Electric drive systems

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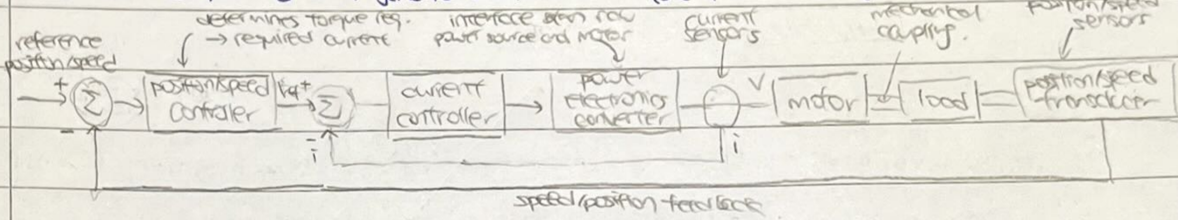
Electric drive systems.

- In general, a drive system is any system which produces controlled motion.
- An electric drive system is a drive system which uses electric power as the source of energy, and an electrical machine as the means of converting that energy into controlled motion.
- The main components of a generic electric drive system is as follows ← not all electric drive systems have all components



The power electronics converter does power conditioning on the i/p power supply, controlled by controllers to drive a motor coupled to mechanical loads. Sensors are used as controller i/ps (as feedback)

Sensors

- There are many types of sensors, - current sensor, position/speed transducer, limit switch, temp. sensor.

① Current sensors

- Current is related to torque → sensing current is equivalent to sensing motor torque.
- Usually, the current sensor is a small temp-stable sense resistor connected in series w/ the armature winding on the GND side of the armature voltage supply
- The pd across the sense resistor is prop. to the current through it, and this voltage is fed to an ADC i/p, so the microcontroller can get the motor torque.

② Position/speed transducers.

- Common position/speed transducers are the resolver and the encoder.
- A resolver is similar to a simple AC generator w/ 2 orthogonal coils. Voltages are induced in these coils.

$$V_1 = \hat{V}\cos\omega t \qquad V_2 = \hat{V}\sin\omega t$$

These o/p voltages are fed to an ADC i/p, where the microcontroller can compute $\hat{V}$ and ω (θ)

$$\theta = \omega t = \arctan\left(\frac{V_2}{V_1}\right) \qquad \hat{V} = \sqrt{V_1^2 + V_2^2}$$

The microprocessor may use a LUT for the trig functions to compute θ (interpolate for intermediate θ)

- Encoders are slotted or lined discs which use optoelectronic devices to detect these slots.

For a simple 1024-line device (10 bits), the microcontroller simply counts the lines to determine the position.

Mechanical load

- The main types of mechanical load include:
  - ↳ Pure inertial load: $T_L = J\frac{d\omega_r}{dt}$
  - ↳ Fan load: $T_L \propto \omega_r^2$
  - ↳ Friction: $T_L = T_F$
  - ↳ Const. power: $T_L \propto \frac{1}{\omega_r}$
  - ↳ Viscous damping: $T_L = K_v \omega_r$
- Many real mechanical loads consist a combination of these basis types.

Controller

- The control of electrical drives is mainly performed by dedicated microcontrollers (PICs, DSP chips etc.) They can perform complex mathematical transformations on sensor information.
- The controller function $K(s)$ often implements the simple PID controller in software.

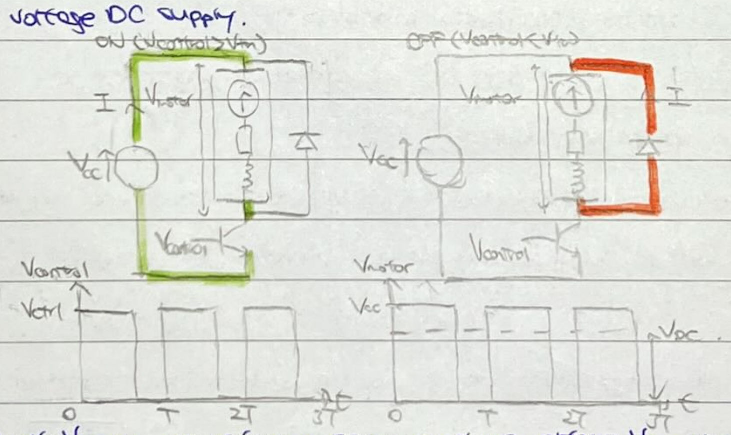
$$K(s) = K_P + {K_i}/{s} + K_d s$$

K_P reduces the observed error; K_i reduces the steady-state error; K_d introduces damping.

- The optimal set of PID parameters can be found by systematic trial and error w/ the Ziegler-Nichols method.

Power electronic converter

- The power electronic converter takes control signals and converts these to larger, more powerful voltage supply req. by the DC motor
- An example is the DC chopper circuit, which gives a variable DC voltage from a typical fixed voltage DC supply.



w/ a PWM signal at $V_{control}$, we get an average armature voltage V_{DC} given by

$$V_{DC} = \frac{1}{T}(pT \cdot V_{cc} + (1-p)T \cdot 0) = pV_{cc}$$

where T is the switching period and $p = \frac{T_{ON}}{T} = \frac{T_{ON}}{T_{ON}+T_{OFF}}$ is the duty cycle

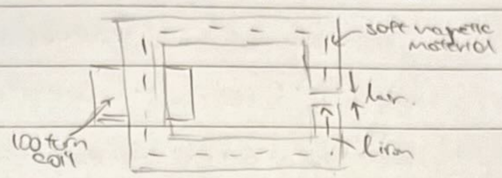
→ the average armature voltage V_{DC} can be varied btwn 0 and V_{cc} by varying p btwn 0 and 1.

Magnetic circuits and PM materials

Soft magnetic materials

Magnetic circuits

- consider a soft magnetic core w/ small airgap wrapped by a coil



A/L : $\oint H \cdot dl = NI$ → $H_{air} l_{air} + H_{iron} l_{iron} = NI$

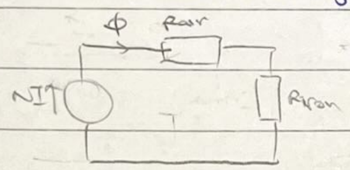
Consv. of flux: $\Phi = \iint_S B \cdot dS$ → $\Phi = BA$

Assuming $A_{air} = A_{iron} = A$, $B_{air} A_{air} = B_{iron} A_{iron}$ → $B_{air} = B_{iron} = B$.

Since $B = \mu_0 \mu_r H$, $H_{air} = \frac{B}{\mu_0}$ $H_{iron} = \frac{B}{\mu_0 \mu_r}$

$$\therefore \quad NI = \left(\frac{l_{iron}}{\mu_0 \mu_r A}\right)\Phi + \left(\frac{l_{air}}{\mu_0 A}\right)\Phi = R_{iron}\Phi + R_{air}\Phi \qquad \boxed{R = \frac{l}{\mu_0 \mu_r A}}$$

We can represent the above as a magnetic circuit



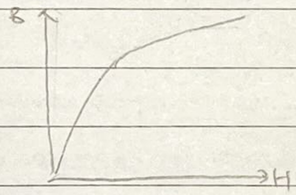
mmf NI ↔ voltage V

flux Φ → current I

reluctance $R = \frac{l}{\mu A}$ ↔ resistance $R = \rho \frac{l}{A}$.

~~Effect~~ of saturation.

- The B-H curve for a typical soft magnetic material (not PM) is as follows:

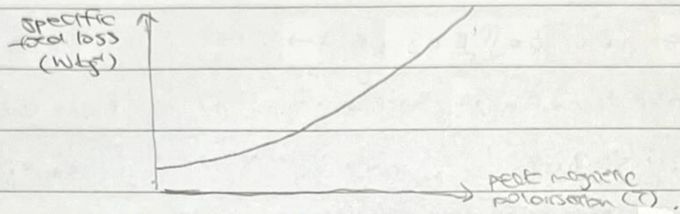


- The curve passes through the origin → absence of applied field H, there is zero flux density B → soft
- Under an applied field H, the material reinforces the effect of the applied field by aligning its own magnetic moments w/ those of the applied field. → achieve higher flux density B
- Beyond a certain applied field, the resulting flux density increases at a reduced rate (saturation) once the magnetic moments that are fully aligned to the applied field cannot contribute further to the flux density produced.

→ Determine the permeability $\mu = \frac{B}{H}$ using the B-H characteristic.
done even for PM materials

Iron losses.

- Iron losses are power losses that occur in the laminations of electrical machines due to (i) eddy currents and (ii) hysteresis.
  * rotor rotates in synchronism w/ stator driven field → not experience alternating B field → no iron loss in rotor
  - ↳ Eddy current loss: Alternating B field → alternating E field → currents → power dissipation as heat.
  - ↳ Hysteresis loss: Work done to reverse the magnetic domains.
- We can quantify iron losses by referring to the specific total loss characteristic of the lamination material. (specific total loss = total iron loss per unit mass).



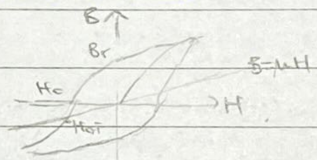
* The characteristic is commonly given for $f=50$ Hz.
- We assume iron losses scale w/ freq. squared, so $\boxed{loss' = \left(\frac{f'}{f}\right)^2 loss}$

loss per unit mass
→ usually find mass w/ $m=\rho V$!

Permanent magnetic (PM) materials

Characteristics and properties of PM materials

- The BH curve for a typical PM material is as follows:



* In the absence of an applied field H, the flux density B is reduced to the remanent flux density Br. To reduce the flux density B to zero, the opposite field req. is the coercive field intensity Hc.
- PMs operate in the second quadrant, and it is important that the operating pt. is not beyond the inflection pt/"knee" of the BH curve as it leads to irreversible loss of magnetisation.
  ← this is the intrinsic coercivity Hci
  (For PM w/ the "knee" in the third quadrant, demagnetisation only occurs when we have excessive field H).
- The max energy product BH_{max} is a figure-of-merit for PM materials — the maximal value of −BH in the second quadrant of the B-H curve. (If the "knee" of the curve is in QII, $BH_{max} = \frac{1}{4}B_rH_c$)
  amount of magnetic energy delivered to ext. circuit
  $B=B_r/2,\ H=H_c/2$
- The greater BH_{max}, the greater the ability to deliver magnetic energy to the ext. magnetic circuit.

Types and choice of magnets.

- The four main types of PM used are NdFeB, SmCo, Alconax and ferrite.
  easily demagnetised as "knee" in QII → very short airgap.
- When choosing the PM material, other than Br, Hc and BH_{max}, these factors should be considered:
  - ↳ Cost of material
  - ↳ Ability to machine or mold it
  - ↳ Magnetisation at high temp. (Curie temp.)
  - ↳ Ability to magnetise in diff. ways (magnetisation in situ)

Machine design

Machine design

Design philosophy and design variables

- The customer wants low cost, high performance, low noise, high reliability and high efficiency.

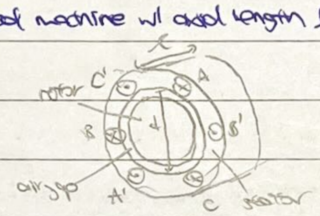
The manufacture wants low part count, ease of manufacture and min. labour

- The designer must perform the balancing act btwn these conflicting constraints

- Common design variables to consider are:
  - ↳ size of machine — frame size, core length
  - ↳ laminations — no. and shape of slots, thickness
  - ↳ windings — no. of turns, winding diameter
  - ↳ thermal — cooling, max. temp-rise
  - ↳ Mechanical — bearings, stress distribution
  - ↳ operating environment

Concentrated and distributed windings.

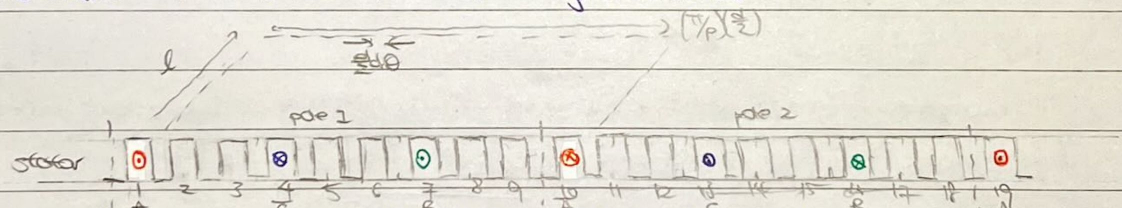
- Consider a three phase electrical machine w/ axial length l and air gap diameter d.



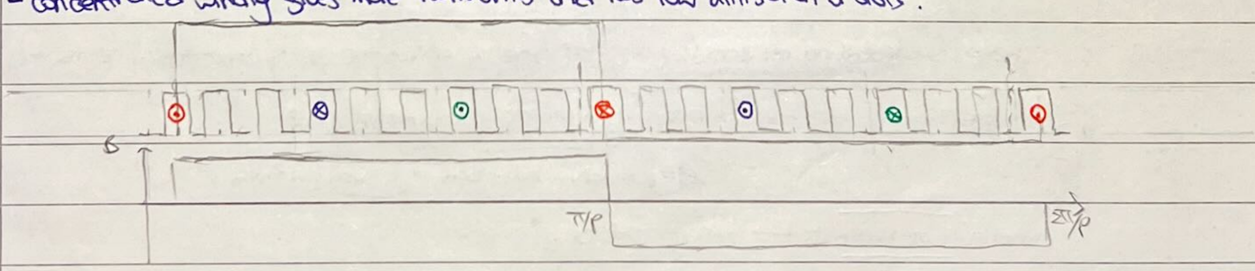
- The resultant magnetic field due to three phase stator currents in the air gap has fundamental component

$$B_g(\theta, t) = \sqrt{2} B_{rms} \cos(\omega t - p\theta)$$

- The distribution of coils for concentrated windings is as follows.



- Concentrated winding gives more harmonics and has low utilisation of slots.



- Considering only the fundamental of quantities (B, E),

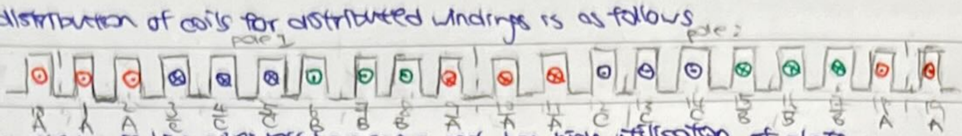
Total flux: $$\phi = \iint_S \underline{B} \cdot d\underline{A} = \int_0^{\pi/p} \sqrt{2} B_{rms} \cos(\omega t - p\theta)\, l \frac{d}{2} d\theta = \frac{\sqrt{2}}{p} B_{rms}\, l d \sin(\omega t)$$

Faraday's law: $$E_{ph} = -N_{ph} \frac{d\phi}{dt} = -\frac{l\omega d}{p} \sqrt{2} N_{ph} B_{rms} \cos(\omega t) = -\sqrt{2} E_{rms} \cos(\omega t)$$

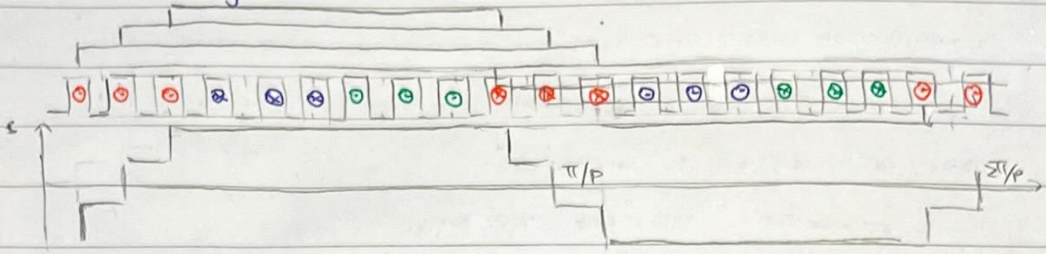
$$\therefore E_{rms} = \frac{l\omega d}{p} N_{ph} B_{rms} = K N_{ph} B_{rms}$$

where N_{ph} is the no. of turns per phase and $\boxed{K = \frac{l\omega d}{p}}$

- The distribution of coils for distributed windings is as follows



- Distributed winding gives less harmonics and has high utilisation of slots.



- However, we expect a lower magnitude of the magnetic field B and the induced emf E.

  This reduction is quantified by the distribution factor k_d, given by

$$\boxed{k_d = \frac{\sin(m\frac{\beta p}{2})}{m\sin(\frac{\beta p}{2})}} \leq 1$$

where $\boxed{m = \frac{\text{total \# of slots}}{\text{\# of phases} \times \text{\# of poles}} = \frac{\text{total \# of slots}}{\text{\# of phases} \times 2p}}$ is the phase band (# of slots used per pole per phase)

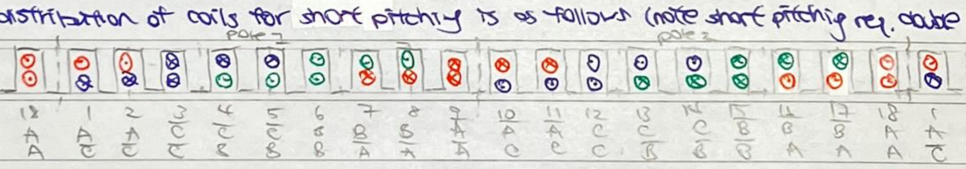
and $\boxed{\beta = \frac{360^\circ}{\text{total \# of slots}}}$ is the physical angle btwn two slots

- considering only the fundamental component of quantities (B, E),

$$E_{rms} = k N_{ph} k_d B_{rms}$$

short pitching.

- short pitching is a winding method that improves on the simple distributed winding, reducing the harmonic content of the air gap field further and allows the length of the phase coils to be shortened.
- short pitching req. more complicated winding, but is widely used for powers above 1kW.
- The distribution of coils for short pitching is as follows (note short pitching req. double layer winding).



- short pitching reduces the flux cut by the coil and therefore the flux voltage is reduced by a factor, the pitch factor k_p, given by

$$\boxed{k_p = \cos\left(\frac{\alpha p}{2}\right)} \leq 1$$

where $\alpha = \text{\# slots short pitched} \times \beta$, is the angle by which the coil is short pitched rel. to full pole pitch.

- considering only the fundamental component of quantities (B, E),

$$E_{rms} = k N_{ph} k_d k_p B_{rms} = k N_{ph} k_w B_{rms}$$

where $\boxed{k_w = k_d k_p}$ is the winding factor.

* Note that $N_{ph} k_w$ is also referred to as the effective turns per phase N_{eff}, i.e. $\boxed{N_{eff} = N_{ph} k_w}$

- If the stator coil (winding) resistance and leakage inductance are small s.t. the voltage drop across them is small compared to supply voltage, then $V_{ph} = E_{rms}$, i.e.

$$\boxed{V_{ph} = k N_{ph} k_w B_{rms} = k N_{eff} B_{rms}}$$

Number of turns per coil N_c

- The no. of turns per coil N_c is given by

$$\boxed{N_c = \frac{N_{ph}}{m \times n \times p}}$$

where n is the no. of layers of winding.

* Note N_c must be an integer → we round up to nearest integer and find the updated N_{ph}.

* N_{ph} is useful for EM induction calculations, N_c is useful for physical assembly.

Machine rating and specific loading.

- The VA rating of a three-phase AC device is well chosen as it indicates the power it may deliver for an indefinite amount of time.

$$S = 3 V_{ph} I_{ph}$$

* VA rating is apparent power — not all VA is for real power as the machine req. reactive power for magnetising currents

- The specific magnetic loading $\bar{B}$ is defined as the average flux density B ⊥ to one pole pitch of the cylindrical surface of circumference of the airgap, and is given by

$$\boxed{\bar{B} = \frac{\Phi_p}{S} = \frac{2\sqrt{2}}{\pi} B_{rms}}$$

- The magnitude of $\bar{B}$ must be limited o/w magnetic saturation will cause excessively large magnetising currents and increased iron losses

- The specific electrical loading $\bar{J}$ is defined as the axial average current per meter at circumference of the airgap, and is given by

$$\boxed{\bar{J} = \frac{\overset{\text{no. of phases}}{3} \times \overset{\text{no. of coil sides}}{2} \times N_{ph} k_w I_{ph}}{\pi d}}$$

- Current in the electrical conductors lead directly to power losses that raise the temp. of the machine, so we need to have greater cooling capabilities for any increase in $\bar{J}$.

- The VA rating S can be expressed as a product of the synchronous speed ω_s, volume of airgap, specific magnetic loading $\bar{B}$ and specific electrical loading $\bar{J}$

$$\boxed{S = \left(\frac{\pi}{\sqrt{2}}\right)\left(\frac{\omega}{p}\right)\left(l \frac{d^2}{4}\pi\right)\bar{J}\bar{B}}$$

This eqn. provides a starting pt. for any motor and shows the major factors that influence the design.

* Any reduction in motor size can only be achieved by increasing $\bar{B}$ and $\bar{J}$

Flux density and saturation

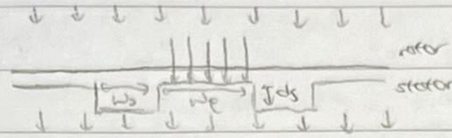
- The specific magnetic loading $\bar{B}$ is the average flux density in the machine. The peak value of the magnetic loading $\hat{B}$ should be below the saturation pt (knee pt of B-H curve)

$$\bar{B} = \frac{2\sqrt{2}}{\pi} B_{rms} = \frac{2}{\pi}\hat{B} \quad \rightarrow \quad \hat{B} = \frac{\pi}{2}\bar{B}$$

The is only a peak value in time, but averaged across the whole machine. Some parts of the machine (tooth and yoke of the core) may have even larger peak values.

① stator and rotor teeth

- Consider the magnetic flux at the stator and rotor teeth.

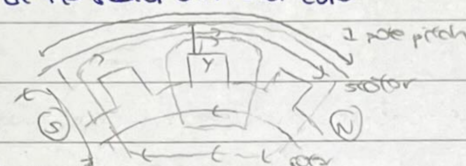


- Assume that the airgap flux over one tooth plus one slot pitch passes only through one tooth.
(reasonable if the slot pitch is small compared to the pole pitch and the flux crosses the airgap radially).
- Denoting the tooth and slot widths w/ w_t and w_s respectively, by cons. of flux,

$$\hat{B}(w_s+w_t)\,l=\hat{B}_t w_t l \rightarrow \boxed{\hat{B}_t=\left(1+\frac{w_s}{w_t}\right)\frac{\pi}{2}\bar{B}}$$

② stator and rotor cores

- consider the magnetic flux at the stator and rotor core



- The flux that passes radially through one magnetic pole splits into two equal components
and travels circumferentially around the stator/rotor iron cores to the next magnetic pole (opp. polarity)
- Denoting the yoke depth w/ Y, by cons. of flux,

$$\hat{B}_c l Y=\frac{1}{2}\frac{\pi d l}{2p}\bar{B} \rightarrow \boxed{\hat{B}_c=\frac{1}{2Y}\frac{\pi d}{2p}\bar{B}}$$

- Typical values lie in the range

$0.45T<\bar{B}<0.81T$ $\quad 1.6T<\hat{B}_t<2.2T$ $\quad 0.8T<\hat{B}_c<1.4T$

→ For economic use of the available material, the teeth operates near saturation.

stator design process.

① use VA rating to find the dimension.

$$S=\left(\frac{\pi}{\sqrt{2}}\right)\left(\frac{\omega}{p}\right)\left(l\frac{d^2}{4}\pi\right)\bar{J}\bar{B}$$

specific magnetic loading $\bar{B}$ determined by magnetic core design, normally given

specific electric loading $\bar{J}$ determined by machine cooling, normally given.

② use voltage to find winding distribution turns per phase N_{ph}, turns per coil N_c and distribution factor.

$$V_{ph}\approx E_{rms}=kN_{ph}k_w B_{rms}=\frac{\pi}{2\sqrt{2}}kN_{ph}k_w\bar{B}$$

* Round up N_c to the nearest integer

③ stator magnetic circuit — tooth and slot.

Calculate w_s, w_t using $\hat{B}_t=\left(1+\frac{w_s}{w_t}\right)\frac{\pi}{2}\bar{B}$, $\boxed{w_s+w_t=\frac{\pi d}{\text{total no. of slots}}}$

Calculate d_s using $\boxed{k_f w_s d_s=S_A N_c}$, $\boxed{S_A=\frac{I_{ph}}{J}}$ current density J directly related to specific electric loading $\bar{J}$

* k_f is the filling factor — the portion of the area filled w/ Cu as the Cu wire has insulation

④ stator magnetic circuit — core

Calculate Y using $\hat{B}_c=\frac{1}{2Y}\frac{\pi d}{2p}\bar{B}$

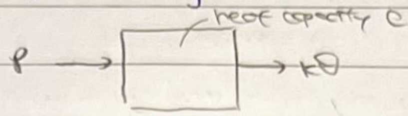
* Check the physical diameter of the stator is in the frame. $\boxed{\text{physical diameter}=d+2d_s+2Y}$

Duty cycle operation

Duty cycle operation

Basic thermal model

- A machine can be represented by the following model:



$$\boxed{P = C\frac{d\theta}{dt} + k\theta}$$

where P is the motor loss ; C is the machine thermal capacity (↑size → ↑C)

θ is the temp. above ambient ; k is the dissipation coeff k (↑cooling → ↑k)

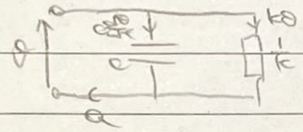
- The 1st order ODE can be solved to give the general sol'n

$$\frac{d\theta}{dt} + \frac{k}{C}\theta = \frac{P}{C} \quad \rightarrow \quad \theta = \frac{P}{k} + Be^{-t/\tau}$$

where $\boxed{\tau = C/k}$ is the thermal time const.

- We get the steady-state temp θ_{max} as $t \to \infty$. $\boxed{\theta_{max} = \lim_{t\to\infty} \frac{P}{k} + Be^{-t/\tau} = \frac{P}{k}}$.

- Note that the thermal network is analogous to the following electrical circuit:



Common operating conditions

① Motor switched on from cold

- Motor switched on from cold → At $t=0$, $\theta=0$

$$\theta = \theta_{max}(1-e^{-t/\tau})$$

② Motor switched off from hot

- Motor switched off from hot → At $t=0$, $\theta=\theta_0>0$

$$\theta = \theta_0 e^{-t/\tau}$$

③ Motor switched on overload from cold

- Denote rated loss w/ P_c and its corresponding steady-state temp. θ_{max}.

For the same dissipation coeff k, if overloaded w/ loss $P_s > P_c$, the new steady-state temp $\theta_{max,s}$ is

$$k = \frac{P_s}{\theta_{max,s}} = \frac{P_c}{\theta_{max}} \quad \rightarrow \quad \boxed{\theta_{max,s} = \frac{P_s}{P_c}\theta_{max}}.$$

$$\therefore \theta = \theta_{max,s}(1-e^{-t/\tau}) = \frac{P_s}{P_c}\theta_{max}(1-e^{-t/\tau})$$

- If overload w/ loss P_s, and the machine takes time T to reach θ_{max}, then

$$\theta_{max} = \frac{P_s}{P_c}\theta_{max}(1-e^{-T/\tau}) \quad \rightarrow \quad T = -\tau\ln\left(1-\frac{P_c}{P_s}\right) \quad \rightarrow \quad \tau = -\frac{T}{\ln(1-P_c/P_s)}$$

→ useful for measuring time const. τ quickly.

* The loss at overload, P_s may be hard to measure → estimate using overload current I_s.

$$\boxed{P_s = P_c\left(\frac{I_s}{I_c}\right)^2}$$

rated current

④ Motor switched on when initially hot

- Motor switched on when initially hot → At $t=0$, $\theta=\theta_0>0$.

$$\theta=\theta_0+(\theta_{max}-\theta_0)(1-e^{-t/\tau})$$

⑤ Repetitive duty cycle operation.

- We can calculate the average temp. θ_{av} from the average loss P_{av}

$$\boxed{\theta_{av}=\frac{P_{av}}{k}}$$

where $P_{av}=\frac{\sum_i P_i T_i}{\sum T_i}$

- The loss can be estimated by the mean of RMS current of each interval and rated loss P_c,

$$\boxed{P_{av}=\frac{\sum_i I_i^2 R T_i}{T}=\frac{P_c}{I_c^2 R}\frac{\sum I_i^2 R T_i}{T}=P_c\frac{\sum_i I_i^2 T_i}{I_c^2 T}}$$

Power loss and motor efficiency

- For an electric motor, the power rating means the mechanical o/p power P_{out} by convention.

- Since the efficiency η is defined as $\eta=\frac{P_{out}}{P_{in}}$, and $P_{loss}=P_{in}-P_{out}$,

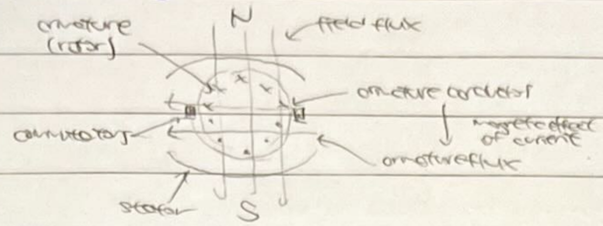
$$\boxed{P_{loss}=P_{in}-P_{out}=P_{out}\left(\frac{P_{in}}{P_{out}}-1\right)=P_{out}\left(\frac{1}{\eta}-1\right)}$$

Brushed and brushless DC motor drives

Brushed DC motor drives

Operation and construction of brushed DC motors

- The DC motor has stationary and rotating members, and the basic layout is as follows:



- The stationary member (stator) has a static magnetic field that crosses from N→S, produced by either (i) DC current in stator windings or (ii) permanent magnets.
- The rotating member (armature) has a set of coils mounted on it, which are connected to the DC power supply via carbon brushes + commutator (ensures the dir. of current flow is as above by changing its dir.)
- The switching of dir. of armature current by the commutator ensures that the armature flux is maintained at 90 electrical degrees wrt the field flux → optimal for torque production.
- The magnetic field in the rotor changes w/ time → rotor made from laminated iron

  The magnetic field in the field poles are static → field poles made from steel.

Armature emf and torque of brushed DC motors

- The average flux density B_{avg} is given by

$$B_{avg} = \frac{\overbrace{\phi \cdot 2p}^{\text{total flux}}}{\underbrace{2\pi r w}_{\text{curved surface area of cylinder}}} = \frac{p\phi}{\pi r w}$$

  where ϕ is the flux per pole, r is the armature radius, w is the axial length of the motor, and p is the no. of pole pairs (# poles ÷ 2).

- The average emf induced in a rotor conductor E_c is therefore

$$E_c = B_{avg} w r \omega_r = \frac{p\phi}{\pi}\omega_r \quad \leftarrow \frac{2\phi}{T},\ \omega = \frac{2\pi}{T}$$

  where ω_r is the rotor speed

- As a coil comprises two conductors one pole apart from each other, the coil emf is $2E_c$. If there are Z conductors on the armature, there will be $Z/2$ coils.
- These coils are all connected in series, so the total emf across the armature winding will be

$$\boxed{E_a = (2E_c)\left(\frac{Z}{2}\right) = \frac{pZ\phi}{\pi}\omega_r = k\phi\omega_r}$$

  where $k = \frac{pZ}{\pi}$ and is a fixed parameter for a given DC motor.

- The average force on one of the armature conductors is

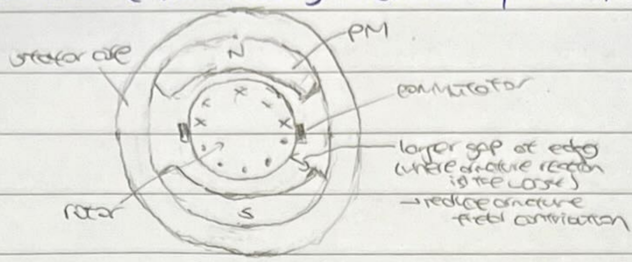
$$F = B_{avg} I_a w = \frac{p\phi}{\pi r w} I_a w = \frac{p\phi}{\pi r} I_a$$

  The torque due to all Z conductors is therefore

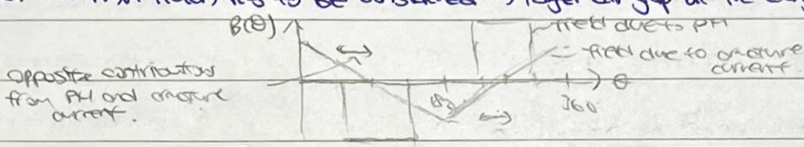
$$\boxed{T = FrZ = \frac{p\phi}{\pi r} I_a r Z = \frac{pZ}{\pi}\phi I_a = k\phi I_a}$$

PM brushed DC motor design

- In a PM brushed DC machine, the field winding is replaced by radially-magnetised PMs.



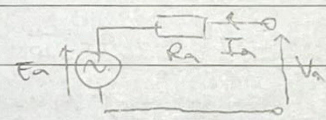
- There are no field winding copper losses → efficiency gains.
- High power and torque density → overall motor dimensions for the equivalent rating are smaller.
  (Although ↓airgap flux density → ↑airgap diameter, we have ↓depth req. by PM in stator)
- The effect of armature reaction — armature currents may demagnetise the PM (particularly at high torque and thus high field) has to be considered → larger air gap at the edges



- The field flux for PM DC motors cannot be varied → we have $K = k\phi$ for the DC machine eqns.

Steady state operation of brushed DC motors.

- Steady state operation means that the motor speed, current, back emf and applied voltage have all settled down to const. values. The equivalent circuit for the DC motor is thus



The DC motor eqn. is thus $\boxed{V_a = E_a + I_a R_a}$

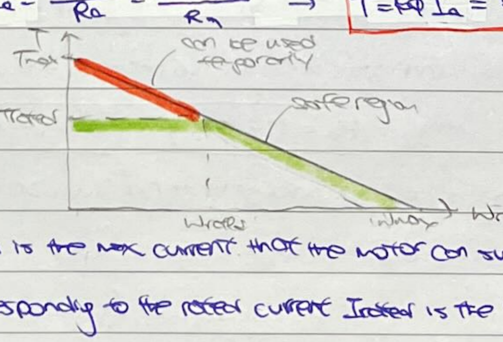
- Multiplying the above by I_a, we have consv. of power

$P_{in} = P_{out} + P_{loss}$, where $\boxed{P_{in} = V_a I_a,\ P_{loss} = I_a^2 R_a,\ P_{out} = P_{in} - P_{loss} = E_a I_a}$

(Note that this is consistent w/ $P_{out} = T\omega_r$: $P_{out} = T\omega_r = (k\phi I_a)\left(\frac{E_a}{k\phi}\right) = E_a I_a$)

- To determine the torque-speed characteristic, consider a fixed armature voltage V_a.

$V_a = E_a + I_a R_a \rightarrow I_a = \frac{V_a - E_a}{R_a} = \frac{V_a - k\phi\omega_r}{R_a} \rightarrow \boxed{T = k\phi I_a = \frac{k\phi V_a}{R_a} - \frac{(k\phi)^2}{R_a}\omega_r}$



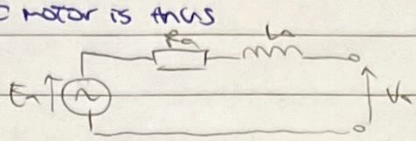
When $\omega = 0$,
$T = T_{max} = \frac{k\phi V_a}{R_a}$
when $T = 0$,
$\omega = \omega_{max} = \frac{V_a}{k\phi}$

- The rated current I_{rated} is the max current that the motor can sustain indefinitely w/o overheating.
  As $T \propto I$ (← for fixed field), the torque corresponding to the rated current I_{rated} is the rated torque T_{rated}.
- The rated torque T_{rated} can only be sustained up to the rated speed ω_{rated}. At $\omega > \omega_{rated}$, we have torque below the rated torque T_{rated}.

* Note that we can have I_{rated} / T_{rated} exceeded temporarily

Transient behaviour of brushed DC motors

- when considering the transient behaviour, we can have changes in demand torque (demand current), thus the armature inductance L_a becomes important (to limit the rate of change of I_a).
- The equivalent circuit for the DC motor is thus



The DC motor eqn. is thus $\boxed{V_a = E_a + I_a R_a + L_a \frac{dI_a}{dt}}$

- Taking Laplace transform of the above eqn, we have

$$V_a(s) = E_a(s) + I_a(s)R_a + sL_a I_a(s)$$

only considering the motor's moment of inertia to quantify the motor itself, $T = J\frac{d\omega_r}{dt}$, and noting $T = K\phi I_a$,

$$T = J\frac{d\omega_r}{dt} = K\phi I_a \rightarrow T(s) = sJ\omega(s) = K\phi I_a(s) \rightarrow I_a(s) = \frac{sJ\omega(s)}{K\phi}$$

substituting $I_a(s)$ and noting that $E_a = K\phi\omega_r \rightarrow E_a(s) = K\phi\omega(s)$, we have

$$V_a(s) = K\phi\omega(s) + s\frac{JR_a}{K\phi}\omega(s) + s^2\frac{JL_a}{K\phi}\omega(s)$$

$$\therefore \boxed{\omega(s) = \left(\frac{1}{K\phi}\right)\frac{1}{s^2\frac{L_a J}{(K\phi)^2} + s\frac{R_a J}{(K\phi)^2} + 1}V_a(s) = \left(\frac{1}{K\phi}\right)\frac{1}{s^2\tau_e\tau_{em} + s\tau_{em} + 1}V_a(s)}$$

where $\boxed{\tau_e = \frac{L_a}{R_a}}$ is the electrical time constant ← only electrical quantities

and $\boxed{\tau_{em} = \frac{R_a J}{(K\phi)^2}}$ is the electro-mechanical time constant. ← both electrical, mechanical quantities.

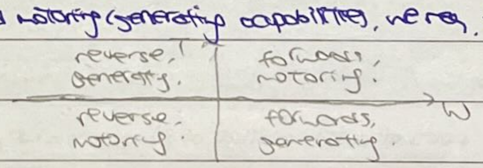
- The electrical time const. τ_e is the time taken for I_a to rise to 63% of its final value when we hold the armature of the DC motor stationary ($\omega = 0$) and applying a step voltage to it.
- The electromechanical time const. τ_{em} is the time taken for ω_r to rises to 63% of its final value when we have an unloaded DC motor ($T = 0$, $I_a = 0$) and applying a step demand in speed.
- Typically $\tau_{em} \gg \tau_e$ ($\frac{\tau_{em}}{\tau_e} \approx 10^3$), so the roots of the characteristic eqn. are approx. $s = -\frac{1}{\tau_{em}}$, $s = -\frac{1}{\tau_e}$.

(However, if we req. very low τ_{em}, J needs to be small → armature is air-cored, and we cannot use the above approximation.)

Four quadrant drive control

If we only req. forward + motoring capabilities, the DC-chopper control circuit is sufficient.

- If we req. forward/reverse and motoring/generating capabilities, we req. four quadrant drive control.



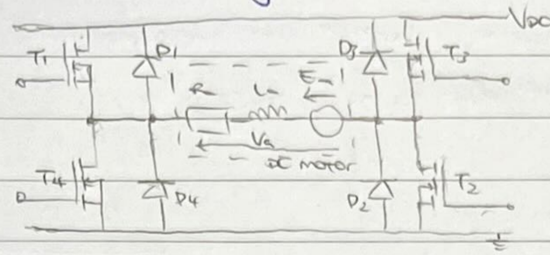
since field flux is fixed.

- Forwards is defined arbitrarily, thus forward vs reverse only depends on the sense of V_a (and thus I_a).
- Motoring vs generating depends on the rel. magnitudes of V_a and E_a.
  - ↳ Motoring: Voltage supply supplies current against armature back emf, i.e. $|V_a| > |E_a|$
  - ↳ Generating: Back emf driving current back through the armature voltage supply, i.e. $|E_a| > |V_a|$.

* Note regenerative braking work poorly at low speeds → we req. conventional braking.

Implementation of four quadrant control

- Four quadrant control can be implemented using a H-bridge circuit.



- For +ve armature voltage V_a, T_1, T_2 are switched ON, T_3, T_4 are switched OFF.
  For -ve armature voltage V_a, T_1, T_2 are switched OFF, T_3, T_4 are switched ON.
  → control forward/reverse .

- To vary the (magnitude) of the armature voltage, we can vary V_{DC} by varying the duty cycle (as in DC chopper circuit)
  → change btwn $|V_a| > |E_a|$ (motoring) and $|E_a| > |V_a|$ (generating) → control motoring/generating.

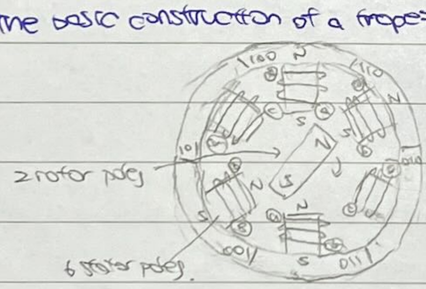
Brushless DC (BLDC) motor drives

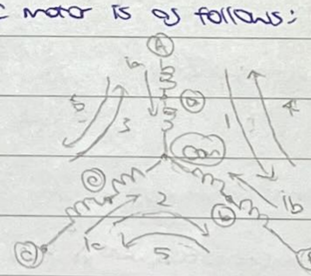
Types of BLDC motors.

- Brushed DC motors suffer from commutator/brush wear, inefficiencies due to field winding and friction at the brushes → BLDC motors avoid these problems.
- There are two main types of BLDC motors:
  ↳ Trapezoidal BLDC: back emf induced in stator winding is trapezoidal
  ↳ Sinusoidal BLDC: back emf induced in stator winding is sinusoidal.
  ↖ also called permanent magnet synchronous machine (PMSM)

Operation and construction of trapezoidal BLDC motors.

- The basic construction of a trapezoidal BLDC motor is as follows:

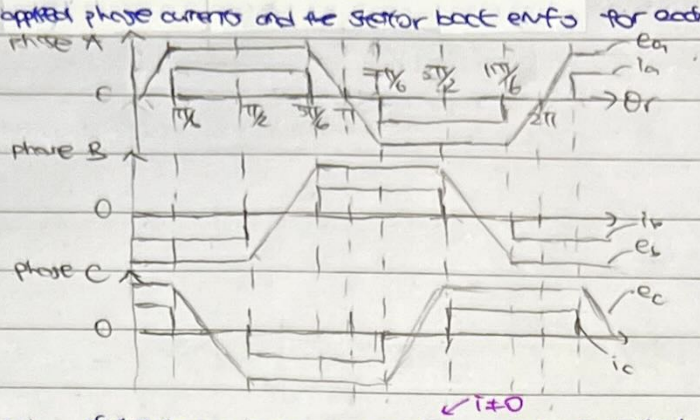




$V_{line} = 2E + IR_{line}$
$= 2E + 2IR_{ph}$

- The rotor has poles in which the PM are radially magnetised w/ adjacent PMs having opposite polarity. The no. of stator poles (multiple of 3) is always diff. to the no. of rotor poles to avoid cogging torque
- Cogging torque (alignment torque/reluctance torque) is a parasitic periodic torque which reflects the tendency of the rotor magnets to align themselves w/ the stator poles in order to minimise the stored magnetic energy.
- The trapezoidal BLDC motor uses concentrated stator windings — the no. of slots per pole per phase (phase band m) is one → stator coils wrapped around individual pole pieces → back emf induced in stator coil is trapezoidal (rather than sinusoidal)

- The plots of the applied phase currents and the stator back emfs for each phase over position is as follows:



- At any pt. in time, two of the three phases are excited — since the neutral pt is not connected to anything else, the same current flows in both of these phases as they are in series. ($R_{line} = 2R_{ph}$)
- During excitation period 1, current flows from A → a → COM → b → B, so stator poles 100, 110 are S; stator poles 011, 001 are N → rotor rotates s.t. its N pole is midway btwn stator poles 100, 110.
- During excitation period 2, current flows from C → c → COM → b → B, similar analysis shows that the rotor rotates s.t. its N pole is midway btwn stator poles 110, 010 → process continues indefinitely.

* Apart from very low speeds, the inertia of the motor makes the resulting motion reasonably smooth.

Armature emf and torque of trapezoidal BLDC motors.

- Notice the phase currents and the back emfs are in phase → rotor and stator fields are 90° wrt to each other (torque angle $\beta = 90°$) → max. torque for a given stator current. (max efficiency).
- Assuming the effective airgap is large, and the speed is low, the stator windings are mainly resistive (i.e. ignore stator inductance). During excitation phase 1, we have:

$$\boxed{V_a = e_a + i_a R_a} \quad [1] \qquad \boxed{V_b = e_b + i_b R_b}$$

$[1] \times i_a + [2] \times i_b$: $\quad V_a i_a + V_b i_b = e_a i_a + i_a^2 R_a + e_b i_b + i_b^2 R_b$

$\rightarrow P_{in} = P_{out} + P_{loss}$, where $P_{in} = V_a i_a + V_b i_b$, $P_{loss} = i_a^2 R_a + i_b^2 R_b$, $P_{out} = P_{in} - P_{loss} = e_a i_a + e_b i_b$

since the rotor field is fixed, the magnitude of the phase induced emf is prop. to the rotor speed.

$$E_a = E_b = E_c = \boxed{E = K\omega_r} \quad \rightarrow \quad e_a(t) = E_A = E \; ; \; e_b(t) = -E_B = -E.$$

and since the phases are connected in series,

$$I_A = I_B = I \quad \rightarrow \quad i_a(t) = I_A = I \; ; \; i_b(t) = -I_B = -I$$

Therefore substituting back into the expression for P_{out}, we have

$$P_{out} = e_a(t) i_a(t) + e_b(t) i_b(t) = (E)(I) + (-E)(-I) = 2EI$$

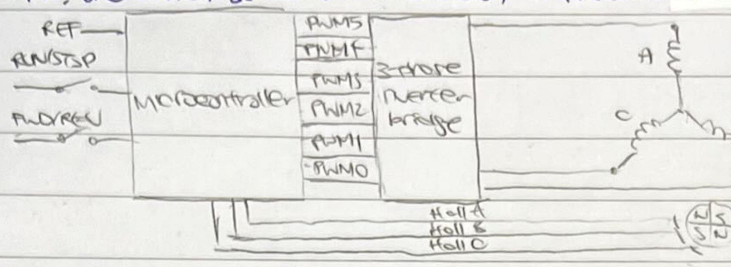
Equating this w/ mechanical power $P = T\omega_r$, we have

$$2EI = T\omega_r \quad \rightarrow \quad 2K\omega_r I = T\omega_r \quad \rightarrow \quad \boxed{T = 2KI}$$

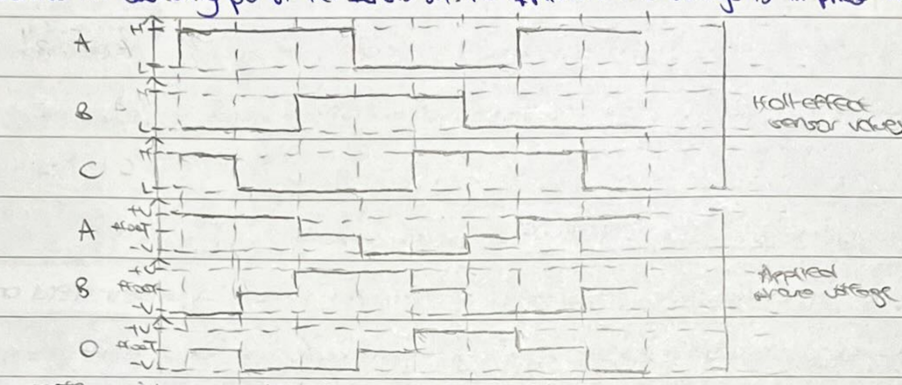
* For periods 2–6, a similar analysis will yield the same result.

Sensored trapezoidal BLDC motor drive system

- A typical BLDC motor drive system uses Hall effect sensors to provide rotor speed and position FB, and current sensor (sense resistors) to provide torque FB to the microcontroller.



- The microcontroller uses sensor FB to determine the switching pts and the req. PWM duty cycle for the IGBTs in the inverter
  position FB ... torque FB
- Typically the software will implement a digital form of PID control for both speed and torque.
- The plot below shows how the Hall-effect sensors are used by the microcontroller to synchronise the switching pts of the IGBTs s.t. the applied phase voltage is in phase w/ the back emf.



commutation period	1	6	5	4	3	2	1	6
sensor o/p code	101	100	110	010	110	100	101	001
A	1	1	–	0	0	–	1	1
B	0	–	1	1	–	0	0	–
C	–	0	0	–	1	1	–	0

we can construct a truth table and implement using combinational logic.

Sensorless trapezoidal BLDC motor drive system

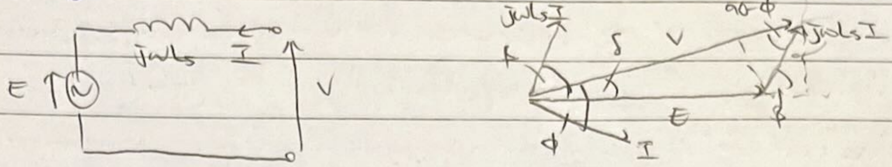
- Sensorless BLDC motor control eliminates the Hall-effect sensors (speed and position FB) by exploiting the fact that at any pt. in time, one of the three phases is floating → we just detect zero crossings.

- We can determine the correct switching instants by detecting the zero-crossing pt. of the floating phase back emf, then add a further delay corresponding to 30 electrical degrees.
- Back emf is prop. to motor speed → method doesn't work at zero speed → need software for starting up.
  To add 30 electrical degrees delay, the period of one rotor revolution is req. → only found when rotor at speed.
- → Sensorless BLDC motor control only works well if the motor is rarely req. to work at low speeds, or in start-stop applications. O/w, sensored control is better.

Operation and construction of sinusoidal BLDC motors.

- The rotor has a fixed magnetic field supplied by PMs, shaped to produce a sinusoidally distributed magnetic field
- The stator has a standard three-phase winding → back emf induced in the stator winding is sinusoidal, and req. a three-phase sinusoidal voltage supply to drive it.
- Using the motoring convention (current flows into machine's terminals), and assuming the machine to be lossless, (i.e. stator winding resistance ignored), the per-phase equivalent circuit and phase diagram are

reasonable esp. for high speeds $\omega L_s \gg R$



Note ϕ is the angle used for $PF = \cos\phi$, δ is the load angle and β is the torque angle.

- The torque angle β is defined as the electrical angle btwn the rotor and stator driven fields.
- As we assumed the motor to be lossless, by consv. of power, $P_{out} = P_{in} = 3VI\cos\phi$

From the phasor diagram, $E\sin\delta = \omega L_s I\cos\phi$, $V\sin\delta = \omega L_s I\sin\beta$

$$\therefore \boxed{P_{out} = \frac{3VE\sin\delta}{\omega L_s} = 3EI\sin\beta}.$$

- Since the rotor field is fixed, the emf induced in the stator windings E is prop. to rotor speed $\omega_r = \omega_s$

$$\boxed{E = k\omega_s}$$

Equating the expression for P_{out} w/ mechanical power $P = T\omega_s$,

$$T\omega_s = 3EI\sin\beta = 3k\omega_s I\sin\beta \quad \rightarrow \quad \boxed{T = 3kI\sin\beta}$$

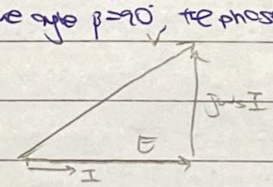
these similarities w/ brushed DC motors give it the name BLDCM.

- If the motor is controlled at the torque angle $\beta = 90°$, it produces max torque for fixed current. It also operates at greatest efficiency as I^2R losses are min. The torque eqn simplifies to $\boxed{T = 3kI}$

Sinusoidal BLDC motor drive system

- Consider the case where the torque angle $\beta = 90°$, the phasor diagram is as follows

phase current I and induced emf E are in phase



- The rated current I_{rated} is the max. current that the motor can sustain indefinitely w/o overheating.

for fixed field

As $T \propto I$, the torque corresponding to the rated current I_{rated} is the rated torque T_{rated}. ($T_{rated} = 3kI_{rated}$)

- Consider operating at rated torque ($I = I_{rated}$), as the rotor speed ω increases, the supply freq. must increase in proportion as $\omega_s = \frac{\omega}{p}$, where p is the no. of pole pairs.
- Thus the phasors $E = k\omega_s$ and $j\omega L_s I$ will both increase in proportion to the rotor speed ω_s → applied voltage V must be controlled st. both its amplitude and freq. are kept prop. to the speed → VVVF control.
- To provide a VVVF three-phase supply to the motor, a three-phase inverter is used (DC → VVVF AC). The DC supply may itself be derived from a fixed voltage, freq. supply using a three-phase full-wave bridge rectifier.

- The main elements of the sinusoidal BLDC motor drive system is as follows:

- The Hall-effect sensors provide speed and position FB and the current sensor providing torque FB to the microcontroller
- The microcontroller uses a PID speed controller to provide a reference value for the amplitude of the BLDC motor phase voltages.
- The microcontroller then determines the phase of the voltages from the rotor position (thus phase of back emf E), speed ω_s and current I.
- The software uses the phasor diagram to calculate the req. amplitude, phase and freq. of phase voltages.

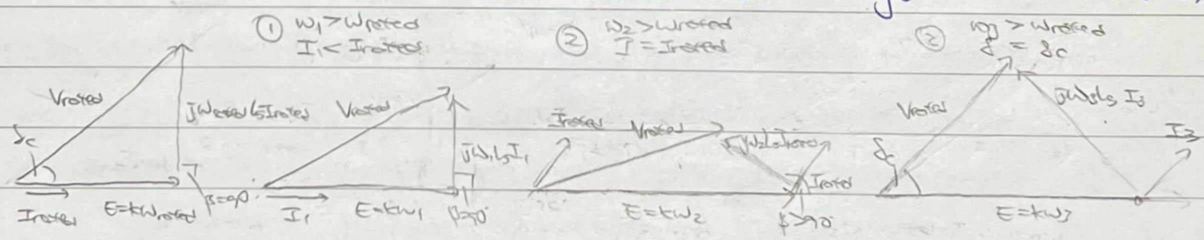
Inverter and motor rating.

f related to ω related to ω_s $\left(\omega_s = \frac{2\pi f}{p}\right)$.

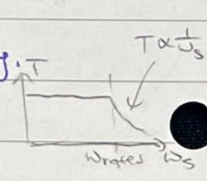
- An inverter has a max voltage, V_{rated}, max current, I_{rated} and max freq. f_{rated}.
- The voltage and current limits are typically matched to the motor.
  - ↳ current limit I_{max} imposes a torque limit T_{rated} to the motor
  - ↳ voltage limit V_{max} gives rise to a rated speed ω_{rated}. ($\uparrow\omega_s \rightarrow \uparrow|E=k\omega_s|, \uparrow|\omega_s L I| \rightarrow \uparrow|V|$, so ω_{rated} corresponds to the speed where $V=V_{rated}$ while keeping $\beta=90$)
- The freq. limit also bounds the max speed the motor can reach (good to check freq. $\leq f_{rated}$)

Reaching speeds above rated speed ω_{rated}.

- Higher speeds are possible but are associated w/ a loss of torque. There are two strategies:
  - ↳ 1) Retain torque angle $\beta=90°$ and accept that we have to reduce the current, thus torque is reduced.
  - ↳ 2) Allow the torque angle $\beta>90°$ so the motor operates at rated voltage and current (torque reduced $T=3kI\sin\beta$).
  - ↳ 3) Allow the torque angle $\beta>90°$ so the motor operates at rated voltage and load angle (const. power, $P=T\omega_s$ $\omega_s\uparrow, T\downarrow$)

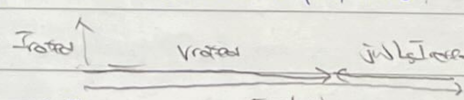


* Note in cases ②,③ $|E|>|V|$ → motor is overexcited. Both are examples of field weakening.



* case ③ is const. power since. $P = \frac{3VE\sin\delta}{X_s} = \frac{3V_{rated}k\omega_s\sin\delta_c}{\omega L_s} = \frac{3V_{rated}k\sin\delta}{pL_s} = \text{const}$.

- The top possible speed is when there is no load torque, $T=0$



* In practice, driving at rated current I_{rated} could demagnetise the rotor PM → limit stator current to lower value.

Electric drive requirements and choices

Requirements for the electric drive

- Common req. for the electric drive are:
  - ↳ Payload and kerb weight
  - ↳ Acceleration and speed
  - ↳ Braking (conventional + regenerative)
  - ↳ Range
  - ↳ Recharging time
  - ↳ Economy and emissions

Requirements for the motor

- The main factors in determining the motor specifications include:
  - ↳ Motor load: The three main components to the load acting on the motor are

    $F_{tot} = F_g + F_d + F_r$; $F_g = Mg\sin\theta$; $F_d = \frac{1}{2} C_d \rho A V^2$; $F_r = C_r N = C_r Mg\cos\theta$.

  - ↳ Motor torque / power: High torque/power is desirable for accelerating heavily or climbing a steep road.

    $T_{wheel} = F_{tot}\left(\frac{d_{wheel}}{2}\right)$; $T_{motor} = \frac{T_w}{N}$ ← gear ratio

  - ↳ Acceleration and top speed.

    At rated torque $a = \frac{F_{tot}}{M}$

    At const. power $a = \frac{V_f - V_i}{\Delta t}$, where $\Delta KE = \frac{1}{2} M (V_f^2 - V_i^2) = P\Delta t$

    Top speed V_{max} $V_{max} = N_{rps,wheel}(\pi d_{wheel}) = \frac{N_{rps,motor}}{N}(\pi d_{wheel})$

  - ↳ Charging.
  - ↳ Economy and carbon footprint.

stepper motors

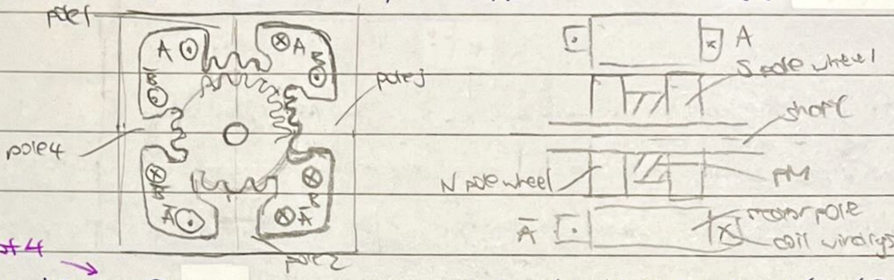
stepper motors

stepper motors.

- A stepper motor is an electrical motor that rotates in a series of small angular steps.
- The rotor's position can be commanded to move and hold at one of these steps w/o any position sensor for FB (open-loop controller), provided it is operated within their limits → precise control.
- stepper motors offer high torque density and can provide a "holding torque" at zero speed.
- There are two main types of stepper motors:
  - ↳ switched-reluctance motors: reluctance torque ← torque due to variations in reluctance.
  - ↳ hybrid stepper motor: reluctance torque + excitation torque ← torque due to interaction btwn PM field and coil field

Operation and construction of hybrid stepper motors

- The basic construction of a standard hybrid stepper motor is as follows.



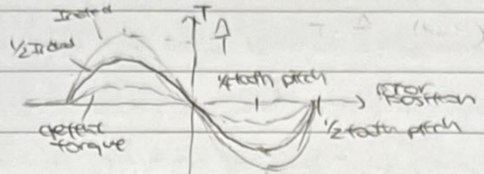
multiples of 4

- This motor has 4 poles, w/ a winding wound around each pole. The coils that make up phase A are connected in series, (similarly for phase B). Each stator pole has a large no. of teeth (eg. 48)
- The rotor is comprised of a pair of wheels made of soft magnetic material, carrying an equal no. of teeth (eg. 50), which are offset from each other by exactly ½ a tooth pitch
- The rotor wheels are sandwiched by an axially-magnetised PM → we have a N pole wheel and a S pole wheel.
- consider phase A excited (phase B off). stator pole 1 becomes S, stator pole 2 becomes N. → N pole wheel aligns its teeth w/ stator pole 1, S pole wheel aligns its teeth w/ stator pole 2. (N/S pole wheel completely misaligned w/ stator pole 2/1)
- wrt. stator poles 3 and 4, the N/S pole wheels are misaligned by ¼ a tooth pitch.
- consider phase B excited (phase A off) stator pole 3 becomes S, stator pole 4 becomes N → N pole wheel aligns its teeth w/ stator pole 3, S pole wheel aligns its teeth w/ stator pole 4.
  → The rotor has moved by ¼ a tooth pitch.
- The cycle of excitation consists of four steps (A, B, −A, −B), and the step angle $\Delta\theta$ is given by

$$\boxed{\Delta\theta = \frac{360^\circ}{4N_t}}$$ where N_t is the no. of rotor teeth per wheel

Torque production

- The torque vs rotor position curves w/ one phase excited is as follows:



- When the rotor is in a full step position, it is stationary → zero torque. Moving the rotor away from this eqm. means rotor torque acts in the opposite dir. to the movement → restoring torque (stable eqm.)
- The restoring torque is max at 1/4 tooth pitch. → when next phase is excited, the rotor is 1/4 tooth pitch away → torque is maximised.
- There is another zero torque pt at 1/2 tooth pitch, but is an unstable eqm.
- When operating at a current below the rated current, I_{rated}, say $I = kI_{rated}$, where $k \in [0,1]$, the max value of static torque is scaled proportionally. The value of static torque T_m is thus

$$\boxed{T_m = -k\hat{T}\sin(N_t\theta)}$$

where $\hat{T}$ is the max value of static torque at rated current I_{rated}.

- Note when we have zero current, the static torque is non-zero — we have detent torque i.e. the rotor position can be "remembered" in the event of power supply failure.

* Note the torque function may not be sinusoidal, but it is a good model

Position error.

- w/ no load torque applied, the rotor and stator teeth align s.t. magnetic energy is min. (full step position)
- If a load torque is applied, the rotor moves away from full step position until the EM torque acting on the rotor exactly balances the load torque, i.e. $\boxed{T_m = T_{load}}$.
- If the load torque exceeds the peak static torque, then the stepper motor is unable to hold its position. Otherwise, the largest position error of the rotor will be 1/4 tooth pitch (1 rotor step)
- Note we can use a speed reduction gearbox btwn the rotor and the load to reduce the rotor step size by the gear ratio → smaller position error

* The rotor step size w/ gearbox of gear ratio N, $\Delta\theta'$ is given by

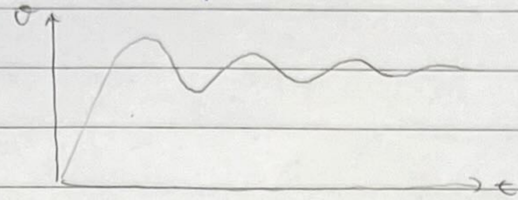
$$\boxed{\Delta\theta' = \frac{\Delta\theta}{N}}$$

where the gear ratio N is defined as

$$\boxed{N = \frac{\text{rotations of driver gear}}{\text{rotations of driven gear}} = \frac{N_t \text{ of driven gear}}{N_t \text{ of driver gear}}}$$

Mechanical resonances

- The transient response of an unloaded stepper motor excited by a single step is as follows:



i.e. a typical lightly damped second order system step response.

- The restoring (EM) torque of the motor is given by $T_m = -\hat{T}\sin(N_t\theta)$. For small displacements about the eqm. position, $\theta = 0$, we can linearise the eqn.

$$\boxed{T_m = -\hat{T}\sin(N_t\theta) \approx -\hat{T}N_t\theta = -k\theta}$$

where k is the torsional stiffness.

- Ignoring damping (lightly damped system), we equate EM torque w/ inertial torque,

$$-k\theta = J\frac{d^2\theta}{dt^2} \rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\omega_0^2\theta, \text{ where } \omega_0 = \sqrt{\frac{k}{J}}}$$

i.e. SHM. However, in reality there will be some damping, so we get the step response above.

* Note for a system w/ speed reduction gearbox, the system's moment of inertia J is

$$\boxed{J = J_{motor} + \frac{J_{load}}{N^2}}$$

- The consequence of this is that we should avoid switching freq. that equal the resonant freq. (or its harmonics) → resonance (increasingly large oscillations) → motor missing steps and uncontrollable.
- The switching freq. we should avoid are $f = nf_0$, $n \in \mathbb{Z}^+$, where the fundamental f_0 is given by

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{J}}}$$

The switching freq. corresponds to 1 rotor step. If there are N_s steps per complete revolution of the rotor,

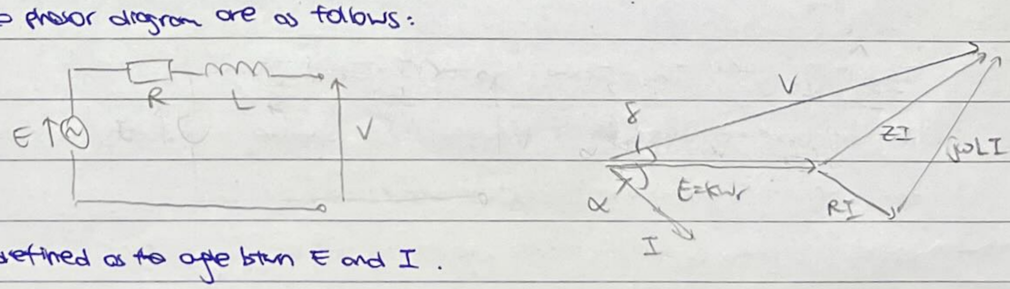
$$\boxed{N_{rps} = \frac{nf_0}{N_s}}$$

($N_s = 4N_t$ for full-stepping mode).

- We can achieve speeds above these problem speeds by accelerating through.

Operation at speed.

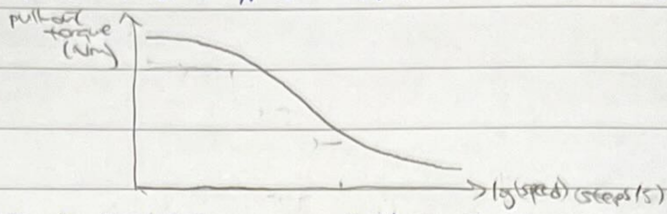
- When operating at speed, the hybrid stepper motor is similar to a sinusoidal BLDCM if all the quantities (V, E, I) are regarded as consisting only of their fundamental freq. components.
- Especially for small stepper motors, the winding resistance is significant. The equivalent circuit and the phasor diagram are as follows:



* α is defined as the angle btwn E and I.

Torque speed characteristic.

- The torque-speed characteristic of a typical stepper motor is as follows:

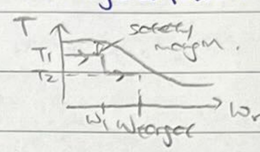


The pullout torque is the min. load torque that would cause the machine to go out of step.
← for open-loop stepper motor
→ rotor position no longer known → drive unstable (position req. to time the phase excitation pattern)

- As the stepper motor at speed is similar to a sinusoidal BLDC motor, $\uparrow\omega_r \rightarrow \uparrow|E|, \uparrow|\omega LI| \rightarrow \uparrow|V|$

The rated speed ω_{rated} corresponds to the speed ω_r when $V = V_{rated}$ (inverter voltage limit).

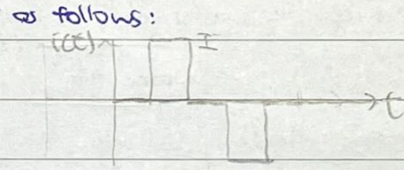
- Increasing the speed above rated speed is possible, but would reduce motor torque.
- To ensure pull-out torque is not exceeded, max acceleration/deceleration rates and max speed is set within the control software.
- Typically, we set T to be lower than $T_{pullout}$ by a safety margin, but as large as possible (within safety range.) to reach the desired step speed as fast as possible.



Excitation strategies

① Full-stepping.

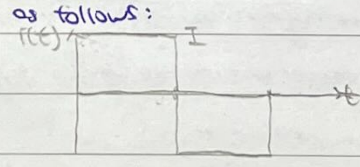
- Full-stepping involves exciting the phases in the sequence {A, B, −A, −B, A ...}
- Rotor step size = ¼ tooth pitch.
- The phase current waveform is as follows:



$I_{rms} = \frac{I}{\sqrt{2}} \rightarrow I_{rms} = I_{rated}$ implies $I = \sqrt{2} I_{rated}$.

② Full-stepping, two phases on at a time

- For extra torque, we excite two phases at a time w/ sequence {(A,B), (B,−A), (−A,−B), (−B,A), (A,B) ...}
- Rotor step size = ¼ tooth pitch (eqm. positions all shifted by ½ tooth pitch)
- The phase current waveform is as follows:

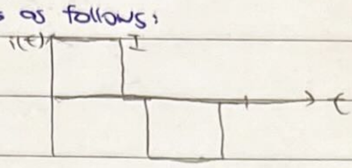


$I_{rms} = I \rightarrow I_{rms} = I_{rated}$ implies $I = I_{rated}$.

* We can operate in mode ① normally, and operate in mode ② w/ $I = \sqrt{2} I_{rated}$ as before temporarily if extra torque is req. for a short period.
(Fine since the power electronics are rated for the larger current).

③ Half stepping.

- Half stepping can be thought of combining mode ① and ②. The phase excitation sequence is $\{A, (A,B), B, (B,-A), -A, (-A,-B), -B, (-B,A), A, AB \dots\}$
- Rotor step size $= \frac{1}{8}$ tooth pitch → smaller position error.
- The phase current waveform is as follows:

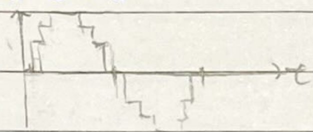


$I_{rms} = \sqrt{\frac{3}{4}}\, I$ → $I_{rms} = I_{rated}$ implies $I = \frac{2}{\sqrt{3}} I_{rated}$.

* We can operate in mode ① normally and operate in mode ② w/ $I = \sqrt{2}\, I_{rated}$ as before temporarily if extra torque is req.

④ Microstepping.

- Microstepping is an extension of half stepping in which more rotation positions are created by exciting pairs of phases w/ diff. proportions of current (achieved via PWM)
- Standard rotor steps are $\frac{1}{16}$ or $\frac{1}{32}$ tooth pitch (but only full/half step positions are accurate)
- The phase current waveform is as follows

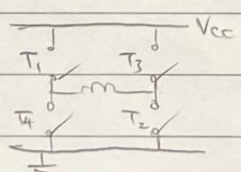


This req. accurate and independent control of phase currents → more complex to implement.

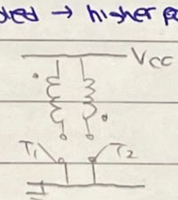
* Microstepping can provide a smoother torque → avoid mechanical resonance.

Drive circuits.

- Large stepper motors req. a full H-bridge per phase. T_1, T_2 ON, T_3, T_4 OFF for current to flow one way, T_1, T_2 OFF, T_3, T_4 ON for current to flow the other way → bipolar switch mode.
- For the standard 2-phase 200 step stepper motor in full step mode, $N_{rps} = 1$ corresponds to 200 steps per second → T_1, T_2 and T_3, T_4 need to switch 100 times per second → can scale up proportionally.
* We typically switch at an even higher freq (e.g. 20kHz) for PWM to vary mean voltage + current (torque) control.
- Small hybrid stepper motor drives often use bifilar windings — applying a +ve voltage to one of the strands results in the current flowing in the opposite dir. to that flowing in the other strand. simpler as only low-side transistors are req.
- T1 ON, T2 OFF for current to flow one way. T1 OFF, T1 ON for current to flow the other way → unipolar drive circuit
makes sense for low power applications
- The reduced drive complexity also reduces the motor efficiency — only half the current-carrying capability used at any one time → effective coil resistance doubled → higher power losses given a fixed current.



H-bridge



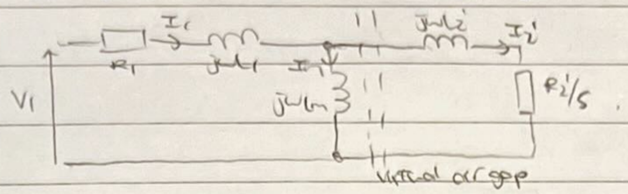
bifilar windings

AC motors

Three-phase induction motors

Three-phase induction motor (IM)

- The equivalent circuit per phase of a three-phase IM is as follows:



R_1: stator winding resistance — L_1 = stator leakage inductance

R_2': Rotor winding resistance — L_2' = Rotor leakage inductance.

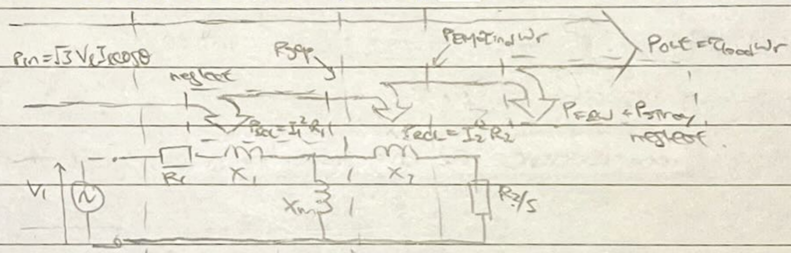
L_m Magnetising inductance — ☆ ignore R_1, X_1 if $X_m \gg R_1, X_1$ at rated flux, ignore X_2

- To produce a torque in an IM, current must flow in the rotor. To induce voltage (to get current) in the rotor, the rotor speed ω_r must be slightly slower than synchronous speed ω_r → rel. movement for flux to get $\frac{d\Phi}{dt} \neq 0$. (usually $\omega_r \sim 0.95\omega_s$)
- The difference btwn the synchronous speed ω_s and the rotor speed ω_r is the slip speed. The ratio of slip speed to synchronous speed is slip, s (usually $s < 0.05$)

$$\boxed{s = \frac{\omega_s - \omega_r}{\omega_s}} \leftrightarrow \boxed{\omega_r = (1-s)\omega_s}, \quad \boxed{s\omega_r = \omega_s - \omega_r}$$

Torque expression for IM.

- If we neglect the stator losses, the power at the airgap equals the i/p power, i.e. $P_{in} = P_{gap}$.
  If we neglect the F&W losses, and stray losses, the o/p mechanical torque equals the EM torque, i.e. $P_{em} = P_{out}$



By conv. of power, $P_{out} = P_{gap} - P_{RCL}$ → $T\omega_r = T\omega_s - 3I_2'^2 R_2'$

$$\therefore \boxed{T = \frac{3 I_2'^2 R_2'}{s\omega_s}}$$

If we neglect the stator resistance R_1 and stator leakage inductance L_1, the voltage across the magnetising branch equals the supply voltage V_1, so the rotor current I_2' is

$$\boxed{I_2' = \frac{V_1}{\sqrt{(R_2'/s)^2 + X_2'^2}}}$$

substituting into the expression for torque above,

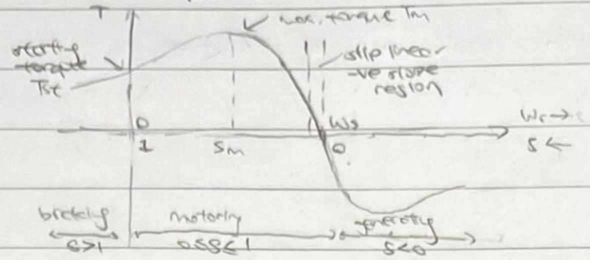
$$\boxed{T = \frac{3 V_1^2 R_2'}{s\omega_s\left((R_2'/s)^2 + X_2'^2\right)}}$$

☆ During typical operation, $s \approx 0$, so $R_2'/s \gg X_2'$, so the torque expression simplifies to

$$T = \frac{3 V_1^2 s}{\omega_s R_2'}$$

Torque-speed characteristic

- The torque-speed characteristic of a typical IM is as follows:



- At normal operation, slip is close to zero, i.e. the steep part of the torque-speed curve
  → a increase in load torque results in a small decrease in speed and vice versa → stable operation.
- Also, when $s \approx 0$, $R_2'/s \gg X_2'$ and the torque is in the slip linear -ve slope region.

$$\boxed{T = \frac{3V_1^2 s}{W_s R_2'} = \frac{3V_1^2}{W_s^2 R_2'}(W_s - W_r)} \rightarrow \boxed{\frac{dT}{dW_r} = -\frac{3V_1^2}{R_2' W_s^2}}$$

- The starting torque T_{st} occurs when $s=1$. We usually want high starting torque to start the machine. The max starting torque can be obtained by shifting the max. torque to $s=1$.

$$T_{st} = \frac{3V_1^2 R_2'}{W_s(R_2'^2 + X_2'^2)}$$

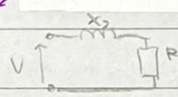
- To find the max torque T_m, we set $\frac{dT}{ds} = 0$, max. torque T_m occurs when slip is

$$\boxed{S_m = \pm\frac{R_2'}{X_2}}$$ depends on R_2, not V_1

and thus the corresponding max. torque is

$$\boxed{T_m = \pm\frac{3V_1^2}{2W_s X_2'}}$$ depends on V_1, not R_2

(Alternatively, apply max. power transfer thm → $R_2'/s = X_2'$

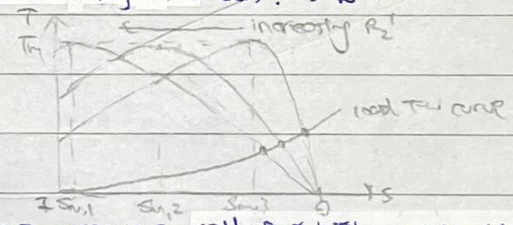


)

Simple methods of speed control

① changing rotor resistance R_2'

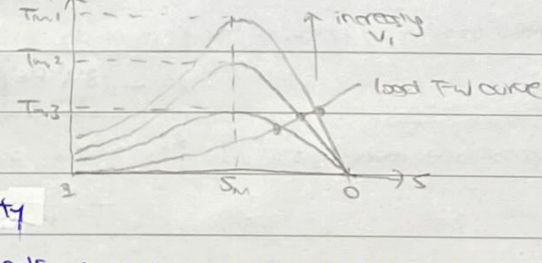
- changing R_2' only changes S_m (while keeping T_m fixed). → $\frac{V_1}{W}$ fixed



- Drawbacks: Increasing R_2' increases rotor loss, small adjustability, not feasible to change R_2' all the time.
  squirrel cage rotor cannot change R_2'

② changing machine phase voltage V_1

- changing V_1 only changes T_m (while keeping S_m fixed) → $\frac{V_1}{W}$ changes
  star-delta switch gives $\sqrt{3}$ factor diff.



- Drawbacks: small adjustability

* These methods are fine for loads where torque and speed change together (↑T when ↑W_r)
  In general, to overcome small adjustability issue, we adjust both V_1 and W. (VVVF)

Speed control using a variable voltage variable frequency (VVVF) supply.

- changing of synchronous speed ω_s is preferable for speed adjustment, which req. changing supply freq. ω. Note that this also changes the stator/rotor leakage reactance X_1, X_2 and the magnetising reactance X_m.
- Neglecting the stator resistance and leakage impedance R_1, X_1 at high supply freq. (since X_m is large), the voltage across the magnetising branch is V_1, so the magnetising current I_m is

$$I_m = \frac{V_1}{\omega L_m}$$

The magnetising current I_m is used to build the airgap flux in the IM,

$$\phi = I_m L_m = \frac{V_1}{\omega}$$

We typically keep the airgap flux in an IM const. (except for very low speeds) → we req. $\frac{V_1}{\omega}$ to be const. Since we are changing the supply freq ω, we need to change the supply voltage's magnitude V_1 as well.

- Since V_1/ω needs to be constant, we can write

$$\boxed{V_1 = k\omega}$$

where k is a const. value determined by the machine design.

- Substituting the above for the torque expression, we get

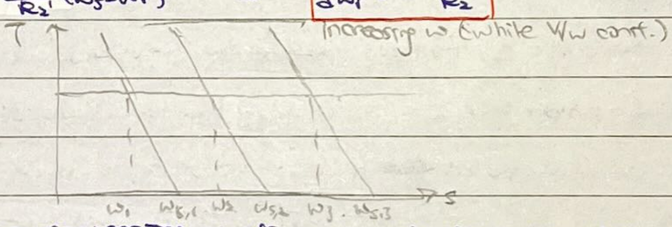
$$T = \frac{3V_1^2 R_2'}{s\omega/p\left(\left(R_2'/s\right)^2 + X_2'^2\right)} = 3pk^2 \frac{s\omega R_2'}{R_2'^2 + (s\omega)^2 L_2'^2} = 3pk^2 \frac{(\omega - p\omega_r)R_2'}{R_2'^2 + (\omega - p\omega_r)^2 L_2'^2}$$

At the linear negative region, the R_2' term dominates, and we have

$$\boxed{T = 3pk^2 \frac{s\omega}{R_2'}}$$ ← useful to group slip freq. $s\omega$ as 1 variable

- In the linear -ve slope region, $T = \frac{3V_1^2 s}{\omega_s R_2'} = \frac{3V_1^2}{\omega_s^2 R_2'}(\omega_s - \omega_r)$. For VVVF, $V_1 = k\omega$, so

$$T = \frac{3pk^2 s}{R_2'}\omega = \frac{3p^2k^2}{R_2'}(\omega_s - \omega_r) \rightarrow \boxed{\frac{dT}{d\omega_r} = -\frac{3p^2k^2}{R_2'}}$$



i.e. we get a group of linear lines for operation at diff. ω → much wider and uniform adjustability.

Operating limits of the VVVF IM drive

① Rotor current limit

- If we neglect stator resistance and leakage impedance R_1, X_1, then $I_2' = \frac{V_1}{\sqrt{(R_2'/s)^2 + X_2'^2}}$. Substitute $V_1 = k\omega$,

$$I_2' = \frac{V_1}{\sqrt{(R_2'/s)^2 + X_2'^2}} = \frac{s\omega k}{\sqrt{R_2'^2 + (s\omega)^2 L_2'^2}}.$$

During typical operation, $s \approx 0$, so $R_2'^2 >> (s\omega)^2 L_2'^2$, and we have

$$I_2' = \frac{s\omega k}{R_2'} \rightarrow R_2' = \frac{s\omega k}{I_2'} \rightarrow \boxed{T = 3pk^2 \frac{s\omega}{R_2'} = 3pk I_2'}$$

- The rotor current I_2' is limited by the rotor cooling arrangement
- The max torque when below base speed ω_b and at the linear -ve slope region occurs when the max rotor current is reached

② stator current limit

- For const. airgap flux, the magnetising current I_m is kept const. → stator current mainly determined by the rotor current. $\boxed{\vec{I_1} = \vec{I_2'} + \vec{I_m}}$

* I_2' is mainly resistive, I_m is mainly inductive → $\vec{I_m}$ leads $\vec{I_2}$ by 90°.

③ Operation below base speed — VVF control

- For an IM drive, we define a base synchronous speed $\omega_{s,b}$, base voltage V_b, base supply freq. ω_b. The base voltage V_b and base supply freq. ω_b are limited by the drive inverter.
- The IM can operate w/ the rated flux (const. airgap flux) below the base values.

$$\boxed{k = \frac{V_b}{\omega_b}} . \qquad \omega_{s,b} = \frac{\omega_b}{p}$$

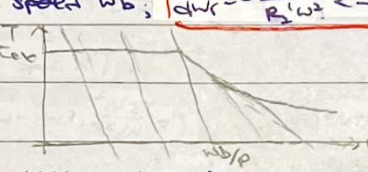
④ Operation above base speed — Field weakening.

- The IM drive inverter o/p voltage is limited to the base voltage V_b, but the max speed can be higher than the base speed ω_b.
- As we increase ω but keep $V = V_b$, we get a smaller $\frac{V}{\omega}$ (since $k = \frac{V_b}{\omega} < \frac{V_b}{\omega_b}$)

This is magnetic field weakening, and will cause compromised torque.

- $T = 3pk^2 \frac{s\omega_b}{R_2'}$, so above base speed ω_b, $(k = \frac{V_b}{\omega})$, $\boxed{T = 3p\frac{sV_b^2}{\omega R_2'} < 3p\frac{sV_b^2}{\omega_b R_2'} = T_{max}}$

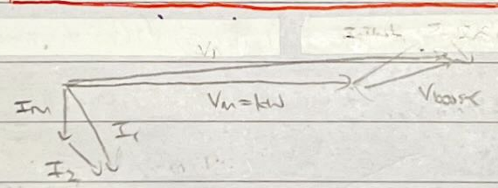
$\frac{dT}{d\omega_r} = -\frac{3p^2V_1^2}{R_2'\omega^2}$, so above base speed ω_b, $\boxed{\frac{dT}{d\omega_r} = -\frac{3p^2V_1^2}{R_2'\omega^2} < -\frac{3p^2V_1^2}{R_2'\omega_b^2} = \left.\frac{dT}{d\omega_r}\right|_{\omega_r=\omega_b}}$



⑤ Operation at very low speeds — Voltage boosting.

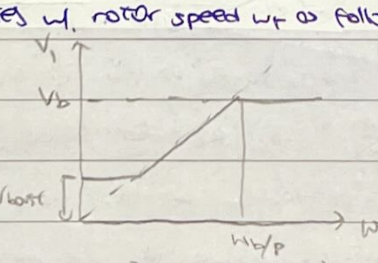
- The const. $\frac{V}{\omega}$ operation assumes the voltage drop across R_1, X_1 to be negligible. At low speeds, the supply freq. ω is low → $X_m = \omega L_m$ is low and the voltage drop cannot be neglected.
- We need to provide a voltage boost to the supply s.t. I_m is constant.

$$\boxed{\vec{V_1} = \vec{V_m} + \vec{V}_{boost} = k\omega\angle\theta + \vec{I_1}(R_1 + j\omega L_1)}$$



* sometimes the question may define $V_{boost} = |V_1| - |V_m|$

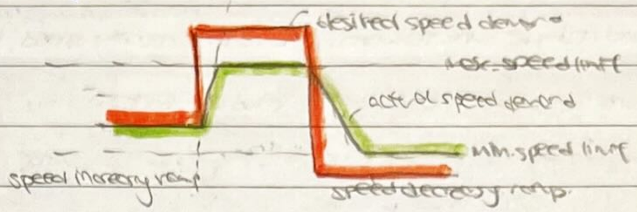
→ The supply voltage V_1 varies w/ rotor speed ω_r as follows:



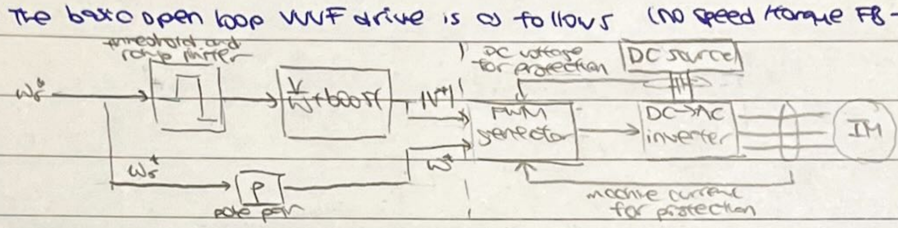
VVVF drive systems.

- In motor control, the demand (torque or speed) is applied under specific conditions due to the dynamic response of the controller. The two main conditions are:

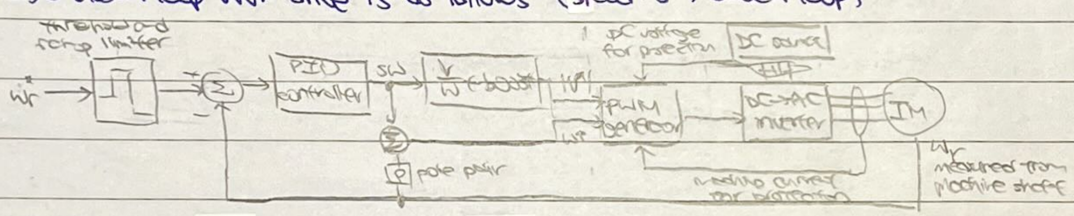
↳ Max. ramp (rate of change) ↳ Max./min. limits.



- This can be implemented in a control system using a threshold and ramp limiter.
- The basic open loop VVVF drive is as follows (no speed/torque FB → open loop)



* The PWM generator takes in DC voltage and machine current for protection (not control).
- Open loop control does not control the slip speed $s\omega_s = \omega_s - \omega_r$ (stays roughly const.) → change in synchronous speed follows change of rotor speed → dynamic response is slow.
- The basic closed loop VVVF drive is as follows (speed FB → closed loop)



- Closed loop control req. rotating speed ω_r feedback → use PID controller to control slip frequency $s\omega$.
  → increased stability, better transient response., but dynamic performance not great
- In practice, we use vector-controlled IM drives — better dynamic performance than closed loop VVVF.

Single-phase induction motors

Magnetic field due to single-phase stator current.

- The magnetic field set up by a phase current I is a square wave, which has fundamental component.

$$B(\theta) = \hat{B}\cos\theta, \quad B \propto I.$$

For an AC current $I = \hat{I}\cos(\omega t)$, the resultant air gap magnetic field due to a single phase stator current

$$B(\theta, t) = \hat{B}'\cos\theta\cos(\omega t)$$

- The pulsating magnetic field can be decomposed into two rotating magnetic fields rotating in opposite dir.

$$B(\theta, t) = \hat{B}'\cos\theta\cos(\omega t) = \frac{\hat{B}'}{2}\cos(\omega t+\theta) + \frac{\hat{B}'}{2}\cos(\omega t-\theta)$$

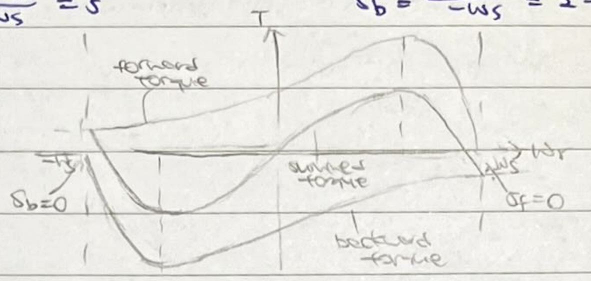
Single-phase induction machine (IM)

- The pulsating magnetic field due to a single phase can be decomposed into two rotating magnetic fields, one rotating in the +ve dir θ, the other rotating in the -ve dir. $-\theta$.
- Each rotating magnetic field can generate a torque as in a three-phase IM.

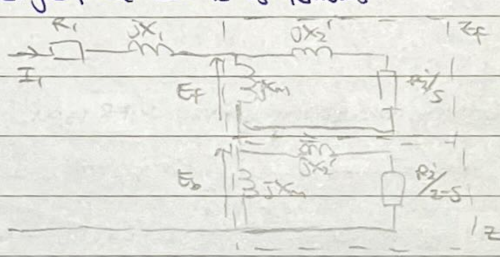
→ resultant torque is the superposition of the forward torque and backward torque.

- There are two synchronous speeds (forward ω_s, backward $-\omega_s$) but only one mechanical speed ω_r, so we have two slips, forward slip S_f and backward slip S_b.

$$S_f = \frac{\omega_s - \omega_r}{\omega_s} = s \qquad S_b = \frac{-\omega_s - \omega_r}{-\omega_s} = 2 - s$$



- The summed torque has zero starting torque → cannot start itself, req. additional starters.
- The summed torque at any speed is reduced than that in three-phase IM since (i) each directional torque is 1/3 of that in three phase, (ii) opp. dir. of torques give reduced summed value.
- The summed torque at synchronous speed is a braking torque ($T<0$) and the no-load speed is lower than synchronous speed.
- The equivalent circuit of a single-phase IM is as follows:



The forward and backward branch impedances Z_f, Z_b are given by

$$Z_f = \frac{jX_m(R_2'/s + jX_2)}{jX_m + (R_2'/s + jX_2)} \qquad Z_b = \frac{jX_m(R_2'/(2-s) + jX_2)}{jX_m + (R_2'/(2-s) + jX_2)}$$

Neglecting stator losses and F&W losses, the forward and backward torques T_f, T_b are given by

$$\boxed{T_f\omega_s = |I_1|^2 Re[Z_f]} \qquad \boxed{T_b\omega_s = |I_1|^2 Re[Z_b]}$$

3-phase IM, $T\omega_s = 3I_2'^2 R_2'/s$

- $T\omega_s$ are power losses, found by # of phase × real power dissipated in "R_2/s"
- The total power loss is

$$T\omega_s = (T_f - T_b)\omega_s$$

and thus the o/p power P_{out} is,

$$\boxed{P_{out} = T\omega_r = T\omega_s(1-s) = (T_f - T_b)\omega_s(1-s)}$$

- The net torque $T = T_f - T_b$ is zero when $T_f = T_b$, i.e.

$$\boxed{Re[Z_f] = Re[Z_b]}$$

Solving this gives $s = 0$ or $s = \pm\sqrt{1 - \left(\frac{R_2'}{X_2 + X_m}\right)^2}$

Starting single-phase IM.

- The magnetic field due to the main winding is pulsating $B_M(\theta,t) = \hat{B}_M \cos(\omega t)\cos\theta$.
- Consider adding a starter winding w/ a temporal displacement ϕ of the current and spatial displacement α of the winding. The magnetic field due to the starter winding is

$$B_S(\theta,t) = \hat{B}_S \cos(\omega t - \phi)\cos(\theta - \alpha)$$

The resultant magnetic field is also pulsating, and can be decomposed into forward/backward components.

$$B(\theta,t) = B_M(\theta,t) + B_S(\theta,t) = \hat{B}_f \cos(\omega t - \theta - \gamma_f) + \hat{B}_b \cos(-\omega t + \theta - \gamma_b)$$

where γ_f and γ_b can be adjusted by ϕ and α for sufficient starting torque.

- The magnitude of each rotating field is given by

$$\hat{B}_f = \sqrt{\hat{B}_M^2 + \hat{B}_S^2 + 2\hat{B}_M\hat{B}_S\cos(\phi - \alpha)} \qquad \hat{B}_b = \sqrt{\hat{B}_M^2 + \hat{B}_S^2 + 2\hat{B}_M\hat{B}_S\cos(\phi + \alpha)}$$

To have max. forward field and min. backward field, we req.

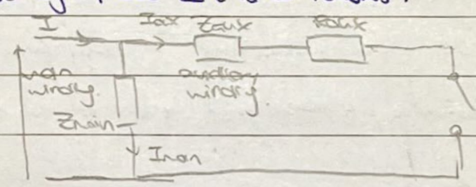
$$\cos(\phi - \alpha) = 1, \ \cos(\phi + \alpha) = 1 \quad \rightarrow \quad \phi - \alpha = 0, \ \phi + \alpha = 180^\circ$$

i.e. we want a $\alpha = 90^\circ$ spatial leading displacement of the starter winding and a $\phi = 90^\circ$ leading phase angle of the current at the starter winding for max starting torque.

- The spatial displacement α is normally fixed once machine is made, and we typically set $\alpha = 90^\circ$.
- To shift the leading phase angle of the starter winding current ϕ from 0° to 90°, we can use (i) split-phase or (ii) capacitor-start.

Split-phase single-phase IM.

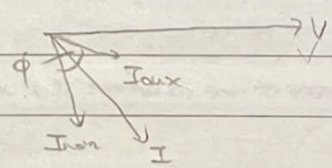
- A diagram of the split-phase single-phase IM is as follows:



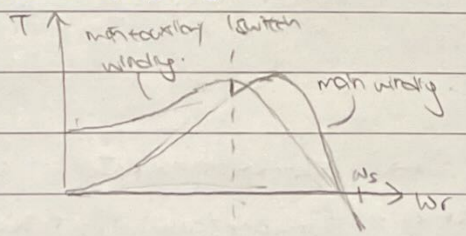
The phase angle btwn I_{aux} and I_{main} is given by.

$$\phi = \angle \frac{V}{Z_{aux} + R_{aux}} - \angle \frac{V}{Z_{main}} < 90^\circ$$

The phasor diagram is as follows:

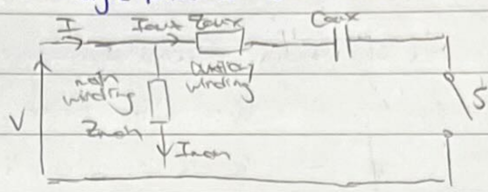


The torque speed characteristic is as follows:



Capacitor-start single-phase IM.

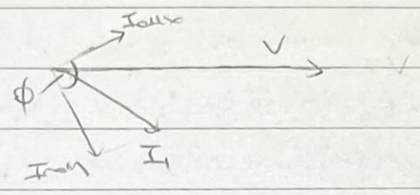
- A diagram of the capacitor-start single phase IM is as follows:



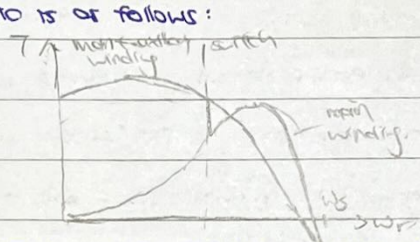
The phase angle btwn I_{aux} and I_{main} is given by

$$\phi = \angle \frac{V}{Z_{aux} + \frac{1}{j\omega C_{aux}}} - \angle \frac{V}{Z_{main}} \approx 90^\circ$$

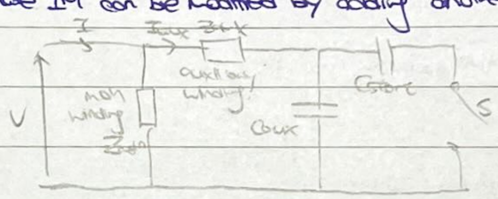
The phasor diagram is as follows:



The torque-speed characteristic is as follows:



- The capacitor-start single-phase IM can be modified by adding another capacitor



The capacitor-start capacitor-stop motor gives greater torque (C_{aux} min. backwards torque) at the expense of greater losses in C_{aux} (Also, we req. Z_{aux}, C_{aux} to endure large currents).

- C_{start} is still needed as we need high capacitance for starting.

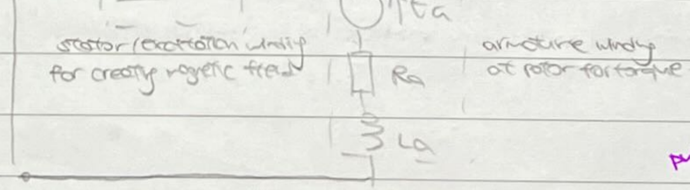
AC series motor / Universal motor

AC series motor / Universal motor

- The AC series motor can be fed by either DC or AC source (so it is also called universal motor)

The working principal of AC series motor is the same as the DC motor.

- The AC series motor has the same structure as the DC machine, and has the equivalent circuit



unidirectional, pulsating at twice supply freq.

$$\phi = L_f I_a \quad \rightarrow \quad E_a = k\phi\omega_r = kL_f I_a \omega_r \ ; \quad T = k\phi I_a = kL_f I_a^2$$

For AC, $I_a = \hat{I}\cos(\omega t - \phi)$: $E_a = kL_f\omega_r \hat{I}\cos(\omega t - \phi)$; $T = kL_f \hat{I}^2 \cos^2(\omega t - \phi) = \frac{1}{2}kL_f\hat{I}^2(1 + \cos 2(\omega t - \phi))$