MONASH University

MONASH
INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

Ian Wern Han Lim
lim.wern.han@monash.edu

Referencing materials by
Rafael Dowsley, Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

MONASH University

Ready?

Agenda

- Circulation with Demands
- Circulation with Demands and Lower Bound

- Applications Example
  - Survey Design
  - Airline Scheduling

MONASH University

Let us begin…

Network Flow

Recall…

- You have learnt Graph
- You have learnt Network Flow

Network Flow

Recall…

- You have learnt Graph
  - How many problems can be modelled as a Graph, then be solved
- You have learnt Network Flow

Network Flow

Recall…

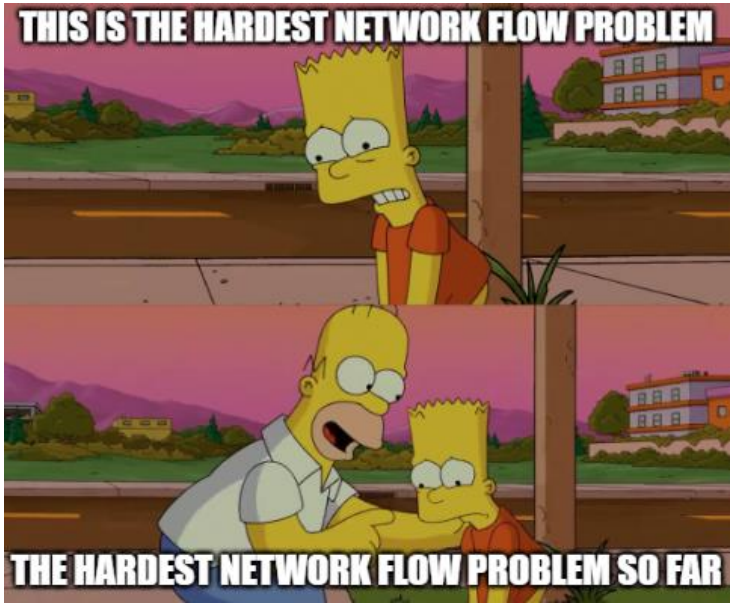
MONASH University

- You have learnt Graph
  - How many problems can be modelled as a Graph, then be solved.
- You have learnt Network Flow
  - Likewise, we have explore the simple Bipartite Matching problem.

Network Flow

Recall…

- You have learnt Graph
  - How many problems can be modelled as a Graph, then be solved.
- You have learnt Network Flow
  - Likewise, we have explore the simple Bipartite Matching problem.
  - … now let us push 1 step further!



MONASH University

Questions?

Circulation with Demands

A Feasibility Problem…

MONASH University

Circulation with Demands

A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow

Circulation with Demands

A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Flow conservation
    - Incoming flow to a vertex == outgoing flow from the vertex

Circulation with Demands

A Feasibility Problem…

MONASH University

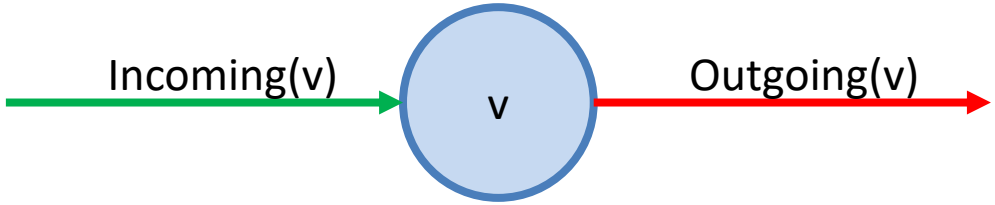
- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Flow conservation
    - Incoming flow to a vertex == outgoing flow from the vertex
    - Now what if we tweak this rule?

Circulation with Demands

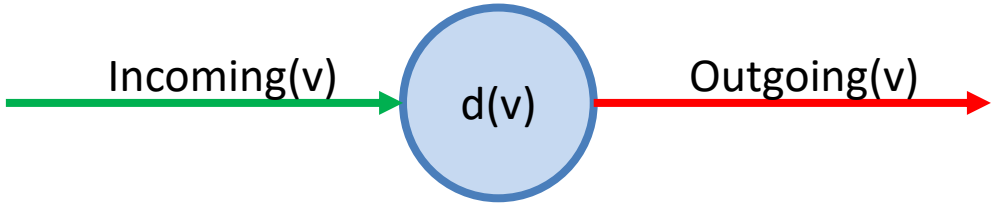
A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Flow conservation
    - Incoming flow to a vertex == outgoing flow from the vertex



- Now what if we tweak this rule?

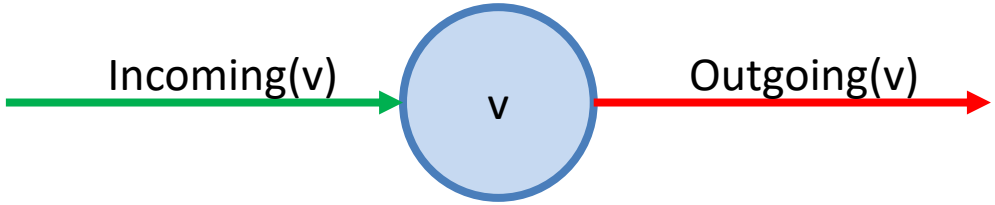


Circulation with Demands

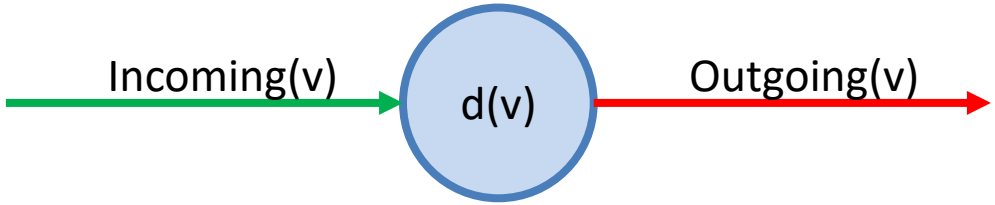
A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Flow conservation
    - Incoming flow to a vertex == outgoing flow from the vertex



- Now what if we tweak this rule? incoming(v) - outgoing(v) = d(v)

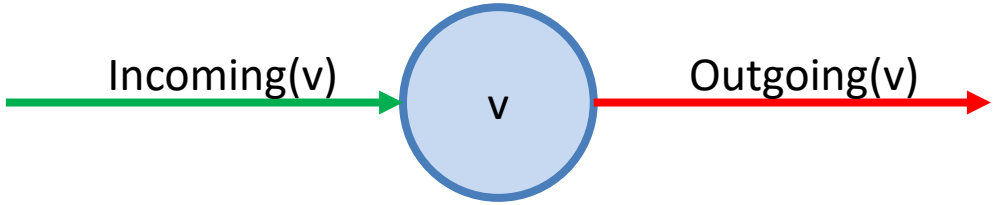


Circulation with Demands

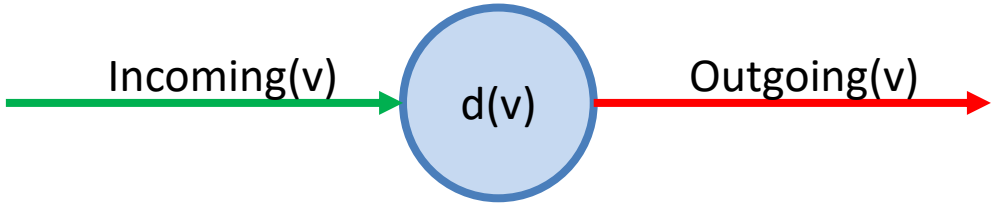
A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Flow conservation
    - Incoming flow to a vertex == outgoing flow from the vertex



- Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)

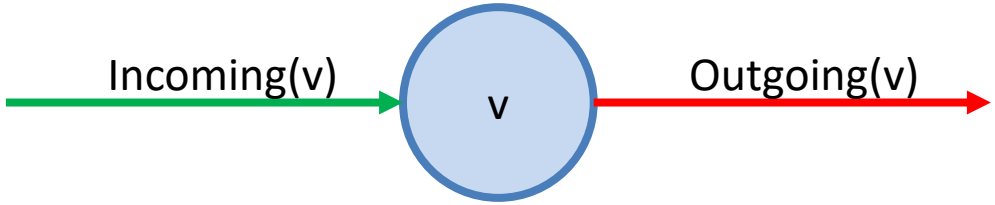


Circulation with Demands

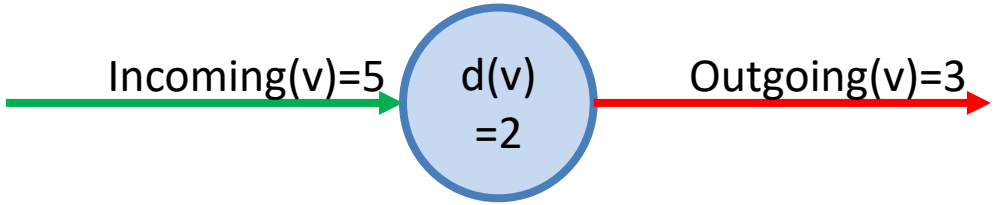
A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Flow conservation
    - Incoming flow to a vertex == outgoing flow from the vertex



- Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)

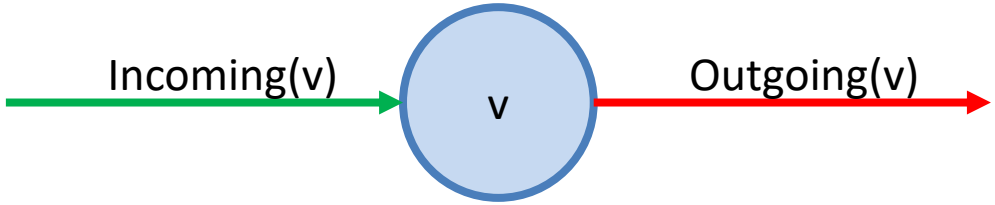


Circulation with Demands

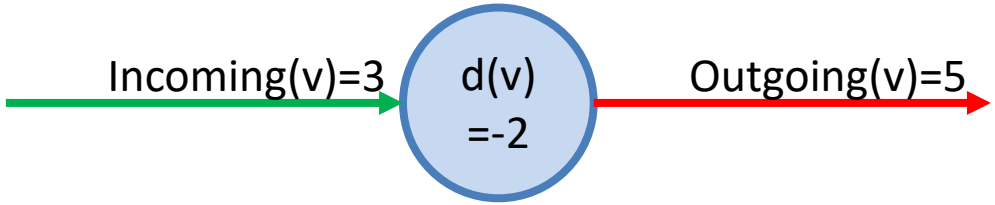
A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Flow conservation
    - Incoming flow to a vertex == outgoing flow from the vertex



- Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)

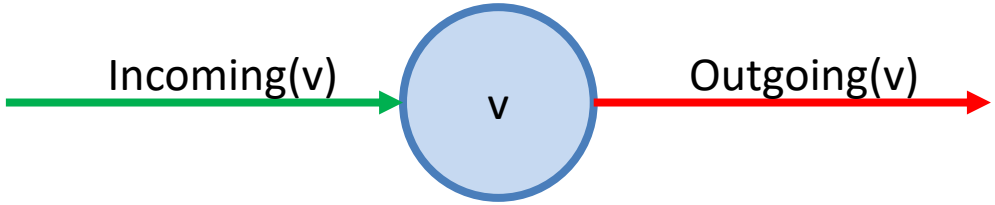


Circulation with Demands

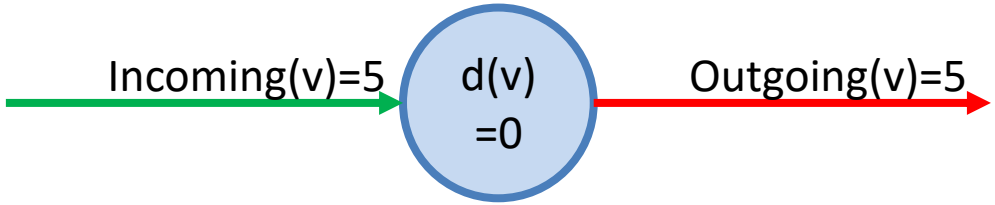
A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Flow conservation
    - Incoming flow to a vertex == outgoing flow from the vertex



- Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)

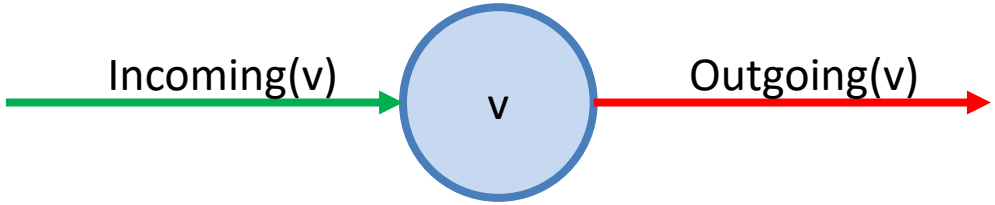


Circulation with Demands

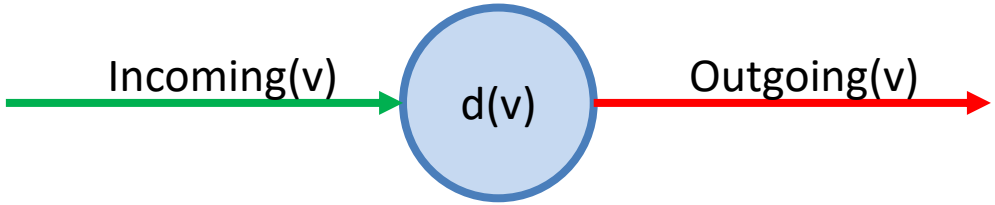
A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - ~~Flow conservation~~ Demand Constraint
    - Incoming flow to a vertex == outgoing flow from the vertex



- Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)

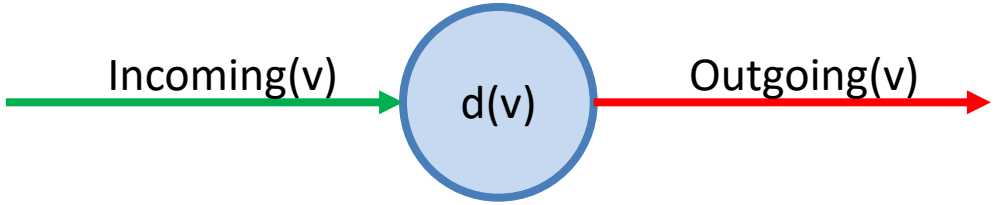


Circulation with Demands

A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Demand Constraint
    - Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)

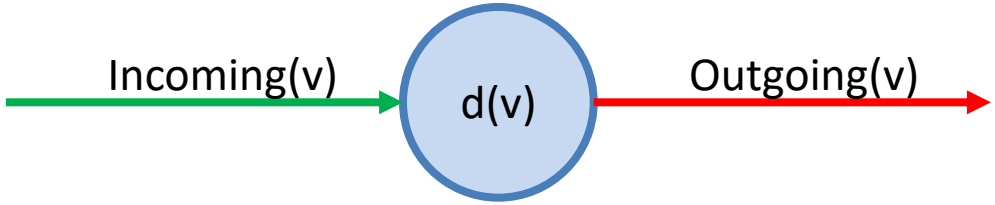


Circulation with Demands

A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Demand Constraint
    - Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)



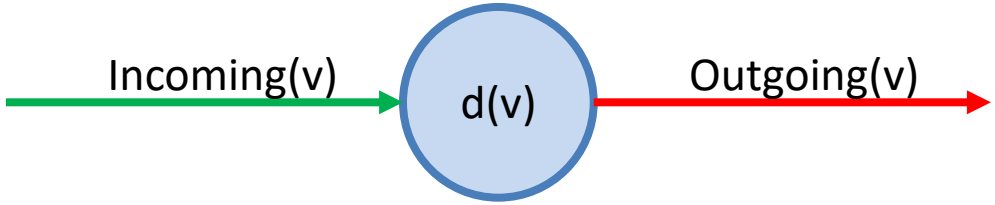
- Circulation with Demands is a feasibility problem

Circulation with Demands

A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Network Flow
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Demand Constraint
    - Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)



- Circulation with Demands is a feasibility problem that satisfy both of the above!

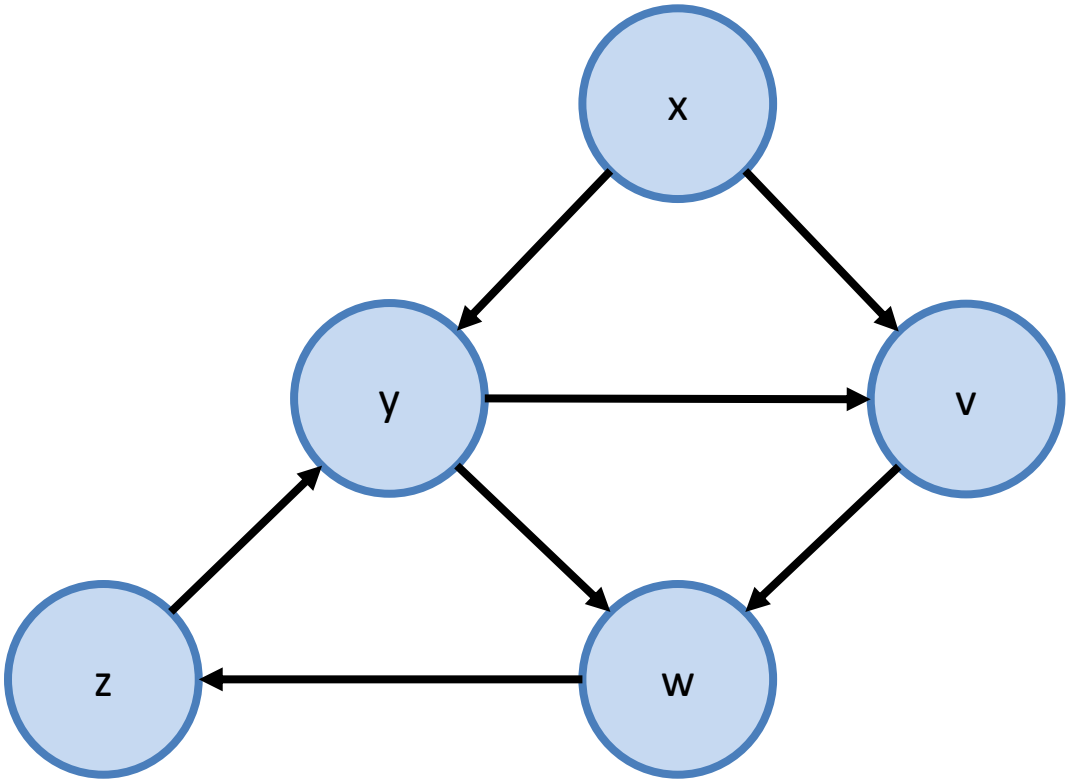
MONASH University

Questions?

Circulation with Demands

A Feasibility Problem…

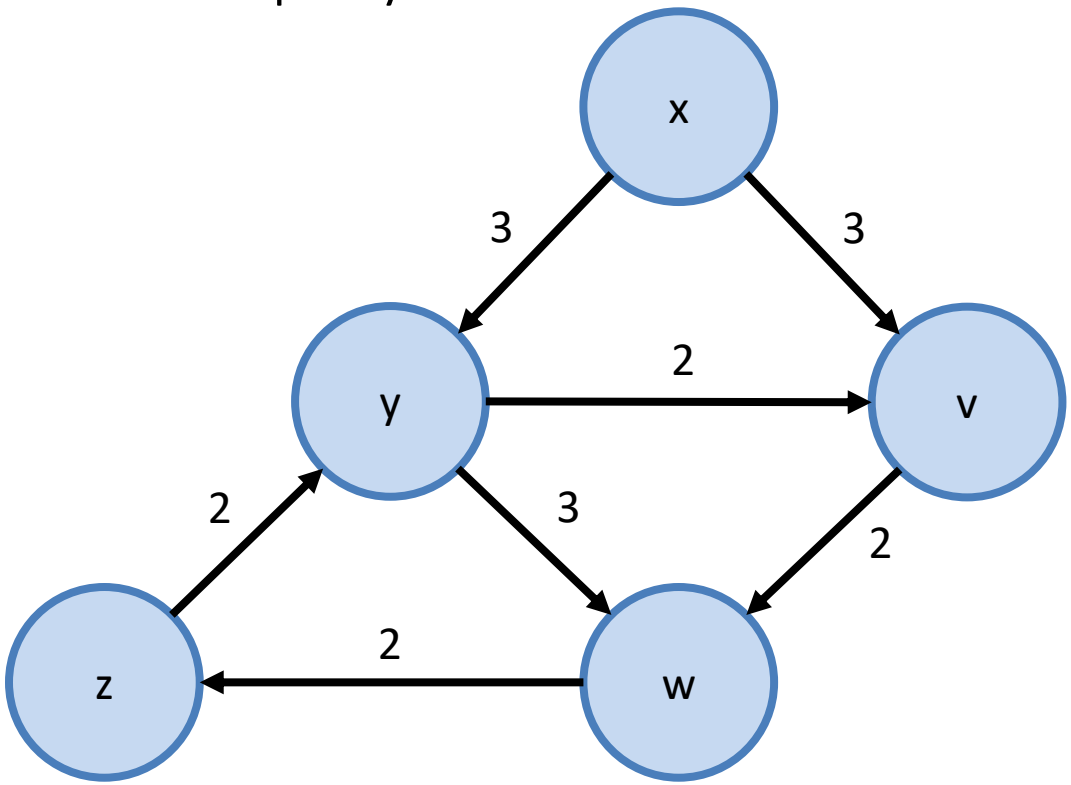
- An example below…



Circulation with Demands

A Feasibility Problem…

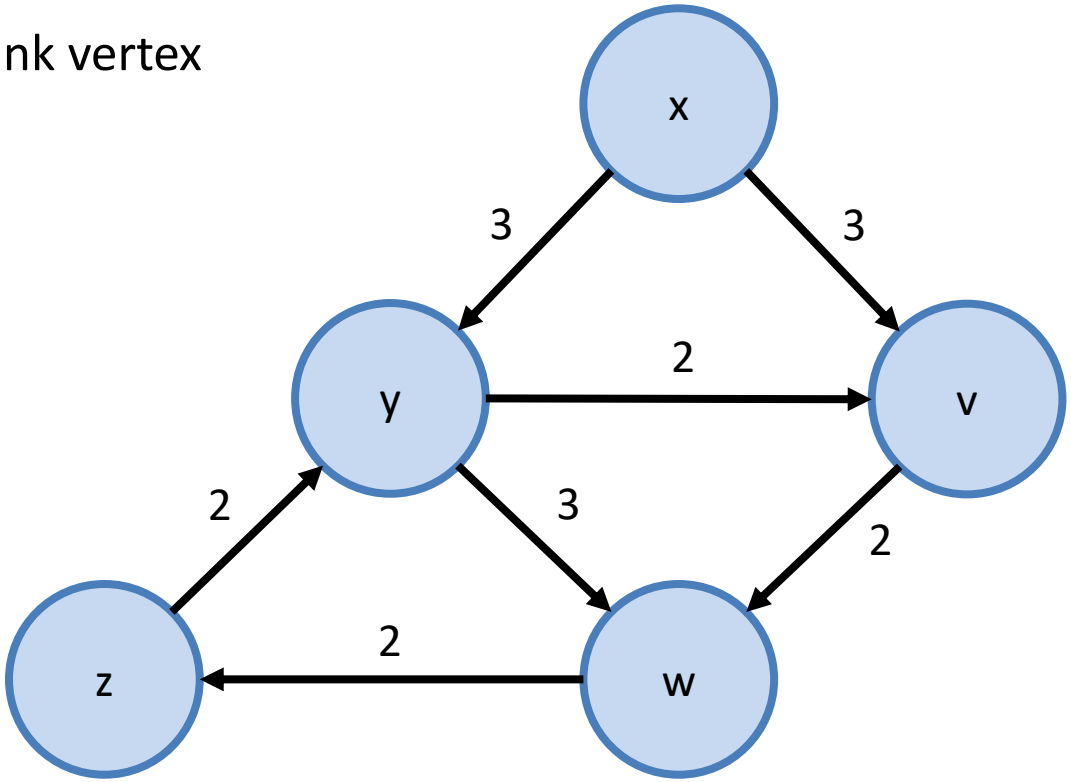
- An example below…
  - With the usual capacity



Circulation with Demands

A Feasibility Problem…

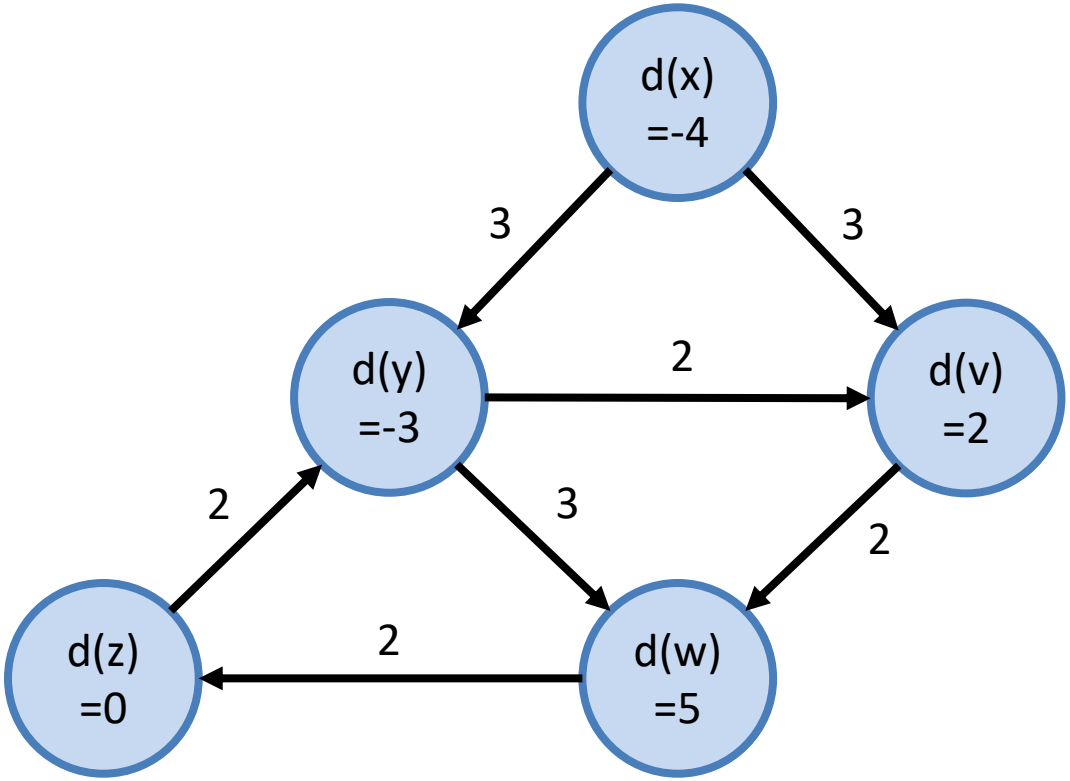
- An example below…
  - No source vertex
  - No sink vertex



Circulation with Demands

A Feasibility Problem…

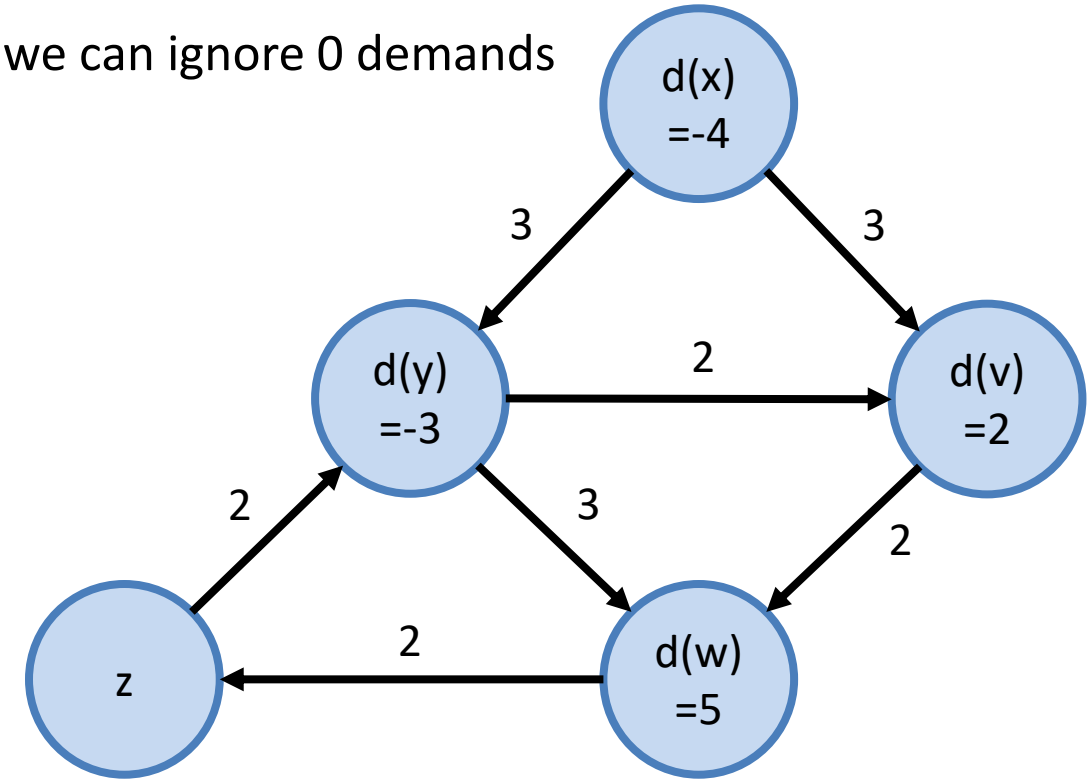
- An example below…
  - Vertex with demands



Circulation with Demands

A Feasibility Problem…

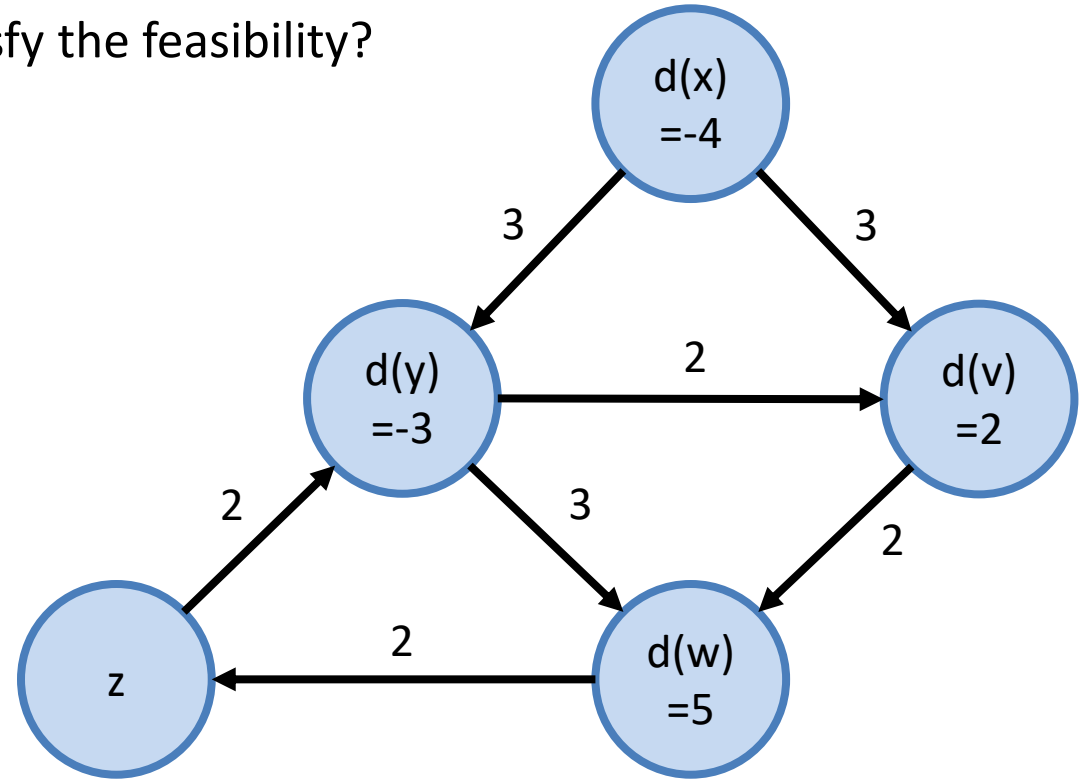
- An example below…
  - Vertex with demands
  - But we can ignore 0 demands



Circulation with Demands

A Feasibility Problem…

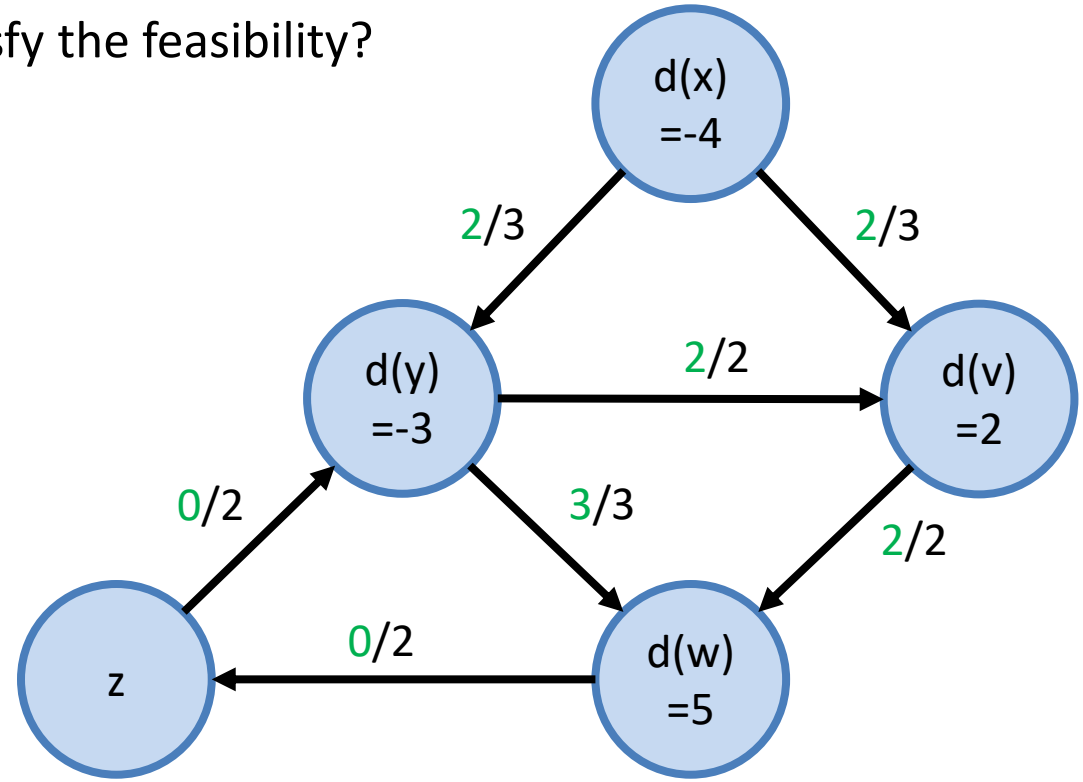
- An example below…
  - Can we have a flow that satisfy the feasibility?



Circulation with Demands

A Feasibility Problem…

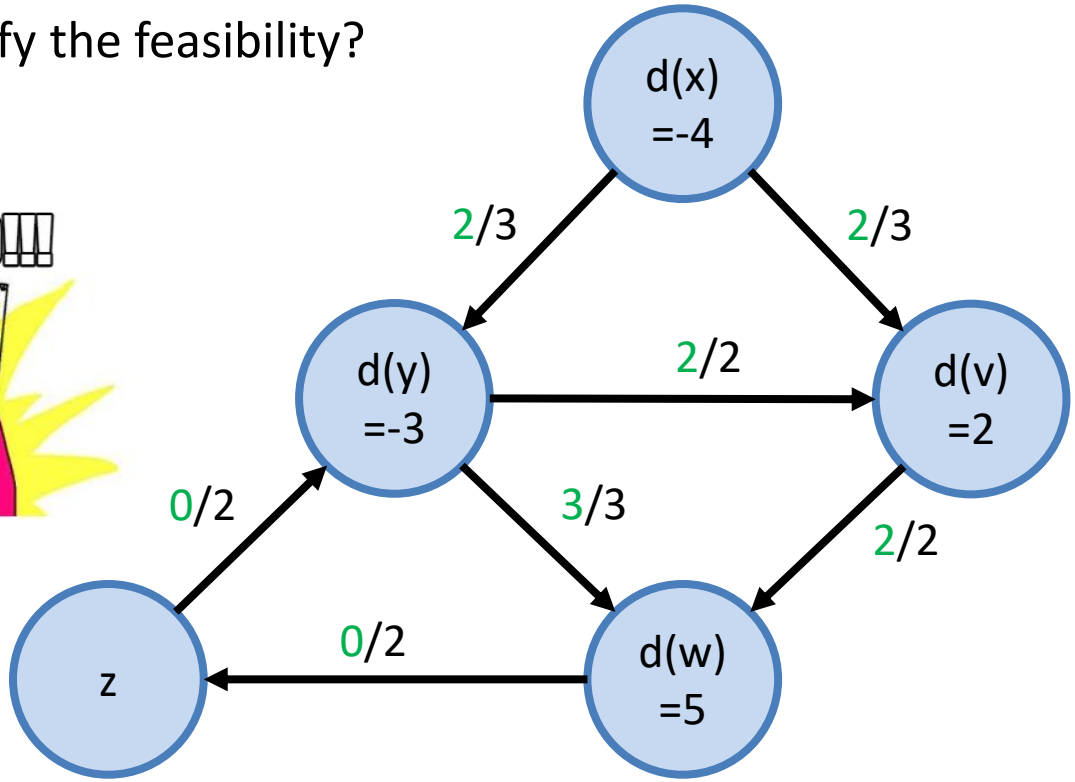
- An example below…
  - Can we have a flow that satisfy the feasibility?



Circulation with Demands

A Feasibility Problem…

- An example below…
  - Can we have a flow that satisfy the feasibility?
  - YES



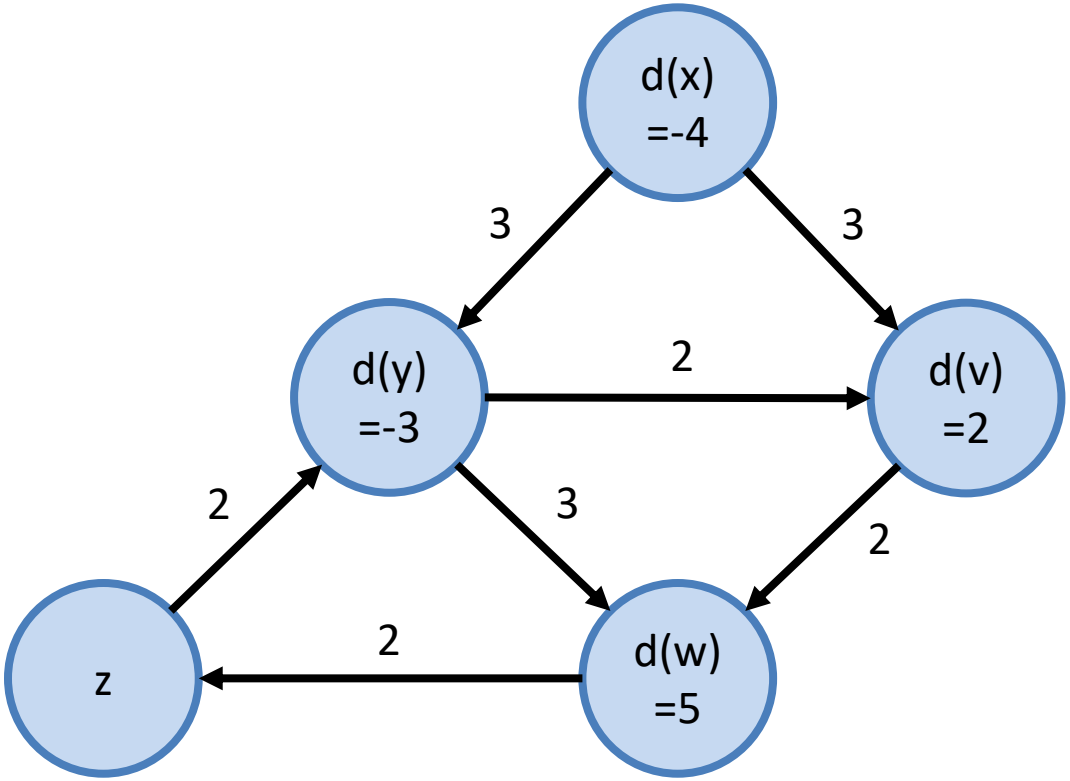
MONASH University

Questions?

Circulation with Demands

How to Check Feasibility…

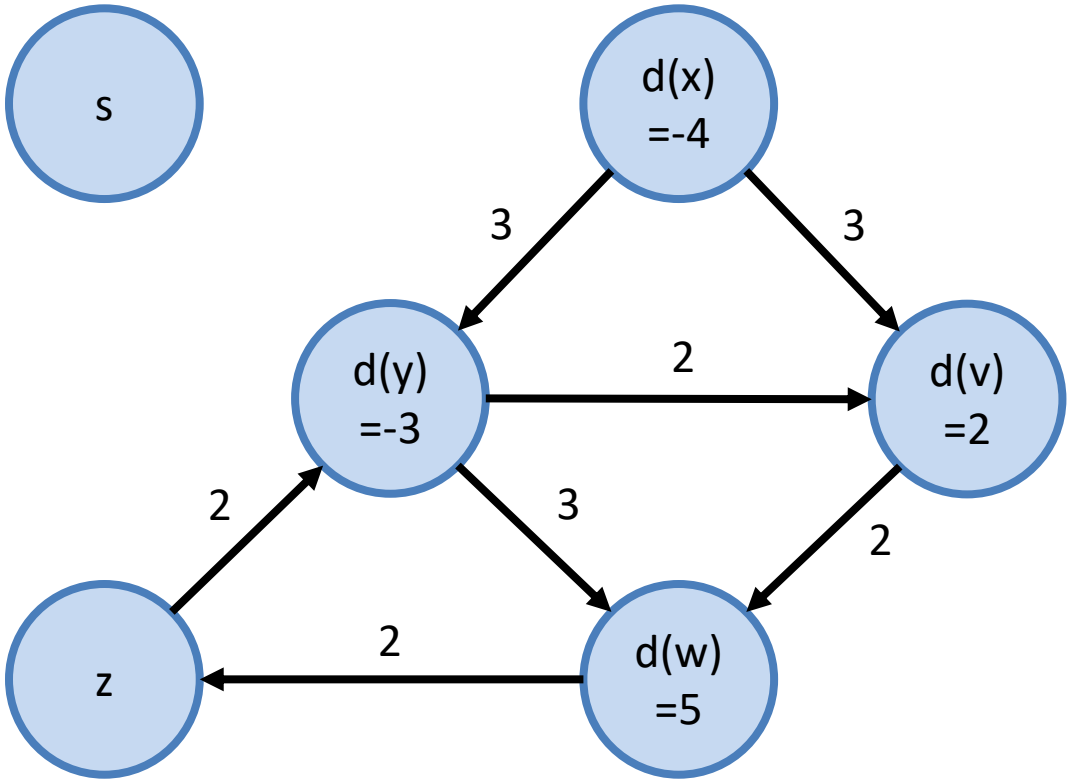
- Given this… How do we solve this?



Circulation with Demands

How to Check Feasibility…

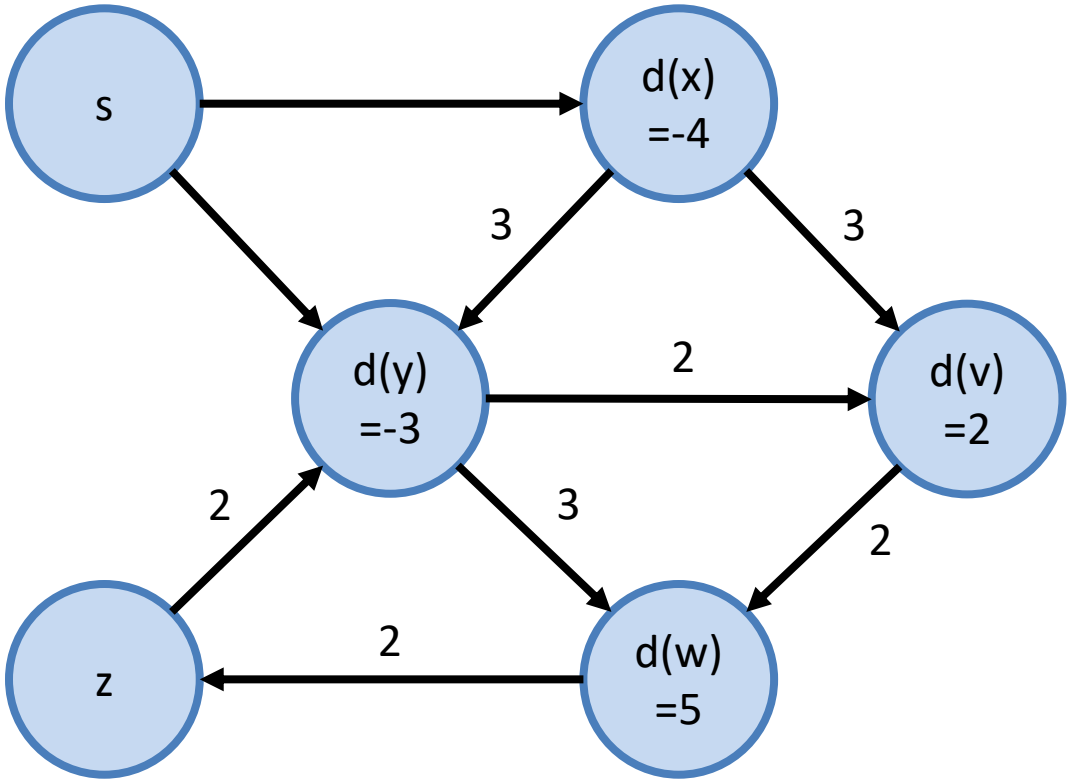
- Make a source



Circulation with Demands

How to Check Feasibility…

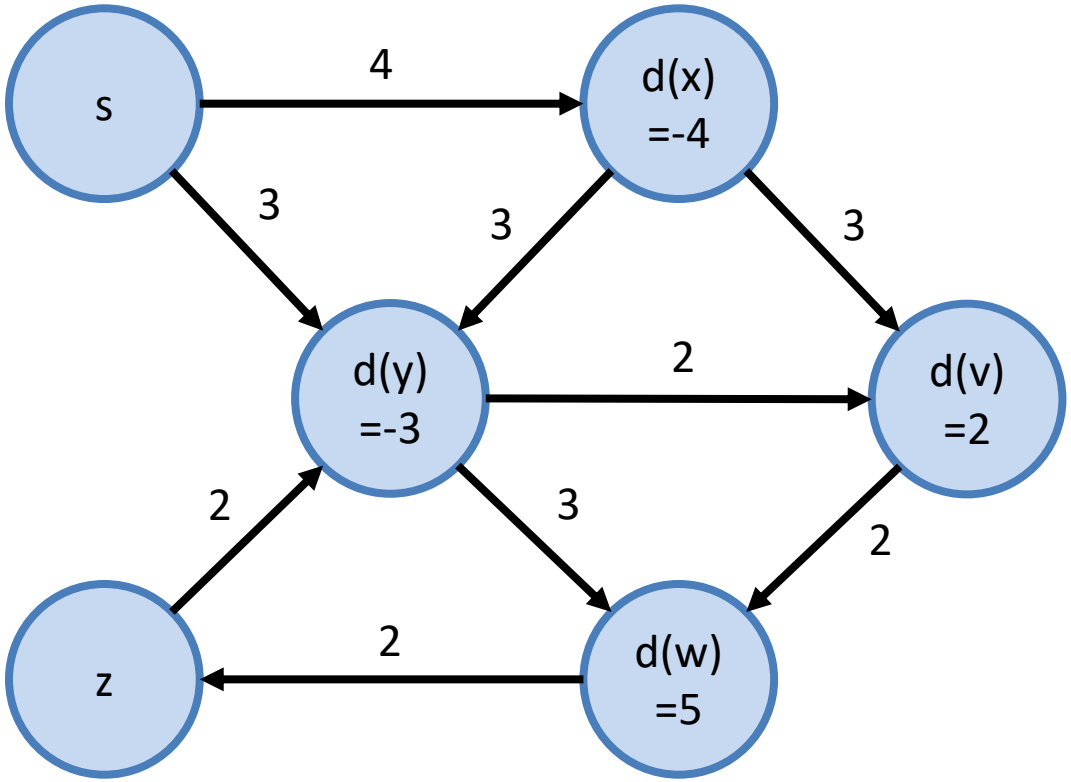
- Make a source, link to all negative demand



Circulation with Demands

How to Check Feasibility…

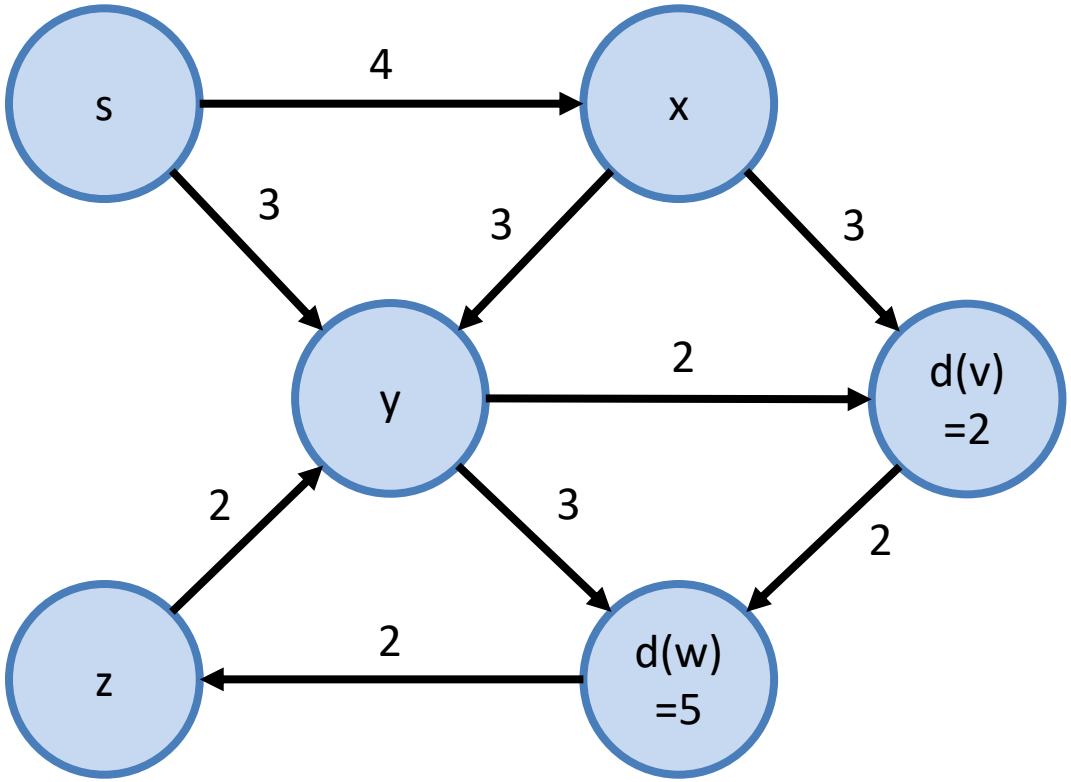
- Make a source, link to all negative demand, weighted



Circulation with Demands

How to Check Feasibility…

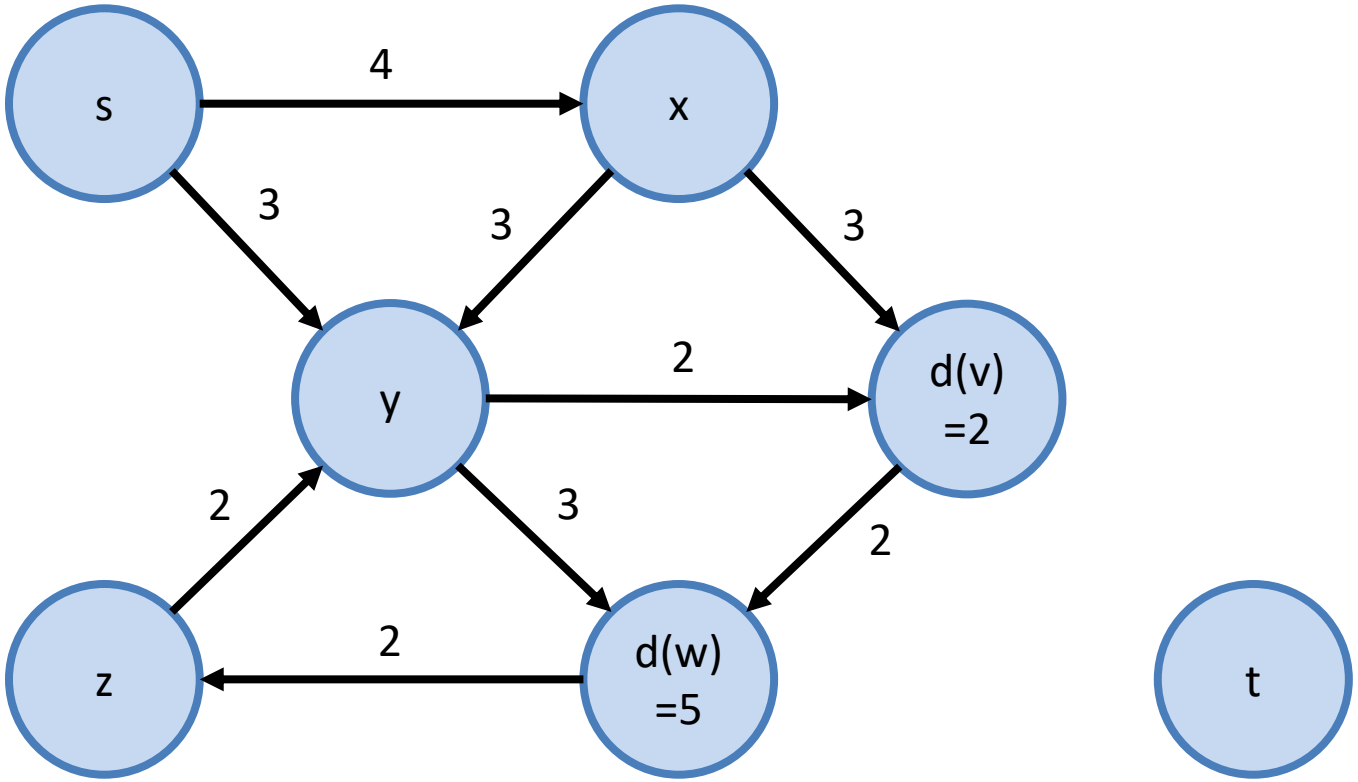
- We are done! for source >.<



Circulation with Demands

How to Check Feasibility…

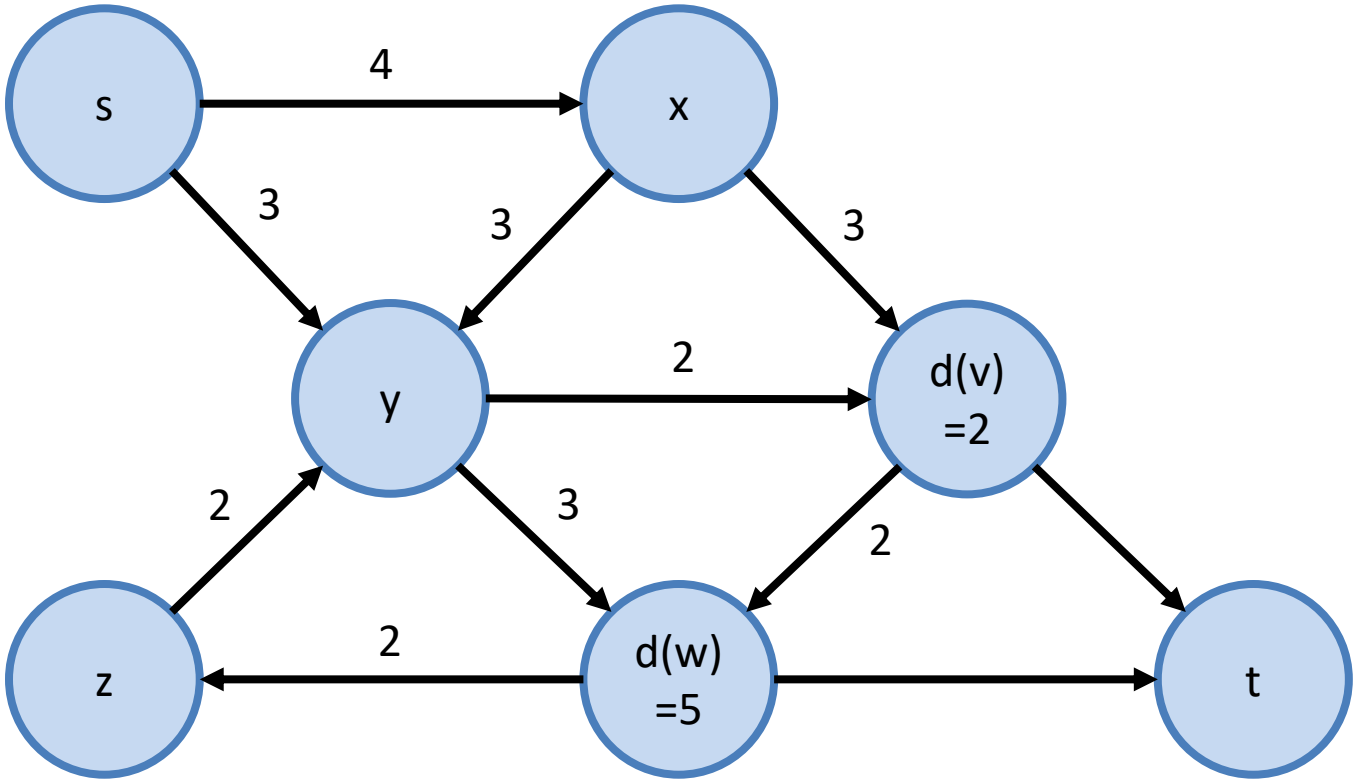
- Make a sink



Circulation with Demands

How to Check Feasibility…

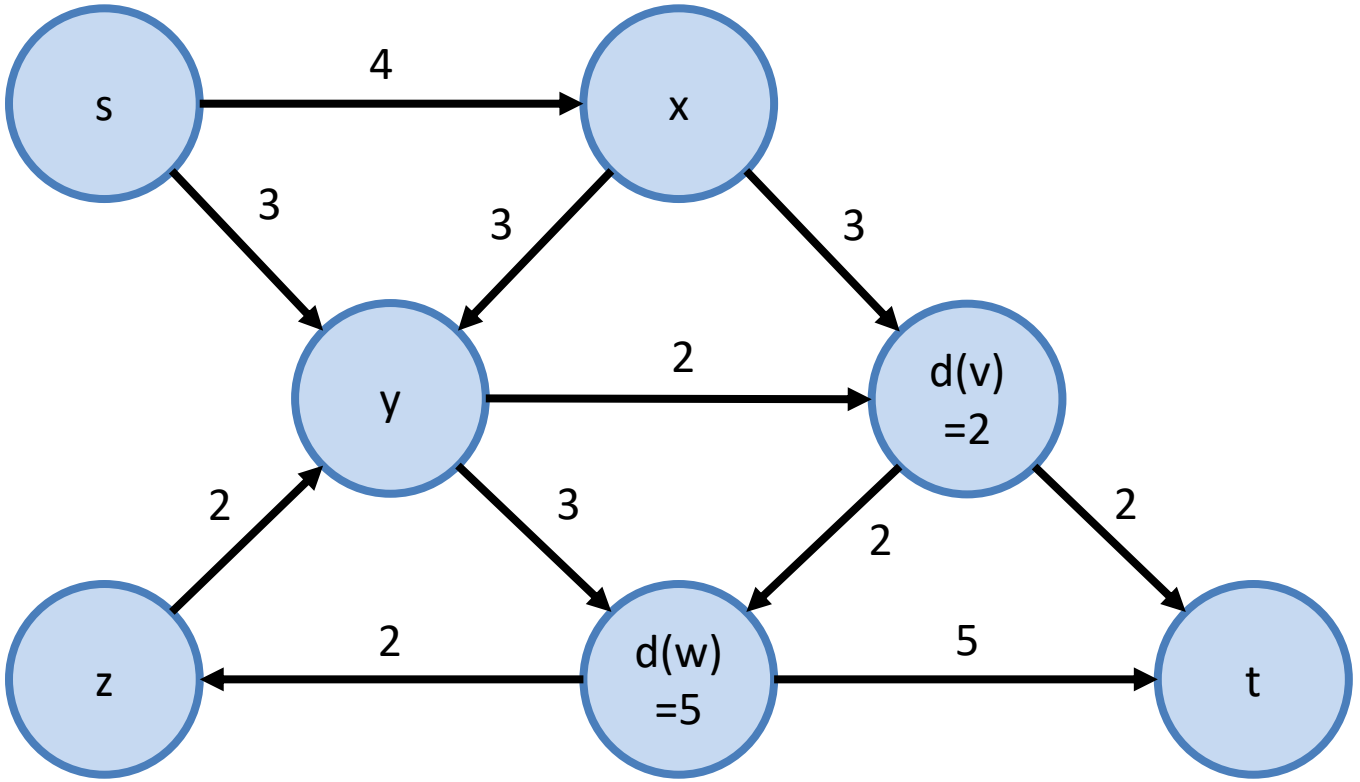
- Make a sink, link from positive demand



Circulation with Demands

How to Check Feasibility…

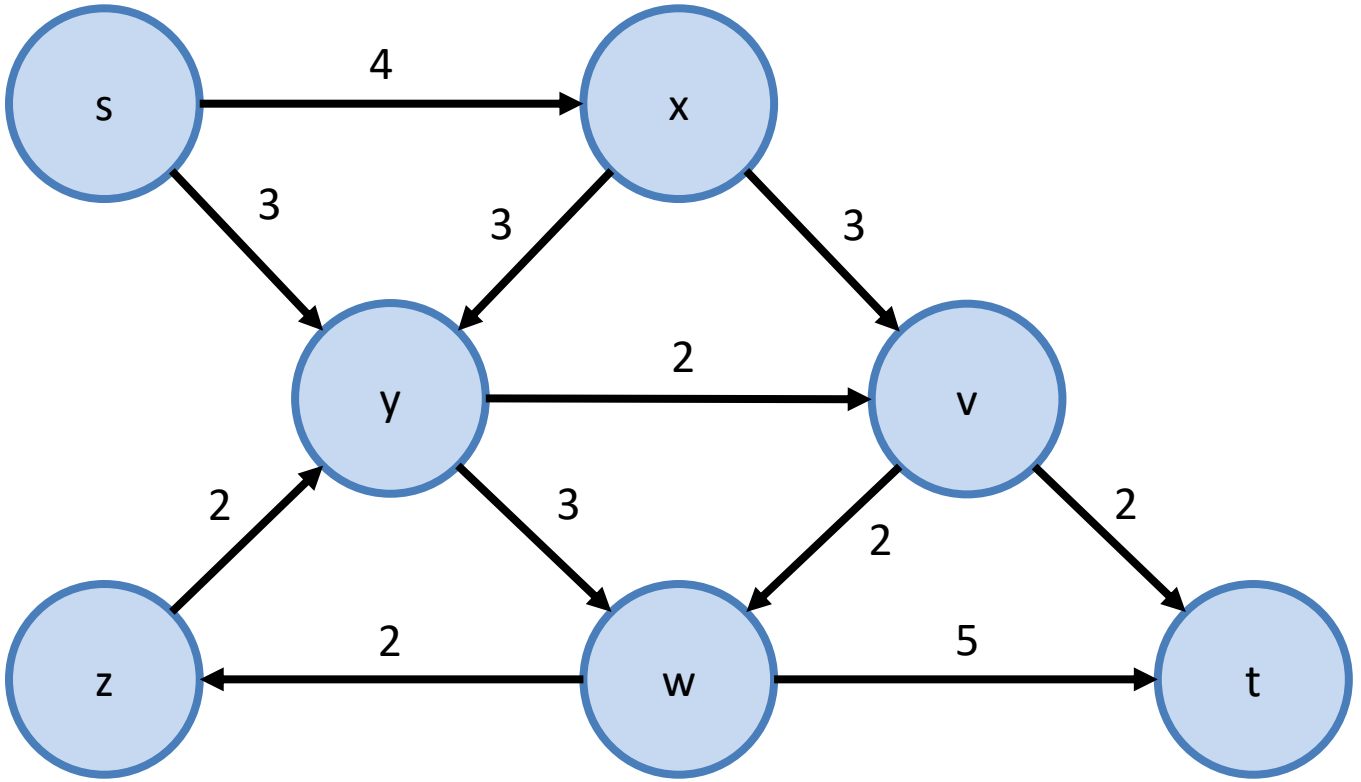
- Make a sink, link from positive demand, weighted



Circulation with Demands

How to Check Feasibility…

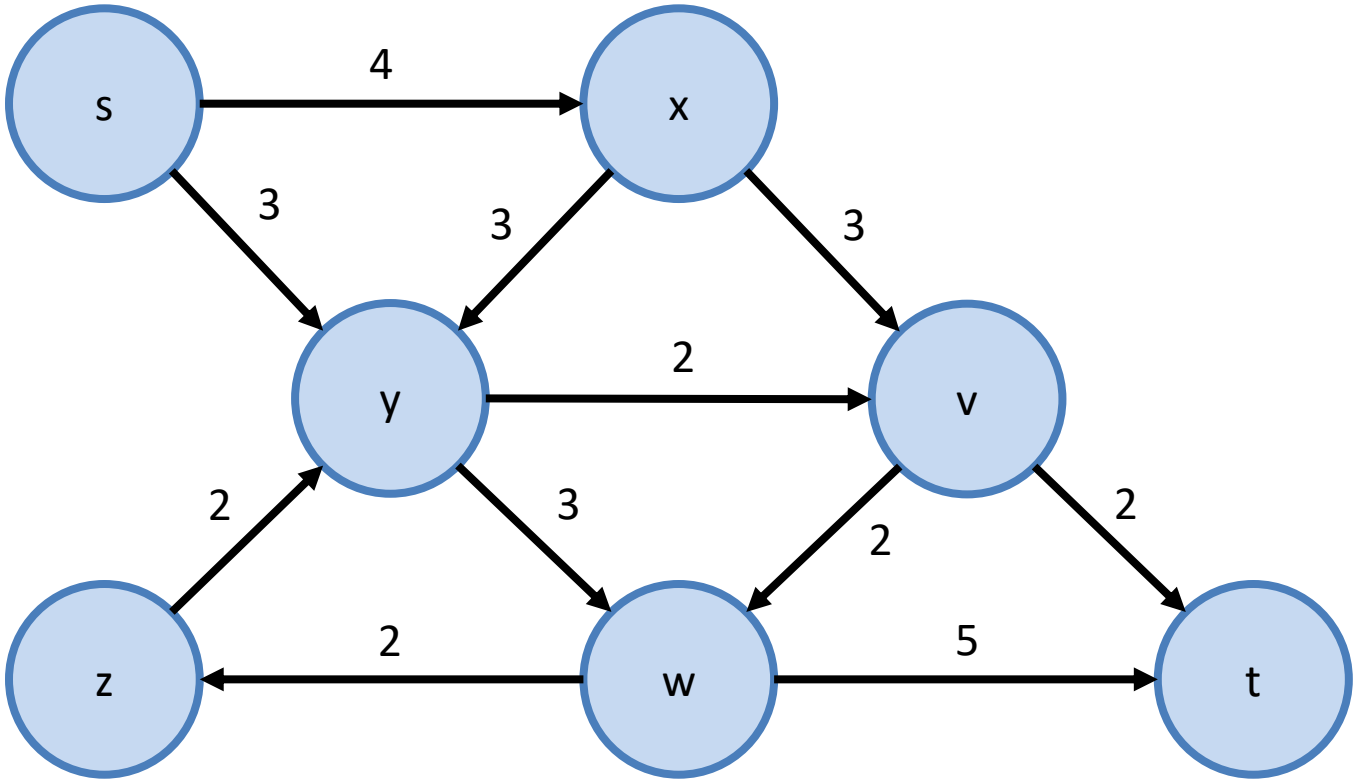
- We are done! for sink >.<



Circulation with Demands

How to Check Feasibility…

- Now same as network flow!



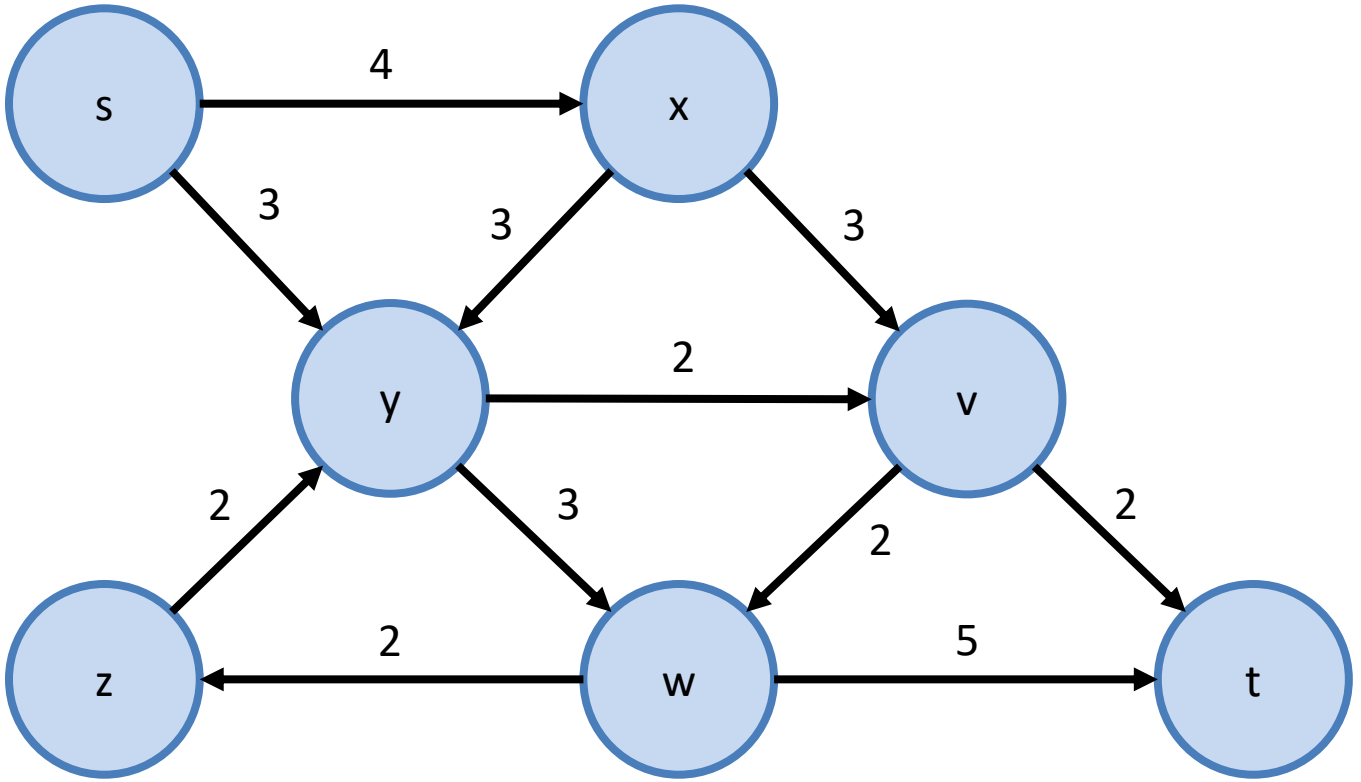
MONASH University

Questions?

Circulation with Demands

How to Check Feasibility…

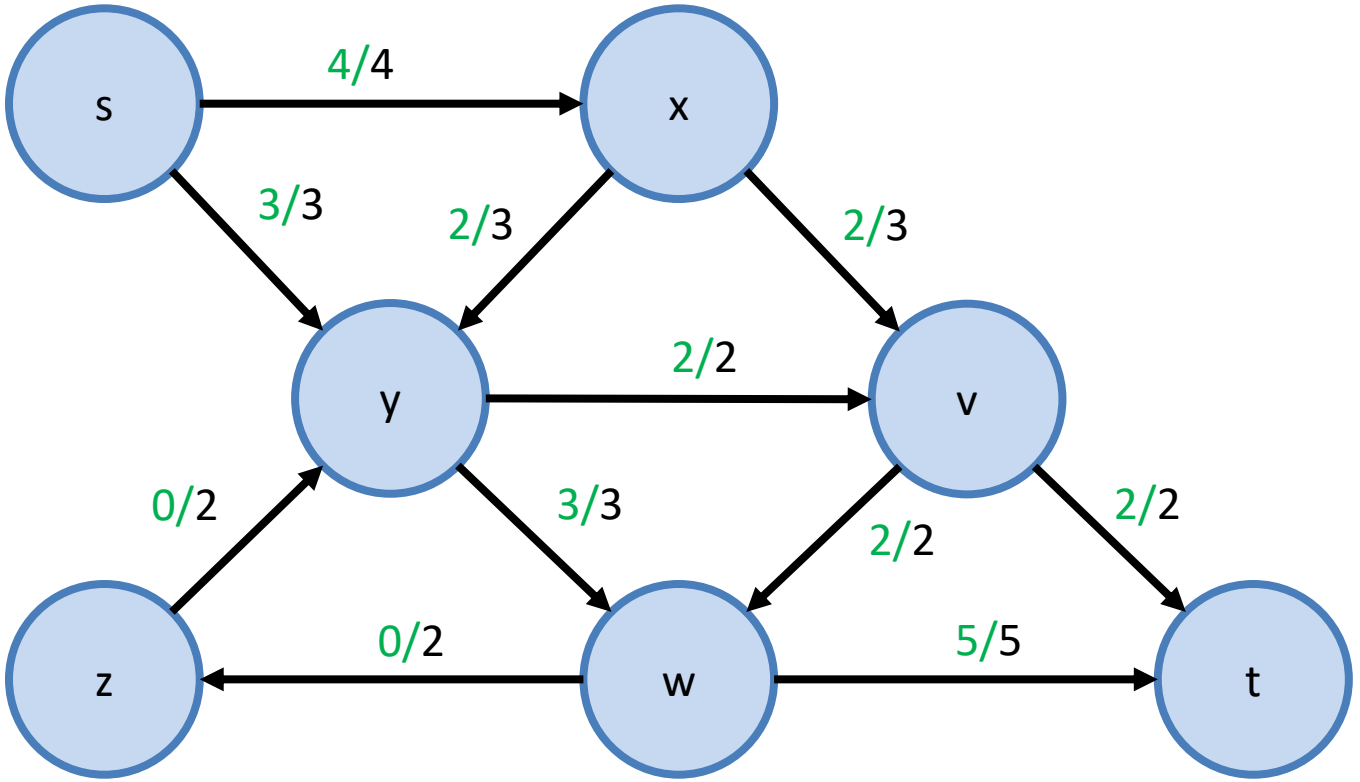
- Then we run Ford-Fulkerson



Circulation with Demands

How to Check Feasibility…

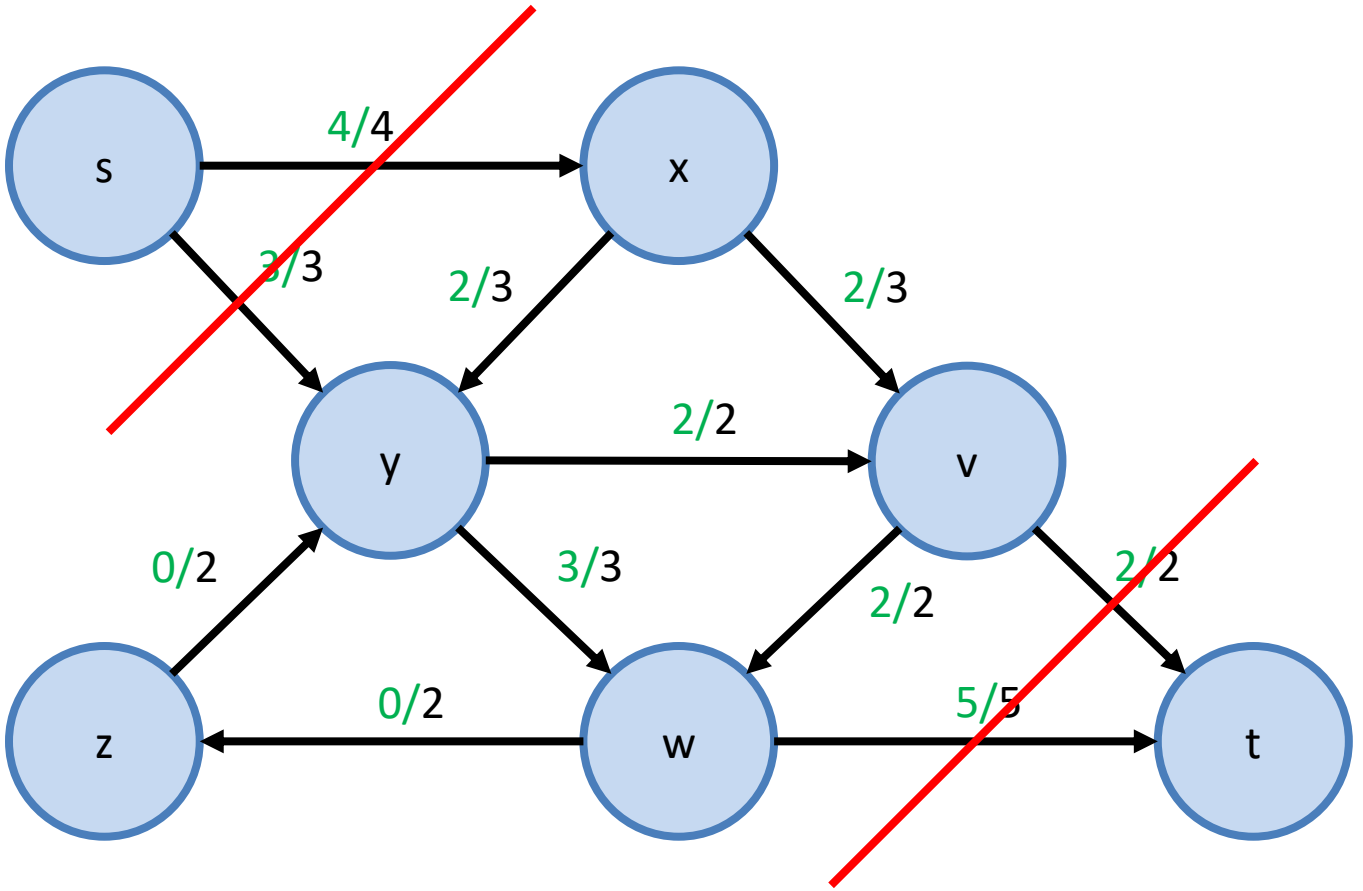
- Then we run Ford-Fulkerson



Circulation with Demands

How to Check Feasibility…

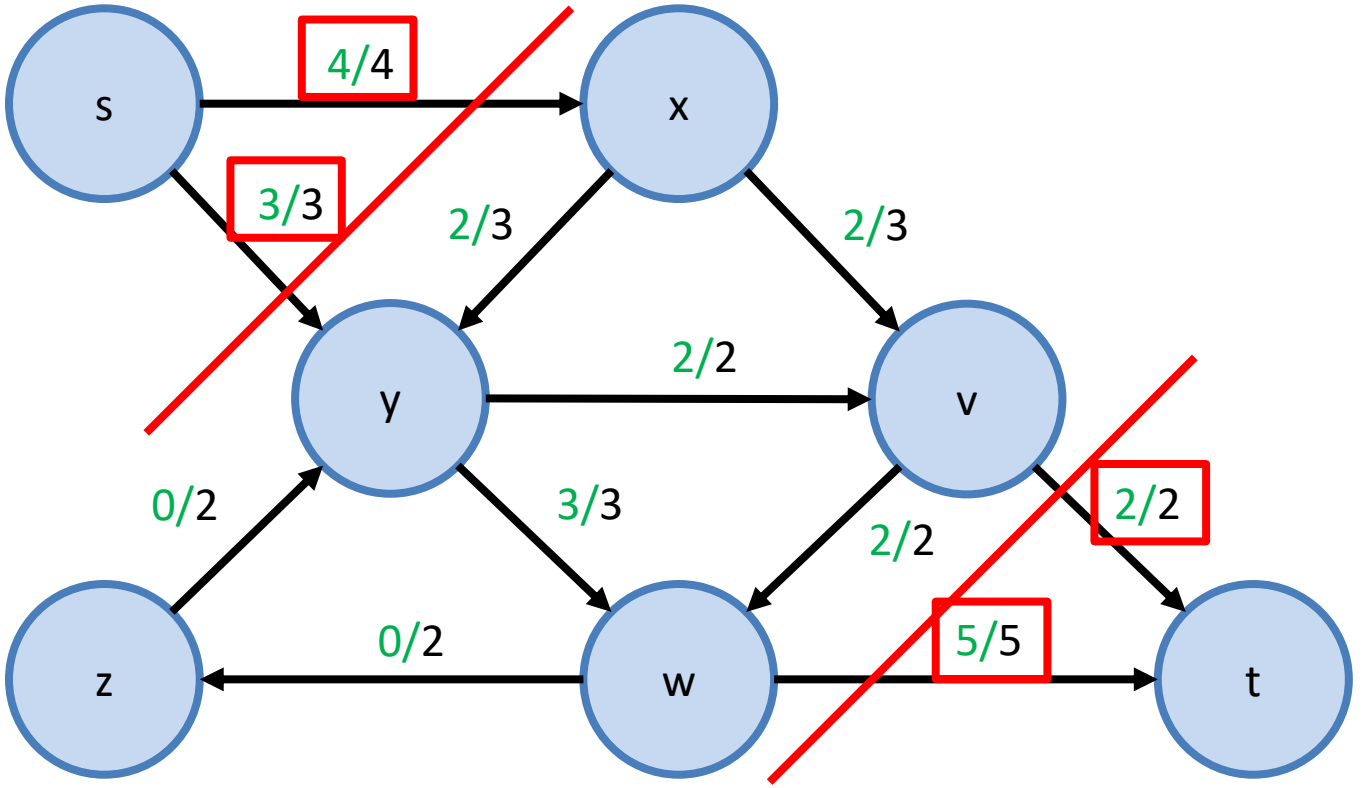
- If we have a cut…



Circulation with Demands

How to Check Feasibility…

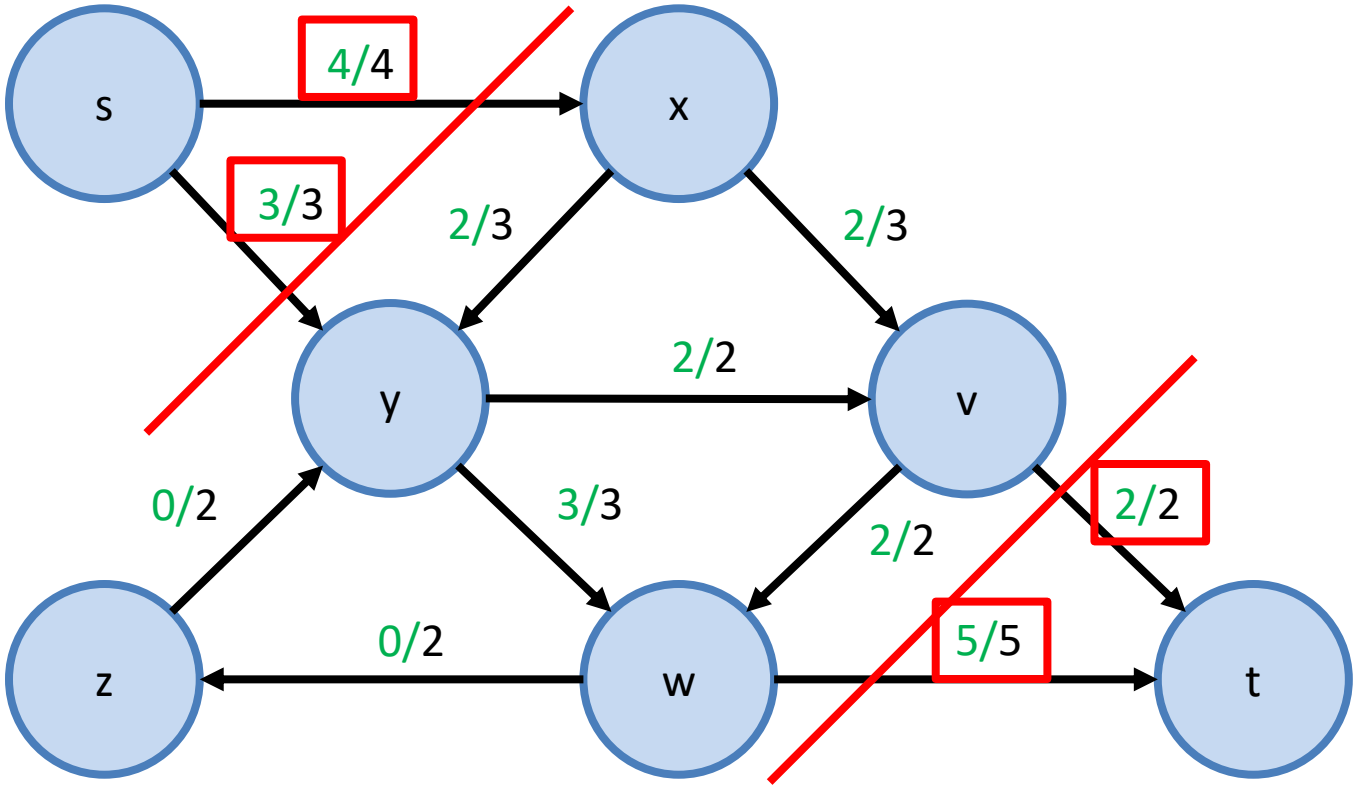
- If we have a cut…



Circulation with Demands

How to Check Feasibility…

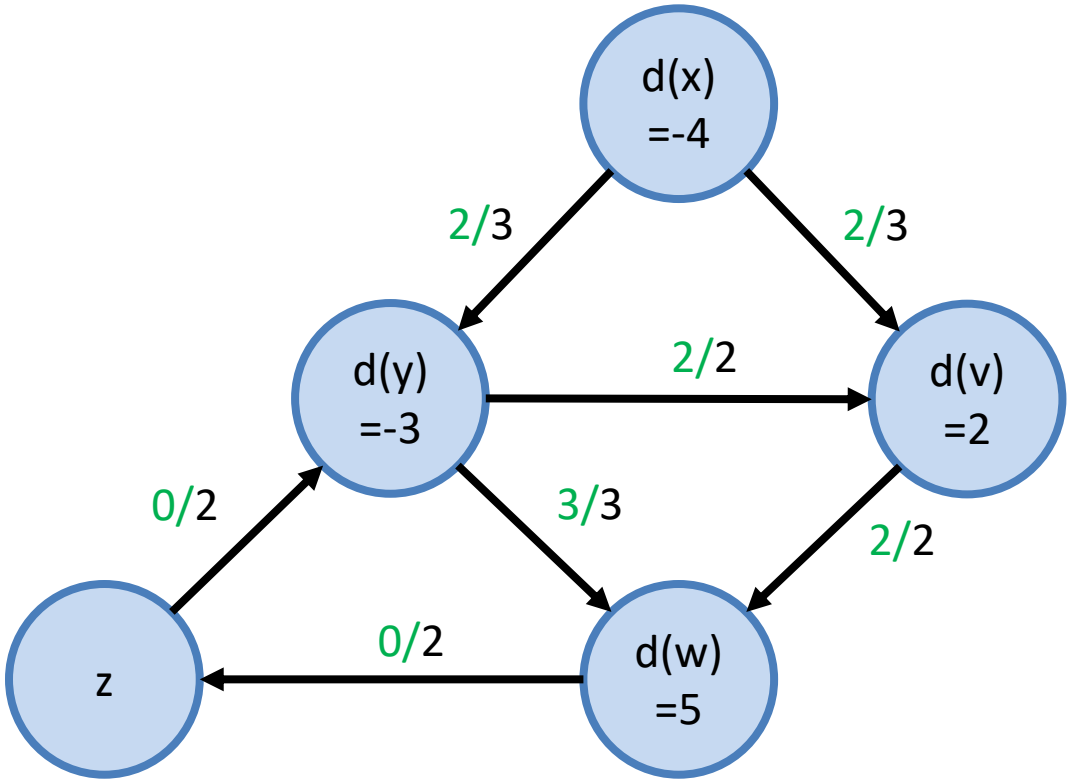
- If we have a cut… it is feasible!



Circulation with Demands

How to Check Feasibility…

- Then we just clean it up as the solution



MONASH University

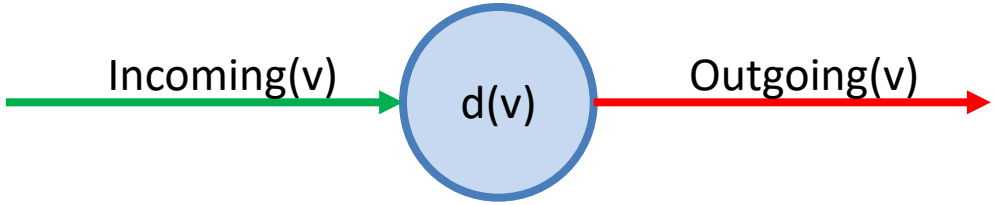
Questions?

Circulation with Demands and Lower Bound

A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Circulation with Demands
  - Capacity Constraint
    - Flow <= Capacity for an edge
  - Demand Constraint
    - Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)



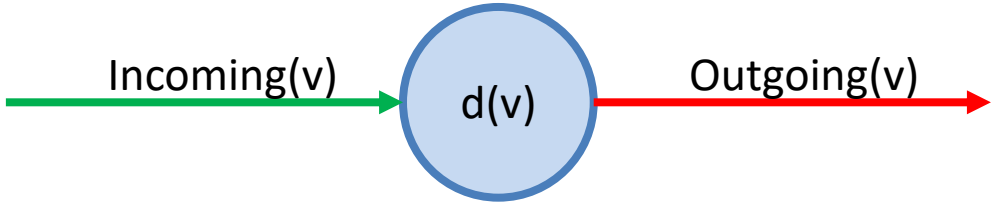
- Circulation with Demands is a feasibility problem that satisfy both of the above!

Circulation with Demands and Lower Bound

A Feasibility Problem…

MONASH University

- Recall the 2 concepts from Circulation with Demands
  - Capacity Constraint
    - Lower bound for an edge <= Flow <= Capacity for an edge
  - Demand Constraint
    - Now what if we tweak this rule? incoming(v) - outgoing(v) = demand(v)

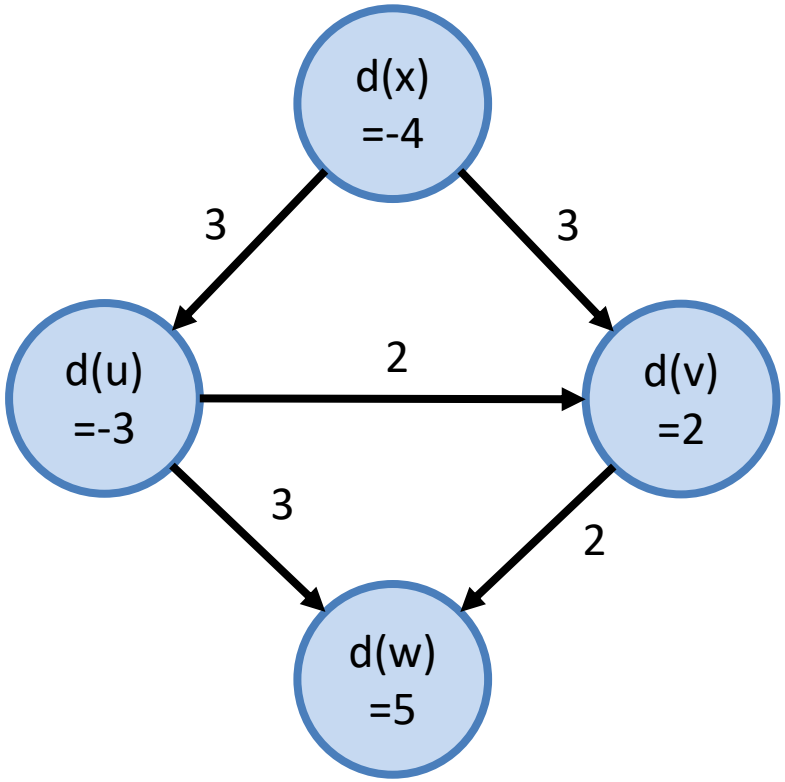


- Circulation with Demands is a feasibility problem that satisfy both of the above!

Circulation with Demands and Lower Bound

A Feasibility Problem…

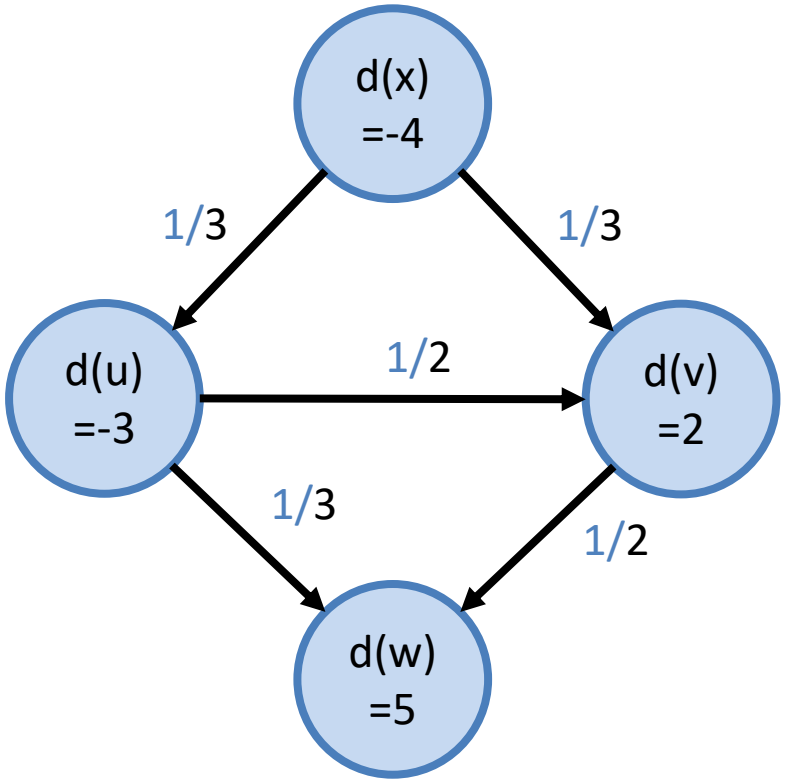
- Consider the following…



Circulation with Demands and Lower Bound

A Feasibility Problem…

- Consider the following… edges have lower bound of 1

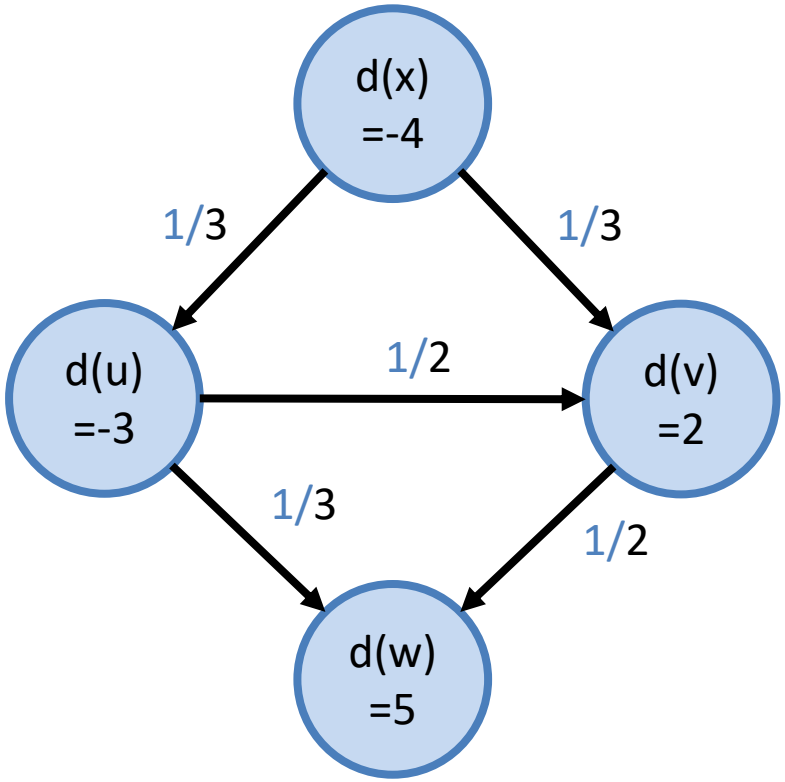


Circulation with Demands and Lower Bound

A Feasibility Problem…

MONASH University

- Can we find a feasible solution?

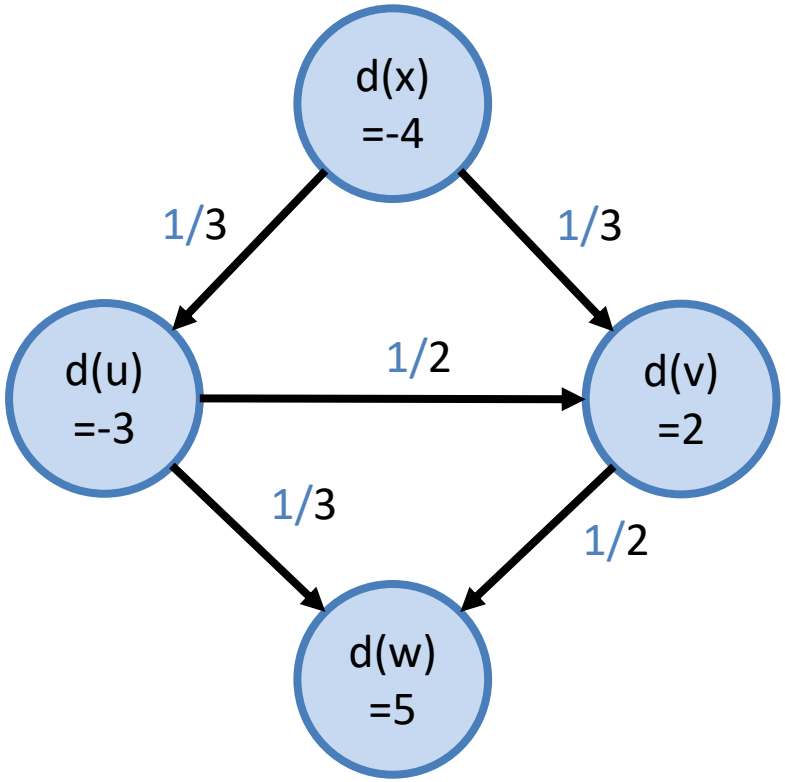


Circulation with Demands and Lower Bound

A Feasibility Problem…

MONASH University

- Can we find a feasible solution? Of course!



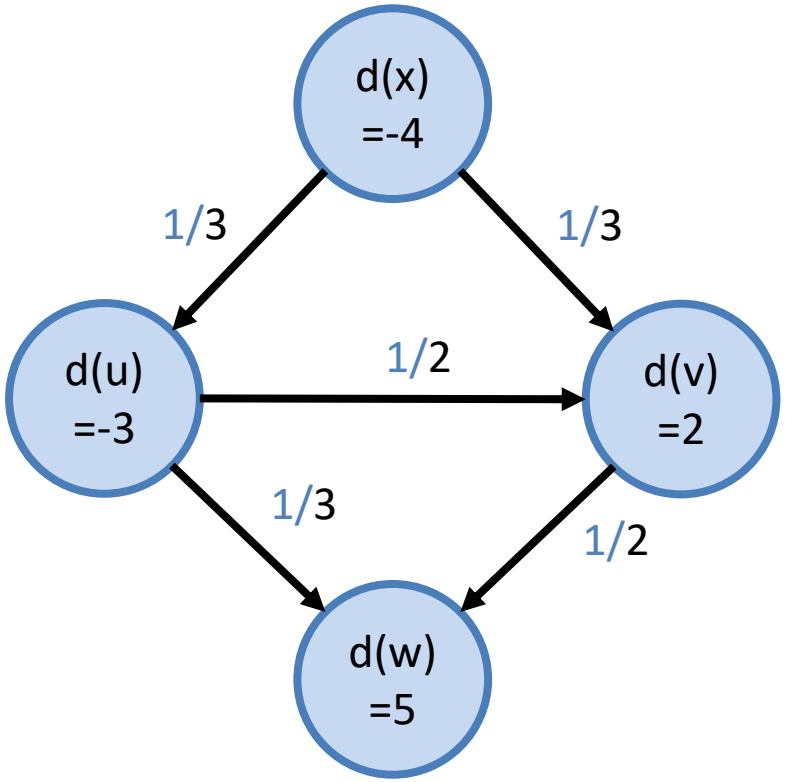
MONASH University

Questions?

Circulation with Demands and Lower Bound

A Feasibility Problem…

- We will need to make some transformation…

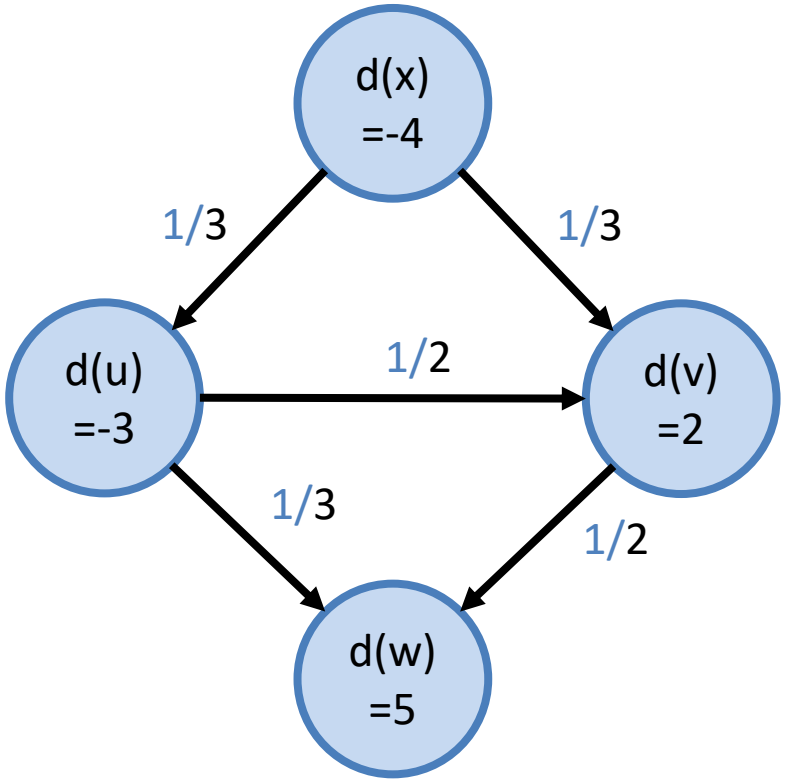


Circulation with Demands and Lower Bound

A Feasibility Problem…

MONASH University

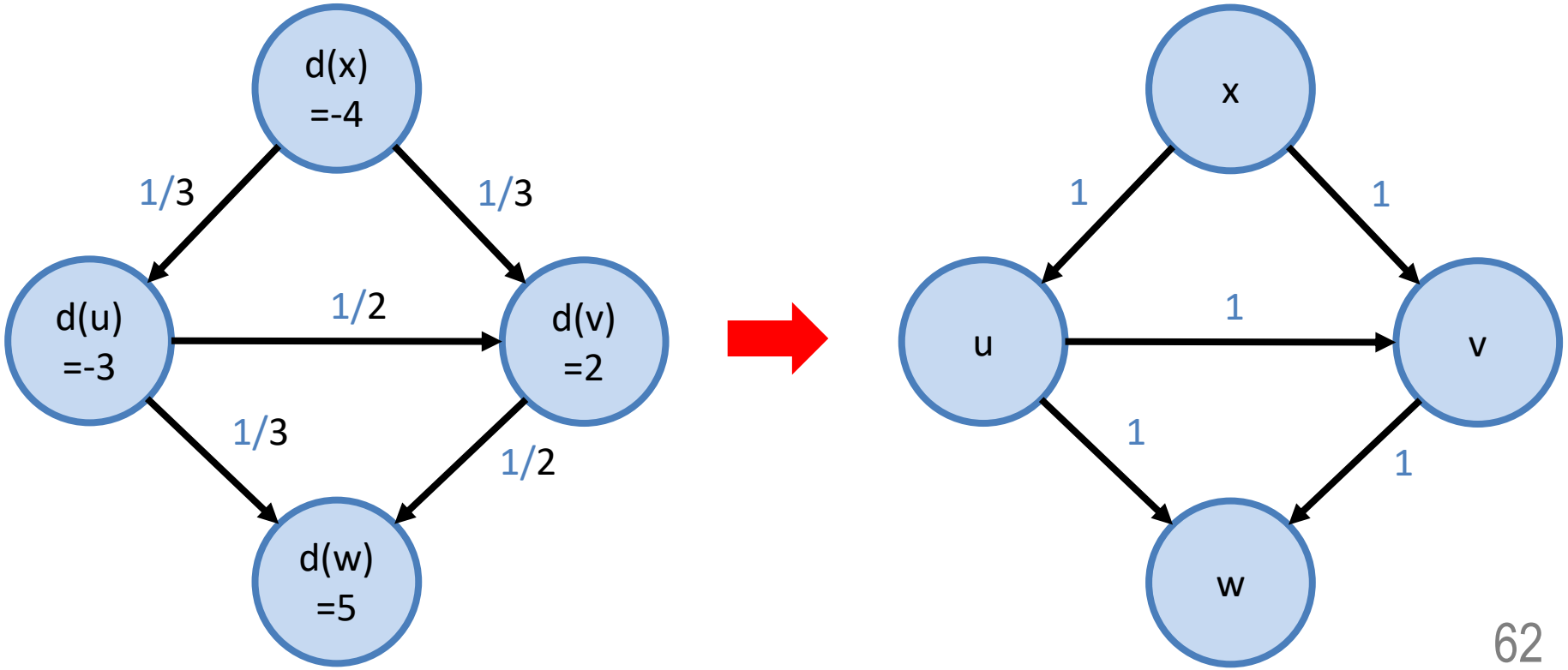
- We will need to make some transformation…



Circulation with Demands and Lower Bound

A Feasibility Problem…

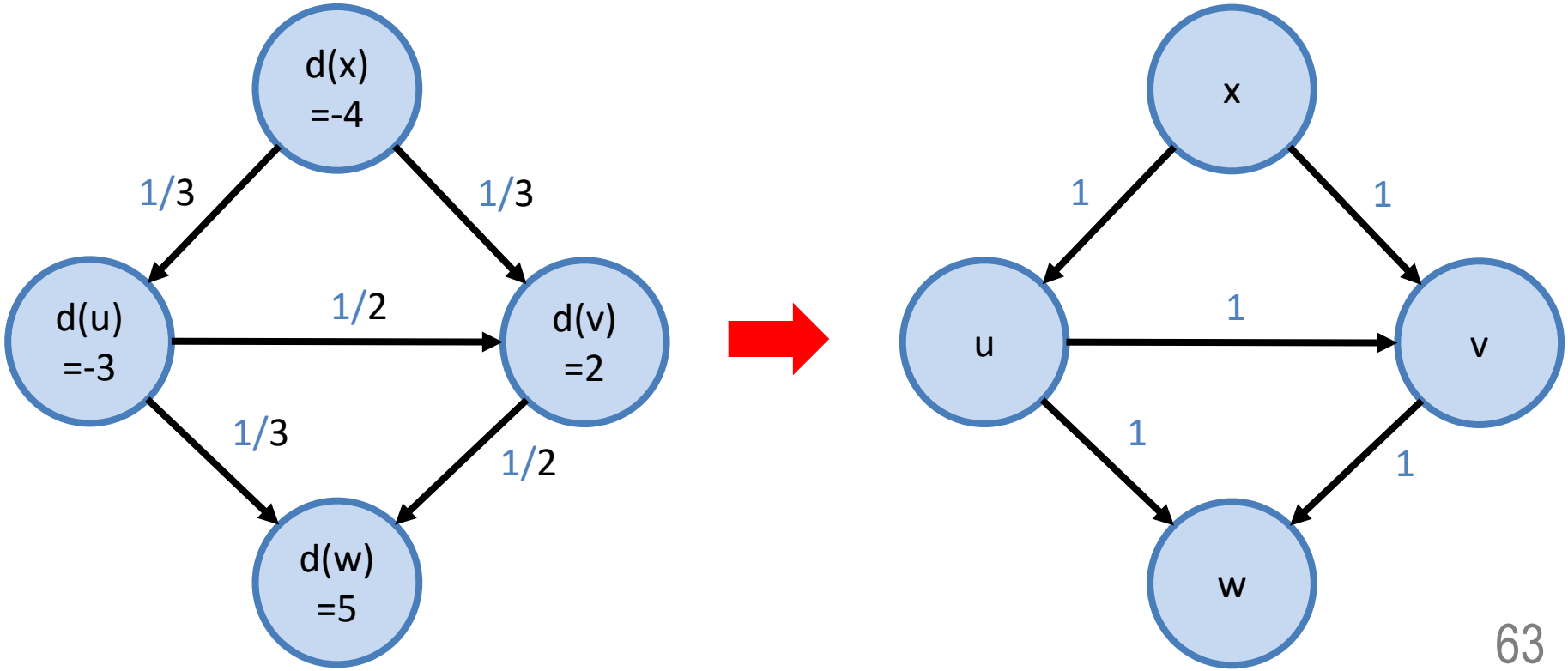
- We will need to make some transformation by removing the lower bound to a temp network…



Circulation with Demands and Lower Bound

A Feasibility Problem…

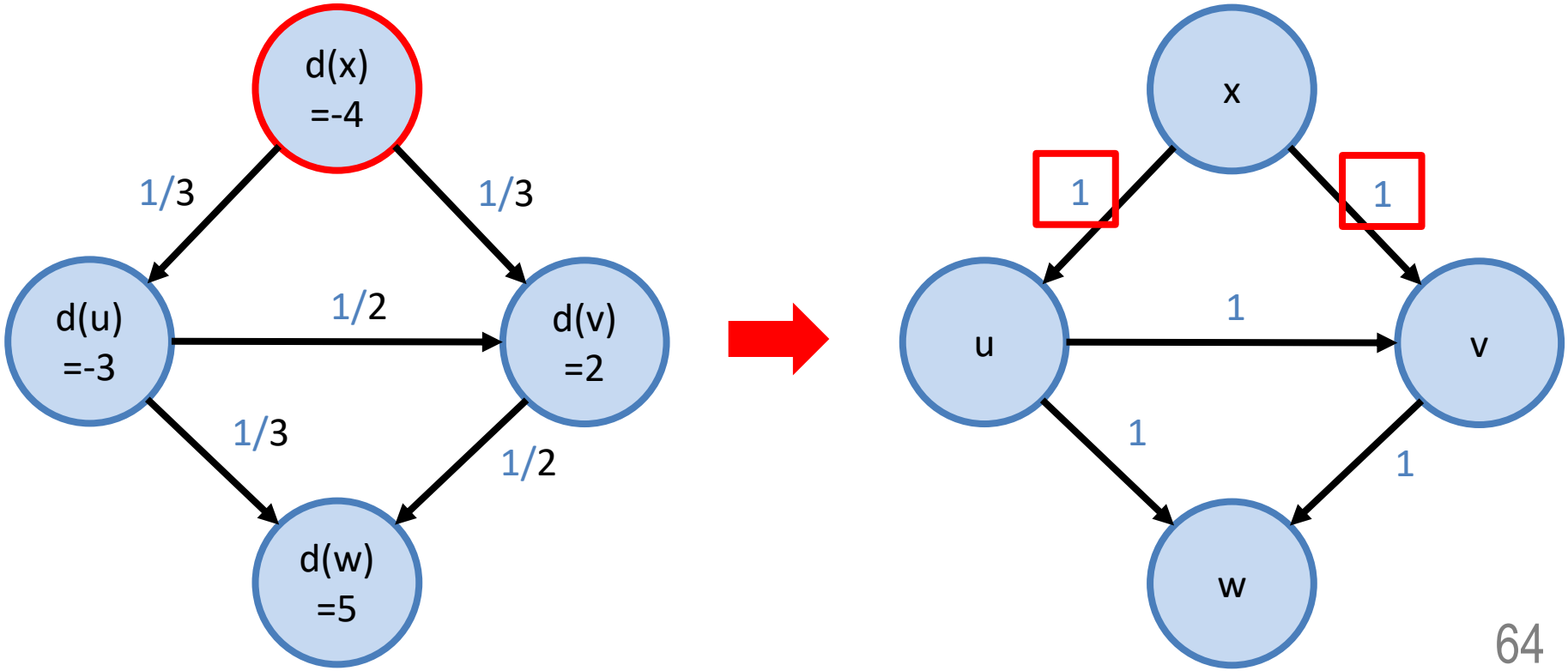
- We will need to make some transformation by removing the lower bound to a temp network…
- Thus, the original reduced…



Circulation with Demands and Lower Bound

A Feasibility Problem…

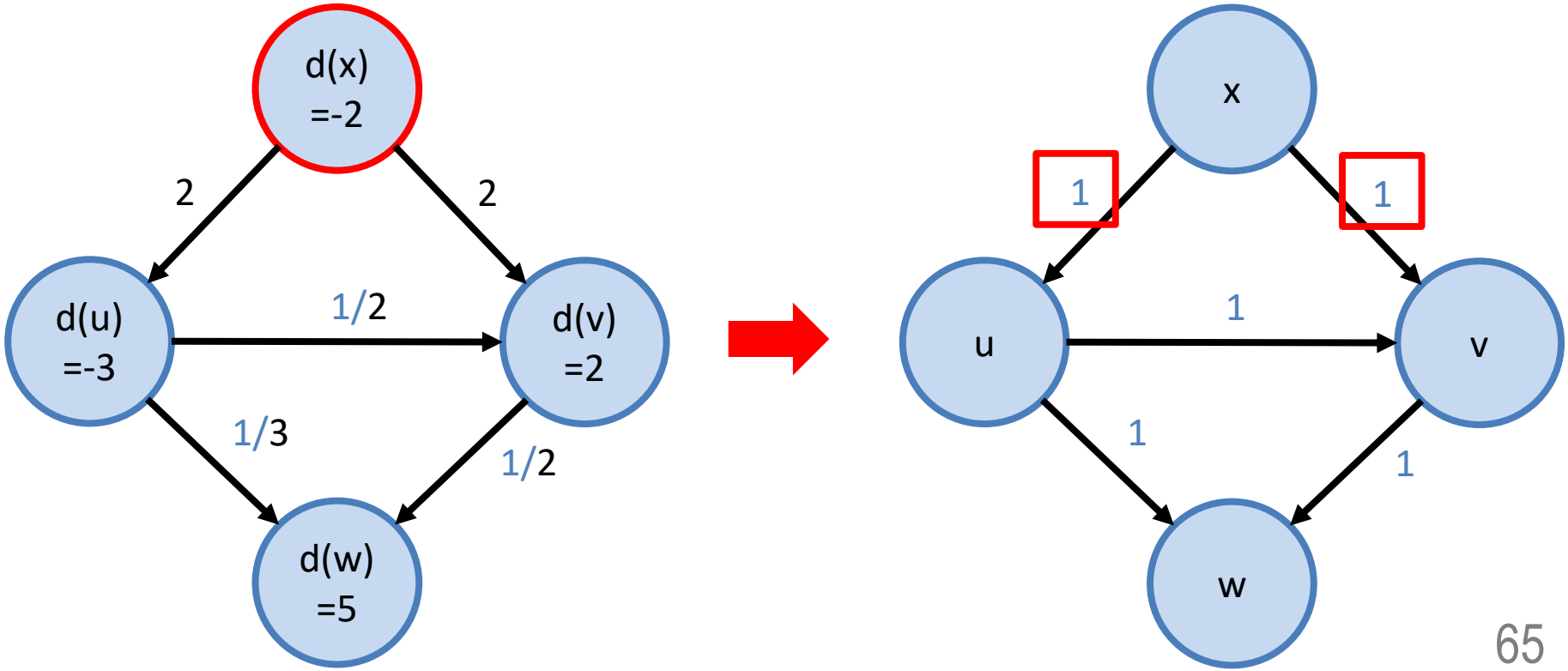
- We will need to make some transformation by removing the lower bound to a temp network…
- Thus, the original reduced…



Circulation with Demands and Lower Bound

A Feasibility Problem…

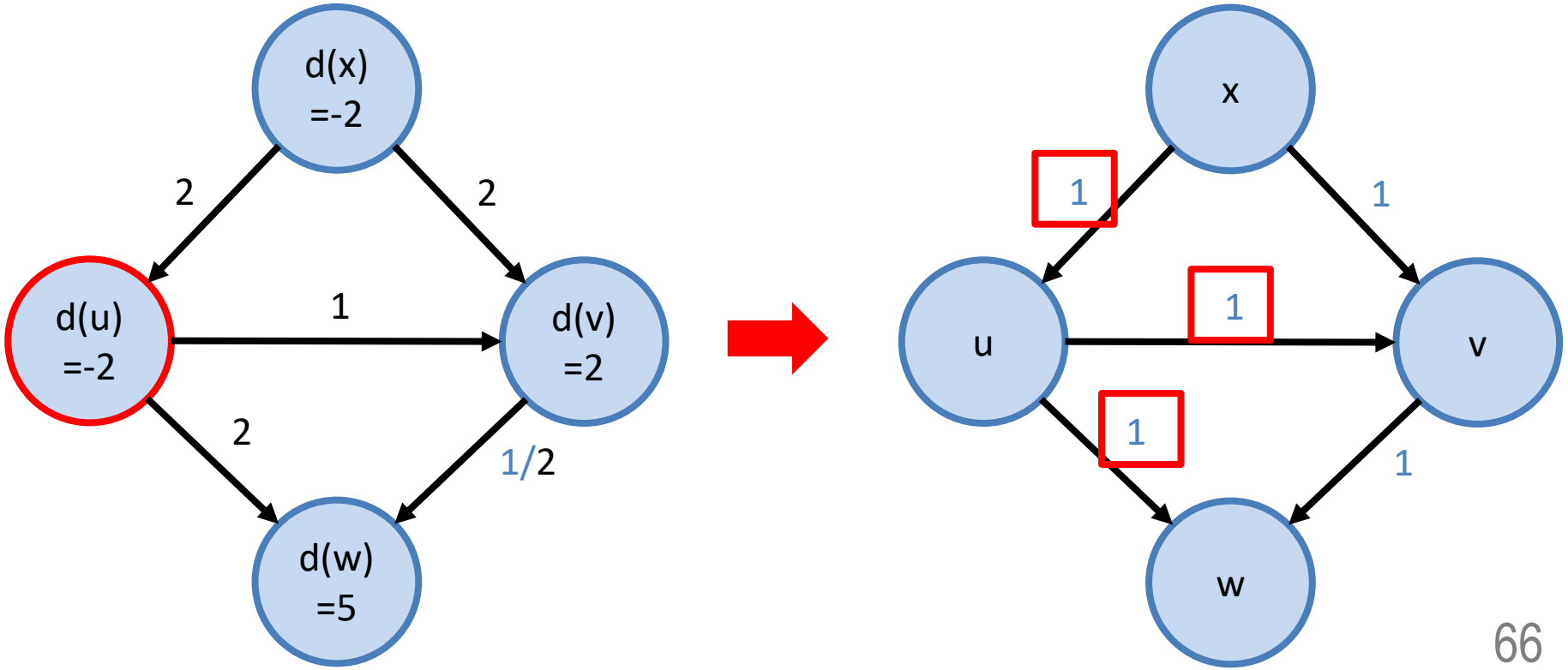
- We will need to make some transformation by removing the lower bound to a temp network…
- Thus, the original reduced…



Circulation with Demands and Lower Bound

A Feasibility Problem…

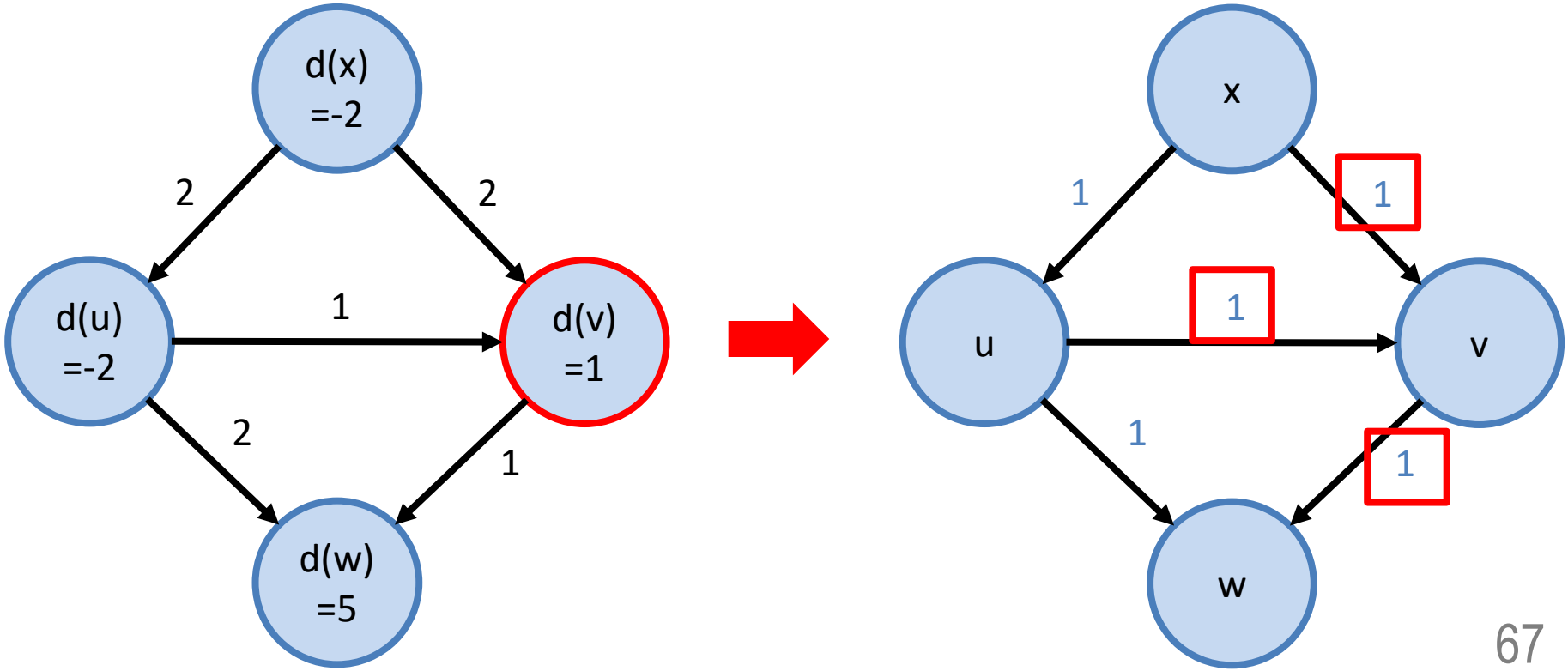
- We will need to make some transformation by removing the lower bound to a temp network…
- Thus, the original reduced…



Circulation with Demands and Lower Bound

A Feasibility Problem…

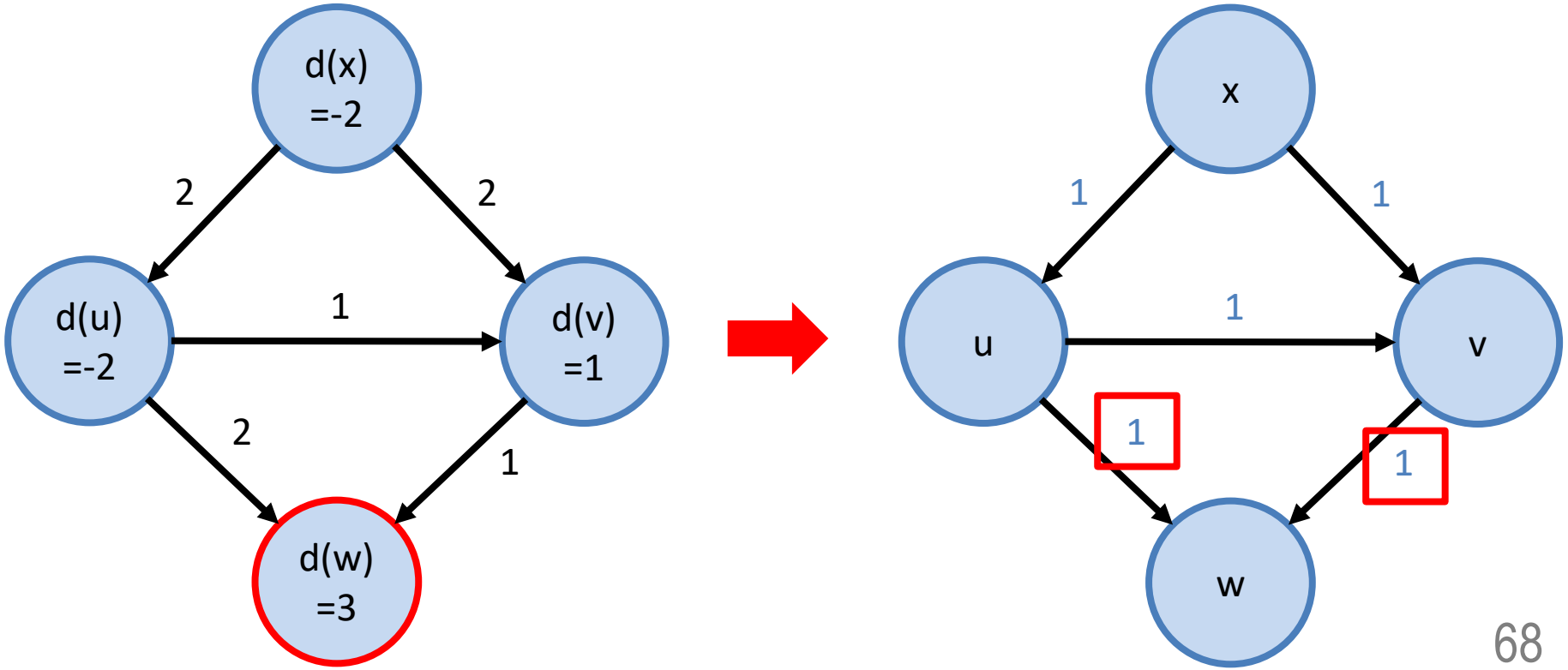
- We will need to make some transformation by removing the lower bound to a temp network…
- Thus, the original reduced…



Circulation with Demands and Lower Bound

A Feasibility Problem…

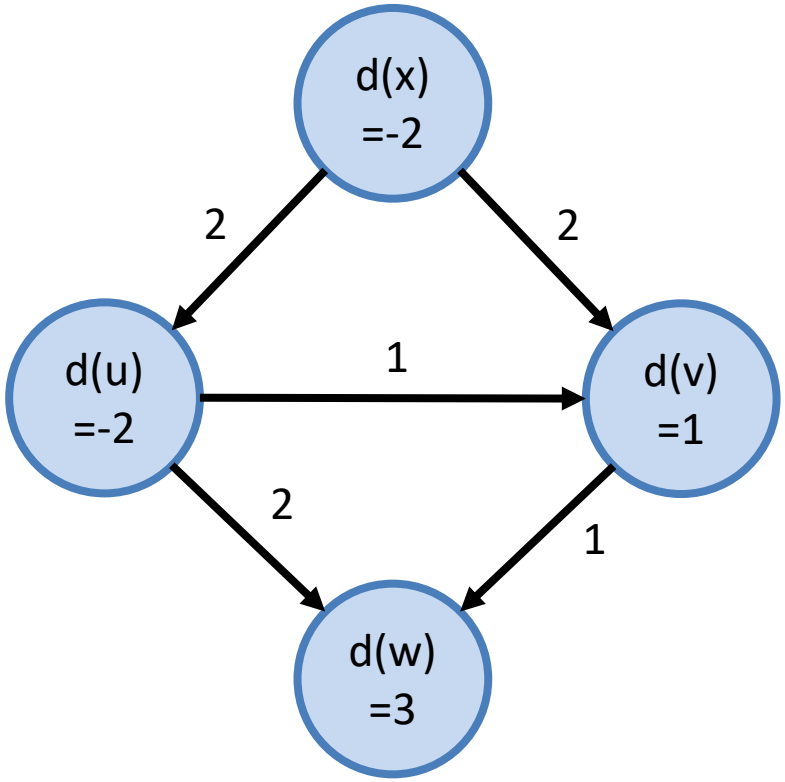
- We will need to make some transformation by removing the lower bound to a temp network…
- Thus, the original reduced…

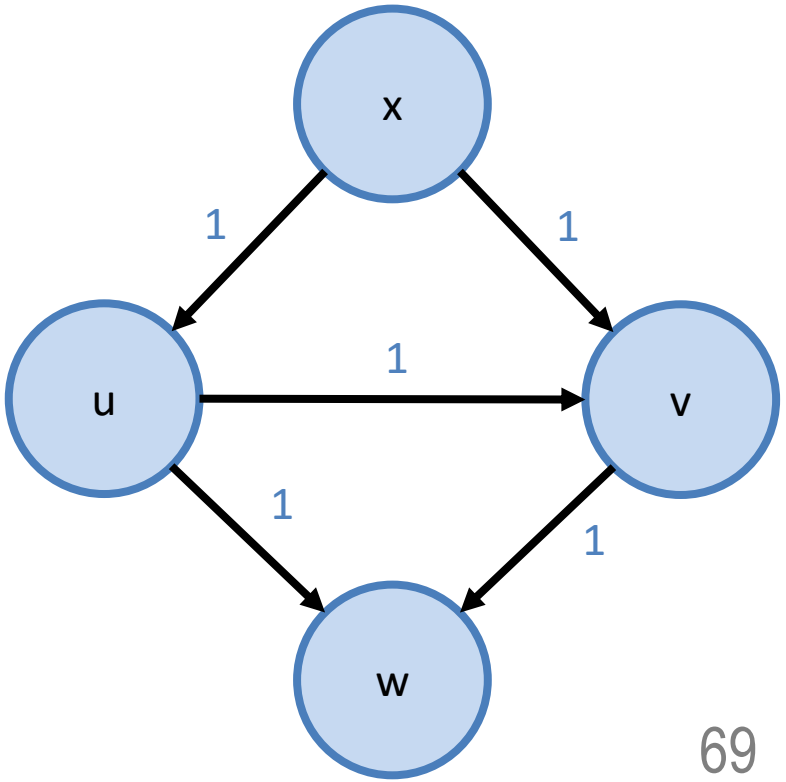


Circulation with Demands and Lower Bound

A Feasibility Problem…

- Thus, the original reduced…

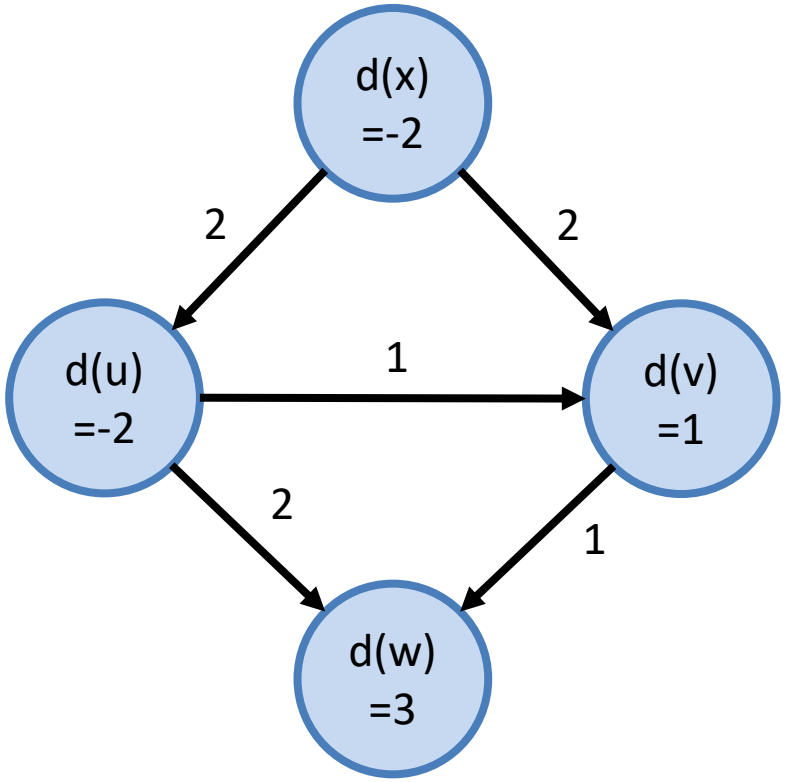


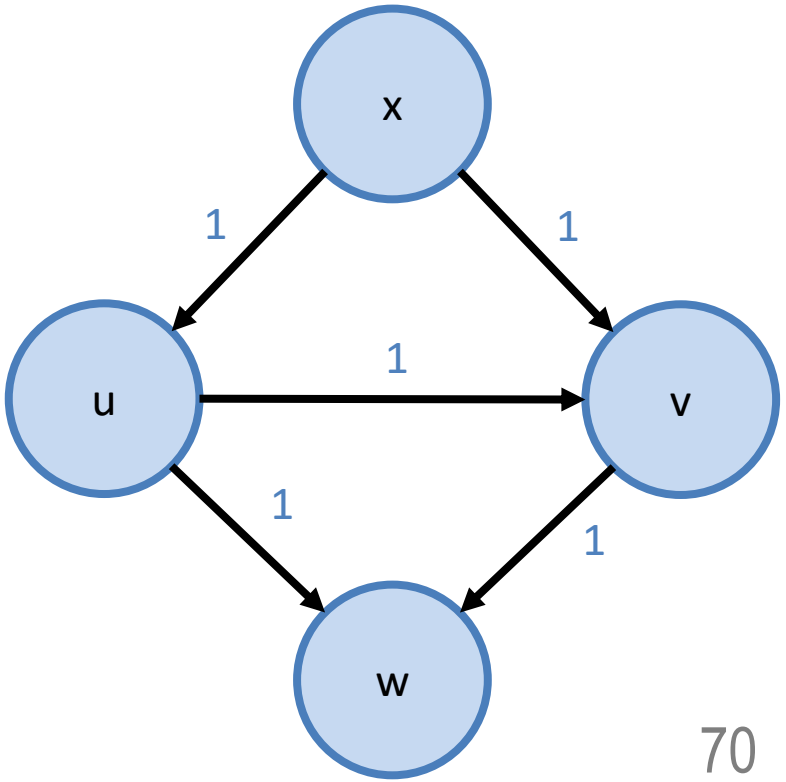


Circulation with Demands and Lower Bound

A Feasibility Problem…

- Thus, the original reduced…





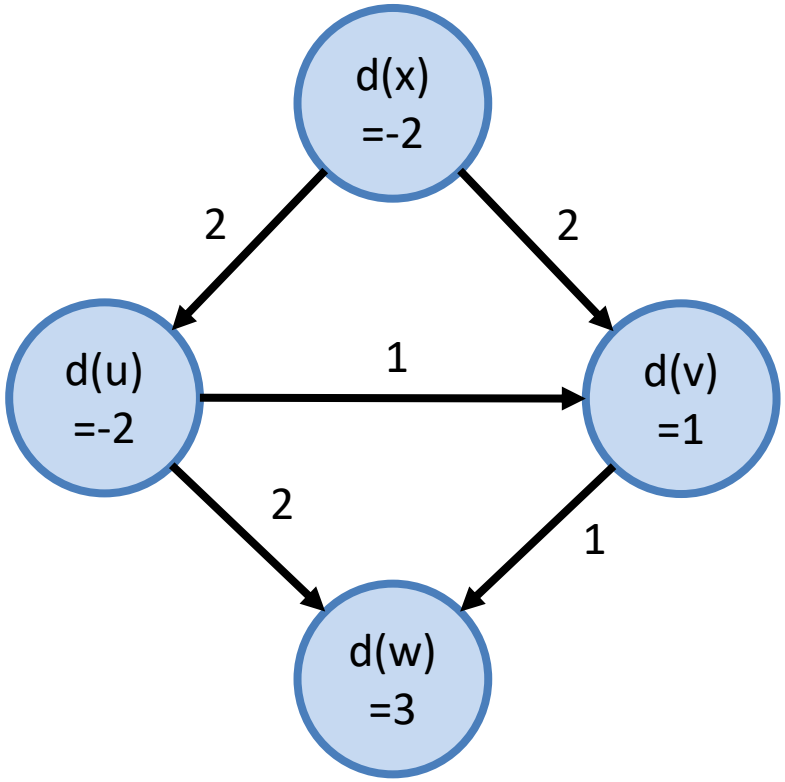
MONASH University

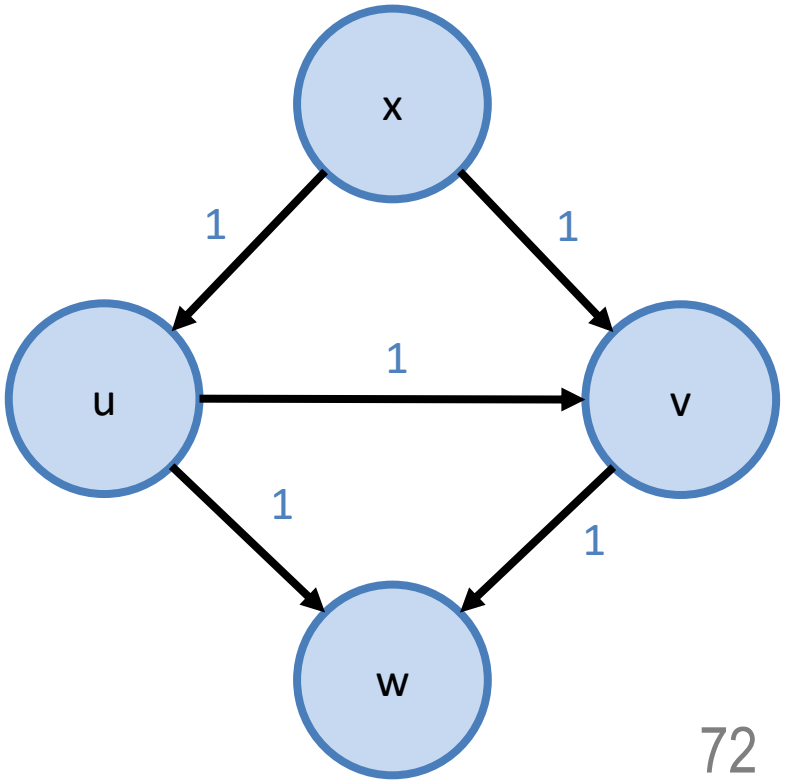
Questions?

Circulation with Demands and Lower Bound

A Feasibility Problem…

- Then we follow the same Circulation with Demands as earlier for the reduced network…

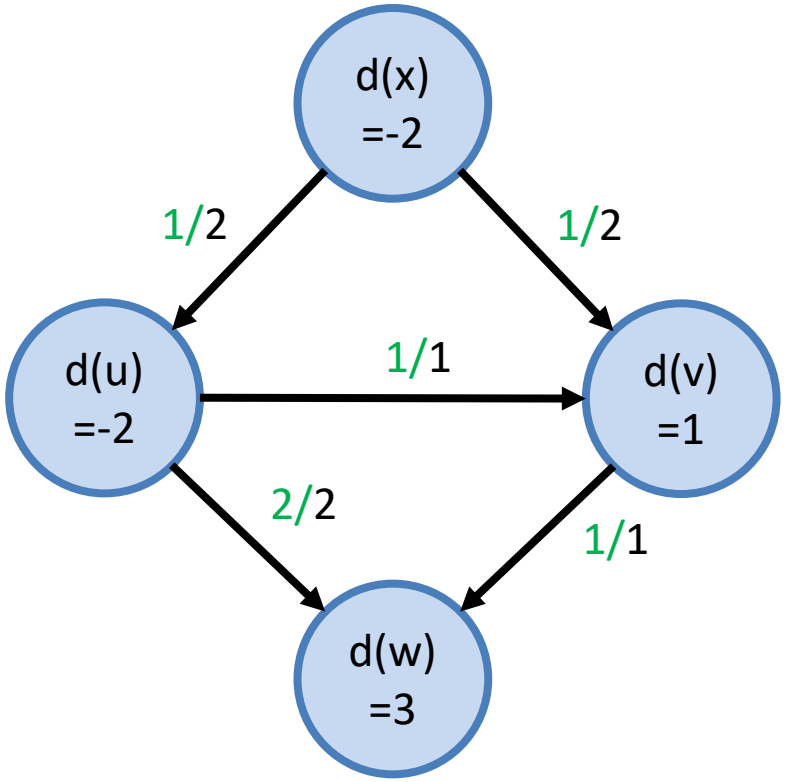


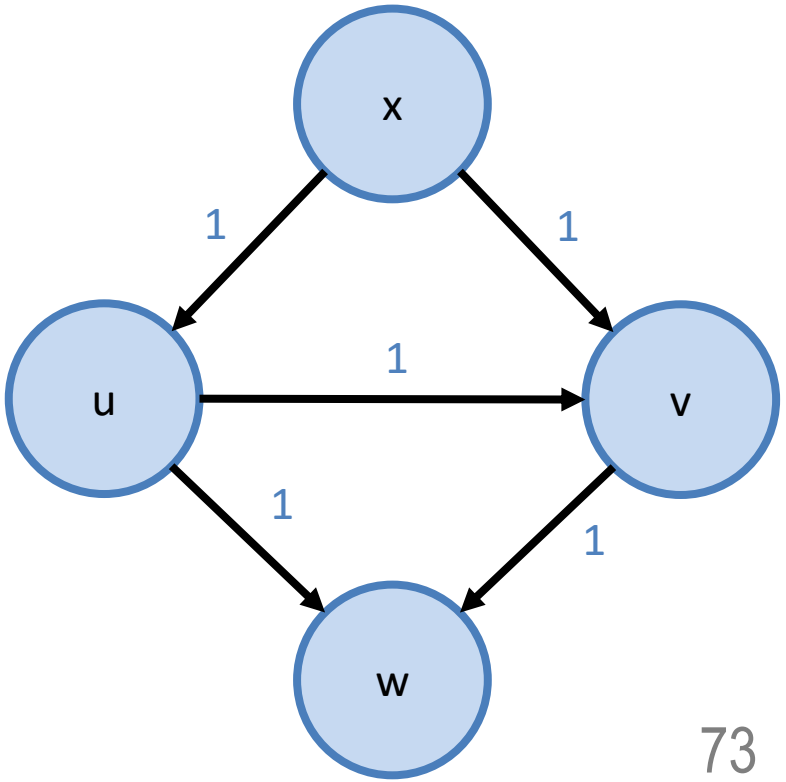


Circulation with Demands and Lower Bound

A Feasibility Problem…

- Then we follow the same Circulation with Demands as earlier for the reduced network…

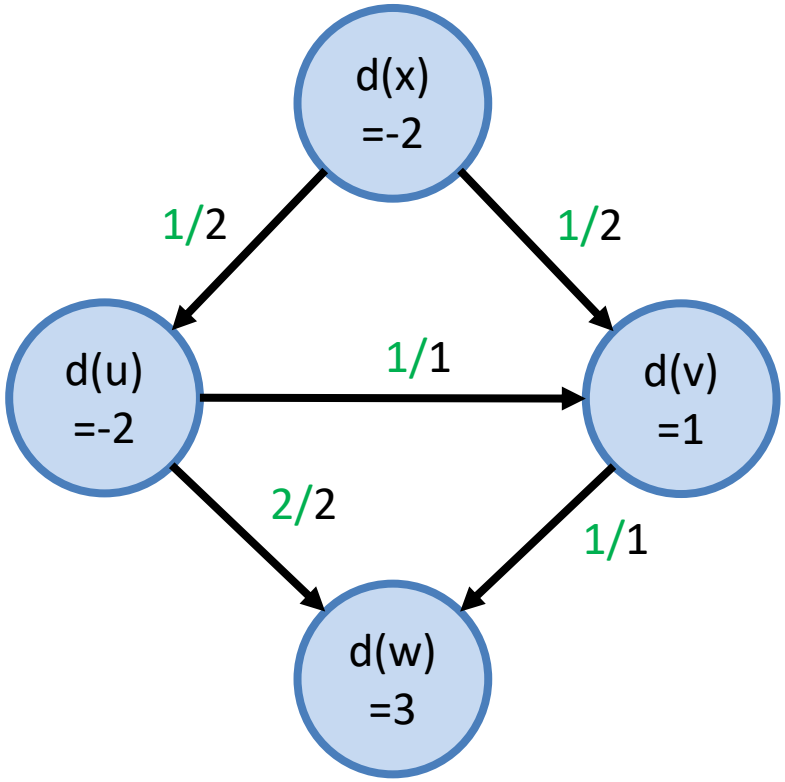


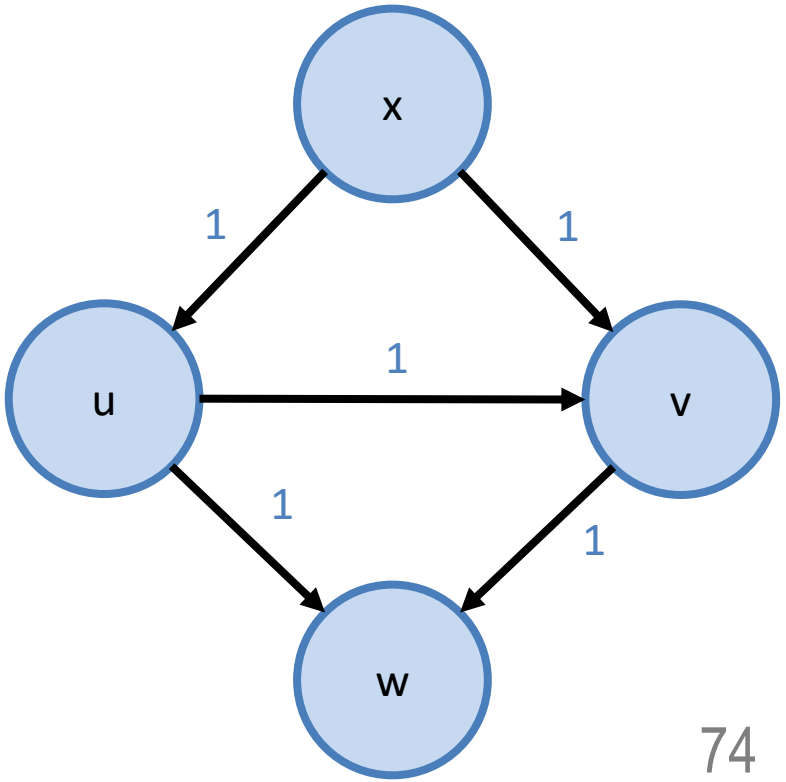


Circulation with Demands and Lower Bound

A Feasibility Problem…

- Then we follow the same Circulation with Demands as earlier for the reduced network… It is feasible!

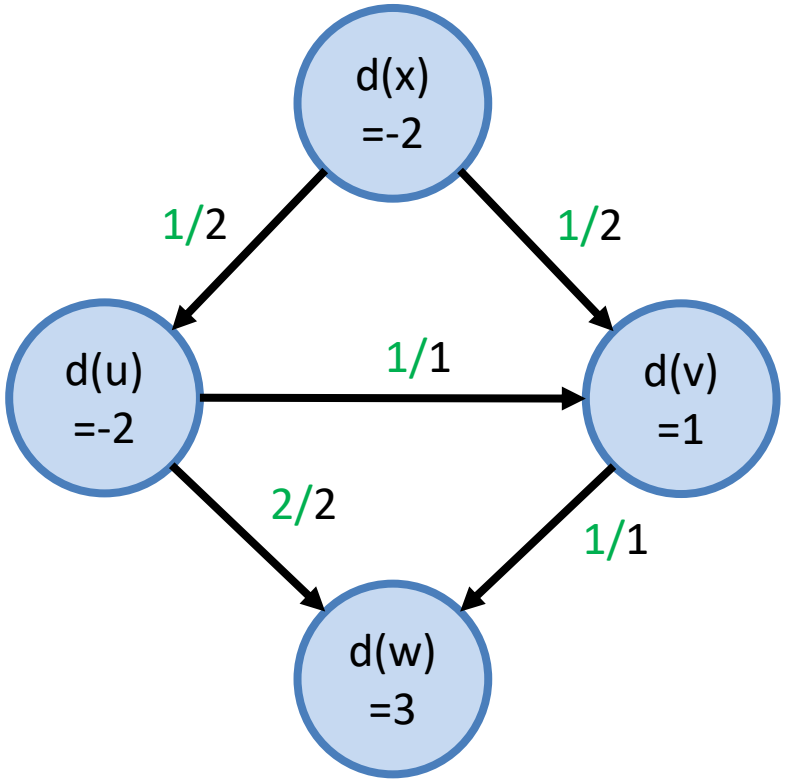


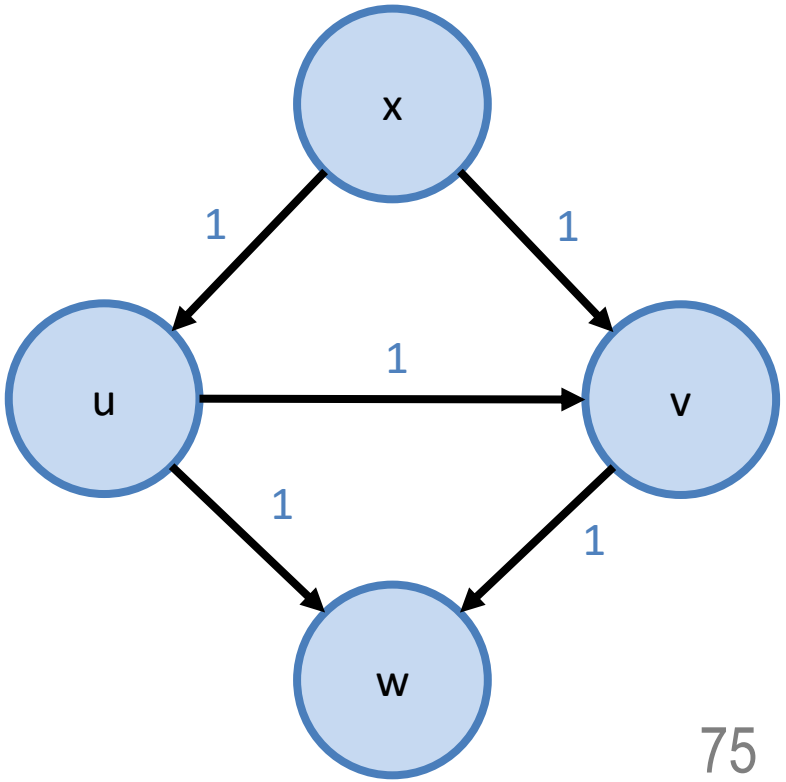


Circulation with Demands and Lower Bound

A Feasibility Problem…

- We finally combine them back but with the additional information using constraint/flow/capacity

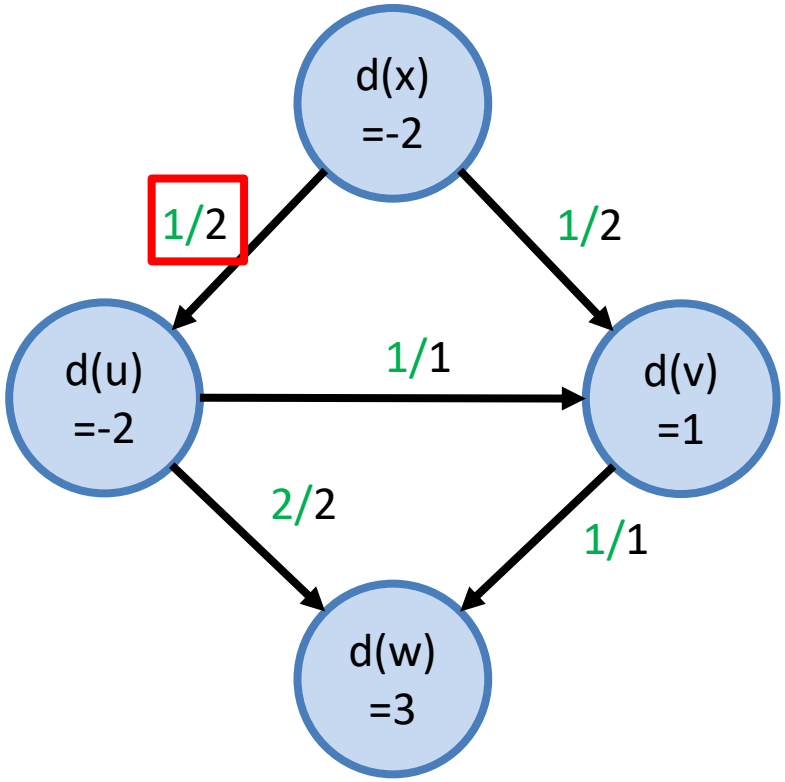


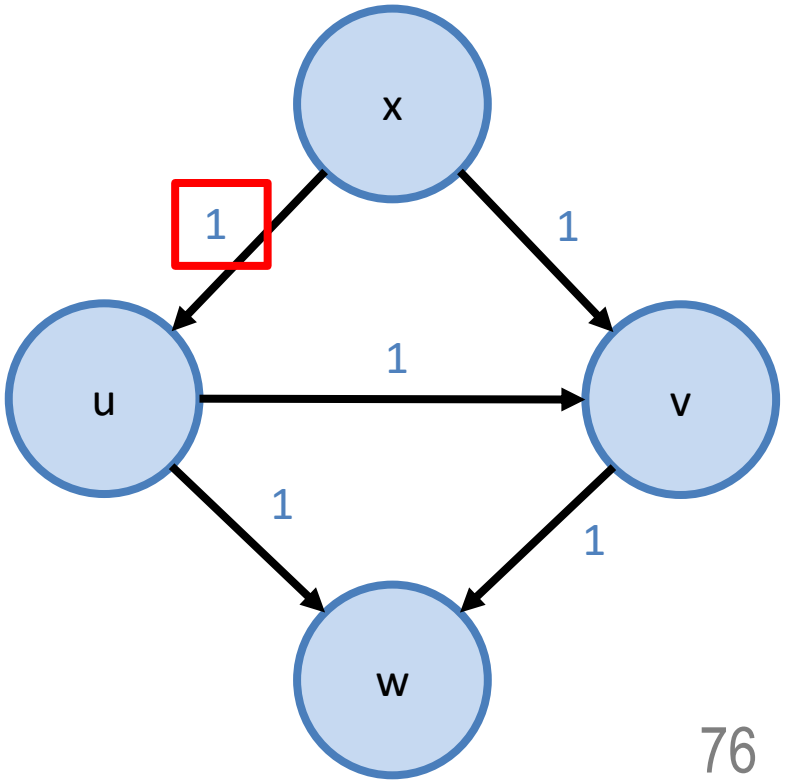


Circulation with Demands and Lower Bound

A Feasibility Problem…

- We finally combine them back but with the additional information using constraint/flow/capacity

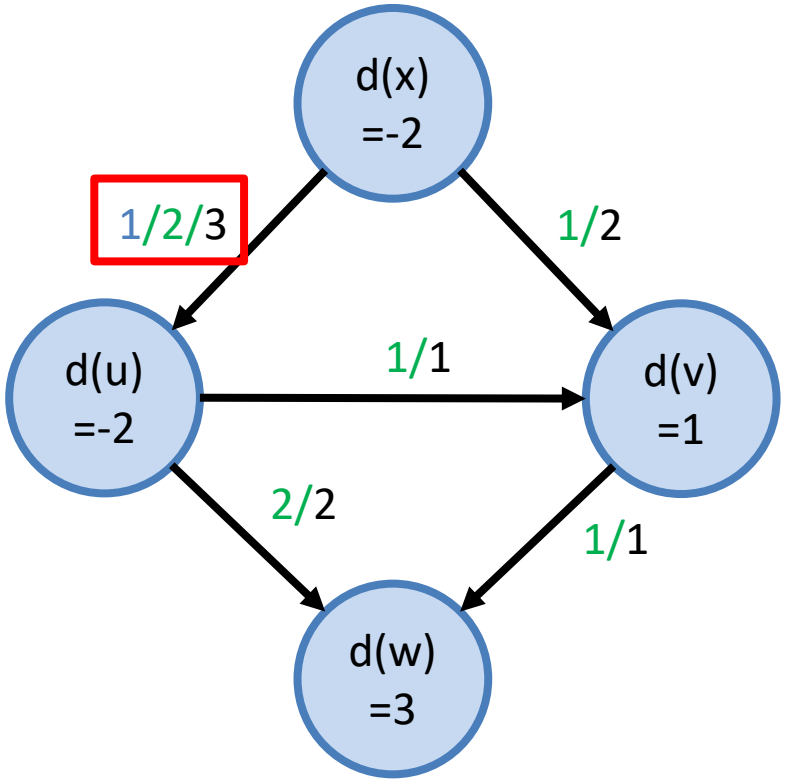


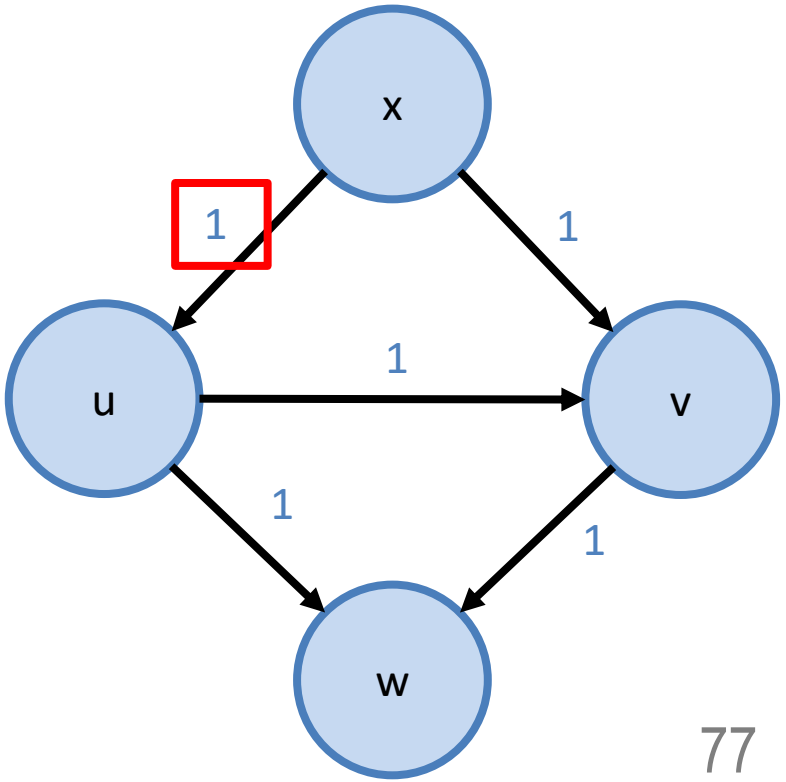


Circulation with Demands and Lower Bound

A Feasibility Problem…

- We finally combine them back but with the additional information using constraint/flow/capacity

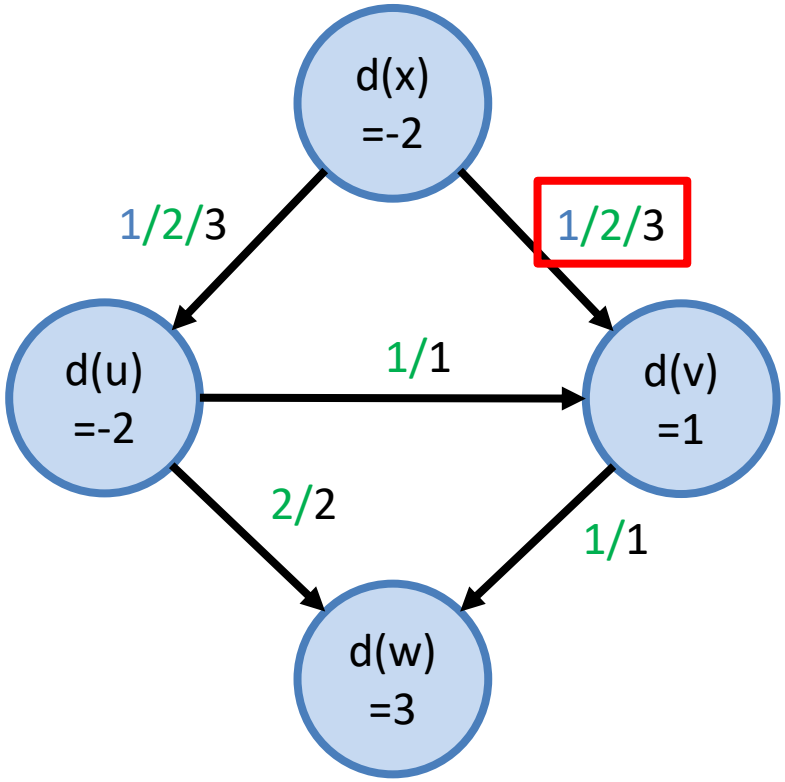


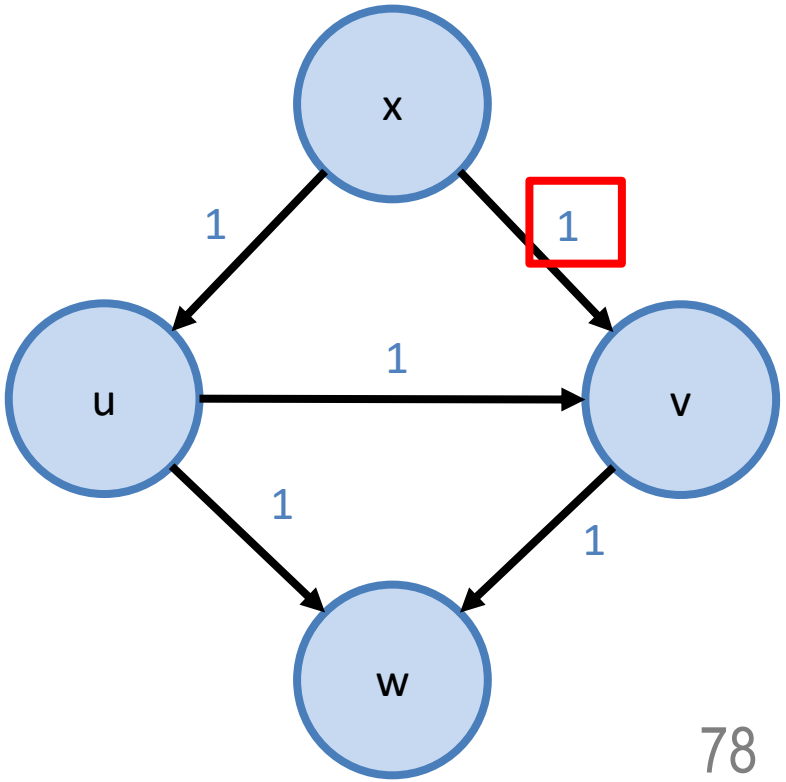


Circulation with Demands and Lower Bound

A Feasibility Problem…

- We finally combine them back but with the additional information using constraint/flow/capacity

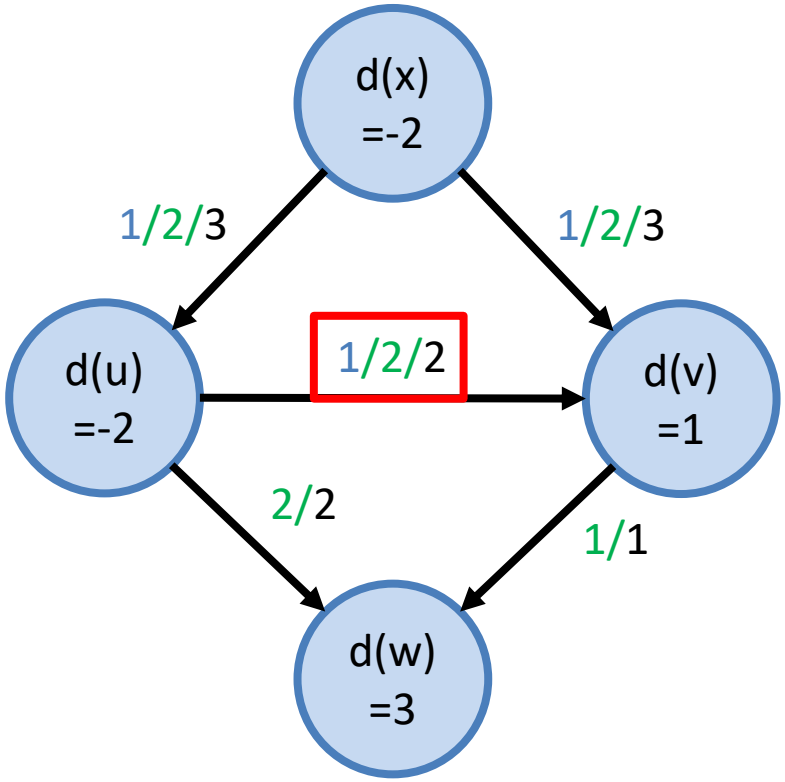


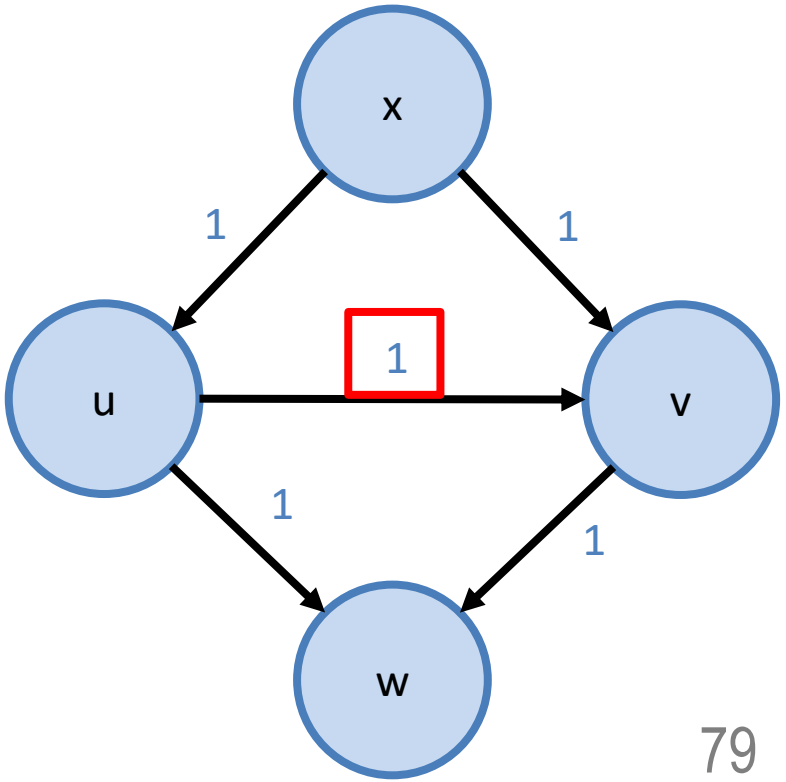


Circulation with Demands and Lower Bound

A Feasibility Problem…

- We finally combine them back but with the additional information using constraint/flow/capacity

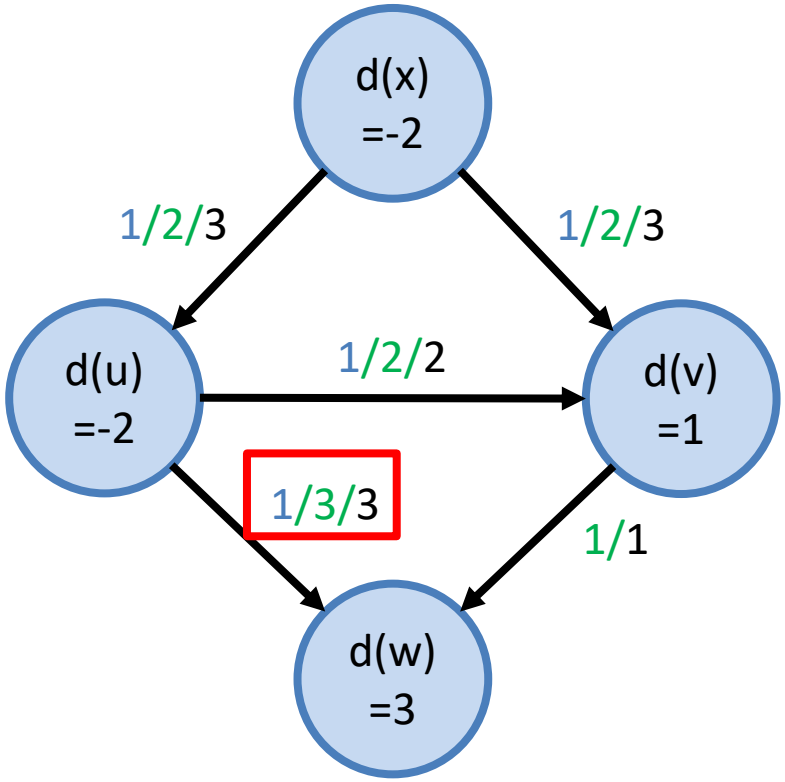


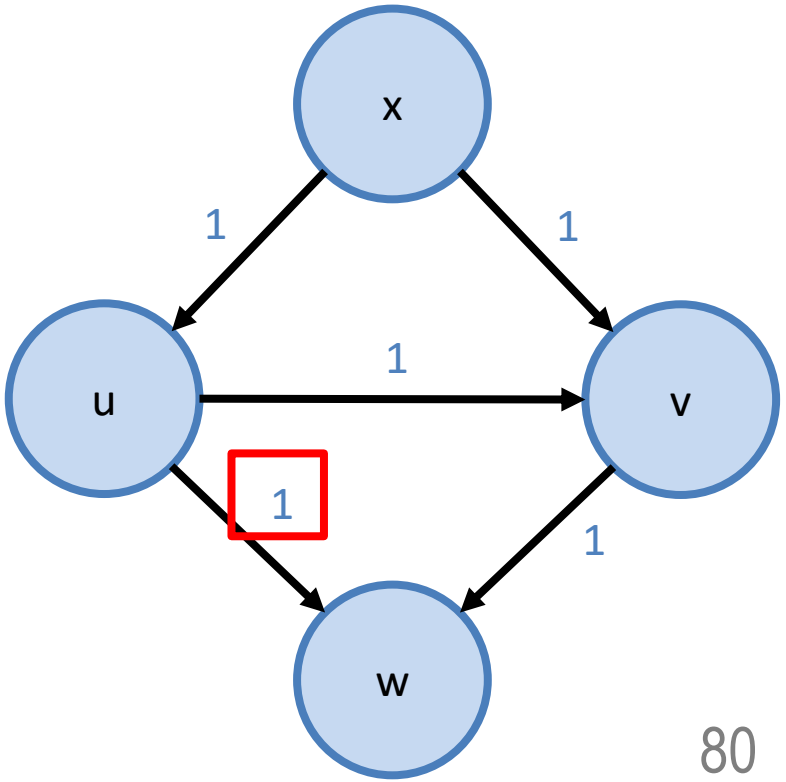


Circulation with Demands and Lower Bound

A Feasibility Problem…

- We finally combine them back but with the additional information using constraint/flow/capacity

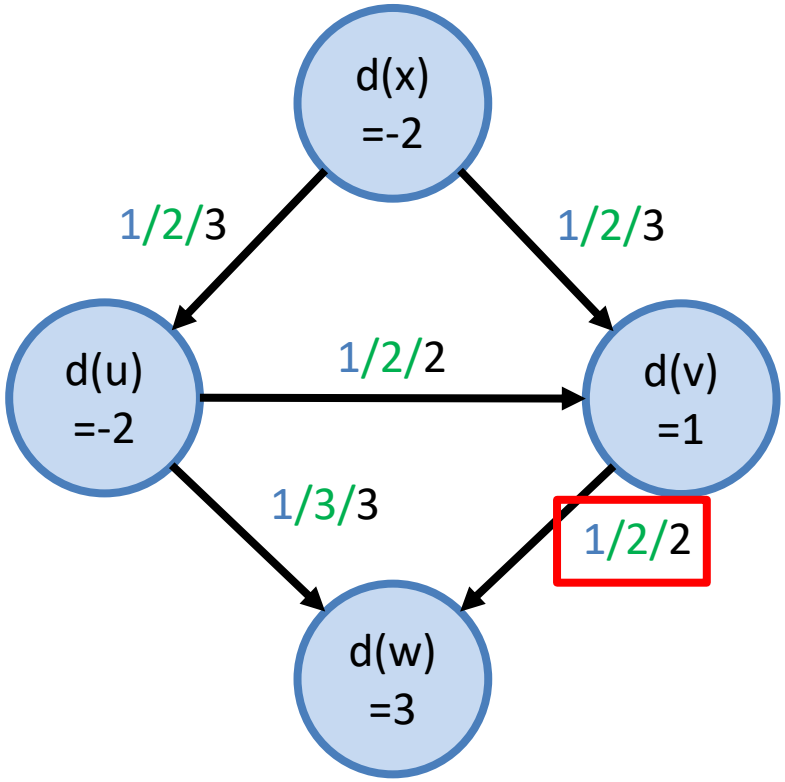


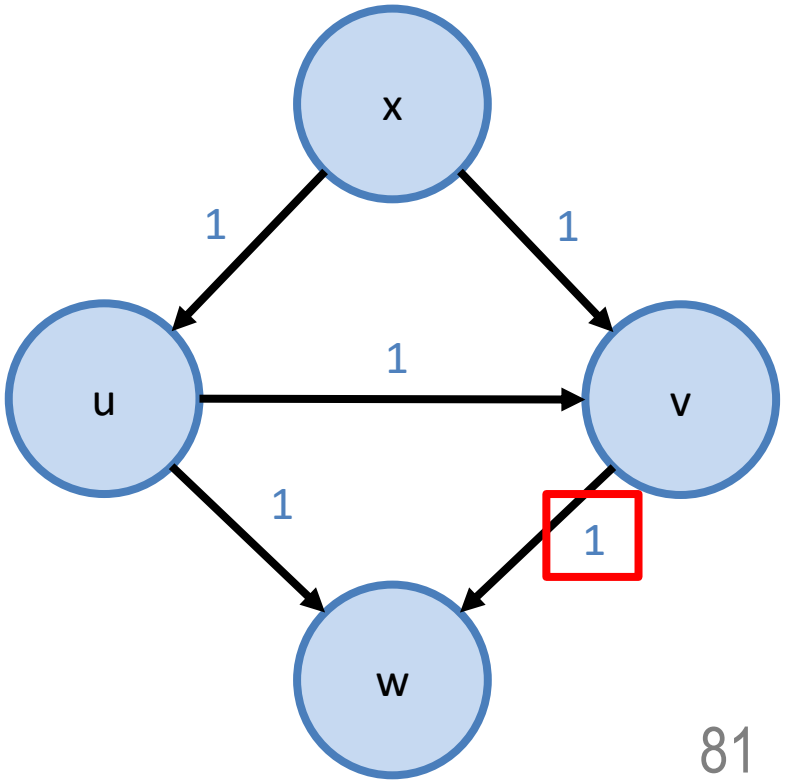


Circulation with Demands and Lower Bound

A Feasibility Problem…

- We finally combine them back but with the additional information using constraint/flow/capacity

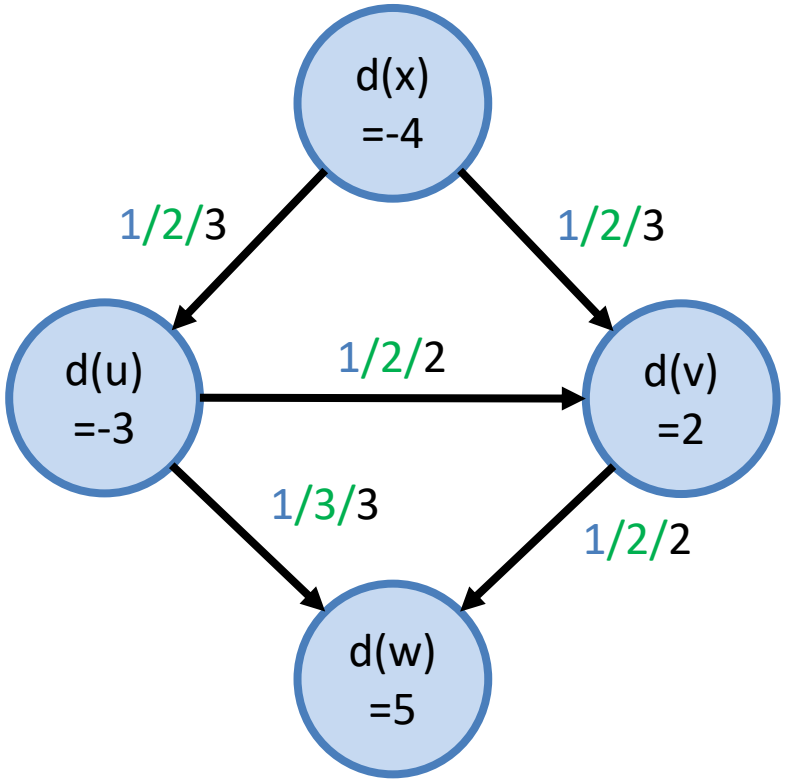


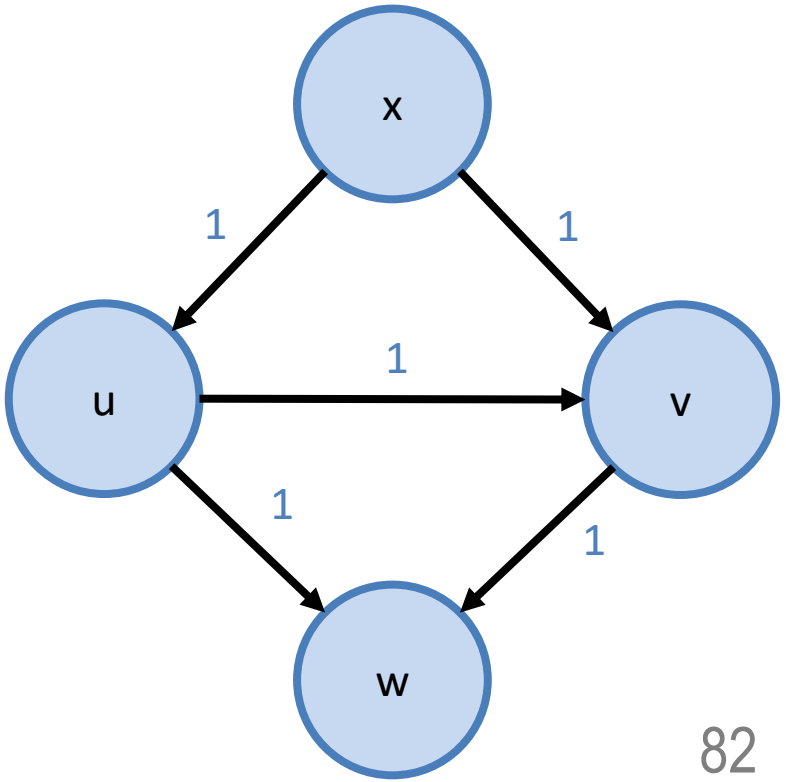


Circulation with Demands and Lower Bound

A Feasibility Problem…

- Don’t forget the demand of the vertices as well!

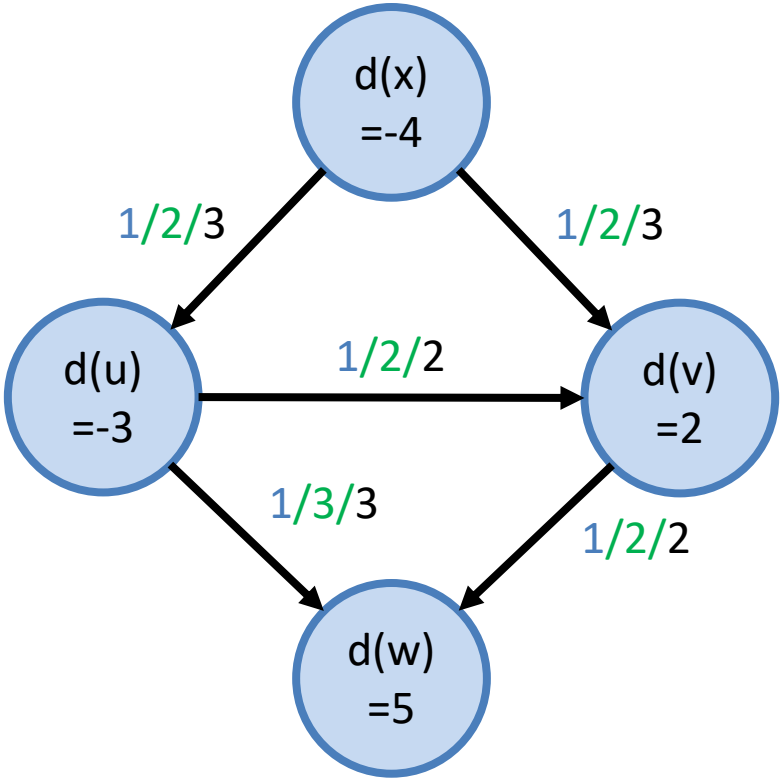




Circulation with Demands and Lower Bound

A Feasibility Problem…

- And we are done!



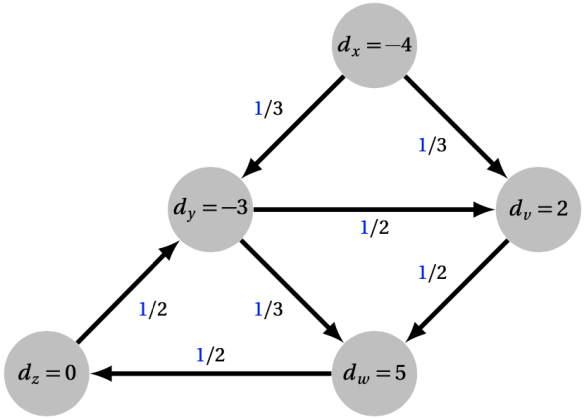
MONASH University

Questions?

Circulation with Demands and Lower Bound

A Feasibility Problem…

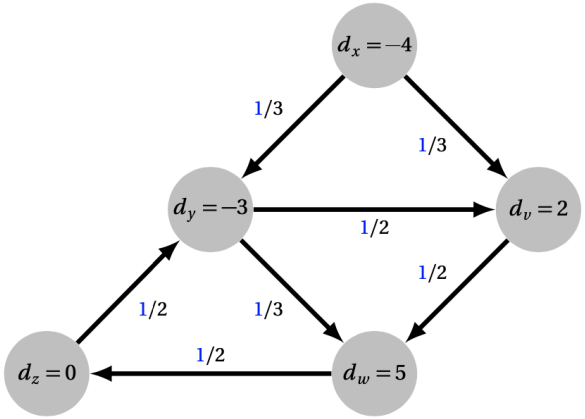
- The following is an example from the Clayton campus



Circulation with Demands and Lower Bound

A Feasibility Problem…

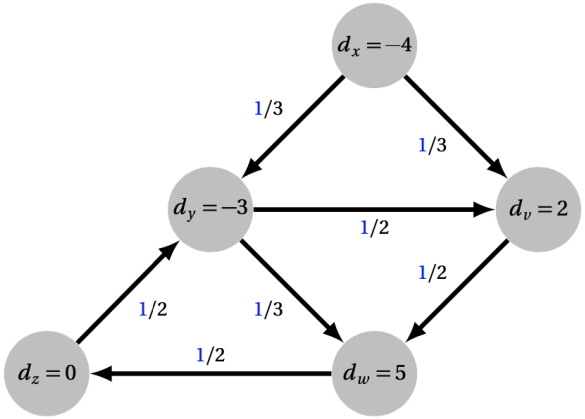
- The following is an example from the Clayton campus
- Work it out on your own to see if it is feasible before we discuss in class

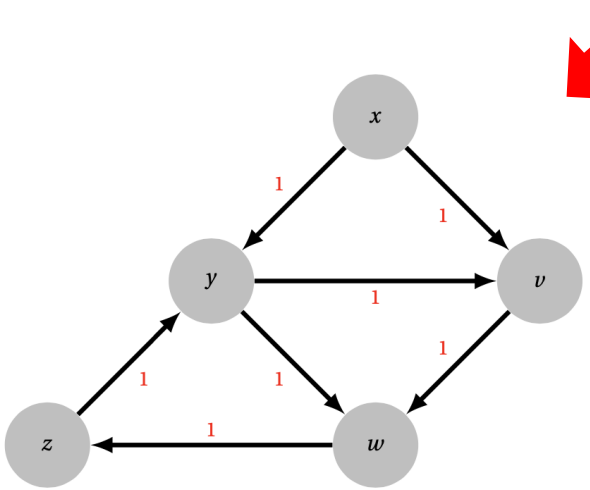


Circulation with Demands and Lower Bound

A Feasibility Problem…

- The following is an example from the Clayton campus
- Work it out on your own to see if it is feasible before we discuss in class

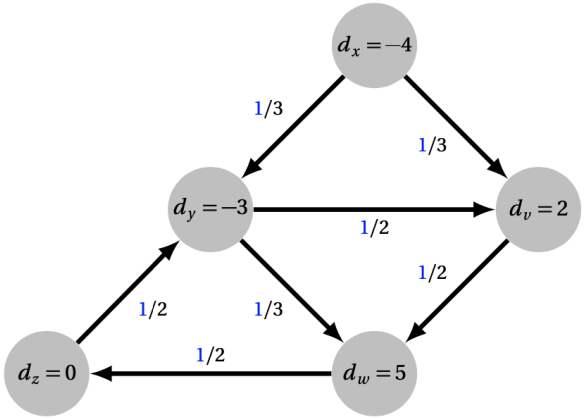


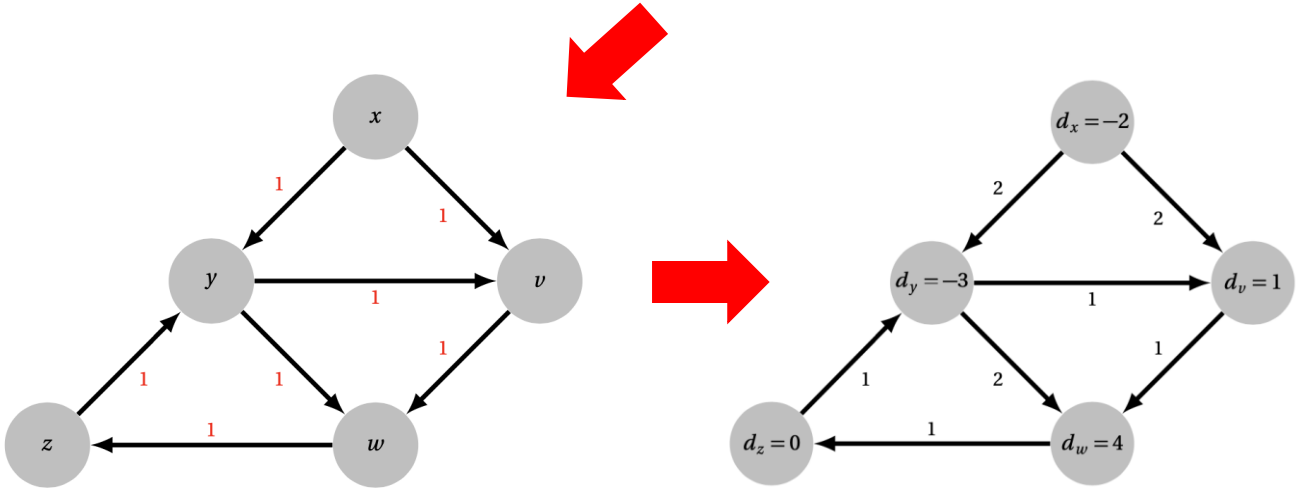


Circulation with Demands and Lower Bound

A Feasibility Problem…

- The following is an example from the Clayton campus
- Work it out on your own to see if it is feasible before we discuss in class.

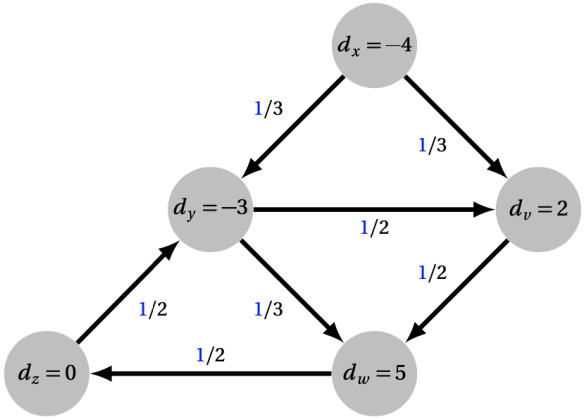


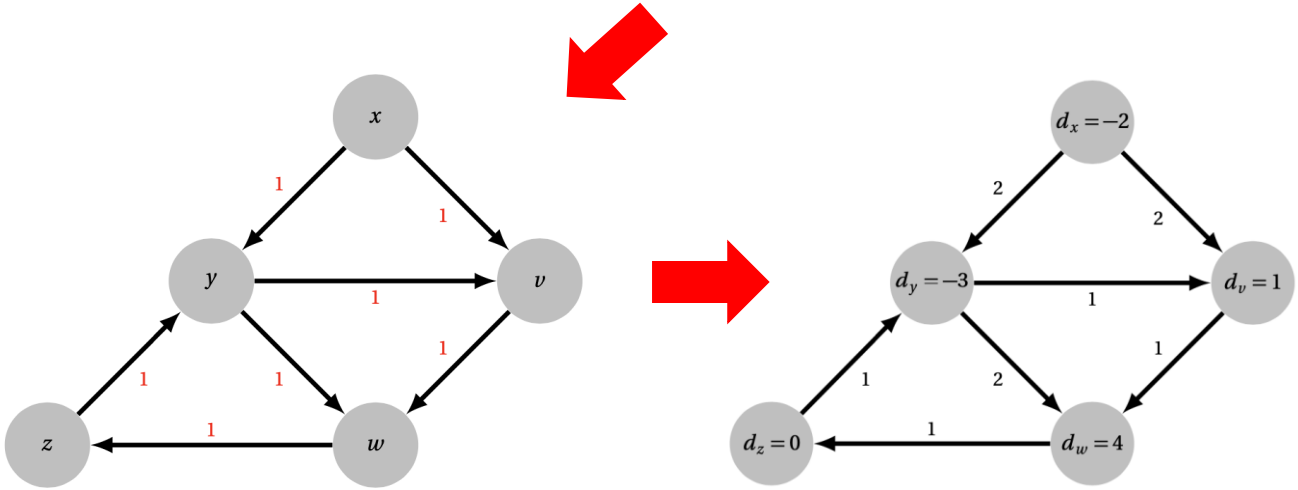


Circulation with Demands and Lower Bound

A Feasibility Problem…

- The following is an example from the Clayton campus
- Work it out on your own to see if it is feasible before we discuss in class. It is it not!

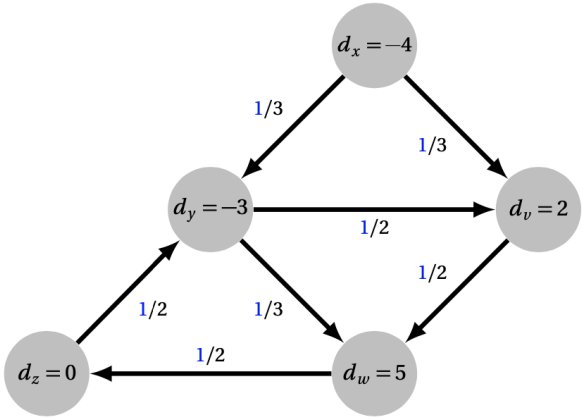


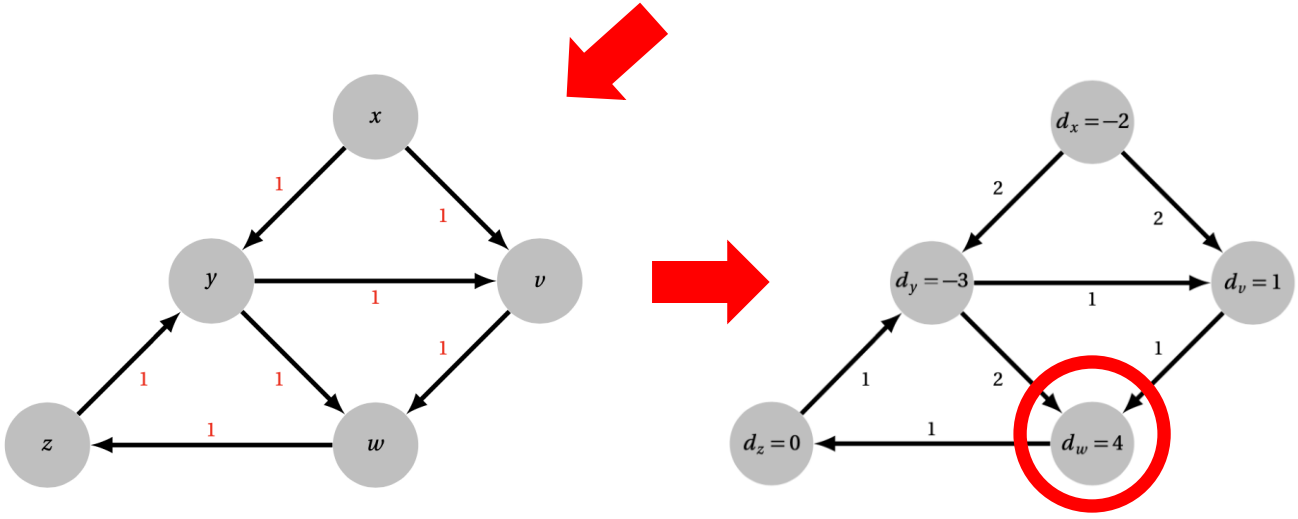


Circulation with Demands and Lower Bound

A Feasibility Problem…

- The following is an example from the Clayton campus
- Work it out on your own to see if it is feasible before we discuss in class. It is it not!





MONASH University

Questions?

Applications with Network Flow

For Real-World Problems

MONASH University

- We know what is a flow network.
- We know how to design flow network for bipartite matching.
- We know how to design flow network for circulation with demands and lower bound?

Applications with Network Flow

For Real-World Problems

MONASH University

- We know what is a flow network.
- We know how to design flow network for bipartite matching.
- We know how to design flow network for circulation with demands and lower bound? We shall see them now…

Applications with Network Flow

For Real-World Problems

MONASH University

- We know what is a flow network.
- We know how to design flow network for bipartite matching.
- We know how to design flow network for circulation with demands and lower bound? We shall see them now…

Applications with Network Flow

For Real-World Problems

MONASH University

- We know what is a flow network.
- We know how to design flow network for bipartite matching.
- We know how to design flow network for circulation with demands and lower bound? We shall see them now…

- Not that we only deal with integers, to make it simpler

MONASH University

Questions?

Applications with Network Flow

Survey Design

Applications with Network Flow

Survey Design

- You have C customers who have used the product
- You have P products

Applications with Network Flow

Survey Design

- You have C customers who have used the product
- You have P products

- You want to conduct a survey, but…

Applications with Network Flow

Survey Design

MONASH University

- You have C customers who have used the product
- You have P products

- You want to conduct a survey, but…
  - You do not want to ask the customer for too many reviews.
  - You do want to ask the customer for at least some reviews.
  - Each product needs to have at least some reviews.
  - Each product do not require more than some reviews.

Applications with Network Flow

Survey Design

MONASH University

- You have C customers who have used the product
- You have P products

- You want to conduct a survey, but…
  - You do not want to ask the customer for too many reviews.
  - You do want to ask the customer for at least some reviews.
  - Each product needs to have at least some reviews.
  - Each product do not require more than some reviews.
  - Of course, each customer can only give a review per product.

Applications with Network Flow

Survey Design

MONASH University

- You have C customers who have used the product
- You have P products

- You want to conduct a survey, but…
  - You do not want to ask the customer for too many reviews.
  - You do want to ask the customer for at least some reviews.
  - Each product needs to have at least some reviews.
  - Each product do not require more than some reviews.
  - Of course, each customer can only give a review per product.

- Let us add some notations

Applications with Network Flow

Survey Design

MONASH University

- You have C customers who have used the product , $c_1, c_2, c_3, \ldots, c_n$
- You have P products, $p_1, p_2, p_3, \ldots, p_m$

- You want to conduct a survey, but…
  - You do not want to ask the customer c_i for too many reviews c_i^+.
  - You do want to ask the customer c_i for at least some reviews c_i^-.
  - Each product p_j needs to have at least some reviews p_j^-.
  - Each product p_j do not require more than some reviews p_j^+.
  - Of course, each customer can only give a review per product.

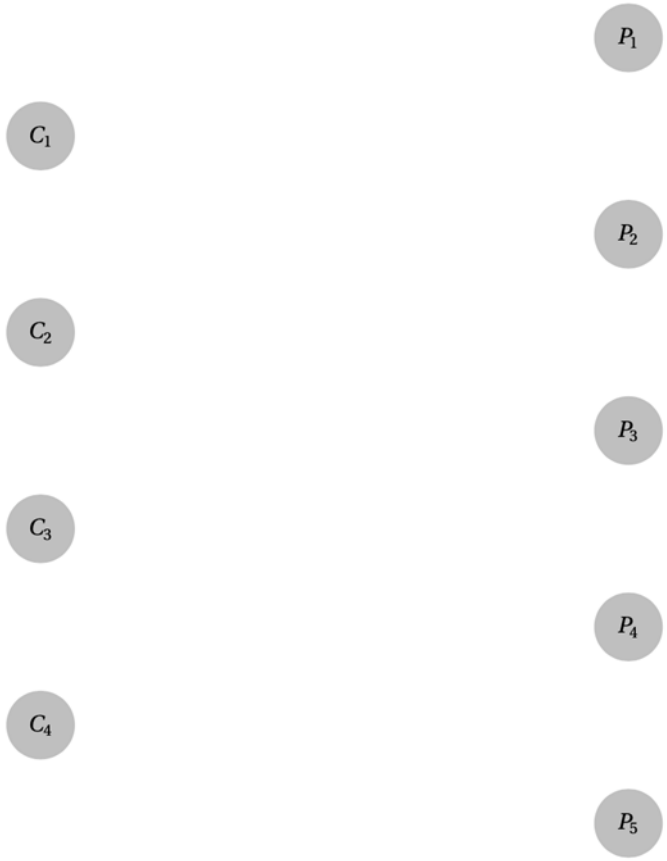
- Let us add some notations

- Now let us go through 1 by 1

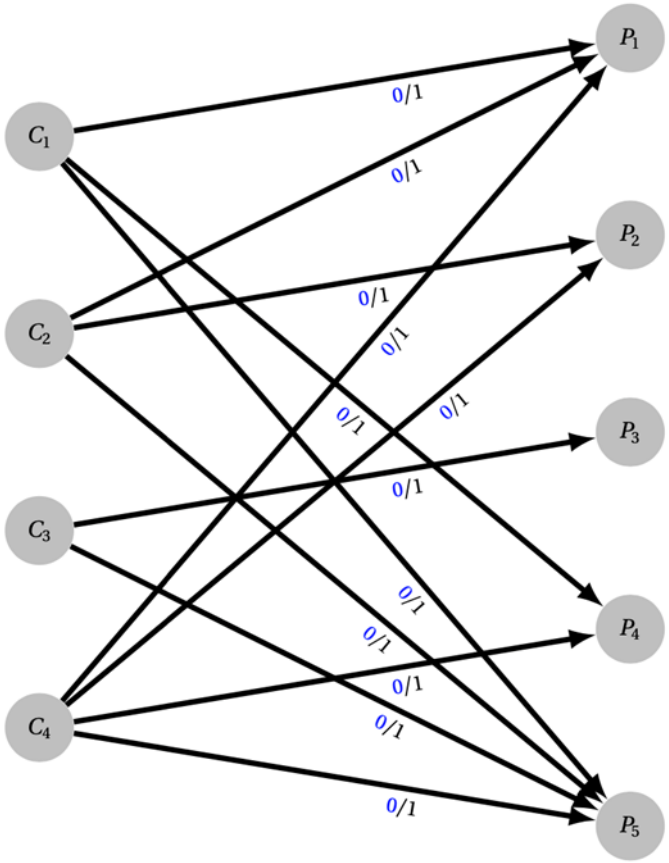
Applications with Network Flow

Survey Design

- The customer and the product.



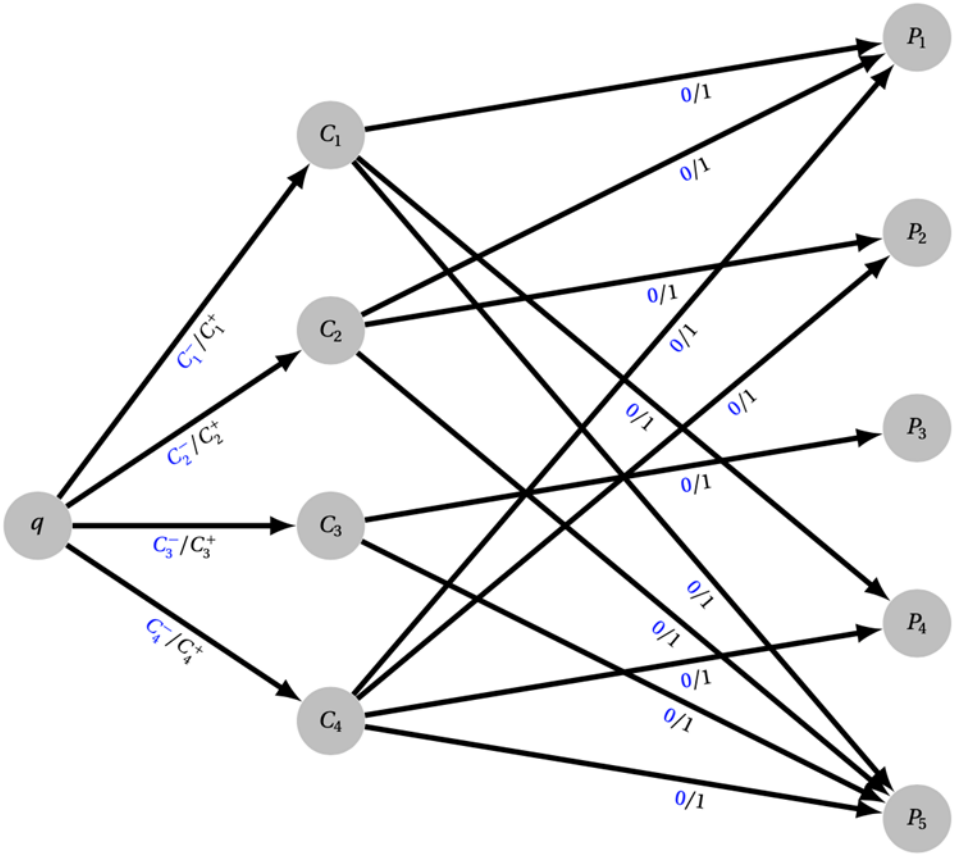
- Each customer can give 0 or 1 review for the product



Applications with Network Flow

Survey Design

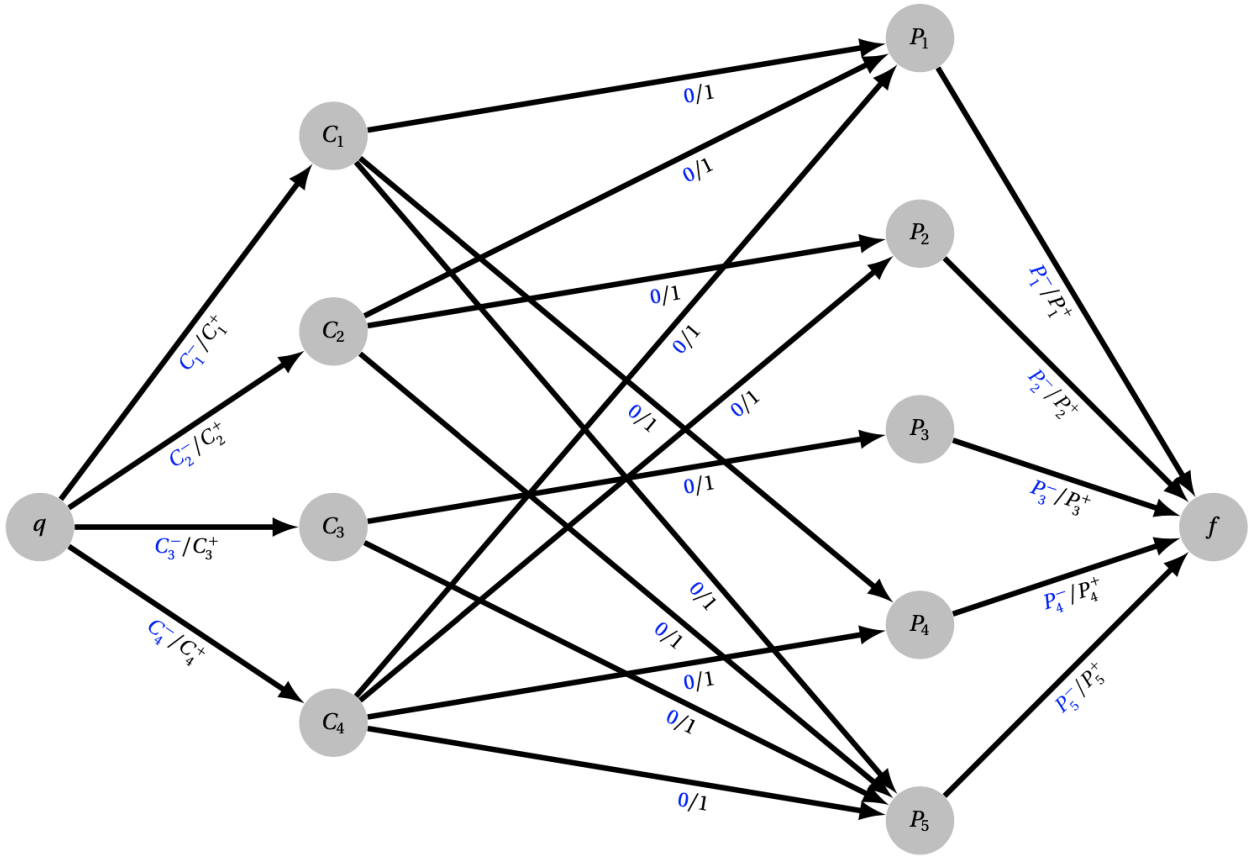
- There is range of review expected from the customer



Applications with Network Flow

Survey Design

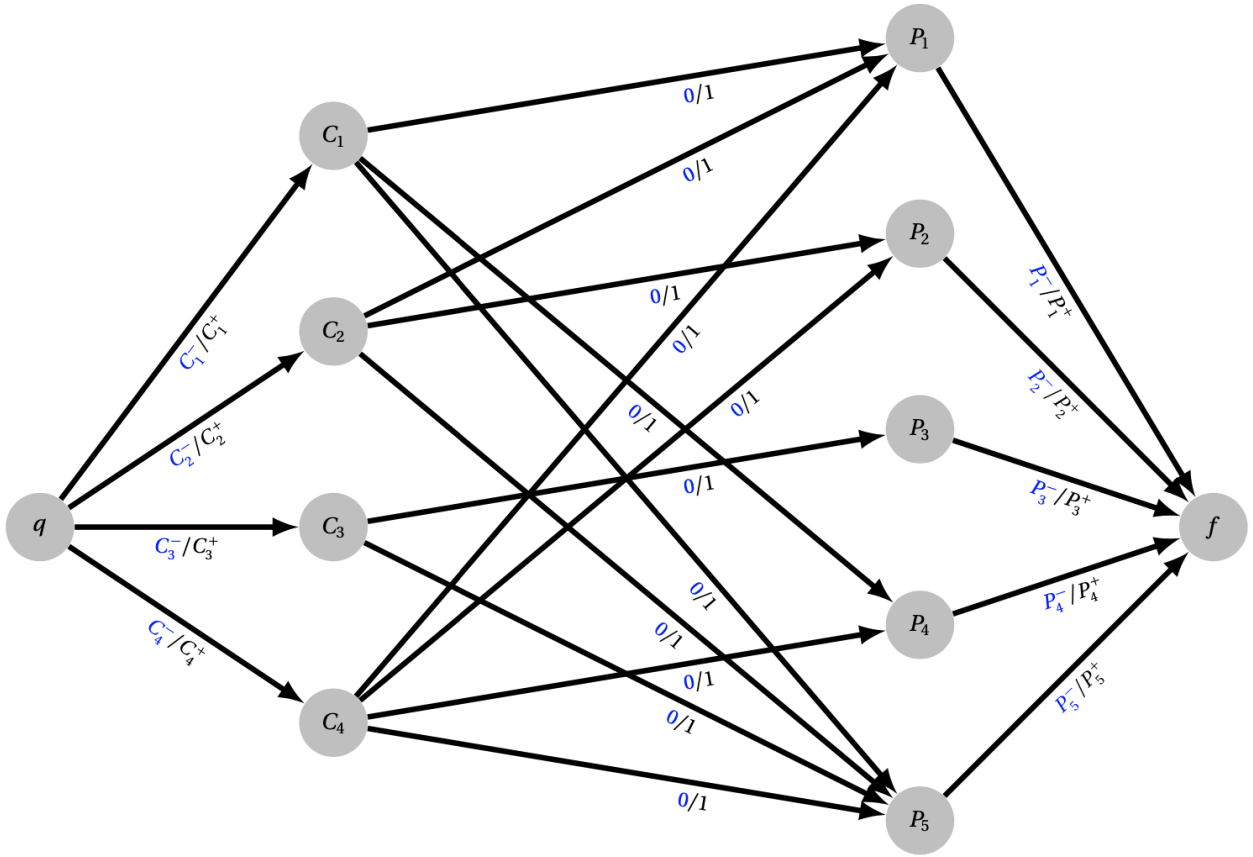
- There is range of review expected from of the product



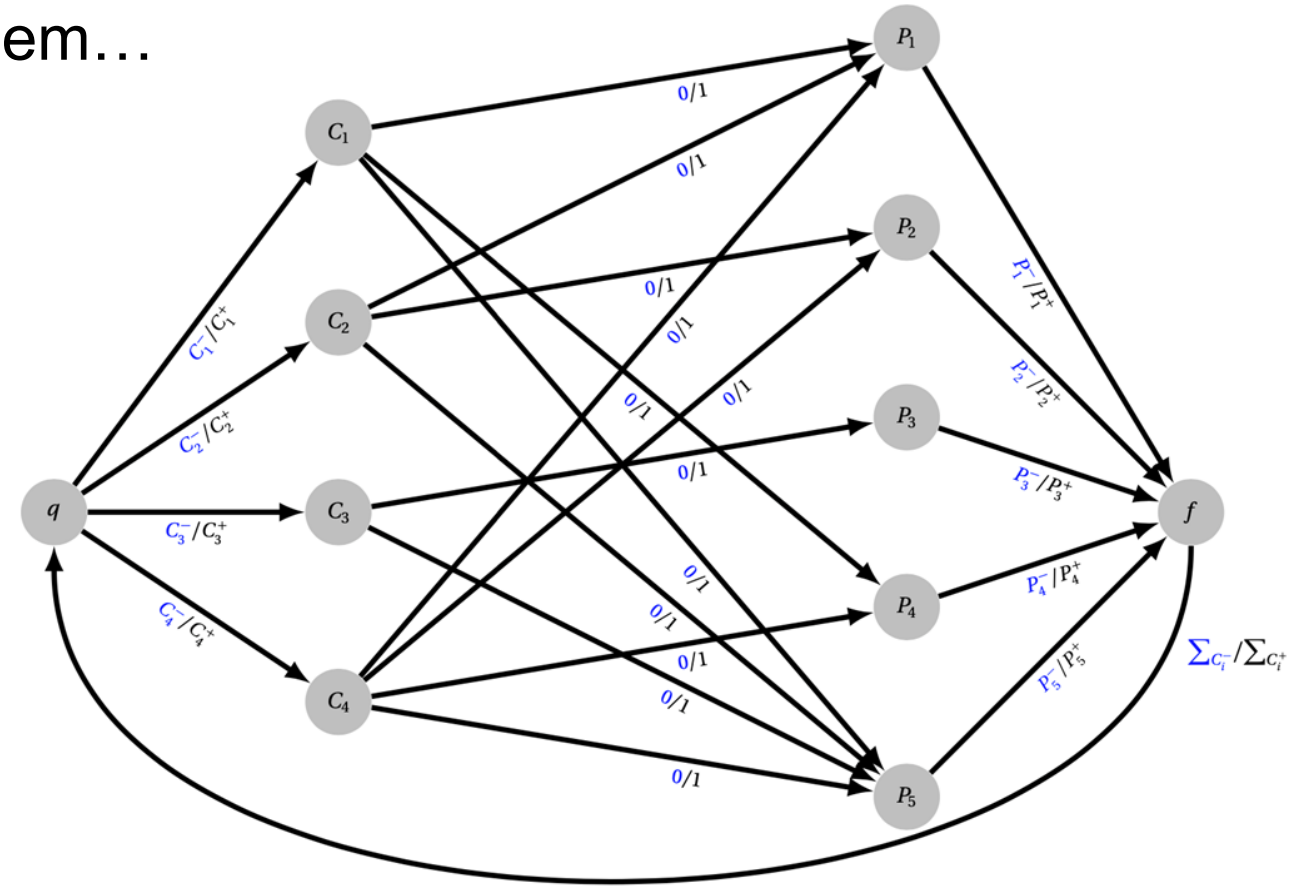
Applications with Network Flow

Survey Design

- In a way, it do look like a bipartite matching problem but with lower bound



- In a way, it do look like a bipartite matching problem but with lower bound, but since it is a circulation problem…



MONASH University

Questions?

Applications with Network Flow

Airline Scheduling

MONASH University

Applications with Network Flow

Airline Scheduling

MONASH University

- You have a collection of airplanes

Applications with Network Flow

Airline Scheduling

MONASH University

- You have a collection of airplanes

- You have a list of routes
- Some of the routes are very profitable, thus you want to fly the routes
  - Departure location
  - Departure time
  - Arrival location
  - Arrival time

Applications with Network Flow

Airline Scheduling

MONASH University

- You have a collection of airplanes

- You have a list of routes
- Some of the routes are very profitable, thus you want to fly the routes
  - Departure location
  - Departure time
  - Arrival location
  - Arrival time

- The airplanes can start flying from any location

Applications with Network Flow

Airline Scheduling

MONASH University

- You have a collection of airplanes, k
- You have a list of routes, $r_1, r_2, \ldots, r_n$
- Some of the routes are very profitable, thus you want to fly the routes
  - Departure location
  - Departure time
  - Arrival location
  - Arrival time
- The airplanes can start flying from any location

Applications with Network Flow

Airline Scheduling

MONASH University

- Imagine you have the following routes:
  - Route 1: SYD 6am – MEL 7am
  - Route 2: CBR 8am – SYD 9am
  - Route 3: MEL 11am – BNE 1pm
  - Route 4: PER 11am – SYD 7pm

Applications with Network Flow

Airline Scheduling

MONASH University

- Imagine you have the following routes:
  - Route 1: SYD 6am – MEL 7am
  - Route 2: CBR 8am – SYD 9am
  - Route 3: MEL 11am – BNE 1pm (can fit route1 and route2 prior)
  - Route 4: PER 11am – SYD 7pm (can fit route1 prior)

Applications with Network Flow

Airline Scheduling

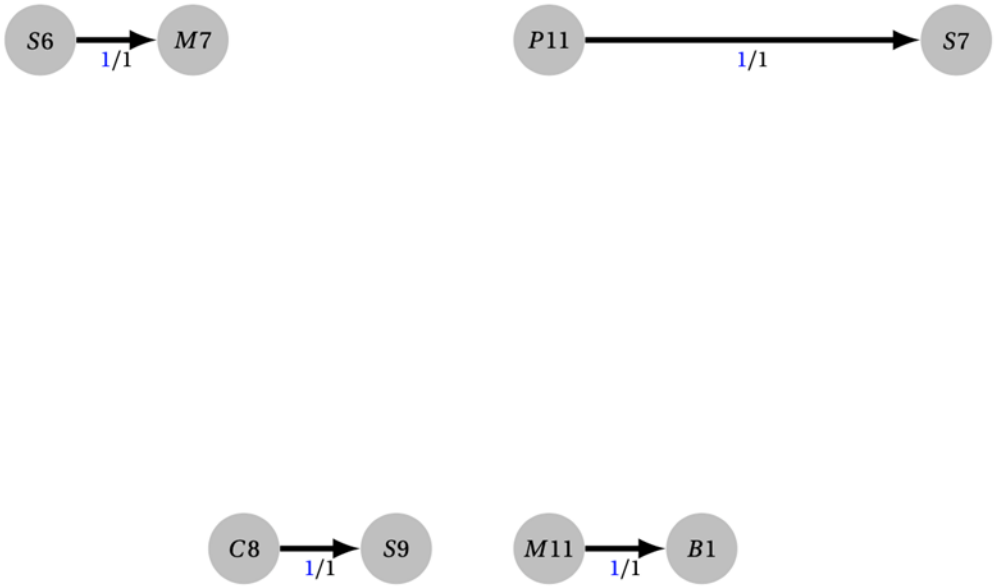
MONASH University

- Imagine you have the following routes:
  - Route 1: SYD 6am – MEL 7am
  - Route 2: CBR 8am – SYD 9am
  - Route 3: MEL 11am – BNE 1pm (can fit route1 and route2 prior)
  - Route 4: PER 11am – SYD 7pm (can fit route1 prior)
- Can you cover these 4 vital routes, using only 2 planes?

Applications with Network Flow

Airline Scheduling

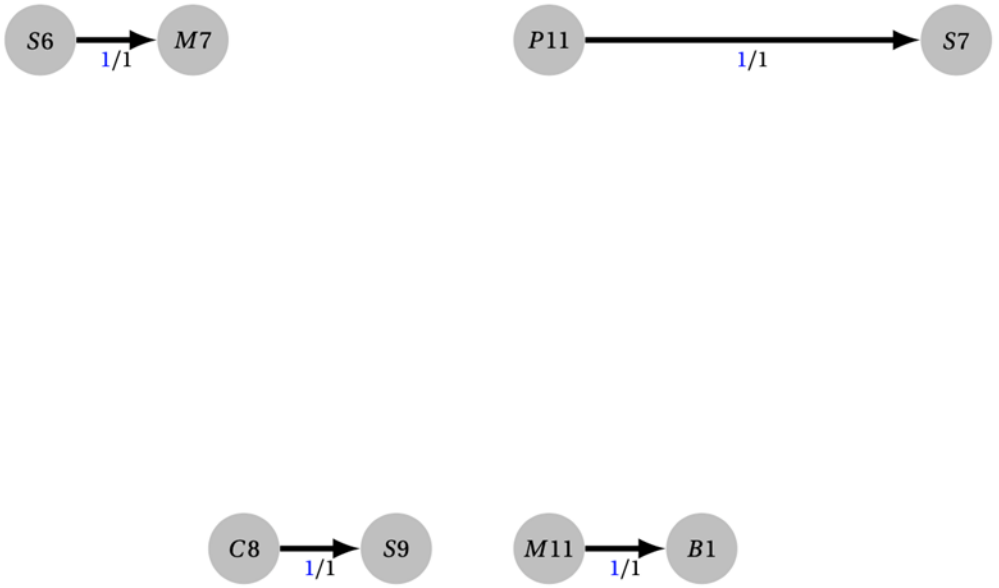
- First, we list down the routes imagine the x-axis as time…



Applications with Network Flow

Airline Scheduling

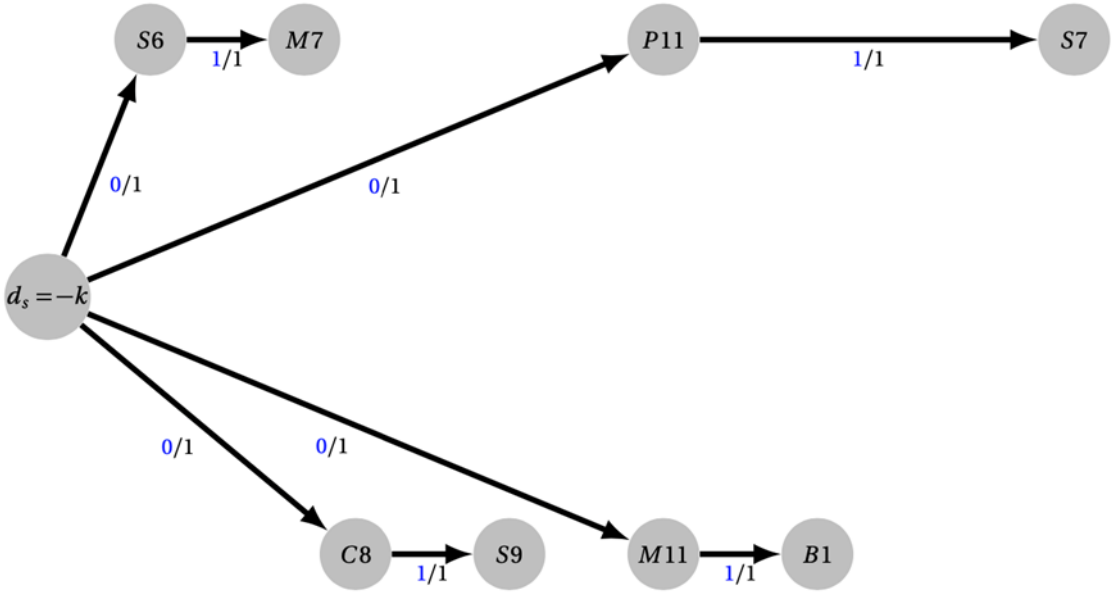
- First, we list down the routes imagine the x-axis as time…
  - Since they are vital flights, we want to always fly and thus lower-bound is set to 1.



Applications with Network Flow

Airline Scheduling

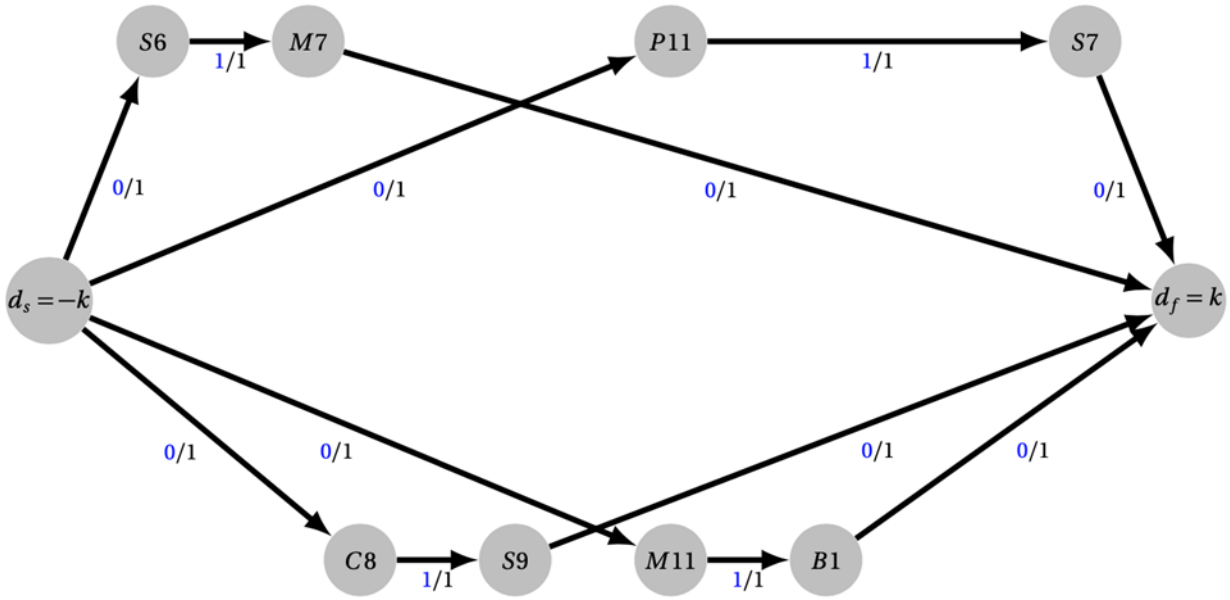
- Then we add a source, which we can place our planes from in any of the locations.
  - Lower bound is 0 because there is not requirement to be placed at which location



Applications with Network Flow

Airline Scheduling

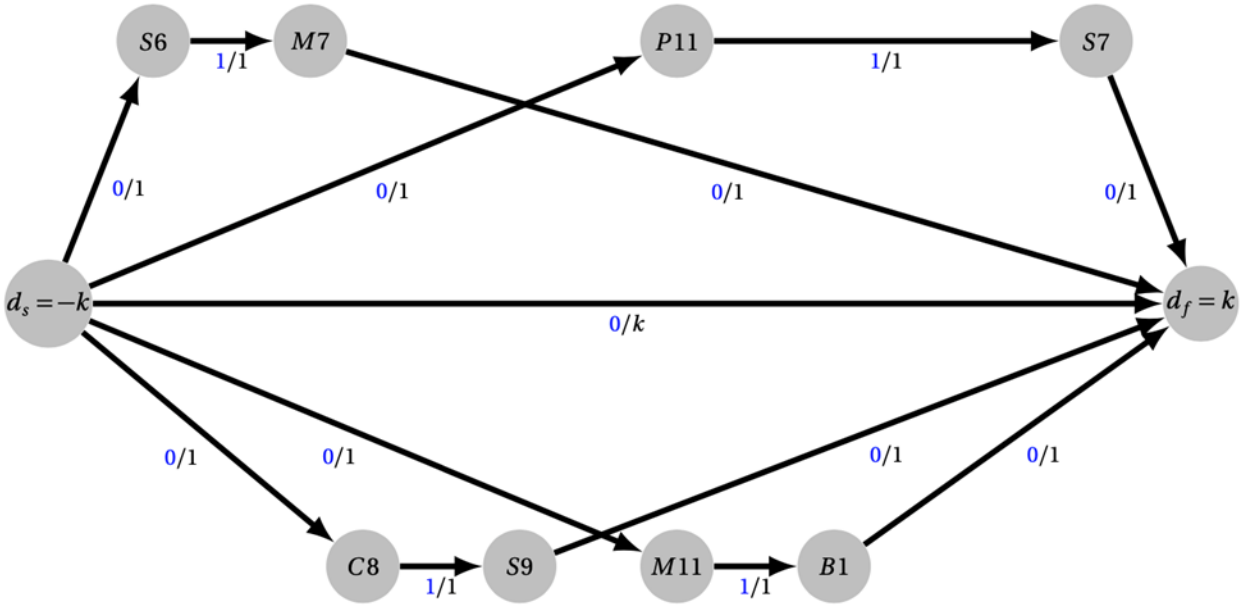
- Next, we add a sink which we can retire our planes at any of the locations.
  - No requirement for the planes to retire from any location



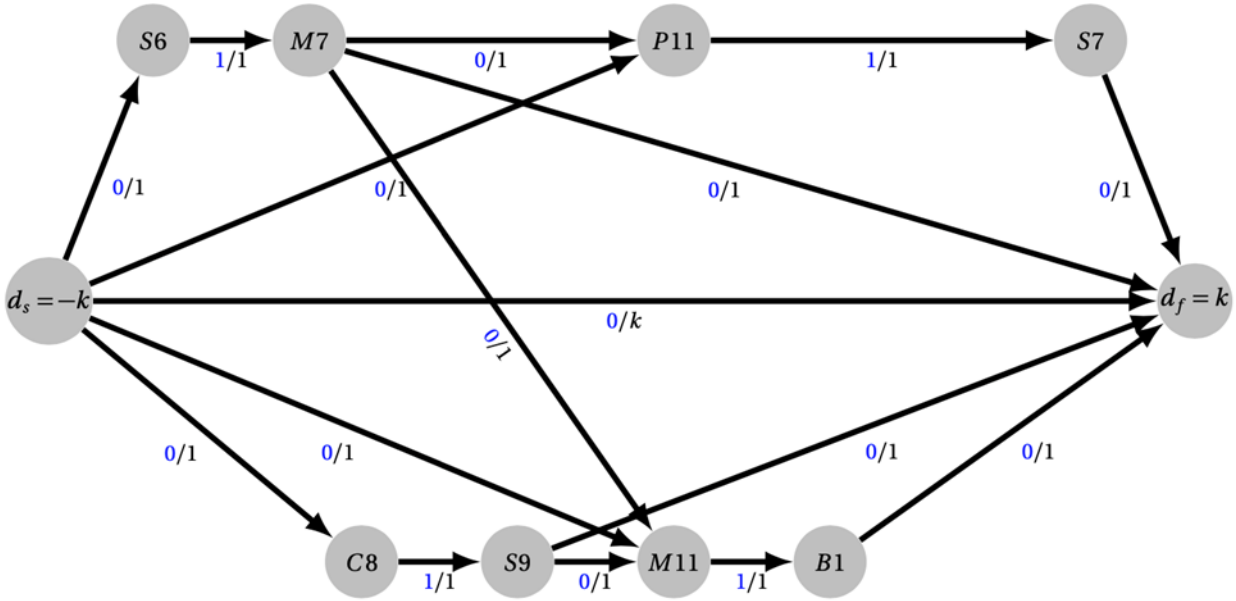
Applications with Network Flow

Airline Scheduling

- But what if we don't need all our planes to cover all vital routes?
  - Thus, they can go from start to retire directly



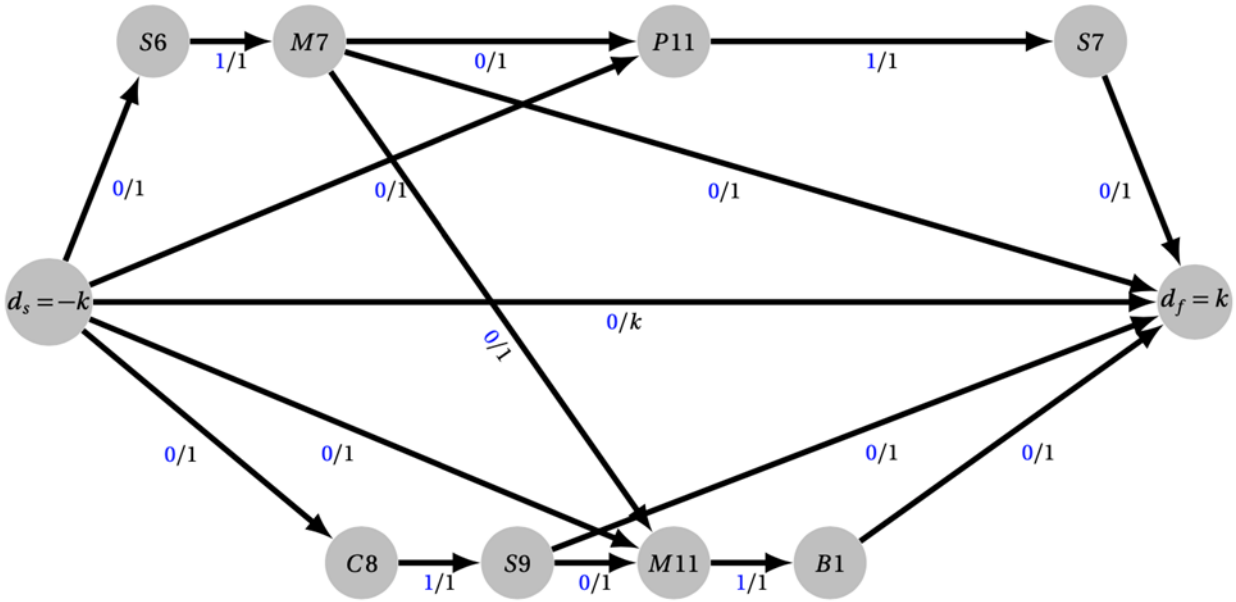
- Since it is possible for a plane to follow a route, then go to another route instead of retiring…
  - We add the edge, again it is optional



Applications with Network Flow

Airline Scheduling

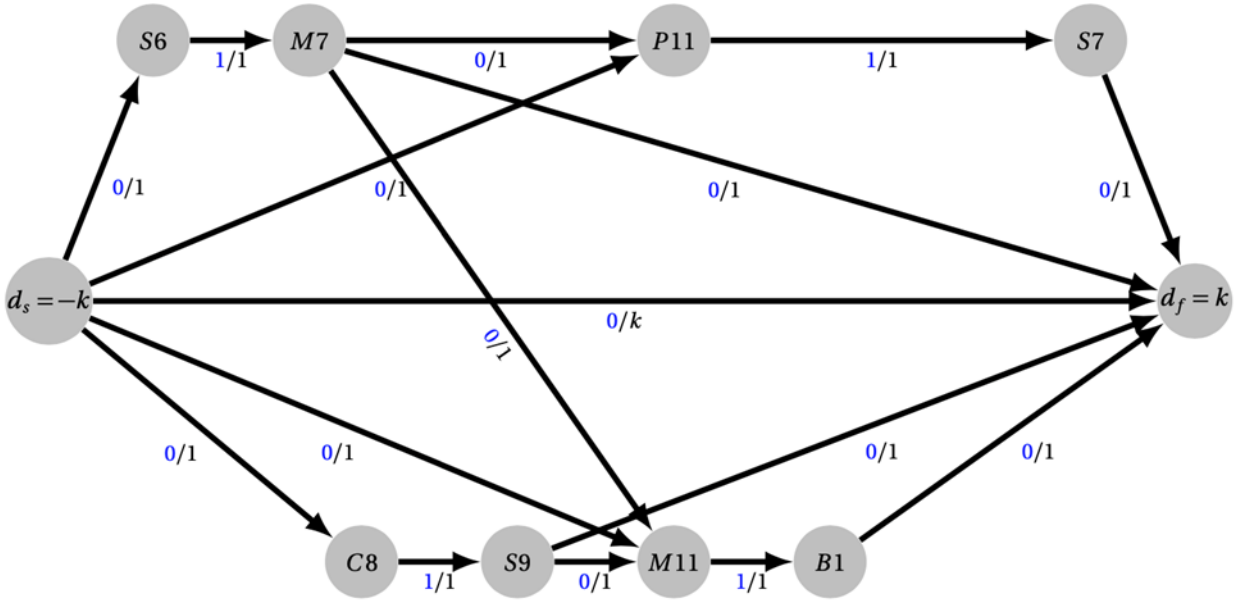
- Then we just solve this as it is!



Applications with Network Flow

Airline Scheduling

- Then we just solve this as it is!
- Answer is yes with 2 planes
  - Plane1: Route1 (S6->M7), then Route4 (P11->S7)
  - Plane2: Route2 (C8->S9), then Route3 (M11->B1)



MONASH University

Questions?

Applications with Network Flow

Other Examples?

MONASH University

Applications with Network Flow

Other Examples?

- Several examples in the studio

Applications with Network Flow

Other Examples?

- Several examples in the studio
  - Choosing profitable projects
  - Determining if teams/ players can progress in a tournament

Applications with Network Flow

Other Examples?

MONASH University

- Several examples in the studio
  - Choosing profitable projects
  - Determining if teams/ players can progress in a tournament
- … and many more
  - Open-pit mining
  - Image segmentation (e.g., background/foreground segmentation)
  - Network connectivity
  - Data mining
  - Distributed computing
  - Network intrusion detection
  - Edge-disjoint paths in graphs
  - Network reliability
  - Multi-camera scene reconstruction
  - Gene function prediction

MONASH University

Questions?

MONASH University

Thank You