Cive 222 Assignment 2 Benjamin Klassen

① $(3+y+2y^2\sin^2 x)dx+(x+2xy-y\sin 2x)dy=0$

$\frac{\partial M}{\partial y}=1+4y\sin^2 x$, $\sin^2 x=\frac{1-\cos 2x}{2}$ $1+4y\left(\frac{1}{2}-\frac{\cos 2x}{2}\right)=1+2y-2y\cos(2x)$

$\frac{\partial N}{\partial x}=1+2y-2y\cos(2x)$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$, $\therefore$ exact de

$(3+y+2y^2\sin^2 x)dx+(x+2xy-y\sin 2x)dy=0$

$\int dx$ $\int dy$ $\frac{4yx}{2y}-y\sin 2x$

$3x+yx$ $2y^2\left(\frac{x}{2}-\frac{1}{4}\sin 2x\right)$

$\therefore u(x,y)=3x+yx+2y^2\left(\frac{x}{2}-\frac{1}{4}\sin 2x\right)=C$

② $(x^2+2x+y)dx + (3x^2y - x)dy = 0$

$\frac{\partial M}{\partial y} = 1$

$\frac{\partial N}{\partial x} = 6xy - 1$ $\Big\rangle \therefore$ not exact

$$\frac{2-6xy}{3x^2y-x} = \frac{-2(3xy-1)}{x(3xy-1)} = \frac{-2}{x}, \therefore \text{Case 1}$$

$$\mu(x) = e^{-2\int \frac{1}{x}}$$

$$\mu(x) = e^{-2\ln|x|} = e^{\ln|x^{-2}|}$$

$$= x^{-2}$$

$$\left(1 + \frac{2}{x} + \frac{y}{x^2}\right)dx + \left(3y - \frac{1}{x}\right)dy = 0$$

$1 \to x$; $\frac{2}{x} \to 2\ln|x|$; $\frac{y}{x^2} \to -y/x$ (via $\frac{-1}{x}$) ; $-\frac{1}{x} \to \frac{3}{2}y^2$

$$\therefore x + 2\ln|x| - y/x + 3/2\,y^2 = C$$

(3) $3(x^2+y^2)dx + x(x^2+3y^2+6y)dy = 0$

$\frac{\partial M}{\partial y} = 6y$

$\frac{\partial N}{\partial x} = 3x^2 + 3y^2 + 6y$ } NEQ, $\therefore$ not exact

$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -3x^2 - 3y^2 = \frac{-3(x^2+y^2)}{3(x^2+y^2)} = -1$, $\therefore$ Case 2

$M(y) = e^{\int 1 dy} = e^y$

$(3x^2e^y + 3y^2e^y)dx + (x^3e^y + 3y^2xe^y + 6xye^y)dy = 0$

$3x^2e^y$ → x^3e^y

$3y^2e^y$ → $3y^2e^yx$

$6yxe^y + 3y^2xe^y$

$\therefore x^3e^y + 3xy^2e^y = C$

$= xe^y(x^2+3y^2) = C$

(4) $(2xy^2 - y)dx + (y^2 + x + y)dy = 0$

$\frac{\partial M}{\partial y} = 4xy - 1$ $\Big\rangle$ NEQ, ∴ not exact

$\frac{\partial N}{\partial x} = 1$

$4xy - 2 = \frac{2(2xy-1)}{y(2xy-1)} = \frac{2}{y}$ ∴ Case 2

$\mu(y) = e^{\int -2/y\,dy}$

$= e^{-2\ln|y|}$, $e^{\ln|y^2|}$

$= y^{-2} = \frac{1}{y^2}$

$\left(2x - \frac{1}{y}\right)dx + \left(1 + \frac{x}{y^2} + \frac{1}{y}\right)dy = 0$

x^2 ; y ; $\frac{-x}{y}$ ($\frac{-1}{y}$) ; $\ln|y|$

$x^2 + y - \frac{x}{y} + \ln|y| = C$