

# Frame determinacy

$b$  = # of members between joints

$r$  = # of external reactions supporting frame

$j$  = # of joints

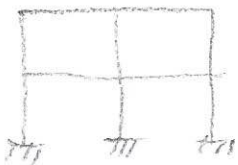
$c$  = # of internal constraints

$$D_{ind} = (3b + r) - (3j + c)$$

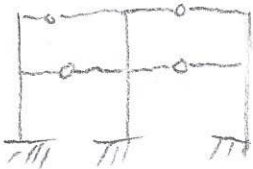
$$3b + r < 3j + c \text{ unstable}$$

$$3b + r = 3j + c \text{ determinate, prov. stable}$$

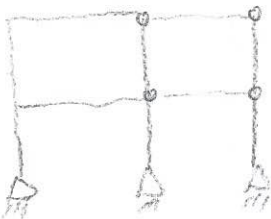
$$3b + r > 3j + c \text{ indeterminate, prov. stable}$$



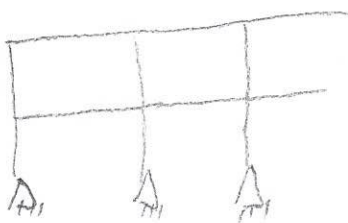
$b=10$   $39 > 27$ , ind and stable  
 $r=9$   
 $j=9$   
 $c=6$



Either ① or ②  
 ①  $b=16$   $51 > 31$   
 $r=9$   
 $j=9$   
 $c=4$   
 ②  $b=14$   $51 > 43$   
 $r=9$   
 $j=13$   
 $c=4$   
 either way, ind 8°



$b=10$   $36 > 38$ , ind 1°  
 $r=6$   
 $j=9$   
 $c=8$



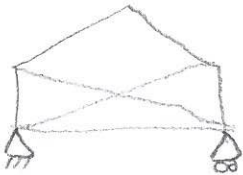
$b=10$   $36 > 27$ , ind 9°  
 $r=6$   
 $c=0$   
 $j=9$

## Truss indeterminacy

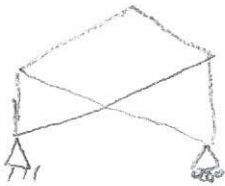
if  $b+r < 2j$ , statically unstable

$b+r = 2j$ , statically determinate (<sup>prov.</sup> ~~also~~ stable)

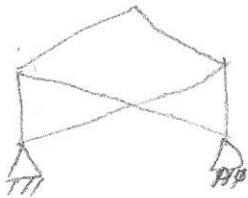
$b+r > 2j$ , indeterminate (<sup>prov.</sup> stable)



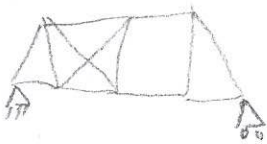
$b=7, r=3, j=5 \therefore 10=10$ , determinate & stable



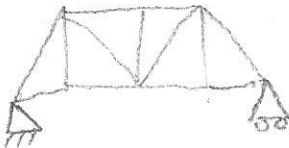
$b=6, r=3, j=5, 9 < 10$  unstable



$b=6, r=4, j=5, 10=10$  stable



$b=13, r=3, j=8, \det(16=16)$ , unstable (rec)



$b=13, r=3, j=8, \det(16=16)$ , stable



$b=13, r=4, j=8, 17 > 16$ , stable ind



$$R=4 \quad C=1$$

let  $R$  = # reactions (forces or moments)

let  $C$  = # internal conditions (internal release)

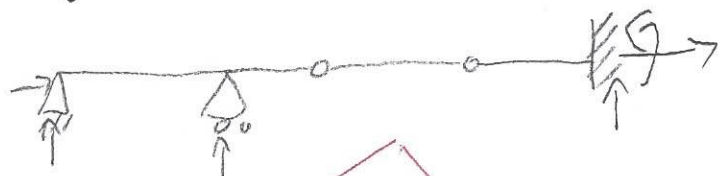
$$\text{degree of indeterminacy} = R - (3 + C)$$

- if 0, det. st. stable

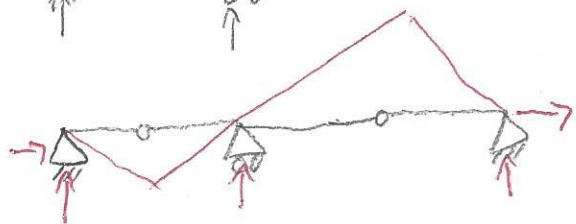
- if < 0, unstable

- if > 0,

Eg.



$$R=6, C=2, 6 - (3+2) = 1^0$$

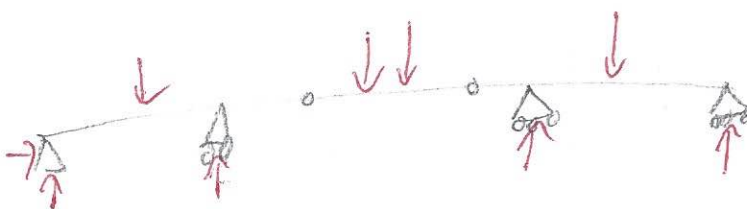


$$R=5, C=2, 5 - (3+2) = 0, \text{ det. unstable}$$

↳ def. 1st, 2nd, 3rd order  
effects



$$R=4, C=1, 4=4, \text{ unstable det.}$$



$$R=6, C=2,$$

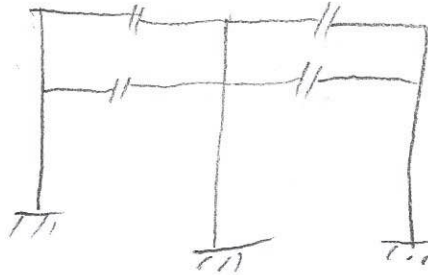
stable det. Break into little pieces



$$R=7, 2^0 \text{ det. Stable}$$

$$C=2$$

Rather than the force method, one can introduce releases to system until it becomes statically stable. The # of releases =  $^{\circ}$  indeterminacy



→ To analyze indeterminate structure, find enough unknowns in system to render determinate (force method and displacement method)

Must satisfy

- linear-elastic behavior
- equilibrium
- compatibility

Force Method (Flexibility)

- redundant forces solved
- solve remainder using equilibrium
- compatibility is explicitly satisfied
- Equilibrium is implicitly satisfied

Displacement Method (Stiffness)

- Equilibrium and force-displacement relationships used to solve for nodal displacement
- Equilibrium and compatibility explicitly satisfied