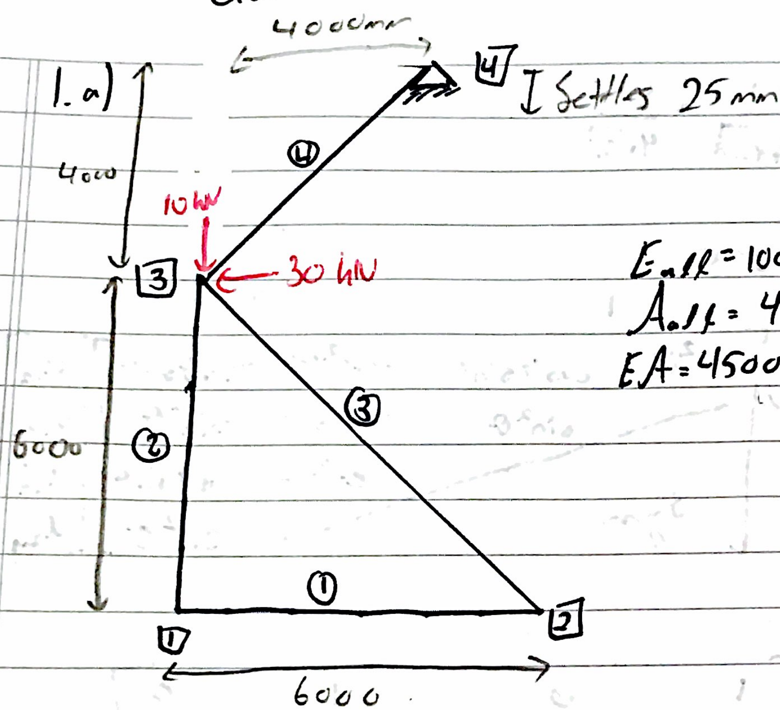
CivE 422 A2

1.a)



$E_{all} = 100\ GPa = 100\ kN/mm^2$

$A_{all} = 450\ mm^2$

$EA = 45000\ kN$

Connectivity

e	n_1	n_2
1	1	2
2	1	3
3	2	3
4	3	4

$$\frac{AE}{L}^{①} = \frac{45000}{6000} = \frac{15}{2}\ kN/mm$$

$$\frac{AE}{L}^{②} = \frac{45000}{6000} = \frac{15}{2}\ kN/mm$$

$$\frac{AE}{L}^{③} = \frac{45000}{\sqrt{6000^2 + 6000^2}} = \frac{15}{2\sqrt{2}}\ kN/mm$$

Date

$$\frac{AE^{①}}{L} = \frac{45000}{\sqrt{4000^2+4000^2}} = \frac{45}{4\sqrt{2}}$$

$\underline{e=1}$, $\theta = 0$

$$\begin{bmatrix} F_{1x}^1 \\ F_{1y}^1 \\ F_{2x}^1 \\ F_{2y}^1 \end{bmatrix} = \frac{AE^{①}}{L} \begin{array}{cc} 1 & 2 \\ \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ & & \cos^2\theta & \cos\theta\sin\theta \\ \text{Sym} & & & \sin^2\theta \end{bmatrix} & \end{array} \begin{bmatrix} u_{1x}^1 \\ u_{1y}^1 \\ u_{2x}^1 \\ u_{2y}^1 \end{bmatrix} \begin{array}{c} 1 \\ \\ 2 \\ \end{array}$$

$$\underset{\sim}{K} = \frac{15}{2} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 15/2 & 0 & -15/2 & 0 \\ 0 & 0 & 0 & 0 \\ -15/2 & 0 & 15/2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\underline{e=2,\ \theta = 90^\circ}$

$$\underset{\sim}{K} = \frac{15}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$
\underline{k} = \begin{array}{c} \\ \\ \\ \\ \end{array}
\begin{array}{cccc|c}
 & 1 & & 3 & \\
\left[\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}\right. & \begin{matrix} 0 \\ 15/2 \\ 0 \\ -15/2 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} & \left.\begin{matrix} 0 \\ -15/2 \\ 0 \\ 15/2 \end{matrix}\right] & \begin{matrix} \\ 1 \\ \\ 3 \end{matrix}
\end{array}
$$

$\underline{e=3 \quad \theta = 135^\circ}$

$$
k = \frac{15}{2\sqrt{2}}
\begin{array}{cccc|c}
2 & & 3 & & \\
\left[\begin{matrix} 0.5 \\ -0.5 \\ -0.5 \\ 0.5 \end{matrix}\right. & \begin{matrix} -0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{matrix} & \begin{matrix} -0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{matrix} & \left.\begin{matrix} 0.5 \\ -0.5 \\ -0.5 \\ 0.5 \end{matrix}\right] & \begin{matrix} 2 \\ \\ 3 \\ \\ \end{matrix}
\end{array}
$$

$$
= \begin{bmatrix}
\frac{15}{4\sqrt{2}} & \frac{-15}{4\sqrt{2}} & \frac{-15}{4\sqrt{2}} & \frac{15}{4\sqrt{2}} \\
\frac{-15}{4\sqrt{2}} & \frac{15}{4\sqrt{2}} & \frac{15}{4\sqrt{2}} & \frac{-15}{4\sqrt{2}} \\
\frac{-15}{4\sqrt{2}} & \frac{15}{4\sqrt{2}} & \frac{15}{4\sqrt{2}} & \frac{-15}{4\sqrt{2}} \\
\frac{15}{4\sqrt{2}} & \frac{-15}{4\sqrt{2}} & \frac{-15}{4\sqrt{2}} & \frac{15}{4\sqrt{2}}
\end{bmatrix}
$$

$e = 4, \quad \theta = 45°$

$$k = \frac{45}{4\sqrt{2}} \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{cccc} 3 & & 4 & \end{array} \\ = & \begin{bmatrix} \frac{45}{8\sqrt{2}} & \frac{45}{8\sqrt{2}} & -\frac{45}{8\sqrt{2}} & -\frac{45}{8\sqrt{2}} \\ \frac{45}{8\sqrt{2}} & \frac{45}{8\sqrt{2}} & -\frac{45}{8\sqrt{2}} & \frac{45}{8\sqrt{2}} \\ -\frac{45}{8\sqrt{2}} & -\frac{45}{8\sqrt{2}} & \frac{45}{8\sqrt{2}} & \frac{45}{8\sqrt{2}} \\ -\frac{45}{8\sqrt{2}} & -\frac{45}{8\sqrt{2}} & \frac{45}{8\sqrt{2}} & \frac{45}{8\sqrt{2}} \end{bmatrix} & \begin{array}{c} 3 \\ \\ 4 \\ \\ \end{array} \end{array}$$

Assemble $\underset{\sim}{K}\underset{\sim}{d} = \underset{\sim}{f} + \underset{\sim}{r}$

$$\begin{bmatrix} \underset{\sim}{K}_{11} & \underset{\sim}{K}_{12} & \underset{\sim}{K}_{13} & \underset{\sim}{K}_{14} \\ & \underset{\sim}{K}_{22} & \underset{\sim}{K}_{23} & \underset{\sim}{K}_{24} \\ & & \underset{\sim}{K}_{33} & \underset{\sim}{K}_{34} \\ & & & \underset{\sim}{K}_{44} \end{bmatrix} = \begin{bmatrix} K_{11}^{(1)} + K_{11}^{(2)} & K_{12}^{(1)} & K_{31}^{(2)} & 0 \\ & K_{22}^{(1)} + K_{22}^{(3)} & K_{32}^{(3)} & 0 \\ & & K_{33}^{(1)} + K_{33}^{(3)} + K_{33}^{(4)} & K_{34}^{(4)} \\ & & & K_{44}^{(4)} \end{bmatrix}$$

$$\underset{8\times 8}{\underset{\sim}{K}} = \begin{array}{c} \begin{array}{cccccccc} & & 3 & & 5 & 6 & & \end{array} \\ \left[\begin{array}{cccccccc} 7.5 & 0 & -7.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7.5 & 0 & 0 & 0 & -15/2 & 0 & 0 \\ -7.5 & 0 & 10.152 & -2.652 & -2.652 & 2.652 & 0 & 0 \\ 0 & 0 & -2.652 & 2.652 & 2.652 & -2.652 & 0 & 0 \\ 0 & 0 & -2.652 & 2.652 & 6.629 & 1.325 & -3.977 & -3.977 \\ 0 & -15/2 & 2.652 & -2.652 & 1.325 & 14.129 & -3.977 & -3.977 \\ 0 & 0 & 0 & 0 & -3.977 & -3.977 & 3.977 & 3.977 \\ 0 & 0 & 0 & 0 & -3.977 & -3.977 & 3.977 & 3.977 \end{array}\right] \end{array} \begin{array}{c} \\ \\ \\ 3 \\ \\ 5 \\ 6 \\ \\ \\ \end{array}$$

$$\underset{\sim}{f} + \underset{\sim}{r} = \begin{bmatrix} f_{1x}+r_{1x} \\ f_{1y}+r_{1y} \\ f_{2x}+r_{2x} \\ f_{2y}+r_{2y} \\ f_{3x}+r_{3x} \\ f_{3y}+r_{3y} \\ f_{4x}+r_{4x} \\ f_{4y}+r_{4y} \end{bmatrix} = \begin{bmatrix} r_{1x} \\ r_{1y} \\ 0 \\ r_{2y} \\ -30 \\ -10 \\ r_{4x} \\ r_{4y} \end{bmatrix}, \quad \underset{\sim}{d} = \begin{bmatrix} 0 \\ 0 \\ d_{2x} \\ 0 \\ d_{3x} \\ d_{3y} \\ 0 \\ 35 \end{bmatrix}$$

Correct

Date

Partitioned, we have:

$$\underset{\sim}{f}_{+c} = \begin{bmatrix} r_{1x} \\ r_{1y} \\ r_{2y} \\ r_{4x} \\ r_{4y} \\ 0 \\ -30 \\ -10 \end{bmatrix}, \quad \underset{\sim}{d} = \begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -25 \\ d_{2x} \\ d_{3x} \\ d_{3y} \end{bmatrix} \end{array} \begin{array}{l} \left.\begin{matrix} \\ \\ \\ \\ \\ \end{matrix}\right] d_E \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right] d_F \end{array},$$

$$K_{EF}' = \begin{bmatrix} -7.5 & 0 & -2.652 & 0 & 0 \\ 0 & 0 & 2.652 & 3.977 & 3.977 \\ 0 & -7.5 & -2.652 & 3.977 & 3.977 \end{bmatrix}$$

$$K_{FF} = \begin{bmatrix} 10.152 & -2.652 & 2.652 \\ -2.652 & 6.629 & 1.325 \\ 2.652 & 1.325 & 14.129 \end{bmatrix}.$$

From MATLAB, $K_{FF}^{-1} = \begin{bmatrix} 0.1196 & 0.0533 & -0.0275 \\ 0.0533 & 0.1775 & -0.0267 \\ -0.0275 & -0.0267 & 0.0784 \end{bmatrix}$

We know $K_{EF}' d_E + K_{FF} d_F = f_F$.

$$d_F = K_{FF}^{-1}(f_F - K_{EF}' d_E)$$

Page

Date

$$\therefore \underset{\sim}{d_F} = \begin{bmatrix} 0.1106 & 0.0533 & -0.0275 \\ 0.0533 & 0.1775 & -0.0267 \\ -0.0275 & -0.0267 & 0.0784 \end{bmatrix} \left(-\begin{bmatrix} 0 \\ 3.977 \\ 3.977 \end{bmatrix} 25 + \begin{bmatrix} 0 \\ -30 \\ -10 \end{bmatrix} \right)$$

$$\underset{\sim}{d_F} = \begin{bmatrix} -3.8992 \\ -20.058 \\ -5.132 \end{bmatrix} \text{mm}$$

$$\therefore d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 25 \\ -3.8992 \\ -20.0582 \\ -5.1318 \end{bmatrix}, \quad f = \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{2y} \\ f_{4x} \\ f_{4y} \\ 0 \\ -30 \\ -10 \end{bmatrix}$$

reactions $= Kd$

$$K_{partitioned} = \begin{bmatrix} 7.5 & 0 & 0 & 0 & 0 & -7.5 & 0 & 0 \\ 0 & 7.5 & 0 & 0 & 0 & 0 & 0 & -7.5 \\ 0 & 0 & 2.652 & 0 & 0 & -2.652 & 2.652 & -2.652 \\ 0 & 0 & 0 & 3.977 & 3.977 & 0 & -3.977 & -3.977 \\ 0 & 0 & 0 & 3.977 & 3.977 & 0 & -3.977 & -3.977 \\ -7.5 & 0 & -2.652 & 0 & 0 & 10.152 & -2.652 & 2.652 \\ 0 & 0 & 2.652 & -3.977 & -3.977 & -2.652 & 6.629 & 1.325 \\ 0 & -7.5 & -2.652 & -3.977 & -3.977 & 2.652 & 1.325 & 14.129 \end{bmatrix}$$

1 Date

$$r_{1x} = [7.5 \quad 0\ 0\ 0\ 0 \quad -7.5\ 0\ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 25 \\ -3.8942 \\ -20.0582 \\ -5.1318 \end{bmatrix} = 29.2442 \text{ kN } [\rightarrow]$$

Similarly, $r_{1y} = [0 \quad 7.5 \quad 0\ 0\ 0 \quad 0\ 0 \quad -7.5] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 25 \\ -3.8942 \\ -20.0582 \\ -5.1318 \end{bmatrix} = 38.4884 \text{ kN } [\uparrow]$

Similarly, $r_{2y} = -29.2442$ kN

$r_{4x} = +0.7558$ kN

$r_{4y} = +0.7558$ kN

Page

Date

$\varepsilon^{(1)} = B^e d_e$

$$B^{1} = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \cos 0 & \sin 0 & 0 & 0 \\ 0 & 0 & \cos 0 & \sin 0 \end{bmatrix} / 6000\,\text{mm}$$

$$d_e = \begin{bmatrix} 0 - 0 \\ 0 - 0 \\ -3.8992 - 0 \\ 0 - 0 \end{bmatrix}$$

$$\therefore \varepsilon^{(1)} = \begin{bmatrix} \frac{-1}{6000} & 0 & 1/6000 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -3.8992 \\ 0 \end{bmatrix}$$

$= -0.00065$

$\sigma^{(1)} = E\varepsilon^{(1)}$

$= 100\,000(-0.00065)$

$= 65$ MPa (Compression)

$\therefore \varepsilon^{(2)} = B^e d_e$

$$B^e d_e = \left\{ \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & \sin 90 & 0 & 0 \\ 0 & 0 & \cos 90 & \sin 90 \end{bmatrix} / 6000 \right\} \begin{bmatrix} 0 - 0 \\ 0 - 0 \\ -20.058 - 0 \\ -5.132 - 0 \end{bmatrix}$$

$$\varepsilon^{(2)} = \begin{bmatrix} 0 & -1/6000 & 0 & 1/6000 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -20.058 - 0 \\ -5.132 \end{bmatrix}$$

$= -0.000855$

$\sigma^{(2)} = E\varepsilon^{(2)}$

$= 85.53$ MPa (Compression)

Date

$\varepsilon^{③}$

$$B^e d^e = \begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}\cos 135 & \sin 135 & 0 & 0 \\ 0 & 0 & \cos 135 & \sin 135\end{bmatrix} \Big/ 6000\,mm \begin{bmatrix}d_{2x} \\ d_{2y} \\ d_{3x} \\ d_{3y}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}-3.8442 \\ 0 \\ -20.058 \\ -5.132\end{bmatrix} \Big/ \sqrt{6000^2+6000^2}$$

$\varepsilon^{③} =$ 0.000919

$\sigma^1 =$ 91.07 MPa [Tension]

$\varepsilon^{④}$, $B^e d^e = \begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}\cos 45 & \sin 45 & 0 & 0 \\ 0 & 0 & \cos 45 & \sin 45\end{bmatrix} \Big/ \sqrt{2(4000^2)} \begin{bmatrix}d_{3x} \\ d_{3y} \\ d_{4x} \\ d_{4y}\end{bmatrix}$

$$= \begin{bmatrix}\frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}-20.0582 \\ -5.132 \\ 0 \\ -25\end{bmatrix} \Big/ \sqrt{2(4000^2)}$$

= 0.00002378

$\sigma^{④} =$ 2.3775 MPa [Tension]

Stiffness Matrix

```
% Linear System Solver
% Solves K d = f+r
% in which
% K is the global stiffness matrix (n x n)
% f is the global nodal external force column matrix (n x 1)
% d is nodal degrees of freedom, (displacements) (n x 1)
% r is nodal residual/reaction column matrix (n x 1)
clc,clear

% inputs
nodes = [0,0;6000,0;0,6000;4000,10000]; % Node location (mm)

% connectivity array - conn(e,i) returns the global node
% number associate  with the ith node of element e.
conn = [1,2;1,3;2,3;3,4]; % Map of connected nodes for elements
fixeddofs = [1,2,4,7,8]; % "E"ssential Degrees of freedoms (dofs)
freedofs = [3,5,6]; % "F"ree Degrees of freedoms (dofs)
fixedvalues = [0,0,0,0,-25]'; %Displacement at fixed point (mm)

% Element properties
nsd = 2; % Number of space dimensions
nn = size(nodes,1); % Number of nodal points
ne = size(conn,1); % Number of elements
nne = 2; % Number of nodes per element
neq = nsd*nn; % Number of equations
A = 450; % Area (mm^2)
E=100; % Young's Modulus (kN/mm^2)
Element_vector=transpose(1:ne);
% Applied Forces
F = [0;0;0;0;-30;-10;0;0]; % Forces applied (kN)

% To determine stiffness matrix
K=zeros(neq,neq);
for e = 1:ne
 n1 = conn(e,1); % ID of node 1 of element e
 n2 = conn(e,2); % ID of node 2 of element e
 % build scatter vector for assembly
 sctr = [2*n1-1,2*n1,2*n2-1,2*n2]; % scatter vector 2*n1-1,2*n1, 2*n2-1, 2*n2
 % get the element's nodal coordiates
 % coord = nodes([n1,n2],:) or...
 x1 = nodes(n1,1); x2 = nodes(n2,1);
 y1 = nodes(n1,2); y2 = nodes(n2,2);
 deltax=x2-x1;
 deltay=y2-y1;
 len = sqrt(deltax^2+deltay^2);
 s=deltay/len;
 c=deltax/len;
 k = A*E/len; % constant coefficient for each truss element
```

```
 Ke = k*[c^2,c*s,-c^2,-c*s;c*s,s^2,-c*s,-s^2;
         -c^2,-c*s,c^2,c*s;-c*s,-s^2,c*s,s^2]; % 1D stiffness matrix
 K(sctr,sctr) = K(sctr,sctr) + Ke; % K Matrix
end
K
```

```
K = 8×8
    7.5000         0   -7.5000         0         0         0         0         0
         0    7.5000         0         0         0   -7.5000         0         0
   -7.5000         0   10.1517   -2.6517   -2.6517    2.6517         0         0
         0         0   -2.6517    2.6517    2.6517   -2.6517         0         0
         0         0   -2.6517    2.6517    6.6291    1.3258   -3.9775   -3.9775
         0   -7.5000    2.6517   -2.6517    1.3258   14.1291   -3.9775   -3.9775
         0         0         0         0   -3.9775   -3.9775    3.9775    3.9775
         0         0         0         0   -3.9775   -3.9775    3.9775    3.9775
```

Displacement and Reaction Determination

```
% this part of the code remains unchanged!
n = length(F); % number of dofs
DOF_vector=1:n;
DOF_labels=strings(n,1);
for i=1:n
    if mod(DOF_vector(i),2)==1
    DOF_labels(i)=join([num2str(round(DOF_vector(i)/2,"TieBreaker","plusinf")),"x"],"");
    else
    DOF_labels(i)=join([num2str(DOF_vector(i)/2),"y"],"");
    end
end
d = zeros(n,1);
r = zeros(n,1);
dE = fixedvalues;
KEE = K(fixeddofs,fixeddofs);
KEF = K(fixeddofs,freedofs);
KFE = K(freedofs,fixeddofs);
KFF = K(freedofs,freedofs);
fE = F(fixeddofs);
fF = F(freedofs);
dF = inv(KFF) * (fF-KFE*dE); % contains unknown dofs/displacements
rE = KEF*dF+KEE*dE-fE; % contains unknown residuals/reactions
% Gather and Display Results
d(fixeddofs) = dE;
d(freedofs) = dF;
r(fixeddofs) = rE;
```

Strain and Stress Determination

```
% Strain and Stress
eps=zeros(ne,1); % A vector to be populated by the for loop to store strain
sigma=zeros(ne,1); % A vector to be populated by the for loop to store stress
for e = 1:ne
 n1 = conn(e,1); % ID of node 1 of element e
 n2 = conn(e,2); % ID of node 2 of element e
```

```
 % get the element's nodal coordiates
 % coord = nodes([n1,n2],:) or...
 x1 = nodes(n1,1); x2 = nodes(n2,1);
 y1 = nodes(n1,2); y2 = nodes(n2,2);
 deltax=x2-x1; % Change in m (mm)
 deltay=y2-y1; % Change in y (mm)
 len = sqrt(deltax^2+deltay^2); % Length of element (mm)
 s=deltay/len; % sin(phi)
 c=deltax/len; % cos(phi)
 T=[c,s,0,0;0,0,c,s]; % Transformation Matrix

% Get the element's nodal displacements from d
    ue1x = d(2*n1-1); ue1y = d(2*n1); % x & y disp of node 1 of element e
    ue2x = d(2*n2-1); ue2y = d(2*n2); % x & y disp of node 2 of element e
    de=[ue1x;ue1y;ue2x;ue2y];  % or de = d(sctr)
% compute B-matrix
    Be = [-1 , 1]*T/len; %B-matrix
    eps(e)= Be*de; %Strain
    sigma(e)=E*eps(e)*1000; % Stress
    %force(e)=sigma(e)*A
end
```

Nicely Formatted Values

```
DOF=transpose(1:n);
table(DOF_labels,d,r,'VariableNames',{'Node/DOF', 'Displacement (mm)','Reaction (kN)'})
```

ans = 8×3 table

	Node/DOF	Displacement (mm)	Reaction (kN)
1	"1x"	0	29.2431
2	"1y"	0	38.4861
3	"2x"	-3.8991	0
4	"2y"	0	-29.2431
5	"3x"	-20.0588	0
6	"3y"	-5.1315	0
7	"4x"	0	0.7569
8	"4y"	-25	0.7569

```
table(Element_vector,eps,sigma,'VariableNames',{'Element Number','Strain','Stress (MPa)'})
```

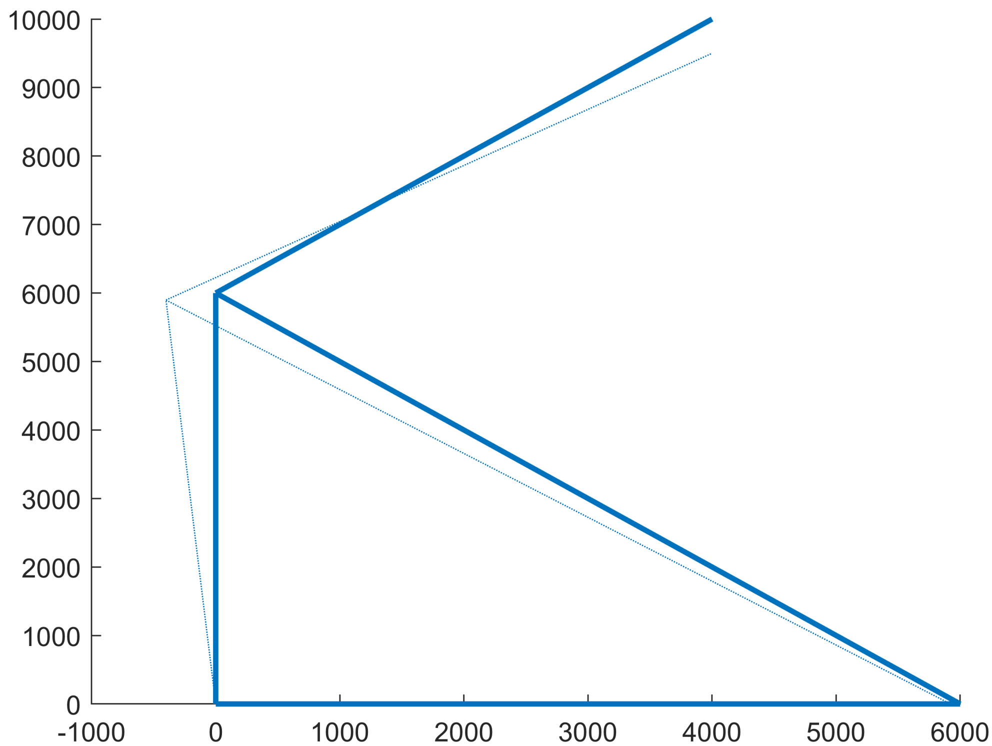
ans = 4×3 table

	Element Number	Strain	Stress (MPa)
1	1	-6.4985e-04	-64.9846
2	2	-8.5525e-04	-85.5248
3	3	9.1902e-04	91.9021

	Element Number	Strain	Stress (MPa)
4	4	2.3788e-05	2.3788

Deflected Shape

```
sf=20; % Scale Factor
for i = 1:ne
        XX = [nodes(conn(i,1),1), nodes(conn(i,2),1)]; % nodes starting x-values
        YY = [nodes(conn(i,1),2),nodes(conn(i,2),2)]; % nodes starting y-values
        line(XX,YY,'LineWidth',2);hold on;
        XX_def = [nodes(conn(i,1),1)+sf*d(2*conn(i,1)-1),
                  nodes(conn(i,2),1)+sf*d(2*conn(i,2)-1)]; % nodes ending x-values
        YY_def = [nodes(conn(i,1),2)+sf*d(2*conn(i,1)),
                  nodes(conn(i,2),2)+sf*d(2*conn(i,2))]; % nodes ending y-values
        line(XX_def,YY_def,'LineStyle',":");hold on;
end
```



Stiffness Matrix

```
% Linear System Solver
% Solves K d = f+r
% in which
% K is the global stiffness matrix (n x n)
% f is the global nodal external force column matrix (n x 1)
% d is nodal degrees of freedom, (displacements) (n x 1)
% r is nodal residual/reaction column matrix (n x 1)
clc,clear

% inputs
conn=[1,2;1,8;2,8;2,3;8,3;8,7;3,7;7,6;3,6;3,4;4,6;4,5;5,6]; % Map of connected nodes for elemen
% connectivity array - conn(e,i) returns the global node
% number associate  with the ith node of element e.
fixeddofs = [1,2,14]; % "E"ssential Degrees of freedoms (dofs)
freedofs = [3,4,5,6,7,8,9,10,11,12,13,15,16]; % "F"ree Degrees of freedoms (dofs)
fixedvalues = [0,0,-89]'; %Displacement at fixed point (mm)
nodes=[0,0;2,3;6,3;8,6;12,6;10,3;8,0;4,0]*1000;% Node location (mm)

% Element properties
nsd = 2; % Number of space dimensions
nn = size(nodes,1); % Number of nodal points
ne = size(conn,1); % Number of elements
nne = 2; % Number of nodes per element
neq = nsd*nn; % Number of equations
A = 600; % Area (mm^2)
E=210; % Young's Modulus (kN/mm^2)
Element_vector=transpose(1:ne);
% Applied Forces
F = [0;0;0;0;0;0;0;0;0;-5000;1000;0;0;0;0;-4000]; % Forces applied (kN)
% To determine stiffness matrix
K=zeros(neq,neq);
for e = 1:ne
 n1 = conn(e,1); % ID of node 1 of element e
 n2 = conn(e,2); % ID of node 2 of element e
 % build scatter vector for assembly
 sctr = [2*n1-1,2*n1,2*n2-1,2*n2]; % scatter vector 2*n1-1,2*n1, 2*n2-1, 2*n2
 % get the element's nodal coordiates
 % coord = nodes([n1,n2],:) or...
 x1 = nodes(n1,1); x2 = nodes(n2,1);
 y1 = nodes(n1,2); y2 = nodes(n2,2);
 deltax=x2-x1;
 deltay=y2-y1;
 len = sqrt(deltax^2+deltay^2);
 s=deltay/len;
 c=deltax/len;
 k = A*E/len; % constant coefficient for each truss element
 Ke = k*[c^2,c*s,-c^2,-c*s;c*s,s^2,-c*s,-s^2;
         -c^2,-c*s,c^2,c*s;-c*s,-s^2,c*s,s^2]; % 1D stiffness matrix
```

```
 K(sctr,sctr) = K(sctr,sctr) + Ke; % K Matrix
end
K
```

```
K = 16×16
   42.2526   16.1290  -10.7526  -16.1290         0         0         0         0···
   16.1290   24.1935  -16.1290  -24.1935         0         0         0         0
  -10.7526  -16.1290   53.0053         0  -31.5000         0         0         0
  -16.1290  -24.1935         0   48.3869         0         0         0         0
         0         0  -31.5000         0   95.2579   16.1290  -10.7526  -16.1290
         0         0         0         0   16.1290   72.5804  -16.1290  -24.1935
         0         0         0         0  -10.7526  -16.1290   53.0053         0
         0         0         0         0  -16.1290  -24.1935         0   48.3869
         0         0         0         0         0         0  -31.5000         0
         0         0         0         0         0         0         0         0
      ⋮
```

Displacement and Reaction Determination

```
% this part of the code remains unchanged!
n = length(F); % number of dofs
DOF_vector=1:n;
DOF_labels=strings(n,1);
for i=1:n
    if mod(DOF_vector(i),2)==1
    DOF_labels(i)=join([num2str(round(DOF_vector(i)/2,"TieBreaker","plusinf")),"x"],"");
    else
    DOF_labels(i)=join([num2str(DOF_vector(i)/2),"y"],"");
    end
end
d = zeros(n,1);
r = zeros(n,1);
dE = fixedvalues;
KEE = K(fixeddofs,fixeddofs);
KEF = K(fixeddofs,freedofs);
KFE = K(freedofs,fixeddofs)
```

```
KFE = 13×3
  -10.7526  -16.1290         0
  -16.1290  -24.1935         0
         0         0   16.1290
         0         0  -24.1935
         0         0         0
         0         0         0
         0         0         0
         0         0         0
         0         0  -16.1290
         0         0  -24.1935
      ⋮
```

```
KFF = K(freedofs,freedofs)
```

```
KFF = 13×13
   53.0053         0  -31.5000         0         0         0         0         0···
         0   48.3869         0         0         0         0         0         0
  -31.5000         0   95.2579   16.1290  -10.7526  -16.1290         0         0
```

```
         0         0   16.1290   72.5804  -16.1290  -24.1935         0         0
         0         0  -10.7526  -16.1290   53.0053         0  -31.5000         0
         0         0  -16.1290  -24.1935         0   48.3869         0         0
         0         0         0         0  -31.5000         0   42.2526   16.1290
         0         0         0         0         0         0   16.1290   24.1935
         0         0  -31.5000         0  -10.7526   16.1290  -10.7526  -16.1290
         0         0         0         0   16.1290  -24.1935  -16.1290  -24.1935
      ⋮
```

```
KFF_inv=inv(KFF)
```

```
KFF_inv = 13×13
    0.0510   -0.0185    0.0352   -0.0132    0.0269   -0.0078    0.0269    0.0032 ···
   -0.0185    0.0433   -0.0079    0.0192    0.0130    0.0052    0.0130   -0.0228
    0.0352   -0.0079    0.0510   -0.0026    0.0587   -0.0078    0.0587   -0.0180
   -0.0132    0.0192   -0.0026    0.0433    0.0391    0.0155    0.0391   -0.0402
    0.0269    0.0130    0.0587    0.0391    0.2189   -0.0367    0.2189   -0.1769
   -0.0078    0.0052   -0.0078    0.0155   -0.0367    0.0554   -0.0367    0.0451
    0.0269    0.0130    0.0587    0.0391    0.2189   -0.0367    0.2506   -0.1981
    0.0032   -0.0228   -0.0180   -0.0402   -0.1769    0.0451   -0.1981    0.2636
    0.0352   -0.0079    0.0510   -0.0026    0.0904   -0.0289    0.0904   -0.0391
   -0.0023   -0.0088   -0.0129   -0.0123   -0.0913    0.0399   -0.0913    0.1163
      ⋮
```

```
fE = F(fixeddofs);
fF = F(freedofs);
dF = KFF\(fF-KFE*dE); % contains unknown dofs/displacements
rE = KEF*dF+KEE*dE-fE; % contains unknown residuals/reactions
% Gather and Display Results
d(fixeddofs) = dE;
d(freedofs) = dF;
r(fixeddofs) = rE;
```

Strain and Stress Determination

```
% Strain and Stress
eps=zeros(ne,1); % A vector to be populated by the for loop to store strain
sigma=zeros(ne,1); % A vector to be populated by the for loop to store stress
for e = 1:ne
 n1 = conn(e,1); % ID of node 1 of element e
 n2 = conn(e,2); % ID of node 2 of element e
 % get the element's nodal coordiates
 % coord = nodes([n1,n2],:) or...
 x1 = nodes(n1,1); x2 = nodes(n2,1);
 y1 = nodes(n1,2); y2 = nodes(n2,2);
 deltax=x2-x1; % Change in m (mm)
 deltay=y2-y1; % Change in y (mm)
 len = sqrt(deltax^2+deltay^2); % Length of element (mm)
 s=deltay/len; % sin(phi)
 c=deltax/len; % cos(phi)
 T=[c,s,0,0;0,0,c,s]; % Transformation Matrix

% Get the element's nodal displacements from d
```

```
    ue1x = d(2*n1-1); ue1y = d(2*n1); % x & y disp of node 1 of element e
    ue2x = d(2*n2-1); ue2y = d(2*n2); % x & y disp of node 2 of element e
    de=[ue1x;ue1y;ue2x;ue2y];  % or de = d(sctr)
% compute B-matrix
    Be = [-1 , 1]*T/len; %B-matrix
    eps(e)= Be*de; %Strain
    sigma(e)=E*eps(e)*1000; % Stress
    %force(e)=sigma(e)*A
end
```

Nicely Formatted Values

```
DOF=transpose(1:n);
table(DOF_labels,d,r,'VariableNames',{'Node/DOF', 'Displacement (mm)','Reaction (kN)'})
```

ans = 16×3 table

	Node/DOF	Displacement (mm)	Reaction (kN)
1	"1x"	0	-1.0000e+03
2	"1y"	0	-875.0000
3	"2x"	137.1612	0
4	"2y"	-55.2740	0
5	"3x"	174.1982	0
6	"3y"	-7.5428	0
7	"4x"	895.0768	0
8	"4y"	-384.7948	0
9	"5x"	1.0009e+03	0
10	"5y"	-1.3364e+03	0
11	"6x"	311.7644	0
12	"6y"	-670.3361	0
13	"7x"	-95.2381	0
14	"7y"	-89	9.8750e+03
15	"8x"	13.2275	0
16	"8y"	-101.7297	0

```
table(Element_vector,eps,sigma,'VariableNames',{'Element Number','Strain','Stress (MPa)'})
```

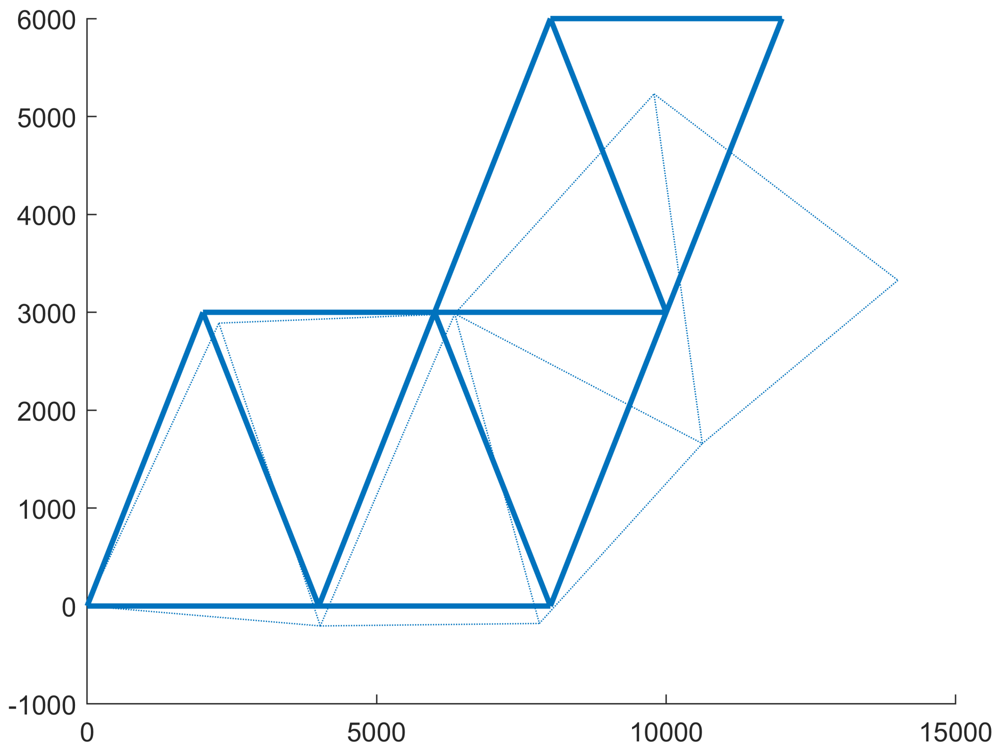
ans = 13×3 table

	Element Number	Strain	Stress (MPa)
1	1	0.0083	1.7527e+03
2	2	0.0033	694.4444
3	3	-0.0083	-1.7527e+03
4	4	0.0093	1.9444e+03

	Element Number	Strain	Stress (MPa)
5	5	0.0465	9.7650e+03
6	6	-0.0271	-5.6944e+03
7	7	-0.0227	-4.7573e+03
8	8	-0.0715	-1.5023e+04
9	9	0.0344	7.2222e+03
10	10	0.0238	5.0077e+03
11	11	-0.0238	-5.0077e+03
12	12	0.0265	5.5556e+03
13	13	-0.0477	-1.0015e+04

Deflected Shape

```
sf=2; % Scale Factor
for i = 1:ne
        XX = [nodes(conn(i,1),1), nodes(conn(i,2),1)]; % nodes starting x-values
        YY = [nodes(conn(i,1),2),nodes(conn(i,2),2)]; % nodes starting y-values
        line(XX,YY,'LineWidth',2);; hold on
        XX_def = [nodes(conn(i,1),1)+sf*d(2*conn(i,1)-1),
                  nodes(conn(i,2),1)+sf*d(2*conn(i,2)-1)]; % nodes ending x-values
        YY_def = [nodes(conn(i,1),2)+sf*d(2*conn(i,1)),
                  nodes(conn(i,2),2)+sf*d(2*conn(i,2))]; % nodes ending y-values
        line(XX_def,YY_def,'LineStyle',":");; hold on
end
```


6000
5000
4000
3000
2000
1000
0
-1000
0
5000
10000
15000