The EA Ising Model via Parallel Tempering and D-Wave

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The Edwards Anderson Ising Model

Given a graph $G = (V, E)$ with variables $x_i \in \{\pm 1\}, i \in V$ and weights $J_{i,j}, (i,j) \in E$, define the hamiltonian

$$H(x) = \sum_{(i,j) \in E} J_{i,j} x_i x_j$$

Finding the global minimum is an NP-complete optimization problem [1].

Parallel Tempering: Idea

Parallel Tempering: Implementation

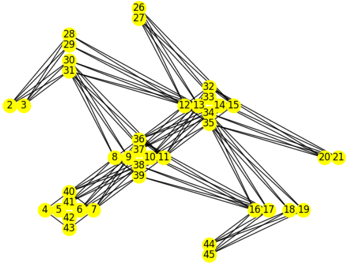
Following basic implementation suggested in [2]

- Start with replicas $x^{(1)}, \ldots, x^{(n)}$ at close, evenly spaced temperatures $T_1 < \ldots < T_n$.
- In parallel, run MCMC with single spin updates for each for n steps.
- Choose i at random, swap replicas $x^{(i)}, x^{(i+1)}$ with probability $\min\{1, e^{(1/T_{i+1} - 1/T_i)(H(x^{(i+1)}) - H(x^{(i)}))}\}$ to satisfy detailed balance for the joint probability distribution

$$P(x^{(1)}, T_1, \ldots, x^{(n)}, T_n) = \prod_{1 \leq i \leq n} P(x^{(i)}, T_i) \propto e^{-\sum_i H(x^{(i)})/T_i}$$

D-Wave Cloud Quantumn Annealer

Consists of a specific sparse graph, Pegasus graph [3]. Vertices are sites that can spin up or down, edge weights can be specified by the user.



D-Wave can minimize arbitrary hamiltonians on subgraphs of Pegasus. To minimize on an arbitrary graph G, first need a minor embedding of G.

Minor Embedding: Idea

Minor Embedding: D-Wave

A *minor embedding* of G_1 in G_2 is a map $\phi : V(G_1) \to 2^{V(G_2)}$ so that

- For each $v \in V(G_1)$, $\phi(v)$ is connected.
- For $v, w \in V(G_1)$, $\phi(v)$ and $\phi(w)$ are disjoint.
- If (v, w) is an edge in G_1, there is at least one edge between $\phi(v)$ and $\phi(w)$.

Minor embedding G_1 into G_2 is NP-hard, but when G is sparse, smaller than Pegasus there are fast heuristic algorithms [4].
When G is not sparse, use a pre-computed minor embedding of complete graph, forget unneeded edges.

Experimental Setup

Graphs considered:

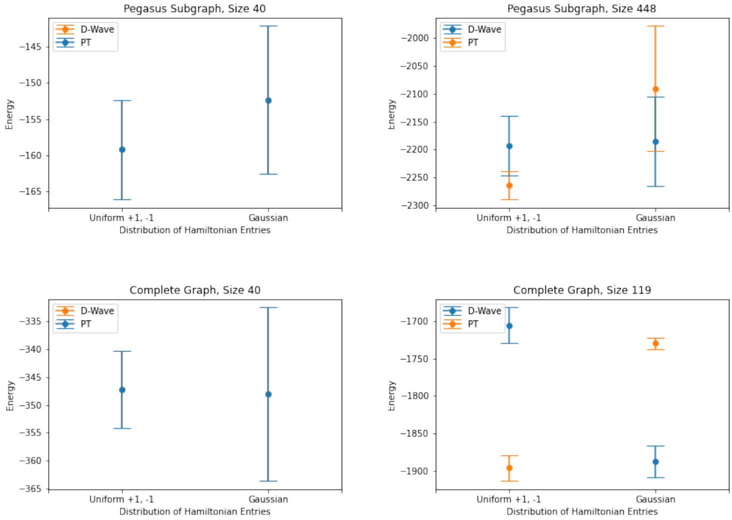
- Subgraphs of Pegasus, of size 40 and 448
- Complete graphs of size 40 and 119 (largest possible for D-Wave)

Ensembles considered

- $J_{ij} \sim \pm 1$ uniformly
- $J_{ij} \sim \mathcal{N}(0, 1)$.

Generated 5 hamiltonians for each graph and distribution of J_{ij} let parallel tempering and D-Wave attempt to solve for the same amount of time.

Results



Pegasus Subgraph, Size 40
D-Wave
PT
Energy
−145
−150
−155
−160
−165
Uniform +1, -1
Gaussian
Distribution of Hamiltonian Entries
Pegasus Subgraph, Size 448
D-Wave
PT
Energy
−2000
−2050
−2100
−2150
−2200
−2250
−2300
Uniform +1, -1
Gaussian
Distribution of Hamiltonian Entries
Complete Graph, Size 40
D-Wave
PT
Energy
−335
−340
−345
−350
−355
−360
−365
Uniform +1, -1
Gaussian
Distribution of Hamiltonian Entries
Complete Graph, Size 119
D-Wave
PT
Energy
−1700
−1750
−1800
−1850
−1900
Uniform +1, -1
Gaussian
Distribution of Hamiltonian Entries

Results

	Peg, Uniform	Peg, Gaussian	Cliq, Uniform	Cliq, Gaussian
Discrepancy	.446	.386	.074	.106

[1] F. Barahona, “On the computational complexity of Ising spin glass models,” *Journal of Physics A Mathematical General*, vol. 15, pp. 3241–3253, Oct. 1982.

[2] D. J. Earl and M. W. Deem, “Parallel tempering: Theory, applications, and new perspectives,” *Phys. Chem. Chem. Phys.*, vol. 7, pp. 3910–3916, 2005.

[3] N. Dattani, S. Szalay, and N. Chancellor, “Pegasus: The second connectivity graph for large-scale quantum annealing hardware,” 2019.

[4] J. Cai, W. G. Macready, and A. Roy, “A practical heuristic for finding graph minors,” 2014.