MONASH
INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

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Ready?

Agenda

- Dynamic Programming

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- Dynamic Programming
  - Brute force (aka the starting point)

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  - Overlapping sub-problems
  - Backtracking

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  - Fibonacci
  - Coin change
  - Knapsack
    - Unbounded
    - 0/1
  - Edit distance

Agenda

- Dynamic Programming
  - Brute force (aka the starting point)
  - Overlapping sub-problems
  - Backtracking for solution reconstruction

  - Fibonacci
  - Coin change
  - Knapsack
    - Unbounded
    - 0/1
  - Edit distance

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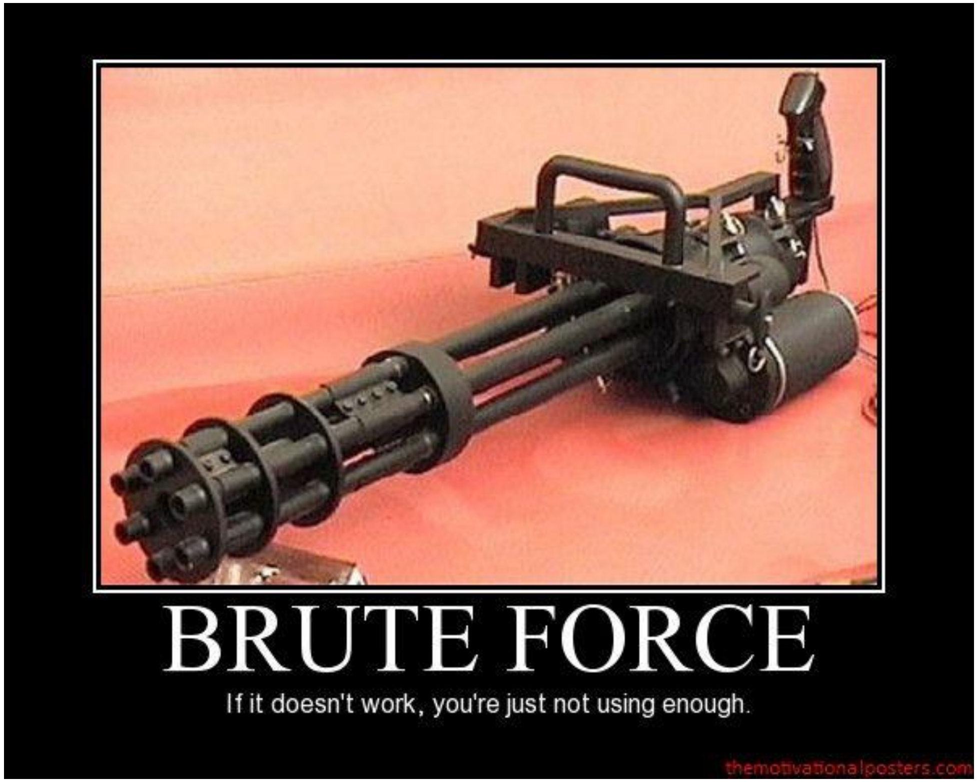
Let us begin…

Dynamic Programming

… a smarter way to brute force

- Can we brute force everything?

- Can we brute force everything?



- Can we brute force everything?
  – So if we can brute force anything, why not just use it?

- Can we brute force everything?
  - So if we can brute force anything, why not just use it?
  - Because brute forcing needs too much effort…

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- Can we brute force everything?
  - So if we can brute force anything, why not just use it?
  - Because brute forcing needs too much effort…

- Consider the Fibonacci problem
  - Find the n-th Fibonacci number

Questions?

Fibonacci

n-th number in the series

- Now if we want the 6th Fibonacci number…
  - How would we get it?

F(6)

Fibonacci

n-th number in the series

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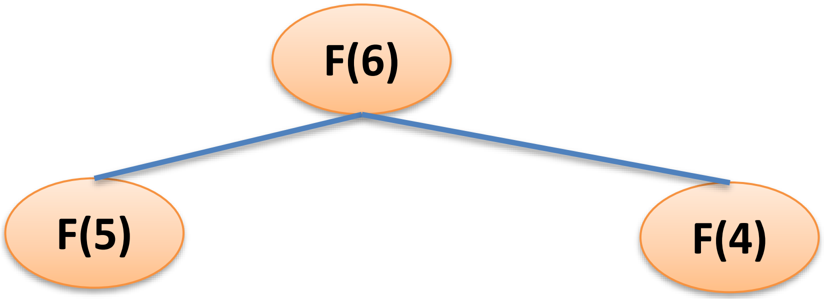

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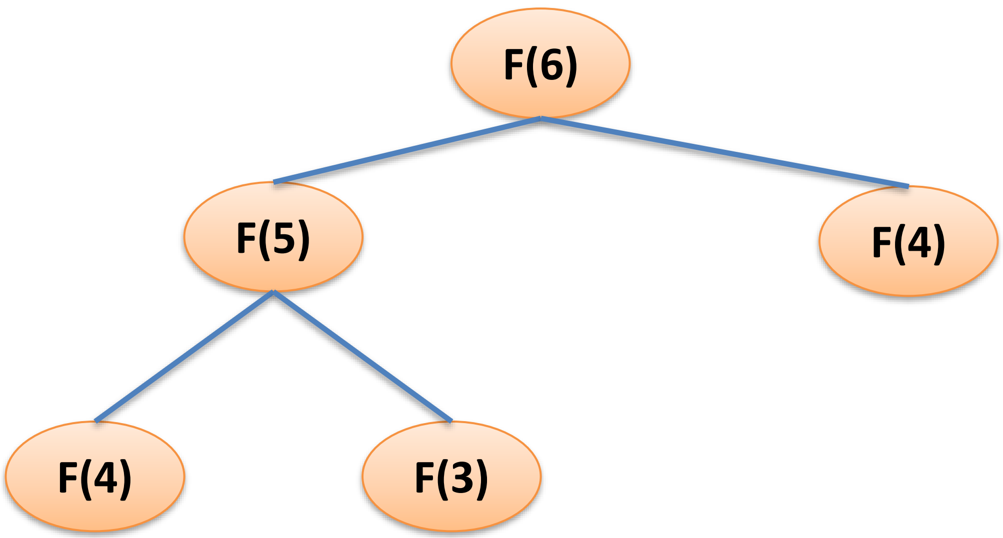

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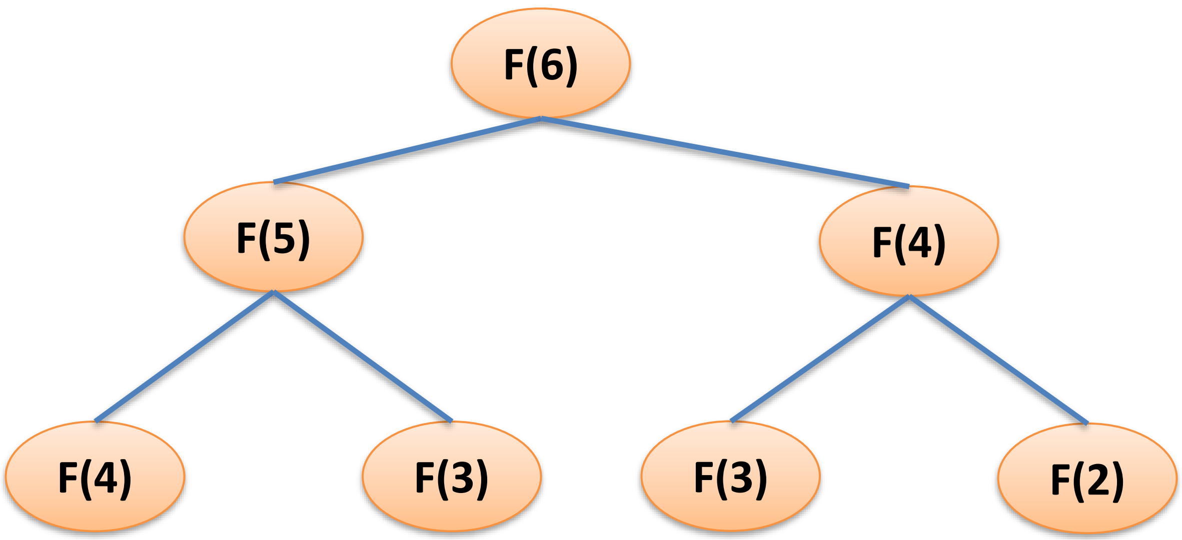
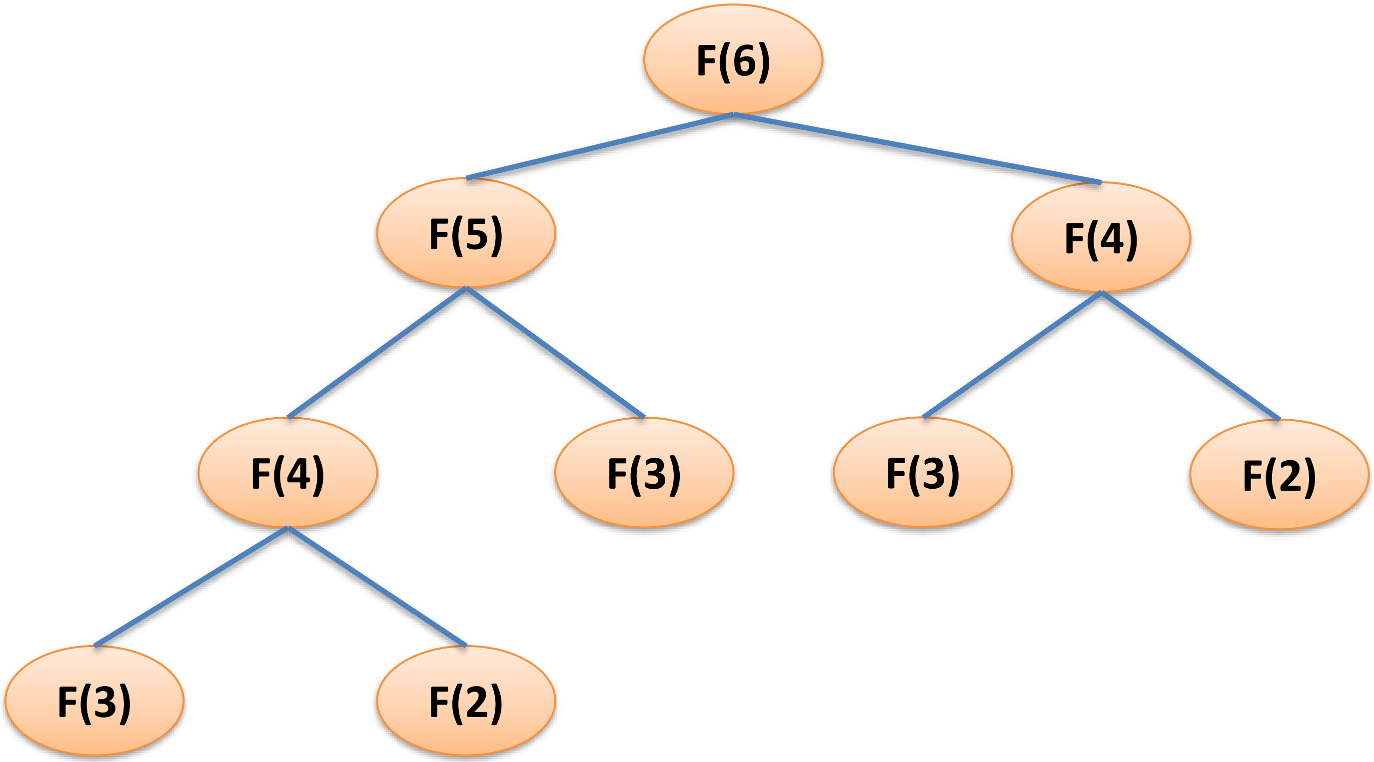


- Now if we want the 6^{th} Fibonacci number…
  - How would we get it?



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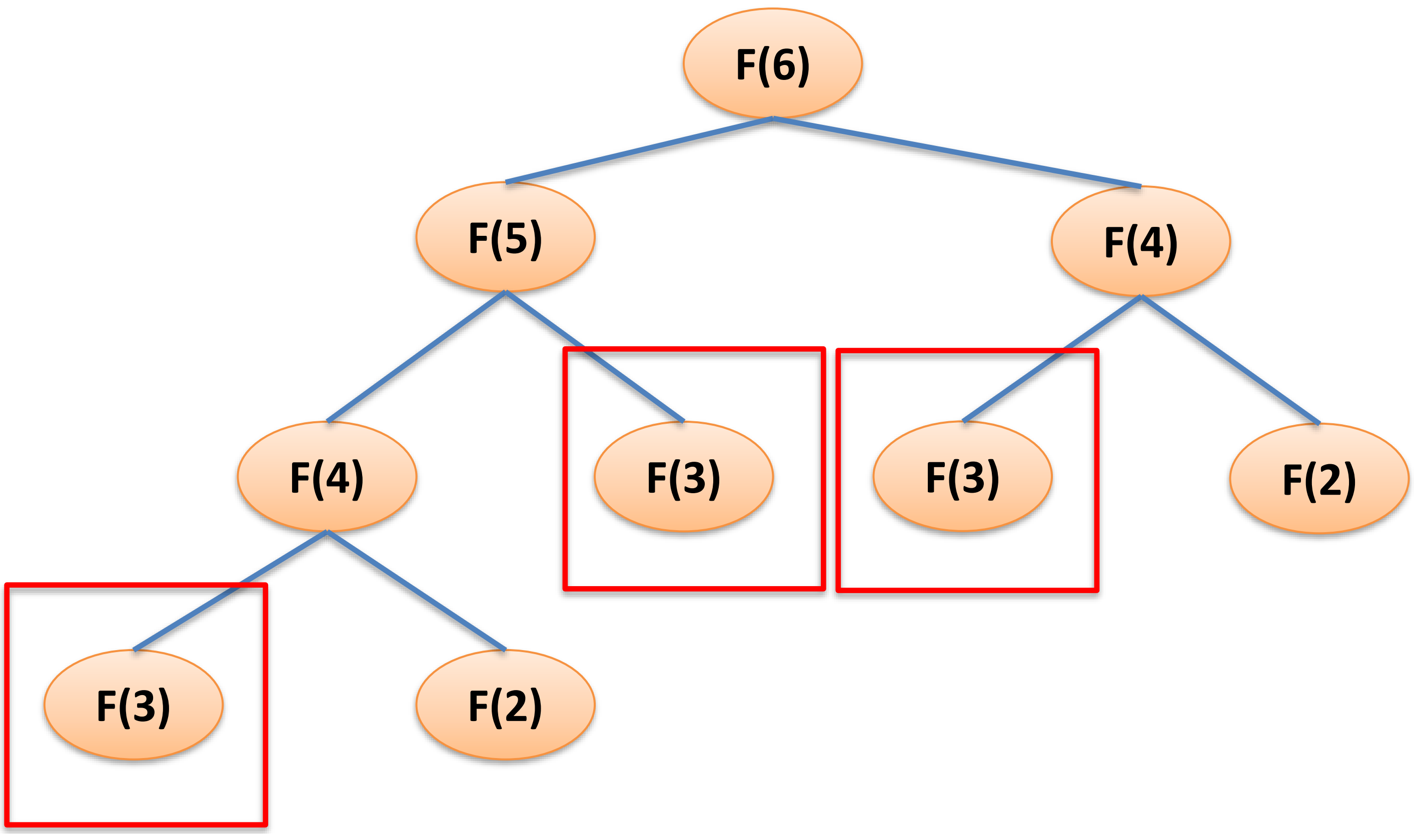
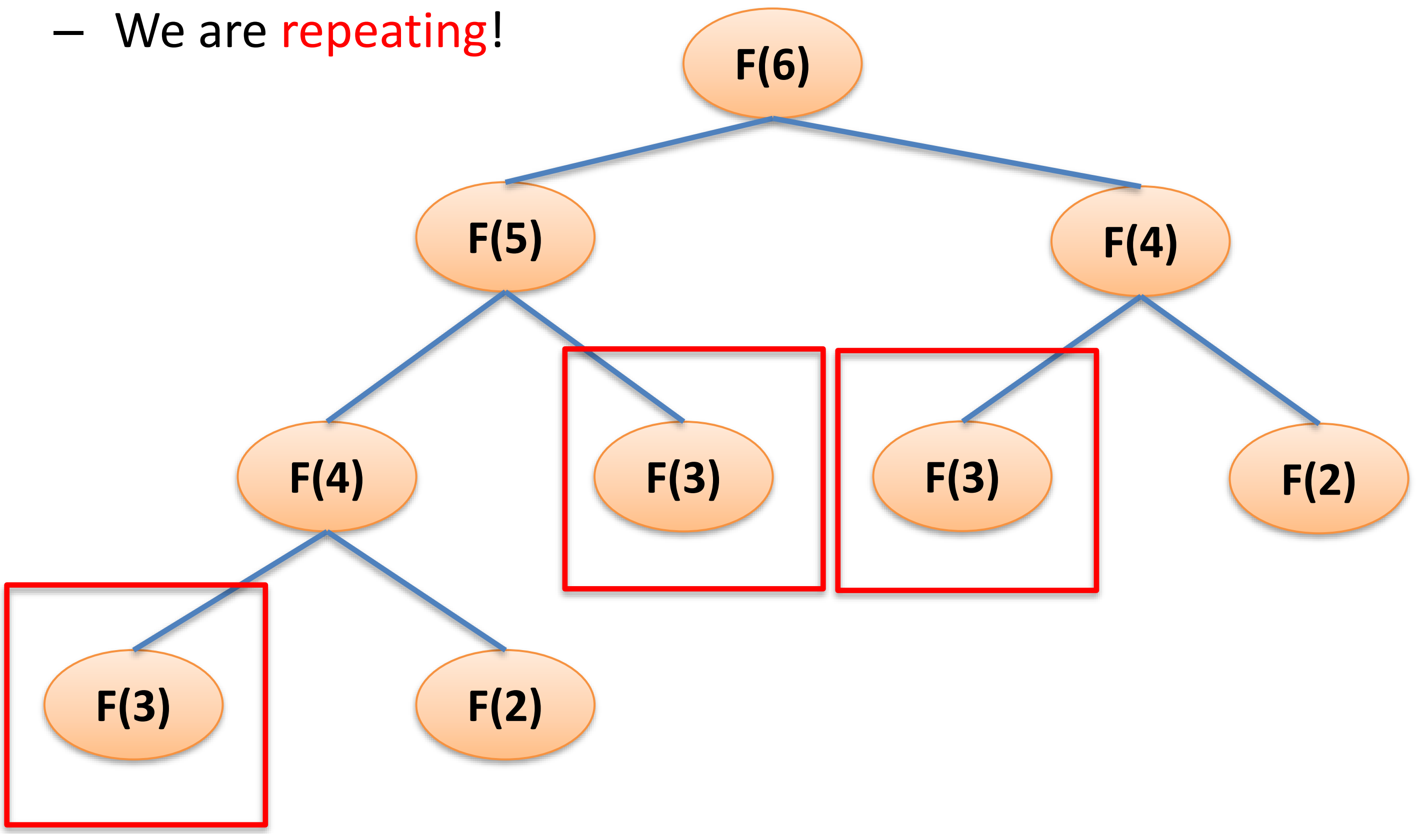
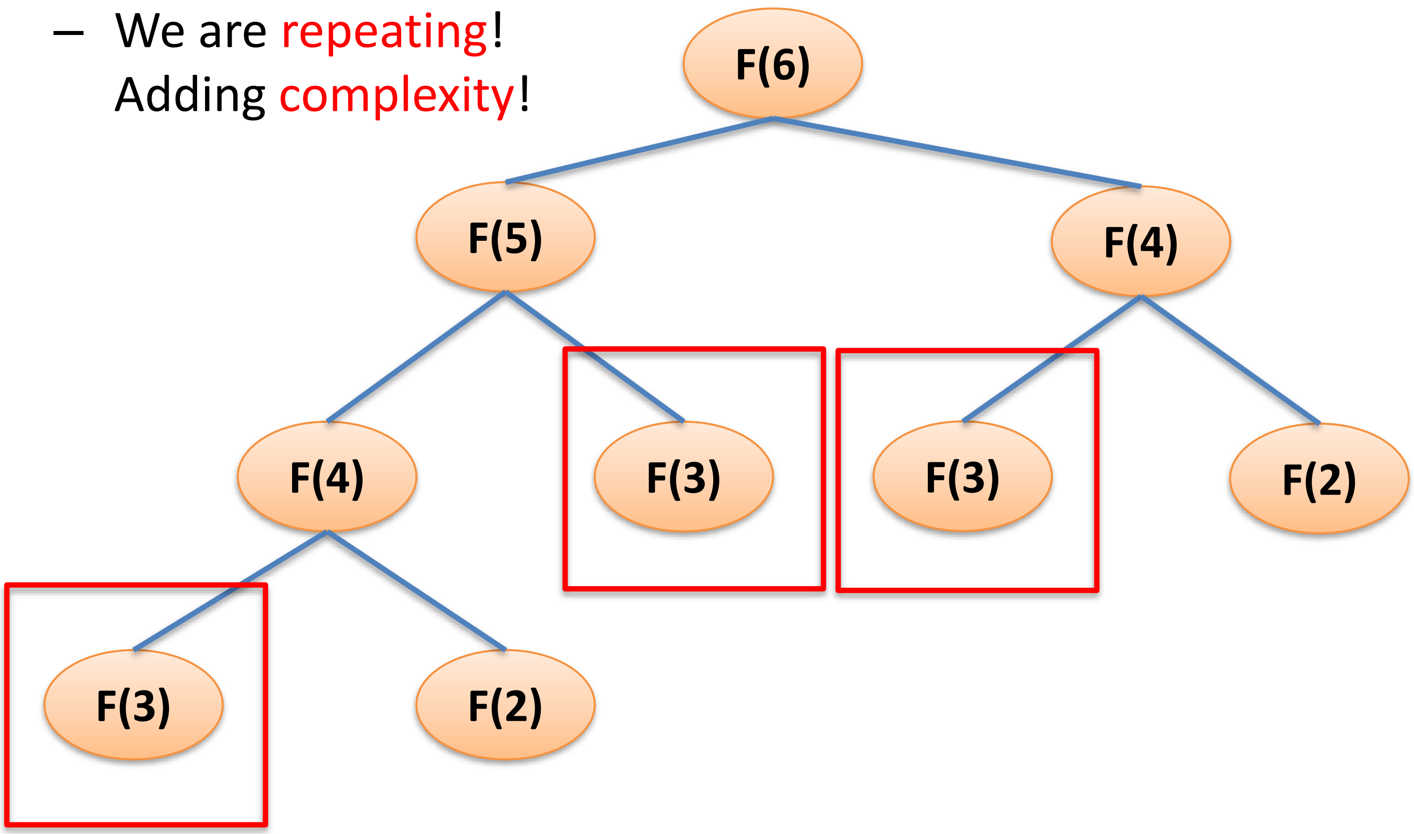


- Now if we want the 6th Fibonacci number…
  - How would we get it?
  - We are repeating!



Fibonacci
n-th number in the series

- Now if we want the 6th Fibonacci number…
  - How would we get it?
  - We are repeating!
    Adding complexity!



Fibonacci

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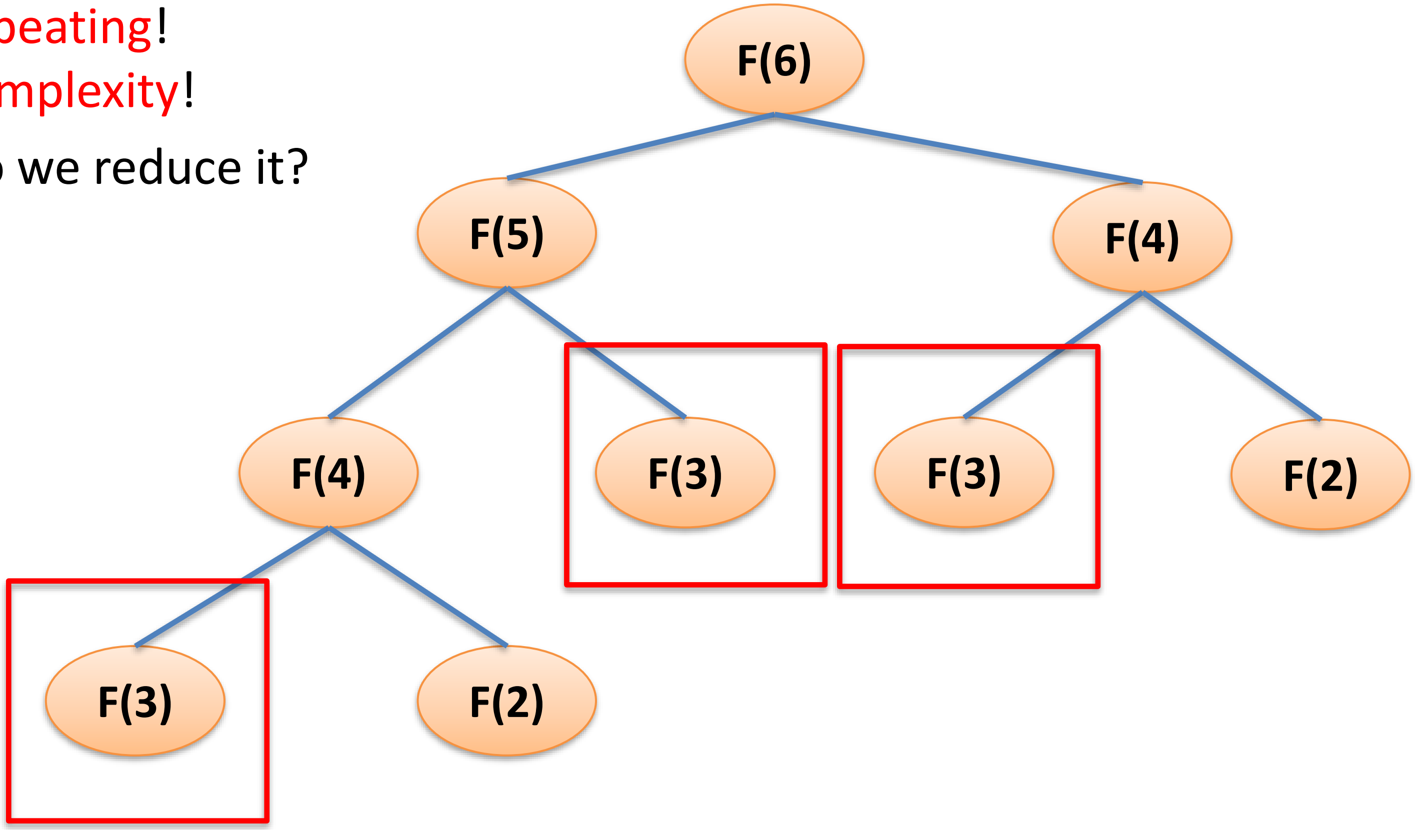
- Now if we want the 6th Fibonacci number…
  - How would we get it?
  - We are repeating!
    Adding complexity!
  - So how do we reduce it?



Fibonacci

n-th number in the series

- Now if we want the 6th Fibonacci number…
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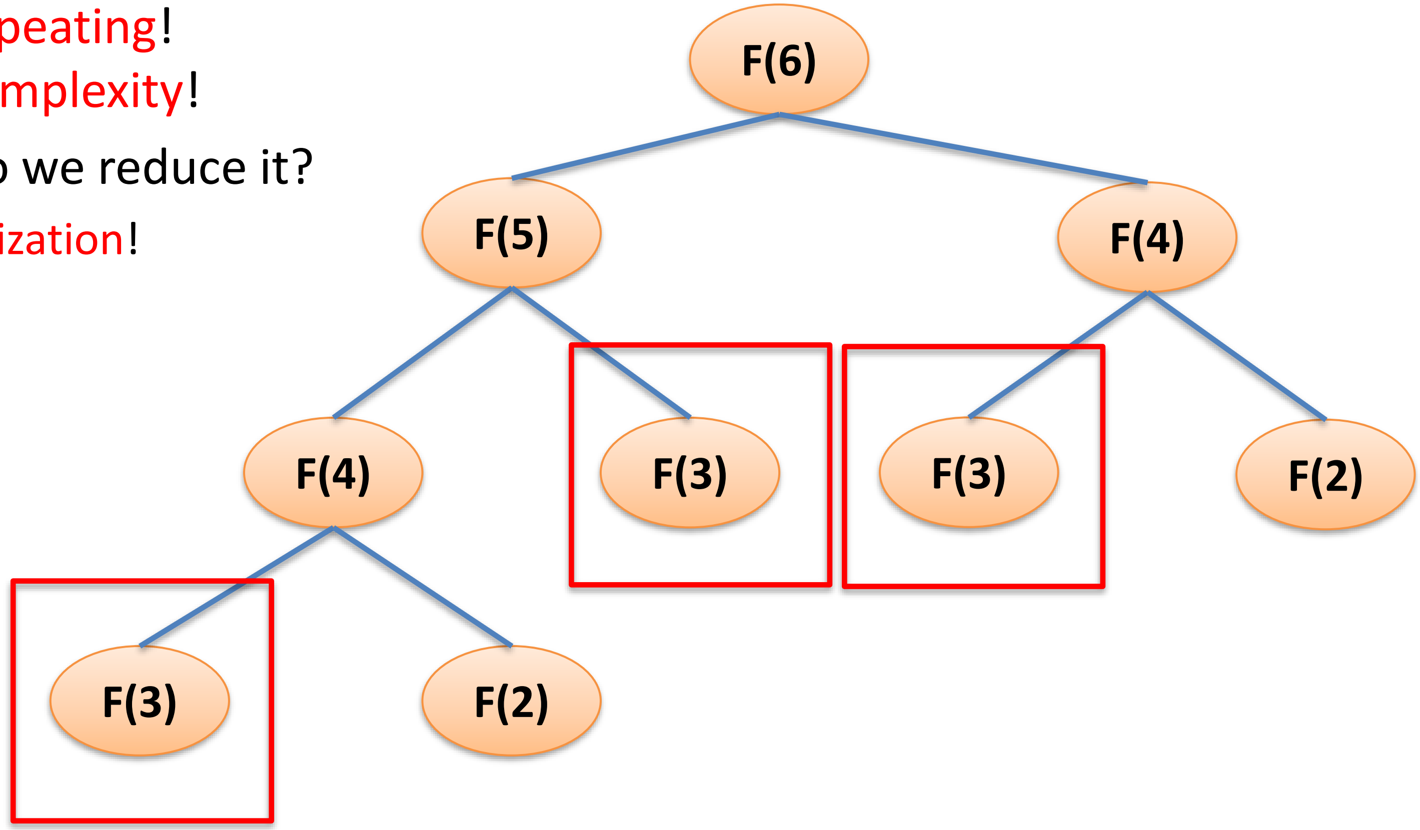
  - We are repeating!
    Adding complexity!
  - So how do we reduce it?
    - Memoization!



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Questions?

Fibonacci
n-th number in the series

- Now if we want the 6^{th} Fibonacci number…
  - How would we get it?
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    Adding complexity!
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    - Memoization!
    - Quick real world examples…

Fibonacci
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    - Memoization!
    - Quick real world examples…
      - What is 12x12?

Fibonacci
n-th number in the series

- Now if we want the 6th Fibonacci number…
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    - Quick real world examples…
      - What is 12x12?
      - What is 12x13?

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Fibonacci
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    - Memoization!
    - Quick real world examples…
      - What is 12x12? Remember this
      - What is 12x13? Used for this…
      - What is 12x14? Used for this…

Fibonacci

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- Now if we want the 6th Fibonacci number…
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    - Quick real world examples…
      - What is 12x12? Remember this
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    - Let's apply to Fibonacci

Questions?

Fibonacci

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- Top-down approach
  - Start from top

Fibonacci
n-th number in the series

- Top-down approach
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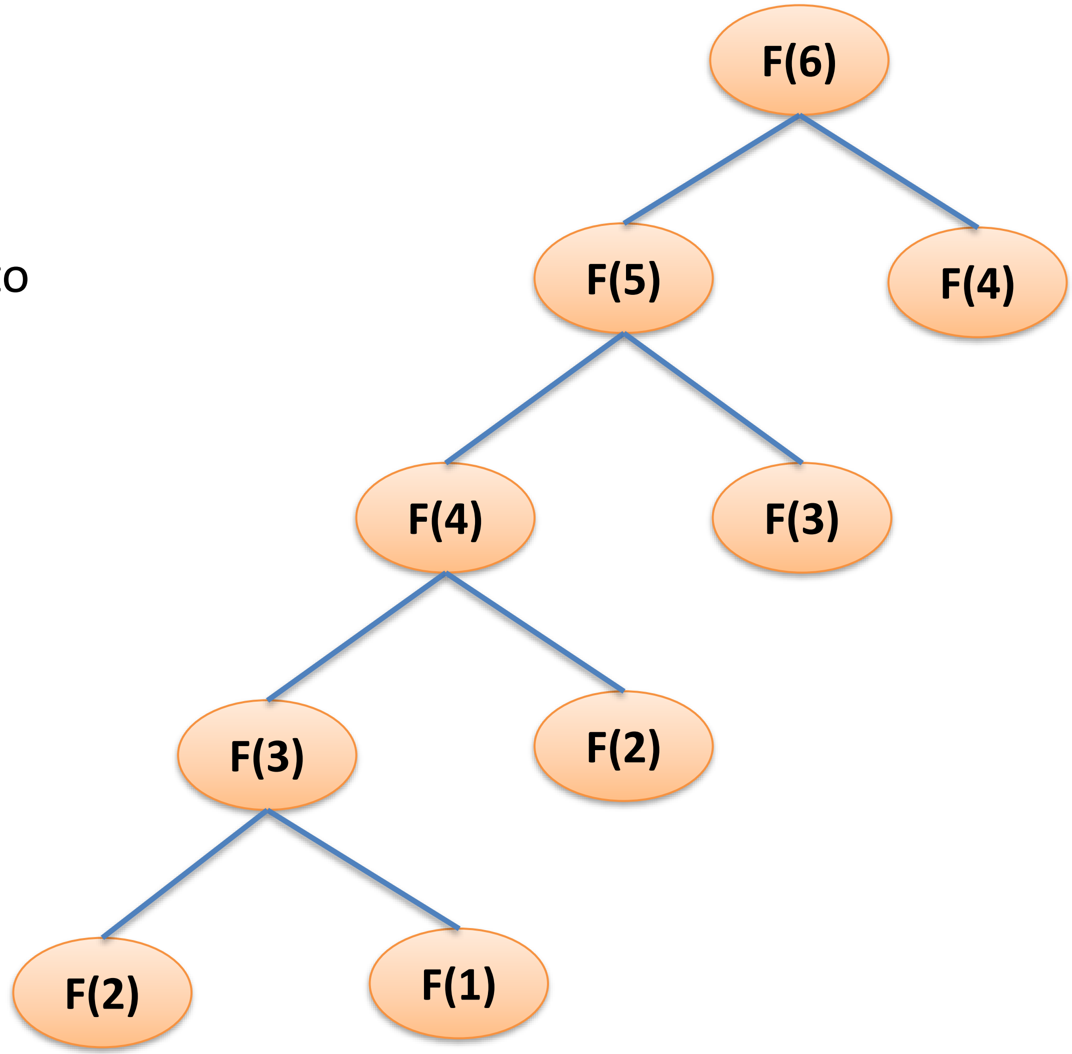
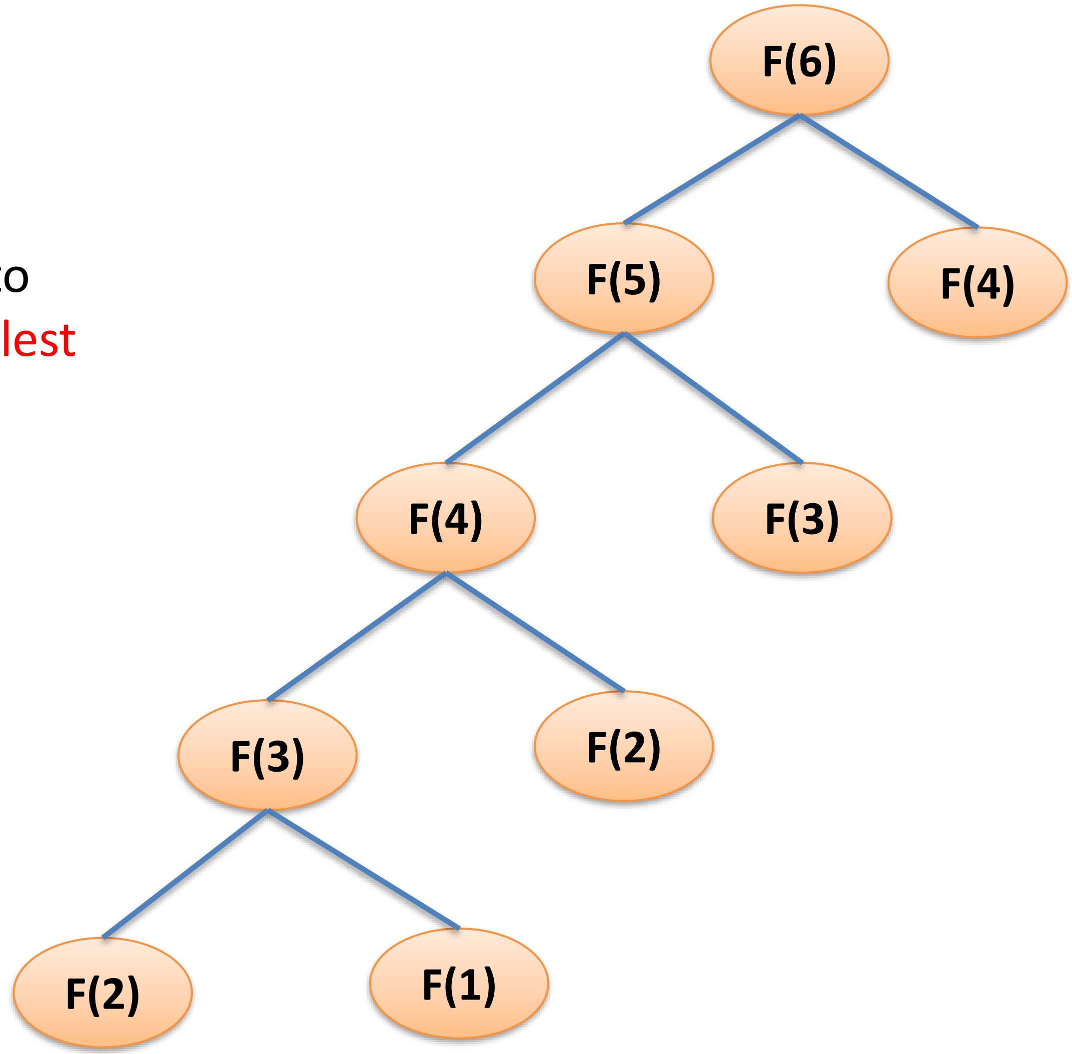
  - Keep on breaking it into smaller problem



Fibonacci
n-th number in the series

- Top-down approach
  - Start from top
  - Keep on breaking it into smaller problem. Smallest is the base case!



Fibonacci

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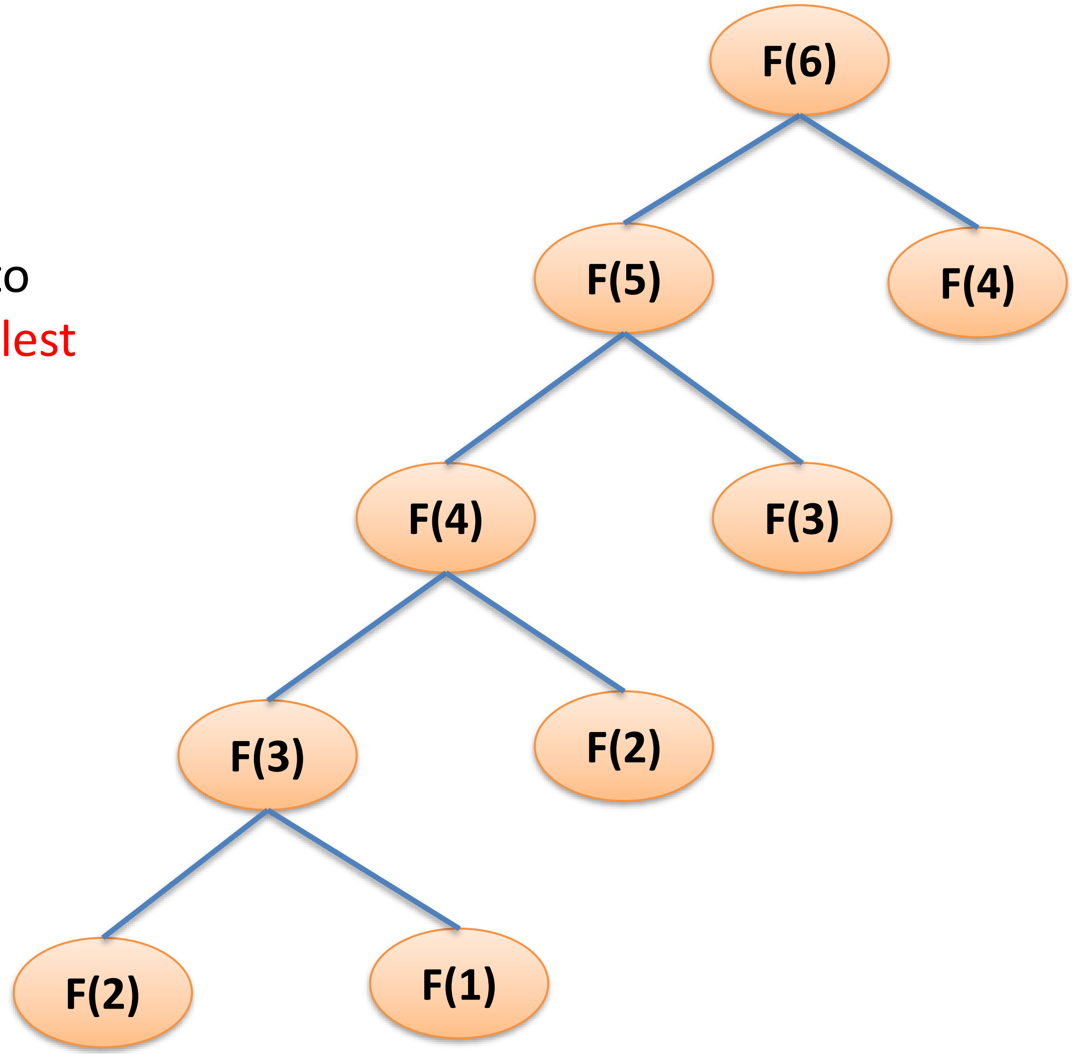
- Top-down approach
  - Start from top
  - Keep on breaking it into smaller problem. Smallest is the base case!
  - Then solve it, reusing the results



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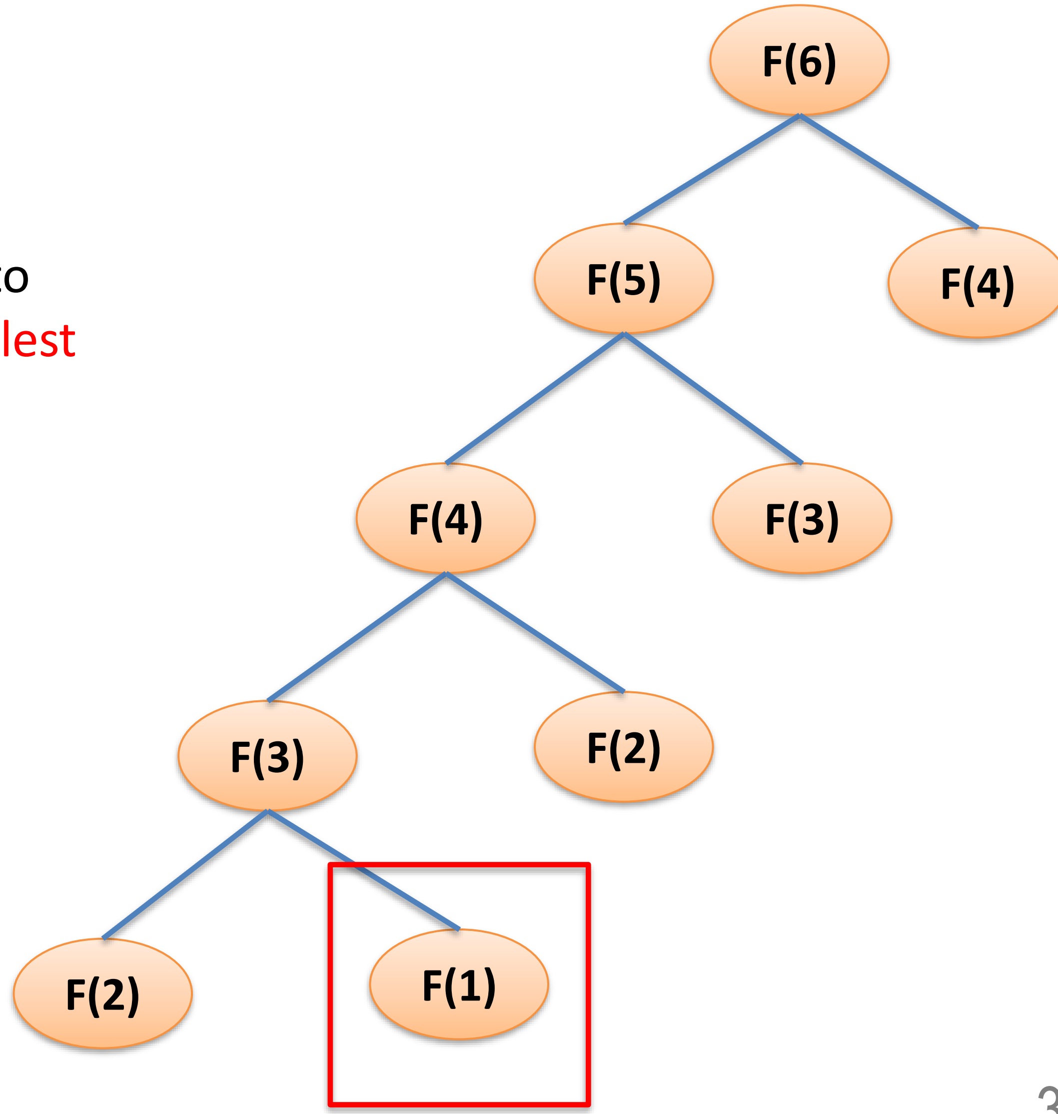

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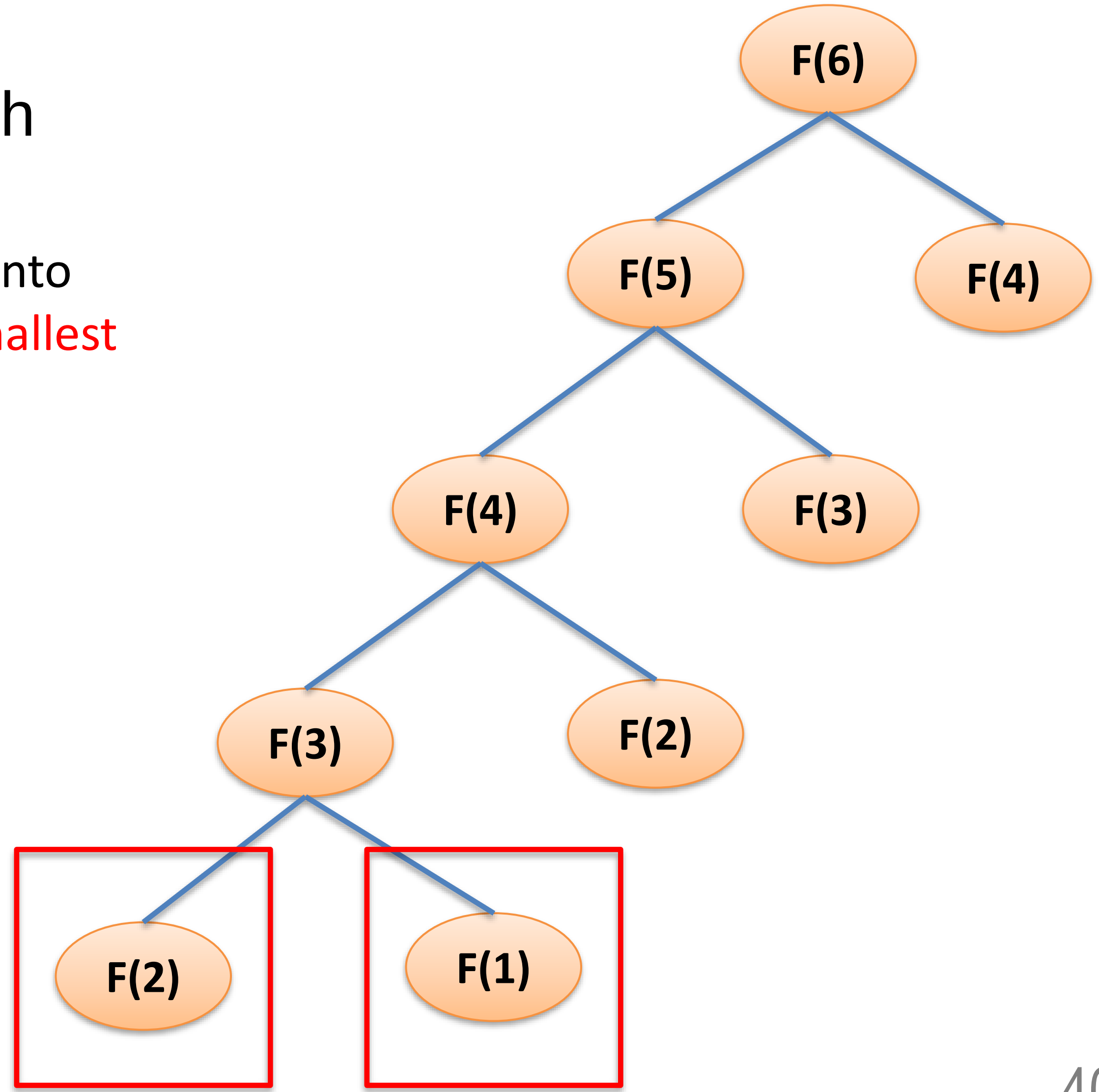

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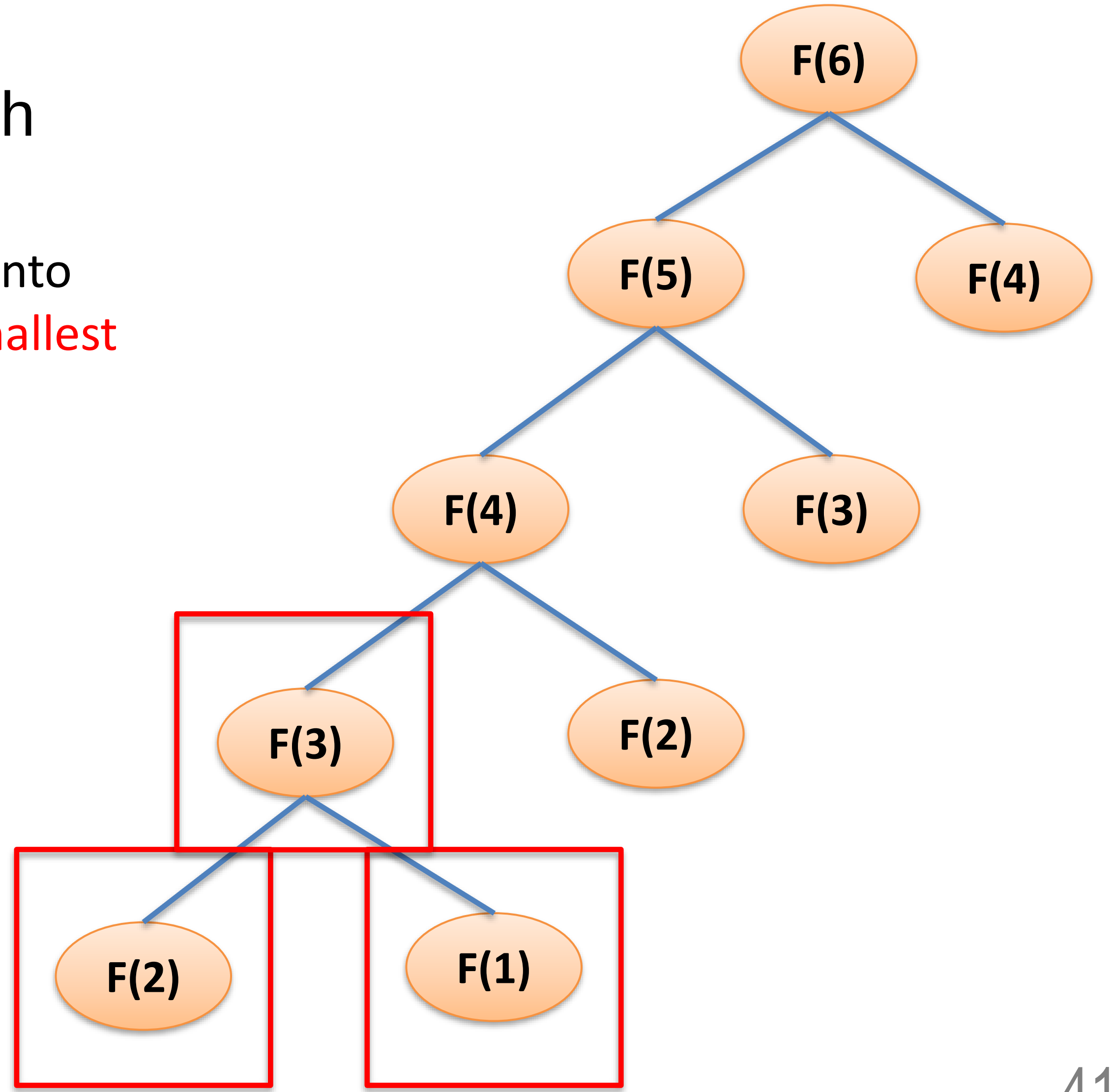

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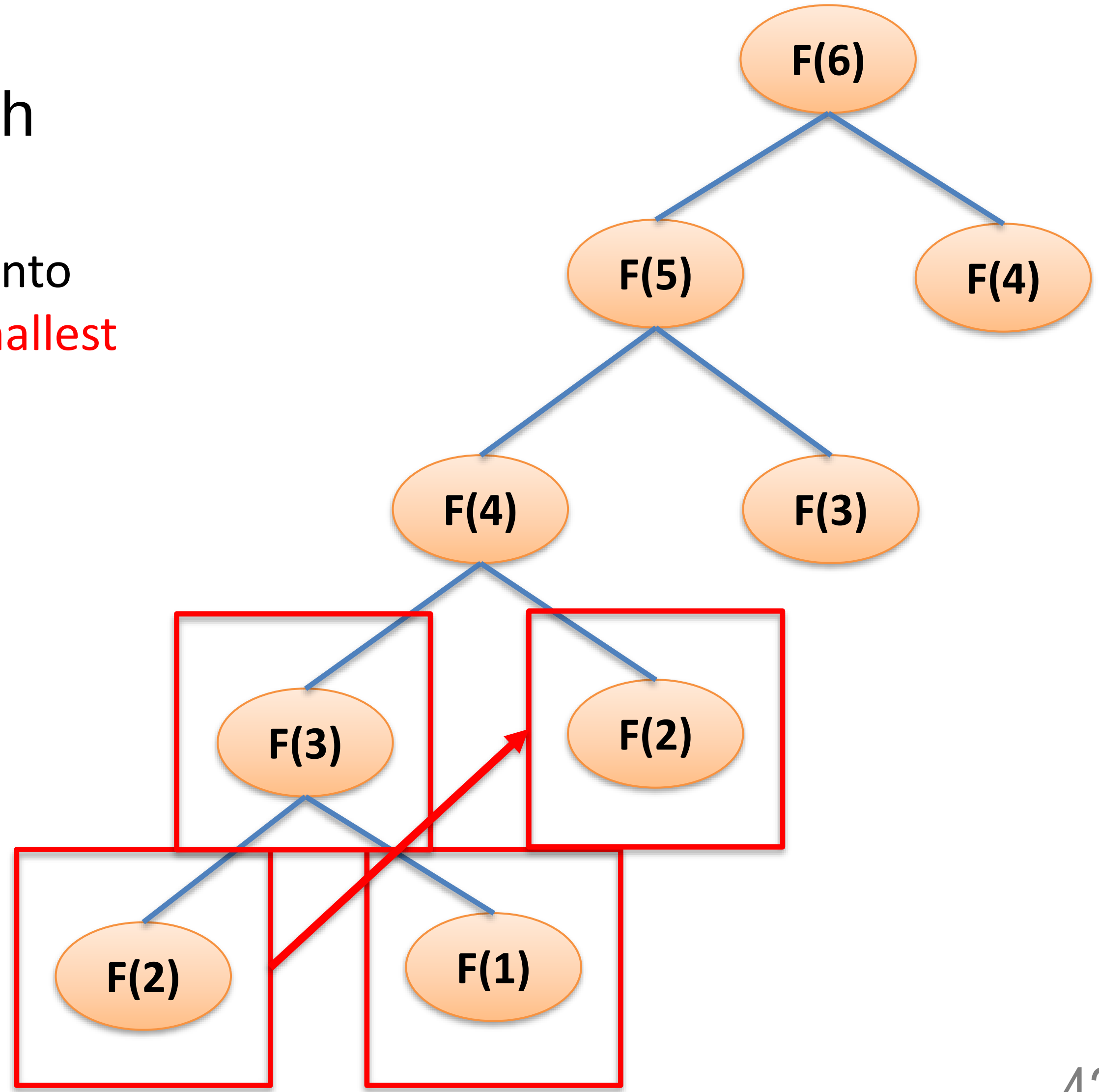

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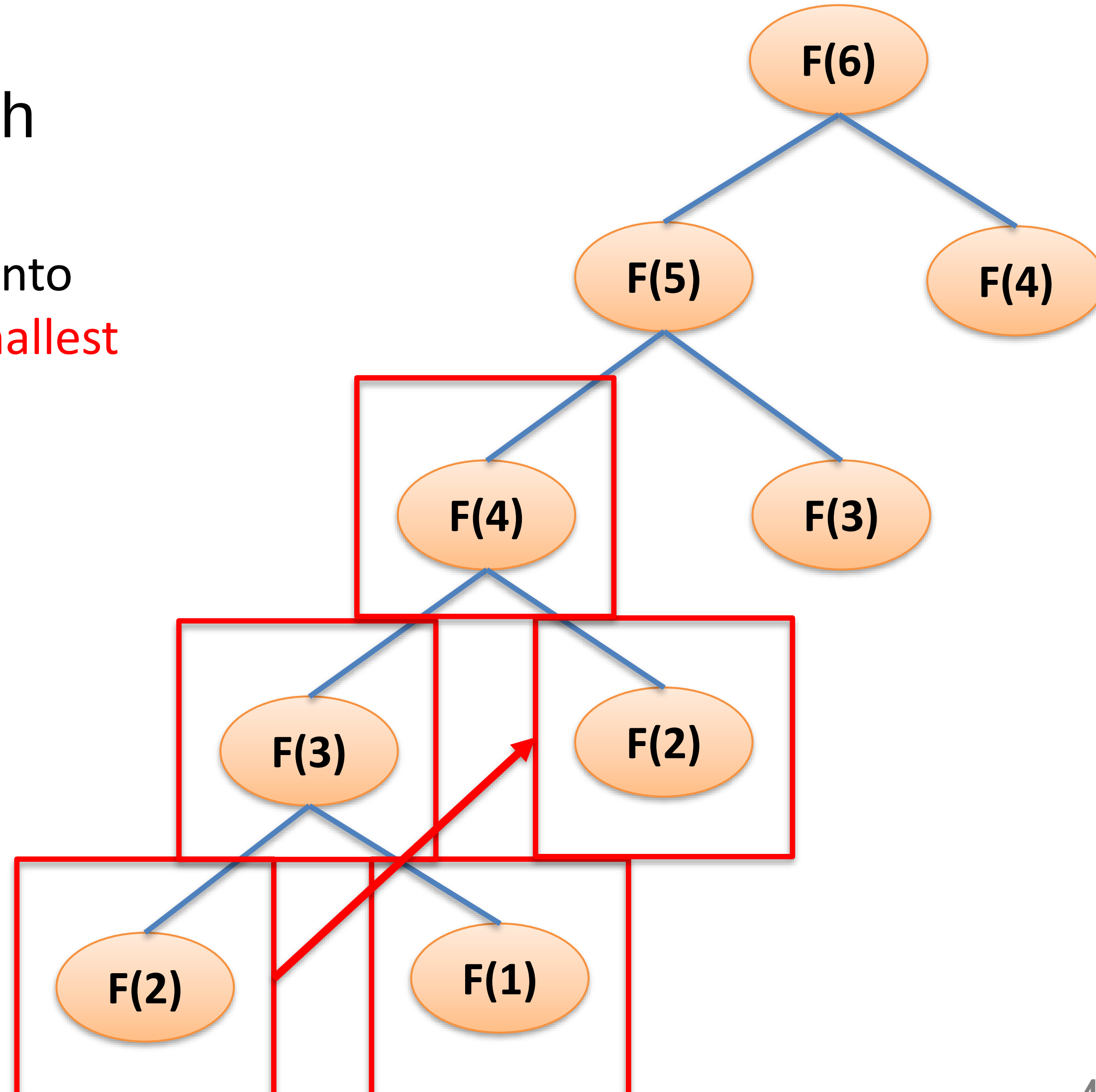

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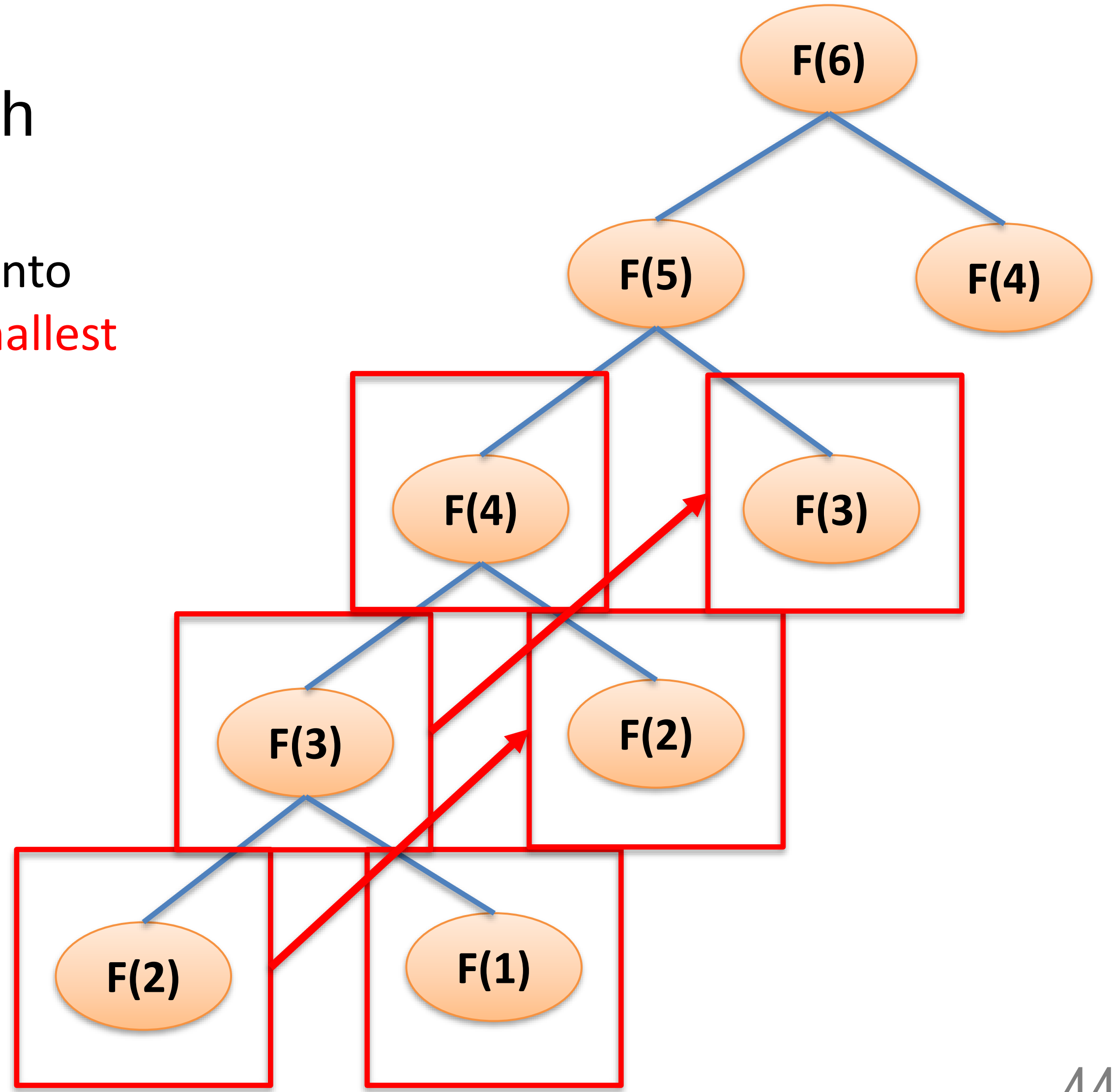

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```
memo = [-1] * N
memo[0] = 0              # in CompSci, we start from 0
memo[1] = 1

def fib_dpa(N):
    if memo[N} != -1:    # if we have computed before
        return memo[N]
    # computer the fibonacci
    memo[N] = fib_dpa(N-1) + fib_dpa(N-2)
    return memo[N]
```

Questions?

- Is it the same?

- Is it the same?
  - Take a big thing
  - Divide to a smaller one
  - Solve the smaller one

Dynamic Programming
vs Divide and Conquer

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Dynamic Programming
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    These solutions are reusable due to overlapping sub problems
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- Let us look at bottom-up to understand better…

Dynamic Programming
vs Divide and Conquer

- Is it the same?
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    These **OPTIMAL** solutions are reusable due to overlapping sub problems
  - Combine solutions up to the big thing…
- NO!... But why?
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Questions?

Fibonacci

n-th number in the series

- Bottom-up approach
  - Can you explain it?

- Bottom-up approach
  - Can you explain it?

Fibonacci
n-th number in the series

- Bottom-up approach
  - Start from the base case
  - Solve it
  - Use it to solve bigger case
  - Until we reach the final one…

Fibonacci

n-th number in the series

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Fibonacci
n-th number in the series

- What is the complexity?
  - Bottom-up
  - Top-down

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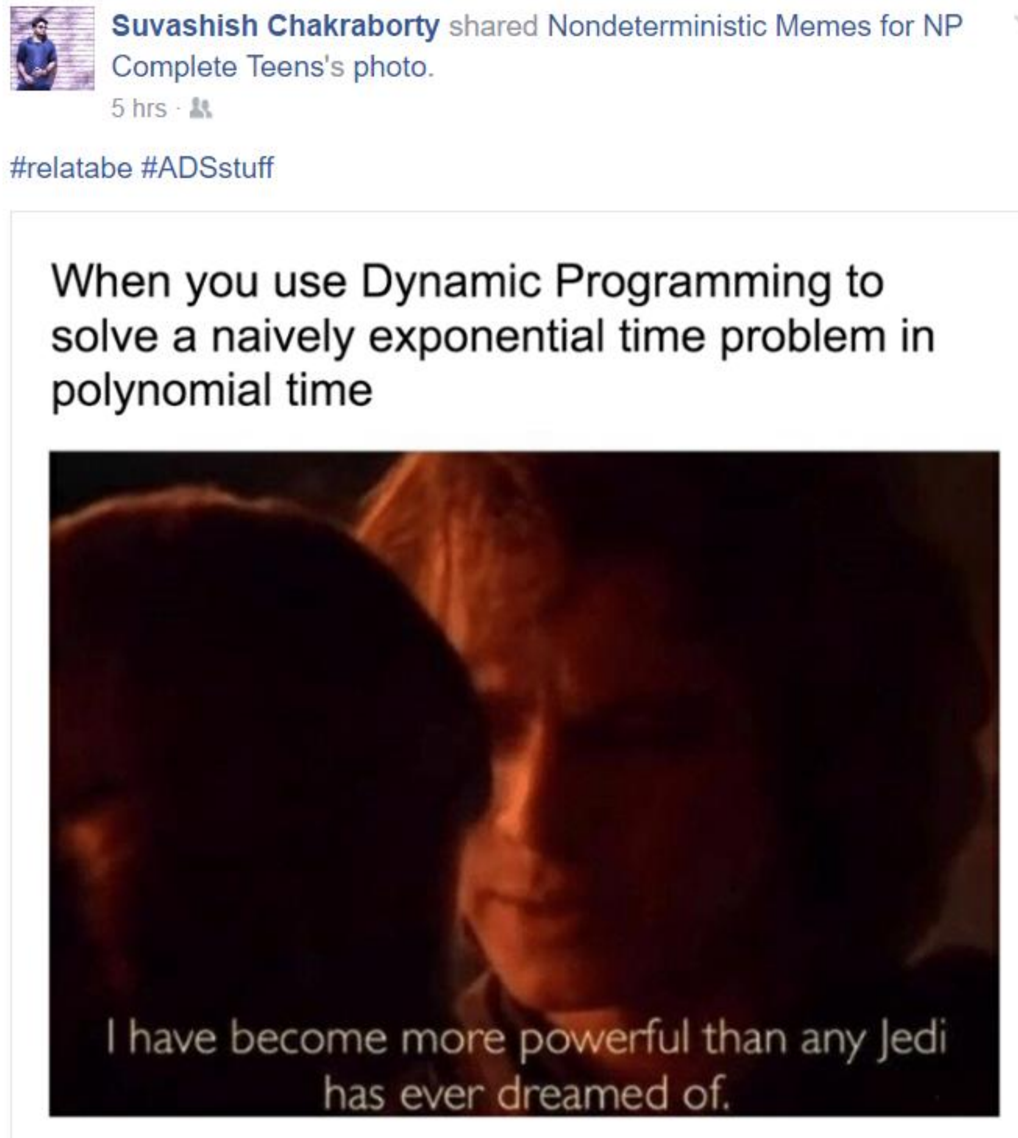

Questions?

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- Can be used to solve a lot of problem
  - Especially finding combinations

The superpower

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  - Popular in interviews
  - Very popular in programming competitions!

Dynamic Programming

The superpower

- Sounds easy right?
  - It isn’t that easy however… Why?

- Can be used to solve a lot of problem
  - Especially finding combinations
  - Popular in interviews
  - Very popular in programming competitions!

- Sounds easy right?
  - It isn't that easy however… Why? Not easy to break problems down

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- So how?

- Sounds easy right?
  - It isn't that easy however… Why? Not easy to break problems down

- So how?
  - Practice

- Sounds easy right?
  - It isn’t that easy however… Why? Not easy to break problems down

- So how?
  - Coin change
  - Knapsack
  - Edit-distance

Questions?

Coin Change problem
The less number of coins…

- Consider the following scenario

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,10,50}

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,10,50}
  - I want 110 doge coin, so what is the possible coin combination?

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,10,50}
  - I want 110 doge coin, so what is the possible coin combination?
    - 2x50 + 1x10
    - 11x10
    - … and many more

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- Consider the following scenario
  - Dogecoin currency is {1,5,10,50}
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    - 2x50 + 1x10 (3 coins)
    - 11x10
    - … and many more
    - But we want the smallest number of coins!

Coin Change problem
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    - … and many more
    - But we want the smallest number of coins!

- Let us now explore the possible solutions…

Questions?

Coin Change problem

The less number of coins…

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- Brute force?

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- Brute force?
  - Try every combination!

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  - Choose the smallest number of coin…

- Consider the following scenario
  - Dogecoin currency is {1,5,10,50}
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- Brute force?
  - Try every combination!
  - Choose the smallest number of coin…
  - Will it work? Of course!

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Questions?

Coin Change problem
The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,10,50}
  - I want 110 doge coin, so what is the possible coin combination?

- Greedy solution?

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,10,50}
  - I want 110 doge coin, so what is the possible coin combination?
- Greedy solution?
  - Try with the biggest number of coin
  - Then fill the balance with the rest #ez

- Consider the following scenario
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  - Doesn't always work (greed is not good)

Coin Change problem
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    {1,5,6,9} and I want 12
    - 1x9 + 3x1 = 12 for 4 coins

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  - Dogecoin currency is {1,5,10,50}
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- Greedy solution?
  - Try with the biggest number of coin
  - Then fill the balance with the rest #ez
  - Doesn’t always work (greed is not good) {1,5,6,9} and I want 12
    - 1x9 + 3x1 = 12 for 4 coins
    - 2x6 = 12 for 2 coins…

Questions?

Coin Change problem
The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,10,50}
  - I want 110 doge coin, so what is the possible coin combination?
- Dynamic solution?
  - Let us try a smaller problem (easier for me to visualize)

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
  - I want 12 doge coin value, so what is the possible coin combination?

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
  - I want 12 doge coin value, so what is the possible coin combination?

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins													

Coin Change problem

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Number of coins	0												

Coin Change problem
The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
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Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

Coin Change problem
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Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

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0+1 = 1

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Number of coins	0	1	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

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Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

- Consider the following scenario
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  - I want 12 doge coin value, so what is the possible coin combination?

-5+5 = 1

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

-5+5 = 1, and so on….

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

Coin Change problem
The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

Repeat the process…

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

1+1=2

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

Coin Change problem
The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

1+1=2, 2 coins as well

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	inf	inf	inf	inf	inf	inf	inf	inf

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

4+1=5, for 5 coins

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	inf	inf	inf	inf	inf	inf	inf	inf

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

4+1=5, for 5 coins

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	5	inf	inf	inf	inf	inf	inf	inf

Coin Change problem
The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

0+5=5, for 1 coin

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	5	inf	inf	inf	inf	inf	inf	inf

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- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

0+5=5, for 1 coin

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	5 vs 1	inf	inf	inf	inf	inf	inf	inf

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

0+5=5, for 1 coin

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	1	inf	inf	inf	inf	inf	inf	inf

Coin Change problem
The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

5+1=6, for 2 coins

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	inf	inf	inf	inf	inf	inf	inf

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

5+1=6, for 2 coins

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	2	inf	inf	inf	inf	inf	inf

Coin Change problem
The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

1+5=6, for 2 coins

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	1	2	inf	inf	inf	inf	inf	inf

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

0+6=6, for 1 coin

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	inf	inf	inf	inf	inf	inf

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?
  - So keep on running it and eventually we would be done

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	inf	inf	inf	inf	inf	inf

Coin Change problem

The less number of coins…

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  - Dogecoin currency is {1,5,6,9}
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Number of coins	0	1	2	3	4	1	1	2	inf	inf	inf	inf	inf

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  - Dogecoin currency is {1,5,6,9}
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Number of coins	0	1	2	3	4	1	1	2	3	inf	inf	inf	inf

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- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?
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Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	4	inf	inf	inf

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?
  - So keep on running it and eventually we would be done

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	1	?	inf	inf

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

  - So keep on running it and eventually we would be done

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	1	2	inf	inf

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- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?
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Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	inf

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

  - So keep on running it and eventually we would be done

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2

Questions?

Coin Change problem
The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

  - So keep on running it and eventually we would be done
    - Complexity?

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	4	2	2	2

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

  - So keep on running it and eventually we would be done
    - Complexity? O(NM)

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	1	1	2	3	4	2	2	2

Coin Change problem

The less number of coins…

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

  - So keep on running it and eventually we would be done
    - Complexity? O(NM) still much faster than brute force…

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	4	2	2	2

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?

  - So keep on running it and eventually we would be done
    - Complexity? O(NM) still much faster than brute force…
      - Note: If the list is sorted, we can terminate earlier on the smaller values

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	1	1	2	3	4	2	2	2

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Questions?

- Consider the following scenario
  - Dogecoin currency is {1,5,6,9}
    - We will loop through this over and over considering the coins…
  - I want 12 doge coin value, so what is the possible coin combination?
  - So keep on running it and eventually we would be done
    - Complexity? O(NM) still much faster than brute force…
    - Can you code it?

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	4	2	2	2

Coin Change problem
The less number of coins…

- I'll give the algorithm here

```
memo = [inf] * (N+1)
memo[0] = 0
for value in range(1,N+1):
    for j in range(M):                          # this is to go through coins
        if coin[j] <= value
            balance = value - coin[j]
            count = 1 + memo[balance]
            if count < memo[value]:             # if we have new optimal
                memo[value] = count
return memo[N]
```

Coin Change problem

The less number of coins…

- I'll give the algorithm here
  - Is this top-down or bottom-up?

```
memo = [inf] * (N+1)
memo[0] = 0
for value in range(1,N+1):
    for j in range(M):                    # this is to go through coins
        if coin[j] <= value
            balance = value - coin[j]
            count = 1 + memo[balance]
            if count < memo[value]:       # if we have new optimal
                memo[value] = count
return memo[N]
```

Coin Change problem
The less number of coins…

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for value in range(1,N+1):
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        if coin[j] <= value
            balance = value - coin[j]
            count = 1 + memo[balance]
            if count < memo[value]:     # if we have new optimal
                memo[value] = count
return memo[N]
```

Coin Change problem
The less number of coins…

- I'll give the algorithm here
  - Is this top-down or bottom-up?
  - The following is the top-down from Aamir (usually recursive)

```
memo = [inf] * (N+1)
memo[0] = 0
for value in range(1,N+1):
    for j in range(M):                # this is to go through coins
        if coin[j] <= value
            balance = value - coin[j]
            count = 1 + memo[balance]
            if count < memo[value]:   # if we have new optimal
                memo[value] = count
return memo[N]
```

Top-down Solution

```
Initialize Memo[ ] to contain -1 for all indices # -1 indicates the solution
for this index has not been computed yet
Memo[0] = 0
Function CoinChange(value)
  if Memo[value] != -1: \\ DISCUSS infinity is incorrect
    return Memo[value]
  else:
    minCoins = Infinity
    for i=1 to N
      if Coins[ i ] <= value
        c = 1 + CoinChange(value - Coins[ i ])
        if c < minCoins
          minCoins = c
    Memo[value] = minCoins
    return Memo[value]
```

Bottom up solution:

1 + Memo[value – Coins[i]]

Coin Change problem

The less number of coins…

- Top-down vs Bottom up
  - Top-down might save some computations
  - Bottom-up might save space
    especially since no recursion

Coin Change problem
The less number of coins…

- Top-down vs Bottom up
  - Top-down might save some computations
  - Bottom-up might save space
    especially since no recursion

  - I only use bottom-up

Coin Change problem

The less number of coins…

- Top-down vs Bottom up
  - Top-down might save some computations
  - Bottom-up might save space
    especially since no recursion

  - I only use bottom-up
    - But some problems could be easier with top-down as it is more intuitive
    - Technically both are interchangeable…

Questions?

Kapsack Problem

Min-max like a boss

- A problem that can be applicable to a lot of real life scenario

Kapsack Problem

Min-max like a boss

- A problem that can be applicable to a lot of real life scenario
  - Given a limitation (cost)

Kapsack Problem

Min-max like a boss

- A problem that can be applicable to a lot of real life scenario
  - Given a limitation (cost)
  - Optimize something (profit)

- A problem that can be applicable to a lot of real life scenario
  - Given a limitation (cost)
  - Optimize something (profit)
  - 18th most popular algorithmic problem

Kapsack Problem

Min-max like a boss

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- A problem that can be applicable to a lot of real life scenario
  - Given a limitation (cost)
  - Optimize something (profit)
  - 18th most popular algorithmic problem

- Given a capacity C and a set of items with weights and value… can you find a combination of item such as the total weight < C but the total value is maximized?

Kapsack Problem

Min-max like a boss

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- A problem that can be applicable to a lot of real life scenario
  - Given a limitation (cost)
  - Optimize something (profit)
  - 18th most popular algorithmic problem
- Given a capacity C and a set of items with weights and value… can you find a combination of item such as the total weight < C but the total value is maximized?
  - Unbounded = items are unlimited
  - Bounded = each item can only be taken once

- Let say you have these items

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Kapsack Problem

Min-max like a boss

- Let say you have these items

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

- And you only have 12 kg… what would you loot?

Kapsack Problem

Min-max like a boss

- Let say you have these items

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	\$550	\$350	\$180	\$40

- And you only have 12 kg… what would you loot?
  - 2xB + 2xD = \$780

Kapsack Problem

Min-max like a boss

- Let say you have these items

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

- And you only have 12 kg… what would you loot?
  - 2xB + 2xD = $780
  - So how do we code it?
    This is very similar to the coin change!

Questions?

- Let us try the bottom-up approach

- Let us try the bottom-up approach
  - Start with 0 weight till 12 weight

Kapsack Problem
Unbounded

- Let us try the bottom-up approach
  - Start with 0 weight till 12 weight
  - Find the optimal for each weight…

Kapsack Problem
Unbounded

- Let us try the bottom-up approach
  - Start with 0 weight till 12 weight
  - Find the optimal for each weight…
  - Use the earlier optimal for the new weight

- Let us try the bottom-up approach
  - Start with 0 weight till 12 weight
  - Find the optimal for each weight… Maximum profit!
  - Use the earlier optimal for the new weight

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0												

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	0	0	0	0	0	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	0	0	0	0	0	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

9kg, too heavy

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	0	0	0	0	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

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- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

5kg, too heavy

Weight	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Profit	0	0	0	0	0	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

6kg, too heavy

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	0	0	0	0	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

1kg, can fit

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	0	0	0	0	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

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- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

1kg, can fit… so we find the optimal of 0kg + 1kg

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	0	0	0	0	0	0	0	0	0	0	0	0

Kapsack Problem

Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

1kg, can fit… so we find the optimal of 0kg + 1kg

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	0	0	0	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	0	0	0	0	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

9kg too heavy

Weight	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Profit	0	40	0	0	0	0	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

5kg too heavy

Weight	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Profit	0	40	0	0	0	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

6kg too heavy

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	0	0	0	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

1kg is OK

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	0	0	0	0	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

1kg is OK… optimal of 1kg + profit from this item

Weight	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Profit	0	40	0	0	0	0	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

1kg is OK... optimal of 1kg + profit from this item

Weight	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Profit	0	40	0	0	0	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

1kg is OK... optimal of 1kg + profit from this item

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	0	0	0	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

We just repeat the process

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	0	0	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

We just repeat the process

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	0	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Now let us try here

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

9kg too heavy…

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

5kg can fit. Optimal 0kg + current item

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	0	0	0	0	0	0	0	0

Kapsack Problem
Unbounded

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- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

5kg can fit. Optimal 0kg + current item

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	0	0	0	0	0	0	0

MONASH University

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

6kg too heavy

Weight	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Profit	0	40	80	120	160	350	0	0	0	0	0	0	0

MONASH University

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

1kg is OK. Optimal 4kg + current item

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

1kg is OK. Optimal 4kg + current item but it is only 200

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	0	0	0	0	0	0	0

Kapsack Problem

Unbounded

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Moving on....

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

heavy

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Optimal 1kg + current item

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	0	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Optimal 1kg + current item

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	390	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Optimal 0kg + current item = 180 only

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	390	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Optimal 5kg + current item = same 390

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	390	0	0	0	0	0	0

- Let us run through it

Item	A	B	C	D
Weight	9kg	5kg	6kg	1kg
Value	$550	$350	$180	$40

Eventually….

Weight	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Profit	0	40	80	120	160	350	390	430	470	550	700	740	780

Questions?

Have a break!

Kapsack Problem
Unbounded

- So what is our algorithm?
  - Let us try to produce it now as part of the class activity!

Kapsack Problem

Unbounded

- So what is our algorithm?
  - Very similar to the coin change except
    - Finding maximum instead of minimum
    - Initialized to 0

Kapsack Problem
Unbounded

- So what is our algorithm?
  - Very similar to the coin change except
    - Finding maximum instead of minimum
    - Initialized to 0
  - You have the 1 array, called items
    - N number of items
    - Items[i].weight for the weight
    - Items[i].profit for the profit

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Questions?

Kapsack Problem

Unbounded

- So what is our algorithm?
  - Very similar to the coin change except
    - Finding maximum instead of minimum
    - Initialized to 0

```
memo = [0] * (N+1)                              # N is the total weight
memo[0] = 0
for bag_weight in range(1,N+1):
    for j in range(M):                          # this is to go through items
        if items[j].weight <= bag_weight:
            balance = bag_weight - items[j].weight
            profit = item[j].profit + memo[balance]
            if profit > memo[bag_weight]:       # if we have new optimal
                memo[bag_weight] = profit
return memo[N]
```

Kapsack Problem
Unbounded

- So what is our algorithm?
  - Very similar to the coin change except
    - Finding maximum instead of minimum
    - Initialized to 0
  - Complexity?

```
memo = [0] * (N+1)                          # N is the total weight
memo[0] = 0
for bag_weight in range(1,N+1):
    for j in range(M):                      # this is to go through items
        if items[j].weight <= bag_weight:
            balance = bag_weight - items[j].weight
            profit = item[j].profit + memo[balance]
            if profit > memo[bag_weight]:   # if we have new optimal
                memo[bag_weight] = profit
return memo[N]
```

Kapsack Problem
Unbounded

- So what is our algorithm?
  - Very similar to the coin change except
    - Finding maximum instead of minimum
    - Initialized to 0
  - Complexity? O(NM)

```
memo = [0] * (N+1)                             # N is the total weight
memo[0] = 0
for bag_weight in range(1,N+1):
    for j in range(M):                         # this is to go through items
        if items[j].weight <= bag_weight:
            balance = bag_weight - items[j].weight
            profit = item[j].profit + memo[balance]
            if profit > memo[bag_weight]:      # if we have new optimal
                memo[bag_weight] = profit
return memo[N]
```

- So what is our algorithm?
  - Very similar to the coin change except
    - Finding maximum instead of minimum
    - Initialized to 0
  - Complexity?
  - Top-down? See Nathan’s slides

Questions?

Kapsack Problem

Unbounded

- But is the code correct?

```
memo = [0] * (N+1)                          # N is the total weight
memo[0] = 0
for bag_weight in range(1,N+1):
    for j in range(M):                      # this is to go through items
        if items[j].weight <= bag_weight:
            balance = bag_weight - items[j].weight
            profit = item[j].profit + memo[balance]
            if profit > memo[bag_weight]:   # if we have new optimal
                memo[bag_weight] = profit
return memo[N]
```

Kapsack Problem
Unbounded

- But is the code correct?
  - Currently, we assume the maximum value is when you find items with a total weight of 12 kg

Eventually….

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	40	80	120	160	350	390	430	470	550	700	740	780

```
memo = [0] * (N+1)                          # N is the total weight
memo[0] = 0
for bag_weight in range(1,N+1):
    for j in range(M):                      # this is to go through items
        if items[j].weight <= bag_weight:
            balance = bag_weight - items[j].weight
            profit = item[j].profit + memo[balance]
            if profit > memo[bag_weight]:   # if we have new optimal
                memo[bag_weight] = profit
return memo[N]
```

Kapsack Problem
Unbounded

- But is the code correct?
  - Currently, we assume the maximum value is when you find items with a total weight of 12 kg
  - What if we can't reach 12 kg?
  - What if the optimal is at 10 kg instead of 12 kg?

Eventually….

Weight	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Profit	0	40	80	120	160	350	390	430	470	550	700	740	780

```
memo = [0] * (N+1)                          # N is the total weight
memo[0] = 0
for bag_weight in range(1,N+1):
    for j in range(M):                      # this is to go through items
        if items[j].weight <= bag_weight:
            balance = bag_weight - items[j].weight
            profit = item[j].profit + memo[balance]
            if profit > memo[bag_weight]:   # if we have new optimal
                memo[bag_weight] = profit
return memo[N]
```

Kapsack Problem
Unbounded

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- But is the code correct?
  - Currently, we assume the maximum value is when you find items with a total weight of 12 kg
  - What if we can't reach 12 kg?
  - What if the optimal is at 10 kg instead of 12 kg?
  - So what must we change?

```
memo = [0] * (N+1)                      # N is the total weight
memo[0] = 0
for bag_weight in range(1,N+1):
    for j in range(M):                  # this is to go through items
        if items[j].weight <= bag_weight:
            balance = bag_weight - items[j].weight
            profit = item[j].profit + memo[balance]
            if profit > memo[bag_weight]:   # if we have new optimal
                memo[bag_weight] = profit
return memo[N]
```

Kapsack Problem

Unbounded

- But is the code correct?
  - Currently, we assume the maximum value is when you find items with a total weight of 12 kg
  - What if we can't reach 12 kg?
  - What if the optimal is at 10 kg instead of 12 kg?
  - So what must we change?
    - memo[i] = memo[i-1]   # copy the previous optimal

```
memo = [0] * (N+1)                          # N is the total weight
memo[0] = 0
for bag_weight in range(1,N+1):
    for j in range(M):                      # this is to go through items
        if items[j].weight <= bag_weight:
            balance = bag_weight - items[j].weight
            profit = item[j].profit + memo[balance]
            if profit > memo[bag_weight]:   # if we have new optimal
                memo[bag_weight] = profit
return memo[N]
```

Kapsack Problem
Unbounded

- But is the code correct?
  - Currently, we assume the maximum value is when you find items with a total weight of 12 kg
  - What if we can't reach 12 kg?
  - What if the optimal is at 10 kg instead of 12 kg?
  - So what must we change?
    - memo[i] = memo[i-1]   # copy the previous optimal
    - or, linear search through memo for the maximum

```
memo = [0] * (N+1)                          # N is the total weight
memo[0] = 0
for bag_weight in range(1,N+1):
    for j in range(M):                      # this is to go through items
        if items[j].weight <= bag_weight:
            balance = bag_weight - items[j].weight
            profit = item[j].profit + memo[balance]
            if profit > memo[bag_weight]:   # if we have new optimal
                memo[bag_weight] = profit
return memo[N]
```

Questions?

Kapsack Problem

0/1 items

- Same problem, but you can’t repeat the item

Kapsack Problem
0/1 items

- Same problem, but you can’t repeat the item
  – So how would we solve it?

Kapsack Problem

0/1 items

- Same problem, but you can't repeat the item
  - So how would we solve it?

- This is where we can see the growing of problems…

Kapsack Problem

0/1 items

- Same problem, but you can't repeat the item
  - So how would we solve it?
- This is where we can see the growing of problems…
  - Grow from 0 weight to N weight

Kapsack Problem

0/1 items

- Same problem, but you can't repeat the item
  - So how would we solve it?

- This is where we can see the growing of problems…
  - Grow from 0 weight to N weight
  - Grow from a set of 0 items till M items

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

Weight	0	1	2	3	4	5	6	7	8	9	10	11	12
Profit	0	?	?	?	?	?	?	?	?	?	?	?	?

Kapsack Problem
0/1 items

- We use a matrix

Increasing weight

	0	1	2	3	4	5	6	7	8	9	10	11	12
0													
A													
B													
C													
D													

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- We use a matrix

Increasing weight

Increasing items in set

	0	1	2	3	4	5	6	7	8	9	10	11	12
0													
A													
B													
C													
D													

Kapsack Problem
0/1 items

- We use a matrix

Increasing weight

Increasing items in set

	0	1	2	3	4	5	6	7	8	9	10	11	12
{}													
{A}													
{A,B}													
{A,B, C}													
{A,B, C,D}													

Kapsack Problem

0/1 items

- We use a matrix
  - So we fill up the matrix

	0	1	2	3	4	5	6	7	8	9	10	11	12
0													
A													
B													
C													
D													

Kapsack Problem

0/1 items

- We use a matrix
  - Base cases

	0	1	2	3	4	5	6	7	8	9	10	11	12
0													
A													
B													
C													
D													

Kapsack Problem
0/1 items

- We use a matrix
  - Base cases
    - No item to choose from…

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A													
B													
C													
D													

Kapsack Problem

0/1 items

- We use a matrix
  - Base cases
    - No item to choose from…
    - Max weight is 0….

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0												
B	0												
C	0												
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0												
B	0												
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0												
B	0												
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0							
B	0												
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	?						
B	0												
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230						
B	0												
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	?					
B	0												
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230					
B	0												
C	0												
D	0												

Kapsack Problem
0/1 items

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- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230				
B	0												
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230			
B	0												
C	0												
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0												
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40											
C	0												
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40										
C	0												
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	\$230	\$40	\$350	\$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40							
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	40						
C	0												
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	40	?					
C	0												
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	40	270					
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	40	270	270				
C	0												
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?
    - Is this correct?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	40	270	270	270	270	270	270
C	0												
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?
    - Is this correct? Here, we can choose not to include B, having only A in bag

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	40	270	270	270	270	270	270
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?
    - Is this correct? Here, we can choose not to include B, having only A in bag

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0												
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	?											
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	?											
D	0												

Kapsack Problem
0/1 items

MONASH University

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40											
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40								
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350							
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390						
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390					
D	0												

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0												

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	?											

Kapsack Problem
0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390				

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550			

Kapsack Problem

0/1 items

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  - So row by row…
    - Start with item A
    - Should we add it?

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Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590		

Kapsack Problem

0/1 items

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    - Should we add it?

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Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	

Kapsack Problem

0/1 items

- We use a matrix
  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	?

Kapsack Problem

0/1 items

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  - So row by row…
    - Start with item A
    - Should we add it?

Item	A	B	C	D
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Value	$230	$40	$350	$550

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

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Questions?

Kapsack Problem

0/1 items

- We use a matrix
  - What is the algorithm/ code?

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

Kapsack Problem

0/1 items

- We use a matrix
  - What is the algorithm/ code?

```
# for every row (item)
for i=1 to M:
    # for every column (weight)
    for j=1 to N:
        # get the excluded at current weight (row above)
        exclude = memo[i-1][j]
        # calculate the include
        include = 0
        if item[i].weight <= j:
            include = item[i].value + memo[i-1][j-item[i].weight]
        memo[i][j] = max(exclude,include)
```

Kapsack Problem

0/1 items

- We use a matrix
  - What is the algorithm/ code?
  - Complexity?

```
# for every row (item)
for i=1 to M:
    # for every column (weight)
    for j=1 to N:
        # get the excluded at current weight (row above)
        exclude = memo[i-1][j]
        # calculate the include
        include = 0
        if item[i].weight <= j:
            include = item[i].value + memo[i-1][j-item[i].weight]
        memo[i][j] = max(exclude,include)
```

Kapsack Problem
0/1 items

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- We use a matrix
  - What is the algorithm/ code?
  - Complexity?
    - O(NM) time from filling matrix
    - O(NM) space for the matrix

```
# for every row (item)
for i=1 to M:
    # for every column (weight)
    for j=1 to N:
        # get the excluded at current weight (row above)
        exclude = memo[i-1][j]
        # calculate the include
        include = 0
        if item[i].weight <= j:
            include = item[i].value + memo[i-1][j-item[i].weight]
        memo[i][j] = max(exclude,include)
```

Kapsack Problem

0/1 items

- We use a matrix
  - What is the algorithm/ code?
  - Complexity?
    - O(NM) time from filling matrix
    - O(NM) space for the matrix… we can reduce this however

```
# for every row (item)
for i=1 to M:
    # for every column (weight)
    for j=1 to N:
        # get the excluded at current weight (row above)
        exclude = memo[i-1][j]
        # calculate the include
        include = 0
        if item[i].weight <= j:
            include = item[i].value + memo[i-1][j-item[i].weight]
        memo[i][j] = max(exclude,include)
```

Kapsack Problem
0/1 items

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- We use a matrix
  - What is the algorithm/ code?
  - Complexity?
    - O(NM) time from filling matrix
    - O(NM) space for the matrix… we can reduce this however
      We realize we do not need the entire matrix! We get the current value by looking at the row above only. So we can just store the latest 2 row…

Kapsack Problem

0/1 items

- We use a matrix
  – What is the algorithm/ code?
  – Complexity?
    - O(NM) time from filling matrix
    - O(NM) space for the matrix… we can reduce this however
      We realize we do not need the entire matrix! We get the current value by looking at the row above only. So we can just store the latest 2 row…
      Reducing complexity to O(2N+M)

- We use a matrix
  - What is the algorithm/ code?
  - Complexity?
    - O(NM) time from filling matrix
    - O(NM) space for the matrix… we can reduce this however
      We realize we do not need the entire matrix! We get the current value by looking at the row above only. So we can just store the latest 2 row… Reducing complexity to O(2N+M)
    - But in reality, we can't do this space saving… because we need it to reconstruct the solution…

Questions?

Take a break!

Kapsack Problem

0/1 items

- So what are the items?

Kapsack Problem
0/1 items

- So what are the items?

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

Kapsack Problem

0/1 items

- So what are the items?
  - Recall that we compare the current value (inclusive) with the value in the row above (exclusive).

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

Kapsack Problem
0/1 items

- So what are the items?
  - Recall that we compare the current value (inclusive) with the value in the row above (exclusive).

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

- So what are the items?
  - Recall that we compare the current value (inclusive) with the value in the row above (exclusive). If same value, means we do not include…

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

Kapsack Problem
0/1 items

- So what are the items?
  - Recall that we compare the current value (inclusive) with the value in the row above (exclusive). If same value, means we do not include…

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

  - {}

Kapsack Problem
0/1 items

- So what are the items?
  - Recall that we compare the current value (inclusive) with the value in the row above (exclusive). If different value, means we include…

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

- {C}

Kapsack Problem

0/1 items

- So what are the items?
  - Recall that we compare the current value (inclusive) with the value in the row above (exclusive). If different value, means we include…
  - Then we update to the suitable weight of the included item

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

  - {C}

Kapsack Problem

0/1 items

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

- So what are the items?
  - Recall that we compare the current value (inclusive) with the value in the row above (exclusive). If different value, means we include…
  - Then we update to the suitable weight of the included item

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

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A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
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0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
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A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
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0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

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Kapsack Problem

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0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

  - {C,B}

Kapsack Problem

0/1 items

Item	A	B	C	D
Weight	6kg	1kg	5kg	9kg
Value	$230	$40	$350	$550

- So what are the items?
  - Recall that we compare the current value (inclusive) with the value in the row above (exclusive). So now do the same…

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

    - {C,B,A}

Kapsack Problem
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	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

    - {C,B,A} makes up the item for total value of 620

Kapsack Problem

0/1 items

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  - Recall that we compare the current value (inclusive) with the value in the row above (exclusive). So now do the same…

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	0	0	0	0	0	0	230	230	230	230	230	230	230
B	0	40	40	40	40	40	230	270	270	270	270	270	270
C	0	40	40	40	40	350	390	390	390	390	390	580	620
D	0	40	40	40	40	350	390	390	390	550	590	590	620

  - {C,B,A} makes up the item for total value of 620
  - This is what we call backtracking!

Questions?

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Backtracking
Reconstruction solution

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  - Knapsack = what are the items?
  - Edit distance = what are the modifications made (insert/delete/replace)

  - When we update the optimal value, we can store the decision we made… But this waste a lot of memory as we need to store decisions/ combinations at every step!
  - So, we leave bread crumbs to backtrack or only the final decision made!

Questions?

- Let say we store our decisions…

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2

- Let say we store our decisions…

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2
The coins	{}	1	1, 1	1, 1, 1	1, 1, 1, 1	5	6	6, 1	6, 1, 1	9	5, 5	6, 5	6, 6

- Let say we store our decisions…
  - Space complexity?

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2
The coins	{}	1	1, 1	1, 1, 1	1, 1, 1, 1	5	6	6, 1	6, 1, 1	9	5, 5	6, 5	6, 6

- Let say we store our decisions…
  - Space complexity? Can be O(N^2)

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2
The coins	{}	1	1, 1	1, 1, 1	1, 1, 1, 1	5	6	6, 1	6, 1, 1	9	5, 5	6, 5	6, 6

- Let say we store our decisions…
  – Space complexity? Can be O(N^2)
  – Improve it further?

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2
The coins	{}	1	1, 1	1, 1, 1	1, 1, 1, 1	5	6	6, 1	6, 1, 1	9	5, 5	6, 5	6, 6

- Let say we store our decisions…
  - Space complexity? Can be O(N^2)
  - Improve it further? Remember the last coin you added

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2
The coins	{}	1	1	1	1	5	6	1	1	9	5	6	6

- Let say we store our decisions…
  - Space complexity? Can be O(N^2)
  - Improve it further? Remember the last coin you added
    So space complexity now?

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2
The coins	{}	1	1	1	1	5	6	1	1	9	5	6	6

- Let say we store our decisions…
  - Space complexity? Can be O(N^2)
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    So space complexity now? O(N)

Value	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2
The coins	{}	1	1	1	1	5	6	1	1	9	5	6	6

- Let say we store our decisions…
  - Space complexity? Can be O(N^2)
  - Improve it further? Remember the last coin you added
    So space complexity now? O(N)
    Coins = {6,6}

Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Number of coins	0	1	2	3	4	1	1	2	3	1	2	2	2
The coins	{}	1	1	1	1	5	6	1	1	9	5	6	6

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Backtracking
vs Decision Array

- Backtracking save space
- Decision array save time

Backtracking
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  - Less auxiliary space
  - Same space complexity
- Decision array save time

Backtracking

vs Decision Array

- Backtracking save space
  - Less auxiliary space
  - Same space complexity
- Decision array save time
  - Faster
  - But time complexity lies in finding the solution still…

Questions?

- Edit-distance
- We will skip this since this is similar to the Knapsack really…

Edit Distance
Cost to convert string

- Edit-distance
- We will skip this since this is similar to the Knapsack really…
  - Will be covered in the tutorial, linking it up with the longest common subsequence (LCS) problem there

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  - Note: Nathan’s slide at the end

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Questions?

Summary

Dynamic Programming Algorithm (DPA)

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- Break it down

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Thank You