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INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
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GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

Ready?

Agenda

- Directed Acyclic Graph (DAG)

Agenda

- Directed Acyclic Graph (DAG)
- Topological Sort
  - Kahn’s algorithm
  - Depth-First Search (DFS) modification

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Let us begin…

Directed Acyclic Graph (DAG)

What is it?

- Graphs are very commonly used to model real world scenario
  - One of which is a DAG

Directed Acyclic Graph (DAG)

What is it?

- Graphs are very commonly used to model real world scenario
  - One of which is a DAG
- What is a DAG?

Directed Acyclic Graph (DAG)

What is it?

- Graphs are very commonly used to model real world scenario
  - One of which is a DAG
- What is a DAG?
  - Directed

Directed Acyclic Graph (DAG)
What is it?

- Graphs are very commonly used to model real world scenario
  - One of which is a DAG
- What is a DAG?
  - Directed
  - Acyclic

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What is it?

- Graphs are very commonly used to model real world scenario
  - One of which is a DAG

- What is a DAG?
  - Directed
  - Acyclic
  - and of course it is a Graph…

Directed Acyclic Graph (DAG)

What is it?

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- Graphs are very commonly used to model real world scenario
  - One of which is a DAG
  - Can you give me an example of a real world DAG?
- What is a DAG?
  - Directed
  - Acyclic
  - and of course it is a Graph…

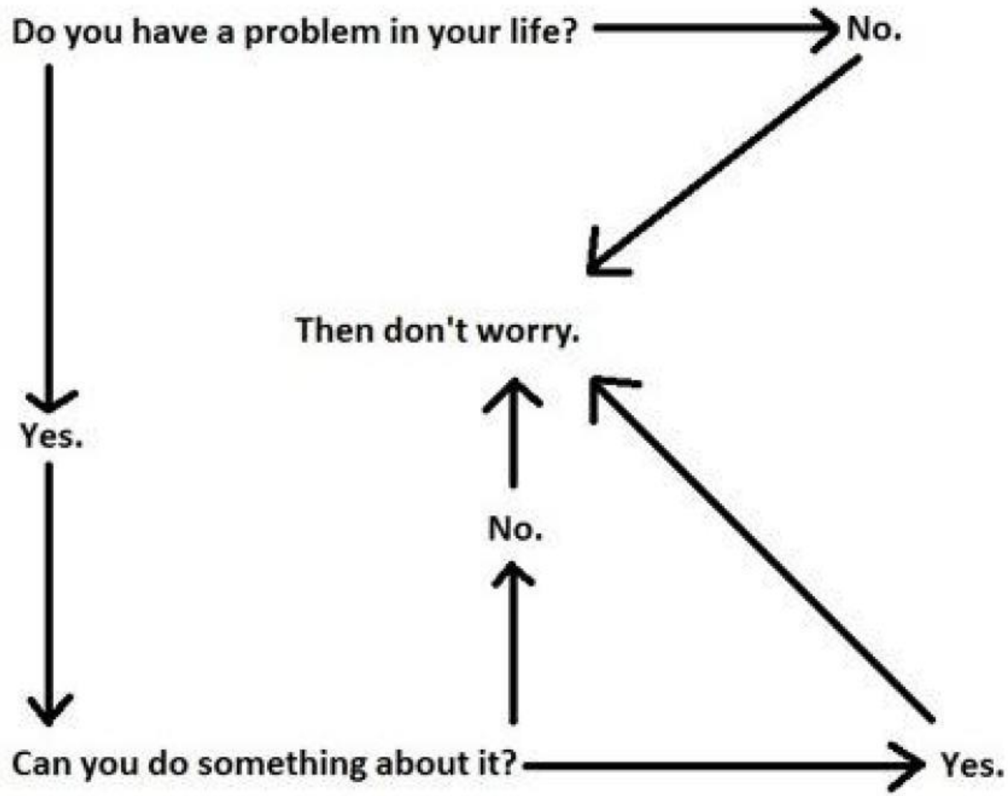
Directed Acyclic Graph (DAG)
What is it?

- Graphs are very commonly used to model real world scenario
  - One of which is a DAG
  - Can you give me an example of a real world DAG?
  - Can you give me an example of a real world non-DAG?

- What is a DAG?
  - Directed
  - Acyclic
  - and of course it is a Graph…

Directed Acyclic Graph (DAG)
What is it?

- A real world DAG



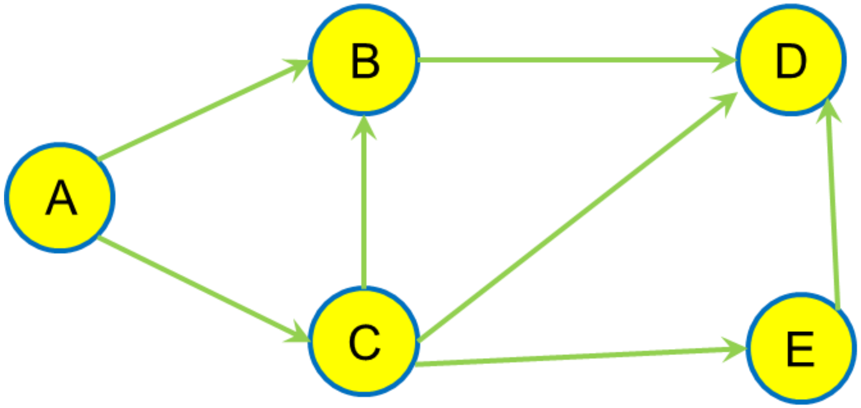
Directed Acyclic Graph (DAG)

What is it?

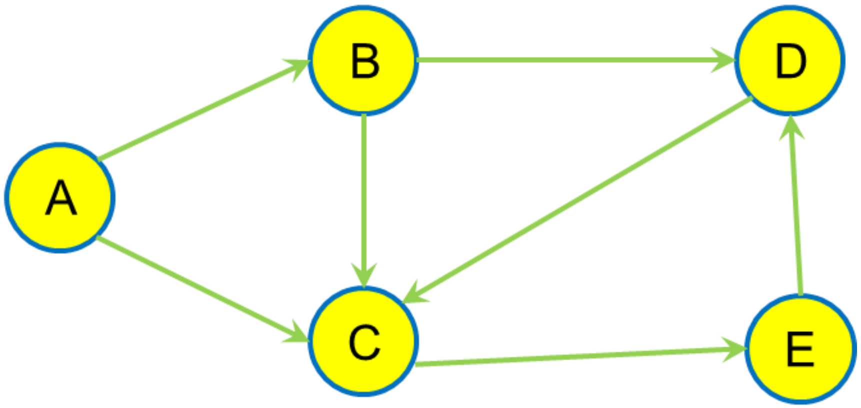
- A real world not DAG

Directed Acyclic Graph (DAG)
What is it?

- Which graph is a DAG?
  - And where is the cycle?



Graph 1



Graph 2

Questions?

Directed Acyclic Graph (DAG)

What is it for?

- Prerequisite mapping

Directed Acyclic Graph (DAG)
What is it for?

- Prerequisite mapping
  - If I have a directed edge from A to B, this means I need A before B

Directed Acyclic Graph (DAG)

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    - Common in project management
    - Common in talent/skill trees!

Directed Acyclic Graph (DAG)

What is it for?

- Prerequisite mapping
  - If I have a directed edge from A to B, this means I need A before B
    - Common in project management
    - Common in talent/skill trees!
    - Your university units!
      - Pass 1045
      - Pass 1008
      - Pass 2004
      - Pass 3155

Directed Acyclic Graph (DAG)

What is it for?

- So for an edge <A,B>

Directed Acyclic Graph (DAG)
What is it for?

- So for an edge <A,B>
  - A is a prerequisite for B
  - A is an ancestor of B
  - A is the subset of B
  - A is ordered before B

Directed Acyclic Graph (DAG)

What is it for?

- So for an edge <A,B>
  - A is a prerequisite for B
  - A is an ancestor of B
  - A is the subset of B
  - A is ordered before B
    - This enable us to sort a DAG

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Questions?

Topological Sort
Ordering of Vertices

- A topological sort
  - Permutation of vertices in a DAG
  - Vertex U will appear before vertex V if we have edge <U,V>

Topological Sort
Ordering of Vertices

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  - Permutation of vertices in a DAG
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Topological Sort
Ordering of Vertices

- A topological sort
  - Permutation of vertices in a DAG
  - Vertex U will appear before vertex V if we have edge <U,V>
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  - But if we don’t have edge <V,W> then V and W are of the same order

Topological Sort
Ordering of Vertices

- A topological sort
  - Permutation of vertices in a DAG
  - Vertex U will appear before vertex V if we have edge <U,V>
    - U<V
  - Vertex U will appear before vertex W is we have edge <U,W>
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  - But if we don’t have edge <V,W> then V and W are of the same order
    - V==W

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  - But if we don’t have edge <V,W> then V and W are of the same order
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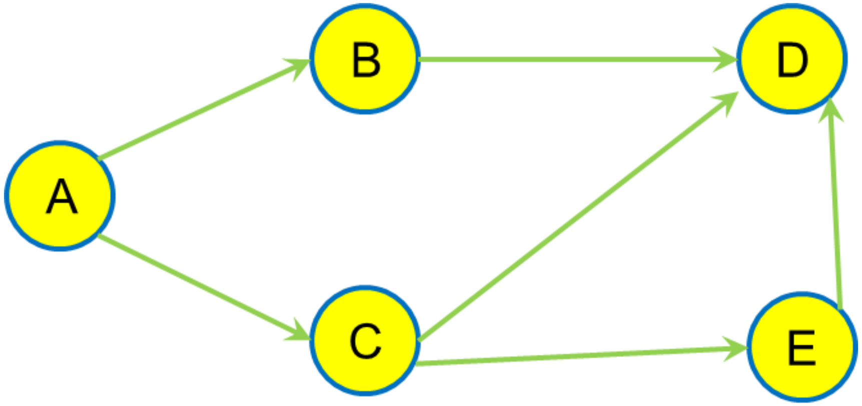
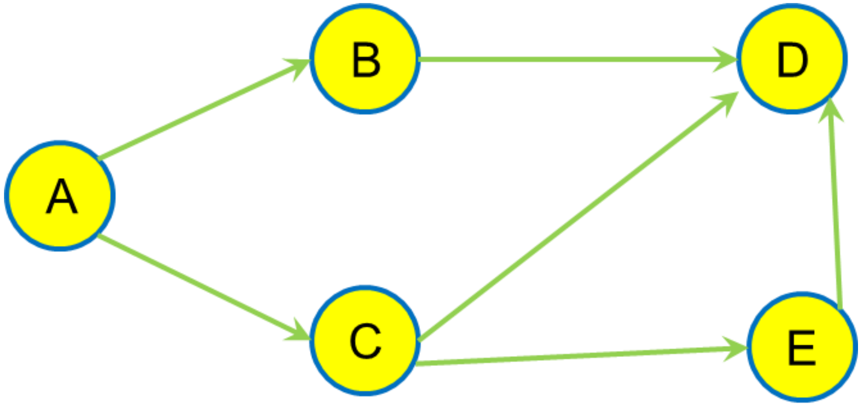
- So we have a DAG of your units
- Topological sort of this DAG gives you the order of units to take!

- Which one of these are not a valid topological sort of the DAG?



Topological Sort
Ordering of Vertices

- Which one of these are not a valid topological sort of the DAG?
  1. A, B, C, E, D
  2. A ,C, B, E, D
  3. A, C, E, B, D
  4. A, B, E, C, D



- Topological sort can be done via
  - Kahn's algorithm
  - A modified DFS

Questions?

Kahn’s Algorithm

For topological sort

- What is the concept like?

Kahn’s Algorithm
For topological sort

- What is the concept like?
  - Start with vertices without incoming edges

Kahn's Algorithm
For topological sort

- What is the concept like?
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Kahn’s Algorithm

For topological sort

- What is the concept like?
  - Start with vertices without incoming edges
  - Delete all outgoing edges from the vertex
  - Add in vertices without incoming edges

- What is the concept like?
  – Start with vertices without incoming edges
  – Delete all outgoing edges from the vertex
  – Add in vertices without incoming edges
  – Repeat!

Kahn’s Algorithm
For topological sort

- Algorithm as follow

```
sorted_list = []
process = []
add all vertices without incoming edges to process

while len(process) > 0:
    vertex_u = process.pop()
    sorted_list.append(vertex_u)
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if graph still has edges:
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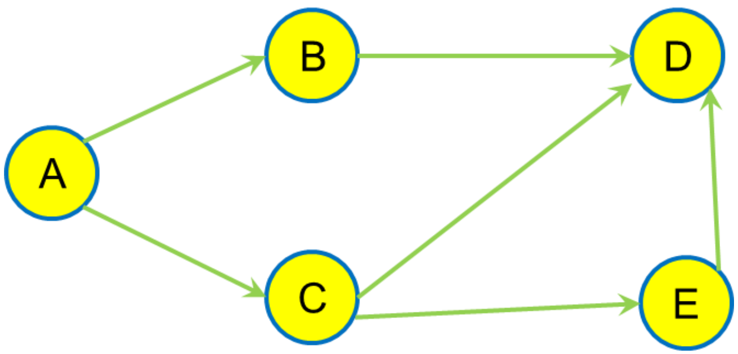

sorted					
process					

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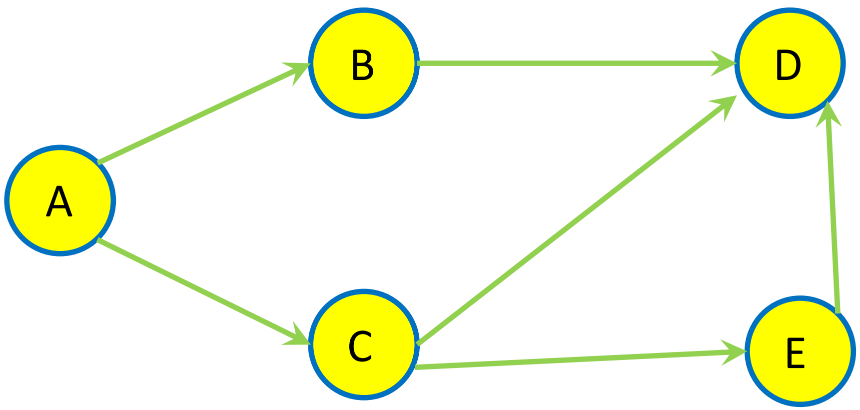

sorted					
process	A				

Kahn's Algorithm
For topological sort

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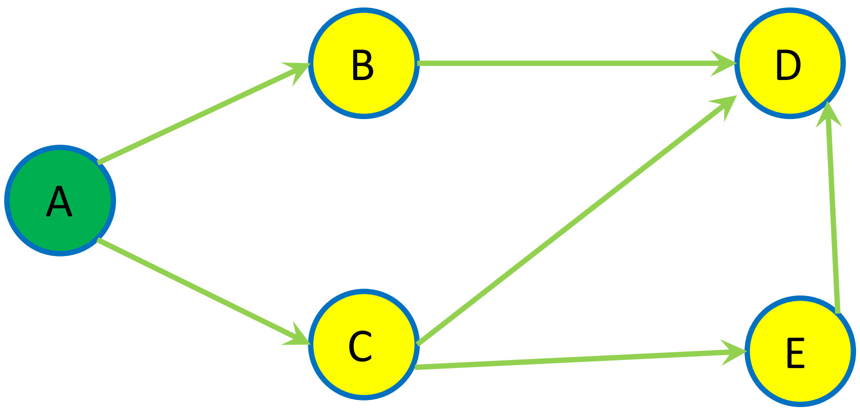

sorted	**A**				
process					

Kahn’s Algorithm

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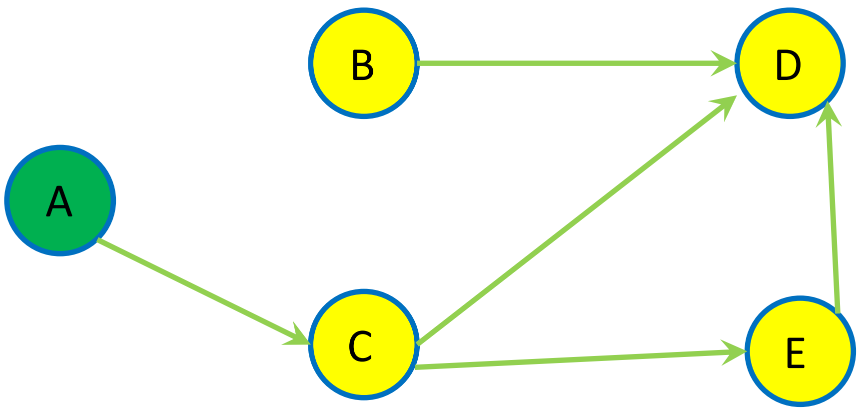

sorted	A				
process					

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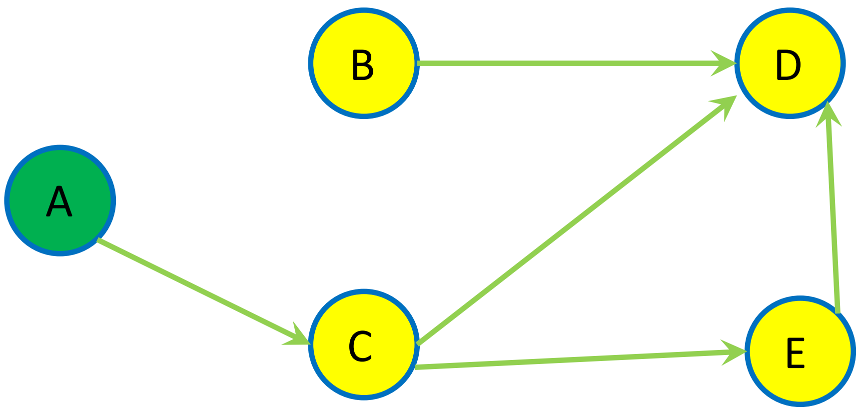

sorted	A				
process	B				

Kahn's Algorithm

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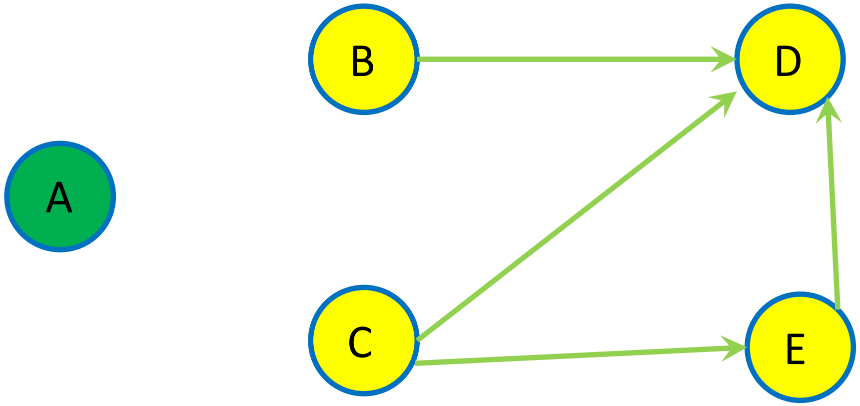

sorted	**A**				
process	B	C			

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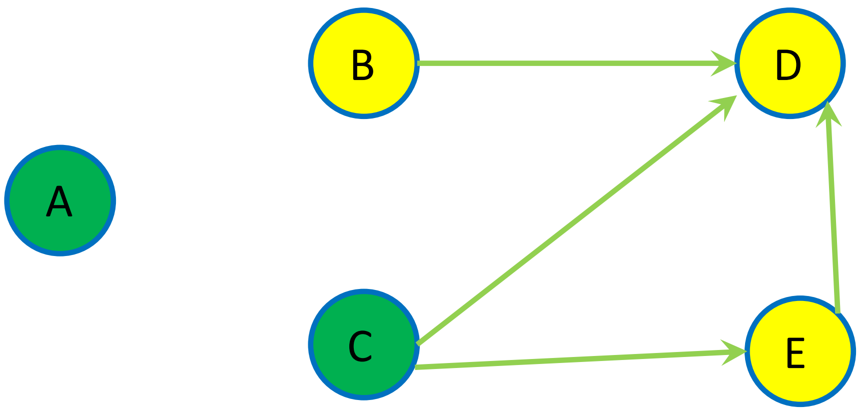

sorted	A	C			
process	B				

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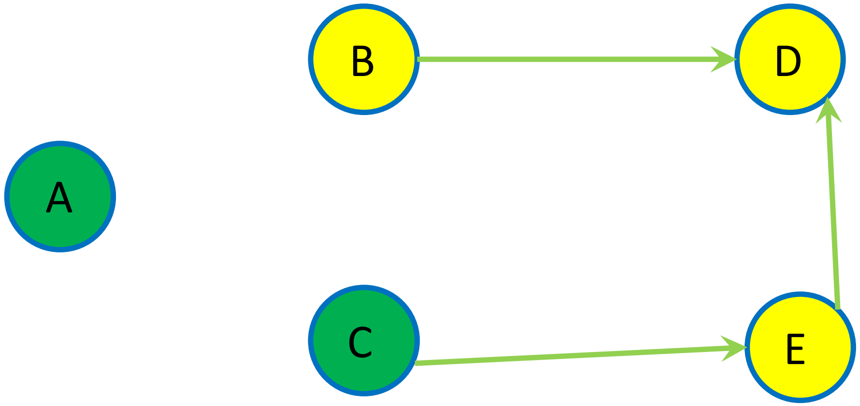

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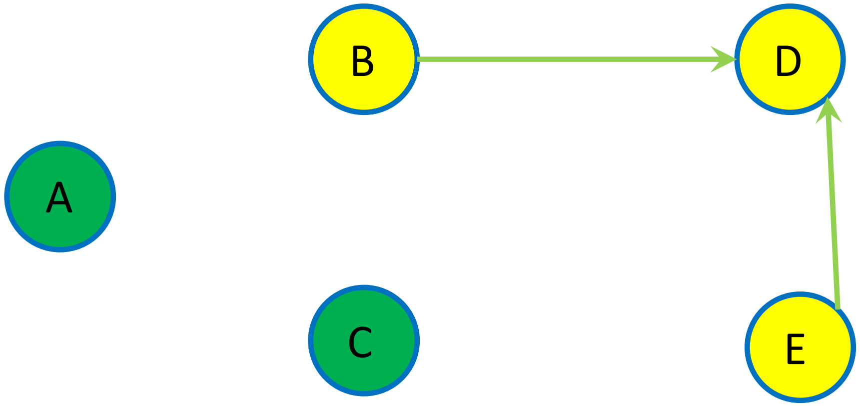

sorted	**A**	**C**			
process	B	E			

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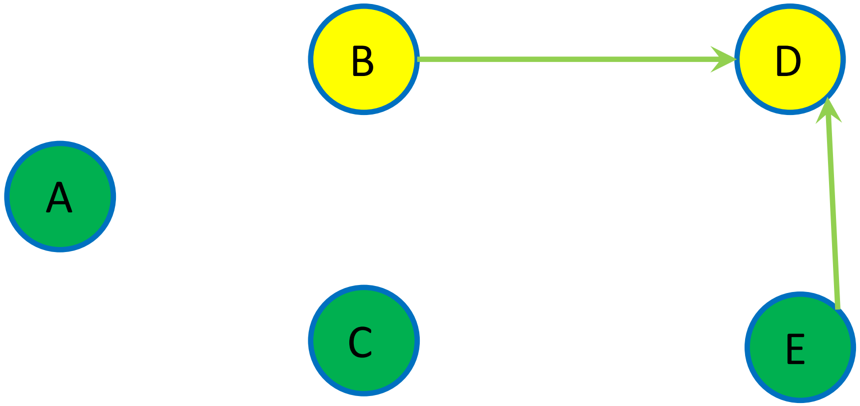

sorted	**A**	**C**	**E**		
process	B				

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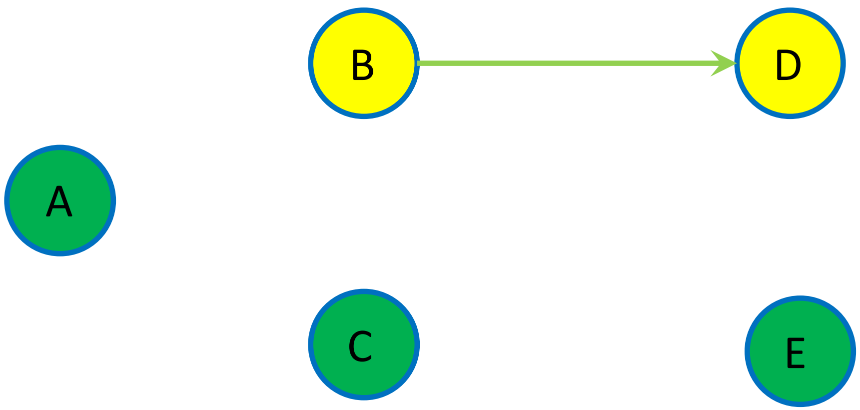

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process	B				

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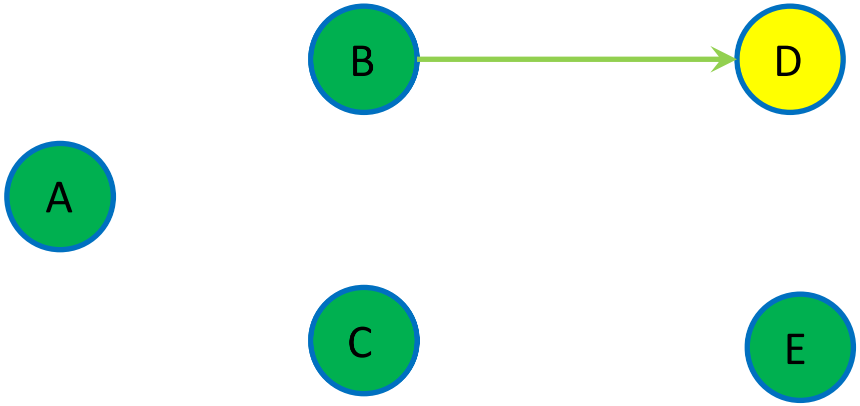

sorted	**A**	**C**	**E**	**B**	
process					

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- Algorithm as follow
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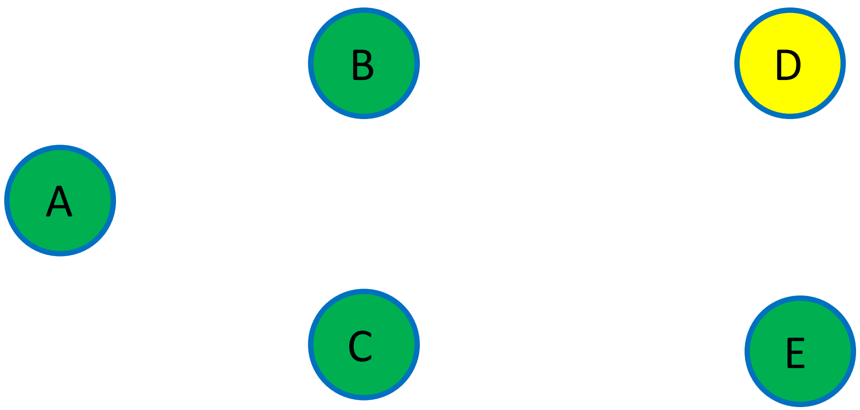

sorted	A	C	E	B	
process	D				

Kahn’s Algorithm

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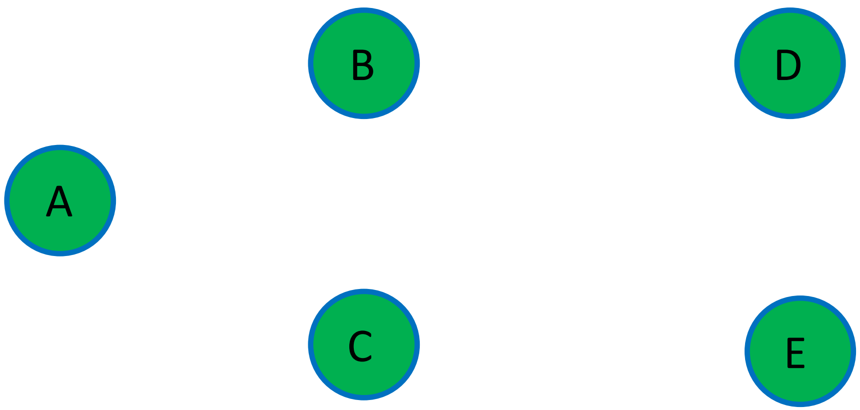

sorted	A	C	E	B	D
process					

Kahn's Algorithm
For topological sort

- Algorithm as follow
  - Seemed simple right?
  - Let us zoomed in to the algorithm more

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if graph still has edges:
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```

Kahn's Algorithm

For topological sort

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```
# for output
sorted_list = []
# tracks number of incoming edges
incoming_edges = [0] * len(vertices)
for edge in graph:          # edge <u,v>
    incoming_edges[vertex_v] += 1
# process queue or stack
process = []
for vertex_v in incoming_edges:
    if incoming_edges[vertex_v] == 0:
        process.append(vertex_v)
# kahn's
while len(process) > 0:
    vertex_u = process.pop()
    sorted_list.append(vertex_u)
    for edge in vertex_u.edges:
        incoming_edges[edge.vertex_v] -= 1
        if incoming_edges[vertex_v] == 0:
            process.append(vertex_v)
# results
if graph still has edges:
    print("Error. Not a cycle")
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Kahn's Algorithm
For topological sort

- Algorithm as follow
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- Complexity?

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Kahn's Algorithm

For topological sort

- Algorithm as follow
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  - Let us zoomed in to the algorithm more
- Complexity?
  - O(V+E) time space

```python
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if graph still has edges:
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```

Questions?

- We saw the complexity of Kahn's being O(V+E)

Depth-First Search (DFS)

Modified for topological sorting

- We saw the complexity of Kahn's being O(V+E)
- Can we modify DFS to do the same?

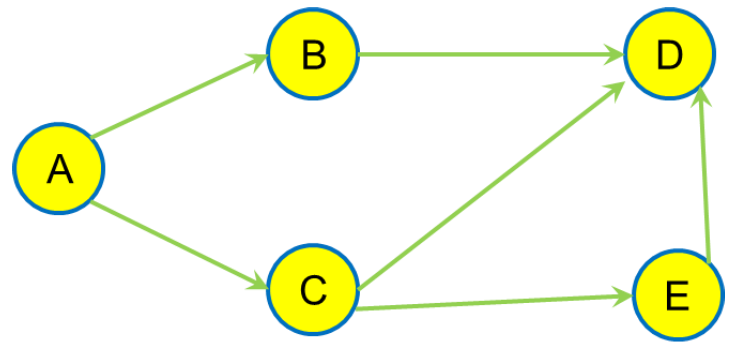
```
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

Depth-First Search (DFS)

Modified for topological sorting

- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see

```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

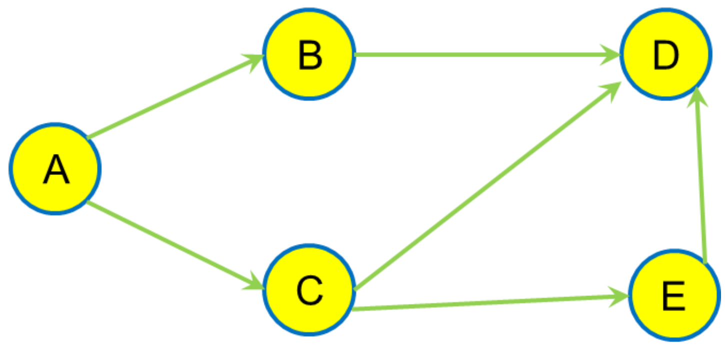


Depth-First Search (DFS)

Modified for topological sorting

```
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A

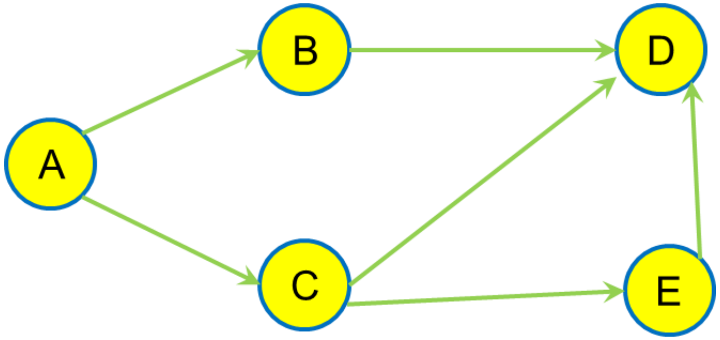


Depth-First Search (DFS)

Modified for topological sorting

- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A
    - Go to B

```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

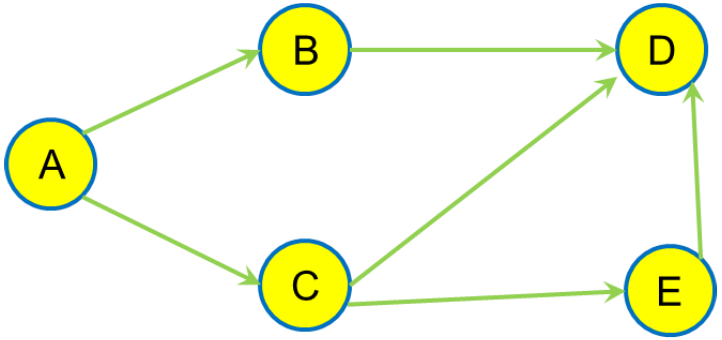


Depth-First Search (DFS)

Modified for topological sorting

- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A
    - Go to B
    - Go to D

```
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```



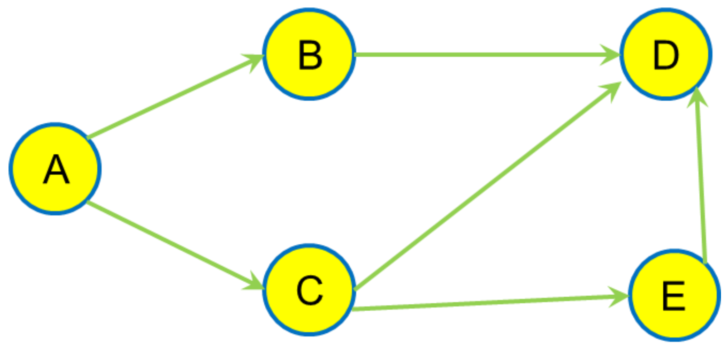
Depth-First Search (DFS)

Modified for topological sorting

MONASH University

- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A
    - Go to B
    - Go to D
    - Go to C

```
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

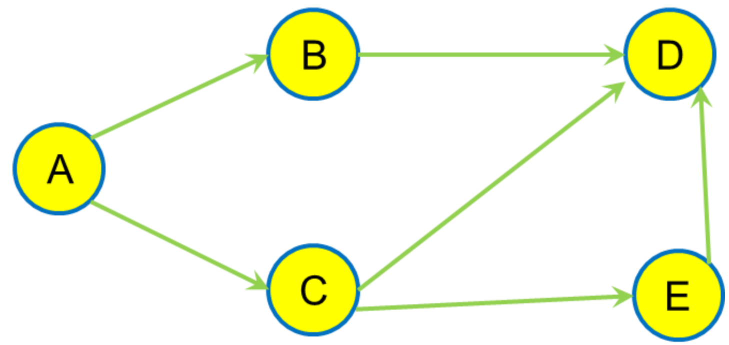


Depth-First Search (DFS)

Modified for topological sorting

- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A
    - Go to B
    - Go to D
    - Go to C
    - Go to E

```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

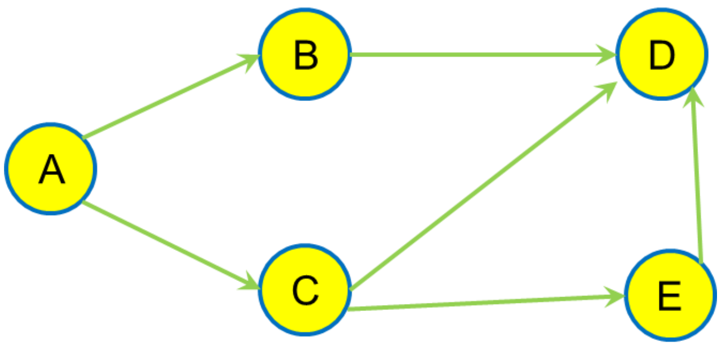


Depth-First Search (DFS)

Modified for topological sorting

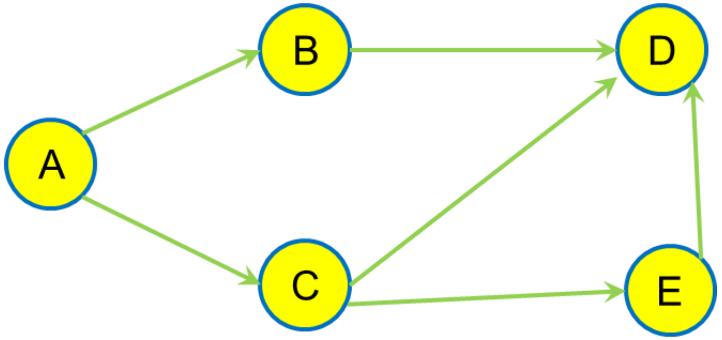
- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A
    - Go to B
    - Go to D
    - Go to C
    - Go to E
    - So we have A, B, D, C, E

```
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```



```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A
    - Go to B
    - Go to D
    - Go to C
    - Go to E
    - So we have A, B, D, C, E
  - Any other DFS order?

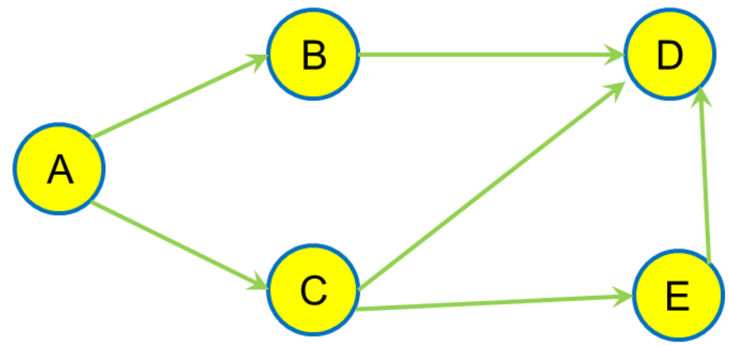


Depth-First Search (DFS)

Modified for topological sorting

- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A
    - Go to B
    - Go to D
    - Go to C
    - Go to E
    - So we have A, B, D, C, E
  - Any other DFS order?
    - A, C, D, E, B

```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

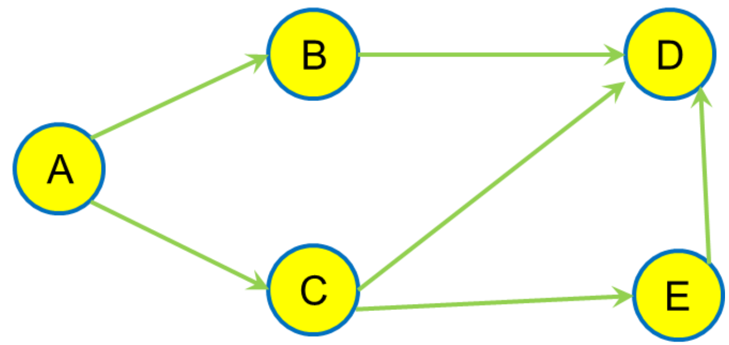


Depth-First Search (DFS)

Modified for topological sorting

```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A
    - Go to B
    - Go to D
    - Go to C
    - Go to E
    - So we have A, B, D, C, E
  - Any other DFS order?
    - A, C, D, E, B
    - A, C, E, D, B



Depth-First Search (DFS)

Modified for topological sorting

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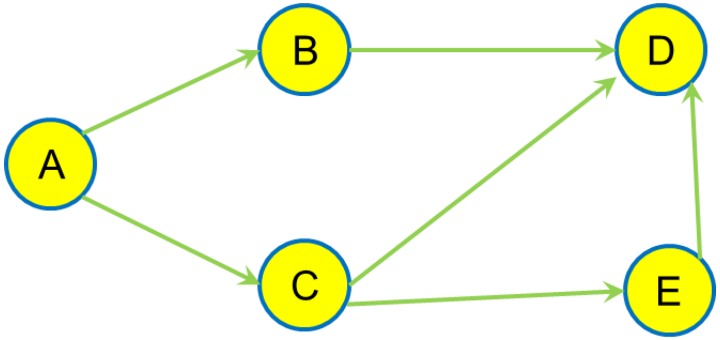
- Can we modify DFS to do the same?
  - Let us run DFS on the following graph and see
    - Start from A
    - Go to B
    - Go to D
    - Go to C
    - Go to E
    - So we have A, B, D, C, E
  - Any other DFS order?
    - A, C, D, E, B
    - A, C, E, D, B
    - A, C, E, B, D?

```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

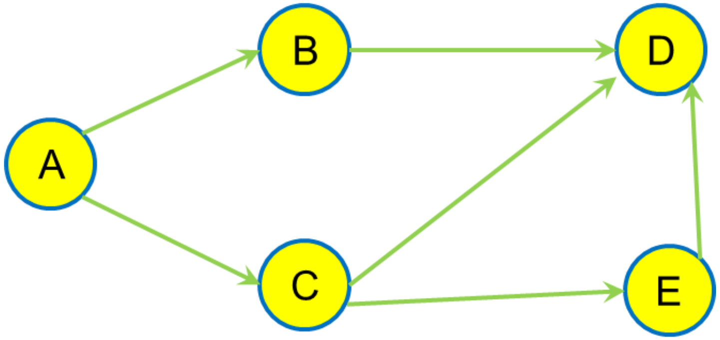


Depth-First Search (DFS)

Modified for topological sorting

- Can we modify DFS to do the same?
  - Any other DFS order?
    - A, B, D, C, E
    - A, C, D, E, B
    - A, C, E, D, B

```
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```



Depth-First Search (DFS)

Modified for topological sorting

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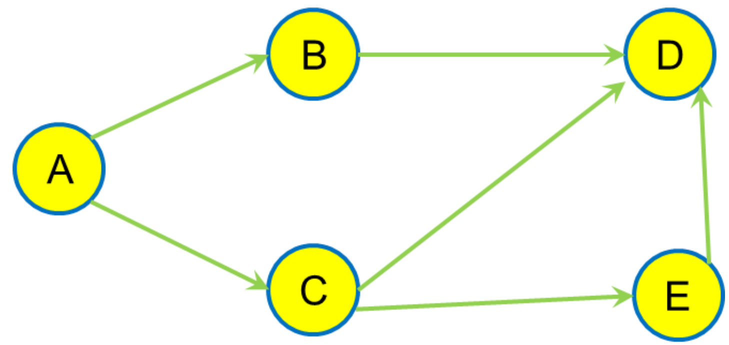
```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

- Can we modify DFS to do the same?
  - Any other DFS order?
    - A, B, D, C, E
    - A, C, D, E, B
    - A, C, E, D, B
  - A possible topological sort
    - A, B, C, E, D
    - A, C, B, E, D



Depth-First Search (DFS)

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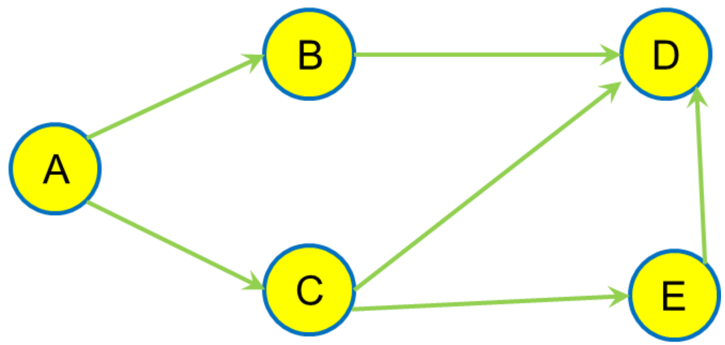
```

- Can we modify DFS to do the same?
  - Any other DFS order?
    - A, B, D, C, E
    - A, C, D, E, B
    - A, C, E, D, B
  - A possible topological sort
    - A, B, C, E, D
    - A, C, B, E, D
  - Notice something?

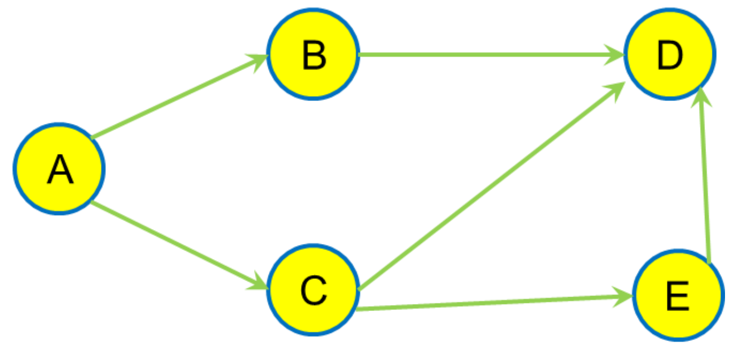


Depth-First Search (DFS)

Modified for topological sorting

```
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

- Can we modify DFS to do the same?
  - Any other DFS order?
    - A, B, D, C, E
    - A, C, D, E, B
    - A, C, E, D, B
  - A possible topological sort
    - A, B, C, E, D
    - A, C, B, E, D
  - Notice something?
    - When we reach the end of the DFS, we go back to an earlier vertex but this vertex should be early in topological sort (such as vertex B or C)

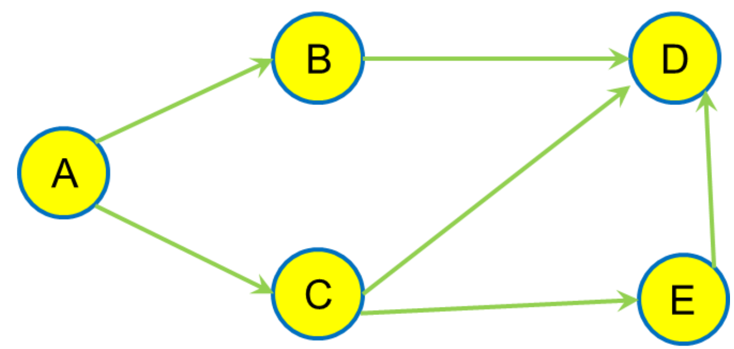


Depth-First Search (DFS)

Modified for topological sorting

```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

- Can we modify DFS to do the same?
  - Any other DFS order?
    - A, B, D, C, E
    - A, C, D, E, B
    - A, C, E, D, B
  - A possible topological sort
    - A, B, C, E, D
    - A, C, B, E, D
  - Notice something?
    - When we reach the end of the DFS, we go back to an earlier vertex but this vertex should be early in topological sort (such as vertex B or C)

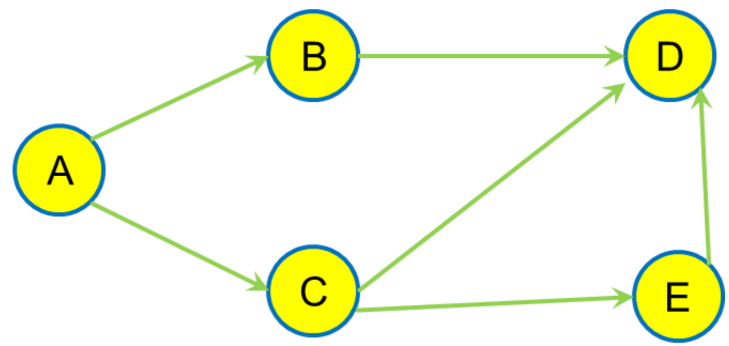


Depth-First Search (DFS)

Modified for topological sorting

```python
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

- Can we modify DFS to do the same?
  - Any other DFS order?
    - A, B, D, C, E
    - A, C, D, E, B
    - A, C, E, D, B
  - A possible topological sort
    - A, B, C, E, D
    - A, C, B, E, D
  - Notice something?
    - When we reach the end of the DFS, we go back to an earlier vertex but this vertex should be early in topological sort (such as vertex B or C)

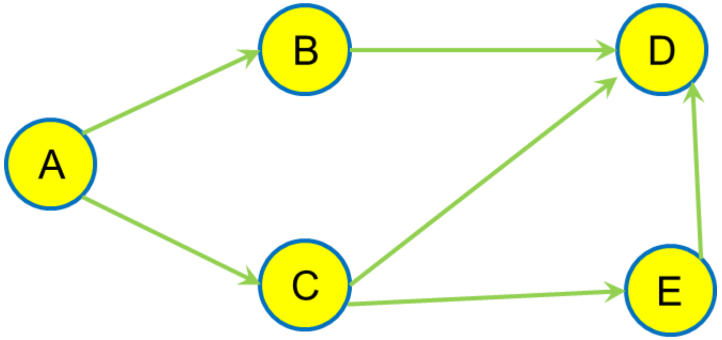


Depth-First Search (DFS)

Modified for topological sorting

```
def dfs_topological(vertex_u):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological(vertex_v)
```

- Can we modify DFS to do the same?
  - Any other DFS order?
    - A, B, D, C, E
    - A, C, D, E, B
    - A, C, E, D, B
  - A possible topological sort
    - A, B, C, E, D
    - A, C, B, E, D
  - Notice something?
    - When we reach the end of the DFS, we go back to an earlier vertex but this vertex should be early in topological sort (such as vertex B or C)
    - So how do we arrange it?



Depth-First Search (DFS)

Modified for topological sorting

- Here's how we modify with a stack!

```python
def dfs_topological(vertex_u):
    # start for result
    stack = []
    # run DFS
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological_aux(vertex_v,stack)
    # output
    print(stack)

def dfs_topological_aux(vertex_u,stack):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological_aux(vertex_v)
    # add to stack
    stack.push(vertex_u)
```

Depth-First Search (DFS)

Modified for topological sorting

- Complexity?

```python
def dfs_topological(vertex_u):
    # start for result
    stack = []
    # run DFS
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological_aux(vertex_v,stack)
    # output
    print(stack)

def dfs_topological_aux(vertex_u,stack):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological_aux(vertex_v)
    # add to stack
    stack.push(vertex_u)
```

Depth-First Search (DFS)

Modified for topological sorting

- Complexity?
  - O(V+E) since we only added a stack

```python
def dfs_topological(vertex_u):
    # start for result
    stack = []
    # run DFS
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological_aux(vertex_v,stack)
    # output
    print(stack)

def dfs_topological_aux(vertex_u,stack):
    vertex_u.visited = True
    for edge in vertex_u.edges:
        if edge.vertex_v.visited == False:
            dfs_topological_aux(vertex_v)
    # add to stack
    stack.push(vertex_u)
```

Questions?

Thank You