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INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

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Ready?

Agenda

- String retrieval

Agenda

- String retrieval
- Tries and suffix tries

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Let us begin…

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- String retrieval is one of the oldest retrieval task in the world…

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- Anything can be represented as a string

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- String retrieval is one of the oldest retrieval task in the world…

- Anything can be represented as a string
  - DNA sequence
  - Images (RGB)
  - Keys
  - … and many more!

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- So how can we search for string very fast?

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- So how can we search for string very fast?
  - Sort the strings
  - Binary search for what you want

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  - What is our complexity?
    - N = number of strings
    - M = average length the string (instead of the longest)

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    - O(MN log N) using merge sort… because O(M) for string comparison
    - O(MN) using radix sort
  - Binary search for what you want

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    - O(MN log N) using merge sort… because O(M) for string comparison
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  - Binary search for what you want
    - O(M log N)… again O(M) for string comparison
  - What is our complexity?
    - N = number of strings
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Questions?

Tries

Efficient string retrieval

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- When we search, we would need to go through every character of a string. Thus, we can use a special data structure that organise it according to characters…

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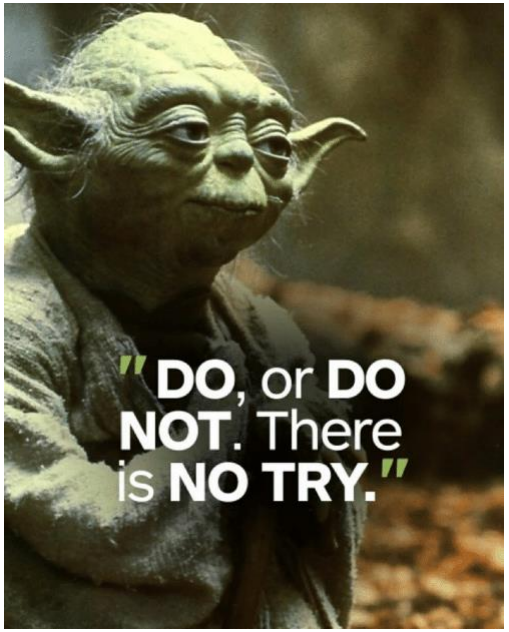

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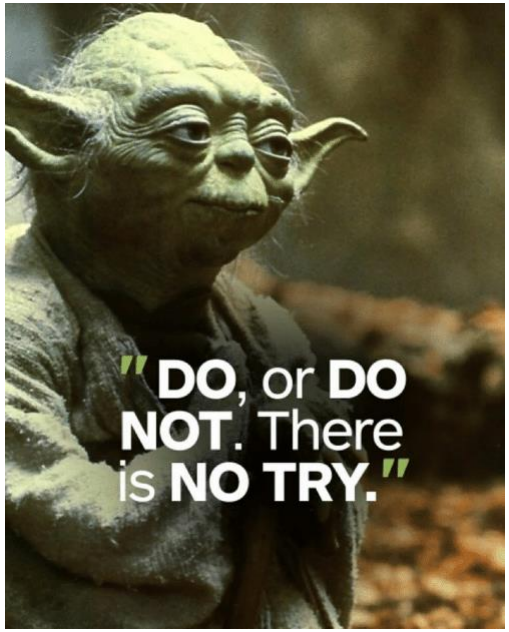
- We use reTRIEval tree
  - A tree
  - M-child per node



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Efficient string retrieval

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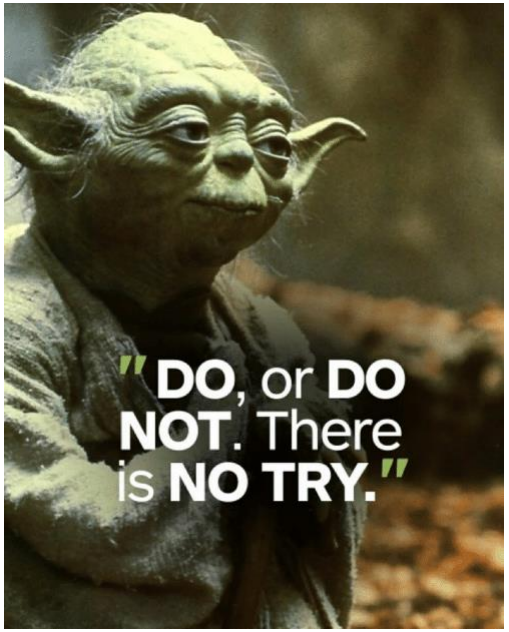
- When we search, we would need to go through every character of a string. Thus, we can use a special data structure that organise it according to characters…

- We use reTRIEval tree
  - A tree
  - M-child per node
    M = number of unique character



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- Let assume we have the following words:
  - Taco
  - Taro
  - Tarot
  - Coco
  - Chobo

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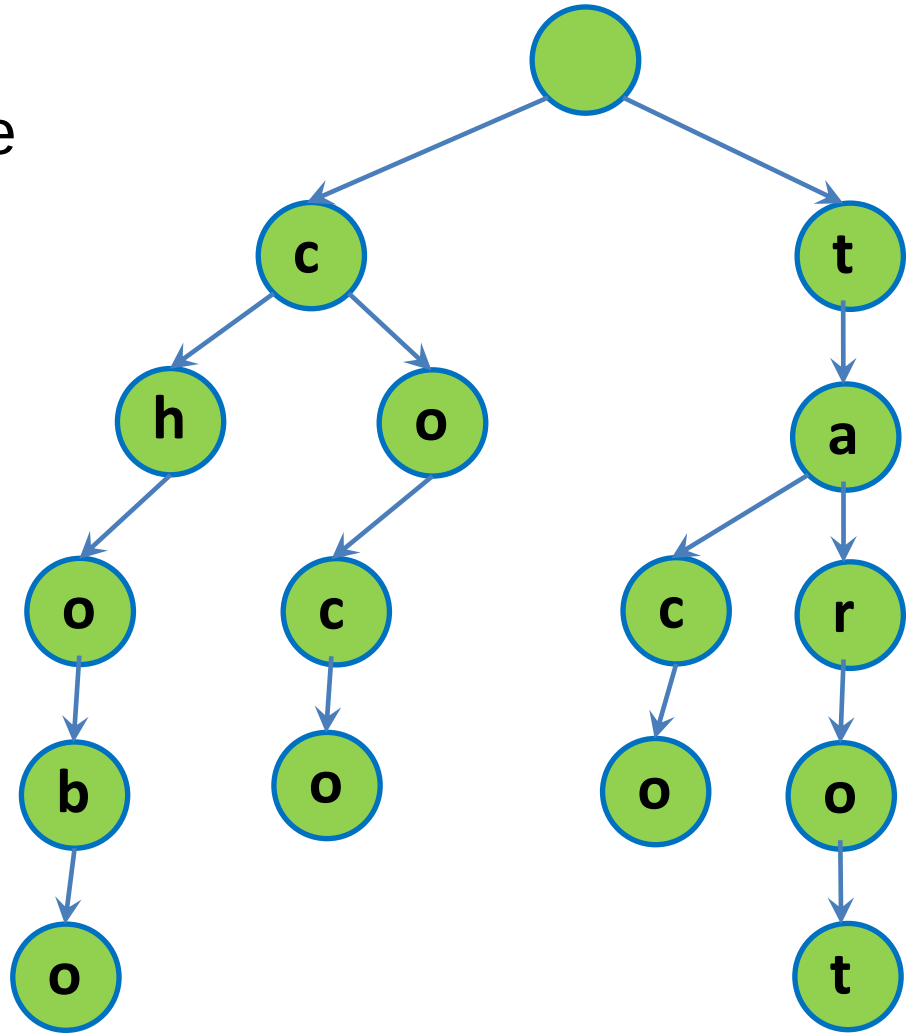

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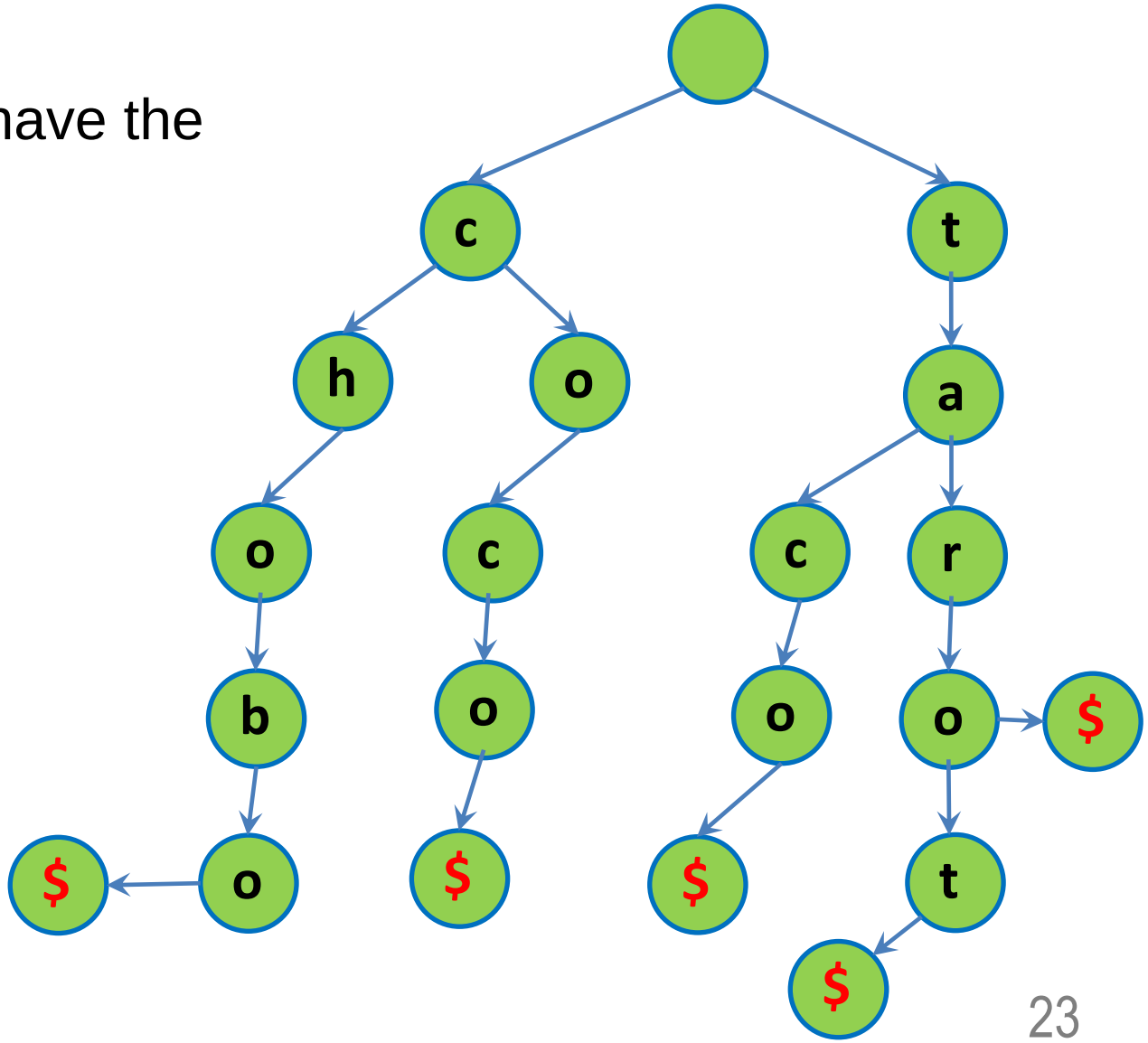

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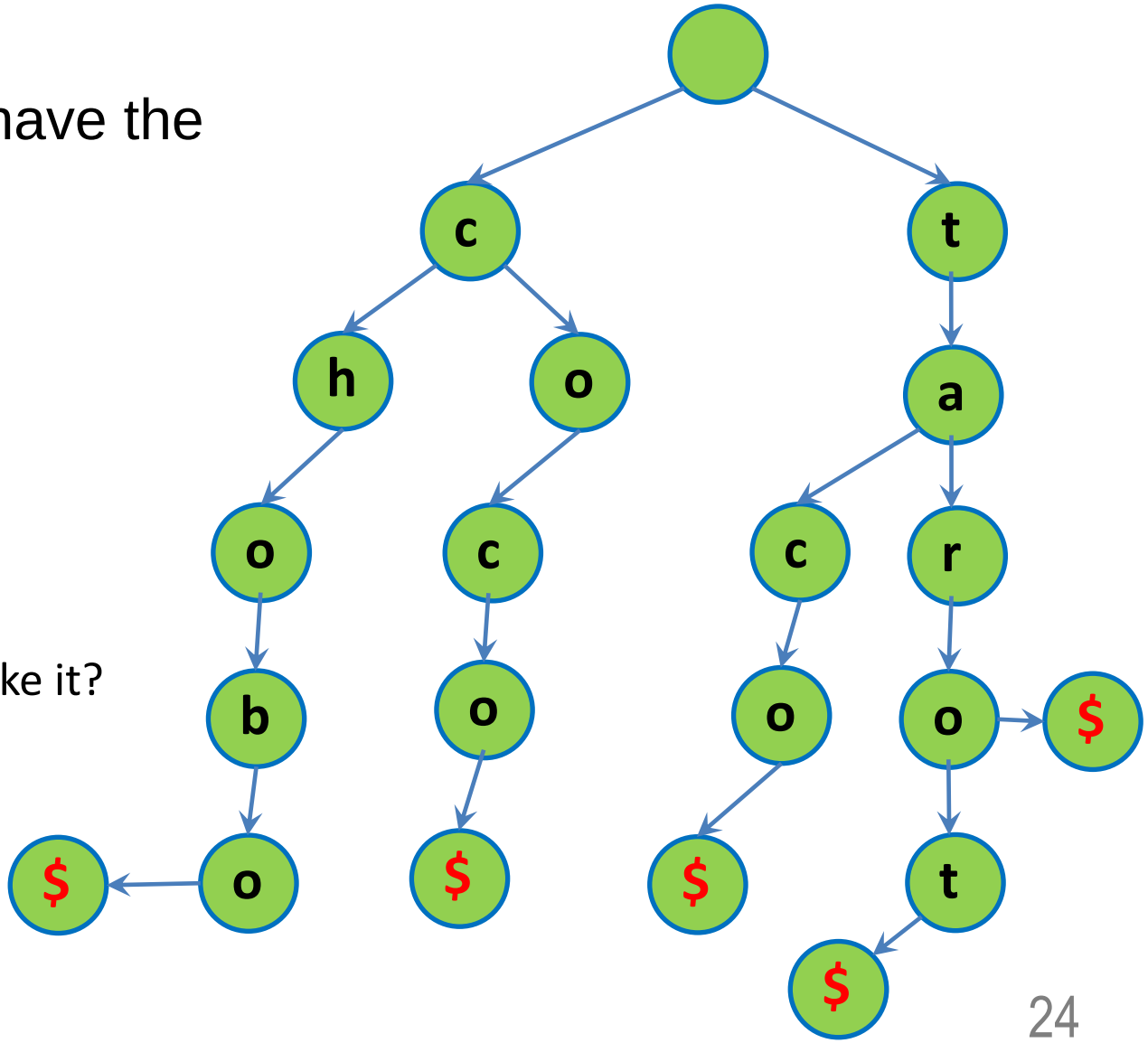

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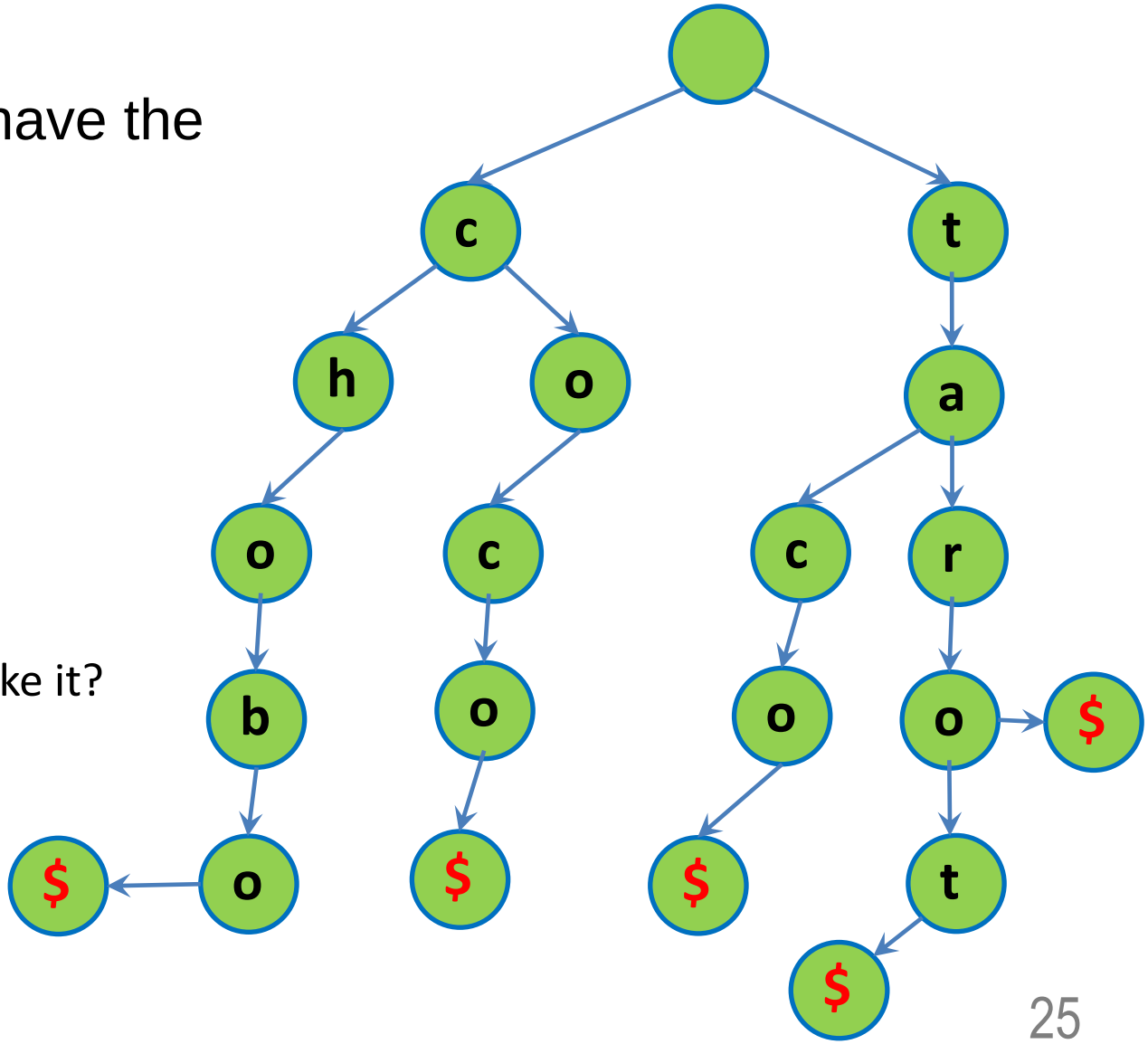
    - Step by step…



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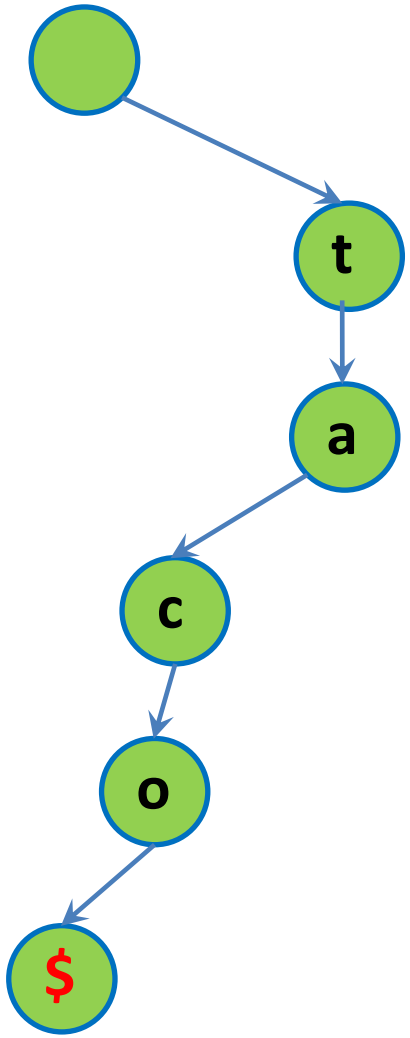

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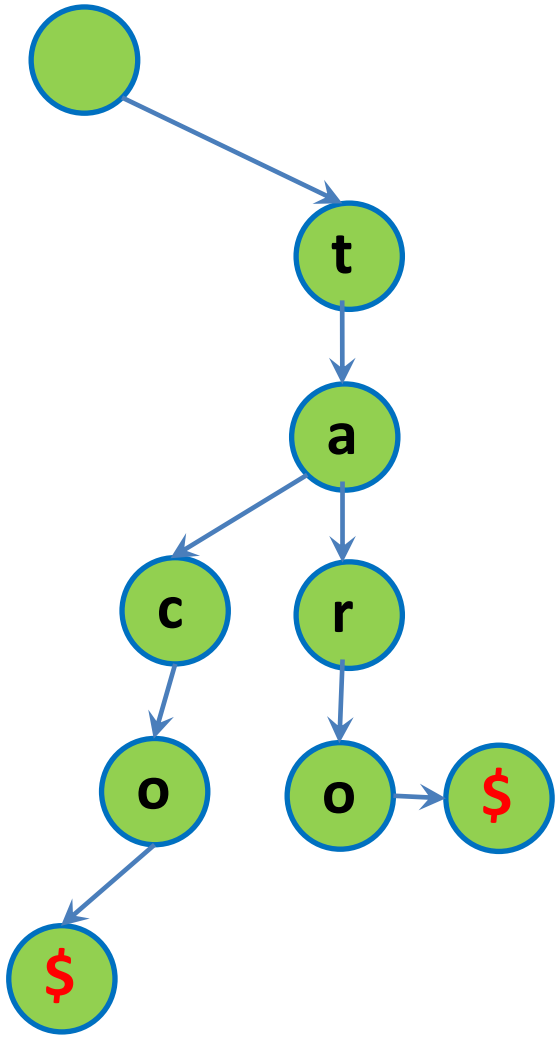

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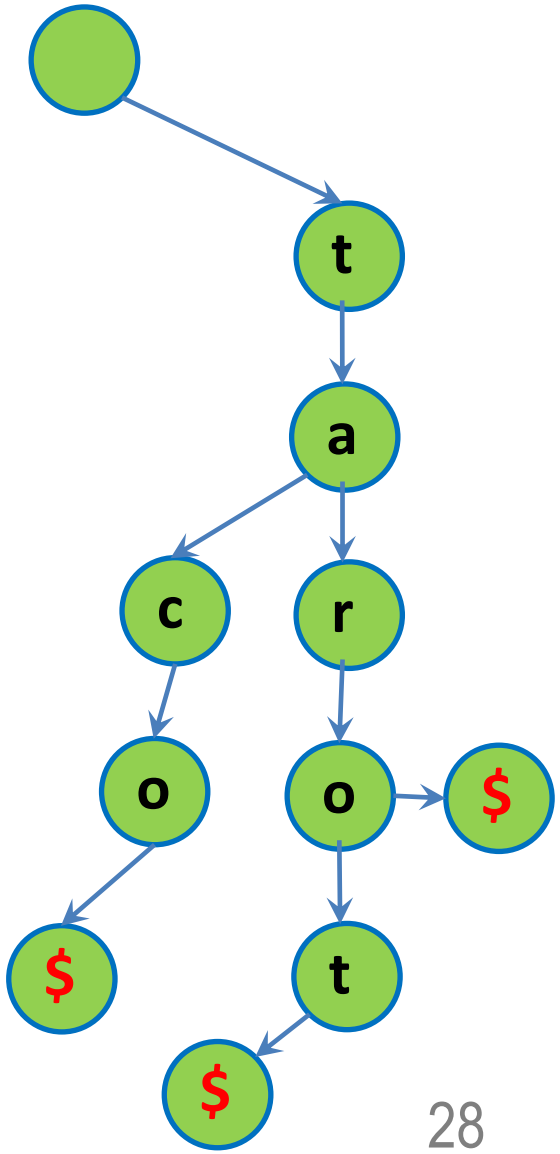

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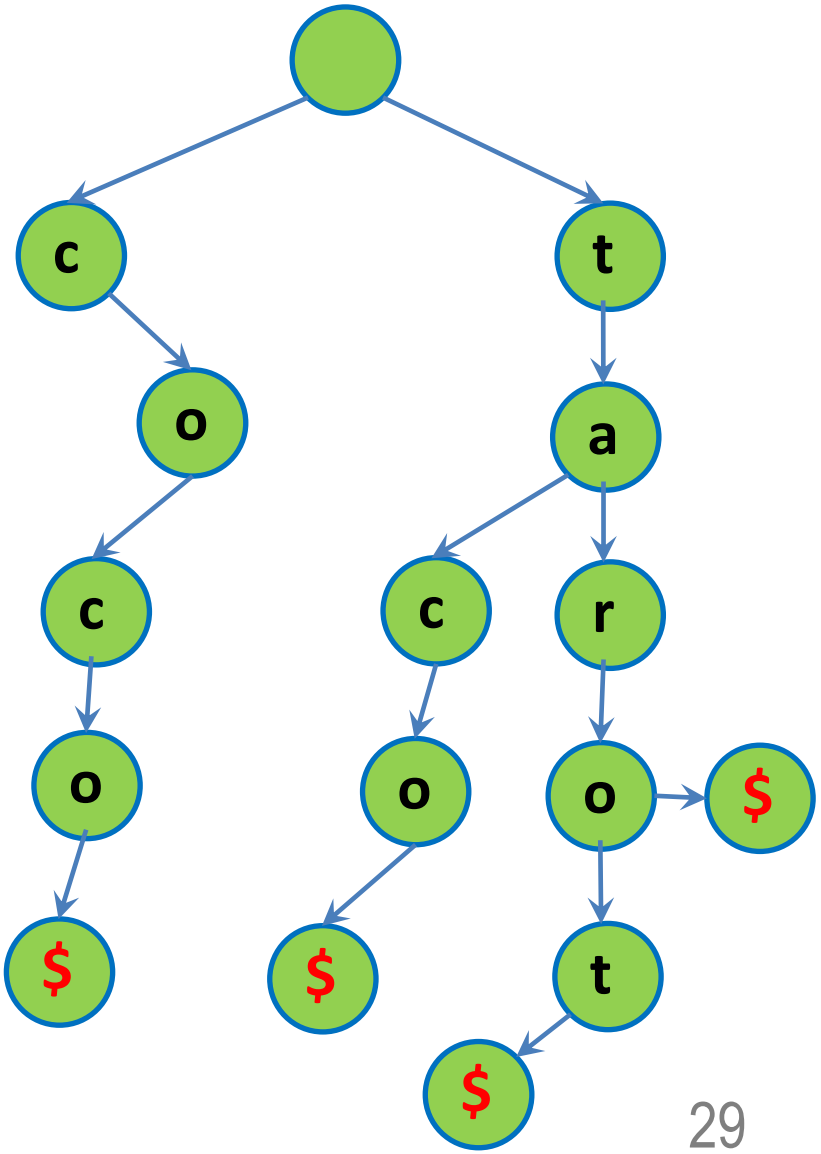

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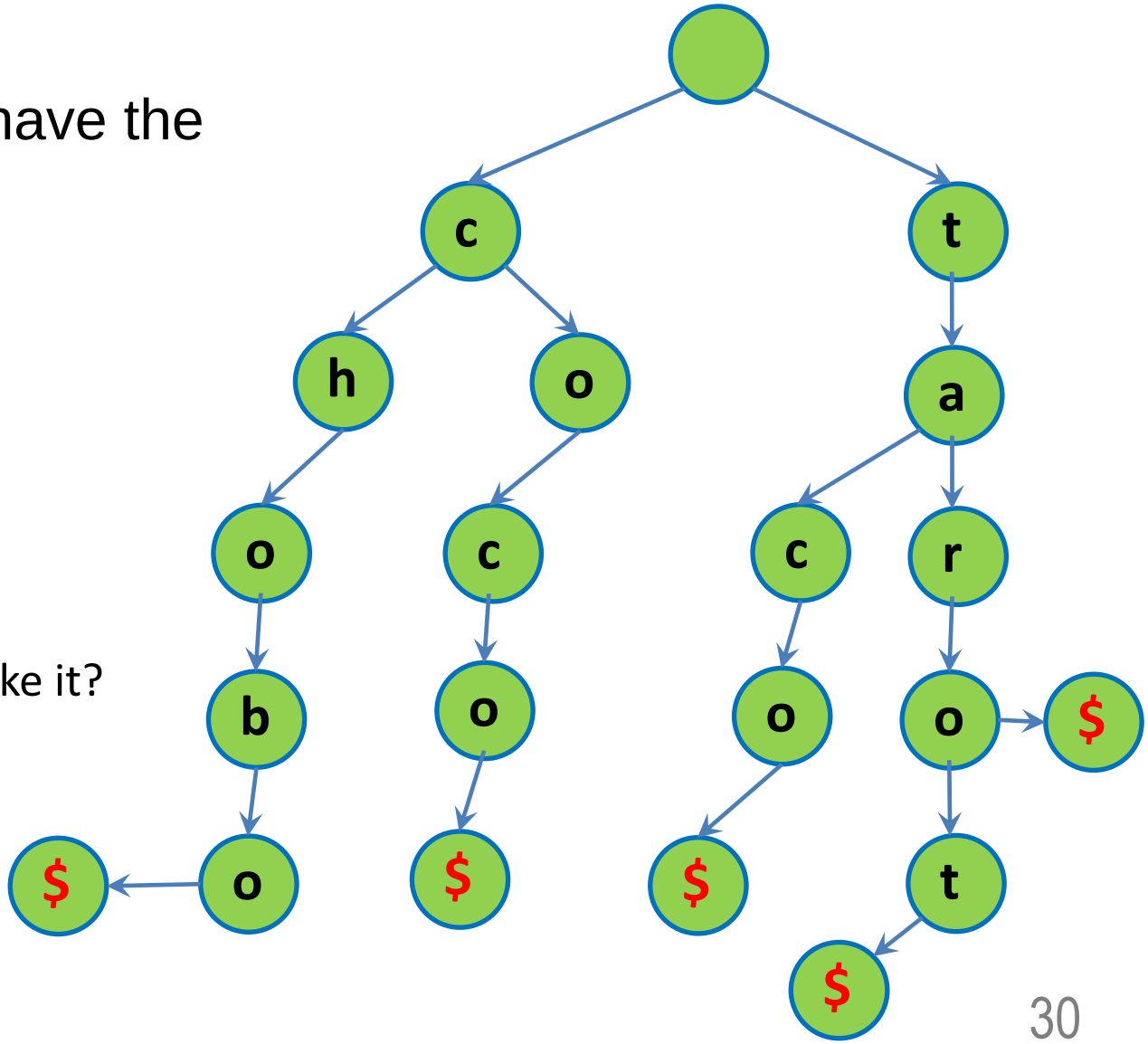

Tries

Efficient string retrieval

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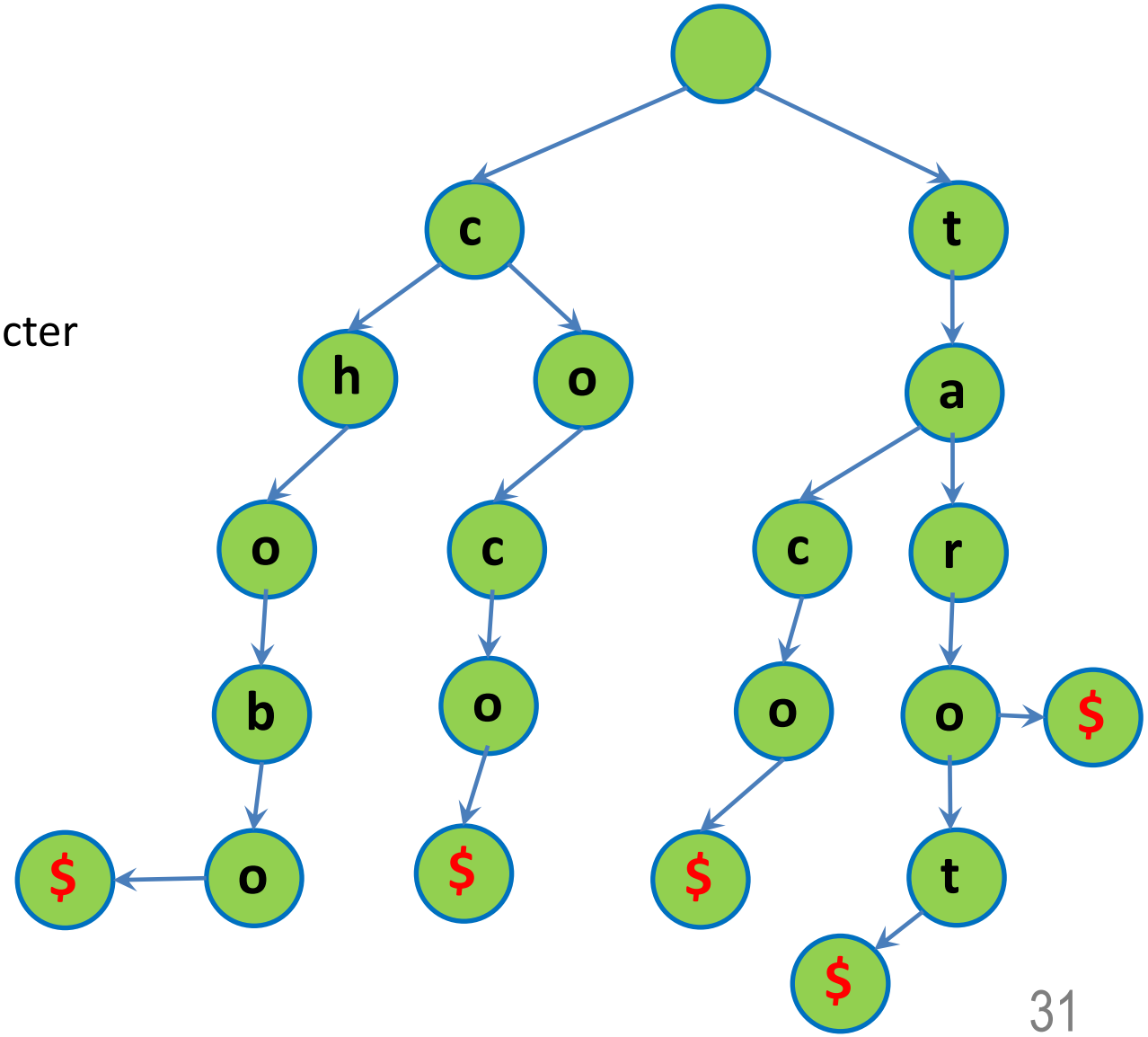
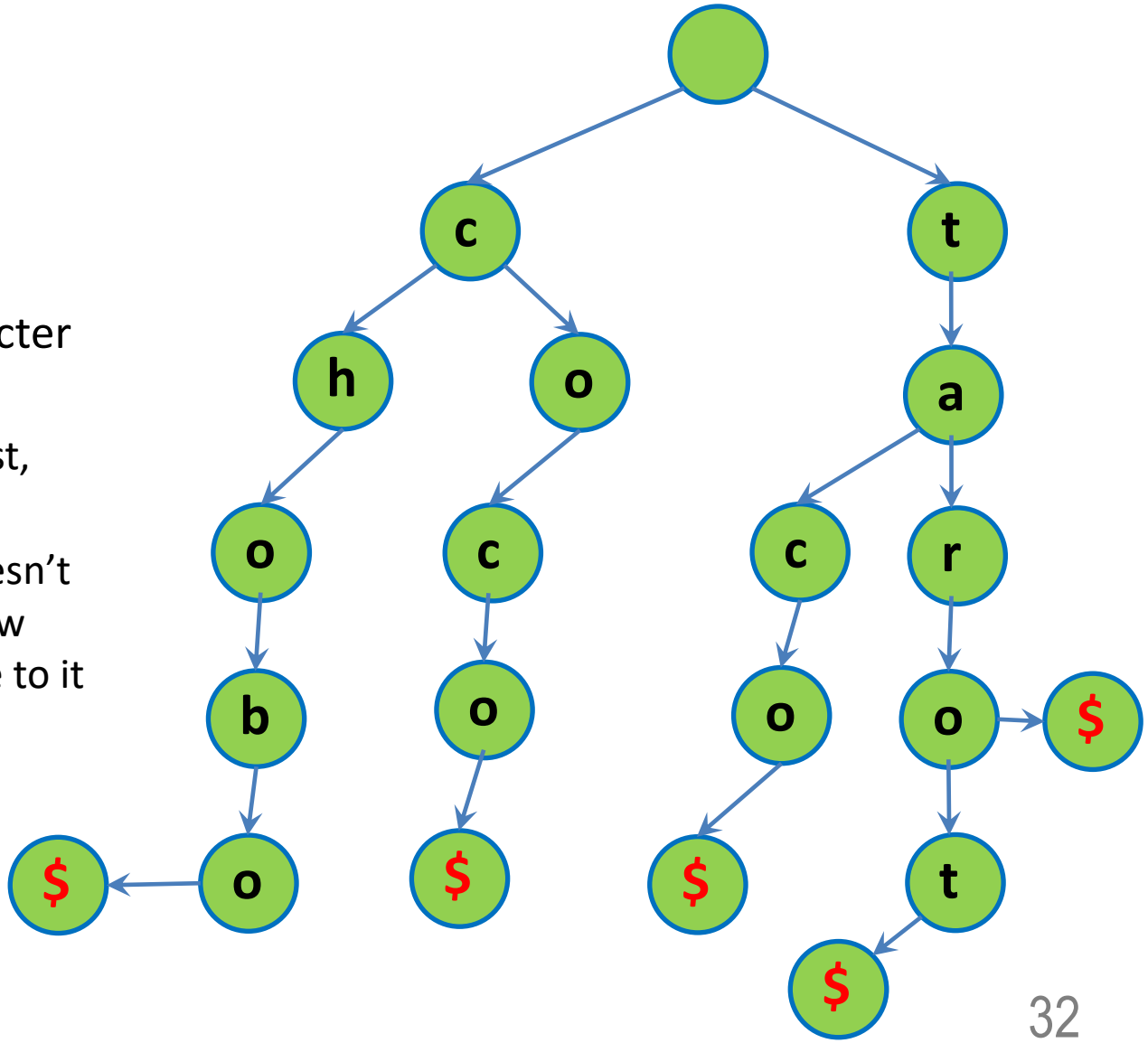
- So steps?
  - For each word
  - Start from root
  - Go through character by character



Tries

Efficient string retrieval

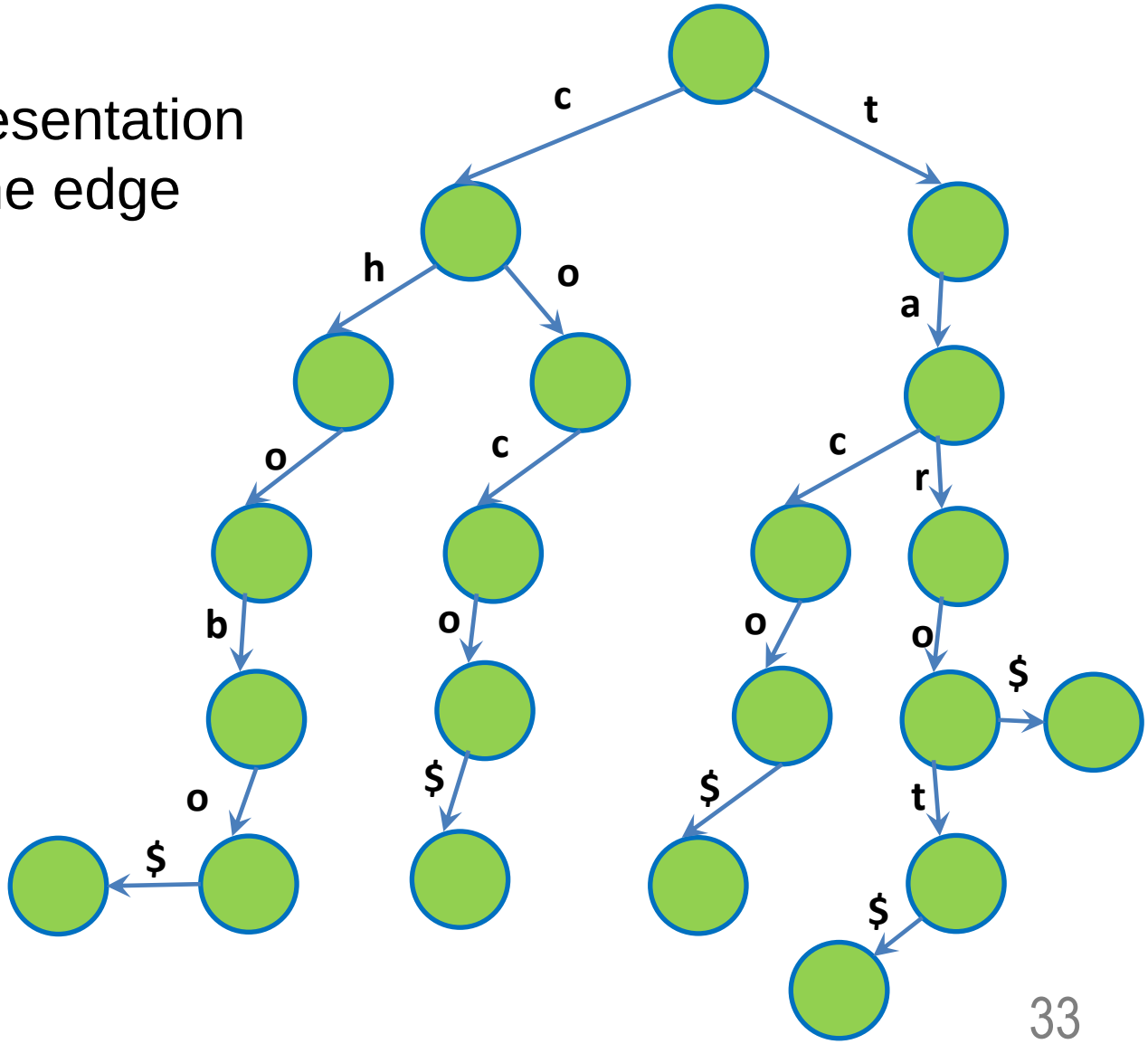
- So steps?
  - For each word
  - Start from root
  - Go through character by character
    - If character exist, follow through
    - If character doesn't exist, create new node and move to it



Tries

Efficient string retrieval

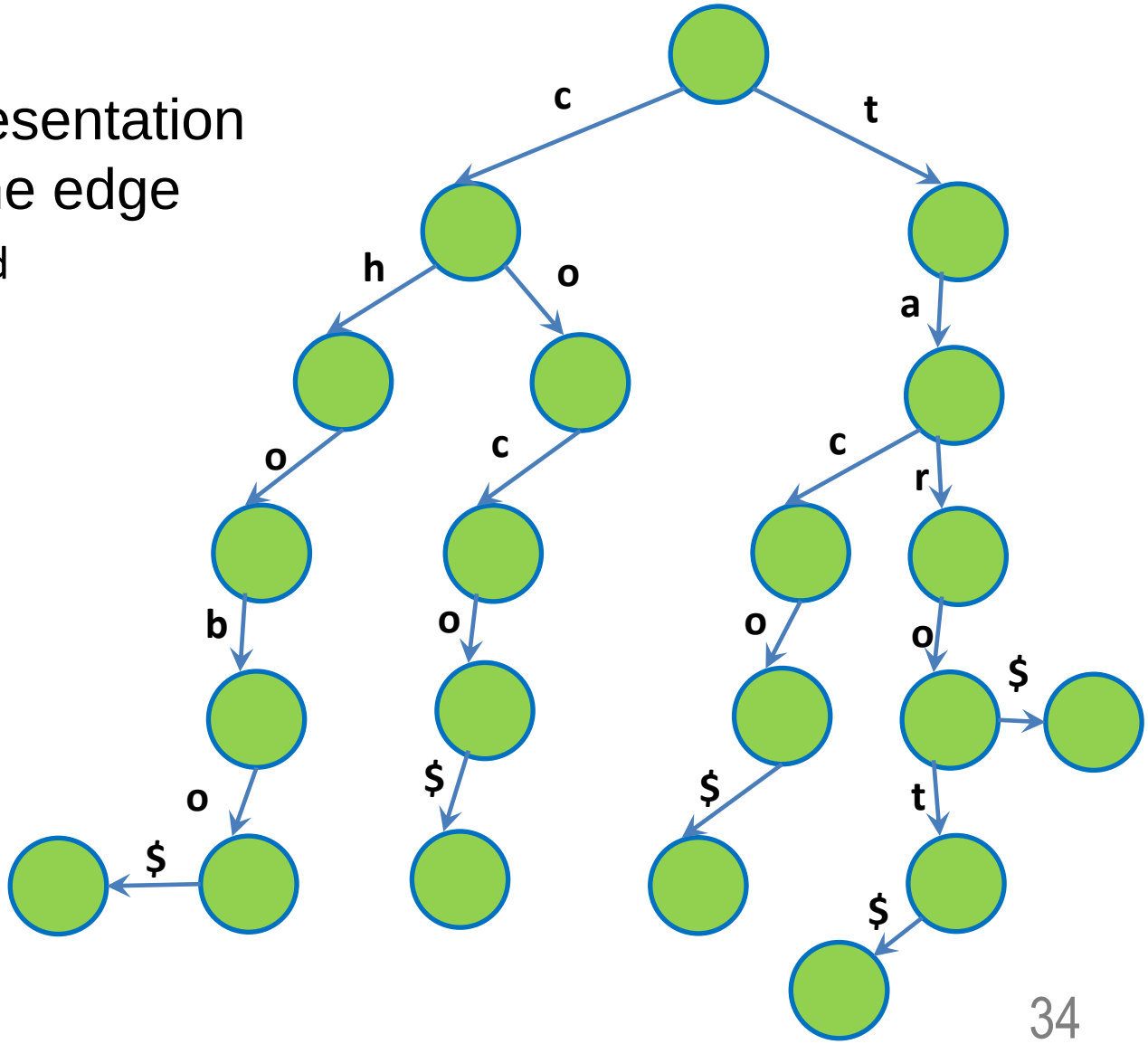
- The proper representation is character at the edge



Tries

Efficient string retrieval

- The proper representation is character at the edge
  - Both are accepted for your exam!



Tries

Efficient string retrieval

- The proper representation is character at the edge
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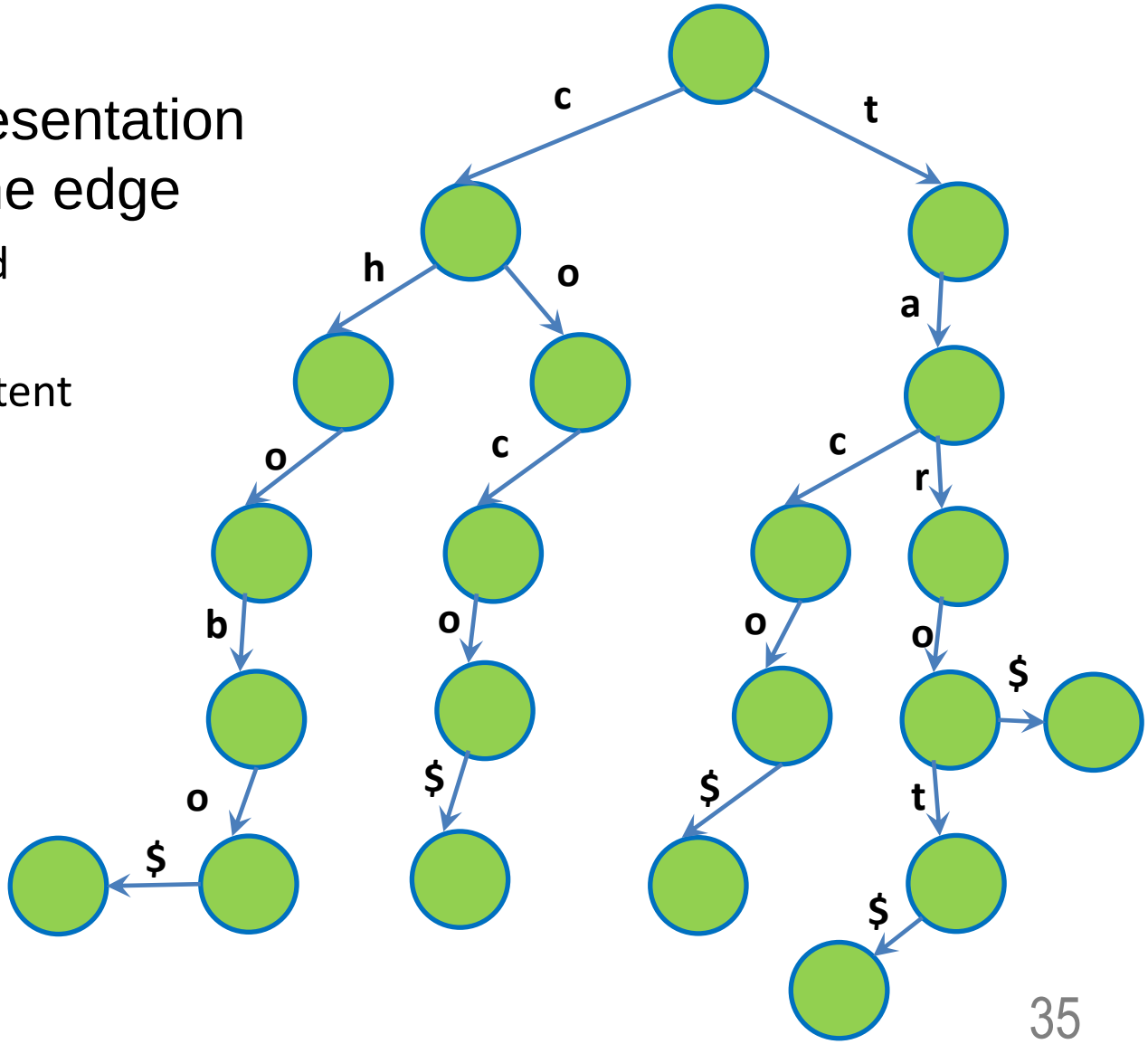
  - This is also consistent with the graph representation



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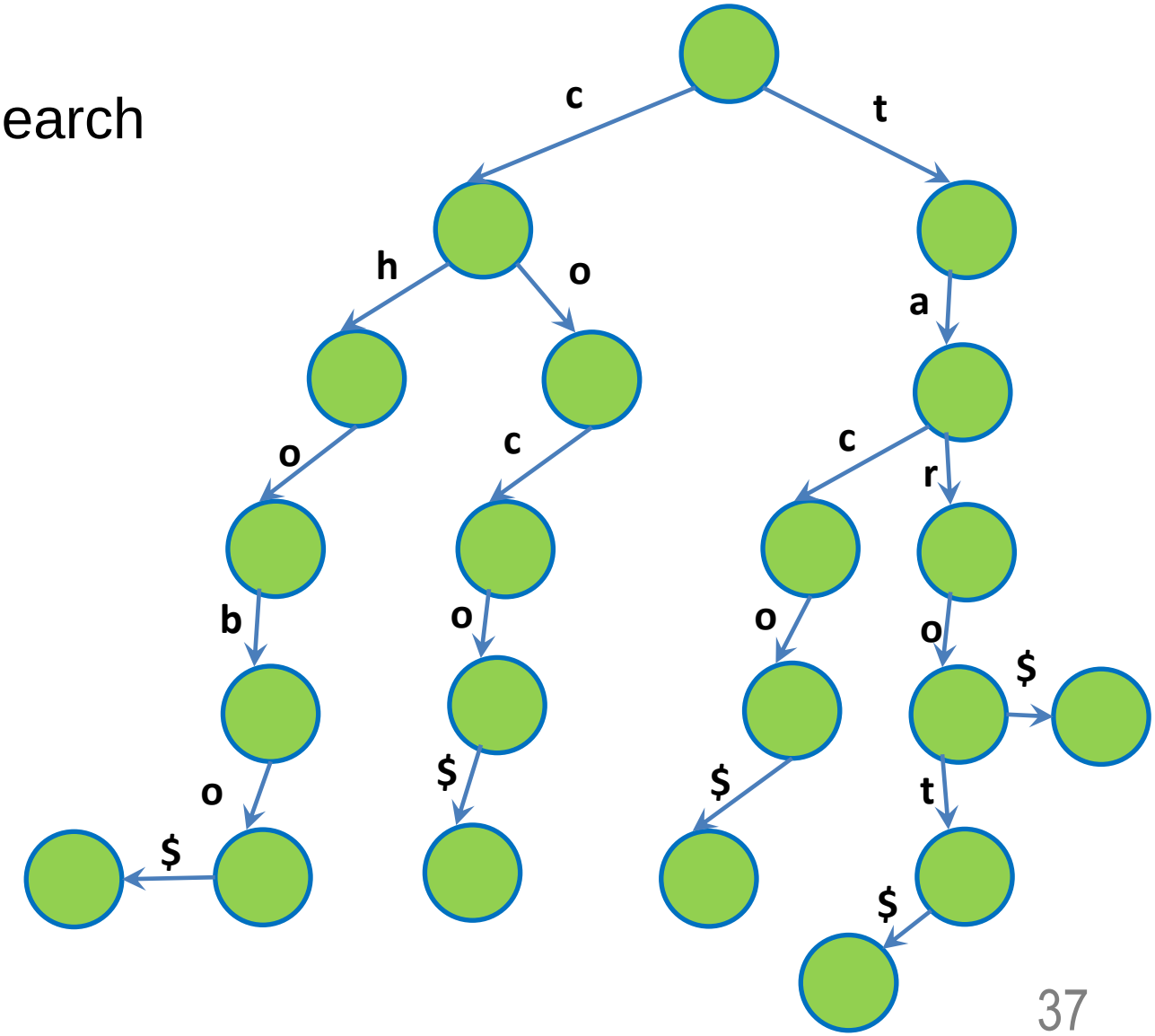
Questions?

Tries

Efficient string retrieval

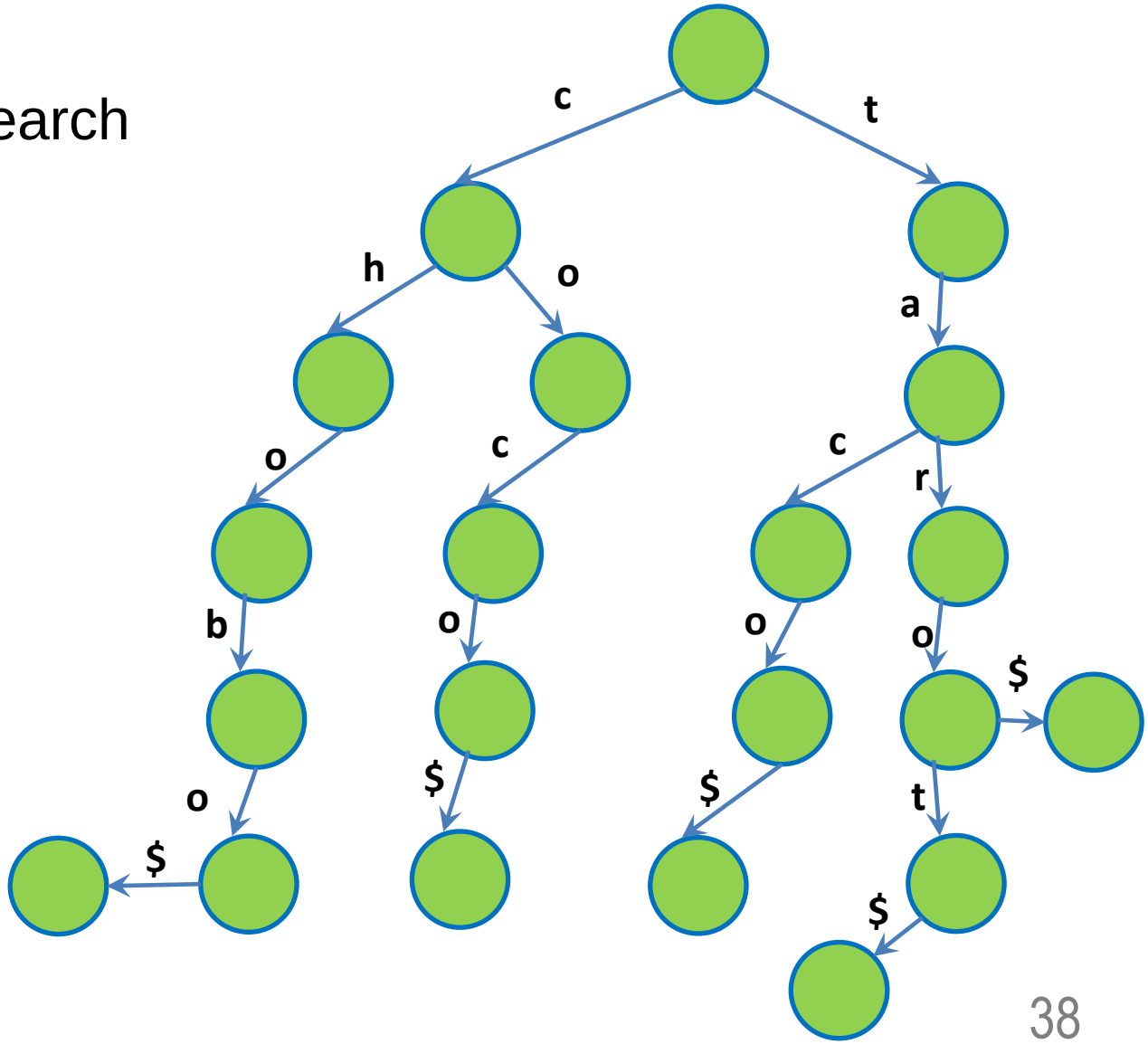
- So how do we search for retrieval?



Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Search for “coco”



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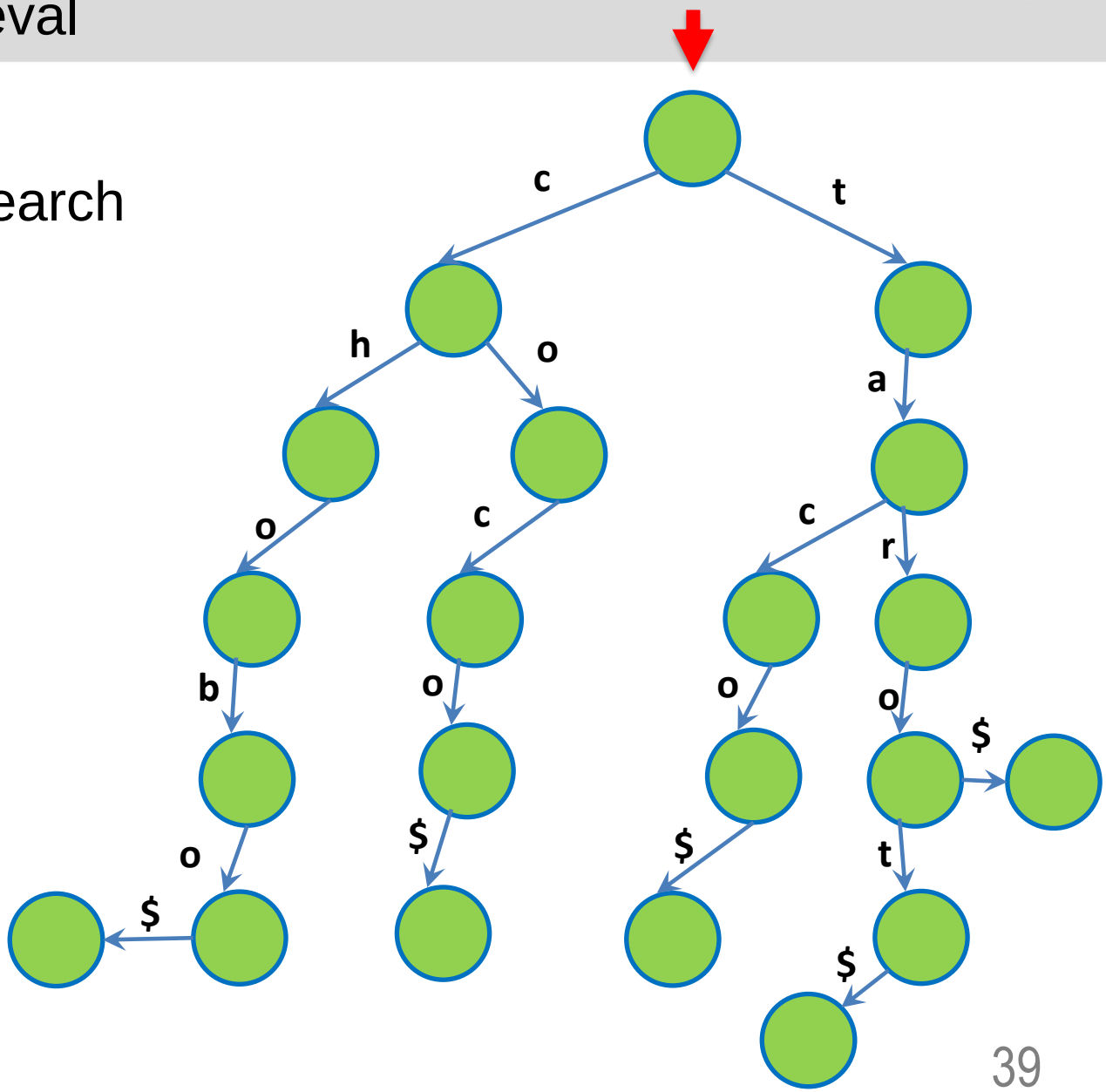

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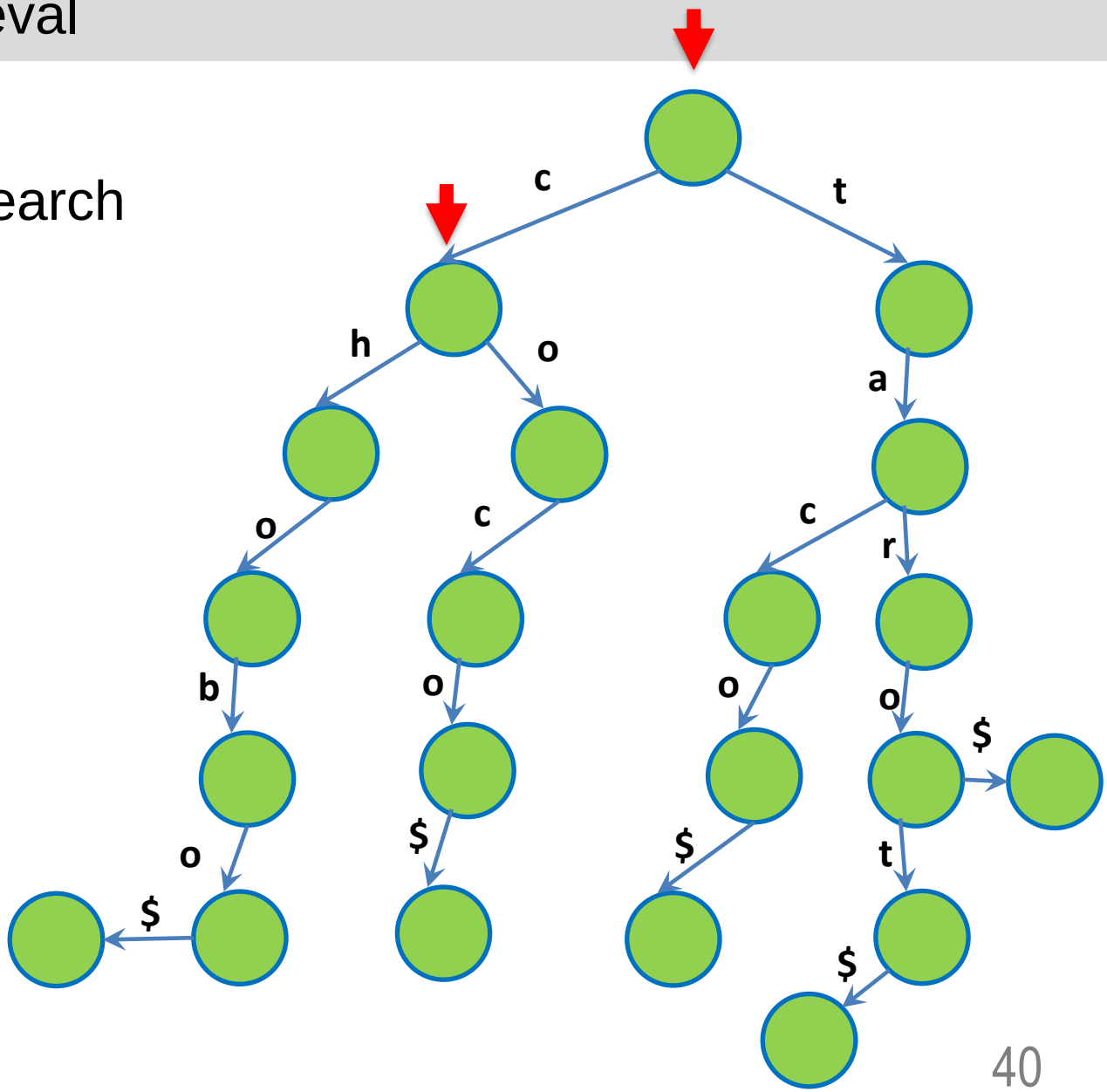

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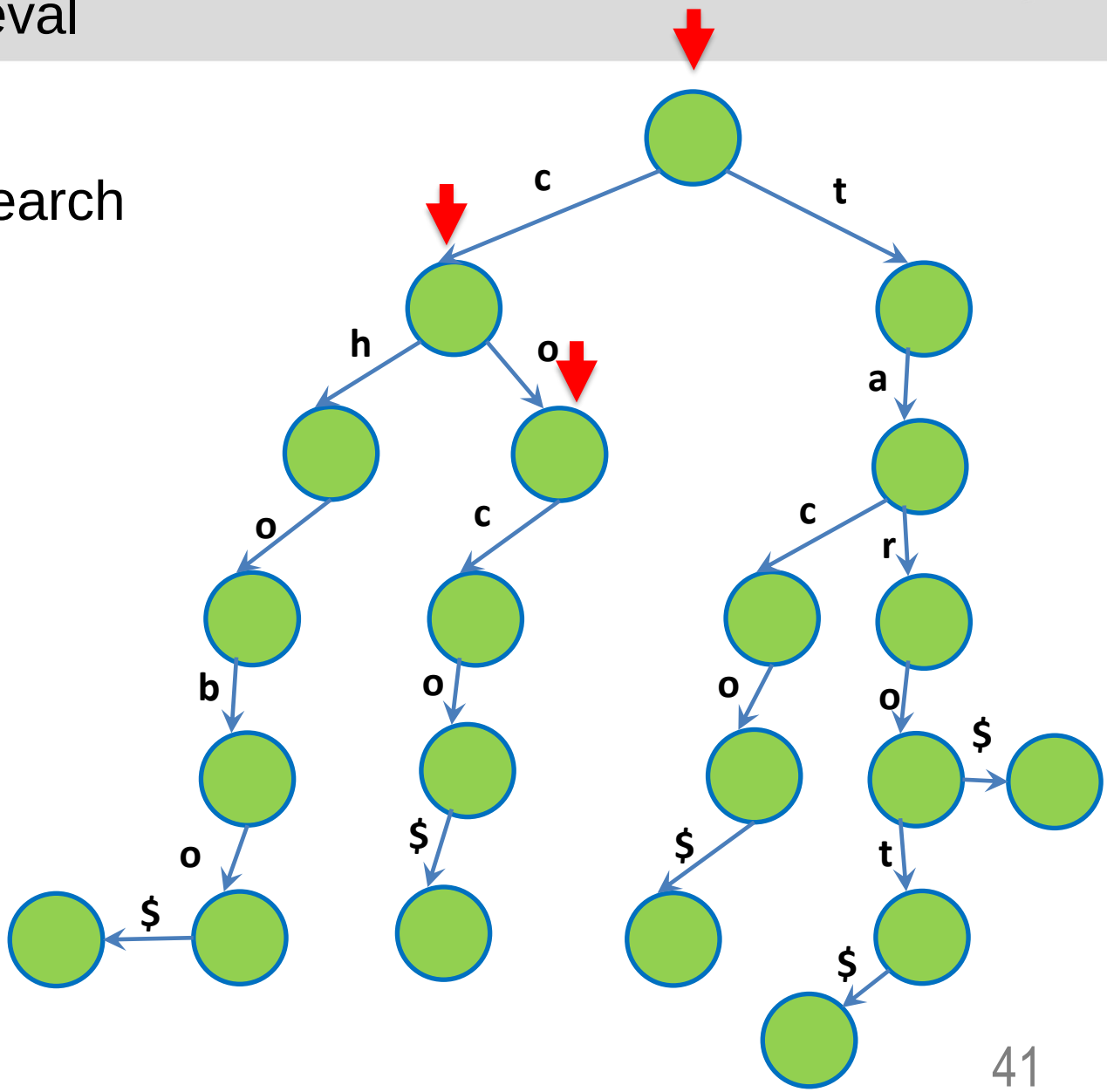

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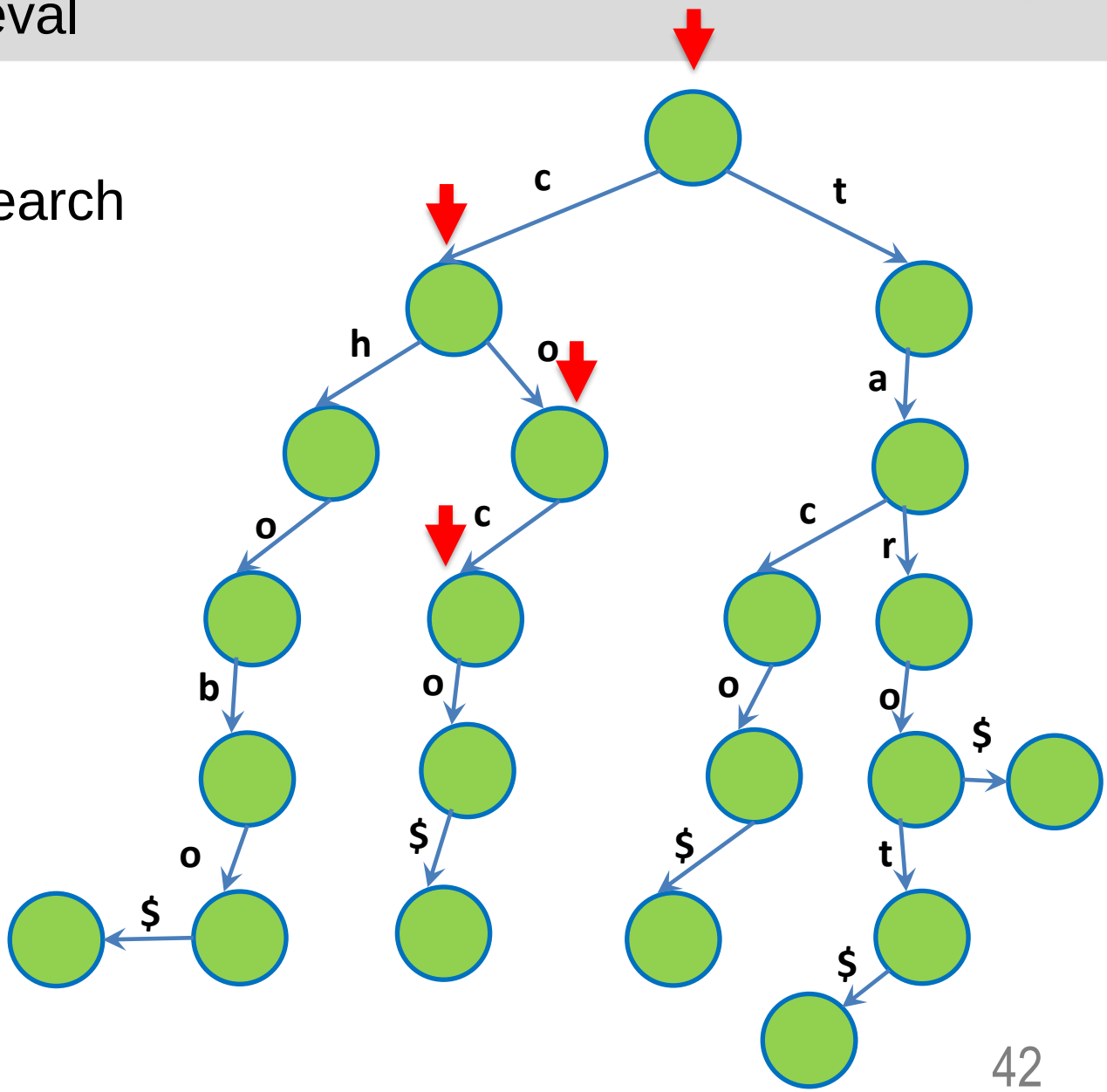

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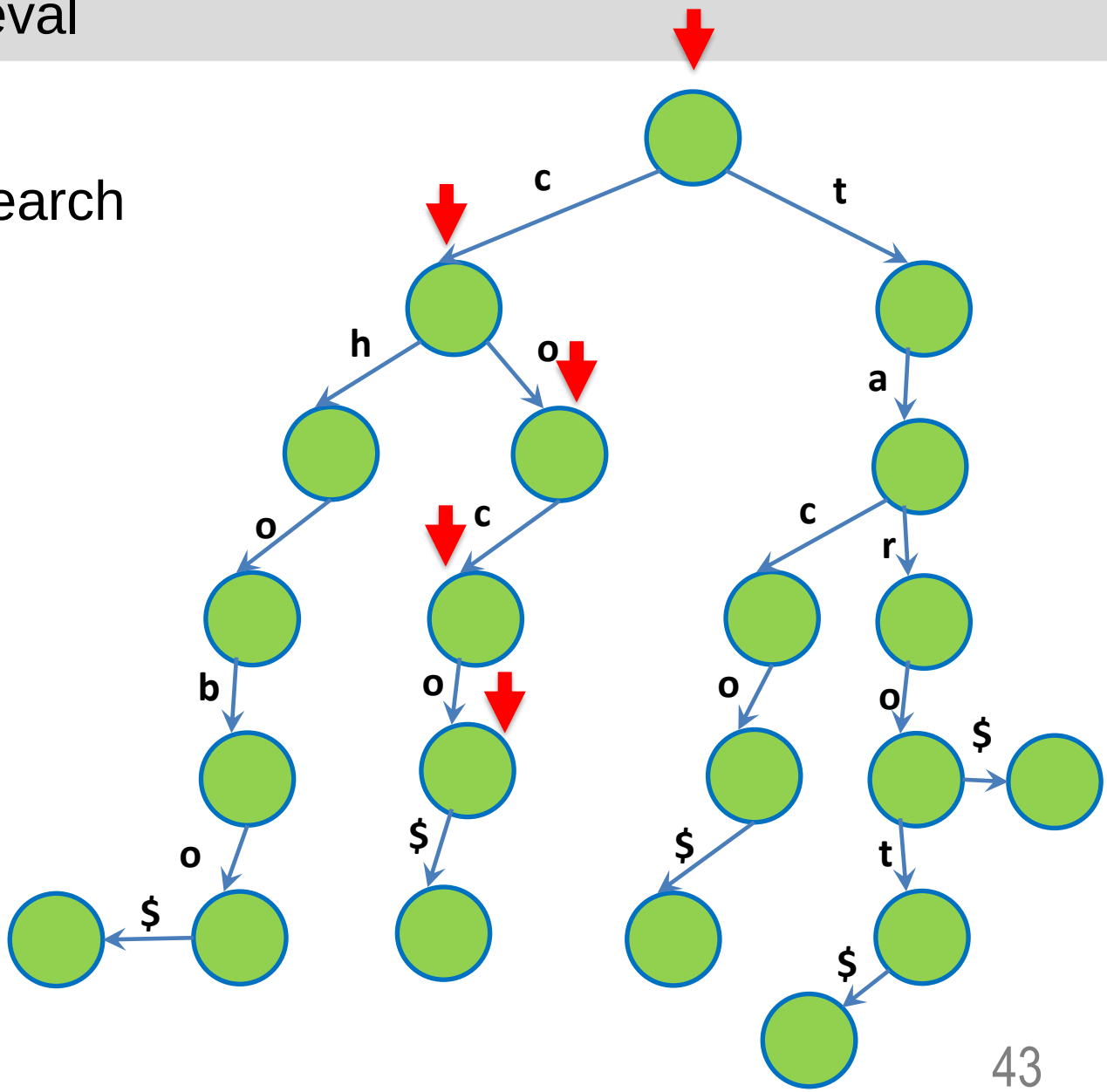

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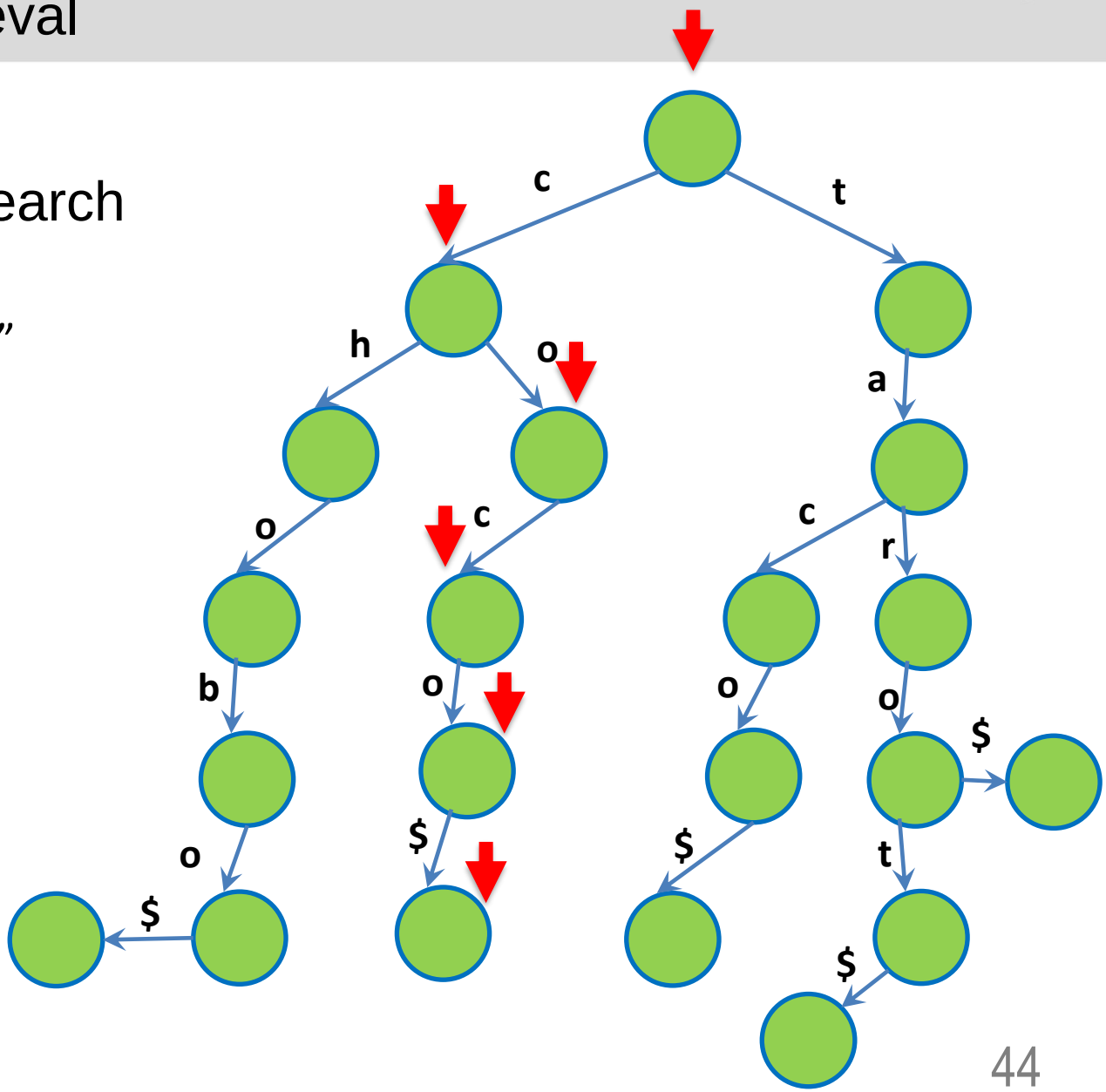

Tries

Efficient string retrieval

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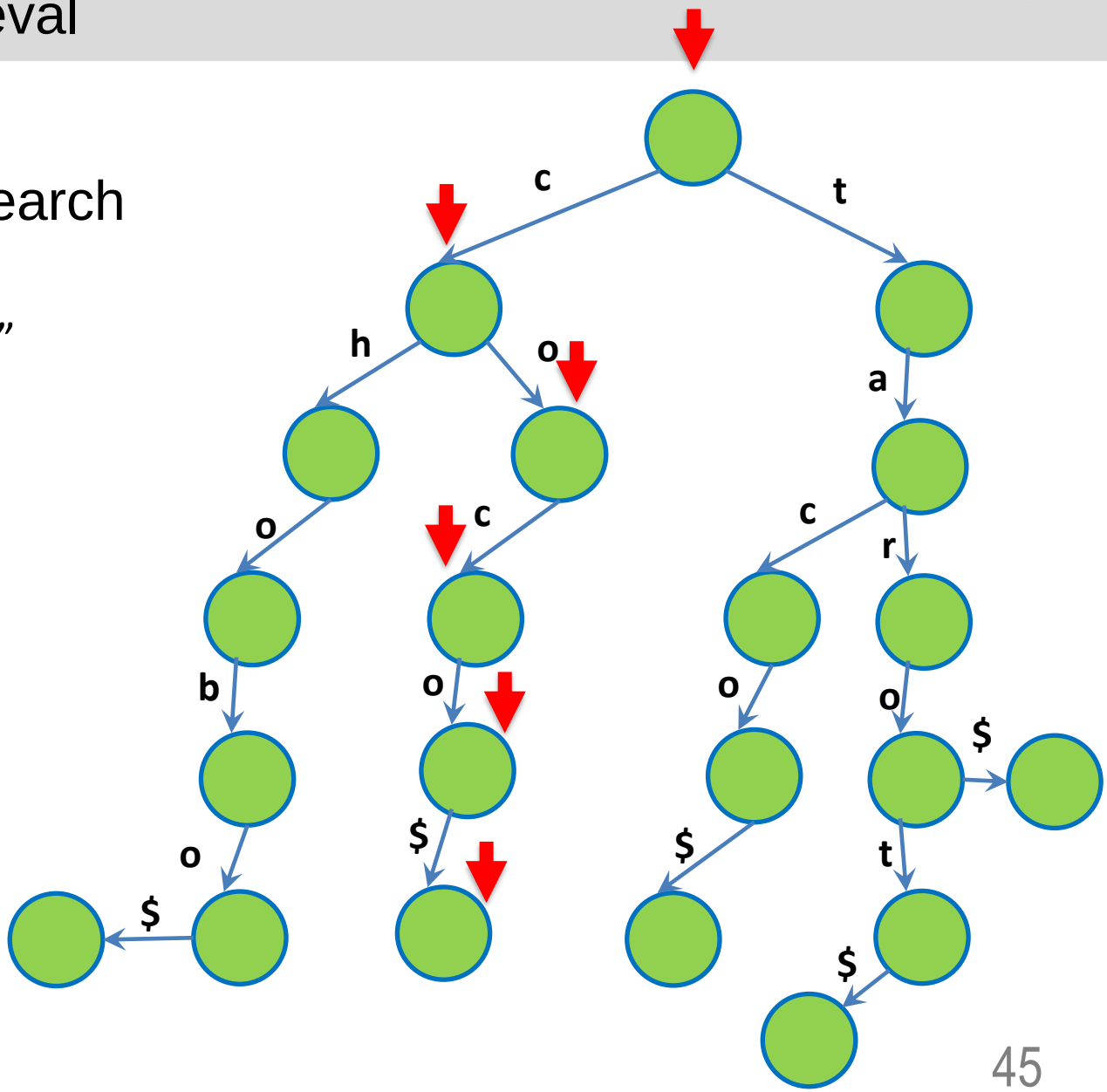
- So how do we search for retrieval?
  - Search for “coco$” so we found it!



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Questions?

Tries

Efficient string retrieval

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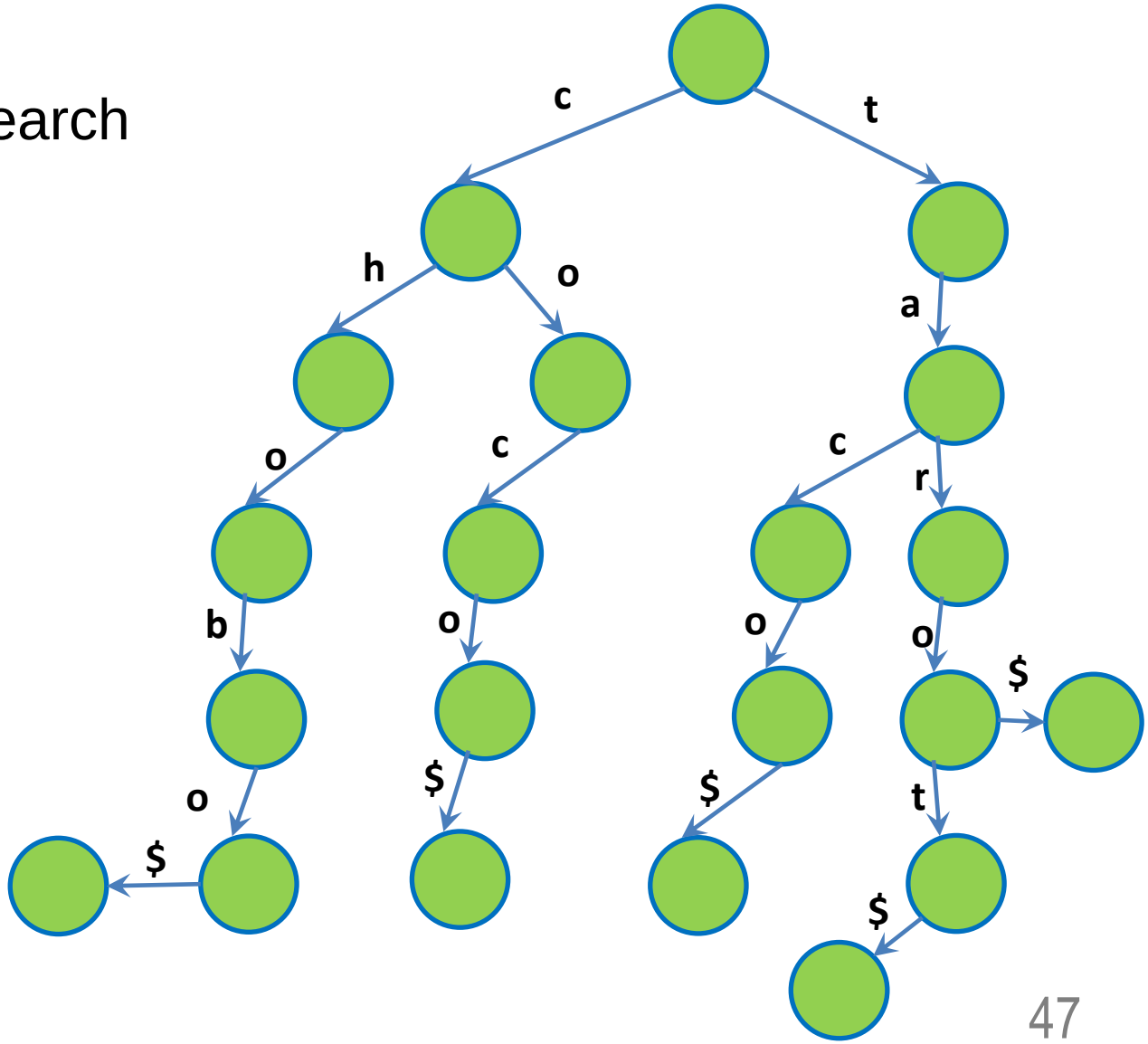
  - Search for “tarot”



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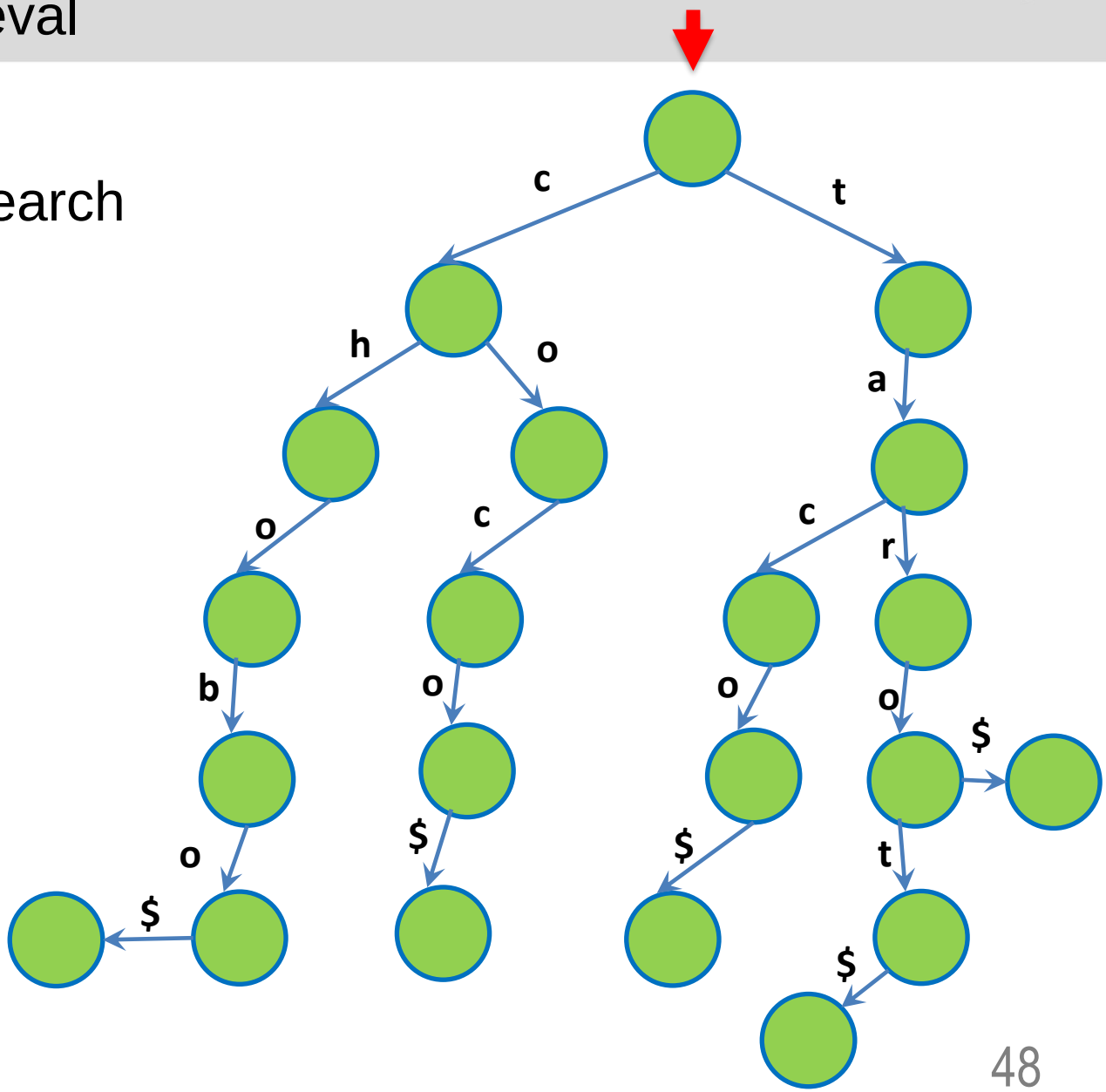

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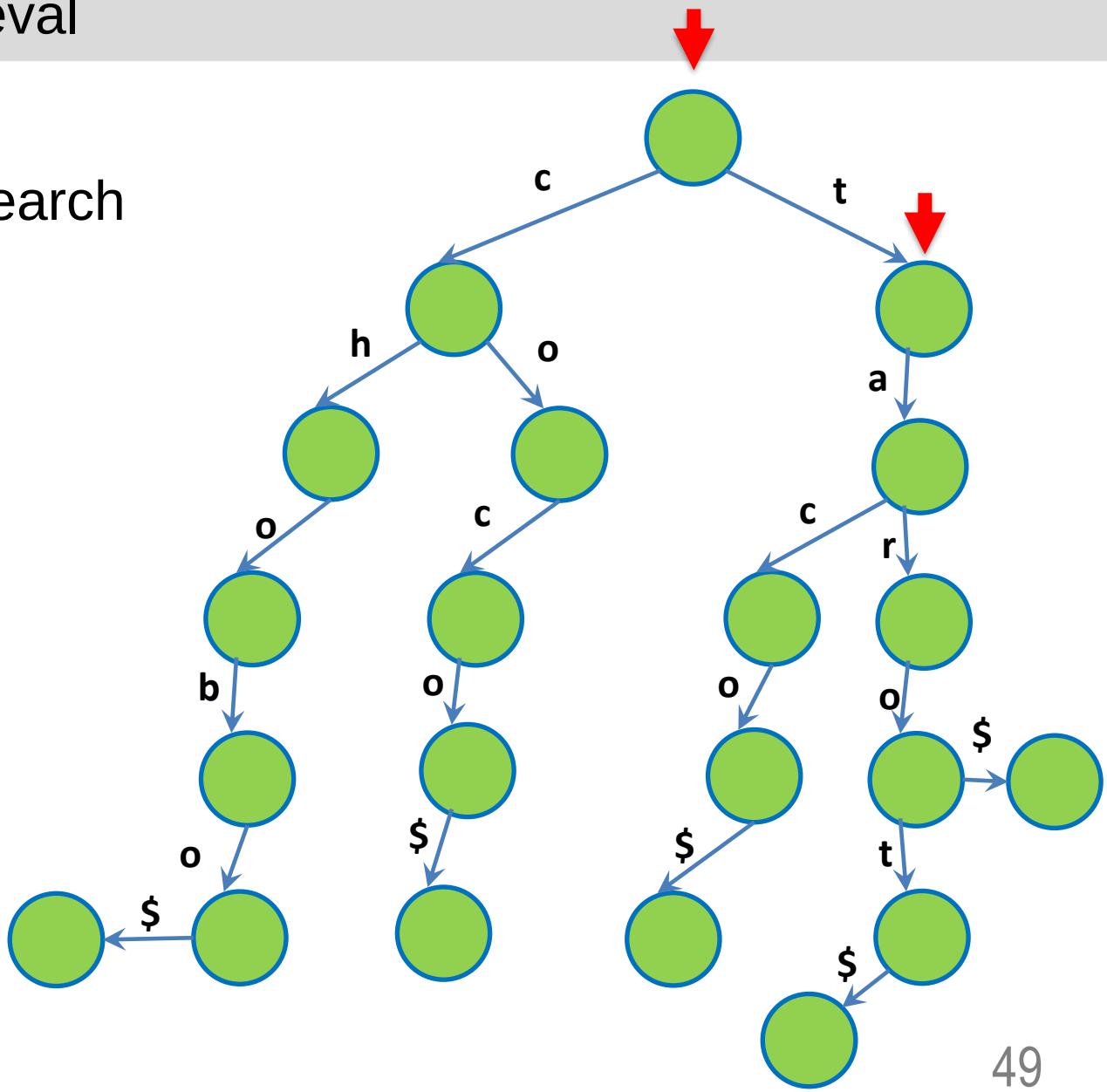

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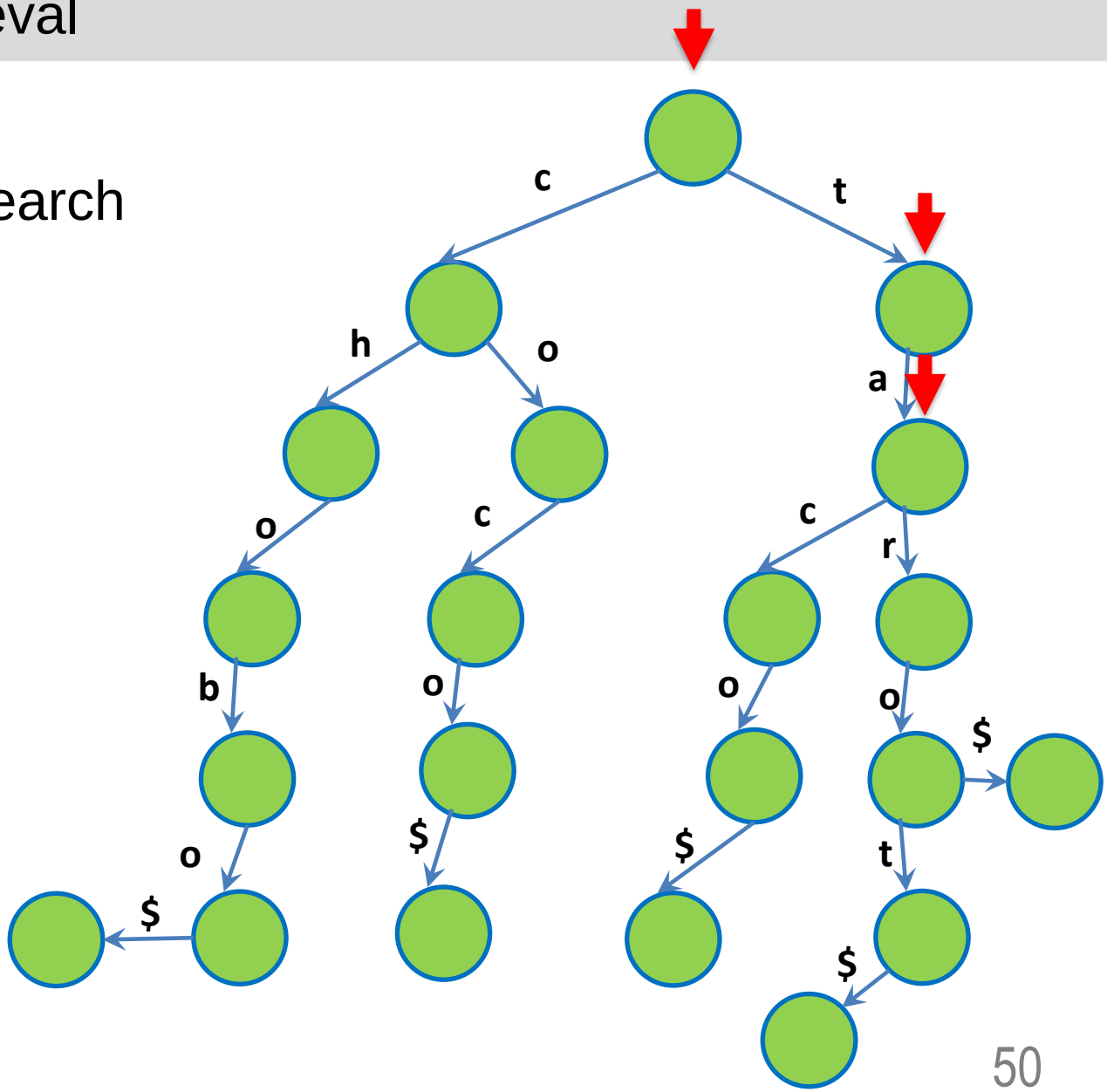

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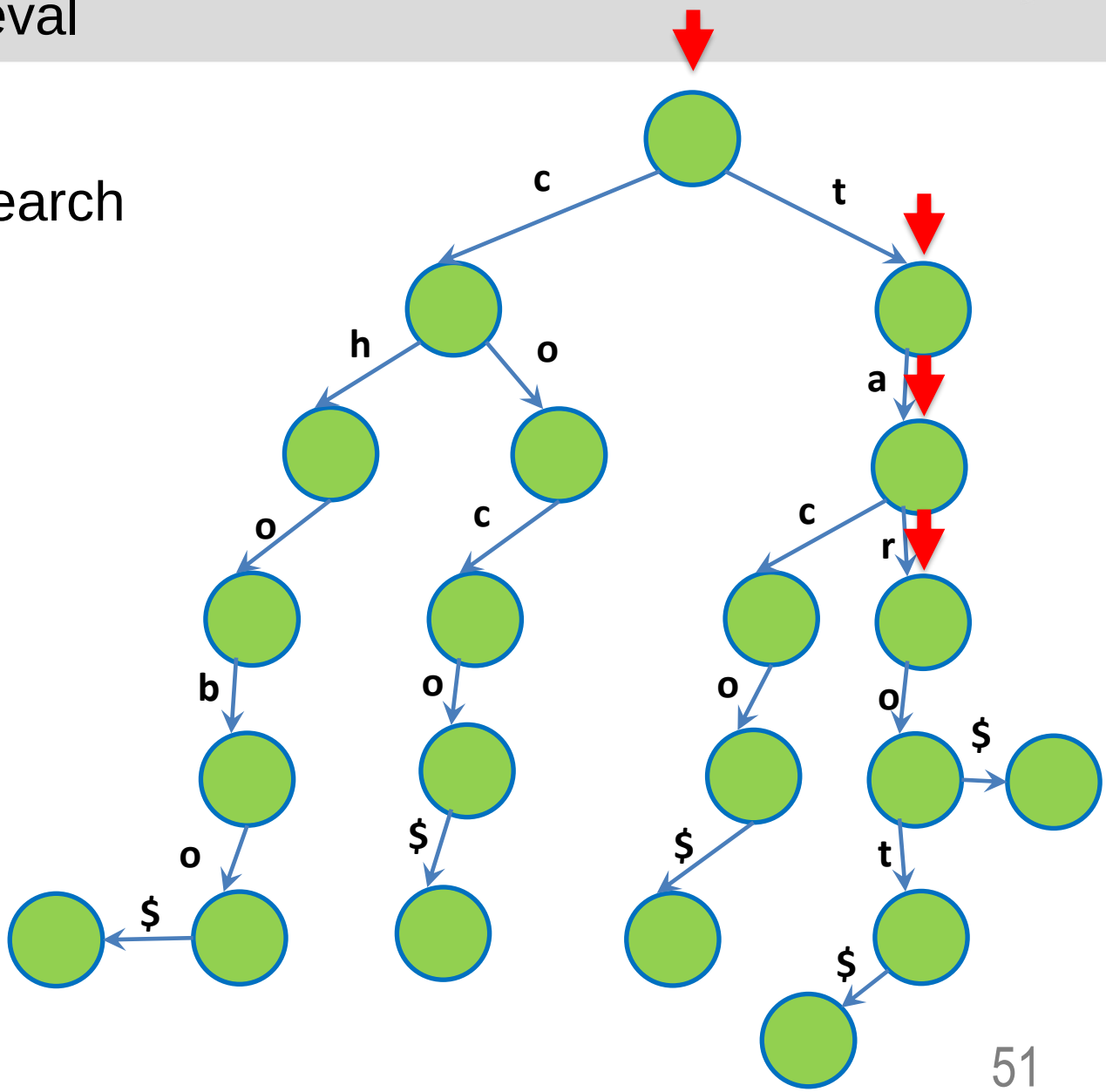

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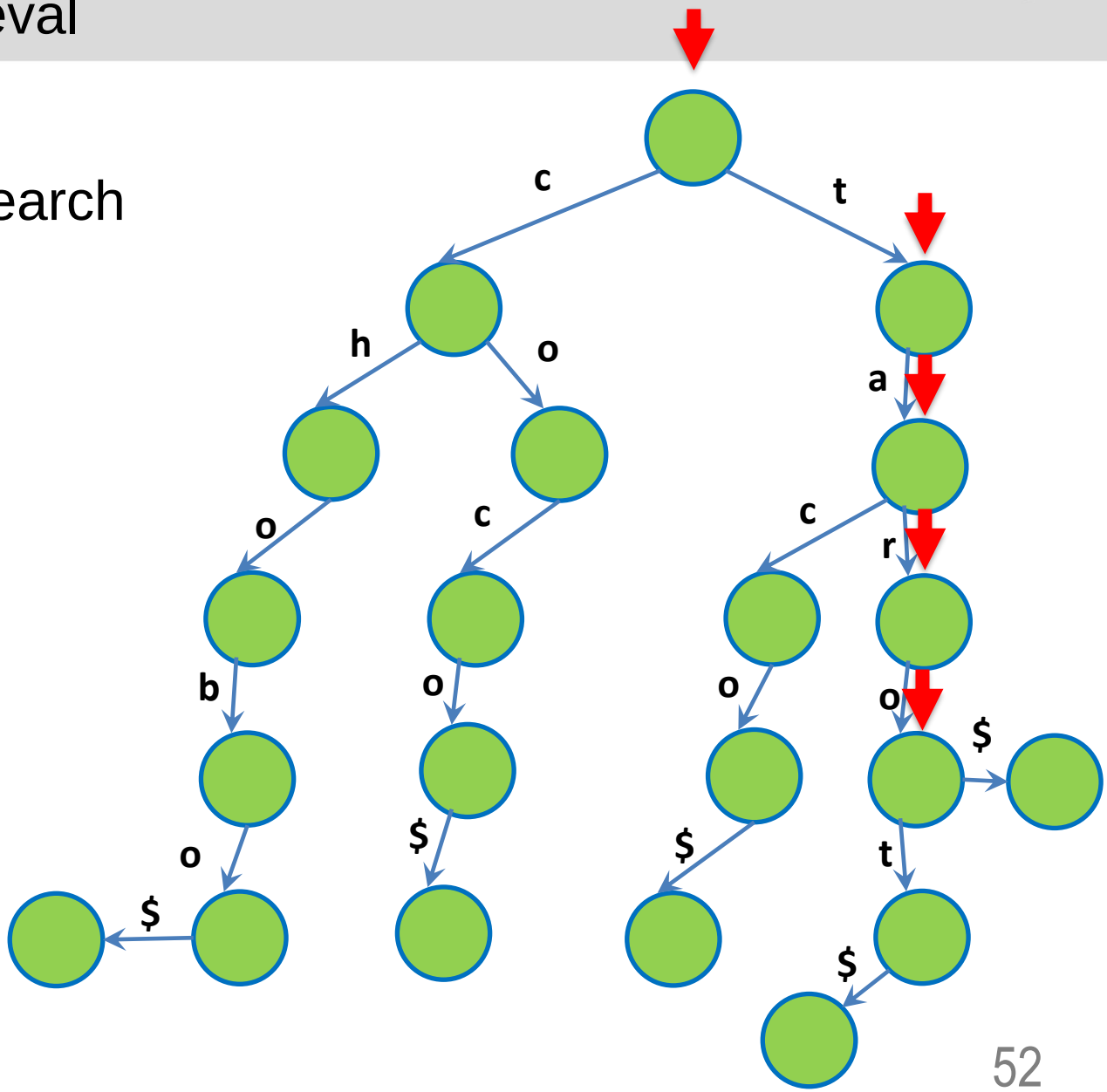

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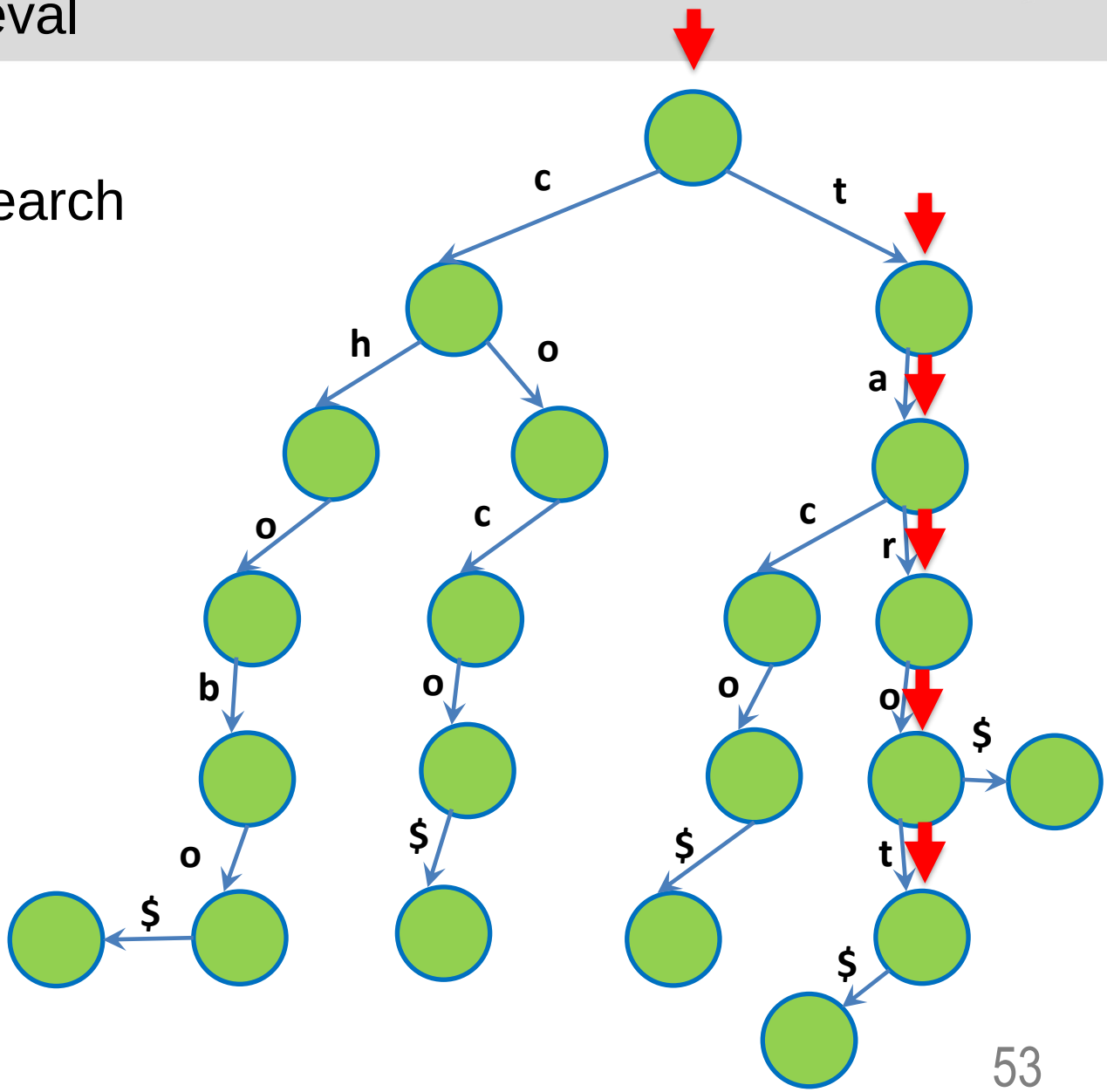

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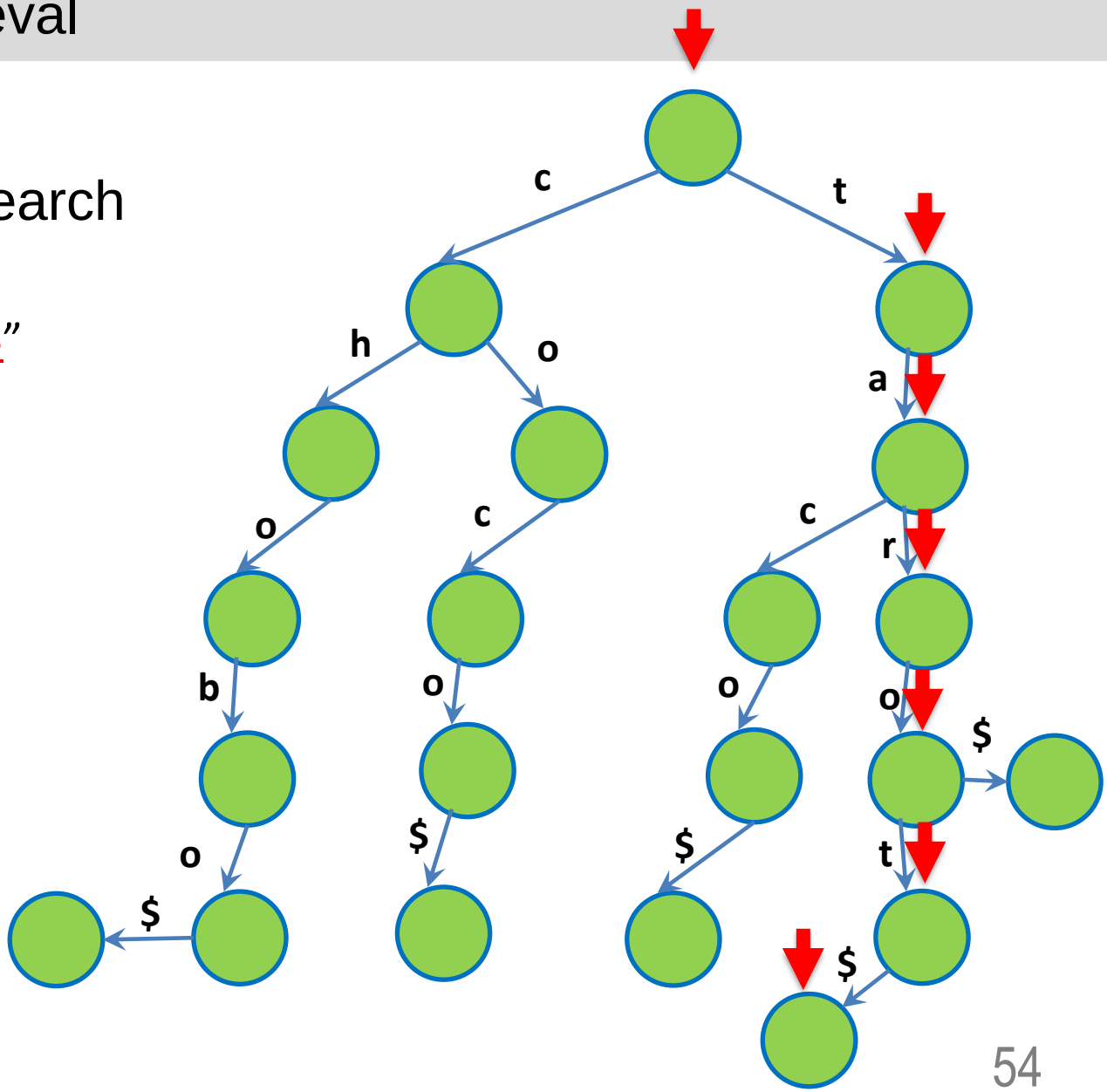
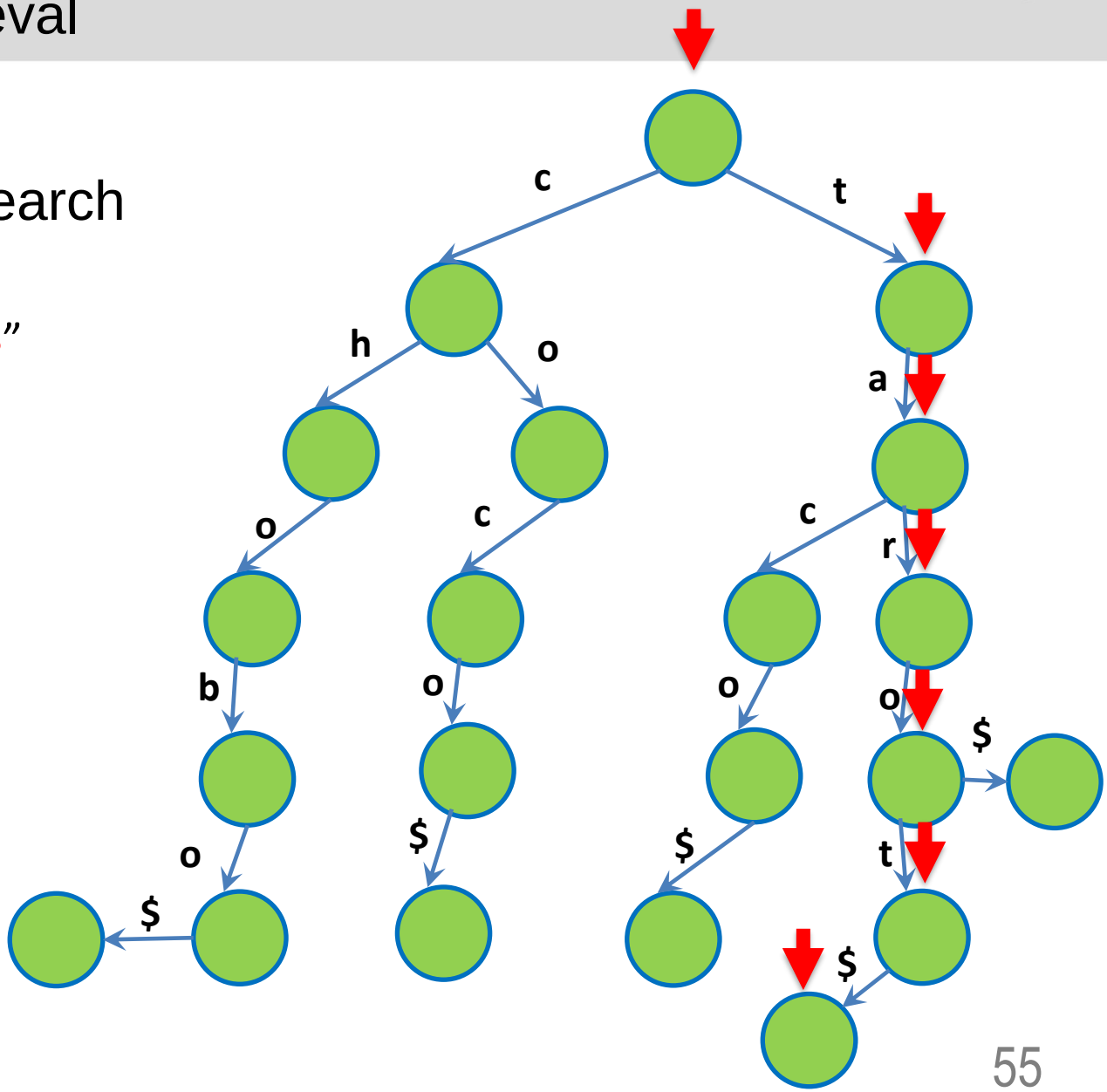


Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Search for “tarot$”
    Found!



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Questions?

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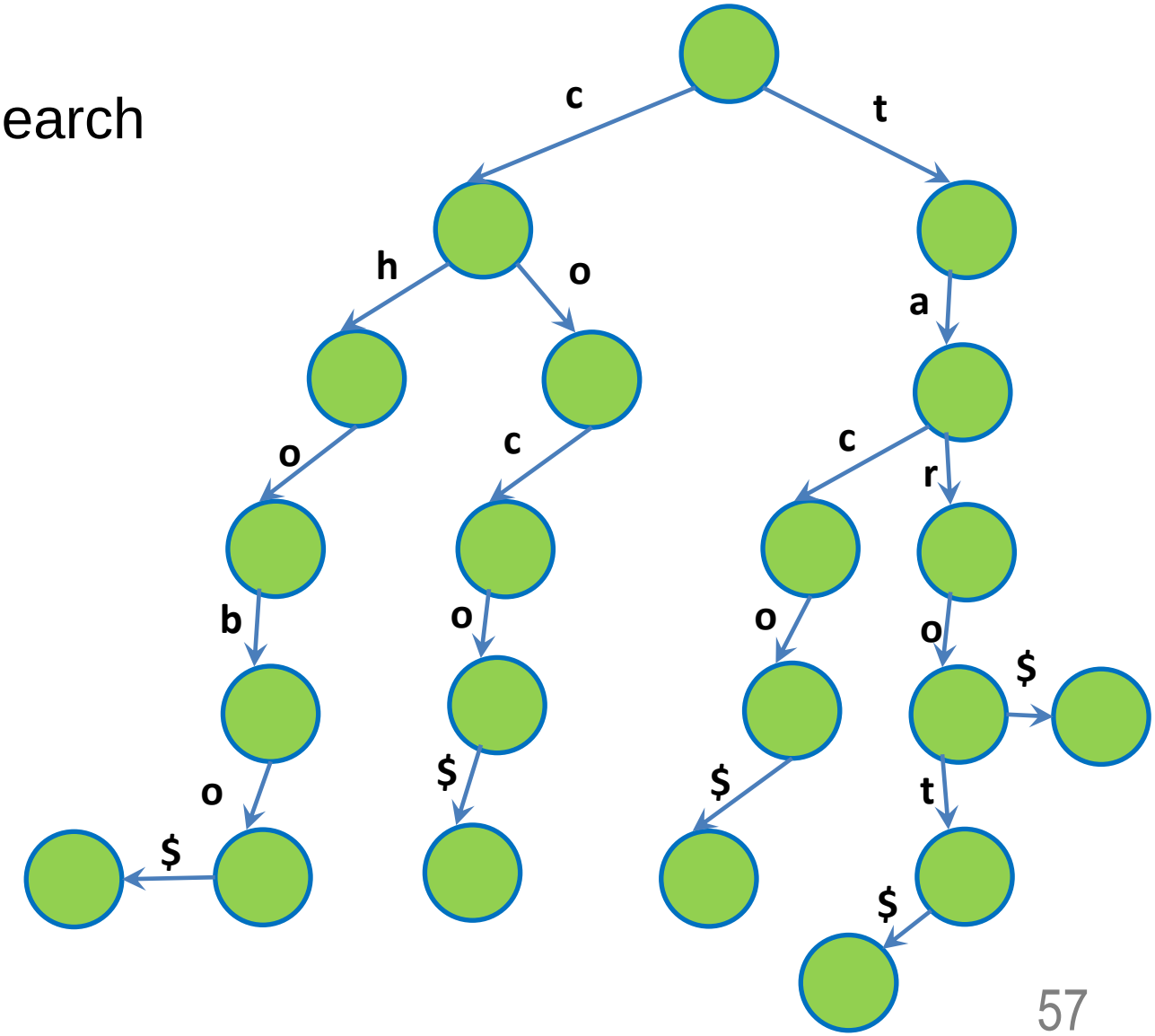
Efficient string retrieval

- So how do we search for retrieval?
  - Search for “cow”



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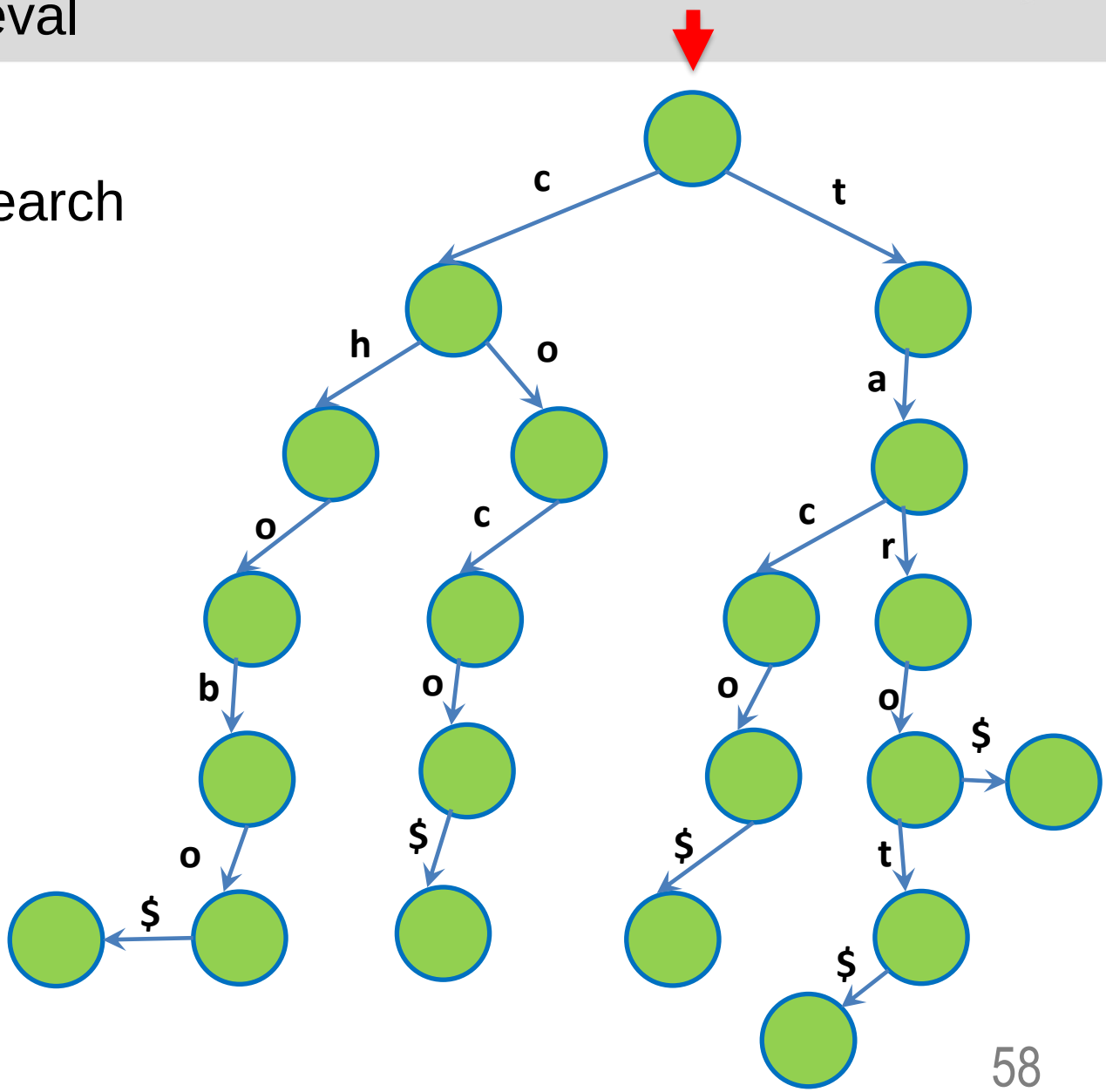

Tries

Efficient string retrieval

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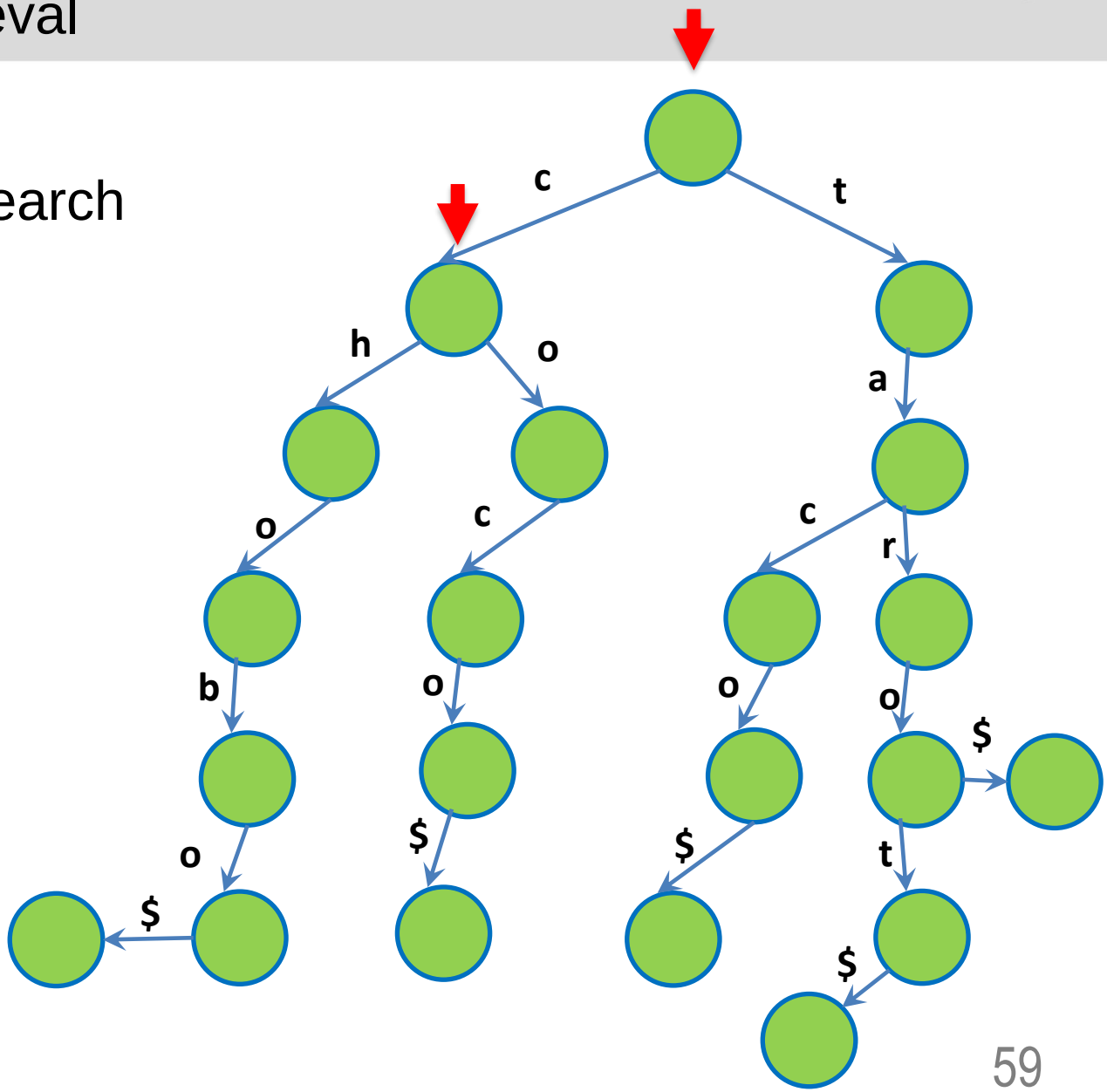

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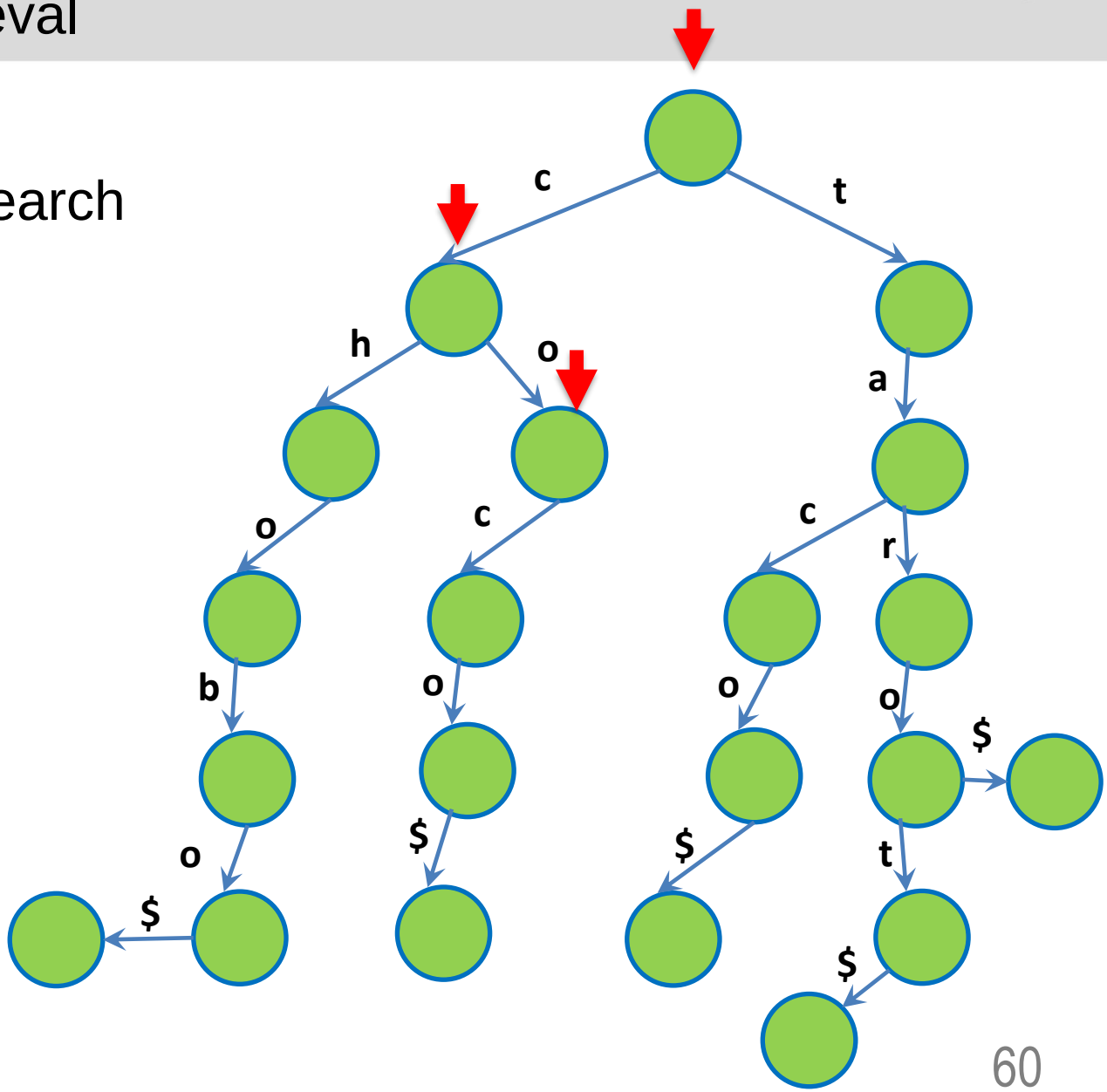

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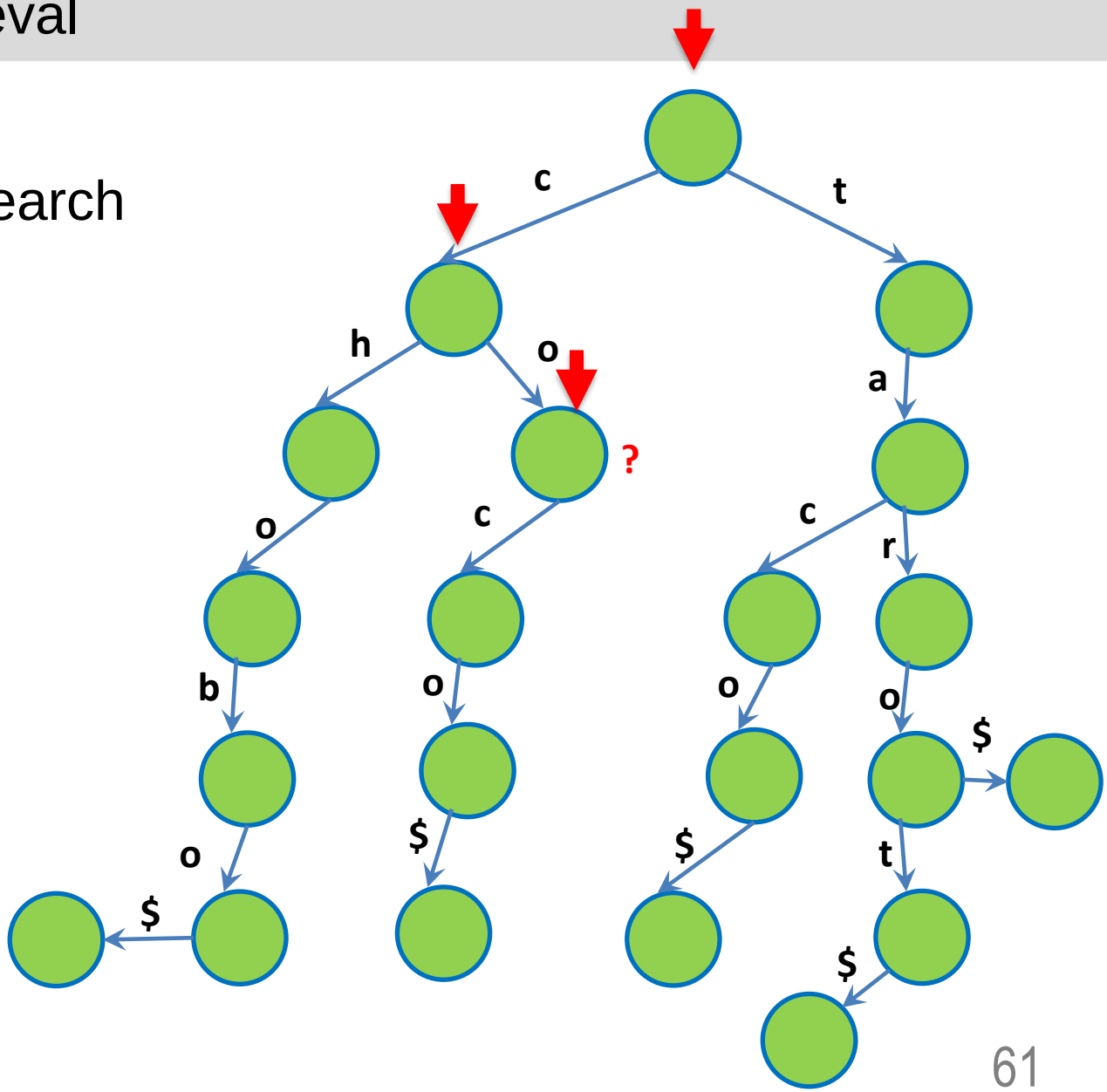

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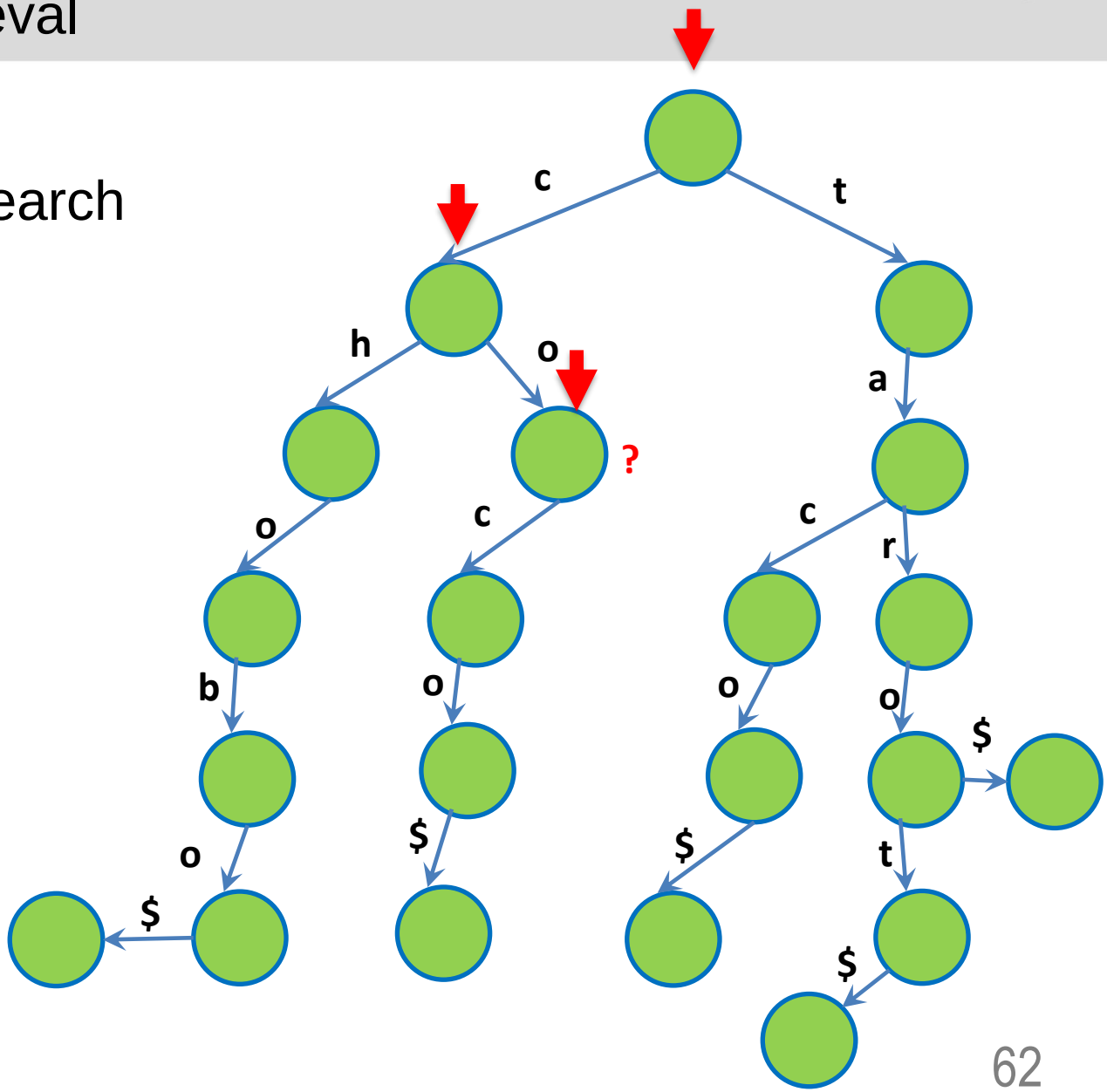

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Questions?

Tries

Efficient string retrieval

- So how do we search for retrieval?
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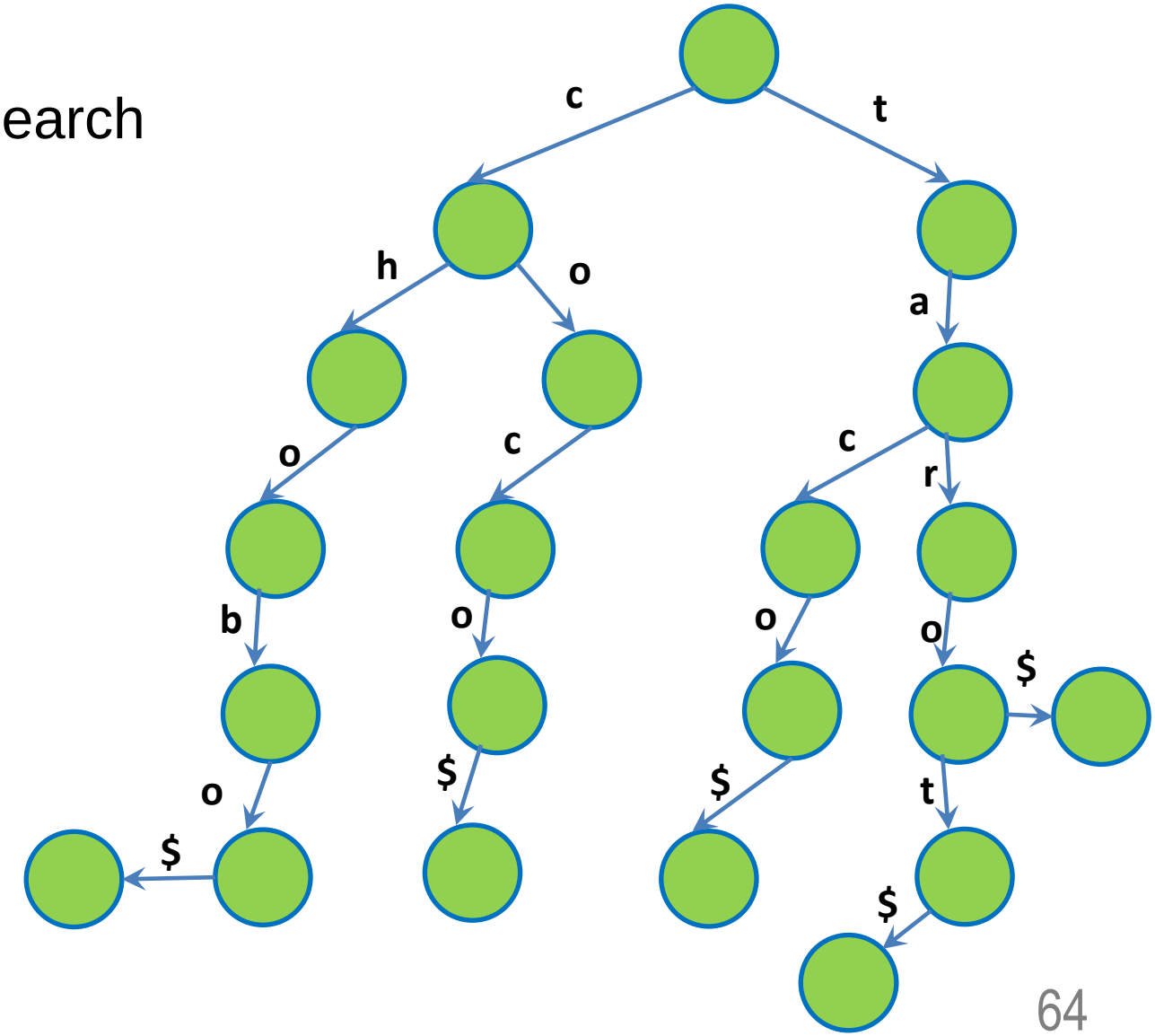

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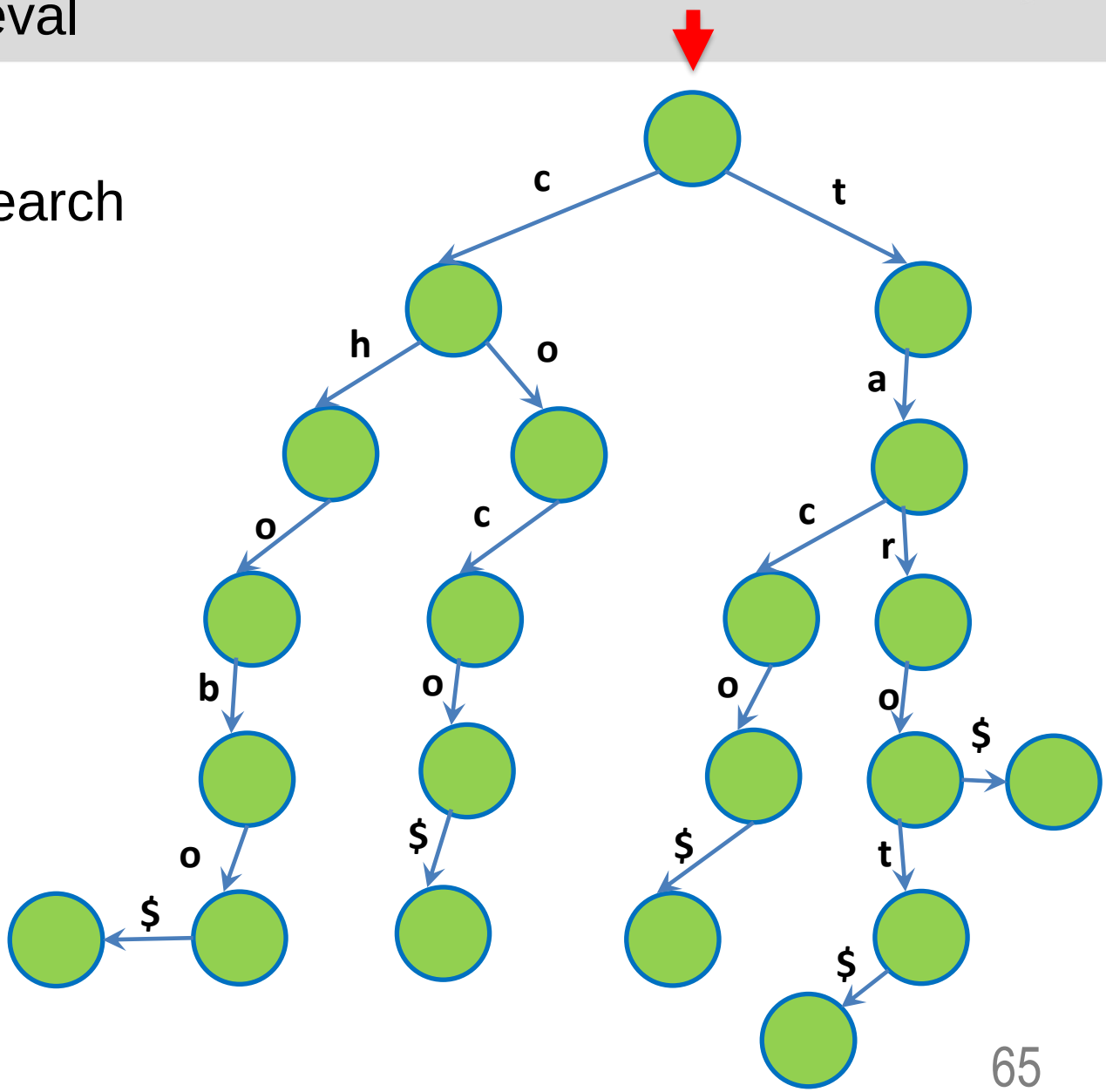

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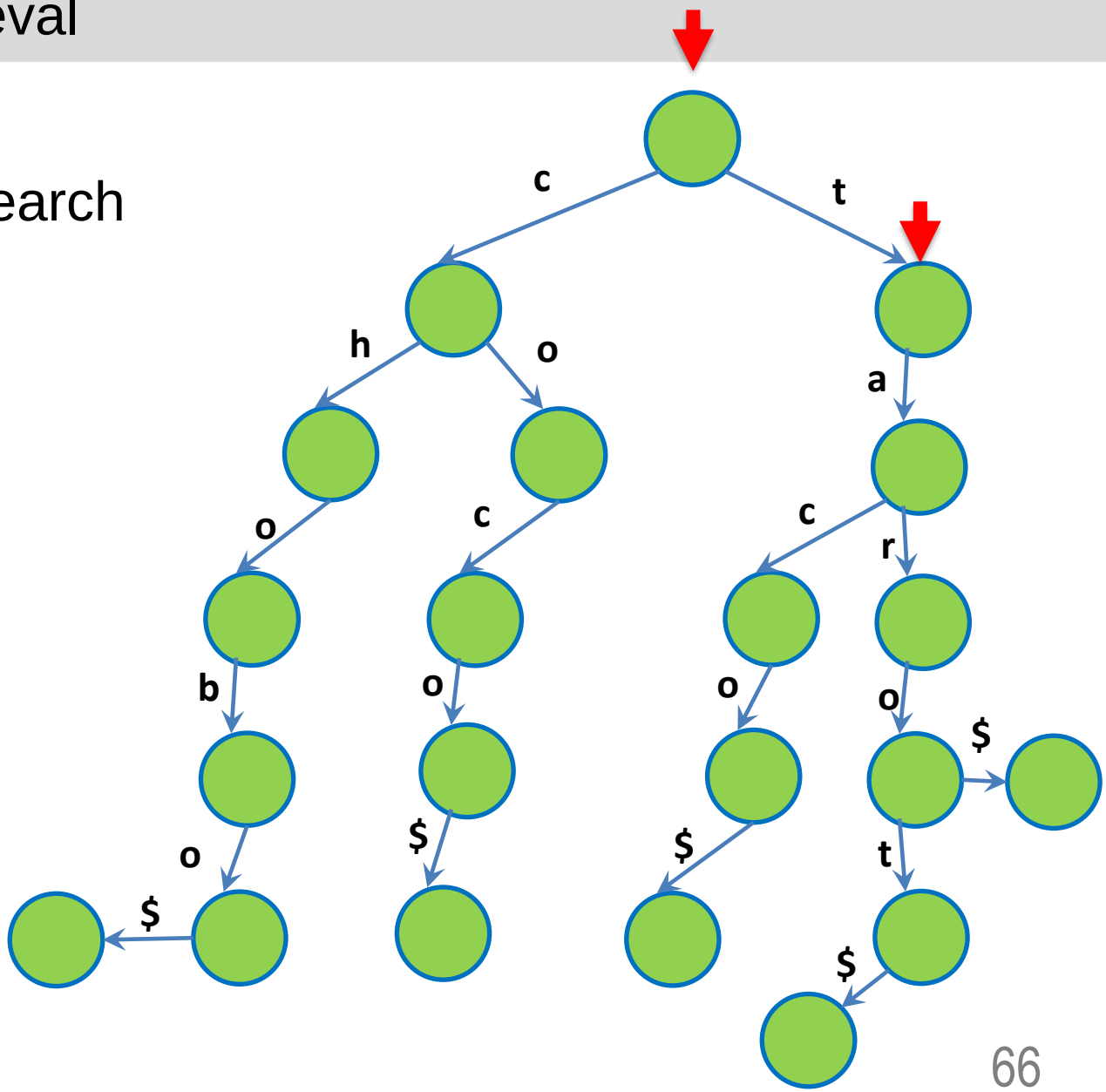

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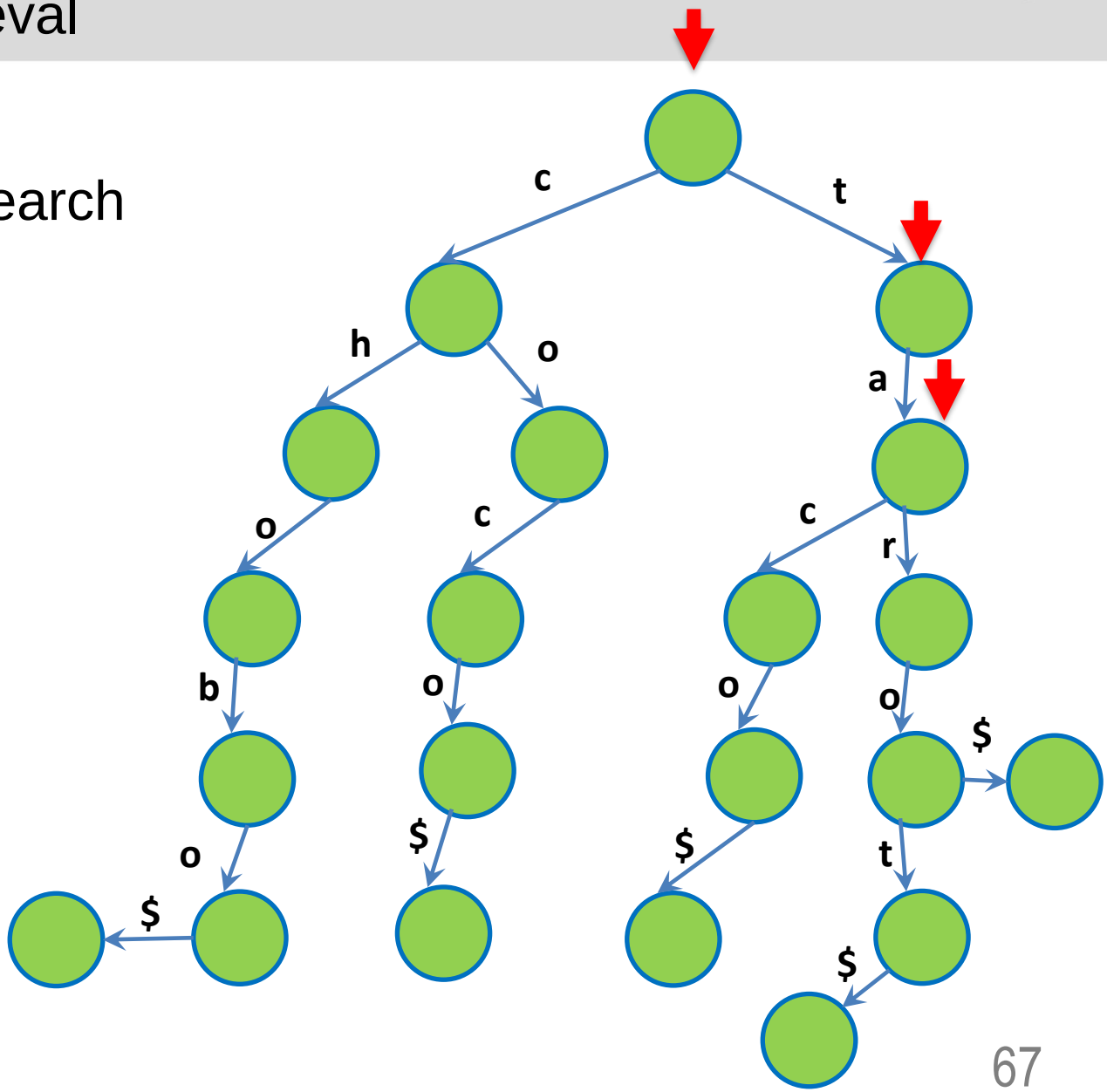

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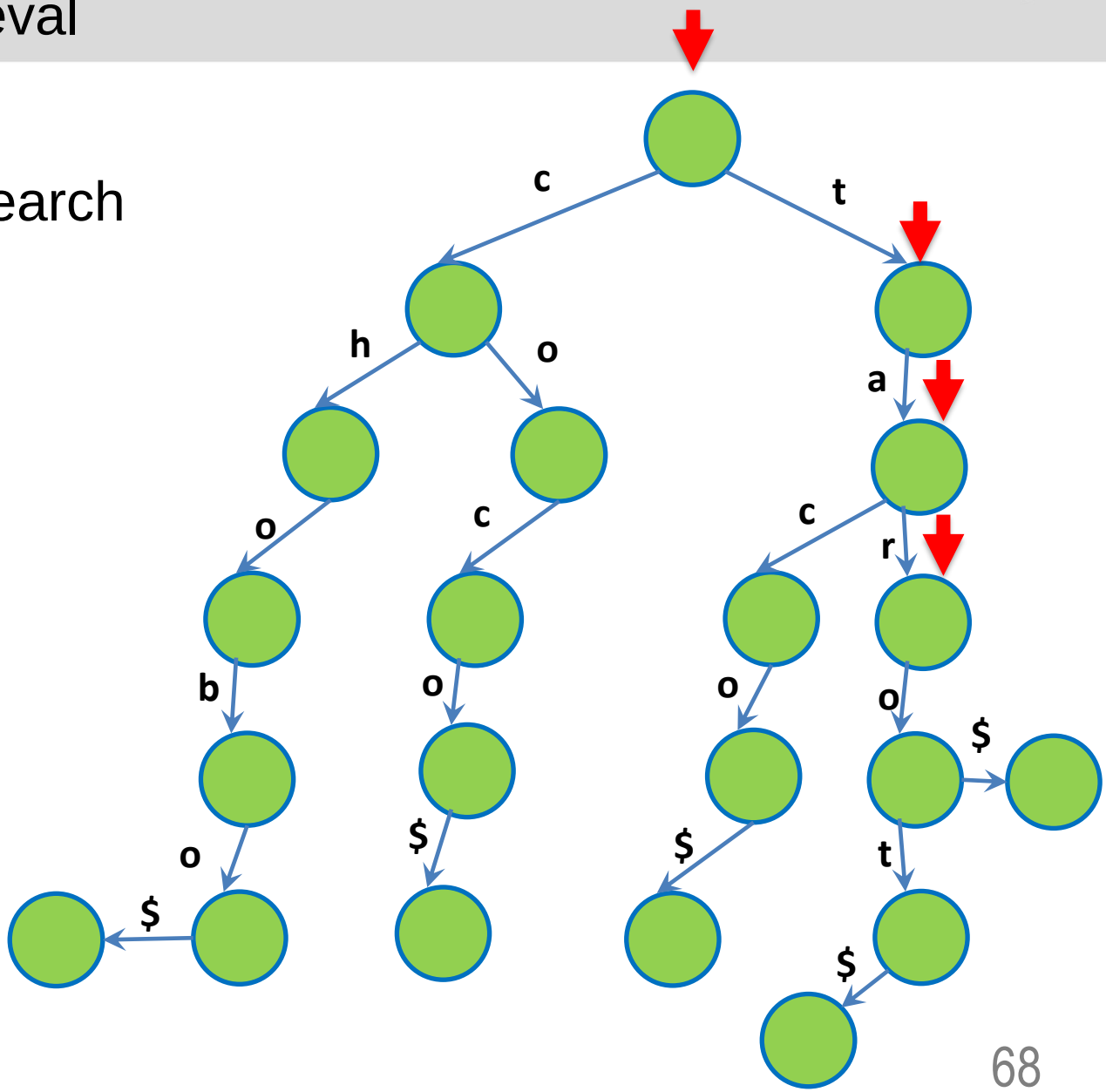

Tries

Efficient string retrieval

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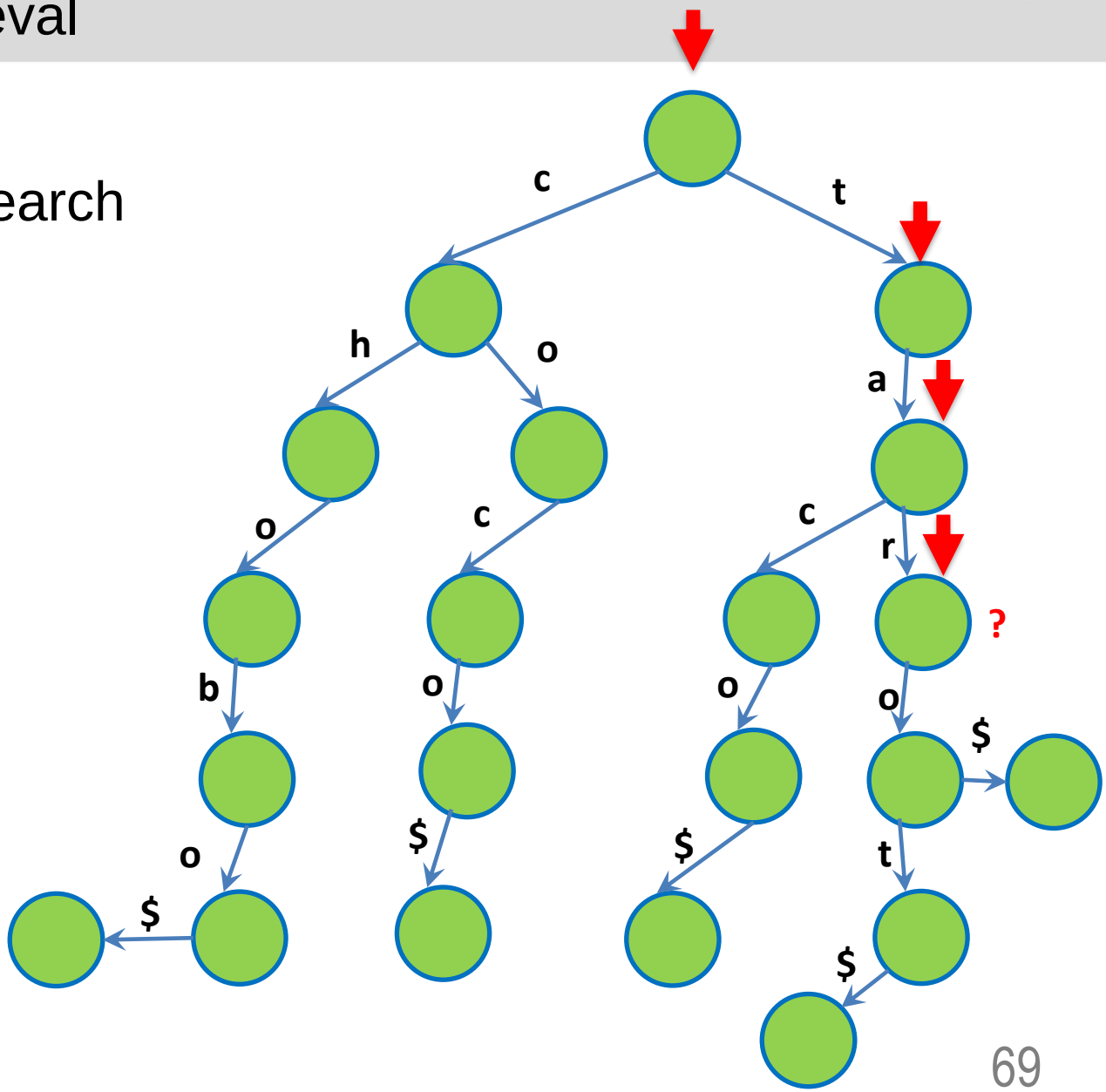

Tries

Efficient string retrieval

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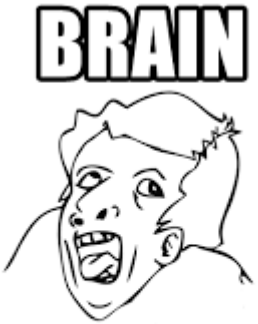
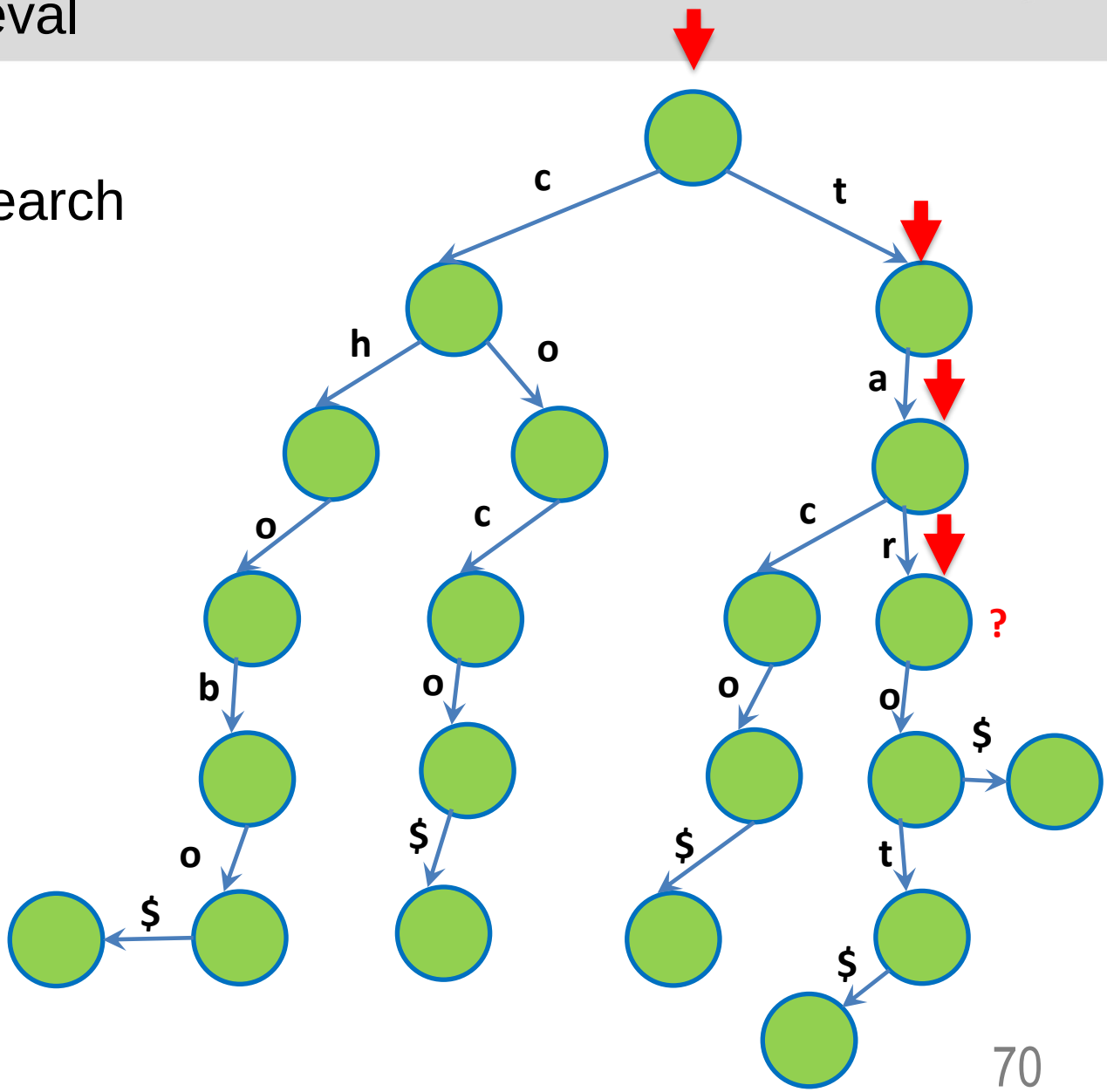
    Not found =(

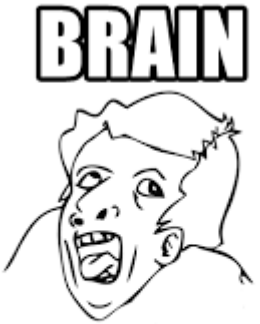


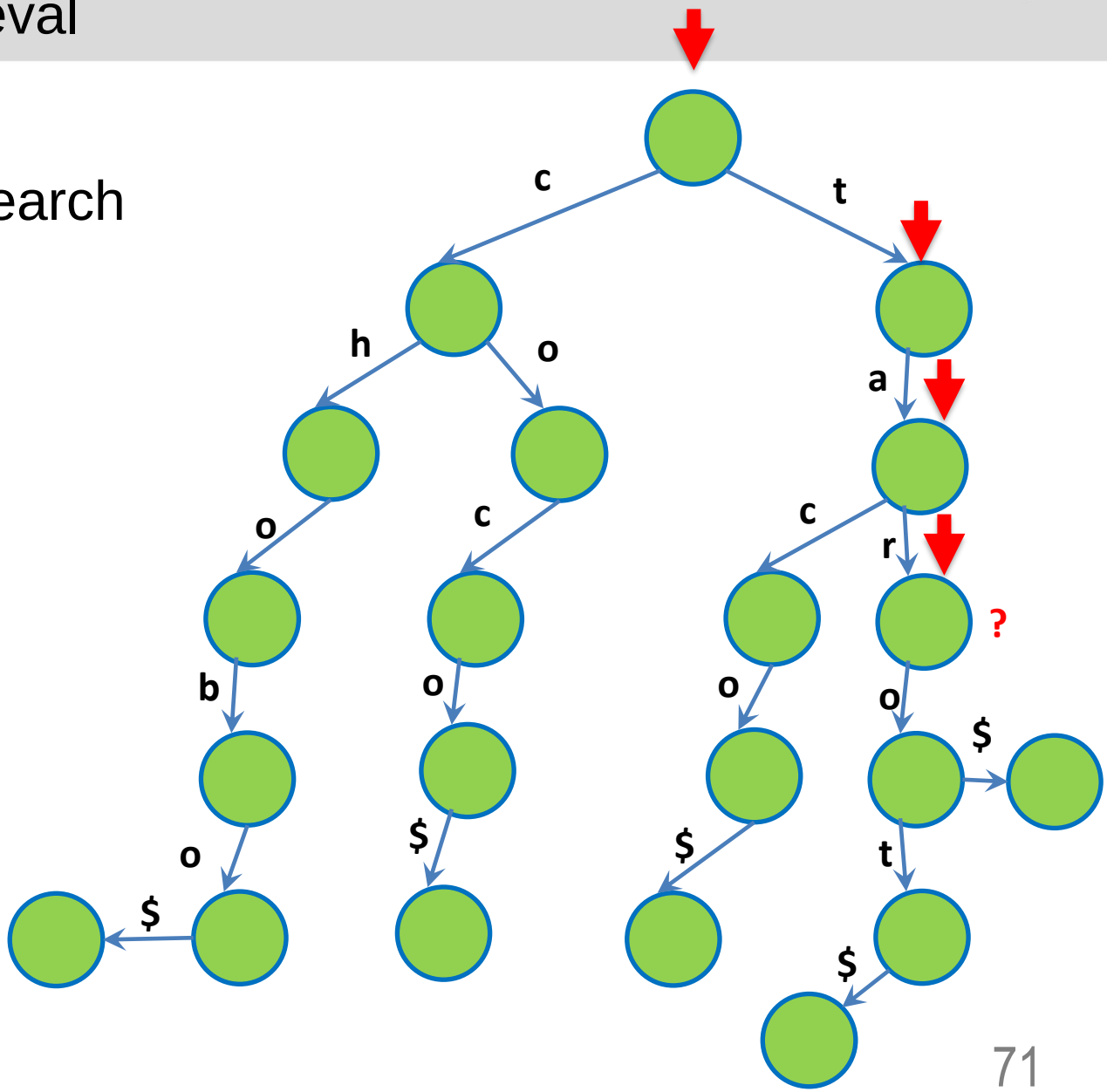


Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Search for “tar$”
    Not found =(
    Need those $$$





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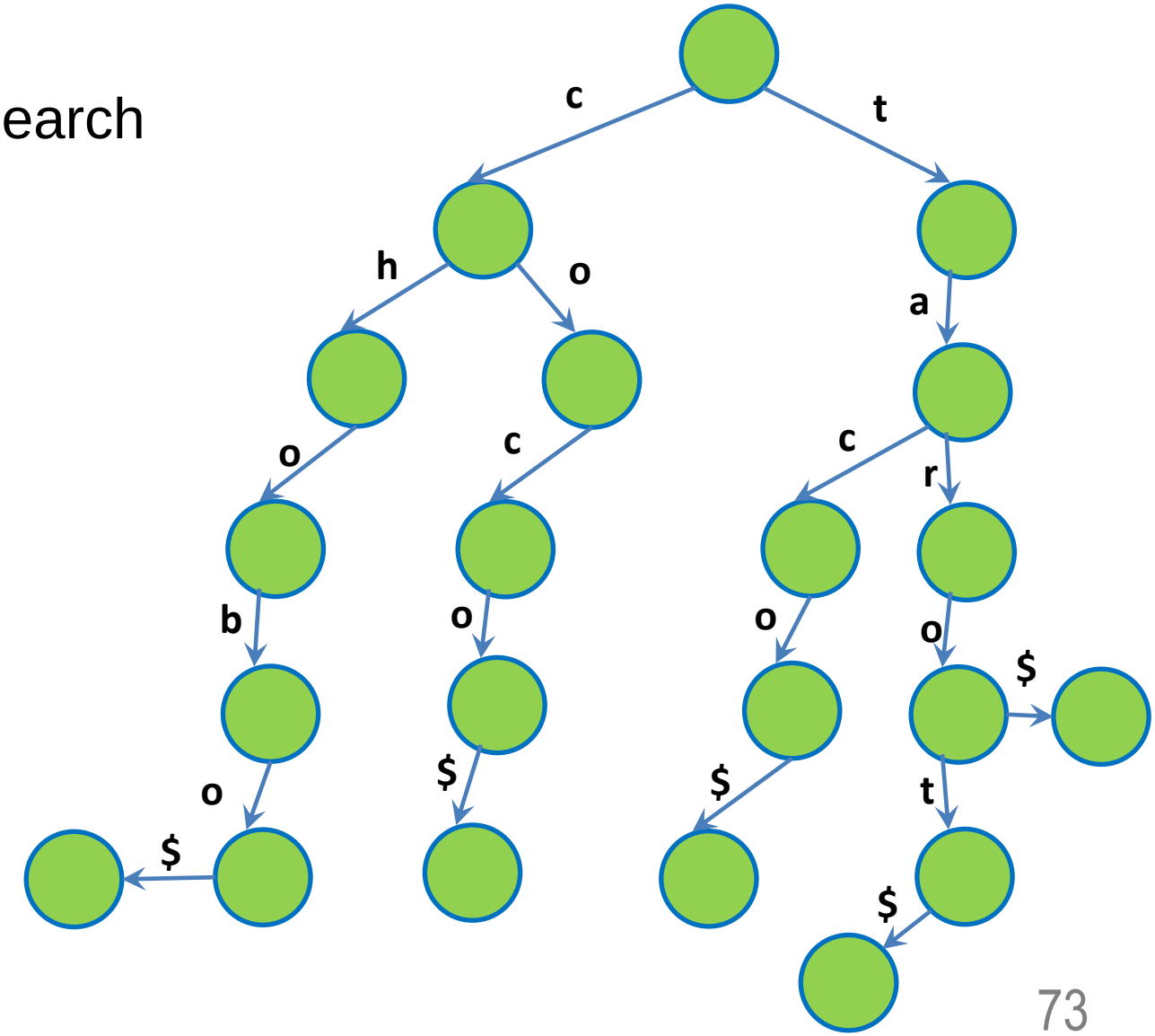
Questions?

Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Complexity?



Tries

Efficient string retrieval

- So how do we search for retrieval?
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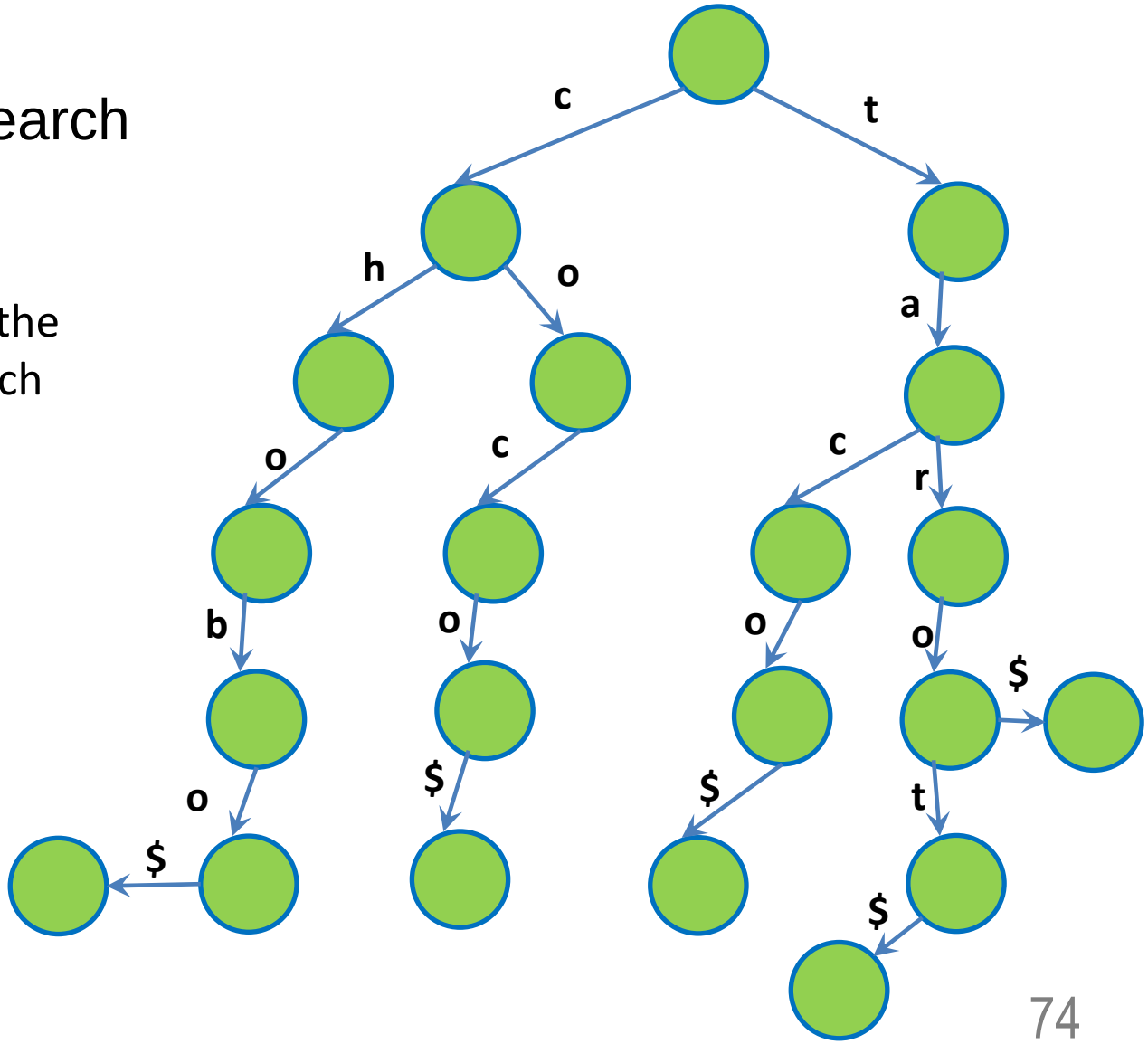
    O(M) where M is the length of the search string…



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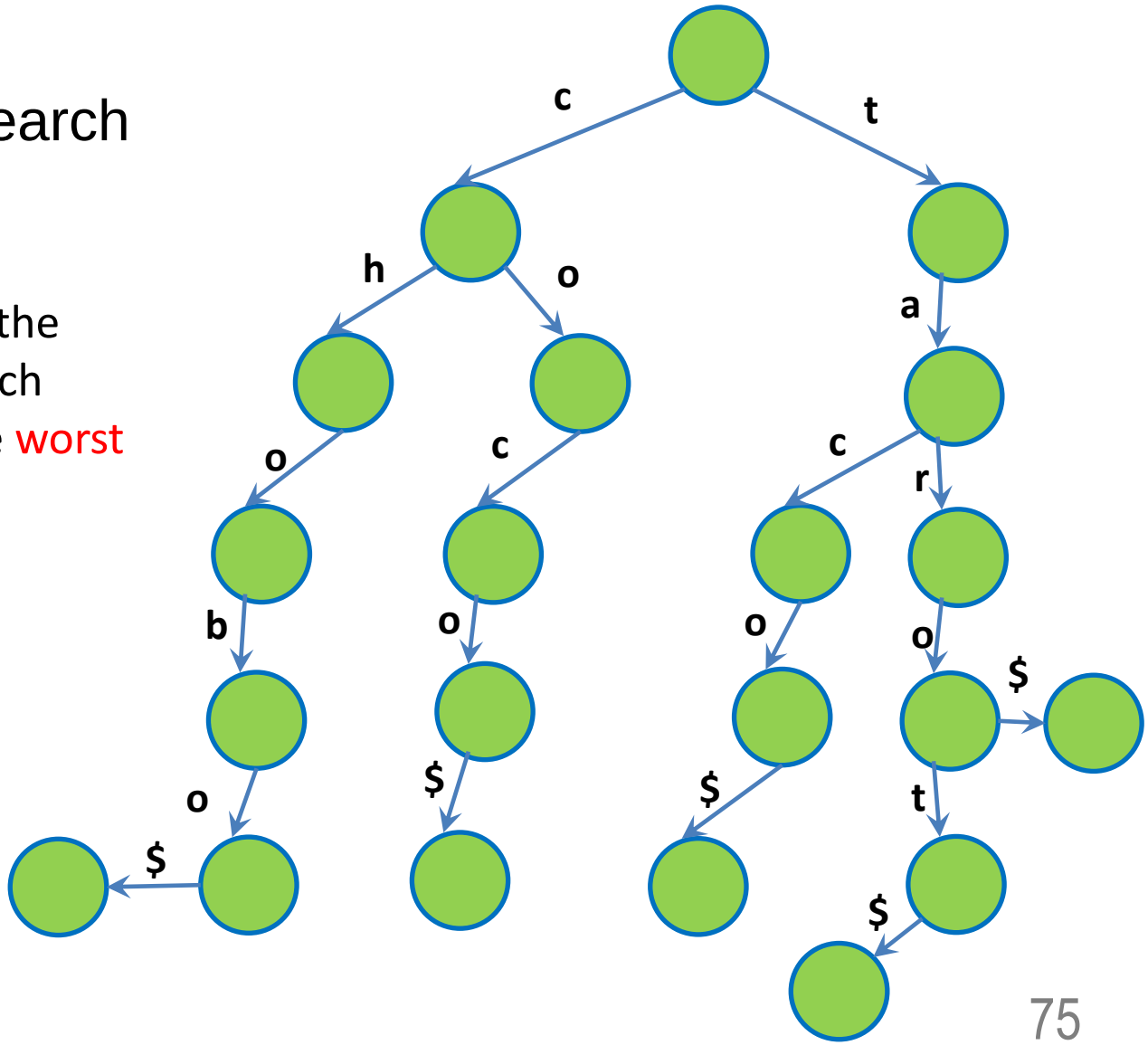
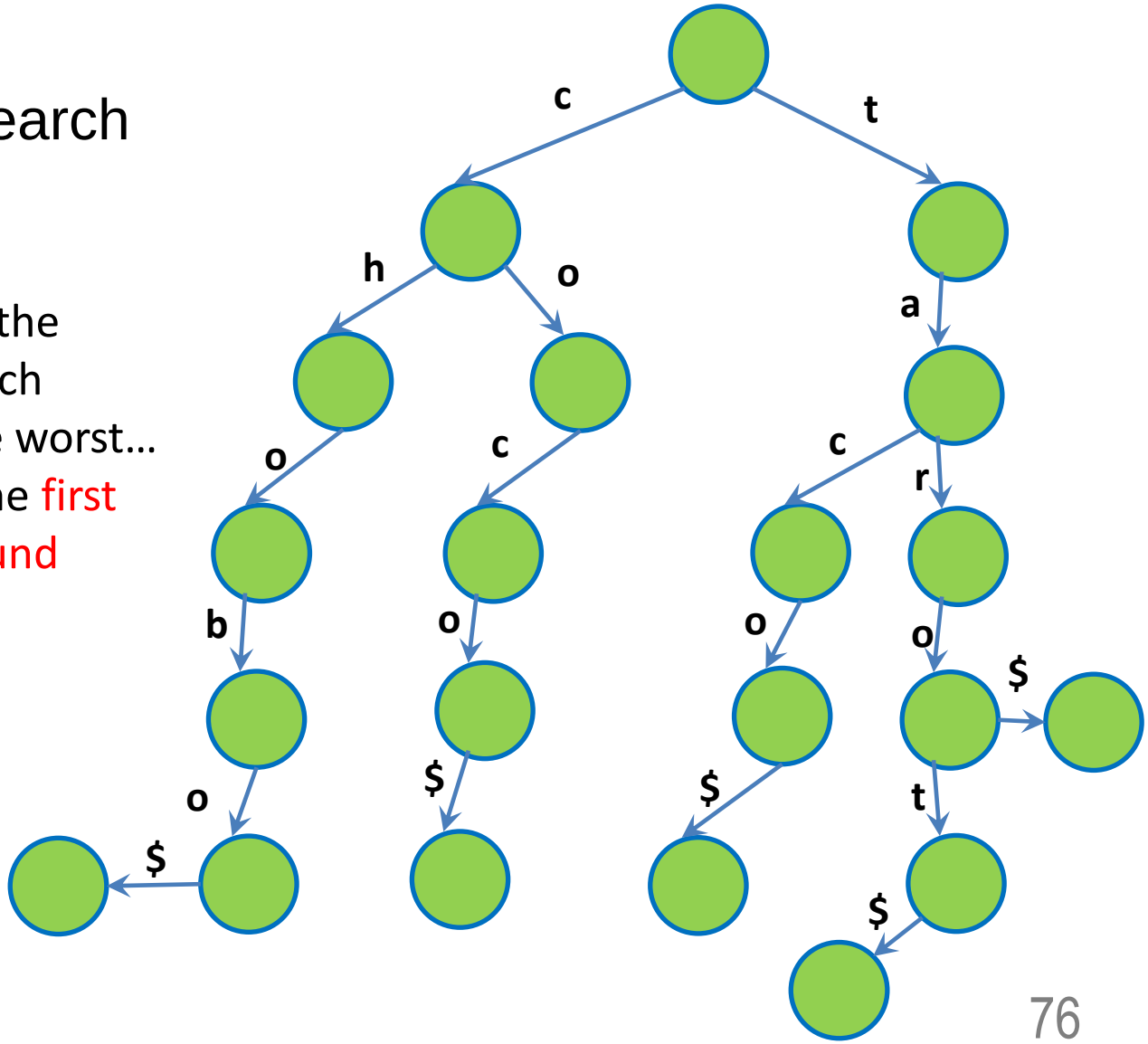


Tries

Efficient string retrieval

- So how do we search for retrieval?
  - Complexity?
    O(M) where M is the length of the search string… This is the worst…
    O(1) best when the first character isn't found



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Questions?

Tries

Efficient string retrieval

- How to implement it?

Tries

Efficient string retrieval

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- How to implement it? With OOP!

Tries

Efficient string retrieval

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- How to implement it? With OOP!
  - Node class

```
1  class Trie:
2      def __init__(self):
3          self.root = Node()
4
5  class Node:
6      def __init__(self, data=None):
7          self.data = data
8          self.links = [None] * 27
```

Tries

Efficient string retrieval

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- How to implement it? With OOP!
  - Node class

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5  class Node:
6      def __init__(self, data=None):
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```

  - Then we need to code the traversal from the root
    - If a link exist, travel through it
      This is O(1) due to the array data structure

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Questions?

Tries

Efficient string retrieval

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- Benefits?

Tries

Efficient string retrieval

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- Benefits?
  - Better for string search than BST/ AVL
  - More versatile than hash table

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    - The edges/ links are in-order (from a to z)
    - This is O(MN)

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  - Search is O(M), where M is length of string
  - Can sort very quickly by traversing the string
    - The edges/ links are in-order (from a to z)
    - This is O(MN)
- Disadvantage?
  - At times can be slower than hash table
  - Wasted space if the self.link array is left empty most of the time

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Questions?

Tries

Usage?

- Height of the trie = length of the longest string

Tries

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Tries

Usage?

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- Height of the trie = length of the longest string
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- We can search for the prefix of strings!
  - Useful for auto correct/ auto complete
  - And many other applications!

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Questions?

Suffix Trie

For suffixes

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- Same as a trie
- But for suffixes

Suffix Trie

For suffixes

- Can you make a suffix trie for apple?

Suffix Trie

For suffixes

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- Can you make a suffix trie for apple?
- List all the suffixes
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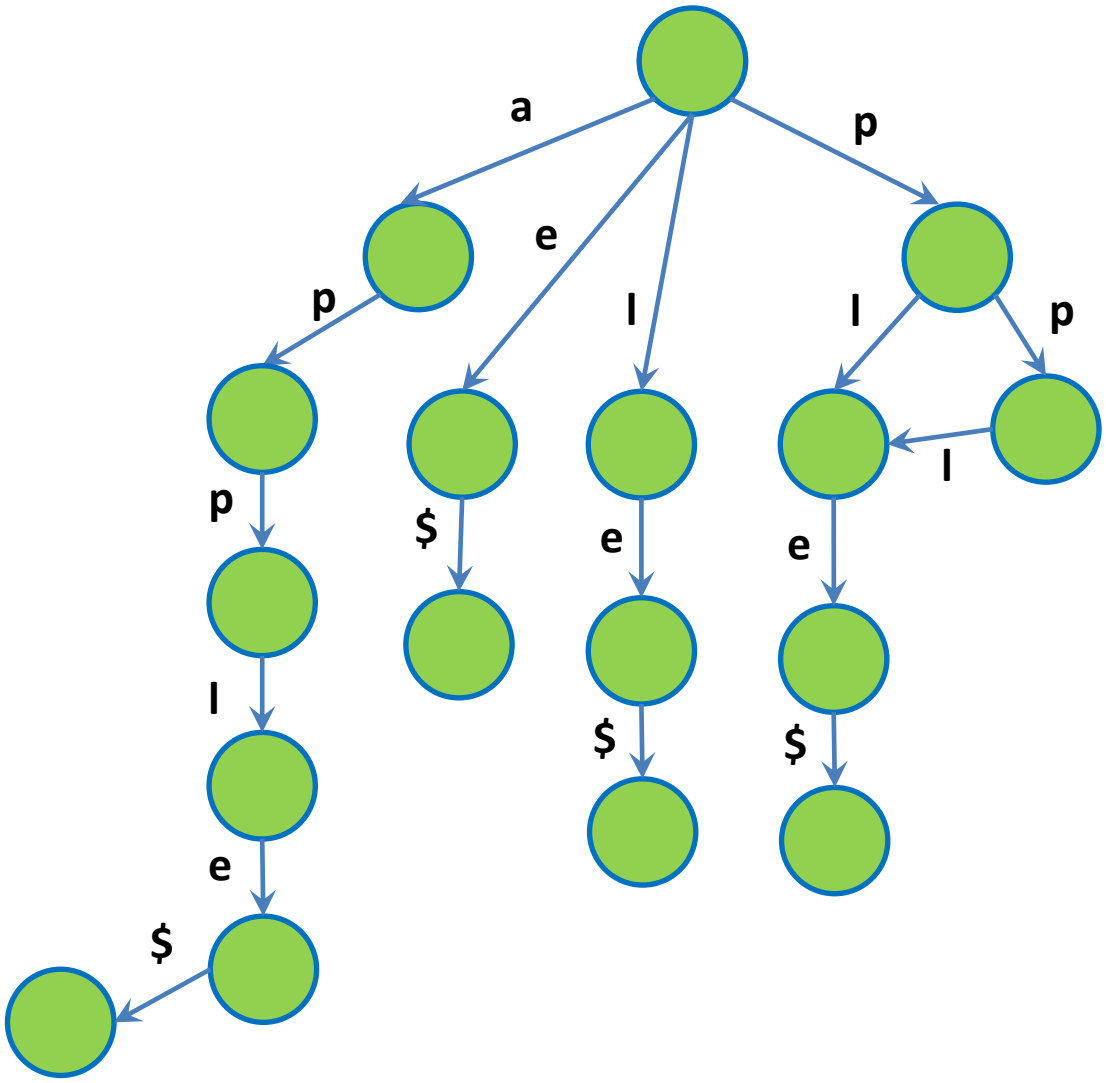

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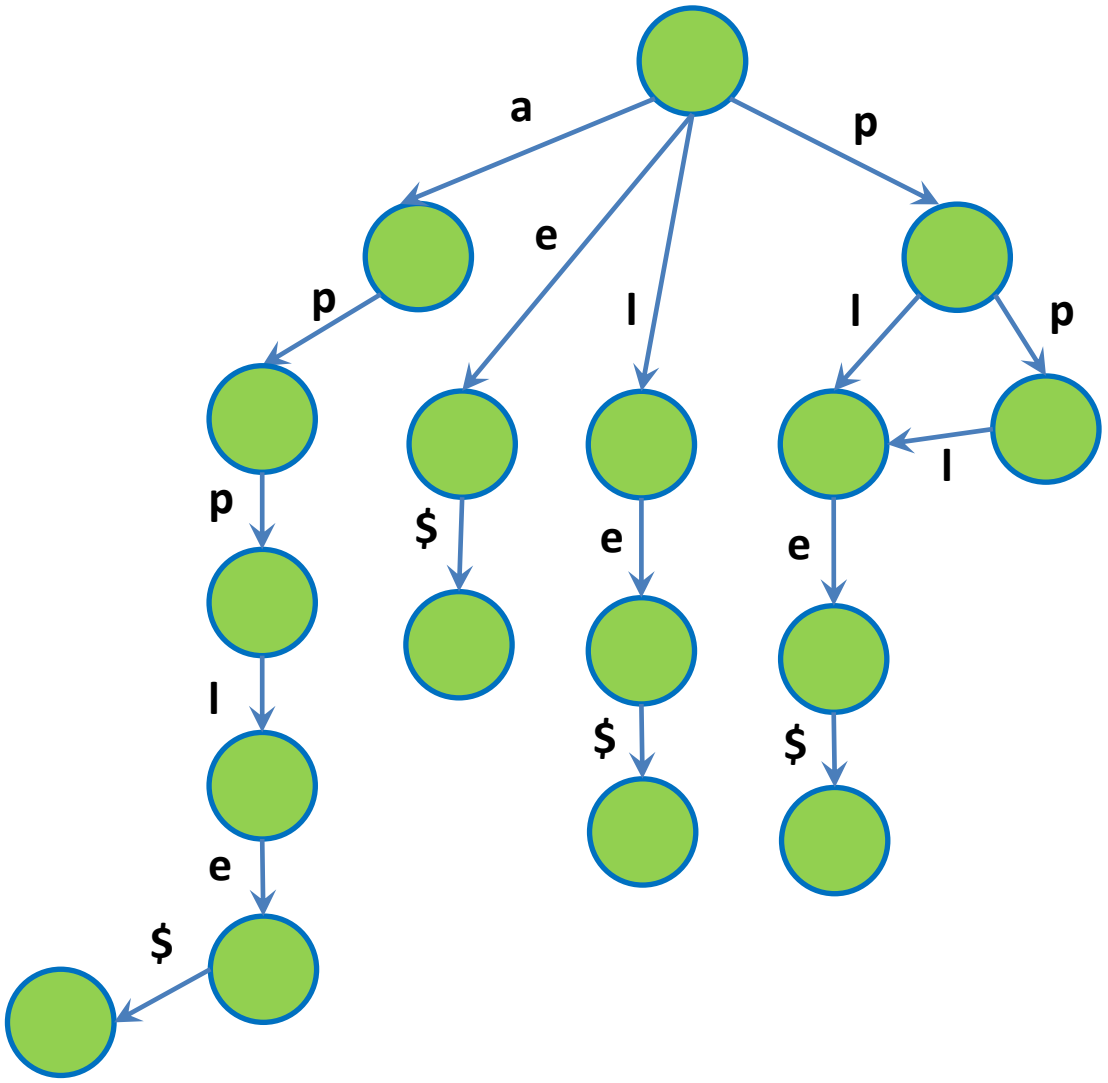
  NO! CYCLE!
  so this is wrong…



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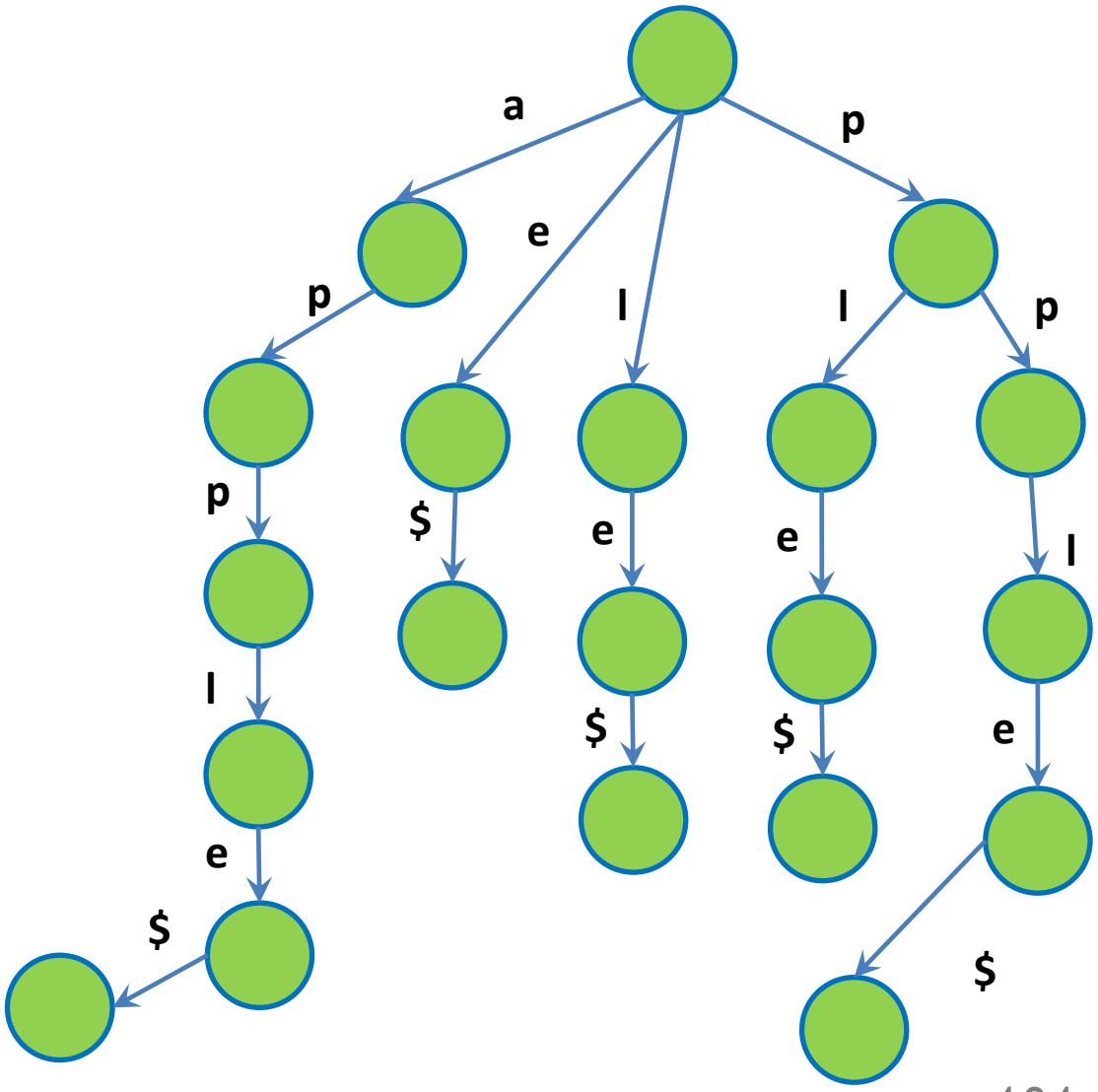

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Questions?

Suffix Trie

Applications?

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- But more goodies now!

Suffix Trie

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- And many more…

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Questions?

Suffix Trie

Applications?

- Space complexity?

Suffix Trie

Applications?

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  - O(N^2)

Suffix Trie

Applications?

- Space complexity?
  - O(N^2)
  - N suffixes, longest suffix is N character

Suffix Trie

Applications?

- Space complexity?
  - O(N^2)
  - N suffixes, longest suffix is N character
  - Have N number of leaves!

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Questions?

Suffix Tree

A tree, not a trie

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- What is a suffix tree?

Suffix Tree

A tree, not a trie

MONASH University

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Suffix Tree

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MONASH University

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Suffix Trie

Suffix Tree

Suffix Tree

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Suffix Tree

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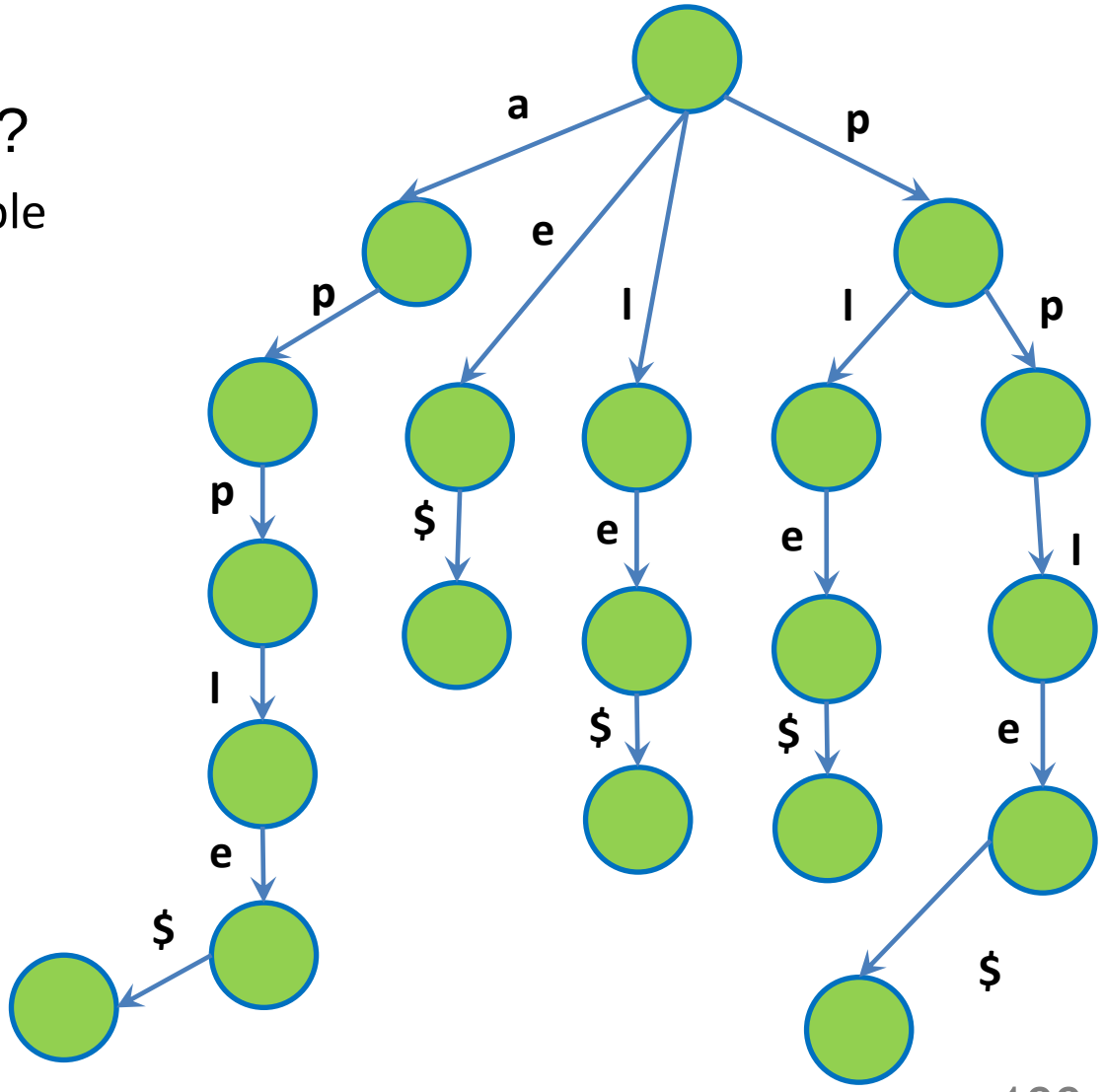

Suffix Tree

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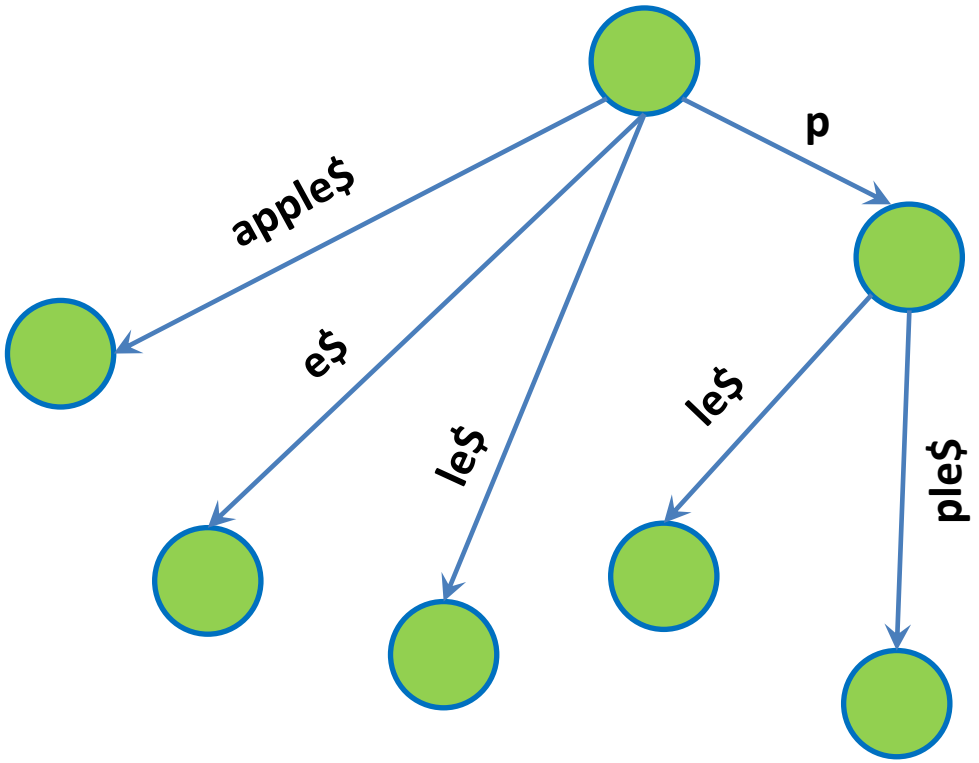

Suffix Tree

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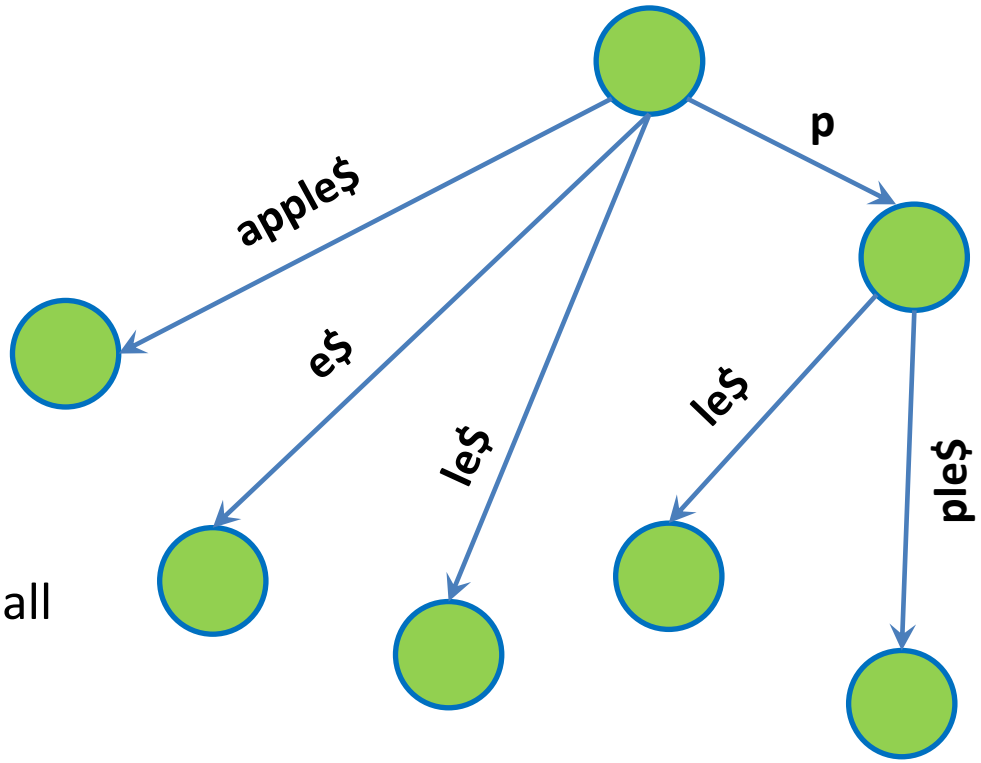

Suffix Tree

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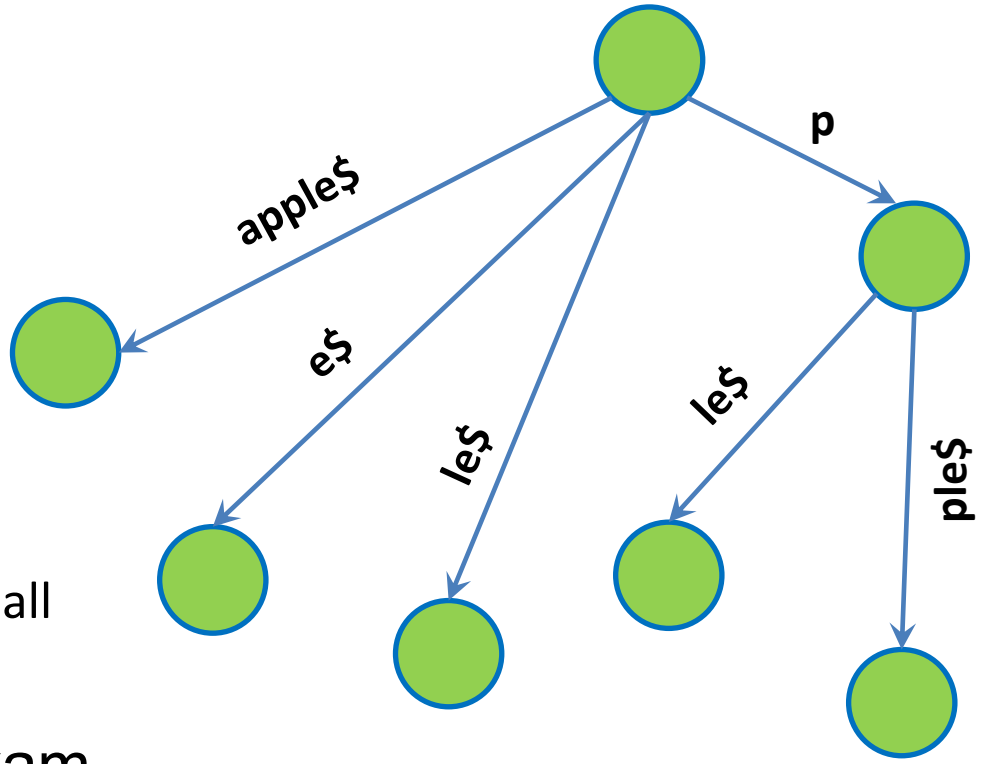
  - Then compress to suffix tree



Suffix Tree

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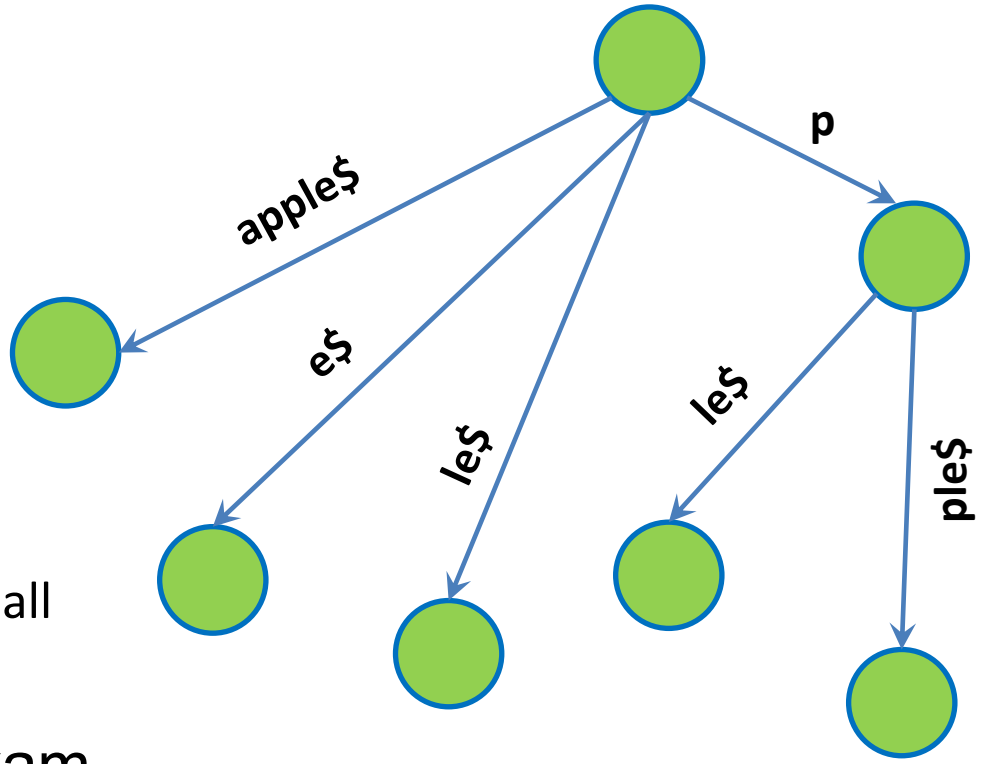
- Note: Some like to separate out the $ node



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Questions?

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- Space complexity O(N^2)

Suffix Tree

A tree, not a trie

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Suffix Tree

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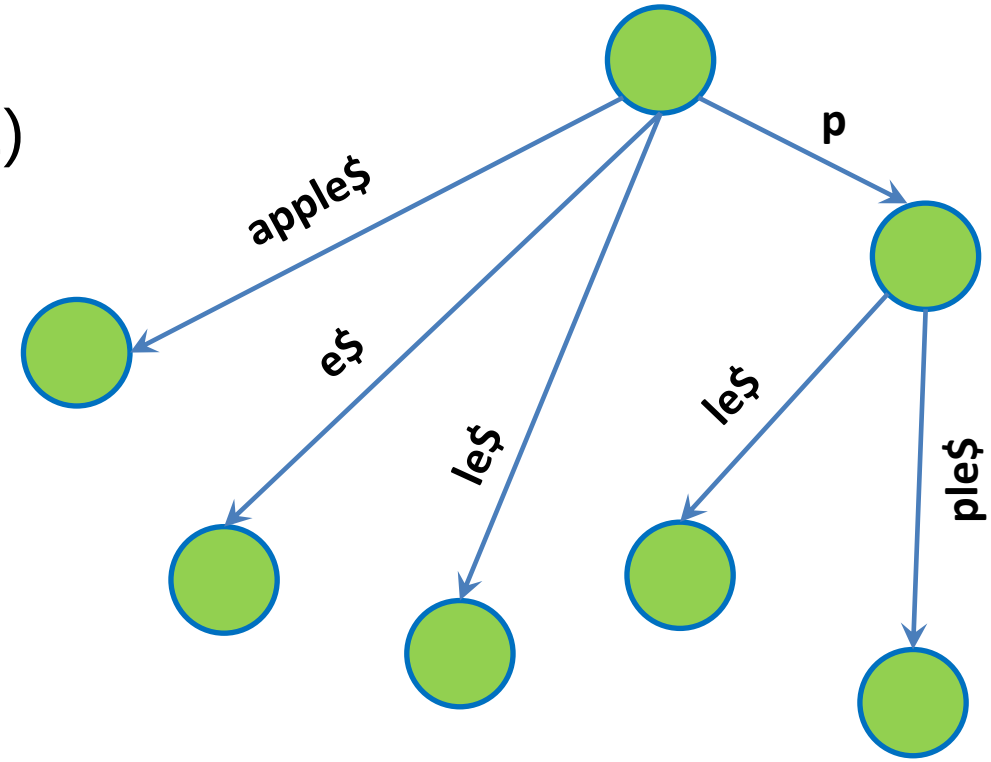

Suffix Tree

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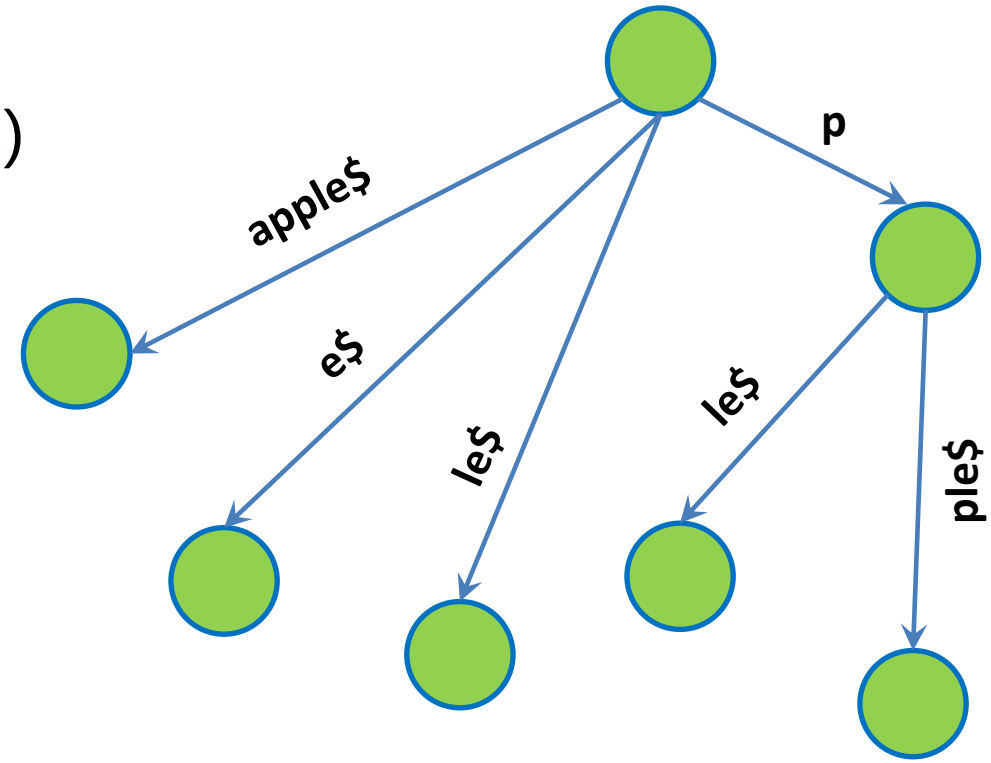
- Our string is apple$



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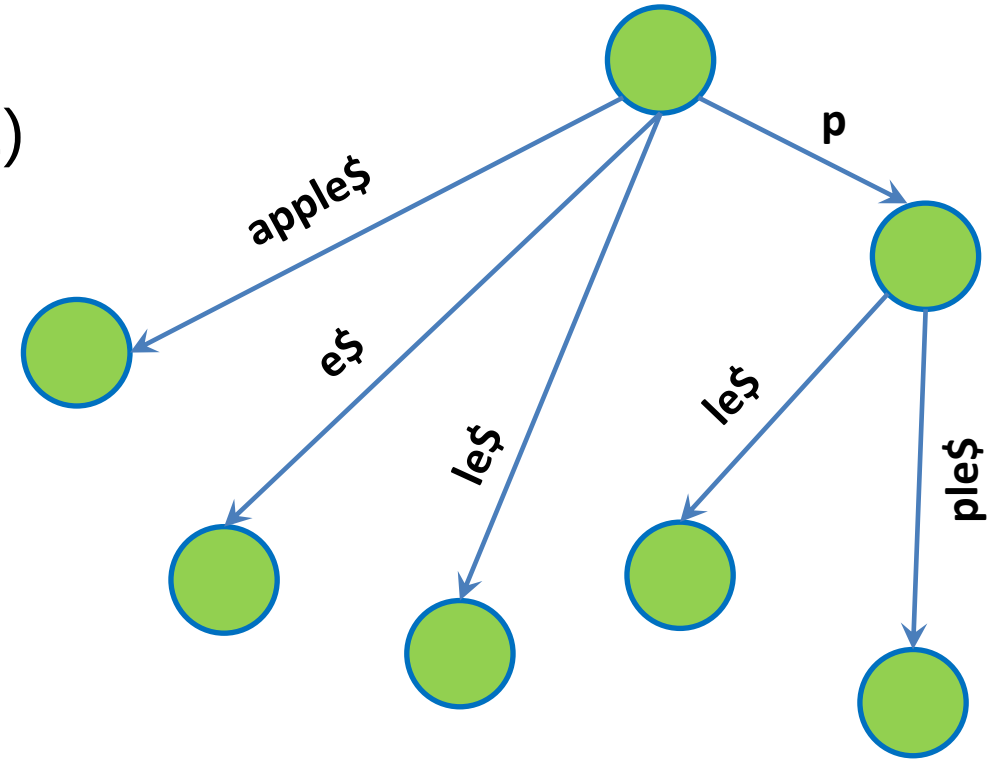

a	p	p	l	e	$
0	1	2	3	4	5

Suffix Tree

A tree, not a trie

- Space complexity O(N^2)
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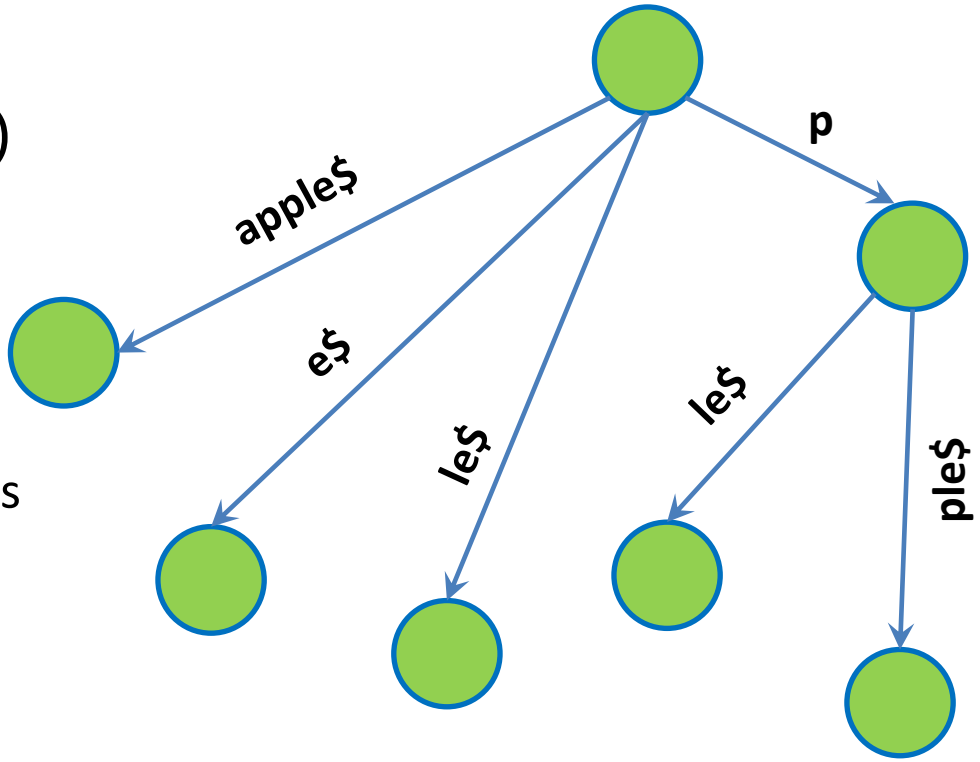

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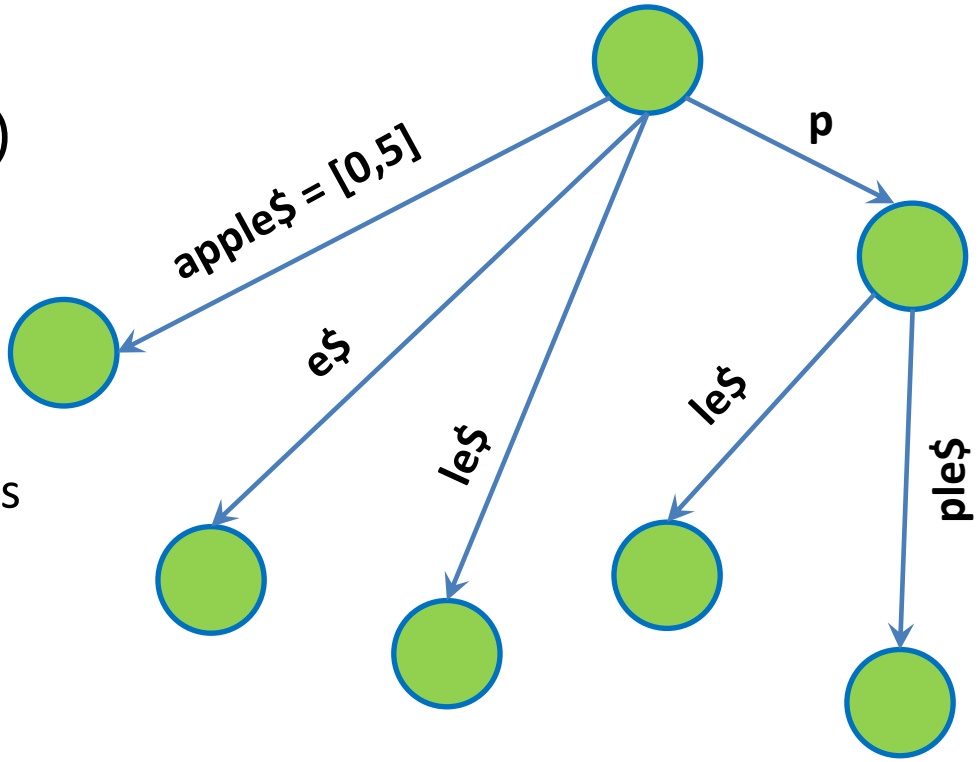

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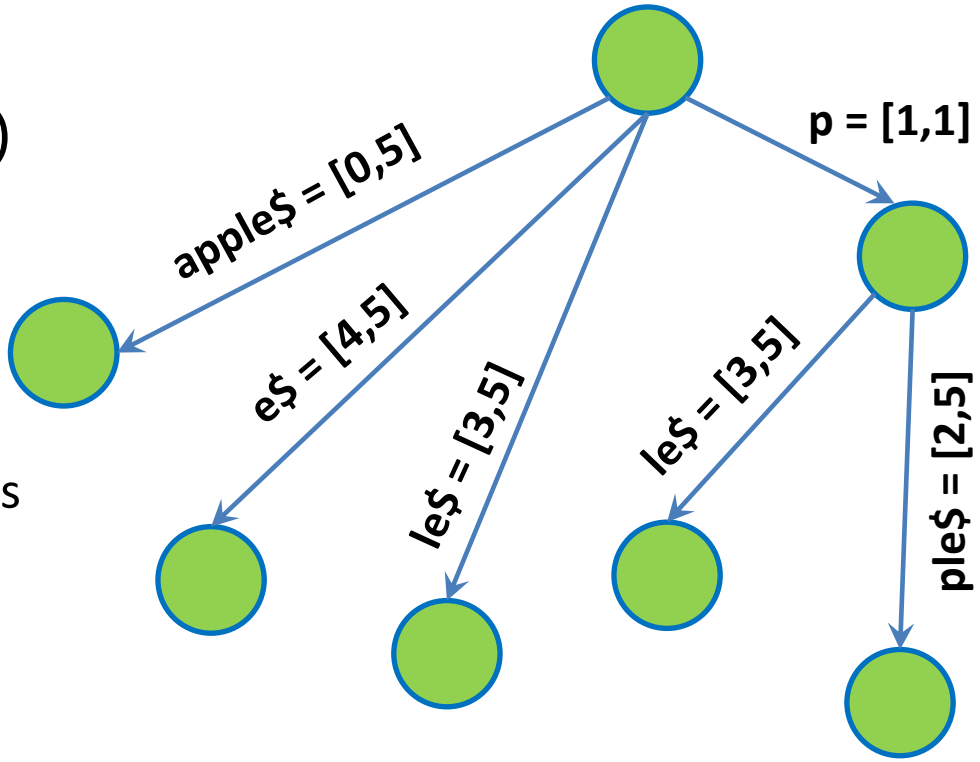

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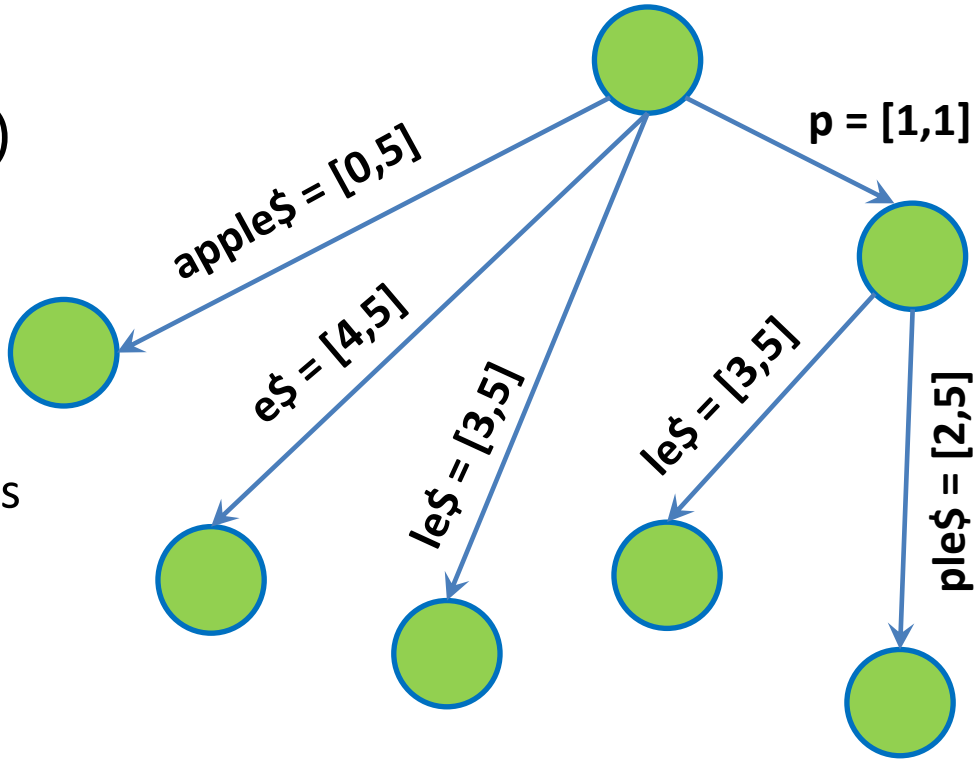

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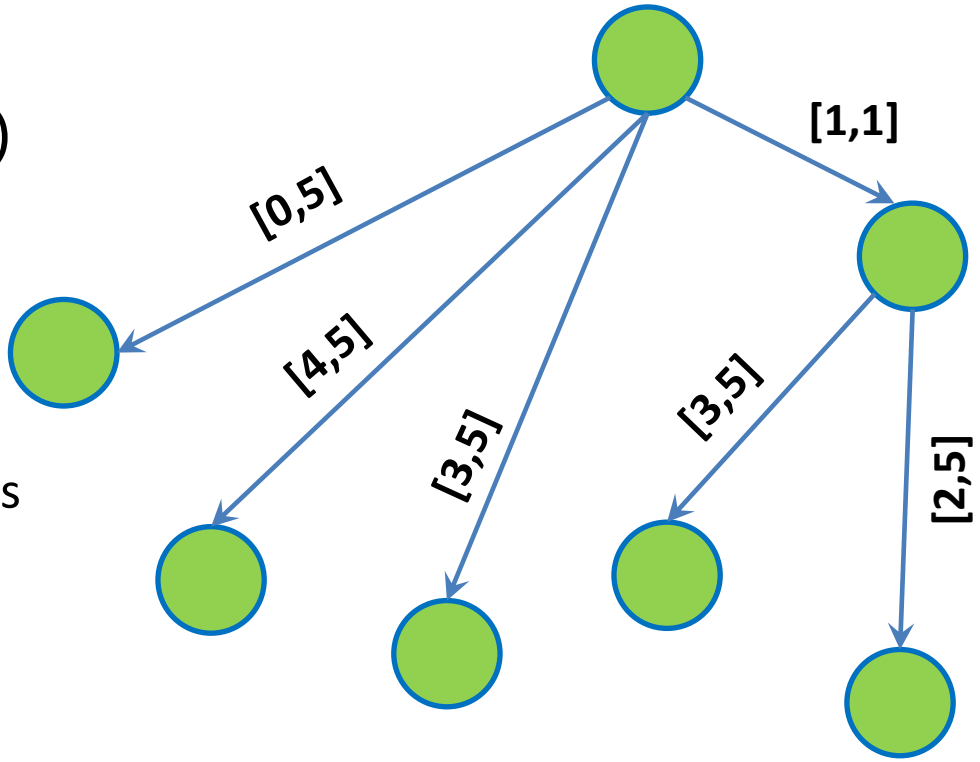

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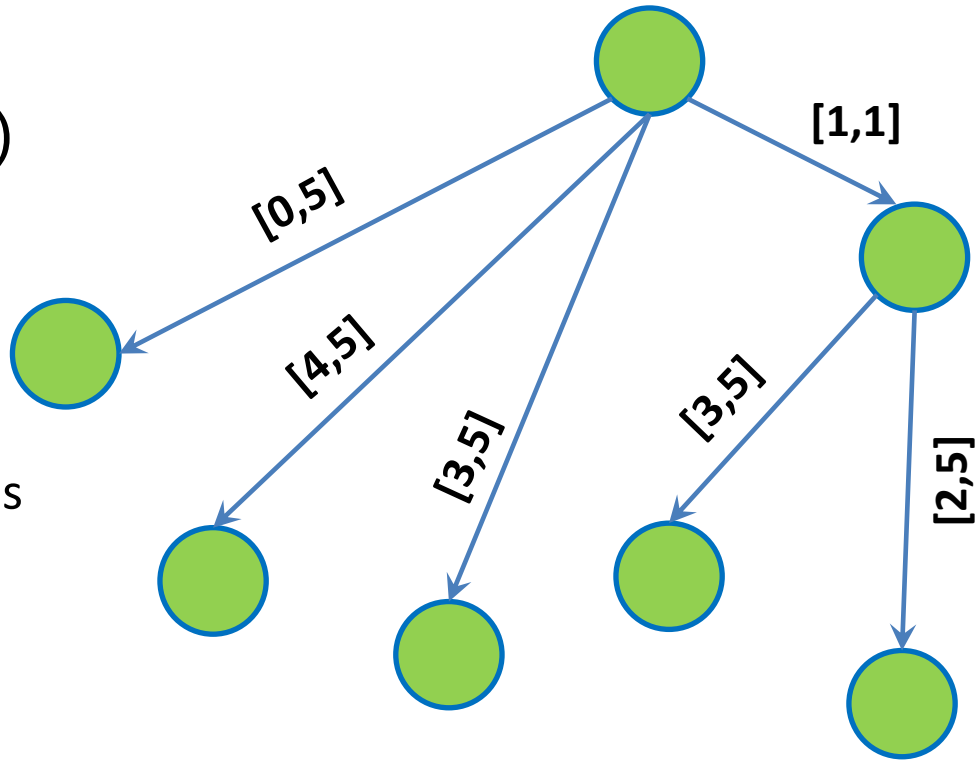

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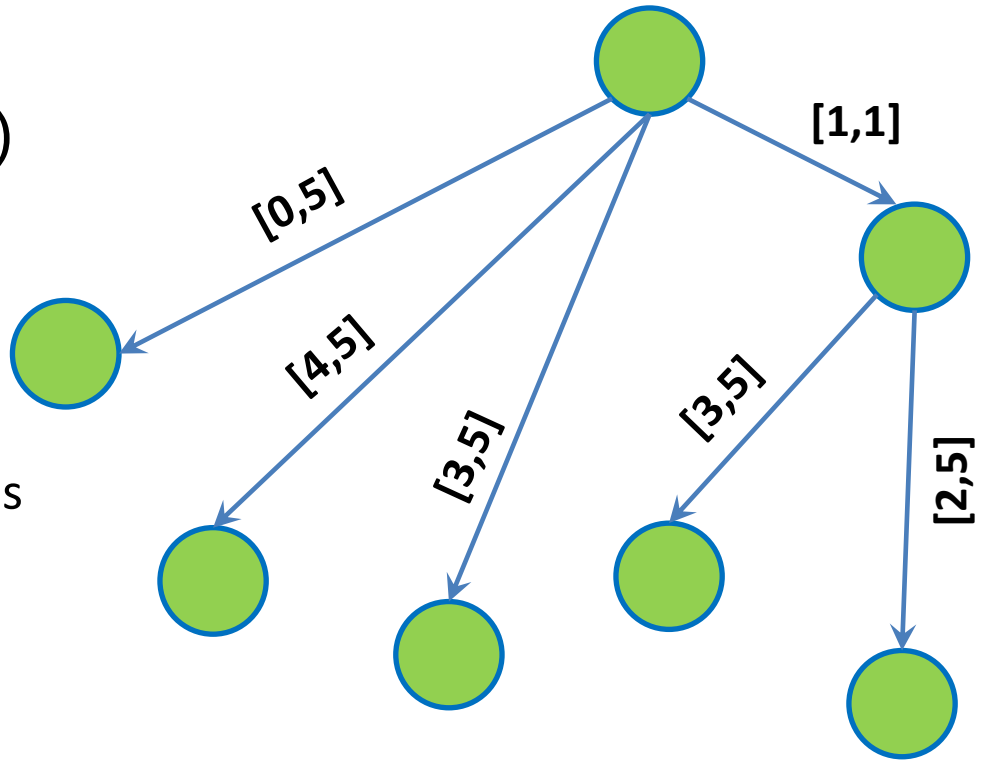
  - O(N)



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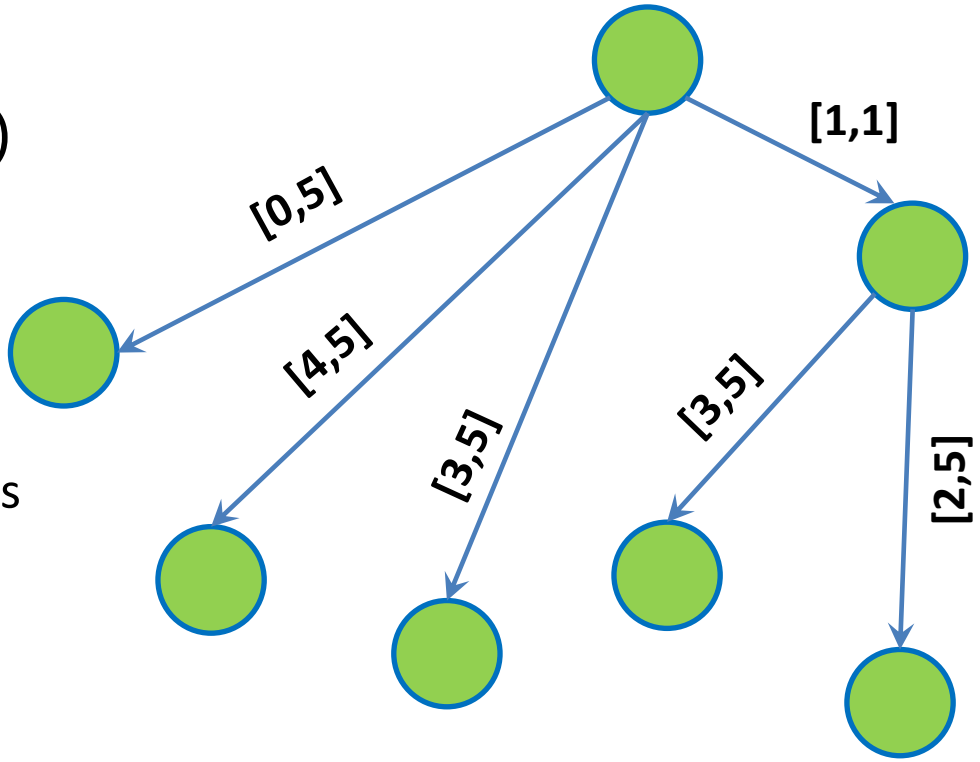

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  - O(N)
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  - O(N)
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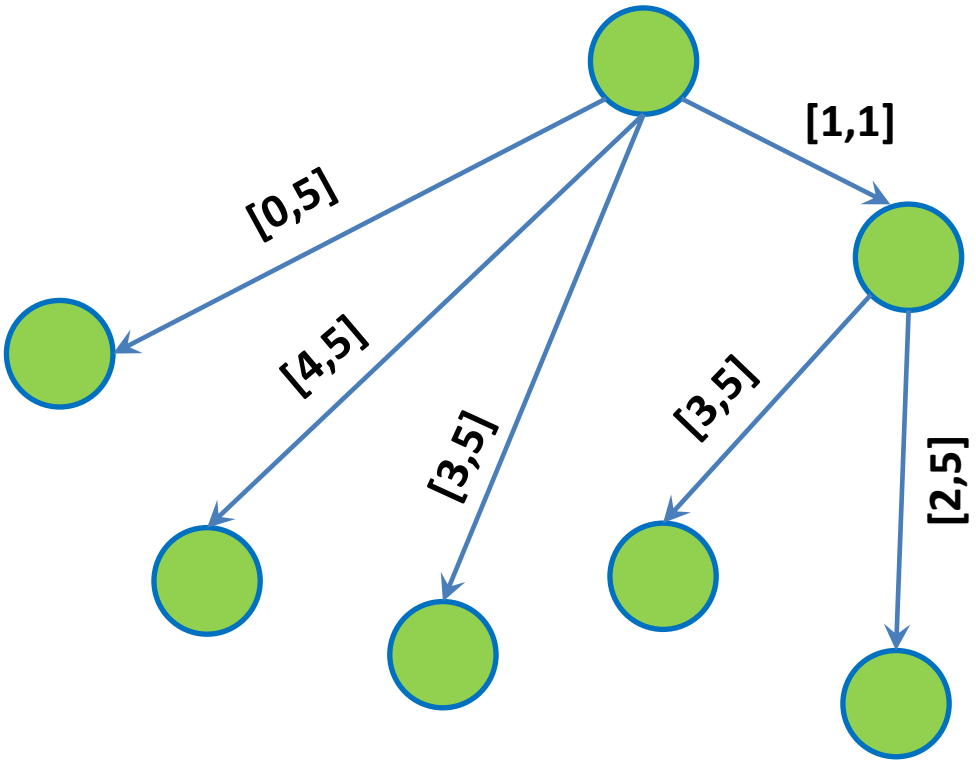
  - Total number of node
    = O(N + N/2 + N/4 + …)
    = O(N)



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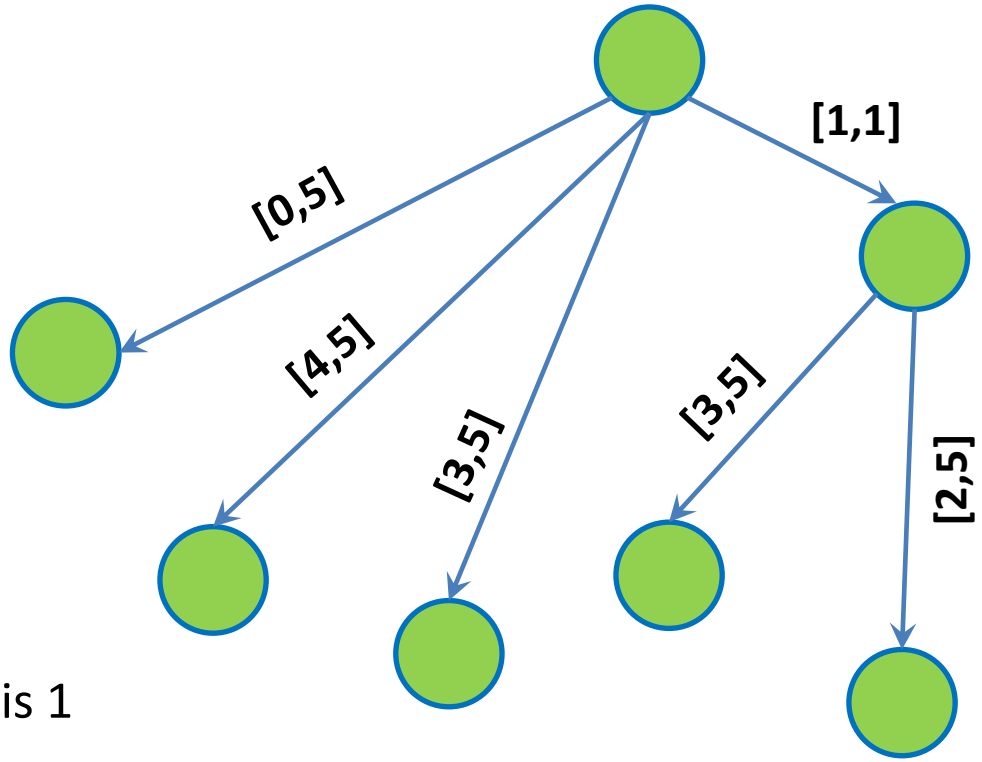
  - Total number of node
    = O(N + N/2 + N/4 + …)
    = O(N)
    * cause we go till root which is 1 from leaves



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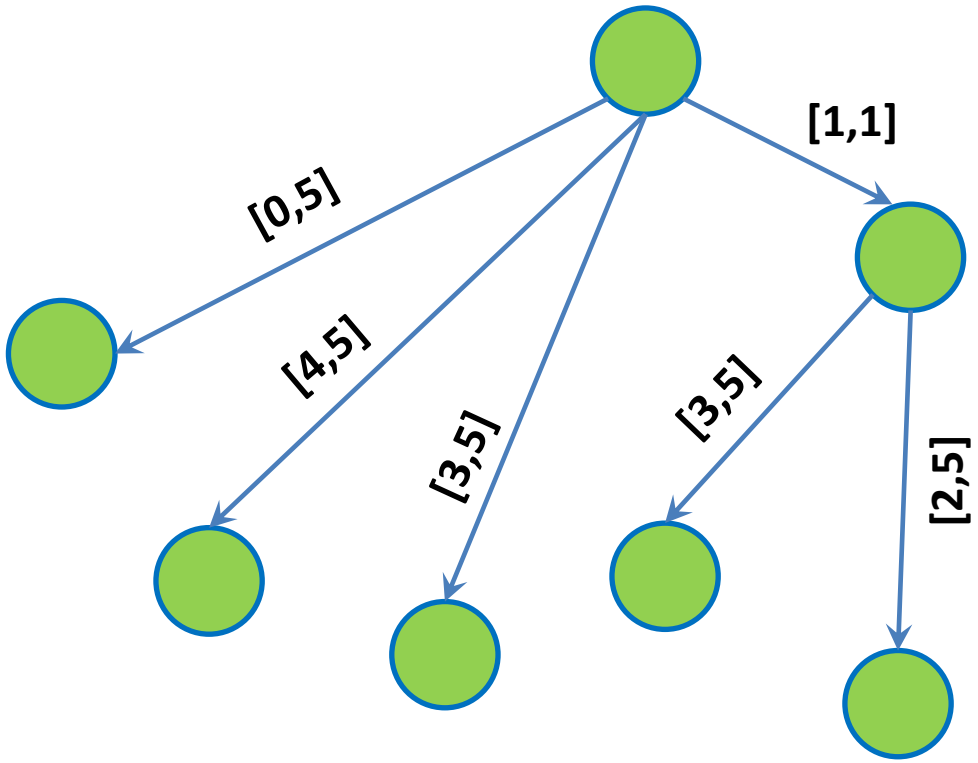

- Time complexity remains O(N^2) as we still need to insert every suffix with N character max

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  - O(N)
  - N leaves
  - Each non-leaf node has at least 2 children
  - Total number of node
    = O(N + N/2 + N/4 + …)
    = O(N)

- Time complexity remains O(N^2) as we still need to insert every suffix with N character max

We learn the hax called
Ukkonen's algorithm (1995)
in FIT3155 to do in O(N)

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Questions?

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- Let us try to implement it!

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- As a class activity
- … and some of the same functions

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- Let us try to implement it!

- As a class activity
- … and some of the same functions
  - Better than you searching online and not understanding what is happening
  - But 2 implementation
    - Iterative
    - Recursive (efficient)

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Questions?

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Thank You