Define $\gamma = \frac{\epsilon c(T)}{(1+\epsilon)n\alpha}$

**Algorithm 1** pseudocode for approximate Nash Equilibrium

```
c(e) ← c(e) − γ   ∀e ∈ T
Run Algorithm 3.3 to attempt to pay for on T under modified cost
while e ∈ T do
    if e is not fully paid then
        Adjust T by replacing T_e with ⋃_{i∈T_e} A_i to get T'
        c(e) ← c(e) − γ   ∀e ∈ T'
        Run Algorithm 3.3 to pay for T' under modified cost
        P(T') ← Σ_i p_i(T')
        p'_i(e) ← p_i(e) + γ p_i(T')/P(T')    for all players and every e ∈ T'
    end if
end while
```

$T^{'}$ is fully paid as $\sum_i p_i^{'}(e) = \sum_i p_i(e) + \gamma$

Whenever $e \in T$ is not fully paid, we form a new tree $T^{'}$, and $c(T^{'}) \leq c(T) - \gamma$. Therefore, we need to reconstruct our Steiner tree most $\frac{c(T)}{\gamma} = \frac{(1+\epsilon)n\alpha}{\epsilon}$ times.Thus the algorithm runs in polynomial time.

**Proof $P^{'}$ being $(1+\epsilon)$ Nash Equilibrium**

$$p_i^{'}(e) = p_i(e) + \gamma \frac{p_i(T^{'})}{P(T^{'})}$$

Suppose $T^{'}$ has $m$ edges:

$$p_i^{'}(T^{'}) = p_i(T^{'}) + \gamma \frac{p_i(T^{'})}{P(T^{'})} m = p_i(T^{'}) + \gamma \frac{p_i(T^{'})}{c(T^{'}) - m\gamma}$$

$$\begin{aligned} p_i^{'}(T^{'}) - p_i(T^{'}) &= \gamma \frac{p_i(T^{'})}{c(T^{'}) - m\gamma} m \\ &= \frac{\epsilon c(T) p_i(T^{'}) m}{(1+\epsilon)n\alpha(c(T^{'}) - m\gamma)} \\ &= \frac{\epsilon c(T) p_i(T^{'})}{(1+\epsilon)\alpha n(\frac{c(T^{'})}{m} - \gamma)} \\ &= \frac{\epsilon c(T) p_i(T^{'})}{(1+\epsilon)\alpha(\frac{n}{m} - \frac{n\gamma}{c(T^{'})})c(T^{'})} \end{aligned}$$

There in the paper, the proof simply goes as

$$\frac{\epsilon c(T) p_i(T^{'}) m}{(1+\epsilon)n\alpha(c(T^{'}) - m\gamma)} \leq \frac{\epsilon c(T) p_i(T^{'})}{(1+\epsilon)\alpha(1-\epsilon)c(T^{'})}$$

which implies that $\frac{n}{m} - \frac{n\gamma}{c(T')} \geq (1-\epsilon)$
However, I believe that $\frac{n}{m}$ would be less or equal to 1. And here I found that $\frac{n\gamma}{c(T')} \geq \epsilon$. This means $\frac{n}{m} - \frac{n\gamma}{c(T')} \leq (1-\epsilon)$. So I'm not sure how they get into this step and also the later part $\frac{\epsilon c(T) p_i(T^{'})}{(1+\epsilon)\alpha(1-\epsilon)c(T')} \leq \epsilon p_i(T^{'})$.

$$\frac{n\gamma}{c(T')} = \frac{\epsilon c(T) n}{(1+\epsilon) n \alpha c(T')} = \frac{\epsilon c(T)}{(1+\epsilon)\alpha c(T')}$$

$$c(T) \geq c(T^{'})$$

$$\frac{n\gamma}{c(T')} \geq \frac{\epsilon}{(1+\epsilon)\alpha}, \alpha \geq 1$$

$$\frac{n\gamma}{c(T')} \geq \epsilon$$