LACROSSE VIRUS DYNAMICS

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1. The Model

S_L	Susceptible Larval (aquatic) stage
I_L	Infected Larval (aquatic) stage
S_M	Susceptible Males
E_M	Exposed Males
I_M	Infected Males

The adult female mosquitoes are in these compartments:

S_B	Susceptible host-seeking females
S_D	Susceptible digesting females
S_O	Susceptible ovipositing females
E_D	Exposed digesting females
E_O	Exposed ovipositing females
I_B	Infected host-seeking females
I_D	Infected digesting females
I_O	Infected ovipositing females

The reservoir species (chipmunk) has the following compartments:

S_C	Susceptible chipmunks
I_C	Infected chipmunks
R_C	Recovered chipmunks

And we have the following table of parameters

γ	Instrinsic growth rate
θ	Vertical transmission rate
K	Carrying capacity
a	Maturation rate
μ_2	Larval death rate
r	Proportion of male offspring
b	Biting rate per capita
β	Transmission rate to vector
p	Mating contact rate
μ_1	Adult death rate
$\tilde{\mu}$	Ovipositing death rate??
τ	Venereal transmission rate
ξ	Tranmission rate to host
φ	Rate from E_D to I_D
ψ	Rate from E_O to I_O
ν	Rate from E_M to I_M
ω	Recovery rate of host

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For the mating success probabilities, we've adopted a more intuitive labeling system: the first subscript describes that disease status of the **female** and the second does the status of the *male*. That brings us this updated table:

α_{SS}	Success between S_D and S_M
α_{SE}	Success between S_D and E_M
α_{SI}	Success between S_D and I_M
α_{ES}	Success between E_D and S_M
α_{EE}	Success between E_D and E_M
α_{EI}	Success between E_D and I_M
α_{IS}	Success between I_D and S_M
α_{IE}	Success between S_D and E_M
α_{II}	Success between I_D and I_M

$$\dot{S}_L = \frac{\gamma}{K}(S_O + (1-\theta)I_O + (1-\theta')E_O)[K - (S_L + I_L)] - aS_L - \mu_2 S_L \tag{1.1}$$

$$\dot{S}_M = raS_L - \mu_1 S_M - \tau p \alpha_{IS} S_M \left(\frac{I_D}{S_D + E_D + I_D}\right) \tag{1.2}$$

$$\dot{S}_B = (1-r)aS_L - bS_B - \mu_1 S_B \tag{1.3}$$

$$\dot{S}_D = bS_B \left(\frac{S_C + R_C + (1-\beta)I_C}{S_C + I_C + R_C}\right) - pS_D \left(\frac{\alpha_{SS}S_M + \alpha_{SE}E_M + \alpha_{SI}I_M}{S_M + E_M + I_M}\right) - \mu_1 S_D \tag{1.4}$$

$$\dot{S}_O = pS_D \left(\frac{\alpha_{SS}S_M + \alpha_{SE}E_M + \alpha_{SI}(1-\tau)I_M}{S_M + E_M + I_M}\right) - \tilde{\mu} S_O \tag{1.5}$$

$$\tag{1.6}$$

$$\dot{E}_M = \tau p \alpha_{IS} S_M \left(\frac{I_D}{S_D + E_D + I_D}\right) - \nu E_M - \mu_1 E_M \tag{1.7}$$

$$\dot{E}_D = \beta b S_B \left(\frac{I_C}{S_C + I_C + R_C}\right) - pE_D \left(\frac{\alpha_{ES}S_M + \alpha_{EE}E_M + \alpha_{EI}I_M}{S_M + E_M + I_M}\right) - \varphi E_D - \mu_1 E_D \tag{1.8}$$

$$\dot{E}_O = \tau p \alpha_{SI} S_D \left(\frac{I_M}{S_M + E_M + I_M}\right) + pE_D \left(\frac{\alpha_{ES}S_M + \alpha_{EE}E_M + \alpha_{EI}I_M}{S_M + E_M + I_M}\right) - \psi E_O - \tilde{\mu} E_O \tag{1.9}$$

$$\tag{1.10}$$

$$\dot{I}_L = \frac{\gamma}{K}(\theta I_O + \theta' E_O)[K - (S_L + I_L)] - aI_L - \mu_2 I_L \tag{1.11}$$

$$\dot{I}_M = raI_L + \nu E_M - \mu_1 I_M \tag{1.12}$$

$$\dot{I}_B = (1-r)aI_L - bI_B - \mu_1 I_B \tag{1.13}$$

$$\dot{I}_D = bI_B + \varphi E_D - pI_D \left(\frac{\alpha_{IS}S_M + \alpha_{IE}E_M + \alpha_{II}I_M}{S_M + E_M + I_M}\right) - \mu_1 I_D \tag{1.14}$$

$$\dot{I}_O = pI_D \left(\frac{\alpha_{IS}S_M + \alpha_{IE}E_M + \alpha_{II}I_M}{S_M + E_M + I_M}\right) + \psi E_O - \tilde{\mu} I_O \tag{1.15}$$

$$\tag{1.16}$$

$$\dot{S}_C = -\xi b S_C \left(\frac{S_B + I_B}{S_C + I_C + R_C}\right)\left(\frac{I_B}{S_B + I_B}\right) \tag{1.17}$$

$$\dot{I}_C = \xi b S_C \left(\frac{S_B + I_B}{S_C + I_C + R_C}\right)\left(\frac{I_B}{S_B + I_B}\right) - \omega I_C \tag{1.18}$$

$$\dot{R}_C = \omega I_C \tag{1.19}$$