

Maximum available power

The different value of work and heat

- consider the following system and CV,

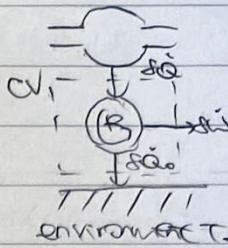
$$\text{1st law: } \dot{Q} = \dot{W} + \dot{Q}_0 \quad [1]$$

$$\text{2nd law: } \frac{\dot{Q}}{T} - \frac{\dot{Q}_0}{T_0} = 0 \quad [2]$$

$$\rightarrow \dot{Q}_0 = T_0 \dot{Q} \quad (\text{2nd law sets the magnitude of } \dot{Q}_0)$$

substituting back into [1], we have. carrot efficiency.

$$\dot{W} = (1 - \frac{T_0}{T}) \dot{Q}$$



environment T_0 .

- The rel. value of heat \dot{Q} compared to work \dot{W} is dependent on the temp. from which it comes T and the temp. of the environment T_0 .

\rightarrow very hot source ($T \rightarrow \infty$): \dot{Q} is as good as \dot{W} ; environment temp. ($T \rightarrow T_0$): \dot{Q} is useless,

Maximum available power in a steady flow device.

- The max. available power can be achieved when the process is reversible.

- consider the following system and CV,

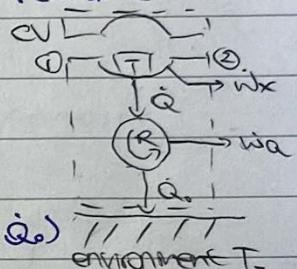
$$\text{1st law: } m(h_2 - h_1) = (-\dot{Q}_0) - (\dot{W}_x + \dot{W}_a) \quad [1]$$

$$\text{2nd law: } m(s_2 - s_1) = \left(-\frac{\dot{Q}_0}{T_0}\right) + \dot{s}_{gen} \quad [2]$$

$$\rightarrow \dot{Q}_0 = mT_0(s_1 - s_2) \quad (\text{2nd law sets the magnitude of } \dot{Q}_0)$$

substituting back into [1], we have

$$\dot{W}_x + \dot{W}_a = m[(h_1 - h_2) - T_0(s_1 - s_2)]$$



where $\dot{W}_x + \dot{W}_a$ is the max. available power of flow from state 1 to 2.

- we can rewrite this as

$$\dot{W}_{max,12} = m(b_1 - b_2) = m[(h_1 - T_0 s_1) - (h_2 - T_0 s_2)]$$

where b is defined as the specific steady flow availability function, $(b_i = h_i - T_0 s_i)$.

- the max. available power that can be extracted between states 1 and 2, per kg mass flow rate is the decrease in b across states 1 and 2.

* Unless T_0 is fixed, b is not a thermodynamic property.

* The availability function b is slightly different from the Gibbs free energy G ,

$$B = H - T_0 S$$

;

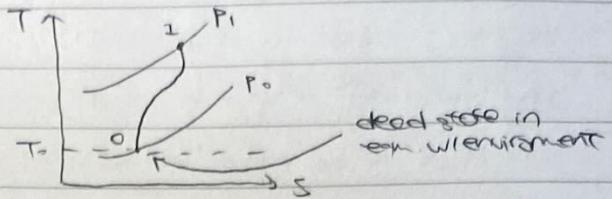
$$G = H - TS$$

For Personal Use Only -bkwk2

The dead state

- when a fluid is in eqm. w/ the environment P_0, T_0 , it is said to be at the dead state

At the dead state, the fluid has no available power.



The maximum available power that can be extracted from a fluid at state 1 is

$$W_{\max} = b_1 - b_0 = (n_1 - T_0 S_1) - (n_0 - T_0 S_0) = e_1$$

where e is the specific steady flow energy ($e_i = b_i - b_0$)

* Note that $\Delta b = \Delta e$. ($b_2 - b_1 = (b_2 - b_0) - (b_1 - b_0) = e_2 - e_1$).

Energy unavailable for work

- from the first law, we see that

$$\dot{W}_{use} = \dot{(H_i - H_o)} - \dot{Q_o}$$

↑ ↗

avail. available energy energy unavailable energy

avail. available power diff. in enthalpy energy flow rate
 flow rate from dead state unavailable for work
 (heat transferred to env.)

- From the second law, we see that

$$Q_o = T_o (S_i - S_o)$$

↓
 energy flow rate
 unavailable for work

↑
 entropy
 flow rate

set by eng.
 w/ the env.

- The energy unavailable for work Q_u is equal to the heat transfer req. to bring the system reversibly w/ an environment of const. temp. and pressure.

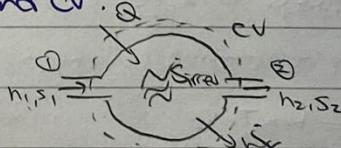
$$Q_0 = T_0 (S_1 - S_0)$$

since S_0 and T_0 are fixed by the law, Q_0 depends on S , while, so we can say that the entropy is the measure of a system's energy that is unavailable for work.

Avalibility balance equation

- The expression for the max. available energy can be written in a diff. form.

Consider the following system and CV.



$$\text{1st law: } m(h_2 - h_1) = \dot{Q} - \dot{W}_X \quad ; \quad \text{2nd law: } m(s_2 - s_1) = \frac{\int dQ}{T} + \dot{S}_{\text{irrev.}}$$

$$\text{Max. available energy : } m(b_2 - b_1) = m[(h_2 - h_1) - T_0(s_2 - s_1)]$$

$$\dot{Q} = \int \dot{V} dP$$

$$= (\dot{Q} - w_k) - T_0 \left(\frac{d\dot{Q}}{T} + \sin \alpha \right)$$

short power
feed-the C

transfer of power into
CV due to heat transfer

loss of available power due to irreversibilities

For Personal Use Only -bkwk2

Transfer of available power due to heat transfer.

- As long as heat is supplied at a temp. above that of the environment, it transfers available power into the CV.
- Recalling the rel. value of heat when compared to work, the power transfer is given by

$$\dot{W} = ((1 - \frac{T_0}{T}) \dot{Q}) \xrightarrow{\text{constant efficiency}} \text{constant efficiency.}$$

$$\dot{W} = \int (1 - \frac{T_0}{T}) d\dot{Q}$$
- If the temp. is constant, then the power transfer is given by

$$\dot{W} = (1 - \frac{T_0}{T}) \dot{Q}$$

Loss of available power due to irreversibilities

- Any process which is irreversible involves a loss in available power, given by

$$\dot{W} = T_0 \dot{S}_{\text{irrev}}$$

- The three irreversible flow process considered in this course are

- ↳ Viscous dissipation in boundary layers
- ↳ Viscous mixing in the freestream of flows w/ diff velocity
- ↳ Heat transfer across a finite temp. diff.

viscous,

thermal

- In each case, we could have, but didn't extract reversibly \rightarrow loss of potential work
 ↳ entropy generation.

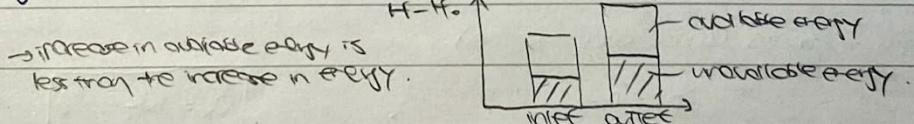
Splitting energy into available energy and unavailable energy.

- It is often insightful to consider the energy of a system as split into two parts - the part available for work and the part unavailable for work.

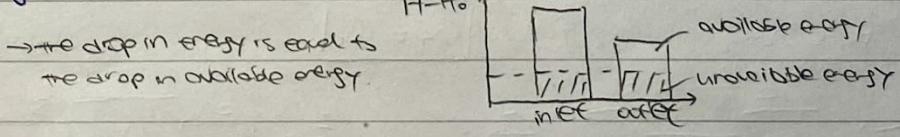
$$\Delta \text{Energy} = \Delta \text{available energy} + \Delta \text{unavailable energy}.$$

$$\Delta H_{1,2} = m \Delta h_{1,2} + \Delta Q_{1,2}$$

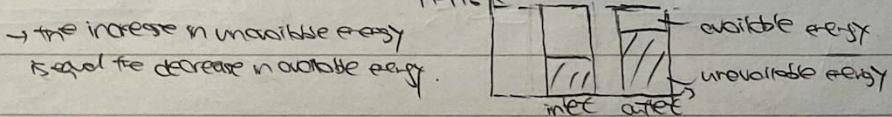
- e.g.: Reversible heat transfer to flow increases both entropy and enthalpy of the flow.



- e.g.: Isentropic turbine reduces the enthalpy of the flow, but the entropy of the flow is unchanged.



- e.g.: Isentropic throttle increases the entropy of the flow, but the entropy of the flow is unchanged.



For Personal Use Only -bkwk2

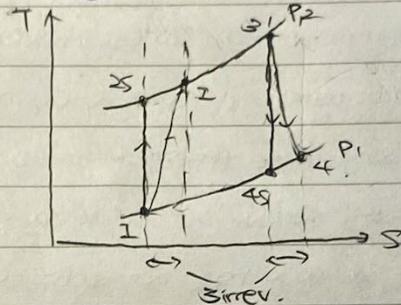
Gas turbines

Real gas turbines

- The air standard Joule cycle is violated in a no. of ways. In a real gas turbine,
 - ↳ there is normally a 2-4% drop in pressure across the combustor,
 - ↳ the combustion process changes the gas composition \rightarrow thermodynamic properties of the working fluid in the turbine is diff. from in the compressor.
 - ↳ within the compressor and turbine, there are irreversible processes that result in the creation in entropy.

Compressor and turbine irreversibilities.

- Compressor and turbine blades have large surface area, contain many leakage jets and involve the mixing of streams of diff. temp. \rightarrow Entropy generation.



- Isentropic compressor efficiency η_c is defined as

$$\eta_c = \frac{(w_c)_{ideal}}{(w_c)_{real}}$$

$$= \frac{\dot{m}(h_2 - h_1)}{\dot{m}(h_2 - h_{1s})} = \frac{CP(T_{2s} - T_1)}{CP(T_2 - T_{1s})} = \frac{T_{2s} - T_1}{T_2 - T_{1s}}$$

- Isentropic turbine efficiency η_t is defined as

$$\eta_t = \frac{(w_t)_{real}}{(w_t)_{ideal}}$$

$$= \frac{\dot{m}(h_3 - h_{4s})}{\dot{m}(h_3 - h_4)} = \frac{CP(T_3 - T_{4s})}{CP(T_3 - T_4)} = \frac{T_3 - T_{4s}}{T_3 - T_4}$$

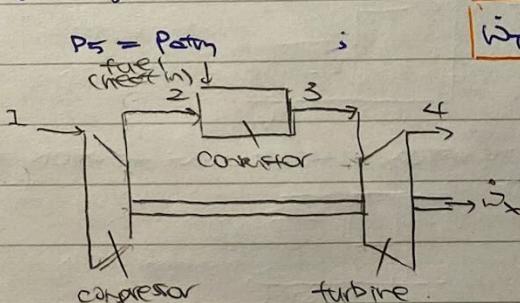
Gas turbine and jet engine.

- For gas turbines (land and marine),

$$P_f = P_{atm}$$

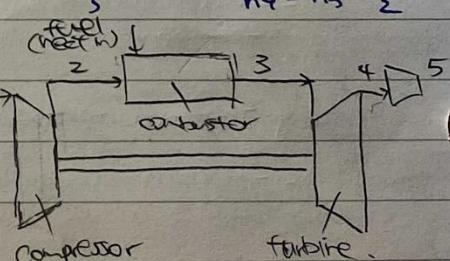
$$\dot{w}_x = \dot{w}_t - \dot{w}_c$$

- For jet engines (aero)



$$\dot{w}_t = \dot{w}_c$$

$$h_4 = h_5 + \frac{V_s^2}{2}$$



For Personal Use Only -bkwk2

First law analysis of gas turbine -

- The first law thermal efficiency $\eta_{\text{first law}}$ is defined as

$$\eta_{\text{first law}} = \frac{\dot{W}_{\text{t}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_{\text{t}} - \dot{W}_{\text{c}}}{\dot{Q}_{\text{in}}}$$

- Consider a reversible compressor, using the isentropic relations we can find T_{2s}

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k}{k-1}} = r_p^{\frac{k}{k-1}} \rightarrow T_{2s} = r_p^{\frac{k}{k-1}} T_1$$

Using the efficiency of the compressor η_c , we can find T_2

$$\eta_c = \frac{h_2 - h_1}{h_{2s} - h_1} = \frac{T_2 - T_1}{T_{2s} - T_1} \rightarrow T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c}$$

Using the SFC, we can find the compressor power \dot{W}_c

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1)$$

- Consider a reversible turbine, using the isentropic relations, we can find T_{4s}

$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_4}\right)^{\frac{k}{k-1}} = r_p^{\frac{k}{k-1}} \rightarrow T_{4s} = r_p^{\frac{k}{k-1}} T_3$$

Using the efficiency of the turbine η_t , we can find T_4

$$\eta_t = \frac{h_2 - h_4}{h_2 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}} \rightarrow T_4 = T_3 + (T_3 - T_{4s})\eta_t$$

Using the SFC, we can find the turbine power \dot{W}_t

$$\dot{W}_t = \dot{m}(h_3 - h_4) = \dot{m}c_p(T_3 - T_4)$$

- Consider the combustor, using the SFC, we can find the rate of heat input \dot{Q}_{in}

$$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_2) = \dot{m}c_p(T_3 - T_2)$$

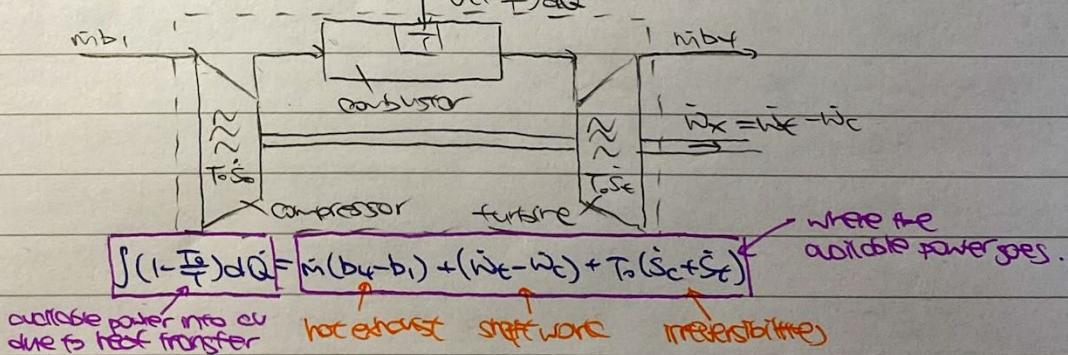
Second law analysis of gas turbine.

a measure of how close we are to the reversible case.

- The second law (rotational efficiency) $\eta_{\text{second law}}$ is defined as

$$\eta_{\text{second law}} = \frac{\dot{W}_{\text{t}}}{\int(1-\frac{T_0}{T})dQ} = \frac{\dot{W}_{\text{t}} - \dot{W}_c}{\int(1-\frac{T_0}{T})dQ}$$

- Consider the gas turbine as a whole. Applying the availability balance eqn,



- consider the combustor alone. Applying the availability balance eqn,

$$\int(1-\frac{T_0}{T})dQ = \dot{m}(b_3 - b_2) = \dot{m}[(h_3 + s_2) - T_0(s_3 - s_2)]$$

$$= \dot{m}[c_p(T_3 - T_2) - T_0(c_p \ln(\frac{T_3}{T_2}) - R M(\frac{T_3}{T_2}))]$$

For Personal Use Only -bkwk2

- The available power in exhaust flow can be found by,

$$\dot{m}(b_2 - b_1) = \dot{m}[(h_{2f} - h_1) - T_0(s_{2f} - s_1)]$$

- For compressor/turbine, it is assumed to be adiabatic ($Q=0$), so by first law, $\Delta S = \Delta S_{rev}$ (w/ same end states). The loss in available power due to irreversibilities is

$$T_0 \dot{S}_c = \dot{m} T_0 (s_2 - s_1) = \dot{m} T_0 (C_p \ln(\frac{T_2}{T_1}) - R \ln(\frac{P_2}{P_1}))$$

$$T_0 \dot{S}_{\sigma} = \dot{m} T_0 (s_4 - s_3) = \dot{m} T_0 (C_p \ln(\frac{T_4}{T_3}) - R \ln(\frac{P_4}{P_3}))$$

Properties of fluids

The state principle.

- In the absence of ext. effects (motion, gravitation, EM, surface tension), the state of a pure substance is fixed by the values of two independent properties.
 - The state principle allows us to say $P = f(T, V)$ [eqn. of state]
 - Eqs. of state depend on the detailed atomic/molecular structure of a substance and cannot be deduced from the laws of thermodynamics \rightarrow obtained via statistical mechanics or experimental experiment (PVT surface).
- * We usually cannot write the eqn. of state in closed form — a major exception is the ideal gas eqn. of state $PV = RT$

Phase changes

- During a phase change, both phases can coexist. In such regions (two-phase region), P and T are not independent properties, i.e. only 1 property needed to fix the state.
- A change in phase is accompanied by absorption/release of energy \rightarrow latent heat, which is a function of p or T.
- The temp./pressure at which a phase change takes place for a given pressure/temp. is the saturation temp./pressure.

Features of the P-V-T surface, P-V/T-V/P-T diagrams.

- Single-phase regions and two-phase regions are separated by saturation lines.
 - \hookrightarrow Liquid / liquid-vapour: saturated liquid line (wet saturated fluid)
 - \hookrightarrow Vapour / liquid-vapour: saturated vapour line (dry saturated fluid).
- Solid-vapour and liquid-vapour regions are separated by the triple pt-line.
- An unsaturated liquid/vapour is known as a subcooled liquid/superheated vapour.
- The critical pt. is the pt. where the saturated liquid and vapour lines meet. It is the max. temp. at which liquid and vapour phases can coexist.
(Above the critical pt, no distinction can be made between liquid and vapour).
- In a P-V diagram, the isotherm that passes through the critical pt. is the critical isotherm.
- In a T-V diagram, the isobar that passes through the critical pt. is the critical isobar.
- The P-T diagram is known as a phase diagram since all three phases are separated by lines.
(Because p,T become linked and constant during a phase change).
- In a P-T diagram, the vapourisation line ends at the critical pt and the triple pt line collapses into a pt. (To reach the triple pt, we start at a pressure and temp., decrease the triple pt. \rightarrow reduce pressure \rightarrow phase change \rightarrow absorb latent heat \rightarrow cool down to triple pt.)

For Personal Use Only -bkwk2

Working fluids

Working fluid.

- Every thermodynamic device relies on a working fluid whose function is to provide a medium for the transfer of heat and work.
- Common working fluids include:
 - ↳ Air - reciprocating engines, gas turbines.
 - ↳ Water - steam engines, steam-turbine power plant
 - ↳ HFCs - heat pumps, refrigeration plant.
- Working fluids that involve a phase change within a cycle permit practical cycles to be closer to the ideal (by having const. T during heat transfer)

Dryness fraction.

- A substance in the liquid-vapour region consists of both liquid and vapour coexisting in equilibrium.
- x is the dryness fraction of the mixture — each 1 kg of mixture contains x kg of vapour and $(1-x)$ kg of liquid (measure how far the phase changes proceed)
- The mass of a mixture m is given by

$$m = m_f + m_g.$$

The volume of the mixture V is given by

$$V = m_f V_f + m_g V_g.$$

by defn
 $x = \frac{m_g}{m_f + m_g}$

The specific volume of the mixture v is then given by

$$v = \frac{V}{m} = \frac{m_f V_f + m_g V_g}{m_f + m_g} = \frac{m_f}{m_f + m_g} V_f + \frac{m_g}{m_f + m_g} V_g$$

$$= (1-x) V_f + x V_g = V_f + x (V_g - V_f) = V_f + x v_{fg}.$$

- The lever rule above applies to all intensive properties (v, u, h, s etc.), so.

$$v = (1-x) V_f + x V_g = V_f + x v_{fg}$$

$$u = (1-x) U_f + x U_g = U_f + x u_{fg}$$

$$h = (1-x) h_f + x h_g = h_f + x h_{fg}$$

$$s = (1-x) S_f + x S_g = S_f + x s_{fg}$$

etc.

* The subscript f means liquid, g means gas.

For Personal Use Only -bkwk2

Tabulated properties.

- P12: combustion
 - P13-14: perfect and imperfect gases.
 - P17: triple pt / critical pt. for steam
 - P18-23: liquid-vapour region for H_2O as a function of temp./pressure.
 - P24-27: region just beyond the liquid-vapour region as a function of temp & pressure.
 - P28-30: transport properties for H_2O , air, CO_2 , H_2 .
 - P32: properties of gases and liquids at sea level and conditions
 - P32-33: International standard atmosphere
 - P37-38: liquid-vapour region + superheated region for refrigerant as a function of temp.
- + sometimes, we may need to perform linear interpolation.

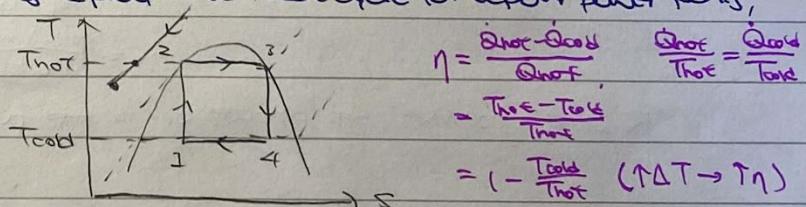
Useful graphs.

- T-S diagram
 - ↳ most common sketched graph for cycles (useful to reference when looking up tables)
 - ↳ good for comparing w/ the Carnot cycle.
- h-s diagram (steam)
 - ↳ much quicker than using tables.
- p-h diagram (refrigerant)
 - ↳ quicker than tables.

Power generation

The Carnot cycle

- Consider the Carnot cycle as a prospective ideal cycle for vapour power plants,



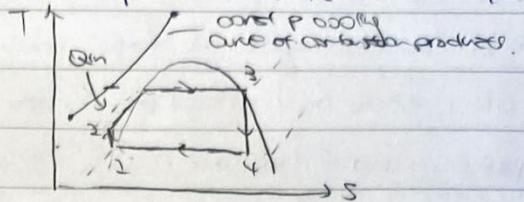
- There are two main problems w/ this:

- ↳ 2 and 3st are in the two-phase region → significant practical problems w/ developing pumps/turbines that operate in the two-phase region (impossible/erosion).
 - ↳ the heat input Q_{in} is from hot combustion products that are at const. pressure, so any potential energy from cooling the products below T_{hot} is lost.
- * In general, the heat addition in steam cycles is limited by the critical pt. (TFC), which limits T_{hot} , and thus the Carnot efficiency.

For Personal Use Only -bkwk2

The Rankine cycle

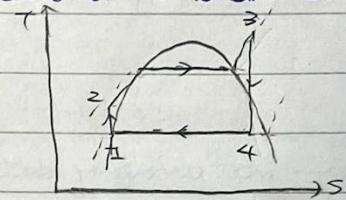
- The Rankine cycle deals w/ some of the problems of the Carnot cycle.



- The pump operates in the single-phase region, and also takes advantage of heat transfer at lower temp.
- However, the turbine still operates in the two-phase region, (erosion of blades)

The superheated Rankine cycle.

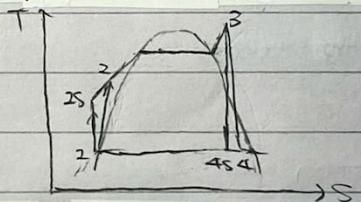
- The superheated Rankine cycle deals w/ the turbine issue of the Rankine cycle.



- The turbine operates less in the two-phase region.
- The average temp. of heat input rises so the Carnot efficiency rises.

The superheated Rankine cycle with irreversibilities

- There may be irreversibilities in the pump and turbine



- As before, the isentropic pump / turbine efficiency η_p/η_t are defined as,

$$\eta_p = \frac{(W_p)_{ideal}}{(W_p)_{real}} = \frac{h_{2S} - h_1}{h_2 - h_1}$$

$$\eta_t = \frac{(W_t)_{ideal}}{(W_t)_{real}} = \frac{h_3 - h_4}{h_3 - h_{2S}}$$

* In this context, the "compressor" is the pump.

For Personal Use Only -bkwk2

First law analysis of the Rankine cycle

- As before the first law efficiency $\eta_{\text{first law}}$ is defined as

$$\eta_{\text{first law}} = \frac{W_p}{Q_{\text{in}}} = \frac{W_e - W_t}{Q_{\text{in}}}$$

h_1, h_2 found
by tables.

- Applying the STEE (Note that we are not working w/ ideal gases so $\Delta h \neq CPdT$)

PUMP 1→2: $W_p = h_2 - h_1$

Boiler 2→3: $q_{\text{in}} = h_3 - h_2$

Turbine 3→4: $W_t = h_3 - h_4$

condenser 4→1: $q_{\text{out}} = h_1 - h_4$

- consider an isentropic feed pump. Using $-W_p = \int_1^2 v dp + (\frac{1}{2} V_1^2 - \frac{1}{2} V_2^2) - (\frac{1}{2} V_3^2 + \frac{1}{2} V_4^2)$

$$W_p = \int_1^2 v dp$$

definition reg.
 $TDS = dq$ (reverside)

Since water is effectively incompressible, $V_1 = V_2 = \text{const}$, so

$$W_p = V_1 (P_2 - P_1)$$

- Consider an isentropic turbine $\rightarrow S_4 = S_3 \rightarrow$ find x_4 .

$$S_4 = S_f + x_4 S_{fg} \quad \rightarrow \quad x_4 = \frac{S_4 - S_f}{S_{fg} - S_f}$$

After finding x_4 , we can then work out h_4

$$h_4 = h_f + x_4 h_{fg}$$

points to note for the Rankine cycle

- $W_p \ll W_e$ in the usual case

- Operating pressures are very high in the boiler and very low in the condenser

→ req. high structural integrity from the pipework

- The temp. at which heat is req. is just above ambient temp. → compensate for the low rel. low T_{hot} (lower than that in gas turbine / diesel engine).

- The specific volume increases greatly through the turbine → volume flow rate at the exit is very high → turbine exit area is very large.

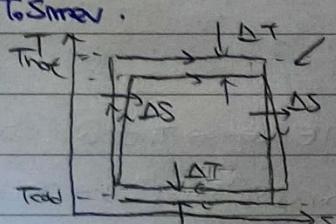
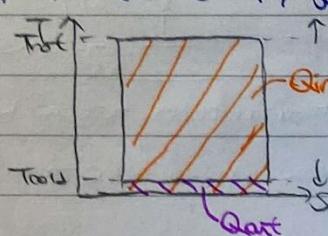
- The steam leaving the turbine typically has dryness fraction $\sim x_4 = 0.9$, → a fog of droplets of diameter about 1 micron → accumulate into larger droplets → erosion.

Raising the efficiency of the Carnot cycle

- To raise the efficiency of the Carnot cycle:

↳ We want to maximise $T_{hot} - T_{cold}$, i.e. increase T_{hot} ($\uparrow Q_{\text{in}}$) and reduce T_{cold} ($\downarrow Q_{\text{out}}$)

↳ Minimise any irreversible entropy generation $T_{\text{carn}} S_{\text{irr}}$.



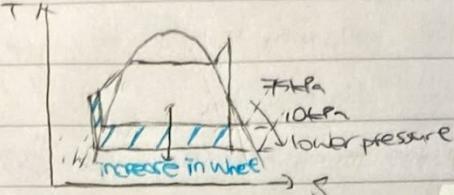
We want to minimise any irreversibilities.

For Personal Use Only -bkwk2

raising the efficiency of the Rankine cycle.

① Lowering condenser pressure.

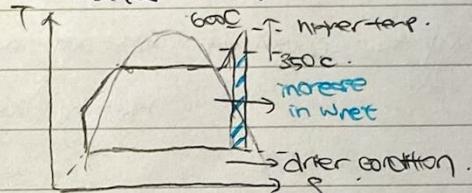
- lowering p lowers T (p and T locked in two-phase region)
- Average T of Q_{in} drops so η_{turb} rises.



- Vapour cycle closed loop $\rightarrow p$ can be below atmospheric pressure (air leak is possible)
- We usually set the pressure s.t. $T_{condenser} - T_{environment} = 10^\circ C$.
- The turbine operates more in the two-phase region.

② Superheat the steam to a higher temp.

- Average T of Q_{in} rises so η_{turb} rises.

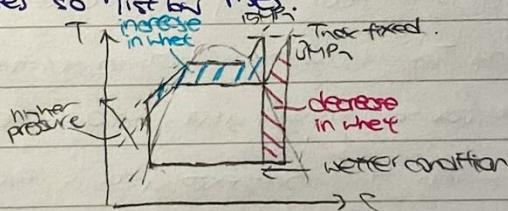


- The turbine operates less in the two-phase region.
- Current steam turbines limited to 600°C by metallurgical considerations (the blades melt)

③ Increasing boiler temperature.

- Increasing p increases T (p and T locked in two-phase region)

- Average T of Q_{in} rises so η_{turb} rises.

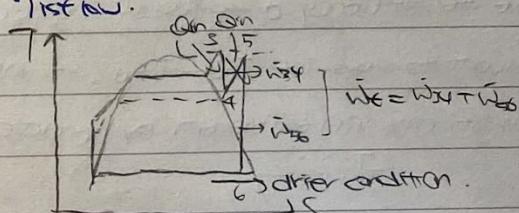


- Since T_{ex} is limited by metallurgical considerations, the turbine operates more in the two-phase region.

④ Reheating.

- Reheat when $p \approx \frac{1}{4} P_{max}$ to lower the moisture content at turbine exit.

- Rise in average T_{in} raises η_{turb} .



- The average T of Q_{in} rises by multiple reheat, but the benefit of the 2nd reheat is around half of the 1st reheat and so on \rightarrow more than 2 reheat is not practical.

For Personal Use Only -bkwk2

Second law analysis of closed cycles.

- consider a closed cycle,

$$m(b_e^0 - b_i^0) = -\dot{W}_x + \int (1 - \frac{T_0}{T}) dQ - T_0 \dot{S}_{irrev}$$

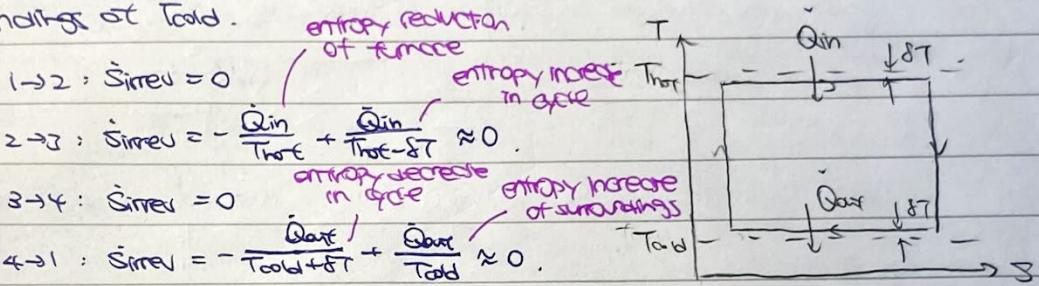
$$\dot{W}_x = \int (1 - \frac{T_0}{T}) dQ - T_0 \dot{S}_{irrev}.$$

shaft power transfer of available power due to heat transfer lost available power due to irreversibilities.

so to maximise work output \dot{W}_x , we must minimise availability destruction (from irreversibilities)

second law analysis of Carnot cycle.

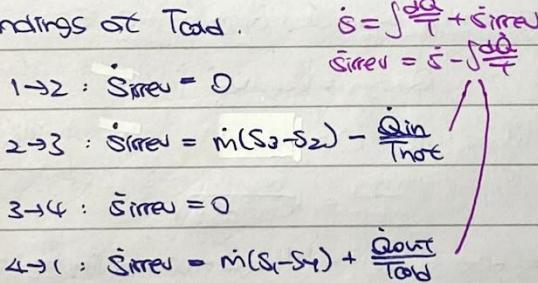
- consider a Carnot cycle w/ Q entering from a furnace at T_{hot} and leaves to surroundings at T_{cold} .



→ destruction of available energy (due to irreversibilities) is zero.

Second law analysis of the Rankine cycle

- Consider a Rankine cycle w/ Q entering from a furnace at T_{hot} and leaves to surroundings at T_{cold} .



→ destruction of available energy (due to irreversibilities) is given by $T_0 \dot{S}_{irrev}$.

& the destruction of available energy for the boiler (2-3) is very large due to

the large temp. diff. w/ the source (furnace).

(could have put reversible heat engine to get shaft work but didn't → entropy generation)

Combined gas-vapour power cycle

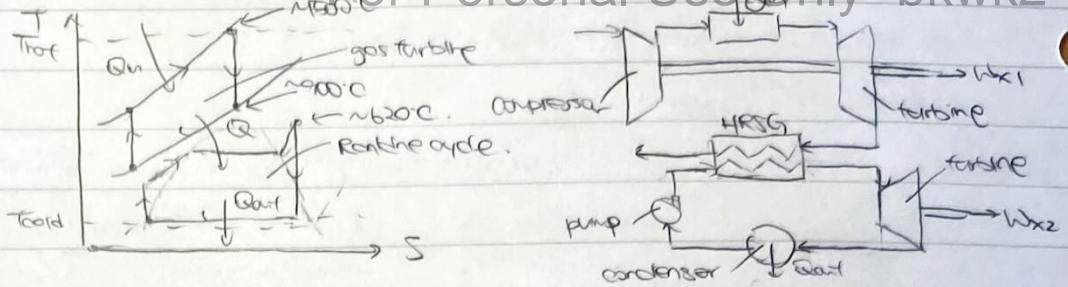
- The efficiency of a gas turbine is limited by its high exhaust temp; Rankine cycle can reject heat to the environment at a low temp.

- The efficiency of a Rankine cycle is limited by its low input temp. if gas turbine can accept heat from the source at a high temp.

→ combine the cycles by connecting the exhaust of the gas turbine w/ the boiler of the Rankine cycle using a heat recovery steam generator (HRSG)

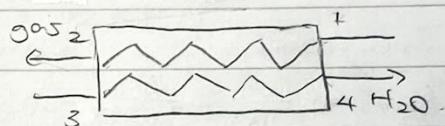
a heat exchanger

For Personal Use Only -bkwk2

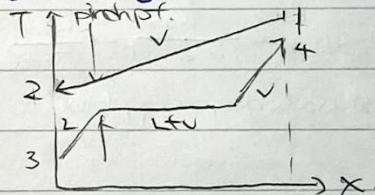


Analysis of HRSG

- The HRSG is a heat-exchanger that exchanges heat from the exhaust gas of the gas turbine to the boiler of the Rankine cycle.
- The HRSG is a counterflow heat-exchanger → minimise the finite temp diff. over which heat transfer occurs → lower irreversible entropy generation.



- Heat-exchangers can be analysed using a distance-temp. diagram.



where x is the fraction of heat transferred (effectively the distance).

- The closest temp. b/w two streams is the pinch pt, usually when the water becomes saturated.
- Equating the heat flow b/wn the two sides of the heat exchanger,

$$\dot{Q}_{out,gas} = \dot{Q}_{in,H2O}$$

$$\dot{m}_{gas}(h_1 - h_2) = \dot{m}_{H2O}(h_4 - h_3)$$

$$\frac{\dot{m}_{gas}}{\dot{m}_{H2O}} = \frac{h_4 - h_3}{C_p(T_1 - T_2)}$$

h_3, h_4 can be found from tables for given T_3, T_4 .

- The destruction of available energy due to irreversibilities in the HRSG is given by

$$S_3, S_4 \text{ can be found from tables for given } T_3, T_4 \quad T_{S,HRSG} = T_o (\dot{m}_{H2O}(S_4 - S_3) + \dot{m}_{gas}(S_2 - S_1)) \\ = T_o (\dot{m}_{H2O}(S_4 - S_3) + \dot{m}_{gas}(C_p \ln(\frac{T_4}{T_1}) - R \ln(\frac{P_4}{P_1})))$$

- Consider a CV from the pinch pt. to the 2/3 end of the HRSG, equating the heat flow b/wn the two sides of the heat exchanger,

$$\dot{Q}_{out,gas} = \dot{Q}_{in,H2O}$$

$$\dot{m}_{gas}(h_{P,g} - h_2) = \dot{m}_{H2O}(h_{P,w} - h_3)$$

$$C_p(T_{P,g} - T_2) = \frac{\dot{m}_{H2O}}{\dot{m}_{gas}} (h_{P,w} - h_3)$$

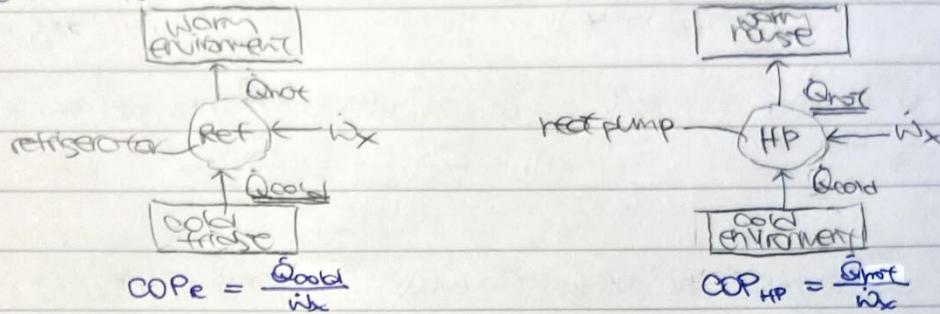
After calculating $T_{P,g}$, and finding $T_{P,w}$ from tables (saturation temp. at the given pressure), we can get the pinch pt temp. diff. $\Delta T_p = T_{P,g} - T_{P,w}$.

For Personal Use Only -bkwk2

Refrigerators and heat pumps

Refrigerator and heat pumps.

- The thermodynamic cycle for refrigerator and heat pumps is the same but what we want is different.

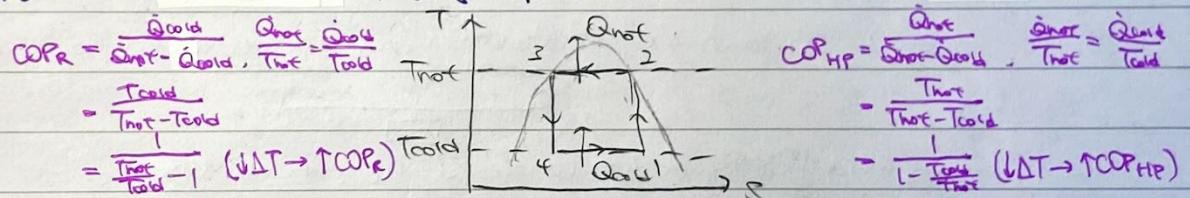


$$NB \quad \dot{Q}_{hot} = \dot{Q}_{cold} + \dot{W}_x \rightarrow COP_{HP} = COP_R + 1.$$

- The best performance of a refrigerator or heat pump is achieved when the cycle is reversible. (Reverse Carnot cycle).

The reversed Carnot cycle.

- Consider the reversed Carnot cycle as a prospective ideal cycle for refrigerators/heat pumps.

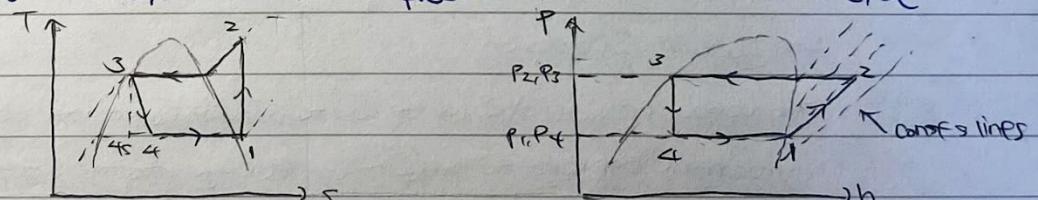


- There is a main problem w/ this:

↳ 1→2 and 3→4 are in the two-phase region → significant practical problems w/ developing compressor/turbines that operate in the two-phase region (impossible erosion)

The real refrigeration cycle.

- The real refrigeration cycle deals w/ the problem of the reversed Carnot cycle

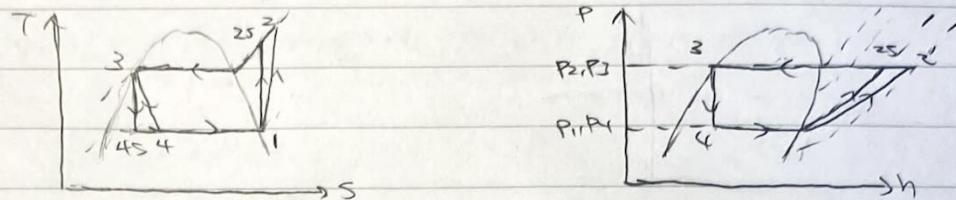


- The starting pt. of the compressor is pushed to the right if it operates in the superheated region
- The turbine is replaced by a throttle valve as it is much cheaper (A throttle valve is isenthalpic but has irreversibilities)

For Personal Use Only -bkwk2

The real refrigeration cycle with irreversibilities.

- There may also be irreversibilities in the compressor.



- As before, the isentropic compressor efficiency η_c is defined as

$$\eta_c = \frac{(W_c)_{ideal}}{(W_c)_{real}} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

First law analysis of the refrigeration cycle.

- As before, the coefficient of performances COP_R/COP_{HP} are defined as

$$COP_R = \frac{\dot{Q}_{load}}{\dot{W}_c}$$

$$COP_{HP} = \frac{\dot{Q}_{heat}}{\dot{W}_c}$$

h_1, h_2 found via tables

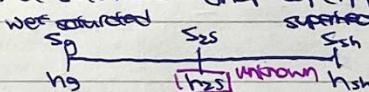
- Applying the STEE (Note that we are not working w/ ideal gases so $\Delta h \neq C_p\Delta T$)

$$\text{Compressor } 1 \rightarrow 2: \quad W_c = h_2 - h_1$$

$$\text{Condenser } 2 \rightarrow 3: \quad \dot{Q}_{heat} = h_2 - h_3$$

$$\text{Evaporator } 4 \rightarrow 1: \quad \dot{Q}_{load} = h_1 - h_4$$

- consider a reversible compressor, we know that $s_{2s} = s_1$, so we can use linear interpolation btwn the wet saturated and superheated pt to find h_{2s} .



Using the efficiency of the compressor η_c , we can find h_2

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \quad \rightarrow \quad h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c}$$

- Since the throttle is isenthalpic, $h_4 = h_3$

choice of refrigerants

- Refrigerants must be cheap, stable, inert and non-toxic
- The pressure range corresponding to the operating temp. range should be small to minimise the work of compression.
- The vapour pressure should be low to reduce condenser costs but should be higher than atmospheric pressure to prevent air leakage into the system.
- The latent heat of evaporation should be high in order to keep the mass flow rate low (so smaller refrigerators / heat pumps).
- Early refrigerants used SO_2 and CH_2Cl → banned as they caused a few credits.
- CFCs were developed ($R-12, R-21$ etc) → banned as they destroyed the ozone layer
- Now use recently-developed $R-134a$ (chlorine free).

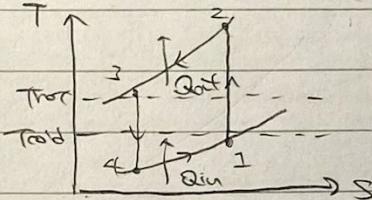
For Personal Use Only -bkwk2

Heat pumps.

- Heat pumps are more expensive to purchase and install than other heating systems, but can save money in the long run.
- the most common heat source is the atmosphere but water and soil are also used (practical COP_{HP} are 1.5-4).
- The biggest problem w/ using the atmosphere is frost → frost disrupts heat transfer into coils → defrost by adding a reverse valve.

Gas refrigeration (Reverse Joule cycle)

- A single-phase refrigeration cycle could be achieved by using the reverse Joule cycle.



- The problem w/ this cycle is that we req. $T_{2,3} > T_{hot}$ and $T_{4,1} < T_{cold}$ so operating temp. range is quite large → inefficient
- Gas refrigeration systems are used for specialist purposes:
 - ↳ Can achieve low temp. ($T_4 < T_1$) → liquefaction of gases
 - ↳ only req. gases as the working fluid → lightweight applications (e.g. aircraft cabin cooling)

Properties of mixtures

Describing mixture composition

- In any fraction of mixture, the no. of moles n_i is given by

$$n_i = \frac{m_i}{M_i}$$

where m_i is the mass of the fraction, M_i is the molecular mass.

- The total mass m and no. of moles n in a mixture is given by.

$$m = \sum m_i$$

$$n = \sum n_i$$

- The mass fraction m_f_i and mole fraction x_i of a mixture is given by.

$$m_f_i = \frac{m_i}{m}$$

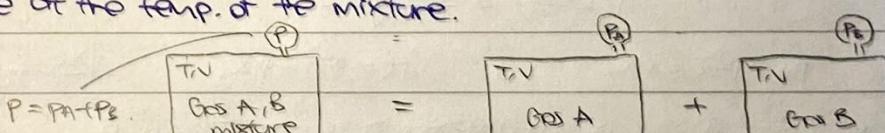
gravimetric analysis

$$x_i = \frac{n_i}{n}$$

volumetric analysis

Dalton's law

- Dalton's law states that the pressure of a mixture of gases is equal to the sum of the pressures of the individual components when each occupies a volume equal to its share of the total volume of the mixture at the same temperature.



Symbolically,

$$P = \sum P_i$$

where P_i is the partial pressure of component i .

- * THIS WORKS SINCE EACH COMPONENT OCCUPIES SUCH A SMALL VOLUME, INDICATES OF EACH ONE FREE TO ROAM THROUGH THE WHOLE VOLUME.

Relating P, V, T for an ideal gas.

- since each of the gases act alone, we can say

$$P_i = \frac{n_i R T}{V}$$

We know for the mixture $P = \frac{n R T}{V}$, so the partial pressure P_i is given by

$$P_i = P \frac{n_i}{n} = P x_i$$

- similarly, the partial volume V_i is given by

$$V_i = V \frac{n_i}{n} = V x_i$$

combine $V_i = \frac{n_i R T}{P}$ and $V = \frac{n R T}{P}$

→ use mole fraction for P, V .

For Personal Use Only -bkwk2

Evaluating U, H, S, Cp, Cv, R.

- The total internal energy of the mixture equals the sum of internal energies in the components.

$$U = m_u = \sum_i M_i u_i$$

$$\therefore u = \frac{1}{m} \sum_i \frac{M_i}{M} u_i$$

- Similarly, for other properties,

$$h = \sum_i \frac{M_i}{M} h_i$$

$$s = \sum_i \frac{M_i}{M} s_i$$

$$C_V = \sum_i \frac{M_i}{M} C_{V,i}$$

$$C_P = \sum_i \frac{M_i}{M} C_{P,i}$$

$$R = \sum_i \frac{M_i}{M} R_i$$

→ Use mass fraction for u, h, s, Cp, Cv, R.

Mixture of gases and vapours.

- A mixture of dry air and water vapour can be treated as a mixture of two component gases.

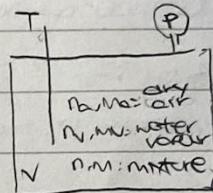
$$n = n_a + n_v$$

$$M = M_a + M_v$$

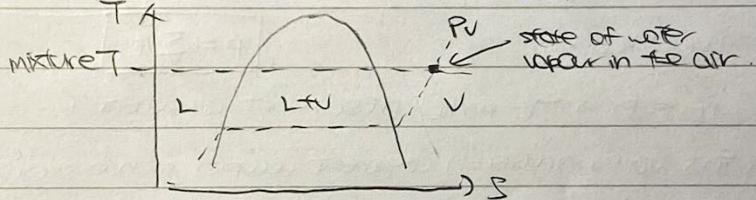
$$P = \frac{nRT}{V} = \frac{(n_a + n_v)RT}{V}$$

$$P_a = P \frac{n_a}{n} = P x_a$$

$$P_v = P \frac{n_v}{n} = P x_v$$



- The water vapour is supercooled steam at T and P.



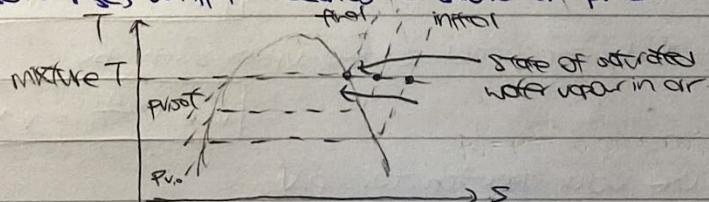
Saturated mixture

- If there is a liquid water in the bottom of the tent, then the water will evaporate until the air becomes saturated and eqm. is reached.

- Consider initially the above mixture of dry air and water vapour, then water is added.

→ the partial pressure of water vapour increases as more water evaporates

→ the partial pressure rises until it reaches the saturation pressure of water at T.



- A mixture can become saturated by

→ evaporation of liquid water → ↑ n_v → ↑ P_v

→ raising the pressure of the mixture → ↑ P → ↑ P_v.

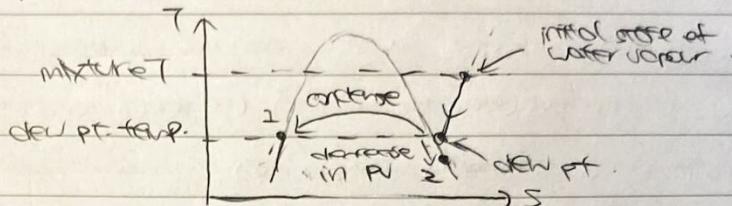
→ lowering the temp. of the mixture.

$$P_v = P \frac{n_v}{n}$$

For Personal Use Only -bkwk2

Dew point temperature

- For an unsaturated air mixture, the water vapour is originally superheated. As we cool the font, the partial pressure P_V is const., and we drop to the dew pt. temp. (keeping volume V const. → keep mixture pressure P const → keep partial pressure P_V const)
- At the dew pt. temp., water starts to condense.



- If we cool beyond the dew pt. temp., we obtain liquid water at the dew pt. temp. (1) since the no. of moles of water vapour n_V decreases, the partial pressure P_V decreases, so we obtain a saturated vapour at a lower pressure/temp. (2)

Specific humidity and relative humidity

- The specific humidity w is defined as

$$w = \frac{m_V}{m_a}$$

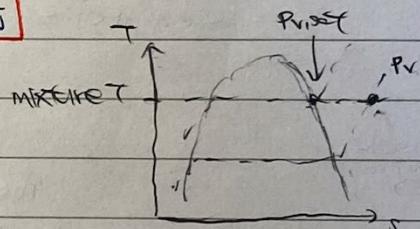
$$w = \frac{m_V}{m_a} = \frac{m_V N_A / R T}{m_a N_A / R T} = \frac{m_V P_V}{m_a P_a} = \frac{0.622 P_V}{26.766 P_a}$$

$$w = 0.622 \frac{P_V}{P_a} = 0.622 \frac{P_V}{P - P_V}$$

- The relative humidity ϕ is defined as

$$\phi = \frac{n_V}{n_{V, \text{sat}}}$$

$$\phi = \frac{n_V}{n_{V, \text{sat}}} = \frac{P_V / R T}{P_{V, \text{sat}} / R T} = \frac{P_V}{P_{V, \text{sat}}}$$



where ϕ in temperate climates are in the range 0.3 - 0.6.

* The diff. defn of humidity w/ϕ are related via the partial pressure of water vapour P_V .

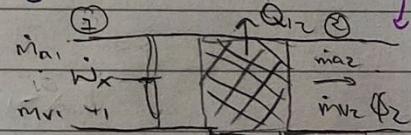
$\phi_2 = 1$ initially but drops as the air warms up

Air Conditioning.

- Consider an air conditioner that passes in moist over a cooling coil which removes water vapour by condensation.

Const. of mass

$$m = m_{V1} + m_{W1} = m_{V2} + m_{W2} + m_{W3}$$



$$\text{Defn of } \phi \text{ (inlet)} \quad \phi_1 = \frac{P_{V1}}{P_{V, \text{sat}}, 1} \rightarrow P_{V1} = \phi_1 P_{V, \text{sat}, 1}$$

$$\text{Defn of } w \text{ (inlet)} \quad w_1 = \frac{m_{V1}}{m_a} = 0.622 \frac{P_{V1}}{P - P_{V1}}$$

$$\text{Defn of } \phi \text{ (outlet)} \quad \phi_2 = \frac{P_{V2}}{P_{V, \text{sat}}, 2} \rightarrow P_{V2} = \phi_2 P_{V, \text{sat}, 2}$$

$$\text{Defn of } w \text{ (outlet)} \quad w_2 = \frac{m_{V2}}{m_a} = 0.622 \frac{P_{V2}}{P - P_{V2}}$$

$$\rightarrow \text{calculate } m_1, m_{V1}, m_{V2}, m_{W2}$$

$$\text{SSEE} \quad Q_{12} = m_{V1} h_{v1} - m_{W3} h_{w3} - m_{V2} h_{v2} + m_a c_p (T_1 - T_2) + \dot{m}_w w_x$$

$$\text{water in water out air}$$

(we approximate $h_{w3} = h_{v1}$, valid since the h-s diagram is approx flat)

V : water vapour
 w : liquid water

a : dry air

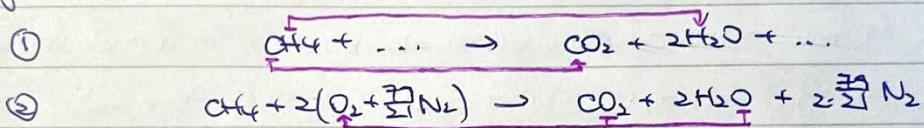
w_{sat} : saturated water vapour

For Personal Use Only -bkwk2

Combustion processes

Stoichiometric combustion.

- We assume air is 21% O₂ and 79% N₂.
- To find the stoichiometric eqn. for combustion
 - ↳ 1) write down fuel and fuel-derived products.
 - ↳ 2) use the no. of oxygen molecules on RHS to set off on LHS.
- N₂ is carried over to the RHS (it takes no part in the combustion but needs to included as it gets warmed up).
- e.g. For the combustion of C₂H₄ in air,



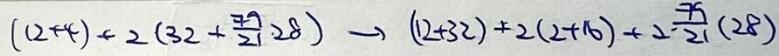
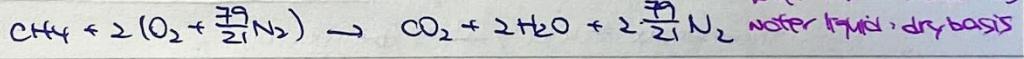
Mass fraction and air fuel ratio.

- The air fuel ratio is defined as the ratio of masses,

$$\boxed{\text{AFR} = \frac{m_{\text{air}}}{m_f}}$$

- To find the mass fraction or air fuel ratio, we convert molar chemical eqn. to masses.

- e.g. For the combustion of C₂H₄ in air. (assume wet basis)



$$\text{so AFR} = \frac{274.67}{16} = 17.2, M_f_{\text{CO}_2} = \frac{44}{16+274.67}, M_f_{\text{H}_2\text{O}} = \frac{36}{16+274.67}, M_f_{\text{N}_2} = \frac{210.67}{16+274.67}$$

here since we assumed wet basis, H₂O is included.
(determine dry/wet basis from temp. of products)

Lambda and equivalence ratio.

- Lambda λ and equivalence ratio ϕ are commonly used to express stoichiometry.

$$\lambda = \frac{\text{AFR}}{\text{AFR}_{\text{stoichiometric}}}$$

$$\phi = \frac{\text{AFR}_{\text{stoichiometric}}}{\text{AFR}}$$

$$\phi = \frac{1}{\lambda}$$

$\lambda > 1 / \phi < 1$: the combustion is lean (excess O₂)

$\lambda = 1 / \phi = 1$: the combustion is stoichiometric

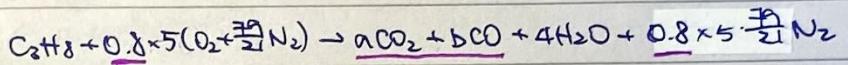
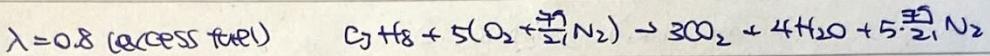
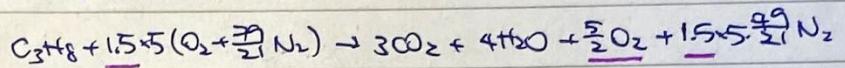
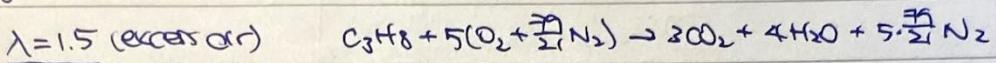
$\lambda < 1 / \phi > 1$: the combustion is rich (excess fuel)

For Personal Use Only -bkwk2

calculations with excess air or excess fuel

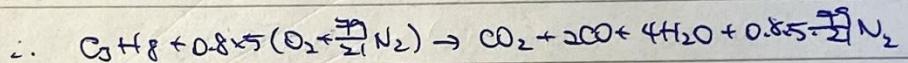
- We always start off w/ the stoichiometric case ($\lambda=1$).
- If $\lambda > 1$, (excess air), then we simply carry over the excess O_2 to the RHS.
- If $\lambda < 1$, (excess fuel), then we change nCO_2 into $aCO + bCO_2$ and solve for a and b by balancing the no. of atoms of C/O on both sides. (Note stoichiometric coefficient of H_2O is unchanged as we assume the hydrogen burns first).

- e.g. For the combustion of C_3H_8 in air.



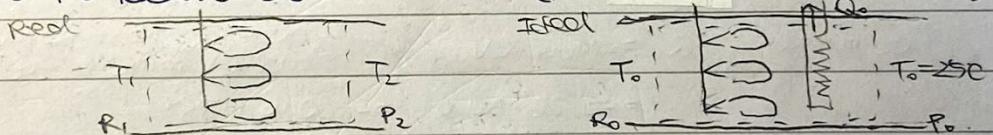
$$C: \quad 3 = a+b \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad a=1$$

$$O: \quad 8 = 2a+b+4 \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad b=2$$



first law applied to combustion

- To consider the general case of combustion (inlet at T_1 , outlet at T_2), we must first consider the idealised case of combustion (inlet and outlet at T_0 , w/ Q_0 out).

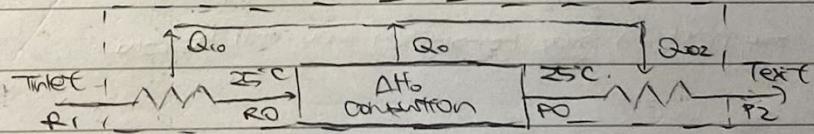


- We split the general case of combustion into three processes.

↳ 1) Heat removed so reactants brought from initial state (R1) to 25°C (R0)

↳ 2) Heat removed so that combustion occurs at constant 25°C (R0 to P0)

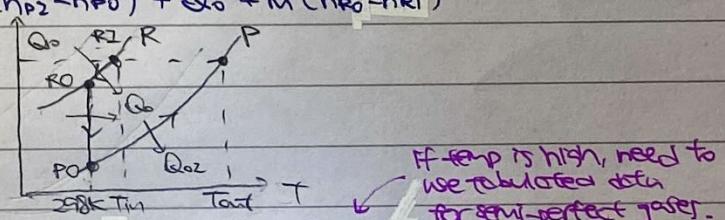
↳ 3) Heat added so combustion products raised to final T (P0 to P2)



Applying QST, $\cancel{\dot{Q}_1^0} - \cancel{\dot{Q}_2^0} = m(h_{P2} - h_{R1}) = m[(h_{P2} - h_{P0}) + (h_{P0} - h_{R0}) + (h_{R0} - h_{R1})]$

$$Q = m(h_{P2} - h_{P0}) + Q_0 + m(h_{R0} - h_{R1})$$

be careful w/ the different masses involved!



$$Q_{10} = m(h_{R0} - h_{R1}) : \text{low temp} - \text{use } \sum m_i C_p (T_{R0} - T_{R1}) (+LH \text{ for liquid})$$

$$Q_{20} = m(h_{P2} - h_{P0}) : \text{high temp} - \text{use tabulated data for semi-perfect gases} [m_h = n_h]$$

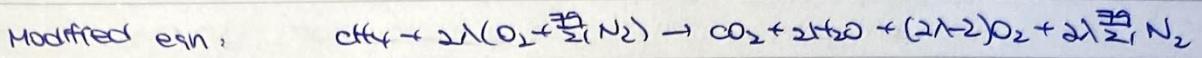
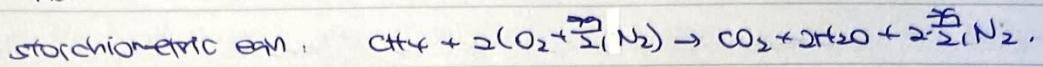
$$Q_0 : \text{use tabulated data for calorific values. } [Q_0 = m \cdot CV]$$

If temp is high, need to use tabulated data for semi-perfect gases.

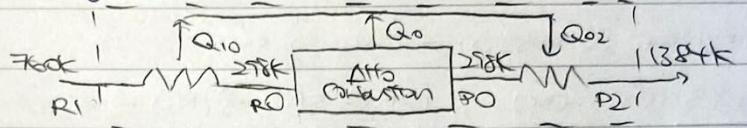
For Personal Use Only -bkwk2

-eg. In a methane-based engine, the compressor exit temp. is 760K and the turbine inlet temp. is 1384K. Calculate the equivalence ratio of the combustion process.

Assume fuel enters at 25°C. and that air is in excess.



consider the following CV.



$$\text{GEE: } \dot{Q}^0 - \dot{W}^0_x = \dot{m}(\bar{h}_{p2} - \bar{h}_{p0}) + \dot{Q}_o + \dot{m}(\bar{h}_{r0} - \bar{h}_{p1})$$

$$\text{Reactants: } \dot{m}(\bar{h}_{r0} - \bar{h}_{p1}) = \sum \dot{m}_i C_{p,i} \Delta T_i$$

$$= \dot{m}_{\text{CH}_4} C_{p,\text{CH}_4} (T_0 - T_1) + \dot{m}_{\text{air}} C_{p,\text{air}} (T_0 - T_1)$$

$$= 2\lambda(137.3)(1.04)(298 - 760)$$

$$= (-132000\lambda) \text{ kJ s}^{-1}$$

$$\begin{aligned} & \leftarrow C_{p,\text{air}} \text{ at } \frac{T_0 + T_1}{2} = 529 \text{ K.} \\ & \theta = \frac{T_0}{T_1} = \frac{250}{760} = 0.33 \\ & \text{cp} = 1.02 \quad 1.04 \quad 1.05 \end{aligned}$$

$$\text{Products: } \dot{m}(\bar{h}_{p2} - \bar{h}_{p0}) = \sum n_i (\bar{h}_{p2,i} - \bar{h}_{p0,i})$$

$$= n_{\text{CO}_2} (\bar{h}_{p2,\text{CO}_2} - \bar{h}_{p0,\text{CO}_2}) + n_{\text{H}_2\text{O}} (\bar{h}_{p2,\text{H}_2\text{O}} - \bar{h}_{p0,\text{H}_2\text{O}})$$

$$+ n_{\text{O}_2} (\bar{h}_{p2,\text{O}_2} - \bar{h}_{p0,\text{O}_2}) + n_{\text{N}_2} (\bar{h}_{p2,\text{N}_2} - \bar{h}_{p0,\text{N}_2})$$

$$= 1(64.39 - 9.77) + 2(52.66 - 9.91) + (2\lambda - 2)(45.05 - 8.56) + 2\frac{7}{2}(43.07 - 8.56)$$

$$= (67.76 + 331.6\lambda) \text{ MJ s}^{-1}$$

$$\text{Combustion: } \dot{Q}_o = -\dot{m}_{\text{CH}_4} \cdot \text{CV}_{\text{eff}} \quad \leftarrow \text{note } \dot{Q}_o \text{ must be -ve and CV in DB is true.}$$

$$= -1.16 \cdot 50.01 = -800.16 \text{ MJ s}^{-1}$$

$$\text{so } -132\lambda + 67.76 + 331.6\lambda - 800.16 = 0 \rightarrow \lambda = 3.67$$

$$\text{and } \phi = \frac{\lambda}{3.67} = 0.272$$

* If we use $C_p \Delta T$ to find enthalpy changes, C_p should be the value at T_{mean} .

* We can use $\Delta h = C_p \Delta T$ even for products when $\lambda \gg 1$ (large excess of air) as the products after combustion may still be cold.