Differential Equations Assignment 1 May 24 2020

① $\frac{dy}{dx} = \frac{x^3 e^{x^2}}{y \ln y}$

$y \ln y \, dy = x^3 e^{x^2} dx$ , $y \neq 0$

$\int y \ln y \, dy = \int x^3 e^{x^2} dx$

LS: $d(y^2) = 2y\,dy$

$\therefore \frac{1}{2} \int \ln y \, d(y^2)$

$= \frac{1}{2}\left[\ln y \cdot y^2 - \int \frac{y^2}{y} dy\right]$

$= \frac{1}{2}\left[\ln y \cdot y^2 - \frac{y^2}{2}\right]$

RS: $d(e^{x^2}) = 2x e^{x^2} dx$

$\therefore \frac{1}{2} \int x^2 d(e^{x^2}) = \frac{1}{2}\left[x^2 e^{x^2} - \int 2x e^{x^2} dx\right]$

$\therefore \frac{1}{2}\left[x^2 e^{x^2} - \int d(e^{x^2})\right]$

$= \left[\frac{1}{2}\left(x^2 e^{x^2} - e^{x^2}\right) + C\right]$

$= \frac{e^{x^2}}{2}(x^2 - 1) + C$

$\ln y \cdot y^2 - \frac{y^2}{2} = e^{x^2}(x^2 - 1) + C$

$y^2\left(\ln y - \frac{1}{2}\right) = e^{x^2}(x^2 - 1) + C$

② $x\cos^2 y\,dx + e^x \tan y\,dy = 0$

$$\frac{\tan y}{\cos^2 y}\,dy = -\frac{x}{e^x}\,dx$$

$$\frac{\frac{\sin y}{\cos y}}{\cos^2 y}\,dy = -\frac{x}{e^x}\,dx$$ , Case 1 implicit since $\tan y$ in numerator, it is clear $\cos y \neq 0$

$$\frac{\sin y}{\cos^3 y}\,dy = -\frac{x}{e^x}\,dx$$

LS: $d(\cos y) = -\sin y\,dy$

$$-\int \frac{1}{\cos^3 y}\,d(\cos y)$$

$$-\int (\cos y)^{-3}\,d(\cos y)$$

$$-\frac{\cos y^{-2}}{-2}$$

$$= \frac{1}{2\cos^2 y}$$

RS:

$$-\int x e^{-x}\,dx = -\left[x(-e^{-x}) - \int e^{-x}\,dx\right]$$

$$= xe^{-x} + (-e^{-x}) + C$$

$$= e^{-x}(x-1) + C$$

$$\frac{1}{2\cos^2 y} = e^{-x}(x-1) + C$$

③ $xy^3 dx + (y+1)e^{-x} dy = 0$

$(y+1)e^{-x} dy = -xy^3 dx$

Case 1

$\int \frac{y+1}{y^3} dy = \int -xe^{x} dx \quad , y \neq 0$

Case 2

LS: $\int \frac{y}{y^3} dy + \int \frac{1}{y^3} dy$ RS: $-\int x\, d(e^x) = -\left[xe^x - e^x\right] + C$

$= \int y^{-2} dy + \int y^{-3} dy$ $= -e^x(x-1) + C$

$= \frac{-1}{y} - \frac{1}{2y^2}$

$\frac{-1}{y} - \frac{1}{2y^2} = -e^x(x-1) + C$

$e^x(x-1) - \frac{1}{y} - \frac{1}{2y^2} = C \quad , y \neq 0$

(4)

$$\left[x\cos^2(y/x) - y\right]dx + x\,dy = 0$$

$$\left[\cos^2(y/x) - y/x\right]dx + dy = 0$$

$$dy = -\left[\cos^2(v) - v\right]dx$$

$$\frac{dy}{dx} = -\left[\cos^2 v - v\right]$$

$$v + x\frac{dv}{dx} = -\cos^2 v + v$$

$$x\frac{dv}{dx} = -\cos^2 v$$

$$\int \frac{1}{\cos^2 v}\,dv = -\int \frac{1}{x}\,dx \quad , \cos^2 v \neq 0$$

$$\int \sec^2 v\,dv = -\ln|x| + C$$

$$\tan v = -\ln|x| + C$$

$$\tan(y/x) = -\ln|x| + C$$

$$\tan(y/x) + \ln|x| = C \quad , \; y \neq 0 \text{ since } \cos y/x \neq 0$$

⑤

$$xy\,dx + (x^2+y^2)\,dy = 0$$

$$\frac{dy}{dx} = -\frac{xy}{x^2+y^2} \div \frac{x^2}{x^2}$$

$$\frac{dy}{dx} = -\frac{y/x}{1+(y/x)^2}$$

$$v + x\frac{dv}{dx} = \frac{-v}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{-v}{1+v^2} - \frac{v(1+v^2)}{(1+v^2)}$$

$$x\frac{dv}{dx} = \frac{-v^3-2v}{(1+v^2)}$$

$$x\frac{dv}{dx} = \frac{-1(v^3+2v)}{(1+v^2)}$$

$$\frac{1+v^2}{v^3+2v}\,dv = \frac{-1}{x}\,dx$$ , $v \neq 0$, $v^2 \neq -2$ (it is impossible) Case 1

LS: $\int \frac{1+v^2}{v(v^2+2)}\,dv$ , $\frac{A}{v} + \frac{Bv+C}{v^2+2}$ $= 2A + Av^2 + Bv^2 + Cv$ , $1 = 2A$, $A = 1/2$, $1 = A+B$, $B = 1/2$ RS: $-\ln|x| + C$

$$= \frac{1}{2}\int \frac{1}{v}\,dv + \frac{1}{2}\int \frac{v}{v^2+2}\,dv$$ , $d(v^2) = 2v\,dv$

$$= \frac{1}{2}\int \frac{1}{v}\,dv + \frac{1}{4}\int \frac{1}{v^2+2}\,d(v^2+2)$$

$$= \frac{1}{2}\ln|v| + \frac{1}{4}\ln|v^2+2| = -\ln|x| + C$$

$$\frac{1}{2}\ln|v| + \frac{1}{4}\ln|v^2+2| + \ln|x| = C$$

$$\frac{1}{2}\ln\left|\frac{y}{x}\right| + \frac{1}{4}\ln\left|\left(\frac{y}{x}\right)^2+2\right| + \ln|x| = C$$ , $v \neq 0$ (Case 1)

⑥ $\frac{dy}{dx} = \frac{2x+y-1}{x-y-2}$

$$\left|\begin{array}{cc|c} 2 & 1 & 1 \\ 1 & -1 & 2 \end{array}\right| \times \tfrac{1}{2} \quad \left|\begin{array}{cc|c} 1 & 1/2 & 1/2 \\ 1 & -1 & 2 \end{array}\right| -R1 \quad \left|\begin{array}{cc|c} 1 & 1/2 & 1/2 \\ 0 & -3/2 & 3/2 \end{array}\right| \times -2/3 \quad \left|\begin{array}{cc|c} 1 & 1/2 & 1/2 \\ 0 & 1 & -1 \end{array}\right| -\tfrac{1}{2}R2 \quad \left|\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right| \therefore P(h,k) = P(1,-1)$$

let $x = X+1$, $y = Y-1$

$$\frac{dy}{dx} = \frac{2(X+1)+(Y-1)-1}{(X+1)-(Y-1)-2} = \frac{2X+Y}{X-Y}$$

$$\frac{dy}{dx} = \frac{2X+Y}{X-Y} \div \frac{X}{X}$$

$$\frac{dy}{dx} = \frac{2+Y/X}{1-Y/X}$$

$$V + X\frac{dV}{dX} = \frac{2+V}{1-V}$$

$$X\frac{dV}{dX} = \frac{2+V}{1-V} - \frac{V(1-V)}{(1-V)}$$

$$X\frac{dV}{dX} = \frac{2+V^2}{1-V}$$

$$\int \frac{1-V}{2+V^2}dV = \int \frac{1}{X}dX \quad , V^2 \neq -2, \text{Case 1 okay}$$

$$① \int \frac{1}{2+v^2}dv - \overset{②}{\int \frac{v}{2+v^2}}dV = \ln|x| + C$$

① let $v = \sqrt{2}u$ ② $d(v^2+2) = 2v\,dv$

$dv = \sqrt{2}du$

$$\sqrt{2}\int \frac{1}{2+2u^2}du - \frac{1}{2}\int \frac{1}{2+v^2}d(2+v^2)$$

$$\frac{\sqrt{2}}{2}\tan^{-1}(u) - \tfrac{1}{2}\ln|2+v^2| = \ln|x| + C$$

$$\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{v}{\sqrt{2}}\right) - 1/2\ln|2+v^2| = \ln|x| + C$$

$$\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{y}{\sqrt{2}x}\right) - \frac{1}{2}\ln\left|2+\left(\frac{y}{x}\right)^2\right| = \ln|x| + C$$

$$\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{y+1}{\sqrt{2}(x-1)}\right) - \frac{1}{2}\ln\left|2+\frac{(y+1)^2}{(x-1)^2}\right| - \ln|x-1| = C$$

$$\sqrt{2}\tan^{-1}\left(\frac{y+1}{\sqrt{2}(x-1)}\right) - \ln\left|2+\frac{(y+1)^2}{(x-1)^2}\right| - \ln|(x-1)^2| = 2C, \text{ let } 2C = A$$

$$\sqrt{2}\tan^{-1}\left(\frac{y+1}{\sqrt{2}(x-1)}\right) - \ln\left|2(x-1)^2+\frac{(y+1)^2(x-1)^2}{(x-1)^2}\right| = A, \text{ let } A = C$$

$$\sqrt{2}\tan^{-1}\left(\frac{y+1}{\sqrt{2}(x-1)}\right) - \ln|2(x-1)^2+(y+1)^2| = C$$