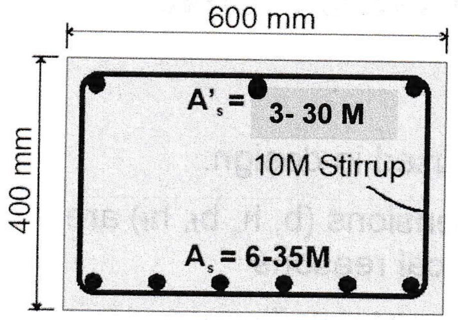
Table 2.1 & 2.2

Example 5: ***Redo Example 3 using Handbook tables.***



Calculate reinforcement ratios:

$$\rho = \frac{A_s}{bd} = \frac{6(1000)}{600(331)} = 0.0302 = 3.02\%$$

$$\rho' = \frac{A'_s}{bd} = \frac{3(700)}{600(331)} = 0.0106 = 1.06\%$$

1. check $\rho < \rho_{max}$

   for $f'_c = 30$ MPa, $f_y = 400$ MPa

   $\rho_{b(singly)} = 2.63\% \times 0.8 = 2.1\%$

   $\rho_{max} = \rho_b + \rho' = 2.1\% + 1.06\% > 3.02\%$ OK

2. Use singly reinforced section Table 2.1:

   $\rho_{equiv(singly)} = \rho - \rho' = 1.96$ $K_r = 5.25$

3. Use Table 2.2:

   $\frac{d'}{d} = \frac{66.3}{331} = 0.2$ $\rho' = 1.06\%$ $K'_r = 2.89$

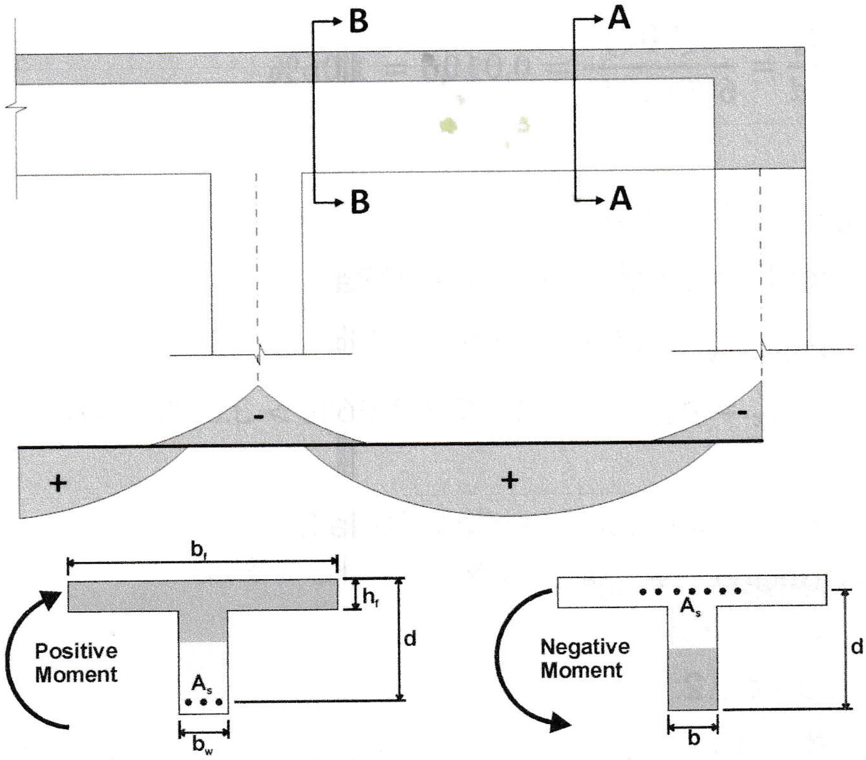
$$M_r = (K_r + K'_r)bd^2 = 10^{-6}(5.25 + 2.89)(600)(331)^2 = 535 \text{ kNm}$$

Doubly-Reinforced Sections Design

➢ Compression steel may be required or used in design:

- To increase M_r where section dimensions (b, h, b_f, h_f) are restricted for architectural or practical reasons
- Where steel is already present to meet detailing requirements
- To increase section ductility

Consider a typical frame/continuous beam:



T-Section
Section A-A

Rectangular Section
Section B-B

➢ Compression reinforcement is normally used at Section B-B:

- May be required to increase M_r due to limited width of compression zone
- Code requires some positive moment steel extended into support

The design process for a doubly-reinforced section can be approached in two possible cases:

Case I: **General case – no initial estimate of compression steel area**

- calculate required <u>tension and compression steel areas</u> to provide $M_r \geq M_f$
- design procedure ensures that tension steel yields
- preferable to have A'_s yielding

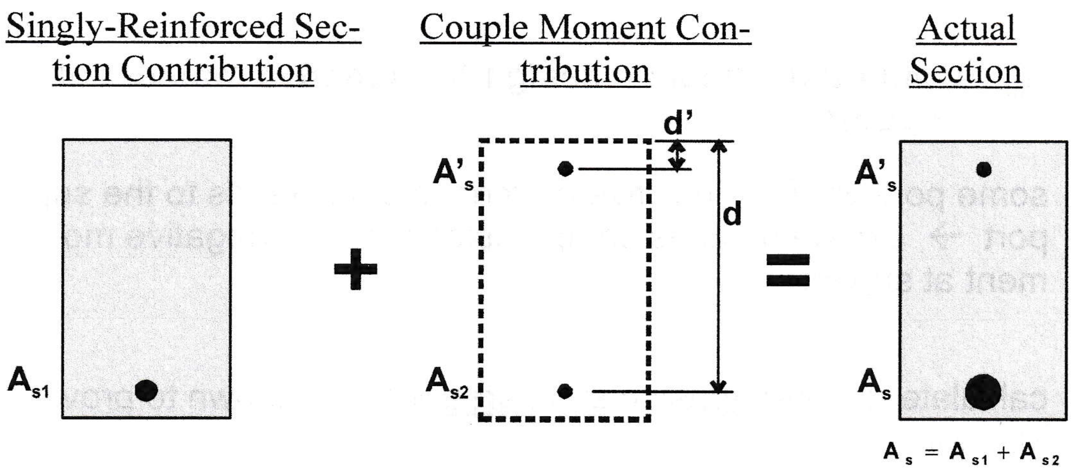
Case II: **Continuous beam – design for negative moment at support**

- some portion of the positive moment steel extends to the support → can be used as compression steel for negative moment at support

- calculate required <u>tension steel area</u> with A'_s known to provide $M_r \geq M_f$
- A'_s may or may not yield
- compression steel area at support may need to be increased to provide $M_r \geq M_f$

Design Procedure: Case I - General Approach

➢ When there is no initial estimate for A'_s, the design of a doubly-reinforced section may be approached as the superposition of a singly-reinforced section and a couple moment:



For singly-reinforced section contribution, M_{r1}:

Need: $A_{s1} \leq A_{sb}$

Assume: $A_{s1} = 60\%\left(A_{sb}\right)$

- gives "practical" amount of reinforcement
- tension steel yields
- A'_s will likely yield

Calculate: $M_{r1} = \phi_s A_{s1} f_y \left(d - a/2\right)$

For couple moment contribution, M_{r2}:

Solve for A'_s and A_{s2} needed to satisfy $M_r = M_{r1} + M_{r2} \geq M_f$

$$\left(M_{r2}\right)_{req'd} = M_f - M_{r1}$$

$$M_{r2} = T_{r2}\left(d - d'\right)$$
$$= \phi_s A_{s2} f_y \left(d - d'\right)$$

$$C_r' = T_{r2}$$
$$\phi_s A_s' f_s' = \phi_s A_{s2} f_y$$

$$A_{s2} = \frac{\left(M_{r2}\right)_{req'd}}{\phi_s f_y \left(d - d'\right)}$$

$$A_s' = \left(\frac{\phi_s f_y}{\phi_s f_s'}\right) A_{s2}$$

General Design Procedure (Case I)

1. Assume section dimensions

overall depth, h	A23.3 Clause 9.8.2.1
width, b	Assume $b_w \approx h/3$ to $h/2$ or based on architect or based on column width
effective depth, d	$d = h - 60\text{mm}$ one layer $d = h - 90\text{mm}$ two layers
effective depth, d'	$d' = 60\text{mm}$ one layer

2. Determine whether doubly-reinforced design is required

Assume singly-reinforced section design:

$$\left(K_r\right)_{req'd} = \frac{M_f}{bd^2}$$ → use Table 2.1 to determine $\rho_{req'd}$

If $\rho_{req'd} < \rho_b$*0.8 then use singly-reinf. design

If $\rho_{req'd} \geq \rho_b$*0.8 then use doubly-reinf. design → go to Step 3.

DOUBLY-REINFORCED SECTION DESIGN APPROACH:

3. Determine Singly-Reinforced Section Contribution

Assume: $\mathbf{A_{s1} = 0.6\rho_b bd}$

Calculate: $\mathbf{a = \dfrac{\phi_s A_{s1} f_y}{\alpha_1 \phi_c f'_c b}}$

$$\mathbf{M_{r1} = \phi_s A_{s1} f_y \left(d - a/2\right)}$$

4. Determine A'_s and A_{s2} based on Couple Moment

$$\mathbf{\left(M_{r2}\right)_{req'd} = M_f - M_{r1}}$$

$$\mathbf{A_{s2} = \frac{\left(M_{r2}\right)_{req'd}}{\phi_s f_y \left(d - d'\right)}} \qquad \mathbf{A'_s = \left(\frac{\phi_s f_y}{\phi_s f'_s}\right) A_{s2}}$$

5. Choose bar size and determine number required

➢ Determine required steel areas:

$$\mathbf{A_s = A_{s1} + A_{s2}} \qquad \mathbf{A'_s = A_{s2}}$$

➢ Ensure bars fit in cross-section

- Check bar spacing requirements
- May need to revise section width, b

➢ Compute "actual" steel areas

➢ Check minimum steel area: Clause 10.5.1.2

6. Check adequacy of section

- Compute M_r
- Use actual steel areas (A_s and A'_s), actual effective depth (d and d') and revised "b" (if applicable)
- Check $M_r \geq M_f$
- Check steel is yielding:

$$\frac{c}{d} \leq \frac{700}{700+f_y} 0.8 \qquad \varepsilon'_s = \left(1-\frac{d'}{c}\right)\varepsilon_{cu} \geq \varepsilon_y$$

Note: if $\varepsilon'_s < \varepsilon_y$, reduce A'_s and reanalyse

- Positive moment section design has been completed
  - Section dimensions have been determined
  - $(A_s)_{positive}$ is known at midspan → some portion of $(A_s)_{positive}$ extends to support
- Use positive moment steel at support as compression steel for negative moment at support

DESIGN PROCEDURE (CASE II – CONTINUOUS BEAM)

→ *Design negative moment section at support*

1. Section dimensions (based on midspan section design)

overall depth, h	Based on positive moment (mid-span) section
width, b	Based on positive moment (mid-span) section
effective depth, d	$d = h - 60mm$ **one layer**
	$d = h - 90mm$ **two layers**
effective depth, d'	$d' = 60mm$ **one layer**

2. Estimate A'$_s$ based on Positive Moment Steel

- Based on positive moment design at midspan, determine amount of steel extended to support section
    - Clause 12.11.1 requires at least ¼ of positive moment A_s to extend to the support of a continuous beam
    - Consider practical cut-off locations and amounts
    -
- Set A'$_s$ = positive moment steel extended to support

3. Calculate A_s required (total tension steel area)

A'$_s$ defines couple moment contribution:

Recall: $$\mathbf{A'_s = \left(\frac{\phi_s f_y}{\phi_s f'_s}\right) A_{s2}}$$

Assume: $\mathbf{f'_s = f_y} \quad \Rightarrow \quad \mathbf{A_{s2} = A'_s}$

Calculate: $\mathbf{M_{r2} = \phi_s A_{s2} f_y (d - d')}$

The remaining tension steel area, A_{s1}, is determined based on the Singly-Reinforced Section Contribution to the section:

Recall: $\mathbf{M_r = M_{r1} + M_{r2} \geq M_f} \quad \Rightarrow \quad \mathbf{(M_{r1})_{req'd} = M_f - M_{r2}}$

$$\mathbf{M_{r1} = \phi_s A_{s1} f_y (d - a/2)}$$

→ Compute required A_{s1} following procedure for singly-reinforced section design

5. Choose bar size and determine number required

- $\mathbf{A'_s}$ → compression steel area and bars are known
- Determine total tension steel required and choose bars:

$$\mathbf{A_s = A_{s1} + A_{s2}}$$

- Ensure bars fit in cross-section
  - Check bar spacing requirements
  - May need to revise section width, b
- Compute "actual" steel areas
- Check minimum steel area: Clause 10.5.1.2

6. Check adequacy of section

- Compute M_r following procedure:
  - Actual steel areas (A_s and A'_s)
  - Actual effective depth (d and d')
  - Revised "b" (if applicable)

- Check $M_r \geq M_f$
- Check steel is yielding:

$$\frac{c}{d} \leq \frac{700}{700+f_y} 0.8 \qquad \varepsilon'_s = \left(1 - \frac{d'}{c}\right)\varepsilon_{cu} \geq \varepsilon_y$$

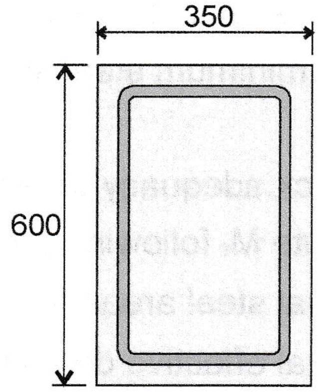
<u>*Note:*</u>

- It may not be possible (practical) to reduce A'_s if the compression steel is not yielding (i.e., $\varepsilon'_s < \varepsilon_y$) since A'_s is based on positive moment steel extended to the support.
- In this case, reanalyse the section considering $\varepsilon'_s < \varepsilon_y$ and verify that $M_r \geq M_f$

Example 6: ***Re-design the beam from Ex. 2 if the super-imposed dead load is increased to 14.25 kN/m and the live load is now 18 kN/m. Keep the same cross-section dimensions as Ex. 2.***

Assume:

- f'_c = 30 MPa & f_y = 400 MPa
- 10M stirrup
- Max. Coarse Agg. = 19 mm
- Assume 25M bars



From Ex. 2:

- Section dimensions as shown
- Concrete cover = 40 mm
- Min. spacing = 35.3 mm

$$w_{self} = 4.94\ kN/m\ based\ on\ provided\ cross-section$$

*we don't have any information about A_s' (not a continuous beam). Therefore, use General Design Procedure

Compute new M_f:

$$w_f = 1.25(4.94 + 14.25) + 1.5(18) = 51.0\ kN/m$$

$$M_f = \frac{w_f L^2}{8} = \frac{(51.0\,kN/m)(9.5m)^2}{8} = 575\ kN/m$$

General Procedure

Step 1: Already done (given)

Step 2: Determine whether A_s' is needed

Assume singly-reinforced

$\therefore d = h - 90 = 510mm$

$$(K_r)_{required} = \frac{M_f}{bd^2} = \frac{575x10^6 Nmm}{(350mm)(510mm)^2} = 6.32MPa$$

Using Table 2.1 $\rho_{required} \approx \rho_b$

Not desirable to design as singly reinforced, therefore add A_s'

Step 3: Singly reinforced section contribution

Assume 2 equal layers of steel (25M bars)

$$d = 600 - 40 - 11.3 - 25.2 - \frac{1}{2}(35.3) = 506mm$$

$$d' = 40 + 11.3 + \frac{1}{2}(25.2) = 63.9mm$$

Assume A_{s1} based on $\rho_1 = 0.6\rho_b$ $[for\ f_c' = 30MPa, \rho_b = 2.63\%]$

$\rho_1 = 0.6(0.0263) = 0.0158$

$A_{s1} = (0.0158)(350)(506) = 2798mm^2$

Determine M_{r1}: Table 2.1

$for\ \rho_1 = 1.58\%$

$\rightarrow K_{r1} = 4.45MPa$

$M_{r1} = (4.45MPa)(350mm)(506mm)^2 \div 10^6$

$= 399kNm < M_f = 575kNm$

$\therefore we\ must\ add\ A_{s2}\ and\ A_s'\ to\ make\ up\ the\ difference$

Step 4: Couple Moment Contribution

$\rightarrow Provide\ A_{s2}\ and\ A_s'\ to\ satisfy\ \Delta M_r = M_f - M_{r1}$

$\Delta M_r = 575kNm - 399kNm = 176kNm$

$\Delta M_r = \phi_s A_{s2} f_y (d - d') \rightarrow A_{s2}\ is\ unknown$

$$\rightarrow A_{s2} = \frac{\Delta M_r}{\phi_s f_y (d - d')} = \frac{176x10^6 Nmm}{(0.85)(400MPa)(506 - 63.9)} = 1171m^2$$

$C_r' = T_{r2} \rightarrow \phi_s A_s' f_s' = \phi_s A_{s2} f_y$

$Assume\ f_s' = f_y \rightarrow A_s' = A_{s2} = 1171mm^2$ **[confirm later]**

Step 5: Choose Bars (already decided to use 25M)

$A_s = A_{s1} + A_{s2} = 2798 + 1171 = 3969mm^2$

Use 8-25M bars $\rightarrow A_s = 4000mm^2\ (2\ layers)$

Check spacing

$$s = \frac{b - 2c - 2d_{st} - nd_b}{n - 1} = \frac{350 - 2(40) - 2(11.3) - 4(25.2)}{4 - 1}$$

$s = 48.9mm\ Clause\ 7.4.1.1$

From Ex. 2 $s > s_{min} \rightarrow OK$

Actual A_s':

$A_s' = 1171mm^2 \rightarrow use\ 3 - 25M\ bars$

$use\ A_s' = 1500mm^2 \rightarrow spacing\ will\ be\ ok$

Step 6: Check $M_r \rightarrow Analysis\ Problem$

For $A_s = 4000mm^2\ and\ A_s' = 1500mm^2$

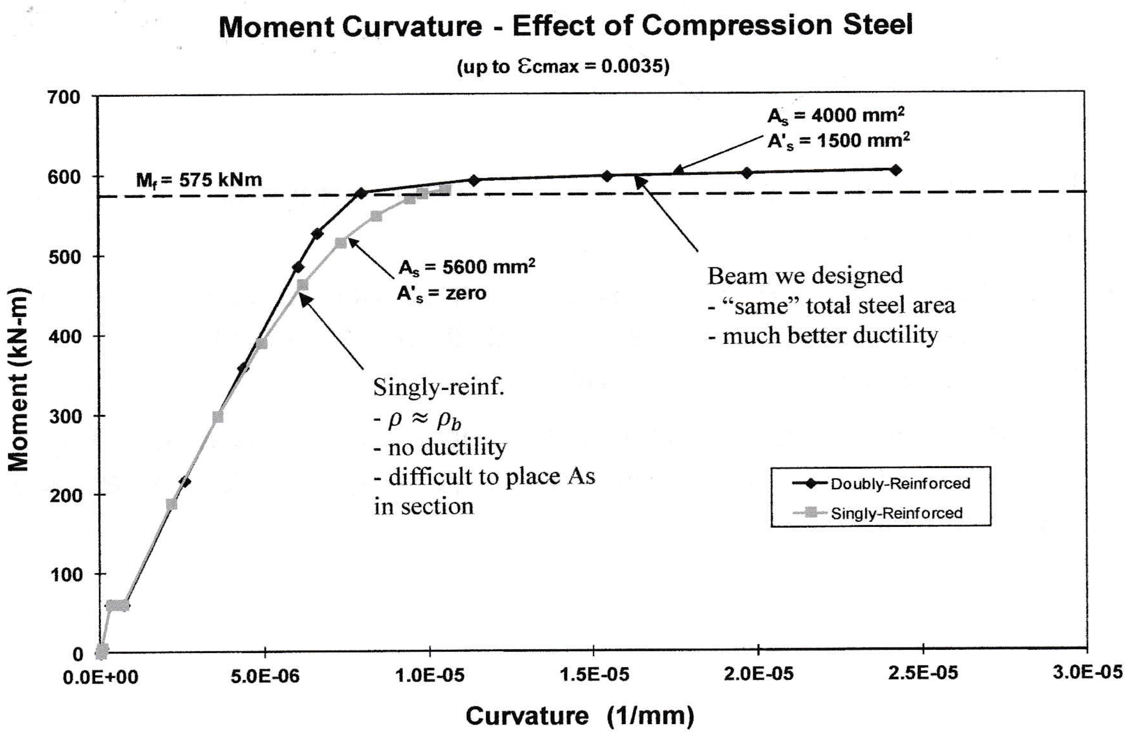
Check $M_r \geq M_f, \varepsilon_s' \geq \varepsilon_y\ and \frac{c}{d} \leq \frac{700}{700+f_y}$

Results:

$$M_r = 590kNm > M_f = 575kNm \rightarrow OK$$

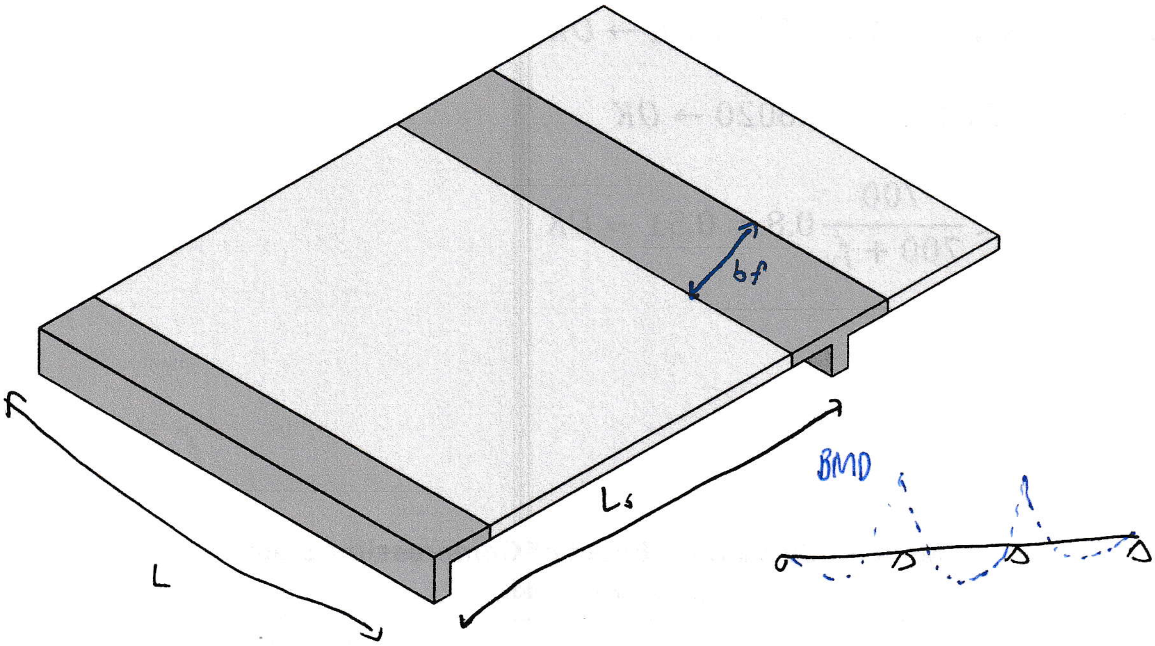
$$\varepsilon_s' = 0.00221 > \varepsilon_y = 0.0020 \rightarrow OK$$

$$\frac{c}{d} = 0.342 < \frac{700}{700 + f_y} 0.8 = 0.51 \rightarrow OK$$



T AND L SECTIONS

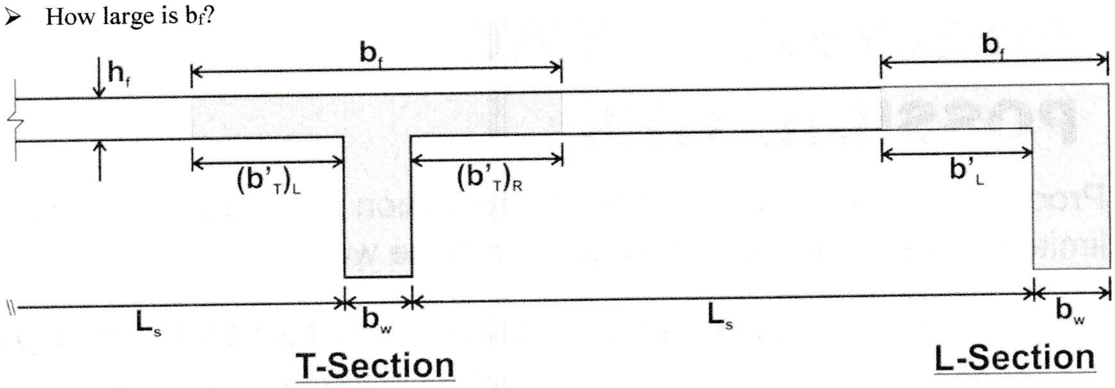
- Floor slabs are typically cast monolithically with the supporting beams.
- A portion of the slab is an integral part of the beam, creating a flanged section → T or L section.



- The portion of the slab that acts as a beam flange is referred to as the effective flange width, b_f.
- The effective flange width depends on several factors, including the level of loading, beam span, slab thickness and spacing between beams (i.e., slab span).
- Due to the significant increase in compression area width, only a very small stress block depth (a) is typically required to balance the tension force in the steel.

EFFECTIVE FLANGE WIDTH

➢ How large is b_f?



T-section → Clause 10.3.3:

Flange overhang:

$$b'_T \le \begin{cases} 0.2L & \text{simple beam} \\ 0.1L & \text{continuous beam} \\ 12h_f & \\ 0.5L_s & \end{cases}$$

beam span

slab span

smallest of all of these taken

- Evaluate flange overhang on each side of web
- Choose smallest b'_T for each side

$$\Rightarrow \quad b_f = (b'_T)_L + b_w + (b'_T)_R$$

L-section → Clause 10.3.4:

Flange overhang:

$$b'_L \le \begin{cases} L/12 \\ 6h_f \\ 0.5L_s \end{cases}$$

$$\Rightarrow \quad b_f = b_w + b'_L$$

T-SECTIONS

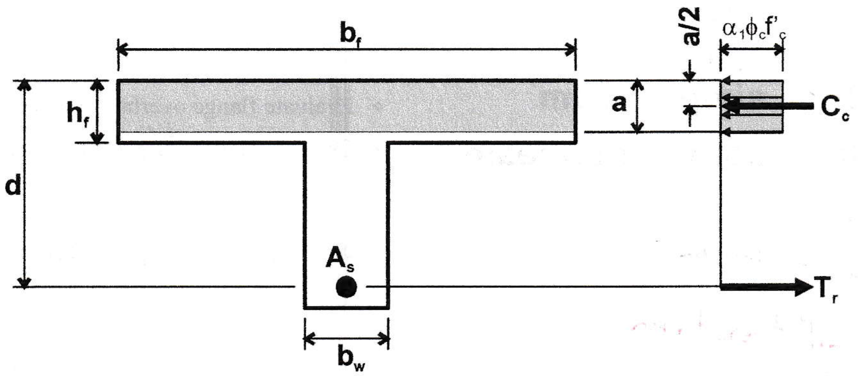
ANALYSIS → TWO POSSIBILITIES

- Procedure depends on whether compression block depth ("a") is limited to the flange, or if it extends into the web.

1) If $a < h_f$ → Compression block limited to flange

Best use of material

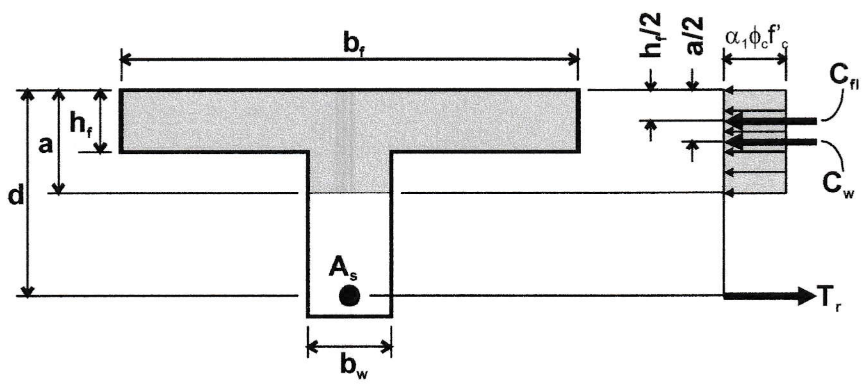
- T-section behaves as rectangular section with very large width b_f → analysis is the same as before.



$$M_r = C_c\left(d - \frac{a}{2}\right) \equiv T_r\left(d - \frac{a}{2}\right)$$

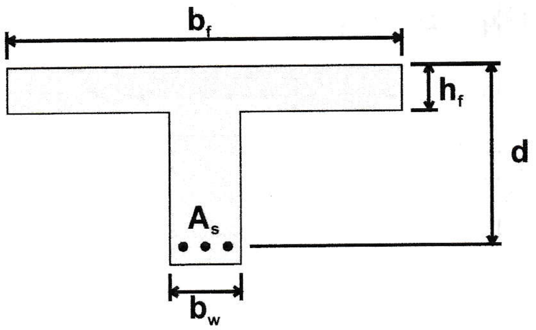
2) If $a > h_f$ → Compression block extends into web

- T-section analysis:



$$M_r = C_{fl}\left(d - \frac{h_f}{2}\right) + C_w\left(d - \frac{a}{2}\right)$$

T-Section Analysis Procedure – Positive Moment



Analysis Procedure

1. Assume compression is limited to flange and compute "a" $\le h_f$

$$a = \frac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b_f}$$

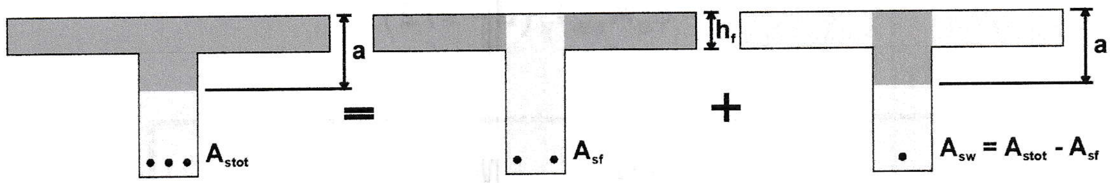
2. ***If a $\le h_f$***, then complete a rectangular section analysis

$$c = a/\beta_1 \qquad \text{check} \qquad \frac{c}{d} \le \frac{700}{700 + f_y} 0.8$$

→ tension reinforcement yielding

$$M_r = \phi_s A_s f_y (d - a/2)$$

3. ***If a > h_f***, then a T-Section analysis is required
Analyse by superposition:



A_{stot} = A_{sf} + A_{sw}

sf = flange   sw = web

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$$M_r \quad = \quad M_{flange} \quad + \quad M_{web}$$

Flange Component

$$T_{fl} = C_{fl}$$

$$\phi_s A_{sf} f_y = \phi_c \alpha_1 f'_c (b_f - b_w) h_f$$

$$A_{sf} = \frac{\phi_c \alpha_1 f'_c (b_f - b_w) h_f}{\phi_s f_y}$$

lever arm $= (d - h_f / 2)$

Thus,

$$\begin{aligned} M_{flange} &= C_{fl}(d - h_f / 2) \\ &= \phi_c \alpha_1 f'_c (b_f - b_w) h_f (d - h_f / 2) \end{aligned}$$

Web Component

- Treat as a rectangular beam with *width* b_w and *reinforced* with $A_{sw} = A_s - A_{sf}$

tot flange

$$T_w = C_w$$

$$\phi_s A_{sw} f_y = \phi_c \alpha_1 f'_c \, a \, b_w$$

$$a = \frac{\phi_s A_{sw} f_y}{\phi_c \alpha_1 f'_c \, b_w}$$

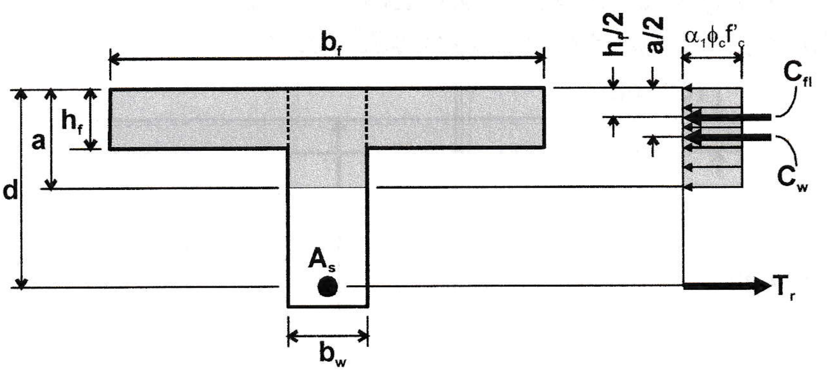
lever arm $= (d - a / 2)$

Thus,

$$\begin{aligned} M_{web} &= T_w (d - a / 2) \\ &= \phi_s A_{sw} f_y (d - a / 2) \end{aligned}$$

$$\boxed{M_r = M_{flange} + M_{web}}$$

Alternate Analysis Approach when $a > h_f$:



For equilibrium:

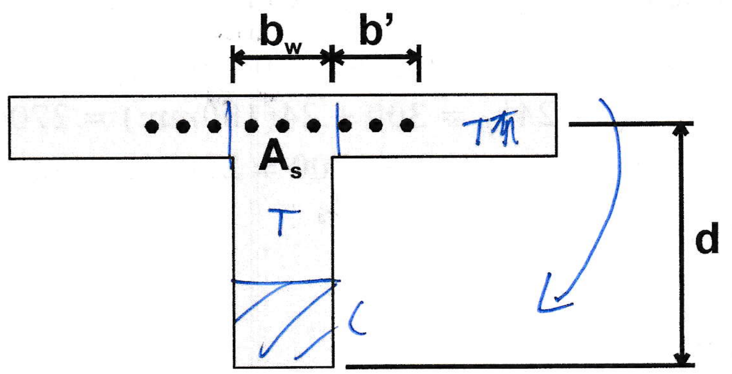
$$\phi_s A_s f_y = \alpha_1 \phi_c f'_c (b_f - b_w) h_f + \alpha_1 \phi_c f'_c b_w a$$

Sum moments about T_r:

$$M_r = C_{fl}(d - h_f / 2) + C_w (d - a / 2)$$

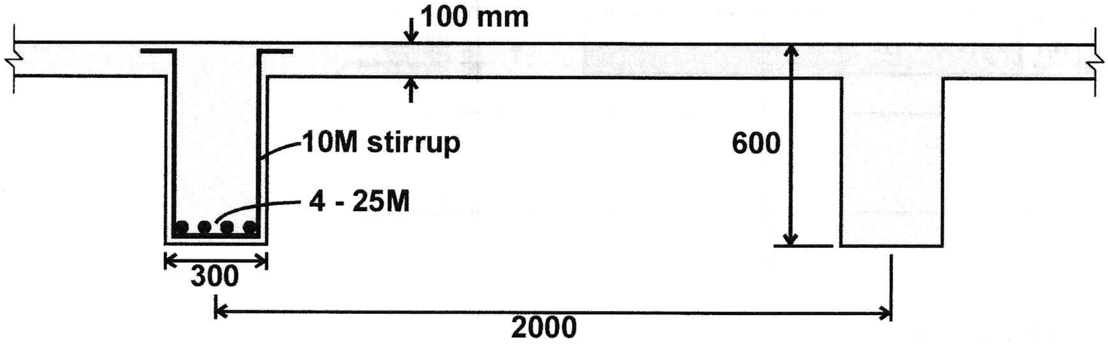
T-SECTION ANALYSIS PROCEDURE – NEGATIVE MOMENT

- Flange is cracked, therefore neglect "b_f" at ultimate
- T-section behaves as a "rectangular section" with width b_w



- Reinforcement placement is not a problem

Example 7: ***Find M_r for the T-beam shown. The beam has a span length of 9.0 m, and is continuous. Take f'_c = 30 MPa and f_y = 400 MPa.***



f'_c = 30 MPa

f_y = 400 MPa

concrete cover = 40 mm

$\alpha_1 = 0.81$

$\beta_1 = 0.90$

$A_s = 4x500mm^2 = 2000mm^2$

$d = 600 - 40 - 11.7 - \frac{25.2}{2} = 536.1mm$

Determine effective flange width (b_f), Clause 10.3.

span

$$b_f = minimum\begin{Bmatrix} b_w + 0.2L = 300 + 0.2(9000mm) = 2100mm \\ b_w + 24h_f = 300 + 24(100mm) = 2700mm \\ b_w + L_s = 300 + 1700 = 2000mm\ [\textbf{governs}] \end{Bmatrix}$$

h_f L_s

Use $b_f = 2000mm$

Assumptions:

- Rectangular Section ($a \le h_f$)
- Steel is yielding

Steel:

$T_r = \phi_s A_s f_y = 0.85(2000mm^2)(400MPa) \div 10^3$

$T_r = 680kN$

Concrete:

$C_c = \alpha_1 \phi_c f_c' a b_f$

$= (0.81)(0.65)(30MPa)a(2000mm) \div 1000^3$

$= 31.6a\ kN$ [**a unknown**]

Equilibrium:

$$T_r = C_r \rightarrow a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f_c' b_f}$$

$$a = \frac{680kN}{31.6} = 21.5mm < h_f = 100mm$$

$a < h_f$ *confirms assumption of rectangular section behaviour*

$$c = \frac{a}{\beta_1} = \frac{21.5mm}{0.90} = 23.9mm$$

$$\frac{c}{d} = \frac{23.9mm}{536.1mm} = 0.045 < \frac{700}{700 + fy} 0.8 = 0.51 \rightarrow steel\ is\ yielding$$

Calculate M_r:

$$M_r = T_r\left(d - \frac{a}{2}\right) = (680kN)(536.1 - \frac{21.5}{2}) \div 10\text{^}3$$

$M_r = 357.2\ kNm$

T-SECTIONS – use of handbook when $a < h_f$

- Analysis same as for rectangular section when $a \le h_f$ → use Handbook Table 2.1. (recall 0.8 factor not included in tables)

Example 8 - redo example 7 using Tables.

$$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b_f} = \frac{(0.85)(2000)(400)}{0.81(0.65)(30)(2000)}$$
$$= 21.5\text{mm} \quad < h_f \quad \Rightarrow \text{behaves as a rectangular section}$$

$$\rho = \frac{A_s}{bd}$$
$$= \frac{2000}{(2000)(536)} = 0.19\%$$

Using Table 2.1

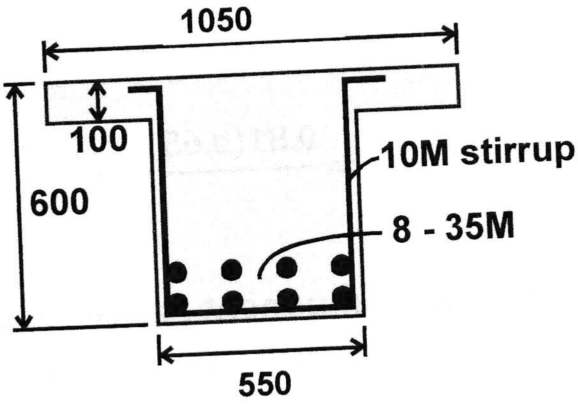
$$K_r = (0.1)\left[\frac{0.19 - 0.18}{0.21 - 0.18}\right] + 0.6$$
$$= 0.633\text{MPa}$$

$$M_r = K_r bd^2$$
$$= 0.633(2000)(536)^2$$
$$= 364\text{kNm}$$

<u>Example 9:</u> ***Find M_r for the T-beam shown.***
Take b_f = 1000 mm, f'_c = 30 MPa and f_y = 400 MPa.
Assume cover = 40 mm and max. C.A. size is 19 mm.

Given:

$$f'_c = 30\ MPa$$
$$\alpha_1 = 0.81$$
$$\beta_1 = 0.90$$
$$A_s = 8 \cdot 1000 = 8000mm^2$$



- Since the layers are equal, the centroid of the steel area lies halfway between the layers.
- If spacing is not known, use CSA A23.3-14 Annex A: Clause 6.6.5.2

$$spacing \geq \begin{Bmatrix} 1.4d_b & = 1.4(35.7) \rightarrow 50mm \\ 1.4\,a_{max} & 1.4(19) \rightarrow 26.6mm \\ 30mm & 30mm \end{Bmatrix}$$

$$d = h - cover - stirrup - d_b - \frac{1}{2}spacing$$

$$d = 600 - 40 - 11.3 - 35.7 - \frac{50}{2} = 488.0mm$$

$$b_f \rightarrow given\ as\ 1050mm$$

Assume: -rectangular section analysis (a<h$_f$)

-steel yields

$$C_r = T_r \rightarrow a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f'_c b} = \frac{0.85(8000mm^2)400MPa}{0.81(0.65)30(1050)}$$

$$a = 164mm > h_f \rightarrow must\ complete\ T - beam\ Analysis$$

3. Flange Component

$$T_{fl} = C_{fl}$$

$$\phi_s A_{sf} f_y = \alpha_1 \phi_c f_c'(b_f - b_w)h_f$$

$$A_{sf} = \frac{0.81(0.65)30(1050 - 550)100}{0.85(400)} = 2323mm^2$$

4. Web Component (solve for a)

$$A_w = A_s - A_{sf} = 8000 - 2323 = 5677mm^2$$

$$T_w = C_w$$

$$\phi_s A_{sw} f_y = \alpha_1 \phi_c f_c' a b_w$$

$$a = \frac{0.85(5677mm^2)400MPa}{0.81(0.65)30(550)} \rightarrow 222.2mm$$

$$a > h_f \rightarrow confirms\ T - beam\ behaviour$$

$$c = \frac{a}{\beta_1} = \frac{222.2}{0.9} = 246.9mm$$

$$\frac{c}{d} = \frac{246.9}{488} = 0.506 < \frac{700}{700 + f_y} 0.8 = 0.51$$

Moment Resistance

$$M_r = C_{fl}\left(d - \frac{h_f}{2}\right) + C_w\left(d - \frac{a}{2}\right)$$

$$C_{fl} = \alpha_1 \phi_c f_c'(b_f - b_w)h_f \rightarrow 0.81(0.65)30(1050 - 550)100$$

$$C_{fl} = 789.8\ kN$$

$$C_w = \alpha_1 \phi_c f_c' a b_w \rightarrow 0.81(0.65)30(222.2)550$$

$$C_w = 1930.3\ kN$$

$$M_r = \left\{789.8\left(488 - \frac{100}{2}\right) + 1930.5\left(488 - \frac{222.2}{2}\right)\right\} 10^{-3}$$

$$M_r = 1073.5\ kN \cdot m$$

$\left(d - \frac{h_f}{2}\right)$ $\left(d - a/2\right)$

flange web

T-SECTIONS – USE OF HANDBOOK WHEN $a > h_f$

➢ Analysis must be separated into two components:

M_{rf} → flange overhangs → A_{sf}

M_{rw} → web → A_{sw}

each represents M_r for a rectangular section with tension reinforcement ➔ Use Table 2.1

M_{rf}: need ρ_f → K_{rf} → $$M_{rf} = K_{rf}(b_f - b_w)d^2$$

M_{rw}: need ρ_w → K_{rw} → $$M_{rw} = K_{rw}(b_w)d^2$$

We know:

$$A_{sf} = \frac{C_{fl}}{\phi_s f_y} = \frac{\phi_c \alpha_1 f'_c (b_f - b_w) h_f}{\phi_s f_y}$$

Define:

$$\rho_f = \frac{A_{sf}}{(b_f - b_w)d} = \frac{\phi_c \alpha_1 f'_c h_f}{\phi_s f_y d}$$

→ function of h_f/d for given f'_c and f_y

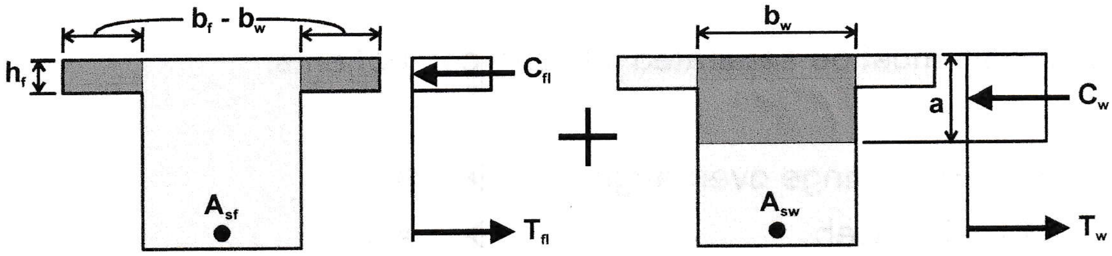
→ Table 2.3

We know:

$$A_{sw} = A_s - A_{sf}$$

$$\rho_w = \frac{A_{sw}}{b_w d}$$

Example 10: Redo Example 9 using Handbook



REDO Using 2019 code (b_w = 550mm, b_f = 1050mm)

Use Table 2.3 to get ρ_f:

For $d/h_f = 488/100 = 4.88 \quad \Rightarrow \quad \rho_f = 0.945\%$

Using Table 2.1: K_{rf} = 2.90 MPa

Determine ρ_w:

$$A_{sf} = \rho_f(b_f - b_w)d = (0.00945)(1000 - 500)(489)$$
$$= 2323 mm^2$$

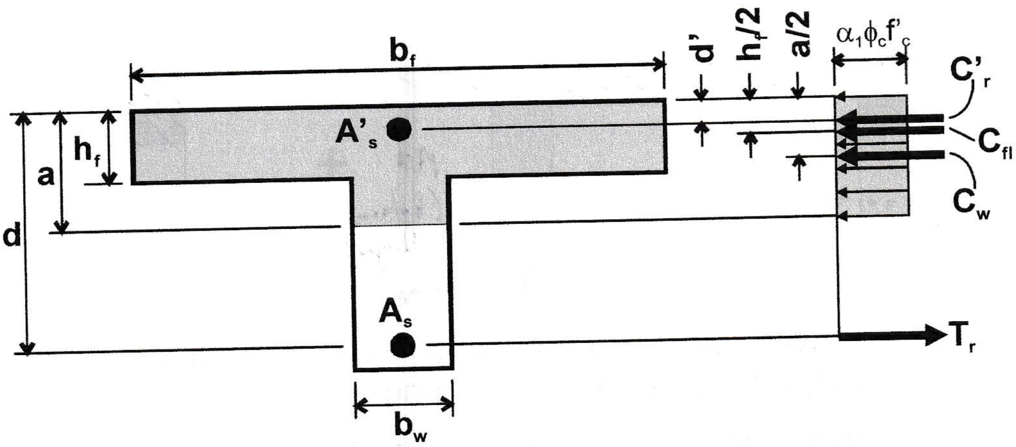
$$A_{sw} = A_s - A_{sf}$$
$$= 8000 - 2323 = 5677 mm^2$$

$$\rho_w = \frac{A_{sw}}{b_w d} = \frac{5677}{550(489)} = 2.1\% < 0.8 \times \rho_{bal} = 0.8 \times 2.63$$

Using Table 2.1: K_{rw} = 5.52 MPa

$$M_r = M_{rf} + M_{rw} = K_{rf}(b_f - b_w)d^2 + K_{rw}b_f d^2$$
$$= \{(2.9)(1050 - 550)488^2 + (5.52)(550)489^2\}10^{-6}$$
$$= 1071 kNm$$

T-SECTION WITH COMPRESSION REINFORCEMENT



For equilibrium:

$$\phi_s A_s f_y = \phi_s A'_s f_y + \alpha_1 \phi_c f'_c (b_f - b_w) h_f + \alpha_1 \phi_c f'_c b_w a$$

➔ Solve for "a"

➔ Check if A_s and A'_s are yielding

Sum moments about tension steel:

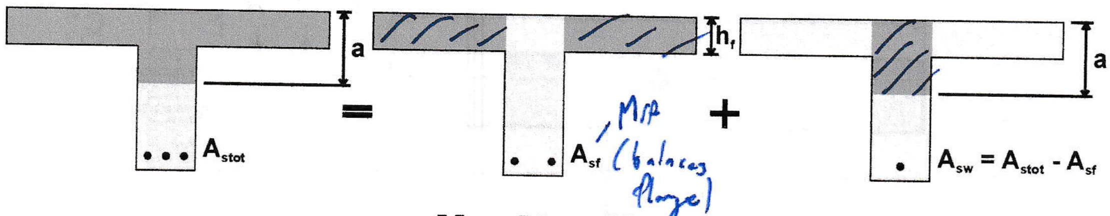
$$M_r = C'_r (d - d') + C_{fl} (d - h_f / 2) + C_w (d - a / 2)$$

For ductile failure:

$$\frac{c}{d} < \frac{700}{700 + f_y} \; 0.8$$

T-SECTIONS DESIGN

- T-Sections are normally singly-reinforced, and the N.A. is located within the flange in most situations → design is identical to design for a rectangular section
- If the N.A. is located below the flange, the design is parallel to the design of a doubly-reinforced section:



$$M_r = M_{rf} + M_{rw}$$

A_{sf} = tensile reinforcement required to balance compression in flange overhangs → "known"

A_{sw} = remaining reinforcement required to ensure $M_r \geq M_f$ → calculate

T-SECTION DESIGN PROCEDURE:

1. Assume section dimensions

overall depth, h	A23.3 Clause 9.8.2.1
flange thickness, h_f	A23.3 Clause 9.8.2.1
effective flange width, b_f	A23.3 Clause 10.3.3
web width, b_w	Assume $b_w \approx h/3$ to $h/2$ <u>or</u> based on architect <u>or</u> based on column width
effective depth, d	**$d = h - 60mm$ one layer** **$d = h - 90mm$ two layers**

can be less, but then requires checking deflections

2. Determine whether T-section design is required

Assume rectangular beam with b = b_f and a = h_f

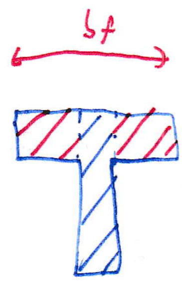
$$M_r^* = \alpha_1 \phi_c f'_c h_f b_f \left(d - \frac{h_f}{2} \right)$$

If $M_r^* \geq M_f$ then use rectangular beam design with b = b_f

If $M_r^* < M_f$ then use T-section design → go to Step 3.

T-section design approach:

3. Determine Flange Contribution



$$A_{sf} = \frac{\phi_c \alpha_1 f'_c (b_f - b_w) h_f}{\phi_s f_y}$$

$$\begin{aligned} M_{rf} &= \phi_s f_y A_{sf} (d - h_f/2) \\ &= \phi_c \alpha_1 f'_c (b_f - b_w) h_f (d - h_f/2) \end{aligned}$$

4. Determine Web Contribution

$$M_{rw} = M_f - M_{rf}$$

Determine A_{sw} to provide M_{rw}

Using Basic Principles:	*Using Design Table 2.1:*
Recall: $M_{rw} = \phi_s A_{sw} f_y (d - a/2)$ $a = \dfrac{\phi_s A_{sw} f_y}{\phi_c \alpha_1 f'_c b_w}$ Thus: $M_{rw} = \phi_s A_{sw} f_y \left(d - \dfrac{\phi_s A_{sw} f_y}{2\phi_c \alpha_1 f'_c b_w} \right)$	Recall: $M_{rw} = K_{rw} b_w d^2$ $(K_{rw})_{req'd} = \dfrac{M_{rw}}{b_w d^2}$ $K_r = \rho \phi_s f_y \left(1 - \dfrac{\rho \phi_s f_y}{2\alpha_1 \phi_c f'_c} \right)$ → Use Table 2.1 to get $\rho_{w,req'd}$ for $(K_{rw})_{req'd}$
→ Solve for A_{sw}	→ $(A_{sw})_{req'd} = \rho_{w,req'd}(b_w d)$

5. Choose bar size and determine number required

➢ Determine total required steel area:

$$A_s = A_{sf} + A_{sw}$$

➢ Ensure bars fit in cross-section
  - Check bar spacing requirements
  - May need to revise web width, b_w

➢ Compute "actual" steel area, A_s, based on bars chosen

➢ Check minimum steel area: Clause 10.5.1.2

$$A_s \geq A_{smin} = \frac{0.2\sqrt{f'_c}}{f_y} b_t h$$

6. Check adequacy of section

- Compute M_r using:
  - Actual steel area
  - Actual "d"
  - Revised "b_w" (if applicable)

M_{rf} and A_{sf} → known

$$A_{sw} = (A_s)_{actual} - A_{sf}$$

Compute M_{rw} based on A_{sw}: - first principles <u>or</u>
- K_r, Table 2.1

$$M_r = M_{rf} + M_{rw}$$

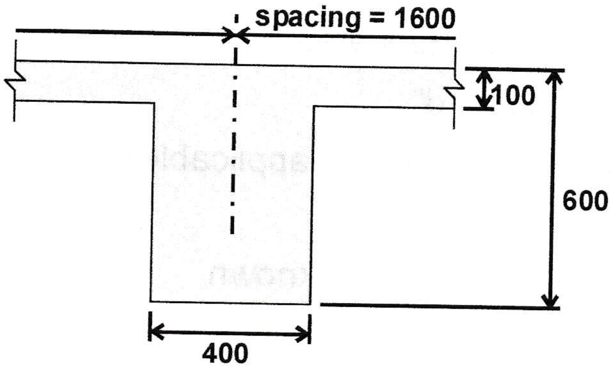
- Check $M_r \geq M_f$

- Check steel is yielding:

$$\frac{c}{d} \leq \frac{700}{700 + f_y} 0.8$$

Example 11: ***Design reinforcement for the simply-supported T-beam shown for M_f = 600 kNm.***

Beam span, L = 10 m



Use:

- f'_c = 30 MPa
- f_y = 400 MPa
- 10M stirrup
- Cover = 40 mm
- Max. C.A. = 19 mm

1. Assume section dimensions:

h, h_f, b_w → given

b_f → Based on Clause 10.3.3

$$b_f \le \begin{cases} b_w + 0.4L \\ b_w + 24h_f \\ b_w + L_s \end{cases} \le \begin{cases} 400 + 0.4(10{,}000) & = 4400\text{mm} \\ 400 + 24(100) & = 2800\text{mm} \\ 400 + (1600 - 400) & = 1600\text{mm} \Rightarrow \textbf{governs} \end{cases}$$

b_f is spacing. Entire slab is taken as T-beam !!!

d → assume two equal layers of 25M bars

$$d = 600 - 40 - 11.3 - 25.2 - \frac{1.4(25.2)}{2} = 506\text{mm}$$

2. Determine whether a T-section design is needed:

Assume rectangular section with $b = b_f$ and $a = h_f$

$$M_r^* = \alpha_1\phi_c f_c' h_f b_f\left(d - \frac{h_f}{2}\right)$$

$$= (0.81)(0.65)(30\,\mathrm{MPa})(100\,\mathrm{mm})(1600\,\mathrm{mm})\left(506 - \frac{100\,\mathrm{mm}}{2}\right) \div 10^6$$

$$= 1152\ \mathrm{kNm}$$

Since $M^*_r > M_f = 600$ kNm, we know that "a" will actually be <u>less than</u> h_f to provide $M_r = 600$ kNm, and thus we can design the beam as a rectangular section with $b = b_f$.

3. <u>Calculate required A_s using rectangular section design</u>

$$K_r = \frac{M_f}{b_f d^2} = \frac{600\times10^6\ \mathrm{Nmm}}{(1600\,\mathrm{mm})(506\,\mathrm{mm})^2} = 1.46\,\mathrm{MPa}$$

Using CAC Handbook Table 2.1 → $\rho = 0.453\%$ (required)

$$A_s = (0.00453)(1600\,\mathrm{mm})(506\,\mathrm{mm})$$ d

$$= 3676\,\mathrm{mm}^2$$

4. <u>Choose bar size and determine number required</u>

Use 8 – 25M $A_s = 4000$ mm² (provided in 2 layers)

Check spacing:

$$s = \frac{b - 2c - 2d_{st} - nd_b}{n-1} = \frac{400 - 2(40) - 2(11.3) - 4(25.2)}{4-1} = 65.53\,\mathrm{mm}$$

$$s \ge \begin{cases} 1.4d_b = 35.3\,\mathrm{mm} & \Rightarrow \text{governs} \\ 1.4\,\mathrm{max.C.A.} = 26.6\,\mathrm{mm} \\ 30\,\mathrm{mm} \end{cases}$$

Since:

s = 65.53 mm > 35.3 mm

→ spacing is OK

Check $A_{s,min}$: $$A_{smin} = \frac{0.2\sqrt{f_c'}}{f_y} b_t h = \frac{0.2\sqrt{30}}{400}(400)(600) = 657\,mm^2$$

$A_s > A_{s,min}$ → OK

5. Check adequacy of section (i.e., $M_r > M_f$)

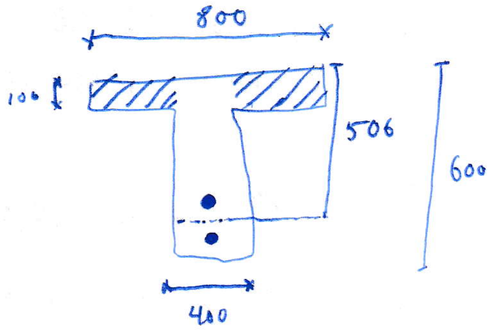
$$a = \frac{\phi_s A_s f_y}{\alpha_1 \phi_c f_c' b_f} = \frac{0.85(4000)(400)}{0.81(0.65)(30)(1600)} = 53.8\text{ mm}$$

$$\frac{c}{d} = \frac{53.8/0.9}{506} = 0.118 < \frac{700}{700+f_y}0.8 = 0.51$$

Steel is yielding

$$\begin{aligned} M_r &= \phi_s A_s f_y (d - a/2) \\ &= 0.85(4000)(400)(506 - 53.8/2) \div 10^6 \\ &= 651.6\text{ kNm} \quad > M_f \Rightarrow \text{OK} \end{aligned}$$

Example 12: ***Repeat Example 11 with b_f = 800 mm.***



1. Assume section dimensions:

   $\overset{600}{h}$, $\overset{100}{h_f}$, $\overset{400}{b_w}$, $\overset{800}{b_f}$ → Given

   d → assume two equal layers of 25M bars

$$d = 600 - 40 - 11.3 - 25.2 - \frac{1.4(25.2)}{2} = 506\,mm$$

2. Determine whether a T-section design is needed:

   Assume rectangular section with b = b_f and a = h_f

$$M_r^* = \alpha_1 \phi_c f_c' h_f b_f \left(d - \frac{h_f}{2}\right)$$

$$= (0.81)(0.65)(30\,MPa)(100\,mm)(800\,mm)\left(506 - \frac{100\,mm}{2}\right) \div 10^6$$

$$= 576.2\ kNm$$

Since $M^*_r < M_f$ = 600 kNm, we know that "a" will have to be <u>greater than</u> h_f to provide $M_r \geq$ 600 kNm, and thus we will need to do a T-section design.

3. Determine flange contribution

$$A_{sf} = \frac{\alpha_1 \phi_c f_c' (b_f - b_w) h_f}{\phi_s f_y} = \frac{(0.81)(0.65)(30)(800 - 400)(100)}{(0.85)(400)}$$

$$= 1858\,mm^2$$

$$M_{rf} = \phi_s A_{sf} f_y (d - h_f/2) = (0.85)(1858)(400)\left(506 - \frac{100}{2}\right) \div 10^6$$

$$= 288.0\ kNm$$

4. Determine web contribution

$$M_{rw} = M_f - M_{rf} = 600 - 288$$
$$= 312 \text{ kNm} \quad \text{(required)}$$

$$K_{rw} = \frac{M_{rw}}{b_w d^2} = \frac{312 \times 10^6 \text{ Nmm}}{(400\text{mm})(506\text{mm})^2} = 3.05 \text{ MPa}$$

Using CAC Handbook Table 2.1 → $\rho_w = 1.01\%$ (required)

$$A_{sw} = (0.0101)(400\text{mm})(506\text{mm})$$
$$= 2044 \text{mm}^2$$

5. Choose bar size and determine number required

$$A_s = A_{sf} + A_{sw} = 1858 + 2044$$
$$= 3902 \text{ mm}^2 \quad \text{(required)}$$

Use 8 – 25M → A_s = 4000 mm² (provided in 2 equal layers)

Check spacing:
$$s = \frac{400 - 2(40) - 2(11.3) - 4(25.2)}{4 - 1}$$
$$= 65.53\text{mm}$$

$$s \geq \begin{cases} 1.4d_b = 35.3\text{mm} & \Rightarrow \text{governs} \\ 1.4\text{max.C.A.} = 26.6\text{mm} \\ 30\text{mm} \end{cases}$$

Since:

s = 65.53 mm > 35.3 mm

→ spacing is OK

Check $A_{s,min}$:
$$A_{smin} = \frac{0.2\sqrt{f'_c}}{f_y} b_t h = \frac{0.2\sqrt{30}}{400}(400)(600) = 657\text{mm}^2$$

$A_s > A_{s,min}$ → OK

6. Check adequacy of section (i.e., $M_r > M_f$)

T-section Analysis will be required (we already know $a > h_f$ from Step 2.):

M_{rf}: The flange contribution is unchanged from Step 3. of the design process:

$$A_{sf} = 1858 \text{ mm}^2$$
$$M_{rf} = 288 \text{ kNm}$$

M_{rw}: The web contribution is determined based on the analysis of a rectangular section with A_{sw} and width b_w:

$$a = \frac{\phi_s A_{sw} f_y}{\alpha_1 \phi_c f'_c b_w} = \frac{0.85(4000-1858)(400)}{0.81(0.65)(30)(400)} = 115.3 \text{ mm}$$

$$\frac{c}{d} = \frac{115.3/0.9}{506} = 0.253 < \frac{700}{700+f_y} 0.8 = 0.51$$

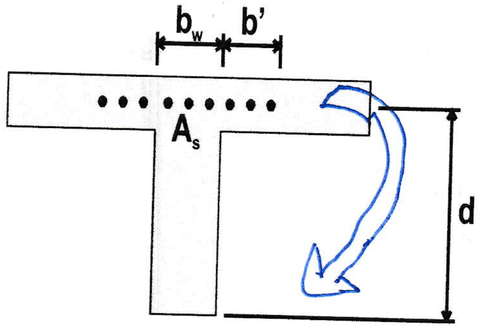
Steel is yielding

$$\begin{aligned} M_{rw} &= \phi_s A_{sw} f_y (d - a/2) \\ &= 0.85(4000-1858)(400)(506-115.3/2) \div 10^6 \\ &= 326.5 \text{ kNm} \end{aligned}$$

M_r:
$$\begin{aligned} M_r &= M_{rf} + M_{rw} = 288 + 326.5 \\ &= 614.5 \text{ kNm} \quad > M_f = 600 \text{ kNm} \Rightarrow \text{OK} \end{aligned}$$

T-SECTION DESIGN – NEGATIVE MOMENT

- Flange is cracked, therefore neglect "b_f" at ultimate
- T-section behaves as a "rectangular section" with width b_w



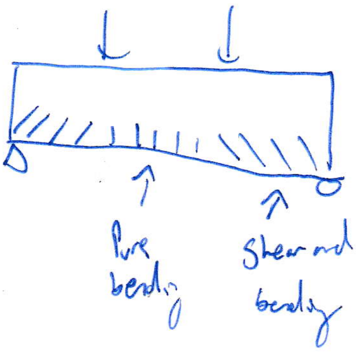
- Reinforcement placement is not a problem

- CSA A23.3 Clause 10.5.3 → a portion of A_s must be placed outside of web width:

  Minimum flexural tension reinforcement in flange overhang

$$A_s = 0.004(b'h_f)$$

$$b' = \text{smaller of} \begin{cases} L/20 \\ b'_T \text{ or } b'_L \end{cases}$$



- Minimum steel area: Clause 10.5.1.2

$$A_{smin} = \frac{0.2\sqrt{f'_c}}{f_y} b_t h$$

b_t = width of tension zone → based on b_f
≤ 2.5 b_w for T-beams
≤ 1.5 b_w for L-beams