MONASH
INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

Faculty of Information Technology, Monash University

Ready?

Agenda

- The Graph data structure
- Graph Traversal algorithms

Agenda

- The Graph data structure
  - Introduction
  - Representation
- Graph Traversal algorithms

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  - Breadth First Search (BFS)
  - Depth First Search (DFS)
  - Dijkstra’s shortest distance

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Basic for many graph-algorithms

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Let us begin…

- Master race of all data structure

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  - Everything can be represented as a graph

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Graph
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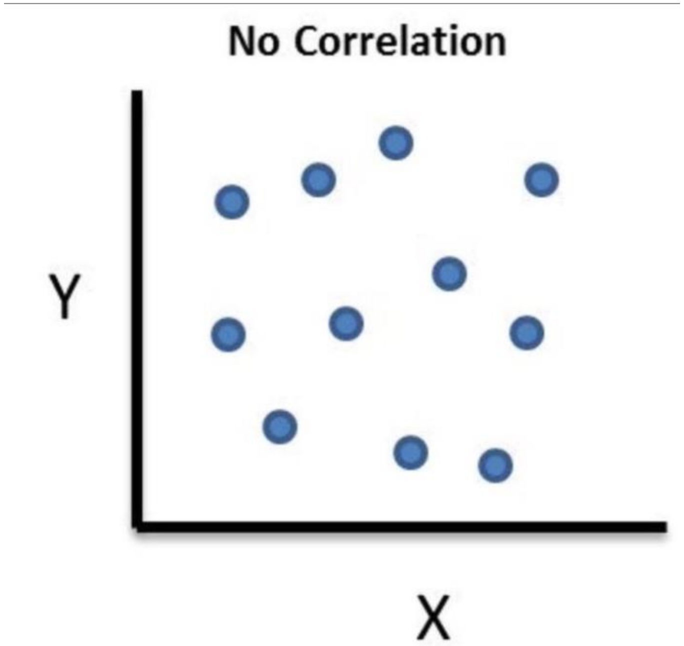

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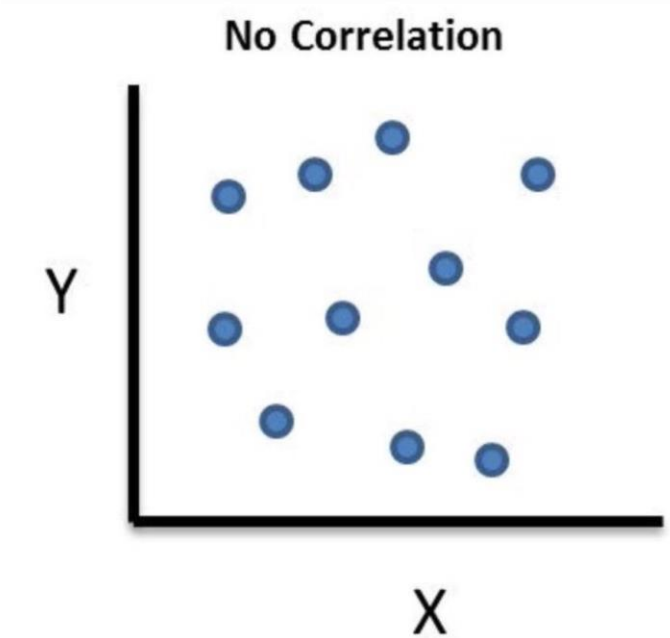

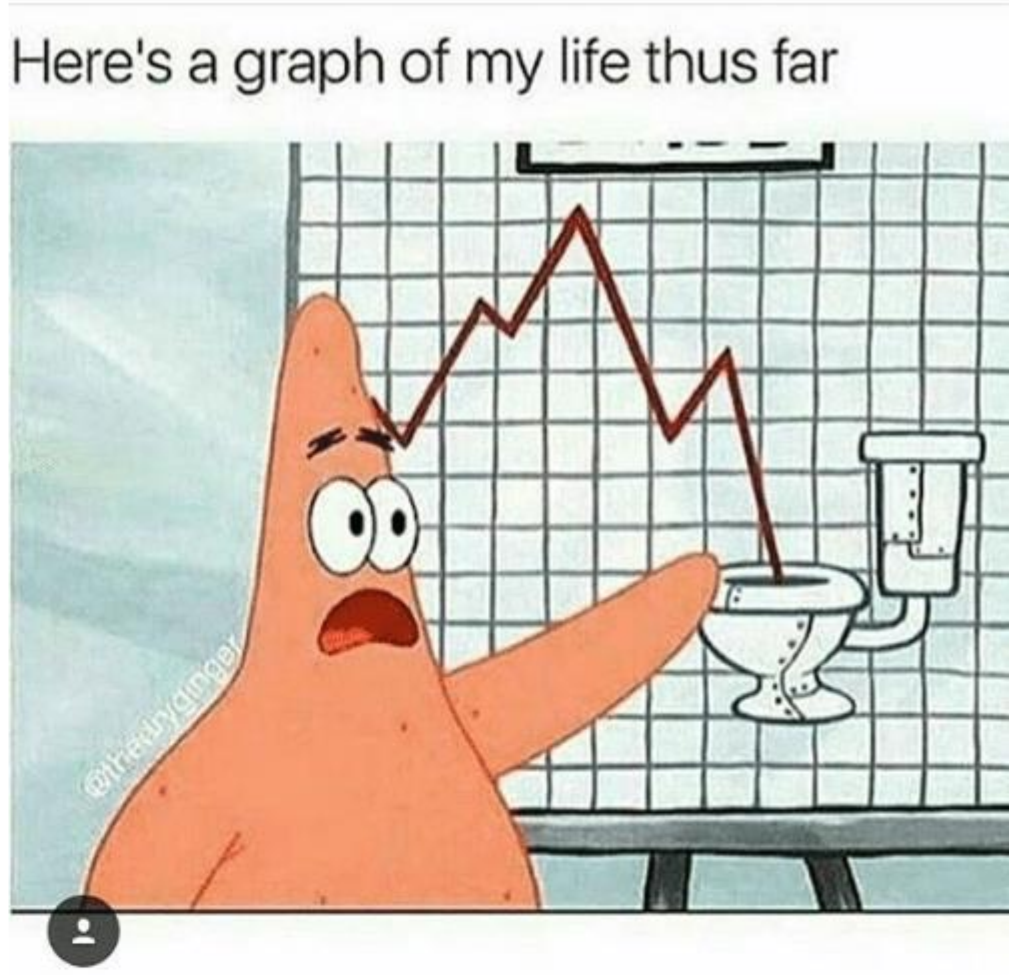

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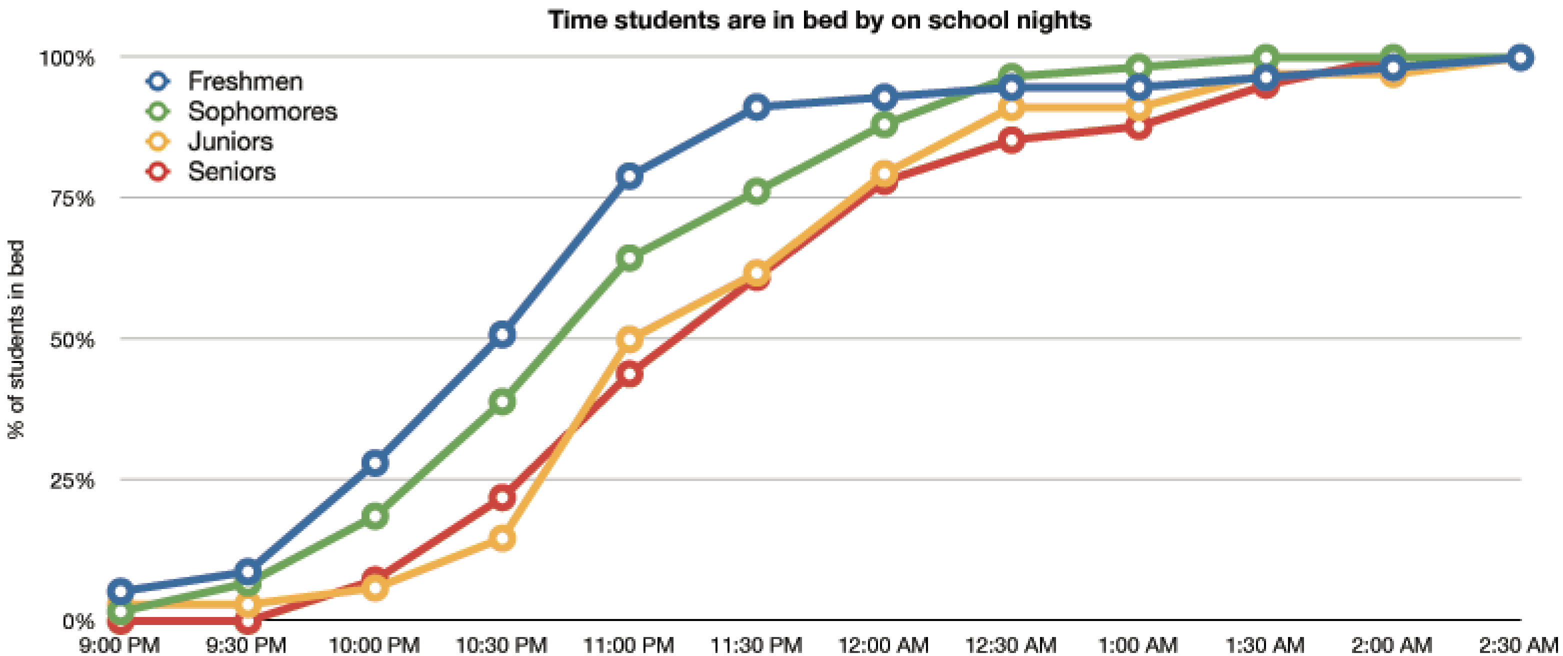

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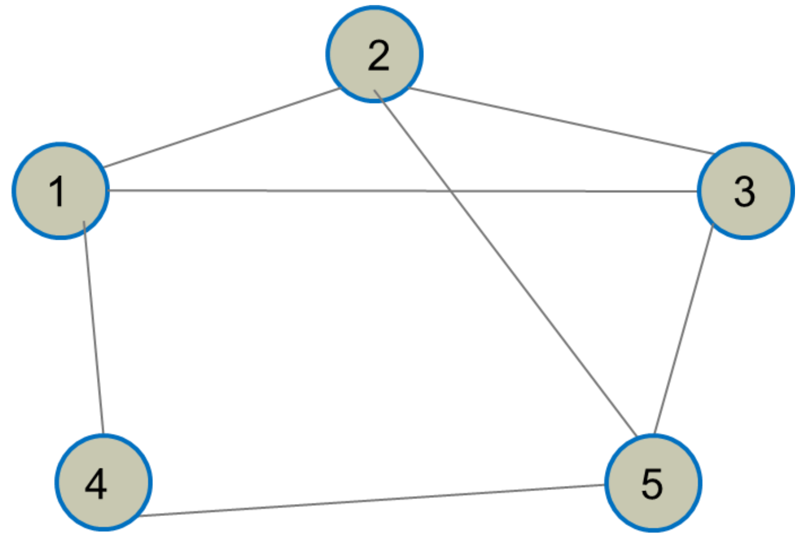

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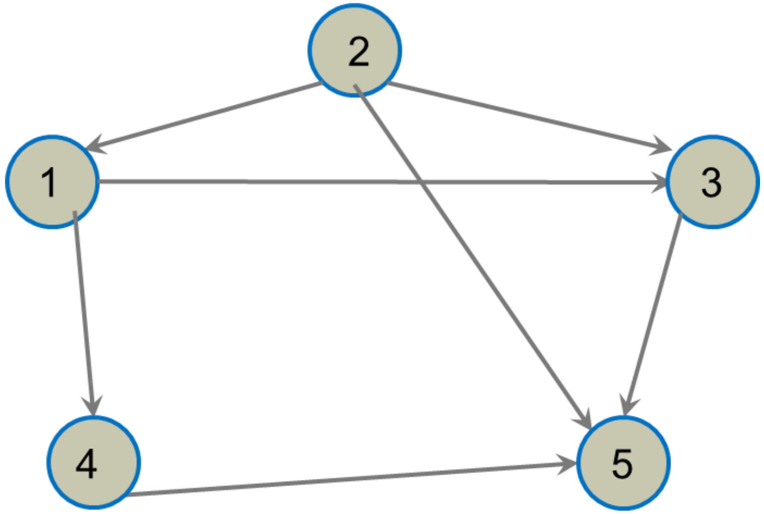

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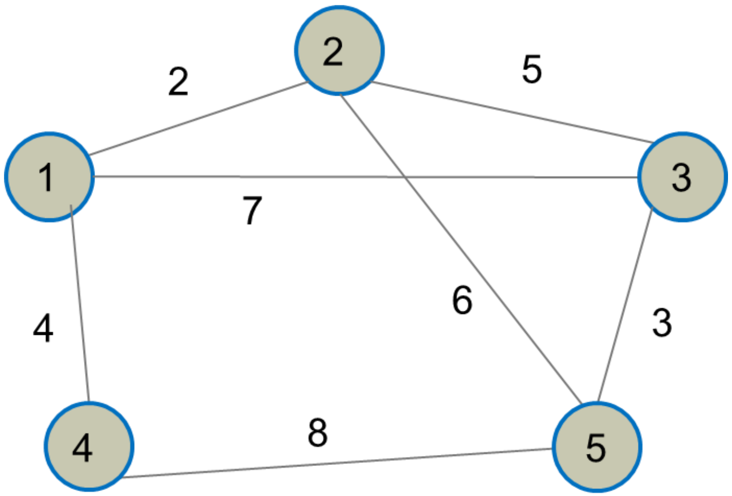

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Graph
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- A graph is called sparse if $E \ll V^2$
- A graph is called dense if $E \approx V^2$

Questions?

Graph

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      - Traversal through webpages and propagate authority
      - You can code it yourself, it is easy!

Questions?

- How do we represent graphs?

- How do we represent graphs? 2 possible way!

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  - Adjacency matrix
  - Adjacency list

- Adjacency matrix
  - Store edge information in a matrix

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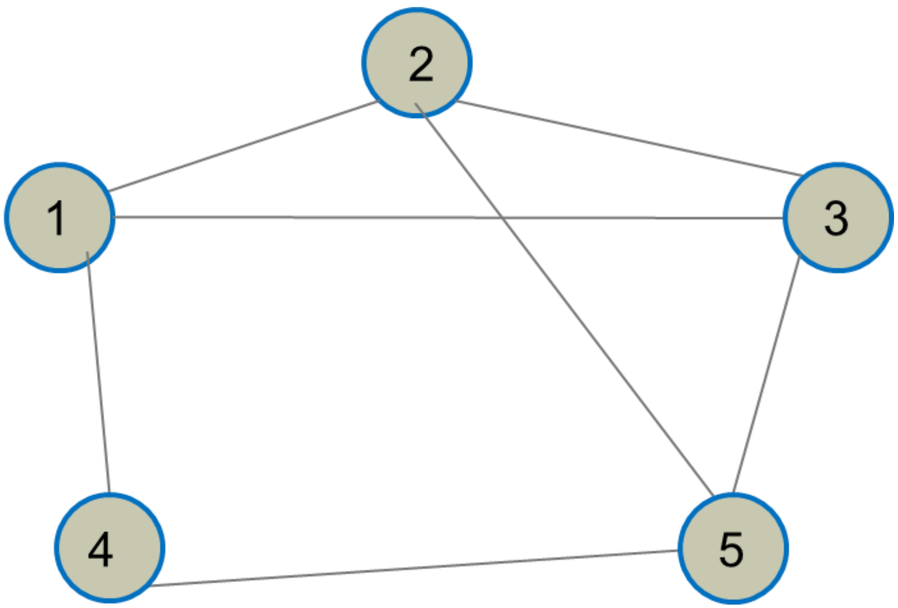
    - True/ False or 1/0 for unweighted

	1	2	3	4	5
1	F	T	T	T	F
2	T	F	T	F	T
3	T	T	F	F	T
4	T	F	F	F	T
5	F	T	T	T	F



Graph
Representation

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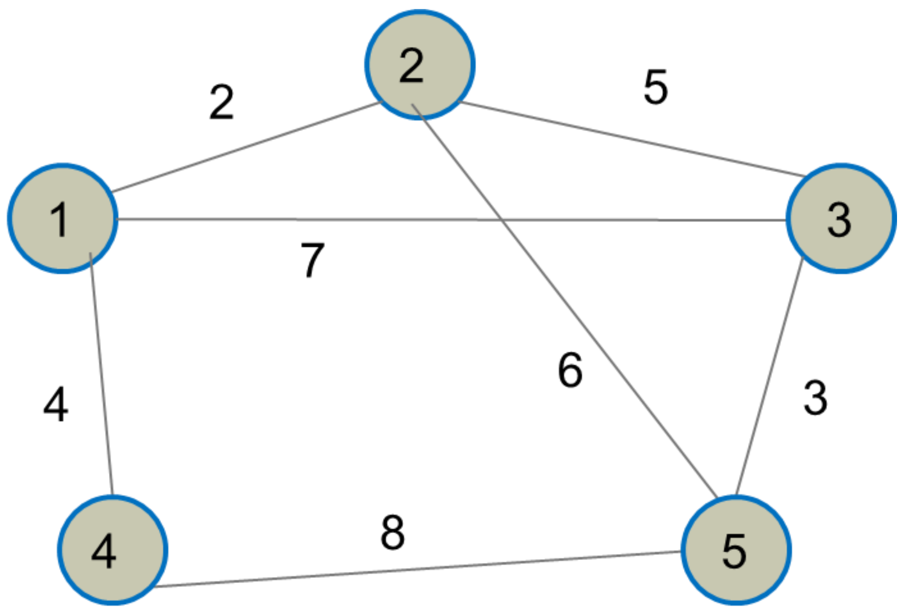
	1	2	3	4	5
1		2	7	4	
2	2		5		6
3	7	5			3
4	4				8
5		6	3	8	



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      - Adjacent = neighbour

- Adjacency list

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  - Array to store all Vertex object

Graph
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4	1	5	
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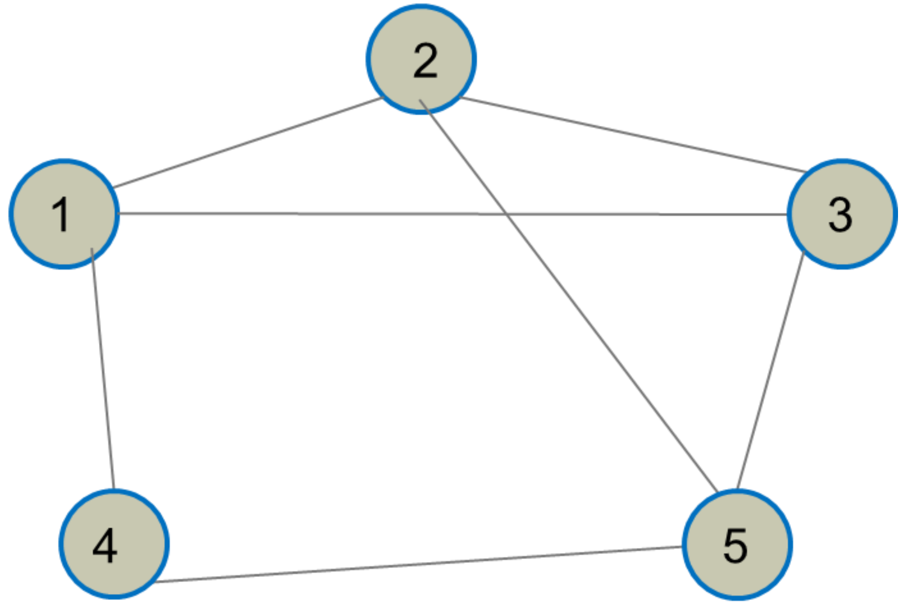
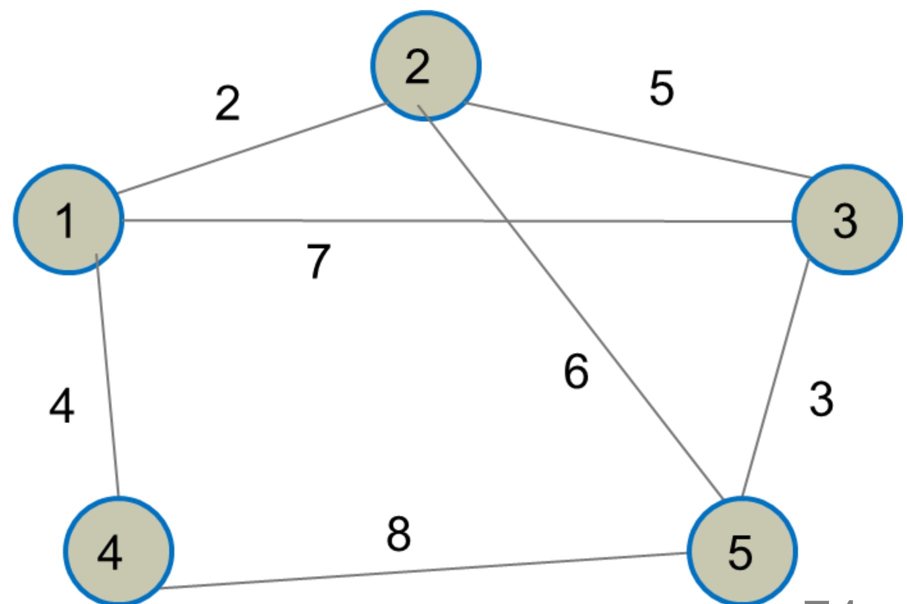


- Adjacency list
  - Array to store all Vertex object
  - Each vertex store a list of edges from it
    - With the weights

1 →	2 (2)	3 (7)	4 (4)
2 →	1 (2)	3 (5)	5 (6)
3 →	1 (7)	2 (5)	5 (3)
4 →	1 (4)	5 (8)	
5 →	2 (6)	3 (3)	4 (8)



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    - O(X) to retrieve all of the adjacent vertices of a vertex

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  - Space complexity?
    - O(V+E). Storing V vertices (as an array) and then total of E edges
  - Time complexity?
    - O(log V) to check if an edge exist if the edges are sorted
      - But you can't use binary search on linked list!
      - So this is still O(X) but you can terminate earlier once you reach a bigger vertex
    - O(X) to retrieve all of the adjacent vertices of a vertex
      - Where X = number of adjacent vertices (output-sensitive complexity)

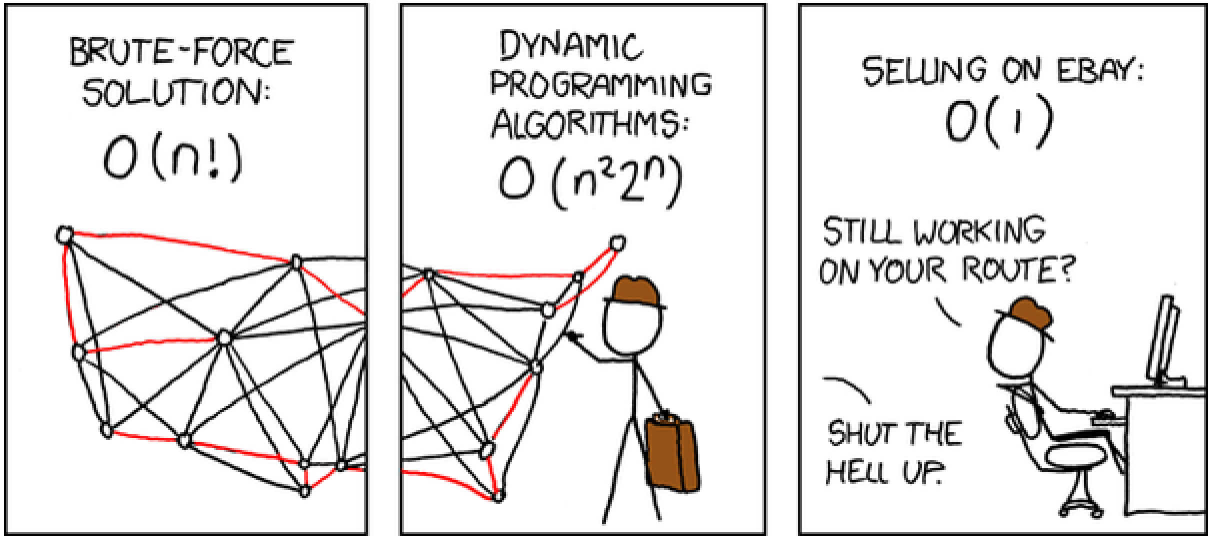
Questions?

- Going from a place to another

Graph
Traversal

- Going from a place (source vertex) to another

- Going from a place (source vertex) to another or everywhere!



Graph
Traversal

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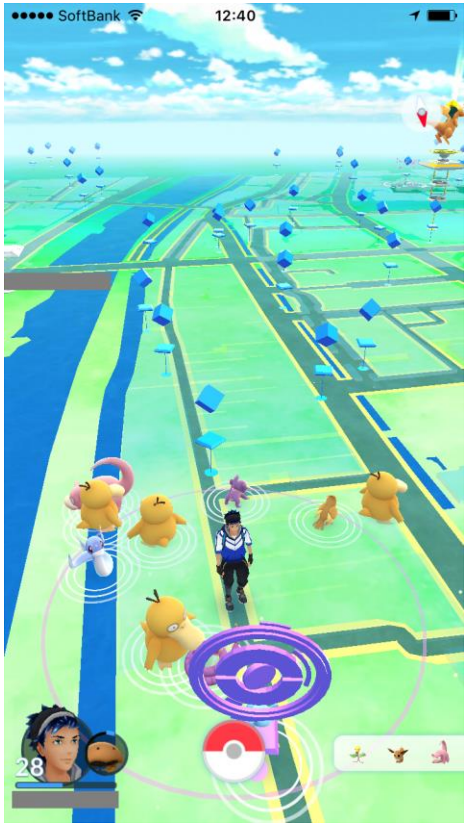

- Breadth-First Search (BFS)
- Depth-First Search (DFS)

Graph
Traversal

- Breadth-First Search (BFS)
  - Going wide
- Depth-First Search (DFS)
  - Going deep

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- Let us begin with a tree first

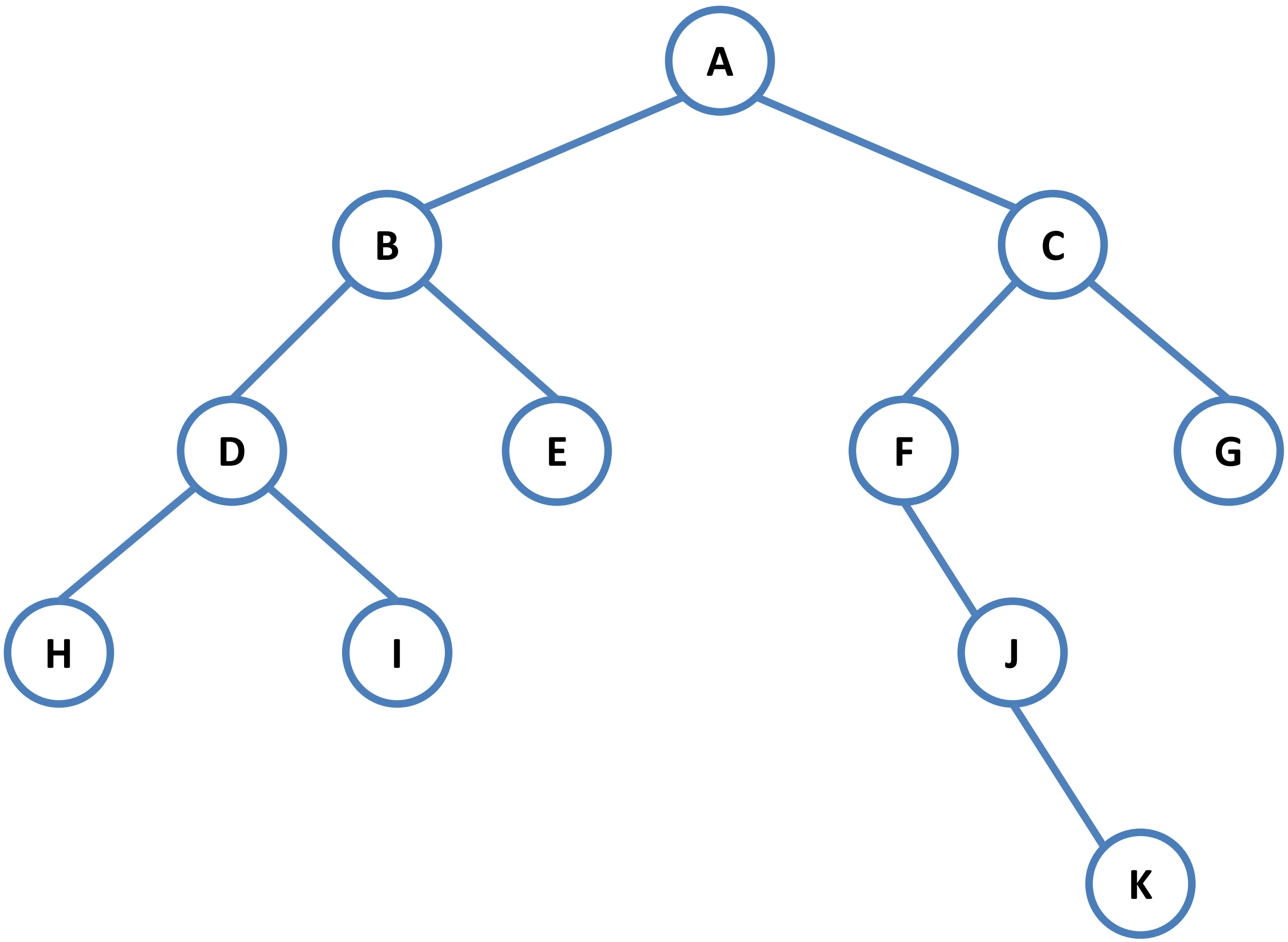
Graph
Traversal

- Breadth-First Search (BFS)
  - Going wide
- Depth-First Search (DFS)
  - Going deep

- Let us begin with a tree first
  - Recall a tree is a graph without cycles

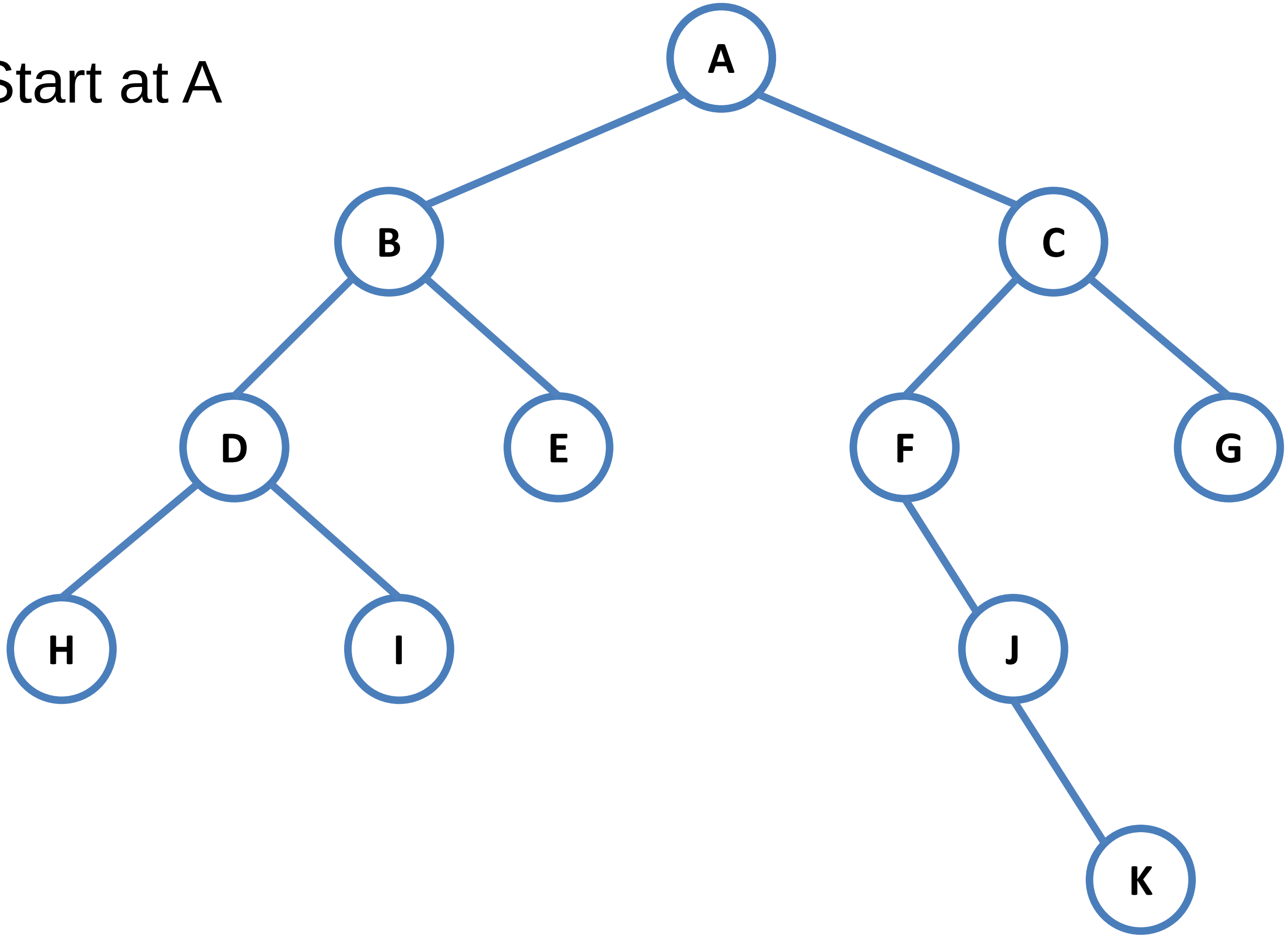
Graph

Traversal



Graph
Traversal

- Start at A



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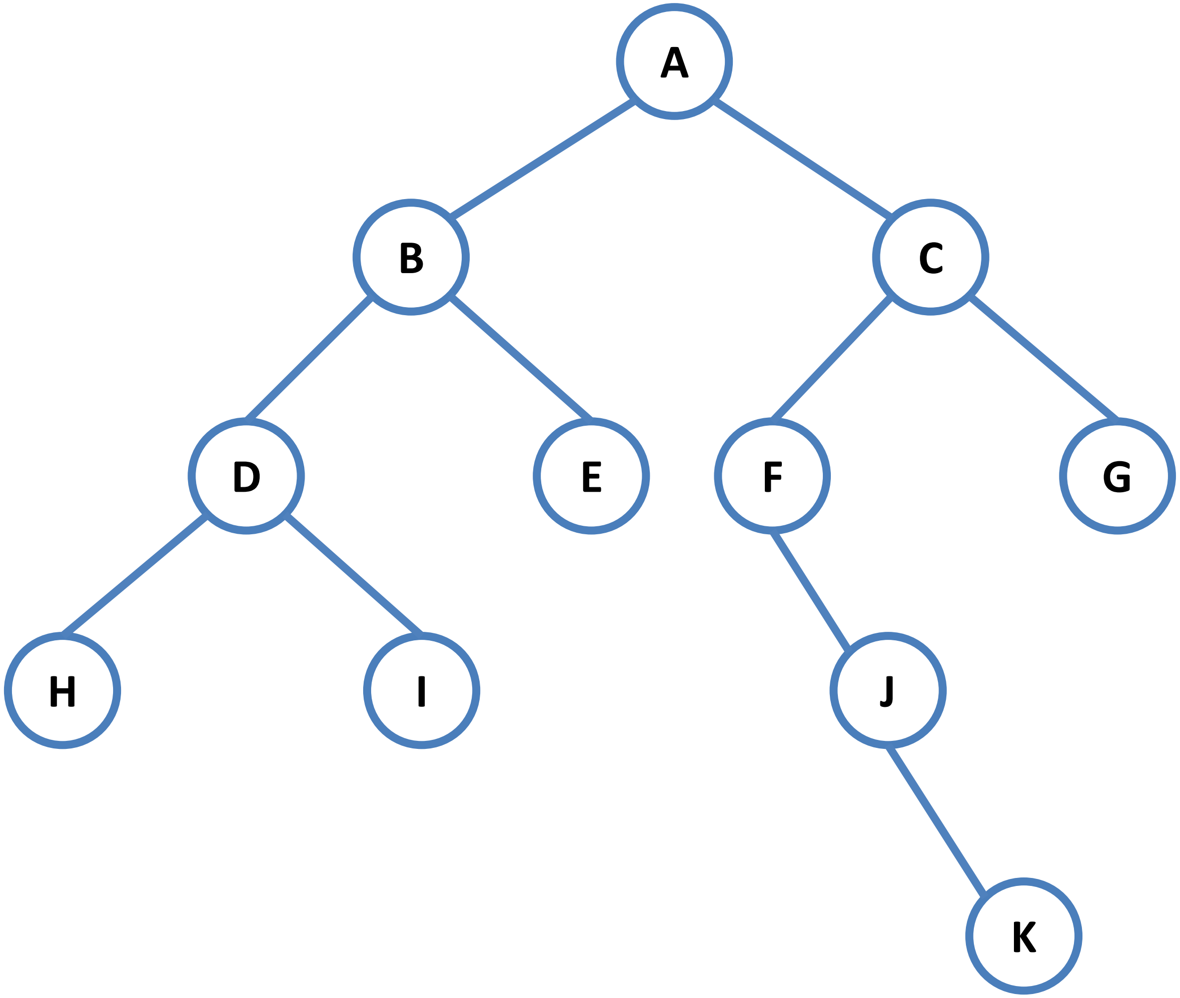
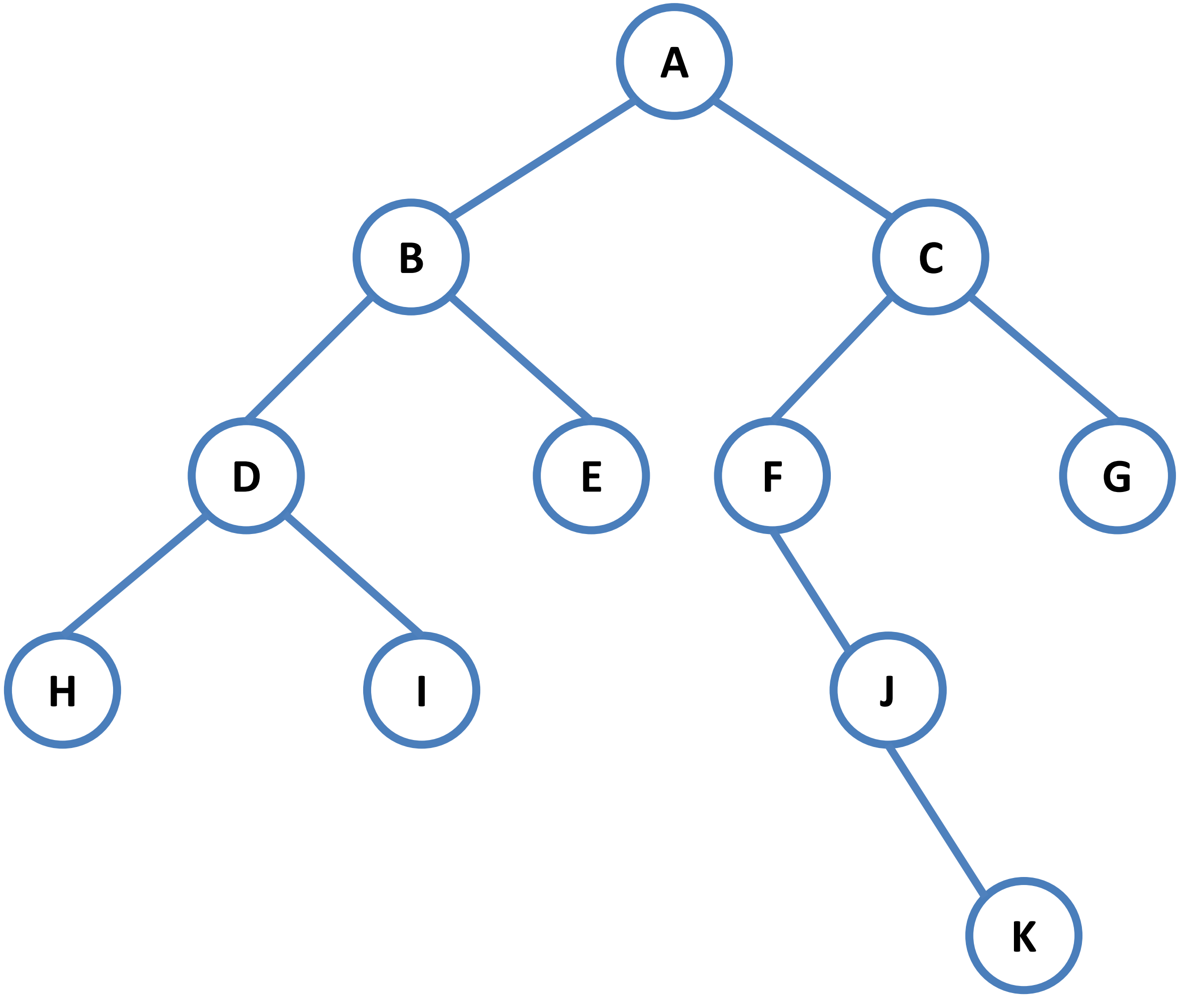
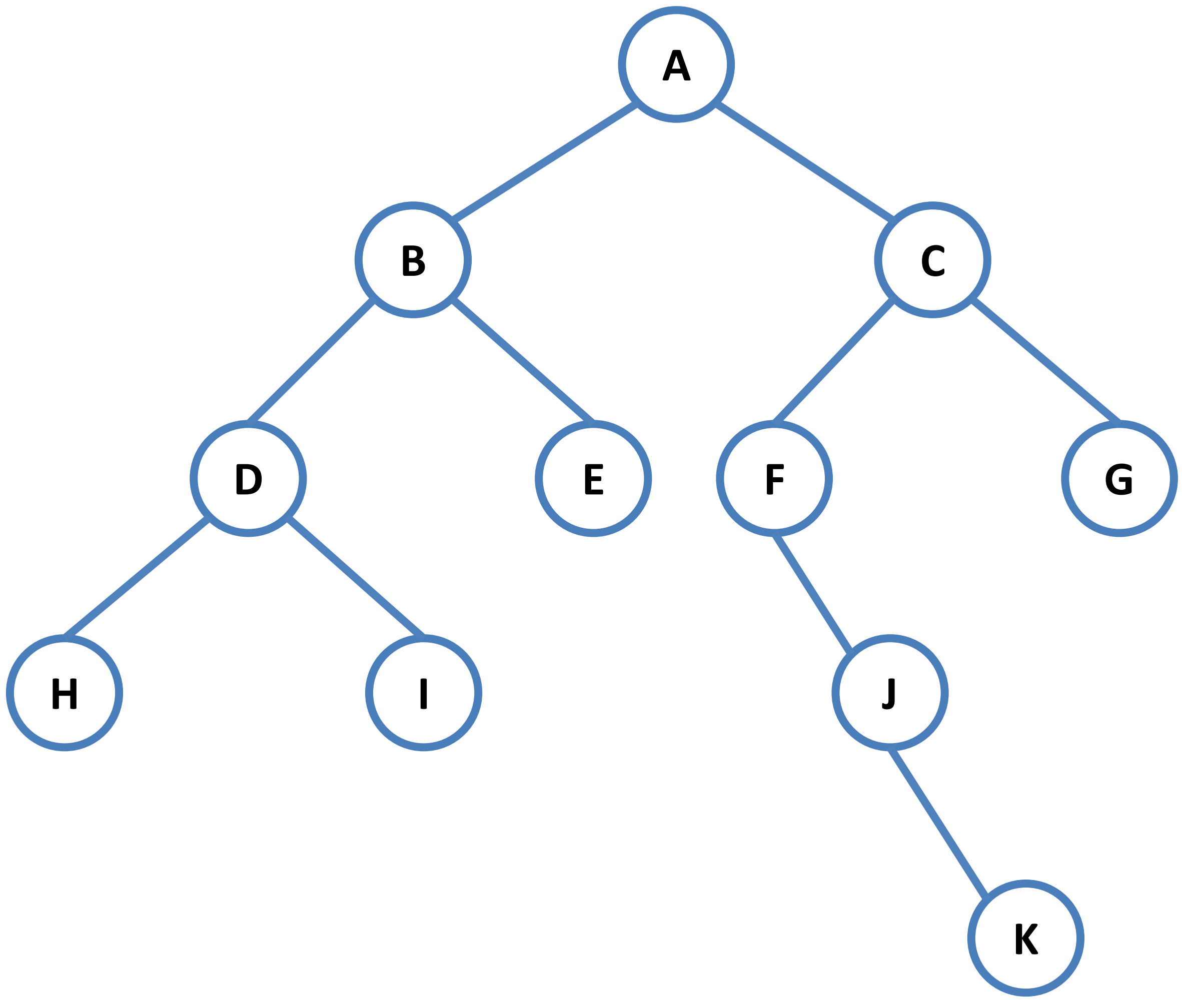


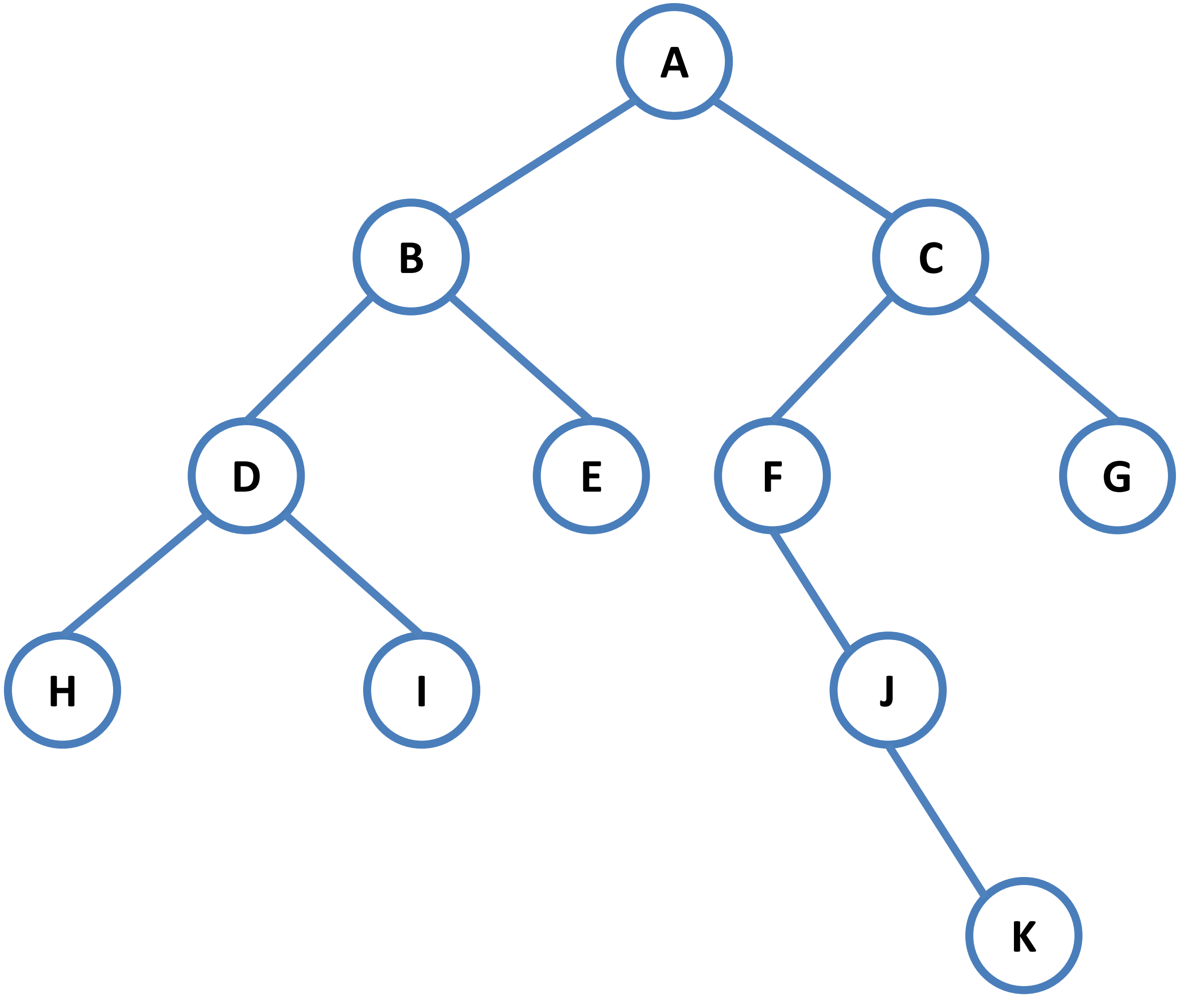
- Start at A, BFS



- Start at A, BFS
- A



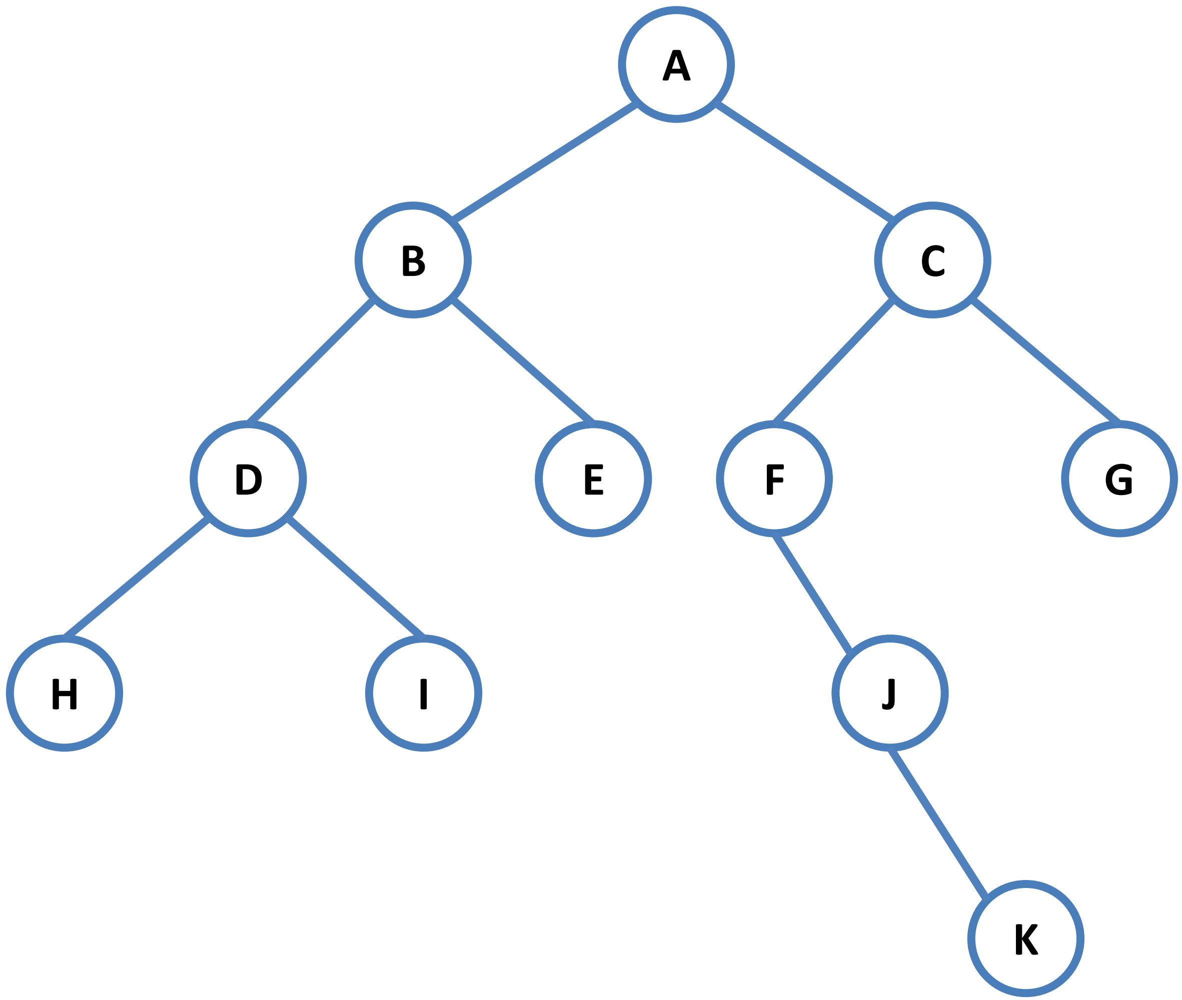
- Start at A, BFS
- A
- B



Graph

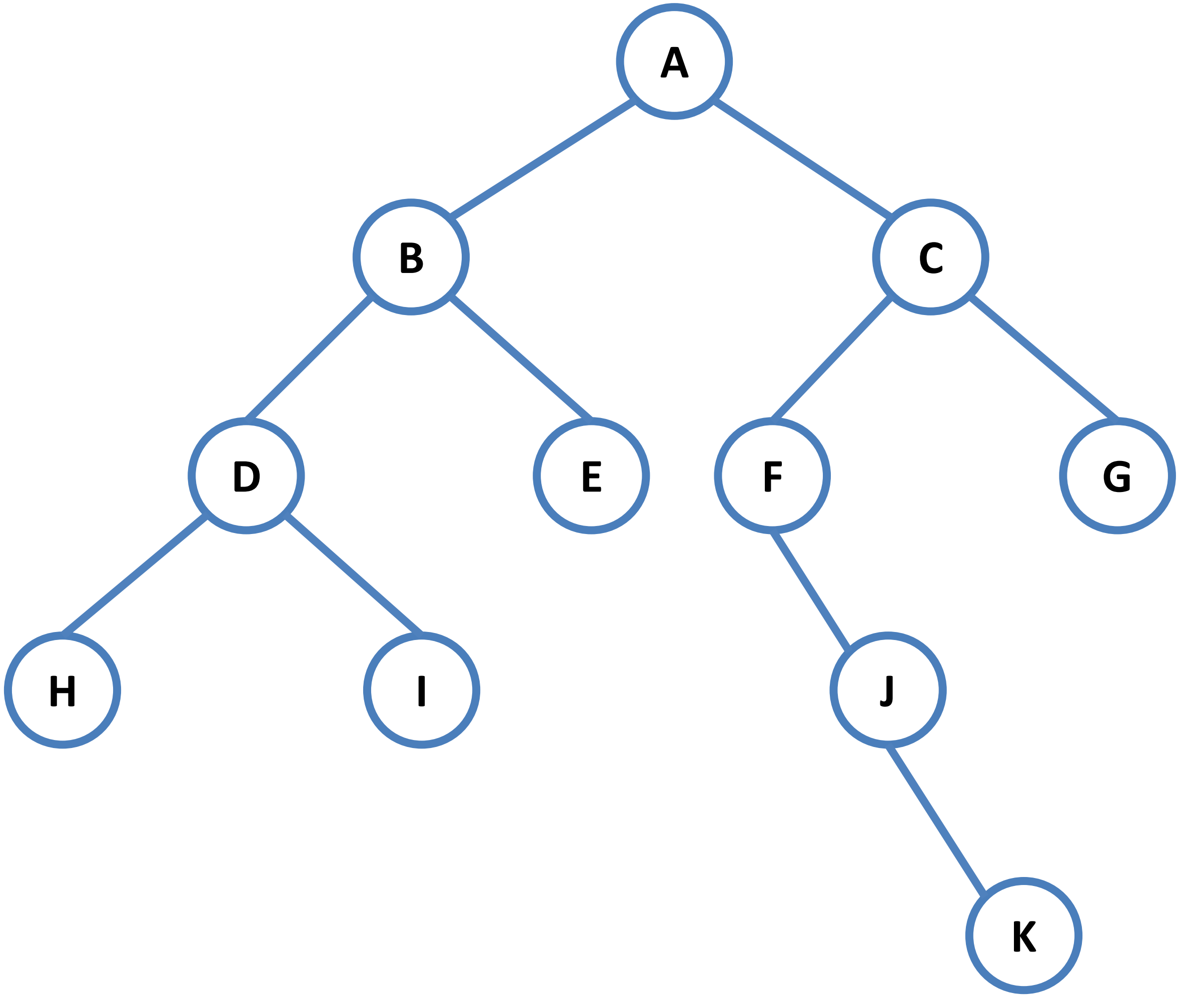
Traversal

- Start at A, BFS
- A
- B
- C

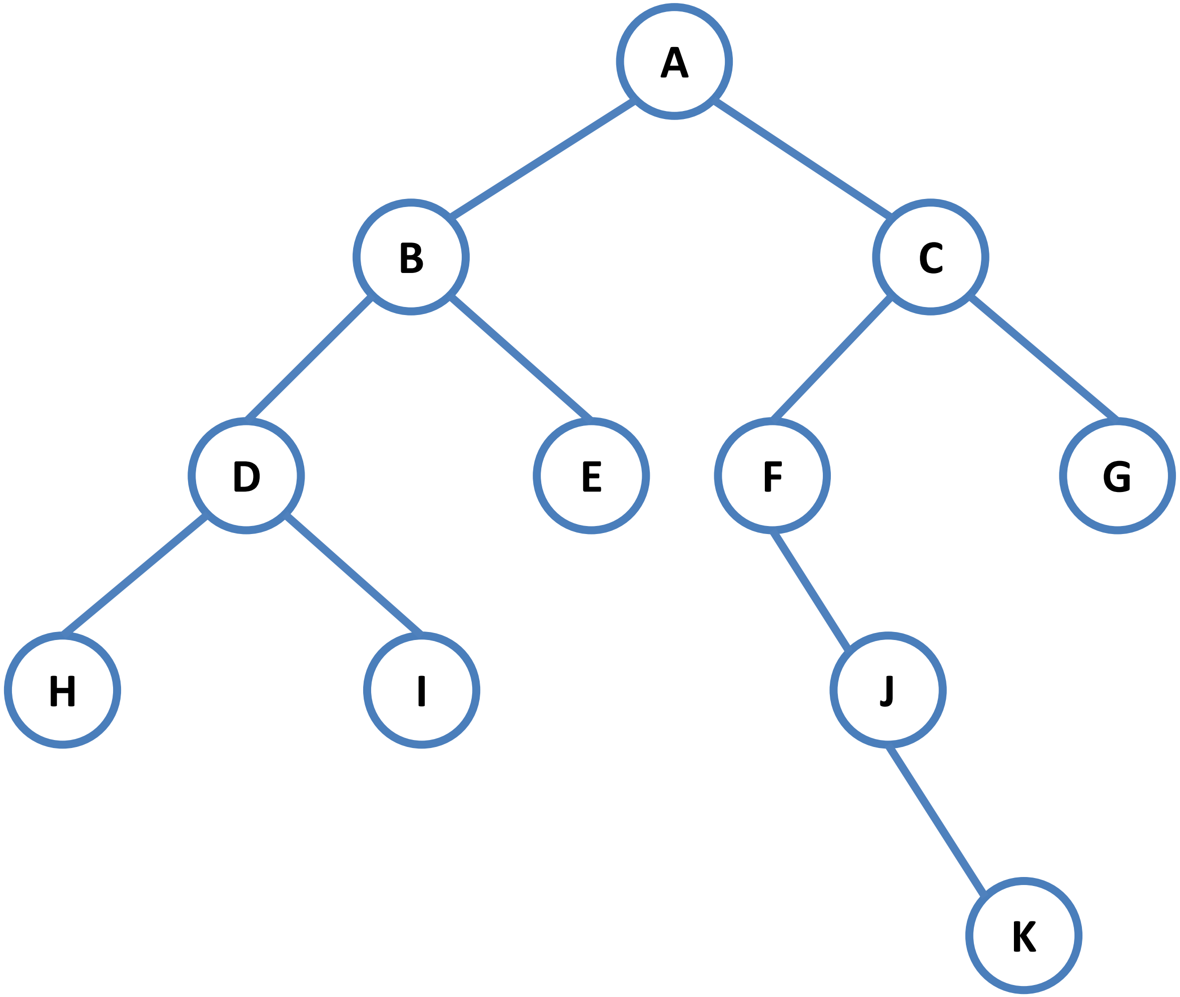


Graph
Traversal

- Start at A, BFS
- A
- B
- C
- D



- Start at A, BFS
- A
- B
- C
- D
- E



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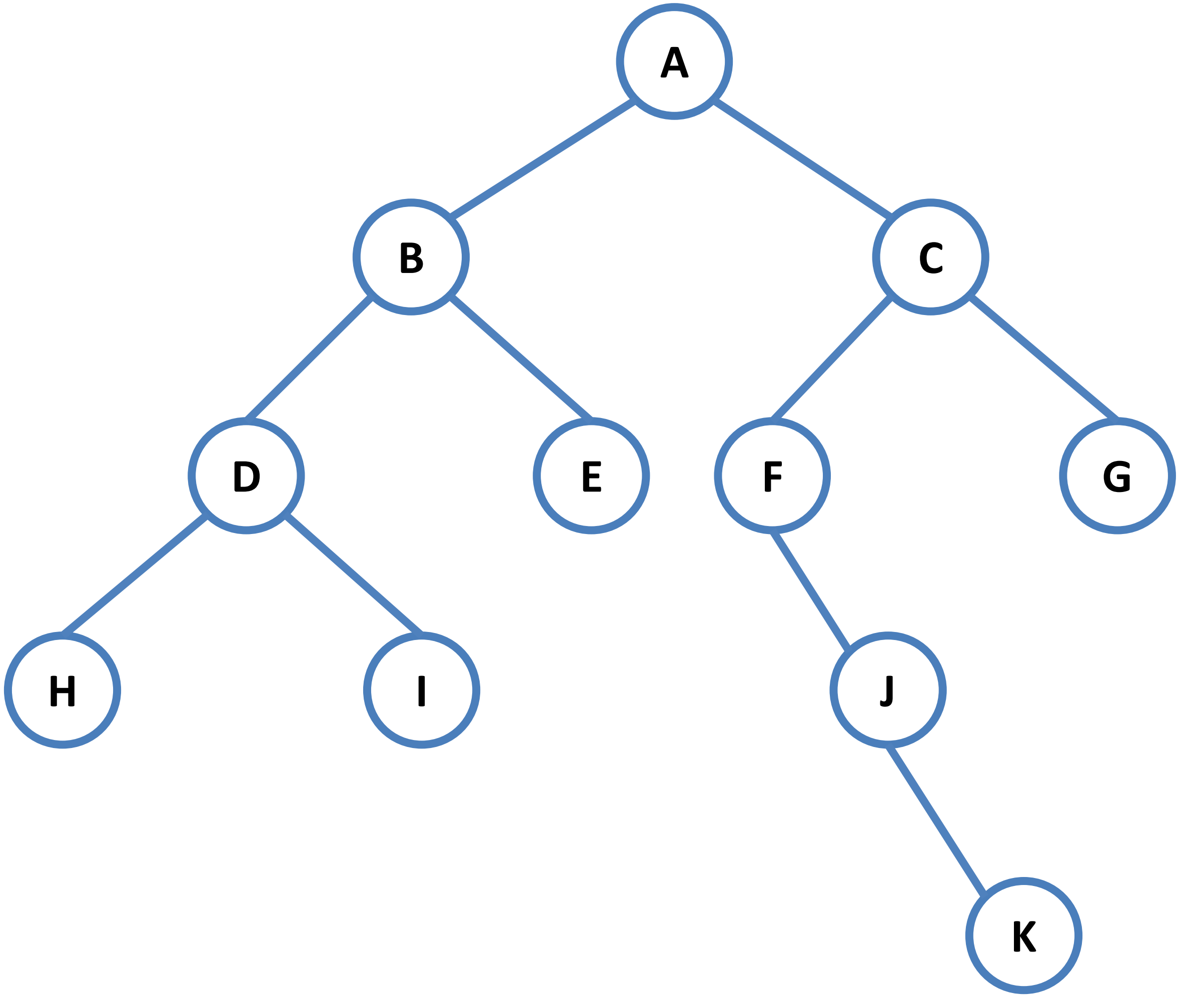
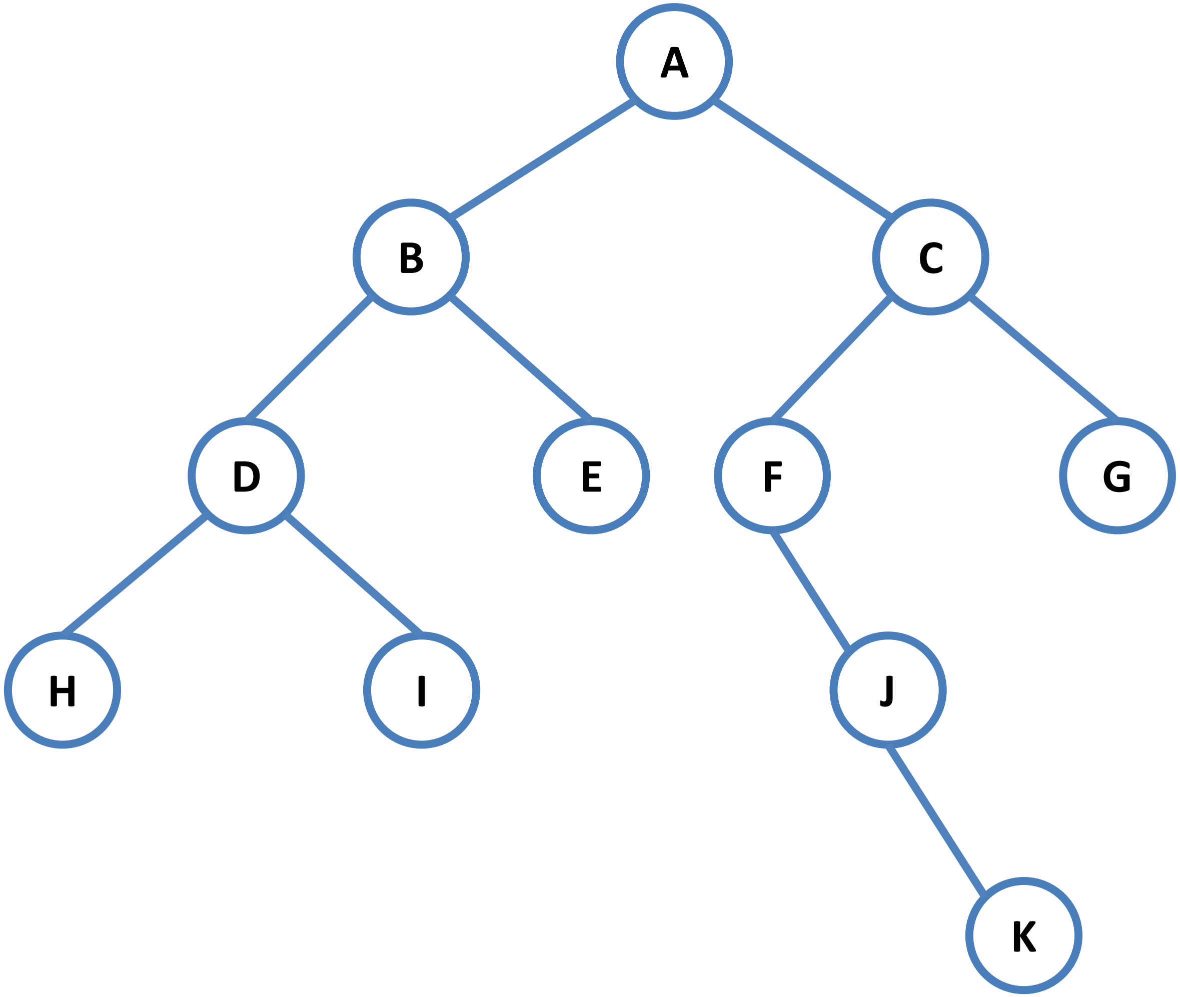
- A
- B
- C
- D
- E
- F



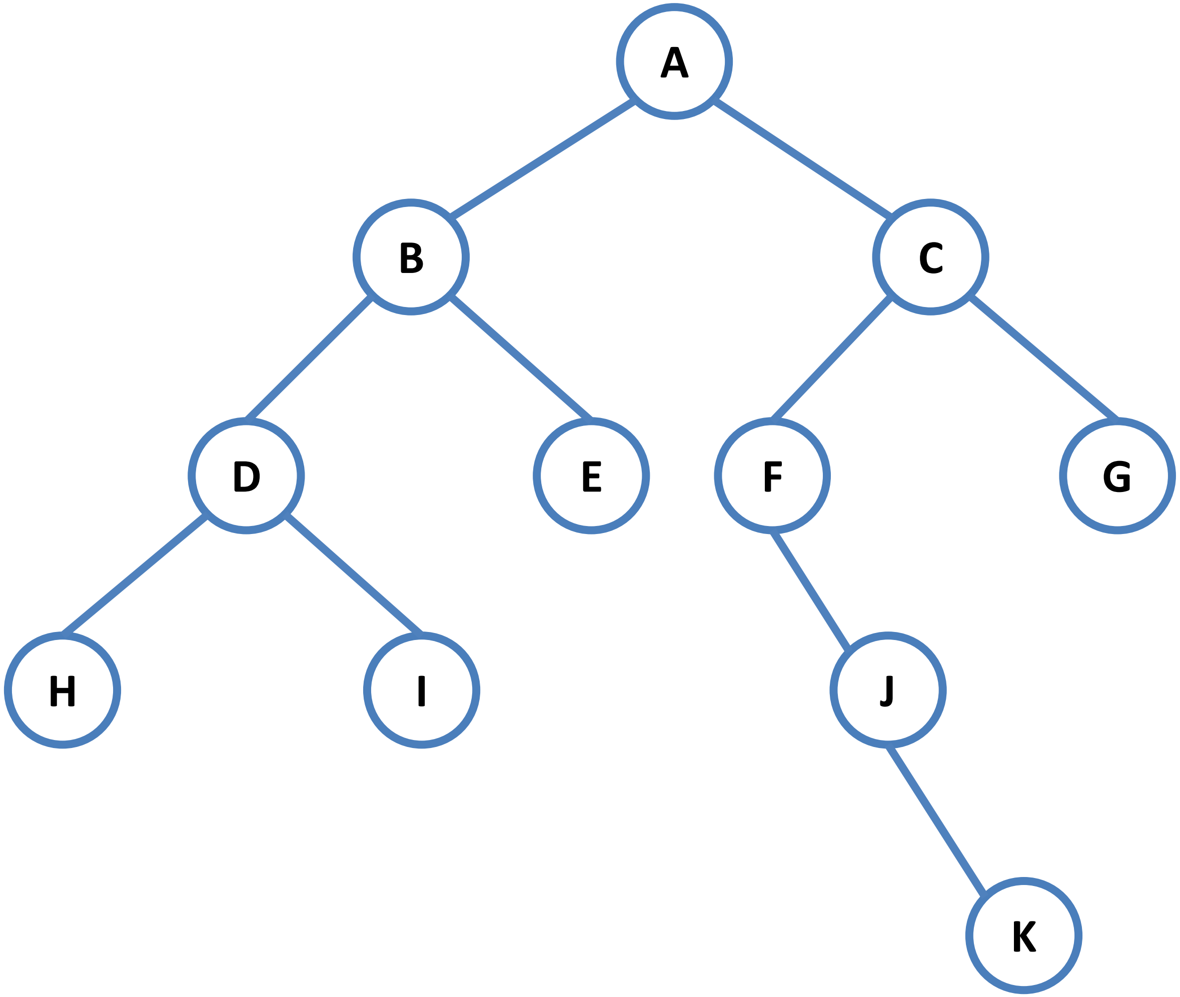
Graph

Traversal

- Start at A, BFS
- A
- B
- C
- D
- E
- F
- G



- Start at A, BFS
- A
- B
- C
- D
- E
- F
- G
- … and so on…



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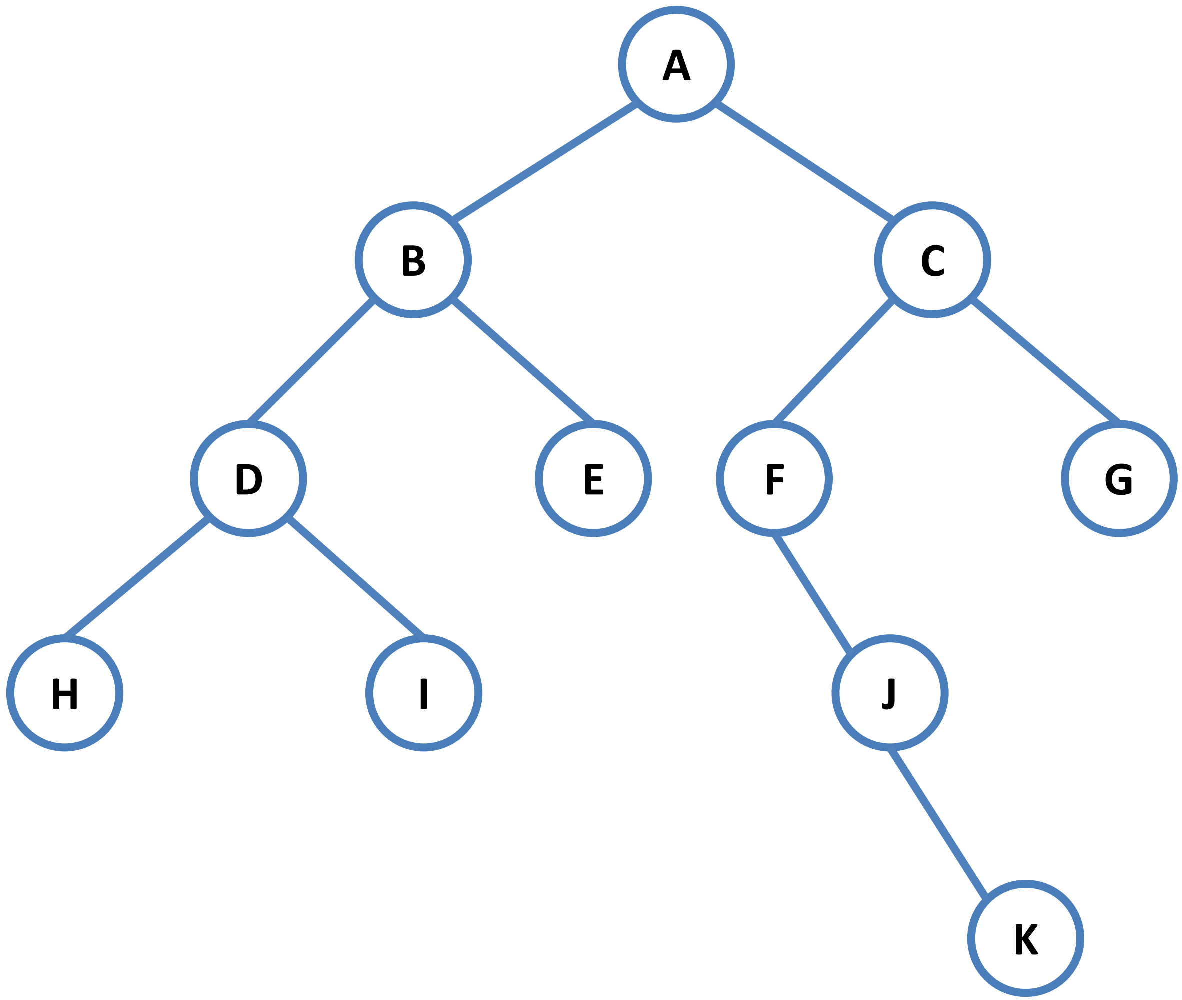
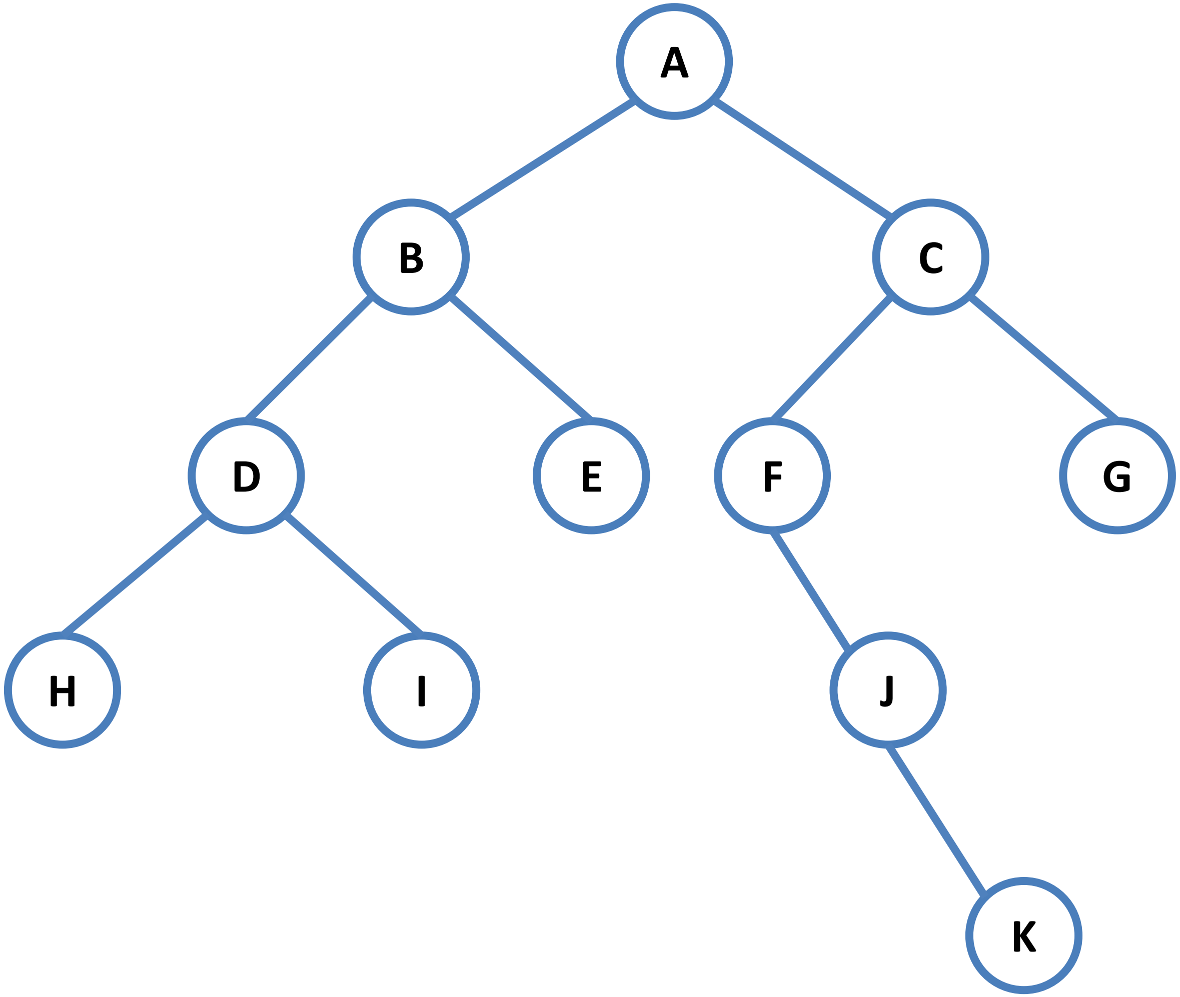
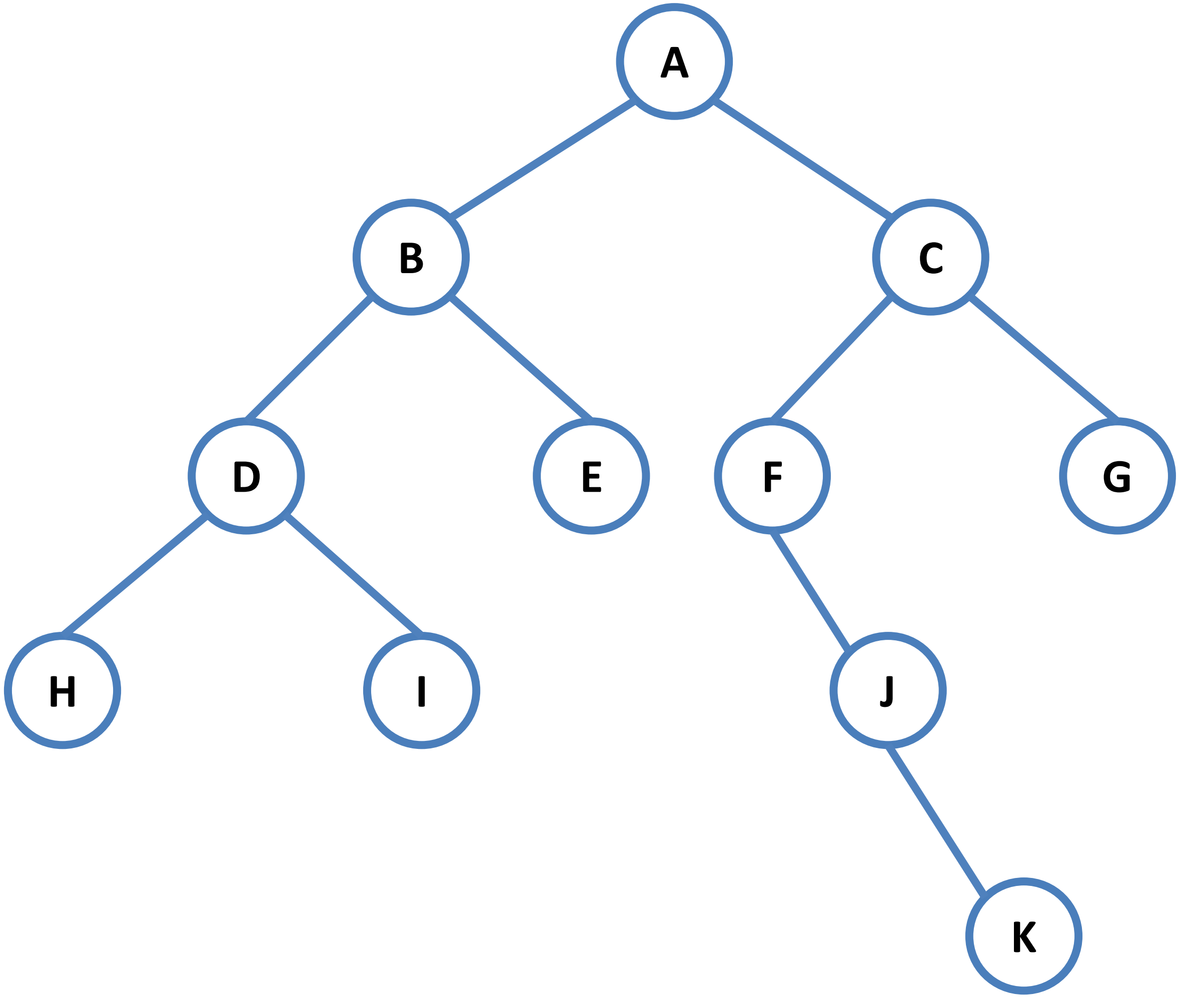


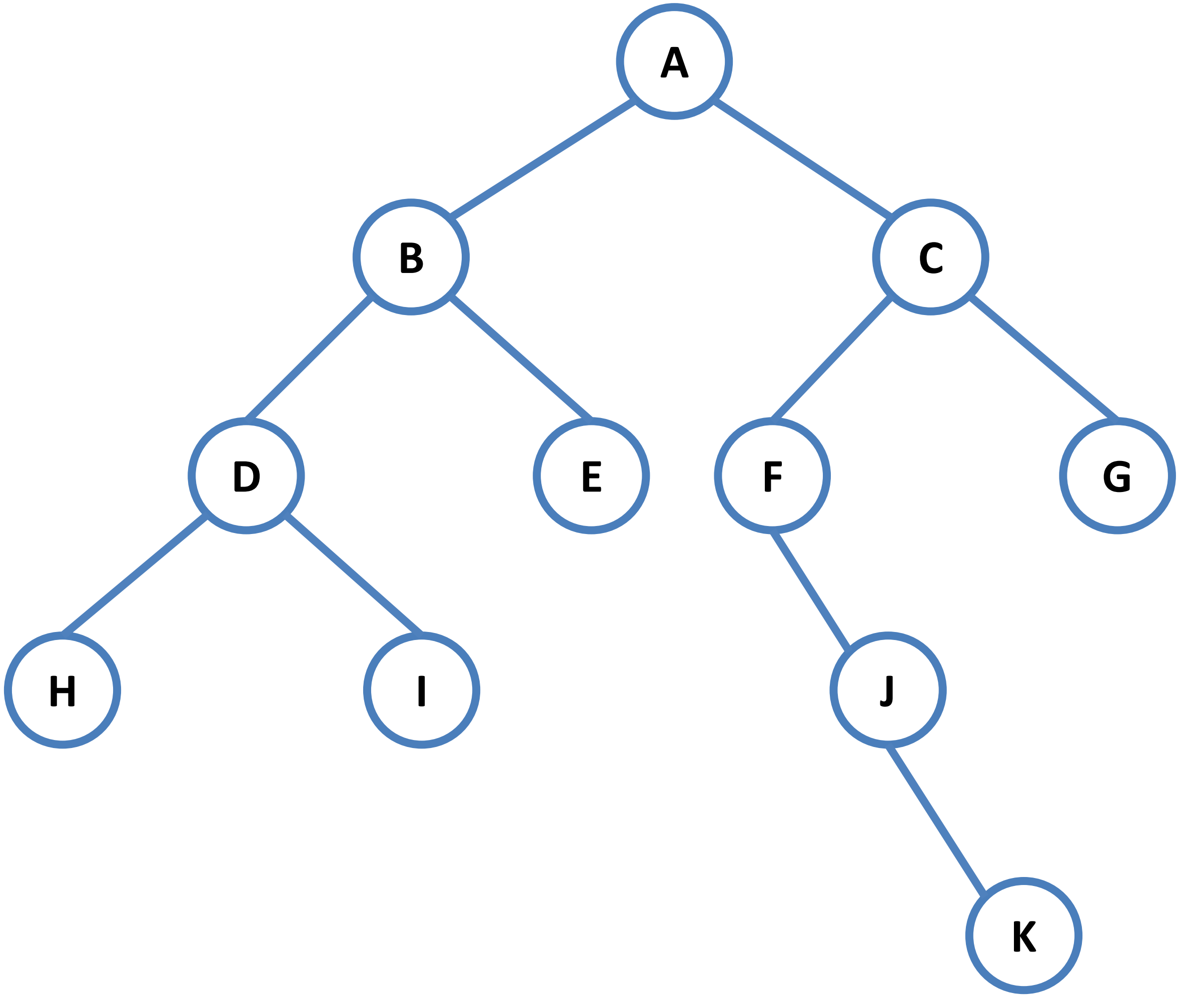
- Start at A, DFS



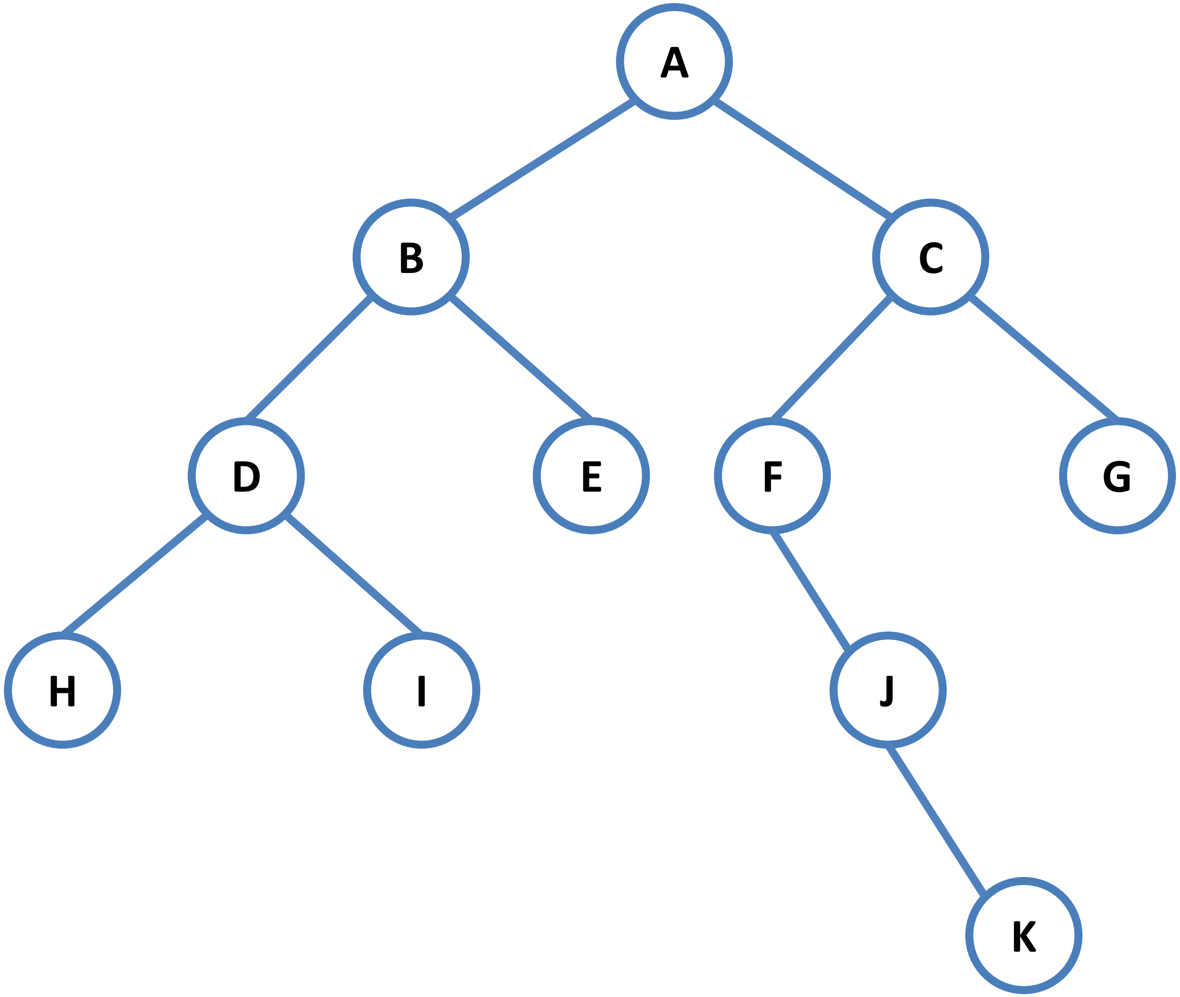
- Start at A, DFS
- A



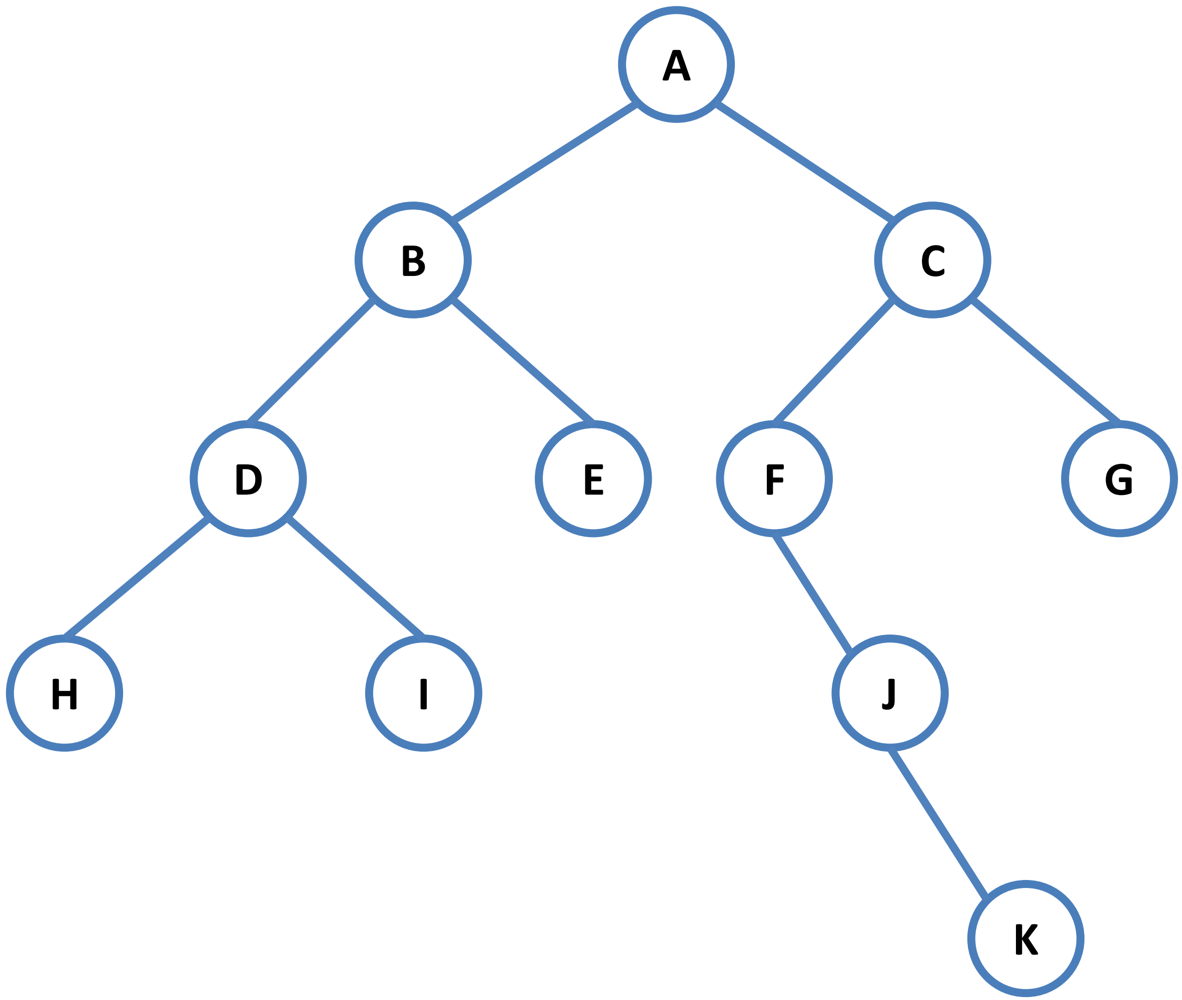
- Start at A, DFS
- A
- B



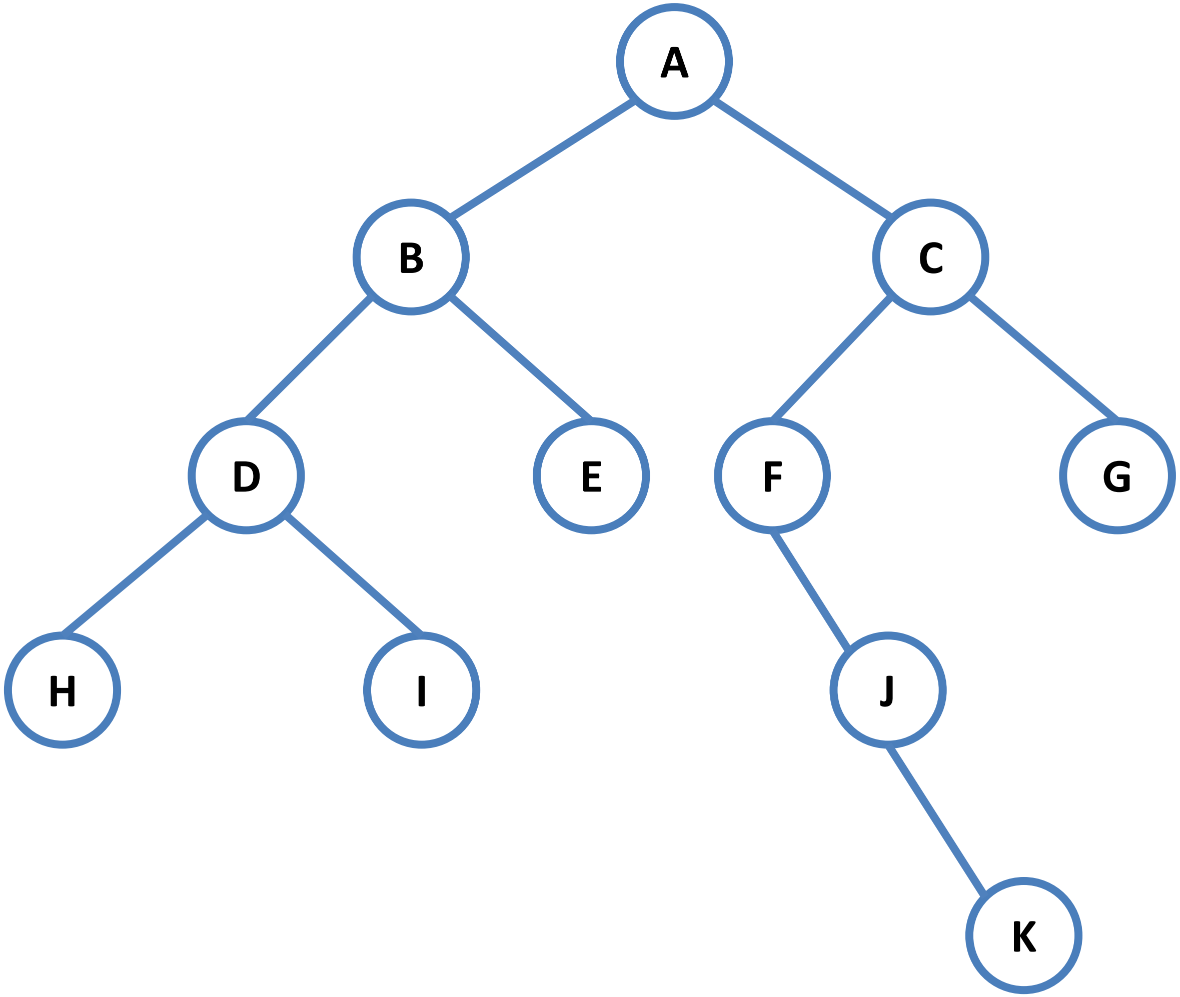
- Start at A, DFS
- A
- B
- D, go deep!



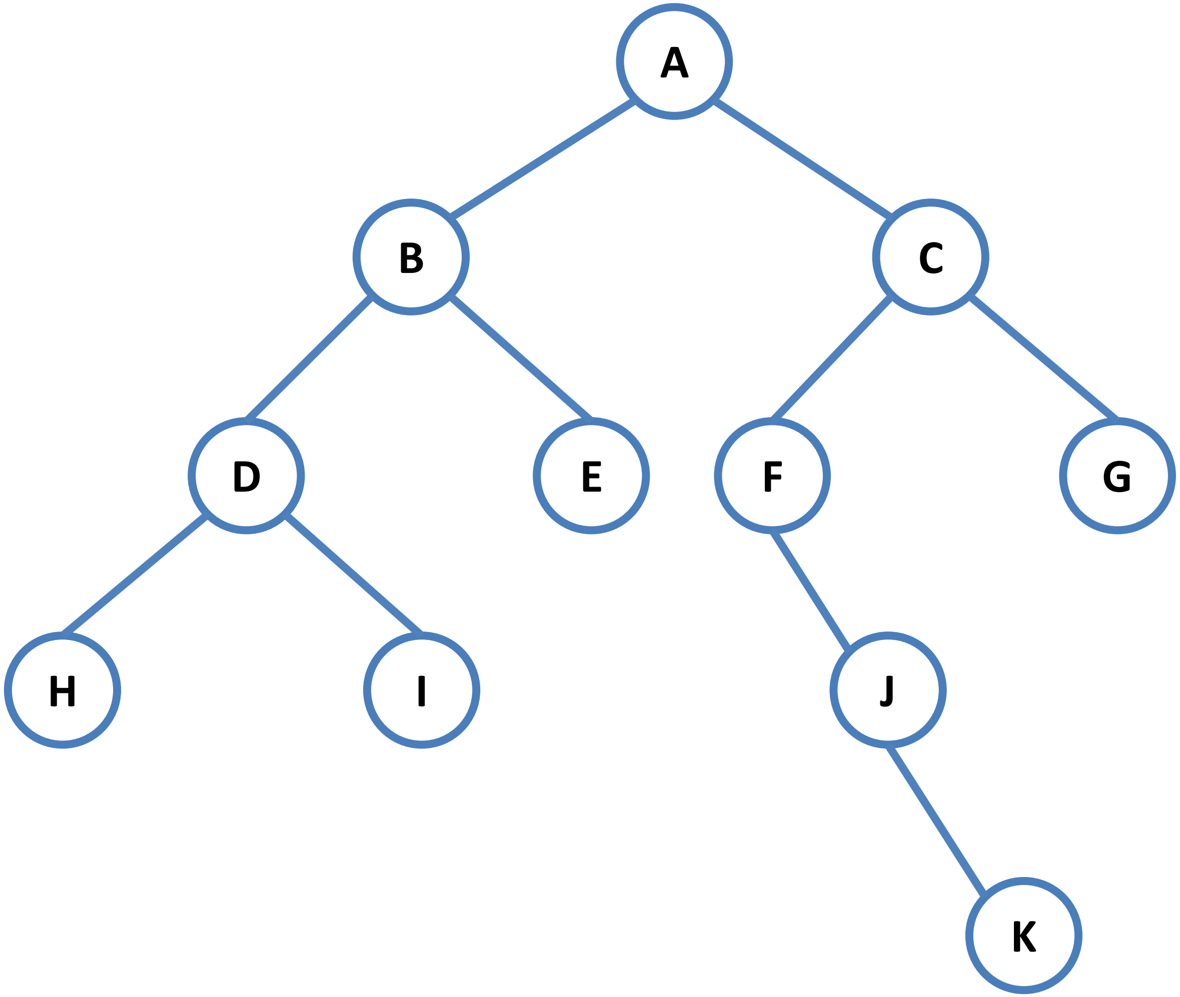
- Start at A, DFS
- A
- B
- D
- H



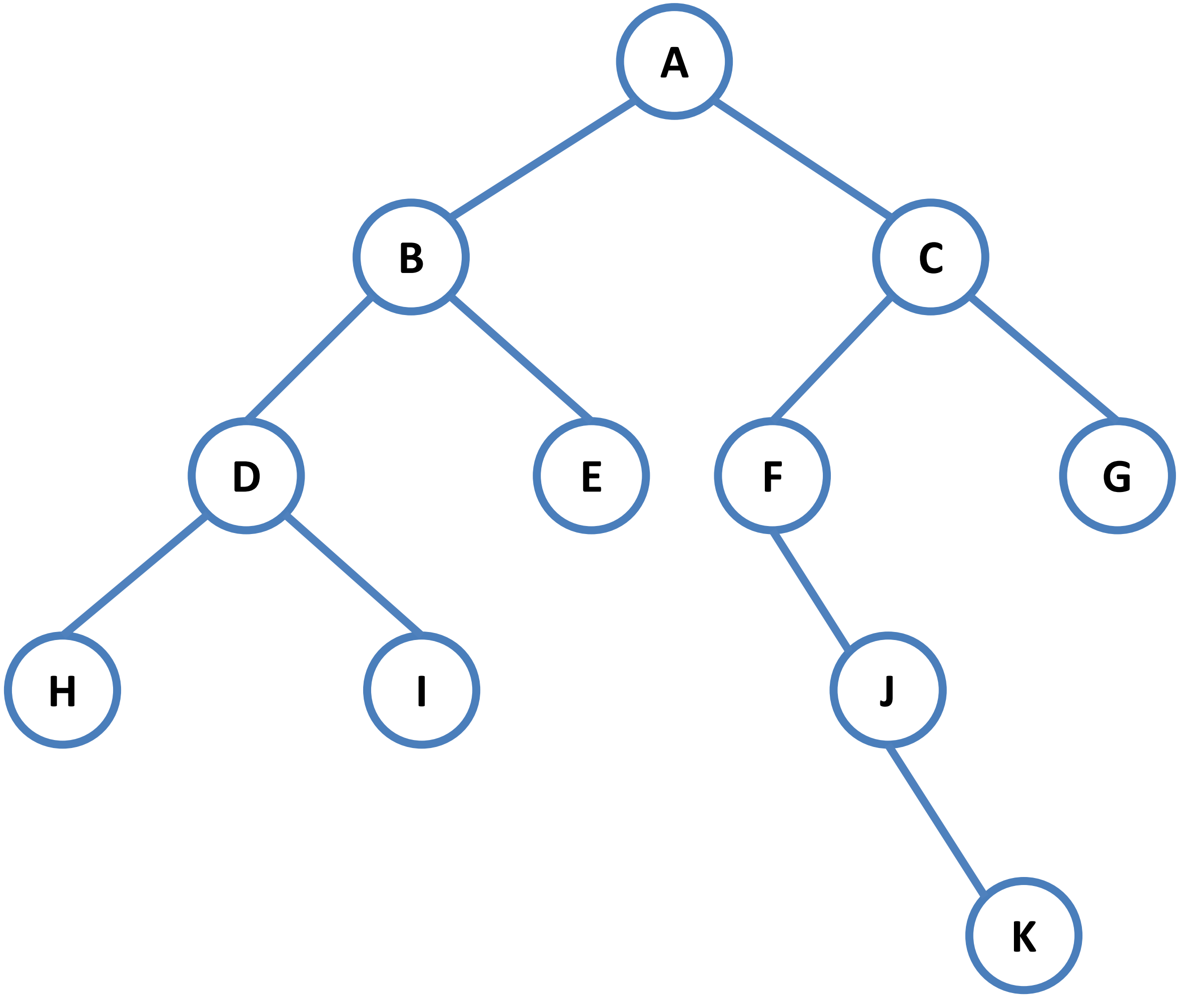
- Start at A, DFS
- A
- B
- D
- H
- can’t go deeper



- Start at A, DFS
- A
- B
- D
- H
- I

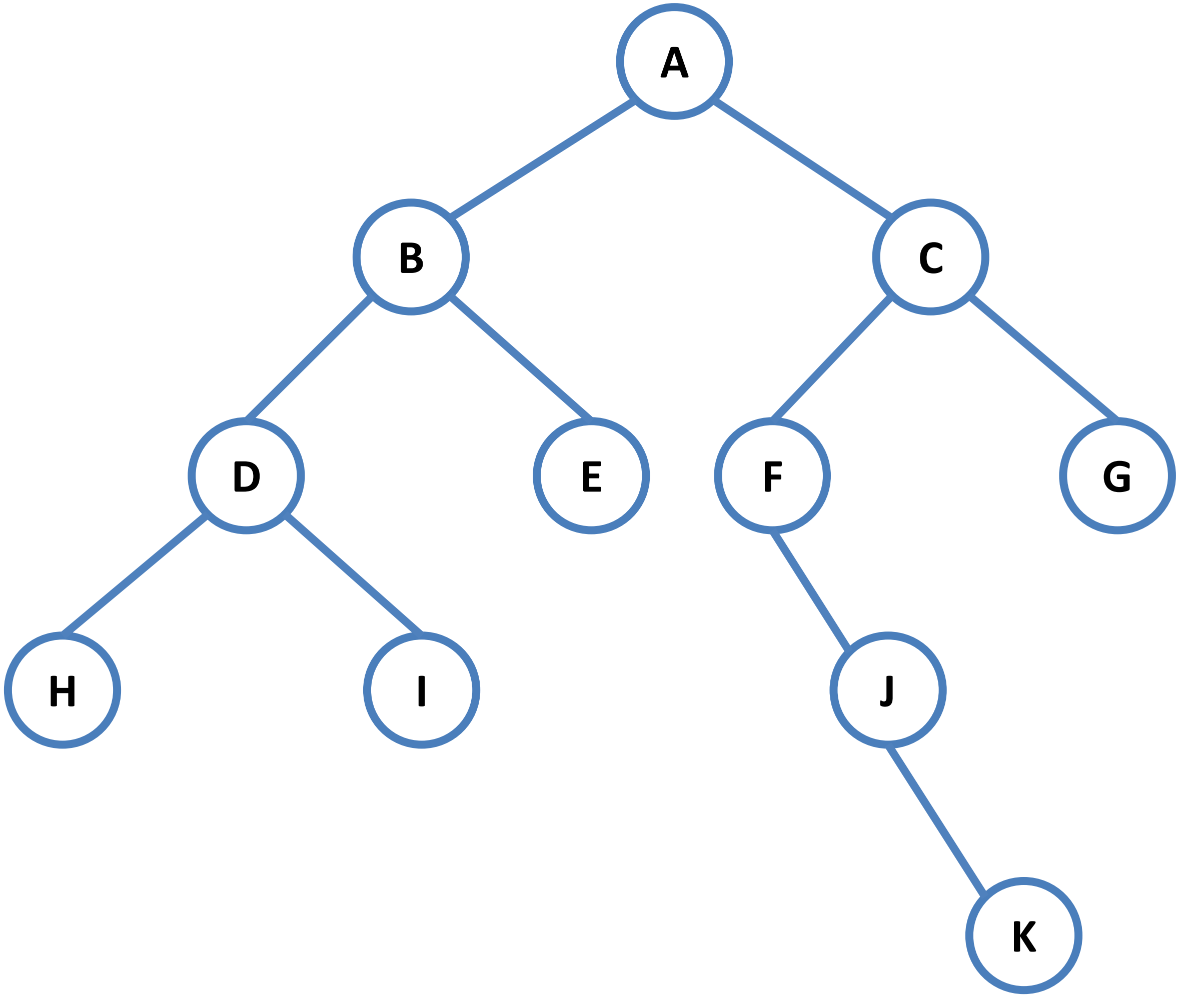


- Start at A, DFS
- A
- B
- D
- H
- I
- E

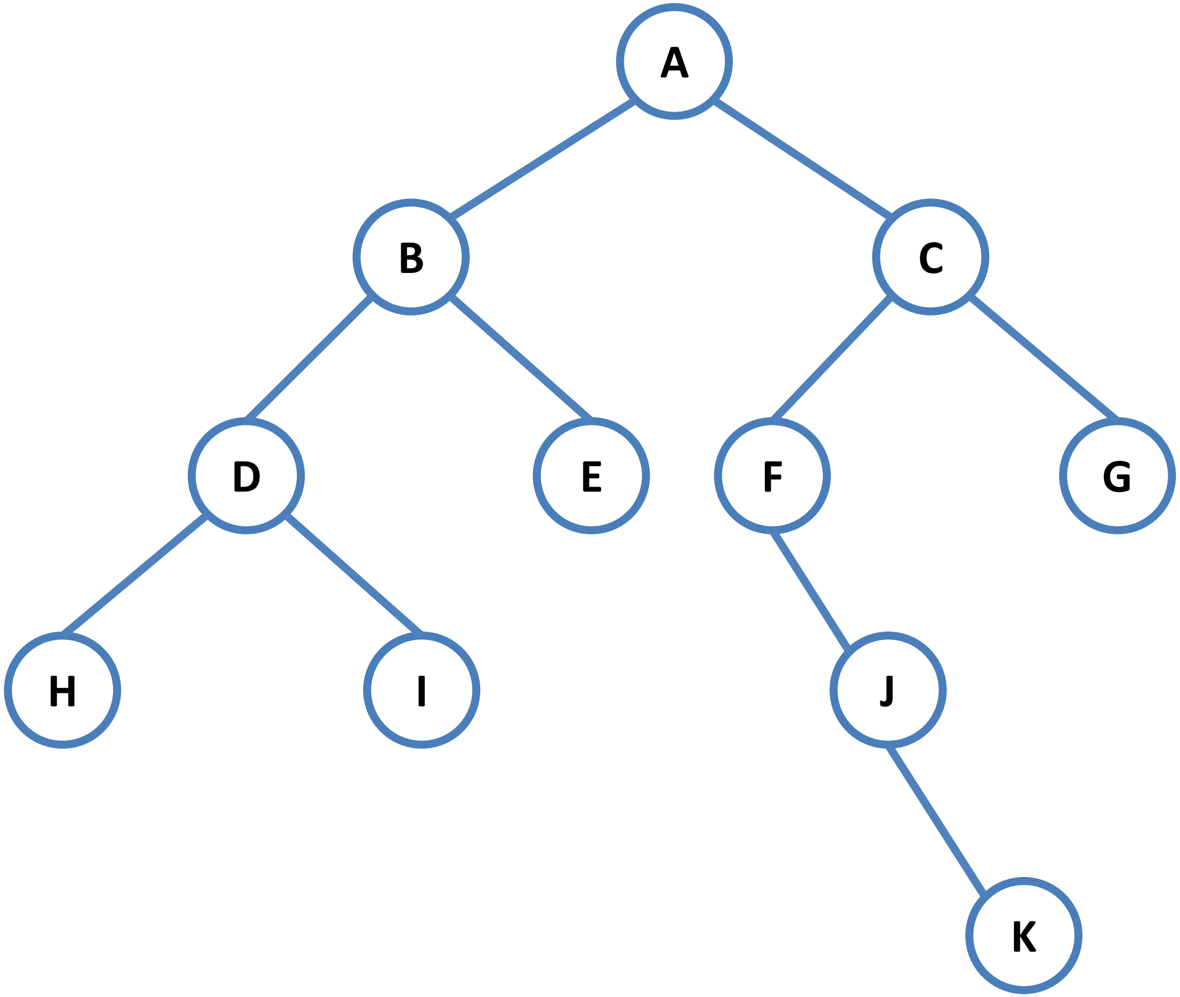


Graph
Traversal

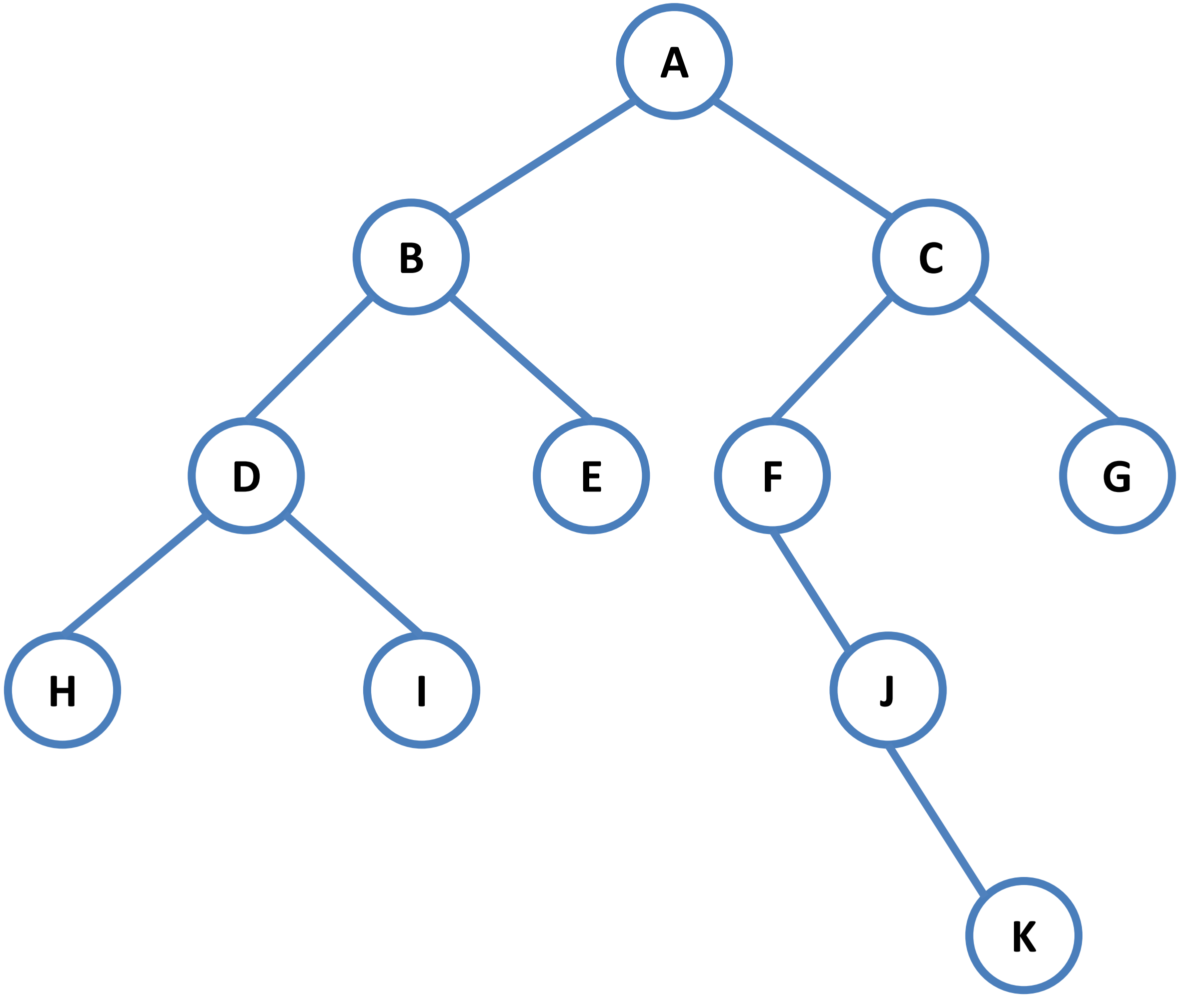
- Start at A, DFS
- A
- B
- D
- H
- I
- E
- C



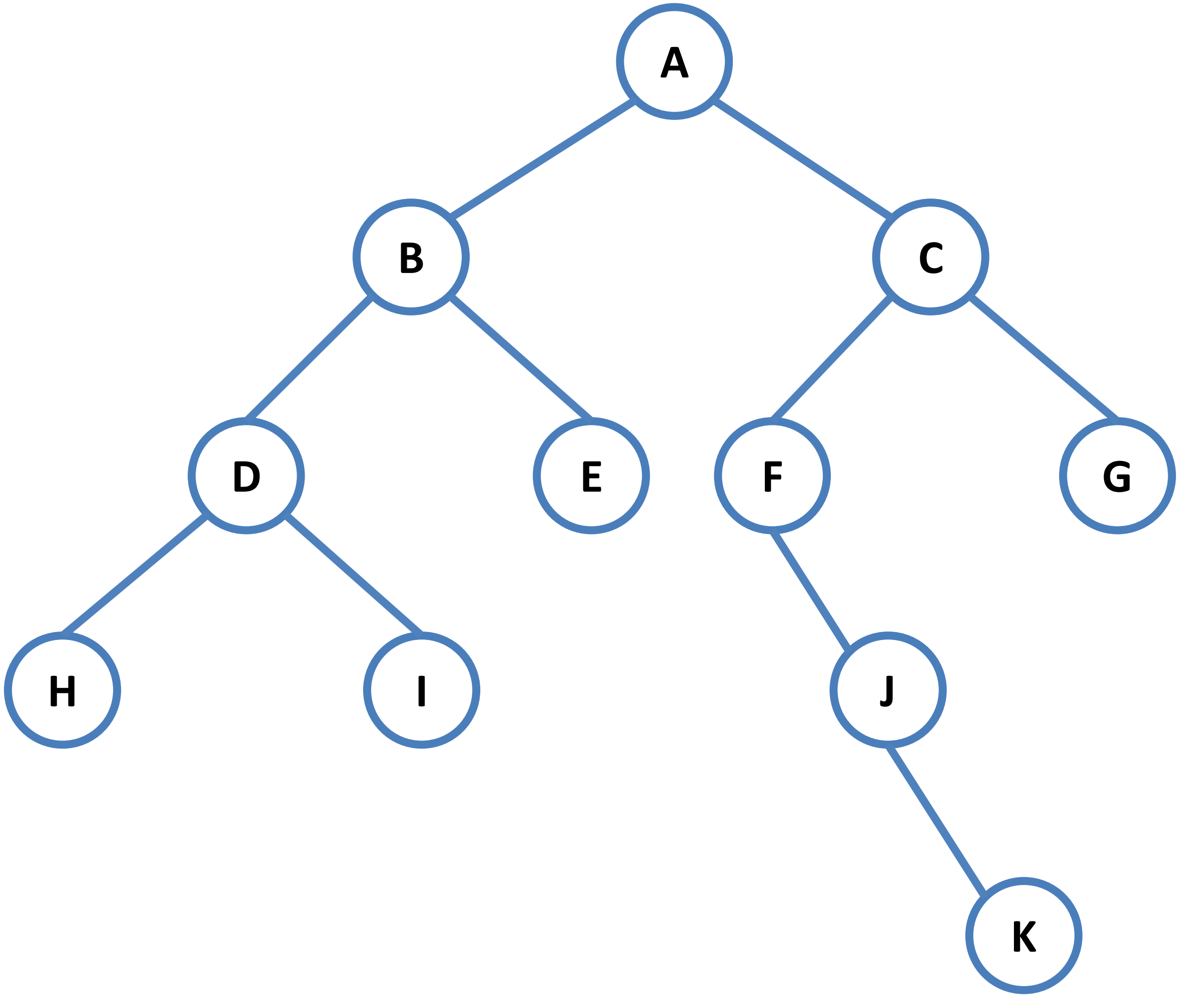
- Start at A, DFS
- A
- B
- D
- H
- I
- E
- C
- F



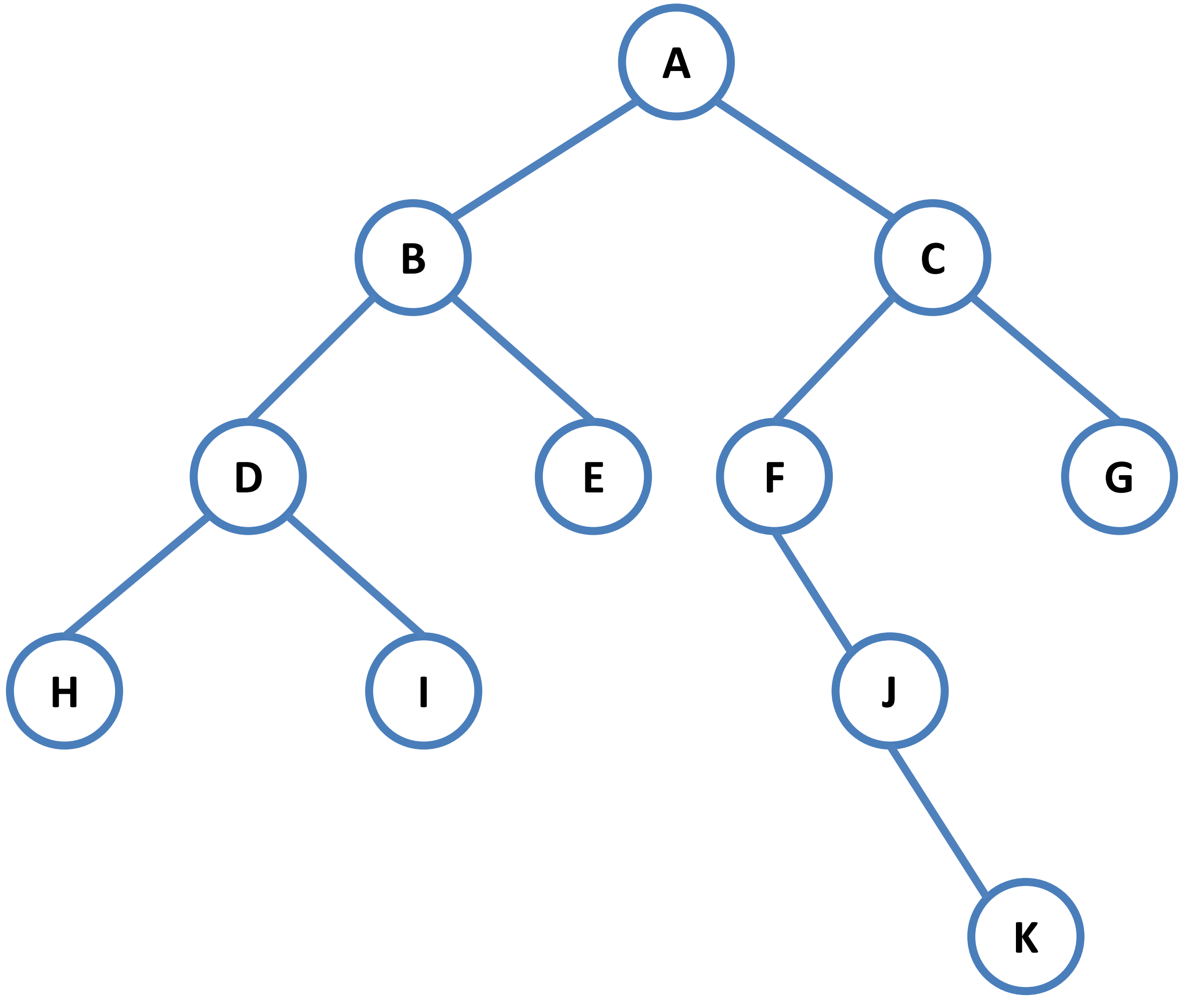
- Start at A, DFS
- A
- B
- D
- H
- I
- E
- C
- F
- J



- Start at A, DFS
- A
- B
- D
- H
- I
- E
- C
- F
- J
- K

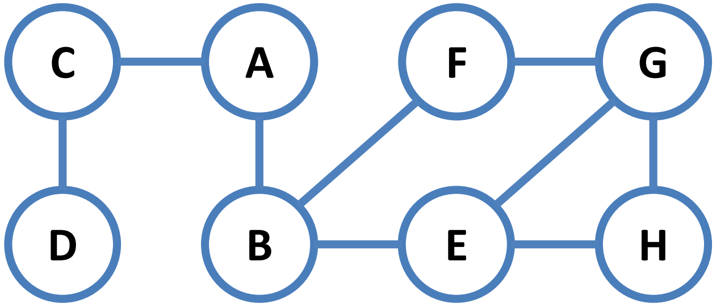


- Start at A, DFS
- B
- D
- H
- I
- E
- C
- F
- J
- K
- G, finally

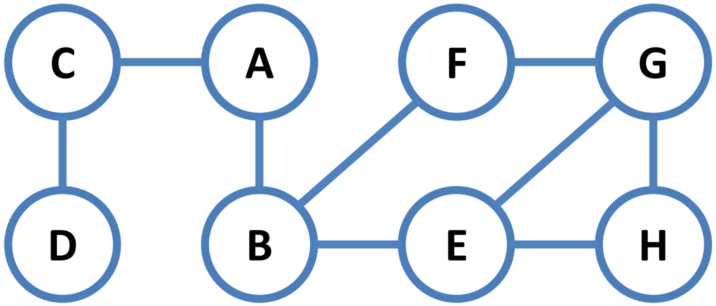


Questions?

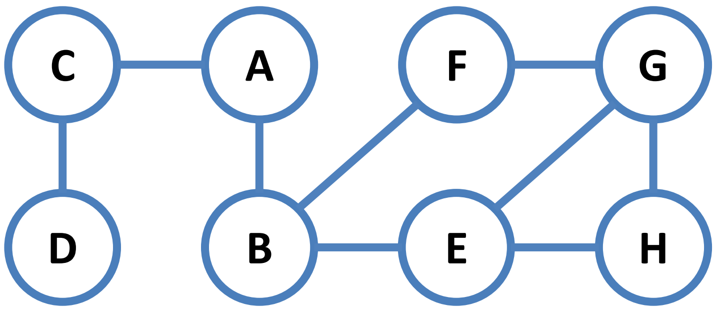
- How would you implement it?



- How would you implement it?
  - Let say we begin from vertex A

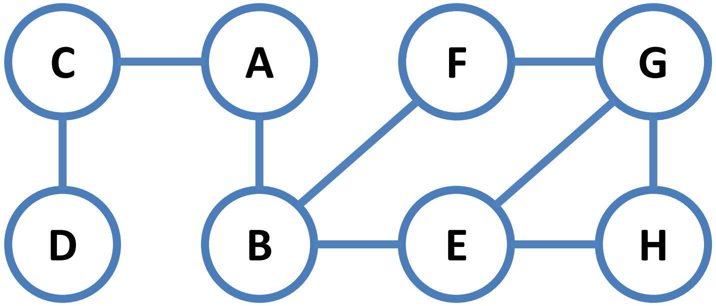


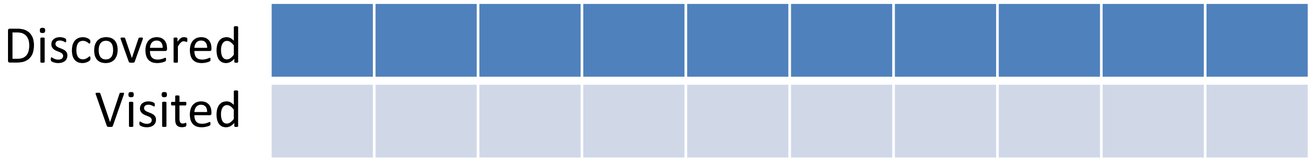
- How would you implement it?
  - Let say we begin from vertex A
  - What is our traversal?



Graph
BFS Implementation

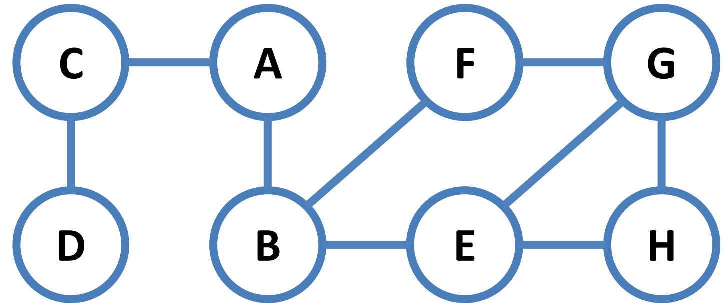
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered



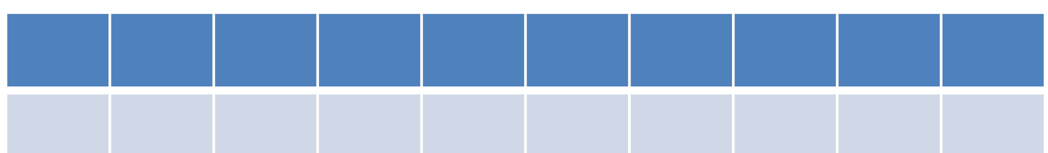


Graph
BFS Implementation

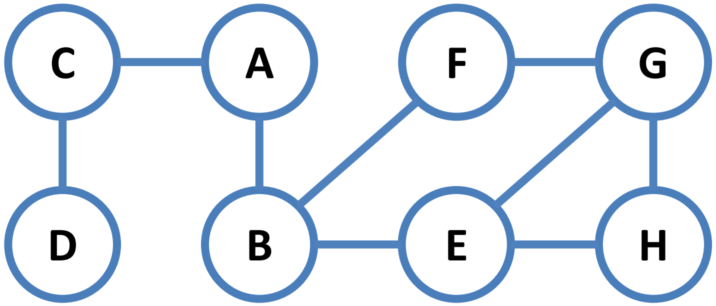
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Start with A



Discovered
Visited



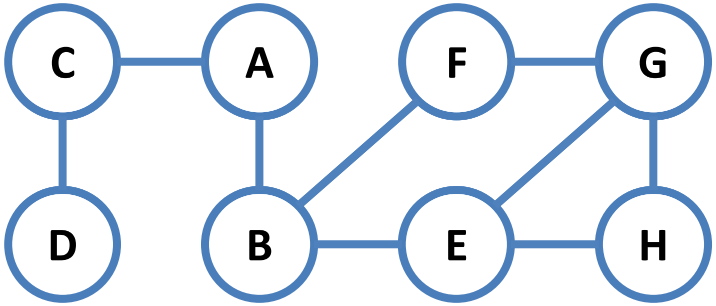
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it



Discovered	A									
Visited										

BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty

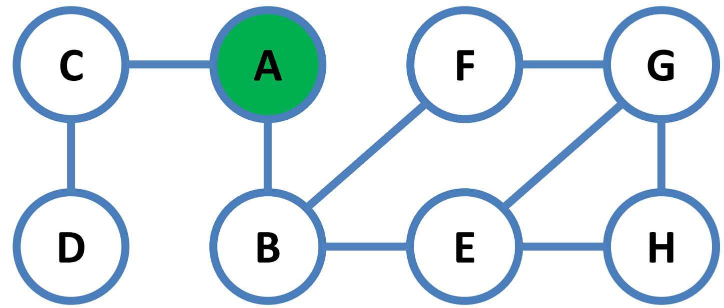


Discovered	A									
Visited										

Graph

BFS Implementation

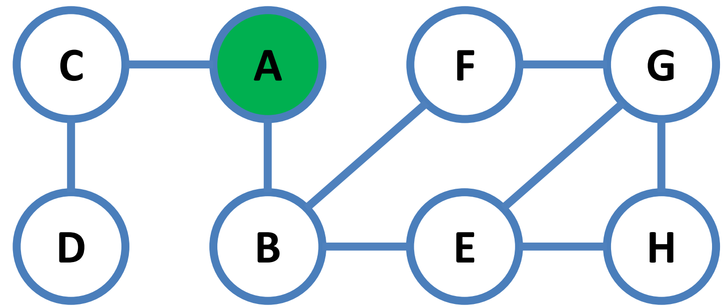
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered



Discovered	A									
Visited										

Graph
BFS Implementation

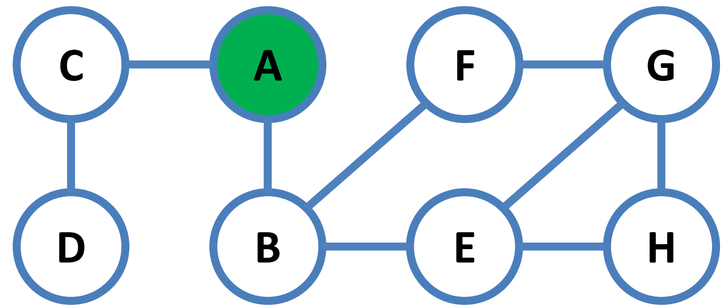
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited



Discovered										
Visited	A									

Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served

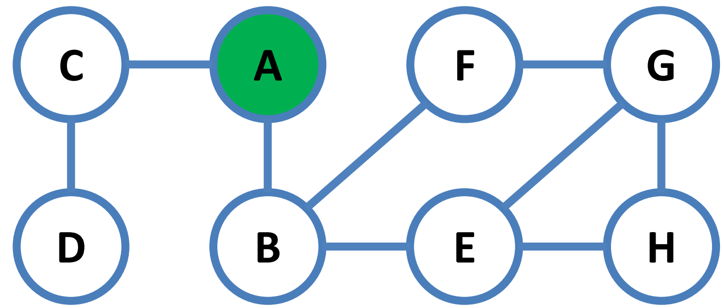


Discovered										
Visited	A									

Graph

BFS Implementation

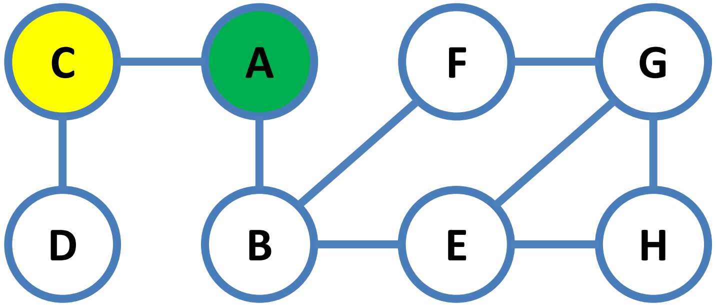
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



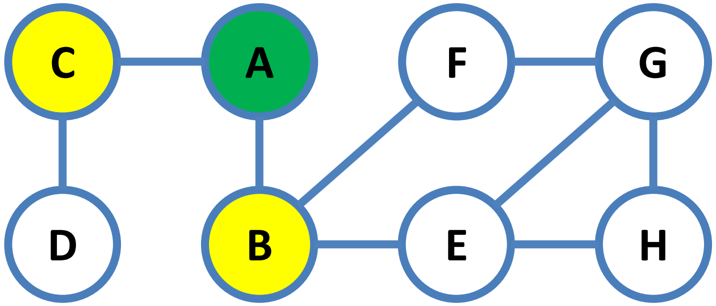
Discovered										
Visited	A									

Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered	C									
Visited	A									



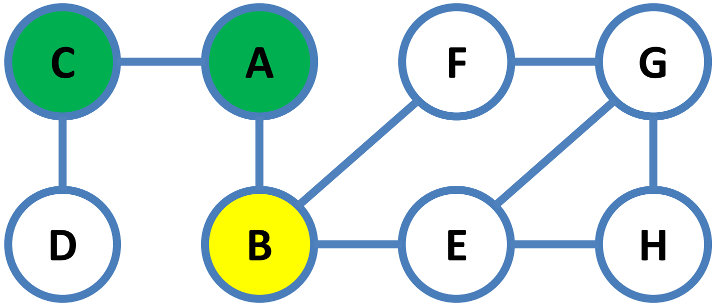
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

Discovered	C	B								
Visited	A									

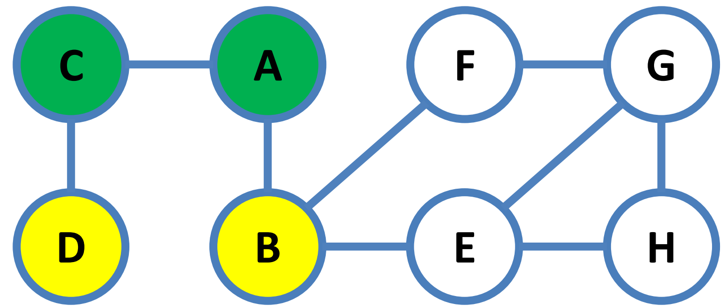
Graph

BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



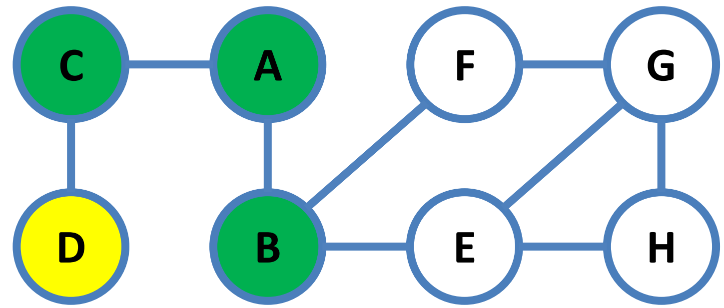
Discovered		**B**								
Visited	A	C								



- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

Discovered		B	D							
Visited	A	C								

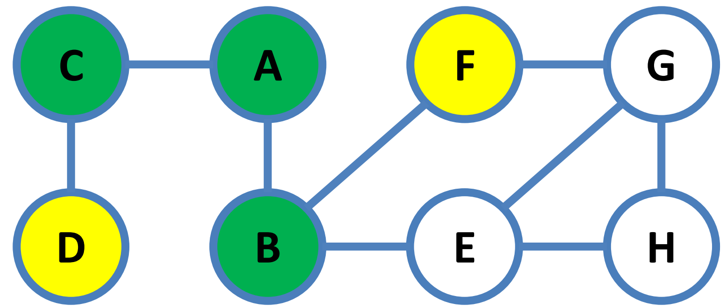
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered			D							
Visited	A	C	B							

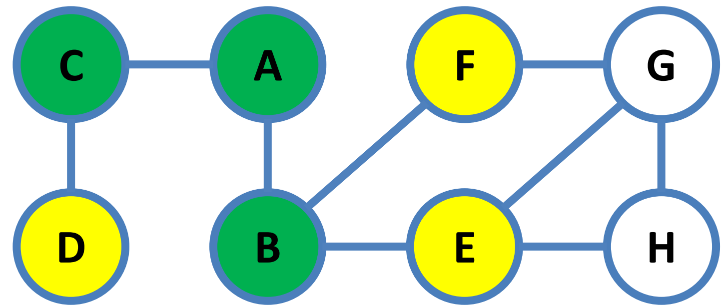
Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



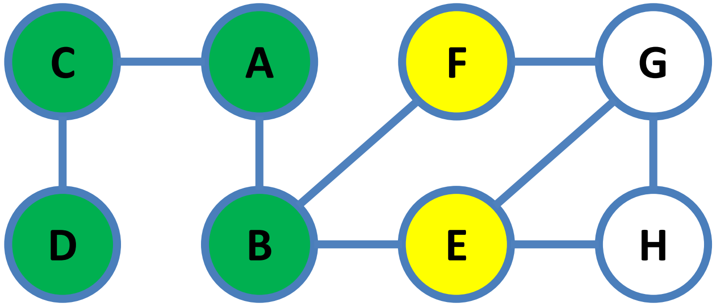
Discovered			**D**	**F**						
Visited	A	C	B							

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered			**D**	**F**	**E**					
Visited	A	C	B							

Graph
BFS Implementation

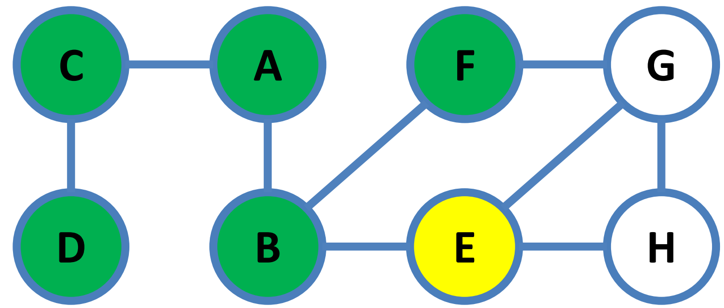


- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

Discovered				**F**	**E**					
Visited	A	C	B	D						

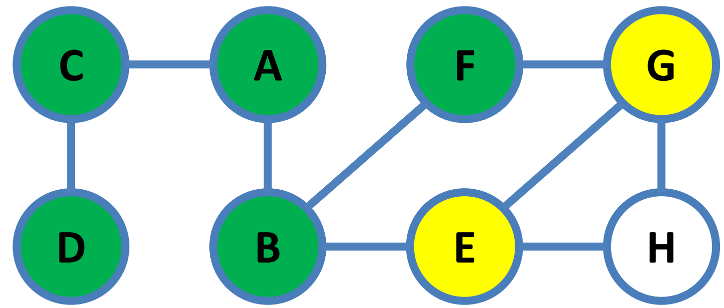
Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered					E					
Visited	A	C	B	D	F					

BFS Implementation



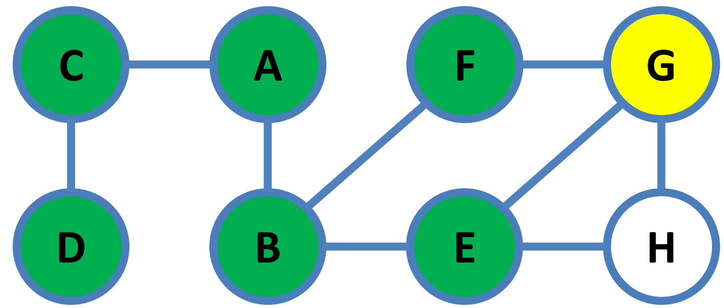
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

Discovered					E	G				
Visited	A	C	B	D	F					

Graph

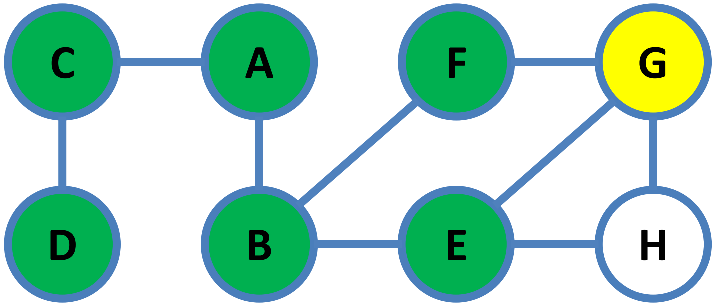
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered						G				
Visited	A	C	B	D	F	E				

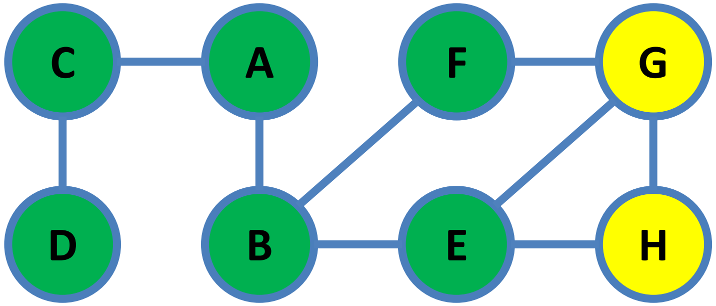
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered						G	G?			
Visited	A	C	B	D	F	E				

Graph
BFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

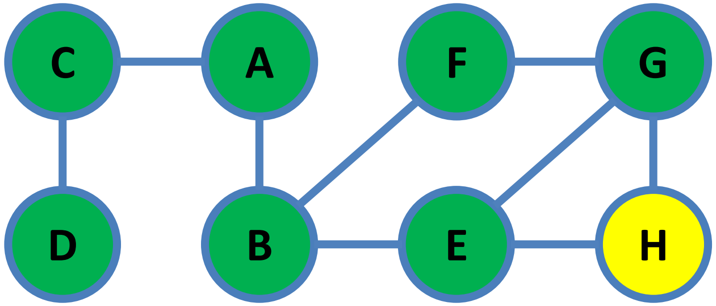


Discovered						G	H			
Visited	A	C	B	D	F	E				

Graph
BFS Implementation

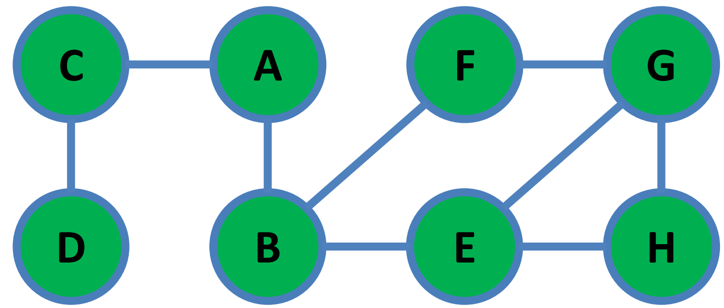
MONASH University

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



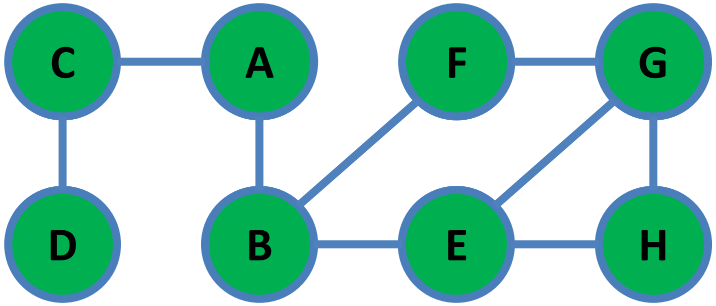
Discovered							H			
Visited	A	C	B	D	F	E	G			

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered										
Visited	A	C	B	D	F	E	G	H		

Graph
BFS Implementation



- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue

- The traversal answer is not unique

Discovered										
Visited	A	C	B	D	F	E	G	H		

BFS Implementation

- Complexity?

- Complexity?
  - Time is O(V+E)

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice

Graph
BFS Implementation

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)
    - V maximum for the discovered queue

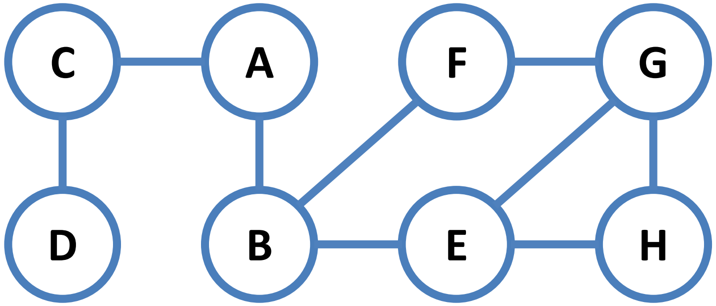
- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)
    - V maximum for the discovered queue
    - E to stored all of the edges (adjacency list)

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)
    - V maximum for the discovered queue
    - E to stored all of the edges (adjacency list)
  - But don’t we need to check the discovered queue for each vertex v?
    - O(V) search through the queue?

- Complexity?
  - Time is O(V+E)
    - Each vertex is visited once
    - Each edge is visited twice
      - For each <u,v> we visit from u and also from v
  - Space is O(V+E)
    - V maximum for the discovered queue
    - E to stored all of the edges (adjacency list)
  - But don’t we need to check the discovered queue for each vertex v?
    - O(V) search through the queue?
    - NO! Implement a Node class with self.discovered = True/ False

Questions?

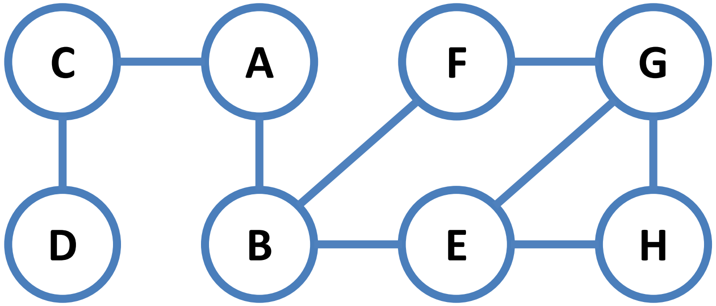
- How would you implement it?
  - Let say we begin from vertex A
  - What is our DFS traversal?



Graph

DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered

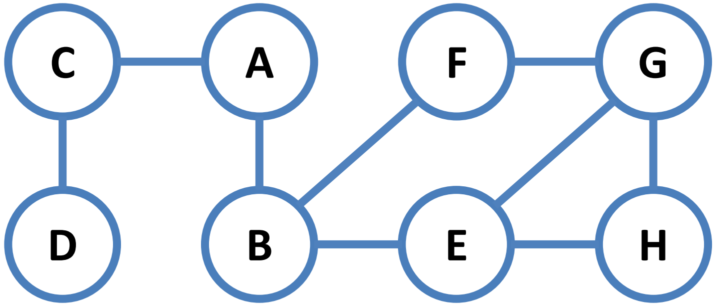


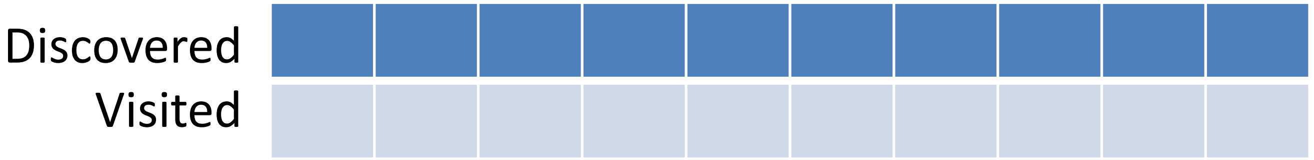
Discovered

Visited

DFS Implementation

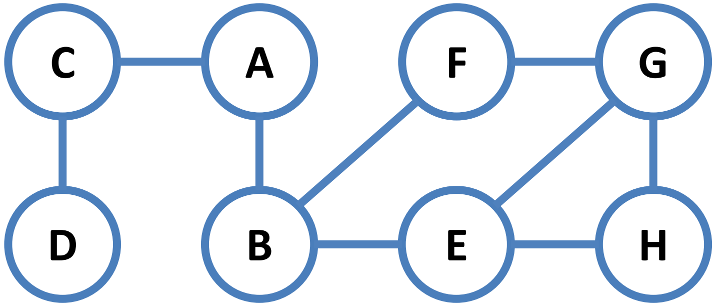
- How would you implement it?
  - Let say we begin from vertex A
  - Have a ~~queue~~ stack for discovered





Graph
DFS Implementation

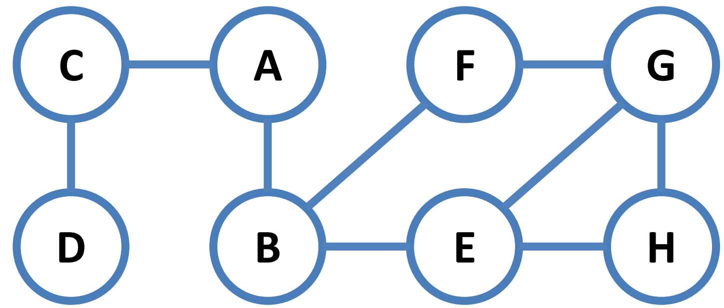
- How would you implement it?
  - Let say we begin from vertex A
  - Have a stack for discovered
    - Push source (A) into it



Discovered	A									
Visited										

Graph
DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a stack for discovered
    - Push source (A) into it
  - While discovered is not empty
    - Pop from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered	A									
Visited										

DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
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    - Push source (A) into it
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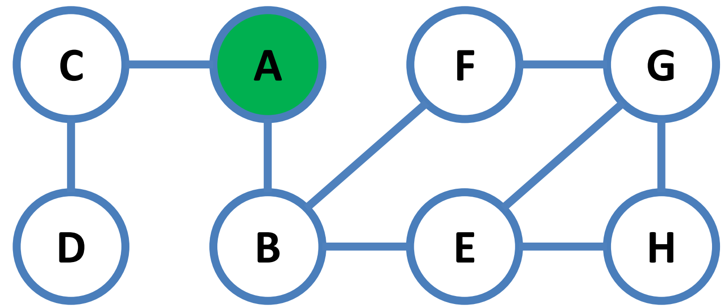

Discovered										
Visited	A									

Graph

DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a stack for discovered
    - Push source (A) into it
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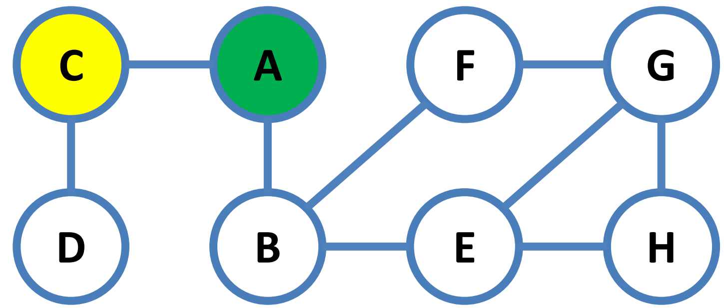

Discovered	C									
Visited	A									

Graph
DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a stack for discovered
    - Push source (A) into it
  - While discovered is not empty
    - Pop from discovered, to visited
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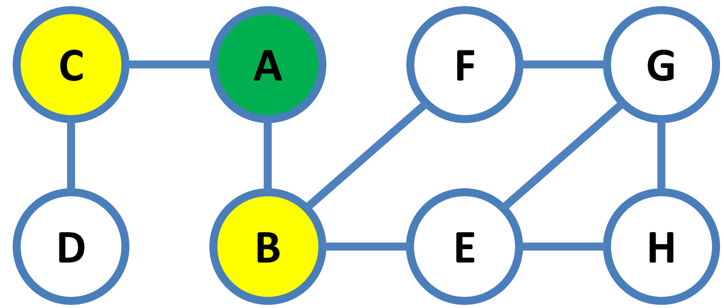

Discovered	C	B								
Visited	A									

Graph
DFS Implementation

MONASH University

- How would you implement it?
  - Let say we begin from vertex A
  - Have a stack for discovered
    - Push source (A) into it
  - While discovered is not empty
    - Pop from discovered, to visited
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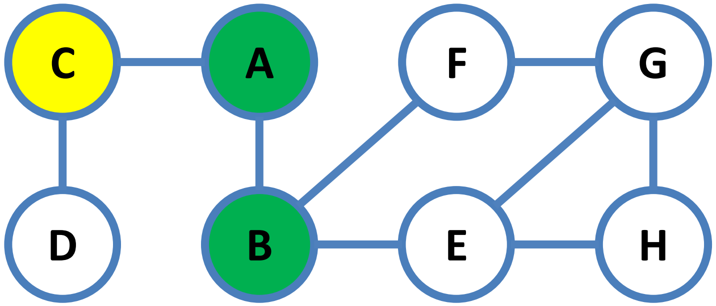

Discovered	C									
Visited	A	B								

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  - Let say we begin from vertex A
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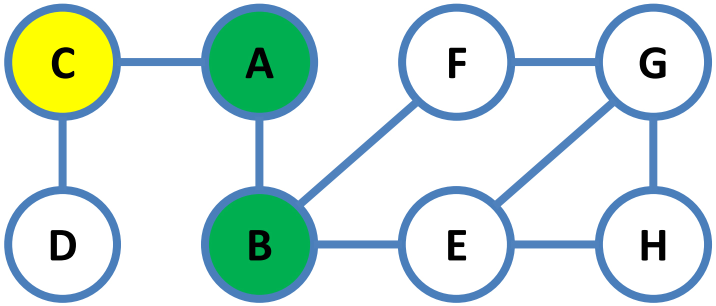
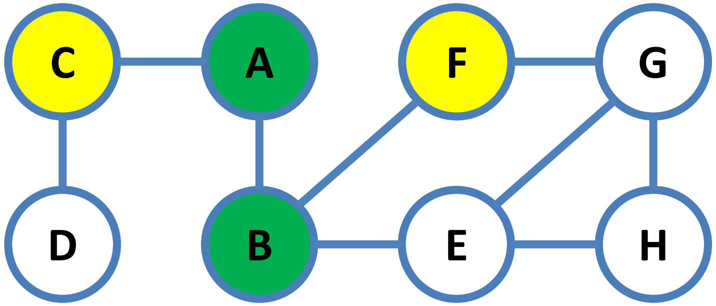


Discovered	**C**	**A?**								
Visited	A	B								



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    - Push source (A) into it
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Discovered	C	F								
Visited	A	B								

Graph

DFS Implementation

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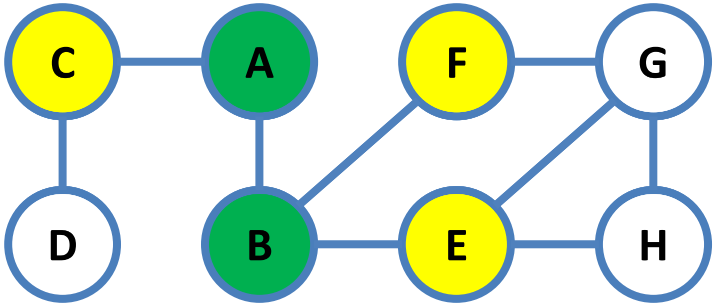
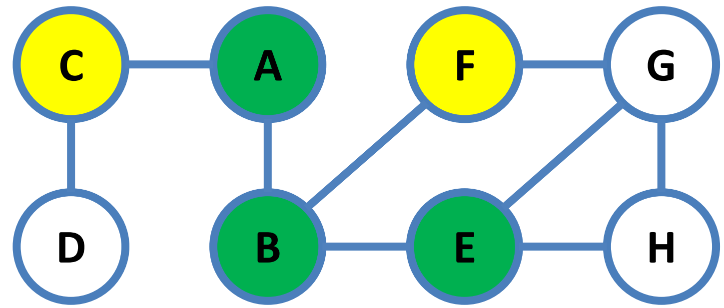


Discovered	**C**	**F**	**E**							
Visited	A	B								

DFS Implementation

MONASH University



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Discovered	C	F								
Visited	A	B	E							

Graph
DFS Implementation

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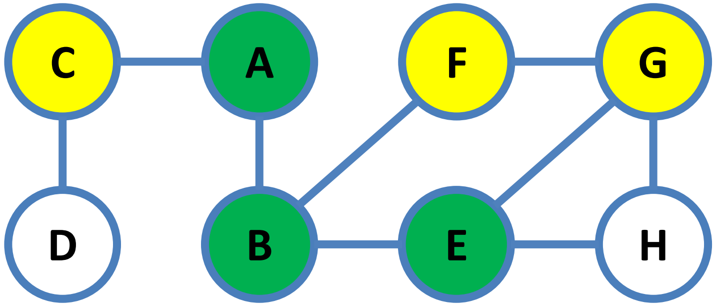

Discovered	C	F	G							
Visited	A	B	E							

Graph
DFS Implementation

- How would you implement it?
  - Let say we begin from vertex A
  - Have a stack for discovered
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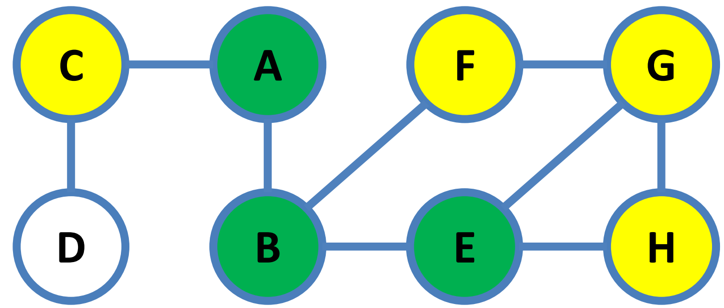

Discovered	**C**	**F**	**G**	**H**						
Visited	A	B	E							

Graph
DFS Implementation

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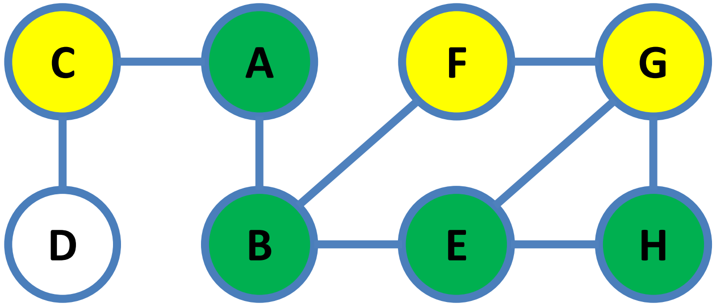

Discovered	**C**	**F**	**G**							
Visited	A	B	E	H						

DFS Implementation

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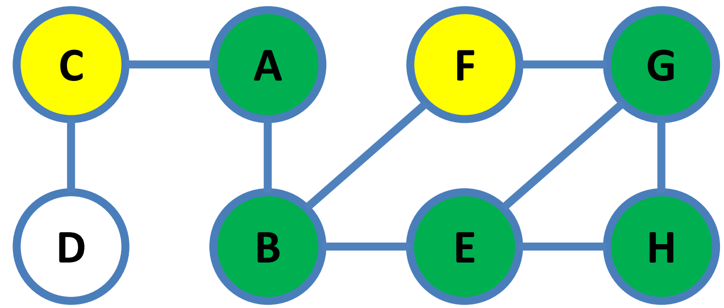

Discovered	**C**	**F**								
Visited	A	B	E	H	G					

Graph

DFS Implementation

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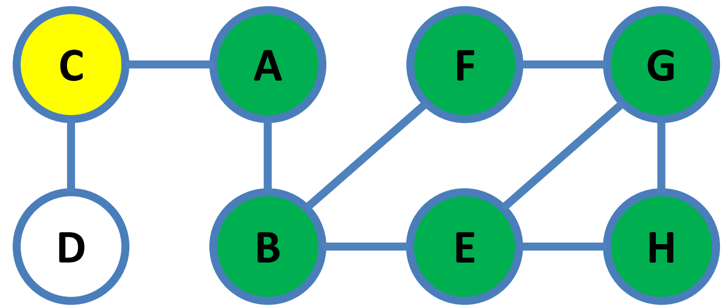

Discovered	C									
Visited	A	B	E	H	G	F				

Graph
DFS Implementation

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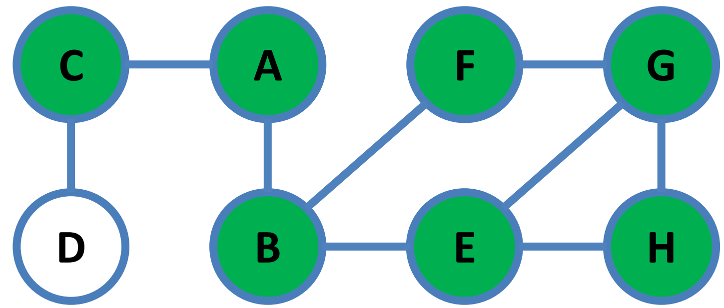

Discovered										
Visited	A	B	E	H	G	F	C			

Graph
DFS Implementation

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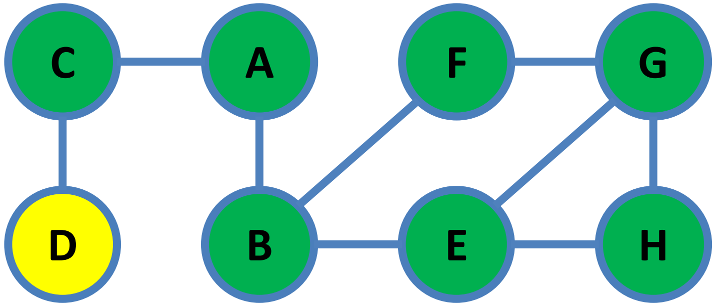

Discovered	**D**									
Visited	A	B	E	H	G	F	C			

Graph
DFS Implementation

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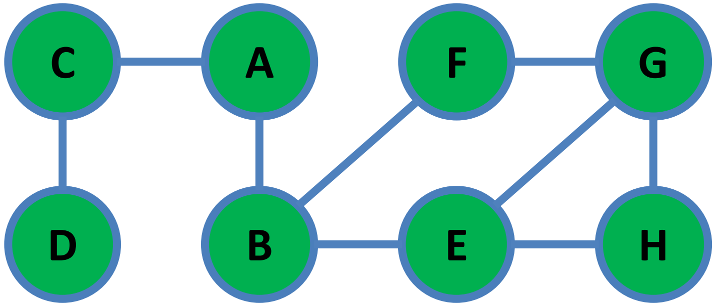

Discovered										
Visited	A	B	E	H	G	F	C	D		

- Complexity?

- Complexity?
  - Time is O(V+E)
    - Explanation same as BFS

DFS Implementation

- Complexity?
  - Time is O(V+E)
    - Explanation same as BFS
  - Space is O(V+E)
    - Explanation same as DFS

- Can you think of another way to implement DFS?

DFS Implementation

- Can you think of another way to implement DFS?
- Recursion!
  - Best way to implement any form of traversal
    - Just like when you implemented tree/ trie traversal

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- Let us just write them all out as a live coding session!

Questions?

- Can you think of another way to implement DFS?
- Recursion!
  - Best way to implement any form of traversal
    - Just like when you implemented tree/ trie traversal

```python
def dfs(current_vertex):
    current_vertex.visited = True
    for next_vertex in current_vertex.adjacent:
        if next_vertex.visited == False:
            dfs(next_vertex)

source_vertex = A
dfs(source_vertex)
```

- Can you think of another way to implement DFS?
- Recursion!
  - Best way to implement any form of traversal
    - Just like when you implemented tree/ trie traversal
  - Make sense because we are going depth-first like how recursion does it!

```python
def dfs(current_vertex):
    current_vertex.visited = True
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        if next_vertex.visited == False:
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```

Questions?

- As mentioned earlier, it is the basic algorithm for many more complex algorithm… the application include:

DFS and BFS application?

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  - Reachability
  - Finding all connected components
  - Finding cycles
  - Shortest path (brute force)
  - Shortest path (non-brute force) on unweighted graph
  - Topological sort (later on)
  - … and many more!

DFS and BFS application?

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  - Reachability
  - Finding all connected components
  - Finding cycles
  - Shortest path (brute force)
  - Shortest path (non-brute force) on unweighted graph
  - Topological sort (later on)
  - … and many more!

  - We will see more in unit notes and tutorials

Questions?

Break!

- Classical problem

- Classical problem
  - Given a set of locations
  - Given routes between locations
  - Given the distance between locations

- Classical problem
  - Given a set of locations
  - Given routes between locations
  - Given the distance between locations
  - Can you find the shortest distance from a source to a destination?

- Classical problem
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  - Given routes between locations
  - Given the distance between locations
  - Can you find the shortest distance from a source to a destination?
  - What is the path?

- Classical problem
  - Given a set of locations as vertices V
  - Given routes between locations as edges E
  - Given the distance between locations as weights W
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- Classical problem
  - Given a set of locations as vertices V
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- If the graph is unweighted?

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- If the graph is unweighted?
  - Use BFS from the source!

- Classical problem
  - Given a set of locations as vertices V
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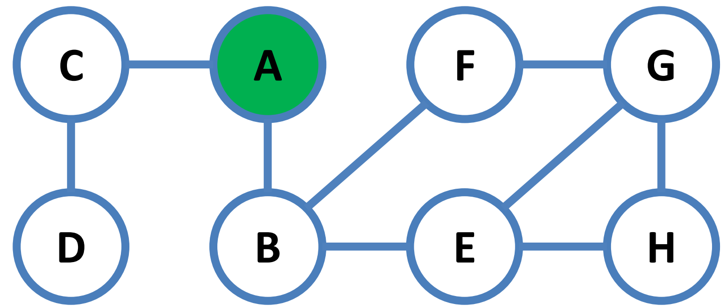
  - Given the distance between locations as weights W
  - Can you find the shortest distance from a source to a destination?
  - What is the path? This would require backtracking

- If the graph is unweighted?
  - Use BFS from the source!
  - Look back our BFS example

Graph
Shortest distance with BFS

- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it with a distance 0
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue



Discovered	A,0									
Visited										

Graph

Shortest distance with BFS

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  - Have a queue for discovered
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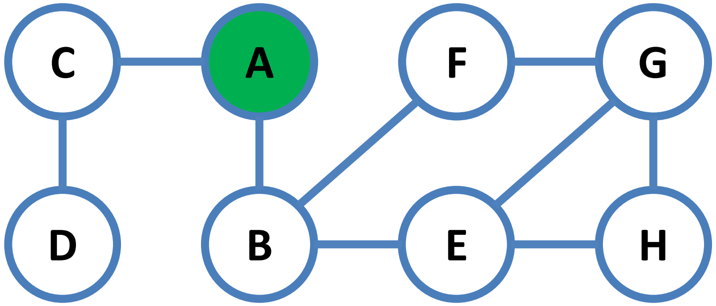
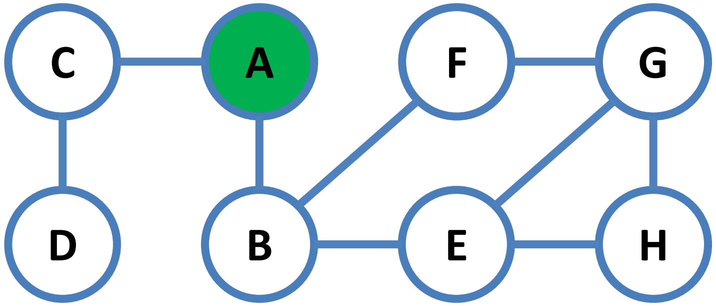


Discovered										
Visited	A,0									

Graph
Shortest distance with BFS



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  - Let say we begin from vertex A
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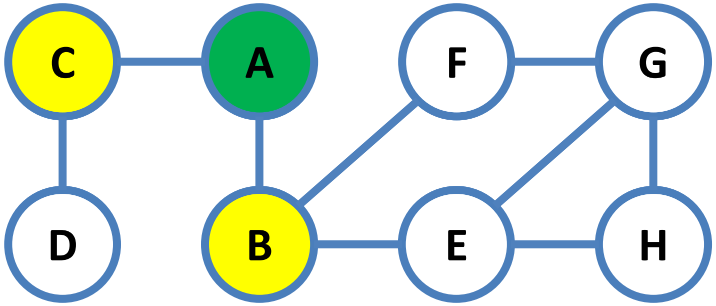
      - If vertex v is not discovered or visited, add to discovered queue
      - v.distance = u.distance + 1

Discovered										
Visited	A,0									



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Discovered	**C,1**	**B,1**								
Visited	A,0									

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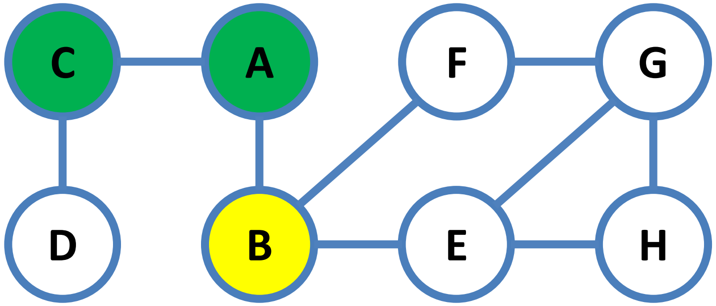

Discovered		**B,1**								
Visited	A,0	C,1								

Graph
Shortest distance with BFS

- How would you implement it?
  - Let say we begin from vertex A
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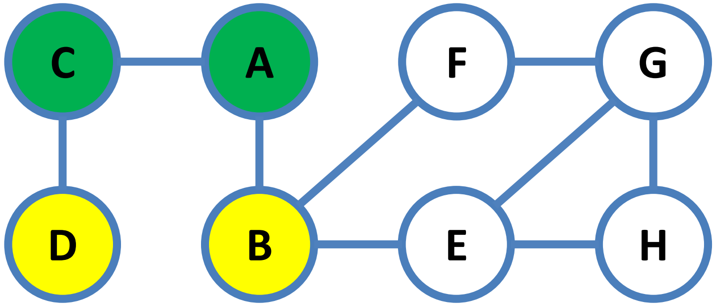
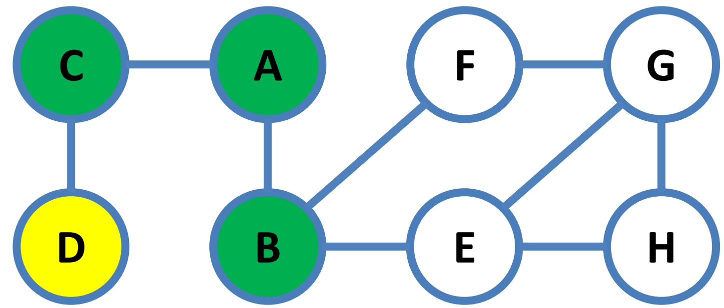


Discovered		**B,1**	**D,2**							
Visited	A,0	C,1								

Graph
Shortest distance with BFS



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    - Serve from discovered, to visited
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Discovered			**D,2**							
Visited	A,0	C,1	B,1							

Graph

Shortest distance with BFS

- How would you implement it?
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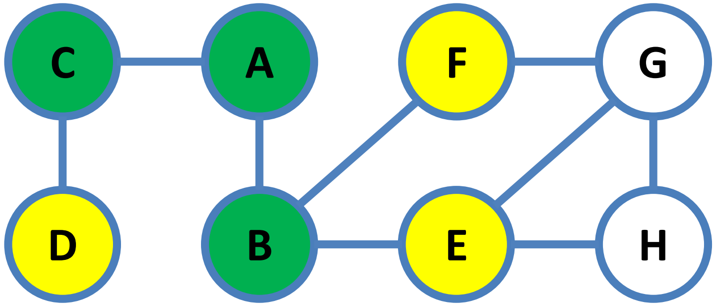

Discovered			**D,2**	**F,2**	**E,2**					
Visited	A,0	C,1	B,1							

Graph
Shortest distance with BFS

- How would you implement it?
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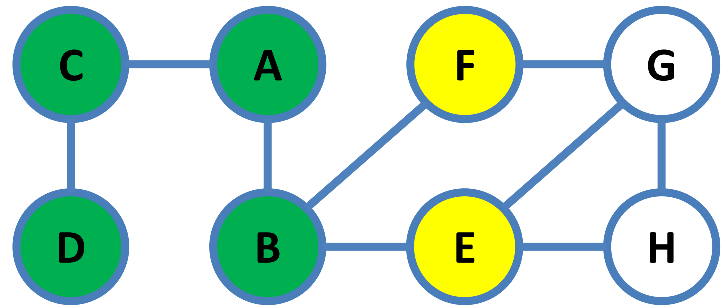
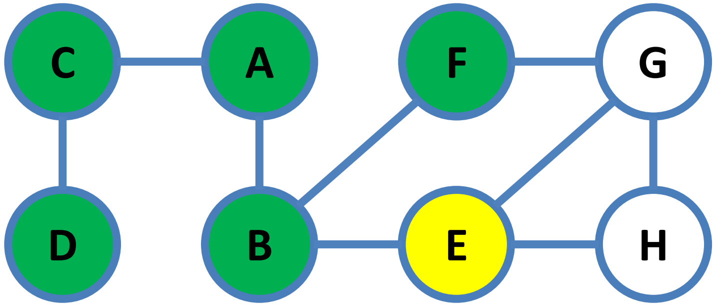


Discovered				F,2	E,2					
Visited	A,0	C,1	B,1	D,2						



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  - While discovered is not empty
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    - For each edge <u,v> where u is the served
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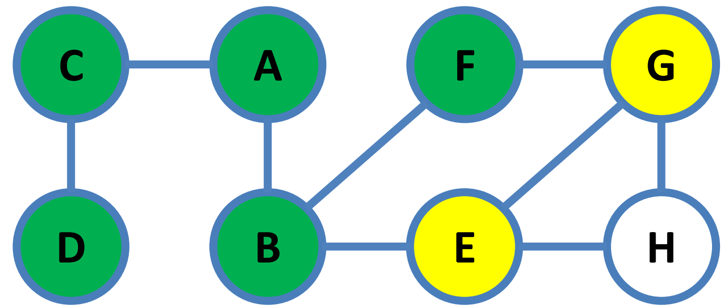
      - v.distance = u.distance + 1

Discovered					**E,2**					
Visited	A,0	C,1	B,1	D,2	F,2					

Graph
Shortest distance with BFS



- How would you implement it?
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Discovered					E,2	G,3				
Visited	A,0	C,1	B,1	D,2	F,2					

Graph
Shortest distance with BFS

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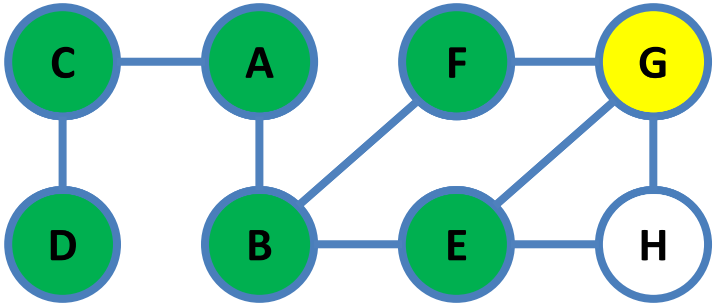

Discovered						G,3				
Visited	A,0	C,1	B,1	D,2	F,2	E,2				

Graph
Shortest distance with BFS

- How would you implement it?
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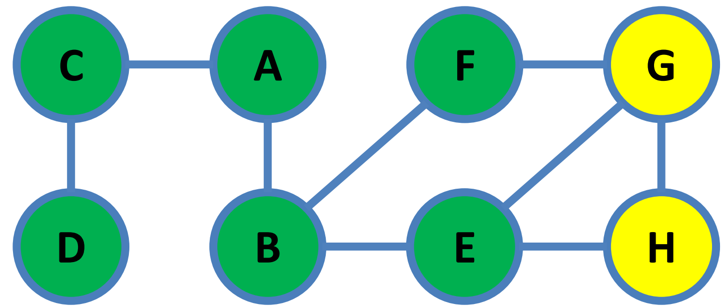
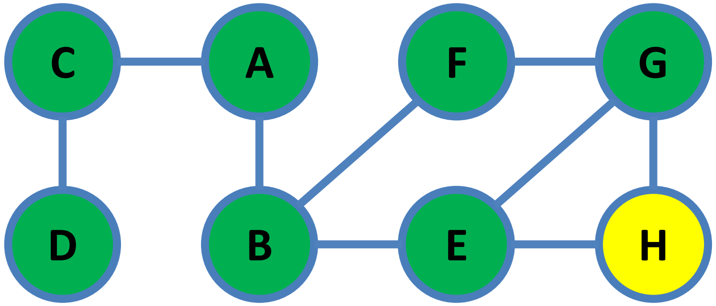


Discovered						G,3	H,3			
Visited	A,0	C,1	B,1	D,2	F,2	E,2				

Graph

Shortest distance with BFS



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  - Have a queue for discovered
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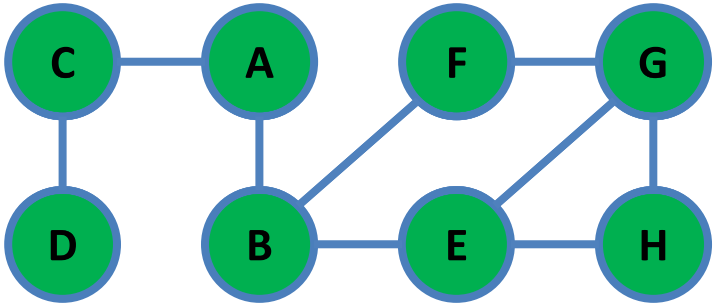
Discovered							**H,3**			
Visited	A,0	C,1	B,1	D,2	F,2	E,2	G,3			

Graph
Shortest distance with BFS



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Discovered										
Visited	A,0	C,1	B,1	D,2	F,2	E,2	G,3	H,3		

Questions?

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- How would you modify the following to find the path?
- How would you implement it?
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it with a distance 0
  - While discovered is not empty
    - Serve from discovered, to visited
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  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue
      - v.distance = u.distance + 1
      - v.previous = u # enable backtracking

Questions?

- What if the graph is weighted?

- What if the graph is weighted?
  - BFS is not able to do it anymore

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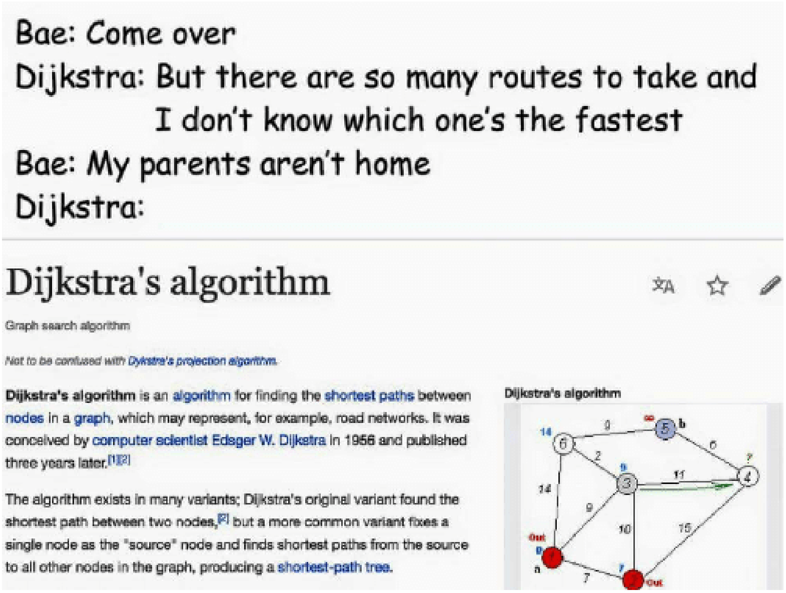
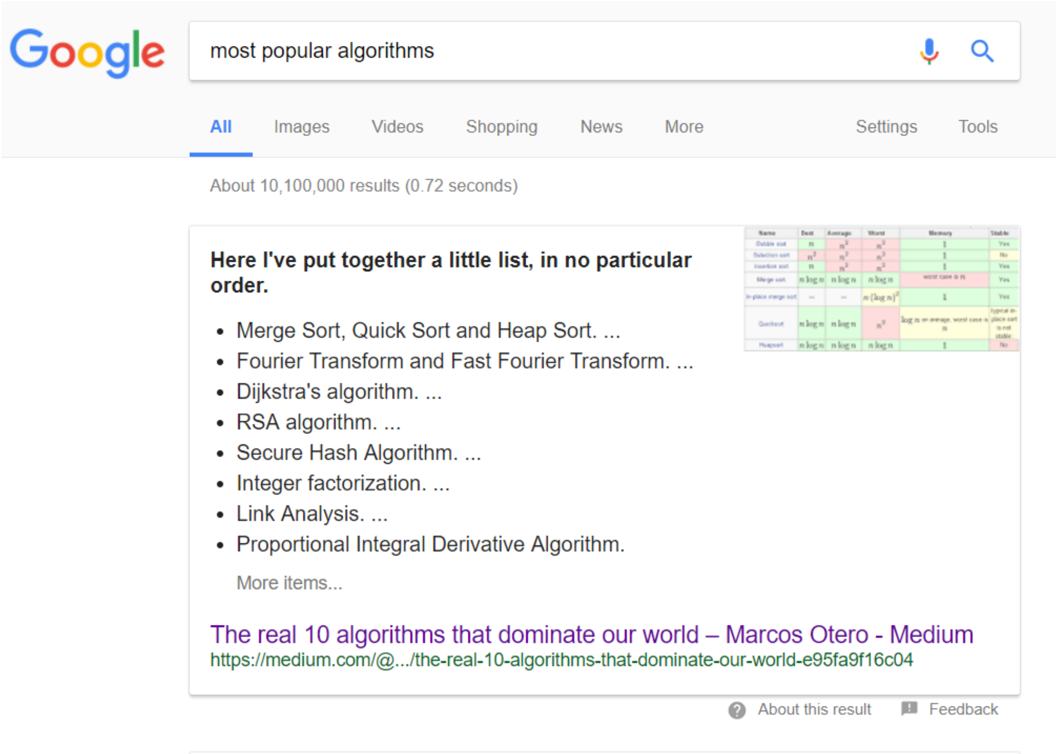
- What if the graph is weighted?
  - BFS is not able to do it anymore
  - Sooo, we call in Dijkstra (the left one)

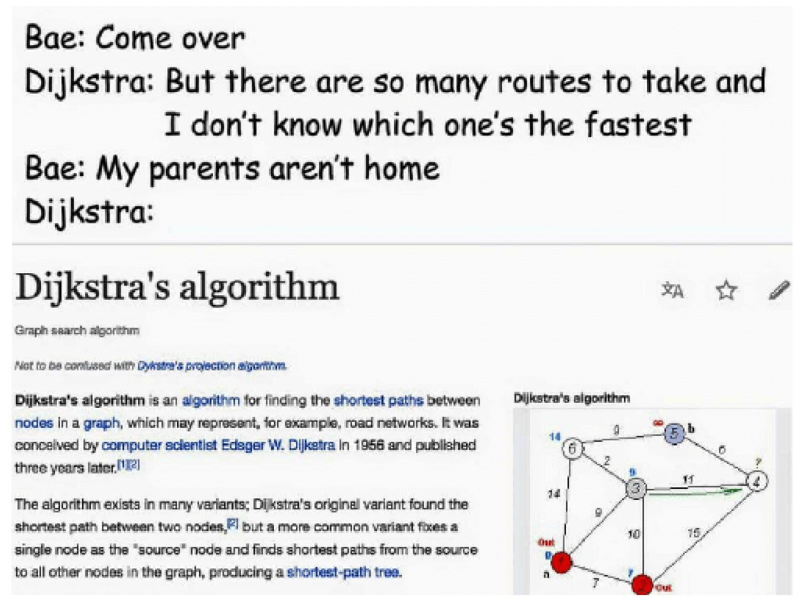
- So Dijkstra came up with the shortest distance algorithm
  - Recall we can backtrack (previous) to get the path



How Dijkstra came up with his algorithm

Graph
Shortest path with Dijkstra





How Dijkstra came up with his algorithm

- It is a combination of 2 algorithms

- It is a combination of 2 algorithms
  - Dynamic programming
  - Greedy

Shortest path with Dijkstra

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- It is a combination of 2 algorithms
  - Dynamic programming
    The minimum distance from A to C can be the minimum of A to B (which we know) and minimum of B to C (which we know as well).
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    If I am at A, I can reach B and C. B is the closest, so I go to B and this is the shortest from A to B. I do not need to check if A to C then C to B (A->C->B) is the shortest anymore.

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Graph

Shortest path with Dijkstra

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  - Thus, Dijkstra doesn’t work for negative edges

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  - Thus, Dijkstra doesn’t work for negative edges
    Note: might work at times when the negative edge isn’t part of a cycle

Questions?

- So how does Dijkstra work?

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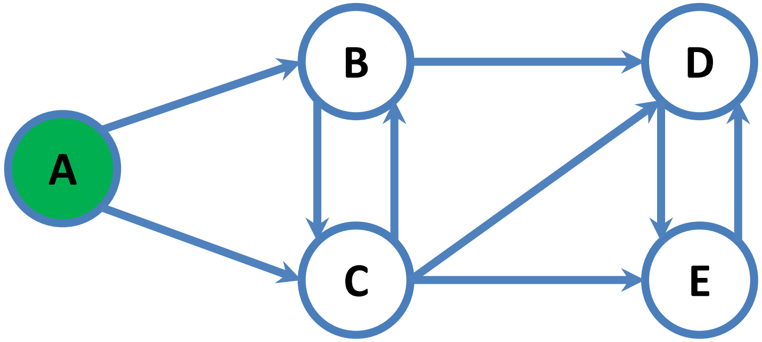
  - Consider the following directed graph



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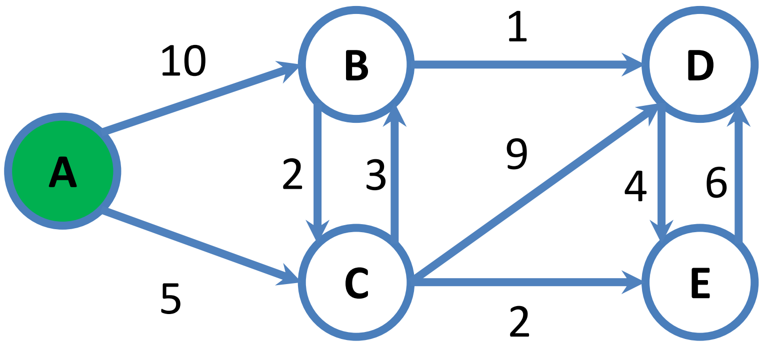
    - Graph is weighted



- So how does Dijkstra work?
  - Consider the following directed graph
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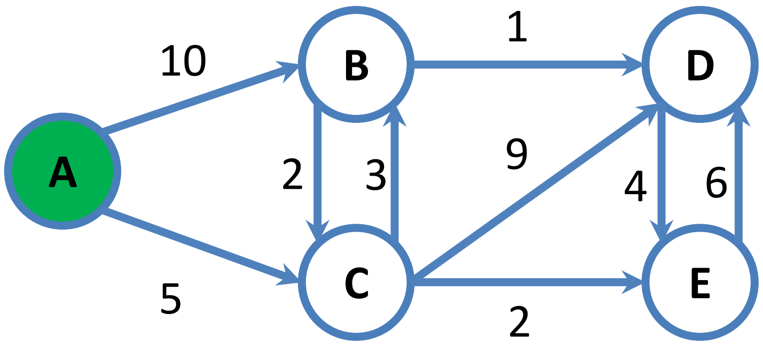
  - So let us begin the algorithm…



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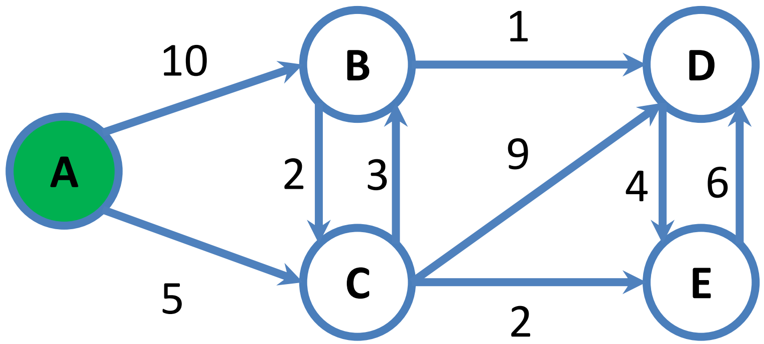
  - We are at A (source), and



Graph

Shortest path with Dijkstra

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Graph
Shortest path with Dijkstra

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Graph
Shortest path with Dijkstra

MONASH University

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  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0
    - B = infinity
    - C = infinity
    - D = infinity
    - E = infinity

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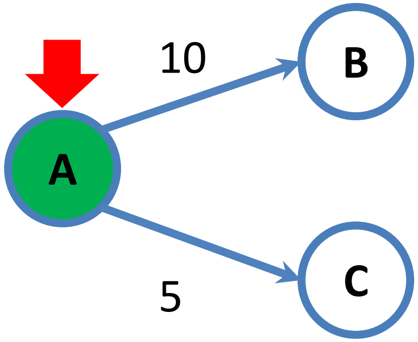

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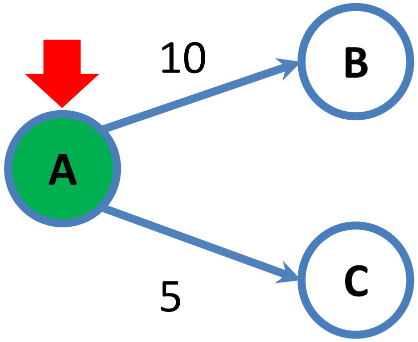

  - So let us begin the algorithm…
  - So what happen is we will slowly wander to the closest point (from A)
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    - B = 10
    - C = 5
    - D = infinity
    - E = infinity

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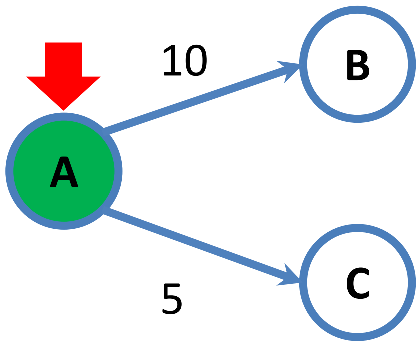

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    - Closest is C, so we move to C

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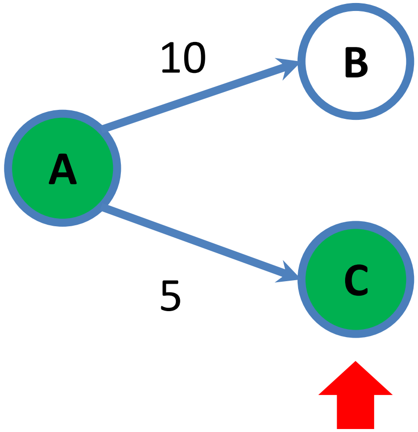

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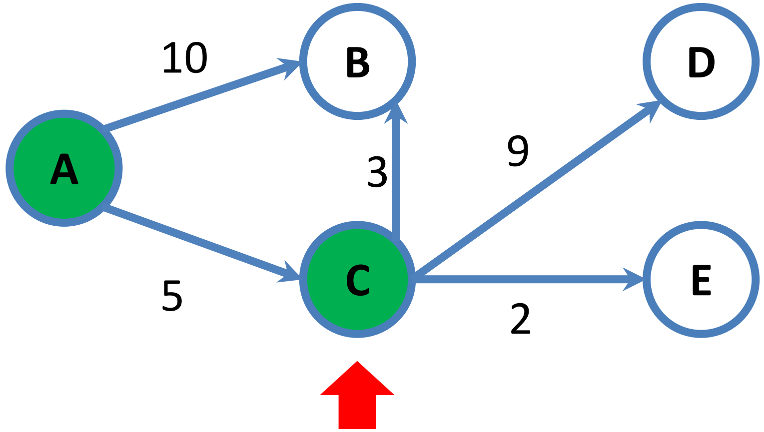

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    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 10
    - C = 5, from here, we can see B, D and E
    - D = infinity
    - E = infinity

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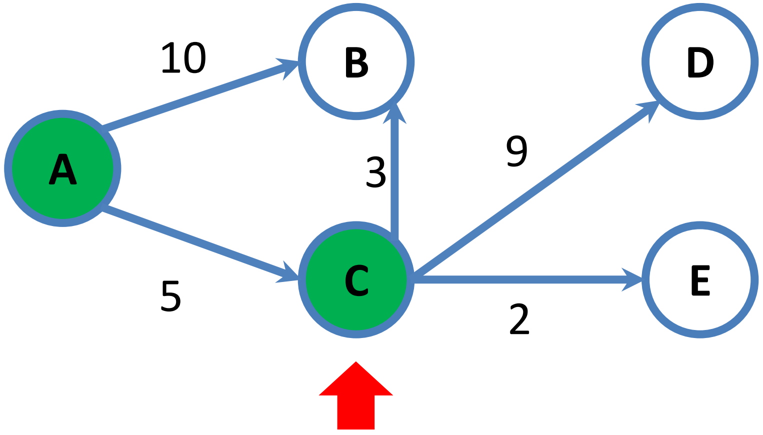

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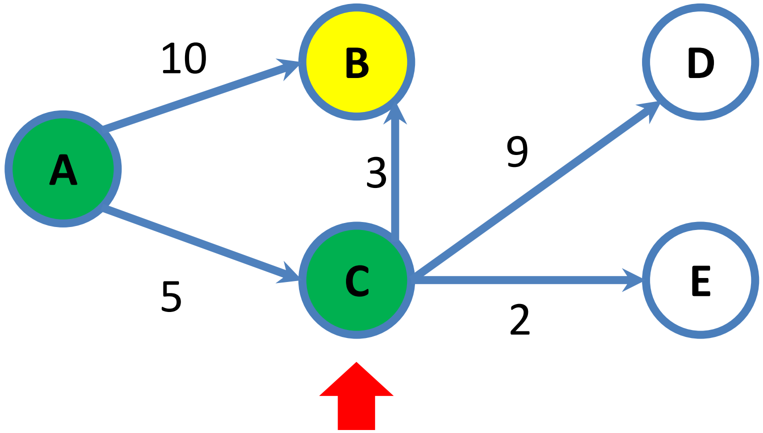

  - So let us begin the algorithm…
  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 10 (A->B) vs 8 (A->C->B)
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = infinity
    - E = infinity

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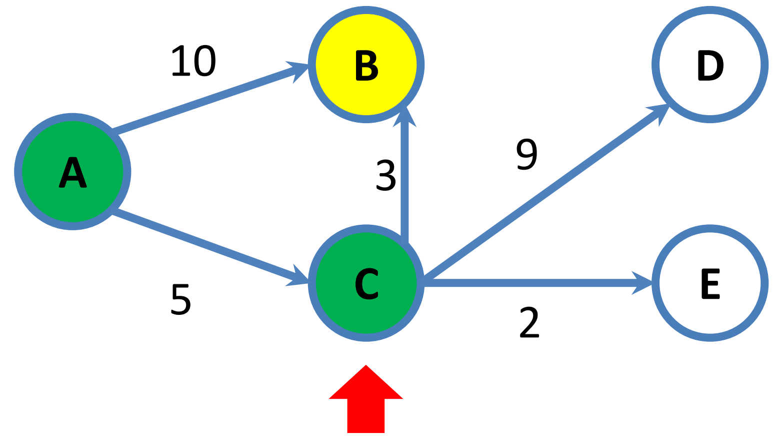

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    - D = infinity
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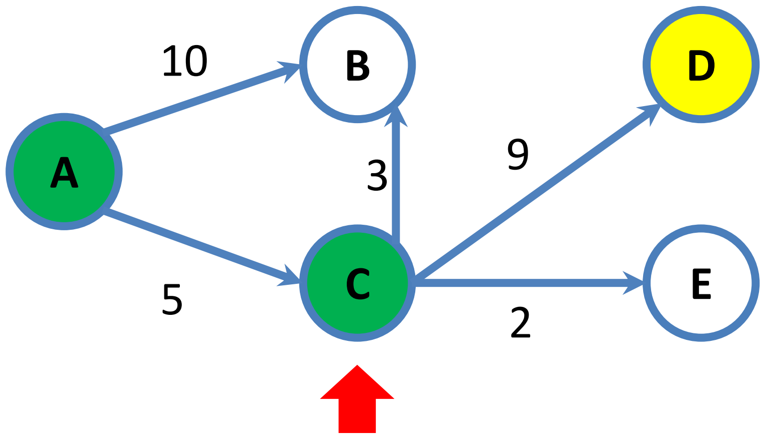

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    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 9?
    - E = infinity

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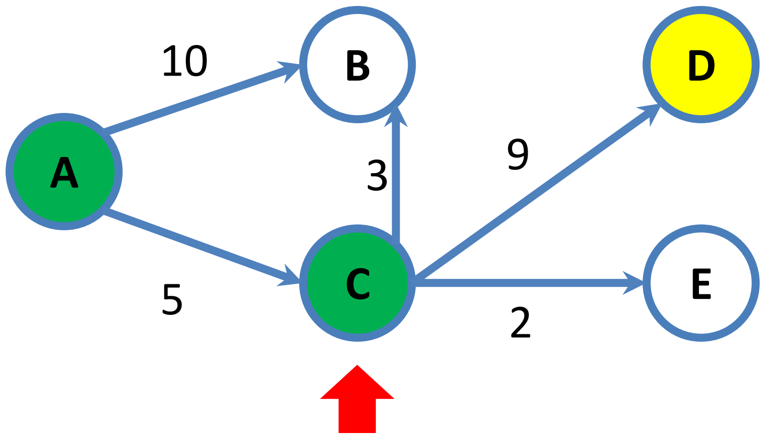

  - So let us begin the algorithm…
  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14 because distance is from A
    - E = infinity

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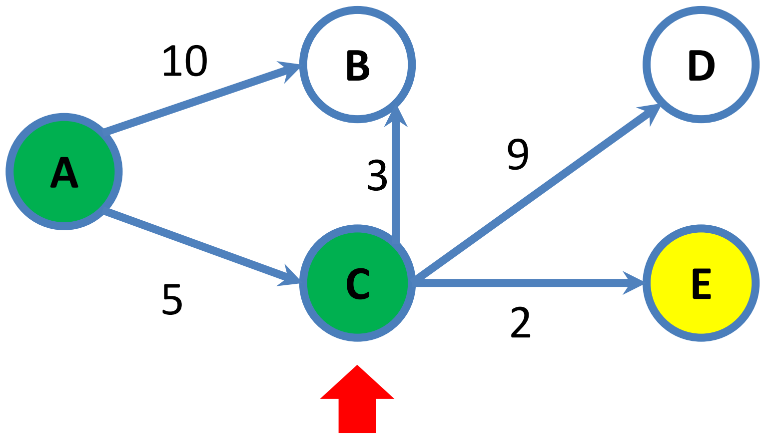

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    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14
    - E = 7

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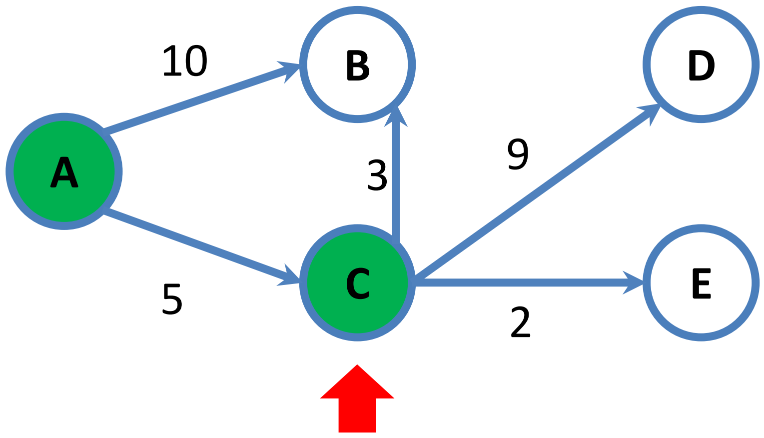

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    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14
    - E = 7
    - Closest is E, so we go E

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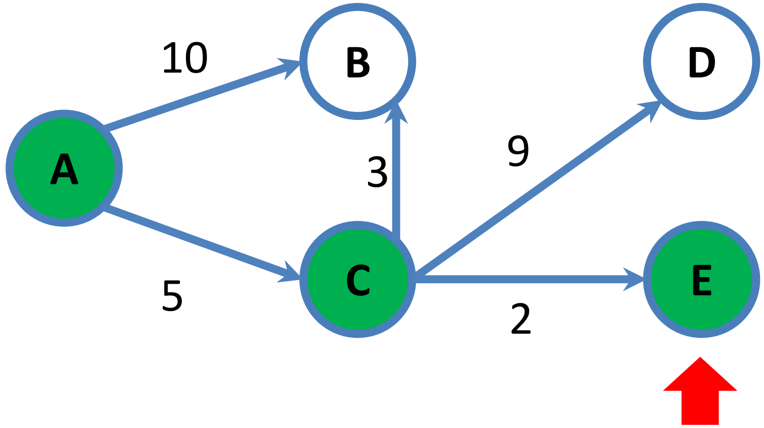

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    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14
    - E = 7
    - Closest is E, so we go E

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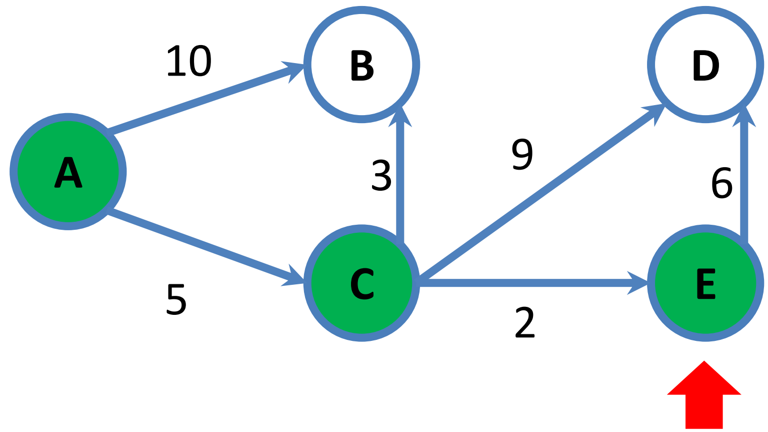

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    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14
    - E = 7, from here, we can see D.

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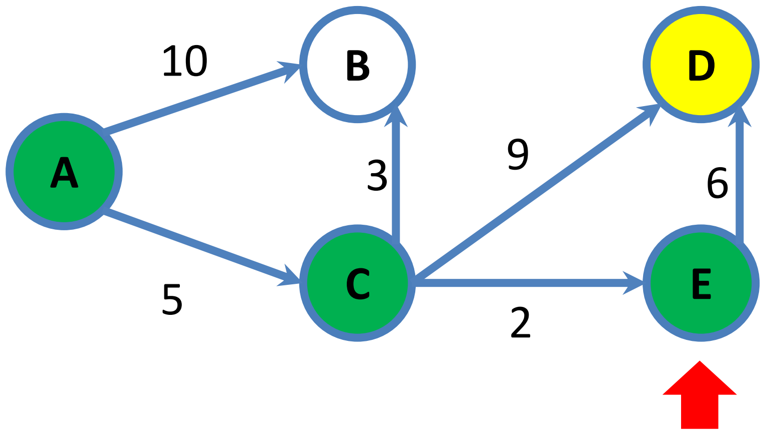

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  - So what happen is we will slowly wander to the closest point (from A)
    - A = 0, from here, we can see B and C (edges from A). Update distance
    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 14 vs 7+6 (A->E->D)
    - E = 7, from here, we can see D. Update the distance

- So how does Dijkstra work?
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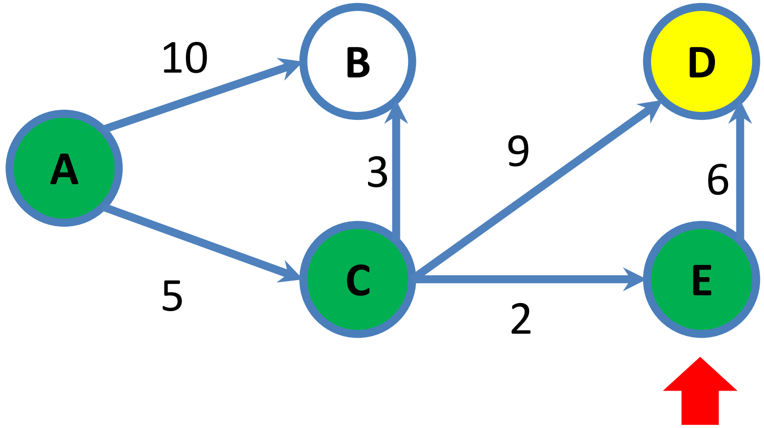

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    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 13
    - E = 7, from here, we can see D. Update the distance

Graph
Shortest path with Dijkstra

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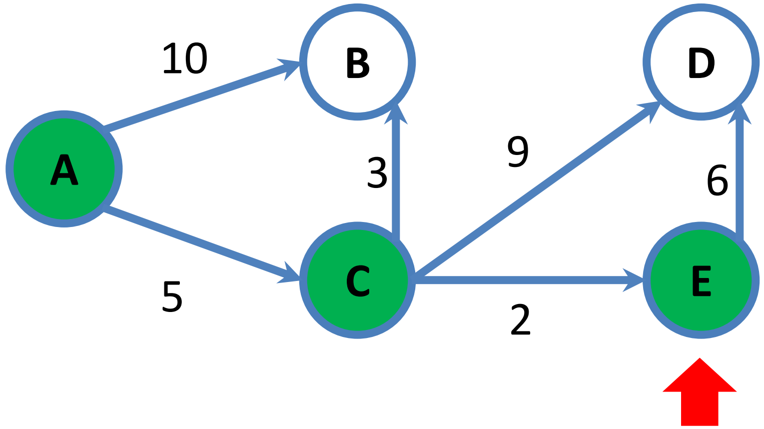

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    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 13
    - E = 7, from here, we can see D. Update the distance
    - Closest is B, so we go B

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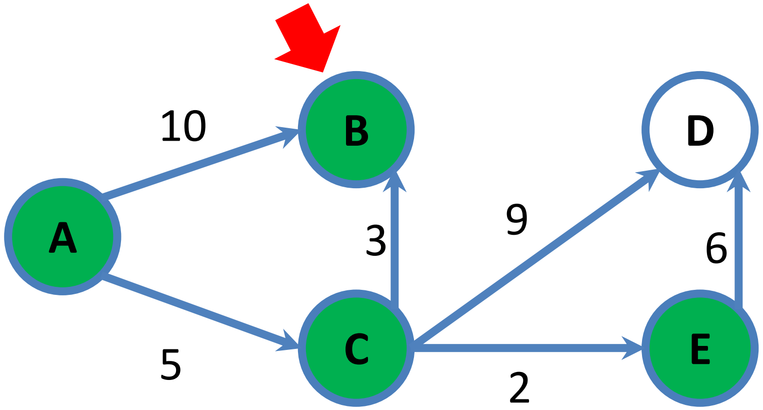

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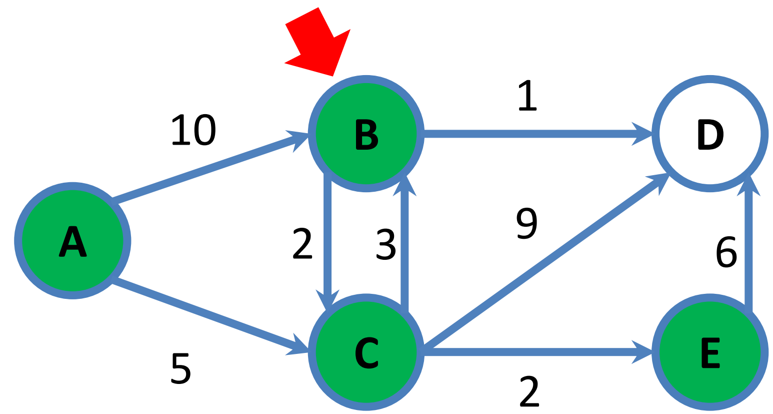
    - B = 8
    - C = 5, from here, we can see B, D and E. Update the distance
    - D = 13
    - E = 7, from here, we can see D. Update the distance
    - Closest is B, so we go B

- So how does Dijkstra work?
  - Consider the following directed graph
    - Graph is weighted



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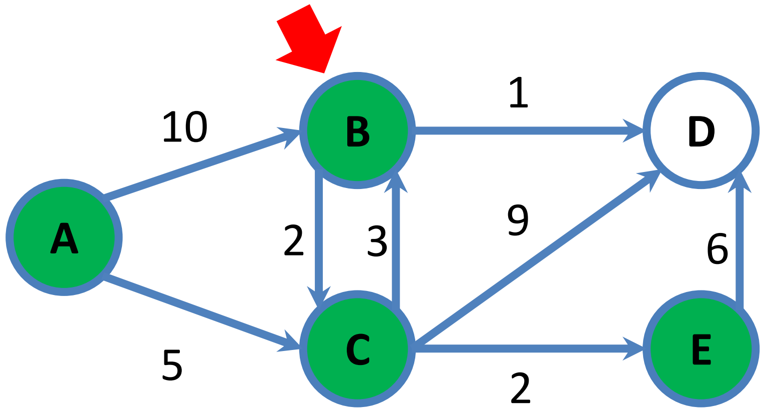

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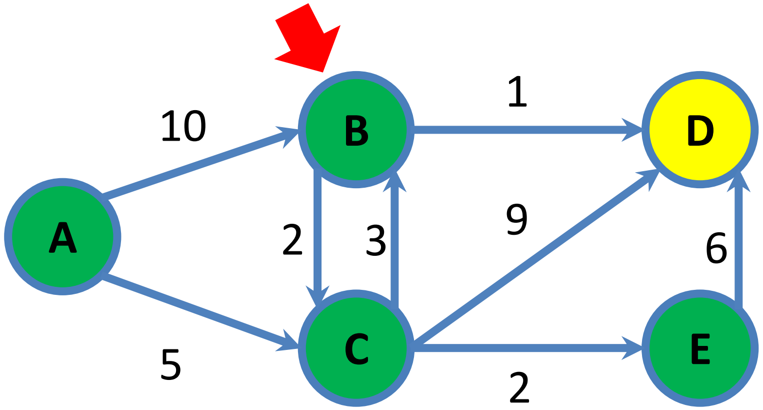

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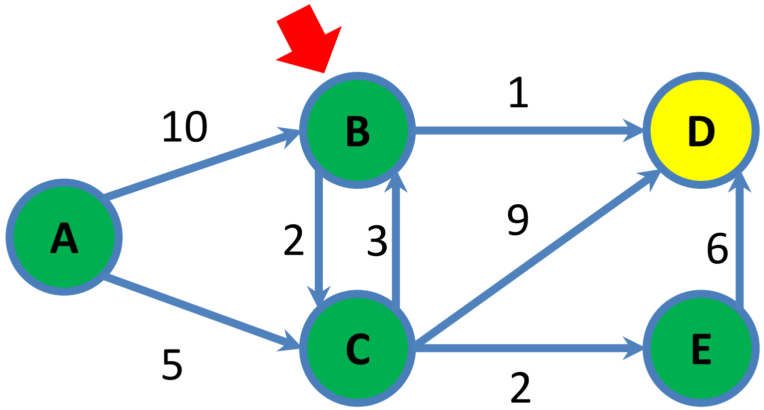

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    - D = 9 (8+1 via A->B->D)
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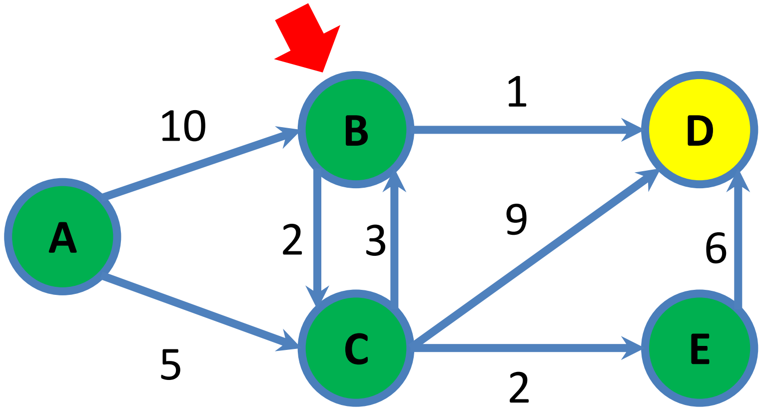

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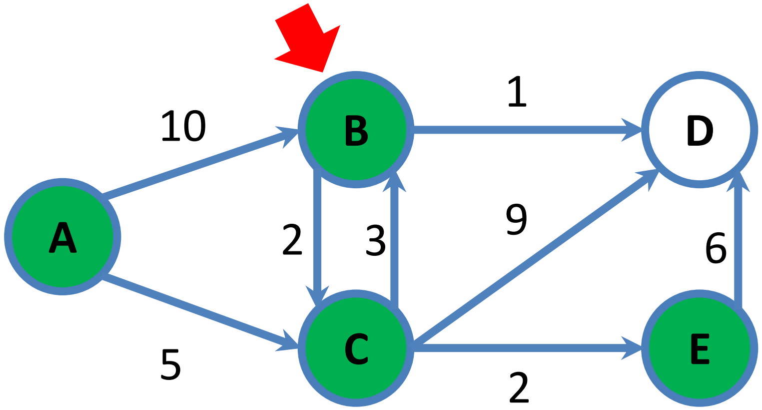

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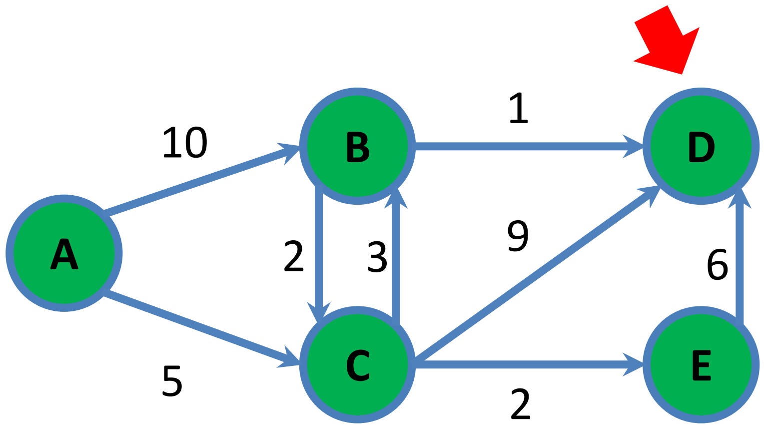

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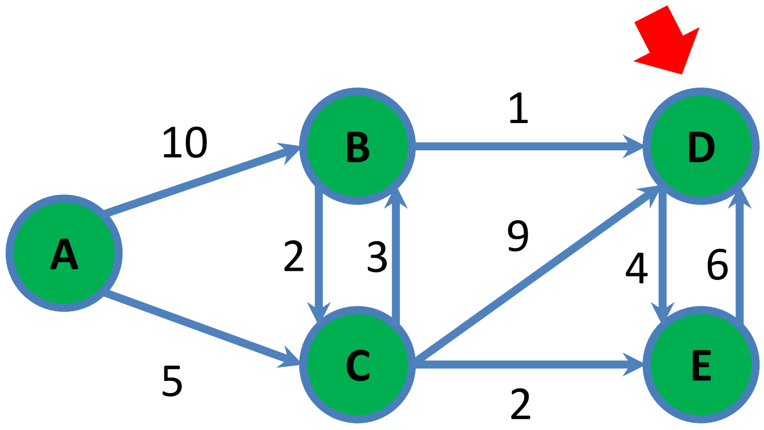

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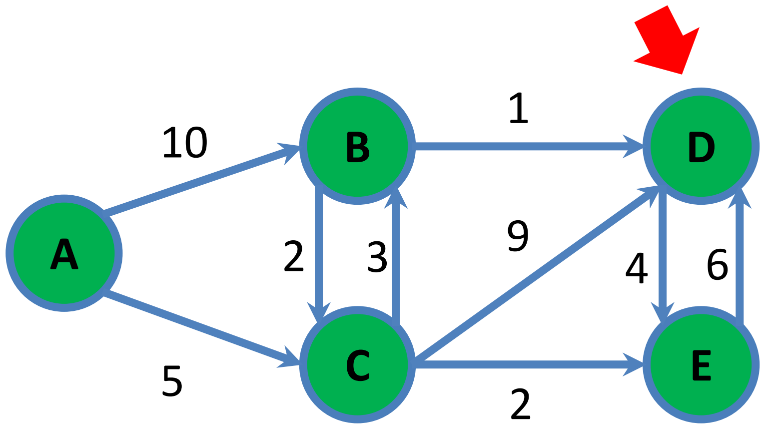

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    - And we are done!

Questions?

- Algorithm?

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    - Serve the closest vertex (not finalized)

- Algorithm? Very similar to the BFS except…
  - Priority queue instead of a normal queue
    - Serve the closest vertex (not finalized)
  - Update the distance if the neighbour vertex is visited but not finalized
    - To the shorter one

- This is the BFS algorithm, we change to Dijkstra now
  - Let say we begin from vertex A
  - Have a queue for discovered
    - Put source (A) into it with a distance 0
  - While discovered is not empty
    - Serve from discovered, to visited
    - For each edge <u,v> where u is the served
      - If vertex v is not discovered or visited, add to discovered queue
      - v.distance = u.distance + 1

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- Try to modify this as part of the in-class activity

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  - We use a min-heap for our priority queue!
    - Note that we need a pointer to the nodes to update distance in O(1)

- Algorithm can be as follows (might differ):

```
discover_queue = MinHeap()
discover_queue.append([source,0])

while discover_queue is not empty:
    u = discover_queue.serve()
    u.visited = True
    for each <u,v,w> in u.edges:
        if v.visited = True:
            pass
        else:
            if v.discovered = False:
                discover_queue.append([v, u.distance+w])
                v.discovered = True
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Questions?

- Complexity?

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Graph

Shortest path with Dijkstra

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O(V)

Graph
Shortest path with Dijkstra

O(V)

Serve: O(log V)

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Graph

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Serve: O(log V)

O(V)

Graph
Shortest path with Dijkstra


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O(V)

Serve: O(log V)

O(V)

Update: O(log V)

Graph

Shortest path with Dijkstra

- Time Complexity?

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O(V)

Serve: O(log V)

O(V)

Update: O(log V)

Graph
Shortest path with Dijkstra

MONASH University

- Time Complexity? O(V^2 log V)

O(V)

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Serve: O(log V)

O(V)

Update: O(log V)

Graph

Shortest path with Dijkstra

- Time Complexity? O(V^2 log V) = O(E log V)

O(V)

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Serve: O(log V)

O(V)

Update: O(log V)

Graph

Shortest path with Dijkstra

- Time Complexity? O(V^2 log V) = O(E log V)
  - Recall for dense graph, E ≈ V^2

O(V)

Serve: O(log V)

O(V)

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Update:
O(log V)

- Time Complexity? O(V^2 log V) = O(E log V)
  - Recall for dense graph, E ≈ V^2

- Note that with Fibonacci heap instead of your binary heap, we can reduce the complexity further to O(E + V log V)

- Time Complexity? $O(V^2 \log V) = O(E \log V)$
  – Recall for dense graph, $E \approx V^2$

- Note that with Fibonacci heap instead of your binary heap, we can reduce the complexity further to $O(E + V \log V) = O(V^2 + V \log V) = O(V^2)$
  – For dense graph

Questions?

- What if we have a single source
  - As usual
- But single target?

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  - As usual
- But single target?
- We can terminate after we have move the target vertex to the visited portion!
  - We would have the shortest distance
  - We can backtrack for the shortest path
    - Via vertex.previous attribute

Questions?

- Why does Dijkstra work?

- Why does Dijkstra work?
  - Let us use Nathan's slides

Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

- Notation:
  - V is the set of vertices
  - Q is the set of vertices in the queue
  - S = V / Q = the set of vertices who have been removed from the queue

Base Case

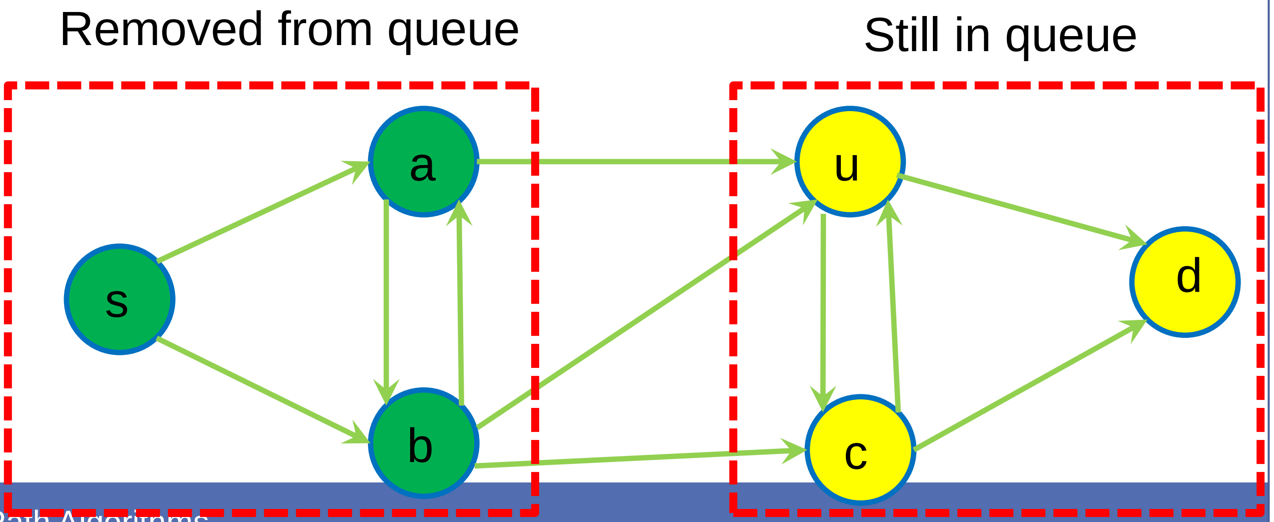
- Dist[s] is initialised to 0, which is the shortest distance from s to s (since there are no negative weights)

Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

Inductive Step:

- Assume that the claim holds for all vertices which have been removed from the queue (S)
- Let u be the next vertex which is removed from the queue
- We will show that dist[u] is correct



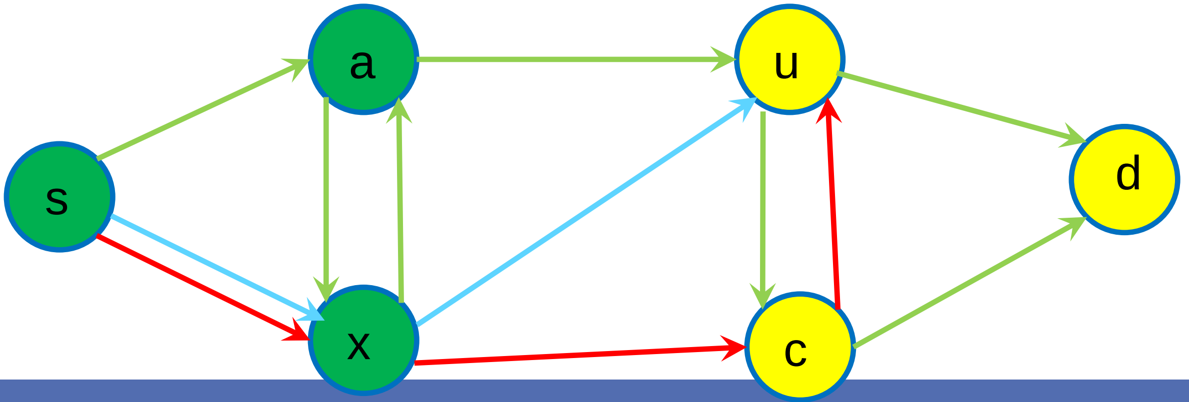
Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

Inductive Step:

- Suppose (for contradiction) there is a shorter path P, s⇝u with len(P) < dist[u]
- Let x be the furthest vertex on P which is in S (i.e. has been finalised)
- By the inductive hypothesis, dist[x] is correct (since it is in S)

Current path
Assumed shorter path (P)

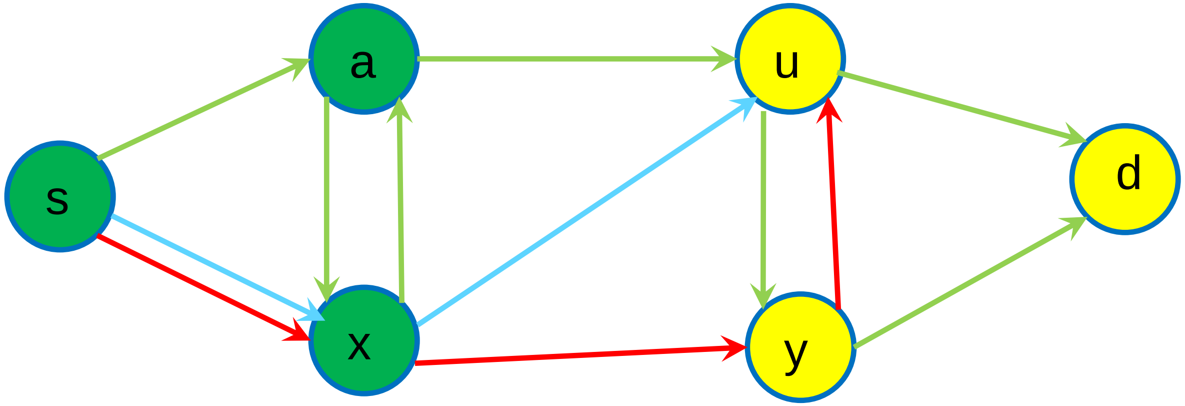


Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

Inductive Step:

- By the inductive hypothesis, dist[x] is correct (since it is in S)
- Let y be the next vertex on P after x
- Len(P) < dist[u] (by assumption)
- Edge weights are non-negative
- Len(s⇝y) <= len(P) < dist[u]

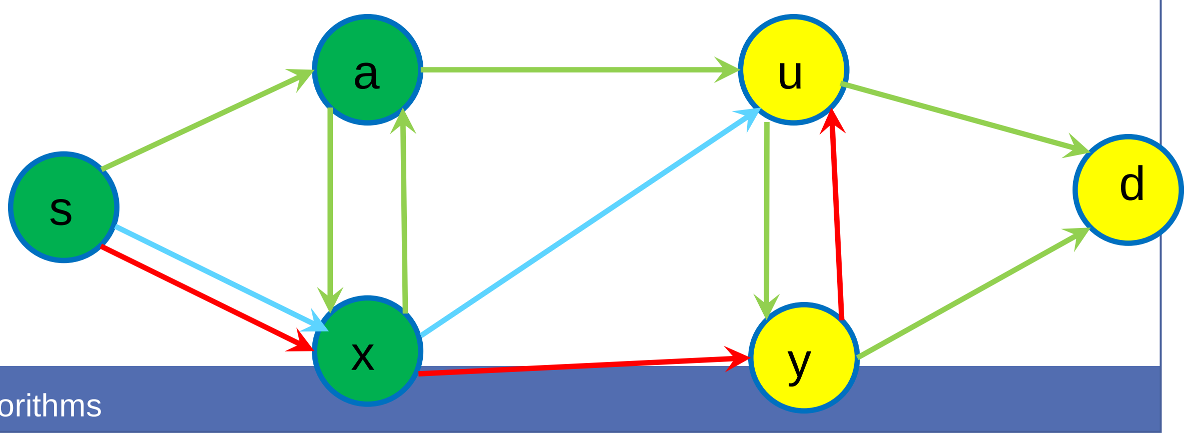


Proof of Correctness

Claim: For every vertex v which has been removed from the queue, dist[v] is correct

Inductive Step:

- Len(s⇝y) <= len(P) < dist[u]
- Since we said that P (via x and y) is a shortest path…
- dist[y] = len(s⇝y) < dist[u]
- So dist[y] < dist[u]…
- If y ≠ u, why didn't y get removed before u???
- If y = u, how can dist[y] <dist[u]???



- Why does Dijkstra work?
  - Let us use Nathan’s slides
  - Or let me just explain it on the whiteboard…
  - Via proof by contradiction!

Questions?

Other shortest path?

- Bellman-Ford
- Floyd-Warshall

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- Bellman-Ford
- Floyd-Warshall
  - With transitive closure

Other shortest path?

- Bellman-Ford
- Floyd-Warshall
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- We see it later in next lectures

Other shortest path?

- Bellman-Ford
  - Single source
  - Can know negative edges
- Floyd-Warshall
  - With transitive closure
  - Single or more sources
  - Can know negative edges

- We see it later in next lectures

Questions?

Thank You