Radio

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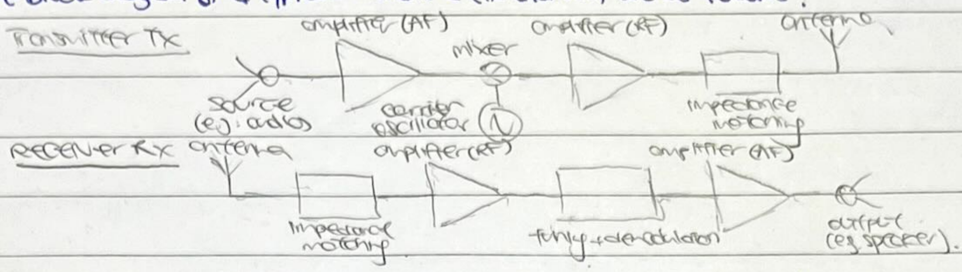
- Radio uses a carrier wave (high freq. signal) which is modulated by an information signal (audio/digital data).

Common modulation schemes include AM, FM, PSK.

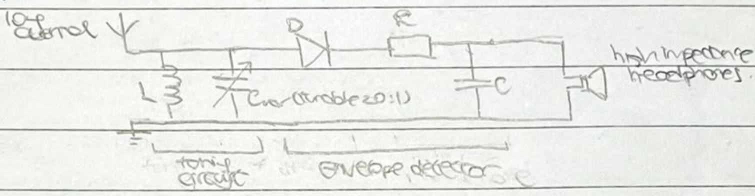
- The block diagram of a typical radio link (Tx and Rx) are as follows.



- To allow efficient use of the EM spectrum, channels are allocated w/ special carrier freq. and BW for diff. applications.

Crystal set receiver.

- The circuit for the crystal set receiver is as follows:



- The radio station is selected by tuning the LC resonant circuit to the right freq, where an AC voltage appears (w/ AM) across them. The envelope (desired audio signal) is then retrieved using the diode + LPF.

- This arrangement has the following drawbacks

↳ Low power o/p : No gain → only a fraction of RF power is available as audio o/p.

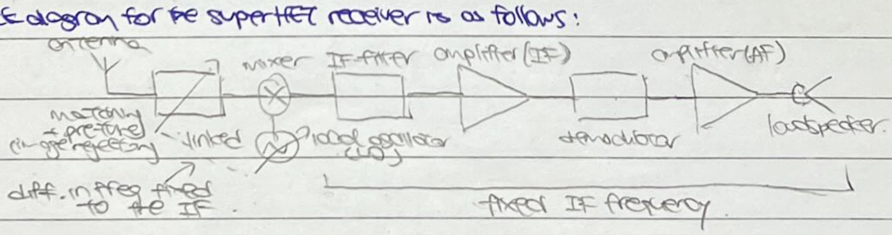
↳ Limited tuning range : C_{var} varies 20 : 1 → f varies $\sqrt{20}$: 1.

↳ Low selectivity : Low Q → can hear neighbouring freq.

↳ BW varies w/ freq : $B = \frac{f_0}{Q}$ → depends on f_0.

Superheterodyne receiver (super HET)

- The block diagram for the super HET receiver is as follows:



- The mixer "moves" the selected carrier freq. down to a fixed IF at which the rest of the circuit operates.

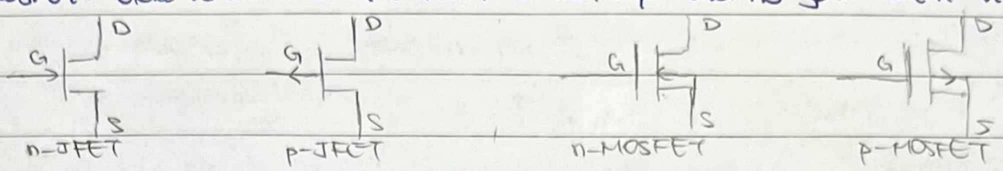
→ gain and BW of the downstream circuitry is const. regardless of the freq. to which the radio is tuned to.

Transistors

Field effect transistors (FET)

Field effect transistors (FET)

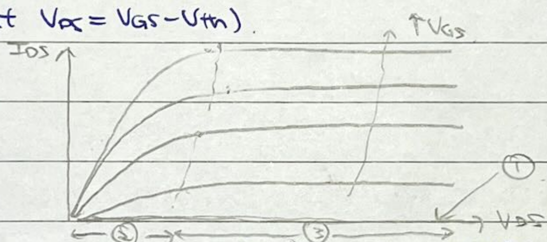
- FETs are voltage-controlled devices, where the E-field from the gate influences the passage of charge carriers along a channel btwn source and drain.
- There are two types of FETs : JFET and MOSFET
  - ↳ JFET: Gate is a reverse-biased junction in normal ops and negligible gate current flows
  - ↳ MOSFET: Gate is isolated by a thin insulation layer and no gate current flows.



FET characteristics

- When V_{GS} rises above a threshold offset voltage V_{th}, then the transistor begins to conduct a current I_{DS} btwn the drain and source, w/ voltage V_{DS} across it.
- There are three regimes of interest: — where $K = \frac{C_i \mu_{eff} W}{2L}$
  - ↳ 1) $V_{GS} - V_{th} < 0$; $I_{DS} = 0$ [Device is off]
  - ↳ 2) $V_{GS} - V_{th} \geq V_{DS}$; $I_{DS} = 2K\left[(V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}\right]$ [Voltage-controlled resistor (linear)]
  - ↳ 3) $V_{GS} - V_{th} \leq V_{DS}$; $I_{DS} = K(V_{GS} - V_{th})^2$ [Voltage-controlled current source (saturation)]

(Note ③ is just ② but $V_{DS} = V_{GS} - V_{th}$).

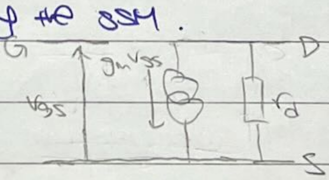


- The FET therefore has a no. of characteristics and applications which differ from BJTs:
  - ↳ High i/p impedance Z_{in}
  - ↳ Used as electrically-programmed variable resistor
  - ↳ Can be used w/ $V_{DS} \approx 0$ → switch
  - ↳ Voltage gain inferior to BJT.

* The IGBT has the i/p characteristics of a FET and o/p characteristics of a BJT → good for medium speed (~μs) and high power switching.

Small signal model [saturation regime, $V_{GS} - V_{th} \leq V_{DS}$].

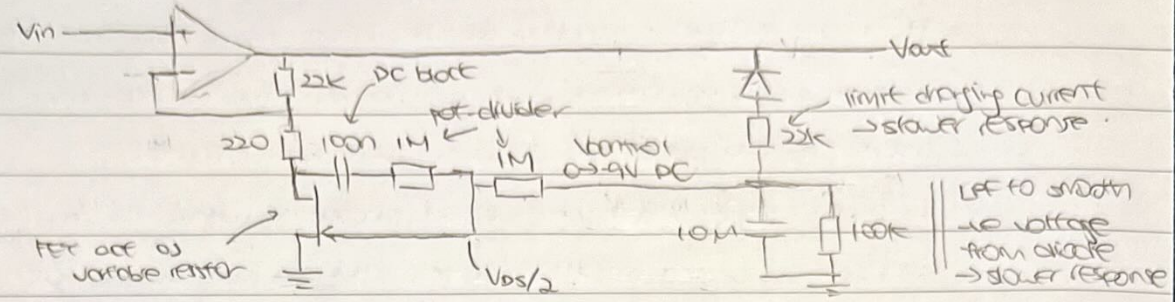
- The FET can be modelled using the SSM.



- The FET is not normally used as a plain amplifier as its gain is less than a BJT w/ similar I_{DS}. However, it has almost infinite i/p impedance Z_{in}, → ideal i/p stage for an amplifier which does not load the circuit it is connected to (pH probes, bio-potential probes)

Voltage-controlled attenuator [linear regime, $V_{GS}-V_{tn} \geq V_{DS}$]

- The circuit for a voltage-controlled attenuator is as follows:



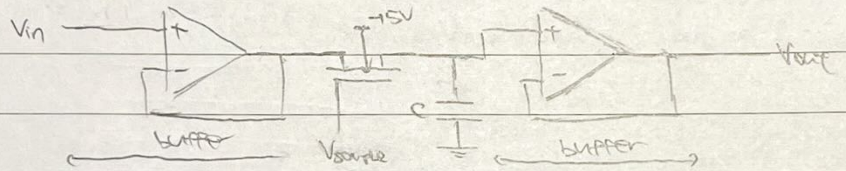
- The FET is in the linear regime and acts as a variable resistor

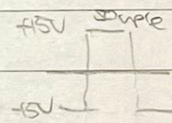
$V_{GS}-V_{tn} \geq V_{DS}$: $\quad R_{DS} = \frac{V_{DS}}{I_{DS}} = \frac{1}{2k\left[(V_{GS}-V_{tn})-\frac{V_{DS}}{2}\right]}$ small non-linearity

- The 1MΩ resistors act as potential dividers to introduce an additional $\frac{V_{DS}}{2}$ at the gate to remove the nonlinearity of the FET, 100nF capacitor acts as a DC block.
- Vcontrol is linked to the o/p via a LPF. $\uparrow|V_{out}| \rightarrow \downarrow V_{control} \rightarrow \downarrow V_{GS}-V_{tn} \rightarrow \uparrow R_{DS} \rightarrow \downarrow$Gain
- The LPF smooths/averages -ve voltage from diode so Vcontrol does not change too quickly and alter the gain within an audio cycle (and distort the waveform).

Sample and hold circuit [linear regime, $V_{GS}-V_{tn} > V_{DS}$]

- The circuit for the sample and hold (analog switch) circuit is as follows:



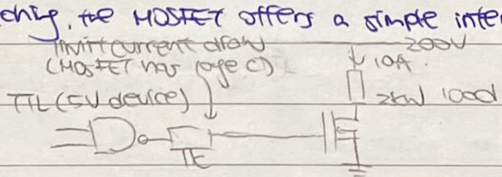


- The FET is in the linear regime and acts as an analog switch
- When Vsample is HIGH, the switch is closed → Vout = Vin

When Vsample is LOW, the switch is open → Vout freezed by capacitor.
- Setting substrate connection to the most -ve potential prevents parasitic latch-up on IC devices.

Interfacing between logic signals and high power load

- For heavy power switching, the MOSFET offers a simple interface btwn TTL and high power load.

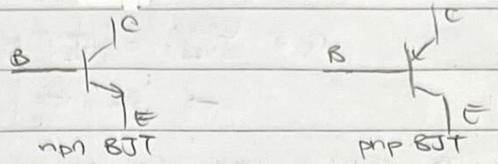


- The FET is in the linear regime and acts as an analog switch
- Note power MOSFETs have large gate capacitance. (High C_{GS} and Miller effect on C_{GD}) ~100 pF → rapid switching from TTL not viable → we would req. specialised IC drivers.

Bipolar Junction transistors (BJT)

Bipolar junction transistors (BJT)

- BJTs are current-controlled devices, where the current at the base influences the passage of charge carriers btwn emitter and collector.



The Ebers-Moll model.

- We can model the BJT using the Ebers-Moll model.

$$\boxed{I_C = I_S\left[\exp\left(\frac{qV_{BE}}{k_BT}\right) - 1\right] \approx I_S\exp\left(\frac{V_{BE}}{V_t}\right)}$$

where I_S is the saturation current (determined by transistor construction)

and V_t is the thermal voltage, given by $V_t = \frac{k_BT}{q}$ ($V_t = 25mV$ at room temp.)

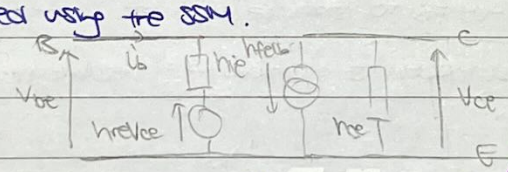
- This is similar in form to the pn diode eqn. For the base-emitter diode,

$$\boxed{I_B = I_S'\left[\exp\left(\frac{qV_{BE}}{k_BT}\right) - 1\right] \approx I_S'\exp\left(\frac{V_{BE}}{V_t}\right)}$$

- Therefore the current gain of the transistor $h_{FE} = \frac{I_C}{I_B} = \frac{I_S'}{I_S}$ is roughly const. (over a limited range) h_{FE} is often ~100-500 for low current, small signal devices.
- The current gain h_{FE} increases as temp. T increases (cannot tell from Ebers-Moll since both I_S and V_t depend on temp. T).
- Due to the exponential nature of Ebers-Moll, a small $\uparrow V_{BE}$ can result in large $\uparrow I_C$ (At room temp. $\uparrow V_{BE}$ by 60mV $\rightarrow \uparrow I_C \times 10$).

Small signal model

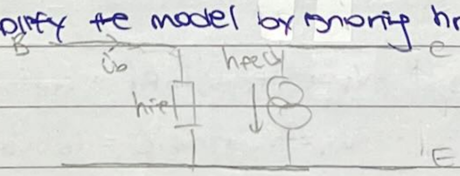
- The BJT can be modelled using the SSM.



due to Early effect

and we would often simplify the model by ignoring h_{re} and h_{oe}

much larger than R_L



* The simplified model only results in a few % error → okay in most cases.

→ e.g. For BC108 @ $I_C = 2mA$, $V_{CE} = 5V$, $h_{fe} = 300$, $h_{ie} = 3\times10^3\Omega$, $h_{re} = 2\times10^{4}$, $h_{oe} = [4\times10^4\Omega]^{-1}$

consider the case $V_{cc} = 10V$, $R_L = 2500\Omega$, $V_{ce} = 1V$

Ignore h_{oe}: $\hat{R} = 2.5k\Omega$, $R = \frac{2.5k \cdot 40k}{2.5k + 40k} = 2.353k\Omega$. so $\% R = \frac{2.5k - 2.353k}{2.5k} = 5.9\%$.

Ignore h_{re}: $V_{ce}h_{re} = 0.2mV$, $V_{be} = i_b h_{ie} \approx \frac{I_e}{h_{fe}} h_{ie} = \frac{V_{ce}}{R_L}\frac{h_{ie}}{h_{fe}} = 4mV$

so $\% V_{be} = \frac{0.2m}{4m} = 5\%$.

Emitter resistance model

- The emitter resistance r_e is defined as

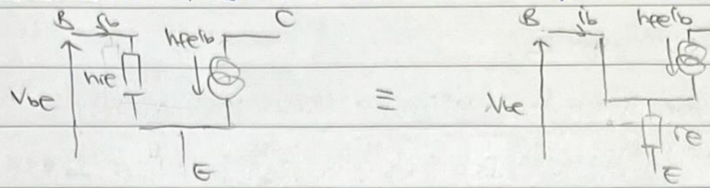
$$r_e = \frac{\partial V_{BE}}{\partial I_C} \simeq \frac{dV_{BE}}{dI_C}$$

- From Ebers Moll eqn, $I_C = I_S\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right] \approx I_S\exp\left(\frac{V_{BE}}{V_t}\right)$, we have

$$r_e = \left(\frac{dI_C}{dV_{BE}}\right)^{-1} = \left[\frac{1}{V_t} I_S\exp\left(\frac{V_{BE}}{V_t}\right)\right]^{-1} = \frac{V_t}{I_C}$$

$$\rightarrow \boxed{r_e = \frac{V_t}{I_C}}$$

- The emitter resistance model is equivalent to the input impedance model



For equivalence of V_{be}, we req. V_{be} and i_c to match in the two models.

$$V_{be} = i_b h_{ie} = h_{fe} i_b r_e$$

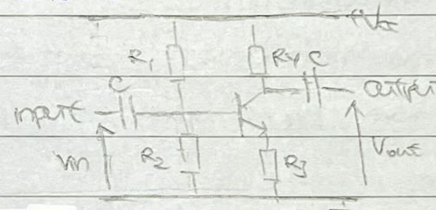
$$\rightarrow \boxed{r_e = \frac{h_{ie}}{h_{fe}}}$$

- Analysis for Z_{in} easier using h_{ie} model ; Analysis for Z_{out} easier using r_e model.

Simple bipolar transistor amplifier design

Standard amplifier circuit.

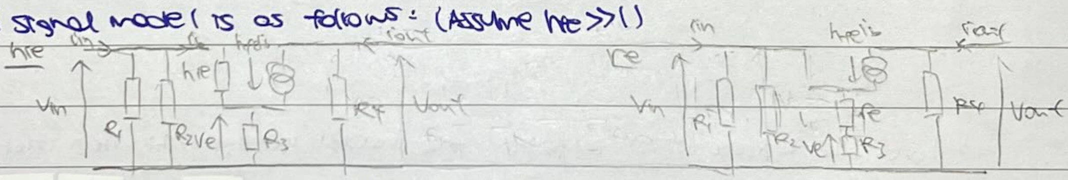
- The standard amplifier circuit is as follows:



↳ C are coupling capacitors (DC block) ↳ R_1, R_2 provide DC base bias current

↳ R_3 gives -ve FB to stabilise bias pt. ↳ R_4 allows for o/p voltage swing

- Small signal model is as follows: (Assume $h_{fe} \gg 1$)



At the input,

$$v_e = (1+h_{fe}) i_b R_3 \simeq h_{fe} i_b R_3$$

$$v_{in} = v_e + h_{ie} i_b = i_b(h_{fe}R_3 + h_{ie})$$

$$\Big] \; \frac{v_e}{v_{in}} = \frac{R_3}{R_3 + r_e}$$

$$R_{in} = R_1 // R_2 // \frac{v_{in}}{i_b} = R_1 // R_2 // (h_{fe}R_3 + h_{ie}) = R_1 // R_2 // h_{fe}(R_3 + r_e)$$

At the output,

$$v_{out} = -(h_{fe} i_b - i_{out}) R_4 = -\frac{h_{fe}R_4 v_{in}}{h_{fe}R_3 + h_{ie}} + R_4 i_{out}$$

$$\text{Gain} = \frac{\partial v_{out}}{\partial v_{in}} = -\frac{h_{fe}R_4}{h_{fe}R_3 + h_{ie}} = -\frac{R_4}{R_3 + r_e}$$

$$R_{out} = \frac{\partial v_{out}}{\partial i_{out}} = R_4$$

$\Rightarrow$ Key results : $\boxed{\text{Gain} = -\frac{R_4}{R_3 + r_e}}$, $\boxed{R_{in} = R_1 // R_2 // h_{fe}(R_3 + r_e)}$, $\boxed{R_{out} = R_4}$, $\boxed{\frac{v_e}{v_{in}} = \frac{R_3}{R_3 + r_e}}$

Designing a multi-stage bipolar transistor amplifier.

① selecting the no. of stages, gain and i/p/o/p impedances.

- The required power gain (in dB) is given by

$$\boxed{\text{Power gain (dB)} = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)}$$

- Assuming each transistor stage gives a gain of 20dB, we can find the no. of transistor stages. N

$$N = \frac{\text{power gain (dB)}}{20}$$

* we round N to the nearest integer.

- The no. of couplings b/wn the source and load is given by N+1. Assuming the i/p and o/p impedances are matched, we lose 50% of the signal voltage at each coupling.

$$\boxed{\text{Total gain product} = \frac{V_{out}}{V_{in}} \times \frac{1}{(0.5)^{N+1}}}$$

if $R_4 \neq R_{out}$ → coupling loss ≠ 0.5, then find via potential divider.

therefore the req. voltage gain of each transistor stage is given by.

$$\text{Voltage gain per stage} = (\text{Total gain product})^{1/N}$$

* The voltage gain calculated is the min. gain → usually ×1.2 to have a margin.

- we choose the i/p and o/p impedance of each stage s.t. the source/load impedances are blended across the N transistor stages.

$$\boxed{\frac{R_{out}}{R_{in}} = \left(\frac{R_{load}}{R_{source}}\right)^{1/N}}$$

* Note $R_{in(1)} = R_{source}$ and $R_{out(N)} = R_{load}$.

② selecting resistor values.

- Consider the following rules of thumb for steady DC voltages req. on the transistors for correct biasing,

$$V_C = \frac{V_{CC}}{2} \qquad V_E \approx \frac{V_{CC}}{20} \qquad V_B = (V_E + \underbrace{0.7}_{V_{BE}})\underbrace{\times 1.2}_{\text{margin for base loading}}$$

Recall for the standard amplifier circuit (one stage), and assume that $R_3 \gg r_e$ ← ignoring r_e is okay, even for the high current stage.

$$\text{Gain} = -\frac{R_4}{R_3 + r_e} \approx -\frac{R_4}{R_3} \qquad R_{in} = R_1 // R_2 // h_{fe}(R_3 + r_e) \approx R_1 // R_2 // h_{fe}R_3 \qquad R_{out} = R_4$$

(i) Find R_{in} and R_{out} for each stage using the ratio $\frac{R_{out}}{R_{in}}$ found above + $R_{in(1)} = R_{source}$ / $R_{out(N)} = R_{load}$.

(ii) Use $R_{out} = R_4$ to find R_4

(iii) Use $\text{Gain} = -\frac{R_4}{R_3}$ to find R_3

(iv) Use $R_2 = 1.5 - 2 \times R_{in}$ to find R_2 (rule of thumb)

(v) Use $V_B = \frac{R_2}{R_1 + R_2} V_{CC}$ to find R_1, where $V_B = (V_E + 0.7) \times 1.2$ and $V_E \times \frac{V_{CC}}{20}$.

* If V_{CC} is not given, then

(i) calculate the o/p voltage swing using $V_o = I_{max} R_{load}$

(ii) we req. V_{CC} at least $2 \times V_o$ to avoid clipping → choose $V_{CC} = 4 \times V_o$ to introduce a safety factor of 2

* If I_C is too low (nA), reduce R_4 to get a more stable bias pt. ← be careful w/ calculating the req. voltage gain as coupling loss ≠ 1/2

* For multi-stage amplifier, we can simply divide the $R_1 - R_4$ values by the $\frac{R_{out}}{R_{in}}$ ratio to avoid recalculating for each stage.

at exam, 3B1 EP1 Q2 more stage

③ selecting capacitor values.

- The series capacitors for AC signal coupling btwn stages are selected in combination w/ the i/p and o/p impedances to give a low freq. response rolloff.

$$\boxed{f_{3dB} = \frac{1}{2\pi(R_{out(n)} + R_{in(n+1)})C}}$$

formula for coupling circuits w/ low freq. response rolloff (high-pass), in databook.

* Note for audio applications, we typically choose $f_{3dB} = 20\text{Hz}$. If the amplifier circuit is req. to operate down to f_{low}, we choose $f_{3dB} = \overset{\text{safety margin}}{\underline{0.5}} \times f_{low}$

* For multi-stage amplifier, as w/ resistor values, we can simply scale using the $\frac{R_{out}}{R_{in}}$ ratio.

④ selecting the transistor.

- we choose the highest h_{fe} device available which can handle the collector current I_C, supply voltage V_{CC} and power dissipation $I_C \cdot \overset{V_{out}}{\frac{V_{CC}}{2}}$, all w/ a safety margin of 50%.

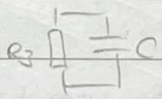
* For Tripos, h_{fe} is usually given and we do not need to select the transistor.

⑤ check the effect of low h_{fe}, particularly for the high current stage.

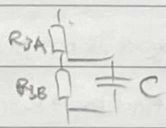
- For the high current stage, the transistor h_{fe} reduces.
  (i) i/p impedance R_{in} decreases ($R_{in} = R_1 // R_2 // h_{fe}R_3$) → signal coupling lower than expected, but is compensated by the margin of the gain.
  (ii) base current i_b req. increases ($\downarrow h_{fe}$, I_C fixed) → decrease R_1, R_2 values to the revised $R_3 h_{fe}$ value.

R_3 as negative feedback.

- The resistor R_3 provides -ve feedback to improve the stability of the circuit (inc. temp. effects and the Early effect (coupling btwn V_{CE} and V_{BE}, i.e. $\Delta V_{CE} \rightarrow \Delta V_{BE}$))
  (e.g. $\downarrow V_B \rightarrow \downarrow I_C \rightarrow \downarrow I_C R_3 \approx V_E \rightarrow V_{BE}$ const., and vice versa → stable bias conditions)
- Since gain $= -\frac{R_4}{R_3}$, the resistor R_3 reduces the gain. However, we can shunt R_3 w/ a capacitor to effectively remove it from SSM at signal freq.



At high freq, the above configuration may not be stable, so we can split the resistor as follows



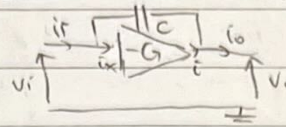
* We choose the capacitor C to give a low freq. response rolloff

$$f_{3dB} = \frac{1}{2\pi(R_3 // r_e)C} \qquad f_{3dB} = \frac{1}{2\pi(R_{3b} // (R_{3a} + r_e))C}$$

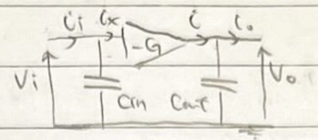
Bipolar transistor frequency response

Miller effect.

- Consider an inverting amplifier (w/ gain −G) w/ some capacitance btwn its i/p and o/p. assume it is ideal, i.e. $i_x = 0$.



we want to find an equivalent capacitor at i/p (to GND) and o/p (to GND)



- summing the currents at the i/p node, (noting $i_x = 0$ for an ideal amplifier)

$$i_i = \frac{V_i - V_o}{1/j\omega C} = \frac{V_i + GV_i}{1/j\omega C} = \frac{V_i}{1/j\omega(1+G)C}$$

$$\therefore Z_{in} = \frac{\partial V_i}{\partial i_i} = \frac{1}{j\omega(1+G)C}$$

i.e. the i/p capacitance is equivalent to $\boxed{C_{in} = (1+G)C}$.

- summing the currents at the o/p node,

$$i_o = i + i_i = i + \frac{V_i - V_o}{1/j\omega C} = i - \frac{V_o/G + V_o}{1/j\omega C} = i - \frac{V_o}{1/j\omega(1+1/G)C}$$

$$\therefore Z_{out} = \frac{\partial V_o}{\partial i_o} = \frac{1}{j\omega(1+1/G)C}$$

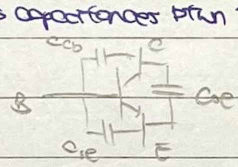
i.e. the o/p capacitance is equivalent to $\boxed{C_{out} = (1+1/G)C \approx C}$.

* The magnification of the capacitance is the Miller effect, and mainly affects the i/p.

$\uparrow C \rightarrow \uparrow \tau = RC \rightarrow$ lower upper 3dB freq. (smaller BW)

Bipolar transistor terminal capacitances.

- In practice, transistors have various capacitances btwn their terminals. For the BJT,



C_{cb} and C_{ce} are due to a reverse-biased junction and are typically 1–10 pF each.

C_{ie} is due to a forward-biased junction and is typically 10–100 pF.

- Actually, C_{ie} depends on the current through the transistor (depletion width in pn junction changes w/ bias), so it is not a single-valued parameter. In fact $C_{ie} \propto I_E \rightarrow C_{ie} = kI_E \approx kI_C$.

- A figure of merit for transistors, f_T, represents the gain-BW product, and is given by,

$$\boxed{f_T = \frac{1}{2\pi C_{ie} r_e}} = \frac{1}{2\pi(kI_E)(0.025/I_C)} = \frac{1}{0.05\pi k} \text{ (const.)}$$

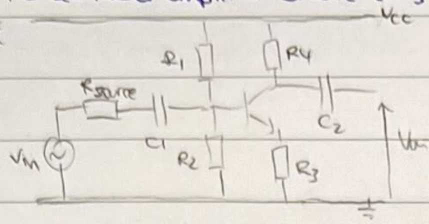
$\rightarrow$ Gain and freq. characteristics of a transistor can be described by just h_{fe}, f_T.

* usually, find I_C using $I_C = \frac{V_{CC}/2}{R_4}$ $\rightarrow$ find r_e using $r_e = \frac{V_T}{I_C}$ $\rightarrow$ find C_{ie} using $f_T = \frac{1}{2\pi C_{ie} r_e}$

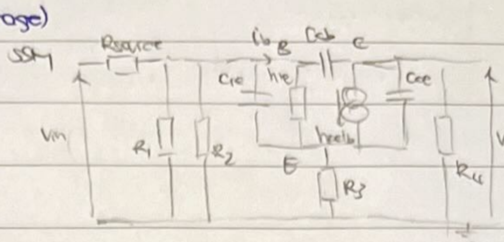
Frequency response of standard amplifier circuit.

- consider the standard amplifier circuit (single stage)

circuit



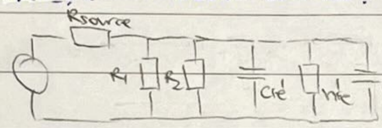
SSM



Note that C_1 and C_2 act as short circuits at SSM signal freq. ∴ C_{cb} and C_{oe} given in datasheet and h_{ie} and C_{ie} need to be calculated ($I_C = \frac{V_{cc}/2}{R_4} \rightarrow r_e = \frac{V_T}{I_C} \rightarrow h_{ie} = h_{fe}r_e$ / $C_{ie} = \frac{1}{2\pi f_T r_e}$)

- We can refer impedances connected btwn B-C / B-E / C-E to B-GND / C-GND using $\frac{V_o}{V_i}, \frac{V_e}{V_i}, \frac{V_e}{V_o}$.
  - ↳ $\frac{V_o}{V_i} = -\frac{R_4}{R_3+r_e} \rightarrow V_{bc} = V_i - V_o = V_i + \frac{R_4}{R_3+r_e}V_i = (1+\frac{R_4}{R_3+r_e})V_i = (1+G)V_i \rightarrow Z_b = \frac{Z_{bc}}{1+G}$
  - ↳ $\frac{V_i}{V_o} = -\frac{R_3+r_e}{R_4} \rightarrow V_{cb} = V_o - V_i = V_o + \frac{R_3+r_e}{R_4}V_o = (1+\frac{R_3+r_e}{R_4})V_o = (1+\frac{1}{G})V_o \rightarrow Z_c = \frac{Z_{cb}}{1+1/G}$
  - ↳ $\frac{V_e}{V_i} = \frac{R_3}{R_3+r_e} \rightarrow V_{be} = V_i - V_e = V_i - \frac{R_3}{R_3+r_e}V_i = \frac{r_e}{R_3+r_e}V_i \rightarrow Z_b = \frac{Z_{be}}{r_e/R_3+r_e}$
  - ↳ $\frac{V_e}{V_o} = \frac{V_e}{V_i}\frac{V_i}{V_o} = -\frac{R_3}{R_4} \rightarrow V_{ce} = V_o - V_e = V_o + \frac{R_3}{R_4}V_o = (1+\frac{R_3}{R_4})V_o \rightarrow Z_c = \frac{Z_{ce}}{1+R_3/R_4}$

- At the i/p, the equivalent SSM circuit becomes.

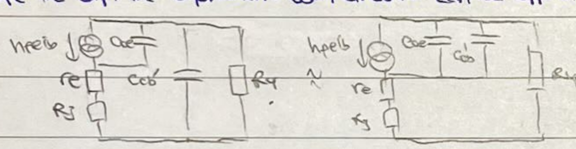


$C_{ie}' = \frac{r_e}{R_3+r_e}C_{ie}$, $h_{ie}' = \frac{h_{ie}}{r_e/R_3+r_e}$, $C_{cb}' = (1+G)C_{cb}$

$R' = R_{source} // R_1 // R_2 // h_{ie}$, $C' = C_{ie}' + C_{cb}'$

The upper freq. response roll-off is then given by $f_{3dB} = \frac{1}{2\pi R'C'}$

- At the o/p, the equivalent SSM circuit can be approx. as (since $R_4 \gg R_3 + r_e$)



$C_{cb}' = (1+1/G)C_{cb}$

$R' = R_3 + r_e + R_4$, $C' = C_{oe} + C_{cb}'$

The upper freq. response roll-off is then given by $f_{3dB} = \frac{1}{2\pi R'C'}$

- The overall upper freq. response is limited by the lower upper 3dB freq, which is typically the input.

Increasing the upper 3dB frequency.

- We can increase the upper 3dB frequency (and thus increase BW) by circumventing the Miller effect.
- To do so, we req. a pair of resistors.
  - (i) T_1 is connected to a high impedance source and has unity gain (↑R, ↓C → small τ)
  - (ii) T_2 is connected to a small impedance source and has a large o/p voltage swing. (↓R, ↑C → small τ)
- Common circuits using the above technique inc. the (modified) differential amplifier and cascade circuits.

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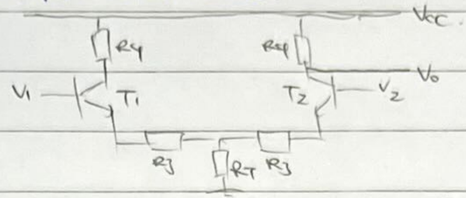
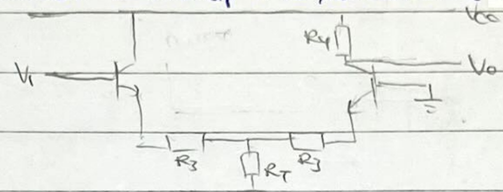
Differential amplifier.

- The circuit for the differential amplifier is as follows:



Note differential gain $= \frac{R_4}{2R_3}$, common-mode gain $= \frac{R_4}{R_3+2R_T}$, CMRR $= \frac{2R_T+R_3}{2R_3} \approx \frac{R_T}{R_3}$ for $R_T \gg R_3$.

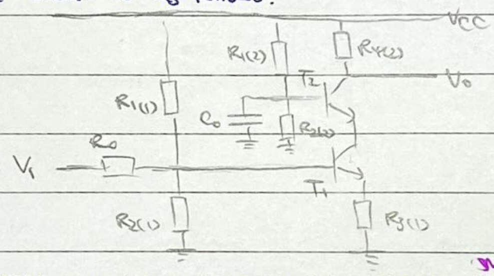
- To circumvent the Miller effect, we ground T2 (small R) and take o/p from its collector (large C), while we take the base of T1 (large R) as the i/p. Since we don't need the o/p of T1, we can short its collector (small C).



* V_1 causes current swing at T1 emitter → modulates through to T2 → voltage swing at T2 collector (o/p).

Cascode circuit.

- The circuit for the cascode circuit is as follows:



The Miller effect is circumvented since T2 is connected to a fixed voltage w/ low impedance (small R), w/ the o/p taken from its collector (large C). The i/p is taken at the base of T1 (large R), which does not have any o/p swing at its collector (small C) ($V_{C(1)}$ fixed. $V_{C(1)} \approx V_{B(2)} - 0.7$).

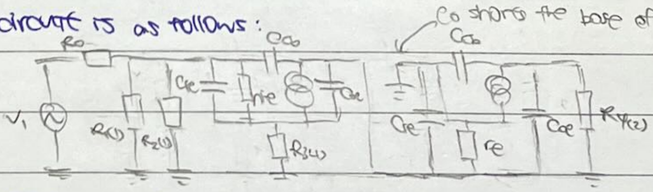
* V_1 causes current swing at T1 collector → modulates through to T2 → voltage swing at T2 collector (o/p).

* C_0 shorts the base of T2 to GND at SSM signal freq, and is chosen using low freq. response roll-off.

$$f_{3dB} = \frac{1}{2\pi(R_{1(2)}//R_{2(2)})C_0}$$

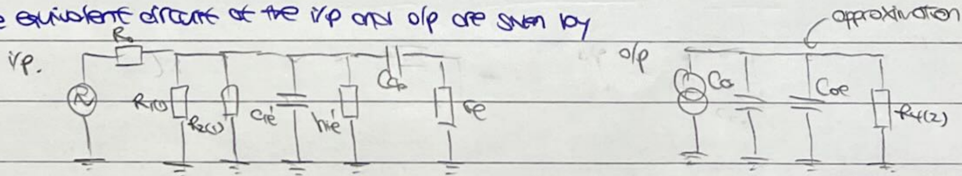
- The SSM for the cascode circuit is as follows:

Co shorts the base of T2 to GND.



check approx

The equivalent circuit of the i/p and o/p are given by



$R' = (R_0 // R_{1(1)} // R_{2(1)} // h_{ie} + r_e)$, $C' = C_{ie}' + C_{cb}$ $\qquad$ $R' = R_{4(2)}$, $C' = C_{cb} + C_{oe}$

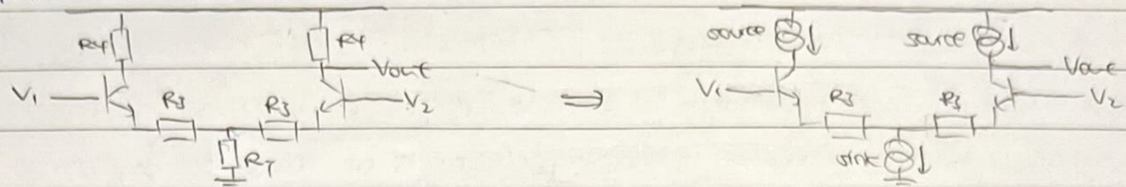
and the upper 3dB freq. is given by $f_{3dB} = \frac{1}{2\pi R'C'}$

Current sources, oscillators and filters

Current sources

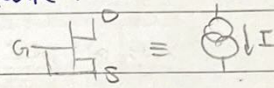
Circuit applications for current sources

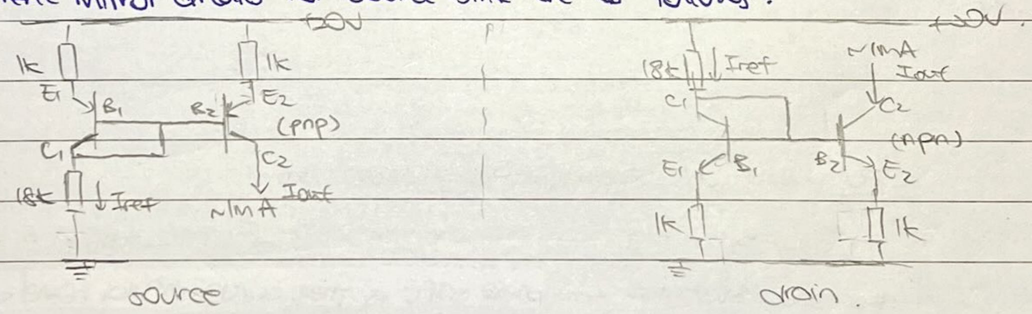
- In a no. of applications, the use of a current source instead of a simple resistor can improve circuit performance – consider the differential amplifier



- The CMRR is given by $CMRR = \frac{R_T}{R_3}$. If R_T is replaced by a current source ($R_T \to \infty$), then $CMRR \to \infty$ (w/ DC bias still applied)
- The differential gain is given by $DG = -\frac{R_4}{2R_3}$. If R_4 is replaced by a current source ($R_4 \to \infty$) then we get a very high gain.

Current mirror circuits

- A simple current source can be made using a simple FET circuit. However, for more accurate current sources, a current mirror can be used. 
- In a current mirror, a reference current is drawn through one of a matched pair of transistors to create a V_{BE} corresponding to that current.
- This V_{BE} is then applied to the other transistor, which aims to supply the same collector current as the first, through its own load.
- The current mirror circuits for source/sink are as follows:



B_1, B_2 connected ; E_1, E_2 same potential (by symmetry) $\rightarrow V_{BE1} = V_{BE2} \rightarrow I_{C1} = I_{C2}$,

Assume large $h_{FE} \rightarrow I_{B1}, I_{B2}$ negligible $\rightarrow I_{ref} = I_{C1} + I_{B1} + I_{B2} = I_{C1}$

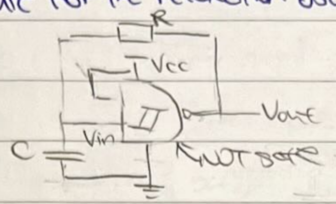
$I_{out} = I_{C2} = I_{C1} = I_{ref}$, where $I_{ref} = \frac{V_{CC} - V_{BE}}{1k + 18k}$ [Assume $V_{BE} = 0.7V$]

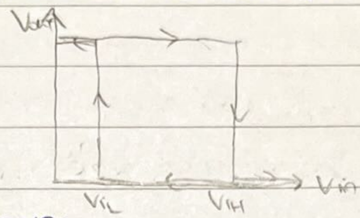
- To design a current source, we consider the current sink counterpart, then invert.
- In SSM analysis of the current mirror, $i_b = 0$ (useful for finding o/p impedance Z_{out}, which is simply the impedance btwn o/p terminal and GND).

Oscillators

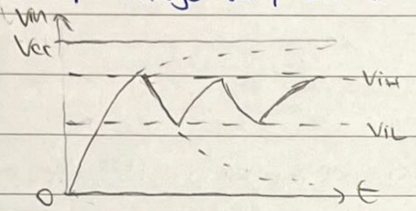
Relaxation oscillator

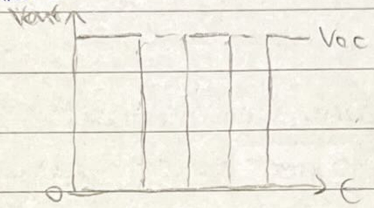
- The relaxation oscillator works by charging up a capacitor through a resistor and subsequently discharging it when a threshold is reached.
- For the circuit to work, the threshold detector must have hysteresis (level to turn it on must be higher than the level to turn it off). We can use the Schmitt trigger gate. *implemented via +ve FB.*
- The circuit for the relaxation oscillator and gate transfer characteristics are as follows:





- The i/p and o/p voltage vary over time as follows:

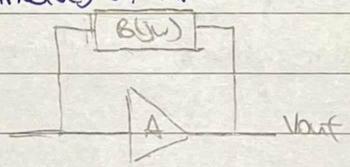




i.e. we get a square wave o/p. — this could be integrated to get a triangular wave o/p or filtered to give a sinusoidal o/p (though not very high precision).

Positive feedback oscillator

- The positive feedback oscillator involves an amplifier A and passive network $B(j\omega)$. *assume const. and does not change the phase*

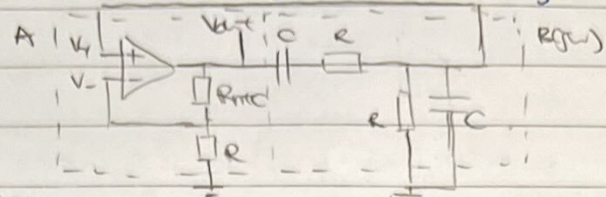


- The circuit will sustain stable oscillations when
  - ↳ (i) $AB(j\omega) = 1$
  - ↳ (ii) $\angle AB(j\omega) = 2k\pi, \ k \in \mathbb{Z}$
- For good stability, the phase Bode plot of $B(j\omega)$ should be a sharp function (small BW).
  The attenuation of the filter at operating freq. should not be too large as we would need large A.
- To start up the oscillations reliably and maintain them when driving a load, the loop gain $AB(j\omega)$ should be >1 when the oscillation amplitude is low (and vice versa).
- We can implement amplitude/gain control using a variable gain attenuator — oscillation amplitude monitored and used to control the gain of the feedback loop.

(To avoid distortion of the o/p sine wave, the amplitude control loop should be much slower than the oscillation frequency. i.e. use LPF smoothing filter w/ large time const τ)

Wien Bridge oscillator

- A common positive-feedback oscillator is the Wien bridge oscillator.



where $A = 1+\frac{R_{NTC}}{R}$ and $B(j\omega) = \frac{1}{3+j\omega CR+\frac{1}{j\omega CR}}$

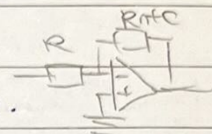
- To satisfy oscillation condition (ii), we req. $\angle B(j\omega) = 0$, which occurs when

$$j\omega CR + \frac{1}{j\omega CR} = 0 \quad \rightarrow \quad \omega = \frac{1}{RC}$$

At this frequency, to satisfy oscillation condition (i), we req $A\cdot\frac{1}{3} = 1$, which occurs when

$$1+\frac{R_{NTC}}{R} = 3 \quad \rightarrow \quad R_{NTC} = 2R.$$

- For amplitude control, a -ve temp. coeff. thermistor is used (↑ oscillation amplitude → ↑ power dissipated → ↑T → ↓R_{NTC} → ↓A, and vice versa).

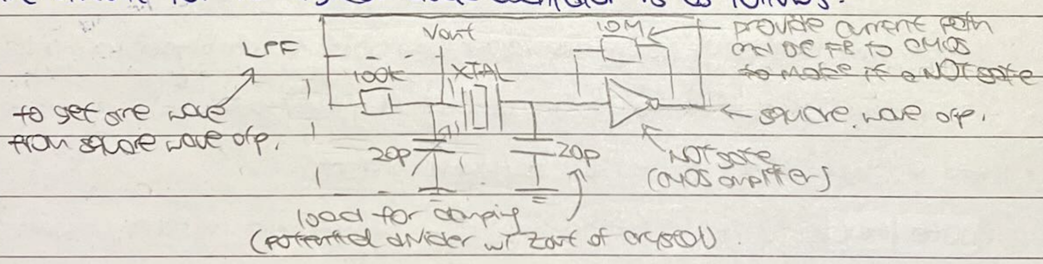


+ In general, to use R_{NTC} to provide -ve FB, place it s.t. gain ∝ R_{NTC}.

(Alternatively, a variable gain attenuator would work as well)

Digital clock oscillator.

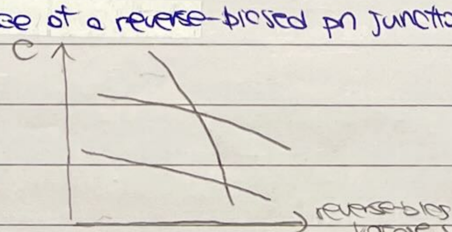
- To build a high stability oscillator for fixed freq. (eg. time references), a quartz crystal is usually used (R, C have non-zero temp. coeff → unstable w/ large temp. variations).
- The device is a specifically shaped piece of quartz which has a mechanical resonance at a particular freq. Since quartz is piezoelectric, we have an electrically resonant circuit.
- The Q-factor of a quartz crystal is typically $\sim 10^3$ and the temp stability is good $\sim 50\,ppm\,°C^{-1}$.
- The circuit for the digital clock oscillator is as follows:



RF oscillator.

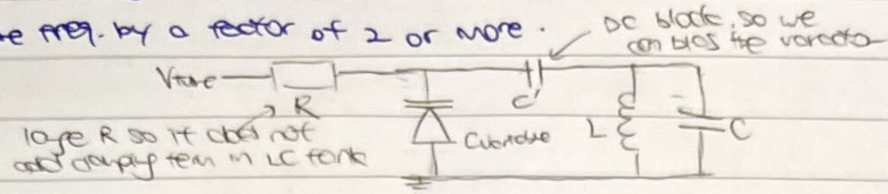
- To tune a radio oscillator (vary RC resonant freq.) electrically, we can use varactors (based on the variable capacitance of a reverse-biased pn junction)

change bias → change width of depletion region → vary C
[Early effect].



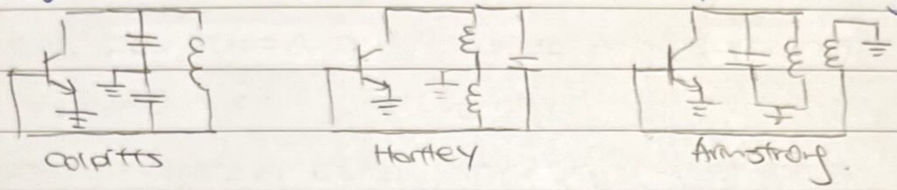
→ we get a voltage-controlled oscillator (VCO)

- By including the capacitance of a varactor in a LC tank circuit, we can tune the freq. by a factor of 2 or more.



so we have $C_{eff} = C + C_{varactor}$ (Note $C' \gg C_{varactor}$ → negligible)

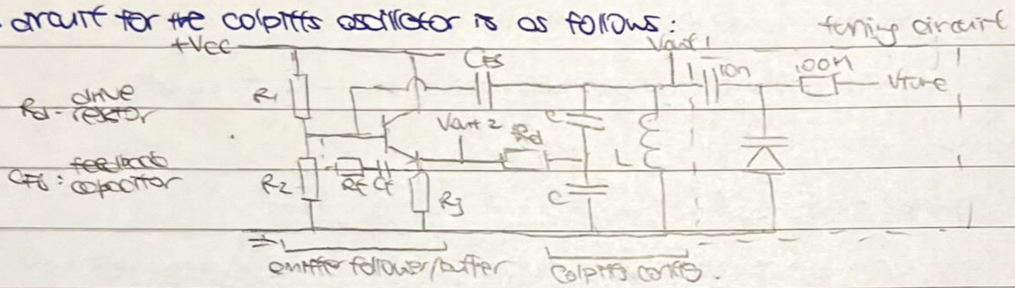
- Popular configurations for RF circuits include Colpitts, Hartley and Armstrong.



All of these can be tuned electronically by coupling a varactor across the LC resonant circuit.

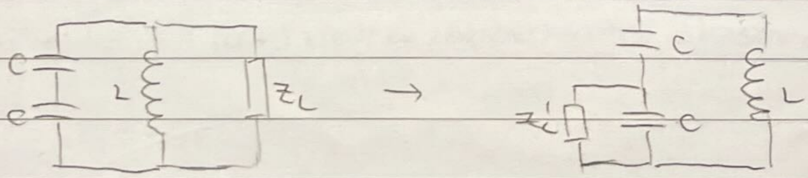
Colpitts oscillator

- The circuit for the Colpitts oscillator is as follows:



The use of two series capacitors across the inductor, to create the LC tank w/ separate drive and FB pts from/to the transistor is the hallmark of the Colpitts design.
(We have an emitter follower (gain ~1) driving the LC network (gain ~2) for the FB loop)

- We can take the o/p from two convenient pts: — reactive loads change resonant freq, resistive loads damp Q / stop oscillation
  - ↳ V_{out1} : min. distortion, but only for high impedance loads (o/w use a buffer)
  - ↳ V_{out2} : higher o/p power and better load isolation, but higher harmonic distortion — minimised using $R_E + C_E$
- If we add a base-emitter load impedance (R_E + DC blocking C_E), we set the emitter swing to ~2/3 supply voltage → less harmonic distortion.
- R_1, R_2 appear across LC tank in SSM → dampen the response, lower Q.
  C_{FB} act as a DC block to prevent L from shorting the base to GND.
- At resonance, the LC network acts as a 2:1 transformer, so a load R_L across the inductor (V_{out1}) referred to the emitter will be $\frac{Z_L}{2^2} = \frac{Z_L}{4}$.



Designing a Colpitts oscillator.

① Power level

- check that $V_{PP} < 60\%$ of V_{CC}.

② select transistor and set DC bias resistors.

- choose a low power NPN transistor w/ $f_T \gg$ req. freq. ($\sim 250\,MHz$)
- set base bias V_B to give $V_E \sim \frac{1}{2}V_{CC}$ for max. voltage swing, $V_E = V_B - 0.7$
- If we use V_{out2}, refer load impedance to emitter V_{out2}, $Z_L' = \frac{Z_L}{2^2} = \frac{Z_L}{4}$.
- choose $R_3 \sim 1.5\text{-}2 \times Z_L'$ (use Z_L referred at emitter V_{out2}) ($\sim 10^2$)
- choose R_1 and $R_2 \sim \underbrace{h_{fe} \times (R_3 // Z_L')}_{\text{impedance looking at the base}}$ and use $V_B = V_{CC}\frac{R_2}{R_1+R_2}$ (Use Z_L referred at emitter V_{out2}) ($\sim 10^5$)

③ select L & C and varactor

- consider total C tuning ratio, $\frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2$ [since $f \propto \frac{1}{\sqrt{LC}}$]
- C_{FB} is large — just to block DC, low impedance at signal freq. range. (e.g. $\sim 10nF$)
- select a varactor (e.g. 1N5476A. $C_{var,max} = 300pF$, $C_{var,min} = 10pF$).

  choose $C \sim 1.5\text{-}2 \times C_{var,max}$, then check $\frac{C_{max}}{C_{min}} = \frac{C/2 + C_{var,max}}{C/2 + C_{var,min}}$ is larger than requirement.
- choose $L = \frac{1}{(2\pi f_{mid})^2 C_{mid}}$, where $f_{mid} = \frac{f_{max}+f_{min}}{2}$, $C_{mid} = \frac{C_{var,max}+C_{var,min}}{2} + \frac{C}{2}$

④. LC network drive resistor.

- select $R_d \leq \frac{1}{5} \times (\text{all resistors} // L) = \frac{1}{5}(R_1 // R_2 // h_{fe}R_3 // h_{fe}Z_L // \omega L Q_L)$ (must be less than 1/4) (~ 50) (ignore $Q_C/\omega C$ term as $Q_C \gg Q_L$)

* Note R_d too small → distortion ; R_d too big → loop gain < 1

e.g. Design an oscillator circuit for a radio tuner. tuning range 595–755 kHz, o/p power 5dBm into 300 Ω, supply 5V. [Note dBm is dB rel to 1mW]

①. $5dBm = 3.2mW = \frac{V_{rms}^2}{Z_L}$, $Z_L = 300\Omega$ → $V_{rms} = 0.95V$ → $V_{PP} = 2\sqrt{2}V_{rms} = 2.68V$.

$V_{PP} = 2.68V < 3V = 0.6V_{CC}$ → OK!

②. set $V_E = \frac{1}{2}V_{CC} = 2.5V$ → $V_B = 2.5 + 0.7 = 3.2V$.

choose $R_3 = 1.75 Z_L = 525\Omega$

choose $R_1, R_2 \sim h_{fe}(R_3 // Z_L) = 47.7k\Omega$, and $V_B = V_{CC}\frac{R_2}{R_1+R_2}$, say $R_1 = 45k\Omega$, $R_2 = 80k\Omega$

* Assume the chosen transistor has $f_T = 250MHz$, $h_{fe} = 250$, for BC182L

③. $\frac{C_{max}}{C_{min}} = \left(\frac{f_{max}}{f_{min}}\right)^2 = 1.78$

choose $C_{FB} = 10nF$, varactor w/ $C_{var,max} = 300pF$, $C_{var,min} = 10pF$.

choose $C = 1.75 \times 300 = 525pF$. → $\frac{C_{max}}{C_{min}} = \frac{C/2 + C_{var,max}}{C/2 + C_{var,min}} = 2.06 > 1.78$ → OK!

$f_{mid} = \frac{f_{max}+f_{min}}{2} = 675kHz$, $C_{mid} = \frac{C_{var,max}+C_{var,min}}{2} + \frac{C}{2} = 417.5pF$.

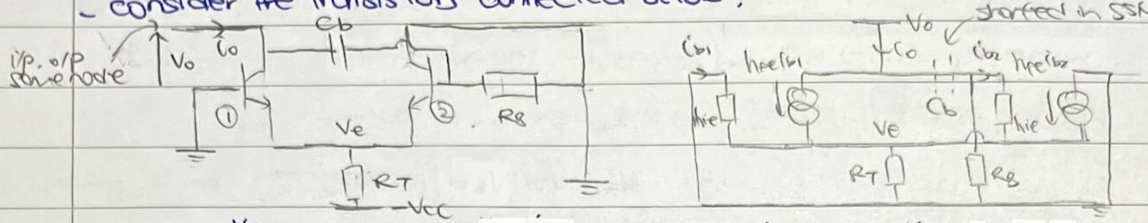
so $L = \frac{1}{(2\pi f_{mid})^2 C_{mid}} = 133\,\mu H$.

④ $R_d \leq \frac{1}{5}(R_1 // R_2 // h_{fe}R_3 // h_{fe}Z_L // \omega L Q_L) = 2.2k\Omega$ → choose $R_d = 2k\Omega$

* choose coupling capacitor from varactor to be $C = 10nF$, varactor resistor to be $10M\Omega$

Negative impedance oscillator

- The negative impedance oscillator is commonly used at high freq. (up to GHz)
- A pair of (fast) transistors can be configured to give an apparent -ve resistance
  → if connected across LC tank circuit, the -ve resistance compensates for the circuit's resistive losses → resonant LC oscillations do not die away (from damping).
- consider the transistors connected below.



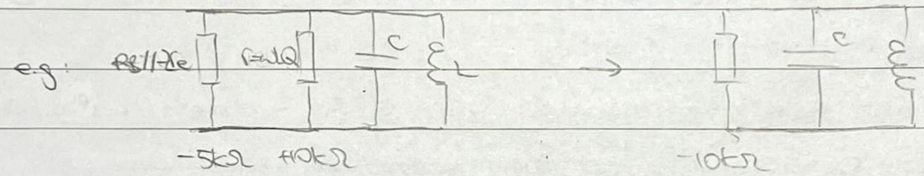
$i_{b1} = -\frac{V_e}{h_{ie}}$; $i_{b2} = \frac{V_o - V_e}{h_{ie}}$; $i_o = h_{fe} i_{b1} + i_{b2} + \frac{V_o}{R_B}$; $V_e = (\cancel{1+}h_{fe})(i_{b1}+i_{b2})R_T$

$\therefore V_e = \frac{h_{fe}}{h_{ie}} R_T(-V_e + V_o - V_e) \rightarrow V_e(\cancel{1}+\frac{2h_{fe}}{h_{ie}}R_T) = \frac{h_{fe}}{h_{ie}} R_T V_o \rightarrow V_e \approx \frac{V_o}{2}$

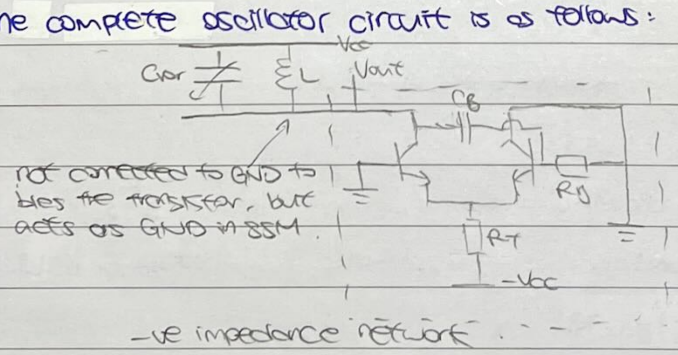
$\therefore i_o = h_{fe}\left(-\frac{V_e}{h_{ie}}\right) + \frac{V_o - V_e}{h_{ie}} + \frac{V_o}{R_B} = -\frac{h_{fe}}{2h_{ie}}V_o + \frac{V_o}{2h_{ie}} + \frac{V_o}{R_B} = \left[\frac{1}{2h_{ie}}(\cancel{1}-h_{fe}) + \frac{1}{R_B}\right]V_o$

$\Rightarrow Z_o = \frac{V_o}{i_o} = \left[\frac{1}{R_B} - \frac{h_{fe}}{2h_{ie}}\right]^{-1} = \left[\frac{1}{R_B} + \frac{1}{-2r_e}\right]^{-1}$, i.e. $Z_o = R_B // -2r_e$.

- Therefore, we can make a negative impedance based oscillator circuit by adding a suitable tuned LC circuit (where resistive parasitic component is cancelled out)



- The LC combination w/ -ve parasitic impedance is inherently unstable → lead to oscillations.
- The amplitude of oscillation grows exponentially, until the voltage swing across the transistors prevents them from making an effective -ve impedance → steady oscillations.
(For transistors, ↑voltage swing → ↓ gain, so steady state).
- The complete oscillator circuit is as follows:



$C_{var} = 12\text{-}60\,pF$
$L = 257\,\mu H$
$C_B = 100\,nF$
$R_B = 10\,k\Omega$
$R_T = 33\,k\Omega$
$V_{CC} = 5V$

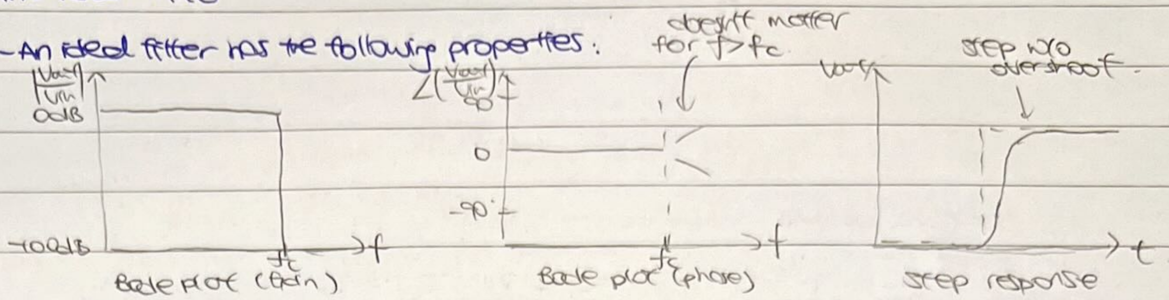
$Z_{out} = 10k // -66k = -810\Omega$

* Intuition for -ve impedance: $\uparrow V_o \rightarrow \uparrow V_{B2} \rightarrow \uparrow V_{E2} \rightarrow \downarrow V_{BE1} \rightarrow \downarrow I_{C1} \rightarrow \downarrow i_o$

Filters

The ideal filter

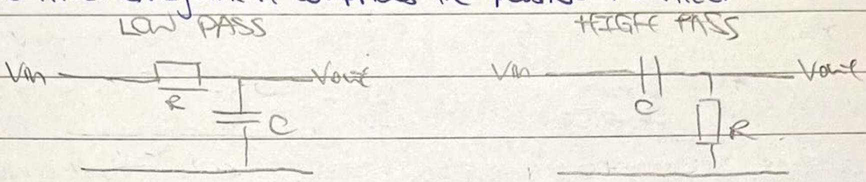
- An ideal filter has the following properties:



This would be ideal for extracting almost any signal from noise, whilst also retaining the original waveform shape. (Real filters have trade-off b/wn freq/time domain performances)

Passive RC filters.

- The simplest filter arrangement comprises the passive RC filter.

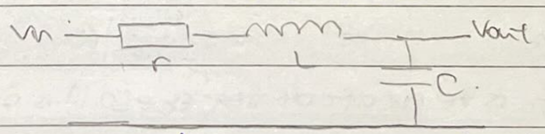


$\frac{V_{out}}{V_{in}} = \frac{1}{1+j2\pi f CR}$ $\qquad$ $\frac{V_{out}}{V_{in}} = \frac{1}{1+\frac{1}{j2\pi f CR}}$

- These are fine for non-demanding applications, where the desired signal freq. is very diff. to that of the background noise / interference. (rare in practice)
- For two freq. fairly close to each other, say 10% diff, $f_2 = 1.1 f_1$, a RC filter set w/ a 3-dB roll-off at f_1 only attenuates the f_2 signal by 5% more (we want >100× attenuation)
(Even cascading 10 stages using buffers only attenuates the f_2 signal by <40% more)

Resonant LC filters.

- The circuit of the resonant LC circuit is as follows:



$\frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{1/j\omega C + r + j\omega L} = \frac{1}{1-\omega^2 LC + j\omega Cr}$, $Q = \frac{1}{\omega_0 Cr} = \frac{\omega_0 L}{r}$, where $\omega_0 = \frac{1}{\sqrt{LC}}$

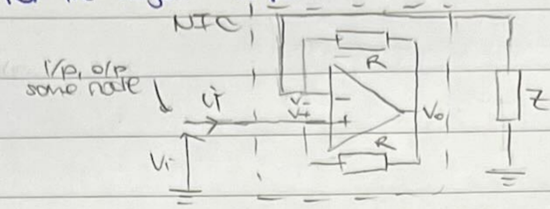
$\rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1-(\omega/\omega_0)^2 + j/Q(\omega/\omega_0)}$ $\quad \therefore$ If $\omega = \omega_0$, $\left|\frac{V_{out}}{V_{in}}\right| = Q$.

(Note if $Q = 100$, for $\omega = 1.1\omega_0$, $\left|\frac{V_{out}}{V_{in}}\right| = 4.76 \approx 0.05Q$ → good selectivity b/wn freq)

- However, there are a few practical problems associated w/ inductors:
  - ↳ Large values (for small ω_0) are physically large and rel. expensive.
  - ↳ They are often lossy, have stray capacitance and are nonlinear
  - ↳ They act as weakly-coupled transformers → pick up interference from stray B fields → noise

Negative impedance circuit

- The circuit for the negative impedance converter (NIC) is as follows:



For ideal OpAmp, $V_+ = V_- \rightarrow V_i = V_-$

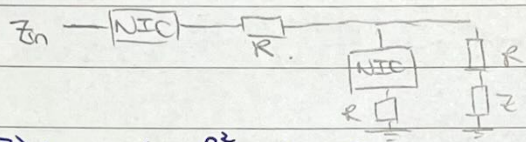
$\sum i_+ : \; i_i - \frac{V_i - V_o}{R} = 0 \quad ; \quad \sum i_- : \; \frac{V_- - V_o}{R} + \frac{V_-}{Z} = 0.$

Eliminating V_o, $\quad \frac{V_i}{R} - i_i = V_i\left(\frac{1}{Z} + \frac{1}{R}\right) \rightarrow Z_{in} = \frac{V_i}{i_i} = -Z$

→ impedance looking into the circuit is the -ve of the impedance Z of the circuit.

* If Z is a capacitor $\frac{1}{j\omega C}$ then $Z_{in} = \frac{j}{\omega C}$ (not quite an inductor as $Z \propto \frac{1}{\omega}$, not $Z \propto \omega$).

- If we want a true inductor, we can connect a pair of NICs as follows (gyrator):



$Z_{in} = -\left(R + \frac{-R(R+Z)}{-R+R+Z}\right) = -\left(R - \frac{R^2}{Z} - R\right) = \frac{R^2}{Z}$

If Z is a capacitor $\frac{1}{j\omega C}$, then $Z_{in} = j\omega CR^2$, i.e. an inductor w/ $L = CR^2$.

- To make an inductor w/ both ends floating, string a pair of gyrator circuits back-to-back, sharing the same series Z at the centre.

ask(!)

* The synthetic inductor only looks like an inductor while the OpAmps are behaving correctly (i.e. within the freq. range / voltage swing of the OpAmp).
(we req. gain control as ↑voltage swing → gain const. → clipping occurs.

Bessel, Butterworth and Chebyshev filters.

$s = j(f/f_c)$

- Three common types of n-pole only filters include:

↑coeff of s^4, faster the roll-off

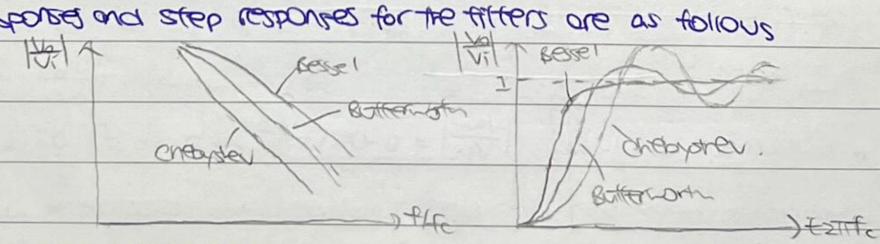
↳ 1) Bessel $\quad \left|\frac{V_o}{V_i}\right| = 10^{-\frac{3}{20}\left(\frac{f}{f_c}\right)^2}$ for $\frac{f}{f_c} < 2 \quad$ eg. $H(s) = \frac{H}{0.0095s^4 + 0.952s^3 + 0.43s^2 + s + 1}$

↳ 2) Butterworth $\quad \left|\frac{V_o}{V_i}\right| = \left(1 + \left(\frac{f}{f_c}\right)^{2n}\right)^{-1/2} \quad$ eg. $H(s) = \frac{H}{s^4 + 2.613s^3 + 3.414s^2 + 2.613s + 1}$

↳ 3) Chebyshev $\quad \left|\frac{V_o}{V_i}\right| = \left(1 + \varepsilon^2 C_n^2\left(\frac{f}{f_c}\right)\right)^{-1/2} \quad$ eg. $H(s) = \frac{H}{3.62s^4 + 3.455s^3 + 5.27s^2 + 2.69s + 1}$

where $\varepsilon \in [0,1]$ is the shape factor; $C_n\left(\frac{f}{f_c}\right)$ is the Chebyshev polynomial of order n for (f/f_c)

- The freq. responses and step responses for the filters are as follows



- The optimum choice of filter for a particular application depends on the rel. importance of freq. and time performance.

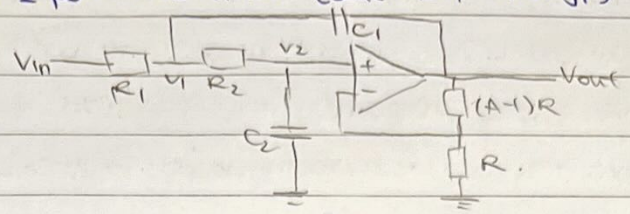
delay all freq. by the same amount → no distortion

↳ Bessel: passing transients undistorted $\qquad$ ↳ Chebyshev: sharp freq. cut-off

↳ Butterworth: fair compromise btwn the two — general all-rounder.

Voltage-controlled voltage source (VCVS) filter.

- consider the low-pass 2-pole VCVS filter (Sallen-Key topology)



For convenience, choose $C_1 = C_2 = C$, $R_1 = R_2 = R$.

$V_o = A V_2$; $V_2 = V_1 \frac{1/j\omega C}{1/j\omega C + R} = V_1 \frac{1}{1+j\omega CR}$; $\frac{V_i - V_1}{R} = \frac{V_1 - V_o}{1/j\omega C} + \frac{V_1 - V_2}{R} \rightarrow V_i = V_1(2+j\omega CR) - V_o j\omega CR - V_2$

$\therefore V_i = \frac{1+j\omega CR}{A} V_o (2+j\omega CR) - V_o j\omega CR - \frac{V_o}{A} = \frac{V_o}{A}\left[1 - (\omega CR)^2 + j\omega CR(3-A)\right]$

$\rightarrow \left|\frac{V_i}{V_o}\right| = A\left[(1-(\omega CR)^2)^2 + (\omega CR)^2(3-A)^2\right]^{-1/2} = A\left[1 + (\omega CR)^2\left[(3-A)^2 - 2\right] + (\omega CR)^4\right]^{-1/2}$

$= A\left[1 + \left(\frac{\omega}{\omega_n}\right)^2 B + \left(\frac{\omega}{\omega_n}\right)^4\right]^{-1/2}$ where $B = (3-A)^2 - 2$, $\omega_n = \frac{1}{CR}$

(Note for $B = 0$, we have the Butterworth filter).

- Therefore, depending on the choice of A and $CR = 1/\omega_n$, a range of filter characteristics can be provided w/ this circuit, w/ higher orders realised by cascading several stages.

(no. of stages $= \frac{\text{no. of poles } n}{\text{poles per stage}} = \frac{n}{2}$).

- The table of req. gain/freq. settings to achieve a particular filter characteristic is as follows:

no. of poles n	Bessel f_n	Bessel A	Butterworth f_n	Butterworth A	Chebyshev f_n	Chebyshev A
2	1.274	1.268	1	1.586	1.231	1.842
4	1.432 1.606	1.084 1.759	1 1	1.152 2.235	0.597 1.031	1.582 2.660
6	1.607 1.692 1.908	1.040 1.364 2.023	1 1 1	1.068 1.586 2.483	0.396 0.708 1.011	1.537 2.448 2.846

where the f_{-3dB} for each stage is given by $f_{-3dB} = \frac{1}{2\pi f_n CR}$ $f_{-3dB} = \frac{1}{2\pi(1/f_n)CR}$

- High pass filters are realised by swapping over Rs and Cs & use $f_n' = \left(\frac{1}{f_n}\right)$ to find CR.

Band pass filters are made by cascading overlapping low pass and high pass stages.

VCVS filter design

benefits and drawbacks of each filter type.

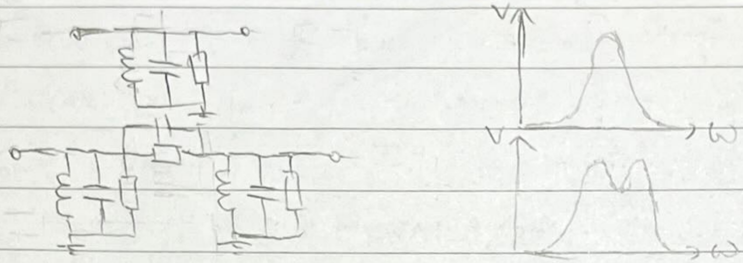
- Determine the type of filter (based on problem req.) and no. of poles n (based on req. SNR / attenuation) → Find f_n and A

- Determine the req. cut-off freq. f_c from the problem (If given the rise time t_{rise}, use $t_{rise} \approx 2.2\tau$, $f_c = \frac{1}{2\pi\tau} \rightarrow f_c = \frac{2.2}{2\pi t_{rise}}$) → Find CR (usually set $R = 10k\Omega$)

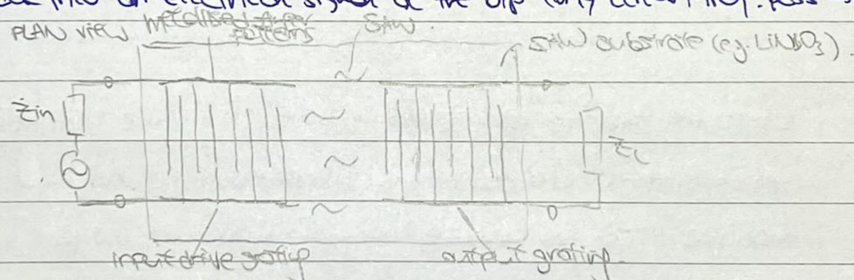
- Design a BPF by combining a LPF and HPF.

ceramic and surface acoustic wave (SAW) filters

- Ceramic and SAW filters rely on the piezoelectric effect. Exotic materials like $LiNbO_3$ and $LiTaO_3$ are used in addition to quartz to achieve high Q factors (~1000)
- ceramic filters are coupled resonator filters — a no. of resonant discs w/. diff. resonant freq. are mechanically coupled together w/ i/p and o/p electrodes deposited onto each end of the stack.
- The operation of a ceramic filter can be modelled as cascading LC circuits.



w/ more stages we can get a flatter top, steeper band edges and wider BW.
- For SAW filters, instead of bulk acoustic waves passing through the piezoceramic, surface waves are propagated in the top few microns along the crystal surface.
- The waves are created and detected by a series of inter-digitated electrode patterns — the E-field btwn the electrodes stress the material to produce a SAW.
- If the pitch (spacing) of the electrodes is correct, then the acoustic signal is converted back into an electrical signal at the o/p (only certain freq. pass → filter)



- To detect/create a wave w/ wavelength λ, the electrodes should have a pitch of λ/2 spacing and λ/4 width [λ found from $v=f\lambda$]
- BW control using variation in grating pitch (increase the pitch gradually, i.e. "chirp")

Mixers and phase-locked loops

Mixers

Mixers as analog multipliers

- The mixer is an analog voltage multiplier, consider the trigonometric relation

$$\sin A \sin B = \tfrac{1}{2}\cos(A-B) - \tfrac{1}{2}\cos(A+B)$$

- For i/ps w/ diff. freq, the o/p contains the sum and diff. terms f_1+f_2, $|f_1-f_2|$

$$A = 2\pi f_1 t,\ B = 2\pi f_2 t \rightarrow \sin A \sin B = \tfrac{1}{2}\cos(2\pi(f_2-f_1)t) - \tfrac{1}{2}\cos(2\pi(f_1+f_2)t)$$

- For i/ps have the same freq, the mixer behaves as a phase detector (DC component in the o/p related to the phase shift btwn the i/ps)

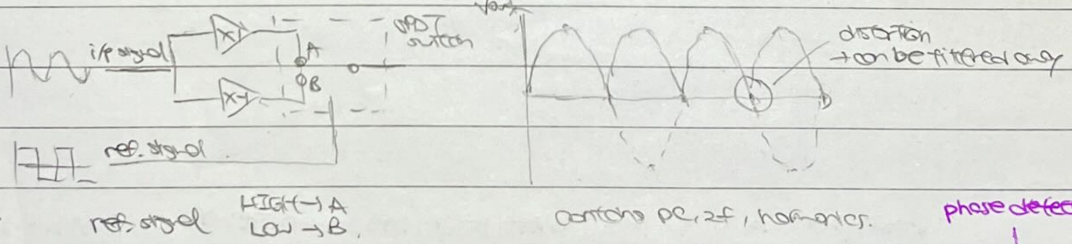
$$A = 2\pi f t,\ B = 2\pi f t + \phi \rightarrow \sin A \sin B = \tfrac{1}{2}\cos\phi - \tfrac{1}{2}\cos(2\pi(2f)t)$$

Ideal mixer properties

- In principle, any nonlinear component can be used as a mixer (applying a sine wave to a device w/ changing slope on I-V curve distorts it → introduces harmonics)
- However, when issues like efficiency, noise, isolation btwn terminals, freq. resp. etc. are considered, then the choice of mixer circuit becomes critical for good performance.
- An ideal mixer has the following properties (in RF applications)
  - ↳ Isolation of LO, IF and RF
  - ↳ No harmonic distortion
  - ↳ Low conversion loss (rel. to ratio of o/p to i/p power)

Inverting/non-inverting amplifier mixer

- An inverting/non-inverting amplifier mixer involves a ref. signal switching the state of the amplifier btwn the two polarities
- If the ref. signal has the same freq. and phase as the i/p signal, then we have active rectification, where the -ve cycles are inverted to be +ve.



phase detector.

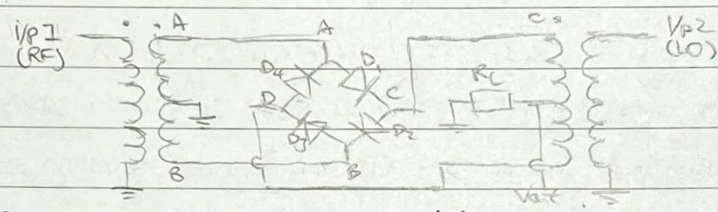
* varying the phase diff. btwn i/ps from 0° to 90° varies DC component from max to 0.
- This circuit is used up to ~10 MHz for phase-sensitive detection, lock-in amplifiers and synchronous detection (ref. signal is a square wave of unity amplitude)
- The XOR gate has the same function where both signals are digital → if o/p of XOR gate is LPFed to remove AC components, we have a square wave phase comparator.

Mixer topologies

- Mixers may be classified by their topology
  - ↳ Unbalanced mixer: in addition to producing a product signal, allowing both i/p signals to pass through and appear as components in the o/p.
  - ↳ Single-balanced mixer: one of its i/ps is applied to a balanced circuit → either LO or RF is suppressed at the o/p, but not both
  - ↳ Double-balanced mixer: both i/ps are applied to a balanced circuit → neither i/p signal (only the product signal) appears at the o/p.

Double balanced mixer

- The circuit for the double balanced mixer is as follows:

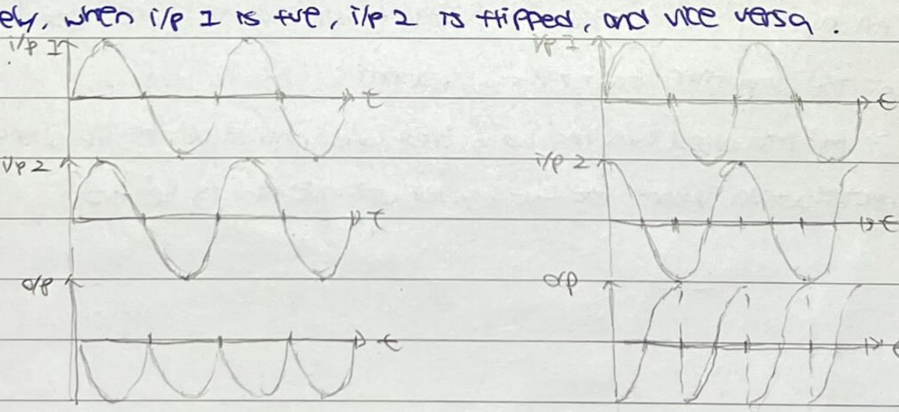


(Note the diodes are connected in a ring NOT like a bridge rectifier)

- When i/p 1 swings into +ve half cycle:
  - ↳ A swings +ve, B swings -ve (by same amt) due to GND at coil midpt.
  - ↳ D_1, D_2 conducts → C is GND; D_3, D_4 blocks → D is floating at high impedance
  - ↳ If i/p 2 is in phase w/ i/p 1 → C swing +ve wrt coil midpt → D swing +ve wrt GND.

(The opposite occurs when i/p swings into -ve half cycle)

- Effectively, when i/p 1 is +ve, i/p 2 is flipped, and vice versa.



- When there is a 90° phase shift btwn i/ps, then the o/p contains no DC.

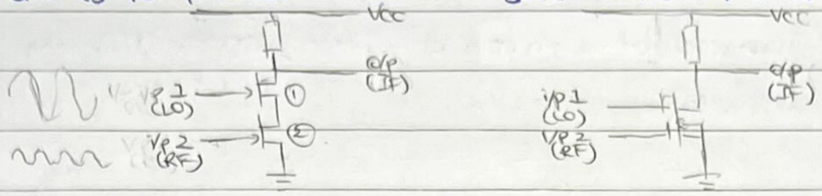
For diff. i/p freq, there is a continuously changing phase diff → DC term becomes beat freq.

(If we have i/p freq. f_1 and f_2, we get o/p freq. $|f_1 - f_2|$ and $f_1 + f_2$)

* Double balanced mixers are rel. expensive as they have transformers.

cascode and dual gate MOSFET mixer (single balanced mixer)

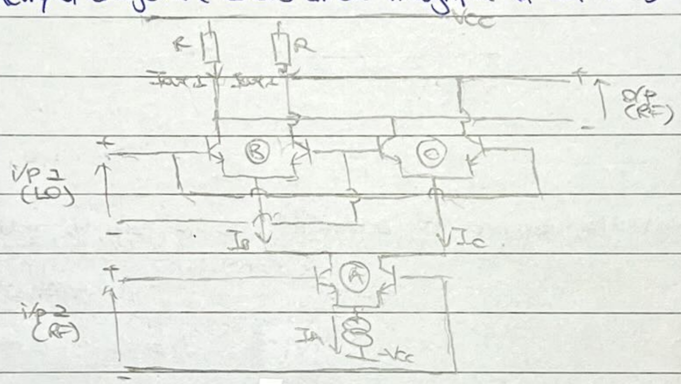
- The circuits for the cascode and dual gate MOSFET mixer are as follows:



- The RF i/p to gate of FET 2 modulates its drain current, which passes through FET 1 as well. The (rel. large) LO i/p to gate of FET 1 changes its resistance
- Since $V = IR$, the voltage o/p is a product of RF and LO signals → mixer.
- The dual-gate MOSFET works the same way but the transistors are in one device w/ two gates.
- Although isolation is good btwn RF/LO i/ps, there is strong leakage of LO i/p to RF o/p (in addition to the sum and difference terms)

Gilbert cell mixer

- The Gilbert cell mixer includes a pair of single balanced mixers, connected differentially w/ the o/ps summed — symmetry of arrangement cancels direct throughput of LO, RF signals.



- The lower diff. amplifier A steers the current I_A from side to side btwn the upper diff. amplifiers B, C.. The upper diff. amplifiers B, C then steer their respective tail currents I_B, I_C btwn their two o/ps, which are cross-connected → neither LO or RF i/p directly couple to IF o/p.
- If the RF i/p steers I_A in the ratio $k_1 : 1-k_1$, and LO i/p steers I_B, I_C in the ratio $k_2 : 1-k_2$,

$$I_B = k_1 I_A \;;\quad I_C = (1-k_1) I_A \;;\quad I_{out,1} = k_2 I_B + (1-k_2) I_C \;;\quad I_{out,2} = (1-k_2) I_B + k_2 I_C$$

$$V_{out} = (I_{out,2} - I_{out,1})R = I_A[k_1(1-k_2) + (1-k_1)k_2 - k_1k_2 - (1-k_1)(1-k_2)] = I_A(2(k_1+k_2) - 4k_1k_2 - 1)$$

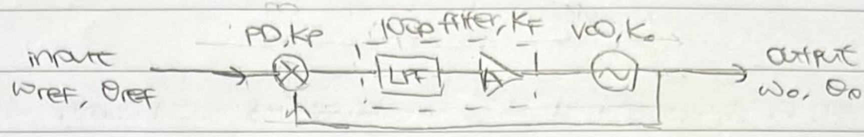
→ If $k_1 = 1/2$ or $k_2 = 1/2$ (no LO/RF signal), then $V_{out} = 0$, as req.

- It is found that for sine wave i/ps, the Gilbert cell mixer o/p is consistent w/ a multiplier.

Phase locked loops (PLL)

phase locked loops (PLL)

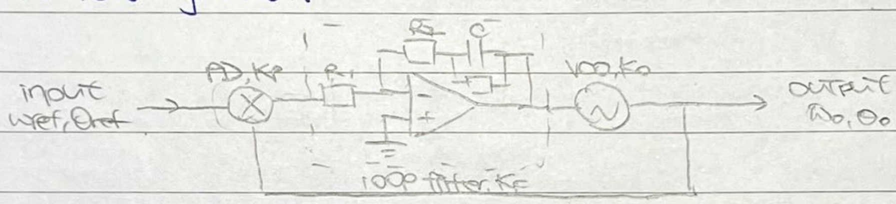
- The PLL is a FB loop consisting of a phase detector (PD), loop filter and voltage-controlled oscillator (VCO)
  mixer   LPF + amplifier



- The PD compares the phase of the i/p w/ the VCO o/p and produces a phase error signal w/ a DC component
- The error signal is filtered and fed to the VCO s.t. the freq. and phase O/ps of the VCO, ω_o, θ_o track and lock on to the i/p signal.
- The PLL has "freq. inertia" — the VCO o/p continues even if the i/p disappears for a while → used in clock recovery, freq. synthesis, FM demod.

Mathematical analysis of PLL

- consider the following circuit



we implicitly assume the oscillating terms θ_{ref}, θ_o both oscillate at the same freq. ω.

$$\theta_o \equiv e^{j(\omega t+\theta)} \qquad \theta_{ref} \equiv e^{j\omega t}$$

- PD $\qquad V_1 = K_P(\theta - \theta_{ref}) \quad [1]$

  VCO $\qquad \omega_o = K_o V_2 \rightarrow j\omega\theta_o = K_o V_2 \quad [2]$

  Loop filter $\qquad \frac{V_2}{V_1} = -\frac{R_2 + 1/j\omega C}{R_1} \rightarrow V_2 = -V_1\left(\frac{R_2}{R_1} + \frac{1}{j\omega C R_1}\right) \quad [3]$

- sub [1], [2] into [3], $\quad \frac{j\omega\theta_o}{K_o} = -K_P(\theta - \theta_{ref})\left(\frac{R_2}{R_1} + \frac{1}{j\omega C R_1}\right)$

$$\theta_o\left[\frac{j\omega}{K_o} + K_P\left(\frac{R_2}{R_1} + \frac{1}{j\omega C R_1}\right)\right] = \theta_{ref}\left[K_P\left(\frac{R_2}{R_1} + \frac{1}{j\omega C R_1}\right)\right]$$

$$\theta_o\left[-\omega^2\frac{CR_1}{K_P K_o} + j\omega C R_2 + 1\right] = \theta_{ref}\left[j\omega C R_2 + 1\right]$$

- converting into std form using $\omega_n^2 = \frac{K_P K_o}{CR_1}$ and $\zeta = \frac{\omega_n C R_2}{2}$,

$$\theta_o\left[\frac{-\omega^2}{\omega_n^2} + 2\zeta\frac{j\omega}{\omega_n} + 1\right] = \theta_{ref}\left[2\zeta\frac{j\omega}{\omega_n} + 1\right]$$

since $\dot\theta_{ref} = j\omega\theta_{ref}$, $\dot\theta_o = j\omega\theta_o$, $\ddot\theta_o = -\omega^2\theta_o$, we have a 2nd order ODE

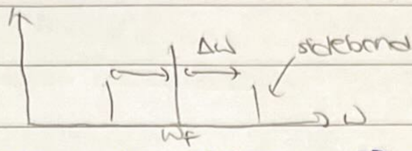
$$\frac{\ddot\theta_o}{\omega_n^2} + \frac{2\zeta\dot\theta_o}{\omega_n} + \theta_o = \frac{2\zeta\dot\theta_{ref}}{\omega_n} + \theta_{ref}$$

(Mechanics 0B P12 case (c): $\frac{\ddot y}{\omega_n^2} + \frac{2\zeta\dot y}{\omega_n} + y = \frac{2\zeta\dot x}{\omega_n} + x$)

* ω is neither the i/p signal ω_{ref} nor the VCO freq. ω_o, but rather the variation btwn the two once the loop is locked (a rate of change of freq.)

Phase locking of PLL.

- When there is a diff. in freq. btwn the i/p and o/p, the PD produces a component at this difference freq. (beat freq.). Some of this gets through the filter to modulate the VCO and give it sidebands in the o/p spectrum.



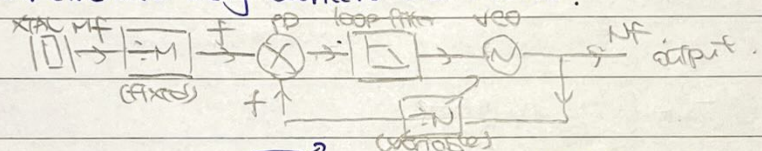
- One of the sidebands is the same as the ref. freq. → PD gives DC component that steers the VCO freq. in the dir. to match the i/p.
- For $\zeta \approx 1$, the time req. for locking is approximately given by

$$t \approx \frac{4\Delta\omega^2}{B^3}$$

where $\Delta\omega$ is the initial freq. diff. and B is the loop BW, given by $B = K_P K_O K_F$.
(If t is large up, achieving a locked phase in unlikely).

Frequency synthesis using PLLs.

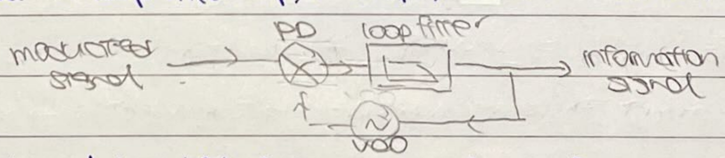
- By introducing digital dividers, we can make a freq. synthesiser w/ PLLs. (divider circuits implemented using counters and MCUs).



where $\boxed{\div M}$ is a fixed divider and $\boxed{\div N}$ is a programmable divider
- By changing the division ratio N, the VCO can be made to run at diff. multiples of the phase comparison freq. f → step freq. is equal to the phase comparison freq.
- A crystal oscillator is usually used as the ref. oscillator, w/ typical freq. $f_{ref} = 10\text{ MHz}$. This is too large a step size for most RF applications → use fixed freq. divider $\boxed{\div M}$

FM demodulation using PLLs.

- By taking the o/p at the loop filter o/p, (VCO i/p), the PLL acts as a FM demodulator.

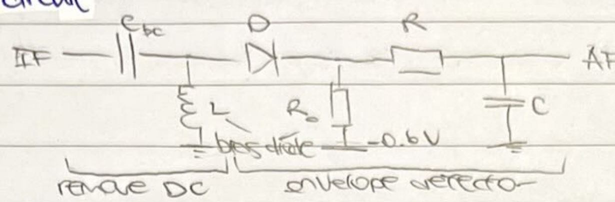


- When the modulated signal has high freq. we expect the information signal (VCO i/p) to be high (since the VCO should give an o/p of high freq. to match the i/p), and vice versa → FM demodulation.

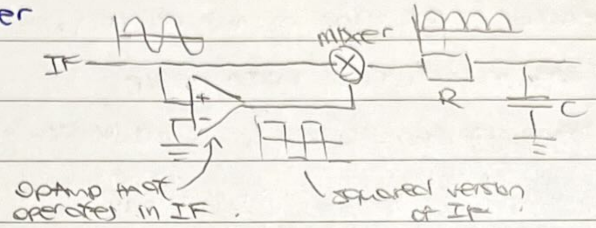
Demodulation circuits

AM demodulation

① simple diode circuit — diodes may not work well at low freq.



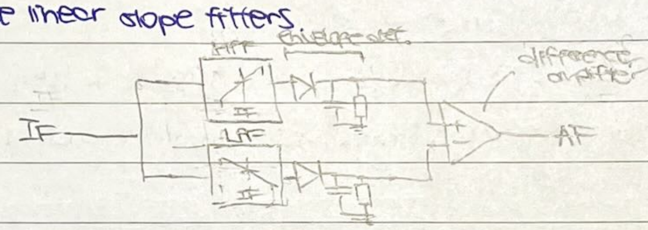
② Active rectifier



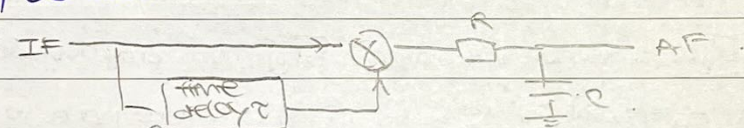
FM demodulation

① PLL (refer to previous section)

② Matched inverse linear slope filters



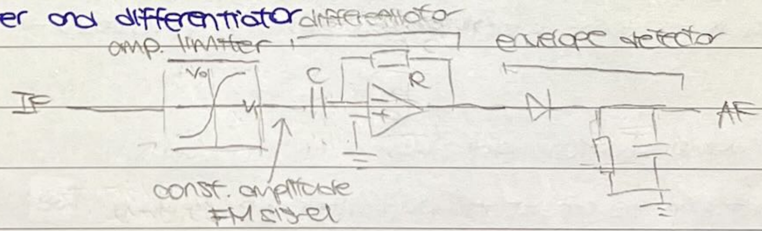
③. Time delay and mixer



set to 90° at nominal IF. (no DC component at nominal IF freq, $\sin\omega t \sin(\omega t - 90) = -\sin\omega t\cos\omega t = -\frac{1}{2}\sin(2\omega t)$)

↑ freq + given same time delay → ↑ phase shift → ↑ DC PD output

④ Amplitude limiter and differentiator



$\frac{d}{dt}[\sin(\omega t)] = \omega\cos(\omega t)$ → amplitude modulated

Imperfect FM signal may have amplitude variation → pass through as noise → use limiter.

Transmission lines, antennas and networks

Transmission lines and impedance matching

Equations for transmission lines and free space waves

- key eqns for t-lines and free space waves are as follows:

	Transmission lines	Free space waves
wave velocity	$v = \frac{1}{\sqrt{L_L C_L}}$	$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8 \, ms^{-1}$.
wavelength	$\lambda = \frac{v}{f}$	$\lambda = \frac{c_0}{f}$
characteristic impedance	$Z_0 = \sqrt{\frac{L_L}{C_L}}$	$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$
phase constant	$\beta = \frac{2\pi}{\lambda}$	$\beta = \frac{2\pi}{\lambda}$
voltage/field reflection coeff.	$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$	$\rho = \frac{\eta - \eta_0}{\eta + \eta_0}$
power reflection coeff.	$P = \rho^2$	$P = \rho^2$.

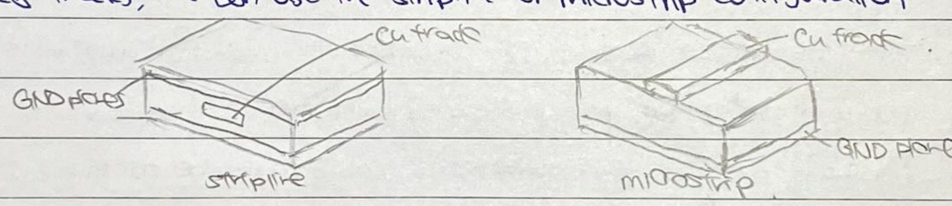
Controlled impedance transmission lines.

- For a coaxial cable w/ inner/outer radius a, b, w/ dielectric of rel. permittivity, ε_r,

$$C_L = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(b/a)} \qquad v = \frac{1}{\sqrt{L_L C_L}} = \frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}}$$

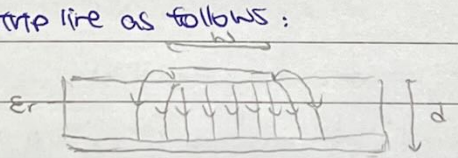
$$\rightarrow Z_0 = \sqrt{\frac{L_L}{C_L}} = \frac{1}{vC_L} = \frac{\sqrt{\mu_0\varepsilon_0\varepsilon_r}\ln(b/a)}{2\pi\varepsilon_0\varepsilon_r} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{\sqrt{\varepsilon_r}}\frac{\ln(b/a)}{2\pi} = \frac{60}{\sqrt{\varepsilon_r}}\ln(b/a) \quad \left[\sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\right].$$

- Dimensions of the coaxial cable (a, b) can be chosen to give the desired Z_0 (e.g. 50Ω, 75Ω)
- For PCB tracks, we can use the stripline or microstrip configuration



↳ stripline: lower loss, connection vias req. ↳ microstrip: easy connections/inspection,

- we can model a microstrip line as follows:



The capacitance per unit length of line may be estimated by $\boxed{C_L = \frac{W_{eff}\varepsilon_0\varepsilon_r}{d} = \frac{(w+2d)\varepsilon_0\varepsilon_r}{d}}$

where we approx $W_{eff} = w+2d$ to account for the fringing field lines (edge effects)

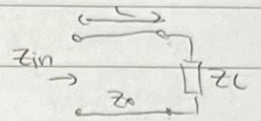
$$\rightarrow \boxed{Z_0 = \sqrt{\frac{L_L}{C_L}} = \frac{1}{vC_L} = \frac{\sqrt{\mu_0\varepsilon_0\varepsilon_r}\, d}{(w+2d)\varepsilon_0\varepsilon_r} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{\sqrt{\varepsilon_r}}\frac{d}{w+2d} = \frac{120\pi}{\sqrt{\varepsilon_r}}\frac{d}{w+2d}} \quad \left[\sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\right]$$

- Dimensions of the microstrip (d, w) can be chosen to give the desired Z_0 (e.g. 50Ω, 75Ω).

+ If $w<0$, then the desired Z_0 is impossible to achieve w/ that particular material ($\sqrt{\varepsilon_r}$) and dielectric thickness d.

Impedance matching using transmission line

- At high freq, if connection length is a large fraction of λ ($\sim \lambda/16$), we should consider the connection as a transmission line if distortions from reflections / phase delays are to be avoided.
- The i/p impedance Z_{in} for a length of t-line (of characteristic impedance Z_0) terminated w/ a given impedance Z_L is given by



$$Z_{in} = Z_0 \frac{Z_L + Z_0 j\tan(\beta L)}{Z_0 + Z_L j\tan(\beta L)}$$

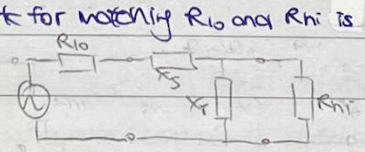
- It is difficult to find req. values of βL to provide a particular Z_{in} → we can solve this graphically using a Smith chart.

Smith chart.

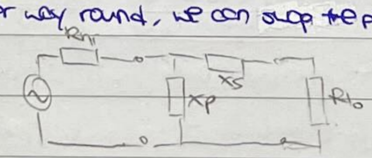
- By consv. of energy, for fixed Z_0, then $|\Gamma| = \rho \leq 1$, i.e unit circle on Smith chart
- Complex impedance is represented on the Smith chart as normalised impedance $\bar{z}$ so that the same chart can be used at diff. reference impedances Z_0, $\bar{z} = \frac{Z}{Z_0}$.
- when $Z = Z_0$, $\Gamma = 0$, so the prime centre represents $\bar{z} = 1 + 0j$
- Lines of const. resistances form a set of circles all converging at $\Gamma = 1$ (open circuit pt.)
  Lines of const. reactances form a set of arcs away from $\Gamma = 1$ (open circuit pt.)
- Lines of const resistances and reactances meet at right angles.
- There are multiple axes and the outside of the chart for phase of reflection, electrical length.
- For a particular pt, radially scaled parameters can be found at the bottom scale.
- Adding a t-line (towards generator / source) would be rotating CW on the Smith chart (Full rotation (360) ↔ $\lambda/2$, scale accordingly for other lengths of t-line).

Impedance matching network (L-network)

- A simple impedance matching network for matching R_{lo} and R_{hi} is as follows.



If R_{hi} and R_{lo} are arranged the other way round, we can swap the position of Xs and Xp

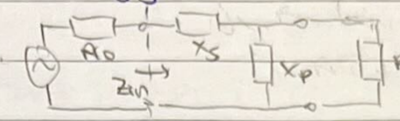


Alternatively, matching must work in both directions, we can simply consider the first circuit.

- X_p, X_s are given by ωL or $-1/\omega C$ as appropriate (depends on LPF or HPF), but we cannot have both X_p/X_s as inductors/capacitors. (LPF: $X_s = \omega L$, $X_p = -\frac{1}{\omega C}$; HPF: $X_s = -\frac{1}{\omega C}$, $X_p = \omega L$)
- The values of ω, C, L can be solved by calculation or using the Smith chart.

① Calculation.

- Consider the first circuit. (since arranging R_{hi} and R_{lo} in both ways yield the same result)



$$Z_{in} = Z_s + R_{hi} // Z_p = Z_s + \frac{R_{hi} Z_p}{R_{hi} + Z_p} = jX_s + \frac{jX_p R_{hi}}{R_{hi} + jX_p} = jX_s + \frac{jX_p R_{hi}^2 + X_p^2 R_{hi}}{R_{hi}^2 + X_p^2}$$

If we choose X_s s.t. $Im[Z_{in}] = 0$, and we req. $Re[Z_{in}] = R_{lo}$, then

$$R_{lo} = \frac{X_p^2 R_{hi}}{R_{hi}^2 + X_p^2} \rightarrow R_{lo}(R_{hi}^2 + X_p^2) = X_p^2 R_{hi} \rightarrow X_p^2 (R_{hi} - R_{lo}) = R_{lo} R_{hi}^2 \rightarrow X_p = \frac{R_{hi}}{Q}$$

$$X_s = -\frac{X_p R_{hi}^2}{R_{hi}^2 + X_p^2} = -\frac{R_{hi}/Q \, R_{hi}^2}{R_{hi}^2 + R_{hi}^2/Q^2} = -\frac{R_{hi}/Q}{1 + 1/Q^2} = -\frac{R_{hi} Q}{Q^2 + 1} = -\frac{R_{hi} Q}{R_{hi}/R_{lo}} = -QR_{lo}$$

where we define the filter Q-factor $Q = \sqrt{\frac{R_{hi} - R_{lo}}{R_{lo}}} = \sqrt{\frac{R_{hi}}{R_{lo}} - 1}$. [so $\boxed{Q = \frac{R_{hi}}{|X_p|} = \frac{|X_s|}{R_{lo}} = \sqrt{\frac{R_{hi}}{R_{lo}} - 1}}$]

- Note that the effective Q factor Q_f is half the filter Q factor Q.

$$\boxed{Q_f = \frac{Q}{2}}$$

- To produce a filter w/ a sharp response (high Q), we can use a pair of matching networks connected back-to-back w/ a hi-lo-hi impedance transition. The half power bandwidth B is given by

$$\boxed{B = \frac{2f_0}{Q} = \frac{f_0}{Q_f}}$$

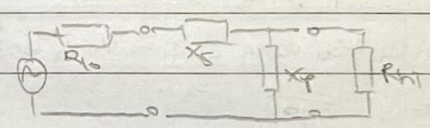
- To obtain a broadband match, a series of matching networks can be cascaded w/ gradually changing intermediate impedances (low Q).

* Note we can obtain expressions for $\frac{R_{hi}}{R_{lo}}$ or ω^2 using $\frac{R_{hi}}{|X_p|} = \frac{|X_s|}{R_{lo}}$ and $\frac{R_{hi}}{|X_p|} = \sqrt{\frac{R_{hi}}{R_{lo}} - 1}$.

* If the load has a reactive component, it can be cancelled out by adding an additional series reactance.

② Smith chart

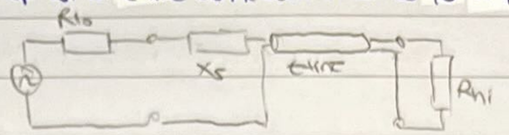
- We can always find an equivalent circuit in the form of the first circuit.



- We can find the req. C, L values using a Smith chart as follows:
  - ↳ 1) Plot the normalised load impedance [pt A]
  - ↳ 2) Plot the reciprocal of pt A and add the admittance $\frac{1}{Z_p}$ (CCW for inductor, CW for capacitor) by drawing an arc w/ const. conductance (real part). [pt. B]
  - ↳ 3) Find the pt. on the arc s.t. when reflected abt the prime centre lands on the unity resistance circle. (Found by drawing circles centred at 3, 1/3 and finding its intersection w/ the arc) [pt. C]
  - ↳ 4) Add the impedance Z_s (CW for inductor, CCW for capacitor) by completing the path from pt C to the prime centre along the unity resistance circle.
  - ↳ 5) Find $\frac{1}{X_p}$, X_s values by finding the reactive component of pt. B, C. → Find C, L

- Note that taking the reciprocal of a pt. on the smith chart is simply rotating around the prime centre by 180° or reflecting abt. the prime centre.

- Actually, we can replace X_p w/ a t-line connected in series for impedance matching.



- We can find the req. C/L value and length of t-line using a smith chart as follows:
  - ↳ 1) Plot the normalised load impedance. [pt.A]
  - ↳ 2) Add a length of t-line by rotating CW s.t. it intersects w/ the unity resistance circle [pt.B]
  - ↳ 3) Add the impedance Z_s (CW for inductor, CCW for capacitor) by completing the path from pt.B to the prime centre along the unity resistance circle.
  - ↳ 4) Find X_s value by finding the reactive component of pt.B → Find C/L.
    Find length of t-line by considering the change in wavelength (towards generator) from pt A to pt.B

(We find wavelength λ using $v=f\lambda$, where $v=\frac{1}{\sqrt{L_l C_l}}$ and $Z_0=\sqrt{\frac{L_l}{C_l}}$)

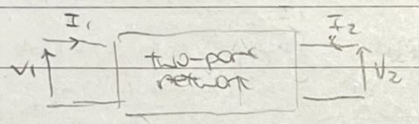
- We replace X_p w/ a t-line when we want to avoid using inductors, which are rel. expensive.
  (so we would use a capacitor for X_s).

S and Z parameters – two port networks

- We can characterise the behaviour of two-port networks using Impedance (Z) or Scattering (S) parameters.

① Z-parameters.

$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$



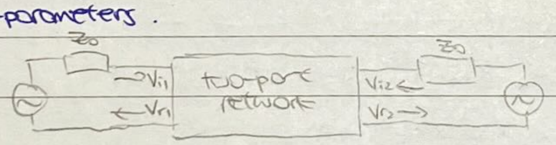
$$\boxed{\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}}$$

↳ Find Z_{11} by setting $I_2=0$ and considering the impedance at port 1

similar steps for Z_{22}, Z_{12}

Find Z_{21} by setting $I_2=0$ and finding the voltage at port 2 rel. to port 1 (e.g. potential divider)

② S-parameters.



$$\boxed{S_{11} = \frac{V_{r1}}{V_{i1}} \quad S_{12} = \frac{V_{r1}}{V_{i2}} \quad S_{21} = \frac{V_{r2}}{V_{i1}} \quad S_{22} = \frac{V_{r2}}{V_{i2}}}$$

↳ For an ideal amplifier of linear gain G w/ load applied, (i) $S_{11}=S_{22}=0$ (no reflections at i/p / o/p)
(ii) $S_{21}=G$ and (iii) $S_{12}=0$ (no coupling back from the load to the i/p).

↳ We can match impedances using S-parameters by considering S_{11}/S_{22}. This can be done by plotting the reflection coefficient S_{11}/S_{22} (reflection coeff magnitude + angle) and match as before

- At low freq., it is often useful to consider Z-parameters as they can be measured easily using open and short circuits on the ports while measuring voltage/current applied.
- At high freq., it is not so easy to make good open and short circuits due to reflections, which may even destroy the DUT, so we use S-parameters instead.

Antennas.

Terms and definitions in antenna engineering.

- The radiation resistance of an antenna R_r is defined s.t.

$$\boxed{P_r = \frac{1}{2} I^2 R_r = I_{rms}^2 R_r}$$, where P_r is the radiated power

- The radiation efficiency of an antenna η is defined as

$$\boxed{\eta = \frac{P_r}{P_{in}}}$$, where $\boxed{P_{in} = \frac{1}{2} I^2 (R_r + R_{ohmic}) = I_{rms}^2 (R_r + R_{ohmic})}$

- The Poynting vector $\underline{N} = \underline{E} \times \underline{H}$ has magnitude equal to the instantaneous power density (power per unit area), and has an average value given by $|\bar{N}|$

$$\boxed{|\bar{N}| = \frac{|E||H|}{2} = E_{rms} H_{rms}}$$

- The gain of an antenna G is defined as

$$\boxed{G = \frac{\text{max. power density}}{\text{power density for ideal isotropic antenna}} = \frac{|\bar{N}|_{max}}{|\bar{N}|_{iso}}}$$, where $\boxed{|\bar{N}|_{iso} = \frac{P_{in}}{4\pi r^2}}$

- The directivity of an antenna D is defined as.

$$\boxed{D = \frac{\text{max. power density}}{\text{power density for lossy isotropic antenna}} = \frac{|\bar{N}|_{max}}{P_r / 4\pi r^2} = \frac{|\bar{N}|_{max}}{|\bar{N}|_{iso} \eta} = \frac{G}{\eta}}$$

- The effective area / aperture of an antenna A_e is defined as

$$\boxed{A_e = \frac{P_{abs}}{|\bar{N}|}}$$, where P_{abs} is the power delivered into a matched load by the antenna.

- The beam angle $d\phi$ can be related to the directivity D.

$$\boxed{D = \frac{P_r / \pi (r d\phi)^2}{P_r / 4\pi r^2} = \frac{4}{(d\phi)^2}}$$

- The Friis transmission eqn. (Antenna eqn) relates the ratio of received / transmitted radiative power $\frac{P_{r,r}}{P_{r,t}}$ to the effective apertures $A_{e,r}, A_{e,t}$ or the gains G_r, G_t of the two antennas.

$$\boxed{\frac{P_{r,r}}{P_{r,t}} = \frac{A_{e,r} A_{e,t}}{r^2 \lambda^2} = G_r G_t \left(\frac{\lambda}{4\pi r}\right)^2}$$

since $A_{e,r} = \frac{P_{r,r}}{G_t P_{r,t} / 4\pi r^2} = \frac{P_{r,r}}{P_{r,t}} \frac{4\pi r^2}{G_t}$ and $A_{e,t} = \frac{P_{r,r}}{G_r P_{r,t} / 4\pi r^2} = \frac{P_{r,r}}{P_{r,t}} \frac{4\pi r^2}{G_r}$

The Antenna eqn. is usually written in the form

$$\boxed{G_r = \frac{4\pi A_{e,r}}{\lambda^2}}$$

usually set depth to $\sim 3\delta$

- The skin depth of a conductor δ is given by

$$\boxed{\delta = \sqrt{\frac{2}{\omega \mu \sigma}}}$$

useful for finding R_{ohmic}.

Size of antennas

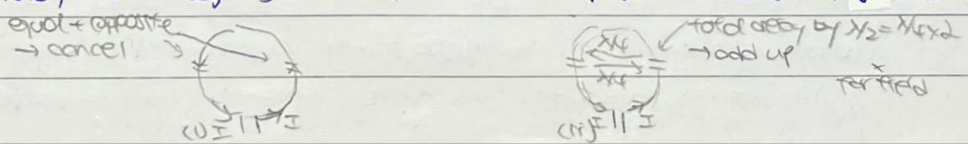
- For efficient radiation, the antenna needs to be comparable to a good fraction of a wavelength in dimension.

(i) Consider a small loop driven by an oscillating current → all parts of the circuit carry a current which is in phase.

At far-field, any current segment w/ $\underline{E}, \underline{H}$ fields has one nearby in antiphase → cancels out → ✗ radiate energy

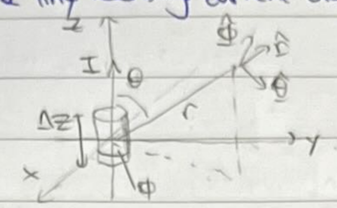
(ii) Consider a larger loop driven by an oscillating current s.t. there is a phase change in the loop ($\sim \lambda/4$ across).

At far field, the current segments now add up due to the extra phase shift in current → ✓ radiate energy.



The ideal dipole

- Consider the ideal dipole – a tiny radiating current element.



$\hat{s} = \hat{\theta} \times \hat{\phi}$

An elemental patch $d\underline{S}$ is given by $d\underline{S} = r^2 \sin\theta \, d\phi \, d\theta \, \hat{s}$

- It can be shown using Maxwell's eqns that the far field, $\underline{E}$ and $\underline{H}$ are given by

$$E = \frac{I\Delta z}{4\pi r} j\omega\mu e^{-j\beta r} \sin\theta \, \hat{\theta} \qquad H = \frac{I\Delta z}{4\pi r} j\beta e^{-j\beta r} \sin\theta \, \hat{\phi}$$

we can find the power radiated P_r by integrating $|\underline{N}| = \frac{1}{2}\underline{E} \times \underline{H}^*$ over a complete sphere.

$$P_r = \frac{1}{2}\iint (\underline{E} \times \underline{H}^*) \cdot d\underline{S} = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 \omega\mu\beta \iint \frac{\sin^2\theta}{r^2}(r^2 \sin\theta)\, d\phi\, d\theta = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 \omega\mu\beta \int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\theta \, d\theta$$

Noting that $\int_0^{\pi} \sin^3\theta \, d\theta = \int_0^{\pi} \frac{1}{4}(3\sin\theta - \sin 3\theta)\, d\theta = \frac{4}{3}$, we have

$$\boxed{P_r = \frac{1}{2}\left(\frac{I\Delta z}{4\pi}\right)^2 \omega\mu\beta (2\pi)\left(\frac{4}{3}\right) = 40\pi^2 I^2 \left(\frac{\Delta z}{\lambda}\right)^2}$$

(where we used $\omega = \frac{2\pi c_0}{\lambda}$, $\beta = \frac{2\pi}{\lambda}$, $c_0 = 3\times10^8\,\mathrm{ms^{-1}}$, $\mu = 4\pi\times10^{-7}\,\mathrm{Hm^{-1}}$).

- Since the radiation resistance R_r is defined s.t. $P_r = \frac{1}{2}I^2 R_r$, for the ideal dipole

$$\boxed{R_r = 80\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2}$$

- $\underline{E}$ and $\underline{H}$ are max. when $\theta = 90^\circ$, and the corresponding peak radiated power density of Poynting is

note $\eta_0 = \frac{|E|}{|H|} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$

$$|\underline{N}|_{max} = \frac{1}{2}|E||H| = \frac{|E|^2}{2\eta_0} = \frac{1}{2\eta_0}\left(\frac{I\Delta z}{4\pi r}\omega\mu\right)^2 = \frac{15\pi}{r^2} I^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

The power density for an ideal isotropic antenna $|\underline{N}|_{iso}$ is given by

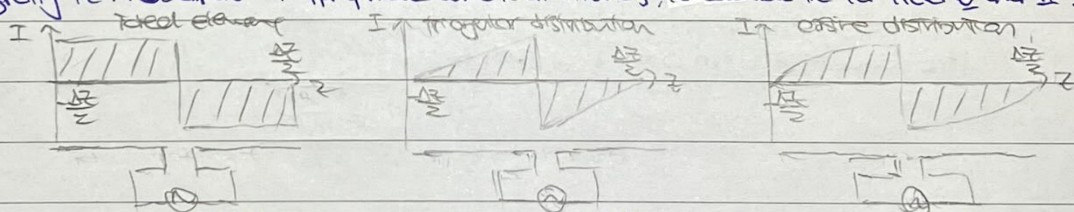
$$|\underline{N}|_{iso} = \frac{P_{in}}{4\pi r^2} = \frac{1/2 I^2 R_r}{4\pi r^2} = \frac{I^2}{8\pi r^2}\left(80\pi^2\left(\frac{\Delta z}{\lambda}\right)^2\right) = \frac{10\pi}{r^2} I^2 \left(\frac{\Delta z}{\lambda}\right)^2$$

Therefore the gain G and directivity D are given by

since $\eta = 1$ for ideal dipole

$$\boxed{D = G = 1.5}$$

- In practice, it is not possible to make an ideal dipole, but any antenna can be made up by integrating the ideal dipole w/ appropriate current distributions, to calculate the far field $\underline{E}$ and $\underline{H}$.

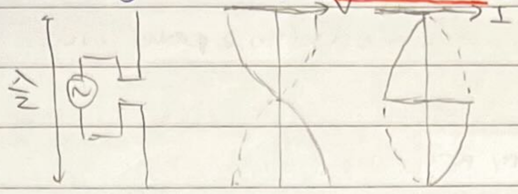


$R_r = 80\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$ $\qquad$ $R_r = 20\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$ $\qquad$ $R_r = 30\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$

- The simplest example is a triangular current distribution, w/ a central peak and dropping to 0 at the ends. Since $R_r \propto (I\Delta z)^2$, R_r for the triangular current distribution is $\left(\frac{1}{2}\right)^2 = \frac{1}{4} \times$ the R_r for the ideal element.
- For $\Delta z \sim \frac{\lambda}{2}$, the triangular current distribution becomes closer to a cosine current distribution, and the corresponding R_r is $R_r = 30\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$
- There is actually a series inductive reactance → trim antenna length slightly → introduce some capacitance to cancel out. (such parasitic reactance are small for large R_r, i.e. large antennas)
- Smaller antennas have low R_r, and thus low efficiency η (since $\eta = \frac{R_r}{R_r + R_{ohmic}}$) → poor at radiating, but alright for receiving. → use larger antennas for transmitting!

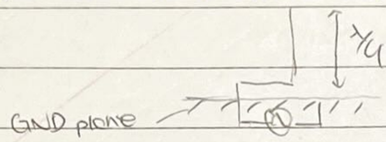
Half-wave dipole

- The half-wave dipole is $\frac{\lambda}{2}$ in length and supports a resonant standing wave along it, giving a cosine current distribution and a sine voltage distribution, w/ $\boxed{R_r = 80\pi^2\left(\frac{\Delta z}{\lambda}\right)^2}$ and $\boxed{D \approx 1.5}$



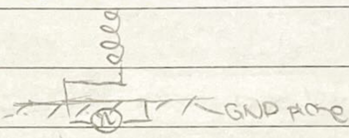
- We can also consider a monopole above a conductive GND plane — a reflection of the monopole gives the same profile as the dipole (radiates half the space → D doubled, R_r halved)

why R_r halved



Helical antenna

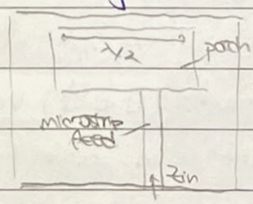
- The size of a half-wave dipole can still be large → helical antenna considered as a more compact version where the length of conductor is coiled up into a spiral.



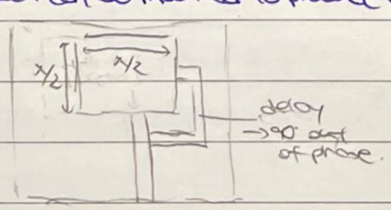
- The helical antenna is less efficient and its structure is not necessarily resonant, but it radiates circularly polarised EM waves. (a pair of linearly polarised EM waves orthogonal and 90° out of phase).
  ↖ will receive signals even if receiver antenna misaligned.

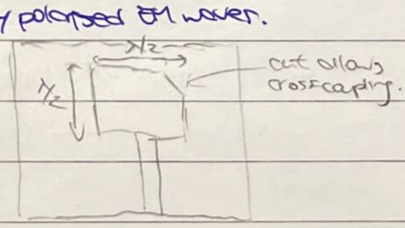
Microstrip patch antenna.

- A length of microstrip (patch) can be made to resonate as a half wave dipole, supporting a standing wave end to end, excited by an off-centre microstrip feed.
- The microstrip feed usually has characteristic impedance Z_0 and is off-centre of the patch so it is impedance matched and can excite the standing wave easier. (hard to do so at the node).



- If the substrate has a high rel. permittivity ε_r, the patch can be very small ($v \propto \frac{1}{\sqrt{\varepsilon_r}}$, $\lambda \propto v$ so $\lambda \propto \frac{1}{\sqrt{\varepsilon_r}}$).
- The microstrip patch can be modified to produce circularly polarised EM waves.



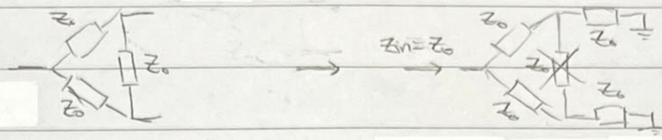


Power dividers, directional couplers and circulators.

Resistor network

- consider the following resistor network, where each port presents a characteristic impedance Z_0.

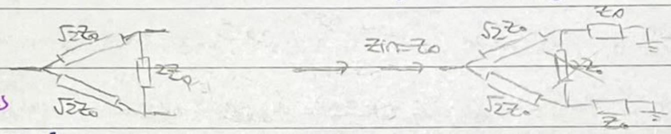
  Viewing from any port is equivalent to two parallel branches of two series Z_0



- Half the power is dissipated in the internal resistors and power is split equally across o/p ports, so each o/p port has ¼× power of i/p port → 6dB splitter.
- This configuration is freq. independent.

Wilkinson coupler

- The Wilkinson coupler comprises a pair of uncoupled lines, each $\frac{\lambda}{4}$ long w/ characteristic impedance $\sqrt{2}Z_0$.



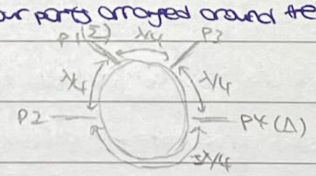
2 parallel branches

Note that $Z_{in} = \frac{1}{2}\left[\frac{(\sqrt{2}Z_0)^2}{Z_0}\right] = Z_0$ → impedance matched

- Power in P1 is split equally to P2 and P3, so each o/p port has ½× power of i/p port → 3dB splitter.
- Power in P2/P3 is only coupled to P1, but not P3/P2, since the contributions via the resistor and $\lambda/2 = \lambda/4 + \lambda/4$ line lengths are in anti-phase and cancel → good isolation (~20dB) btwn P2, P3.

Hybrid ring coupler (rat-race coupler)

- The hybrid ring coupler is a four-port 3dB directional coupler consisting of a $3\lambda/2$ ring of t-line of characteristic impedance $\sqrt{2}Z_0$, w/ four ports arranged around the ring.

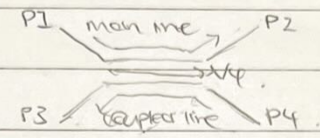


- Power in P1 splits and travels both ways around the ring.
  - ↳ P2/P3 : signals arrive in phase and add ($\frac{\lambda}{4}$ vs $\frac{5\lambda}{4}$)
  - ↳ P4 : signals arrive in anti-phase and cancel ($\frac{\lambda}{2}$ vs λ)

  → power in P1 is split equally to P2 and P3, so each o/p port has ½× power of i/p port → 3dB splitter.
- If power is directed into P2 and P3, then
  - ↳ P1 : signals arrive in phase and add ($\lambda/4$ vs $\frac{\lambda}{4}$) → sum signal (Σ)
  - ↳ P4 : signals arrive in anti-phase and cancel ($\frac{3\lambda}{4}$ vs $\frac{\lambda}{4}$) → difference signal (Δ).

Directional coupler

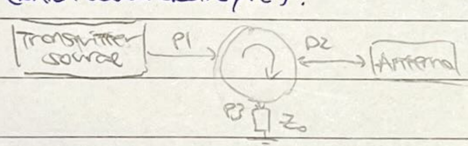
- Instead of splitting the power equally, the directional coupler can just tap off a fraction of the power being directed into a system (e.g. power meter to monitor signals + reflections of an antenna).
- This can be achieved by running parallel microstrip lines close to each other for a short length. The fields couple across and tap off a fraction of the power (↓distance → ↑coupling → ↑power tapped off).



- Most of the power input to P1 is routed through to P2, but a fraction is tapped off to P3 (but none to P4), and vice versa for power input to P2.
- Using signal detectors at P3, P4, we can monitor how much power is going to P2 / reflected back.

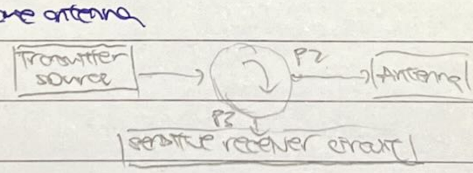
Circulators.

- Circulators are useful when we req. non-reciprocal operation (e.g. unidirectional routing of a signal). All power into P1 goes to P2, all power into P2 goes to P3 etc..
- If P3 is simply a matched load, then we have an isolator — a device which protects the transmitter source from reflections (which could destroy it).



The isolator only routes power from P1 to P2, but not the other way around.

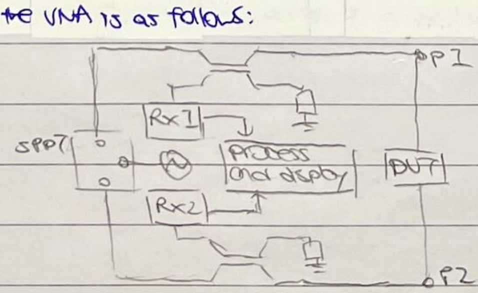
- The circulator could also protect sensitive receiver circuitry from power transmitted signals, which is also connected to the same antenna.



- Circulators are limited in BW and typically have fairly low isolation of ~20dB.

Vector Network Analyser (VNA)

- A VNA can separate forward and reflected waves → we can measure S-parameters.
- The circuit/block diagram for the VNA is as follows:



- When the oscillator is applied to P1, signal at Rx1 gives S_{11} and signal at Rx2 gives S_{21}