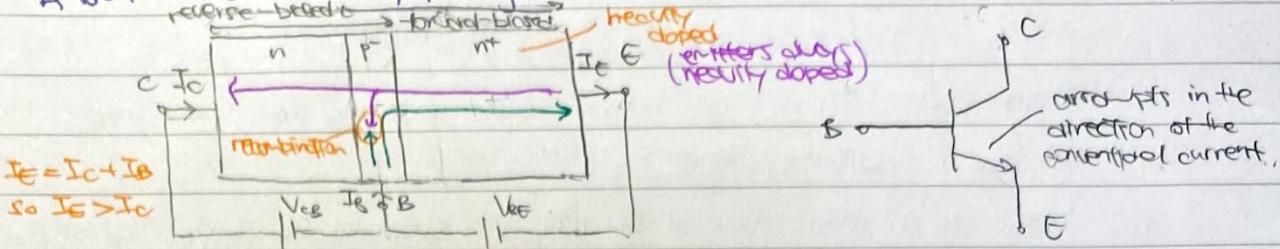


# Bipolar Junction Transistor For Personal Use Only -bkwk2

## The bipolar junction transistor

The bipolar junction transistor (BJT).

- A BJT can be made by putting two diodes back to back (npn or pnp).



- For an npn transistor, the base-emitter diode is forward biased and the collector-base diode is reverse biased.
- As the base is very thin and the collector is +ve, most e<sup>-</sup>s cross the base and flow to the collector, w/ a small fraction of e<sup>-</sup> stop and recombine in the base.
- The base current I<sub>b</sub> is the combination of a very small hole current from the base to the emitter and the hole current that recombines w/ the e<sup>-</sup> from the base.
- The depletion region of the reverse-biased p-n junction is unable to stop the flow of e<sup>-</sup> because the depletion region is very thin, and there is a e<sup>-</sup> conc. gradient (n<sup>+</sup>, n).
- In summary, a small change in base voltage V<sub>b</sub>, and hence base current I<sub>b</sub> causes a large change in collector current I<sub>c</sub>.
- Since some e<sup>-</sup> from the emitter recombines at the base, I<sub>e</sub> > I<sub>c</sub>. As only a small fraction of e<sup>-</sup> recombine, I<sub>b</sub> ≪ I<sub>e</sub>, therefore I<sub>e</sub> ≈ I<sub>c</sub>.
- The currents I<sub>c</sub>, I<sub>e</sub> and I<sub>b</sub> can be related as follows :

$$I_c = \alpha I_e$$

where  $\alpha$  is the common base current gain and usually ranges from 0.9 to 0.999.

By Kirchoff's current law,  $I_e = I_c + I_b$ , so

$$I_c = \frac{\alpha}{1+\alpha} I_b = h_{FE} I_b$$

where  $h_{FE}$  is the forward current gain and usually ranges from 20 to 500. high power high speed.

- $h_{FE}$  is practically constant in the operating region there may be variations from one transistor to another due to manufacturing tolerance and design variations.
- The transistor is "bipolar" as both holes and e<sup>-</sup>s take part in current flow. However, in an npn transistor, e<sup>-</sup>s dominate the current flow and vice versa.
- For a given size of device, an npn transistor performs better than a pnp transistor as e<sup>-</sup>s are more mobile than holes. (e<sup>-</sup> move at c ; holes move at speed of sound)

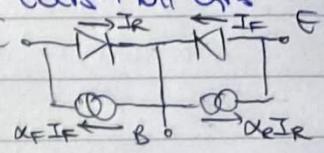
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## Characteristics of the bipolar transistor

- The DC operation of the transistor is governed by the Ebers-Moll eqns

$$I_E = I_{Es} \left[ \exp\left(\frac{V_{BE}}{V_T}\right) - 1 \right] - \alpha_E I_C \left[ \exp\left(\frac{V_{CE}}{V_T}\right) - 1 \right]$$

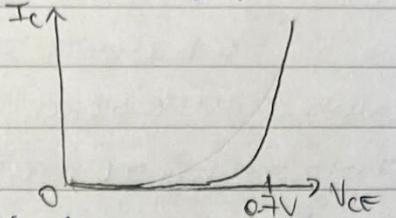
$$I_C = \alpha_F I_{Es} \left[ \exp\left(\frac{V_{BE}}{V_T}\right) + 1 \right] - I_{Cs} \left[ \exp\left(\frac{V_{CE}}{V_T}\right) - 1 \right]$$



- The Ebers-Moll eqns are beyond the scope of this module  $\rightarrow$  graphical approach.

### ① Input characteristics

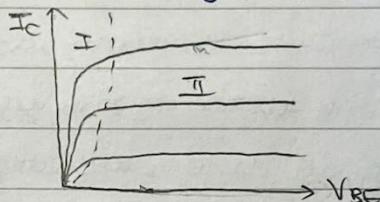
- The I/P characteristics of the BJT is a graph of  $I_C$  vs  $V_{CE}$  w/ fixed  $V_{BE}$ .



- The base-emitter junction starts to conduct at  $V_{BE} \approx 0.7V$ .

### ② Output characteristics

- The O/P characteristics of the BJT is a graph of  $I_C$  vs  $V_{CE}$  w/ fixed  $V_{BE}$



- In region I,  $I_C$  strongly depends on  $V_{BE}$  (non-linear region  $\rightarrow$  bad)

In region II,  $I_C$  depends mainly on  $I_B$  and hardly on  $V_{BE}$  (linear region  $\rightarrow$  good).

\* We want to control  $I_C$  using  $I_B$ , not  $V_{BE}$ .

## Performance limits of the bipolar transistor

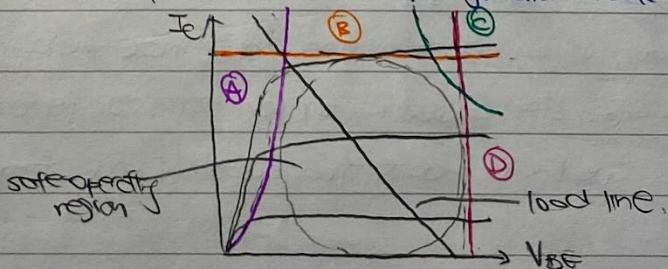
- The BJT characteristic sets the safe operating region (or forbidden zones) for the circuit based on the following limiting factors

↳ ①: Non-linear (for  $I_C, I_E$ ) region

↳ ②: Max. limit of  $I_C$

↳ ③: Max. power dissipation limit,  $(V_{CE}I_C)_{max}$ . If  $V_{CE}I_C > (V_{CE}I_C)_{max}$ , the BJT becomes very hot.

↳ ④: Max. value of  $V_{CE}$ . The reverse-biased p-n junction breaks down



- Ideally, the load line should cross diagonally through the safe operating region and the operating pt. should be as central as possible (to allow for max swing abt. the operating pt.)

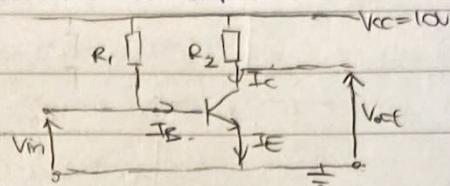
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## DC operating point and AC small signal model

DC operating point for the bipolar transistor

- We can find the rep. resistance values after choosing the op. pt. ( $V_{CE}$ ,  $I_C$ ,  $I_S$ ).

- As an example, consider the common emitter amplifier.

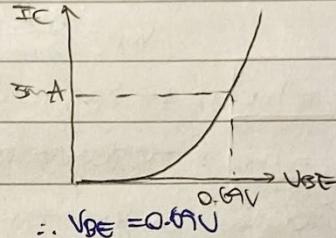


Say we choose an operating pt. of  $V_{CE} = 5V$ ,  $I_C = 5mA$ ,  $I_S = 30nA$ .

Applying ohm's law, we have .

$$R_1 = \frac{V_{CC} - V_{BE}}{I_B} \quad R_2 = \frac{V_{CC} - V_{CE}}{I_C}$$

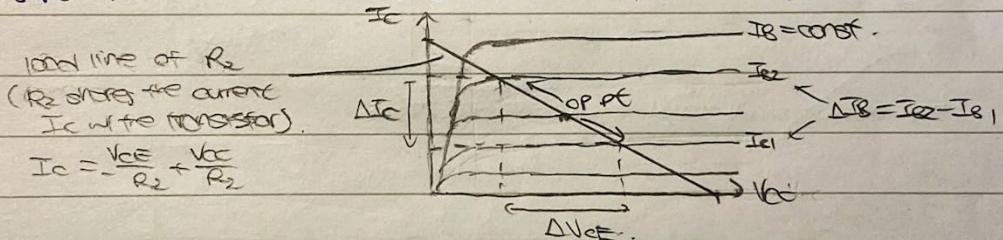
Using the input characteristics, we can find  $V_{BE}$  .



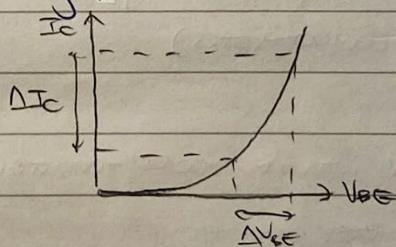
We can now substitute values in the eqns above and find that  $R_1 = 27k\Omega$ ,  $R_2 = 1k\Omega$ .

- We can graphically estimate the current/voltage gains using both the I/p and o/p characteristics.

First superimpose the load line of  $R_2$  on top of the o/p characteristic of the transistor.



Then we can find  $\Delta V_{CE}$  corresponding to  $\Delta I_C$ .



After finding  $\Delta I_C$ ,  $\Delta I_B$ ,  $\Delta V_{CE}$ ,  $\Delta V_{BE}$  graphically, we can find the current/voltage gains.

current gain =	$\frac{\Delta I_c}{\Delta I_B}$
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voltage gain =	$\frac{\Delta V_{CE}}{\Delta V_{BE}}$
----------------	---------------------------------------

- The graphical method for finding the gain is tedious/inaccurate → use AC small signal model.

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AC small signal model for the bipolar transistor:

- At the output, a small change in  $I_C$  due to  $I_B$  and  $V_{CE}$  gives us.

$$I_C = \frac{\partial I_C}{\partial I_B} I_B + \frac{\partial I_C}{\partial V_{CE}} V_{CE}$$

where

$$\left. \frac{\partial I_C}{\partial I_B} \right|_{V_{CE}} = h_{FE}$$

$h_{FE}$  is the current gain measured at const  $V_{CE}$

and

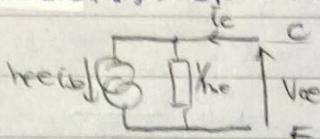
$$\left. \frac{\partial I_C}{\partial V_{CE}} \right|_{I_B} = h_{RE}$$

$h_{RE}$  is the output admittance required at const  $I_B$ .

- The expression for  $I_C$  can therefore be written as,

$$I_C = h_{FE} I_B + h_{RE} V_{CE}$$

We can represent this using the small-signal equivalent circuit as follows.



- At the input, a small change in  $V_{BE}$  due to  $I_B$  and  $V_{CE}$  gives us.

$$V_{BE} = \frac{\partial V_{BE}}{\partial I_B} I_B + \frac{\partial V_{BE}}{\partial V_{CE}} V_{CE}$$

where

$$\left. \frac{\partial V_{BE}}{\partial I_B} \right|_{V_{CE}} = h_{IE}$$

$h_{IE}$  is the input resistance measured at const  $V_{CE}$

and

$$\left. \frac{\partial V_{BE}}{\partial V_{CE}} \right|_{I_B} = h_{RE}$$

$h_{RE}$  is the reverse voltage transfer ratio at const  $I_B$ .

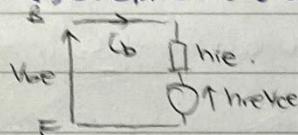
"Resistance" of the forward-biased PN junction (RE).

usually negligible as most of the voltage drop is across the reverse-biased PN junction (CB).

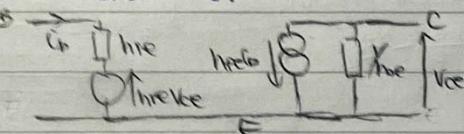
- The expression for  $V_{BE}$  can therefore be written as

$$V_{BE} = h_{IE} I_B + h_{RE} V_{CE}$$

We can represent this using the small-signal equivalent circuit as follows.



- The overall small signal equivalent circuit is.

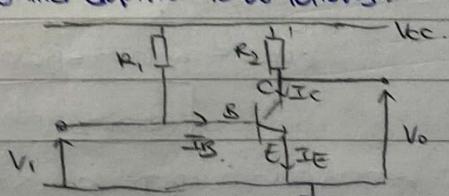


- The parameters change w/ operating pt. ( $h_{FE}$ ,  $h_{IE}$  can be found graphically from the transistor i/p and o/p characteristics)

Circuit design with the bipolar transistor.

The common emitter amplifier.

- The circuit of the common emitter amplifier is as follows:



DC analysis

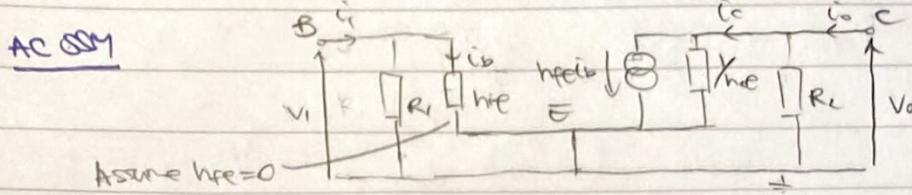
choose  $V_{CE}$ ,  $I_C$ ,  $I_B$  to select the operating pt. ( $V_{CE} \approx \frac{1}{2} V_{CC}$ ).

$$R_1 = \frac{V_{CC} - V_{CE}}{I_B}$$

$$R_2 = \frac{V_{CC} - V_{CE}}{I_C}$$

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AC SSM



At the input (base)

$$V_i = h_{fe} C_b$$

At the output (collector)

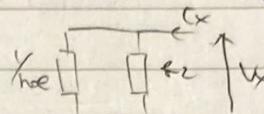
$$V_o = -h_{fe} C_b \left( R_L // \frac{1}{h_{fe}} \right) = -h_{fe} C_b \left( \frac{R_L}{R_L + h_{fe}} \right)$$

$$\therefore \frac{V_o}{V_i} = -\frac{h_{fe}}{h_{fe}} \left( \frac{R_L}{R_L + h_{fe}} \right)$$

Input resistance

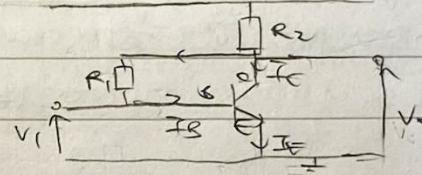
$$R_{in} = \frac{V_i}{C_b} = R_f // h_{fe} = \frac{R_f h_{fe}}{R_f + h_{fe}}$$

Output resistance - short to input and find  $\frac{V_o}{C_x}$ .



$$R_{out} = \frac{V_o}{C_x} = h_{fe} R_2 = \frac{R_2}{1+h_{fe}}$$

- In practice, the transistor parameters can vary quite a bit  $\rightarrow$  using the common emitter amplifier would result in unacceptable variations in operating PT.
- We can improve the stability of the circuit by connecting  $R_L$  to the collector rather than  $V_{cc}$ . Introducing feedback.

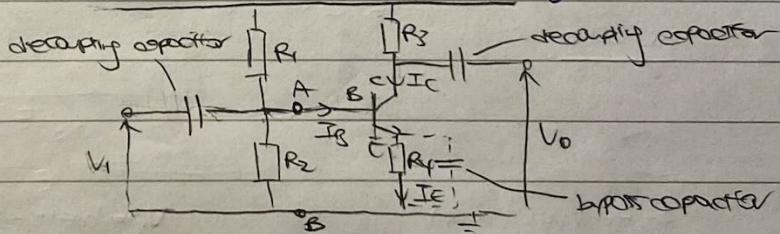


If  $h_{fe}$  is above/below nominal,  $V_{ce}$  is lower/higher, which reduces/increases  $I_B$ , giving a degree of self-compensation

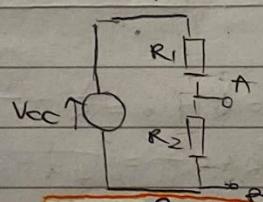
\* THIS circuit is acceptable for audio/vocal applications but there is a penalty as the small-signal input resistance is reduced.

Improved circuit.

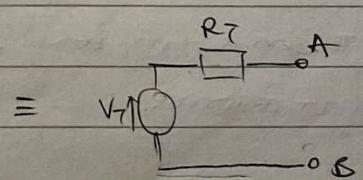
- Including a resistor in the emitter circuit enables a stable operating PT to be achieved.



- The potential divider formed by  $R_1, R_2$  sets  $V_P$  const. if  $I_B$  is small compared to the current through the divider. For analysis, we can replace the potential divider w/ its Thevenin equivalent.



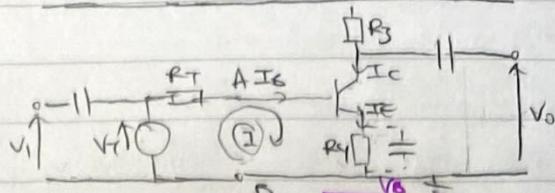
$$V_T = \frac{R_2}{R_1 + R_2} V_{cc}$$



$$f_T = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

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- The circuit then becomes .



DC analysis

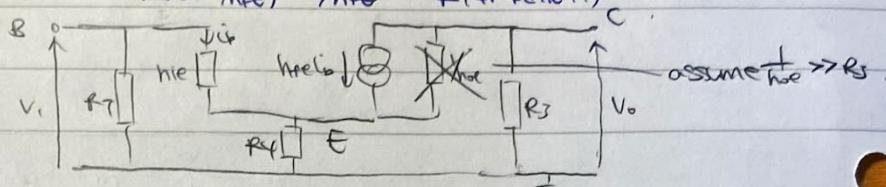
$$\textcircled{1} \text{ DC: } V_T = I_B R_T + V_{BE} + I_E R_4$$

$$= I_B R_T + V_{BE} + (I_B + I_C) R_4.$$

$$= \frac{I_C}{h_{FE}} R_T + V_{BE} + I_C \left( \frac{1}{h_{FE}} + 1 \right) R_4$$

$$\therefore I_C = \frac{V_T - V_{BE}}{R_4 (1 + h_{FE}) + R_T h_{FE}} = \frac{h_{FE} (V_T - V_{BE})}{R_T + R_4 (h_{FE} + 1)}$$

AC SSM



At the input (base)

$$V_i = h_{ie} i_b + (1 + h_{fe}) i_b R_T$$

At the output (collector)

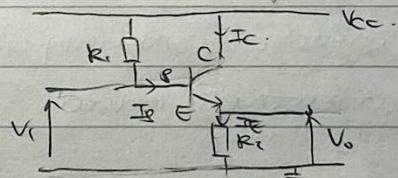
$$V_o = -h_{fe} i_b R_L$$

$$\therefore \frac{V_o}{V_i} = -\frac{h_{fe} R_L}{h_{ie} + (1 + h_{fe}) R_T}$$
extra term reduces the gain.

- Having the extra resistor in the emitter circuit reduces the gain, but this can be overcome by bypassing  $R_E$  w/ a bypass capacitor. (At high frequencies, we effectively have short circuit).

The emitter follower — a buffer.

- The circuit of the emitter follower is as follows :



DC analysis

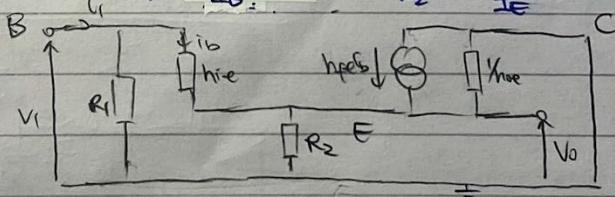
choose  $V_{CE}$ ,  $I_E$ ,  $I_C$  to select the operating pt. ( $V_{CE} \approx V_{CE}$ )

$$V_B = V_{CC} - V_{CE} + V_{BE}$$

$$R_1 = \frac{V_{CC} - V_B}{I_B}$$

$$R_2 = \frac{V_{CC} - V_{CE}}{I_E}$$

AC SSM



At the input (base)

$$V_i = h_{ie} i_b + V_o$$

At the output (emitter)

$$(1 + h_{fe}) i_b = \frac{V_o}{R_2} + \frac{V_o}{h_{oe}}$$

$$\therefore \frac{V_o}{V_i} = \frac{1 + h_{fe}}{1 + h_{fe} + h_{oe} R_2 + h_{oe} h_{ie}}$$

\* The gain is slightly below 1.

Input resistance .

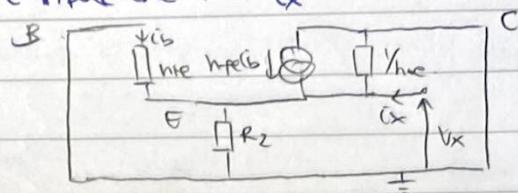
$$i_r = \frac{V_i}{R_1} + i_b = \frac{V_i}{R_1} + \frac{V_i - V_o}{h_{ie}} = V_i \left( \frac{1}{R_1} + \frac{1}{h_{ie}} \right)$$

$$\therefore R_{in} = \frac{V_i}{i_r} = \frac{1}{\frac{1}{R_1} + \frac{1}{h_{ie}}}$$

$$G = \frac{V_o}{V_i}$$

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Output resistance - short to input and find  $\frac{V_x}{C_x}$



Find the base current

$$i_B = \frac{Q - V_x}{h_{fe}} = -\frac{V_x}{h_{fe}}$$

$\Sigma I$  at emitter

$$(1+h_{fe})i_B + C_x = \frac{V_x}{R_L} + \frac{V_x}{h_{fe}}$$

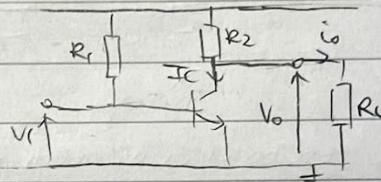
$$\therefore R_{out} = \frac{V_x}{C_x} = \frac{1}{R_L + h_{fe} + \left(\frac{h_{fe}}{h_{fe}}\right)}$$

i.e.  $R_L$ ,  $h_{fe}$  and  $\frac{h_{fe}}{h_{fe}}$  in parallel.

## Power amplifiers and large signals.

The bipolar transistor as a power amplifier. [class A operation],

- Power amplifiers can be constructed w/ both FETs and BJTs (predominantly BJTs)
- Consider a common emitter amplifier w/ a resistive load connected.



- For a sinusoidal wave output w/ amplitude  $V_o, C_0$ , the AC power out is given by

$$P = \frac{V_o}{2} \cdot \frac{C_0}{2} = \frac{V_o^2}{2R_L}$$

- Ignoring the effects of saturation, the max. amplitude of voltage swing is  $V_o = \frac{V_{cc}}{2}$ , so

$$P_{max} = \frac{V_{cc}^2}{8R_L}$$

- The DC power in is given by the average current drawn multiplied by the supply voltage.

$$P = I_{dc} V_{cc}$$

current varies sinusoidally about  $I_{dc} \rightarrow \text{average} \neq I_{dc}$ .

where  $I_{dc}$  is the steady state current at the operating pt.

- The efficiency of the amplifier is therefore.

$$\eta = \frac{\text{AC power out}}{\text{DC power in}} = \frac{\frac{V_o^2}{2R_L}}{I_{dc} V_{cc}} = \frac{\frac{V_o^2}{2R_L}}{\left(\frac{V_{cc}}{2}\right) V_{cc}} = \frac{V_o^2}{V_{cc}^2}$$

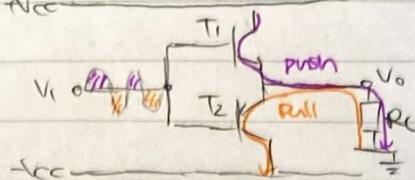
and is max. when  $V_o = \frac{V_{cc}}{2}$ , so  $\eta_{max} = 1/4$

- THIS circuit is inefficient as there is a DC current flowing all the time (QSS) when  $V_I=0$ .

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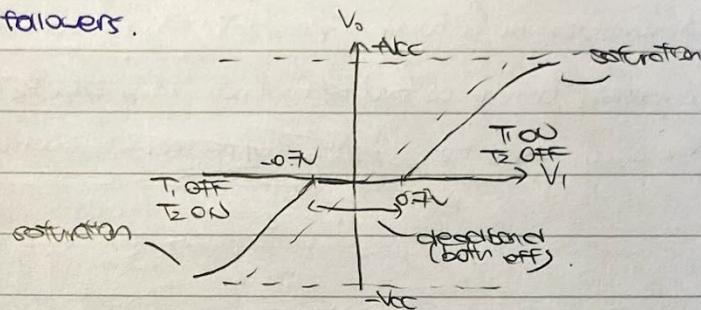
The complementary emitter follower. [class B operation]

- The circuit of the complementary emitter follower is as follows.



Here, T<sub>1</sub> is a NPN transistor and T<sub>2</sub> is a PNP transistor

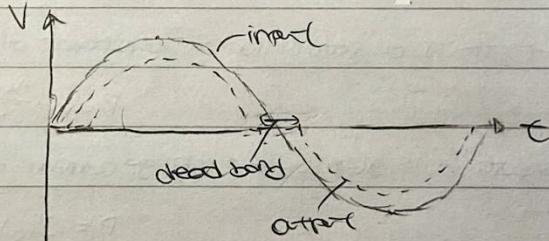
- w/ no input signal, T<sub>1</sub> and T<sub>2</sub> are non-conducting. ↗ good for the efficiency.
- As V<sub>i</sub> starts to become +ve/-ve, T<sub>1</sub>/T<sub>2</sub> starts to conduct and behaves as an emitter follower, whereas T<sub>2</sub>/T<sub>1</sub> remains non-conducting.
- the voltage gain of the circuit when either transistor is conductor is just under 1 as they act as emitter followers.



- There exists a deadband as (V<sub>i</sub>) needs to reach 0.7V for the transistors to conduct.

Above |V<sub>i</sub>| = 0.7V, the output voltage V<sub>o</sub> increases linearly w/ the input voltage V<sub>i</sub>, until saturation.

- The effect on a sinusoidal wave input is as follows:

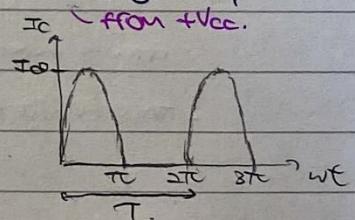


- There is an absence of output for |V<sub>i</sub>| < 0.7V, which is known as crossover distortion.

This is unsatisfactory in audio amplifiers as it leads to higher order harmonics (unpleasant sounding).

- Assuming a sinusoidal output signal V<sub>o</sub>(t) = V<sub>o</sub> sin(ωt), the DC power is given by

$$\begin{aligned} P &= \frac{1}{T} \int_{0}^{T/2} V_{oC} I_C dt \\ &= \frac{1}{T} \int_{0}^{T/2} V_o \frac{\sin(\omega t)}{R_L} dt \\ &= \frac{V_o^2}{2R_L T} \end{aligned}$$



- The efficiency of the amplifier is therefore

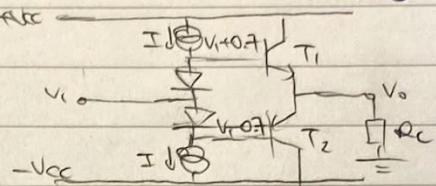
$$\eta = \frac{\text{AC power out}}{\text{DC power in}} = \frac{V_o^2 / 2R_L}{\text{dc bias } V_o} = \frac{\pi}{4} \frac{V_o}{V_{CC}}$$

and is max when V<sub>o</sub> = V<sub>CC</sub>, so  $\eta_{\max} = \frac{\pi}{4}$

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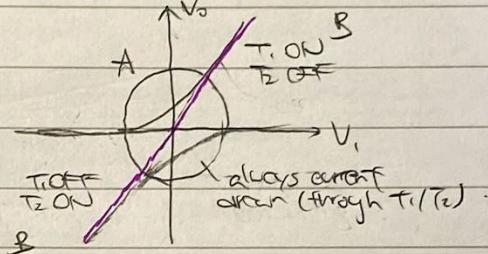
Complementary emitter follower with biasing. [class AB operation]

- To avoid crossover distortion, we can bias the circuit to the edge of conduction in the state of zero input signal.
- We can use a voltage source to fix the voltage drop across diodes, to set the bias for the two transistors (the diodes provide the correct biasing voltage as they are also made of Si).
- The circuit of the complementary emitter follower w/ biasing is as follows:



where the current sources could be current mirrors.

- A composite characteristic can be constructed from the characteristics of the individual transistors.



## Amplifier classes

- Amplifiers can be categorised into the following classes
  - ↳ class A: Draws a current from the supply, irrespective of signal level.
  - ↳ class B: The transistors conduct for half the cycle  $\rightarrow$  zero power dissipation for zero input, so higher efficiency, but has crossover distortion.
  - ↳ class AB: The transistors conduct for more than half the cycle but less than the whole.  
 $\rightarrow$  compromise between linearity, power output and efficiency.

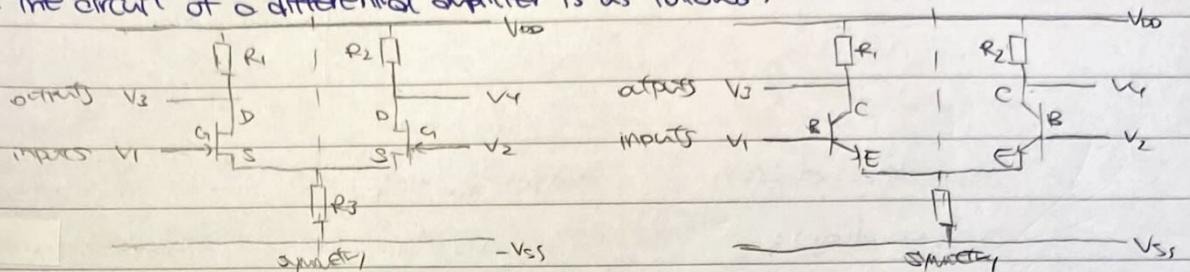
## The differential amplifier

# For Personal Use Only -bkwk2

### The differential amplifier

#### The differential amplifier

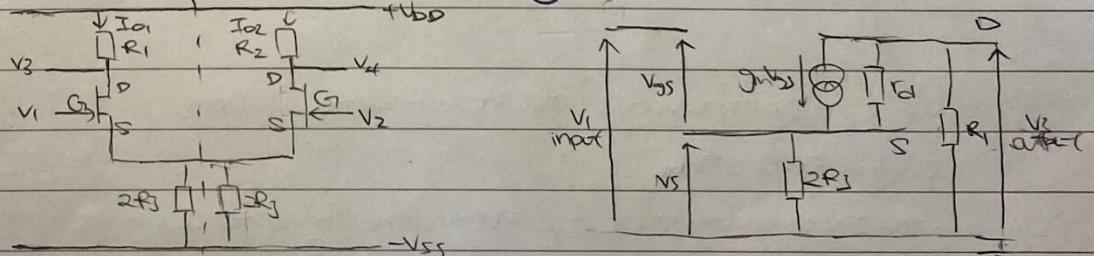
- The differential amplifier/bi-fet pair is the input circuit of an OpAmp.
- It uses a source (FET) or emitter (BJT) coupled pair of identical transistors to amplify the difference b/w the voltages at the two inputs.
- The circuit of a differential amplifier is as follows:



- For common mode input ( $V_1 = V_2$ ), if  $V_1$  and  $V_2$  increase together, then the currents  $I_{D1}, I_{D2}$  will rise, and so will the current through  $R_3 \rightarrow V_3$  (FET) /  $V_5$  (BJT) increase.
- The gate-source (FET) / base-emitter (BJT) voltage drops  $\rightarrow$  small outputs  $V_3, V_4$ , so the common mode gain is small.
- For differential mode input ( $V_1 = -V_2$ ), if  $V_1$  increases, and  $V_2$  decreases, then the current  $I_{D1}$  will rise, the current  $I_{D2}$  will drop, and so the current through  $R_3$  is constant  $\rightarrow V_3$  (FET) /  $V_5$  (BJT) remains constant.
- The gate-source (FET) / base-emitter (BJT) voltage drops w/ input voltage  $\rightarrow$  large outputs  $V_3, V_4$ , so the differential gain is large.
- \* The analysis for FET / BJT-based differential amplifiers is the same, only the sum of the transistor charges. (Note that BJT-based ones are less effective  $\rightarrow$  FET-based ones in notes)

#### Common mode gain.

- As  $V_1 = V_2$ , the same changes are occurring in on both sides of the circuit, so we can use the half-circuit approach (after splitting  $R_3$  into two // resistors each of value  $2R_3$ ).



Sum currents of source :  $g_m V_{GS} + \frac{V_3 - V_S}{R_d} + \frac{0 - V_S}{2R_3} = 0$

Sum currents of drain :  $g_m V_{GS} + \frac{V_3 - V_S}{R_d} + \frac{V_2 - 0}{R_1} = 0$

$$V_S = -\frac{2R_3}{R_d} V_3$$

Consider voltages :

$$V_{GS} = V_1 - V_S$$

Similar expression for  $\frac{V_2}{V_1}$

$\rightarrow$  The common mode gain is

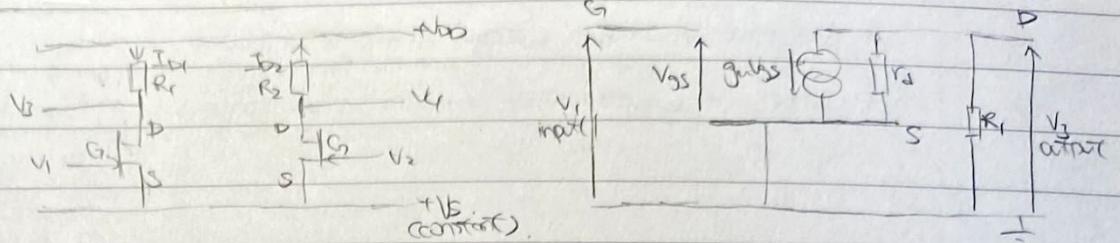
$$\frac{V_3}{V_1} = -\frac{g_m R_1}{1 + 2g_m R_3 + 2R_3/R_d + R_d/R_1}$$

\* Note that increasing  $R_3$  reduces the common mode gain.

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## Differential mode gain

- At  $V_1 = -V_2$ , only charge in current through one transistor is matched by an equal and opposite charge through the other  $\rightarrow$  current through  $R_3$  is constant.
- Therefore  $V_s(\text{FET}) / V_e(\text{BJT})$  is constant  $\rightarrow$  effectively at earth potential for SSM.



$$\text{Sum currents of drain: } g_m V_{gs} + \frac{V_2 - 0}{R_2} + \frac{V_2 - 0}{R_1} = 0.$$

Consider voltages:

$\rightarrow$  the differential mode gain is:

$$V_{gs} = V_1$$

$$\frac{V_2}{V_1} = -\frac{g_m}{Y_{Gd} + Y_{Fd}}$$

Similar expression for  $\frac{V_2}{V_1}$

\* The expression can also be obtained by putting  $R_2 = 0$  in the expression for common mode gain.

## Common mode rejection ratio (CMRR)

- The CMRR is a measure of performance of a differential amplifier and is defined as:

$$\boxed{\text{CMRR} = \frac{\text{differential mode gain}}{\text{common mode gain}}}$$

- Since we want the differential mode gain to be as large as possible and the common mode gain to be as small as possible, the larger the CMRR, the better the differential amplifier.

- The CMRR for the circuit above is:

$$\text{CMRR} = \frac{-\frac{g_m}{Y_{Gd} + Y_{Fd}}}{\frac{g_m}{2R_2}} = \frac{1 + 2g_m R_2 + 2R_2 Y_{Gd} + R_2 Y_{Fd}}{1 + R_2 Y_{Fd}}$$

$\rightarrow$  we can approximate the CMRR to be  $\sim 2g_m R_2$

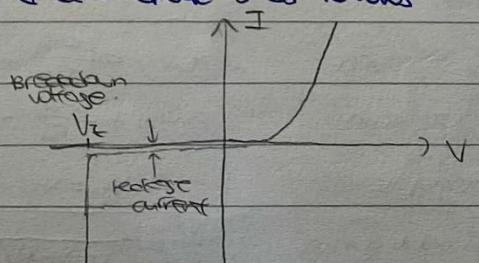
-  $R_3$  plays a critical role in setting the CMRR and has to be as large as possible, so it is given the name "long-tailed pair". (long-tailed = high voltage)

- To maintain a reasonable DC biasing current  $I_0$ , the negative supply voltage ( $-V_{ss}$ ) needs to increase w/ increasing  $R_3$ .

## Use of current sources with the differential amplifier.

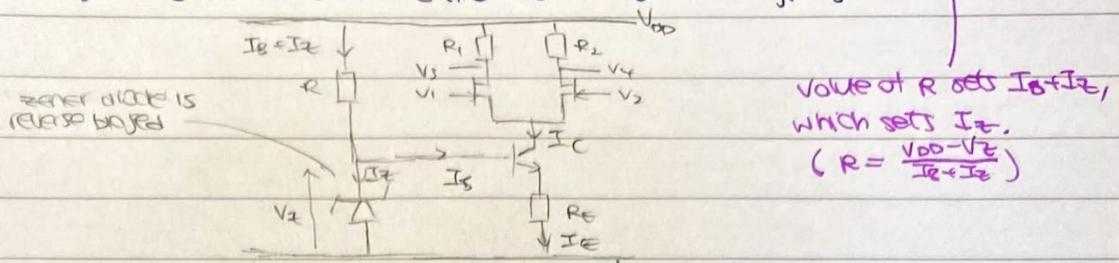
Using a Zener diode.

- The I-V characteristics of a Zener diode is as follows



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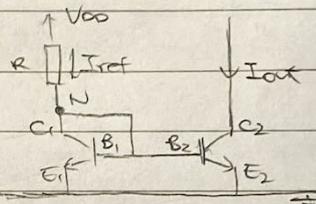
- If the voltage across the Zener diode  $I_Z$  is greater than a threshold value, the voltage across the Zener diode will be fixed to be  $V_Z \rightarrow$  voltage regulator.



- Since  $V_B = V_Z$  is fixed,  $V_E = V_B - V_{BE}$  is also fixed, so  $I_E$  is constant. As  $I_B \ll I_E$ ,  $I_C \approx I_E$  so we can say  $I_C$  is constant.
- \* For this to work, the transistor must be operating in the linear region.

## The current mirror circuit

- In ICs, it is inconvenient to include Zener diodes  $\rightarrow$  use the current mirror to provide a current source. The circuit for a current mirror is as follows:



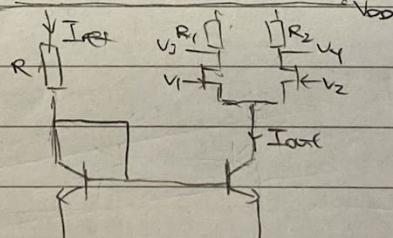
- Provided both transistors are identical and operate in the linear region, then  $I_{out} \approx I_{ref}$ .  
 (The value of  $I_{ref}$  is set by  $R$ )

$$B_1, B_2 \text{ connected, } E_1, E_2 \text{ grounded} \rightarrow V_{BE1} = V_{BE2} \rightarrow I_{C1} = I_{C2}$$

$$\text{Assume large } h_{FE} \rightarrow V_{BE1}, V_{BE2} \text{ negligible} \rightarrow I_{ref} = I_{C1} + I_{B1} + I_{B2} = I_{C1}$$

$$\rightarrow I_{out} = I_{C2} = I_{C1} = I_{ref}, \text{ where } I_{ref} = \frac{V_{DD} - V_{BE1}}{R}$$

- Applying the current mirror circuit as a current source in the differential amplifier circuit



Negative feedback theoryNegative feedback .

- The negative feedback reduces gain but offers other benefits :

↳ Stabilisation of gain (reduce the effect of a change in open-loop gain)

↳ Reduced distortion

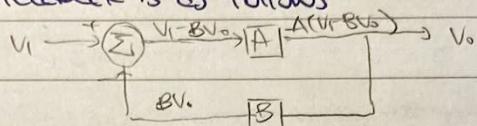
↳ Increased bandwidth (increase upper 3dB freq, decrease lower 3dB freq)

↳ For voltage amplifiers, increase i/p resistance and decrease o/p resistance

↳ For current amplifiers, decrease i/p resistance and increase o/p resistance

ideal for  
voltage  
amplifiers

- In general, negative feedback is as follows



ideal for  
current  
amplifiers

where  $A$  is the forward/open-loop gain and  $B$  is the fraction of the output fed back.

$$V_o = A(V_i - BV_o)$$

$$V_o(1+AB) = AV_i$$

→ closed loop gain  $G$  is

$$G = \frac{V_o}{V_i} = \frac{A}{1+AB}$$

$AB$  is the loop gain  
(return ratio)

i.e. the (closed-loop) gain is decreased by a factor of  $\frac{1}{1+AB}$

- For large  $A$ , as is often the case w/ OpAmps, the closed-loop gain is  $\frac{1}{B}$  i.e. the closed-loop gain is determined by the feedback network and not the properties of the amplifier itself.

Stability of gain

- without feedback a change in gain  $\delta A$  results in a fractional change of gain  $\frac{\delta A}{A}$
- with feedback, the (closed-loop) gain  $G$  is given by

$$G = \frac{A}{1+AB}$$

$$\frac{dG}{dA} = \frac{1+AB-AB}{(1+AB)^2} = \frac{1}{(1+AB)^2}$$

The small change in (closed-loop) gain  $\delta G$  arising from a small change in (open-loop) gain  $\delta A$  is

$$\delta G = \frac{dG}{dA} \delta A$$

so the fractional change of (closed-loop) gain  $\frac{\delta G}{G}$  is

$$\frac{\delta G}{G} = \frac{1}{G} \frac{dG}{dA} \delta A = \frac{1}{A} \cdot \frac{1}{(1+AB)^2} \cdot \delta A$$

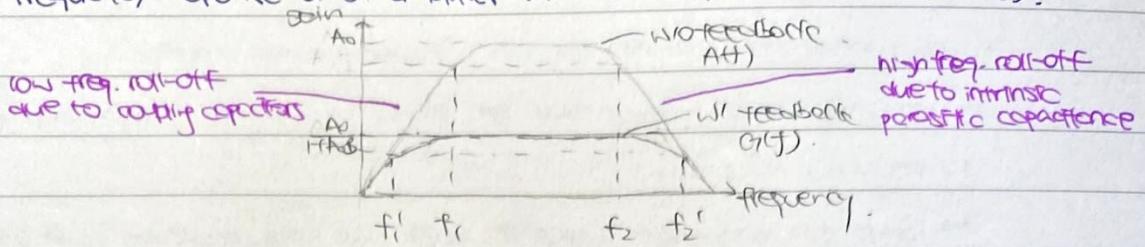
$$\frac{\delta G}{G} = \frac{\delta A}{A} \frac{1}{1+AB}$$

i.e. the fractional change in (closed-loop) gain is reduced by a factor of  $(1+AB)$

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## Effect on bandwidth

- The frequency response of an amplifier w/o and w/ feedback is as follows.



- The frequency-dependent gain,  $A(f)$  can be expressed as.

$$A(f) = (\text{low freq. cut off}) \times A_o \times (\text{high freq. cut off})$$

$$A(f) = \frac{1}{(1+f/f_1)} A_o \frac{1}{(1+f/f_2)} \quad \begin{cases} f/f_1 \ll 1 & \rightarrow 0 \text{ for } f \ll f_1 \\ f/f_2 \ll 1 & \rightarrow 0 \text{ for } f \gg f_2 \end{cases}$$

where  $A_o$  is the gain at mid-band frequencies.

- With feedback, the frequency-dependent gain,  $G(f)$  can be expressed as.

$$G(f) = \frac{A(f)}{1+A(f)B}$$

$$G(f) = \frac{A_o}{(1+f/f_1)(1+f/f_2) + A_o B}$$

(i) At mid-band frequencies, i.e.  $f \gg f_1$  and  $f \ll f_2$ ,  $(1+f/f_1 \approx 1, 1+f/f_2 \approx 1)$

$$G = \frac{A_o}{1+A_o B}$$

i.e. the (closed-loop) gain is reduced by a factor of  $(1+A_o B)$

(ii) At low frequencies, i.e.  $f \ll f_2$  ( $1+f/f_2 \approx 1$ )

$$G(f) = \left( \frac{A_o}{1+A_o B} \right) \left( \frac{1}{1+f/f_1(1+A_o B)} \right)$$

$$f_1' = \frac{f_1}{1+A_o B}$$

i.e. the lower 3dB frequency has been reduced by a factor of  $(1+A_o B)$

(iii) At high frequencies, i.e.  $f \gg f_1$ , ( $1+f/f_1 \approx 1$ )

$$G(f) = \left( \frac{A_o}{1+A_o B} \right) \left( \frac{1}{1+f/f_2(1+A_o B)} \right)$$

$$f_2' = f_2(1+A_o B)$$

i.e. the upper 3dB frequency has been increased by a factor of  $(1+A_o B)$

→ total bandwidth  $f_2 - f_1$  has increased.

*broadband here refers to the upper 3dB frequency*

## Gain-bandwidth product

- Many opamps are designed to give stable DC performance  $\rightarrow f_1=0$  so  $BW = f_2 - f_1^0 = f_2$ .

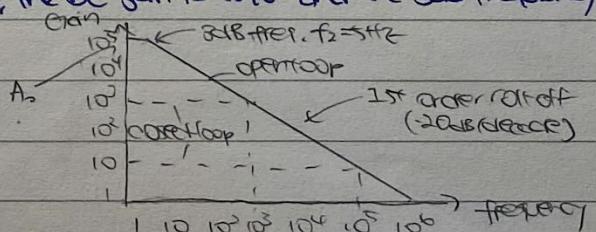
- As seen from the analysis above, adding -ve feedback:

↳ decreases the gain by a factor of  $(1+A_o B)$

↳ increases the upper 3dB freq (BW here) by a factor of  $(1+A_o B)$

]  
gain-bandwidth product stays constant.

- For the type 741 Opamp, the DC gain is  $\sim 10^5$  and the 3dB frequency  $\sim 5\text{Hz}$ .

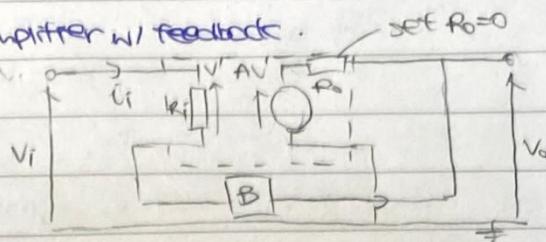


\* Gain-bandwidth product is  $1\text{MHz} [10^3 \times 10^3 \text{ and } 10 \times 10^5]$

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EFFECT ON INPUT IMPEDANCE (VOLTAGE AMPLIFIER)

- consider a voltage amplifier w/ feedback.



The input resistance is given by  $R_{in} = \frac{Vi}{I_i}$

without feedback

$$R_{in} = R_i$$

with feedback.

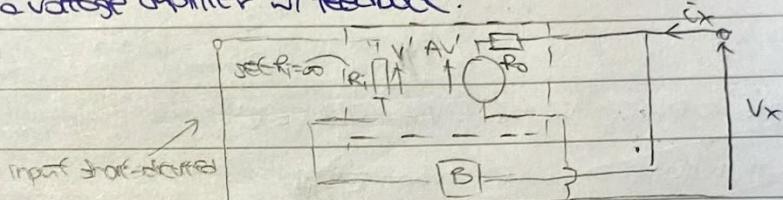
$$V_o = \frac{A}{1+AB} V_i, \quad A = \frac{V_i - B V_o}{R_i} \rightarrow$$

$$\boxed{R_{in} = R_i(1+AB)}$$

i.e. the input resistance is increased by a factor of  $(1+AB)$

EFFECT ON OUTPUT RESISTANCE (VOLTAGE AMPLIFIER)

- consider a voltage amplifier w/ feedback.



The output resistance is given by  $R_{out} = \frac{V_x}{I_x}$ .

Without feedback

$$R_{out} = R_o$$

With feedback

$$I_x = \frac{V_x + ABV_x}{R_o} \rightarrow \boxed{R_{out} = \frac{R_o}{1+AB}}$$

i.e. the output resistance is decreased by a factor of  $(1+AB)$

## CHARACTERISTICS OF REAL OPERATIONAL AMPLIFIERS

### The ideal operational amplifier

- the choice of i/p transistor has a major effect on the characteristics of an OpAmp (FET  $\rightarrow$  high  $R_{in}$ ;BJT  $\rightarrow$  high  $A$ , more stable)

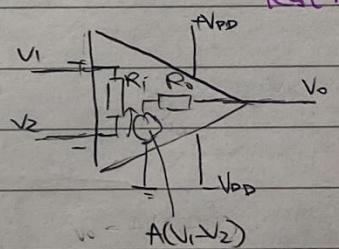
- To ensure as ideal a performance as possible for the OpAmp circuit,

$\hookrightarrow$  1) choose a suitable OpAmp — best  $A$ ,  $R_i$ ,  $R_o$ ,  $BW$ ,  $V_{offset}$  etc. for a given price.

$\hookrightarrow$  2) choose suitable resistors — avoid designing circuits w/  $R$  values close to  $R_i$  and  $R_o$ .

$\rightarrow$  if rated performance can be obtained, we can apply the ideal OpAmp model. Ideally or least K/D

Ideal	Real
Open-loop gain $A$	Infinite $A = \infty$
Input resistance $R_i$	Finite but v. large $R_i = 10^6 - 10^{12} \Omega$
Output resistance $R_o$	Zero $R_o = 0$



- FOR REAL OPAMPS:

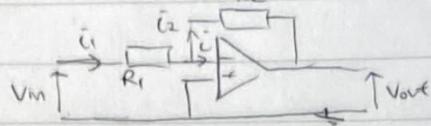
$\hookrightarrow$  Voltage rule:  $\therefore A = \infty \rightarrow$  o/p attempts to make  $(V_1 - V_2) = 0$

$\hookrightarrow$  Current rule:  $\therefore R_i = \infty \rightarrow$  i/p draws no current.  $I_1 = I_2 = 0$ .

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## Inverting operational amplifier

- The circuit of an inverting opamp is as follows:



$$\text{Voltage rule: } V_+ = V_- = 0 \rightarrow V_- \text{ is referred to as virtual ground}$$

$$\text{Current rule: } i_+ = i_- = 0 \rightarrow i_1 = i_2 + I^0$$

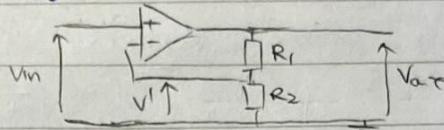
$$\frac{V_{in} - 0}{R_1} = i_1 = i_2 = \frac{0 - V_{out}}{R_2}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

inverting as the closed-loop gain is -ve.

## Non-inverting operational amplifier

- The circuit of a non-inverting opamp is as follows:



$$\text{Voltage rule: } V_+ = V_- \rightarrow V_{in} = V'$$

$$\text{Potential divider, } V' = V_{out} \cdot \frac{R_1}{R_1 + R_2}$$

$$V_{in} = V_{out} \frac{R_2}{R_1 + R_2}$$

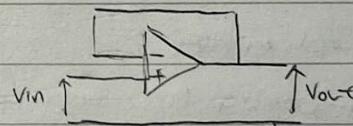
$$\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

non-inverting as the closed-loop gain is +ve.

## Voltage follower (buffer)

- The voltage follower is a non-inverting amplifier w/  $R_1 = 0$  and  $R_2 = \infty$ .

- The circuit of a voltage follower is as follows:



$$\text{Voltage rule: } V_+ = V_- \rightarrow V_{in} = V_{out} .$$

$$\frac{V_{out}}{V_{in}} = 1$$

Input impedance:

$$R_{in} = A R_i \sim \infty$$

Output impedance:

$$R_{out} = \frac{R_o}{A} \sim 0 .$$

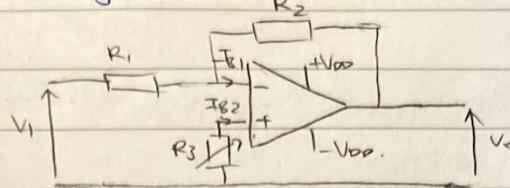
- (closed-loop) gain of 1, high input impedance, low output impedance  $\rightarrow$  buffer.

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Deviations from ideal behaviour in operational amplifiers

## ① Input bias current and input bias voltage.

- Consider an OpAmp where the i/p stage is made usingBJTs. The base currents  $I_{B1}, I_{B2}$  are needed to set the operating pt of the i/p transistors (For 741 amplifier,  $I_B \sim 100\text{nA}$ ).



$I_{B1}, I_{B2}$  are there due to the bipolar power supply

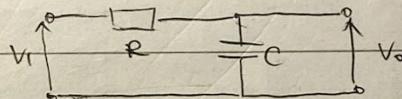
- $I_{B1}$  flows through  $R_1$  and  $R_2$ , so even w/  $V_i = 0$ , there is some unwanted output  $V_o = I_{B1}R_2$  (for 741 amplifier, w/  $R_1 = 5\text{k}\Omega, R_2 = 500\text{k}\Omega \rightarrow V_o = 0.05\text{V}$ ) → choose OpAmp w/ low  $I_B$ .
- Also, as the i/p transistors are never perfectly matched (manufacturing defects), the two bias currents are not always equal (For 741 amplifier, the offset current  $|I_{B1} - I_{B2}| \sim 20\text{nA}$ ). → eliminate unwanted offset voltage by adding a (variable) resistor  $R_3$ /choose OpAmp w/ low  $I_B$ .
- Applying zero volts to both i/p's of an OpAmp does not produce zero o/p. The input offset voltage  $V_{OS}$  is the voltage needed b/w the two i/p's to drive the o/p to zero. (For 741 amplifier,  $V_{OS} \sim 2\text{mV}$ ) → null the input offset voltage using the "offset null" pin.

## ② Frequency response of OpAmps.

capacitor added in circuit for gain compensation

- The gain of an OpAmp falls w/ frequency due to stray capacitances / gain compensation.

The effect can be modelled as an RC network.



- The gain of the RC network is given by

$$\frac{V_o}{V_i} = \frac{1}{1+j\omega CR}$$

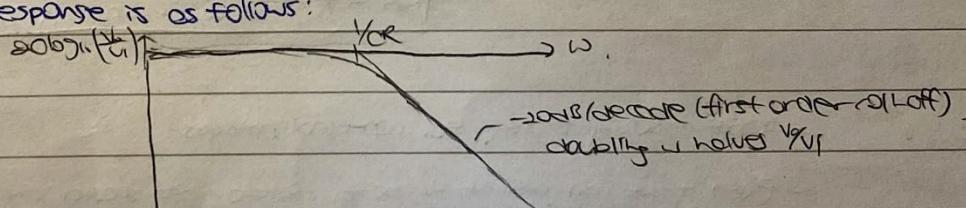
At low frequencies,  $\omega CR \ll 1$

$$\frac{V_o}{V_i} = 1$$

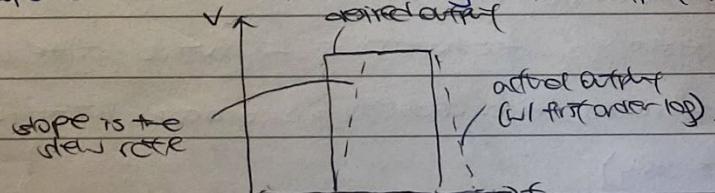
At high frequencies,  $\omega CR \gg 1$

$$\frac{V_o}{V_i} = \frac{1}{\omega CR}$$

The frequency response is as follows:



- The frequency response results in a first order log of the output.

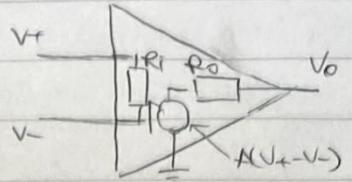


The slow rate is finite due to the first order log.

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The non-ideal operational amplifier model

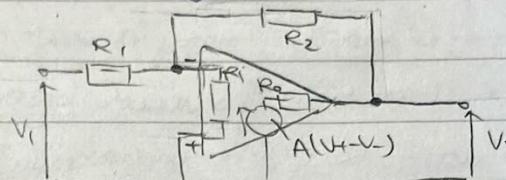
- The overall non-ideal model is as follows:



- Problems can use any subset of  $A, R_i, R_o$  from the fully ideal, to one or any of them as a practical constraint.

Inverting amplifier with a non-ideal operational amplifier.

- The circuit of a non-ideal inverting opamp is as follows:



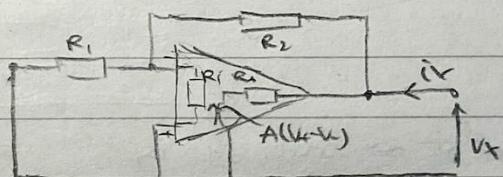
$$\sum I @ i/p : \frac{V_1 - V_-}{R_1} = \frac{V_- - V_0}{R_2} \leftarrow \frac{V_- - V_+}{R_i} \rightarrow \frac{V_1}{R_1} + \frac{V_0}{R_2} = V_- \left( \frac{1}{R_i} + \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\sum I @ o/p : \frac{A(V_+ - V_-) - V_0}{R_o} = \frac{V_0 - V_-}{R_2} \rightarrow -V_0 \left( \frac{1}{R_o} + \frac{1}{R_2} \right) = V_- \left( \frac{A}{R_o} - \frac{1}{R_2} \right)$$

$$R_i, R_2 \text{ not comparable to } R_o, R_1 \text{ then } \frac{V_0}{V_1} = -\frac{R_2}{R_i} \quad \therefore \frac{V_1}{R_1} + \frac{V_0}{R_2} = \frac{V_0}{R_2} \left( \frac{R_o + R_2}{R_o + A R_2} \right) \left( \frac{1}{R_i} + \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\text{ideab case: } \frac{V_0}{V_1} = -\frac{R_2}{R_i} \left[ \frac{1 - R_o/A R_2}{1 + 1/A (1 + R_2/R_1 + R_2/R_i + R_o/R_i - R_o/R_2)} \right]$$

- To find the output resistance, we short the input



$$\sum I @ i/p : \frac{V_- - 0}{R_1} + \frac{V_- - V_+}{R_i} + \frac{V_- - V_X}{R_2} = 0 \rightarrow \frac{V_X}{R_2} = V_- \left( \frac{1}{R_i} + \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\sum I @ o/p : \frac{A(V_+ - V_-) - V_X}{R_o} + i_X = \frac{V_X - V_-}{R_2} \rightarrow i_X - V_X \left( \frac{1}{R_o} + \frac{1}{R_2} \right) = V_- \left( \frac{A}{R_o} - \frac{1}{R_2} \right)$$

$$\therefore i_X - V_X \left( \frac{1}{R_o} + \frac{1}{R_2} \right) = V_X \left( \frac{1}{1 + A R_i + A R_2} \right) \left( \frac{A}{R_o} - \frac{1}{R_2} \right)$$

$$R_{out} = \frac{V_X}{i_X} = \frac{V_X}{R_o R_2 + (A R_2 - R_o) \left( \frac{R_o R_2}{R_i R_1 + R_i R_2 + R_i R_o} \right)}$$

\* Problem-solving strategy for non-ideal opamps.

- Even if we can assume  $R_i = \infty$  or  $R_o = 0$ , we should still keep it there. (do not jump to the conclusion  $V_+ = V_-$ !)

- We generally sum the currents at the input or at the output or both.

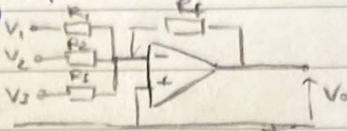
(The algebra for the non-ideal opamp is complicated but the numerical values can be found quite easily)

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## Practical uses of operational amplifiers

### Summing amplifier.

- The circuit of an inverting summing amplifier is as follows:



$\Sigma I @ \text{input} :$

$$\frac{V_1 - 0}{R_1} + \frac{V_2 - 0}{R_2} + \frac{V_3 - 0}{R_3} = \frac{0 - V_o}{R_f}$$

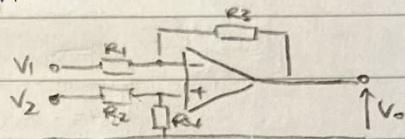
$$\therefore V_o = -R_f \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

If  $R_i = R_r$ ,

$$V_o = -\frac{R_f}{R} \sum V_i$$

### Difference amplifier

- The circuit of a difference amplifier is as follows:



Voltage rule

$$V_- = V_+ \rightarrow V_- = V_2 \cdot \frac{R_f}{R_2 + R_f}$$

current rule

$$i_+ = i_- = 0 \rightarrow \frac{V_+ - V_-}{R_1} = \frac{V_- - V_o}{R_f}$$

$$\therefore V_o = -\frac{R_f}{R_1} V_1 + \left( \frac{R_f R_1}{R_1 + R_f} \right) V_2$$

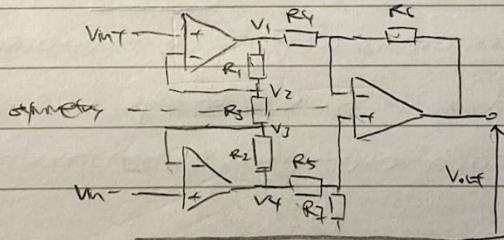
If  $R_1 = R_2, R_f = R_4$ ,

$$V_o = \frac{R_2}{R_1} (V_2 - V_1)$$

### Three opamp difference circuit.

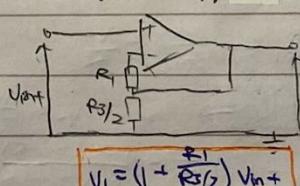
- The three opamp difference circuit is the difference amplifier above, w/ two additional opamps as an input stage.

- The three opamp difference circuit is as follows:

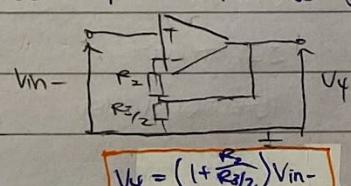


- For common mode signals ( $V_{in+} = V_{in-}$ ),  $V_1 = V_4 \rightarrow V_1 = V_2 = V_3 = V_4 \rightarrow$  the two left opamps act as voltage followers/buffers, and  $V_1 = V_{in+} = V_4 = V_{in-}$

- For differential mode signals ( $V_{in+} = -V_{in-}$ ),  $V_1 = -V_4 \rightarrow V_2 = -V_3 \rightarrow$  potential of the midpoint of  $R_3$  stays at zero potential,  $\rightarrow$  the input circuit can be analysed as noninverting amplifiers:



$$V_1 = \left( 1 + \frac{R_1}{R_3/2} \right) V_{in+}$$



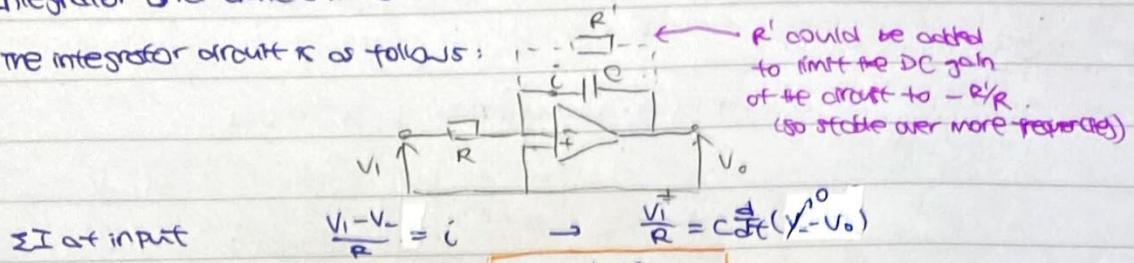
$$V_4 = \left( 1 + \frac{R_2}{R_5/2} \right) V_{in-}$$

- The three opamp difference circuit features high input resistance and an excellent CMRR  
 $\rightarrow$  widely used in instrumentation.

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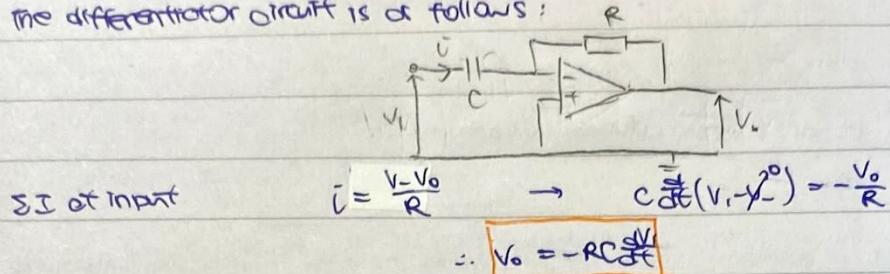
## Integrator and differentiator.

- The integrator circuit is as follows:



- The circuit can also act as a LPF ( $\frac{V_o}{V_1} = -\frac{R_C R'}{R}$ )

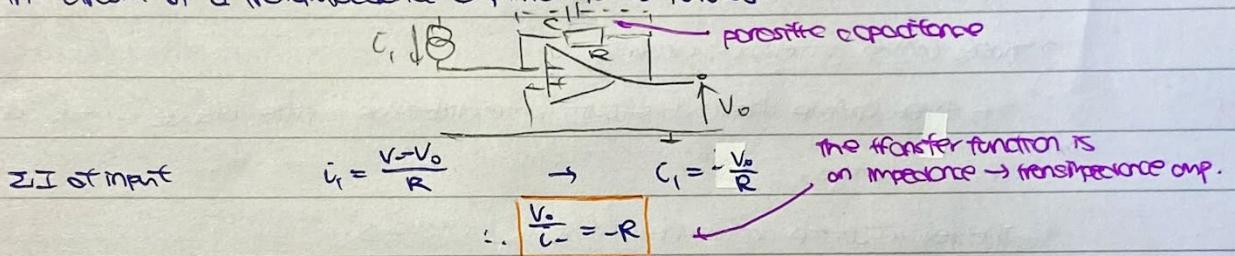
- The differentiator circuit is as follows:



- The circuit can also act as a HPF ( $\frac{V_o}{V_1} = -\frac{R}{\Sigma C}$ )

## Transimpedance amplifier

- The circuit of a transimpedance amplifier is as follows:

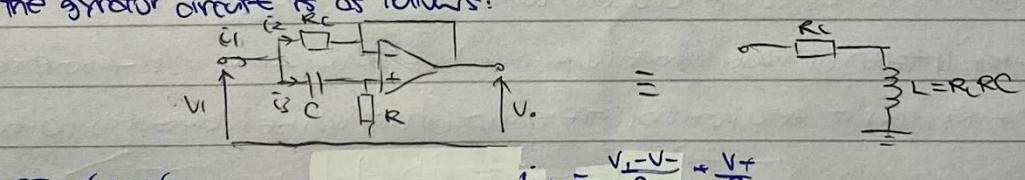


- Practically, the current source comes from a reverse-biased photodiode

- Typically  $C_1 \approx nA$ , so we req. R to be large  $\sim M\Omega$ , which will have parasitic capacitance, leading to a 1st order roll-off in gain for high frequencies.

## Gyrorator.

- The gyrorator circuit is as follows:



Potential divider

$$V_L = \frac{R}{R + j\omega C} V_1 \rightarrow V_+ = \frac{j\omega CR}{j\omega CR + R} V_1$$

$$\therefore i_1 = V_1 \left[ \frac{1}{R_L} + j\omega C \frac{(R_L + R)}{R_L(R + j\omega CR)} \right]$$

$$Z_L = \frac{V_1}{i_1} = \frac{R_L + j\omega C R R_L}{1 + j\omega C R L}$$

"Gyro" on inductor using a capacitor

For large R, the denominator dominates,  $\infty Z_L = \frac{V_1}{i_1} = R_L + j\omega C R R_L = R_L + j\omega L$

- The circuit is useful when a value of L is req. but the actual physical size of the component is not practical  $\rightarrow$  use capacitor/resistor (mimic) instead.

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## Oscillators

Oscillators using linear circuits

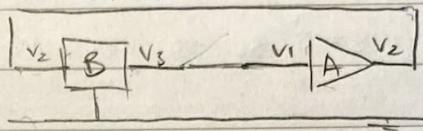
- The closed loop gain  $G_1$  is given by

$$G_1 = \frac{A}{1+AB}$$

- Nyquist's stability theorem in short states that the return ratio  $AB$  must not exceed a magnitude of 1 when its phase is  $-180^\circ$  for a stable system (and we have the feedback).

- If  $AB = -1$  (i.e.,  $|AB| = 1$ ,  $\angle AB = -180^\circ$ ), then  $G_1$  becomes infinite and oscillations will occur. The circuit will produce an output w/o any external circuit.

- consider an amplifier A and a passive network B. (RLC circuit)



↑ this reg. oscillator, which in practice will be noise.  
we still have a power supply for active components

We assume the gain of A is constant and shows no change in phase, whereas the response of the passive network varies w/ frequency, so

$$\frac{V_2}{V_1} = A$$

$$\frac{V_3}{V_2} = B(j\omega)$$

- For oscillations to occur, we req.

↳ Loop phase of 0 or multiples of  $2\pi$

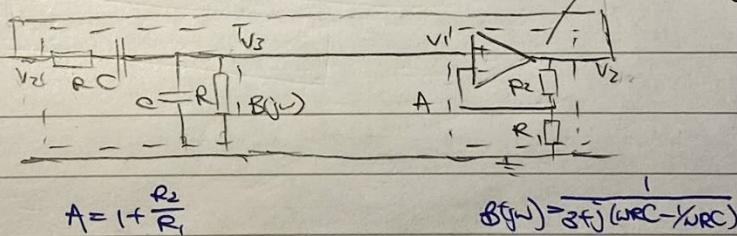
↳ Loop gain of unity.

$$1 + AB(j\omega) = 0 \quad \text{or} \quad 2k\pi$$

$$|AB(j\omega)| = 1$$

## Wien bridge oscillator

- The circuit of a Wien bridge oscillator is as follows : non-inverting amplifier.



$$A = 1 + \frac{R_2}{R_1}$$

$$B(j\omega) = \frac{1}{2j\omega(C_1 + C_2)}$$

The conditions for oscillation are satisfied if  $\angle B(j\omega) = 0$  or  $\pi$ , which occurs when

$$\omega R C - \frac{1}{\omega R' C'} = 0 \rightarrow \omega = \frac{1}{RC}$$

At this frequency,  $B = 1/3$ , so A must be 3.

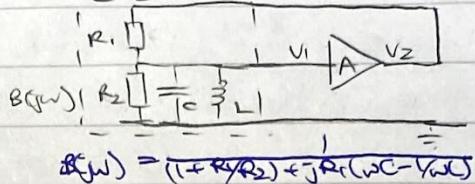
- In actual circuits, we must stabilize the amplitude of oscillations to avoid overdriving the amplifier and consequent distortion. → use a thermistor in place of  $R_1/R_2$ .

- Using a thermistor w/  $\text{d}\alpha/\text{d}T > 0$ , in place of  $R_2$ , any increase in temp increases the power dissipated in the resistor, which heats it up and decreases the resistance → reduces the gain.

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oscillators using tuned circuits.

- A tuned circuit oscillator can be represented as a frequency-dependent tuned network and a linear amplifier as follows:



If the amplifier is non-inverting (+ve gain), the conditions for oscillation are satisfied

If  $\angle B(j\omega) = 0$  or  $2\pi v$ , which occurs when

$$\omega C - \frac{1}{\omega L} = 0 \rightarrow \omega = \frac{1}{\sqrt{LC}}$$

At this frequency,  $B = \frac{R_2}{R_1 + R_2}$ , so A must be  $\frac{R_1 R_2}{R_2}$