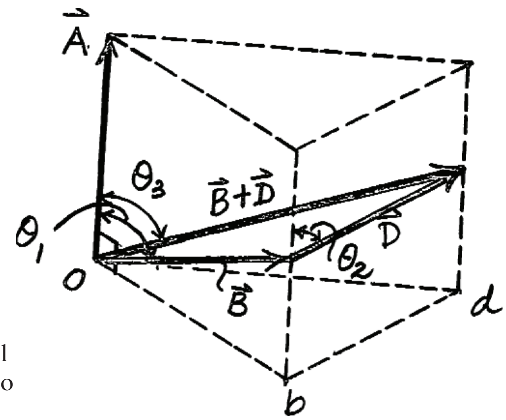
3–1.

Prove the distributive law for the vector cross product, i.e., $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$.

SOLUTION



Consider the three vectors, with **A** vertical.

Note *obd* is perpendicular to **A**.

$$od = |\mathbf{A} \times (\mathbf{B} + \mathbf{D})| = |\mathbf{A}||\mathbf{B} + \mathbf{D}| \sin \theta_3$$

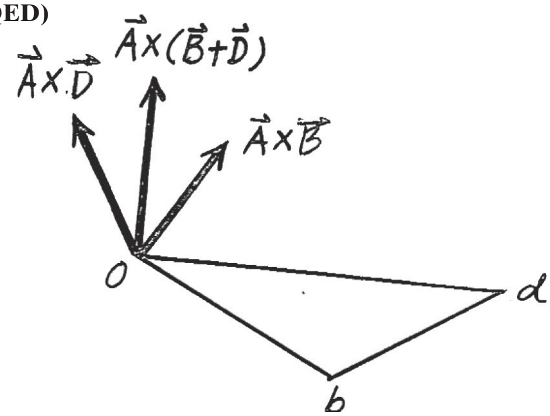
$$ob = |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin \theta_1$$

$$bd = |\mathbf{A} \times \mathbf{D}| = |\mathbf{A}||\mathbf{D}| \sin \theta_2$$

Also, these three cross products all lie in the plane *obd* since they are all perpendicular to **A**. As noted, the magnitude of each cross product is proportional to the length of each side of the triangle.

The three vector cross products also form a closed triangle $o'b'd'$, which is similar to triangle *obd*. Thus from the figure,

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \textbf{(QED)}$$



Note also,

$$\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k}$$

$$\mathbf{B} = B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k}$$

$$\mathbf{D} = D_x\,\mathbf{i} + D_y\,\mathbf{j} + D_z\,\mathbf{k}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix}$$

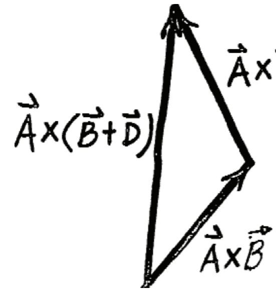
$$= [A_y(B_z + D_z) - A_z(B_y + D_y)]\mathbf{i}$$

$$- [A_x(B_z + D_z) - A_z(B_x + D_x)]\mathbf{j}$$

$$+ [A_x(B_y + D_y) - A_y(B_x + D_x)]\mathbf{k}$$

$$= [(A_y B_z - A_z B_y)\mathbf{i} - (A_x B_z - A_z B_x)]\mathbf{j} + (A_x B_y - A_y B_x)\mathbf{k}$$

$$+ [(A_y D_z - A_z D_y)\mathbf{i} - (A_x D_z - A_z D_x)\mathbf{j} + (A_x D_y - A_y D_x)\mathbf{k}$$



$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix}$$

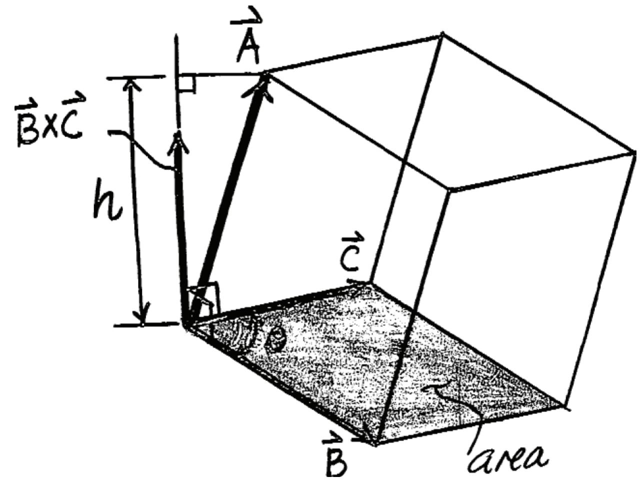
$$= (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \textbf{(QED)}$$

Ans:
N/A

3–2.

Prove the triple scalar product identity $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$.

SOLUTION



As shown in the figure,

$$\text{Area} = B(C \sin \theta) = |\mathbf{B} \times \mathbf{C}|$$

Thus,

$$\text{Volume of parallelepiped is } |\mathbf{B} \times \mathbf{C}||h|$$

But

$$|h| = |\mathbf{A} \cdot \mathbf{u}_{(\mathbf{B}\times\mathbf{C})}| = \left|\mathbf{A} \cdot \left(\frac{\mathbf{B} \times \mathbf{C}}{|\mathbf{B} \times \mathbf{C}|}\right)\right|$$

Thus,

$$\text{Volume} = |\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})|$$

Since $|(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}|$ represents this same volume then

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} \qquad \textbf{(QED)}$$

Also,

$$LHS = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$$

$$= (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k}) \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$= A_x\,(B_yC_z - B_zC_y) - A_y\,(B_xC_z - B_zC_x) + A_z\,(B_xC_y - B_yC_x)$$

$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

$$RHS = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \cdot (C_x\,\mathbf{i} + C_y\,\mathbf{j} + C_z\,\mathbf{k})$$

$$= C_x(A_y\,B_z - A_zB_y) - C_y(A_xB_z - A_zB_x) + C_z(A_xB_y - A_yB_x)$$

$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

Thus, $LHS = RHS$

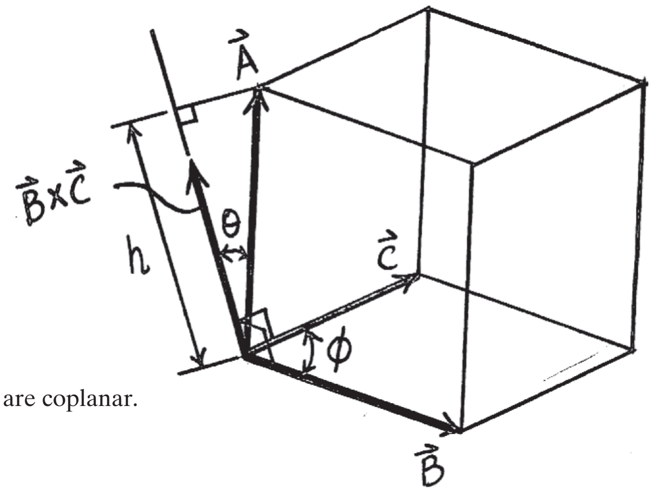
$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} \qquad \textbf{(QED)}$$

Ans:
N/A

3–3.

Given the three nonzero vectors **A**, **B**, and **C**, show that if $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, the three vectors *must* lie in the same plane.

SOLUTION



Consider

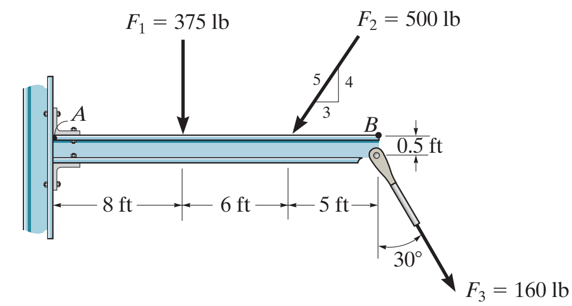
$$
\begin{aligned}
|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})| &= |\mathbf{A}|\,|\mathbf{B} \times \mathbf{C}\,| \cos\theta \\
&= (|\mathbf{A}| \cos\theta)|\mathbf{B} \times \mathbf{C}| \\
&= |h|\,|\mathbf{B} \times \mathbf{C}| \\
&= BC\,|h| \sin\phi \\
&= \text{volume of parallelepiped}
\end{aligned}
$$

If $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, then the volume equals zero, so that **A, B,** and **C** are coplanar.

Ans:
N/A

***3–4.**

Determine the moment about point A of each of the three forces acting on the beam.



SOLUTION

$$\zeta + (M_{F_1})_A = -375(8)$$

$$= -3000 \text{ lb}\cdot\text{ft} = 3.00 \text{ kip}\cdot\text{ft} \quad \textit{(Clockwise)}$$ **Ans.**

$$\zeta + (M_{F_2})_A = -500\left(\frac{4}{5}\right)(14)$$

$$= -5600 \text{ lb}\cdot\text{ft} = 5.60 \text{ kip}\cdot\text{ft} \quad \textit{(Clockwise)}$$ **Ans.**

$$\zeta + (M_{F_3})_A = -160(\cos 30^\circ)(19) + 160 \sin 30^\circ(0.5)$$

$$= -2593 \text{ lb}\cdot\text{ft} = 2.59 \text{ kip}\cdot\text{ft} \quad \textit{(Clockwise)}$$ **Ans.**

Ans:

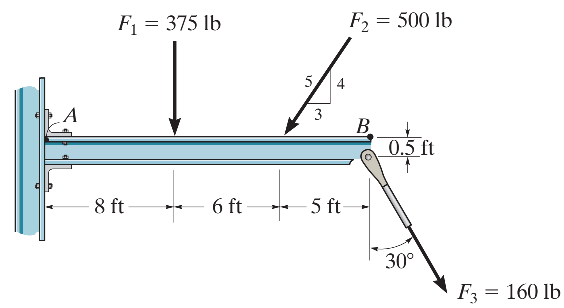
$(M_{F_1})_A = 3.00$ kip·ft ***(Clockwise)***

$(M_{F_2})_A = 5.60$ kip·ft ***(Clockwise)***

$(M_{F_3})_A = 2.59$ kip·ft ***(Clockwise)***

3–5.

Determine the moment about point B of each of the three forces acting on the beam.



SOLUTION

$$\circlearrowleft + (M_{F_1})_B = 375(11)$$

$$= 4125 \text{ lb} \cdot \text{ft} = 4.125 \text{ kip} \cdot \text{ft} \quad \textit{\textbf{(Counterclockwise)}}$$ **Ans.**

$$\circlearrowleft + (M_{F_2})_B = 500\left(\frac{4}{5}\right)(5)$$

$$= 2000 \text{ lb} \cdot \text{ft} = 2.00 \text{ kip} \cdot \text{ft} \quad \textit{\textbf{(Counterclockwise)}}$$ **Ans.**

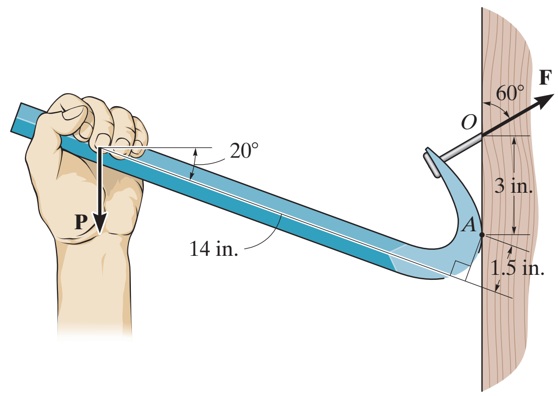
$$\circlearrowleft + (M_{F_3})_B = 160 \sin 30°(0.5) - 160 \cos 30°(0)$$

$$= 40.0 \text{ lb} \cdot \text{ft} \quad \textit{\textbf{(Counterclockwise)}}$$ **Ans.**

Ans:
$(M_{F_1})_B = 4.125 \text{ kip} \cdot \text{ft} \circlearrowleft$
$(M_{F_2})_B = 2.00 \text{ kip} \cdot \text{ft} \circlearrowleft$
$(M_{F_3})_B = 40.0 \text{ lb} \cdot \text{ft} \circlearrowleft$

3–6.

The crowbar is subjected to a vertical force of $P = 25$ lb at the grip, whereas it takes a force of $F = 155$ lb at the claw to pull the nail out. Find the moment of each force about point A and determine if **P** is sufficient to pull out the nail.



SOLUTION

$\circlearrowleft + M_P = 25(14 \cos 20° + 1.5 \sin 20°) = 341 \text{ in.} \cdot \text{lb}$ ***(Counterclockwise)***

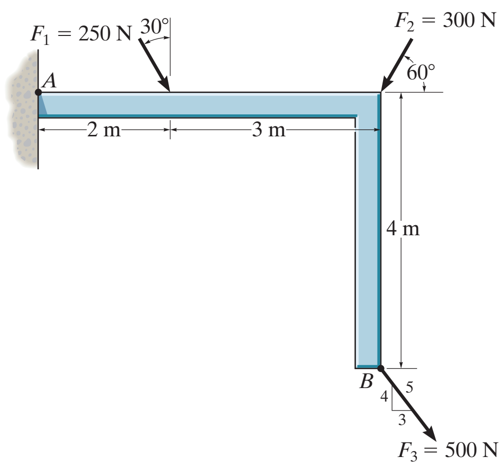
$\circlearrowright + M_F = 155 \sin 60°(3) = 403 \text{ in.} \cdot \text{lb}$ ***(Clockwise)***

Since $M_F > M_P$, $P = 25$ lb is **not sufficient** to pull out the nail. **Ans.**

Ans:
$M_P = 341 \text{ in.} \cdot \text{lb} \circlearrowleft$
$M_F = 403 \text{ in.} \cdot \text{lb} \circlearrowright$
Not sufficient

3–7.

Determine the moment of each of the three forces about point A.



SOLUTION

The moment arm measured perpendicular to each force from point A is

$$d_1 = 2 \sin 60° = 1.732 \text{ m}$$

$$d_2 = 5 \sin 60° = 4.330 \text{ m}$$

$$d_3 = 2 \sin 53.13° = 1.60 \text{ m}$$

Using each force where $M_A = Fd$, we have

$\circlearrowleft + (M_{F_1})_A = -250(1.732)$

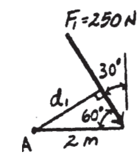
$= -433 \text{ N} \cdot \text{m} = 433 \text{ N} \cdot \text{m}$ ***(Clockwise)*** **Ans.**

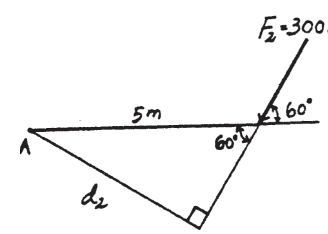
$\circlearrowleft + (M_{F_2})_A = -300(4.330)$

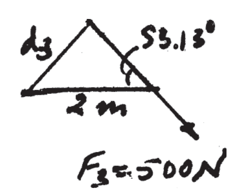
$= -1299 \text{ N} \cdot \text{m} = 1.30 \text{ kN} \cdot \text{m}$ ***(Clockwise)*** **Ans.**

$\circlearrowleft + (M_{F_3})_A = -500(1.60)$

$= -800 \text{ N} \cdot \text{m} = 800 \text{ N} \cdot \text{m}$ ***(Clockwise)*** **Ans.**



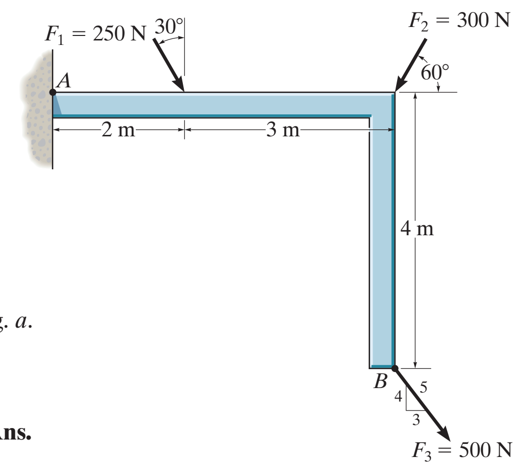




Ans:
$(M_{F_1})_A = 433 \text{ N} \cdot \text{m} \circlearrowright$
$(M_{F_2})_A = 1.30 \text{ kN} \cdot \text{m} \circlearrowright$
$(M_{F_3})_A = 800 \text{ N} \cdot \text{m} \circlearrowright$

***3–8.**

Determine the moment of each of the three forces about point B.



SOLUTION

The forces are resolved into horizontal and vertical component as shown in Fig. *a*. For $\mathbf{F}_1$,

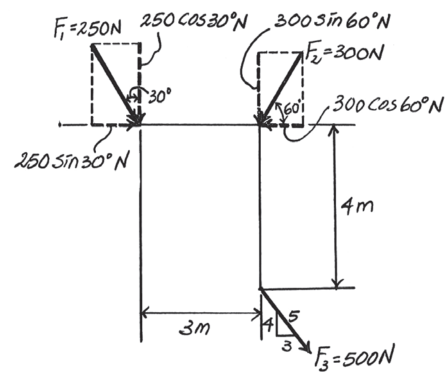
$$\circlearrowleft + M_B = 250 \cos 30°(3) - 250 \sin 30°(4)$$
$$= 149.51 \text{ N}\cdot\text{m} = 150 \text{ N}\cdot\text{m} \circlearrowright \qquad \textbf{Ans.}$$

For $\mathbf{F}_2$,

$$\circlearrowleft + M_B = 300 \sin 60°(0) + 300 \cos 60°(4)$$
$$= 600 \text{ N}\cdot\text{m} \circlearrowright \qquad \textbf{Ans.}$$

Since the line of action of $\mathbf{F}_3$ passes through B, its moment arm about point B is zero. Thus

$$M_B = 0 \qquad \textbf{Ans.}$$

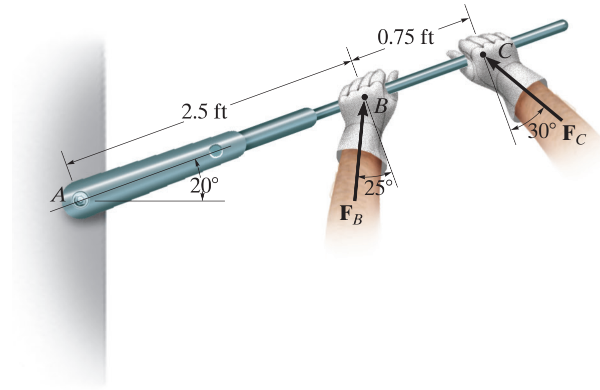


(a)

Ans:
$M_B = 150 \text{ N}\cdot\text{m} \circlearrowright$
$M_B = 600 \text{ N}\cdot\text{m} \circlearrowright$
$M_B = 0$

3–9.

Determine the moment of each force about the bolt at A. Take $F_B = 40$ lb, $F_C = 50$ lb.



SOLUTION

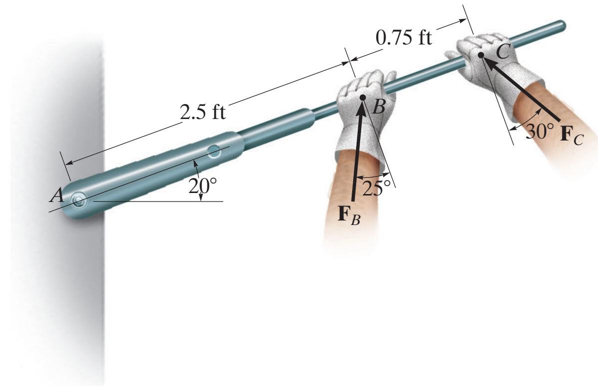
$$\circlearrowleft + M_B = 40 \cos 25°(2.5) = 90.6 \text{ lb} \cdot \text{ft} \circlearrowleft \qquad \textbf{Ans.}$$

$$\circlearrowleft + M_C = 50 \cos 30°(3.25) = 141 \text{ lb} \cdot \text{ft} \circlearrowleft \qquad \textbf{Ans.}$$

Ans:
$M_B = 90.6 \text{ lb} \cdot \text{ft} \circlearrowleft$
$M_C = 141 \text{ lb} \cdot \text{ft} \circlearrowleft$

3–10.

If $F_B = 30$ lb and $F_C = 45$ lb, determine the resultant moment about the bolt at A.



SOLUTION

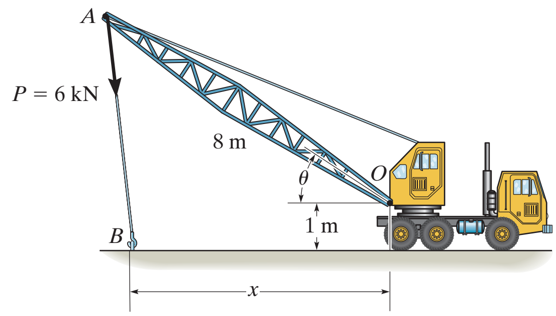
$$\zeta + M_A = 30 \cos 25°(2.5) + 45 \cos 30°(3.25)$$

$$= 195 \text{ lb} \cdot \text{ft} \circlearrowleft$$

Ans:
$M_A = 195 \text{ lb} \cdot \text{ft} \circlearrowleft$

3–11.

The cable exerts a force of $P = 6$ kN at the end of the 8-m-long crane boom. If $\theta = 30°$, determine the placement x of the hook at B so that this force creates a maximum moment about point O. What is this moment?



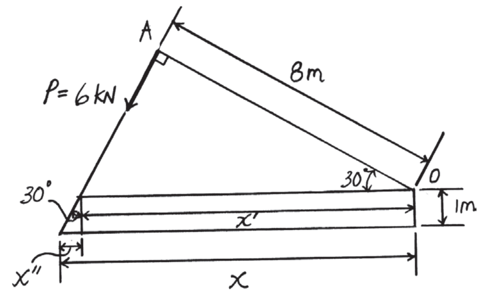
SOLUTION

In order to produce the maximum moment about point O, **P** must act perpendicular to the boom's axis OA as shown in Fig. a. Thus,

$$\circlearrowleft + (M_O)_{max} = 6\,(8) = 48.0 \text{ kN} \cdot \text{m (counterclockwise)} \qquad \textbf{Ans.}$$

Referring to the geometry of Fig. a,

$$x = x' + x'' = \frac{8}{\cos 30°} + \tan 30° = 9.814 \text{ m} = 9.81 \text{ m} \qquad \textbf{Ans.}$$

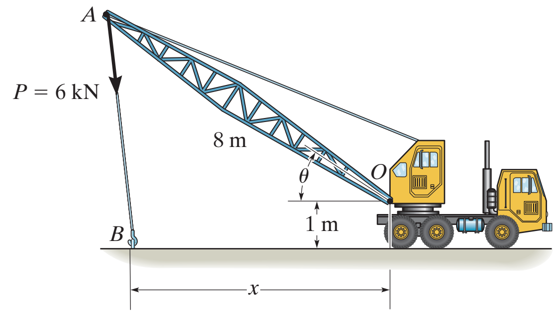


(a)

Ans:
$(M_O)_{max} = 48.0 \text{ kN} \cdot \text{m} \circlearrowleft$
$x = 9.81$ m

***3–12.**

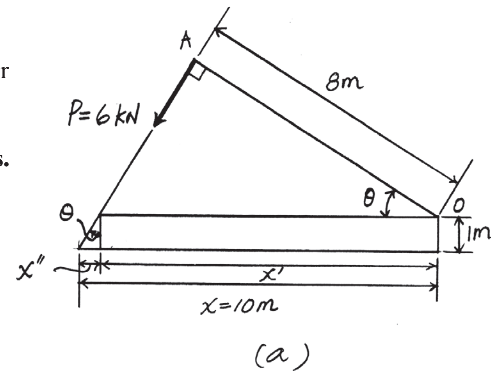
The cable exerts a force of $P = 6\text{ kN}$ at the end of the 8-m-long crane boom. If $x = 10\text{ m}$, determine the angle θ of the boom so that this force creates a maximum moment about point O. What is this moment?



SOLUTION

In order to produce the maximum moment about point O, **P** must act perpendicular to the boom's axis OA as shown in Fig. a. Thus,

$$\circlearrowleft + (M_O)_{max} = 6\,(8) = 48.0\text{ kN}\cdot\text{m (counterclockwise)} \qquad \textbf{Ans.}$$



(a)

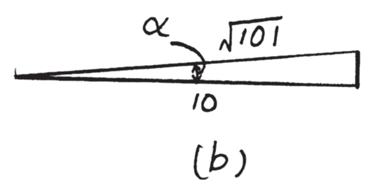
Referring to the geometry of Fig. a,

$$x = x' + x''; \qquad 10 = \frac{8}{\cos\theta} + \tan\theta$$

$$10 = \frac{8}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$10\cos\theta - \sin\theta = 8$$

$$\frac{10}{\sqrt{101}}\cos\theta - \frac{1}{\sqrt{101}}\sin\theta = \frac{8}{\sqrt{101}} \qquad \textbf{(1)}$$



(b)

From the geometry shown in Fig. b,

$$\alpha = \tan^{-1}\left(\frac{1}{10}\right) = 5.711°$$

$$\sin\alpha = \frac{1}{\sqrt{101}} \qquad \cos\alpha = \frac{10}{\sqrt{101}}$$

Then Eq (1) becomes

$$\cos\theta\cos 5.711° - \sin\theta\sin 5.711° = \frac{8}{\sqrt{101}}$$

Referring that $\cos(\theta + 5.711°) = \cos\theta\cos 5.711° - \sin\theta\sin 5.711°$

$$\cos(\theta + 5.711°) = \frac{8}{\sqrt{101}}$$

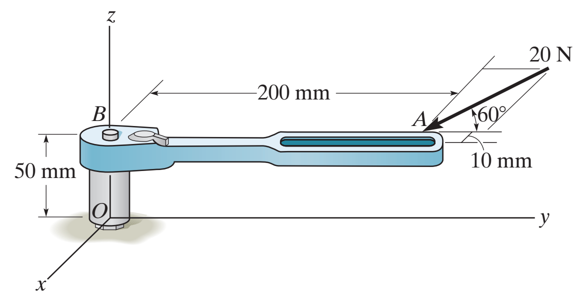
$$\theta + 5.711° = 37.247°$$

$$\theta = 31.54° = 31.5° \qquad \textbf{Ans.}$$

Ans:
$(M_O)_{max} = 48.0\text{ kN}\cdot\text{m (counterclockwise)}$
$\theta = 31.5°$

3–13.

The 20-N horizontal force acts on the handle of the socket wrench. What is the moment of this force about point B. Specify the coordinate direction angles α, β, γ of the moment axis.

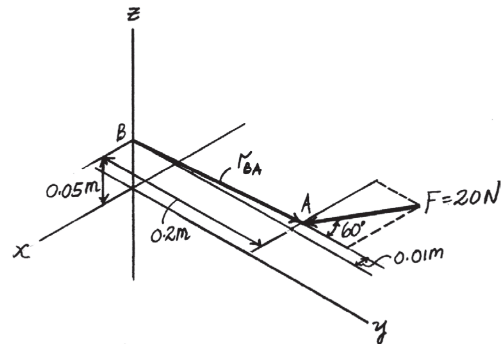


SOLUTION

Force Vector And Position Vector. Referring to Fig. *a*,

$$F = 20\,(\sin 60°\mathbf{i} - \cos 60°\mathbf{j}) = \{17.32\mathbf{i} - 10\mathbf{j}\}\text{ N}$$

$$\mathbf{r}_{BA} = \{-0.01\mathbf{i} + 0.2\mathbf{j}\}\text{ m}$$



(a)

Moment of Force *F* about point *B*.

$$\mathbf{M}_B = \mathbf{r}_{BA} \times F$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.01 & 0.2 & 0 \\ 17.32 & -10 & 0 \end{vmatrix}$$

$$= \{-3.3641\,\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{-3.36\,\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Here the unit vector for $\mathbf{M}_B$ is $\mathbf{u} = -\mathbf{k}$. Thus, the coordinate direction angles of $\mathbf{M}_B$ are

$$\alpha = \cos^{-1} 0 = 90° \qquad \textbf{Ans.}$$

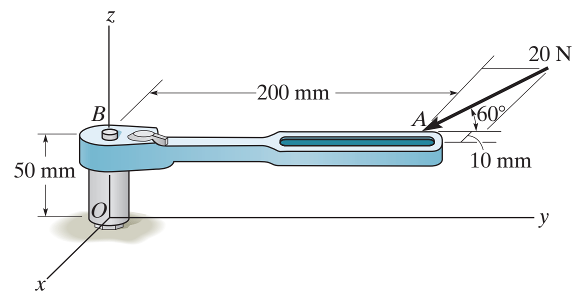
$$\beta = \cos^{-1} 0 = 90° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}(-1) = 180° \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_B = \{-3.36\,\mathbf{k}\}\text{ N}\cdot\text{m}$
$\alpha = 90°$
$\beta = 90°$
$\gamma = 180°$

3–14.

The 20-N horizontal force acts on the handle of the socket wrench. Determine the moment of this force about point O. Specify the coordinate direction angles α, β, γ of the moment axis.

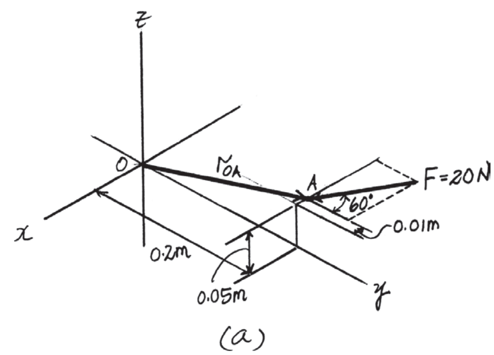


SOLUTION

Force Vector And Position Vector. Referring to Fig. *a*,

$$\mathbf{F} = 20\,(\sin 60°\mathbf{i} - \cos 60°\mathbf{j}) = \{17.32\mathbf{i} - 10\mathbf{j}\}\text{ N}$$

$$\mathbf{r}_{OA} = \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}\text{ m}$$



Moment of *F* About point *O*.

$$\mathbf{M}_O = \mathbf{r}_{OA} \times F$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.01 & 0.2 & 0.05 \\ 17.32 & -10 & 0 \end{vmatrix}$$

$$= \{0.5\mathbf{i} + 0.8660\mathbf{j} - 3.3641\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{0.5\mathbf{i} + 0.866\mathbf{j} - 3.36\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The magnitude of $\mathbf{M}_O$ is

$$M_O = \sqrt{(M_O)_x^2 + (M_O)_y^2 + (M_O)_z^2} = \sqrt{0.5^2 + 0.8660^2 + (-3.3641)^2}$$

$$= 3.5096\text{ N}\cdot\text{m}$$

Thus, the coordinate direction angles of $\mathbf{M}_O$ are

$$\alpha = \cos^{-1}\left[\frac{(M_O)_x}{M_O}\right] = \cos^{-1}\left(\frac{0.5}{3.5096}\right) = 81.81° = 81.8° \qquad \textbf{Ans.}$$

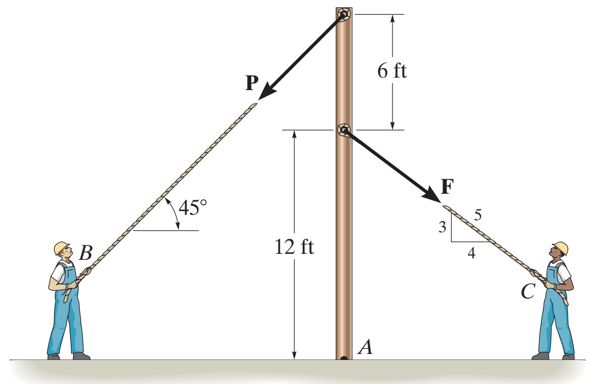
$$\beta = \cos^{-1}\left[\frac{(M_O)_y}{M_O}\right] = \cos^{-1}\left(\frac{0.8660}{3.5096}\right) = 75.71° = 75.7° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(M_O)_z}{M_O}\right] = \cos^{-1}\left(\frac{-3.3641}{3.5096}\right) = 163.45° = 163° \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_O = \{0.5\mathbf{i} + 0.866\mathbf{j} - 3.36\mathbf{k}\}\text{ N}\cdot\text{m}$
$\alpha = 81.8°$
$\beta = 75.7°$
$\gamma = 163°$

3–15.

Two men exert forces of $F = 80$ lb and $P = 50$ lb on the ropes. Determine the moment of each force about A. Which way will the pole rotate, clockwise or counterclockwise?



SOLUTION

$$\curvearrowright + (M_A)_C = 80\left(\frac{4}{5}\right)(12) = 768 \text{ lb} \cdot \text{ft} \curvearrowright$$ **Ans.**

$$\curvearrowleft + (M_A)_B = 50\,(\cos 45°)(18) = 636 \text{ lb} \cdot \text{ft} \curvearrowleft$$ **Ans.**

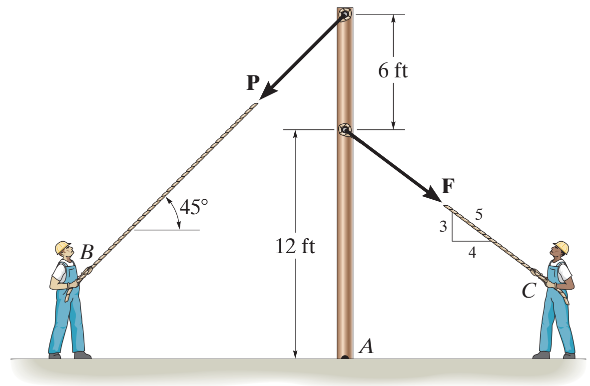
Since $(M_A)_C > (M_A)_B$

Clockwise **Ans.**

Ans:
$(M_A)_C = 768 \text{ lb} \cdot \text{ft} \curvearrowright$
$(M_A)_B = 636 \text{ lb} \cdot \text{ft} \curvearrowleft$
Clockwise

***3–16.**

If the man at B exerts a force of $P = 30$ lb on the rope, determine the magnitude of the force **F** the man at C must exert to prevent the pole from rotating, i.e., so the resultant moment about A of both forces is zero.



SOLUTION

$$\circlearrowleft + \qquad 30\,(\cos 45°)(18) = F\left(\frac{4}{5}\right)(12) = 0$$

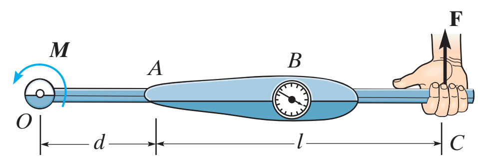
$$F = 39.8 \text{ lb} \qquad \textbf{Ans.}$$

Ans:

$F = 39.8$ lb

3–17.

The torque wrench ABC is used to measure the moment or torque applied to a bolt when the bolt is located at A and a force is applied to the handle at C. The mechanic reads the torque on the scale at B. If an extension AO of length d is used on the wrench, determine the required scale reading if the desired torque on the bolt at O is to be M.



SOLUTION

Moment at $A = m = Fl$

Moment at $O = M = (d + l)F$

$$M = (d + l)\frac{m}{l}$$

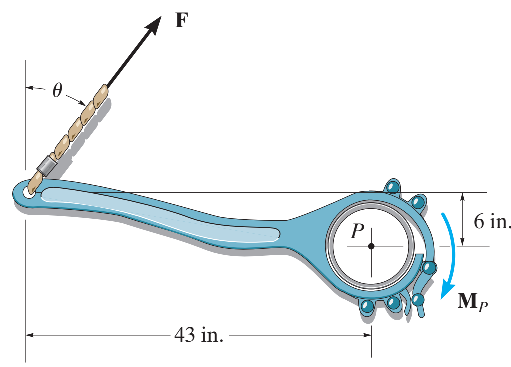
$$m = \left(\frac{l}{d + l}\right)M \qquad \textbf{Ans.}$$

Ans:

$$m = \left(\frac{l}{d + l}\right)M$$

3–18.

The tongs are used to grip the ends of the drilling pipe. Determine the torque (moment) M_P that the applied force $F = 150$ lb exerts on the pipe about point P as a function of θ. Plot this moment M_P versus θ for $0° \leq \theta \leq 90°$.



SOLUTION

$$M_P = 150 \cos \theta(43) + 150 \sin \theta(6)$$

$$= (6450 \cos \theta + 900 \sin \theta) \text{ lb} \cdot \text{ in.}$$

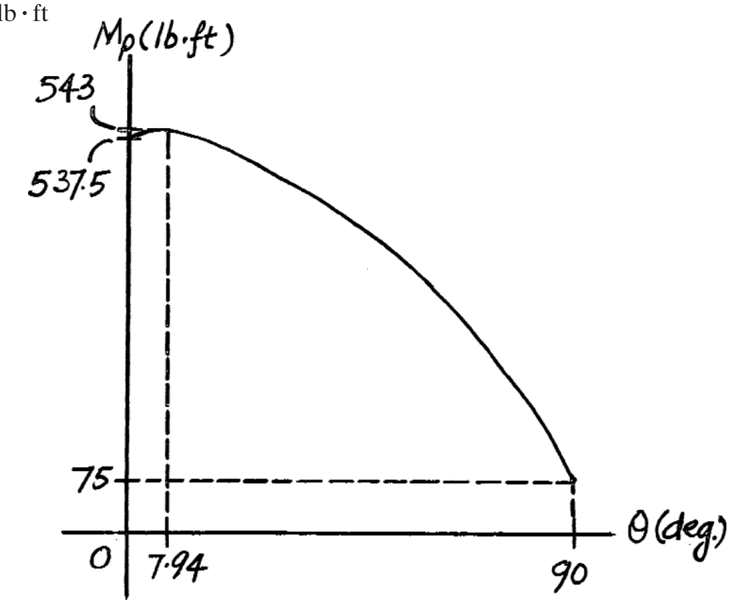
$$= (537.5 \cos \theta + 75 \sin \theta) \text{ lb} \cdot \text{ft}$$ **Ans.**

$$\frac{dM_P}{d\theta} = -537.5 \sin \theta + 75 \cos \theta = 0 \qquad \tan \theta = \frac{75}{537.5} \qquad \theta = 7.943°$$

At $\theta = 7.943°$, M_P is maximum.

$$(M_P)_{max} = 538 \cos 7.943° + 75 \sin 7.943° = 543 \text{ lb} \cdot \text{ft}$$

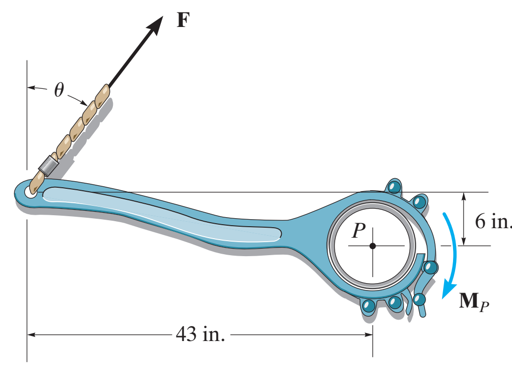
$$\text{Also } (M_P)_{max} = 150 \text{ lb}\left(\left(\frac{43}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right)^{\frac{1}{2}} = 543 \text{ lb} \cdot \text{ft}$$



Ans:
$M_P = (537.5 \cos \theta + 75 \sin \theta) \text{ lb} \cdot \text{ft}$

3–19.

The tongs are used to grip the ends of the drilling pipe. If a torque (moment) of $M_P = 800 \text{ lb} \cdot \text{ft}$ is needed at P to turn the pipe, determine the cable force F that must be applied to the tongs. Set $\theta = 30^\circ$.



SOLUTION

$$M_P = F\cos 30^\circ(43) + F\sin 30^\circ(6) \qquad \text{Set } M_P = 800(12)\ \text{lb}\cdot\text{in.}$$

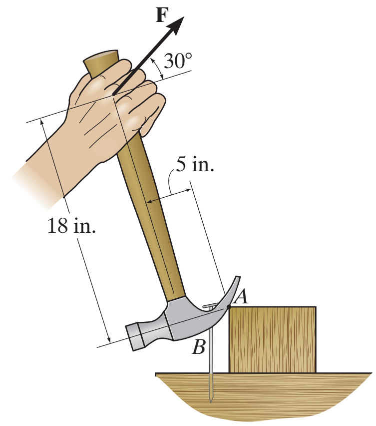
$$800(12) = F\cos 30^\circ(43) + F\sin 30^\circ(6)$$

$$F = 239\ \text{lb}$$ **Ans.**

Ans:
$F = 239$ lb

***3–20.**

The handle of the hammer is subjected to the force of $F = 20$ lb. Determine the moment of this force about the point A.

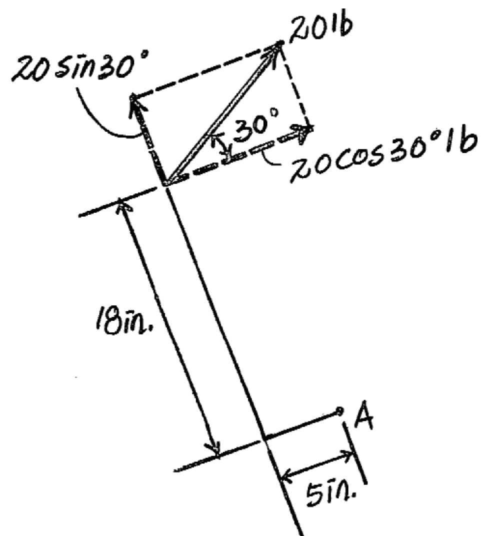


SOLUTION

Resolving the 20-lb force into components parallel and perpendicular to the hammer, Fig. a, and applying the principle of moments,

$$\zeta + M_A = -20\cos 30°(18) - 20\sin 30°(5)$$

$$= -361.77\ \text{lb}\cdot\text{in} = 362\ \text{lb}\cdot\text{in.}\ \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$

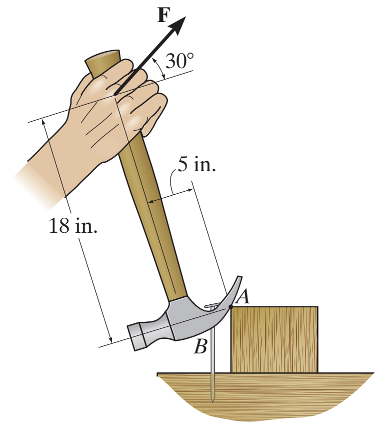


(a)

Ans:
$M_A = 362$ lb · in. **(*Clockwise*)**

3–21.

In order to pull out the nail at B, the force $\mathbf{F}$ exerted on the handle of the hammer must produce a clockwise moment of 500 lb · in. about point A. Determine the required magnitude of force $\mathbf{F}$.

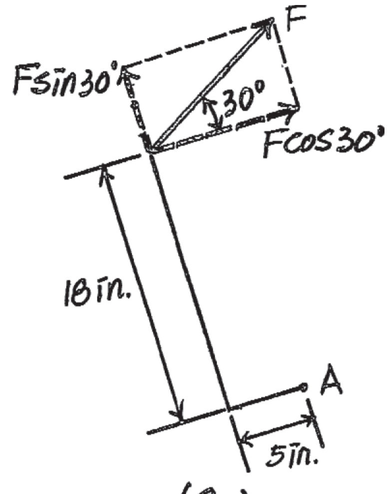


SOLUTION

Resolving force $\mathbf{F}$ into components parallel and perpendicular to the hammer, Fig. a, and applying the principle of moments,

$$\circlearrowleft +M_A = -500 = -F \cos 30°(18) - F \sin 30°(5)$$

$$F = 27.6 \text{ lb} \qquad \textbf{Ans.}$$

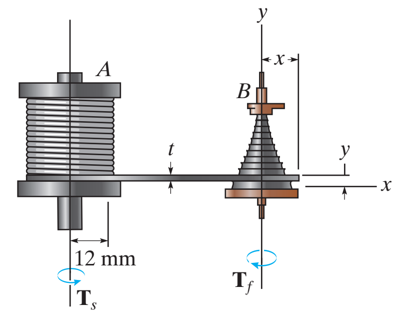


(a)

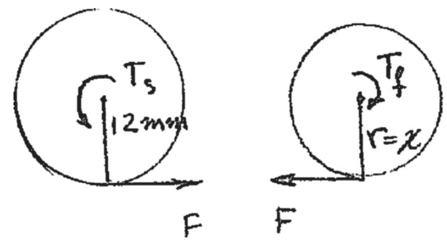
Ans:
$F = 27.6$ lb

3–22.

Old clocks were constructed using a *fusee* B to drive the gears and watch hands. The purpose of the fusee is to increase the leverage developed by the mainspring A as it uncoils and thereby loses some of its tension. The mainspring can develop a torque (moment) $T_s = k\theta$, where $k = 0.015\ \text{N}\cdot\text{m/rad}$ is the torsional stiffness and θ is the angle of twist of the spring in radians. If the torque T_f developed by the fusee is to remain constant as the mainspring winds down, and $x = 10$ mm when $\theta = 4$ rad, determine the required radius of the fusee when $\theta = 3$ rad.



SOLUTION



When $\theta = 4$ rad, $r = 10$ mm

$T_s = 0.015(4) = 0.06\ \text{N}\cdot\text{m}$

$F = \dfrac{0.06}{0.012} = 5\ \text{N}$

$T_f = 5(0.010) = 0.05\ \text{N}\cdot\text{m}$ (constant)

When $\theta = 3$ rad,

$T_s = 0.015(3) = 0.045\ \text{N}\cdot\text{m}$

$F = \dfrac{0.045}{0.012} = 3.75\ \text{N}$

For the fusee require

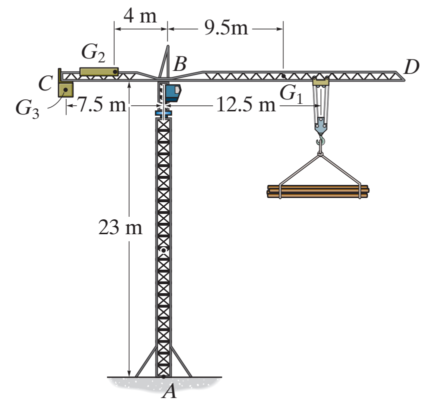
$0.05 = 3.75\, r$

$r = 0.0133\ \text{m} = 13.3\ \text{mm}$ **Ans.**

Ans:
$r = 13.3$ mm

3–23.

The tower crane is used to hoist the 2-Mg load upward at constant velocity. The 1.5-Mg jib BD, 0.5-Mg jib BC, and 6-Mg counterweight C have centers of mass at G_1, G_2, and G_3, respectively. Determine the resultant moment produced by the load and the weights of the tower crane jibs about point A and about point B.



SOLUTION

Since the moment arms of the weights and the load measured to points A and B are the same, the resultant moments produced by the load and the weight about points A and B are the same.

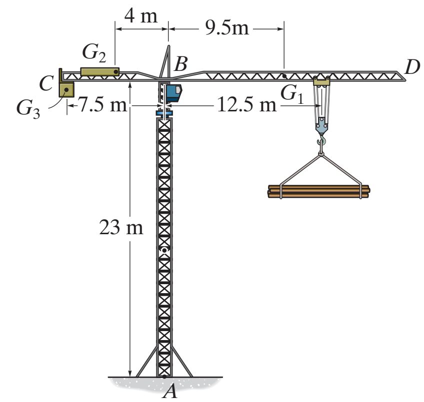
$\zeta + (M_R)_A = (M_R)_B = \Sigma Fd;$ $\quad (M_R)_A = (M_R)_B = 6000(9.81)(7.5) + 500(9.81)(4) - 1500(9.81)(9.5)$
$-2000(9.81)(12.5) = 76\,027.5 \text{ N}\cdot\text{m} = 76.0 \text{ kN}\cdot\text{m}$ ***(Counterclockwise)***

Ans.

Ans:
$(M_R)_A = (M_R)_B = 76.0 \text{ kN}\cdot\text{m}$ ↺

***3–24.**

The tower crane is used to hoist a 2-Mg load upward at constant velocity. The 1.5-Mg jib BD and 0.5-Mg jib BC have centers of mass at G_1 and G_2, respectively. Determine the required mass of the counterweight C so that the resultant moment produced by the load and the weight of the tower crane jibs about point A is zero. The center of mass for the counterweight is located at G_3.



SOLUTION

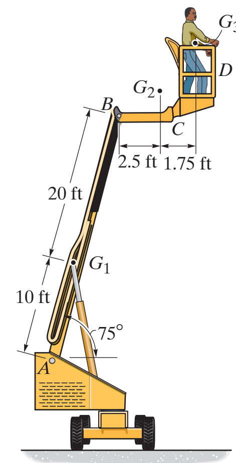
$\circlearrowleft + (M_R)_A = \Sigma Fd;\qquad 0 = M_C(9.81)(7.5) + 500(9.81)(4) - 1500(9.81)(9.5) - 2000(9.81)(12.5)$

$M_C = 4966.67\text{ kg} = 4.97\text{ Mg}$ **Ans.**

Ans:
$M_C = 4.97\text{ Mg}$

3–25.

If the 1500-lb boom AB, the 200-lb cage BCD, and the 175-lb man have centers of gravity located at points G_1, G_2, and G_3, respectively, determine the resultant moment produced by each weight about point A.



SOLUTION

Moment of the weight of boom AB about point A:

$$\circlearrowleft + (M_{AB})_A = -1500(10\cos 75°) = -3882.29\ \text{lb}\cdot\text{ft}$$

$$= 3.88\ \text{kip}\cdot\text{ft}\ \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$

Moment of the weight of cage BCD about point A:

$$\circlearrowleft + (M_{BCD})_A = -200(30\cos 75° + 2.5) = -2052.91\ \text{lb}\cdot\text{ft}$$

$$= 2.05\ \text{kip}\cdot\text{ft}\ \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$

Moment of the weight of the man about point A:

$$\circlearrowleft + (M_{\text{man}})_A = -175(30\cos 75° + 4.25) = -2102.55\ \text{lb}\cdot\text{ft}$$

$$= 2.10\ \text{kip}\cdot\text{ft}\ \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$

Ans:

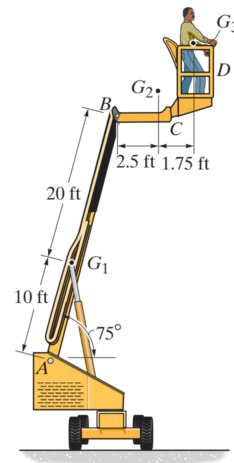
$(M_{AB})_A = 3.88\ \text{kip}\cdot\text{ft} \circlearrowright$

$(M_{BCD})_A = 2.05\ \text{kip}\cdot\text{ft} \circlearrowright$

$(M_{\text{man}})_A = 2.10\ \text{kip}\cdot\text{ft} \circlearrowright$

3–26.

If the 1500-lb boom AB, the 200-lb cage BCD, and the 175-lb man have centers of gravity located at points G_1, G_2, and G_3, respectively, determine the resultant moment produced by all the weights about point A.

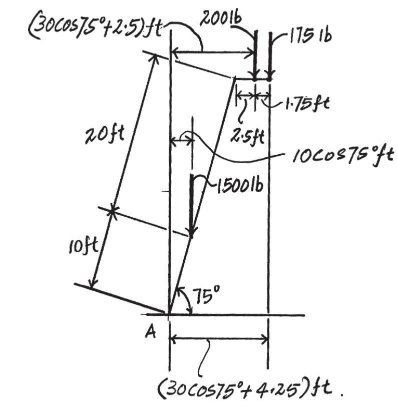


SOLUTION

Referring to Fig. *a*, the resultant moment of the weight about point A is given by

$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad (M_R)_A = -1500(10\cos 75^\circ) - 200(30\cos 75^\circ + 2.5) - 175(30\cos 75^\circ + 4.25)$$

$$= -8037.75\ \text{lb}\cdot\text{ft} = 8.04\ \text{kip}\cdot\text{ft}\ \textbf{(Clockwise)} \qquad \textbf{Ans.}$$

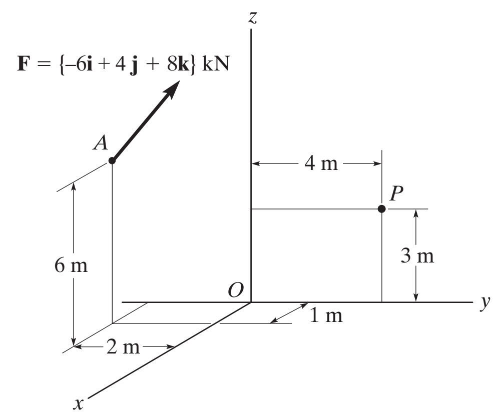


(a)

Ans:
$(M_R)_A = 8.04\ \text{kip}\cdot\text{ft} \circlearrowright$

3–27.

Determine the moment of the force **F** about point O. Express the result as a Cartesian vector.



SOLUTION

Position Vector. The coordinates of point A are $(1, -2, 6)$ m.

Thus,

$$\mathbf{r}_{OA} = \{\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}\} \text{ m}$$

The moment of *F* About Point *O*.

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$

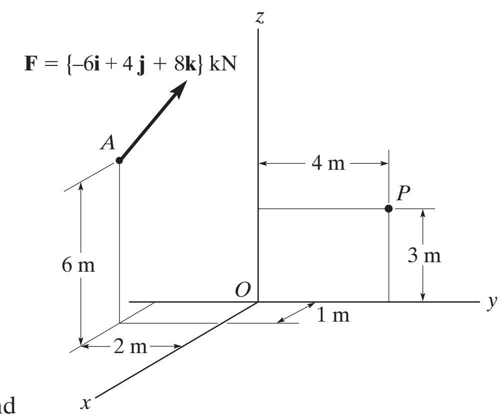
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 6 \\ -6 & 4 & 8 \end{vmatrix}$$

$$= \{-40\mathbf{i} - 44\mathbf{j} - 8\mathbf{k}\} \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_O = \{-40\mathbf{i} - 44\mathbf{j} - 8\mathbf{k}\} \text{ kN} \cdot \text{m}$

***3–28.**

Determine the moment of the force **F** about point P. Express the result as a Cartesian vector.



SOLUTION

Position Vector. The coordinates of points A and P are $A\,(1, -2, 6)$ m and $P\,(0, 4, 3)$ m, respectively. Thus,

$$\mathbf{r}_{PA} = (1 - 0)\mathbf{i} + (-2 - 4)\mathbf{j} + (6 - 3)\mathbf{k}$$

$$= \{\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\ \text{m}$$

The moment of *F* About Point *P*.

$$M_P = \mathbf{r}_{PA} \times \mathbf{F}$$

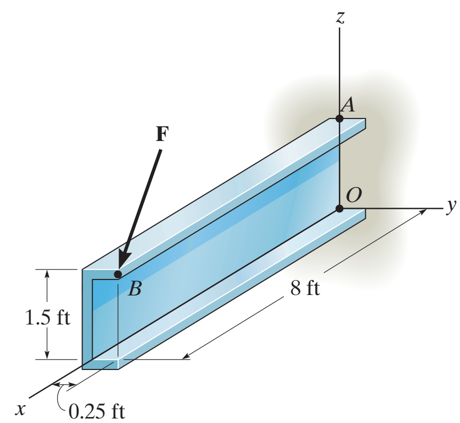
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -6 & 3 \\ -6 & 4 & 8 \end{vmatrix}$$

$$= \{-60\mathbf{i} - 26\mathbf{j} - 32\mathbf{k}\}\ \text{kN} \cdot \text{m} \qquad \textbf{Ans.}$$

Ans:
$M_P = \{-60\mathbf{i} - 26\mathbf{j} - 32\mathbf{k}\}\ \text{kN} \cdot \text{m}$

3–29.

The force $\mathbf{F} = \{400\mathbf{i} - 100\mathbf{j} - 700\mathbf{k}\}$ lb acts at the end of the beam. Determine the moment of this force about point O.



SOLUTION

Position Vector. The coordinates of point B are $B(8, 0.25, 1.5)$ ft.

Thus,

$$\mathbf{r}_{OB} = \{8\mathbf{i} + 0.25\mathbf{j} + 1.5\mathbf{k}\}\text{ ft}$$

Moments of *F* About Point *O*.

$$M_O = \mathbf{r}_{OB} \times \mathbf{F}$$

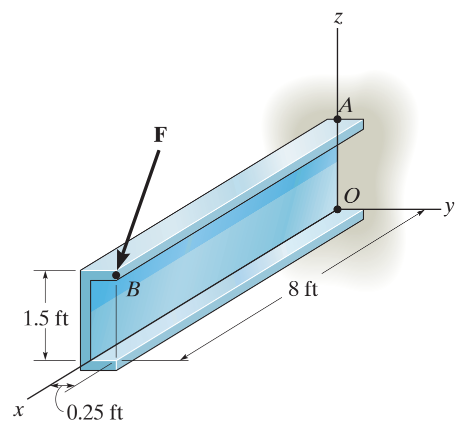
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0.25 & 1.5 \\ 400 & -100 & -700 \end{vmatrix}$$

$$= \{-25\mathbf{i} + 6200\mathbf{j} - 900\mathbf{k}\}\text{ lb}\cdot\text{ft}$$ **Ans.**

Ans:
$M_O = \{-25\mathbf{i} + 6200\mathbf{j} - 900\mathbf{k}\}$ lb · ft

3–30.

The force $\mathbf{F} = \{400\mathbf{i} - 100\mathbf{j} - 700\mathbf{k}\}$ lb acts at the end of the beam. Determine the moment of this force about point A.



SOLUTION

Position Vector. The coordinates of points A and B are $A\,(0, 0, 1.5)$ ft and $B\,(8, 0.25, 1.5)$ ft, respectively. Thus,

$$\mathbf{r}_{AB} = (8 - 0)\mathbf{i} + (0.25 - 0)\mathbf{j} + (1.5 - 1.5)\mathbf{k}$$
$$= \{8\mathbf{i} + 0.25\mathbf{j}\}\text{ ft}$$

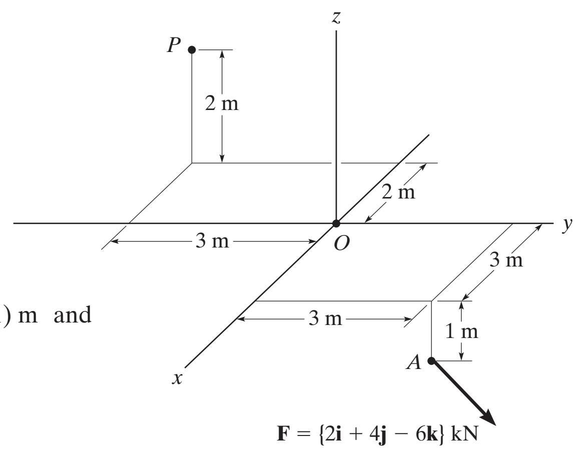
Moment of *F* About Point *A*.

$$M_A = \mathbf{r}_{AB} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0.25 & 0 \\ 400 & -100 & -700 \end{vmatrix}$$
$$= \{-175\mathbf{i} + 5600\mathbf{j} - 900\mathbf{k}\}\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

Ans:
$M_A = \{-175\mathbf{i} + 5600\mathbf{j} - 900\mathbf{k}\}\text{ lb}\cdot\text{ft}$

3–31.

Determine the moment of the force **F** about point P. Express the result as a Cartesian vector.



SOLUTION

Position Vector. The coordinates of points A and P are $A\,(3, 3, -1)$ m and $P\,(-2, -3, 2)$ m, respectively. Thus,

$$\mathbf{r}_{PA} = [3 - (-2)]\mathbf{i} + [3 - (-3)]\mathbf{j} + (-1 - 2)\mathbf{k}$$

$$= \{5\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

Moment of *F* About Point *P*.

$$M_P = \mathbf{r}_{AP} \times \mathbf{F}$$

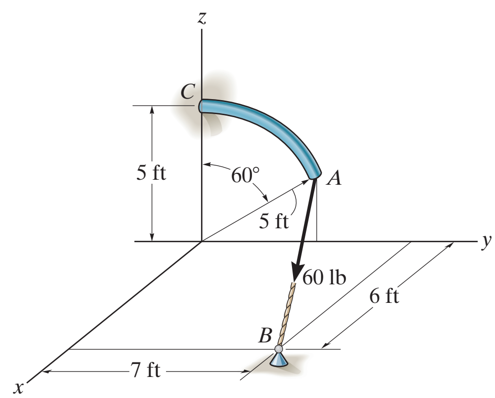
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 6 & -3 \\ 2 & 4 & -6 \end{vmatrix}$$

$$= \{-24\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}\}\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M_P = \{-24\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}\}\text{ kN}\cdot\text{m}$

***3–32.**

The curved rod has a radius of 5 ft. If a force of 60 lb acts at its end as shown, determine the moment of this force about point *C*.



SOLUTION

Position Vector and Force Vector:

$$\mathbf{r}_{CA} = \{(5 \sin 60° - 0)\mathbf{j} + (5 \cos 60° - 5)\mathbf{k}\} \text{ m}$$

$$= \{4.330\mathbf{j} - 2.50\mathbf{k}\} \text{ m}$$

$$\mathbf{F}_{AB} = 60\left(\frac{(6-0)\mathbf{i} + (7 - 5\sin 60°)\mathbf{j} + (0 - 5\cos 60°)\mathbf{k}}{\sqrt{(6-0)^2 + (7 - 5\sin 60°)^2 + (0 - 5\cos 60°)^2}}\right) \text{lb}$$

$$= \{51.231\mathbf{i} + 22.797\mathbf{j} - 21.346\mathbf{k}\} \text{ lb}$$

Moment of Force $\mathbf{F}_{AB}$ About Point C: Applying Eq. 4–7, we have

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$

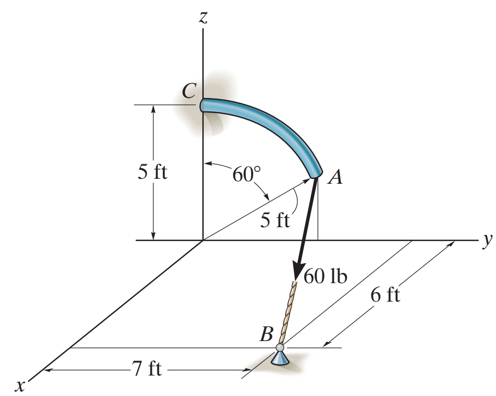
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.330 & -2.50 \\ 51.231 & 22.797 & -21.346 \end{vmatrix}$$

$$= \{-35.4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\} \text{ lb} \cdot \text{ft}$$ **Ans.**

Ans:
$\mathbf{M}_C = \{-35.4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\} \text{ lb} \cdot \text{ft}$

3–33.

Determine the smallest force F that must be applied along the rope in order to cause the curved rod to fail at the support C. This requires a moment of $M = 80\ \text{lb}\cdot\text{ft}$ to be developed at C.



SOLUTION

Position Vector and Force Vector:

$$\mathbf{r}_{CA} = \{(5\sin 60° - 0)\mathbf{j} + (5\cos 60° - 5)\mathbf{k}\}\ \text{m}$$
$$= \{4.330\mathbf{j} - 2.50\,\mathbf{k}\}\ \text{m}$$

$$\mathbf{F}_{AB} = F\left(\frac{(6-0)\mathbf{i} + (7 - 5\sin 60°)\mathbf{j} + (0 - 5\cos 60°)\mathbf{k}}{\sqrt{(6-0)^2 + (7 - 5\sin 60°)^2 + (0 - 5\cos 60°)^2}}\right)\ \text{lb}$$
$$= 0.8539F_{\mathbf{i}} + 0.3799F_{\mathbf{j}} - 0.3558F_{\mathbf{k}}$$

Moment of Force $\mathbf{F}_{AB}$ About Point C:

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.330 & -2.50 \\ 0.8539F & 0.3799F & -0.3558F \end{vmatrix}$$
$$= -0.5909F_{\mathbf{i}} - 2.135\mathbf{j} - 3.697\mathbf{k}$$

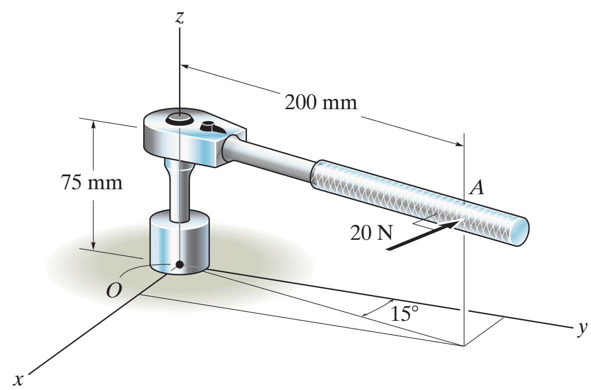
Require

$$80 = \sqrt{(0.5909)^2 + (-2.135)^2 + (-3.697)^2}\,F$$
$$F = 18.6\ \text{lb.} \qquad \textbf{Ans.}$$

Ans:
$F = 18.6\ \text{lb}$

3–34.

A 20-N horizontal force is applied perpendicular to the handle of the socket wrench. Determine the magnitude and the coordinate direction angles of the moment created by this force about point O.



SOLUTION

$$\mathbf{r}_A = 0.2 \sin 15°\mathbf{i} + 0.2 \cos 15°\mathbf{j} + 0.075\mathbf{k}$$

$$= 0.05176\mathbf{i} + 0.1932\mathbf{j} + 0.075\mathbf{k}$$

$$\mathbf{F} = -20 \cos 15°\mathbf{i} + 20 \sin 15°\mathbf{j}$$

$$= -19.32\mathbf{i} + 5.176\mathbf{j}$$

$$M_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.05176 & 0.1932 & 0.075 \\ -19.32 & 5.176 & 0 \end{vmatrix}$$

$$= \{-0.3882\mathbf{i} - 1.449\mathbf{j} + 4.00\mathbf{k}\}\ \text{N}\cdot\text{m}$$

$$M_O = 4.272 = 4.27\ \text{N}\cdot\text{m}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-0.3882}{4.272}\right) = 95.2°$$ **Ans.**

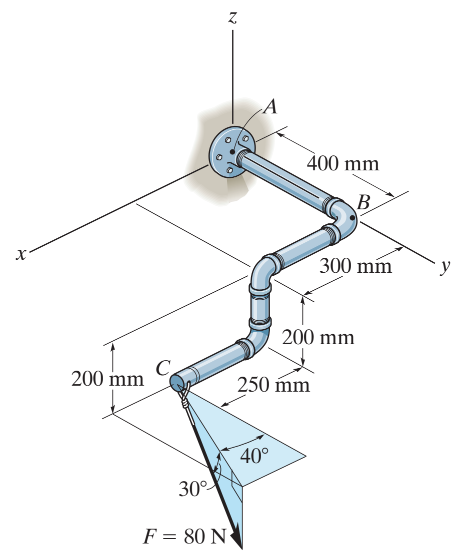
$$\beta = \cos^{-1}\left(\frac{-1.449}{4.272}\right) = 110°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{4}{4.272}\right) = 20.6°$$ **Ans.**

Ans:
$M_O = 4.27\ \text{N}\cdot\text{m}$
$\alpha = 95.2°$
$\beta = 110°$
$\gamma = 20.6°$

3–35.

The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point A.



SOLUTION

Position Vector And Force Vector:

$$\mathbf{r}_{AC} = \{(0.55 - 0)\mathbf{i} + (0.4 - 0)\mathbf{j} + (-0.2 - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.55\mathbf{i} + 0.4\mathbf{j} - 0.2\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 80(\cos 30^\circ \sin 40^\circ\mathbf{i} + \cos 30^\circ \cos 40^\circ\mathbf{j} - \sin 30^\circ\mathbf{k})\text{ N}$$

$$= (44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\}\text{ N}$$

Moment of Force* F *About Point A: Applying Eq. 4–7, we have

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$

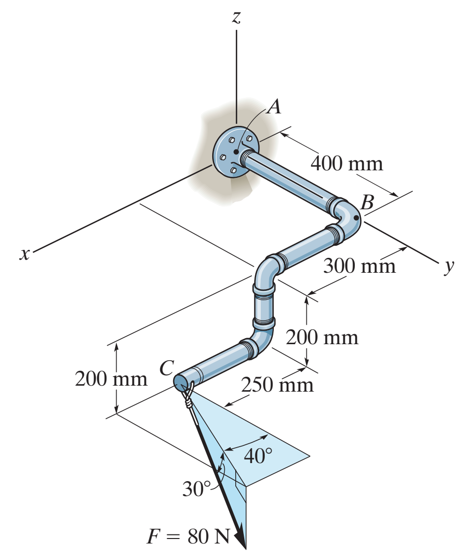
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_A = \{-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}\}\text{ N}\cdot\text{m}$

***3–36.**

The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point B.



SOLUTION

Position Vector And Force Vector:

$$\mathbf{r}_{BC} = \{(0.55 - 0)\,\mathbf{i} + (0.4 - 0.4)\mathbf{j} + (-0.2 - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.55\mathbf{i} - 0.2\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 80\,(\cos 30^\circ \sin 40^\circ\mathbf{i} + \cos 30^\circ \cos 40^\circ\mathbf{j} - \sin 30^\circ\mathbf{k})\text{ N}$$

$$= (44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\}\text{ N}$$

Moment of Force* F *About Point B: Applying Eq. 4–7, we have

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F}$$

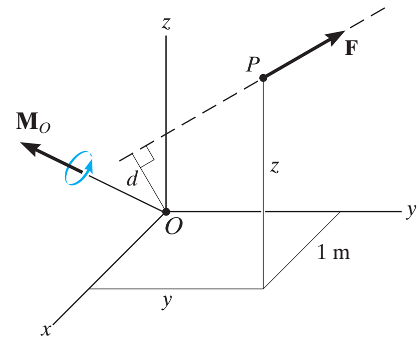
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{10.6\mathbf{i} + 13.1\mathbf{j} + 29.2\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

Ans:
$\mathbf{M}_B = \{10.6\mathbf{i} + 13.1\mathbf{j} + 29.2\mathbf{k}\}\text{ N}\cdot\text{m}$

3–37.

A force of $\mathbf{F} = \{6\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}$ kN produces a moment of $\mathbf{M}_O = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\}$ kN·m about the origin, point O. If the force acts at a point having an x coordinate of $x = 1$ m, determine the y and z coordinates. *Note*: The figure shows $\mathbf{F}$ and $\mathbf{M}_O$ in an arbitrary position.



SOLUTION

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & -2 & 1 \end{vmatrix}$$

$$4 = y + 2z$$

$$5 = -1 + 6z$$

$$-14 = -2 - 6y$$

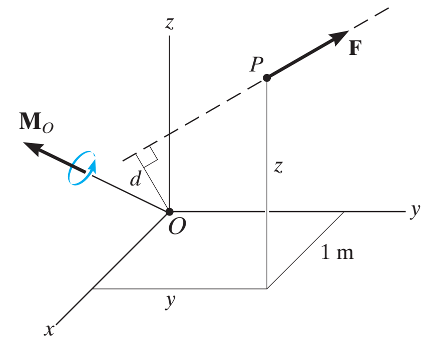
$$y = 2 \text{ m}$$ **Ans.**

$$z = 1 \text{ m}$$ **Ans.**

Ans:
$y = 2$ m
$z = 1$ m

3–38.

The force $\mathbf{F} = \{6\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}\}$ N creates a moment about point O of $\mathbf{M}_O = \{-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\}\ \text{N}\cdot\text{m}$. If the force passes through a point having an x coordinate of 1 m, determine the y and z coordinates of the point. Also, realizing that $M_O = Fd$, determine the perpendicular distance d from point O to the line of action of $\mathbf{F}$. *Note*: The figure shows $\mathbf{F}$ and $\mathbf{M}_O$ in an arbitrary position.



SOLUTION

$$-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & 8 & 10 \end{vmatrix}$$

$$-14 = 10y - 8z$$

$$8 = -10 + 6z$$

$$2 = 8 - 6y$$

$$y = 1\ \text{m} \qquad \textbf{Ans.}$$

$$z = 3\ \text{m} \qquad \textbf{Ans.}$$

$$M_O = \sqrt{(-14)^2 + (8)^2 + (2)^2} = 16.25\ \text{N}\cdot\text{m}$$

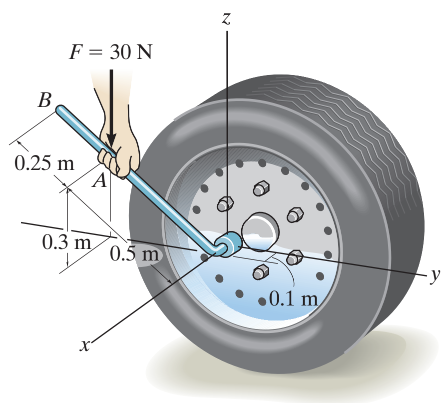
$$F = \sqrt{(6)^2 + (8)^2 + (10)^2} = 14.14\ \text{N}$$

$$d = \frac{16.25}{14.14} = 1.15\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$y = 1$ m
$z = 3$ m
$d = 1.15$ m

3–39.

The lug nut on the wheel of the automobile is to be removed using the wrench and applying the vertical force of $F = 30$ N at A. Determine if this force is adequate, provided 14 N · m of torque about the x axis is initially required to turn the nut. If the 30-N force can be applied at A in any other direction, will it be possible to turn the nut?



SOLUTION

$M_x = 30\left(\sqrt{(0.5)^2 - (0.3)^2}\right) = 12\text{ N}\cdot\text{m} < 14\text{ N}\cdot\text{m}$, No **Ans.**

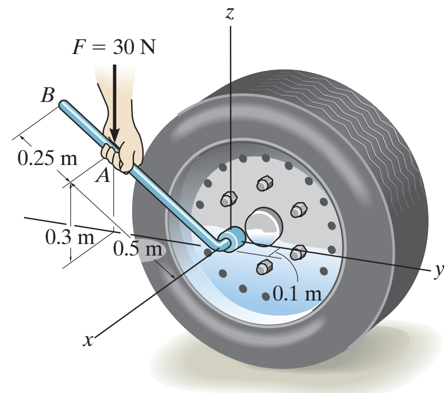
For $(M_x)_{max}$, apply force perpendicular to the handle and the x - axis.

$(M_x)_{max} = 30\,(0.5) = 15\text{ N}\cdot\text{m} > 14\text{ N}\cdot\text{m}$, Yes **Ans.**

Ans:
No
Yes

***3–40.**

Solve Prob. 3–39 if the cheater pipe AB is slipped over the handle of the wrench and the 30-N force can be applied at any point and in any direction on the assembly.



SOLUTION

$M_x = 30\,(0.75)\left(\frac{4}{5}\right) = 18\text{ N}\cdot\text{m} > 14\text{ N}\cdot\text{m},$ Yes **Ans.**

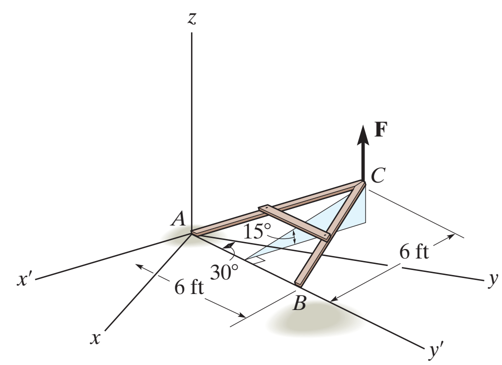
$(M_x)_{max}$ occurs when force is applied perpendicular to both the handle and the x - axis.

$(M_x)_{max} = 30(0.75) = 22.5\text{ N}\cdot\text{m} > 14\text{N}\cdot\text{m},$ Yes **Ans.**

Ans:
Yes
Yes

3–41.

The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the y' axis passing through points A and B when the frame is in the position shown.



SOLUTION

Scalar analysis :

$M_{y'} = 80\,(6\cos 15°) = 464\ \text{lb}\cdot\text{ft}$

Vector analysis :

$\mathbf{u}_{AB} = \cos 60°\,\mathbf{i} + \cos 30°\,\mathbf{j}$

Coordinates of point C :

$x = 3\sin 30° - 6\cos 15°\cos 30° = -3.52\ \text{ft}$

$y = 3\cos 30° + 6\cos 15°\sin 30° = 5.50\ \text{ft}$

$z = 6\sin 15° = 1.55\ \text{ft}$

$\mathbf{r}_{AC} = -3.52\,\mathbf{i} + 5.50\,\mathbf{j} + 1.55\,\mathbf{k}$

$\mathbf{F} = 80\ \text{k}$

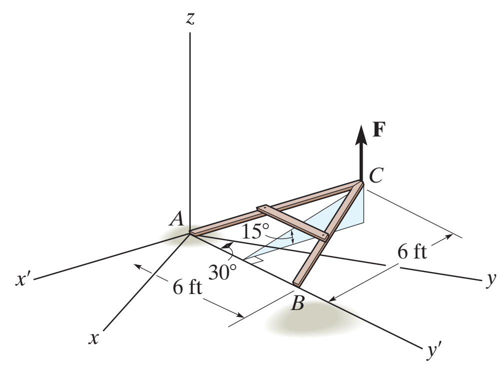
$$M_{y'} = \begin{vmatrix} \sin 30° & \cos 30° & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix}$$

$M_{y'} = 464\ \text{lb}\cdot\text{ft}$ **Ans.**

Ans:
$M_{y'} = 464\ \text{lb}\cdot\text{ft}$

3–42.

The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the x axis when the frame is in the position shown.



SOLUTION

Using x', y', z :

$\mathbf{u}_x = \cos 30° \, \mathbf{i}' + \sin 30° \, \mathbf{j}'$

$\mathbf{r}_{AC} = -6 \cos 15° \, \mathbf{i}' + 3 \, \mathbf{j}' + 6 \sin 15° \, \mathbf{k}$

$\mathrm{F} = 80 \, \mathbf{k}$

$$M_x = \begin{vmatrix} \cos 30° & \sin 30° & 0 \\ -6 \cos 15° & 3 & 6 \sin 15° \\ 0 & 0 & 80 \end{vmatrix} = 207.85 + 231.82 + 0$$

$M_x = 440 \text{ lb} \cdot \text{ft}$

Also, using x, y, z,

Coordinates of point C :

$x = 3 \sin 30° - 6 \cos 15° \cos 30° = -3.52 \text{ ft}$

$y = 3 \cos 30° + 6 \cos 15° \sin 30° = 5.50 \text{ ft}$

$z = 6 \sin 15° = 1.55 \text{ ft}$

$\mathbf{r}_{AC} = -3.52 \, \mathbf{i} + 5.50 \, \mathbf{j} + 1.55 \, \mathbf{k}$

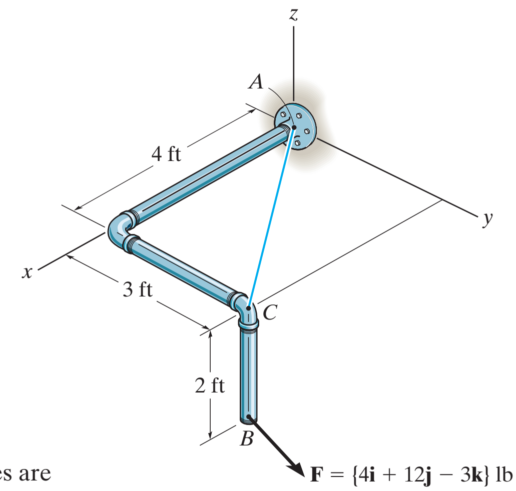
$\mathbf{F} = 80 \, \mathbf{k}$

$$M_x = \begin{vmatrix} 1 & 0 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 440 \text{ lb} \cdot \text{ft}$$ **Ans.**

Ans:
$M_x = 440 \text{ lb} \cdot \text{ft}$

3–43.

Determine the magnitude of the moment of the force **F** about the x, the y, and the z axis. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.



SOLUTION

(a) ***Vector Analysis***

Position Vector:

$$\mathbf{r}_{AB} = \{(4 - 0)\,\mathbf{i} + (3 - 0)\mathbf{j} + (-2 - 0)\mathbf{k}\}\text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

Moment of Force* F *About x, y, and z Axes: The unit vectors along x, y, and z axes are **i**, **j**, and **k**, respectively. Applying Eq. 4–11, we have

$$M_x = \mathbf{i}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0\text{ lb}\cdot\text{ft}$$ **Ans.**

$$M_y = \mathbf{j}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00\text{ lb}\cdot\text{ft}$$ **Ans.**

$$M_z = \mathbf{k}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 0 + 1[4(12) - (4)(3)] = 36.0\text{ lb}\cdot\text{ft}$$ **Ans.**

(b) ***ScalarAnalysis***

$M_x = \Sigma M_x;\quad M_x = 12(2) - 3(3) = 15.0\text{ lb}\cdot\text{ft}$ **Ans.**

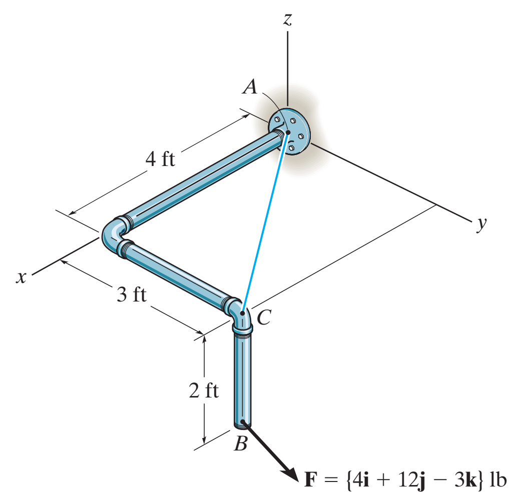
$M_y = \Sigma M_y;\quad M_y = -4(2) + 3(4) = 4.00\text{ lb}\cdot\text{ft}$ **Ans.**

$M_z = \Sigma M_z;\quad M_z = -4(3) + 12(4) = 36.0\text{ lb}\cdot\text{ft}$ **Ans.**

Ans:
$M_x = 15.0\text{ lb}\cdot\text{ft}$
$M_y = 4.00\text{ lb}\cdot\text{ft}$
$M_z = 36.0\text{ lb}\cdot\text{ft}$

***3–44.**

Determine the moment of force **F** about an axis extending between A and C. Express the result as a Cartesian vector.



SOLUTION

Position Vector:

$$\mathbf{r}_{CB} = \{-2\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

Unit Vector Along AC Axis:

$$\mathbf{u}_{AC} = \frac{(4-0)\mathbf{i} + (3-0)\mathbf{j}}{\sqrt{(4-0)^2 + (3-0)^2}} = 0.8\mathbf{i} + 0.6\mathbf{j}$$

Moment of Force F About AC Axis: With $\mathbf{F} = \{4\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}\}$ lb, applying Eq. 4–7, we have

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{CB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 0 & 0 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(0)(-3) - 12(-2)] - 0.6[0(-3) - 4(-2)] + 0$$

$$= 14.4\text{ lb}\cdot\text{ft}$$

Or

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(3)(-3) - 12(-2)] - 0.6[4(-3) - 4(-2)] + 0$$

$$= 14.4\text{ lb}\cdot\text{ft}$$

Expressing $\mathbf{M}_{AC}$ as a Cartesian vector yields

$$\mathbf{M}_{AC} = M_{AC}\,\mathbf{u}_{AC}$$

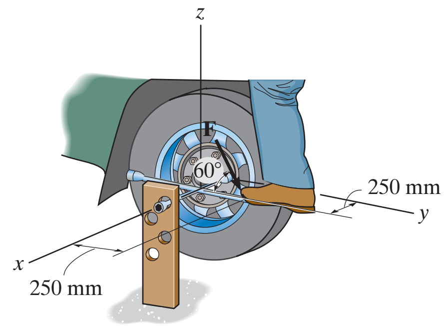
$$= 14.4(0.8\mathbf{i} + 0.6\mathbf{j})$$

$$= \{11.5\mathbf{i} + 8.64\mathbf{j}\}\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_{AC} = \{11.5\mathbf{i} + 8.64\mathbf{j}\}\text{ lb}\cdot\text{ft}$

3–45.

The board is used to hold the end of the cross lug wrench in the position shown when the man applies a force of $F = 100$ N. Determine the magnitude of the moment produced by this force about the x axis. Force $\mathbf{F}$ lies in a vertical plane.



SOLUTION

Vector Analysis

Moment About the x Axis: The position vector $\mathbf{r}_{AB}$, Fig. a, will be used to determine the moment of $\mathbf{F}$ about the x axis.

$$\mathbf{r}_{AB} = (0.25 - 0.25)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{0.25\mathbf{j}\}\text{ m}$$

The force vector $\mathbf{F}$, Fig. a, can be written as

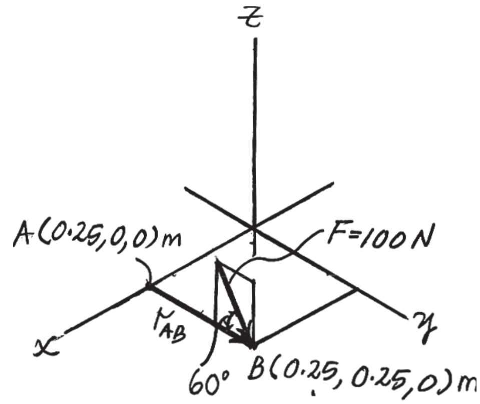
$$\mathbf{F} = 100(\cos 60°\mathbf{j} - \sin 60°\mathbf{k}) = \{50\mathbf{j} - 86.60\mathbf{k}\}\text{ N}$$

Knowing that the unit vector of the x axis is $\mathbf{i}$, the magnitude of the moment of $\mathbf{F}$ about the x axis is given by

$$M_x = \mathbf{i}\cdot\mathbf{r}_{AB}\times\mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 50 & -86.60 \end{vmatrix}$$

$$= 1[0.25(-86.60) - 50(0)] + 0 + 0$$

$$= -21.7\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

The negative sign indicates that M_x is directed towards the negative x axis.

Scalar Analysis

This problem can be solved by summing the moment about the x axis.

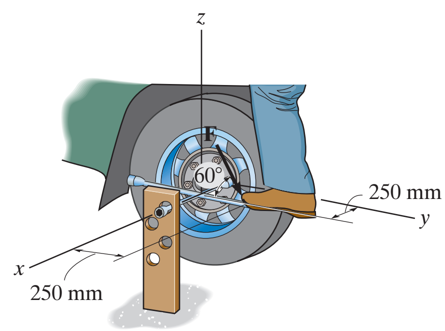
$$M_x = \Sigma M_x; \qquad M_x = \left|-100\ \sin 60°(0.25) + 100\ \cos 60°(0)\right|$$

$$= 21.7\text{ N}\cdot\text{m} \quad \textbf{Ans.}$$

Ans:
$M_x = 21.7\text{ N}\cdot\text{m}$

3–46.

The board is used to hold the end of the cross lug wrench in the position shown. If a torque of 30 N · m about the x axis is required to tighten the nut, determine the required magnitude of the force **F** needed to turn the wrench. Force **F** lies in a vertical plane.



SOLUTION

Vector Analysis

Moment About the x Axis: The position vector $\mathbf{r}_{AB}$, Fig. a, will be used to determine the moment of **F** about the x axis.

$$\mathbf{r}_{AB} = (0.25 - 0.25)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{0.25\mathbf{j}\}\text{ m}$$

The force vector **F**, Fig. a, can be written as

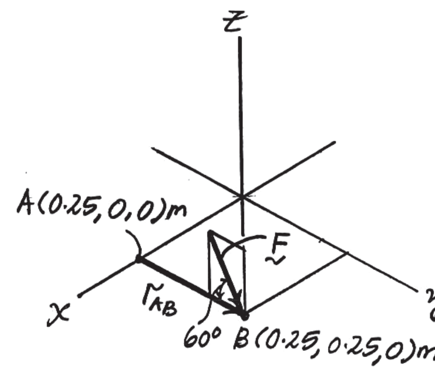
$$\mathbf{F} = F(\cos 60°\mathbf{j} - \sin 60°\mathbf{k}) = 0.5F\mathbf{j} - 0.8660F\mathbf{k}$$

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

$$M_x = \mathbf{i} \cdot \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0.5F & -0.8660F \end{vmatrix}$$

$$= 1[0.25(-0.8660F) - 0.5F(0)] + 0 + 0$$

$$= -0.2165F \qquad \textbf{Ans.}$$



(a)

The negative sign indicates that M_x is directed towards the negative x axis. The magnitude of **F** required to produce $M_x = 30$ N · m can be determined from

$$30 = 0.2165F$$

$$F = 139\text{ N} \qquad \textbf{Ans.}$$

Scalar Analysis

This problem can be solved by summing the moment about the x axis.

$$M_x = \Sigma M_x; \qquad -30 = -F\sin 60°(0.25) + F\cos 60°(0)$$

$$F = 139\text{ N} \qquad \textbf{Ans.}$$

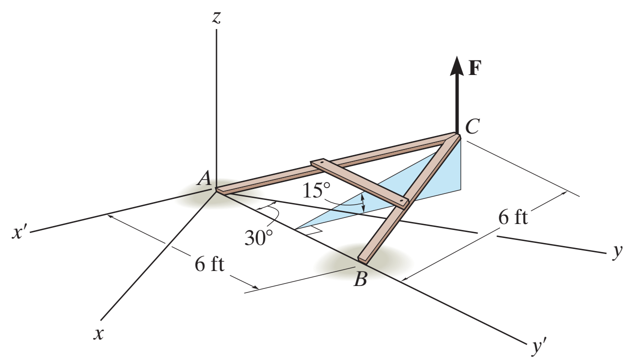
Ans:

$M_x = -0.2165F$

$F = 139$ N

3–47.

The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the y axis when the frame is in the position shown.



SOLUTION

Using x', y', z:

$$\mathbf{u}_y = -\sin 30^\circ\, \mathbf{i}' + \cos 30^\circ\, \mathbf{j}'$$

$$\mathbf{r}_{AC} = -6\cos 15^\circ \mathbf{i}' + 3\,\mathbf{j}' + 6\sin 15^\circ\, \mathbf{k}$$

$$\mathbf{F} = 80\,\mathbf{k}$$

$$M_y = \begin{vmatrix} -\sin 30^\circ & \cos 30^\circ & 0 \\ -6\cos 15^\circ & 3 & 6\sin 15^\circ \\ 0 & 0 & 80 \end{vmatrix} = -120 + 401.53 + 0$$

$$M_y = 282 \text{ lb}\cdot\text{ft}$$ **Ans.**

Also, using x, y, z:

Coordinates of point C:

$$x = 3\sin 30^\circ - 6\cos 15^\circ \cos 30^\circ = -3.52 \text{ ft}$$

$$y = 3\cos 30^\circ + 6\cos 15^\circ \sin 30^\circ = 5.50 \text{ ft}$$

$$z = 6\sin 15^\circ = 1.55 \text{ ft}$$

$$\mathbf{r}_{AC} = -3.52\,\mathbf{i} + 5.50\,\mathbf{j} + 1.55\,\mathbf{k}$$

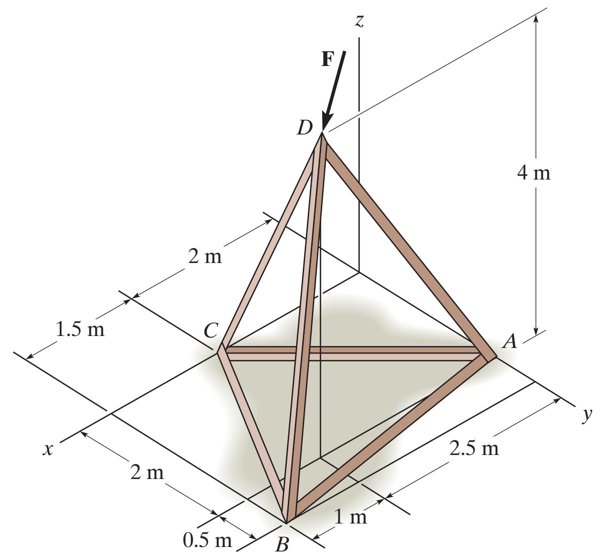
$$\mathbf{F} = 80\,\mathbf{k}$$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 282 \text{ lb}\cdot\text{ft}$$ **Ans.**

Ans:
$M_y = 282 \text{ lb}\cdot\text{ft}$

***3–48.**

Determine the magnitude of the moment of the force $\mathbf{F} = \{50\mathbf{i} - 20\mathbf{j} - 80\mathbf{k}\}$ N about member AB of the tripod.



SOLUTION

$$\mathbf{u}_{AB} = \frac{\{3.5\mathbf{i} + 0.5\mathbf{j}\}}{\sqrt{(3.5)^2 + (0.5)^2}}$$

$$\mathbf{u}_{AB} = \{0.9899\mathbf{i} + 0.1414\mathbf{j}\}$$

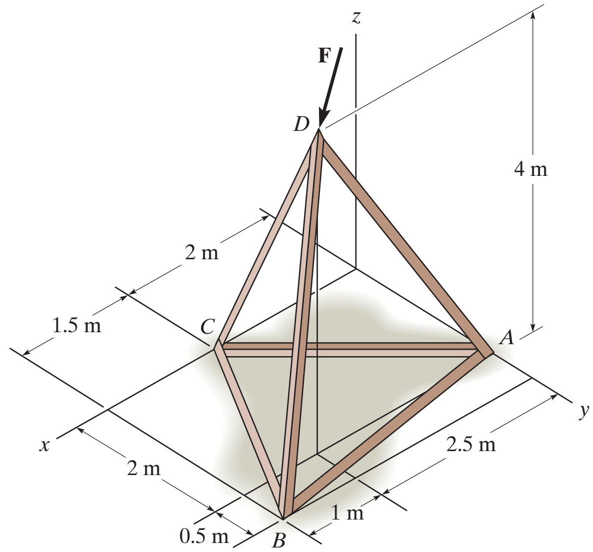
$$M_{AB} = \mathbf{u}_{AB} \cdot (\mathbf{r}_{AD} \times \mathbf{F}) = \begin{vmatrix} 0.9899 & 0.1414 & 0 \\ 2.5 & 0 & 4 \\ 50 & -20 & -80 \end{vmatrix}$$

$$M_{AB} = 136 \text{ N} \cdot \text{m}$$ **Ans.**

Ans:
$M_{AB} = 136 \text{ N} \cdot \text{m}$

3–49.

Determine the magnitude of the moment of the force $\mathbf{F} = \{50\mathbf{i} - 20\mathbf{j} - 80\mathbf{k}\}$ N about member BC of the tripod.



SOLUTION

$$\mathbf{u}_{BC} = \frac{\{-1.5\mathbf{i} - 2.5\mathbf{j}\}}{\sqrt{(-1.5)^2 + (-2.5)^2}}$$

$$\mathbf{u}_{BC} = \{-0.5145\mathbf{i} - 0.8575\mathbf{j}\}$$

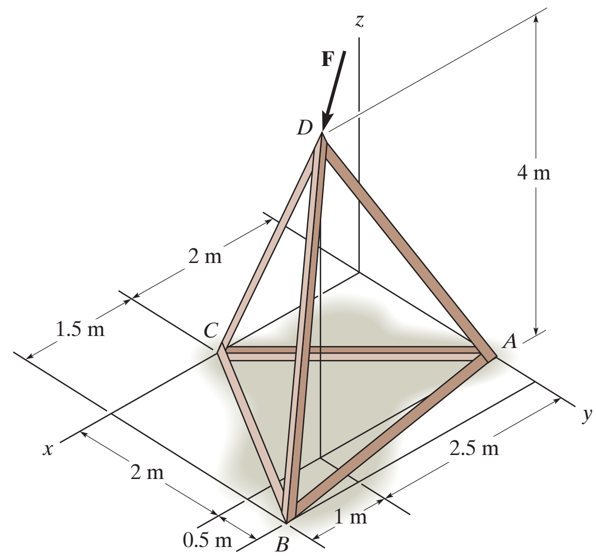
$$M_{BC} = \mathbf{u}_{BC} \cdot (\mathbf{r}_{CD} \times \mathbf{F}) = \begin{vmatrix} -0.5145 & -0.8575 & 0 \\ 0.5 & 2 & 4 \\ 50 & -20 & -80 \end{vmatrix}$$

$M_{BC} = 165\ \text{N} \cdot \text{m}$ **Ans.**

Ans:
$M_{BC} = 165\ \text{N} \cdot \text{m}$

3–50.

Determine the magnitude of the moment of the force $\mathbf{F} = \{50\mathbf{i} - 20\mathbf{j} - 80\mathbf{k}\}$ N about member CA of the tripod.



SOLUTION

$$\mathbf{u}_{CA} = \frac{\{-2\mathbf{i} + 2\mathbf{j}\}}{\sqrt{(-2)^2 + (2)^2}}$$

$$\mathbf{u}_{CA} = \{-0.707\mathbf{i} + 0.707\mathbf{j}\}$$

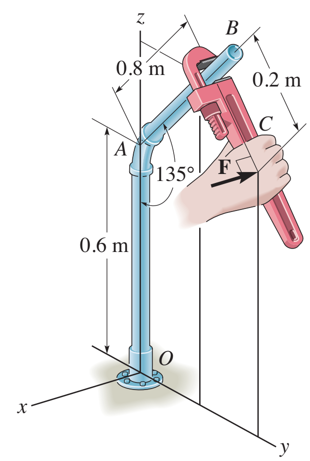
$$M_{CA} = \mathbf{u}_{CA} \cdot (\mathbf{r}_{AD} \times \mathbf{F}) = \begin{vmatrix} -0.707 & 0.707 & 0 \\ 2.5 & 0 & 4 \\ 50 & -20 & -80 \end{vmatrix}$$

$$M_{CA} = 226 \text{ N} \cdot \text{m}$$ **Ans.**

Ans:
$M_{CA} = 226 \text{ N} \cdot \text{m}$

3–51.

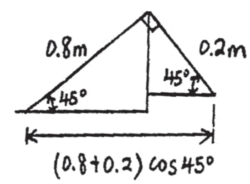
A horizontal force of $\mathbf{F} = \{-50\mathbf{i}\}$ N is applied perpendicular to the handle of the pipe wrench. Determine the moment that this force exerts along the axis OA (z axis) of the pipe assembly. Both the wrench and pipe assembly, $OABC$, lie in the y–z plane. *Suggestion:* Use a scalar analysis.



SOLUTION

$$M_z = 50(0.8 + 0.2)\cos 45° = 35.36\ \text{N}\cdot\text{m}$$

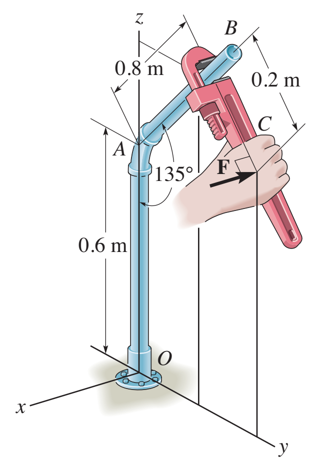
$$M_z = \{35.4\ \mathbf{k}\}\ \text{N}\cdot\text{m}$$ **Ans.**



Ans:
$M_z = \{35.4\ \mathbf{k}\}\ \text{N}\cdot\text{m}$

***3–52.**

Determine the magnitude of the horizontal force $\mathbf{F} = -F\mathbf{i}$ acting on the handle of the pipe wrench so that this force produces a component of the moment along the OA axis (z axis) of the pipe assembly of $\mathbf{M}_z = \{4\mathbf{k}\}\ \text{N}\cdot\text{m}$. Both the wrench and the pipe assembly, $OABC$, lie in the y–z plane. *Suggestion*: Use a scalar analysis.



SOLUTION

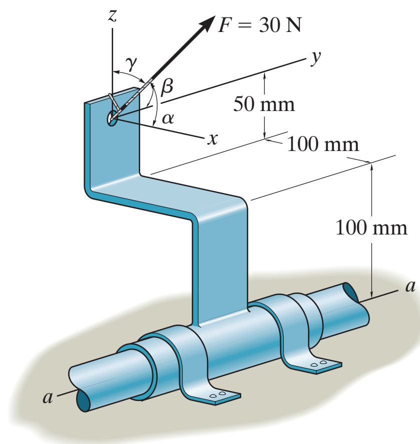
$M_z = F(0.8 + 0.2)\cos 45° = 4$

$F = 5.66\ \text{N}$ **Ans.**

Ans:
$F = 5.66\ \text{N}$

3–53.

Determine the moment of the force about the $a-a$ axis of the pipe if $\alpha = 60°$, $\beta = 60°$, and $\gamma = 45°$. Also, determine the coordinate direction angles of F in order to produce the maximum moment about the $a-a$ axis. What is this moment?



SOLUTION

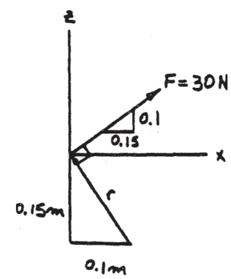
$$\mathbf{F} = 30\,(\cos 60°\,\mathbf{i} + \cos 60°\,\mathbf{j} + \cos 45°\,\mathbf{k})$$

$$= \{15\,\mathbf{i} + 15\,\mathbf{j} + 21.21\,\mathbf{k}\}\text{ N}$$

$$\mathbf{r} = \{-0.1\,\mathbf{i} + 0.15\,\mathbf{k}\}\text{ m}$$

$$\mathbf{u} = \mathbf{j}$$

$$M_a = \begin{vmatrix} 0 & 1 & 0 \\ -0.1 & 0 & 0.15 \\ 15 & 15 & 21.21 \end{vmatrix} = 4.37\text{ N}\cdot\text{m}$$ **Ans.**



F must be perpendicular to **u** and **r**.

$$\mathbf{u}_F = \frac{0.15}{0.1803}\mathbf{i} + \frac{0.1}{0.1803}\mathbf{k}$$

$$= 0.8321\mathbf{i} + 0.5547\mathbf{k}$$

$$\alpha = \cos^{-1} 0.8321 = 33.7°$$ **Ans.**

$$\beta = \cos^{-1} 0 = 90°$$ **Ans.**

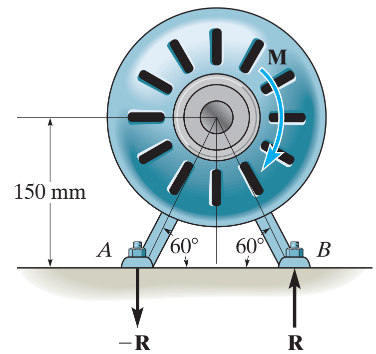
$$\gamma = \cos^{-1} 0.5547 = 56.3°$$ **Ans.**

$$M = 30\,(0.1803) = 5.41\text{ N}\cdot\text{m}$$ **Ans.**

Ans:
$M_a = 4.37\text{ N}\cdot\text{m}$
$\alpha = 33.7°$
$\beta = 90°$
$\gamma = 56.3°$
$M = 5.41\text{ N}\cdot\text{m}$

3–54.

A clockwise couple $M = 5\ \mathrm{N \cdot m}$ is resisted by the shaft of the electric motor. Determine the magnitude of the reactive forces $-\mathbf{R}$ and $\mathbf{R}$ which act at supports A and B so that the resultant of the two couples is zero.



SOLUTION

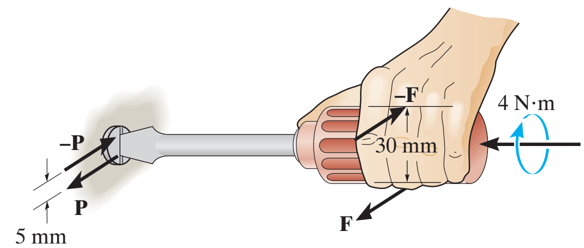
$\circlearrowleft + M_C = -5 + R\,(2(0.15)/\tan 60°) = 0$

$R = 28.9\ \mathrm{N}$ **Ans.**

Ans:
$R = 28.9\ \mathrm{N}$

3–55.

A twist of 4 N · m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces **F** exerted on the handle and **P** exerted on the blade.



SOLUTION

For the handle

$M_C = \Sigma M_x;$ $\quad F(0.03) = 4$

$F = 133\text{ N}$ **Ans.**

For the blade,

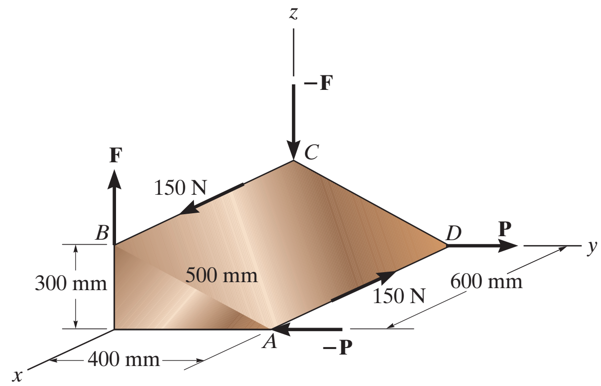
$M_C = \Sigma M_x;$ $\quad P(0.005) = 4$

$P = 800\text{ N}$ **Ans.**

Ans:
$F = 133\text{ N}$
$P = 800\text{ N}$

***3–56.**

If the resultant couple of the three couples acting on the triangular block is to be zero, determine the magnitude of forces **F** and **P**.



SOLUTION

$BA = 0.5\text{ m}$

The couple created by the 150 - N forces is

$M_{C1} = 150\,(0.5) = 75\text{ N}\cdot\text{m}$

Then

$$\mathbf{M}_{C1} = 75\left(\frac{3}{5}\right)\mathbf{j} + 75\left(\frac{4}{5}\right)\mathbf{k}$$

$$= 45\,\mathbf{j} + 60\,\mathbf{k}$$

$$\mathbf{M}_{C2} = -P\,(0.6)\,\mathbf{k}$$

$$\mathbf{M}_{C3} = -F\,(0.6)\,\mathbf{j}$$

Require

$$\mathbf{M}_{C1} + \mathbf{M}_{C2} + \mathbf{M}_{C3} = \mathbf{0}$$

$$45\,\mathbf{j} + 60\,\mathbf{k} - P\,(0.6)\,\mathbf{k} - F\,(0.6)\,\mathbf{j} = \mathbf{0}$$

Equate the **j** and **k** components

$$45 - F\,(0.6) = 0$$

$F = 75\text{ N}$ **Ans.**

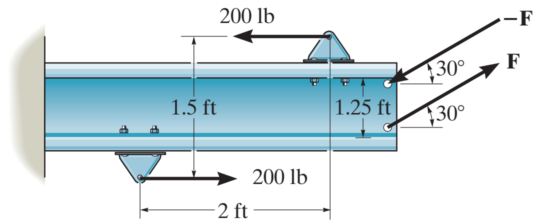
$$60 - P\,(0.6) = 0$$

$P = 100\text{ N}$ **Ans.**

Ans:
$F = 75\text{ N}$
$P = 100\text{ N}$

3–57.

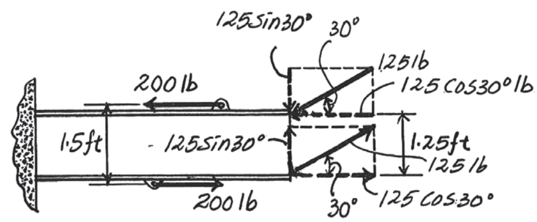
If $F = 125$ lb, determine the resultant couple moment.



SOLUTION

125 lb couple is resolved in to their horizontal and vertical components as shown in Fig. *a*.

$$\circlearrowleft + (M_R)_C = 200(1.5) + 125\cos 30°(1.25)$$
$$= 435.32 \text{ lb}\cdot\text{ft} = 435 \text{ lb}\cdot\text{ft} \circlearrowleft$$ **Ans.**

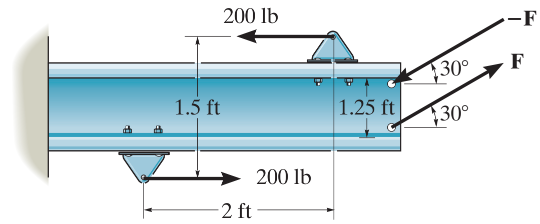


(a)

Ans:
$(M_R)_C = 435$ lb·ft $\circlearrowleft$

3–58.

Determine the magnitude of **F** so that the resultant couple moment is 450 lb · ft, counterclockwise. Where on the beam does the resultant couple moment act?



SOLUTION

$$\circlearrowleft + M_R = \Sigma M; \qquad 450 = 200(1.5) + F\cos 30°(1.25)$$

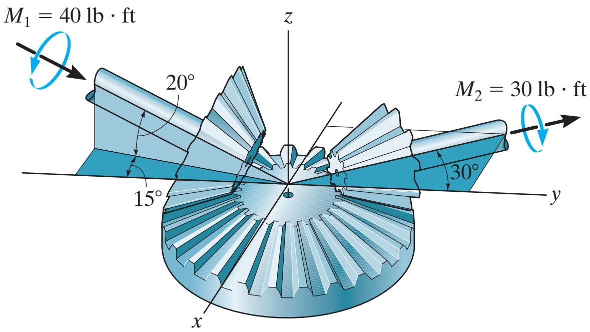
$$F = 139 \text{ lb} \qquad \textbf{Ans.}$$

The resultant couple moment is a free vector. It can act at any point on the beam.

Ans:
$F = 139$ lb
Anywhere

3–59.

Determine the magnitude and coordinate direction angles of the resultant couple moment.



SOLUTION

$$\mathbf{M}_1 = 40 \cos 20° \sin 15° \,\mathbf{i} + 40 \cos 20° \cos 15° \,\mathbf{j} - 40 \sin 20° \,\mathbf{k}$$

$$= 9.728 \,\mathbf{i} + 36.307 \,\mathbf{j} - 13.681 \,\mathbf{k}$$

$$\mathbf{M}_2 = -30 \sin 30° \,\mathbf{i} + 30 \cos 30° \,\mathbf{j}$$

$$= -15 \,\mathbf{i} + 25.981 \,\mathbf{j}$$

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2 = -5.272 \,\mathbf{i} + 62.288 \,\mathbf{j} - 13.681 \,\mathbf{k}$$

$$M_R = \sqrt{(-5.272)^2 + (62.288)^2 + (-13.681)^2} = 63.990 = 64.0 \text{ lb} \cdot \text{ft}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-5.272}{63.990}\right) = 94.7°$$ **Ans.**

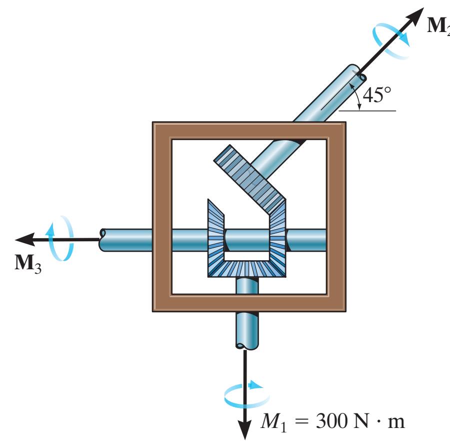
$$\beta = \cos^{-1}\left(\frac{62.288}{63.990}\right) = 13.2°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-13.681}{63.990}\right) = 102°$$ **Ans.**

Ans:
$M_R = 64.0$ lb · ft
$\alpha = 94.7°$
$\beta = 13.2°$
$\gamma = 102°$

***3–60.**

Determine the required magnitude of the couple moments $\mathbf{M}_2$ and $\mathbf{M}_3$ so that the resultant couple moment is zero.



SOLUTION

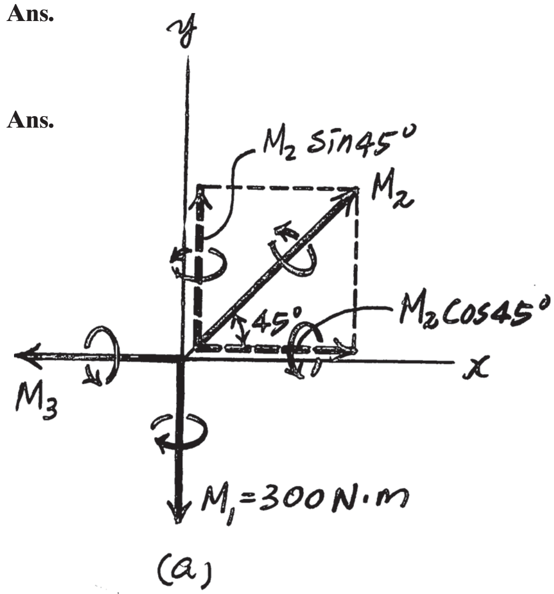
Since the couple moment is the free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ can be simplified as shown in Fig. *a*. Since the resultant of $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ is required to be zero,

$(M_R)_y = \Sigma M_y\,;$ $\quad 0 = M_2 \sin 45° - 300$

$M_2 = 424.26\ \text{N} \cdot \text{m} = 424\ \text{N} \cdot \text{m}$ **Ans.**

$(M_R)_x = \Sigma M_x\,;$ $\quad 0 = 424.26 \cos 45° - M_3$

$M_3 = 300\ \text{N} \cdot \text{m}$ **Ans.**

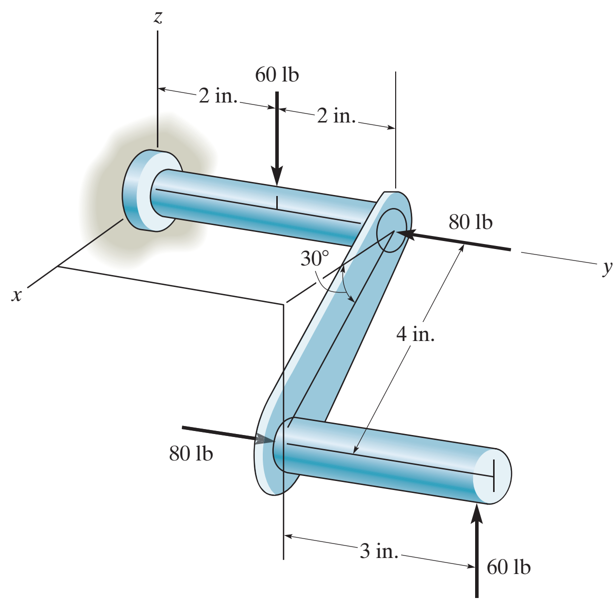


(a)

Ans:
$M_2 = 424\ \text{N} \cdot \text{m}$
$M_3 = 300\ \text{N} \cdot \text{m}$

3–61.

Determine the resultant couple moment of the two couples that act on the assembly. Specify its magnitude and coordinate direction angles.



SOLUTION

$$\mathbf{M}_R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4\cos 30^\circ & 5 & -4\sin 30^\circ \\ 0 & 0 & 60 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4\cos 30^\circ & 0 & -4\sin 30^\circ \\ 0 & 80 & 0 \end{vmatrix}$$

$$= 300\,\mathbf{i} - 207.85\,\mathbf{j} + 160\,\mathbf{i} + 277.13\,\mathbf{k}$$

$$= \{460\,\mathbf{i} - 207.85\,\mathbf{j} + 277.13\,\mathbf{k}\}\text{ lb}\cdot\text{in.}$$

$$M_R = \sqrt{(460)^2 + (-207.85)^2 + (277.13)^2} = 575.85 = 576\text{ lb}\cdot\text{in.}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{460}{575.85}\right) = 37.0^\circ$$ **Ans.**

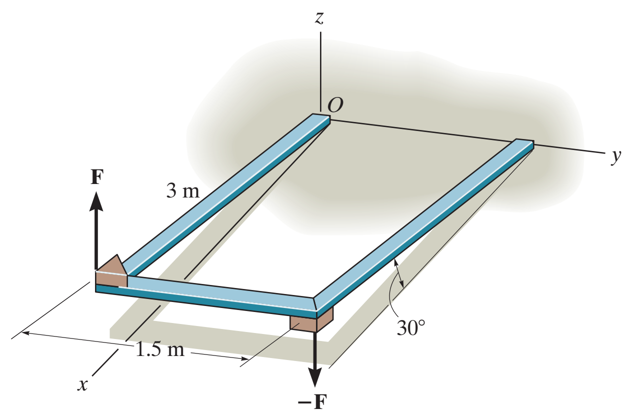
$$\beta = \cos^{-1}\left(\frac{-207.85}{575.85}\right) = 111^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{277.13}{575.85}\right) = 61.2^\circ$$ **Ans.**

Ans:
$M_R = 576\text{ lb}\cdot\text{in.}$
$\alpha = 37.0^\circ$
$\beta = 111^\circ$
$\gamma = 61.2^\circ$

3–62.

Express the moment of the couple acting on the frame in Cartesian vector form. The forces are applied perpendicular to the frame. What is the magnitude of the couple moment? Take $F = 50$ N.

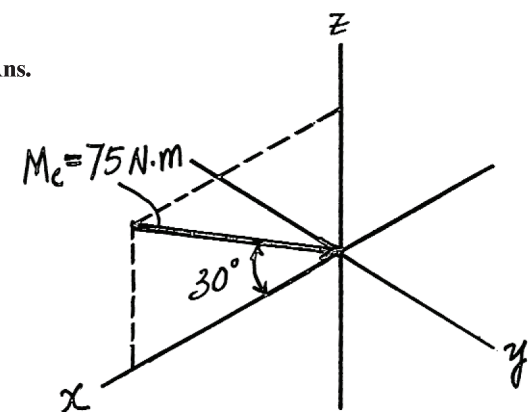


SOLUTION

$M_C = 80(1.5) = 75\ \text{N}\cdot\text{m}$ **Ans.**

$M_C = -75(\cos 30^\circ\ \mathbf{i} + \cos 60^\circ\ \mathbf{k})$

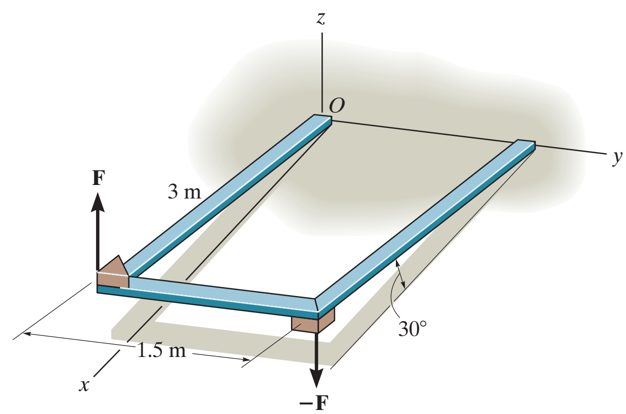
$= \{-65.0\mathbf{i} - 37.5\mathbf{k}\}\ \text{N}\cdot\text{m}$ **Ans.**



Ans:
$M_C = \{-65.0\mathbf{i} - 37.5\mathbf{k}\}\ \text{N}\cdot\text{m}$

3–63.

In order to turn over the frame, a couple moment is applied as shown. If the component of this couple moment along the x axis is $\mathbf{M}_x = \{-20\mathbf{i}\}$ N · m, determine the magnitude F of the couple forces.



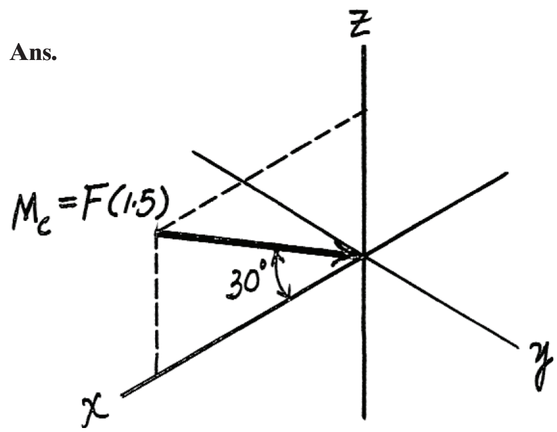
SOLUTION

$M_C = F\,(1.5)$

Thus

$$20 = F\,(1.5)\cos 30^\circ$$

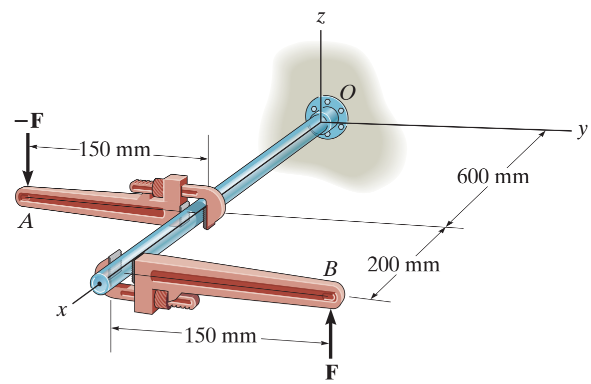
$$F = 15.4\text{ N}$$ **Ans.**



Ans:
$F = 15.4$ N

***3–64.**

Express the moment of the couple acting on the pipe in Cartesian vector form. What is the magnitude of the couple moment? Take $F = 125$ N.



SOLUTION

$$\mathbf{M}_C = \mathbf{r}_{AB} \times (125\,\mathbf{k})$$

$$\mathbf{M}_C = (0.2\mathbf{i} + 0.3\mathbf{j}) \times (125\,\mathbf{k})$$

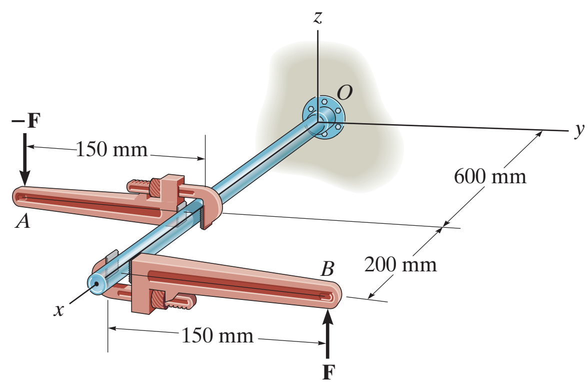
$$\mathbf{M}_C = \{37.5\mathbf{i} - 25\mathbf{j}\}\ \text{N}\cdot\text{m}$$

$$M_C = \sqrt{(37.5)^2 + (-25)^2} = 45.1\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:
$M_C = 45.1$ N·m

3–65.

If the couple moment acting on the pipe has a magnitude of 300 N · m, determine the magnitude of the forces applied to the wrenches.



SOLUTION

$$\mathbf{M}_C = \mathbf{r}_{AB} \times (F\,\mathbf{k})$$

$$= (0.2\mathbf{i} + 0.3\mathbf{j}) \times (F\,\mathbf{k})$$

$$= \{0.2F\mathbf{i} - 0.3F\mathbf{j}\}\ \text{N} \cdot \text{m}$$

$$M_C = F\sqrt{(0.2F)^2 + (-0.3F)} = 0.3606\,F$$

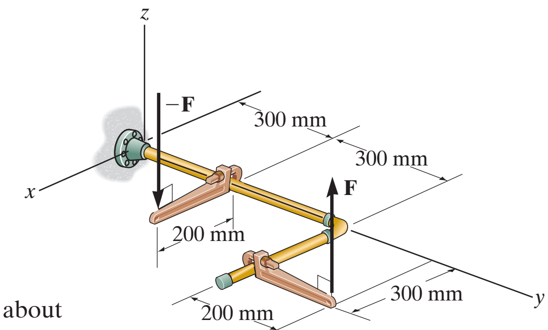
$$300 = 0.3606\,F$$

$$F = 832\ \text{N}$$ **Ans.**

Ans:
$F = 832$ N

3–66.

If $F = 80$ N, determine the magnitude and coordinate direction angles of the couple moment. The pipe assembly lies in the x–y plane.



SOLUTION

It is easiest to find the couple moment of **F** by taking the moment of **F** or –**F** about point A or B, respectively, in Fig. a. Here, the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3 - 0.2)\mathbf{i} + (0.8 - 0.3)\mathbf{j} + (0 - 0)\mathbf{k} = [0.1\mathbf{i} + 0.5\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{BA} = (0.2 - 0.3)\mathbf{i} + (0.3 - 0.8)\mathbf{j} + (0 - 0)\mathbf{k} = [-0.1\mathbf{i} - 0.5\mathbf{j}]\text{ m}$$

The force vectors **F** and –**F** can be written as

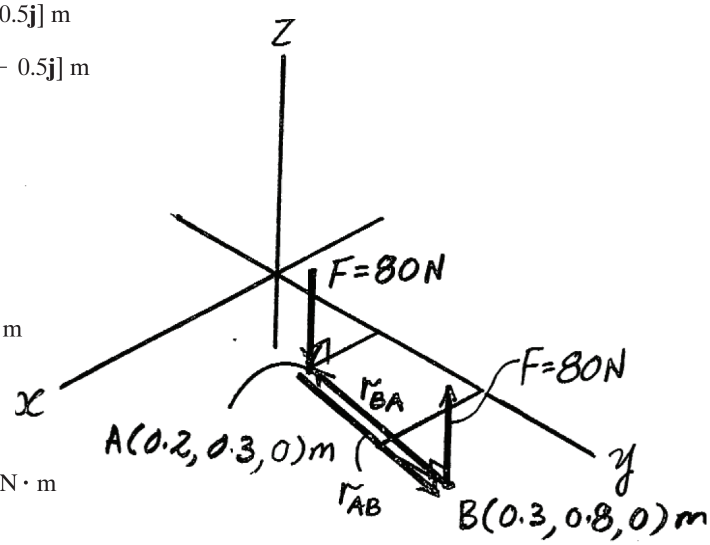
$$\mathbf{F} = \{80\,\mathbf{k}\}\text{ N and } -\mathbf{F} = [-80\,\mathbf{k}]\text{ N}$$

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\text{ N}\cdot\text{m}$$

or

$$\mathbf{M}_c = \mathbf{r}_{BA} \times -\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.1 & -0.5 & 0 \\ 0 & 0 & -80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\text{ N}\cdot\text{m}$$



(a)

The magnitude of M_c is given by

$$M_c = \sqrt{M_x{}^2 + M_y{}^2 + M_z{}^2} = \sqrt{40^2 + (-8)^2 + 0^2} = 40.79\text{ N}\cdot\text{m} = 40.8\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $\mathbf{M}_c$ are

$$\alpha = \cos^{-1}\left(\frac{M_x}{M}\right) = \cos\left(\frac{40}{40.79}\right) = 11.3° \qquad \textbf{Ans.}$$

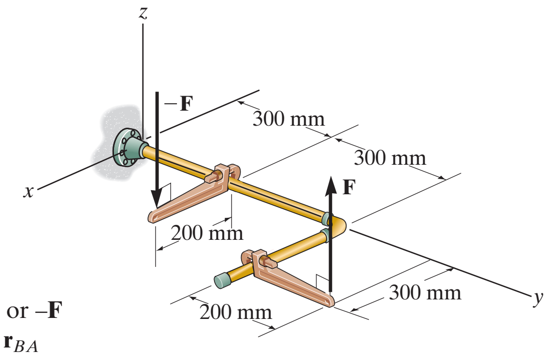
$$\beta = \cos^{-1}\left(\frac{M_y}{M}\right) = \cos\left(\frac{-8}{40.79}\right) = 101° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{M_z}{M}\right) = \cos\left(\frac{0}{40.79}\right) = 90° \qquad \textbf{Ans.}$$

Ans:
$M_c = 40.8$ N·m
$\alpha = 11.3°$
$\beta = 101°$
$\gamma = 90°$

3–67.

If the magnitude of the couple moment acting on the pipe assembly is 50 N · m, determine the magnitude of the couple forces applied to each wrench. The pipe assembly lies in the x–y plane.



SOLUTION

It is easiest to find the couple moment of **F** by taking the moment of either **F** or –**F** about point A or B, respectively, in Fig. a. Here, the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3 - 0.2)\mathbf{i} + (0.8 - 0.3)\mathbf{j} + (0 - 0)\mathbf{k} = [0.1\mathbf{i} + 0.5\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{BA} = (0.2 - 0.3)\mathbf{i} + (0.3 - 0.8)\mathbf{j} + (0 - 0)\mathbf{k} = [-0.1\mathbf{i} - 0.5\mathbf{j}]\text{ m}$$

The force vectors **F** and –**F** can be written as
$\mathbf{F} = \{F\mathbf{k}\}$ N and $-\mathbf{F} = [-F\mathbf{k}]$N

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & F \end{vmatrix} = 0.5F\mathbf{i} - 0.1F\mathbf{j}$$

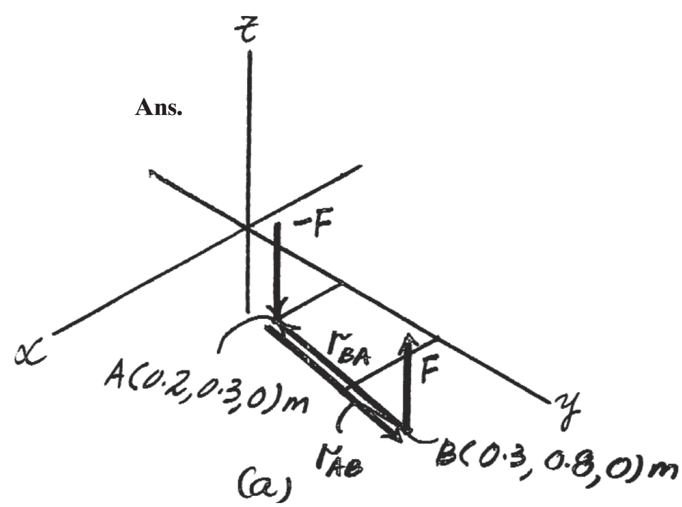
The magnitude of M_c is given by

$$M_c = \sqrt{M_x^{\,2} + M_y^{\,2} + M_z^{\,2}} = \sqrt{(0.5F)^2 + (0.1F)^2 + 0^2} = 0.5099F$$

Since M_c is required to equal 50 N · m,

$$50 = 0.5099F$$

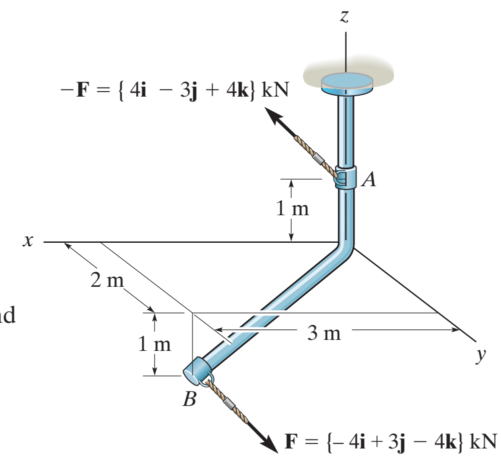
$$F = 98.1\text{ N}$$ **Ans.**



Ans:
$F = 98.1$ N

***3–68.**

Express the moment of the couple acting on the rod in Cartesian vector form. What is the magnitude of the couple moment?



SOLUTION

Position Vector. The coordinates of points A and B are $A\,(0, 0, 1)$ m and $B\,(3, 2, -1)$ m, respectively. Thus,

$$\mathbf{r}_{AB} = (3 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (-1 - 1)\mathbf{k} = \{3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}\}\text{ m}$$

Couple Moment.

$$\mathbf{M}_C = \mathbf{r}_{AB} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -2 \\ -4 & 3 & -4 \end{vmatrix}$$

$$= \{-2\mathbf{i} + 20\mathbf{j} + 17\mathbf{k}\}\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The magnitude of $\mathbf{M}_C$ is

$$M_C = \sqrt{(M_C)_x^2 + (M_C)_y^2 + (M_C)_z^2}$$

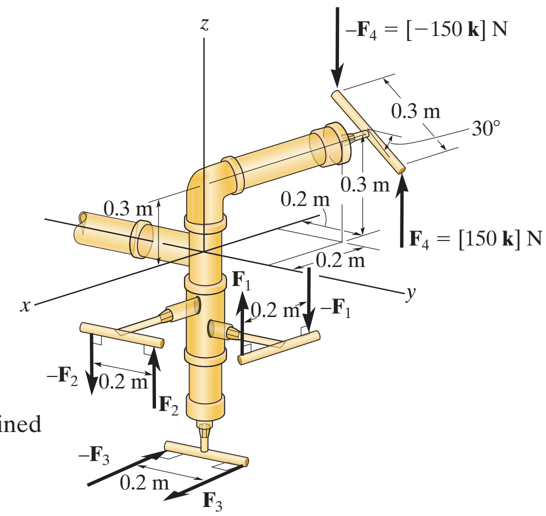
$$= \sqrt{(-2)^2 + 20^2 + 17^2}$$

$$= 26.32\text{ kN}\cdot\text{m} = 26.3\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_C = \{-2\mathbf{i} + 20\mathbf{j} + 17\mathbf{k}\}$ kN·m
$M_C = 26.3$ kN·m

3–69.

If $F_1 = 100$ N, $F_2 = 120$ N, and $F_3 = 80$ N, determine the magnitude and coordinate direction angles of the resultant couple moment.



SOLUTION

Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, and $\mathbf{r}_4$, Fig. *a*, must be determined first.

$\mathbf{r}_1 = \{0.2\mathbf{i}\}$ m $\mathbf{r}_2 = \{0.2\mathbf{j}\}$ m $\mathbf{r}_3 = \{0.2\mathbf{j}\}$ m

From the geometry of Figs. *b* and *c*, we obtain

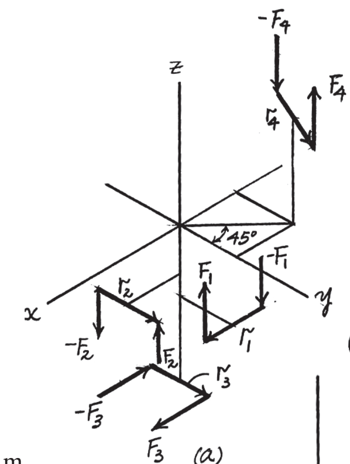
$$\mathbf{r}_4 = 0.3\cos 30^\circ \cos 45^\circ\mathbf{i} + 0.3\cos 30^\circ \sin 45^\circ\mathbf{j} - 0.3\sin 30^\circ\mathbf{k}$$
$$= \{0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}\}\text{ m}$$

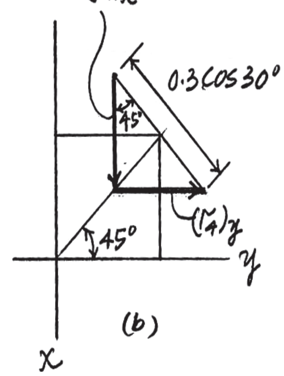
The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

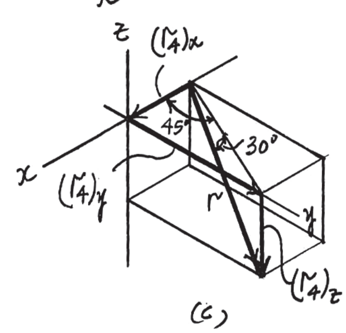
$\mathbf{F}_1 = \{100\mathbf{k}\}$ N $\mathbf{F}_2 = \{120\mathbf{k}\}$ N $\mathbf{F}_3 = \{80\mathbf{i}\}$ N

Thus,

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (0.2\mathbf{i}) \times (100\mathbf{k}) = \{-20\mathbf{j}\}\text{ N}\cdot\text{m}$$
$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (0.2\mathbf{j}) \times (120\mathbf{k}) = \{24\mathbf{i}\}\text{ N}\cdot\text{m}$$
$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (0.2\mathbf{j}) \times (80\mathbf{i}) = \{-16\mathbf{k}\}\text{ N}\cdot\text{m}$$
$$\mathbf{M}_4 = \mathbf{r}_4 \times \mathbf{F}_4 = (0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}) \times (150\mathbf{k}) = \{27.56\mathbf{i} - 27.56\mathbf{j}\}\text{ N}\cdot\text{m}$$







Resultant Moment: The resultant couple moment is given by

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}_c; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$
$$= (-20\mathbf{j}) + (24\mathbf{i}) + (-16\mathbf{k}) + (27.56\mathbf{i} - 27.56\mathbf{j})$$
$$= \{51.56\mathbf{i} - 47.56\mathbf{j} - 16\mathbf{k}\}\text{ N}\cdot\text{m}$$

The magnitude of the couple moment is

$$(\mathbf{M}_c)_R = \sqrt{[(\mathbf{M}_c)_R]_x^2 + [(\mathbf{M}_c)_R]_y^2 + [(\mathbf{M}_c)_R]_z^2}$$
$$= \sqrt{(51.56)^2 + (-47.56)^2 + (-16)^2}$$
$$= 71.94\text{ N}\cdot\text{m} = 71.9\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $(\mathbf{M}_c)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_c)_R]_x}{(M_c)_R}\right) = \cos\left(\frac{51.56}{71.94}\right) = 44.2^\circ \qquad \textbf{Ans.}$$

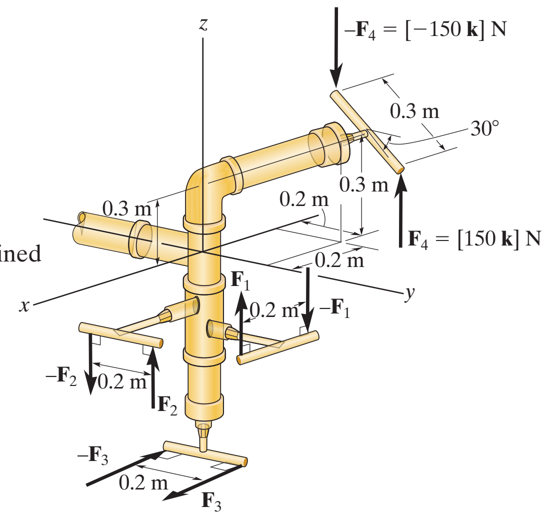
$$\beta = \cos^{-1}\left(\frac{[(M_c)_R]_y}{(M_c)_R}\right) = \cos\left(\frac{-47.56}{71.94}\right) = 131^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_c)_R]_z}{(M_c)_R}\right) = \cos\left(\frac{-16}{71.94}\right) = 103^\circ \qquad \textbf{Ans.}$$

Ans:
$(M_C)_R = 71.9$ N·m
$\alpha = 44.2^\circ$
$\beta = 131^\circ$
$\gamma = 103^\circ$

3–70.

Determine the required magnitude of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ so that the resultant couple moment is $(\mathbf{M}_c)_R = [50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k}]$ N·m.



SOLUTION

Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, and $\mathbf{r}_4$, Fig. *a*, must be determined first.

$\mathbf{r}_1 = \{0.2\mathbf{i}\}$ m $\qquad \mathbf{r}_2 = \{0.2\mathbf{j}\}$ m $\qquad \mathbf{r}_3 = \{0.2\mathbf{j}\}$ m

From the geometry of Figs. *b* and *c*, we obtain

$$\mathbf{r}_4 = 0.3\cos 30^\circ \cos 45^\circ\mathbf{i} + 0.3\cos 30^\circ \sin 45^\circ\mathbf{j} - 0.3\sin 30^\circ\mathbf{k}$$
$$= \{0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}\}\text{ m}$$

The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = F_1\mathbf{k} \qquad \mathbf{F}_2 = F_2\mathbf{k} \qquad \mathbf{F}_3 = F_3\mathbf{i}$

Thus,

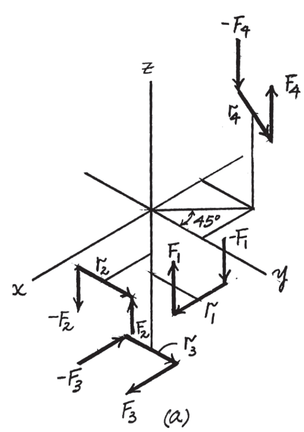
$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (0.2\mathbf{i}) \times (F_1\mathbf{k}) = -0.2\,F_1\mathbf{j}$$
$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (0.2\mathbf{j}) \times (F_2\mathbf{k}) = 0.2\,F_2\mathbf{i}$$
$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (0.2\mathbf{j}) \times (F_3\mathbf{i}) = -0.2\,F_3\mathbf{k}$$
$$\mathbf{M}_4 = \mathbf{r}_4 \times \mathbf{F}_4 = (0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}) \times (150\mathbf{k}) = \{27.56\mathbf{i} - 27.56\mathbf{j}\}\text{ N}\cdot\text{m}$$

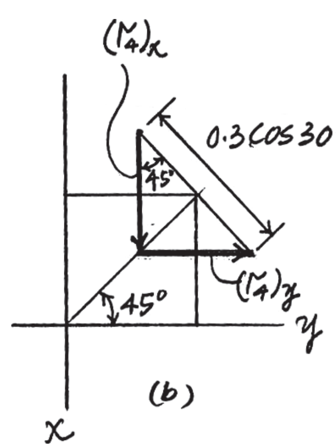
Resultant Moment: The resultant couple moment is required to equal $(\mathbf{M}_c)_R = \{50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k}\}$ N·m. Thus,

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}_c; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$
$$50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k} = (-0.2F_1\mathbf{j}) + (0.2F_2\mathbf{i}) + (-0.2F_3\mathbf{k}) + (27.56\mathbf{i} - 27.56\mathbf{j})$$
$$50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k} = (0.2F_2 + 27.56)\mathbf{i} + (-0.2F_1 - 27.56)\mathbf{j} - 0.2F_3\mathbf{k}$$

Equating the **i**, **j**, and **k** components yields

$50 = 0.2F_2 + 27.56$	$F_2 = 112$ N	**Ans.**
$-45 = -0.2F_1 - 27.56$	$F_1 = 87.2$ N	**Ans.**
$-20 = -0.2F_3$	$F_3 = 100$ N	**Ans.**

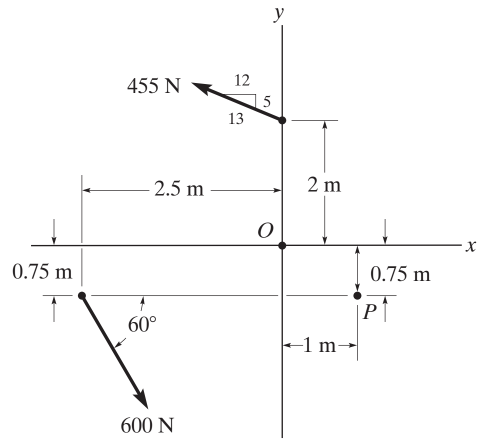




Ans:
$F_2 = 112$ N
$F_1 = 87.2$ N
$F_3 = 100$ N

3–71.

Replace the force system by an equivalent resultant force and couple moment at point O.



SOLUTION

Equivalent Resultant Force And Couple Moment At *O*.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 600 \cos 60° - 455\left(\frac{12}{13}\right) = -120 \text{ N} = 120 \text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 455\left(\frac{5}{13}\right) - 600 \sin 60° = -344.62 \text{ N} = 344.62 \text{ N}\downarrow$$

As indicated in Fig. *a*,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{120^2 + 344.62^2} = 364.91 \text{ N} = 365 \text{ N} \qquad \textbf{Ans.}$$

And

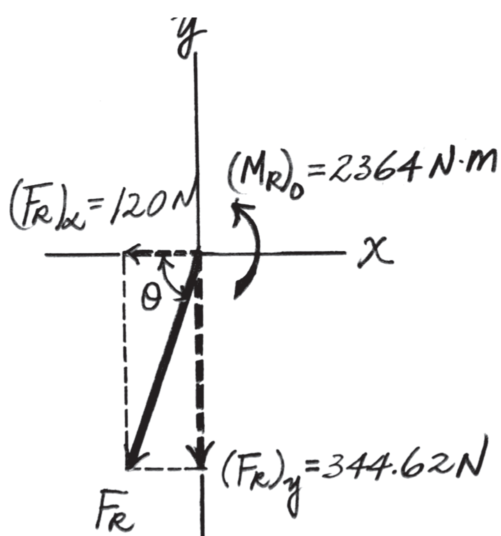
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{344.62}{120}\right) = 70.80° = 70.8° \qquad \textbf{Ans.}$$

Also,

$$\circlearrowleft +(M_R)_O = \Sigma M_O; \; (M_R)_O = 455\left(\frac{12}{13}\right)(2) + 600 \cos 60° \,(0.75) + 600 \sin 60° \,(2.5)$$

$$= 2364.04 \text{ N} \cdot \text{m}$$

$$= 2364 \text{ N} \cdot \text{m (\textbf{counterclockwise})} \qquad \textbf{Ans.}$$

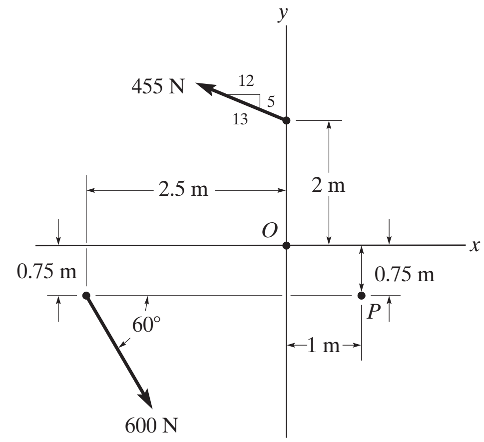


(a)

Ans:
$F_R = 365$ N
$\theta = 70.8°$
$(M_R)_O = 2364$ N · m (**counterclockwise**)

***3–72.**

Replace the force system by an equivalent resultant force and couple moment at point P.



SOLUTION

Equivalent Resultant Force And Couple Moment At P.

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 600 \cos 60° - 455\left(\frac{12}{13}\right) = -120 \text{ N} = 120 \text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 455\left(\frac{5}{13}\right) - 600 \sin 60° = -344.62 \text{ N} = 344.62 \text{ N} \downarrow$$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{120^2 + 344.62^2} = 364.91 \text{ N} = 365 \text{ N} \qquad \textbf{Ans.}$$

And

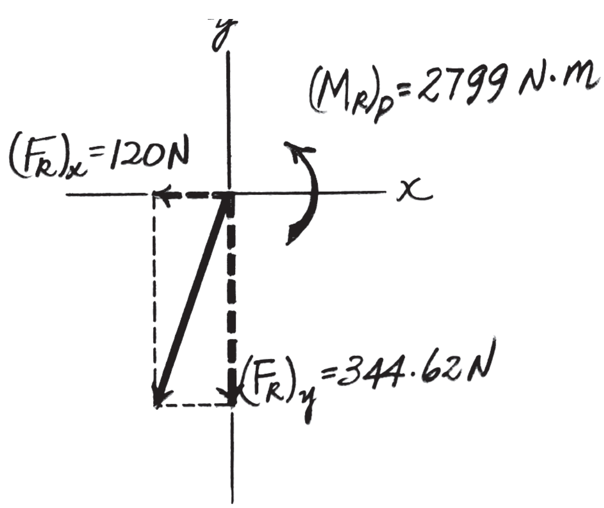
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{344.62}{120}\right) = 70.80° = 70.8° \measuredangle \qquad \textbf{Ans.}$$

Also,

$$\circlearrowleft +(M_R)_P = \Sigma M_P; \quad (M_R)_P = 455\left(\frac{12}{13}\right)(2.75) - 455\left(\frac{5}{13}\right)(1) + 600 \sin 60° \,(3.5)$$

$$= 2798.65 \text{ N} \cdot \text{m}$$

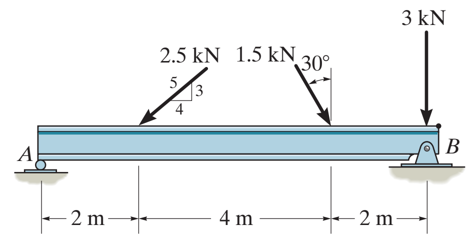
$$= 2799 \text{ N} \cdot \text{m } (\textbf{counterclockwise}) \qquad \textbf{Ans.}$$



Ans:
$F_R = 365$ N
$\theta = 70.8° \measuredangle$
$(M_R)_P = 2799$ N · m (**counterclockwise**)

3–73.

Replace the loading acting on the beam by an equivalent force and couple moment at point A.



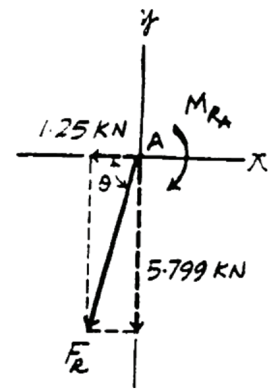
SOLUTION

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 1.5 \sin 30^\circ - 2.5\left(\frac{4}{5}\right)$$

$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -1.5 \cos 30^\circ - 2.5\left(\frac{3}{5}\right) - 3$$

$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \qquad \textbf{Ans.}$$

and

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8^\circ \qquad \textbf{Ans.}$$

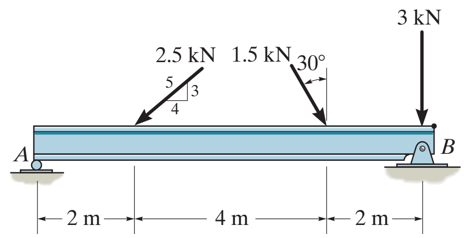
$$\circlearrowleft + M_{R_A} = \Sigma M_A; \qquad M_{R_A} = -2.5\left(\frac{3}{5}\right)(2) - 1.5 \cos 30^\circ(6) - 3(8)$$

$$= -34.8 \text{ kN}\cdot\text{m} = 34.8 \text{ kN}\cdot\text{m} \ \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$

Ans:
$F_R = 5.93$ kN
$\theta = 77.8^\circ$
$M_{R_A} = 34.8$ kN·m $\circlearrowright$

3–74.

Replace the loading acting on the beam by an equivalent force and couple moment at point B.



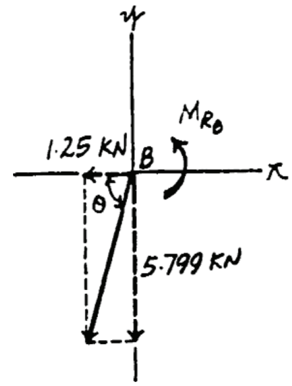
SOLUTION

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 1.5 \sin 30° - 2.5\left(\frac{4}{5}\right)$$

$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -1.5 \cos 30° - 2.5\left(\frac{3}{5}\right) - 3$$

$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \qquad \textbf{Ans.}$$

and

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8° \qquad \textbf{Ans.}$$

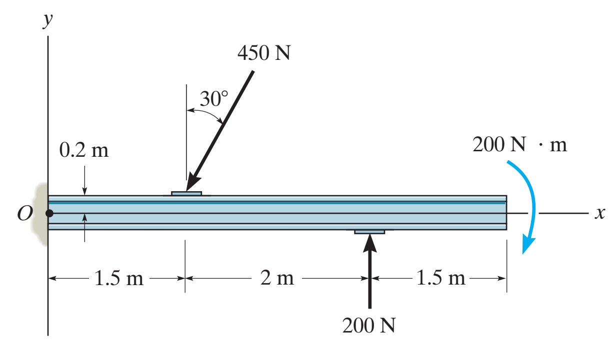
$$\circlearrowleft + M_{R_B} = \Sigma M_{R_B}; \qquad M_B = 1.5 \cos 30°(2) + 2.5\left(\frac{3}{5}\right)(6)$$

$$= 11.6 \text{ kN} \cdot \text{m} \quad \textbf{(\textit{Counterclockwise})} \qquad \textbf{Ans.}$$

Ans:
$F_R = 5.93$ kN
$\theta = 77.8°$
$M_B = 11.6$ kN · m **(*Counterclockwise*)**

3–75.

Replace the loading acting on the beam by an equivalent resultant force and couple moment at point O.



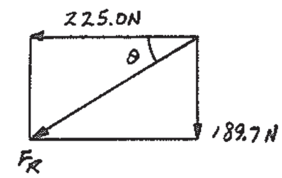
SOLUTION

$\overset{+}{\leftarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 450 \sin 30° = 225.0$

$+\downarrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 450 \cos 30° - 200 = 189.7$

$F_R = \sqrt{(225)^2 + (189.7)^2} = 294 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{189.7}{225}\right) = 40.1°$ **Ans.**



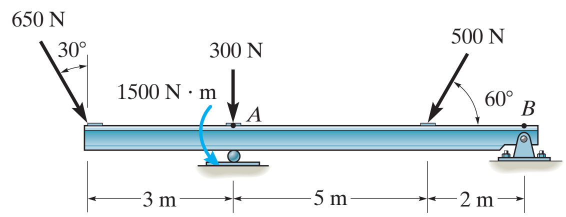
$\circlearrowleft + M_{RO} = \Sigma M_O; \quad M_{RO} = 450 \cos 30° (1.5) - 450 (\sin 30°)(0.2) - 200 (3.5) + 200$

$M_{RO} = 39.6 \text{ N} \cdot \text{m} \circlearrowright$ **Ans.**

Ans:
$F_R = 294$ N
$\theta = 40.1°$
$M_{RO} = 39.6 \text{ N} \cdot \text{m} \circlearrowright$

***3–76.**

Replace the loading acting on the post by an equivalent resultant force and couple moment at point A.



SOLUTION

Equivalent Resultant Force And Couple Moment at Point A.

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 650 \sin 30° - 500 \cos 60° = 75 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -650 \cos 30° - 300 - 500 \sin 60°$$

$$= -1295.93 \text{ N} = 1295.93 \text{ N} \downarrow$$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{75^2 + 1295.93^2} = 1298.10 \text{ N} = 1.30 \text{ kN} \quad \textbf{Ans.}$$

And

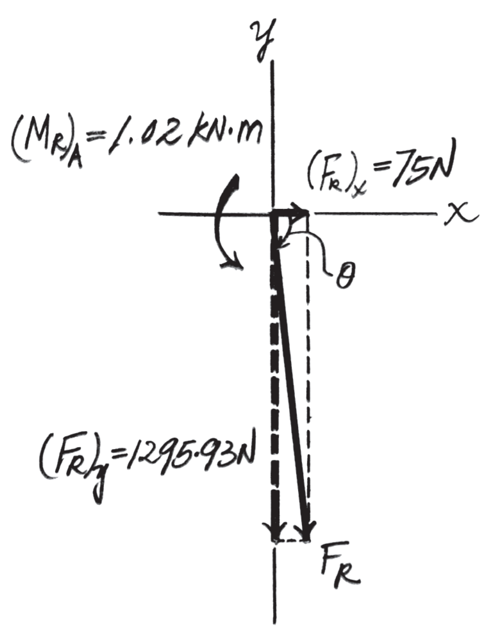
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{1295.93}{75}\right) = 86.69° = 86.7° \measuredangle \quad \textbf{Ans.}$$

Also,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad (M_R)_A = 650 \cos 30° (3) + 1500 - 500 \sin 60° (5)$$

$$= 1023.69 \text{ N} \cdot \text{m}$$

$$= 1.02 \text{ kN} \cdot \text{m (\textbf{counterclockwise})} \quad \textbf{Ans.}$$

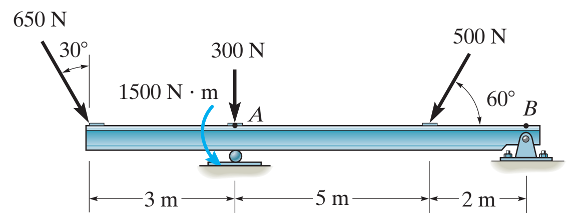


(a)

Ans:
$F_R = 1.30$ kN
$\theta = 86.7° \measuredangle$
$(M_R)_A = 1.02$ kN · m (**counterclockwise**)

3–77.

Replace the loading acting on the post by an equivalent resultant force and couple moment at point B.



SOLUTION

Equivalent Resultant Force And Couple Moment At Point *B*.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 650 \sin 30° - 500 \cos 60° = 75 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -650 \cos 30° - 300 - 500 \sin 60°$$

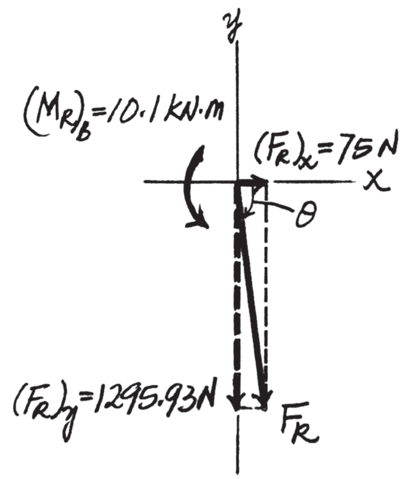
$$= -1295.93 \text{ N} = 1295.93 \text{ N}\downarrow$$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{75^2 + 1295.93^2} = 1298.10 \text{ N} = 1.30 \text{ kN} \quad \textbf{Ans.}$$

And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{1295.93}{75}\right) = 86.69° = 86.7° \quad \textbf{Ans.}$$



Also,

$$\circlearrowleft + (M_R)_B = \Sigma M_B; \quad (M_R)_B = 650 \cos 30° (10) + 300(7) + 500 \sin 60°(2) + 1500$$

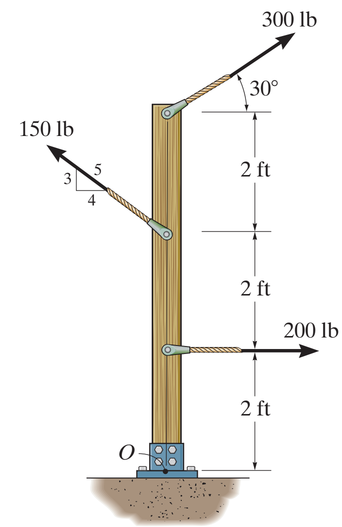
$$= 10{,}095.19 \text{ N} \cdot \text{m}$$

$$= 10.1 \text{ kN} \cdot \text{m } (\textbf{counterclockwise}) \quad \textbf{Ans.}$$

Ans:
$F_R = 1.30$ kN
$\theta = 86.7°$
$(M_R)_B = 1.01$ kN · m (**counterclockwise**)

3–78.

Replace the loading acting on the post by a resultant force and couple moment at point O.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes, we have

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 300\cos 30° - 150\left(\frac{4}{5}\right) + 200 = 339.81 \text{ lb} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 300\sin 30° + 150\left(\frac{3}{5}\right) = 240 \text{ lb} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{339.81^2 + 240^2} = 416.02 \text{ lb} = 416 \text{ lb} \qquad \textbf{Ans.}$$

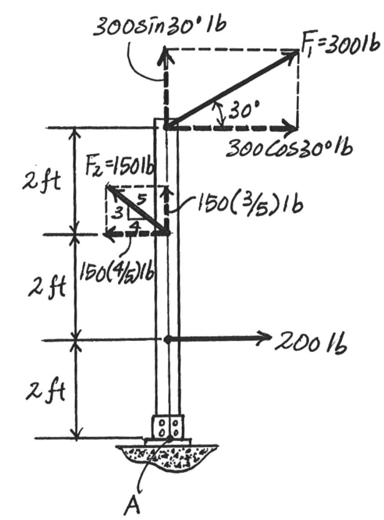
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{240}{339.81}\right] = 35.23° = 35.2° \measuredangle \qquad \textbf{Ans.}$$

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A, we can write

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad (M_R)_A = 150\left(\frac{4}{5}\right)(4) - 200(2) - 300\cos 30°(6)$$

$$= -1478.85 \text{ lb}\cdot\text{ft} = 1.48 \text{ kip}\cdot\text{ft } \textbf{(\textit{Clockwise}) Ans.}$$

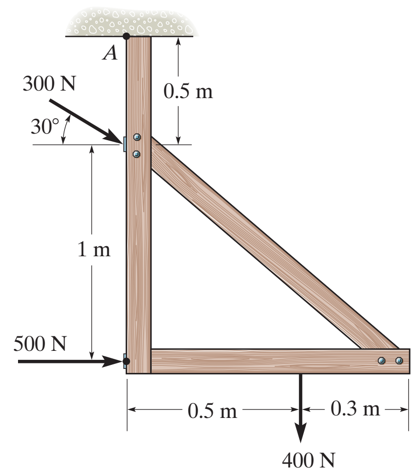


(a)

Ans:
$F_R = 416$ lb
$\theta = 35.2° \measuredangle$
$(M_R)_A = 1.48$ kip · ft ***(Clockwise)***

3–79.

Replace the loading acting on the frame by an equivalent resultant force and couple moment acting at point A.



SOLUTION

Equivalent Resultant Force And Couple Moment At *A*.

$\xrightarrow{+}$ $(F_R)_x = \Sigma F_x$; $\quad (F_R)_x = 300 \cos 30° + 500 = 759.81 \text{ N} \rightarrow$

$+\uparrow$ $(F_R)_y = \Sigma F_y$; $\quad (F_R)_y = -300 \sin 30° - 400 = -550 \text{ N} = 550 \text{ N} \downarrow$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{759.81^2 + 550^2} = 937.98 \text{ N} = 938 \text{ N}$$ **Ans.**

And

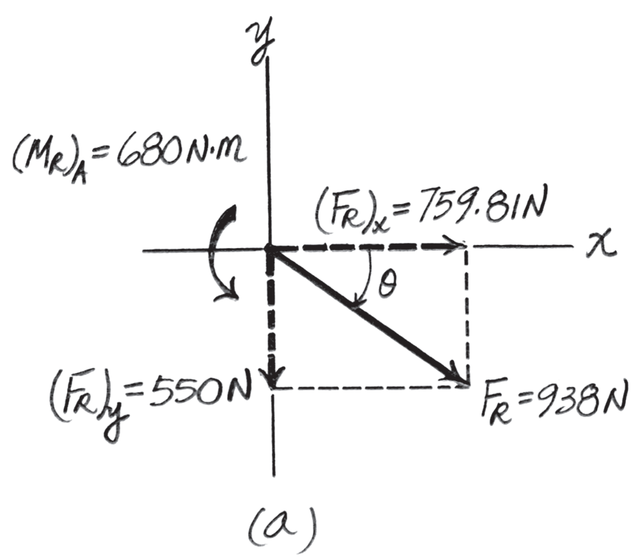
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{550}{759.81}\right) = 35.90° = 35.9° \measuredangle$$ **Ans.**

Also,

$\circlearrowleft +(M_R)_A = \Sigma M_A$; $\quad (M_R)_A = 300 \cos 30°(0.5) + 500(1.5) - 400(0.5)$

$= 679.90 \text{ N} \cdot \text{m}$

$= 680 \text{ N} \cdot \text{m}$ (**counterclockwise**) **Ans.**

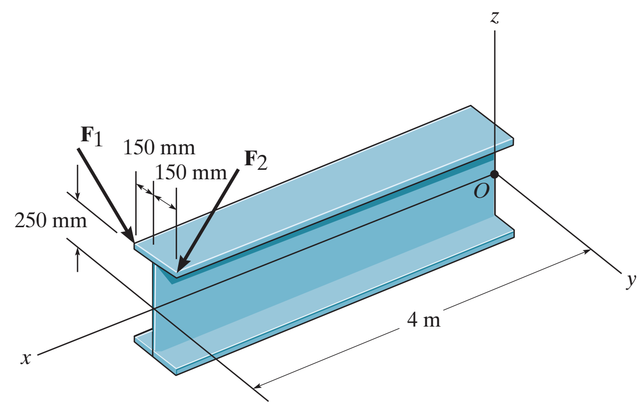


(a)

Ans:
$F_R = 938$ N
$\theta = 35.9° \measuredangle$
$(M_R)_A = 680$ N · m (**counterclockwise**)

***3–80.**

The forces $\mathbf{F}_1 = \{-4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\}$ kN and $\mathbf{F}_2 = \{3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}\}$ kN act on the end of the beam. Replace these forces by an equivalent force and couple moment acting at point O.



SOLUTION

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{-1\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}\}$ kN **Ans.**

$\mathbf{M}_{RO} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -0.15 & 0.25 \\ -4 & 2 & -3 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0.15 & 0.25 \\ 3 & -4 & -2 \end{vmatrix}$$

$$= (-0.05\mathbf{i} + 11\mathbf{j} + 7.4\mathbf{k}) + (0.7\mathbf{i} + 8.75\mathbf{j} - 16.45\mathbf{k})$$

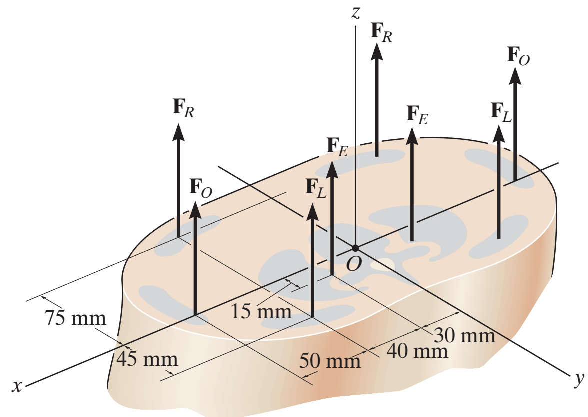
$$= (0.65\mathbf{i} + 19.75\mathbf{j} - 9.05\mathbf{k})$$

$\mathbf{M}_{RO} = \{0.650\mathbf{i} + 19.75\mathbf{j} - 9.05\mathbf{k}\}$ kN·m **Ans.**

Ans:
$\mathbf{M}_{RO} = \{0.650\mathbf{i} + 19.75\mathbf{j} - 9.05\mathbf{k}\}$ kN·m

3–81.

A biomechanical model of the lumbar region of the human trunk is shown. The forces acting in the four muscle groups consist of $F_R = 35$ N for the rectus, $F_O = 45$ N for the oblique, $F_L = 23$ N for the lumbar latissimus dorsi, and $F_E = 32$ N for the erector spinae. These loadings are symmetric with respect to the y–z plane. Replace this system of parallel forces by an equivalent force and couple moment acting at the spine, point O. Express the results in Cartesian vector form.



SOLUTION

$\mathbf{F}_R = \Sigma \mathbf{F}_z;\qquad \mathbf{F}_R = \{2(35 + 45 + 23 + 32)\mathbf{k}\} = \{270\mathbf{k}\}\text{ N}$ **Ans.**

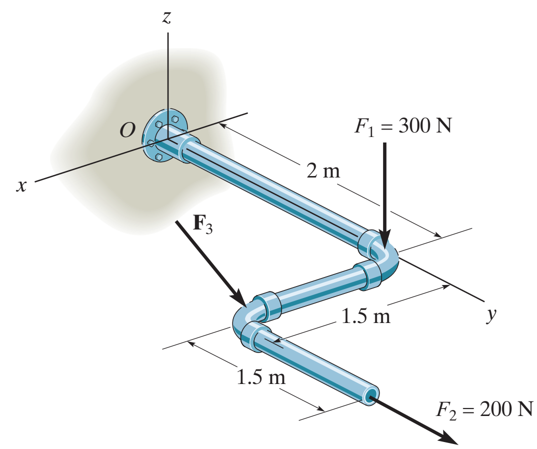
$\mathbf{M}_{RO_x} = \Sigma \mathbf{M}_{O_x};\qquad \mathbf{M}_{RO} = [-2(35)(0.075) + 2(32)(0.015) + 2(23)(0.045)]\mathbf{i}$

$\mathbf{M}_{RO} = \{-2.22\mathbf{i}\}\text{ N}\cdot\text{m}$ **Ans.**

Ans:
$\mathbf{F}_R = \{270\mathbf{k}\}\text{ N}$
$\mathbf{M}_{RO} = \{-2.22\mathbf{i}\}\text{ N}\cdot\text{m}$

3–82.

Replace the loading by an equivalent resultant force and couple moment at point O. Take $\mathbf{F}_3 = \{-200\mathbf{i} + 500\mathbf{j} - 300\mathbf{k}\}$ N.



SOLUTION

Position And Force Vectors.

$$\mathbf{r}_1 = \{2\mathbf{j}\}\text{ m} \qquad \mathbf{r}_2 = \{1.5\mathbf{i} + 3.5\mathbf{j}\} \qquad \mathbf{r}_3 = \{1.5\mathbf{i} + 2\mathbf{j}\}\text{ m}$$

$$\mathbf{F}_1 = \{-300\mathbf{k}\}\text{ N} \quad \mathbf{F}_2 = \{200\mathbf{j}\}\text{ N} \qquad \mathbf{F}_3 = \{-200\mathbf{i} + 500\mathbf{j} - 300\mathbf{k}\}\text{ N}$$

Equivalent Resultant Force And Couple Moment At Point *O*.

$$\mathbf{F}_R = \Sigma F; \qquad F_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (-300\mathbf{k}) + 200\mathbf{j} + (-200\mathbf{i} + 500\mathbf{j} - 300\mathbf{k})$$

$$= \{-200\mathbf{i} + 700\mathbf{j} - 600\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O; \quad (\mathbf{M}_R)_O = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 0 & 0 & -300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 3.5 & 0 \\ 0 & 200 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2 & 0 \\ -200 & 500 & -300 \end{vmatrix}$$

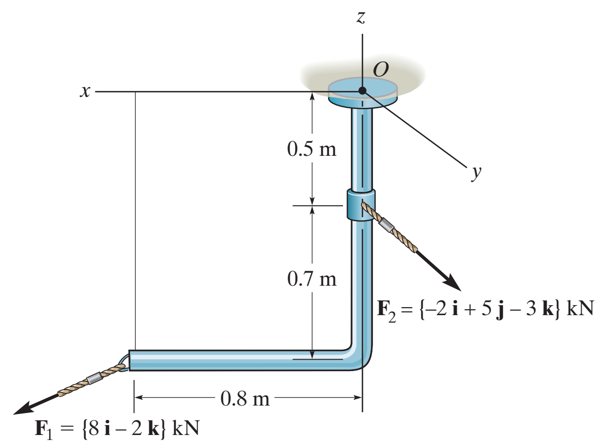
$$= (-600\mathbf{i}) + (300\mathbf{k}) + (-600\mathbf{i} + 450\mathbf{j} + 1150\mathbf{k})$$

$$= \{-1200\mathbf{i} + 450\mathbf{j} + 1450\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = \{-200\mathbf{i} + 700\mathbf{j} - 600\mathbf{k}\}$ N
$(\mathbf{M}_R)_O = \{-1200\mathbf{i} + 450\mathbf{j} + 1450\mathbf{k}\}$ N · m

3–83.

Replace the loading by an equivalent resultant force and couple moment at point O.



SOLUTION

Position Vectors. The required position vectors are

$$\mathbf{r}_1 = \{0.8\mathbf{i} - 1.2\mathbf{k}\}\text{ m} \qquad \mathbf{r}_2 = \{-0.5\mathbf{k}\}\text{ m}$$

Equivalent Resultant Force And Couple Moment At Point *O*.

$$\mathbf{F}_R = \Sigma\mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (8\mathbf{i} - 2\mathbf{k}) + (-2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$$

$$= \{6\mathbf{i} + 5\mathbf{j} - 5\mathbf{k}\}\text{ kN}$$ **Ans.**

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O; \qquad (\mathbf{M}_R)_O = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.8 & 0 & -1.2 \\ 8 & 0 & -2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.5 \\ -2 & 5 & -3 \end{vmatrix}$$

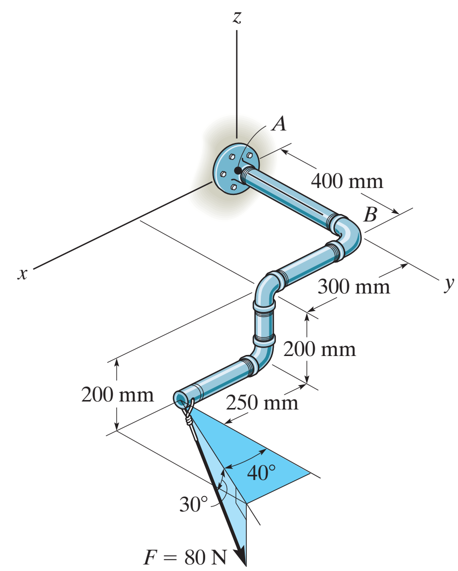
$$= (-8\mathbf{j}) + (2.5\mathbf{i} + \mathbf{j})$$

$$= \{2.5\mathbf{i} - 7\mathbf{j}\}\text{ kN}\cdot\text{m}$$ **Ans.**

Ans:
$\mathbf{F}_R = \{6\mathbf{i} + 5\mathbf{j} - 5\mathbf{k}\}$ kN
$(\mathbf{M}_R)_O = \{2.5\mathbf{i} - 7\mathbf{j}\}$ kN·m

***3–84.**

Replace the force of $F = 80$ N acting on the pipe assembly by an equivalent resultant force and couple moment at point A.



SOLUTION

$\mathbf{F}_R = \Sigma \mathbf{F}$;

$$\mathbf{F}_R = 80 \cos 30° \sin 40° \,\mathbf{i} + 80 \cos 30° \cos 40° \,\mathbf{j} - 80 \sin 30° \,\mathbf{k}$$

$$= 44.53 \,\mathbf{i} + 53.07 \,\mathbf{j} - 40 \,\mathbf{k}$$

$$= \{44.5 \,\mathbf{i} + 53.1 \,\mathbf{j} - 40 \,\mathbf{k}\} \text{ N}$$ **Ans.**

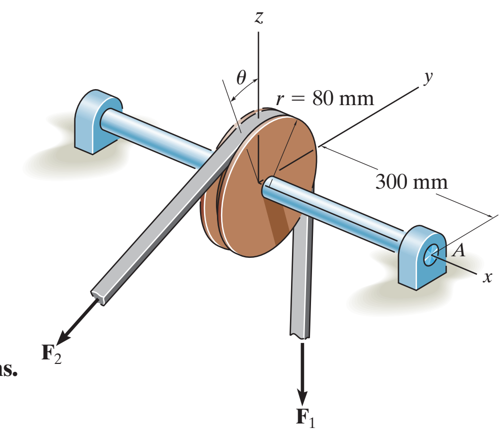
$$\mathbf{M}_{RA} = \Sigma \mathbf{M}_A; \quad \mathbf{M}_{RA} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40 \end{vmatrix}$$

$$= \{-5.39 \,\mathbf{i} + 13.1 \,\mathbf{j} + 11.4 \,\mathbf{k}\} \text{ N} \cdot \text{m}$$ **Ans.**

Ans:
$\mathbf{F}_R = \{44.5 \,\mathbf{i} + 53.1 \,\mathbf{j} - 40 \,\mathbf{k}\}$ N
$\mathbf{M}_{RA} = \{-5.39 \,\mathbf{i} + 13.1 \,\mathbf{j} + 11.4 \,\mathbf{k}\}$ N · m

3–85.

The belt passing over the pulley is subjected to forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Set $\theta = 0°$ so that $\mathbf{F}_2$ acts in the $-\mathbf{j}$ direction.



SOLUTION

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = \{-40\mathbf{j} - 40\mathbf{k}\}\ \text{N}$$ **Ans.**

$$\mathbf{M}_{RA} = \Sigma(\mathbf{r} \times \mathbf{F})$$

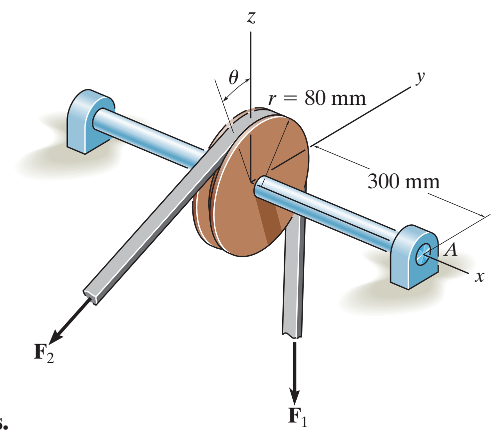
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0 & 0.08 \\ 0 & -40 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix}$$

$$\mathbf{M}_{RA} = \{-12\mathbf{j} + 12\mathbf{k}\}\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:
$\mathbf{F}_R = \{-40\mathbf{j} - 40\mathbf{k}\}$ N
$\mathbf{M}_{RA} = \{-12\mathbf{j} + 12\mathbf{k}\}$ N·m

3–86.

The belt passing over the pulley is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Take $\theta = 45°$.



SOLUTION

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

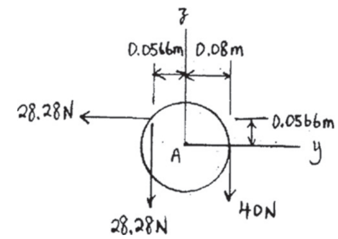
$$= -40\cos 45°\mathbf{j} + (-40 - 40\sin 45°)\mathbf{k}$$

$$\mathbf{F}_R = \{-28.3\mathbf{j} - 68.3\mathbf{k}\}\ \text{N} \qquad \textbf{Ans.}$$

$$\mathbf{r}_{AF1} = \{-0.3\mathbf{i} + 0.08\mathbf{j}\}\ \text{m}$$

$$\mathbf{r}_{AF2} = -0.3\mathbf{i} - 0.08\sin 45°\mathbf{j} + 0.08\cos 45°\mathbf{k}$$

$$= \{-0.3\mathbf{i} - 0.0566\mathbf{j} + 0.0566\mathbf{k}\}\ \text{m}$$



$$\mathbf{M}_{RA} = (\mathbf{r}_{AF1} \times \mathbf{F}_1) + (\mathbf{r}_{AF2} \times \mathbf{F}_2)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & -0.0566 & 0.0566 \\ 0 & -40\cos 45° & -40\sin 45° \end{vmatrix}$$

$$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Also,

$$M_{RA_x} = \Sigma M_{A_x}$$

$$M_{RA_x} = 28.28(0.0566) + 28.28(0.0566) - 40(0.08)$$

$$M_{RA_x} = 0$$

$$M_{RA_y} = \Sigma M_{A_y}$$

$$M_{RA_y} = -28.28(0.3) - 40(0.3)$$

$$M_{RA_y} = -20.5\ \text{N}\cdot\text{m}$$

$$M_{RA_z} = \Sigma M_{A_z}$$

$$M_{RA_z} = 28.28(0.3)$$

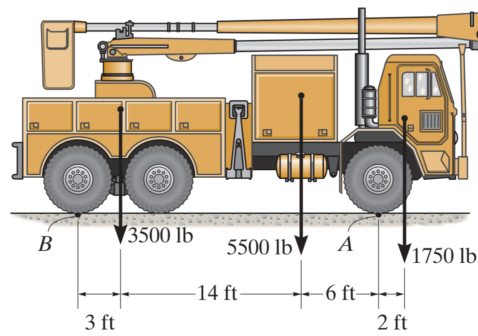
$$M_{RA_z} = 8.49\ \text{N}\cdot\text{m}$$

$$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_R = \{-28.3\mathbf{j} - 68.3\mathbf{k}\}$ N
$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}$ N·m

3–87.

The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location, measured from B.



SOLUTION

$$+\uparrow F_R = \Sigma F_y; \qquad F_R = -1750 - 5500 - 3500$$

$$= -10\,750 \text{ lb} = 10.75 \text{ kip}\downarrow \qquad \textbf{Ans.}$$

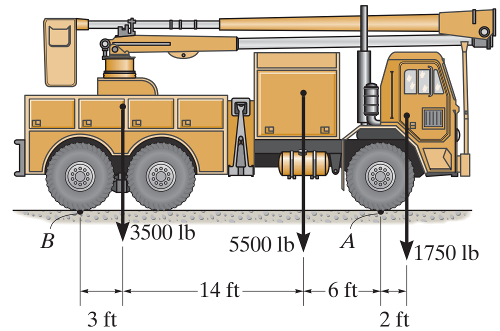
$$\circlearrowleft + M_{R_A} = \Sigma M_A; \quad -10\,750d = -3500(3) - 5500(17) - 1750(25)$$

$$d = 13.7 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 10.75$ kip$\downarrow$
$d = 13.7$ ft

***3–88.**

The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location, measured from point A.



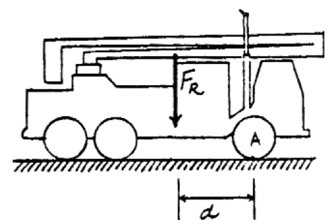
SOLUTION

Equivalent Force:

$$+\uparrow F_R = \Sigma F_y; \qquad F_R = -1750 - 5500 - 3500$$

$$= -10\,750 \text{ lb} = 10.75 \text{ kip}\downarrow \qquad \textbf{Ans.}$$

Location of Resultant Force From Point A:



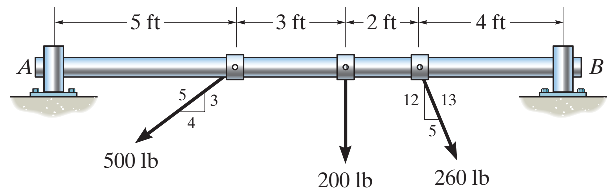
$$\circlearrowleft + M_{R_A} = \Sigma M_A; \qquad 10\,750(d) = 3500(20) + 5500(6) - 1750(2)$$

$$d = 9.26 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 10.75 \text{ kip}\downarrow$
$d = 9.26$ ft

3–89.

Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end A.



SOLUTION

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = -500\left(\frac{4}{5}\right) + 260\left(\frac{5}{13}\right) = -300 \text{ lb} = 300 \text{ lb} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -500\left(\frac{3}{5}\right) - 200 - 260\left(\frac{12}{13}\right) = -740 \text{ lb} = 740 \text{ lb} \downarrow$$

$$F = \sqrt{(-300)^2 + (-740)^2} = 798 \text{ lb} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{740}{300}\right) = 67.9^\circ \qquad \textbf{Ans.}$$

$$\circlearrowleft +M_{RA} = \Sigma M_A; \qquad 740(x) = 500\left(\frac{3}{5}\right)(5) + 200(8) + 260\left(\frac{12}{13}\right)(10)$$

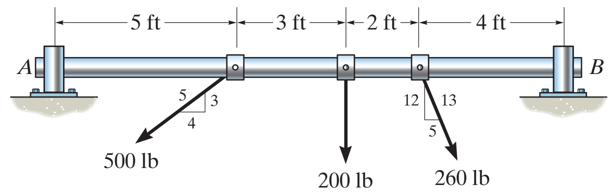
$$740(x) = 5500$$

$$x = 7.43 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F = 798$ lb
$\theta = 67.9^\circ$
$x = 7.43$ ft

3–90.

Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end B.



SOLUTION

$$\overset{+}{\rightarrow} \Sigma F_{R_x} = \Sigma F_x; \qquad F_{R_x} = -500\left(\frac{4}{5}\right) + 260\left(\frac{5}{13}\right) = -300 \text{ lb} = 300 \text{ lb} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -500\left(\frac{3}{5}\right) - 200 - 260\left(\frac{12}{13}\right) = -740 \text{ lb} = 740 \text{ lb} \downarrow$$

$$F = \sqrt{(-300)^2 + (-740)^2} = 798 \text{ lb} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{740}{300}\right) = 67.9^\circ \qquad \textbf{Ans.}$$

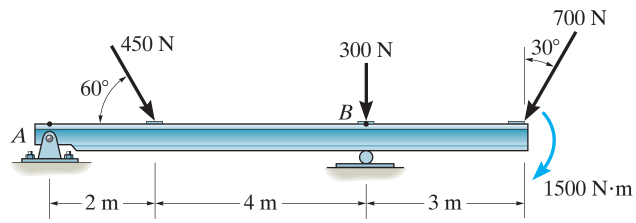
$$\circlearrowleft + M_{RB} = \Sigma M_B; \qquad 740(x) = 500\left(\frac{3}{5}\right)(9) + 200(6) + 260\left(\frac{12}{13}\right)(4)$$

$$x = 6.57 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F = 798$ lb
$\theta = 67.9^\circ$
$x = 6.57$ ft

3–91.

Replace the loading by a single resultant force. Specify where the force acts, measured from end A.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{1296}{125}\right) = 84.5°$ **Ans.**

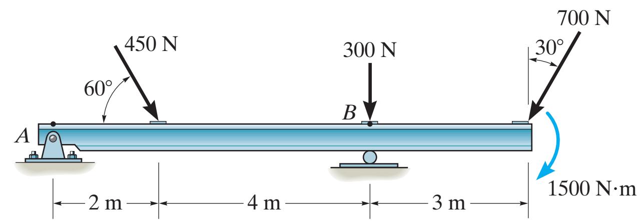
$\circlearrowleft + M_{RA} = \Sigma M_A;\qquad 1296(x) = 450 \sin 60°(2) + 300(6) + 700 \cos 30°(9) + 1500$

$x = 7.36 \text{ m}$ **Ans.**

Ans:
$F = 1302 \text{ N}$
$\theta = 84.5°$
$x = 7.36 \text{ m}$

***3–92.**

Replace the loading by a single resultant force. Specify where the force acts, measured from B.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{1296}{125}\right) = 84.5°$ **Ans.**

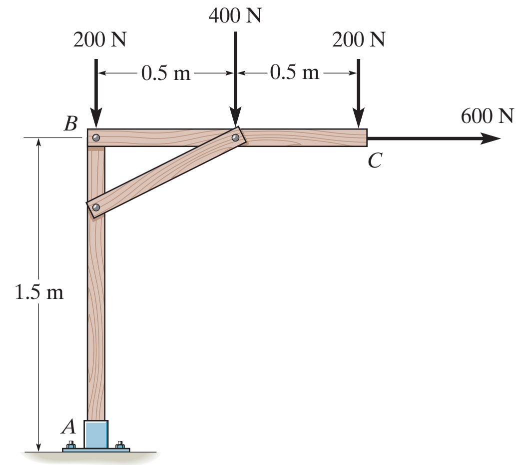
$\circlearrowleft + M_{RB} = \Sigma M_B; \qquad 1296(x) = -450 \sin 60°(4) + 700 \cos 30°(3) + 1500$

$x = 1.36 \text{ m (to the right)}$ **Ans.**

Ans:
$F = 1302 \text{ N}$
$\theta = 84.5°$
$x = 1.36 \text{ m (to the right)}$

3–93.

Replace the loading by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.



SOLUTION

Equivalent Resultant Force. Referring to Fig. *a*,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 600 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -200 - 400 - 200 = -800 \text{ N} = 800 \text{ N}\downarrow$$

As indicated in Fig. *a*,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{600^2 + 800^2} = 1000 \text{ N} \qquad \textbf{Ans.}$$

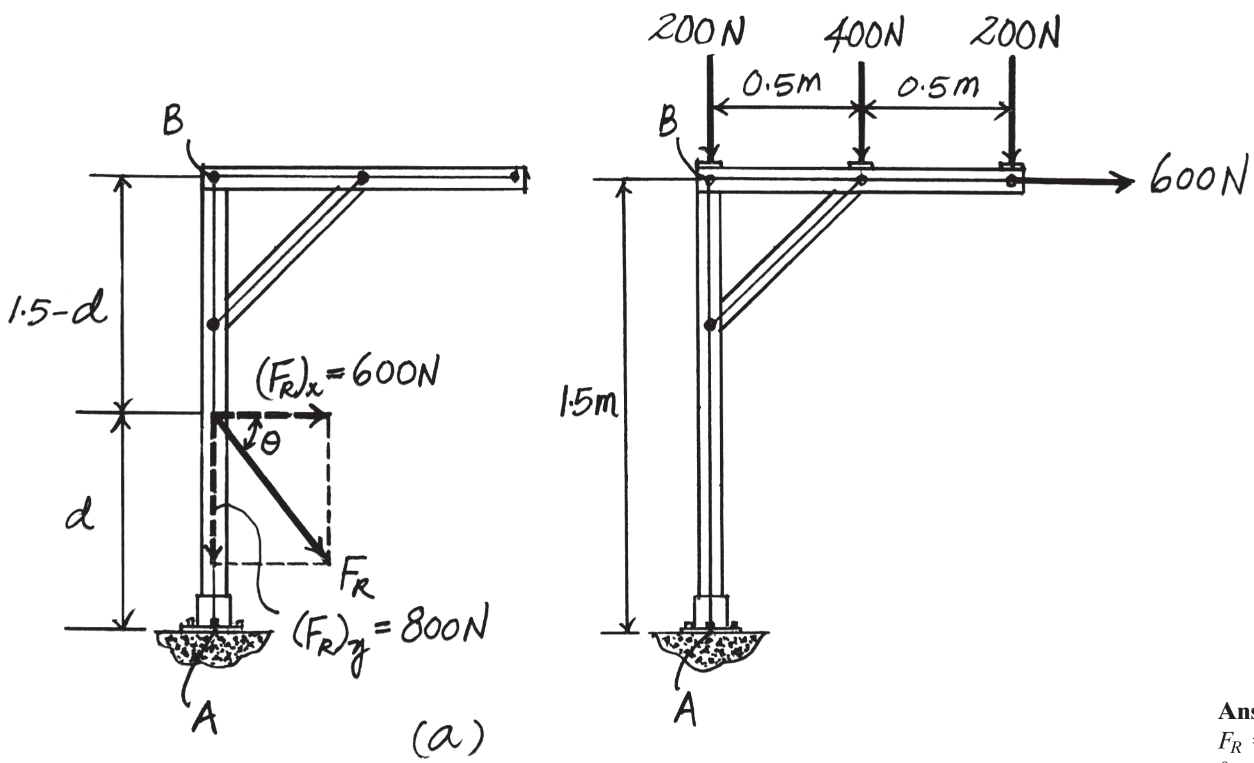
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{800}{600}\right) = 53.13° = 53.1° \qquad \textbf{Ans.}$$

Location of Resultant Force. Along AB,

$$\circlearrowleft + (M_R)_B = \Sigma M_B; \qquad 600(1.5 - d) = -400(0.5) - 200(1)$$

$$d = 2.1667 \text{ m} = 2.17 \text{ m} \qquad \textbf{Ans.}$$

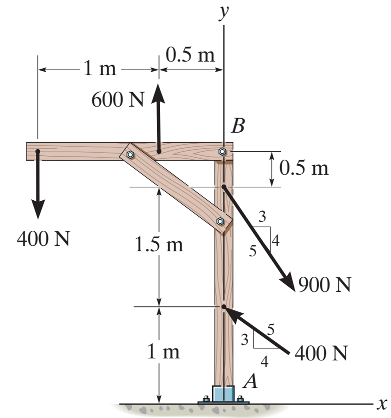


(a)

Ans:
$F_R = 1000$ N
$\theta = 53.1°$
$d = 2.17$ m

3–94.

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.



SOLUTION

Equivalent Resultant Force. Referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900\left(\frac{3}{5}\right) - 400\left(\frac{4}{5}\right) = 220\text{ N} \quad \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 + 400\left(\frac{3}{5}\right) - 400 - 900\left(\frac{4}{5}\right)$$

$$= -280\text{ N} = 280\text{ N} \downarrow$$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{220^2 + 280^2} = 356.09\text{ N} = 356\text{ N} \qquad \textbf{Ans.}$$

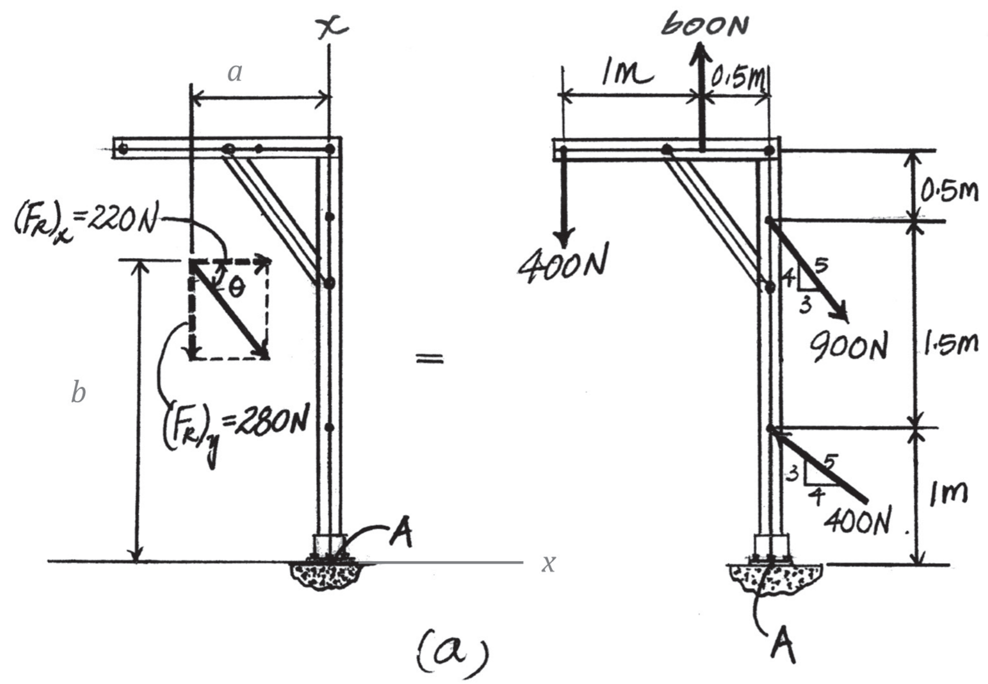
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{280}{220}\right) = 51.84° = 51.8° \qquad \textbf{Ans.}$$

Location of Resultant Force. Referring to Fig. a,

$$\zeta + (M_R)_A = \Sigma M_A; \qquad 280\,a - 220\,b = 400(1.5) - 600(0.5) - 900\left(\frac{3}{5}\right)(2.5) + 400\left(\frac{4}{5}\right)(1)$$

$$220\,b - 280\,a = 730 \qquad \textbf{(1)}$$



(a)

3–94. Continued

Along AB, $a = 0$. Then Eq. (1) becomes

$$220\,b - 280(0) = 730$$

$$b = 3.318 \text{ m}$$

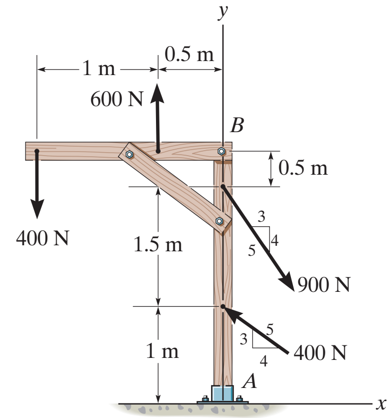
Thus, the intersection point of line of action of $\mathbf{F}_R$ on AB measured upward from point A is

$$d = b = 3.32 \text{ m}$$ **Ans.**

Ans:
$F_R = 356$ N
$\theta = 51.8°$
$d = b = 3.32$ m

3–95.

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a horizontal line along member CB, measured from end C.



SOLUTION

Equivalent Resultant Force. Referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900\left(\frac{3}{5}\right) - 400\left(\frac{4}{5}\right) = 220\text{ N} \quad \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 + 400\left(\frac{3}{5}\right) - 400 - 900\left(\frac{4}{5}\right)$$

$$= -280\text{ N} = 280\text{ N} \downarrow$$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{220^2 + 280^2} = 356.09\text{ N} = 356\text{ N} \qquad \textbf{Ans.}$$

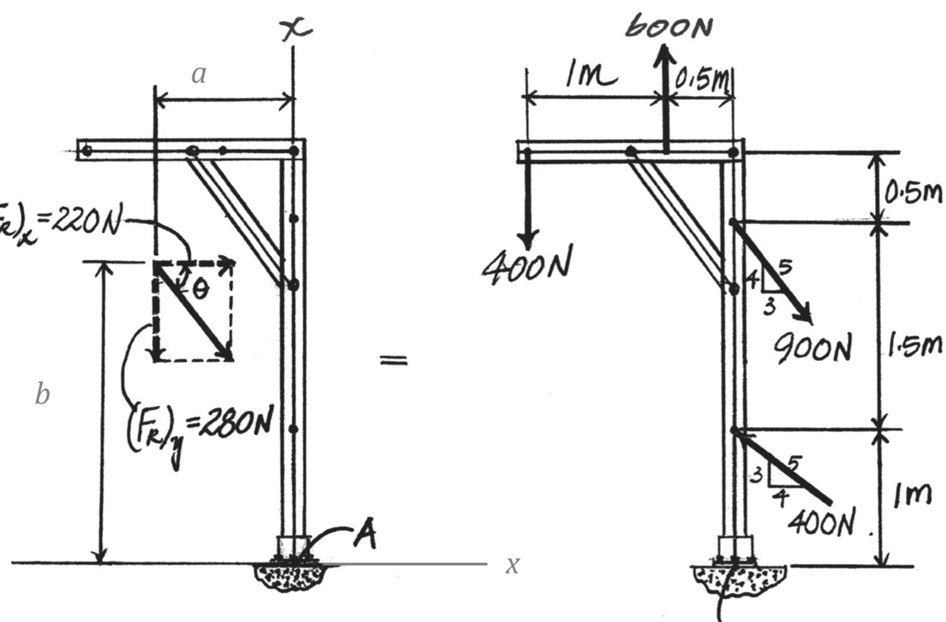
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{280}{220}\right) = 51.84° = 51.8° \qquad \textbf{Ans.}$$

Location of Resultant Force. Referring to Fig. a,

$$\zeta + (M_R)_A = \Sigma M_A; \qquad 280\,a - 220\,b = 400(1.5) - 600(0.5) - 900\left(\frac{3}{5}\right)(2.5) + 400\left(\frac{4}{5}\right)(1)$$

$$220\,b - 280\,a = 730 \qquad \textbf{(1)}$$



(a)

3–95. Continued

Along BC, $b = 3$ m. Then Eq. (1) becomes

$$220(3) - 280\,a = 730$$

$$a = -0.25 \text{ m}$$

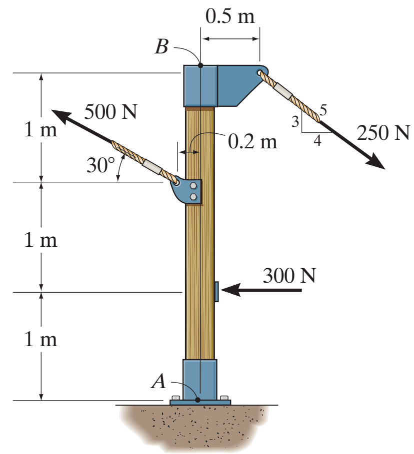
Thus, the intersection point of line of action of $\mathbf{F}_R$ on CB measured to the right of point C is

$$d = 1.5 - (-0.25) = 1.75 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 356$ N
$\theta = 51.8°$
$d = 1.75$ m

***3–96.**

Replace the loading acting on the post by a resultant force, and specify where its line of action intersects the post AB, measured from point A.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. *a*. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30^\circ - 300 = -533.01\text{ N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 500\sin 30^\circ - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N} \qquad \textbf{Ans.}$$

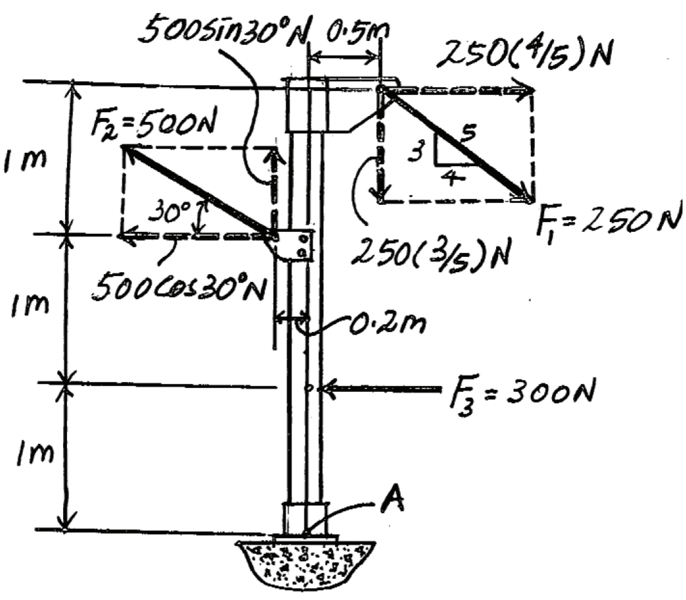
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63^\circ = 10.6^\circ \qquad \textbf{Ans.}$$

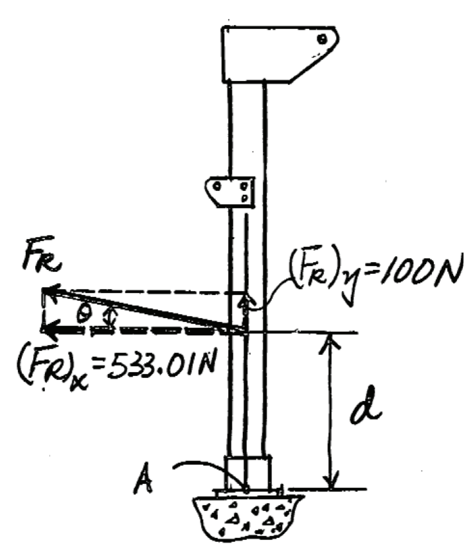
Location of the Resultant Force: Applying the principle of moments, Figs. *a* and *b*, and summing the moments of the force components algebraically about point A,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad 533.01(d) = 500\cos 30^\circ(2) - 500\sin 30^\circ(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 250\left(\frac{4}{5}\right)(3) + 300(1)$$

$$d = 0.8274\text{ mm} = 827\text{ mm} \qquad \textbf{Ans.}$$



(a)

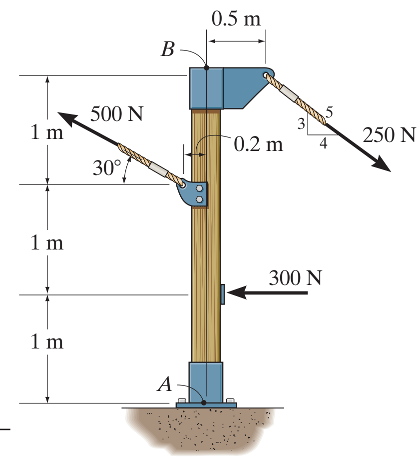


(b)

Ans:
$F_R = 542$ N
$\theta = 10.6^\circ$
$d = 0.827$ m

3–97.

Replace the loading acting on the post by a resultant force, and specify where its line of action intersects the post AB, measured from point B.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30° - 300 = -533.01\text{N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 500\sin 30° - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N} \qquad \textbf{Ans.}$$

The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63° = 10.6° \qquad \textbf{Ans.}$$

Location of the Resultant Force: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point B,

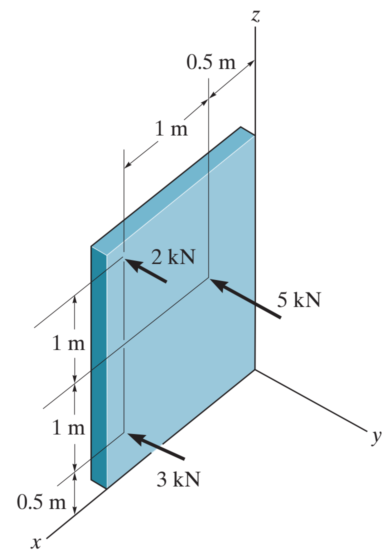
$$\circlearrowleft +(M_R)_B = \Sigma M_b; \qquad -533.01(d) = -500\cos 30°(1) - 500\sin 30°(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 300(2)$$

$$d = 2.17\text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 542$ N
$\theta = 10.6°$
$d = 2.17$ m

3–98.

Replace the parallel force system acting on the plate by a resultant force and specify its location on the x–z plane.



SOLUTION

Resultant Force: Summing the forces acting on the plate,

$(F_R)_y = \Sigma F_y;\qquad F_R = -5\text{ kN} - 2\text{ kN} - 3\text{ kN}$

$= -10\text{ kN}$ **Ans.**

The negative sign indicates that $\mathbf{F}_R$ acts along the negative y axis.

Resultant Moment: Using the right-hand rule, and equating the moment of $\mathbf{F}_R$ to the sum of the moments of the force system about the x and z axes,

$(M_R)_x = \Sigma M_x;\qquad (10\text{ kN})(z) = (3\text{ kN})(0.5\text{ m}) + (5\text{ kN})(1.5\text{ m}) + 2\text{ kN}(2.5\text{ m})$

$z = 1.40\text{ m}$ **Ans.**

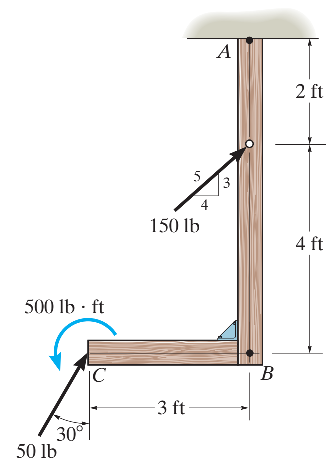
$(M_R)_z = \Sigma M_z;\qquad -(10\text{ kN})(x) = -(5\text{ kN})(0.5\text{ m}) - (2\text{ kN})(1.5\text{ m}) - (3\text{ kN})(1.5\text{ m})$

$x = 1.00\text{ m}$ **Ans.**

Ans:
$F_R = -10\text{ kN}$
$x = 1.00\text{ m}$
$z = 1.40\text{ m}$

3–99.

Replace the loading acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member AB, measured from A.



SOLUTION

$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 150\left(\frac{4}{5}\right) + 50 \sin 30° = 145 \text{ lb}$$

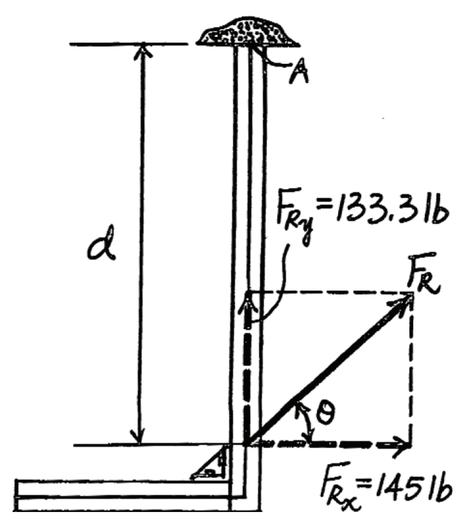
$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 50 \cos 30° + 150\left(\frac{3}{5}\right) = 133.3 \text{ lb}$$

$$F_R = \sqrt{(145)^2 + (133.3)^2} = 197 \text{ lb} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{133.3}{145}\right) = 42.6° \qquad \textbf{Ans.}$$

$$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 145\, d = 150\left(\frac{4}{5}\right)(2) - 50 \cos 30° (3) + 50 \sin 30° (6) + 500$$

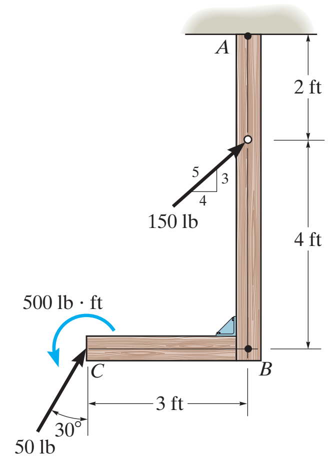
$$d = 5.24 \text{ ft} \qquad \textbf{Ans.}$$



Ans:
$F_R = 197$ lb
$\theta = 42.6°$
$d = 5.24$ ft

***3–100.**

Replace the loading acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member BC, measured from B.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 150\left(\frac{4}{5}\right) + 50 \sin 30° = 145 \text{ lb}$

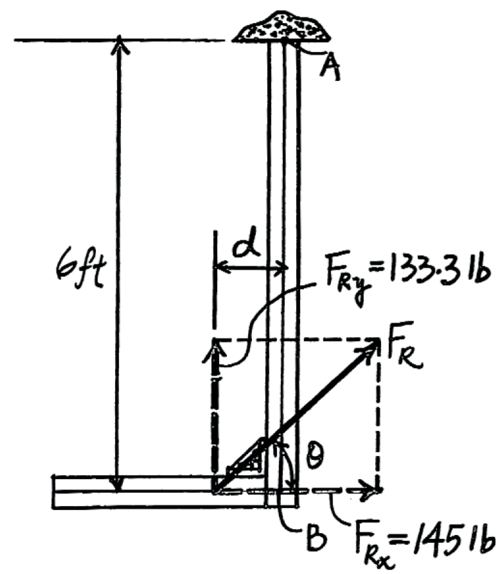
$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 50 \cos 30° + 150\left(\frac{3}{5}\right) = 133.3 \text{ lb}$

$F_R = \sqrt{(145)^2 + (133.3)^2} = 197 \text{ lb}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{133.3}{145}\right) = 42.6°$ **Ans.**

$\circlearrowleft + M_{RA} = \Sigma M_A;\quad 145\,(6) - 133.3\,(d) = 150\left(\frac{4}{5}\right)(2) - 50 \cos 30°\,(3) + 50 \sin 30°\,(6) + 500$

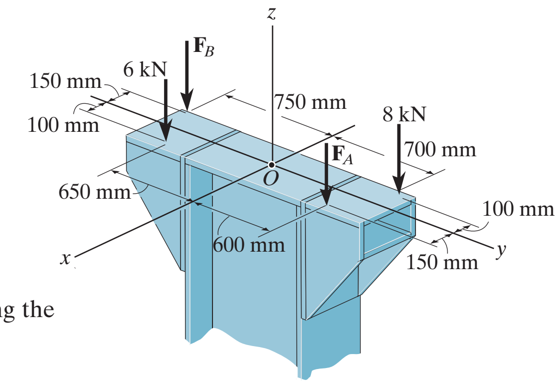
$d = 0.824 \text{ ft}$ **Ans.**



Ans:
$F_R = 197 \text{ lb}$
$\theta = 42.6°$
$d = 0.824 \text{ ft}$

3–101.

If $F_A = 7$ kN and $F_B = 5$ kN, represent the force system by a resultant force, and specify its location on the x–y plane.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces in Fig. *a* along the z axis to the resultant force $\mathbf{F}_R$, Fig. *b*,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -6-5-7-8$$

$$F_R = 26 \text{ kN} \qquad \textbf{Ans.}$$

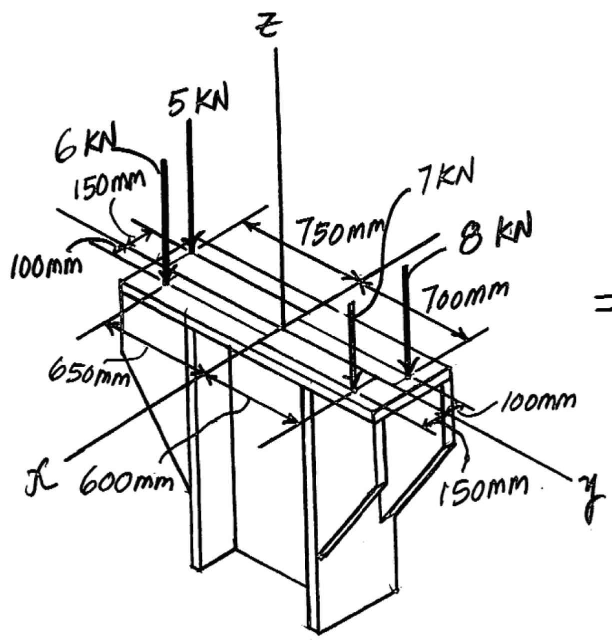
Point of Application: By equating the moment of the forces shown in Fig. *a* and $\mathbf{F}_R$, Fig. *b*, about the x and y axes,

$$(M_R)_x = \Sigma M_x; \qquad -26(y) = 6(650) + 5(750) - 7(600) - 8(700)$$

$$y = 82.7 \text{ mm} \qquad \textbf{Ans.}$$

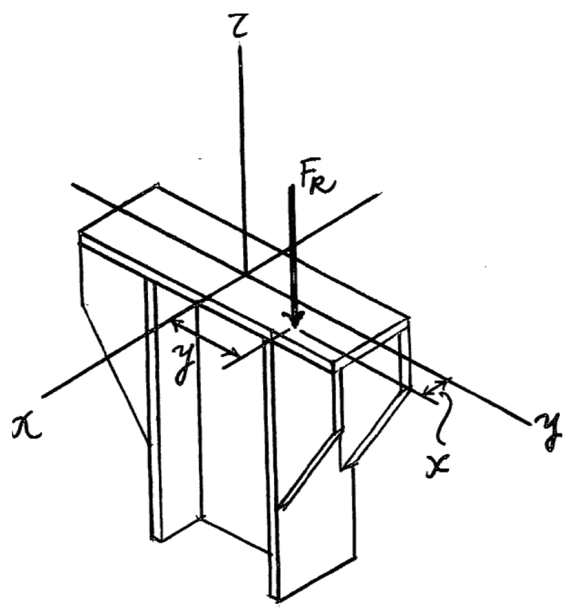
$$(M_R)_y = \Sigma M_y; \qquad 26(x) = 6(100) + 7(150) - 5(150) - 8(100)$$

$$x = 3.85 \text{ mm} \qquad \textbf{Ans.}$$



(a)

=

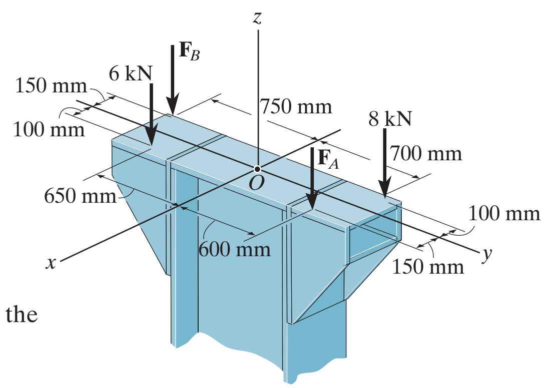


(b)

Ans:
$F_R = 26$ kN
$y = 82.7$ mm
$x = 3.85$ mm

3–102.

Determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$ so that the resultant force passes through point O.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces in Fig. *a* along the z axis to the resultant force $\mathbf{F}_R$, Fig. *b*,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -F_A - F_B - 8 - 6$$

$$F_R = F_A + F_B + 14 \qquad \textbf{(1)}$$

Point of Application: Since $\mathbf{F}_R$ is required to pass through point O, the moment of $\mathbf{F}_R$ about the x and y axes are equal to zero. Thus,

$$(M_R)_x = \Sigma M_x; \qquad 0 = F_B(750) + 6(650) - F_A(600) - 8(700)$$

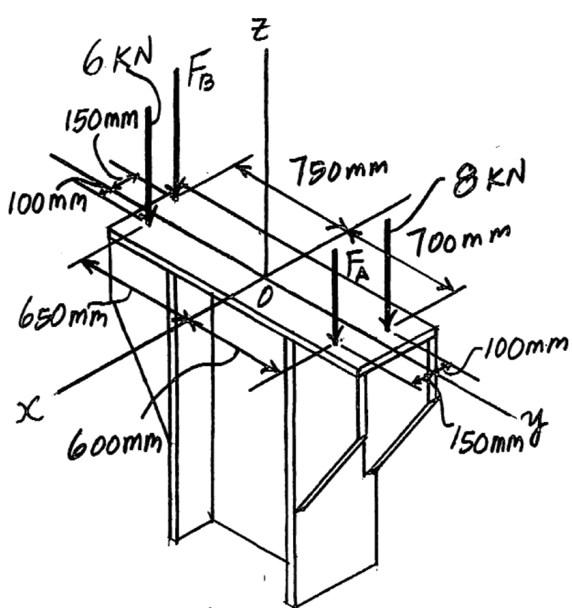
$$750F_B - 600F_A - 1700 = 0 \qquad \textbf{(2)}$$

$$(M_R)_y = \Sigma M_y; \qquad 0 = F_A(150) + 6(100) - F_B(150) - 8(100)$$

$$159F_A - 150F_B + 200 = 0 \qquad \textbf{(3)}$$

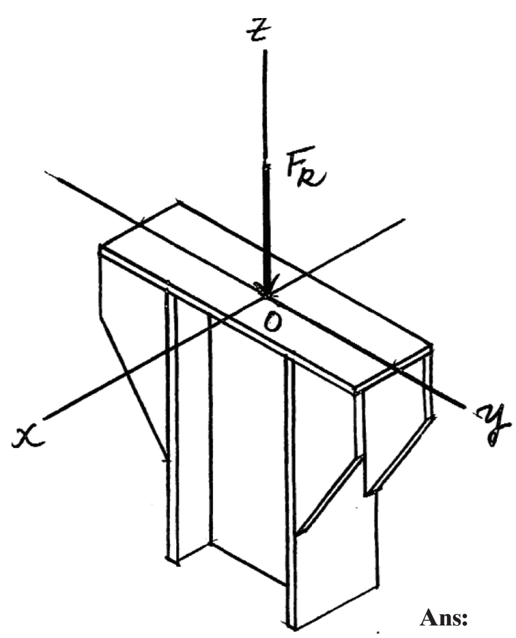
Solving Eqs. (1) through (3) yields

$F_A = 18.0 \text{ kN}$ $\qquad F_B = 16.7 \text{ kN}$ $\qquad F_R = 48.7 \text{ kN}$ **Ans.**



(a)

=

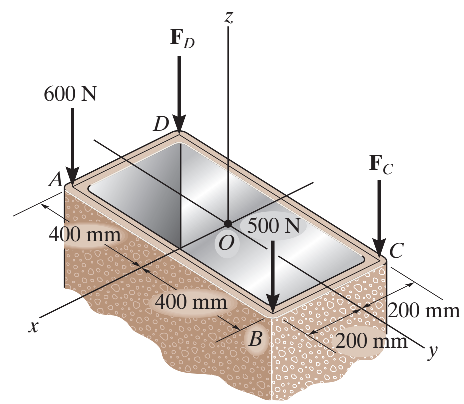


(b)

Ans:
$F_A = 18.0 \text{ kN}$
$F_B = 16.7 \text{ kN}$
$F_R = 48.7 \text{ kN}$

3–103.

The tube supports the four parallel forces. Determine the magnitudes of forces $\mathbf{F}_C$ and $\mathbf{F}_D$ acting at C and D so that the equivalent resultant force of the force system acts through the midpoint O of the tube.



SOLUTION

Since the resultant force passes through point O, the resultant moment components about x and y axes are both zero.

$$\Sigma M_x = 0; \qquad F_D(0.4) + 600(0.4) - F_C(0.4) - 500(0.4) = 0$$

$$F_C - F_D = 100 \qquad \textbf{(1)}$$

$$\Sigma M_y = 0; \qquad 500(0.2) + 600(0.2) - F_C(0.2) - F_D(0.2) = 0$$

$$F_C + F_D = 1100 \qquad \textbf{(2)}$$

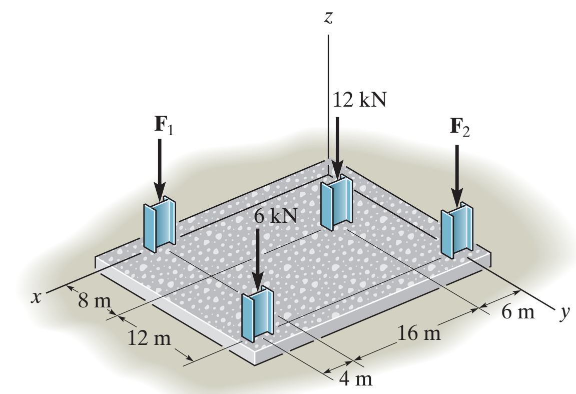
Solving Eqs. (1) and (2) yields:

$$F_C = 600\text{ N} \qquad F_D = 500\text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_C = 600$ N
$F_D = 500$ N

***3–104.**

The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 8$ kN and $F_2 = 9$ kN.



SOLUTION

Equivalent Resultant Force. Sum the forces along the z axis by referring to Fig. a.

$+\uparrow (F_R)_z = \Sigma F_z; \qquad -F_R = -8 - 6 - 12 - 9 \qquad F_R = 35 \text{ kN}$ **Ans.**

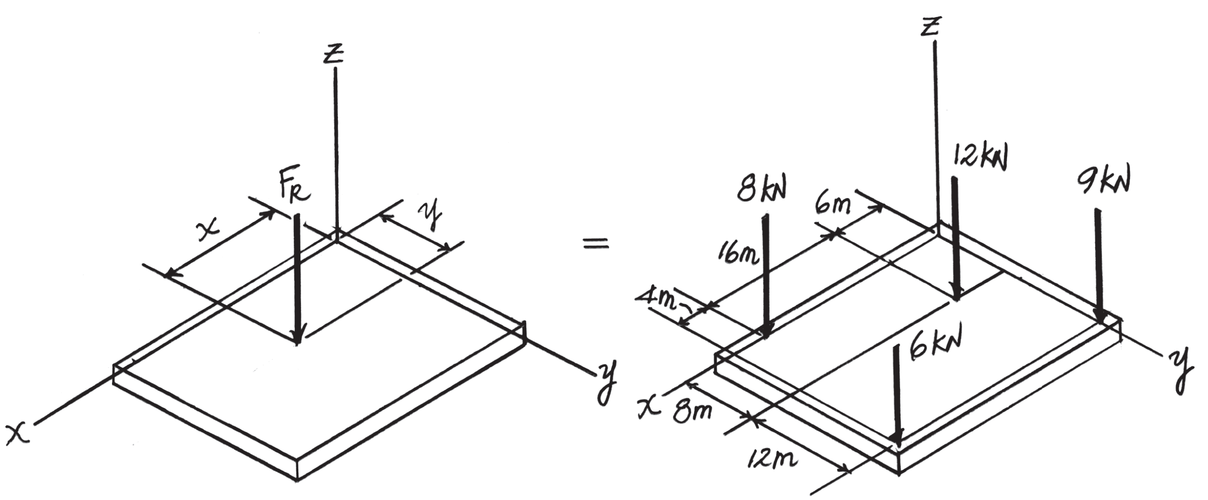
Location of the Resultant Force. Sum the moments about the x and y axes by referring to Fig. a.

$$(M_R)_x = \Sigma M_x; \qquad -35\,y = -12(8) - 6(20) - 9(20)$$

$$y = 11.31 \text{ m} = 11.3 \text{ m} \qquad \textbf{Ans.}$$

$$(M_R)_y = \Sigma M_y; \qquad 35\,x = 12(6) + 8(22) + 6(26)$$

$$x = 11.54 \text{ m} = 11.5 \text{ m} \qquad \textbf{Ans.}$$

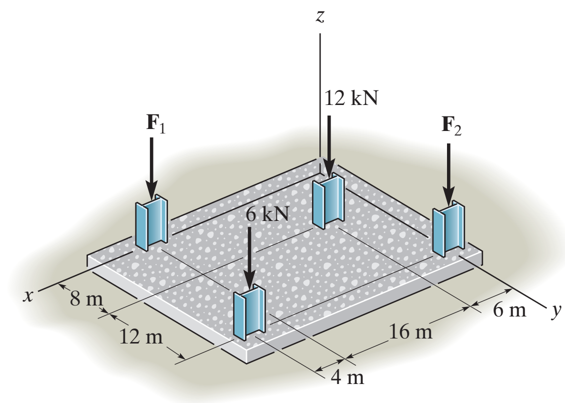


(a)

Ans:
$F_R = 35$ kN
$y = 11.3$ m
$x = 11.5$ m

3–105.

The building slab is subjected to four parallel column loadings. Determine $\mathbf{F}_1$ and $\mathbf{F}_2$ if the resultant force acts through point (12 m, 10 m).



SOLUTION

Equivalent Resultant Force. Sum the forces along z axis by referring to Fig. a.

$+\uparrow (F_R)_z = \Sigma F_z; \qquad -F_R = -F_1 - F_2 - 12 - 6 \qquad F_R = F_1 + F_2 + 18$

Location of the Resultant Force. Sum the moments about the x and y axes by referring to Fig. a.

$$(M_R)_x = \Sigma M_x; \qquad -(F_1 + F_2 + 18)(10) = -12(8) - 6(20) - F_2(20)$$

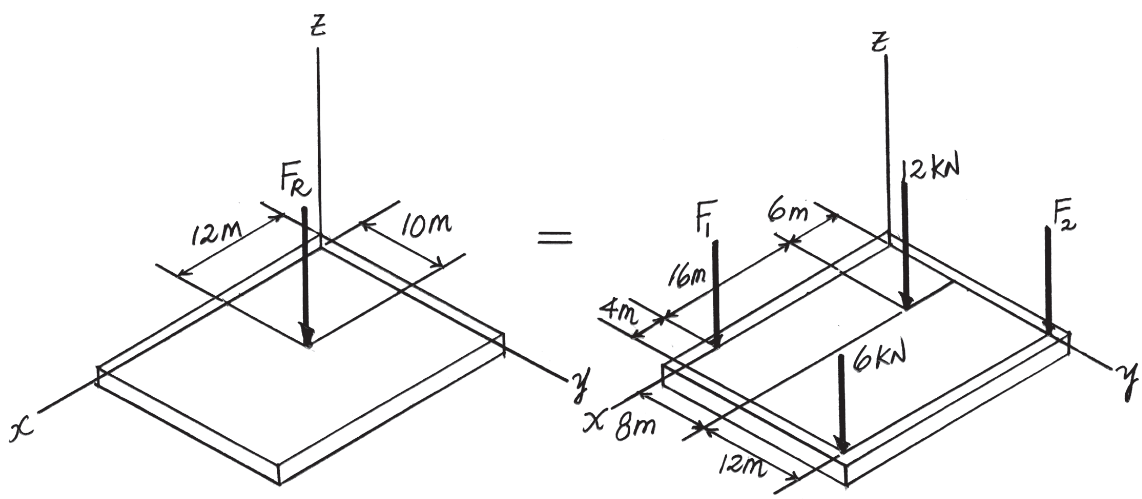
$$10F_1 - 10F_2 = 36 \qquad \textbf{(1)}$$

$$(M_R)_y = \Sigma M_y; \qquad (F_1 + F_2 + 18)(12) = 12(6) + 6(26) + F_1(22)$$

$$12F_2 - 10F_1 = 12 \qquad \textbf{(2)}$$

Solving Eqs (1) and (2),

$$F_1 = 27.6 \text{ kN} \qquad F_2 = 24.0 \text{ kN} \qquad \textbf{Ans.}$$

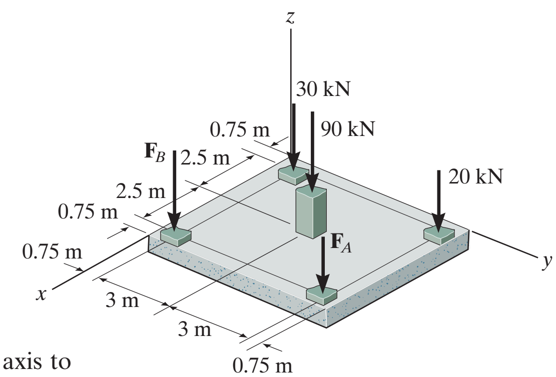


(a)

Ans:
$F_1 = 27.6$ kN
$F_2 = 24.0$ kN

3–106.

If $F_A = 40$ kN and $F_B = 35$ kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$, Fig. b,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -30 - 20 - 90 - 35 - 40$$

$$F_R = 215 \text{ kN} \qquad \textbf{Ans.}$$

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

$$(M_R)_x = \Sigma M_x; \qquad -215(y) = -35(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - 40(6.75)$$

$$y = 3.68 \text{ m} \qquad \textbf{Ans.}$$

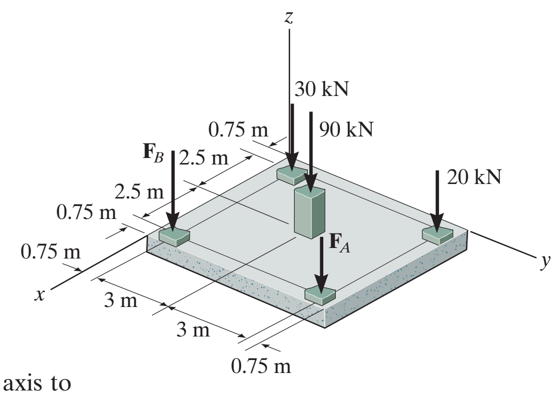
$$(M_R)_y = \Sigma M_y; \qquad 215(x) = 30(0.75) + 20(0.75) + 90(3.25) + 35(5.75) + 40(5.75)$$

$$x = 3.54 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 215$ kN
$y = 3.68$ m
$x = 3.54$ m

3–107.

If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $\mathbf{F}_A$ and $\mathbf{F}_B$ and the magnitude of the resultant force.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -30 - 20 - 90 - F_A - F_B$$

$$F_R = 140 + F_A + F_B \qquad \textbf{(1)}$$

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

$$(M_R)_x = \Sigma M_x; \qquad -F_R(3.75) = -F_B(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - F_A(6.75)$$

$$F_R = 0.2F_B + 1.8F_A + 132 \qquad \textbf{(2)}$$

$$(M_R)_y = \Sigma M_y; \qquad F_R(3.25) = 30(0.75) + 20(0.75) + 90(3.25) + F_A(5.75) + F_B(5.75)$$

$$F_R = 1.769F_A + 1.769F_B + 101.54 \qquad \textbf{(3)}$$

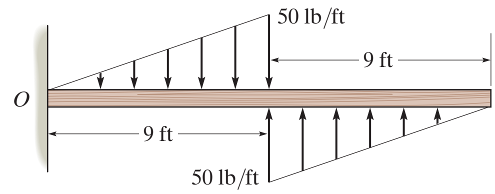
Solving Eqs.(1) through (3) yields

$F_A = 30\,\text{kN}$ $\qquad F_B = 20\text{ kN}$ $\qquad F_R = 190\,\text{kN}$ $\qquad$ **Ans.**

Ans:
$F_A = 30\text{ kN}$
$F_B = 20\text{ kN}$
$F_R = 190\text{ kN}$

***3–108.**

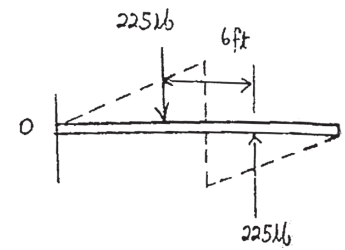
Replace the loading by an equivalent resultant force and couple moment acting at point O.



SOLUTION

$+\uparrow F_R = \Sigma F;$ $\quad F_R = 0$ **Ans.**

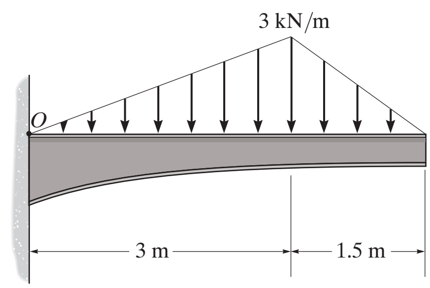
$\circlearrowleft + M_{RO} = \Sigma M_O;$ $\quad M_{RO} = 225(6) = 1350 \text{ lb} \cdot \text{ft} = 1.35 \text{ kip} \cdot \text{ft}$ **Ans.**



Ans:
$F_R = 0$
$M_{RO} = 1.35 \text{ kip} \cdot \text{ft}$

3–109.

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam, measured from point O.



SOLUTION

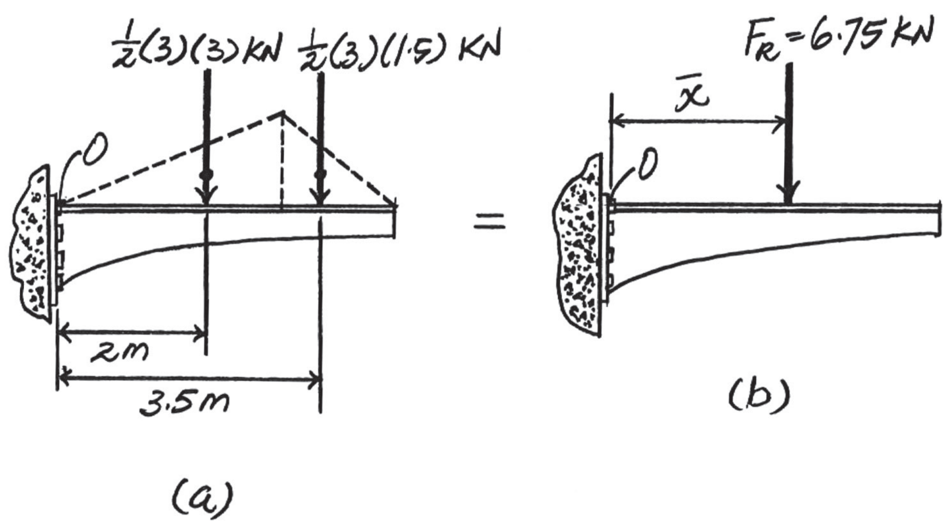
Loading: The distributed loading can be divided into two parts as shown in Fig. *a*.
Equations of Equilibrium: Equating the forces along the *y* axis of Figs. *a* and *b*, we have

$$+\downarrow F_R = \Sigma F; \qquad F_R = \frac{1}{2}(3)(3) + \frac{1}{2}(3)(1.5) = 6.75 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

If we equate the moment of F_R, Fig. *b*, to the sum of the moment of the forces in Fig. *a* about point O, we have

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad -6.75(\bar{x}) = -\frac{1}{2}(3)(3)(2) - \frac{1}{2}(3)(1.5)(3.5)$$

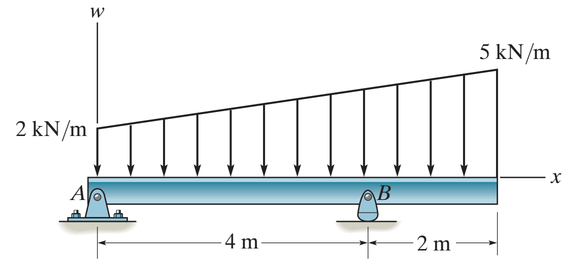
$$\bar{x} = 2.5 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$F_R = 6.75$ kN
$\bar{x} = 2.5$ m

3–110.

Replace the loading by an equivalent resultant force and specify its location on the beam, measured from A.



SOLUTION

Equivalent Resultant Force. Summing the forces along the y axis by referring to Fig. a,

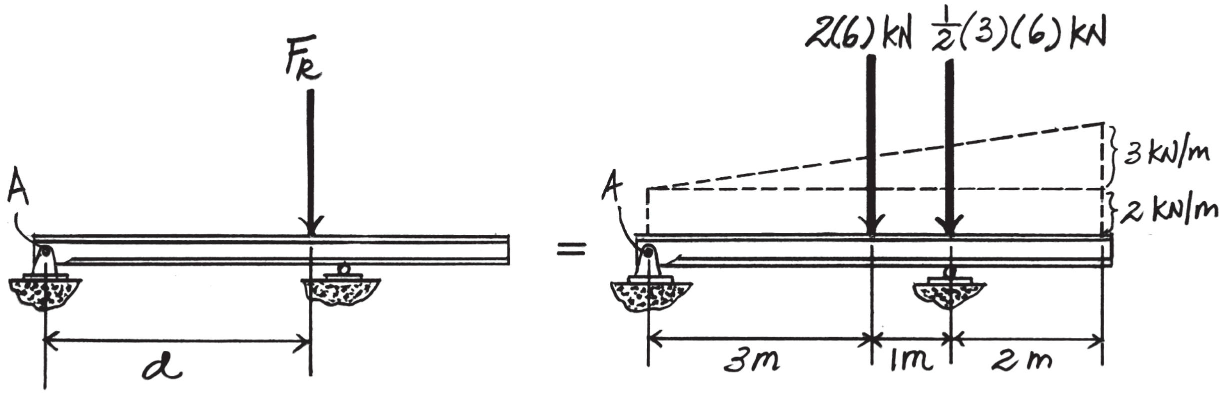
$$+\uparrow \ (F_R)_y = \Sigma F_y; \quad -F_R = -2(6) - \frac{1}{2}(3)(6)$$

$$F_R = 21.0 \text{ kN}\downarrow \qquad \textbf{Ans.}$$

Location of the Resultant Force. Summing the moments about point A,

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -21.0(d) = -2(6)(3) - \frac{1}{2}(3)(6)(4)$$

$$d = 3.429 \text{ m} = 3.43 \text{ m} \qquad \textbf{Ans.}$$

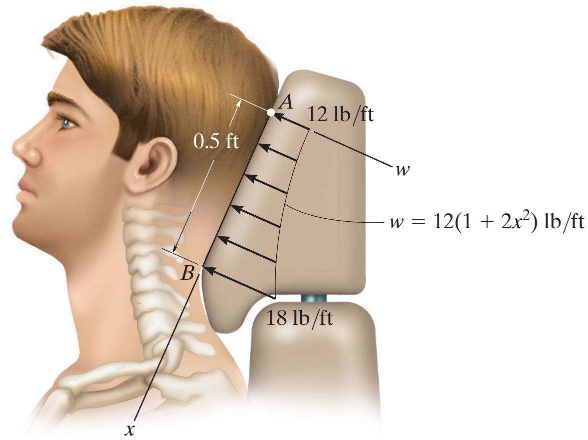


(a)

Ans:
$F_R = 21.0$ kN
$d = 3.43$ m

3–111.

Currently eighty-five percent of all neck injuries are caused by rear-end car collisions. To alleviate this problem, an automobile seat restraint has been developed that provides additional pressure contact with the cranium. During dynamic tests the distribution of load on the cranium has been plotted and shown to be parabolic. Determine the equivalent resultant force and its location, measured from point A.



SOLUTION

$$F_R = \int w(x)\,dx = \int_0^{0.5} 12\left(1 + 2x^2\right)dx = 12\left[x + \frac{2}{3}x^3\right]_0^{0.5} = 7\text{ lb}$$ **Ans.**

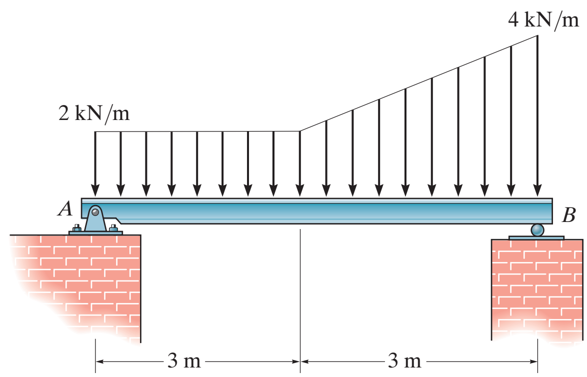
$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)dx} = \frac{\int_0^{0.5} x(12)\left(1 + 2x^2\right)dx}{7} = \frac{12\left[\frac{x^2}{2} + (2)\frac{x^4}{4}\right]_0^{0.5}}{7}$$

$$\bar{x} = 0.268\text{ ft}$$ **Ans.**

Ans:
$F_R = 7$ lb
$\bar{x} = 0.268$ ft

***3–112.**

Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at A.



SOLUTION

Equivalent Resultant Force. Summing the forces along the y axis by referring to Fig. a,

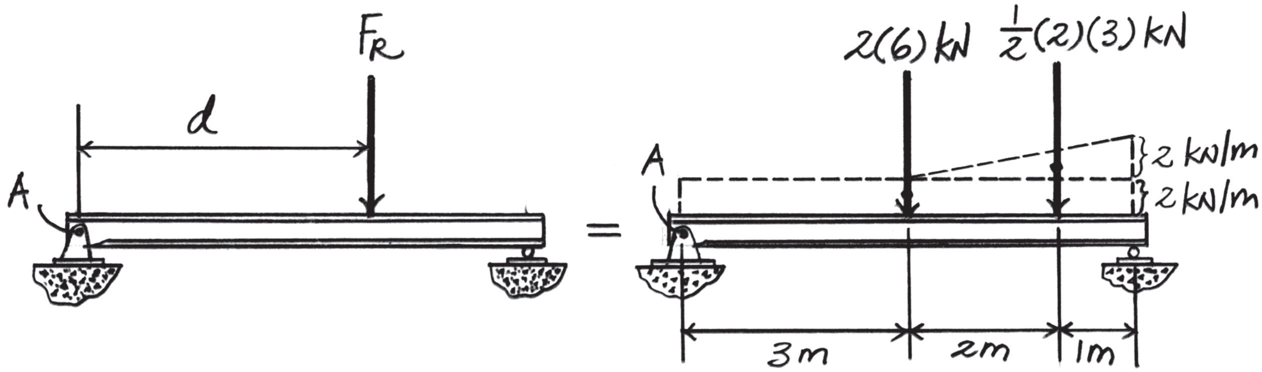
$$+\uparrow (F_R)_y = \Sigma F_y; \quad -F_R = -2(6) - \frac{1}{2}(2)(3)$$

$$F_R = 15.0 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

Location of the Resultant Force. Summing the moments about point A,

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -15.0(d) = -2(6)(3) - \frac{1}{2}(2)(3)(5)$$

$$d = 3.40 \text{ m} \qquad \textbf{Ans.}$$

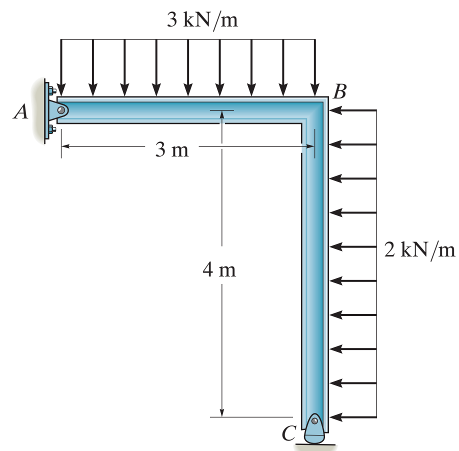


(a)

Ans:
$F_R = 15.0$ kN
$d = 3.40$ m

3–113.

Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a horizontal line along member AB, measured from A.



SOLUTION

Equivalent Resultant Force. Summing the forces along the x and y axes by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -2(4) = -8 \text{ kN} = 8 \text{ kN} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -3(3) = -9 \text{ kN} = 9 \text{ kN} \downarrow$$

Then

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{8^2 + 9^2} = 12.04 \text{ kN} = 12.0 \text{ kN} \qquad \textbf{Ans.}$$

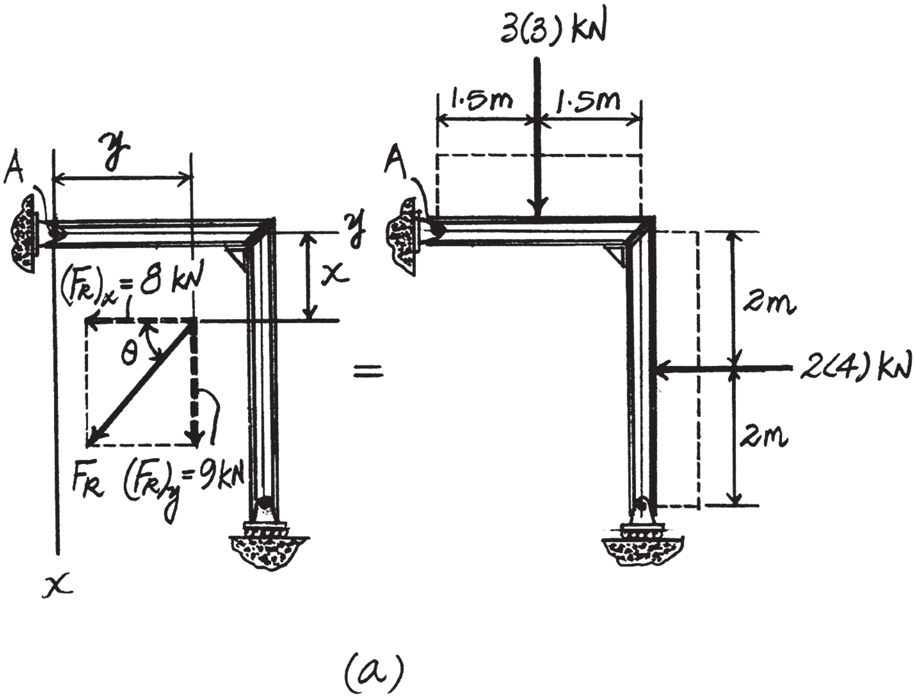
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{9}{8}\right) = 48.37° = 48.4° \measuredangle \qquad \textbf{Ans.}$$

Location of the Resultant Force. Summing the moments about point A, by referring to Fig. a,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad -8x - 9y = -3(3)(1.5) - 2(4)(2)$$

$$8x + 9y = 29.5 \qquad (1)$$



(a)

3–113. Continued

Along AB, $x = 0$. Then Eq (1) becomes

$$8(0) + 9y = 29.5$$

$$y = 3.278 \text{ m}$$

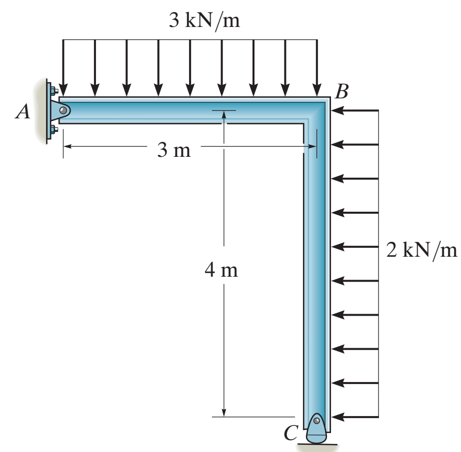
Thus, the intersection point of line of action of $\mathbf{F}_R$ on AB measured to the right from point A is

$$d = y = 3.28 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 12.0$ kN
$\theta = 48.4°$
$d = 3.28$ m

3–114.

Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a vertical line along member *BC*, measured from *C*.



SOLUTION

Equivalent Resultant Force. Summing the forces along the *x* and *y* axes by referring to Fig. *a*,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -2(4) = -8\text{ kN} = 8\text{ kN} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -3(3) = -9\text{ kN} = 9\text{ kN} \downarrow$$

Then

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{8^2 + 9^2} = 12.04\text{ kN} = 12.0\text{ kN} \qquad \textbf{Ans.}$$

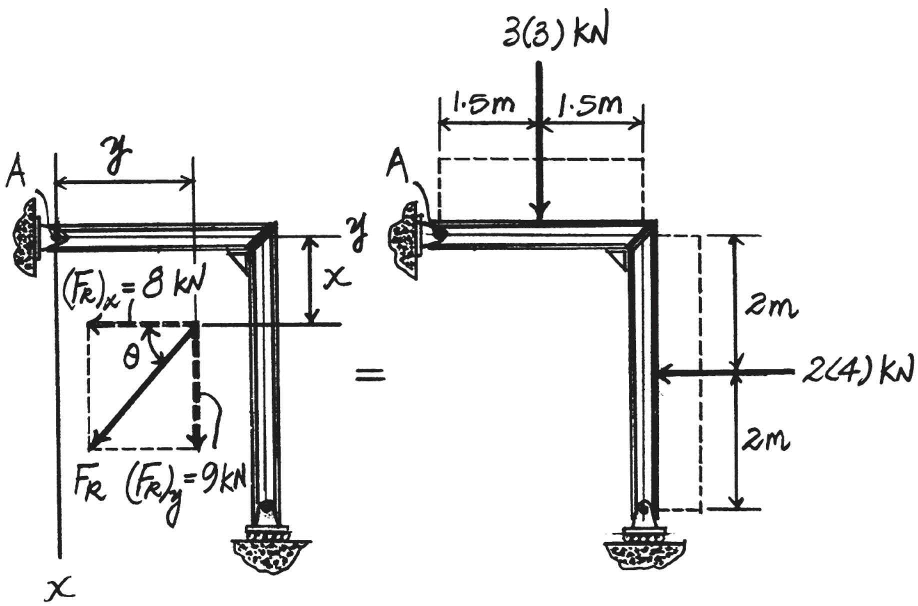
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{9}{8}\right) = 48.37^\circ = 48.4^\circ \qquad \textbf{Ans.}$$

Location of the Resultant Force. Summing the moments about point *A*, by referring to Fig. *a*,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad -8x - 9y = -3(3)(1.5) - 2(4)(2)$$

$$8x + 9y = 29.5 \qquad (1)$$



(a)

3–114. Continued

Along BC, $y = 3$ m. Then Eq (1) becomes

$$8x + 9(3) = 29.5$$

$$x = 0.3125 \text{ m}$$

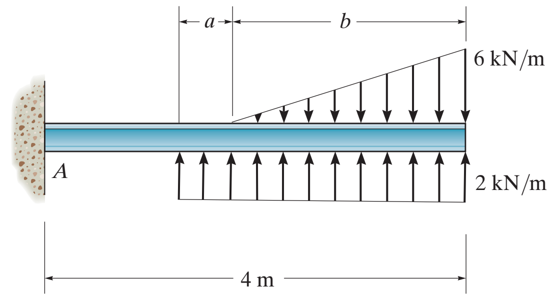
Thus, the intersection point of line of action of $\mathbf{F}_R$ on BC measured upward from point C is

$$d = 4 - x = 4 - 0.3125 = 3.6875 \text{ m} = 3.69 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 12.0$ kN
$\theta = 48.4°$ ⦣
$d = 3.69$ m

3–115.

Determine the length b of the triangular load and its position a on the beam so that the equivalent resultant force is zero and the resultant couple moment is 8 kN·m clockwise.



SOLUTION

Equivalent Resultant Force And Couple Moment At Point *A*. Summing the forces along the y axis by referring to Fig. a, with the requirement that $\mathbf{F}_R = 0$,

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad 0 = 2(a + b) - \frac{1}{2}(6)(b)$$

$$2a - b = 0 \qquad (1)$$

Summing the moments about point A, with the requirement that $(M_R)_A = 8\ \text{kN}\cdot\text{m}$,

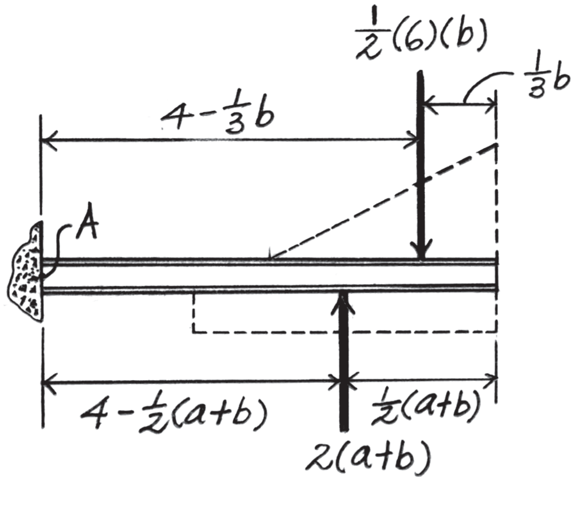
$$\circlearrowleft + (M_R)_A = \Sigma M_A; \ -8 = 2(a + b)\left[4 - \frac{1}{2}(a + b)\right] - \frac{1}{2}(6)(b)\left(4 - \frac{1}{3}b\right)$$

$$-8 = 8a - 4b - 2ab - a^2 \qquad (2)$$

Solving Eqs (1) and (2),

$$a = 1.264\ \text{m} = 1.26\ \text{m} \qquad \textbf{Ans.}$$

$$b = 2.530\ \text{m} = 2.53\ \text{m} \qquad \textbf{Ans.}$$

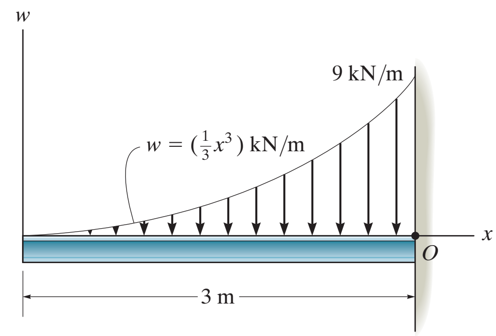


(a)

Ans:
$a = 1.26$ m
$b = 2.53$ m

***3–116.**

Determine the equivalent resultant force and couple moment at point O.



SOLUTION

Equivalent Resultant Force And Couple Moment About Point *O*. The differential force indicated in Fig. *a* is $dF_R = w\,dx = \dfrac{1}{3}x^3dx$. Thus, summing the forces along the y axis,

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad F_R = -\int dF_R = -\int_0^{3\text{m}} \frac{1}{3}x^3dx$$

$$= -\frac{1}{12}x^4 \Big|_0^{3\text{m}}$$

$$= -6.75\text{ kN} = 6.75\text{ kN}\downarrow \qquad \textbf{Ans.}$$

Summing the moments about point O,

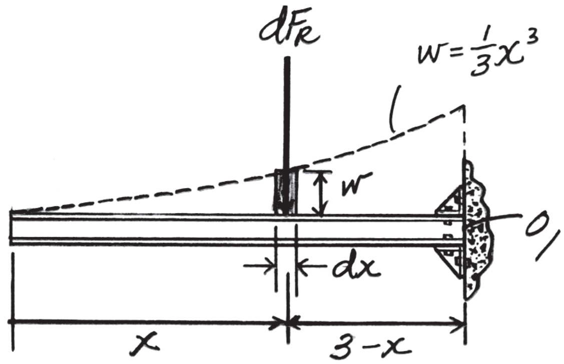
$$\circlearrowleft+(M_R)_O = \Sigma M_O; \quad (M_R)_O = \int (3-x)dF_R$$

$$= \int_0^{3\text{m}} (3-x)\left(\frac{1}{3}x^3dx\right)$$

$$= \int_0^{3\text{m}} \left(x^3 - \frac{1}{3}x^4\right)dx$$

$$= \left(\frac{x^4}{4} - \frac{1}{15}x^5\right)\Big|_0^{3\text{m}}$$

$$= 4.05\text{ kN}\cdot\text{m (\textbf{counterclockwise})} \qquad \textbf{Ans.}$$

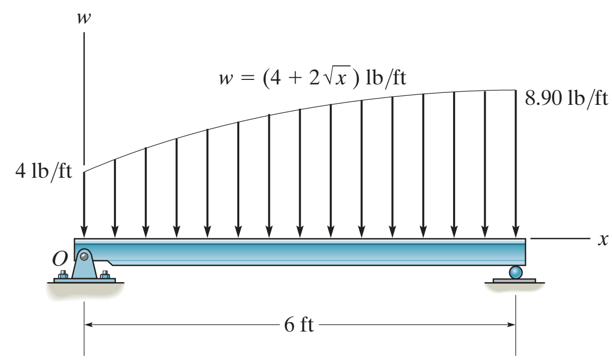


(a)

Ans:
$F_R = 6.75\text{ kN}\downarrow$
$(M_R)_O = 4.05\text{ kN}\cdot\text{m}$ (**counterclockwise**)

3–117.

Determine the magnitude of the equivalent resultant force and its location, measured from point O.



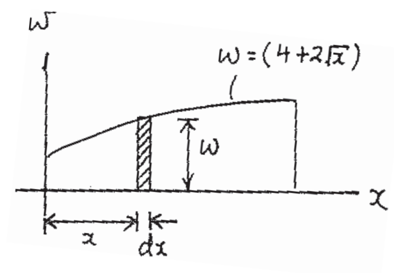
SOLUTION

$dA = wdx$

$$F_R = \int dA = \int_0^6 (4 + 2\sqrt{x})\,dx$$

$$= \left[4x + \frac{4}{3}x^{\frac{3}{2}}\right]_0^6$$

$$F_R = 43.6 \text{ lb}$$ **Ans.**



$$\int \bar{x}dF = \int_0^6 (4x + 2x^{\frac{3}{2}})\,dx$$

$$= \left[2x^2 + \frac{4}{5}x^{\frac{5}{2}}\right]_0^6$$

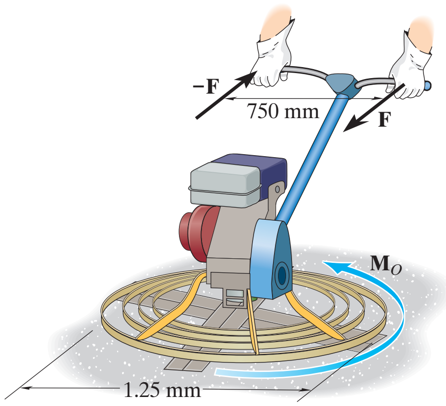
$$= 142.5 \text{ lb} \cdot \text{ft}$$

$$\bar{x} = \frac{142.5}{43.6} = 3.27 \text{ ft}$$ **Ans.**

Ans:
$F_R = 43.6$ lb
$\bar{x} = 3.27$ ft

***R3–4.**

Friction on the concrete surface creates a couple moment of $M_O = 100\ \text{N}\cdot\text{m}$ on the blades of the trowel. Determine the magnitude of the couple forces so that the resultant couple moment on the trowel is zero. The forces lie in a horizontal plane and act perpendicular to the handle of the trowel.



SOLUTION

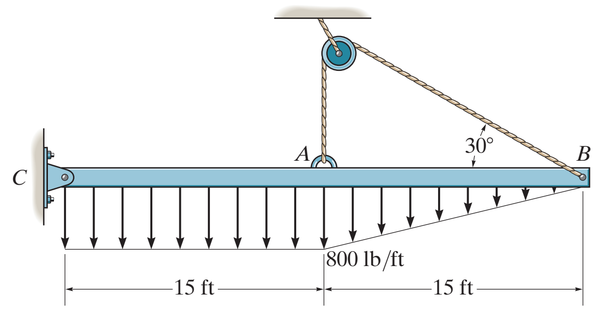
Couple Moment: The couple moment of **F** about the vertical axis is $M_C = F(0.75) = 0.75F$. Since the resultant couple moment about the vertical axis is required to be zero, we can write

$(M_c)_R = \Sigma M_z; \qquad 0 = 100 - 0.75F \qquad F = 133\ \text{N}$ **Ans.**

Ans:
$F = 133\ \text{N}$

***R3–8.**

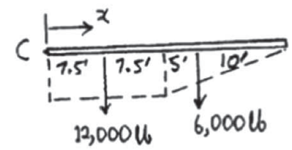
Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at C.



SOLUTION

$+\downarrow F_R = \Sigma F;\qquad F_R = 12\,000 + 6000 = 18\,000 \text{ lb}$

$F_R = 18.0 \text{ kip} \downarrow$ **Ans.**



$\circlearrowleft + M_{RC} = \Sigma M_C;\qquad 18\,000x = 12\,000(7.5) + 6000(20)$

$x = 11.7 \text{ ft}$ **Ans.**

Ans:
$F_R = 18.0 \text{ kip} \downarrow$
$x = 11.7 \text{ ft}$