To determine ω_n

1. Solve eigenvalue problem
2. Find roots
3. Plug in roots to solve for mode shape vectors (3.74)

$$\begin{bmatrix} -\omega_n^2 M + K & K \\ K & -\omega_n^2 M + K \end{bmatrix} \begin{bmatrix} 1 \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad , \quad \begin{bmatrix} 1 \\ B \end{bmatrix} = \psi$$

To determine mass normalized modes $\phi = [\phi_1, \phi_2]$

1. $\phi_1 = \alpha \psi$, $\phi_2 = \beta \psi$, ... 3.81, 3.83

2. $\phi_1^T M \phi_1 = 1$, $\phi_2^T M \phi_2 = 1$ 3.77

To determine displacement response

1. Solve for ω_n and mass-normalized modes
2. Relate initial displacement/velocity to y

$y_0 = \phi^T M x_0$ (3.103)

$\dot{y}_0 = \phi^T M \dot{x}_0$ (3.104)

3. Since $\phi^T M \phi = I$

$$\phi^T K \phi = \begin{bmatrix} \omega_{n,1}^2 & 0 & 0 \\ 0 & \omega_{n,2}^2 & 0 \\ 0 & 0 & \omega_{n,3}^2 \end{bmatrix} = \Lambda \quad (3.94)$$

$$\phi^T C \phi = \begin{bmatrix} 2\zeta_1\omega_{n,1} & 0 & 0 \\ 0 & 2\zeta_2\omega_{n,2} & 0 \\ 0 & 0 & 2\zeta_3\omega_{n,3} \end{bmatrix} \quad (3.126)$$

4. For undamped SDOF w/ free vibration, $y(t) = y_{0,1}\cos\omega_{n,1}t + \dfrac{\dot{y}_{0,1}\sin(\omega_{n,1}t)}{\omega_{n,1}}$ (3.99)

For damped SDOF w/ free vibration, $y(t) = e^{-\zeta_1\omega_{n,1}t}\left(y_{0,1}\cos\omega_{d,1}t + \dfrac{\dot{y}_{0,1} + y_{0,1}\zeta_1\omega_{n,1}}{\omega_{d,1}}\sin\omega_d t\right)$ (3.147)

$\omega_{d,n} = \omega_{n,i}\sqrt{1-\zeta_i^2}$, $\zeta = \frac{1}{2}\left[\frac{\omega_2 - \omega_1}{\omega_n}\right]$, if crit damp given $= \zeta$

For undamped SDOF w/ forced vibration (if impulse), $y(t) = \frac{1}{m\omega_n}\int_0^t P(\tau)\sin\omega_n(t-\tau)$ (2.222), $\ddot{y} + 2\zeta_n\omega_n\dot{y}_n + \omega_n^2 y_n = \phi_n^T F(t)$

For damped SDOF w/ forced vibration $y(t) = \frac{1}{m\omega_D}\int_0^t P(\tau)e^{-\zeta\omega_n(t-\tau)}\sin\omega_D(t-\tau)\,d\tau$ (2.223), alternatively look at solns

5. $x = \phi y$ (3.100)

↳ If both initial conditions, add response to damped duty

$\omega_n = \sqrt{\frac{k}{m}}$

$\omega_D = \omega_n\sqrt{1-\zeta^2}$

$\zeta = \frac{c}{2\sqrt{km}} = \frac{c_{eff}}{2m\omega_n}$

$x(t) = e^{-\zeta\omega_n t}\left[x_0 \cos\omega_D t + \frac{v_0 + \zeta\omega_n x_0}{\omega_D}\sin\omega_D t\right]$ ← Viscously damped free vibration

$\delta = \frac{1}{N}\ln\left(\frac{x_i}{x_{i+N}}\right) = 2\pi\zeta$

$I = \frac{mL^2}{12} + mr^2$; if only particle, mr^2

To find $\ddot{\theta}$?, $\Sigma M = 0$ ↓ w/o considering motion etc.

$m_{eff}\ddot{x}(t) + c_{eff}\dot{x}(t) + k_{eff}x(t) = P(t)$, $k(u - u_{st})$, $u_{st} = \frac{\theta mg}{k}$

$x(t) = \int_0^t \frac{P(\tau)\sin\omega_n(t-\tau)d\tau}{m\omega_n}$, $d\left(\frac{\cos\omega_n(t-\tau)d\tau}{\omega_n}\right) = \sin\omega_n(t-\tau)$

↳ $u = \frac{1}{m\omega_n^2}$

$x_{max} = x_{st}\left[1 + \frac{2\sin(\omega_n t_1/2)}{\omega_n t_1}\right]$

$T = \frac{2\pi}{\omega_n}$, $\omega_n = 2\pi f_n$

(s) (Hz)

$x(t) = A_1\cos\omega_n t + A_2\sin\omega_n t + \frac{P_0}{k[1-(\omega/\omega_n)^2]}\sin(\omega t)$

- If $x(0) = x_0$, $A_1 = x_0$
- If $\dot{x}(0) = v_0$, $A_2 = \frac{v_0}{\omega_n} - \frac{P_0\,\omega/\omega_n}{k[1-(\omega/\omega_n)^2]}$

$|AF| = \frac{1}{1-(\omega/\omega_n)^2}$ (undamped)

True if at resonance

$: \frac{1}{2\zeta\sqrt{1-\zeta^2}}$ (Damped) $= \frac{x_p}{x_{st}}$

$x_p(t) = \frac{x_{g0}\,\beta^2\left[(1-\beta^2+4\zeta^2\beta^2)\sin\omega t - 2\zeta\beta^3\cos\theta\right]}{(1-\beta^2)^2 + 4\zeta^2\beta^2}$

$TR = \sqrt{\frac{1+4\zeta^2\beta^2}{(1-\beta^2)^2+4\zeta^2\beta^2}} = \frac{z_{max}+z_0}{z_0}$

$x(t) = e^{-\zeta\omega_n t}(A\cos\omega_D t + D\sin\omega_D t) + x_{st}\left[\frac{1-\beta^2}{(1-\beta^2)^2+4\zeta^2\beta^2}\sin\omega t - \frac{2\zeta\beta}{(1-\beta^2)^2+4\zeta^2\beta^2}\cos\omega t\right]$

↑ $\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{P_0\,\omega_n^2\sin\omega t}{k}$

$FTR = P_0\,TR$

Complementary $x_c = A\cos\omega_n t + B\sin\omega_n t$

Particular, if D operator, $D^2 = -\beta^2$

1. Find $|AF|_{max}$
2. $\frac{|AF|}{\sqrt{2}}$, find frequencies @ this line
3. $\zeta = \frac{f_b - f_a}{2\omega_n}$

$m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_g(t)$, assuming $x_g(t) = x_{g0}\sin(\omega t)$

$m\ddot{x} + c\dot{x} + kx = m x_{g0}\,\omega^2\sin(\omega t)$

$x(t) = \frac{1}{m\omega_n}\int_0^t P(\tau)\sin\omega_n(t-\tau)$

2.240

1. a) What is mode normalization and why is it necessary? **[3]**

Mode normalization: modes are normalized first to a unit displacement in 1 mode, as only the relative displacement is known. this is convenient for plotting

The modal matrix is further mass normalized to make the first term in equation of motion, $M_{ii} = \phi^T M \phi = 1$ for convenience

b) What is mode truncation and when is it applicable? **[3]**

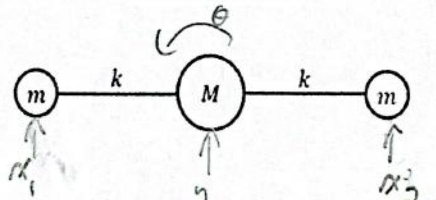
Mode truncation is truncating certain mode shape vectors from the modal matrix.

It is applicable when those modes don't contribute much to the ~~displace~~ equation of motion, as well as if the omission doesn't produce a large margin of error

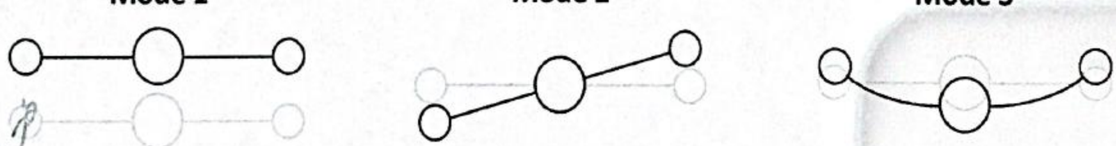
2. An airplane modelled as a three mass (3-DOF) system has the three natural modes illustrated below. Comment on the first two modes. Why do these modes occur and what are the corresponding natural frequencies? **[6]**

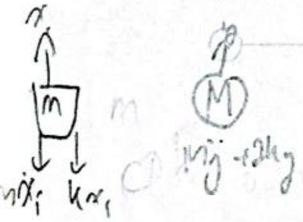
Physical System

3-DOF Model



Mode 1 Mode 2 Mode 3





$$\begin{bmatrix} m & & \\ & m & \\ & & M\end{bmatrix}\begin{Bmatrix}\ddot{x}_1\\ \ddot{x}_2\\ \ddot{y}\end{Bmatrix} + \begin{bmatrix} k & 0 & 0\\ 0 & k & 0\\ 0 & 0 & 2k\end{bmatrix}\begin{Bmatrix} x_1\\ x_2\\ y\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix}$$

UNIVERSITY OF
WATERLOO

Department of Civil and
Environmental Engineering

CIVE 505
Structural Dynamics

Quiz 1

June 15, 2022

Benjamin Klassen
Name

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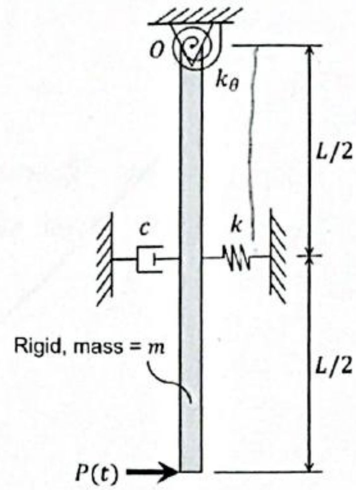
ID

Notes:

- Duration: 90 minutes
- Pages: 8 (including cover)
- You may refer to your course notes, but no other materials are permitted.
- A handheld calculator may be used but no other electronic resources (e.g., computers, tablets, phones) are allowed.
- Neatly show all your work. Illegible or unexplained work will receive no credit.

Good Luck!

1. A rigid bar with mass m is hanging vertically with a linear spring and viscous damper at mid-height and a rotational spring at the support at point O. A horizontal force $P(t)$ is applied at the free end of the bar.

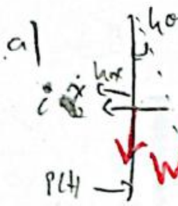


   a) Assuming small rotations, derive the equation of motion for the rigid bar in terms of its rotation θ. Comment on the effect of the bar's self-weight. **[8]**

   b) Given $m = 20$ kg, $k = 1$ kN/m, $k_\theta = 2$ kN · m/rad, and $L = 2$ m, determine the undamped natural period of the system. **[3]**

   c) Using the parameters in b), design the damper (i.e., determine c) such that the system has a damping ratio $\zeta = 0.05$. **[3]**

a)



$, m = mLg$, $\theta = \frac{x}{L/2} = \frac{2x}{L}$, $x = \frac{\theta L}{2}$

$\dot{x} = \frac{\dot{\theta} L}{2}$, x

$\Sigma M_O = 0, \; P(t)L - c\dot{x}(L/2) - k(x - x_{st})L/2 - \left(\frac{mL^2}{12} + m(L/2)^2\right)\ddot{\theta} - k_\theta \theta + mLg x$

$P(t)L \cancel{\;} = c\frac{\dot{\theta}L^2}{4} + k\left(\frac{\theta L}{2} - x_{st}\right)L/2 + \frac{mL^2}{3}\ddot{\theta} + k_\theta\theta - \frac{mL^2 g\theta}{2}$

To find x_{st}, equilibrium $x_{st} = 0$, because it is directly below the rotation point and thus it doesn't have an initial displacement

$\Sigma M = 0, \; kx_{st}(L/2) - P(t)L + k_\theta\theta = 0,$

$x_{st} = \frac{2P(t)L - 2k\theta}{kL}$

$\therefore k\left(\frac{\theta L}{2}\right.$

$\therefore P(t) = \frac{c\dot{\theta}L}{4} + \frac{k\theta L}{4} + \frac{mL^2}{3}\ddot{\theta} + \frac{k_\theta\theta}{L} - \frac{mLg\theta}{2}$

a) $\frac{mL^2}{3}\ddot{\theta} + \frac{cL^2}{4}\dot{\theta} + \theta\left(\frac{kL^2}{4} + \frac{k_\theta}{\cancel{4}} + \frac{mLg}{2}\right) = P(t)$

m_{eff} c_{eff} k_{eff}

−1

comment on the effect of the bar's self-weight? #1

b) $m = 20$, $k = 1$, $k_\theta = \frac{2kN\cdot m}{rad}$, $L = 2m$

$\omega_n = \sqrt{\frac{\frac{1000\cdot 2}{4} + \frac{2000}{2} - \frac{20\cdot 2\cdot 9.81}{2}}{20}}$ this ok but for your Keff.

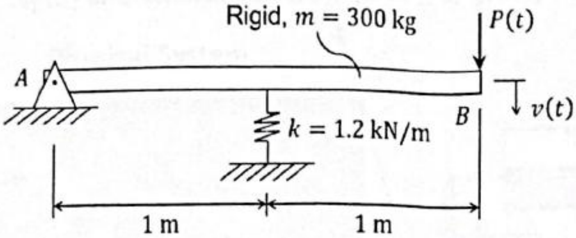
$= 8.074$ rads/s X 10.948

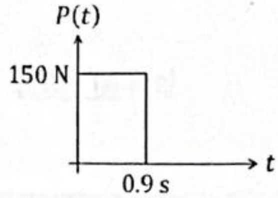
$T = \frac{2\pi}{8.074} = 0.778$ s X

c) $\zeta = \frac{c_{eff}}{2m\omega_n} = \frac{cL/4}{2\cdot 20\cdot 8.074} = 0.05$, $c = 32.296$ X $\frac{Ns}{m}$

you applied the correct equations but since your Keff is slightly different (due to − in $\frac{mLg}{2}$) your results are different. − 1

2. Consider the SDOF system shown below where the motion of the system is described by $v(t)$. The system is at rest when a rectangular pulse load is applied at point B.





$, P(t) = P_0$

Assuming small displacements, the equation of motion is given in terms of $v(t)$ as

$$\underbrace{100}_{m_{eff}}\ddot{v}(t) + \underbrace{300}_{k_{eff}}v(t) = P(t) \qquad (100 D^2 + 300)\, v = P(t)$$

a) Determine the maximum displacement response. [4]

b) For the given pulse, an engineer suggests the maximum displacement can be reduced by increasing m. Is this true? Explain. [3]

Maximum over steady-state era

a) $x(t) = \frac{1}{m\omega_n}\int_0^t P_0 \sin\omega_n(t-\tau)\,d\tau$, $\omega_n = \sqrt{\frac{1200}{300}} = 2$ **-1**

X $\omega_n = \sqrt{\frac{K_{eff}}{m_{eff}}} = \sqrt{\frac{300}{100}}$

$= \frac{1}{m\omega_n} P_0 \left[\frac{\cos\omega_n(t-\tau)}{\omega_n}\right]_0^t$ @ $t = 0.9$, " max. forcing

$= \frac{P_0}{m\omega_n^2}\left[\cos\omega_n(t-0.9) - \cos\omega_n t\right]$

$= \frac{P_0}{m\omega_n^2}\left[1 - \cos\omega_n t\right]$ **X** not need, this is found on your notes Sec. 2.10.2

$x(0.9) = \frac{150}{300 \cdot 2^2}\left[1 - \cos(2 \cdot 0.9)\right]$ **X** Max disp Eq. 2.213

$= 0.1534$ m

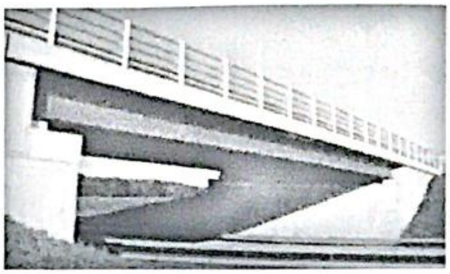
$= 153.4$ mm **-2**

b) Not necessarily the maximum displacement; the mass contributes to x_{st}, so total displacement will increase. However, it will decrease the displacement response by Duhamel's integral, as m is on denominator.

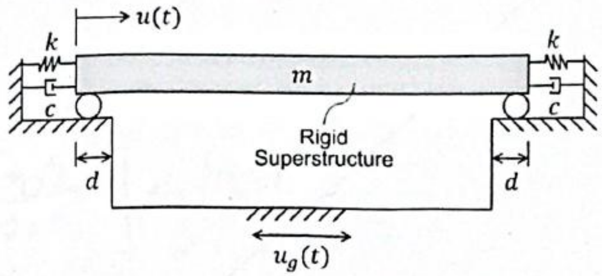
X **+2**

3. A single-span bridge subjected to an earthquake ground motion is being modelled as a SDOF system as shown below. The support provided by the abutments at the ends of the rigid superstructure ($m = 1.5 \times 10^6$ kg) is represented using linear springs ($k = 3 \times 10^8$ N/m) and viscous dampers ($c = 3 \times 10^6$ N·s/m).

Physical System



SDOF Model



The ground displacement $u_g(t)$ is identical at both ends of the bridge and can be represented as $u_g(t) = A \sin \omega t$ where $A = 0.4$ m and $\omega = 15$ rad/s. $u(t)$ is the *relative* horizontal displacement of the bridge with respect to the ground. Assume the bridge is initially at rest.

a) Write the equation of motion and the steady-state displacement response of the bridge. **[6]**

b) What is the maximum *total* steady-state displacement of the rigid superstructure? **[3]**

c) In an earthquake, the superstructure could become "unseated" (i.e., fall off the supports) if the relative displacement $u(t)$ of the superstructure is greater than the seat width d. If $d = 60$ cm, will this occur as a result of the ground motion in this problem? Only the steady-state response needs to be considered. **[3]**

a) $m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$ w/ u being relative motion (2.182).

$\therefore$ w/ $x_g = A\sin(\omega t)$

$m\ddot{u} + 2c\dot{x} + 2kx = mA\omega^2 \sin(\omega t)$

$1.5E6\ddot{u} + 3E6\dot{u} + 3E8u = 1.5E6 \cdot 0.4 \cdot 15^2 \cdot \sin(15t)$ X -1

$= 135000000 \sin(15t)$, divide all by mass

$\ddot{u} + 2\dot{u} + 200u = 90\sin(15t)$ X

Steady-state response = similar to 2.149, $\therefore$ 2.151 applies

$x_p(t) = M\sin(\omega t) + N\cos(\omega t)$ (2.151)

$M = \frac{1-(\omega/\omega_n)^2}{(1-\beta^2)^2 + 4\zeta^2\beta^2} x_{st}$, $N = \frac{-2\zeta\beta}{(1-\beta^2)^2 + 4\zeta^2\beta^2} x_{st}$

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{3E6}{2\sqrt{5E8 \cdot 1.5E6}} = 0.0707 \sim 0.071$$ ✓/✗

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{200} = 14.1421, \quad \omega = 15 \text{ rads/sec}$$ ✗ but OK for your Keff.

$$\therefore \beta = 1.06066 \sim 1.061$$

$$\therefore x_p(t) = \left[\frac{1-1.061^2}{(1-1.061^2)^2+4 \cdot 0.071^2 \cdot 1.061^2} \cdot \frac{90}{200}\right] \sin(15t)$$

$$- \frac{2 \cdot 0.071 \cdot 1.061}{(1-1.061^2)^2+4 \cdot 0.071^2 \cdot 1.061^2} \cdot \frac{90}{200}\Big] \cos(15t)$$

$$= -1.4693 \sin(15t) - 1.7608 \cos(15t)$$

b) From 2.188,

$$x_{t\,max} = A\sqrt{\frac{1+4\zeta^2\beta^2}{(1-\beta^2)^2+4\zeta^2\beta^2}}$$ +1 for applying the right Eq.

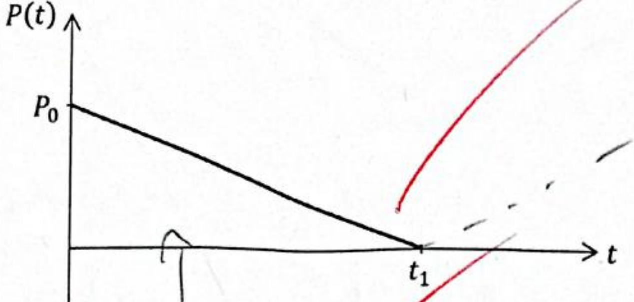
$$= 0.4\sqrt{\frac{1+4 \cdot 0.071^2 \cdot 1.061^2}{(1-1.061^2)^2+4 \cdot 0.071^2 \cdot 1.061^2}}$$

$$= 2.0615 \text{ m}$$ ✗

−2

c) $\frac{15}{\sqrt{2}} = 10.61$, $10.61 < 14.14 < 15$, $\therefore$ the frequency is capable of causing resonance. As a result, the displacement, as solved in b), is plausible and is ~~greater than 60cm~~, $\therefore$ the structure may fall off the seat.

4. Determine the response of an undamped linear SDOF system to the dynamic load shown below. Assume the system is initially at rest. **[12]**



$\frac{P_0 t}{t_1}$

$P(t) = P_0\left(1 - \frac{t}{t_1}\right)$

From $0 \to t_1$

$$x(t) = \frac{P_0}{m\omega_n}\left[\int_0^t \sin(\omega_n(t-\tau))\,d\tau - \int_0^t (\tau/t_1)\sin(\omega_n(t-\tau))\,d\tau\right], \quad d\left(\frac{\cos[\omega_n(t-\tau)]}{\omega_n}\right) = \sin\omega_n(t-\tau)$$

$$= \frac{P_0}{m\omega_n}\left[\frac{\cos[\omega_n(t-\tau)]}{\omega_n}\Big|_0^t - \frac{1}{t_1}\int_0^t \tau\, d\left(\frac{\cos[\omega_n(t-\tau)]}{\omega_n}\right)\right]$$

$$= \frac{P_0}{m\omega_n^2}\left[\cos[\omega_n(t-\tau)]\Big|_0^t - \left\{\frac{\tau\cos[\omega_n(t-\tau)]}{t_1} - \frac{1\,\omega_n}{t_1}\int\frac{\cos[\omega_n(t-\tau)]}{\omega_n}\,d\tau\right\}\Big|_0^t\right]$$

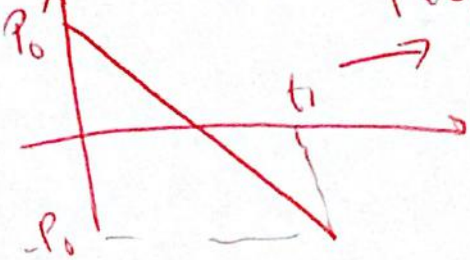
(1) $$= \frac{P_0}{m\omega_n^2}\left[\cos[\omega_n(t-\tau)]\Big|_0^t - \frac{\tau\cos[\omega_n(t-\tau)]}{t_1}\Big|_0^t - \frac{1}{t_1\omega_n}\sin[\omega_n(t-\tau)]\Big|_0^t\right]$$

$$= \frac{P_0}{m\omega_n^2}\left[1 - \cos[\omega_n t] - \frac{t}{t_1} + \frac{\sin\omega_n t}{t_1\omega_n}\right]$$

From $t > t_1$

② Applying a negative Duhamel's Integral beyond t_1, we have:

$\bar{P}(\tau) = 0$!!



$$\cancel{①\Big|_0^{t_1}} + \frac{1}{m\omega_n}\int_{t_1}^{t} \frac{P t}{t_1} \sin[\omega_n(t-\tau)]$$

$$= ①\Big|_0^{t_1} + \frac{P_0}{m\omega_n^2 t_1} \int t \sin[\omega_n(t-\tau)]\,d\tau ,$$

as previously shown, $\frac{1}{\omega_n}\int_{t_1}^{t} \frac{t}{t_1} \sin[\omega_n(t-\tau)]\,d\tau$

$$② = \frac{\tau}{t_1}\cos[\omega_n(t-\tau)]\Big|_0^{t} + \frac{1}{t_1\omega_n}\sin[\omega_n(t-\tau)]\Big|_0^{t}$$

$\therefore$, we have $①\Big|_0^{t_1} + \cancel{②\Big|_{t_1}^{t}}$ 0

$$\frac{P_0}{m\omega_n^2}\left[\cos[\omega_n(t-\tau)]\Big|_0^{t_1} - \frac{\tau\cos[\omega_n(t-\tau)]}{t_1}\Big|_0^{t_1} - \frac{1}{t_1\omega_n}\sin[\omega_n(t-\tau)]\Big|_0^{t_1}\right.$$

$$\left. + \cancel{\frac{\tau}{t_1}\cos[\omega_n(t-\tau)]\Big|_{t_1}^{t} + \frac{1}{t_1\omega_n}\sin[\omega_n(t-\tau)]\Big|_{t_1}^{t}}\right]$$

$$= \frac{P_0}{m\omega_n^2}\left[\cancel{\cos[\omega_n(t-t_1)]} - \cos[\omega_n(t)] - \cancel{\cos[\omega_n(t-t_1)]} - \frac{1}{t_1\omega_n}\sin[\omega_n(t-t_1)] + \frac{1}{t_1\omega_n}\sin[\omega_n(t)]\right.$$

$$\left. + \frac{t}{t_1} - \cos[\omega_n(t-t_1)] + - \frac{1}{t_1\omega_n}\sin[\omega_n(t-t_1)]\right]$$

-3

$\frac{P_0}{m\omega_n^2}$ [

UNIVERSITY OF
WATERLOO

Department of Civil and
Environmental Engineering

CIVE 505
Structural Dynamics

Quiz 2

July 8, 2022

Benjamin Ulissen 20761178

Name **ID**

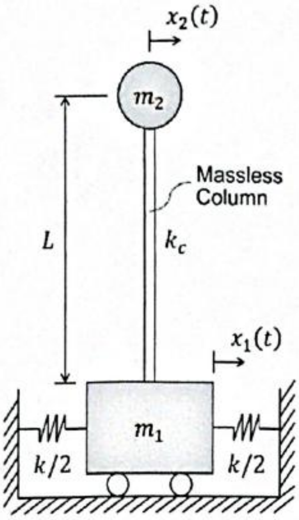
Notes:

- Duration: 60 minutes
- Pages: 5 (including cover)
- You may refer to your course notes, but no other materials are permitted.
- A handheld calculator may be used but no other electronic resources (e.g., computers, tablets, phones) are allowed.
- Neatly show all your work. Illegible or unexplained work will receive no credit.

Good Luck!

1. Consider the 2-DOF model of the Calgary Tower shown below. The base and foundation of the structure is idealized as a lumped mass m_1 while the restaurant and observation deck at the top is idealized as lumped mass m_2. The column supporting m_2 is assumed to be *massless* with a lateral stiffness k_c. The stiffness of the base and the support provided by the soil are modelled by the linear springs with stiffness $k/2$.

Calgary Tower



2-DOF Model

a) Write the *undamped* equation of motion in matrix form. The effect of gravity on m_2 can be neglected. **[5]**

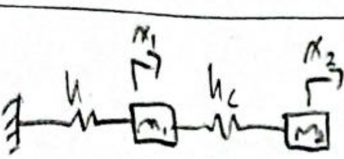
b) Given $m_1 = 50 \times 10^6$ kg, $m_2 = 3 \times 10^6$ kg, $k = 7 \times 10^8$ N/m, $k_c = 1 \times 10^8$ N/m, and $L = 140$ m, determine the undamped natural periods and mass normalized mode shapes. Sketch the mode shapes. **[18]**

c) Assuming stiffness proportional damping, determine the modal damping matrix $\boldsymbol{\Phi}^T \boldsymbol{C} \boldsymbol{\Phi}$ with 10% critical damping in Mode 1. **[3]**

d) Suppose the top of the tower (m_2) is subjected to a sudden gust of wind that can be characterized as an *impulsive force* $F_2(t) = A\delta(t)$ where $A = 20 \times 10^6$ N. Assume the system is initially at rest and there are no other applied loads. Determine the displacement response of the system with damping as defined in part c). **[10]**

*(**Note**: The response of a damped SDOF system to a unit impulse is given in Equation 2.220 of the course notes.)*

e) Comment on the validity of the 2-DOF model. How accurately do you think the model represents the true behaviour of Calgary Tower under wind loads? How can the model be improved? **[4]**

1.a) 

how you derive this? your free body diagrams?

$\therefore$, from 3.7, $\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k+k_c & -k_c \\ -k_c & k_c \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} P_1(t) \\ P_2(t) \end{Bmatrix}$

b) $m_1 = 50E6\ kg$
$m_2 = 3E6\ kg$
$k = 7E8\ N/m$
$k_c = 1E8\ N/m$
$L = 140$

$\therefore\ M = \begin{bmatrix} 50E6 & \\ & 3E6 \end{bmatrix}$, $K = \begin{bmatrix} 8E8 & -1E8 \\ -1E8 & 1E8 \end{bmatrix}$

From 3.74, $\begin{vmatrix} -\omega_n^2 M + K & K \\ K & -\omega_n^2 m + k \end{vmatrix}\begin{bmatrix} 1 \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{vmatrix} -\lambda^2 50E6 + 8E8 & -1E8 \\ -1E8 & -\lambda^2 3E6 + 1E8 \end{vmatrix} = 0$, $\lambda^4(50E6 \cdot 3E6) - \lambda^2(50E6 \cdot 1E8 + 8E8 \cdot 3E6) + 1E8 \cdot 8E8 - (1E8)^2$

$= \lambda^2 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 36.5737, 12.7590$

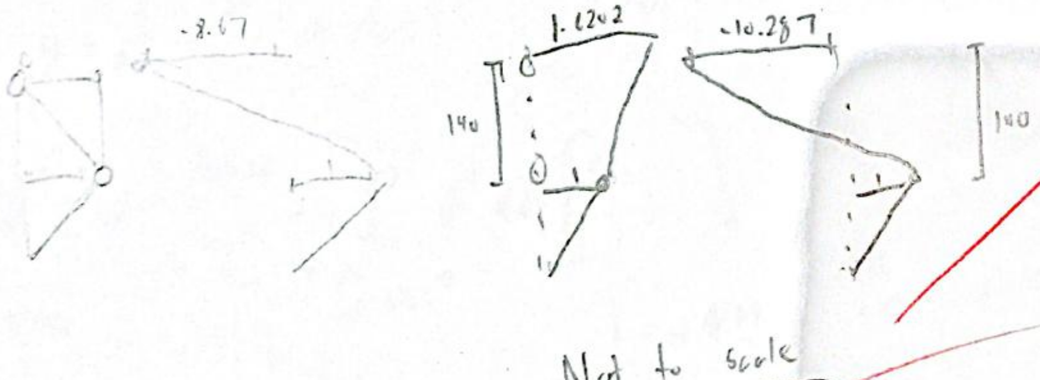
$\omega_n = 3.572, 6.0476$

Periods?? -1

$(-12.7590 \cdot 50E6 + 8E8)(1) - 1E8\ \beta$, $\beta = 1.6202$

$(-36.5737 \cdot 50E6 + 8E8)(1) - 1E8\ \beta$, $\beta = -10.287$

$\therefore\ \phi = \begin{array}{cc} \text{Mode 1} & \text{Mode 2} \end{array} \begin{bmatrix} 1 & 1 \\ 1.6202 & -10.287 \end{bmatrix}$



Not to scale

$\phi = \begin{bmatrix} 1 & 1 \\ 1.6202 & -10.287 \end{bmatrix}$, let $\phi_1 = \alpha\phi_1$, $\phi_2 = \beta\phi_2$

$[\alpha \;\; 1.62022] \begin{bmatrix} 50E6 & \\ & 3E6 \end{bmatrix} \begin{bmatrix} \alpha \\ 1.62022 \end{bmatrix} = 1$, $(50E6 + 7875144.12)\alpha^2 = 1$

$\alpha = 0.000131$

$[\beta \;\; -10.287\beta] \begin{bmatrix} 50E6 & \\ & 3E6 \end{bmatrix} \begin{bmatrix} \beta \\ -10.287\beta \end{bmatrix} = 1$, $(50E6 + 317467107)\alpha^2 = 1$

$\beta = 0.000052166$

$\phi_{n,n} = \begin{bmatrix} 0.000131 & 0.000052 \\ 0.000212 & -0.000537 \end{bmatrix}$

c) $\beta = \frac{2\zeta_1}{\omega_{n,1}} = \frac{2 \cdot 0.1}{3.572} = 0.055991$

$C = \beta K = 0.055991 \begin{bmatrix} 8E8 & -1E8 \\ -1E8 & 1E8 \end{bmatrix} = \begin{bmatrix} 44792833.15 & -5599104.14 \\ -5599104.14 & 5599104.14 \end{bmatrix}$

−1

This is ok but this part ask you for $\phi^T C \phi$, please carefully read the statement

d) $F_2 = A\delta(t)$,

$= 20E6\ \delta t$

(2220) $y_1 = \frac{1}{m\omega_D} e^{-\zeta\omega_n t} \sin \omega_D t$; ζ_2 unknown

$\therefore \phi^T C \phi = \begin{bmatrix} 2\zeta_1\omega_n & \\ & \ddots \end{bmatrix}$, $\phi^T K \phi = \begin{bmatrix} \omega_n^2 & \\ & \ddots \end{bmatrix}$

ah ok!!

still where are the values of your $\phi^T C \phi$??

$\begin{bmatrix} 2 \cdot 0.1 \cdot 3.572 & \\ & 2\zeta_2 \cdot 6.0476 \end{bmatrix} = \beta \begin{bmatrix} 3.572^2 & \\ & 6.0476^2 \end{bmatrix} + \alpha\phi^T M \phi$, assuming $\alpha = 0$

$2\zeta_2 \cdot 6.0476 = 0.055991 \cdot 6.0476^2$

$\zeta_2 = 0.1693$

$\therefore \omega_{D_1} = 3.572\sqrt{1-0.1^2} = 3.554$

$\omega_{D_2} = 6.0472\sqrt{1-0.1093^2} = 5.9599$

$\therefore y_i = \frac{1}{m\omega_D} e^{-\xi\omega_n t} \sin\omega_D t$, m=1 b/c mass normalized, $\times$ 20E6 b/c not unit force

$= \frac{4240}{3.554} e^{-0.1 \cdot 3.572t} \sin 3.554t$ $\phi^T[F] = 0.000212 \cdot 20E6 = 4240$

$= 1193.02\, e^{-0.3572t} \sin 3.554t$

Similarly, $y_2 = \frac{-10740}{5.9599} e^{-0.1 \cdot 6.0472t} \sin 5.9599t$ $\phi^T[F] = -0.000537 \cdot 20E6 = -10740$

$= 1802.04\, e^{-0.6047t} \sin 5.9599t$

From 3.100, $x = \phi y$

$$x_1 = \begin{bmatrix} 0.00431 & 0.000652 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0.000212 & -0.000537 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

I don't know what you wrote here difficult to read.

e) I do not believe that the model is very accurate. Due to the height of 140 m, one would think that the column mass would be accounted for. The model could be improved by considering this mass, potentially through 3-4 additional degrees of freedom.

Additionally, the damping matrix could use rayleigh or direct damping to consider the mass term and become more accurate.