CivE 495 Assignment 4

① $LL = 1.9\ kN/m^2$, $DL = 0.5\ kN/m^2$, SPF #1/#2, sheathing top and bottom, 2×d, 300 c/c, $E_s = E(K_T K_{SE})$

$d = ?$

$\Delta_{max_{cantilever}} = \frac{wL^4}{8EI}$, $\Delta_{max_{span}} = \frac{5wL^4}{384EI}$, $M_r = \phi F_b S K_{Zb} K_L$

$V_r = \phi F_v \frac{2A_n}{3} K_{Zv}$

w_f Total load $= (0.5^{\times 1.25} + 1.9^{\times 1.5})(0.3) = 1.0425\ kN/m$

w_L Transient $= 1.9(0.3) \times 1.5 = 0.855\ kN/m$

$w_d = 0.5 \times 0.3 \times 1.25 = 0.1875\ kN/m$

$w_{L\ unfactored} = 1.9 \times 0.3 = 0.57\ kN/m$

$w_{d\ unfactored} = 0.5 \times 0.3 = 0.15\ kN/m$

Span

$\Delta_{max} = \frac{3400}{360} = 9.44mm$ (LL only)

$\Delta_{max} = \frac{3400}{180} = 18.89mm$ (Total load)

Cantilever

$\Delta_{max} = \frac{1400}{180} = 7.78mm$ (LL only)

$\Delta_{max} = \frac{1400}{90} = 15.56mm$ (total load)

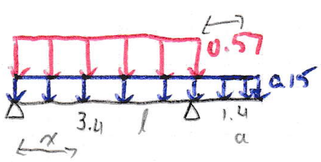
There are 3 Load Cases of interest:

i ↳ DL along both spans + LL along long span

ii ↳ DL along both spans + LL along short span

iii ↳ DL along both spans + LL along both spans

Case i - Use formulas from reference sections 30 and 33 - pg. 955 of WDM



Moment between supports from DL $= \frac{wx}{2l}(l^2 - a^2 - xl)$

Moments between supports from LL $= \frac{wx}{2}(l - x)$

$\therefore M_f = \frac{0.1875}{2 \cdot 3.4} x(3.4^2 - 1.4^2 - 3.4x) + \frac{0.855}{2} x(3.4 - x)$

$= 0.319x - 0.054x - 0.094x^2 + 1.4535x - 0.4275x^2$

$M = -0.5215x^2 + 1.7182x$

$\frac{dM}{dx} = -1.043x + 1.7182 = 0$, Max moment @ $x = 1.647\ m$

$M_f = 1.425\ kN \cdot m$

Deflection on overhang (case i)

Cantilever Δ from DL $= \frac{w x_1}{24EI}(4a^2 l - l^3 + 6a^2 x_1 - 4a x_1^2 + x_1^3)$

$$\frac{0.15}{24EI}\left[x(4a^2 l - l^3) + x^2(6a^2) - x^3(4a) + x^4\right]$$

$$= \frac{1}{EI}\left[-0.07905x_1 + 0.0735x_1^2 - 0.035x_1^3 + 0.00625x_1^4\right]$$

Cantilever Δ from LL $= \frac{1}{EI}\left[\frac{w l^3 x_1}{24}\right]$

$$= \frac{1}{EI}\left[\frac{0.57 \cdot 3.4^3}{24} x_1\right]$$

$$= \frac{1}{EI}\left[0.93347x_1\right]$$

LL only, $\Delta = 0.00778$ m

$EI = \frac{1}{0.00778}\left[0.93347x_1\right]$, max when $x_1 = 1.4$

$EI_{req} = 167.977$ kN·m²

DL + LL, $\Delta = 0.01556$ m

$$EI = \frac{1}{0.01556}\left[0.85442x + 0.0735x^2 - 0.035x^3 + 0.00625x_1^4\right]$$

$= 83.638x_1 + 5.905x_1^2 - 2.812x_1^3 + 0.502x_1^4$, max @ $x_1 = 1.4$

$EI_{req} = 81.508$ kN·m²

V_x between supports from DL $= R_1 - wx$, $R_1 = \frac{w}{2l}(l^2 - a^2)$

V_x between supports from LL $= w(l/2 - x)$

$$V_f = \frac{0.1875}{2 \cdot 3.4}(3.4^2 - 1.4^2) - 0.1875x + \frac{0.855 \cdot 3.4}{2} - 0.855x$$

$= 1.7182 - 1.0425x$

@ $x = 0$, $V = 1.7182$ @ $x = 3.4$, $V = -1.8283$

$\therefore V_f = 1.826$ kN . Note: did not remove member depths from calculation for 1) conservation and bc 2) unknown. Will use if fails

Deflection between supports

Δ between supports from DL $= \frac{wx}{24EIl}(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2)$

$$= \frac{0.135}{24EI}\left[x(l^3 - 2a^2l) + x^3\left(\frac{2a^2}{l} - 2l\right) + x^4\right]$$

$$= \frac{1}{EI}\left[0.16235x - 0.035219x^3 + 0.005625x^4\right]$$

Δ between supports from LL $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

$$\frac{0.56}{24EI}\left[xl^3 - 2lx^3 + x^4\right]$$

$$= \frac{1}{EI}\left[0.93347 - 0.1615x^3 + 0.02375x^4\right]$$

LL-only, $\Delta = 0.00944$ m

$$EI = \frac{1}{0.00944}\left[0.93347x - 0.1615x^3 + 0.02375x^4\right]$$

$$= 98.885x - 17.108x^3 + 2.515x^4$$

$$\frac{dEI}{dx} = 98.885x - 51.324x^2 + 10.06x^3 = 0$$, From cubic solver

$\cancel{x = -1.24}$

$\cancel{x = 4.65}$

$x = 1.70$

$EI_{req'd} = 105.058$ kN.m^2

DL+LL, $\Delta = 0.01889$ m

$$EI = \frac{1}{0.01889}\left[1.09582x - 0.19679x^3 + 0.03x^4\right]$$

$$EI = 58.011x - 10.418x^3 + 1.588x^4$$

$$\frac{dEI}{dx} = 58.011 - 31.254x^2 + 6.352x^3 = 0,$$ From cubic solver,

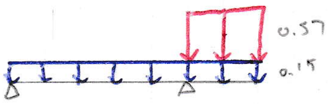
$x_1 = \cancel{-1.22}$

$x_2 = \cancel{4.46}$

$x_3 = 1.678$ m

$EI_{req'd} = 60.71$ kN.m^2

Case ii - Use formulas from reference 30 & 31



Moment max (@ R2) $= \frac{wa^2}{2}$

$= \frac{1.0425 \cdot 1.4^2}{2}$

$M_f = 1.022$ kN.m

Shear max (@ R2) $= wa$

$= 1.0425(1.4)$

$= 1.46$ kN , did not remove bearing depth for conservation

LL only deflection , $\Delta = 0.00778$ m

Δ from LL on overhang $= \frac{wx_1}{24EI}\left[4a^2 l + 6a^2 x_1 - 4ax_1^2 + x_1^3\right]$, biggest at $x_1 = 1.4$

$EI = \frac{0.57 \cdot 1.4}{24 \cdot 0.00778}\left[4 \cdot 1.4^2 \cdot 3.4 + 6 \cdot 1.4^2 \cdot 1.4 - 4 \cdot 1.4^3 + 1.4^3\right]$

$EI_{req} = 149.104$ kN.m

LL+DL deflection , $\Delta = 0.01556$ m

LL component: $\frac{wx}{24EI}\left[4a^2 l + 6a^2 x - 4ax^2 + x^3\right]$

$= \frac{0.57x}{24EI}\left[4 \cdot 1.4^2 \cdot 3.4 + 6 \cdot 1.4^2 \cdot x - 4 \cdot 1.4 \cdot x^2 + x^3\right]$

$= \frac{1}{EI}\left[0.63308x_1 + 0.2793x_1^2 - 0.133x_1^3 + 0.02375x_1^4\right]$

DL component: $\frac{1}{EI}\left[-0.07905x_1 + 0.0735x_1^2 - 0.035x_1^3 + 0.00625x_1^4\right]$ (from before)

$EI = \frac{1}{0.01556}\left[0.55403x_1 + 0.3528x_1^2 - 0.168x_1^3 + 0.03x_1^4\right]$

$= 35.606x_1 + 22.674x_1^2 - 10.797x_1^3 + 1.928x^4$

$\frac{dEI}{dx} = 35.606 + 45.348x_1 - 32.391x_1^2 + 7.712x^3 = 0$, From cubic solver

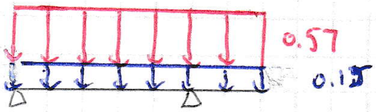
$x = -0.545$

$x_2 = 2.37$

$\therefore$ Max at end (1.4),

$EI_{req'd} = 72.069$ kN.m^2

<u>Case iii</u>



$$M_1 = \frac{w}{8l^2}(l+a)^2(l-a)^2$$

$$= \frac{1.0425}{8 \cdot 3.4^2}(3.4+1.4)^2(3.4-1.4)^2$$

$$= 1.0389 \text{ kNm}$$

$M_2 = \frac{wa^2}{2}$, Same as Case ii

$$V_3 = \frac{w}{2l}(l^2+a^2)$$

$$= \frac{1.0425}{2 \cdot 3.4}(3.4^2+1.4^2)$$

$= 2.073$ kN (did not remove depth from L for conservation)

V_2 = Same as Case ii

$$V_1 = \frac{w}{2l}(l^2-a^2)$$

$$= \frac{1.0425}{2 \cdot 3.4}(3.4^2-1.4^2)$$

$$= 1.472 \text{ kN}$$

Globally, we have :

- $M_f = 1.42$ kN·m
- $V_f = 2.073$ kN
- $EI_{req} = 167.977$ kN·m^2

We know

- $K_T = 1$
- $K_S = 1$
- $K_L = 1$
- $K_D = 1$
- $K_N = 1$
- $K_L = 1$ (Supported by drywall and sheathing) ,∴ Table OK to use

From pg. 43, try 38×184 . We know case 2

∴ $M_r = 5.35 > 1.42$, OK

$V_r = 10.6 > 2.073$, OK

$E_s I = 207 > 167.977$, OK

Can't use 38×140 bc $EI = 104$ kN·m^2

∴ Use 2×"8" joist

b) The beam has areas subject to positive and negative bending. Also, the ratio 184/38 = 5:1, ∴, intermediate support is necessary along the compressive edge. Because there is floor sheathing on top, the beam is supported in positive bending. This is what currently governs the design. However, removing the drywall would remove the support in negative bending, this would introduce lateral-torsional buckling, and could lead to the negative moment governing the design.

② Span = 2m, T_w = 3m, DL = 1.5 kPa, LL = 2.4 kPa, 2x6 stud wall, shallowest 2x SPF #1,2

size / # plies?

$w_D = 1.5(3) = 4.5 \text{ kN/m} \times 1.25 = 5.625 \text{ kN/m}$

$w_L = 2.4(3) = 7.2 \text{ kN/m} \times 1.5 = 10.8 \text{ kN/m}$ $\quad w_f = 10.8 + 5.625 = 16.425 \text{ kN/m}$

$$M_f = \frac{wl^2}{8} = \frac{16.425(2)^2}{8} = 8.2125 \text{ kNm}$$

$$V_f = \frac{wl}{2} = \frac{16.425(2)}{2} = 16.425 \text{ kN}$$

$$\Delta_{max} = \frac{2\,0}{360} = \frac{5wL^4}{384EI} = \frac{5(7.2)(2)^4}{384EI}$$

$EI_{req} = 270 \text{ kN m}^2$

$K_T = 1$
$K_N = 1$
$K_S = 1$
$K_D = 1$

From CL 6.5.3.2.4, K_L doesn't necessarily equal 1. However, because there is likely "lateral" support from purlins, and having multiple plies lowers the d:w ratio, K_L taken as unity.

We know max plies is 3 bc 38x3<140, w/ 140 being depth of 2x6 stud wall

Try 3-ply 2x8

$M_r = 9.02 > 8.2125$ ✓

$V_r = 24.9 > 16.425$ ✓

$EI = 562 > 207$ ✓

∴ 3-ply 2"x8" works. Since 2x6 does not pass, 2"x8" is best.

$M_r = \phi F_b S K_L K_{Zb}$, $F_b = f_b(K_D K_H K_{Sb} K_T)$, $\phi = 0.9$, $K_T = K_L = K_D = K_{Sb} = 1$

K_H	1.1	Table 6.12
f_b	11.8	Table 6.4
K_{Zb}	1.2	Table 6.13

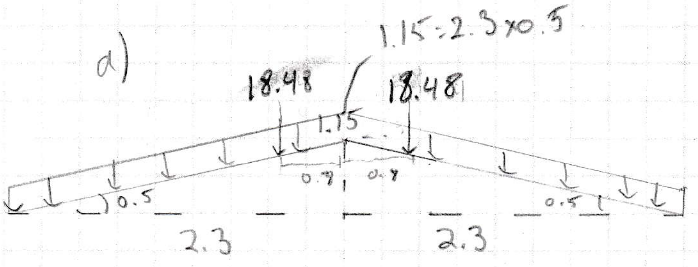
$$S = \frac{bd^2}{6} = \frac{38 \cdot 184^2}{6} = \frac{643264}{3} \times 3 \text{ plies} = 643264 \text{ mm}^3$$

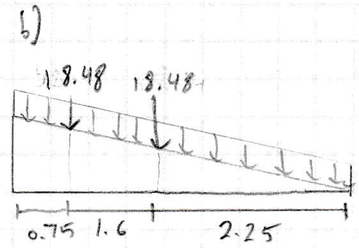
$$M_r = 0.9 \cdot 11.8 \cdot 1.1 \cdot 1.2 \cdot 643264 \times \frac{1}{1000} \times \frac{1}{1000}$$
$$= 9.02 \text{ kN}\cdot\text{m}$$

③ Length = 4.6 m, T_w = 2m, K_0 = 1, 1.7L+1.2D+0.4(0.7) Axle, = 1.98L+1.2D

UDL = 1.5 kPa × 1.2 × 2m
= 3.6 kN/m

LL = 1.98(28) - 1/3
= 18.48 kN





The axle width is the same as the tributary width. Thus, in the scenario that both axle's are within the tributary width, there will be load-sharing between the beams. In comparison, only having the back axle on the beam is a worse case.

Also, since moment maximized in centre, there needs to be a wheel on either side of the centre to maximize its load. Furthermore, since shear is maximized at the support, the axle loads need to be as close as possible - in this case, 0.75 m away.

The calculations below follow influence lines a) and b)

$$M_{max} = 18.48\left[\frac{1.15}{2.3}(2.3-0.8)\right]\times 2 + 3.6\times 4.6\times 1.15\times\frac{1}{2}$$

$$= 13.86 + 9.522$$

$$= 23.382 \text{ kN·m}$$

depths subtracted because significant

$$V_{max} = 18.48\left[\frac{4.6-0.75}{4.6}\right] + 18.48\left[\frac{2.25}{4.6}\right] + 3.6\left[4.6-2(0.0853)\right]\times\frac{1}{2}$$

$$= 15.45 + 9.04 + 15.95$$

$$= 40.44$$

$12 \times 12 = 292 \times 292$

$V_r = \phi F_v \cdot \frac{2 A_n}{3} K_{Zv}$, $M_r = \phi F_b S K_L K_{Zb}$, $F_v = f_v K_D K_H K_S K_T$

$F_b = f_b K_D K_H K_S K_T$

$V\phi = 0.9 = M\phi$

$K_T = 1$ (Large cross section)

$f_b = 18.3$ (Table 6.7)

$f_v = 1.5$

$K_D = 1$

$K_{Zv} = 1.1$

$K_S = 1$

$K_H = 1$

$K_L = 1$ (1:1 ratio)

$S = \frac{bd^2}{6}$

$= \frac{292^3}{6} = 4149514.667 \ mm^3$

$V_r = 0.9 \times 1.5 \times \frac{1kN}{1000N} \times 292^2 \times \frac{2}{3} \times 1.1$

$= 84.41 \ kN$

$M_r = 0.9 \times 18.3 \times \frac{1kN}{1000N} \times \frac{1m}{1000mm} \times \frac{292^3}{6} \times 1.1$

$= 75.18 \ kNm$

Since $M_r > M_f$ beam is sufficient

$V_r > V_f$