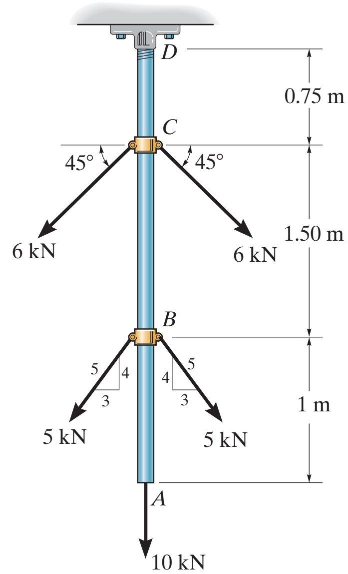
9–1.

The A992 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 80 mm^2, determine the displacement of B and A. Neglect the size of the couplings at B and C.



SOLUTION

Normal Force And Stress: For segments AB, BC and CD, referring to the FBDs of the lower segments shown in Fig. a, b and c, respectively,

$$+\uparrow\Sigma F_y = 0; \qquad P_{AB} - 10 = 0 \qquad P_{AB} = 10.0\text{ kN}$$

$$+\uparrow\Sigma F_y = 0; \qquad P_{BC} - 2\left[5\left(\frac{4}{5}\right)\right] - 10 = 0 \qquad P_{BC} = 18.0\text{ kN}$$

$$+\uparrow\Sigma F_y = 0; \qquad P_{CD} - 2(6\sin 45°) - 2\left[5\left(\frac{4}{5}\right)\right] - 10 = 0 \qquad P_{CD} = 26.485\text{ kN}$$

Since the rod has a constant cross-section and segment CD is subjected to the greatest normal force, this segment will develop the greatest average normal stress.

$$\sigma_{\max} = \sigma_{CD} = \frac{N_{CD}}{A} = \frac{26.485(10^3)}{80(10^{-6})} = 331.07(10^6)\text{ Pa} = 331.07\text{ MPa}$$

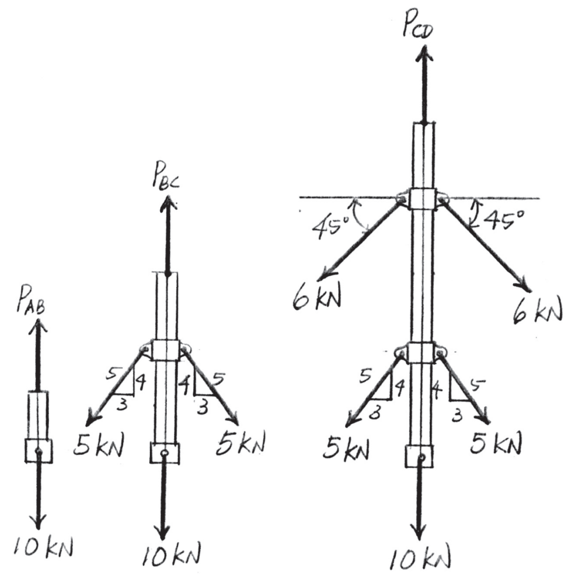
Displacement: For A992 steel, $\sigma_y = 345$ MPa and $E = 200$ GPa. Since $\sigma_{\max} < \sigma_y$,

$$\delta_B = \Sigma\frac{NL}{AE} = \frac{1}{AE}(P_{BC}L_{BC} + P_{CD}L_{CD}) = \frac{1}{80(10^{-6})[200(10^9)]}[18.0(10^3)(1.5) + 26.485(10^3)(0.75)]$$

$$= 0.002929\text{ m} = 2.93\text{ mm}\downarrow \qquad \textbf{Ans.}$$

$$\delta_A = \frac{P_{AB}L_{AB}}{AE} + \delta_B = \frac{10.0(10^3)(1)}{80(10^{-6})[200(10^9)]} + 0.002929$$

$$= 0.003554\text{ m} = 3.55\text{ mm}\downarrow \qquad \textbf{Ans.}$$

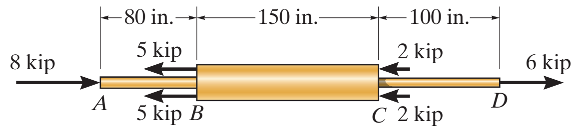


(a) (b) (c)

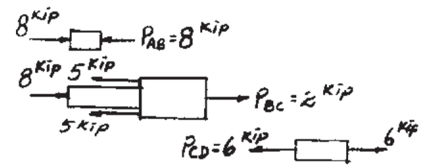
Ans:
$\delta_B = 2.93$ mm $\downarrow$,
$\delta_A = 3.55$ mm $\downarrow$

9–2.

The copper shaft is subjected to the axial loads shown. Determine the displacement of end A with respect to end D if the diameters of each segment are $d_{AB} = 0.75$ in., $d_{BC} = 1$ in., and $d_{CD} = 0.5$ in. Take $E_{cu} = 18(10^3)$ ksi.



SOLUTION



$$\delta_{A/D} = \Sigma\frac{PL}{AE} = \frac{-8(80)}{\frac{\pi}{4}(0.75)^2(18)(10^3)} + \frac{2(150)}{\frac{\pi}{4}(1)^2(18)(10^3)} + \frac{6(100)}{\frac{\pi}{4}(0.5)^2(18)(10^3)}$$

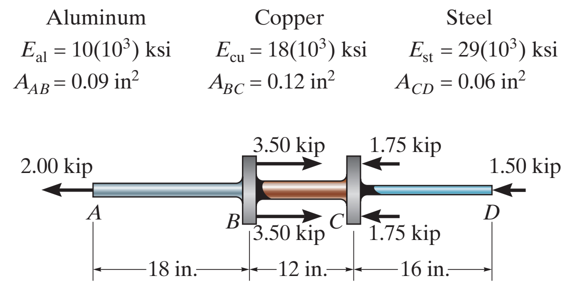
$$= 0.111 \text{ in.}$$ **Ans.**

The positive sign indicates that end A moves away from end D.

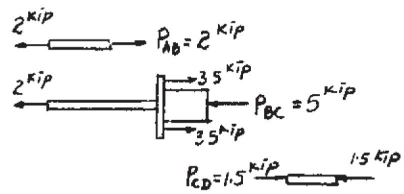
Ans:
$\delta_{A/D} = 0.111$ in. away from end D

9–3.

The composite shaft, consisting of aluminum, copper, and steel sections, is subjected to the loading shown. Determine the displacement of end A with respect to end D and the normal stress in each section. The cross-sectional area and modulus of elasticity for each section are shown in the figure. Neglect the size of the collars at B and C.



SOLUTION



$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{2}{0.09} = 22.2 \text{ ksi} \qquad \text{(T)} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{-5}{0.12} = -41.7 \text{ ksi} \qquad \text{(C)} \qquad \textbf{Ans.}$$

$$\sigma_{CD} = \frac{P_{BC}}{A_{BC}} = \frac{-1.5}{0.06} = -25.0 \text{ ksi} \qquad \text{(C)} \qquad \textbf{Ans.}$$

$$\delta_{AD} = \Sigma\frac{PL}{AE} = \frac{2(18)}{(0.09)(10)(10^3)} + \frac{(-5)(12)}{(0.12)(18)(10^3)} + \frac{(-1.5)(16)}{(0.06)(29)(10^3)}$$

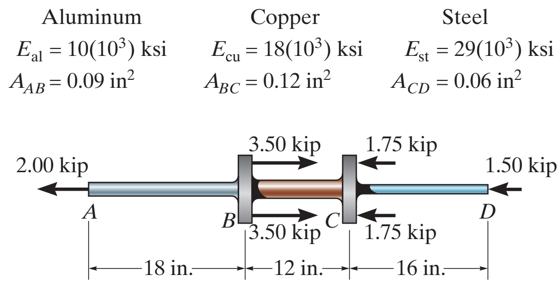
$$= -0.00157 \text{ in.} \qquad \textbf{Ans.}$$

The negative sign indicates end A moves towards end D.

Ans:
$\sigma_{AB} = 22.2$ ksi (T), $\sigma_{BC} = 41.7$ ksi (C),
$\sigma_{CD} = 25.0$ ksi (C),
$\delta_{A/D} = 0.00157$ in. towards end D

***9–4.**

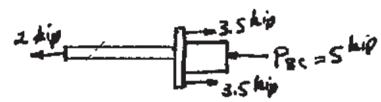
The composite shaft, consisting of aluminum, copper, and steel sections, is subjected to the loading shown. Determine the displacement of B with respect to C. The cross-sectional area and modulus of elasticity for each section are shown in the figure. Neglect the size of the collars at B and C.



SOLUTION

$$\delta_{B/C} = \frac{PL}{AE} = \frac{(-5)(12)}{(0.12)(18)(10^3)} = -0.0278 \text{ in.}$$ **Ans.**

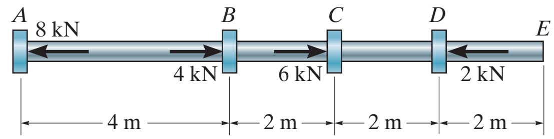
The negative sign indicates end B moves towards end C.



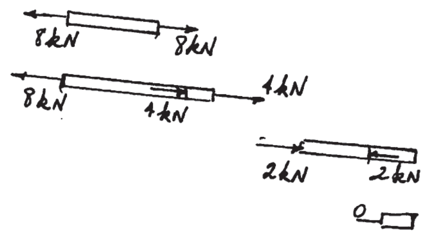
Ans:
$\delta_{B/C} = -0.0278$ in. B moves towards end C

9–5.

The 2014-T6 aluminum rod has a diameter of 30 mm and supports the load shown. Determine the displacement of end A with respect to end E. Neglect the size of the couplings.



SOLUTION



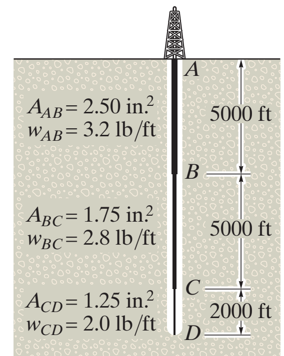
$$\delta_{A/E} = \Sigma\frac{PL}{AE} = \frac{1}{AE}[8(4) + 4(2) - 2(2) + 0(2)](10^3)$$

$$= \frac{36(10^3)}{\frac{\pi}{4}(0.03)^2(73.1)(10^9)} = 0.697\,(10^{-3}) = 0.697\text{ mm}$$ **Ans.**

Ans:
$\delta_{A/E} = 0.697$ mm

9–6.

The A-36 steel drill shaft of an oil well extends 12 000 ft into the ground. Assuming that the pipe used to drill the well is suspended freely from the derrick at A, determine the maximum average normal stress in each pipe string and the elongation of its end D with respect to the fixed end at A. The shaft consists of three different sizes of pipe, AB, BC, and CD, each having the length, weight per unit length, and cross-sectional area indicated.



SOLUTION

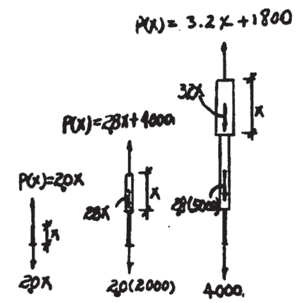
$$\sigma_A = \frac{P}{A} = \frac{3.2(5000) + 18000}{2.5} = 13.6 \text{ ksi}$$ **Ans.**

$$\sigma_B = \frac{P}{A} = \frac{2.8(5000) + 4000}{1.75} = 10.3 \text{ ksi}$$ **Ans.**

$$\sigma_C = \frac{P}{A} = \frac{2(2000)}{1.25} = 3.2 \text{ ksi}$$ **Ans.**

$$\delta_D = \Sigma \int \frac{P(x)\,dx}{A(x)\,E} = \int_0^{2000} \frac{2x\,dx}{(1.25)(29)(10^6)} + \int_0^{5000} \frac{(2.8x + 4000)dx}{(1.75)(29)(10^6)} + \int_0^{5000} \frac{(3.2x + 18000)dx}{(2.5)(29)(10^6)}$$

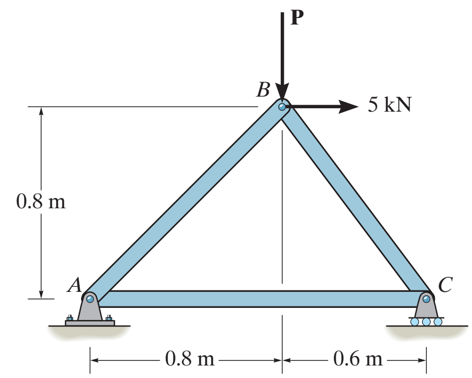
$$= 2.99 \text{ ft}$$ **Ans.**



Ans:
$\sigma_A = 13.6$ ksi, $\sigma_B = 10.3$ ksi,
$\sigma_C = 3.2$ ksi, $\delta_D = 2.99$ ft

9–7.

The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm^2. Determine the horizontal displacement of the roller at C when $P = 8$ kN.

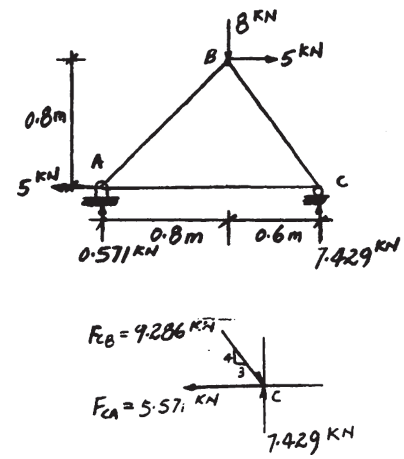


SOLUTION

By observation the horizontal displacement of roller C is equal to the displacement of point C obtained from member AC.

$F_{CA} = 5.571$ kN

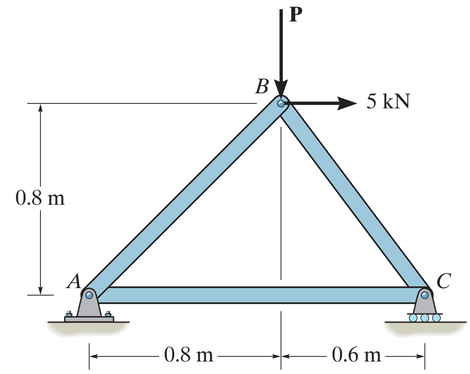
$$\delta_C = \frac{F_{CA}L}{AE} = \frac{5.571(10^3)(1.40)}{(400)(10^{-6})(200)(10^6)} = 0.0975 \text{ mm} \rightarrow$$ **Ans.**



Ans:
$\delta_C = 0.0975$ mm $\rightarrow$

***9–8.**

The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm^2. Determine the magnitude P required to displace the roller to the right 0.2 mm.



SOLUTION

$\circlearrowleft + M_A = 0;$ $\qquad -P(0.8) - 5(0.8) + C_y(1.4) = 0$

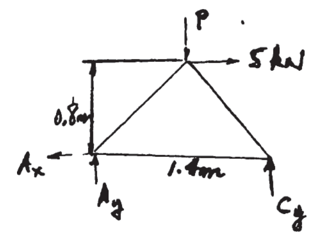
$$C_y = 0.5714\,P + 2.857$$

$+\uparrow \Sigma F_y = 0;$ $\qquad C_y - F_{BC}\left(\frac{4}{5}\right) = 0$

$$F_{BC} = 1.25\,C_y$$

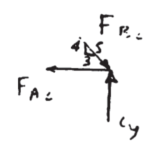
$\xrightarrow{+} \Sigma F_x = 0;$ $\qquad -F_{AC} + 1.25\,C_y\,(0.6) = 0$

$$F_{AC} = 0.75\,C_y = 0.4286\,P + 2.14286$$



Require

$$\delta_{C_A} = 0.0002 = \frac{(0.4286\,P + 2.14286)(10^3)(1.4)}{(400)(10^{-6})(200)(10^9)}$$

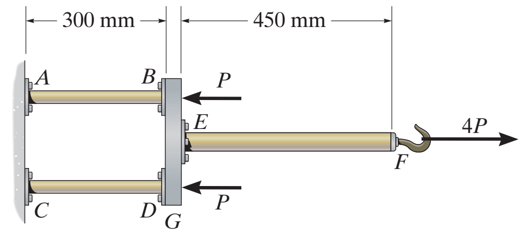


$P = 21.7$ kN **Ans.**

Ans:
$P = 21.7$ kN

9–9.

The assembly consists of two 10-mm diameter red brass C83400 copper rods AB and CD, a 15-mm diameter 304 stainless steel rod EF, and a rigid bar G. If $P = 5$ kN, determine the horizontal displacement of end F of rod EF.



SOLUTION

Internal Loading: The normal forces developed in rods EF, AB, and CD are shown on the free-body diagrams in Figs. a and b.

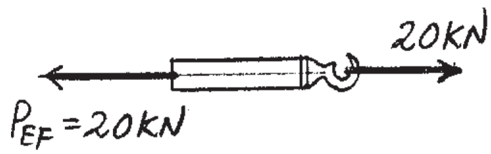
Displacement: The cross-sectional areas of rods EF and AB are $A_{EF} = \frac{\pi}{4}(0.015^2) = 56.25(10^{-6})\pi \text{ m}^2$ and $A_{AB} = \frac{\pi}{4}(0.01^2) = 25(10^{-6})\pi \text{ m}^2$.

$$\delta_F = \Sigma\frac{PL}{AE} = \frac{P_{EF}L_{EF}}{A_{EF}E_{st}} + \frac{P_{AB}L_{AB}}{A_{AB}E_{br}}$$

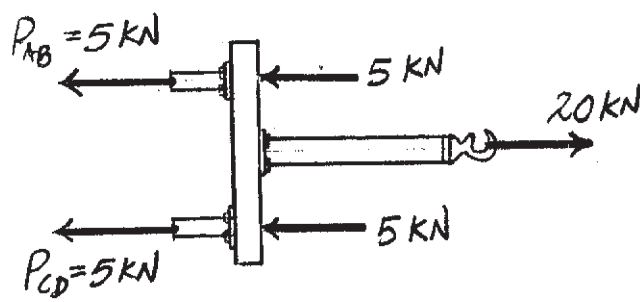
$$= \frac{20(10^3)(450)}{56.25(10^{-6})\pi(193)(10^9)} + \frac{5(10^3)(300)}{25(10^{-6})\pi(101)(10^9)}$$

$$= 0.453 \text{ mm}$$ **Ans.**

The positive sign indicates that end F moves away from the fixed end.



(a)

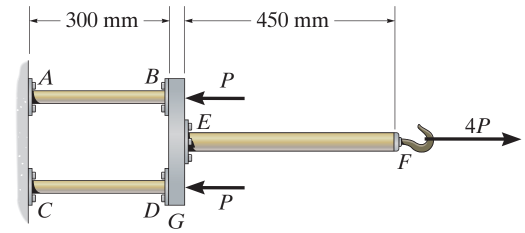


(b)

Ans:
$\delta_F = 0.453$ mm

9–10.

The assembly consists of two 10-mm diameter red brass C83400 copper rods AB and CD, a 15-mm diameter 304 stainless steel rod EF, and a rigid bar G. If the horizontal displacement of end F of rod EF is 0.45 mm, determine the magnitude of P.



SOLUTION

Internal Loading: The normal forces developed in rods EF, AB, and CD are shown on the free-body diagrams in Figs. *a* and *b*.

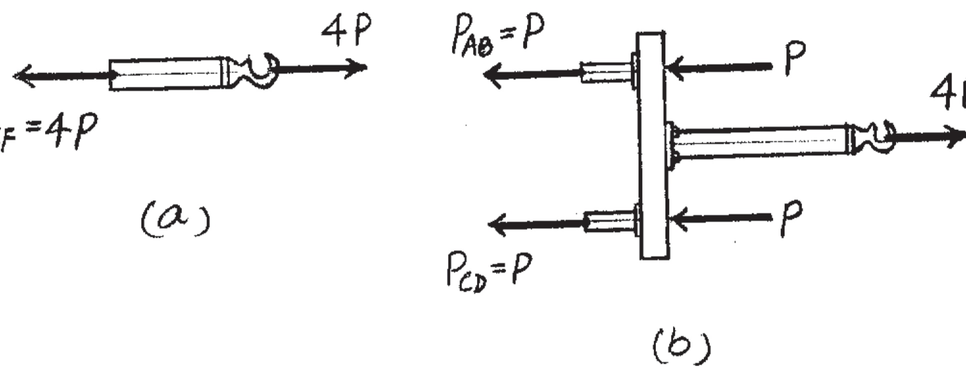
Displacement: The cross-sectional areas of rods EF and AB are $A_{EF} = \frac{\pi}{4}(0.015^2) = 56.25(10^{-6})\pi \text{ m}^2$ and

$$A_{AB} = \frac{\pi}{4}(0.01^2) = 25(10^{-6})\pi \text{ m}^2.$$

$$\delta_F = \Sigma\frac{PL}{AE} = \frac{P_{EF}L_{EF}}{A_{EF}E_{\text{st}}} + \frac{P_{AB}L_{AB}}{A_{AB}E_{\text{br}}}$$

$$0.45 = \frac{4P(450)}{56.25(10^{-6})\pi(193)(10^9)} + \frac{P(300)}{25(10^{-6})\pi(101)(10^9)}$$

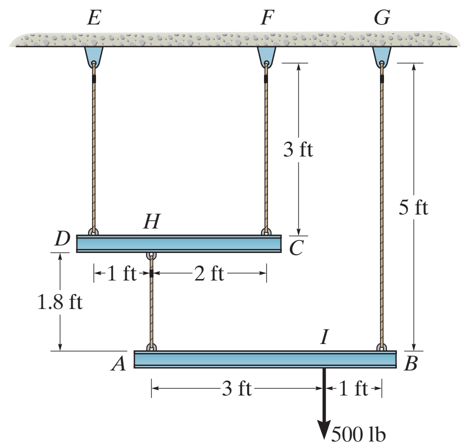
$P = 4967 \text{ N} = 4.97 \text{ kN}$ **Ans.**



Ans:
$P = 4.97$ kN

9–11.

The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the vertical displacement of the 500-lb load if the members were originally horizontal when the load was applied. Each wire has a cross-sectional area of 0.025 in^2.



SOLUTION

Internal Forces in the wires:

FBD (b):

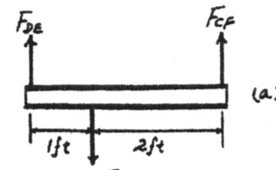
$\zeta+\Sigma M_A = 0;\quad F_{BC}(4) - 500(3) = 0 \quad F_{BC} = 375.0 \text{ lb}$

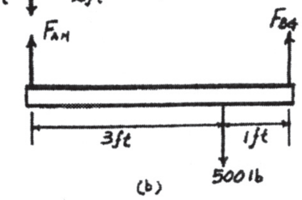
$+\uparrow\Sigma F_y = 0;\quad F_{AH} + 375.0 - 500 = 0 \quad F_{AH} = 125.0 \text{ lb}$

FBD (a):

$\zeta+\Sigma M_D = 0;\quad F_{CF}(3) - 125.0(1) = 0 \quad F_{CF} = 41.67 \text{ lb}$

$+\uparrow\Sigma F_y = 0;\quad F_{DE} + 41.67 - 125.0 = 0 \quad F_{DE} = 83.33 \text{ lb}$



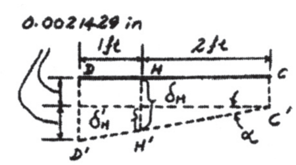


Displacement:

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{83.33(3)(12)}{0.025(28.0)(10^6)} = 0.0042857 \text{ in.}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{41.67(3)(12)}{0.025(28.0)(10^6)} = 0.0021429 \text{ in.}$$

$$\frac{\delta'_H}{2} = \frac{0.0021429}{3}; \qquad \delta'_H = 0.0014286 \text{ in.}$$



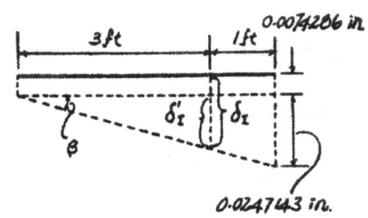
$$\delta_H = 0.0014286 + 0.0021429 = 0.0035714 \text{ in.}$$

$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{125.0(1.8)(12)}{0.025(28.0)(10^6)} = 0.0038571 \text{ in.}$$

$$\delta_A = \delta_H + \delta_{A/H} = 0.0035714 + 0.0038571 = 0.0074286 \text{ in.}$$

$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{375.0(5)(12)}{0.025(28.0)(10^6)} = 0.0321428 \text{ in.}$$

$$\frac{\delta'_l}{3} = \frac{0.0247143}{4}; \qquad \delta'_l = 0.0185357 \text{ in.}$$

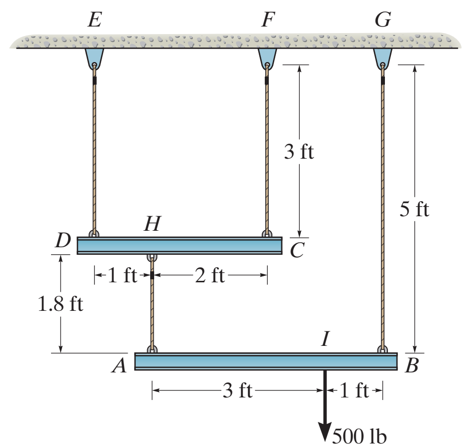


$$\delta_l = 0.0074286 + 0.0185357 = 0.0260 \text{ in.} \qquad \textbf{Ans.}$$

Ans:
$\delta_l = 0.0260$ in.

***9–12.**

The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the angle of tilt of each member after the 500-lb load is applied. The members were originally horizontal, and each wire has a cross-sectional area of 0.025 in^2.



SOLUTION

Internal Forces in the wires:

FBD (b):

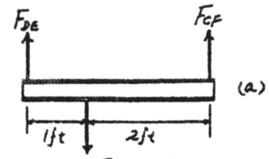
$$\circlearrowleft + \Sigma M_A = 0; \quad F_{BG}(4) - 500(3) = 0 \quad F_{BG} = 375.0 \text{ lb}$$

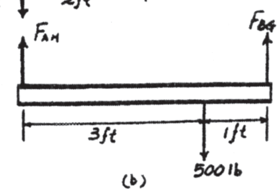
$$+\uparrow \Sigma F_y = 0; \quad F_{AH} + 375.0 - 500 = 0 \quad F_{AH} = 125.0 \text{ lb}$$

FBD (a):

$$\circlearrowleft + \Sigma M_D = 0; \quad F_{CF}(3) - 125.0(1) = 0 \quad F_{CF} = 41.67 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{DE} + 41.67 - 125.0 = 0 \quad F_{DE} = 83.33 \text{ lb}$$





Displacement:

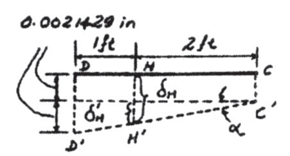
$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{83.33(3)(12)}{0.025(28.0)(10^6)} = 0.0042857 \text{ in.}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{41.67(3)(12)}{0.025(28.0)(10^6)} = 0.0021429 \text{ in.}$$

$$\frac{\delta'_H}{2} = \frac{0.0021429}{3}; \qquad \delta'_H = 0.0014286 \text{ in.}$$

$$\delta_H = \delta'_H + \delta_C = 0.0014286 + 0.0021429 = 0.0035714 \text{ in.}$$

$$\tan \alpha = \frac{0.0021429}{36}; \qquad \alpha = 0.00341° \qquad \textbf{Ans.}$$

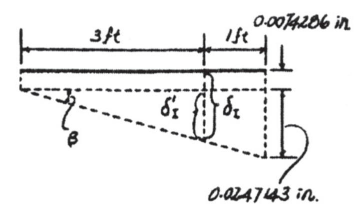


$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{125.0(1.8)(12)}{0.025(28.0)(10^6)} = 0.0038571 \text{ in.}$$

$$\delta_A = \delta_H + \delta_{A/H} = 0.0035714 + 0.0038571 = 0.0074286 \text{ in.}$$

$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{375.0(5)(12)}{0.025(28.0)(10^6)} = 0.0321428 \text{ in.}$$

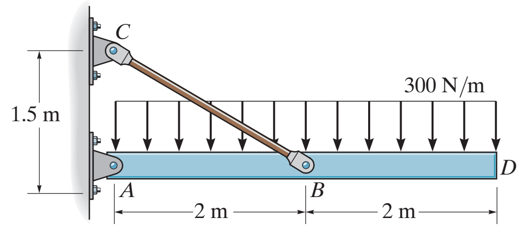
$$\tan \beta = \frac{0.0247143}{48}; \qquad \beta = 0.0295° \qquad \textbf{Ans.}$$



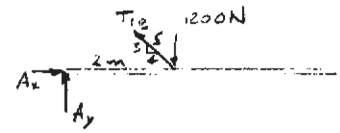
Ans:
$\alpha = 0.00341°$,
$\beta = 0.0295°$

9–13.

The rigid bar is supported by the pin-connected rod CB that has a cross-sectional area of 14 mm^2 and is made from 6061-T6 aluminum. Determine the vertical deflection of the bar at D when the distributed load is applied.



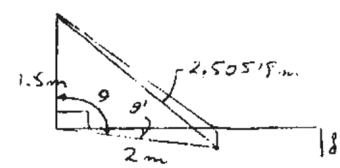
SOLUTION



$$\circlearrowleft + \Sigma M_A = 0; \quad 1200(2) - T_{CB}(0.6)(2) = 0$$

$$T_{CB} = 2000 \text{ N}$$

$$\delta_{B/C} = \frac{PL}{AE} = \frac{(2000)(2.5)}{14(10^{-6})(68.9)(10^9)} = 0.0051835$$



$$(2.5051835)^2 = (1.5)^2 + (2)^2 - 2(1.5)(2)\cos\theta$$

$$\theta = 90.248°$$

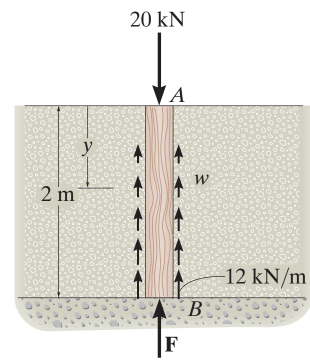
$$\theta = 90.248° - 90° = 0.2478° = 0.004324 \text{ rad}$$

$$\delta_D = \theta\, r = 0.004324(4000) = 17.3 \text{ mm}$$ **Ans.**

Ans:
$\delta_D = 17.3$ mm

9–14.

The post is made of Douglas fir and has a diameter of 100 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance distributed around the post that is triangular along its sides; that is, it varies from $w = 0$ at $y = 0$ to $w = 12\text{ kN/m}$ at $y = 2\text{ m}$, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the entire post, Fig. *a*,

$$+\uparrow\Sigma F_y = 0; \qquad F + \frac{1}{2}(12)(2) - 20 = 0 \qquad F = 8.00\text{ kN} \qquad \textbf{Ans.}$$

Normal Force: Referring to the FBD of the upper segment of the post sectioned at arbitrary distance y, Fig. *b*,

$$+\uparrow\Sigma F_y = 0; \qquad \frac{1}{2}(6y)(y) - 20 - P(y) = 0 \qquad Py = \left(3y^2 - 20\right)\text{ kN}$$

Displacement: For Douglas Fir, $E = 13.1$ GPa.

$$\delta_{A/B} = \int_0^L \frac{N(y)dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{ meters}} \left(3y^2 - 20\right)dy$$

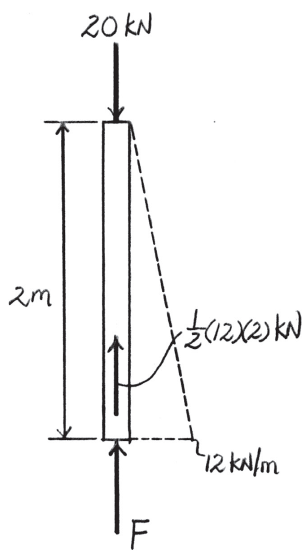
$$= \frac{1}{AE}\left(y^3 - 20y\right)\Big|_0^{2\text{ meters}}$$

$$= -\frac{32\text{ kN}\cdot\text{m}}{AE}$$

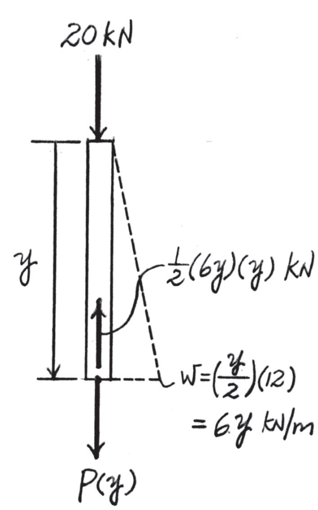
$$= -\frac{32\left(10^3\right)}{\frac{\pi}{4}\left(0.1^2\right)\left[13.1\left(10^9\right)\right]}$$

$$= -0.3110\left(10^{-3}\right)\text{ m} = -0.311\text{ mm} \qquad \textbf{Ans.}$$

The sign indicates that end A moves toward end B.



(a)

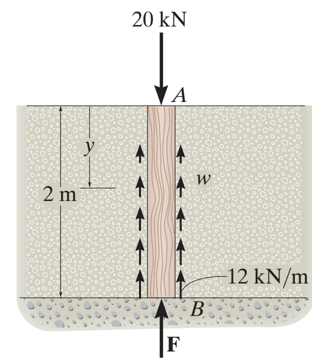


(b)

Ans:
$F = 8.00$ kN,
$\delta_{A/B} = -0.311$ mm

9–15.

The post is made of Douglas fir and has a diameter of 100 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance that is distributed along its length and varies linearly from $w = 4$ kN/m at $y = 0$ to $w = 12$ kN/m at $y = 2$ m, determine the force $\mathbf{F}$ at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the entire post, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F + \frac{1}{2}(4 + 12)(2) - 20 = 0 \qquad F = 4.00 \text{ kN} \qquad \textbf{Ans.}$$

Normal Force: Referring to the FBD of the upper segment of the post sectioned at arbitrary distance y, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \quad (4 + 2y)y - 20 - P(y) = 0 \qquad P(y) = \left(2y^2 + 4y - 20\right) \text{ kN}$$

Displacement: For Douglas Fir, $E = 13.1$ GPa.

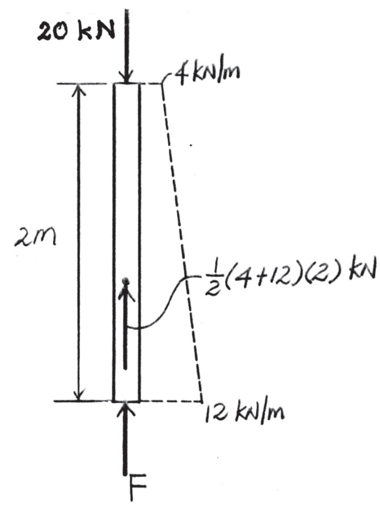
$$\delta_{A/B} = \int_0^L \frac{N(y)dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{ m}} \left(2y^2 + 4y - 20\right)dy$$

$$= \frac{1}{AE}\left(\frac{2}{3}y^3 + 2y^2 - 20y\right)\Bigg|_0^{2\text{ m}}$$

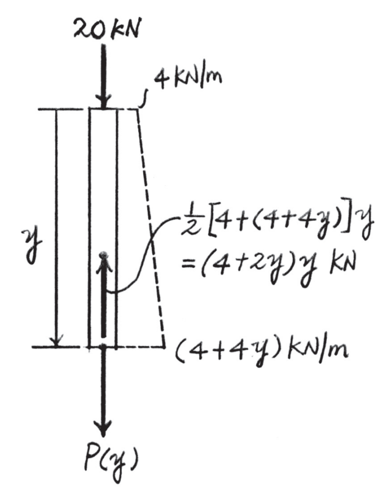
$$= -\frac{80 \text{ kN}\cdot\text{m}}{3\,AE}$$

$$= -\frac{80(10^3)}{3\left[\frac{\pi}{4}\left(0.1^2\right)\right]\left[13.1\left(10^9\right)\right]}$$

$$= -0.2592\left(10^{-3}\right) \text{ m} = -0.259 \text{ mm} \qquad \textbf{Ans.}$$



(a)

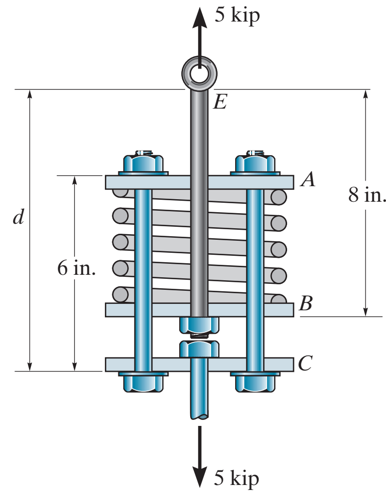


(b)

Ans:
$F = 4.00$ kN,
$\delta_{A/B} = -0.259$ mm

***9–16.**

The coupling rod is subjected to a force of 5 kip. Determine the distance d between C and E accounting for the compression of the spring and the deformation of the bolts. When no load is applied the spring is unstretched and $d = 10$ in. The material is A-36 steel and each bolt has a diameter of 0.25 in. The plates at A, B, and C are rigid and the spring has a stiffness of $k = 12$ kip/in.

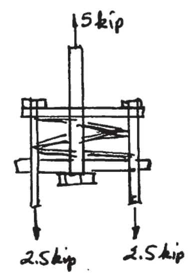


SOLUTION

$$\delta_{\text{center bolt}} = \frac{PL}{AE} = \frac{5(10^3)(8)}{\frac{\pi}{4}(0.25)^2(29)(10^6)} = 0.028099 \text{ in. } \uparrow$$

$$\delta_{\text{side bolts}} = \frac{PL}{AE} = \frac{2.5(10^3)(6)}{\frac{\pi}{4}(0.25)^2(29)(10^6)} = 0.010537 \text{ in. } \downarrow$$

$$\delta_{sp} = \frac{P}{k} = \frac{5}{12} = 0.41667 \text{ in. } \uparrow$$



$$\delta d = 0.41667 + 0.028099 + 0.010537$$

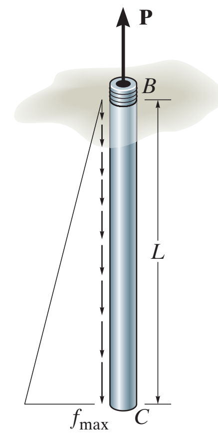
$$\delta d = 0.455 \text{ in.}$$

$$d = 10 + 0.455 = 10.455 \text{ in.}$$ **Ans.**

Ans:
$d = 10.455$ in.

9–17.

The pipe is stuck in the ground so that when it is pulled upward the frictional force along its length varies linearly from zero at B to f_{max} (force/length) at C. Determine the initial force P required to pull the pipe out and the pipe's elongation just before it starts to slip. The pipe has a length L, cross-sectional area A, and the material from which it is made has a modulus of elasticity E.



SOLUTION

From FBD (a),

$$+\uparrow \Sigma F_y = 0; \qquad P - \frac{1}{2}(F_{max}\,L) = 0$$

$$P = \frac{F_{max}\,L}{2} \qquad \textbf{Ans.}$$

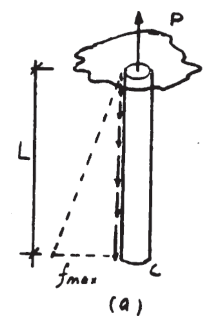
From FBD (b),

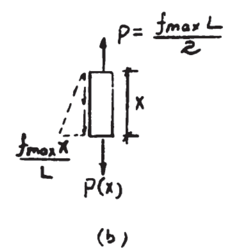
$$+\downarrow \Sigma F_y = 0; \qquad P(x) + \frac{1}{2}\left(\frac{F_{max}\,x}{L}\right)x - \frac{F_{max}\,L}{2} = 0$$

$$P(x) = \frac{F_{max}\,L}{2} - \frac{F_{max}\,x^2}{2L}$$

$$\delta = \int_0^L \frac{P(x)\,dx}{A(x)E} = \int_0^L \frac{F_{max}\,L}{2AE}dx - \int_0^L \frac{F_{max}\,x^2}{2AEL}dx$$

$$= \frac{F_{max}\,L^2}{3AE} \qquad \textbf{Ans.}$$





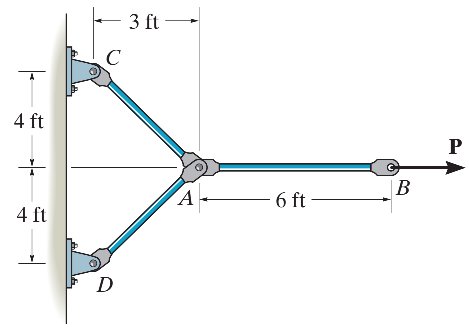
Ans:

$$P = \frac{F_{max}\,L}{2},$$

$$\delta = \frac{F_{max}\,L^2}{3AE}$$

9–18.

The linkage is made of three pin-connected A992 steel members, each having a diameter of $1\frac{1}{4}$ in. If a horizontal force of $P = 60$ kip is applied to the end B of member AB, determine the displacement of point B.

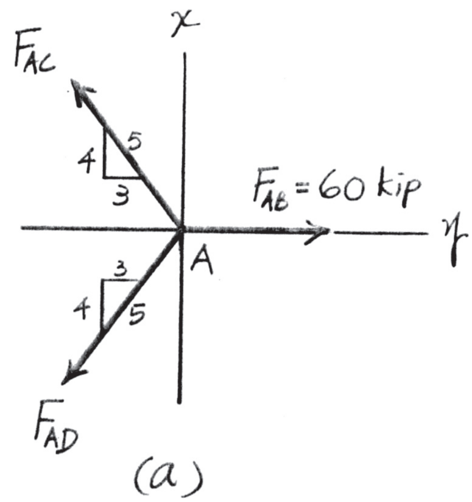


SOLUTION

Normal Forces And Stresses: Consider the equilibrium of joint A, Fig. *a*.

$$+\uparrow\Sigma F_y = 0; \qquad F_{AC}\left(\frac{4}{5}\right) - F_{AD}\left(\frac{4}{5}\right) = 0 \qquad F_{AC} = F_{AD} = F$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 60 - 2\left[F\left(\frac{3}{5}\right)\right] = 0 \qquad F = 50.0 \text{ kip}$$



Since the cross-sectional areas of each of the members are the same, member AB, which is subjected to the greatest normal force, will develop the maximum normal stress.

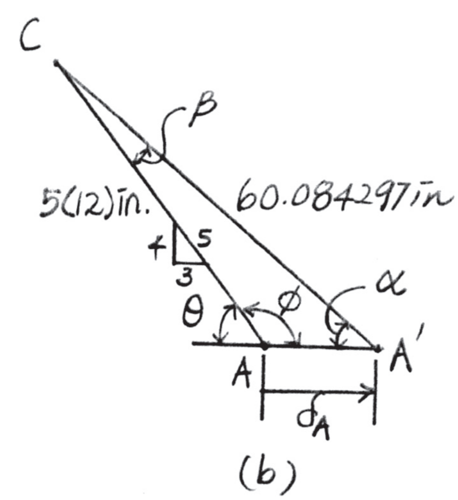
$$\sigma_{\max} = \sigma_{AB} = \frac{60}{\frac{\pi}{4}(1.25^2)} = 48.89 \text{ ksi}$$

Displacement: For A992 steel, $E = 29.0(10^3)$ ksi and $\sigma_y = 50$ ksi.

Since $\sigma_{\max} < \sigma_y$,

$$\delta_{AC} = \frac{FL_{AC}}{AE} = \frac{50.0[5(12)]}{\frac{\pi}{4}(1.25^2)[29.0(10^3)]} = 0.084297 \text{ in.}$$

$$\delta_{B/A} = \frac{F_{AB}L_{AB}}{AE} = \frac{60[6(12)]}{\frac{\pi}{4}(1.25^2)[29.0(10^3)]} = 0.121388 \text{ in.}$$



Referring to the geometry shown in Fig. *b*,

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13010^\circ \quad \phi = 180^\circ - \theta = 126.86990^\circ$$

$$L_{AC}' = L_{AC} + \delta_{AC} = 5(12) + 0.084297 = 60.084297 \text{ in.}$$

Applying the sine law,

$$\frac{\sin\alpha}{5(12)} = \frac{\sin 126.86990^\circ}{60.084297}; \quad \alpha = 53.023056^\circ$$

$$\beta = 180^\circ - 126.86990^\circ - 53.023056^\circ = 0.107047^\circ$$

Then

$$\frac{\delta_A}{\sin 0.107047^\circ} = \frac{5(12)}{\sin 53.023056^\circ}; \quad \delta_A = 0.140321 \text{ in.}$$

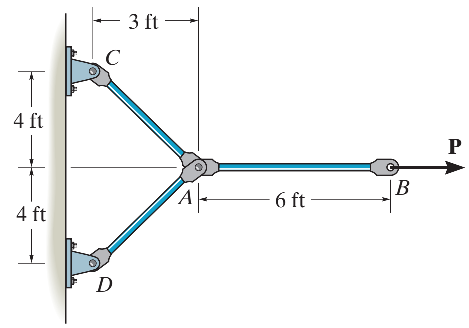
Thus,

$$\delta_B = \delta_A + \delta_{B/A} = 0.140321 + 0.121388 = 0.26171 \text{ in.} = 0.262 \text{ in.} \rightarrow$$ **Ans.**

Ans:
$\delta_B = 0.262$ in. $\rightarrow$

9–19.

The linkage is made of three pin-connected A992 steel members, each having a diameter of $1\frac{1}{4}$ in. Determine the magnitude of the force **P** needed to displace point B 0.25 in. to the right.

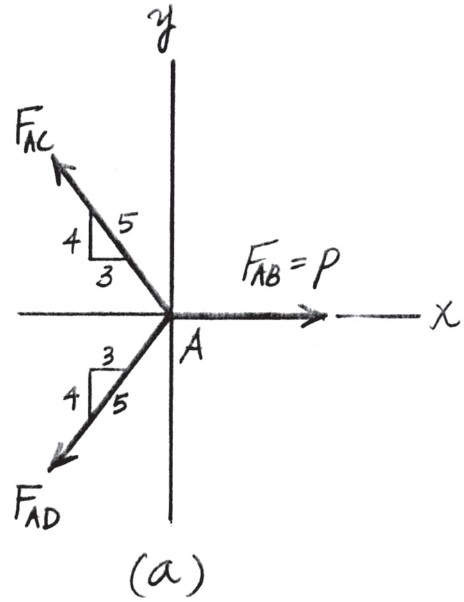


SOLUTION

Normal Forces: Consider the equilibrium of joint A, Fig. a.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{4}{5}\right) - F_{AD}\left(\frac{4}{5}\right) = 0 \qquad F_{AC} = F_{AD} = F$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 2\left[F\left(\frac{3}{5}\right)\right] = 0 \qquad F = 0.8333\,P$$



Displacement: For A992 steel, $E = 29.0(10^3)$ ksi and $\sigma_y = 50$ ksi.

$$\delta_{AC} = \frac{FL_{AC}}{AE} = \frac{0.8333P[5(12)]}{\frac{\pi}{4}(1.25^2)\left[29.0(10^3)\right]} = 1.40495(10^{-3})\,P$$

$$\delta_{B/A} = \frac{F_{AB}L_{AB}}{AE} = \frac{P[6(12)]}{\frac{\pi}{4}(1.25^2)\left[29.0(10^3)\right]} = 2.02313(10^{-3})\,P \rightarrow$$

Referring to the geometry shown in Fig. b, since δ_A is very small compared to L_{AC}, $\cos\alpha \approx \cos\theta = \frac{3}{5}$. Thus,

$$\delta_A = \frac{\delta_{AC}}{\cos\alpha} = \frac{1.40495(10^{-3})\,P}{3/5} = 2.34159(10^{-3})\,P$$

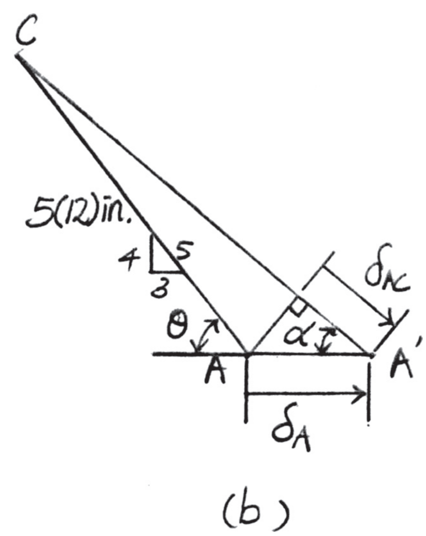
It is required that $\delta_B = 0.25$ in $\rightarrow$. Thus,

$$\xrightarrow{+} \delta_B = \delta_A + \delta_{B/A}; \qquad 0.25 = 2.34159(10^{-3})\,P + 2.02313(10^{-3})\,P$$

$$P = 57.28 \text{ kip} = 57.3 \text{ kip} \qquad \textbf{Ans.}$$

Since the cross-sectional areas of each of the members are the same, member AB, which is subjected to the greatest normal force, will develop maximum normal stress.

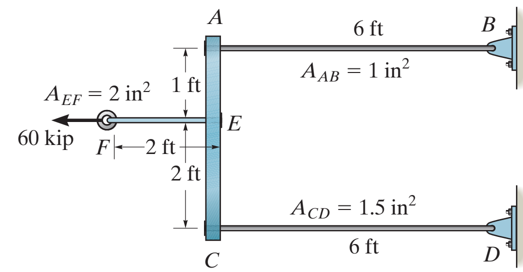
$$\sigma_{max} = \sigma_{AB} = \frac{57.28}{\frac{\pi}{4}(1.25^2)} = 46.67 \text{ ksi} < \sigma_y \qquad \textbf{(O.K)}$$



Ans:
$P = 57.3$ kip

***9–20.**

The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar AC. The cross-sectional area of each rod is given in the figure. If a force of 60 kip is applied to the ring F, determine the horizontal displacement of point F.



SOLUTION

Normal Forces And Stresses: Referring to the FBD of the rigid bar, Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(3) - 60(1) = 0 \quad F_{CD} = 20.0\text{ kip}$

$\circlearrowleft + \Sigma M_C = 0; \quad 60(2) - F_{AB}(3) = 0 \quad F_{AB} = 40.0\text{ kip}$

Thus,

$$\sigma_{EF} = \frac{F_{EF}}{A_{EF}} = \frac{60}{2} = 30.0\text{ ksi}; \sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{20.0}{1.5} = 13.33\text{ ksi}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40.0}{1} = 40.0\text{ ksi}$$

Displacement: For Ti-6AI-4V titanium, $E = 17.4(10^3)$ ksi and $\sigma_y = 134$ ksi. Since the normal stress in each rod $\sigma < \sigma_y$,

$$\delta_A = \frac{F_{AB}L_{AB}}{A_{AB}E} = \frac{40.0[6(12)]}{1[17.4(10^3)]} = 0.16552\text{ in.} \leftarrow$$

$$\delta_C = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{20.0[6(12)]}{1.5[17.4(10^3)]} = 0.5517\text{ in.} \leftarrow$$

$$\delta_{F/E} = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{60[2(12)]}{2[17.4(10^3)]} = 0.04138\text{ in.} \leftarrow$$

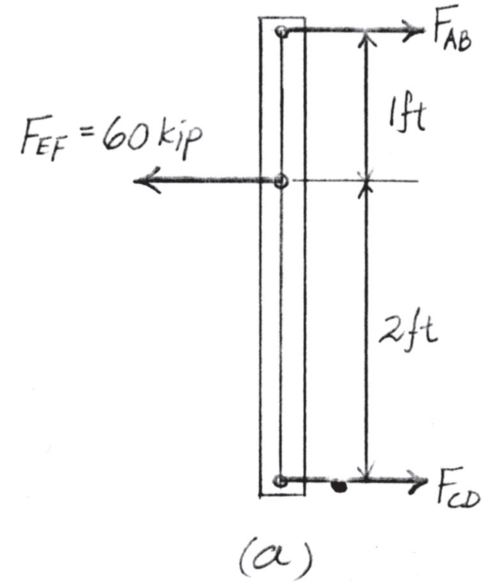
Referring to the geometry shown in Fig. *b*,

$$\frac{\delta'}{2} = \frac{0.16552 - 0.05517}{3}; \delta' = 0.07356\text{ in.}$$

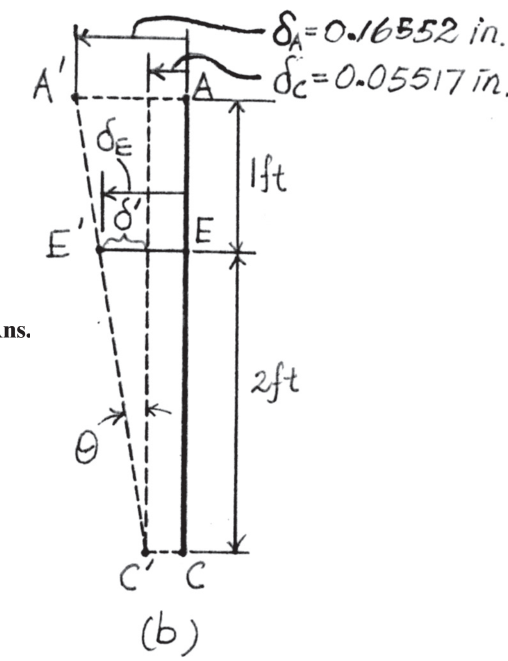
$$\delta_E = \delta_C + \delta' = 0.05517 + 0.07356 = 0.12874\text{ in.} \leftarrow$$

Then

$$\overset{+}{\leftarrow} \delta_F = \delta_E + \delta_{F/E} = 0.12874 + 0.04138 = 0.170\text{ in.} \leftarrow \qquad \textbf{Ans.}$$



(a)

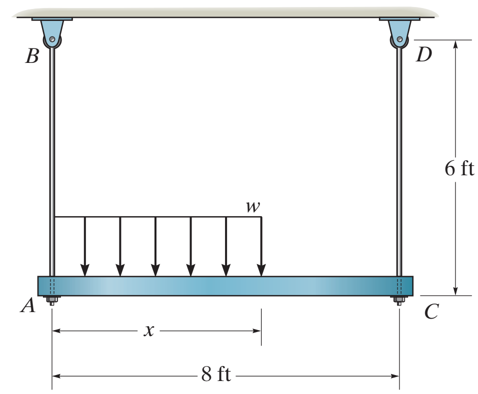


(b)

Ans:
$\delta_F = 0.170$ in.

9–21.

The rigid beam is supported at its ends by two A-36 steel tie rods. If the allowable stress for the steel is $\sigma_{allow} = 16.2$ ksi, the load $w = 3$ kip/ft, and $x = 4$ ft, determine the smallest diameter of each rod so that the beam remains in the horizontal position when it is loaded.

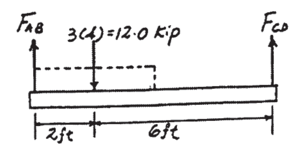


SOLUTION

Internal Force in the Rods:

$$+\Sigma M_A = 0; \qquad F_{CD}(8) - 12.0(2) = 0 \quad F_{CD} = 3.00 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + 3.00 - 12.0 = 0 \quad F_{AB} = 9.00 \text{ kip}$$



Displacement: To maintain the rigid beam in the horizontal position, the elongation of both rods AB and CD must be the same.

$$\delta_{AB} = \delta_{CD}$$

$$\frac{9.00(6)(12)}{\frac{\pi}{4}d_{AB}^2 E} = \frac{3.00(6)(12)}{\frac{\pi}{4}d_{CD}^2 E};$$

$$9d_{CD}^2 = 3d_{AB}^2; \qquad d_{AB} = \sqrt{3}\, d_{CD} \qquad \textbf{(1)}$$

Allowable Normal Stress: Assume failure of rod AB.

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \qquad 16.2 = \frac{9.00}{\frac{\pi}{4}d_{AB}^2}$$

$$d_{AB} = 0.841 \text{ in.} \qquad \textbf{Ans.}$$

From Eq. (1), $\quad d_{CD} = 0.486$ in. **Ans.**

Assume failure of rod CD.

$$\sigma_{allow} = \frac{F_{CD}}{A_{AB}}; \qquad 16.2 = \frac{3.00}{\frac{\pi}{4}d_{CD}^2}$$

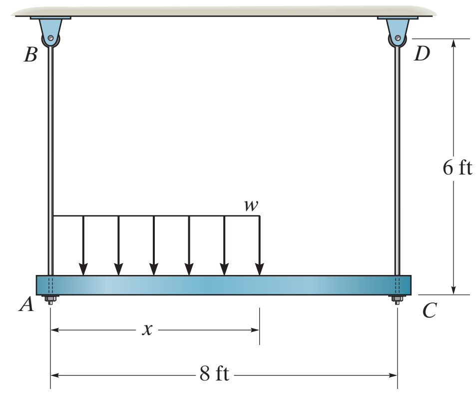
$$d_{CD} = 0.486 \text{ in.} \qquad \textbf{Ans.}$$

From Eq. (1), $\quad d_{AB} = 0.841$ in. **Ans.**

Ans:
$d_{AB} = 0.841$ in.,
$d_{CD} = 0.486$ in.

9–22.

The rigid beam is supported at its ends by two A-36 steel tie rods. The rods have diameters $d_{AB} = 0.5$ in. and $d_{CD} = 0.3$ in. If the allowable stress for the steel is $\sigma_{allow} = 16.2$ ksi, determine the largest intensity of the distributed load w and its length x on the beam so that the beam remains in the horizontal position when it is loaded.



SOLUTION

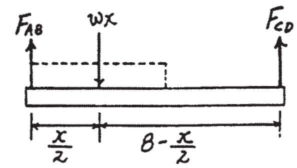
Internal Force in the Rods:

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD}(8) - wx\left(\frac{x}{2}\right) = 0$$

$$8F_{CD} - \frac{wx^2}{2} = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -F_{AB}(8) + wx\left(8 - \frac{x}{2}\right) = 0$$

$$8wx - \frac{wx^2}{2} - 8F_{AB} = 0 \qquad \textbf{(2)}$$



Displacement: To maintain the rigid beam in the horizontal position, both elongations of rods AB and CD must be the same.

$$\delta_{AB} = \delta_{CD}$$

$$\frac{F_{AB}(6)(12)}{\frac{\pi}{4}(0.5^2)E} = \frac{F_{CD}(6)(12)}{\frac{\pi}{4}(0.3^2)E}$$

$$F_{CD} = 0.360\, F_{AB} \qquad \textbf{(3)}$$

Allowable Normal Stress: Assume failure of rod AB.

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \qquad 16.2 = \frac{F_{AB}}{\frac{\pi}{4}(0.5^2)} \qquad F_{AB} = 3.1809 \text{ kip}$$

Using $F_{AB} = 3.1809$ kip and solving Eqs. (1) to (3) yields :

$$F_{CD} = 1.1451 \text{ kip}$$

$$x = 4.24 \text{ ft} \qquad \textbf{Ans.}$$

$$w = 1.02 \text{ kip/ft} \qquad \textbf{Ans.}$$

Assume failure of rod CD.

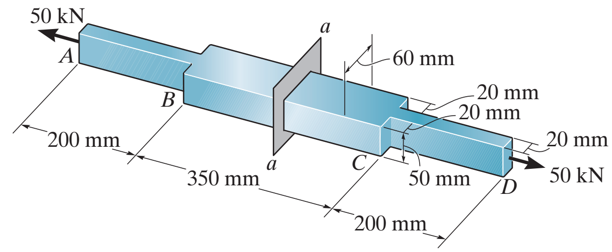
$$\sigma_{allow} = \frac{F_{CD}}{A_{CD}}; \qquad 16.2 = \frac{F_{CD}}{\frac{\pi}{4}(0.3^2)} \qquad F_{CD} = 1.1451 \text{ kip}$$

Therefore, rods AB and CD fail simultaneously.

Ans:
$x = 4.24$ ft,
$w = 1.02$ kip/ft

9–23.

The steel bar has the original dimensions shown in the figure. If it is subjected to an axial load of 50 kN, determine the change in its length and its new cross-sectional dimensions at section a–a. $E_{st} = 200$ GPa, $\nu_{st} = 0.29$.



SOLUTION

$$\delta_{A/D} = \Sigma\frac{PL}{AE} = \frac{2(50)(10^3)(200)}{(0.02)(0.05)(200)(10^9)} + \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)}$$

$$= 0.129 \text{ mm}$$ **Ans.**

$$\delta_{B/C} = \frac{PL}{AE} = \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)} = 0.02917 \text{ mm}$$

$$\epsilon_{BC} = \frac{\delta_{B/C}}{L_{BC}} \frac{0.02917}{350} = 0.00008333$$

$$\epsilon_{lat} = -\nu\,\epsilon_{long} = -(0.29)(0.00008333) = -0.00002417$$

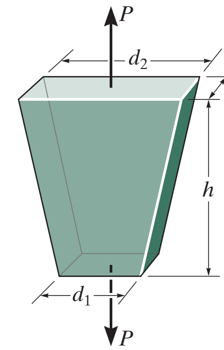
$$h' = 50 - 50(0.00002417) = 49.9988 \text{ mm}$$ **Ans.**

$$w' = 60 - 60(0.00002417) = 59.9986 \text{ mm}$$ **Ans.**

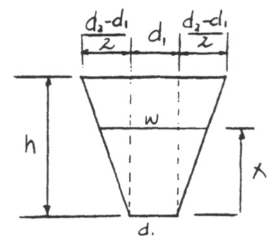
Ans:
$\delta_{A/D} = 0.129$ mm,
$h' = 49.9988$ mm,
$w' = 59.9986$ mm

***9–24.**

Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load P.



SOLUTION



$$w = d_1 + \frac{d_2 - d_1}{h}x = \frac{d_1 h + (d_2 - d_1)x}{h}$$

$$\delta = \int \frac{P(x)\,dx}{A(x)E} = \frac{P}{E}\int_0^h \frac{dx}{\frac{[d_1 h + (d_2 - d_1)x]t}{h}}$$

$$= \frac{Ph}{E\,t}\int_0^h \frac{dx}{d_1 h + (d_2 - d_1)x}$$

$$= \frac{Ph}{E\,t\,d_1 h}\int_0^h \frac{dx}{1 + \frac{d_2 - d_1}{d_1 h}x} = \frac{Ph}{E\,t\,d_1 h}\left(\frac{d_1 h}{d_2 - d_1}\right)\left[\ln\left(1 + \frac{d_2 - d_1}{d_1 h}x\right)\right]\Bigg|_0^h$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(1 + \frac{d_2 - d_1}{d_1}\right)\right] = \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(\frac{d_1 + d_2 - d_1}{d_1}\right)\right]$$

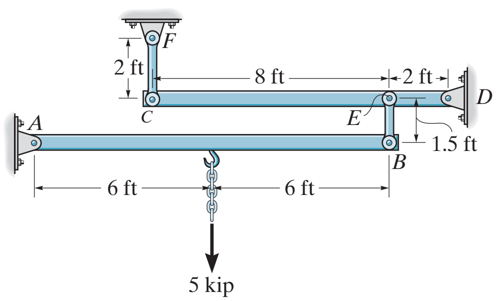
$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right] \qquad \textbf{Ans.}$$

Ans:

$$\delta = \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right]$$

9–25.

The assembly consists of two rigid bars that are originally horizontal. They are supported by pins and 0.25-in.-diameter A-36 steel rods. If the vertical load of 5 kip is applied to the bottom bar AB, determine the displacement at C, B, and E.



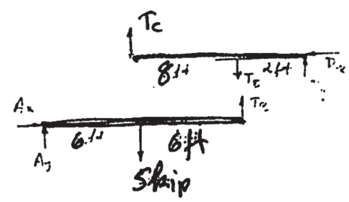
SOLUTION

$$\zeta+\Sigma M_A = 0; \qquad T_B(12) - 5(6) = 0$$

$$T_B = 2.5 \text{ kip}$$

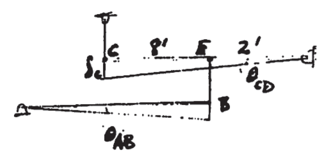
$$\zeta+\Sigma M_D = 0; \qquad 2.5(2) - T_C(10) = 0$$

$$T_C = 0.5 \text{ kip}$$



$$\delta_{B/E} = \frac{PL}{AE} = \frac{2.5(1.5)(12)}{\frac{\pi}{4}(0.25)^2(29)(10^3)} = 0.0316 \text{ in.}$$

$$\delta_C = \frac{PL}{AE} = \frac{0.5(2)(12)}{\frac{\pi}{4}(0.25)^2(29)(10^3)} = 0.0084297 \text{ in.} = 0.00843 \text{ in.} \qquad \textbf{Ans.}$$



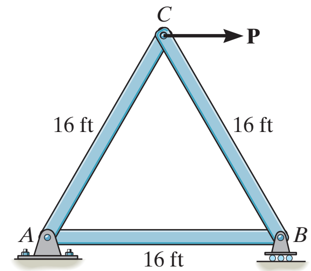
$$\delta_E = \left(\frac{2}{10}\right)\delta_C = \frac{2}{10}(0.0084297) = 0.00169 \text{ in.} \qquad \textbf{Ans.}$$

$$\delta_B = \delta_E + \delta_{B/E} = 0.00169 + 0.0316 = 0.0333 \text{ in.} \qquad \textbf{Ans.}$$

Ans:
$\delta_C = 0.00843$ in.,
$\delta_E = 0.00169$ in.,
$\delta_B = 0.0333$ in.

9–26.

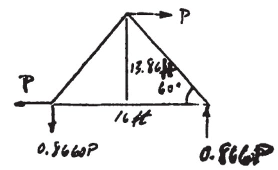
The truss consists of three members, each made from A-36 steel and having a cross-sectional area of 0.75 in^2. Determine the greatest load P that can be applied so that the roller support at B is not displaced more than 0.03 in.

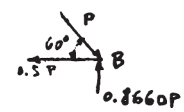


SOLUTION

$$\delta_{B_A} = 0.03 \text{ in.} = \frac{(0.5)P(16)(12)}{(0.75)(29)(10^6)}$$

$$P = 6.80 \text{ kip} \qquad \textbf{Ans.}$$

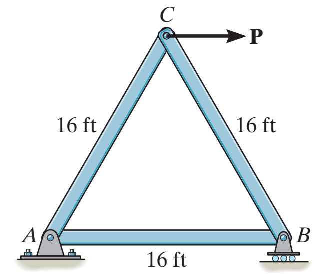




Ans:
$P = 6.80$ kip

9–27.

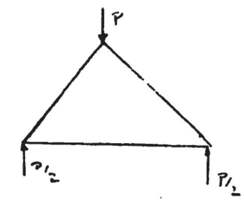
Solve Prob. 9–26 when the load **P** acts vertically downward at C.

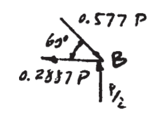


SOLUTION

$$\delta_{B_A} = 0.03 \text{ in.} = \frac{0.2887\,P(16)(12)}{(0.75)(29)(10^6)}$$

$$P = 11.8 \text{ kip}$$ **Ans.**

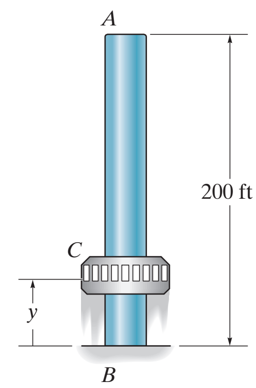




Ans:
$P = 11.8$ kip

***9–28.**

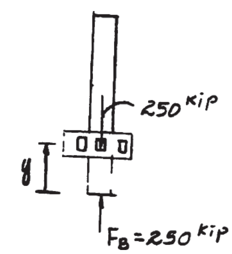
The observation cage C has a weight of 250 kip and through a system of gears, travels upward at constant velocity along the A-36 steel column, which has a height of 200 ft. The column has an outer diameter of 3 ft and is made from steel plate having a thickness of 0.25 in. Neglect the weight of the column, and determine the average normal stress in the column at its base, B, as a function of the cage's position y. Also, determine the displacement of end A as a function of y.



SOLUTION

$$\sigma_B = \frac{P}{A} = \frac{250}{\frac{\pi}{4}\left(36^2 - 35.5^2\right)} = 8.90 \text{ ksi}$$

σ_B is independent of y.

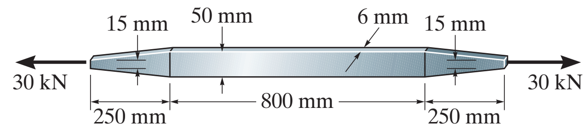


$$\delta_A = \frac{PL}{AE} = \frac{250y}{\frac{\pi}{4}(36^2 - 35.5^2)(29)\left(10^3\right)} = [0.307\left(10^{-3}\right)y] \text{ ft}$$ **Ans.**

Ans:
$\delta_A = [0.307\left(10^{-3}\right)y]$ ft

9–29.

Determine the elongation of the aluminum strap when it is subjected to an axial force of 30 kN. $E_{al} = 70$ GPa.



SOLUTION

$$\delta = (2)\frac{Ph}{Et(d_2 - d_1)}\ln\frac{d_2}{d_1} + \frac{PL}{AE}$$

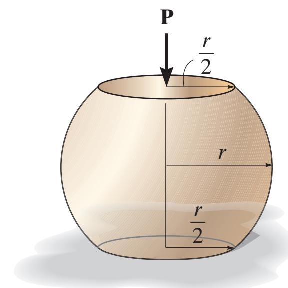
$$= \frac{2(30)(10^3)(250)}{(70)(10^9)(0.006)(0.05 - 0.015)}\left(\ln\frac{50}{15}\right) + \frac{30(10^3)(800)}{(0.006)(0.05)(70)(10^9)}$$

$$= 2.37 \text{ mm}$$ **Ans.**

Ans:
$\delta = 2.37$ mm

9–30.

The ball is truncated at its ends and is used to support the bearing load **P**. If the modulus of elasticity for the material is E, determine the decrease in the ball's height when the load is applied.

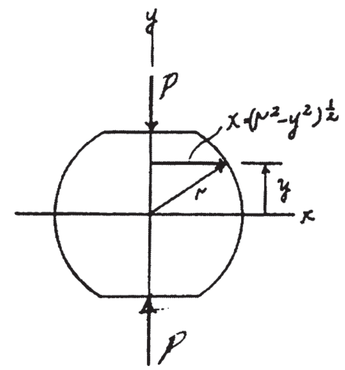


SOLUTION

Displacement:

Geometry:

$$A(y) = \pi x^2 = \pi\left(r^2 - y^2\right)$$



Displacement: When $x = \frac{r}{2}$, $\quad y = \pm\frac{\sqrt{3}}{2}r$

$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= \frac{P}{\pi E}\int_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r} \frac{dy}{r^2 - y^2}$$

$$= \frac{P}{\pi E}\left[\frac{1}{2r}\ln\frac{r+y}{r-y}\right]\Bigg|_{-\frac{\sqrt{3}}{2}r}^{\frac{\sqrt{3}}{2}r}$$

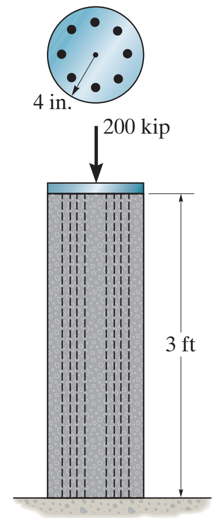
$$= \frac{P}{2\pi\, r\, E}\left[\ln 13.9282 - \ln 0.07180\right]$$

$$= \frac{2.63\,P}{\pi r E}$$ **Ans.**

Ans:
$\delta = \frac{2.63\,P}{\pi r E}$

9–31.

The column is constructed from high-strength concrete and eight A992 steel reinforcing rods. If the column is subjected to an axial force of 200 kip, determine the average normal stress in the concrete and in each rod. Each rod has a diameter of 1 in.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the upper segment of the column sectioned at an arbitrary distance, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad P_{con} + 8P_{st} - 200 = 0 \qquad \textbf{(1)}$$

Compatibility: Since the steel bows and concrete are bonded,

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}L}{A_{st}E_{st}} = \frac{P_{con}L}{A_{con}E_{con}}$$

For A992 steel, $E_{st} = 29.0(10^3)$ ksi and $\sigma_y = 50$ ksi. Also, $E_{con} = 4.20(10^3)$ ksi for high-strength concrete. Thus,

$$\frac{P_{st}(3)(12)}{\frac{\pi}{4}(1^2)\left[29.0(10^3)\right]} = \frac{P_{con}(3)(12)}{\left[\frac{\pi}{4}(8^2) - 8\left(\frac{\pi}{4}\right)(1^2)\right]\left[4.20(10^3)\right]}$$

$$P_{st} = 0.1233\, P_{con} \qquad \textbf{(2)}$$

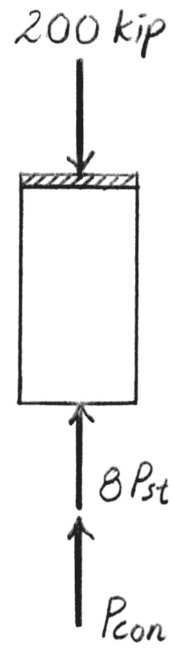
Solving Eqs. (1) and (2),

$$P_{con} = 100.68 \text{ kip} \qquad P_{st} = 12.41 \text{ kip}$$

Average Normal Stress:

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{100.68}{\left[\frac{\pi}{4}(8^2) - 8\left(\frac{\pi}{4}\right)(1^2)\right]} = 2.29 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{12.41}{\frac{\pi}{4}(1^2)} = 15.8 \text{ ksi} \qquad \textbf{Ans.}$$

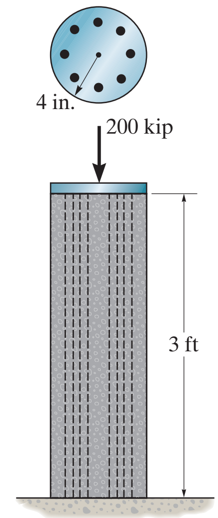


(a)

Ans:
$\sigma_{con} = 2.29$ ksi,
$\sigma_{st} = 15.8$ ksi

***9–32.**

The column is constructed from high-strength concrete and eight A992 steel reinforcing rods. If the column is subjected to an axial force of 200 kip, determine the required diameter of each rod so that 60% of the axial force is carried by the concrete.



SOLUTION

Equilibrium: The axial force of 200 kip is required to distribute in such a manner that 60% is carried by concrete and 40% is carried by steel. Thus,

$P_{con} = 0.6(200) = 120 \text{ kip} \qquad P_{st} = 0.4(200) = 80 \text{ kip}$

Compatibility: Since the steel and concrete are bonded, then

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}L}{A_{st}E_{st}} = \frac{P_{con}L}{A_{con}E_{con}}$$

$$A_{st} = \left(\frac{P_{st}}{P_{con}}\right)\left(\frac{E_{con}}{E_{st}}\right)A_{con}$$

For A992 steel, $E_{st} = 29.0(10^3)$ ksi and $\sigma_y = 50$ ksi. Also, $E_{con} = 4.20(10^3)$ ksi for high-strength concrete. Thus,

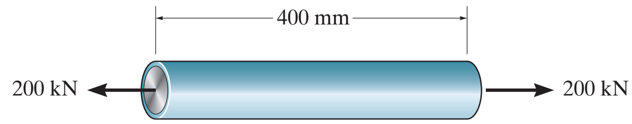
$$8\left(\frac{\pi}{4}d^2\right) = \left(\frac{80}{120}\right)\left[\frac{4.20(10^3)}{29.0(10^3)}\right]\left[\frac{\pi}{4}(8^2) - 8\left(\frac{\pi}{4}d^2\right)\right]$$

$d = 0.839$ in. **Ans.**

Ans:
$d = 0.839$ in.

9–33.

The A-36 steel pipe has a 6061-T6 aluminum core. It is subjected to a tensile force of 200 kN. Determine the average normal stress in the aluminum and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm.



SOLUTION

Equations of Equilibrium:

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad P_{al} + P_{st} - 200 = 0 \qquad \textbf{(1)}$$

Compatibility:

$$\delta_{al} = \delta_{st}$$

$$\frac{P_{al}(400)}{\frac{\pi}{4}\left(0.07^2\right)(68.9)\left(10^9\right)} = \frac{P_{st}(400)}{\frac{\pi}{4}\left(0.08^2 - 0.07^2\right)(200)\left(10^9\right)}$$

$$P_{al} = 1.125367\, P_{st} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$P_{st} = 94.10 \text{ kN} \qquad P_{al} = 105.90 \text{ kN}$$

Average Normal Stress:

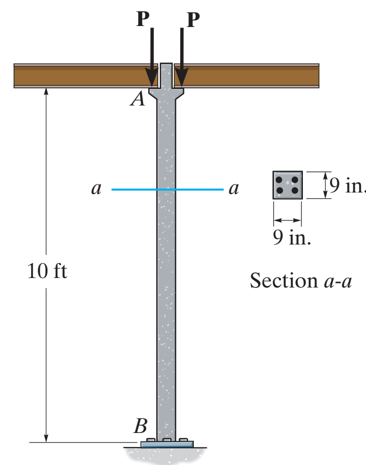
$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{105.90\left(10^3\right)}{\frac{\pi}{4}\left(0.07^2\right)} = 27.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{94.10\left(10^3\right)}{\frac{\pi}{4}\left(0.08^2 - 0.07^2\right)} = 79.9 \text{ MPa} \qquad \textbf{Ans.}$$

Ans:
$\sigma_{al} = 27.5$ MPa,
$\sigma_{st} = 79.9$ MPa

9–34.

If column AB is made from high strength precast concrete and reinforced with four $\frac{3}{4}$ in. diameter A-36 steel rods, determine the average normal stress developed in the concrete and in each rod. Set $P = 75$ kip.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the cut part of the concrete column shown in Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad P_{con} + 4P_{st} - 2(75) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Since the steel bars and the concrete are firmly bonded, their deformation must be the same. Thus,

$$\delta_{con} = \delta_{st}$$

$$\frac{P_{con}(10)(12)}{\left[(9)(9) - 4\left(\frac{\pi}{4}\right)\left(\frac{3}{4}\right)^2\right](4.20)(10^3)} = \frac{P_{st}(10)(12)}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2(29)(10^3)}$$

$$P_{con} = 25.974P_{st} \qquad \textbf{(2)}$$

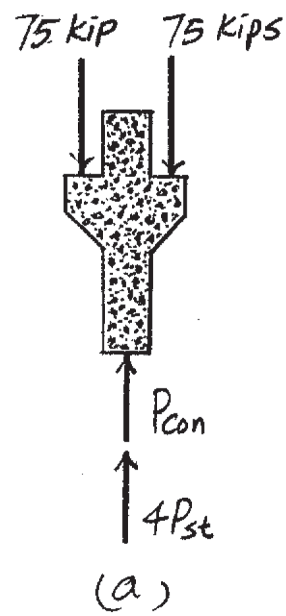
Solving Eqs. (1) and (2),

$$P_{st} = 5.0043 \text{ kip} \qquad P_{con} = 129.98 \text{ kip}$$

Normal Stress: Applying Eq. (1-6),

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{129.98}{(9)(9) - 4\left(\frac{\pi}{4}\right)\left(\frac{3}{4}\right)^2} = 1.64 \text{ ksi} \qquad \textbf{Ans.}$$

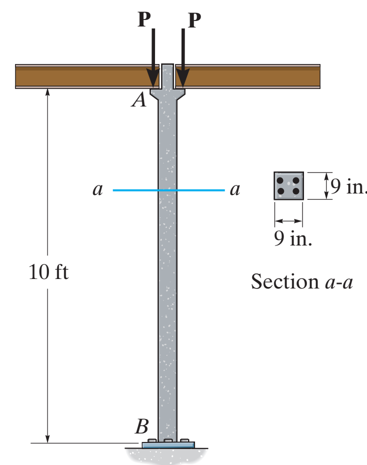
$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{5.0043}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2} = 11.3 \text{ ksi} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{con} = 1.64$ ksi,
$\sigma_{st} = 11.3$ ksi

9–35.

If column AB is made from high strength precast concrete and reinforced with four $\frac{3}{4}$ in. diameter A-36 steel rods, determine the maximum allowable floor loadings **P**. The allowable normal stresses for the concrete and the steel are $(\sigma_{\text{allow}})_{\text{con}} = 2.5$ ksi and $(\sigma_{\text{allow}})_{\text{st}} = 24$ ksi, respectively.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the cut part of the concrete column shown in Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad P_{\text{con}} + 4P_{\text{st}} - 2P = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Since the steel bars and the concrete are firmly bonded, their deformation must be the same. Thus,

$$\delta_{\text{con}} = \delta_{\text{st}}$$

$$\frac{P_{\text{con}}(10)(12)}{\left[(9)(9) - 4\left(\frac{\pi}{4}\right)\left(\frac{3}{4}\right)^2\right](4.20)(10^3)} = \frac{P_{\text{st}}(10)(12)}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2(29.0)(10^3)}$$

$$P_{\text{con}} = 25.974P_{\text{st}} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$P_{\text{st}} = 0.06672P \qquad P_{\text{con}} = 1.7331P$$

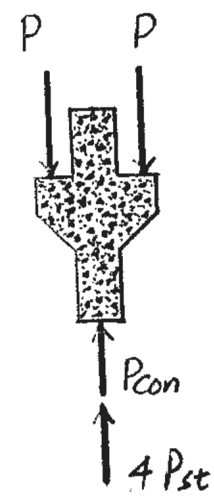
Allowable Normal Stress:

$$(\sigma_{\text{con}})_{\text{allow}} = \frac{P_{\text{con}}}{A_{\text{con}}}; \qquad 2.5 = \frac{1.7331P}{(9)(9) - 4\left(\frac{\pi}{4}\right)\left(\frac{3}{4}\right)^2}$$

$$P = 114.29 \text{ kip} = 114 \text{ kip (controls)} \qquad \textbf{Ans.}$$

$$(\sigma_{\text{st}})_{\text{allow}} = \frac{P_{\text{st}}}{A_{\text{st}}}; \qquad 24 = \frac{0.06672P}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2}$$

$$P = 158.91 \text{ kip}$$

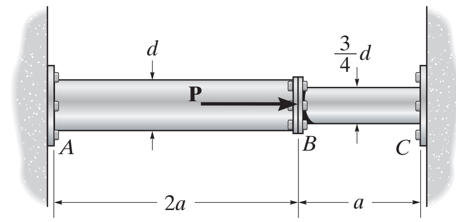


(a)

Ans:
$P = 114$ kip

***9–36.**

Determine the support reactions at the rigid supports A and C. The material has a modulus of elasticity of E.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_A - F_C = 0$ **(1)**

Compatibility Equation: Using the method of superposition, Fig. *b*,

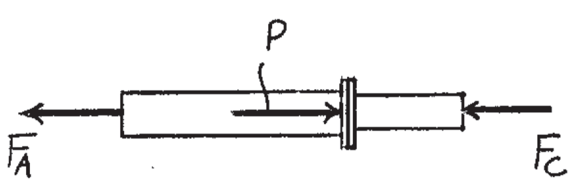
$(\xrightarrow{+}) \qquad \delta = \delta_P - \delta_{F_C}$

$$0 = \frac{P(2a)}{\left(\frac{\pi}{4}d^2\right)E} - \left[\frac{F_C a}{\frac{\pi}{4}\left(\frac{3}{4}d\right)^2 E} + \frac{F_C(2a)}{\left(\frac{\pi}{4}d^2\right)E}\right]$$

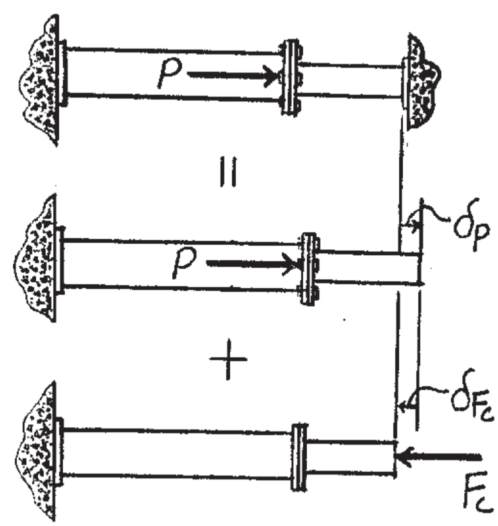
$$F_C = \frac{9}{17}P$$ **Ans.**

Substituting this result into Eq. (1),

$$F_A = \frac{8}{17}P$$ **Ans.**



(a)

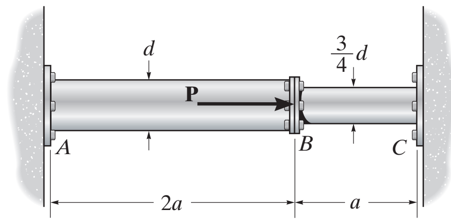


(b)

Ans:
$F_C = \frac{9}{17}P,$
$F_A = \frac{8}{17}P$

9–37.

If the supports at A and C are flexible and have a stiffness k, determine the support reactions at A and C. The material has a modulus of elasticity of E.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_A - F_C = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. b,

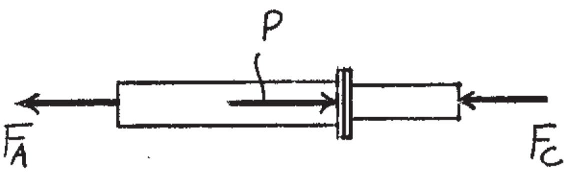
$$(\xrightarrow{+}) \qquad \delta_C = \delta_P - \delta_{F_C}$$

$$\frac{F_C}{k} = \left[\frac{P(2a)}{\left(\frac{\pi}{4}d^2\right)E} + \frac{P}{k}\right] - \left[\frac{F_C a}{\frac{\pi}{4}\left(\frac{3}{4}d\right)^2 E} + \frac{F_C(2a)}{\left(\frac{\pi}{4}d^2\right)E} + \frac{F_C}{k}\right]$$

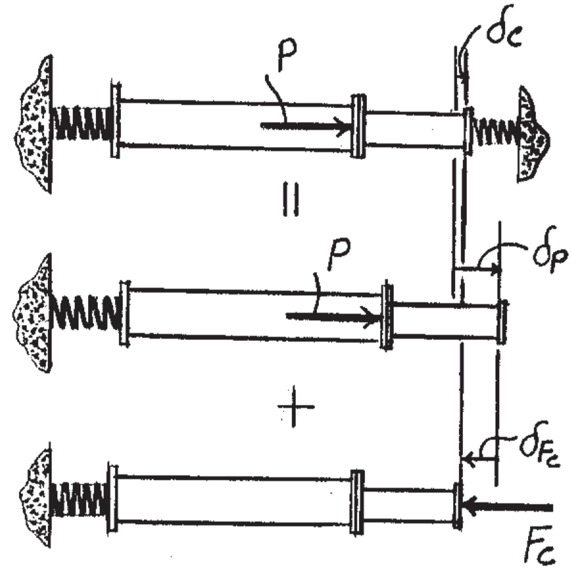
$$F_C = \left[\frac{9(8ka + \pi d^2 E)}{136ka + 18\pi d^2 E}\right]P \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = \left(\frac{64ka + 9\pi d^2 E}{136ka + 18\pi d^2 E}\right)P \qquad \textbf{Ans.}$$



(a)



(b)

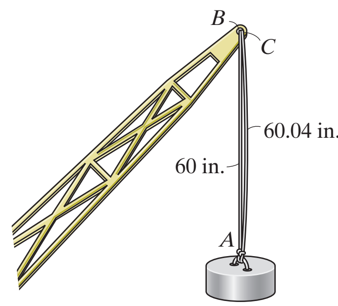
Ans:

$$F_C = \left[\frac{9(8ka + \pi d^2 E)}{136ka + 18\pi d^2 E}\right]P,$$

$$F_A = \left(\frac{64ka + 9\pi d^2 E}{136ka + 18\pi d^2 E}\right)P$$

9–38.

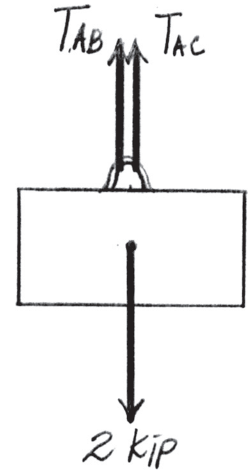
The load of 2000 lb is to be supported by the two vertical steel wires for which $\sigma_Y = 70$ ksi. Originally wire AB is 60 in. long and wire AC is 60.04 in. long. Determine the force developed in each wire after the load is suspended. Each wire has a cross-sectional area of 0.02 in^2. $E_{st} = 29.0(10^3)$ ksi.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the load, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad T_{AB} + T_{AC} - 2 = 0 \qquad \textbf{(1)}$$



(a)

Compatibility: Referring to the deformation diagram shown in Fig. *b*,

$$(+\downarrow) \qquad \delta_{AB} = \delta_{AC} + 0.04$$

$$\frac{T_{AB}L_{AB}}{AE_{st}} = \frac{T_{AC}L_{AC}}{AE_{st}} + 0.04$$

$$\frac{T_{AB}(60)}{(0.02)\left[29.0\left(10^3\right)\right]} = \frac{T_{AC}(60.04)}{(0.02)\left[29.0\left(10^3\right)\right]} + 0.04$$

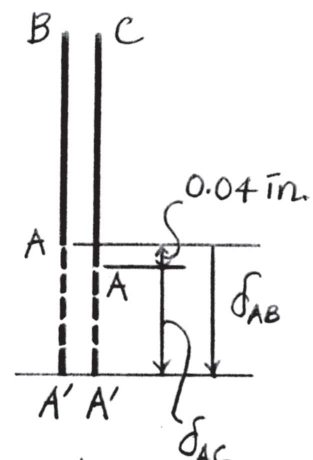
$$60\,T_{AB} = 60.04\,T_{AC} + 23.2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_{AC} = 0.8064 \text{ kip} = 0.806 \text{ kip} \qquad T_{AB} = 1.1936 \text{ kip} = 1.19 \text{ kip} \qquad \textbf{Ans.}$$

Average Normal Stress:

$$\sigma_{max} = \sigma_{AB} = \frac{T_{AB}}{A} = \frac{1.1936}{0.02} = 59.68 \text{ ksi} < \sigma_y = 70 \text{ ksi} \qquad \textbf{(O.K!)}$$

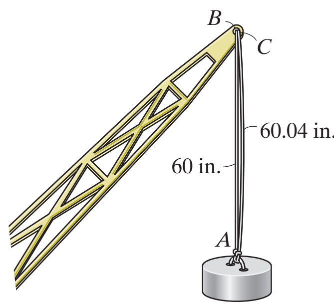


(b)

Ans:
$T_{AC} = 0.806$ kip,
$T_{AB} = 1.19$ kip

9–39.

The load of 2000 lb is to be supported by the two vertical steel wires for which $\sigma_Y = 70$ ksi. Originally wire AB is 60 in. long and wire AC is 60.04 in. long. Determine the cross-sectional area of AB if the load is to be shared equally between both wires. Wire AC has a cross-sectional area of 0.02 in^2. $E_{st} = 29.0(10^3)$ ksi.



SOLUTION

Equilibrium: The force of 2 kip must be shared equally by the two wires. Hence,

$$T_{AB} = T_{AC} = \frac{2}{2} = 1.00 \text{ kip}$$

Compatibility: Referring to the deformation diagram shown in Fig. *a*,

$$(+\downarrow) \qquad \delta_{AB} = \delta_{AC} + 0.04$$

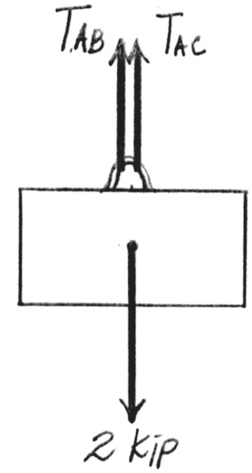
$$\frac{T_{AB}L_{AB}}{A_{AB}E_{st}} = \frac{T_{AC}L_{AC}}{A_{AC}E_{st}} + 0.04$$

$$\frac{1.00(60)}{A_{AB}\left[29.0(10^3)\right]} = \frac{1.00(60.04)}{(0.02)\left[29.0(10^3)\right]} + 0.04$$

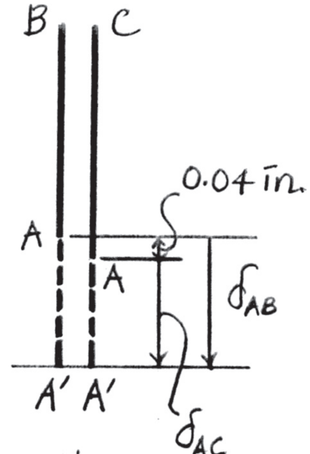
$$A_{AB} = 0.01442 \text{ in}^2 = 0.0144 \text{ in}^2$$ **Ans.**

Average Normal Stress:

$$\sigma_{\max} = \sigma_{AB} = \frac{1.00}{0.01442} = 69.37 \text{ ksi} < \sigma_y = 70 \text{ ksi}$$ **(O.K!)**



(a)

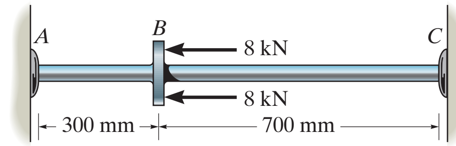


(b)

Ans:
$A_{AB} = 0.0144 \text{ in}^2$

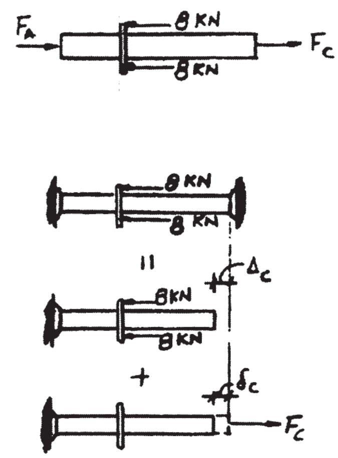
***9–40.**

The A-36 steel pipe has an outer radius of 20 mm and an inner radius of 15 mm. If it fits snugly between the fixed walls before it is loaded, determine the reaction at the walls when it is subjected to the load shown.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_A + F_C - 16 = 0 \qquad \textbf{(1)}$



By superposition:

$$(\overset{+}{\rightarrow}) \qquad 0 = -\Delta_C + \delta_C$$

$$0 = \frac{-16\,(300)}{AE} + \frac{F_C\,(1000)}{AE}$$

$F_C = 4.80\text{ kN}$ **Ans.**

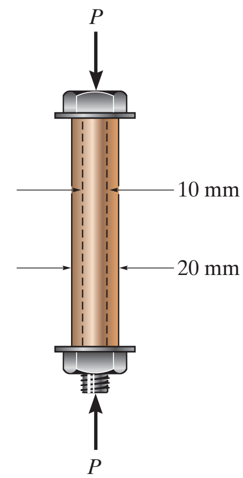
From Eq. (1),

$F_A = 11.2\text{ kN}$ **Ans.**

Ans:
$F_C = 4.80\text{ kN}$,
$F_A = 11.2\text{ kN}$

9–41.

The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the yield stress for the steel is $(\sigma_Y)_{st} = 640$ MPa, and for the bronze $(\sigma_Y)_{br} = 520$ MPa, determine the magnitude of the largest elastic load P that can be applied to the assembly. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.



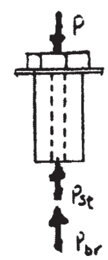
SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - P = 0 \qquad$ **(1)**

Assume failure of bolt:

$$P_{st} = (\sigma_Y)_{st}(A) = 640(10^6)\left(\frac{\pi}{4}\right)(0.01^2)$$

$$= 50265.5 \text{ N}$$



$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\, P_{br}$$

$$50\,265.5 = 0.6667\, P_{br}$$

$$P_{br} = 75398.2 \text{ N}$$

From Eq. (1),

$$P = 50265.5 + 75398.2$$

$$= 125663.7 \text{ N} = 126 \text{ kN} \qquad \text{(controls)}$$ **Ans.**

Assume failure of sleeve:

$$P_{br} = (\sigma_Y)_{br}(A) = 520(10^6)\left(\frac{\pi}{4}\right)(0.02^2 - 0.01^2) = 122\,522.11 \text{ N}$$

$$P_{st} = 0.6667\, P_{br}$$

$$= 0.6667(122\,522.11)$$

$$= 81\,681.4 \text{ N}$$

From Eq. (1),

$$P = 122\,522.11 + 81\,681.4$$

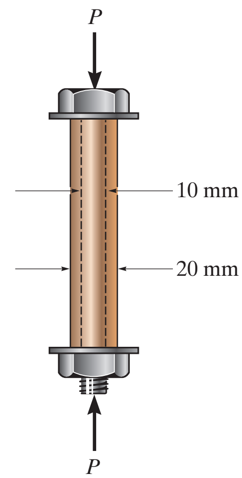
$$= 204\,203.52 \text{ N}$$

$$= 204 \text{ kN}$$

Ans:
$P = 126$ kN

9–42.

The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the bolt is subjected to a compressive force of $P = 20$ kN, determine the average normal stress in the steel and the bronze. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 20 = 0 \qquad \textbf{(1)}$

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

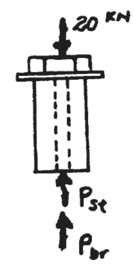
$$P_{st} = 0.6667\,P_{br} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$P_{st} = 8\text{ kN} \qquad P_{br} = 12\text{ kN}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{8(10^3)}{\frac{\pi}{4}(0.01^2)} = 102\text{ MPa} \qquad \textbf{Ans.}$$

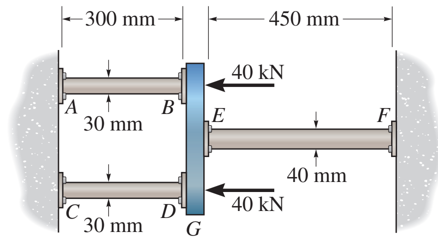
$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{12(10^3)}{\frac{\pi}{4}(0.02^2 - 0.01^2)} = 50.9\text{ MPa} \qquad \textbf{Ans.}$$



Ans:
$\sigma_{st} = 102$ MPa,
$\sigma_{br} = 50.9$ MPa

9–43.

The assembly consists of two red brass C83400 copper rods AB and CD of diameter 30 mm, a stainless 304 steel alloy rod EF of diameter 40 mm, and a rigid cap G. If the supports at A, C, and F are rigid, determine the average normal stress developed in the rods.



SOLUTION

Equation of Equilibrium: Due to symmetry, $F_{AB} = F_{CD} = F$. Referring to the free-body diagram of the assembly shown in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F + F_{EF} - 2\left[40\left(10^3\right)\right] = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. b,

$$(\overset{+}{\rightarrow}) \quad 0 = -\delta_P + \delta_{EF}$$

$$0 = -\frac{40(10^3)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)} + \left[\frac{F_{EF}(450)}{\frac{\pi}{4}(0.04^2)(193)(10^9)} + \frac{\left(F_{EF}/2\right)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)}\right]$$

$$F_{EF} = 42\,483.23 \text{ N}$$

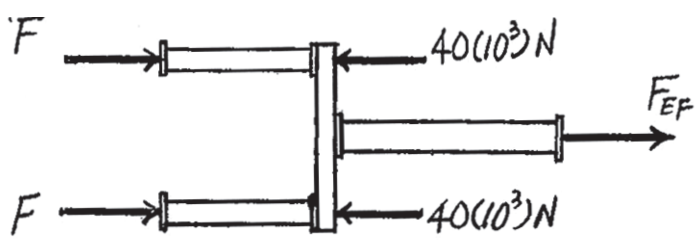
Substituting this result into Eq. (1),

$$F = 18\,758.38 \text{ N}$$

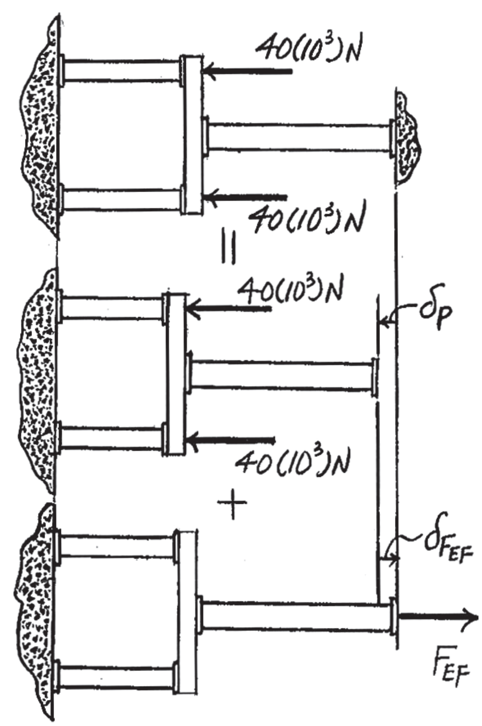
Normal Stress: We have

$$\sigma_{AB} = \sigma_{CD} = \frac{F}{A_{CD}} = \frac{18\,758.38}{\frac{\pi}{4}(0.03^2)} = 26.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{EF} = \frac{F_{EF}}{A_{EF}} = \frac{42\,483.23}{\frac{\pi}{4}(0.04^2)} = 33.8 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

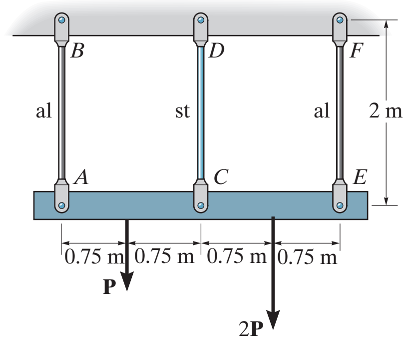


(b)

Ans:
$\sigma_{AB} = \sigma_{CD} = 26.5$ MPa,
$\sigma_{EF} = 33.8$ MPa

***9–44.**

The rigid beam is supported by the three suspender bars. Bars AB and EF are made of aluminum and bar CD is made of steel. If each bar has a cross-sectional area of 450 mm^2, determine the maximum value of P if the allowable stress is $(\sigma_{\text{allow}})_{\text{st}} = 200$ MPa for the steel and $(\sigma_{\text{allow}})_{\text{al}} = 150$ MPa for the aluminum. $E_{\text{st}} = 200$ GPa, $E_{\text{al}} = 70$ GPa.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the rigid beam Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(1.5) + F_{EF}(3) - P(0.75) - 2P(2.25) = 0$$

$$1.5F_{CD} + 3F_{EF} = 5.25P \tag{1}$$

$$\circlearrowleft + \Sigma M_E = 0; \quad 2P(0.75) + P(2.25) - F_{CD}(1.5) - F_{AB}(3) = 0$$

$$1.5F_{CD} + 3F_{AB} = 3.75P \tag{2}$$

Compatibility: Referring to the displacement diagram of the rigid beam, Fig. *b*,

$$\frac{\delta_{CD} - \delta_{AB}}{1.5} = \frac{\delta_{EF} - \delta_{AB}}{3}$$

$$2\delta_{CD} = \delta_{EF} + \delta_{AB}$$

$$2\left(\frac{F_{CD}L}{A\left[200(10^9)\right]}\right) = \frac{F_{EF}L}{A\left[70(10^9)\right]} + \frac{F_{AB}L}{A\left[70(10^9)\right]}$$

$$F_{CD} = \frac{10}{7}(F_{EF} + F_{AB}) \tag{3}$$

Solving Eqs. (1), (2) and (3),

$$F_{EF} = 0.8676\,P \qquad F_{AB} = 0.3676\,P \qquad F_{CD} = 1.7647\,P$$

Assume that bar EF fails. Then

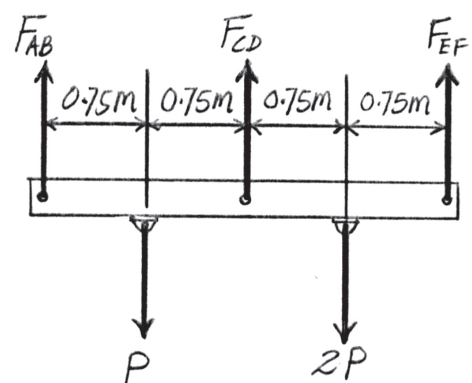
$$(\sigma_{\text{allow}})_{\text{al}} = \frac{F_{EF}}{A}; \qquad 150(10^6) = \frac{0.8676\,P}{450(10^{-6})}$$

$$P = 77.80(10^3)\text{ N} = 77.80\text{ kN}$$

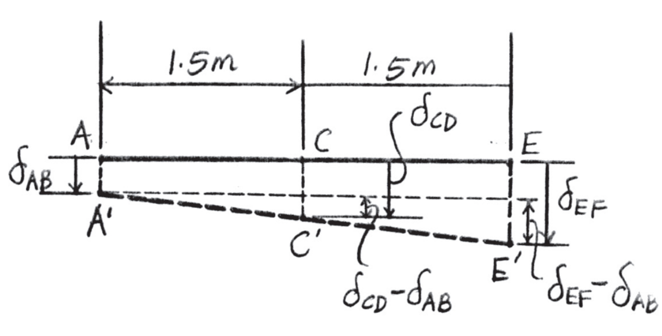
Assume that bar CD fails. Then

$$(\sigma_{\text{allow}})_{\text{st}} = \frac{F_{CD}}{A}; \qquad 200(10^6) = \frac{1.7647\,P}{450(10^{-6})}$$

$$P = 51.0(10^3)\text{ N} = 51.0\text{ kN} \qquad \text{(control!)}\ \textbf{Ans.}$$



(a)

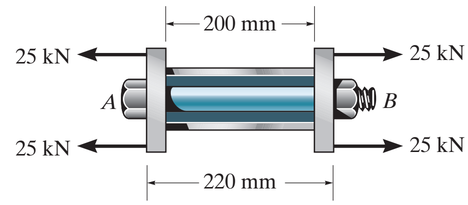


(b)

Ans:
$P = 51.0$ kN

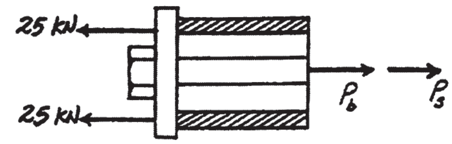
9–45.

The bolt AB has a diameter of 20 mm and passes through a sleeve that has an inner diameter of 40 mm and an outer diameter of 50 mm. The bolt and sleeve are made of A-36 steel and are secured to the rigid brackets as shown. If the bolt length is 220 mm and the sleeve length is 200 mm, determine the tension in the bolt when a force of 50 kN is applied to the brackets.



SOLUTION

Equation of Equilibrium:



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad P_b + P_s - 25 - 25 = 0$

$$P_b + P_s - 50 = 0 \qquad \textbf{(1)}$$

Compatibility:

$$\delta_b = \delta_s$$

$$\frac{P_b(220)}{\frac{\pi}{4}(0.02^2)200(10^9)} = \frac{P_s(200)}{\frac{\pi}{4}(0.05^2 - 0.04^2)(200)(10^9)}$$

$$P_b = 0.40404\, P_s \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

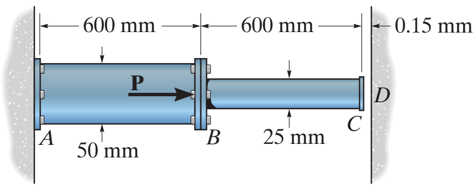
$$P_s = 35.61 \text{ kN}$$

$$P_b = 14.4 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P_b = 14.4$ kN

9–46.

If the gap between C and the rigid wall at D is initially 0.15 mm, determine the support reactions at A and D when the force $P = 200$ kN is applied. The assembly is made of solid A-36 steel cylinders.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 200(10^3) - F_D - F_A = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

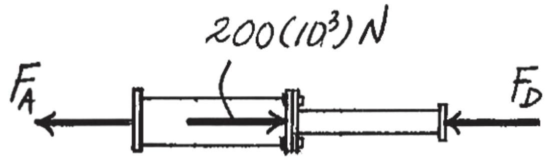
$$(\xrightarrow{+}) \qquad \delta = \delta_P - \delta_{F_D}$$

$$0.15 = \frac{200(10^3)(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} - \left[\frac{F_D\,(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} + \frac{F_D\,(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}\right]$$

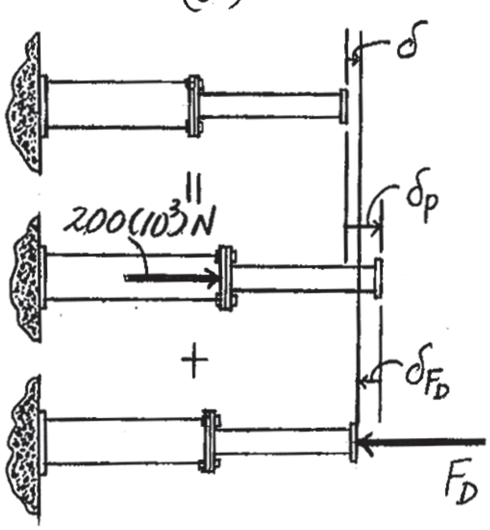
$$F_D = 20\,365.05 \text{ N} = 20.4 \text{ kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = 179\,634.95 \text{ N} = 180 \text{ kN} \qquad \textbf{Ans.}$$



(a)

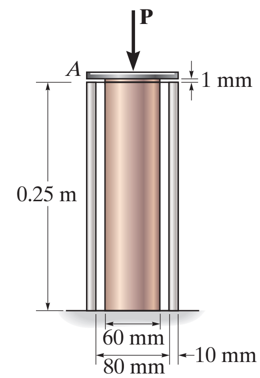


(b)

Ans:
$F_D = 20.4$ kN,
$F_A = 180$ kN

9–47.

The support consists of a solid red brass C83400 copper post surrounded by a 304 stainless steel tube. Before the load is applied the gap between these two parts is 1 mm. Given the dimensions shown, determine the greatest axial load that can be applied to the rigid cap A without causing yielding of any one of the materials.

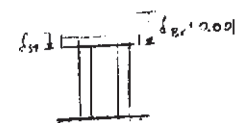


SOLUTION

Require

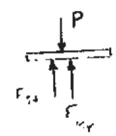
$$\delta_{st} = \delta_{br} + 0.001$$

$$\frac{F_{st}(0.25)}{\pi[(0.05)^2 - (0.04)^2]193(10^9)} = \frac{F_{br}(0.25)}{\pi(0.03)^2(101)(10^9)} + 0.001$$



$$0.45813\,F_{st} = 0.87544\,F_{br} + 10^6 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad F_{st} + F_{br} - P = 0 \qquad \textbf{(2)}$$



Assume brass yields, then

$$(F_{br})_{max} = \sigma_\gamma A_{br} = 70(10^6)(\pi)(0.03)^2 = 197\,920.3\text{ N}$$

$$(\epsilon_\gamma)_{br} = \sigma_\gamma/E = \frac{70.0(10^6)}{101(10^9)} = 0.6931(10^{-3})\text{ mm/mm}$$

$$\delta_{br} = (\epsilon_\gamma)_{br}L = 0.6931(10^{-3})(0.25) = 0.1733\text{ mm} < 1\text{ mm}$$

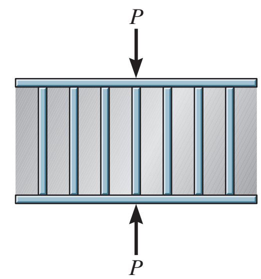
Thus, only the brass is loaded.

$$P = F_{br} = 198\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P = 198$ kN

***9–48.**

The specimen represents a filament-reinforced matrix system made from plastic (matrix) and glass (fiber). If there are n fibers, each having a cross-sectional area of A_f and modulus of E_f, embedded in a matrix having a cross-sectional area of A_m and modulus of E_m, determine the stress in the matrix and in each fiber when the force P is applied on the specimen.

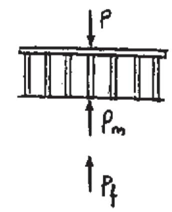


SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad -P + P_m + P_f = 0 \qquad \textbf{(1)}$$

$$\delta_m = \delta_f$$

$$\frac{P_m L}{A_m E_m} = \frac{P_f L}{n A_f E_f}; \qquad P_m = \frac{A_m E_m}{n A_f E_f} P_f \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$P_m = \frac{A_m E_m}{n A_f E_f + A_m E_m} P; \qquad P_f = \frac{n A_f E_f}{n A_f E_f + A_m E_m} P$$

Normal stress:

$$\sigma_m = \frac{P_m}{A_m} = \frac{\left(\dfrac{A_m E_m}{n A_f E_f + A_m E_m} - P\right)}{A_m} = \frac{E_m}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans.}$$

$$\sigma_f = \frac{P_f}{n A_f} = \frac{\left(\dfrac{n A_f E_f}{n A_f E_f + A_m E_m} P\right)}{n A_f} = \frac{E_f}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans.}$$

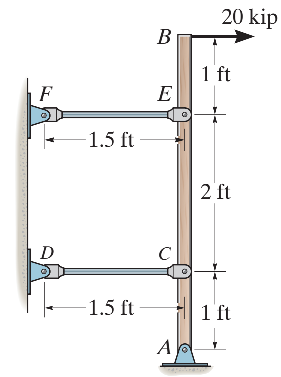
Ans:

$$\sigma_m = \frac{E_m}{n A_f E_f + A_m E_m} P,$$

$$\sigma_f = \frac{E_f}{n A_f E_f + A_m E_m} P$$

9–49.

The rigid bar is pinned at A and supported by two aluminum rods, each having a diameter of 1 in., a modulus of elasticity $E_{al} = 10(10^3)$ ksi, and yield stress of $(\sigma_Y)_{al} = 40$ ksi. If the bar is initially vertical, determine the displacement of the end B when the force of 20 kip is applied.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the rigid bar, Fig. *a*,

$$\circlearrowleft+\Sigma M_A = 0; \qquad F_{EF}(3) + F_{CD}(1) - 20(4) = 0 \qquad \textbf{(1)}$$

Compatibility: Referring to the displacement diagram of the rigid bar, Fig. *b*,

$$\frac{\delta_E}{3} = \frac{\delta_C}{1}$$

$$\delta_E = 3\delta_C$$

$$\frac{F_{EF}L}{AE} = 3\left[\frac{F_{CD}L}{AE}\right]$$

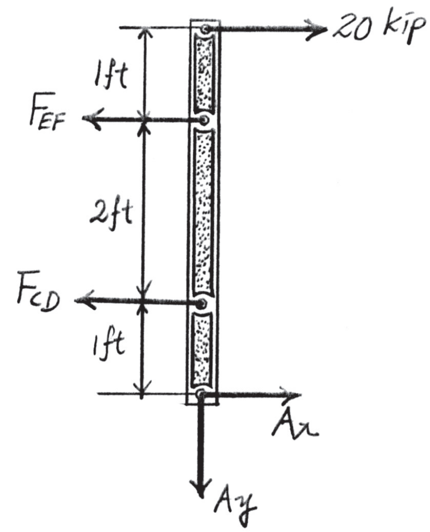
$$F_{EF} = 3F_{CD} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$F_{CD} = 8.00 \text{ kip} \qquad F_{EF} = 24.0 \text{ kip}$$

Here,

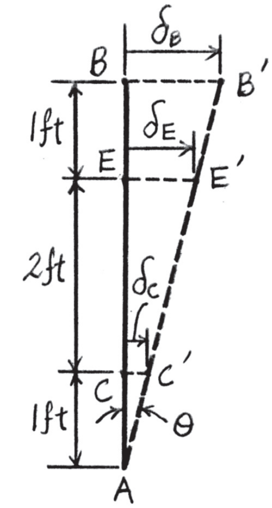
$$\sigma_{max} = \sigma_{EF} = \frac{F_{EF}}{A} = \frac{24.0}{\frac{\pi}{4}(1^2)} = 30.56 \text{ ksi} < (\sigma_Y)_{al} = 40 \text{ ksi} \qquad \textbf{(O.K!)}$$



(a)

Displacement: Again, referring to Fig. *b*,

$$\frac{\delta_B}{4} = \frac{\delta_E}{3}; \qquad \delta_B = \frac{4}{3}\delta_E = \frac{4}{3}\left(\frac{F_{EF}L}{AE}\right) = \frac{4}{3}\left\{\frac{24.0[1.5(12)]}{\frac{\pi}{4}(1^2)\left[10(10^3)\right]}\right\} = 0.0733 \text{ in.} \qquad \textbf{Ans.}$$

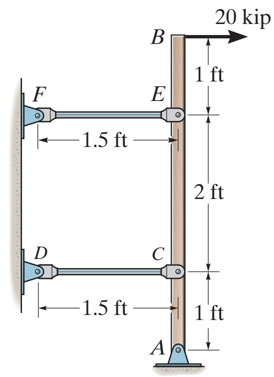


(b)

Ans:
$\delta_B = 0.0733$ in.

9–50.

The rigid bar is pinned at A and supported by two aluminum rods, each having a diameter of 1 in., a modulus of elasticity $E_{al} = 10(10^3)$ ksi, and yield stress of $(\sigma_Y)_{al} = 40$ ksi. If the bar is initially vertical, determine the angle of tilt of the bar when the 20-kip load is applied.



SOLUTION

Equation of Equilibrium: Referring to the FBD of the rigid bar, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{EF}(3) + F_{CD}(1) - 20(4) = 0 \qquad \textbf{(1)}$$

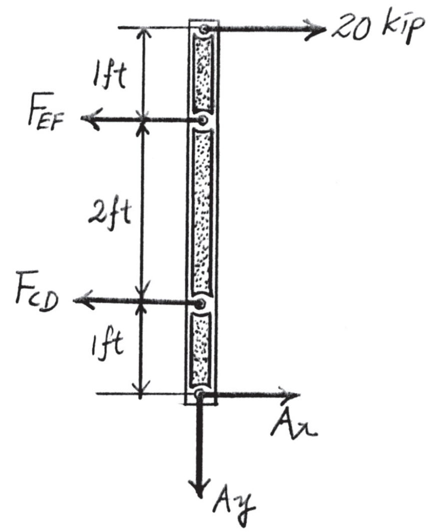
Compatibility: Referring to the displacement diagram of the rigid bar, Fig. *b*,

$$\frac{\delta_E}{3} = \frac{\delta_C}{1}$$

$$\delta_E = 3\delta_C$$

$$\frac{F_{EF}L}{AE} = 3\left[\frac{F_{CD}L}{AE}\right]$$

$$F_{EF} = 3F_{CD} \qquad \textbf{(2)}$$



(a)

Solving Eqs. (1) and (2),

$$F_{CD} = 8.00 \text{ kip} \qquad F_{EF} = 24.0 \text{ kip}$$

Here,

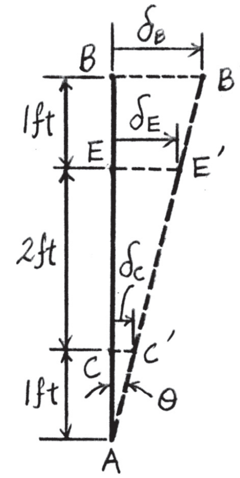
$$\sigma_{max} = \sigma_{EF} = \frac{F_{EF}}{A} = \frac{24.0}{\frac{\pi}{4}(1^2)} = 30.56 \text{ ksi} < (\sigma_Y)_{al} = 50 \text{ ksi} \qquad \textbf{(O.K!)}$$

Displacement: Again, referring to Fig. *b*,

$$\delta_E = \frac{F_{EF}L}{AE} = \frac{24.0[1.5(12)]}{\frac{\pi}{4}(1^2)\left[10(10^3)\right]} = 0.05500 \text{ in.}$$

Thus,

$$\theta = \frac{\delta_E}{L_{AE}} = \frac{0.05500}{3(12)} = 0.0015279 \text{ rad} = 0.0875° \qquad \textbf{Ans.}$$

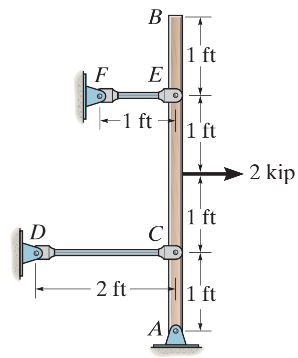


(b)

Ans:
$\theta = 0.0875°$

9–51.

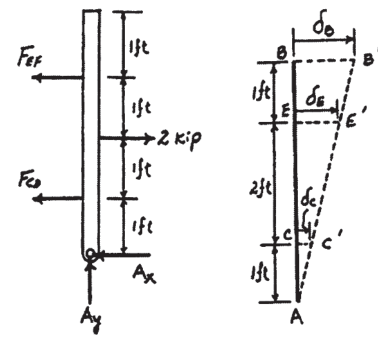
The rigid bar is pinned at A and supported by two aluminum rods, each having a diameter of 1 in. and a modulus of elasticity $E_{al} = 10(10^3)$ ksi. If the bar is initially vertical, determine the displacement of the end B when the force of 2 kip is applied.



SOLUTION

Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(1) + F_{EF}(3) - 2(2) = 0 \qquad \textbf{(1)}$$



Compatibility:

$$\delta_C = \frac{\delta_E}{3}$$

$$\frac{F_{CD}(2)(12)}{AE} = \frac{1}{3}\left[\frac{F_{EF}(1)(12)}{AE}\right]$$

$$F_{EF} = 6F_{CD} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$F_{CD} = 0.21053 \text{ kip} \qquad F_{EF} = 1.2632 \text{ kip}$$

Displacement: Point B

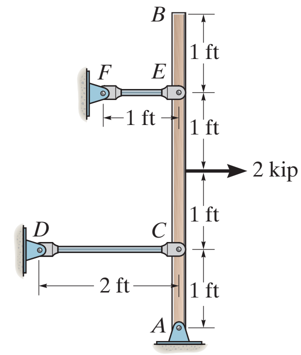
$$\frac{\delta_B}{4} = \frac{\delta_E}{3}$$

$$\delta_B = \frac{4}{3}\delta_E = \frac{4}{3}\left[\frac{1.2632(1)(12)}{\frac{\pi}{4}(1^2)(10)(10^3)}\right] = 0.00257 \text{ in.} \qquad \textbf{Ans.}$$

Ans:
$\delta_B = 0.00257$ in.

***9–52.**

The rigid bar is pinned at A and supported by two aluminum rods, each having a diameter of 1 in. and a modulus of elasticity $E_{al} = 10(10^3)$ ksi. If the bar is initially vertical, determine the force in each rod when the 2-kip load is applied.



SOLUTION

Equations of Equilibrium:

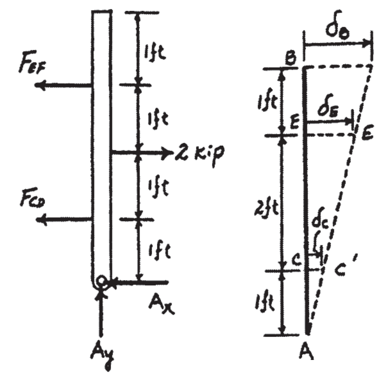
$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(1) + F_{EF}(3) - 2(2) = 0$ **(1)**

Compatibility:

$$\delta_C = \frac{\delta_E}{3}$$

$$\frac{F_{CD}(2)(12)}{AE} = \frac{1}{3}\left[\frac{F_{EF}(1)(12)}{AE}\right]$$

$$F_{EF} = 6F_{CD} \quad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields:

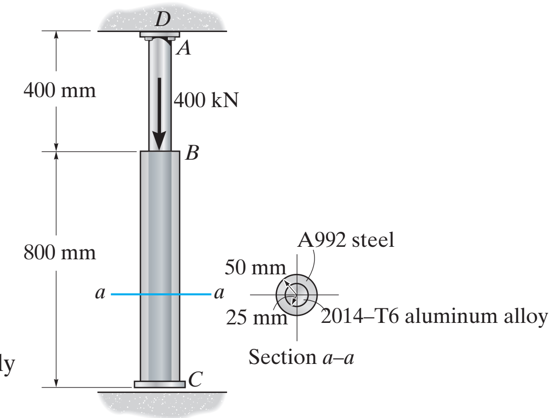
$F_{CD} = 0.211$ kip **Ans.**

$F_{EF} = 1.26$ kip **Ans.**

Ans:
$F_{CD} = 0.211$ kip,
$F_{EF} = 1.26$ kip

9–53.

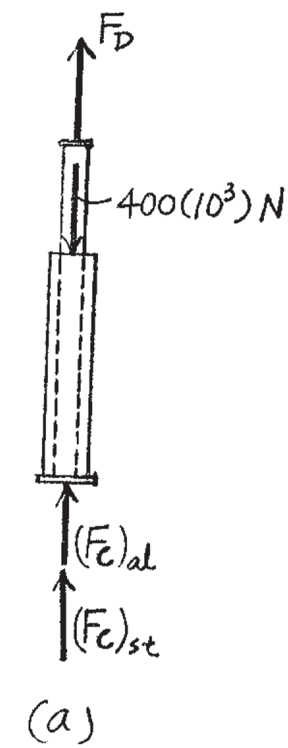
The 2014-T6 aluminum rod AC is reinforced with the firmly bonded A992 steel tube BC. If the assembly fits snugly between the rigid supports so that there is no gap at C, determine the support reactions when the axial force of 400 kN is applied. The assembly is attached at D.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad F_D + (F_C)_{al} + (F_C)_{st} - 400(10^3) = 0 \qquad \textbf{(1)}$$



Compatibility Equation: Using the method of superposition, Fig. b,

$$(+\downarrow) \qquad 0 = \delta_p - \delta_{FC}$$

$$0 = +\frac{400(10^3)(400)}{\pi(0.025^2)(73.1)(10^9)} - \left[\frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)} + \frac{[(F_C)_{al} + (F_C)_{st}](400)}{\pi(0.025^2)(73.1)(10^9)}\right]$$

$$400(10^3) = 3(F_C)_{al} + (F_C)_{st} \qquad \textbf{(2)}$$

Also, since the aluminum rod and steel tube of segment BC are firmly bonded, their deformation must be the same. Thus,

$$(\delta_{BC})_{st} = (\delta_{BC})_{al}$$

$$\frac{(F_C)_{st}(800)}{\pi(0.05^2 - 0.025^2)(200)(10^9)} = \frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)}$$

$$(F_C)_{st} = 8.2079(F_C)_{al} \qquad \textbf{(3)}$$

Solving Eqs. (1) and (2),

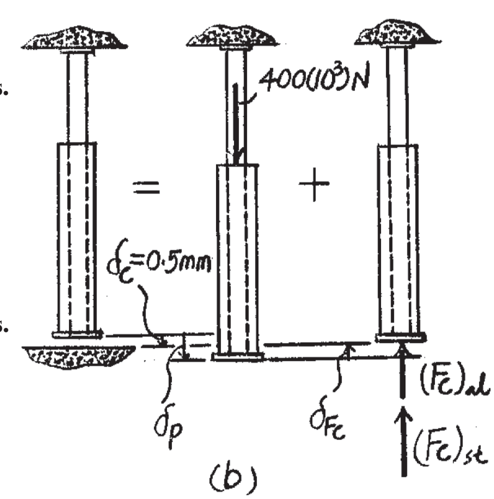
$$(F_C)_{al} = 35.689 \text{ kN} \qquad (F_C)_{st} = 292.93 \text{ kN}$$

Substituting these results into Eq. (1),

$$F_D = 71.4 \text{ kN} \qquad \textbf{Ans.}$$

Also,

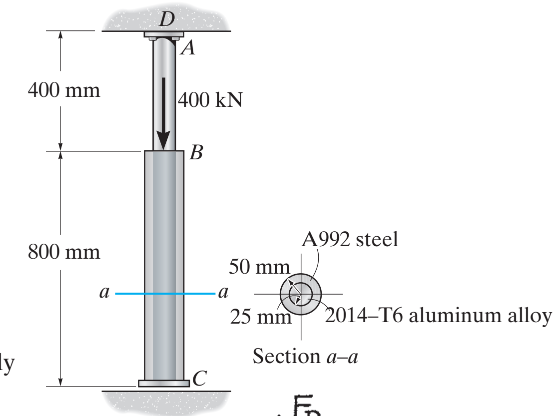
$$F_C = (F_C)_{st} + (F_C)_{al}$$
$$= 35.689 + 292.93$$
$$= 329 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$F_D = 71.4$ kN, $F_C = 329$ kN

9–54.

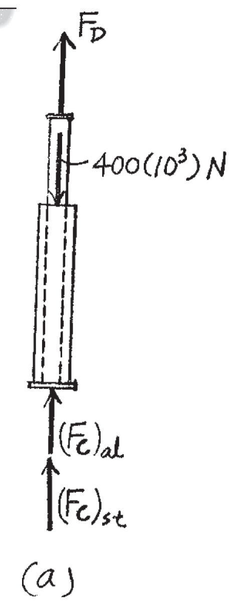
The 2014-T6 aluminum rod AC is reinforced with the firmly bonded A992 steel tube BC. When no load is applied to the assembly, the gap between end C and the rigid support is 0.5 mm. Determine the support reactions when the axial force of 400 kN is applied.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad F_D + (F_C)_{al} + (F_C)_{st} - 400(10^3) = 0 \qquad \textbf{(1)}$$



Compatibility Equation: Using the method of superposition, Fig. b,

$$(+\downarrow) \qquad \delta_C = \delta_P - \delta_{FC}$$

$$0.5 = +\frac{400(10^3)(400)}{\pi(0.025^2)(73.1)(10^9)} - \left[\frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)} + \frac{[(F_C)_{al} + (F_C)_{st}](400)}{\pi(0.025^2)(73.1)(10^9)}\right]$$

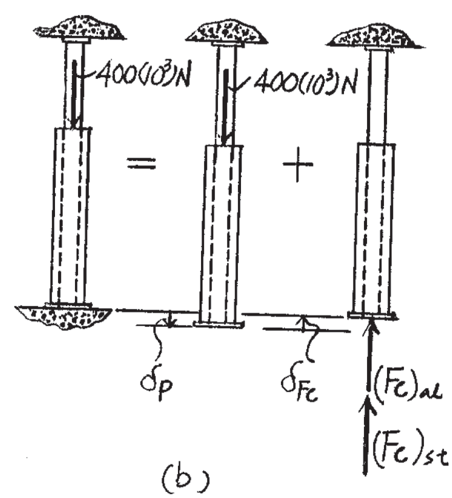
$$220.585(10^3) = 3(F_C)_{al} + (F_C)_{st} \qquad \textbf{(2)}$$

Also, since the aluminum rod and steel tube of segment BC are firmly bonded, their deformation must be the same. Thus,

$$(\delta_{BC})_{st} = (\delta_{BC})_{al}$$

$$\frac{(F_C)_{st}(800)}{\pi(0.05^2 - 0.025^2)(200)(10^9)} = \frac{(F_C)_{al}(800)}{\pi(0.025^2)(73.1)(10^9)}$$

$$(F_C)_{st} = 8.2079(F_C)_{al} \qquad \textbf{(3)}$$



Solving Eqs. (2) and (3),

$$(F_C)_{al} = 19.681 \text{ kN} \qquad (F_C)_{st} = 161.54 \text{ kN}$$

Substituting these results into Eq. (1),

$$F_D = 218.777 \text{ kN} = 219 \text{ kN} \qquad \textbf{Ans.}$$

Also,

$$F_C = (F_C)_{al} + (F_C)_{st}$$

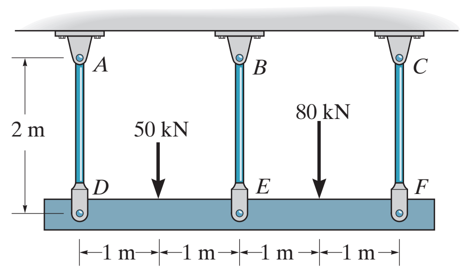
$$= 19.681 + 161.54$$

$$= 181 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_D = 219$ kN,
$F_C = 181$ kN

9–55.

The three suspender bars are made of A992 steel and have equal cross-sectional areas of 450 mm^2. Determine the average normal stress in each bar if the rigid beam is subjected to the loading shown.



SOLUTION

Referring to the *FBD* of the rigid beam, Fig. *a*,

$$+\uparrow\Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 50(10^3) - 80(10^3) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad F_{BE}(2) + F_{CF}(4) - 50(10^3)(1) - 80(10^3)(3) = 0 \qquad \textbf{(2)}$$

Referring to the geometry shown in Fig. *b*,

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{4}\right)(2)$$

$$\delta_{BE} = \frac{1}{2}\left(\delta_{AD} + \delta_{CF}\right)$$

$$\frac{F_{BE}\cancel{L}}{\cancel{AE}} = \frac{1}{2}\left(\frac{F_{AD}\cancel{L}}{\cancel{AE}} + \frac{F_{CF}\cancel{L}}{\cancel{AE}}\right)$$

$$F_{AD} + F_{CF} = 2\,F_{BE} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

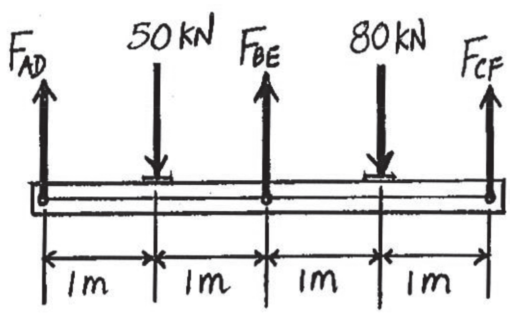
$$F_{BE} = 43.33(10^3)\text{ N} \qquad F_{AD} = 35.83(10^3)\text{ N} \qquad F_{CF} = 50.83(10^3)\text{ N}$$

Thus,

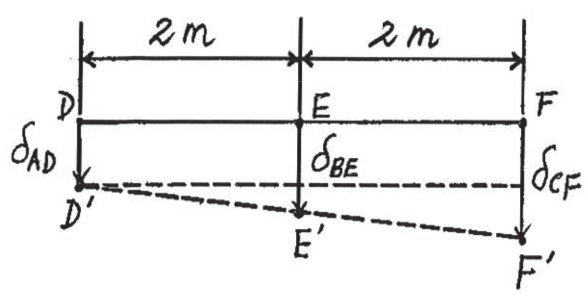
$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{43.33(10^3)}{0.45(10^{-3})} = 96.3\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{35.83(10^3)}{0.45(10^{-3})} = 79.6\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{CF} = 113\text{ MPa} \qquad \textbf{Ans.}$$



(a)

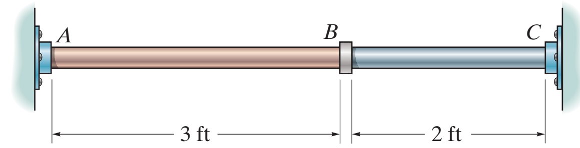


(b)

Ans:
$\sigma_{BE} = 96.3$ MPa,
$\sigma_{AD} = 79.6$ MPa,
$\sigma_{CF} = 113$ MPa

***9–56.**

The C83400-red-brass rod AB and 2014-T6-aluminum rod BC are joined at the collar B and fixed connected at their ends. If there is no load in the members when $T_1 = 50°\text{F}$, determine the average normal stress in each member when $T_2 = 120°\text{F}$. Also, how far will the collar be displaced? The cross-sectional area of each member is $1.75\ \text{in}^2$.



SOLUTION

$\Sigma F_x = 0; \qquad F_{br} = F_{al} = F$

$$\delta_{N/C} = 0$$

$$-\frac{F_{br}\,L_{AB}}{A_{AB}\,E_{br}} + \alpha_B \Delta T\, L_{AB} - \frac{F_{al} L_{BC}}{A_{BC} E_{al}} + \alpha_{al} \Delta T\, L_{BC} = 0$$

$$-\frac{F(3)(12)}{(1.75)(14.6)(10^6)} + 9.80(10^{-6})(120 - 50)(3)(12)$$

$$-\frac{F(2)(12)}{1.75(10.6)(10^6)} + 12.8(10^{-6})(120 - 50)(2)(12) = 0$$

$$F = 17\,093.4\ \text{lb}$$

$$\sigma_{br} = \sigma_{al} = \frac{17\,093.4}{1.75} = 9.77\ \text{ksi}$$ **Ans.**

$9.77\ \text{ksi} < (\sigma\gamma)_{al}$ and $(\sigma\gamma)_{br}$ **OK**

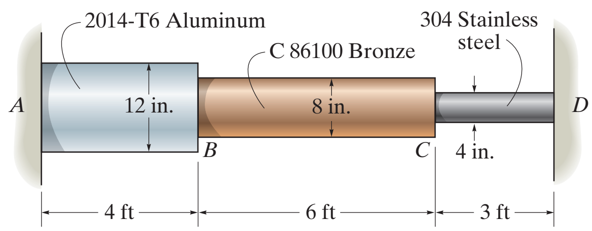
$$\delta_B = -\frac{17\,093.4(3)(12)}{1.75(14.6)(10^6)} + 9.80(10^{-6})(120 - 50)(3)(12)$$

$$\delta_B = 0.611(10^{-3})\ \text{in.} \rightarrow$$ **Ans.**

Ans:
$\sigma_{br} = \sigma_{al} = 9.77\ \text{ksi}$
$\delta_B = 0.611(10^{-3})\ \text{in.} \rightarrow$

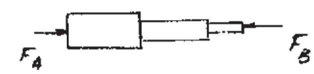
9–57.

The assembly has the diameters and material indicated. If it fits securely between its fixed supports when the temperature is $T_1 = 70°\text{F}$, determine the average normal stress in each material when the temperature reaches $T_2 = 110°\text{F}$.



SOLUTION

$\Sigma F_x = 0; \qquad F_A = F_B = F$



$$\delta_{A/D} = 0; \qquad -\frac{F(4)(12)}{\pi(6)^2(10.6)(10^6)} + 12.8(10^{-6})(110 - 70)(4)(12)$$

$$-\frac{F(6)(12)}{\pi(4)^2(15)(10^6)} + 9.60(10^{-6})(110 - 70)(6)(12)$$

$$-\frac{F(3)(12)}{\pi(2)^2(28)(10^6)} + 9.60(10^{-6})(110 - 70)(3)(12) = 0$$

$$F = 277.69 \text{ kip}$$

$\sigma_{al} = \dfrac{277.69}{\pi(6)^2} = 2.46 \text{ ksi}$ **Ans.**

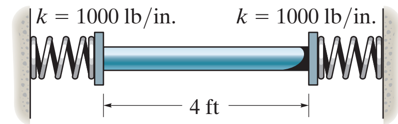
$\sigma_{br} = \dfrac{277.69}{\pi(4)^2} = 5.52 \text{ ksi}$ **Ans.**

$\sigma_{st} = \dfrac{277.69}{\pi(2)^2} = 22.1 \text{ ksi}$ **Ans.**

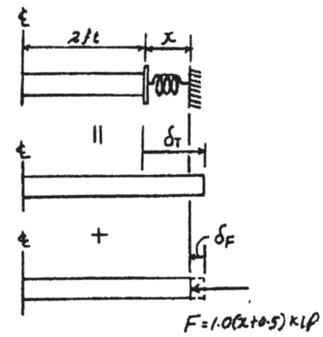
Ans:
$\sigma_{al} = 2.46$ ksi,
$\sigma_{br} = 5.52$ ksi,
$\sigma_{st} = 22.1$ ksi

9–58.

The rod is made of A992 steel and has a diameter of 0.25 in. If the rod is 4 ft long when the springs are compressed 0.5 in. and the temperature of the rod is $T = 40°\text{F}$, determine the force in the rod when its temperature is $T = 160°\text{F}$.



SOLUTION



Compatibility:

$$(\overset{+}{\rightarrow}) \qquad x = \delta_T - \delta_F$$

$$x = 6.60(10^{-6})(160 - 40)(2)(12) - \frac{1.00(x + 0.5)(2)(12)}{\frac{\pi}{4}(0.25^2)(29.0)(10^3)}$$

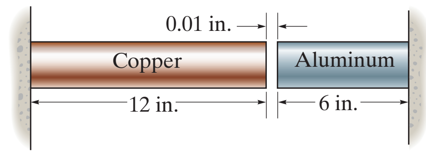
$$x = 0.01040 \text{ in.}$$

$$F = 1.00(0.01040 + 0.5) = 0.510 \text{ kip}$$ **Ans.**

Ans:
$F = 0.510$ kip

9–59.

The two cylindrical rod segments are fixed to the rigid walls such that there is a gap of 0.01 in. between them when $T_1 = 60°\text{F}$. What larger temperature T_2 is required in order to just close the gap? Each rod has a diameter of 1.25 in. Determine the average normal stress in each rod if $T_2 = 300°\text{F}$. Take $\alpha_{al} = 13(10^{-6})/°\text{F}$, $E_{al} = 10(10^3)$ ksi, $(\sigma_Y)_{al} = 40$ ksi, $\alpha_{cu} = 9.4(10^{-6})/°\text{F}$, $E_{cu} = 15(10^3)$ ksi, and $(\sigma_Y)_{cu} = 50$ ksi.



SOLUTION

Thermal Expansion: To just close the gap,

$$\delta_T = \alpha_{al}\Delta T L_{al} + \alpha_{cu}\Delta T L_{cu}$$

$$0.01 = 13\left(10^{-6}\right)(T_2 - 60)(6) + 9.4\left(10^{-6}\right)(T_2 - 60)(12)$$

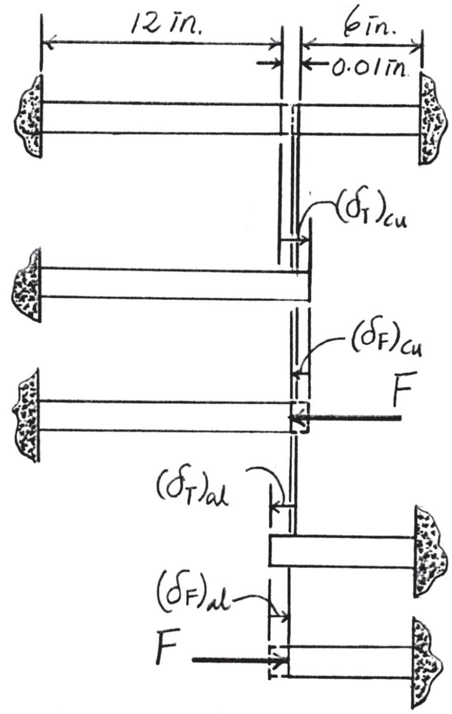
$$T_2 = 112.41°\text{F} = 112°\text{F}$$ **Ans.**

Compatibility: Referring to the deformation diagram shown in Fig. *a*,

$$0.01 = (\delta_T)_{cu} - (\delta_F)_{cu} + (\delta_T)_{al} - (\delta_F)_{al}$$

$$0.01 = 9.4\left(10^{-6}\right)(300 - 60)(12) - \frac{F(12)}{\frac{\pi}{4}\left(1.25^2\right)\left[15\left(10^3\right)\right]} + 13\left(10^{-6}\right)(300 - 60)(6) - \frac{F(6)}{\frac{\pi}{4}\left(1.25^2\right)\left[10\left(10^3\right)\right]}$$

$$F = 31.37 \text{ kip}$$



(a)

Average Normal Stress:

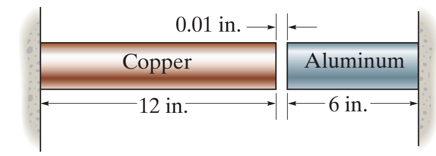
$$\sigma_{al} = \sigma_{cu} = \frac{F}{A} = \frac{31.37}{\frac{\pi}{4}\left(1.25^2\right)} = 25.57 \text{ ksi} = 25.6 \text{ ksi}$$ **Ans.**

Since $\sigma_{al} < (\sigma_\gamma)_{al} = 40$ ksi and $\sigma_{cu} < (\sigma_\gamma)_{cu} = 50$ ksi, the solution is valid.

Ans:
$T_2 = 112°\text{F}$,
$\sigma_{al} = \sigma_{cu} = 25.6$ ksi

***9–60.**

The two cylindrical rod segments are fixed to the rigid walls such that there is a gap of 0.01 in. between them when $T_1 = 60°\text{F}$. Each rod has a diameter of 1.25 in. Determine the average normal stress in each rod if $T_2 = 400°\text{F}$, and also calculate the new length of the aluminum segment. Take $\alpha_{al} = 13(10^{-6})/°\text{F}$, $E_{al} = 10(10^3)$ ksi, $(\sigma_Y)_{al} = 40$ ksi, $\alpha_{cu} = 9.4(10^{-6})/°\text{F}$, $(\sigma_Y)_{cu} = 50$ ksi, and $E_{cu} = 15(10^3)$ ksi.



SOLUTION

Compatibility: Referring to the deformation diagram shown in Fig. *a*,

$$0.01 = (\delta_T)_{cu} - (\delta_F)_{cu} + (\delta_T)_{al} - (\delta_F)_{al}$$

$$0.01 = 9.4\left(10^{-6}\right)(400 - 60)(12) - \frac{F(12)}{\frac{\pi}{4}\left(1.25^2\right)\left[15\left(10^3\right)\right]}$$

$$+ 13\left(10^{-6}\right)(400 - 60)(6) - \frac{F(6)}{\frac{\pi}{4}\left(1.25^2\right)\left[10\left(10^3\right)\text{ ksi}\right]}$$

$$F = 48.10 \text{ kip}$$

Average Normal Stress:

$$\sigma_{al} = \sigma_{cu} = \frac{F}{A} = \frac{48.10}{\frac{\pi}{4}\left(1.25^2\right)} = 39.19 \text{ ksi} = 39.2 \text{ ksi} \qquad \textbf{Ans.}$$

Displacement: Since $\sigma_{al} < (\sigma_y)_{al} = 40$ ksi and $\sigma_{cu} < (\sigma_y)_{cu} = 50$ ksi, the solution is valid.

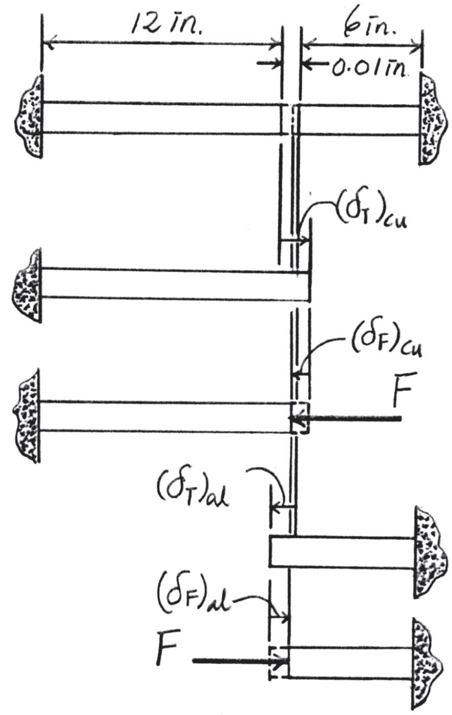
$$\delta_{al} = (\delta_T)_{al} - (\delta_F)_{al}$$

$$= 13\left(10^{-6}\right)(400 - 60)(6) - \frac{48.10(6)}{\frac{\pi}{4}\left(1.25^2\right)\left[10\left(10^3\right)\right]}$$

$$= 0.0030034 \text{ in.}$$

Thus,

$$L_{al} = (L_{al})_0 + \delta_{al} = 6 + 0.0030034 = 6.00300 \text{ in.} \qquad \textbf{Ans.}$$

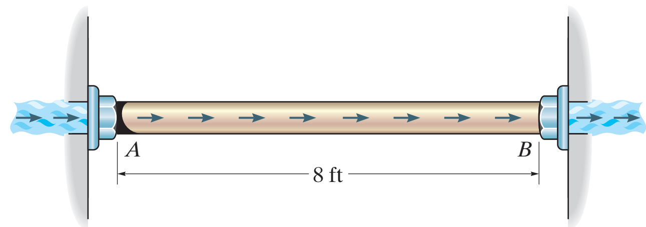


(a)

Ans:
$\sigma_{al} = \sigma_{cu} = 39.2$ ksi,
$L_{al} = 6.00300$ in.

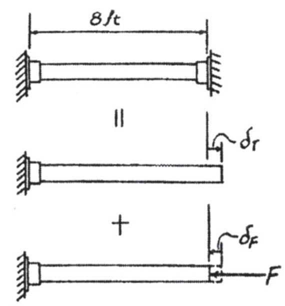
9–61.

The pipe is made of A992 steel and is connected to the collars at A and B. When the temperature is 60°F, there is no axial load in the pipe. If hot gas traveling through the pipe causes its temperature to rise by $\Delta T = (40 + 15x)$°F, where x is in feet, determine the average normal stress in the pipe. The inner diameter is 2 in., the wall thickness is 0.15 in.



SOLUTION

Compatibility:



$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int_0^L \alpha\, \Delta T\, dx$$

$$0 = 6.60(10^{-6}) \int_0^{8\text{ ft}} (40 + 15\,x)\, dx - \frac{F(8)}{A(29.0)(10^3)}$$

$$0 = 6.60(10^{-6}) \left[40(8) + \frac{15(8)^2}{2} \right] - \frac{F(8)}{A(29.0)(10^3)}$$

$F = 19.14\,A$

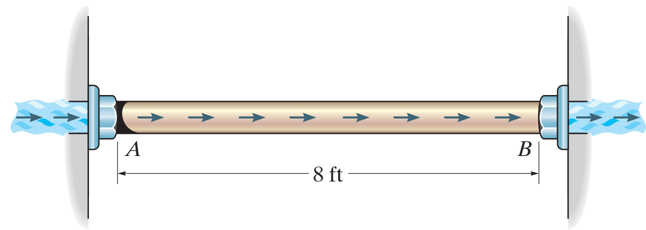
Average Normal Stress:

$$\sigma = \frac{19.14\,A}{A} = 19.1 \text{ ksi}$$ **Ans.**

Ans:
$\sigma = 19.1$ ksi

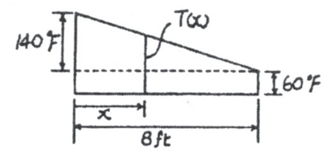
9–62.

The bronze C86100 pipe has an inner radius of 0.5 in. and a wall thickness of 0.2 in. If the gas flowing through it changes the temperature of the pipe uniformly from $T_A = 200°\text{F}$ at A to $T_B = 60°\text{F}$ at B, determine the axial force it exerts on the walls. The pipe was fitted between the walls when $T = 60°\text{F}$.



SOLUTION

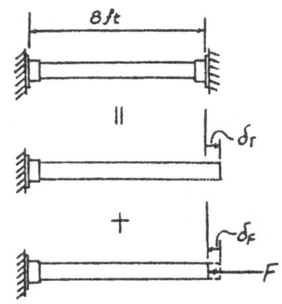
Temperature Gradient:



$$T(x) = 60 + \left(\frac{8 - x}{8}\right)140 = 200 - 17.5x$$

Compatibility:

$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int \alpha \Delta T dx$$



$$0 = 9.60\left(10^{-6}\right)\int_0^{8\text{ ft}} [(200 - 17.5x) - 60]\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)\, 15.0(10^3)}$$

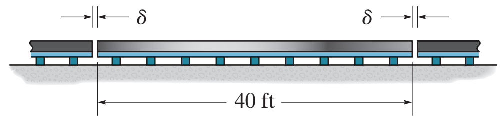
$$0 = 9.60\left(10^{-6}\right)\int_0^{8\text{ ft}} (140 - 17.5x)\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)\, 15.0(10^3)}$$

$F = 7.60$ kip **Ans.**

Ans:
$F = 7.60$ kip

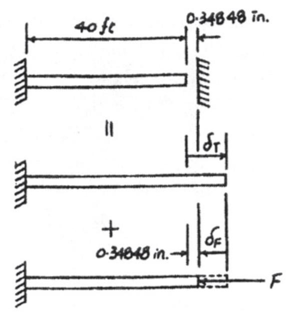
9–63.

The 40-ft-long A-36 steel rails on a train track are laid with a small gap between them to allow for thermal expansion. Determine the required gap δ so that the rails just touch one another when the temperature is increased from $T_1 = -20°\text{F}$ to $T_2 = 90°\text{F}$. Using this gap, what would be the axial force in the rails if the temperature rises to $T_3 = 110°\text{F}$? The cross-sectional area of each rail is 5.10 in^2.



SOLUTION

Thermal Expansion: Note that since adjacent rails expand, each rail will be required to expand $\frac{\delta}{2}$ on each end, or δ for the entire rail.



$$\delta = \alpha \Delta T L = 6.60(10^{-6})[90 - (-20)](40)(12)$$

$$= 0.34848 \text{ in.} = 0.348 \text{ in.} \qquad \textbf{Ans.}$$

Compatibility:

$$(\overset{+}{\rightarrow}) \qquad 0.34848 = \delta_T - \delta_F$$

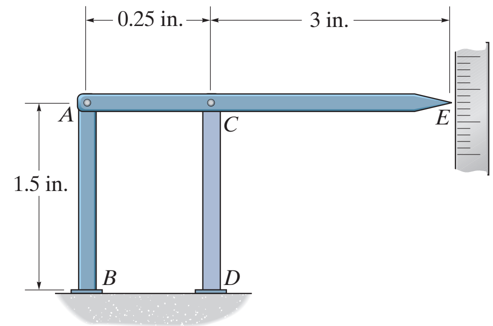
$$0.34848 = 6.60(10^{-6})[110 - (-20)](40)(12) - \frac{F(40)(12)}{5.10(29.0)(10^3)}$$

$$F = 19.5 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$\delta = 0.348$ in.,
$F = 19.5$ kip

***9–64.**

The device is used to measure a change in temperature. Bars AB and CD are made of A-36 steel and 2014-T6 aluminum alloy, respectively. When the temperature is at 75°F, ACE is in the horizontal position. Determine the vertical displacement of the pointer at E when the temperature rises to 150°F.



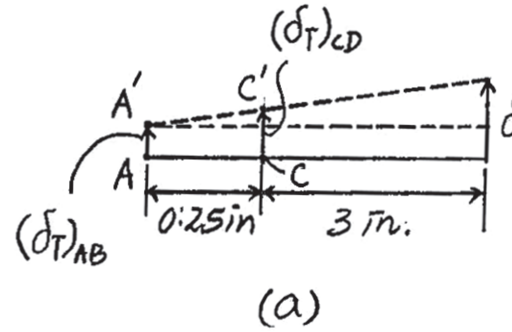
SOLUTION

Thermal Expansion:

$$\left(\delta_T\right)_{CD} = \alpha_{\text{al}}\Delta T L_{CD} = 12.8(10^{-6})(150 - 75)(1.5) = 1.44(10^{-3})\text{ in.}$$

$$\left(\delta_T\right)_{AB} = \alpha_{\text{st}}\Delta T L_{AB} = 6.60(10^{-6})(150 - 75)(1.5) = 0.7425(10^{-3})\text{ in.}$$

From the geometry of the deflected bar AE shown, Fig. a,



$$\delta_E = \left(\delta_T\right)_{AB} + \left[\frac{\left(\delta_T\right)_{CD} - \left(\delta_T\right)_{AB}}{0.25}\right](3.25)$$

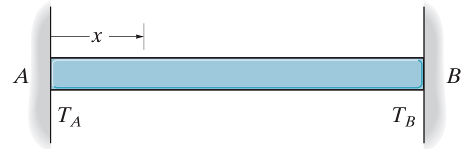
$$= 0.7425(10^{-3}) + \left[\frac{1.44(10^{-3}) - 0.7425(10^{-3})}{0.25}\right](3.25)$$

$$= 0.00981\text{ in.} \uparrow$$ **Ans.**

Ans:
$\delta_E = 0.00981\text{ in.} \uparrow$

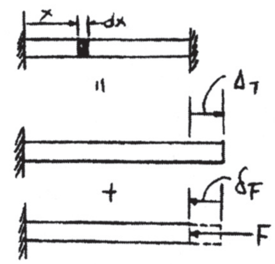
9–65.

The bar has a cross-sectional area A, length L, modulus of elasticity E, and coefficient of thermal expansion α. The temperature of the bar changes uniformly along its length from T_A at A to T_B at B so that at any point x along the bar $T = T_A + x(T_B - T_A)/L$. Determine the force the bar exerts on the rigid walls. Initially no axial force is in the bar and the bar has a temperature of T_A.



SOLUTION

$\overset{+}{\rightarrow}$ $0 = \Delta_T - \delta_F$ **(1)**



However,

$$d\Delta_T = \alpha\Delta_T\,dx = \alpha\left(T_A + \frac{T_B - T_A}{L}x - T_A\right)dx$$

$$\Delta_T = \alpha\int_0^L \frac{T_B - T_A}{L}x\,dx = \alpha\left[\frac{T_B - T_A}{2L}x^2\right]\Bigg|_0^L$$

$$= \alpha\left[\frac{T_B - T_A}{2}L\right] = \frac{\alpha L}{2}(T_B - T_A)$$

From Eq. (1),

$$0 = \frac{\alpha L}{2}(T_B - T_A) - \frac{FL}{AE}$$

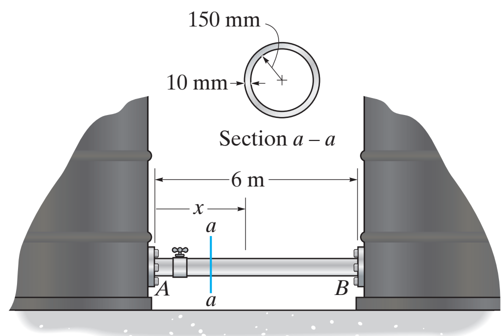
$$F = \frac{\alpha AE}{2}(T_B - T_A)$$ **Ans.**

Ans:

$$F = \frac{\alpha AE}{2}(T_B - T_A)$$

9–66.

When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, the temperatures at ends A and B rise to 130°C and 80°C, respectively. If the temperature drop along the pipe is linear, determine the average normal stress developed in the pipe. Assume each tank provides a rigid support at A and B.



SOLUTION

Temperature Gradient: Since the temperature varies linearly along the pipe, Fig. *a*, the temperature gradient can be expressed as a function of x as

$$T(x) = 80 + \frac{50}{6}(6 - x) = \left(130 - \frac{50}{6}x\right)°\text{C}$$

Thus, the change in temperature as a function of x is

$$\Delta T = T(x) - 30° = \left(130 - \frac{50}{6}x\right) - 30 = \left(100 - \frac{50}{6}x\right)°\text{C}$$

Compatibility Equation: If the pipe is unconstrained, it will have a free expansion of

$$\delta_T = \alpha \int \Delta T dx = 12(10^{-6}) \int_0^{6\text{m}} \left(100 - \frac{50}{6}x\right)dx = 0.0054 \text{ m} = 5.40 \text{ mm}$$

Using the method of superposition, Fig. *b*,

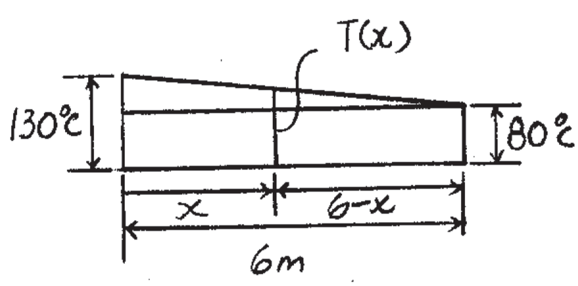
$$(\overset{+}{\rightarrow}) \qquad 0 = \delta_T - \delta_F$$

$$0 = 5.40 - \frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^9)}$$

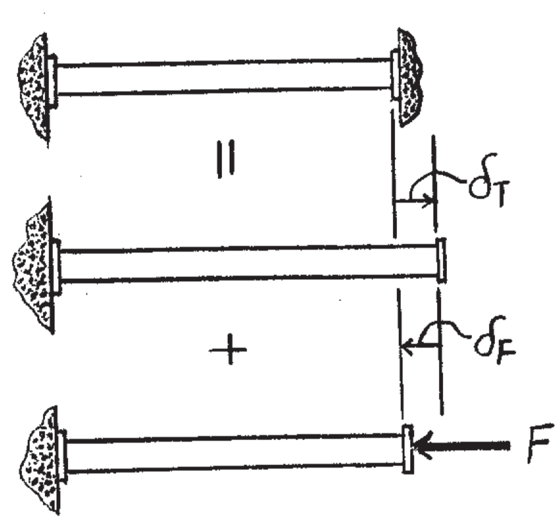
$$F = 1\,753\,008 \text{ N}$$

Normal Stress:

$$\sigma = \frac{F}{A} = \frac{1\,753\,008}{\pi(0.16^2 - 0.15^2)} = 180 \text{ MPa}$$ **Ans.**



(a)

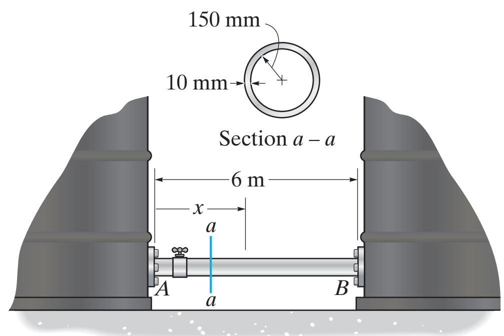


(b)

Ans:
$\sigma = 180$ MPa

9–67.

When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, the temperatures at ends A and B rise to 130°C and 80°C, respectively. If the temperature drop along the pipe is linear, determine the average normal stress developed in the pipe. Assume the walls of each tank act as a spring, each having a stiffness of $k = 900$ MN/m.



SOLUTION

Temperature Gradient: Since the temperature varies linearly along the pipe, Fig. *a*, the temperature gradient can be expressed as a function of x as

$$T(x) = 80 + \frac{50}{6}(6 - x) = \left(130 - \frac{50}{6}x\right)°\text{C}$$

Thus, the change in temperature as a function of x is

$$\Delta T = T(x) - 30° = \left(130 - \frac{50}{6}x\right) - 30 = \left(100 - \frac{50}{6}x\right)°\text{C}$$

Compatibility Equation: If the pipe is unconstrained, it will have a free expansion of

$$\delta_T = \alpha \int \Delta T dx = 12(10^{-6}) \int_0^{6\text{ m}} \left(100 - \frac{50}{6}x\right) dx = 0.0054 \text{ m} = 5.40 \text{ mm}$$

Using the method of superposition, Fig. *b*,

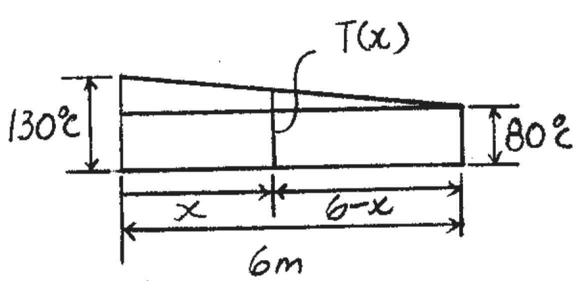
$$(\overset{+}{\rightarrow}) \qquad \delta = \delta_T - \delta_F$$

$$\frac{F}{900(10^6)}(1000) = 5.40 - \left[\frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^9)} + \frac{F}{900(10^6)}(1000)\right]$$

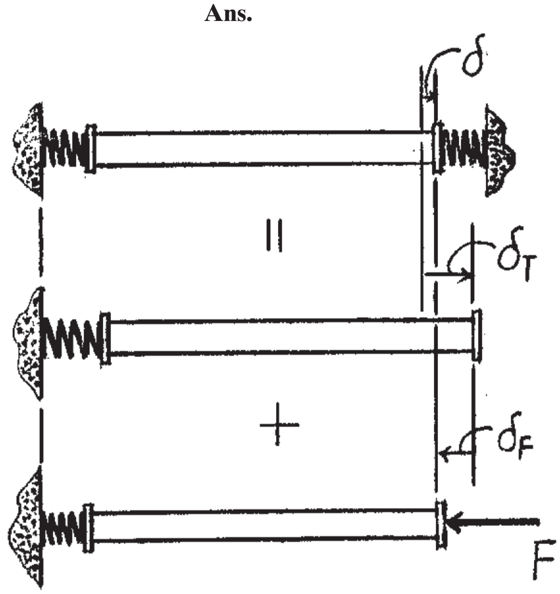
$$F = 1\,018\,361 \text{ N}$$

Normal Stress:

$$\sigma = \frac{F}{A} = \frac{1\,018\,361}{\pi(0.16^2 - 0.15^2)} = 105 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

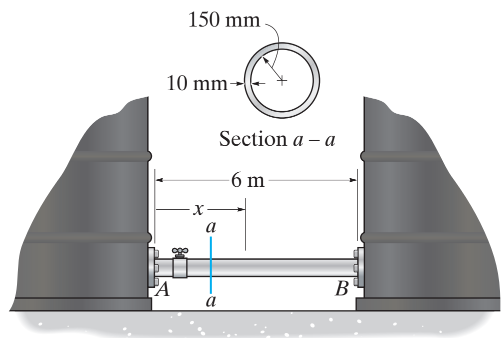


(b)

Ans:
$\sigma = 105$ MPa

***9–68.**

When the temperature is at 30°C, the A-36 steel pipe fits snugly between the two fuel tanks. When fuel flows through the pipe, it causes the temperature to vary along the pipe as $T = (\frac{5}{3}x^2 - 20x + 120)$°C, where x is in meters. Determine the normal stress developed in the pipe. Assume each tank provides a rigid support at A and B.



SOLUTION

Compatibility Equation: The change in temperature as a function of x is $\Delta T = T - 30° = \left(\frac{5}{3}x^2 - 20x + 120\right) - 30 = \left(\frac{5}{3}x^2 - 20x + 90\right)$°C. If the pipe is unconstrained, it will have a free expansion of

$$\delta_T = \alpha \int \Delta T dx = 12(10^{-6}) \int_0^{6\text{ m}} \left(\frac{5}{3}x^2 - 20x + 90\right) dx = 0.0036 \text{ m} = 3.60 \text{ mm}$$

Using the method of superposition, Fig. b,

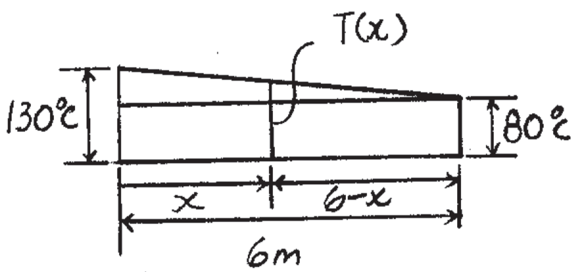
$$(\overset{+}{\rightarrow}) \qquad 0 = \delta_T - \delta_F$$

$$0 = 3.60 - \frac{F(6000)}{\pi(0.16^2 - 0.15^2)(200)(10^9)}$$

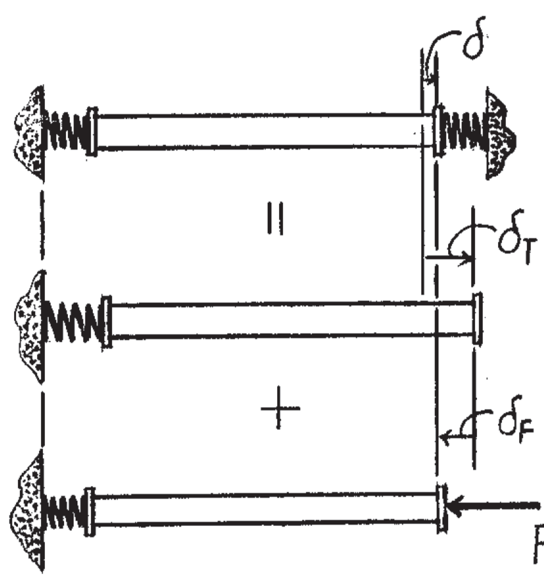
$$F = 1\,168\,672.47 \text{ N}$$

Normal Stress:

$$\sigma = \frac{F}{A} = \frac{1\,168\,672.47}{\pi\left(0.16^2 - 0.15^2\right)} = 120 \text{ MPa}$$ **Ans.**



(a)

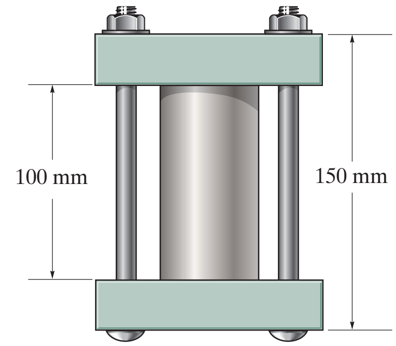


(b)

Ans:
$\sigma = 120$ MPa

9–69.

The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 20°$ C. If the 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the force in the cylinder when the temperature rises to $T_2 = 130°\text{C}$.

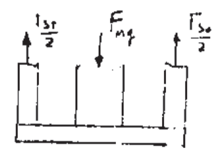


SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad F_{st} = F_{mg} = F$

$$\delta_{mg} = \delta_{st}$$

$$\alpha_{mg} L_{mg} \Delta T - \frac{F_{mg} L_{mg}}{E_{mg} A_{mg}} = \alpha_{st} L_{st} \Delta T + \frac{F_{st} L_{st}}{E_{st} A_{st}}$$



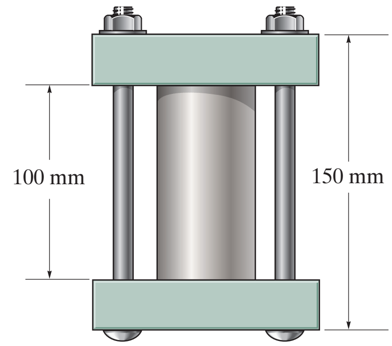
$$26(10^{-6})(0.1)(110) - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2} = 17(10^{-6})(0.150)(110) + \frac{F(0.150)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2}$$

$F = 904\text{ N}$ **Ans.**

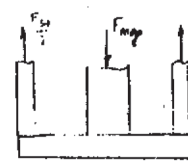
Ans:
$F = 904\text{ N}$

9–70.

The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 15°\text{C}$. If the two 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the temperature at which the average normal stress in either the magnesium or the steel first becomes 12 MPa.



SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad F_{st} = F_{mg} = F$

$$\delta_{mg} = \delta_{st}$$

$$\alpha_{mg} L_{mg}\,\Delta T - \frac{F_{mg}L_{mg}}{E_{mg}A_{mg}} = \alpha_{st}L_{st}\Delta T + \frac{F_{st}L_{st}}{E_{st}A_{st}}$$

$$26(10^{-6})(0.1)(\Delta T) - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2} = 17(10^{-6})(0.150)(\Delta T) + \frac{F(0.150)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2}$$

The steel has the smallest cross-sectional area.

$$F = \sigma A = 12(10^6)(2)\left(\frac{\pi}{4}\right)(0.01)^2 = 1885.0\text{ N}$$

Thus,

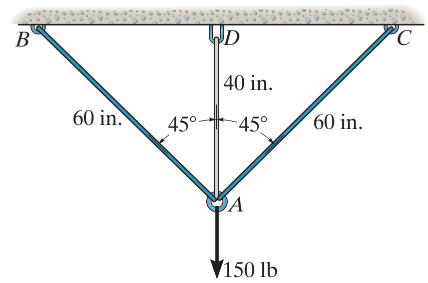
$$\Delta T = 229°$$

$$T_2 = 229° + 15° = 244°\text{ C} \qquad \textbf{Ans.}$$

Ans:
$T_2 = 244°\text{ C}$

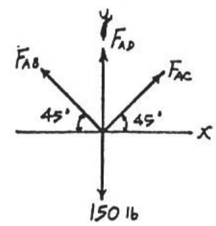
9–71.

The wires AB and AC are made of steel, and wire AD is made of copper. Before the 150-lb force is applied, AB and AC are each 60 in. long and AD is 40 in. long. If the temperature is increased by 80°F, determine the force in each wire needed to support the load. Each wire has a cross-sectional area of 0.0123 in^2. Take $E_{st} = 29(10^3)$ ksi, $E_{cu} = 17(10^3)$ ksi, $\alpha_{st} = 8(10^{-6})/°F$, $\alpha_{cu} = 9.60(10^{-6})/°F$.



SOLUTION

Equations of Equilibrium:

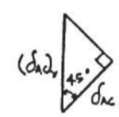


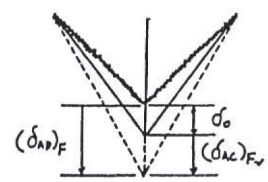
$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC} \cos 45° - F_{AB} \cos 45° = 0$$

$$F_{AC} = F_{AB} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F \sin 45° + F_{AD} - 150 = 0 \qquad \textbf{(1)}$$

Compatibility:





$$(\delta_{AC})_T = 8.0(10^{-6})(80)(60) = 0.03840 \text{ in.}$$

$$(\delta_{AC})_{T_2} = \frac{(\delta_{AC})_T}{\cos 45°} = \frac{0.03840}{\cos 45°} = 0.05431 \text{ in.}$$

$$(\delta_{AD})_T = 9.60(10^{-6})(80)(40) = 0.03072 \text{ in.}$$

$$\delta_0 = (\delta_{AC})_{T_2} - (\delta_{AD})_T = 0.05431 - 0.03072 = 0.02359 \text{ in.}$$

$$(\delta_{AD})_F = (\delta_{AC})_{F_r} + \delta_0$$

$$\frac{F_{AD}(40)}{0.0123(17.0)(10^6)} = \frac{F(60)}{0.0123(29.0)(10^6)\cos 45°} + 0.02359$$

$$0.1913F_{AD} - 0.2379F = 23.5858 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields:

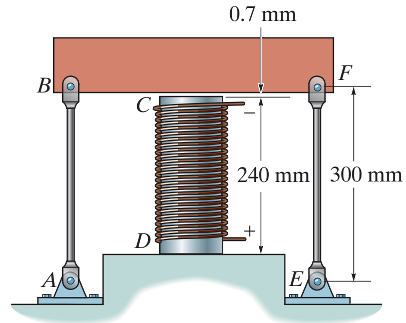
$F_{AC} = F_{AB} = F = 10.0$ lb **Ans.**

$F_{AD} = 136$ lb **Ans.**

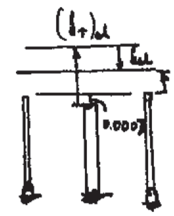
Ans:
$F_{AC} = F_{AB} = 10.0$ lb,
$F_{AD} = 136$ lb

***9–72.**

The cylinder CD of the assembly is heated from $T_1 = 30°\text{C}$ to $T_2 = 180°\text{C}$ using electrical resistance. At the lower temperature T_1 the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm². CD is made of aluminum and has a cross-sectional area of 375 mm². $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, and $\alpha_{al} = 23(10^{-6})/°\text{C}$.



SOLUTION

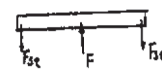


$$\delta_{st} = (\delta_\gamma)_{al} - \delta_{al} - 0.0007$$

$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^9)} = 23(10^{-6})(150)(0.24) - \frac{F(0.24)}{(375)(10^{-6})(70)(10^9)} - 0.0007$$

$$12F_{st} = 128\,000 - 9.1428F \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F - 2F_{st} = 0 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields,

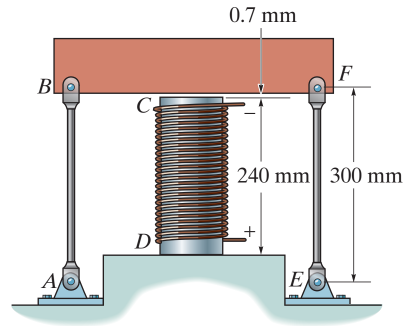
$F_{AB} = F_{EF} = F_{st} = 4.23$ kN **Ans.**

$F_{CD} = F = 8.45$ kN

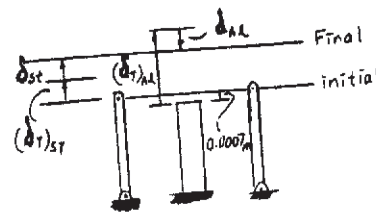
Ans:
$F_{AB} = F_{EF} = 4.23$ kN

9–73.

The cylinder CD of the assembly is heated from $T_1 = 30°\text{C}$ to $T_2 = 180°\text{C}$ using electrical resistance. Also, the two end rods AB and EF are heated from $T_1 = 30°\text{C}$ to $T_2 = 50°\text{C}$. At the lower temperature T_1 the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm^2. CD is made of aluminum and has a cross-sectional area of 375 mm^2. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 12(10^{-6})/°\text{C}$, and $\alpha_{al} = 23(10^{-6})/°\text{C}$.



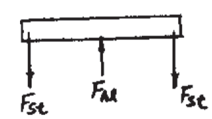
SOLUTION



$$\delta_{st} + (\delta_T)_{st} = (\delta_T)_{al} - \delta_{al} - 0.0007$$

$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^9)} + 12(10^{-6})(50 - 30)(0.3)$$

$$= 23(10^{-6})(180 - 30)(0.24) - \frac{F_{al}(0.24)}{375(10^{-6})(70)(10^9)} - 0.0007$$



$$12.0F_{st} + 9.14286F_{al} = 56000 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{al} - 2F_{st} = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

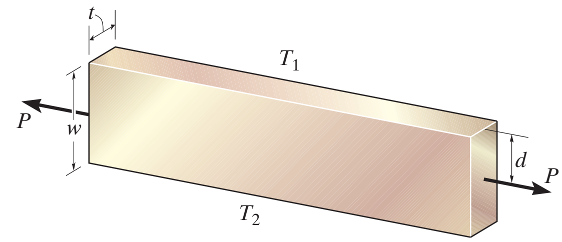
$$F_{AB} = F_{EF} = F_{st} = 1.85 \text{ kN} \qquad \textbf{Ans.}$$

$$F_{CD} = F_{al} = 3.70 \text{ kN}$$

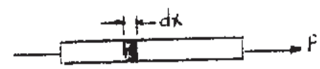
Ans:
$F_{AB} = F_{EF} = 1.85$ kN

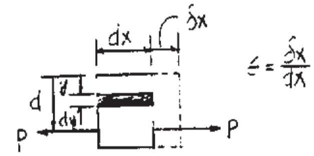
9–74.

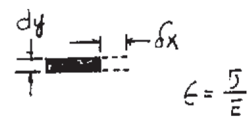
The metal strap has a thickness t and width w and is subjected to a temperature gradient T_1 to T_2 ($T_1 < T_2$). This causes the modulus of elasticity for the material to vary linearly from E_1 at the top to a smaller amount E_2 at the bottom. As a result, for any vertical position y, measured from the top surface, $E = [(E_2 - E_1)/w]y + E_1$. Determine the position d where the axial force P must be applied so that the bar stretches uniformly over its cross section.



SOLUTION







$$\epsilon = \text{constant} = \epsilon_0$$

$$\epsilon_0 = \frac{\sigma}{E} = \frac{\sigma}{\left(\left(\frac{E_2 - E_1}{w}\right)y + E_1\right)}$$

$$\sigma = \epsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - \int_A \sigma\, dA = 0$$

$$P = \int_0^w \sigma t\, dy = \int_0^m \epsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right) t\, dy$$

$$P = \epsilon_0 t\left(\frac{E_2 - E_1}{2} + E_1 w\right) = \epsilon_0 t\left(\frac{E_2 + E_1}{2}\right)w$$

$$\circlearrowleft + \Sigma M_0 = 0: \quad P(d) - \int_A y\,\sigma\, dA = 0$$

$$\epsilon_0 t\left(\frac{E_2 + E_1}{2}\right)wd = \int_0^w \epsilon_0\left(\left(\frac{E_2 - E_1}{w}\right)y^2 + E_1 y\right) t\, dy$$

$$\epsilon_0 t\left(\frac{E_2 + E_1}{2}\right)wd = \epsilon_0 t\left(\frac{E_2 - E_1}{3}w^2 + \frac{E_1}{2}w^2\right)$$

$$\left(\frac{E_2 + E_1}{2}\right)d = \frac{1}{6}(2E_2 + E_1)w$$

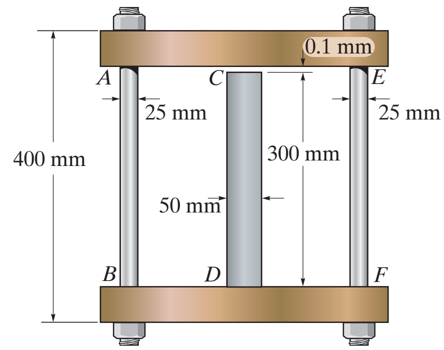
$$d = \left(\frac{2E_2 + E_1}{3(E_2 + E_1)}\right)w \qquad \textbf{Ans.}$$

Ans:

$$d = \left(\frac{2E_2 + E_1}{3(E_2 + E_1)}\right)w$$

R9–1.

The assembly consists of two A992 steel bolts AB and EF and an 6061-T6 aluminum rod CD. When the temperature is at 30° C, the gap between the rod and rigid member AE is 0.1 mm. Determine the normal stress developed in the bolts and the rod if the temperature rises to 130° C. Assume BF is also rigid.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the rigid cap shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad F_r - 2F_b = 0 \qquad \textbf{(1)}$$

Compatibility Equation: If the bolts and the rod are unconstrained, they will have a free expansion of $(\delta_T)_b = \alpha_{st}\Delta T L_b = 12(10^{-6})(130 - 30)(400) = 0.48$ mm and $(\delta\gamma)_r = \alpha_{al}\Delta T L_r = 24(10^{-6})(130 - 30)(300) = 0.72$ mm. Referring to the initial and final position of the assembly shown in Fig. *b*,

$$(\delta_T)_r - \delta_{Fr} - 0.1 = (\delta_T)_b + \delta_{Fb}$$

$$0.72 - \frac{F_r(300)}{\frac{\pi}{4}(0.05^2)(68.9)(10^9)} - 0.1 = 0.48 + \frac{F_b(400)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}$$

$$F_b + 0.5443F_r = 34361.17 \qquad \textbf{(2)}$$

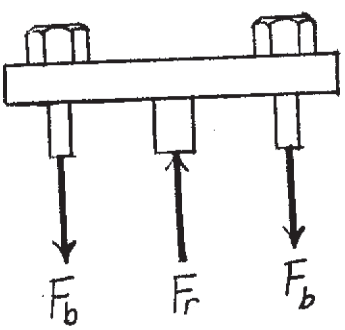
Solving Eqs. (1) and (2),

$$F_b + 16\,452.29 \text{ N} \qquad F_r = 32\,904.58 \text{ N}$$

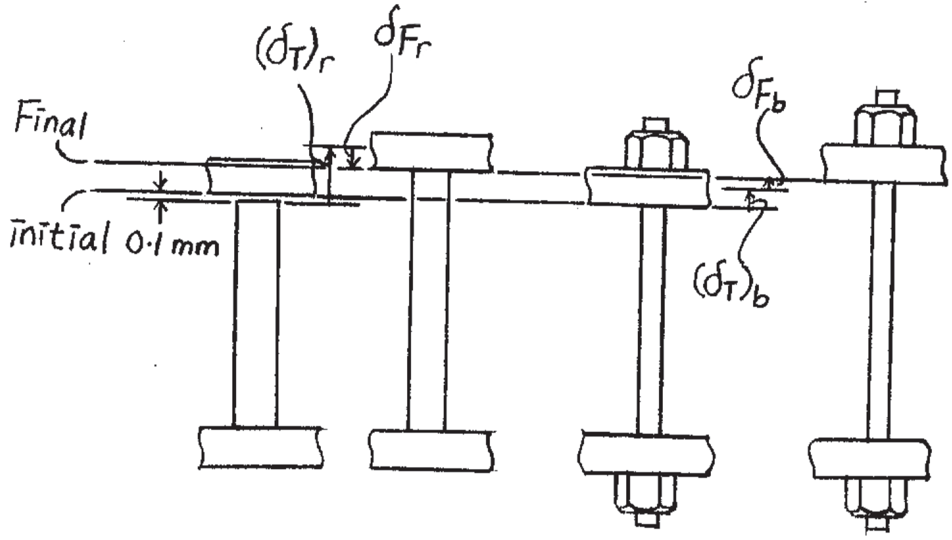
Normal Stress:

$$\sigma_b = \frac{F_b}{A_b} = \frac{16\,452.29}{\frac{\pi}{4}(0.025^2)} = 33.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_r = \frac{F_r}{A_r} = \frac{32\,904.58}{\frac{\pi}{4}(0.05^2)} = 16.8 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

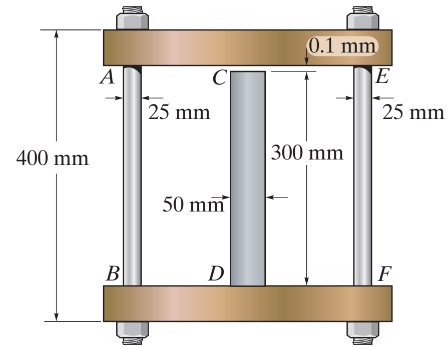


(b)

Ans:
$\sigma_b = 33.5$ MPa,
$\sigma_r = 16.8$ MPa

R9–2.

The assembly shown consists of two A992 steel bolts AB and EF and an 6061-T6 aluminum rod CD. When the temperature is at 30° C, the gap between the rod and rigid member AE is 0.1 mm. Determine the highest temperature to which the assembly can be raised without causing yielding either in the rod or the bolts. Assume BF is also rigid.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of the rigid cap shown in Fig. a,

$$+\uparrow \Sigma F_y = 0; \quad F_p - 2F_b = 0 \qquad \textbf{(1)}$$

Normal Stress: Assuming that the steel bolts yield first, then

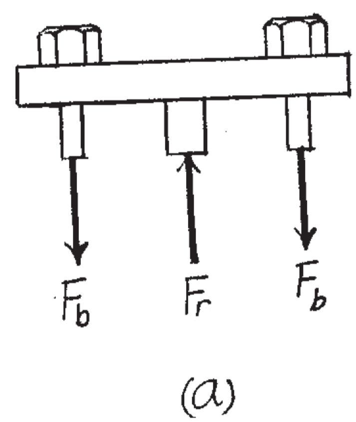
$$F_b = (\sigma_Y)_{st} A_b = 250(10^6)\left[\frac{\pi}{4}(0.025^2)\right] = 122\,718.46 \text{ N}$$

Substituting this result into Eq. (1),

$$F_p = 245\,436.93 \text{ N}$$

Then,

$$\sigma_p = \frac{F_p}{A_p} = \frac{245\,436.93}{\frac{\pi}{4}(0.05^2)} = 125 \text{ MPa} < (\sigma_Y)_{al} \qquad \textbf{(O.K!)}$$

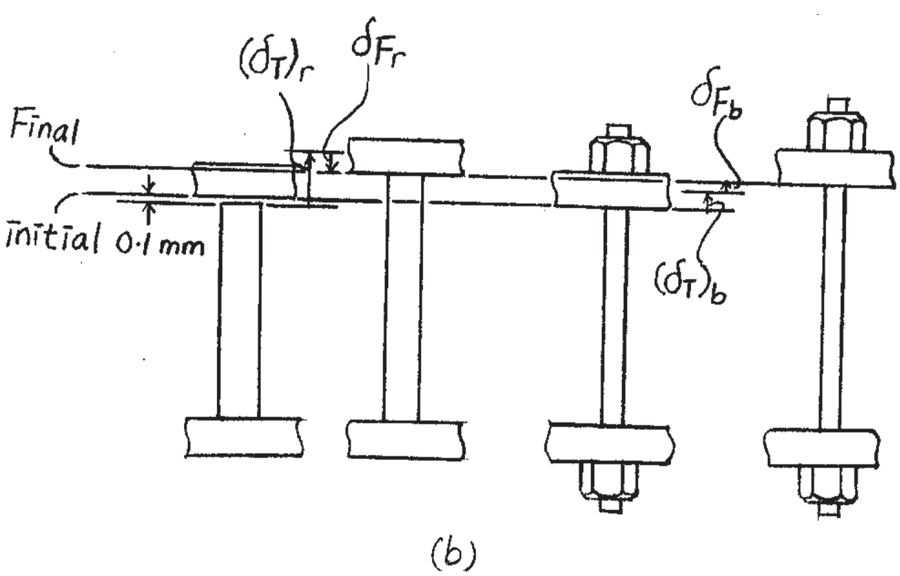


Compatibility Equation: If the assembly is unconstrained, the bolts and the post will have free expansion of $(\delta_T)_b = \alpha_{st}\Delta T L_b = 12(10^{-6})(T - 30)(400) = 4.8(10^{-3})(T - 30)$ and $(\delta_T)_p = \alpha_{al}\Delta T L_p = 24(10^{-6})T - 30)(300) = 7.2(10^{-3})(T - 30)$. Referring to the initial and final position of the assembly shown in Fig. b,

$$(\delta_T)_p - \delta_{Fp} - 0.1 = (\delta_T)_b + \delta_{Fb}$$

$$7.2(10^{-3})(T-30) - \frac{245\,436.93(300)}{\frac{\pi}{4}(0.05^2)(68.9)(10^9)} - 0.1 = 4.8(10^{-3})(T-30) + \frac{122\,718.46(400)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}$$

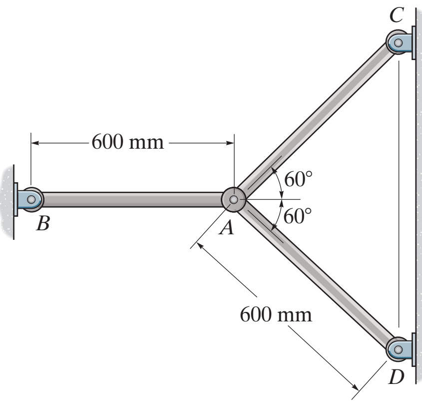
$$T = 506.78° \text{ C} = 507°\text{C} \qquad \textbf{Ans.}$$



Ans:
$T = 507°\text{C}$

R9–3.

The rods each have the same 25-mm diameter and 600-mm length. If they are made of A992 steel, determine the forces developed in each rod when the temperature increases by 50° C.



SOLUTION

Equation of Equilibrium: Referring to the free-body diagram of joint A shown in Fig. a,

$+\uparrow\Sigma F_x = 0;\qquad F_{AD}\sin 60^\circ - F_{AC}\sin 60^\circ = 0\qquad F_{AC} = F_{AD} = F$

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad F_{AB} - 2F\cos 60^\circ = 0$

$$F_{AB} = F \qquad \textbf{(1)}$$

Compatibility Equation: If AB and AC are unconstrained, they will have a free expansion of $\left(\delta_T\right)_{AB} = \left(\delta_T\right)_{AC} = \alpha_{\text{st}}\Delta TL = 12(10^{-6})(50)(600) = 0.36$ mm. Referring to the initial and final position of joint A,

$$\delta_{F_{AB}} - \left(\delta_T\right)_{AB} = \left(\delta_T{}'\right)_{AC} - \delta_{F_{AC}}{}'$$

Due to symmetry, joint A will displace horizontally, and $\delta_{AC}{}' = \dfrac{\delta_{AC}}{\cos 60^\circ} = 2\delta_{AC}$. Thus, $\left(\delta_T{}'\right)_{AC} = 2(\delta_T)_{AC}$ and $\delta_{F_{AC}}{}' = 2\delta_{F_{AC}}$. Thus, this equation becomes

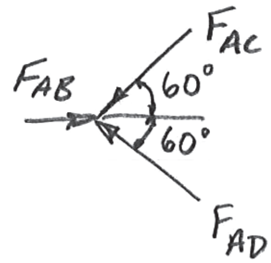
$$\delta_{F_{AB}} - \left(\delta_T\right)_{AB} = 2\left(\delta_T\right)_{AC} - 2\delta_{AC}$$

$$\frac{F_{AB}(600)}{\frac{\pi}{4}\left(0.025^2\right)(200)(10^9)} - 0.36 = 2(0.36) - 2\left[\frac{F(600)}{\frac{\pi}{4}\left(0.025^2\right)(200)(10^9)}\right]$$

$$F_{AB} + 2F = 176\,714.59 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$F_{AB} = F_{AC} = F_{AD} = 58\,904.86\text{ N} = 58.9\text{ kN (C)} \qquad \textbf{Ans.}$$

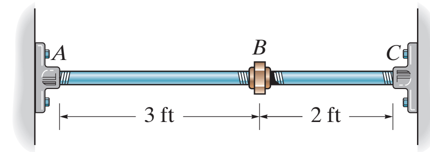


(a)

Ans:
$F_{AB} = F_{AC} = F_{AD} = 58.9$ kN (C)

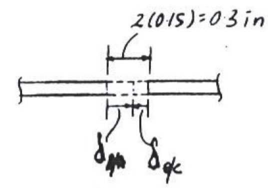
***R9–4.**

Two A992 steel pipes, each having a cross-sectional area of $0.32\ \text{in}^2$, are screwed together using a union at B. Originally the assembly is adjusted so that no load is on the pipe. If the union is then tightened so that its screw, having a lead of 0.15 in., undergoes two full turns, determine the average normal stress developed in the pipe. Assume that the union and couplings at A and C are rigid. Neglect the size of the union. *Note:* The lead would cause the pipe, when *unloaded*, to shorten 0.15 in. when the union is rotated one revolution.



SOLUTION

The loads acting on both segments AB and BC are the same since no external load acts on the system.



$$0.3 = \delta_{B/A} + \delta_{B/C}$$

$$0.3 = \frac{P(3)(12)}{0.32(29)(10^3)} + \frac{P(2)(12)}{0.32(29)(10^3)}$$

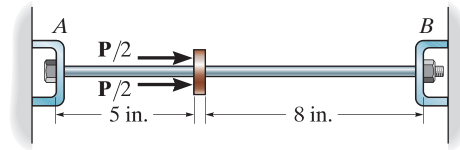
$$P = 46.4\ \text{kip}$$

$$\sigma_{AB} = \sigma_{BC} = \frac{P}{A} = \frac{46.4}{0.32} = 145\ \text{ksi} \qquad \textbf{Ans.}$$

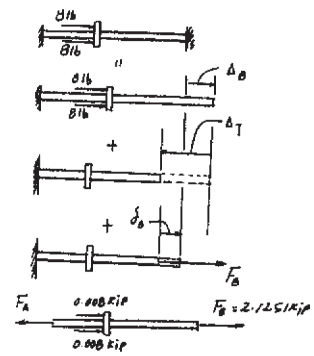
Ans:
$\sigma_{AB} = \sigma_{BC} = 145\ \text{ksi}$

R9–5.

The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at A and B when $T_1 = 70°\text{F}$. If the temperature becomes $T_2 = -10°\text{F}$, and an axial force of $P = 16$ lb is applied to the rigid collar as shown, determine the reactions at the rigid supports A and B.



SOLUTION



$$\overset{+}{\rightarrow} 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{0.016(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[70° - (-10°)](13) + \frac{F_B(13)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)}$$

$$F_B = 2.1251 \text{ kip} = 2.13 \text{ kip} \qquad \textbf{Ans.}$$

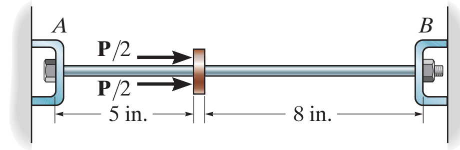
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 2(0.008) + 2.1251 - F_A = 0$$

$$F_A = 2.14 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$F_B = 2.13$ kip,
$F_A = 2.14$ kip

R9–6.

The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at A and B when $T_1 = 70°F$. Determine the force P that must be applied to the collar so that, when $T = 0°F$, the reaction at B is zero.

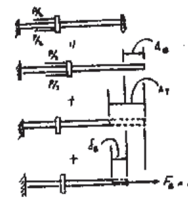


SOLUTION

$\overset{+}{\rightarrow} \quad = \Delta_B - \Delta_T + \delta_B$

$$0 = \frac{P(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[(70)(13)] + 0$$

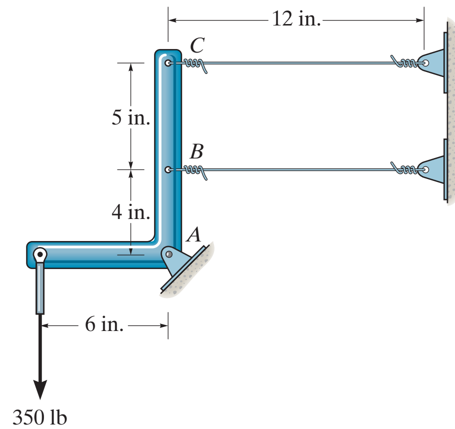
$P = 4.85$ kip **Ans.**



Ans:
$P = 4.85$ kip

R9–7.

The rigid link is supported by a pin at A and two A-36 steel wires, each having an unstretched length of 12 in. and cross-sectional area of 0.0125 in^2. Determine the force developed in the wires when the link supports the vertical load of 350 lb.



SOLUTION

Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \qquad -F_C(9) - F_B(4) + 350(6) = 0 \qquad \textbf{(1)}$$

Compatibility:

$$\frac{\delta_B}{4} = \frac{\delta_C}{9}$$

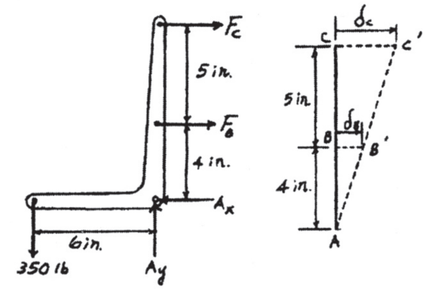
$$\frac{F_B(L)}{4AE} = \frac{F_C(L)}{9AE}$$

$$9F_B - 4F_C = 0, \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$F_B = 86.6 \text{ lb} \qquad \textbf{Ans.}$$

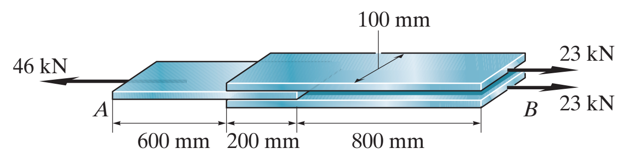
$$F_C = 195 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$F_B = 86.6$ lb,
$F_C = 195$ lb

***R9–8.**

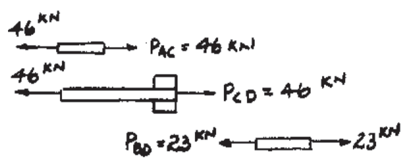
The joint is made from three A992 steel plates that are bonded together at their seams. Determine the displacement of end *A* with respect to end *B* when the joint is subjected to the axial loads. Each plate has a thickness of 5 mm.



SOLUTION

$$\delta_{A/B} = \Sigma\frac{PL}{AE} = \frac{46(10^3)(600)}{(0.005)(0.1)(200)(10^9)} + \frac{46(10^3)(200)}{3(0.005)(0.1)(200)(10^9)} + \frac{23(10^3)(800)}{(0.005)(0.1)(200)(10^9)}$$

$= 0.491$ mm **Ans.**



Ans:
$\delta_{A/B} = 0.491$ mm