MONASH
INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

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Ready?

Agenda

- Sorting Algorithms
  - Comparison based
    - Selection
    - Insertion
  - Non-comparison based (the IMBA ones)
    - Counting
    - Radix

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Let us begin…

Sorting

- We are back to sorting!
  - Bubble
  - Insertion
  - Selection
  - Merge
  - Quick

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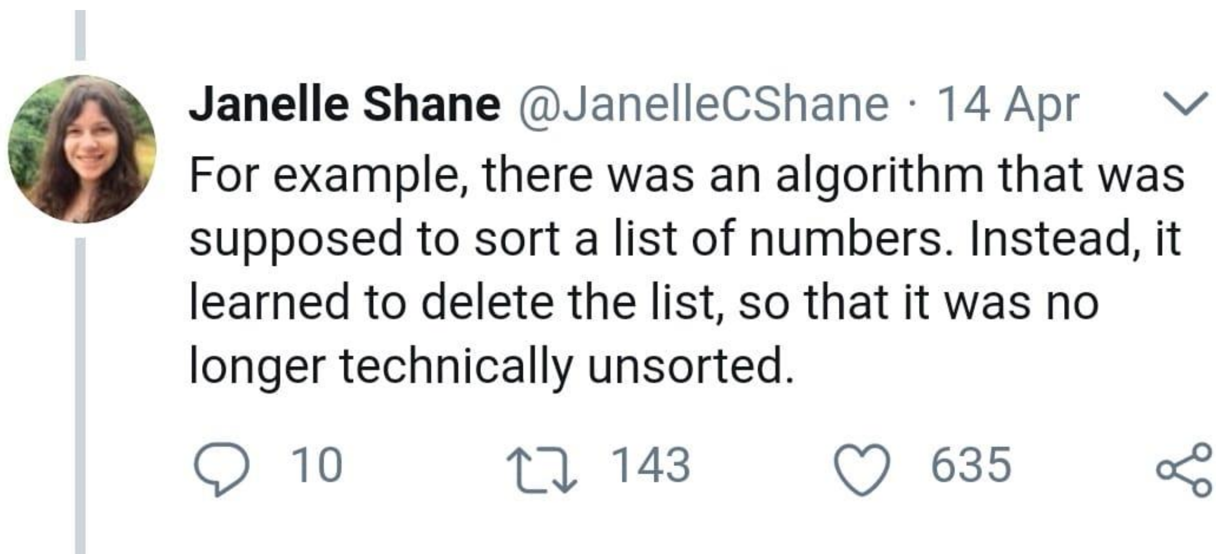

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- All of these are known as comparison based sorting. Why? Because we compare between items to know if $a < b$ or $b > a$
- Now let us analyze them based on what we have learnt!

Questions?

- Correctness
- Complexity

- Correctness
  - Loop invariant
  - Termination
- Complexity
  - Time
  - Space

Selection Sort

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- Correctness
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    - my_list[0…i-1] is sorted
    - my_list[0…i-1] <= my_list[i…N]
  - Termination

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  - Termination
    - i and j always increment and both reach the end of the list
  - So why is it working then?
    - i keep increment till n and we know from invariant 0…i-1 is sorted, thus we will sort the entire list!

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      - Like comparing between words, you need to compare the alphabets

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      - Like comparing between words, you need to compare the alphabets
      - We know complexity is based on comparison O(N^2) comparisons…
      - So our final complexity is O(kN^2)

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  - Minimum is 1, so we swap

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  - [4a, 2, 3, 4b, 1]
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  - [1, 2, 3, 4b, 4a]

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  - [4a, 2, 3, 4b, 1]
  - Minimum is 1, so we swap
  - [1, 2, 3, 4b, 4a]
  - Now we see that 4a is behind 4b!

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Problem 1. Write psuedocode for insertion sort, except instead of sorting the elements into non-decreasing order, sort them into non-increasing order. Identify a useful invariant of this algorithm.

Sorting

Insertion Sort

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    for i in range(1, len(my_list)):
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        while j >= 0 and key < my_list[j]:
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Sorting

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    - my_list[0…i-1] sorted
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    - Each loop keep look left, compare and swap till beginning of list
  - So if O(k) is the comparison cost, when we have O(kN^2) worst case!
  - What about space?
    - O(N) for the input list
    - O(1) auxiliary cause it is in-place

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  - Most shifting will ensure stability

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Questions?

	Best	Worst	Average	Stable?	In-place?
Selection Sort	$O(N^2)$	$O(N^2)$	$O(N^2)$	No	Yes
Insertion Sort	$O(N)$	$O(N^2)$	$O(N^2)$	Yes	Yes
Heap Sort	$O(N \log N)$	$O(N \log N)$	$O(N \log N)$	No	Yes
Merge Sort	$O(N \log N)$	$O(N \log N)$	$O(N \log N)$	Yes	No
Quick Sort	$O(N \log N)$	$O(N^2)$ – can be made $O(N \log N)$	$O(N \log N)$	Depends	No

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Selection Sort	$O(N^2)$	$O(N^2)$	$O(N^2)$		
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Heap Sort	O(N log N)	O(N log N)	O(N log N)		
Merge Sort	O(N log N)	O(N log N)	O(N log N)	Yes	No
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Auxiliary for Recursion

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  - If each recursion is k, then my total space is O(k log N)!!!

Auxiliary for Recursion

- The recursion stack takes up memory!!!
  - So that is why it isn't in-place!
    - Iterative is easier to get in-place
  - If I have recursion log N times, then I take O(log N) space for the recursion alone!
  - If each recursion is k, then my total space is O(k log N)!!!

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Merge Sort	$O(N \log N)$	$O(N \log N)$	$O(N \log N)$	Yes	No
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- So… what is the lower bound for the sorting algorithms that we have learnt?
  - Bubble
  - Insertion
  - Selection
  - Quick
  - Merge

- These are all comparison based
- $\Omega(N \log N)$
- We will see more of this later

Questions?

Thank you