[-2\langle x - a\rangle^3 + 4\langle x - 2a\rangle^3 + a^2x] \qquad \textbf{Ans.}$$

$(v_{max})_{AB}$ occurs at $x = 1.4082a$, where $\frac{dv}{dx} = 0$. Thus,

$$(v_{max})_{AB} = v|_{x=1.4082a} = \frac{P}{12EI}[-2\langle 1.4082a - a\rangle^3 + 0 + a^2(1.4082a)]$$

$$= \frac{0.106Pa^3}{EI} \qquad \textbf{Ans.}$$

At $C, x = 3a$. Thus,

$$v_C = v|_{x=3a} = \frac{P}{12EI}[-2\langle 3a - a\rangle^3 + 4(3a - 2a)^3 + a^2(3a)]$$

$$= -\frac{3Pa^3}{4EI} \qquad \textbf{Ans.}$$

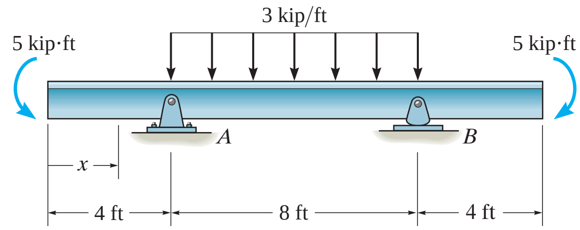
Ans:

$v = \frac{P}{12EI}[-2\langle x - a\rangle^3 + 4\langle x - 2a\rangle^3 + a^2x]$,

$(v_{max})_{AB} = \frac{0.106Pa^3}{EI}, v_C = -\frac{3Pa^3}{4EI}$

16–35.

The beam is subjected to the load shown. Determine the equation of the elastic curve. EI is constant.



SOLUTION

$$M = -5\langle x-0\rangle^0 - (-12)\langle x-4\rangle - \frac{3}{2}\langle x-4\rangle^2 - (-12)\langle x-12\rangle - \left(\frac{-3}{2}\right)\langle x-12\rangle^2$$

$$M = -5 + 12\langle x-4\rangle - \frac{3}{2}\langle x-4\rangle^2 + 12\langle x-12\rangle + \frac{3}{2}\langle x-12\rangle^2$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -5 + 12\langle x-4\rangle - \frac{3}{2}\langle x-4\rangle^2 + 12\langle x-12\rangle + \frac{3}{2}\langle x-12\rangle^2$$

$$EI\frac{dv}{dx} = -5x + 6\langle x-4\rangle^2 - \frac{1}{2}\langle x-4\rangle^3 + 6\langle x-12\rangle^2 + \frac{1}{2}\langle x-12\rangle^3 + C_1 \qquad (1)$$

$$EIv = \frac{-5}{2}x^2 + 2\langle x-4\rangle^3 - \frac{1}{8}\langle x-4\rangle^4 + 2\langle x-12\rangle^3 + \frac{1}{8}\langle x-12\rangle^4 + C_1x + C_2 \qquad (2)$$

Boundary Conditions:

$v = 0$ at $x = 4$ ft

From Eq. (2),

$$0 = -40 + 0 - 0 + 0 + 0 + 4C_1 + C_2$$

$$4C_1 + C_2 = 40 \qquad (3)$$

$v = 0$ at $x = 12$ ft

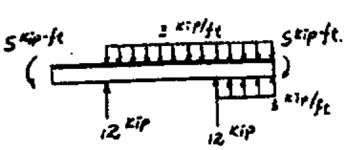
$$0 = -360 + 1024 - 512 + 0 + 0 + 12C_1 + C_2$$

$$12C_1 + C_2 = -152 \qquad (4)$$

Solving Eqs. (3) and (4) yields:

$C_1 = -24$ $\qquad C_2 = 136$

$$v = \frac{1}{EI}[-2.5x^2 + 2\langle x-4\rangle^3 - \frac{1}{8}\langle x-4\rangle^4 + 2\langle x-12\rangle^3 + \frac{1}{8}\langle x-12\rangle^4 - 24x + 136] \text{ kip}\cdot\text{ft}^3 \quad \textbf{Ans.}$$

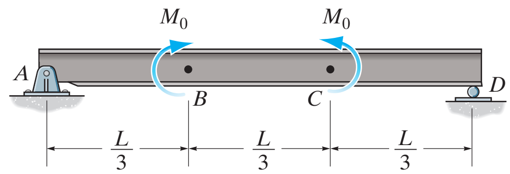


Ans:

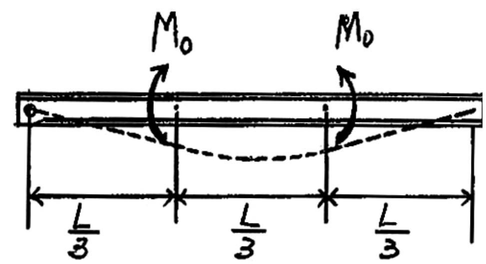
$$v = \frac{1}{EI}\Big[-2.5x^2 + 2\langle x-4\rangle^3 - \frac{1}{8}\langle x-4\rangle^4 + 2\langle x-12\rangle^3 + \frac{1}{8}\langle x-12\rangle^4 - 24x + 136\Big] \text{ kip}\cdot\text{ft}^3$$

***16–36.**

Determine the equation of the elastic curve, the slope at A, and the deflection at B. EI is constant.



SOLUTION



(a)

Support Reactions and Elastic Curve: As shown in Fig. *a*.

Moment Function:

$$M = -(-M_0)\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$= M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$EI\frac{dv}{dx} = M_0\left\langle x - \frac{L}{3}\right\rangle - M_0\left\langle x - \frac{2}{3}L\right\rangle + C_1 \qquad \textbf{(1)}$$

$$EIv\,\frac{M_0}{2}\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{M_0}{2}\left\langle x - \frac{2}{3}L\right\rangle^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: Due to symmetry, $\dfrac{dv}{dx} = 0$ at $x = \dfrac{L}{2}$. Then Eq. (1) gives

$$EI(0) = M_0\left(\frac{L}{2} - \frac{L}{3}\right) - 0 + C_1 \qquad C_1 = -\frac{M_0L}{6}$$

At $x = 0, v = 0$. Then, Eq. (2) gives

$$EI(0) = 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{M_0}{6EI}\left[6\left\langle x - \frac{L}{3}\right\rangle - 6\left\langle x - \frac{2}{3}L\right\rangle - L\right]$$

At $A, x = 0$. Thus,

$$\theta_A = \frac{dv}{dx}\bigg|_{x=0} = \frac{M_0}{6EI}\left[6(0) - 6(0) - L\right] = -\frac{M_0L}{6EI} \qquad \textbf{Ans.}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right] \qquad \textbf{Ans.}$$

At $B, x = \dfrac{L}{3}$. Thus,

$$v_B = v|_{x=\frac{L}{3}} = \frac{M_0}{6EI}\left[3(0) - 3(0) - L\left(\frac{L}{3}\right)\right]$$

$$= -\frac{M_0L^2}{18EI} \qquad \textbf{Ans.}$$

Ans:

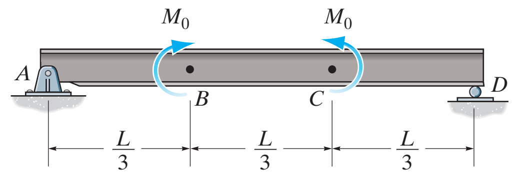
$$\theta_A = -\frac{M_0L}{6EI},$$

$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right],$$

$$v_B = -\frac{M_0L^2}{18EI}$$

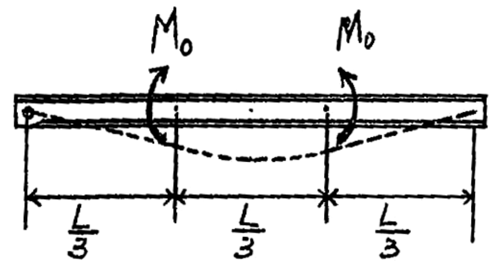
16–37.

Determine the equation of the elastic curve and the maximum deflection of the simply supported beam. EI is constant.



SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. *a*.



(a)

Moment Function:

$$M = -(-M_0)\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$= M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = M_0\left\langle x - \frac{L}{3}\right\rangle^0 - M_0\left\langle x - \frac{2}{3}L\right\rangle^0$$

$$EI\frac{dv}{dx} = M_0\left\langle x - \frac{L}{3}\right\rangle - M_0\left\langle x - \frac{2}{3}L\right\rangle + C_1 \qquad \textbf{(1)}$$

$$EIv\frac{M_0}{2}\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{M_0}{2}\left\langle x - \frac{2}{3}L\right\rangle^2 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions: Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then Eq. (1) gives

$$EI(0) = M_0\left(\frac{L}{2} - \frac{L}{3}\right) - 0 + C_1 \qquad C_1 = -\frac{M_0L}{6}$$

At $x = 0$, $v = 0$. Then, Eq. (2) gives

$$EI(0) = 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right] \qquad \textbf{Ans.}$$

v_{max} occurs at $x = \frac{L}{2}$, where $\frac{dv}{dx} = 0$. Then,

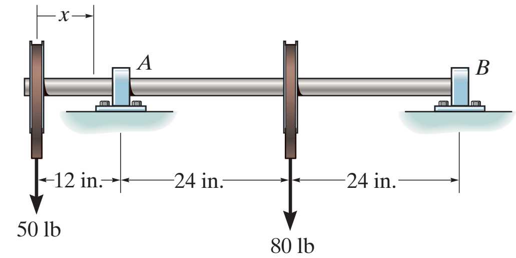
$$v_{max} = v|_{x=\frac{L}{2}} = \frac{M_0}{6EI}\left[3\left(\frac{L}{2} - \frac{L}{3}\right)^2 - 0 - L\left(\frac{L}{2}\right)\right]$$

$$= -\frac{5M_0L^2}{72EI} \qquad \textbf{Ans.}$$

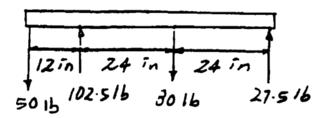
Ans:
$$v = \frac{M_0}{6EI}\left[3\left\langle x - \frac{L}{3}\right\rangle^2 - 3\left\langle x - \frac{2}{3}L\right\rangle^2 - Lx\right],$$
$$v_{max} = -\frac{5M_0L^2}{72EI}$$

16–38.

The shaft supports the two pulley loads. Determine the equation of the elastic curve. EI is constant.



SOLUTION



$$M = -50\langle x - 0\rangle - (-102.5)\langle x - 12\rangle - 80\langle x - 36\rangle$$

$$= -50x + 102.5\langle x - 12\rangle - 80\langle x - 36\rangle$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -50x + 102.5\langle x - 12\rangle - 80\langle x - 36\rangle$$

$$EI\frac{dv}{dx} = -25x^2 + \frac{102.5}{2}\langle x - 12\rangle^2 - 40\langle x - 36\rangle^2 + C_1 \qquad \textbf{(1)}$$

$$EIv = -\frac{25}{3}x^3 + \frac{102.5}{6}\langle x - 12\rangle^3 - \frac{40}{3}\langle x - 36\rangle^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

At $x = 12$ in., $v = 0$.

From Eq. (2),

$$0 = -\frac{25}{3}(12)^3 + 0 - 0 + 12C_1 + C_{2\theta}$$

$$12C_1 + C_2 = 14\,400 \qquad \textbf{(3)}$$

At $x = 60$ in., $v = 0$.

$$0 = -\frac{25}{3}(60)^3 + \frac{102.5}{6}(60 - 12)^3 - \frac{40}{3}(60 - 36)^3 + 60C_1 + C_2$$

$$60C_1 + C_2 = 95\,040 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$C_1 = 1680$ $\qquad$ $C_2 = -5760$

The elastic curve; from Eq. (2),

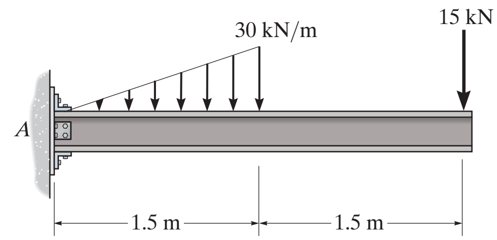
$$v = \frac{1}{EI}[-8.33x^3 + 17.1\langle x - 12\rangle^3 - 13.3\langle x - 36\rangle^3 + 1680x - 5760]\ \text{lb}\cdot\text{in}^3 \quad \textbf{Ans.}$$

Ans:

$$v = \frac{1}{EI}[-8.33x^3 + 17.1\langle x - 12\rangle^3 - 13.3\langle x - 36\rangle^3 + 1680x - 5760]\ \text{lb}\cdot\text{in}^3$$

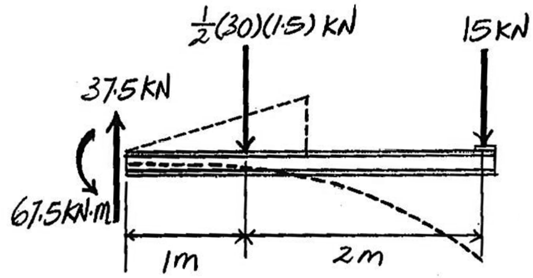
16–39.

Determine the maximum deflection of the cantilevered beam. Take $E = 200$ GPa and $I = 65.0(10^6)\ \text{mm}^4$.



SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. *a*.



(a)

Moment Function: From Fig. *b*, we obtain

$$M = -(-37.5)\langle x - 0\rangle - 67.5\langle x - 0\rangle^0 - \frac{20}{6}\langle x - 0\rangle^3$$

$$-\left(-\frac{20}{6}\right)\langle x - 1.5\rangle^3 - \left(-\frac{30}{2}\right)\langle x - 1.5\rangle^2$$

$$= 37.5x - 67.5 - \frac{10}{3}x^3 + \frac{10}{3}\langle x - 1.5\rangle^3 + 15\langle x - 1.5\rangle^2$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = 37.5x - 67.5 - \frac{10}{3}x^3 + \frac{10}{3}\langle x - 1.5\rangle^3 + 15\langle x - 1.5\rangle^2$$

$$EI\frac{dv}{dx} = 18.75x^2 - 67.5x - \frac{5}{6}x^4 + \frac{5}{6}\langle x - 1.5\rangle^4 + 5\langle x - 1.5\rangle^3 + C_1 \qquad \textbf{(1)}$$

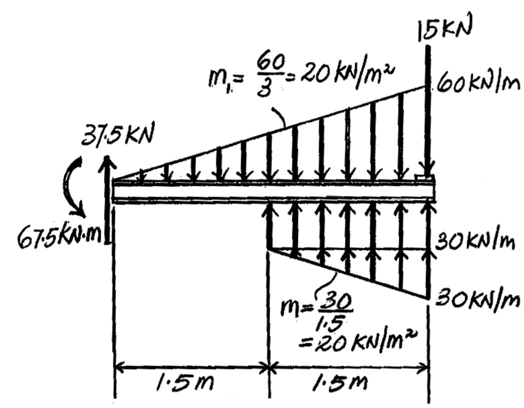
$$EIv = 6.25x^3 - 33.75x^2 - \frac{1}{6}x^5 + \frac{1}{6}\langle x - 1.5\rangle^5 + \frac{5}{4}\langle x - 1.5\rangle^4 + C_1x + C_2 \ \textbf{(2)}$$

Boundary Conditions: At $x = 0, \dfrac{dv}{dx} = 0$. Then Eq. (1) gives

$$0 = 0 - 0 - 0 + 0 + 0 + C_1 \qquad C_1 = 0$$

At $x = 0, v = 0$. Then Eq. (2) gives

$$0 = 0 - 0 - 0 + 0 + 0 + 0 + C_2 \qquad C_2 = 0$$



(b)

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[6.25x^3 - 33.75x^2 - \frac{1}{6}x^5 + \frac{1}{6}\langle x - 1.5\rangle^5 + \frac{5}{4}\langle x - 1.5\rangle^4\right]$$

v_{max} occurs at $x = 3$ m. Thus,

$$v_{max} = v|_{x=3\text{ m}}$$

$$= \frac{1}{EI}\left[6.25\left(3^3\right) - 33.75\left(3^2\right) - \frac{1}{6}\left(3^5\right) + \frac{1}{6}(3 - 1.5)^5 + \frac{5}{4}(3 - 1.5)^4\right]$$

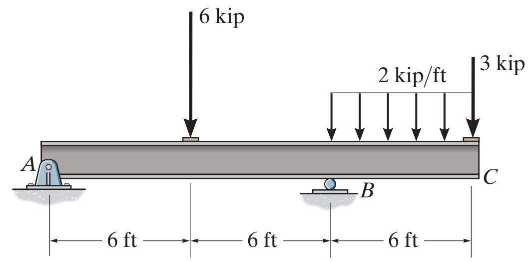
$$= -\frac{167.91\ \text{kN}\cdot\text{m}^3}{EI} = -\frac{167.91\left(10^3\right)}{200\left(10^9\right)\left[65.0\left(10^{-6}\right)\right]}$$

$$= -0.01292\ \text{m} = -12.9\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$v_{max} = -12.9$ mm

***16–40.**

Determine the slope at A and the deflection of end C of the overhang beam. Take $E = 29(10^3)$ ksi and $I = 204$ in^4.



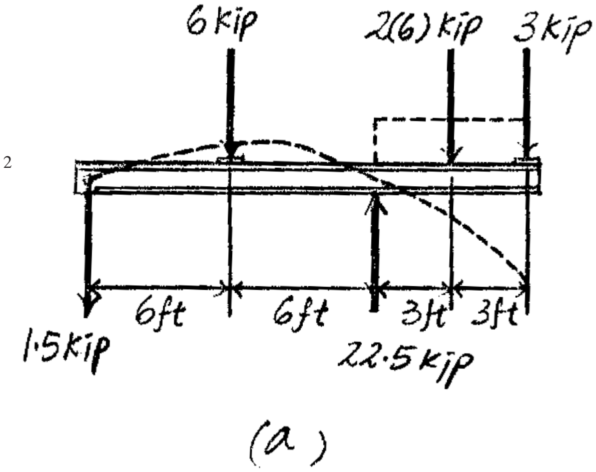
SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. *a*.

Moment Function: From Fig. *a*, we obtain

$$M = -1.5\langle x - 0\rangle - 6\langle x - 6\rangle - (-22.5)\langle x - 12\rangle - \frac{2}{2}\langle x - 12\rangle^2$$

$$= -1.5x - 6\langle x - 6\rangle + 22.5\langle x - 12\rangle - \langle x - 12\rangle^2$$



Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -1.5x - 6\langle x - 6\rangle + 22.5\langle x - 12\rangle - \langle x - 12\rangle^2$$

$$EI\frac{dv}{dx} = -0.75x^2 - 3\langle x - 6\rangle^2 + 11.25\langle x - 12\rangle^2 - \frac{1}{3}\langle x - 12\rangle^3 + C_1 \quad \textbf{(1)}$$

$$EIv = -0.25x^3 - \langle x - 6\rangle^3 + 3.75\langle x - 12\rangle^3 - \frac{1}{12}\langle x - 12\rangle^4 + C_1x + C_2 \quad \textbf{(2)}$$

Boundary Conditions: At $x = 0, v = 0$. Then Eq. (2) gives

$$0 = -0 - 0 + 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 12\,\text{ft}, v = 0$. Then Eq. (2) gives

$$0 = -0.25(12^3) - (12 - 6)^3 + 0 - 0 + C_1(12) + 0 \qquad C_1 = 54\,\text{kip}\cdot\text{ft}^2$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[-0.75x^2 - 3\langle x - 6\rangle^2 + 11.25\langle x - 12\rangle^2 - \frac{1}{3}\langle x - 12\rangle^3 + 54\right]$$

At $A, x = 0$. Thus,

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = \frac{1}{EI}[-0 - 0 + 0 - 0 + 54]$$

$$= \frac{54\,\text{kip}\cdot\text{ft}^2}{EI} = \frac{54(12^2)}{29(10^3)(204)} = 0.00131\text{ rad} \qquad \textbf{Ans.}$$

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[-0.25x^3 - \langle x - 6\rangle^3 + 3.75\langle x - 12\rangle^3 - \frac{1}{12}\langle x - 12\rangle^4 + 54x\right] \qquad \textbf{Ans.}$$

***16–40. Continued**

At C, $x = 18$ ft. Thus,

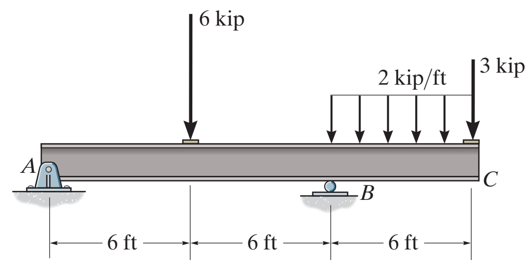
$$v_C = v\big|_{x=18\,\text{ft}} = \frac{1}{EI}\left[-0.25(18^3) - (18 - 6)^3 + 3.75(18 - 12)^3 - \frac{1}{12}(18 - 12)^4 + 54(18)\right]$$

$$= -\frac{1512\,\text{kip}\cdot\text{ft}^3}{EI} = -\frac{1512(12^3)}{29(10^3)(204)} = -0.442\ \text{in.} \qquad \textbf{Ans.}$$

Ans:
$\theta_A = 0.00131$ rad,
$v = \frac{1}{EI}\left[-0.25x^3 - \langle x - 6\rangle^3 + 3.75\langle x - 12\rangle^3 - \frac{1}{12}\langle x - 12\rangle^4 + 54x\right]$,
$v_C = -0.442$ in.

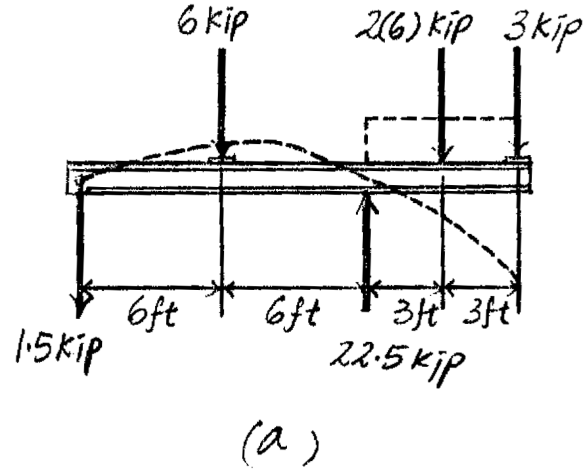
16–41.

Determine the maximum deflection in region AB of the overhang beam. Take $E = 29(10^3)$ ksi and $I = 204\ \text{in}^4$.



SOLUTION

Support Reactions and Elastic Curve: As shown in Fig. *a*.



Moment Function: From Fig. *a*, we obtain

$$M = -1.5\langle x - 0\rangle - 6\langle x - 6\rangle - (-22.5)\langle x - 12\rangle - \frac{2}{2}\langle x - 12\rangle^2$$

$$= -1.5x - 6\langle x - 6\rangle + 22.5\langle x - 12\rangle - \langle x - 12\rangle^2$$

Equations of Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -1.5x - 6\langle x - 6\rangle + 22.5\langle x - 12\rangle - \langle x - 12\rangle^2$$

$$EI\frac{dv}{dx} = -0.75x^2 - 3\langle x - 6\rangle^2 + 11.25\langle x - 12\rangle^2 - \frac{1}{3}\langle x - 12\rangle^3 + C_1 \qquad \textbf{(1)}$$

$$EIv = -0.25x^3 - \langle x - 6\rangle^3 + 3.75\langle x - 12\rangle^3 - \frac{1}{12}\langle x - 12\rangle^4 + C_1x + C_2 \quad \textbf{(2)}$$

Boundary Conditions: At $x = 0$, $v = 0$. Then Eq. (2) gives

$$0 = -0 - 0 + 0 - 0 + C_1(0) + C_2 \qquad C_2 = 0$$

At $x = 12$ ft, $v = 0$. Then Eq. (2) gives

$$0 = -0.25(12^3) - (12 - 6)^3 + 0 - 0 + C_1(12) + 0 \qquad C_1 = 54\ \text{kip}\cdot\text{ft}^2$$

Substituting the value of C_1 into Eq. (1),

$$\frac{dv}{dx} = \frac{1}{EI}\left[-0.75x^2 - 3\langle x - 6\rangle^2 + 11.25\langle x - 12\rangle^2 - \frac{1}{3}\langle x - 12\rangle^3 + 54\right]$$

Assuming that $\frac{dv}{dx} = 0$ occurs in the region 6 ft $< x <$ 12 ft, then

$$\frac{dv}{dx} = 0 = \frac{1}{EI}\left[-0.75x^2 - 3(x - 6)^2 + 0 - 0 + 54\right]$$

$$-0.75x^2 - 3(x - 6)^2 + 54 = 0$$

$$3.75x^2 - 36x + 54 = 0$$

Solving for the root 6 ft $< x <$ 12 ft,

$$x = 7.7394\ \text{ft}$$

16–41. Continued

Substituting the values of C_1 and C_2 into Eq. (2),

$$v = \frac{1}{EI}\left[-0.25x^3 - \langle x - 6\rangle^3 + 3.75\langle x - 12\rangle^3 - \frac{1}{12}\langle x - 12\rangle^4 + 54x\right]$$

$$(v_{\max})_{AB} = v\big|_{x=7.7394\,\text{ft}} = \frac{1}{EI}\left[-0.25(7.7394^3) - (7.7394 - 6)^3 + 0 - 0 + 54(7.7394)\right]$$

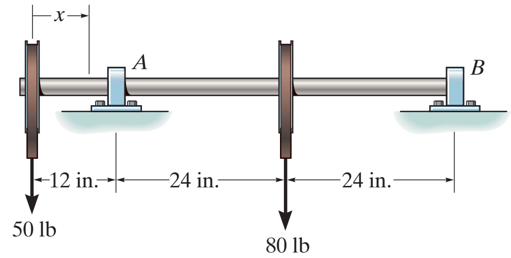
$$= \frac{296.77\,\text{kip}\cdot\text{ft}^3}{EI} = \frac{296.77(12^3)}{29(10^3)(204)}$$

$$= 0.0867\text{ in.}$$ **Ans.**

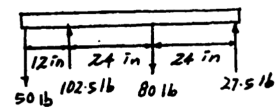
Ans:
$(v_{\max})_{AB} = 0.0867$ in.

16–42.

The shaft supports the two pulley loads shown. Determine the slope of the shaft at A and B. The bearings exert only vertical reactions on the shaft. EI is constant.



SOLUTION



$$M = -50\langle x - 0\rangle - (-102.5)\langle x - 12\rangle - 80\langle x - 36\rangle$$

$$= -50x + 102.5\langle x - 12\rangle - 80\langle x - 36\rangle$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -50x + 102.5\langle x - 12\rangle - 80\langle x - 36\rangle$$

$$EI\frac{dv}{dx} = -25x^2 + \frac{102.5}{2}\langle x - 12\rangle^2 - 40\langle x - 36\rangle^2 + C_1 \qquad \textbf{(1)}$$

$$EIv = -\frac{25}{3}x^3 + \frac{102.5}{6}\langle x - 12\rangle^3 - \frac{40}{3}\langle x - 36\rangle^3 + C_1x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

At $x = 12$ in., $v = 0$.

From Eq. (2),

$$0 = -\frac{25}{3}(12)^3 + 0 - 0 + 12C_1 + C_2$$

$$12C_1 + C_2 = 14\,400 \qquad \textbf{(3)}$$

At $x = 60$ in., $v = 0$.

$$0 = -\frac{25}{3}(60)^3 + \frac{102.5}{6}(60 - 12)^3 - \frac{40}{3}(60 - 36)^3 + 60C_1 + C_2$$

$$60C_1 + C_2 = 95\,040 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields:

$C_1 = 1680 \qquad C_2 = -5760$

$$EI\frac{dv}{dx} = -25x^2 + 51.25\langle x - 12\rangle^2 - 40\langle x - 36\rangle^2 + 1680$$

At $x = 12$ in.,

$$\theta_A = \frac{1}{EI}(-25(12^2) + 0 - 0 + 1680)$$

$$\theta_A = -\frac{1920}{EI} \qquad \textbf{Ans.}$$

At $x = 60$ in.,

$$\theta_B = \frac{1}{EI}(-25(60^2) + 51.25(60 - 12)^2 - 40(60 - 36)^2 + 1680)$$

$$\theta_B = \frac{6720\ \text{lb}\cdot\text{in}^2}{EI} \qquad \textbf{Ans.}$$

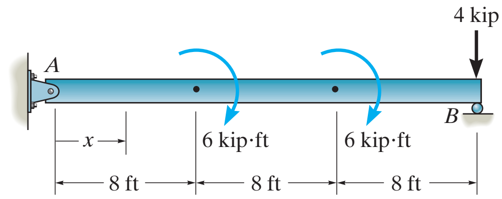
Ans:

$\theta_A = -\dfrac{1920}{EI}$,

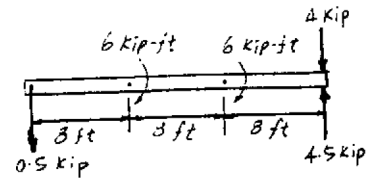
$\theta_B = \dfrac{6720\ \text{lb}\cdot\text{in}^2}{EI}$

16–43.

Determine the equation of the elastic curve. *EI* is constant.



SOLUTION



$$M = -0.5\langle x - 0\rangle - (-6)\langle x - 8\rangle^0 - (-6)\langle x - 16\rangle^0$$

$$= -0.5x + 6\langle x - 8\rangle^0 + 6\langle x - 16\rangle^0$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -0.5x + 6\langle x - 8\rangle^0 + 6\langle x - 16\rangle^0$$

$$EI\frac{dv}{dx} = -0.25x^2 + 6\langle x - 8\rangle + 6\langle x - 16\rangle + C_1 \qquad \textbf{(1)}$$

$$EIv = -\frac{0.25}{3}x^3 + 3\langle x - 8\rangle^2 + 3\langle x - 16\rangle^2 + C_1 x + C_2 \qquad \textbf{(2)}$$

Boundary Conditions:

At $x = 0$, $v = 0$.

From Eq. (2),

$0 = -0 + 0 + 0 + 0 + C_2$; $\qquad C_2 = 0$

At $x = 24$ ft, $v = 0$.

$$0 = -\frac{0.25}{3}(24)^3 + 3(24 - 8)^2 + 3(24 - 16)^2 + 24C_1; \qquad C_1 = 8.0$$

The Elastic Curve:

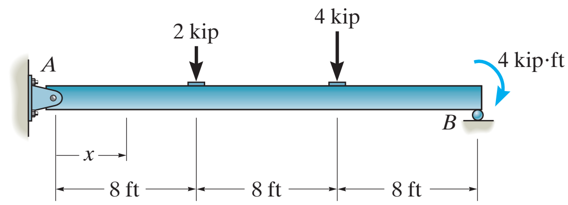
$$v = \frac{1}{EI}[-0.0833x^3 + 3\langle x - 8\rangle^2 + 3\langle x - 16\rangle^2 + 8.00x]\ \text{kip}\cdot\text{ft}^3$$ **Ans.**

Ans:

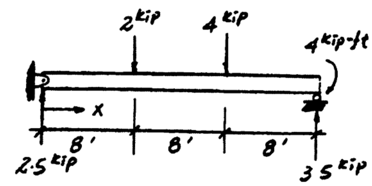
$$v = \frac{1}{EI}[-0.0833x^3 + 3\langle x - 8\rangle^2 + 3\langle x - 16\rangle^2 + 8.00x]\ \text{kip}\cdot\text{ft}^3$$

***16–44.**

Determine the equation of the elastic curve. EI is constant.



SOLUTION



$M = -(-2.5)\langle x - 0\rangle - 2\langle x - 8\rangle - 4\langle x - 16\rangle$

$M = 2.5x - 2\langle x - 8\rangle - 4\langle x - 16\rangle$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M = 2.5x - 2\langle x - 8\rangle - 4\langle x - 16\rangle$$

$$EI\frac{dv}{dx} = 1.25x^2 - \langle x - 8\rangle^2 - 2\langle x - 16\rangle^2 + C_1$$

$$EIv = 0.417x^3 - 0.333\langle x - 8\rangle^3 - 0.667\langle x - 16\rangle^3 + C_1 x + C_2 \qquad \textbf{(1)}$$

Boundary Conditions:

$v = 0 \quad$ at $\quad x = 0$

From Eq. (1), $\quad C_2 = 0$

$v = 0 \quad$ at $\quad x = 24$ ft

$0 = 5760 - 1365.33 - 341.33 + 24C_1$

$C_1 = -169$

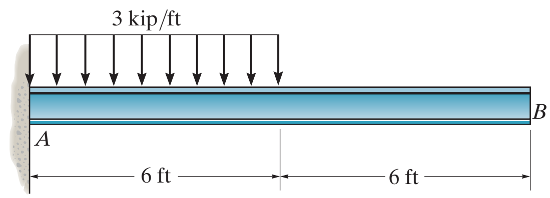
$$v = \frac{1}{EI}[0.417x^3 - 0.333\langle x - 8\rangle^3 - 0.667\langle x - 16\rangle^3 - 169x]\ \text{kip}\cdot\text{ft}^3 \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{1}{EI}[0.417x^3 - 0.333\langle x - 8\rangle^3 - 0.667\langle x - 16\rangle^3 - 169x]\ \text{kip}\cdot\text{ft}^3$$

16–45.

The W10 × 15 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the slope and displacement at its end B.



SOLUTION

Here,

$$\theta_B = \left|\theta_{B/A}\right| = \frac{1}{3}\left(\frac{54}{EI}\right)(6)$$

$$= -\frac{108\ \text{kip}\cdot\text{ft}^2}{EI}$$

For $W\,10 \times 15$, $I = 68.9\ \text{in}^4$, and for A36 steel $E = 29.0\left(10^3\right)$ ksi. Thus,

$$\theta_B = -\frac{108\left(12^2\right)}{29\left(10^3\right)(68.9)}$$

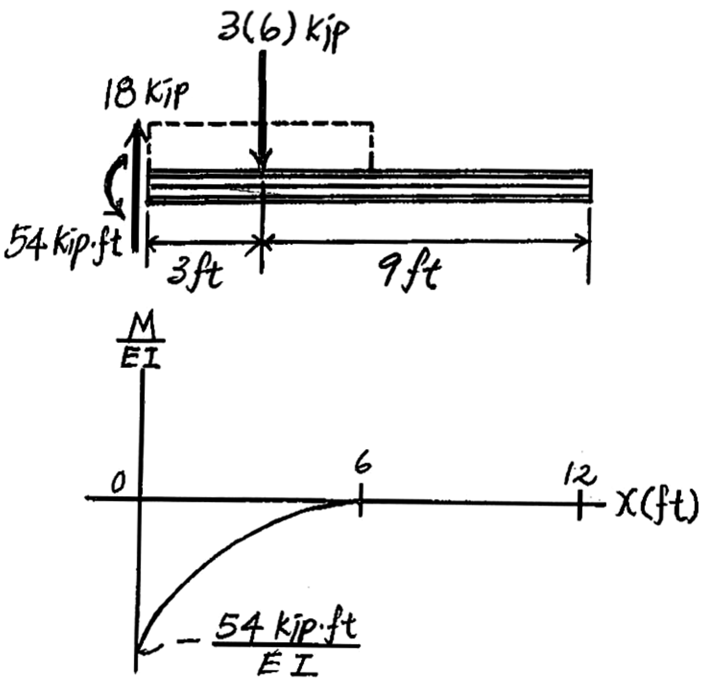
$$= -0.00778\ \text{rad}$$ **Ans.**

$$v_B = \left|t_{B/A}\right| = \left[\frac{3}{4}(6) + 6\right]\left[\frac{1}{3}\left(\frac{54}{EI}\right)(6)\right]$$

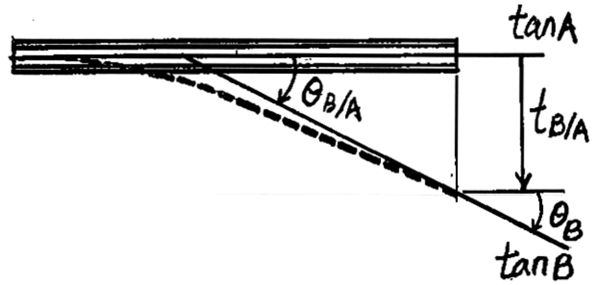
$$= \frac{1134\ \text{kip}\cdot\text{ft}^3}{EI}$$

$$= \frac{1134\left(12^3\right)}{29\left(10^3\right)(68.9)}$$

$$= 0.981\ \text{in.}\ \downarrow$$ **Ans.**



(a)

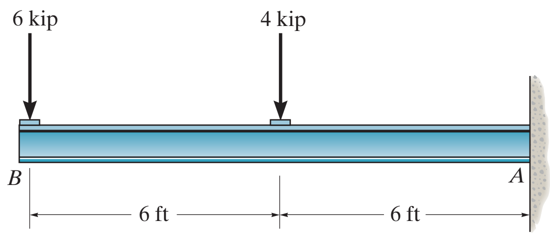


(b)

Ans:
$\theta_B = -0.00778$ rad, $v_B = 0.981$ in.↓

16–46.

The W10 × 15 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the displacement at *B* and the slope at *B*.



SOLUTION

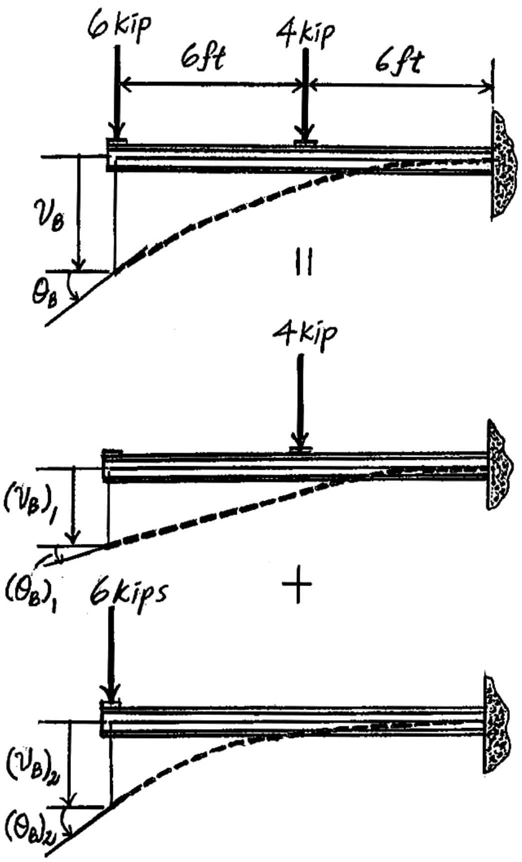
Using the table in the appendix, the required slopes and deflections for each load case are computed as follows:

$$(v_B)_1 = \frac{5PL^3}{48EI} = \frac{5(4)\left(12^3\right)}{48EI} = \frac{720\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$

$$(\theta_B)_1 = \frac{PL^2}{8EI} = \frac{4\left(12^2\right)}{8EI} = \frac{72\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$(v_B)_2 = \frac{PL^3}{3EI} = \frac{6\left(12^3\right)}{3EI} = \frac{3456\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$

$$(\theta_B)_2 = \frac{PL^2}{2EI} = \frac{6\left(12^2\right)}{2EI} = \frac{432\ \text{kip}\cdot\text{ft}^2}{EI}$$



(a)

Then the slope and deflection at *B* are

$$\theta_B = (\theta_B)_1 + (\theta_B)_2$$

$$= \frac{72}{EI} + \frac{432}{EI}$$

$$= \frac{504\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$v_B = (v_B)_1 + (v_B)_2$$

$$= \frac{720}{EI} + \frac{3456}{EI}$$

$$= \frac{4176\ \text{kip}\cdot\text{ft}^3}{EI}$$

For A36 steel $W10 \times 15, I = 68.9\ \text{in}^4$ and $E = 29.0\left(10^3\right)$ ksi.

$$\theta_B = \frac{504(144)}{29.0\left(10^3\right)(68.9)}$$

$$= 0.0363\ \text{rad} = 2.08°$$ **Ans.**

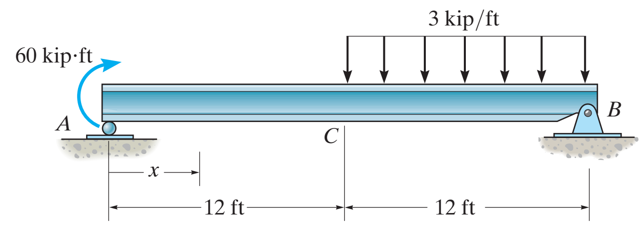
$$v_B = \frac{4176(1728)}{29.0\left(10^3\right)(68.9)}$$

$$= 3.61\ \text{in.}\downarrow$$ **Ans.**

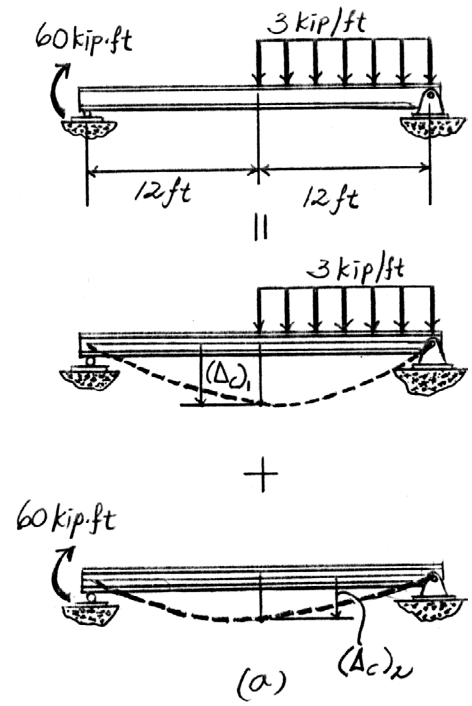
Ans:
$\theta_B = 2.08°$,
$v_B = 3.61$ in. ↓

16–47.

The W14 × 43 simply supported beam is made of A992 steel and is subjected to the loading shown. Determine the deflection at its center C.



SOLUTION



Elastic Curve: The elastic curves for the partial uniform distributed load and couple moment are drawn separately as shown in Fig. *a*.

Method of Superposition: Using the table in Appendix C, the required displacements shown in Fig. *a* are

$$(v_C)_1 = \frac{5wL^4}{768EI} = \frac{5(3)(24^4)}{768EI} = \frac{6480\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$

$$(v_C)_2 = \frac{M_O x}{6EIL}\left(x^2 - 3Lx + 2L^2\right)$$

$$= \frac{60(12)}{6EI(24)}\left[12^2 - 3(24)(12) + 2\left(12^2\right)\right]$$

$$= -\frac{2160\ \text{kip}\cdot\text{ft}^3}{EI} = \frac{2160\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$

$$v_C = (v_C)_1 + (v_C)_2 = \frac{6480\ \text{kip}\cdot\text{ft}^3}{EI} + \frac{2160\ \text{kip}\cdot\text{ft}^3}{EI} = \frac{8640\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$

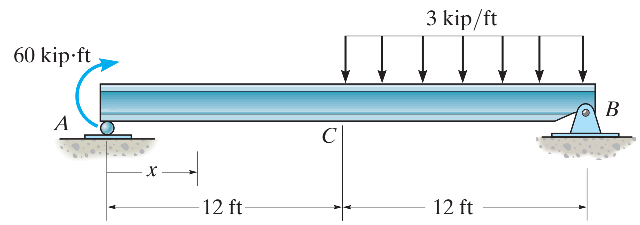
For W14 × 43 wide-flange section, $I = 428\ \text{in}^4$. Also, $E = 29\left(10^3\right)$ ksi for A992 steel. Then

$$v_C = \frac{8640\left(12^3\right)}{29\left(10^3\right)(428)} = 1.203\ \text{in.} = 1.20\ \text{in.}\downarrow \qquad \textbf{Ans.}$$

Ans:
$v_C = 1.20\ \text{in.}\downarrow$

***16–48.**

The W14 × 43 simply supported beam is made of A992 steel and is subjected to the loading shown. Determine the slope at *A* and *B*.



SOLUTION

Elastic Curve: The elastic curves for the partial uniform distributed load and couple moment are drawn separately as shown in Fig. *a*.

Method of Superposition: Using the table in Appendix C, the required slopes shown in Fig. *a* are

$$(\theta_A)_1 = \frac{7wL^3}{384EI} = \frac{7(3)(24^3)}{384EI} = \frac{756\ \text{kip}\cdot\text{ft}^2}{EI}$$

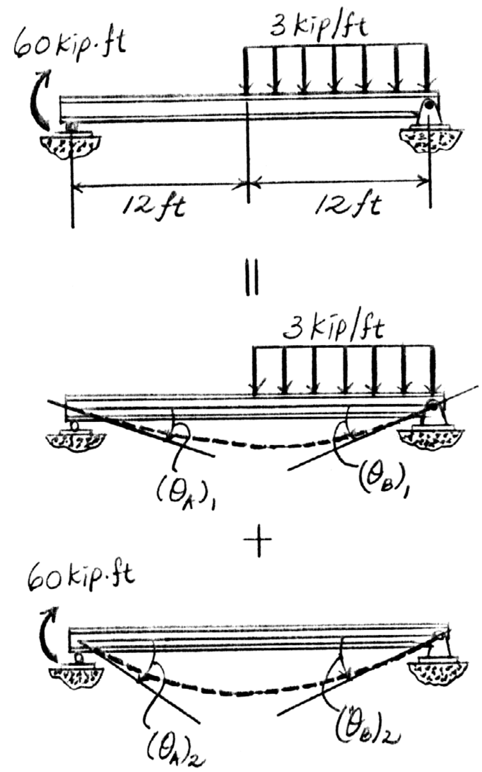
$$(\theta_A)_2 = \frac{M_O L}{3EI} = \frac{60(24)}{3EI} = \frac{480\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$(\theta_B)_1 = \frac{3wL^3}{128EI} = \frac{3(3)(24^3)}{128EI} = \frac{972\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$(\theta_B)_2 = \frac{M_O L}{6EI} = \frac{60(24)}{6EI} = \frac{240\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$\theta_A = (\theta_A)_1 + (\theta_A)_2 = \frac{756\ \text{kip}\cdot\text{ft}^2}{EI} + \frac{480\ \text{kip}\cdot\text{ft}^2}{EI} = \frac{1236\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$\theta_B = (\theta_B)_1 + (\theta_B)_2 = \frac{972\ \text{kip}\cdot\text{ft}^2}{EI} + \frac{240\ \text{kip}\cdot\text{ft}^2}{EI} = \frac{1212\ \text{kip}\cdot\text{ft}^2}{EI}$$



(a)

For W14 × 43 wide-flange section, $I = 428\ \text{in}^4$. Also, $E = 29(10^3)$ ksi for A992 steel. Then

$$\theta_A = -\frac{1236(12^2)}{29(10^3)(428)}$$

$$= -0.01434\ \text{rad} = -0.822° \qquad \textbf{Ans.}$$

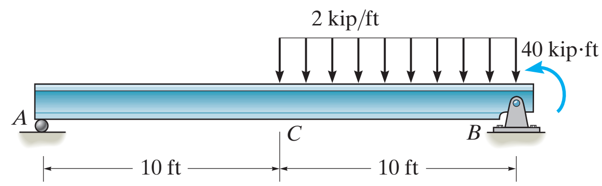
$$\theta_B = \frac{1212(12^2)}{29(10^3)(428)}$$

$$= 0.01406\ \text{rad} = 0.806° \qquad \textbf{Ans.}$$

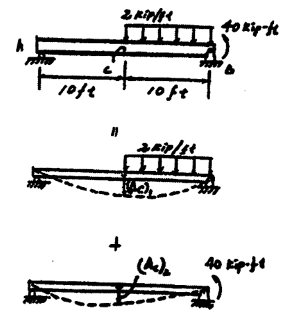
Ans:
$\theta_A = -0.822°, \theta_B = 0.806°$

16–49.

The W14 × 43 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center *C*.



SOLUTION



$$(v_C)_1 = \frac{5wL^4}{768EI} = \frac{5(2)(20^4)}{768EI} = \frac{2083.33}{EI}\downarrow$$

$$(v_C)_2 = \frac{Mx}{6EIL}(x^2 - 3Lx + 2L^2) = \frac{40(10)}{6(20)EI}\left[10^2 - 3(20)(10) + 2(20)^2\right]$$

$$= \frac{1000}{EI}\downarrow$$

$$v_C = (v_C)_1 + (v_C)_2 = \frac{2083.33}{EI} + \frac{1000}{EI}$$

$$= \frac{3083.33}{EI}\text{kip}\cdot\text{ft}^3$$

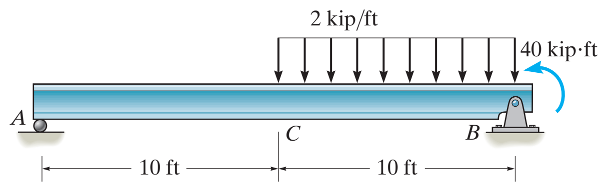
Numerical substitution for W14 × 43: $I_x = 428 \text{ in}^4$

$$v_C = \frac{3083.33(12^3)}{29(10^3)(428)} = 0.429 \text{ in.}\downarrow$$ **Ans.**

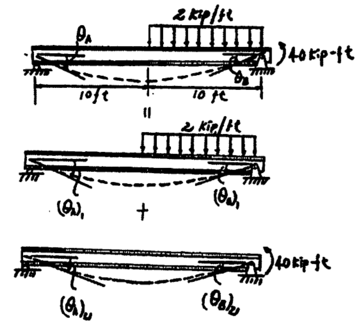
Ans:
$v_C = 0.429 \text{ in.}\downarrow$

16–50.

The W14 × 43 simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the slope at A and B.



SOLUTION



$$|\theta_A| = \theta_{A_1} + \theta_{A_2}$$

$$= \frac{7wL^3}{384EI} + \frac{ML}{6EI}$$

$$= \frac{\frac{7(2)}{12}(240^3)}{384EI} + \frac{40(12)(240)}{6EI} = \frac{61{,}200}{29(10^3)(428)}$$

$$\theta_A = -0.00493 \text{ rad} = -0.283°$$ **Ans.**

$$\theta_B = \theta_{B_1} + \theta_{B_2}$$

$$= \frac{3wL^3}{128EI} + \frac{ML}{3EI}$$

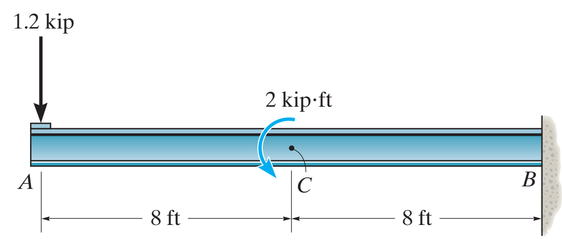
$$= \frac{\frac{3(2)}{12}(240^3)}{128EI} + \frac{40(12)(240)}{3EI} = \frac{92{,}400}{29(10^3)(428)}$$

$$= 0.007444 \text{ rad} = 0.427°$$ **Ans.**

Ans:
$\theta_A = -0.283°$,
$\theta_B = 0.427°$

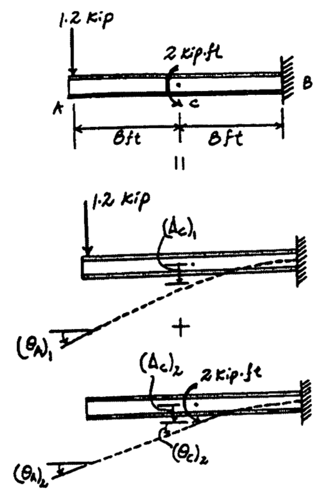
16–51.

The W8 × 48 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the displacement at C and the slope at A.



SOLUTION

Elastic Curve: The elastic curves for the concentrated load and couple moment are drawn separately as shown.



Method of Superposition: Using the table in Appendix C, the required slope and displacement are

$$(v_C)_1 = \frac{Px^2}{6EI}(3L_{AB} - x) = \frac{1.2(8^2)}{6EI}[3(16) - 8]$$

$$= \frac{512 \text{ kip} \cdot \text{ft}^3}{EI} \downarrow$$

$$(v_C)_2 = \frac{M_0 L_{BC}^2}{2EI} = \frac{2(8^2)}{2EI} = \frac{64.0 \text{ kip} \cdot \text{ft}^3}{EI} \downarrow$$

$$(\theta_A)_1 = \frac{PL_{AB}^2}{2EI} = \frac{1.2(16^2)}{2EI} = \frac{153.6 \text{ kip} \cdot \text{ft}^2}{EI}$$

$$(\theta_A)_2 = (\theta_C)_2 = \frac{M_0 L_{BC}}{EI} = \frac{2(8)}{EI} = \frac{16.0 \text{ kip} \cdot \text{ft}^2}{EI}$$

The slope at A is

$$\theta_A = (\theta_A)_1 + (\theta_A)_2$$

$$= \frac{153.6}{EI} + \frac{16.0}{EI}$$

$$= \frac{169.6 \text{ kip} \cdot \text{ft}^2}{EI}$$

$$= \frac{169.6\,(144)}{29.0(10^3)(184)} = 0.00458 \text{ rad}$$ **Ans.**

The displacement at C is

$$v_C = (v_C)_1 + (v_C)_2$$

$$= \frac{512}{EI} + \frac{64.0}{EI}$$

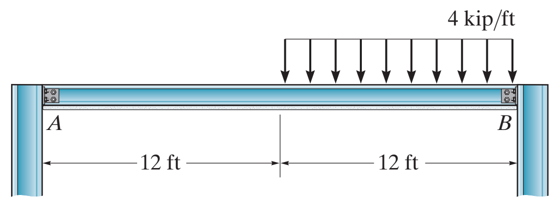
$$= \frac{576 \text{ kip} \cdot \text{ft}^3}{EI}$$

$$= \frac{576(1728)}{29.0(10^3)\,(184)} = 0.187 \text{ in.} \downarrow$$ **Ans.**

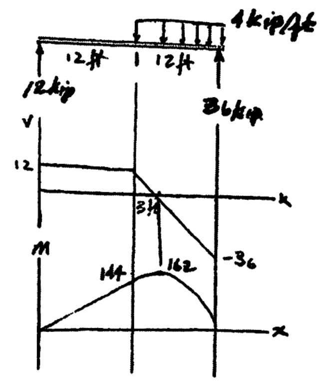
Ans:
$\theta_A = 0.00458$ rad,
$v_C = 0.187$ in.↓

***16–52.**

The beam supports the loading shown. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi. Assume A is a roller and B is a pin.



SOLUTION



$$V_{max} = 36 \text{ kip}$$

$$M_{max} = 162 \text{ kip} \cdot \text{ft}$$

Strength Criterion:

$$\sigma_{allow} = \frac{M}{S_{req'd}}$$

$$24 = \frac{162(12)}{S_{req'd}}$$

$$S_{req'd} = 81 \text{ in}^3$$

Choose W16 × 50: $S = 81.0 \text{ in}^3, \quad t_w = 0.380 \text{ in.}, \quad d = 16.26 \text{ in.}, \quad I_x = 659 \text{ in}^4$

Check Shear:

$$\tau_{allow} = \frac{V}{A_{web}}$$

$$14 \geq \frac{36}{(16.26)(0.380)} = 5.83 \text{ ksi} \qquad \text{OK}$$

Deflection Criterion:

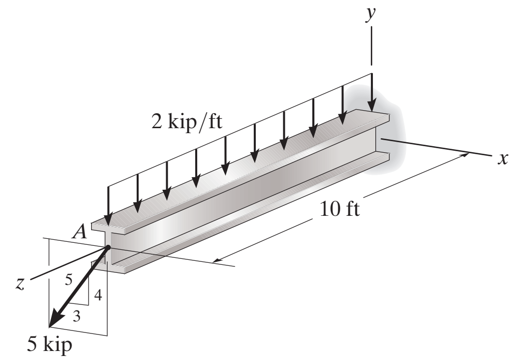
$$v_{max} = 0.006563\frac{wL^4}{EI} = 0.006563\left(\frac{(4)(24)^4(12)^3}{29(10^3)(659)}\right) = 0.7875 \text{ in.} < \frac{1}{360}(24)(12) = 0.800 \quad \textbf{OK}$$

Use W16 × 50. **Ans.**

Ans:
Use W16 × 50.

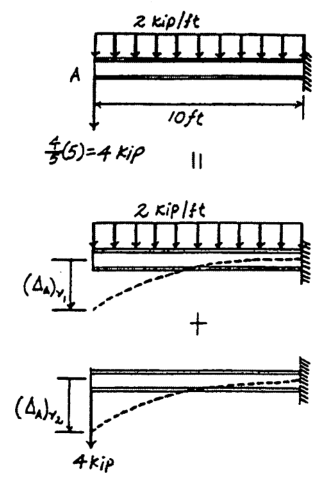
16–53.

The W24 × 104 A-36 steel beam is used to support the uniform distributed load and a concentrated force which is applied at its end. If the force acts at an angle with the vertical as shown, determine the horizontal and vertical displacement at A.



SOLUTION

Method of Superposition: Using the table in Appendix C, the required vertical displacements are



$$(v_A)_{v_1} = \frac{wL^4}{8EI_x} = \frac{2(10^4)}{8EI_x} = \frac{2500 \text{ kip}\cdot\text{ft}^3}{EI_x}\downarrow$$

$$(v_A)_{v_2} = \frac{P_y L^3}{3EI_x} = \frac{\frac{4}{5}(5)(10^3)}{3EI_x} = \frac{1333.33 \text{ kip}\cdot\text{ft}^3}{EI_x}\downarrow$$

The vertical displacement at A is

$$(v_A)_v = (v_A)_{v_1} + (v_A)_{v_2}$$

$$= \frac{2500}{EI_x} + \frac{1333.33}{EI_x}$$

$$= \frac{3833.33 \text{ kip}\cdot\text{ft}^3}{EI_x}$$

$$= \frac{3833.33(1728)}{29.0(10^3)(3100)} = 0.0737 \text{ in.}$$ **Ans.**

The horizontal displacement at A is

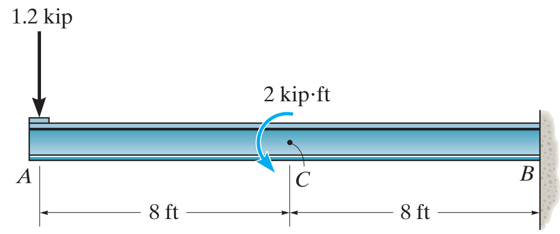
$$(v_A)_k = \frac{P_x L^3}{3EI_y}$$

$$= \frac{\frac{3}{5}(5)(10^3)}{3EI_y} = \frac{1000 \text{ kip}\cdot\text{ft}^3}{EI_y} = \frac{1000(1728)}{29.0(10^3)(259)} = 0.230 \text{ in.}$$ **Ans.**

Ans:
$(v_A)_v = 0.0737$ in.,
$(v_A)_k = 0.230$ in.

16–54.

The W8 × 48 cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the displacement at its end *A*.



SOLUTION

Elastic Curve: The elastic curves for the concentrated load and couple moment are drawn separately as shown.

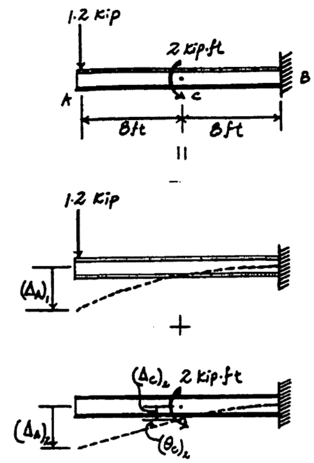
Method of Superposition: Using the table in Appendix C, the required slope and displacement are

$$(v_A)_1 = \frac{PL_{AB}^3}{3EI} = \frac{1.2(16^3)}{3EI} = \frac{1638.4\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$

$$(v_C)_2 = \frac{M_0\,L_{BC}^2}{2EI} = \frac{2(8^2)}{2EI} = \frac{64.0\ \text{kip}\cdot\text{ft}^3}{EI}$$

$$(\theta_C)_2 = \frac{M_0\,L_{BC}}{EI} = \frac{2(8)}{EI} = \frac{16.0\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$(v_A)_2 = (v_C)_2 + (\theta_C)_2\,L_{AC} = \frac{64.0}{EI} + \frac{16.0}{EI}(8) = \frac{192\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$



The displacement at *A* is

$$v_A = (v_A)_1 + (v_A)_2$$

$$= \frac{1638.4}{EI} + \frac{192}{EI}$$

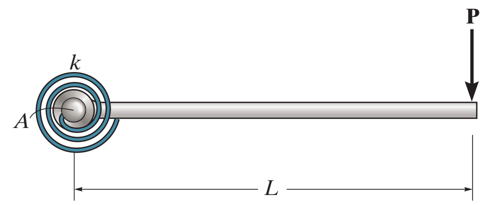
$$= \frac{1830.4\ \text{kip}\cdot\text{ft}^3}{EI}$$

$$= \frac{1830.4(1728)}{29.0(10^3)(184)} = 0.593\ \text{in.}\downarrow$$ **Ans.**

Ans:
$v_A = 0.593\ \text{in.}\downarrow$

16–55.

The rod is pinned at its end A and attached to a torsional spring having a stiffness k, which measures the torque per radian of rotation of the spring. If a force $\mathbf{P}$ is always applied perpendicular to the end of the rod, determine the displacement of the force. EI is constant.



SOLUTION

In order to maintain equilibrium, the rod has to rotate through an angle θ.

$$\circlearrowleft + \Sigma M_A = 0; \quad k\theta - PL = 0; \quad \theta = \frac{PL}{k}$$

Hence,

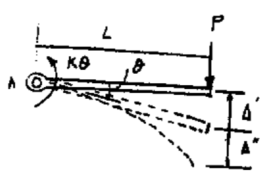
$$v' = L\theta = L\left(\frac{PL}{k}\right) = \frac{PL^2}{k}$$

Elastic deformation:

$$v'' = \frac{PL^3}{3EI}$$

Therefore,

$$v = v' + v'' = \frac{PL^2}{k} + \frac{PL^3}{3EI} = PL^2\left(\frac{1}{k} + \frac{L}{3EI}\right)$$ **Ans.**

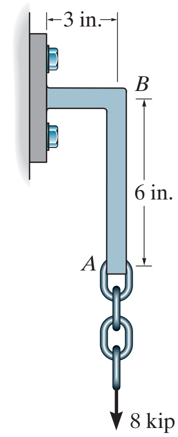


Ans:

$$v = PL^2\left(\frac{1}{k} + \frac{L}{3EI}\right)$$

***16–56.**

Determine the vertical deflection and the change in angle at the end A of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment AB. EI is constant.

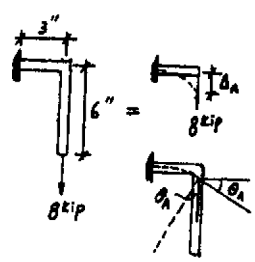


SOLUTION

Assume that the deflection is small enough so that the 8-kip force remains in line with segment AB and then bending of segment of AB can be neglected.

$$v_A = \frac{PL^3}{3EI} = \frac{8(3)^3}{3EI} = \frac{72}{EI}\downarrow$$ **Ans.**

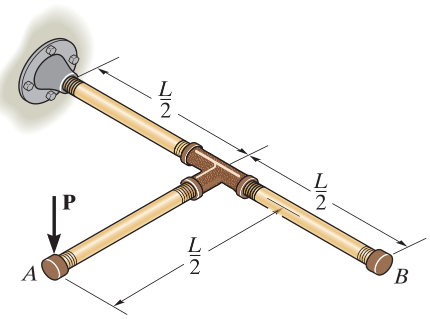
$$\theta_A = \frac{PL^2}{2EI} = \frac{8(3^2)}{2EI} = \frac{36}{EI}$$ **Ans.**



Ans:
$v_A = \frac{72}{EI}\downarrow, \theta_A = \frac{36}{EI}$

16–57.

The pipe assembly consists of three equal-sized pipes with flexibility stiffness EI and torsional stiffness GJ. Determine the vertical deflection at A.



SOLUTION

$$v_D = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

$$(v_A)_1 = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

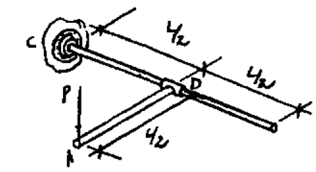
$$\theta = \frac{TL}{GJ} = \frac{(PL/2)\left(\frac{L}{2}\right)}{GJ} = \frac{PL^2}{4GJ}$$

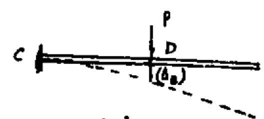
$$(v_A)_2 = \theta\left(\frac{L}{2}\right) = \frac{PL^3}{8GJ}$$

$$v_A = v_D + (v_A)_1 + (v_A)_2$$

$$= \frac{PL^3}{24EI} + \frac{PL^3}{24EI} + \frac{PL^3}{8GJ}$$

$$= PL^3\left(\frac{1}{12EI} + \frac{1}{8GJ}\right)\downarrow \qquad \textbf{Ans.}$$



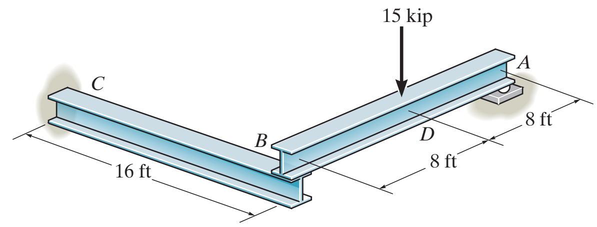


$$\Delta_D = \frac{P\left(\frac{L}{2}\right)^3}{3EI} = \frac{PL^3}{24EI}$$

Ans:
$$v_A = PL^3\left(\frac{1}{12EI} + \frac{1}{8GJ}\right)\downarrow$$

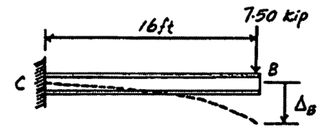
16–58.

The assembly consists of a cantilevered beam *CB* and a simply supported beam *AB*. If each beam is made of A-36 steel and has a moment of inertia about its principal axis of $I_x = 118\ \text{in}^4$, determine the displacement at the center *D* of beam *BA*.



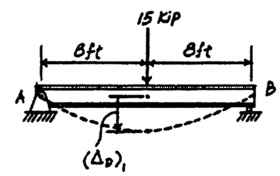
SOLUTION

Method of Superposition: Using the table in Appendix C, the required slopes and displacements are



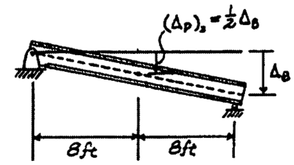
$$\Delta_B = \frac{PL_{BC}^3}{3EI} = \frac{7.50(16^3)}{3EI} = \frac{10240\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$

$$(\Delta_D)_1 = \frac{PL_{AB}^3}{48EI} = \frac{15(16^3)}{48EI} = \frac{1280\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$



$$(\Delta_D)_2 = \frac{1}{2}\Delta_B = \frac{5120\ \text{kip}\cdot\text{ft}^3}{EI}\downarrow$$

The vertical displacement at *A* is



$$\Delta_D = (\Delta_D)_1 + (\Delta_D)_2$$

$$= \frac{1280}{EI} + \frac{5120}{EI}$$

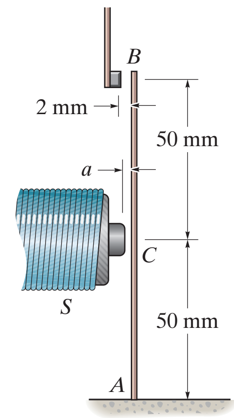
$$= \frac{6400\ \text{kip}\cdot\text{ft}^3}{EI}$$

$$= \frac{6400\ (1728)}{29.0(10^3)(118)} = 3.23\ \text{in.}\downarrow$$ **Ans.**

Ans:
$\Delta_D = 3.23\ \text{in.}\downarrow$

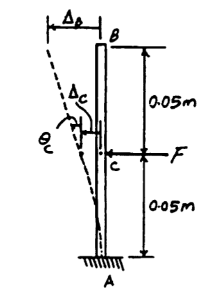
16–59.

The relay switch consists of a thin metal strip or armature AB that is made of red brass C83400 and is attracted to the solenoid S by a magnetic field. Determine the smallest force F required to attract the armature at C in order that contact is made at the free end B. Also, what should the distance a be for this to occur? The armature is fixed at A and has a moment of inertia of $I = 0.18(10^{-12})\ \mathrm{m}^4$.



SOLUTION

Elastic Curve: As shown.



Method of Superposition: Using the table in Appendix C, the required slopes and displacements are

$$\theta_C = \frac{PL_{AC}^2}{2EI} = \frac{F(0.05^2)}{2EI} = \frac{0.00125F\ \mathrm{m}^2}{EI}$$

$$\Delta_C = \frac{PL_{AC}^3}{3EI} = \frac{F(0.05^3)}{3EI} = \frac{41.667\left(10^{-6}\right)F\ \mathrm{m}^3}{EI} \qquad \textbf{(1)}$$

$$\Delta_B = \Delta_C + \theta_C L_{CB}$$

$$= \frac{41.667\left(10^{-6}\right)F}{EI} + \frac{0.00125\left(10^{-6}\right)F}{EI}(0.05)$$

$$= \frac{104.167\left(10^{-6}\right)F\ \mathrm{m}^3}{EI} \qquad \textbf{(2)}$$

Required the displacement $\Delta_B = 0.002$ m. From Eq. (2),

$$0.002 = \frac{104.167\left(10^{-6}\right)F}{101\left(10^9\right)(0.18)\left(10^{-12}\right)}$$

$$F = 0.349056\ \mathrm{N} = 0.349\ \mathrm{N} \qquad \textbf{Ans.}$$

From Eq. (1),

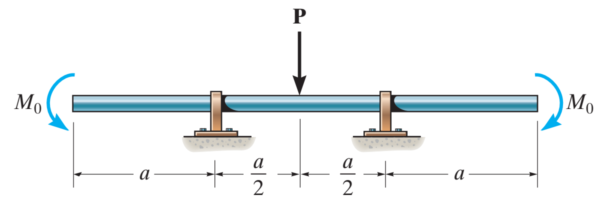
$$a = \Delta_C = \frac{41.667\left(10^{-6}\right)(0.349056)}{101\left(10^9\right)(0.18)\left(10^{-12}\right)}$$

$$= 0.800\left(10^{-3}\right)\ \mathrm{m} = 0.800\ \mathrm{mm} \qquad \textbf{Ans.}$$

Ans:
$F = 0.349$ N,
$a = 0.800$ mm

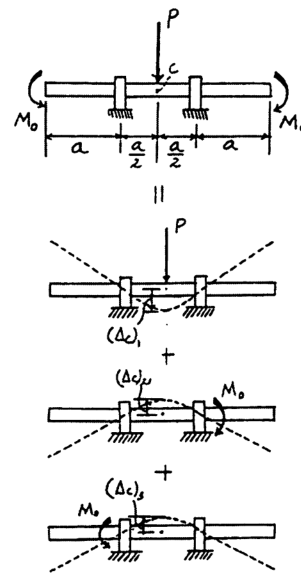
***16–60.**

Determine the moment M_0 in terms of the load P and dimension a so that the deflection at the center of the shaft is zero. EI is constant.



SOLUTION

Elastic Curve: The elastic curves for the concentrated load and couple moment are drawn separately as shown.



Method of Superposition: Using the table in Appendix C, the required slope and displacement are

$$(\Delta_C)_1 = \frac{Pa^3}{48EI} \downarrow$$

$$(\Delta_C)_2 = (\Delta_C)_3 = \frac{M_0 x}{6EIL}\left(x^2 - 3Lx + 2L^2\right)$$

$$= \frac{M_0\left(\frac{a}{2}\right)}{6EIa}\left[\left(\frac{a}{2}\right)^2 - 3(a)\left(\frac{a}{2}\right) + 2a^2\right]$$

$$= \frac{M_0 a^2}{16EI} \uparrow$$

Require the displacement at C to equal zero.

$$(+\uparrow) \qquad \Delta_C = 0 = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$0 = -\frac{Pa^3}{48EI} + \frac{M_0 a^2}{16EI} + \frac{M_0 a^2}{16EI}$$

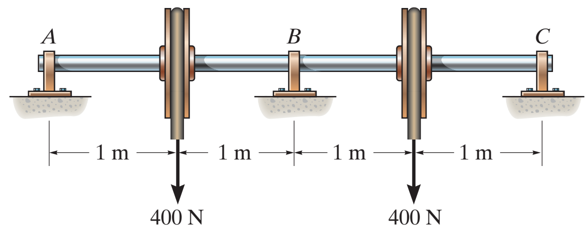
$$M_0 = \frac{Pa}{6}$$ **Ans.**

Ans:

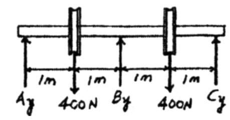
$M_0 = \dfrac{Pa}{6}$

16–61.

Determine the reactions at the journal bearing supports A, B, and C of the shaft, then draw the shear and moment diagrams. EI is constant.



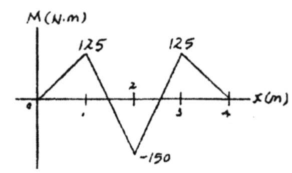
SOLUTION



Support Reactions: FBD(a).

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 800 = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y\,(2) + C_y\,(4) - 400(1) - 400(3) = 0 \qquad \textbf{(2)}$$



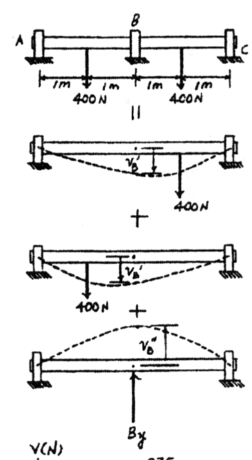
Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B' = \frac{Pbx}{6EIL}\left(L^2 - b^2 - x^2\right)$$

$$= \frac{400(1)(2)}{6EI(4)}\left(4^2 - 1^2 - 2^2\right)$$

$$= \frac{366.67\ \text{N}\cdot\text{m}^3}{EI} \quad \downarrow$$

$$v_B'' = \frac{PL^3}{48EI} = \frac{B_y\left(4^3\right)}{48EI} = \frac{1.3333B_y\,\text{m}^3}{EI} \quad \uparrow$$



The compatibility condition requires

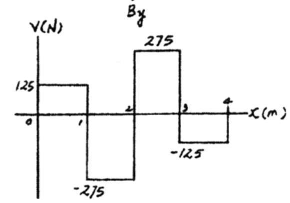
$$(+\downarrow) \qquad 0 = 2v_B' + v_B''$$

$$0 = 2\left(\frac{366.67}{EI}\right) + \left(-\frac{1.3333B_y}{EI}\right)$$

$$B_y = 550\ \text{N} \qquad \textbf{Ans.}$$

Substituting B_y into Eqs. (1) and (2) yields

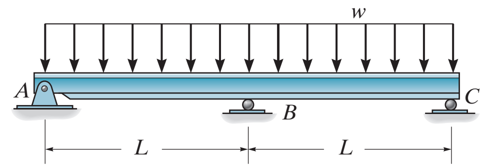
$$A_y = 125\ \text{N} \qquad C_y = 125\ \text{N} \qquad \textbf{Ans.}$$



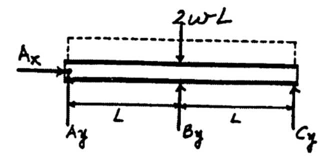
Ans:
$B_y = 550$ N, $A_y = 125$ N, $C_y = 125$ N

16–62.

Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.



SOLUTION



Support Reactions: FBD(a).

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

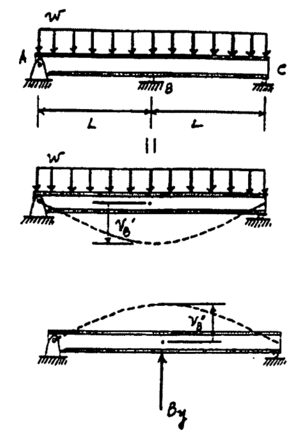
$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y + C_y - 2wL = 0$ **(1)**

$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(L) + C_y(2L) - (2wL)(L) = 0$ **(2)**

Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B' = \frac{5wL_{AC}^4}{384EI} = \frac{5w(2L)^4}{384EI} = \frac{5wL^4}{24EI}\downarrow$$

$$v_B'' = \frac{PL_{AC}^3}{48EI} = \frac{B_y(2L)^3}{48EI} = \frac{B_yL^3}{6EI}\uparrow$$



The compatibility condition requires

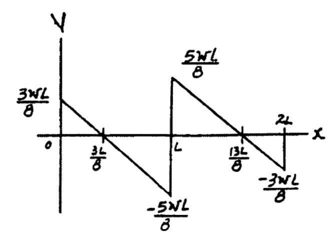
$$(+\downarrow) \qquad 0 = v_B' + v_B''$$

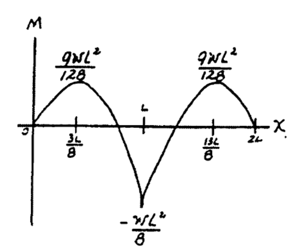
$$0 = \frac{5wL^4}{24EI} + \left(-\frac{B_yL^3}{6EI}\right)$$

$$B_y = \frac{5wL}{4}$$ **Ans.**

Substituting the value of B_y into Eqs. (1) and (2) yields

$$C_y = A_y = \frac{3wL}{8}$$ **Ans.**

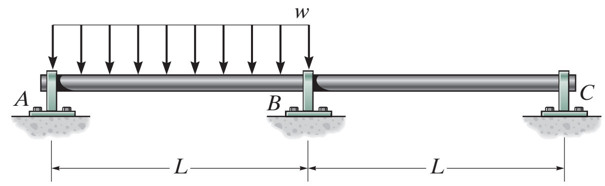




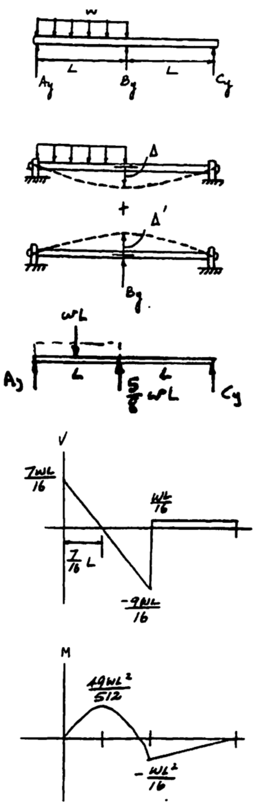
Ans:
$A_x = 0,$
$B_y = \dfrac{5wL}{4},$
$C_y = \dfrac{3wL}{8}$

16–63.

Determine the reactions at the supports, then draw the shear and moment diagrams. *EI* is constant.



SOLUTION



$$\Delta = \frac{5w(2L)^4}{768EI} = \frac{5wL^4}{48EI} \downarrow$$

$$\Delta' = \frac{B_y(2L)^4}{48EI} = \frac{B_yL^3}{6EI} \uparrow$$

Require:

$$(+\downarrow) \qquad 0 = \Delta - \Delta'$$

$$0 = \frac{5wL^4}{48EI} - \frac{B_yL^3}{6EI}$$

$$B_y = \frac{5}{8}wL \uparrow \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad wL\left(\frac{L}{2}\right) - \frac{5}{8}wL(L) - C_y(2L) = 0$$

$$C_y = -\frac{wL}{16} = \frac{wL}{16} \downarrow \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad -wL - \frac{wL}{16} + \frac{5}{8}wL + A_y = 0$$

$$A_y = \frac{7}{16}wL \uparrow \qquad \textbf{Ans.}$$

Ans:

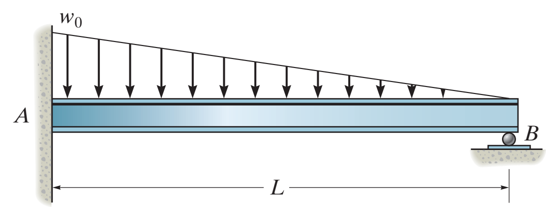
$B_y = \frac{5}{8}wL \uparrow$,

$C_y = \frac{wL}{16} \downarrow$,

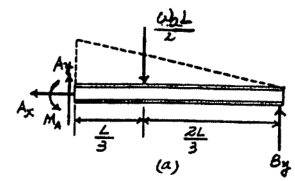
$A_y = \frac{7}{16}wL \uparrow$

***16–64.**

Determine the reactions at the supports A and B. EI is constant.



SOLUTION

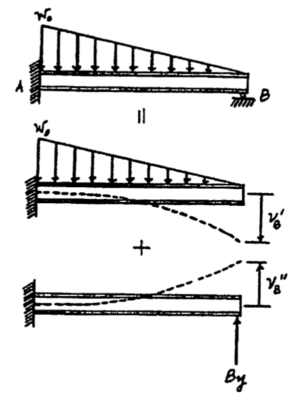


Support Reactions: FBD(a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad \textbf{(2)}$$



Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B{}' = \frac{w_0 L^4}{30EI} \downarrow \qquad v_B{}'' = \frac{B_y L^3}{3EI} \uparrow$$

The compatibility condition requires

$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

$$0 = \frac{w_0 L^4}{30EI} + \left(-\frac{B_y L^3}{3EI}\right)$$

$$B_y = \frac{w_0 L}{10} \qquad \textbf{Ans.}$$

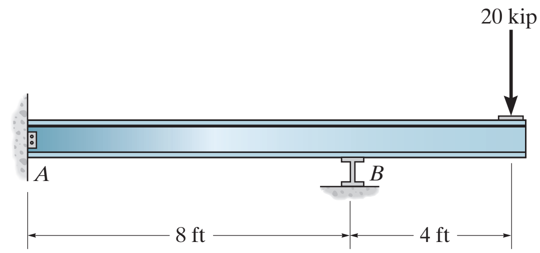
Substituting B_y into Eqs. (1) and (2) yields

$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15} \qquad \textbf{Ans.}$$

Ans:
$A_x = 0,$
$B_y = \frac{w_0 L}{10},$
$A_y = \frac{2w_0 L}{5},$
$M_A = \frac{w_0 L^2}{15}$

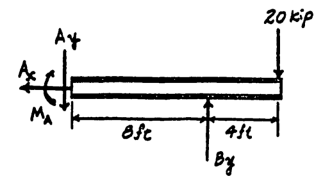
16–65.

The beam is used to support the 20-kip load. Determine the reactions at the supports. Assume A is fixed and B is a roller.



SOLUTION

Support Reactions: FBD(a).



$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -A_y + B_y - 20 = 0$ **(1)**

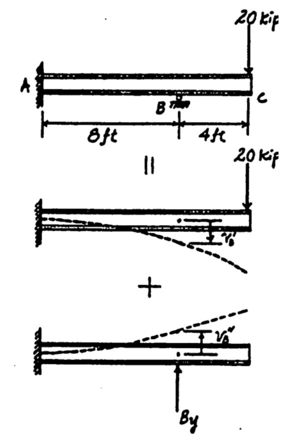
$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(8) - M_A - 20(12) = 0$ **(2)**

Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B{}' = \frac{Px^2}{6EI}(3L - x)$$

$$= \frac{20(8^2)}{6EI}[(3(12) - 8)] = \frac{5973.33 \text{ kip}\cdot\text{ft}^3}{EI}\downarrow$$

$$v_B{}'' = \frac{PL_{AB}^3}{3EI} = \frac{B_y(8^3)}{3EI} = \frac{170.67B_y \text{ ft}^3}{EI}\uparrow$$



The compatibility condition requires

$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

$$0 = \frac{5973.33}{EI} + \left(-\frac{170.67B_y}{EI}\right)$$

$B_y = 35.0 \text{ kip}$ **Ans.**

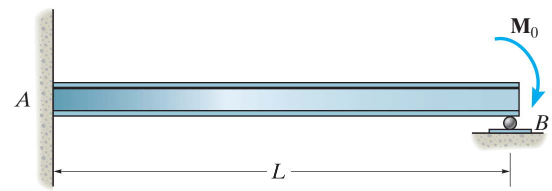
Substituting B_y into Eqs. (1) and (2) yields

$A_y = 15.0 \text{ kip} \qquad M_A = 40.0 \text{ kip}\cdot\text{ft}$ **Ans.**

Ans:
$A_x = 0$,
$B_y = 35.0$ kip,
$A_y = 15.0$ kip,
$M_A = 40.0$ kip · ft

16–66.

Determine the reactions at the supports A and B. EI is constant.



SOLUTION

Referring to the FBD of the beam, Fig. *a*,

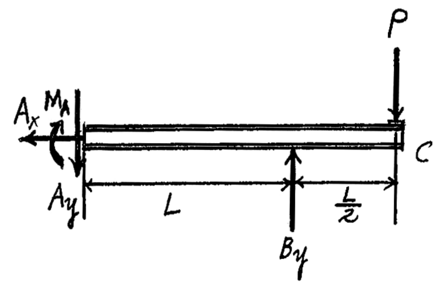
$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad B_y - P - A_y = 0$

$$A_y = B_y - P \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \; -M_A + B_y L - P\left(\frac{3}{2}L\right) = 0$$

$$M_A = B_y L - \frac{3}{2}PL \qquad \textbf{(2)}$$



(a)

Referring to Fig. *b* and the table in appendix, the necessary deflections are computed as follows:

$$v_P = \frac{Px^2}{6EI}(3L_{AC} - x)$$

$$= \frac{P(L^2)}{6EI}\left[3\left(\frac{3}{2}L\right) - L\right]$$

$$= \frac{7PL^3}{12EI}\downarrow$$

$$v_{B_y} = \frac{PL_{AB}^3}{3EI} = \frac{B_y L^3}{3EI}\uparrow$$

The compatibility condition at support B requires that

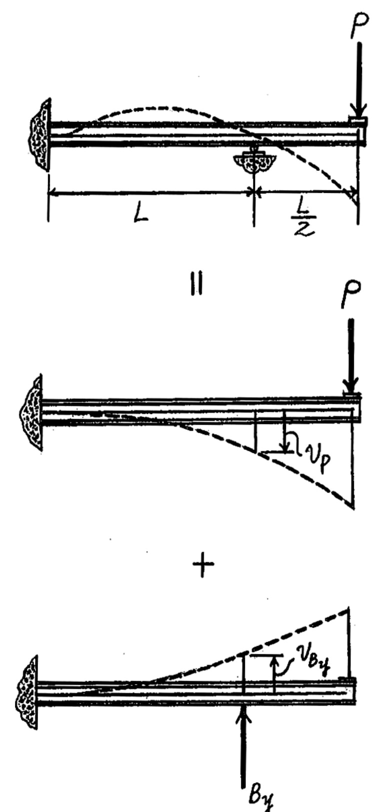
$$(+\downarrow) \qquad 0 = v_P + v_{B_y}$$

$$0 = \frac{7PL^3}{12EI} + \left(\frac{-B_y L^3}{3EI}\right)$$

$$B_y = \frac{7P}{4} \qquad \textbf{Ans.}$$

Substitute this result into Eqs. (1) and (2).

$$A_y = \frac{3P}{4} \qquad M_A = \frac{PL}{4} \qquad \textbf{Ans.}$$

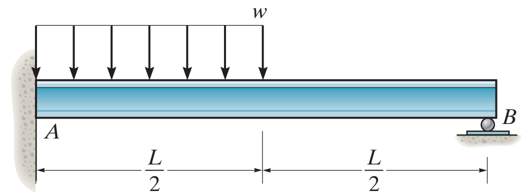


(b)

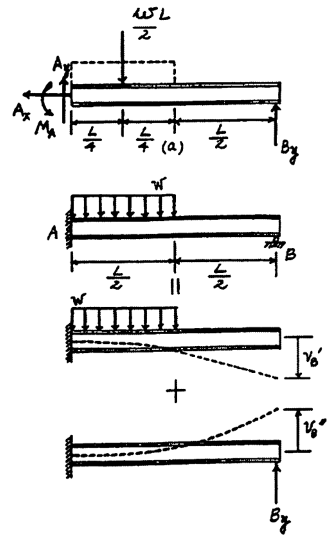
Ans:
$A_x = 0,$
$B_y = \frac{7P}{4},$
$A_y = \frac{3P}{4},$
$M_A = \frac{PL}{4}$

16–67.

Determine the reactions at the supports A and B. EI is constant.



SOLUTION



Support Reactions: FBD(a).

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y + B_y - \frac{wL}{2} = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(L) + M_A - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0 \qquad \textbf{(2)}$$

Method of Superposition: Using the table in Appendix C, the required displacements are

$$v_B{}' = \frac{7wL^4}{384EI}\downarrow \qquad v_B{}'' = \frac{PL^3}{3EI} = \frac{B_yL^3}{3EI}\uparrow$$

The compatibility condition requires

$$(+\downarrow) \qquad 0 = v_B{}' + v_B{}''$$

$$0 = \frac{7wL^4}{384EI} + \left(-\frac{B_yL^3}{3EI}\right)$$

$$B_y = \frac{7wL}{128} \qquad \textbf{Ans.}$$

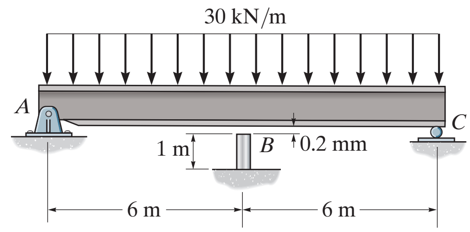
Substituting B_y into Eqs. (1) and (2) yields

$$A_y = \frac{57wL}{128} \qquad M_A = \frac{9wL^2}{128} \qquad \textbf{Ans.}$$

Ans:
$A_x = 0$,
$B_y = \frac{7wL}{128}$,
$A_y = \frac{57wL}{128}$,
$M_A = \frac{9wL^2}{128}$

***16–68.**

Before the uniform distributed load is applied to the beam, there is a small gap of 0.2 mm between the beam and the post at B. Determine the support reactions at A, B, and C. The post at B has a diameter of 40 mm, and the moment of inertia of the beam is $I = 875(10^6)\ \text{mm}^4$. The post and the beam are made of material having a modulus of elasticity of $E = 200$ GPa.



SOLUTION

Equations of Equilibrium: Referring to the free-body diagram of the beam, Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y + F_B + C_y - 30(12) = 0$ **(1)**

$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(6) + C_y(12) - 30(12)(6) = 0$ **(2)**

Method of Superposition: Referring to Fig. *b* and the table in the appendix, the necessary deflections are

$$(v_B)_1 = \frac{5wL^4}{384EI} = \frac{5(30)\left(12^4\right)}{384EI} = \frac{8100\ \text{kN}\cdot\text{m}^3}{EI}\ \downarrow$$

$$(v_B)_2 = \frac{PL^3}{48EI} = \frac{F_B\left(12^3\right)}{48EI} = \frac{36F_B}{EI}\ \uparrow$$

The deflection of point B is

$$v_B = 0.2\left(10^{-3}\right) + \frac{F_B L_B}{AE} = 0.2\left(10^{-3}\right) + \frac{F_B(1)}{AE}\ \downarrow$$

The compatibility condition at support B requires

$$(+\downarrow) \qquad v_B = (v_B)_1 + (v_B)_2$$

$$0.2\left(10^{-3}\right) + \frac{F_B\,(1)}{AE} = \frac{8100}{EI} + \left(-\frac{36F_B}{EI}\right)$$

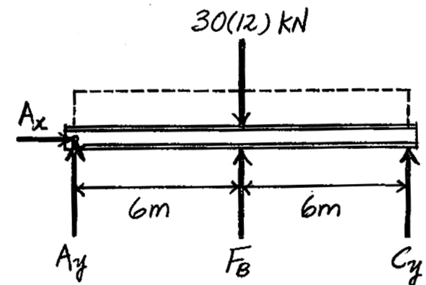
$$0.2\left(10^{-3}\right)E + \frac{F_B}{A} = \frac{8100}{I} - \frac{36F_B}{I}$$

$$\frac{F_B}{\frac{\pi}{4}\left(0.04^2\right)} + \frac{36F_B}{875\left(10^{-6}\right)} = \frac{8100}{875\left(10^{-6}\right)} - \frac{0.2\left(10^{-3}\right)\left[200\left(10^9\right)\right]}{1000}$$

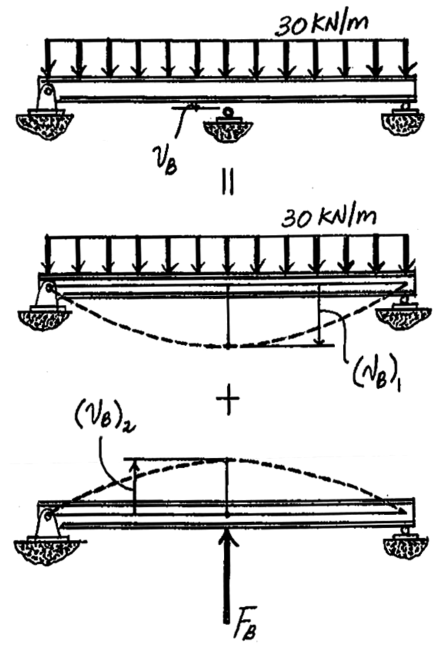
$F_B = 219.78\ \text{kN} = 220\ \text{kN}$ **Ans.**

Substituting the result of F_B into Eqs. (1) and (2),

$A_y = C_y = 70.11\ \text{kN} = 70.1\ \text{kN}$ **Ans.**



(a)

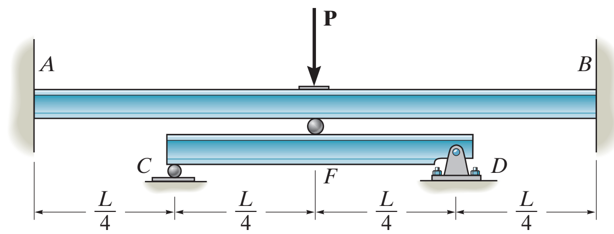


(b)

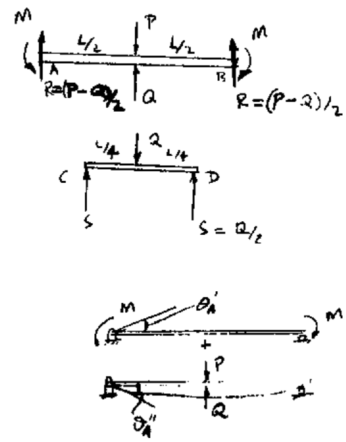
Ans:
$A_x = 0$,
$F_B = 220$ kN,
$A_y = C_y = 70.1$ kN

16–69.

The fixed supported beam AB is strengthened using the simply supported beam CD and the roller at F which is set in place just before application of the load **P**. Determine the reactions at the supports if EI is constant.



SOLUTION



$\delta_F =$ Deflection of top beam at F

$\delta'_F =$ Deflection of bottom beam at F

$$\delta_F = \delta'_F$$

$$(+\downarrow)\ \frac{(P-Q)(L^3)}{48EI} - \frac{2M\left(\frac{L}{2}\right)}{6\,EIL}\left[L^2 - \left(\frac{L}{2}\right)^2\right] = \frac{Q\left(\frac{L}{2}\right)^3}{48EI}$$

$$\frac{(P-Q)L}{48} - \frac{1}{6}M\frac{3}{4} = \frac{QL}{48(8)}$$

$$8PL - 48M = 9QL \qquad \textbf{(1)}$$

$$\theta_A = \theta'_A + \theta''_A = 0$$

$$\circlearrowleft+ \quad -\frac{ML}{6EI} - \frac{ML}{3EI} + \frac{(P-Q)L^2}{16EI} = 0$$

$$8M = (P-Q)L \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2):

$$M = QL/16$$

$$Q = 2P/3$$

$$S = P/3$$

$$R = P/6$$

$$M = PL/24$$

Thus,

$$M_A = M_B = \frac{1}{24}PL \qquad \textbf{Ans.}$$

$$A_y = B_y = \frac{1}{6}P \qquad \textbf{Ans.}$$

$$C_y = D_y = \frac{1}{3}P \qquad \textbf{Ans.}$$

$$D_x = 0 \qquad \textbf{Ans.}$$

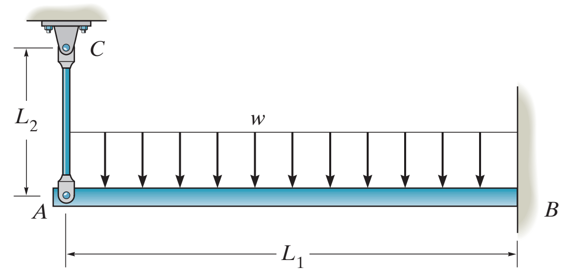
Ans:

$M_A = M_B = \frac{1}{24}PL,\ A_y = B_y = \frac{1}{6}P,$

$C_y = D_y = \frac{1}{3}P,\ D_x = 0$

16–70.

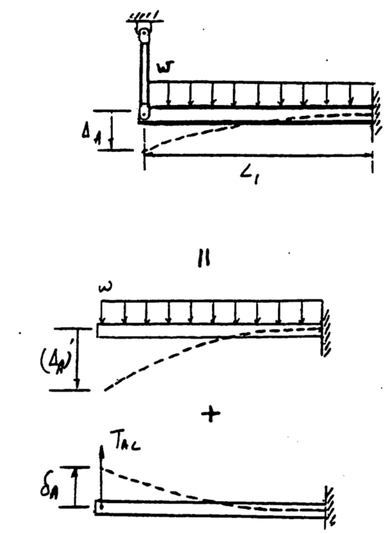
The beam has a constant E_1I_1 and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.



SOLUTION

$$(\Delta_A)' = \frac{wL_1^4}{8E_1I_1}; \qquad \Delta_A = \frac{T_{AC}L_2}{A_2E_2}$$

$$\delta_A = \frac{T_{AC}L_1^3}{3E_1I_1}$$



By Superposition:

$$(+\downarrow) \qquad \Delta_A = (\Delta_A)' - \delta_A$$

$$\frac{T_{AC}L_2}{A_2E_2} = \frac{wL_1^4}{8E_1I_1} - \frac{T_{AC}L_1^3}{3E_1I_1}$$

$$T_{AC}\left(\frac{L_2}{A_2E_2} + \frac{L_1^3}{3E_1I_1}\right) = \frac{wL_1^4}{8E_1I_1}$$

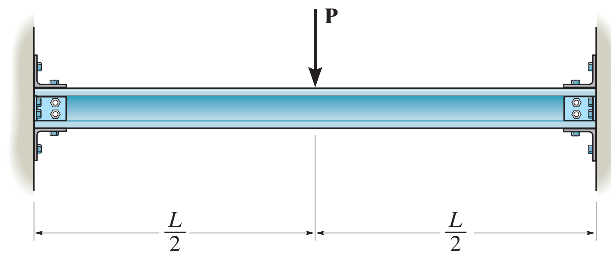
$$T_{AC} = \frac{3wA_2E_2L_1^4}{8(3E_1I_1L_2 + A_2E_2L_1^3)} \qquad \textbf{Ans.}$$

Ans:

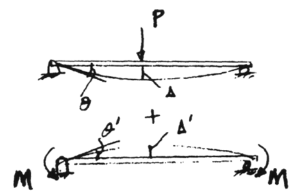
$$T_{AC} = \frac{3wA_2E_2L_1^4}{8(3E_1I_1L_2 + A_2E_2L_1^3)}$$

16–71.

The beam is supported by the bolted supports at its ends. When loaded these supports initially do not provide an actual fixed connection, but instead allow a slight rotation α before becoming fixed after the load is fully applied. Determine the moment at the supports and the maximum deflection of the beam.



SOLUTION



$$\theta - \theta' = \alpha$$

$$\frac{PL^2}{16EI} - \frac{ML}{3EI} - \frac{ML}{6EI} = \alpha$$

$$ML = \left(\frac{PL^2}{16EI} - \alpha\right)(2EI)$$

$$M = \frac{PL}{8} - \frac{2EI}{L}\alpha \qquad \textbf{Ans.}$$

$$\Delta_{max} = \Delta - \Delta' = \frac{PL^3}{48EI} - 2\left[\frac{M(\frac{L}{2})}{6EIL}\left[L^2 - (L/2)^2\right]\right]$$

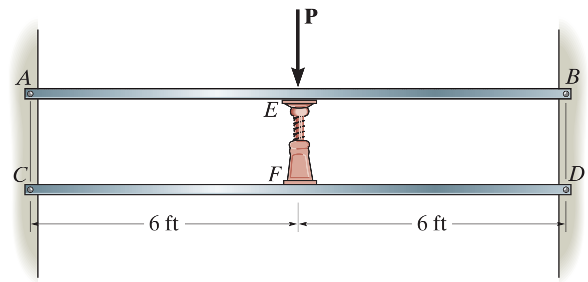
$$\Delta_{max} = \frac{PL^3}{48EI} - \frac{L^2}{8EI}\left(\frac{PL}{8} - \frac{2EI\alpha}{L}\right)$$

$$\Delta_{max} = \frac{PL^3}{192EI} + \frac{\alpha L}{4} \qquad \textbf{Ans.}$$

Ans:
$M = \dfrac{PL}{8} - \dfrac{2EI}{L}\alpha,\ \Delta_{max} = \dfrac{PL^3}{192EI} + \dfrac{\alpha L}{4}$

***16–72.**

Each of the two members is made from 6061-T6 aluminum and has a square cross section 1 in. × 1 in. They are pin connected at their ends and a jack is placed between them and opened until the force it exerts on each member is 50 lb. Determine the greatest force P that can be applied to the center of the top member without causing either of the two members to yield. For the analysis neglect the axial force in each member. Assume the jack is rigid.



SOLUTION

The jack force will cause a spread, Δ, between the bars. After P is applied, this spread is the difference between δ_E and δ_F.

$$\Delta = \delta_F - \delta_E$$

Let R be the final reaction force of the jack on the bar above and the bar below. From Appendix C,

$$2\left(\frac{50L^3}{48EI}\right) = \frac{RL^3}{48EI} - \frac{(P - R)L^3}{48EI}$$

$$R = \frac{P}{2} + 50$$

The bottom member will yield first, since it will be subject to greater deformation after P is applied. The moment due to the support reactions, $R/2$ at each end, is greatest in the middle:

$$M_{\max} = \frac{R}{2}\left(\frac{L}{2}\right) = \left(\frac{P}{4} + 25\right)(6)(12) = 18P + 1800$$

$$\sigma_{\max} = \frac{Mc}{I}$$

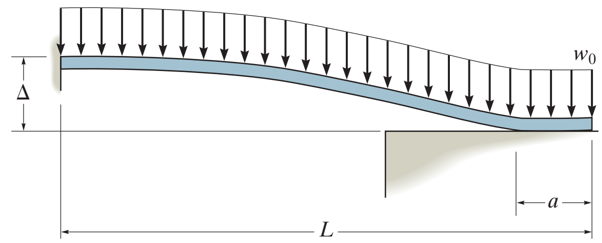
$$37(10^3) = \frac{(18P + 1800)\left(\frac{1}{2}\right)}{\frac{1}{12}(1)(1^3)}$$

$P = 243$ lb **Ans.**

Ans:
$P = 243$ lb

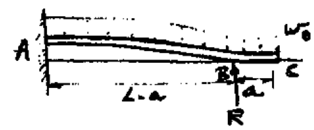
16–73.

The beam is made from a soft linear elastic material having a constant EI. If it is originally a distance Δ from the surface of its end support, determine the length a that rests on this support when it is subjected to the uniform load w_0, which is great enough to cause this to happen.



SOLUTION

The curvature of the beam in region BC is zero; therefore, there is no bending moment in the region BC. The reaction R is at B where it touches the support. The slope is zero at this point and the deflection is Δ where



$$\Delta = \frac{w_0(L-a)^4}{8EI} - \frac{R(L-a)^3}{3EI}$$

$$\theta_x = 0 = \frac{w_0(L-a)^3}{6EI} - \frac{R(L-a)^2}{2EI}$$

Thus,

$$R = \frac{w_0(L-a)}{3}$$

$$\Delta = \frac{w_0(L-a)^4}{(72EI)}$$

$$L - a = \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}}$$

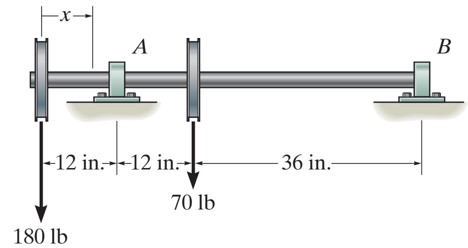
$$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}}$$ **Ans.**

Ans:

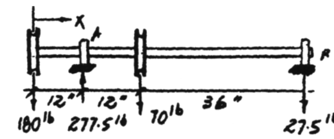
$$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{1/4}$$

R16–1.

Determine the equation of the elastic curve. Use discontinuity functions EI is constant.



SOLUTION



$$M = -180\langle x - 0\rangle - (-277.5)\langle x - 12\rangle - 70\langle x - 24\rangle$$

$$M = -180x + 277.5\langle x - 12\rangle - 70\langle x - 24\rangle$$

Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M = -180x + 277.5\langle x - 12\rangle - 70\langle x - 24\rangle$$

$$EI\frac{dv}{dx} = -90x^2 + 138.75\langle x - 12\rangle^2 - 35\langle x - 24\rangle^2 + C_1$$

$$EIv = -30x^3 + 46.25\langle x - 12\rangle^3 - 11.67\langle x - 24\rangle^3 + C_1x + C_2 \qquad \textbf{(1)}$$

Boundary Conditions:

$v = 0$ at $x = 12$ in.

From Eq. (1)

$$0 = -51{,}840 + 12C_1 + C_2$$

$$12C_1 + C_2 = 51\,840 \qquad \textbf{(2)}$$

$v = 0$ at $x = 60$ in.

From Eq. (1)

$$0 = -6\,480\,000 + 5\,114\,880 - 544\,320 + 60C_1 + C_2$$

$$60C_1 + C_2 = 1909440 \qquad \textbf{(3)}$$

Solving Eqs. (2) and (3) yields:

$$C_1 = 38\,700 \qquad C_2 = -412\,560$$

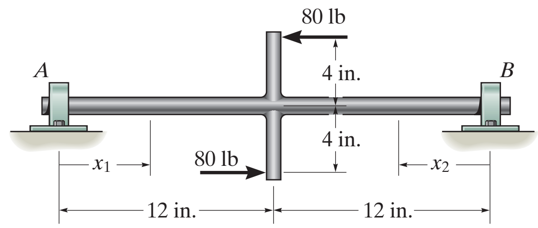
$$v = \frac{1}{EI}[-30x^3 + 46.25\langle x - 12\rangle^3 - 11.7\langle x - 24\rangle^3$$
$$+ 38\,700x - 412\,560]\ \text{lb}\cdot\text{in}^3 \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{1}{EI}\left[-30x^3 + 46.25\langle x - 12\rangle^3 - 11.7\langle x - 24\rangle^3 + 38{,}700x - 412{,}560\right]\ \text{lb}\cdot\text{in}^3$$

R16–2.

Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. Use the method of integration. EI is constant.



SOLUTION

For $M_1(x) = 26.67\,x_1$

$$EI\frac{d^2v_1}{dx_1^2} = 26.67x_1$$

$$EI\frac{dv_1}{dx_1} = 13.33x_1^2 + C_1 \qquad \textbf{(1)}$$

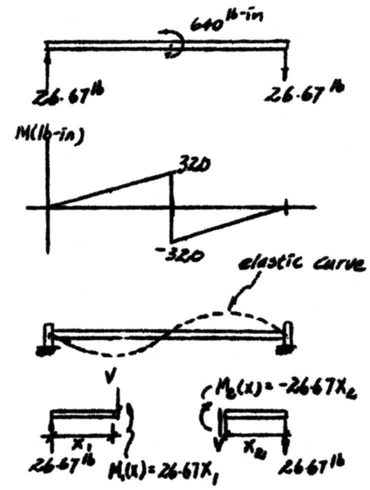
$$EIv_1 = 4.44x_1^3 + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = -26.67x_2$

$$EI\frac{d^2v_2}{dx_2^2} = -26.67x_2$$

$$EI\frac{dv_2}{dx_2} = -13.33x_2^2 + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = -4.44x_2^3 + C_3x_2 + C_4 \qquad \textbf{(4)}$$



Boundary Conditions:

$v_1 = 0 \quad$ at $\quad x_1 = 0$

From Eq. (2)

$C_2 = 0$

$v_2 = 0 \quad$ at $\quad x_2 = 0$

$C_4 = 0$

Continuity Conditions:

$$\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = 12$$

From Eqs. (1) and (3)

$$1920 + C_1 = -(-1920 + C_3)$$

$$C_1 = -C_3 \qquad \textbf{(5)}$$

$v_1 = v_2 \quad$ at $\quad x_1 = x_2 = 12$

$$7680 + 12C_1 = -7680 + 12C_3$$

$$C_3 - C_1 = 1280 \qquad \textbf{(6)}$$

Solving Eqs. (5) and (6) yields:

$C_3 = 640 \qquad C_1 = -640$

$$v_1 = \frac{1}{EI}\left(4.44x_1^3 - 640x_1\right)\ \text{lb}\cdot\text{in}^3 \qquad \textbf{Ans.}$$

$$v_2 = \frac{1}{EI}\left(-4.44x_2^3 + 640x_2\right)\ \text{lb}\cdot\text{in}^3 \qquad \textbf{Ans.}$$

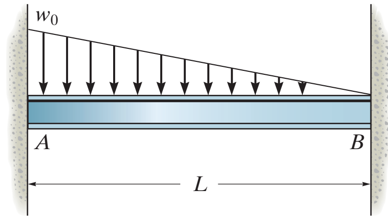
Ans:

$v_1 = \dfrac{1}{EI}(4.44x_1^3 - 640x_1)\ \text{lb}\cdot\text{in}^3,$

$v_2 = \dfrac{1}{EI}(-4.44x_2^3 + 640x_2)\ \text{lb}\cdot\text{in}^3$

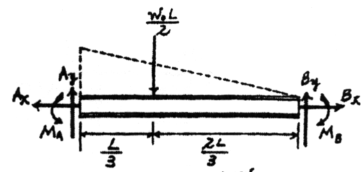
R16–3.

Determine the moment reactions at the supports A and B. Use the method of integration. EI is constant.



SOLUTION

Support Reactions: FBD(a).

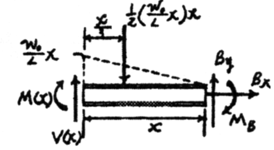


(a)

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \tag{1}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - M_B - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \tag{2}$$

Moment Function: FBD(b).



(b)

$$\circlearrowleft + \Sigma M_{\text{NA}} = 0; \qquad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) - M_B + B_y x = 0$$

$$M(x) = B_y x - \frac{w_0}{6L}x^3 - M_B$$

Slope and Elastic Curve:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - \frac{w_0}{6L}x^3 - M_B$$

$$EI\frac{dv}{dx} = \frac{B_y}{2}x^2 - \frac{w_0}{24L}x^4 - M_B x + C_1 \tag{3}$$

$$EI\,v = \frac{B_y}{6}x^3 - \frac{w_0}{120L}x^5 - \frac{M_B}{2}x^2 + C_1 x + C_2 \tag{4}$$

Boundary Conditions:

At $x = 0, \dfrac{dv}{dx} = 0$ From Eq. (3), $C_1 = 0$

At $x = 0, v = 0.$ From Eq. (4), $C_2 = 0$

At $x = L, \dfrac{dv}{dx} = 0.$ From Eq. (3),

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} - M_B L$$

$$0 = 12B_y L - w_0 L^2 - 24M_B \tag{5}$$

At $x = L, v = 0.$ From Eq. (4),

$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} - \frac{M_B L^2}{2}$$

$$0 = 20B_y L - w_0 L^2 - 60M_B \tag{6}$$

R16–3. Continued

Solving Eqs. (5) and (6) yields,

$$M_B = \frac{w_0 L^2}{30}$$ **Ans.**

$$B_y = \frac{3w_0 L}{20}$$

Substituting B_y and M_B into Eqs. (1) and (2) yields,

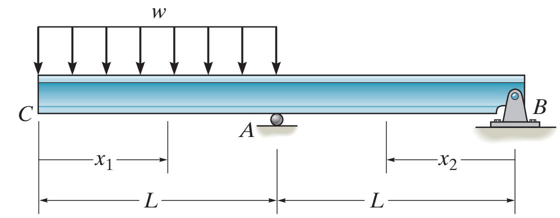
$$M_A = \frac{w_0 L^2}{20}$$ **Ans.**

$$A_y = \frac{7w_0 L}{20}$$

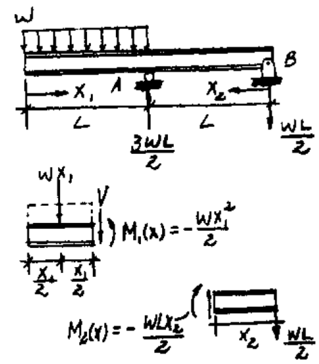
Ans:
$M_B = \dfrac{w_0 L^2}{30}$, $M_A = \dfrac{w_0 L^2}{20}$

***R16–4.**

Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. Use the method of integration. EI is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \dfrac{-wx_1^1}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{-wx_1^4}{24} + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = \dfrac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_2^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_3^2}{4} + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = \frac{-wLx_2^3}{12} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_2 = 0$ at $x_2 = 0$

From Eq. (4):

$C_4 = 0$

$v_2 = 0$ at $x_2 = L$

From Eq. (4):

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0$ at $x_1 = L$

From Eq. (2):

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad \textbf{(5)}$$

***R16–4. Continued**

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2} \qquad \text{at} \qquad x_1 = x_2 = L$$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = -\frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad \textbf{(6)}$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1 = L} = -\left.\frac{dv_2}{dx_2}\right|_{x_2 = L} = \frac{wL^3}{6EI} \qquad \textbf{Ans.}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + BL^3x_1 - 7L^4) \qquad \textbf{Ans.}$$

$$(v_1)_{\max} = \frac{-7wL^4}{24EI} \qquad (x_1 = 0)$$

The negative sign indicates downward displacement.

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad \textbf{(7)} \qquad \textbf{Ans.}$$

$$(v_2)_{\max} \text{ occurs when } \frac{dv_2}{dx_2} = 0$$

From Eq. (6)

$$L^3 - 3Lx_2^2 = 0$$

$$x_2 = \frac{L}{\sqrt{3}}$$

Substitute x_2 into Eq. (7),

$$(v_2)_{\max} = \frac{wL^4}{18\sqrt{3}EI} \qquad \textbf{(8)}$$

$$v_{\max} = (v_1)_{\max} = -\frac{7wL^4}{24EI} \qquad \textbf{(9)} \qquad \textbf{Ans.}$$

Ans:

$\theta_A = \dfrac{wL^3}{6EI}$,

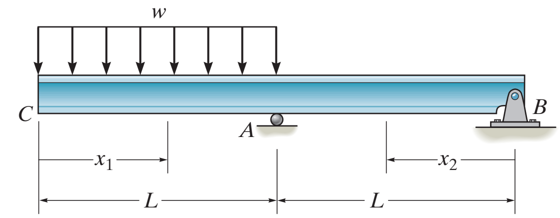
$v_1 = \dfrac{w}{24EI}(-x_1^4 + BL^3x_1 - 7L^4)$,

$v_2 = \dfrac{wL}{12EI}(L^2x_2 - x_2^3)$,

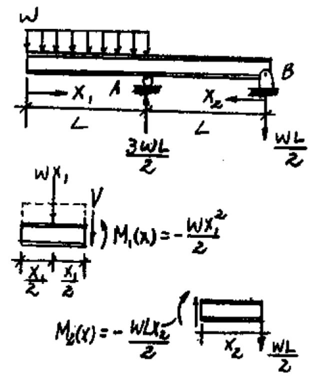
$v_{\max} = -\dfrac{7wL^4}{24EI}$

R16–5.

Determine the maximum deflection between the supports A and B. Use the method of integration. EI is constant.



SOLUTION



Elastic Curve and Slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \dfrac{-wx_1^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad \textbf{(1)}$$

$$EIv_1 = \frac{-wx_1^4}{24} + C_1x_1 + C_2 \qquad \textbf{(2)}$$

For $M_2(x) = \dfrac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_2^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_2^2}{4} + C_3 \qquad \textbf{(3)}$$

$$EIv_2 = \frac{-wLx_2^3}{12} + C_3x_2 + C_4 \qquad \textbf{(4)}$$

Boundary Conditions:

$v_2 = 0 \quad$ at $\quad x_2 = 0$

From Eq. (4):

$C_4 = 0$

$v_2 = 0 \quad$ at $\quad x_2 = L$

From Eq. (4):

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0 \quad$ at $\quad x_1 = L$

From Eq. (2):

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad \textbf{(5)}$$

R16–5. Continued

Continuity Conditions:

$$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2} \qquad \text{at} \qquad x_1 = x_2 = L$$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = -\frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad \textbf{(6)}$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1 = L} = -\left.\frac{dv_2}{dx_2}\right|_{x_2 = L} = \frac{wL^3}{6EI}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + 8L^3x_1 - 7L^4)$$

$$(v_1)_{\max} = \frac{-7wL^4}{24EI} \qquad (x_1 = 0)$$

The negative sign indicates downward displacement.

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad \textbf{(7)}$$

$(v_2)_{\max}$ occurs when $\dfrac{dv_2}{dx_2} = 0$

From Eq. (6)

$$L^3 - 3Lx_2^2 = 0$$

$$x_2 = \frac{L}{\sqrt{3}}$$

Substitute x_2 into Eq. (7),

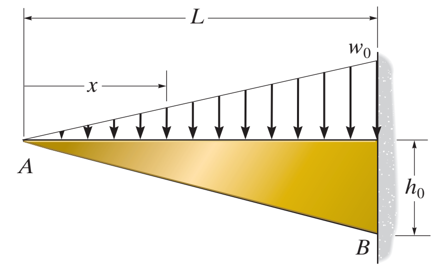
$$(v_2)_{\max} = \frac{wL^4}{18\sqrt{3}EI} \qquad \textbf{Ans.}$$

Ans:

$$(v_2)_{\max} = \frac{wL^4}{18\sqrt{3}EI}$$

R16–6.

If the cantilever beam has a constant thickness t, determine the deflection at end A. The beam is made of material having a modulus of elasticity E.



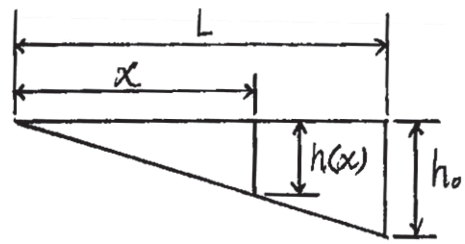
SOLUTION

Section Properties: Referring to the geometry shown in Fig. *a*,

$$\frac{h(x)}{x} = \frac{h_0}{L}; \qquad h(x) = \frac{h_0}{L}x$$

Thus, the moment of inertia of the tapered beam as a function of x is

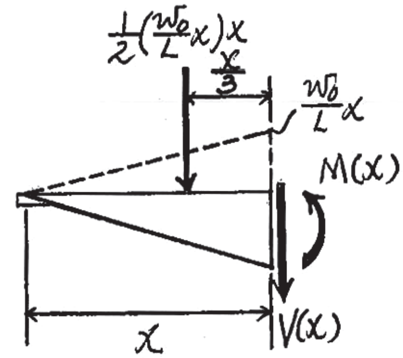
$$I(x) = \frac{1}{12}t\left[h(x)\right]^3 = \frac{1}{12}t\left(\frac{h_0}{L}x\right)^3 = \frac{t{h_0}^3}{12L^3}x^3$$



(a)

Moment Function. Referring to the free-body diagram of the beam's segment, Fig. *b*,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x) + \left[\frac{1}{2}\left(\frac{w_0}{L}x\right)x\right]\left(\frac{x}{3}\right) = 0 \qquad M(x) = -\frac{w_0}{6L}x^3$$



(b)

Equations of slope and Elastic Curve.

$$E\frac{d^2v}{dx^2} = \frac{M(x)}{I(x)}$$

$$E\frac{d^2v}{dx^2} = \frac{-\frac{w_0}{6L}x^3}{\frac{t{h_0}^3}{12L^3}x^3} = -\frac{2w_0L^2}{t{h_0}^3}$$

$$E\frac{dv}{dx} = -\frac{2w_0L^2}{t{h_0}^3}x + C_1 \qquad (1)$$

$$Ev = -\frac{w_0L^2}{t{h_0}^3}x^2 + C_1x + C_2 \qquad (2)$$

Boundary conditions. At $x = L, \frac{dv}{dx} = 0$. Then Eq. (1) gives

$$0 = -\frac{2w_0L^2}{t{h_0}^3}(L) + C_1 \qquad C_1 = \frac{2w_0L^3}{t{h_0}^3}$$

At $x = L, v = 0$. Then Eq. (2) gives

$$0 = -\frac{w_0L^2}{t{h_0}^3}\left(L^2\right) + \frac{2w_0L^3}{t{h_0}^3}(L) + C_2 \qquad C_2 = -\frac{w_0L^4}{t{h_0}^3}$$

Substituting the results of C_1 and C_2 into Eq. (2),

$$v = \frac{w_0L^2}{Et{h_0}^3}\left(-x^2 + 2Lx - L^2\right)$$

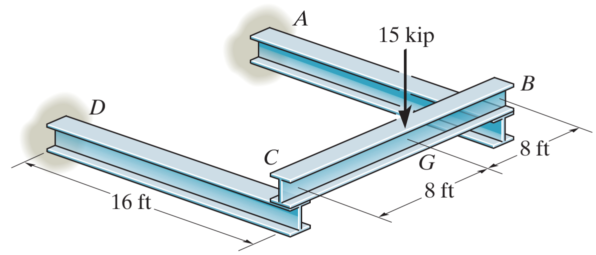
At $A, x = 0$. Then

$$v_A = v\big|_{x=0} = -\frac{w_0L^4}{Et{h_0}^3} = \frac{w_0L^4}{Et{h_0}^3}\downarrow \qquad \textbf{Ans.}$$

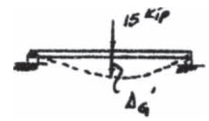
Ans:
$v_A = \frac{w_0L^4}{Eth_0^3}\downarrow$

R16–7.

The framework consists of two A-36 steel cantilevered beams CD and BA and a simply supported beam CB. If each beam is made of steel and has a moment of inertia about its principal axis of $I_x = 118\text{ in}^4$, determine the deflection at the center G of beam CB. ***Use the method of superposition***.



SOLUTION



$$\Delta_C = \frac{PL^3}{3EI} = \frac{7.5(16^3)}{3EI} = \frac{10.240}{EI}\ \downarrow$$

$$\Delta'_G = \frac{PL^3}{48EI} = \frac{15(16^3)}{48EI} = \frac{1.280}{EI}\ \downarrow$$

$$\Delta_G = \Delta_C + \Delta'_G$$

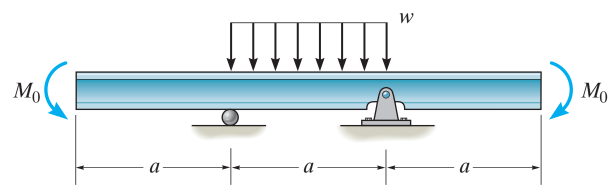
$$= \frac{10{,}240}{EI} + \frac{1{,}280}{EI} = \frac{11{,}520}{EI}$$

$$= \frac{11{,}520(1728)}{29(10^3)(118)} = 5.82\text{ in.}\downarrow$$ **Ans.**

Ans:
$\Delta_G = 5.82\text{ in.}\downarrow$

***R16–8.**

Using the method of superposition, determine the magnitude of $\mathbf{M}_0$ in terms of the distributed load w and dimension a so that the deflection at the center of the beam is zero. EI is constant.



SOLUTION

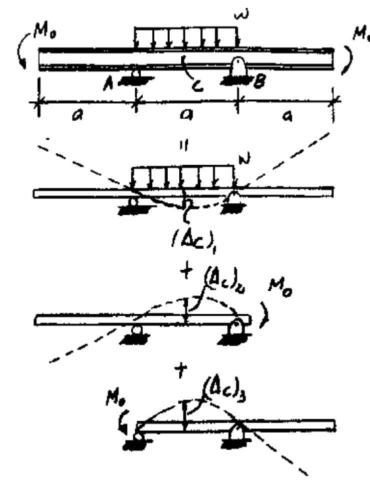
$$(\Delta_C)_1 = \frac{5wa^4}{384EI} \downarrow$$

$$(\Delta_C)_2 = (\Delta_C)_3 = \frac{M_0 a^2}{16EI} \uparrow$$

$$\Delta_C = 0 = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$+\uparrow \qquad 0 = \frac{-5wa^4}{384EI} + \frac{M_0 a^2}{8EI}$$

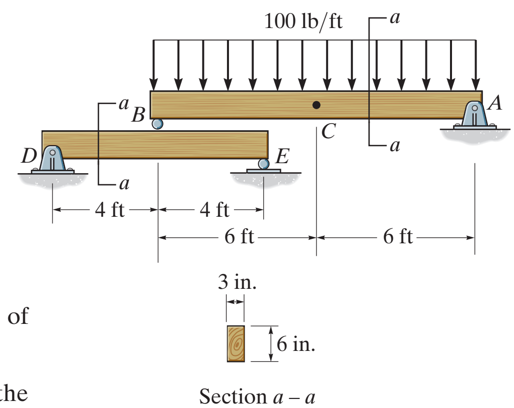
$$M_0 = \frac{5wa^2}{48}$$ **Ans.**



Ans:
$M_0 = \dfrac{5wa^2}{48}$

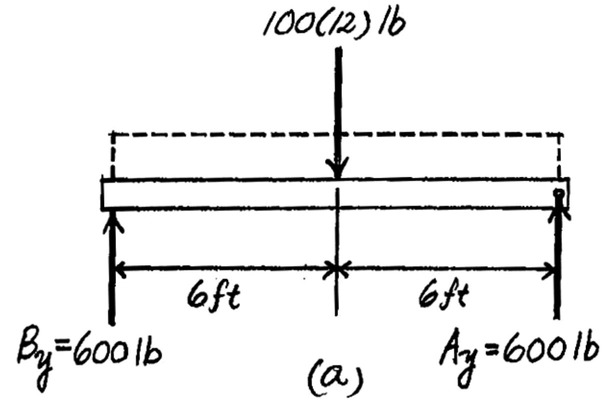
R16–9.

Using the method of superposition, determine the deflection at C of beam AB. The beams are made of wood having a modulus of elasticity of $E = 1.5(10^3)$ ksi.



SOLUTION

Support Reactions: The reaction at B is shown on the free-body diagram of beam AB, Fig. a.



Method of superposition. Referring to Fig. b and the table in the appendix, the deflection of point B is

$$\Delta_B = \frac{PL_{DE}{}^3}{48EI} = \frac{600\left(8^3\right)}{48EI} = \frac{6400 \text{ lb}\cdot\text{ft}^3}{EI} \downarrow$$

Subsequently, referring to Fig. c,

$$(\Delta_C)_1 = \Delta_B\left(\frac{6}{12}\right) = \frac{6400}{EI}\left(\frac{6}{12}\right) = \frac{3200 \text{ lb}\cdot\text{ft}^3}{EI} \downarrow$$

$$(\Delta_C)_2 = \frac{5wL^4}{384EI} = \frac{5(100)\left(12^4\right)}{384EI} = \frac{27000 \text{ lb}\cdot\text{ft}^3}{EI} \downarrow$$

Thus, the deflection of point C is

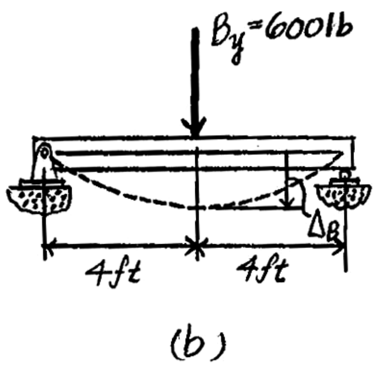
$$(+\downarrow) \qquad \Delta_C = (\Delta_C)_1 + (\Delta_C)_2$$

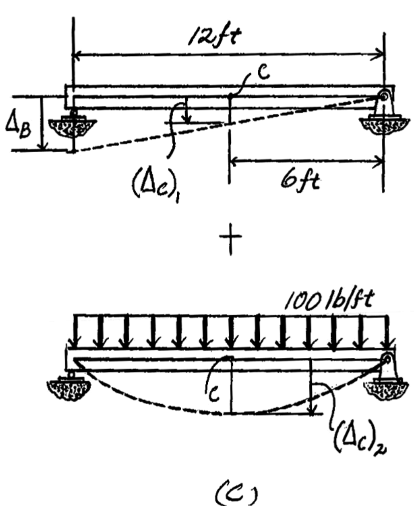
$$= \frac{3200}{EI} + \frac{27000}{EI}$$

$$= \frac{30200 \text{ lb}\cdot\text{ft}^3}{EI} = \frac{30200\left(12^3\right)}{1.5\left(10^6\right)\left[\frac{1}{12}(3)\left(6^3\right)\right]}$$

$$= 0.644 \text{ in.} \downarrow$$

Ans.





Ans:
$\Delta_C = 0.644 \text{ in.}\downarrow$