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INFORMATION
TECHNOLOGY

FIT2004
Algorithms and Data Structures

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Referencing materials by
Nathan Companez, Aamir Cheema, Arun Konagurthu and Lloyd Allison

GROUP OF EIGHT AUSTRALIA

Faculty of Information Technology, Monash University

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Ready?

Agenda

- Binary Search Tree (BST)

Agenda

- Binary Search Tree (BST)
  - AVL Tree

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Let us begin…

Tree
And Binary Search Tree (BST)

- Our lord and saviour

Tree
And Binary Search Tree (BST)

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- What is a binary search tree?

Tree
And Binary Search Tree (BST)

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- What is a binary search tree?
  - Empty tree is BST

Tree
And Binary Search Tree (BST)

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- What is a binary search tree?
  - Empty tree is BST
  - All elements in left subtree < root
  - All elements in right subtree > root

Tree
And Binary Search Tree (BST)

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- What is a binary search tree?
  - Empty tree is BST
  - All elements in left subtree < root
  - All elements in right subtree > root
  - What about ==?

Tree
And Binary Search Tree (BST)

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- What is a binary search tree?
  - Empty tree is BST
  - All elements in left subtree < root
  - All elements in right subtree > root
  - What about ==? Key are unique!

Tree
And Binary Search Tree (BST)

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- What is a binary search tree?
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  - All elements in left subtree < root
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  - What about ==? Key are unique!
  - Left subtree is BST
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Tree
And Binary Search Tree (BST)

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- What is a binary search tree?
  - Empty tree is BST
  - All elements in left subtree < root
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  - What about ==? Key are unique!
  - Left subtree is BST
  - Right subtree is BST

} Recursive definition

Tree
And Binary Search Tree (BST)

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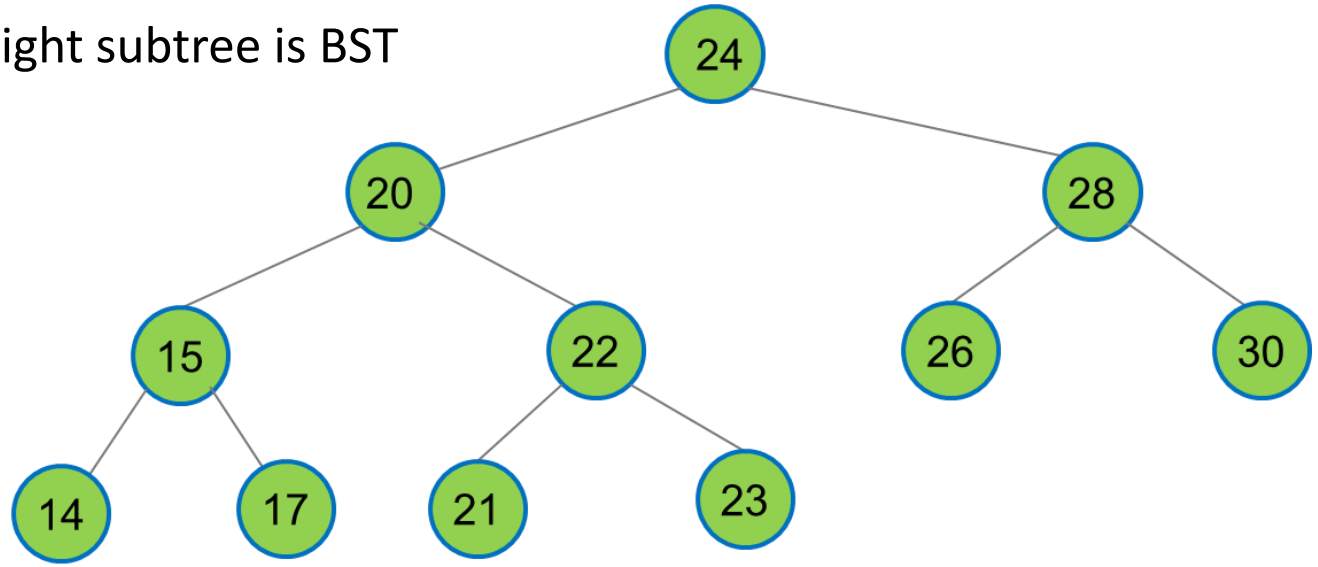
- What is a binary search tree?
  - Empty tree is BST
  - All elements in left subtree < root
  - All elements in right subtree > root
  - What about ==? Key are unique!
  - Left subtree is BST
  - Right subtree is BST



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Questions?

Tree
And Binary Search Tree (BST)

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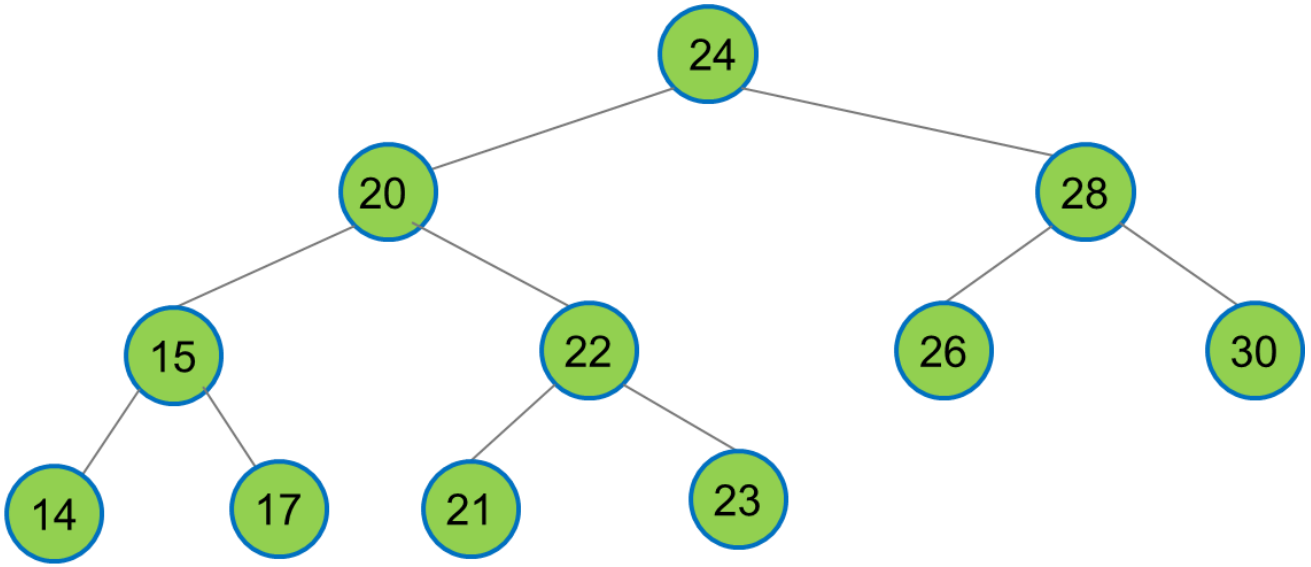
- The main functions?
  - Insert
  - Delete



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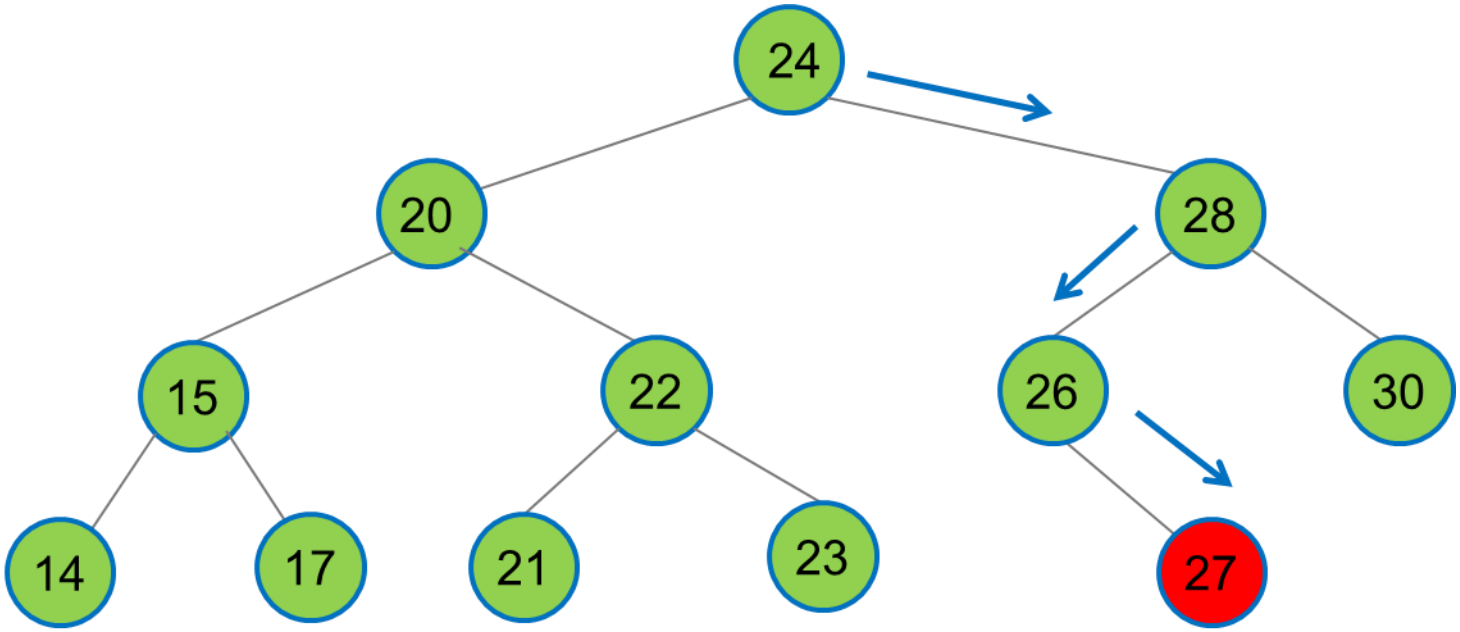
- The main functions?
  - Insert 27



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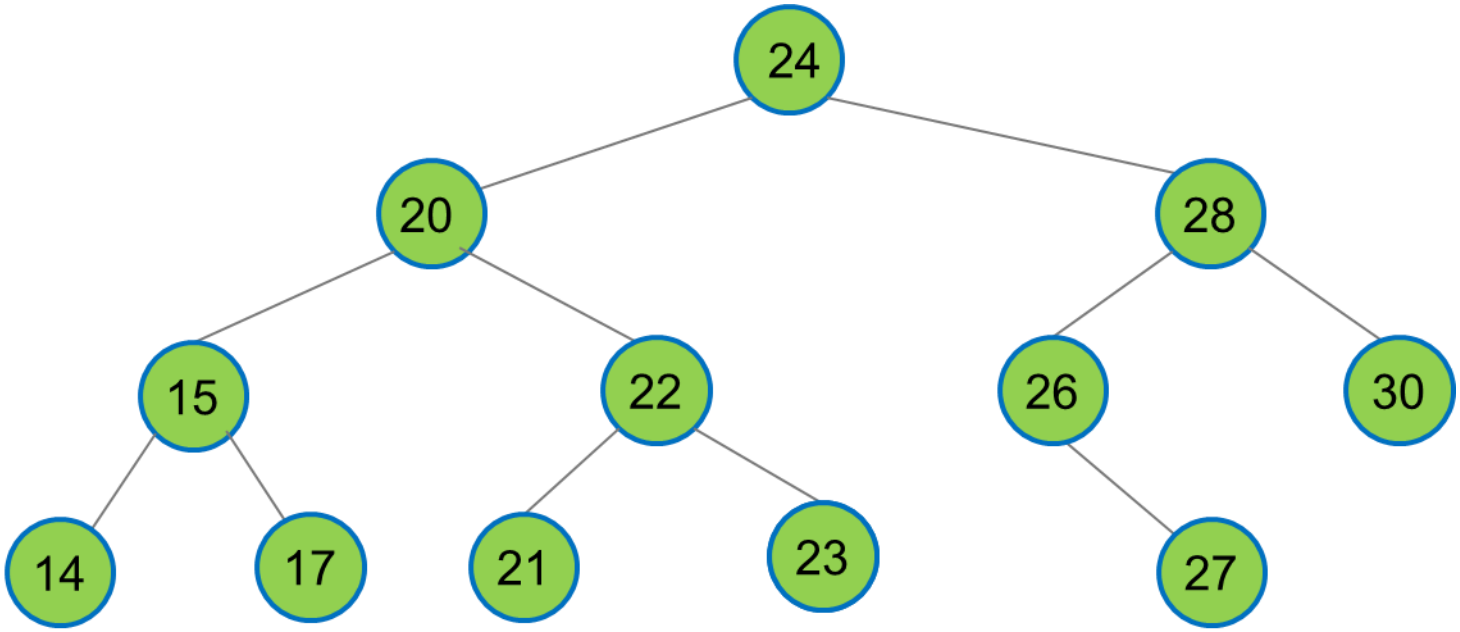
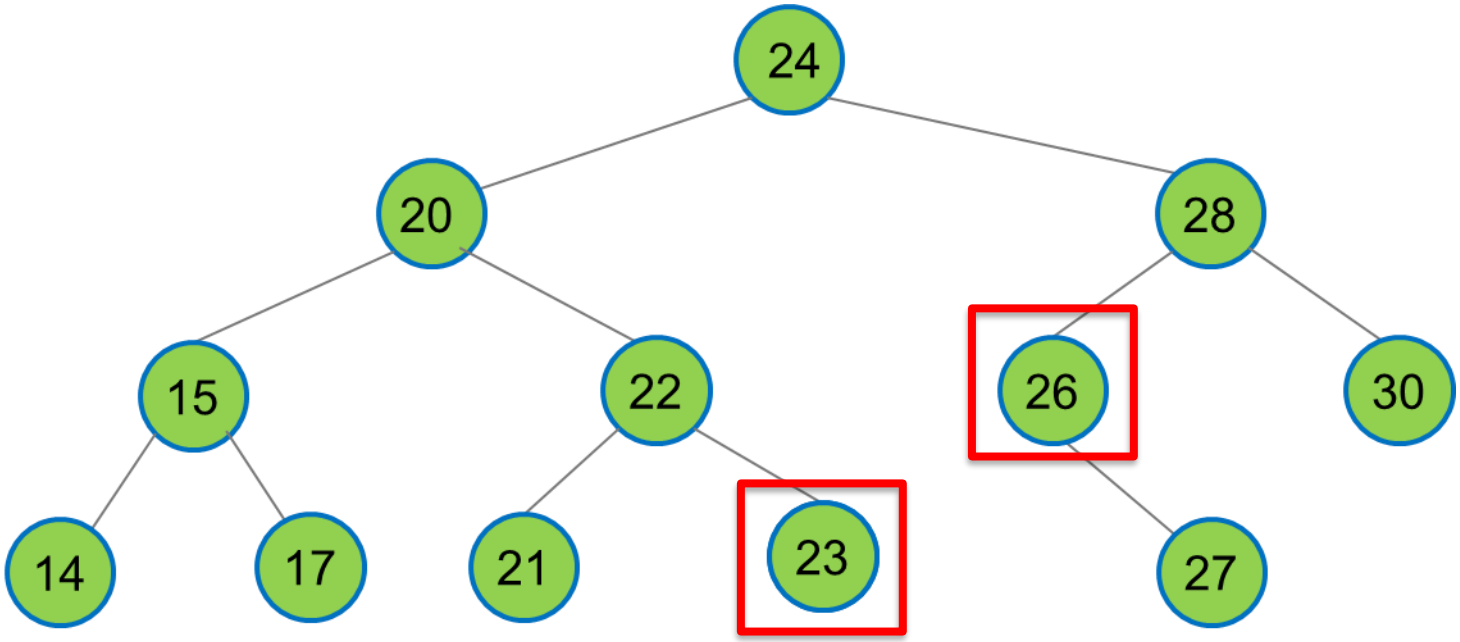
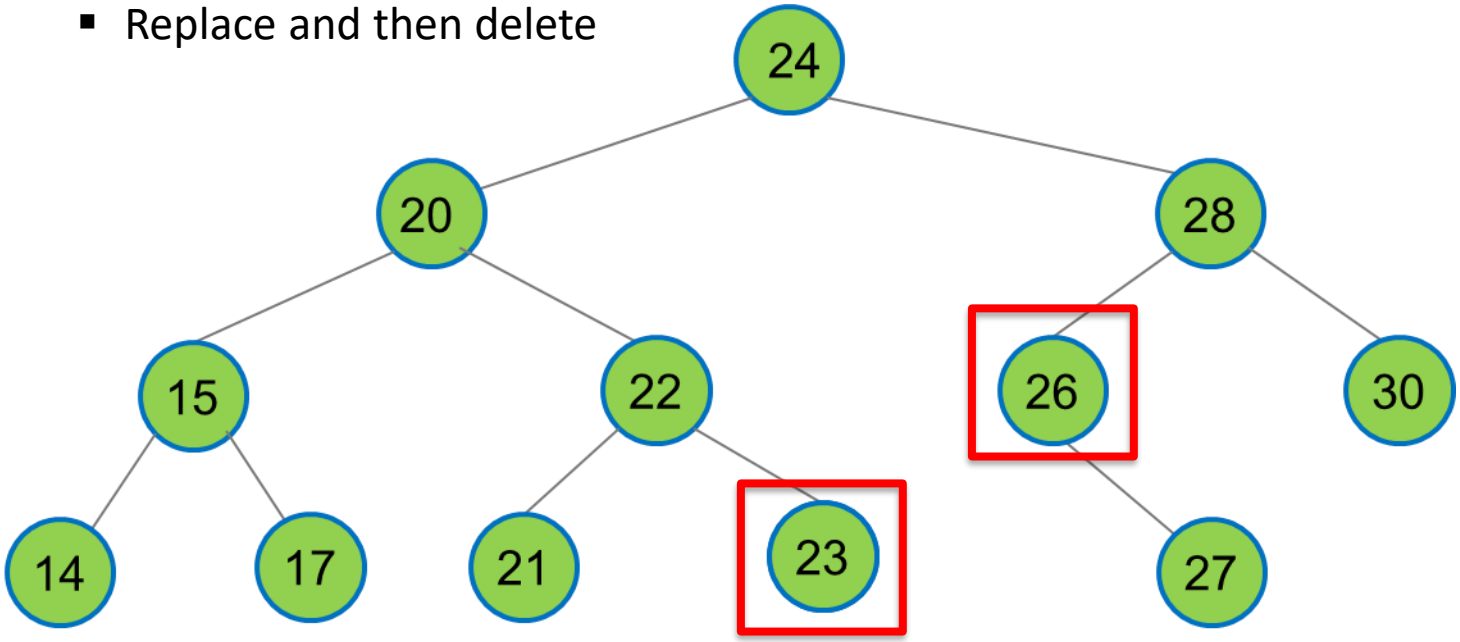
- The main functions?
  - Delete 24



- The main functions?
  - Delete 24
    - Find the biggest left child
    - Find the smallest right child



- The main functions?
  - Delete 24
    - Find the biggest left child
    - Find the smallest right child
    - Replace and then delete



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Questions?

Tree
And Binary Search Tree (BST)

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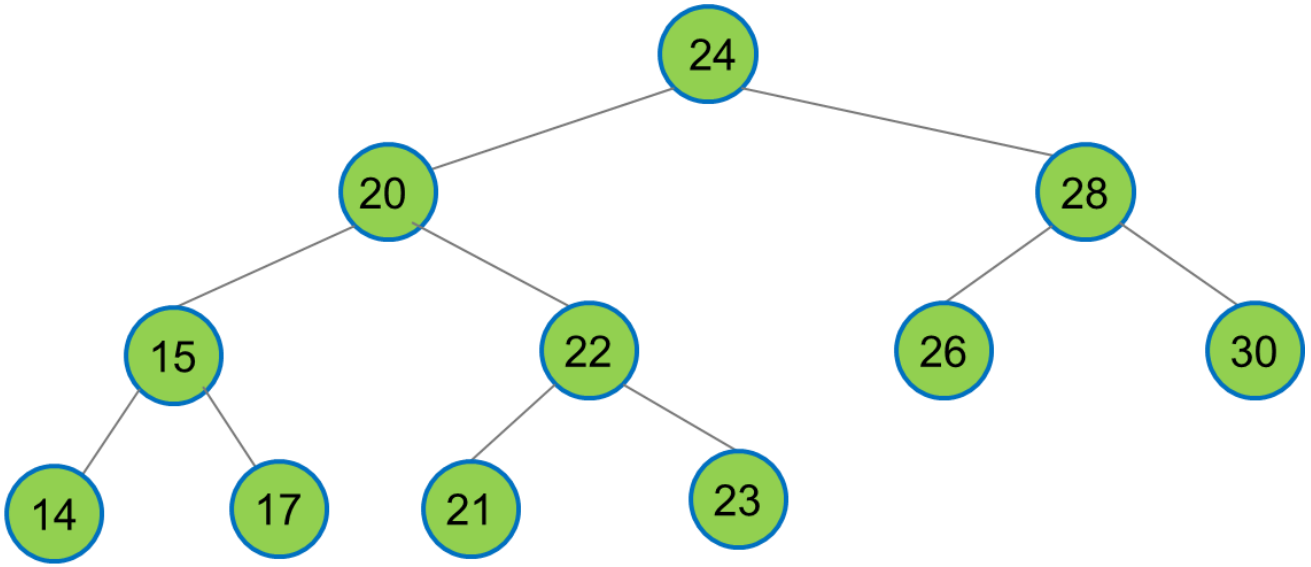
- The main functions?
  - Search
  - Traversal



Tree
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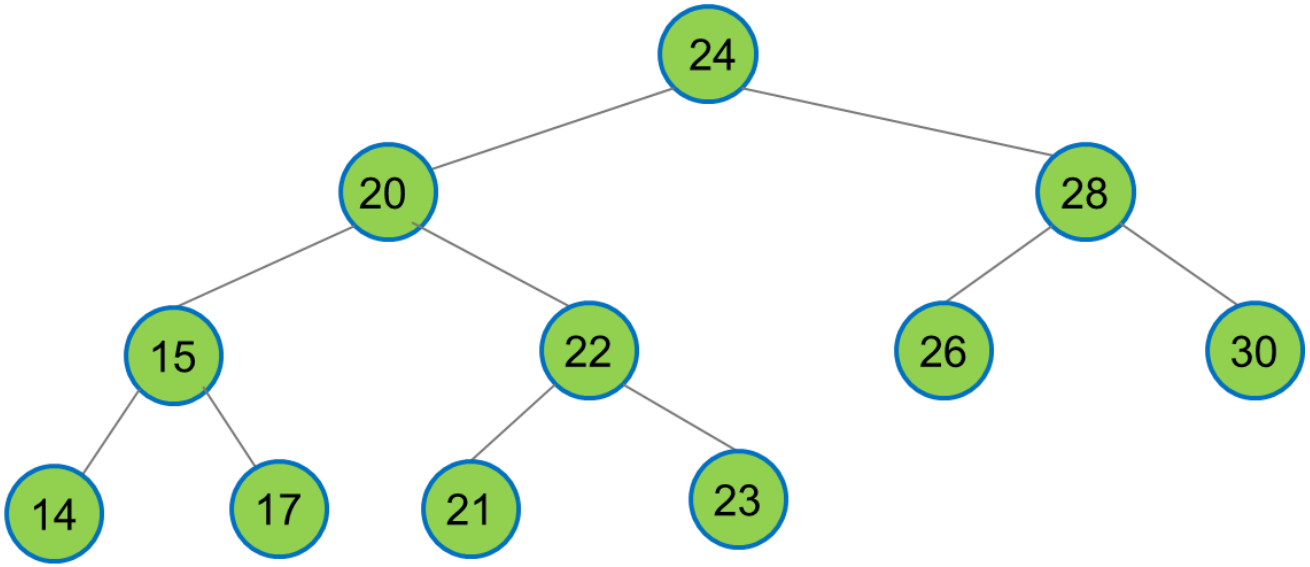
- The main functions?
  - Search 24



Tree
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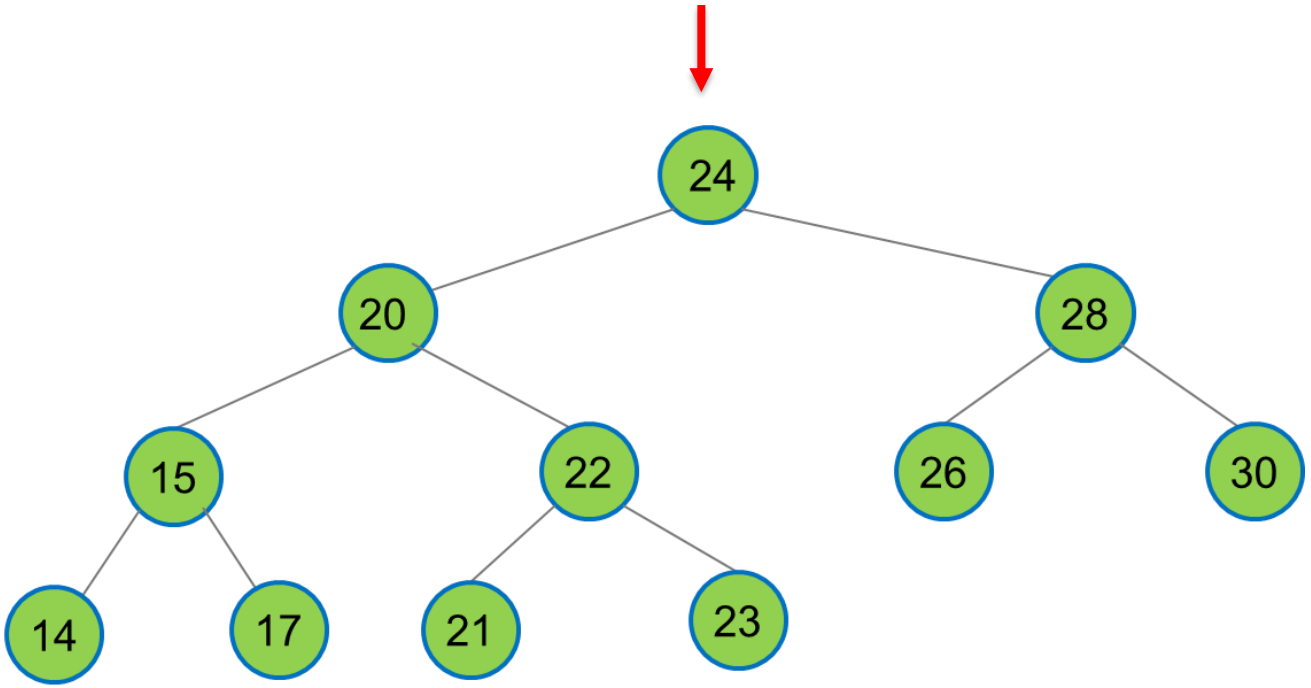

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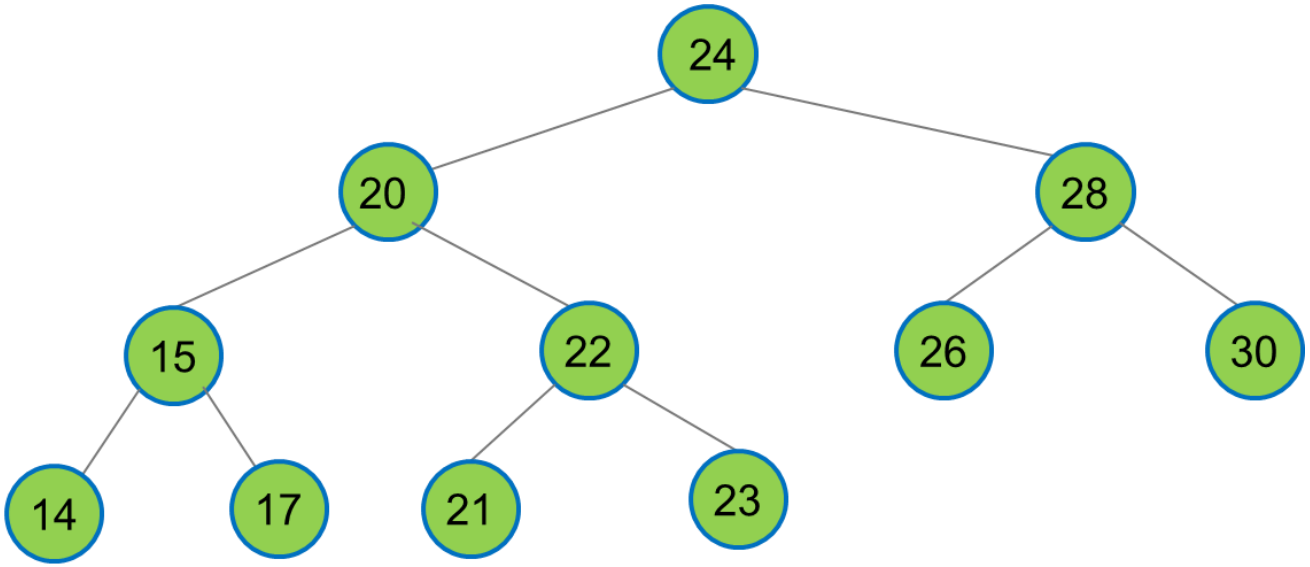
- The main functions?
  - Search 17



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- The main functions?
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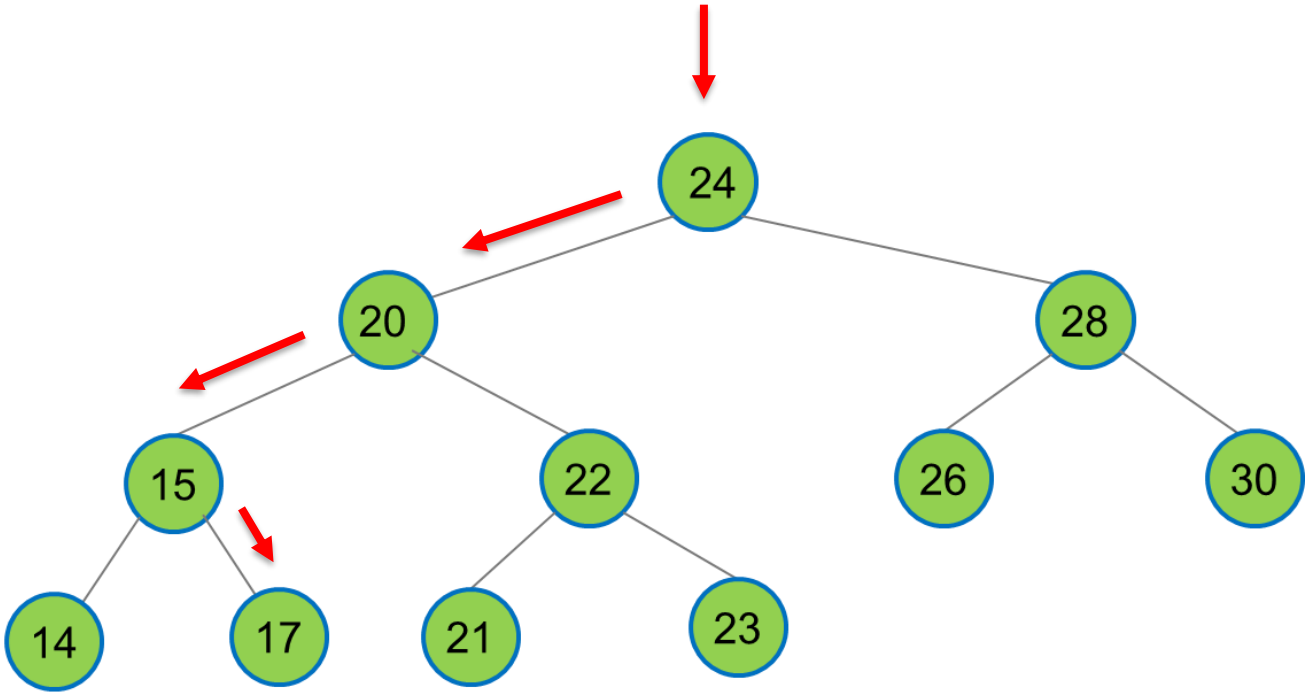

Tree
And Binary Search Tree (BST)

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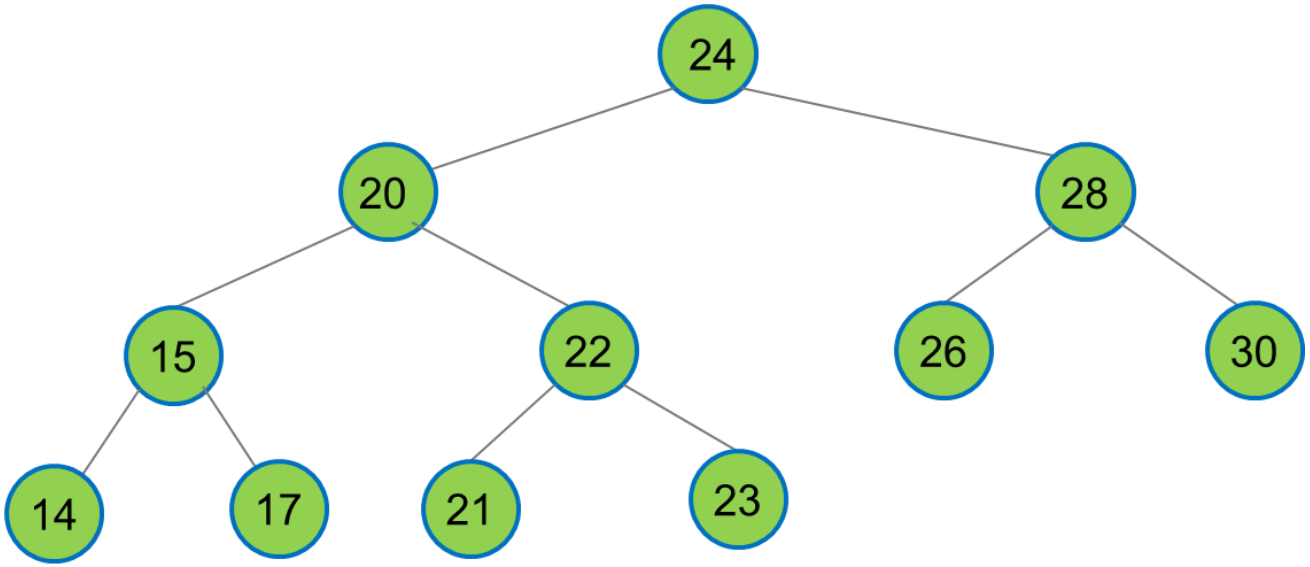
- The main functions?
  - Search 26



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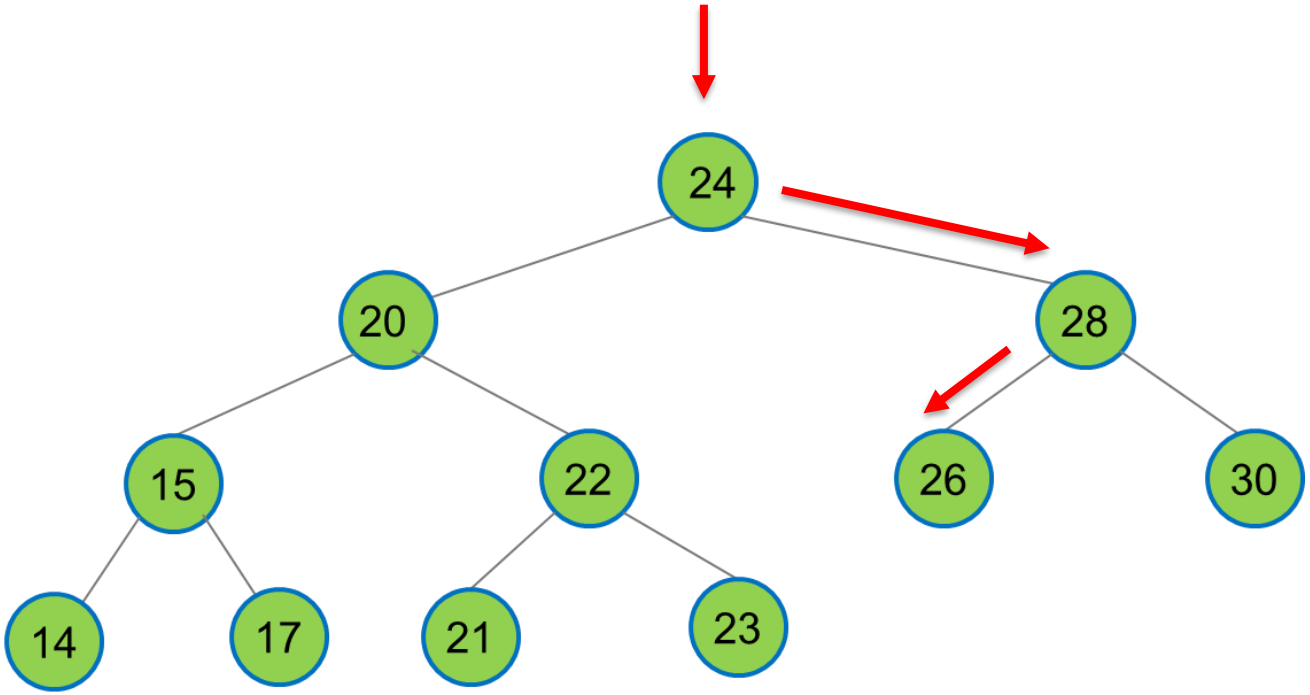
- The main functions?
  - Search 26



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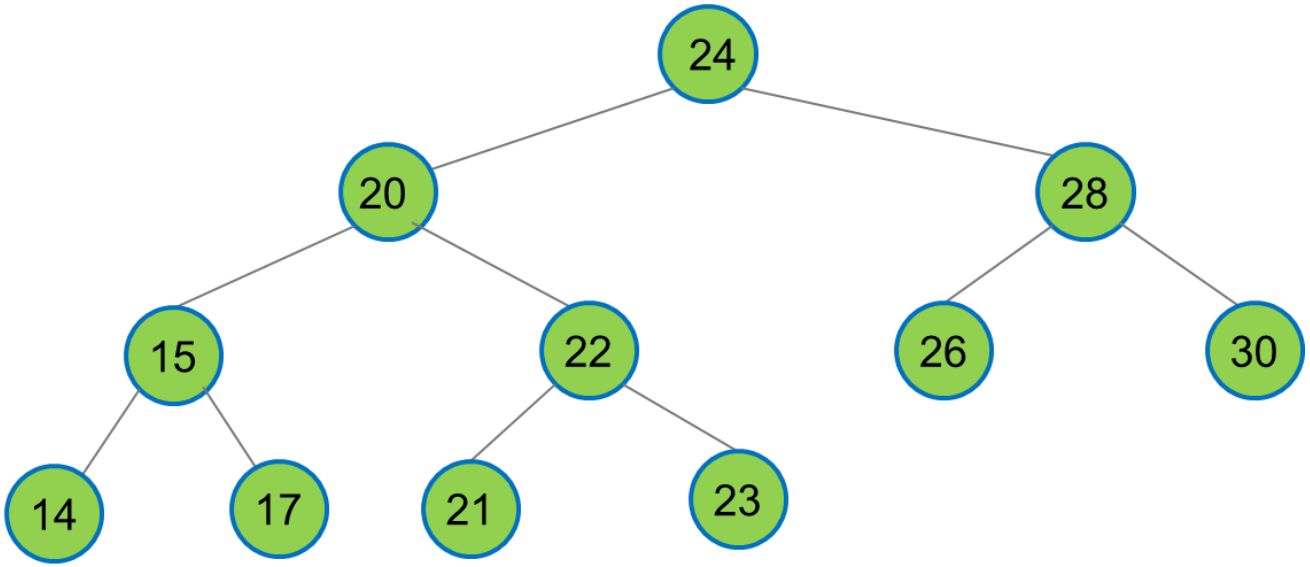
- The main functions?
  - Search 19



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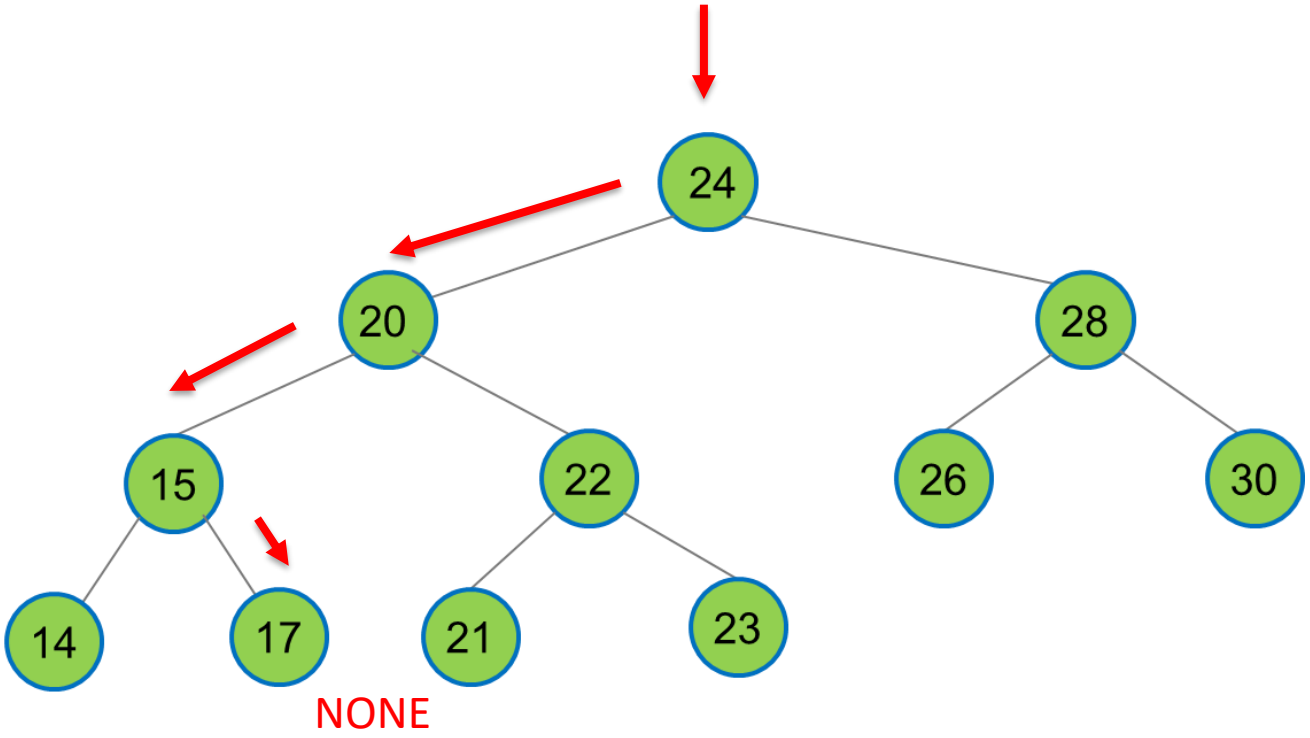
- The main functions?
  - Search 19



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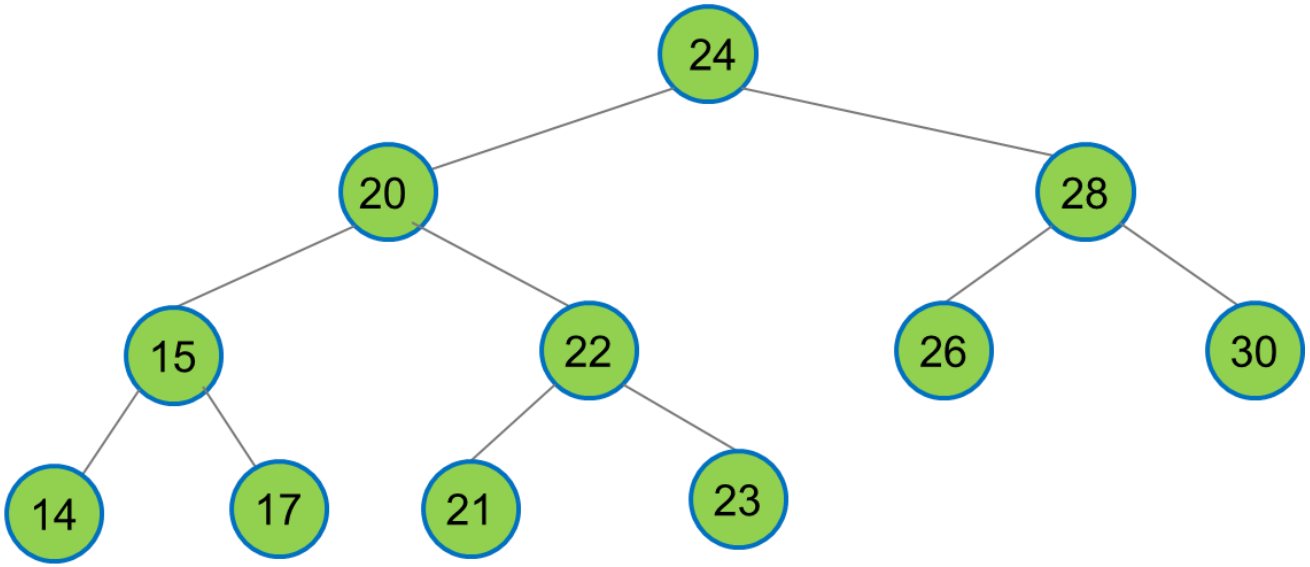
- The main functions?
  - Search for smallest number in BST



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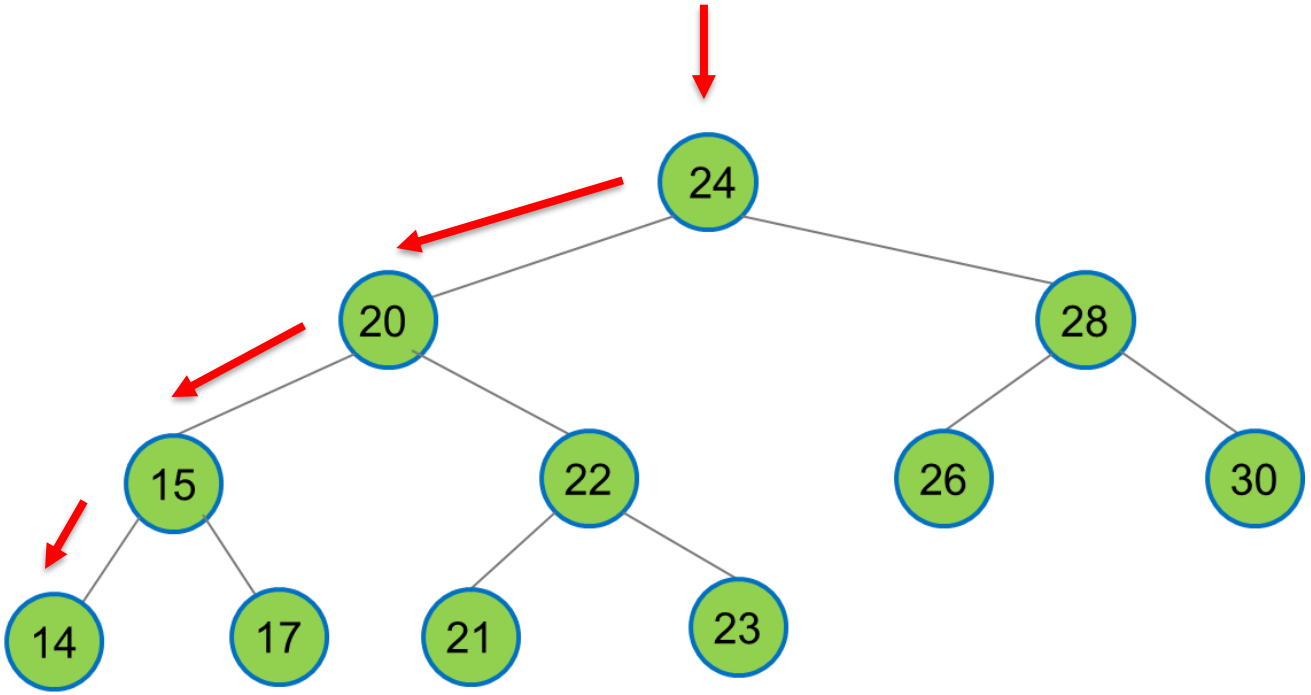

Tree
And Binary Search Tree (BST)

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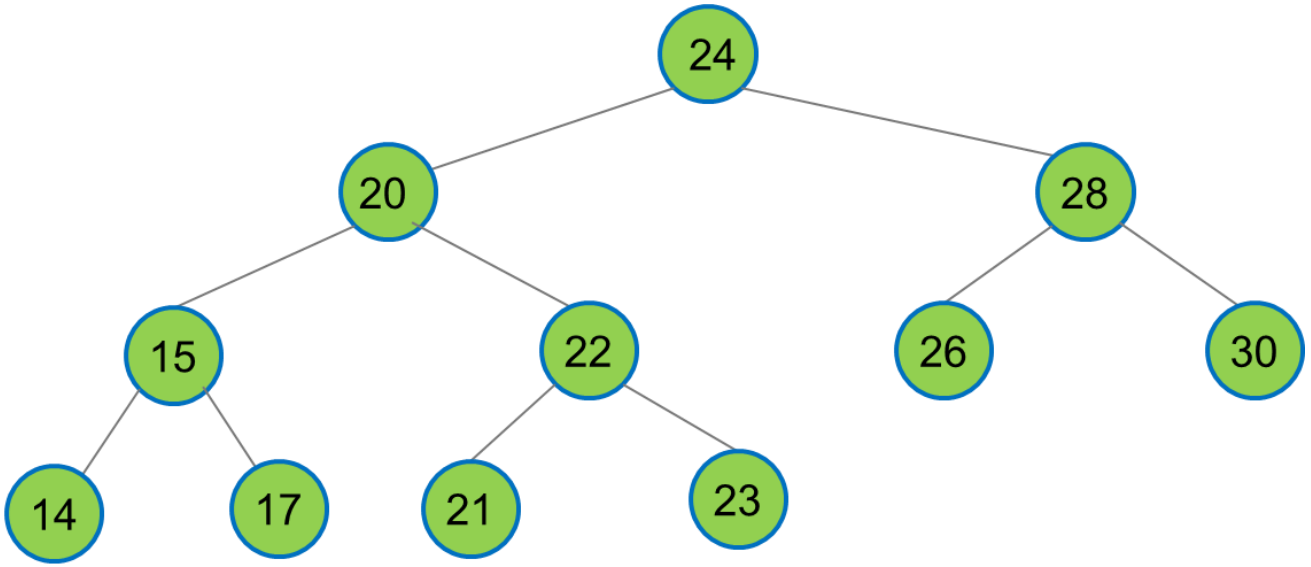
- The main functions?
  - Search complexity?



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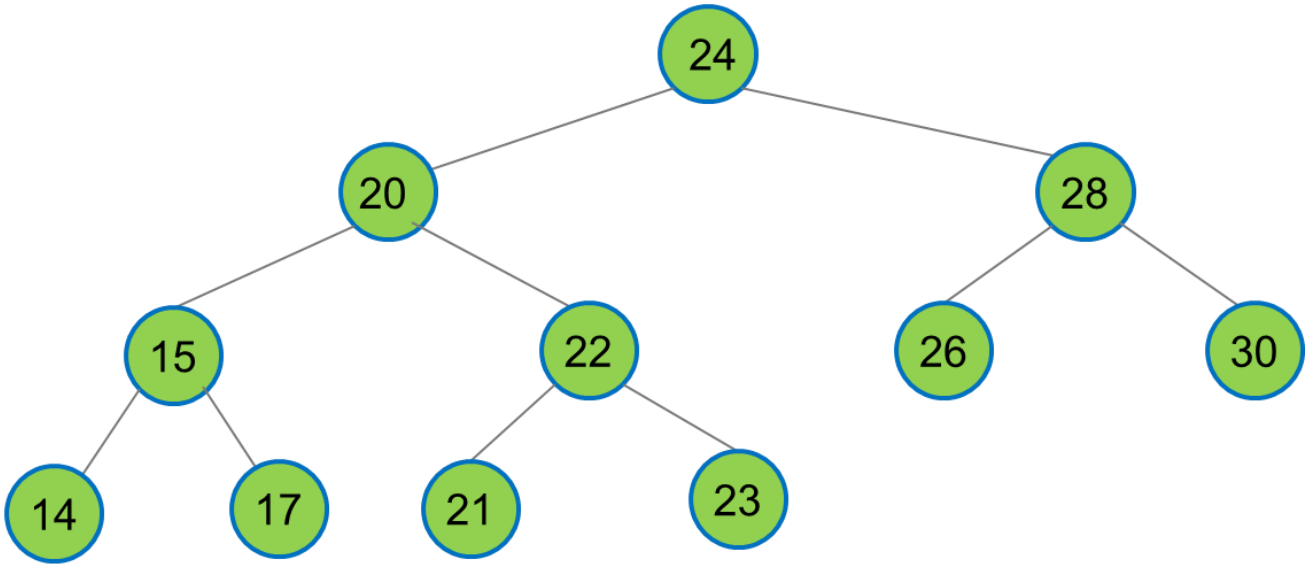
- The main functions?
  - Search complexity? O(log N)?



Tree
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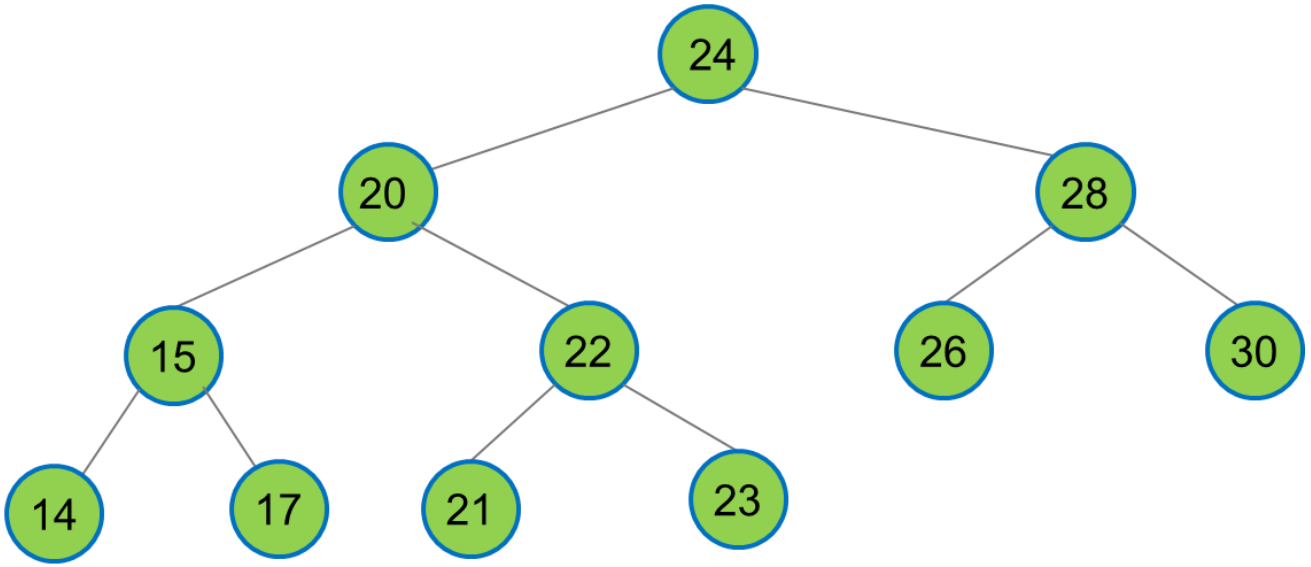

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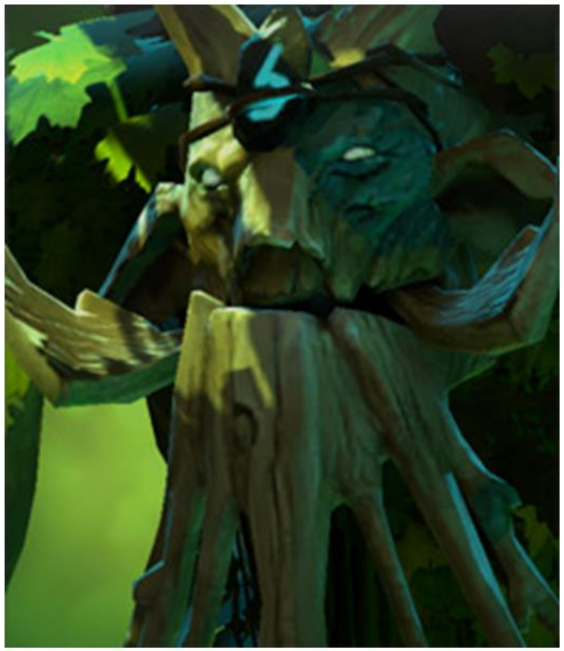
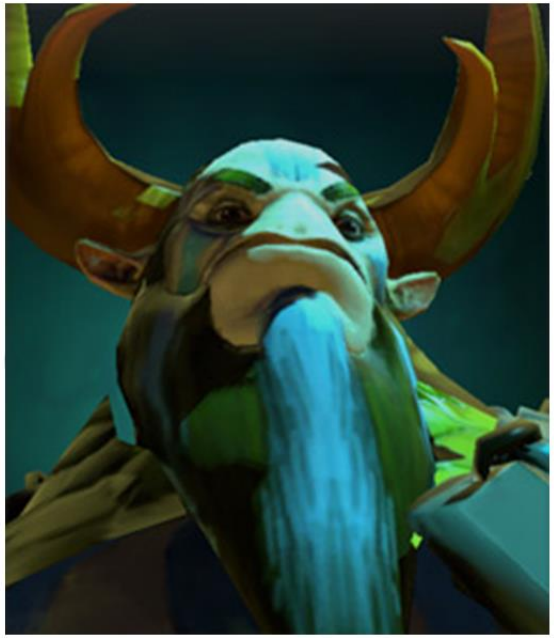
- The main functions?
  - Search complexity? O(N)
    - Complexity is high when tree is imbalanced!

Non-balanced



Balanced



Tree
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- The main functions?
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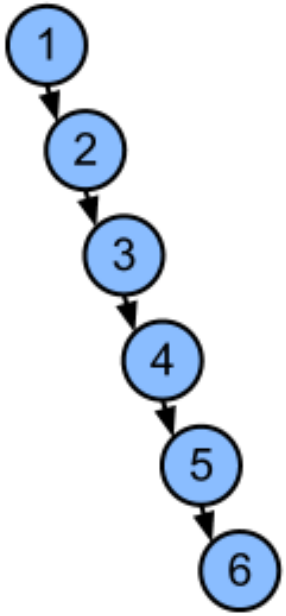
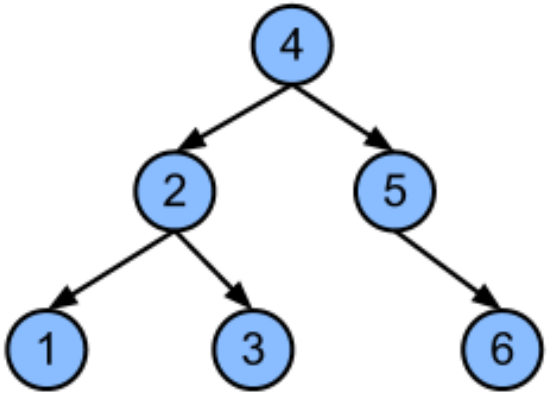


Balanced



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Questions?

AVL Tree

The better BST!

- This is BST

AVL Tree

The better BST!

- This is BST

This is AVL

AVL Tree

The better BST!

- This is BST

This is Adelson-Velskii Landis (AVL)

AVL Tree

The better BST!

- AVL trees are self balancing…

AVL Tree

The better BST!

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- AVL trees are self balancing…
- Height balanced
  - absolute(Height(left) – Height(right)) <= 1
  - Ensure a maximum of O(log N) height

AVL Tree

The better BST!

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- AVL trees are self balancing…
- Height balanced
  - absolute(Height(left) – Height(right)) <= 1
  - Ensure a maximum of O(log N) height
  - True for each subtree as well

AVL Tree

The better BST!

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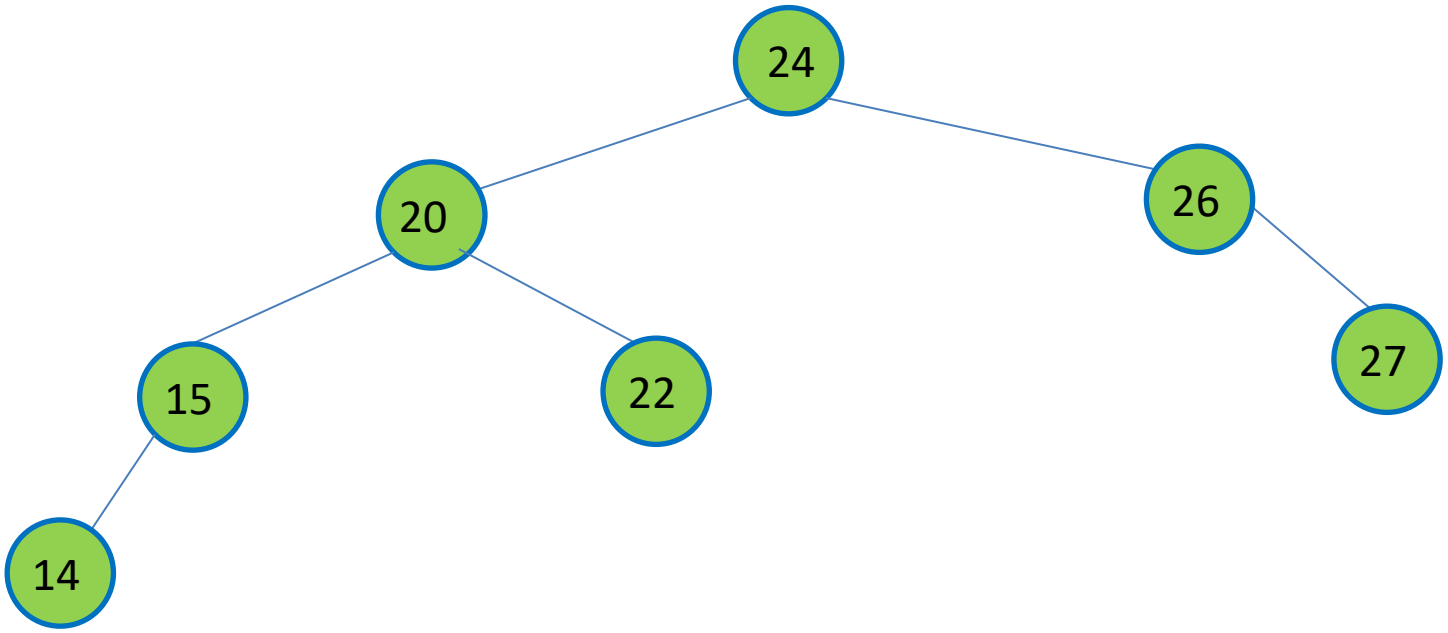
- AVL trees are self balancing…
- Height balanced
  - absolute(Height(left) – Height(right)) <= 1
  - Ensure a maximum of O(log N) height
  - True for each subtree as well
  - Now let us see the trees

AVL Tree

The better BST!

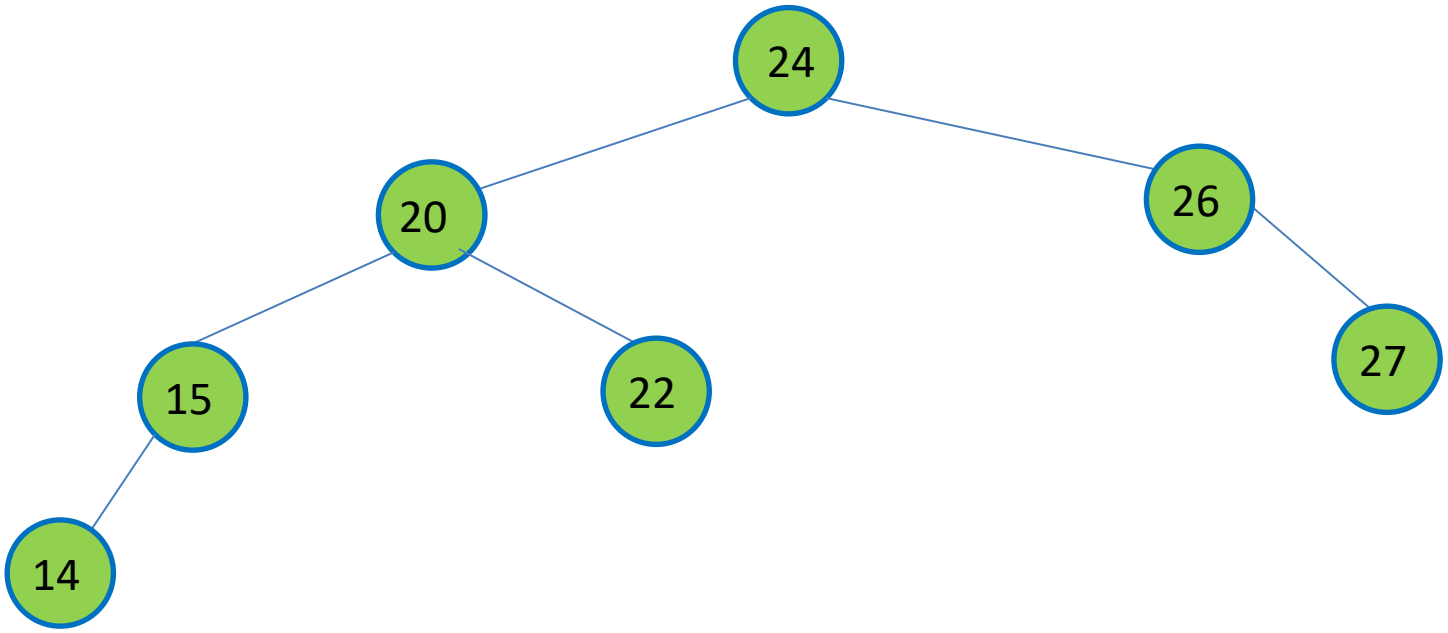
- What is the height?



AVL Tree

The better BST!

- What is the height?
  - Let us do it by hand



AVL Tree

The better BST!

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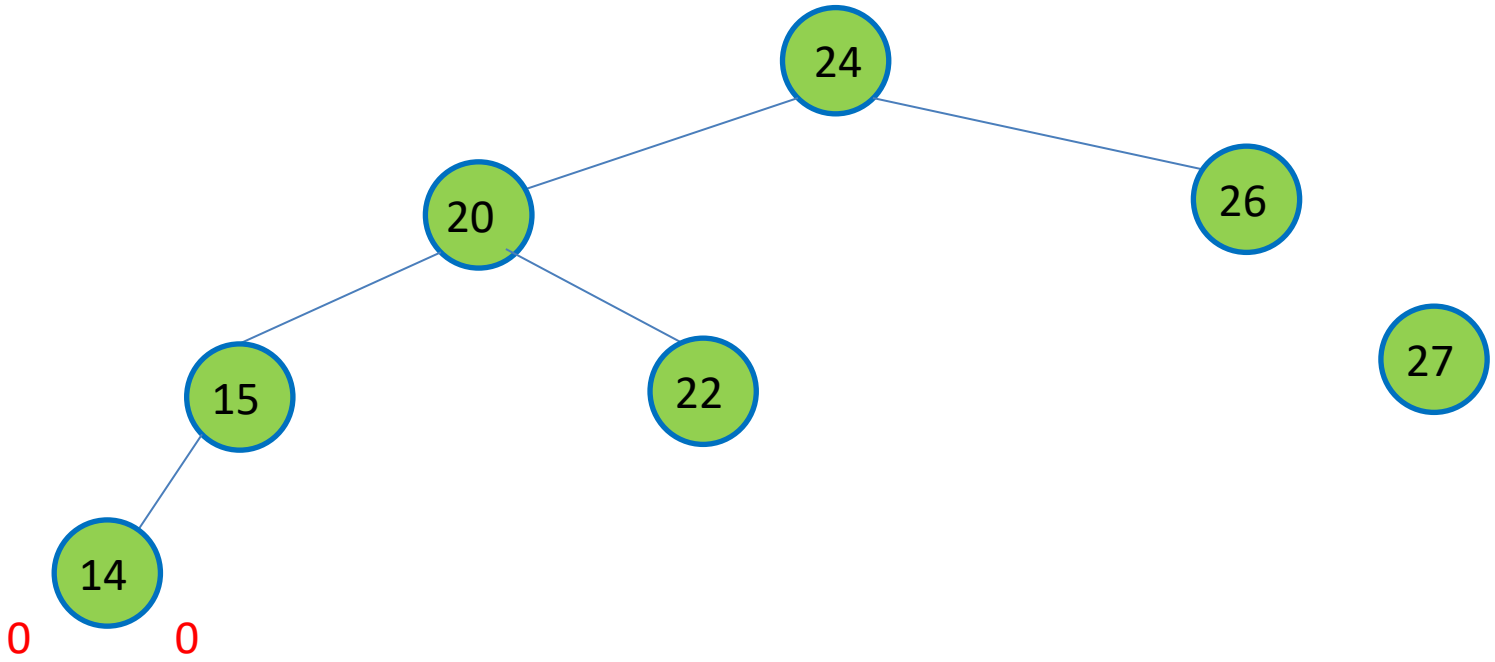

AVL Tree
The better BST!

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- What is the height?
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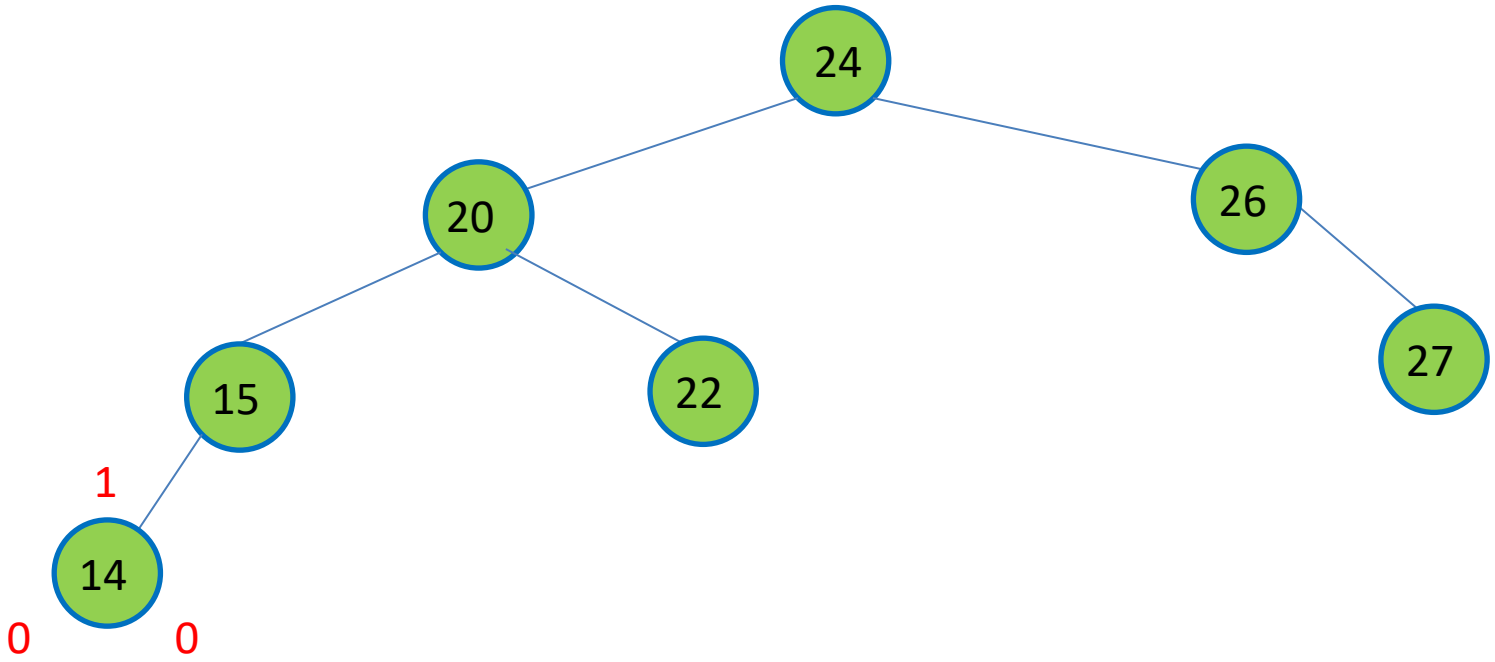

AVL Tree

The better BST!

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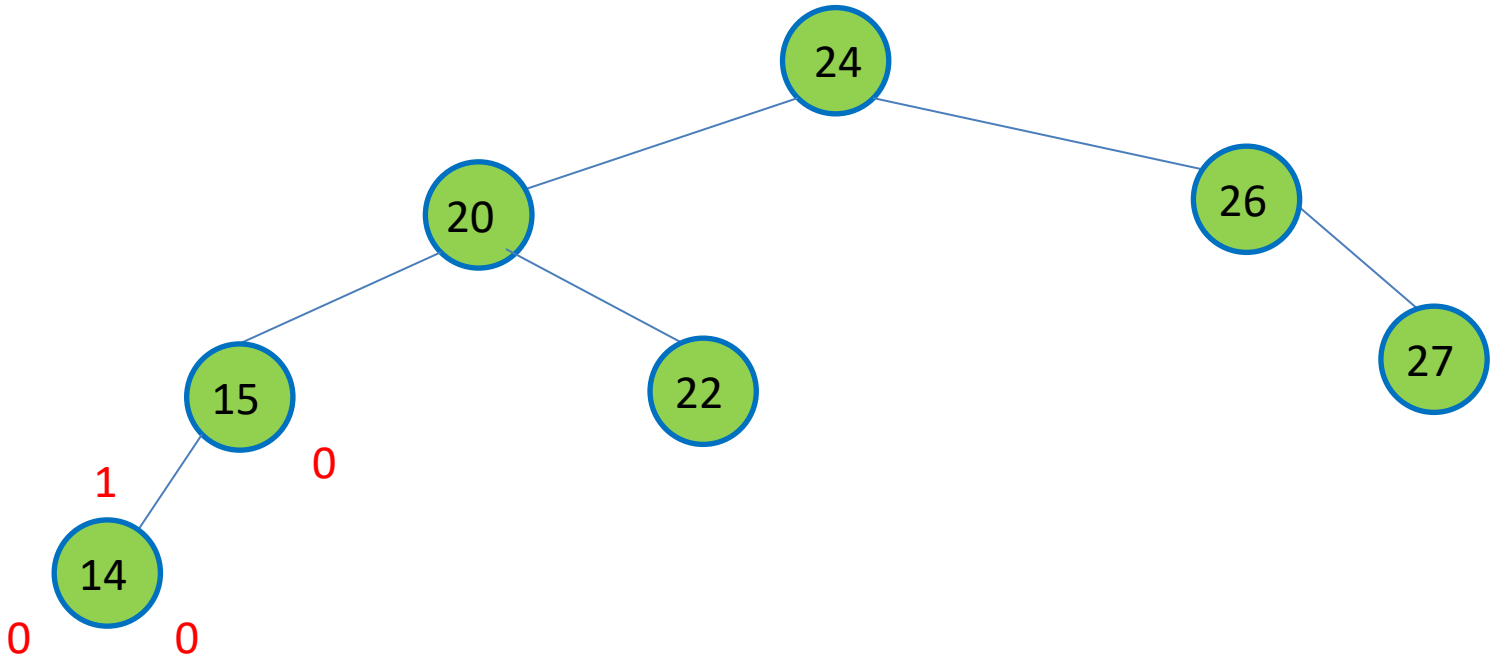

AVL Tree

The better BST!

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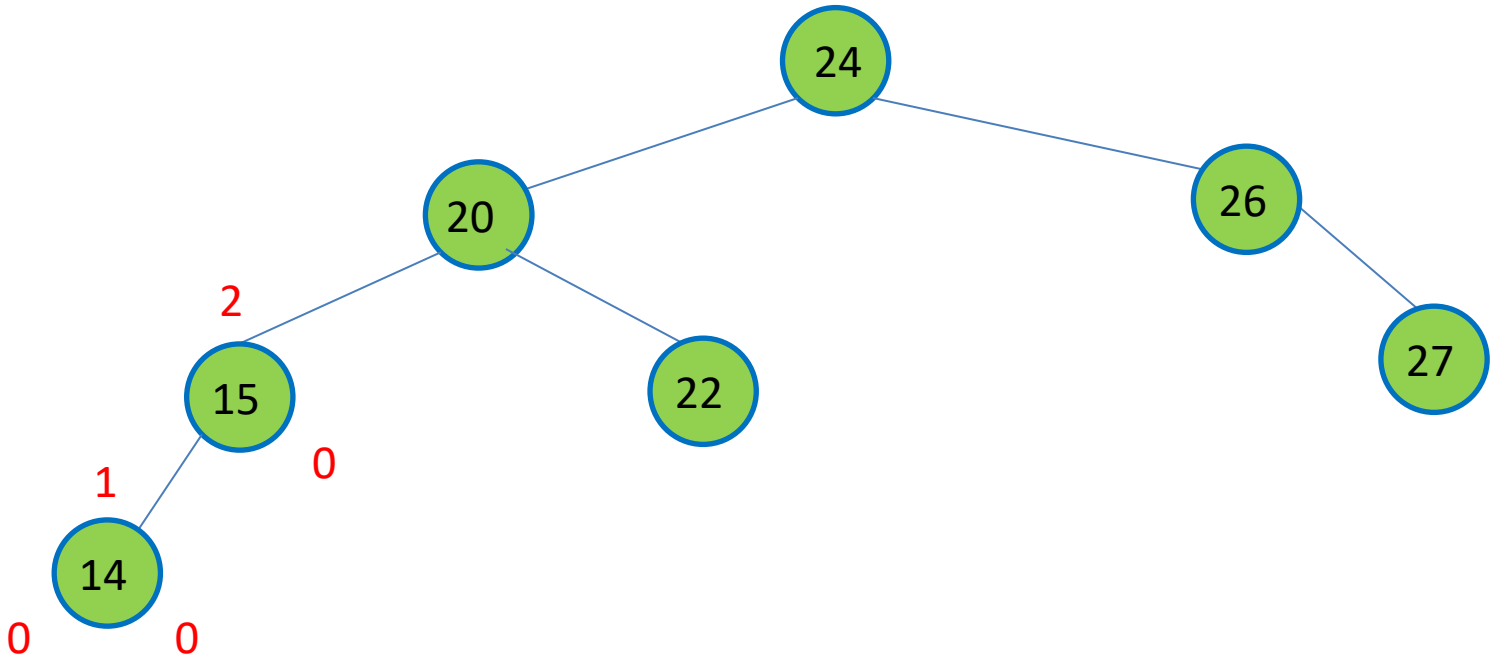

AVL Tree

The better BST!

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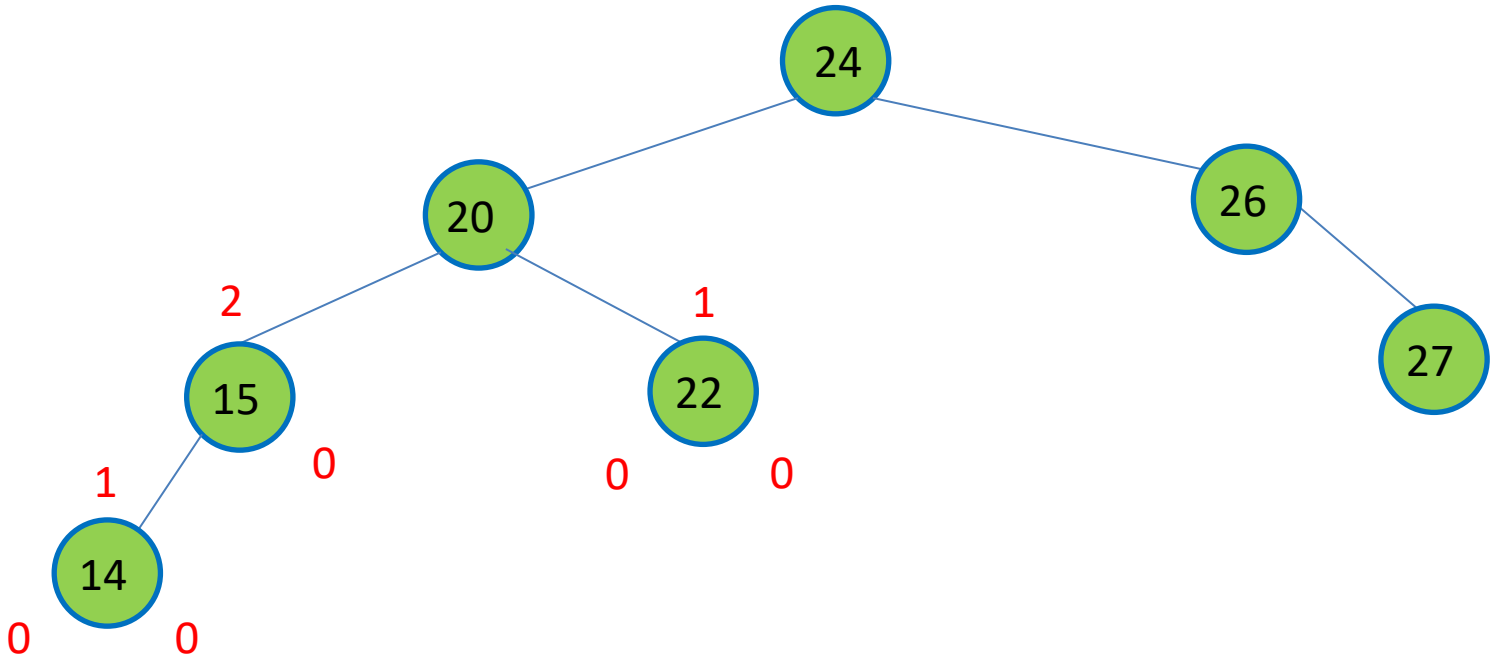

AVL Tree

The better BST!

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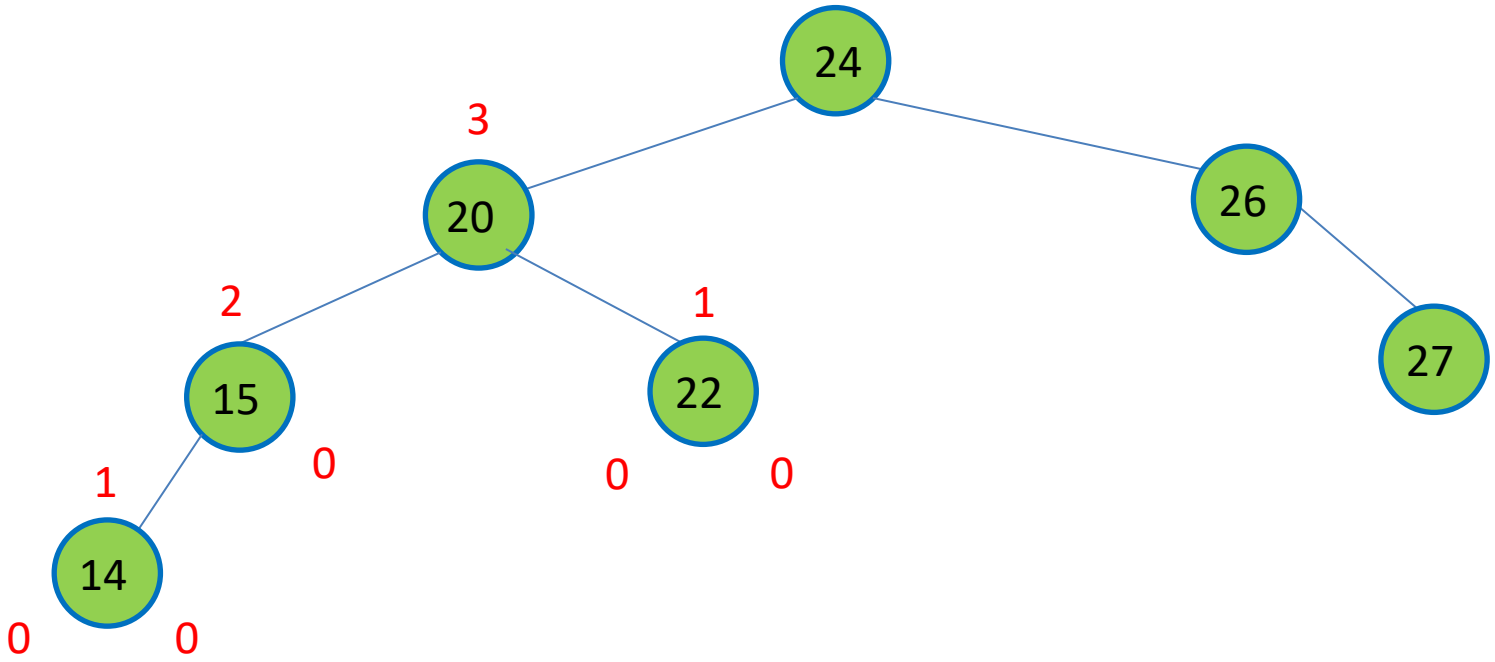

AVL Tree

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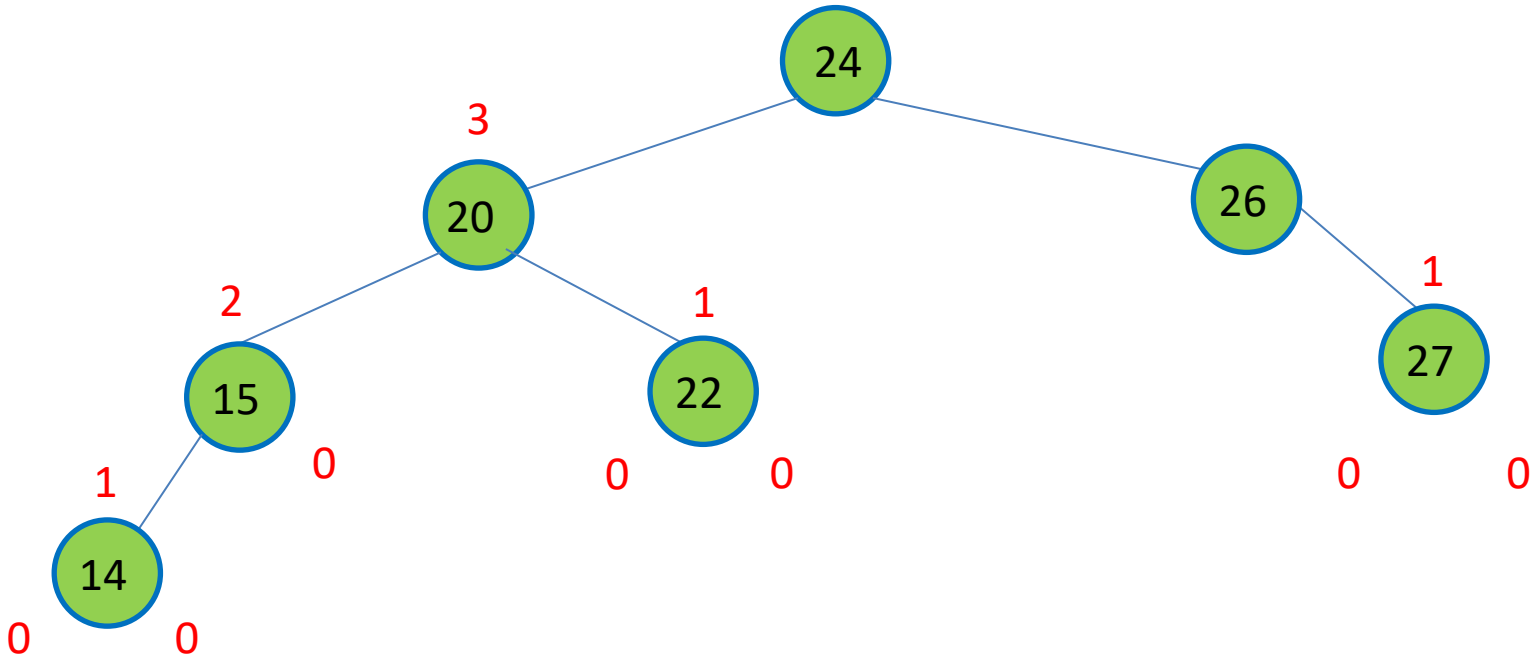

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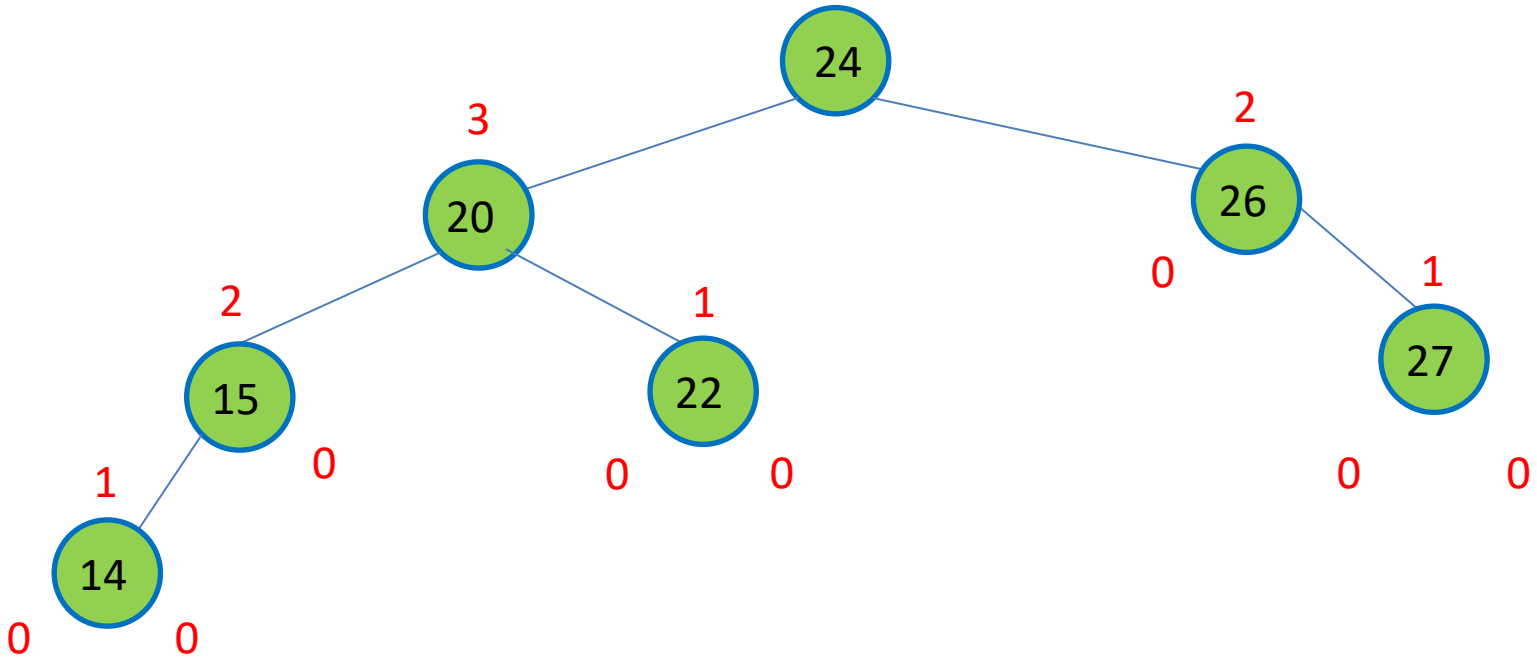

AVL Tree

The better BST!

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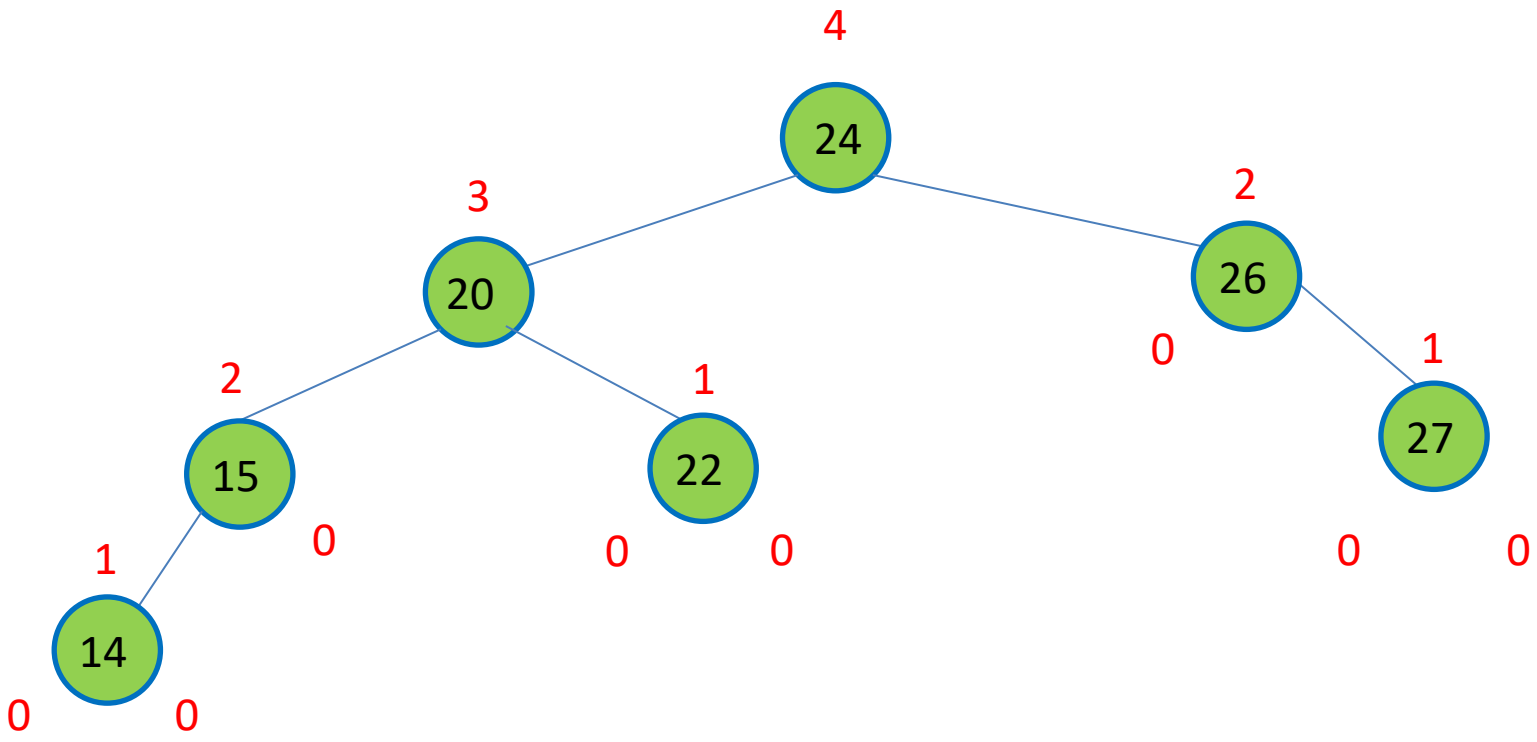
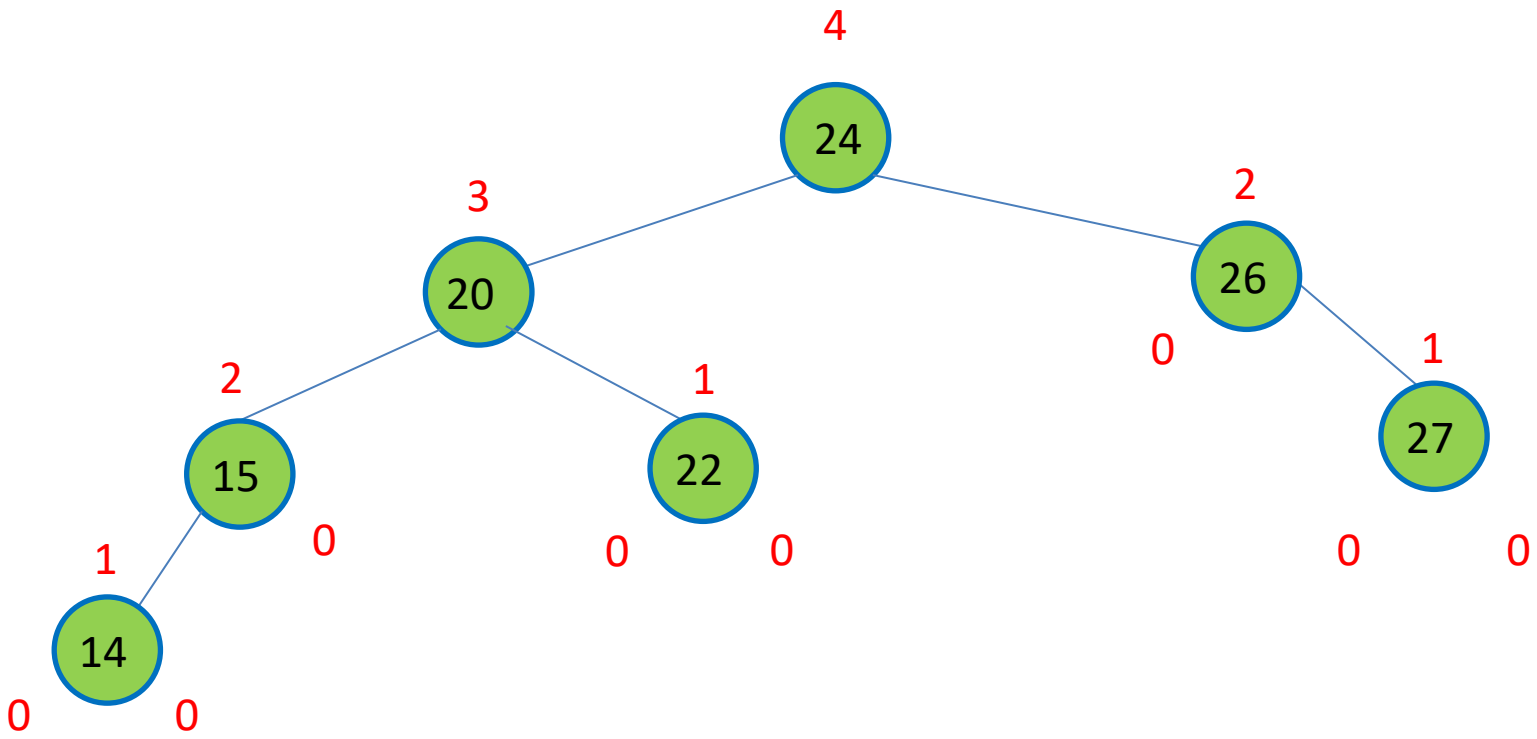


AVL Tree

The better BST!

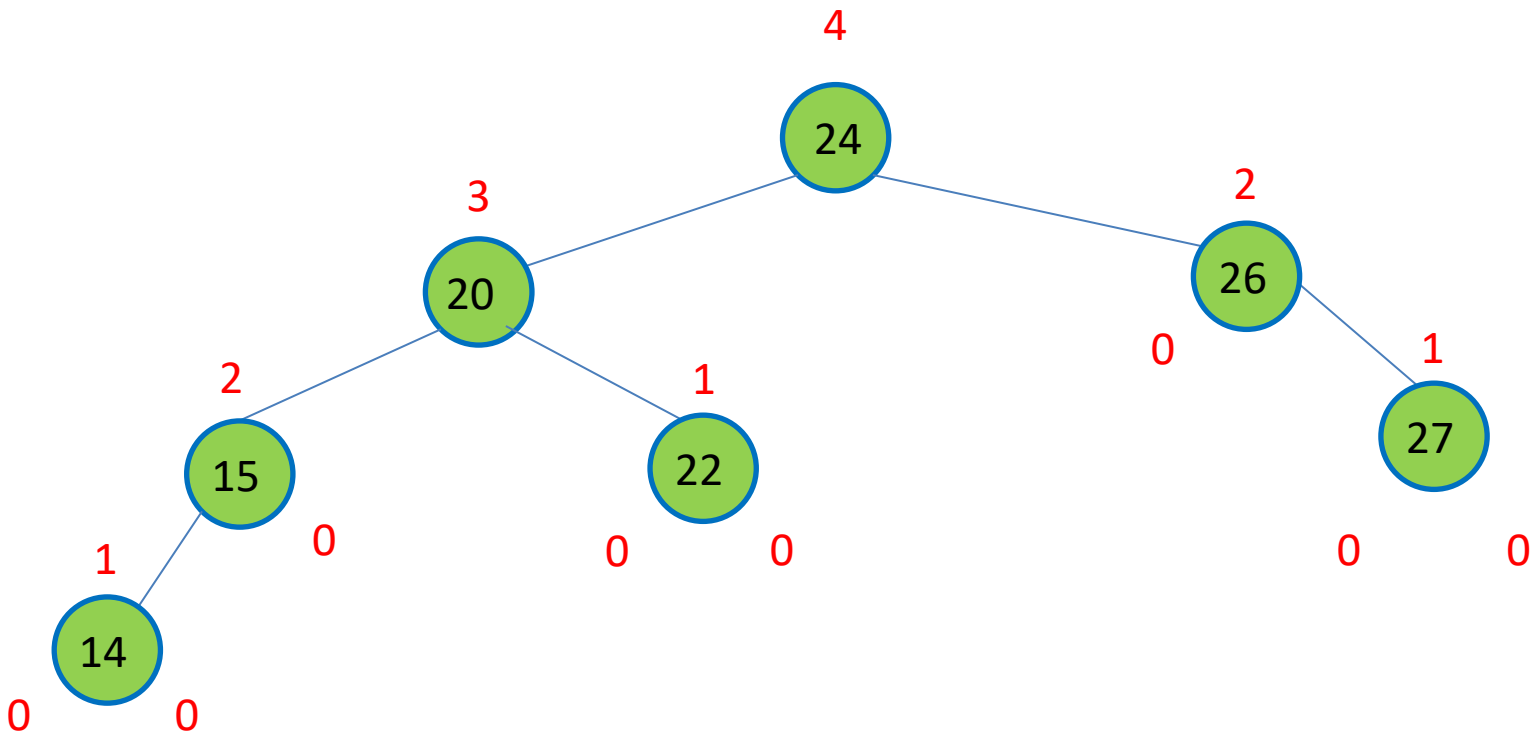
- What is the height?
- Is the tree balanced?



AVL Tree

The better BST!

- What is the height?
- Is the tree balanced?
  - Balance factor = {-1, 0, 1}



AVL Tree

The better BST!

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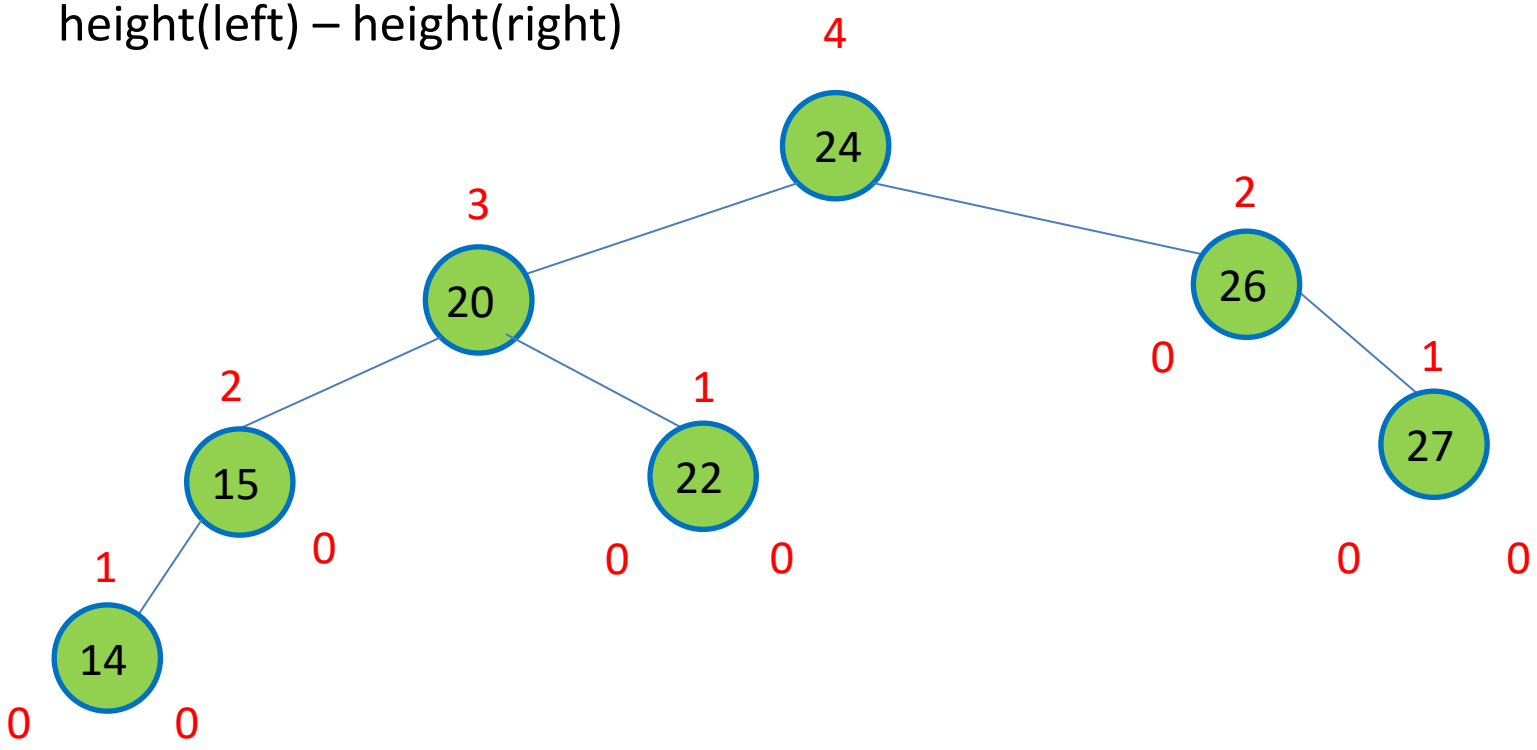
- What is the height?
- Is the tree balanced?
  - Balance factor = {-1, 0, 1}
    height(left) – height(right)



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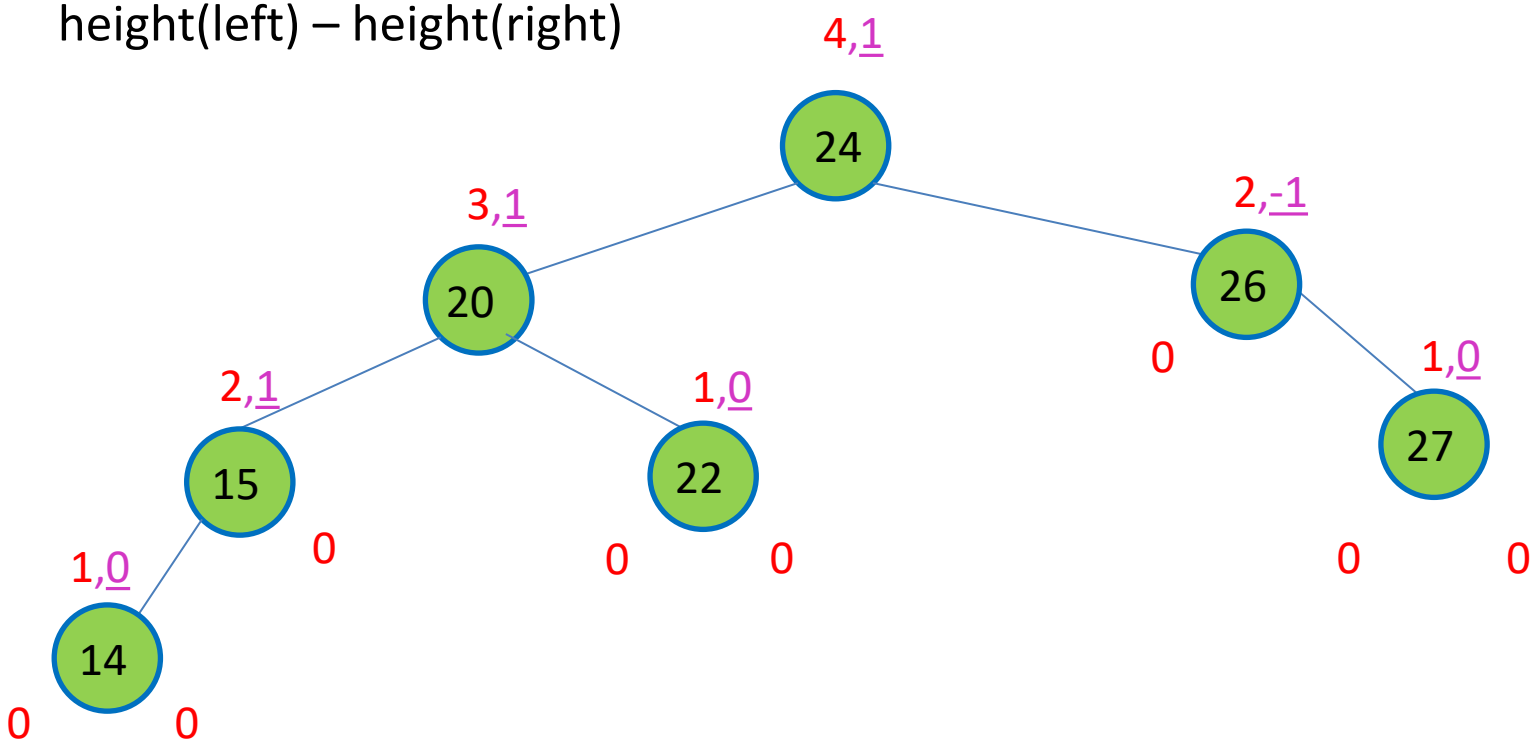
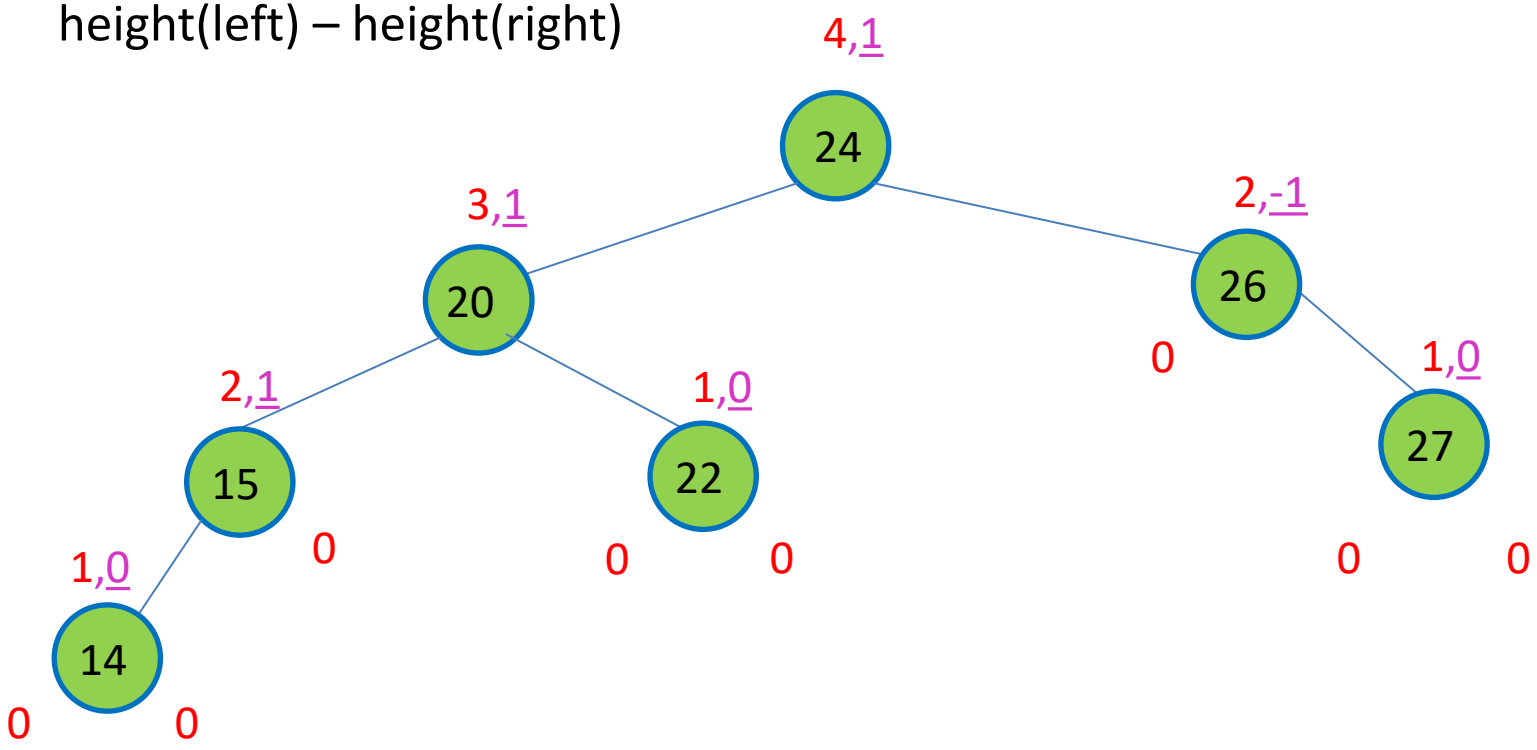


AVL Tree

The better BST!

- What is the height?
- Is the tree balanced? YES
  - Balance factor = {-1, 0, 1}
    height(left) – height(right)



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Questions?

AVL Tree

The better BST!

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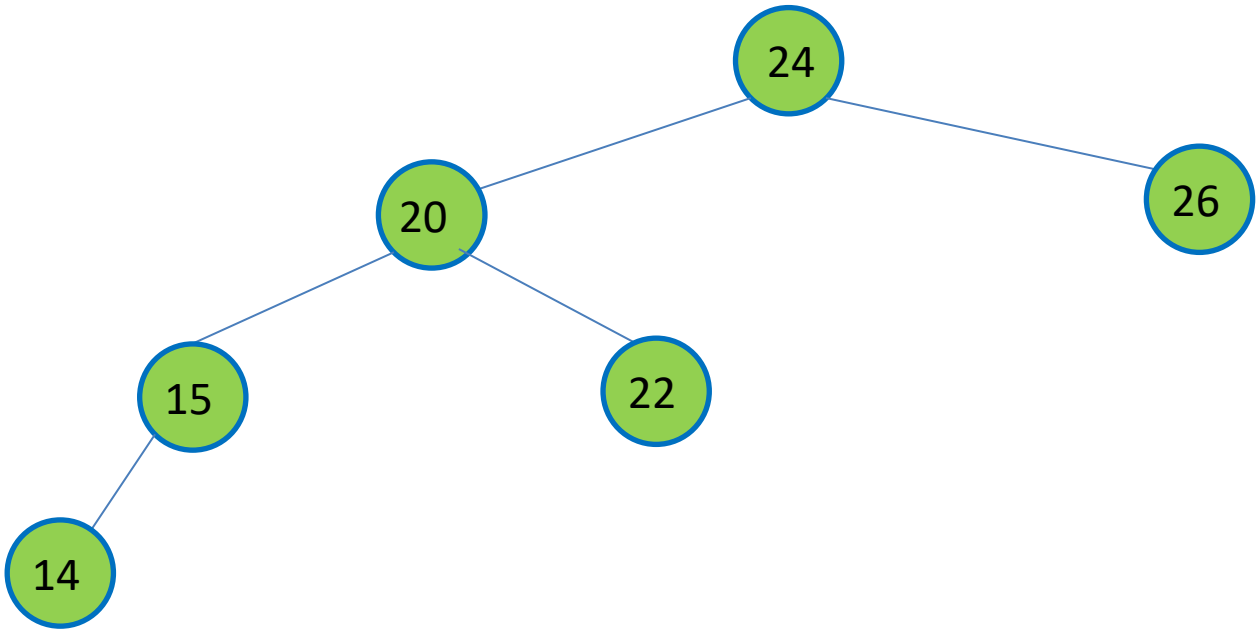
- Let's try a new tree…



AVL Tree

The better BST!

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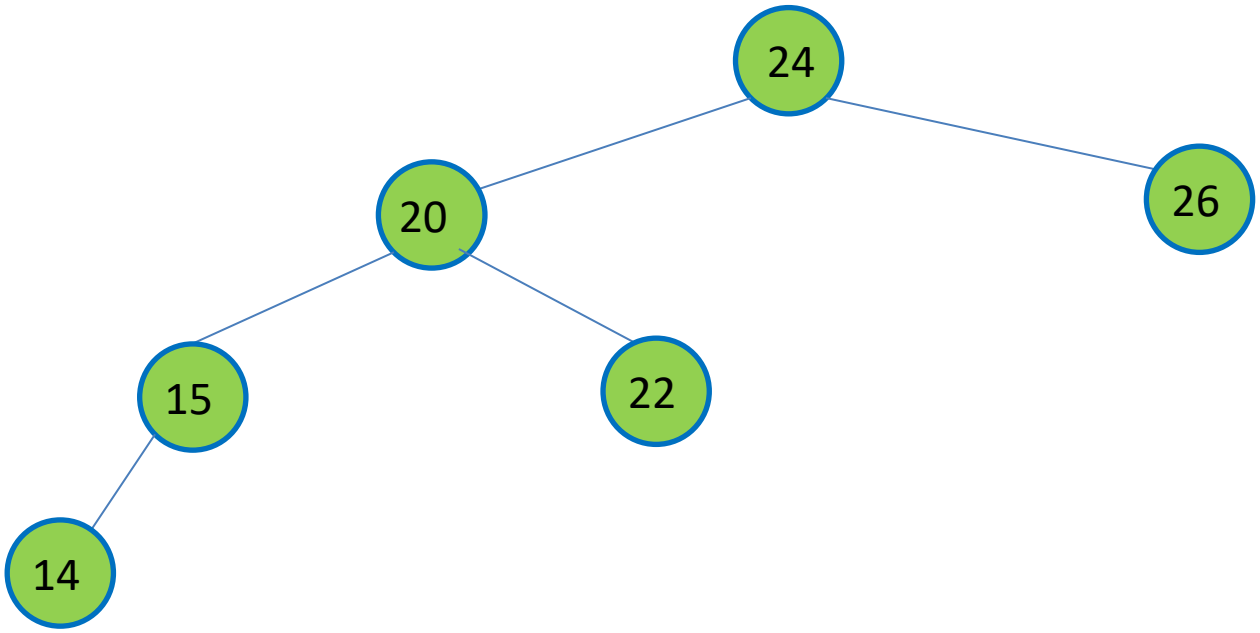
- Let's try a new tree…
  - Is this balanced?



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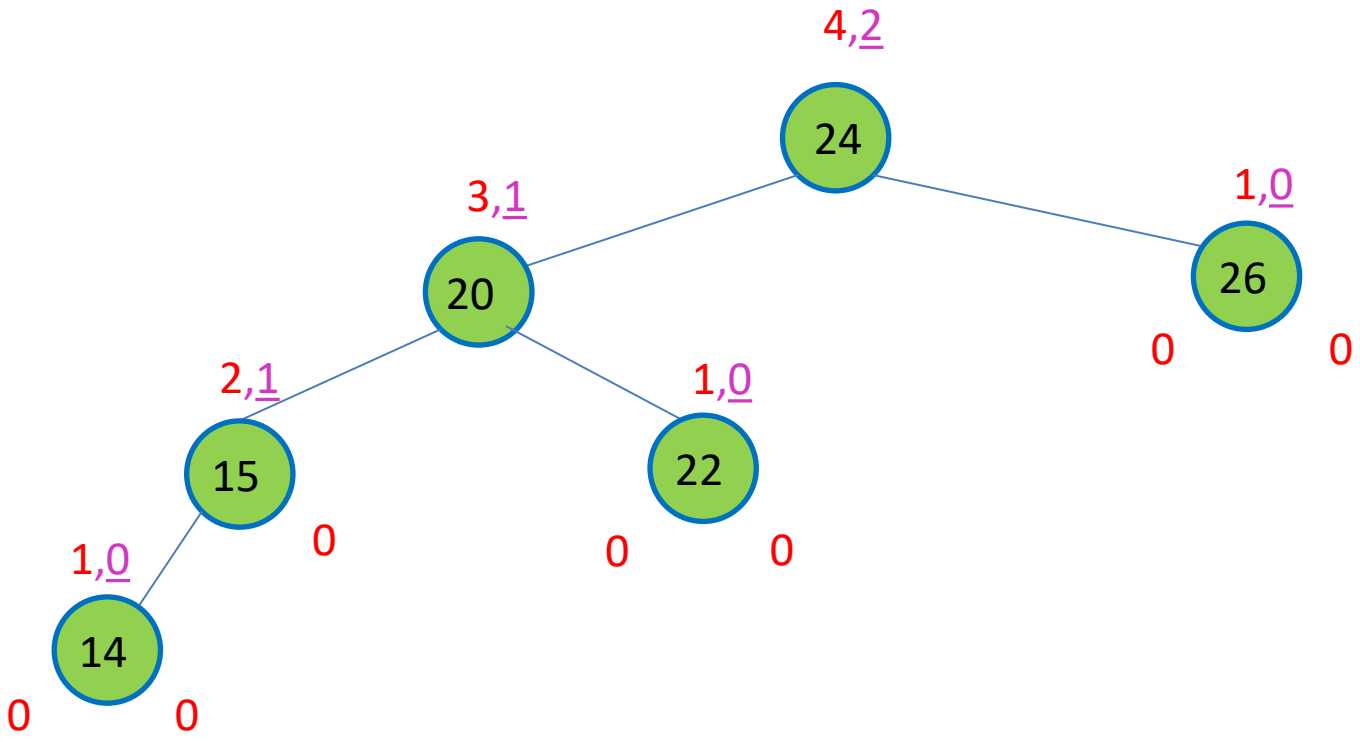

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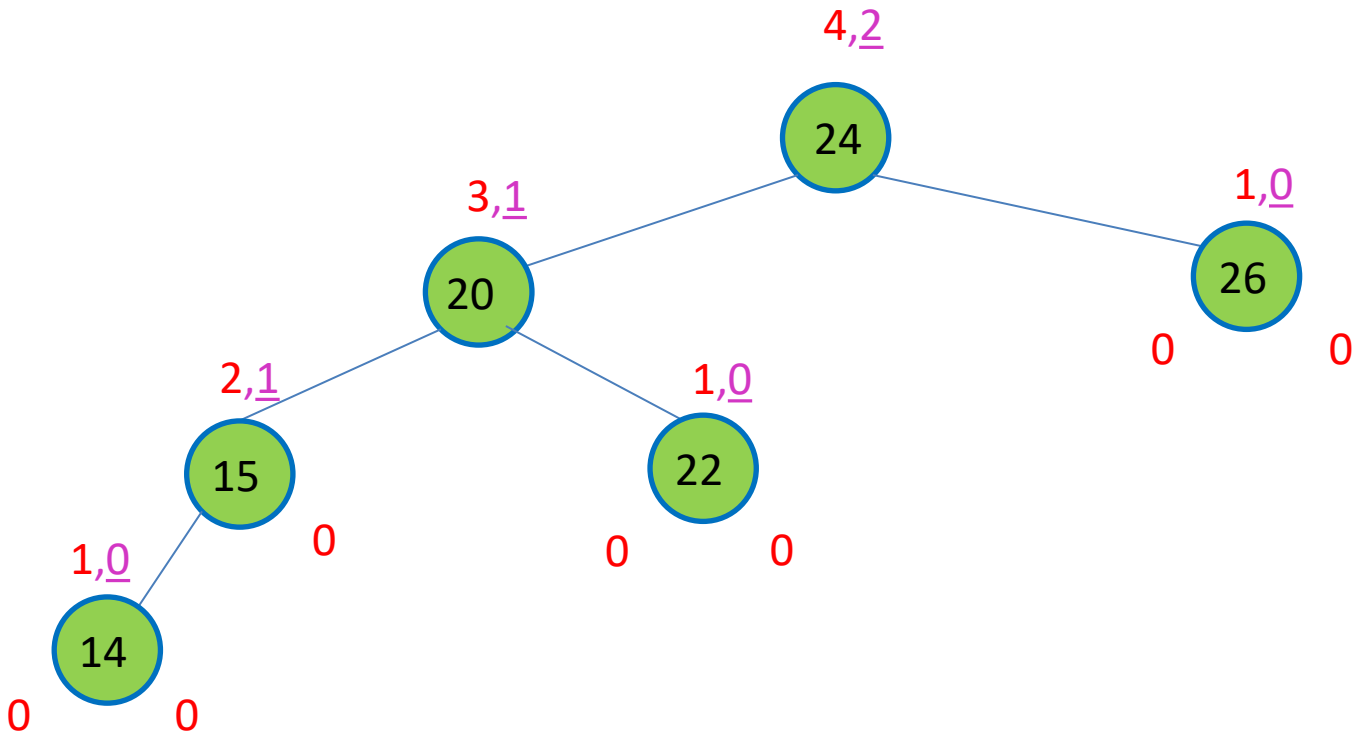

AVL Tree
The better BST!

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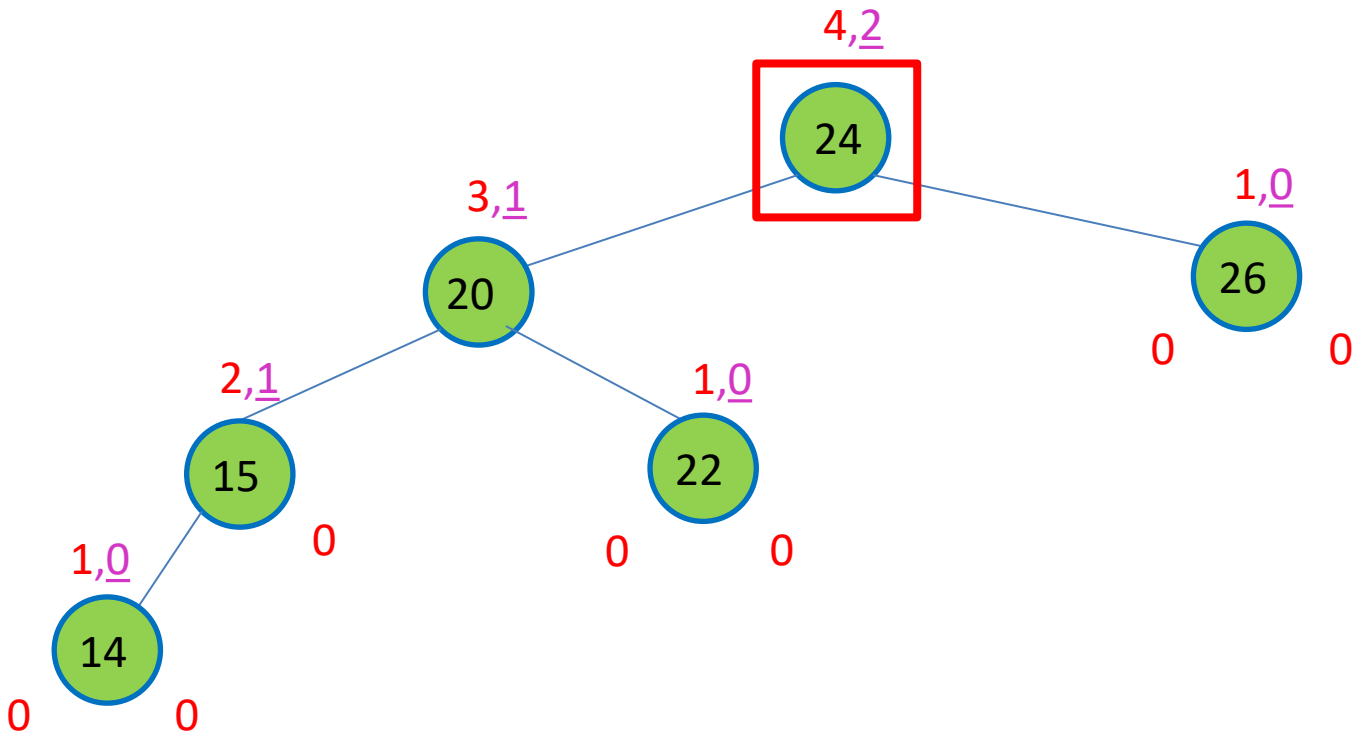
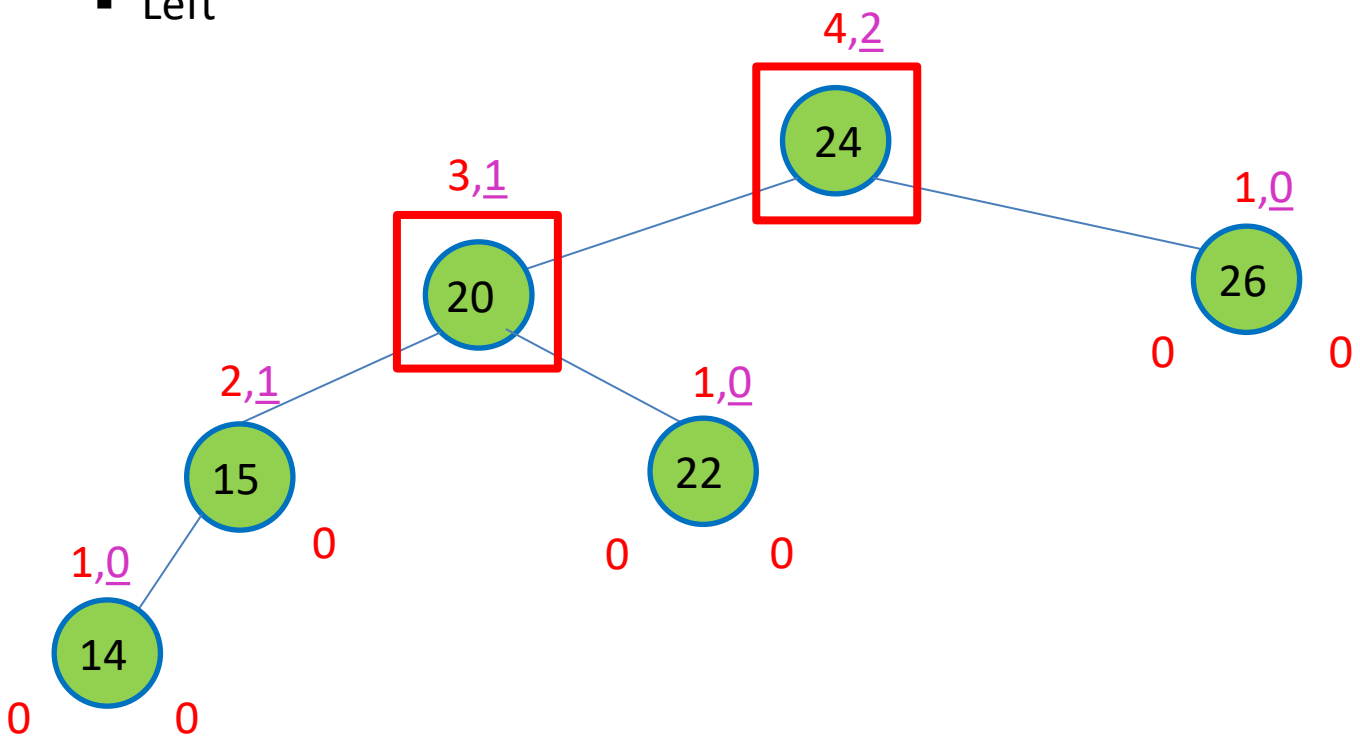
- Let's try a new tree…
  - Is this balanced?
    - No



AVL Tree
The better BST!

MONASH University

- Let's try a new tree…
  - Is this balanced?
    - No
    - Left

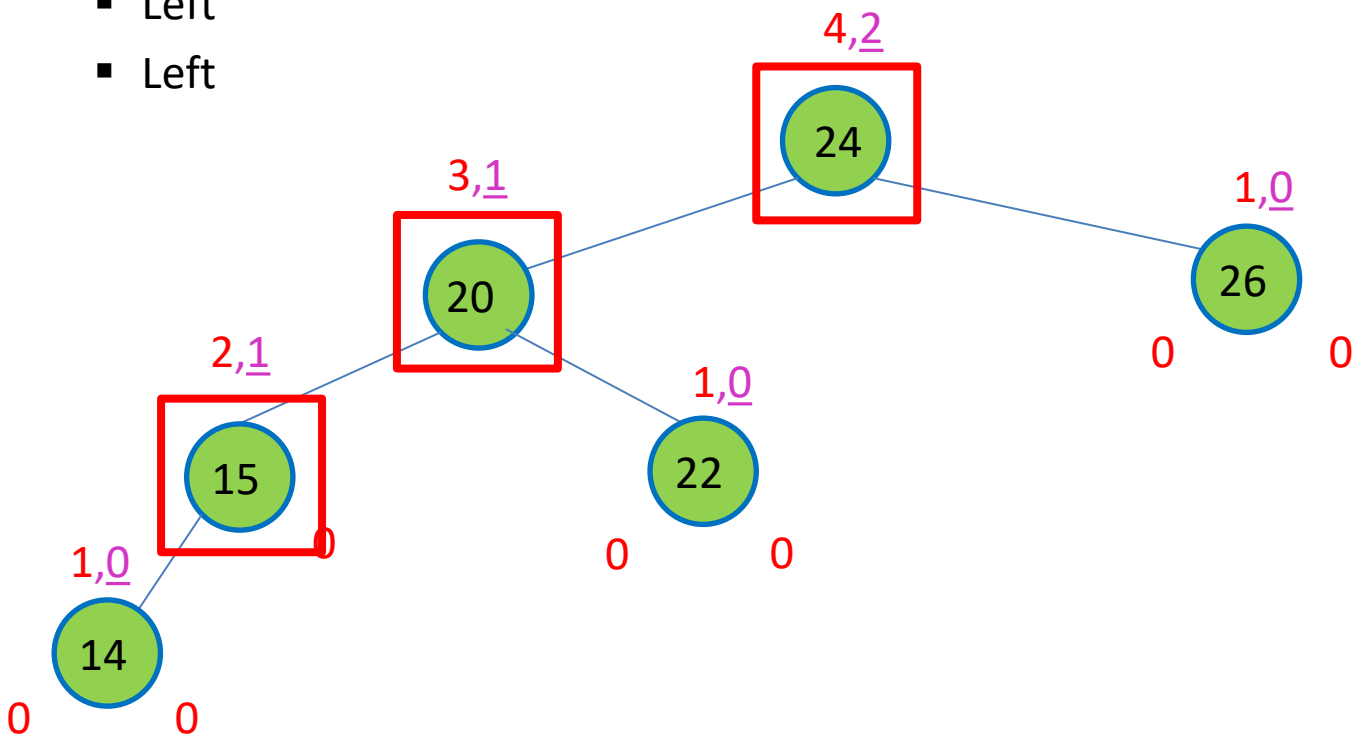


AVL Tree

The better BST!

MONASH University

- Let's try a new tree…
  - Is this balanced?
    - No
    - Left
    - Left

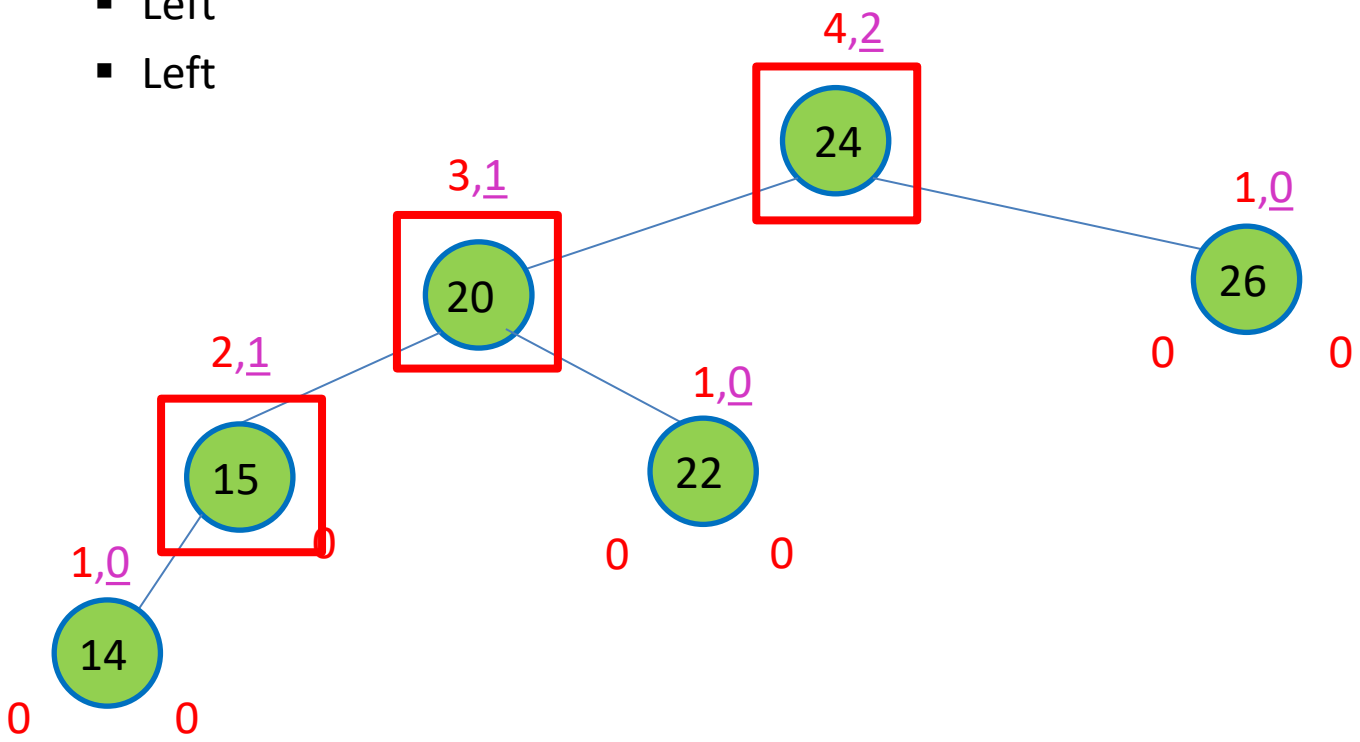


AVL Tree

The better BST!

MONASH University

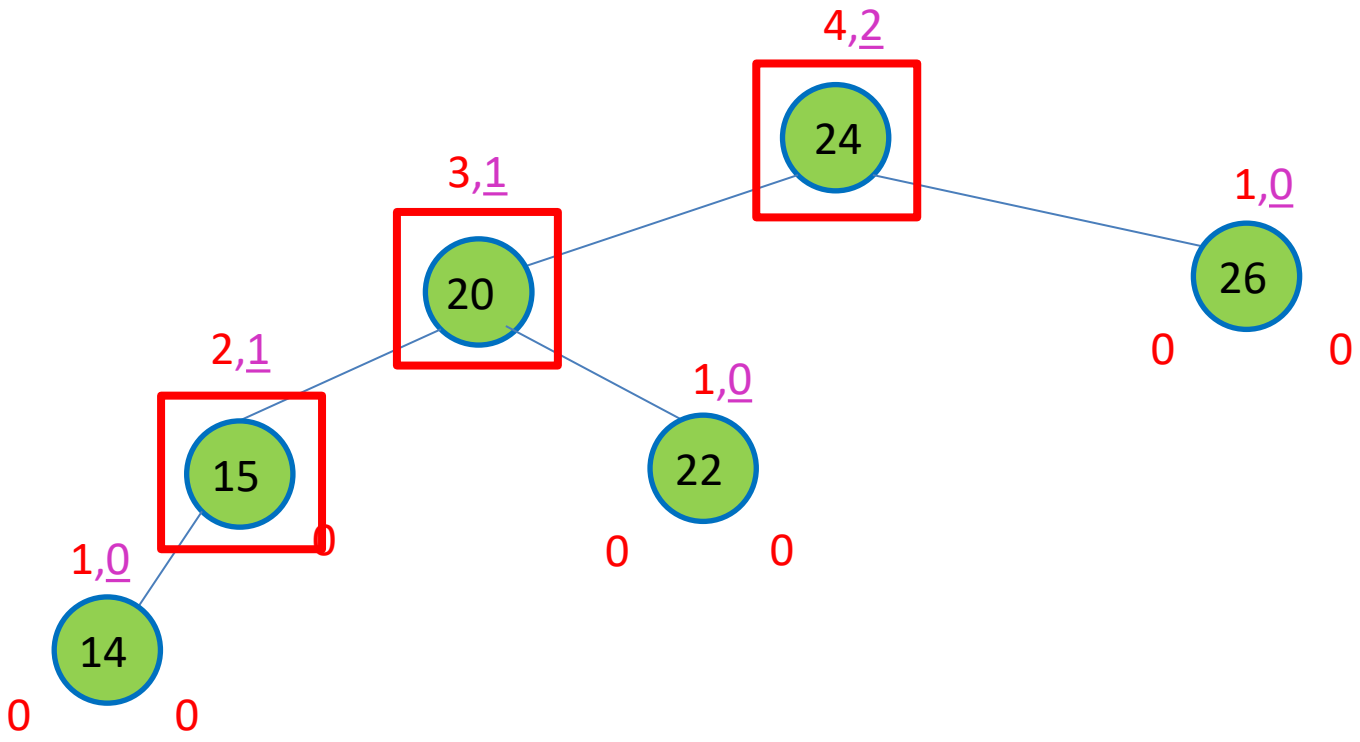
- Let's try a new tree…
  - Is this balanced? Left-Left imbalanced
    - No
    - Left
    - Left



AVL Tree

The better BST!

- How to balance?
  - If left-left imbalanced, rotate right



AVL Tree

The better BST!

- How to balance?
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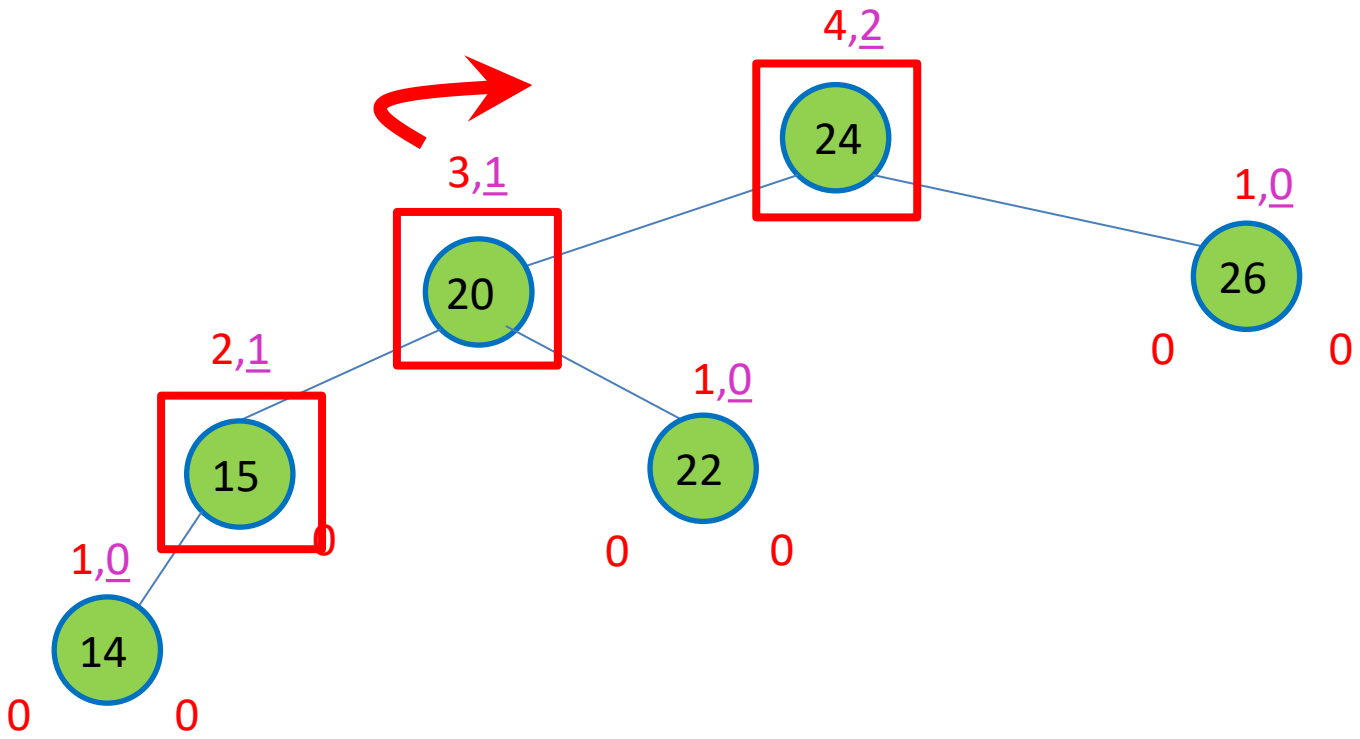
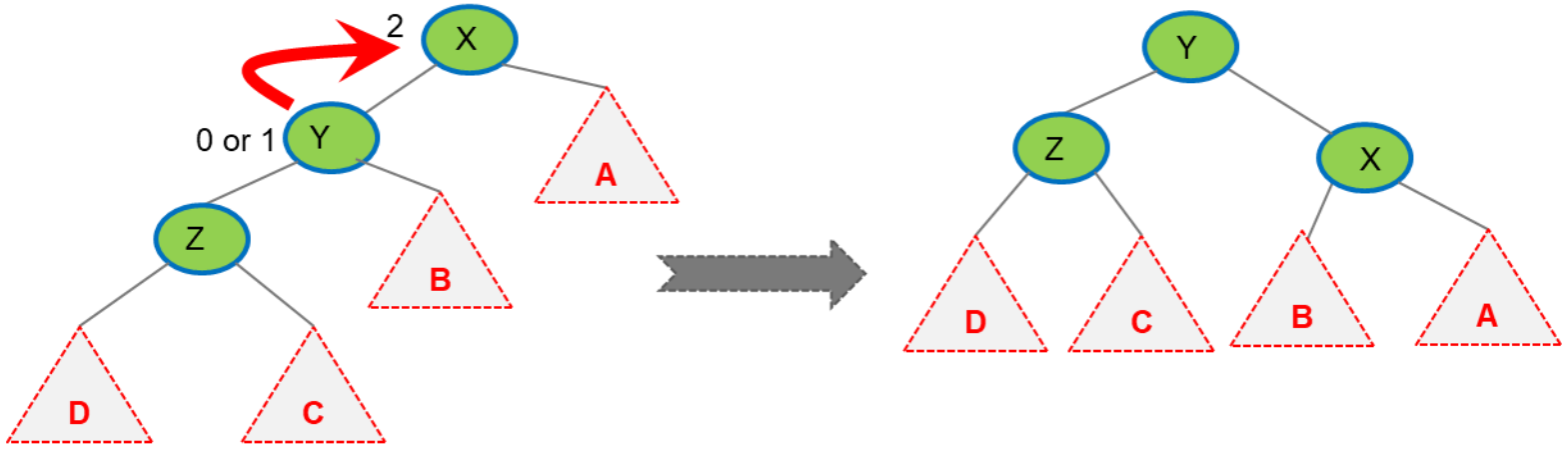


AVL Tree

The better BST!

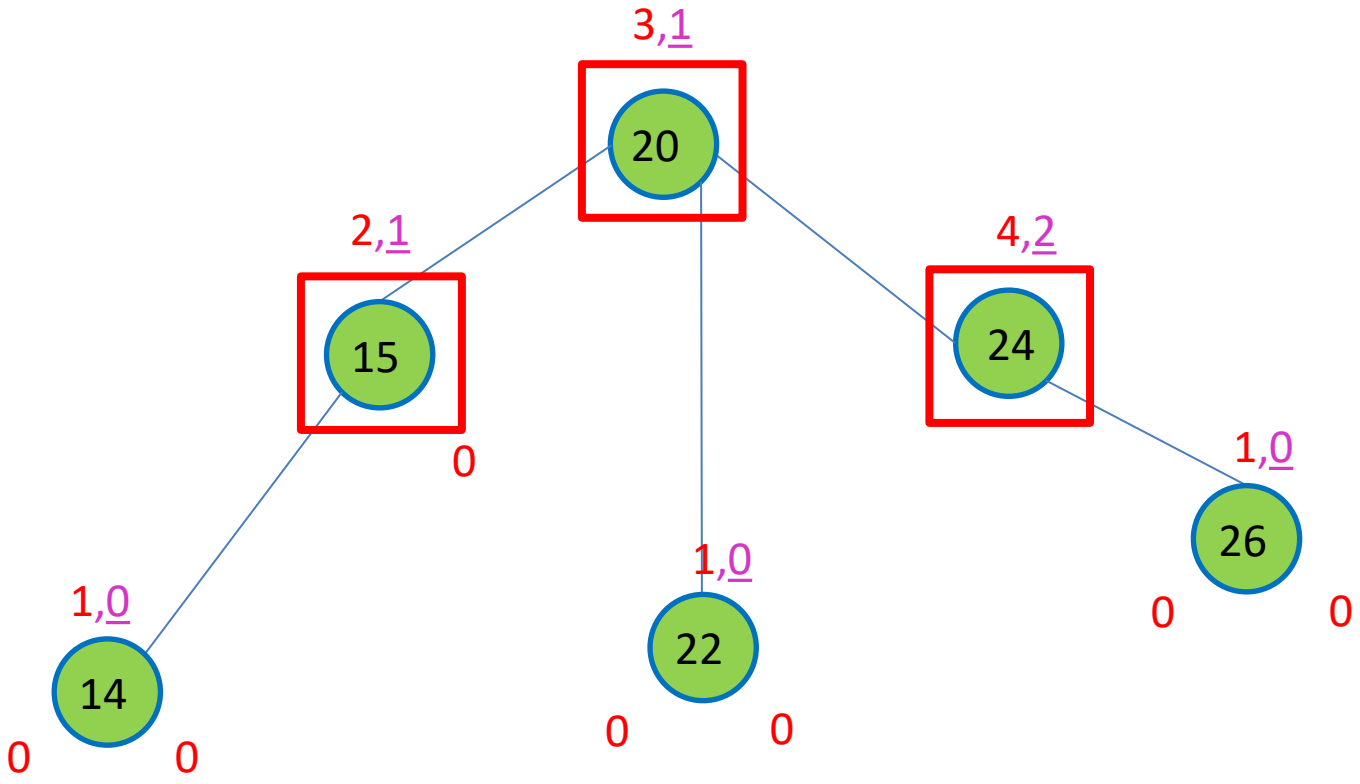
- How to balance?
  - If left-left imbalanced, rotate right
  - If left-left, we rotate right



AVL Tree

The better BST!

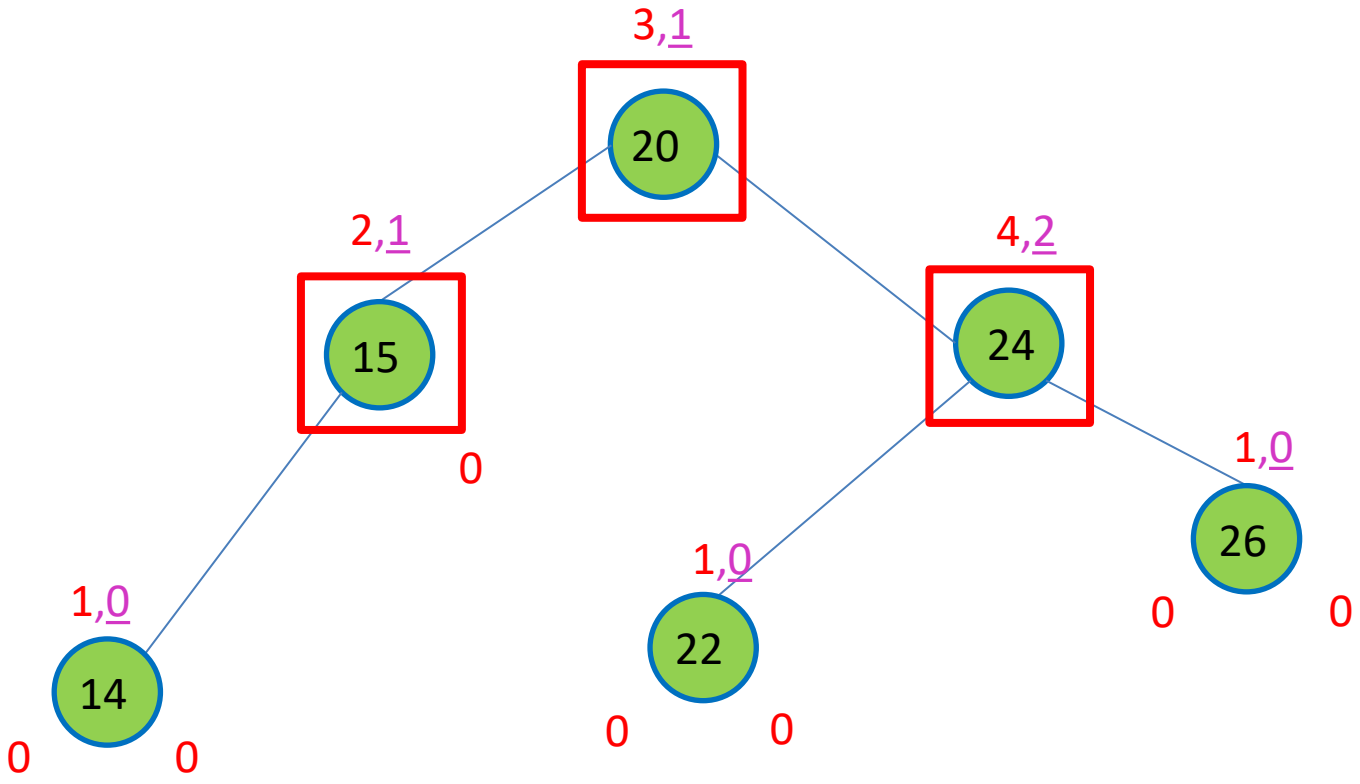
- How to balance?
  - If left-left imbalanced, rotate right



AVL Tree

The better BST!

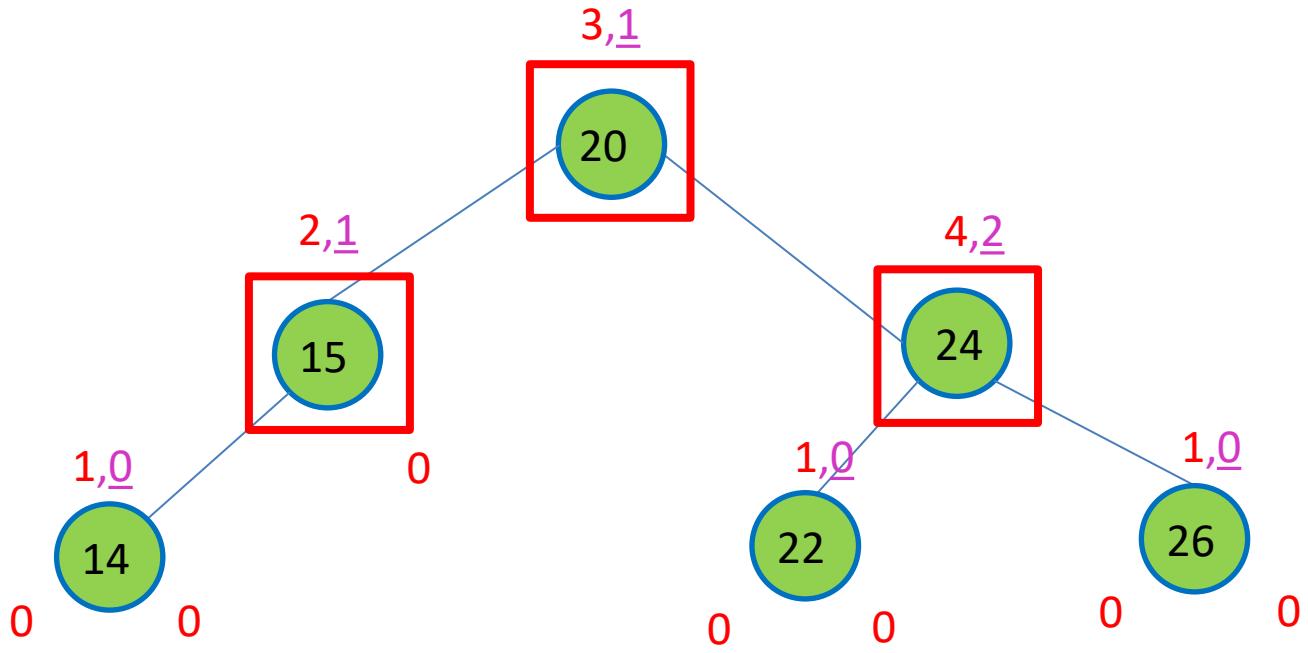
- How to balance?
  - If left-left imbalanced, rotate right



AVL Tree

The better BST!

- How to balance?
  - If left-left imbalanced, rotate right



AVL Tree

The better BST!

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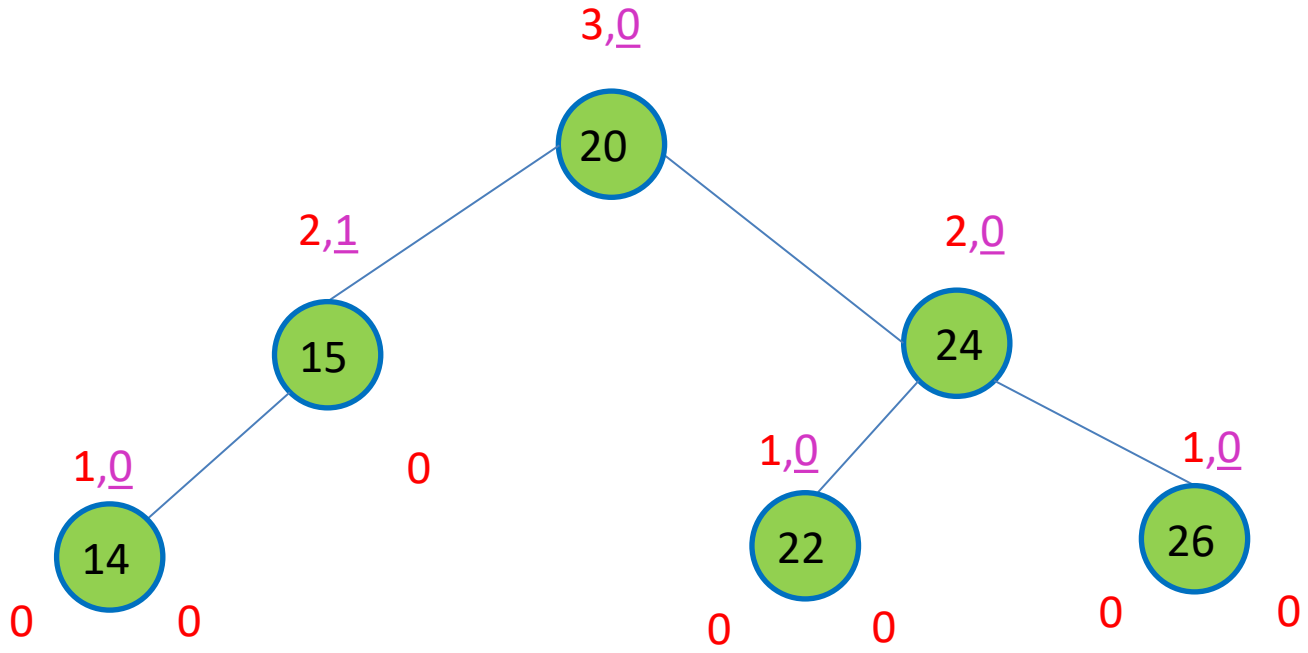
- How to balance?
  - If left-left imbalanced, rotate right
  - Recalculate



AVL Tree

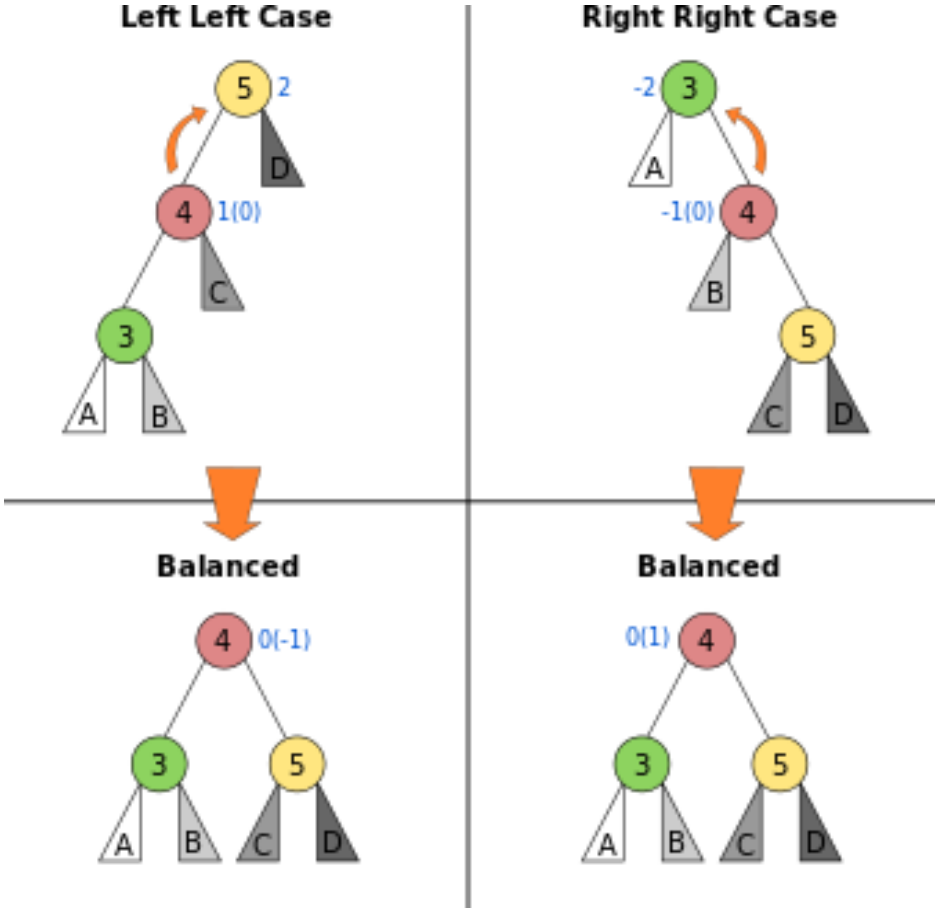
The better BST!

- How to balance?
  - If left-left imbalanced, rotate right
  - Recalculate

AVL Tree

The better BST!

- How to balance?
  - If left-left imbalanced, rotate right
  - If right-right, rotate left



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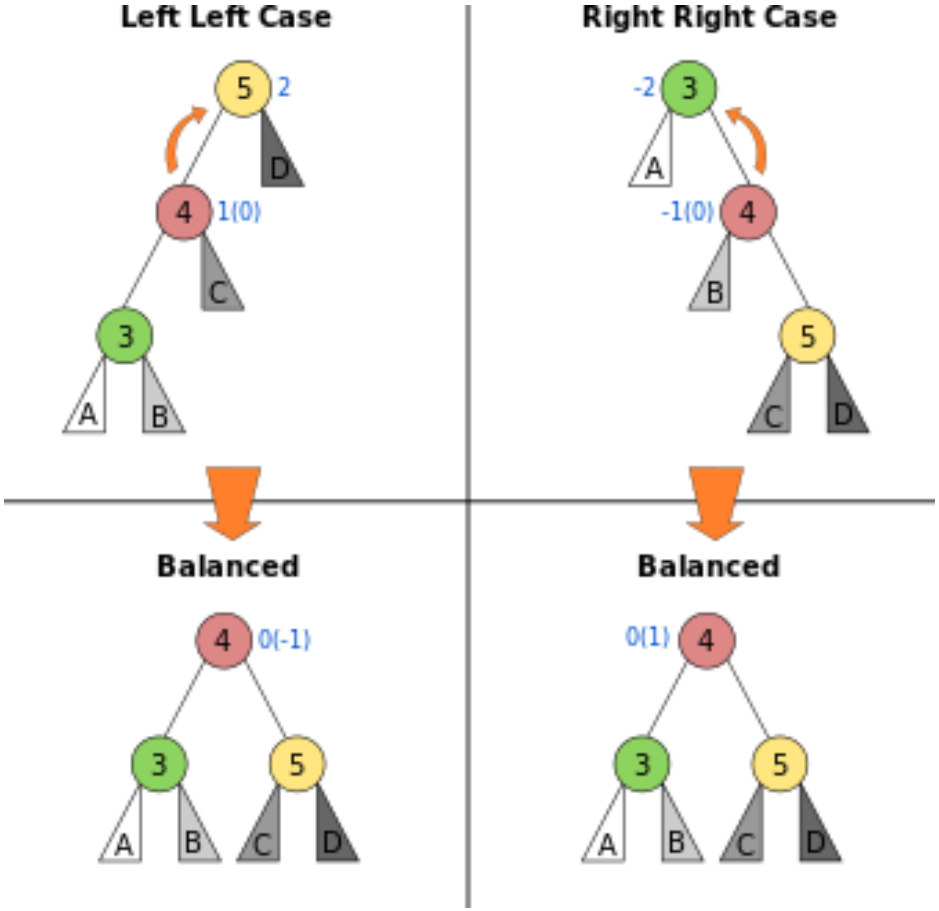
Questions?

AVL Tree

The better BST!

- How to balance?
  - If left-left imbalanced, rotate right
  - If right-right, rotate left
- But what if
  - Left-right?
  - Right-left?



AVL Tree

The better BST!

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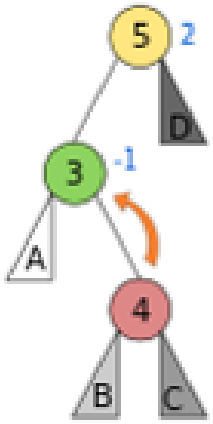

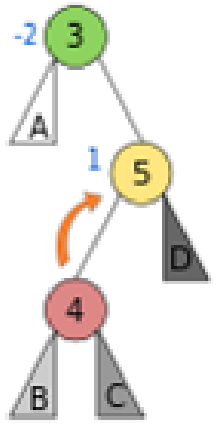

AVL Tree

The better BST!

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  - If right-right, rotate left
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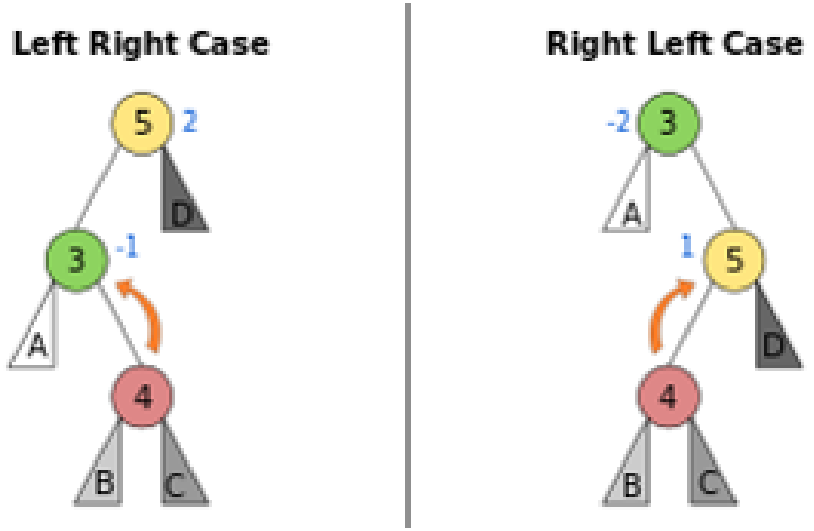
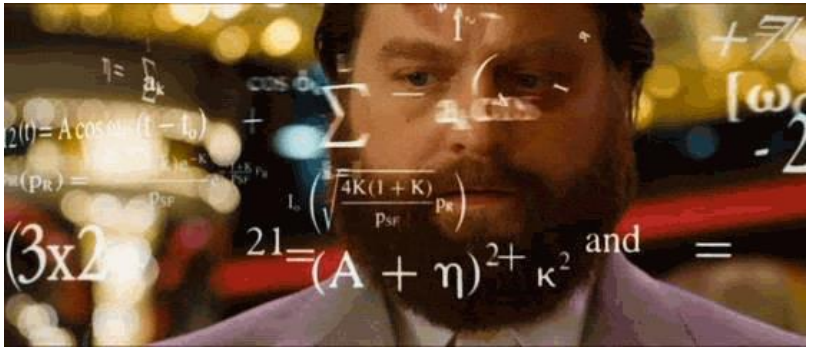
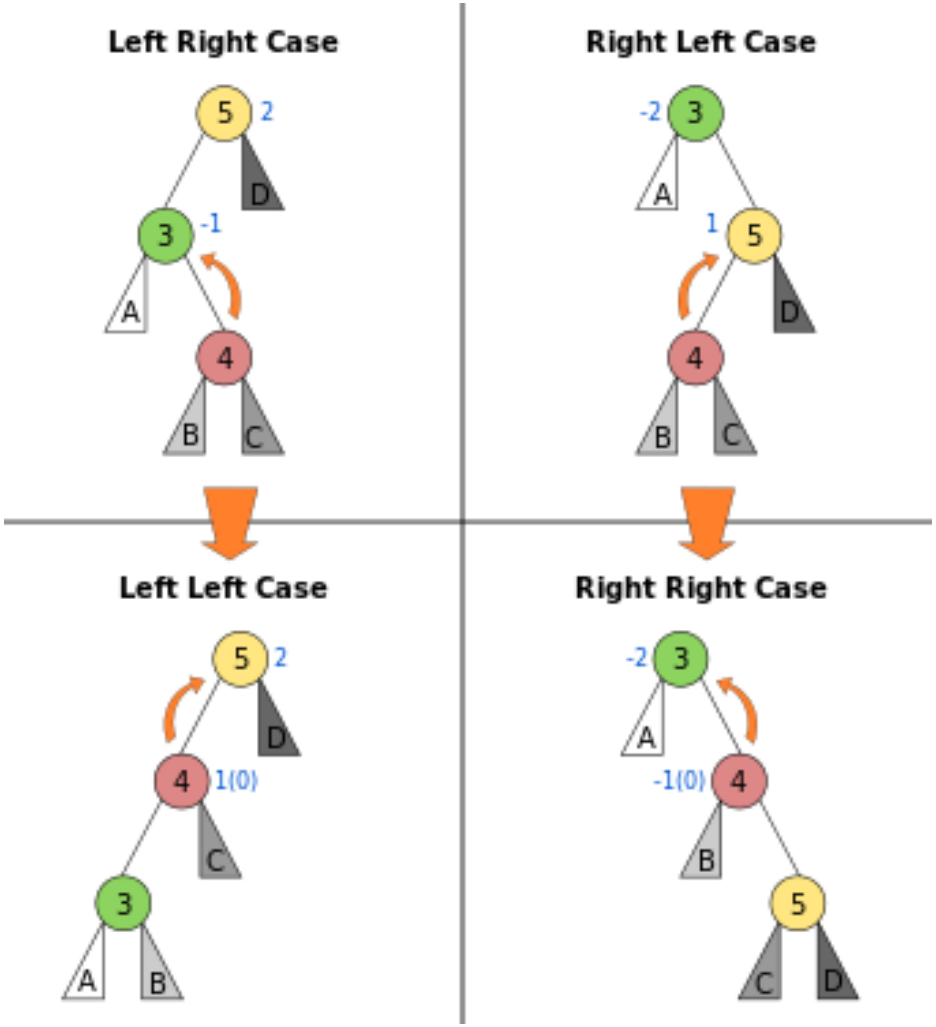




AVL Tree

The better BST!

- How to balance?
  - If left-left imbalanced, rotate right
  - If right-right, rotate left
- But what if
  - Left-right?
  - Right-left?
  - We do double rotation



AVL Tree

The better BST!

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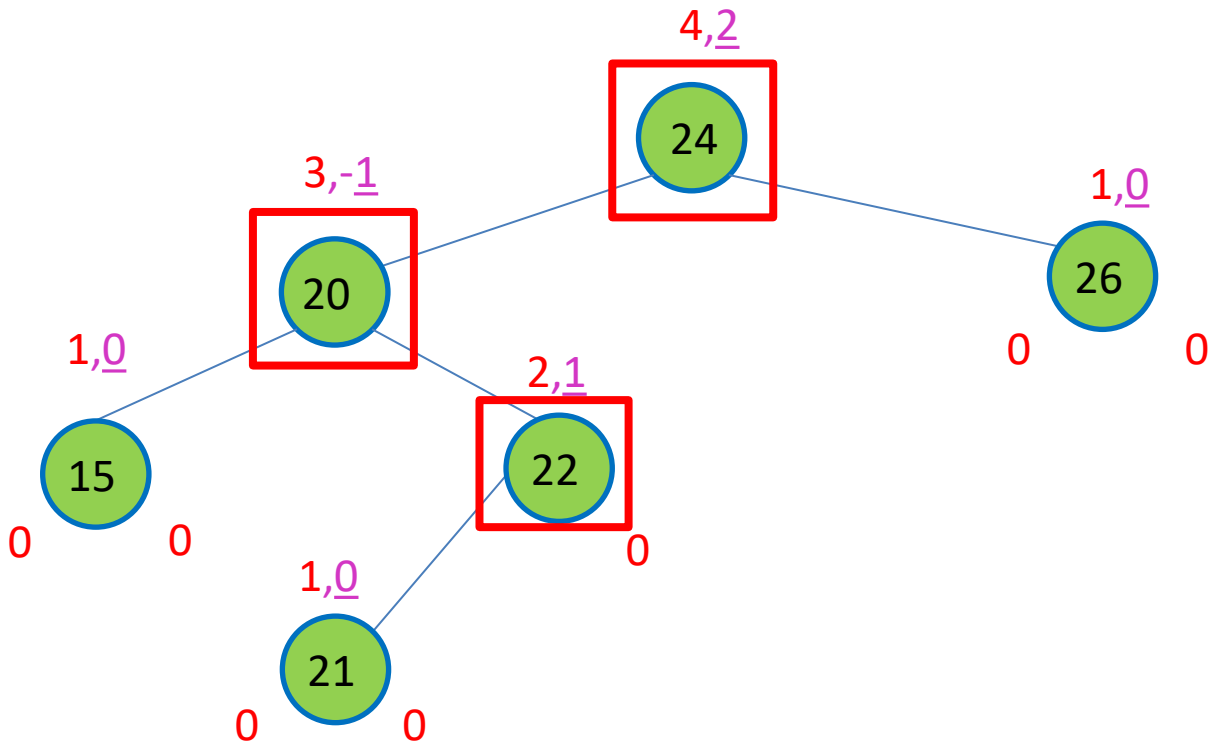
- Left-right imbalanced example



AVL Tree

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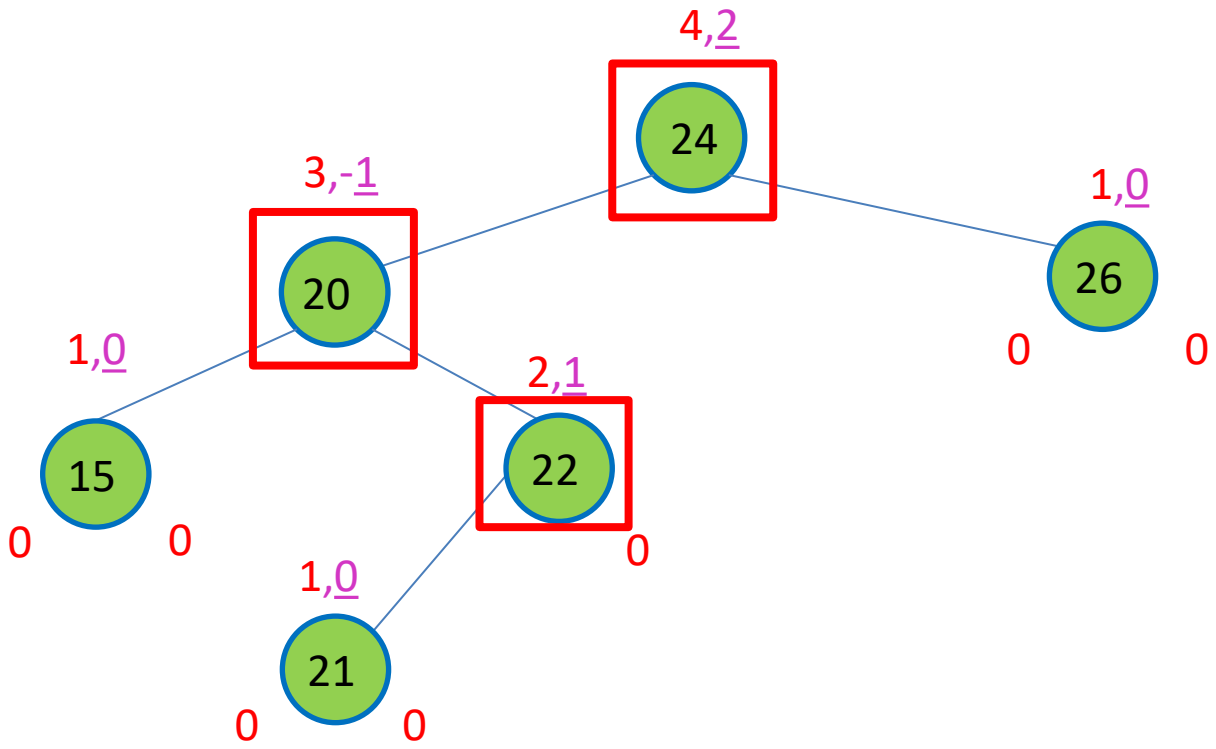
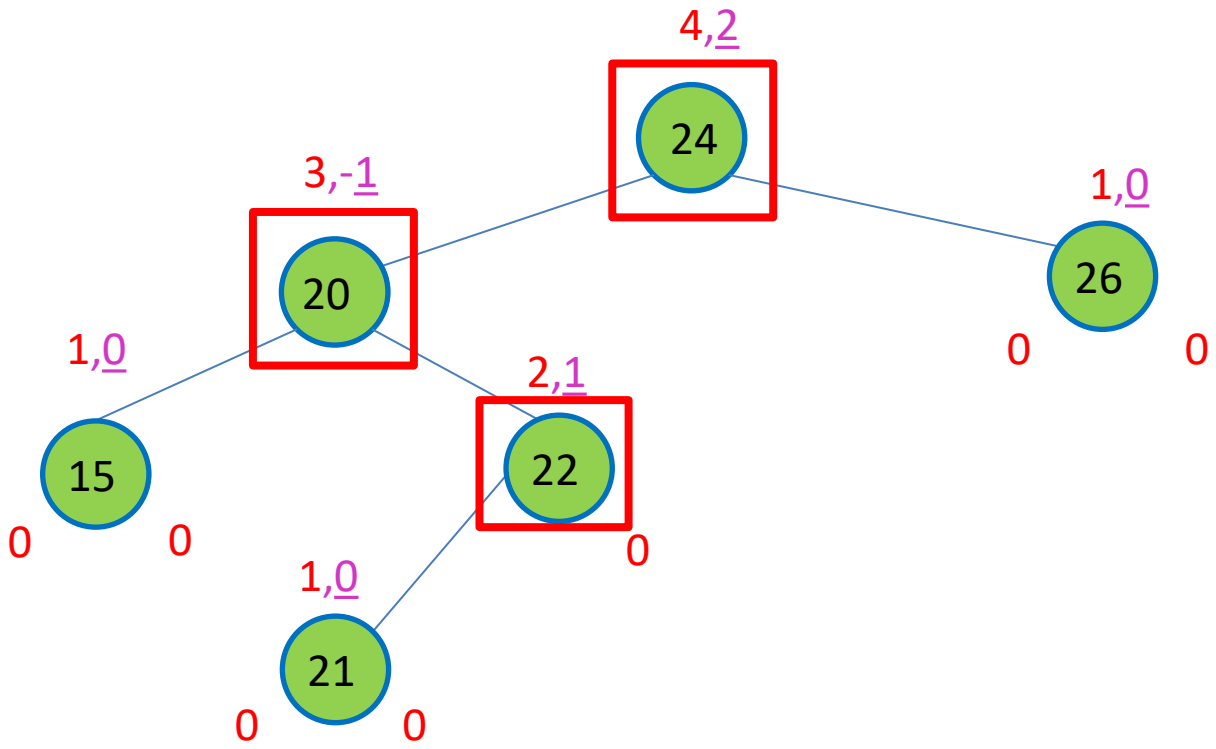
- Left-right imbalanced example
  - How do we balance?



- Left-right imbalanced example
  - How do we balance? Rotate left, so it become left-left imbalanced



AVL Tree

The better BST!

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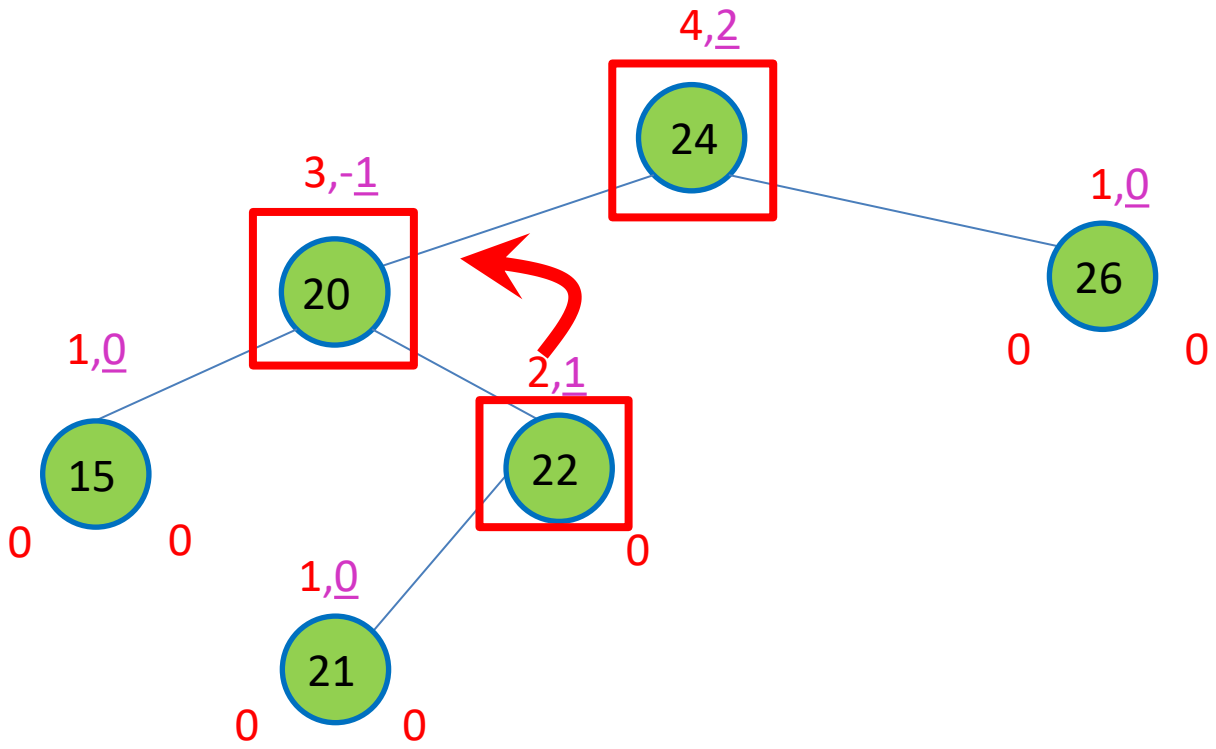

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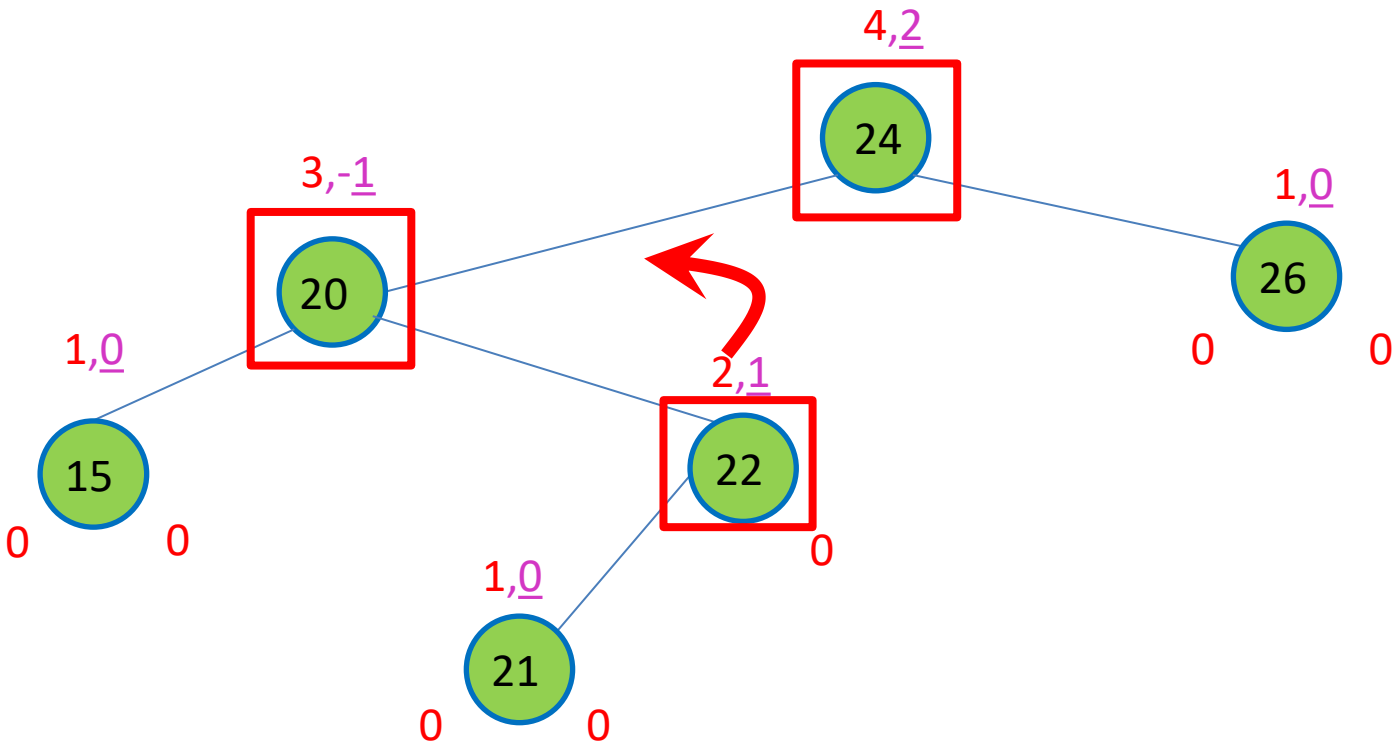

AVL Tree

The better BST!

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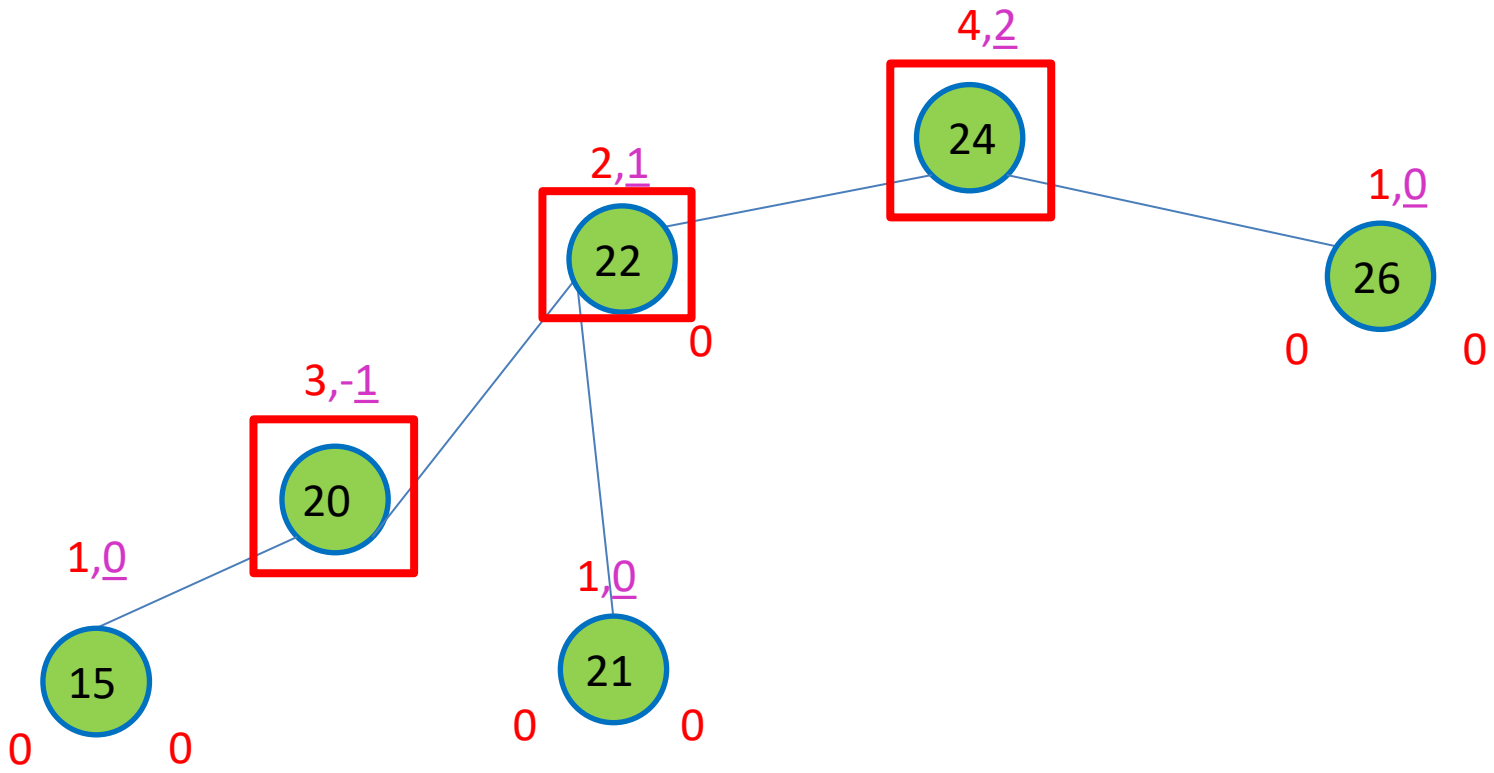

AVL Tree

The better BST!

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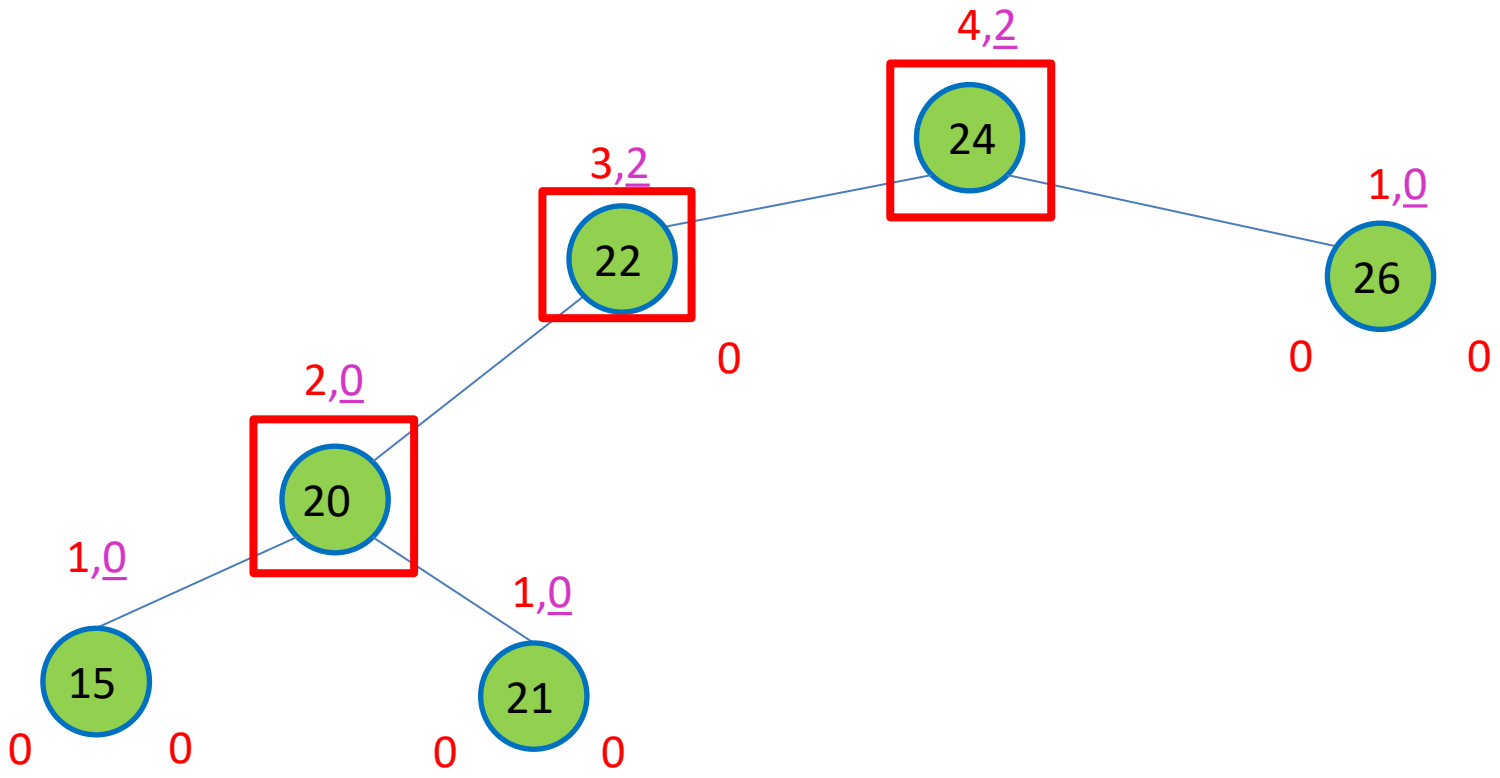
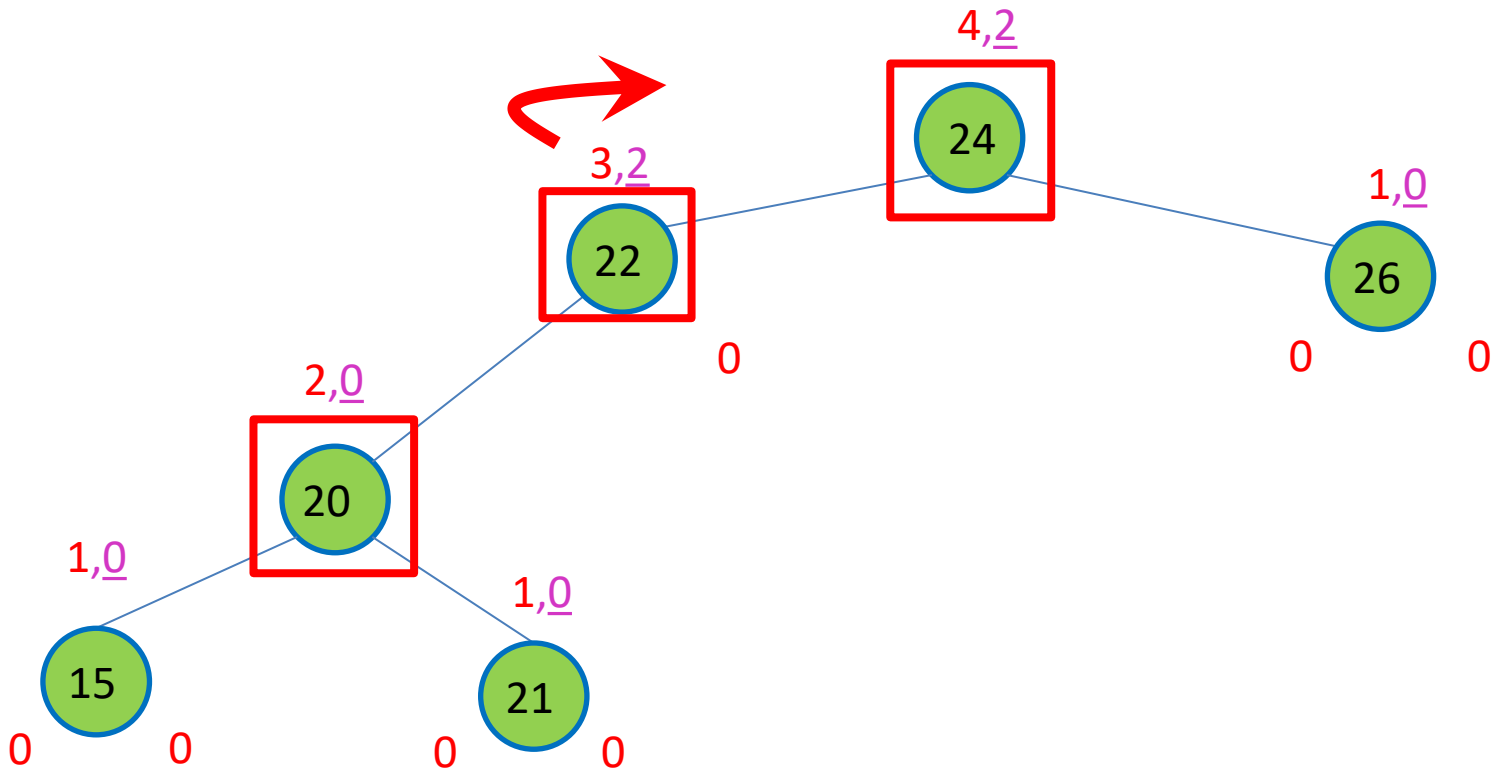


AVL Tree
The better BST!

- Left-right imbalanced example
  - How do we balance? Rotate left, so it become left-left imbalanced
    - Then rotate right



AVL Tree
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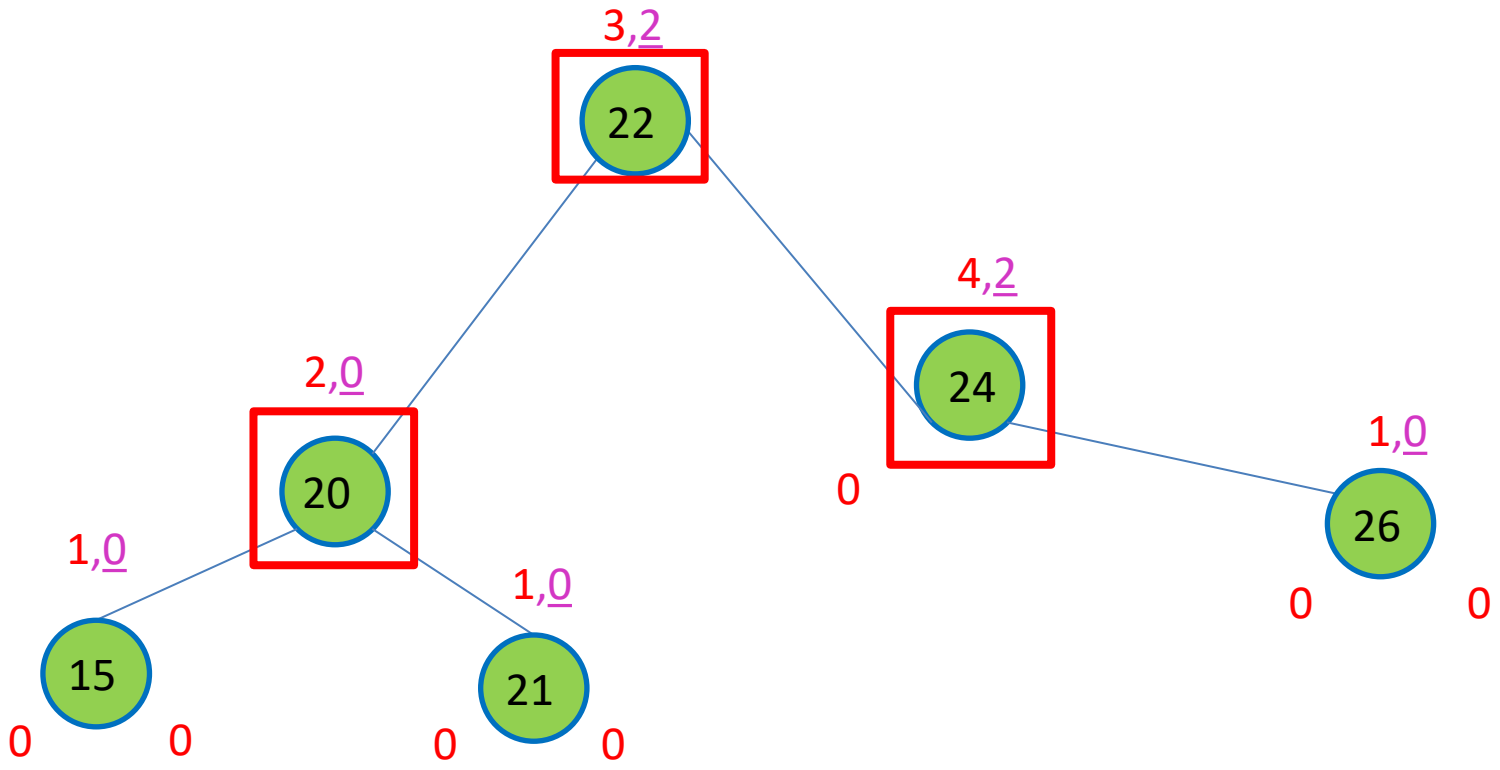

AVL Tree
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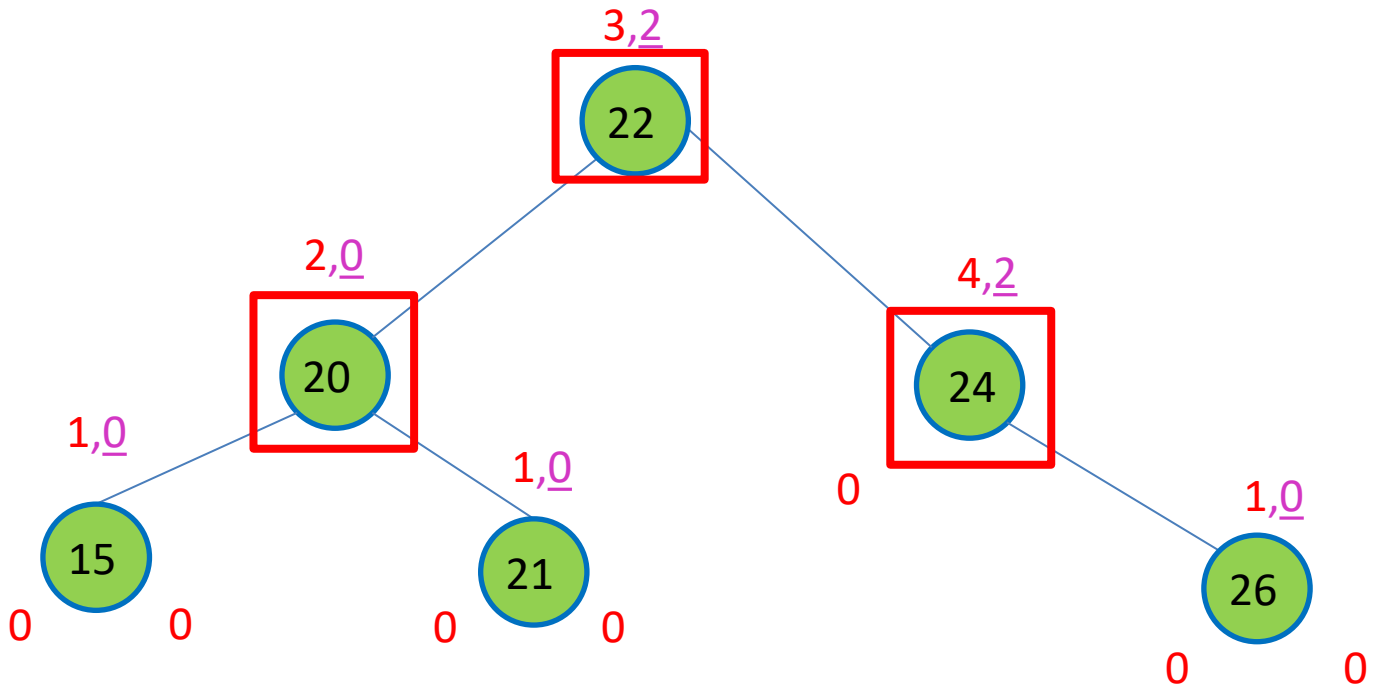

AVL Tree

The better BST!

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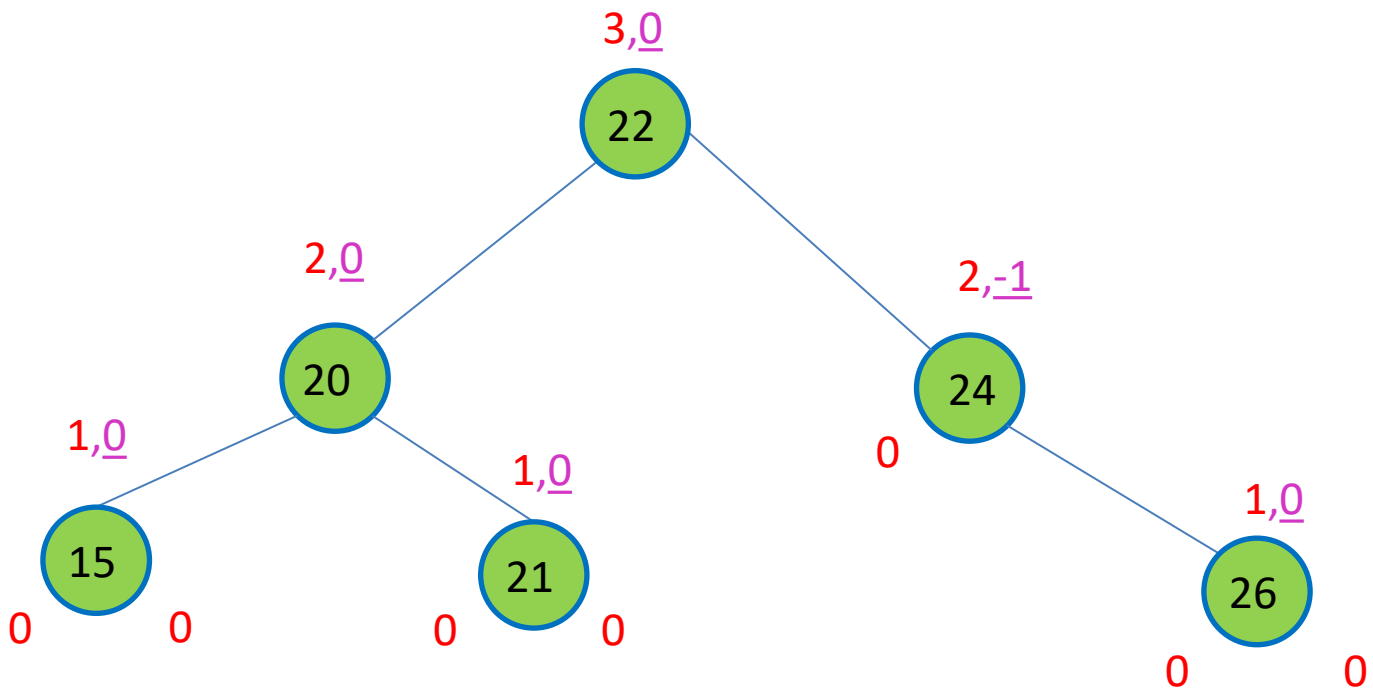
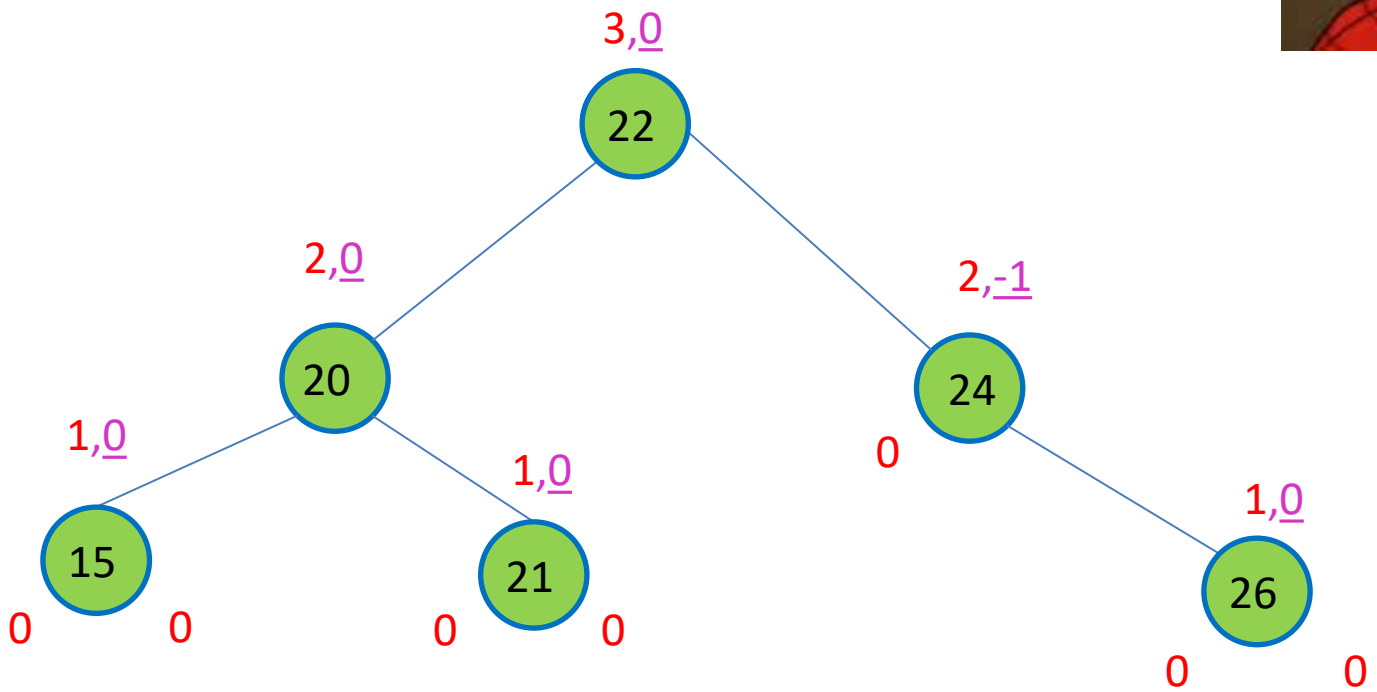
- Left-right imbalanced example
  - Let us check balance again



AVL Tree

The better BST!

- Left-right imbalanced example
  - Let us check balance again



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Questions?

AVL Tree
Complexity?

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- Everything is the same as BST
  - Insert
  - Delete
  - Search

AVL Tree
Complexity?

MONASH University

- Everything is the same as BST
  - Insert
  - Delete
  - Search

} Check balance after that

AVL Tree
Complexity?

MONASH University

- Everything is the same as BST
  - Insert
  - Delete
  - Search

  } Check balance after that

- So what is the balancing cost?

AVL Tree
Complexity?

MONASH University

- Everything is the same as BST
  - Insert
  - Delete
  - Search

  } Check balance after that

- So what is the balancing cost?
  - Check balance from the bottom
  - If we find imbalance, we balance (at most 2 rotations – a constant)

AVL Tree
Complexity?

MONASH University

- Everything is the same as BST
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  - So this would be O(log N)

AVL Tree
Complexity?

MONASH University

- Everything is the same as BST
  - Insert
  - Delete
  - Search

  } Check balance after that

- So what is the balancing cost?
  - Check balance from the bottom
    - Done via recursion…
  - If we find imbalance, we balance (at most 2 rotations – a constant)
  - So this would be O(log N)

AVL Tree
Complexity?

MONASH University

- Everything is the same as BST
  - Insert
  - Delete
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  } Check balance after that

- So what is the balancing cost?
  - Check balance from the bottom
    - Done via recursion…
    - Note: This isn't O(N) because we would just update and not calculate from scratch as how we do by hand
  - If we find imbalance, we balance (at most 2 rotations – a constant)
  - So this would be O(log N)

AVL Tree
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MONASH University

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  - Delete
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  - If we find imbalance, we balance (at most 2 rotations – a constant)
  - So this would be O(log N)
  - Note: Analyzed further in tutorial supplementary question

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Questions?

AVL Tree
Ian's way

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- Now let us look at Ian's way…

AVL Tree
Ian's way

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- Now let us look at Ian's way…
  - Let us model it like how exams would ask

AVL Tree
Ian's way

MONASH University

- Now let us look at Ian's way…
  - Let us model it like how exams would ask
  - Give me a list of numbers…

AVL Tree
Ian's way

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- Now let us look at Ian's way…
  - Let us model it like how exams would ask
  - Give me a list of numbers…
  - I will insert these into a AVL tree, checking balance

AVL Tree
Ian's way

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- Now let us look at Ian's way…
  - Let us model it like how exams would ask
  - Give me a list of numbers…
  - I will insert these into a AVL tree, checking balance
  - Then give me a list of numbers…
  - I will delete these from the AVL tree, checking balance

AVL Tree
Ian's way

MONASH University

- Now let us look at Ian's way…
  - Let us model it like how exams would ask
  - Give me a list of numbers…
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  - Then give me a list of numbers…
  - I will delete these from the AVL tree, checking balance
- This will be our active learning activity!
  - And I won't repeat in the tutorial >.<

AVL Tree
Ian's way

- Now let us look at Ian's way…
  - Let us model it like how exams would ask
  - Give me a list of numbers…
  - I will insert these into a AVL tree, checking balance
  - Then give me a list of numbers…
  - I will delete these from the AVL tree, checking balance
- This will be our active learning activity!
  - And I won't repeat in the tutorial >.<
- Since we don't have face to face, we use RNG

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Questions?

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Thank You