MONASH University

MONASH
INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

Ian Wern Han Lim
lim.wern.han@monash.edu

Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

Ready?

Agenda

- Quick-select

Agenda

- Quick-select
  - K-th order statistics

Agenda

- Quick-select
  - K-th order statistics
  - Using it to find the median

Agenda

- Quick-select
  - K-th order statistics
  - Using it to find the median
    - For Quick sort pivot?
    - Median of median?

MONASH University

Let us begin…

K-the Order Statistics

What is it?

K-the Order Statistics

What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array

K-the Order Statistics

What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array

- Example: 21,84,16,14,79,51,66,21,54,32

K-the Order Statistics

What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array

- Example: 21,84,16,14,79,51,66,21,54,32
- If k=1, we want the smallest item
  - 14

K-the Order Statistics

What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array

- Example: 21,84,16,14,79,51,66,21,54,32
- If k=1, we want the smallest item
  - 14
- If k=2, want the 2 smallest items
  - 16,14 (note: order is not important)

K-the Order Statistics
What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array

- Example: 21,84,16,14,79,51,66,21,54,32
- If k=1, we want the smallest item
  - 14
- If k=2, want the 2 smallest items
  - 16,14 (note: order is not important)
- If k=5, we want the 5 smallest items
  - 21,16,14,21,32

K-the Order Statistics
What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array

- Example: 21,84,16,14,79,51,66,21,54,32
- If k=1, we want the smallest item
  – 14
- If k=2, want the 2 smallest items
  – 16,14 (note: order is not important)
- If k=5, we want the 5 smallest items
  – 21,16,14,21,32
  – Isn't this partition?

K-the Order Statistics

What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array
  - First quartile (Q1); k=N/4
  - Median; k=N/2
  - Third quartile (Q3); k= 3N/4

K-the Order Statistics

What is it?

- Given an unsorted array
- Find the k-th smallest elements in the array
  - First quartile (Q1); k=N/4
  - Median; k=N/2
  - Third quartile (Q3) = k= 3N/4

- But how can we get it?

Questions?

- Sort the list

K-the Order Statistics

Getting it

- Sort the list
- Then slice the list for the k-th we want

K-the Order Statistics

Getting it

- Sort the list
- Then slice the list for the k-th we want

K-the Order Statistics

Getting it

- Sort the list
- Then slice the list for the k-th we want
- Complexity is high!

K-the Order Statistics

Getting it

- Sort the list
  - Sorting gives us O(NM log N)
  - Where N is number of item in list
  - Where M is the comparison cost
- Then slice the list for the k-th we want
- Complexity is high!

K-the Order Statistics

Getting it

- Sort the list
  - Sorting gives us O(NM log N)
  - Where N is number of item in list
  - Where M is the comparison cost
  - We can't be sure counting sort work because item_max can be large!
- Then slice the list for the k-th we want
- Complexity is high!

K-the Order Statistics
Getting it

- Sort the list
  - Sorting gives us O(NM log N)
  - Where N is number of item in list
  - Where M is the comparison cost
  - We can’t be sure counting sort work because item_max can be large!
  - We can’t be sure radix sort work well as well…
- Then slice the list for the k-th we want
- Complexity is high!

K-the Order Statistics

Getting it

- Sort the list
  - Sorting gives us O(NM log N)
  - Where N is number of item in list
  - Where M is the comparison cost
  - We can’t be sure counting sort work because item_max can be large!
  - We can’t be sure radix sort work well as well…
- Then slice the list for the k-th we want
  - O(k) to slice
- Complexity is high!

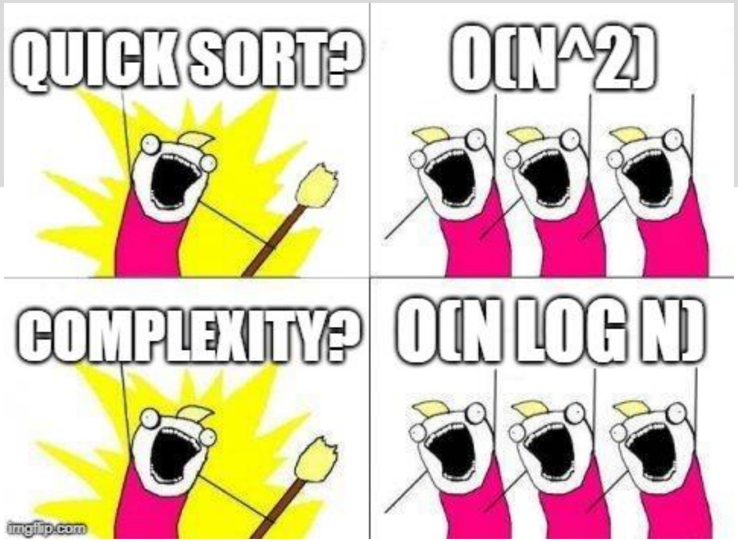
K-the Order Statistics

Getting it

- Sort the list
  - Sorting gives us O(NM log N)
  - Where N is number of item in list
  - Where M is the comparison cost
  - We can't be sure counting sort work because item_max can be large!
  - We can't be sure radix sort work well as well…
- Then slice the list for the k-th we want
  - O(k) to slice
- Complexity is high!

K-the Order Statistics

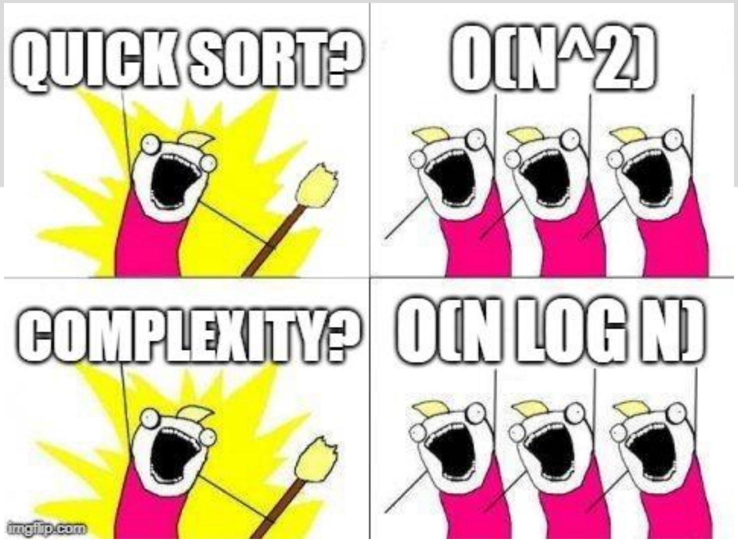
Getting it



- Sort the list
  - Sorting gives us O(NM log N)
  - Where N is number of item in list
  - Where M is the comparison cost
  - We can't be sure counting sort work because item_max can be large!
  - We can't be sure radix sort work well as well...
- Then slice the list for the k-th we want
  - O(k) to slice
- Complexity is high!

K-the Order Statistics

Getting it



- Sort the list
  - Sorting gives us O(NM log N)
  - Where N is number of item in list
  - Where M is the comparison cost
  - We can’t be sure counting sort work because item_max can be large!
  - We can’t be sure radix sort work well as well…
- Then slice the list for the k-th we want
  - O(k) to slice
- Complexity is high!

Questions?

Quick Select
What is it?

- In a nutshell, it is like quick sort

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?

Quick Select

What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want?

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want?
    - Index < what we want?

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want?
    - Index < what we want?
    - Index = what we want?

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!

20	80	90	10	30	50	70	60

20	10	30	50	80	90	70	60

Pivot	X
In Sorted position	X
Others	X

In sorted position (at index 4, i.e., 4th smallest)

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!

20	80	90	10	30	50	70	60

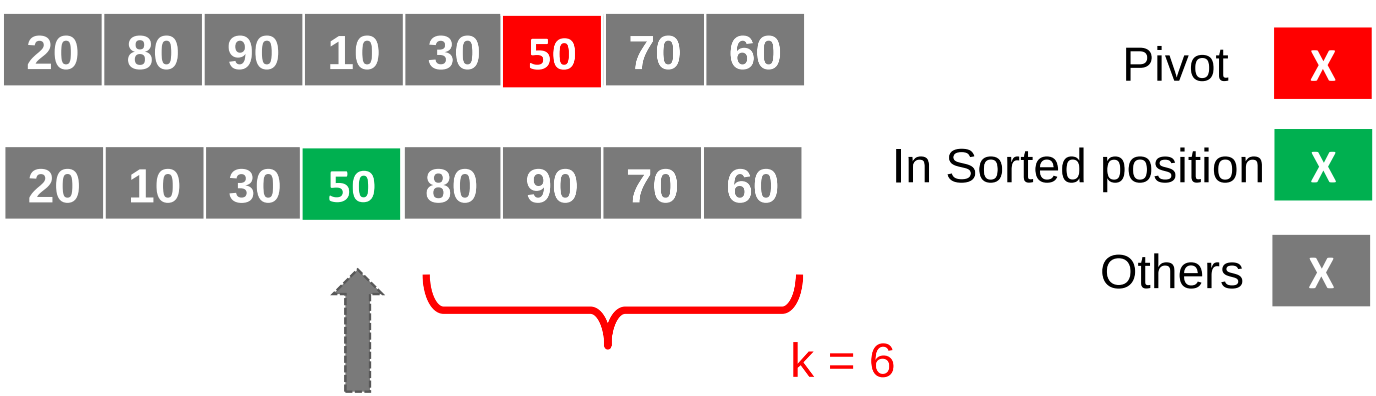
20	10	30	50	80	90	70	60

Pivot	X
In Sorted position	X
Others	X

k = 2

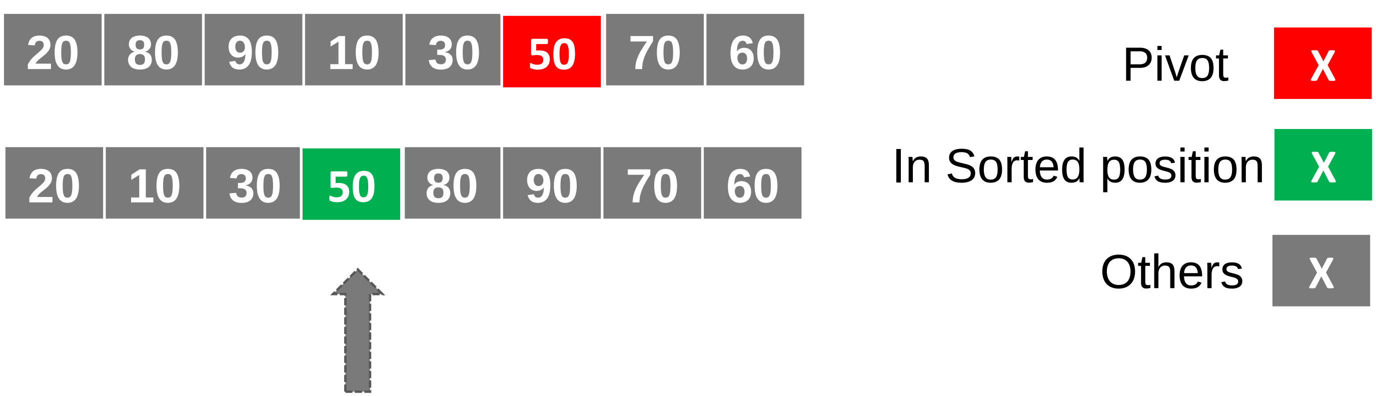
In sorted position (at index 4, i.e., 4th smallest)

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!



In sorted position (at index 4, i.e., 4th smallest)

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!



In sorted position (at index 4, i.e., 4^{th} smallest), return everything on the left of the index…

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!

  - So it is really just like implementing a quick sort except we changed…

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!
  - So it is really just like implementing a quick sort except we changed…
    - Go left or go right

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!
  - So it is really just like implementing a quick sort except we changed…
    - Go left or go right… vs go both for quicksort

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!
  - So it is really just like implementing a quick sort except we changed…
    - Go left or go right… vs go both for quicksort
  - Complexity?

Quick Select
What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!

  - So it is really just like implementing a quick sort except we changed…
    - Go left or go right… vs go both for quicksort
  - Complexity?
    - Best case O(N) cause we still need to partition once!

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!

  - So it is really just like implementing a quick sort except we changed…
    - Go left or go right… vs go both for quicksort
  - Complexity?
    - Best case O(N) cause we still need to partition once!
    - Worst case O(N^2) cause our pivot always fail!!!

Quick Select

What is it?

- In a nutshell, it is like quick sort
  - Select a pivot
  - Perform partition
  - What is the index of the pivot?
    - Index > what we want? Repeat for left
    - Index < what we want? Repeat for right
    - Index = what we want? Return, we found the item!
  - So it is really just like implementing a quick sort except we changed…
    - Go left or go right… vs go both for quicksort
  - Complexity?
    - Best case O(N) cause we still need to partition once!
    - Worst case O(N^2) cause our pivot always fail!!!

- Allow us to find what we want without sorting!

Questions?

Quick Sort

With quick select

- So we will now
  - Use quick select to find the median as a pivot
  - Use quick sort to sort

Quick Sort
With quick select

- So we will now
  - Use quick select to find the median as a pivot
    - Note: Since quick-select do perform the partition as well, we can avoid doing partition in the quick sort phase itself!
  - Use quick sort to sort

Quick Sort
With quick select

- So we will now
  - Use quick select to find the median as a pivot
    - Worst case complexity of O(N^2) when pivot != k till the last final iteration…
    - Note: Since quick-select do perform the partition as well, we can avoid doing partition in the quick sort phase itself!
  - Use quick sort to sort

Quick Sort

With quick select

Quick Sort

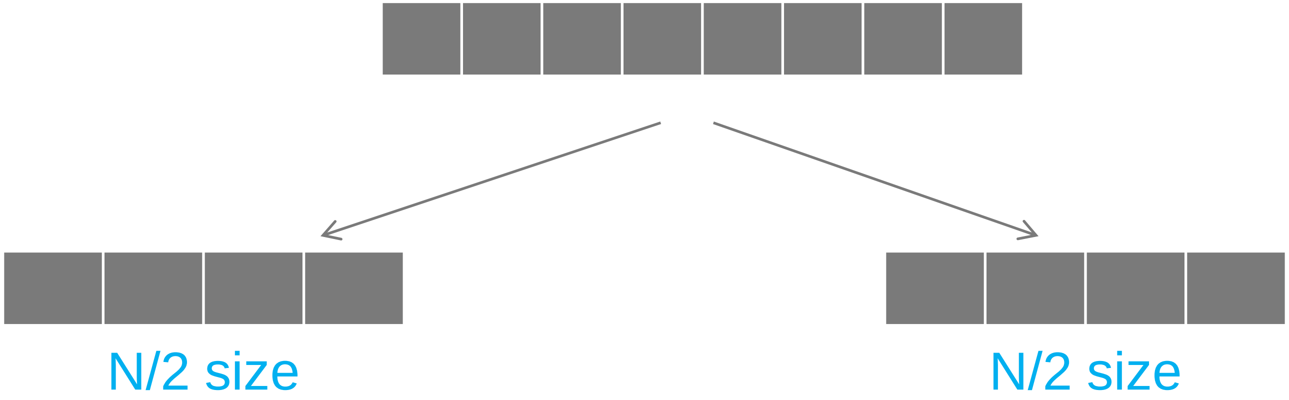
With quick select

N^2

Quick Sort

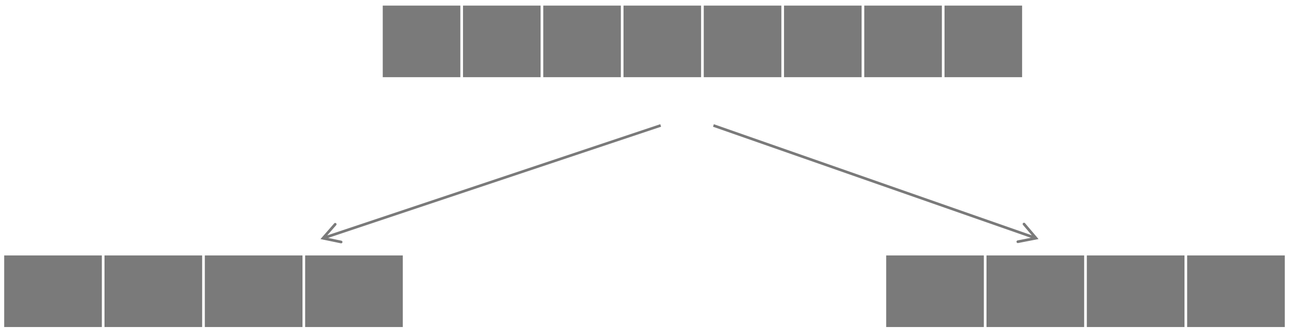
With quick select

N^2



Quick Sort

With quick select



N^2

N/2 size

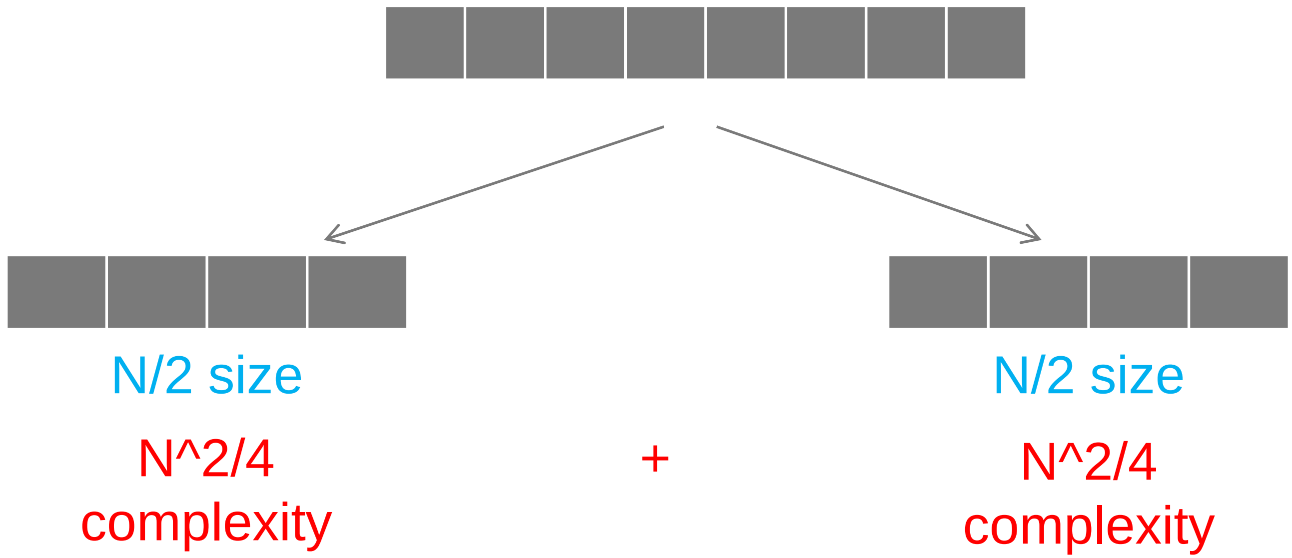
N^2/4 complexity

N/2 size

N^2/4 complexity

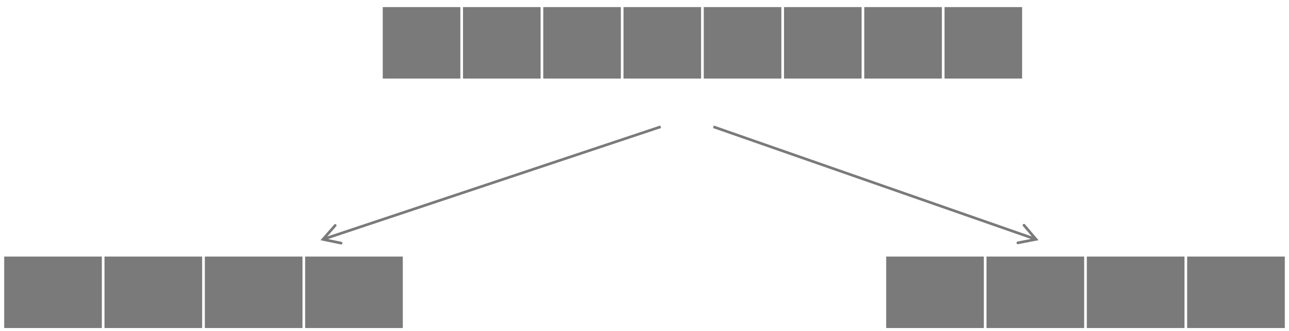
Quick Sort
With quick select

N^2



Quick Sort

With quick select



N^2

$N^2/2$

N/2 size

N^2/4 complexity

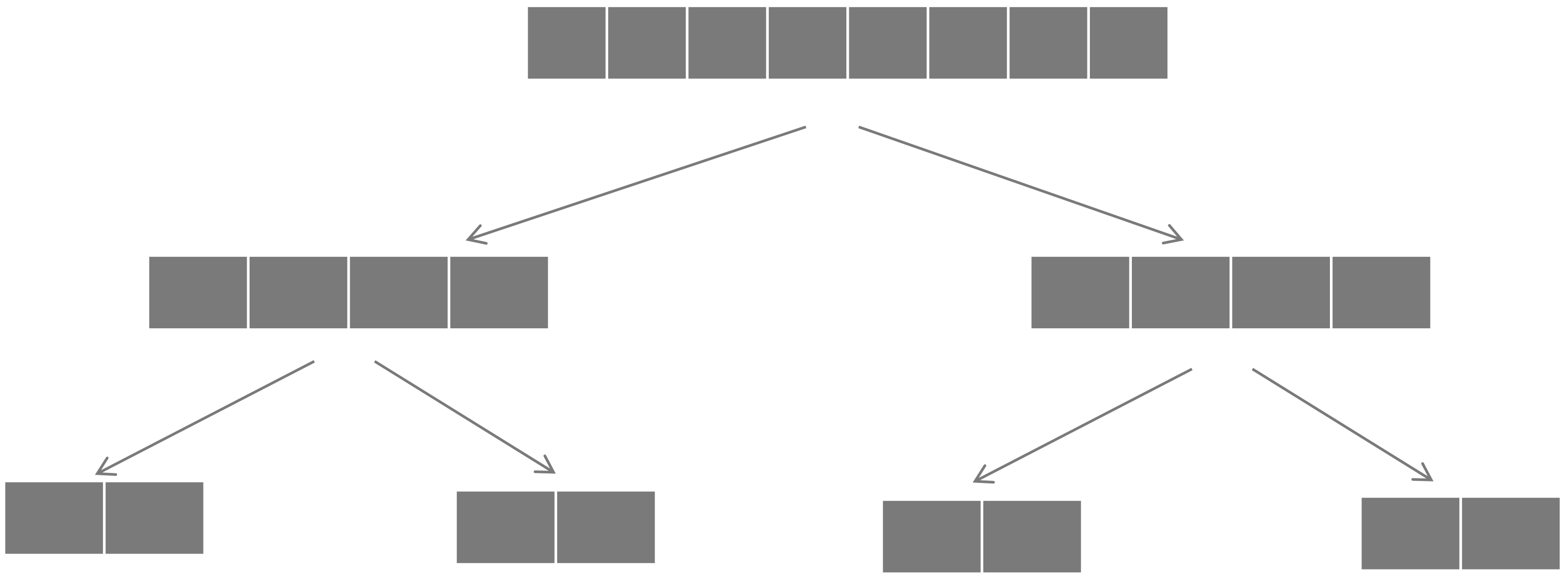
+

N/2 size

N^2/4 complexity

Quick Sort

With quick select

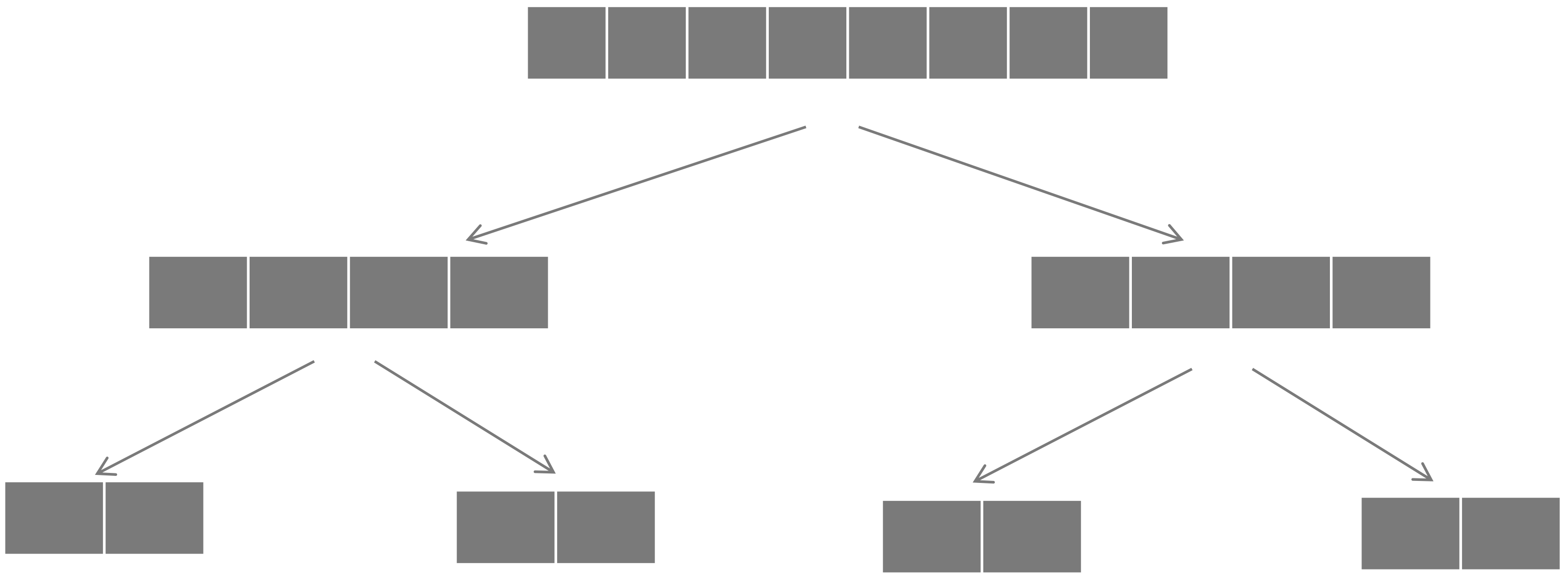


N^2

N^2/2

Quick Sort

With quick select



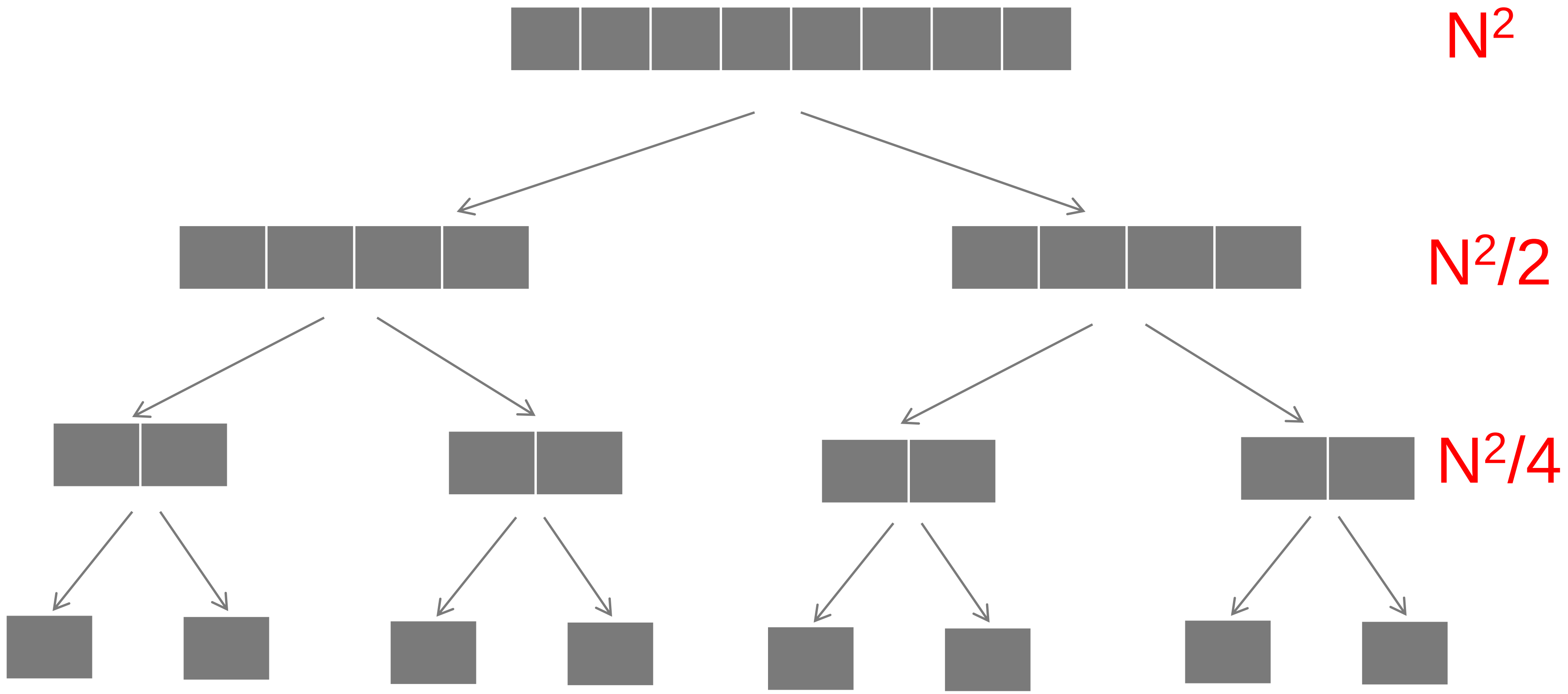
N^2

$N^2/2$

$N^2/4$

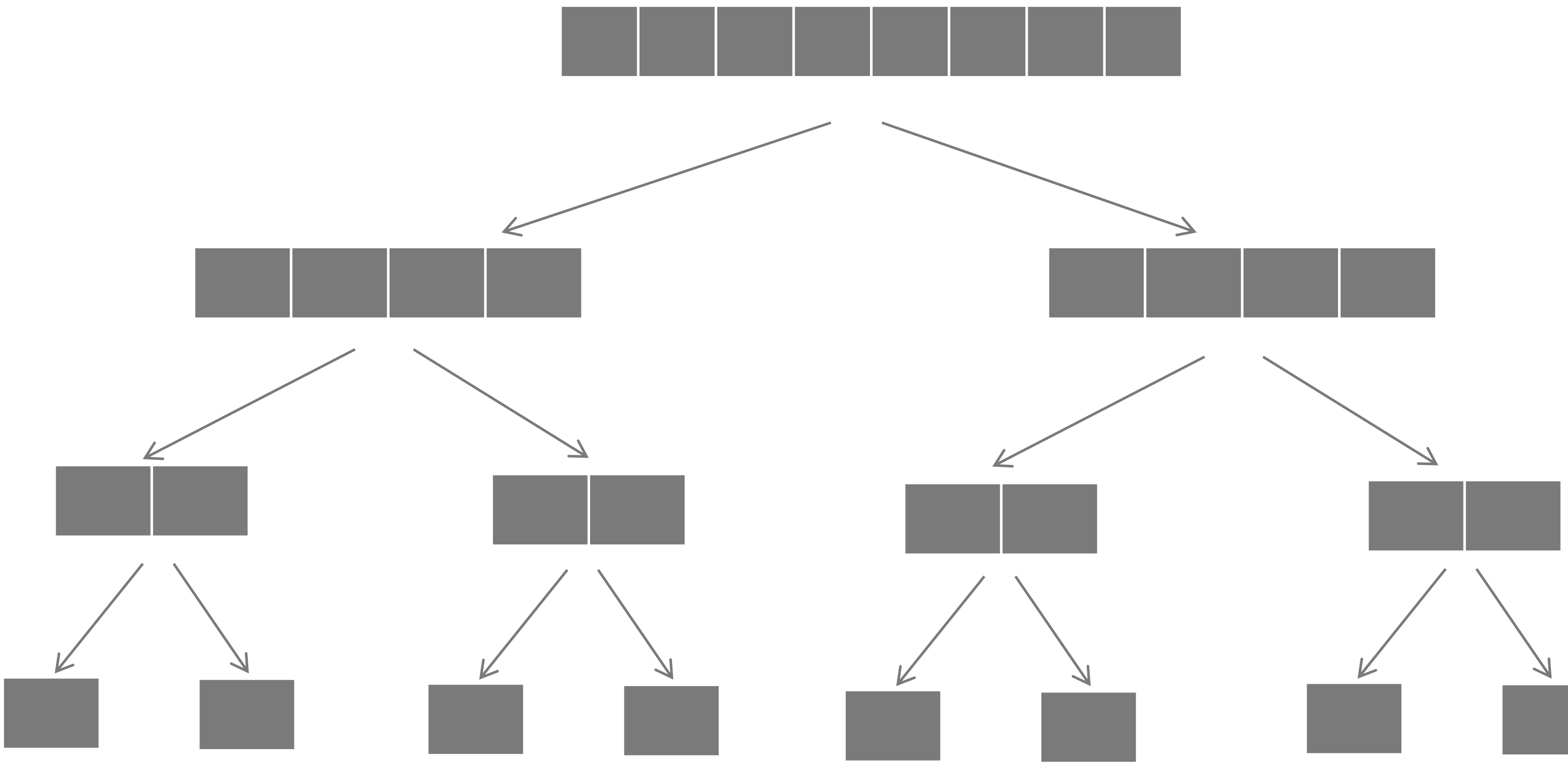
Quick Sort

With quick select



Quick Sort

With quick select



N^2

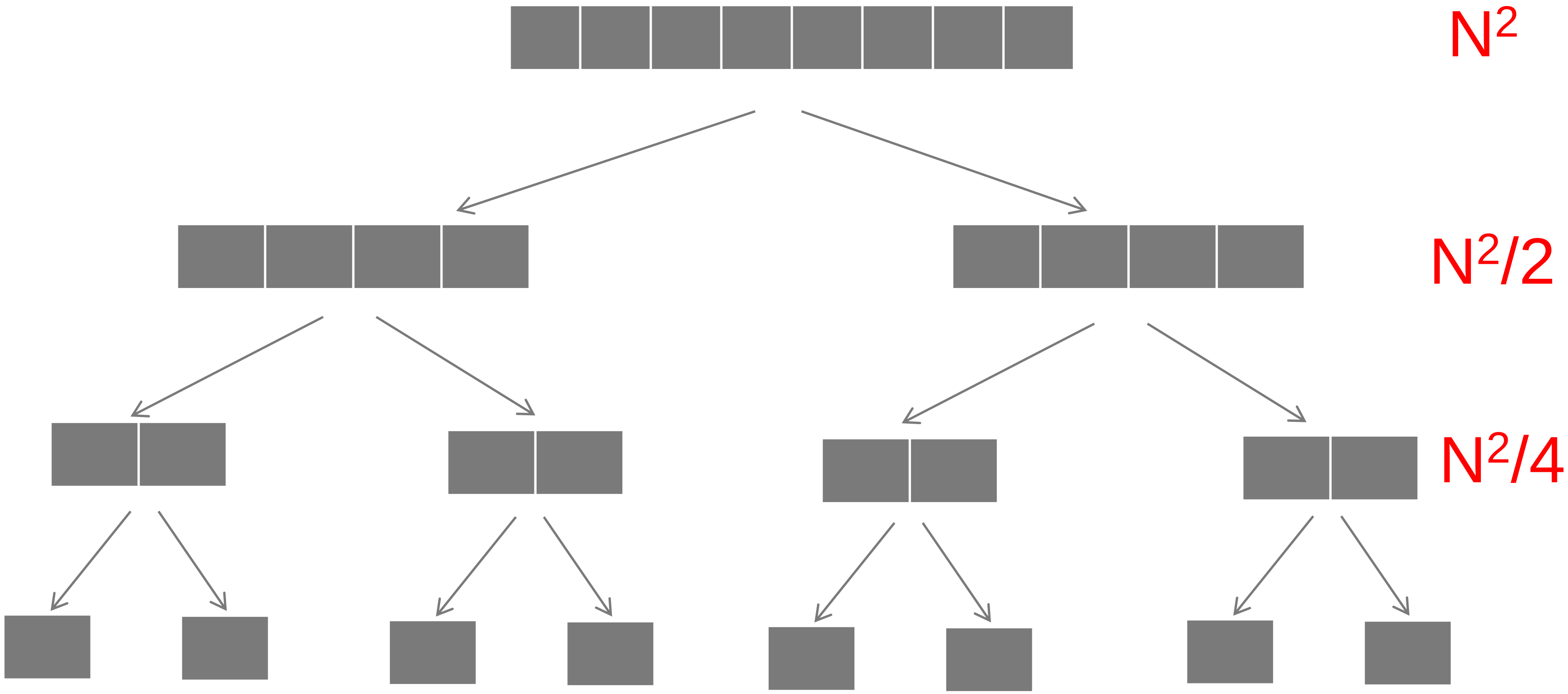
$N^2/2$

$N^2/4$

Worst-case cost at level k: $N^2/2^k$

Quick Sort

With quick select

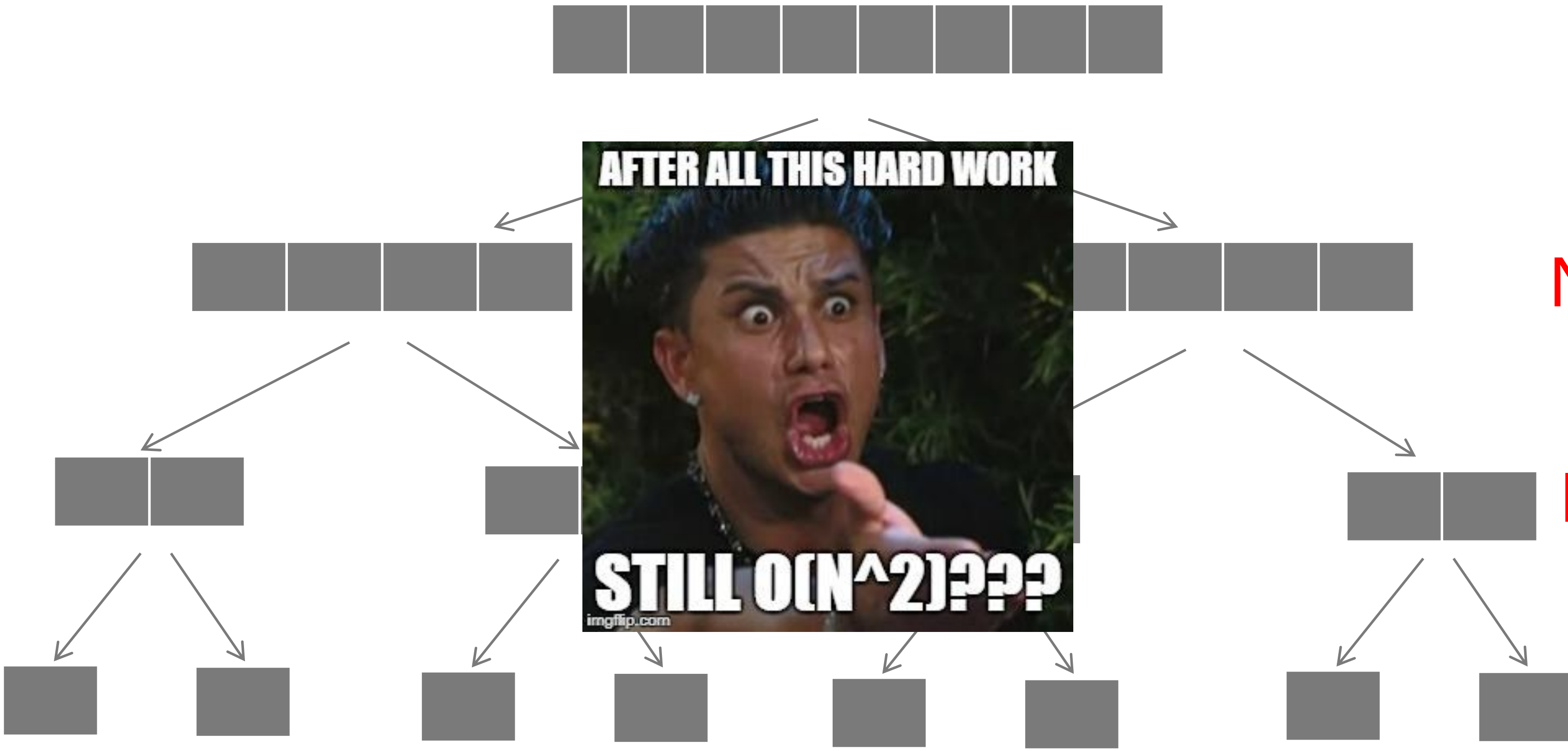


Worst-case cost at level k: $N^2/2^k$

Total cost: $N^2 + N^2/2 + N^2/4 + \ldots + 1 = N^2(1 + ½ + ¼ + \ldots) =$ $O(N^2)$

Quick Sort

With quick select



N^2

$N^2/2$

$N^2/4$

Worst-case cost at level k: $N^2/2^k$

Total cost: $N^2 + N^2/2 + N^2/4 + \ldots + 1 = N^2(1 + ½ + ¼ + \ldots) =$ $O(N^2)$

Quick Sort
With quick select

- So what is the complexity?
  - Worst case…
  - O(N^2)

Questions?

Quick Sort
With quick select

- So what is the complexity?
  - Worst case…
  - O(N^2)
  - … so is it useless?

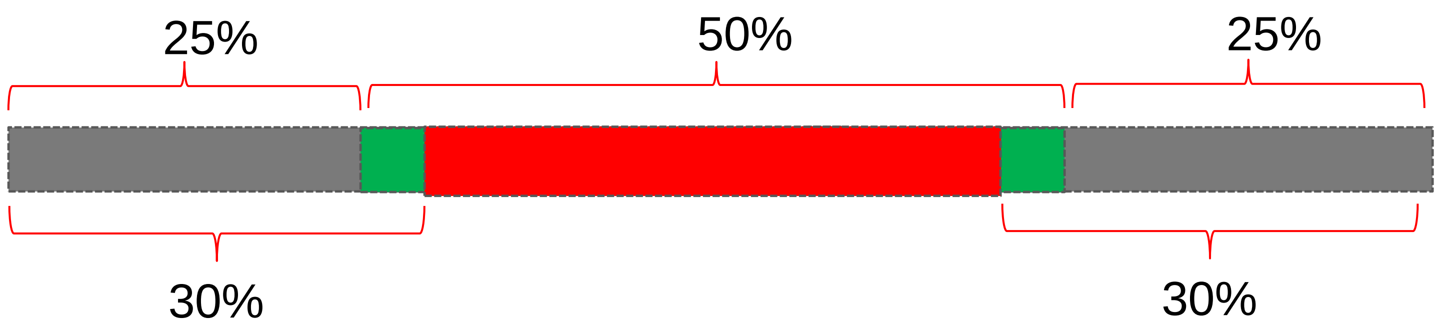
Quick Sort
With quick select

- So what is the complexity?
  - Worst case…
  - O(N^2)
  - … so is it useless?

  - Why not we adjust it now, to not find the median but find just "good enough" known as the median of median…

- So what is the complexity?
  - Worst case…
  - O(N^2)
  - … so is it useless?

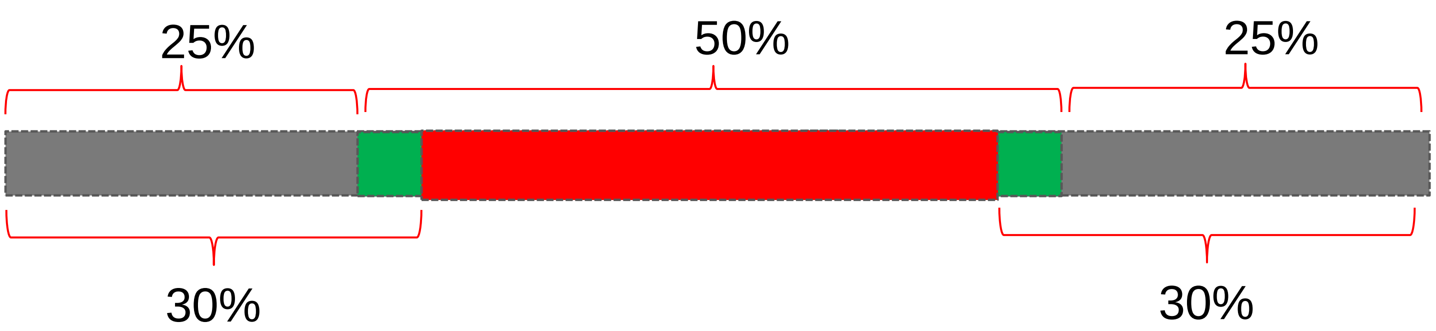
  - Why not we adjust it now, to not find the median but find just “good enough” known as the median of median…



Quick Sort

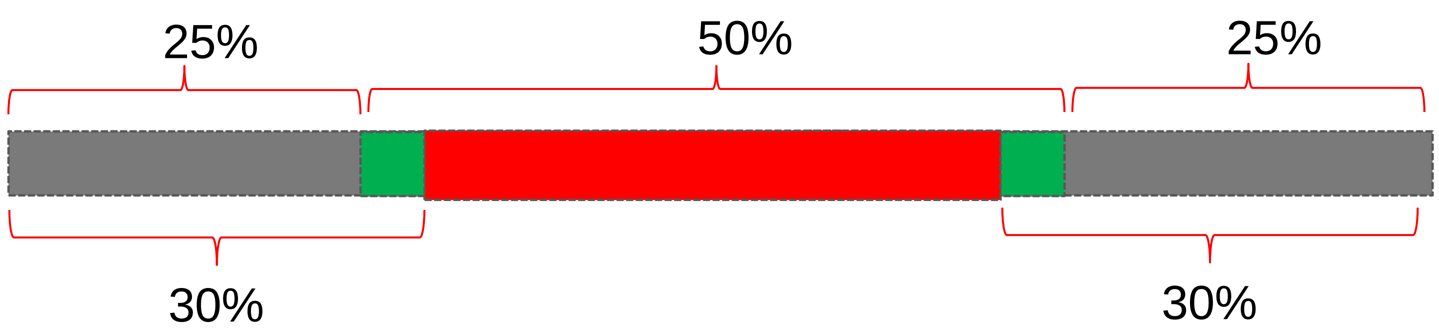
With quick select

- So what is the complexity?
  - Why not we adjust it now, to not find the median but find just “good enough” known as the median of median…
    - Find pivot within the green area using quick select
    - Do normal quick sort



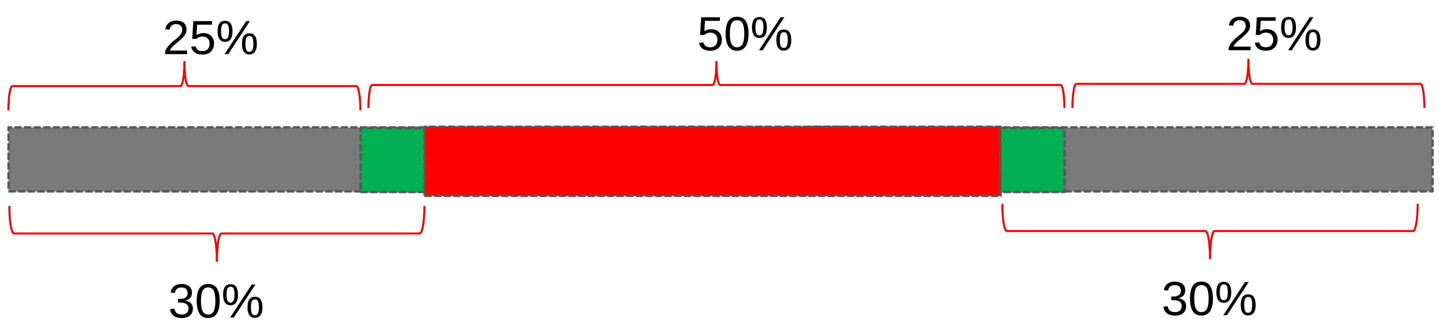
Quick Sort
With quick select

- So what is the complexity?
  - Why not we adjust it now, to not find the median but find just "good enough" known as the median of median…
    - Find pivot within the green area using quick select O(N)
    - Do normal quick sort

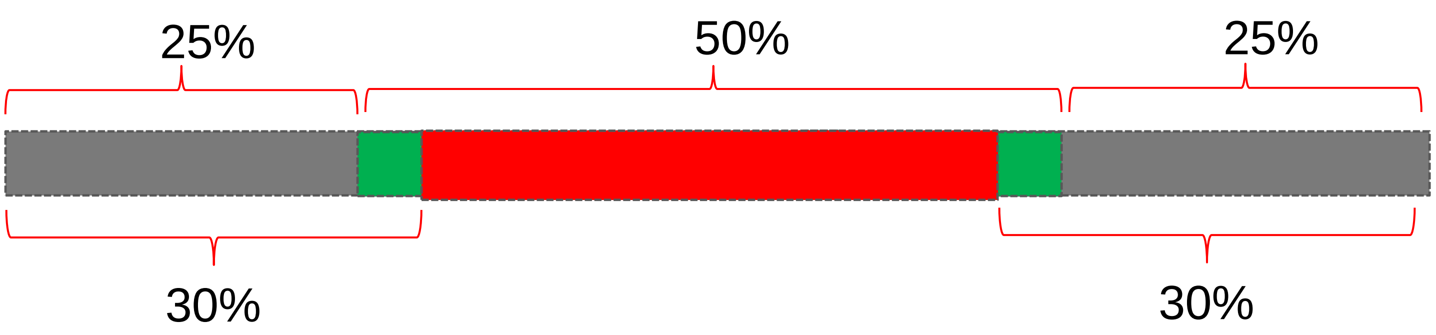


Quick Sort
With quick select

- So what is the complexity?
  - Why not we adjust it now, to not find the median but find just "good enough" known as the median of median…
    - Find pivot within the green area using quick select O(N)
    - Do normal quick sort O(1) why?



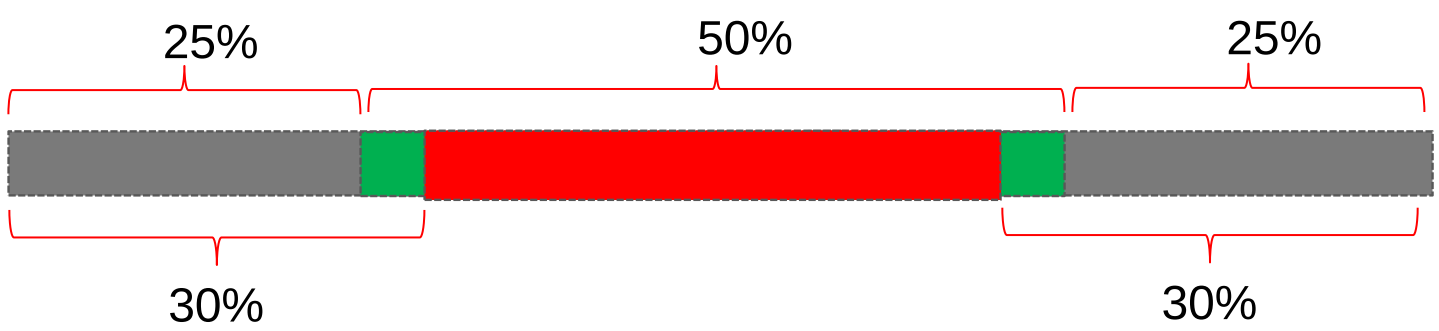
- So what is the complexity?
  - Why not we adjust it now, to not find the median but find just "good enough" known as the median of median…
    - Find pivot within the green area using quick select O(N)
    - Do normal quick sort O(1) why?
    - Maximum height is roughly log n



Quick Sort

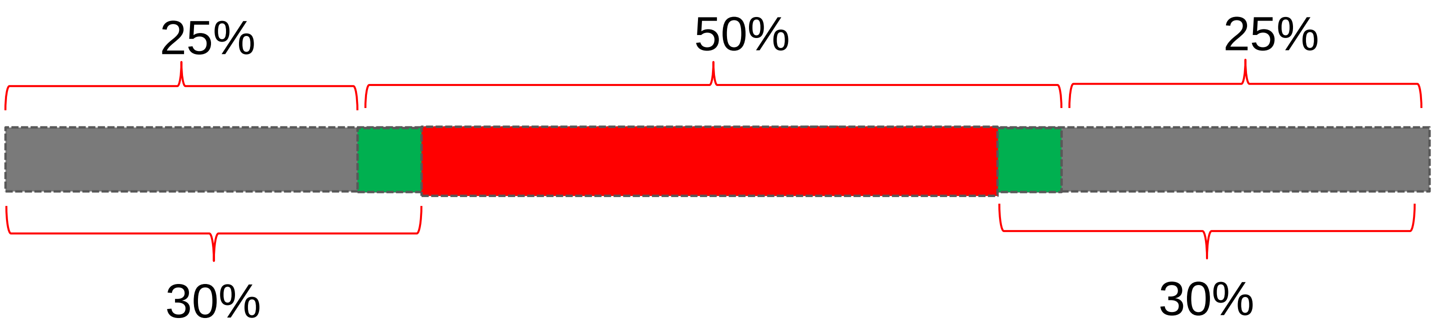
With quick select

- So what is the complexity?
  - Why not we adjust it now, to not find the median but find just “good enough” known as the median of median…
    - Find pivot within the green area using quick select O(N)
    - Do normal quick sort O(1) why?
    - Maximum height is roughly log n
    - Final complexity? O(N log N) still lol



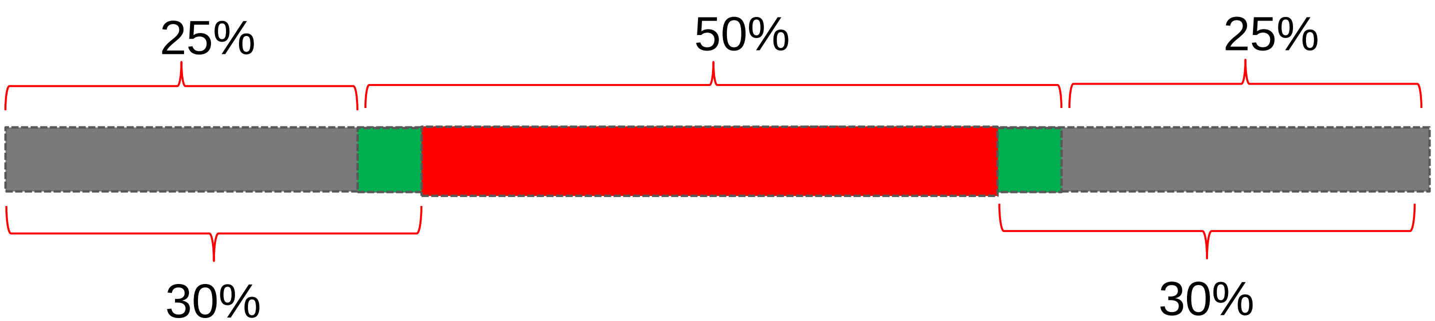
Quick Sort
With quick select

- So what is the complexity?
  - Why not we adjust it now, to not find the median but find just "good enough" known as the median of median…
    - Find pivot within the green area using quick select O(N)
    - Do normal quick sort O(1) why?
    - Maximum height is roughly log n
    - Final complexity? O(N log N) still lol



Quick Sort
With quick select

- So what is the complexity?
  - Why not we adjust it now, to not find the median but find just "good enough" known as the median of median…
    - Find pivot within the green area using quick select O(N)
    - Do normal quick sort O(1) why?
    - Maximum height is roughly log n
    - Final complexity? O(N log N) still lol



- In reality, random pivot works well due to probability…

Questions?

- Now a way to make quick select better is via median-of-medians

Quick Sort

With quick select median of medians

- Now a way to make quick select better is via median-of-medians
  - This is not examinable
  - I will be using Nathan's slides

Quick Sort
With quick select median of medians

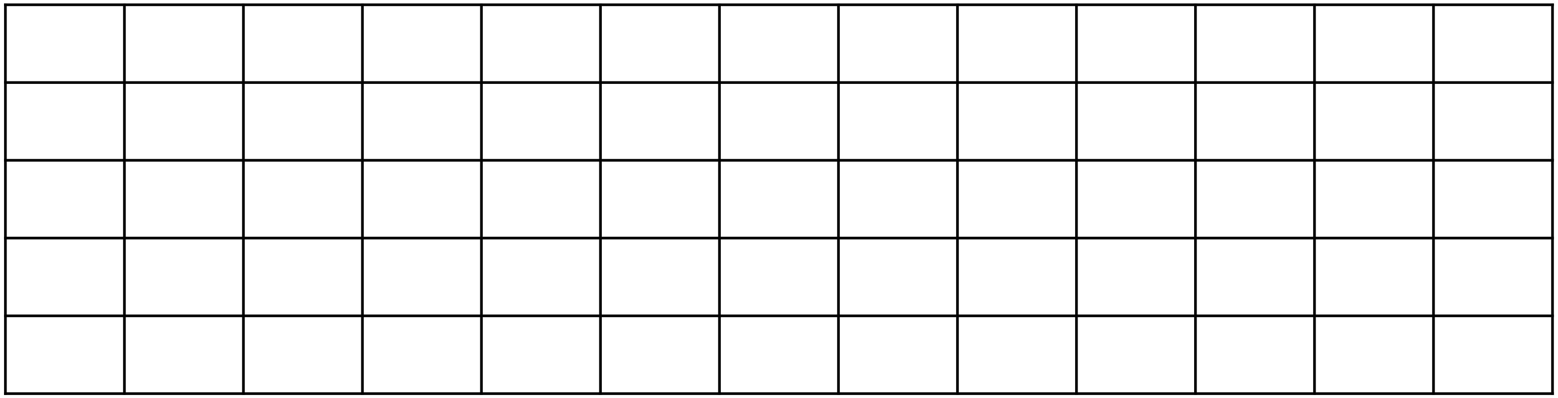
- Now a way to make quick select better is via median-of-medians
  - This is not examinable
  - I will be using Nathan's slides

- And this ensure the worst case or quick-sort is O(N log N)

Median of medians (not examinable)

- Sort groups of size five

Bigger

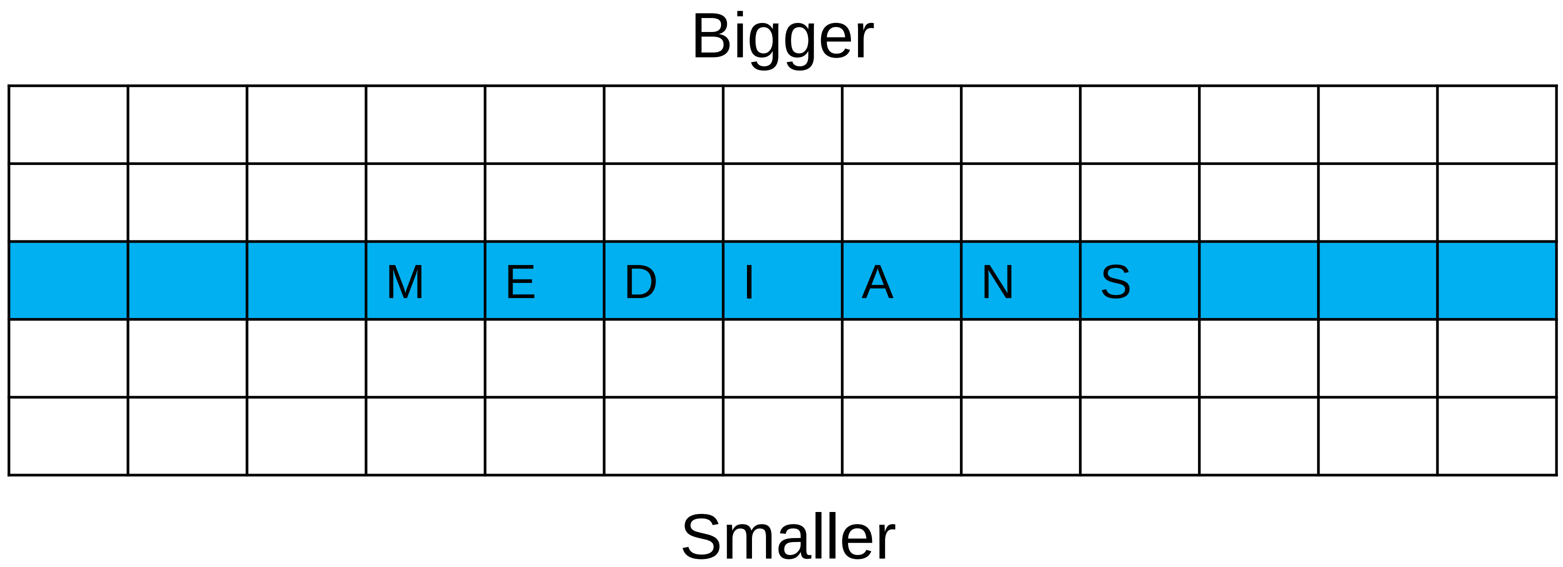


Smaller

Median of medians (not examinable)

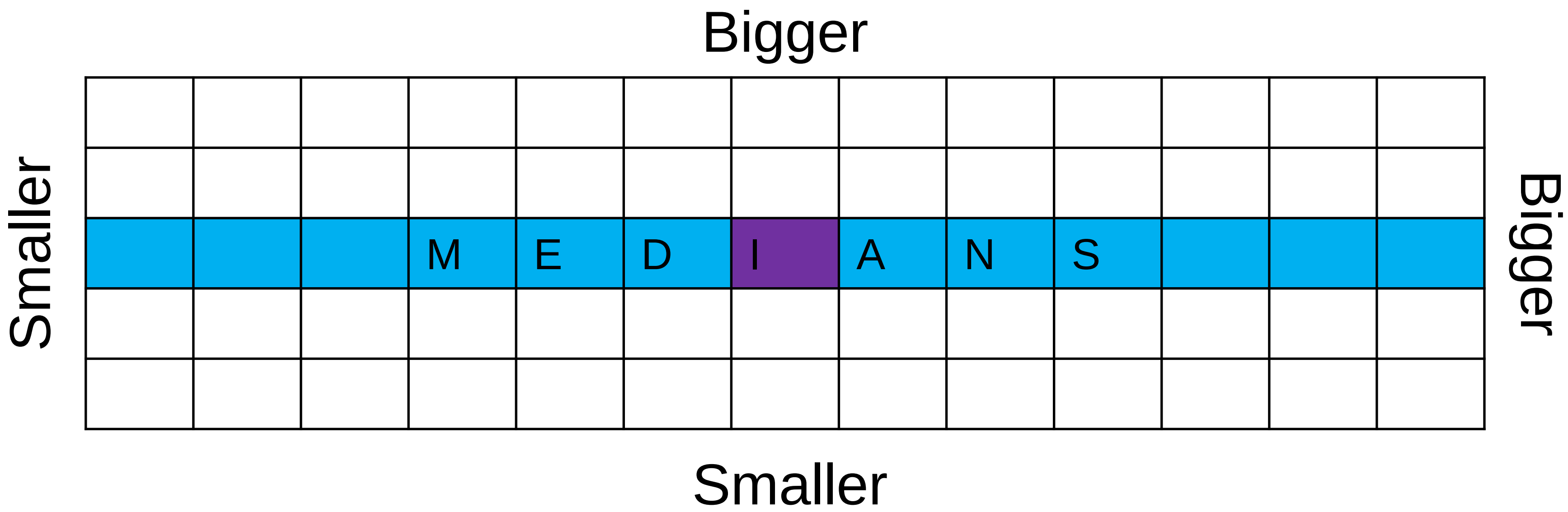
Sort groups of size five

Find the medians



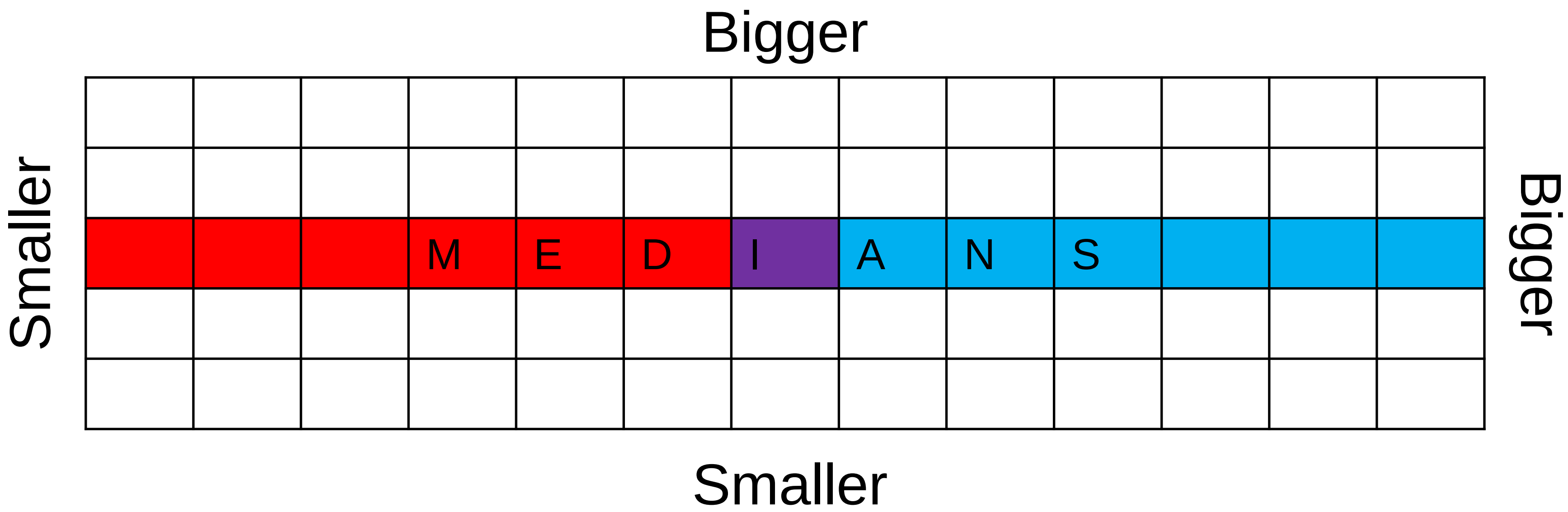
Median of medians (not examinable)

- Sort groups of size five
- Find the medians
- Find the median of those!
- (Note that the groups of 5 are not actually sorted, just shown here in sorted order for clarity)



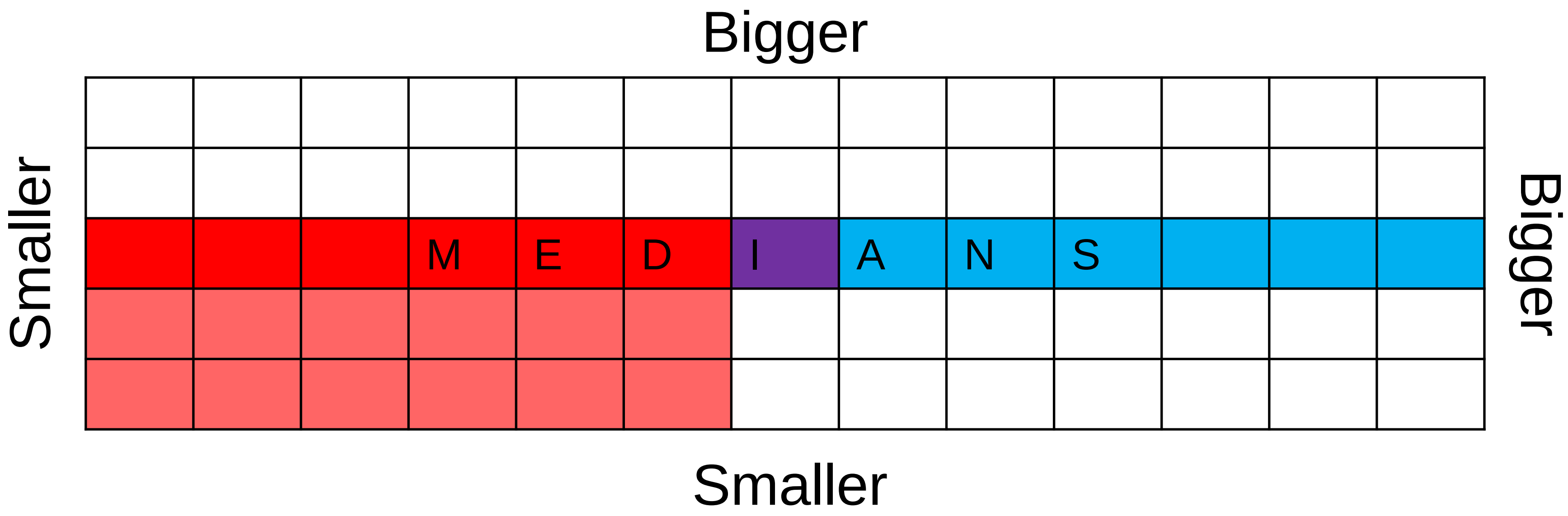
Median of medians (not examinable)

- Median of medians is bigger than half the medians



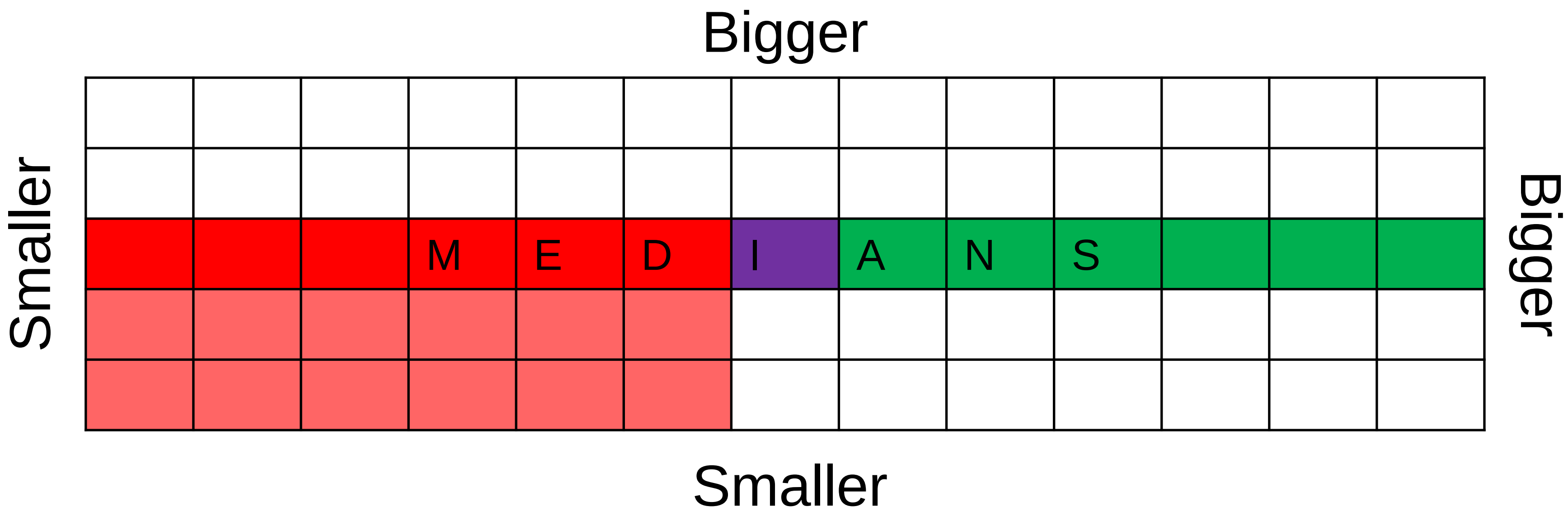
Median of medians (not examinable)

- Median of medians is bigger than half the medians
- So it is bigger than all the red values as well



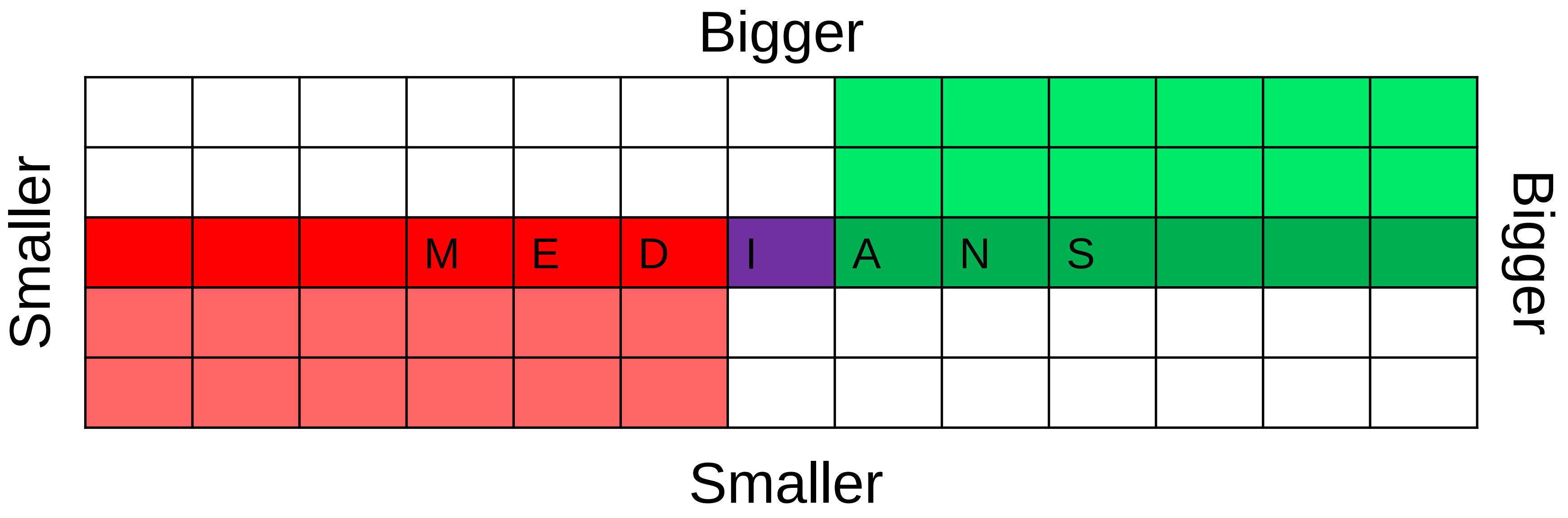
Median of medians (not examinable)

- Median of medians is smaller than half the medians



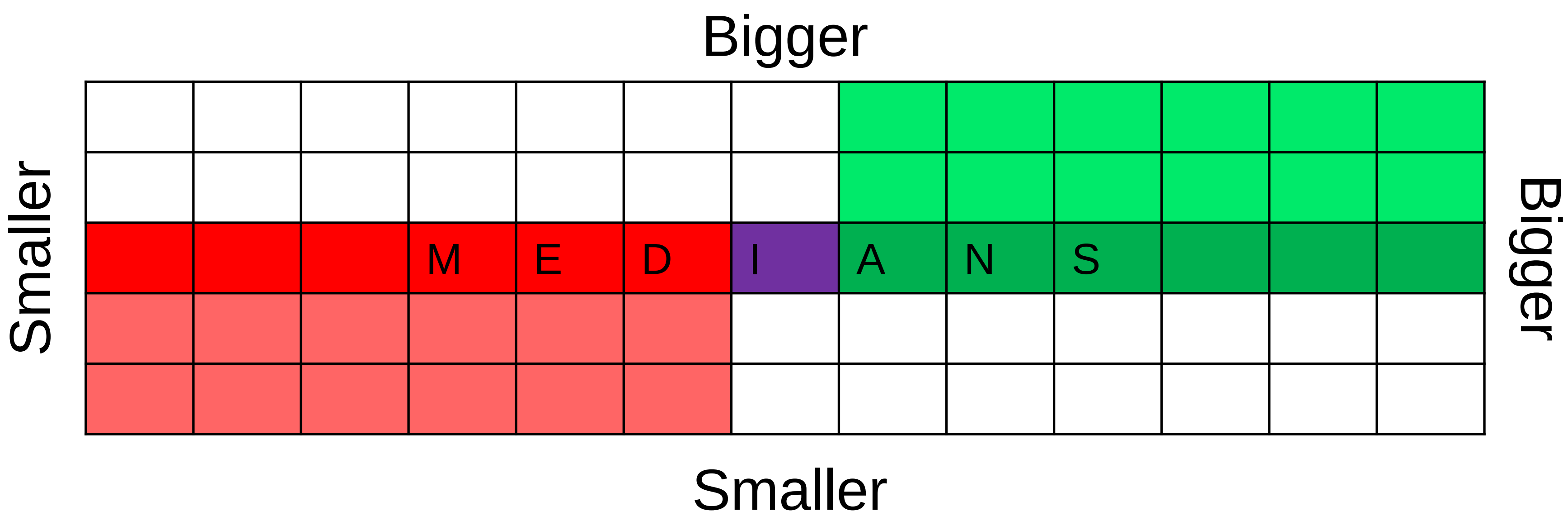
Median of medians (not examinable)

- Median of medians is smaller than half the medians
- So it is smaller than the green values as well



Median of medians (not examinable)

- Median of medians is greater than 30% and also less than 30%, so its in the middle 40%
- The worst split we can get using the MoM is 70:30!
- However, we did need to find the exact median of n/5 items… how?



Quicksort with O(N log N) in worst-case

Median_of_medians(list[1..n])

divide into sublists of size 5

medians = [median of each sublist]

use quickselect to find the median of **medians**

Quicksort with O(N log N) in worst-case

```
Median_of_medians(list[1..n])
    if n <= 5
        use insertion sort to find the median, and return it
    divide into sublists of size 5
    medians = [median of each sublist]
    use quickselect to find the median of medians
```

Quicksort with O(N log N) in worst-case

```
Median_of_medians(list[1..n])
    if n <= 5
        use insertion sort to find the median, and return it
    divide into sublists of size 5
    medians = [median of each sublist]
    return quickselect(medians, (len(medians)+1)/2)
```

Quicksort with O(N log N) in worst-case

This call uses quickselect!
But with a weaker pivot

Quickselect(list, lo, hi, k)

    if lo > hi
        return array[k]
    pivot = **median_of_medians**(list, lo, hi, k)
    mid = *partition*(array, lo, hi, pivot)
    if mid > k
        return *quickselect*(array, lo, mid-1, k)
    elif k > mid
        return *quickselect*(array, mid+1, hi, k)
    else
        return array[k]

Quick Sort
with O(N log N) in worst-case

- Wait what?
  - Median of median calls quick select
  - Quick select calls median of median

Quick Sort

with O(N log N) in worst-case

- Wait what?
  - Median of median calls quick select
  - Quick select calls median of median

  - This is called co-recursion…

Quicksort with O(N log N) in worst-case

This call uses quickselect!
But with a weaker pivot

```
Quickselect(list, lo, hi, k)
    if lo > hi
        return array[k]
    pivot = median_of_medians(list, lo, hi, k) (
    mid = partition(array, lo, hi, pivot) (70:30 pivot in worst)
    if mid > k
        return quickselect(array, lo, mid-1, k) (n/7 in worst)
    elif k > mid
         return quickselect(array, mid+1, hi, k) (n/7 in worst)
    else
        return array[k]
```

Quicksort with O(N log N) in worst-case

Quickselect time complexity recurrence

$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + an$$

- $T\left(\frac{n}{5}\right)$ for recursing on the list of the medians of groups of 5 (inside the call to median of medians)
- $T\left(\frac{7n}{10}\right)$ for the main recursive call, which is guaranteed to have split the list at least 30:70 (because the pivot was selected by MoM)
- an for the linear time partition algorithm + time to find medians of groups of five

Solving this give linear time!

So armed with a linear time quickselect, we can now quicksort in NlogN worst case…

Questions?

Quick Select

and average case complexity

- Note: NOT EXAMINABLE for math approach

Quick Select
and average case complexity

- What algorithm is quick select?

Quick Select
and average case complexity

- What algorithm is quick select?
  - Partial sorting with partition
  - Divide and conquer

Quick Select
and average case complexity

- What algorithm is quick select?
  - Partial sorting with partition
  - Divide and conquer
  - Recursion

Quick Select
and average case complexity

- What algorithm is quick select?
  - Partial sorting with partition
  - Divide and conquer
  - Recursion

$$T_N = N + 1 + \begin{cases} T(N-i) & \text{if } i < k, \\ 1 & \text{if } i = k, \\ T(i-1) & \text{if } i > k \end{cases} .$$

Quick Select
and average case complexity

- What algorithm is quick select?
  - Partial sorting with partition
  - Divide and conquer
  - Recursion

$$T_N = N + 1 + \begin{cases} T(N-i) & \text{if } i < k, \\ 1 & \text{if } i = k, \\ T(i-1) & \text{if } i > k \end{cases}.$$

  - N is the size of list
  - i is the rank of the pivot
  - k is the k-th element we are looking for

Quick Select
and average case complexity

- What algorithm is quick select?
  - Partial sorting with partition
  - Divide and conquer
  - Recursion

$$T_N = N + 1 + \begin{cases} T(N-i) & \text{if } i < k, \\ 1 & \text{if } i = k, \\ T(i-1) & \text{if } i > k \end{cases}.$$

  - N is the size of list
  - i is the rank of the pivot
  - k is the k-th element we are looking for
  - With this, we can now figure out the average complexity

Quick Select
and average case complexity

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

  - What does this means?

Quick Select
and average case complexity

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

  - What does this means?
    - Every pivot case
    - Sum it all up
    - Get the average

MONASH University

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = \boxed{N} + 1 + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

Quick Select
and average case complexity

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + \boxed{1} + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

Quick Select

and average case complexity

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \boxed{\frac{1}{N}}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

Quick Select
and average case complexity

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \frac{1}{N}\left(\boxed{\sum_{i=1}^{k-1} T(N-i)} + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

Quick Select
and average case complexity

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \boxed{\sum_{i=k+1}^{N} T(i-1)} + 1\right)$$

Quick Select
and average case complexity

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) \boxed{+ 1}\right)$$

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

  - Now we try to solve this

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

  - Now we try to solve this
    - Multiply both side with N

$$NT_N = N^2 + N + \sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + \frac{1}{N}$$

Quick Select
and average case complexity

- What algorithm is quick select?
  - With this, we can now figure out the average complexity
  - This is the expected run time on average

$$T_N = N + 1 + \frac{1}{N}\left(\sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + 1\right)$$

  - Now we try to solve this
    - Multiply both side with N

$$NT_N = N^2 + N + \sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + \boxed{\frac{1}{N}}$$

Quick Select
and average case complexity

- So now we have

$$NT_N = N^2 + N + \sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + \frac{1}{N}$$

- How do we solve it?
  - Not easy

- So now we have

$$NT_N = N^2 + N + \sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + \frac{1}{N}$$

- How do we solve it?
  – Not easy
  – We create another equation, instead of N it is N-1

$$(N-1)T_{N-1} = (N-1)^2 + (N-1) + \sum_{i=2}^{k} T(N-i) + \sum_{i=k+1}^{N+1} T(i-1) + \frac{1}{N-1}$$

- So now we have

$$NT_N = N^2 + N + \sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + \frac{1}{N}$$

- How do we solve it?
  - Not easy
  - We create another equation, instead of N it is N-1. Why?

$$(N-1)T_{N-1} = (N-1)^2 + (N-1) + \sum_{i=2}^{k} T(N-i) + \sum_{i=k+1}^{N+1} T(i-1) + \frac{1}{N-1}$$

Quick Select

and average case complexity

- So now we have

$$NT_N = N^2 + N + \sum_{i=1}^{k-1} T(N-i) + \sum_{i=k+1}^{N} T(i-1) + \frac{1}{N}$$

- How do we solve it?
  - Not easy
  - We create another equation, instead of N it is N-1. Why? N – (N-1) = 1!!!

$$(N-1)T_{N-1} = (N-1)^2 + (N-1) + \sum_{i=2}^{k} T(N-i) + \sum_{i=k+1}^{N+1} T(i-1) + \frac{1}{N-1}$$

Quick Select
and average case complexity

- Doing the N – (N-1), we have

$$NT_N - (N-1)T_{N-1} = N^2 + N - (N-1)^2 - (N-1) - T(N-1) + T(N-1) + \frac{1}{N} - \frac{1}{N-1}$$

- Doing the N – (N-1), we have

$$NT_N - (N-1)T_{N-1} = N^2 + N - (N-1)^2 - (N-1) - T(N-1) + T(N-1) + \frac{1}{N} - \frac{1}{N-1}$$

- We then clean it

$$NT_N = (N-1)T_{N-1} + 2N + \frac{1}{N(N-1)}$$

Quick Select
and average case complexity

- Doing the N – (N-1), we have

$$NT_N - (N-1)T_{N-1} = N^2 + N - (N-1)^2 - (N-1) - T(N-1) + T(N-1) + \frac{1}{N} - \frac{1}{N-1}$$

- We then clean it

$$NT_N = (N-1)T_{N-1} + 2N + \frac{1}{N(N-1)}$$

- Then have the left side of T_N

$$T_N = \frac{N-1}{N}T_{N-1} + 2 + \frac{1}{N^2(N-1)}$$

- Finally our average case complexity is from the equation:

$$T_N = \frac{N-1}{N} T_{N-1} + 2 + \frac{1}{N^2(N-1)}$$

Quick Select
and average case complexity

- Finally our average case complexity is from the equation:

$$T_N = \frac{N-1}{N} T_{N-1} + 2 + \frac{1}{N^2(N-1)}$$

- What is the complexity?

- Finally our average case complexity is from the equation:

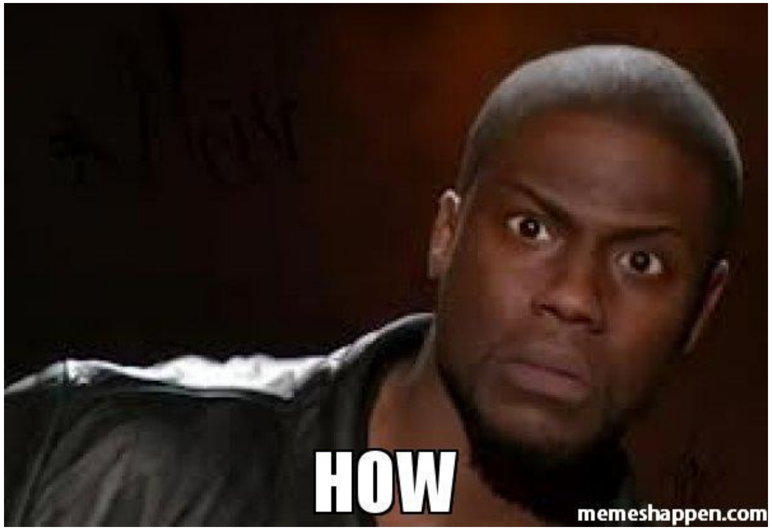
$$T_N = \frac{N-1}{N} T_{N-1} + 2 + \frac{1}{N^2(N-1)}$$

- What is the complexity? O(N)

- Finally our average case complexity is from the equation:

$$T_N = \frac{N-1}{N} T_{N-1} + 2 + \frac{1}{N^2(N-1)}$$

- What is the complexity? O(N)



Quick Select
and average case complexity

- Finally our average case complexity is from the equation:

$$T_N = \boxed{\frac{N-1}{N}} T_{N-1} + 2 + \boxed{\frac{1}{N^2(N-1)}}$$

- What is the complexity? O(N)
  - Both red boxes are < 1

Quick Select

and average case complexity

- Finally our average case complexity is from the equation:

$$T_N = \boxed{\frac{N-1}{N}} T_{N-1} + 2 + \boxed{\frac{1}{N^2(N-1)}}$$

- What is the complexity? O(N)
  - Both red boxes are < 1

$$T_N < T_{N-1} + 3$$

  - And for complexity, we are only concerns with bounds!

Quick Select
and average case complexity

- Finally our average case complexity is from the equation:

$$T_N = \boxed{\frac{N-1}{N}} T_{N-1} + 2 + \boxed{\frac{1}{N^2(N-1)}}$$

- What is the complexity? O(N)
  - Both red boxes are < 1

$$T_N < T_{N-1} + 3$$

  - And for complexity, we are only concerns with bounds!

$$T_N < 3N = O(N)$$

Questions?

- What are online algorithms?

- What are online algorithms?
  – Let say I give you a list of number, ask you to sort it…

- What are online algorithms?
  - Let say I give you a list of number, ask you to sort it…
  - Then I say I left out a number, and give it to you…

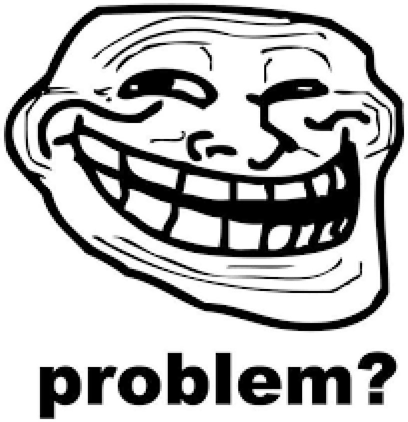
- What are online algorithms?
  - Let say I give you a list of number, ask you to sort it…
  - Then I say I left out a number, and give it to you…
  - Then you re-sort it…

Bonus

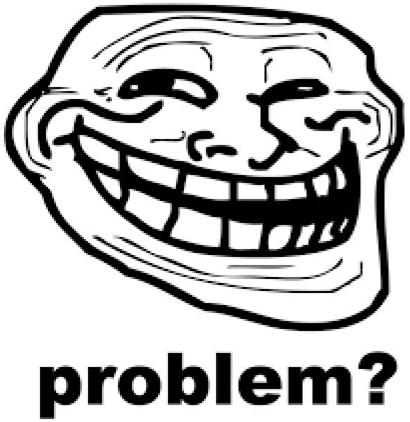
Online algorithms

- What are online algorithms?
  - Let say I give you a list of number, ask you to sort it…
  - Then I say I left out a number, and give it to you…
  - Then you re-sort it…
  - Then oops, I forgot another number lol

- What are online algorithms?
  - Let say I give you a list of number, ask you to sort it…
  - Then I say I left out a number, and give it to you…
  - Then you re-sort it…
  - Then oops, I forgot another number lol

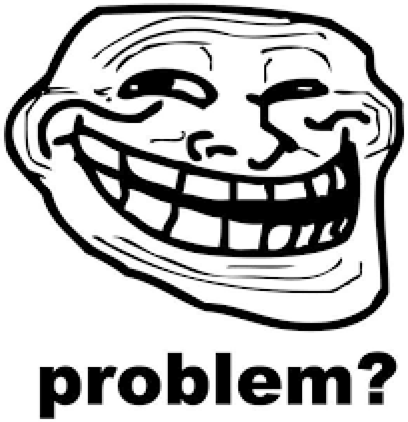


- What are online algorithms?
  - Let say I give you a list of number, ask you to sort it…
  - Then I say I left out a number, and give it to you…
  - Then you re-sort it…
  - Then oops, I forgot another number lol
  - Algorithms that can process new information without re-processing the old one



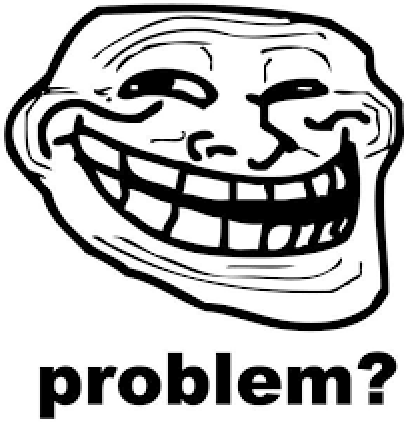
- What are online algorithms?
  - Let say I give you a list of number, ask you to sort it…
  - Then I say I left out a number, and give it to you…
  - Then you re-sort it…
  - Then oops, I forgot another number lol

  - Algorithms that can process new information without re-processing the old one
    - Insertion sort

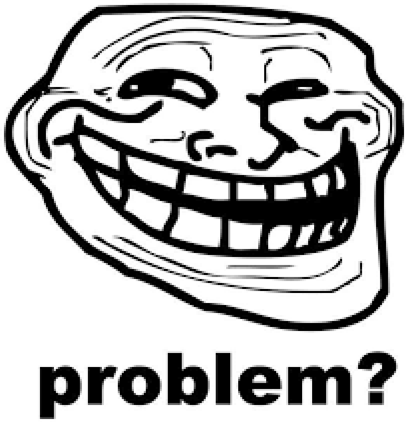


- What are online algorithms?
  – Let say I give you a list of number, ask you to sort it…
  – Then I say I left out a number, and give it to you…
  – Then you re-sort it…
  – Then oops, I forgot another number lol

  – Algorithms that can process new information without re-processing the old one
    - Insertion sort
    - What about k-th order statistic?



- What are online algorithms?
  - Let say I give you a list of number, ask you to sort it…
  - Then I say I left out a number, and give it to you…
  - Then you re-sort it…
  - Then oops, I forgot another number lol
  - Algorithms that can process new information without re-processing the old one
    - Insertion sort
    - What about k-th order statistic?
      - Does quick select still work?



- From your earlier studio
  - Note: Question number changes between semester

Problem 8. Devise an efficient online algorithm[1] that finds the smallest k elements of a sequence of integers. Write psuedocode for your algorithm. [Hint: Use a data structure that you have learned about in a previous unit]

Bonus
Online algorithms

- From your Tutorial 03 Question 08
  - Using quick select?
  - Using a new approach?

Problem 8. Devise an efficient online algorithm[1] that finds the smallest k elements of a sequence of integers. Write psuedocode for your algorithm. [Hint: Use a data structure that you have learned about in a previous unit]

Questions?

Thank You