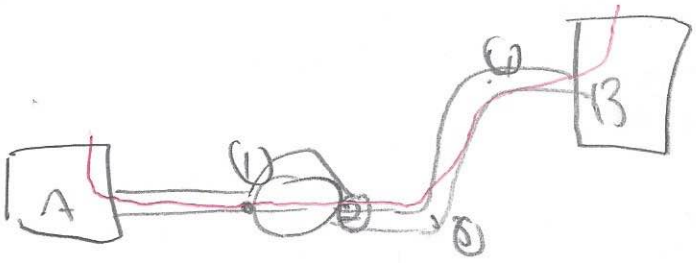
Pump Curve, $h_p = 100 - 8000Q^2$, $f = 0.02$, $\phi = 15cm$, $l = 100m$



$K_{entrance} = 0.5$

$K_{bend} = 0.25$

$K_{exit} = 1$

a) Find Q, i.e. find operating point. Determine system curve

System Curve: E eqn from A → B

$$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + z_A + h_p = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + z_B + \sum h_L$$

$$h_p = 50 + \left[K_e + 2K_B + K_d + \frac{fL}{d}\right]\frac{V^2}{2g} \qquad V^2 = \frac{Q^2}{A^2} = \frac{16Q^2}{\pi^2 d^4}$$

$$= 50 + \left[0.5 + 2(0.25) + 1 + \frac{0.02(100)}{0.15}\right]\frac{8Q^2}{g\pi^2(0.15)^4}$$

$$= 50 + 2503\,Q^2$$

$$100 - 8000Q^2 = 50 + 2503Q^2$$

$$Q = 0.0690\ m^3/s$$

$$\therefore 50 + 2503(0.069)^2 = 61.92m \quad \text{(operating point)}$$

b) Find h_p provided by pump to the water → P_{hyd}

$$P_{hyd} = \gamma Q h_p = 9790(0.0690)(61.42) = 41928W \times \frac{1hp}{745.7W} = 56.1\ hp$$

c) Location of lowest and highest pipe pressure

P_{min} just before the pump (P_1)

P_{max} just after the pump (P_2)

E equation from A → ①

$$z_A = z_{datum} + \frac{P_1}{\gamma} + \sum h_L + \frac{V^2}{2g}$$

$$\frac{P_1}{\gamma} = z_A - \frac{V^2}{2g} - \sum h_L$$

$$\frac{P_1}{\gamma} = 4 - \frac{V^2}{2g} - \left[K_e + \frac{fL}{d}\right]\frac{V^2}{2g}$$

$$= 4 - \left[0.5 + 1 + \frac{0.02(25)}{0.15}\right]0.7772$$

$$\frac{P_1}{\gamma} = 0.244$$

Q at operating point

$Q_{max} = 0.064\ m^3/s$

$\therefore V = 3.905\ m/s$

$\therefore \frac{V^2}{2g} = 0.7772$

From ① → ②

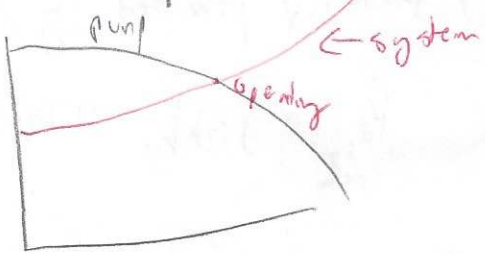
pump curve includes this

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 + h_p = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 + h_L$$

- V before and after pump are equal

$$\frac{P_2}{\gamma} = \frac{P_1}{\gamma} + h_p = 0.244 + 61.92 = 62.16\ m$$

d)



1. System Curve

a) Apply Energy Equation from (A) → (B)

$$\frac{V_A^2}{2g} + \frac{P_A}{\gamma} + z_A + h_p = \frac{V_B^2}{2g} + \frac{P_B}{\gamma} + z_B + \sum h_{L_{A\to B}}$$

$$h_p = z_B - z_A + \sum h_L$$

$$= 30 + \left(K_E + K_0 + K_0 + f\frac{L}{D}\right)\frac{1}{2g}\left(\frac{Q}{A}\right)^2$$

$$= 30 + \left(0.5 + 0.35 + 1 + 0.015 \cdot \frac{1000}{0.4}\right)\frac{1}{2(9.81)} \cdot \frac{Q^2}{(0.1257)^2}$$

$$= 30 + \frac{(1.85 + 37.5)Q^2}{0.31} = 30 + 126.9Q^2$$

= System Curve

Pump Curve, $h_p = 65 - 400Q^2$ (2)

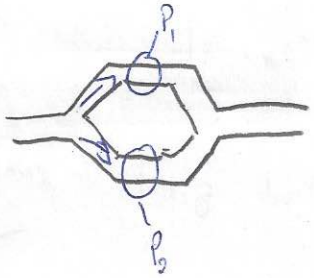
Operating Point, set (1) = 2

$$30 + 126.9Q^2 = 65 - 400Q^2$$

$$Q = \sqrt{\frac{35}{526.9}} = 0.2577 \text{ m}^3/\text{s}$$

$$\therefore h_p = 65 - 400(0.2577)^2 = 38.44$$

b) What if 2 identical pipes in parallel, same System Curve,
→ so no new losses



$$h_p = h_{p_1} = h_{p_2}$$

$$Q = Q_1 + Q_2$$

Pump curve for 1 pump, $h_p = 65 - 400Q^2$, $Q = \sqrt{\frac{65-h_p}{400}}$

2 Pumps in parallel: $Q = Q_1 + Q_2 = \sqrt{\frac{65-h_p}{400}} + \sqrt{\frac{65-h_p}{400}} = 2\sqrt{\frac{65-h_p}{400}}$

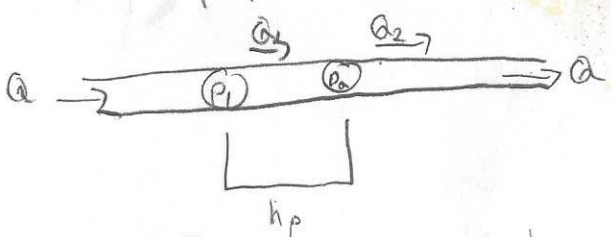
Alternatively, $h_p = 65 - 100Q^2$ (2b)

From before, system curve $= 30 + 126.9Q^2$

$\therefore 30 + 126.9Q^2 = 65 - 100Q^2$

$Q = \sqrt{\frac{35}{226.9}} = 0.3928\ m^3/s$ → $h_p = 65 - 100(0.3928)^2 = 49.57\ m$ (Operating Point)

2 identical pumps in series → Same system curve, $h_p = 65 - 400Q^2$



Single pump curve, $h_p = 65 - 400Q^2$

2 pumps in series, $h_p = h_{p_1} + h_{p_2} = (65 - 400Q^2) + (65 - 400Q^2)$

$h_p = 130 - 800Q^2$ (2c)

$30 + 126.9Q^2 = 130 - 800Q^2$

$Q = \sqrt{\frac{100}{926.9}} = 0.3285\ m^3/s$ → $h_p = 130 - 800(0.3285)^2 = 43.67\ m$

∴ higher flow w/ parallel

hydraulic power $P_o = \gamma Q h_p$ ... Power lower in series, Δhead greater in parallel

Eg. Pump 10 ft above tank, $Q = 0.5\ ft^3/s$

NPSH = 15 ft

$K_L = 20$

$\phi = 4\ in$

Determine whether $NPSH_A \geq NPSH_R = 15\ ft$

Water @ 80°F

$\gamma_w = 62.22\ \frac{lb}{ft^3}$

$P_v = 0.5069\ psia$

1) Apply E eqn from 1 → 2

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_s^2}{2g} + \frac{P_2}{\gamma} + z_s + \sum h_{L_{1-2}}$$

$$\frac{P_{atm}}{\gamma} + z_1 = \frac{V_s^2}{2g} + \frac{P_s}{\gamma} + K_L\frac{V^2}{2g}$$

$$\frac{V_s^2}{2g} + \frac{P_s}{\gamma} = \frac{P_{atm}}{\gamma} + z_1 - \frac{K_L Q^2}{2gA^2}$$

$$\frac{V_s^2}{2g} + \frac{P_s}{\gamma} - \frac{P_{vap}}{\gamma} = \frac{P_{atm}}{\gamma} - \frac{P_{vap}}{\gamma} + z_1 - \frac{K_L Q^2}{2gA^2}$$

$$NPSH_A = \frac{(14.7 - 0.5069)\ \frac{lb}{in^2}}{62.22\ lb/ft^3} \times \frac{12\ in^2}{1\ ft^2} - 10\ ft - \frac{20\,(0.5\ ft/s)^2}{2(32.2\ ft/s^2)\left\{\pi/4\,(4\ in \cdot 1\ ft/12\ in)^2\right\}^2}$$

$$= 32.8\ ft - 10\ ft - 10.2\ ft = 12.6\ ft$$

∴ $NPSH_A < NPSH_R$; Pump may not perform as expected

1) If not given, suction pressure assumed = vapor pressure

$P_s < P_{vap}$

Lec 23 – Pump Coeff

Eg 1. A pump w/ D = 10" ops @ 3560 rpm

↳ Pumps under under head of 300 ft

a) what speed operated @ for max efficiency?

b) Discharge?

i) Use pump-scaling relationships to determine the 2nd pump characteristics

Pump 1	Pump 2
D = 10"	D = 10"
N = 3560 rpm	h_p = 300 ft
max efficiency (BEP)	max efficiency (BEP)

↳ Using performance curve, Q = 400 GPM @ H_p = 60, h_p = 370 ft (falls in 65% efficiency center)

Assume geometrically similar pumps

$$C_{H_1} = C_{H_2}$$

$$C_{Q_1} = C_{Q_2}$$

$$\therefore C_{H_1} = C_{H_2}, \left(\frac{gh_1}{\omega^2 D^2}\right)_1 = \left(\frac{gh_2}{\omega^2 D^2}\right)_2 \quad \frac{\omega_2^2}{\omega_1^2} = \frac{h_2}{h_1}, \quad \frac{\omega_2}{\omega_1} = \sqrt{\frac{h_2}{h_1}}$$

$$\omega = N \cdot \frac{2\pi}{\text{rotation}} \times \frac{1\ \text{min}}{60\ \text{sec}}$$

$$\therefore \frac{\omega_2}{\omega_1} = \frac{N_2 \cdot \frac{2\pi}{60}}{N_1 \cdot \frac{2\pi}{60}} = \sqrt{\frac{h_2}{h_1}}$$

$$N_2 = N_1 \sqrt{\frac{h_2}{h_1}} = 3560 \sqrt{\frac{300}{370}} = 3122\ \text{rpm}$$

b) $C_{Q_1} = C_{Q_2}$, $\left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2$

$Q_2 = Q_1 \frac{\omega_2}{\omega_1} = Q_1 \frac{N_2}{N_1} = 400 \text{ GPM} \cdot \frac{3122}{3360}$

$Q_2 = 350.8 \text{ gal/min}$

∴ Pump 2 should run @ N = 3122 rpm

↳ Check w/ figure 9.18 on p. 8-8

pt $Q = 351$ gpm & $h_p = 300$ ft lies on max efficiency line just below N = 3200 rpm

Eg. 2. 500 L/s, inlet head 10m, $N_S = 2000$ rpm

Type of pump?

1. Convert rpm to ω
2. Calculate specific speed
3. Check whether radial, axial, or mixed flow pump

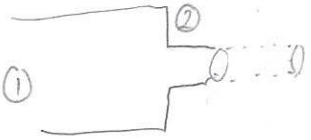
$\omega = N \cdot \frac{\pi}{30} = \frac{200}{3}\pi$

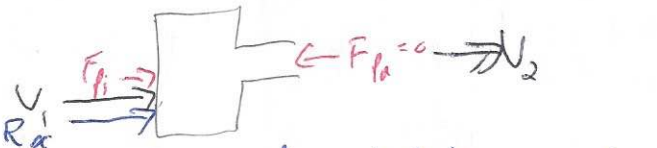
Specific Speed, $N_S = \frac{\omega Q^{0.5}}{(gh_p)^{0.75}} = \frac{209.4(0.5)^{0.5}}{[9.81 \times 10]^{0.75}} = 4.75$

∴ Axial $N_S > 3.7$. Axial flow pump required most efficient

Momentum Conservation

Assume steady-state flow
↳ liquid = water, ∴ incompressible





↳ External horiz. force req'd to balance momentum

① $\phi = 3$ in
$V_1 = 25$ ft/s
$p_1 = 75$ psi

② $V_2 = 100$ ft/s
$p_2 = 0$ psi

Flow rate: $Q = V_1 A_1 = 25 \frac{ft}{s} \cdot \frac{\pi}{4}\left(\frac{3}{12} ft\right)^2 = 25 \frac{ft}{s}(0.04909 ft^2)$

$Q = 1.227 \frac{ft^3}{s}$

$$\Sigma F_x = (\rho Q V_x)_{out} - (\rho Q V_x)_{in}$$

momentum out/in signs

$$R_x + F_{p_1} - F_{p_2}^{\downarrow 0} = \rho Q [+V_2 - V_1]$$

velocity signs w.r.t. x axis

$$R_x + 75 \frac{lb}{in^2} \times 144 \frac{in^2}{ft^2} \times 0.04909 - 0 = 1.94 \frac{slug}{ft^3}\left(1.227 \frac{ft^3}{s}\right)[+100 - (+25)] ft/s$$

p A

$R_x + 530.14 lb = 178.5 lb$

$R_x = -351.6$ lb →

$\therefore R_x = 351.6$ lb ←, force acting on water to satisfy momentum balance due to the nozzle

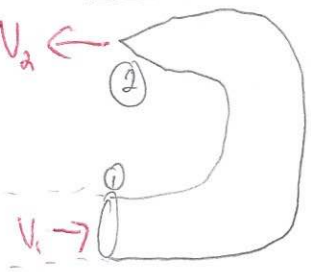
F_{weld} = F to keep nozzle attached to the pipe

$F_{weld} = R_x = 351.6$ lb

b) is flow ideal? Check using Eqn from ① → ②

$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_L$, $z_1 = z_2$ ∴ $h_L = 27.5$ ft ∴ flow is not ideal

Momentum Conservation



Assume steady-state water flow
- incompressible
- constant density

Is flow ideal? → If ideal, no friction along walls
↳ h_L calc after

$P_2 = 0$
$P_1 = 700$ kPa
$V_1 = 2$ m/s
$A_1 = 300$ mm ∅
$A_2 = 160$ mm ∅

1. Continuity. $Q = V_1 A_1 = 2 \cdot \frac{\pi}{4}(0.3)^2$
$= 0.1414$ m³/s

$V_2 = \frac{Q}{A_2} = \frac{0.1414}{\pi/4(0.16)^2} = 7.033$ m/s

$\Sigma F_x = (\rho Q V_x)_{out} - (\rho Q V_x)_{in}$

$R_x + F_{P_1} = \rho Q (V_2 - V_1)$

$R_x + 100000 \text{Pa}\left(\frac{\pi}{4} \cdot 0.3^2\right) = 998\left(0.1414 \frac{m^3}{s}\right)\left((-7.033) - (+2)\right)$ (− direction)

$R_x + 7068.6\text{N} = -1274.7$ N

$R_x = -8343$N

$R_x = 8343$N ←

Is flow ideal? $\rightarrow h_L = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = 7.9 \neq 0$, ∴ not ideal

Can we solve if P_1 not given?

$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 + h_L$ → 0 (assumed ideal flow)

$\frac{P_1}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$

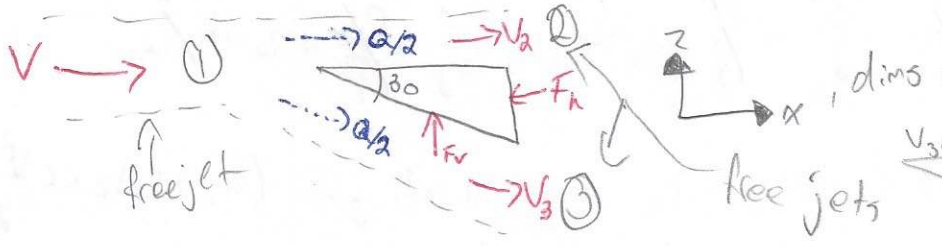
Alternatively... info to calculate h_L given, then $\frac{P_1}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L$

Eg. free jet of ideal fluid (inviscid, density constant)

$Q = 10\ L/s$ ← steady-state flow

$V = 10\ m/s$

half flow one way, half other way. Wedge weighs 5 N



, dims of jets & wedge not given

E balance from ①→② & ①→③

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 + h_{L1-2} = \frac{V_3^2}{2g} + \frac{P_3}{\gamma} + z_3 + h_{L1-3}$$, Assume $z_1 \sim z_2 \sim z_3$

$$\therefore |V_1| = |V_2| = |V_3| = \underline{|V|}$$

Momentum Balance

$$\Sigma F_x = \Sigma(\rho Q V_x)_{out} - \Sigma(\rho Q V_x)_{in}$$

$$-F_H = \rho\frac{Q}{2}[V_2] + \rho\frac{Q}{2}[V_3\cos 30] - \rho Q[V_1]$$

$$= \rho Q V\left[\frac{1}{2} + \frac{\cos 30}{2} - 1\right] = 998(0.01)(10)[-0.567]$$

$$= 6.69\ N \leftarrow$$

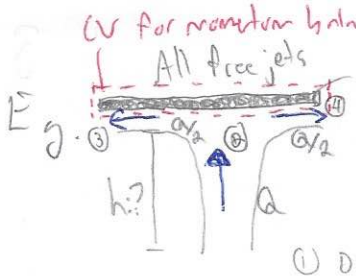
$$\Sigma F_y = \Sigma(\rho Q V_y)_{out} - \Sigma(\rho Q V_y)_{in}$$

assume small size, dims not given; reg dir ↙

$$F_V + W_{wedge} + W_{water} = \rho\frac{Q}{2}[(-V_3\sin 30) - 0]$$

$$F_V - 5 = 998\left(\frac{0.01}{2}\right)(-10\sin 30) = -24.95\ N$$

$$F_V = -19.95\uparrow \quad , \quad F_V = 30\ N\downarrow$$



Assume steady-state flow

$Q = V_1 A_1 = 10 \cdot \frac{\pi}{4}(0.02)^2$

$= 3.14 \times 10^{-3}\ m^3/s$

* assume jet splits into 2 equal sub-jets

Energy Balance from ① → ② or ① → ③ or ① → ④

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + z_2 + h_{L,1-2} = \frac{V_3^2}{2g} + \frac{P_3}{\gamma} + z_3 + h_{L,1-3} = \frac{V_4^2}{2g} + \frac{P_4}{\gamma} + z_4 + h_{L,1-4}$$

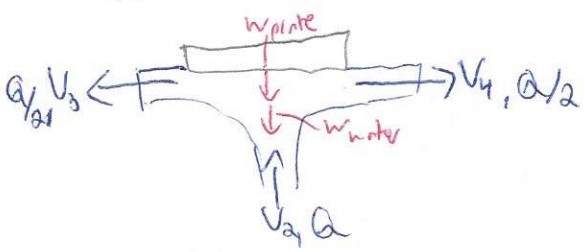
(z_1 = Datum = 0; $h_{L,1-2}$, $h_{L,1-3}$, $h_{L,1-4}$ = 0)

↳ Assume h_L are small & $z_2 \sim z_3 \sim z_4$ since no info given (h_L and $\Delta z \ll \frac{V^2}{2g}$)

$$\frac{10^2}{2g} = \frac{V_2^2}{2g} + h = \frac{V_3^2}{2g} + h = \frac{V_4^2}{2g} + h$$

$$\therefore h = \frac{10^2}{2g} - \frac{V_2^2}{2g} = 5.1 - \frac{V_2^2}{2g} \quad (1)$$

Vertical Momentum Balance @ CV drawn on last slide



$\Sigma F_z = (\rho Q V_2)_{out} - (\rho Q V_2)_{in}$

$-w_{plate} - \cancel{w_{water}}^{\text{small}} = 0 - \rho Q V_2$

(small → no dims)

$$V_2 = \frac{w_{plate}}{\rho Q} = \frac{mg}{\rho Q} = \frac{1.5\,kg\,(9.81\ m/s^2)}{998\ kg/m^3\,(3.14E\text{-}3\ m^3/s)}$$

$= 14.696$ m/s

$$\therefore h = 5.1\,m - \frac{(14.696\ m/s)^2}{2(9.81\ m/s^2)} = 3.98\,m$$

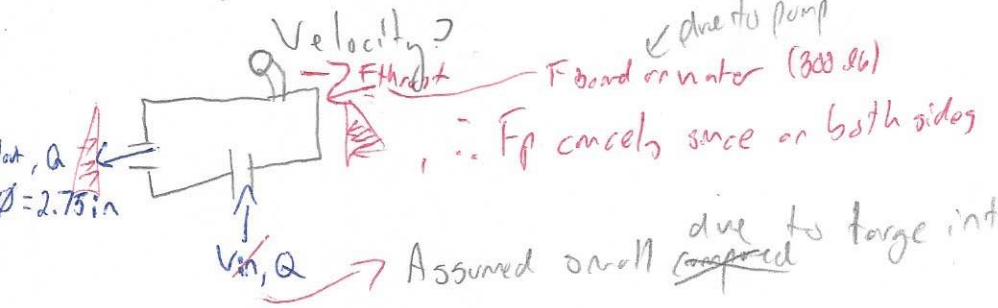
Eg. $\phi = 2.75$ in → 300 lb

Q, V_{jet}

Thrust = 300 lb = $F_{water\ on\ board}$

jet flow rate = ?

Velocity?



→ Assumed overall ~~compared~~ due to large intake area

→ x, hor momentum balance:

$$\Sigma F_x = (\rho Q V_x)_{out} - (\rho Q V_x)_{in}$$

$$-300 = \rho (V_{jet} A_{jet})(-V_{jet}) - 0$$

$$300\ lb = \rho A_{jet} V_{jet}^2$$

$$V_{jet} = \sqrt{\frac{300.}{1.94 \frac{slug}{ft^3} \times \frac{\pi}{4}(2.75\ in \times \frac{1\ ft}{12\ in})^2}} = 61.24\ ft/s$$

$$Q = V_{jet} A_{jet} = 61.24 \cdot \frac{\pi}{4} \cdot (2.75\ in \times \frac{1\ ft}{12\ in})^2 = 2.526\ \frac{ft^3}{s}$$

Momentum Correction Factor

$\rho Q V = \rho V A V = \rho A V^2 = \rho \int v^2 dA$. If not uniform, $\beta \rho A V^2$ used.

β is correcto. factor