Safety and limit states design for reinforced concrete[1]

J. G. MACGREGOR
Department of Civil Engineering, University of Alberta, Edmonton, Alta., Canada T6G 2E1

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This state-of-art paper reviews the concept of limit states design. Following a brief review of statistical definitions the sources of variability in reinforced concrete structures are reviewed. Methods of defining structural safety are reviewed. Following a derivation of the procedures used to compute load and ϕ factors, a series of ϕ factors compatible with the 1975 National Building Code of Canada load factors are computed. With the exception of the value for shear the new ϕ factors are lower than the current American Concrete Institute and Canadian Standards Association values by about the amount of the ratio of load factors in National Building Code of Canada and American Concrete Institute. The computed ϕ for shear is considerably lower than the corresponding value from the American Concrete Institute Code. An Appendix traces the development of the American Concrete Institute load and ϕ factors.

Cet article examine le concept du calcul aux états limites et se veut un exposé à jour des études réalisées sur cette question. Après un bref rappel de définitions statistiques, il passe en revue les causes de la variabilité dans les structures de béton armé. Puis les auteurs exposent les méthodes de calcul des charges et des coefficients ϕ, et pour ces derniers est établie une série de valeurs conformes aux règles C.N.B. 1975. Sauf dans le cas du cisaillement, les nouveaux coefficients ϕ ont des valeurs inférieures à celles des règles A.C.I. et ACNOR dans un rapport approximativement égal au quotient des coefficients C.N.B. et A.C.I. La valeur calculée du coefficient ϕ pour le cisaillement est considérablement plus faible que la valeur correspondante tirée du règlement A.C.I. Un appendice retrace les étapes qui ont conduit à la définition des coefficients de charge et de pondération ϕ tels qu'ils apparaissent dans le règlement A.C.I.

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1. Introduction

1.1 *Development of Limit States Design in Canada*

In 1963 the American Concrete Institute Building Code (ACI 1963) pioneered the North American use of split load factors to account for overloads and understrength members. The procedure followed in deriving the ACI safety provisions involved: (i) selection of the statistical parameters involved; (ii) statistical estimate of the probability of failure; and (iii) a liberal amount of committee compromise as outlined in Appendix A.

The introduction of the ACI safety provisions coincided with an upsurge in the interest in, and the development of, safety theories. In the 13 years since the adoption of the 1963 ACI Code major advances have occurred in all aspects of safety and sophisticated procedures have evolved for estimating load factors and resistance or ϕ factors. In today's attempts to evaluate safety provisions, however, the three steps listed earlier are still required and, as shown in section 6.3.6 of this paper, the original ACI ϕ factors and load factors are remarkably similar to those currently being developed.

The chapter on Structural Loads and Procedures of the 1975 National Building Code of Canada (NRC 1975) included rules for both working stress design (section 4.1.3) and limit states design (section 4.1.4). The limit states design concept is discussed more fully in section 2 of this paper. The safety provisions for limit states design, as proposed for use in Canada, involve common load factors, presented in section 4.1.4 of the National Building Code, to account for overloads, and resistance, performance or ϕ factors, to be presented in the various material structural specifications, to account for possible understrength of the structural members.

The decision to adopt common load factors

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for all materials is an attempt to reach uniform levels of safety and to unify and simplify structural design. With the increased use of mixed structural systems such as steel frames braced by concrete shear walls and supported by concrete footings resting on soil, flat plate floors supported by steel columns, precast floors supported by masonry walls, etc., common load factors become highly desirable.

The load factors in section 4.1.4 of NBC (1975) were developed by the Canadian Standards Association/National Building Code (CSA/NBC) Joint Liaison Committee on Limit States Design on the basis of an initial proposal from the CSA Committee on Steel Structures who at the time were developing a new standard, CSA S16.1 (1974). The load factors are common to all construction materials and will be applied to each building material as soon as appropriate resistance or ϕ factors compatible with the 1975 load factors have been derived for that material. Because this has not yet been done for concrete structures, section 4.1.4.2(6) of NBC (1975) requires the use of the load factors and ϕ factors currently in ACI 318-71 (1971) or CSA A23.3 (CSA 1973) as an interim measure. The committee responsible for the Canadian reinforced concrete code has formed a subcommittee chaired by the writer to develop new ϕ factors for concrete structures. This paper reviews some of the problems involved in this task.

1.2 *Research on Structural Safety*

Early in 1975 the Task Group on Safety Criteria for Limit States Design, charged with co-ordinating the development of load factors and ϕ factors for future editions of NBC and future CSA structural standards, proposed a research program to develop the necessary safety provisions in a unified manner. Although this research proposal was enthusiastically supported by structural specification committees, it has not been funded and hence little work has been carried out.

The necessary research and developmental work on the safety of reinforced concrete structures is currently underway at the University of Alberta. This work is specifically aimed at developing resistance factors for design office use. Although, similar research for timber is underway at the University of British Columbia and the Western Forest Products Laboratories, no major work is underway on other materials. Furthermore, the essential aspect of interdisciplinary co-ordination is absent. For this reason it is very important that funding be found for the major research program described earlier.

1.3 *Scope of Report*

This report is intended to be an introductory survey of the concepts involved in establishing load and ϕ factors for concrete structures. Section 2 introduces the concept of limit states design and compares current design procedures to the limit states design philosophy. A number of basic statistical definitions are presented in section 3 for use later in the report. Section 4 reviews the reasons why load and resistance factors are introduced in structural design. Three basic methods for estimating the safety of structures are presented in section 5 followed by a discussion of the safety provisions in the ACI (1971) and CSA A23.3 (1973) Codes and the latest proposals by the European Concrete Committee for European design regulations. Perhaps the most important part of this report is section 6 which outlines one procedure currently under consideration for establishing load and ϕ factors and presents a few typical example calculations. The final section serves as a summary. The derivation of the ACI (1971) and CSA A23.3 (1973) safety procedures is reviewed in Appendix A.

It should be noted that the load factors and ϕ factors presented in this report are far from being finalized for design and should not be used until incorporated in the necessary material structural standards.

2. Limit States and Limit States Design

An engineer designing a new type of structure using a new material would follow a procedure which includes most of the following steps:

1. Select a structural system for the particular purpose.
2. Identify all modes of failure or ways in which the structure might fail to fulfil its intended purpose.
3. Determine mathematical relationships between the loads and each of the potential modes of failure.

4. Determine the properties of the materials and loads which are required in these relationships.
5. Select a reasonable level of safety.
6. Analyze an idealized model of the structure.
7. Interpret the results of the structural analysis in terms of the actual structure.
8. Proportion the structural members.

In utilizing a well established material such as reinforced concrete, all of these steps are necessary, but several would have been done by persons other than the designer. Thus, the equations in step 3 and the material properties in step 4 are obtained in engineering research centers while the level of safety in step 5 will have been established by a building code committee. All too often step 7 is forgotten, especially when computer analyses are carried out.

When a structure or structural element becomes unfit for its intended use it is said to have reached a *limit state.*

Limit states design is a design process that involves:

1. Identification of all potential modes of failure (limit states).
2. Determination of acceptable levels of safety against occurrence of each limit state.
3. Consideration by the designer of the significant limit states.

2.1 *List of Limit States for Reinforced Concrete Structures*

Limit states can be divided into three basic groups:

1. *Ultimate Limit States*—related to a structural collapse of part or all of the structure. Such a limit state should have a very low probability of occurrence since it may lead to loss of life and major financial losses.

(a) Loss of equilibrium of a part or all of the structure when considered as a rigid body (tipping or sliding).
(b) Rupture of critical parts of the structure leading to collapse or progressive collapse.
(c) Formation of a plastic mechanism.
(d) Instability due to deformations of the structure.
(e) Collapse due to corrosion, deterioration, fatigue, or brittle collapse.
(f) Structural effects of fire or explosions.

2. *Damage Limit States*—related to damage of the structure such as cracking or spalling of the concrete. Since there is less danger of loss of life, a higher probability of occurrence can be tolerated than in the case of the ultimate limit states.

(a) Premature or excessive cracking.
(b) Excessive deformations leading to damage to non-structural elements or changes in the distribution of forces.
(c) Permanent inelastic deformations.

3. *Serviceability or Functional Limit States*—related to disruption of the functional use of the structure.

(a) Excessive deformations for normal service.
  (i) Sensory acceptability (visual, auditory, tactile)
  (ii) Serviceability (drainage, malfunction of machinery)
(b) Undesirable vibrations.

Traditionally, reinforced concrete has been designed by either working stress design or ultimate strength design. Each of these procedures explicitly considers a few, but not all, of the critical limit states. Thus, working stress design prevents the attainment of permanent inelastic deformations (limit state 2(c)) by limiting the stresses under working loads. The use of allowable stresses can be used to ensure that limit states 1(b), 1(c), 1(d), and 1(e) do not occur at service load levels. The designer does not know what the safety against collapse really is, however, particularly if the stresses due to the major loads counteract each other.

Ultimate strength design, on the other hand, checks limit states 1(b) and 1(d) directly. If the critical cross sections have adequate ductility, current design procedures which employ elastic analysis for forces and moments and ultimate strength design of sections will give a lower bound solution to the plastic mechanism load, 1(c), if stability failures are prevented. The ACI (1971) and CSA (1973) concrete codes attempt to limit the reinforcement ratio so that beams are always ductile in flexure. As pointed out in section 4.3(e) of this paper, these limits may not always guarantee ductility.

Both design procedures consider the limit states involving deformations, severe cracking, and vibrations by requiring a separate check of these cases. Both design procedures prevent overall loss of equilibrium, although at different load levels. Checking overall equilibrium at the service load level may not be adequate if the major loads counteract each other or increase at different rates.

The difference between these two current design procedures and limits states design stems from the fact that in limit states design the designer is expected to identify all the critical limit states and consider them either explicitly by design checks or implicitly by satisfying certain detailing requirements or minimum reinforcement requirements. Ideally, the limit states would be expressed in terms of performance requirements which are essentially independent of the structural material. Hence, the basic deflection limits, etc. should be presented in Chapter 4.1 of the National Building Code (National Research Council of Canada 1975) and should be similar for all materials.

With the reduction of safety factors against ultimate limit states, the damage and serviceability limit states become much more critical and may govern designs. Thus, for example, crack width provisions may govern the limit states design of medium to long span bridges since crack width is proportional to reinforcement stress and the dead load stresses represent a major part of the total stresses in the reinforcement.

3. Brief Review of Statistical Definitions

3.1 *General Definitions and Concepts*[2]

If a large number of individual sampled values are available for a particular variable, they can be plotted in a *histogram* or *frequency diagram* as shown in Fig. 1. The total number of observations is N (in Fig. 1, $N = 243$), the number at any value of x is n_x.

$$[1] \quad \sum_{x=-\infty}^{\infty} n_x = N$$

$$= \text{'area under the histogram'}$$

The total data have an arithmetic average or *mean*, $\bar{x}$. The dispersion of the data is measured by the *sample standard deviation*, σ_x:

$$[2] \qquad \sigma_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N-1}}$$

[2]For more detailed information on probability and statistics see a textbook on the subject such as Benjamin and Cornell (1970).

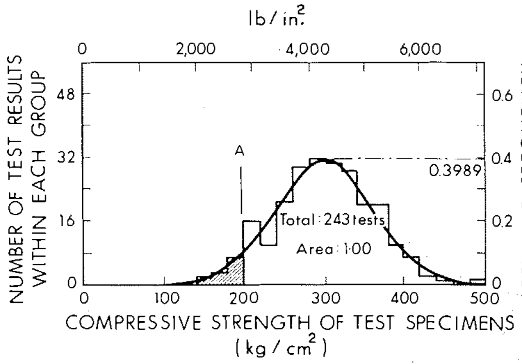

FIG. 1. Distribution of concrete compressive strength.

This is essentially the root mean square deviation of the values from the mean. Frequently it is more convenient to express the standard deviation as a fraction or percentage of the mean. This is called the *sample coefficient of variation*, V_x:

$$[3] \quad V_x = \sigma_x/\bar{x} \quad (\text{or } \sigma_x/\bar{x} \times 100\%)$$

The histogram is really only applicable to the particular number of pieces of observations considered (N). A more universally applicable diagram is obtained by dividing the vertical ordinates by N to get a *frequency diagram* as shown by the right hand scale in Fig. 1. The area under the frequency diagram is $N/N = 1.0$.

3.2 *Properties of Normal and Log-Normal Distributions*

Frequently it is possible to represent the data with a standard statistical distribution corresponding to the probabilities involved in a particular type of statistical process. Two commonly used distributions in load factor theory are the normal distribution and the log-normal distribution.

If a random variable is the sum of a number of independent causes, the distribution of the sum will tend to approach a normal distribution. Because many natural random variables appear to have a normal distribution, this distribution is widely used even when the mathematical conditions are not strictly satisfied. A normal distribution has been fitted to the concrete strength data in Fig. 1 and as can be seen it represents this set of data reasonably well. The normal distribution extends

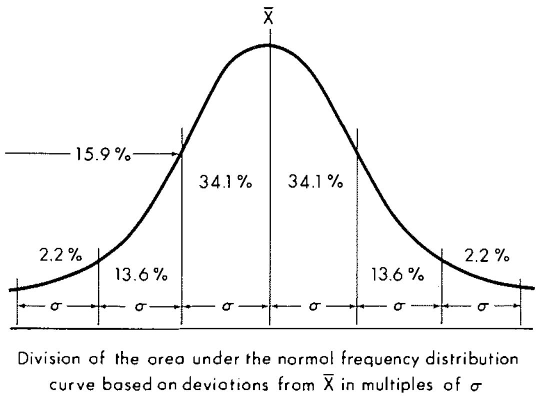

FIG. 2. Properties of a normal distribution.

from $-\infty$ to ∞ and is symmetrical about the mean, as shown in Figs. 1 and 2.

The area under the shaded part of the curve in Fig. 1 is given by:

[4] $$P(x \leqslant A) = \int_{-\infty}^{A} f_x \, dx$$

This is a fraction less than one and represents the probability that a data point will fall to the left of A (concrete strength less than 200 kg/cm^2 in this case) in a single trial. Tables of $P(x \leqslant A)$ where A is specified as being $\beta\sigma_x$ above or below the mean, are available for standard probability distributions. Table 1 is an example. Later in this paper β will be used as the 'safety index' since it can be used to estimate the probability of under-strength values.

For β greater than about 3, the values of $P(x \leqslant A)$ or P_f may be approximated by:

[5] $$P_f \simeq 460e^{-4.3\beta}$$

TABLE 1. Values of $P(x \leqslant A)$ for $A = (\bar{x} - \beta\sigma_x)$ normal distribution

β	$P(x \leqslant A)$
1.0	0.1587
1.28	0.1
1.64	0.05
2.0	0.0227
2.32	0.01
3.0	1.35×10^{-3}
3.1	1×10^{-3}
3.5	1.1×10^{-4}
4.0	3.2×10^{-5}
4.5	3×10^{-6}

In a log-normal distribution, the function $y = \ln x$ is normally distributed. This results in a skewed distribution of x as shown in Fig. 3 which shows yield strength data from almost 20 000 tests of structural steel plates and shapes produced in Sweden (Alpsten 1972). The probability of a given value of y falling more than β standard deviations below y can be obtained directly from Table 1. In theory a log-normal distribution will be applicable if the function in question results from the product of a large number of independent random variables. In many cases in practice, however, it has been adopted because the observed data are skewed and a reasonably good fit can be obtained with this relatively simple transformation of a normal distribution.

3.3 *Combination of Normal Distributions*

Frequently it is necessary to combine the effects of a number of normally distributed variables to determine the overall effect of the combination. These combinations may be 'additive' or 'multiplicative'. The strength of an axially loaded concrete column, $P_o = P_c + P_s$, is an additive combination as is $U = D + L$. On the other hand, the load carried by the steel in the axially loaded column is a multiplicative combination, $P_s = A_{st} \cdot f_y$.

Generally when a number of discrete variables are combined additively the combination will tend to be closer to a normal distribution than the distributions that are combined.

The procedures for combining statistical dis-

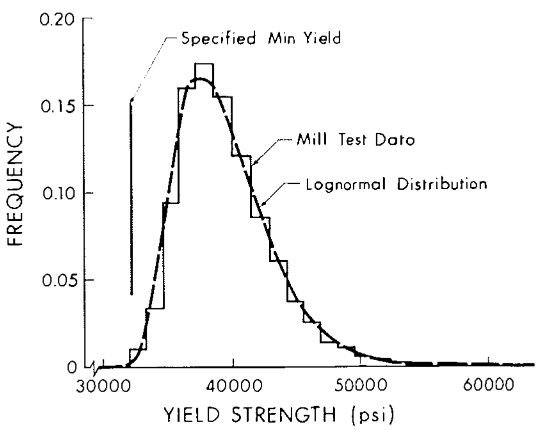

FIG. 3. Distribution of structural steel yield strengths (Alpsten 1972).

tributions depend on whether the combination is additive or multiplicative. In the following, A and B are independent random variables with means $\bar{A}$ and $\bar{B}$, standard deviations σ_A and σ_B, and coefficients of variation V_A and V_B.

(a) *Additive Combinations*

Let

[6a] $$X = A - B \text{ (or } A + B)$$

then

[6b] $$\bar{X} = \bar{A} - \bar{B} \text{ (or } \bar{A} + \bar{B})$$

and

[6c] $$\sigma_X = \sqrt{\sigma_A^2 + \sigma_B^2}$$

An example of this type of combination is given in section 6.2.3.

(b) *Multiplicative Combinations*

Let

[7a] $$Y = A \times B$$

then

[7b] $$\bar{Y} \simeq \bar{A} \times \bar{B}$$

and

[7c] $$V_Y \simeq \sqrt{V_A^2 + V_B^2 + V_A^2 V_B^2}$$

Since V_A and V_B are generally less than 0.3 in most situations only a small error in V_Y (5% at most) is introduced by neglecting the product term and it will be assumed that:

[7d] $$V_Y = \sqrt{V_A^2 + V_B^2}$$

An example of this type of combination is given in section 6.2.2.

4. Reasons for Requiring Load and Resistance Factors in Structural Design

There are three primary reasons for including safety factors of some sort in structural design: (a) the strengths of materials or elements may be less than expected, (b) overloads may occur, and (c) the consequences of a failure may be very severe. Each of these will be reviewed in the remainder of this section.

4.1 *Understrength Materials or Elements*

In calculations this effect will be referred to by the factor R (for resistance) with mean $\bar{R}$, standard deviation σ_R and coefficient of variation V_R. R, in turn, can be subdivided into a number of individual factors which contribute to variations in the resistance, R.

4.1.1. *Material strengths may have both a systematic and a random difference from that assumed in design*—Factor M, $\bar{M}$, σ_M, and V_M

(a) *Variability in strengths*

The compression strength of concrete is variable as shown in Fig. 1 and is generally assumed to be normally distributed. The standard deviation depends on the degree of control and is affected by the strength of the concrete itself. Values of the standard deviation from a large number of jobs are plotted in Fig. 4 (Rackwitz 1973). For $f_c' \leqslant 4000$ psi (27.6 MPa), average control corresponds roughly to a constant coefficient of variation of about $V = 0.15$ while for $f_c' > 4000$ psi (27.6 MPa), average control corresponds roughly to a constant standard deviation of about $\sigma = 600$ psi (4.1 MPa) (Rackwitz 1973). Poor control and good control, respectively, correspond to V and σ about one-third greater or smaller than these values.

Similarly, the yield strength and ultimate tensile strength of reinforcement are variable. The shape of the distribution is variously described as being normal, log-normal or extremal. The results of almost 20 000 mill tests on structure steel plates and shapes with a nominal yield strength of 31 300 psi (220 MPa) are compared to a log-normal distribution in Fig. 3 and fit this distribution quite well (Alpsten 1972). On the other hand, a number of investigators have recommended the use of a normal distribution for higher strength structural steels and for reinforcing bars (Allen 1970). Data from mill tests of grade

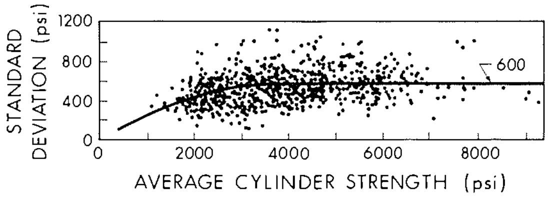

FIG. 4. Relationship between average cylinder strength of concrete and standard deviation of test series (Rackwitz 1973).

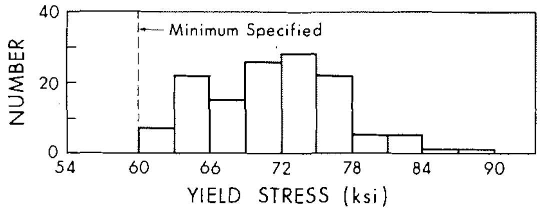

FIG. 5. Distribution of steel yield strengths for grade 60 reinforcement (Allen 1972).

60 reinforcing bars were plotted in Fig. 5 (Allen 1972). These data are slightly skewed and probably could be represented equally well by a normal or log-normal distribution.

The mean yield of reinforcing bars is relatively constant for bar sizes up to No. 11 but drops for larger bar sizes as shown in Fig. 6 (Grant 1976).

(b) *Effect of Speed of Testing*

The strengths of both concrete and steel are affected by the rate of loading. Under extremely slowly applied loads or very high sustained loads the compression strength of concrete drops to about 75 to 80% of the short time strength (Rüsch 1960). This is offset, however, by the maturing of the concrete. Concrete subjected to sustained loads less than this critical value, followed by rapidly applied loads, will not be weakened by the sustained load.

In a mill test on reinforcing bars, the load is applied at a very high rate. In a structure, the loading rate is very much lower under such loadings as dead and live load. Based on tests

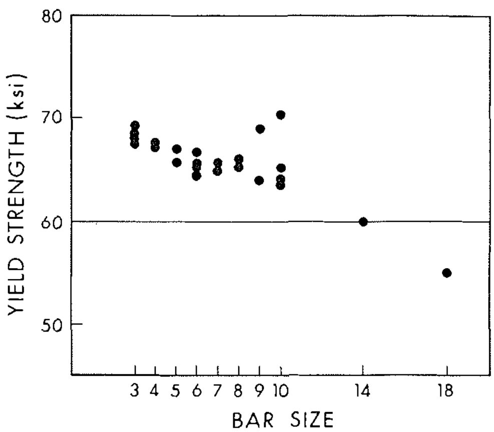

FIG. 6. Variation in mill test yield strength with bar size—grade 60 reinforcement (Grant 1976).

of reinforcing bars, Allen (1972) has suggested that the mean mill test yield strength is about 4 ksi (27.6 MPa) higher than the mean static yield strength.

(c) *In situ Strengths vs. Specimen Strengths*

The strength of concrete in a structure will differ somewhat from the strength of the same concrete in a control specimen for several reasons. These include the different stress regimes the specimen and structure, different placing procedures, different curing conditions, the effect of vertical water migration during the placing of concrete in deep members and the greater compaction of concrete near the bottom of such members due to the weight of the concrete higher in the forms. In general, high strength concrete is more affected by this than low strength concrete.

The reduction in strength due to these causes is partially offset by the fact that the ACI code (1971) and CSA A23.3 (1973) require the mean strength of concrete to be higher than the specified values. Thus, for average control, the mean strength of the control cylinders will range from 700 to 900 psi (4.8 to 6.2 MPa) greater than the specified strength. Based on this, and on equations and data from Allen (1970), Petersons (1964), and Bloem (1968), the *mean* 28 day strength of concrete in a structure cured with minimum acceptable curing can be taken as:

$$[8] \quad \bar{f}_{c\,(\mathrm{structure})} = (0.675\, f_c' + 1.1)\ \mathrm{ksi}$$

but not more than $1.15\, f_c'$

Changes in material strength due to maturing of the concrete or deterioration could also be included here if desired.

(d) *Effect of Variability of Shrinkage Stresses or Residual Stresses*

In members in which cracking is a critical limit state, the variability of the residual

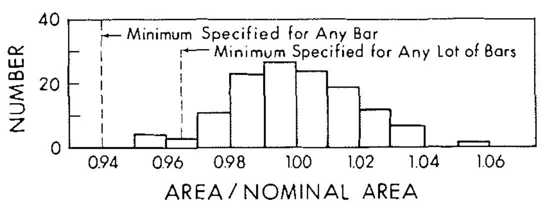

FIG 7. Ratio of actual bar area to nominal area (Allen 1972).

stresses due to shrinkage may affect the cracking load. Similarly the transfer of compression loading from concrete to steel due to creep and shrinkage in columns may lead to premature yielding of the compression steel. This may be significant in stability failures of slender columns with small amounts of reinforcement.

4.1.2. *Members may vary from assumed due to fabrication errors*—Factor F, $\bar{F}$, σ_F, and V_F

(a) *Rolling Tolerances in Reinforcing Bars*

As the rolls used to produce the reinforcing bars wear out or go out of adjustment, the shape of the deformations and the bar area change slightly. For bars larger than No. 3, CSA and American Society of Testing and Materials (ASTM) specifications allow up to 6% underweight on any individual bar. Areas measured on 102 specimens from five sizes are compared to their nominal areas in Fig. 7 (Allen 1972). Based on this and other data, Allen has concluded that the mean area should be taken as 0.975 times the nominal area with a coefficient of variation of 1.6%.

(b) *Geometrical errors in cross section and errors in placement of reinforcement*

Relatively common variations in dimensions can significantly affect the size and hence the strength of concrete members as shown in Fig. 8 (Birkeland and Westhoff 1972). Much of the current data on geometrical errors in concrete construction have been obtained in Sweden although a limited amount of data are available from North American sources.

Measurements of approximately 6000 cast-in-place slabs roughly 5 to 8 in. (12.7 to 20.3 cm) in thickness (Fiorato 1973) showed the average thickness approximately 0.05 in. (1.3 mm) thicker than the designed thickness with a standard deviation of about 0.3 in. (0.8 mm). The effective depth, d, in the positive moment region averaged about 0.25 in. (0.6 mm) less than specified with $\sigma = 0.3$ in. (0.8 mm). In the negative moment region d averaged about 0.75 in. (1.9 mm) less than specified with $\sigma = 0.5$ in. (1.3 mm). For precast slabs the average error in overall thickness and the d for positive moment was approximately zero with standard deviations of about half those for cast-in-place members.

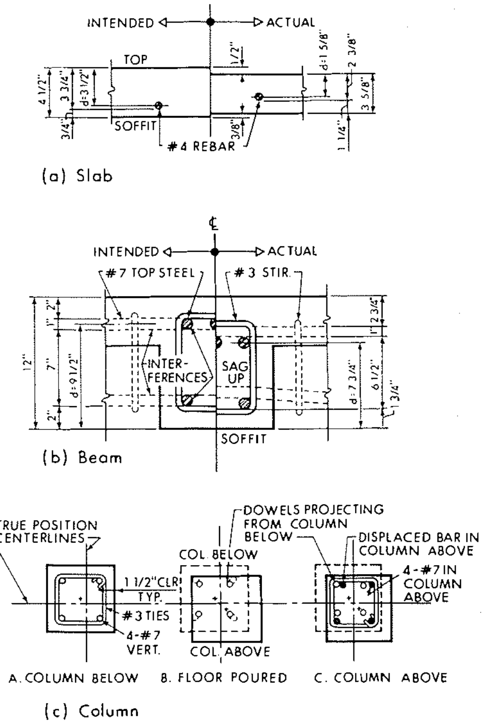

FIG. 8. Effects on size and strength by variations in dimensions (Birkeland and Westhoff 1972).

Measurements of the width and thickness of 299 columns ranging from 12 in. to 30 in. (30.5 to 76.2 cm) in width from eight buildings are shown in Fig. 9 (Tso and Zelman 1970). This data showed a mean error of +0.06 in. (1.5 mm) and a standard deviation of 0.28 in. (0.8 mm). For Swedish precast columns the mean error was about the same but the standard deviation was about half as much (Fiorato 1973).

4.1.3. *Simplified assumptions and equations may lead to systematic or random errors*—Factor P, $\bar{P}$, σ_P, and V_P

The use of such simplifications as the rectangular stress-block and the crushing strain, ϵ_u, introduce both systematic and random errors. Figures 10 and 11 are from Mattock *et al.* (1961) who compare the strengths calculated using the rectangular stress block to the strengths measured in laboratory tests. A portion of the variation shown in these figures is due to variations between the strengths of the control cylinders, etc. and the actual strengths of the materials in the test specimens.

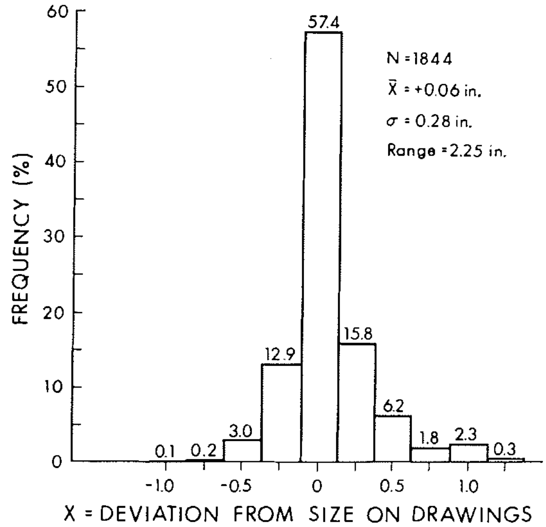

FIG 9. Difference between actual widths of columns and the sizes shown on drawings (Tso and Zelman 1970).

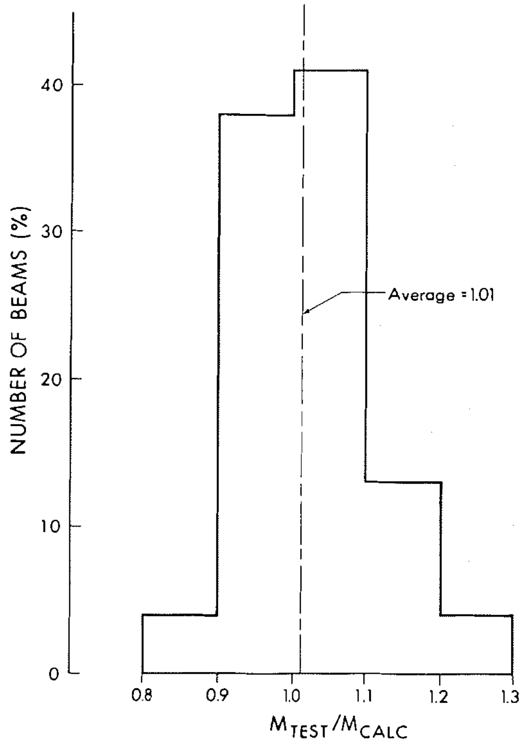

FIG. 10 Comparison of strengths calculated using rectangular stress block to strength measured in laboratory tests (Mattock *et al.* 1961).

4.1.4 *The use of discrete sizes leads to variations in the actual capacity of members*—Factor B, $\bar{B}$, σ_B, and V_B

Because the reinforcement in a beam or column must be some combination of whole bars, the area of steel actually provided in a member may differ from that found to be necessary in the calculations (Lind 1976). This is illustrated in Fig. 12 which shows the practical bar choices available for an 18 in. by 18 in. (45.7 cm by 45.7 cm) tied column. Based on allowing up to 5% underdesign, the mean area provided in this case would be about 1.02 times the calculated area with a coefficient of variation of 5%.

4.2 *Overloads*

In calculations the maximum load to come on a structure during its life will be referred to by the factor U with mean $\bar{U}$, standard deviation σ_U and coefficient of variation V_U. The factor U can be subdivided into a number of individual factors which contribute to variations in the total load U.

4.2.1. *The magnitudes of the loads may vary from those assumed*—Factor S, $\bar{S}$, σ_S, and V_S

Although dead loads, D, are known more accurately than any other loads except possibly fluid loads, they can vary due to:

1. Variations in size of members.
2. Variations in density of material due to different types of aggregate, different moisture content, etc.
3. Structural and non-structural alterations.

Thus, the dead load varies randomly from structure to structure in a population of structures and also changes from time to time in a given structure due to renovations, etc. The lifetime maximum dead load will be assumed to have an average value equal to the design value with a coefficient of variation of 0.07 (Allen 1975). This implies that the lifetime maximum dead load will not exceed 114% of the design value in 97% of all structures.

Live load, L, varies considerably from time to time and from building to building. Loading surveys suggest, however, that the occupancy in a given part of a building will change from 5 to 20 times during the life of a building and hence the probability of having a high load during the life of the building is fairly high (Mitchell and Woodgate 1971; Allen 1975). In

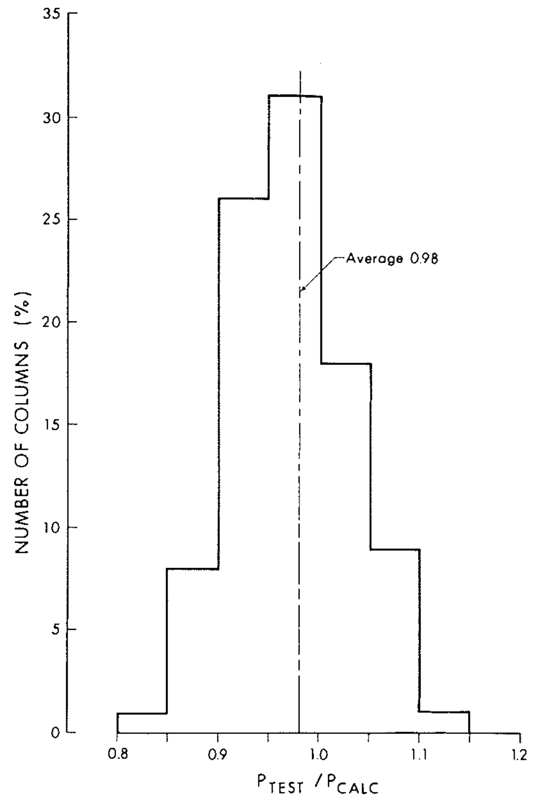

FIG. 11. Comparison of strengths calculated using rectangular stress block to strengths measured in laboratory tests (Mattock *et al.* 1961).

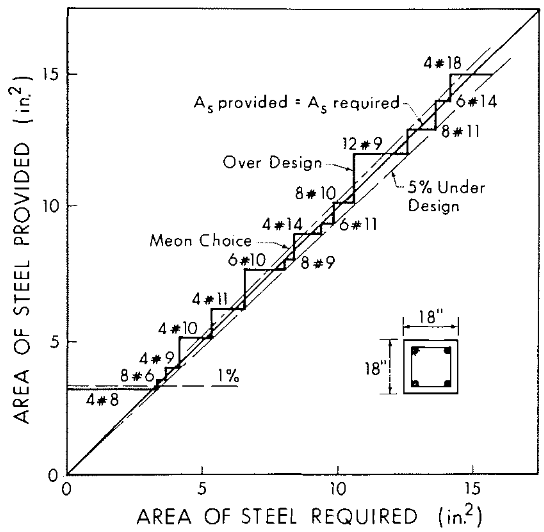

FIG. 12. Effect of selection of discrete bar sizes on choice of reinforcement in a tied column.

addition, the larger the area considered, the smaller the scatter in the maximum load. Figure 13 presents probability distributions for the maximum live loads to occur in a given area during the 30 year life of an office building (Comité Européen Du Béton 1973). The office live loads and live load reduction factors for floor members in the 1975 National Building Code of Canada (NRC 1975) given by the dashed line agree quite closely with the 95th percentile loads from these distributions.

Allen (1975) has suggested that the mean lifetime maximum office live load is about 70% of the loads specified in loading tables with a coefficient of variation of 0.3. This value is used in calculations in this paper.

4.2.2 *Uncertainties in calculation of load effects*—Factor E, $\bar{E}$, σ_E, and V_E

The assumptions of stiffnesses, span lengths, etc. and the inaccuracies involved in modelling three dimensional structures for structural analysis lead to variations between the stress resultants which actually occur in a building and those estimated in the designer's analysis. Allen (1975) has suggested that the mean values from an analysis should be about equal to 1.0 times the real values. For statically determinate members Allen suggests $V_E = 0.07$. For more complex structures Lind (1976) suggests $V_E = 0.20$ for slender columns in which the moments are largely due to compatibility of deformations and hence depend on more variables than the beam moments.

Galambos and Ravindra (1973) have further subdivided E into one term dealing with

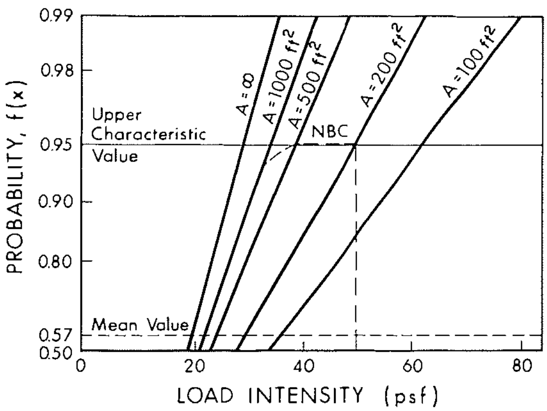

FIG. 13. Distributions of maximum office floor loads expected during 50 year life of a building.

the structural analysis itself which is assumed to have a mean of 1.0 and $V = 0.05$, and two additional terms A and B dealing with the uncertainties and approximations involved in idealizing the dead and live load for the analysis. The latter account for such things as considering live loads to be uniformly distributed, static loads, idealizing localized loads as point loads, etc. Galambos and Ravindra (1973) have assumed that these have a mean of 1.0 and $V_A = 0.04$ and $V_B = 0.2$ for factors affecting the idealization of the dead and live load, respectively.

The analyses in this report are based on the assumption that the structural analysis effects will be larger for live loads than for dead loads. No attempt will be made to distinguish between the type of structure or member, however. This is based on the assumption that, although the accuracy of an analysis of an indeterminate structure is probably less than that of a statically determinate beam, there is more potential for load redistribution in such a structure and this offsets much of the loss of safety due to possible inaccuracies in the analysis. The factor E will be assumed as follows: for dead load $\overline{E}_D = 1.0$ and $V_{ED} = 0.08$, for live load $\overline{E}_L = 1.0$ and $V_{EL} = 0.20$.

4.3 *Consequences of Failure*

A number of subjective values must be considered in establishing load factors. These include:

(a) *Cost of replacing the building.* Generally, one consequence of a failure is that the building must be repaired or replaced. This cost is relatively easy to evaluate. It should be noted that the cost of the building may be several times the cost of the structural system in the building. In addition demolition costs may add to the replacement costs.

(b) *Potential loss of life.* It is much more difficult to rationally account for risk to occupants in any safety factor theory. Not only is it morally difficult to put a value on a human life but the socio-economic value of the loss depends on the type and magnitude of the accident. Thus, for example, 200 accidents each with one fatality have much less impact and are much less newsworthy than one accident with 200 fatalities. The aversion function in Fig. 14 has been used to measure the socio-

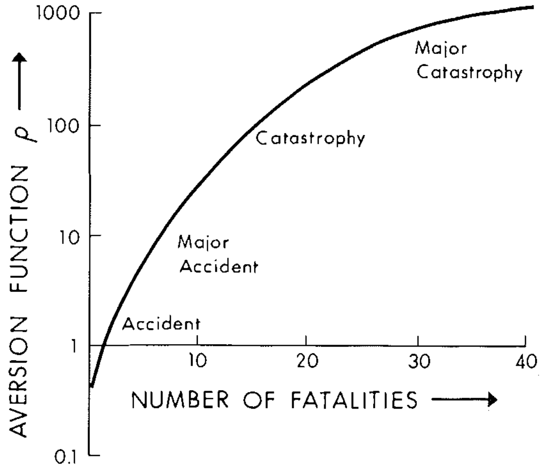

FIG. 14. Schematic representation of impact of various types of accident.

economic effects of various types of accidents (Lind and Basler 1972).

(c) *Costs to society in lost time, lost revenue, or indirect loss of life or property due to failure.* The loss of a major urban bridge may well cost the residents of the city much more in time lost in traffic jams, etc. than the actual cost of the structure. Similarly, the collapse of a fire station or hospital during a disaster, may lead to severe loss of life due to the inability to fight fires or treat people during the disaster.

An importance factor is included in the load factor equation in section 4.1.4 of the 1975 National Building Code of Canada (National Research Council of Canada 1975) to account for the consequences of failure. This factor is 1.0 for buildings of normal human occupancy and 0.8 for buildings such as farm or storage sheds. For post disaster buildings, importance factors greater than 1.0 are applied to wind and earthquake loads since these are the loads causing the disaster.

(d) *The importance of the structural element in the structure.* The collapse of a roof beam will generally be less critical than the collapse of a lower floor column in a tall building because the failure of a column is more apt to affect a larger area than that of a beam. Thus, the CSA (1973) and ACI (1971) concrete codes penalize columns relative to beams.

(e) *The type of failure, warning of failure, and existence of alternative load paths.* If the occupants of a building have warning of impending failure as would occur if severe cracking or excessive deflections developed prior to failure, the probability of loss of life is less. For this reason the ACI (1971) and CSA (1973) concrete codes have limitations favoring ductile structures. Thus, for example, the steel percentage in beams must not exceed three-quarters of that corresponding to the onset of brittle compression failures. Similarly, tied columns are penalized relative to spiral columns since the latter tend to be more ductile.

It is interesting to note in passing that Allen (1970) has shown that there is still a danger of compression failures in shallow beams with the maximum percentage of reinforcement allowed by the ACI and CSA due to the random variations of materials and dimensions. This is illustrated in Fig. 15. The shaded area represents beams failing in compression in a random sample of beams with $\rho = 0.75\ \rho_b$.

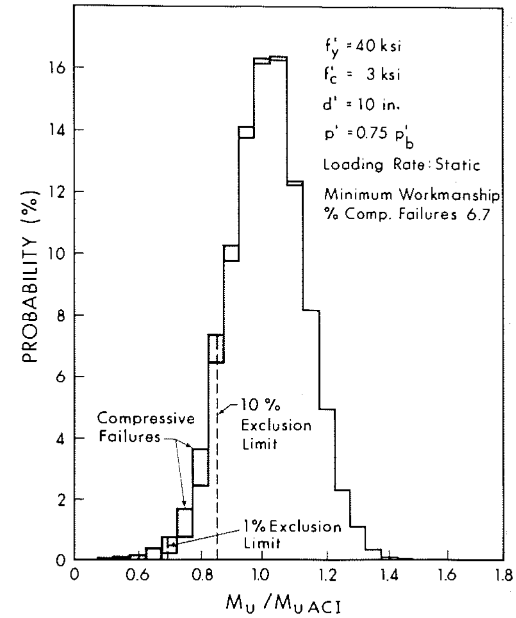

FIG. 15. Comparison of ACI design equation for beams to the strengths of a randomly generated sample of beams (Allen 1970).

5. Methods of Defining Safety for Structural Design

Three basic methods of defining structural safety will be reviewed briefly to show their differences, strengths, and shortcomings. This will be followed by a review of the procedures which are currently used to define the safety of reinforced concrete structures in a number of selected codes.

5.1 *Basic Procedures for Defining Safety*

5.1.1 *Factor of Safety or Working Stress Design Format*

The factor of safety can be defined as:

$$[9] \quad \text{Factor of Safety} = \frac{\text{Ultimate Resistance}}{\text{Service Load}} = \frac{R}{U}$$

This implies that both of these quantities are well defined each with a unique value. As we have seen, however, the resistance, R, is affected by a number of variables and is variable itself as shown by the frequency diagram for R in Fig. 16. Similarly, the maximum load, U, that the structure will receive in its lifetime is also a variable. As a result the definition of the factor of safety given in [9] lacks clarity. Two possible restatements of [9] are:

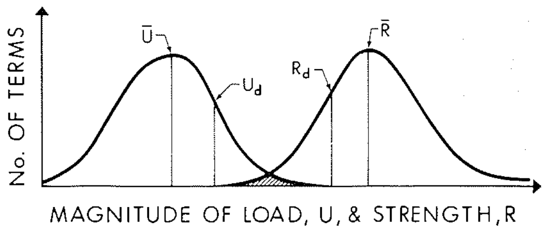

FIG. 16. Variation of loads on structures and the strengths of the same structures.

$$[10] \quad \text{Central Factor of Safety} = \frac{\text{Mean Resistance}}{\text{Mean Load}} = \frac{\bar{R}}{\bar{U}}$$

or

$$[11] \quad \text{Nominal Factor of Safety} = \frac{\text{Design Resistance}}{\text{Service Load}} = \frac{R_d}{U_d}$$

where R_d is the capacity computed according to the design code and U_d is the service load given in the local or national building code. The intersection of the frequency diagrams for R and U, show shaded in Fig. 16, suggests that there is definitely a probability that failure will occur under some possible combination of strength and load. It should be noted, however, that this area is not equal to the probability of failure.

In working stress of reinforced concrete the factor of safety has been assumed to be closely related to:

$$[12a] \quad \text{Factor of Safety} = \frac{\text{Yield Strength of Reinforcement}}{\text{Allowable Steel Stress}}$$

or

$$[12b] \quad \text{Factor of Safety} = \frac{\text{Concrete Strength}}{\text{Allowable Concrete Stress}}$$

This method of defining safety has four drawbacks:

1. It does not adequately account for the variability of loadings and resistances. Two extreme cases are compared in Fig. 17. In Fig. 17a the control of loading and resistance are both very good and there is relatively little probability of failure as evidenced by the small overlap of the curves. Figure 17b corresponds to a case with poor control of loadings and resistance. Although the central factor of safety is the same for both cases, the probability of failure is much higher in the second case. In traditional working stress design for reinforced concrete the greater variability of concrete was recognized in part by using a slightly higher safety factor in computing the allowable stress for concrete.

2. It does not adequately account for variations in loadings which increase at different rates or have different signs. The factor of safety or working stress format assumes that all loadings will increase at approximately the same rate. This becomes serious in the case where a highly variable load such as wind, earthquake, or soil pressure causes forces opposite in sign to those resulting from relatively constant loads such as dead load or prestressing forces. The stresses due to an overload may be opposite in sign to those at service loads and the reinforcement provided for service load conditions may not be adequate to prevent failure. As an example, the calculated stresses in the reinforcement at the location where failure is believed to have started in the Ferrybridge cooling towers was $0.5f_y$ under $1D + 1W$ which was felt to be satisfactory since a working stress design was used. The reinforcement stresses rose to $1.0f_y$ under $1D + 1.15W$ and to the ultimate tensile strength under $1D + 1.3W$ (Goode 1976, pers. commun.).

3. There is no attempt to evaluate the ultimate load capacity. In working stress design it is assumed that the ratio between service load capacity and ultimate capacity is the same as the ratio between allowable stresses and material strengths. This relationship has not been checked adequately for the high strength materials currently in use.

4. There is no rational method of considering such things as consequences of failure or type of failure.

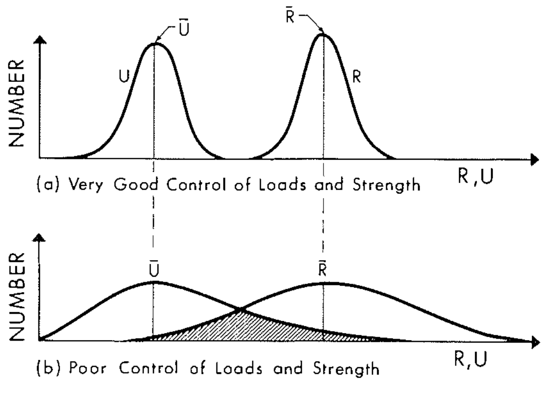

FIG. 17. Effect of dispersion of loads and strengths on probability of failure for constant central factor of safety.

5.1.2 *Maximum Probability of Failure Method*

If R, $\bar{R}$, and σ_R represent the distribution of strengths and U, $\bar{U}$, and σ_U represent the loads, any given structure will fail if $U > R$. Thus, the probability of failure is the probability that $U > R$ or:

$$[13] \quad P_f = P[(R - U) < 0]$$

or alternatively:

$$[14] \quad P_f = P[R/U < 1.0]$$

or since ln 1.0 = 0:,

$$[15] \quad P_f = P[\ln(R/U) < 0]$$

If we know $\bar{R}$, σ_R, $\bar{U}$, and σ_U, we can define a new function $Y = R - U$ (Fig. 18) with mean $\bar{Y}$ and standard deviation σ_Y calculated according to the procedures presented in section 3.3 of this paper. The function Y repre-

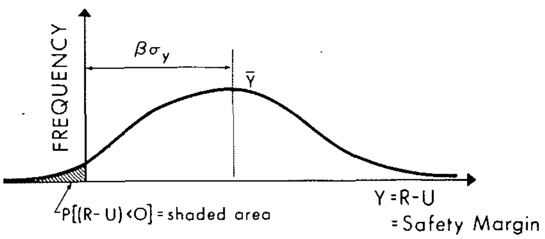

FIG. 18. Definition of probability of failure and safety index, β.

sents the 'margin of safety' for a given structure. The probability of failure is the probability that a particular structure will fall in the shaded area in Fig. 18:

[16] $P_f = P[(R - U) < 0]$ = shaded area

For normal distribution or other standard distributions this probability can be calculated or obtained from tables as a function of the type of distribution and the value of β.

This procedure is not generally used in this form because of the work involved in evaluating the probabilities of failure for every structure. However, as will be seen in section 6.1, it forms the basis for computing load and resistance (ϕ) factors and hence, indirectly it is of considerable importance to designers. The major problems in the use of probabilities of failure to define safety involve the choice of acceptable probabilities of failure and the need for statistical data on many aspects of loading and construction. On the other hand, this philosophy leads to a rational method for estimating safety factors.

5.1.3. *Minimum Cost Structure Including Cost of Failure*

If it is possible to estimate the probability of failure, then one can calculate the total cost of a structure including the costs of failure. The various items involved in the total cost of a building are plotted schematically in Fig. 19 (Rüsch and Rackwitz 1972). The total lifetime cost of a building, T, can be represented as the sum of the original construction costs, C, the maintenance costs, M, and the insurance costs, I. The insurance costs are related to the probability of failure and rise rapidly as the probability of failure increases. Such a calculation provides an estimate of the optimum probability of failure.

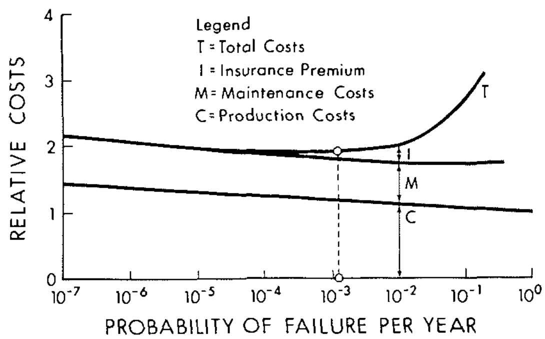

FIG. 19. Evaluation of optimum probability of failure (hypothetical) (Rüsch and Rackwitz 1972).

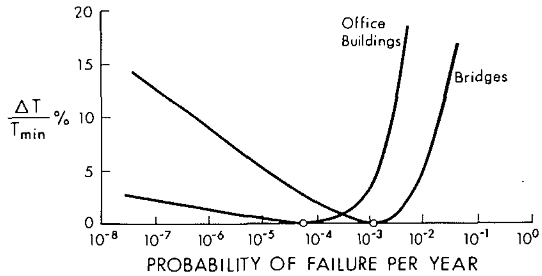

FIG. 20. Variation in total cost of structure with probability of failure (hypothetical) (Rüsch and Rackwitz 1972).

As shown in Fig. 20, it is possible, in theory, to compute an optimum failure probability for a given type of structure. The optimum probability of failure will vary with the type of structure as shown in Fig. 20. Allen (1968) has estimated that the structural portion of a reinforced concrete building with a probability of failure of 10^{-4} in 30 years will cost about 9% more than that of a building with a probability of failure of 10^{-2} in 30 years. Since the cost of the structure is only about one-third of the total cost of a building, the additional cost of this increase in safety will be 3% of the total cost of the building. As shown in Fig. 20, the total costs $T = C + M + I$ of office buildings are relatively insensitive to the probability of failure below the optimum. This is less true for bridges since the structure accounts for much of the cost of a bridge.

5.2 *Current Code Procedures for Defining Safety*

5.2.1 *ACI Code and CSA Standard A23.3*

The ACI (1971) and CSA (1973) design

requirements for reinforced concrete are based on an underlying assumption that if the probability of understrength members is roughly 1 in 100 and the probability of overload is roughly 1 in 1000, the probability of overload on an understrength structure is about 1 in 100 000. Load factors were derived to achieve this probability of overload. Based on values of concrete and steel strength corresponding to probability of 1 in 100 of understrength, the strengths of a number of typical sections were computed. The ratio of the strength based on these values to the strength based on nominal strengths of a number of typical sections were arbitrarily adjusted to allow for the consequences of failure and the mode of failure of a particular type of member, and for a number of other sources of variation in strength. The appendix traces the history of the development of the current ACI and CSA load factors. Although the original derivation was semi-rational and represented the state-of-the-art in the late fifties, subsequent modifications and compromises greatly reduced the rationality.

A basic statistical error in the procedure used by the ACI to estimate the probability of failure is illustrated in Fig. 21. This figure shows the relationship between a population of loads, U, and a corresponding population of strengths, R. The 45° line in the figure represents the situation where the load U equals the strength R. Combinations of U and R falling above this line result in failure such as, for example, load U_1 in Fig. 21(a) acting on structure R_1. Load U_2 acting on structure R_2 represents a safe combination. Thus, regions A, B, and C in Fig. 21(b) represent failure conditions while region D represents the domain of safe conditions.

The shaded area in the frequency diagrams in Fig. 21(b) represent 'overloads' and 'understrengths' as defined in the derivation of the ACI Code safety provisions. The probability of such an overload occurring on such an understrength structure, shown by region B in Fig. 21(b) is 1 in 10 000 as assumed by the ACI. This definition of the probability of failure ignores potential failures in regions A and C in Fig. 21(b) and hence underestimates the actual probability of failure. Computations of the actual level of safety in the 1971 ACI Code estimate that it gives a probability of

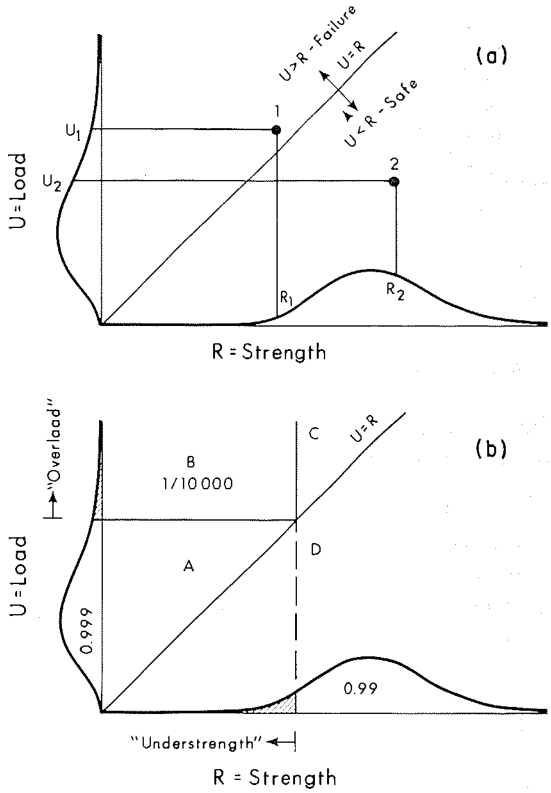

FIG. 21. (a) Definition of failure. (b) ACI definition of probability of failure.

failure for flexure of 1.3×10^{-5} rather than 1×10^{-5} (see section 6.2). It should be noted that [13], [14] or [15] will give a correct estimate of the probability of failure.

5.2.2 *European Concrete Committee—October 1975*[3]

Design is based on characteristic strengths, f_{ck} and f_{yk}, which are estimators of the 5th percentile strengths of the concrete and steel, and on characteristic loads, U_k. The characteristic dead load is taken as the mean dead load.

In design the designer ensures that:

[17] $R^* \geqslant$ Effects of

$$\left\{\gamma f_D D_k + \gamma_{f_{L_1}} L_i + \sum_{i=2}^{n} \gamma_{f_i} \psi_i L_{f_i}\right\}$$

where R^* is the capacity of a section calculated using design material strengths equal to

[3]Comité Européen du Béton (1975).

f_{ck}/γ_{m_c} and f_{yk}/γ_{m_s} and dimensions increased or decreased by the allowable tolerance; γ_{m_c} and γ_{m_s} are the material understrength factors for concrete and steel; γ_{f_d}, etc. are the overload factors for the particular loads concerned; and, $\psi_i L_{f_i}$ is the 'frequent value' of a variable load, where ψ has a value of about 0.5 to 0.7.

The material understrength factors γ_{m_c} and γ_{m_s} were originally found by combining γ_{m1}, γ_{m2}, γ_{m3}, γ_{m4}, and γ_{m5} where:

γ_{m1} takes account of variations in the strength of the materials themselves.

γ_{m2} accounts for the variations between the strength and dimensions from those essumed in the design, the degree of control on site and the possibility of deviations from the assumed resistance model.

γ_{m3} accounts for possible inaccurate assessment of the strengths which may depend on the structural material.

γ_{m4} accounts for the consequences of failure.

γ_{m5} accounts for the type of failure.

For normal control average inspection and normal consequences of failure for the case of flexure in an under-reinforced beam, γ_m would be 1.5 for the concrete and 1.15 for the steel.[4]

Similarly the load factors, γ_f, can be considered as the product of γ_{f1}, γ_{f2}, and γ_{f3} where:

γ_{f1} accounts for the possibility of variations in the loads.

γ_{f2} is a load combination factor and accounts for the reduced probability of all loads acting at once.

γ_{f3} accounts for errors in the structural analysis which are independent of the structural material.

The value of γ_f would normally be 1.4 on dead load and 1.5 on live load.[4]

For serviceability and progressive collapse limit states the designer must ensure that R based on the characteristic strengths and the specified dimensions exceeds $D + \Sigma\, \psi_i\, L_i$.

5.2.3 *Comparison of ACI and CEB Safety Provisions*

There is a major difference in philosophy between the ACI (1971) and Comité Européen du Béton (CEB 1975) procedures for defining load factors. The ACI combines all the member understrength terms into one term, ϕ, which is intended to reflect the probability of the member being understrength plus the consequences of failure, etc. On the other hand, the CEB uses reduced material strengths in design to reflect the probability that the materials will be understrength. In addition, design is based on reduced geometric dimensions to include the effect of construction tolerances. The effect of errors in the design equations, consequences of failure and type of failure are included in the γ_m term by means of γ_{m2}, γ_{m4}, and γ_{m5}.

The major advantage of the ACI procedure is slightly increased simplicity in application. Thus, one need consider only ϕ instead of separate γ_m values for steel and for concrete. In addition the ACI ϕ factors may be slightly more pleasing statistically, from the standpoint of properly combining the variations of one particular set of variables to come up with a factor reflecting the combined effect of that entire family of variables.

On the other hand, when the resistance of a member depends partly on the strength of concrete, which may have a coefficient of variation of 0.15 to 0.2, and partly on steel with a coefficient of variation half as big, the effect of varying the portion of the load assigned to the steel and concrete cannot be accounted for nearly as well as with a single ϕ factor, as it can be when separate γ_m values are used. Thus, for example, the variation in concrete strength has a significant effect on the variability of eccentrically loaded columns failing in compression but a

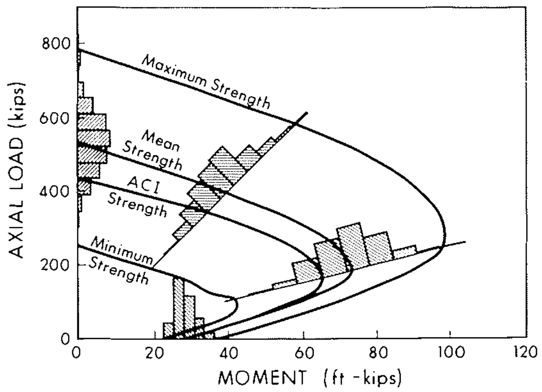

FIG. 22. Dispersion of strengths of eccentrically loaded columns in a randomly generated sample of 1000 columns (Grant 1976).

[4]The numerical values of the coefficients are still under discussion and may change.

very much smaller effect on the variability of columns failing in tension as shown in Fig. 22 (Grant 1976). This is adequately treated by the CEB but not by ACI. If, however, the member strength is not linearly related to the material strength as, for example, in $v_c = 2\sqrt{f_c'}$, the required safety may not be attained by the use of f_c/γ_m in design since the effect will only be $\sqrt{1/\gamma_m}$. For this reason the CEB uses $\sqrt{f_c}/\gamma_m$ in such cases. All things considered, however, the CEB procedure probably gives a more uniform estimate of the understrength of a member than the ACI and CSA ϕ factors do.

6. Derivation of Load Factors and Resistance Factors

The following sections present the bases of the derivation of load and resistance factors for use in design. It is assumed that a single ϕ factor will be used rather than separate γ_m values for steel and concrete. The procedures used in this chapter were developed largely by Cornell (1969) and Lind (1971).

6.1 *Basic Theory*

As explained in section 5.1.2, the probability of failure can be expressed as:

[14] $$P_f = P[R/U < 1.0]$$

or since ln 1.0 = 0:

[15] $$P_f = P[\ln(R/U) < 0]$$

Both of these are true regardless of the actual frequency distributions of R and U. We shall define:

[18] $$Y = \ln(R/U)$$

If we assume that Y is normally distributed, R/U will be log-normally distributed. A log-normal distribution of R/U has been assumed because U tends to be skewed, those parts of R dependent on steel strength tend to be skewed, and because theoretically a log-normal distribution better represents the products of random variables (R/U). Perhaps more important, however, it is relatively simple to implement and gives reasonable results.

The mean and standard deviation of Y are:

[19] $$\overline{Y} = \overline{\ln(R/U)} = \ln(R/U)$$

[20] $$\sigma_Y^2 = \sigma^2(\ln R/U) = \sigma^2(\ln R) + \sigma^2(\ln U)$$

The function Y is plotted in Fig. 18. As stated by [15], the probability of failure can be expressed as the probability that Y is less than zero. This probability is represented by the shaded area in Fig. 18. The probability of failure can thus be defined by the number of standard deviations, $\beta\sigma_Y$, that the mean value of Y, Y, is above zero. This allows us to write:

[21] $$\overline{\ln(R/U)} \geqslant \beta\,\sigma(\ln R/U)$$

or

[22] $$\overline{\ln(R/U)} \geqslant \beta\sqrt{\sigma^2(\ln R) + \sigma^2(\ln U)}$$

For a log-normal distribution:

[23] $$V_R = (e^{\sigma(\ln R)^2} - 1)^{1/2}$$

For $V_R \leqslant 0.6$ it is an acceptable approximation to write:

[24] $$V_R^2 \simeq \sigma^2(\ln R)$$

The error in this approximation is less than 2% for $V_R = 0.3$ rising to about 10% for $V_R = 0.6$. Thus, we can rewrite [22] as:

[25] $$\overline{\ln(R/U)} \geqslant \beta\sqrt{V_R^2 + V_U^2}$$

Lind (1971) has shown that:

[26] $$\sqrt{A^2 + B^2} \simeq \alpha A + \alpha B$$

where α is a 'separation function' having values between 0.707 and 1.0. Values of α are plotted in Figure 23 (Lind 1971). For A/B between 1/3 and 3, $\alpha = 0.75 \pm 0.06$. Thus, the separation function α can be used to simplify [25] giving:

[27] $$\ln(\overline{R}/\overline{U}) \geqslant \beta\,\alpha\,V_R + \beta\,\alpha\,V_U$$

or

[28] $$\overline{R}/\overline{U} \geqslant e^{(\beta\alpha V_R + \beta\alpha V_U)}$$

Rearranging this gives

[29] $$\overline{R}[e^{-\beta\alpha V_R}] \geqslant \overline{U}[e^{+\beta\alpha V_U}]$$

This resembles the current ACI (1971) and CSA (1973) code format in that the average strength $\overline{R}$ is multiplied by a factor less than 1.0 and the average load $\overline{U}$ is multiplied by a factor greater than 1.0. However, when the designer uses the code design equations and the specified strengths, he computes the design strength, R rather than the mean strength, R. Similarly, design is based on values of U speci-

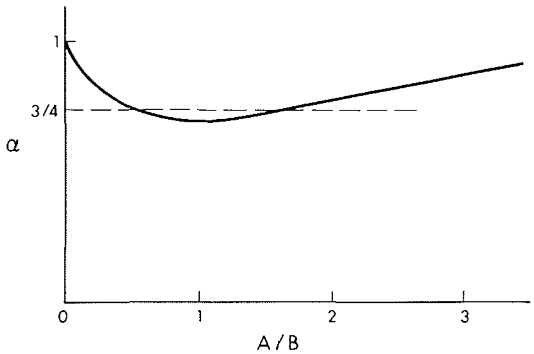

FIG. 23. Variation in separation function α (Lind 1971).

fied in code loading tables. We shall define γ_R and γ_U such that:

[30] $$\overline{R} = R_{\gamma R}$$

[31] $$\overline{U} = U_{\gamma U}$$

Then

[32] $$R_{\gamma_R}[e^{-\beta\alpha V_R}] \geqslant U_{\gamma_U}[e^{\beta\alpha V_U}]$$

or

[33] $$R\,\phi \geqslant U\,\lambda$$

where ϕ is a 'resistance factor' and λ is a 'load factor'. Thus:

[34] $$\phi = \gamma_R\, e^{-\beta\alpha V_R}$$

and

[35] $$\lambda = \gamma_U\, e^{\beta\alpha V_U}$$

Before we can proceed to derive values of ϕ and λ it is necessary to choose an appropriate level of safety defined by the safety index β and we must estimate γ_R, V_R, γ_U, and V_U. The choice of β and the calculation of these terms will be discussed in the next few sections.

This procedure is known as a *second moment probabilistic method.* The method is probabilistic because it considers the random nature of the variables. It is called a second moment probabilistic method because it considers only two statistical parameters, the mean and coefficient of variation, to describe the distribution of the variables.

6.2 *Choice of Acceptable Probability of Failure*

Engineers have attempted to estimate the magnitude of an acceptable probability of failure in two major ways. Using relationships related to [34] and [35] one can calculate the values of β corresponding to the load factors and ϕ factors in the current codes. If these β values and the related probabilities of failure are felt to be realistic, they can be used to derive new values of ϕ and λ for use in the new code format. If not, more appropriate target values of β can be selected on the basis of the performance of the current code or engineering judgement. This approach is called 'calibration' since the new code is calibrated or made to agree with a target established by a study of the old code. This assumes that the load factors in the old code had been developed over a long period and represent a good engineering estimate of the required safety. Using this technique Siu *et al.* (1975) have estimated the weighted average β values in current Canadian design specifications to be:

Reinforced concrete	— Flexure	$\beta = 4.2$,	$P_f \simeq 1.3 \times 10^{-5}$
	— Tied columns	$\beta = 5.22$,	$P_f \simeq 2 \times 10^{-7}$
	— Shear	$\beta = 3.64$,	$P_f \simeq 1.3 \times 10^{-4}$
Structural steel	— Tensile yielding	$\beta = 3.86$,	$P_f \simeq 5.8 \times 10^{-5}$
	— Columns	$\beta = 4.69$,	$P_f \simeq 1.4 \times 10^{-6}$

These values vary widely from one structural member to another. It does not seem reasonable, for example, that the probability of shear failures in concrete beams should be 10 times that of flexural failures. Nor does it seem reasonable that the probability of failure of a steel column should be ten times that of a concrete column.

Shortcomings of calibration as the sole means of setting the value of β or P_f for future codes include:

(1) The levels of safety in the current code will generally vary widely from one type of member to another as shown by the values listed in the previous paragraph.

(2) The levels of safety may vary from code

TABLE 2. Risk of death for various activities*

Activity	Yearly death rate per person per year: For those concerned	Yearly death rate per person per year: For the total population
Motorcycle racing	5×10^{-4}	
Mountain climbing	5×10^{-3}	
Mining	7×10^{-4}	
Swimming	1×10^{-4}	2×10^{-5}
Automobile travel		3.6×10^{-4}
Airplane travel	1×10^{-4}	
Fire in buildings		2×10^{-5}
Poisoning		1.1×10^{-5}
Lightning		5×10^{-7}
Vaccinations and innoculations		1×10^{-8}
Structural collapse		
During construction	3×10^{-5}	
All others		2×10^{-7}

*Data from Allen (1968), Otway *et al.* (1970), and Rüsch and Rackwitz (1972).

to code depending on the interests and motivation of the code writing body. Thus, the levels of safety in the industry sponsored American Institute of Steel Construction Code may well differ from those in the ACI Code which is produced by an independent committee with a majority of its members representing designers or users.

(3) Due to rapid code changes in recent years, there has not been sufficient experience with current codes to know whether they provide adequate safety (Rüsch and Rackwitz 1972).

It is important, therefore, to critically review the β values obtained from code calibration before selecting the target values. If this is done, some of the shortcomings listed can be alleviated.

An alternative to calibration is to select a probability of failure which is comparable to the risks people are prepared to accept in other activities. The risks involved in a number of activities are given in Table 2 and Fig. 24. Because people are not equally exposed to all these risks they have been expressed in terms of the 'average' risk to the average person. For some of the more dangerous activities the risk is expressed in terms of the average person who is actually involved in the activity under consideration.

As explained in the following paragraphs from Otway *et al.* (1970) the acceptance of risks depends on the level of risk involved.

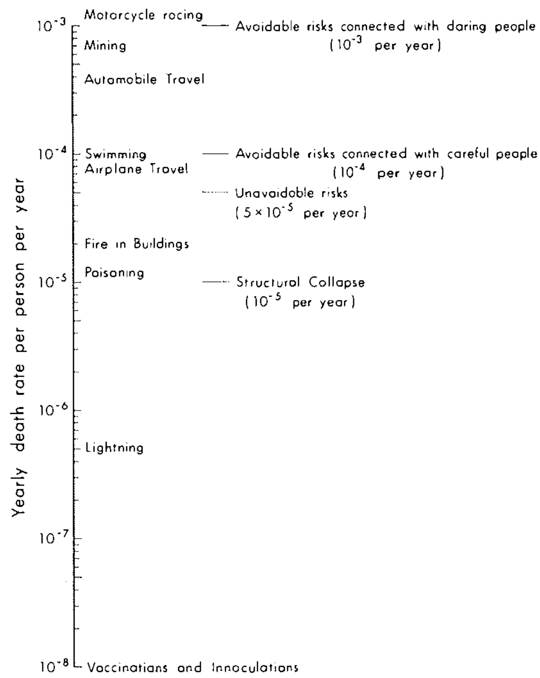

FIG. 24. Risks of various activities.

"Accidents providing hazards in the order of 10^{-3} per person per year are uncommon. When a risk approaches this level, immediate action is taken to reduce the hazard. This level of risk is unacceptable to everyone.

At an accident level of 10^{-4} per person per year, people spend money, especially public money, to control the cause. Money is spent for traffic signs and control, and police and fire departments are maintained with public funds. Safety slogans popularized for accidents in this category show an element of fear; *e.g.*, 'The life you save may be your own.'

Risks at the level of 10^{-5} per person per year are still considered by society. Mothers warn their children about most of these hazards (playing with fire, drowning, firearms, poisons), and some people accept a degree of inconvenience, such as not travelling by air, to avoid them. Safety slogans for these risks have a precautionary ring: 'Never swim alone', 'Never point a gun at another person', 'Keep medicine out of children's reach.'

Accidents with a probability of about 10^{-6} per person per year are not of great concern to the average person. He may be aware of them but he feels they will never happen to him. Phrases associated with these occurrences have an element of resignation: 'Lightning never strikes twice', 'An act of God'."

Based on similar reasoning, Rüsch and Rackwitz (1972) have suggested that the levels of acceptable risk can be summarized as follows: avoidable risks connected with daring people – 10^{-3} per year, avoidable risks connected with careful people – 10^{-4} per year, and unavoidable risks – 5×10^{-5} per year. Since the occupant of a building would consider a structural collapse to be an unavoidable risks, the probability of failure of structure through collapse which results in one death should be about 5×10^{-5} per year. If more deaths result, as might be the case in the failure of a dam or a tall building, the probability of failure should probably be even smaller.

A German study (Rüsch and Rackwitz 1972) of roofs which failed under snow loads and the roofs in randomly selected satisfactory buildings from the same regions, showed that the calculated values of failure probability were between 10^{-3} and 10^{-5} per year for the buildings that failed. For structures which did not fail, the annual probability of failure was always less than 10^{-5}.

This evidence suggests that the probability of failure should not be less than about 10^{-5} per year. This corresponds to about 3×10^{-4} during the 30 year life of a normal building. For a normal distribution, this probability corresponds to $(\bar{Y} - \beta\sigma_Y)$ with $\beta = 3.45$. A value of $\beta = 3.5$ will be used in this paper for ductile structures with normal consequences of failure. This will be increased to $\beta = 4.0$ if either the consequences of failure become severe or the failure occurs in a brittle manner. This should yield roughly a probability of failure of 10^{-4} in 30 years for ductile structures and 10^{-5} in 30 years for brittle structures.

6.3 *Derivation of Load Factors and Resistance Factors for Reinforced Concrete Beams and Columns*

The calculation of load factors and resistance or ϕ factors requires a choice of β, as discussed in section 6.1, and requires that γ_R, V_R, γ_U, and V_U be calculated. These quantities will be evaluated in the following subsections and used to derive load and ϕ factors for flexure and shear in a reinforced concrete beam and for axial load in a tied column.

6.3.1 *Selection of Statistical Properties of Variables*

The properties assumed in the calculations are listed in Table 3.

(i) *Concrete*—The specified concrete strength, f_c', will be assumed to be 4000 psi (27.9 MPa). The concrete strength will be assumed to be normally distributed and the standard deviation of the control cylinders will be assumed to be 600 psi (4.1 MPa) ($V = 0.15$) corresponding to average control (ACI 1965). A mean control cylinder strength of 4900 psi (33.8 MPa) is required to satisfy section 4.2.2.1 of ACI 317–71 (ACI 1971). The mean strength of the concrete in the structure itself, $\bar{f}_c$, calculated using [8], will be taken as 3800 psi (26.2 MPa). The coefficient of variation of the concrete in the structure, V_c, will be taken as 0.18, calculated by combining $V = 0.15$ for the control cylinders and the coefficient of variation of the ratio between the strength in the structure and that in the control cylinders, assumed to be 0.10 (Bloem 1968). The tensile strength will also be assumed to have a coefficient of variation of 0.18.

(ii) *Reinforcement*—Grade 60 reinforcement will be used with a specified yield strength of 60 ksi (414 MPa) and a mean mill test yield strength of 66 ksi (455 MPa). The mean static yield, $\bar{f}_y$, will be used in these calculations and will be assumed to be 62 ksi (428 MPa). The coefficient of variation of the yield strength, V_s, will be assumed to be 0.07 (Allen 1972).

TABLE 3. Statistical distributions assumed in calculations

	Specified	Mean *in situ*	Mean *in situ* / Specified	σ	V
Material Strengths, M					
Concrete strength	4000 psi	3800 psi	—	—	0.18
Concrete tensile strength	—	—	—	—	0.18
Yield strength	60 ksi	62 ksi	—	—	0.07
Dimensions, F					
b–beam, column (in.)	12	12.05	1.004	0.3	0.025
d–beam (in.)	18	17.85	0.992	0.45	0.025
h–column (in.)	12	12.05	1.004	0.3	0.025
s–stirrups (in.)	9	9.0	1.00	0.25	0.03
A_s–beam (in.^{2})	3.24	—	1.00	—	0.06
A_s–column (in.^{2})	2.16	—	1.00	—	0.06
A_v–stirrups (in.^{2})	0.22	0.22	1.00	—	0.06
Accuracy of code equations, P					
M_u–under-reinforced beams	—	—	1.06	—	0.04
P_u–axially loaded columns	—	—	0.98	—	0.05
V_c–shear carried by concrete	—	—	1.10	—	0.15
V_s–shear carried by stirrups	—	—	1.20	—	0.15
Loadings, S					
Dead load	—	—	1.0	—	0.07
Maximum floor load in 30 year life	—	—	0.7	—	0.30
Structural analysis, E					
Dead load effects	—	—	1.0	—	0.08
Live load effects	—	—	1.0	—	0.20

(iii) *Dimensions*—The dimensions chosen by the designer are: beam width, b = 12 in. (30.5 cm); beam effective depth, H = 18 in. (45.7 cm); column width and overall depth, $b = h$ = 12 in. (30.5 cm). In the absence of Canadian data, the average width and overall depth of the beam and column will be assumed to be 0.05 in. (1.3 mm) greater than the design values with a standard deviation of 0.30 in. (0.8 mm) (Fiorato 1973). The mean effective depth of the beam will be assumed to be 0.15 in. (0.4 mm) less than the design value with a standard deviation of 0.4 in. (1.0 cm) (Fiorato 1973). The mean widths, $\bar{b}$, etc. are given in Table 3, along with coefficients of variation, V_b, etc. calculated by dividing the standard deviations given above by the mean values.

The beam and column will both be assumed to have a calculated longitudinal steel percentage of 1.5% giving a calculated steel area A_s = 3.24 in.^{2} (20.9 cm^2) in the beam and A_{st} = 2.16 in.^{2} (13.9 cm^2) in the column. In addition, the beam has No. 3 U-stirrups at 9 in. (22.9 cm) on centers.

Since it is unlikely that bars of exactly this area will be available, the area of the bars selected by the designer will be assumed to average 1.02 times that required with a coefficient of variation of 0.05 (Fig. 12). Finally, the rolling tolerances are such that the mean area of a given reinforcing bar or group of bars will be assumed to be 0.98 times the nominal area chosen by the designer with a coefficient of variation of 0.03 (Allen 1972). Thus, the mean area of steel will be assumed to be $1.02 \times 0.98 = 1.00$ times that chosen by the designer and the coefficient of variation of the steel area will be assumed to be $\sqrt{0.05^2 + 0.03^2}$ or 0.06.

(iv) *Accuracy of ACI Design Equations*—Due to the use of the rectangular stress block, the limiting strains, and the neglect of strain hardening, the strengths calculated using the ACI (1971) Code and CSA A23.3 (1973) may differ from the actual strength even if the measured strengths of the concrete and steel in the member from control specimens are used in the calculations. For under-reinforced beams, Mattock *et al.* (1961) suggest average of measured to calculated strength ranging from 1.04 to 1.11 with coefficients of variation ranging from 0.05 to 0.10. Some of this variation will be due to differences between the control specimens and the material strengths in the members. For beams we shall assume the mean strength to be $\bar{P} = 1.06$ with a coefficient of variation $V_P = 0.04$. For tied columns, Mattock *et al.* (1961) found mean ratios of test to calculated strengths ranging from 0.97 to 1.00 with coefficients of variation ranging from 0.046 to 0.074 including possible in-test variations. In this study the mean ratio of actual strength to design strength will be taken as $\bar{P} = 0.98$ and the coefficient of variation as $V_P = 0.05$.

The 1962 Report of ACI Committee 326 (ACI–ASCE 1962) indicated that ACI–ASCE Equation 11.4 for v_c had a mean ratio of measured to computed capacity ranging from 1.03 to 1.30 with an overall average of 1.18 with $V = 0.16$. For beams with stirrups the mean ratio was 1.37 with $V = 0.205$. For the purposes of this comparison we shall assume that $\bar{P} = 1.10$ for the shear carried by the concrete and $\bar{P} = 1.20$ for that carried by stirrups. In both cases we shall assume $V = 0.15$.

6.3.2 *Computation of ϕ for Flexure of a Reinforced Concrete Beam*

(i) *Design Strength, R*

Using the assumptions in the ACI (1971) or CSA (1973) codes, the designer would calculate the beam strength (neglecting ϕ) as:

$$\begin{aligned} M_U &= A_s f_y(d - a/2) \\ &= 3.24 \\ &\quad \times 60\left(18 - \frac{3.24 \times 60}{2 \times 0.85 \times 4 \times 12}\right)\Big/ 12 \\ &= 253 \text{ ftK} \end{aligned}$$

Thus, the strength calculated by the designer would be $R = 253$ *ftK.*

(ii) *Mean Strength, R*

The mean strength can be computed using the mean strengths and dimensions in the equation for M_U:

$$\bar{M}_U = \bar{A}_s \bar{f}_y(\bar{d} - \bar{a}/2)$$

$$\bar{a}/2 = \frac{3.24 \times 62}{2.085 \times 3.8 \times 12.05} = 2.58$$

$$\bar{M}_U = 3.24 \times 62(17.85 - 2.58)/12 = 256 \text{ ftK}$$

This value of $\bar{M}_U$ must be corrected to allow for errors in the equation itself. Thus:

$$\bar{R} = \bar{M}_U \times \bar{P} = 256 \times 1.06 = 271 \text{ ftK}$$

$$\gamma_R \text{ is the ratio } \frac{\bar{R}}{R} = 1.071$$

(iii) *Coefficient of Variation, V_R*

The coefficient of variation will be computed in a number of stages:

$V_{a/2}$—Since $a/2$ is the product of a number of variables, [7c] will be used to compute $V_{a/2}$:

$$\begin{aligned} V_{a/2} &= \sqrt{V_{A_s}^2 + V_{f_y}^2 + V_{f_c}^2 + V_b^2} \\ &= \sqrt{0.06^2 + 0.07^2 + 0.18^2 + 0.025^2} = 0.204 \end{aligned}$$

Thus $V_{a/2} = 0.204$.

$V_{(d-a/2)}$—Since this is the sum of two variables, it is necessary to use [6c] to compute the standard deviation of this term and then convert that to a coefficient of variation using [3]. Thus

$$\sigma_{(d-a/2)} = \sqrt{\sigma_d^2 + \sigma_{a/2}^2}$$

where

$$\sigma_{a/2} = (\bar{a}/2) \cdot V_{a/2} = 2.58 \times 0.204 = 0.526 \text{ in.}$$

$$\sigma_{(d-a/2)} = \sqrt{0.45^2 + 0.526^2} = 0.69 \text{ in.}$$

and

$$V_{(d-a/2)} = \frac{\sigma_{(d-a/2)}}{\bar{d} - \bar{a}/2} = \frac{0.69}{15.27} = 0.045$$

V_{M_U}—Because M_U is calculated as the product of A_s, f_y, and $(d - a/2)$, V_{M_U} can be calculated as:

$$\begin{aligned} V_{M_U} &= \sqrt{V_{A_s}^2 + V_{f_y}^2 + V_{(d-a/2)}^2} \\ &= \sqrt{0.06^2 + 0.07^2 + 0.045^2} = 0.103 \end{aligned}$$

(Since $a/2$ is small compared to d, V_d could be used rather than $V_{(d-a/2)}$ in this calculation for simplicity. The resulting V_{M_U} would be 0.096 which is close enough in most cases.)

V_R—Finally, it is necessary to include the effect of the accuracy of the design equation in the coefficient of variation:

$$\begin{aligned} V_R &= \sqrt{V_{M_U}^2 + V_P^2} \\ &= \sqrt{0.103^2 + 0.04^2} = 0.11 \end{aligned}$$

(iv) *Summary*—For the reinforced concrete beam considered: $R = 253$ ftK, $\bar{R} = 271$ ftK, $\gamma_R = 1.071$, and $V_R = 0.11$.

(v) *Computation of ϕ for Flexure*—Equation [34] gives ϕ as:

$$\phi = \gamma_R e^{-\beta\alpha V_R}$$

Since a flexural failure of an under-reinforced reinforced concrete beam is a ductile failure, generally with normal consequences of failure, we shall use $\beta = 3.5$ (see section 6.2). The term α will be taken to 0.75 as explained in section 6.1. Thus:

$$\phi = 1.071\ e^{-3.5 \times 0.75 \times 0.11} = 0.802$$

The value of ϕ for this particular problem is 0.802. This value of ϕ will correspond to the load factors to be developed in section 6.3.5. By calculating ϕ values for a range of different properties, a weighted average value can be obtained.

6.3.3 *Computation of ϕ for an Axially Loaded Tied Column*

(i) *Design Strength, R*

The design strength will be assumed to be given by the traditional addition law:

$$P_o = [0.85\,f_c'\,(A_g - A_{st})] + (A_{st}f_y)$$
$$= [0.85 \times (144 - 2.16)] + (2.16 \times 60) = 612 \text{ kips}$$

(ii) *Mean Strength, $\bar{R}$*

Based on the mean strengths and dimensions the mean axial load capacity is:

$$\bar{P}_o = [0.85 \times 3.8\,(12.05^2 - 2.16)] + (2.16 \times 62) = 596 \text{ kips}$$

Again, this value must be corrected to allow for errors in the equation itself. Thus:

$$\bar{R} = \bar{P}_o \times \bar{P} = 596 \times 0.98 = 584 \text{ kips}$$

and

$$\gamma_R = \frac{\bar{R}}{P_o} = \frac{584}{612} = 0.955$$

(iii) *Coefficient of Variation, V_R*

The strength of the axially loaded column is the sum of the load carried by the concrete, P_c, and the steel, P_s. The coefficients of variation of P_c and P_s will be evaluated separately using [7c] and combined using [6c].

V_{P_c} and σ_{P_c}—The load carried by the concrete is the product of the variables f_c', b, and h, thus:

$$V_{P_c} = \sqrt{V_{f_c}^2 + V_b^2 + V_h^2} = 0.183$$

The standard deviation of the load carried by the concrete is:

$$\sigma_{P_c} = V_{P_c} \times \bar{P}_c = 0.183 \times 462 \text{ kips} = 84.5 \text{ kips}$$

V_{P_s} and σ_{P_s}—Similarly, the load carried by the reinforcement is the produce of f_y and A_{st}:

$$V_{P_s} = \sqrt{_{f_y}{}^2 + V_{A_{st}}{}^2} = 0.092$$

and

$$\sigma_{P_s} = 0.092 \times 134 \text{ kips} = 12.4 \text{ kips}$$

V_{P_o} and σ_{P_o}—Since P_o is a sum we must combine σ values to get σ_{P_o}:

$$\sigma_{P_o} = \sqrt{\sigma_{P_c}{}^2 + \sigma_{P_s}{}^2} = \sqrt{84.5^2 + 12.4^2} = 85.4 \text{ kips}$$

Thus

$$\bar{V}_{P_o} = \frac{\sigma_{P_o}}{\bar{P}_o} = \frac{85.4}{596} = 0.143$$

Again, this must be adjusted to allow for errors in the equation used to compute P_o: $V_R = \sqrt{V_{P_o}{}^2 + V_P{}^2}$ and $V_R = 0.152$.

(iv) *Summary*—For this particular axially loaded column: $R = 612$ kips, $\bar{R} = 584$ kips, $\gamma_R = 0.955$, and $V_R = 0.152$.

(v) *Computation of ϕ for Axially Loaded Column*

Since the failure of an axially loaded tied column will be brittle and may have serious consequences, we shall use $\beta = 4.0$ in evaluating ϕ.

$$\phi = \gamma_R\ e^{-\beta\alpha V_R}$$
$$= 0.955\ e^{-4.0 \times 0.75 \times 0.152} = 0.606$$

Thus the value of ϕ for this particular case is 0.606. Before a final value can be chosen, a number of different column cross sections must be studied. In addition, ϕ values must be calculated for eccentrically loaded columns. The variability of a randomly generated set of 12 in. (30.5 cm) square eccentrically loaded columns with a total steel percentage of 1% is shown in Fig. 22. The dispersion in the compression failure range corresponds reasonably well with that calculated in this section for pure axial load. The dispersion in the tension failure range approaches that for pure flexure.

The line labelled ACI Strength in Fig. 22 represents the strength based on the ACI design assumptions for $\phi = 1.0$ and ignoring load factors. When the ACI load factors and ϕ factors are included, the design strength is found to be safe relative to the minimum strength plotted in this figure.

6.3.4 *Computation of ϕ for Shear in a Beam*

(i) *Design Strength, R*

The design strength will be assumed to be given by:

$$V_U = 2\sqrt{f_c'}\,bd + \frac{A_v f_y d}{s}$$

$$= \frac{2\sqrt{4000} \times 12 \times 18}{1000} + \frac{0.22 \times 60 \times 18}{9}$$

$$R = V_U = 53.7 \text{ kips}$$

(ii) *Mean Strength, $\bar{R}$*

$$V_U = \frac{2\sqrt{3800} \times 12.05 \times 17.85}{1000} + \frac{0.22 \times 62 \times 17.85}{9}$$

$$= 26.5 + 27.1 = 53.6 \text{ kips}$$

Correcting this for errors in the equation for V_U gives:

$$\bar{R} = 1.1 \times 26.5 + 1.2 \times 27.1 = 61.7 \text{ kips}$$

$$\gamma_R = \frac{61.7}{53.7} = 1.151$$

(iii) *Coefficient of Variation, V_R*

V_{V_c} and σ_{V_c} —The shear carried by the concrete is the product of $\sqrt{f_c'}$ (assumed to have $V = 0.18$), b and d. Thus:

$$V_{V_c} = \sqrt{V_{f_c}{}^2 + V_b{}^2 + V_d{}^2} = 0.183$$

$$\sigma_{V_c} = 0.183 \times 26.5 = 4.86 \text{ kips}$$

V_{V_s} and σ_{V_s}—

$$V_{V_s} = \sqrt{V_{f_y}{}^2 + V_{A_v}{}^2 + {+}_d 2 + V_s{}^2} = 0.10$$

$$\sigma_{V_s} = 0.10 \times 27.1 = 2.71 \text{ kips}$$

V_{V_U} and σ_{V_U} —

$$\sigma_{V_U} = \sqrt{\sigma_{V_c}{}^2 + \sigma_{V_s}{}^2} = 5.57$$

$$\bar{V}_{V_U} = \frac{\sigma_{V_U}}{V_U} = 0.104$$

Correcting this for errors in the basic strength equation gives

$$V_R = 0.182$$

(iv) *Summary*—For this particular beam in shear: $R = 53.7$ kips, $\bar{R} = 61.7$ kips, $\gamma_R = 1.51$, and $V_R = 0.182$.

(v) *Computation of ϕ for Shear*

Since a shear failure will generally tend to be brittle we shall use $\beta = 4.0$.

$$\phi = \gamma_R e^{-\beta\alpha V_R} = 1.151\ e^{-4\times 0.75\times 0.182} = 0.667$$

Thus, for this beam which had roughly $2\sqrt{f_c'}$ resisted by V_c and $2\sqrt{f_c'}$ resisted by stirrups, the computed ϕ was 0.667.

6.3.5 *Computation of Load Factors for Live and Dead Loads*

The loads and structural analysis terms will be assumed to be represented by the statistical distributions given in Table 3. The values given are documented by Allen (1975) and were used to derive load factors for steel structures. For simplicity, the derivation will be limited to the combination of dead plus live loads. Similar analyses are required for other load combinations. The load and resistance factors can be expressed as:

$$[33] \qquad \phi R \geqslant \lambda U$$

where λ is the load factor. The derivation of λ will be based on [34]:

$$[34] \qquad \lambda = \gamma_U e^{\beta\alpha V_U}$$

But, $U = D + L$ where both D and L are separate variables. Because the coefficient of variation of D is much smaller than that of L, it is desirable to separate these. For derivation of code values of λ the procedures followed by Allen (1975) or Siu *et al.* (1975) should be followed. For the purposes of this paper, however, approximations to λ_D and λ_L can be derived in the following manner. Using [31] and [34] the left hand side of [33] becomes:

$$[35] \qquad \lambda U = \bar{U}^{\beta\alpha V U}$$

Using the first two terms of the series expansion for e^x gives

$$[36] \quad \bar{U}\, e^{\beta\alpha V_U} = (\bar{D} + \bar{L})(1 + \beta\alpha \bar{V}_U)$$

$$= (\bar{D} + \bar{L})\left(1 + \frac{\beta\alpha\sqrt{\bar{D}^2 V_D{}^2 + \bar{L}^2 V_L{}^2}}{\bar{D} + \bar{L}}\right)$$

$$\simeq (\bar{D} + \bar{L})\left(1 + \frac{\beta\alpha^2 \bar{D} V_D + \beta\alpha^2 \bar{L} V_L}{\bar{D} + \bar{L}}\right)$$

or

[37] $\bar{U}e^{\beta\alpha V_U} \simeq \bar{D}(1 + \beta\alpha^2 V_D) + \bar{L}(1 + \beta\alpha^2 V_L)$

The terms in the brackets in [37] can be rewritten in exponential form for consistency giving separate load factors for D and L:

[38] $\lambda_D = \gamma_D e^{\beta\alpha^2 V_D}$

[39] $\lambda_L = \gamma_L e^{\beta\alpha^2 V_L}$

Two values of each of λ_D and γ_L will be derived in the following sections, one for ductile failures, based on $\beta = 3.5$, and a second for brittle failures, based on $\beta = 4.0$.

(i) *Derive* γ_D

$$\lambda_D = \gamma_D e^{\beta\alpha^2 V_D}$$

The term V_D is affected by variations in the load, V_{SD}, and variations due to the structural analysis, V_{ED}. Thus:

$$V_D = \sqrt{V_{SD}^2 + V_{ED}^2} = \sqrt{0.07^2 + 0.08^2} = 0.106$$

For a ductile member $\beta = 3.5$ and:

$$\lambda_D = 1.0e^{(3.5\times 0.75^2 \times 0.106)} \simeq 1.23$$

For a brittle member $\beta = 4$ and:

$$\lambda_D = 1.0e^{(4\times 0.75^2 \times 0.106)} = 1.27$$

(ii) *Derive* λ_L

$$\lambda_L = \gamma_L e^{\beta\alpha^2 V_L}$$

where

$$V_L = \sqrt{V_{SL}^2 + V_{EL}^2} = \sqrt{0.3^2 + 0.2^2} = 0.36$$

For a ductile member:

$$\lambda_L = 0.7e^{3.5\times 0.75^2 \times 0.36} = 1.42$$

For a brittle member:

$$\lambda_L = 0.7e^{(4\times 0.75^2 \times 0.36)} = 1.576$$

(iii) *Summary*

For a ductile member:

$$\phi R \geqslant 1.23D = 1.42L$$

For a brittle member:

$$\phi R \geqslant 1.27D + 1.58L$$

6.3.6 *Modification of Load and* ϕ *Factors for Code Presentation*

Since a given structure will normally have both ductile and brittle members, it is desirable to have the same load factors for the entire structure and modify the ϕ factors slightly to account for the changes required to obtain common λ values: Thus:

For a ductile member:

$$1.035\ \phi\ R \geqslant 1.25D + 1.5L$$

For flexure in under-reinforced beams:

$$1.035\ \phi = 1.035 \times 0.802 = 0.83$$

or:

$$0.83\ M_U \geqslant \text{moment due to } (1.25D + 1.5L)$$

For a brittle member:

$$0.965\ \phi\ R \geqslant 1.25D + 1.5L$$

For axially loaded columns:

$$0.965\ \phi = 0.965 \times 0.606 = 0.59$$

or:

$$0.59P_0 \geqslant \text{axial force due to } (1.25D + 1.5L)$$

For shear in beams:

$$0.965 \times 0.667 = 0.64$$

or:

$$0.65\ V_U \geqslant \text{shear due to } (1.25D + 1.5L)$$

It is interesting to compare these preliminary results to the current ACI (1971) and CSA A23.3 (1973) safety requirements. If both sides of the ACI and CSA safety equations are divided by 1.125 the following equation results:

$$\frac{\phi}{1.125} R \geqslant 1.244D + 1.511L$$

The values of $\phi/1.125$ are 0.8 for flexure, 0.62 for tied columns and 0.76 for shear. Thus, the ϕ factors derived in this report lead to member strengths within 5% of those in the 1971 ACI code except for shear in which the new ϕ factor would require 19% additional strength.

Much more extensive study of various sizes of members, reinforcing ratios, live to dead to wind load ratios is required before final ϕ factors can be proposed for the building code. However, it is expected that the final values will be similar to the ones derived in this section.

6.4 *Proposed Procedures for Defining Resistance Factors for Concrete Structures*

It is envisaged that the following steps will be required in the derivation of load and ϕ factors for concrete structures:

1. *Collect Data on Statistical Distribution of Parameters*

Extensive data of the type summarized in section 6.3.1 and Table 3 must be collected for each component affecting the strength of reinforced and prestressed concrete structures.

In addition, data must be obtained from designers about typical dead to live to wind load ratios in reinforced concrete structures, and typical concrete strengths, steel percentages, etc. so that the factors can be optimized for the most commonly used cases.

2. *Theoretical and Design Strength Equations*

Procedures for calculating the theoretical member strengths must be selected and compared to available tests. The design equations should be those in the code. The theoretical equations should be as exotic as necessary to accurately estimate the true member strength. See, for example, Allen (1970).

3. *Calculate γ_R, V_R for Various Structural Actions*

Values of γ_R, V_R must be calculated for flexure, column cross sections in combined bending and axial load, slender columns, shear, bond, prestressed concrete, deflections, and possibly cracking. For some members these terms can be computed using direct statistical methods similar to those outlined in section 6.3.2. For other members such as column cross sections or slender columns the interaction of the variables is more complex and a Monte-Carlo simulation technique must be employed to estimate Y_R and V_R (Allen 1970). Figure 22 was obtained in this way.

4. *Calculate Resistance Factors*

Once a format has been selected for the resistance factors (either ϕ factors as in ACI and CSA or reduced material strengths f_c* and f_y* as in CEB) a linear programming solution can be used to compute the optimum values of the resistance (Siu *et al.* 1975). The input required for this solution includes the desired β value, the load factors from section 4.1.4 of NBC (1975) and weighting factors on the various values of ρ, f_c', D/L, and D/W to be considered in the solution.

The resistance factors and load factors should also be compared or calibrated to those currently used to see if wide divergences exist.

5. *Trial Designs*

Finally, a number of typical structures should be designed to compare the results of using the new and old design procedures.

6.5 *Shortcomings of Procedure Used to Calculate Load and Resistance Factors*

There are three major shortcomings of the second moment probabilistic procedure used to calculate load and ϕ factors:

1. The procedure is only as good as the data used in the solution. This is a problem to all procedures. Statistical data of the type required are not widely available.

2. The procedure assumes that specific probabilities of failure can be evaluated. Since this calculation depends on a knowledge of the extreme ranges of the strength distributions which are not adequately known, the computed probabilities of failure could differ from the actual values by as much as a factor of 10.

3. Failure of the structure is assumed to occur when one cross section or element reaches its capacity. For a weakest-link structure such as a truss, failure will occur when the weakest of many elements is overloaded. For a ductile indeterminate structure, loads will be distributed from section to section before the entire structure fails. Our solution will tend to overestimate the safety of the first case and underestimate the second.

4. Only failure by known overloads has been considered. Such causes of failure as gross errors, fire, explosion, etc. have not been considered. Allen (1975) and Knoll (1976) discuss this problem.

In spite of these shortcomings, the second moment probabilistic procedure used here to compute safety parameters will be used to derive code ϕ factors because it provides a rational procedure for estimating safety factors.

7. Summary and Conclusions

The process of limit states design of concrete structures has been examined and compared to existing design procedures. Neither working stress design nor ultimate strength design considered all the necessary limit states adequately. In the future, designers should consider the various limit states more explicitly.

The reasons for requiring safety provisions have been summarized in section 4 and proposed statistical descriptions of the important variables were established in section 6.3.1 and Table 3.

A number of techniques for establishing safety provisions for structures were reviewed in section 5. Of these, procedures based on attaining a specified probability of failure were found to be most satisfactory. The derivation of a second moment probabilistic procedure for computing ϕ factors and λ factors was presented in section 6.1 and used in section 6.3 to derive ϕ factors for reinforced concrete beams and columns. Finally a procedure for evaluating ϕ factors for future codes was outlined.

The values of the apparent safety factor (load factor/ϕ factor) derived in this paper were close to those currently used in ACI 318-71 (1971) or CSA A23.3 (1973) except for shear. The value derived for shear was considerably lower than the ACI value suggesting that the level of safety against shear failure may not be adequate in those codes.

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Appendix. Derivation of Load Factors in ACI Code

The load factors in the ACI Code evolved through a series of stages. The load and resistance factors in the proposed revisions to the ACI Code published in 1962 had a very different format than the load factors in the Appendix to the 1956 Code and the load and ϕ factors adopted in 1963 were different again. The following explanation is based partly on unpublished reports and partly on discussions with persons originally involved in setting the safety factors.

(a) *1956 ACI Code (ACI 1956a)*

The report of the ASCE–ACI Joint Committee on Ultimate Strength Design (ACI–ASCE 1955) recommended load factors of:

[A1] $$U = 1.2\,(D + T) + 2.4\,L$$

[A2] $$U = K\,(D + T + L)$$

where $K = 2$ for columns, 1.8 for beams and T = temperature, shrinkage, and similar effects. There were no ϕ factors. Other equations considered uplift loadings and wind but these will not be considered in this discussion.

The basis of the load factors is explained in the closure (ACI 1956*b*) to the discussion of the 1956 ACI Code which included the Joint Committee load factors in its Appendix on ultimate strength design: ". . . The joint committee found little difficulty in reconciling various ultimate strength formulas or derivations proposed and all available test results, but committee judgment was required to establish recommended load factors. The factors finally recommended are actually conservative; they were agreeable to the most conservative members of the committee after comparison to practice in other countries using the ultimate strength method and comparison to U.S. practice using the straightline method. In extreme cases they give more than the same factor of safety required for straightline design and in the usual case about the same."

The introduction of ultimate strength design in the 1956 Code was severely criticized by a reinforcing bar producer who apparently believed the proposed code would favor the use of hard grade steel which they did not produce.

(b) *1962 Proposed Revisions to ACI-318-56 (ACI 1962a)*

Between 1956 and 1962 extensive work was done by a subcommittee of 318 to codify the ultimate strength design procedures. This subcommittee was chaired by Prof. G. Winter. Much of the development of the safety provisions was done by Mr. T. F. Collier.

The basis of the development of the safety provisions was the assumption that 1 in 100 000 was an acceptable probability of failure. It was assumed that if there was a probability of 1 in 1000 of understrength and 1 in 100 of overload, the probability of failure would be the product of these two or 1 in 100 000. This is not quite correct statistically since the variables should be combined using [6c] and [7c], but it is adequate as a starting point. In the statistical calculations all variables were assumed to be normally distributed.

For average control, 1 in 1000 concrete compression tests would be expected to fall below $f_c^* = 0.67 f_c'$ provided that the concrete met the ACI control requirements that no more than 1 in 10 tests fell below f_c'. In a similar manner, 1 in 1000 steel tension tests would be expected to fall below 0.9 f_y. This was lowered to $f_y^* = 0.8 f_y$ to allow for dimensional tolerances, etc.

Calculations of the ultimate capacities, M_U, P_U, V_U, etc. were to be carried out using f_c^* and f_y^*. The probability of understrength of a cross section would then be 1 in 1000 if the strength depended entirely on either concrete or steel and somewhat smaller if the strength was affected by both materials.

Overload factors were expressed in terms of the basic equation

[A3] $$U = K_1D + K_2 (L + A)$$

where K_1 and K_2 are load factors and A was an overload allowance of $0.2L$ but not less than 20 psf (958 N/m^2). The factor A was intended to reflect the much higher probability that a lightly loaded area would be overloaded.

Based on the assumptions that the dead load had a coefficient of variation of 8 to 10%, load factors of 1.18 to 1.24 would be required if the probability of overload was to be 1 in 100. The code committee arbitrarily rounded this off to 1.3.

Setting load factors for live load was more difficult because live loads, especially small live loads, tend to be more variable and harder to predict than dead loads. In essence, it was assumed that a load of $(L + A)$ would be exceeded about 1 time in 10. To reduce this to the desired 1 in 100, load factors of 1.25 and 1.35 would be required if the coefficient of variation of the loads was $33\frac{1}{3}$ and 50%, respectively. Based on all of this ACI Committee 318 chose $K_2 = 1.3$ giving:

[A4] $$U = 1.3D + 1.3 (L + A)$$

Finally, the designer was required to increase the axial load, P, and the moment, M, by 10% for all columns to recognize the importance of columns in a structure. For tied columns an additional 10% increase was required because of the brittle failure of such columns.

(c) *Discussions of the 1962 proposed Revisions (ACI 1962b)*

The load factors proposed in the 1962 Proposed Revisions were vigorously opposed on a number of grounds by several groups. This criticism centered on five main points:

1. A number of discussors were concerned about the effect of the A term for prestressed roof members in parts of the country where the specified roof loads were small. It was pointed out that for roof loads less than 43 psf (2.06 MPa) the 1962 Proposed Revisions led to a larger section than the Appendix to ACI 318-56.

2. Several discussors pointed out that since it was common practice to pretension tendons to an initial stress of 80% of the ultimate, the value of $f_y^* = 0.8 f_y$ seemed low. They also felt that plant control should be recognized in selecting the understrength factors for concrete.

3. The probability of having both $f_c^* = 0.67 f_c'$ and $f_y^* = 0.8 f_y$ in the same member was questioned.

4. Engineers would not feel comfortable with live load factors as low as 1.3.

5. The use of $f_c^* = 0.67 f_c'$ and $f_y^* = 0.8 f_y$ implied that concrete was a less dependable material than steel and the resulting public reaction would force concrete out of the market.

(d) *1963 ACI Building Code (ACI 1963)*

The presentation and philosophy of the safety factors was completely changed between the publication of the 1962 Proposed Revisions and the publication of the 1963 ACI Code eleven months later. These changes were carried out under pressure from a number of interest groups and are not well documented. Although some of the following is based on discussions with committee members, other parts are speculation based largely on the committee's closure to the discussion of the Proposed Revisions.

Three major changes were made. First, as an interim step in the development, the term 1.3 $(L + A)$ was replaced with 1.3 $(1.2L) = 1.56L$. Second, the probability of overload was increased to roughly 1 in 1000 by arbitrarily multiplying both load factors by 1.15. At the same time the probability of understrength was reduced to about 1 in 100 giving $f_c^* \simeq 0.85 f_c'$ and $f_y^* \simeq 0.9 f_y$. Finally, the ratio of the strength of a cross section based on the new values of f_c^* and f_y^* to that based on f_c' and f_y was evaluated and called ϕ. The values of ϕ for spiral and tied columns were adjusted to account for the additional 10 to 20% safety margin required in the 1962 Proposed Revisions.

(e) *1971 ACI Code (ACI 1971)*

The basic load factors in the 1971 Code were reduced about 6% from those in the 1963 Code. This was an arbitrary decision by the Code Committee which they explained as follows (ACI 1970).

"Note that, because of the more comprehensive Code provisions, additional research and experience, improved concrete and steel quality control, load factors have been decreased from 1.5 to 1.4 and 1.8 to 1.7 for an average reduction in the neighborhood of 6 percent."